As per the
Latest Syllabus
of JNTU
Kakinada

Electrical Circuit Analysis-1

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A Sudhakar is presently working as Principal at RVR & JC College of Engineering, Guntur, Andhra Pradesh. He obtained his B.Tech degree (1978-83) from V R Siddartha Engineering College affiliated to Acharya Nagarjuna University, and ME from Jadavpur University, Kolkata(1983-85). Thereafter, he obtained his PhD from Andhra University, Visakhapatnam, in 2002. He has 29 years of teaching and research experience in the field of Electronics and Communication Engineering and has also worked as Associate Professor at AIMST, Malaysia, during 2003–2005. Presently, he is a member of Planning and Monitoring Board and Senate Member of Acharya Nagarjuna University. He is a member of various professional societies like IEEE, SEMCEI, ISTE and Fellow in IETE. He visited several countries—Malaysia, Singapore, Russia, Sweden—to present papers in International conferences. He has published more than 50 papers in various national and international journals and has co-authored several books, all published by McGraw-Hill education Ltd. He has completed and working on various research and sponsored projects funded by AICTE and UGC, Govt. of India. His fields of interest include Circuits, Signals and systems and Array Antennas. He received the Prathibha Puraskar from TR Mahesh Charitable Trust, Guntur, for his services in the field of Higher Education, Andhra Pradesh, in July 2014. He has received a Certificate of Excellence for his outstanding services, achievements and contributions to the field of Electronics and Communication Engineering by the IETE Vijayawada centre on the occasion of IETE Foundation Day in November 2014.

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He has over 34 years of teaching experience and has taught various courses in electrical and electronics engineering. Some of his papers were also published in national and international journals. He has co-authored books on networks and circuits for JNTU, Anna University, UP Technological University and Vishveshwaraiah Technological University. Presently, he is pursuing his research work in Andhra University under the guidance of Dr K A Gopalarao. His areas of interest include Control Systems, Power Systems, and Operation and Control.

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Dedicated to Our
Parents
and
Students

Contents

Preface

This book is exclusively designed for use as a text for an introductory course in Electrical Circuit Analysis-1 offered to first-year undergraduate engineering students of Jawaharlal Nehru Technological University, Kakinada. The primary goal of this text is to enable the student to have a firm grasp over the basic principles of electric circuits, and develop an understanding of circuits and the ability to design practical circuits that perform the desired operations. Emphasis is placed on basic laws, theorems and techniques which are used to develop a working knowledge of the methods of analysis used most frequently in further topics of electrical engineering.

Each chapter begins with principles and theorems together with illustrative and other descriptive material. A large number of solved examples showing students the step-by-step processes for applying the techniques are presented in the text. Several questions in worked examples have been selected from university question papers. As an aid to both the instructor and the student, objective questions and tutorial problems provided at the end of each chapter progress from simple to complex. Due care is taken to see that the reader can easily start learning the analysis of Electric Circuit Analysis without prior knowledge of mathematics.

Salient Features

- 100% coverage of JNTU Kakinada latest syllabus
- Individual topics very well supported by solved examples
- Roadmap to the syllabus provided for systematic reading of the text
- University questions incorporated at appropriate places in the text
- Excellent pedagogy:
    - Illustrations: ***847***
    - Solved Examples: ***468***
    - Practice Problems: ***165***
    - Objective Type Questions: ***162***

The book is organized in 6 chapters. All the elements with definitions, basic laws and configurations of the resistive circuits, capacitive and inductive elements are introduced in *Chapter 1*. This chapter also includes Kirchhoff's laws, network reduction techniques, star–delta transformations, nodal and mesh analysis. Single-Phase AC circuits are discussed in *Chapter 2*. Resonance phenomena, bandwidth, quality factor and locus diagrams are presented in *Chapter 3*. Chapter 4 deals with magnetic circuits, Faraday's laws, concept of self- and mutual inductance. Network topology is taken up in *Chapter 5*. Graph theory has been written in an easy-to-understand manner in this unit. Network theorems, with dc and ac, are presented in *Chapter 6*. Theorems like Superposition, Thevenin's, Norton's, Maximum Power Transfer, Millman's, Reciprocity and Compensation are discussed in this chapter.

Questions that have appeared in University Examinations are included at the appropriate places which will serve to enhance understanding.

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A Sudhakar
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ROADMAP TO THE SYLLABUS

UNIT I: Introduction to Electrical Circuits

Passive components and their V-I relations; Sources (dependent and independent); Kirchhoff's laws; Network reduction techniques(series, parallel, series-parallel, star-to-delta and delta-to-star transformation); Source transformation technique;Nodal analysis and mesh analysis.

GO TO **Chapter 1** ***Introduction to Electrical Circuits***

UNIT II: Single-Phase AC Systems

Periodic waveforms (determination of rms, average value and form factor); Concept of phase angle and phase difference; Complex and polar forms of representations;Steady-state analysis of R, L and C circuits; Power factor and its significance—Real, Reactive power and Apparent power.

GO TO **Chapter 2** ***Single-Phase AC Circuits***

UNIT III: Resonance

Locus diagrams for various combination of R, L and C; Resonance;Concept of bandwidth and quality factor.

GO TO **Chapter 3** ***Locus Diagrams and Resonance***

UNIT IV: Magnetic Circuit

Basic definition of MMF, flux and reluctance; Analogy between electrical and magnetic circuits; Faraday's laws of electromagnetic induction; Concept of self- and mutual inductances; Dot convention—coefficient of coupling and composite magnetic circuit; Analysis of series and parallel magnetic circuits.

GO TO **Chapter 4** ***Magnetic Circuits***

UNIT V: Network Topology

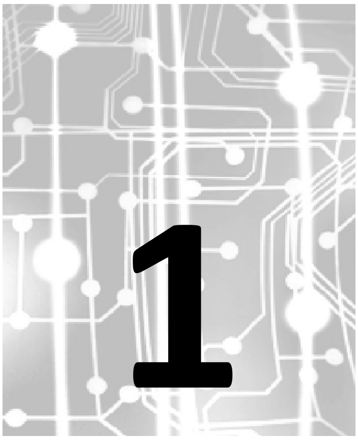
Definitions of graph and tree; Basic cutset and tieset matrices for planar networks; Loop and nodal methods of analysis of networks with dependent and independent voltage and current sources; Duality and dual networks.

GO TO **Chapter 5** ***Network Topology***

UNIT VI: Network Theorems (DC & AC Excitations)

Superposition theorem; Thevenin's theorem; Norton's theorem; Maximum power transfer theorem; Reciprocity theorem; Millman's theorem and compensation theorem.

GO TO **Chapter 6** ***Network Theorems***



1 Introduction to Electrical Circuits

1.1 INTRODUCTION TO ELECTRICAL CIRCUITS

1.1.1 Voltage

According to the structure of an atom, we know that there are two types of charges: positive and negative. A force of attraction exists between these positive and negative charges. A certain amount of energy (work) is required to overcome the force and move the charges through a specific distance. All opposite charges possess a certain amount of potential energy because of the separation between them. The difference in potential energy of the charges is called the *potential difference.*

Potential difference in electrical terminology is known as voltage, and is denoted either by V or υ. It is expressed in terms of energy (W) per unit charge (Q); i.e.

$$V = \frac{W}{Q} \quad \text{or} \quad v = \frac{dw}{dq}$$

dw is the small change in energy, and

dq is the small change in charge.

where energy (W) is expressed in joules (J), charge (Q) in coulombs (C), and voltage (V) in volts (V). One volt is the potential difference between two points when one joule of energy is used to pass one coulomb of charge from one point to the other.

Example 1.1 *If 70 J of energy is available for every 30 C of charge, what is the voltage?*

Solution

$$V = \frac{W}{Q} = \frac{70}{30} = 2.33\,\text{V}$$

Example 1.2 *A resistor with a current of 3 A through it converts 500 J of electrical energy to heat energy in 12 s. What is the voltage across the resistor?*

Solution

$$V = \frac{W}{Q}$$

$$Q = I \times t$$

$$= 3 \times 12 = 36 \text{ C}$$

$$V = \frac{500}{36} = 13.88 \text{ V}$$

1.1.2 Current

There are free electrons available in all semiconductive and conductive materials. These free electrons move at random in all directions within the structure in the absence of external pressure or voltage. If a certain amount of voltage is applied across the material, all the free electrons move in one direction depending on the polarity of the applied voltage, as shown in Fig. 1.1.

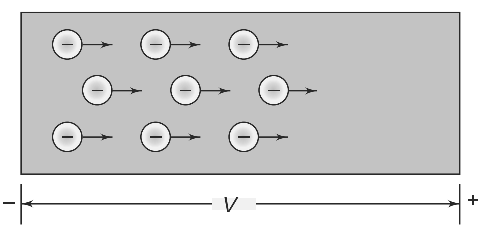


Fig. 1.1

This movement of electrons from one end of the material to the other end constitutes an electric current, denoted by either I or i. The conventional direction of current flow is opposite to the flow of – ve charges, i.e. the electrons.

Current is defined as the rate of flow of electrons in a conductive or semiconductive material. It is measured by the number of electrons that flow past a point in unit time. Expressed mathematically,

$$I = \frac{Q}{t}$$

where I is the current, Q is the charge of electrons, and t is the time, or

$$i = \frac{dq}{dt}$$

where dq is the small change in charge, and dt is the small change in time.

In practice, the unit *ampere* is used to measure current, denoted by A. One ampere is equal to one coulomb per second. One coulomb is the charge carried by 6.25×10^{18} electrons. For example, an ordinary 80 W domestic ceiling fan on 230 V supply takes a current of approximately 0.35 A. This means that electricity is passing through the fan at the rate of 0.35 coulomb every second, i.e. 2.187×10^{18} electrons are passing through the fan in every second; or simply, the current is 0.35 A.

Example 1.3 *Five coulombs of charge flow past a given point in a wire in 2 s. How many amperes of current is flowing?*

Solution

$$I = \frac{Q}{t} = \frac{5}{2} = 2.5\,\text{A}$$

1.1.3 Power and Energy

Energy is the capacity for doing work, i.e. energy is nothing but stored work. Energy may exist in many forms such as mechanical, chemical, electrical and so on. Power is the rate of change of energy, and is denoted by either P or p. If certain amount of energy is used over a certain length of time, then

$$\text{Power } (P) = \frac{\text{energy}}{\text{time}} = \frac{W}{t}, \text{ or}$$

$$p = \frac{dw}{dt}$$

where dw is the change in energy and dt is the change in time.

We can also write
$$p = \frac{dw}{dt} = \frac{dw}{dq} \times \frac{dq}{dt}$$
$$= v \times i = vi \quad \text{W}$$

Energy is measured in joules (J), time in seconds (s), and power in watts (W).

By definition, one watt is the amount of power generated when one joule of energy is consumed in one second. Thus, the number of joules consumed in one second is always equal to the number of watts. Amounts of power less than one watt are usually expressed in fraction of watts in the field of electronics; viz. milliwatts (mW) and microwatts (μW). In the electrical field, kilowatts (kW) and megawatts (MW) are common units. Radio and television stations also use large amounts of power to transmit signals.

Example 1.4 *What is the power in watts if energy equal to 50 J is used in 2.5 s?*

Solution

$$P = \frac{\text{energy}}{\text{time}} = \frac{50}{2.5} = 20\text{ W}$$

Example 1.5 *A 5 Ω resistor has a voltage rating of 100 V. What is its power rating?*

Solution

$$P = VI$$
$$I = V/R$$
$$P = \frac{V^2}{R} = \frac{(100)^2}{5} = 2000\text{ W} = 2\text{ kW}$$

1.1.4 The Circuit

Simply an electric circuit consists of three parts: (1) energy source, such as battery or generator, (2) the load or sink, such as lamp or motor, and (3) connecting wires as shown in Fig. 1.2. This arrangement represents a simple circuit. A battery is connected to a lamp with two wires. The purpose of the circuit is to transfer energy from source (battery) to the load (lamp). And this is accomplished by the passage of electrons through wires around the circuit.

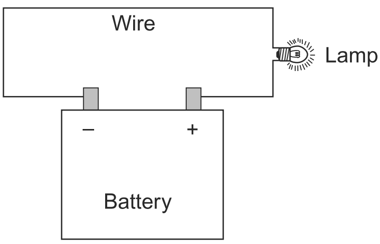


Fig. 1.2

The current flows through the filament of the lamp, causing it to emit visible light. The current flows through the battery by chemical action. A closed circuit is defined as a circuit in which the current has a complete path to flow. When the current path is broken so that current cannot flow, the circuit is called an open circuit.

More specifically, interconnection of two or more simple circuit elements (viz. voltage sources, resistors, inductors and capacitors) is called an electric network. If a network contains at least one closed path, it is called an electric circuit. By definition, a simple circuit element is the mathematical model of two terminal electrical devices, and it can be completely characterised by its voltage and current. Evidently then, a physical circuit must provide means for the transfer of energy.

Broadly, network elements may be classified into four groups, viz.

1. Active or passive
2. Unilateral or bilateral
3. Linear or nonlinear
4. Lumped or distributed

1.1.5 Active and Passive *[JNTU Jan. 2010, Nov. 2011]*

Energy sources (voltage or current sources) are active elements, capable of delivering power to some external device. Passive elements are those which are capable only of receiving power. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element. More specifically, an active element is capable of delivering an average power greater than zero to some external device over an infinite time interval. For example, ideal sources are active elements. A passive element is defined as one that cannot supply average power that is greater than zero over an infinite time interval. Resistors, capacitors and inductors fall into this category.

1.1.6 Bilateral and Unilateral

In the bilateral element, the voltage-current relation is the same for current flowing in either direction. In contrast, a unilateral element has different relations between voltage and current for the two possible directions of current. Examples of bilateral elements are elements made of high conductivity materials in general. Vacuum diodes, silicon diodes, and metal rectifiers are examples of unilateral elements.

1.1.7 Linear and Nonlinear Elements

An element is said to be linear, if its voltage-current characteristic is at all times a straight line through the origin. For example, the current passing through a resistor is proportional to the voltage applied through it, and the relation is expressed as $V \propto I$ or $V = IR$. A linear element or network is one which satisfies the principle of superposition, i.e. the principle of homogeneity and additivity. An element which does not satisfy the above principle is called a nonlinear element.

1.1.8 Lumped and Distributed

Lumped elements are those elements which are very small in size and in which simultaneous actions takes place for any given cause at the same instant of time. Typical lumped elements are capacitors, resistors, inductors and transformers. Generally the elements are considered as lumped when their size is very small compared to the wave length of the applied signal. Distributed elements, on the other hand, are those which are not electrically separable for analytical purposes. For example, a transmission line which has distributed resistance, inductance and capacitance along its length may extend for hundreds of miles.

1.2 PASSIVE COMPONENTS

1.2.1 Resistance Parameter – Ohm's law

When a current flows in a material, the free electrons move through the material and collide with other atoms. These collisions cause the electrons to lose some of their energy. This loss of energy per unit charge is the drop in potential across the material. The amount of energy lost by the electrons is related to the physical property of the material. These collisions restrict the movement of electrons. The property of a material to restrict the flow of electrons is called resistance, denoted by R. The symbol for the resistor is shown in Fig. 1.3.

Fig. 1.3

The unit of resistance is ohm (Ω). Ohm is defined as the resistance offered by the material when a current of one ampere flows between two terminals with one volt applied across it.

According to Ohm's law, the current is directly proportional to the voltage and inversely proportional to the total resistance of the circuit, i.e.

$$I = \frac{V}{R}$$

$$\text{or} \quad i = \frac{v}{R}$$

We can write the above equation in terms of charge as follows.

$$V = R\frac{dq}{dt}, \quad \text{or} \quad i = \frac{v}{R} = Gv$$

where G is the conductance of a conductor. The units of resistance and conductance are ohm (Ω) and mho (℧) respectively.

When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat. The power absorbed by the resistor is given by

$$P = vi = (iR)\, i = i^2 R$$

where i is the current in the resistor in amps, and v is the voltage across the resistor in volts. Energy lost in a resistance in time t is given by

$$W = \int_0^t p\,dt = pt = i^2 Rt = \frac{v^2}{R} t$$

where v is the volts
R is in ohms
t is in seconds and
W is in joules

Example 1.6 *A 10 Ω resistor is connected across a 12 V battery. How much current flows through the resistor?*

Solution $V = IR$

$$I = \frac{V}{R} = \frac{12}{10} = 1.2 \text{ A}$$

1.2.2 Inductance Parameter

[JNTU June 2009 and May/June 2008]

A wire of certain length, when twisted into a coil becomes a basic inductor. If current is made to pass through an inductor, an electromagnetic field is formed. A change in the magnitude of the current changes the electromagnetic field. Increase in current expands the fields, and decrease in current reduces it. Therefore, a change in current produces change in the electromagnetic field, which induces a voltage across the coil according to Faraday's law of electromagnetic induction.

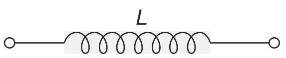


Fig. 1.4

The unit of inductance is *henry*, denoted by *H*. By definition, the inductance is one henry when current through the coil, changing at the rate of one ampere per second, induces one volt across the coil. The symbol for inductance is shown in Fig. 1.4.

The current-voltage relation is given by

$$v = L\frac{di}{dt}$$

where v is the voltage across inductor in volts, and i is the current through inductor in amps.

We can rewrite the above equations as

$$di = \frac{1}{L} v \, dt$$

Integrating both sides, we get

$$\int_0^t di = \frac{1}{L}\int_0^t vdt$$

$$i(t) - i(0) = \frac{1}{L}\int_0^t vdt$$

$$i(t) = \frac{1}{L}\int_0^t vdt + i(0)$$

From the above equation we note that the current in an inductor is dependent upon the integral of the voltage across its terminals and the initial current in the coil, $i(0)$.

The power absorbed by inductor is

$$P = vi = Li\frac{di}{dt} \text{ watts}$$

The energy stored by the inductor is

$$W = \int_0^t pdt$$

$$= \int_0^t Li\frac{di}{dt}dt = \frac{Li^2}{2}$$

From the above discussion, we can conclude the following.

1. The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.
2. A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor the current cannot change abruptly.
3. The inductor can store finite amount of energy, even if the voltage across the inductor is zero, and
4. A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

Example 1.7 *The current in a 2 H inductor varies at a rate of 2 A/s. Find the voltage across the inductor and the energy stored in the magnetic field after 2 s.*

Solution

$$v = L\frac{di}{dt}$$
$$= 2\times 4 = 8 \text{ V}$$
$$W = \frac{1}{2}Li^2$$
$$= \frac{1}{2}\times 2\times (4)^2 = 16 \text{ J}$$

Example 1.8 *Find the inductance of a coil through which flows a current of 0.2 A with an energy of 0.15 J.*

Solution

$$W = \frac{1}{2}LI^2$$
$$L = \frac{2\times W}{I^2} = \frac{2\times 0.15}{(0.2)^2} = 7.5 \text{ H}$$

Example 1.9 *Find the inductance of a coil in which a current increases linearly from 0 to 0.2 A in 0.3 s, producing a voltage of 15 V.*

Solution

$$v = L\frac{di}{dt}$$

Current in 1 second $= \dfrac{0.2}{0.3} = 0.66 \text{ A}$

The current changes at a rate of 0.66 A/s,

$$\therefore \quad L = \frac{v}{\left(\dfrac{di}{dt}\right)}$$
$$L = \frac{15 \text{ V}}{0.66 \text{ A/s}} = 22.73 \text{ H}$$

Example 1.10 *A current of 1 A is supplied by a source to an inductor of 1 H. Calculate the energy stored in the inductor. What happens to this energy if the source is short circuited?*

Solution

$$\text{Energy stored } \frac{1}{2}\text{LI}^2 = \frac{1}{2}1\times 1^2 = 0.5 \text{ Joules}$$

If the inductor has an internal resistance, the stored energy is dissipated in the resistance after the short circuit as per the time constant (1/r) of the coil.

If the coil is a pertect inductor. The current would circulate through the shorted coil continuously.

Example 1.11 *Derive the expression for the energy stored in an ideal inductor?*

Solution *Expression for Energy Stored in an ideal inductor*

Let 'L' be the co-efficient of self inductance and i be the current flowing through it.

Let 'dw' be the small amount of work to be expended to over come self induced emf.

$$\therefore \quad dw = Ei\,dt$$

$$dw = L\frac{di}{dt}i\,dt \quad \left[\because E = L\frac{di}{dt}\right] \text{ from lenz law}$$

$$dw = Li\,di \tag{1}$$

Hence total work to be done in establishing a maximum current i_0 is given by integrating (1) from 0 to i_0.

$$\therefore \quad w = \int_0^{i_0} dw = \int_0^{i_0} Li\,di = L\int_0^{i_0} i\,di$$

$$= L\left[\frac{1}{2}\frac{i_0^2}{1}\right]$$

$$w = \frac{1}{2}Li_0^2$$

$\therefore$ Energy stored in an inductor $w = \frac{1}{2}Li_0^2$

1.2.3 Capacitance Parameter

Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor. The conducting surfaces are called *electrodes*, and the insulating medium is called *dielectric*. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges, and is concentrated within the dielectric. The amount of charge per unit voltage that is capacitor can store is its capacitance, denoted by C. The unit of capacitance is *Farad* denoted by F. By definition, one Farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates. The symbol for capacitance is shown in Fig. 1.5.

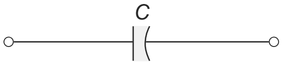


Fig. 1.5

A capacitor is said to have greater capacitance if it can store more charge per unit voltage and the capacitance is given by

$$C = \frac{Q}{V}, \quad \text{or} \quad C = \frac{q}{v}$$

(lower case letters stress instantaneous values)

We can write the above equation in terms of current as

$$i = C\frac{dv}{dt} \qquad \left(\because\ i = \frac{dq}{dt}\right)$$

where υ is the voltage across capacitor, *i* is the current through it

$$dv = \frac{1}{C} idt$$

Integrating both sides, we have

$$\int_0^t dv = \frac{1}{C}\int_0^t idt$$

$$v(t) - v(0) = \frac{1}{C}\int_0^t idt$$

$$v(t) = \frac{1}{C}\int_0^t idt + v(0)$$

where $v(0)$ indicates the initial voltage across the capacitor.

From the above equation, the voltage in a capacitor is dependent upon the integral of the current through it, and the initial voltage across it.

The power absorbed by the capacitor is given by

$$p = vi = vC\frac{dv}{dt}$$

The energy stored by the capacitor is

$$W = \int_0^t pdt = \int_0^t vC\frac{dv}{dt}dt$$

$$W = \frac{1}{2}Cv^2$$

From the above discussion we can conclude the following

1. The current in a capacitor is zero if the voltage across it is constant; that means, the capacitor acts as an open circuit to dc.
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly.
3. The capacitor can store a finite amount of energy, even if the current through it is zero, and
4. A pure capacitor never dissipates energy, but only stores it; that is why it is called *non-dissipative passive element*. However, physical capacitors dissipate power due to internal resistance.

Example 1.12 *A capacitor having a capacitance 2 μF is charged to a voltage of 1000 V. Calculate the stored energy in joules.*

Solution $$W = \frac{1}{2}Cv^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (1000)^2 = 1 \text{ J}.$$

Example 1.13 *When a dc voltage is applied to a capacitor, the voltage across its terminals is found to build up in accordance with $V_C = 50(1 - e^{-100t})$. After a lapse of 0.01 s, the current flow is equal to 2 mA.*

(a) Find the value of capacitance in microfarads.

(b) How much energy is stored in the electric field at this time?

Solution

(a) $$i = C\frac{dv_C}{dt}$$

where $\upsilon_C = 50(1 - e^{-100t})$

$$i = C\frac{d}{dt}50\left(1 - e^{-100t}\right)$$

$$= C \times 50 \times 100e^{-100t}$$

At $t = 0.01$ s, $i = 2$ mA

$$C = \frac{2 \times 10^{-3}}{50 \times 100 \times e^{-100 \times 0.01}} = 1.089 \ \mu F$$

(b) $$W = \frac{1}{2}Cv_C^2$$

At $t = 0.01$ s, $\upsilon_C = 50(1 - e^{-100 \times 0.01}) = 31.6$ V

$$W = \frac{1}{2} \times 1.089 \times 10^{-6} \times (31.6)^2$$

$$= 0.000543 \text{ J}$$

1.3 VOLTAGE—CURRENT RELATIONSHIP FOR PASSIVE ELEMENTS

In this section, we discuss about the voltage current relationship of passive elements for different input signals. Table 1.1 shows the voltage current relations of three circuit elements resistor R, inductor L and capacitor C.

Table 1.1 *V–I relation of circuit elements*

Circuit element	*Voltage (V)*	*Current (A)*	*Power (W)*
Resistor R (Ohms Ω)	$v = Ri$	$i = \frac{v}{R}$	$P = i^2 R$
Inductor L (Henry H)	$v = L\frac{di}{dt}$	$i = \frac{1}{2}\int v dt + i_0$ where i_0 is the initial current in inductor	$P = L\frac{di}{dt}$
Capacitor C (Farad F)	$v = \frac{1}{C}\int i dt + v_0$ where v_0 is the initial voltage across capacitor	$i = c\frac{dv}{dt}$	$P = cv\frac{dv}{dt}$

1.3.1 Resistive Element

[JNTU Nov. 2011]

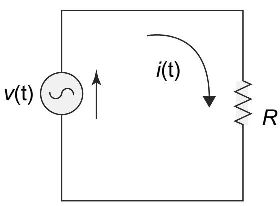


Fig. 1.6

Consider the voltage function is applied to a resistor R as shown in Fig. 1.6. The current $i(t)$ is flowing through the circuit.

The relation between $v(t)$ and $i(t)$ is

$$v(t) = R\, i(t)$$

Now, let us determine the relation between voltage and current for various input signals through following examples.

Example 1.14 *The voltage function v(t) is a repeating square wave shown in Fig. 1.7, applied to a pure resistor of 10 ohms. Plot current i(t) and power p(t).*

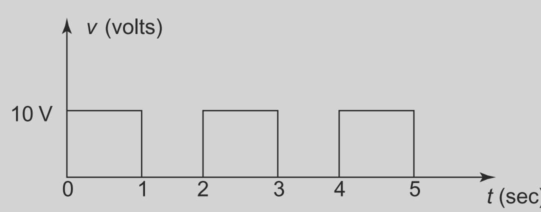


Fig. 1.7

Solution Since $v(t) = R\ i(t)$, the voltage varies directly as the current. The maximum value of current is

$$i_{max} = \frac{v_{max}}{R} = \frac{10}{10} = 1\text{ A}$$

Since power $P = vi$, the maximum value of power is

$$P_{max} = v_{max}\, i_{max} = 10(1) = 10\text{ W}$$

The resultant current and power waveforms are shown in Figs 1.8 (a) and (b) respectively.

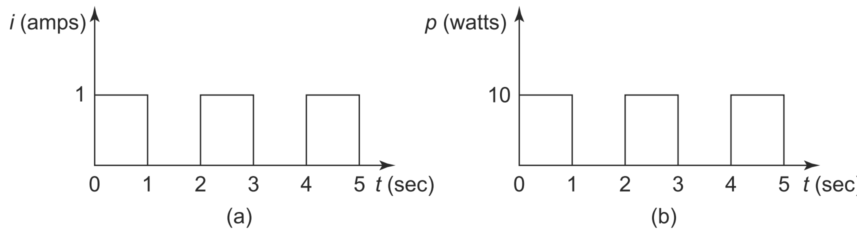


Fig. 1.8

Example 1.15 *A single pure resistance of 20 ohms passes a current of the waveform shown in Fig. 1.9. Determine and sketch the voltage V(t) and the instantaneous power p(t).*

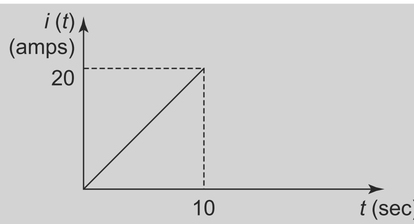


Fig. 1.9

Solution From the Fig. 1.9, the instantaneous current $i(t)$ is given by $i(t) = 2t$ amperes

The corresponding voltage is

$$v(t) = R\,i(t)$$

$$= 20 \times 2t = 40\,t \text{ volts.}$$

The corresponding instantaneous power is

$$p(t) = v(t)\,i(t)$$

$$= 40t \times 2t = 80t^2 \text{ watts.}$$

The resultant voltage and power waveforms are shown in Figs 1.10 (a) and (b) respectively.

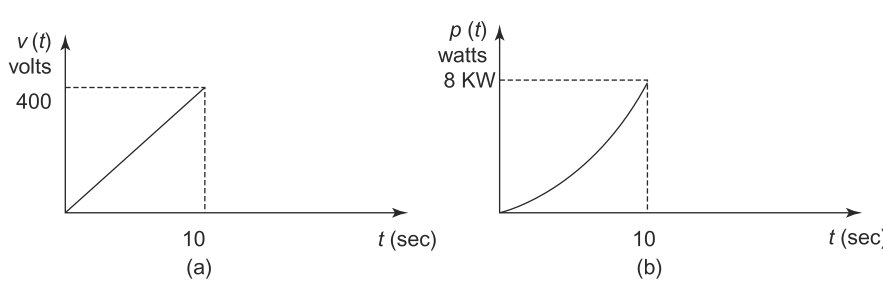


Fig. 1.10

Example 1.16 *The current function shown in Fig. 1.11 below is a repeating sawtooth and exists in a pure resister of 8Ω. Find the voltage v(t) and instantaneous power p(t).*

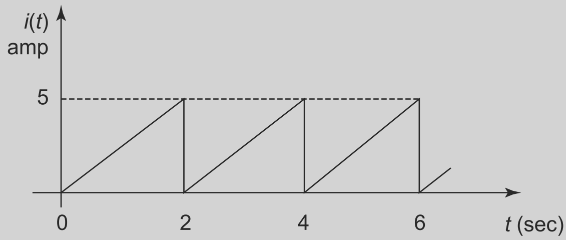


Fig. 1.11

Solution Since $v(t) = R\,i(t)$

$$v_{max} = R\,i_{max} = (8)\,(5) = 40\text{ V}$$

when $0 < t < 2\text{s},\ i(t) = \frac{5}{2}t = 2.5\,t$ amperes

Then, voltage $v(t) = R\,i(t)$

$$= 8(2.5\,t) = 20\,t \text{ volts}$$

Instantaneous power $p(t) = v(t)\,i(t)$

$$= 20\,t \times 2.5\,t$$

$$= 50\,t^2 \text{ watts.}$$

Therefore, the voltage $v(t)$ and power $p(t)$ waveforms are shown in Figs 1.12 (a) and (b) respectively.

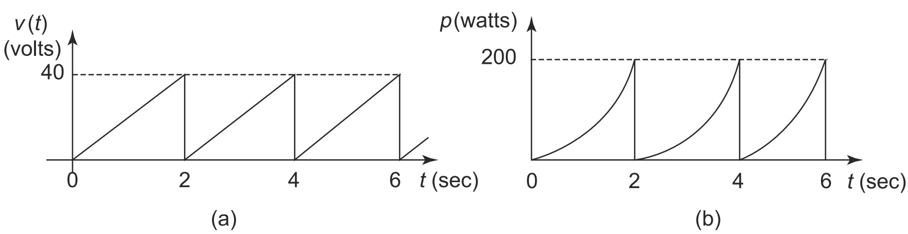


Fig. 1.12

Example 1.17 *The voltage waveform shown in Fig. 1.13 is applied to a pure resistor of 10 Ω. Sketch the current waveform and instantaneous power.*

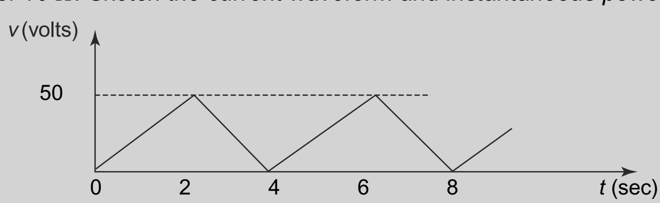


Fig. 1.13

Solution Since $v(t) = R\,i(t)$,

$$i_{max} = \frac{v_{max}}{R} = \frac{50}{10} = 5 \text{ amperes.}$$

when $0 \le t \le 2\text{s}$, $v = 25\,t$ volts,

then $i = \dfrac{25t}{10} = 2.5\,t$ amps.

when $2\text{s} \le t \le 4\text{s}$, $v = -25\,t$ volts

then $i = \dfrac{-25t}{10} = -2.5\,t$ amps

The instantaneous power $p(t) = v(t)\,i(t)$

when $0 < t < 2\text{s}$, $p = vi$

$$= 25\,t \times 2.5\,t$$
$$= 62.5\,t^2 \text{ watts}$$

when $2\text{s} < t < 4\text{s}$, $p = vi$

$$= -25\,t \times -2.5\,t$$
$$= 62.5\,t^2 \text{ watts}$$

Therefore, the current and power waveforms are shown in Figs 1.14 (a) and (b) respectively.

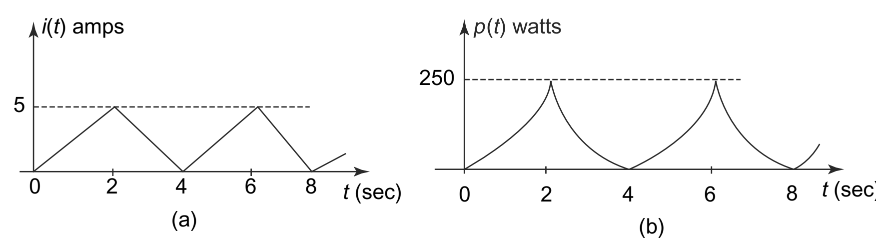


Fig. 1.14

1.3.2 Capacitive Element *[JNTU Nov. 2011]*

Consider a capacitive element shown in Fig. 1.15.
The capacitance c is given by the voltage – current relationship

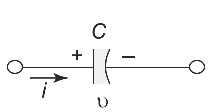


Fig. 1.15

$$i = c\frac{dv}{dt}$$

or

$$v = \frac{1}{c}\int i\,dt + v_0$$

where v_0 is the initial voltage across the capacitor.

Now let us determine the response of pure capacitor for various input waveforms through following examples.

Example 1.18 *The current waveform shown in Fig. 1.16 is applied to capacitor of 5 μF. Find the voltage across the capacitor.*

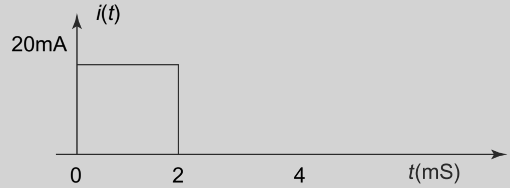


Fig. 1.16

Solution Assume initial voltage across the capacitor is zero.

The voltage across the capacitor is

$$v(t) = \frac{1}{c}\int i(t)\,dt + v_0$$

$$i(t) = 20 \text{ mA}, 0 \leq t \leq 2\text{ms}$$
$$= 0, t > 2 \text{ ms}$$

$$\therefore v(t) = \frac{1}{5\times 10^{-6}}\int 20\times 10^{-3}\,dt + v_0$$

since $v_0 = 0$

$v(t) = 4000\,t \quad 0 \leq t \leq 2\text{ms}$

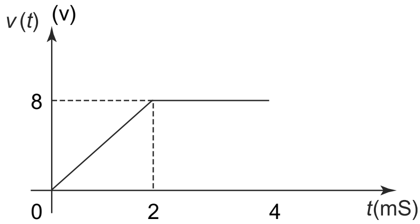


Fig. 1.17

At $t \geq 2$ ms, the voltage across the capacitor

$v(t) = 8$ volts.

The resultant waveform is shown in Fig. 1.17.

Example 1.19 *A current of the waveform shown in Fig. 1.18 flows through a capacitance C = 100 μF. Sketch the voltage waveform and determine V_m.*

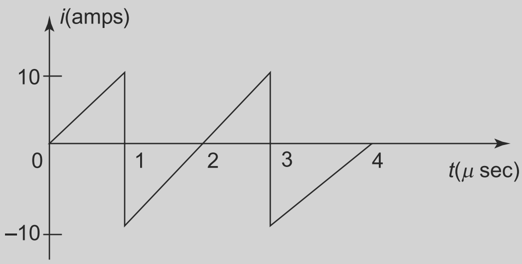


Fig. 1.18

Solution Assume initial voltage across the capacitor is zero.

The voltage across the capacitor is

$$v(t) = \frac{1}{c}\int i(t)\,dt + v_0$$

$$i(t) = 10 \times 10^6\, t\,;\ 0 \le t \le 1\mu s$$
$$= -20 + 10 \times 10^6 t;\ 1\mu s \le t \le 2\ \mu s$$

Since $v_0 = 0$

$$v(t) = \frac{1}{100 \times 10^{-6}} \int 10 \times 10^6 t\ dt;\quad 0 \le t \le 1\mu s$$

and

$$v(t) = \frac{1}{100 \times 10^{-6}} \int [-20 + 10 \times 10^6 t] dt;\quad 1\ \mu s \le t \le 2\ \mu s$$

Therefore,

$$v(t) = \frac{10 \times 10^6}{100 \times 10^{-6}} \frac{t^2}{2};\quad 0 \le t \le 1\ \mu s$$
$$v(t) = \frac{1}{100 \times 10^{-6}} \left[-20t + 10 \times 10^6 \frac{t^2}{2} \right];\quad 1\ \mu s \le t \le 2\ \mu s$$

The voltage waveform is shown in Fig. 1.19

The maximum voltage $V_m = 0.05$ V.

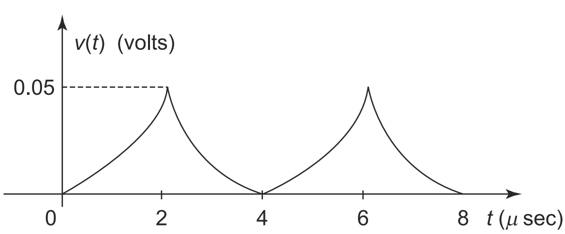


Fig. 1.19

Example 1.20 *The voltage waveform shown in Fig. 1.20 is applied to a pure capacitor of 50 μF. Sketch i(t), p(t), and determine I_m and p_m.*

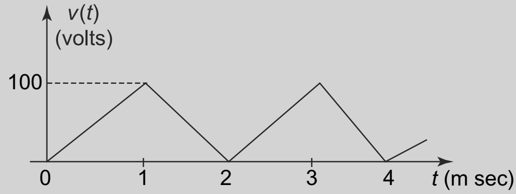


Fig. 1.20

Solution Since $i = c\dfrac{dv}{dt}$

From the voltage waveform $v(t) = 100 \times 10^3 t;\ 0 \le t \le 1\text{ms}$

$= 200 - 100 \times 10^3 t;\ 1\text{ms} \le t \le 2\text{ ms}$

Therefore, the current

$$i(t) = c\frac{dv(t)}{dt}$$

$$= 50 \times 10^{-6}\left[\frac{d}{dt}(100 \times 10^3 t)\right]$$

$$= 5\text{ A}; \qquad 0 \le t \le 1\text{ ms}$$

$$i(t) = c\frac{d}{dt}[v(t)]$$

$$= 50 \times 10^{-6}\left[\frac{d}{dt}(200 - 100 \times 10^3 t)\right]$$

$$= -5\text{ A}; \qquad 1\text{ ms} \le t \le 2\text{ ms}$$

The instantaneous power $p(t) = v(t)\, i(t)$

$= 100 \times 10^3 t \times 5$

$= 500 \times 10^3 t;\ 0 \le t \le 1\text{ ms}$

and $p(t) = [\,200 - 100 \times 10^3 t][-5]$

$= 500 \times 10^3 t - 10^3,\ 1\text{ms} \le t \le 2\text{ms}.$

The max value of current

$I_m = 5\text{ A}$

and the max value of power

$P_m = 500\text{ watts}$

The current and instantaneous power waveforms are shown in Figs 1.21 (a) and (b) respectively.

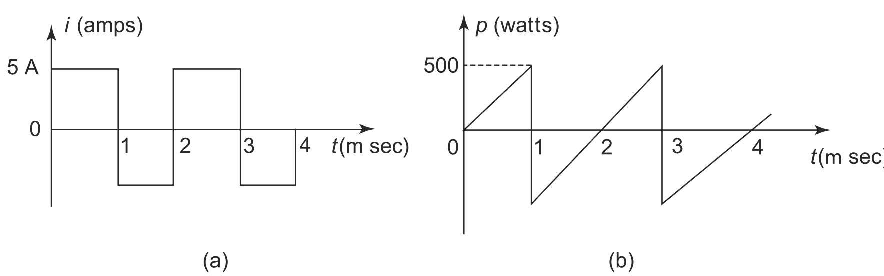


Fig. 1.21

Example 1.21 *A capacitor is charged to 1 volt at t = 0. A resistor of 1 ohm is connected across its terminals. The current is known to be of the form* $i(t) = e^{-t}$ *amperes for t > 0. At a particular time the current drops to 0.37A at that instant determine.*

(i) At what rate is the voltage across the capacitor changing?
(ii) What is the value of the charge on the capacitor?
(iii) What is the voltage across the capacitor?
(iv) How much energy is stored in the electric field of the capacitor?
(v) What is the voltage across the resistor?

Solution

(i) The current equation is given as $i(t) = i(0^+)\, e^{-t/RC}$; given $i(t) = e^{-t/RC}$

$i(0^+) = 1\text{A};\ RC = 1;\ C = 1\text{F}$

When $i(t) = 0.37$ amperes

$$i(t) = 0.37 = e^{-t/1}$$

$$-t \log_e e = \log_e 0.37$$

$$t = 0.9942 \text{ sec}$$

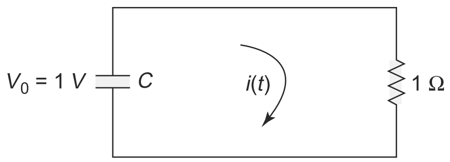


Fig. 1.22

$$i(t) = C\frac{dV(t)}{dt} \Rightarrow \frac{dV(t)}{dt} = \frac{i(t)}{C} = \frac{0.37}{1} = 0.37 \text{ V/sec}$$

or $$V_i(t) = \frac{1}{C}\int_0^t i(t)dt + V_0$$

$$= -\frac{1}{C}\int_0^t e^{-t}dt + V_0 \qquad [\therefore\ i(t) = -(t)]$$

$$= \frac{-1}{1}\frac{e^{-t}}{(-1)} + 1 = e^{-t}$$

$$= V_c(t) = e^{-t} \text{ for } t > 0$$

$$\therefore \quad \frac{dV_C(t)}{dt} = -e^{-t} = -e^{-0.9942} = -0.37 \text{ V/sec}$$

(ii) Charge on the capacitor

$$Q = C\,V_c = 1.e^{-t} = 0.37 \text{ coulombs}$$

(iii) Voltage across the capacitor

$$V_c(t) = e^{-t} = 0.37 \text{ volts}$$

(iv) Energy stored in the capacitor

$$W_C = \frac{1}{2}CV_c^2 = \frac{1}{2}1(e^{-t})^2 = \frac{e^{-2t}}{2} = 0.06845 \text{ joules}$$

(v) Voltage across the resistor at $t = 0.9942$ sec

$$V_R = i(t), R = e^{-t} = 0.37 \text{ V}$$

1.3.3 Inductive Element

[JNTU Nov. 2011]

Consider an inductive element shown in Fig. 1.23

The inductance L is given by the voltage current relationship.

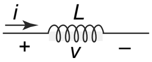


Fig. 1.23

$$v = L\frac{di}{dt}$$

or

$$i = \frac{1}{L}\int v dt + i_0$$

where i_0 is initial current flowing through inductor.

Example 1.22 *The current waveform shown in Fig. 1.24 is the current in a pure inductor L = 0.5 H. Sketch the voltage waveform.*

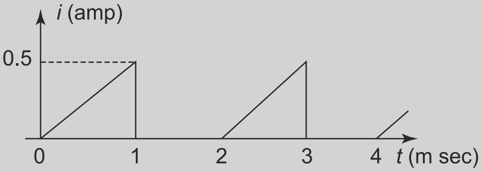


Fig. 1.24

Solution The relation between voltage and current in an inductor is given by

$$v = L\frac{di}{dt}$$

From the Fig. 1.51, the current

$$i(t) = 0.5 \times 10^3 t\,;\, 0 \le t \le 1\,\text{ms}$$

$$= 0;\, 1\text{ms} \le t \le 2\,\text{ms}$$

The voltage across inductor

$$v = 0.5 \times \frac{d}{dt}(0.5 \times 10^3 t)$$

$$= 0.5 \times 0.5 \times 10^3 = 250\,\text{V}$$

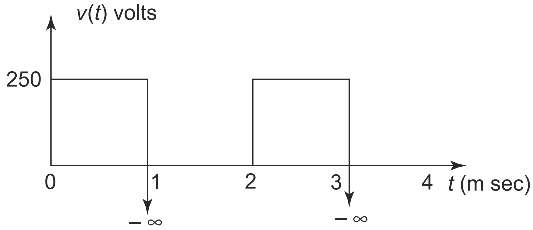


Fig. 1.25

Practically, the current in an inductor never the discontinuous function as shown at 1 ms and 3 ms. The derivative has an infinite negative value at the points of discontinuity, there will be negative infinite spikes on the voltage waveform at these points.

The voltage waveform is shown in Fig. 1.25.

Example 1.23 *An indictor element 10 mH passes a current i(t) of waveform shown in Fig. 1.26. Find the voltage across the element. Also sketch the voltage waveform.*

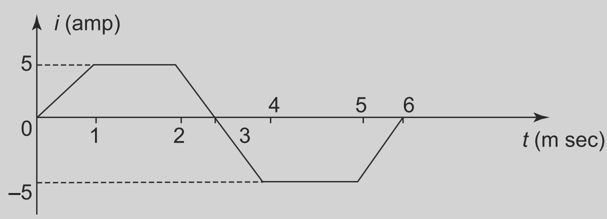


Fig. 1.26

Solution The voltage across inductor is given by

$$v = L\frac{di}{dt}$$

$$i(t) = 5 \times 10^3 t \text{ amp; } 0 \le t \le 1 \text{ m sec}$$

$$v(t) = (10 \times 10^{-3})\,(5 \times 10^3) = 50\text{V}$$

$$i(t) = 5\text{A; } 1\text{ms} \le t \le 2\text{ms}$$

$$v(t) = 10 \times 10^{-3}\,(0) = 0$$

$$i(t) = -5 \times 10^3 t + 15;\ 2\text{ms} \le \text{t} \le 4\text{ms}$$

$$v(t) = (10 \times 10^{-3})\,(-5 \times 10^3) = -50\text{V}$$

$$i(t) = -5\text{A; } 4\text{ms} \le \text{t} \le 6\text{ms}$$

$$v(t) = 10 \times 10^{-3}(0) = 0$$

$$i(t) = 5 \times 10^3\, t - 30\ ;\ 5\text{ms} \le t \le 6\text{ms}$$

$$\therefore \quad v(t) = 10 \times 10^{-3} \times 5 \times 10^3 = 50\text{V}$$

Therefore, the voltage waveform is shown in Fig. 1.27.

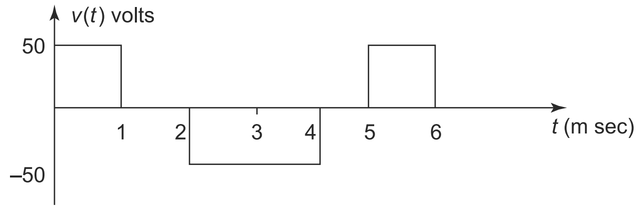


Fig. 1.27

Example 1.24 *A pure inductance of 0.01 H has an applied voltage with a waveform shown in Fig. 1.28. Sketch the corresponding current waveform and determine the expression for i in the first interval* $0 < t < 1$ *ms.*

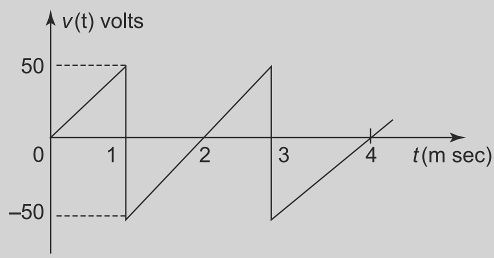


Fig. 1.28

Solution The voltage current relation in an inductor is given by

$$i(t) = \frac{1}{L}\int v(t)\,dt + i_0$$

Since $i_0 = 0$

$$i = \frac{1}{0.01}\int v(t)\,dt$$

$$v(t) = 50 \times 10^3 t;\ 0 \le t \le 1\,\text{ms}$$

$$= -100 + 50 \times 10^3 t\ ;\ 1\,\text{ms} \le t \le 2\,\text{ms}.$$

Therefore, the current equation

$$i(t) = \frac{1}{0.01}\int 50 \times 10^3 t \cdot dt$$

$$i(t) = 25 \times 10^5 t^2\ ;\ 0 \le t \le 1\,\text{ms}$$

and $$i(t) = \frac{1}{0.01}\int [-100 + 50 \times 10^3 t]dt$$

$$= \frac{1}{0.01}\left[-100\,t + 50 \times 10^3 \frac{t^2}{2}\right];1\text{ ms} \le t \le 2\text{ ms}$$

The current waveform is shown in Fig. 1.29.

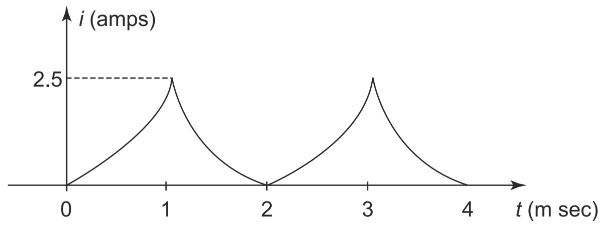


Fig. 1.29

Example 1.25 *The following current wavefrom i(t) is passed through a series RL circuit with R = 2 ; L = 2 mH. Find the voltage across each element and sketch the same. (See Fig. 1.30)* *[JNTU April/May 2003]*

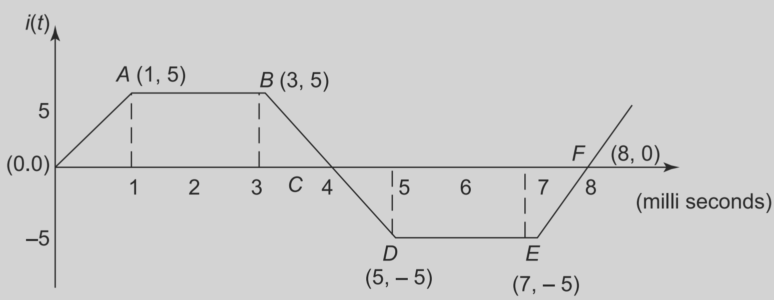


Fig. 1.30

Solution

For line $OA, m = \dfrac{5}{1}$

$$i(t) - 0 = \frac{5}{1}(t - 0)$$

$$i(t) = 5t$$

For line AB, $i(t) = 5$

For line BD,

$$(i(t) - 5) = \frac{-5-5}{5-3}(t-3)$$

$$i(t) - 5 = -5(t - 3)$$

$$i(t) = -5t + 20$$

For line DE, $i(t) = -5$

For line EF,

$$(i(t) + 5) = \frac{5}{1}(t - 7)$$

$$i(t) = 5t - 40$$

Voltage induced in the inductor

Along OA

$$V_{OA} = L\frac{di}{dt} = 2\times10^{-3}\times\frac{d(5t)}{dt} = 2\times10^{-3}\times5\times10^{-3} = 10\ \mu\text{V}$$

Along AB

$$V_{AB} = L\frac{di}{dt} = 0$$

Along BD

$$V_{Bd} = L\frac{di}{dt} = 2\times10^{-3}\times\frac{d(-5t+20)}{dt} = -10\times10^{-6}\ \text{V} = -10\ \mu\text{V}$$

Along DE

$$V_{DE} = L\frac{di}{dt} = 0$$

Along EF

$$V_{EF} = L\frac{di}{dt} = 2\times10^{-3}\times\frac{d(5t+40)\times10^{-3}}{dt} = 10\ \mu\text{V}$$

The wave is shown in Fig. 1.31.

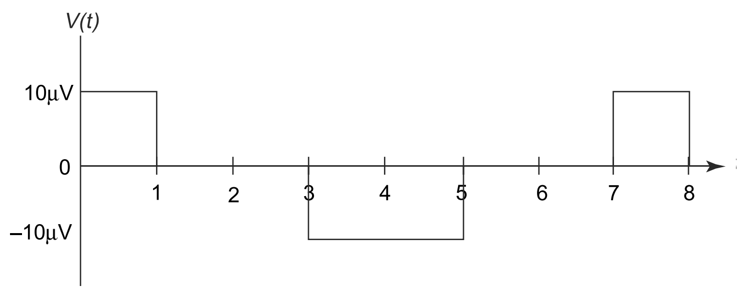


Fig. 1.31

Voltage waveform across the resistor is the same as current through the circuit as shown in Fig. 1.32.

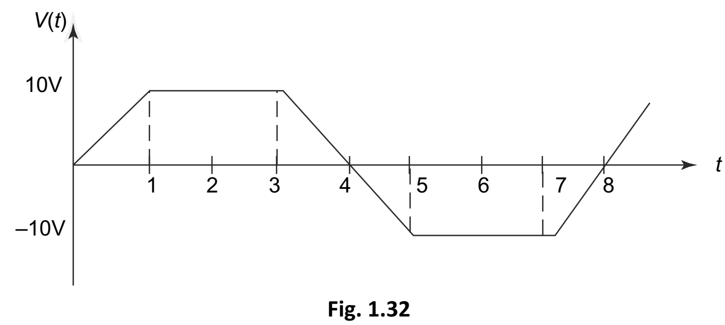


Fig. 1.32

Example 1.26 *Describe the Volt-ampere relations for R, L and C Parameters.*

Solution *Volt-ampere Relations for R, L and C Parameters*

The passive elements R, L, C are defined by the way in which the current and voltage are related for individual element.

(i) If the current 'I' and voltage 'V' are related by a constant for a single element then the element is a resistance 'R'. The Resistance 'R' represents the constant of proportionality.

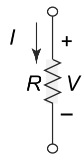


Fig. 1.33

$\therefore$ Voltage, $V = RI$ (ohms law)

Current $I = \dfrac{V}{R}$

Power, $P = VI = I^2R$

The units of resistance 'R' is ohms (Ω).

(ii) If the current and voltage are related such that the voltage is the time derivative of current, then the element is an inductance 'L'. The inductance 'L' represents the constant of proportionality.

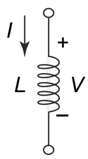


Fig. 1.34

$\therefore$ Voltage, $V = L\dfrac{dI}{dt}$

Current, $I = \dfrac{1}{L}\int V\,dt + K_1 \quad [K_1 = \text{constant}]$

Power, $P = VI = LI\dfrac{dI}{dt}$

The units of inductance 'L' is Henry (H).

(iii) If the voltage and current are related such that the current is the time derivative of the voltage, then the element is a capacitance 'C'. The capacitance 'C' is the constant of proportionality.

$\therefore$ Current, $I = C\dfrac{dV}{dt}$

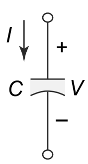


Fig. 1.35

Voltage, $V = \dfrac{1}{C}\int I\,dt + K_2 \quad [K_2 = \text{constant}]$

Power, $P = VI = VC\dfrac{dV}{dt}$

The units of capacitance 'C' is Farads (F).

1.3.4 Combination of Inductances and capacitances

Inductors in Series

Consider a voltage source is applied to the series combination of N inductors as shown in Fig. 1.36.

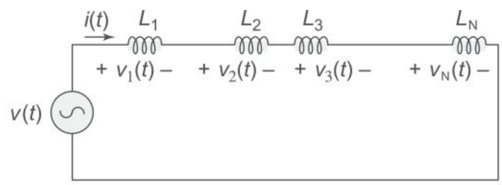


Fig. 1.36

In the circuit, the current passing through each inductive element is same. Also, the source voltage applied to the circuit $v(t)$ is equal to the sum of the individual voltages.

ie $v(t) = v_1(t) + v_2(t) + v_3(t) + \dots\dots\dots + v_N(t)$

$$v(t) = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \dots + L_N\frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \dots + L_N)\frac{di}{dt}$$

$$v(t) = L_{eq}\frac{di}{dt}$$

Therefore, the equivalent inductance is

$$L_{eq} = L_1 + L_2 + L_3 + \dots\dots\dots\dots + L_N$$

The equivalent inductance of any number of inductors connected in series is the sum of the individual inductances.

Example 1.27 *Three inductances are connected as shown in Fig. 1.37. What is the equivalent inductance?*

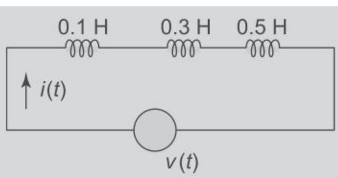


Fig. 1.37

Solution Since the current passing through each inductance is same, the three inductances are connected in series.

The equivalent inductance $L_{eq} = (0.1 + 0.3 + 0.5)$ H

$L_{eq} = 0.9$ H.

Inductors in Parallel

Consider the circuit shown in Fig. 1.38. The current source $i(t)$ is applied to circuit. Assume a voltage $v(t)$ exists across the parallel combination and let the currents in L_1, L_2 ….., L_N be $i_1(t)$, $i_2(t)$ ……, $i_N(t)$ respectively.

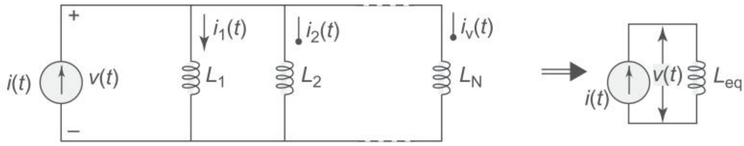


Fig. 1.38

Since the total current i_T is the sum of the branch currents.

$$i(t) = i_1(t) + i_2(t) + \ldots\ldots\ldots + i_N(t)$$

or

$$\frac{1}{L_{eq}}\int v(t)dt = \frac{1}{L_1}\int v(t)dt + \frac{1}{L_2}\int v(t)dt + \ldots.. + \frac{1}{L_N}\int v(t)dt$$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots\ldots.. + \frac{1}{L_N}$$

Therefore, the reciprocal of the equivalent inductance of any number of inductors in parallel is the sum of the reciprocals of the individual inductances.

Example 1.28 *There pure inductances are connected as shown in Fig. 1.39. What equivalent inductance L_{eq} may replace this circuit?*

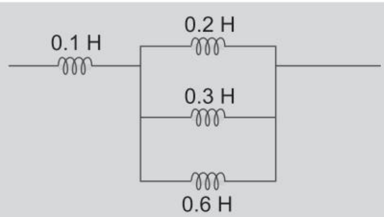


Fig. 1.39

Solution Equivalent inductance of parallel combination is

$$\frac{1}{L_{\text{eqp}}} = \frac{1}{0.2} + \frac{1}{0.3} + \frac{1}{0.6} = 10$$

$$L_{\text{eqp}} = 0.1 \text{ H}$$

The required equivalent inductance

$$L_{\text{eq}} = 0.1 \text{ H} + L_{\text{eqp}}$$

$$L_{\text{eq}} = 0.2 \text{ H}$$

Capacitors in Series

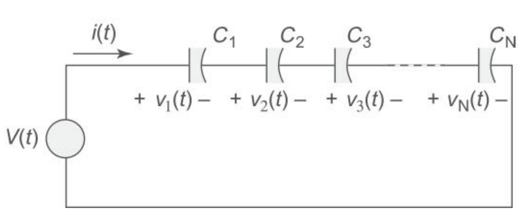


Fig. 1.40

Consider a circuit consists of N capacitors in series as shown in Fig. 1.40.

In the circuit, the total voltage applied to the circuit is equal to sum of the voltages across individual capacitive elements.

$\therefore \quad v(t) = v_1(t) + v_2(t) + v_3(t) + \ldots\ldots\ldots + v_N(t)$

Assuming zero initial voltage across each capacitor

$$v(t) = \frac{1}{C_1}\int i(t)dt + \frac{1}{C_2}\int i(t)dt + \frac{1}{C_3}\int i(t)dt + \ldots\ldots\ldots + \frac{1}{C_N}\int i(t)dt$$

where $\quad v(t) = \dfrac{1}{C_{\text{eq}}}\int i(t)dt$

$$\therefore \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots + \frac{1}{C_{\text{N}}}$$

The reciprocal of the equivalent capacitance of any number of capacitors connected in series is the sum of the reciprocals of the individual capacitances.

Example 1.29 *The two capacitors shown in Fig. 1.41 are connected in series. Find the equivalent capacitance of the circuit.*

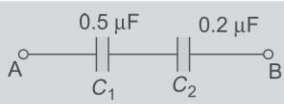


Fig. 1.41

Solution The equivalent capacitance of the circuit shown in Fig. 1.41 is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{0.5\times10^{-6}\times0.2\times10^{-6}}{0.7\times10^{-6}}$$

$\therefore \quad C_{\text{eq}} = 0.143\ \mu\text{F}.$

Capacitors in Parallel

Consider the circuit shown in Fig. 1.42 consists of N parallel capacitors. A current source is applied to the circuit. The total current applied to the circuit is the sum of the individual currents flowing in the circuit.

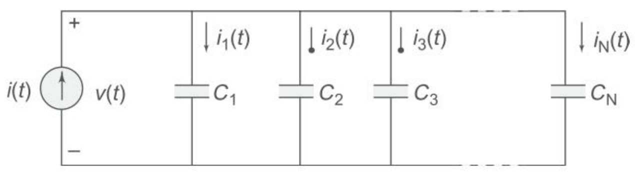


Fig. 1.42

$i(t) = i_1(t) + i_2(t) + i_3(t) + \ldots\ldots\ldots + i_N(t)$

$$C_{eq}\frac{dv(t)}{dt} = C_1\frac{dv(t)}{dt} + C_2\frac{dv(t)}{dt} + C_3\frac{dv(t)}{dt} + \ldots + C_N\frac{dv(t)}{dt}$$

From the above equation, we get

$$C_{eq} = C_1 + C_2 + C_3 + \ldots\ldots\ldots + C_N$$

The resultant capacitance of any number of capacitors in parallel is the sum of the individual capacitances.

Example 1.30 *Find the equivalent capacitance C_e of the combination of capacitors shown in Fig. 1.43*

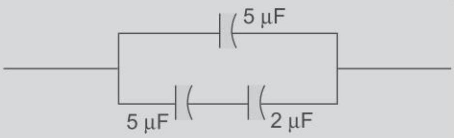


Fig. 1.43

Solution Equivalent capacitance of series branch is

$$C_s = \frac{C_1C_2}{C_1 + C_2} = \frac{5\times 2\times 10^{-6}}{5+2} = 1.43\ \mu\text{F}$$

The required equivalent capacitance is

$$C_{eq} = C_s + 5\ \mu\text{F}$$

$$C_{eq} = 1.43 + 5 = 6.43\ \mu\text{F}$$

Example 1.31 *Find the total equivalent capacitance, total energy stored if the applied voltage is 100 V for the circuit as shown in Fig. 1.44* *[JNTU Jan 2010]*

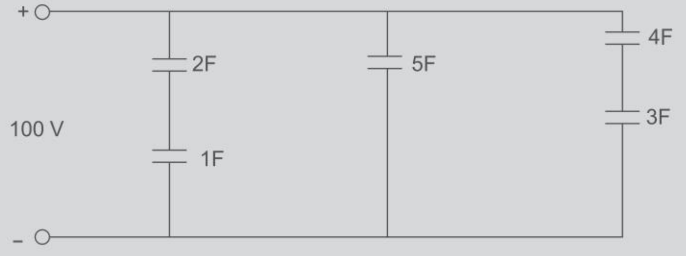


Fig. 1.44

Solution 4F and 3F in series

$$\text{C}_{eq} = \frac{4\times 3}{4+3} = \frac{12}{4}\ \text{F}$$

$\frac{12}{7}$ F in parallel with 5 F

$$\therefore\ \text{C}_{eq} = \frac{12}{7} + 5 = \frac{35+12}{7} = \frac{47}{7}\ \text{F}$$

$\therefore \quad C_{eq} = 2\text{ F} \ \& \ 1\text{ F in series}$

$$C_{eq} = \frac{2\times 1}{2+1} = \frac{2}{3}\text{F}$$

$\therefore \quad \frac{2}{3}\text{F in parallel with } \frac{47}{7}\text{ F}$

$$\therefore \quad C_{eq} = \frac{2}{3} + \frac{47}{7} = \frac{14+141}{21} = \frac{155}{21}\text{F}$$

$$\therefore \quad E = \frac{1}{2}CV^2$$

$$\therefore \quad E = \frac{1}{2}\times\frac{155}{21}\times 100\times 100$$

$$E = 36900\text{ J}$$

1.4 VOLTAGE AND CURRENT SOURCES—INDEPENDENT AND DEPENDENT SOURCES *[JNTU Nov. 2011]*

According to their terminal voltage-current characteristics, electrical energy sources are categorised into ideal voltage sources and ideal current sources. Further they can be divided into independent and dependent sources.

An ideal voltage source is a two-terminal element in which the voltage υ_s is completely independent of the current i_s through its terminals. The representation of ideal constant voltage source is shown in Fig. 1.45(a).

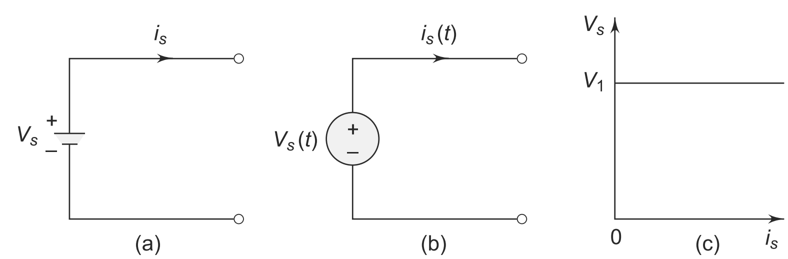


Fig. 1.45

If we observe the $\upsilon - i$ characteristics for an ideal voltage source as shown in Fig. 1.45(c) at any time, the value of the terminal voltage υ_s is constant with respect to the value of current i_s. Whenever $\upsilon_s = 0$, the voltage source is the same as that of a short circuit. Voltage sources need not have constant magnitude; in many cases the specified voltage may be time-dependent like a sinusoidal waveform. This may be represented as shown in Fig. 1.45(b). In many practical voltage sources, the internal resistance is represented in series with the source as shown in Fig. 1.46(a). In this, the voltage across the terminals falls as the current through it increases, as shown in Fig. 1.46(b).

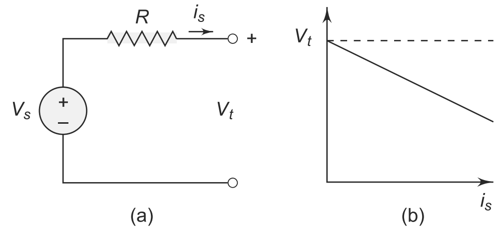


Fig. 1.46

The terminal voltage υ_t depends on the source current as shown in Fig. 1.46(b), where $\upsilon_t = \upsilon_s - i_s R$.

An ideal constant current source is a two-terminal element in which the current i_s completely independent of the voltage υ_s across its terminals. Like voltage sources we can have current sources of constant magnitude i_s or sources whose current varies with time $i_s(t)$. The representation of an ideal current source is shown in Fig. 1.47(a).

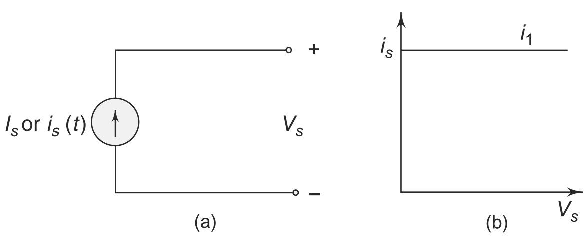


Fig. 1.47

If we observe the $\upsilon - i$ characteristics for an ideal current source as shown in Fig. 1.48(b), at any time the value of the current i_s is constant with respect to the voltage across it. In many practical current sources, the resistance is in parallel with a source as shown in Fig. 1.48(a). In this the magnitude of the current falls as the voltage across its terminals increases. Its terminal $\upsilon - i$ characteristics is shown in Fig. 1.48(b). The terminal current is given by $i_t = i_s - (\upsilon_s/R)$, where R is the internal resistance of the ideal current source.

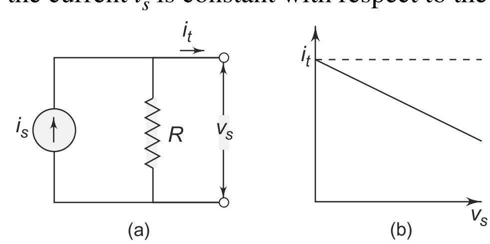


Fig. 1.48

The two types of ideal sources we have discussed are independent sources for which voltage and current are independent and are not affected by other parts of the circuit. In the case of dependent sources, the source voltage or current is not fixed, but is dependent on the voltage or current existing at some other location in the circuit.

Dependent or controlled sources are of the following types:

(i) voltage controlled voltage source (VCVS)
(ii) current controlled voltage source (CCVS)
(iii) voltage controlled current source (VCCS)

(iv) current controlled current source (CCCS)

These are represented in a circuit diagram by the symbol shown in Fig. 1.49. These types of sources mainly occur in the analysis of equivalent circuits of transistors.

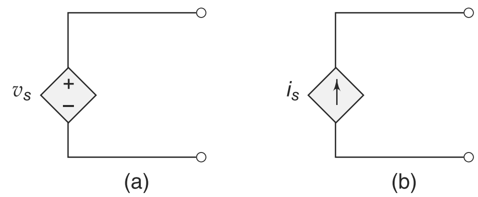


Fig. 1.49

1.5 KIRCHHOFF'S LAWS — NETWORK REDUCTION TECHNIQUES — SERIES, PARALLEL, SERIES PARALLEL

[JNTU May/June 2008, Jan 2010]

1.5.1 Kirchhoff's Voltage Law

[JNTU Nov. 2011]

Kirchhoff's voltage law states that the algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time. When the current passes through a resistor, there is a loss of energy and, therefore, a voltage drop. In any element, the current always flows from higher potential to lower potential. Consider the circuit in Fig. 1.50. It is customary to take the direction of current *I* as indicated in the figure, i.e. it leaves the positive terminal of the voltage source and enters into the negative terminal.

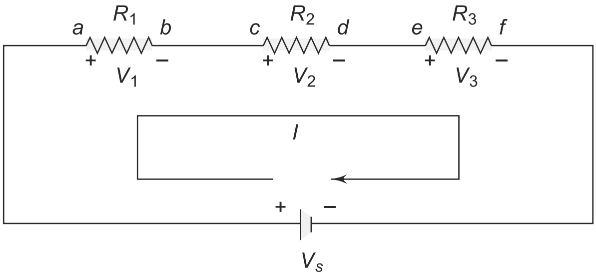


Fig. 1.50

As the current passes through the circuit, the sum of the voltage drop around the loop is equal to the total voltage in that loop. Here the polarities are attributed to the resistors to indicate that the voltages at points *a*, *c* and *e* are more than the voltages at *b*, *d* and *f*, respectively, as the current passes from *a* to *f*.

$$\therefore \quad V_s = V_1 + V_2 + V_3$$

Consider the problem of finding out the current supplied by the source *V* in the circuit shown in Fig. 1.51.

Our first step is to assume the reference current direction and to indicate the polarities for different elements. (See Fig. 1.52).

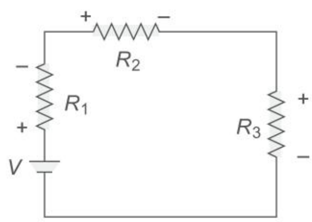


Fig. 1.51

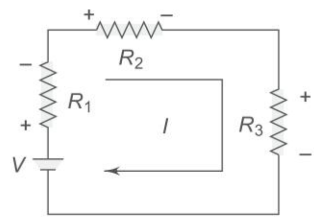


Fig. 1.52

By using Ohm's law, we find the voltage across each resistor as follows.

$$V_{R1} = IR_1,\ V_{R2} = IR_2,\ V_{R3} = IR_3$$

where V_{R1}, V_{R2} and V_{R3} are the voltages across R_1, R_2 and R_3, respectively. Finally, by applying Kirchhoff's law, we can form the equation

$$V = V_{R1} + V_{R2} + V_{R3}$$

$$V = IR_1 + IR_2 + IR_3$$

From the above equation the current delivered by the source is given by

$$I = \frac{V}{R_1 + R_2 + R_3}$$

Example 1.32 *For the circuit shown in Fig. 1.53, determine the unknown voltage drop V_1.*

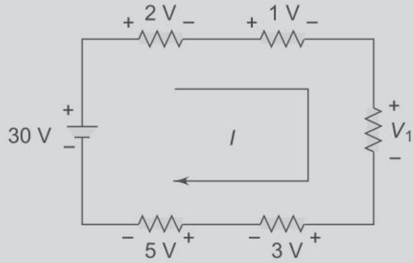


Fig. 1.53

Solution According to Kirchhoff's voltage law, the sum of the potential drops is equal to the sum of the potential rises;

Therefore, $30 = 2 + 1 + V_1 + 3 + 5$

or $V_1 = 30 - 11 = 19$ V

Example 1.33 *What is the current in the circuit shown in Fig. 1.54? Determine the voltage across each resistor.*

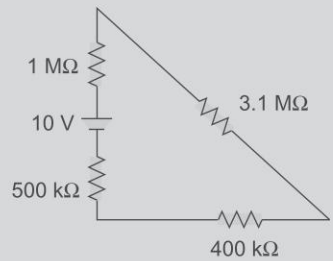


Fig. 1.54

Solution We assume current I in the clockwise direction and indicate polarities (Fig. 1.55). By using Ohm's law, we find the voltage drops across each resistor.

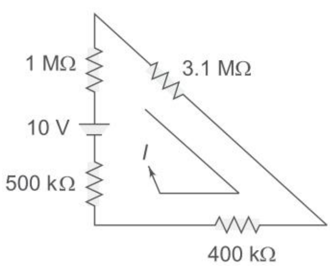


Fig. 1.55

$$V_{1M} = I, \quad V_{3.1M} = 3.1\,I$$

$$V_{500\,K} = 0.5\,I, \quad V_{400\,K} = 0.4\,I$$

Now, by applying Kirchhoff's voltage law, we form the equation.

$$10 = I + 3.1\,I + 0.5\,I + 0.4\,I$$

or $5\,I = 10$

or $I = 2\ \mu A$

$\therefore$ Voltage across each resistor is as follows:

$$V_{1M} = 1 \times 2 = 2.0\text{ V}$$

$$V_{3.1M} = 3.1 \times 2 = 6.2\text{ V}$$

$$V_{400\,K} = 0.4 \times 2 = 0.8\text{ V}$$

$$V_{500\,K} = 0.5 \times 2 = 1.0\text{ V}$$

Example 1.34 *In the circuit given in Fig. 1.56, find (a) the current I, and (b) the voltage across 30 Ω.*

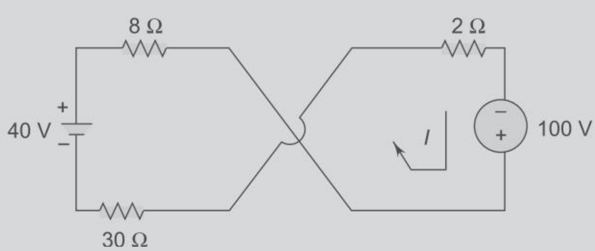


Fig. 1.56

Solution We redraw the circuit as shown in Fig. 1.57 and assume current direction and indicate the assumed polarities of resistors

By using Ohm's law, we determine the voltage across each resistor as

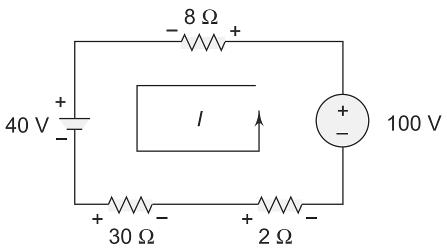


Fig. 1.57

$$V_8 = 8I,\ V_{30} = 30I,\ V_2 = 2I$$

By applying Kirchhoff's law, we get

$$100 = 8I + 40 + 30I + 2I$$

$$40\,I = 60 \text{ or } I = \frac{60}{40} = 1.5\,\text{A}$$

∴ Voltage drop across 30 Ω = V_{30} = 30 × 1.5 = 45 V

Example 1.35 *State Ohm's law.*

[JNTU May/June 2008]

Solution Ohm's law: Ohm's law states that the voltage across any element is proportional to current flowing through the element.

$$V \alpha I$$

$$V = RI$$

R is the proportionality constant and is defined as resistance. Its unit is (Ω).

1.5.2 Voltage Division

The series circuit acts as a voltage divider. Since the same current flows through each resistor, the voltage drops are proportional to the values of resistors. Using this principle, different voltages can be obtained from a single source, called a voltage divider. For example, the voltage across a 40 Ω resistor is twice that of 20 Ω in a series circuit shown in Fig. 1.58.

In general, if the circuit consists of a number of series resistors, the total current is given by the total voltage divided by equivalent resistance. This is shown in Fig. 1.59.

Fig. 1.58

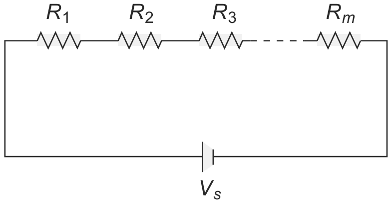


Fig. 1.59

The current in the circuit is given by $I = V_s/(R_1 + R_2 + \ldots + R_m)$. The voltage across any resistor is nothing but the current passing through it, multiplied by that particular resistor.

Therefore,
$$V_{R1} = IR_1$$
$$V_{R2} = IR_2$$
$$V_{R3} = IR_3$$
$$\vdots$$
$$V_{Rm} = IR_m$$

or
$$V_{Rm} = \frac{V_s(R_m)}{R_1 + R_2 + \ldots + R_m}$$

From the above equation, we can say that the voltage drop across any resistor, or a combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage, i.e.

$$V_m = \frac{R_m}{R_T} V_s$$

where V_m is the voltage across mth resistor, R_m is the resistance across which the voltage is to be determined and R_T is the total series resistance.

Example 1.36 *What is the voltage across the 10 Ω resistor in Fig. 1.60.*

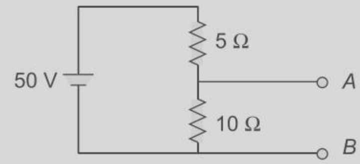


Fig. 1.60

Solution Voltage across $10\ \Omega = V_{10} = 50 \times \frac{10}{10+5} = \frac{500}{15} = 33.3\text{ V}$

Example 1.37 *Find the voltage between A and B in a voltage divider network shown in Fig. 1.61.*

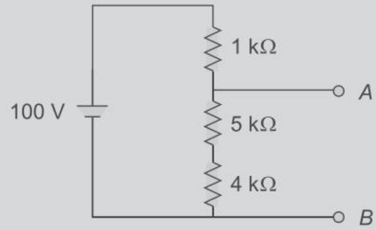


Fig. 1.61

Solution Voltage across $9\text{ k}\Omega = V_9 = V_{AB} = 100 \times \frac{9}{10} = 90\text{ V}$

Example 1.38 *What is the voltage across A and B in the circuit shown in Fig. 1.62.*

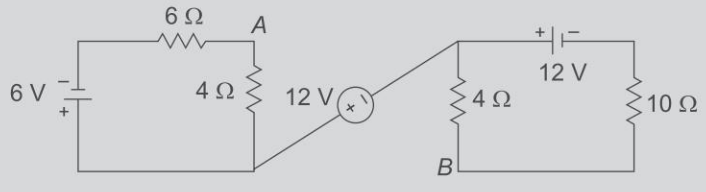


Fig. 1.62

Solution The above circuit can be redrawn as shown in Fig. 1.63.

Assume loop currents I_1 and I_2 as shown in Fig. 1.63.

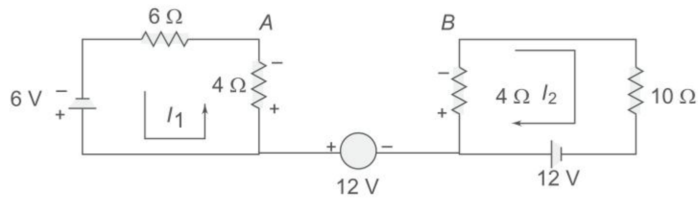


Fig. 1.63

$$I_1 = \frac{6}{10} = 0.6\,\text{A}$$

$$I_2 = \frac{12}{14} = 0.86\,\text{A}$$

V_A = Voltage drop across 4 Ω resistor = 0.6 × 4 = 2.4 V

V_B = Voltage drop across 4 Ω resistor = 0.86 × 4 = 3.44 V

The voltage between points *A* and *B* is the sum of voltages as shown in Fig. 1.64.

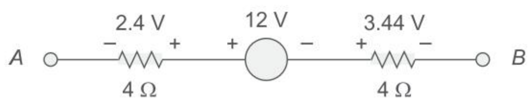


Fig. 1.64

$\therefore \quad V_{AB} = -2.4 + 12 + 3.44$

$= 13.04$ V

1.5.3 Power In Series Circuit

The total power supplied by the source in any series resistive circuit is equal to the sum of the powers in each resistor in series, i.e.

$$P_S = P_1 + P_2 + P_3 + \ldots + P_m$$

where m is the number of resistors in series, P_S is the total power supplied by source and P_m is the power in the last resistor in series. The total power in the series circuit is the total voltage applied to a circuit, multiplied by the total current. Expressed mathematically,

$$P_S = V_s I = I^2 R_T = \frac{V_s^2}{R_T}$$

where V_s is the total voltage applied, R_T is the total resistance, and I is the total current.

Example 1.39 *Determine the total amount of power in the series circuit in Fig. 1.65.*

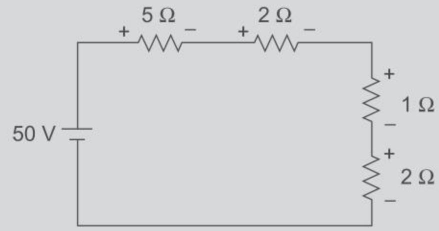


Fig. 1.65

Solution Total resistance = 5 + 2 + 1 + 2 = 10 Ω

We know $$P_S = \frac{V_s^2}{R_T} = \frac{(50)^2}{10} = 250 \text{ W}$$

Check We find the power absorbed by each resistor

$$\text{Current} = \frac{50}{10} = 5 \text{ A}$$

$$P_5 = (5)^2 \times 5 = 125 \text{ W}$$

$$P_2 = (5)^2 \times 2 = 50 \text{ W}$$

$$P_1 = (5)^2 \times 1 = 25 \text{ W}$$

$$P_2 = (5)^2 \times 2 = 50 \text{ W}$$

The sum of these powers gives the total power supplied by the source $P_S = 250$ W.

Example 1.40 *A 20 V battery with an internal resistance of 5 ohms is connected to a resistor of x ohms. If an additional resistance of 6 Ω is connected across the battery, find the value of x, so that the external power supplied by the battery remain the same.*

Solution Power supplied to x by battery $= \left(\frac{20}{5+x}\right)^2 x = P_1$

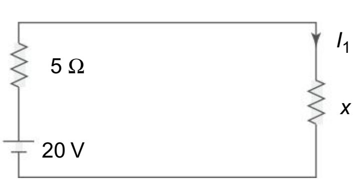


Fig. 1.66

$$I_2 = \frac{20}{5 + \frac{6x}{6+x}} = \frac{120}{30+11x}$$

$$\text{Power supplied to } x = \left(\frac{120}{30+11x}\right)^2 x = P_2$$

$$P_1 = P_2 \Rightarrow \frac{20}{5+x} = \frac{120}{11x+30}$$

$$x = 0$$

1.5.4 Kirchhoff's Current Law

[JNTU Nov. 2011]

Kirchhoff's current law states that the sum of the currents entering into any node is equal to the sum of the currents leaving that node. The node may be an interconnection of two or more branches. In any parallel circuit, the node is a junction point of two or more branches. The total current entering into a node is equal to the current leaving that node. For example, consider the circuit shown in Fig. 1.67, which contains two nodes A and B. The total current I_T entering node A is divided into I_1, I_2 and I_3. These currents flow out of node A. According to Kirchhoff's current law, the current into node A is equal to the total current out of node A: that is, $I_T = I_1 + I_2 + I_3$. If we consider node B, all three currents I_1, I_2, I_3 are entering B, and the total current I_T is leaving node B, Kirchhoff's current law formula at this node is therefore the same as at node A.

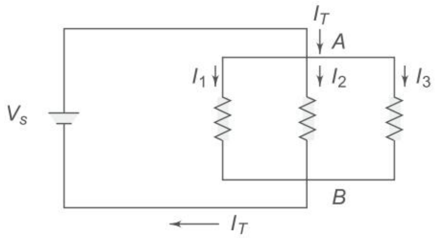


Fig. 1.67

$$I_1 + I_2 + I_3 = I_T$$

In general, sum of the currents entering any point or node or junction equal to sum of the currents leaving from that point or node or junction as shown in Fig. 1.68.

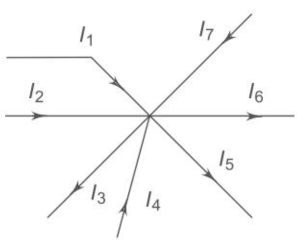


Fig. 1.68

$$I_1 + I_2 + I_4 + I_7 = I_3 + I_5 + I_6$$

If all of the terms on the right side are brought over to the left side, their signs change to negative and a zero is left on the right side, i.e.

$$I_1 + I_2 + I_4 + I_7 - I_3 - I_5 - I_6 = 0$$

This means that the algebraic sum of all the currents meeting at a junction is equal to zero.

Example 1.41 *Determine the current in all resistors in the circuit shown in Fig. 1.69.*

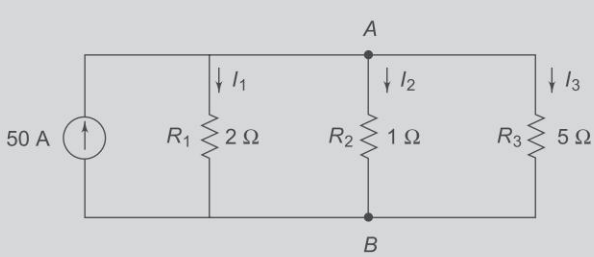


Fig. 1.69

Solution The above circuit contains a single node 'A' with reference node 'B'. Our first step is to assume the voltage V at node A. In a parallel circuit the same voltage is applied across each element. According to Ohm's law, the currents passing through each element are $I_1 = V/2, I_2 = V/1, I_3 = V/5$.

By applying Kirchhoff's current law, we have

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{2} + \frac{V}{1} + \frac{V}{5}$$

$$50 = V\left[\frac{1}{2} + \frac{1}{1} + \frac{1}{5}\right] = V[0.5 + 1 + 0.2]$$

$$V = \frac{50}{1.7} = \frac{500}{17} = 29.41 \text{ V}$$

Once we know the voltage V at node A, we can find the current in any element by using Ohm's law.

The current in the 2 Ω resistor is $I_1 = 29.41/2 = 14.705$ A.

Similarly $$I_2 = \frac{V}{R_2} = \frac{V}{1} = 29.41 \text{ A}$$

$$I_3 = \frac{29.41}{5} = 5.882 \text{ A}$$

$\therefore$ $$I_1 = 14.7 \text{ A}, I_2 = 29.4 \text{ A, and } I_3 = 5.88 \text{ A}$$

Example 1.42 *For the circuit shown in Fig. 1.70, find the voltage across the 10 Ω resistor and the current passing through it.*

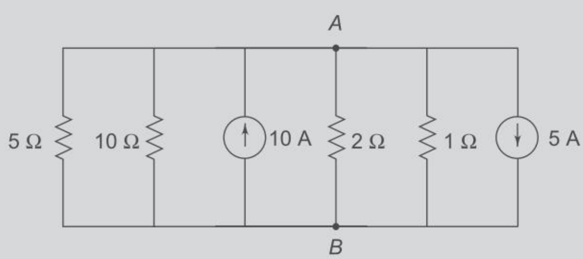


Fig. 1.70

Solution The circuit shown above is a parallel circuit, and consists of a single node A. By assuming voltage V at the node A w.r.t. B, we can find out the current in the 10 Ω branch. (See Fig. 1.71)

According to Kirchhoff's current law,

$$I_1 + I_2 + I_3 + I_4 + 5 = 10$$

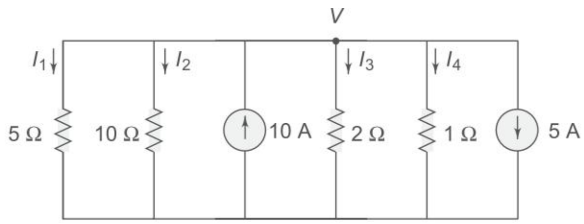


Fig. 1.71

By using Ohm's law we have

$$I_1 = \frac{V}{5};\ I_2 = \frac{V}{10},\ I_3 = \frac{V}{2},\ I_4 = \frac{V}{1}$$

$$\frac{V}{5} + \frac{V}{10} + \frac{V}{2} + V + 5 = 10$$

$$V\left[\frac{1}{5} + \frac{1}{10} + \frac{1}{2} + 1\right] = 5$$

$$V[0.2 + 0.1 + 0.5 + 1] = 5$$

$$V = \frac{5}{1.8} = 2.78\,\text{V}$$

∴ The voltage across the 10 Ω resistor is 2.78 V and the current passing through it is

$$I_2 = \frac{V}{10} = \frac{2.78}{10} = 0.278\,\text{A}$$

Example 1.43 *Determine the current through resistance R_3 in the circuit shown in Fig. 1.72.*

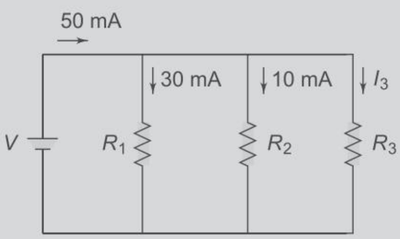


Fig. 1.72

Solution According to Kirchhoff's current law,

$$I_T = I_1 + I_2 + I_3$$

where I_T is the total current and I_1, I_2 and I_3 are the currents in resistances R_1, R_2 and R_3 respectively.

$$\therefore \quad 50 = 30 + 10 + I_3$$

$$\text{or} \quad I_3 = 10\ \text{mA}$$

Example 1.44 *Using Kirchhoff's current law, find the values of the currents i_1 and i_2 in the circuit shown in Fig. 1.73.*

[JNTU May/June 2006]

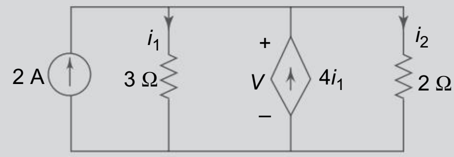


Fig. 1.73

Solution Applying KCL at node V and also $i_1 = \frac{V}{3}$

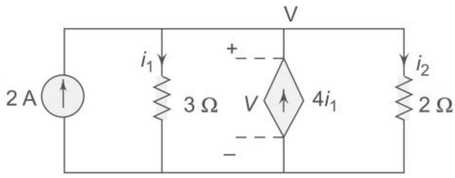


Fig. 1.74

$$\frac{V}{3} + \frac{V}{2} = 2 + 4i_1 \Rightarrow \frac{V}{3} + \frac{V}{2} = \frac{4V}{3} + 2$$

from which $V = -4$ volts

$$i_1 = -\frac{4}{3}, \quad i_2 = -2\text{ A}$$

1.5.5 Parallel Resistance

When the circuit is connected in parallel, the total resistance of the circuit decreases as the number of resistors connected in parallel increases. If we consider m parallel branches in a circuit as shown in Fig. 2.75, the current equation is

$$I_T = I_1 + I_2 + \ldots + I_m$$

The same voltage is applied across each resistor. By applying Ohm's law, the current in each branch is given by

$$I_1 = \frac{V_s}{R_1}, I_2 = \frac{V_s}{R_2}, \ldots I_m = \frac{V_s}{R_m}$$

According to Kirchhoff's current law,

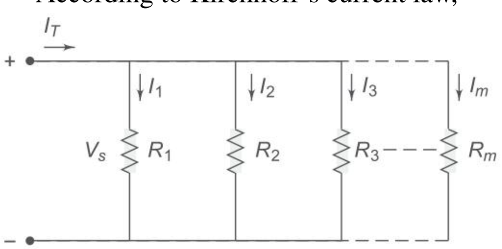


Fig. 1.75

$$I_T = I_1 + I_2 + I_3 + \ldots + I_m$$

$$\frac{V_s}{R_T} = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \ldots + \frac{V_s}{R_m}$$

From the above equation, we have

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_m}$$

Example 1.45 *Determine the parallel resistance between points A and B of the circuit shown in Fig. 1.76.*

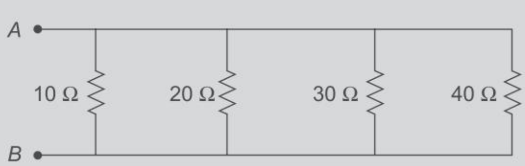


Fig. 1.76

Solution $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$$

$$= 0.1 + 0.05 + 0.033 + 0.025 = 0.208$$

or $R_T = 4.8\ \Omega$

Example 1.46 *Determine the total current in the circuit shown in Fig. 1.77.*

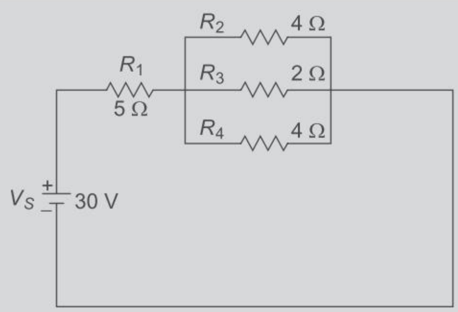


Fig. 1.77

Solution Resistances R_2, R_3 and R_4 are in parallel

$\therefore$ Equivalent resistance $R_5 = R_2 \parallel R_3 \parallel R_4$

$$= \frac{1}{1/R_2 + 1/R_3 + 1/R_4}$$

$$\therefore \quad R_5 = 1\ \Omega$$

R_1 and R_5 are in series,

$\therefore$ Equivalent resistance $R_T = R_1 + R_5 = 5 + 1 = 6\ \Omega$

And the total current $I_T = \dfrac{V_s}{R_T} = \dfrac{30}{6} = 5\text{ A}$

1.5.6 Current Division

In a parallel circuit, the current divides in all branches. Thus, a parallel circuit acts as a current divider. The total current entering into the parallel branches is divided into the branches currents according to the resistance values. The branch having higher resistance allows lesser current, and the branch with lower resistance allows more current. Let us find the current division in the parallel circuit shown in Fig. 1.78.

The voltage applied across each resistor is V_s. The current passing through each resistor is given by

$$I_1 = \frac{V_s}{R_1}, \; I_2 = \frac{V_s}{R_2}$$

If R_T is the total resistance, which is given by $R_1R_2/(R_1 + R_2)$,

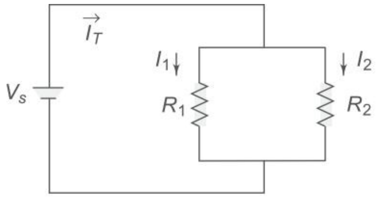


Fig. 1.78

$$\text{Total current } I_T = \frac{V_s}{R_T} = \frac{V_s}{R_1 R_2}(R_1 + R_2)$$

$$\text{or } I_T = \frac{I_1 R_1}{R_1 R_2}(R_1 + R_2) \quad \text{since } V_s = I_1R_1$$

$$I_1 = I_T \cdot \frac{R_2}{R_1 + R_2}$$

Similarly,

$$I_2 = I_T \cdot \frac{R_1}{R_1 + R_2}$$

From the above equations, we can conclude that the current in any branch is equal to the ratio of the opposite branch resistance to the total resistance value, multiplied by the total current in the circuit. In general, if the circuit consists of *m* branches, the current in any branch can be determined by

$$I_i = \frac{R_T}{R_i + R_T} I_T$$

where I_i represents the current in the *i*th branch
R_i is the resistance in the *i*th branch
R_T is the total parallel resistance to the *i*th branch and
I_T is the total current entering the circuit.

Example 1.47 *Determine the current through each resistor in the circuit shown in Fig. 1.79.*

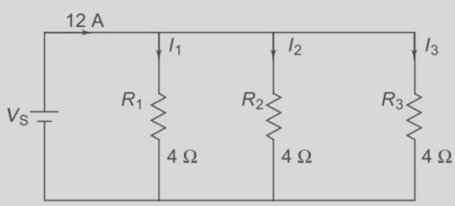


Fig. 1.79

Solution
$$I_1 = I_T \times \frac{R_T}{(R_1 + R_T)}$$

where
$$R_T = \frac{R_2 R_3}{R_2 + R_3} = 2\ \Omega$$

∴
$$R_1 = 4\ \Omega$$
$$I_T = 12\text{ A}$$
$$I_1 = 12 \times \frac{2}{2+4} = 4\text{ A}$$

Similarly,
$$I_2 = 12 \times \frac{2}{2+4} = 4\text{ A}$$

and $$I_3 = 12 \times \frac{2}{2+4} = 4\text{ A}$$

Since all parallel branches have equal values of resistance, they share current equally.

Example 1.48 *Determine the current delivered by the source in the circuit shown in Fig. 1.80.*

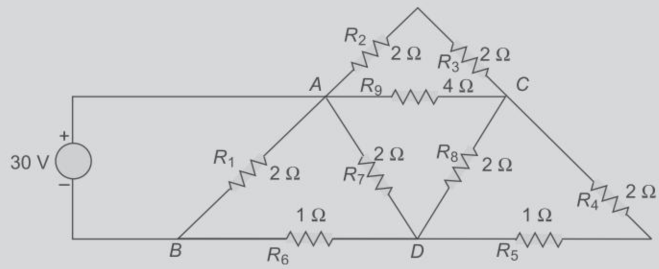


Fig. 1.80

Solution The circuit can be modified as shown in Fig. 1.81, where R_{10} is the series combination of R_2 and R_3.

$\therefore \quad R_{10} = R_2 + R_3 = 4\ \Omega$

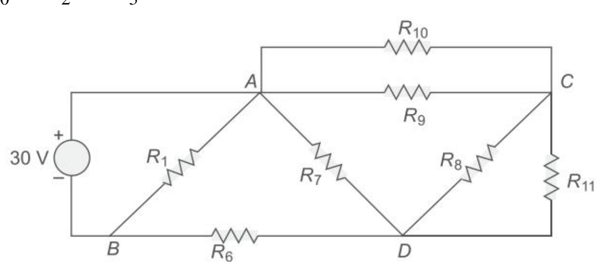


Fig. 1.81

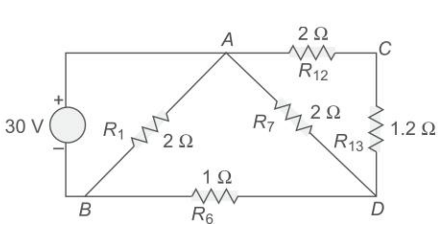


Fig. 1.82

R_{11} is the series combination of R_4 and R_5

$\therefore \quad R_{11} = R_4 + R_5 = 3\ \Omega$

Further simplification of the circuit leads to Fig. 1.82 where R_{12} is the parallel combination of R_{10} and R_9.

$\therefore \quad R_{12} = (R_{10} \,||\, R_9) = (4 \,||\, 4) = 2\ \Omega$

Similarly, R_{13} is the parallel combination of R_{11} and R_8

$\therefore \quad R_{13} = (R_{11} \,||\, R_8) = (3 \,||\, 2) = 1.2\ \Omega$

In Fig. 1.81 as shown, R_{12} and R_{13} are in series, which is in parallel with R_7 forming R_{14}. This is shown in Fig. 1.82.

$\therefore \quad R_{14} = [(R_{12} + R_{13})//R_7]$

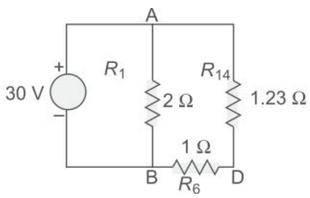


Fig. 1.83

$= [(2 + 1.2)//2] = 1.23\ \Omega$

Further, the resistances R_{14} and R_6 are in series, which is in parallel with R_1 and gives the total resistance

$$R_T = [(R_{14} + R_6)//R_1]$$

$$= [(1 + 1.23)//(2)] = 1.05\ \Omega$$

The current delivered by the source = 30/1.05 = 28.57 A

Example 1.49 *Determine the current in the 10 Ω resistance and find V_s in the circuit shown in Fig. 1.84.*

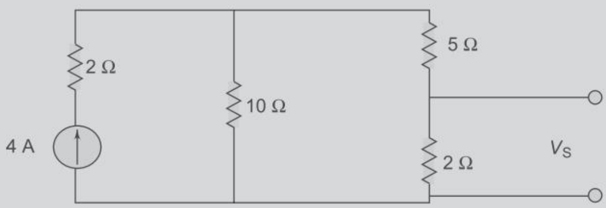


Fig. 1.84

Solution The current in 10 Ω resistance

$$I_{10} = \text{total current} \times (R_T)/(R_T + R_{10})$$

where R_T is the total parallel resistance.

$$I_{10} = 4 \times \frac{7}{17} = 1.65\text{ A}$$

Similarly, the current in resistance R_5 is

$$I_5 = 4 \times \frac{10}{10+7} = 2.35\text{ A}$$

or $\quad 4 - 1.65 = 2.35$ A

The same current flows through the 2 Ω resistance.

$\therefore$ Voltage across 2 Ω resistance, $V_s = I_5 \times 2 = 2.35 \times 2 = 4.7$ V

Example 1.50 *Determine the value of resistance R and current in each branch when the total current taken by the circuit shown in Fig. 1.85 is 6 A.*

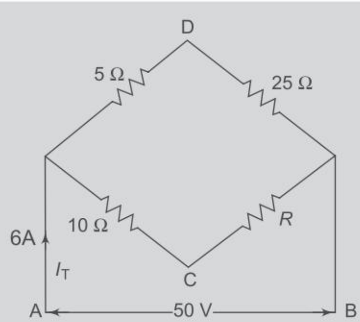


Fig. 1.85

Solution The current in branch *ADB*

$$I_{30} = 50/(25 + 5) = 1.66 \text{ A}$$

The current in branch *ACB* $I_{10} + {}_R = 50/(10 + R)$.

According to Kirchhoff's current law

$$I_T = I_{30} + I_{(10+R)}$$

$$6\text{A} = 1.66 \text{ A} + I_{10+R}$$

$$\therefore \quad I_{10+R} = 6 - 1.66 = 4.34 \text{ A}$$

$$\therefore \quad \frac{50}{10+R} = 4.34$$

$$10+R = \frac{50}{4.34} = 11.52$$

$$R = 1.52\ \Omega$$

1.5.7 Power in a Parallel Circuit

The total power supplied by the source in any parallel resistive circuit is equal to the sum of the powers in each resistor in parallel, i.e.

$$P_S = P_1 + P_2 + P_3 + \dots + P_m$$

where m is the number of resistors in parallel, P_S is the total power and P_m is the power in the last resistor.

Example 1.51 *Find the current in the 10 Ω resistance, V_1, and source voltage V_s in the circuit shown in Fig. 1.86.*

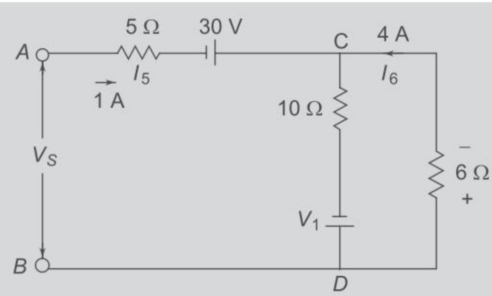


Fig. 1.86

Solution Assume voltage at node $C = V$

By applying Kirchhoff's current law, we get the current in the 10 Ω resistance

$$I_{10} = I_5 + I_6 = 4 + 1 = 5 \text{ A}$$

The voltage across the 6 Ω resistor is $V_6 = 24$ V

$\therefore$ Voltage at node *C* is $V_C = -24$ V.

The voltage across branch *CD* is the same as the voltage at node *C*.

Voltage across 10 Ω only = 10 × 5 = 50 V

So

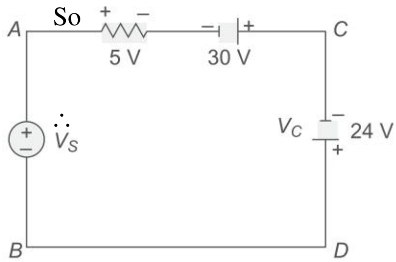


Fig. 1.87

$$V_C = V_{10} - V_1$$

$$-24 = 50 - V_1$$

$$V_1 = 74 \text{ V}$$

Now, consider the loop CABD shown in Fig. 1.87.

If we apply Kirchhoff's voltage law we get

$$V_s = 5 - 30 - 24 = -49 \text{ V}$$

Example 1.52 *Find the power delivered by the source in the circuit shown in Fig. 1.88.*

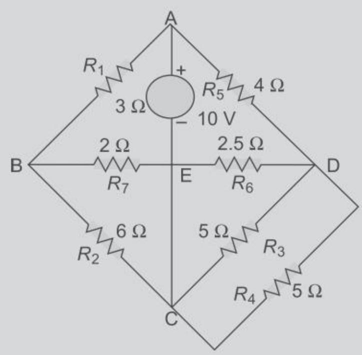


Fig. 1.88

Solution Between points $C(E)$ and D, resistances R_3 and R_4 are in parallel, which gives

$$R_8 = (R_3//R_4) = 2.5 \ \Omega$$

Between points B and $C(E)$, resistances R_2 and R_7 are in parallel, which gives

$$R_9 = (R_2 \parallel R_7) = 1.5 \ \Omega$$

Between points $C(E)$ and D, resistances R_6 and R_8 are in parallel and gives

$$R_{10} = (R_6 \parallel R_8) = 1.25 \ \Omega$$

The series combination of R_1 and R_9 gives

$$R_{11} = R_1 + R_9 = 3 + 1.5 = 4.5 \ \Omega$$

Similarly, the series combination of R_5 and R_{10} gives

$$R_{12} = R_5 + R_{10} = 5.25 \ \Omega$$

The resistances R_{11} and R_{12} are in parallel, which gives

Total resistance $= (R_{11} \parallel R_{12}) = 2.42$ ohms

These reductions are shown in Figs. 1.89 (a), (b), (c) and (d).

$$\text{Current delivered by the source} = \frac{10}{2.42} = 4.13 \text{ A}$$

$$\text{Power delivered by the source} = VI$$

$$= 10 \times 4.13 = 41.3 \text{ W}$$

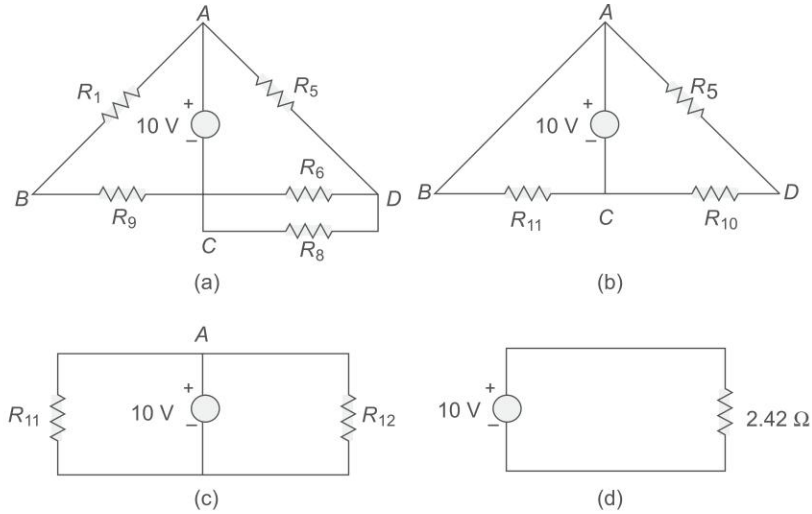


Fig. 1.89 (a, b, c and d)

Example 1.53 *Determine the voltage drop across the 10 Ω resistance in the circuit as shown in Fig. 1.90.*

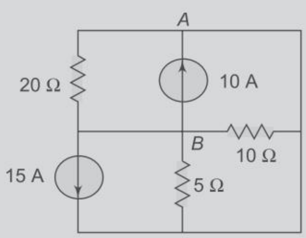


Fig. 1.90

Solution The circuit is redrawn as shown in Fig. 1.91.

This is a single node pair circuit. Assume voltage V_A at node A. By applying Kirchhoff's current law at node A, we have

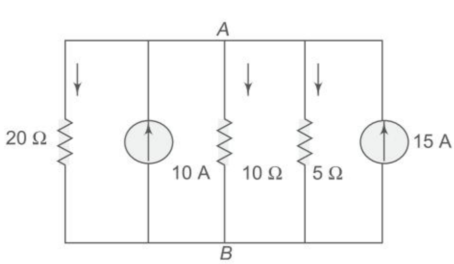


Fig. 1.91

$$\frac{V_A}{20}+\frac{V_A}{10}+\frac{V_A}{5}=10+15$$

$$V_A\left[\frac{1}{20}+\frac{1}{10}+\frac{1}{5}\right]=25\,\text{A}$$

$$V_A(0.05+0.1+0.2)=25\,\text{A}$$

$$V_A=\frac{25}{0.35}=71.42\,\text{V}$$

The voltage across 10 Ω is nothing but the voltage at node A.

$$\therefore \quad V_{10}=V_A=71.42\text{ V}$$

Example 1.54 *In the circuit shown in Fig. 1.92 what are the values of R_1 and R_2, when the current flowing through R_1 is 1 A and R_2 is 5 A? What is the value of R_2 when the current flowing through R_1 is zero?*

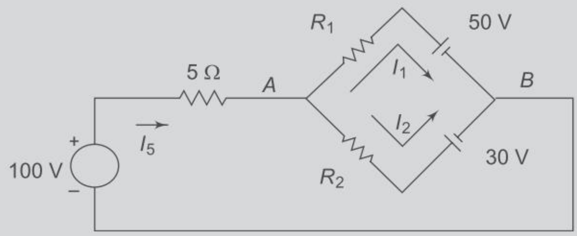


Fig. 1.92

Solution The current in the 5 Ω resistance

$$I_5 = I_1 + I_2 = 1 + 5 = 6\text{ A}$$

Voltage across resistance 5 Ω is $V_5 = 5 \times 6 = 30$ V

The voltage at node A, $V_A = 100 - 30 = 70$ V

$$\therefore \qquad I_2 = \frac{V_A - 30}{R_2} = \frac{70-30}{R_2}$$

$$R_2 = \frac{70-30}{I_2} = \frac{40}{5} = 8\,\Omega$$

Similarly, $R_1 = \dfrac{70-50}{I_1} = \dfrac{20}{1} = 20\,\Omega$

When $V_A = 50$ V, the current I_1 in resistance R_1 becomes zero

$$\therefore \qquad I_2 = \frac{50-30}{R_2}$$

where I_2 becomes the total current

$$\therefore \qquad I_2 = \frac{100 - V_A}{5}$$

$$= \frac{100-50}{5} = 10\text{ A}$$

$$\therefore \qquad R_2 = \frac{20}{I_2}$$

$$= \frac{20}{10} = 2\,\Omega$$

Example 1.55 *Determine the output voltage V_{out} in the circuit shown in Fig. 1.93.*

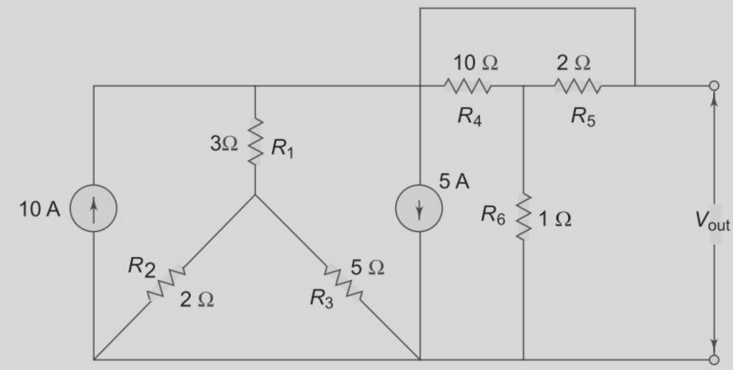


Fig. 1.93

Solution The circuit shown in Fig. 1.93 can be redrawn as shown in Fig. 1.94.

In Fig. 1.94, R_2 and R_3 are in parallel, R_4 and R_5 are in parallel. The complete circuit is a single node pair circuit. Assuming voltage V_A at node A and applying Kirchhoff's current law in the circuit, we have

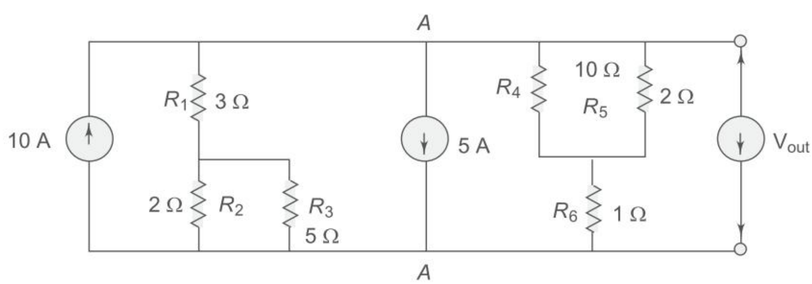


Fig. 1.94

$$10\text{A} - \frac{V_A}{4.43} - 5\text{A} - \frac{V_A}{2.67} = 0$$

$\therefore$ $$V_A\left[\frac{1}{4.43} + \frac{1}{2.67}\right] = 5\,\text{A}$$

$$V_A[0.225 + 0.375] = 5$$

$\therefore$ $$V_A = \frac{5}{0.6} = 8.33\,\text{V}$$

$$V_{\text{out}} = V_A = 8.33\text{ V}$$

Example 1.56 *Determine the voltage V_{AB} in the circuit shown in Fig. 1.95.*

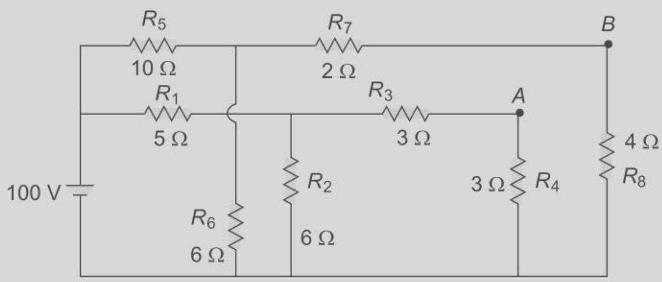


Fig. 1.95

Solution The circuit in Fig. 1.95 can be redrawn as shown in Fig. 1.96 (a).
At node 3, the series combination of R_7 and R_8 are in parallel with R_6, which gives $R_9 = [(R_7 + R_8)//R_6] = 3\ \Omega$.
At node 2, the series combination of R_3 and R_4 are in parallel with R_2, which gives $R_{10} = [(R_3 + R_4)//R_2] = 3\ \Omega$.

It is further reduced and is shown in Fig. 1.96 (b).

Simplifying further we draw it as shown in Fig. 1.96 (c).

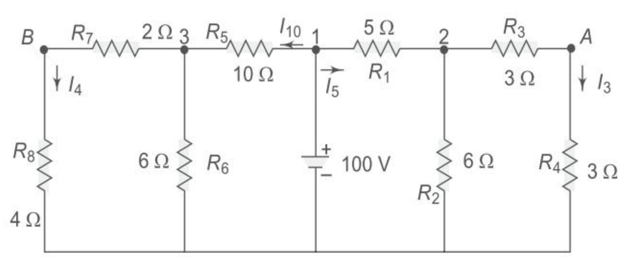


Fig. 1.96 (a)

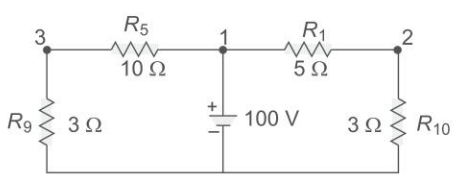


Fig. 1.96 (b)

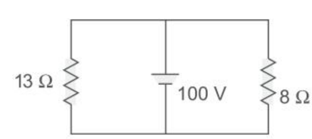


Fig. 1.96 (c)

Total current delivered by the source $= \dfrac{100}{R_T}$

$$= \frac{100}{(13//8)} = 20.2\ \text{A}$$

Current in the 8 Ω resistor is $I_8 = 20.2 \times \dfrac{13}{13+8} = 12.5\ \text{A}$

Current in the 13 Ω resistor is $I_{13} = 20.2 \times \dfrac{8}{13+8} = 7.69\ \text{A}$

So $I_5 = 12.5$ A, and $I_{10} = 7.69$ A

Current in the 4 Ω resistance $I_4 = 3.845$ A

Current in the 3 Ω resistance $I_3 = 6.25$ A

$$V_{AB} = V_A - V_B$$

Where $$V_A = I_3 \times 3\ \Omega = 6.25 \times 3 = 18.75 \text{ V}$$

$$V_B = I_4 \times 4\ \Omega = 3.845 \times 4 = 15.38 \text{ V}$$

∴ $$V_{AB} = 18.75 - 15.38 = 3.37 \text{ V}$$

Example 1.57 *Determine the value of R in the circuit shown in Fig. 1.97, when the current is zero in the branch CD.*

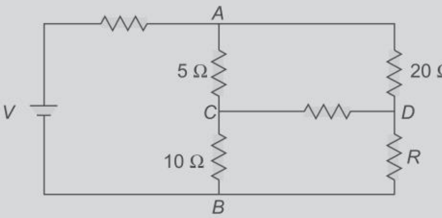


Fig. 1.97

Solution The current in the branch *CD* is zero, if the potential difference across *CD* is zero.

That means, voltage at point *C* = voltage at point *D*.

Since no current is flowing, the branch *CD* is open circuited. So the same voltage is applied across *ACB* and *ADB*

$$V_{10} = V_A \times \frac{10}{15}$$

$$V_R = V_A \times \frac{R}{20+R}$$

∴ $$V_{10} = V_R$$

and $$V_A \times \frac{10}{15} = V_A \times \frac{R}{20+R}$$

∴ $$R = 40\ \Omega$$

Example 1.58 *Find the power absorbed by each element in the circuit shown in Fig. 1.98.*

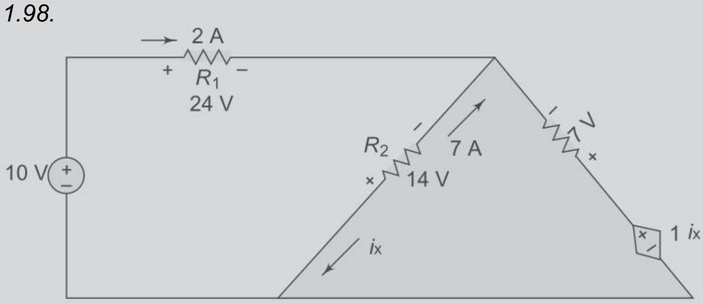


Fig. 1.98

Solution Power absorbed by any element = VI
where V is the voltage across the element and I is the current passing through that element

Here potential rises are taken as (–) sign.
Power absorbed by 10 V source = –10 × 2 = –20 W
Power absorbed by resistor R_1 = 24 × 2 = 48 W
Power absorbed by resistor R_2 = 14 × 7 = 98 W
Power absorbed by resistor R_3 = –7 × 9 = –63 W
Power absorbed by dependent voltage source = (1 × –7) × 9 = –63 W

Example 1.59 *Show that the algebraic sum of the five absorbed power values in Fig. 1.99 is zero.*

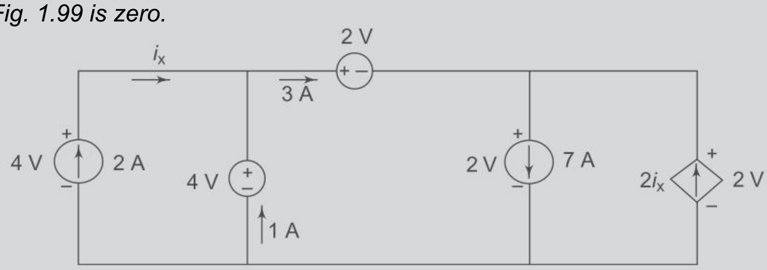


Fig. 1.99

Solution Power absorbed by 2 A current source = (–4) × 2 = –8 W
Power absorbed by 4 V voltage source = (–4) × 10 = –4 W
Power absorbed by 2 V voltage source = (2) × 3 = 6 W
Power absorbed by 7 A current source = (7) × 2 = 14 W
Power absorbed by $2i_x$ dependent current source = (–2) × 2 × 2 = –8 W
Hence, the algebraic sum of the five absorbed power values is zero.

Example 1.60 *For the circuit shown in Fig. 1.100, find the power absorbed by each of the elements.*

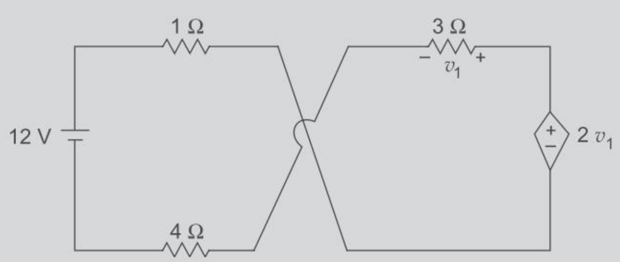


Fig. 1.100

Solution The above circuit can be redrawn as shown in Fig. 1.101.
Assume loop current I as shown in Fig. 1.101.
If we apply Kirchhoff's voltage law, we get

$$-12 + I - 2\upsilon_1 + \upsilon_1 + 4I = 0$$

The voltage across 3 Ω resistor is $\upsilon_1 = 3I$

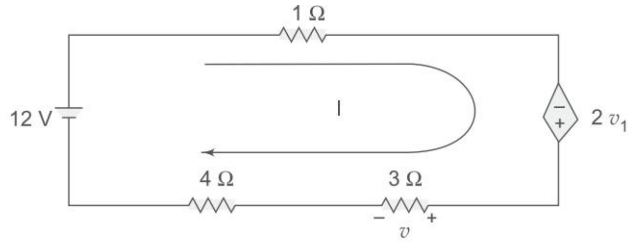


Fig. 1.101

Substituting v_1 in the loop equation, we get $I = 6$ A

Power absorbed by the 12 V source = (–12) × 6 = –72 W

Power absorbed by the 1 Ω resistor = 6 × 6 = 36 W

Power absorbed by $2v_1$ dependent voltage source

$$= (2v_1)I = 2 \times 3 \times 6 \times 6 = -216 \text{ W}$$

Power absorbed by 3 Ω resistor = $v_1 \times I$ = 18 × 6 = 108 W

Power absorbed by 4 Ω resistor = 4 × 6 × 6 = 144 W

Example 1.61 *For the circuit shown in Fig. 1.102, find the power absorbed by each element.*

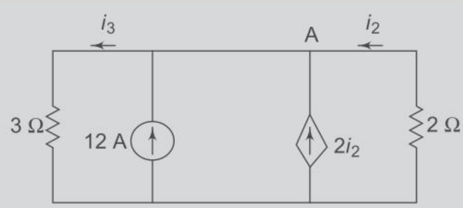


Fig. 1.102

Solution The circuit shown in Fig. 1.102 is a parallel circuit and consists of a single node *A*. By assuming voltage *V* at node A, we can find the current in each element.

According to Kirchhoff's current law

$$i_3 - 12 - 2i_2 - i_2 = 0$$

By using Ohm's law, we have

$$i_3 = \frac{V}{3}, \; i_2 = \frac{-V}{2}$$

$$V\left[\frac{1}{3} + 1 + \frac{1}{2}\right] = 12$$

$$\therefore \quad V = \frac{12}{1.83} = 6.56$$

$$i_3 = \frac{6.56}{3} = 2.187\text{A}; \; i_2 = \frac{-6.56}{2} = -3.28\text{ A}$$

Power absorbed by the 3 Ω resistor = (+6.56) (2.187) = 14.35 W

Power absorbed by 12 A current source = (–6.56) 12 = –78.72 W

Power absorbed by $2i_2$ dependent current source

$$= (-6.56) \times 2 \times (-3.28) = 43.03 \text{ W}$$

Power absorbed by 2 Ω resistor = (–6.52) (–3.28) = 21.51 W

Example 1.62 *Find the value of E in the network shown in Fig. 1.103.*

[JNTU April/May 2007]

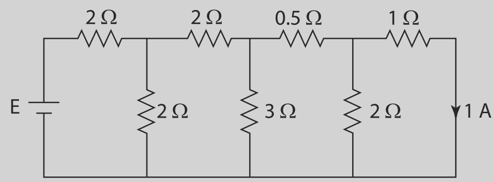


Fig. 1.103

Solution Calculating current through all branches

$$E = 2 \times I + 5.916$$
$$E = 2 \times 5.04 + 5.916$$
$$E = 15.99 \text{ V}$$
$$E = 16 \text{ V}$$

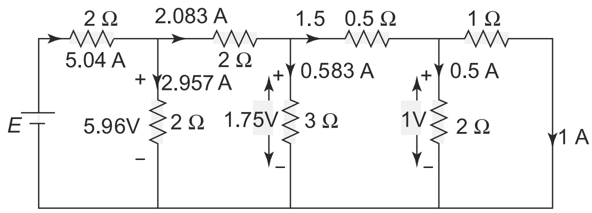


Fig. 1.104

Example 1.63 *Determine the current through the 6-Ω resistor and the power supplied by the current source for the circuit shown in Fig. 1.105.*

[JNTU April/May 2006]

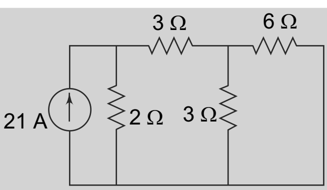


Fig. 1.105

Solution Current through 6-Ω resistor and power supplied by the current source

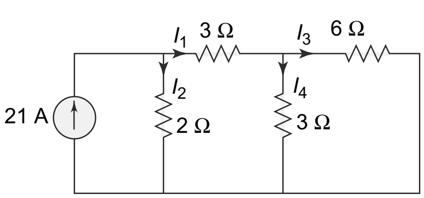


Fig. 1.106

$$I_1 + I_2 = 21 \text{ A}$$
$$I_2 = \frac{21 \times 5}{7} = 15 \text{ A}$$
$$I_1 = 6 \text{ A}$$
$$I_1 = I_3 + I_4 = 6 \text{ A}$$
$$I_4 = \frac{6 \times 6}{9} = 4 \text{ A}, I_3 = 2 \text{ A}$$

Current through 6-Ω resistor is $= I_3 = 2$ A

$$I_3 = 2 \text{ A}$$

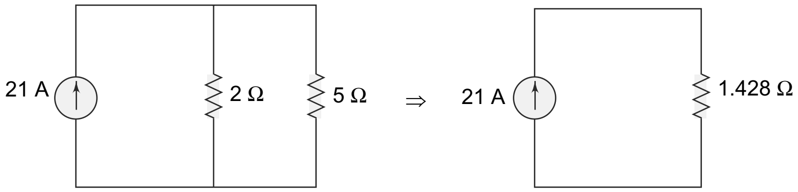


Fig. 1.107

Power supplied by the current source.

Power supplied by current source = Power consumed in the resistor.

$$= I^2R = (21)^2 \times 1.428$$

$$P = 629.748 \text{ W}$$

Example 1.64 *A circuit consisting of three resistances 12 Ω, 18Ω and 36 Ω respectively joined in parallel is connected in series with a fourth resistance. The whole circuit is applied with 60 V and it is found that the power dissipated in the 12 Ω resistor is 36 W. Determine the value of the fourth resistance and the total power dissipated in the circuit.* *[JNTU May/June 2008]*

Solution Given that 12 Ω, 18 Ω and 36 Ω respectively joined in parallel to each other. Let the fourth resistance be R Ω which is in series with the parallel combination as shown in the Fig. 1.108.

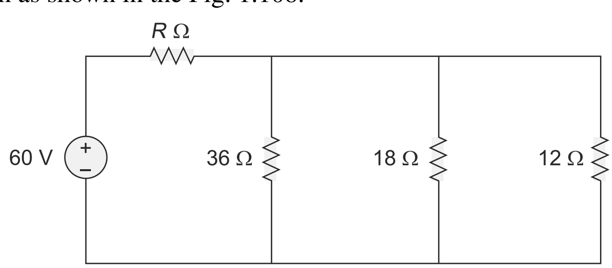


Fig. 1.108

Equivalent resistance of parallel combination

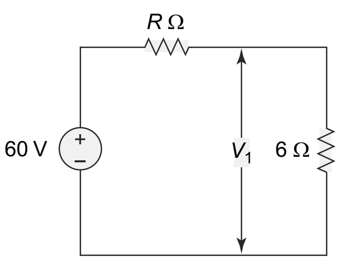


Fig. 1.109

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{18} + \frac{1}{36}$$

$$R_{eq} = 6 \ \Omega$$

The figure reduces to

Voltage across 6 Ω resistor , $V_1 = \dfrac{60 \times 6}{6 + R}$ [voltage division]

As the voltage across 12 Ω is also V_1 and it is given that power dissipated by 12 Ω is 36 W

$$V_1^2/R = 36 \text{ W}$$

$$\frac{(60\times 6)^2}{(6+R)^2\times 12}=36$$

$$(6 + R)^2 = 60 \times 5$$

$$R^2 + 12R + 36 = 300$$

$$R^2 + 12R - 264 = 0$$

$$R=-12\pm\sqrt{\frac{144+4(264)}{2}}=11.32\ \Omega$$

The current I flowing in the circuit is

$$I=\frac{60}{6+11.32}=3.464\text{ A}$$

Total power dissipated in the corcuit $P = VI$

$$= 60 \times 3.464 = 207.852\text{ W}$$

Example 1.65 *A circuit consists of three resistors of 3 ohms, 4 ohms and 6 ohms in parallel and a fourth resistor of 4 ohms in series. A battery of 12 V emf and an internal resistance of 6 ohms is connected across the circuit. Find the total current in the circuit and terminal voltage across the battery.*

[JNTU May/June 2008]

Solution Three resistors of 3 Ω, 4 Ω and 6 Ω are in parallel and a fourth resistor of 4 Ω is in series.

The 12 V battery has a internal resistance of 6 Ω.

The circuit can be taken as

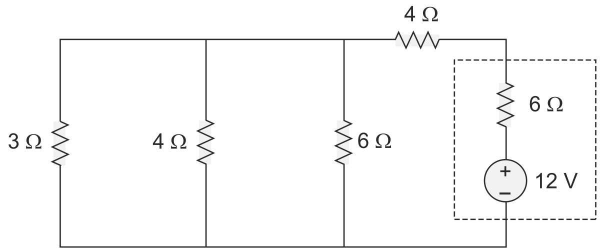


Fig. 1.110

The circuit can be reduced to as shown in Fig. 1.111

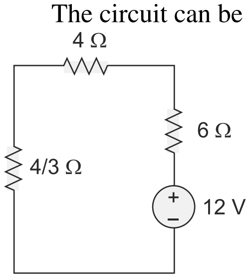


Fig. 1.111

The current I flowing in the circuit

$$=\frac{12}{10+4/3}=1.0588\text{ A}$$

Terminal voltage = battery voltage − drop due to internal resistance

$$12 - 6 \times 1.0588 = 5.647\text{ V}$$

Example 1.66 *A 50 ohm resistor is in parallel with a 100 ohm resistor. The current in a 50 ohm resistor is 7.2 A. What is the value of the third resistance to be added in parallel to make the line current as 12.1 A?.* [JNTU May/June 2008]

Solution A 50 Ω resistor is in parallel with 100 Ω. The current in 50 Ω is 7.2 Ω. Let the third resistance be R Ω.

The line current is 12.1 A.

The circuit is

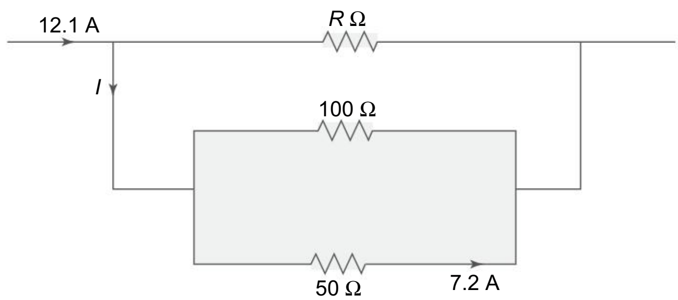


Fig. 1.112

Let I be the current flowing through parallel combination of 100 and 50 Ω. The current I flowing through 50 Ω resistor is

$$\frac{I \times 100}{150} = 7.2 \text{ [current division]}$$

$$I = 10.8\ A$$

The current through R Ω is = 12.1 − 10.8 = 1.3 A.

Thus, by current division

$$1.3 = \frac{12.1 \times 33.33}{R + 33.33}$$

$$1.3\ R + 1.3 \times 33.33 = 12.1 \times 33.33$$

$$1.3\ R = 359.99$$

$$R = 276.92\ \Omega$$

Example 1.67 *Find the current delivered by the source for the network shown in Fig. 1.113 using network reductions technique.* [JNTU June 2009]

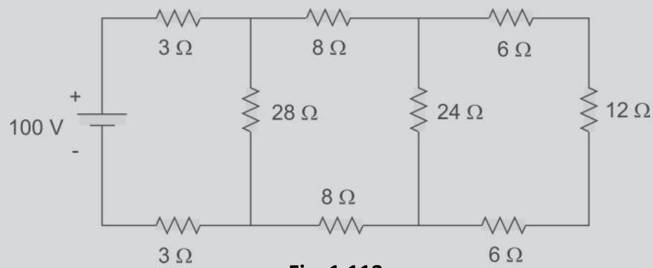


Fig. 1.113

Solution

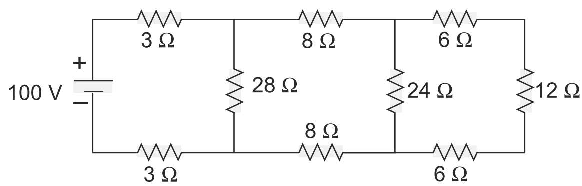


Fig. 1.114

Replacing series combination of $6\,\Omega$, $12\,\Omega$ and $6\,\Omega$ by $(6 + 12 + 6)\,\Omega = 24\,\Omega$

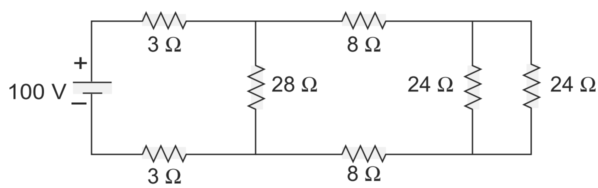


Fig. 1.115(a)

Replacing parallel combination of $24\,\Omega$ and $24\,\Omega$ by $\dfrac{24\times 24}{24+24}\Omega = 12\,\Omega$

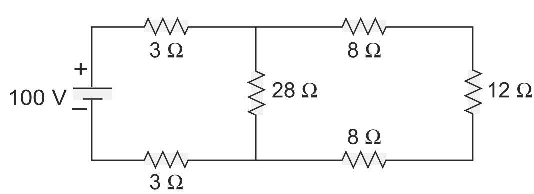


Fig. 1.115(b)

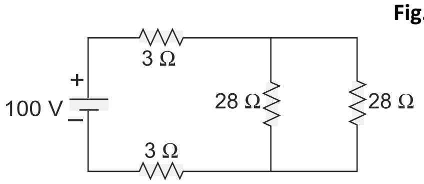


Fig. 1.115(c)

Replacing series combination of $8\,\Omega$, $12\,\Omega$ and $8\,\Omega$ by $(8 + 12 + 8)\,\Omega = 28\,\Omega$

Replacing parallel combination of $28\,\Omega$ and $28\,\Omega$ by $\dfrac{28\times 28}{28+28}\Omega = 14\,\Omega$

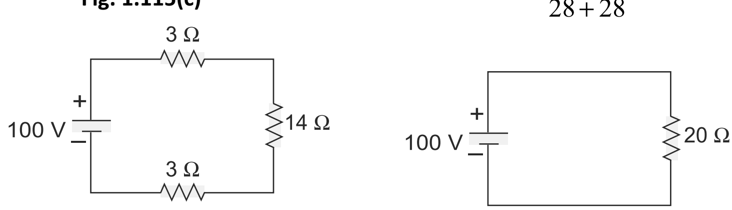


Fig. 1.115(d)

Replacing series combination of $3\,\Omega$, $14\,\Omega$ and $3\,\Omega$ by $(3 + 14 + 3)\,\Omega = 20\,\Omega$

$\therefore$ Current delivered by the source

$$= \frac{100}{20} \text{ amp}$$

$$= 5 \text{ amp}$$

Example 1.68 *Find the value of applied d.c. voltage for the network, shown in Fig. 1.116.* *[JNTU June 2009]*

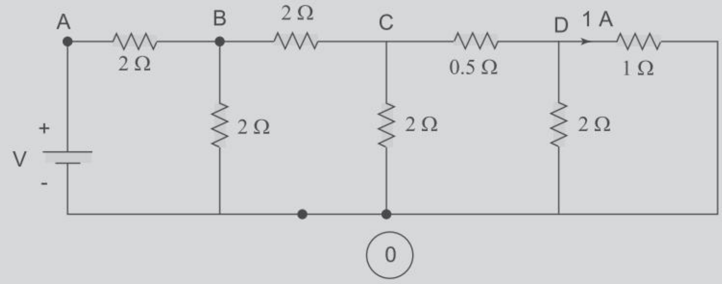


Fig. 1.116

Solution Replacing parallel combination of 1Ω and

$$2\ \Omega \text{ by } \frac{2\times 1}{2+1}\Omega = \frac{2}{3}\Omega$$

Replacing series combination of 0.5 Ω and 2/3 Ω by $\left(\frac{1}{2}+\frac{2}{3}\right)\Omega = \frac{7}{6}\Omega$

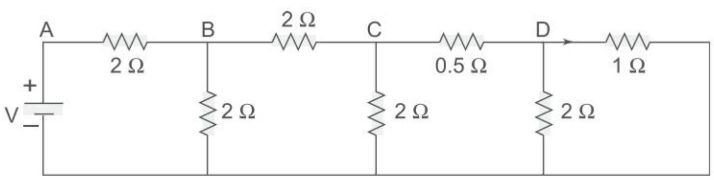


Fig. 1.117

Replacing parallel combination of 2 Ω and 7/6 Ω by $\frac{2\times(7/6)}{2+(7/6)}\Omega = (14/19)\Omega$

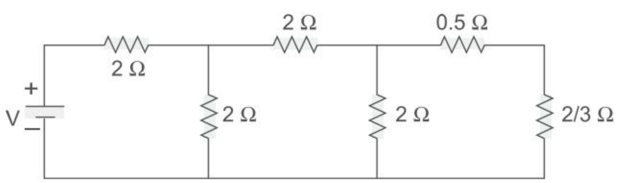


Fig. 1.118(a)

Replacing series combination of 2 Ω and 14/19 Ω by (2 + (14/19)) Ω = (52/19) Ω

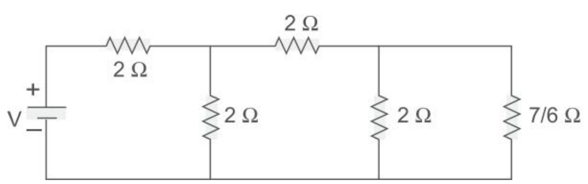


Fig. 1.118(b)

Replacing parallel combination of 2 Ω and (52/19) Ω by

$$\frac{2\times(52/19)}{2+(52/19)}\Omega=(52/45)\Omega$$

Replacing series combination of 2 Ω and (52/45) Ω by (2 + (52/45)) Ω = (142/45) Ω

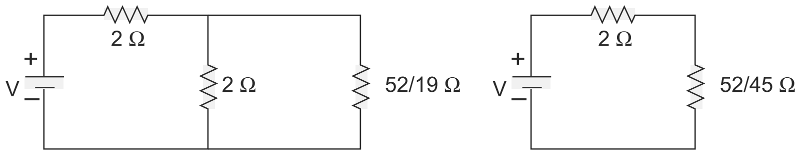


Fig. 1.118(c)

$\therefore$ Total current $= \dfrac{V}{(142/45)}$

$\therefore$ Current through 1 Ω (DO) resistance

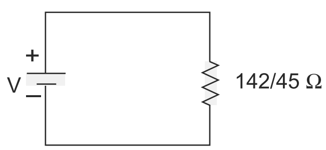


Fig. 1.118(d)

$$=\frac{V}{(142/45)}\times\frac{2}{2+(52/19)}\times\frac{2}{2+(7/6)}\times\frac{2}{2+1}$$

$$=\frac{4V}{71}=1 \quad \text{(according to the question)}$$

$$\therefore V=\frac{71}{4}\text{volt}=17.75\,\text{volt}.$$

Example 1.69 *Three resistances are connected is parallel having the ratio of 1:2:3 the total power consumed is 100 W when 10 V is applied to the combinations, find the values of the resitances.* *[JNTU June 2009]*

Solution Let, the resistances be R, $2R$ and $3R$.

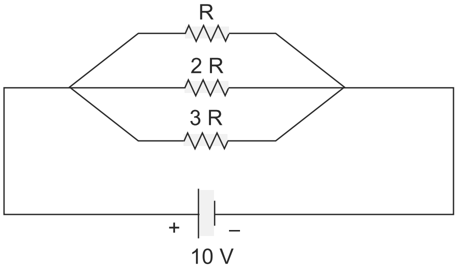


Fig. 1.119

$$Total\ power\ V\,I=100$$

$$Tolta\ I\ current\ I=\frac{100}{V}=10A$$

$$I=I_R+I_{2R}+I_{3R}$$

Then according to the question

or, $$\frac{10}{R}+\frac{10}{2R}+\frac{10}{3R}=10$$

$$\frac{1}{R}\left(1+\frac{1}{2}+\frac{1}{3}\right)=1$$

or, $$R=1+\frac{1}{2}+\frac{1}{3}=\frac{6+3+2}{6}=\frac{11}{6}\text{ ohm}$$

$\therefore$ The resistances are $\dfrac{11}{6}$ ohm, $\dfrac{11}{3}$ ohm and $\dfrac{11}{2}$ ohm.

Example 1.70 *Find the current through each element and the total power delivered by the source for the network as shown in Fig. 1.120. [JNTU Jan 2009]*

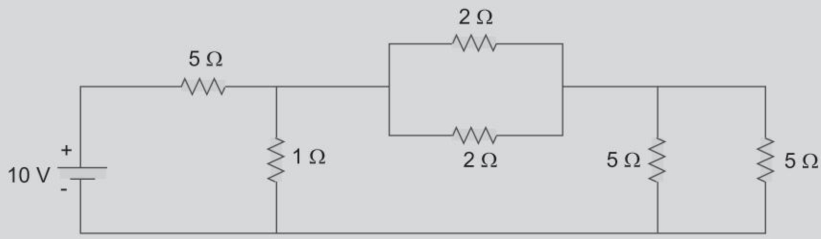


Fig. 1.120

Solution The circuit of Fig. 1.120 can be reduced to the circuit shown in Fig. 1.122

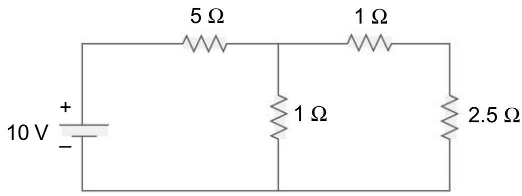


Fig. 1.121

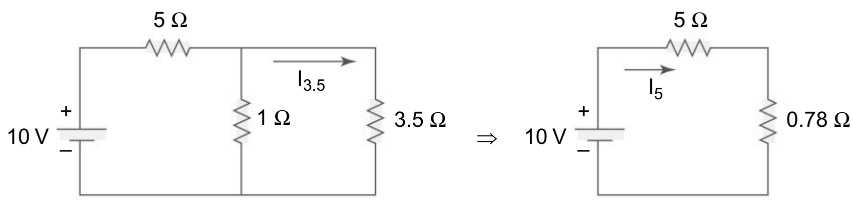


Fig. 1.122

The current through 5Ω resistor

$$I_5 = \frac{10}{5.78} = 1.73\text{A}$$

Total power delivered by 10*V* source = 1.73 × 10

= 17.3 watts

Current through 3.5Ω resistance

$$= I_5 \times \frac{1}{1+3.5} = \frac{1.73}{4.5} = 0.385\text{A}$$

Current in 1Ω is same as the current in 3.5Ω
Current in 2Ω is divided equally

∴ $I_2 = 0.1925\text{A}$

Current in 5Ω is divided equaly

∴ $I_5 = 0.1925\text{A}$

Example 1.71 *Obtain the potential difference V_{AB} in the circuit shown in Fig. 1.123 using Kirchhoff's laws.* *[JNTU Jan 2010]*

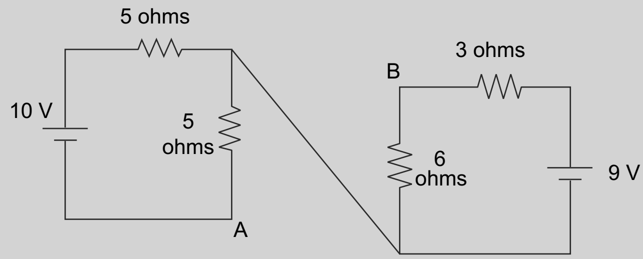


Fig. 1.123

Solution

Using KVL,

For loop-1, $10 = (5 + 5)\, i_1$ or $i_1 = 1$ amp

For loop-2, $9 = (6 + 3)\, i_2$ or $i_2 = 1$ amp

$\therefore$ Voltage drop across 6 Ω (BO) $= 6i_2 = 6$ volt

$\therefore$ Voltage drop across 5 Ω (OA) $= 5i_1 = 5$ volt

Voltage drop across $\quad V_{BA} = V_{BO} + V_{BA} = 11$ volt

$\therefore \quad V_{AB} = -11$ volt

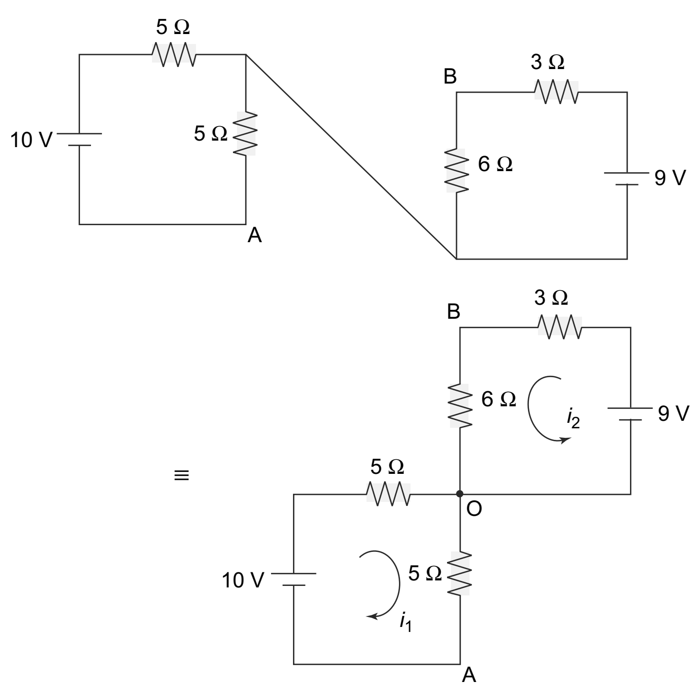


Fig. 1.124

Example 1.72 *In the circuit as shown in Fig. 1.125 find the currents in all the resistors. Also calculate the supply voltage and power supplied by the source.* *[JNTU Jan 2010]*

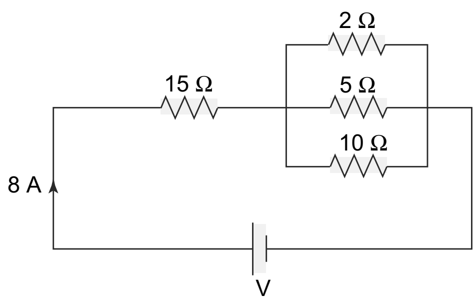


Fig. 1.125

Solution Let current through 10 Ω = i amp

$\therefore$ Current through 5Ω = $2i$ amp

$\therefore$ Current through 2Ω = $5i$ amp

$\therefore$ According to the question:

$i + 2i + 5i = 8i = 8$ amp

$\therefore$ $i = 1$amp

$\therefore$ Current through 10 ohm = 1 amp

Current through 5 ohm = 2 amp

Current through 2 ohm = 5 amp

$$\text{Equivalent impedance} = \left(15 + 1 \Big/ \left[\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right] \text{ohm}\right)$$

$$= 16.25 \text{ ohm}$$

$\therefore$ According to the question,

$$\frac{V}{16.25} = 8$$

or, $V = 130$ volt

$\therefore$ Power supplied by the source = (130×8) watt

= 1040 watt

1.6 THE STAR-DELTA TRANSFORMATION

In the preceding chapter, a simple technique called the *source transformation technique* has been discussed. The star delta transformation is another technique useful in solving complex networks. Basically, any three circuit elements, i.e. resistive, inductive or capacitive, may be connected in two different ways. One way of connecting these elements is called the star connection, or the Y connection. The other way of connecting these elements is called the delta (Δ) connection. The circuit is said to be in star connection, if three elements are connected as shown in Fig. 1.126(a), when it appears like a star (Y). Similarly, the circuit is said to be in delta connection, if three elements are connected as shown in Fig. 1.126(b), when it appears like a delta (Δ).

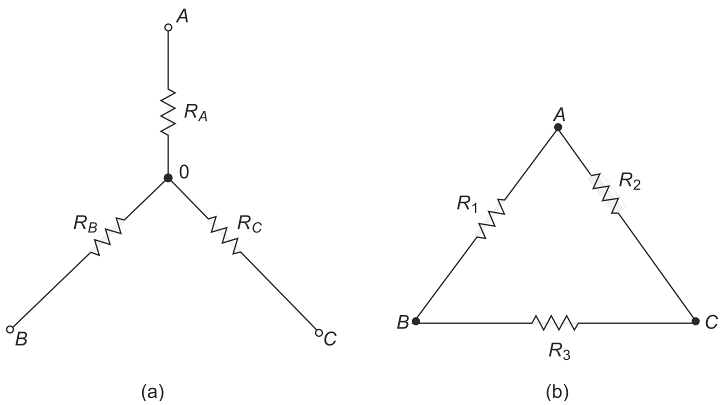


Fig. 1.126

The above two circuits are equal if their respective resistances from the terminals AB, BC and CA are equal. Consider the star connected circuit in Fig. 1.126(a); the resistance from the terminals AB, BC and CA respectively are

$$R_{AB}(Y) = R_A + R_B$$
$$R_{BC}(Y) = R_B + R_C$$
$$R_{CA}(Y) = R_C + R_A$$

Similarly, in the delta connected network in Fig. 1.126(b), the resistances seen from the terminals AB, BC and CA, respectively, are

$$R_{AB}(\Delta) = R_1 \,||\, (R_2 + R_3) = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

$$R_{BC}(\Delta) = R_3 \,||\, (R_1 + R_2) = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

$$R_{CA}(\Delta) = R_2 \,||\, (R_1 + R_3) = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3}$$

Now, if we equate the resistances of star and delta circuits, we get

$$R_A + R_B = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \quad (1.1)$$

$$R_B + R_C = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \quad (1.2)$$

$$R_C + R_A = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} \quad (1.3)$$

Subtracting Eq. 1.2 from Eq. 1.1, and adding Eq. 1.3 to the resultant, we have

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} \quad (1.4)$$

Similarly $$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} \quad (1.5)$$

and $$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3} \quad (1.6)$$

Thus, a delta connection of R_1, R_2 and R_3 may be replaced by a star connection of R_A, R_B and R_C as determined from Eqs 1.4, 1.5 and 1.6. Now if we multiply the Eqs 1.4 and 1.55, 1.5 and 1.6, 1.6 and 1.4, and add the three, we get the final equation as under:

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1^2 R_2 R_3 + R_3^2 R_1 R_2 + R_2^2 R_1 R_3}{(R_1 + R_2 + R_3)^2} \quad (1.7)$$

In Eq. 1.7 dividing the LHS by R_A, gives R_3; dividing it by R_B gives R_2, and doing the same with R_C, gives R_1.

Thus $$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

and $$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

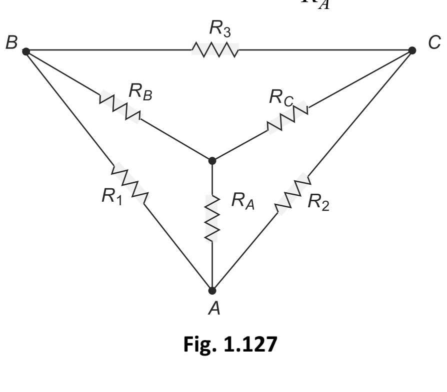


Fig. 1.127

From the above results, we can say that a star connected circuit can be transformed into a delta connected circuit and vice-versa.

From Fig. 1.127 and the above results, we can conclude that any resistance of the delta circuit is equal to the sum of the products of all possible pairs of star resistances divided by the opposite resistance of the star circuit. Similarly, any resistance of the star circuit is equal to the product of two adjacent resistances in the delta connected circuit divided by the sum of all resistances in delta connected circuit.

Example 1.73 *Obtain the star connected equivalent for the delta connected circuit shown in Fig. 1.128.*

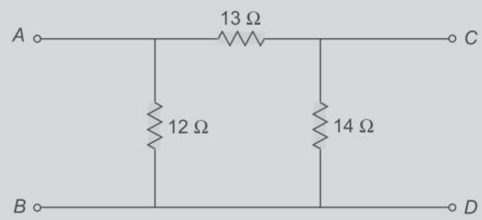


Fig. 1.128

Solution The above circuit can be replaced by a star connected circuit as shown in Fig. 1.129(a).

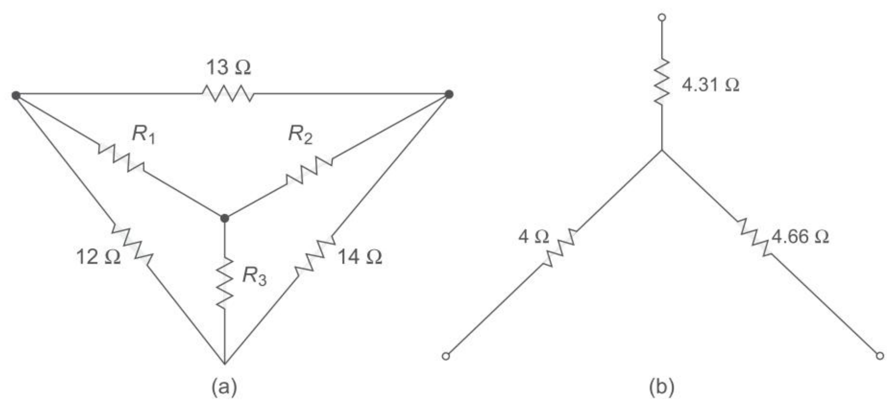


Fig. 1.129

Performing the Δ to Y transformation, we obtain

$$R_1 = \frac{13\times12}{14+13+12}, \quad R_2 = \frac{13\times14}{14+13+12}$$

and $$R_3 = \frac{14\times12}{14+13+12}$$

$\therefore$ $$R_1 = 4\ \Omega, R_2 = 4.66\ \Omega, R_3 = 4.31\ \Omega$$

The star-connected equivalent is shown in Fig. 1.129 (b).

Example 1.74 *Obtain the delta-connected equivalent for the star-connected circuit shown in Fig. 1.130.*

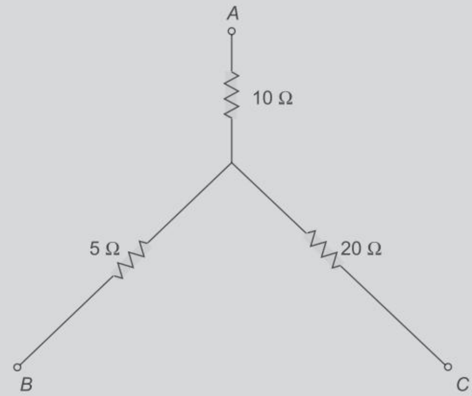


Fig. 1.130

Solution The above circuit can be replaced by a delta-connected circuit as shown in Fig. 1.131(a).

Performing the Y to Δ transformation, we get from the Fig. 1.131 (a)

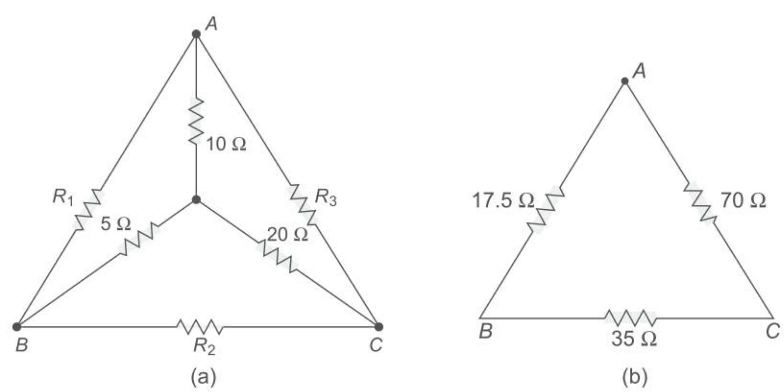


Fig. 1.131

$$R_1 = \frac{20\times10+20\times5+10\times5}{20} = 17.5\,\Omega$$

$$R_2 = \frac{20\times10+20\times5+10\times5}{10} = 35\,\Omega$$

and $$R_3 = \frac{20\times10+20\times5+10\times5}{5} = 70\,\Omega$$

The equivalent delta circuit is shown in Fig. 1.131 (b).

Example 1.75 *Determine the current drawn by the circuit shown in Fig. 1.132.*

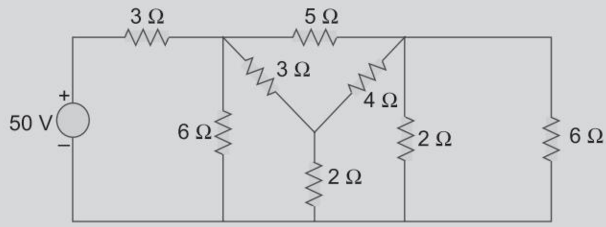


Fig. 1.132

Solution To simplify the network, the star circuit in Fig. 1.132 is converted into a delta circuit as shown under.

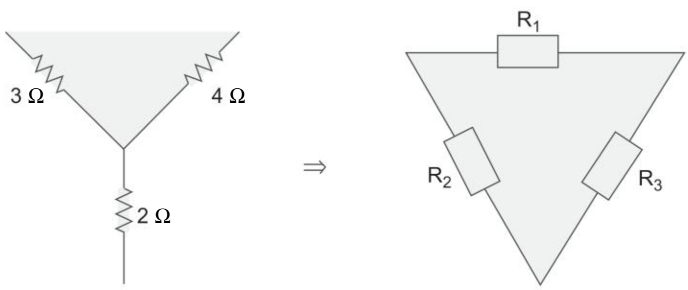


Fig. 1.133

$$R_1 = \frac{4\times3+4\times2+3\times2}{2} = 13\ \Omega$$
$$R_2 = \frac{4\times3+4\times2+3\times2}{4} = 6.5\ \Omega$$
$$R_3 = \frac{4\times3+4\times2+3\times2}{3} = 8.7\ \Omega$$

The original circuit is redrawn as shown in Fig. 1.134.

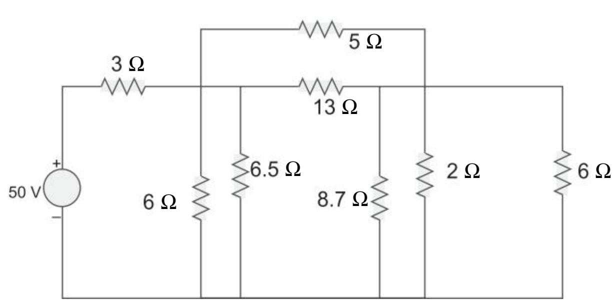


Fig. 1.134

It is further simplified as shown in Fig. 1.135(c). Here the resistors 5 Ω and 13 Ω are in parallel, 6 Ω and 6.5 Ω are in parallel, and 8.7 Ω and 2 Ω are in parallel.

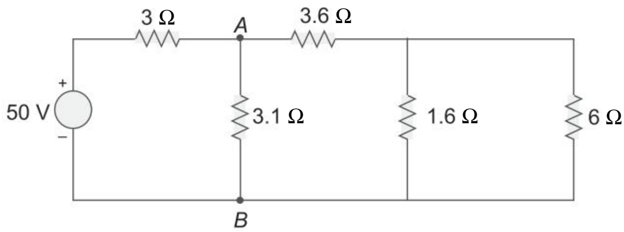


Fig. 1.135

In the above circuit the resistors 6 Ω and 1.6 Ω are in parallel, the resultant of which is in series with 3.6 Ω resistor and is equal to $\left[3.6+\frac{6\times1.6}{7.6}\right]=4.9\ \Omega$ as shown in Fig. 1.136(a).

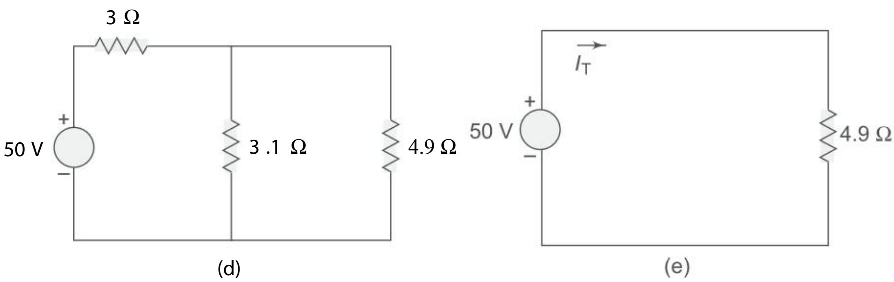


Fig. 1.136(a) and (b)

In the above circuit 4.9 Ω and 3.1 Ω resistors are in parallel, the resultant of which is in series with 3 Ω resistor.

Therefore, the total resistance $R_T = 3+\frac{3.1\times4.9}{8} = 4.9\ \Omega$

The current drawn by the circuit I_T = 50/4.9 = 10.2 A (See Fig. 1.136 (b)).

Example 1.76 *In Fig. 1.137 determine the equivalent resistance by using star-delta transformation.*

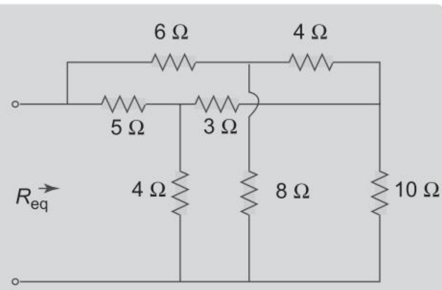


Fig. 1.137

Solution In Fig. 1.137, we have two star circuits, one consisting of 5 Ω, 3 Ω and 4 Ω resistors, and the other consisting of 6 Ω, 4 Ω and 8 Ω resistors. We convert the star circuits into delta circuits, so that the two delta circuits are in parallel.

In Fig. 1.138(a)

$$R_1 = \frac{5\times3+4\times3+5\times4}{4} = 11.75\ \Omega$$

$$R_2 = \frac{5\times3+4\times3+5\times4}{4} = 15.67\ \Omega$$

$$R_3 = \frac{5\times3+4\times3+5\times4}{5} = 9.4\ \Omega$$

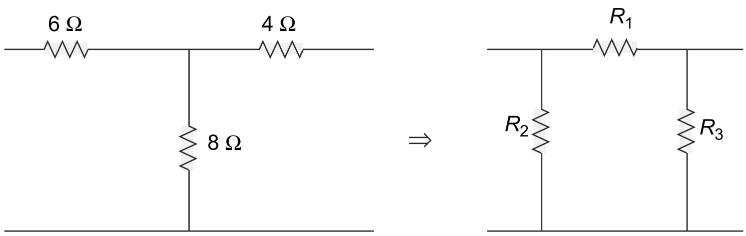


Fig. 1.138(a)

Similarly, in Fig. 1.138(b)

$$R_1 = \frac{6\times4+4\times8+8\times6}{8} = 13\ \Omega$$

$$R_2 = \frac{6\times4+4\times8+8\times6}{4} = 26\ \Omega$$

$$R_3 = \frac{6\times4+4\times8+8\times6}{6} = 17.3\ \Omega$$

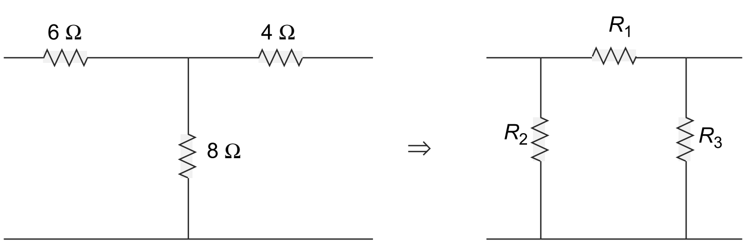


Fig. 1.138(b)

The simplified circuit is shown in Fig. 1.138(c)

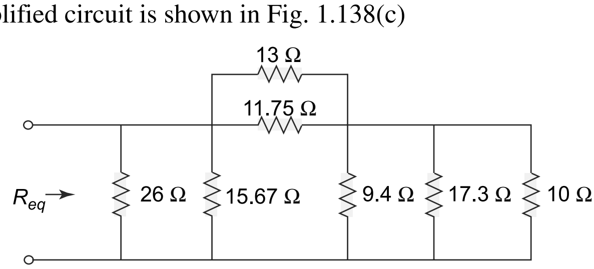


Fig. 1.138(c)

In the above circuit, the three resistors 10 Ω, 9.4 Ω and 17.3 Ω are in parallel.
Equivalent resistance = (10 || 9.4 || 17.3) = 3.78 Ω

Resistors 13 Ω and 11.75 Ω are in parallel
Equivalent resistance = (13 || 11.75) = 6.17 Ω
Resistors 26 Ω and 15.67 Ω are in parallel

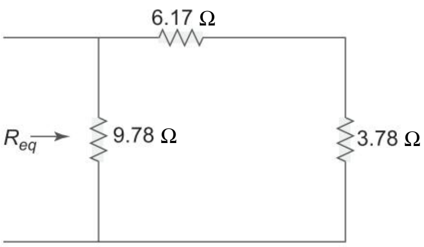


Fig. 1.138(d)

Equivalent resistance = (26 || 15.67) = 9.78 Ω

The simplified circuit is shown in Fig. 1.138(d)

From the above circuit, the equivalent resistance is given by

R_{eq} = (9.78) || (6.17 + 3.78)

= (9.87) || (9.95) = 4.93 Ω

Example 1.77 *Find the voltage to be applied across AB in order to drive a current of 5A into the circuit by using star-delta transformation. Refer Fig. 1.139.*
[JNTU May/June 2006]

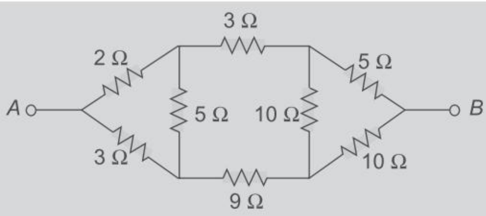


Fig. 1.139

Solution

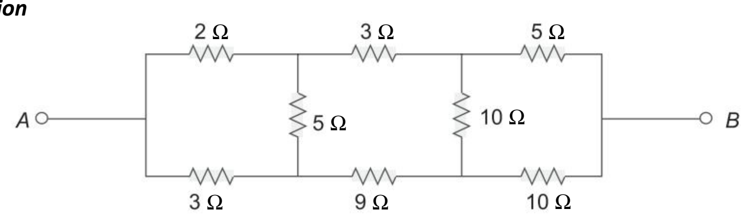


Fig. 1.140

Using star-delta transformation

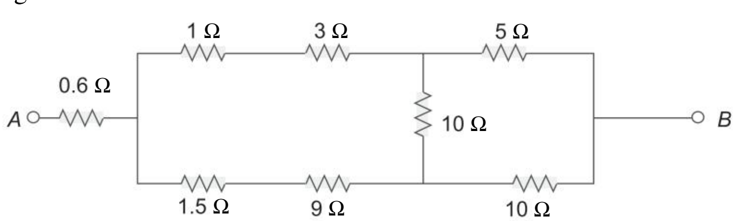


Fig. 1.141(a)

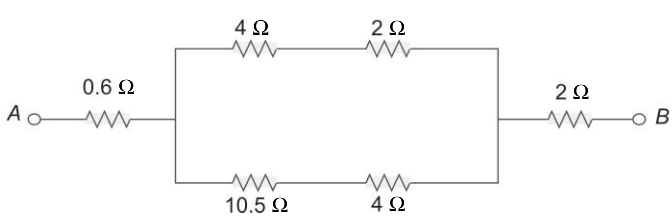
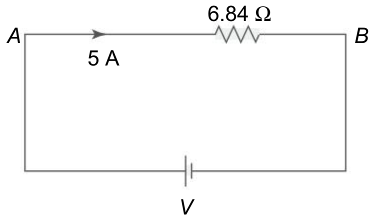


Fig. 1.141(b)

Fig. 1.141(c)



The voltage applied across VAB

$V_{AB} = 5 \times 6.84$
$= 34.2v$

Fig. 1.141(d)

Example 1.78 *Determine the voltage appearing across terminals y_Z, when it a d.c. voltage of 100 V is appplied across x-y terminals in the figure below.*

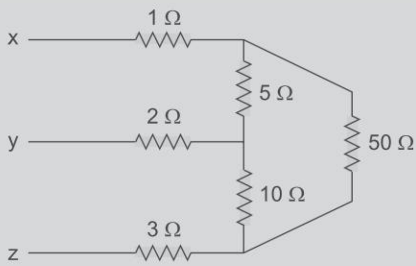


Fig. 1.142

Solution Converting delta network to star network

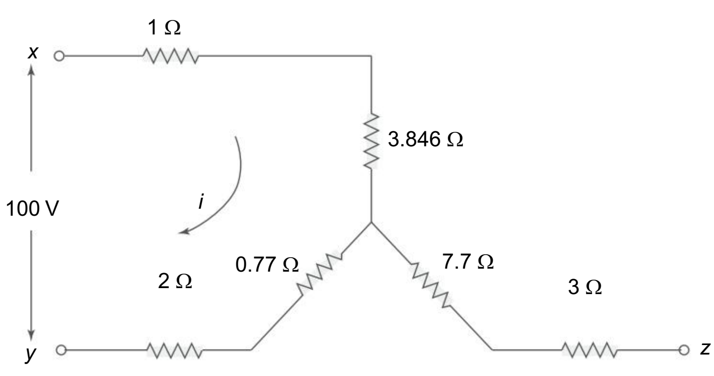


Fig. 1.143

$$\text{Current, } i = \frac{100}{1+3.846+0.77+2} = \frac{100}{7.616} = 13.13A$$

$$\text{Voltage across } y_z^N, V_z = -13.13 \times (2+0.77)$$
$$= -36.37V$$

Example 1.79 *Find equivalent resistance between AB in the circuit shown in the Fig. 1.144. All resistances are equal to R.*

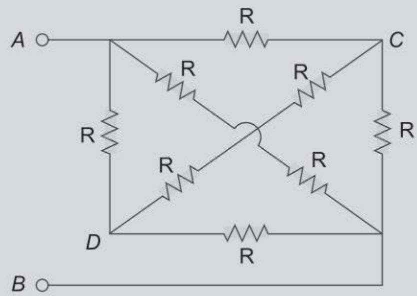


Fig. 1.144

Solution Converting the Star point C into Δ.

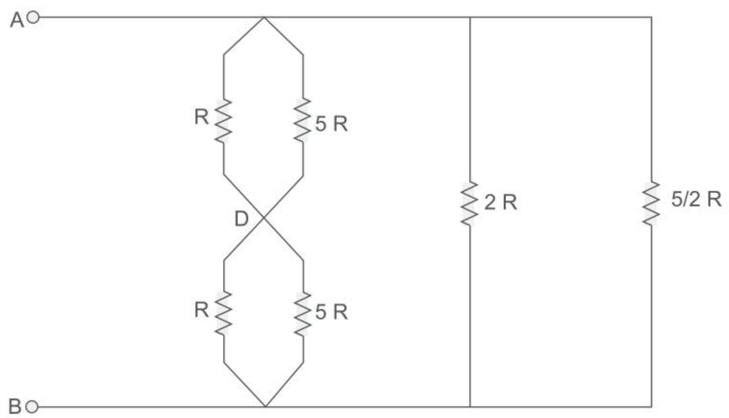


Fig. 1.145

Further reducing the circuit shown in Fig. 1.145 between terminals AB

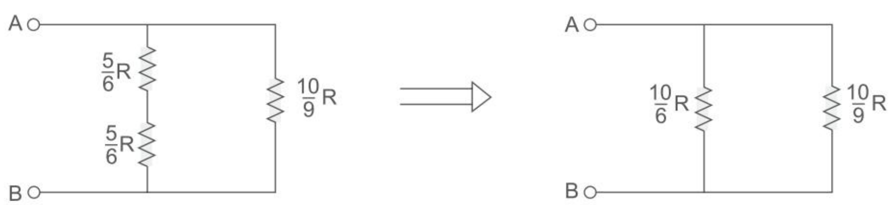


Fig. 1.146

Resistance between terminals AB

$$R_{AB} = \left(\frac{10}{6}R\right) \Big/ \left(\frac{10}{9}R\right)$$

$$= \frac{10}{15}R = 0.667R$$

1.7 SOURCE TRANSFORMATION TECHNIQUE *[JNTU Nov. 2011]*

In solving networks to find solutions one may have to deal with energy sources. It has already been discussed that basically, energy sources are either voltage sources or current sources. Sometimes it is necessary to convert a voltage source to a current source and vice-versa. Any practical voltage source consists of an ideal voltage source in series with an internal resistance. Similarly, a practical current source consists of an ideal current source in parallel with an internal resistance as shown in Fig. 1.147. R_v and R_i represent the internal resistances of the voltage source V_s, and current source I_s, respectively.

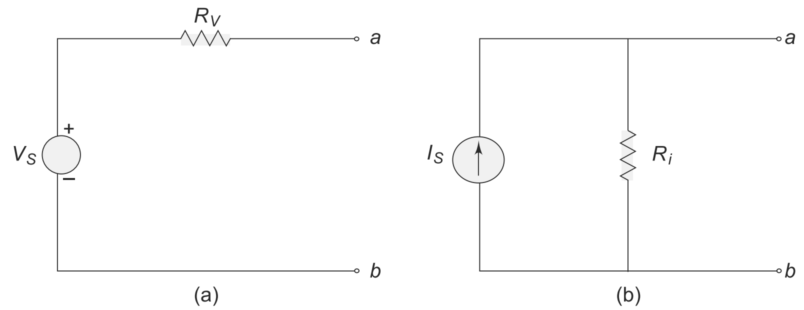


Fig. 1.147

Any source, be it a current source or a voltage source, drives current through its load resistance, and the magnitude of the current depends on the value of the load resistance. Figure 1.148 represents a practical voltage source and a practical current source connected to the same load resistance R_L.

From Fig. 1.148(a), the load voltage can be calculated by using Kirchhoff's voltage law as

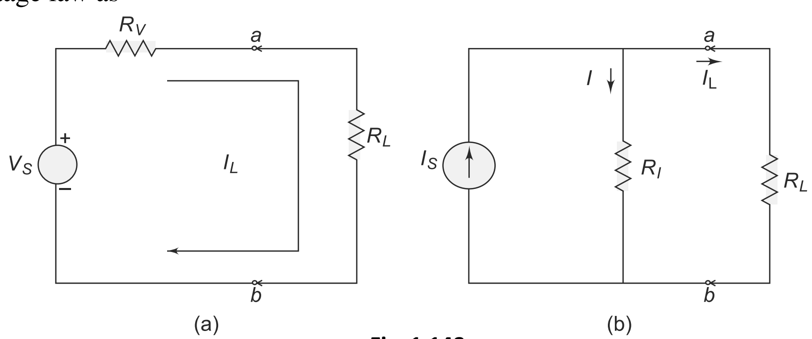


Fig. 1.148

$$V_{ab} = V_s - I_L R_v$$

The open circuit voltage $V_{OC} = V_s$

The short circuit current $I_{SC} = \dfrac{V_s}{R_v}$

From Fig. 1.148 (b),

$$I_L = I_S - I = I_S - \frac{V_{ab}}{R_I}$$

The open circuit voltage $V_{OC} = I_S R_I$

The short circuit current $I_{SC} = I_S$

The above two sources are said to be equal, if they produce equal amounts of current and voltage when they are connected to identical load resistances. Therefore, by equating the open circuit voltages and short circuit currents of the above two sources we obtain

$$V_{OC} = I_s R_I = V_S$$

$$I_{SC} = I_S = \frac{V_s}{R_v}$$

It follows that $R_I = R_V = R_s \quad \therefore \quad V_s = I_S R_S$
where R_S is the internal resistance of the voltage or current source. Therefore, any practical voltage source, having an ideal voltage V_S and internal series resistance R_S can be replaced by a current source $I_S = V_S/R_S$ in parallel with an internal resistance R_S. The reverse transformation is also possible. Thus, a practical current source in parallel with an internal resistance R_S can be replaced by a voltage source $V_S = I_s R_s$ in series with an internal resistance R_S.

Example 1.80 *Determine the equivalent voltage source for the current source shown in Fig. 1.149.*

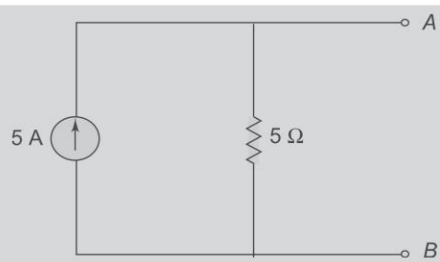


Fig. 1.149

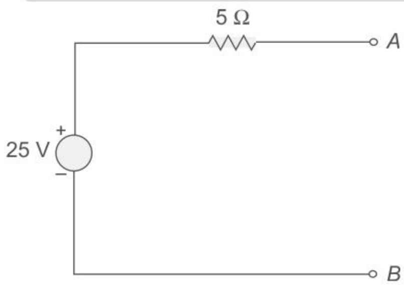


Fig. 1.150

Solution The voltage across terminals A and B is equal to 25 V. Since the internal resistance for the current source is 5 Ω, the internal resistance of the voltage source is also 5 Ω. The equivalent voltage source is shown in Fig. 1.150.

Example 1.81 *Determine the equivalent current source for the voltage source shown in Fig. 1.151.*

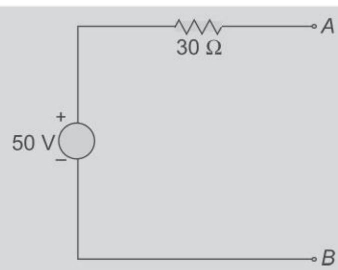


Fig. 1.151

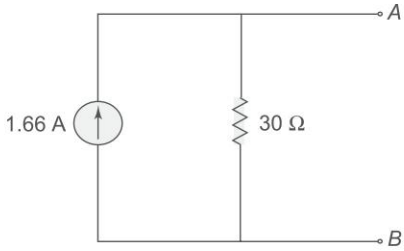


Fig. 1.152

Solution The short circuit current at terminals A and B is equal to

$$I = \frac{50}{30} = 1.66\,\text{A}$$

Since the internal resistance for the voltage source is 30 Ω, the internal resistance of the current source is also 30 Ω. The equivalent current source is shown in Fig. 1.152.

Example 1.82 *Using source transformation, find the power delivered by the 50 V voltage source in the circuit shown in Fig. 1.153.*

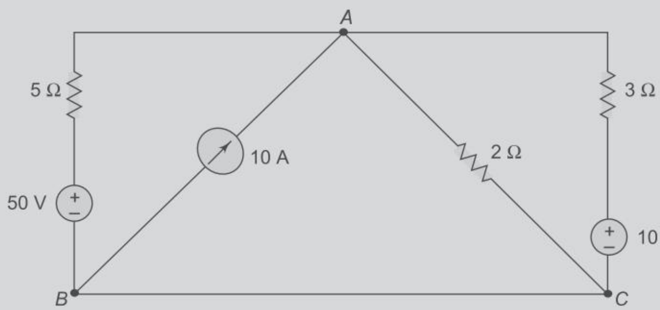


Fig. 1.153

Solution The current source in the circuit in Fig. 1.153 can be replaced by a voltage source as shown in Fig. 1.154.

$$\frac{V-50}{5} + \frac{V-20}{2} + \frac{V-10}{3} = 0$$

$$V\,[0.2 + 0.5 + 0.33] = 23.33$$

$$\text{or} \quad V = \frac{23.33}{1.03} = 22.65\,\text{V}$$

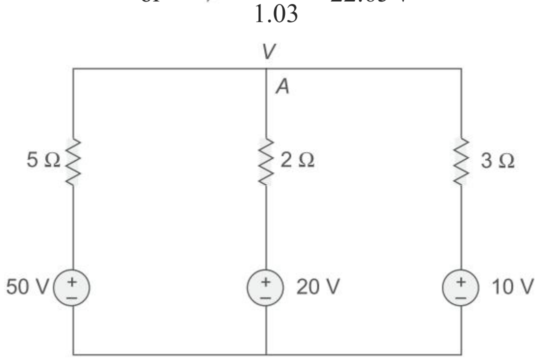


Fig. 1.154

∴ The current delivered by the 50 V voltage source is $(50 - V)/5$

$$= \frac{50 - 22.65}{5} = 5.47\ \text{A}$$

Hence, the power delivered by the 50 V voltage source = 50 × 5.47 = 273.5 W.

Example 1.83 *By using source transformation, source combination and resistance combination convert the circuit shown in Fig. 1.155 into a single voltage source and single resistance.*

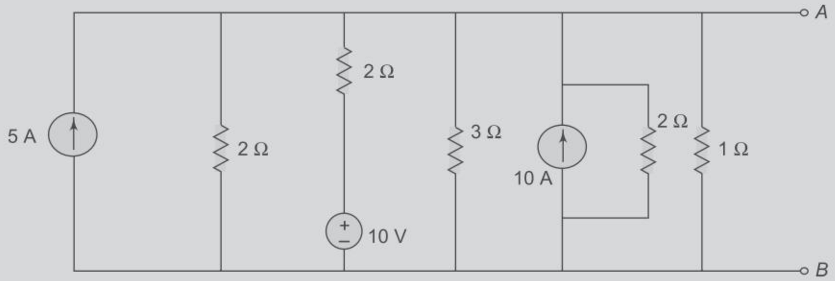


Fig. 1.155

Solution The voltage source in the circuit of Fig. 1.155 can be replaced by a current source as shown in Fig. 1.156(a).

Here the current sources can be combined into a single source. Similarly, all the resistances can be combined into a single resistance, as shown in Fig. 1.156(b).

Figure 1.156(b) can be replaced by single voltage source and a series resistance as shown in Fig. 1.156(c).

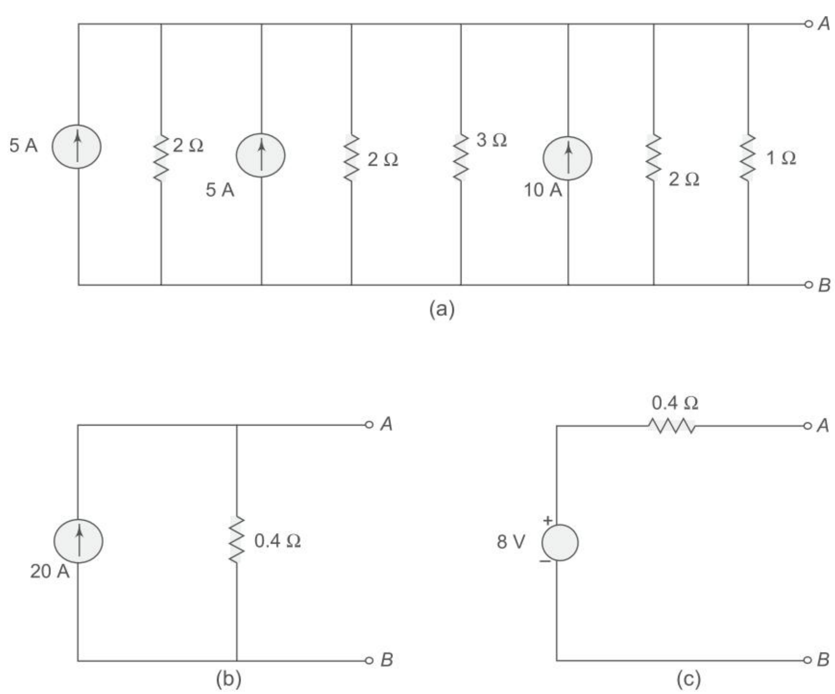


Fig. 1.156

Example 1.84 *Find the voltage and current source equivalent representation of the following network across AB, as shown in Fig. 1.157*

[JNTU Jan 2010]

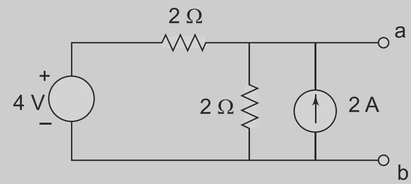


Fig. 1.157

Solution Voltage and current source equivalent representation of the following network across AB.

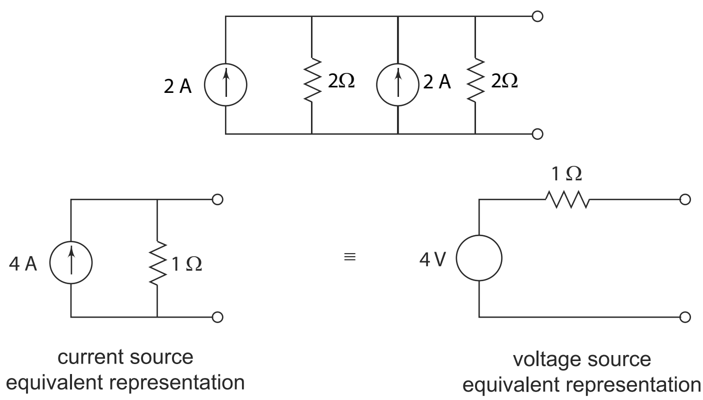


Fig 1.158

Example 1.85 *Find the value of current I_i in Fig. 1.159.*

[JNTU April/May 2007]

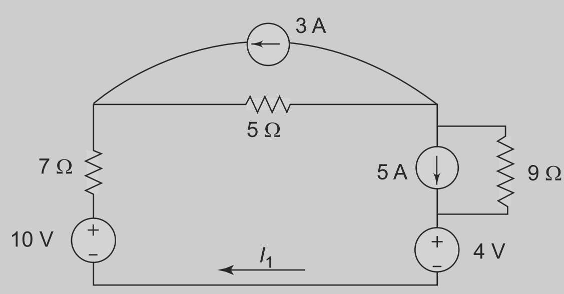


Fig. 1.159

Solution Converting current source into equivalent voltage source

By applying KVL

$$10 - 7I_1 - 15 - 5I_1 - 9I_1 + 45 - 4 = 0$$

$$36 = 21I_1$$

$$I_1 = \frac{36}{21} = 1.714 \text{ A}$$

$$I_1 = 1.714 \text{ A}$$

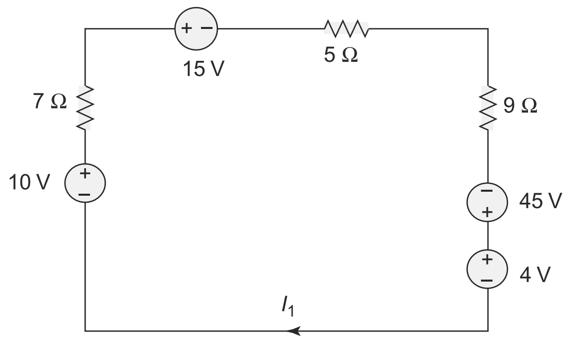


Fig. 1.160

Example 1.86 *Using source Transformation, reduce the network between A and B into an equivalent voltage source. (Fig. 1.161)*

[JNTU May/June 2006]

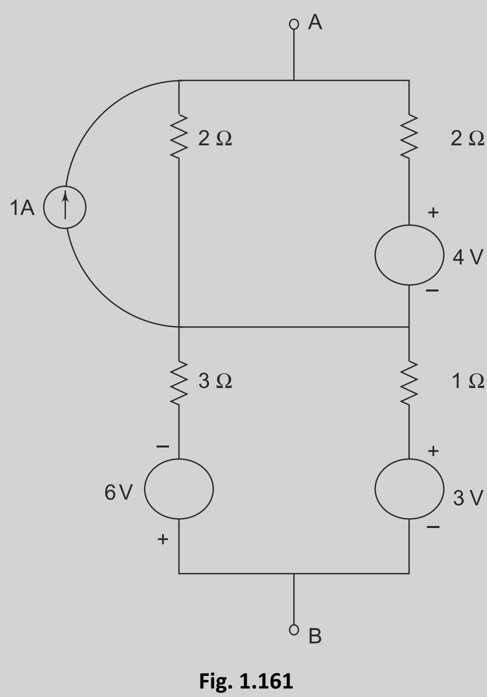


Fig. 1.161

Solution Given circuit

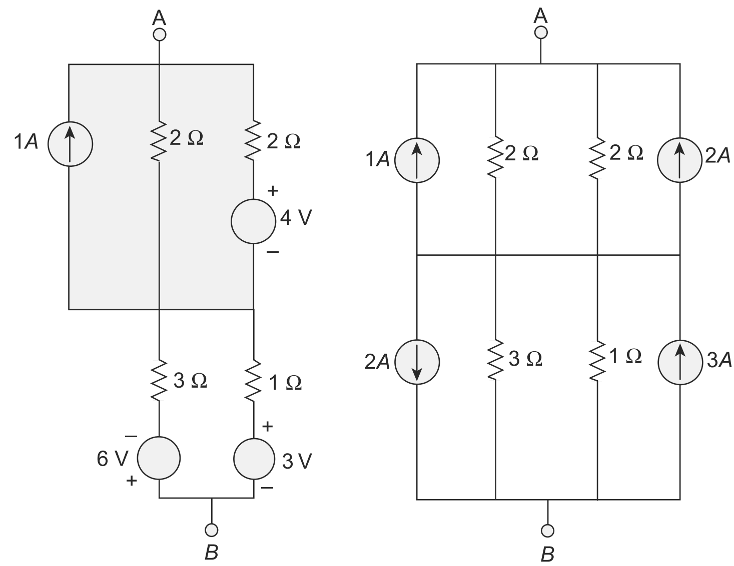


Fig. 1.162(a)

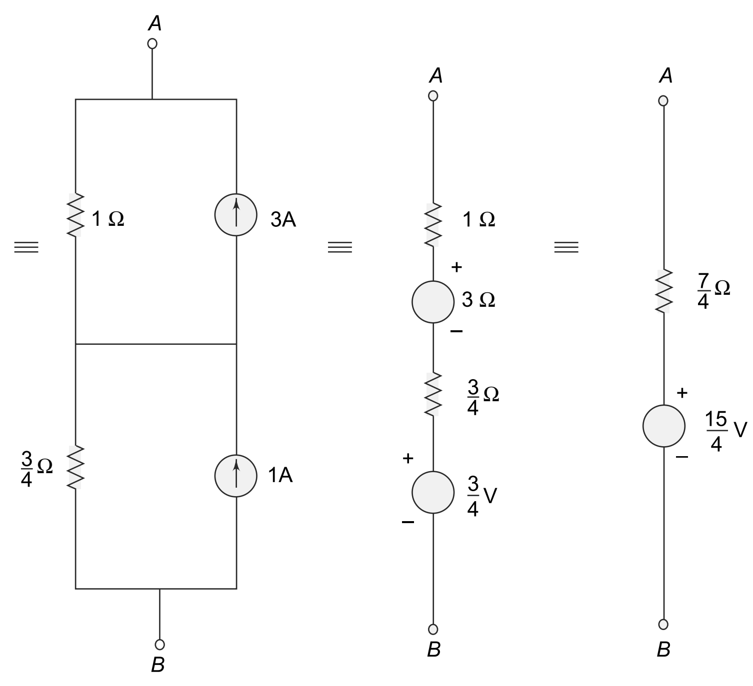


Fig. 1.162(b)

Example 1.87 *Reduce the network shown in Fig. 1.163, to a single loop network by successive source transformation, to obtain the current in the 12 Ω resistor.*

[JNTU May/June 2006]

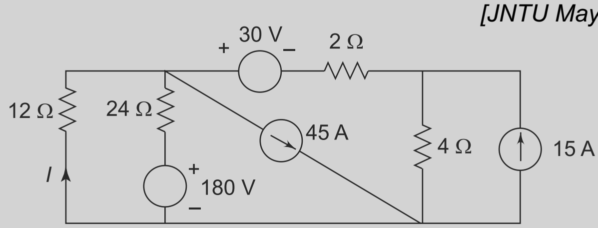


Fig. 1.163

Solution By source transformation $I = 22.5 \times \dfrac{4.8}{16.8} = 6.428$ A.

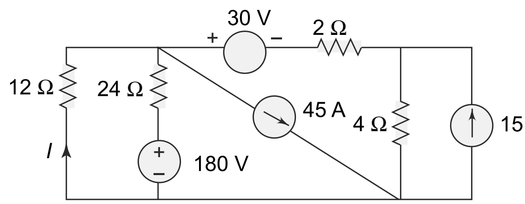


Fig. 1.164

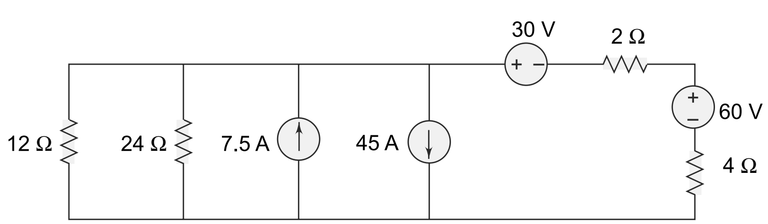


Fig. 1.165

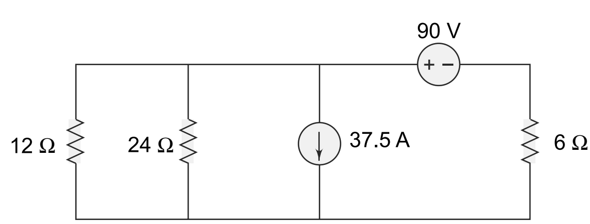


Fig. 1.166

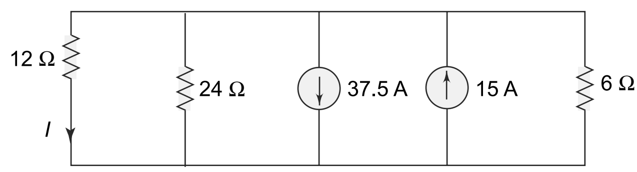


Fig. 1.167

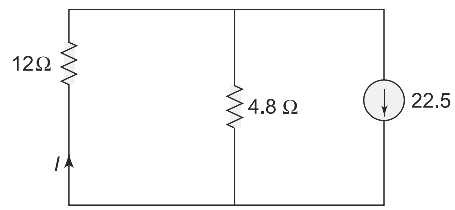


Fig. 1.168

Practice Problems

1.1 (i) Determine the current in each of the following cases

(a) 75 C in 1 s (b) 10 C in 0.5 s (c) 5 C in 2 s

(ii) How long does it take 10 C to flow past a point if the current is 5 A?

1.2 A resistor of 30 Ω has a voltage rating of 500 V; what is its power rating?

1.3 A resistor with a current of 2 A through it converts 1000 J of electrical energy to heat energy in 15 s. What is the voltage across the resistor?

1.4 The filament of a light bulb in the circuit has a certain amount of resistance. If the bulb operates with 120 V and 0.8 A of current, what is the resistance of its filament?

1.5 Find the capacitance of a circuit in which an applied voltage of 20 V gives an energy store of 0.3 J.

1.6 A 6.8 kΩ resistor has burned out in a circuit. It has to be replaced with another resistor with the same ohmic value. If the resistor carries 10 mA, what should be its power rating?

1.7 If you wish to increase the amount of current in a resistor from 100 mA to 150 mA by changing the 20 V source, by how many volts should you change the source? To what new value should you set it?

1.8 A 12 V source is connected to a 10 Ω resistor.

(a) How much energy is used in two minutes?

(b) If the resistor is disconnected after one minute, does the power absorbed in resistor increase or decrease?

1.9 A capacitor is charged to 50 μC. The voltage across the capacitor is 150 V. It is then connected to another capacitor four times the capacitance of the first capacitor. Find the loss of energy.

1.10 The voltage across two parallel capacitors 5 μF and 3 μF changes uniformly from 30 to 75 V in 10 ms. Calculate the rate of change of voltage for (i) each capacitor, and (ii) the combination.

1.11 The voltage waveform shown in Fig. 1.169 is applied to a pure capacitor of 60 μF. Sketch $i(t)$, $p(t)$ and determine I_m and P_m.

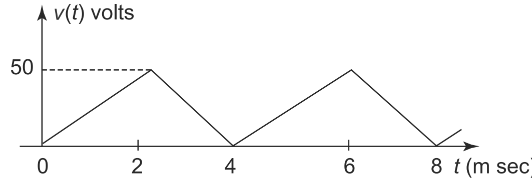


Fig. 1.169

1.12 Determine an expression for the current if the voltage across a pure capacitor is given as

$$v = V_m \left[wt - \frac{(wt)^3}{3!} + \frac{(wt)^5}{5!} - \frac{(wt)^7}{7!} + \dots \right]$$

1.13 A 2 μF capacitor has a charge function $q = 100\ [1 \times e^{-5\times10^4 t}]$ μc. Determine the corresponding voltage and current functions.

1.14 A pure inductance of 0.05 H has an applied voltage with the waveform shown in Fig. 1.170. Sketch the corresponding current waveform and determine the expression for i in the first internal.

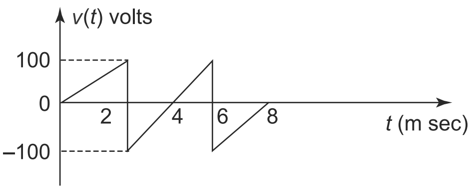


Fig. 1.170

1.15 An inductor of 0.004 H contains a current with a waveform shown in Fig. 1.171. Sketch the voltage waveform.

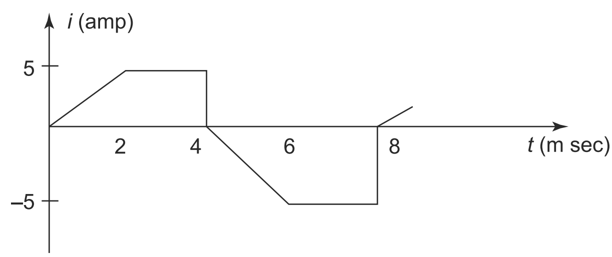


Fig. 1.171

1.16 A single pure inductance of 3 mH passes a current of the waveform shown in Fig. 1.172. Determine and sketch the voltage υ(t) and the instantaneous power $p(t)$.

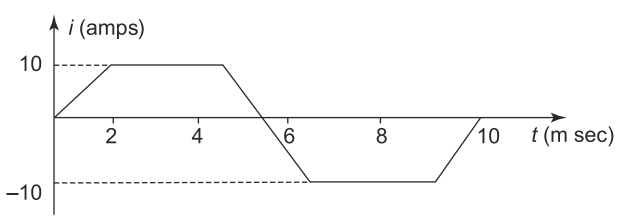


Fig. 1.172

1.17 Simplify the circuit shown in Fig. 1.173 using series parallel combinations.

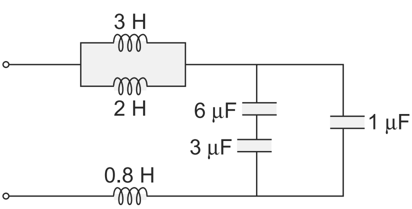


Fig. 1.173

1.18 Determine the equivalent capaci- t ance of the circuit shown in Fig. 1.174 if all the capacitors are 10 μF.

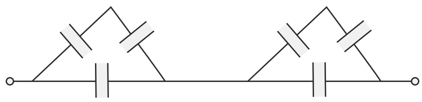


Fig. 1.174

1.19 Reduce the circuit shown in Fig. 1.78 to a single equivalent capacitance across terminals *a* and *b*.

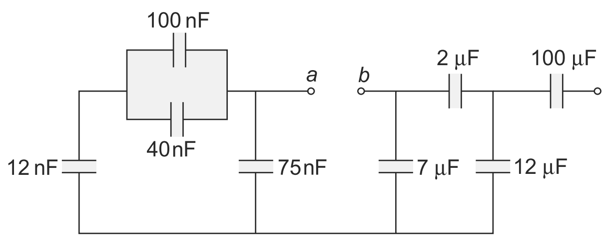


Fig. 1.175

1.20 For the circuit shown in Fig. 1.176, find the equivalent inductance.

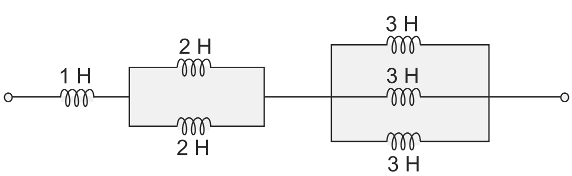


Fig. 1.176

1.21 The following voltage drops are measured across each of three resistors in series: 5.5 V, 7.2 V and 12.3 V. What is the value of the source voltage to which these resistors are connected? If a fourth resistor is added to the circuit with a source voltage of 30 V. What should be the drop across the fourth resistor?

1.22 What is the voltage V_{AB} across the resistor shown in Fig. 1.177?

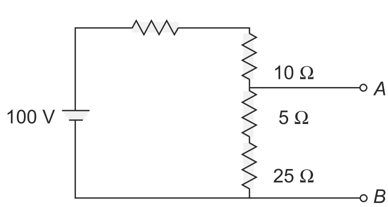


Fig. 1.177(a)

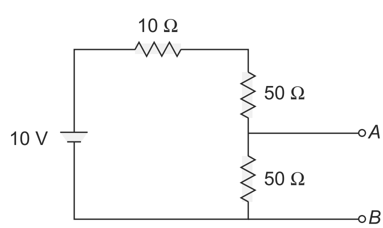


Fig. 1.177(b)

1.23 The source voltage in the circuit shown in Fig. 1.178 is 100 V. How much voltage does each metre read?

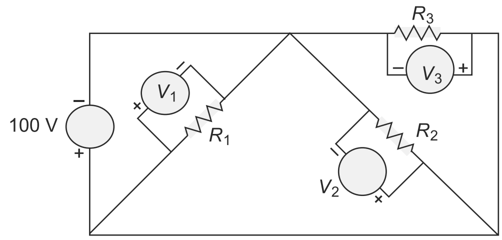


Fig. 1.178

1.24 Using the current divider formula, determine the current in each branch of the circuit shown in Fig. 1.179.

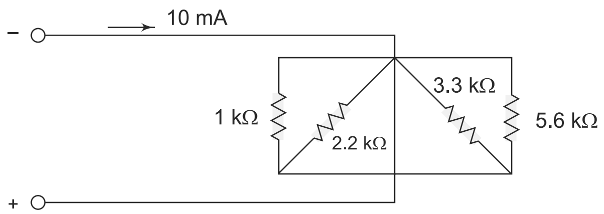


Fig. 1.179

1.25 Six light bulbs are connected in parallel across 110 V. Each bulb is rated at 75 W. How much current flows through each bulb, and what is the total current?

1.26 For the circuit shown in Fig. 1.180, find the total resistance between terminals *A* and *B*; the total current drawn from a 6 V source connected from *A* to *B*; and the current through 4.7 kΩ; voltage across 3 kΩ.

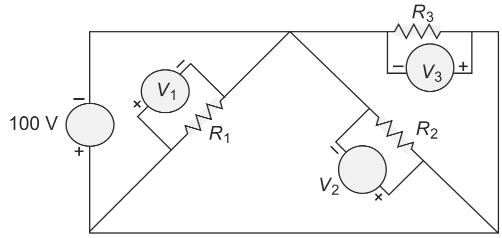


Fig. 1.180

1.27 For the circuit shown in Fig. 1.181, find the total resistance.

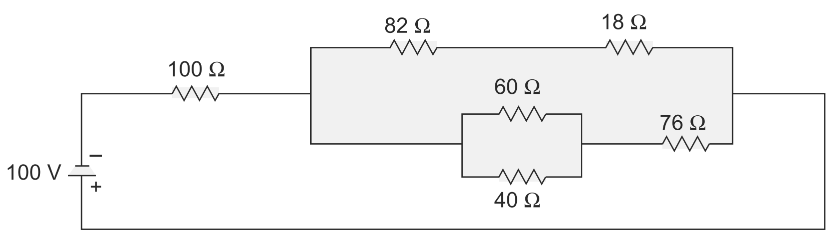


Fig. 1.181

1.28 The current in the 5 Ω resistance of the circuit shown in Fig. 1.182 is 5 A. Find the current in the 10 Ω resistor. Calculate the power consumed by the 5 Ω resistor.

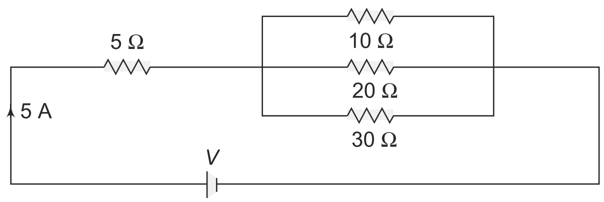


Fig. 1.182

1.29 A battery of unknown emf is connected across resistances as shown in Fig. 1.183. The voltage drop across the 8 Ω resistor is 20 V. What will be the current reading in the ammeter? What is the emf of the battery.

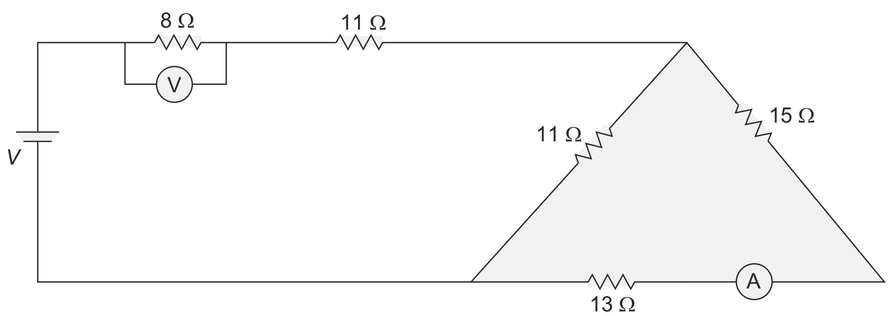


Fig. 1.183

1.30 An electric circuit has three terminals *A*, *B*, *C*. Between *A* and *B* is connected a 2 Ω resistor, between *B* and *C* are connected a 7 Ω resistor and 5 Ω resistor in parallel and between *A* and *C* is connected a 1 Ω resistor. A battery of 10 V is then connected between terminals *A* and *C*. Calculate (a) total current drawn from the battery (b) voltage across the 2 Ω resistor (c) current passing through the 5 Ω resistor.

1.31 Use Ohm's law and Kirchhoff's laws on the circuit given in Fig. 1.184, find V_{in}, V_s and power provided by the dependent source.

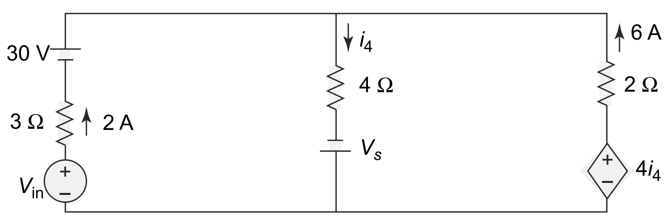


Fig. 1.184

1.32 Use Ohm's law and Kirchhoff's laws on the circuit given in Fig. 1.185, find all the voltages and currents.

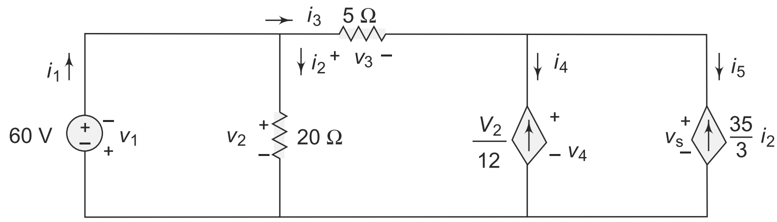


Fig. 1.185

1.33 Find the power absorbed by each element and show that the algebraic sum of powers is zero in the circuit shown in Fig. 1.186.

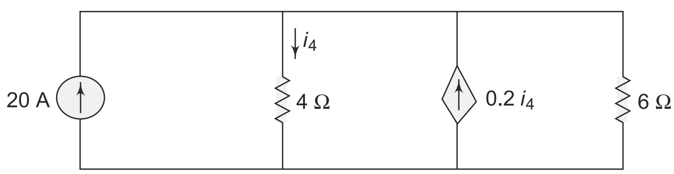


Fig. 1.186

1.34 Find the power absorbed by each element in the circuit shown in Fig. 1.187.

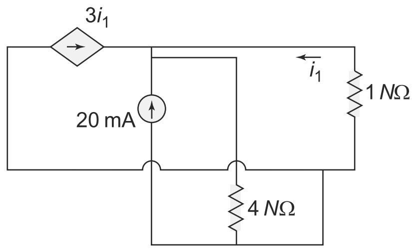


Fig. 1.187

Objective Type Questions

1.1 How many coulombs of charge do 50×10^{31} electrons possess?

(a) 80×10^{12} C (b) 50×10^{31} C
(c) 0.02×10^{-31} C (d) $1/80 \times 10^{12}$ C

1.2 Determine the voltage of 100 J/25 C.

(a) 100 V (b) 25 V
(c) 4 V (d) 0.25 V

1.3 What is the voltage of a battery that uses 800 J of energy to move 40 C of charge through a resistor?

(a) 800 V (b) 40 V
(c) 25 V (d) 20 V

1.4 Determine the current if a 10 coulomb charge passes a point in 0.5 seconds.
(a) 10 A (b) 20 A
(c) 0.5 A (d) 2 A

1.5 If a resistor has 5.5 V across it and 3 mA flowing through it, what is the power?
(a) 16.5 mW (b) 15 mW
(c) 1.83 mW (d) 16.5 W

1.6 Identify the passive element among the following.
(a) Voltage source (b) Current source
(c) Inductor (d) Transistor

1.7 If a resistor is to carry 1 A of current and handle 100 W of power, how many ohms must it be? Assume that voltage can be adjusted to any required value.
(a) 50 Ω (b) 100 Ω
(c) 1 Ω (d) 10 Ω

1.8 A 100 Ω resistor is connected across the terminals of a 2.5 V battery. What is the power dissipation in the resistor?
(a) 25 W (b) 100 W
(c) 0.4 W (d) 6.25 W

1.9 Determine total inductance of a parallel combination of 100 mH, 50 mH and 10 mH.
(a) 7.69 mH (b) 160 mH
(c) 60 mH (d) 110 mH

1.10 How much energy is stored by a 100 mH inductance with a current of 1 A?
(a) 100 J (b) 1 J
(c) 0.05 J (d) 0.01 J

1.11 Five inductors are connected in series. The lowest value is 5 μH. If the value of each inductor is twice that of the preceding one, and if the inductors are connected in order ascending values. What is the total inductance?
(a) 155 μH (b) 155 H
(c) 155 mH (d) 25 μH

1.12 Determine the charge when $C = 0.001$ μF and $\upsilon = 1$ KV.
(a) 0.001 C (b) 1 μC
(c) 1 C (d) 0.001 C

1.13 If the voltage across a given capacitor is increased, does the amount of stored charge
(a) increase (b) decrease
(c) remain constant (d) is exactly doubled

1.14 A 1 μF, a 2.2 μF and a 0.05 μF capacitors are connected in series. The total capacitance is less than

(a) 0.07 (b) 3.25
(c) 0.05 (d) 3.2

1.15 How much energy is stored by a 0.05 μF capacitor with a voltage of 100 V?

(a) 0.025 J (b) 0.05 J
(c) 5 J (d) 100 J

1.16 Which one of the following is an ideal voltage source?

(a) voltage independent of current
(b) current independent of voltage
(c) both (a) and (b)
(d) none of the above

1.17 The following voltage drops are measured across each of three resistors in series: 5.2 V, 8.5 V and 12.3 V. What is the value of the source voltage to which these resistors are connected?

(a) 8.2 V (b) 12.3 V (c) 5.2 V (d) 26 V

1.18 A certain series circuit has a 100 Ω, a 270 Ω, and a 330 Ω resistor in series. If the 270 Ω resistor is removed, the current

(a) increases (b) becomes zero
(c) decrease (d) remain constant

1.19 A series circuit consists of a 4.7 kΩ, 5.6 kΩ, 9 kΩ and 10 kΩ resistor. Which resistor has the most voltage across it?

(a) 4.7 kΩ (b) 5.6 kΩ (c) 9 kΩ (d) 10 kΩ

1.20 The total power in a series circuit is 10 W. There are five equal value resistors in the circuit. How much power does each resistor dissipate?

(a) 10 W (b) 5 W (c) 2 W (d) 1 W

1.21 When a 1.2 kΩ resistor, 100 Ω resistor, 1 kΩ resistor and 50 Ω resistor are in parallel, the total resistance is less than

(a) 100 Ω (b) 50 Ω (c) 1 kΩ (d) 1.2 kΩ

1.22 If a 10 V battery is connected across the parallel resistors of 3 Ω, 5 Ω, 10 Ω and 20 Ω, how much voltage is there across 5 Ω resistor?

(a) 10 V (b) 3 V (c) 5 V (d) 20 V

1.23 If one of the resistors in a parallel circuit is removed, what happens to the total resistance?

(a) decreases (b) increases
(c) remain constant (d) exactly doubles

1.24 The power dissipation in each of three parallel branches is 1 W. What is the total power dissipation of the circuit?

(a) 1 W (b) 4 W
(c) 3 W (d) zero

1.25 In a four branch parallel circuit, 10 mA of current flows in each branch. If one of the branch opens, the current in each of the other branches

(a) increases (b) decreases
(c) remains unaffected (d) doubles

1.26 Four equal value resistors are connected in parallel. Five volts are applied across the parallel circuit, and 2.5 mA are measured from the source. What is the value of each resistor?

(a) 4 Ω (b) 8 Ω
(c) 2.5 Ω (d) 5 Ω

1.27 Six light bulbs are connected in parallel across 110 V. Each bulb is related at 75 W. How much current flows through each bulb?

(a) 0.682 A (b) 0.7 A
(c) 75 A (d) 110 A

1.28 A 330 Ω resistor is in series with the parallel combination of four 1 kΩ resistors. A 100 V source is connected to the circuit. Which resistor has the most current through it.

(a) 330 Ω resistor
(b) parallel combination of three 1 kΩ resistors
(c) parallel combination of two 1 kΩ resistors
(d) 1 kΩ resistor

1.29 The current i_4 in the circuit shown in Fig. 1.188 is equal to

(a) 12 A (b) −12 A
(c) 4 A (d) None of the above

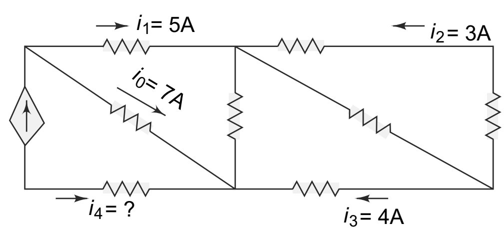


Fig. 1.188

1.30 The voltage V in Fig. 1.189 is equal to

(a) 3 V
(b) −3 V
(c) 5 V
(d) None of the above

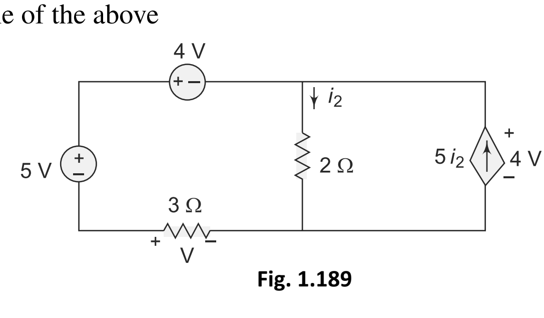


Fig. 1.189

1.31 The voltage *V* in Fig. 1.190 is always equal to

(a) 9 V
(b) 5 V
(c) 1 V
(d) None of the above

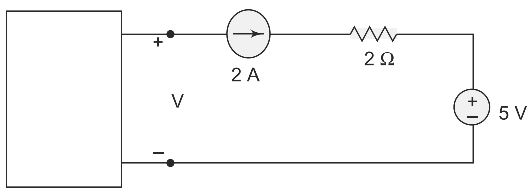


Fig. 1.190

1.32 The voltage *V* in Fig. 1.191 is

(a) 10 V
(b) 15 V
(c) 5 V
(d) None of the above

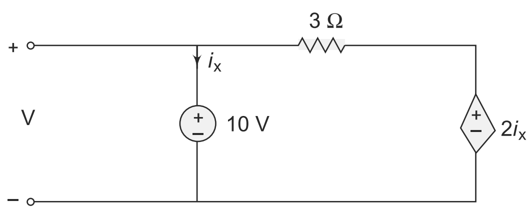
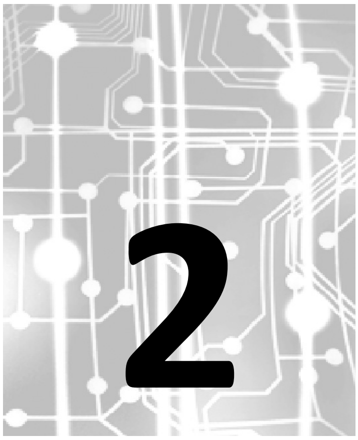


Fig. 1.191



2 Single-Phase ac Circuits

2.1 PERIODIC WAVEFORMS (DETERMINATION ON RMS, AVERAGE VALUE AND FORM FACTOR

Many a time, alternating voltages and currents are represented by a sinusoidal wave, or simply a sinusoid. It is a very common type of alternating current (ac) and alternating voltage. The sinusoidal wave is generally referred to as a sine wave. Basically an alternating voltage (current) waveform is defined as the voltage (current) that fluctuates with time periodically, with change in polarity and direction. In general, the sine wave is more useful than other waveforms, like pulse, sawtooth, square, etc. There are a number of reasons for this. One of the reasons is that if we take any second order system, the response of this system is a sinusoid. Secondly, any periodic waveform can be written in terms of sinusoidal function according to Fourier theorem. Another reason is that its derivatives and integrals are also sinusoids. A sinusoidal function is easy to analyse. Lastly, the sinusoidal function is easy to generate, and it is more useful in the power industry. The shape of a sinusoidal waveform is shown in Fig. 2.1.

The waveform may be either a current waveform, or a voltage waveform. As seen from the Fig. 2.1, the wave changes its magnitude and direction with time. If we start at time $t = 0$, the wave goes to a maximum value and returns to zero, and then decreases to a negative maximum value before returning to zero. The sine wave changes with time in an orderly manner. During the positive portion of voltage, the current flows in one direction; and during the negative portion of voltage, the current flows in the opposite direction.

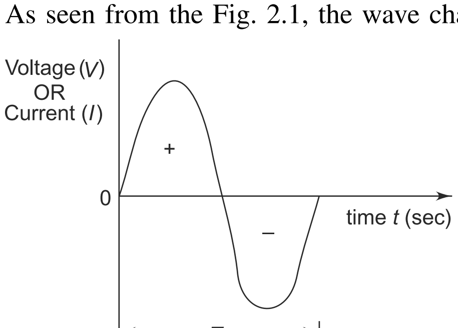


Fig. 2.1

The complete positive and negative portion of the wave is one cycle of the sine wave. Time is designated by t. The time taken for any wave to complete one full cycle is called the period (T). In general, any periodic wave constitutes a number of such cycles. For example, one cycle of a sine wave repeats a number of times as shown in Fig. 2.2. Mathematically it can be represented as $f(t) = f(t + T)$ for any t.

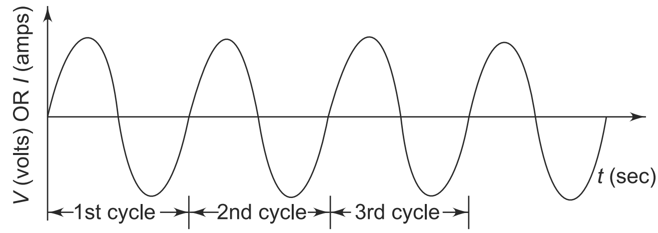


Fig. 2.2

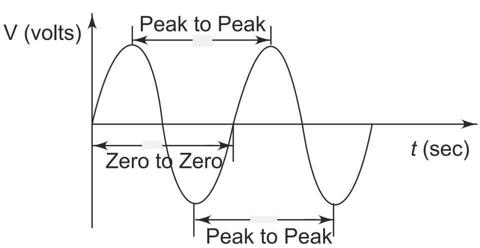


Fig. 2.3

The period can be measured in the following different ways (See Fig. 2.3).

From zero crossing of one cycle to zero crossing of the next cycle.

1. From positive peak of one cycle to positive peak of the next cycle, and
2. From negative peak of one cycle to negative peak of the next cycle.

The frequency of a wave is defined as the number of cycles that a wave completes in one second.

In Fig. 2.4 the sine wave completes three cycles in one second. Frequency is measured in hertz. One hertz is equivalent to one cycle per second, 60 hertz is 60 cycles per second and so on. In Fig. 2.4, the frequency denoted by f is 3 Hz, that is three cycles per second. The relation between time period and frequency is given by

$$f = \frac{1}{T}$$

A sine wave with a longer period consists of fewer cycles than one with a shorter period.

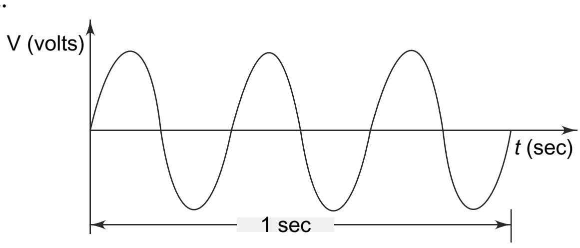


Fig. 2.4

Example 2.1 *What is the period of sine wave shown in Fig. 2.5?*

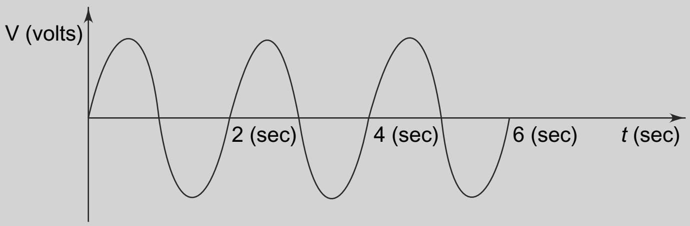


Fig. 2.5

Solution From Fig. 2.5, it can be seen the sine wave takes two seconds to complete one period in each cycle

$$T = 2\,\text{s}$$

Example 2.2 *The period of a sine wave is 20 milliseconds. What is the frequency?*

Solution

$$f = \frac{1}{T}$$

$$= \frac{1}{20\,\text{ms}} = 50\,\text{Hz}$$

Example 2.3 *The frequency of a sine wave is 30 Hz. What is its period?*

Solution

$$T = \frac{1}{f}$$

$$= \frac{1}{30} = 0.03333\ \text{s}$$

$$= 33.33\ \text{ms}$$

Example 2.4 *Calculate the frequency for each of the following values of time period.*

(a) 2 ms (b) 100 ms (c) 5 ms (d) 5 s

Solution The relation between frequency and period is given by

$$f = \frac{1}{T}\,\text{Hz}$$

(a) Frequency $f = \dfrac{1}{20\times10^{-3}} = 50\ \text{Hz}$

(b) Frequency $f = \dfrac{1}{100\times10^{-3}} 10\ \text{Hz}$

(c) Frequency $f = \dfrac{1}{5\times10^{-6}} = 200$ kHz

(d) Frequency $f = \dfrac{1}{5} = 0.2$ Hz

Example 2.5 *Calculate the period for each of the following values of frequency.*

(a) 50 Hz (b) 100 kHz (c) 1 Hz (d) 2 MHz

Solution The relation between frequency and period is given by

$$f = \frac{1}{T} \text{ Hz}$$

(a) Time period $T = \dfrac{1}{f} = \dfrac{1}{50} = 0.02$ s

(b) Time period $T = \dfrac{1}{f} = \dfrac{1}{100\times10^{3}} = 10\ \mu\text{s}$

(c) Time period $T = \dfrac{1}{f} = \dfrac{1}{1} = 1$ s

(d) Time period $T = \dfrac{1}{f} = \dfrac{1}{2\times10^{6}} = 0.5\ \mu\text{s}$

Example 2.6 *A sine wave has a frequency of 50 kHz. How many cycles does it complete in 20 ms?*

Solution The frequency of sine wave is 50 kHz.

That means in 1 second, a sine wave goes through 50×10^3 cycles.

In 20 ms the number of cycles $= 20 \times 10^{-3} \times 50 \times 10^3$

$= 1$ kHz

That means in 20 ms the sine wave goes through 10^3 cycles.

2.1.1 Angular Relation of a Sinusoidal Wave

A sine wave can be measured along the *X*-axis on a time base which is frequency- dependent. A sine wave can also be expressed in terms of an angular measurement. This angular measurement is expressed in degrees or radians. A radian is defined as the angular distance measured along the circumference of a circle which is equal to the radius of the circle. One radian is equal to 57.3°. In a 360° revolution, there are 2π radians. The angular measurement of a sine wave is based on 360° or 2π radians for a complete cycle as shown in Figs 2.6 (a) and (b).

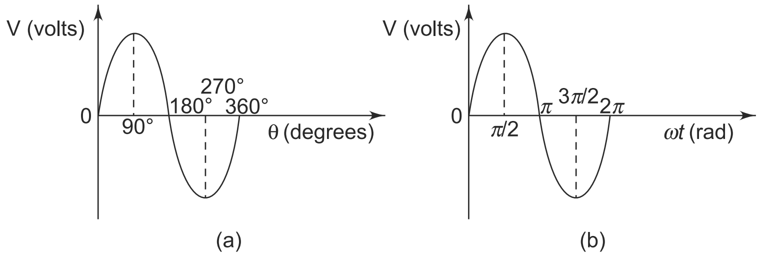


Fig. 2.6

A sine wave completes a half cycle in 180° or π radians; a quarter cycle in 90° or π/2 radians, and so on.

2.1.2 Phase of a Sinusoidal Wave

The phase of a sine wave is an angular measurement that specifies the position of the sine wave relative to a reference. The wave shown in Fig. 2.7 is taken as the reference wave.

When the sine wave is shifted left or right with reference to the wave shown in Fig. 2.7, there occurs a phase shift. Figure 2.8 shows the phase shifts of a sine wave.

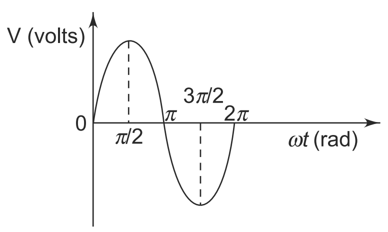


Fig. 2.7

In Fig. 2.8(a), the sine wave is shifted to the right by 90° (π/2 rad) shown by the dotted lines. There is a phase angle of 90° between *A* and *B*. Here the waveform *B* is lagging behind waveform *A* by 90°. In other words, the sine wave *A* is leading the waveform *B* by 90°. In Fig. 2.8(b) the sine wave *A* is lagging behind the waveform *B* by 90°. In both cases, the phase difference is 90°.

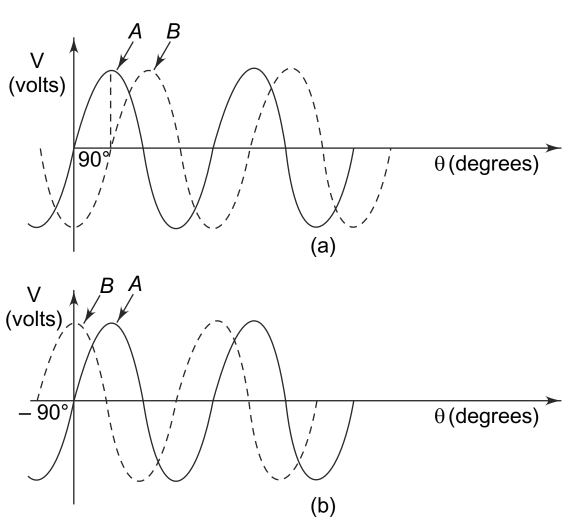


Fig. 2.8

Example 2.7 *What are the phase angles between the two sine waves shown in Figs 2.9 (a) and (b)?*

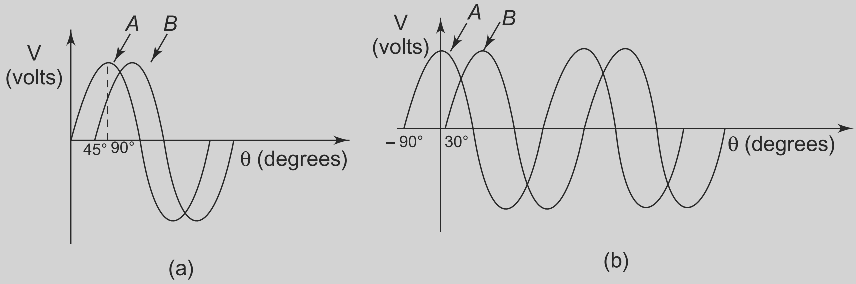


Fig. 2.9

Solution In Fig. 2.9(a), sine wave *A* is in phase with the reference wave; sine wave *B* is out of phase, which lags behind the reference wave by 45°. So we say that sine wave *B* lags behind sine wave *A* by 45°.

In Fig. 2.9(b), sine wave *A* leads the reference wave by 90°; sine wave *B* lags behind the reference wave by 30°. So the phase difference between *A* and *B* is 120°, which means that sine wave *B* lags behind sine wave *A* by 120°. In other words, sine wave *A* leads sine wave *B* by 120°.

Example 2.8 *Sine wave 'A' has a positive going zero crossing at 45°. Sine wave 'B' has a positive going zero crossing at 60°. Determine the phase angle between the signals. Which of the signal lags behind the other?*

Solution The two signals drawn are shown in Fig. 2.10.

From Fig. 2.10, the signal *B* lags behind signal *A* by 15°. In other words, signal *A* leads signal *B* by 15°.

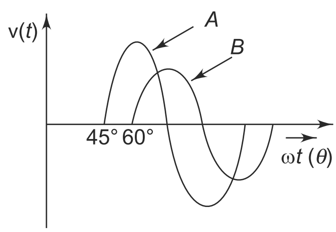


Fig. 2.10

Example 2.9 *One sine wave has a positive peak at 75°, and another has a positive peak at 100°. How much is each sine wave shifted in phase from the 0° reference? What is the phase angle between them?*

Solution The two signals are drawn as shown in Fig. 2.11.

The signal *A* leads the reference signal by 15°.

The signal *B* lags behind the reference signal by 10°.

The phase angle between these two signals is 25°.

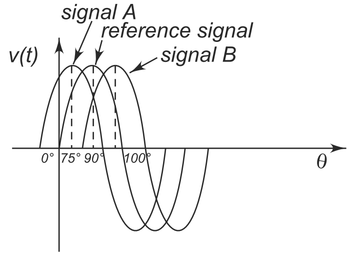


Fig. 2.11

2.1.3 The Sinusoidal Wave Equation

A sine wave is graphically represented as shown in Fig. 2.12(a). The amplitude of a sine wave is represented on vertical axis. The angular measurement (in degrees or radians) is represented on horizontal axis. Amplitude A is the maximum value of the voltage or current on the Y-axis.

In general, the sine wave is represented by the equation

$$v(t) = V_m \sin \omega t$$

The above equation states that any point on the sine wave represented by an instantaneous value $v(t)$ is equal to the maximum value times the sine of the angular frequency at that point. For example, if a certain sine wave voltage has peak value of 20 V, the instantaneous voltage at a point $\pi/4$ radians along the horizontal axis can be calculated as

$$v(t) = V_m \sin \omega t$$

$$= 20 \sin\left(\frac{\pi}{4}\right) = 20 \times 0.707 = 14.14\,\text{V}$$

When a sine wave is shifted to the left of the reference wave by a certain angle ϕ, as shown in Fig. 2.12(b), the general expression can be written as

$$v(t) = V_m \sin(\omega t + \phi)$$

When a sine wave is shifted to the right of the reference wave by a certain angle ϕ, as shown in Fig. 2.12(c), the general expression is

$$v(t) = V_m \sin(\omega t - \phi)$$

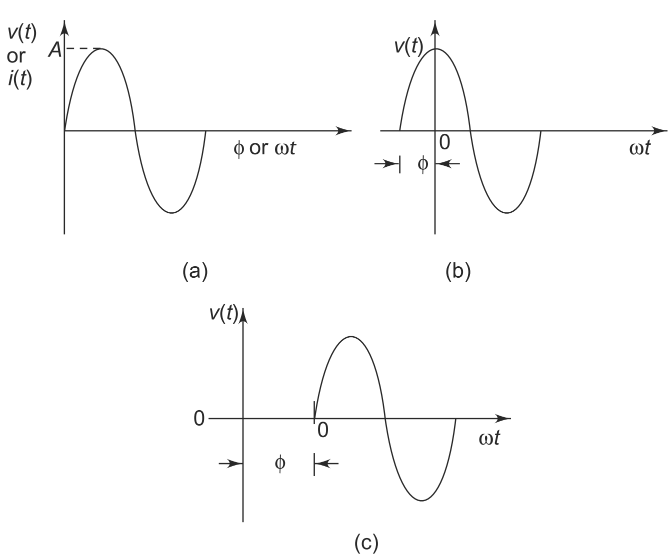


Fig. 2.12

Example 2.10 *Determine the instantaneous value at the 90° point on the X-axis for each sine wave shown in Fig. 2.13.*

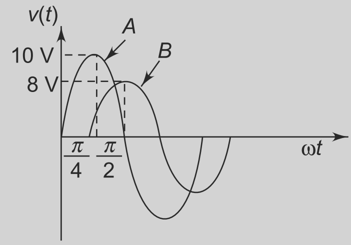


Fig. 2.13

Solution From Fig. 2.13, the equation for the sine wave *A*

$$v(t) = 10 \sin \omega t$$

The value at π/2 in this wave is

$$v(t) = 10 \sin \frac{\pi}{2} = 10\,\text{V}$$

The equation for the sine wave *B*

$$v(t) = 8 \sin(\omega t - \pi/4)$$

At $\omega t = \pi/2$

$$v(t) = 8 \sin\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= 8 \sin 45^\circ$$

$$= 8(0.707)$$

$$= 5.66\,\text{V}$$

As the magnitude of the waveform is not constant, the waveform can be measured in different ways. These are instantaneous, peak, peak to peak, root mean square (rms) and average values.

2.1.4 Instantaneous Value

Consider the sine wave shown in Fig. 2.14. At any given time, it has some instantaneous value. This value is different at different points along the waveform.

In Fig. 2.14 during the positive cycle, the instantaneous values are positive and during the negative cycle, the instantaneous values are negative. In Fig. 2.14 shown at time 1 ms, the value is 4.2 V; the value is 10 V at 2.5 ms, –2 V at 6 ms and –10 V at 7.5 and so on.

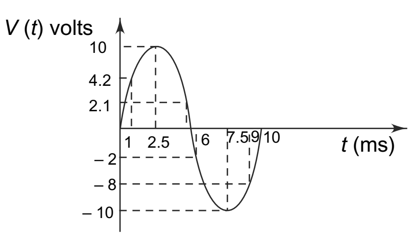


Fig. 2.14

2.1.5 Peak Value

The peak value of the sine wave is the maximum value of the wave during positive half cycle, or maximum value

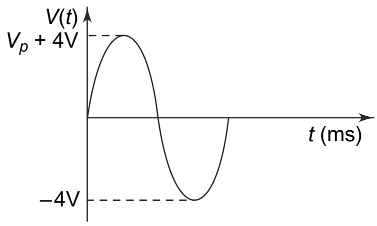


Fig. 2.15

of wave during negative half cycle. Since the value of these two are equal in magnitude, a sine wave is characterised by a single peak value. The peak value of the sine wave is shown in Fig. 2.15; here the peak value of the sine wave is 4 V.

2.1.6 Peak to Peak Value

The peak to peak value of a sine wave is the value from the positive to the negative peak as shown in Fig. 2.16. Here the peak to peak value is 8 V.

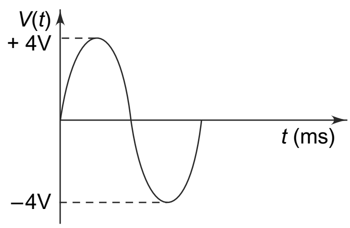


Fig. 2.16

2.1.7 Average Value

[JNTU May/June 2006, Jan 2010, Nov 2011]

In general, the average value of any function $v(t)$, with period T is given by

$$v_{av} = \frac{1}{T}\int_0^T v(t)dt$$

That means that the average value of a curve in the X-Y plane is the total area under the complete curve divided by the distance of the curve. The average value of a sine wave over one complete cycle is always zero. So the average value of a sine wave is defined over a half-cycle, and not a full cycle period.

The average value of the sine wave is the total area under the half-cycle curve divided by the distance of the curve.

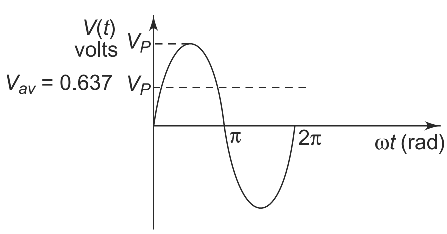


Fig. 2.17

The average value of the sine wave $v(t) = V_P \sin \omega t$ is given by

$$v_{av} = \frac{1}{\pi}\int_0^{\pi} V_P \sin \omega t \; d(\omega t)$$

$$= \frac{1}{\pi}\left[-V_P \cos \omega t\right]_0^{\pi}$$

$$= \frac{2V_P}{\pi} = 0.637\, V_P$$

The average value of a sine wave is shown by the dotted line in Fig. 2.17.

Example 2.11 *Find the average value of a cosine wave f(t) = cos ωt shown in Fig. 2.18.*

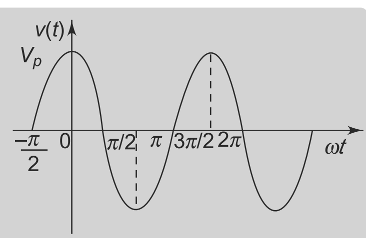


Fig. 2.18

Solution The average value of a cosine wave

$$v(t) = V_P \cos \omega t$$

$$V_{av} = \frac{1}{\pi}\int_{\pi/2}^{3\pi/2} V_P \cos \omega t \; d(\omega t)$$

$$= \frac{1}{\pi} V_P (-\sin \omega t)_{\pi/2}^{3\pi/2}$$

$$= \frac{-V_P}{\pi}\left[-1-1\right] = \frac{2V_P}{\pi} = 0.637\ V_P$$

2.1.8 Root Mean Square Value or Effective Value *[JNTU May/June 2006, Jan 2010, Nov 2011]*

The root mean square (rms) value of a sine wave is a measure of the heating effect of the wave. When a resistor is connected across a dc voltage source as shown in Fig. 2.19(a), a certain amount of heat is produced in the resistor in a given time. A similar resistor is connected across an ac voltage source for the same time as shown in Fig. 2.19(b). The value of the ac voltage is adjusted such that the same amount of heat is produced in the resistor as in the case of the dc source. This value is called the rms value.

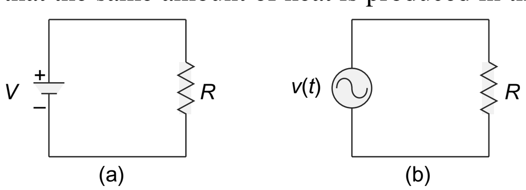


Fig. 2.19

That means the rms value of a sine wave is equal to the dc voltage that produces the same heating effect. In general, the rms value of any function with period T has an effective value given by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T \overline{v(t)}^2\, dt}$$

Consider a function $v(t) = V_P \sin \omega t$

$$\text{The rms value, } V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_P \sin \omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{T}\int_0^{2\pi} V_P^2 \left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)}$$

$$= \frac{V_P}{\sqrt{2}} = 0.707\ V_P$$

If the function consists of a number of sinusoidal terms, that is

$$v(t) = V_0 + (V_{c1} \cos\ \omega t + V_{c2} \cos 2\ \omega t + \cdots) + (V_{s1} \sin \omega t + V_{s2} \sin 2\ \omega t + \cdots)$$

The rms, or effective value is given by

$$V_{\text{rms}} = \sqrt{V_0^2 + \frac{1}{2}(V_{c1}^2 + V_{c2}^2 + \cdots) + \frac{1}{2}(V_{s1}^2 + V_{s2}^2 + \cdots)}$$

Example 2.12 *A wire is carrying a direct current of 20 A and a sinusoidal alternating current of peak value 20 A. Find the rms value of the resultant current in the wire.*

Solution The rms value of the combined wave

$$= \sqrt{20^2 + \frac{20^2}{2}}$$

$$= \sqrt{400 + 200} = \sqrt{600} = 24.5\,\text{A}$$

Example 2.13 *Find the RMS value of the voltage wave whose equation v(t) = 10 + 200 sin (wt − 30°) + 100 cos 3 wt − 50 sin (5wt + 60°).*

Solution

$$V_{\text{rms}} = \sqrt{10^2 + \frac{(200)^2}{2} + \frac{(100)^2}{2} + \frac{(50)^2}{2}}$$

$$= \sqrt{100 + 20000 + 5000 + 1250}$$

$$= 162.327\,\text{V}$$

Example 2.14 *Find R.M.S. and average value of the following waveform.*

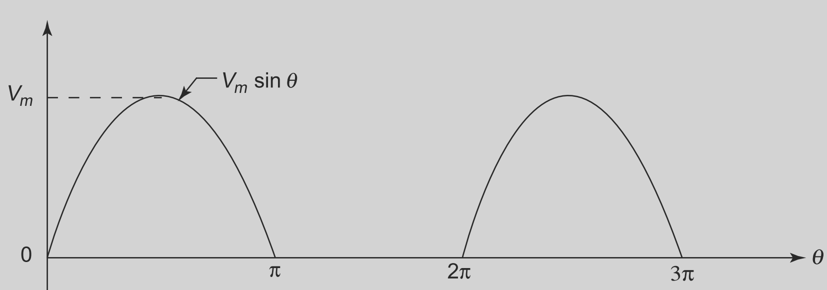


Fig. 2.20

Solution

R.M.S. value, $$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\frac{(1-\cos 2\theta)}{2}\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \frac{V_m}{2}$$

Average value, $V_{\text{ave}} = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta \, d\theta = \frac{V_m}{2\pi}[-\cos\theta]_0^{2\pi} = \frac{V_m}{\pi}$

Example 2.15 *For the trapezoidal waveform shown in the Fig. 2.21, determine average value.* *[JNTU Jan 2010]*

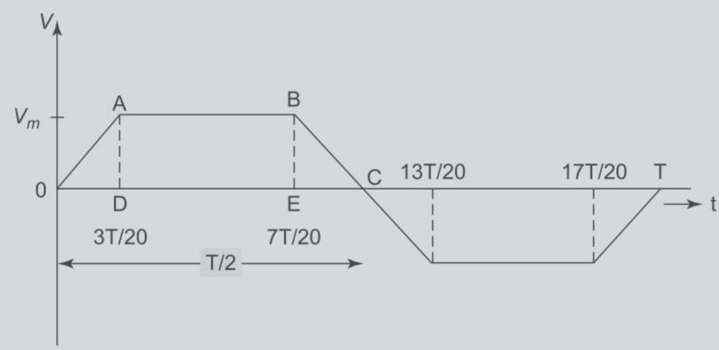


Fig. 2.21

Solution

$$V_{AVG} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{T}\left[\int_0^{3T/20}\frac{20V_m \cdot t}{3T}dt + \int_{3T/20}^{7T/20} V_m \cdot dt + \int_{7T/20}^{13T/20} V_m - \frac{20V_m}{3T}\left(t - \frac{7T}{20}\right)dt\right.$$

$$\left. - \int_{13T/20}^{17T/20} V_m \; dt + \int_{17T/20}^{T}\frac{20V_m}{3T}(t-T)dt\right]$$

$$= \frac{1}{T}\left[\left(\frac{20V_m}{3T\times 2\times 20\times 20}9T^2\right) + \left(V_m \times \frac{4T}{20}\right) + \left(\frac{10V_m}{3}\times\frac{6T}{20}\right) - \frac{20V_m}{3T\times 2}\left(\frac{13T}{20}\right)^2 - \left(\frac{7T}{20}\right)\right.$$

$$\left. - \left(V_m \times \frac{4T}{20}\right) - \left(\frac{20V_m}{30}\times\frac{3T}{20}\right) + \frac{20V_m}{3T\times 2}\left(T^2 - \left(\frac{17T}{20}\right)^2\right)\right]$$

$$= \frac{20V_m T^2}{T\times 3T\times 2}[0.0225 - 0.0225] = 0$$

2.1.9 Peak Factor

The peak factor of any waveform is defined as the ratio of the peak value of the wave to the rms value of the wave.

$$\text{Peak factor } = \frac{V_P}{V_{\text{rms}}}$$

$$\text{Peak factor of the sinusoidal waveform } = \frac{V_P}{V_P/\sqrt{2}} = \sqrt{2} = 1.414$$

2.1.10 Form Factor

[JNTU May/June 2006, Nov 2011]

Form factor of a waveform is defined as the ratio of rms value to the average value of the wave.

$$\text{Form factor } = \frac{V_{\text{rms}}}{V_{\text{av}}}$$

Form factor of a sinusoidal waveform can be found from the above relation.

$$\text{For the sinusoidal wave, the form factor } = \frac{V_P/\sqrt{2}}{0.637 V_P} = 1.11$$

Example 2.16 *Find the form factor for the following waveform.*

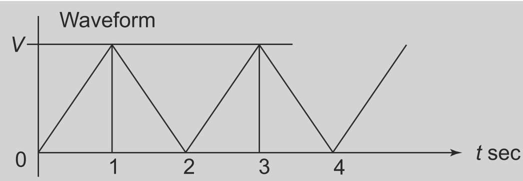


Fig. 2.22

Solution

$$\text{Form factor} = \frac{\text{R.M.S. value}}{\text{Average value}}$$

Average value of the triangular waveform 0 to 2 sec

$$V_{\text{av}} = \frac{1}{2}\left[\int_0^1 V.t\, dt + \int_1^2 -V(t-2)\, dt\right]$$

$$= \frac{1}{2}\left[V\frac{t^2}{2}\Big|_0^1 + -V\frac{t^2}{2}\Big|_1^2 + 2V.t\Big|_1^2\right]$$

$$= \frac{1}{2}\left[\frac{V}{2} - \frac{3}{2}V + 2V\right] = \frac{V}{2}$$

$$\text{R.M.S. value, } (V_{\text{rms}}) = \left[\frac{1}{2}\int_0^1 V^2 t^2 dt + \int_1^2 V^2 (t-2)^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{2} \left\{ V^2 \frac{t^3}{3} \Big|_0^1 + V^2 \frac{t^3}{3} \Big| + 4V^2 t \Big|_1^2 - 4V^2 \frac{t^2}{2} \Big|_1^2 \right\} \right]^{1/2}$$

$$= \left[\frac{1}{2} \left\{ \frac{V^2}{3} - \frac{7V^2}{3} - 2V^2 \right\} \right]^{1/2}$$

$$= \left[\frac{1}{2} \left\{ \frac{8V^2 - 6V^2}{3} \right\} \right]^{1/2} = \frac{V}{\sqrt{3}}$$

Form factor = $V/\sqrt{3} / V/2 = \frac{2}{\sqrt{3}} = 1.155$

Example 2.17 *A sinusoidal current wave is given by i = 50 sinl00 πt. Determine*

(a) The greatest rate of change of current.
(b) Derive average and rms values of current.
(c) The time interval between a maximum value and the next zero value of current. *[JNTU Jan 2010]*

Solution (a) $i = 50 \sin 100\, \pi\, t$

$\therefore$ $\sin \omega t \therefore \omega = 2\pi f$

$\therefore$ $2\pi f = 100\,\pi$

$$f = \frac{100\pi}{2\pi} = 50\,\text{Hz}$$

$$\therefore \quad \frac{di}{dt} = 50 \times 100\pi \cos 100\pi t = 5000\pi \cos 100\pi t$$

$$\therefore \quad \left(\frac{di}{dt}\right)_{\max} = 5000\pi$$

(b) Average

$$I_{av} = \frac{2I_m}{\pi} = \frac{2 \times 50}{3.142} = 31.826\,\text{A}$$

rms $$I = \frac{I_m}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.35\,\text{A}$$

(c) Time Interval

$$t = \frac{1}{f} = \frac{1}{50} = 0.023 = 20\ \text{ms}$$

$\therefore$ Time Interval $\Rightarrow \frac{20}{4} = 5\,\text{ms}$

Example 2.18 *A sine wave has a peak value of 25 V. Determine the following values.*

(a) rms *(b) peak to peak* *(c) average*

Solution (a) rms value of the sine wave

$$V_{rms} = 0.707\ V_P = 0.707 \times 25 = 17.68\text{ V}$$

(b) peak to peak value of the sine wave $V_{PP} = 2V_P$

$$V_{PP} = 2 \times 25 = 50\text{ V}$$

(c) average value of the sine wave

$$V_{av} = 0.637\ V_P = (0.637)25 = 15.93\text{ V}$$

Example 2.19 *A sine wave has a peak value of 12 V. Determine the following values.*

(a) rms *(b) average* *(c) crest factor* *(d) form factor*

Solution (a) rms value of the given sine wave

$$= (0.707)12 = 8.48\text{ V}$$

(b) average value of the sine wave $= (0.637)12 = 7.64$ V

(c) crest factor of the sine wave $= \dfrac{\text{Peak value}}{\text{rms value}}$

$$= \frac{12}{8.48} = 1.415$$

(d) Form factor $= \dfrac{\text{rms value}}{\text{average value}} = \dfrac{8.48}{7.64} = 1.11$

Example 2.20 *Find the form factor of the half-wave rectified sine wave shown in Fig. 2.23.*

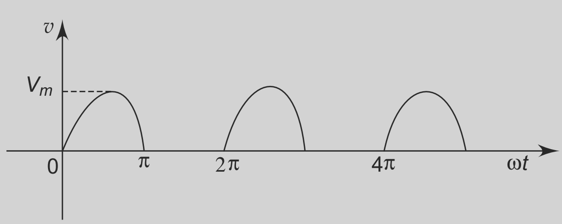


Fig. 2.23

Solution $v = V_m \sin \omega t$, for $0 < \omega t < \pi$

$= 0$, for $\pi < \omega t < 2\pi$

the period is 2π.

$$V_{av} = \frac{1}{2\pi}\left\{\int_0^{\pi} V_m \sin\omega t\, d(\omega t) + \int_{\pi}^{2\pi} 0\, d(\omega t)\right\}$$

Average value $= 0.318\, V_m$

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{\pi} (V_m \sin\omega t)^2\, d(\omega t)$$

$$= \frac{1}{4} V_m^2$$

$$V_{rms} = \frac{1}{2} V_m$$

Form factor $= \dfrac{V_{rms}}{V_{av}} = \dfrac{0.5\, V_m}{0.318\, V_m} = 1.572$

Example 2.21 *Find the average and effective values of the saw tooth wave-form shown in Fig. 2.24 below.*

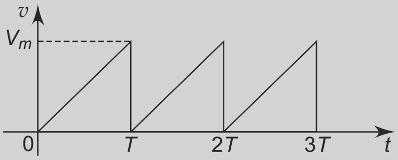


Fig. 2.24

Solution From Fig. 2.24 shown, the period is T.

$$V_{av} = \frac{1}{T}\int_0^{T} \frac{V_m}{T} t\, dt$$

$$= \frac{1}{T}\frac{V_m}{T}\int_0^{T} t\, dt$$

$$= \frac{V_m}{T^2}\frac{T^2}{2} = \frac{V_m}{2}$$

Effective value $V_{rms} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^{T} v^2\, dt}$

$$= \sqrt{\frac{1}{T}\int_0^{T}\left[\frac{V_m}{T} t\right]^2 dt}$$

$$= \frac{V_m}{\sqrt{3}}$$

Example 2.22 *Find the average and rms value of the full wave rectified sine wave shown in Fig. 2.25.*

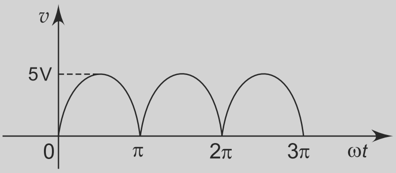


Fig. 2.25

Solution Average value

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} 5\sin\omega t\, d(\omega t)$$

$$= 3.185$$

Effective value or rms value

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi}(5\sin\omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{25}{2}} = 3.54$$

Example 2.23 *The full wave rectified sine wave shown in Fig. 2.26 has a delay angle of 60°. Calculate V_{av} and V_{rms}.*

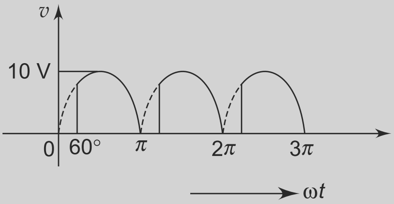


Fig. 2.26

Solution Average value

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} 10\sin(\omega t)\, d(\omega t)$$

$$= \frac{1}{\pi}\int_{60^\circ}^{\pi} 10\sin\omega t\, d(\omega t)$$

$$V_{av} = \frac{10}{\pi}(-\cos\omega t)_{60}^{\pi} = 4.78$$

Effective value

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{60^\circ}^{\pi}(10\sin\omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{100}{\pi}\int_{60^\circ}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)}$$

$$= 6.33$$

Example 2.24 *Find the form factor of the square wave as shown in Fig. 2.27.*

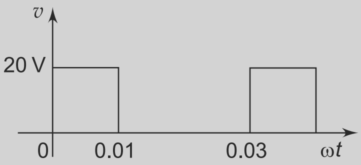


Fig. 2.27

Solution $v = 20$ for $0 < t < 0.01$

$= 0$ for $0.01 < t < 0.03$

The period is 0.03 sec.

Average value $$V_{av} = \frac{1}{0.03}\int_0^{0.01} 20\,dt$$

$$= \frac{20(0.01)}{0.03} = 6.66$$

Effective value $$V_{eff} = \sqrt{\frac{1}{0.03}\int_0^{0.01} (20)^2\,dt} = 66.6 = 0.816$$

Form factor $$= \frac{0.816}{6.66} = 0.123$$

Example 2.25 *Find the form factor of the following waveform shown in Fig. 2.28.*
[JNTU April/May 2007]

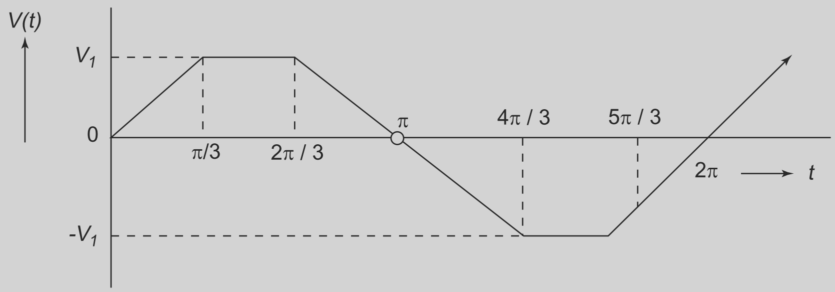


Fig. 2.28

Solution From 0 to π/3, $V = \frac{3V_1}{\pi}t$

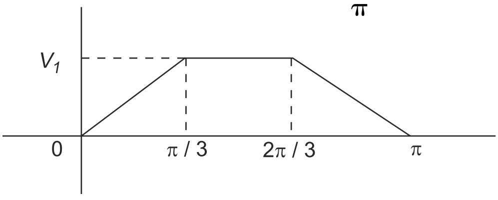


Fig. 2.29

From π/3 to 2π/3 $V = V_1$

From 2π/3 to π

$$V = 3V_1 - \frac{3V_1}{\pi}t$$

Form factor $= \dfrac{V_{rms}}{V_{avg}}$

$$V_{avg} = \frac{1}{T}\int_0^T V(t)\ dt$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/3} \frac{3V_1}{\pi}\, t\ dt + \int_{\pi/3}^{2\pi/3} V_1\ dt + \int_{2\pi/3}^{\pi} 3V_1 - \frac{3V_1}{\pi}\, t\ dt\right]$$

$$= \frac{1}{\pi}\left[\frac{3V_1}{\pi}\cdot\left(\frac{\pi}{3}\right)\frac{1}{2} + V_1\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) + 3V_1\left(\pi - \frac{2\pi}{3}\right) - \frac{3V_1}{\pi}\cdot\frac{1}{2}\left[\pi^2 - \frac{4\pi^2}{9}\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{V_1}{6}\cdot\pi + \frac{V_1}{3}\pi + V_1\cdot\pi - \frac{5}{6}V_1\right] = \frac{2}{3}V_1$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T [V(t)]^2\ dt}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/3}\left(\frac{3V_1}{\pi}t\right)^2 dt + \int_{\pi/3}^{2\pi/3}(V_1)^2\ dt + \int_{2\pi/3}^{\pi}\left(3V_1 - \frac{3V_1}{\pi}t\right)^2 dt\right]}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/3}\frac{9V_1^2}{\pi^2}t^2\ dt + \int_{\pi/3}^{2\pi/3}V_1^2\ dt + \int_{2\pi/3}^{\pi} 9V_1^2 + \frac{9V_1^2}{\pi^2}t^2 - \frac{18V_1}{\pi}t\ dt\right]}$$

$$= Sqrt\left\{\frac{1}{\pi}\left[\frac{9V_1^2}{\pi^2}\cdot\frac{1}{3}\cdot\left(\frac{\pi}{3}\right)^3 + V_1^2\left(\frac{2\pi}{3} - \frac{\pi}{3}\right) + 9V_1^2\left(\pi - \frac{2\pi}{3}\right)\right.\right.$$

$$\left.\left. + \frac{9V_1^2}{\pi^2}\cdot\frac{1}{3}\cdot\left(\pi^3 - \frac{8\pi^3}{27}\right) - \frac{18V_1}{\pi}\cdot\frac{1}{2}\left(\pi^2 - \frac{4\pi^2}{9}\right)\right]\right\}$$

$$= \sqrt{\frac{1}{\pi}\left[\frac{9V_1^2}{\pi^2}\cdot\frac{1}{3}\cdot\frac{\pi^3}{27} + V_1^2\cdot\frac{\pi}{3} + 9V_1^2\cdot\frac{\pi}{3} + \frac{3V_1^2}{\pi^2}\cdot\frac{19\pi^3}{27} - \frac{9V_1^2}{\pi}\cdot\frac{5\pi^2}{9}\right]}$$

$$= \sqrt{\frac{1}{\pi}\left[\frac{\pi}{9}V_1^2 + \frac{\pi}{3}V_1^2 + 3\pi V_1^2 + \frac{19}{9}\pi V_1^2 - 5\pi V_1^2\right]}$$

$$= \sqrt{\frac{5}{9}V_1^2} = \frac{\sqrt{5}}{3}V_1$$

$$\text{Form factor } = \frac{V_{rms}}{V_{avg}}$$

$$= \frac{\frac{\sqrt{5}}{3}V_1}{\frac{2}{3}V_1} = \frac{\sqrt{5}}{2} = 1.12$$

Example 2.26 *Find the average and effective values of a full wave rectified sine wave shown in Fig. 2.30.*

[JNTU May/June 2002]

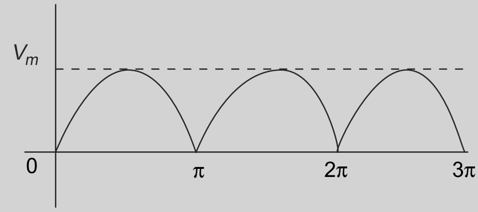


Fig. 2.30

Solution Average value

$$V_{avg} = \frac{1}{T}\int_0^T V_m \sin\theta \, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta \, d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{2V_m}{\pi}$$

Effective value

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \sin^2\theta d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta d\theta}$$

$$= V_m\sqrt{\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta}$$

$$= V_m\sqrt{\frac{2}{\pi}\times\frac{1}{2}\times\frac{\pi}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

Example 2.27 *Determine the RMS value of a half-wave rectified sinusoidal voltage of peak value, V_m.* *[JNTU May/June 2002]*

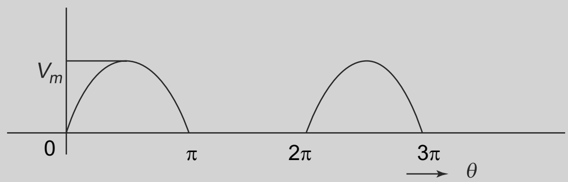


Fig. 2.31

Solution

$$\text{RMS value} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)_0^{\pi}}$$

$$= \frac{V_m^2}{2\pi} \times \frac{\pi}{2} = \frac{V_m}{2}$$

$$\therefore \quad V_{\text{rms}} = \frac{V_m}{2}$$

Example 2.28 *Find R.M.S. and average value of the following waveform.*

[JNTU May/June 2004]

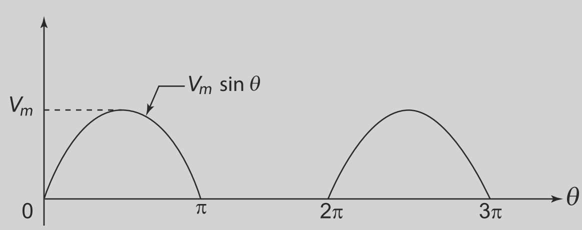


Fig. 2.32

Solution

$$\text{R.M.S. value, } V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta \; d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \frac{(1-\cos 2\theta)}{2} d\theta}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \frac{V_m}{2}$$

$$\text{Average value, } V_{\text{ave}} = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta \, d\theta$$

$$= \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{2\pi} = \frac{V_m}{\pi}$$

Example 2.29 *Derive expression for r.m.s. and average value of a sinusoidal alternating quantity.* *[JNTU May/June 2008]*

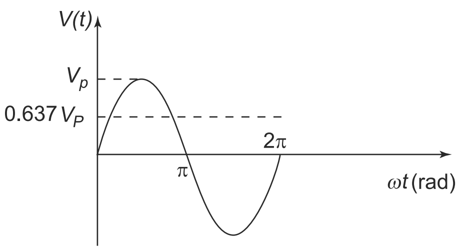


Fig. 2.33

Solution *Average value of a sine wave*

The average value of a curve in the $X-Y$ plane is the total area under the complete curve divided by the distance of the curve. The average value of a sine wave over one complete cycle is always zero. So the average value of a sine wave is defined over a half-cycle, and not a full-cycle period.

The average value of sine wave

$$V(t) = V_P \sin \omega t$$

$$V_{\text{av}} = \frac{1}{\pi}\int_0^{\pi} V_P \sin\omega t \; d(\omega t) = \frac{1}{\pi}[-V_P \; \cos\omega t]_0^{\pi} = \frac{2V_p}{\pi} = 0.637 \; V_P$$

rms value of a sine wave

The root mean square (rms) value of a sine wave is a measure of the heating effect of the wave.

R.M.S value of any waveform is determined by using

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T (V(t))^2 \; dt}$$

Let the function $V(t)$ be $V_P \sin \omega t$.

$$= \sqrt{\frac{1}{T}\int_0^T (V_p \sin\omega t)^2 \; d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_p^2 \sin^2 \omega t \; d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_p^2 \left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)} = \sqrt{\frac{1}{2\pi}\left[\tfrac{1}{2}(\omega t) - \frac{\sin 2\omega t}{4}\right]_0^{2\pi} \times V_p^2}$$

$$= \sqrt{\frac{1}{2\pi}\left[\frac{2\pi}{2} - 0\right] V_p^2}$$

$$V_{\text{rms}} = \frac{V_p}{\sqrt{2}} = 0.707 \; V_p.$$

Example 2.30 *Define*

(i) frequency,
(ii) phase,
(iii) form factor, and
(iv) peak factor. *[JNTU May/June 2008]*

Solution (i) *Frequency*: The frequency of a wave is defined as the number of cycles that a wave completes in one second. The unit of frequency is hertz.

One hertz is equivalent to one cycle per second.

(ii) *Phase*: The phase of a sine wave is an angular measurement that specifies the position of sine wave relative to reference.

When the sine wave is shifted left or right with reference to wave shown in Figure, there occurs a phase shift.

(iii) *Form Factor*: Form factor of a wave is defined as the ratio of rms value to average value of the wave.

$$\text{Form factor} = \frac{V_{RMS}}{V_{avg}}$$

$$\text{For sine wave} = \frac{V_p/\sqrt{2}}{0.637 V_p} = 1.11$$

(iv) *Peak Factor*: The peak factor of any waveform is defined as the ratio of peakvalue of the wave to the rms value of the wave

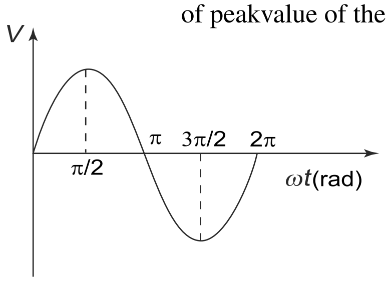


Fig. 2.34

$$\text{Peak factor} = \frac{V_p}{V_{RMS}}$$

For sine wave

$$\text{Peak factor} = \frac{V_p}{V_p/\sqrt{2}} = \sqrt{2} = 1.414$$

2.2 CONCEPT OF PHASE ANGLE AND PHASE DIFFERENCE

[JNTU Nov. 2011]

A phasor diagram can be used to represent a sine wave in terms of its magnitude and angular position. Examples of phasor diagrams are shown in Fig. 2.35.

In Fig. 2.35(a), the length of the arrow represents the magnitude of the sine wave; angle θ represents the angular position of the sine wave. In Fig. 2.35(b), the magnitude of the sine wave is one and the phase angle is 30°. In Fig. 2.35(c) and (d), the magnitudes are four and three, and phase angles are 135° and 225°, respectively. The position of a phasor at any instant can be expressed as a positive or negative angle. Positive angles are measured counterclockwise from 0°, whereas negative angles are measured clockwise from 0°. For a given positive angle θ, the corresponding negative angle is θ−360°. This is shown in Fig. 2.36(a). In Fig. 2.36(b), the positive angle 135° of vector *A* can be represented by a negative angle −225°, (135°−360°).

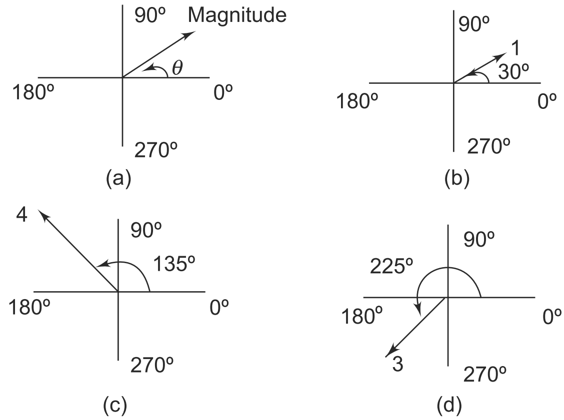


Fig. 2.35

A phasor diagram can be used to represent the relation between two or more sine waves of the same frequency. For example, the sine waves shown in Fig. 2.37(a) can be represented by the phasor diagram shown in Fig. 2.37(b).

In the above Figure, sine wave *B* lags behind sine wave *A* by 45°; sine wave *C* leads sine wave *A* by 30°. The length of the phasors can be used to represent peak, rms, or average values.

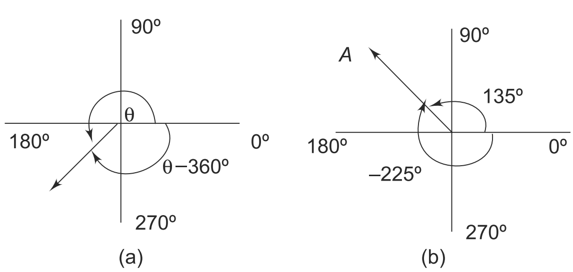


Fig. 2.36

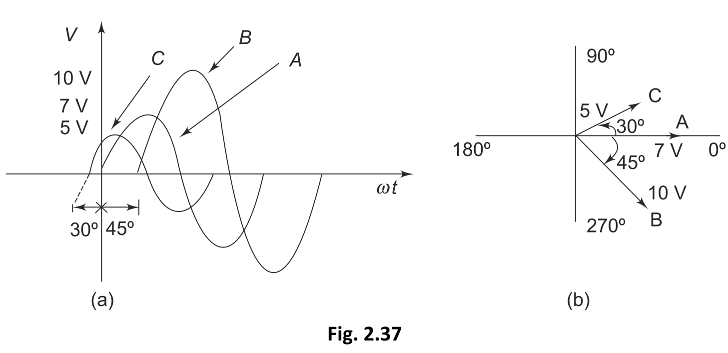


Fig. 2.37

Example 2.31 *Draw the phasor diagram to represent the two sine waves shown in Fig. 2.38.*

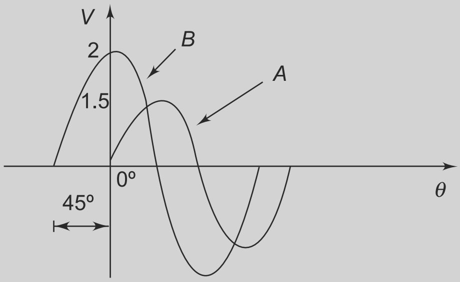


Fig. 2.38

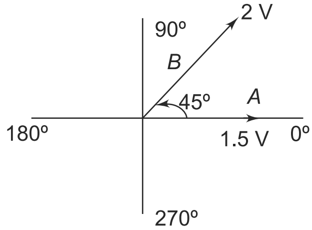


Fig. 2.39

Solution The phasor diagram representing the sine waves is shown in Fig. 2.39. The length of the each phasor represents the peak value of the sine wave

Example 2.32 *Explain the term phase difference.*

Solution The difference in phase between two waves is called phase difference. In the figure below the sine wave is shifted to the right by 90° shown by the dotted lines.

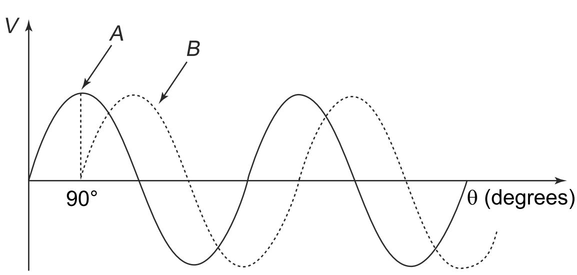


Fig. 2.40

There is a phase difference of 90° between *A* and *B*.

The waveform *B* is lagging behind waveform *A* by 90° or in other words, wave *A* is leading the waveform *B* by 90°.

2.2.1 *j*-Notation *[JNTU Nov. 2011]*

j is used in all electrical circuits to denote imaginary numbers. Alternate symbol for *j* is $\sqrt{-1}$, and is known as *j* factor or *j* operator.

Thus

$$\sqrt{-1} = \sqrt{(-1)(1)} = j(1)$$
$$\sqrt{-2} = \sqrt{(-1)2} = j\sqrt{2}$$
$$\sqrt{-4} = \sqrt{(-1)4} = j2$$
$$\sqrt{-5} = \sqrt{(-1)5} = j\sqrt{5}$$

Since j is defined as $\sqrt{-1}$, it follows that $(j)(j) = j^2 = \left(\sqrt{-1}\right)\left(\sqrt{-1}\right) = -1$

$\therefore \quad (j3)(j3) = j^2 3^2$

Since $\quad j^2 = -1$

$$(j3)\,(j3) = -9$$

(i.e.) the square root of −9 is $j3$

Therefore $j3$ is a square root of −9

The use of j factor provides a solution to an equation of the form $x^2 = -4$

Thus $\quad x = \sqrt{-4} = \sqrt{(-1)4}$

$$x = \left(\sqrt{-1}\right)2$$

With $\quad j = \sqrt{-1}, x = j2$

The real number 9 when multiplied three times by j becomes $-j9$.

$$(j)\,(j)\,(j) = (j)^2\, j = (-1)j = -j$$

Finally when real number 10 is multiplied four times by j, it becomes 10

$$j = +j$$
$$j^2 = (j)\,(j) = -1$$
$$j^3 = (j^2)\,(j) = (-1)j = -j$$
$$j^4 = (j^2)\,(j)^2 = (-1)\,(-1) = +1$$

Example 2.33 *Express the following imaginary numbers using the j factor*

(a) $\sqrt{-13}$ *(b)* $\sqrt{-9}$ *(c)* $\sqrt{-29}$ *(d)* $\sqrt{-49}$

Solution

(a) $\sqrt{-13} = \sqrt{(-1)(13)} = j\sqrt{13}$

(b) $\sqrt{-9} = \sqrt{(-1)9} = j3$

(c) $\sqrt{-29} = \sqrt{(-1)29} = j\sqrt{29}$

(d) $\sqrt{-49} = \sqrt{(-1)(49)} = j7$

2.3 COMPLEX AND POLAR FORMS OF REPRESENTATION

A complex number $(a + jb)$ can be represented by a point whose coordinates are (a, b).

Thus, the complex number $3 + j4$ is located on the complex plane at a point having rectangular coordinates (3, 4).

This method of representing complex numbers is known as the rectangular form. In ac analysis, impedances, currents and voltages are commonly represented by complex numbers that may be either in the rectangular form or in the polar form. In Fig. 2.41 the complex number in the polar form is represented. Here R is the magnitude of the complex number and ϕ is the angle of the complex number. Thus, the polar form of the complex number is $R \angle\phi$. If the rectangular coordinates (a, b) are known, they can be converted into polar form. Similarly, if the polar coordinates (R, ϕ) are known, they can be converted into rectangular form.

In Fig. 2.41, a and b are the horizontal and vertical components of the vector R, respectively. From Fig. 2.41, R can be found as $R = \sqrt{a^2 + b^2}$.

Also from Fig. 2.41,

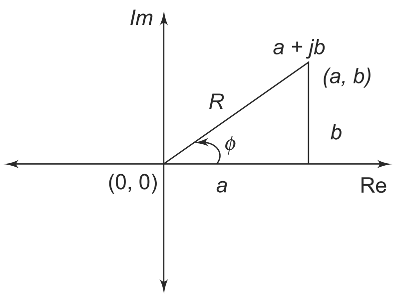


Fig. 2.41

$$\sin\phi = \frac{b}{R}$$

$$\cos\phi = \frac{a}{R}$$

$$\tan\phi = \frac{b}{a}$$

$$\phi = \tan^{-1}\frac{b}{a}$$

$$R = \sqrt{a^2 + b^2}$$

Example 2.34 *Express $10 \angle 53.1°$ in rectangular form.*

Solution

$$a + jb = R(\cos\phi + j\sin\phi)$$

$$R = 10;\ \angle\phi = \angle 53.1°$$

$$a + jb = R\cos\phi + jR\sin\phi$$

$$R\cos\phi = 10\cos 53.1° = 6$$

$$R\sin\phi = 10\sin 53.1° = 8$$

$$a + jb = 6 + j8$$

Example 2.35 *Express $3 + j4$ in polar form.*

Solution

$$R\cos\phi = 3 \quad (1)$$

$$R\sin\phi = 4 \quad (2)$$

Squaring and adding the above equations, we get

$$R^2 = 3^2 + 4^2$$

$$R = \sqrt{3^2 + 4^2} = 5$$

From (1) and (2), $\tan\phi = 4/3$

$$\phi = \tan^{-1}\frac{4}{3} = 53.13°$$

Hence the polar form is $5\ \angle 53.13°$

2.3.1 Operations with Complex Numbers

The basic operations such as addition, subtraction, multiplication and division can be performed using complex numbers.

Addition It is very easy to add two complex numbers in the rectangular form. The real parts of the two complex numbers are added and the imaginary parts of the two complex numbers are added. For example,

$$(3 + j4) + (4 + j5) = (3 + 4) + j(4 + 5) = 7 + j9$$

Subtraction Subtraction can also be performed by using the rectangular form. To subtract, the sign of the subtrahand is changed and the components are added. For example, subtract $5 + j3$ from $10 + j6$:

$$10 + j6 - 5 - j3 = 5 + j3$$

Multiplication To multiply two complex numbers, it is easy to operate in polar form. Here we multiply the magnitudes of the two numbers and add the angles algebraically. For example, when we multiply $3\ \angle 30°$ with $4\ \angle 20°$, it becomes $(3)(4)\ \angle 30° + 20° = 12\ \angle 50°$.

Division To divide two complex numbers, it is easy to operate in polar form. Here we divide the magnitudes of the two numbers and subtract the angles. For example, the division of

$$9\angle 50° \text{ by } 3\angle 15° = \frac{9\angle 50°}{3\angle 15°} = 3\angle 50° - 15° = 3\angle 35°.$$

2.3.2 Phase Relation in Pure Resistance

When a sinusoidal voltage of certain magnitude is applied to a resistor, a certain amount of sine wave current passes through it. We know the relation between $v(t)$ and $i(t)$ in the case of a resistor. The voltage/current relation in case of a resistor is linear,

i.e. $$v(t) = i(t)R$$

Consider the function

$$i(t) = I_m \sin \omega t = IM\left[I_m e^{j\omega t}\right] \text{ or } I_m\ \angle 0°$$

If we substitute this in the above equation, we have

$$v(t) = I_m R \sin \omega t = V_m \sin \omega t$$

$$= IM\left[V_m e^{j\omega t}\right] \text{ or } V_m \angle 0°$$

where $$V_m = I_m R$$

If we draw the waveform for both voltage and current as shown in Fig. 2.42, there is no phase difference between these two waveforms. The amplitudes of the waveform may differ according to the value of resistance.

As a result, in pure resistive circuits, the voltages and currents are said to be in phase. Here the term impedance is defined as the ratio of voltage to current function. With ac voltage applied to elements, the ratio of exponential voltage to the corresponding current (impedance) consists of magnitude and phase angles. Since the phase difference is zero in case of a resistor, the phase angle is zero. The impedance in case of resistor consists only of magnitude, i.e.

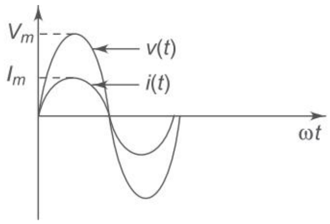


Fig. 2.42

$$Z = \frac{V_m \angle 0°}{I_m \angle 0°} = R$$

Example 2.36 *A sinusoidal voltage is applied to the resistive circuit shown in Fig. 2.43. Determine the following values.*

(a) I_{rms} (b) I_{av} (c) I_P (d) I_{PP}

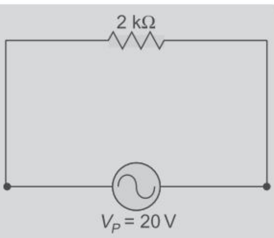


Fig. 2.43

Solution The function given to the circuit shown is

$$v(t) = V_P \sin \omega t = 20 \sin \omega t$$

The current passing through the resistor

$$i(t) = \frac{v(t)}{R}$$

$$i(t) = \frac{20}{2 \times 10^3} \sin \omega t$$

$$= 10 \times 10^{-3} \sin \omega t$$

$$I_p = 10 \times 10^{-3} \text{ A}$$

The peak value $I_P = 10$ mA

Peak to peak value $I_{PP} = 20$ mA

rms value $I_{rms} = 0.707\, I_P$

$= 0.707 \times 10 \text{ mA} = 7.07$ mA

Average value $I_{av} = (0.637)\, I_P$

$= 0.637 \times 10 \text{ mA} = 6.37$ mA

2.3.3 Phase Relation in a Pure Inductor

As discussed earlier in Chapter 1, the voltage current relation in the case of an inductor is given by

$$v(t) = L\frac{di}{dt}$$

Consider the function $i(t) = I_m \sin\omega t = IM\left[I_m e^{j\omega t}\right]$ or $I_m\angle 0°$

$$v(t) = L\frac{d}{dt}(I_m \sin\omega t)$$

$$= L\omega I_m \cos\omega t = \omega L I_m \cos\omega t$$

$$v(t) = V_m \cos\omega t, \quad \text{or} \quad V_m \sin(\omega t + 90°)$$

$$= IM\left[V_m e^{j(\omega t + 90°)}\right] \quad \text{or} \quad V_m\angle 90°$$

where $V_m = \omega L I_m = X_L I_m$

and $e^{j90°} = j = 1\angle 90°$

If we draw the waveforms for both, voltage and current, as shown in Fig. 2.43, we can observe the phase difference between these two waveforms.

As a result, in a pure inductor the voltage and current are out of phase. The current lags behind the voltage by 90° in a pure inductor as shown in Fig. 2.44.

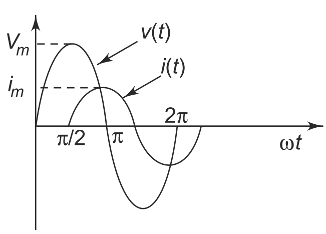


Fig. 2.44

The impedance which is the ratio of exponential voltage to the corresponding current, is given by

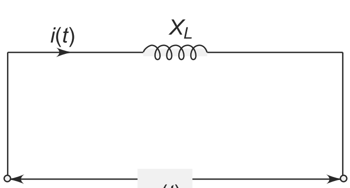


Fig. 2.45

$$Z = \frac{V_m \sin(\omega t + 90°)}{I_m \sin\omega t}$$

where $V_m = \omega L I_m$

$$= \frac{I_m \omega L \sin(\omega t + 90°)}{I_m \sin\omega t}$$

$$= \frac{\omega L I_m \angle 90°}{I_m \angle 0°}$$

$\therefore$ $Z = j\omega L = jX_L$

where $X_L = \omega L$ and is called the inductive reactance.

Hence, a pure inductor has an impedance whose value is ωL.

Example 2.37 *A sinusoidal voltage is applied to the circuit shown in Fig. 2.46. The frequency is 3 kHz. Determine the inductive reactance.*

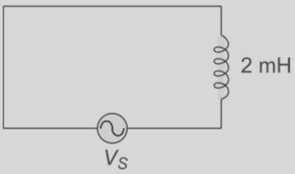


Fig. 2.46

Solution $X_L = 2\pi fL$

$$= 2\pi \times 3 \times 10^3 \times 2 \times 10^{-3}$$

$$= 37.69\ \Omega$$

Example 2.38 *Determine the rms current in the circuit shown in Fig. 2.47.*

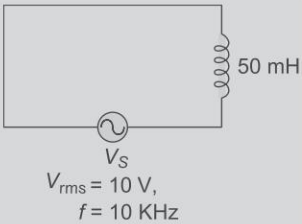


Fig. 2.47

Solution

$$X_L = 2\pi fL$$

$$= 2\pi \times 10 \times 10^3 \times 50 \times 10^{-3}$$

$$X_L = 3.141\ \text{k}\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L}$$

$$= \frac{10}{3.141 \times 10^3} = 3.18\ \text{mA}$$

2.3.4 Phase Relation in Pure Capacitor

As discussed in Chapter 1, the relation between voltage and current is given by

$$v(t) = \frac{1}{C}\int i(t)\, dt$$

Consider the function $i(t) = I_m \sin \omega t = IM\left[I_m e^{j\omega t}\right]$ or $I_m \angle 0°$

$$v(t) = \frac{1}{C}\int I_m \sin \omega t\, d(t)$$

$$= \frac{1}{\omega C} I_m[-\cos \omega t]$$

$$= \frac{I_m}{\omega C} \sin(\omega t - 90^\circ)$$

$$\therefore \quad v(t) = V_m \sin(\omega t - 90^\circ)$$

$$= IM\left[I_m e^{j(\omega t - 90^\circ)}\right] \text{ or } V_m \angle -90^\circ$$

where $V_m = \dfrac{I_m}{\omega C}$

$$\therefore \quad \frac{V_m \angle -90^\circ}{I_m \angle 0^\circ} = Z = \frac{-j}{\omega C}$$

Hence, the impedance is $Z = \dfrac{-j}{\omega C} = -jX_C$

where $X_C = \dfrac{1}{\omega C}$ and is called the capacitive reactance.

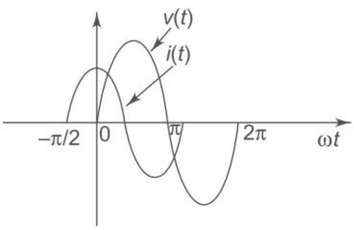


Fig. 2.48

If we draw the waveform for both, voltage and current, as shown in Fig. 2.48, there is a phase difference between these two waveforms.

As a result, in a pure capacitor, the current leads the voltage by 90°. The impedance value of a pure capacitor

$$X_C = \frac{1}{\omega C}$$

Example 2.39 *A sinusoidal voltage is applied to a capacitor as shown in Fig. 2.49. The frequency of the sine wave is 2 kHz. Determine the capacitive reactance.*

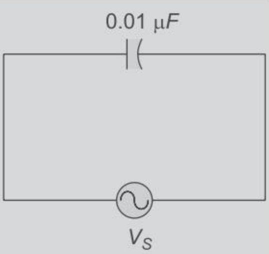


Fig. 2.49

Solution

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 2 \times 10^3 \times 0.01 \times 10^{-6}}$$

$$= 7.96 \text{ k}\Omega$$

Example 2.40 *Determine the rms current in the circuit shown in Fig. 2.50.*

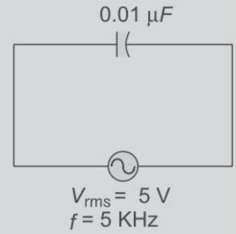


Fig. 2.50

Solution

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 5 \times 10^3 \times 0.01 \times 10^{-6}}$$

$$= 3.18\ \text{k}\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{5}{3.18\ \text{K}} = 1.57\ \text{mA}$$

2.3.5 Concept of Impedance, Reactance, Susceptance and Admittance

So far our discussion has been confined to resistive circuits. Resistance restricts the flow of current by opposing free electron movement. Each element has some resistance; for example, an inductor has some resistance; a capacitance also has some resistance. In the resistive element, there is no phase difference between the voltage and the current. In the case of pure inductance, the current lags behind the voltage by 90 degrees, whereas in the case of pure capacitance, the current leads the voltage by 90 degrees. Almost all electric circuits offer impedance to the flow of current. Impedance is a complex quantity having real and imaginary parts; where the real part is the resistance and the imaginary part is the reactance of the circuit.

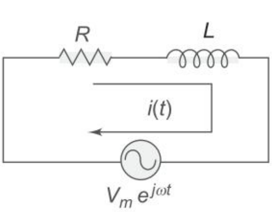


Fig. 2.51

Consider the *RL* series circuit shown in Fig. 2.51. If we apply the real function $V_m \cos \omega t$ to the circuit, the response may be $I_m \cos \omega t$. Similarly, if we apply the imaginary function $jV_m \sin \omega t$ to the same circuit, the response is $jI_m \sin \omega t$. If we apply a complex function, which is a combination of real and imaginary functions, we will get a complex response.

This complex function is $V_m\ e^{j\omega t} = V_m\ (\cos \omega t + j \sin \omega t)$. Applying Kirchhoff's law to the circuit shown in Fig. 2.51,

we get $$V_m\, e^{j\omega t} = Ri(t) + L\frac{di}{dt}$$

The solution of this differential equation is

$$i(t) = I_m e^{j\omega t}$$

By substituting $i(t)$ in the above equation, we get

$$V_m e^{j\omega t} = R I_m e^{j\omega t} + L\frac{d}{dt}(I_m e^{j\omega t})$$

$$V_m e^{j\omega t} = RI_m e^{j\omega t} + LI_m j\omega e^{j\omega t}$$

$$V_m = (R + j\omega L)I_m$$

Impedance is defined as the ratio of the voltage to current function

$$Z = \frac{V_m e^{j\omega t}}{\frac{V_m}{R + j\omega L} e^{j\omega t}} = R + j\omega L$$

Complex impedance is the total opposition offered by the circuit elements to ac current, and can be displayed on the complex plane. The impedance is denoted by Z. Here the resistance R is the real part of the impedance, and the reactance X_L is the imaginary part of the impedance. The resistance R is located on the real axis. The inductive reactance X_L is located on the positive j axis. The resultant of R and X_L is called the complex impedance.

Figure 2.52 is called the impedance diagram for the RL circuit. From Fig. 2.52, the impedance $Z = \sqrt{R^2 + (\omega L)^2}$, and angle $\theta = \tan^{-1} \omega L/R$. Here, the impedance is the vector sum of the resistance and inductive reactance. The angle between impedance and resistance is the phase angle between the current and voltage applied to the circuit.

Similarly, if we consider the RC series circuit, and apply the complex function $V_m e^{j\omega t}$ to the circuit in Fig. 2.53, we get a complex response as follows.

Applying Kirchhoff's law to the above circuit, we get

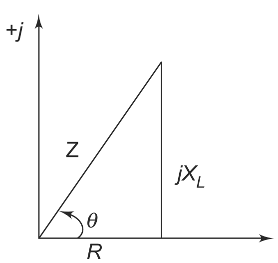


Fig. 2.52

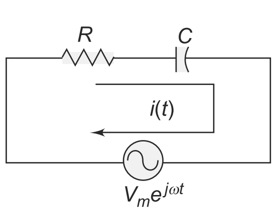


Fig. 2.53

$$V_m e^{j\omega t} = Ri(t) + \frac{1}{C}\int i(t)\,dt$$

Solving this equation we get,

$$i(t) = I_m e^{j\omega t}$$

$$V_m e^{j\omega t} = R I_m e^{j\omega t} + \frac{1}{C} I_m \left(\frac{+1}{j\omega}\right) e^{j\omega t}$$

$$= \left[RI_m - \frac{j}{\omega C} I_m\right] e^{j\omega t}$$

$$V_m = \left(R - \frac{j}{\omega C}\right) I_m$$

The impedance

$$Z = \frac{V_m e^{j\omega t}}{V_m / [R - j/\omega C] e^{j\omega t}}$$

$$= [R - (j/\omega C)]$$

Here impedance Z consists of resistance (R), which is the real part, and capacitive reactance ($X_C = 1/\omega C$), which is the imaginary part of the impedance. The resistance, R, is located on the real axis, and the capacitive reactance X_C is located on the negative j axis in the impedance diagram in Fig. 2.54.

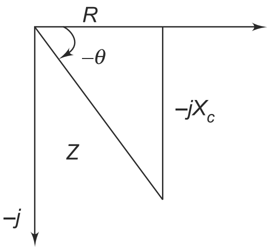


Fig. 2.54

From Fig. 2.54, impedance $Z = \sqrt{R^2 + X_C^2}$ or $\sqrt{R^2 + (1/\omega C)^2}$ and angle $\theta = \tan^{-1}(1/\omega CR)$. Here, the impedance, Z, is the vector sum of resistance and capacitive reactance. The angle between resistance and impedance is the phase angle between the applied voltage and current in the circuit.

The impedance, Z is composed of real and imaginary parts

$$Z = R + jX$$

where R is the resistance, measured in ohms

X is the reactance, measured in ohms

The admittance (Y) is the inverse of the impedance (Z).

$$Y = Z^{-1} = \frac{1}{Z}$$

where Y is the admittance, measured in Siemens.

Admittance is a measure of how easily a circuit will allow a current to flow.

$$Y = Z^{-1} = \frac{1}{R + jX} = \left(\frac{R}{R^2 + X^2}\right) + j\left(\frac{-X}{R^2 + X^2}\right)$$

Admittance is a complex number

$$Y = G + jB$$

where G (conductance) and B (susceptance) are given by

$$G = \frac{R}{R^2 + X^2}$$

$$B = \frac{-X}{R^2 + X^2}$$

The magnitude and phase of the admittance are given by

$$|Y| = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}$$

$$\angle Y = \arctan\left(\frac{B}{G}\right) = \arctan\left(\frac{-X}{R}\right)$$

where G is the conductance, measured in Siemens.
where B is the susceptance, measured in Siemens.

2.4 STEADY STATE ANALYSIS OF R,L AND C CIRCUITS

2.4.1 Series Circuit

The impedance diagram is a useful tool for analysing series ac circuits. Basically we can divide the series circuits as RL, RC and RLC circuits. In the analysis of series ac circuits, one must draw the impedance diagram. Although the impedance diagram usually is not drawn to scale, it does represent a clear picture of the phase relationships.

If we apply a sinusoidal input to an RL circuit, the current in the circuit and all voltages across the elements are sinusoidal. In the analysis of the RL series circuit, we can find the impedance, current, phase angle and voltage drops. In Fig. 2.55(a) the resistor voltage (V_R) and current (I) are in phase with each other, but lag behind the source voltage (V_S). The inductor voltage (V_L) leads the source voltage (V_S). The phase angle between current and voltage in a pure inductor is always 90°. The amplitudes of voltages and currents in the circuit are completely dependent on the values of elements (i.e. the resistance and inductive reactance). In the circuit shown, the phase angle is somewhere between zero and 90° because of the series combination of resistance with inductive reactance, which depends on the relative values of R and X_L.

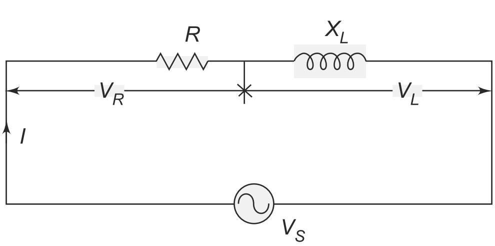


Fig. 2.55(a)

The phase relation between current and voltages in a series RL circuit is shown in Fig. 2.55(b).

Here V_R and I are in phase. The amplitudes are arbitrarily chosen. From Kirchhoff's voltage law, the sum of the voltage drops must equal the applied voltage. Therefore, the source voltage V_S is the phasor sum of V_R and V_L.

$$\therefore V_S = \sqrt{V_R^2 + V_L^2}$$

The phase angle between resistor voltage and source voltage is

$$\theta = \tan^{-1}(V_L/V_R)$$

where θ is also the phase angle between the source voltage and the current. The phasor diagram for the series RL circuit that represents the waveforms in Fig. 2.55(c).

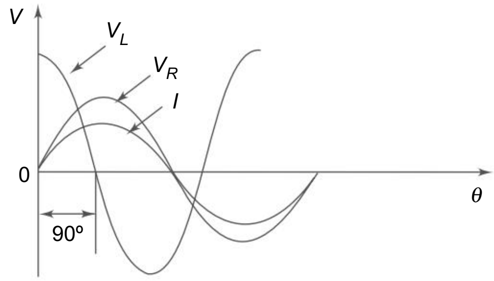


Fig. 2.55(b)

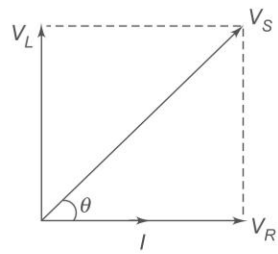


Fig. 2.55(c)

Example 2.41 *To the circuit shown in Fig. 2.56, consisting a 1 kW resistor connected in series with a 50 mH coil, a 10 V rms, 10 kHz signal is applied. Find impedance Z, current I, phase angle θ, voltage across resistance V_R, and the voltage across inductance V_L.*

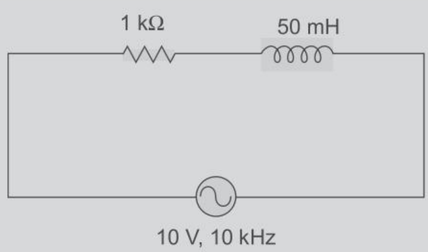


Fig. 2.56

Solution Inductive reactance $X_L = \omega L$

$$= 2\pi f L = (6.28)\,(10^4)\,(50 \times 10^{-3}) = 3140\ \Omega$$

In rectangular form,

Total impedance $Z = (1000 + j3140)\ \Omega$

$$= \sqrt{R^2 + X_L^2}$$

$$= \sqrt{(1000)^2 + (3140)^2} = 3295.4\ \Omega$$

Current $I = V_S/Z = 10/3295.4 = 3.03$ mA

Phase angle $\theta = \tan^{-1}(X_L/R) = \tan^{-1}(3140/1000) = 72.33°$

Therefore, in polar form total impedance $Z = 3295.4 \angle 72.33°$

Voltage across resistance $V_R = IR = 3.03 \times 10^{-3} \times 1000 = 3.03$ V

Voltage across inductive reactaVnce $V_L = IX_L = 3.03 \times 10^{-3} \times 3140 = 9.51$ V

Example 2.42 *Determine the source voltage and the phase angle, if voltage across the resistance is 70 V and voltage across the inductive reactance is 20 V as shown in Fig. 2.57.*

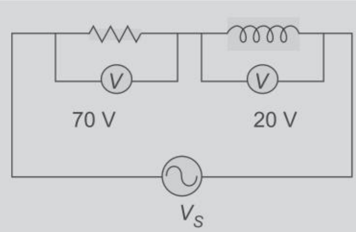


Fig. 2.57

Solution In Fig. 2.57, the source voltage is given by

$$V_S = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(70)^2 + (20)^2} = 72.8 \text{ V}$$

The angle between current and source voltage is

$$\theta = \tan^{-1} (V_L/V_R) = \tan^{-1} (20/70) = 15.94°$$

Example 2.43 *A signal generator supplies a 30 V, 100 Hz signal to a series circuit shown in Fig. 2.58. Determine the impedance, the line current and phase angle in the given circuit.*

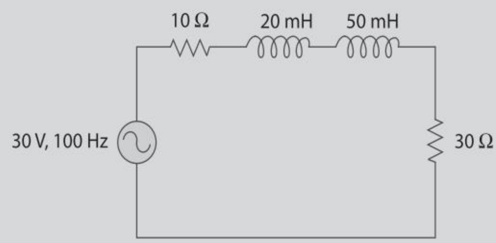


Fig. 2.58

Solution In Fig. 2.59, the resistances and inductive reactances can be combined.

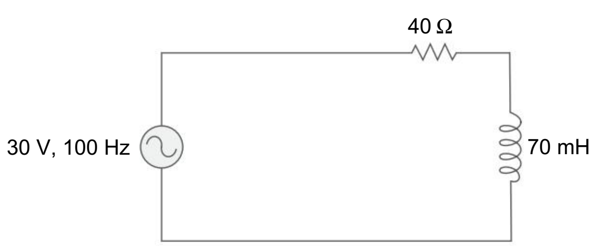


Fig. 2.59

First, we find the inductive reactance

$$X_L = 2\pi f L = 2\pi \times 100 \times 70 \times 10^{-3} = 43.98 \ \Omega$$

In rectangular form, the total impedance is

$$Z_T = (40 + j43.98) \ \Omega$$

Current $I = \dfrac{V_S}{Z_T} = \dfrac{30\angle 0^\circ}{40 + j43.98}$

Here we are taking source voltage as the reference voltage

$$\therefore \quad I = \frac{30\angle 0^\circ}{59.45\angle +47.7^\circ} = 0.5\angle -47.7^\circ \text{ A}$$

The current lags behind the applied voltage by 47.7°

Hence, the phase angle between voltage and current

$$\theta = 47.7°$$

Example 2.44 *For the circuit shown in Fig. 2.60, find the effective voltages across resistance and inductance, and also determine the phase angle.*

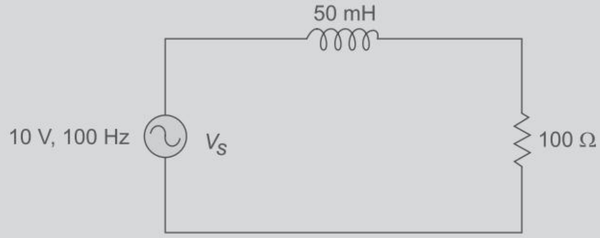


Fig. 2.60

Solution In rectangular form,

Total impedance $Z_T = R + jX_L$

where $X_L = 2\pi f L$

$= 2\pi \times 100 \times 50 \times 10^{-3} = 31.42\ \Omega$

$\therefore$ $Z_T = (100 + j31.42)\ \Omega$

Current $$I = \frac{V_S}{Z_T} = \frac{10\angle 0^\circ}{(100 + j31.42)} = \frac{10\angle 0^\circ}{104.8\angle 17.44^\circ} = 0.095\angle -17.44^\circ$$

Therefore, the phase angle between voltage and current

$\theta = 17.44°$

Voltage across resistance is $V_R = IR = 0.095 \times 100 = 9.5$ V

Voltage across inductive reactance is $V_L = IX_L = 0.095 \times 31.42 = 2.98$ V

When a sinusoidal voltage is applied to an RC series circuit, the current in the circuit and voltages across each of the elements are sinusoidal. The series RC circuit is shown in Fig. 2.61(a).

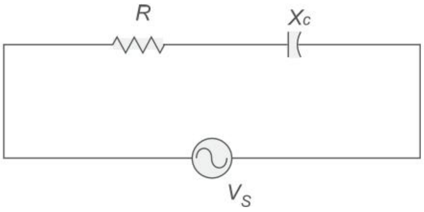


Fig. 2.61(a)

Here the resistor voltage and current are in phase with each other. The capacitor voltage lags behind the source voltage. The phase angle between the current and the capacitor voltage is always 90°. The amplitudes and the phase relations between the voltages and current depend on the ohmic values of the resistance and the capacitive reactance. The circuit is a series combination of both resistance and capacitance; and the phase angle between the applied voltage and the total current is somewhere between zero and 90°, depending on the relative values of the resistance and reactance. In a series RC circuit, the current is the same through the resistor and the capacitor. Thus, the resistor voltage is in phase with the current, and the capacitor voltage lags behind the current by 90° as shown in Fig. 2.61(b).

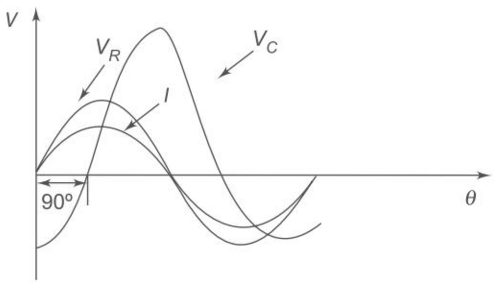


Fig. 2.61(b)

Here, I leads V_C by 90°. V_R and I are in phase. From Kirchhoff's voltage law, the sum of the voltage drops must be equal to the applied voltage. Therefore, the source voltage is given by

$$V_S = \sqrt{V_R^2 + V_C^2}$$

The phase angle between the resistor voltage and the source voltage is

$$\theta = \tan^{-1}(V_C/V_R)$$

Since the resistor voltage and the current are in phase, θ also represents the phase angle between the source voltage and current. The voltage phasor diagram for the series RC circuit, voltage and current phasor diagrams represented by the waveforms in Fig. 2.61(b) are shown in Fig. 2.61(c).

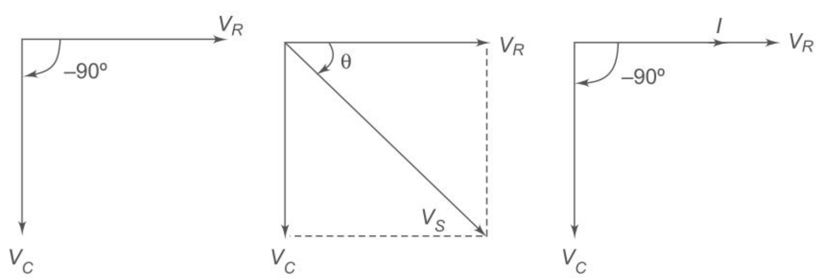


Fig. 2.61(c)

Example 2.45 *A sine wave generator supplies a 500 Hz, 10 V rms signal to a 2 kΩ resistor in series with a 0.1 μF capacitor as shown in Fig. 2.62. Determine the total impedance Z, current I, phase angle θ, capacitive voltage V_C, and resistive voltage V_R.*

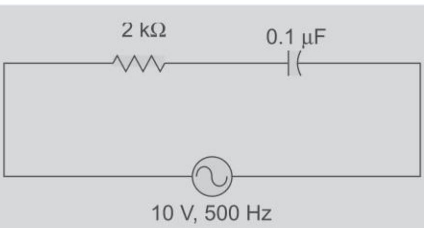


Fig. 2.62

Solution To find the impedance Z, we first solve for X_C

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 500 \times 0.1 \times 10^{-6}}$$

$$= 3184.7\ \Omega$$

In rectangular form,

Total impedance $Z = (2000 - j3184.7)\ \Omega$

$$Z = \sqrt{(2000)^2 + (3184.7)^2}$$

$$= 3760.6\ \Omega$$

Phase angle $\theta = \tan^{-1}(-X_C/R) = \tan^{-1}(-3184.7/2000) = -57.87°$

Current $I = V_S/Z = 10/3760.6 = 2.66$ mA

Capacitive voltage $V_C = IX_C$

$$= 2.66 \times 10^{-3} \times 3184.7 = 8.47\ \text{V}$$

Resistive voltage $V_R = IR$

$$= 2.66 \times 10^{-3} \times 2000 = 5.32\ \text{V}$$

The arithmetic sum of V_C and V_R does not give the applied voltage of 10 volts. In fact, the total applied voltage is a complex quantity. In rectangular form,

Total applied voltage $V_S = 5.32 - j8.47$ V

In polar form

$$V_S = 10 \angle -57.87°\ \text{V}$$

The applied voltage is complex, since it has a phase angle relative to the resistive current.

Example 2.46 *Determine the source voltage and phase angle when the voltage across the resistor is 20 V and the capacitor is 30 V as shown in Fig. 2.63.*

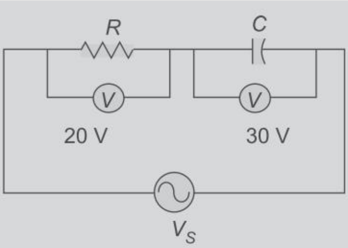


Fig. 2.63

Solution Since V_R and V_C are 90° out of phase, they cannot be added directly. The source voltage is the phasor sum of V_R and V_C.

$$\therefore\ V_S = \sqrt{V_R^2 + V_C^2} = \sqrt{(20)^2 + (30)^2} = 36\ \text{V}$$

The angle between the current and source voltage is

$$\theta = \tan^{-1}(V_C/V_R) = \tan^{-1}(30/20) = 56.3°$$

Example 2.47 *A resistor of 100 Ω is connected in series with a 50 μF capacitor. Find the effective voltage applied to the circuit at a frequency of 50 Hz. The effective voltage across the resistor is 170 V. Also determine voltage across the capacitor and phase angle. (See Fig. 2.64)*

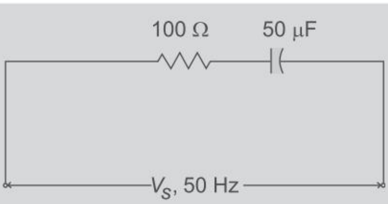


Fig. 2.64

Solution Capacitive reactance $X_C = \dfrac{1}{2\pi fC}$

$$= \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\,\Omega$$

Total impedance $Z_T = (100 - j63.66)\ \Omega$

Voltage across 100 Ω resistor is $V_R = 170$ V

Current in resistor, $I = \dfrac{170}{100} = 1.7$ A

Since the same current passes through capacitive reactance, the effective voltage across the capacitive reactance is

$$V_C = IX_C$$
$$= 1.7 \times 63.66 = 108.22 \text{ V}$$

The effective applied voltage to the circuit

$$V_S = \sqrt{V_R^2 + V_C^2}$$
$$= \sqrt{(170)^2 + (108.22)^2} = 201.5\text{ V}$$

Total impedance in polar form

$$Z_T = 118.54\ \angle{-32.48°}$$

Therefore, the current leads the applied voltage by 32.48°.

Example 2.48 *For the circuit shown in Fig. 2.65, determine the value of impedance when a voltage of (30 + j50)V is applied to the circuit and the current flowing is (−5 + j15) A. Also determine the phase angle.*

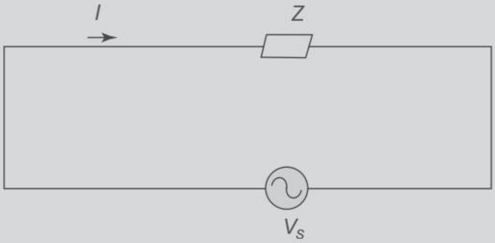


Fig. 2.65

Solution Impedance $Z = \dfrac{V_S}{I} = \dfrac{30 + j50}{-5 + j15}$

$$= \frac{58.31\angle 59°}{15.81\angle 108.43°} = 3.69\angle -49.43°$$

In rectangular form, the impedance $Z = 2.4 - j2.8$

Therefore, the circuit has a resistance of 2.4 Ω in series with capacitive reactance 2.8 Ω.

Phase angle between voltage and current is $\theta = 49.43°$. Here, the current leads the voltage by 49.43°.

A series RLC circuit is the series combination of resistance, inductance and capacitance. If we observe the impedance diagrams of series RL and series RC circuits as shown in Fig. 2.66(a) and (b), the inductive reactance, X_L, is displayed on the $+j$ axis and the capacitive reactance, X_C, is displayed on the $-j$ axis. These reactance are 180° apart and tend to cancel each other.

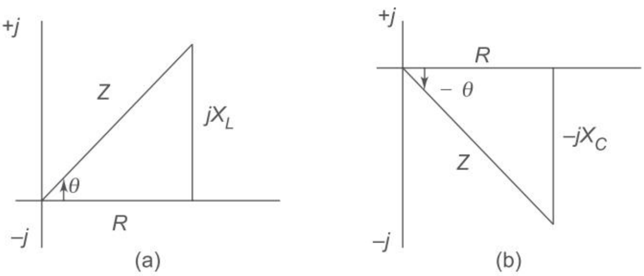


Fig. 2.66

The magnitude and type of reactance in a series RLC circuit is the difference of the two reactance. The impedance for an RLC series circuit is given by $Z=\sqrt{R^2+(X_L-X_C)^2}$. Similarly, the phase angle for an RLC circuit is

$$\theta=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

Example 2.49 *In the circuit shown in Fig. 2.67, determine the total impedance, current I, phase angle θ, and the voltage across each element.*

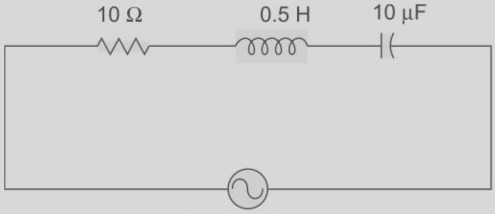


Fig. 2.67

Solution To find impedance Z, we first solve for X_C and X_L

$$X_C=\frac{1}{2\pi fC}=\frac{1}{6.28\times 50\times 10\times 10^{-6}}$$

$$= 318.5\ \Omega$$

$$X_L = 2\pi fL = 6.28 \times 0.5 \times 50 = 157\ \Omega$$

Total impedance in rectangular form

$$Z = (10 + j157 - j\,318.5)\ \Omega$$

$$= 10 + j(157 - 318.5)\ \Omega = 10 - j161.5\ \Omega$$

Here, the capacitive reactance dominates the inductive reactance.

$$Z = \sqrt{(10)^2 + (161.5)^2}$$
$$= \sqrt{100 + 26082.2} = 161.8\,\Omega$$
$$\text{Current } I = V_S / Z = \frac{50}{161.8} = 0.3\text{ A}$$

Phase angle $\theta = \tan^{-1} [(X_L - X_C)/R] = \tan^{-1} (-161.5/10) = -86.45°$

Voltage across the resistor $V_R = IR = 0.3 \times 10 = 3$ V

Voltage across the capacitive reactance $= IX_C = 0.3 \times 318.5 = 95.55$ V

Voltage across the inductive reactance $= IX_L = 0.3 \times 157 = 47.1$ V

2.4.2 Parallel Circuits

The complex number system simplifies the analysis of parallel ac circuits. In series circuits, the current is the same in all parts of the series circuit. In parallel ac circuits, the voltage is the same across each element.

The voltages for an RC series circuit can be expressed using complex numbers, where the resistive voltage is the real part of the complex voltage and the capacitive voltage is the imaginary part. For parallel RC circuits, the voltage is the same across each component. Here the total current can be represented by a complex number. The real part of the complex current expression is the resistive current; the capacitive branch current is the imaginary part.

Example 2.50 *A signal generator supplies a sine wave of 20 V, 5 kHz to the circuit shown in Fig. 2.68. Determine the total current I_T, the phase angle and total impedance in the circuit.*

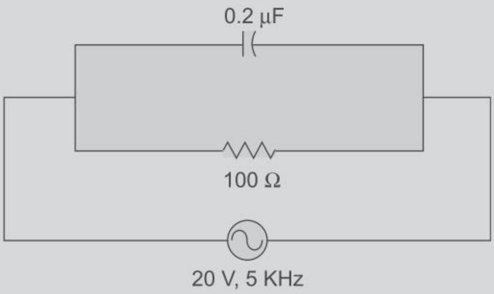


Fig. 2.68

Solution Capacitive reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{6.28 \times 5 \times 10^3 \times 0.2 \times 10^{-6}} = 159.2\,\Omega$$

Since the voltage across each element is the same as the applied voltage, we can solve for the two branch currents.

∴ Current in the resistance branch

$$I_R = \frac{V_S}{R} = \frac{20}{100} = 0.2\text{ A}$$

and current in the capacitive branch

$$I_C = \frac{V_S}{X_C} = \frac{20}{159.2} = 0.126\,\text{A}$$

The total current is the vector sum of the two branch currents.

∴ Total current $I_T = (I_R + jI_C)$ A = (0.2 + j0.13) A

In polar form $I_T = 0.24 \angle 33°$

So the phase angle θ between applied voltage and total current is 33°. It indicates that the total line current is 0.24 A and leads the voltage by 33°. Solving for impedance, we get

$$Z = \frac{V_S}{I_T} = \frac{20\angle 0°}{0.24\angle 33°} = 83.3\angle -33°\ \Omega$$

Example 2.51 *Determine the impedance and phase angle in the circuit shown in Fig. 2.69.*

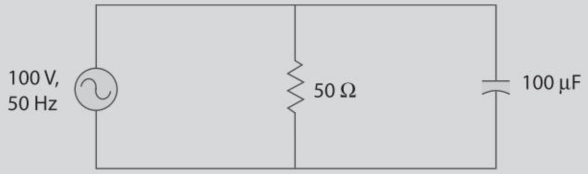


Fig. 2.69

Solution Capacitive reactance $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\,\Omega$

Capacitive susceptance $B_C = \frac{1}{X_C}$

$$= \frac{1}{31.83} = 0.031\,\text{S}$$

Conductance G $= \frac{1}{R} = \frac{1}{50} = 0.02\,\text{S}$

Total admittance $Y = \sqrt{G^2 + B_C^2}$

$$= \sqrt{(0.02)^2 + (0.031)^2}$$

$$= 0.037\,\text{S}$$

Total impedance $Z = \frac{1}{Y} = \frac{1}{0.037} = 27.02\,\Omega$

Phase angle $\theta = \tan^{-1}\left(\frac{R}{X_C}\right) = \tan^{-1}\left(\frac{50}{31.83}\right)$

$$\theta = 57.52°$$

Example 2.52 *For the parallel circuit in Fig. 2.70, find the magnitude of current in each branch and the total current. What is the phase angle between the applied voltage and total current?*

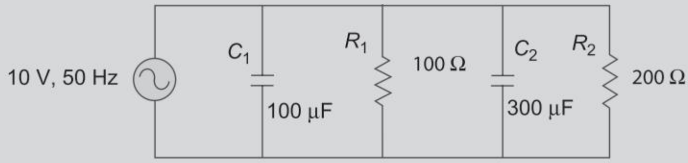


Fig. 2.70

Solution First let us find the capacitive reactances.

$$X_{C_1} = \frac{1}{2\pi f C_1}$$

$$= \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$$

$$X_{C_2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 50 \times 300 \times 10^{-6}}$$

$$= 10.61\Omega$$

Here the voltage across each element is the same as the applied voltage.

Current in the 100 μF capacitor $I_C = \frac{V_S}{X_{C_1}}$

$$= \frac{10\angle 0^\circ}{31.83\angle -90^\circ} = 0.31\angle 90^\circ \text{ A}$$

Current in the 300 μF capacitor $I_{C_2} = \frac{V_S}{X_{C_2}}$

$$= \frac{10\angle 0^\circ}{10.61\angle -90^\circ} = 0.94\angle 90^\circ \text{ A}$$

Current in the 100 Ω resistor is $I_{R_1} = \frac{V_S}{R_1} = \frac{10}{100} = 0.1\text{ A}$

Current in the 200 Ω resistor is $I_{R_2} = \frac{V_S}{R_2} = \frac{10}{200} = 0.05\text{ A}$

Total current $I_T = I_{R_1} + I_{R_2} + j(I_{C_1} + I_{C_2})$

$$= 0.1 + 0.05 + j(0.31 + 0.94)$$

$$= 1.26\angle 83.2^\circ \text{ A}$$

The circuit shown in Fig. 2.70 can be simplified into a single parallel RC circuit as shown in Fig. 2.71.

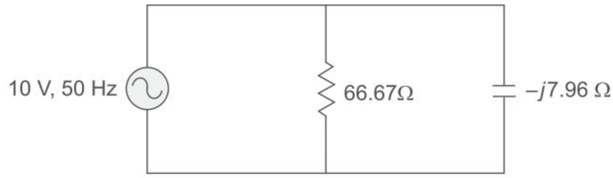


Fig. 2.71

In Fig. 2.71, the two resistances are in parallel and can be combined into a single resistance. Similarly, the two capacitive reactances are in parallel and can be combined into a single capacitive reactance.

$$R = \frac{R_1 R_2}{R_1 + R_2} = 66.67\ \Omega$$

$$X_C = \frac{X_{C_1} X_{C_2}}{X_{C_1} + X_{C_2}} = 7.96\ \Omega$$

Phase angle θ between voltage and current is

$$\theta = \tan^{-1}\left(\frac{R}{X_C}\right) = \tan^{-1}\left(\frac{66.67}{7.96}\right) = 83.19°$$

Here the current leads the applied voltage by 83.19°.

In a parallel RL circuit, the inductive current is imaginary and lies on the $-j$ axis. The current angle is negative when the impedance angle is positive. Here also the total current can be represented by a complex number. The real part of the complex current expression is the resistive current; and inductive branch current is the imaginary part.

Example 2.53 *A 50 Ω resistor is connected in parallel with an inductive reactance of 30 Ω. A 20V signal is applied to the circuit. Find the total impedance and line current in the circuit shown in Fig. 2.72.*

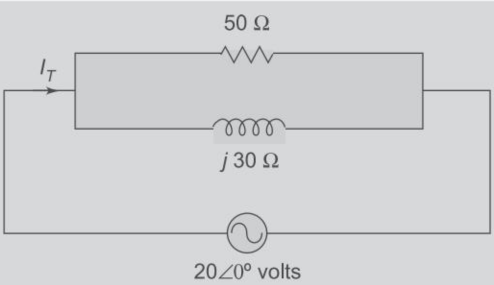


Fig. 2.72

Solution Since the voltage across each element is the same as the applied voltage, current in the resistive branch,

$$I_R = \frac{V_s}{R} = \frac{20\angle 0^\circ}{50\angle 0^\circ} = 0.4\,\text{A}$$

current in the inductive branch

$$I_L = \frac{V_s}{X_L} = \frac{20\angle 0^\circ}{30\angle 90^\circ} = 0.66\angle -90^\circ$$

Total current is $I_T = 0.4 - j0.66$

In polar form, $I_T = 0.77 \angle -58.8°$

Here the current lags behind the voltage by 58.8°

Total impedance $$Z = \frac{V_s}{I_T} = \frac{20\angle 0^\circ}{0.77\angle -58.8^\circ} = 25.97\angle 58.8^\circ\ \Omega$$

Example 2.54 *For the circuit shown in Fig. 2.73, determine the total current, impedance Z and phase angle.*

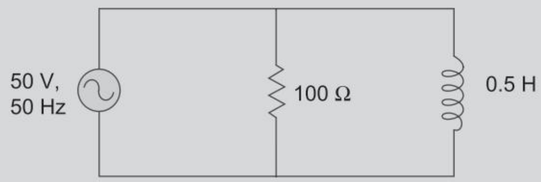


Fig. 2.73

Solution Here, the voltage across each element is the same as the applied voltage.

Current in resistive branch $$I_R = \frac{V_S}{R} = \frac{50}{100} = 0.5\ \text{A}$$

Inductive reactance $$X_L = 2\pi f L$$

$$= 2\pi \times 50 \times 0.5 = 157.06\ \Omega$$

Current in inductive branch

$$I_L = \frac{V_S}{X_L} = \frac{50}{157.06} = 0.318\ \text{A}$$

Total current $$I_T = \sqrt{I_R^2 + I_L^2}$$

or $(0.5 - j0.318)\text{A} = 0.59 \angle -32.5°$

For parallel RL circuits, the inductive susceptance is

$$B_L = \frac{1}{X_L} = \frac{1}{157.06} = 0.0064\ \text{S}$$

Conductance $$G = \frac{1}{100} = 0.01\ \text{S}$$

$\therefore$ $$\text{Admittance} = \sqrt{G^2 + B_L^2} = \sqrt{(0.01)^2 + (0.0064)^2}$$

$$= 0.0118\ \text{S}$$

Converting to impedance, we get

$$Z = \frac{1}{Y} = \frac{1}{0.012} = 83.33\ \Omega$$

Phase angle $$\theta = \tan^{-1}\left(\frac{R}{X_L}\right) = \tan^{-1}\left(\frac{100}{157.06}\right) = 32.48°$$

2.4.3 Compound Circuits

In many cases, ac circuits to be analysed consist of a combination of series and parallel impedances. Circuits of this type are known as series-parallel, or compound circuits. Compound circuits can be simplified in the same manner as a series-parallel dc circuit consisting of pure resistances.

Example 2.55 *For the circuit shown in Fig. 2.74, find the current I drawn from the source.*

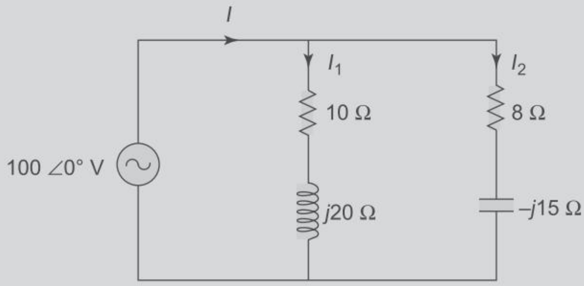


Fig. 2.74

Solution The impedance as seen by the source is

$$Z = (10 + j20) \| (8 - j15)$$

$$\frac{380 + j10}{18 + 5j} = 19.742 - j4.928$$

$\therefore$ Current drawn from source $I = \dfrac{V}{Z} = \dfrac{100}{19.742 - j4.928}$

$$= 4.768 + j1.19$$

$$= 4.914 \underline{|14.01^\circ}$$

or

$$I_1 = \frac{100}{10 + j20} = 2 - j4$$

$$I_2 = \frac{100}{8 - j15} = 2.768 + j5.1903$$

$$I = I_1 + I_2 = 4.768 + j1.19$$

$$= 4.914 \underline{|14.01^\circ}$$

Example 2.56 *Determine the equivalent impedance of Fig. 2.75.*

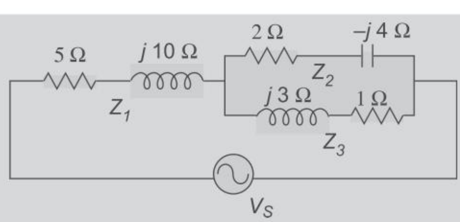


Fig. 2.75

Solution In the circuit, Z_1 is in series with the parallel combination of Z_2 and Z_3

where $Z_1 = (5 + j10)\ \Omega,$

$Z_2 = (2 - j4)\ \Omega,$

$Z_3 = (1 + j3)\ \Omega$

The total impedance

$$Z_T = Z_1 + \frac{z_2 z_3}{z_2 + z_3}$$

$$= (5 + j10) + \frac{(2 - j4)(1 + j3)}{(2 - j4) + (1 + j3)}$$

$$= (5 + j10) + \frac{4.47\angle -63.4^\circ \times 3.16\angle +71.5^\circ}{3 - j1}$$

$$= (5 + j10) + \frac{14.12\angle 81^\circ}{3 - j1}$$

$$= (5 + j10 + \frac{14.12\angle 81^\circ}{3.16\angle -18^\circ}$$

$$= 5 + j10 + 4.46\ \angle 26.1^\circ$$

$$= 5 + j10 + 4 + j1.96$$

$$= 9 + j11.96$$

The equivalent circuit for the compound circuit shown in Fig. 2.75 is a series circuit containing 9 Ω of resistance and 11.96 Ω of inductive reactance. In polar form,

$$Z = 14.96\ \angle 53.03^\circ$$

The phase angle between current and applied voltage is

$$\theta = 53.03^\circ$$

Example 2.57 *In the circuit of Fig. 2.76, determine the values of the following (a) Z_T (b) I_T (c) θ.*

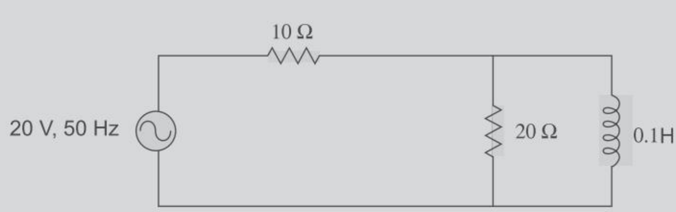


Fig. 2.76

Solution First, the inductive reactance is calculated.

$$X_L = 2\pi f L$$

$$= 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

In Fig. 2.77, the 10 Ω resistance is in series with the parallel combination of 20 Ω and $j31.42\ \Omega$

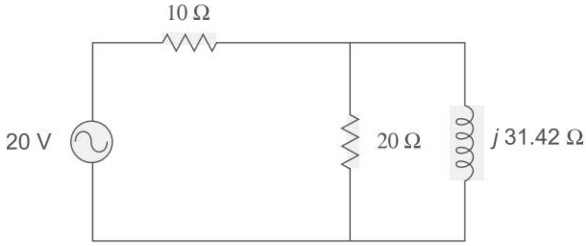


Fig. 2.77

$$\therefore \qquad Z_T = 10 + \frac{(20)(j31.42)}{(20 + j31.42)}$$

$$= 10 + \frac{628.4\angle 90^\circ}{37.24\angle 57.52^\circ} = 10 + 16.87\angle 32.48^\circ$$

$$= 10 + 14.23 + j9.06 = 24.23 + j9.06$$

In polar form, $Z_T = 25.87\ \angle 20.5°$

Here the current lags behind the applied voltage by 20.5°

Total current $$I_T = \frac{V_s}{Z_T}$$

$$= \frac{20}{25.87\angle 20.5^\circ} = 0.77\angle -20.5^\circ$$

The phase angle between voltage and current is

$$\theta = 20.5°$$

Example 2.58 *For the circuit shown in Fig. 2.78, determine the total impedance, total current and phase angle.*

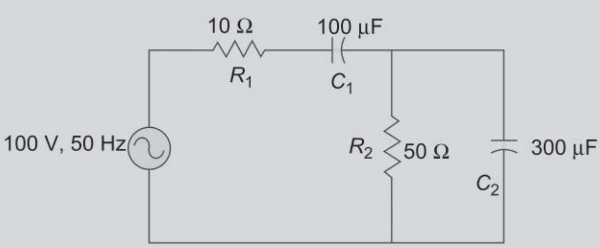


Fig. 2.78

Solution First, we calculate the magnitudes of the capacitive reactances.

$$X_{C_1} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$$

$$X_{C_2} = \frac{1}{2\pi \times 50 \times 300 \times 10^{-6}} = 10.61\ \Omega$$

We find the impedance of the parallel portion by finding the admittance.

$$G_2 = \frac{1}{R_2} = \frac{1}{50} = 0.02\,\text{S}$$

$$B_{C_2} = \frac{1}{X_{C_2}} = \frac{1}{10.61} = 0.094\,\text{S}$$

$$Y_2 = \sqrt{G_2^2 + B_{C_2}^2} = \sqrt{(0.02)^2 + (0.094)^2} = 0.096\,\text{S}$$

$$Z_2 = \frac{1}{Y_2} = \frac{1}{0.096} = 10.42\,\Omega$$

The phase angle associated with the parallel portion of the circuit

$$\theta_P = \tan^{-1}(R_2/X_{C_2}) = \tan^{-1}(50/10.61) = 78.02°$$

The series equivalent values for the parallel portion are

$$R_{\text{eq}} = Z_2 \cos\theta_P = 10.42 \cos(78.02°) = 2.16\,\Omega$$

$$X_{C(\text{eq})} = Z_2 \sin\theta_P = 10.42 \sin(78.02°) = 10.19\,\Omega$$

The total resistance

$$R_T = R_1 + R_{\text{eq}} = (10 + 2.16) = 12.16\,\Omega$$

$$X_{C_T} = X_{C_1} + X_{C(\text{eq})} = (31.83 + 10.19) = 42.02\,\Omega$$

Total impedance

$$Z_T = \sqrt{R_T^2 + X_{C_T}^2} = \sqrt{(12.16)^2 + (42.02)^2} = 43.74\,\Omega$$

We can also find the total current by using Ohm's law

$$I_T = \frac{V_S}{Z_T} = \frac{100}{43.74} = 2.29\,\text{A}$$

The phase angle

$$\theta = \tan^{-1}\left(\frac{X_{C_T}}{R_T}\right) = \tan^{-1}\left(\frac{42.02}{12.16}\right) = 73.86°$$

Example 2.59 *Determine the voltage across each element of the circuit shown in Fig. 2.79 and draw the voltage phasor diagram.*

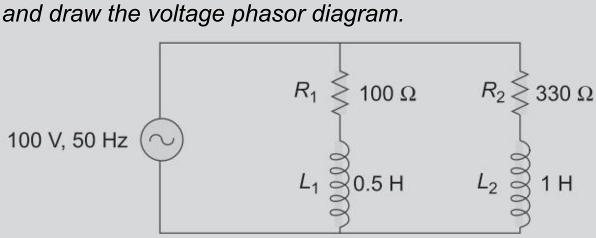


Fig. 2.79

Solution First we calculate X_{L_1} and X_{L_2}

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.5 = 157.08\ \Omega$$
$$X_{L_2} = 2\pi f L_2 = 2\pi \times 50 \times 1.0 = 314.16\ \Omega$$

Now we determine the impedance of each branch

$$Z_1 = \sqrt{R_1^2 + X_{L_1}^2} = \sqrt{(100)^2 + (157.08)^2} = 186.2\ \Omega$$
$$Z_2 = \sqrt{R_2^2 + X_{L_2}^2} = \sqrt{(330)^2 + (314.16)^2} = 455.63\ \Omega$$

The current in each branch

$$I_1 = \frac{V_S}{Z_1} = \frac{100}{186.2} = 0.537\ \text{A}$$

and $$I_2 = \frac{V_S}{Z_2} = \frac{100}{455.63} = 0.219.$$

The voltage across each element

$$V_{R_1} = I_1 R_1 = 0.537 \times 100 = 53.7\ \text{V}$$
$$V_{L_1} = I_1 X_{L_1} = 0.537 \times 157.08 = 84.35\ \text{V}$$
$$V_{R_2} = I_2 R_2 = 0.219 \times 330 = 72.27\ \text{V}$$
$$V_{L_2} = I_2 X_{L_2} = 0.219 \times 314.16 = 68.8\ \text{V}$$

The angles associated with each parallel branch are now determined.

$$\theta_1 = \tan^{-1}\left(\frac{X_{L_1}}{R_1}\right) = \tan^{-1}\left(\frac{157.08}{100}\right) = 57.52°$$
$$\theta_2 = \tan^{-1}\left(\frac{X_{L_2}}{R_2}\right) = \tan^{-1}\left(\frac{314.16}{330}\right) = 43.59°$$

i.e. I_1 lags behind V_S by 57.52° and I_2 lags behind V_S by 43.59°

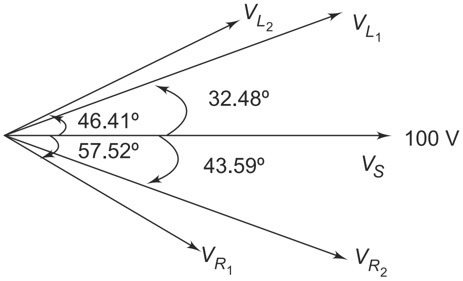


Fig. 2.80

Here, V_{R_1} and I_1 are in phase and therefore, lag behind V_S by 57.52°

V_{R_2} and I_2 are in phase, and therefore lag behind V_S by 43.59°

V_{L_1} leads I_1 by 90°, so its angle is 90° – 57.52° = 32.48°

V_{L_2} leads I_2 by 90°, so its angle is 90° – 43.59° = 46.41°

The phase relations are shown in Fig. 2.80.

Example 2.60 *In the series parallel circuit shown in Fig. 2.81, the effective value of voltage across the parallel parts of the circuits is 50 V. Determine the corresponding magnitude of V.*

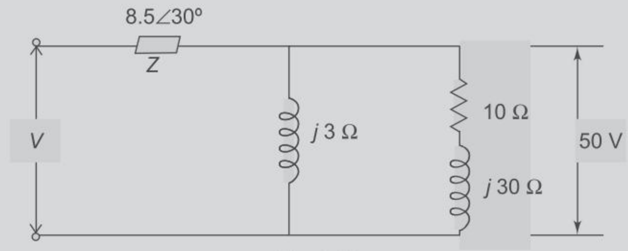


Fig. 2.81

Solution Here we can determine the current in each branch of the parallel part.

Current in the $j3\ \Omega$ branch, $I_1 = \dfrac{50}{3} = 16.67\text{ A}$

Current in $(10 + j30)\ \Omega$ branch, $I_2 = \dfrac{50}{31.62} = 1.58\text{ A}$

Total current $I_T = 16.67 + 1.58 = 18.25\text{ A}$

Total impedance

$$Z_T = 8.5\angle 30° + \frac{3\angle 90° \times (10 + j30)}{(10 + j30) + 3\angle 90°}$$

$$= 8.5\angle 30° + \frac{3\angle 90° \times 31.62\angle 71.57°}{10 + j33}$$

$$= 7.36 + j4.25 + \frac{94.86\angle 161.57°}{34.48\angle 73.14°}$$

$$= 7.36 + j4.25 + 2.75\angle 88.43°$$

$$= 7.36 + j4.25 + 0.075 + j2.75$$

$$= (7.435 + j7)\ \Omega$$

$$= 10.21\angle 43.27°$$

In polar form, total impedance is $Z_T = 10.21\angle 43.27°$

The magnitude of applied voltage $V = I \times Z_T = 18.25 \times 10.21 = 186.33\text{ V}.$

Example 2.61 *For the series parallel circuit shown in Fig. 2.82, determine (a) the total impedance between the terminals a, b and state if it is inductive or capacitive (b) the voltage across in the parallel branch, and (c) the phase angle.*

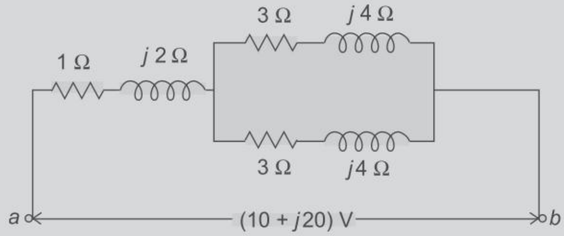


Fig. 2.82

Solution Here the parallel branch can be combined into a single branch

$$Z_P = (3 + j4) \parallel (3 + j4) = (1.5 + j2)\ \Omega$$

Total impedance $Z_T = 1 + j2 + 1.5 + j2 = (2.5 + j4)\ \Omega$

Hence the total impedance in the circuit is inductive

Total current in the circuit

$$I_T = \frac{V_S}{Z_T} = \frac{10 + j20}{2.5 + j4}$$

$$= \frac{22.36\angle 63.43°}{4.72\angle 57.99°}$$

$$\therefore \quad I_T = 4.74\ \angle 5.44°\ \text{A}$$

i.e. the current lags behind the voltage by 57.99°

Phase angle $\theta = 57.99°$

Voltage across in the parallel branch

$$V_P = (1.5 + j2)\ 4.74\ \angle 5.44°$$

$$= 2.5 \times 4.74\angle(5.44° + 53.13°)$$

$$= 11.85\angle 58.57°\ \text{V}$$

Example 2.62 *In the series parallel circuit shown in Fig. 2.83, the two parallel branches A and B are in series with C. The impedances are $Z_A = 10 + j8$, $Z_B = 9 - j6$, $Z_C = 3 + j2$ and the voltage across the circuit is $(100 + j0)$ V. Find the currents I_A, I_B and the phase angle between them.*

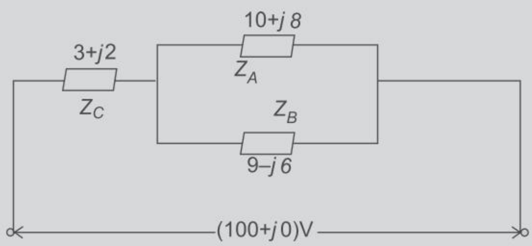


Fig. 2.83

Solution Total parallel branch impedance,

$$Z_P = \frac{Z_A Z_B}{Z_A + Z_B}$$

$$= \frac{(10 + j8)(9 - j6)}{19 + j2}$$

$$= \frac{12.8\angle 38.66° \times 10.82\angle -33.7}{19.1\angle 6°} = 7.25\angle -1.04°$$

In rectangular form,

Total parallel impedance $Z_P = 7.25 - j0.13$

This parallel impedance is in series with Z_C

Total impedance in the circuit

$$Z_T = Z_P + Z_C = 7.25 - j0.13 + 3 + j2 = (10.25 + j1.87)\ \Omega$$

Total current $I_T = \dfrac{V_S}{Z_T}$

$$= \frac{(100 + j0)}{(10.25 + j1.87)}$$

$$= \frac{100\angle 0^\circ}{10.42\angle 10.34^\circ}$$

$$= 9.6\angle -10.34^\circ$$

The current lags behind the applied voltage by 10.34°

Current in branch A is

$$I_A = I_T \frac{Z_B}{Z_A + Z_B}$$

$$= 9.6\angle -10.34^\circ \times \frac{(9 - j6)}{19 + j2}$$

$$= \frac{9.6\angle -10.34^\circ \times 10.82\angle -33.7^\circ}{19.1\angle 6^\circ}$$

$$= 5.44\angle -50.04^\circ\ \text{A}$$

Current in branch B is I_B

$$I_B = I_T \times \frac{Z_A}{Z_A + Z_B}$$

$$= 9.6\angle -10.34^\circ \times \frac{10 + j8}{19 + j2}$$

$$= \frac{9.6\angle -10.34^\circ \times 12.8\angle 38.66^\circ}{19.1\angle 6^\circ}$$

$$= 6.43\angle 22.32^\circ\ \text{A}$$

The angle between I_A and I_B,

$$\theta = (50.04^\circ + 22.32^\circ) = 72.36^\circ$$

Example 2.63 *A series circuit of two pure elements has the following applied voltage and resulting current.*

$$V = 15\cos(200t - 30^\circ)\ \text{volts}$$

$$I = 8.5\cos(200t + 15)\ \text{volts}$$

Find the elements comprising the circuit.

Solution By inspection, the current leads the voltage by 30° + 15° = 45°. Hence the circuit must contain resistance and capacitance.

$$\tan 45 = \frac{1}{\omega CR}$$

$$1 = \frac{1}{\omega CR}, \quad \therefore \frac{1}{\omega C} = R$$

$$\frac{V_m}{I_m} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + R^2} = \sqrt{2}\, R$$

$$\therefore \qquad R = \frac{15}{8.5 \times \sqrt{2}} = 1.248\ \Omega$$

$$\frac{1}{\omega C} = 1.248\ \Omega$$

and $$C = \frac{1}{200 \times 1.248} = 4 \times 10^{-3}\ \text{F}$$

Example 2.64 *A resistor having a resistance of R = 10 Ω and an unknown capacitor are in series. The voltage across the resistor is V_R = 50 sin (1000 t + 45°) volts. If the current leads the applied voltage by 60° what is the unknown capacitance C?*

Solution Here, the current leads the applied voltage by 60°.

$$\tan 60° = \frac{1}{\omega CR}$$

Since $$R = 10\ \Omega$$

$$\omega = 1000 \text{ radians}$$

$$\tan 60° = \frac{1}{\omega CR}$$

$$C = \frac{1}{\tan 60 \times 1000 \times 10} = 57.7\ \mu\text{F}$$

Example 2.65 *A series circuit consists of two pure elements has the following current and voltage.*

v = 100 sin (2000 t + 50°) V

i = 20 cos (2000 t + 20°) A

Find the elements in the circuit.

Solution We can write i = 20 sin (2000 t + 20° + 90°)

Since cos θ = sin (θ + 90°)

Current i = 20 sin (2000 t + 110°) A

The current leads the voltage by 110° – 50° = 60°

and the circuit must consist of resistance and capacitance.

$$\tan\theta = \frac{1}{\omega CR}$$

$$\frac{1}{\omega C} = R\tan 60° = 1.73\text{ R}$$

$$\frac{V_m}{I_m} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{100}{20}$$

$$R\sqrt{1+(1.73)^2} = \frac{100}{20}$$

$$R(1.99) = 5$$

$$R = 2.5\,\Omega$$

and $$C = \frac{1}{\omega(1.73\text{R})} = 115.6\,\mu\text{F}$$

Example 2.66 *A two branch parallel circuit with one branch of R = 100 Ω and a single unknown element in the other branch has the following applied voltage and total current.*

$v = 2000\cos(1000\,t + 45°)$ V

$I_T = 45\sin(1000\,t + 135°)$ A

Find the unknown element.

Solution Here, the voltage applied is same for both elements.

Current passing through resistor is $i_R = \dfrac{v}{R}$

$\therefore$ $$i_R = 20\cos(1000\,t + 45°)$$

Total current $i_T = i_R + i_X$

Where i_x is the current in unknown element.

$$i_x = i_T - i_R$$

$$= 45\sin(1000\,t + 135°) - 20\cos(1000\,t + 45°)$$

$$= 45\sin(1000\,t + 135°) - 20\sin(1000\,t + 135°)$$

Current passing through the unknown element.

$$i_x = 25\sin(1000\,t + 135°)$$

Since the current and voltage are in phase, the element is a resistor.
And the value of resistor

$$R = \frac{v}{i_X} = \frac{2000}{25} = 80\,\Omega$$

Example 2.67 *Find the total current to the parallel circuit with L = 0.05 H and C = 0.667 μF with an applied voltage of $v = 200\sin 5000t$ V.*

Solution Current in the inductor $i_L = \frac{1}{L}\int v dt$

$$i_L = \frac{1}{0.05}\int 200 \sin 5000 t$$

$$= \frac{-200 \cos 5000 t}{0.05 \times 5000}$$

$$i_L = -0.8 \cos 5000 t$$

Current in the capacitor $i_C = C\frac{dv}{dt}$

$\therefore$ $i_C = 0.667 \times 10^{-6} \frac{d}{dt}(200 \sin 5000 t)$

$$i_C = 0.667 \cos 5000 t$$

Total current $i_T = i_L + i_C$

$$= 0.667 \cos 5000 t - 0.8 \cos 5000 t$$

$$= -0.133 \cos (5000 t)$$

Total current $i_T = 0.133 \sin (5000 t - 90°)$ A

Example 2.68 *A current $i = 4 \sin (314 t - 100)$ produces a voltage drop $v = 220 \sin (314 t - 200)$ in a circuit. Find the values of the circuit parameters assuming a series circuit.* *[JNTU May/June 2008]*

Solution Current $i = 4 \sin (314 t - 100)$

$$V = 220 \sin (314 t - 200)$$

$$I = 4\angle -100$$

$$V = 220\angle -200$$

As it is a series circuit

Impedance $Z = V/I$

$$= \frac{220\angle -200}{4\angle -100} = 55\angle -100$$

$$Z = 55\angle -100$$

$$= -9.55 - j54.164$$

$$= -(9.55 + j54.164)$$

$$R = 9.55\,\Omega \qquad X_L = 54.164\,\Omega$$

$$\omega L = 54.164\,\Omega$$

$$L = \frac{54.164}{314} = 0.172\ H$$

Example 2.69 *A parallel circuit has two branches. One branch consists of a resistance of 10 Ω and an inductance of 35 mH connected in series. If the circuit is connected across a 230 V, 50 Hz supply, what capacitance should be connected in the second branch such that the current drawn from the supply is in phase with the supply voltage?* *[JNTU June 2009]*

Solution If current i has to be inphase with the voltage, then the reactance of the circuit should be equal to 0.

The impedance

$$Z = (10 + 0.035S) \parallel \frac{1}{SC}$$

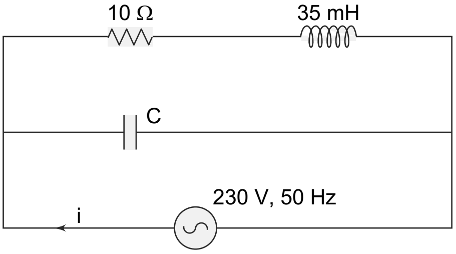


Fig. 2.84

$$= \frac{(10+0.035S)\frac{1}{SC}}{\left(10+0.035S+\frac{1}{SC}\right)}$$

$$= \left(\frac{\frac{10}{SC}+\frac{0.035}{C}}{10+0.035S+\frac{1}{SC}}\right) = \frac{(10+0.035S)}{(10SC+0.035S^2C+1)}$$

$$= \frac{(10+0.035S)(1+0.035S^2C-10SC)}{\left|10SC+0.035S^2C+1\right|^2}$$

Reactance parts;

$$\frac{-100SC+(0.035S)(1+0.035S^2C)}{\left|10SC+0.035S^2C+1\right|^2} = 0$$

$\therefore$ $$-100SC + 0.035S + 0.035^2S^3C = 0$$

or $$-100C + 0.035 + 0.035S^2C = 0$$

or $$-100C + 0.035 + 0.035S^2C = 0$$

or where $S = jw$ $$-100C + 0.035 - 120.7801\,C = 0$$

$\therefore$ $$C = \frac{0.035}{220.7801} = 158.53\,\mu F$$

2.5 POWER FACTOR AND ITS SIGNIFICANCE

2.5.1 Instantaneous Power

In a purely resistive circuit, all the energy delivered by the source is dissipated in the form of heat by the resistance. In a purely reactive(inductive or capacitive) circuit, all the energy delivered by the source is stored by the inductor or capacitor in its magnetic or electric field during a portion of the voltage cycle, and then is returned to the source during another portion of the cycle, so that no net energy is transferred. When there is complex impedance in a circuit, part of the energy is alternately stored and returned by

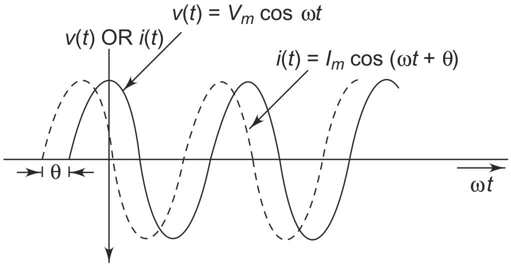


Fig. 2.85

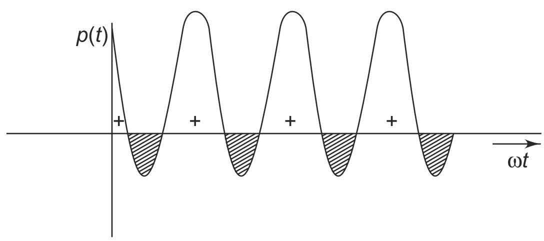


Fig. 2.86

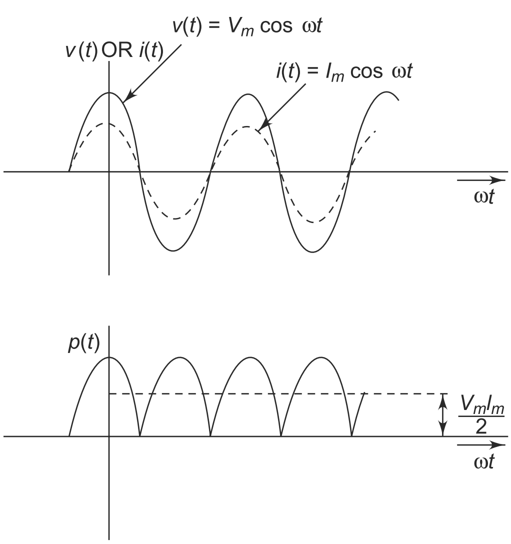


Fig. 2.87

the reactive part, and part of it is dissipated by the resistance. The amount of energy dissipated is determined by the relative values of resistance and reactance.

Consider a circuit having complex impedance. Let $v\ (t) = V_m \cos \omega t$ be the voltage applied to the circuit and let $i(t) = I_m \cos (\omega t + \theta)$ be the corresponding current flowing through the circuit. Then the power at any instant of time is

$$P(t) = v\ (t)\ i\ (t) = V_m \cos \omega t\ I_m \cos (\omega t + \theta) \qquad (2.1)$$

From Eq. 2.1, we get

$$P(t) = \frac{V_m I_m}{2} \left[\cos (2\omega t + \theta) + \cos \theta\right] \qquad (2.2)$$

Equation 2.2 represents *instantaneous power*. It consists of two parts. One is a fixed part, and the other is time-varying which has a frequency twice that of the voltage or current waveforms. The voltage, current and power waveforms are shown in Figs 2.85 and 2.86.

Here, the negative portion (hatched) of the power cycle represents the power returned to the source. Figure 2.86 shows that the instantaneous power is negative whenever the voltage and current are of opposite sign. In Fig. 2.86, the positive portion of the power is greater than the negative portion of the power; hence the average power is always positive, which is almost equal to the constant part of the instantaneous power (Eq. 2.2). The positive portion of the power cycle varies with the phase angle between the voltage and current waveforms. If the circuit is pure resistive, the phase angle between voltage and current is zero; then there is no negative cycle in the $P(t)$ curve. Hence, all the power delivered by the source is completely dissipated in the resistance.

If θ becomes zero in Eq. 2.1, we get

$$P(t) = v(t)\, i(t)$$

$$= V_m I_m \cos^2 \omega t$$

$$= \frac{V_m I_m}{2}(1+\cos 2\omega t) \tag{2.3}$$

The waveform for Eq. 2.3, is shown in Fig. 2.87, where the power wave has a frequency twice that of the voltage or current. Here the average value of power is $V_m I_m/2$.

When phase angle θ is increased, the negative portion of the power cycle increases and lesser power is dissipated. When θ becomes π/2, the positive and negative portions of the power cycle are equal. At this instant, the power dissipated in the circuit is zero, i.e. the power delivered to the load is returned to the source.

2.5.2 Average Power

To find the average value of any power function, we have to take a particular time interval from t_1 to t_2; by integrating the function from t_1 to t_2 and dividing the result by the time interval $t_2 - t_1$, we get the average power.

$$P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} P(t)\, dt \tag{2.4}$$

In general, the average value over one cycle is

$$P_{\text{av}} = \frac{1}{T}\int_0^T P(t)\, dt \tag{2.5}$$

By integrating the instantaneous power $P(t)$ in Eq. 2.5 over one cycle, we get average power

$$P_{\text{av}} = \frac{1}{T}\int_0^T \left\{\frac{V_m I_m}{2}\left[\cos(2\omega t+\theta)+\cos\theta\right]dt\right\}$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\left[\cos(2\omega t+\theta)\right]dt + \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos\theta\, dt \tag{2.6}$$

In Eq. 2.6, the first term becomes zero, and the second term remains. The average power is therefore

$$P_{\text{av}} = \frac{V_m I_m}{2}\cos\theta \text{ W} \tag{2.7}$$

We can write Eq. 2.7 as

$$P_{\text{av}} = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta \tag{2.8}$$

In Eq. 2.8, $V_m/\sqrt{2}$ and $I_m/\sqrt{2}$ are the effective values of both voltage and current.

$\therefore \qquad P_{av} = V_{eff} I_{eff} \cos\theta$

To get average power, we have to take the product of the effective values of both voltage and current multiplied by cosine of the phase angle between voltage and the current.

If we consider a purely resistive circuit, the phase angle between voltage and current is zero. Hence, the average power is

$$P_{av} = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R$$

If we consider a purely reactive circuit (i.e. purely capacitive or purely inductive), the phase angle between voltage and current is 90°. Hence, the average power is zero or $P_{av} = 0$.

If the circuit contains complex impedance, the average power is the power dissipated in the resistive part only.

Example 2.70 *A voltage of $v(t) = 100 \sin \omega t$ is applied to a circuit. The current flowing through the circuit is $i(t) = 15 \sin (\omega t - 30°)$. Determine the average power delivered to the circuit.*

Solution The phase angle between voltage and current is 30°.

Effective value of the voltage $V_{eff} = \dfrac{100}{\sqrt{2}}$

Effective value of the current $I_{eff} = \dfrac{15}{\sqrt{2}}$

Average power $\qquad P_{av} = V_{eff} I_{eff} \cos\theta$

$$= \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \cos 30°$$

$$= \frac{100 \times 15}{2} \times 0.866 = 649.5 \text{ W}$$

Example 2.71 *Determine the average power delivered to the circuit consisting of an impedance $Z = 5 + j8$ when the current flowing through the circuit is $I = 5\angle 30°$.*

Solution The average power is the power dissipated in the resistive part only.

or $\qquad P_{av} = \dfrac{I_m^2}{2} R$

Current $\qquad I_m = 5\text{A}$

$\therefore \qquad P_{av} = \dfrac{5^2}{2} \times 5 = 62.5 \text{W}$

2.5.3 Apparent Power, Power Factor and Signifiacnce *[JNTU Nov. 2011]*

The power factor is useful in determining useful power (true power) transferred to a load. The highest power factor is 1, which indicates that the current to a load is in phase with the voltage across it (i.e. in the case of resistive load). When the power factor is 0, the current to a load is 90° out of phase with the voltage (i.e. in case of reactive load).

Consider the following equation

$$P_{av} = \frac{V_m I_m}{2} \cos\theta \text{ W} \tag{2.9}$$

In terms of effective values

$$P_{av} = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta$$

$$= V_{eff} I_{eff} \cos\theta \text{ W} \tag{2.10}$$

The average power is expressed in watts. It means the useful power transferred from the source to the load, which is also called true power. If we consider a dc source applied to the network, true power is given by the product of the voltage and the current. In case of sinusoidal voltage applied to the circuit, the product of voltage and current is not the true power or average power. This product is called *apparent power*. The apparent power is expressed in volt amperes, or simply VA.

$\therefore$ Apparent power $= V_{eff} I_{eff}$

In Eq. 2.10, the average power depends on the value of cos θ; this is called the *power factor* of the circuit.

$$\therefore \text{ Power factor } (pf) = \cos\theta = \frac{P_{av}}{V_{eff} I_{eff}}$$

Therefore, power factor is defined as the ratio of average power to the apparent power, whereas apparent power is the product of the effective values of the current and the voltage. Power factor is also defined as the factor with which the volt amperes are to be multiplied to get true power in the circuit.

In the case of sinusoidal sources, the power factor is the cosine of the phase angle between voltage and current

$$pf = \cos\theta$$

As the phase angle between voltage and total current increases, the power factor decreases. The smaller the power factor, the smaller the power dissipation. The power factor varies from 0 to 1. For purely resistive circuits, the phase angle between voltage and current is zero, and hence the power factor is unity. For purely reactive circuits, the phase angle between voltage and current is 90°, and hence the power factor is zero. In an RC circuit, the power factor is referred to as *leading* power factor because the current leads the voltage. In an RL circuit, the power factor is referred to as lagging power factor because the current lags behind the voltage.

Example 2.72 *A sinusoidal voltage $v = 50 \sin \omega t$ is applied to a series RL circuit. The current in the circuit is given by $i = 25 \sin (\omega t - 53°)$. Determine (a) apparent power (b) power factor and (c) average power.*

Solution (a) Apparent power $P = V_{eff} I_{eff}$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$= \frac{50 \times 25}{2} = 625 \text{ VA}$$

(b) Power factor = cos θ

where θ is the angle between voltage and current

$$\theta = 53°$$

∴ power factor = $\cos\theta = \cos 53° = 0.6$

(c) Average power $P_{av} = V_{eff} I_{eff} \cos\theta$

$$= 625 \times 0.6 = 375 \text{ W}$$

Example 2.73 *Find the total current and the power consumed by the circuit.*

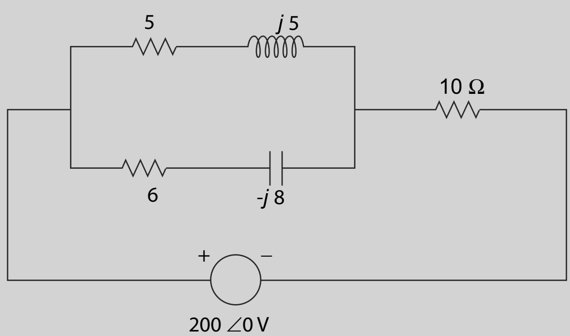


Fig. 2.88

Solution Total impedance of the circuit,

$$Z_T = (5 + j5) \parallel (6 - j8) + 10$$

$$Z_T = 16.15 + j0.769$$

$$I = \frac{V}{Z} = \frac{200\angle 0}{16.15 + j0.769} = 12.35 - j0.588 \text{ A}$$

$$= 12.36\angle -2.72°$$

Power consumed = $I^2 R$

$$= (12.36)^2 \times 16.15 = 2467\text{W}$$

or $VI\cos\theta = 200 \times 12.36 \times \cos(-2.72) = 2467$ W.

2.5.4 Real and Reactive Power

We know that the average power dissipated is

$$P_{av} = V_{eff}[I_{eff} \cos \theta] \tag{2.11}$$

From the impedance triangle shown in Fig. 2.89

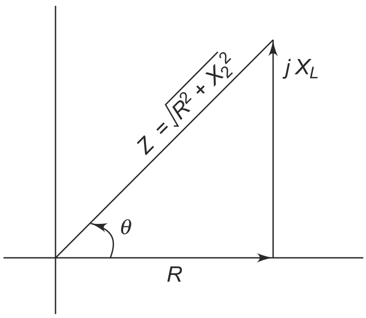


Fig. 2.89

$$\cos\theta = \frac{R}{|Z|} \tag{2.12}$$

and $$V_{eff} = I_{eff} Z \tag{2.13}$$

If we substitute Eqs (2.12) and (2.13) in Eq. (2.11), we get

$$P_{av} = I_{eff}\, Z\left[I_{eff}\,\frac{R}{Z}\right]$$

$$= I_{eff}^2 R \text{ watts} \tag{2.14}$$

This gives the average power dissipated in a resistive circuit.

If we consider a circuit consisting of a pure inductor, the power in the inductor

$$P_r = iv_L \tag{2.15}$$

$$= iL\frac{di}{dt}$$

Consider $$i = I_m \sin(\omega t + \theta)$$

Then $$P_r = I_m^2 \sin(\omega t + \theta)\, L\omega \cos(\omega t + \theta)$$

$$= \frac{I_m^2}{2}(\omega L)\sin 2(\omega t + \theta)$$

$\therefore$ $$P_r = I_{eff}^2(\omega L) \sin 2(\omega t + \theta) \tag{2.16}$$

From the above equation, we can say that the average power delivered to the circuit is zero. This is called *reactive* power. It is expressed in volt-amperes reactive (VAR).

$$P_r = I_{eff}^2 X_L \text{ VAR} \tag{2.17}$$

From Fig. 2.89, we have

$$X_L = Z \sin\theta \tag{2.18}$$

Substituting Eq. 2.18 in Eq. 2.17, we get

$$P_r = I_{eff}^2 Z \sin\theta$$

$$= (I_{eff} Z) I_{eff} \sin\theta$$

$$= V_{eff} I_{eff} \sin\theta \text{ VAR}$$

2.5.5 Complex Power

A generalised impedance phase diagram is shown in Fig. 2.90. A phasor relation for power can also be represented by a similar diagram because of the fact that true power P_{av} and reactive power P_r differ from R and X by a factor I^2_{eff}, as shown in Fig. 2.90.

The resultant power phasor $I^2_{eff} Z$, represents the apparent power P_a.
At any instant in time, P_a is the total power that appears to be transferred between the source and reactive circuit. Part of the apparent power is true power and part of it is reactive power. Absolute value of complex power is called apparent power.

$$\therefore \qquad P_a = I^2_{eff} Z$$

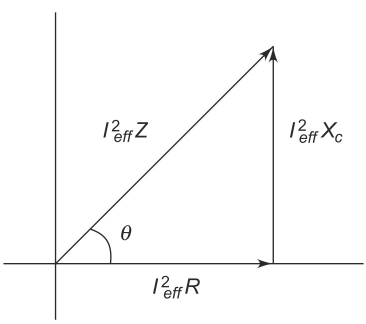


Fig. 2.90

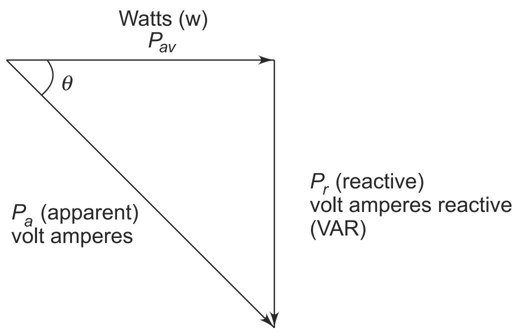


Fig. 2.91

The power triangle is shown in Fig. 2.91.

From Fig. 2.90, we can write

$$P_{true} = P_a \cos \theta$$

or average power $P_{av} = P_a \cos \theta$

and reactive power $P_r = P_a \sin \theta$

Example 2.74 *In an electrical circuit R, L and C are connected in parallel. R = 10Ω L = 0.1 H, C = 100 μF. The circuit is energized with a supply at 230 V, 50 Hz. Calculate*

(a) Impedance
(b) Current taken from supply
(c) p.f. of the circuit
(d) Power consumed by the circuit

Solution The circuit is as shown in Fig. 2.92.

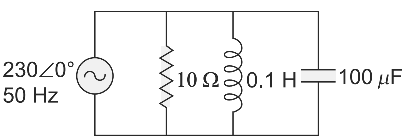


Fig. 2.92

The impedance of 3 branches are

$$Z_1 = 10\,\Omega$$

$$Z_2 = j2\pi f L = 2 \times \pi \times 50 \times 0.1 = j31.41\,\Omega$$

$$Z_3 = \frac{-j}{2\pi fc} = \frac{-j}{2\times\pi\times 50\times 100\times 10^{-6}} = -j31.84\ \Omega$$

(a) Impedance of circuit $Z = \left[\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right]^{-1}$

$$= \left[\frac{1}{10}+\frac{1}{j31.41}+\frac{1}{-j31.84}\right]^{-1}$$

$$\approx 10\ \Omega$$

(b) Current taken from supply $I = \frac{V}{Z} = \frac{230\angle 0°}{10} = 23A.$ i.e. $23\angle 0°$A

(c) p.f. of the circuit = cos θ = 1

(d) Power consumed by the circuit

Real power consumed = I^2R = $23^2\times 10$ = 5.3 kW

Reactive power consumed = 0 KVAR

Example 2.75 *A coil of resistance I0 V and an inductance of 0.1 H is connected in series with a capacitor of capacitance 150 μF a cross at 200 V, 50 Hz supply. Calculate (i) Impedance (ii) Current (iii) Power and power factor of the circuit.*

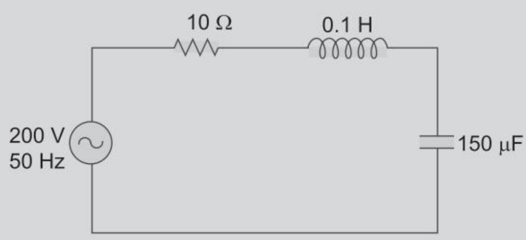


Fig. 2.93

Solution (i) Total impedance

$$Z = R + j\omega L - \frac{j}{\omega c}$$

$$= 10 + j31.45 - j21.22$$

$$= 10 + j10.194$$

$$= 14.279\underline{|45.55}$$

(ii) Current $I = \frac{V}{Z}$

$$= \frac{200\underline{|0°}}{14.279\underline{|45.55°}}$$

$$= 14\underline{|-45.55°}$$

(iii) Power factor = cos (45.55°)

= 0.7 lagging

Real power = $VI \cos \theta$

$= 200 \times 14 \times 0.7$

$= 1.9$ kW

Reactive power = $VI \sin \phi$

$= 200 \times 14 \times \sin(-45.55)$

$= -1.998$ KVAR

"−1" Sign indicates that it absorbs the reactive power.

Example 2.76 *Two coils A and B have resistance of 12V and 6V and inductances of 0.02 and 0.03 H respectively. These are connected in parallel and a voltage of 200 V at 50 Hz is applied to their combination. Find*

(a) Current in the each coil.
(b) The total current and the
(c) The power factor of the circuit.
(d) Power consumed by each coil and total power. [JNTU June 2009]

Solution

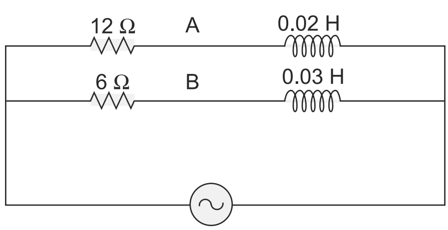


Fig. 2.94

Impedance of coil A = $(12 + j \times 50 \times 0.02 \times 2\pi)\ \Omega = (12 + j6.28)\ \Omega$.

Impedance of coil B = $(6 + j \times 50 \times 0.03 \times 2\pi)\ \Omega = (6 + j9.42)\ \Omega$.

(a) ∴ Current in coil A $= \dfrac{200}{12 + j6.28}$ amp $= 14.767\angle -27.63^\circ$ amp

$= (13.083 - j6.848)$ amp

∴ Current in coil B $= \dfrac{200}{6 + j9.42}$ amp $= 17.907\angle -57.51^\circ$ amp

$= (9.619 - j15.104)$ amp

(b) ∴ The total current = $[(13.083 + 9.619) - j(6.848 + 15.104)]$ amp

$= (22.702 - j21.952)$ amp

$= 31.579\angle -44.04^\circ$ amp

(c) ∴ Power factor = $\cos(-44.04^\circ) = 0.719$

(d) ∴ Real power consumed by coil A = 200 × 14.769 × cos (27.63°) watt

= 2616.59 watt

Real power consumed by coil B = 200 × 17.907 × cos(−57.51°) watt

= 1923.76 watt

Real power consumed by the total network

= 200 × 31.579 × cos (44.04°) watt

= 4540.14 watt

Reactive power consumed by coil A = 200 × 14.767 × sin (27.63°) VAR

= 1369.67 VAR

Reactive power consumed by coil B = 200 × 17.907 × sin(−57.51°) VAR

= −3020.85 VAR

Reactive power consumed by the total network

= 200 × 31.579 × sin (44.04°) VAR

= 4390.49 VAR

Example 2.77 *Find complex power in the following circuit.* *[JNTU May/June 2004]*

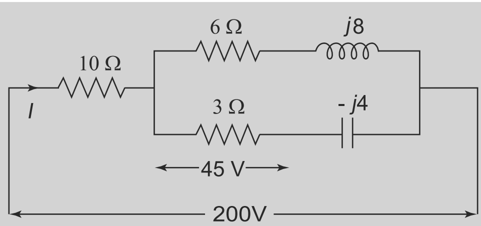


Fig. 2.95

Solution Taking the source voltage as reference

$V = 200\angle 0$ V

$$I = \frac{200\angle 0}{10 + \dfrac{(6+j8)(3-j4)}{(9+j4)}} = 13.396 + j1.886 = 13.52\angle 8°$$

Complex power = VI^*

$= (200 \angle 0)(13.52 \angle -8°)$

$S = VI^* = 2704 \angle -8°$ VA

Complex power $(P + jQ) = 2704 \angle -8° = (2677.68 - j376.32)$

$P = 2677.68\ W$; $Q = 376.32$ VAR leading.

Example 2.78 *In the circuit shown in Fig. 2.96, a voltage of v(t) = 50 sin (ωt + 30°) is applied. Determine the true power, reactive power and power factor.*

[JNTU Nov. 2011]

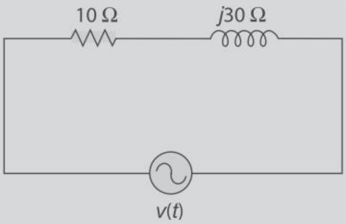


Fig. 2.96

Solution The voltage applied to the circuit is

$$v(t) = 50 \sin(\omega t + 30°)$$

The current in the circuit is

$$I = \frac{V}{Z} = \frac{50\angle 30^o}{10 + j30} = \frac{50\angle 30^o}{31.6\angle 71.56^o}$$

$$= 1.58\ \angle -41.56°\ \text{A}$$

The phasor diagram is shown in Fig. 2.97.

The phase angle between voltage and current $\theta = 71.56°$

Power factor $= \cos\theta = \cos 71.56° = 0.32$

True power or average power

$$P_{av} = V_{eff} I_{eff} \cos\theta$$

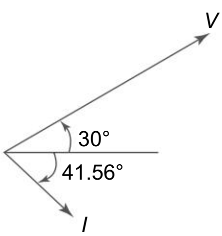


Fig. 2.97

$$= \frac{50 \times 1.58}{\sqrt{2} \times \sqrt{2}} \cos 71.56°$$

$$= 12.49\ \text{W}$$

$$\text{Reactive power} = V_{eff} I_{eff} \sin\theta$$

$$= \frac{50 \times 1.58}{\sqrt{2} \times \sqrt{2}} \sin 71.56°$$

$$= 37.47\ \text{VAR}$$

Example 2.79 *Determine the circuit constants in the circuit shown in Fig. 2.98, if the applied voltage to the circuit v(t) = 100 sin (50t + 20°). The true power in the circuit is 200 W and the power factor is 0.707 lagging.*

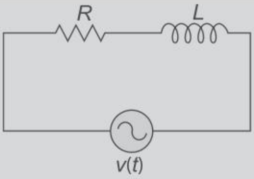


Fig. 2.98

Solution Power factor $= \cos\theta = 0.707$

∴ The phase angle between voltage and current

$$\theta = \cos^{-1} 0.707 = 45°$$

Here the current lags behind the voltage by 45°.

Hence, the current equation is $i(t) = I_m \sin(50t - 25°)$

True power $= V_{eff} I_{eff} \cos\theta = 200$ W

$$I_{eff} = \frac{200}{V_{eff}\cos\theta}$$

$$= \frac{200}{(100/\sqrt{2})\times 0.707} = 4\text{ A}$$

$$I_m = 4\times\sqrt{2} = 5.66\text{ A}$$

∴ The current equation is $i(t) = 5.66 \sin(50t - 25°)$

The impedance of the circuit

$$Z = \frac{V}{I} = \frac{(100/\sqrt{2})\angle 20^\circ}{(5.66/\sqrt{2})\angle -25^\circ}$$

∴ $Z = 17.67\angle 45° = 12.5 + j12.5$

Since $Z = R + jX_L = 12.5 + j12.5$

∴ $R = 12.5$ ohms, $X_L = 12.5$ ohms

$$X_L = \omega L = 12.5$$

$$L = \frac{12.5}{50} = 0.25\text{ H}$$

Example 2.80 *A voltage v(t) = 150 sin 250t is applied to the circuit shown in Fig. 2.99. Find the power delivered to the circuit and the value of inductance in Henrys.*

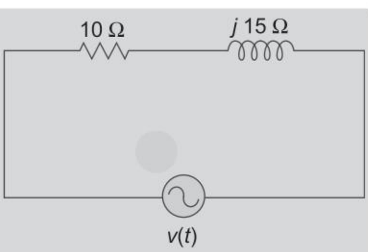


Fig. 2.99

Solution $Z = 10 + j15\ \Omega$

The impedance $Z = 18\angle 56.3°$

The impedance of the circuit $Z = \dfrac{V}{I}$

$$18\angle 56.3° = \frac{(150/\sqrt{2})\angle 0^\circ}{I}$$

∴ Phasor current $$I = \frac{150/\sqrt{2}}{18\angle 56.3^\circ} = 5.89\angle -56.3°$$

The current equation is $i(t) = 5.89\sqrt{2}\sin(250t - 56.3°)$

$$= 8.33 \sin (250t - 56.3°)$$

The phase angle between the current and the voltage

$$\theta = 56.3°$$

The power delivered to the circuit

$$P_{av} = VI\cos\theta$$

$$= \frac{150}{\sqrt{2}} \times \frac{8.33}{\sqrt{2}} \cos 56.3°$$

$$= 346.6 \text{ W}$$

The inductive impedance $X_L = 15\ \Omega$

$\therefore \quad \omega L = 15$

$\therefore \quad L = \dfrac{15}{250} = 0.06\text{ H}$

Example 2.81 *Determine the power factor, true power, reactive power and apparent power in the circuit in Fig. 2.100.*

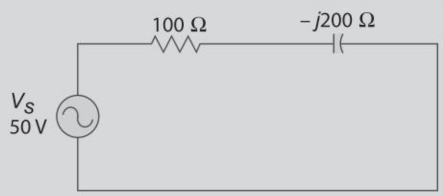


Fig. 2.100

Solution The impedance of the circuit

$$Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{(100)^2 + (200)^2} = 223.6\ \Omega$$

The current $I = \dfrac{V_S}{Z} = \dfrac{50}{223.6} = 0.224$

The phase angle

$$\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$= \tan^{-1}\left(\frac{-200}{100}\right) = -63.4°$$

$\therefore$ The power factor $pf = \cos\theta = \cos(63.4°) = 0.448$

The true power $P_{av} = VI\cos\theta$

$$= 50 \times 0.224 \times 0.448 = 5.01 \text{ W}$$

The reactive power $P_v = I^2 X_C$

$$= (0.224)^2 \times 200 = 10.03 \text{ VAR}$$

The apparent power

$$P_a = I^2 Z = (0.224)^2 \times 223.6 = 11.21 \text{ VA}$$

Example 2.82 *In a certain RC circuit, the true power is 300 W and the reactive power is 1000 W. What is the apparent power?*

Solution The true power P_{true} or $P_{\text{av}} = VI \cos\theta$

$$= 300 \text{ W}$$

The reactive power $P_r = VI \sin\theta$

$$= 1000 \text{ W}$$

From the above results

$$\tan\theta = \frac{1000}{300} = 3.33$$

The phase angle between voltage and current, $\theta = \tan^{-1} 3.33 = 73.3°$

The apparent power $P_a = VI = \dfrac{300}{\cos 73.3°} = 1043.9 \text{ VA}$

Example 2.83 *A sine wave of v(t) = 200 sin 50t is applied to a 10 Ω resistor in series with a coil. The reading of a voltmeter across the resistor is 120 V and across the coil, 75 V. Calculate the power and reactive volt-amperes in the coil and the power factor of the circuit.*

Solution The rms value of the sine wave

$$V = \frac{200}{\sqrt{2}} = 141.4 \text{ V}$$

Voltage across the resistor, $V_R = 120$ V

Voltage across the coil, $V_L = 75$ V

$\therefore$ $IR = 120$ V

The current in resistor, $I = \dfrac{120}{10} = 12 \text{ A}$

Since $IX_L = 75$ V

$\therefore$ $X_L = \dfrac{75}{12} = 6.25\ \Omega$

Power factor, $pf = \cos\theta = \dfrac{R}{Z}$

where $Z = 10 + j6.25 = 11.8 \angle 32°$

$\therefore$ $\cos\theta = \frac{R}{Z} = \frac{10}{11.8} = 0.85$

True power $P_{true} = I^2 R = (12)^2 \times 10 = 1440$ W

Reactive power $P_r = I^2 X_L = (12)^2 \times 6.25 = 900$ VAR

Example 2.84 *For the circuit shown in Fig. 2.101, determine the true power, reactive power and apparent power in each branch. What is the power factor of the total circuit?*

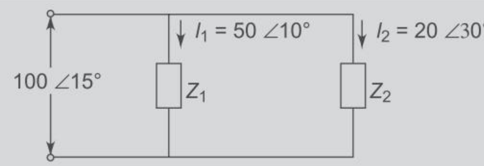


Fig. 2.101

Solution In the circuit shown in Fig. 2.100, we can calculate Z_1 and Z_2.

Impedance $Z_1 = \frac{100\angle 15^o}{50\angle 10^o} = 2\angle 5^o = (1.99 + j0.174)\ \Omega$

Impedance $Z_2 = \frac{100\angle 15^o}{20\angle 30^o} = 5\angle -15^o = (4.83 - j1.29)\ \Omega$

True power in branch Z_1 is $P_{t_1} = I_1^2 R = (50)^2 \times 1.99 = 4975$ W

Reactive power in branch Z_1, $P_{r_1} = I_1^2 X_L$

$= (50)^2 \times 0.174 = 435$ VAR

Apparent power in branch Z_1, $P_{a_1} = I_1^2 Z_1$

$= (50)^2 \times 2$

$= 2500 \times 2 = 5000$ VA

True power in branch Z_2, $P_{t_2} = I_2^2 R$

$= (20)^2 \times 4.83 = 1932$ W

Reactive power in branch Z_2, $P_{r_2} = I_2^2 X_C$

$= (20)^2 \times 1.29 = 516$ VAR

Apparent power in branch Z_2, $P_{a_2} = I_2^2 Z_2$

$= (20)^2 \times 5 = 2000$ VA

Total impedance of the circuit, $Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$

$$= \frac{2\angle 5^\circ \times 5 \times \angle -15^\circ}{1.99 + j0.174 + 4.83 - j1.29}$$

$$= \frac{10\angle -10^\circ}{6.82 - j1.116}$$

$$= \frac{10\angle -10^\circ}{6.9\angle -9.29^\circ} = 1.45\angle -0.71^\circ$$

The phase angle between voltage and current, $\theta = 0.71^\circ$

$\therefore$ Power factor $pf = \cos\theta$

$= \cos 0.71^\circ = 0.99$ leading

Example 2.85 *A voltage of $v(t) = 141.4 \sin \omega t$ is applied to the circuit shown in Fig. 2.102. The circuit dissipates 450 W at a lagging power factor, when the voltmeter and ammeter readings are 100 V and 6 A, respectively. Calculate the circuit constants.*

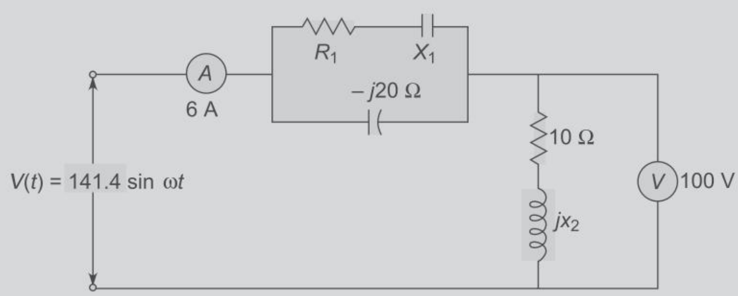


Fig. 2.102

Solution The magnitude of the current passing through $(10 + jX_2)\ \Omega$ is

$$I = 6\text{ A}$$

The magnitude of the voltage across the $(10 + jX_2)$ ohms, $V = 100$ V. The magnitude of impedance $(10 + jX_2)$ is V/I.

Hence $$\sqrt{10^2 + X_2^2} = \frac{100}{6} = 16.67\,\Omega$$

$\therefore$ $$X_2 = \sqrt{(16.67)^2 - (10)^2} = 13.33\,\Omega$$

Total power dissipated in the circuit $= VI\cos\theta = 450$ W

$\therefore$ $$V = \frac{141.4}{\sqrt{2}} = 100\text{ V}$$

$$I = 6\text{ A}$$

$$100 \times 6 \times \cos\theta = 450$$

The power factor $pf = \cos\theta = \dfrac{450}{600} = 0.75$

$\theta = 41.4°$

The current lags behind the voltage by 41.4°

The current passing through the circuit, $I = 6\angle -41.4°$

The voltage across $(10 + j13.33)\ \Omega$, $V = 6\angle -41.4° \times 16.66\angle 53.1°$

$= 100\angle 11.7°$

The voltage across parallel branch, $V_1 = 100\angle 0° - 100\angle 11.7°$

$= 100 - 97.9 - j20.27$

$= (2.1 - j20.27)\text{V} = 20.38\angle -84.08°$

The current in $(-j20)$ branch, $I_2 = \dfrac{20.38\angle -84.08°}{20\angle -90°} = 1.02\angle +5.92°$

The current in $(R_1 - jX_1)$ branch, I_1

$= 6\angle -41.4° - 1.02\angle 5.92° = 4.5 - j3.97 - 1.01 - j0.1$

$= 3.49 - j4.07 = 5.36\angle -49.39°$

The impedance $Z_1 = \dfrac{V_1}{I_1} = \dfrac{20.38\angle -84.08°}{5.36\angle -49.39°}$

$= 3.8\angle -34.69° = (3.12 - j2.16)\ \Omega$

Since $R_1 - jX_1 = (3.12 - j2.16)\ \Omega$

$R_1 = 3.12\ \Omega$

$X_1 = 2.16\ \Omega$

Example 2.86 *Determine the value of the voltage source and power factor in the following network if it delivers a power of 100 W to the circuit shown in Fig. 2.103. Find also the reactive power drawn from the source.*

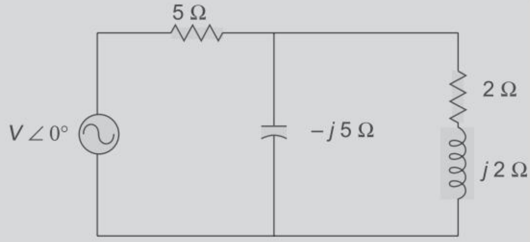


Fig. 2.103

Solution Total impedance in the circuit,

$$Z_{eq} = 5 + \frac{(2+j2)(-j5)}{2+j2-j5}$$

$$= 5 + \frac{10-j10}{2-j3} = 5 + \frac{14.14\angle -45°}{3.6\angle -56.3°} = 5 + 3.93\angle 11.3°$$

$$= 5 + 3.85 + j0.77 = 8.85 + j0.77 = 8.88\angle 4.97°$$

Power delivered to the circuit, $P_T = I^2 R_T = 100$ W

$\therefore \quad I^2 \times 8.85 = 100$

Current in the circuit, $I = \sqrt{\dfrac{100}{8.85}} = 3.36\,\text{A}$

Power factor $pf = \cos\theta = \dfrac{R}{Z}$

$$= \frac{8.85}{8.88} = 0.99$$

Since $VI \cos\theta = 100$ W

$$V \times 3.36 \times 0.99 = 100$$

$$\therefore \quad V = \frac{100}{3.36 \times 0.99} = 30.06\,\text{V}$$

The value of the voltage source, $V = 30.06$ V

Reactive power $P_r = VI \sin\theta$

$$= 30.06 \times 3.36 \times \sin(4.97°)$$

$$= 30.06 \times 3.36 \times 0.087 = 8.8 \text{ VAR}$$

Example 2.87 *For the circuit shown in Fig. 2.104, determine the circuit constants when a voltage of 100 V is applied to the circuit, and the total power absorbed is 600 W. The circuit constants are adjusted such that the currents in the parallel branches are equal and the voltage across the inductance is equal and in quadrature with the voltage across the parallel branch.*

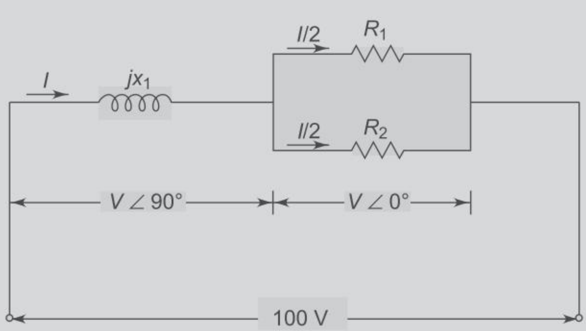


Fig. 2.104

Solution Since the voltages across the parallel branch and the inductance are in quadrature, the total voltage becomes $100\angle 45°$ as shown in Fig. 2.105.

Total voltage is $100 \angle 45° = V + j0 + 0 + jV$

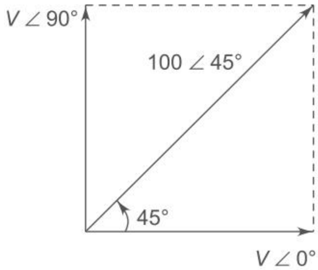


Fig. 2.105

From the above result, $70.7 + j70.7 = V + jV$

$$\therefore \qquad V = 70.7$$

If we take current as the reference, then current passing through the circuit is $I \angle 0°$. Total power absorbed by the circuit $= VI \cos\theta = 600$ W

or $\qquad 100 \times I \times \cos 45° = 600$ W

$$\therefore \qquad I = 8.48 \text{ A}$$

Hence, the inductance, $X_1 = \dfrac{V\angle 90^\circ}{I\angle 0^\circ} = \dfrac{70.7\angle 90^\circ}{8.48} = 8.33\angle 90^\circ$

$$\therefore \qquad X_1 = 8.33\ \Omega$$

Current through the parallel branch, R_1 is $I/2 = 4.24$ A

Resistance, $\qquad R_1 = \dfrac{V\angle 0}{I/2\angle 0} = \dfrac{70.7}{4.24} = 16.6\,\Omega$

Current through parallel branch R_2 is $I/2 = 4.24$ A

Resistance is $\qquad R_2 = \dfrac{70.7}{4.24} = 16.67\,\Omega$

Example 2.88 *Determine the average power delivered by the $500\angle 0^\circ$ voltage source in Fig. 2.106 and also dependent source.*

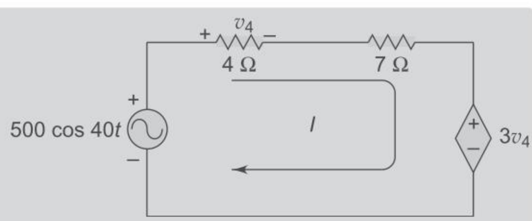


Fig. 2.106

Solution The current I can be determined by using Kirchhoff's voltage law.

$$I = \frac{500\angle 0^\circ - 3v_4}{7+4}$$

where $\qquad v_4 = 4I$

$$I = \frac{500\angle 0^\circ}{11} - \frac{12I}{11}$$

$$I = 21.73\angle 0^\circ$$

Power delivered by the $500\angle 0°$ voltage source $= \dfrac{500 \times 21.73}{2} = 5.432\,\text{kW}$

Power delivered by the dependent voltage source

$$= \frac{3v_4 \times I}{2} = \frac{3\times 4I\times I}{2} = 2.833\,\text{kW}$$

Example 2.89 *Find the average power delivered by the dependent voltage source in the circuit shown in Fig. 2.107.*

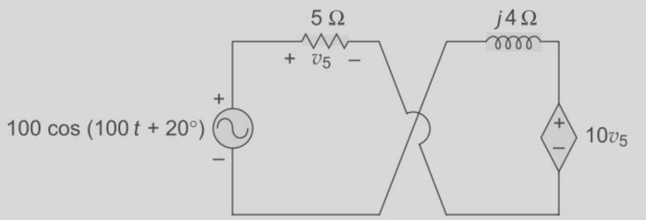


Fig. 2.107

Solution The circuit is redrawn as shown in Fig. 2.108.

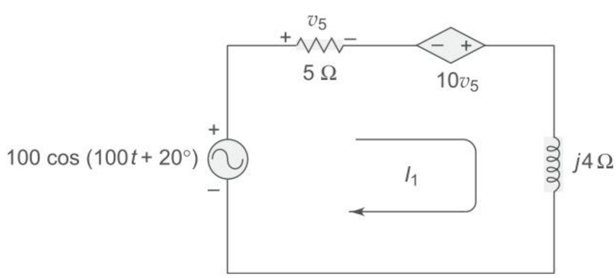


Fig. 2.108

Assume current I_1 flowing in the circuit.

The current I_1 can be determined by using Kirchhoff's voltage law.

$$I_1 = \frac{100\angle 20^\circ + 10\times 5 I_1}{5 + j4}$$

$$I_1 - \frac{50\, I_1}{5+j4} = \frac{100\angle 20^\circ}{5+j4}$$

$$I_1 = 2.213\ \angle -154.9^\circ$$

Average power delivered by the dependent source

$$= \frac{V_m\, I_m}{2} = \cos\theta$$

$$= \frac{10 V_5\, I_1}{2}\cos\theta$$

$$= \frac{50\times(2.213)^2}{2} = 122.43\,\text{W}$$

Example 2.90 *For the circuit shown in Fig. 2.109, find the average power delivered by the voltage source.*

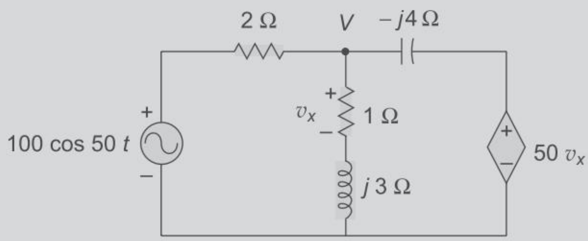


Fig. 2.109

Solution Applying Kirchhoff's current law at node

$$\frac{V-100\angle 0^\circ}{2}+\frac{V}{1+j3}+\frac{V-50V_x}{-j4}=0$$

$$V_x=\frac{V}{1+j3}\text{ volts}$$

Substituting in the above equation, we get

$$\frac{V-100\angle 0^\circ}{2}+\frac{V}{1+j3}+\frac{V}{-j4}-\frac{50\,\text{V}}{(1+j3)(-j4)}=0$$

$$V = 14.705\ \angle 157.5^\circ$$

$$I=\frac{V-100\angle 0^\circ}{2}=\frac{14.705\angle 157.5^\circ-100\angle 0^\circ}{2}=56.865\angle 177.18^\circ$$

Power delivered by the source $=\dfrac{100\times 56.865\cos 177.18^\circ}{2}$

$= 2.834$ kW

Example 2.91 *For the circuit shown in Fig. 2.110, find the average power delivered by the dependent current source.*

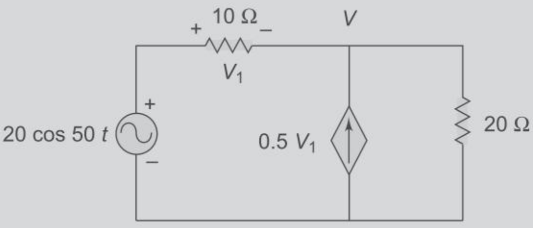


Fig. 2.110

Solution Applying Kirchhoff's current law at node

$$\frac{V-20\angle 0^\circ}{10}-0.5V_1+\frac{V}{20}=0$$

where $V_1 = 20\ \angle 0^\circ - V$

Substituting V_1 in the above equation, we get

$$V = 18.46 \angle 0°$$

$$V_1 = 1.54 \angle 0°$$

Average power delivered by the dependent source

$$\frac{V_m I_m \cos\theta}{2} = \frac{18.46 \times 0.5 \times 1.54}{2} = 7.107 \text{ W}$$

Example 2.92 *In the circuit shown in Fig. 2.111, calculate,*

(i) The total impedance
(ii) The total current
(iii) Power factor
(iv) The total S, P and Q
(v) The total admittance. Also, draw vector diagram. [JNTU May/June 2006]

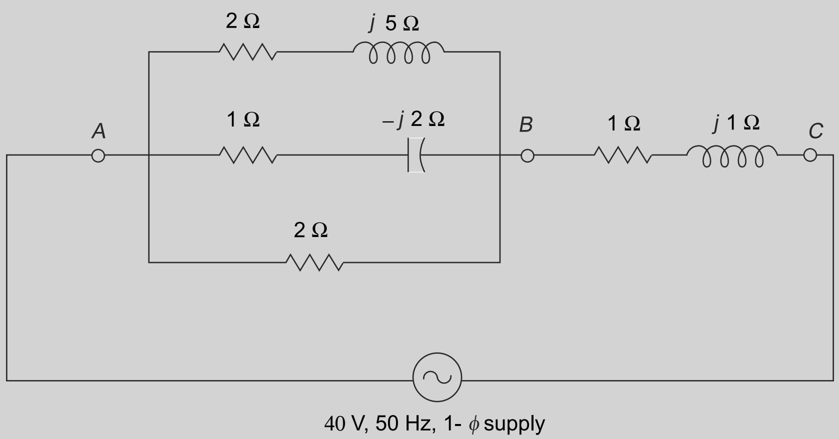


Fig. 2.111

Solution (i)

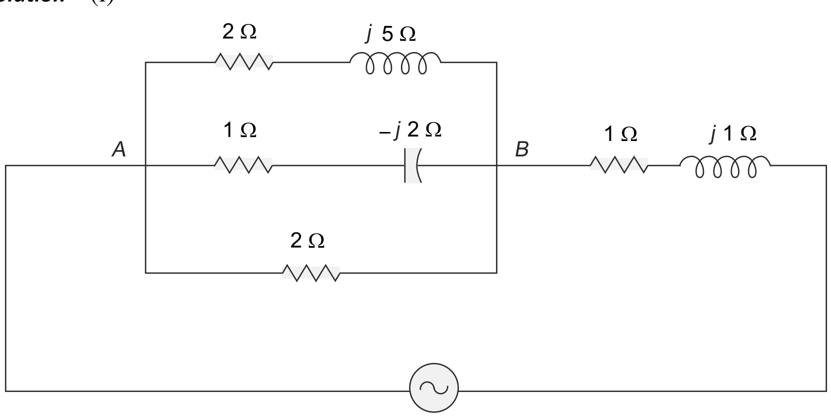


Fig. 2.112

Admittance between A and B is

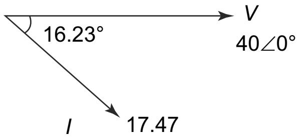


Fig. 2.113

$$\frac{1}{2+j5}+\frac{1}{1-j2}+\frac{1}{2}$$

$$=\frac{1}{5.38\angle 68.2^\circ}+\frac{1}{2.24\angle -63.4^\circ}+0.5$$

$$=0.069-j0.17+0.199+j0.399+0.5$$

$$=0.768+j0.229=0.8\angle 16.6^\circ$$

Impedance between A and B $=\dfrac{1}{0.8\angle 16.6^\circ}=1.25\angle -16.6^\circ$

Total impedance $=1+j1+1.198-j0.36=2.29\angle 16.23^\circ\,\Omega$

(ii) Total current $=\dfrac{40}{2.29\angle 16.23^\circ}=17.47\angle -16.23^\circ$ A

(iii) Power factor $=\cos 16.23=0.96$ lagging

(iv) $P=VI\cos\varphi$

$$=40\times 17.47\cos 16.23^\circ=670.95\text{ W}$$

$$Q=VI\sin\varphi$$

$$=40\times 17.47\sin 16.23^\circ=195.31\text{ VAR}$$

$$S=P+jQ=640.95+j195.31$$

$$=698.798\angle 16.23^\circ\text{ VA.}$$

(v) Total admittance $=\dfrac{1}{2.29\angle 16.23^\circ}=0.43\angle -16.23^\circ\nu$

Example 2.93 *The voltage of a circuit is $V = 200\sin(\omega t + 30^\circ)$ and the current is $I = 50\sin(\omega t + 60^\circ)$. Calculate*

(i) the average power, reactive volt-amperes and apparent power

(ii) the circuit elements if $\omega = 100\pi$ rad/sec *[JNTU April/May 2007]*

Solution $V=200\sin(\omega t+30^\circ)$

$$i=50\sin(\omega t+60^\circ)$$

(i) Avg. power $=V_m I_m\cos\theta$

$$=\frac{200}{\sqrt{2}}\times\frac{50}{\sqrt{2}}\cos(60-30)$$

$$P_{av}=4330.127\text{ W.}$$

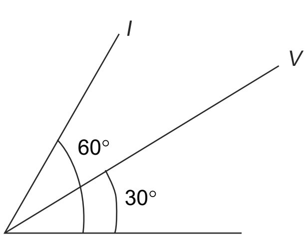


Fig. 2.114

Reactive volt ampere $=V_m I_m\sin\theta$

$$=\frac{200}{\sqrt{2}}\cdot\frac{50}{\sqrt{2}}\sin(60-30)$$

$$P_r=2500\text{ VAR}$$

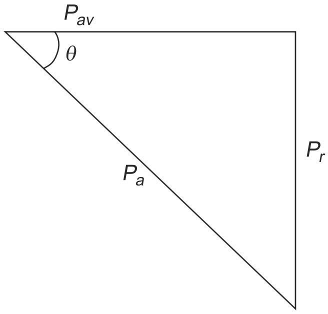


Fig. 2.115

Apparent power

$$= \frac{p_{av}}{\cos\theta} = \frac{4330.127}{\cos 30^\circ} = 5000 \text{ VA}$$

(ii) The current leads the voltage by 30°. Hence the circuit must contain R and C.

$$\tan\theta = \frac{1}{\omega RC} \Rightarrow \tan 30^\circ = \frac{1}{100\pi \times RC}$$

$$\Rightarrow RC = 0.0055 \Rightarrow C = \frac{0.0055}{R}$$

$$Z = \frac{Vm}{I_m} = \sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2}$$

$$\frac{200}{50} = \sqrt{R^2 + \left(\frac{R}{100\pi \times 0.0055}\right)^2}$$

$$4 = 1.155\text{R} \Rightarrow R = \frac{4}{1.155} = 3.46\ \Omega$$

and

$$C = \frac{0.0055}{3.46} = 1.59 \text{ mF}$$

Example 2.94 *In the circuit shown in Fig. 2.116, what 50-Hz voltage is to be applied across A B terminals so that a current of 10 A will flow in the capacitor.*

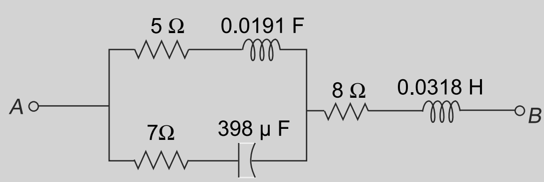


Fig. 2.116

Solution

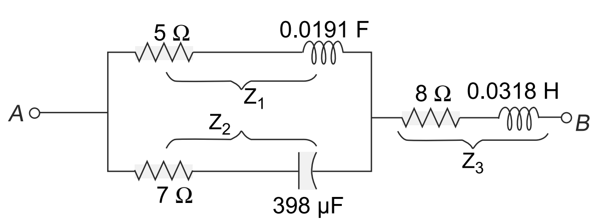


Fig.2.117

$$Z_1 = 5 + j2\pi \times 50 \times 0.0191 = 5 + j6\ \Omega$$

$$Z_2 = 7 + \frac{1}{j2\pi \times 50 \times 398\mu} = 7 - j8\ \Omega$$

$$Z_3 = 8 + j2\pi \times 50 \times 0.0318 = 8 + j10\ \Omega$$

Given that current through the capacitor is 10 A $= I_2$. Hence voltage across Z_2 is

$$V_1 = 10 \times Z_2 = 10\,(7 - j8) = 70 - j80 \text{ V}$$

The current through the other branch is

$$I_1 = \frac{V_1}{Z_1}$$

$$= \frac{70 - j80}{5 + j6} = -2.13 - j13.44 \text{ A}$$

Total current in the network is

$$I = I_1 + I_2$$
$$= -2.13 - j13.44 + 10$$
$$= 7.87 - j13.44\, A$$

Let V_2 be the voltage across Z_3.

$$V_2 = IZ_3$$
$$= (7.87 - j13.44)\,(8 + j10)$$
$$= 197.38 - j28.85 \text{ V}$$

The voltage to be applied across AB terminals so that a current of 10 A will flow in the capacitor $V = V_1 + V_2$

$$= 70 - j80 + 197.38 - j28.85$$
$$= 267.38 - j108.85$$
$$= 288.68 \angle{-22.15°} \text{ V.}$$

Example 2.95 *Find the values of R and C in the circuit shown in Fig. 2.118 so that $V_b = 4V_a$ and V_a and V_b are in phase quadrature.*

[JNTU May/June 2002]

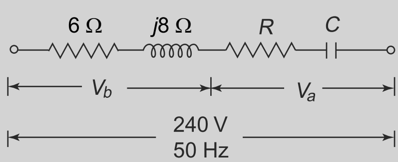


Fig. 2.118

Solution

$$V_b = I\sqrt{6^2 + 8^2} = 10\, I$$

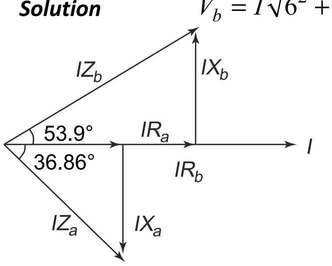


Fig.2.119

$$Z_b = 6 + j8 - 10\angle 53.13°, \quad Z_a = \sqrt{R^2 + X_a^2}$$

$$V_a = I\sqrt{R^2 + X_a^2}, \quad V_b = 4\, V_a$$

$$10\, I = 4 \times I\sqrt{R^2 + X_a^2} \Rightarrow \sqrt{R^2 + X_a^2} - 2.5$$

$$R^2 + X_a^2 = 6.25$$

Let Z_a be at an angle θ with reference. Given that V_a and V_b are in phase quadrature.

$$\therefore \qquad \theta + 53.13^\circ = 90^\circ \Rightarrow \theta = 36.87^\circ$$

$$R = Z_a \cos\phi = 2$$

$$X_a = Z_a \sin\phi = 1.5\ \Omega$$

$$\Rightarrow \quad C = \frac{1}{2\pi f X_a} = \frac{1}{2\pi \times 50 \times 1.5} = 2.12\,\text{mF}$$

Example 2.96 *Determine the branch and total active and reactive powers in the parallel circuit shown in Fig. 2.120. Use j notation* *[JNTU May/June 2002]*

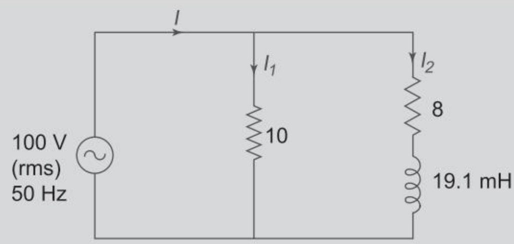


Fig. 2.120

Solution *Branch I*

$I_1 = 10\text{ A } \angle 0°$

Active power

$= I^2R = 10^2\,(10)$

$= 1$ KW

Reactive power $= 0$

Branch II

$$I_2 = \frac{100}{8 + j6} = 10\angle -36.86° = 8 - j6$$

Complex power $= VI^* = 100\,(8 + j6)$

$= 800 + j\,600$

Active power $= 0.8$ KW

Reactive power $= 0.6$ KVAR

$\therefore$ Total active power in the circuit $= 1.8$ KW

$\therefore$ Total reactive power in the circuit $= 0.6$ KVAR

Example 2.97 *Find the branch currents, total current and the total power in the circuit shown below.* *[JNTU May/June 2004]*

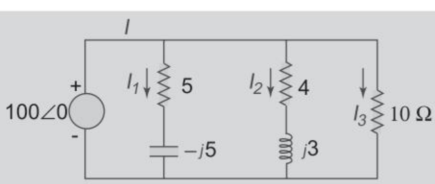


Fig. 2.121

Solution Branch currents $I_1 = \dfrac{100 + j0}{5 - j5} = 10 + j10$

$$I_2 = \frac{100 + j0}{4 + j3} = 16 - j12$$

$$I_3 = \frac{100 + j0}{10} = 10 + j0$$

$$\text{Total current } (I) = I_1 + I_2 + I_3$$
$$= 36 - j2$$
$$= 36.055\angle -3.179^\circ$$

$$\text{Total power} = VI \times \cos\phi$$
$$= 100 \times 36.055 \times \cos 3.179^\circ$$
$$= 3599.95 \text{ watts.}$$

Example 2.98 *Find the total current and the power consumed by the circuit.*

[JNTU May/June 2004]

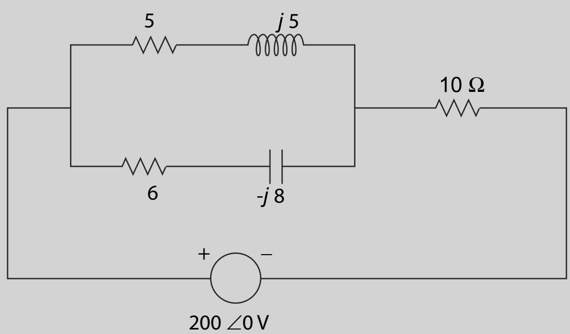


Fig. 2.122

Solution Total impedance of the circuit,

$$Z_T = (5 + j5) \parallel (6 - j8) + 10$$

$$Z_T = 16.15 + j0.769$$

$$I = \frac{V}{Z_T} = \frac{200\angle 0}{16 + 5 + j0.769}$$
$$= 12.35 - j0.588 \text{ A}$$
$$= 12.36\angle -2.72^\circ$$

$$\text{Power consumed} = I^2R$$
$$= (12.36)^2 \times 16.15 = 2467 \text{ W}$$

or $\quad VI\cos\theta = 200 \times 12.36 \times \cos(-2.72)$
$$= 2467 \text{ W.}$$

Example 2.99 *Find the value of R_1 and X_1 when a lagging current in the circuit gives a power of 2 kW.* *[JNTU May/June 2004]*

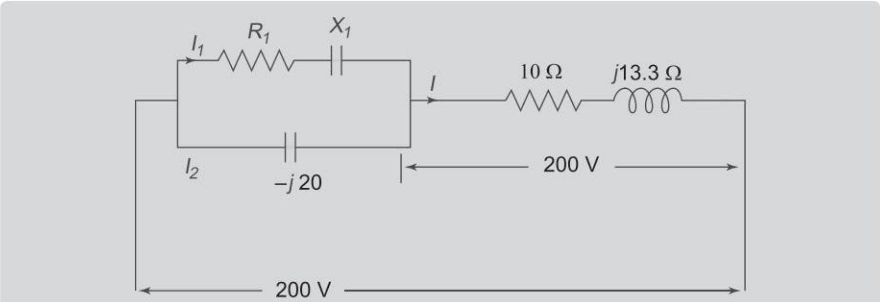


Fig. 2.123

Solution Let us take the voltage across (10 + j13.3 Ω) impedance as reference and calculate the total current I.

$$I = \frac{200\angle 0}{10 + j13.3} = 7.223 - j9.606 = 12.02\angle -53.06°\text{A}$$

Let us assume the phase angle between supply voltage and total current as φ which is equal to (θ + 53.06°).

Hence, real power in the circuit 2000 = 200 × 12.02 cos (θ + 53.06)

Therefore, θ = −19.5° and source voltage V = 200 ∠ −19.5°

Voltage across $R_1 + jX_1$ = 200 ∠ −19.5° − 200 ∠ 0°

$$= -11.47 - j\,66.76$$

$$I_2 = \frac{-11.47 - j66.76}{-j20} = 3.338 - j0.5735$$

$$I_1 = I - I_2$$

$$= 7.223 - j9.606 - 3.338 + j0.5735$$

$$= 9.8325\angle -66.72^{v}$$

$$Z_1 = \frac{V}{I_1} = \frac{-11.47 - j66.76}{9.8325\angle -66.72}$$

$$= 5.776 - j3.7543$$

Thus, R_1 = 5.776 Ω and X_1 = 3.7543 Ω.

Example 2.100 *A metal filament lamp, rated at 750 watts, 100 V is to be connected in series with a capacitor across a 230 V, 60 Hz supply. Calculate*

(a) the capacitance required, and

(b) the power factor. *[JNTU May/June 2008]*

Solution Given power across metal filament lamp = 750 watts

Voltage across metal filament lamp = 100 V

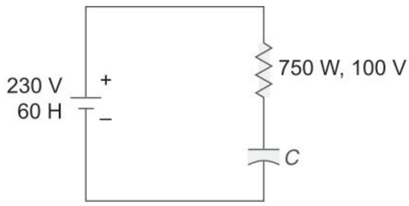


Fig. 2.124

Supply voltage = 230 V

Supply frequency = 60 Hz

Resistance across metal filament lamp =

$$R = \frac{V^2}{P} = \frac{(100)^2}{750} = 13.33\ \Omega$$

Current through metal filament lamp = $I = \frac{V}{R} = \frac{100}{13.33} = 7.5\text{ Amp}$

Impedance in the circuit

$$Z = \sqrt{R^2 + X_C^2} = \frac{V}{I}$$

$$\sqrt{R^2 + X_C^2} = \frac{230}{7.5} = 30.66$$

$$\sqrt{(13.33)^2 + X_C^2} = 30.66$$

$$X_C = 2.7.617 \Rightarrow \frac{1}{\omega c} = 27.617$$

$$\Rightarrow \qquad C = 9.605 \times 10^{-5}\, F$$

$\therefore$ Capacitance $C = 96.05\ \mu F$

$$\cos\theta = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{13.33}{30.66}$$

Power factor = $\cos\theta = 0.43468$

Example 2.101 *In the circuit (Fig. 2.125) shown, determine the voltage V_{AB} to be applied to the circuit if a current of 2.5 A is required to flow in the capacitor. Determine also total power factor and total active and reactive powers. Draw the phasor diagram.* *[JNTU May/June 2006]*

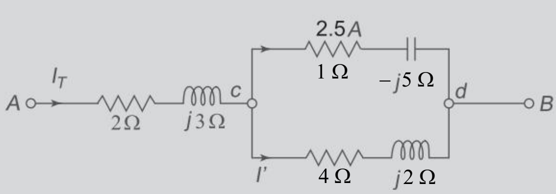


Fig. 2.125

Solution $V_{cd} = 2.5\,(1 - j5) = I'\,(4 + j2)$

(Assuming "I" is the current through '4 + j2' Ω)

$$I' = \frac{2.5(1-j5)}{4+j2} = 2.85\angle-105.25°$$

$$= -0.75 - j2.75$$

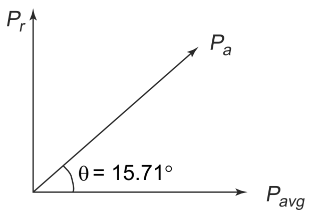


Fig. 2.126

$$\therefore \quad I_T = 2.5 - 0.75 - j2.75 \qquad (\because I_T = 2.5 + I')$$

$$= 1.75 - j2.75 = 3.26\angle-57.53°$$

$$V_{AB} = I_T(2+j3) + 2.5(1-j5)$$

$$= (1.75 - 2.75j)(2+j3) + 2.5(1-j5)$$

$$= 14.25 - 12.75j = 19.12\angle-41.82°$$

$$Z_{AB} = \frac{V_{AB}}{I_T} = \frac{19.12\angle-41.82°}{3.26\angle-57.53°} = 5.865\angle15.71°$$

$$\theta = 15.71°$$

Total power factor $= \cos\theta = \cos 15.71° = 0.962$

Total active power $(P_{avg}) = V_{AB} I_T \cos\theta$

$$= 19.12 \times 3.26 \times 0.962 = 59.96 \text{ W}$$

Total reactive power $(P_r) = V_{AB} I_T \sin\theta$

$$= 19.12 \times 3.26 \times \sin 15.71^o = 16.87 \text{ VAR}$$

Apparent power $P_a = V_{AB} I_T = 19.12 \times 3.26 = 62.3112 VA$

Example 2.102 *A current of 5 A flows through a non-inductive resistance in series with a chocking coil when supplied at 250v, 50 Hz. If the voltage across the non inductive resistance is 125 V and that across that coil 200 V, calculate the Impedance, Reactance and Resistance of the coil, power absorbed by the coil and the total power draw the phasor diagram.* [JNTU May/June 2006]

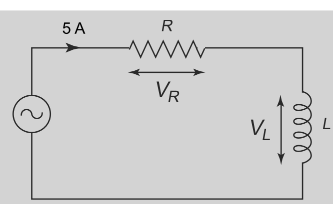


Fig. 2.127

Solution Given

$$|V_R| = 125 \text{ V}$$

$$|V_L| = 200 \text{ V}$$

$$|I| = 5 \text{ A}$$

$$|V_R| = |I|R = 125 \text{ V} \qquad \Rightarrow \quad R = \frac{125}{5} = 25\,\Omega \quad (\nu\ I = 5\text{A})$$

$$|V_L| = |I|X_L = |I|(j\omega L) \quad \therefore \quad |V_L| = 200\text{ V}$$

$$\Rightarrow |X_L| = 40 \qquad \Rightarrow \quad 5(2\pi \times 50)L = 200$$

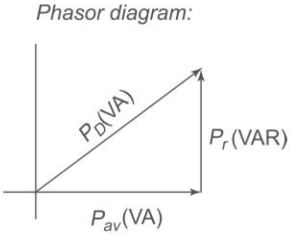


Fig. 2.128

$$\Rightarrow L = \frac{200}{500\pi} = 127.3 \text{ mH}$$

$$Z = 25 + j40 = 47.16\angle 57.99°$$

$$\text{Power absorbed by coil } = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.1273 \times 25 = 1.59 \text{ watts}$$

$$\text{True power } P_{av} = VI\cos\theta = 250 \times 5 \times \cos 57.99° = 662.58 \text{ watts}$$

$$\text{Reactive power, } P_r = I^2 X_L = 25 \times 40 = 1000 \text{ VAR}$$

$$\text{Apparent power, } P_a = I^2 Z = 25 \times 47.16 = 1179 \text{ VA}$$

Example 2.103 *In the following circuit (Fig. 2.129), when 220 V A.C. is applied across A and B, Current drawn is 20 Amps and power input is 3000 w. Find the value of Z and its parameters.* *[JNTU May/June 2006]*

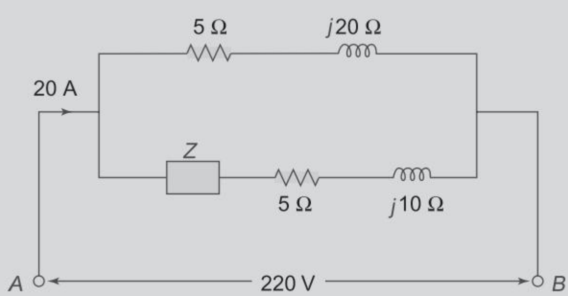


Fig. 2.129

Solution

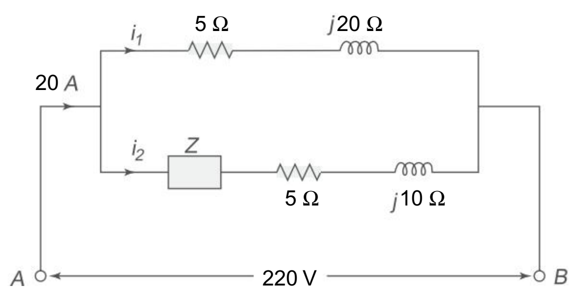


Fig. 2.130

$$i_1 = \frac{220}{5+j20} \text{ A}$$

But $\quad i_1 + i_2 = 20 \text{ A}$

$$i_2 = 20 - \frac{220}{5+j20} \tag{1}$$

$$\text{Also, } i_2 = \frac{220}{Z+5+j10} \tag{2}$$

From (1) and (2)

$$20 - \frac{220}{5+j20} = \frac{220}{Z+5+j20}$$

$$\frac{-120+j400}{5+j20} = \frac{220}{5+Z+j20}$$

$$Z = \frac{5700+j3600}{-120+j400}$$

$$Z = -4.33 + j15.55$$

$$Z = 16.14\angle 105.56°$$

Example 2.104 *When a voltage of 220 V A.C. supply connected across the AB terminals, the total power input is 3.25 kw and the current is 20 Amps. Find the current through Z_3. (Fig. 2.131)*

[JNTU May/June 2006]

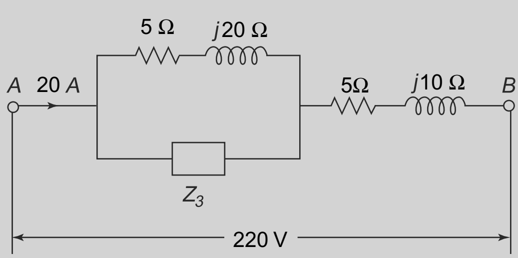


Fig. 2.131

Solution Voltage across (5 + j10) Ω branch

$$V = 20\,(5 + j10) = 223.6\angle 63.43° = 100 + j200$$

$$I\,(5 + j20) + 100 + j200 = 220.$$

(Let I be the current through $5 + j20\ \Omega$ branch)

$$I = \frac{120 - j200}{5+j20} = -8 - 8i$$

$$I_{Z_3} = 20 - I = 28 + 8i = 29.12\angle 15.9°$$

Example 2.105 *What is complex power? Explain in detail.*

[JNTU May/June 2006]

Solution *Complex power*

Active power (P):

The active power or real power in an a.c. circuit is given by the product of voltage, current and cosine of the phase angle. It is always positive

$$P = VI \cos \theta \text{ watts}$$

Reactive power (Q):

The reactive power in an a.c. circuit is given by the product of voltage, current and sine of the phase angle θ.
If θ is leading then reactive power is taken as +ve and it is capacitive.
If θ is lagging then reactive power is taken as −ve and it is inductive

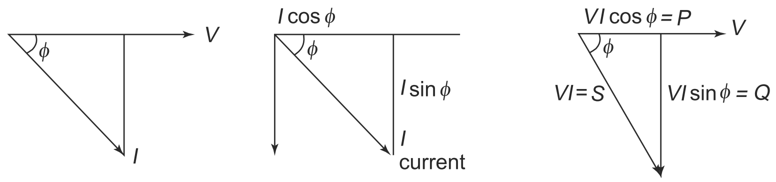
$$Q = VI \sin \theta \text{ VARs.}$$

Apparent power:

The apparent power in an a.c. circuit is the product of voltage and current. It is measured in voltamps.

$$S = VI \text{ volt amps.}$$

The component *I cos* θ = Active component or real component or in phase component of a current.
The product of voltage and the above component (active component) gives active power. The component *I sin* θ = Reactive component or quadrature component of current.



(a) Phasor diagram (b) Current (c) Power

Fig. 2.132

The produce of this component with voltage V gives the reactive power.

$$\text{Power factor } \cos \phi = \frac{\text{Real power}}{\text{Apparent power}}$$

The factor sin θ is called the reactive factor.
Complex power = (Active power) + *j* (Reactive power)

Example 2.106 *The current in a given circuit is I = (12 – j5) A when the applied voltage is V = (160 – j120)V. Determine*

(i) The complex expression for power
(ii) Power factor of the circuit
(iii) The complex expression for impedance of the circuit
(iv) Draw the phasor diagram.

[JNTU May/June 2006]

Solution (i) $P_a = V_{eff} I_{eff}\ VA$

$$P_{ar} = V_{eff} I_{eff} \cos\theta \text{ watts}$$

$$P_r = V_{eff} I_{eff} \sin\theta \text{ VAR}$$

$$Z = \frac{V}{I} = \frac{160 - j120}{12 - j5} = 14.91 - j3.786$$

$$|I| = 13\text{ A}$$

$$= 15.38\angle{-14.25°}$$

$$\therefore \quad P_{avg} = I^2 R = 2519.79\text{ W}$$

$$P_r = I^2 X = 639.834\text{ VAR}$$

$$P_a = I^2 Z = 2599.22\text{ VA}$$

Complex power = 2519.79 + j 639.834

(ii) $Pf = \cos\theta = \cos(-14.25°) = 0.969$

(iii) $Z = 14.91 - j3.786$

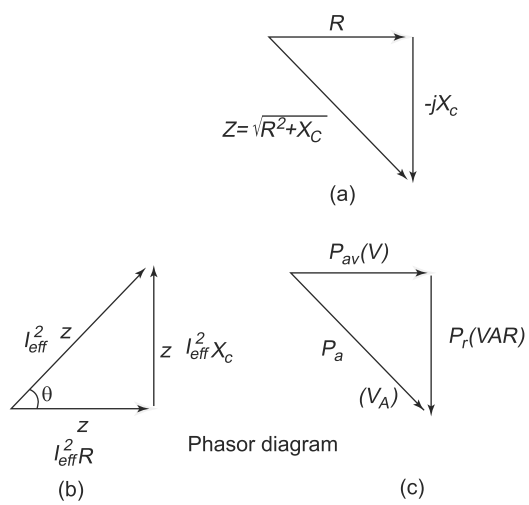


Fig. 2.133

Example 2.107 *A series RLC circuit consists of resistor of 100 Ω, an inductor of 0.318 H and a capacitor of unknown value. When this circuit is energised by a 230 V, 50 Hz ac supply, the current was found to be 23 A. Find the value of capacitor and the total power consumed.* *[JNTU June 2009]*

Solution The circuit is series RLC and is shown in Fig. 2.133

$$X_L = 2\pi f L$$

$$= 2\pi \times 50 \times 0.318 = 99.9\ \Omega$$

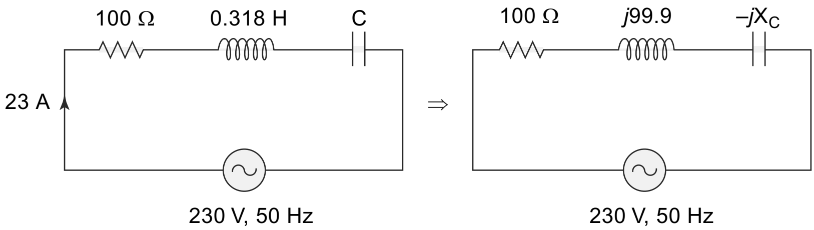


Fig. 2.134

Total impedance Z = $100 + j(99.9 - X_C)$

$$|Z| = \frac{V}{I} = \frac{230}{23} = 10\,\Omega$$

$$|Z| = \sqrt{(100)^2 + (99.9 - X_C)^2} = 10$$

$$X_C = 0.41\,\Omega$$

$$\frac{1}{\omega c} = 0.41\,\Omega$$

$$C = 7.76\,mF$$

Power Consumed = $I^2R = (23)^2 \times 100$

= 52900 W

= 52.9 KW

Example 2.108 *Two circuits, the impedances of which $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 + j8)\ \Omega$ are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch?*

[JNTU Jan 2010]

Solution

Equivalent impedance $= \dfrac{Z_1 Z_2}{Z_1 + Z_2} = \dfrac{(10 + j5)(6 + j8)}{10 + j5 + 6 + 8j} = \dfrac{(60 - 40) + j(30 + 80)}{(10 + 6) + j(5 + 8)}$ ohm

$$= \frac{20 + j110}{16 + j13} = 5.42\angle 40.60°\ \text{ohm}$$

$\therefore$ Voltage across the network = $(15 \times 5.42\angle 40.60°)$ volt

= $81.3\angle 40.60°$ volt

$\therefore$ Current through $Z_1 = \dfrac{Z_2}{(Z_1 + Z_2)} \times 15\ \text{amp}$

$$= \frac{6 + j8}{16 + j13} \times 15\ \text{amp} = 7.28\angle 14.04°\text{amp}$$

$\therefore$ Current through $Z_2 = \dfrac{Z_1}{(Z_1 + Z_2)} \times 15\ \text{amp}$

$$= \frac{10 + j5}{16 + j13} \times 15\ \text{amp} = 8.13\angle -12.53°\text{amp}$$

$\therefore$ Power taken by $Z_1 = 81.3 \times 7.28 \times \cos 26.2°$ watt
$= 529.38$ watt

$\therefore$ Power taken by $Z_2 = 81.3 \times 8.13 \times \cos 53.13°$ watt
$= 396.58$ watt

Practice Problems

2.1 Calculate the frequency of the following values of period.

(a) 0.2 s (b) 50 ms
(c) 500 μs (d) 10 μs

2.2 Calculate the period for each of the values of frequency.

(a) 60 Hz (b) 500 Hz
(c) 1 kHz (d) 200 kHz
(e) 5 MHz

2.3 A certain sine wave has a positive going zero crossing at 0° and an rms value of 20 V. Calculate its instantaneous value at each of the following angles.

(a) 33° (b) 110°
(c) 145° (d) 325°

2.4 For a particular 0° reference sinusoidal current, the peak value is 200 mA; determine the instantaneous values at each of the following.

(a) 35° (b) 190°
(c) 200° (d) 360°

2.5 Sine wave A lags sine wave B by 30°. Both have peak values of 15 V. Sine wave A is the reference with a positive going crossing at 0°. Determine the instantaneous value of sine wave B at 30°, 90°, 45°, 180° and 300°.

2.6 Find the average values of the voltages across R_1 and R_2. In Fig. 2.135 values shown are rms.

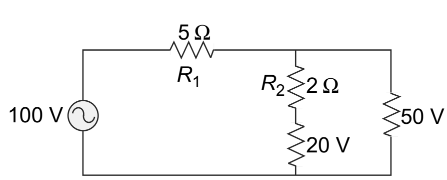


Fig. 2.135

2.7 A sinusoidal voltage is applied to the circuit shown in Fig. 2.136, determine rms current, average current, peak current, and peak to peak current.

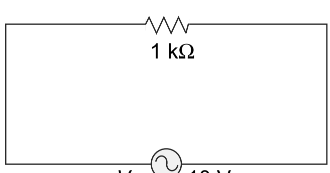


Fig. 2.136

2.8 A sinusoidal voltage of $v(t) = 50 \sin (500t)$ applied to a capacitive circuit. Determine the capacitive reactance, and the current in the circuit.

2.9 A sinusoidal voltage source in series with a dc source as shown in Fig. 2.137.

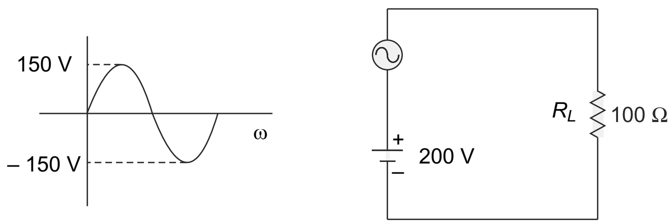


Fig. 2.137

Sketch the voltage across R_L. Determine the maximum current through R_L and the average voltage across R_L.

2.10 Find the effective value of the resultant current in a wire which carries a direct current of 10 A and a sinusoidal current with a peak value of 15 A.

2.11 An alternating current varying sinusoidally, with a frequency of 50 Hz, has an rms value of 20 A. Write down the equation for the instantaneous value and find this value at (a) 0.0025 s (b) 0.0125 s after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A?

2.12 Determine the rms value of the voltage defined by

$$v = 5 + 5 \sin (314t + \pi/6)$$

2.13 Find the effective value of the function $v = 100 + 50 \sin \omega t$.

2.14 A full wave rectified sine wave is clipped at 0.707 of its maximum value as shown in Fig. 2.138. Find the average and effective values of the function.

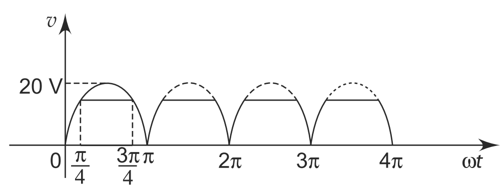


Fig. 2.138

2.15 Find the rms value of the function shown in Fig. 2.139 and described as follows

$0 < t < 0.1 \quad v = 40 (1 - e^{-100t})$

$0.1 < t < 0.2 v = 40\, e^{-50(t\, -\, 0.1)}$

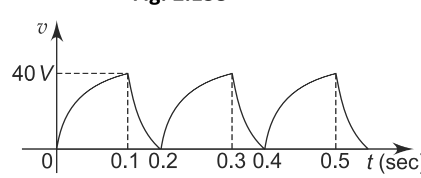


Fig. 2.139

2.16 Calculate average and effective values of the waveform shown in Fig. 2.140 and hence find from factor.

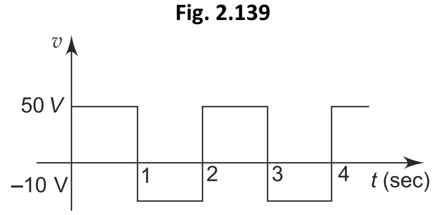


Fig. 2.140

2.17 A full wave rectified sine wave is clipped such that the effective value is 0.5 V_m as shown in Fig. 2.141. Determine the amplitude at which the waveform is clipped.

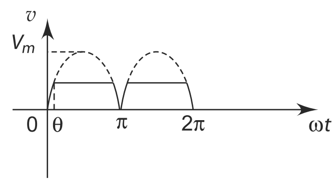


Fig. 2.141

2.18 A delayed full wave rectified sine wave has an average value of half the maximum value as shown in Fig. 2.142. Find the angle θ.

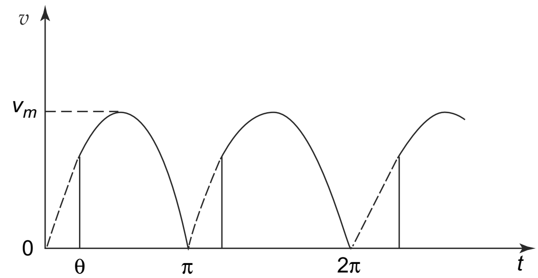


Fig. 2.142

2.19 For the circuit shown in Fig. 2.143, determine the impedance, phase angle and total current.

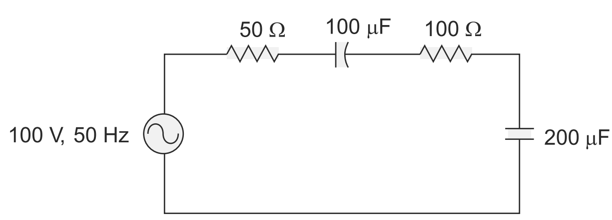


Fig. 2.143

2.20 Calculate the total current in the circuit in Fig. 2.144, and determine the voltage across resistor V_R, and across capacitor V_C.

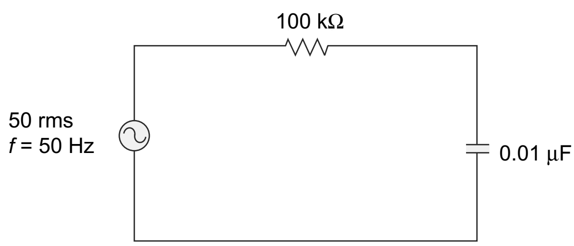


Fig. 2.144

2.21 Determine the impedance and phase angle in the circuit shown in Fig. 2.145.

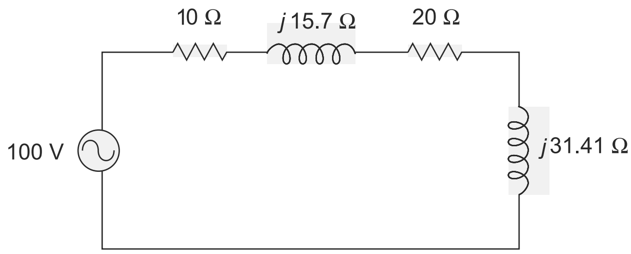


Fig. 2.145

2.22 Calculate the impedance at each of the following frequencies; also determine the current at each frequency in the circuit shown in Fig. 2.146.

(a) 100 Hz (b) 3 kHz

2.23 A signal generator supplies a sine wave of 10 V, 10 kHz, to the circuit shown in Fig. 2.147. Calculate the total current in the circuit. Determine the phase angle θ for the circuit. If the total inductance in the circuit is doubled, does θ increase or decrease, and by how many degrees?

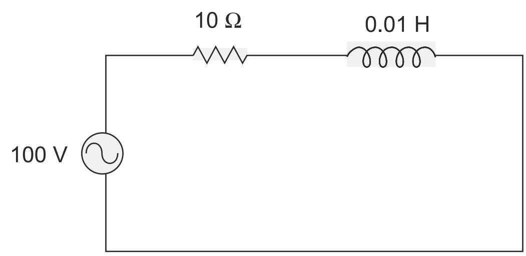


Fig. 2.146

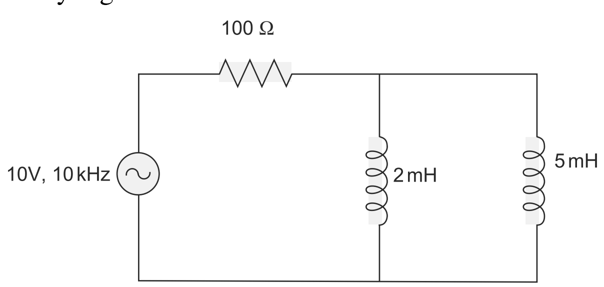


Fig. 2.147

2.24 For the circuit shown in Fig. 2.148, determine the voltage across each element. Is the circuit predominantly resistive or inductive? Find the current in each branch and the total current.

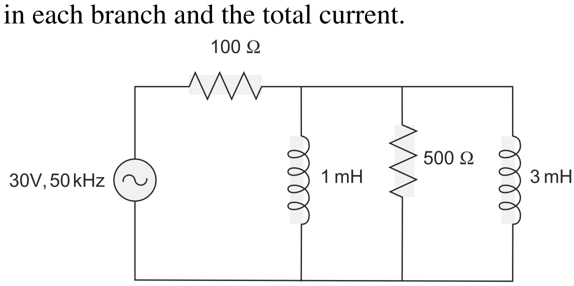


Fig. 2.148

2.25 Determine the total impedance Z_T, the total current I_T, phase angle θ, voltage across inductor L, and voltage across resistor R_3 in the circuit shown in Fig. 2.149.

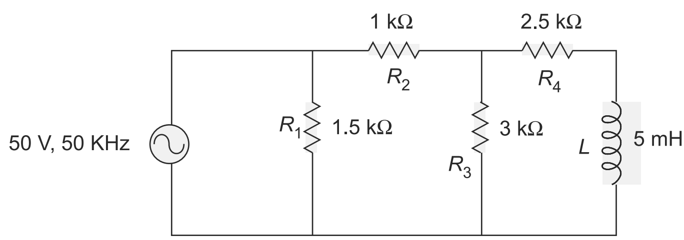


Fig. 2.149

2.26 For the circuit shown in Fig. 2.150, determine the value of frequency of supply voltage when a 100 V, 50 A current is supplied to the circuit.

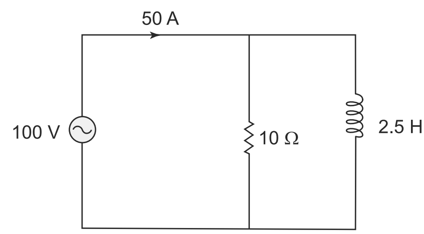


Fig. 2.150

2.27 A sine wave generator supplies a signal of 100 V, 50 Hz to the circuit shown in Fig. 2.151. Find the current in each branch, and total current. Determine the voltage across each element and draw the voltage phasor diagram.

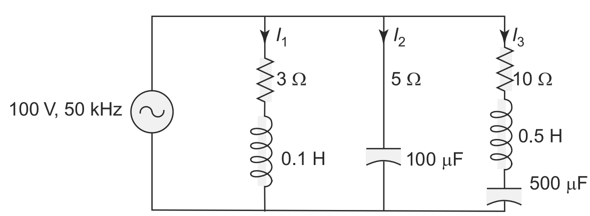


Fig. 2.151

2.28 Determine the voltage across each element in the circuit shown in Fig. 2.152. Convert the circuit into an equivalent series form. Draw the voltage phasor diagram.

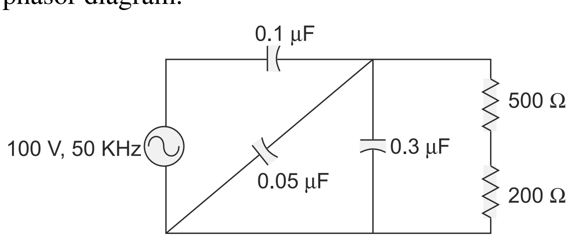


Fig. 2.152

2.29 For the circuit shown in Fig. 2.153, determine the total current I_T, phase angle θ and voltage across each element.

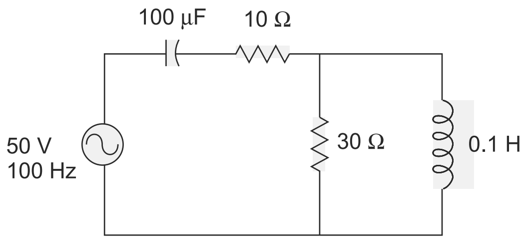


Fig. 2.153

2.30 For the circuit shown in Fig. 2.154, the applied voltage $v = V_m \cos \omega t$. Determine the current in each branch and obtain the total current in terms of the cosine function.

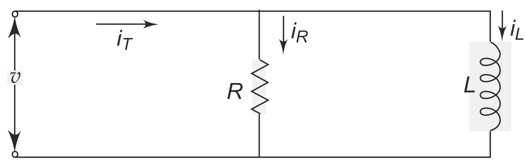


Fig. 2.154

2.31 For the circuit shown in Fig. 2.155, the voltage across the inductor is $v_L = 15 \sin 200\, t$. Find the total voltage and the angle which the current lags the total voltage.

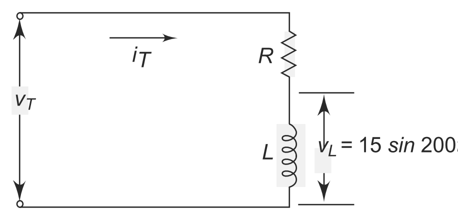


Fig. 2.155

2.32 In a parallel circuit having a resistance $R = 5\,\Omega$ and $L = 0.02$ H, the applied voltage is $v = 100 \sin (1000\ t + 50°)$ volts. Find the total current and the angle by which the current lags the applied voltage.

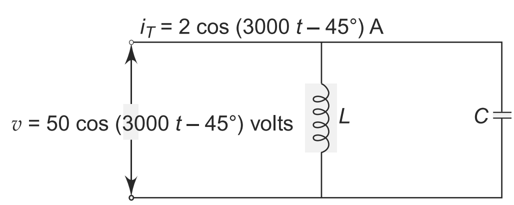


Fig. 2.156

2.33 In the parallel circuit shown in Fig. 2.156, the current in the inductor is five times greater than the current in the capacitor. Find the element values.

2.34 In the parallel circuit shown in Fig. 2.157, the applied voltage is $v = 100 \sin 5000\ t$ V. Find the currents in each branch and also the total current in the circuit.

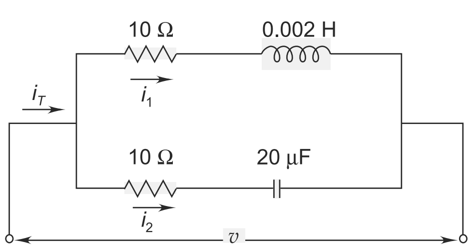


Fig. 2.157

2.35 For the circuit shown in Fig. 2.158, find the total current and the magnitude of the impedance.

2.36 Solve for current I using PSpice in the circuit shown in Fig. 2.159.

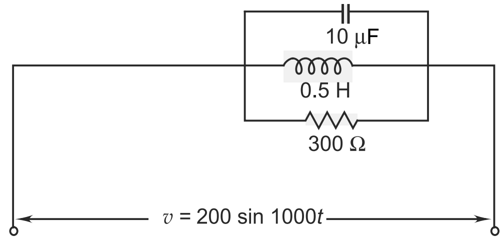


Fig. 2.158

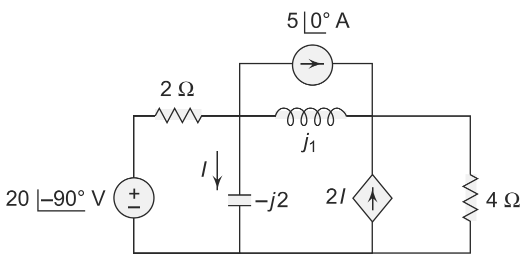


Fig. 2.159

2.37 The voltmeter reads 45 volts across the 5 Ω resistor in the circuit shown in Fig. 2.160 below. Find the ammeter reading.

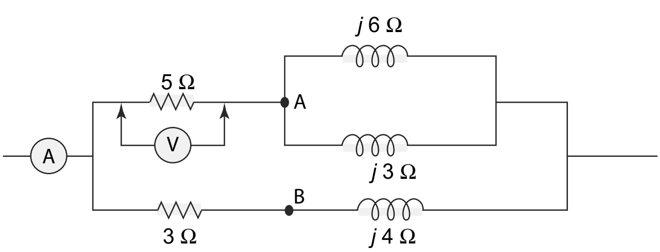


Fig. 2.160

2.38 For the circuit shown in Fig. 2.161, a voltage of 250 sin ωt is applied. Determine the power factor of the circuit, if the voltmeter readings are V_1 = 100 V, V_2 = 125 V, V_3 = 150 V and the ammeter reading is 5 A.

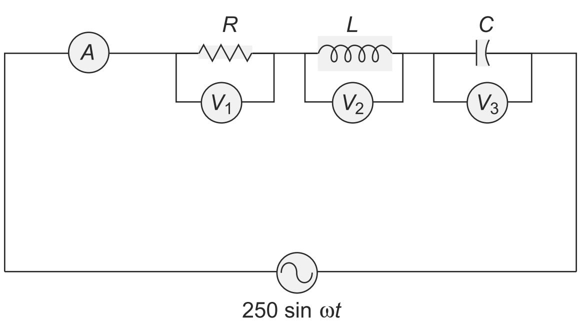


Fig. 2.161

2.39 For the circuit shown in Fig. 2.162, a voltage $v(t)$ is applied and the resulting current in the circuit $i(t) = 15 \sin(\omega t + 30°)$ amperes. Determine the active power, reactive power, power factor, and the apparent power.

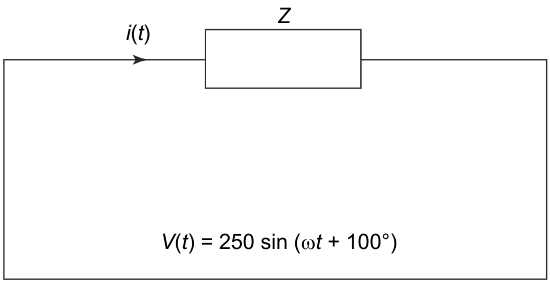


Fig. 2.162

2.40 A series RL circuit draws a current of $i(t) = 8 \sin(50t + 45°)$ from the source. Determine the circuit constants, if the power delivered by the source is 100 W and there is a lagging power factor of 0.707.

2.41 Two impedances, $Z_1 = 10 \angle -60° \ \Omega$ and $Z_2 = 16 \angle 70° \ \Omega$ are in series and pass an effective current of 5 A. Determine the active power, reactive power, apparent power and power factor.

2.42 For the circuit shown in Fig. 2.163, determine the value of the impedance if the source delivers a power of 200 W and there is a lagging power factor of 0.707. Also find the apparent power.

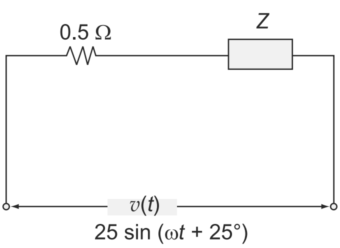


Fig. 2.163

2.43 A voltage of $v(t) = 100 \sin 500\, t$ is applied across a series R-L-C circuit where $R = 10\ \Omega$, $L = 0.05$ H and $C = 20\ \mu$F. Determine the power supplied by the source, the reactive power supplied by the source, the reactive power of the capacitor, the reactive power of the inductor, and the power factor of the circuit.

2.44 For the circuit shown in Fig. 2.164, determine the power dissipated and the power factor of the circuit.

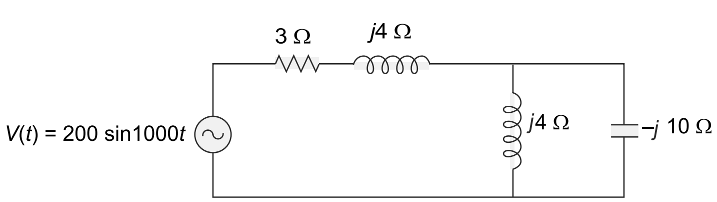


Fig. 2.164

2.45 For the circuit shown in Fig. 2.165, determine the power factor and the power dissipated in the circuit.

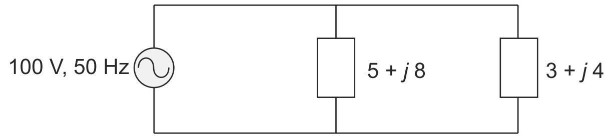


Fig. 2.165

2.46 For the circuit shown in Fig. 2.166, determine the power factor, active power, reactive power and apparent power.

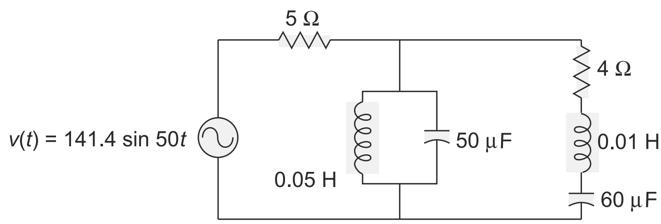


Fig. 2.166

2.47 In the parallel circuit shown in Fig. 2.167, the power in the 5 Ω resistor is 600 W and the total circuit takes 3000 VA at a leading power factor of 0.707. Find the value of impedance *Z*.

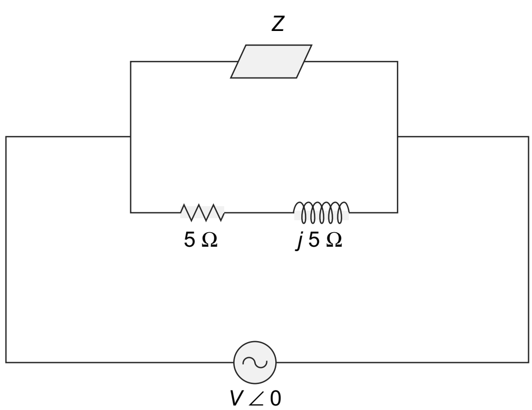


Fig. 2.167

2.48 For the parallel circuit shown in Fig. 2.168, the total power dissipated is 1000 W. Determine the apparent power, the reactive power, and the power factor.

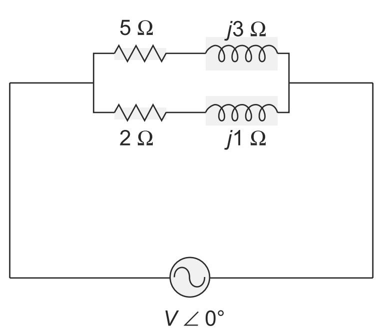


Fig. 2.168

2.49 A voltage source $v(t) = 150 \sin \omega t$ in series with 5 Ω resistance is supplying two loads in parallel, $Z_A = 60\angle 30°$, and $Z_B = 50\angle -25°$. Find the average power delivered to Z_A, the average power delivered to Z_B, the average power dissipated in the circuit, and the power factor of the circuit.

2.50 For the circuit shown in Fig. 2.169, determine the true power, reactive power and apparent power in each branch. What is the power factor of the total circuit?

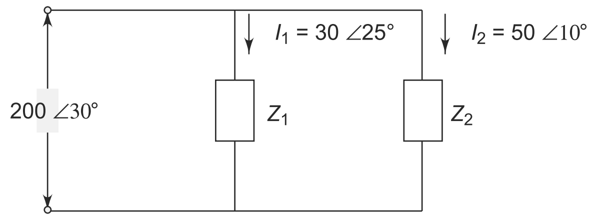


Fig. 2.169

2.51 Determine the value of the voltage source, and the power factor in the network shown in Fig. 2.170 if it delivers a power of 500 W to the circuit shown in Fig. 2.168. Also find the reactive power drawn from the source.

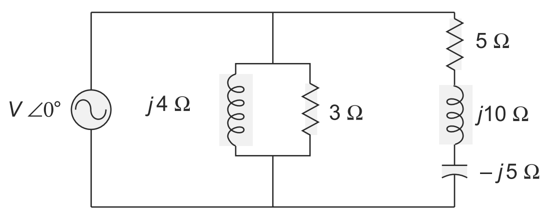


Fig. 2.170

2.52 Find the average power dissipated by the 500 Ω resistor shown in Fig. 2.177.

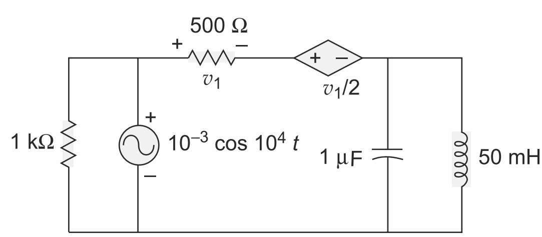


Fig. 2.171

2.53 Find the power dissipated by the voltage source shown in Fig. 2.172.

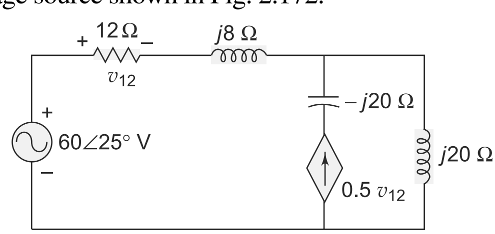


Fig. 2.172

2.54 Find the power delivered by current source shown in Fig. 2.173.

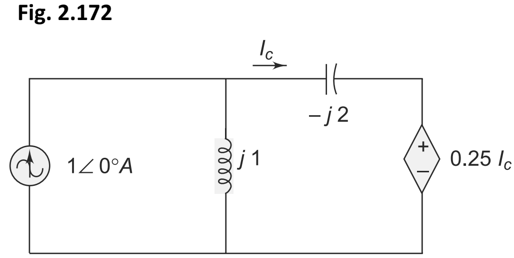


Fig. 2.173

2.55 For the circuit shown in Fig. 2.174, determine the power factor, active power, reactive power and apparent power.

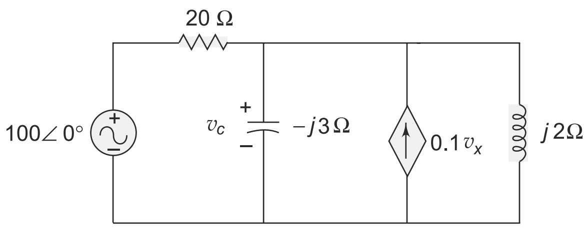


Fig. 2.174

2.56 For the circuit shown in Fig. 2.175, find:

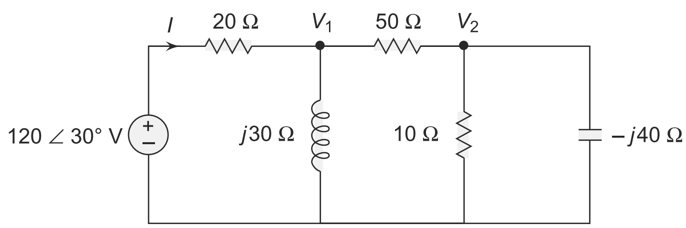


Fig. 2.175

(a) real power dissipated by each element
(b) the total apparant power supplied by the circuit
(c) the power factor of the circuit.

Objective Type Questions

2.1 One sine wave has a period of 2 ms, another has a period of 5 ms, and other has a period of 10 ms. Which sine wave is changing at a faster rate?

(a) sine wave with period 2 ms (b) sine wave with period of 5 ms
(c) all are at the same rate (d) sine wave with period of 10 ms

2.2 How many cycles does a sine wave go through in 10 s when its frequency is 60 Hz?

(a) 10 cycles (b) 60 cycles (c) 600 cycles (d) 6 cycles

2.3 If the peak value of a certain sine wave voltage is 10 V, what is the peak to peak value?

(a) 20 V (b) 10 V (c) 5 V (d) 7.07 V

2.4 If the peak value of a certain sine wave voltage is 5 V, what is the rms value?

(a) 0.707 V (b) 3.535 V (c) 5 V (d) 1.17 V

2.5 What is the average value of a sine wave over a full cycle?

(a) V_m (b) $\frac{V_m}{\sqrt{2}}$ (c) zero (d) $\sqrt{2}V_m$

2.6 A sinusoidal current has peak value of 12 A. What is its average value?

(a) 7.64 A (b) 24 A (c) 8.48 A (d) 12 A

2.7 Sine wave *A* has a positive going zero crossing at 30°. Sine wave *B* has a positive going zero crossing at 45°. What is the phase angle between two signals?

(a) 30° (b) 45° (c) 75° (d) 15°

2.8 A sine wave has a positive going zero crossing at 0° and an rms value of 20 V. What is its instantaneous value at 145°?

(a) 7.32 V (b) 16.22 V (c) 26.57 V (d) 21.66 V

2.9 In a pure resistor, the voltage and current are

(a) out of phase (b) in phase
(c) 90° out of phase (d) 45° out of phase

2.10 The rms current through a 10 kΩ resistor is 5 mA. What is the rms voltage drop across the resistor?

(a) 10 V (b) 5 V (c) 50 V (d) zero

2.11 In a pure capacitor, the voltage

(a) is in phase with the current (b) is out of phase with the current
(c) lags behind the current by 90° (d) leads the current by 90°

2.12 A sine wave voltage is applied across a capacitor; when the frequency of the voltage is increased, the current

(a) increases (b) decreases (c) remains the same (d) is zero

2.13 The current in a pure inductor

(a) lags behind the voltage by 90° (b) leads the voltage by 90°
(c) is in phase with the voltage (d) lags behind the voltage by 45°

2.14 A sine wave voltage is applied across an inductor; when the frequency of voltage is increased, the current

(a) increases (b) decreases (c) remains the same (d) is zero

2.15 The rms value of the voltage for a voltage function $v = 10 + 5 \cos (628t + 30°)$ volts through a circuit is

(a) 5 V (b) 10 V (c) 10.6 V (d) 15 V

2.16 For the same peak value, which is of the following wave will have the highest rms value

(a) sine wave (b) square wave
(c) triangular wave (d) half wave rectified sine wave

2.17 For 100 volts rms value triangular wave, the peak voltage will be

(a) 100 V (b) 111 V (c) 141 V (d) 173 V

2.18 The form factor of dc voltage is

(a) zero (b) infinite (c) unity (d) 0.5

2.19 For the half wave rectified sine wave shown in Fig. 2.176, the peak factor is

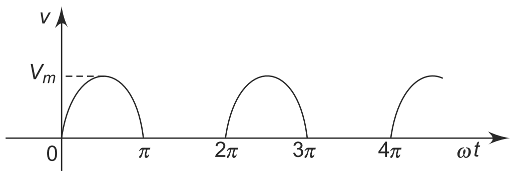


Fig. 2.176

(a) 1.41 (b) 2.0 (c) 2.82 (d) infinite

2.20 For the square wave shown in Fig. 2.177, the form factor is

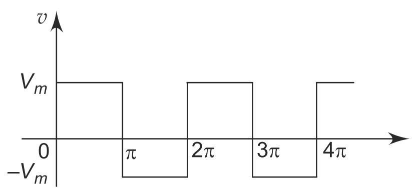


Fig. 2.177

(a) 2.0 (b) 1.0 (c) 0.5 (d) zero

2.21 The power consumed in a circuit element will be least when the phase difference between the current and voltage is

(a) 0°(b) 30° (c) 90° (d) 180°

2.22 The voltage wave consists of two components: A 50 V dc component and a sinusoidal component with a maximum value of 50 volts. The average value of the resultant will be

(a) zero (b) 86.6 V (c) 50 (d) none of the above

2.23 A 1 kHz sinusoidal voltage is applied to an RL circuit, what is the frequency of the resulting current?

(a) 1 kHz (b) 0.1 kHz (c) 100 kHz (d) 2 kHz

2.24 A series RL circuit has a resistance of 33 kΩ, and an inductive reactance of 50 kΩ. What is its impedance and phase angle?

(a) 56.58 Ω, 59.9° (b) 59.9 kΩ, 56.58°
(c) 59.9 Ω, 56.58° (d) 5.99 Ω, 56.58°

2.25 In a certain *RL* circuit, $V_R = 2$ V and $V_L = 3$ V. What is the magnitude of the total voltage?

(a) 2 V (b) 3 V (c) 5 V (d) 3.61 V

2.26 When the frequency of applied voltage in a series RL circuit is increased what happens to the inductive reactance?

(a) decreases (b) remains the same
(c) increases (d) becomes zero

2.27 In a certain parallel RL circuit, $R = 0\ \Omega$, and $X_L = 75\ \Omega$. What is the admittance?

(a) 0.024 S (b) 75 S (c) 50 S (d) 1.5 S

2.28 What is the phase angle between the inductor current and the applied voltage in a parallel RL circuit?

(a) 0°(b) 45° (c) 90° (d) 30°

2.29 When the resistance in an RC circuit is greater than the capacitive reactance, the phase angle between the applied voltage and the total current is closer to

(a) 90° (b) 0° (c) 45° (d) 120°

2.30 A series RC circuit has a resistance of 33 kΩ, and a capacitive reactance of 50 kΩ. What is the value of the impedance.

(a) 50 kΩ (b) 33 kΩ (c) 20 kΩ (d) 59.91 Ω

2.31 In a certain series RC circuit, $V_R = 4$ V and $V_C = 6$ V. What is the magnitude of the total voltage?

(a) 7.2 V (b) 4 V (c) 6 V (d) 52 V

2.32 When the frequency of the applied voltage in a series RC circuit is increased what happens to the capacitive reactance?

(a) it increases (b) it decreases (c) it is zero (d) remains the same

2.33 In a certain parallel RC circuit, $R = 50\ \Omega$ and $X_C = 75\ \Omega$. What is Y?

(a) 0.01 S (b) 0.02 S (c) 50 S (d) 75 S

2.34 The admittance of an RC circuit is 0.0035 S, and the applied voltage is 6 V. What is the total current?

(a) 6 mA (b) 20 mA (c) 21 mA (d) 5 mA

2.35 What is the phase angle between the capacitor current and the applied voltage in a parallel RC circuit?

(a) 90° (b) 0° (c) 45° (d) 180°

2.36 In a given series RLC circuit, X_C is 150 Ω, and X_L is 80 Ω, what is the total reactance? What is the type of reactance?

(a) 70 Ω, inductive (b) 70 Ω, capacitive
(c) 70 Ω, resistive (d) 150 Ω, capacitive

2.37 In a certain series RLC circuit $V_R = 24$ V, $V_L = 15$ V, and $V_C = 45$ V. What is the source voltage?

(a) 38.42 V (b) 45 V (c) 15 V (d) 24 V

2.38 When $R = 10\ \Omega$, $X_C = 18\ \Omega$ and $X_L = 12\ \Omega$, the current

(a) leads the applied voltage (b) lags behind the applied voltage
(c) is in phase with the voltage (d) is none of the above

2.39 A current $i = A \sin 500\,t$ A passes through the circuit shown in Fig. 2.178. The total voltage applied will be

(a) $B \sin 500\,t$ (b) $B \sin (500\,t - \theta°)$
(c) $B \sin (500\,t + \theta°)$ (d) $B \cos (200\,t + \theta°)$

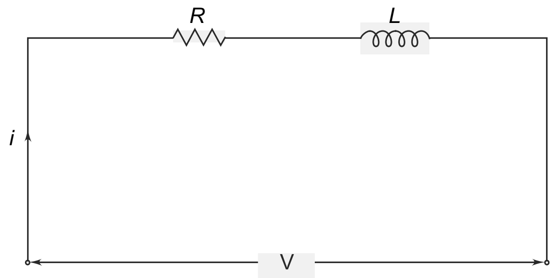


Fig. 2.178

2.40 A current of 100 mA through an inductive reactance of 100 Ω produces a voltage drop of

(a) 1 V (b) 6.28 V (c) 10 V (d) 100 V

2.41 When a voltage $v = 100 \sin 5000\, t$ volts is applied to a series circuit of $L = 0.05$ H and unknown capacitance, the resulting current is $i = 2 \sin (5000\, t + 90°)$ amperes. The value of capacitance is

(a) 66.7 pF (b) 6.67 pF (c) 0.667 μF (d) 6.67 μF

2.42 A series circuit consists of two elements has the following current and applied voltage.

$$i = 4 \cos (2000\, t + 11.32°) \text{ A}$$
$$v = 200 \sin (2000\, t + 50°) \text{ V}$$

The circuit elements are

(a) resistance and capacitance (b) capacitance and inductance
(c) inductance and resistance (d) both resistances

2.43 A pure capacitor of $C = 35$ μF is in parallel with another single circuit element. The applied voltage and resulting current are

$$v = 150 \sin 300\, t \text{ V}$$
$$i = 16.5 \sin (3000\, t + 72.4°) \text{ A}$$

The other element is

(a) capacitor of 30 μF (b) inductor of 30 mH
(c) resistor of 30 Ω (d) none of the above

2.44 The phasor combination of resistive power and reactive power is called

(a) true power (b) apparent power
(c) reactive power (d) average power

2.45 Apparent power is expressed in

(a) volt-amperes (b) watts
(c) volt-amperes or watts (d) VAR

2.46 A power factor of '1' indicates

(a) purely resistive circuit,
(b) purely reactive circuit
(c) combination of both, (a) and (b)
(d) none of these

2.47 A power factor of 0 indicates
(a) purely resistive element (b) purely reactive element
(c) combination of both (a) and (b) (d) none of the above

2.48 For a certain load, the true power is 100 W and the reactive power is 100 VAR. What is the apparent power?
(a) 200 VA (b) 100 VA (c) 141.4 VA (d) 120 VA

2.49 If a load is purely resistive and the true power is 5 W, what is the apparent power?
(a) 10 VA (b) 5 VA (c) 25 VA (d) $\sqrt{50}$ VA

2.50 True power is defined as
(a) $VI \cos \theta$ (b) VI (c) $VI \sin \theta$ (d) none of these

2.51 In a certain series RC circuit, the true power is 2 W, and the reactive power is 3.5 VAR. What is the apparent power?
(a) 3.5 VA (b) 2 VA (c) 4.03 VA (d) 3 VA

2.52 If the phase angle θ is 45°, what is the power factor?
(a) cos 45° (b) sin 45° (c) tan 45° (d) none of these

2.53 To which component in an RC circuit is the power dissipation due?
(a) capacitance (b) resistance (c) both (d) none

2.54 A two element series circuit with an instantaneous current $I = 4.24 \sin (5000\, t + 45°)$ A has a power of 180 watts and a power factor of 0.8 lagging. The inductance of the circuit must have the value.
(a) 3 H (b) 0.3 H
(c) 3 mH (d) 0.3 mH

2.55 In the circuit shown in Fig. 2.179, if branch A takes 8 KVAR, the power of the circuit will be
(a) 2 kW (b) 4 kW
(c) 6 kW (d) 8 kW

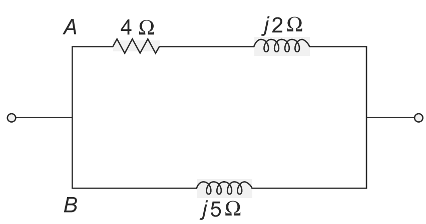


Fig. 2.179

2.56 In the circuit shown in Fig. 2.180, the voltage across 30 Ω resistor is 45 volts. The reading of the ammeter A will be

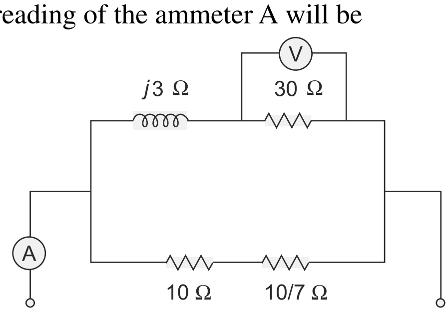


Fig. 2.180

(a) 10 A (b) 19.4 A
(c) 22.4 A (d) 28 A

2.57 In the circuit shown in Fig. 2.181, v_1 and v_2 are two identical sources of $10\angle 90°$. The power supplied by V_1 is

(a) 6 W (b) 8.8 W (c) 11 W (d) 16 W

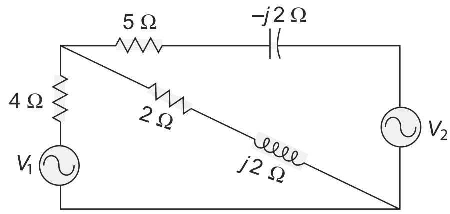


Fig. 2.181

3 Locus Diagrams and Resonance

3.1 LOCUS DIAGRAMS

A phasor diagram may be drawn and is expanded to develop a curve; known as a locus. Locus diagrams are useful in determining the behaviour or response of an RLC circuit when one of its parameters is varied while the frequency and voltage kept constant. The magnitude and phase of the current vector in the circuit depends upon the values of R, L, and C and frequency at the fixed source voltage. The path traced by the terminus of the current vector when the parameters R, L or C are varied while f and v are kept constant is called the current locus.

The term circle diagram identifies locus plots that are either circular or semicircular loci of the terminus (the tip of the arrow) of a current phasor or voltage phasor. Circle diagrams are often employed as aids in analysing the operating characteristics of circuits like equivalent circuit of transmission lines and some types of AC machines.

Locus diagrams can be also drawn for reactance, impedance, susceptance and admittance when frequency is variable. Loci of these parameters furnish important information for use in circuit analysis. Such plots are particularly useful in the design of electric wave filters.

3.1.1 Series R-L, R-C, R-L-C Circuits *[JNTU Nov. 2011]*

To discuss the basis of representing a series circuit by means of a circle diagram consider the circuit shown in Fig. 3.1(a). The analytical procedure is greatly simplified by assuming that inductance elements have no resistance and that capacitors have no leakage current.

The circuit under consideration has constant reactance but variable resistance. The applied voltage will be assumed with constant rms voltage V. The power factor angle is designated by θ. If $R = 0$, I_L is obviously equal to $\dfrac{V}{X_L}$ and has maximum value. Also I lags V by 90°. This is shown in Fig. 3.1(b). If R is increased from zero value, the magnitude of I becomes less than $\frac{V}{X_L}$ and θ becomes less than 90° and finally when the limit is reached, i.e. when R equals to infinity, I equals to zero and θ equals to zero. It is observed that the tip of the current vector represents a semicircle as indicated in Fig. 3.1(b).

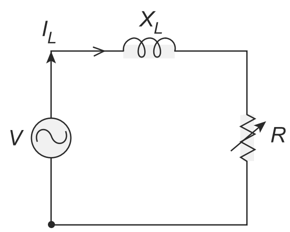


Fig. 3.1(a)

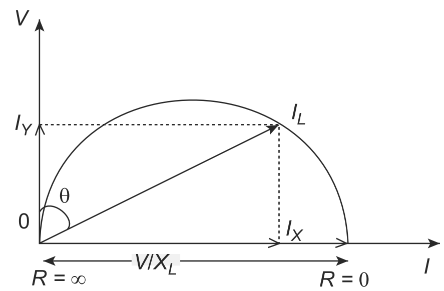


Fig. 3.1(b)

In general

$$I_L = \frac{V}{Z}$$

$$\sin\theta = \frac{X}{Z}$$

or

$$Z = \frac{X_L}{\sin\theta}$$

$$I = \frac{V}{X_L}\sin\theta \tag{3.1}$$

For constant V and X, Eq. 3.1 is the polar equation of a circle with diameter $\frac{V}{X_L}$. Figure 3.1(b) shows the plot of Eq. 3.1 with respect to V as reference.

The active component of the current I_L in Fig. 3.1(b) is $OI_L \cos\theta$ which is proportional to the power consumed in the RL circuit. In a similar way we can draw the loci of current if the inductive reactance is replaced by a capacitive reactance as shown in Fig. 3.1(c). The current semicircle for the RC circuit with variable R will be on the left-hand side of the voltage vector OV with diameter $\frac{V}{X_L}$ as shown in Fig. 3.1(d). The current vector OI_C leads V by θ°. The active component of the current I_cX in Fig. 3.1(d) is $OI_C \cos\theta$ which is proportional to the power consumed in the RC circuit.

Circle Equations for an R_L Circuit

(a) Fixed reactance and variable resistance The X-co-ordinate and Y-co-ordinate of I_L in Fig. 3.1(b) respectively are $I_X = I_L \sin\theta$; $I_Y = I_L \cos\theta$.

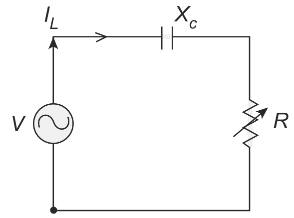


Fig. 3.1 (c)

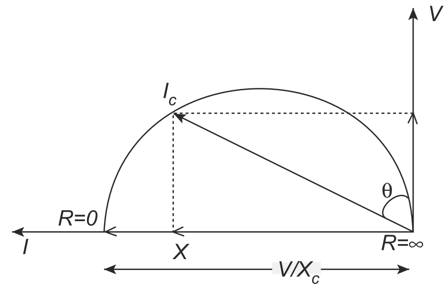


Fig. 3.1 (d)

Where $I_L = \dfrac{V}{Z}$; $\sin\theta = \dfrac{X_L}{Z}$; $\cos\theta = \dfrac{R}{Z}$; $Z = \sqrt{R^2 + X_L^2}$

$$I_X = \frac{V}{Z}\cdot\frac{X_L}{Z} = V\cdot\frac{X_L}{Z^2} \tag{3.2}$$

$$I_Y = \frac{V}{Z}\cdot\frac{R}{Z} = V\cdot\frac{R}{Z^2} \tag{3.3}$$

Squaring and adding Eqs 3.2 and 3.3, we obtain

$$I_X^2 + I_Y^2 = \frac{V^2}{R^2 + X_L^2} \tag{3.4}$$

From Eq. 3.2

$$Z^2 = R^2 + X_L^2 = V\cdot\frac{X_L}{I_X}$$

∴ Equation 3.4 can be written as $I_X^2 + I_Y^2 = \dfrac{V}{X_L}\cdot I_X$

or
$$I_X^2 + I_Y^2 - \frac{V}{X_L}\cdot I_X = 0$$

Adding $\left(\dfrac{V}{2X_L}\right)^2$ to both sides the above equation can be written as

$$\left(I_X - \frac{V}{2X_L}\right)^2 + I_Y^2 = \left(\frac{V}{2X_L}\right)^2 \tag{3.5}$$

Equation 3.5 represents a circle whose radius is $\dfrac{V}{2X_L}$ and the co-ordinates of the centre are $\dfrac{V}{2X_L}, 0$.

In a similar way we can prove that for a series R_C circuit as in Fig. 3.1(c), with variable R, the locus of the tip of the current vector is a semi-circle and is given by

$$\left(I_X + \frac{V}{2X_C}\right)^2 + I_Y^2 = \frac{V^2}{4X_C^2} \tag{3.6}$$

The centre has co-ordinates of $-\dfrac{V}{2X_L}, 0$ and radius as $\dfrac{V}{2X_L}$.

(b) Fixed resistance, variable reactance Consider the series RL circuit with constant resistance R but variable reactance X_L as shown in Fig. 3.2(a).

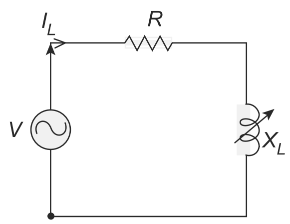


Fig. 3.2(a)

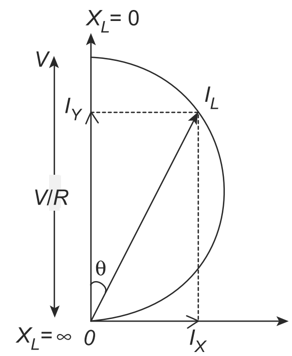


Fig. 3.2(b)

When $X_L = 0$; I_L assumes maximum value of $\dfrac{V}{R}$ and $\theta = 0$, the power factor of the circuit becomes unity; as the value X_L is increased from zero, $I_L$is reduced and finally when X_L is α, current becomes zero and θ will be lagging behind the voltage by 90° as shown in Fig. 3.2(b). The current vector describes a semicircle with diameter $\dfrac{V}{R}$ and lies in the right-hand side of voltage vector OV. The active component of the current $OI_L \cos\theta$ is proportional to the power consumed in the RL circuit.

Equation of circle

Consider Eq. 3.4 $\quad I_X^2 + I_Y^2 = \dfrac{V^2}{R^2 + X_L^2}$

From Eq. 3.3 $\quad Z^2 = R^2 + X_L^2 = \dfrac{VR}{I_Y} \qquad (3.7)$

Substituting Eq. 3.7 in Eq. 3.4

$$I_X^2 + I_Y^2 = \frac{V}{R} I_Y \qquad (3.8)$$

$$I_X^2 + I_Y^2 - \frac{V}{R} I_Y = 0$$

Adding $\left(\dfrac{V}{2R}\right)^2$ to both sides the above equation can be written as

$$I_X^2 + \left(I_Y - \frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2 \qquad (3.9)$$

Equation 3.9 represents a circle whose radius is $\dfrac{V}{2R}$ and the co-ordinates of the centre are $0, \dfrac{V}{2R}$.

Let the inductive reactance in Fig. 3.2(a) be replaced by a capacitive reactance as shown in Fig. 3.3(a).

The current semicircle of a RC circuit with variable X_C will be on the left-hand side of the voltage vector OV with diameter $\frac{V}{R}$. The current vector OI_c leads V by θ°. As before, it may be proved that the equation of the circle shown in Fig. 3.3(b) is

$$I_X^2 + \left(I_Y - \frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2$$

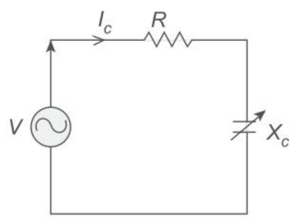


Fig. 3.3(a)

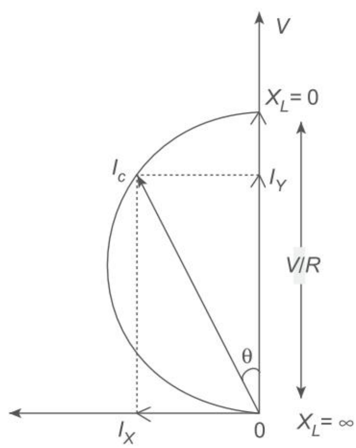


Fig. 3.3(b)

Example 3.1 *For the circuit shown in Fig. 3.4(a) plot the locus of the current, mark the range of I for maximum and minimum values of R, and the maximum power consumed in the circuit. Assume X_L = 25 Ω and R = 50 Ω. The voltage is 200 V; 50 Hz.*

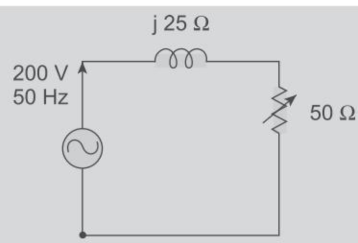


Fig. 3.4(a)

Solution Maximum value of current

$$I_{max} = \frac{200}{25} = 8\,\text{A};\ \theta = 90°$$

Minimum value of current

$$I_{min} = \frac{200}{\sqrt{(50)^2 + (25)^2}} = 3.777\text{A};$$

$$\theta = 27.76°$$

The locus of the current is shown in Fig. 3.4(b).

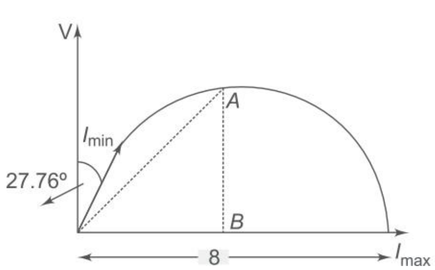


Fig. 3.4(b)

Power consumed in the circuit is proportional to $I \cos \theta$ for constant V. The maximum ordinate possible in the semicircle (AB in Fig. 3.4(b)) represents the maximum power consumed in the circuit. This is possible when $\theta = 45°$, under the condition power factor $\cos\theta = \cos 45° = \frac{1}{\sqrt{2}}$.

Hence, the maximum power consumed in the circuit $= V \times AB = V \times \frac{I_{max}}{L}$

$$I_{max} = \frac{V}{X_L} = 84\,\text{A}$$

$$P_{max} = \frac{V^2}{2X_L} = \frac{(200)^2}{2 \times 25} = 800\,\text{W}$$

Example 3.2 *For the circuit shown in Fig. 3.4(a), if the reactance is variable plot the range of I for maximum and minimum values of X_L and maximum power consumed in the circuit.*

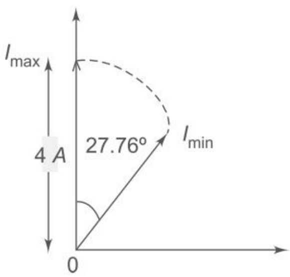


Fig. 3.5

Solution

Maximum value of current $I_{max} = \frac{200}{50} = 4\,\text{A}; \theta = 0$

Minimum value of current

$$I_{min} = \frac{200}{\sqrt{(50)^2 + (25)^2}}$$
$$= 3.777\text{A}; \theta = 27.76°$$

The locus of current is shown in Fig. 3.5. Maximum power will be when $I = 4$ A. Hence $P_{max} = 4 \times 200 = 800$ W.

Example 3.3 *For the circuit shown in Fig. 3.6(a) draw the locus of the current. Mark the range of I for maximum and minimum values. Assume $X_C = 50\ \Omega$; $R = 10\ \Omega$; $V = 400\ \Omega$.*

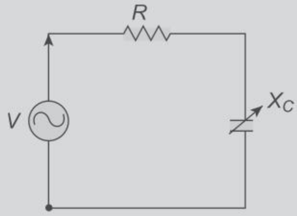


Fig. 3.6(a)

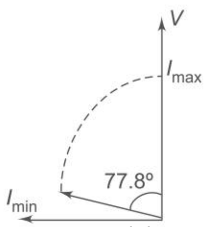


Fig. 3.6(b)

Solution $I_{max} = \frac{400}{10} = 40\,\text{A}; \theta = 0°$

$$I_{min} = \frac{400}{\sqrt{(50)^2 + (10)^2}} = 7.716\text{A}.\ \theta = \tan^{-1} 5$$
$$= 77.8°$$

The locus of the current is shown in Fig. 3.6(b).

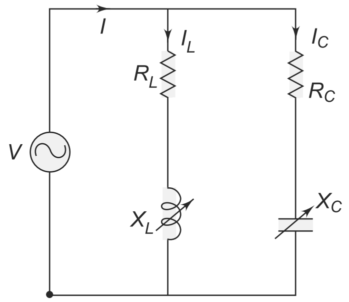


Fig. 3.7(a)

3.1.2 Parallel Circuits *[JNTU Nov. 2011]*

(a) Variable X_L Locus plots are drawn for parallel branches containing RLC elements in a similar way as for series circuits. Here we have more than one current locus. Consider the parallel circuit shown in Fig. 3.7(a). The quantities that may be varied are X_L, X_C, R_L and R_C for a given voltage and frequency.

Let us consider the variation of X_L from zero to ∞. Let *OV* shown in Fig. 3.7(b), be the voltage vector, taken as reference. A current, I_C, will flow in the condenser branch whose parameters are held constant and leads *V* by an angle $\theta_C = \tan^{-1}\left(\frac{X_C}{R_C}\right)$, when $X_L = 0$, the current l_L through the inductive branch is maximum and is given by $\frac{V}{R_L}$ and it is in phase with the applied voltage. When X_L is increased from zero, the current through the inductive branch I_L decreases and lags *V* by $\theta_L = \tan^{-1}\frac{X_L}{R_L}$ as shown in Fig. 3.7(b). For any value of I_L the $I_L R_L$ drop and $I_L X_L$ drop must add at right angles to give the applied voltage. These drops are shown as *OA* and *AV* respectively. The locus of I_L is a semicircle, and the locus *of* $I_L R_L$ drop is also a semicircle. When $X_L = 0$, i.e. I_L is maximum, I_L coincides with the diameter of its semicircle and $I_L R_L$ drop also coincides with the diameter of its semi-circle as shown in the figure; both these semicircles are shown with dotted circles as OI_LB and *OAV* respectively.

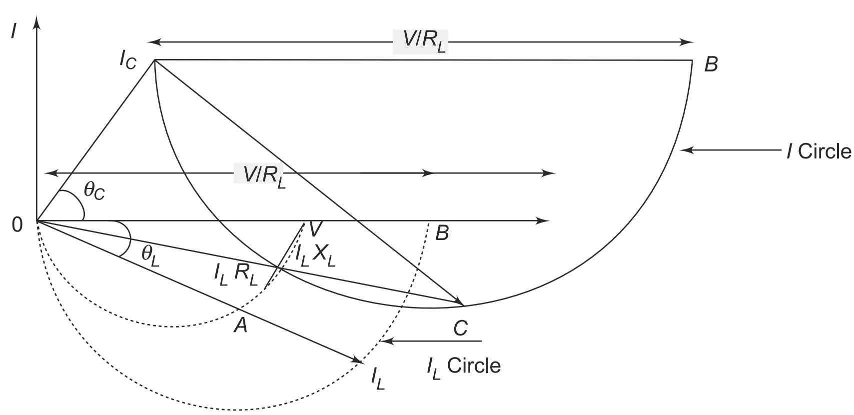


Fig. 3.7(b)

Since the total current is $I_C + I_L$. For example, a particular value of I_C and I_L the total current is represented by *OC* on the total current semicircle. As X_L is varied, the locus of the resultant current is therefore, the circle I_C *CB* as shown with thick line in the Fig. 3.7(b).

(b) Variable X_C A similar procedure can be adopted as outlined above to draw the locus plots of I_1 and I when X_C is varying while R_L, R_C, X_L, V and f are held constant. The curves are shown in Fig. 3.7(c).

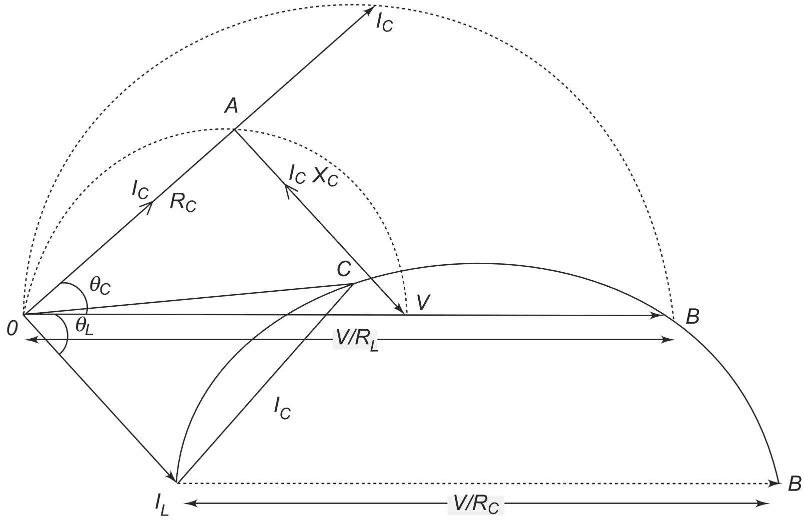


Fig. 3.7(c)

OV presents the voltage vector, *OB* is the maximum current through *RC* branch when $X_L = 0$; OI_L is the current through the R_L branch lagging *OV by* an angle $\theta_L = \tan^{-1}\dfrac{C_L}{R_L}$. As X_C is increased from zero, the current through the capacitive branch I_C decreases and leads V by $\theta_C = \tan^{-1}\dfrac{X_C}{R_C}$. For a particular I_C, the resultant current $I = I_L + I_C$ and is given by *OC*. The dotted semicircle OI_CB is the locus of the I_C, thick circle I_LCB is the locus of the resultant current.

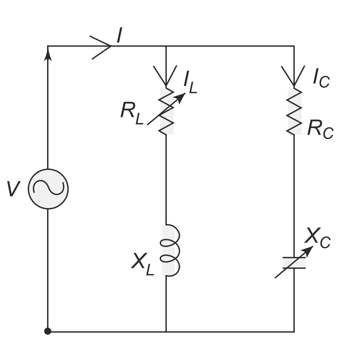


Fig. 3.8(a)

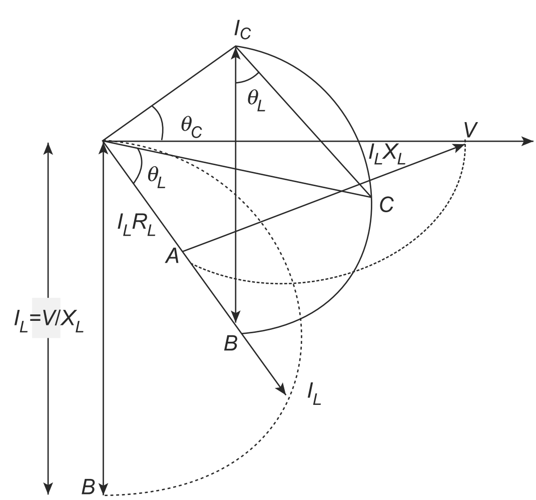


Fig. 3.8(b)

(c) Variable R_L The locus of current for the variation of R_L in Fig. 3.8(a) is shown in Fig. 3.8(b). *OV* represents the reference voltage, OI_LB represents the locus of I_L and I_C *CB* represents the resultant current locus. Maximum $I_L = \dfrac{V}{X_L}$ is represented by *OB*.

(d) Variable R_C The locus of currents for the variation of R_C in Fig. 3.9(a) is plotted in Fig. 3.9(b) where *OV* is the source voltage and semicircle *OAB* represents the locus of I_C The resultant current locus is given by BCI_L.

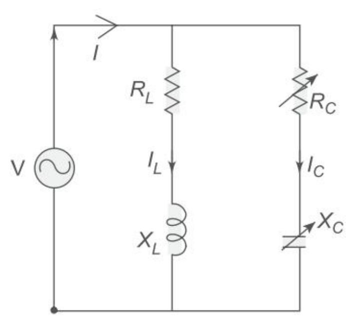


Fig. 3.9(a)

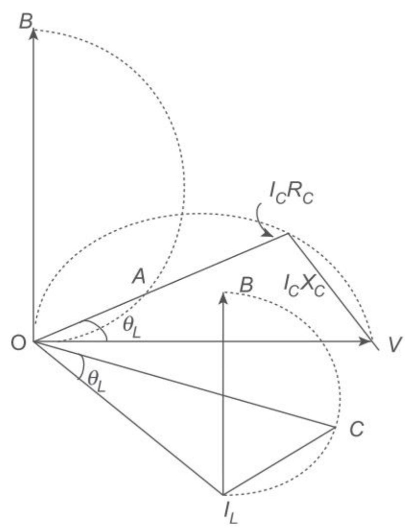


Fig. 3.9(b)

Example 3.4 *For the parallel circuit shown in Fig. 3.10(a), draw the locus of I_1 and I. Mark the range of values for R_1 between 10 Ω and 100 Ω. Assume X_L = 25 Ω and R_2 = 25 Ω. The supply voltage is 200 V and frequency is 50 Hz, both held constant.*

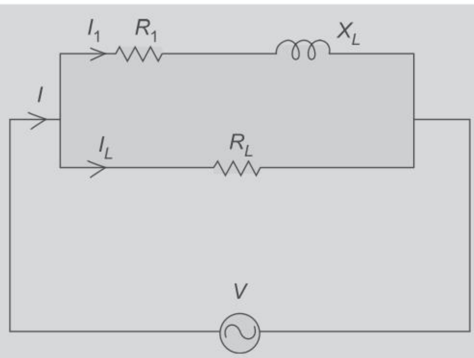


Fig. 3.10(a)

Solution Let us take voltage as reference; on the positive *X*-axis. I_2, is given by $I_2 = \dfrac{200}{25} = 8\text{A}$ and is in phase with *V.*

$$\text{When } R_1 = 10\ \Omega\ \ I_1 = \frac{200}{\sqrt{(100+625)}} = 7.42\text{ A};\ \theta_1 = \tan^{-1}\frac{25}{10} = 68.19°$$

when $R_1 = 100\ \Omega$ $I_1 = \dfrac{200}{\sqrt{(10000+625)}} = 1.94\text{ A}; \theta_2 = \tan^{-1}\dfrac{25}{100} = 14.0°$

The variation of I_1 and I are shown in Fig. 3.10(b).

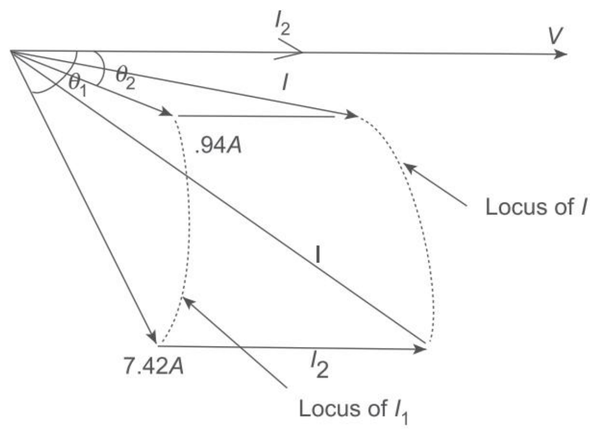


Fig. 3.10(b)

Example 3.5 *Draw the locus of I_2 and I for the parallel circuit shown in Fig. 3.11(a).*

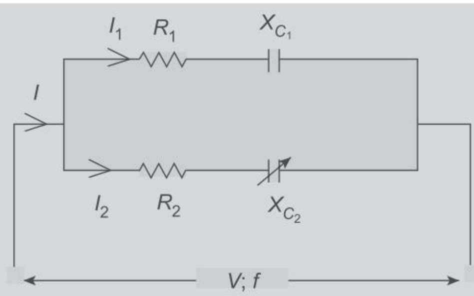


Fig. 3.11(a)

Solution

I_1 leads the voltage by a fixed angle θ_1 given by $\tan^{-1}\dfrac{X_C}{R_1}$

I_2 varies according to the value of X_{C_2}

I_2 is maximum when $X_{C_2} = 0$ and is in phase with V

I_2 is zero when $X_{C_2} = \infty$ as shown in Fig. 3.11(b).

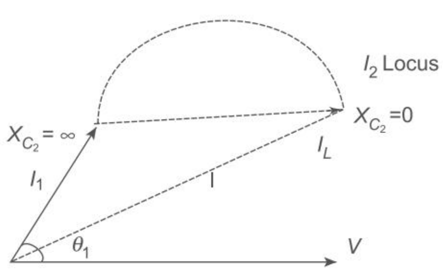


Fig. 3.11(b)

Example 3.6 *For a parallel circuit shown in Fig. 3.12(a) plot the locus of currents.*

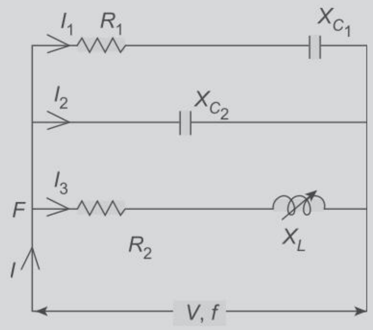


Fig. 3.12(a)

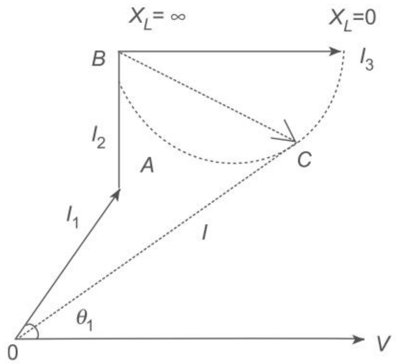


Fig. 3.12(b)

Solution Current I_1, leads the voltage by a fixed angle θ_1 given by $\tan^{-1}\dfrac{X_C}{R_1}$, current I_2 leads the voltage by 90°. I_3 varies according to the value of X_L, when $X_L = 0$, I_3 is maximum and is given by $\dfrac{V}{R_L}$; is in phase with *V;* when $X_L = \infty$, I_3 is zero. Both these extremities are shown in Fig. 3.12(b). For a particular value of I_3 the total current I is given by $I_1 + I_2 + I_3 = OA + AB + BC$.

Example 3.7 *For the circuit shown in the Fig.3.13 draw the locus of the total current vector I*

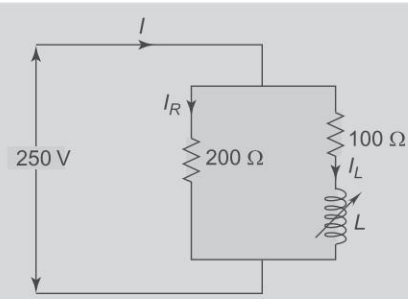


Fig. 3.13

Solution Total current $I = I_R + I_L$

$$I_R = \frac{250}{200} = 1.25\,\text{A} \quad \text{[fixed value] shown with vector OA.}$$

I_L varies from minimum value when x_L is maximum.
To maximum value when x_l is zero.
The locus diagram is shown in the figure below.

The maximum value of $I_L = \dfrac{250}{100} = 2.5\,\text{A}$ [shown with the vector AB]

The maximum current vector OB = 3.75 A is inphase with V. For any intermediate value of X_L, the vector AC represents I_L and the total current vector is OCϕ represents the PF angle of the circuit. The locus is shown in Fig. 3.14.

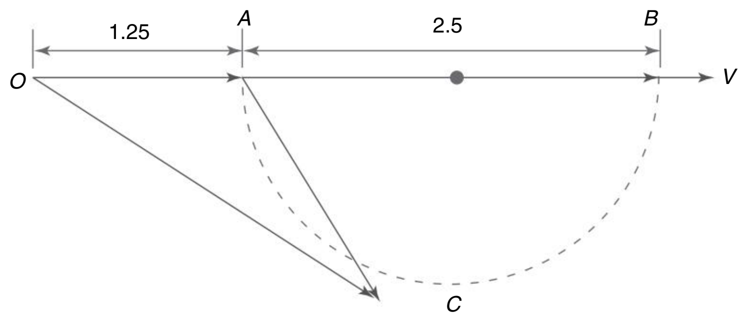


Fig. 3.14

Example 3.8 *Obtain the locus of the total current for the circuit shown in the Fig.3.15. What is the minimum current?*

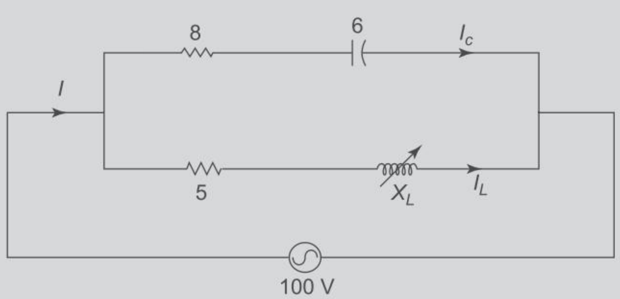


Fig. 3.15

Solution Let the current in the RL branch be I_L and in the RC branch be I_C, the total current I is the algebraic sum of the two currents.

$$I = I_L + I_C$$

When $X_L = 0$

$$I_L = \frac{100\angle 0}{5} = 20\angle 0$$

$$I_C = \frac{100\angle 0}{8 - j6} = \frac{100\angle 0}{10\angle -36.86} = 10\angle 36.86^\circ$$

$$I = 20\angle 0 + 10\angle 36.86^\circ$$
$$= 20 + 8 + j6 = 28 + j6 = 28.63\angle 12^\circ$$

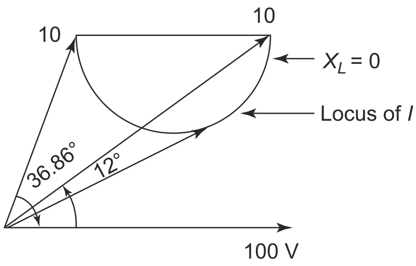


Fig. 3.16

when $x_L \to \infty$,

$I_L \to 0$ with $-90°$ with voltage reference

$$I_C = 10 \angle 36.86°$$

$\therefore$ The total current I is $10 \angle 36.86°$ which is also the minimum current in the circuit. The locus of the total current is in the Fig. 3.16.

3.2 SERIES RESONANCE *[JNTU Nov. 2011]*

Frequency response analysis is important to us for two primary reasons. First, if we know the frequency response then we can predict the response of the circuit to any input. Sinusoidal waveforms have the elegent property that they can be combined to form other (non-sinusoidal) waveforms. Therefore the frequency response allows us to understand a circuits response to more complex inputs. Second, we are often interested in designing circuits with particular frequency characteristics. For example, in the design of an audio 3-way loud speaker system, we would like to direct low frequency signals to the woofers, high frequency signals to the tweets, and mid frequency signals to the mid range speakers. Therefore we would need a circuit that is capable of passing certain frequencies of a signal and rejecting others. The concept of resonance is highly useful in the design of basic filtering circuits for use in everyday applications such as an audio amplifiers.

Consider an AC circuit with a single voltage source and any number of resistors, capacitors and inductors. If the frequency of the source is fixed, then a complete analysis in either the time domain or the frequency domain is possible. In the time domain, a differential is extracted from the circuit and solved. In general, the order of the differential equation is equal to the number of energy storage elements in the circuit. A much easier method is to solve the circuit using phasor analysis in the frequency domain. The analysis is easier in the frequency domain because differentiation in time transforms to multiplication by $j\omega$. As a result, an algebraic equation arises rather than a differential equation. Algebraic equations are easier to solve the differential equations. If the frequency of the voltage source is varied, the impedance of each storage element changes, as the response of the circuit varies as a function of input frequency. The frequency response of a circuit is a quantitative description of its behaviour in the frequency domain.

In many electrical circuits, resonance is a very important phenomenon. The study of resonance is very useful, particularly in the area of communications. For example, the ability of a radio receiver to select a certain frequency, transmitted by a station and to eliminate frequencies from other stations is based on the principle of resonance. In a series RLC circuit, the current lags behind, or leads the applied voltage depending upon the values of X_L and $X_C \cdot X_L$ causes the total current to lag behind the applied voltage, while X_C causes the total current to lead the applied voltage. When $X_L > X_C$, the circuit is predominantly inductive, and when $X_C > X_L$ the circuit is

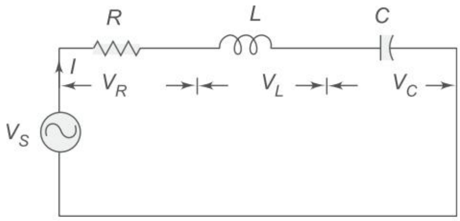


Fig. 3.17

predominantly capacitive. However, if one of the parameters of the series RLC circuit is varied in such a way that the current in the circuit is in phase with the applied voltage, then the circuit is said to be in resonance.

Consider the series RLC circuit shown in Fig. 3.17.

The total impedance for the series RLC circuit is

$$Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

It is clear from the circuit that the current $I = V_S/Z$

The circuit is said to be in resonance if the current is in phase with the applied voltage. In a series RLC circuit, series resonance occurs when $X_L = X_C$. The frequency at which the resonance occurs is called the *resonant frequency*.

Since $X_L = X_C$, the impedance in a series RLC circuit is purely resistive. At the resonant frequency, f_r, the voltages across capacitance and inductance are equal in magnitude. Since they are 180° out of phase with each other, they cancel each other and, hence zero voltage appears across the *LC* combination.

At resonance

$$X_L = X_C \text{ i.e. } \omega L = \frac{1}{\omega C}$$

Solving for resonant frequency, we get

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

In a series RLC circuit, resonance may be produced by varying the frequency, keeping *L* and *C* constant; otherwise, resonance may be produced by varying either *L* or *C* for a fixed frequency.

Example 3.9 *For the circuit shown in Fig. 3.18, determine the value of capacitive reactance and impedance at resonance.*

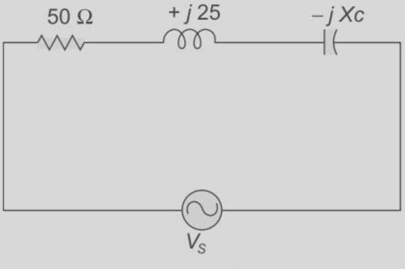


Fig. 3.18

Solution At resonance

$$X_L = X_C$$

Since $X_L = 25\ \Omega$

$$X_L = 25\ \Omega \quad \therefore \frac{1}{\omega C} = 25$$

The value of impedance at resonance is

$$Z = R$$

$$\therefore \quad Z = 50\ \Omega$$

Example 3.10 *Determine the resonant frequency for the circuit shown in Fig. 3.19.*

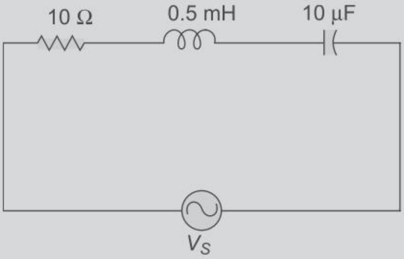


Fig. 3.19

Solution The resonant frequency is $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{10\times10^{-6}\times0.5\times10^{-3}}}$$

$$f_r = 2.25\,\text{kHz}$$

Example 3.11 *A 50 Ω resistor is connected in series with an inductor having internal resistance, a capacitor and 100 V variable frequency supply as shown in Fig. 3.20. At a frequency of 200 Hz, a maximum current of 0.7 A flows through the circuit and voltage across the capacitor is 200 V. Determine the circuit constants.*

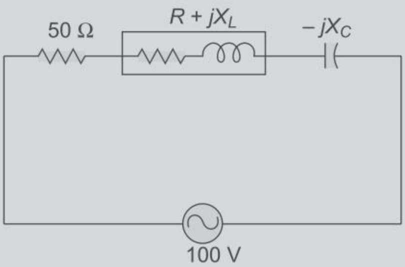


Fig. 3.20

Solution At resonance, current in the circuit is maximum

$$I = 0.7\ \text{A}$$

Voltage across capacitor is $V_C = IX_C$

Since $V_C = 200, I = 0.7$

$$X_C = \frac{1}{\omega C}$$

$$\omega C = \frac{0.7}{200}$$

$$\therefore \quad C = \frac{0.7}{200 \times 2\pi \times 200}$$
$$= 2.785\,\mu\text{F}$$

At resonance

$$X_L - X_C = 0$$

$$\therefore \quad X_L = X_C$$

Since $X_C = \dfrac{1}{\omega C} = \dfrac{200}{0.7} = 285.7\ \Omega$

$$X_L = \omega L = 285.7\ \Omega$$

$$\therefore \quad L = \frac{285.7}{2\pi \times 200} = 0.23\,\text{H}$$

At resonance, the total impedance

$$Z = R + 50$$

$$\therefore \quad R + 50 = \frac{V}{I} = \frac{100}{0.7}$$

$$R + 50 = 142.86\ \Omega$$

$$\therefore \quad R = 92.86\ \Omega$$

3.3 IMPEDANCE AND PHASE ANGLE OF A SERIES RESONANT CIRCUIT *[JNTU Nov. 2011]*

The impedance of a series RLC circuit is

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The variation of X_C and X_L with frequency is shown in Fig. 3.21.

At zero frequency, both X_C and Z are infinitely large, and X_L is zero because at zero frequency the capacitor acts as an open circuit and the inductor acts as a short circuit. As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L, at frequencies below the resonant frequency f_r, Z decreases along with X_C. At resonant frequency $X_C = X_L$, and $Z = R$. At frequencies above the resonant frequency f_r, X_L is larger than X_C, causing Z to increase. The phase angle as a function of frequency is shown in Fig. 3.22.

At a frequency below the resonant frequency, current leads the source voltage because the capacitive reactance is greater than the inductive reactance. The phase angle decreases as the frequency approaches the resonant value, and is 0°

at resonance. At frequencies above resonance, the current lags behind the source voltage, because the inductive reactance is greater than capacitive reactance. As the frequency goes higher, the phase angle approaches 90°.

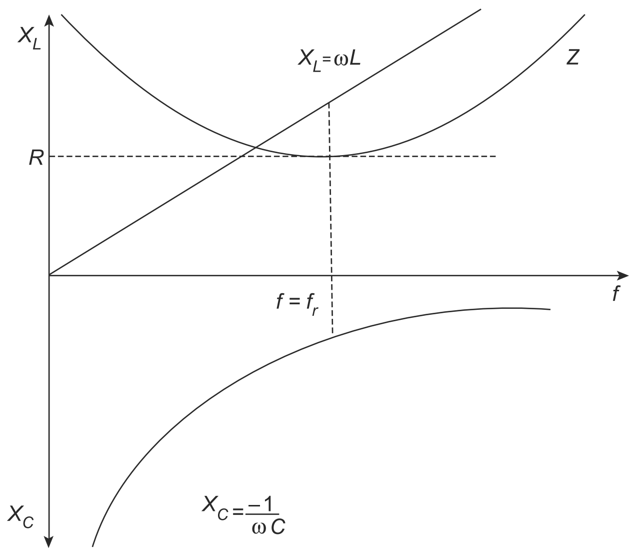


Fig. 3.21

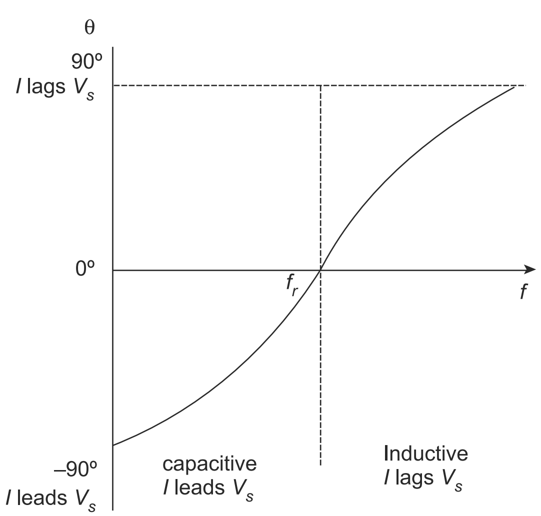


Fig. 3.22

Example 3.12 *For the circuit shown in Fig. 3.23, determine the impedance at resonant frequency, 10 Hz above resonant frequency, and 10 Hz below resonant frequency.*

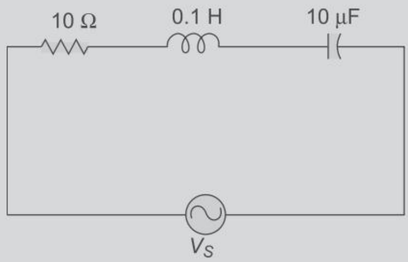


Fig. 3.23

Solution Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{0.1\times 10\times 10^{-6}}} = 159.2\,\text{Hz}$$

At 10 Hz below f_r = 159.2 – 10 = 149.2 Hz
At 10 Hz below f_r = 159.2 + 10 = 169.2 Hz
Impedance at resonance is equal to R

$\therefore \quad Z = 10\,\Omega$

Capacitive reactance at 149.2 Hz is

$$X_{C_1} = \frac{1}{\omega_1 C} = \frac{1}{2\pi\times 149.2\times 10^{-6}\times 10}$$

$$\therefore \quad X_{C_1} = 106.6\,\Omega$$

Capacitive reactance at 169.2 Hz is

$$X_{C_2} = \frac{1}{\omega_2 C} = \frac{1}{2\pi\times 169.2\times 10\times 10^{-6}}$$

$$\therefore \quad X_{C_2} = 94.06\,\Omega$$

Inductive reactance at 149.2 Hz is

$$X_{L_1} = \omega_2 L = 2\pi\times 149.2\times 0.1 = 93.75\,\Omega$$

Inductive reactance at 169.2 Hz is

$$X_{L_2} = \omega_2 L = 2\pi\times 169.2\times 0.1 = 106.31\,\Omega$$

Impedance at 149.2 Hz is

$$|Z| = \sqrt{R^2 + \left(X_{L_1} - X_{C_1}\right)^2}$$

$$= \sqrt{(10)^2 + (93.75 - 106.6)^2} = 16.28\,\Omega$$

Here X_{C_1} is greater than X_{L_1}, so Z is capacitive.
Impedance at 169.2 Hz is

$$|Z| = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2}$$

$$= \sqrt{(10)^2 + (106.31 - 94.06)^2} = 15.81\,\Omega$$

Here X_{L_1} is greater than X_{C_1}, so Z is inductive.

Example 3.13 *A series RLC circuit consists of resistance R = 20 Ω, inductance, L = 0.01H and capacitance, C = 0.04 μF. Calculate the frequency at resonance. If a 10 Volts of frequency equal to the frequency of resonance is applied to this circuit, calculate the values of V_C and V_L across C and L respectively. Find the frequencies at which these voltages V_C and V_L are maximum?* *[JNTU June 2006]*

Solution $R = 20\ \Omega;\ L = 0.01\ \text{H};\ C = 0.04\ \mu\text{F}$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 0.04 \times 10^{-6}}} = 7.957\,\text{kHz}$$

At resonance $I = \dfrac{V}{R} = \dfrac{10}{20} = 0.5\text{A}$

The voltage drop across the inductor is

$$V_L = I\,X_L$$

$$= \frac{\omega L V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

$$= \frac{2\pi \times 7.957 \times 10^3 \times 0.01 \times 10}{\sqrt{(20)^2 + \left(2\pi \times 7.957 \times 10^3 \times 0.01 - \dfrac{1}{2\pi \times 7.957 \times 10^3 \times 0.04 \times 10^{-6}}\right)^2}}$$

$$= 250V$$

$$V_C = I\,X_C = \frac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} \times \frac{1}{\omega C}$$

$$= \frac{10 \times \dfrac{1}{2\pi \times 7.957 \times 10^3 \times 0.04 \times 10^{-6}}}{\sqrt{(20)^2 + \left(2\pi \times 7.957 \times 10^3 \times 0.01 - \dfrac{1}{2\pi \times 7.957 \times 10^3 \times 0.04 \times 10^{-6}}\right)^2}}$$

$$V_C = 250\ \text{V}$$

The frequency at which the voltage across inductor maximum

$$f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1-\frac{R^2C}{2L}}}$$

$$= \frac{1}{2\pi\sqrt{0.01\times 0.04\times 10^{-6}}}\sqrt{\frac{1}{1-\frac{(20)^2\times 0.04\times 10^{-6}}{2\times 0.01}}}$$

$$f_L = 7960 \text{ Hz}$$

The frequency at which the voltage across capacitor maximum

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{2L}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.01\times 0.04\times 10^{-6}}-\frac{(20)^2}{2\times 0.01}}$$

$$\therefore \quad = 7949 \text{ Hz}$$

The maximum voltage across the capacitor occurs below resonant frequency, and the maximum voltage across the inductor occurs above the resonant frequency.

Example 3.14 *A series circuit comprising R, L and C is supplied at 220 V, 50 Hz. At resonance, the voltage across the capacitor is 550 V. The current at resonance is 1 A. Determine the circuit parameters R, L and C.* *[JNTU May 2006]*

Solution At resonance

$$X_L = X_C$$

$$\text{Current at resonance} = I = \frac{V}{R+j(X_L - X_C)} = \frac{V}{R}$$

$$I = \frac{220}{R}$$

$$\therefore \quad R = 220\ \Omega$$

$$V_C = I_O X_C$$

$$550 = 1\times\frac{1}{\omega_o c}$$

$$C = \frac{1}{550\times 2\pi f} = \frac{1}{550\times 2\times\pi\times 50}$$

$$C = 5.78\ \mu\text{F}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$LC = \left(\frac{1}{2\pi f_o}\right)^2$$

$$L = \frac{1}{C}\left(\frac{1}{2\pi f_o}\right)^2$$

$$= \frac{1}{5.78\times 10^{-6}}\left(\frac{1}{100\pi}\right) = 1.750\,\text{H}$$

∴ Circuit elements at resonance are

$$R = 220\ \Omega,\ L = 1.75\ \text{H},\ C = 5.78\ \mu\text{F}$$

3.4 VOLTAGES AND CURRENTS IN A SERIES RESONANT CIRCUIT *[JNTU Nov. 2011]*

The variation of impedance and current with frequency is shown in Fig. 3.24.

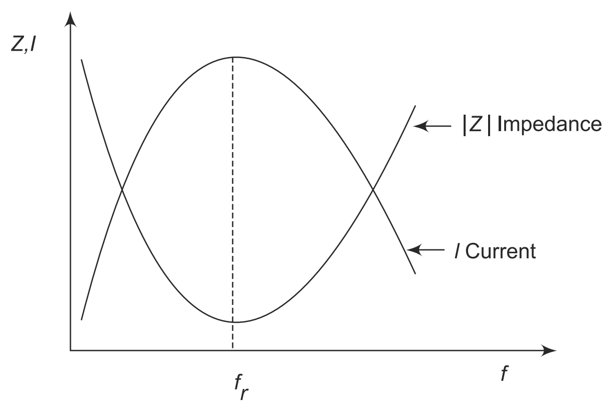


Fig. 3.24

At resonant frequency, the capacitive reactance is equal to inductive reactance, and hence the impedance is minimum. Because of minimum impedance, maximum current flows through the circuit. The current variation with frequency is plotted.

The voltage drop across resistance, inductance and capacitance also varies with frequency. At $f = 0$, the capacitor acts as an open circuit and blocks current. The complete source voltage appears across the capacitor. As the frequency increases, X_C decreases and X_L increases, causing total reactance $X_C - X_L$ to decrease. As a result, the impedance decreases and the current increases. As the current increases, V_R also increases, and both V_C and V_L increase.

When the frequency reaches its resonant value f_r, the impedance is equal to R, and hence, the current reaches its maximum value, and V_R is at its maximum value.

As the frequency is increased above resonance, X_L continues to increase and X_C continues to decrease, causing the total reactance, $X_L - X_C$ to increase. As a result there is an increase in impedance and a decrease in current. As the current decreases, V_R also decreases, and both V_C and V_L decrease. As the frequency becomes very high, the current approaches zero, both V_R and V_C approach zero, and V_L approaches V_S.

The response of different voltages with frequency is shown in Fig. 3.25.

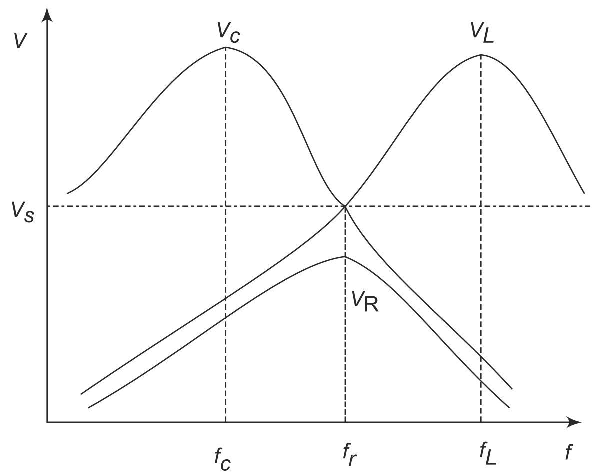


Fig. 3.25

The drop across the resistance reaches its maximum when $f = f_r$. The maximum voltage across the capacitor occurs at $f = f_c$. Similarly, the maximum voltage across the inductor occurs at $f = f_L$.

The voltage drop across the inductor is

$$V_L = IX_L$$

where $$I = \frac{V}{Z}$$

$$\therefore \quad V_L = \frac{\omega L V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

To obtain the condition for maximum voltage across the inductor, we have to take the derivative of the above equation with respect to frequency, and make it equal to zero.

$$\therefore \quad \frac{dV_L}{d\omega} = 0$$

If we solve for ω, we obtain the value of ω when V_L is maximum.

$$\frac{dV_L}{d\omega} = \frac{d}{d\omega}\left\{\omega L V\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2}\right\}$$

$$\frac{LV\left(R^2+\omega^2L^2-\frac{2L}{C}+\frac{1}{\omega^2C^2}\right)^{-1/2} - \frac{\omega LV}{2}\left(R^2+\omega^2L^2-\frac{2L}{C}+\frac{1}{\omega^2C^2}\right)\left(2\omega L^2-\frac{2}{\omega^3C^2}\right)}{R^2+\omega^2L^2-\frac{2L}{C}+\frac{1}{\omega^2C^2}}=0$$

From this

$$R^2-\frac{2L}{C}+2/\omega^2C^2=0$$

$$\therefore \qquad \omega L=\sqrt{\frac{2}{2LC-R^2C^2}}=\frac{1}{\sqrt{LC}}\sqrt{\frac{2}{2-\frac{R^2C}{L}}}$$

$$f_L=\frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1-\frac{R^2C}{2L}}}$$

Similarly, the voltage across the capacitor is

$$V_C=IX_C=\frac{I}{\omega C}$$

$$\therefore V_C=\frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}\times\frac{1}{\omega C}$$

To get maximum value $\frac{dV_C}{d\omega}=0$

If we solve for ω, we obtain the value of ω when V_C is maximum.

$$\frac{dV_C}{d\omega}=\omega C\frac{1}{2}\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]^{-1/2}\left[2\left(\omega L-\frac{1}{\omega C}\right)\left(L+\frac{1}{\omega^2C}\right)\right]$$
$$+\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}\,C=0$$

From this

$$\omega_C^2=\frac{1}{LC}-\frac{R^2}{2L}$$

$$\omega_C = \sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$\therefore \qquad f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

The maximum voltage across the capacitor occurs below the resonant frequency; and the maximum voltage across the inductor occurs above the resonant frequency.

3.5 BANDWIDTH OF AN RLC CIRCUIT

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency, and it is denoted by *BW*. Figure 3.26 shows the response of a series RLC circuit.

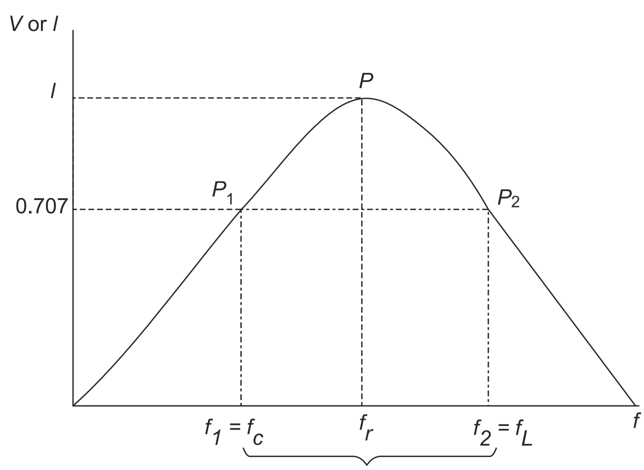


Fig. 3.26

Here the frequency f_1 is the frequency at which the current is 0.707 times the current at resonant value, and it is called the lower cut-off frequency. The frequency f_2 is the frequency at which the current is 0.707 times the current at resonant value (i.e. maximum value), and is called the *upper cut-off frequency*. The bandwidth, or *BW*, is defined as the frequency difference between f_2 and f_1.

$$\therefore \qquad BW = f_2 - f_1$$

The unit of *BW* is hertz (Hz).

If the current at P_1 is 0.707 I_{max}, the impedance of the circuit at this point is $\sqrt{2}\,R$, and hence

$$\frac{1}{\omega_1 C} - \omega_1 L = R \tag{3.10}$$

Similarly, $$\omega_2 L - \frac{1}{\omega_2 C} = R \tag{3.11}$$

If we equate both the above equations, we get

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right) \tag{3.12}$$

From Eq. 3.12, we get

$$\omega_1 \omega_2 = \frac{1}{LC}$$

we have $$\omega_r^2 = \frac{1}{LC}$$

$\therefore$ $$\omega_r^2 = \omega_1 \omega_2 \tag{3.13}$$

If we add Eqs 3.10 and 3.11, we get

$$\frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C} = 2R$$

$$(\omega_2 - \omega_1)L + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = 2R \tag{3.14}$$

Since

$$C = \frac{1}{\omega_r^2 L}$$

and

$$\omega_1 \omega_2 = \omega_r^2$$

$$(\omega_2 - \omega_1)L + \frac{\omega_r^2 L(\omega_2 - \omega_1)}{\omega_r^2} = 2R \tag{3.15}$$

From Eq. 3.15, we have

$$\omega_2 - \omega_1 = \frac{R}{L} \tag{3.16}$$

$$f_2 - f_1 = \frac{R}{2\pi L} \tag{3.17}$$

or $$BW = \frac{R}{2\pi L}$$

From Eq. 3.17, we have

$$f_2 - f_1 = \frac{R}{2\pi L}$$

$$\therefore \quad f_r - f_1 = \frac{R}{4\pi L}$$

$$f_2 - f_r = \frac{R}{4\pi L}$$

The lower frequency limit $f_1 = f_r - \dfrac{R}{4\pi L}$ (3.18)

The upper frequency limit $f_2 = f_r + \dfrac{R}{4\pi L}$ (3.19)

If we divide the equation on both sides by f_r, we get

$$\frac{f_2 - f_1}{f_r} = \frac{R}{2\pi f_r L} \tag{3.20}$$

Here an important property of a coil is defined. It is the ratio of the reactance of the coil to its resistance. This ratio is defined as the Q of the coil. Q is known as a figure of merit, it is also called quality factor and is an indication of the quality of a coil.

$$Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R} \tag{3.21}$$

If we substitute Eq. 3.20 in Eq. 3.21, we get

$$\frac{f_2 - f_1}{f_r} = \frac{1}{Q} \tag{3.22}$$

The upper and lower cut-off frequencies are sometimes called the *half-power frequencies*. At these frequencies the power from the source is half of the power delivered at the resonant frequency.

At resonant frequency, the power is

$P_{max} = I^2_{max} R$

At frequency f_1, the power is $P_1 = \left(\dfrac{I_{max}}{\sqrt{2}}\right)^2 R = \dfrac{I^2_{max} R}{2}$

Similarly, at frequency f_2, the power is

$$P_2 = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$$

$$= \frac{I^2_{max} R}{2}$$

The response curve in Fig. 3.26 is also called the *selectivity curve* of the circuit. Selectivity indicates how well a resonant circuit responds to a certain frequency and eliminates all other frequencies. The narrower the bandwidth, the greater the selectivity.

3.6 THE QUALITY FACTOR (*Q*) AND ITS EFFECT ON BANDWIDTH

The quality factor, Q, is the ratio of the reactive power in the inductor or capacitor to the true power in the resistance in series with the coil or capacitor.

The quality factor

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

In an inductor, the max energy stored is given by $\dfrac{LI^2}{2}$

Energy dissipated per cycle $= \left(\dfrac{I}{\sqrt{2}}\right)^2 R \times T = \dfrac{I^2 RT}{2}$

$\therefore$ Quality factor of the coil $Q = 2\pi \times \dfrac{\frac{1}{2} LI^2}{\frac{I^2 R}{2} \times \frac{1}{f}} = \dfrac{2\pi f L}{R} = \dfrac{\omega L}{R}$

Similarly, in a capacitor, the max energy stored is given by $\dfrac{CV^2}{2}$

The energy dissipated per cycle $= (I/\sqrt{2})^2 R \times T$

The quality factor of the capacitance circuit

$$Q = \frac{2\pi \frac{1}{2} C \left(\frac{I}{\omega C}\right)^2}{\frac{I^2}{2} R \times \frac{1}{f}} = \frac{1}{\omega CR}$$

In series circuits, the quality factor $Q = \dfrac{\omega L}{R} = \dfrac{1}{\omega CR}$

We have already discussed the relation between bandwidth and quality factor, which is $Q = \dfrac{f_r}{BW}$.

A higher value of circuit Q results in a smaller bandwidth. A lower value of Q causes a larger bandwidth.

3.7 MAGNIFICATION IN SERIES RESONANCE

If we assume that the voltage applied to the series RLC circuit is V, and the current at resonance is I, then the voltage across L is $V_L = IX_L = (V/R)\,\omega_r L$

Similarly, the voltage across C

$$V_C = I\,X_C = \frac{V}{R\omega_r C}$$

Since $Q = 1/\omega_r CR = \omega_r L/R$

where ω_r is the frequency at resonance.

Therefore $V_L = VQ$

$$V_C = VQ$$

The ratio of voltage across either L or C to the voltage applied at resonance can be defined as magnification.

$\therefore$ Magnification $= Q = V_L/V$ or V_C/V

Example 3.15 *A series circuit with R = 10 Ω, L = 0.1 H and C = 50 μF has an applied voltage V = 50 ∠0° with a variable frequency. Find the resonant frequency, the value of frequency at which maximum voltage occurs across the inductor and the value of frequency at which maximum voltage occurs across the capacitor.*

Solution The frequency at which maximum voltage occurs across the inductor is

$$f_L = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{\left(1-\frac{R^2C}{2L}\right)}}$$

$$= \frac{1}{2\pi\sqrt{0.1\times 50\times 10^{-6}}}\sqrt{\frac{1}{1-\left(\frac{(10)^2\times 50\times 10^{-6}}{2\times 0.1}\right)}}$$

$$= 72.08 \text{ Hz}$$

Similarly, $f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{2L}}$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1\times 50\times 10^{-6}}-\frac{(10)^2}{2\times 0.1}}$$

$$= \frac{1}{2\pi}\sqrt{200000-500}$$

$$= 71.08 \text{ Hz}$$

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.1\times 50\times 10^{-6}}} = 71.18\,\text{Hz}$

It is clear that the maximum voltage across the capacitor occurs below the resonant frequency and the maximum inductor voltage occurs above the resonant frequency.

Example 3.16 *A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor when the capacitor is set to 500 PF, the current has the max. value, while it is reduced to one half when capacitance 600 PF, find (i) resistance (ii) inductance (iii) Q factor of inductor.*

Solution Given f = 1 MHz

Let the max. current be I_{max}

Given at 1 MHz, for C = 500 Pf

$$I = I_{max}$$

∴ Imaginary part of impedance is zero, i.e. $X_L = X_C$

$$2\pi f L = \frac{1}{2\pi fc}$$

$$6.283 \times 10^6 \times L = 318.31$$

$$L = 50.66\,\mu\text{H}$$

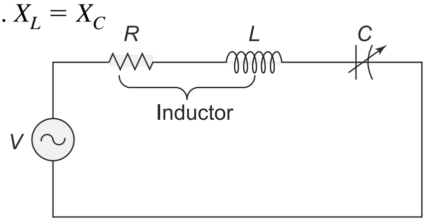


Fig. 3.27

Now also given $I = \dfrac{I_{max}}{2}$ at C = 600 PF

$$I = \frac{I_{max}}{2} = \frac{V}{R + j(6.283\times 10^6 L - 265.25)} \qquad (3.23)$$

$$\left(\because \quad X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi\times 10^6 \times 600\times 10^{-12}} = 265.25\right)$$

$$\text{and } I_{max} = \frac{V}{R} \qquad (3.24)$$

Dividing Eq. 3.24 by Eq. 3.23

$$Z = \frac{R + j(6.283\times 10^6 L - 265.25)}{R}$$

$$\Rightarrow \quad 2R = R + j\,(6.283 \times 10^6 L - 265.25)$$

$$R = j\,(318.31 - 265.25)$$

$$R = 53.06\,\Omega$$

∴ (i) $R = 53.06\,\Omega$

(ii) $L = 50.66\,\mu\text{H}$

(iii) $Q = \dfrac{\omega L}{R} = 5.999 \approx 6$

Example 3.17 *Obtain the expression for the frequency at which maximum voltage occurs across the capacitance in series resonance circuit in terms of the Q-factor and resonance frequency.*

Solution The frequency at which V_c is maximum is given by

$$f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$f_c = \frac{1}{2\pi}\left[\sqrt{\frac{1}{LC}\left[1 - \frac{R^2C}{2L}\right]}\right]$$

$$= \frac{1}{2\pi}\left[\sqrt{\frac{R^2}{LC}\left(\frac{1}{R^2} - \frac{C}{2L}\right)}\right]$$

$$= \frac{1}{2\pi}\frac{R}{\sqrt{LC}}\left[\sqrt{\frac{1}{R^2} - \frac{C}{2L}}\right]$$

$$= \frac{1}{2\pi}\frac{R}{\sqrt{LC}}\left[\sqrt{\frac{C}{L}\left[\frac{L}{CR^2} - \frac{1}{2}\right]}\right]$$

$$= \frac{1}{2\pi\sqrt{LC}}.R\sqrt{\frac{C}{L}}\left[\frac{L}{CR^2} - \frac{1}{2}\right]^{1/2}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}};\; Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow \frac{1}{Q} = R\sqrt{\frac{C}{L}}$$

$$\therefore \qquad f_c = \frac{f_o}{Q}\left[\frac{L}{CR^2} - \frac{1}{2}\right]^{1/2}$$

Example 3.18 *In a series RLC circuit if the applied voltage is 10 V, and resonance frequency is 1 kHz, and Q factor is 10, what is the maximum voltage across the inductance.*

Solution Resonance freq. $(f_r) = \dfrac{1}{2\pi\sqrt{LC}} = 1000$ (3.25)

Quality factor $(Q) = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = 10$ (3.26)

$$\sqrt{LC} = \frac{1}{2\pi \times 1000} = 6283.18$$

$$LC = 39.47 \times 10^6$$

From 3.25, $\dfrac{1}{2\pi} = \sqrt{LC}\; 1000$ (3.27)

From 3.26, $\dfrac{1}{R} = \sqrt{\dfrac{C}{L}}\, 10$ (3.28)

From 3.27 and 3.28

$$\frac{1}{2\pi R} = 10^4 \sqrt{LC}\sqrt{\frac{C}{L}}$$

$$\frac{1}{2\pi RC} = 10000$$

$$RC = 1.59154 \times 10^{-5} \simeq 1.6 \times 10^{-5}$$

The maximum voltage across the inductance occurs at frequency greater than the resonance frequency which is given by

$$f_L = \frac{1}{2\pi\sqrt{LC - \dfrac{(RC)^2}{2}}}$$

$$f_L = \frac{1}{2\pi\sqrt{39.47 \times 10^6 - \dfrac{(1.6 \times 10^{-5})^2}{2}}} = 1002.5$$

It can be observed that, the above frequency is approximately equal to resonance frequency,

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{39.47 \times 10^6}}$$

Hence we can take the voltage across the inductor $= Q \times V$

$= 10 \times 10$

$= 100$ volts

Example 3.19 *A series RLC circuit is connected across a variable frequency supply and has R = 12 ohms, L = l mH and C = l000 pF. Calculate*

(a) Resonant frequency.

(b) Q factor and

(c) Half power frequencies. Derive the formulae used. *[JNTU Jan 2010]*

Solution (a) Resonant frequency $= \dfrac{1}{2\pi\sqrt{\text{LC}}} = \dfrac{1}{2\pi\sqrt{1 \times 10^{-3} \times 1000 \times 10^{-12}}}$ Hz

$= 159.155$ KHz

(b) Q-factor $= \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

$$= \frac{1}{12}\sqrt{\frac{1 \times 10^{-3}}{1000 \times 10^{-12}}} = 83.33$$

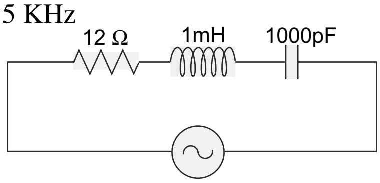


Fig. 3.28

(c) Half Power Frequencies are given as,

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] = 158.203 \text{ KHz}$$

and
$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] = 160.113 \text{ KHz}$$

Example 3.20 *Determine the quality factor of a coil for the series circuit consisting of R = 10 Ω, L = 0.1 H and C = 10 μF.*

Solution Quality factor $Q = \frac{f_r}{BW}$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1\times10\times10^{-6}}} = 159.2\,\text{Hz}$$

At lower half power frequency, $X_C > X_L$

$$\frac{1}{2\pi f_1 C} - 2\pi f_1 L = R$$

From which $f_1 = \frac{-R+\sqrt{R^2+4L/C}}{4\pi L}$

At upper half power frequency $X_L > X_C$

$$2\pi f_2 L - \frac{1}{2\pi f_2 C} = R$$

From which $f_2 = \frac{+R+\sqrt{R^2+4L/C}}{4\pi L}$

Bandwidth $BW = f_2 - f_1 = \frac{R}{2\pi L}$

Hence $Q_0 = \frac{f_r}{BW} = \frac{2\pi f_r L}{R} = \frac{2\times\pi\times159.2\times0.1}{10}$

$$Q_0 = \frac{f_r}{BW} = 10$$

Example 3.21 *A voltage v(t) = 10 sin ωt is applied to a series RLC circuit. At the resonant frequency of the circuit, the maximum voltage across the capacitor is found to be 500 V. Moreover, the bandwidth is known to be 400 rad/sec and the impedance at resonance is 100 . Find the resonant frequency. Also find the values of L and C of the circuit.*

Solution The applied voltage to the circuit is

$$V_{max} = 10\text{ V}$$

$$V_{rms} = \frac{10}{\sqrt{2}} = 7.07\text{ V}$$

The voltage across capacitor $V_C = 500\text{ V}$

The magnification factor $Q = \frac{V_C}{V} = \frac{500}{7.07} = 70.7$

The bandwidth $BW = 400\text{ rad/sec}$

$$\omega_2 - \omega_1 = 400\text{ rad/sec}$$

The impedance at resonance $Z = R = 100\ \Omega$

Since $$Q = \frac{\omega_r}{\omega_2 - \omega_1}$$

$$\omega_r = Q(\omega_2 - \omega_1) = 28280\text{ rad/sec}$$

$$f_r = \frac{28280}{2\pi} = 4499\text{ Hz}$$

The bandwidth $$\omega_2 - \omega_1 = \frac{R}{L}$$

$$L = \frac{R}{\omega_2 - \omega_1} = \frac{100}{400} = 0.25\text{ H}$$

Since $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{(2\pi f_r)^2 \times L} = \frac{1}{2\pi \times (4499)^2 \times 0.25} = 5\,n\text{F}$$

Example 3.22 *A series RLC circuit consists of a 50 Ω. resistance, 0.2 H inductance and 10 μF capacitor with an applied voltage of 20 V. Determine the resonant frequency. Find the Q factor of the circuit. Compute the lower and upper frequency limits and also find the bandwidth of the circuit.*

Solution Resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 10 \times 10^{-6}}} = 112.5\text{ Hz}$$

Quality factor $Q = \frac{\omega L}{R} = \frac{2\pi \times 112.5 \times 0.2}{50} = 2.83$

Lower frequency limit

$$f_1 = f_r - \frac{R}{4\pi L} = 112.5 - \frac{50}{4\times\pi\times 0.2} = 92.6\,\text{Hz}$$

Upper frequency limit

$$f_2 = f_r + \frac{R}{4\pi L} = 112.5 + \frac{50}{4\pi\times 0.2} = 112.5 + 19.89 = 132.39\,\text{Hz}$$

Bandwidth of the circuit

$$BW = f_2 - f_1 = 132.39 - 92.6 = 39.79\ \text{Hz}$$

Example 3.23 *Determine the Quality factor, bandwidth and the half power frequencies of a series resonant circuit with R = 5 Ω, L = 0.05 H and C = 5 μf.*

Solution Resonance frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.05\times 5\times 10^{-6}}}$$

$$= 318.3\,\text{Hz}$$

Quality factor $$Q = \frac{W_r L}{R} = \frac{2\pi(318.3)(0.05)}{5}$$

$$= 20$$

Bandwidth $$= \frac{f_r}{Q} = \frac{318.3}{20} = 15.915\,\text{Hz}$$

$$f_r = \sqrt{f_1 f_2}$$

$$f_r^2 = f_1 f_2 \Rightarrow f_1 = \frac{f_r^2}{f_2}$$

Also $f_2 - f_1 = 15.915$ Hz

$$f_2 - \frac{f_r^2}{f_2} = 15.915$$

$$f_2^2 - f_r^2 - 15.915 f_2 = 0$$

$$\Rightarrow \quad f_2^2 - 15.915 f_2 - 10.13\times 10^4 = 0$$

$$f_2 = 326\ \text{Hz}$$

$$f_1 = 310\ \text{Hz}$$

Half power points can also be calculated using

$$f_1 = f_r - \frac{R}{4\pi L} = 318.3 - \frac{5}{4\pi \times 0.05} = 310\,\text{Hz}$$

$$f_2 = f_r + \frac{R}{4\pi L} = 318.3 + \frac{5}{4\pi \times 0.05} = 326\,\text{Hz}$$

Example 3.24 *A series RLC circuit with Q = 250 is resonant at 1.5 MHz. Find the frequencies at half power points and also bandwidth.*

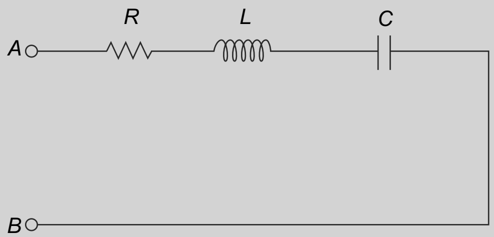


Fig. 3.29

Solution Given $Q = 250$

$$Q = \frac{\omega_o L}{R}$$

$$250 = \frac{2\pi \times f_o \times L}{R} \Rightarrow \frac{R}{L} = \frac{2\pi \times 1.5 \times 10^6}{250} = 37.7 \times 10^3$$

Lower half power frequency $f_1 = f_r - \dfrac{R}{4\pi L}$

$$= 1.5 \times 10^6 - \frac{37.7 \times 10^3}{4\pi}$$

$$= 1.5 \times 10^6 - 3 \times 10^3$$

$$= 1.496\ \text{MHz}$$

Upper half power frequency $f_2 = f_r + \dfrac{R}{4\pi L}$

$$= 1.5 \times 10^6 + \frac{37.7 \times 10^3}{4\pi}$$

$$= 1.5\,\text{M} + 3k = 1.503\ \text{MHz}$$

Bandwidth $= f_2 - f_1 = 1.503\,\text{M} - 1.496\ \text{M} = 7\ \text{kHz}$

Example 3.25 *For the circuit shown in Fig. 3.30, determine the value of Q at resonance and bandwidth of the circuit.*

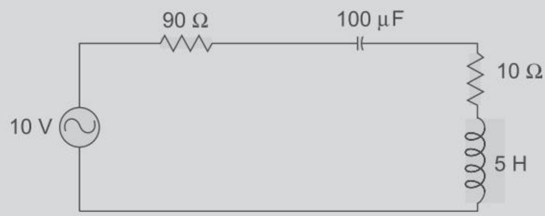


Fig. 3.30

Solution The resonant frequency,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{5\times100\times10^{-6}}}$$

$$= 7.12\,\text{Hz}$$

Quality factor $\qquad Q = X_L/R = 2\pi f_r L/R$

$$= \frac{2\pi\times7.12\times5}{100} = 2.24$$

Bandwidth of the circuit is $BW = \dfrac{f_r}{Q} = \dfrac{7.12}{2.24} = 3.178\,\text{Hz}$

Example 3.26 *For the circuit shown in Fig. 3.31, determine the frequency at which the circuit resonates. Also find the voltage across the inductor at resonance and the Q factor of the circuit.*

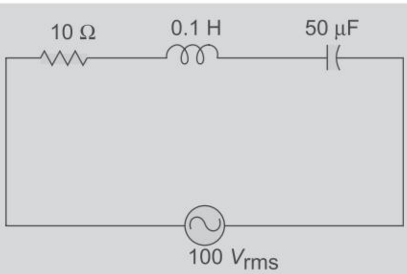


Fig. 3.31

Solution The frequency of resonance occurs when $X_L = X_C$

$$\omega L = \frac{1}{\omega C}$$

$$\therefore \qquad \omega = \frac{1}{\sqrt{LC}}\text{ radians/sec} = \frac{1}{\sqrt{0.1\times50\times10^{-6}}}$$

$$= 447.2\text{ radians/sec}$$

$$f_r = \frac{1}{2\pi}(447.2) = 71.17\text{ Hz}$$

The current passing through the circuit at resonance,

$$I = \frac{V}{R} = \frac{100}{10} = 10\text{ A}$$

The voltage drop across the inductor

$$V_L = IX_L = I\omega L$$

$$= 10 \times 447.2 \times 0.1 = 447.2\text{ V}$$

The quality factor $Q = \dfrac{\omega L}{R}$

$$= \frac{447.2 \times 0.1}{10} = 4.47$$

Example 3.27 *A series RLC circuit has a quality factor of 5 at 50 rad/sec. The current flowing through the circuit at resonance is 10 A and the supply voltage is 100 V. The total impedance of the circuit is 20 Ω. Find the circuit constants.*

Solution The quality factor $Q = 5$

At resonance the impedance becomes resistance.

The current at resonance is $I = \dfrac{V}{R}$

$$\therefore \quad 10 = \frac{100}{R}$$

$$R = 10\ \Omega$$

$$Q = \frac{\omega L}{R}$$

Since $Q = 5, R = 10$

$$\omega L = 50$$

$$\therefore \quad L = \frac{50}{\omega} = 1\text{H}$$

$$Q = \frac{1}{\omega CR}$$

$$C = \frac{1}{Q\omega R}$$

$$= \frac{1}{5 \times 50 \times 10}$$

$$C = 400\ \mu F$$

Example 3.28 *In the circuit shown in Fig. 3.32 a maximum current of 0.1 A flows through the circuit when the capacitor is at 5 μF with a fixed frequency and a voltage of 5 V. Determine the frequency at which the circuit resonates, the bandwidth, the quality factor Q and the value of resistance at resonant frequency.*

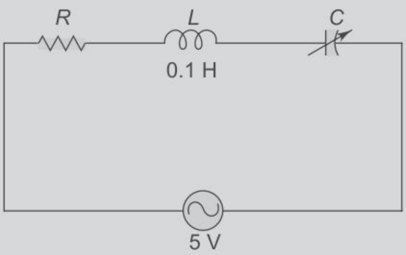


Fig. 3.32

Solution At resonance, the current is maximum in the circuits

$$I = \frac{V}{R}$$

$$\therefore \qquad R = \frac{V}{I} = \frac{5}{0.1} = 50\,\Omega$$

The resonant frequency is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 \times 5 \times 10^{-6}}} = 1414.2 \text{ rad/sec}$$

$$f_r = \frac{1414.2}{2\pi} = 225\,\text{Hz}$$

The quality factor is

$$Q = \frac{\omega L}{R} = \frac{1414.2 \times 0.1}{50} = 2.8$$

Since $$\frac{f_r}{BW} = Q$$

The bandwidth $BW = \dfrac{f_r}{Q} = \dfrac{225}{2.8} = 80.36\,\text{Hz}$

Example 3.29 *In the circuit shown in Fig. 3.33, determine the circuit constants when the circuit draws a maximum current at 10 μF with a 10 V, 100 Hz supply. When the capacitance is changed to 12 μF, the current that flows through the circuit becomes 0.707 times its maximum value. Determine Q of the coil at 900 rad/sec. Also find the maximum current that flows through the circuit.*

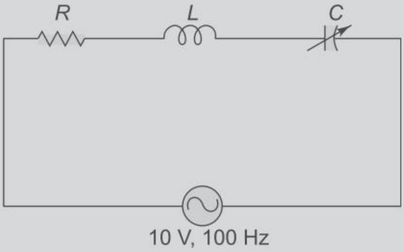


Fig. 3.33

Solution At resonant frequency, the circuit draws maximum current. So, the resonant frequency $f_r = 100$ Hz

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{C \times (2\pi f_r)^2}$$

$$= \frac{1}{10 \times 10^{-6}(2\pi \times 100)^2} = 0.25 \text{ H}$$

We have $$\omega L - \frac{1}{\omega C} = R$$

$$900 \times 0.25 - \frac{1}{900 \times 12 \times 10^{-6}} = R$$

$\therefore$ $$R = 132.4\ \Omega$$

The quality factor $Q = \frac{\omega L}{R} = \frac{900 \times 0.25}{132.4} = 1.69$

The maximum current in the circuit is $I = \frac{10}{132.4} = 0.075$ A

Example 3.30 *In the circuit shown in Fig. 3.34 the current is at its maximum value with capacitor value C = 20 μF and 0.707 times its maximum value with C = 30 μF. Find the value of Q at ω = 500 rad/sec, and circuit constants.*

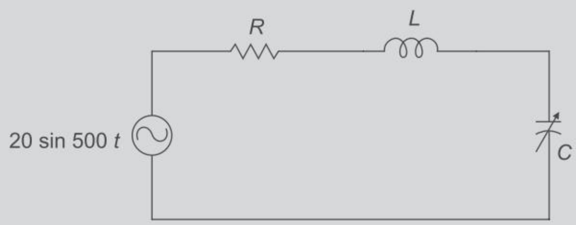


Fig. 3.34

Solution The voltage applied to the circuit is $V = 20$ V. At resonance, the current in the circuit is maximum. The resonant frequency $\omega_r = 500$ rad/sec.

Since $$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad L = \frac{1}{\omega_r^2 C} = \frac{1}{(500)^2 \times 20 \times 10^{-6}} = 0.2\,\text{H}$$

Since we have $$\omega L - \frac{1}{\omega C} = R$$

$$500 \times 0.2 - \frac{1}{500 \times 30 \times 10^{-6}} = R$$

$$\therefore R = 100 - 66.6 = 33.4$$

The quality factor is $Q = \frac{\omega L}{R} = \frac{500 \times 0.2}{33.4} = 2.99$

Example 3.31 *A coil having R = 15 Ω and L = 40 mH is connected in series with a capacitor across a 240 V source resonates at 350 Hz. Find the value of*

(a) capacitance *(b) power dissipated in the coil*

(c) Q factor *(d) voltage across the capacitor and coil*

Solution (a) At $f = f_r$, $X_L = X_C$

$$\Rightarrow \quad C = \frac{1}{4\pi^2 L f_r^2} = \frac{1}{4 \times \pi^2 \times 40 \times 10^{-3} \times (350)^2}$$

$$= 5.17\,\mu\text{F}$$

(b) At resonance $I = \frac{V}{R} = \frac{240}{15} = 16\text{ A}$

Power dissipated $= I^2 R = (16)^2 \times 15 = 4.84$ kW

(c) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{15}\sqrt{\frac{40 \times 10^{-3}}{5.17 \times 10^{-6}}} = 5.863$

(d) $V_c = -jQV = 5.863 \times 240 = 1407.12 \angle -90^\circ\ V$

Let the voltage across the inductance of the coil be $V_L = V_C$ in magnitude

$\therefore \quad V_L = 1407.12 \angle 90^\circ$

Let V_R is the voltage across the resistance of the coil then

$$V_R = V = 240 \angle 0^\circ$$

The voltage across the coil $V_{coil} = V_L + V_R$

$$= 1407.12 \angle 90^\circ + 240 \angle 0^\circ$$

$$= 240 + j1407.12$$

$$= 1427.44 \angle 80.32^\circ$$

Example 3.32 *With respect to a (resonant circuit), i.e., series resonant circuit, prove that the bandwidth is inversely proportional to the Q-factor at resonance*

Solution The bandwidth of any system is the range of frequencies for which the current (or) the output voltage equals to 70.7% of it's value at resonance.

Bandwidth $= f_2 - f_1$

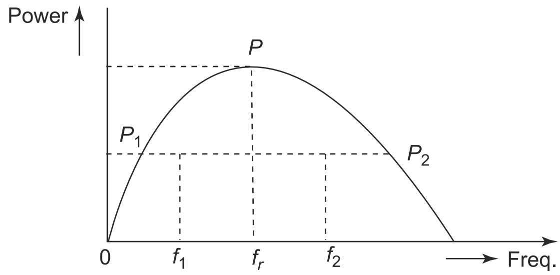


Fig. 3.35

If the current at P_1 is 0.707 Imax, the impedance of the circuit at this point is $\sqrt{2}\,R$.

$$\frac{1}{\omega_1 c} - \omega_1 L = R \tag{3.29}$$

$$\omega_2 L - \frac{1}{\omega_2 c} = R \tag{3.30}$$

By equating 3.29 and 3.30 we get,

$$\frac{1}{c}\left[\frac{\omega_1 + \omega_2}{\omega_1\omega_2}\right] = L(\omega_1 + \omega_2)$$

$$\omega_1\omega_2 = \frac{1}{LC}$$

we have $\omega_r^2 = \dfrac{1}{LC} \Rightarrow \omega_r^2 = \omega_1\,\omega_2$

Adding the equations 3.31 and 3.32

$$\Rightarrow \quad \frac{1}{\omega_1 c} - \frac{1}{\omega_2 c} + L(\omega_2 - \omega_1) = 2R$$

$$\frac{1}{c}\left[\frac{\omega_1 - \omega_2}{\omega_1\omega_2}\right] + L(\omega_2 - \omega_1) = 2R$$

Since $c = \dfrac{1}{\omega_r^{\,2} L}, \omega_1\,\omega_2 = \omega_r^{\,2}$

$$(\omega_2 - \omega_1)\,L + L(\omega_2 - \omega_1) = 2R$$

$$L\,(\omega_2 - \omega_1) = R$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$\Rightarrow \quad f_2 - f_1 = \frac{R}{2\pi L}$$

$$\therefore \quad \frac{f_2 - f_1}{f_r} = \frac{R}{2\pi f_r L} = \frac{1}{Q}$$

$$\therefore \quad Q = \frac{f_r}{BW}$$

Example 3.33 *A series R-L-C circuit with R = 100 V, L = 0.5 H and C = 40 μF has an applied voltage of 50 V with variable frequency. Calculate*

(a) Resonance frequency
(b) Current at resonance
(c) Voltage across R, L and C
(d) Upper and Lower half frequencies
(e) Bandwidth
(f) Q-factor of the circuit *[JNTU May 2007]*

Solution $R = 100\ \Omega, L = 0.5\ H, C = 40\ \mu\text{F}, V = 50\ V$

(a) Resonance frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.5 \times 40 \times 10^{-6}}}$

$$f_r = 35.58 \text{ Hz}$$

(b) Current at resonance, $I = \dfrac{V}{Z} = \dfrac{V}{R}$

$$I = \frac{V}{R} = \frac{50}{100} = 0.5 \text{ A}$$

(c) Voltage across resistanc e, $V_R = I_R = 0.5 \times 100 = 50$ volts

Voltage across inductance, $V_L = \omega L = 2\pi \times 0.5 \times 35.58 = 111.8$ volts

Voltage across capacitan

$$V_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 40 \times 10^{-6} \times 35.58} = 111.8 \text{ volts}$$

(d) $f_r - f_1 = \dfrac{R}{4\pi L} \Rightarrow 35.59 - f_1 = \dfrac{100}{4\pi \times 0.5}$

$\therefore$ Lower-half frequency, $f_1 = 19.6745$ Hz

$$f_2 - f_r = \frac{R}{4\pi L} \Rightarrow f_2 - 35.59 = \frac{100}{4\pi \times 0.5}$$

$\therefore$ Upper-half frequency, $f_2 = 51.5055$ Hz

(e) Bandwidth, $BW = \dfrac{R}{2\pi L} = \dfrac{100}{2\pi \times 0.5} = 31.831\text{Hz}$

(f) Q-factor, $Q = \dfrac{f_r}{BW} = \dfrac{35.59}{31.831} = 1.1181$

3.8 PARALLEL RESONANCE *[JNTU June 2009]*

Basically, parallel resonance occurs when $X_C = X_L$. The frequency at which resonance occurs is called the *resonant* frequency. When $X_C = X_L$, the two branch currents are equal in magnitude and 180° out of phase with each other. Therefore, the two currents cancel each other out, and the total current is zero. Consider the circuit shown in Fig. 3.36. The condition for resonance occurs when $X_L - X_C$.

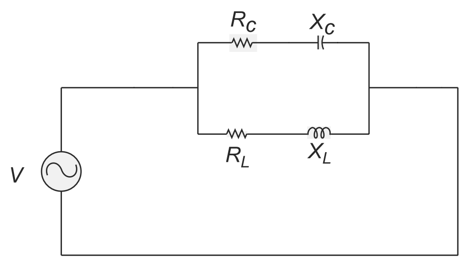


Fig. 3.36

In Fig. 3.36, the total admittance

$$Y = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - (j/\omega C)}$$

$$= \frac{R_L + j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + (j/\omega C)}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

$$= \frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} + j\left\{\left[\frac{1/\omega C}{R_C^2 + \frac{1}{\omega^2 C^2}}\right] - \left[\frac{\omega L}{R_L^2 + \omega^2 L^2}\right]\right\} \quad (3.31)$$

At resonance the susceptance part becomes zero

$$\frac{\omega_r L}{R_L^2 + \omega_r^2 L^2} = \frac{\frac{1}{\omega_r C}}{R_C^2 + \frac{1}{\omega_r^2 C^2}} \quad (3.32)$$

$$\omega_r L\left[R_C^2 + \frac{1}{\omega_r^2 C^2}\right] = \frac{1}{\omega_r C}\left[R_L^2 + \omega_r^2 L^2\right]$$

$$\omega_r^2\left[R_C^2 + \frac{1}{\omega_r^2 C^2}\right] = \frac{1}{LC}\left[R_L^2 + \omega_r^2 L^2\right]$$

$$\omega_r^2 R_C^2 - \frac{\omega_r^2 L}{C} = \frac{1}{LC} R_L^2 - \frac{1}{C^2}$$

$$\omega_r^2\left[R_C^2 - \frac{L}{C}\right] = \frac{1}{LC}\left[R_L^2 - \frac{L}{C}\right]$$

$$\omega_r = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}} \quad (3.33)$$

The condition for resonant frequency is given by Eq. 3.33. As a special case, if $R_L = R_C$, then Eq. 3.33 become

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Therefore

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Example 3.34 *Find the resonant frequency in the ideal parallel LC circuit shown in Fig. 3.37.*

Fig. 3.37

Solution $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{50\times10^{-3}\times0.01\times10^{-6}}} = 7117.6\,\text{Hz}$

Example 3.35 *For the parallel circuit shown in the Fig. 3.38. Find the resonance frequency at* $R_L = R_C$

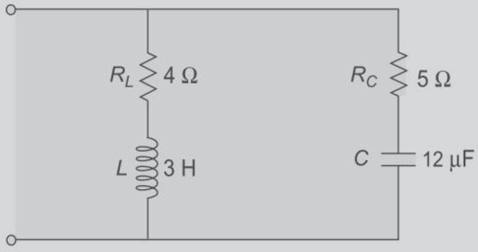


Fig. 3.38

Solution

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2-(L/C)}{R_C^2-(L/C)}}$$

$$= \frac{1}{2\pi\sqrt{3\times12\times10^{-6}}}\sqrt{\frac{(4)^2-(3/12\times10^{-6})}{(5)^2-(3/12\times10^{-6})}}$$

$$= 26\sqrt{\frac{249984}{249975}} = 26\text{ Hz}$$

Example 3.36 *Compare series and parallel resonant circuits.*

[JNTU June 2009]

Solution

Series Resonant Circuit	Parallel Resonant Circuit
1. The applied voltage and the resulting current are in phase which also mean that the p.f. of RLC series resonant circuit is unity.	1. Power factor is unity.
2. The net reactance is zero at resonance and the impedance does have the resistive part only.	2. Net impedance at resonance of the parallel circuit is maximum and equal to (L/CR) Ω.
3. The current in the circuit is maximum and is (V/R) A. Since at resonance, the line current in the series LCR circuit is maximum hence it is called acceptor circuit.	3. Current at resonance is [V/(L/CR)] and is in phase with the applied voltage. The value of current at resonance is minimum.
4. At resonance the circuit has got minimum impedance and maximum admittance.	4. The admittance is minimum and the net susceptance is zero at resonance.
5. Frequency of resonance is given by is given by $f_o = \frac{1}{2\pi\sqrt{LC}}$ Hz.	5. The resonance frequency of this circuit is $f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$.

3.9 RESONANT FREQUENCY FOR A TANK CIRCUIT

[JNTU June 2009, Nov 2011]

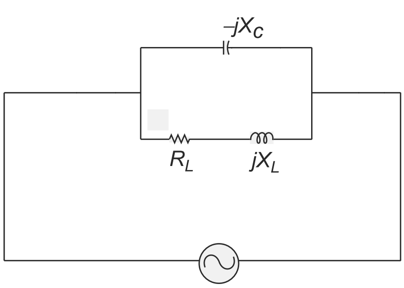


Fig. 3.39

The parallel resonant circuit is generally called a tank circuit because of the fact that the circuit stores energy in the magnetic field of the coil and in the electric field of the capacitor. The stored energy is transferred back and forth between the capacitor and coil and vice–versa. The tank circuit is shown in Fig. 3.39. The circuit is said to be in resonant condition when the susceptance part of admittance is zero.

The total admittance is

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{-jX_C} \qquad (3.34)$$

Simplifying Eq. 3.34, we have

$$Y = \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{j}{X_C} = \frac{R_L}{R_L^2 + X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

To satisfy the condition for resonance, the susceptance part is zero.

$$\therefore \quad \frac{1}{X_C} = \frac{X_L}{R_L^2 + X_L^2} \qquad (3.35)$$

$$\left(\omega C = \frac{\omega L}{R_L^2 + \omega^2 L^2}\right) \qquad (3.36)$$

From Eq. 3.36, we get

$$R_L^2 + \omega^2 L^2 = \frac{L}{C}$$

$$\omega^2 L^2 = \frac{L}{C} - R_L^2$$

$$\omega^2 = \frac{1}{LC} - \frac{R_L^2}{L^2}$$

$$\therefore \quad \omega = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \qquad (3.37)$$

The resonant frequency for the tank circuit is

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} \qquad (3.38)$$

Example 3.37 *In a parallel resonance circuit shown in figure find the resonance frequency, dynamic resistance and bandwidth.*

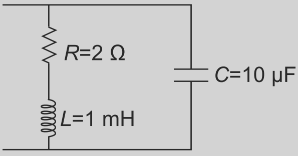


Fig. 3.40

Solution The circuit shown in the above figure is the most common form of parallel resonant circuit in practical use and is also called the tank circuit.
The admittance of the circuit is

$$Y = \frac{1}{Z} = \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$Y = \frac{1}{-jX_C} + \frac{1}{R + jX_L}$$
$$= j\omega C + \frac{1}{R + j\omega L}$$
$$= j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$
$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right)$$

At resonance the susceptance part is zero.

Hence at $\omega = \omega_r, C = \frac{L}{R^2 + \omega_r^2 L^2} = 0$

$$R^2 + \omega_r^2 L^2 = \frac{L}{C}$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2 \Rightarrow \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{3.39}$$

Resonance frequency, $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ (3.40)

$$\therefore \quad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$
$$= \frac{1}{2\pi \times 1 \times 10^{-3}}\sqrt{\frac{1 \times 10^{-3}}{10 \times 10^{-6}} - 4}$$
$$= 1559.4 \text{ Hz}$$

Dynamic impedance:

The input admittance at resonance is given by

$$Y_r = \frac{R}{R^2 + \omega_r^2 L^2}$$

The impedance at resonance is

$$Z_r = \frac{1}{y_r} = \frac{R^2 + \omega_r^2 L^2}{R} = R + \frac{\omega_r^2 L^2}{R}$$

Substituting $\omega_r^2 L^2$ from Eq. 3.39 gives,

$$Z_r = R + \frac{\frac{L}{C} - R^2}{R} = R + \frac{L}{CR} - R$$

$Z_r = \frac{L}{CR}$ which is called dynamic impedance.

This is a pure resistance because it is independent of the frequency.

Here, dynamic resistance $= \dfrac{1\times10^{-3}}{10\times10^{-6}\times2} = 50\ \Omega$

Bandwidth of the parallel resonance circuit $= \dfrac{\omega_r}{Q}$

$$\omega_r = \frac{1}{L}\sqrt{\frac{L}{C} - R^2} = 9797.95$$

$$Q_o = \frac{\omega o L}{R} = \frac{9797.95\times1\times10^{-3}}{2} = 4.898$$

Bandwidth $= \dfrac{9797.5}{4.898} = 2000.4$

Example 3.38 *For the tank circuit shown in Fig. 3.41, find the resonant frequency.*

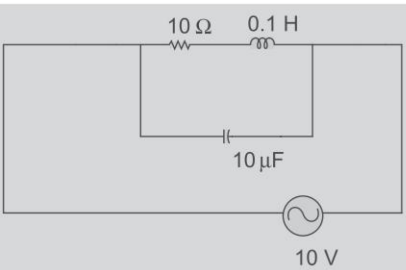


Fig. 3.41

Solution The resonant frequency

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.1\times10\times10^{-6}} - \frac{(10)^2}{(0.1)^2}}$$

$$= \frac{1}{2\pi}\sqrt{(10)^6 - (10)^2} = \frac{1}{2\pi}(994.98) = 158.35\,\text{Hz}$$

Example 3.39 *Find the value of L at which the circuit resonates at a frequency of 1000 rad/sec in the circuit shown in Fig. 3.42*

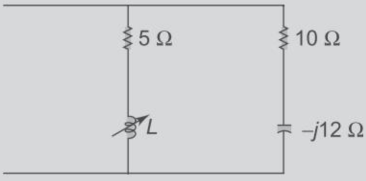


Fig. 3.42

Solution $Y = \dfrac{1}{10 - j12} + \dfrac{1}{5 + jX_L}$

$$Y = \frac{10 + j12}{10^2 + 12^2} + \frac{5 - jX_L}{25 + X_L^2}$$

$$= \frac{10}{10^2 + 12^2} + \frac{5}{25 + X_L^2} + j\left[\frac{12}{10^2 + 12^2} - \frac{X_L}{25 + X_L^2}\right]$$

At resonance the susceptance becomes zero.

Then $$\frac{X_L}{25 + X_L^2} = \frac{12}{10^2 + 12^2}$$

$$12X_L^2 - 244\, X_L + 300 = 0$$

From the above equation

$$X_L^2 - 20.3\, X_L + 25 = 0$$

$$X_L = \frac{+20.3 \pm \sqrt{(20.3)^2 - 4 \times 25}}{2}$$

$$= \frac{20.3 + \sqrt{412 - 100}}{2} \text{ or } \frac{20.3 - \sqrt{412 - 100}}{2}$$

$$= 18.98\ \Omega \text{ or } 1.32\ \Omega$$

$\therefore$ $$X_L = \omega L = 18.98 \text{ or } 1.32\ \Omega$$

$$L = \frac{18.98}{1000} \text{ or } \frac{1.32}{1000}$$

$$L = 18.98 \text{ mH or } 1.32 \text{ mH}$$

Example 3.40 *Two impedances $Z_1 = 20 + j10$ and $Z_2 = 10 - j30$ are connected in parallel and this combination is connected in series with $Z_3 = 30 + jX$. Find the value of X which will produce resonance.*

Solution Total impedance is

$$Z = Z_3 + (Z_1 \| Z_2)$$

$$= (30 + jX) + \left\{\frac{(20 + j10)(10 - j30)}{20 + j10 + 10 - j30}\right\}$$

$$= (30 + jX) + \frac{200 - j600 + j100 + 300}{30 - j20}$$

$$= 30 + jX + \left(\frac{500 - j500}{30 - j20}\right)$$

$$= 30 + jX + \left[\frac{500(1 - j)(30 + j20)}{(30)^2 + (20)^2}\right]$$

$$= (30 + jX) + \left[\frac{500(30 + j20 - j30 + 20)}{900 + 400}\right]$$

$$= 30 + jX + \frac{5}{13}(50 - j10)$$

$$= \left(30 + \frac{5}{13} \times 50\right) + j\left(X - \frac{5}{13} \times 10\right)$$

At resonance, the imaginary part is zero

$$\therefore \quad X - \frac{50}{13} = 0$$

$$X = \frac{50}{13} = 3.85\ \Omega$$

Example 3.41 *For the circuit shown in Fig. 3.43, find the value of capacitance which results in resonance, when $f_r = 2000/\pi$*

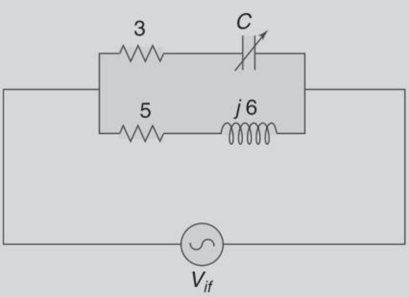


Fig. 3.43

Solution At resonance, the imaginary part of the admittance is zero. Hence, the complex admittance is a real number

$$Y = \frac{1}{5 + j6} + \frac{1}{3 - jx_c}$$

$$= \frac{5 - j6}{61} + \frac{3 + jx_c}{(3 - jx_c)(3 + jx_c)}$$

$$= \frac{5 - j6}{61} + \frac{3 + jx_c}{9 + x_c^2}$$

Separating the real and imaginary parts

$$Y = \left(\frac{5}{61} + \frac{3}{9 + x_c^2}\right) + j\left(\frac{x_c}{9 + x_c^2} - \frac{6}{61}\right)$$

Equating the j term to zero.

$$\frac{x_c}{9 + x_c^2} = \frac{6}{61}$$

$$6x_c^2 - 61x_c + 54 = 0$$

From which X_c = 9.18 (or) 0.979 Ω

$$\frac{1}{\omega C} = 9.18 \text{ (or) } \frac{1}{\omega C} = 0.979$$

Example 3.42 *An impedance Z_1 = 10 + j10 Ω is connected in parallel with another impedance of 8.5 Ω resistance and a variable capacitance connected in series. Find C such that the circuit is in resonance at 5 kHz.*

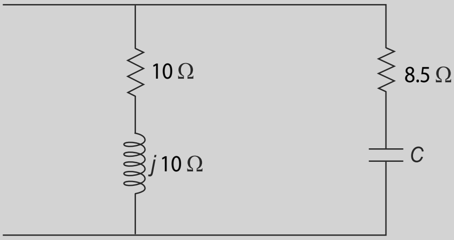


Fig. 3.44

Solution Considering the admittance

$$Y = \frac{1}{10 + j10} + \frac{1}{8.5 - jX_c}$$

$$= \frac{10 - j10}{10^2 + 10^2} + \frac{8.5 + jX_c}{(8.5)^2 + X_c^2}$$

$$= \frac{10}{10^2 + 10^2} + \frac{8.5}{(8.5)^2 + X_c^2} + j\left(\frac{X_c}{(8.5)^2 + X_c^2} - \frac{10}{200}\right)$$

At resonance the susceptance becomes zero.

$$\frac{X_C}{(8.5)^2 + X_c^2} = \frac{1}{20}$$

$$20\, X_c = X_c^2 + 72.25$$

$$X_c^2 - 20X_c + 72.25 = 0$$

$$X_c = 15.267 \text{ or } 4.732$$

$$\frac{1}{\omega c} = X_c = 15.267 \text{ or } 4.732$$

$$c = \frac{1}{2 \times \pi \times 5000 \times 15.267} \text{ or } \frac{1}{2 \times \pi \times 5000 \times 4.732}$$

c = 2.084 μF or 6.726 μF.

Example 3.43 *For the parallel network shown below, determine the value of R at 10 Ω resonance.*

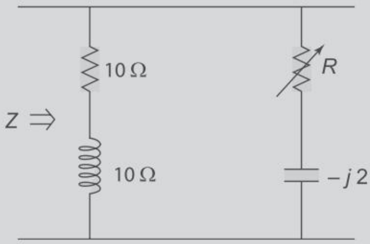


Fig. 3.45

Solution $Z = (10 + j10) \parallel (R - j2)$

$$= \frac{(10+j10)(R-j2)}{10+j10+R-j2}$$

$$= \frac{10R - j20 + j10R + 20)}{10+R+8j}$$

$$= \frac{10R + 20 + j(10R - 20)}{10+R+8j}$$

$$= \frac{[(10R+20)+j(10R-20)][10+R-j8]}{(10+R)^2+64}$$

$$= [(10R+20)(10+R)+8(10R-20) - j8(10R+20) + j(10+R)(10R-20)]\frac{1}{(10+R)^2+64}$$

At resonance imaginary part = 0

$$\Rightarrow \quad 8\,(10R + 20) - (10 + R)\,(10R - 20) = 0$$

$$10R^2 = 360$$

$$R = 6\ \Omega$$

Example 3.44 *An impedance $Z_1 = 10 + j10\ \Omega$ is connected in parallel with another impedance of resistance 8.5 Ω and a variable capacitance connected in series. Find C such that the circuit is in resonance at 5 KHz.*

[JNTU Jan 2010]

Solution

$$Z_1 = 10 + j10\ \Omega$$
$$Z_2 = 8.5 - jX_C$$

$$X_C^2\,(R_2^2 + X_C^2) = X_L\,(R_1^2 + X_C^2)$$

or, $$X_C^2\,(10^2 + 10^2) = 10\,(8.5^2 + X_C^2)$$

or, $$200X_C^2 = 722.5 + 10X_C^2$$

or, $X_C = 3.8$

or, $2 \times \pi \times 5 \times 1000 \times C = \dfrac{1}{3.8}$

or, $C = 8.5\ \mu F$

3.10 VARIATION OF IMPEDANCE WITH FREQUENCY

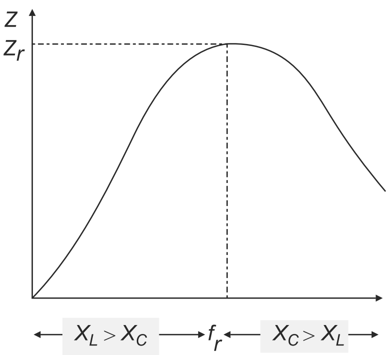


Fig. 3.46

The impedance of a parallel resonant circuit is maximum at the resonant frequency and decreases at lower and higher frequencies as shown in Fig. 3.46.

At very low frequencies, X_L is very small and X_C is very large, so the total impedance is essentially inductive. As the frequency increases, the impedance also increases, and the inductive reactance dominates until the resonant frequency is reached. At this point $X_L = X_C$ and the impedance is at its maximum. As the frequency goes above resonance, capacitive reactance dominates and the impedance decreases.

Example 3.45 *Determine the value of the capacitance C in order that the circuit in the Fig. 3.47 is resonant at 6366 Hz.*

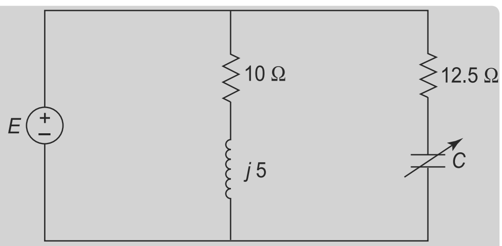


Fig. 3.47

Solution The admittance considered is

$$Y = \frac{1}{10 + j5} + \frac{1}{12.5 - jX_C}$$

$$= \frac{10 - j5}{10^2 + 5^2} + \frac{12.5 + jX_C}{(12.5)^2 + X_C^2}$$

$$= \frac{10}{10^2 + 5^2} + \frac{12.5}{12.5^2 + X_C^2} + j\left(\frac{X_C}{(12.5)^2 + X_C^2} - \frac{5}{102 + 52}\right)$$

At resonance the susceptance becomes zero.

$$\frac{X_C}{(12.5)^2 + X_C^2} = \frac{5}{10^2 + 5^2}$$

$$5X_c^2 + 5(12.5)^2 = (10^2 + 5^2)X_C$$

$$5X_c^2 - 125X_C + 781.25 = 0$$

$$X_C = 125 \pm \frac{\sqrt{(125)^2 - 4(781.25)5}}{2 \times 5} = 125$$

$$\frac{1}{\omega C} = 12.5$$

$$C = \frac{1}{2 \times \pi \times 6366 \times 12.5}$$

$$= 2 \times 10^{-6}\, F = 2\, \mu F$$

3.11 Q FACTOR OF PARALLEL RESONANCE [JNTU Jan 2010]

Consider the parallel RLC circuit shown in Fig. 3.48.

In the circuit shown, the condition for resonance occurs when the susceptance part is zero.

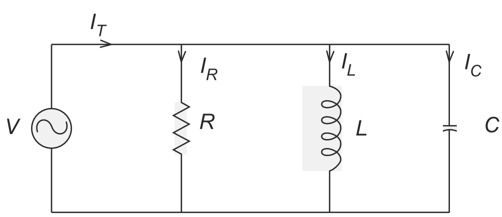


Fig. 3.48

Admittance $Y = G + jB$ (3.41)

$$= \frac{1}{R} + j\omega C + \frac{1}{j\omega L} \quad (3.42)$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

The frequency at which resonance occurs is

$$\omega_r C - \frac{1}{\omega_r L} = 0 \quad (3.43)$$

$$\omega_r = \frac{1}{\sqrt{LC}} \quad (3.44)$$

The voltage and current variation with frequency is shown in Fig. 3.49. At resonant frequency, the current is minimum.

The bandwidth, $BW = f_2 - f_1$

For parallel circuit, to obtain the lower half power frequency,

$$\omega_1 C - \frac{1}{\omega_1 L} = -\frac{1}{R} \quad (3.45)$$

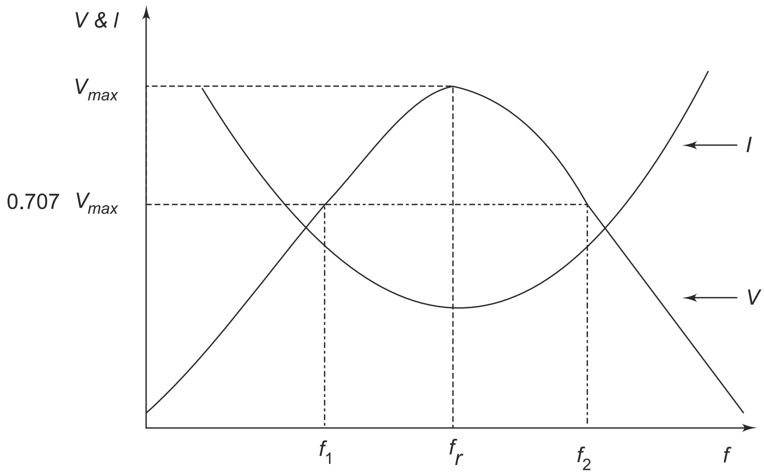


Fig. 3.49

From Eq. 3.45, we have

$$\omega_1^2 + \frac{\omega_1}{RC} - \frac{1}{LC} = 0 \tag{3.46}$$

If we simplify Eq. 3.46, we get

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \tag{3.47}$$

Similarly, to obtain the upper half power frequency

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R} \tag{3.48}$$

From Eq. 3.48, we have

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \tag{3.49}$$

Bandwidth

$$BW = \omega_2 - \omega_1 = \frac{1}{RC}$$

The quality factor is defined as $Q_r = \dfrac{\omega_r}{\omega_2 - \omega_1}$

$$Q_r = \frac{\omega_r}{1/RC} = \omega_r RC$$

In other words,

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated/cycle}}$$

In the case of an inductor,

The maximum energy stored $= \frac{1}{2} LI^2$

Energy dissipated per cycle $= \left(\frac{I}{\sqrt{2}}\right)^2 \times R \times T$

The quality factor $Q = 2\pi \times \frac{1/2(LI^2)}{\frac{I^2}{2} R \times \frac{1}{f}}$

$$\therefore \qquad Q = 2\pi \times \frac{\frac{1}{2} L \left(\frac{V}{\omega L}\right)^2 R}{\frac{V^2}{2} \times \frac{1}{f}} = \frac{2\pi f LR}{\omega^2 L^2} = \frac{R}{\omega L}$$

For a capacitor, maximum energy stored $= 1/2\,(CV^2)$

Energy dissipated per cycle $= P \times T = \frac{V^2}{2 \times R} \times \frac{1}{f}$

The quality factor $Q = 2\pi \times \frac{1/2(CV^2)}{\frac{V^2}{2R} \times \frac{1}{f}} = 2\pi f CR = \omega CR$

3.12 MAGNIFICATION IN PARALLEL RESONANCE

Current magnification occurs in a parallel resonant circuit. The voltage applied to the parallel circuit, $V = IR$

Since $I_L = \frac{V}{\omega_r L} = \frac{IR}{\omega_r L} = IQ_r$

For the capacitor, $I_C = \frac{V}{1/\omega_r C} = IR\omega_r C = IQ_r$

Therefore, the quality factor $Q_r = I_L/I$ or I_C/I

3.13 REACTANCE CURVES IN PARALLEL RESONANCE

The effect of variation of frequency on the reactance of the parallel circuit is shown in Fig. 3.46.

The effect of inductive susceptance,

$$B_L = \frac{-1}{2\pi f L}$$

Inductive susceptance is inversely proportional to the frequency or ω. Hence it is represented by a rectangular hyperbola, MN. It is drawn in fourth quadrant, since

B_L is negative. Capacitive susceptance, $B_C = 2\pi fC$. It is directly proportional to the frequency f or ω. Hence it is represented by OP, passing through the origin. Net susceptance $B = B_C - B_L$. It is represented by the curve JK, which is a hyperbola. At point ω_r, the total susceptance is zero, and resonance takes place. The variation of the admittance Y and the current I is represented by curve VW. The current will be minimum at resonant frequency.

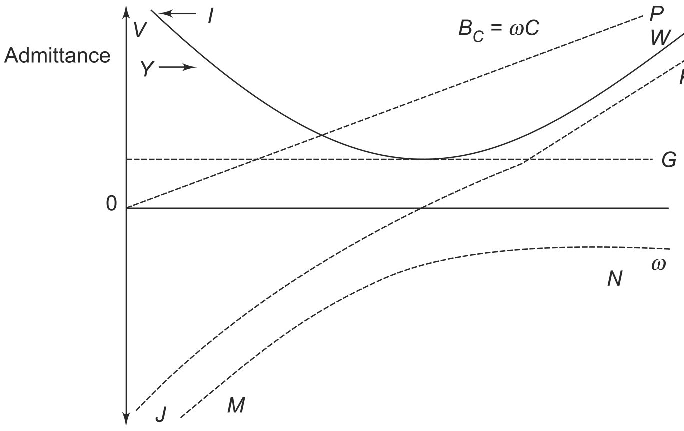


Fig. 3.50

Example 3.46 *From the parallel circuit shown in the Fig. 3.51*

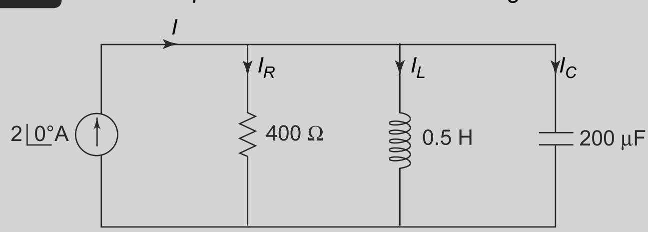


Fig. 3.51

(a) Find the resonance frequency
(b) Find the currents in all the branches at resonance
(c) Quality factor

Solution (a) $$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.5\times 200\times 10^{-6}}} = 15.91\text{ Hz}$$

(b) At resonance, the current through the resistance is same as the current from the source

$\therefore \quad I_R = I = 2\text{ A}$

The voltage across the parallel branch $= I_R R$

$\Rightarrow \quad V(t) = 2\times 400 = 800\,\underline{|0^\circ}$

$$\therefore \quad I_L(t) = \frac{800\angle 0}{JWL} = \frac{800\angle 0}{100\times 0.5\angle 90} = 16\angle -90^\circ$$

$$I_C(t) = \frac{800\angle 0}{-J/WC} = \frac{800\angle 0}{50\angle -90^\circ} = 16\angle 90^\circ$$

(c) The quality factor $= \frac{I_L}{I}$ (or) $\frac{I_C}{I} = \frac{16}{2} = 8$

Example 3.47 *In the circuit shown in Fig. 3.52, an inductance of 0.1 H having a Q of 5 is in parallel with a capacitor. Determine the value of capacitance and coil resistance at resonant frequency of 500 rad/sec.*

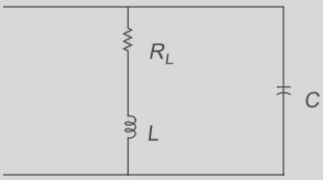


Fig. 3.52

Solution The quality factor $Q = \frac{\omega_r L}{R}$

Since $L = 0.1$ H, $Q = 5$ and

$\omega_r = 500$ rad/sec

$$Q = \frac{500\times 0.1}{R}$$

$$\therefore \quad R = \frac{500\times 0.1}{5} = 10\,\Omega$$

Since $\omega_r^2 = \frac{1}{LC}$

$$(500)^2 = \frac{1}{0.1\times C}$$

$\therefore$ The capacitance value $C = \frac{1}{0.1\times(500)^2} = 40\ \mu$F

Example 3.48 *In the parallel resonant circuit, determine the resonance frequency, dynamic resistance and bandwidth for the circuit shown in the Fig.3.53*

[JNTU May/June 2006]

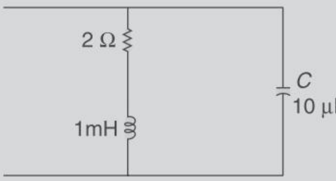


Fig. 3.53

Solution Total admittance (tank circuit)

$$Y = \frac{1}{R + j\omega L} + \frac{1}{-j/\omega C}$$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$= \frac{R}{R^2 + \omega^2 C^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

At resonance, the susceptance part (B) becomes zero.
Reactance

$$Y = G + jB \qquad Z = R + jX$$

Conductance (G), Susceptance (B), Resistance (R)

$$\omega_r C = \frac{\omega_r L}{R^2 + \omega_r^2 L^2}$$

$$R^2 + \omega^2{}_r L^2 = \frac{L}{C} \Rightarrow \omega^2{}_r = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right)$$

$$\Rightarrow \quad \omega^2{}_r = \frac{1}{LC} - \frac{R^2}{L^2} \Rightarrow \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Here $R = 2\ \Omega$, $L = 1\,\text{mH}$, $C = 10\ \mu\text{F}$

$$\omega_r = \sqrt{\frac{1}{10^{-8}} - \frac{4}{10^{-6}}} = \sqrt{10^6 \times 96} = 9.79 \times 10^3\,\text{Hz}$$

$$f_r = \frac{\omega_r}{2\pi} = 1.559\ \text{kHz}$$

Dynamic resistance $(R) = \dfrac{R^2 + \omega^2{}_r L^2}{R}$

$$= \left.\frac{R^2 + \omega^2 L^2}{R}\right|_{\omega = \omega_r} = 2 + \frac{96 \times 10^6 \times 10^{-6}}{2} = 50\ \Omega$$

Bandwidth $= \dfrac{1}{RC}$ (for lid resonant ckt)

$$= \frac{1}{50 \times 10\,\mu f} = 2\,\text{kHz}$$

$$BW = \frac{R}{L} = \frac{2}{1\,mH} = 2\,\text{kHz}.$$

Practice Problems

3.1 For the circuit shown in Fig. 3.54 determine the frequency at which the circuit resonates. Also find the voltage across the capacitor at resonance, and the Q factor of the circuit.

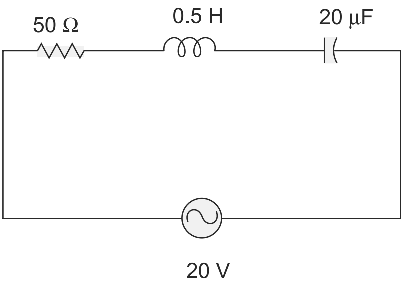


Fig. 3.54

3.2 A series RLC circuit has a quality factor of 10 at 200 rad/sec. The current flowing through the circuit at resonance is 0.5 A and the supply voltage is 10 V. The total impedance of the circuit is 40 Ω. Find the circuit constants.

3.3 The impedance $Z_1 = (5 + j3)$ Ω, and $Z_2 = (10 - j30)$ Ω. are connected in parallel as shown in Fig. 3.55 Find the value of X_3 which will produce resonance at the terminals *a* and *b*.

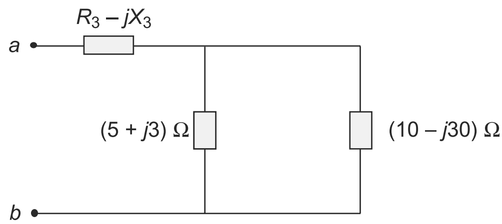


Fig. 3.55

3.4 A RLC series circuit is to be chosen to produce a magnification of 10 at 100 rad/sec. The source can supply a maximum current of 10 A and the supply voltage is 100 V. The power frequency impedance of the circuit should not be more than 14.14 Ω. Find the values of *R*, *L* and *C*.

3.5 A voltage $v(t) = 50 \sin \omega t$ is applied to a series RLC circuit. At the resonant frequency of the circuit, the maximum voltage across the capacitor is found to be 400 V. The bandwidth is known to be 500 rad/sec and the impedance at resonance is 100 Ω. Find the resonant frequency, and compute the upper and lower limits of the bandwidth. Determine the values of *L* and *C* of the circuit.

3.6 A current source is applied to the parallel arrangement of *R*, *L* and *C* where $R = 12$ Ω, L = 2 H and C = 3 μF. Compute the resonant frequency in rad/sec. Find the quality factor. Calculate the value of bandwidth. Compute the lower and upper frequency of the bandwidth. Compute the voltage appearing across the parallel elements when the input signal is $i(t) = 10 \sin 1800\, t$.

3.7 For the circuit shown in Fig. 3.56 determine the value of Rc for which the given circuit resonates.

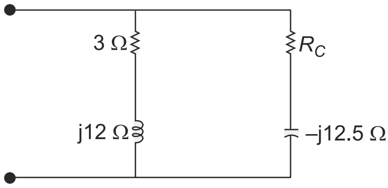


Fig. 3.56

3.8 For the circuit shown in Fig. 3.57 the applied voltage $v(t) = 15$ sin

1800t. Determine the resonant frequency. Calculate the quality factor and bandwidth. Compute the lower and upper limits of the bandwidth.

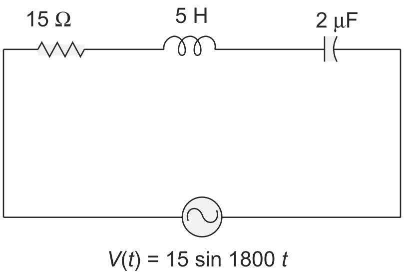


Fig. 3.57

3.9 In the circuit shown in Fig. 3.58 the current is at its maximum value with inductor value L = 0.5 H, and 0.707 times of its maximum value with L = 0.2 H. Find the value of Q at ω = 200 rad/sec and circuit constants.

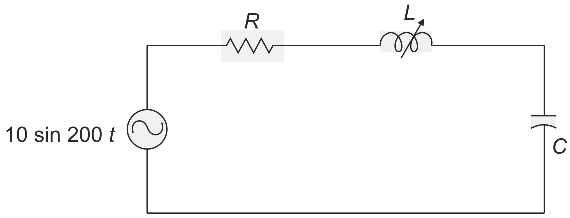


Fig. 3.58

3.10 The voltage applied to the series RLC circuit is 5 V, The Q of the coil is 25 and the value of the capacitor is 200 PF. The resonant frequency of the circuit is 200 kHz. Find the value of inductance, the circuit current and the voltage across the capacitor.

Objective Type Questions

3.1 What is the total reactance of a series RLC circuit at resonance?

(a) equal to X_L (b) equal to X_c (c) equal to R (d) zer

3.2 What is the phase angle of a series RLC circuit at resonance?

(a) zero (b) 90° (c) 45° (d) 30°

3.3 In a series circuit of L = 15 mH and C = 0.015 μF and R = 80 Ω, what is the impedance at the resonant frequency?

(a) (15 mH) ω (b) (0.015 F) ω (c) 80 Ω (d) l/(ω × (0.015))

3.4 In a series RLC circuit operating below the resonant frequency, the current

(a) I leads V_S (b) I lags behind V_S (c) I is in phase with V_S

3.5 In a series RLC circuit, if C is increased, what happens to the resonant frequency?

(a) It increases (b) It decreases
(c) It remains the same (d) It is zero

3.6 In a certain series resonant circuit, V_c = 150 V, V_L = 150 V and V_R = 50 V. What is the value of the source voltage?

(a) zero (b) 50 V (c) 150 V (d) 200 V

3.7 A certain series resonant circuit has a bandwidth of 1000 Hz. If the existing coil is replaced by a coil with a lower Q, what happens to the bandwidth?

(a) It increases (b) It decreases
(c) It is Zero (d) It remains the same

3.8 In a parallel resonance circuit, why does the current lag behind the source voltage at frequencies below resonance?

(a) because the circuit is predominantly resistive
(b) because the circuit is predominantly inductive
(c) because the circuit is predominantly capacitive
(d) none of the above

3.9 In order to tune a parallel resonant circuit to a lower frequency, the capacitance must

(a) be increased (b) be decreased
(c) be zero (d) remain the same

3.10 What is the impedance of an ideal parallel resonant circuit without resistance in either branch?

(a) zero (b) inductive (c) capacitive (d) infinite

3.11 If the lower cut: Off frequency is 2400 Hz and the upper cut-off frequency is 2800 Hz, what is the bandwidth?

(a) 400 Hz (b) 2800 Hz (c) 2400 Hz (d) 5200 Hz

3.12 What values of L and C should be used in a tank circuit to obtain a resonant frequency of 8 kHz? The bandwidth must be 800 Hz. The winding resistance of the coil is 10 V.

(a) 2mH, 1 μF (b) 10 H, 0.2 μF
(c) 1.99 mH, 0.2 μF (d) 1.99 mH, 10 μF

4 Magnetic Circuits

4.1 MAGNETIC CIRCUITS *[JNTU Nov. 2011]*

4.1.1 Basic Definition of MMF, Flux and Reluctance

The presence of charges in space or in a medium creates an electric field, similarly the flow of current in a conductor sets up a magnetic field. Electric field is represented by electric flux lines, magnetic flux lines are used to describe the magnetic field. The path of the magnetic flux lines is called the magnetic circuit. Just as a flow of current in the electric circuit requires the presence of an electromotive force, so the production of magnetic flux requires the presence of magneto-motive force (mmf). We now discuss some properties related to magnetic flux.

(i) Flux density (B) The magnetic flux lines start and end in such a way that they form closed loops. Weber (Wb) is the unit of magnetic flux (ϕ). Flux density (B) is the flux per unit area. Tesla (T) or Wb/m^2 is the unit of flux density.

$$B = \frac{\Phi}{A} \text{ Wb/m}^2 \text{ or Tesla}$$

where B is a quantity called magnetic flux density in teslas, ϕ is the total flux in webers and A is the area perpendicular to the lines in m^2.

(ii) Magneto-motive force MMF ($\Im$) A measure of the ability of a coil to produce a flux is called the *magneto-motive force*. It may be considered as a magnetic pressure, just as emf is considered as an electric pressure. A coil with N turns which is carrying a current of I amperes constitutes a magnetic circuit and produces an mmf of NI ampere turns. The source of flux (ϕ) in the magnetic circuit is the mmf. The flux produced in the circuit depends on mmf and the length of the circuit.

(iii) Magnetic field strength (H) The magnetic field strength of a circuit is given by the mmf per unit length.

$$\text{H} = \frac{\Im}{l} = \frac{NI}{l} \text{ AT/m}$$

The magnetic flux density (B) and its intensity (field strength) in a medium can be related by the following equation

$$B = \mu H$$

where $\mu = \mu_0 \mu_r$ is the permeability of the medium in Henrys/metre (H/m),

μ_0 = absolute permeability of free space and is equal to $4\pi \times 10^{-7}$ H/m

and μ_r = relative permeability of the medium.

Relative permeability is a non-dimensional numeric which indicates the degree to which the medium is a better conductor of magnetic flux as compared to free space. The value of $\mu_r = 1$ for air and non-magnetic materials. It varies from 1,000 to 10,000 for some types of ferro-magnetic materials.

(iv) Reluctance **($\mathfrak{R}$)** It is the property of the medium which opposes the passage of magnetic flux. The magnetic reluctance is analogous to resistance in the electric circuit. Its unit is AT/Wb. Air has a much higher reluctance than does iron or steel. For this reason, magnetic circuits used in electrical machines are designed with very small air gaps.

According to definition, reluctance $= \dfrac{\text{mmf}}{\text{flux}}$

The reciprocal of reluctance is known as permeance $\dfrac{1}{\mathfrak{R}} = \dfrac{\phi}{\mathfrak{I}}$

Thus reluctance is *a* measure of the opposition offered by a magnetic circuit to the setting up of the flux. The reluctance of the magnetic circuit is given by

$$\mathfrak{R} = \frac{1}{\mu}\frac{l}{a}$$

where l is the length, a is the cross-sectional area of the magnetic circuit and μ is the permeability of the medium.

From the above equations

$$\frac{1}{\mu} \cdot \frac{l}{a} = \frac{\mathfrak{I}}{\phi}$$

or $$\frac{\mathfrak{I}}{1} = \frac{1}{\mu} \cdot \frac{\phi}{a}$$

$$\frac{NI}{l} = \frac{1}{\mu} \cdot B$$

$$H = \frac{1}{\mu} \cdot B$$

or $$B = \mu H$$

Example 4.1 *A cast steel electromagnet has an air gap of length 2 mm and an iron path of length 30 cm. Find the MMF needed to produce a flux density of 0.8T in the air gap. The relative permeability of the steel core at this flux density is 1000. Neglect leakage and fringing.*

Solution Air-gap length $l_g = 2 \text{ mm} = 2 \times 10^{-3}\text{ m}$

Iron path length $l_i = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$

Flux density $B = 0.8\text{T} = 0.8 \text{ Wb/m}^2$

$$\mu_r = 1000$$

$$\text{Total A.T} = \text{mmf} = H_i l_i + H_g l_g$$

$$\frac{B \times l_i}{\mu_0 \mu_g} + \frac{B}{\mu_0} l_g = \frac{0.8 \times 30 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000} + \frac{0.8 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} = 1464 \text{ A.T.}$$

Hence, total MMF required to produce a flux density of 0.8T = 1464 A.T.

Example 4.2 *(a) Two similar coils connected in series gave a total inductance of 600 mH and when one of the coil is reversed, the total inductance is 300 mH. Determine the mutual inductance between the coils and coefficient of coupling?*

(b) A cast steel structure is made of a rod of square section 2.5 cm × 2.5 cm as shown in Fig. 4.1 What is the current that should be passed in a 500 turn coil on the left limb, so that a flux of 2.5 mwb is made to pass in the right limb. Assume permeability as 750 and neglect leakage.

[JNTU June 2006]

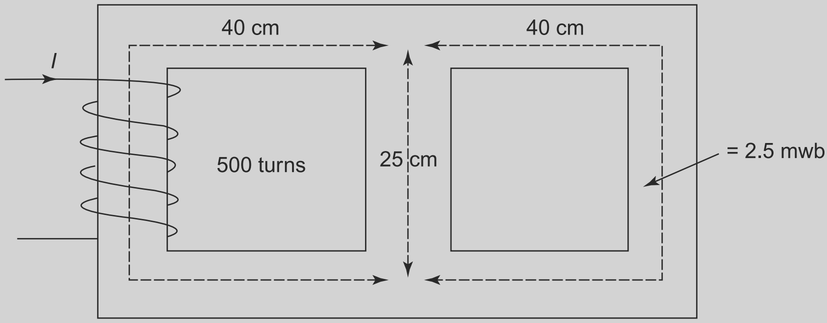


Fig. 4.1

Solution (a) $L_1 + L_2 + 2\text{ M} = 600 \text{ mH}$ (4.1)

$L_1 + L_2 - 2\text{ M} = 390 \text{ mH}$ (4.2)

$(4.1) - (4.2) \Rightarrow 4\text{ M} = 300 \text{ mH}$

$\text{M} = 75 \text{ mH}$

$L_1 = L_2 = L$

From Eq. (4.2), $2L - 2 \times 75 = 300$

$L = 225$

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{75}{\sqrt{(225)^2}} = \frac{1}{3}$$

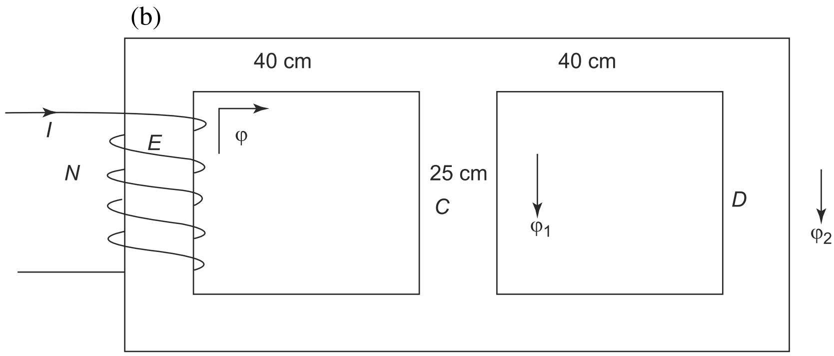


Fig. 4.2

$$\varphi = \varphi_1 + \varphi_2$$

Also mmf in C = mmf in D

$$\therefore \quad \varphi_1 \frac{25}{\mu A} = 2.5 \times 10^{-3} \frac{40}{\mu A}$$

$$\varphi_1 = 4 \text{mmf}$$

$$\varphi = \varphi_1 + \varphi_2 = 6.5 \text{mmf}$$

Total A. T. for the hole circuit is

(i) that required for path E and

(ii) that required for path C or D.

$$\text{Flux density in } E = \frac{6.5 \times 10^{-3}}{(2.5)^2 \times 10^{-4}} = 10.4 \text{ web/m}^2$$

$$\text{A.T. in } E = \frac{10.4 \times 0.4}{4\pi \times 10^{-7} \times 750} = 4414 \text{ A.T.}$$

$$\text{Flux density in path } D = \frac{2.5 \times 10^{-3}}{(2.5)^2 \times 10^{-4}} = 40 \text{ web/m}^2$$

$$\text{A.T. in } D = \frac{40 \times 0.4}{4\pi \times 10^{-2} \times 750} = 1698 \text{ A.T.}$$

$$\text{Total A.T.} = 4414 + 1698 + 6112$$

$$\text{Current needed} = \frac{6112}{500} = 12.224 \text{ A}$$

Example 4.3 *Define Magneto Motive Force, Magnetic Flux, and Reluctance of a Magnetic circuit. Specify the unit for the above quantities, sate the relation between the above quantities.* *[JNTU June 2006]*

Solution *Magneto Motive Force* (MMF)

Magneto Motive Force (MMF) is the measure of the ability of a coil to produce a flux. As EMF is considered to be an electric pressure, MMF is also considered to be a magnetic pressure. A coil with N turns carrying a current of 'I' ampere's represents a magnetic circuit producing an MMF of 'NI' ampere turns.

$\therefore$ MMF = NI Ampere Turns.

The MMF is the source of flux (ϕ) in the magnetic circuit. The length of the circuit and the MMF determines the amount of flux produced in the circuit.

Units of MMF = Ampere Turns (AT)

Reluctance (S)

It is the property of the medium which opposes the passage of magnetic flux. The reluctance in the magnetic circuit is similar to the resistance in the electric circuit.

$$\therefore \quad \text{Reluctance} = \frac{\text{MMF}}{\text{flux}}$$

$$S = \frac{\text{MMF}}{\phi}$$

Units of Reluctance is AT/wb.

The reluctance is the measure of the opposing offered to the set up of the flux by a magnetic circuit.

$$\therefore \quad S = \frac{\text{MMF}}{\phi} = \frac{NI}{\phi} \qquad [\because \phi = B \times A]$$

$$\therefore \quad S = \frac{NI}{B \times A} = \frac{NI}{\mu_0 \mu_r + 1 \times A} \qquad [\because B = \mu_0 \mu_r H]$$

$$\therefore \quad S = \frac{NI}{\mu_0 \mu_r \frac{NI}{l} \times A} \qquad \left[\because H = \frac{NI}{L}\right]$$

$$\therefore \quad S = \frac{l}{\mu_o \mu_r A} \text{ AT/wb}$$

$$\therefore \quad S = \frac{L}{\mu A} \text{ AT/wb}$$

where l = length of magnetic path; A = Area of cross section of magnetic circuit; and $\mu = \mu_0 \mu_r$ = Permeability of Medium.

Magnetic Flux (ϕ)

The total number of lines of induction passing normally through a surface is called Magnetic flux (ϕ).

Flux does not actually flow in a magnetic circuit.

Magnetic flux is directly proportional to the pole strength of the magnet.

i.e. $\phi \propto m$

(or) $\phi = \mu m$

where μ = Permeability of medium.

Units of magnetic flux is weber (wb).

Relation between MMF, S and ϕ

The Relation between MMF, Magnetic flux and Reluctance of a magnetic circuit is given as

$$\text{Magnetic flux} = \frac{\text{Magneto Motive Force}}{\text{Reluctance}}$$

$$\text{i.e. } \phi = \frac{\text{MMF}}{S}$$

$$\text{i.e. } \phi = \frac{NI}{\frac{L}{MA}}$$

4.2 ANALOGY BETWEEN ELECTRICAL AND MAGNETIC CIRCUITS

A series electric and magnetic circuits are shown in Fig. 4.3 (a) and (b) respectively.

Figure 4.3 (a) represents an electric circuit with three resistances connected in series, the dc source E drives the current I through all the three resistances whose voltage drops are V_1, V_2 and V_3. Hence, $E = V_1 + V_2 + V_3$, also $E = I(R_1 + R_2 + R_3)$. We also know that $R = \frac{\rho l}{a}$, where ρ is the specific resistance of the material, l is the length of the wire of the resistive material and a is the area of cross-section of the wire.

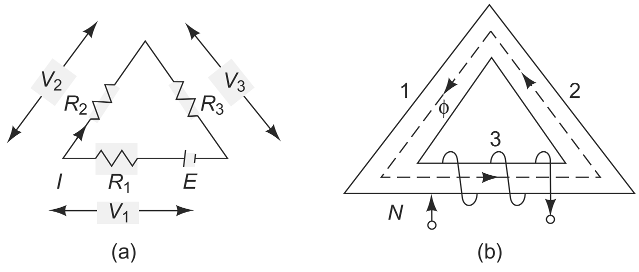


Fig. 4.3

The drop across each resistor $V = RI = \rho l \frac{I}{a}$

or $$\frac{V}{l} = \rho \frac{I}{a}$$

Voltage drop per unit length = specific resistance × current density.

Let us consider the magnetic circuit in Fig. 4.3 (b). The MMF of the circuit is given by $\Im = NI$, drives the flux ϕ around the three parts of the circuit which are in series. Each part has a reluctance $\Re = \frac{1}{\mu} \cdot \frac{l}{a}$, where l is the length and a is the area of cross-section of each arm. The mmf of the magnetic circuit is given by $\Im = \Im_1 + \Im_2 + \Im_3 . \Im = \phi\ (\Re_1 + \Re_2 + \Re_3)$ where $\Re_1$, $\Re_2$ and $\Re_3$ are the reluctances of the portion 1, 2 and 3 respectively.

Also
$$\Im = \frac{1}{\mu} \cdot \frac{l}{a} \cdot \phi$$
$$\frac{\Im}{l} = \frac{1}{\mu} \cdot \frac{\phi}{a}$$
$$H = \frac{1}{\mu} \cdot B$$

$\frac{1}{\mu}$ can be termed as *reluctance* of a cubic metre of magnetic material from which, the above equation gives the mmf per unit length (intensity) which is analogous to the voltage per unit length. Parallels between electric-circuit and magnetic-circuit quantities are shown in Table 4.1.

Thus, it is seen that the magnetic reluctance is analogous to resistance, mmf is analogous to emf and flux is analogous to current. These analogies are useful in magnetic circuit calculations. Though we can draw many parallels between the two circuits, the following differences do exists.

The electric current is a true flow but there is no flow in a magnetic flux. For a given temperature, ρ is independent of the strength of the current, but μ is not independent of the flux.

In an electric circuit energy is expended so long as the current flows, but in a magnetic circuit energy is expended only in creating the flux, and not in maintaining it. Parallels between the quantities are shown in Table 4.1.

Table 4.1 *Analogy between magnetic and electric circuit*

Electric circuit	*Magnetic circuit*
Exciting force = emf in volts Response = current in amps Voltage drop = VI volts	mmf in AT flux in webers mmf drop = $\Re\phi$ AT
Electric field density $= \frac{V}{l}$ volts/m	Magnetic field intensity $= \frac{\Im}{1}$ AT/m
Current $(I) = \frac{E}{R} A$	Flux $(\phi) = \frac{\Im}{R}$ Web
Current density$(J) = \frac{I}{a}$ Amp/m^2	Flux density $(B) = \frac{\phi}{A}$ Web/m^2
Resistance $(R) = \frac{\rho_l}{a}$ ohm	Reluctance $(\Re) = \frac{1}{\mu} \cdot \frac{l}{a}$ AT/Web
Conductance $(G) = \frac{1}{R}$ Mho	Permeance $= \frac{1}{\Re} = \frac{\mu a}{\mu} \cdot \frac{l}{a}$ Web/AT

4.3 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

4.3.1 Electromagnetic Induction

The majority of electrical machines are electromagnetic devices, work on electromagnetic induction principles. An emf can be induced (produced) in a circuit, when there is a net change in the magnetic flux linking with the circuit. This fact was demonstrated by Michael Faraday and summed up into two famous laws known as Faraday's laws electromagnetic induction.

4.3.2 Faraday's Laws *[JNTU June 2006, Nov 2011]*

Faraday's first law of electromagnetic induction states that an emf is induced in a coil when the magnetic flux linking the coil changes with time.

Faraday's second law states that this included voltage is proportional to the time rate of change of the current which produced the magnetic field, or the magnitude of the induced emf is proportional to the time rate of change of flux linkages. It is expressed as

$$e = \frac{d\psi}{dt} = -N\frac{d\varphi}{dt}$$

where e denotes the emf induced in the coil or circuit

ψ - flux linkages = $N\varphi$

N - no. of turns

φ - flux in Weber's

The minus sign takes into account the sense of the induced emf dictated by Lenz's law.

An induced emf can be either (i) dynamically induced, or (ii) statically induced. In the former case, the field is stationary and conductors are moving—this arrangement is used in dc generators and motors. In the second case, the conductor or coil is stationary and the flux linking with it changes. This arrangement is used in transformers and alternators.

Example 4.4 *What are statically and dynamically induced emfs.*

Solution Statically induced EMF:

EMF induced in a coil due to the change of its own flux linked with it or emf induced in one coil by the influence of the other coil is known as statically induced emf.

Dynamically induced EMF:

When a coil with certain number of turns or a conductor is rotated in a magnetic filed (as in d.c. generator's), an emf is induced in it which is known as dynamically induced emf.

4.3.3 Dynamically Induced emf or Motional emf *[JNTU Nov. 2011]*

When a single conductor of length 1 meters moves with a velocity of v m/sec at right angles to uniform magnetic field of flux density B tesla between N and S poles, the emf induced in the conductor is given by $e = Blv$ volts.

If the conductor moves at an angle q to the direction of the magnetic field, the emf induced in the conductor is given by $e = Blv \sin \theta$.

If the conductor moves parallel to the flux lines, the emf induced in the conductor = 0.

Motional emf is associated with energy conversion from electrical to mechanical or mechanical to electrical.

4.3.4 Statically Induced emf

Statically induced emf or transformer emf does not involve any rotation of the conductor or coil, hence is not associated with energy conversion, it is however associated with energy transfer.

The magnitude of this emf can be obtained by Faraday's law $e = \frac{-N\, d\phi}{dt}$.

4.3.5 Fleming's Right-hand Rule

The direction of the dynamically induced emf or current can be determined by Fleming's right-hand rule. Accordingly, there exists a definite relation between the direction of the induced current or voltage, flux and the direction of motion of conductor, when the thumb, forefinger and middle finger of the right hand are held mutually perpendicular to each other. If the thumb points to the direction of the motion, and the forefinger to the direction of the field, then the middle finger will point in the direction of the induced emf.

4.3.6 Lenz's Law

The emf induced by variation of flux or magnetic field is termed as statically induced emf. A statically induced emf may be (i) mutually induced, or (ii) self-induced.

The direction of statically induced current or voltage may be found by Lenz's law formulated by the Russian physicist Heinrich Lenz. According to Lenz's law, the induced current always develops a flux that opposes the change responsible for inducing this current, or the counter emf or back emf, always has a polarity which opposes the force that created it.

The induced emf is given a minus sign in order to take into consideration the fact that the counter emf opposes the change in flux thus

$$e = \frac{d\psi}{dt} = -N\frac{d\varphi}{dt}$$

4.4 CONCEPT OF SELF AND MUTAL INDUCTANCES *[JNTU Nov. 2011]*

4.4.1 Introduction

Two circuits are said to be 'coupled' when energy transfer takes place from one circuit to the other when one of the circuits is energised. There are many types of couplings like conductive coupling as shown by the potential divider in Fig. 4.4 (a) inductive or magnetic coupling as shown by a two winding transformer in Fig. 4.4 (b) or conductive and inductive coupling as shown by an auto transformer in Fig. 4.4 (c). A majority of the electrical circuits in practice are conductively or electromagnetically coupled. Certain coupled elements are frequently used in network analysis and synthesis. Transformer, transistor and electronic pots, etc. are some among these circuits. Each of these elements may be represented as a two port network as shown in Fig. 4.4 (d).

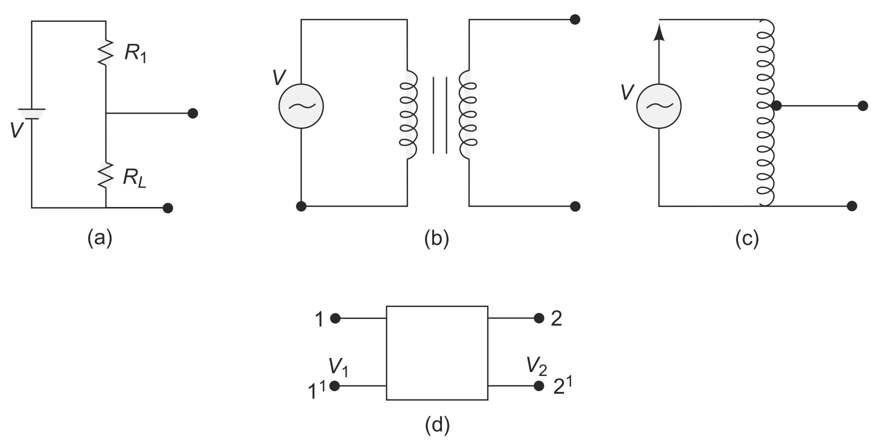


Fig. 4.4

4.4.2 Conductively Coupled Circuit and Mutual Impedance

A conductively coupled circuit which does not involve magnetic coupling is shown in Fig. 4.5 (a).

In the circuit shown the impedance Z_{12} or Z_{21} common to loop 1 and loop 2 is called *mutual impedance*. It may consists of a pure resistance, a pure inductance, a pure capacitance or a combination of any of these elements. Mesh analysis, nodal analysis or Kirchhoff's laws can be used to solve these type of circuits as described in Chapter 2.

The general definition of mutual impedance is explained with the help of Fig. 4.5 (b).

The network in the box may be of any configuration of circuit elements with two ports having two pairs of terminals 1-1′ and 2-2′ available for measurement. The mutual impedance between port 1 and 2 can be measured at 1-1′ or 2-2′. If it is measured at 2-2′. It can be defined as the voltage developed (V_2) at 2-2′ per unit current (I_1) at port 1-1′. If the box contains linear bilateral elements, then the mutual impedance measured at 2-2′ is same as the impedance measured at 1-1′ and is defined as the voltage developed (V_1) at 1-1′ per unit current (I_2) at port 2-2′.

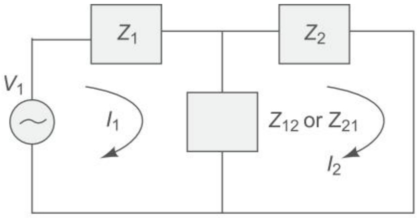


Fig. 4.5 (a)

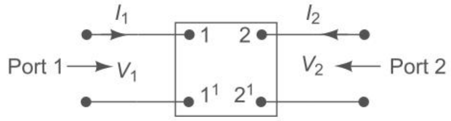


Fig. 4.5 (b)

Example 4.5 *Find the mutual impedance for the circuit shown in Fig. 4.6.*

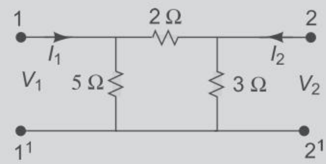


Fig. 4.6

Solution Mutual impedance is given by

$$\frac{V_2}{I_1} \text{ or } \frac{V_1}{I_2}$$

$$V_2 = \frac{3}{2} I_1 \text{ or } \frac{V_1}{I_1} = 1.5\,\Omega$$

$$V_1 = 5 \times I_2 \times \frac{3}{10} \text{ or } \frac{V_2}{I_2} = 1.5\,\Omega$$

4.4.3 Self inductance and Mutual Inductance

The property of inductance of a coil was introduced in Section 1.6. A voltage is induced in a coil when there is a time rate of change of current through it. The inductance parameter L, is defined in terms of the voltage across it and the time rate of change of current through it $v(t) = L\frac{di(t)}{dt}$ where, $v(t)$ is the voltage across the coil, $I(t)$ is the current through the coil and L is the inductance of the coil. Strictly speaking, this definition is of self-inductance and this is considered as a circuit element with a pair of terminals. Whereas a circuit element "mutual inductor" does not exist. Mutual inductance is a property associated with two or more coils or inductors which are in close proximity and the presence of common magnetic flux which links the coils. A transformer is such a device whose operation is based on mutual inductance.

Let us consider two coils, L_1, and L_2 as shown in Fig. 4.7 (a), which are sufficiently close together, so that the flux produced by i_1 in coil L_1, also link coil L_2. We assume that the coils do not move with respect to one another, and the medium in which the flux is established has a constant permeability. The two coils may be also arranged on a common magnetic core, as shown in Fig. 4.7 (b). The two coils are said to be magnetically coupled, but act as a separate circuits. It is possible to relate the voltage induced in one coil to the time rate of change of current in the other coil. When a voltage v_1 is applied across L_1, a current i_1 will start flowing in this coil, and produce a flux ϕ. This flux also links coil L_2. If i_1 were to change with respect to time, the flux 'ϕ' would also change with respect to time. The time-varying flux surrounding the second coil, L_2 induces an emf, or voltage, across the terminals of L_2; this voltage is proportional to the time rate of change of current flowing through the first coil L_1. The two coils, or circuits, are said to be inductively coupled, because of this property they are called coupled elements or coupled circuits and the induced voltage, or emf is called the voltage/emf of mutual induction and is given by $v_2(t) = M_1 \dfrac{di_1(t)}{dt}$ volts, where v_2 is the voltage induced in coil L_2 and M_1 is the coefficient of proportionality, and is called the coefficient of mutual inductance, or simple mutual inductance.

If current i_2 is made to pass through coil L_2 as shown in Fig. 4.7 (c) with coil L_1 open, a change of i_2 would cause a voltage v_1 in coil L_1, given by $v_1(t) = M_2 \dfrac{di_2(t)}{dt}$.

In the above equation, another coefficient of proportionality M_2 is involved. Though it appears that two mutual inductances are involved in determining the mutually induced voltages in the two coils, it can be shown from energy considerations that the two coefficients are equal and, therefore, need not be represented by two different letters. Thus $M_1 = M_2 = M$.

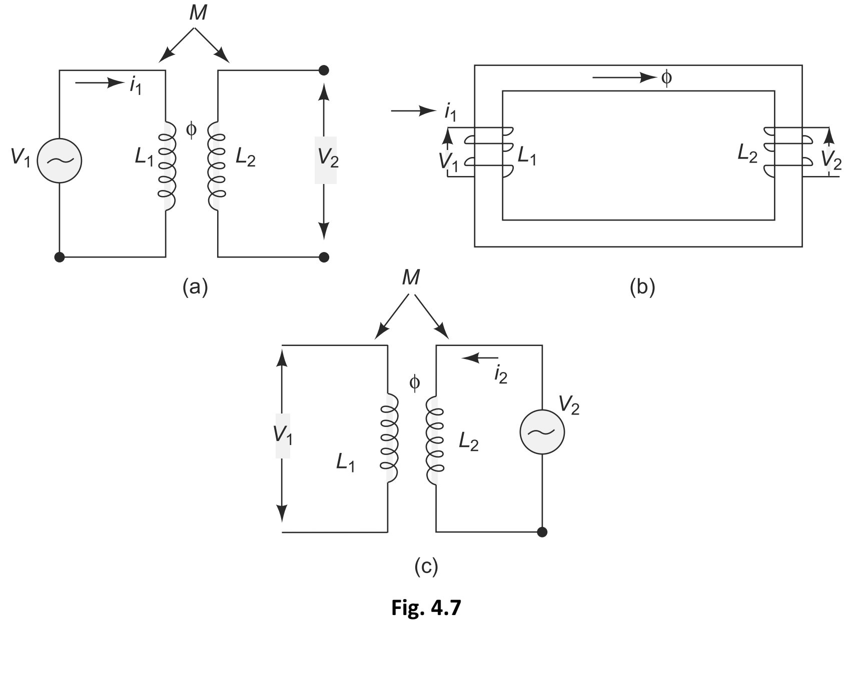


Fig. 4.7

$$\therefore \qquad v_2(t) = M\frac{di_1(t)}{dt} \text{ Volts} \tag{4.3}$$

$$v_1(t) = M\frac{di_2(t)}{dt} \text{ Volts} \tag{4.4}$$

In general, in a pair of linear time invariant coupled coils or inductors, a non-zero current in each of the two coils produces a mutual voltage in each coil due to the flow of current in the other coil. This mutual voltage is present independently of, and in addition to, the voltage due to self induction. Mutual inductance is also measured in Henrys and is positive, but the mutually induced voltage, $M\frac{di}{dt}$ may be either positive or negative, depending on the physical construction of the coil and reference directions. To determine the polarity of the mutually induced voltage (i.e. the sign to be used for the mutual inductance), the dot convention is used.

4.5 DOT CONVENTION

Dot convention is used to establish the choice of correct sign for the mutually induced voltages in coupled circuits.

Circular dot marks and/or special symbols are placed at one end of each of two coils which are mutually coupled to simplify the diagrammatic representation of the windings around its core.

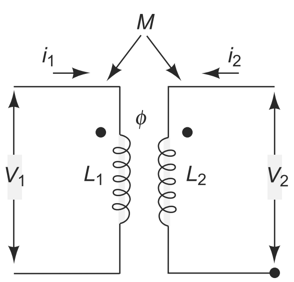


Fig. 4.8

Let us consider Fig. 4.8 which shows a pair of linear, time invariant, coupled inductors with self inductances L_1 and L_2 and a mutual inductance M. If these inductors form a portion of a network, currents i_1 and i_2 are shown, each arbitrarily assumed entering at the dotted terminals, and voltages v_1 and v_2 are developed across the inductors. The voltage across L_1 is, thus composed of two parts and is given by

$$v_1(t) = L_1\frac{di_1(t)}{dt} \pm M\frac{di_2(t)}{dt} \tag{4.5}$$

The first term on the RHS of the above equation is the self induced voltage due to i_1, and the second term represents the mutually induced voltage due to i_2.

$$\text{Similarly, } v_2(t) = L_2\frac{di_2(t)}{dt} \pm M\frac{di_1(t)}{dt} \tag{4.6}$$

Although the self-induced voltages are designated with positive sign, mutually induced voltages can be either positive or negative depending on the direction of the winding of the coil and can be decided by the presence of the *dots* placed at one end of each of the two coils. The convention is as follows.

If two terminals belonging to different coils in a coupled circuit are marked identically with dots then for the same direction of current relative to like terminals, the magnetic flux of self and mutual induction in each coil add together. The physical basis of the dot convention can be verified by examining Fig. 4.9. Two coils *ab* and *cd* are shown wound on a common iron core.

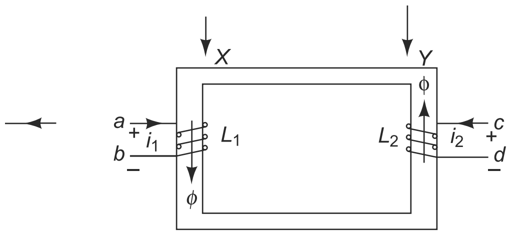


Fig. 4.9

It is evident from Fig. 4.9 that the direction of the winding of the coil *ab* is clock-wise around the core as viewed at *X*, and that of *cd* is anti-clockwise as viewed at *Y*. Let the direction of current i_1 in the first coil be from *a* to *b*, and increasing with time. The flux produced by i_1 in the core has a direction which may be found by right hand rule, and which is downwards in the left limb of the core. The flux also increases with time in the direction shown at *X*. Now suppose that the current i_2 in the second coil is from *c* to *d*, and increasing with time. The application of the right hand rule indicates that the flux produced by i_2 in the core has an upward direction in the right limb of the core. The flux also increases with time in the direction shown at *Y*. The assumed currents i_1 and i_2 produce flux in the core that are additive. The terminals *a* and *c* of the two coils attain similar polarities simultaneously. The two simultaneously positive terminals are identified by two dots by the side of the terminals as shown in Fig. 4.10.

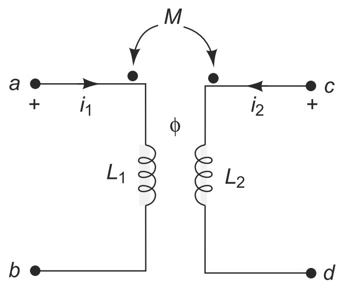


Fig. 4.10

The other possible location of thedots is the other ends of the coil to get additive fluxes in the core, i.e. at *b* and *d*. It can be concluded that the mutually induced voltage is positive when currents i_1 and i_2 both enter (or leave) the windings by the dotted terminals. If the current in one winding enters at the dot-marked terminals and the current in the other winding leaves at the dot-marked terminal, the voltages due to self and mutual induction in any coil have opposite signs.

Example 4.6 *Using dot convention, write voltage equations for the coils shown in Fig. 4.11.*

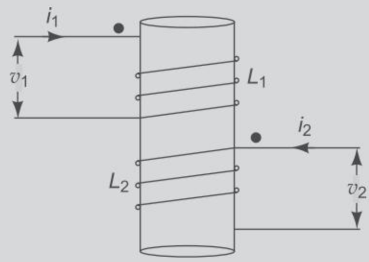


Fig. 4.11

Solution Since the currents are entering at the dot marked terminals the mutually induced voltages or the sign of the mutual inductance is positive; using the sign convention for the self-inductance, the equations for the voltages are

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Example 4.7 *Write the equation for voltage v_0 for the circuits shown in Fig. 4.12.*

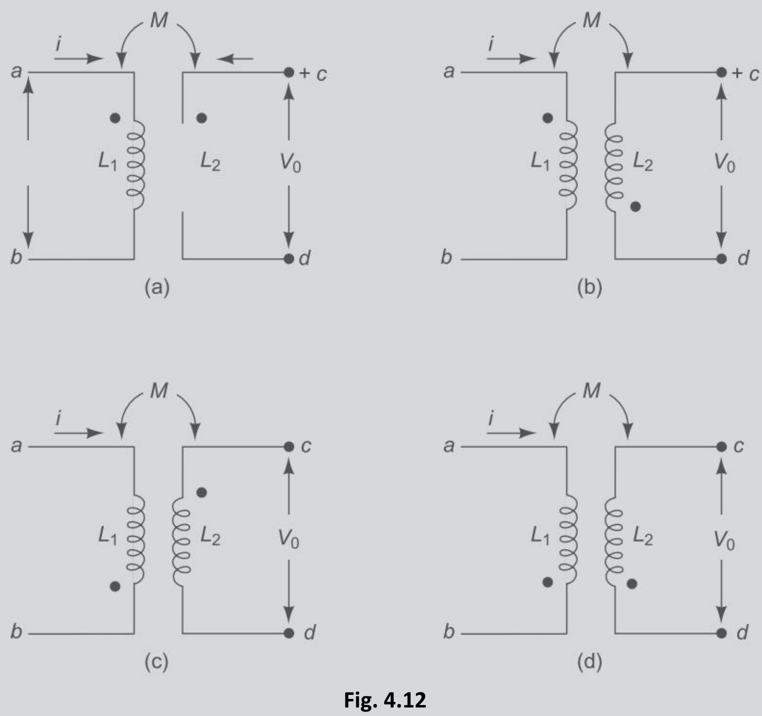


Fig. 4.12

Solution v_0 is assumed positive with respect to terminal C and the equation is given by

(a) $v_0 = M\dfrac{di}{dt}$ (b) $v_0 = -M\dfrac{di}{dt}$

(c) $v_0 = -M\dfrac{di}{dt}$ (d) $v_0 = M\dfrac{di}{dt}$

Example 4.8 *Formulate the loop equation for the network shown in the Fig. 4.13.* *[JNTU 2004]*

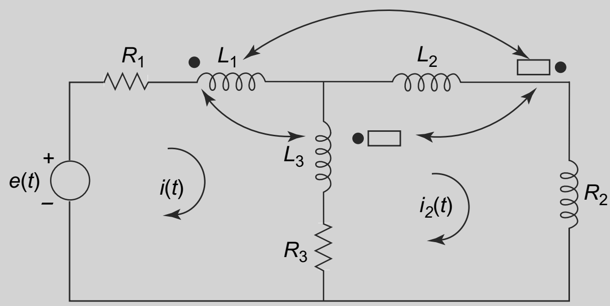


Fig. 4.13

Solution For the loop (1)

$$e(t) = i_l R_1 + L_1\frac{dl_i}{dt} + M_{31}\frac{d}{dt}(i_1 - i_2) + M_{21}\frac{d}{dt}(-i_2) + L_3\frac{d}{dt}(i_1 - i_2)$$

$$+ M_{13}\frac{d}{dt}(i_1) + M_{23}\frac{d}{dt}(-i_2) + (i_1 - i_2)R_3$$

$$e(t) = i_1 R_1 + L_1 \mathrm{S} i_1 + M_{31}\mathrm{S}(i_1 - i_2) + M_{21}(-\mathrm{S}i_2) + L_3\mathrm{S}(i_1 - i_2)$$

$$+ M_{13}\mathrm{S}i_1 + M_{23}(-\mathrm{S}i_2) + (i_1 - i_2)R_3$$

$$e(t) = i_1\left[R_1 + R_3 + \mathrm{S}(L_1 + L_3 + M_{31} + M_{13})\right] - i_1$$

$$\left[\mathrm{S}(M_{31} + M_{21}L_3 + M_{23}) + R_3\right]$$

For the loop (2)

$$L_2\frac{di_2}{dt} + M_{12}\frac{d}{dt}(-i_1) + M_{32}\frac{d}{dt} - (i_1 - i_2) + i_2 R_2 + (i_2 - i_1)R_3$$

$$+ L_3\frac{di}{dt}(i_2 - i_1) + M_{13}\frac{d}{dt}(-i_1) + M_{2/3}\frac{d}{dt}i_2 = 0$$

$$i_2\left[S(L_2+M_{32}+M_{23}+R_3+L_3)+R_2+R_3\right]$$

$$-i_1\left[S(M_{12}+M_{32}+M_{13}+L_3)+R_3\right]=0$$

Example 4.9 *In the circuit shown in Fig. 4.14, write the equation for the voltages across the coils ab and cd; also mention the polarities of the terminals.*

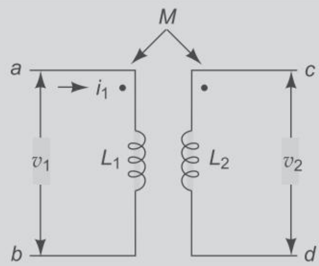


Fig. 4.14

Solution Current i_1 is only flowing in coil *ab*, whereas coil *cd* is open. Therefore, there is no current in coil *cd*. The emf due to self induction is zero on coil *cd*.

$$\therefore \qquad v_2(t)=M\frac{di_1(t)}{dt} \text{ with } C \text{ being positive}$$

Similarly the emf due to mutual induction in coil *ab* is zero.

$$\therefore \qquad v_1(t)=L\frac{di_1(t)}{dt}$$

Example 4.10 *In the circuit shown in Fig. 4.15, write the equation for the voltages v_1 and v_2. L_1 and L_2 are the coefficients of self inductances of coils 1 and 2, respectively, and M is the mutual inductance.*

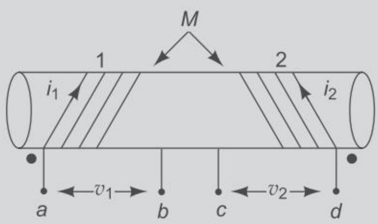


Fig. 4.15

Solution In the figure, *a* and *d* are like terminals.

Currents i_1 and i_2 are entering at dot marked terminals.

$$v_1=L_1\frac{di_1(t)}{dt}+\frac{M\,di_2(t)}{dt}$$

$$v_2=L_2\frac{di_2(t)}{dt}+\frac{M\,di_1(t)}{dt}$$

Example 4.11 *For the circuit shown in Fig. 4.16, write the mesh equations.*

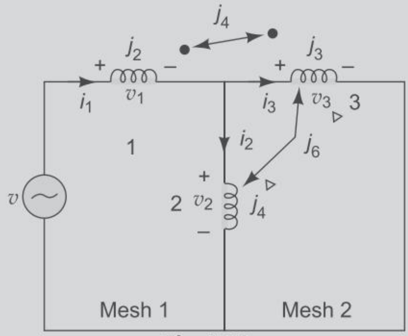


Fig. 4.16

Solution There exists mutual coupling between coil 1 and 3, and 2 and 3. Assuming branch currents i_1, i_2 and i_3 in coils 1, 2 and 3, respectively, the equation for mesh 1 is

$$v = v_1 + v_2$$

$$v = i_1 j_2 - i_3 j_4 + i_2 j_4 - i_3 j_6 \tag{4.7}$$

$j_4\, i_3$ is the mutual inductance drop between coils (1) and (3), and is considered negative according to dot convention and $i_3 j_6$ is the mutual inductance drop between coils 2 and 3.

For the 2nd mesh $0 = -v_2 + v_3 = -(j_4 i_2 - j_6 i_3) + j_3 i_3 - j_6 i_2 - j_4 i_1$ (4.8)

$$= -j_4 i_1 - j_{10} i_2 + j_9 i_3 \tag{4.9}$$

$$i_1 = i_3 + i_2$$

Example 4.12 *Explain the Dot Convention for mutually coupled coils.*

[JNTU June 2006]

Solution *Dot Convention*

Mutual inductance is the ability of one inductor to induce voltage across the neighbouring inductor measured in Henrys (H).

The mutually induced emf $M\dfrac{di}{dt}$ may be positive (or) negative but M is always positive.

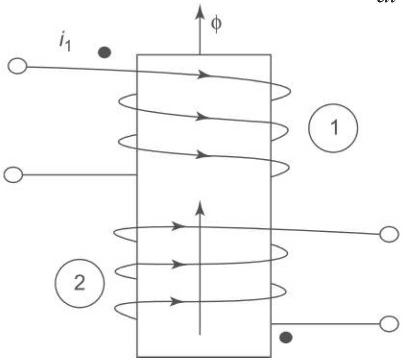


Fig. 4.17

We apply dot convention to determine the polarity of the induced emf. Consider two coils (1) and (2) as shown.

1. Place a dot at one end of coil (1) and assume that the current enters at that dotted end in coil (1).
2. Place another dot at one of the ends of coil (2) such that the current entering at that end in coil (2) establishes magnetic flux in the same direction.

In order that the flux produced by I_2 flowing in coil (2) produce flux in the same upward direction it should enter at lower end of coil (2). Hence place a dot at that end of coil (2).

4.6 COEFFICIENT OF COUPLING *[JNTU June 2009]*

4.6.1 Coefficient of Coupling

The amount of coupling between the inductively coupled coils is expressed in terms of the coefficient of coupling, which is defined as

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

where M = mutual inductance between the coils
L_1 = self inductance of the first coil, and
L_2 = self inductance of the second coil

Coefficient of coupling is always less than unity, and has a maximum value of 1 (or 100%). This case, for which $K = 1$, is called perfect coupling, when the entire flux of one coil links the other. The greater the coefficient of coupling between the two coils, the greater the mutual inductance between them, and vice-versa. It can be expressed as the fraction of the magnetic flux produced by the current in one coil that links the other coil.

For a pair of mutually coupled circuits shown in Fig. 4.18, let us assume initially that i_1, i_2 are zero at $t = 0$

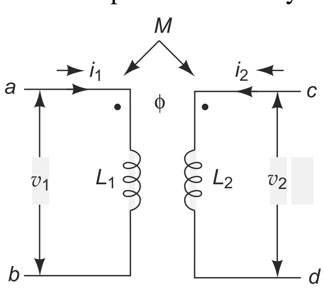


Fig. 4.18

then $v_1(t) = L_1 \dfrac{di_1(t)}{dt} + M\dfrac{di_2(t)}{dt}$

and $v_2(t) = L_2 \dfrac{di_2(t)}{dt} + M\dfrac{di_1(t)}{dt}$

Initial energy in the coupled circuit at $t = 0$ is also zero. The net energy input to the system shown in Fig. 4.18 at time t is given by

$$W(t) = \int_0^t \left[v_1(t)\ i_1(t) + v_2(t)\ i_2(t) \right] dt$$

Substituting the values of $v_1(t)$ and $v_2(t)$ in the above equation yields

$$W(t) = \int_0^t \left[L_1 i_1(t)\frac{di_1(t)}{dt} + L_2 i_2(t)\frac{di_2(t)}{dt} + M(i_1(t))\frac{di_2(t)}{dt} + i_2(t)\frac{di_1(t)}{dt} \right] dt$$

From which we get

$$W(t) = \frac{1}{2}L_1\left[i_1(t)\right]^2 + \frac{1}{2}L_2\left[i_2(t)\right]^2 + M\left[i_1(t) i_2(t)\right]$$

If one current enters a dot-marked terminal while the other leaves a dot marked terminal, the above equation becomes

$$W(t) = \frac{1}{2}L_1[i_1(t)]^2 + \frac{1}{2}L_2[i_2(t)]^2 - M[i_1(t)i_2(t)]$$

According to the definition of passivity, the net electrical energy input to the system is non-negative. $W(t)$ represents the energy stored within a passive network, it cannot be negative.

∴ $W(t) \geq 0$ for all values of i_1, i_2; L_1, L_2 or M

The statement can be proved in the following way. If i_1 and i_2 are both positive or negative, $W(t)$ is positive. The other condition where the energy equation could be negative is

$$W(t) = \frac{1}{2}L_1[i_1(t)]^2 + \frac{1}{2}L_2[i_2(t)]^2 - M[i_1(t)i_2(t)] \tag{4.10}$$

The above equation can be rearranged as

$$W(t) = \frac{1}{2}\left(\sqrt{L_1}i_1 - \frac{M}{\sqrt{L_1}}i_2\right)^2 + \frac{1}{2}\left(L_2 - \frac{M^2}{L_1}\right)i_2^2$$

The first term in the parenthesis of the right side of the above equation is positive for all values of i_1 and i_2, and, thus, the last term cannot be negative; hence

$$L_2 - \frac{M^2}{L_1} \geq 0 \tag{4.11}$$

$$\frac{L_1 L_2 - M^2}{L_1} \geq 0 \tag{4.12}$$

$$L_1 L_2 - M^2 \geq 0 \tag{4.13}$$

$$\sqrt{L_1 L_2} \geq M \tag{4.14}$$

$$M \leq \sqrt{L_1 L_2} \tag{4.15}$$

Obviously the maximum value of the mutual inductance is $\sqrt{L_1 L_2}$. Thus, we define the coefficient of coupling for the coupled circuit as

$$K = \frac{M}{\sqrt{L_1 L_2}} \tag{4.16}$$

The coefficient, K, is a non negative number and is independent of the reference directions of the currents in the coils. If the two coils are a great distance apart in space, the mutual inductance is very small, and K is also very small. For iron-core coupled circuits, the value of K may be as high as 0.99, for air-core coupled circuits, K varies between 0.4 to 0.8.

Example 4.13 *Two inductively couple d coils have self inductances L_1 = 50 mH and L_2 = 200 mH. If the coefficient of coupling is 0.5 (i), find the value of mutual inductance between the coils, and (ii) what is the maximum possible mutual inductance?*

Solution (i) $M = K\sqrt{L_1 L_2}$

$$= 0.5\sqrt{50\times10^{-3}\times200\times10^{-3}} = 50\times10^{-3}\,\text{H}$$

(ii) Maximum value of the inductance when $K = 1$,

$$M = \sqrt{L_1 L_2} = 100\ \text{mH}$$

Example 4.14 *Derive the Expression for coefficient coupling between pair of magnetically coupled coils.* *[JNTU June 2006]*

Solution *Coefficient of Coupling*

It is a measure of the flux linkages between the two coils.

The coefficient of coupling is defined as the fraction of the total flux produced by one coil linking with another and it is denoted by 'k'.

Let $\phi_1 \Rightarrow$ flux produced by coil -1

$\phi_2 \rightarrow$ flux produced by coil -2

$\phi_{12} \rightarrow$ flux produced by coil $-$ 1 linking with coil $-$ 2

$\phi_{21} \rightarrow$ flux produced by coil $-$ 2 linking with coil $-$ 1

$\therefore$ Coefficient of coupling $k = \dfrac{\phi_{12}}{\phi_1} = \dfrac{\phi_{21}}{\phi_2}$

k value lies between 0 and 1.

we know that $$M_{12} = \frac{M_2\phi_{12}}{i_1},\ M_{21} = \frac{M_1\phi_{21}}{i_2}$$

$$M_{12}\times M_{21} = \frac{M_2\phi_{12}\times M_1\phi_{21}}{i_1 i_2}$$

$$M^2 = \frac{M_2\times k\phi_1}{i_1}\times\frac{M_1\times k\phi_2}{i_2}$$

$$M^2 = k^2\frac{M_1\phi_1}{i_1}\times\frac{M_2\phi_2}{i_2} = k^2 L_1 L_2$$

$$\Rightarrow k = \frac{M}{\sqrt{L_1 L_2}}$$

Example 4.15 *Obtain the equivalent 'T' for magnetically coupled circuit shown in Fig. 4.19.* *[JNTU June 2006]*

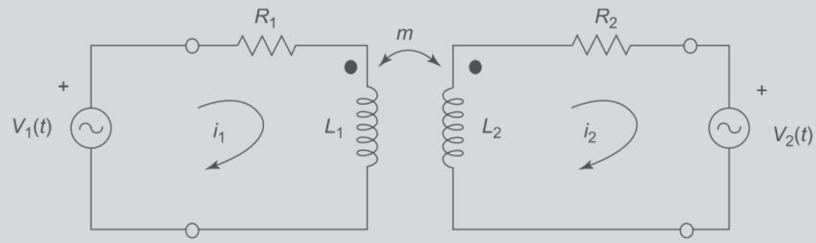


Fig. 4.19

Solution

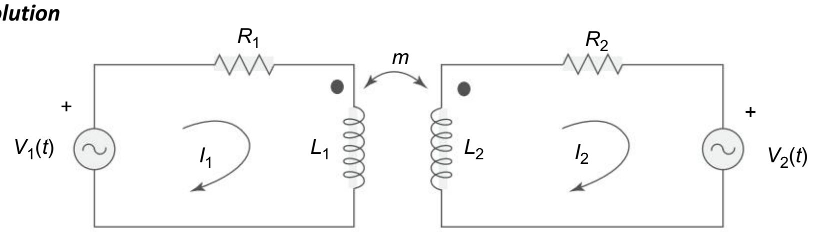


Fig. 4.20

$$V_1(t) = I_1R_1 + L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt}$$

$$V_2(t) = I_2R_2 + L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt}$$

The equivalent 'T' for magnetically coupled circuit is

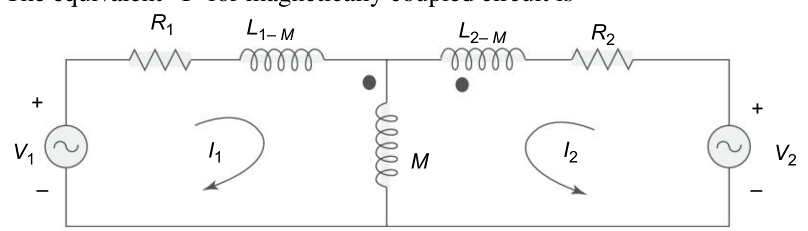


Fig. 4.21

Example 4.16 *Write down the loop equations for the network shown in Fig. 4.22.* *[JNTU June 2006]*

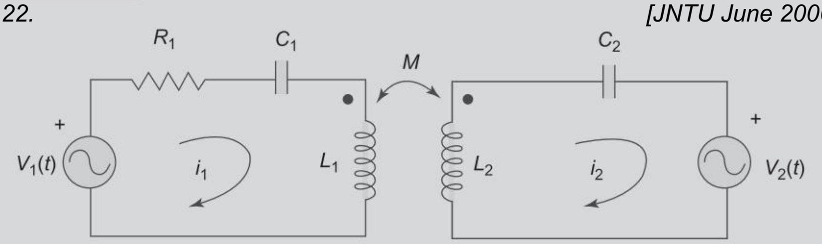


Fig. 4.22

Solution As i_1 is entering at the dot terminal, and i_2 is leaving the dot terminal, sign of M (mutual inductance) is $-$ve

$i_1(R_1 - j/\omega C_1 + jwL_1) - i_2 jwM = V_1(t)$

is loop equation for 1st mesh.

$I_2(jwL_2 - j/wC_2) - i_1(jwM) = -V_2(t)$

is loop equation for 2nd mesh

Example 4.17 *Obtain the equivalent 'T' for a magnetically coupled circuit shown in Fig. 4.23.* *[JNTU May 2007]*

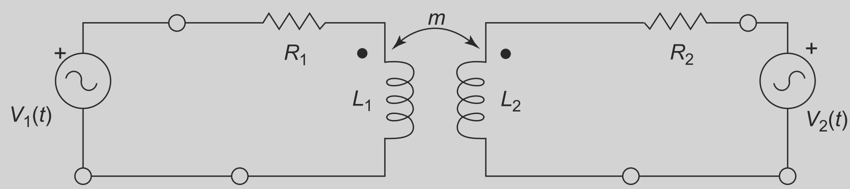


Fig. 4.23

Solution The equivalent for 'T' the given magnetically coupled circuit is

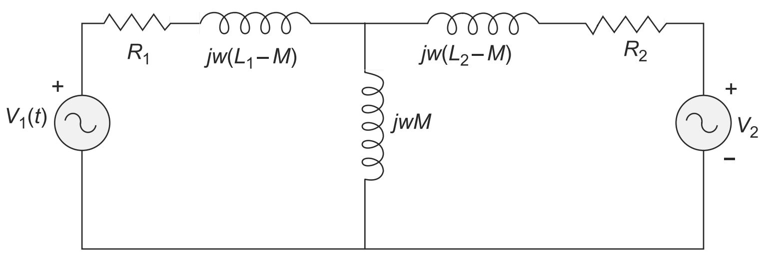


Fig. 4.24

Example 4.18 *Write down the loop equations for the network shown in Fig. 4.25.* *[JNTU May 2007]*

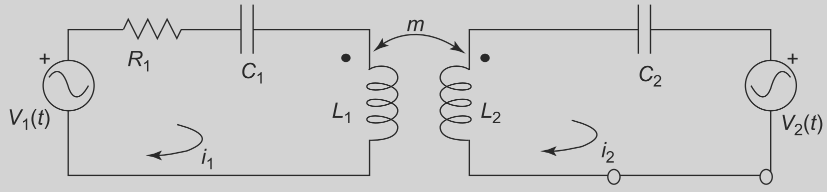


Fig. 4.25

Solution The loop equations for the given network is

$$V_1 = I_1(R_1 + jwL_1) + \frac{1}{jwc_1}(I_1) - jwMI_2$$

$$jwL_2 I_2 + \frac{1}{jwc_2}(I_2) - I_1(jwM) + V_2 = 0.$$

4.6.2 Ideal Transformer

Transfer of energy from one circuit to another circuit through mutual induction is widely utilised in power systems. This purpose is served by transformers. Most often, they transform energy at one voltage (or current) into energy at some other voltage (or current).

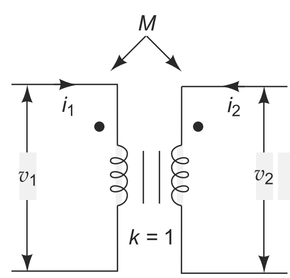
A transformer is a static piece of apparatus, having two or more windings or coils arranged on a common magnetic core. The transformer winding to which the supply source is connected is called the *primary*, while the winding connected to load is called the *secondary*. Accordingly, the voltage across the primary is called the primary voltage, and that across the secondary, the secondary voltage. Correspondingly i_1 and i_2 are the currents in the primary and secondary windings. One such transformer is shown in Fig. 4.26 (a). In circuit diagrams, ideal transformers are represented by Fig. 4.26 (b). The vertical lines between the coils represent the iron core; the currents are assumed such that the mutual inductance is positive. An ideal transformer is characterised by assuming (i) zero power dissipation in the primary and secondary windings, i.e. resistances in the coils are assumed to be zero, (ii) the self inductances of the primary and secondary are extremely large in comparison with the load impedance, and (iii) the coefficient of coupling is equal to unity, i.e. the coils are tightly coupled without having any leakage flux. If the flux produced by the current flowing in a coil links all the turns, the self inductance of either the primary or secondary coil is proportional to the square of the number of turns of the coil. This can be verified from the following results.

The magnitude of the self induced emf is given by

$$v = L\frac{di}{dt} \tag{4.17}$$

If the flux linkages of the coil with N turns and current are known, then the self induced emf can be expressed as

(a)



(b) $N_1 : N_2$

Fig. 4.26

$$L = N\frac{d\phi}{dt} \tag{4.18}$$

$$L\frac{di}{dt} = N\frac{d\phi}{dt} \tag{4.19}$$

$$L = N\frac{d\phi}{dt}$$

But $$\phi = \frac{Ni}{\text{reluctance}}$$

$\therefore$ $$L = N\frac{d}{di}\left(\frac{Ni}{\text{reluctance}}\right)$$

$$L = \frac{N^2}{\text{reluctance}}$$

$$L \alpha N^2 \tag{4.20}$$

From the above relation it follows that

$$\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2 \tag{4.21}$$

where $a = N_2/N_1$ is called the *turns ratio* of the transformer. The turns ratio, a, can also be expressed in terms of primary and secondary voltages. If the magnetic permeability of the core is infinitely large then the flux would be confined to the core. If ϕ is the flux through a single turn coil on the core and N_1, N_2 are the number of turns of the primary and secondary, respectively, then the total flux through windings 1 and 2, respectively, are

$$\phi_1 = N_1\,\phi;\ \phi_2 = N_2\,\phi$$

Also we have $$v_1 = \frac{d\phi_1}{dt}, \text{ and } v_2 = \frac{d\phi_2}{dt} \tag{4.22}$$

so that $$\frac{V_2}{V_1} = \frac{N_2\dfrac{d\phi}{dt}}{N_1\dfrac{d\phi}{dt}} = \frac{N_2}{N_1} \tag{4.23}$$

Figure 4.26 shows an ideal transformer to which the secondary is connected to a load impedance Z_L. The turns ratio $\dfrac{N_2}{N_1} = a$.

The ideal transformer is a very useful model for circuit calculations, because with few additional elements like R, L and C, the actual behaviour of the physical transformer can be accurately represented. Let us analyse this transformer with sinusoidal excitations. When the excitations are sinusoidal voltages or currents, the steady state response will also be sinusoidal. We can use phasors for representing these voltages and currents. The input impedance of the transformer can be determined by writing mesh equations for the circuit shown in Fig. 4.27.

$$V_1 = j\omega L_1 L_1 - j\omega M I_2 \tag{4.24}$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2)I_2 \tag{4.25}$$

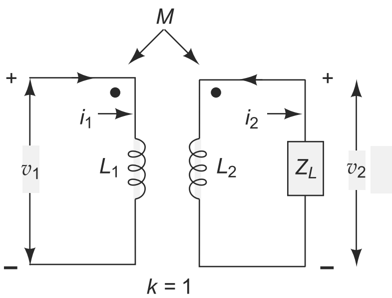


Fig. 4.27

where V_1, V_2 are the voltage phasors, and I_1, I_2 are the current phasors in the two windings. $j\omega L_1$ and $j\omega L_2$ are the impedances of the self inductances and $j\omega M$ is the impedance of the mutual inductance, ω is the angular frequency.

From Eq. 4.25 $I_2 = \dfrac{j\omega M I_1}{(Z_L + j\omega L_2)}$

Substituting in Eq. 4.24, we have

$$V_1 = I_1 j\omega L_1 + \frac{I_1 \omega^2 M^2}{Z_L + j\omega L_2}$$

The input impedance $Z_{in} = \dfrac{V_1}{I_1}$

$$\therefore \qquad Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{(Z_L + j\omega L_2)}$$

When the coefficient of coupling is assumed to be equal to unity,

$$M = \sqrt{L_1 L_2}$$

$$\therefore \qquad Z_{in} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{(Z_L + j\omega L_2)}$$

We have already established that $\dfrac{L_2}{L_1} = a^2$

where a is the turns ratio N_2/N_1

$$\therefore \qquad Z_{in} = j\omega L_1 + \frac{\omega^2 L_1^2 a^2}{(Z_L + j\omega L_2)}$$

Further simplication leads to

$$Z_{in} = \frac{(Z_L + j\omega L_2)\, j\omega L_1 + \omega^2 L_1^2 a^2}{(Z_L + j\omega L_2)}$$

$$Z_{in} = \frac{j\omega L_1 Z_L}{(Z_L + j\omega L_2)}$$

As L_2 is assumed to be infinitely large compared to Z_L

$$Z_{in} = \frac{j\omega L_1 Z_L}{j\omega a^2 L_1} = \frac{Z_L}{a^2} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

The above result has an interesting interpretation, that is the ideal transformers change the impedance of a load, and can be used to match circuits with different impedances in order to achieve maximum power transfer. For example, the input impedance of a loudspeaker is usually very small, say 3 to 12 Ω for connecting directly to an amplifier. The transformer with proper turns ratio can be placed between the output of the amplifier and the input of the loudspeaker to match the impedances as shown in Fig. 4.28.

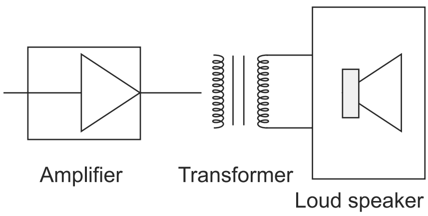


Fig. 4.28

Example 4.19 *An ideal transformer has $N_1 = 10$ turns, and $N_2 = 100$ turns. What is the value of the impedance referred to as the primary, if a 1000 Ω resistor is placed across the secondary?*

Solution The turns ratio $a = \dfrac{100}{10} = 10$

$$Z_{\text{in}} = \frac{Z_L}{a^2} = \frac{1000}{100} = 10\,\Omega$$

The primary and secondary currents can also be expressed in terms of turns ratio.

From Eq. 4.25, we have

$$I_1 jwM = I_2 (Z_L + jwL_2)$$

$$\frac{I_1}{I_0} = \frac{Z_L + j\omega L_2}{j\omega M}$$

When L_2, is very large compared to Z_L,

$$\frac{I_1}{I_2} = \frac{j\omega L_2}{j\,\omega\,M} = \frac{L_2}{M}$$

Substituting the value of $M = \sqrt{L_1 L_2}$ in the above equation $\dfrac{I_1}{I_2} = \dfrac{L_2}{M}$

$$\frac{I_1}{I_2} = \frac{L_2}{\sqrt{L_1 L_2}} = \sqrt{\frac{L_2}{L_1}}$$

$$\frac{I_1}{I_2} = \sqrt{\frac{L_2}{L_1}} = a = \frac{N_2}{N_1}$$

Example 4.20 *An amplifier with an output impedance of 1936 Ω is to feed a loudspeaker with an impedance of 4 Ω.*

(a) Calculate the desired turns ratio for an ideal transformer to connect the two systems.

(b) An rms current of 20 mA at 500 Hz is flowing in the primary. Calculate the rms value of current in the secondary at 500 Hz.

(c) What is the power delivered to the load?

Solution (a) To have maximum power transfer the output impedance of the

$$\text{amplifier} = \frac{\text{Load impedence}}{a^2}$$

$$\therefore \qquad 1936 = \frac{4}{a^2}$$

$$\therefore \qquad a = \sqrt{\frac{4}{1936}} = \frac{1}{22}$$

$$\text{or} \qquad \frac{N_2}{N_1} = \frac{1}{22}$$

(b) $I_1 = 20$ mA

We have $\dfrac{I_1}{I_2} = a$

RMS value of the current in the secondary winding

$$= \frac{I_1}{a} = \frac{20 \times 10^{-3}}{1/22} = 0.44 \text{ A}$$

(c) The power delivered to the load (speaker)

$$= (0.44)^2 \times 4 = 0.774 \text{ W}$$

The impedance changing properties of an ideal transformer may be utilised to simplify circuits. Using this property, we can transfer all the parameters of the primary side of the transformer to the secondary side, and *vice-versa*. Consider he coupled circuit shown in Fig. 4.29 (a).

To transfer the secondary side load and voltage to the primary side, the secondary voltage is to be divided by the ratio, a, and the load impedance is to be divided by a^2. The simplified equivalent circuits shown in Fig. 4.29 (b).

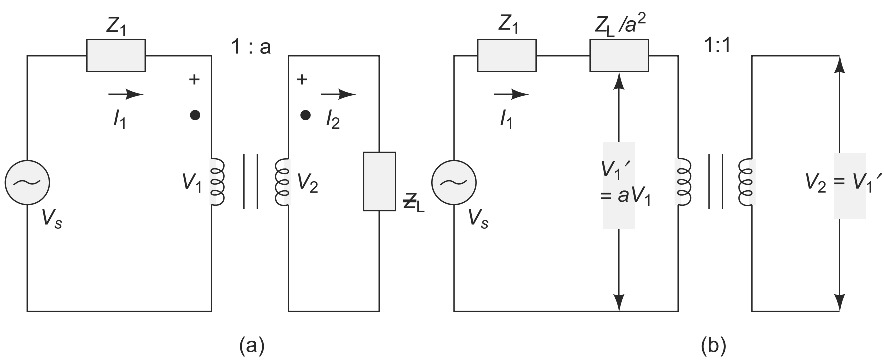


Fig. 4.29

Example 4.21 *For the circuit shown in Fig. 4.30 with turns ratio, a = 5, draw the equivalent circuit referring (a) to primary and (b) secondary. Take source resistance as 10 Ω.*

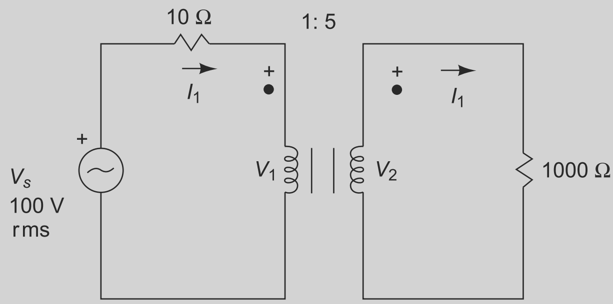


Fig. 4.30

Solution (a) Equivalent circuit referred to primary is as shown in Fig. 4.31 (a).

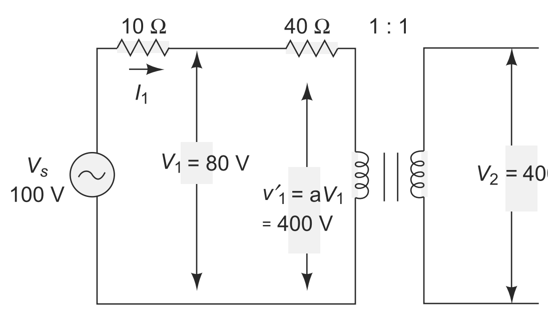


Fig. 4.31 (a)

(b) Equivalent circuit referred to secondary is as shown in Fig. 4.31 (b).

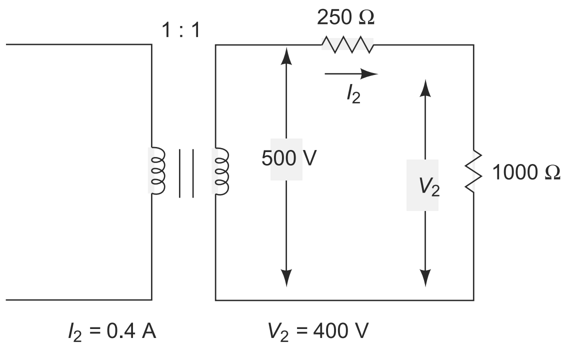


Fig. 4.31 (b)

Example 4.22 *In Fig. 4.32 $L_1 = 4H$; $L_2 = 9_1, H, K = 0.5, i_1 = 5\cos(50t - 30°)$ A, $i_2 = 2\cos(50t - 30°)$ A. Find the values of (a) v_1, (b) v_2, and (c) the total energy stored in the system at t = 0.*

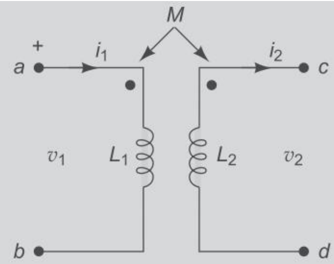


Fig. 4.32

Solution Since the current in coil *ab* is entering at the dot marked terminal, whereas in coil *cd* the current is leaving, we can write the equations as

$$v_1 = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$v_2 = -M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

$$M = K\sqrt{L_1 L_2} = 0.5\sqrt{36} = 3$$

(a) $$v_1 = 4\frac{d}{dt}\left[5\cos(50t - 30°) - 3\frac{d}{dt}[2\cos(50t - 30°)]\right]$$

$$v_1 = 20[-\sin(50t - 30°)\times 50] - 6[-\sin(50t - 30°)50]$$

$$v_1 = 500 - 150 = 350 \text{ V}$$

(b) $$v_2 = -3\frac{d}{dt}[5\cos(50t - 30°)] + 9\frac{d}{dt}[2\cos(50t - 30°)]$$

$$= -15[-\sin(50t - 30°)\times 50] + 18[-\sin(50t - 30°)50]$$

at $t = 0$

$$v_2 = -375 + 450 = 75 \text{ V}$$

(c) The total energy stored in the system

$$W(t) = \frac{1}{2}L_1[i_1(t)]^2 + \frac{1}{2}L_2[i_2(t)]^2 - M[i_1(t)i_2(t)]$$

$$= \frac{1}{2}\times 4[5\cos(50t - 30°)]^2 + \frac{1}{2}\times 9[2\cos(50t - 30°)]^2$$
$$-3[5\cos(50t - 30°)\times 2\cos(50t - 30°)]$$

at $t = 0$ W(t) = 28.5 j

Example 4.23 *Solve for the currents I_1, and I_2 in the circuit shown in Fig.4.33. Also, find the ratio of V_2/V_1.* *[JNTU June 2006]*

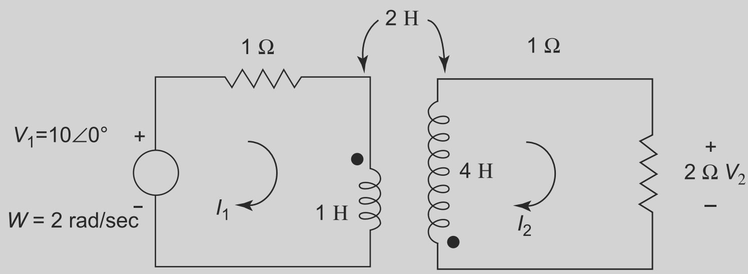


Fig. 4.33

Solution $w = 2$ rad/sec

$J \times L_1 = J_2\ \Omega$

$J \times L_2 = J(4 \times 2) = J8$

KVL to Loop 1

$$M = J_4$$

$$I_1\,(1 + J_2) + (J4)I_2 = V_1 \tag{4.26}$$

KVL to Loop 2

$$(J_4)I_1 + (2 + J_8)I_2 = 0$$

So the mesh equation are

$$(1 + J2)I_1 + (J4)I_2 = V_1 = 10$$

$$(J4)I_1 + (2 + J8)I_2 = 0$$

$$\begin{bmatrix} 1+J2 & J4 \\ J4 & 2+J8 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 10 & J4 \\ 0 & 2+J8 \end{vmatrix}}{\Delta} \qquad I_2 = \frac{\begin{vmatrix} 1+J2 & 10 \\ J4 & 0 \end{vmatrix}}{\Delta}$$

$$\Delta = \begin{vmatrix} 1+J2 & J4 \\ J4 & 2+J8 \end{vmatrix} = 2+12i$$

$$I_1 = \frac{20+80i}{2+12i} \qquad I_2 = \frac{-40i}{2+12i}$$

$I_1 = 6.75 - 0.540i \quad I_2 = -3.243 - 0.540i$

$V_2 = 2I_2 \quad I_2 = 3.287\angle -170.53°$ A

Ratio $$\frac{V_2}{V_1} = \frac{2\times(3.287\angle{-170.53°})}{10\angle 0°}$$

$$\frac{V_2}{V_1} = 0.657\angle{-170.537°}$$

4.6.3 Analysis Of Multi-Winding Coupled Circuits

Inductively coupled multi-mesh circuits can be analysed using Kirchhoff's laws and by loop current methods. Consider Fig. 4.34, where three coils are inductively coupled. For such a system of inductors we can define a inductance matrix L as

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

where L_{11}, L_{22} and L_{33} are self inductances of the coupled circuits, and $L_{12} = L_{21}$; $L_{23} = L_{32}$ and $L_{13} = L_{31}$ are mutual inductances. More precisely, L_{12} is the mutual inductance between coils 1 and 2, L_{13} is the mutual inductance between coils 1 and 3, and L_{23} is the mutual inductance between coils 2 and 3. The inductance matrix has its order equal to the number of inductors and is symmetric. In terms of voltages across the coils, we have a voltage vector related to i by

$$[v] = [L]\left[\frac{di}{dt}\right]$$

where v and i are the vectors of the branch voltages and currents, respectively. Thus the branch volt-ampere relationships of the three inductors are given by

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \\ di_3/dt \end{bmatrix}$$

Using KVL and KCL, the effective inductances can be calculated. The polarity for the inductances can be determined by using passivity criteria, whereas the signs of the mutual inductances can be determined by using the dot convention.

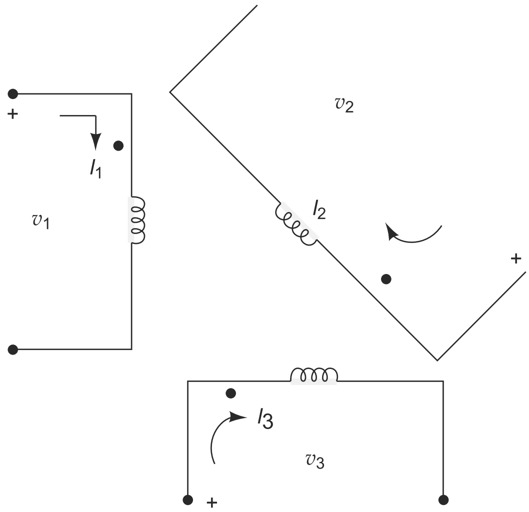


Fig. 4.34

Example 4.24 *For the circuit shown in Fig. 4.35, write the inductance matrix.*

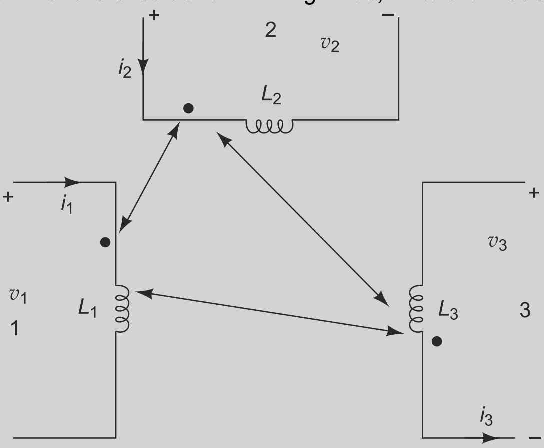


Fig. 4.35

Solution Let L_1, L_2 and L_3 be the self inductances, and $L_{12} = L_{21}$, $L_{23} = L_{32}$ and $L_{13} = L_{31}$ be the mutual inductances between coils, 1, 2, 2, 3 and 1, 3, respectively. $L_{12} = L_{21}$ is positive, as both the currents are entering at dot marked terminals, whereas $L_{13} = L_{31}$, and $L_{23} = L_{32}$ are negative.

$\therefore$ The inductance matrix is $L = \begin{bmatrix} L_1 & L_{12} & -L_{13} \\ L_{21} & L_2 & -L_{23} \\ -L_{31} & -L_{32} & L_3 \end{bmatrix}$

4.6.4 Series Connection of Coupled Inductors

Let there be two inductors connected in series, with self inductances L_1 and L_2 and mutual inductance of M. Two kinds of series connections are possible; series aiding as in Fig. 4.36 (a), and series opposition as in Fig. 4.36 (b).

In the case of series aiding connection, the currents in both inductors at any instant of time are in the same direction relative to like terminals as shown in Fig. 4.36 (a). For this reason, the magnetic fluxes of self induction and of mutual induction linking with each element add together.

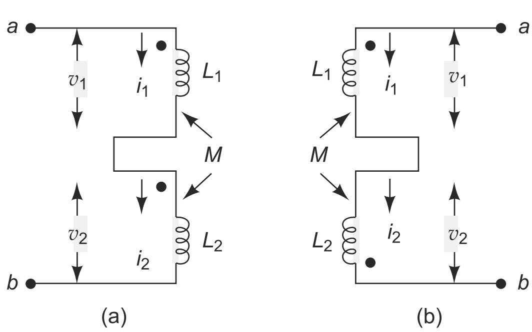


Fig. 4.36

In the case of series opposition connection, the currents in the two inductors at any instant of time are in opposite direction relative to like terminals as shown in Fig. 4.36 (b). The inductance of an element is given by $L = \phi/i$ where ϕ is the flux produced by the inductor.

$$\therefore \quad \phi = Li$$

For the series aiding circuit, if ϕ_1, and ϕ_2 are the flux produced by the coils 1 and 2, respectively, then the total flux

$$\phi = \phi_1 + \phi_2$$

where $\phi_1 = L_1 i_1 + M i_2$

$$\phi_2 = L_2 i_2 + M i_1$$

$\therefore$ $\phi = Li = L_1 i_1 + M i_2 + L_2 i_2 + M i_1$

Since $i_1 = i_2 = i$

$$L = L_1 + L_2 + 2M$$

Similarly, for the series opposition

$$\phi = \phi_1 + \phi_2$$

where $\phi_1 = L_1 i_1 - M i_2$

$$\phi_2 = L_2 i_2 - M i_2$$

$$\phi = Li = L_1 i_1 - M i_2 + L_2 i_2 - M i_1$$

Since $i_1 = i_2 = i$

$$L = L_1 + L_2 + 2M$$

In general, the inductance of two inductively coupled elements in series is given by $L = L_1, + L_2 \pm 2M$.

Positive sign is applied to the series aiding connection, and negative sign to the series opposition connection.

Example 4.25 *Two coils connected in series have an equivalent inductance of 0.4 H when connected in aiding, and an equivalent inductance 0.2 H when the connection is opposing. Calculate the mutual inductance of the coils.*

Solution When the coils are arranged in aiding connection, the inductance of the combination is $L_1 + L_2 + 2M = 0.4$; and for opposing connection, it is $L_1 + L_2 - 2M = 0.2$. Solving the two equations, we get

$$4M = 0.2 \text{ H}$$

$$M = 0.05 \text{ H}$$

Example 4.26 *Calculate the effective inductance of the current shown in the Fig. 4.37.*

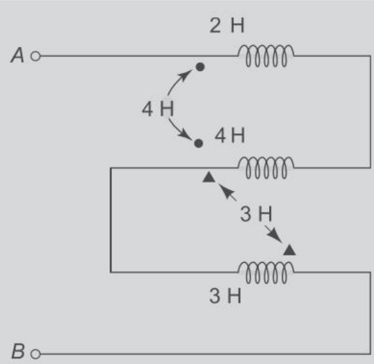


Fig. 4.37

Solution Let '*i*' be the current from A to B and *v* be the voltage across AB.

$$v = \frac{di}{dt}[2+4+3-4-4+3+3]$$

The first three terms are self-induced terms and the later four terms are mutual terms.

$$\therefore \qquad v = 7\frac{di}{dt}$$

$$L = 7\text{H}$$

Example 4.27 *Calculate the effective inductance of the circuit shown in Fig. 4.38 across terminals a and b.*

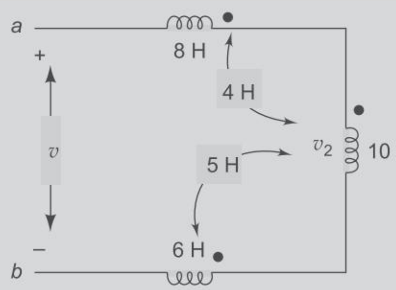


Fig. 4.38

Solution Let the current in the circuit be *i*

$$v = 8\frac{di}{dt} - 4\frac{di}{dt} + 10\frac{di}{dt} - 4\frac{di}{dt} + 5\frac{di}{dt} + 6\frac{di}{dt} + 5\frac{di}{dt}$$

or $$\frac{di}{dt}[34-8] = 26\frac{di}{dt} = v$$

Let L be the effective inductance of the circuit across *ab*. Then the voltage across $ab = v = L\frac{di}{dt} = 26\frac{di}{dt}$

Hence, the equivalent inductance of the circuit is given by 26 H.

Example 4.28 *Write down the voltage equation for the following, and determine the effective inductance.* *[JNTU June 2006]*

Solution Apply KVL in the given loop

$$V(t) = L_1\frac{di(t)}{dt} + L_2\frac{di(t)}{dt} + M_A\frac{di(t)}{dt} + M_A\frac{di(t)}{dt} + L_3\frac{di(t)}{dt} - M_B\frac{di(t)}{dt} - M_B\frac{di(t)}{dt} - M_C\frac{di(t)}{dt} - M_C\frac{di(t)}{dt}$$

$$\therefore \qquad V(t) = [L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C]\frac{di(t)}{dt}$$

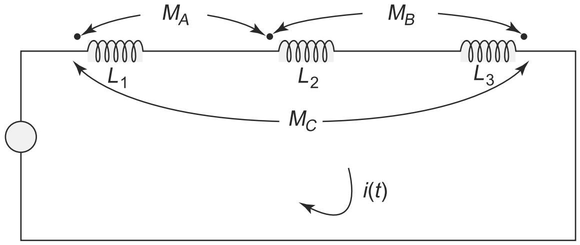


Fig. 4.39

is the required voltage equation.

We have $V(t) = L\dfrac{di(t)}{dt}$

$$L\frac{di(t)}{dt} = [L_1 + L_2 + L_3 + 2M_A - 2M_B - 2M_C]\frac{di(t)}{dt}$$

$\therefore$ $L = L_1+L_2+L_3+2M_A - 2M_B - 2M_C$ is the equivalent inductance.

Example 4.29 *Two identical coils connected in series gave an inductance of 800 mH and when one of the coils is reversed gave an inductance of 400 mH. Determine self-inductance, mutual inductance between the coils and the co-efficient of coupling.* *[JNTU June 2006]*

Solution Let 'L' be the self inductance of the coils and M be the mutual inductance between the coils.

Given data

Two identical coils connected in series gave an inductance of 800 mH

i.e. $L+L+2M = 800$ [$\because$ identical coils $L_1 = L_2 = L$]

$2L+2M = 800$

When one of the coils is reversed gave an inductance of 400 mH

i.e. $L+L - 2M = 400$

$2L - 2M = 400$

Add (1) and (2) we get $4L = 1200$

$L = 300$ mH

Subtracting (2) from (1) we get $4M = 400$ mH

$M = 100$ mH

$\therefore$ Self inductance of each coil $= L = 300$ mH

Mutual inductance between the coils $= M = 100$ mH

Co-efficient of coupling $= K = \dfrac{M}{\sqrt{L_1L_2}}$

$\therefore$ $K = \dfrac{M}{\sqrt{LL}}$ [$\because L_1 = L_2 = L$]

$$\therefore \quad K = \frac{M}{\sqrt{L^2}} = \frac{M}{L}$$

$$\therefore \quad K = \frac{100\,\text{mH}}{300\,\text{mH}}$$

$$\therefore \quad K = 1/3$$

∴ Co-efficient of coupling = 1/3.

Example 4.30 *In the circuit shown in Fig. 4.40 find the voltage across the terminals A and B if the current changes at the rate of 100 A/sec. The value of L_1, L_2 and M are 1 H, 2 H, and 0.5 H respectively. [JNTU May 2007]*

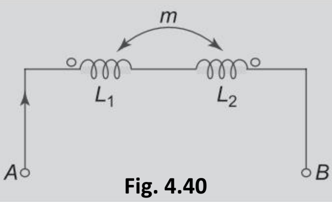


Fig. 4.40

Solution

$$V_{AB} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt}$$

$$= (L_1 + L_2 - 2M)\frac{di}{dt}$$

$$V_{AB} = (1 + 2 - 2(0.5))\,100$$

$$V_{AB} = 200 \text{ volts}$$

Example 4.31 *A 15 mH coil is connected in series with another coil. The total inductance is 70 mH. When one of the coils is reversed, the total inductance is 30 mH. Find the inductance of second coil, mutual inductance and coefficient of coupling. Derive the expression used. [JNTU June 2009]*

Solution Total inductance = $L_1 + L_2 + 2\text{M}$

$$= 15\,\text{mH} + x + 2\text{M} = 70\,\text{mH} \quad (4.27)$$

Total inductance = $L_1 + L_2 - 2\text{M}$

$$= 15\,\text{mH} + x - 2\text{M} = 30\,\text{mH} \quad (4.28)$$

So inductance of 2nd coil:

(4.27) + (4.28)

$$15\,\text{mH} + x + 2\,\text{M} = 70\,\text{mH}$$

$$15\,\text{mH} + x - 2\,\text{M} = 30\,\text{mH}$$

or

$$30\,\text{mH} + 2x = 100\,\text{mH}$$

$$\therefore \quad x = 35\,\text{mH}$$

Now putting this in (4.27)

$$15\,\text{mH} + 35\,\text{mH} + 2\,\text{M} = 70\,\text{mH}$$

$$2\text{M} = 20\,\text{mH}$$

$$\therefore \quad \text{M} = 10\,\text{mH}$$

$$\text{M} = k\sqrt{L_1 L_2}$$

$$\therefore \quad 10 = k\sqrt{35 \times 15}$$

$$\therefore \quad k = 0.436$$

4.6.5 Parallel Connection of Coupled Coils

Consider two inductors with self inductances L_1 and L_2 connected parallel which are mutually coupled with mutual inductance M as shown in Fig. 4.41 (a) and (b).

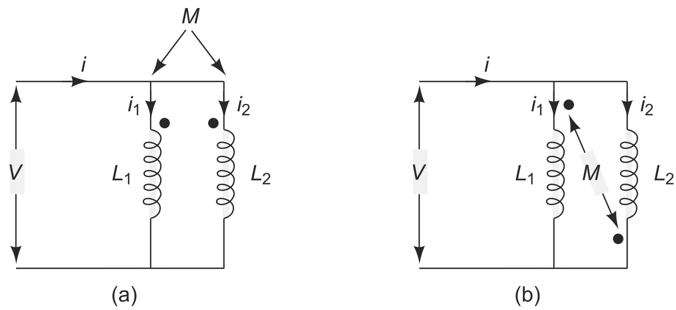


Fig. 4.41

Let us consider Fig. 4.41 (a) where the self induced emf in each coil assists the mutually induced emf as shown by the dot convention.

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \tag{4.29}$$

The voltage across the parallel branch is given by

$$v = L_1\frac{di_2}{dt} + M\frac{di_2}{dt} \quad \text{or} \quad L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

also $$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\frac{di_1}{dt}(L_1 - M) = \frac{di_2}{dt}(L_2 - M)$$

$$\therefore \quad \frac{di}{dt} = \frac{di_2}{dt}\frac{(L_2 - M)}{(L_1 - M)} \tag{4.30}$$

Substituting Eq. 4.30 in Eq. 4.31, we get

$$\frac{di}{dt} = \frac{di_2}{dt}\frac{(L_2 - M)}{(L_1 - M)} + \frac{di_2}{dt} = \frac{di_1}{dt}\left[\frac{(L_2 - M)}{L_1 - M} + 1\right] \tag{4.31}$$

If L_{eq} is the equivalent inductance of the parallel circuit in Fig. 4.41 (a) then v is given by

$$v = L_{eq}\frac{di}{dt}$$

$$L_{eq}\frac{di}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L_{eq}}\left[L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right]$$

Substituting Eq. 4.32 in the above equation we get

$$\frac{di}{dt} = \frac{1}{L_{eq}}\left[L_1\frac{di_2(L_2 - M)}{dt(L_2 - M)} + M\frac{di_2}{dt}\right]$$

$$= \frac{1}{L_{eq}}\left[L_1\frac{(L_2 - M)}{L_1 - M} + M\right]\frac{di_2}{dt} \tag{4.32}$$

Equating Eq. 4.32 and Eq. 4.31, we get

$$\frac{L_2 - M}{L_2 - M} + 1 = \frac{1}{L_{eq}}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]$$

Rearranging and simplifying the above equation results in

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

If the voltage induced due to mutual inductance oppose the self induced emf in each coil as shown by the dot convention in Fig. 4.32 (b), the equivalent inductance its given by

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

Example 4.32 *For the circuit shown in Fig. 4.42, find the ratio of output voltage to the source voltage.*

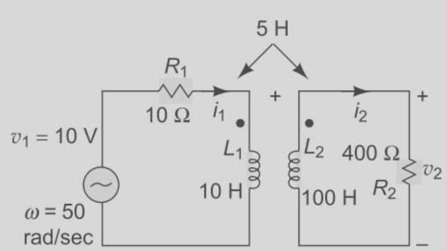


Fig. 4.42

Solution Let us consider i_1 and i_2 as mesh currents in the primary and secondary windings.
As the current i_1 is entering at the dot marked terminal, and current i_2 is leaving the dot marked terminal, the sign of the mutual inductance is to be negative. Using Kirchhoff's voltage law, the voltage equation for the first mesh is

$$i_1(R_1 + j\omega L_1) - i_2 j\omega M = v_1$$

$$i_1(10 + j500) - i_2 j250 = 10 \tag{4.33}$$

Similarly, for the 2nd mesh

$$i_1(R_2 + j\omega L_2) - i_1 j\omega M = 0$$

$$i_2(400 + j5000) - i_1 j250 = 0 \tag{4.34}$$

$$i_2 = \frac{\begin{vmatrix} (10+j500) & 10 \\ -j250 & 0 \end{vmatrix}}{\begin{vmatrix} (10+j500) & -j250 \\ -j250 & (400+j5000) \end{vmatrix}}$$

$$i_2 = 0.00102 \angle -84.13^\circ$$

$$v_2 = i_2 \times R_2$$

$$= 0.00102 \angle -84.13^\circ \times 400$$

$$= 0.408 \angle -84.13^\circ$$

$$\frac{v_2}{v_1} = \frac{0.408}{10} \angle -84.13^\circ$$

$$\frac{v_2}{v_1} = 40.8 \times 10^{-3} \angle -84.13^\circ$$

Example 4.33 *Calculate the effective inductance of the circuit shown in Fig. 4.43 across AB.*

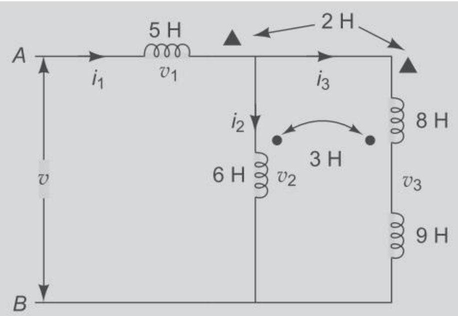


Fig. 4.43

Solution The inductance matrix is

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} = \begin{bmatrix} 5 & 0 & -2 \\ 0 & 6 & -3 \\ -2 & -3 & 17 \end{bmatrix}$$

From KVL $v = v_1 + v_2$ (4.35)

and $v_2 = v_3$ (4.36)

From KCL $i_1 = i_2 + i_3$ (4.37)

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & -2 \\ 0 & 6 & -3 \\ -2 & -3 & 17 \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \\ di_3/dt \end{bmatrix}$$

$$v_1 = 5\frac{di_1}{dt} - 2\frac{di_3}{dt} \tag{4.38}$$

and
$$v_2 = 6\frac{di_2}{dt} - 3\frac{di_3}{dt} \tag{4.39}$$

$$v_3 = -2\frac{di_1}{dt} - 3\frac{di_2}{dt} + 17\frac{di_3}{dt} \tag{4.40}$$

From Eq. 4.35, we have

$$v = v_1 + v_2$$

$$= 5\frac{di_1}{dt} - 2\frac{di_3}{dt} + 6\frac{di_2}{dt} - 3\frac{di_3}{dt}$$

$$v = 5\frac{di_1}{dt} + 6\frac{di_2}{dt} - 5\frac{di_3}{dt} \tag{4.41}$$

From Eq. 4.37

$$\frac{di_1}{dt} = \frac{di_2}{dt} + \frac{di_3}{dt} \tag{4.42}$$

Substituting Eq. 4.42 in Eq. 4.43, we have

$$v_3 = -2\left[\frac{di_2}{dt} + \frac{di_3}{dt}\right] - 3\left[\frac{di_2}{dt}\right] + 17\left[\frac{di_3}{dt}\right]$$

or
$$-5\frac{di_2}{dt} + 15\frac{di_3}{dt} = v_3 \tag{4.43}$$

Multiplying Eq. 4.39 by 5, we get

$$30\frac{di_2}{dt} - 15\frac{di_3}{dt} = 5v_2 \tag{4.44}$$

Adding Eqs (4.43) and (4.44), we get

$$25\frac{di_2}{dt} = v_3 + 5v_2$$

$$25\frac{di_2}{dt} = 6v_2$$

$$= 6v_3, \text{ since } v_2 = v_3$$

or
$$v_2 = \frac{25}{6}\frac{di_2}{dt}$$

From Eq. 4.41

$$\frac{25}{6}\frac{di_2}{dt} = 6\frac{di_2}{dt} - 3\frac{di_3}{dt}$$

from which $\dfrac{di_2}{dt} = \dfrac{18}{11}\dfrac{di_3}{dt}$

From Eq. 4.44

$$\frac{di_2}{dt} = \frac{di_2}{dt} + \frac{11}{18}\frac{di_2}{dt} = \frac{29}{18}\frac{di_2}{dt}$$

Substituting the values of $\dfrac{di_2}{dt}$ and $\dfrac{di_3}{dt}$ in Eq. 4.41 yields

$$v = 5\frac{di_1}{dt} + 6\frac{18}{29}\frac{di_1}{dt} - 5\frac{11}{18}\frac{di_2}{dt}$$

$$= 5\frac{di_2}{dt} + \frac{108}{29}\frac{di_1}{dt} - \frac{55}{18}\frac{18}{29}\frac{di_1}{dt}$$

$$v = \frac{198}{29}\frac{di_1}{dt} = 6.827\frac{di_1}{dt}$$

∴ equivalent inductance across AB = 6.827 H

Example 4.34 *Write the mesh equations for the network shown in Fig. 4.44.*

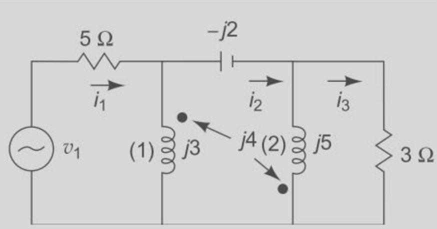


Fig. 4.44

Solution The circuit contains three meshes. Let us assume three loop currents i_1 i_2 and i_3.

For the first mesh

$$5i_1 + j_3(i_1 - i_2) + j_4(i_3 - i_2) = v_1 \tag{4.45}$$

The drop due to self inductance is $j3(i_1 - i_2)$ is written by considering the: Current $(i_1 - i_2)$ entering at dot marked terminal in the first coil, $j4(i_3 - i_2)$ is the nutually induced voltage in coil 1 due to current $(i_3 - i_2)$ entering at dot marked terminal of coil 2.

Similarly, for the 2nd mesh,

$$j3(i_2 - i_1) + j5(i_2 - i_3) - j2i_2 + j4(i_2 - i_3) + j4(i_2 - i_1) = 0 \tag{4.46}$$

$j4(i_2 - i_1)$ is the mutually induced voltage in coil 2 due to the current in coil 1, and $j4(i_2 - i_3)$ is the mutually induced voltage in coil 1 due to the current in coil 2.

For the third mesh,

$$3i_3 + j5(i_3 - i_2) + j4(i_1 - i_2) = 0 \tag{4.47}$$

Further simplification of Eqs 4.45, 4.46 and 4.47 leads to

$$(5 + j3)i_1 - j7i_2 + j4i_3 = v_1 \tag{4.48}$$

$$-j7i_1 + j14i_2 - j9i_3 = 0 \tag{4.49}$$

$$j4i_1 - j9i_2 + (3 + j5)i_3 = 0 \tag{4.50}$$

Example 4.35 *The inductance matrix for the circuit of three series connect coupled coils is given in Fig. 4.45. Find the inductances, and indicate the dots for the coils.*

$$L = \begin{bmatrix} 4 & -4 & 1 \\ -4 & 2 & -3 \\ 1 & -3 & 6 \end{bmatrix}$$

All elements are in henrys.

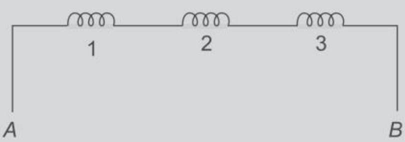


Fig. 4.45

Solution The diagonal elements (4, 2, 6) in the matrix represent the self inductances of the three coils 1, 2 and 3, respectively. The second element in the 1st row (−4) is the mutual inductance between coil 1 and 2, the negative sign indicates that the current in the first coil enters the dotted terminal, and the current in the second coil enters at the undotted terminal. Similarly, the remaining elements are fixed. The values of inductances and the dot convention is shown in Fig. 4.46.

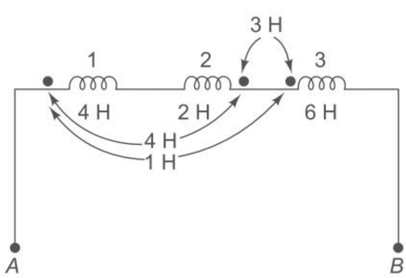


Fig. 4.46

Example 4.36 *Find the voltage across the 10 Ω resistor for the network shown in Fig. 4.47.*

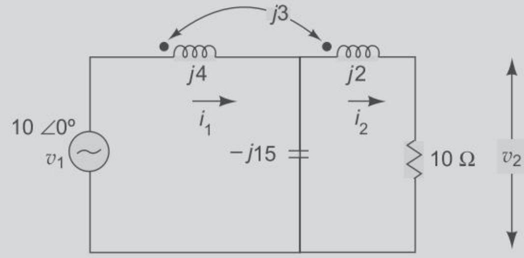


Fig. 4.47

Solution From Fig. 4.46 is clear that

$$v_2 = i_2 10 \tag{4.51}$$

Mesh equation for the first mesh is

$$j4i_1 - j15\,(i_1 - i_2) + j3i_2 = 10\angle 0°$$

$$-j11i_1 + j18i_2 = 10\angle 0° \tag{4.52}$$

Mesh equation for the 2nd mesh is

$$j2i_2 + 10i_2 - j15(i_2 - i_1) + j3i_1 = 0$$

$$j18i_1 - j13i_2 + 10i_2 = 0$$

$$j18i_1 + i_2\,(10 - j13) = 0 \tag{4.53}$$

Solving for i_2 from Eqs 4.52 and 4.53, we get

$$i_2 = \begin{bmatrix} -j11 & 10\angle 0° \\ j18 & 0 \end{bmatrix} \bigg/ \begin{bmatrix} -j11 & j18 \\ j18 & 10 - j3 \end{bmatrix}$$

$$= \frac{-180\angle 90°}{291 - j110}$$

$$= \frac{-180\angle 90°}{311\angle 20.70°} = -0.578\angle 110.7°$$

$$v_2 = i_2\,10 = -5.78\angle 110.7°$$

$$|v_2| = 5.78$$

Example 4.37 *Write the loop equations for the coupled circuit shown in Fig. 4.48.* *[JNTU June 2006]*

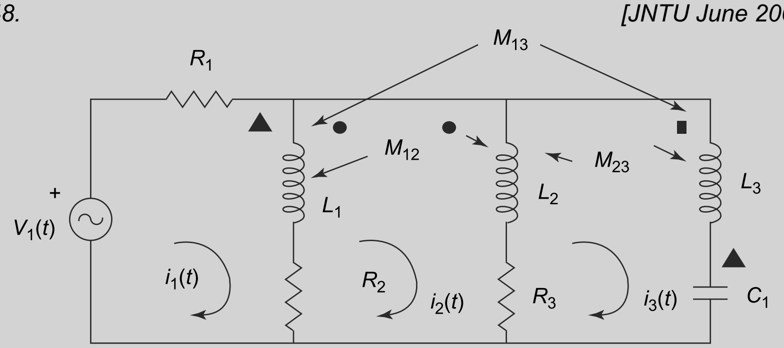


Fig. 4.48

Solution

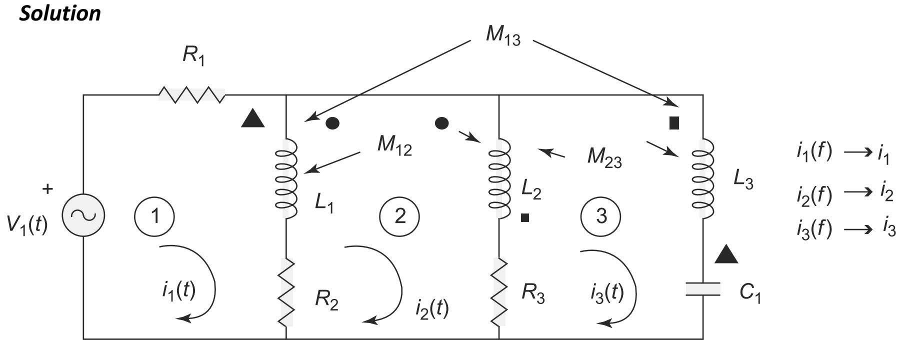


Fig. 4.49

Loop Equations: (*By Dot Rule Convention*)

$$(1) \Rightarrow V_1(t) = i_1(t)(R_1 + R_2) + L_1 \frac{di_1(t)}{dt} - i_2(t)R_2 + M_{12} \frac{di_2(t)}{dt}$$

$$- M_{13} \frac{di_3(t)}{dt} - L_1 \frac{di_2(t)}{dt} - M_{12} \frac{di_3(t)}{dt}$$

$$(2) \Rightarrow R_2(i_2(t) - i_1(t)) + L_1\left(\frac{di_2(t)}{dt} - \frac{di_1(t)}{dt}\right) - M_{12}\left(\frac{di_2(t)}{dt} - \frac{di_3}{dt}\right)$$

$$+ M_{13} \frac{di_3}{dt} + L_2\left(\frac{di_2}{dt} - \frac{di_3}{dt}\right) - M_{12}\left(\frac{di_2}{dt} - \frac{di_1}{dt}\right) - M_{23} \frac{di_3}{dt}$$

$$+ R_3(i_2 - i_3) = 0$$

$$(3) \Rightarrow R_3(i_3 - i_2) + L_2\left(\frac{di_3}{dt} - \frac{di_2}{dt}\right) - M_{12}\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) + M_{23} \frac{di_3}{dt}$$

$$+ L_3 \frac{di_3}{dt} - M_{13}\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right) + M_{23}\left(\frac{di_3}{dt} - \frac{di_2}{dt}\right) + \frac{1}{C_1}\int i_3 dt = 0.$$

Example 4.38 *Write the loop equations for the coupled circuits shown in Fig. 4.50.* *[JNTU May 2007]*

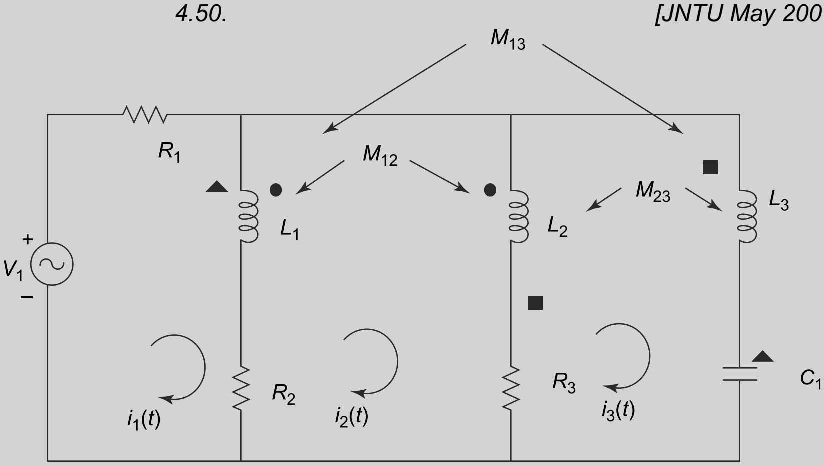


Fig. 4.50

Solution Given circuit is

The loop equations are

$$V_1(t) = R_1 i_1(t) + L_1 \frac{d}{dt}[i_1(t) - i_2(t)] - M_{12}\frac{d}{dt}[i_2(t) - i_3(t)] \tag{4.54}$$

$$- M_{13}\frac{d}{dt}[i_3(t)] + R_2[i_1(t) - i_2(t)]$$

Loop 2

$$R_2[i_2(t) - i_1(t)] + L_1\left[\frac{di_2(t)}{dt} - \frac{di_i(t)}{dt}\right] - M_{12}\frac{d}{dt}[i_2(t) - i_3(t)]$$

$$+ M_{13}\frac{di_3(t)}{dt} + L_2\frac{d[i_2(t) - i_3(t)]}{dt} - M_{12}\left[\frac{di_2(t)}{dt} - \frac{di_i(t)}{dt}\right]$$

$$- M_{23}\frac{di_3(t)}{dt} + R_3(i_2 - i_3) = 0$$

Loop 3

$$R_3(i_3 - i_2) + L_2\frac{d(i_3 - i_2)}{dt} - M_{12}\frac{d(i_l - i_2)}{dt} + M_{23}\frac{di_3}{dt}$$

$$+ L_3\frac{di_3}{dt} - M_{13}\frac{d}{dt} - M_{23}\frac{d(i_3 - i_2)}{dt} + \frac{1}{C_1}\int i_3 dt = 0$$

4.6.6 Tuned Circuits

Tuned circuits are, in general, single tuned and double tuned. Double tuned circuits are used in radio receivers to produce uniform response to modulated signals over a specified bandwidth; double tuned circuits are very useful in communication system.

Single Tuned Circuit

Consider the circuit in Fig. 4.51. A tank circuit (i.e. a parallel resonant circuit) on the secondary side is inductively coupled to coil (1) which is excited by a source, v_i. Let R_s be the source resistance and R_1, R_2 be the resistances of coils, 1 and 2, respectively. Also let L_1, L_2 be the self inductances of the coils, 1 and 2, respectively.

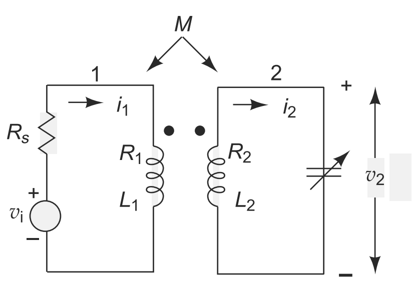


Fig. 4.51

Let $R_s + R_1 + j\omega L_1 = R_s$ with the assumption that $R_s \gg R_1 \gg j\omega L_1$

The mesh equations for the circuit shown in Fig. 4.50 are

$$i_1 R_s - j\omega M i_2 = v_i$$

$$-j\omega M i_1 + \left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right) i_2 = 0$$

$$i_2 = \begin{vmatrix} R_s & v_i \\ -j\omega M & 0 \end{vmatrix} \Bigg/ \begin{vmatrix} R_s & (-j\omega M) \\ (-j\omega M) & \left(R_2 + j\omega L_2 - \frac{i}{\omega C}\right) \end{vmatrix}$$

or
$$i_2 = \frac{j v_i \omega M}{R_s\left(R_2 + j\omega L_2 - \frac{j}{\omega C}\right) + \omega^2 M^2}$$

The output voltage $v_o = i_2 \cdot \dfrac{1}{j\omega C}$

$$v_o = \frac{jv_i\omega M}{j\omega C\left\{R_s\left[R_2 + \left(j\omega L_2 - \frac{1}{\omega C}\right)\right] + \omega^2\ M^2\right\}}$$

The voltage transfer function, or voltage amplification, is given by

$$\frac{v_o}{v_i} = A = \frac{M}{C\left\{R_s\left[R_2 + \left(j\omega L_2 - \frac{1}{\omega C}\right)\right] + \omega^2\ M^2\right\}}$$

When the secondary side is tuned, i.e. when the value of the frequency ω is such that $\omega L_2 = 1/\omega C$, or at resonance frequency ω_r, the amplification is given by

$$A = \frac{v_o}{v_i} = \frac{M}{C\left[R_s R_2 + \omega_r^2\ M^2\right]}$$

the current i_2 at resonance $i_2 = \dfrac{jv_i\omega_r M}{R_s R_2 + \omega_r^2\ M^2}$

Thus, it can be observed that the output voltage, current and amplification depends on the mutual inductance M at resonance frequency, when $M = K\sqrt{L_1 L_2}$.

The maximum output voltage or the maximum amplification depends on M. To get the condition for maximum output voltage, make $dv_o/dM = 0$.

$$\frac{dv_o}{dM} = \frac{d}{dM}\left[\frac{v_i M}{C\left[R_s R_2 + \omega_r^2 M^2\right]}\right]$$

$$= 1 - 2M^2\omega_r^2\left[R_s R_2 + \omega_r^2 M^2\right]^{-1} = 0$$

From which, $R_s R_2 = \omega_r^2 M^2$

or $$M = \sqrt{\frac{R_s R_2}{\omega_r}}$$

From the above value of M, we can calculate the maximum output voltage.

Thus $$v_{oM} = \frac{v_i}{2\omega_r C\sqrt{R_s R_2}},$$

or the maximum amplification is given by

$$A_m = \frac{1}{2\omega_r C\sqrt{R_s R_2}} \quad \text{and} \quad i_2 = \frac{jv_i}{2\sqrt{R_s R_2}}$$

The variation of the amplification factor or output voltage with the coefficient of coupling is shown in Fig. 4.52.

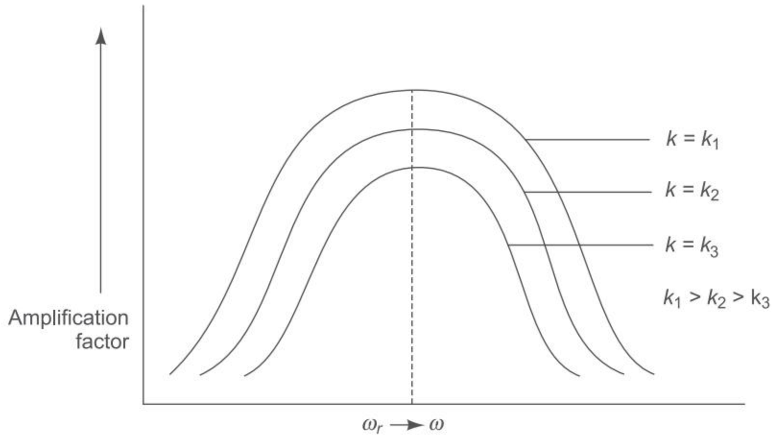


Fig. 4.52

Example 4.39 *Consider the single tuned circuit shown in Fig. 4.53 and determine (i) the resonant frequency (ii) the output voltage at resonance and (iii) the maximum output voltage. Assume $R_s >> w_r L_1$, and K = 0.9.*

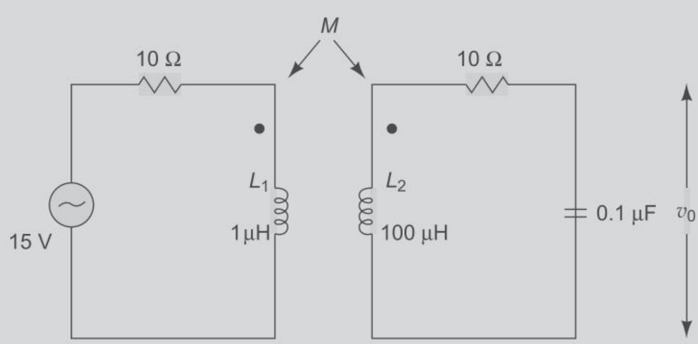


Fig. 4.53

Solution

$$M = K\sqrt{L_1 L_2}$$
$$= 0.9\sqrt{1\times10^{-6}\times100\times10^{-6}}$$
$$= 9\,\mu\text{H}$$

(i) Resonance frequency

$$\omega_r = \frac{1}{\sqrt{L_2 C}} = \frac{1}{\sqrt{100\times10^{-6}\times0.1\times10^{-6}}}$$
$$= \frac{10^6}{\sqrt{10}}\ \text{rad/sec.}$$

or $$f_r = 50.292\ \text{KHz}$$

The value of $\omega_r L_1 = \dfrac{10^6}{\sqrt{10}} 1\times10^{-6} = 0.316$

Thus the assumption that $\omega_r L_1 R_s \ll$ is justified,

(ii) Output voltage

$$v_o = \frac{Mv_i}{C\left[R_s R_2 + \omega_r^2 M\right]}$$

$$= \frac{9\times10^{-6}\times15}{0.1\times10^{-6}\left[10\times10+\left(\frac{10^6}{\sqrt{10}}\right)^2\times9\times10^{-6}\right]} = 1.5\,\text{mV}$$

(iii) Maximum value of output voltage

$$v_{oM} = \frac{\omega_i}{2\omega_r C\sqrt{R_s R_2}}$$

$$= \frac{15}{2\times\frac{10^6}{\sqrt{10}}\times0.1\times10^{-6}\sqrt{100}}$$

$$v_{oM} = 23.7\,\text{V}$$

Example 4.40 *The resonant frequency of the tuned circuit shown in Fig. 4.54 is 1000 rad/sec. Calculate the self-inductances of the two coils and the optimum value of the mutual inductance.*

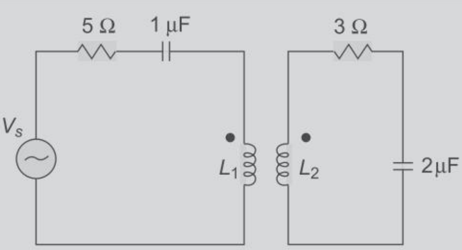


Fig. 4.54

Solution We know that

$$\omega_r^2 = \frac{1}{L_1C_1} = \frac{1}{L_2C_2}$$

$$L_1 = \frac{1}{\omega_r^2 C_1} = \frac{1}{(1000)^2 1\times10^{-6}} = 1\text{ H}$$

$$L_2 = \frac{1}{\omega_r^2 C_2} = \frac{1}{(1000)^2\times2\times10^{-6}} = 0.5\text{ H}$$

Optimum value of the mutual inductance is given by

$$M_{\text{optimum}} = \frac{\sqrt{R_1R_2}}{\omega_r}$$

where R_1 and R_2 are the resistances of the primary and secondary coils

$$M = \frac{\sqrt{15}}{1000} = 3.87\,\text{mH}$$

Double Tuned Coupled Circuits

Figure 4.55 shows a double tuned transformer circuit involving two series resonant circuits.

For the circuit shown in the figure, a special case where the primary and secondary resonate at the same frequency ω_r, is considered here,

i.e, $$\omega_r^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}$$

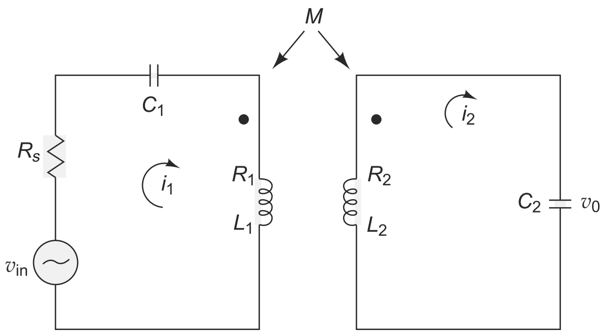


Fig. 4.55

The two mesh equations for the circuit are

$$v_{\text{in}} = i_1\left(R_s + R_1 + j\omega L_1 - \frac{j}{\omega C_1}\right) - i_2 j\omega M$$

$$0 = -j\omega M i_1 + i_2\left(R_2 + j\omega L_2 - \frac{j}{\omega C_2}\right)$$

From which

$$i_2 = \frac{V_{\text{in}} j\omega M}{\left[(R_s + R_1) + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)\right]\left[R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right)\right] + \omega^2 M^2}$$

also $$\omega_r = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}} \text{ at resonance}$$

$$v_o = \frac{V_{\text{in}} M}{C_2\left[(R_s + R_1)R_2 + \omega_r^2 M^2\right]}$$

or $$v_o = A v_{in}$$

where A is the amplification factor given by

$$A = \frac{M}{C_2\left[(R_1 + R_s)R_2 + \omega_r^2 M^2\right]}$$

The maximum amplification or the maximum output voltage can be obtained by taking the first derivative of v_o with respect to M, and equating it to zero.

$$\therefore \qquad \frac{dV_o}{dM} = 0, \text{ or } \frac{dA}{dM} = 0$$

$$\frac{dA}{dM} = (R_1 + R_s)R_2 + \omega_r^2 M^2 - 2M^2\omega_r^2 = 0$$

$$\omega_r^2 M^2 = R_2(R_1 + R_s)$$

$$M_c = \frac{\sqrt{R_2(R_1 + R_s)}}{\omega_r}$$

where M_c is the critical value of mutual inductance. Substituting the value of M_c in the equation of v_o, we obtain the maximum output voltage as

$$|v_o| = \frac{V_{\text{in}}}{2\omega_r^2 C_2 M_c}$$

$$= \frac{V_{\text{in}}}{2\omega_r C_2\sqrt{R_2(R_1 + R_s)}}$$

and $$|i_2| = \frac{V_{\text{in}}}{2\omega_r M_c} = \frac{V_{\text{in}}}{2\sqrt{R_2(R_1 + R_s)}}$$

By definition, $M = K\sqrt{L_1 L_2}$, the coefficient of coupling, K at $M = M_c$ is called the critical coefficient of coupling, and is given by $K_c = M_c/\sqrt{L_2 L_1}$.

The critical coupling causes the secondary current to have the maximum possible value. At resonance, the maximum value of amplification is obtained by changing M, or by changing the coupling coefficient for a given value of L_1 and L_2. The variation of output voltage with frequency for different coupling coefficients is shown in Fig. 4.56.

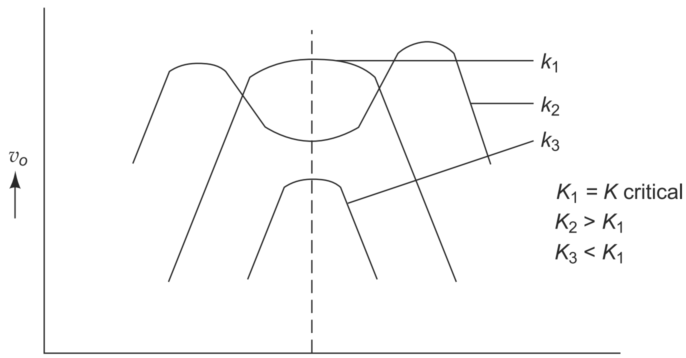


Fig. 4.56

Example 4.41 *The tuned frequency of a double tuned circuit shown in Fig. 4.57 is 10^4 rad/sec. If the source voltage is 2 V and has a resistance of 0.1Ω; calculate the maximum output voltage at resonance if $R_1 = 0.01\ \Omega$, $L_1 = 2\ \mu H$; $R_2 = 0.1\Omega$, and $L_2 = 25\ \mu H$.*

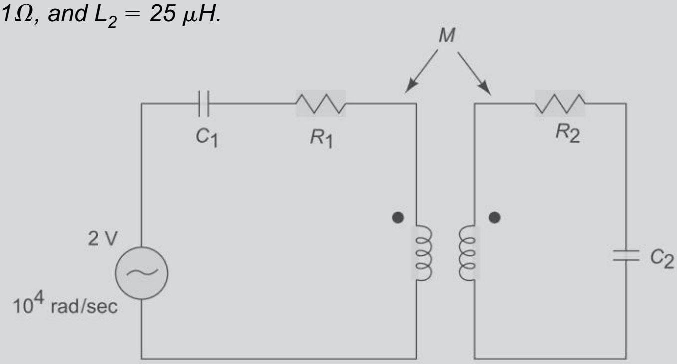


Fig. 4.57

Solution The maximum output voltage $v_o = \dfrac{v_i}{2\omega_r^2 C_2 M_c}$

where M_c is the critical value of the mutual inductance given by

$$M_c = \frac{\sqrt{R_2(R_1 + R_s)}}{\omega_r}$$

$$M_c = \frac{\sqrt{0.1(0.01 + 0.1)}}{10^4} = 10.48\ \mu\text{H}$$

At resonance

$$\omega_r^2 = \frac{1}{L_2 C_2}$$

$$C_2 = \frac{1}{\omega_r^2 L_2} = \frac{1}{(10^4)^2 \times 25 \times 10^{-6}} = 0.4 \times 10^{-3}\ \text{F}$$

$$v_0 = \frac{2}{2(10^4)^2 \times 0.4 \times 10^{-3} \times 10.48 \times 10^{-6}}$$

$$= 2.385\ \text{V}$$

4.7 ANALYSIS OF MAGNETIC CIRCUITS

4.7.1 Analysis of Magnetic Circuits

A series magnetic circuit is analogous to a series electric circuit. Kirchhoff's laws are applicable to magnetic circuits also. Consider a ring specimen having a magnetic path of l meters, area of cross-section $(A)\,m^2$ with a mean radius of R meters having a coil of N turns carrying I amperes wound uniformly as shown in Fig. 4.58. MMF is responsible for the establishment of flux in the magnetic medium. This mmf acts along the magnetic lines of force. The flux produced by the circuit is given by

$$\phi = \frac{\text{MMF}}{\text{Reluctance}}$$

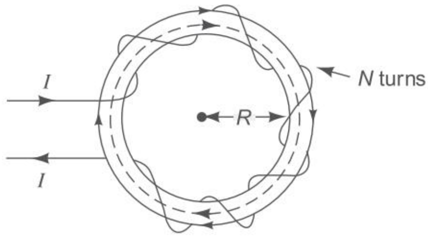


Fig. 4.58

The magnetic field intensity of the ring is given by $H = \frac{\text{mmf}}{l} = \frac{NI}{l} = \text{AT/m}$

where l is the mean length of the magnetic path and is given by $2\pi R$.

Flux density $B = \mu_o \mu_r H = \mu_o \mu_r \frac{NI}{l} \text{Wb/m}^2$

$$\text{Flux } \phi = \mu HA \text{ Webers}$$

$$= \mu_0 \mu_r \frac{NI}{l} \times A \text{ Wb}$$

$$\phi = \frac{NI}{l/\mu_o \mu_r A} \text{ Wb}$$

NI is the mmf of the magnetic circuit, which is analogous to emf in electric circuit. $l/\mu_o \mu_r A$ is the reluctance of the magnetic circuit which is analogous to resistance in electric circuit.

Example 4.42 *A circular ring consists of semicircular sections or cast Iron and cast steel with an air gap of 0.25 mm as shown in the Fig. 4.59. The mean path of the Iron and steel are 0.35 m and 0.25 m respectively. Find the ampere-turns required to produce a flux of 5 mwB. The relative permeability's of iron and steel are 170 and 800 respectively. The cross sectional area of the ring is 5 cm².*

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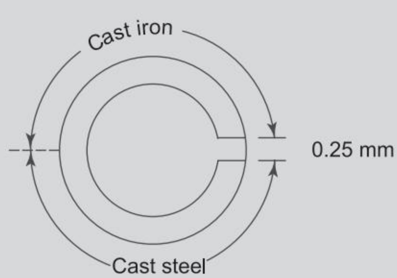


Fig. 4.59

Solution $B = \frac{\phi}{A} = \frac{5 \times 10^{-3}}{10 \times 10^{-4}} = 5\ wb/m^2$

Air gap: $H = \frac{B}{\mu_o} = \frac{5}{4\pi \times 10^{-7}} = \text{AT/m}$

$$AT_g = \frac{5}{4\pi \times 10^{-7}} \times 0.25 \times 10^{-3} \text{ AT}$$

Cast Iron path: $H = \frac{B}{\mu_o \mu_r} = \frac{5}{4\pi \times 10^{-7} \times 170}$

$$\text{Ampere turns for Iron path} = \frac{5}{4\pi \times 10^{-7} \times 170} \times 0.35 \text{ AT}$$

$$\text{Cast steel path:} \quad H = \frac{5}{4\pi \times 10^{-7} \times 180} \times 0.25 \text{ AT}$$

$$\text{Total Ampere turns} = \frac{5}{4\pi \times 10^{-7}}\left[0.25 \times 10^{-3} + \frac{0.35}{170} + \frac{0.25}{800}\right]$$

$$= 1044 \text{ AT}$$

4.7.2 Magnetic Leakage And Fringing

Figure 4.60 shows a magnetised iron ring with a narrow air gap, and the flux which crosses the gap can be regarded as useful flux. Some of the total flux produced by the ring does not cross the air gap, but instead takes a shorter route, as shown in Fig. 4.60 and is known as *leakage flux*. The flux while crossing the air gap bulges outwards due to variation in reluctance. This is known as *fringing*. This is because the lines of force repel each other when passing through the air as a result the flux density in the air gap decreases. For the purpose of calculation it is assumed that the iron carries the whole of the total flux throughout its length. The ratio of total flux to useful flux is called the *leakage coefficient* or leakage factor.

Leakage factor = Total flux/useful flux.

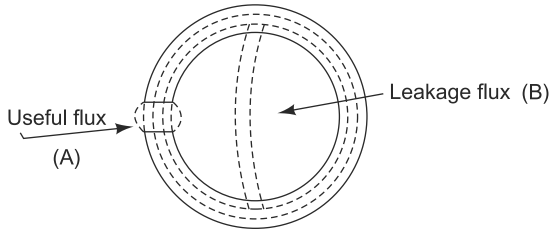


Fig. 4.60

Example 4.43 *A coil of 100 turns is wound uniformly over a insulator ring with a mean circumference of 2 m and a uniform sectional area of 0.025 cm². If the coil is carrying a current of 2 A. Calculate (a) the mmf of the circuit, (b) magnetic field intensity (c) flux density (d) the total flux.*

Solution (a) mmf = NI = 100 × 2 = 2000 AT

(b) $H = \frac{\text{mmf}}{l} = \frac{2000}{2} = 1000 \text{ AT/m}$

(c) $B = \mu_0 H = 4\pi \times 10^{-7} \times 1000 = 1.2565 \text{ mWb/m}^2$

(d) $\phi = B \times A = 1.2565 \times 10^{-3} \times 0.025 \times 10^{-4} = 0.00314 \times 10^{-6}$ Wb.

Example 4.44 *Calculate the mmf required to produce a flux of 5 mWb across an air gap of 2.5 mm of length having an effective area of 100 cm² of a cast steel ring of mean iron path of 0.5 m and cross-sectional area of 150 cm² as shown in Fig. 4.61. The relative permeability of the cast steel is 800. Neglect leakage flux.*

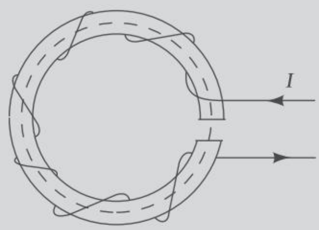


Fig. 4.61

Solution Area of the gap = 100×10^{-4} m²

$$\text{Flux density of the gap} = \frac{5\times10^{-3}\times10^{4}}{100} = 0.5 \text{ T}$$

$$H \text{ of the gap} = \frac{B}{\mu_0} = \frac{0.5}{4\pi\times10^{-7}}$$

$$= 0.39 \times 10^{6} \text{ A/m}$$

Length of the gap = 2.5×10^{-3} m

mmf required for the gap = $0.39 \times 10^{6} \times 2.5 \times 10^{-3}$ = 975 AT

$$\text{Flux density in the cast steel ring is} = \frac{\phi}{\text{Area}}$$

$$= \frac{5\times10^{-3}\times10^{4}}{100}$$

$$= 0.333 \text{ T}$$

$$\therefore \quad H = \frac{B}{\mu_0\mu_r} = \frac{0.333}{4\pi10^{-7}\times800} = 332 \text{ AT/m}$$

Length of the cast steel path = 0.5 m

The required mmf for the cast steel to produce the necessary flux = 0.5×332 = 166 AT

Therefore total mmf = 975 + 166 = 1141 AT.

4.7.3 Composite Magnetic Circuit

Consider a toroid composed of three different magnetic materials of different permeabilities, areas and lengths excited by a coil of N turns.

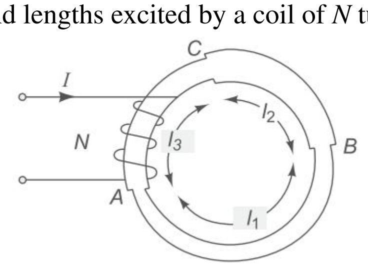


Fig. 4.62

With a current of I amperes as shown in Fig. 4.62. The lengths of sections AB, BC and CA are l_1, l_2 and l_3 respectively. Each section will have its own reluctance and permeability. Since all of them are joined in series, the total reluctance of the combined magnetic circuit is given by

$$\Re_{\text{Total}} = \frac{1}{\mu A}$$

$$= \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3}$$

The flux produced in the circuit is given by

$$\phi = \frac{\text{mmf}}{\text{Total reluctance}} \text{ Wb}$$

$$\phi = \left[\frac{NI}{\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3}} \right] \text{Wb}$$

Example 4.45 *A circular ring having a cross-sectional area of 5 cm² and a length of 4π cm in iron has an air gap of 0.1π cm made as a saw cut. The relative permeability of iron is 800. The ring is wound with a coil of 2000 turns and carries a current of 100 mA. Determine the air gap flux. Neglect leakage and fringing.*

Solution Cross section area of the Iron ring, $I_i = 5 \times 10^{-4}$ m²

Length of iron ring $l_i = 4\pi \times 10^{-2}$ m

Length of air gap, $lg = 0.1\pi \times 10^{-2}$ m

$\mu_r = 800$

No. of turns, $N = 2000$

$i = 100$ mA

Total ampere turns $(MMF) = NX_i$

$= 2000 \times 100 \times 10^{-3}$

$= 200$ AT

$$\text{Total reluctance } R = \frac{l_i}{a_i \mu_0 \mu_r} + \frac{lg}{ag\,\mu_0}$$

$$= \frac{4\pi}{5\times10^{-4} \times 4\pi \times 10^{-7} \times 800} + \frac{0.1\pi \times 10^{-2}}{5\times10^{-4} \times 4\pi \times 10^{-7}}$$

$$\text{Air gap flux} = \frac{\text{Total MMF}}{\text{Reluctance}} = \frac{200}{5.25\times10^{-6}}$$

$$\phi_g = 38\ \mu wb.$$

Example 4.46 *An iron ring 10 cm dia and 15 cm² in cross-section is wound with 250 turns of wire for a flux density of 1.5 Web/m² and permeability 500. Find the exciting current, the inductance and stored energy. Find corresponding quantities when there is a 2 mm air gap.*

Solution (a) *Without air gap*

$$\text{Length of the flux path} = \pi D = \pi \times 10 = 31.41\,\text{cm} = 0.3141\text{ m}$$

$$\text{Area of flux path} = 15\,\text{cm}^2 = 15 \times 10^{-4}\text{ m}^2$$

$$\text{mmf} = \text{A.T}$$

$$A = \frac{\text{mmf}}{T}$$

$$H = \frac{B}{\mu_0 \mu_r} = \frac{15}{4\pi \times 10^{-7} \times 500} = 2387$$

$$\text{mmf} = H \times l = 2387 \times 0.3141 = 750\text{ A}$$

$$\text{Exciting current} = \frac{\text{mmf}}{T} = \frac{750}{250} = 3\text{ A}$$

$$\text{Reluctance} = \frac{l}{\mu_0 \mu_r A} = \frac{0.3141}{4\pi 10^{-7} \times 500 \times 15 \times 10^{-4}}$$

$$= 333270$$

$$\text{Self Inductance} = \frac{N^2}{\text{Reluctance}} = \frac{(250)^2}{333270} = 0.1875\text{ H}$$

$$\text{Energy} = \frac{1}{2} L I^2 = \frac{1}{2} \times 0.1875 \times (3)^2$$

$$= 0.843\text{ Joules}$$

(b) *With air gap*

$$\text{Reluctance of the gap} = \frac{l}{\mu_0 A} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 15 \times 10^{-4}}$$

$$= 1.06 \times 10^6\text{ A/Wb}$$

$$\text{Total reluctance} = (0.333 + 1.06)\,10^6 = 1.393 \times 10^6\text{ A/Wb}$$

$$\text{mmf} = \phi \times \text{reluctance}$$

$$= 1.5 \times 15 \times 10^{-4} \times 1.393 \times 10^6$$

$$= 3134\text{ AT}$$

$$\text{Exciting current} = \frac{3134}{250} = 12.536\text{ A}$$

$$L = \frac{N^2}{\Re} = \frac{(250)^2}{1.393 \times 10^6} = 44.8 \text{ mH}$$

$$\text{Energy } = \frac{1}{2} L I^2$$

$$= \frac{1}{2} \times 44.8 \times 10^{-3} \times (12.536)^2$$

$$= 3.52 \text{ Joules}$$

Example 4.47 *A 700 turn coil is wound on the central limb of the cast steel frame as shown in Fig. 4.63. A total flux of 1.8 m Wb is required in the gap. What is the current required? Assume that the gap density is uniform and that all lines pass straight across the gap. All dimensions are in centimeters. Assume μ_r as 600.*

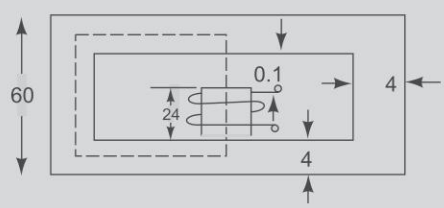


Fig. 4.63

Solution Each of the side limbs carry half the total flux as their reluctances are equal. Total mmf required is equal to the sum of the mmf required for gap, central limb and side limb.

Reluctance of gap and central limb are in series and they carry the same flux.

Air gap

$$\phi_g = 1.8 \times 10^{-3} \text{ Wb}$$

$$A_g = 4 \times 4 \times 10^{-4} \text{ m}^2$$

$$B_c = \frac{1.8 \times 10^{-3}}{16 \times 10^{-4}} = 1.125 \text{ Wb/m}^2$$

$$H_c = \frac{B_c}{\mu_0} = \frac{1.125}{4\pi \times 10^{-7}} = 8.95 \times 10^5 \text{ AT/m}$$

$$\text{Required mmf for the gap } = H_g l_g$$

$$= 8.95 \times 10^5 \times 0.001 = 895 \text{ AT}$$

Central Limb

$$\phi_c = 1.8 \times 10^{-3} \text{ Wb}$$

$$A_c = 4 \times 4 \times 10^{-4} \text{ m}^2$$

$$B_c = 1.125 \text{ Wb/m}^2$$

$$H_c = \frac{B_c}{\mu_0 \mu_r} = \frac{1.125}{4\pi \times 10^{-7} \times 600} = 1492 \text{ AT/m}$$

Required mmf for central limb = $H_c l_c$

$= 1492 \times 0.24 = 358$ AT

Side Limb

$$\phi_s = \frac{1}{2} \times \text{flux in central limb} = \frac{1}{2} \times 1.8 \times 10^{-3} = 0.9 \times 10^{-3} \text{ Wb}$$

$$B_s = \frac{0.9 \times 10^{-3}}{16 \times 10^{-4}} = 0.5625 \text{ Wb/m}^2$$

$$H_s = \frac{B_s}{\mu_0 \mu_r} = \frac{0.5625}{4\pi \times 10^{-7} \times 600} = 746 \text{ AT/m}$$

Required mmf for side limb = $H_s\, l_s$

$= 746 \times 0.6 = 447.6 + 448$

Total mmf = 895 + 358 + 448 = 1701 AT

$$\text{Required current} = \frac{1701}{700} = 2.43 \text{ A}$$

Example 4.48 *A cast steel ring has a circular cross section 3 cm in diameter and a mean circumference of 80 cm. The ring is uniformly wound with 600 turns.*

(i) Estimate the current required to produce a flux of 0.5 mcob in the ring.

(ii) If a 2-mm wide saw cut is made in the ring, find approximately the flux produced by the current found (i).

(iii) Find the current value which will give the same flux as in (i). Assume the gap density to be the same as in the iron and neglect fringing.

[JNTU June 2006]

Solution (i) Length of the flux path = Mean circumference

$= 80 \times 10^{-2}$ m

$$A = \text{area} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(3 \times 10^{-2})^2 = 7.068 \times 10^{-4} \text{m}^2$$

$$\text{H} = \frac{B}{\mu_0 \mu_r} = \frac{\phi}{A \times \mu_0 \mu_r}$$

ϕ = *flux* = 0.5 *m Wb*

$\mu_0 = 4\pi \times 10^{-7}$

μ_r = 600 for cast steel iron

$$H = \frac{0.5\times10^{-3}}{7.068\times10^{-4}\times p\times10^{-7}\times60}$$
$$= 9382.36$$

$$mmf = H \times l$$
$$= 9382.36 \times 80 \times 10^{-2}$$
$$= 7505.89 \text{ AT}$$
$$N = \text{no. of turns} = 600$$

$$\therefore \quad \text{exciting current} = \frac{mmf}{N}$$
$$= \frac{7505.89}{600}$$
$$= 12.5 \text{ A}$$

$$\therefore \quad i = 12.5 \text{ A}$$

(ii) $$\text{Reluctance} = \frac{l}{\pi_0 \pi_r A} = \frac{80\times10^{-12}}{4\pi\times10^{-17}\times600\times7.068\times10^{-4}}$$
$$= 1.500 \times 10^6 \, A/Wb$$

$$\text{Reluctance of air gap} = \frac{1}{\mu_0 A}$$
$$= \frac{3\times10^{-3}}{4\pi\times10^{-7}\times7.068\times10^{-4}}$$
$$= 2.25 \times 10^6 \text{ A/Wb}$$

$$\text{Total reluctance} = (1.5 + 2.25)10^6$$
$$= 3.75 \times 10^6 \text{ A/Wb}$$
$$mmf = \varphi \times \text{reluctance}$$
$$\varphi = \frac{7505.89}{3.75\times10^6} = 2 \text{ mWb}$$

(iii) For $\varphi = 0.5 \, mWb$

$$\text{Total reluctance} = 3.75 \times 10^6 \text{ A/Wb}$$
$$mmf = \varphi \times \text{reluctance}$$
$$= 0.5 \times 10^{-3} \times 3.75 \times 10^6$$
$$= 1.875 \times 10^3$$
$$= 1875 \text{ AT}$$

$$\text{Exciting current} = \frac{mmf}{no. of\ turns}$$

$$\text{no. of turns} = 600$$

$$\therefore \quad \text{exciting current} = \frac{1875}{600}$$

$$= 3.125\,A.$$

Example 4.49 *A coil of 500 turns is wound uniformly over a wooden ring having a mean circumference of 50 cm and a cross sectional area of 500 mm². If the current through the coil is 3 Amps, calculate*

(i) The magnetic field strength

(ii) The flux density and

(iii) The total flux. *[JNTU June 2006]*

Solution Given $N = 500, I = 3\text{A}$

$$A = 500 \times 10^{-6}\ \text{m}^2$$

Mean circumference (Magnetic path)

$$l = 50 \times 10^{-2}\ \text{m}$$

(i) $H = \dfrac{mmt}{l}$

But $\text{mmf} = NI = 1500\ \text{AT}$

and $l = 50 \times 10^{-2}$

Magnetic field strength, $\text{H} = 3000\ \text{AT/m}$

(ii) $B = \mu_0 m = 4\pi \times 10^{-7} \times 3000 = 3.769\ \text{mwb/m}^2$

$\therefore$ Flux density $(B) = 3.769\ \text{mwb/m}^2$

(iii) $\phi = B \times A = 3.769 \times 10^{-3} \times 500 \times 10^{-6}$

$$= 1.8845 \times 10^{-6}\ wb$$

$\therefore$ Total flux $(\phi) = 1.8845 \times 10^{-6}\ wb$

Example 4.50 *Two coils having 30 and 600 turns are wound side by side on a closed iron circuit of 100 cm² cross section and mean length 150 cm. Calculate:*

(i) The self-inductance of the two coils and mutual inductance if relative permeability of iron is 2000. Assume no magnetic leakage.

(ii) 0 to 10 A steadily in 0.01 sec *[JNTU May 2007]*

Solution $N_1 = 30,\ a = 100\ \text{cm}^2 = 10 \times 10^{-4}\ \text{m}^2,\ N_2 = 600$

$l = 150\,\text{cm} = 1.5\ \text{m},\ \mu_r = 2000$

$$\text{Reluctance} = \frac{l}{\mu_0 \mu_r \cdot a}$$

$$= \frac{1.5}{4\pi \times 10^{-7} \times 2000 \times 100 \times 10^{-4}}$$

$$= 0.05968 \times 10^6$$

$$L_1 = \frac{N_1^2}{\text{Reluctance}} = \frac{(30)^2}{0.05968 \times 10^6}$$

$$= 15 \text{ mH}$$

$$L_2 = \frac{N_2^2}{\text{Reluctance}}$$

$$= \frac{(600)^2}{0.05968 \times 10^6}$$

$$= 6 \text{ H}$$

If there is no magnetic leakage

$$\mu = \sqrt{L_1 L_2} = \sqrt{15 \times 10^{-3} \times 6} = 0.3 \text{ H}$$

Example 4.51 *A coil of 500 turns is wound uniformly over a wooden ring having a mean circumference of 50 cm and a cross-sectional area of 500 mm². If the current through the coil is 3 A, calculate*

(i) the magnetic field strength
(ii) the flux density, and
(iii) the total flux. *[JNTU May 2007]*

Solution No. of turns = 500 (n)

Mean circumference = 50 cm (l)
Cross-sectional area = 500 mm² (A)

Current through the coil = 3A (I)

(i) Magnetic field strength, $\text{H} = \frac{mmf}{l}$ where $l \rightarrow$ circumference of ring and $mmf = NI$

$$H = \frac{NI}{l} = \frac{500 \times 3}{50 \times 10^{-2}} = 3000 \text{ AT/m}$$

(ii) The flux density

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 3000 = 3.77 \text{ m wb/m}^2.$$

(iii) The total flux

$$\phi = B \times A = 500 \times 10^{-6} \times 3.77 \times 10^{-3}$$

$$= 1.885 \times 10^{-6} \text{ wb.}$$

Example 4.52 *An iron ring has a mean diameter of 25 cm and an area of c.s. of 5 cm² and is wound with a coil of 1000 turns. Determine the current in the coil to establish a flux density of 0.8 wb/m² in the ring. Take the relative permeability of iron as 500. In case if an iron gap of 2 mm is cut in the ring, what is the current in the coil to establish the same flux density?* *[JNTU May 2007]*

Solution Mean diameter of ring = 25 cm (D)

$\therefore$ Circumference of ring $= \pi D = \pi \times 25 = 78.054$ cm (5.55)

Flux density, $B = 0.8\ Wb/m^2$

$$B = \mu_0 \mu_r H = \mu_0 \mu_r \frac{NI}{l}$$

$$\Rightarrow I = \frac{BI}{\mu_0 \mu_r N} = \frac{0.8 \times 78.54 \times 10^{-2}}{4\pi \times 10^{-7} \times 500 \times 1000} = 1\text{ A}$$

Since *a* 2-mm iron gap is cut and 50 *l* becomes

$$l = 78.54\text{ cm} - 2\text{ mm} = 0.7834$$

$$I = \frac{Bl}{\mu_0 \mu_r N} = \frac{0.8 \times 78.34 \times 10^{-2}}{4\pi \times 10^{-7} \times 500 \times 1000} = 0.997\text{ A}$$

Example 4.53 *Two coils A & B are wound on same Ferromagnetic core. There are 300 turns on A and 2800 turns on B. A current of 4 A through coil A produces a flux of 800 μwb in the core. If this current is reversed in 20 ms, find the average emf induced in coils A and B.* *[JNTU June 2009]*

Solution

Coil A	Coil B
$N = 300$ T	$N = 2800$ T
$I = 4$A	
$\phi = 800\ \mu$wb	
$T = 20$ ms	

$$\text{Induced Emf in coil } A = N_A \frac{d\phi}{dt}$$

$$= 300 \times \frac{800 \times 10^{-6} - (-800 \times 10^{-6})}{20 \times 10^{-3}}$$

$$= 48\text{ V}$$

$$\text{Induced Emf in Coil B} = N_B \frac{d\phi}{dt}$$

$$= 2800 \times 0.16$$
$$= 448\text{ V}$$

Example 4.54 *A torroid is made of steel rod of 2 cm diameter. The mean radius of torroid is 20 cm relative permeability of steel is 2000. Compute the current required to produce 1 m web of flux and 1000 turns in the torroid.*

Solution Length of the flux path $= \pi D = \pi \times 20 = 62.83$ cm $= 0.6283$ m

$$\text{Area of flux path} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2)^2 = 3.141 \text{ cm}^2$$

$$\text{Magnetic field intensity } H = \frac{B}{\mu_o \mu_r}$$

$$B = \frac{\phi}{\text{Area}} = \frac{10^{-3}}{3.141\times10^{-4}} 3.1 \text{ web/m}^2$$

$$H = \frac{3.1}{4\pi\times10^{-7}\times2000} = 1233.45 \text{ AT/m}$$

$$mmf = H \times l = 1233.45 \times 0.6283$$

$$= 775 \text{ A.T.}$$

$$\text{Exciting current} = \frac{mmf}{T}$$

$$\frac{775}{1000} = 0.775\text{A}$$

Example 4.55 *An iron ring of mean length 50 cm has an air gap of 1 mm and a winding of 200 turns. If the permeability of iron is 400 when a current of 1.25 A flows through the coil. Find the flux density.*

Solution AT_1 required for iron path in the ring $= H_i \times l_i = \dfrac{B}{\mu_o \mu_r} \times l_i$

$$= \frac{B}{4\pi\times10^{-7}\times400}\times0.5$$

AT_2 required for air gap of 1 mm $\quad H_g l_g = \dfrac{B}{\mu_o}\times l_g$

$$= \frac{B}{4\pi\times10^{-7}}\times1\times10^{-3}$$

Total ampere turns $= AT_1 + AT_2$

$$200\times1.25 = \left[\frac{B\times0.5}{4\pi\times10^{-7}\times400} + \frac{B}{4\pi\times10^{-7}}\times10^{-3}\right]$$

$$250 = \frac{B}{4\pi\times10^{-7}}[1.25\times10^{-3} + 10^{-3}]$$

$$B = 0.314 \text{ web/m}^2$$

Example 4.56 *An iron ring 15 cms in diameter and 10 cm² in area of cross section is wound with a coil of 200 turns. Determine the current in the coil to establish a flux density of 1 wb/m² if the relative permeability of iron is 500. In case if an air gap of 2 mm is cut in the ring, what is the current in the coil to establish the same flux density.*

Solution (i) *Without air gap*

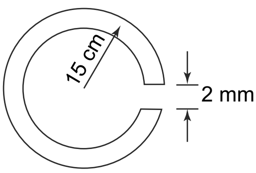


Fig. 4.64

Diameter of Iron ring = 15 (cm) = 15×10^{-2} m

Area of Iron ring = 10 cm² = 10×10^{-4} m²

Number of turns (N) = 200

$$\text{Reluctance of Iron ring } (\Re_i) = \frac{l_i}{\mu_o \mu_r . A}$$

$$\text{Length of Iron path } (l_i) = \pi . d = \pi \times 15 \times 10^{-2} \text{ m}$$

$$(\Re_i) = \frac{15\pi \times 10^{-2}}{4\pi \times 10^{-7} \times 500 \times 10 \times 10^{-4}} = 7.5 \times 105 \text{ AT/Wb}$$

mmf = Flux × reluctance

$I \times 200 = B.A.\ \Re i$

$$I = \frac{1 \times 10 \times 10^{-4} \times 7.5 \times 10^{5}}{200} = 3.75\text{A}$$

(ii) *With 2 mm air gap cut in the iron ring reluctance of air gap*

$$(\Re_g) = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$$

$$= 15.915 \times 10^5 \text{ AT/Wb}$$

With 2 mm air gap the length of the Iron path is reduced by 2 mm.

$\therefore$ $l_i = 15\pi \times 10^{-2} - 2 \times 10^{-3}$

But this is negligibly small.

$\therefore$ Total reluctance = $\Re_i + \Re g = 23.415 \times 10^5$ AT/Wb

$$\therefore \quad I = \frac{\phi \Re}{N} = \frac{B.A.\Re}{N}$$

$$= \frac{1 \times 10 \times 10^{-4} \times 23.415 \times 10^{5}}{200}$$

Required current (I) = 11.707A

If the gap length is taken into consideration:

$$\text{Total emf} = \frac{B_i l_i}{\mu_o \mu_r} + \frac{B_i l_g}{\mu_o}$$

$$= \frac{1(\pi \times 15 \times 10^{-2} - 2 \times 10^{-3})}{4\pi \times 10^{-7} \times 500} \frac{1 \times 2 \times 10^{-3}}{4\pi \times 10^{-7}} = 338.35 \text{ AT}$$

$$\therefore \qquad I = \frac{2338.35}{200} = 11.691\text{A}$$

Example 4.57 *An iron ring of cross sectional area 800 m² and of mean radius 170 mm has two windings connected in series, one of 500 turns and the other of 700 turns. If the relative permeability of iron is 1200 find,*

(a) The self inductance of each coil.

(b) The mutual inductance, assume that there is no leakage. Derive the formulae used. *[JNTU Jan 2010]*

Solution $A = 800 \text{ m}^2$ $\quad N_1 = 500$ $\quad N_2 = 700$ $\quad r = 170$ mm

$C = l = 2\pi r$ $\quad \mu r = 1200$

(a) Reluctance (S), $\dfrac{l}{\mu_0 \mu_r A} = \dfrac{2\pi \times 170 \times 10^{-3}}{4\pi \times 10^{-7} \times 1200 \times 800}$

$= 0.8854$

$$\text{Self Inductance } (L) = \frac{N^2}{S} = \frac{(500)^2}{0.8854}$$

$$h = 282.35 \times 10^3 \text{ H}$$

(b) Mutual Inductance

$$m = \frac{N_1 N_2}{S} = \frac{500 \times 700}{0.8854}$$

$$m \Rightarrow 553.422 \times 10^3 \text{ H}$$

4.7.4 Analysis of Parallel Magnetic Circuits

We have seen that a series magnetic circuit carries the same flux and the total mmf required to produce a given quantity of flux is the sum of the mmf's for the separate parts. In a parallel magnetic circuit, different parts of the circuit are in parallel. For such circuits the Kirchhoff's laws, in their analogous magnetic form can be applied for the analysis. Consider an iron core having three limbs *A*, *B* and *C* as shown in Fig. 4.65 (a). *A* Coil with *N* turns is arranged around limb *A* which carries a current *I* amperes. The flux produced by the coil in limb *A*. ϕ_A is divided between limbs *B* and *C* and each equal to $\phi_A/2$. The reluctance offered by the two parallel paths is equal to the half the reluctance of each path (Assuming equal lengths and cross sectional areas). Similar to Kirchhoff's current law in an electric circuit, the total magnetic flux directed towards a junction in a magnetic circuit is equal to the sum of the magnetic fluxes directed away from that junction. Accordingly $\phi_A = \phi_B + \phi_C$ or $\phi_A - \phi_B - \phi_C = 0$. The electrical equivalent of the above circuit is shown in Fig. 4.65 (b). Similar to Kirchhoff's second law, in a closed magnetic circuit, the resultant mmf is equal to the algebraic sum of the products of field strength and the length of each part in the closed path. Thus applying the law to the first loop in Fig. 4.65 (a), we get

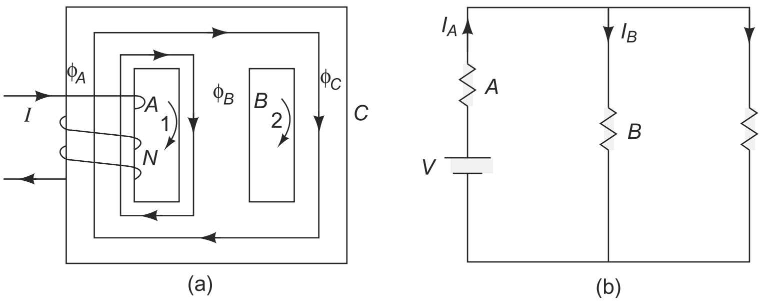


Fig. 4.65

$$NI = H_A\, l_A + H_B\, l_B$$

or $$NI = \phi_A \Re_A + \phi_B \Re_B$$

The mmf across the two parallel paths is identical.
Therefore *NI* is also equal to

$$NI = \phi_A\, \Re_A + \phi_C \Re_C$$

Practice Problems

4.1 Using the dot convention, write the voltage equations for the coils shown in Fig. 4.66.

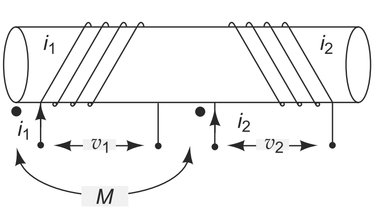


Fig. 4.66

4.2 Two inductively coupled coils have self inductances $L_1 = 40$ mH and $L_2 = 150$ mH. If the coefficient of coupling is 0.7, (i) find the value of mutual inductance between the coils, and (ii) the maximum possible mutual inductance.

4.3 For the circuit shown in Fig. 4.67 write the inductance matrix.

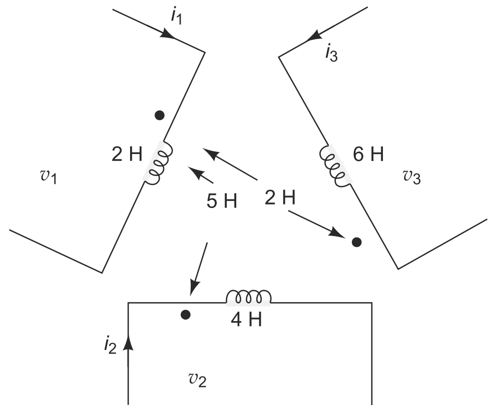


Fig. 4.67

4.4 Two coils connected in series have an equivalent inductance of 0.8 H when connected in aiding, and an equivalent inductance of 0.5 H when the connection is opposing. Calculate the mutual inductance of the coils.

4.5 In Fig. 4.68, L_1, = 2 H; L_2 = 6 H; K = 0.5; i_1 = 4 sin (40t − 30°) A; i_2 = 2 sin (40t − 30°) A. Find the values of (i) v_1, and (ii) v_2.

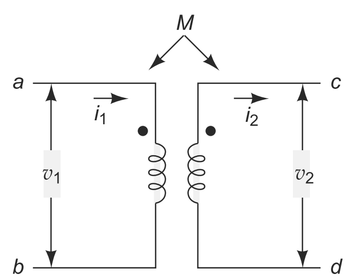


Fig. 4.68

4.6 For the circuit shown in Fig. 4.69, write the mesh equations.

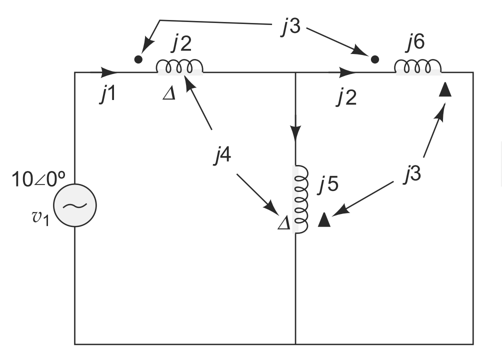


Fig. 4.69

4.7 Calculate the effective inductance of the circuit shown in Fig. 4.70 across *XY*.

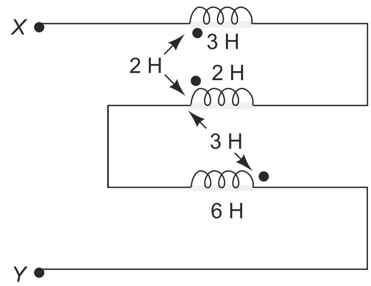


Fig. 4.70

4.8 For the circuit shown in Fig. 4.71, find the ratio of output voltage to the input voltage.

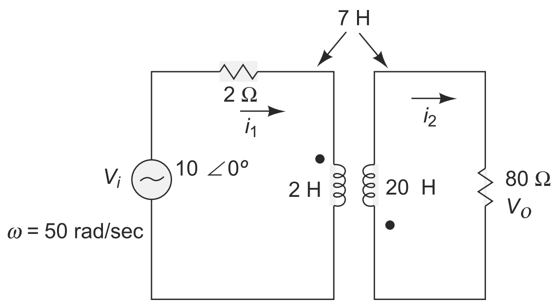


Fig. 4.71

4.9 Calculate the effective inductance of the circuit shown in Fig. 4.72.

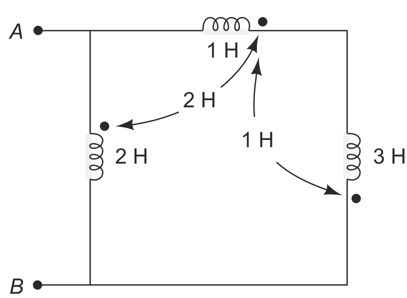


Fig. 4.72

4.10 Write the mesh equations for the network shown in Fig. 4.73.

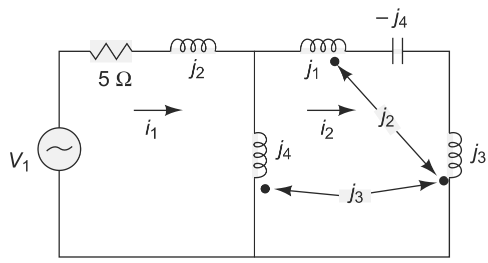


Fig. 4.73

4.11 Find the source voltage if the voltage across the 100 ohms is 50 V for the network in the Fig. 4.74.

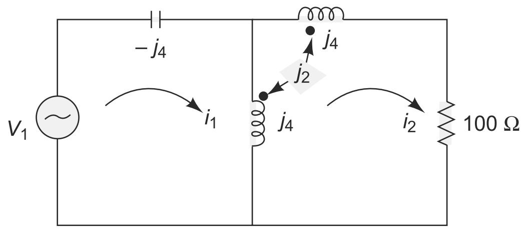


Fig. 4.74

4.12 The inductance matrix for the circuit of a three series connected coupled coils is given below. Find the inductances and indicate the dots for the coils.

$$L = \begin{bmatrix} 8 & -2 & 1 \\ -2 & 4 & -6 \\ 1 & -6 & 6 \end{bmatrix}$$

Objective Type Questions

4.1 Mutual inductance is a property associated with
(a) only one coil
(b) two or more coils
(c) two or more coils with magnetic coupling

4.2 Dot convention in coupled circuits is used
(a) to measure the mutual inductance
(b) to determine the polarity of the mutually induced voltage in coils
(c) to determine the polarity of the self induced voltage in coils

4.3 Mutually induced voltage is present independently of, and in addition to, the voltage due to self induction.
(a) true (b) false

4.4 Two terminals belonging to different coils are marked identically with dots, if for the different direction of current relative to like terminals the magnetic flux of self and mutual induction in each circuit add together.
(a) true (b) false

4.5 The maximum value of the coefficient of coupling is
(a) 100% (b) more than 100% (c) 90%

4.6 The case for which the coefficient of coupling $K = 1$ is called perfect coupling
(a) true (b) false

4.7 The maximum possible mutual inductance of two inductively coupled coils with self inductances $L_1 = 25$ mH and $L_2 = 100$ mH is given by

(a) 125 mH (b) 75 mH (c) 50 mH

4.8 The value of the coefficient of coupling is more for aircored coupled circuits compared to the iron core coupled circuits.

(a) true (b) false

4.9 Two inductors are connected as shown in Fig. 4.75. What is the value of the effective inductance of the combination.

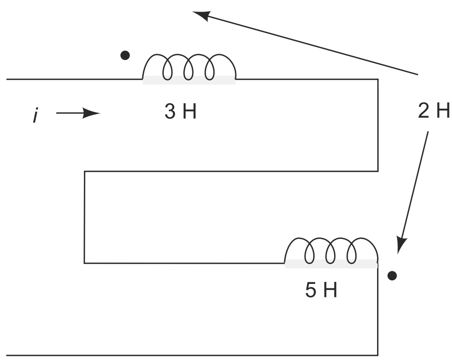


Fig. 4.75

(a) 8 H (b) 10 H (c) 4 H

4.10 Two coils connected in series have an equivalent inductance of 3 H when connected in aiding. If the self inductance of the first coil is 1 H, what is the self inductance of the second coil (Assume $M = 0.5$ H)

(a) 1 H (b) 2 H (c) 3 H

4.11 For Fig. 4.76 shown below, the inductance matrix is given by

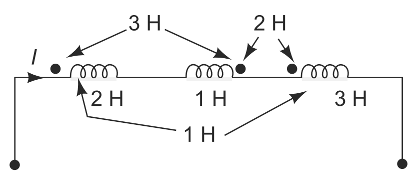


Fig. 4.76

(a) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -3 & 1 \\ -3 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$

5 Network Topology

5.1 DEFINITION OF GRAPH AND TREE

5.1.1 Graph – Planar and Non-planar Graphs *[JNTU Nov. 2011]*

A division of mathematics called topology or graph theory deals with graphs of networks and provides information that helps in the formulation of network equations. In circuit analysis, all the elements in a network must satisfy Kirchhoff's laws, besides their own characteristics. Based on these laws, we can form a number of equations. These equations can be easily written by converting the network into a graph. Certain aspects of network behaviour are brought into better perspective if a graph of the network is drawn. If each element or a branch of a network is represented on a diagram by a line irrespective of the characteristics of the elements, we get a graph. Hence, network topology is network geometry. A network is an interconnection of elements in various branches at different nodes as shown in Fig. 5.1. The corresponding graph is shown in Fig. 5.2 (a).

The graphs shown in Figs 5.2 (b) and (c) are also graphs of the network in Fig. 5.1.

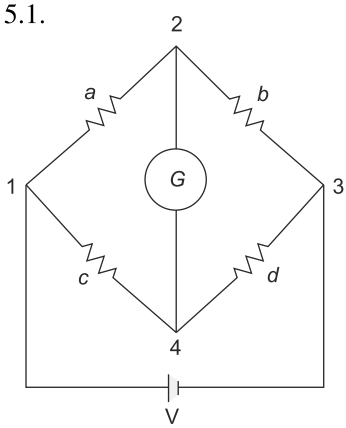


Fig. 5.1

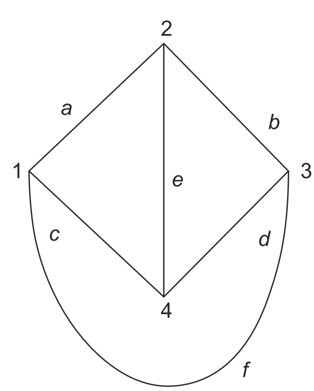


Fig. 5.2 (a)

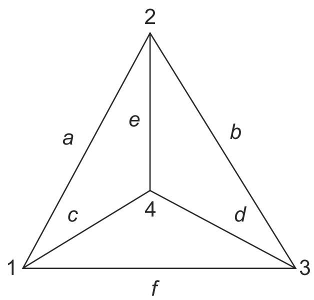


Fig. 5.2 (b)

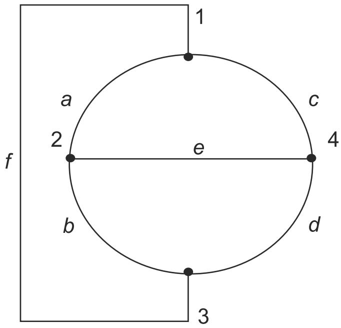


Fig. 5.2 (c)

It is interesting to note that the graphs shown in Fig. 5.2 (a), (b) and (c) may appear to be different but they are topologically equivalent. A branch is represented by a line segment connecting a pair of nodes in the graph of a network. A node is a terminal of a branch, which is represented by a point. Nodes are the end points of branches. All these graphs have identical relationships between branches and nodes.

The three graphs in Fig. 5.2 have six branches and four nodes. These graphs are also called undirected. If every branch of a graph has a *direction* as shown in Fig. 5.3, then the graph is called a *directed graph*.

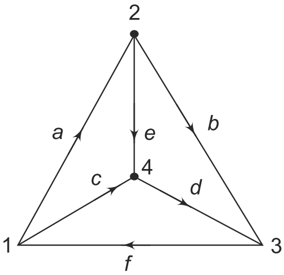


Fig. 5.3

A node and a branch are incident if the node is a terminal of the branch. Nodes can be incident to one or more elements. The number of branches incident at a node of a graph indicates the degree of the node. For example, in Fig. 5.3 the degree of node 1 is three. Similarly, the degree of node 2 is three. If each element of the connected graph is assigned a direction as shown in Fig. 5.3 it is then said to be oriented. A graph is connected if and only if there is a path between every pair of nodes. A path is said to exist between any two nodes, for example 1 and 4 of the graph in Fig. 5.3, if it is possible to reach node 4 from node 1 by traversing along any of the branches of the graph. A graph can be drawn if there exists a path between any pair of nodes. A loop exists, if there is more than one path between two nodes.

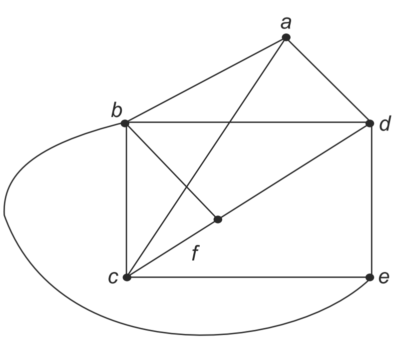


Fig. 5.4

Planar and Non-Planar Graphs

A graph is said to be planar if it can be drawn on a plane surface such that no two branches cross each other as shown in Fig 5.2. On the other hand in a non-planar graph there will be branches which are not in the same plane as others, i.e., a non-planar graph cannot be drawn on a plane surface without a crossover. Figure 5.4 illustrates a non-planar graph.

5.1.2 Tree and Co-Tree

[JNTU Nov. 2011]

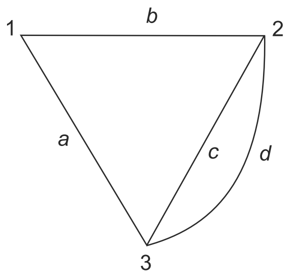


Fig. 5.5

A tree is a connected subgraph of a network which consists of all the nodes of the original graph but no closed paths. The graph of a network may have a number of trees. The number of nodes in a graph is equal to the number nodes in the tree. The number of branches in a tree is less than the number of branches in a graph. A graph is a tree if there is a unique path between any pair of nodes. Consider a graph with four branches and three nodes as shown in Fig. 5.5.

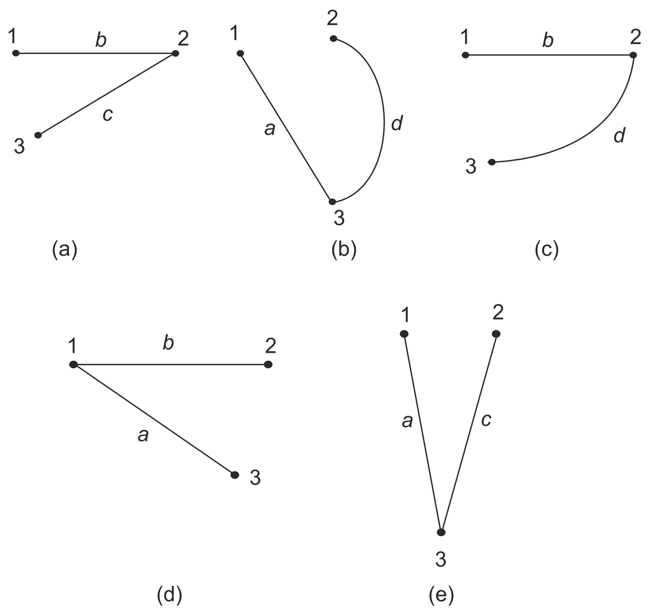


Fig. 5.6

Five open-ended graphs based on Fig. 5.5 are represented by Figs 5.6 (a) to (e). Since each of these open-ended graphs satisfies all the requirements of a tree, each graph in Fig. 5.6 is a tree corresponding to Fig. 5.5.

In Fig. 5.6, there is no closed path or loop; the number of nodes $n = 3$ is the same for the graph and its tree, where as the number of branches in the tree is only two. In general, if a tree contains n nodes, then it has $(n - 1)$ branches.

In forming a tree for a given graph, certain branches are removed or opened. The branches thus opened are called links or *link branches*. The links for Fig. 5.6 (a) for example are *a* and *d* and Fig. for 5.6 (b) are *b* and *c*. The set of all links of a given tree is called the co-tree of the graph. Obviously, the branches *a*, *d* are a co-tree for Fig. 5.6 (a) and b, c are the co-tree. Similarly, for the tree in Fig. 5.6 (b), the branches *b*, *c* are the co-tree. Thus the link branches and the tree branches combine to form the graph of the entire network.

Example 5.1 *For the given graph shown in Fig. 5.7 draw the number of possible trees.*

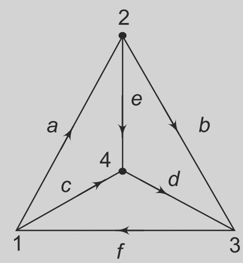


Fig. 5.7

Solution The number of possible trees for Fig. 5.7 are represented by Figs 5.8 (a) – (l).

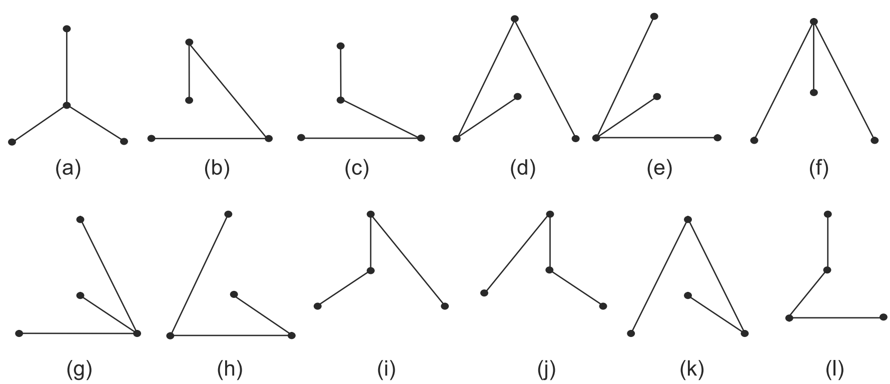


Fig. 5.8

Example 5.2 *For the network shown in Fig. 5.9 draw oriented graph and draw all possible trees.*

[JNTU Jan 2010]

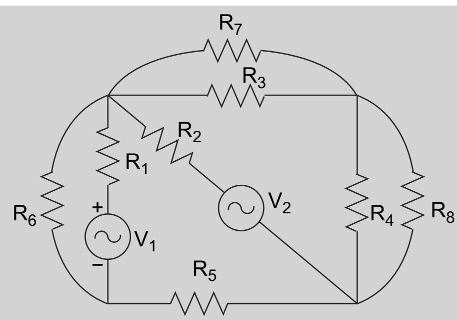
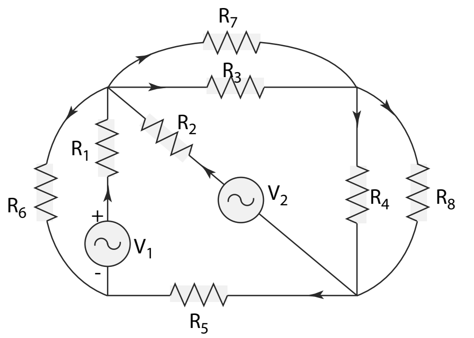


Fig. 5.9



Solution Short circuiting the voltage sources, we obtain the oriented graph as, we obtain the trees as

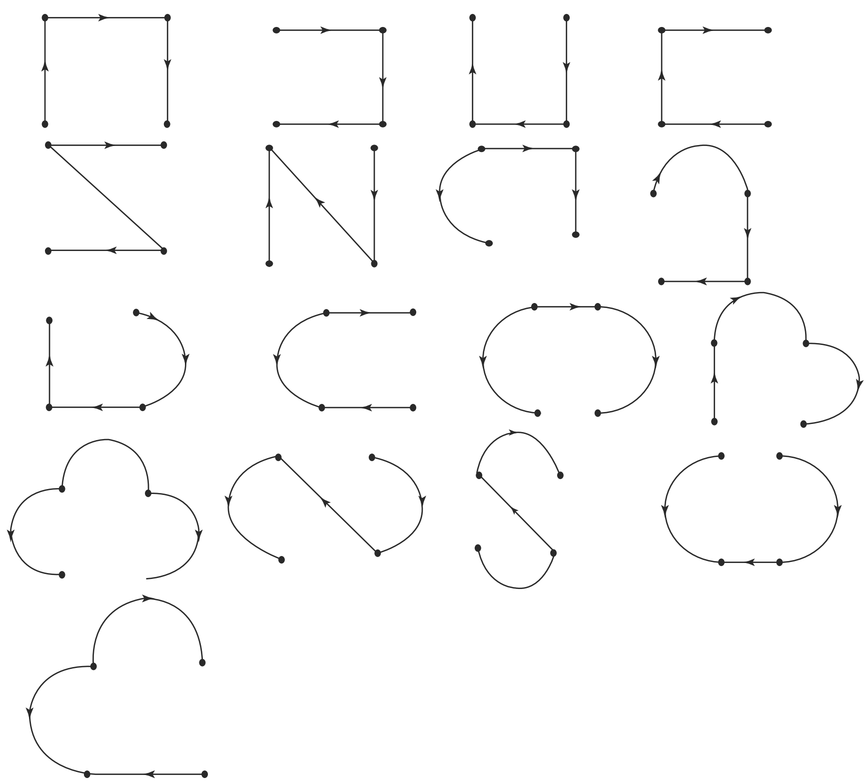


Fig. 5.10

5.1.3 Twigs and Links

The branches of a tree are called its 'twigs'. For a given graph, the complementary set of branches of the tree is called the co-tree of the graph. The branches of a co-tree are called links, i.e., those elements of the connected graph that are not included in the tree links and form a subgraph. For example, the set of branches (b, d, f) represented by dotted lines in Fig. 5.13 form a co-tree of the graph in Fig. 5.11 with respect to the tree in Fig. 5.12.

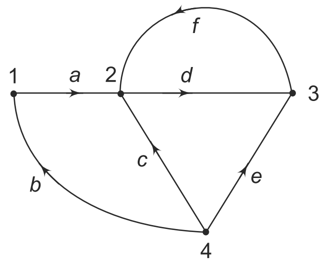


Fig. 5.11

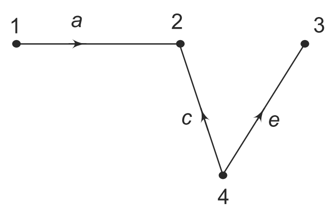


Fig. 5.12

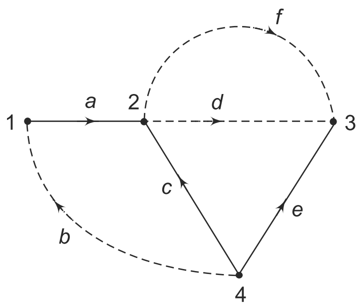


Fig. 5.13

The branches *a*, *c* and *e* are the twigs while the branches *b*, *d* and *f* are the links of this tree. It can be seen that for a network with *b* branches and *n* nodes, the number of twigs for a selected tree is $(n - 1)$ and the number of links I with respect to this tree is $(b - n + 1)$. The number of twigs $(n - 1)$ is known as the tree value of the graph. It is also called the *rank* of the tree. If a link is added to the tree, the resulting graph contains one closed path, called a loop. The addition of each subsequent link forms one or more additional loops. Loops which contain only one link are independent and are called basic loops.

5.1.4 Incidence Matrix (*A*)

The incidence of elements to nodes in a connected graph is shown by the element node incidence matrix (*A*). Arrows indicated in the branches of a graph result in an oriented or a directed graph. These arrows are the indication for the current flow or voltage rise in the network. It can be easily identified from an oriented graph regarding the incidence of branches to nodes. It is possible to have an analytical description of an oriented-graph in a matrix form. The dimensions of the matrix A is $n \times b$ where n is the number of nodes and b is number of branches. For a graph having n nodes and b branches, the complete incidence matrix A is a rectangular matrix of order $n \times b$.

In matrix A with n rows and b columns an entry a_{ij} in the i^{th} row and j^{th} column has the following values.

$a_{ij} = 1$, if the j^{th} branch is incident to and oriented away from the i^{th} node.
$a_{ij} = -1$, if the j^{th} branch is incident to and oriented towards the i^{th} node. (5.1)
$a_{ij} = 0$, if the j^{th} branch is not incident to the i^{th} node.

Figure 5.12 shows a directed graph.

Following the above convention its incidence matrix A is given by

$$\begin{matrix} \text{nodes} & & & \text{branches}\rightarrow & & & \\ \downarrow & a & b & c & d & e & f \end{matrix}$$

$$A = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & +1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix}$$

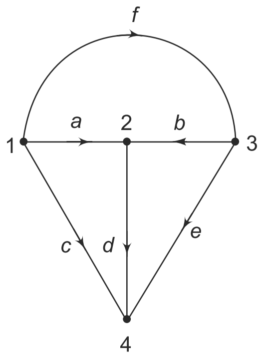


Fig. 5.14

The entries in the first row indicates that three branches *a*, *c* and *f* are incident to node 1 and they are oriented away from node 1 and therefore the entries a_{11};a_{13} and a_{16} are +1. Other entries in the 1st row are zero as they are not connected to node 1. Likewise, we can complete the incidence matrix for the remaining nodes 2, 3 and 4.

Example 5.3 *Construct the incident matrix for the graph shown in Fig. 5.15.*

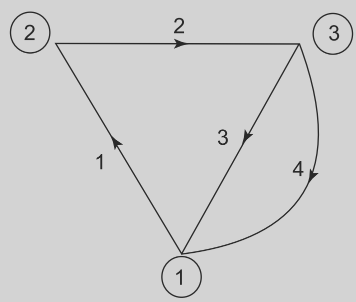


Fig. 5.15

Solution The dimensions of incidence matrix 'A' is $n \times b$ where n is number of nodes and b is number of branches, hence the dimensions of the incidence matrix for the above graph is 3×4.

Incidence matrix

n - nodes

b- branches

A =

n \ b	1	2	3	4
1	1	0	−1	−1
2	−1	1	0	0
3	0	−1	1	1

The incidence matrix is given by

$$A = \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

5.1.5 Properties of Incidence Matrix (A)

Following properties are some of the simple conclusions from incidence matrix A.

1. Each column representing a branch contains two non-zero entries $+1$ and -1; the rest being zero. The unit entries in a column identify the nodes of the branch between which it is connected.
2. The unit entries in a row identify the branches incident at a node. Their number is called the degree of the node.
3. A degree of 1 for a row means that there is one branch incident at the node. This is commonly possible in a tree.
4. If the degree of a node is two, then it indicates that two branches are incident at the node and these are in series.
5. Columns of A with unit entries in two identical rows correspond to two branches with same end nodes and hence they are in parallel.
6. Given the incidence matrix A the corresponding graph can be easily constructed since A is a complete mathematical replica of the graph.
7. If one row of A is deleted the resulting $(n - 1) \times b$ matrix is called the reduced incidence matrix A_1. Given A_1, A is easily obtained by using the first property.

It is possible to find the exact number of trees that can be generated from a given graph if the reduced incidence matrix A_1 is known and the number of possible trees is given by Det $(A_1 A_1^T)$ where A_1^T is the transpose of the matrix A_1.

Example 5.4 *Draw the graph corresponding to the given incidence matrix.*

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & +1 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & +1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ +1 & +1 & +1 & +1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution There are five rows and eight columns which indicate that there are five nodes and eight branches. Let us number the columns from a to h and rows as 1 to 5.

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{matrix} a & b & c & d & e & f & g & h \end{matrix} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Mark the nodes corresponding to the rows 1, 2, 3, 4 and 5 as dots as shown in Fig. 5.16 (a). Examine each column of A and connect the nodes (unit entries) by a branch; label it after marking an arrow.

For example, examine the first column of A. There are two unit entries one in the first row and 2nd in the last row, hence connect branch a between node 1 and 5. The entry of A_{11} is – ve and that of A_{51} is + ve. Hence the orientation of the branch is away from node 5 and towards node 1 as per the convention. Proceeding in this manner we can complete the entire graph as shown in Fig. 5.16 (b).

From the incidence matrix A, it can be verified that branches c and d are in parallel (property 5) and branches e and f are in series (property 4).

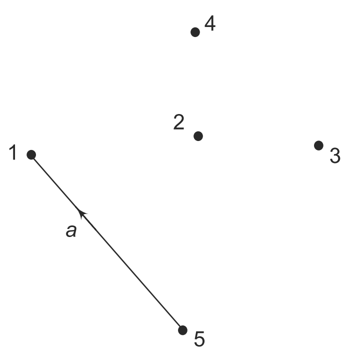


Fig. 5.16 (a)

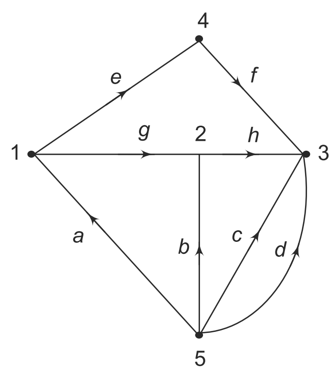


Fig. 5.16 (b)

Example 5.5 *Obtain the incidence matrix A from the following reduced incidence matrix A_1 and draw its graph.*

$$[A_1] = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Solution There are five rows and seven columns in the given reduced incidence matrix $[A_1]$. Therefore, the number of rows in the complete incidence matrix A will be $5 + 1 = 6$. There will be six nodes and seven branches in the graph. The dimensions of matrix A is 6×7. The last row in A, i.e., 6th row for the matrix A can be obtained by using the first property of the incidence matrix. It is seen that the first column of $[A_1]$ has a single non-zero element –1. Hence, the first element in the 6th row will be +1 $(-1 + 1 = 0)$. Second column of A_1 has two non-zero elements +1 and –1, hence the 2nd element in the 6th row will be 0. Proceeding in this manner we can obtain the 6th row. The complete incidence matrix can therefore be written as

$$[A] = \begin{matrix} a \\ b \\ c \\ d \\ e \\ f \end{matrix} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

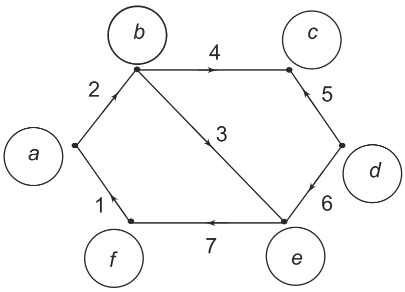


Fig. 5.17

We have seen that any one of the rows of a complete incidence matrix can be obtained from the remaining rows. Thus it is possible to delete any one row from A without loosing any information in A_1. Now the oriented graph can be constructed from the matrix A. The nodes may be placed arbitrarily. The number of nodes to be marked will be six. Taking node 6 as reference node the graph is drawn as shown in Fig. 5.17.

5.1.6 Incidence Matrix and KCL

[JNTU June 2009]

Kirchhoff's current law (KCL) of a graph can be expressed in terms of the reduced incidence matrix as $A_1 I = 0$.

A_1, I is the matrix representation of KCL, where I represents branch current vectors $I_1, I_2 \ldots I_6$.

Consider the graph shown in Fig. 5.18. It has four nodes a, b, c and d.

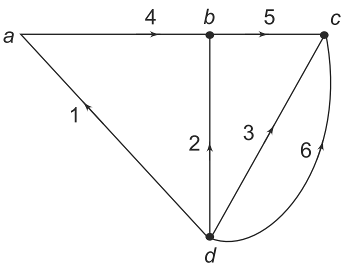


Fig. 5.18

Let node d be taken as the reference node. The positive reference direction of the branch currents corresponds to the orientation of the graph branches. Let the branch currents be $i_1, i_2, \ldots i_6$. Applying KCL at nodes a, b and c.

$$-i_1 + i_4 = 0$$

$$-i_2 - i_4 + i_5 = 0$$

$$-i_3 = i_5 - i_6 = 0$$

These equations can be written in the matrix form as follows:

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_1 I_b = 0 \tag{5.2}$$

Here, I_b represents column matrix or a vector of branch currents.

$$I_b = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_b \end{bmatrix}$$

A_1 is the reduced incidence matrix of a graph with n nodes and b branches. And it is a $(n - 1) \times b$ matrix obtained from the complete incidence matrix of A deleting one of its rows. The node corresponding to the deleted row is called the reference node or datum node. It is to be noted that $A_1 I_b = 0$ gives a set of $n - 1$ linearly independent equations in branch currents $I_1, I_2, \ldots I_6$. Here $n = 4$. Hence, there are three linearly independent equations.

Example 5.6 *For the network shown in Fig. 5.19 obtain the incidence matrix and mesh equations- Obtain the current through 20 Ω.*

[JNTU Jan 2010]

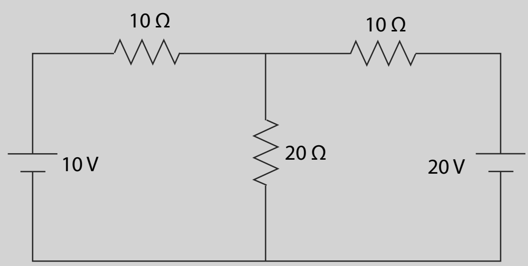


Fig. 5.19

Solution

The graph obtained

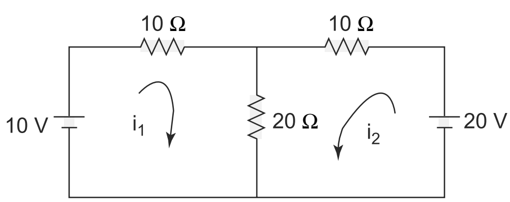


Fig. 5.20

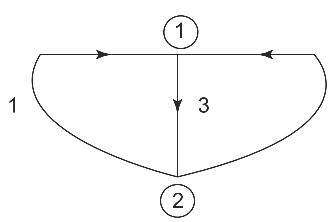


Fig. 5.21

Incidence matrix is given as

$$\begin{array}{cc} \text{Nodes} & \text{Branches} \rightarrow \\ \downarrow & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} -1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \end{array}$$

Mesh equations are given as

$$10 = 30i_1 + 20i_2 \Rightarrow 1 = 3i_1 + 2i_2$$

$$20 = 20i_1 + 30i_2 \Rightarrow 2 = 2i_1 + 3i_2$$

$\therefore \quad i_1 = -0.2$ amp

$\therefore \quad i_2 = 0.8$ amp

$\therefore$ Current through 20 V $= i_1 + i_2 = 0.6$ amp

5.2 BASIC CUT-SET FOR PLANAR NETWORKS

A cut-set is a minimal set of branches of a connected graph such that the removal of these branches causes the graph to be cut into exactly two parts. The important property of a cut-set is that by restoring anyone of the branches of the cut-set the graph should become connected. A cut-set consists of one and only one branch of the network tree, together with any links which must be cut to divide the network into two parts.

Consider the graph shown in Fig. 5.22 (a).

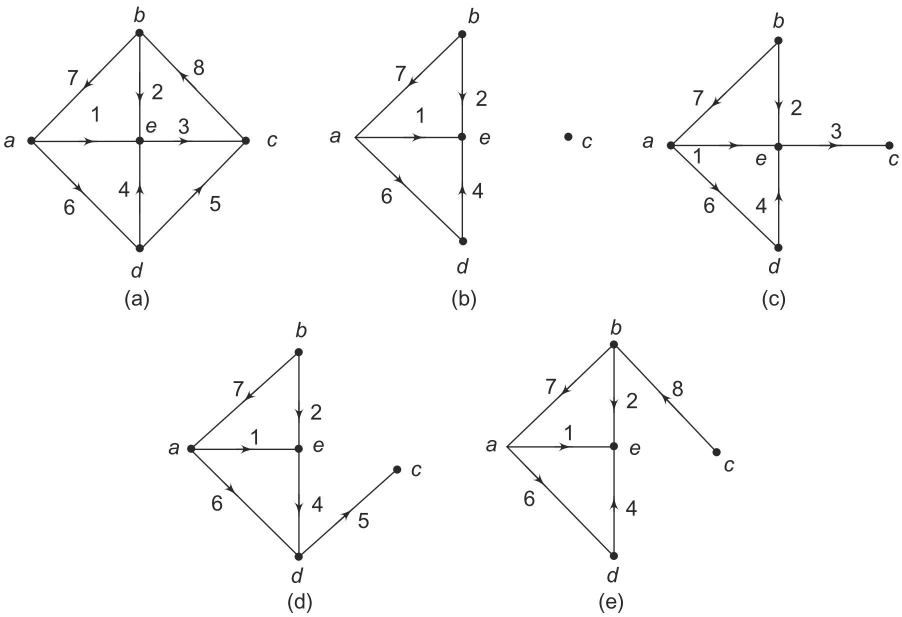


Fig. 5.22

If the branches 3, 5 and 8 are removed from the graph, we see that the connected graph of Fig. 5.22 (a) is separated into two distinct parts, each of which is connected as shown in Fig. 5.22 (b). One of the parts is just an isolated node. Now suppose the removed branch 3 is replaced, all others still removed. Fig. 5.22 (c) shows the resultant graph. The graph is now connected. Likewise replacing the removed branches 5 and 8 of the set {3, 5, 8} one at a time, all other ones remaining removed, we obtain the resulting graphs as shown in Figs 5.22 (d) and (e). The set formed by the branches 3, 5 and 8 is called the cut-set of the connected graph of Fig. 5.22 (a).

5.2.1 Cut-Set Orientation

A cut-set is oriented by arbitrarily selecting the direction. A cut-set divides a graph into two parts. In the graph shown in Fig. 5.23, the cut-set is {2, 3}. It is represented by a dashed line passing through branches 2 and 3. This cut-set separates the graph into two parts shown as part-1 and part-2. We may take the orientation either from part-1 to part-2 or from part-2 to part-1.

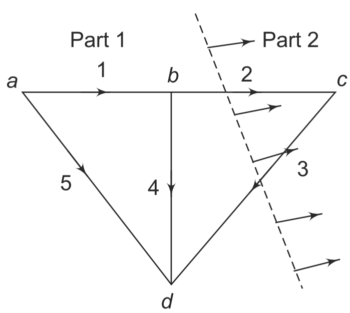


Fig. 5.23

The orientation of some branches of the cut-set may coincide with the orientation of the cut-set while some branches of the cut-set may not coincide. Suppose we choose the orientation of the cut-set {2, 3} from part-1 to part-2 as indicated in Fig. 5.55, then the orientation of branch 2 coincides with the cut-set, whereas the orientation of the branch 3 is opposite.

5.2.2 Cut-Set Matrix and KCL for Cut-Sets

KCL is also applicable to a cut-set of a network. For any lumped electrical network, the algebraic sum of all the cut-set branch currents is equal to zero. While writing the KCL equation for a cut-set, we assign positive sign for the current in a branch if its direction coincides with the orientation of the cut-set and a negative sign to the current in a branch whose direction is opposite to the orientation of the cut-set. Consider the graph shown in Fig. 5.24. It has five branches and four nodes. The branches have been numbered 1 through 5 and their orientations are also marked. The following six cut-sets are possible as shown in Fig. 5.25 (a)-(f).

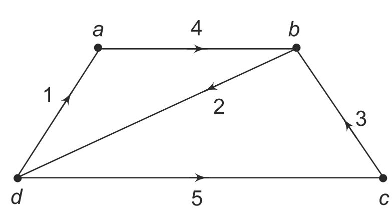


Fig. 5.24

Cut-set C_1: {1, 4}; cut-set C_2: {4, 2, 3}

Cut-set C_3: {3, 5}; cut-set C_4: {1, 2, 5}

Cut-set C_5: {4, 2, 5}; cut-set C_6: {1, 2, 3}

Applying KCL for each of the cut-set we obtain the following equations. Let $i_1, i_2 \dots i_6$ be the branch currents.

$$\left.\begin{aligned} C_1 &: i_1 - i_4 = 0 \\ C_2 &: -i_2 + i_3 + i_4 = 0 \\ C_3 &: -i_3 + i_5 = 0 \\ C_4 &: i_1 - i_2 + i_5 = 0 \\ C_5 &: -i_2 + i_4 + i_5 = 0 \\ C_6 &: i_1 - i_2 + i_3 = 0 \end{aligned}\right\} \tag{5.3}$$

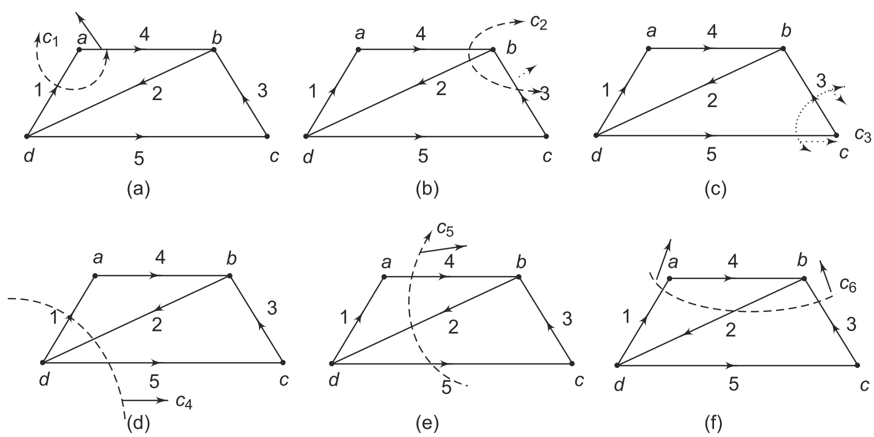


Fig. 5.25 (a to f)

These equations can be put into matrix form as

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

or

$$QI_b = 0 \tag{5.4}$$

where the matrix Q is called augmented cut-set matrix of the graph or all cut-set matrix of the graph. The matrix I_b is the branch-current vector.

The all cut-set matrix can be written as $Q_{\text{th}} = [q_{ij}]$.
Where q_{ij} is the element in the i^{th} row and j^{th} column. The order of Q is number of cut-sets $\times$ number of branch as in the graph.

q_{ij} = 1, if branch j in the cut-set i and the orientation coincides with each other
q_{ij} = –1, if branch j is in the cut-set i and the orientation is opposite.
q_{ij} = 0, if branch j is not present in cut-set i. (5.5)

Example 5.7 *For the network-graph shown in Fig. 5.26 (a) with given orientation obtain the all cut-set (augmented cut-set) matrix.*

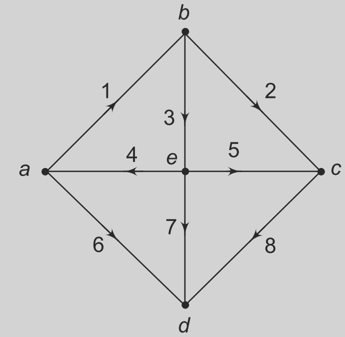


Fig. 5.26 (a)

Solution The graph has four nodes and eight branches. There are in all 12 possible cut-sets as shown with dashed lines in Figs 5.26 (b) and (c). The orientation of the cut-sets has been marked arbitrarily. The cut-sets are

C_1: {1, 46}; C_2: {1, 2, 3}; C_3: {2, 5, 8}
C_4: {6, 7, 8}; C_5: {1, 3, 5, 8}; C_6: {1, 4, 7, 8}
C_7: {2, 5, 6, 7}; C_8: {2, 3, 4, 6} C_9: {1, 4, 7, 5, 2}
C_{10}: {2, 3, 4, 7, 8}; C_{11}: {6, 4, 3, 5, 8}; C_{12}: {1, 3, 5, 7, 6}

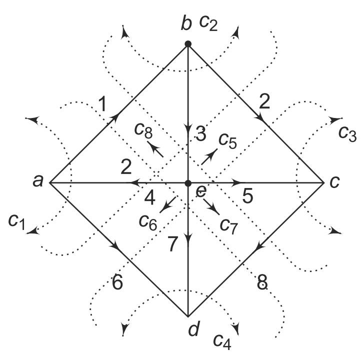


Fig. 5.26 (b)

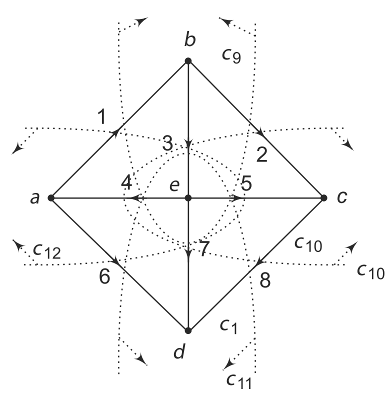


Fig. 5.26 (c)

Eight cut-sets C_1 to C_8 are shown if Fig. 5.26 (b) and four cut-sets C_9 to C_{11} are shown in Fig. 5.26 (c) for clarity.

As explained in section 5.2.2 with the help of Eq. 5.5, the all cut-set matrix Q is given by

$$
\begin{array}{c} \text{cut-sets} \\ \downarrow \end{array}
\quad \begin{array}{c} \text{branches} \rightarrow \end{array}
$$

$$
Q = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \end{array}
\begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
-1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & -1 & 0 & 1 & 0 & 0 & -1 \\
-1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & -1 & -1 & 1 & 0 & -1 & 0 & 0 \\
1 & -1 & 0 & -1 & -1 & 0 & -1 & 0 \\
0 & 1 & 1 & -1 & 0 & 0 & -1 & -1 \\
0 & 0 & 1 & -1 & -1 & 1 & 0 & 1 \\
-1 & 0 & 1 & 0 & -1 & -1 & -1 & 0
\end{bmatrix} \end{array}
$$

Matrix Q is a 12×8 matrix since there are 12 cut-sets and eight branches in the graph.

Example 5.8 *Determine the basic Cut-set matrix for the oriented graph given in the Fig. 5.27 where the elements 1, 2 and 3 are tree branches.*

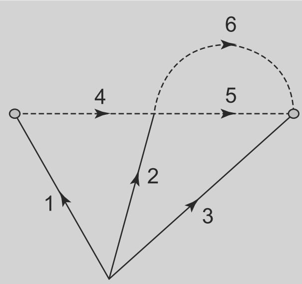


Fig. 5.27

Solution Branches 1, 2 and 3 are the twigs of the tree. The remaining branches 4, 5 and 6 are called links. Let us consider a tree as in Fig. 5.28.

For each twig, there will be a basic Cut-set. Therefore, for a network graph with r nodes and 'b' branches there will be $(n - 1)$ number of basic Cut-sets.

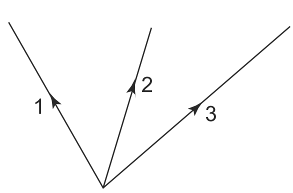


Fig. 5.28

The link that must be added to twig 1 to form a Cut- set 1 is 4. Thus Corresponding to twig 1 the basic Cut-set {1, 4} as shown.

As a Convention the orientation of a Cut-set is chosen to consider with that of its defining twig similarly, other Cut-sets C_2 and C_3 corresponding to twigs 2 and 3 are also shown in the Figs 5.29 (b) and (c).

$C_1 = \{1, 4\}$ Corresponding to twig 1

$C_2 = \{2, 4, 5, 6\}$ Corresponding to twig 2

$C_3 = \{3, 5, 6\}$ Corresponding to twig 3

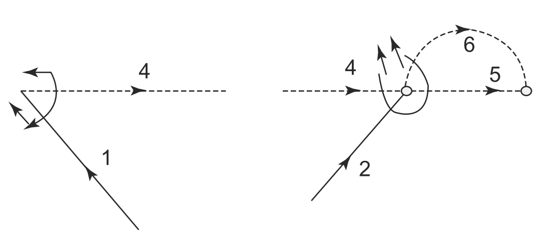


Fig. 5.29

The basic Cut-set matrix Q_f of a graph with n nodes and b branches corresponds to a tree T is an $(n - 1) \times b$ matrix.

Thus the basic Cut-set Matrix is given by

$$\begin{array}{c} f \text{ Cut-sets} \quad \text{branches} \rightarrow \\ \begin{array}{cc} & \downarrow \; 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\ Q_f = \begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} & \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array} \end{array}$$

Example 5.9 *For the given network Fig. 5.30, draw the oriented graph and choose one possible tree and construct the basic cutest schedule. Write down the network equations from the above matrix.*

[JNTU June 2006]

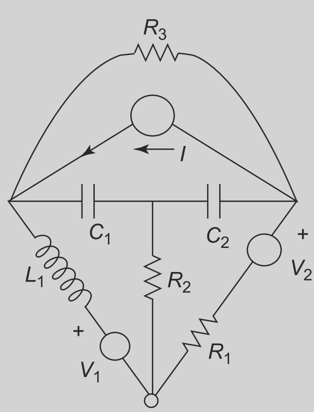


Fig. 5.30

Solution The oriented graph for the given network can be as shown in figure.

$$C_1: i_1 - i_5 + i_6 + i_7 = 0$$

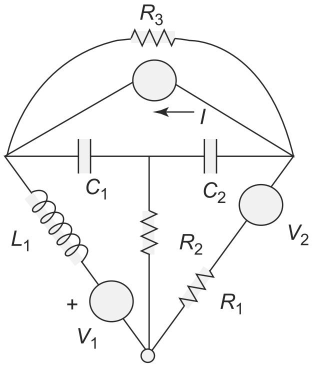


Fig. 5.31

$$C_2: i_2 - i_4 + i_6 + i_7 = 0$$

$$C_3: i_3 + i_4 - i_5 = 0$$

$$\begin{array}{lc} & \text{branches} \\ f\text{-cut-sets} & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{l} [7,6,\overline{2},4] \\ [7,6,\overline{1},4] \\ [3,4,\overline{5}] \end{array} & \begin{array}{c} a \\ b \\ c \end{array}\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

Example 5.10 *Draw the Graph for network shown obtain a tree. What is the number of mesh currents required for network?* *[JNTU June 2009]*

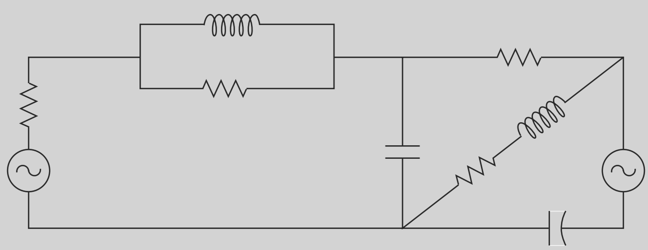
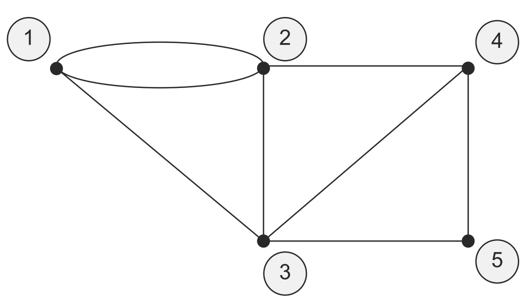


Fig. 5.32

Solution

Graph



Tree

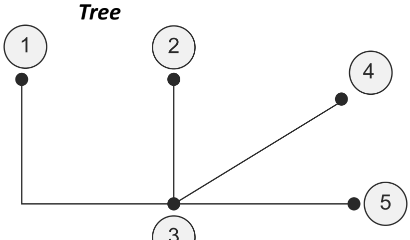


Fig. 5.33

No. of Mesh currents = 4

5.2.3 Fundamental Cut-Sets

Observe the set of Eq. 5.3 in Section 5.2.2 with respect to the graph in Fig. 5.57. Only first three equations are linearly independent, remaining equations can be obtained as a linear combination of the first three. The concept of fundamental cut-set (*f*-cut-set) can be used to obtain a set of linearly independent equations in branch current variables. The *f*-cut-sets are defined for a given tree of the graph. From a connected graph, first a tree is selected, and then a twig is selected. Removing this twig from the tree separates the tree into two parts. All the links which go from one part of the disconnected tree to the other, together with the twig of the selected tree will constitute a cut-set. This cut-set is called a fundamental cut-set or *f*-cut-set or the graph. Thus a fundamental cut-set of a graph with respect to a tree is a cut-set that is formed by one twig and a unique set of links. For each branch of the tree, i.e. for each twig, there will be a *f*-cut-set. So, for a connected graph having n nodes, there will be $(n - 1)$ twigs in a tree, the number of *f*-cut-sets is also equal to $(n - 1)$.

Fundamental cut-set matrix Q_f is one in which each row represents a cut-set with respect to a given tree of the graph. The rows of Q_1 correspond to the fundamental cut-sets and the columns correspond to the branches of the graph. The procedure for obtaining a fundamental cut-set matrix is illustrated in Example 5.6.

Example 5.11 *Obtain the fundamental cut-set matrix Q_f for the network graph shown in Fig. 5.34 (a).*

Solution A selected tree of the graph is shown in Fig. 5.34 (a).

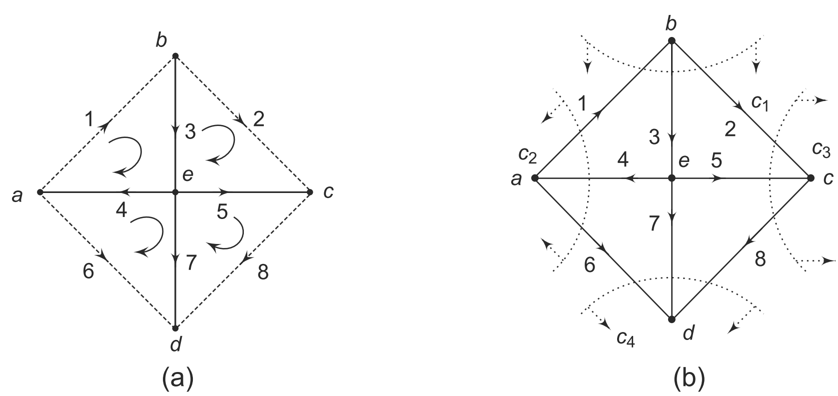


Fig. 5.34

The twigs of the tree are {3, 4, 5, 7}. The remaining branches 1, 2, 6 and 8 are the links, corresponding to the selected tree. Let us consider twig 3. The minimum number of links that must be added to twig 3 to form a cut-set C_1 is {1, 2}. This set is unique for C_1. Thus corresponding to twig 3. The *f*-cut-set C_1 is {1, 2, 3}. This is shown in Fig. 5.34 (b). As a convention the orientation of a cut-set is chosen to coincide with that of its defining twig. Similarly, corresponding to twig 4, the *f*-cut-set

C_2 is {1, 4, 6} corresponding to twig 5, the *f*-cut-set C_3 is {2, 5, 8} and corresponding to twig 7, the *f*-cut-set is {6, 7, 8}. Thus the *f*-cut-set matrix is given by

$$Q_f = \begin{matrix} \text{f-cut-sets} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{matrix} \begin{matrix} \text{branches} \\ \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & +1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{matrix} \tag{5.6}$$

5.2.4 Tree Branch Voltages and *f*-Cut-Set Matrix

From the cut-set matrix the branch voltages can be expressed in terms of tree branch voltages. Since all tree branches are connected to all the nodes in the graph, it is possible to trace a path from one node to any other node by traversing through the tree-branches.

Let us consider Example 5.11, there are eight branches. Let the branch voltages be $V_1, V_2, \ldots V_8$. There are, four twigs, let the twig voltages be V_{t3}, V_{t4}, V_{t5} and V_{t7} for twigs 3, 4, 5 and 7 respectively.

We can express each branch voltage in terms of twig voltages as follows.

$$\begin{aligned}
V_1 &= -V_3 - V_4 = -V_{t3} - V_{t4} \\
V_2 &= +V_3 + V_5 = +V_{t3} + V_{t5} \\
V_3 &= V_{t3} \\
V_4 &= V_{t4} \\
V_5 &= V_{t5} \\
V_6 &= V_7 - V_4 = V_{t7} - V_{t4} \\
V_7 &= V_{t7} \\
V_8 &= V_7 - V_5 = V_{t7} - V_{t5}
\end{aligned}$$

The above equations can be written in matrix form as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 0 \\ +1 & 0 & +1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_{t_3} \\ V_{t_4} \\ V_{t_5} \\ V_{t_7} \end{bmatrix} \tag{5.7}$$

The first matrix on the right hand side of Eq. 5.7 is the transpose of the *f*-cut-set matrix Q_f given in Eq. 5.6 in Example 5.9. Hence, the Eq. 5.2 can be written as

$$V_b = Q_f^T V_t. \tag{5.8}$$

Where V_b is the column matrix of branch-voltages V_t is the column matrix of twig voltages corresponding to the selected tree and Q_f^T in the transpose of f-cut-set matrix.

Equation 5.8 shows that each branch voltage can be expressed as a linear combination of the tree-branch voltages. For this purpose fundamental cut-set (f-cut-set) matrix can be used without writing loop equations.

Example 5.12 *Find the fundamental tie-set and cut-set matrix for the graph and for the tree shown in the Fig. 5.35.*

[JNTU June 2006]

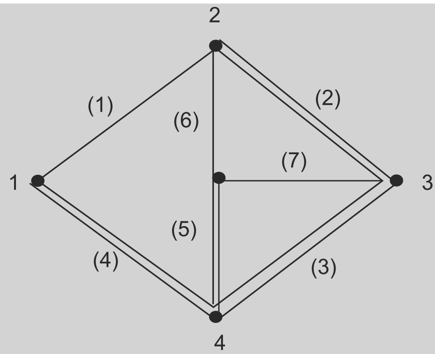


Fig. 5.35

Solution Three are 5 nodes, $n = 5$

There are 7 branches $= b = 7$

No. of twigs or tree branches

$= n - 1 = 4$ (2, 3, 4, 5)

No. of link branches

$= b - (n - 1) = 3$ (1, 6, 7)

The Tie-sets are shown below

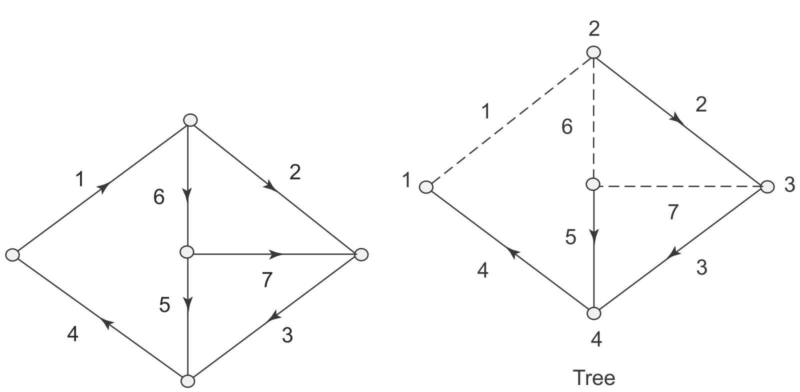


Fig. 5.36

$$V_1 + V_2 + V_3 + V_4 = 0$$
$$V_2 + V_3 - V_5 - V_6 = 0$$
$$V_3 - V_5 + V_7 = 0$$

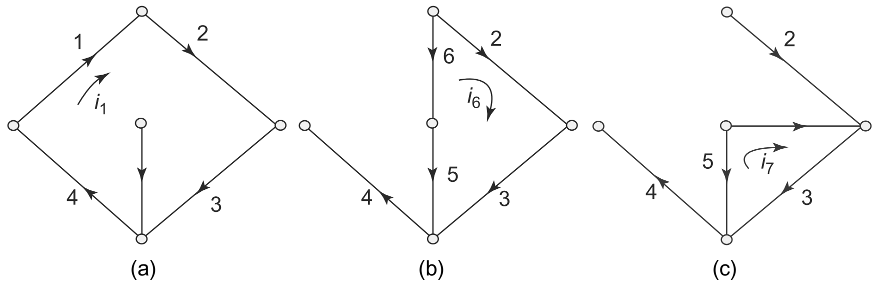


Fig. 5.37

Tie set matrix loop; Branches →

$$\begin{array}{c} \downarrow \\ i_1 \\ i_2 \\ i_3 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}$$

The required Tie-set matrix is given by

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 \end{bmatrix}$$

Cut-set

For the given Tree there are four fundamental cut-sets each for one twig and given by

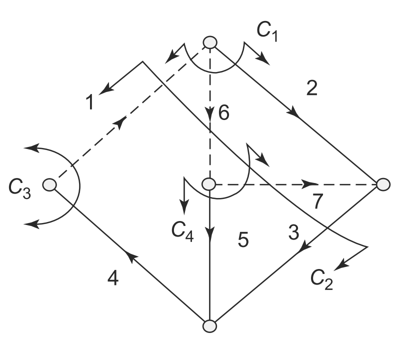


Fig. 5.37(d)

Twig 2; *f*-cut-set [1,2,6]

Twig 3; *f*-cut-set [1, 3, 6, 7]

Twig 4; *f*-cut-set [1, 4]

Twig 5; *f*-cut-set [5, 7]

The cut-sets are formed as shown

f-cut set matrix

$$\begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Example 5.13 *Draw the oriented graph of the network shown in Fig. 5.38 and write the cut-set matrix.* *[JNTU June 2006]*

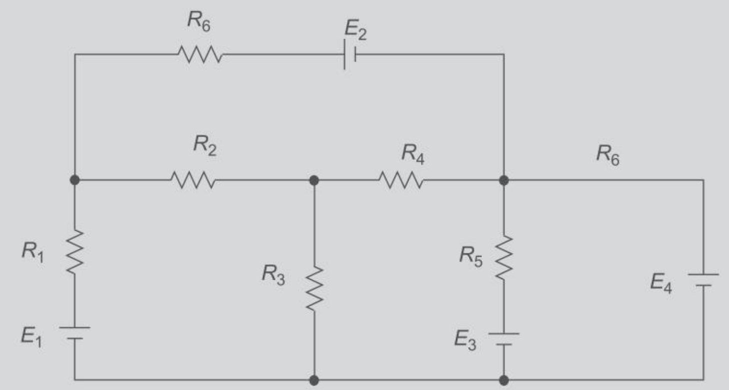


Fig. 5.38

Solution The oriented graph of the network is shown in figure. An orbitrary tree is selected to form fundamental cut-set (*f*-cut-set) matrix. The tree branches (Twigs) are shown with thick lines and the line branches are shown with dashed lines.

No.of branches = 7

No.of nodes (n) = 4

Twigs $= n - 1 = 3$ (2, 3, 6)

No.of links (l) $= b - (n - 1) = 4$(1, 4, 5, 7)

For twig 2; *f*-cut-set $C_1 \rightarrow (1, 2, 5)$

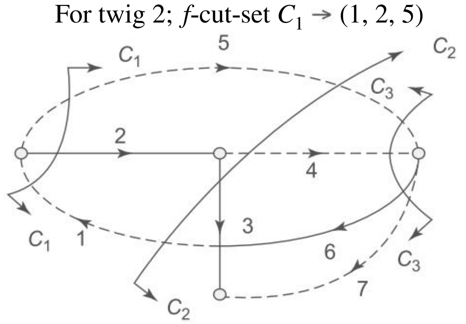


Fig. 5.39

For twig 3; *f*-cut-set $C_2 \rightarrow (1, 3, 4, 5)$

For twig 6; *f*-cut-set $C_3 \rightarrow (4, 5, 6, 7)$

Fundamental cut-set matrix

f-cut-set	Branches 1	2	3	4	5	6	7
C_1	−1	1	0	0	1	0	0
C_2	−1	0	1	1	1	0	0
C_3	0	0	0	−1	−1	1	1

Example 5.14 *Obtain the fundamental loop and fundamental cut-set matrices for the graph shown in Fig. 5.40.*

[JNTU May 2007]

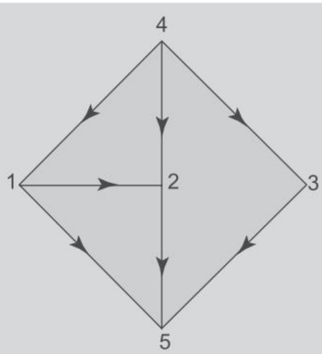
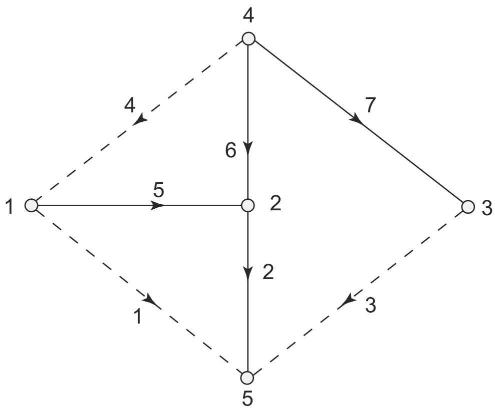


Fig. 5.40

Solution For the given graph, an arbitrary tree is chosen for which the no. of nodes $n = 5$



Tree

Fig. 5.41

No. of branches $b = 7$

No. of tree branches of twigs $(n - 1) = 4(2, 5, 6, 7)$

No. of link branches $l = b - (n - 1)$
$= 3(1, 3, 4)$

For a given tree of a graph, addition of each link between any two nodes forms a loop called the fundamental loop, (*f*-loop) or a tie-set. By adding links 1, 3 and 4, we can form three fundamental loops as shown in the figure. By convention, a fundamental loop is marked with the same orientation as its defining link current.

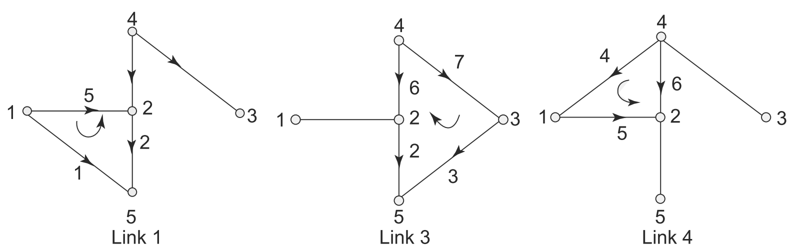


Fig. 5.42

Tie – sets

Tie-set schedule (Fundamental loop matrix)

Link no	*Branch No*						
	1	2	3	4	5	6	7
1	1	−1	0	0	−1	0	0
3	0	−1	1	0	0	−1	1
4	0	0	0	1	1	−1	0

Cut-set

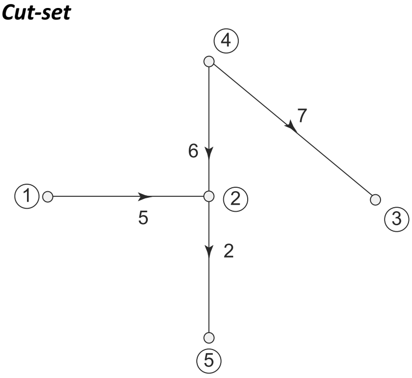


Fig. 5.43(a)

Consider the tree of the graph shown in figure with 5 nodes 1–5 and four tree branches.

The following are the fundamental cut-sets

f-cut-set corresponding to twig 2; $C_1 = \{1, 2, 3\}$

f-cut-set corresponding to twig 5; $C_2 = \{1, 4, 5\}$

f-cut-set corresponding to twig 6; $C_3 = \{3, 4, 6\}$

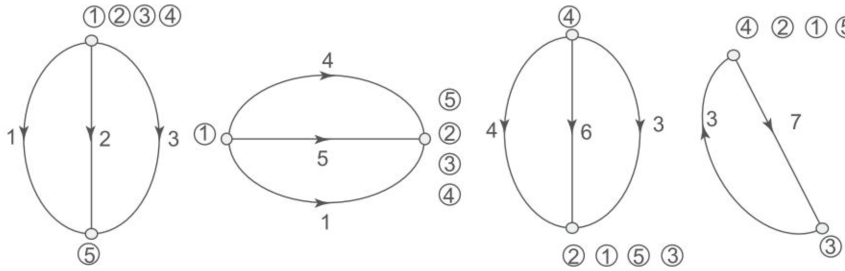


Fig. 5.43(b)

f-cut-set corresponding to twig 7; $C_4 = \{3, 7\}$

Thus, the *f*-cut-set matrix is given by *f*-cut-sets.

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Example 5.15 *Obtain the fundamental cut-set matrices for the network shown in Fig. 5.44.* *[JNTU May 2007]*

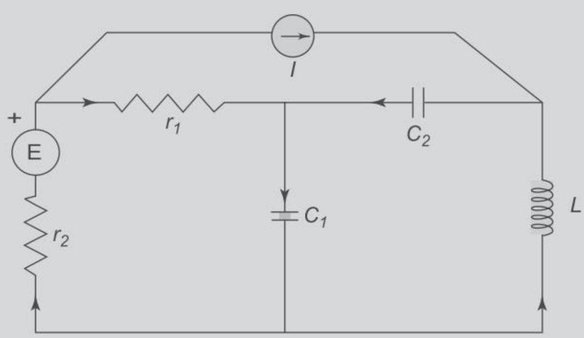


Fig. 5.44

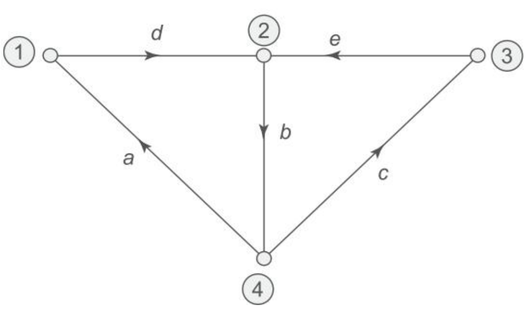
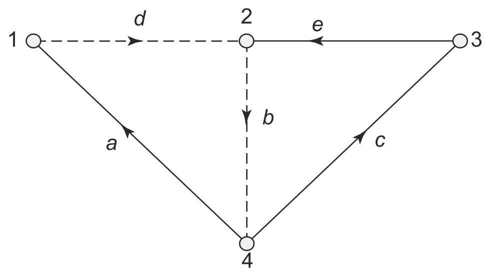


Fig. 5.45

Solution By short circuiting voltage source and open circuiting current source, the oriented graph can be drawn as shown.

The number of nodes are 4 and branches are five. An arbitrary tree is chosen as shown, with twig branches as *a*, *c*, *e* and links as *d* and *b*.



Tree
Fig. 5.46

f-loop matrix

$$\begin{array}{c} \text{Branches} \\ \begin{array}{cc} \text{links} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} b \\ d \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & -1 & 1 & -1 \end{bmatrix} \end{array} \end{array}$$

The cut-sets are given by

$C_1 = \{a, d\}$

$C_2 = \{b, c, d\}$

$C_3 = \{b, d, e\}$

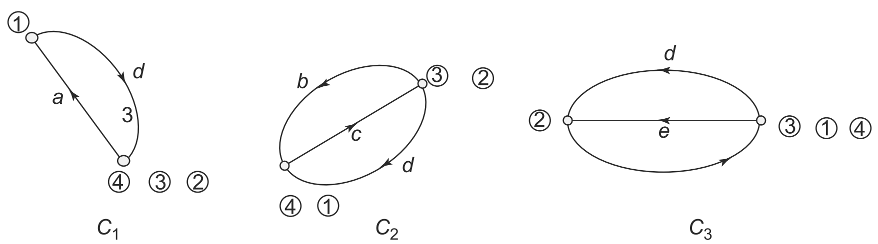


Fig. 5.47

f-cut-set matrix

$$\begin{array}{c} \text{Branches} \\ \begin{array}{cc} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} & \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 \end{bmatrix} \end{array} \end{array}$$

Example 5.16 *For the given network Fig. 5.48 graph, Construct the Basic Tieset incidence matrix, tracking elements 1, 6, 8, 3 as tree branches. Express the link branch Voltage in terms of tree branch voltages*

[JNTU June 2006]

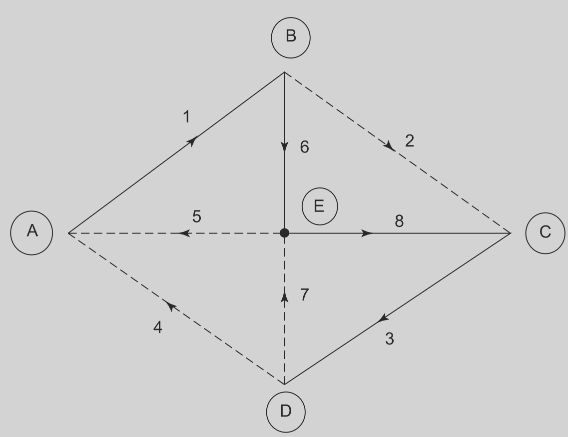


Fig. 5.48

Solution

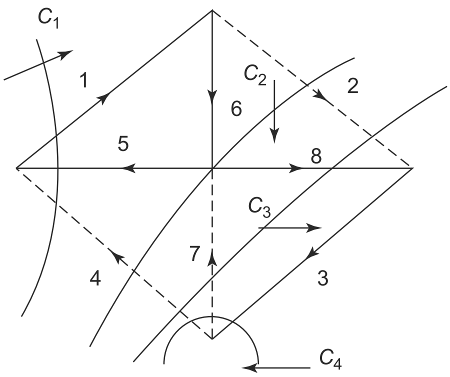


Fig. 5.49

(a) Cut-set incidence matrix is

$$Q = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

The link branch voltage in terms of tree branch voltages is given by

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}$$

(b) Write the Tieset matrix for the graph shown in figure taking the tree consisting of branches 2, 3, 4.

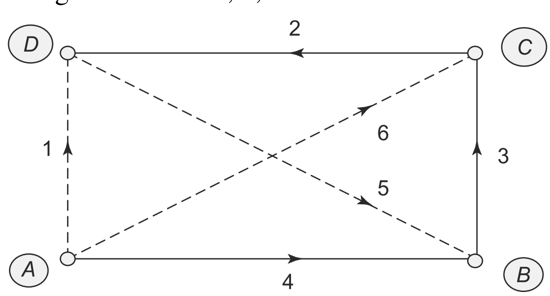


Fig. 5.50

Solution

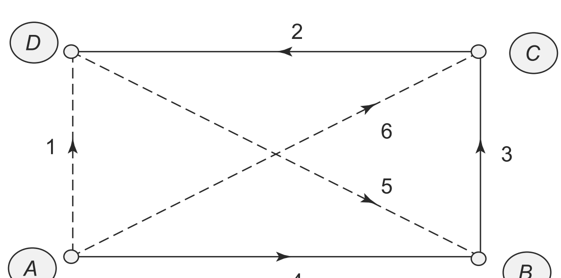


Fig. 5.51

Basic tiesets \ e	1	2	3	4	5	6
(5, 3, 2)	0	−1	−1	0	1	0
(6, 3, 4)	0	0	−1	−1	0	1
(1, 2, 3, 4)	1	−1	−1	−1	0	0

Example 5.17 *For the given network (Fig. 5.52) graph, Construct the basic cutest incidence matrix, tracking elements 1, 6, 8, 3 as tree branches. Express the link branch voltage in terms of tree branch voltages.*

[JNTU May 2007]

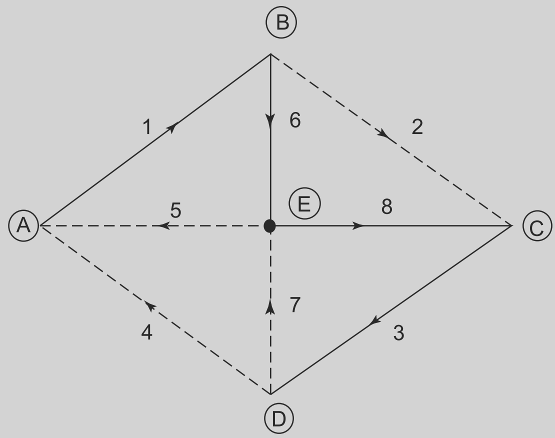


Fig. 5.52

Solution The incidence matrix is given by

Nodes \ Elements	1	2	3	4	5	6	7	8
A	1	0	0	−1	−1	0	0	0
B	−1	1	0	0	0	1	0	0
C	0	−1	1	0	0	0	0	−1
D	0	0	−1	1	0	0	1	0
E	0	0	0	0	1	−1	−1	1

Cut-set matrix is given by

Cutset Branch \ Elements	1	2	3	4	5	6	7	8
C_1	−1	0	0	1	1	0	0	0
C_2	0	0	1	−1	0	0	−1	0
C_3	0	−1	0	1	1	−1	0	0
C_4	0	1	0	−1	0	0	−1	1

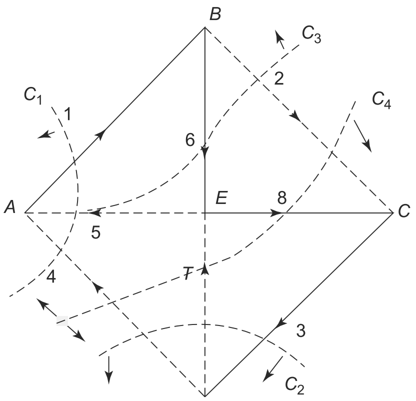


Fig. 5.53

We have $V_1 + V_6 + V_5 = 0$

$\Rightarrow V_5 = -(V_1 + V_6)$

$V_2 - V_8 - V_6 = 0 \Rightarrow V_2 = V_6 + V_8$

$V_8 + V_3 + V_7 = 0 \Rightarrow V_7 = -(V_3 + V_8)$

$V_5 + V_7 - V_4 = 0$

$\Rightarrow V_4 = V_5 + V_7 = -V_1 - V_6 - V_3 - V_8$

$V_4 = -(V_1 + V_3 + V_6 + V_8)$

Example 5.18 *For the network shown in Fig. 5.54 draw the oriented graph and draw all possible trees.* *[JNTU June 2008]*

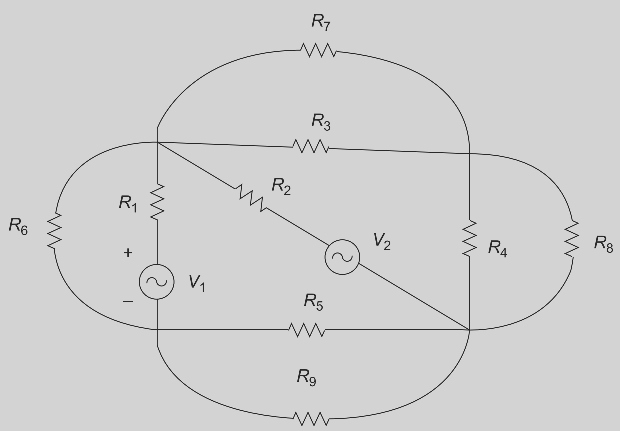


Fig. 5.54

Solution Replace the network with a graph. The voltage sources have been short-circuited.

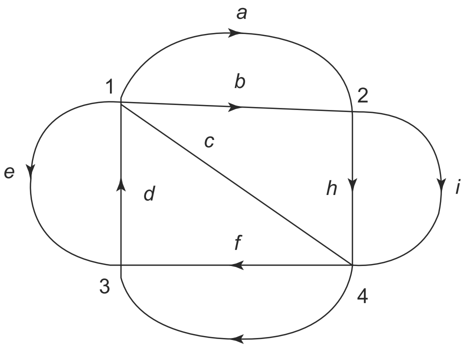


Fig. 5.55

Some possible trees are

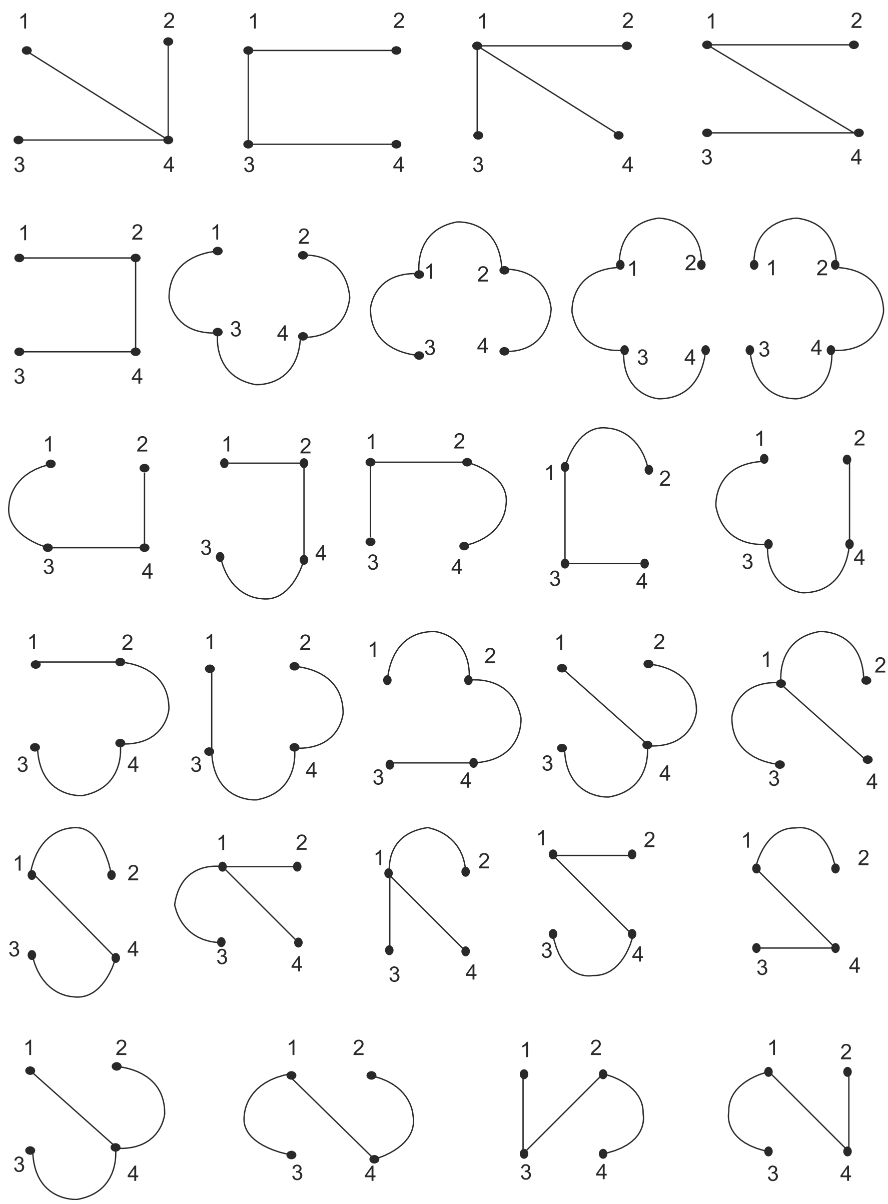


Fig. 5.56

Example 5.19 *For the graph in the Fig. 5.57, write the cut set schedule and obtain the relation between tree branch voltages and branch voltages.*

[JNTU June 2008]

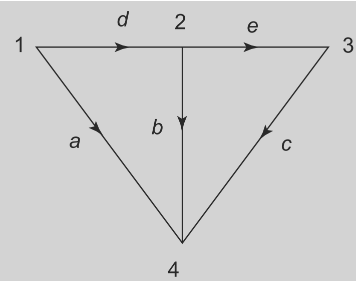


Fig. 5.57

Solution 6 cut-sets are possible for the graph.

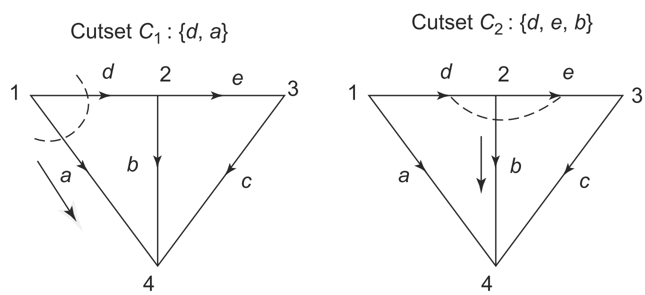


Fig. 5.58

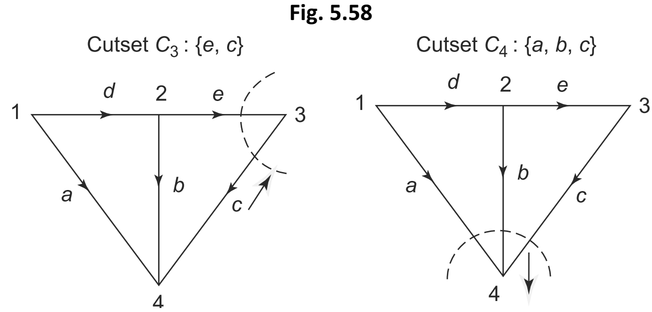


Fig. 5.59

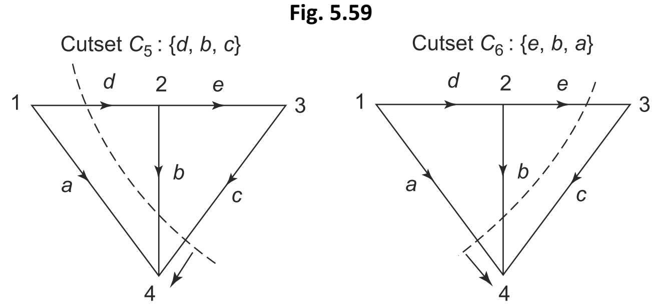


Fig. 5.60

Applying KCL for each cut set we get the following equations.

$$C_1: i_d + i_a = 0$$

$$C_2: i_b + i_e - i_d = 0$$

$$C_3: i_e - i_c = 0$$

$C_4: i_a + i_b + i_c = 0$
$C_5: i_b + i_e - i_d = 0$
$C_6: i_a + i_b + i_e = 0.$

The equations can be put into matrix form.

$$Q_f = \begin{array}{c} \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In order to find the relation between branch voltage and tree branch voltage. Let us consider the tree

There are 5 branches. Let the branch voltages be V_a, V_b, V_c, V_d and V_e.

There are 3 twigs the twig–voltages be V_{td}, V_{te}, V_{tb}.

We can express branch voltages in terms of twig voltages.

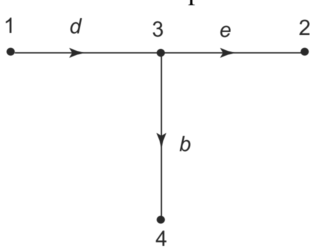


Fig. 5.61

$V_d + V_b = V_a$ $\qquad$ $V_a = V_{td} + V_{tb}$

$V_e + V_c = V_b$ $\qquad$ $V_c = V_{tb} - V_{te}$

$V_b = V_{tb}$

$V_d = V_{td}$

$V_e = V_{te}.$

The above equations can be written in matrix form as

$$V_b = Q_f^T V_t$$

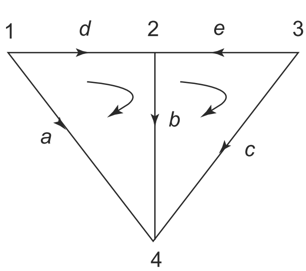


Fig. 5.62

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix} = V_b \qquad \begin{bmatrix} V_{td} \\ V_{te} \\ V_{tb} \end{bmatrix} = V_t$$

V_b is column matrix of branch–voltages and V_t is column matrix of twig voltages.

∴ The relation can be expressed as.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{td} \\ V_{te} \\ V_{tb} \end{bmatrix}$$

Example 5.20 *For the graph shown in Fig. 5.63, find the tie-set and cut-set matrices.*

[JNTU June 2009]

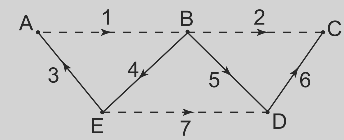


Fig. 5.63

Solution

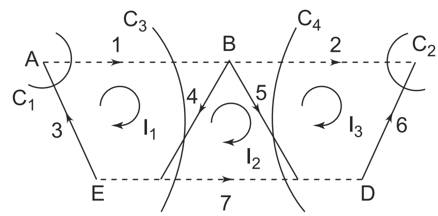


Fig. 5.64

Tie-set Matrix

Loop currents ↓ Branches →

$$\begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \end{array}$$

Cut-set Matrix

Cut sets ↓ Branches →

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

Example 5.21 *Obtain the cut-set matrix for the given network, as shown in Fig. 5.65.*

[JNTU Jan 2010]

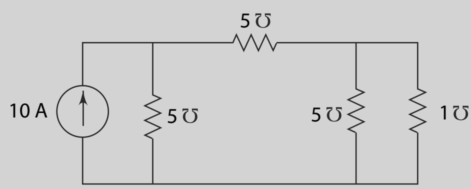


Fig. 5.65

Solution Cut-Set Matrix

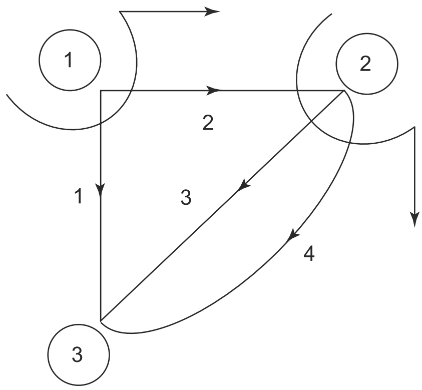


Fig. 5.66

$$
\begin{array}{cc}
\text{Cut-sets} & \text{Branches} \\
\downarrow & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\
\begin{matrix} C_1 \\ C_2 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}
\end{array}
$$

Example 5.22 *Find the cut-set matrix of the network as shown in Fig. 5.67 and obtain relationship between the branch currents.*

[JNTU Jan 2010]

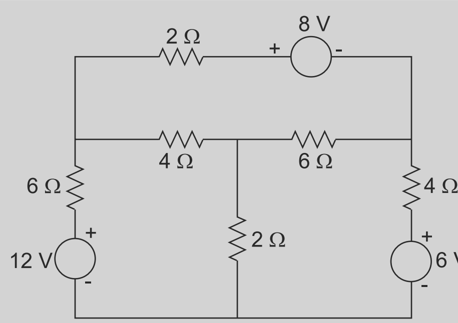


Fig. 5.67

Solution

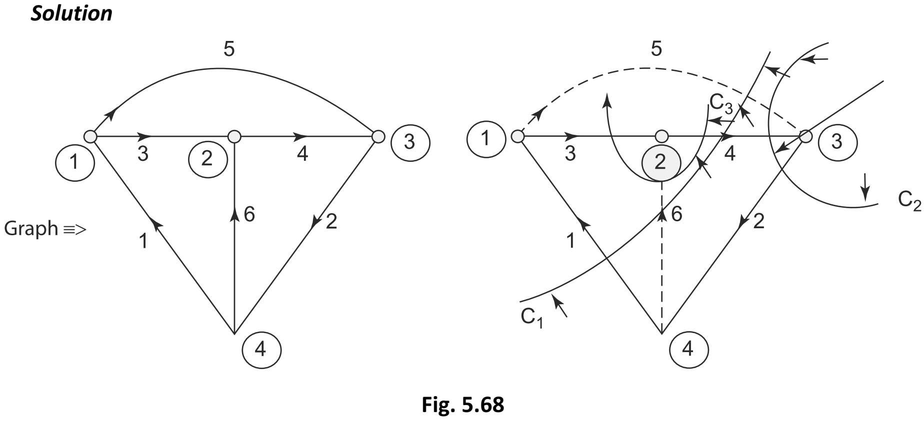


Fig. 5.68

$$\begin{array}{c} \text{Cut set} \\ \downarrow \\ C_1(1,6,4) \\ C_2(2,4,5) \\ C_3(3,6,4) \end{array} \begin{array}{c} \text{Branches} \rightarrow \\ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & +1 \end{bmatrix} \end{array}$$

KCL equations

$$i_1 - i_4 + i_6 = 0$$
$$i_2 - i_4 - i_5 = 0$$
$$i_3 - i_4 + i_6 = 0$$

Example 5.23 *Obtain the cut-set matrix for the network as shown in Fig. 5.69.* *[JNTU Jan 2010]*

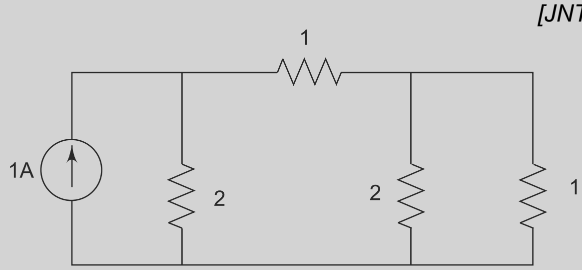


Fig. 5.69

Solution Graph obtained by open circuiting current source.

Fundamental cut set matrix is formed as

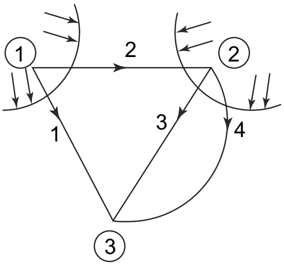


Fig. 5.70

$$\begin{array}{c} \text{Cut sets} \\ \downarrow \\ C_1 \\ C_2 \end{array} \begin{array}{c} \text{Branches} \rightarrow \\ \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

5.3 BASIC TIE-SET MATRICES FOR PLANAR NETWORKS

For a given tree of a graph, addition of each link between any two nodes forms a loop called the fundamental loop. In a loop there exists a closed path and a circulating current, which is called the link current. The current in any branch of a graph can be found by using link currents.

The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link. This loop is also called *f*-loop or a tie-set.

Consider a connected graph shown in Fig. 5.71 (a). It has four nodes and six branches. One of its trees is arbitrarily chosen and is shown in Fig. 5.71 (b).

The twigs of this tree are branches 4, 5 and 6. The links corresponding to this tree are branches 1, 2 and 3. Every link defines a fundamental loop of the network.

No. of nodes $n = 4$

No. of branches $b = 6$

No. of tree branches or twigs $= n - 1 = 3$

No. of link branches $I = b - (n - 1) = 3$

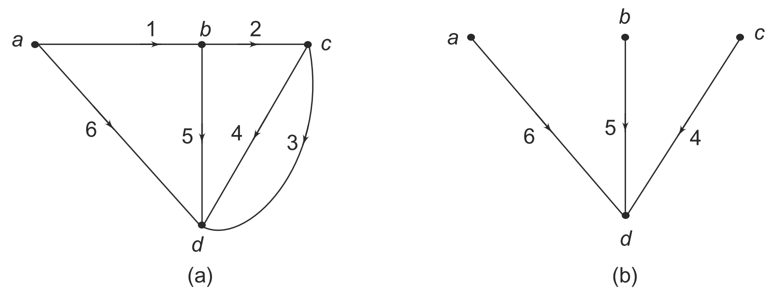


Fig. 5.71

Let $i_1, i_2, \ldots i_6$ be the branch currents with directions as shown in Fig. 5.71 (a). Let us add a link in its proper place to the tree as shown in 5.71 (c). It is seen that a loop I_1 is formed by the branches 1, 5 and 6. There is a formation of link current, let this current be I_1. This current passes through the branches 1, 5 and 6. By convention a fundamental loop is given the same orientation as its defining link, i.e., the link current I_1 coincides with the branch current direction i_1 in *ab*. A tie set can also be defined as the set of branches that forms a closed loop in which the link current flows. By adding the other link branches 2 and 3, we can form two more fundamental loops or *f*-loops with link currents I_2 and I_3 respectively as shown in Figs 5.71 (d) and (e).

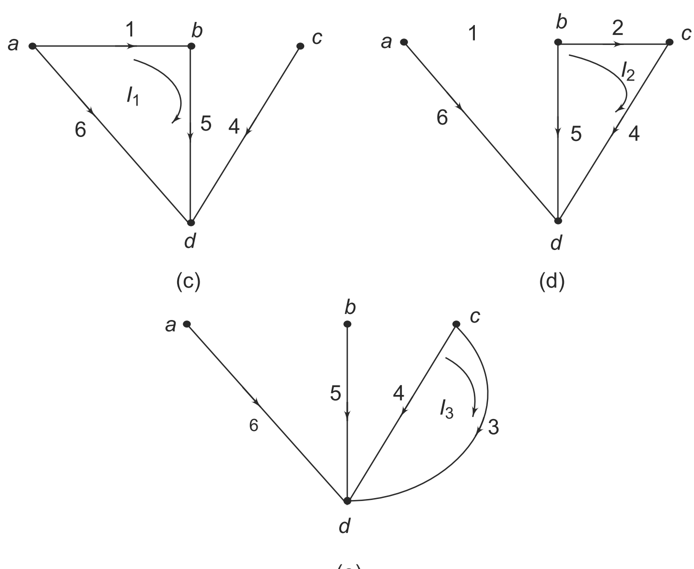


Fig. 5.72

5.3.1 Tie-Set Matrix

Kirchhoff's voltage law can be applied to the f-loops to get a set of linearly independent equations. Consider Fig. 5.73.

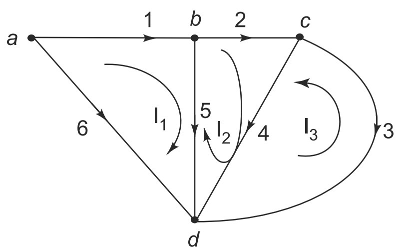


Fig. 5.73

There are three fundamental loops I_1, I_2 and I_3 corresponding to the link branches 1, 2 and 3 respectively. If V_1, V_2, … V_6 are the branch voltages the KVL equations for the three f-loops can be written as

$$\begin{aligned} V_1 + V_5 - V_6 &= 0 \\ V_2 + V_4 - V_5 &= 0 \\ V_3 - V_4 &= 0 \end{aligned} \tag{5.9}$$

In order to apply KVL to each fundamental loop, we take the reference direction of the loop which coincides with the reference direction of the link defining the loop.

The above equation can be written in matrix form as

$$\begin{array}{c} \text{loop branches} \rightarrow \qquad 3\times 6 \\ \begin{array}{c} \downarrow \\ I_1 \\ I_2 \\ I_3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{array}$$

$$B\,V_b = 0 \tag{5.10}$$

where B is an $I \times b$ matrix called the tie-set matrix or fundamental loop matrix and V_b is a column vector of branch voltages.

The tie set matrix B is written in a compact form as $B\,[b_{ij}]$ (5.11)

The element b_{ij} of B is defined as

$b_{ij} = 1$ when branch b_j is in the f-loop I_i (loop current) and their reference directions coincide.

$b_{ij} = -1$ when branch b_j is in the f-loop I_i (loop current) and their reference directions are opposite.

$b_{ij} = 0$ when branch b_j is not in the f-loop I_i.

5.3.2 Tie-set Matrix and Branch Currents

It is possible to express branch currents as a linear combination of link current using matrix B.

If I_B and I_l represents the branch current matrix and loop current matrix respectively and B is the tie-set matrix, then

$$[I_b] = [B^T]\,[I_L] \tag{5.12}$$

where $[B^T]$ is the transpose of the matrix $[B]$. Equation (5.12) is known as link current transformation equation.

Consider the tie-set matrix of Fig. 5.21.

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

The branch current vector $[I_b]$ is a column vector.

$$[I_b] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$$

The loop current vector $[I_L]$ is a column vector

$$[I_L] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Therefore the link current transformation equation is given by $[I_b] = [B^T]\,[I_L]$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The branch currents are

$$i_1 = I_1$$
$$i_2 = I_2$$
$$i_3 = I_3$$
$$i_4 = I_2 - I_3$$
$$i_5 = I_1 - I_2$$
$$i_6 = -I_1$$

Example 5.24 *For the electrical network shown in Fig. 5.74 draw its topological graph and write its incidence matrix, tie-set matrix, link current transformation equation and branch currents.*

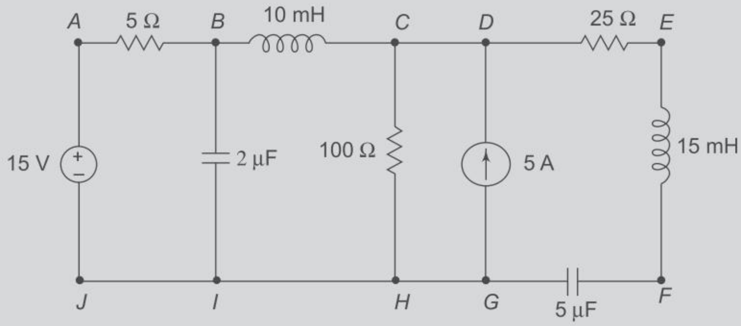


Fig. 5.74

Solution Voltage source is short circuited, current source is open circuited, the points which are electrically at same potential are combined to form a single node. The graph is shown in Fig. 5.75.

Combining the simple nodes and arbitrarily selecting the branch current directions the oriented graph is shown in Fig. 5.76. The simplified consists of three nodes. Let them be x, y and z and five branches 1, 2, 3, 4 and 5. The complete incidence matrix is given by

$$\text{nodes} \downarrow \quad \text{branches} \rightarrow$$

$$A = \begin{matrix} \\ x \\ y \\ z \end{matrix} \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & 1 \end{bmatrix}$$

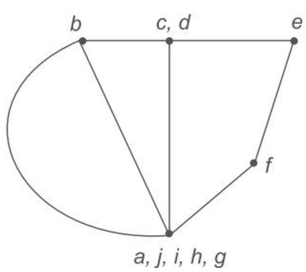


Fig. 5.75

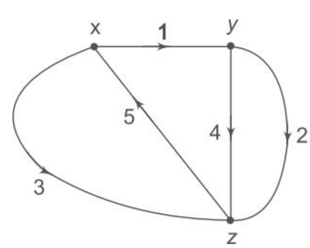


Fig. 5.76

Let us choose node z as the reference or datum node for writing the reduced incidence matrix A_1 or we can obtain A_1 by deleting the last row elements in A.

$$\text{nodes} \downarrow \quad \text{branches} \rightarrow$$

$$A_1 = \begin{matrix} x \\ y \end{matrix} \overset{\begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix}}{\begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ -1 & 1 & 0 & 1 & 0 \end{bmatrix}}$$

For writing the tie-set matrix, consider the tree in the graph in Fig. 5.76.

No. of nodes $n = 3$

No. of branches $= 5$

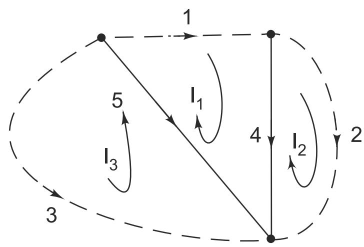


Fig. 5.77

No. of tree branches or twigs $= n - 1 = 2$

No. of link branches $I = b - (n - 1) = 5 - (3 - 1) = 3$

The tree shown in Fig. 5.77 consists of two branches 4 and 5 shown with solid lines and the link branches of the tree are 1, 2 and 3 shown with dashed lines. The tie-set matrix or fundamental loop matrix is given by

loop branches →

$$\begin{array}{cc} \downarrow & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \end{array}$$

$$B = \begin{array}{c} I_1 \\ I_2 \\ I_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

To obtain the link current transformation equation and thereby branch currents the transpose of B should be calculated.

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The equation $[I_b] = [B^T]\,[I_L]$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The branch currents are given by

$$i_1 = I_1$$

$$i_2 = I_2$$

$$i_3 = I_3$$

$$i_4 = I_1 - I_2$$

$$i_5 = I_1 + I_3$$

Example 5.25 *Write the tie-set matrix for the graph shown in the Fig. 5.78, taking the tree consisting of branches 2, 4, 5.*

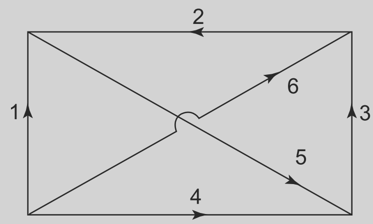


Fig. 5.78

Solution The twigs of the tree are 2, 4 and 5. The links corresponding to the tree are 1, 3 and 6 as shown in the Fig. 5.79.

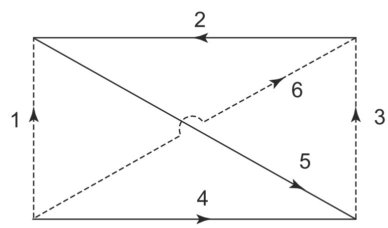


Fig. 5.79

Number of nodes $n = 4$

Number of branches $b = 6$

Number of tree branches of twigs

$= n - 1 = 3$

Number of link branches $= b - (n - 1) = 3$

For writing the tie-set matrix consider the three links one at a time. The tie-set matrix of fundamental loop matrix is given by

$$B = \begin{array}{r} \text{branches} \\ \text{Loops } I_1 \\ I_2 \\ I_3 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

The tie-sets are shown in the Figs 5.80 (a), (b) and (c)

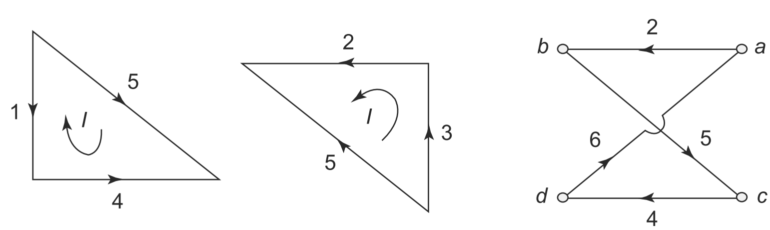


Fig. 5.80

Example 5.26 *For the given graph and tree shown in the Fig. 5.81, write the tie-set matrix and obtain the relationship between the branch currents and link currents.*

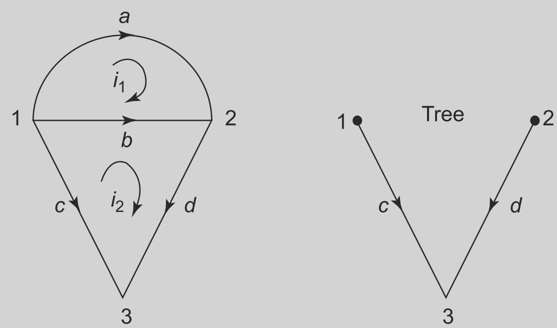


Fig. 5.81

Solution Number of link branches = $b (n - 1)$

where b is number of branches and n is number of nodes

$\therefore$ Link branches = 4 − (3 − 1) = 2

The link branches are a and b

Let the branch currents are i_a, i_b, i_c and i_d.

The two links currents are i_1 and i_2 as shown in the Fig. 5.82.

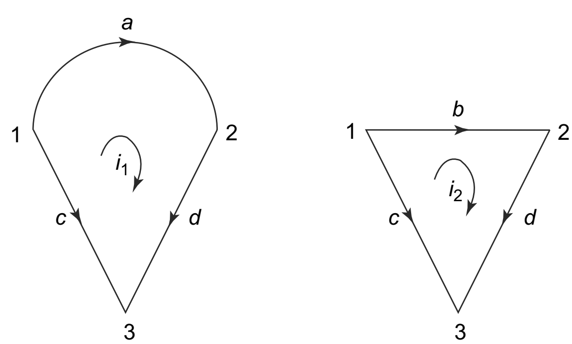


Fig. 5.82

There are two fundamental loops corresponding to the link branches a and b. If V_a and V_b are the branch voltages, the KVL equations for the two f-loops can be written as

$$V_a + V_d - V_c = 0$$

$$V_b + V_d - V_c = 0$$

The above equation can be written as

$$\begin{array}{c} \text{Loop} \quad \text{branches} \\ \text{Currents} \end{array}$$

$$\begin{matrix} \downarrow \\ i_1 \\ i_2 \end{matrix} \begin{matrix} \text{a} & \text{b} & \text{c} & \text{d} \\ \end{matrix} \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & +1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = 0$$

Example 5.27 *For the topological graph shown in the Fig. 5.83, obtain the fundamental tie-set matrix choosing the tree containing the two elements 5 and 6.*
[JNTU May/June 2004]

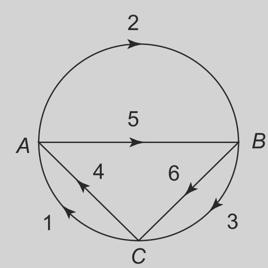


Fig. 5.83

Solution The tree of the graph is shown with solid lines (5 and 6) and the links are shown with dashed lines (1, 2, 3, 4) as in Fig. 5.84.

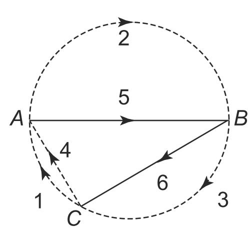


Fig. 5.84

For a given tree of a graph, addition of each link between any two nodes forms a loop called the fundamental loop. In a loop, there exists a closed path and a circulating current, which is called the link current.

The fundamental loop formed by one link at a time, has a unique path in the tree joining. The two nodes of the link. This loop is also called f-loop or a tie-set. Every link defines a fundamental loop of the network.

No. of nodes in the graph $n = 3 = (A, B, C)$

No. of branches $b = 6 = (1, 2, 3, 4, 5, 6)$

No. of tree branches or twigs $n - 1 = 2 = (5, 6)$

No. of link branches, $l = b - (n - 1) = 4$ (1, 2, 3, 4)

Tie-sets are formed as shown in Fig. 5.85

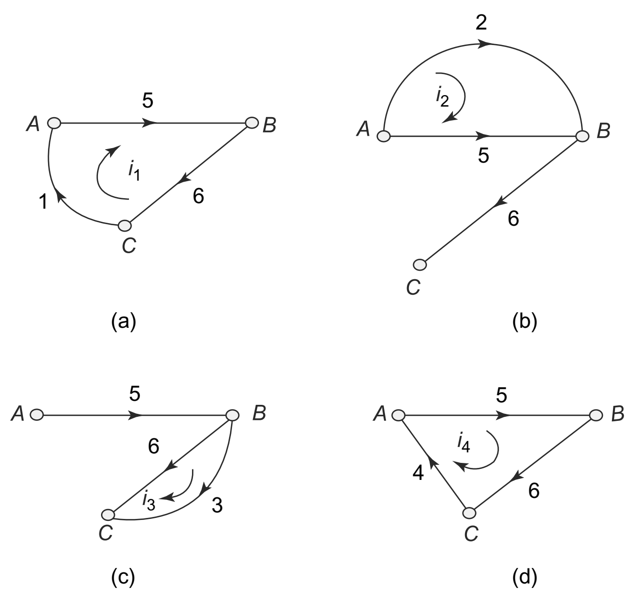


Fig. 5.85

The KVL equations for the three f-loops can be written as

$$V_1 + V_5 + V_6 = 0$$
$$V_2 - V_5 = 0$$
$$V_3 - V_6 = 0$$
$$V_4 + V_5 + V_6 = 0$$

In order to apply KVL to each loop, we take the reference direction of the loop which coincides with the reference direction of the link defining the loop.

The above equations can be written as

$[B]\,[V_b] = 0$, where B is a 4×6 tie-set Matrix.

Loops Branches →

$$\begin{matrix} \downarrow & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}$$

$$\begin{matrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{matrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, tie set matrix, $B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

Example 5.28 *Draw the oriented graph of the network shown in Fig. 5.86 and write the incidence matrix.* *[JNTU May 2007]*

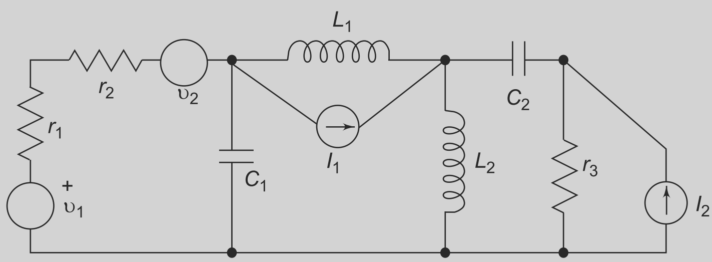


Fig. 5.86

Solution Directions of currents are arbitrarily assumed as shown in the circuit of Fig. 5.87.

Ideal voltage sources and current sources do not appear in the graph of a linear network. Ideal voltage source is represented by short circuit and an ideal current source is replaced by an open circuit. The nodes that appear in the graph are numbered (1) (2) (3) (4) and (5); branches as a, b, c, d, e, f and g. The graph is as shown in the Fig. 5.88

For a graph with n nodes and b branches, the order of the incidence matrix is $(n - 1) \times b$. Choose node (5) as reference (or datum) node for writing incidence matrix. The required incidence matrix is given by

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

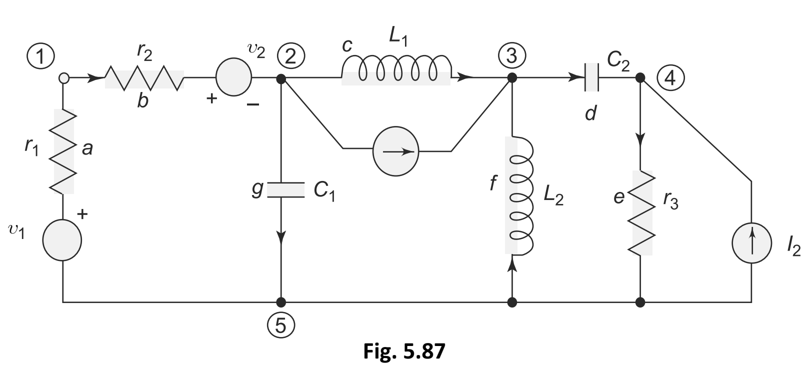


Fig. 5.87

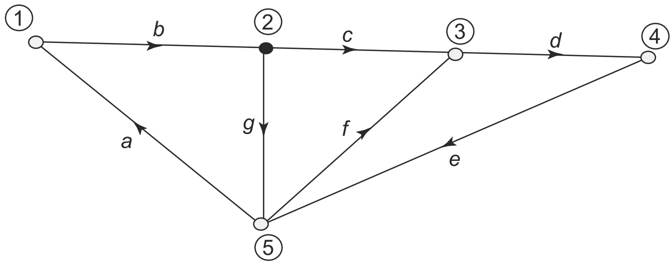


Fig. 5.88

Example 5.29 *Draw the oriented graph of the network shown in Fig. 5.89.*
[JNTU May 2007]

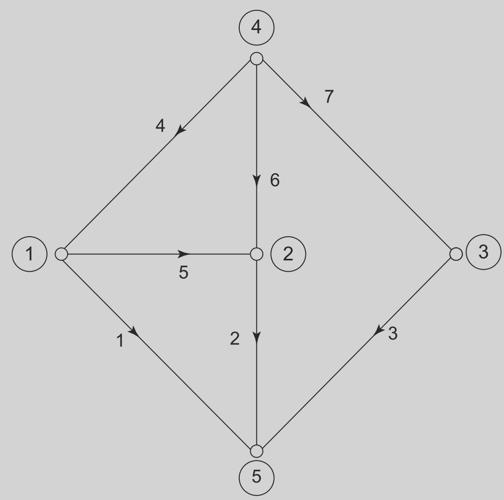
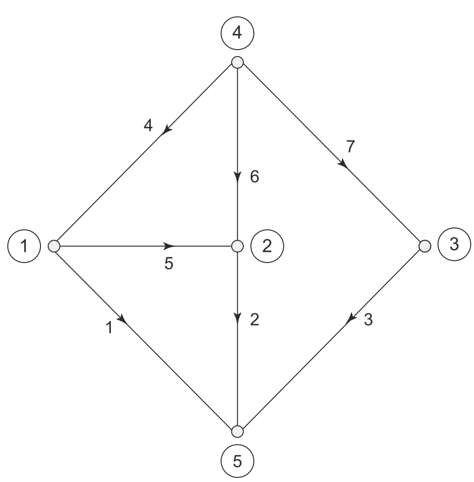


Fig. 5.89

Solution The graph represented in figure itself represents the oriented graph in which (1)-(5) are nodes and 1-7 are branches.



Oriented graph

Fig. 5.90

Example 5.30 *Write the Tie-set matrix for the graph shown in Fig. 5.91, taking the tree consisting of branches.*

[JNTU May 2006]

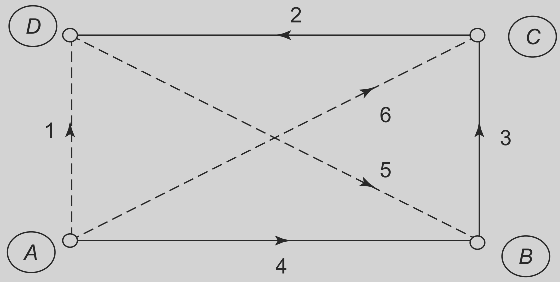


Fig. 5.91

Solution

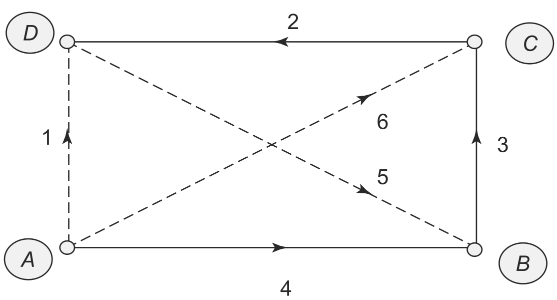


Fig. 5.92

Basictiesets \ e	1	2	3	4	5	6
(5, 3, 2)	0	−1	−1	0	1	0
(6, 3, 4)	0	0	−1	−1	0	1
(1, 2, 3, 4)	1	−1	−1	−1	0	0

Example 5.31 *For the network shown in Fig. 5.93 Find the tie-set matrix loop current.*

[JNTU June 2008]

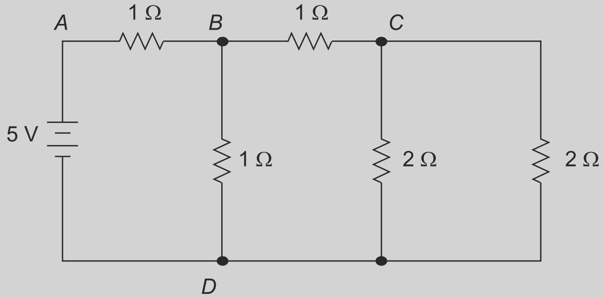


Fig. 5.93

Solution First replace the circuit with the network graph.

I_1, I_2, I_3 are loop currents corresponding to the branches.

There are three f-loops. We can apply KVL for this f-loops.

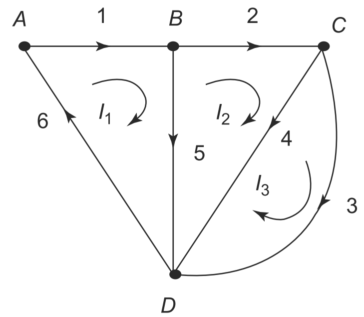


Fig. 5.94

$$V_1 + V_5 + V_6 = 0$$
$$V_2 + V_4 - V_5 = 0$$
$$V_3 - V_4 = 0$$

The above equations can be written in matrix form as

loop branches $\rightarrow$

$$\begin{matrix} \downarrow & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}$$
$$\begin{matrix} I_1 \\ I_2 \\ I_3 \end{matrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

It is possible to express branch currents as a linear combination of link currents using matrix B.

Let I_b represents branch current matrix.

I_L represents loop current matrix.

$$I_b = [B^T]\,[I_L]$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$I_b = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} \qquad I_L = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$I_1 = i_1$ Loop currents are I_1, I_2, I_3

$I_2 = i_2$

$I_3 = i_3$

Example 5.32 *Write the matrix loop equation for the network shown in Fig. 5.95 and determine the loop currents.*

[JNTU June 2008]

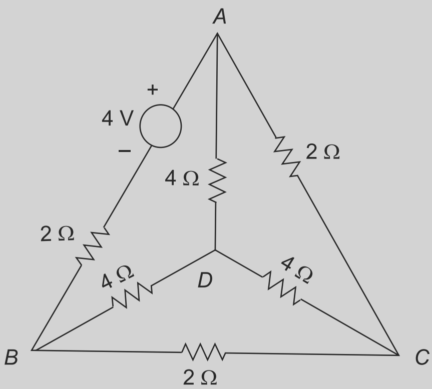


Fig. 5.95

Solution The graph for the following circuit is

This can be represented in matrix form as follows.

$$\begin{array}{c} \text{nodes} \\ \downarrow \end{array} \quad \text{branches} \rightarrow$$

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

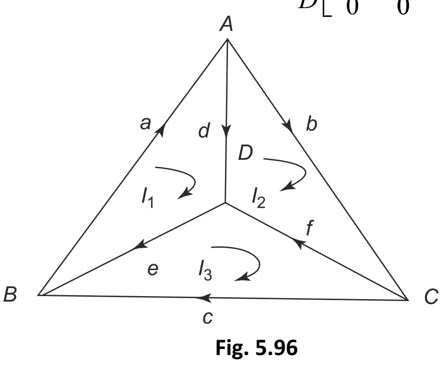


Fig. 5.96

Consider the loop equations

$$V_a + V_d + V_e = 0$$

$$V_b + V_f - V_d = 0$$

$$-V_e + V_c - V_f = 0$$

In order to find loop currents, we can apply mesh analysis.

Applying KVL to each loops.

$$10\,I_1 - 4\,I_2 - 4\,I_3 = 4 \qquad (5.13)$$

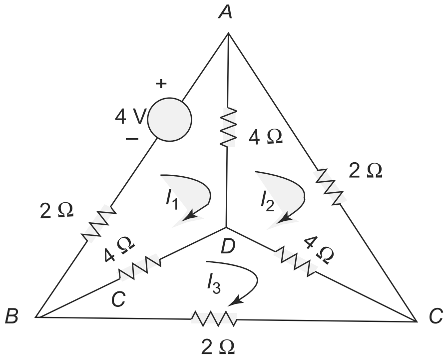


Fig. 5.97

$$10\,I_2 - 4\,I_1 - 4\,I_3 = 0 \tag{5.14}$$

$$10\,I_3 - 4\,I_1 - 4\,I_2 = 0 \tag{5.15}$$

From (5.15) we have

$$I_3 = 4/10\,(I_1 + I_2)$$

Substituting this in equation (5.14),

$$10I_2 - 4I_1 - \frac{16}{10}(I_1 + I_2) = 0$$

$$I_2\,(10 - 1.6) - 5.6\,I_1 = 0$$

$$-5.6\,I_1 + 8.4\,I_2 = 0 \tag{5.16}$$

The first equation reduces

$$8.4\,I_1 - 5.6\,I_2 = 4 \tag{5.17}$$

By solving I_1 and I_2 (5.16) and (5.17), we get

$$I_1 = 0.857\,A$$

$$I_2 = 0.57\,A$$

$$I_3 = 4/10\,(I_1 + I_2) = 0.5708\,A$$

Example 5.33 *For the resistive network as shown in Fig. 5.98 write a tie-set schedule and equilibrium equations, on current basis. Determine the branch currents and branch voltages.*

[JNTU June 2009]

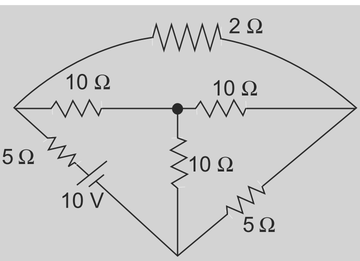


Fig. 5.98

Solution In order to determine tie-set schedule, we must draw graph of given network, and to draw the graph, we have to replace all the resistors by line segments where as the voltage source must be replaced with short ckt.

Graph

There are 4 nodes, 6 branches.

Tree contains 4 nodes, 3 branches

d, e, f → Twigs

a, b, c → links

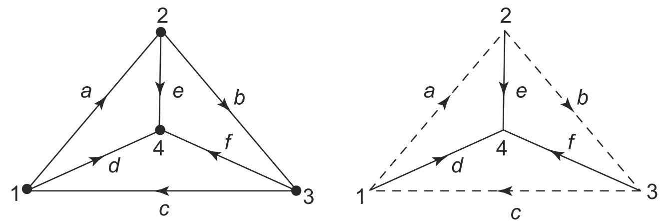


Fig. 5.99

Tie sets

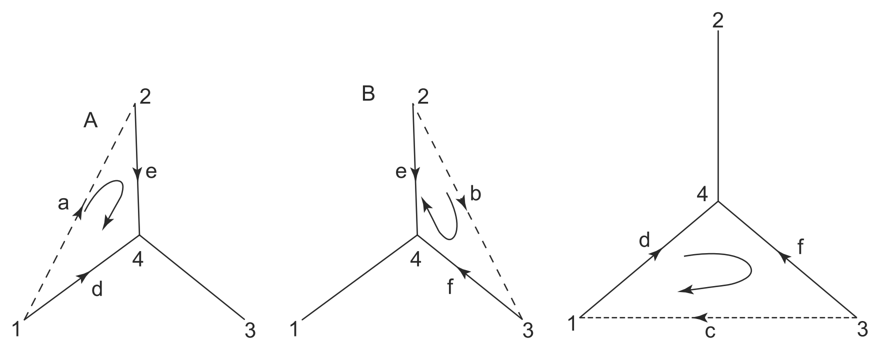


Fig. 5.100

Tie set schedule

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Example 5.34 *Draw the oriented network graph from the incidence matrix given below.* *[JNTU June 2009]*

Nodes	*Branches*					
	1	*2*	*3*	*4*	*5*	*6*
A	−1	0	0	1	−1	0
B	1	−1	0	0	0	−1
C	0	1	−1	0	1	0
D	0	0	+1	−1	0	+1

Solution

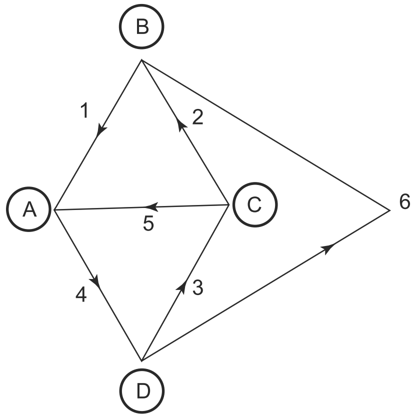


Fig. 5.101

Example 5.35 *For the n/w shown in Fig. 5.102, draw the oriented graph, select a tree and obtain a tie-set matrix. Write down the KVL equations from the tie-set matrix.* *[JNTU June 2009]*

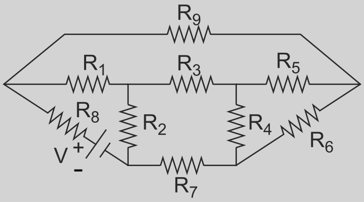


Fig. 5.102

Solution

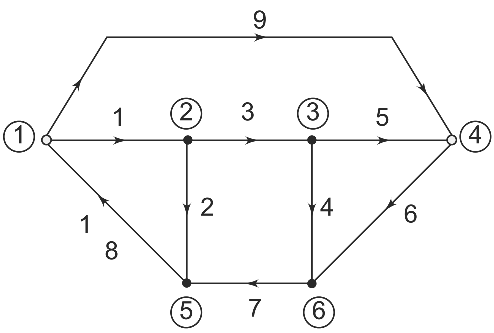


Fig. 5.103

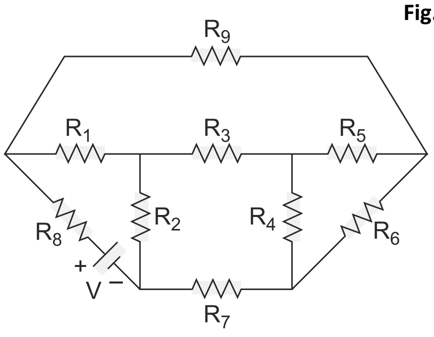


Fig. 5.104

Replacing voltage source by a short circuit, we obtain the graph as

The chosen tree

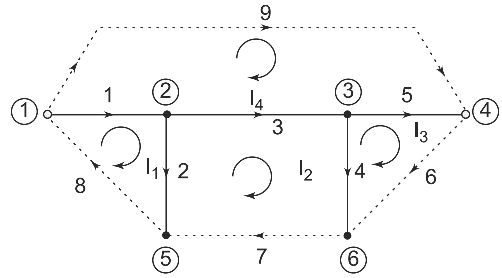


Fig. 5.105

The tie-set matrix is given as

Loop current ↓ Branches →

Loop current	1	2	3	4	5	6	7	8	9
I_1	1	1	0	0	0	0	0	1	0
I_2	0	–1	1	1	0	0	1	0	0
I_3	0	0	0	–1	1	1	0	0	0
I_4	–1	0	–1	0	–1	0	0	0	1

The KVL equations are as follows

$$v_1 + v_2 + v_8 = 0$$
$$-v_2 + v_3 + v_4 + v_7 = 0$$
$$-v_4 + v_5 + v_6 = 0$$
$$-v_1 - v_3 - v_5 + v_9 = 0$$

Example 5.36 *For the given graph shown in Figs. 5.106 and 5.107 write the tie-set schedule and obtain the relation between branch currents and link currents.* *[JNTU June 2009]*

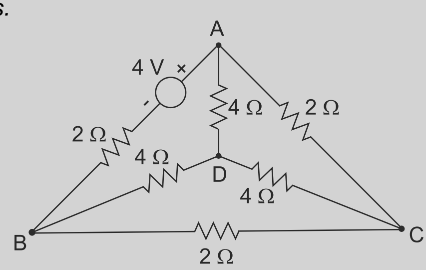


Fig. 5.106

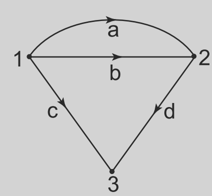


Fig. 5.107

Solution

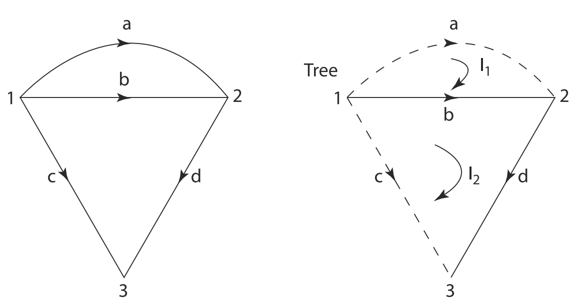


Fig. 5.108

Tie-Set Matrix

Loop currents ↓ Branches →

Loop currents	a	b	c	d
I_1	1	–1	0	0
I_2	0	1	–1	1

Relation between Link Current (I_L) and Branch Current (i_b)

$$I_1 = i_a$$
$$-I_1 + I_2 = i_b$$
$$-I_2 = i_c$$
$$I_2 = i_d$$

Example 5.37 *Write the tie-set schedule and write tie-set matrices also. Write the relationship between the branch current and link currents of the given Fig. 5.109.*

[JNTU Jan 2010]

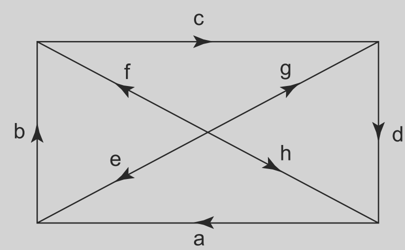


Fig. 5.109

Solution

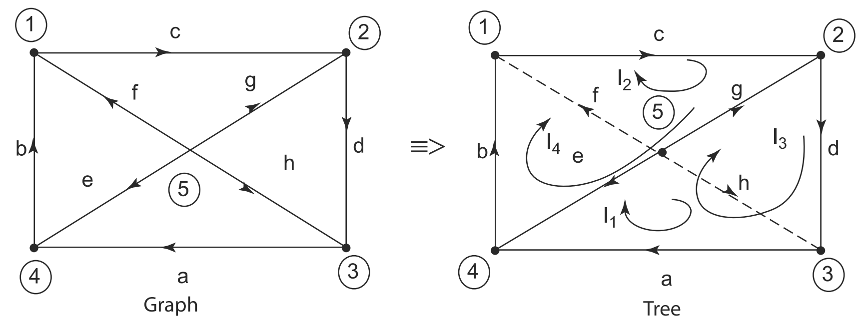


Fig. 5.110

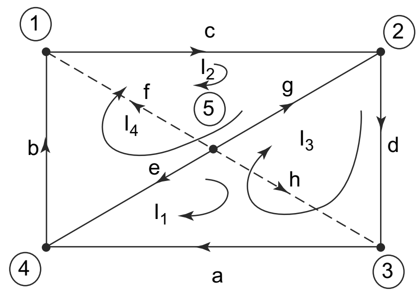


Fig. 5.111

$$
\begin{array}{c} \text{Loop Current} \\ \downarrow \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array}
\begin{array}{c} \text{Branches} \rightarrow \\ \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \\ \text{Tie-set matrix} \end{array}
$$

Relation between
Branch & loop current

$$j_a = I_1 + I_3 \qquad j_b = I_4 \qquad j_c = I_2 + I_4, \qquad j_d = I_3$$
$$j_e = -(I_1 + I_3), \qquad i_j = I_2, \qquad j_g = I_3 - (I_2 + I_4), \qquad j_h = I_1$$

($j_a, j_b, j_c \ldots$ .are the branch currents)

5.4 LOOP AND NODAL METHODS OF ANALYSIS OF NET WORKS WITH INDEPENDENT AND DEPENDENT VOLTAGE AND CURRENT SOURCES

5.4.1 Mesh (Loop) Analysis

Mesh and nodal analysis are two basic important techniques used in finding solutions for a network. The suitability of either mesh or nodal analysis to a particular problem depends mainly on the number of voltage sources or current sources. If a network has a large number of voltage sources, it is useful to use mesh analysis; as this analysis requires that all the sources in a circuit be voltage sources. Therefore, if there are any current sources in a circuit they are to be converted into equivalent voltage sources, if, on the other hand, the network has more current sources, nodal analysis is more useful.

Mesh analysis is applicable only for planar networks. For non-planar circuits mesh analysis is not applicable. A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.

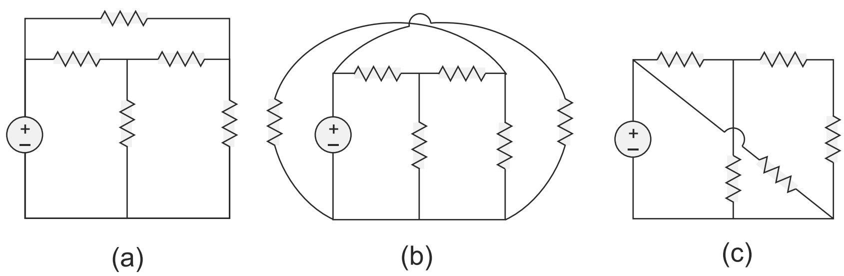


Fig. 5.112

Figure 5.112(a) is a planar circuit. Figure 5.112(b) is a non-planar circuit and Fig. 5.112(c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loop is a closed path. A mesh is defined as a loop which does not contain any other loops within it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.

Observation of the Fig. 5.113 indicates that there are two loops *abefa*, and *bcdeb* in the network. Let us assume loop currents I_1 and I_2 with directions as indicated in the figure. Considering the loop *abefa* alone, we observe that current I_1 is passing through R_1, and $(I_1 - I_2)$ is passing through R_2. By applying Kirchhoff's voltage law, we can write

$$V_s = I_1R_1 + R_2(I_1 - I_2)$$

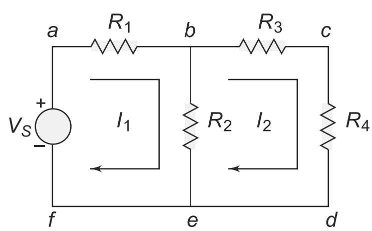


Fig. 5.113

Similarly, if we consider the second mesh *bcdeb*, the current I_2 is passing through R_3 and R_4, and $(I_2 - I_1)$ is passing through R_2. By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2(I_2 - I_1) + R_3 I_2 + R_4 I_2 = 0$$

By rearranging the above equations, the corresponding mesh current equations are

$$I_1(R_1 + R_2) - I_2 R_2 = V_s$$

$$-I_1 R_2 + (R_2 + R_3 + R_4) I_2 = 0$$

By solving the above equations, we can find the currents I_1 and I_2. If we observe Fig. 5.113, the circuit consists of five branches and four nodes, including the reference node. The number of mesh currents is equal to the number of mesh equations.

And the number of equations = branches – (nodes – 1). In Fig. 5.113, the required number of mesh currents would be 5 – (4 – 1) = 2.

In general, if we have B number of branches and N number of nodes including the reference node then the number of linearly independent mesh equations $M = B - (N - 1)$.

Example 5.38 *Write the mesh current equations in the circuit shown in Fig. 5.114, and determine the currents.*

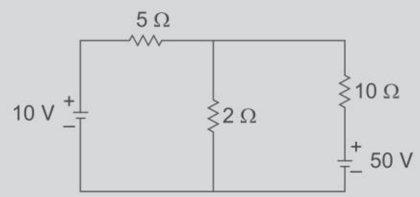


Fig. 5.114

Solution Assume two mesh currents in the direction as indicated in Fig. 5.115.

The mesh current equations are

$$5I_1 + 2(I_1 - I_2) = 10$$

$$10I_2 + 2(I_2 - I_1) + 50 = 0$$

We can rearrange the above equations as

$$7I_1 - 2I_2 = 10$$

$$-2I_1 + 12I_2 = -50$$

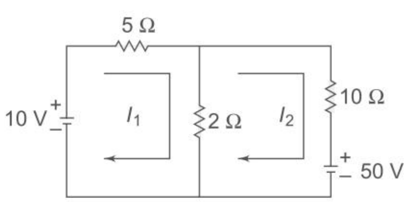


Fig. 5.115

By solving the above equations, we have $I_1 = 0.25\ A$, and $I_2 = -4.125\ A$

Here the current in the second mesh, I_2, is negative; that is the actual current I_2 flows opposite to the assumed direction of current in the circuit of Fig. 5.115.

Example 5.39 *Determine the mesh current I_1 in the circuit shown in Fig. 5.116.*

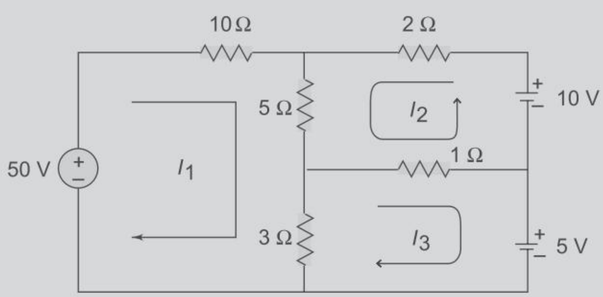


Fig. 5.116

Solution From the circuit, we can form the following three mesh equations

$$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50$$

$$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10$$

$$3(I_3 - I_1) + 1(I_3 + I_2) = -5$$

Rearranging the above equations we get

$$18I_1 + 5I_2 - 3I_3 = 50$$

$$5I_1 + 8I_2 + I_3 = 10$$

$$-3I_1 + I_2 + 4I_3 = -5$$

According to Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 50 & 5 & -3 \\ 10 & 8 & 1 \\ -5 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{1175}{356}$$

or $I_1 = 3.3$ A

Similarly,

$$I_2 = \frac{\begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ -3 & -5 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{-355}{356}$$

or $I_2 = -0.997\ A$

$$I_3 = \frac{\begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}} = \frac{525}{356}$$

or $I_3 = 1.47$ A

∴ $I_1 = 3.3$ A, $I_2 = -0.997$ A, $I_3 = 1.47$ A

5.4.2 Mesh Equations by Inspection Method

The mesh equations for a general planar network can be written by inspection without going through the detailed steps. Consider a three mesh networks as shown in Fig. 5.117.

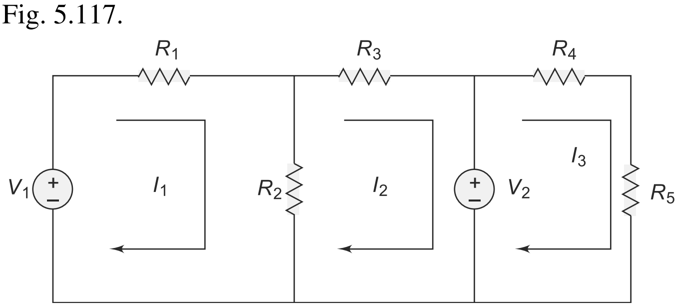


Fig. 5.117

The loop equations are

$$I_1R_1 + R_2(I_1 - I_2) = V_1 \tag{5.18}$$

$$R_2(I_2 - I_1) + I_2R_3 = -V_2 \tag{5.19}$$

$$R_4I_3 + R_5I_3 = V_2 \tag{5.20}$$

Reordering the above equations, we have

$$(R_1 + R_2)\, I_1 - R_2I_2 = V_1 \tag{5.21}$$

$$-R_2I_1 + (R_2 + R_3)I_2 = -V_2 \tag{5.22}$$

$$(R_4 + R_5)I_3 = V_2 \tag{5.23}$$

The general mesh equations for three mesh resistive network can be written as

$$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a \tag{5.24}$$

$$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b \tag{5.25}$$

$$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c \tag{5.26}$$

By comparing the Eqs 5.21, 5.22 and 5.23 with Eqs 5.24, 5.25, and 5.26 respectively, the following observations can be taken into account.

1. The self resistance in each mesh.
2. The mutual resistances between all pairs of meshes and
3. The algebraic sum of the voltages in each mesh.

The self resistance of loop 1, $R_{11} = R_1 + R_2$, is the sum of the resistances through which I_1 passes.

The mutual resistance of loop 1, $R_{12} = -R_2$, is the sum of the resistances common to loop currents I_1 and I_2. If the directions of the currents passing through the common resistance are the same, the mutual resistance will have a positive sign; and if the directions of the currents passing through the common resistance are opposite then the mutual resistance will have a negative sign.

$V_a = V_1$ is the voltage which drives loop one. Here, the positive sign is used if the direction of the current is the same as the direction of the source. If the current direction is opposite to the direction of the source, then the negative sign is used.

Similarly, $R_{22} = (R_2 + R_3)$ and $R_{33} = R_4 + R_5$ are the self resistances of loops two and three, respectively. The mutual resistances $R_{13} = 0$, $R_{21} = -R_2$, $R_{23} = 0$, $R_{31} = 0$, $R_{32} = 0$ are the sums of the resistances common to the mesh currents indicated in their subscripts.

$V_b = -V_2$, $V_c = V_2$ are the sum of the voltages driving their respective loops.

Example 5.40 *Write the mesh equations for the circuit shown in Fig. 5.118.*

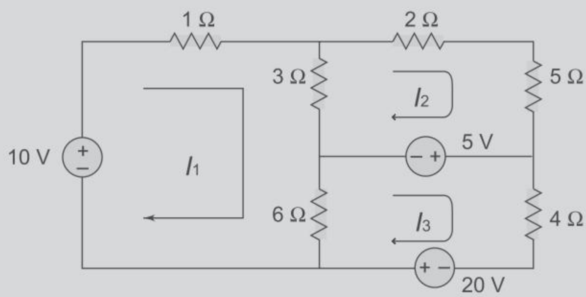


Fig. 5.118

Solution The general equations for three mesh network are

$$R_{11}I_1 \pm R_{12}I_2 \pm R_{13}I_3 = V_a \tag{5.27}$$

$$\pm R_{21}I_1 + R_{22}I_2 \pm R_{23}I_3 = V_b \tag{5.28}$$

$$\pm R_{31}I_1 \pm R_{32}I_2 + R_{33}I_3 = V_c \tag{5.29}$$

Consider Eq. (5.27),

R_{11} = self resistance of loop 1 = (1 Ω + 3 Ω + 6 Ω) = 10 Ω

R_{12} = the mutual resistance common to loop 1 and loop 2 = –3 Ω

Here, the negative sign indicates that the currents are in opposite direction

R_{13} = the mutual resistance common to loop 1 and 3 = –6 Ω

V_a = +10 V, the voltage driving the loop 1.

Here, the positive sign indicates the loop current I_1 is in the same direction as the source element.

Therefore, Eq. (5.27) can be written as

$$10I_1 - 3I_2 - 6I_3 = 10 \text{ V} \tag{5.30}$$

Consider Eq. (5.28),

R_{21} = mutual resistance common to loop 1 and loop 2 = –3 Ω

R_{22} = self resistance of loop 2 = (3 Ω + 2 Ω + 5 Ω) = 10 Ω

R_{23} = 0, there is no common resistance between loop 2 and loop 3.

V_b = –5 V, the voltage driving the loop 2.

Therefore, Eq. (5.28) can be written as

$$-3I_1 + 10I_2 = -5 \text{ V} \tag{5.31}$$

Consider Eq. (5.29),

R_{31} = mutual resistance common to loop 3 and loop 1 = –6 Ω

R_{32} = mutual resistance common to loop 3 and loop 2 = 0

R_{33} = self resistance of loop 3 = (6 Ω + 4 Ω) = 10 Ω

V_c = the algebraic sum of the voltages driving loop 3

= (5 V + 20 V) = 25 V

Therefore, Eq. (5.29) can be written as

$$-6I_1 + 10I_3 = 25 \text{ V} \tag{5.32}$$

The three mesh equation are

$$10I_1 - 3I_2 - 6I_3 = 10 \text{ V}$$
$$-3I_1 + 10I_2 = -5 \text{ V}$$
$$-6I_1 + 10I_3 = 25 \text{ V}$$

Example 5.41 *Determine the power dissipation in the 4 Ω resistor of the circuit shown in Fig. 5.119 by using mesh analysis.*

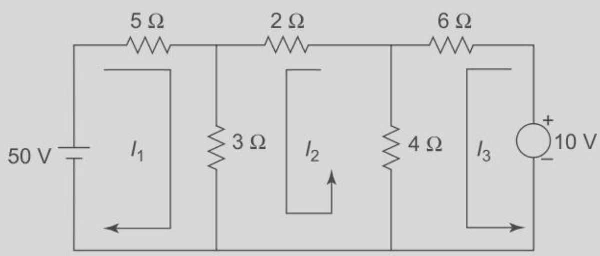


Fig. 5.119

Solution Power dissipated in the 4 Ω resistor is $P_4 = 4(I_2 - I_3)^2$

By using mesh analysis, we can find the currents I_2 and I_3.

From Fig. 5.119, we can form three equations.

From the given circuit in Fig. 5.119, we can obtain three mesh equations in terms of I_1, I_2 and I_3

$$8I_1 + 3I_2 = 50$$
$$3I_1 + 9I_2 - 4I_3 = 0$$
$$-4I_2 + 10I_3 = 10$$

By solving the above equations we can find I_1, I_2 and I_3.

$$I_2 = \frac{\begin{vmatrix} 8 & 50 & 0 \\ 3 & 0 & -4 \\ 0 & 10 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & +3 & 0 \\ 3 & 9 & -4 \\ 0 & -4 & 10 \end{vmatrix}} = \frac{-1180}{502} = -2.35 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 8 & 3 & 50 \\ 3 & 9 & 0 \\ 0 & -4 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & 3 & 0 \\ 3 & 9 & -4 \\ 0 & -4 & 10 \end{vmatrix}} = \frac{30}{502} = 0.06 \text{ A}$$

The current in the 4 Ω resistor = $(I_2 - I_3)$

$$= (-2.35 - 0.06)\text{A} = -2.41 \text{ A}$$

Therefore, the power dissipated in the 4 Ω resistor, $P_4 = (2.41)^2 \times 4 = 23.23$ W.

Example 5.42 *Using mesh analysis, determine the voltage V_S which gives a voltage of 50 V across the 10 Ω resistor as shown in Fig. 5.120.*

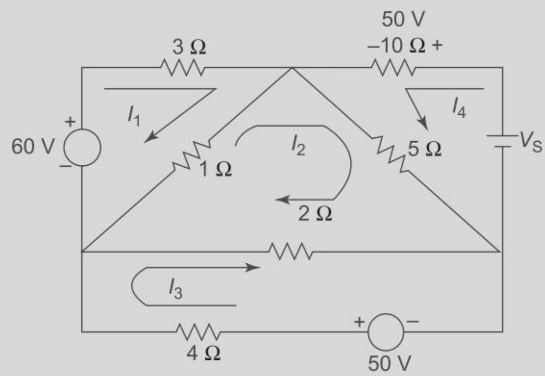


Fig. 5.120

Solution Since the voltage across the 10 Ω resistor is 50 V, the current passing through it is $I_4 = 50/10 = 5$ A.

From Fig. 5.120, we can form four equations in terms of the currents I_1, I_2, I_3 and I_4, as

$$4I_1 - I_2 = 60$$

$$-I_1 + 8I_2 - 2I_3 + 5I_4 = 0$$

$$-2I_2 + 6I_3 = 50$$

$$5I_2 + 15I_4 = VS$$

Solving the above equations, using Cramer's rule, we get

$$I_4 = \frac{\begin{vmatrix} 4 & -1 & 0 & 60 \\ -1 & 8 & -2 & 0 \\ 0 & -2 & 6 & 50 \\ 0 & 5 & 0 & V_S \end{vmatrix}}{\begin{vmatrix} 4 & -1 & 0 & 0 \\ -1 & 8 & -2 & 5 \\ 0 & -2 & 6 & 0 \\ 0 & 5 & 0 & 15 \end{vmatrix}}$$

$$\Delta = 4\begin{vmatrix} 8 & -2 & 5 \\ -2 & 6 & 0 \\ 5 & 0 & 15 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 & 5 \\ 0 & 6 & 0 \\ 0 & 0 & 15 \end{vmatrix}$$

$$= 4\{8(90) + 2(-30) + 5(-30)\} + 1\{-1(90)\}$$

$$\Delta = 1950$$

Example 5.43 *Determine the voltage V which causes the current I_1 to be zero for the circuit shown in Fig. 5.121. Use Mesh analysis.*

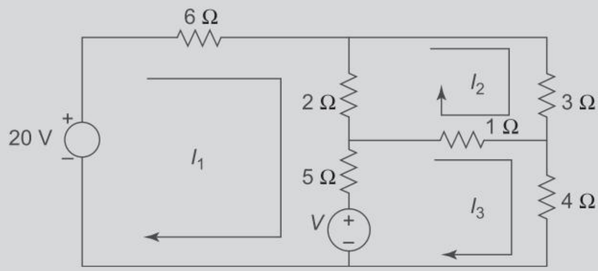


Fig. 5.121

Solution From Fig. 5.121 we can form three loop equations in terms of I_1, I_2, I_3 and V, as follows

$$13I_1 - 2I_2 - 5I_3 = 20 - V$$
$$-2I_1 + 6I_2 - I_3 = 0$$
$$-5I_1 - I_2 + 10I_3 = V$$

Using Cramer's rule, we get

$$I_1 = \frac{\begin{vmatrix} 20 & -V & -2 & -5 \\ & 0 & 6 & -1 \\ & V & -1 & +10 \end{vmatrix}}{\begin{vmatrix} 13 & -2 & -5 \\ -2 & +6 & -1 \\ -5 & -1 & +10 \end{vmatrix}}$$

$$\Delta_1 = (20 - V)(+60 - 1) + 2(V) - 5(-6\,V)$$
$$= 1180 - 27\,V$$

we have $\Delta = 557$

$$I_1 = \frac{\Delta_1}{557}$$

$\therefore$ $\Delta_1 = 0$

$$-27\,V + 1180 = 0$$

$\therefore$ $V = 43.7\text{ V}$

$$\Delta_4 = 4\begin{vmatrix} 8 & -2 & 0 \\ -2 & 6 & 50 \\ 5 & 0 & V_S \end{vmatrix} + 1\begin{vmatrix} -1 & -2 & 0 \\ 0 & 6 & 50 \\ 0 & 0 & V_S \end{vmatrix} - 60\begin{vmatrix} -1 & 8 & -2 \\ 0 & -2 & 6 \\ 0 & 5 & 0 \end{vmatrix}$$

$$= 4\{8(6\,V_S) + 2(-2V_S - 250)\} + 1\{-1(6V_S)\} - 60\,\{-1\,(-30)\}$$

$$= 170\,V_S - 3800$$

$$I_4 = \frac{170V_S - 3800}{1950}$$

$$\therefore \quad V_S = \frac{1950 \times I_4 + 3800}{170} = 79.7 \text{ V}$$

Example 5.44 *Write and solve the equation for Mesh Current in the network shown.*

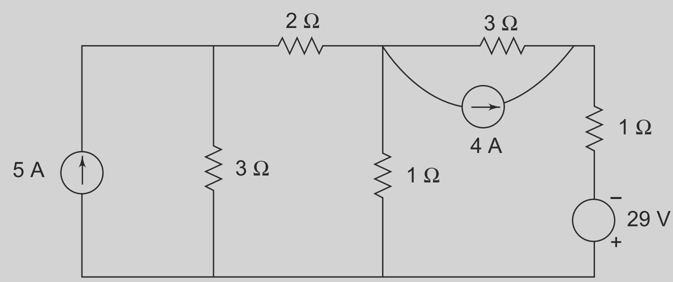


Fig. 5.122

Solution By source transformation technique transform 5A and 4A current sources into voltage sources.

5 A current source in parallel with 3 Ω can be transformed to 15 V in series with 3 Ω and 4 A current source in parallel with 3 Ω can be transformed to 12 volts in series with 3 Ω. The equivalent circuit is as shown below:

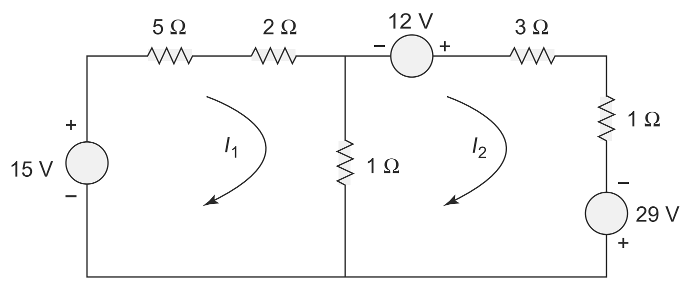


Fig. 5.123

The mesh equations are

$$2I_1 + 5I_1 + 1(I_1 - I_2) = 15$$

$$1(I_2 - I_1) + 4I_2 = 41$$

$$\Rightarrow \quad 8I_1 - I_2 = 15 \quad (1)$$

$$5I_2 - I_1 = 41 \quad (2)$$

on solving equations (1) and (2) we get

$$I_1 = 2.97 \text{ Amps}$$

$$I_2 = 8.74 \text{ Amps}$$

Example 5.45 *Determine the current in all branches of the following network and the voltage across for resistors using loop method.*

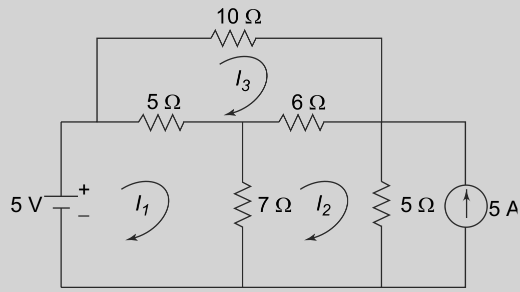


Fig. 5.124

Solution Applying mesh equation to the loops (1), (2) and (3) We get

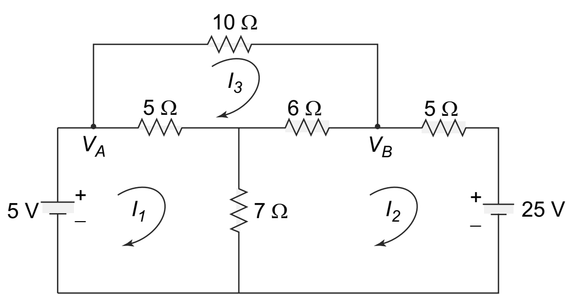


Fig. 5.125

$$5(I_1 - I_3) + 7(I_1 - I_2) = 5$$

$$12I_1 - 7I_2 - 5I_3 = 5 \quad (1)$$

$$7(I_2 - I_1) + 6(I_2 - I_3) + 5I_2 = -25$$

$$-7I_1 + 18I_2 - 6I_3 = -25 \quad (2)$$

$$10I_3 + 5(I_3 - I_1) + 6(I_3 - I_2) = 0$$

$$-5I_1 - 6I_2 + 21I_3 = 0 \quad (3)$$

By solving above 3 equations, we get

$$I_1 = -1.231 \text{ A}$$

$$I_2 = -2.172 \text{ A}$$

$$I_3 = -0.9138 \text{ A}$$

Current in 5 Ω resistor is −0.3172 A

7 Ω resistor is 0.941 A
6 Ω resistor is 1.2582 A
10 Ω resistor is −0.9138 A
5 Ω resistor is −2.172 A

Example 5.46 *Write the matrix loop equation for the given network and determine the loop currents, as shown in figure and find the current through each element in the network.* *[JNTU June 2009]*

Loop equations

$$4 = 10\,I_1 - 4\,I_2 - 4\,I_3$$
$$0 = -\,4\,I_1 + 10\,I_2 - 4\,I_3$$
$$0 = -\,4\,I_1 - 4\,I_2 + 10\,I_3$$

Matrix loop equation

$$\begin{bmatrix} 10 & -4 & -4 \\ -4 & 10 & -4 \\ -4 & -4 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

Let $\Delta = \begin{bmatrix} 10 & -4 & -4 \\ -4 & 10 & -4 \\ -4 & -4 & 10 \end{bmatrix} = 840 - 224 - 224 = 392$

$$\Delta_1 = \begin{bmatrix} 4 & -4 & -4 \\ 0 & 10 & -4 \\ 0 & -4 & 10 \end{bmatrix} = 336$$

$$\Delta_2 = \begin{bmatrix} 10 & 4 & -4 \\ -4 & 0 & -4 \\ -4 & 0 & 10 \end{bmatrix} = 224$$

$$\Delta_3 = \begin{bmatrix} 10 & -4 & 4 \\ -4 & 10 & 0 \\ -4 & -4 & 0 \end{bmatrix} = 224$$

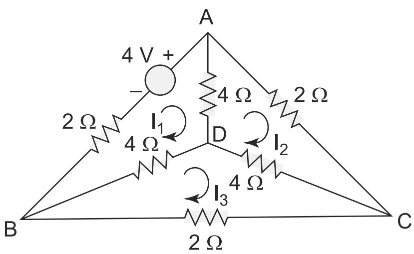


Fig. 5.126

$$\therefore\ I_1 = \frac{\Delta_1}{\Delta} = \frac{336}{392}\text{ amp} = \frac{6}{7}\text{ amp}$$

$$\therefore\ I_2 = \frac{\Delta_2}{\Delta} = \frac{224}{392}\text{ amp} = \frac{4}{7}\text{ amp}$$

$$\therefore\ I_3 = \frac{\Delta_3}{\Delta} = \frac{224}{392}\text{ amp} = \frac{4}{7}\text{ amp}$$

$\therefore$ Current through AB $= I_1 = \frac{6}{7}$ amp

$\therefore$ Current through AC $= I_2 = \frac{4}{7}$ amp

$\therefore$ Current through BC $= I_3 = \frac{4}{7}$ amp

$\therefore$ Current through AD $= I_1 - I_2 = \left(\frac{6}{7} - \frac{4}{7}\right)$ amp $= \frac{2}{7}$ amp

$\therefore$ Current through BD $= I_1 - I_3 = \left(\frac{6}{7} - \frac{4}{7}\right)$ amp $= \frac{2}{7}$ amp

$\therefore$ Current through CD $= I_2 - I_3 = \left(\frac{4}{7} - \frac{4}{7}\right)$ amp $= 0$ amp

Example 5.47 *Find the power dissipated in 1 ohms resistor & voltage drop across each resistor as shown in Fig. 5.127.*

[JNTU Jan 2010]

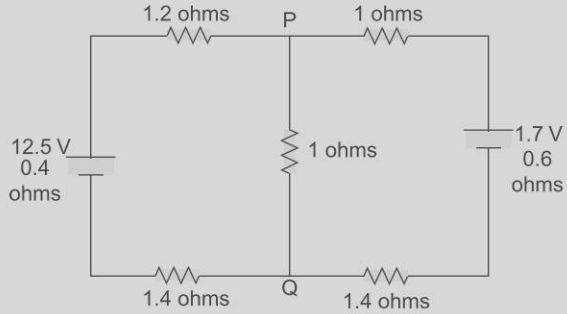


Fig. 5.127

Solution

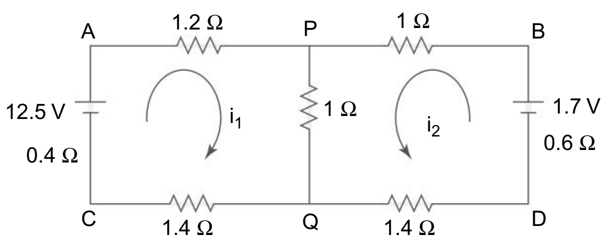


Fig. 5.128

Using KVL

$$12.5 = 4i_1 + i_2$$

$$1.7 = 4i_2 + i_1$$

$\therefore$ $i_1 = 3.22$ amp

$\therefore$ $i_2 = -0.38$ amp

$\therefore$ Current through PQ $= (3.22 - 0.38)$ amp $= 2.84$ amp

$\therefore$ Power dissipation in PQ $= (2.84^2 \times 1)$ watt

$= 8.0656$ watt

Voltage drop across AP $= (1.2 \times 3.22)$ volt $= 3.864$ V

Voltage drop across PB $= -(1 \times 0.38)$ volt $= -0.38$ V

Voltage drop across PQ $= (1 \times 2.84)$ volt $= 2.84$ V

Voltage drop across QC $= (1.4 \times 3.22)$ volt $= 4.508$ V

Voltage drop across QD $= -(1.4 \times 0.38)$ volt $= -0.532$ V

Example 5.48 *In the circuit shown in figure, determine the current through the 2 Ω resistor and the total current delivered by the battery. Use Kirchhoff's laws.*

[JNTU Jan 2010]

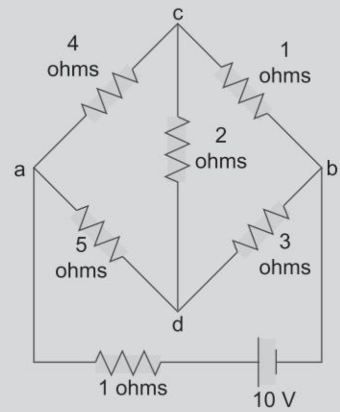


Fig. 5.129

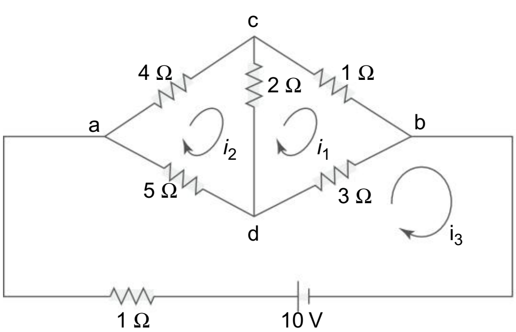


Fig. 5.130

Solution

$$11i_1 - 2i_2 - 5i_3 = 0$$

$$-2i_1 + 6i_2 - 3i_3 = 0$$

$$-5i_1 - 3i_2 + 9_3 = 10$$

$$\therefore \begin{bmatrix} 11 & -2 & -5 \\ -2 & 6 & -3 \\ -5 & -3 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\therefore \ \Delta = \begin{bmatrix} 11 & -2 & -5 \\ -2 & 6 & -3 \\ -5 & -3 & 9 \end{bmatrix} = 495 - 66 - 180 = 249$$

$$\therefore \ \Delta_1 = \begin{bmatrix} 0 & -2 & -5 \\ 0 & 6 & -3 \\ 10 & -3 & 9 \end{bmatrix} = 366$$

$$\therefore \ \Delta_2 = \begin{bmatrix} 11 & 0 & -5 \\ -2 & 0 & -3 \\ -5 & 10 & 9 \end{bmatrix} = 430$$

$$\therefore \ \Delta_3 = \begin{bmatrix} 11 & -2 & 0 \\ -2 & 6 & 0 \\ -5 & -3 & 10 \end{bmatrix} = 620$$

$$\therefore\ i_1 = \frac{\Delta_1}{\Delta} = 1.45\,\text{amp},\ i_2 = \frac{\Delta_2}{\Delta} = 1.73\,\text{amp},\ i_3 = \frac{\Delta_3}{\Delta} = 2.49\ \text{amp}$$

$\therefore$ Current through $2\Omega = -i_1 + i_2$ a= 0.28 amp

Total current $= i_3 = 2.49$ amp

Example 5.49 *What is the value of R such that the power supplied by both the sources are equal?* *[JNTU April/May 2003]*

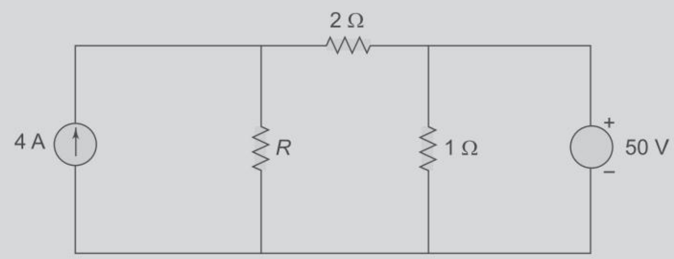


Fig. 5.131

Solution Converting current source into voltage source

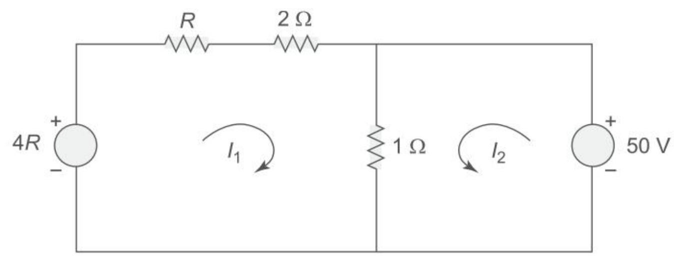


Fig. 5.132

Applying KVL for both the meshes

$$4R = (R + 3)\, i_1 + i_2 \tag{1}$$

$$50 = i_1 + i_2 \tag{2}$$

The power supplied by both the source are equal

$$4R\, i_1 = 50 i_2$$

$$R = 12.5 \frac{i_2}{i_1} \tag{3}$$

From eq (1)

$$4R - i_1 R - 3i_1 - i_2 = 0$$

$$R(4 - i_1) - 3i_1 - i_2 = 0 \tag{4}$$

Substituting equation 3 in 4

$$12.5\frac{i_2}{i_1}(4-i_1)-3i_1-i_2=0 \tag{5}$$

$$50\frac{i_2}{i_1}-13.5i_2-3i_1=0 \tag{6}$$

From equation 2. $\quad i_2 = 50 - i_1$ (7)

Substuting equation 7 in 6

$$50\left(\frac{50-i_1}{i_1}\right)-13.5(50-i_1)-3i_1=0 \tag{8}$$

$$10.5i_1^2 - 725\ i_1 + 2500 = 0 \tag{9}$$

from which $i_1 = \dfrac{725 \pm 717.72}{21} = 68.7 \text{ or } 0.347 \text{ A}$

If $\quad i_1 = 68.7$ A:

from equation (2) $\quad i_2 = -15.407$ A

and $\quad R = \dfrac{12.5(-18.7)}{68.7} = -3.4\,\Omega$

If $\quad i_1 = 0.347$ A

$i_2 = 46.3598$ A

and $\quad R = 12.5 \times \dfrac{46.3598}{3.6402} = 1788.6\,\Omega$

Considering positive value of $R = 1768.6\ \Omega$
Power supplied by current source

$$= 4 \times 1788.6 \times 0.347 = 2482.65 \text{ W}$$

Power supplied by voltage source

$$= 50 \times 49.653 = 2482.65 \text{ W}$$

The value of $R = 1788.6\ \Omega$

5.4.3 Supermesh Analysis

Suppose any of the branches in the network has a current source, then it is slightly difficult to apply mesh analysis straight forward because first we should assume an unknown voltage across the current source, writing mesh equations as before, and then relate the source current to the assigned mesh currents. This is generally a difficult approach. One way to overcome this difficulty is by applying the supermesh technique. Here we have to choose the kind of supermesh. A supermesh is constituted by two adjacent loops that have a common current source. As an example, consider the network shown in Fig. 5.133.

Here, the current source I is in the common boundary for the two meshes 1 and 2. This current source creates a supermesh, which is nothing but a combination of meshes 1 and 2.

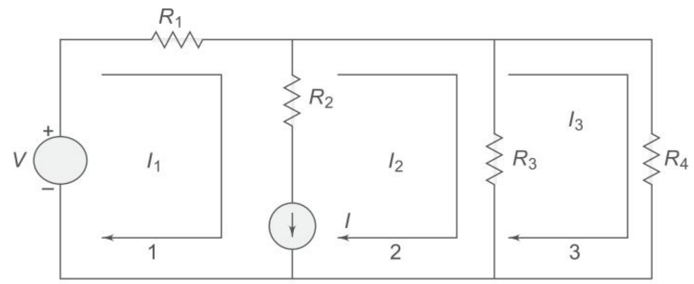


Fig. 5.133

$$R_1I_1 + R_3(I_2 - I_3) = V$$

or $$R_1I_1 + R_3I_2 - R_4I_3 = V$$

Considering mesh 3, we have

$$R_3(I_3 - I_2) + R_4I_3 = 0$$

Finally, the current I from current source is equal to the difference between two mesh currents, i.e.

$$I_1 - I_2 = I$$

We have, thus, formed three mesh equations which we can solve for the three unknown currents in the network.

Example 5.50 *Determine the current in the 5 Ω resistor in the network given in Fig. 5.134.*

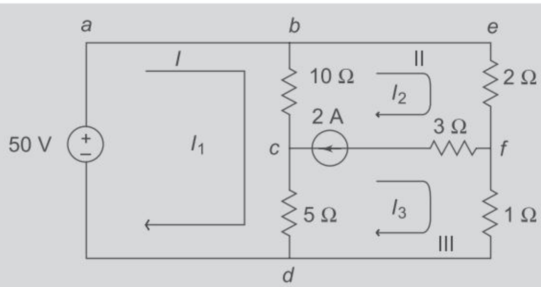


Fig. 5.134

Solution From the first mesh, i.e. *abcda*, we have

$$50 = 10(I_1 - I_2) + 5(I_1 - I_3)$$

or $$15I_1 - 10I_2 - 5I_3 = 50 \qquad (5.33)$$

From the second and third meshes, we can form a supermesh

$$10(I_2 - I_1) + 2I_2 + I_3 + 5(I_3 - I_1) = 0$$

or $$-15I_1 + 12I_2 + 6I_3 = 0 \quad (5.34)$$

The current source is equal to the difference between II and III mesh currents, i.e.

$$I_2 - I_3 = 2\text{A} \quad (5.35)$$

Solving 2.16, 2.17 and 2.18, we have

$$I_1 = 19.99 \text{ A}, I_2 = 17.33 \text{ A, and } I_3 = 15.33 \text{ A}$$

The current in the 5 Ω resistor = $I_1 - I_3$

$$= 19.99 - 15.33 = 4.66 \text{ A}$$

∴ The current in the 5 Ω resistor is 4.66 A.

Example 5.51 *Write the mesh equations for the circuit shown in Fig. 5.135 and determine the currents, I_1, I_2 and I_3.*

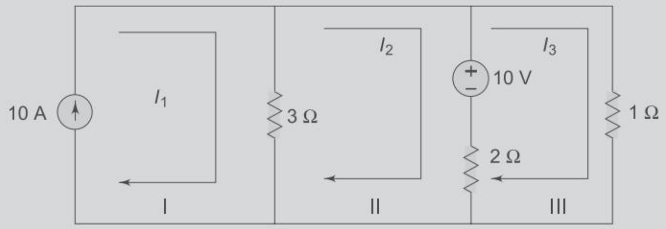


Fig. 5.135

Solution In Fig. 5.135, the current source lies on the perimeter of the circuit, and the first mesh is ignored. Kirchhoff's voltage law is applied only for second and third meshes.

From the second mesh, we have

$$3(I_2 - I_1) + 2(I_2 - I_3) + 10 = 0$$

or $$-3I_1 + 5I_2 - 2I_3 = -10 \quad (5.36)$$

From the third mesh, we have

$$I_3 + 2(I_3 - I_2) = 10$$

or $$-2I_2 + 3I_3 = 10 \quad (5.37)$$

From the first mesh,

$$I_1 = 10 \text{ A} \quad (2.38)$$

From the above three equations, we get

$$I_1 = 10 \text{ A}, \; I_2 = 7.27 \text{ A}, I_3 = 8.18 \text{ A}$$

Example 5.52 *Determine the loop currents for the circuit shown in Fig. 5.136 by using mesh analysis.*

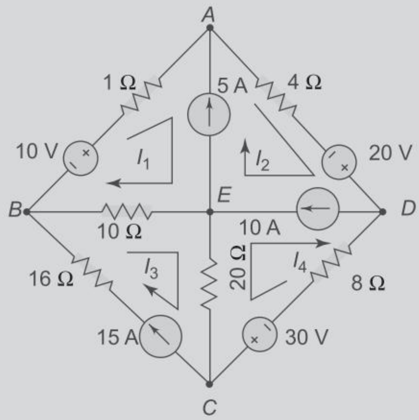


Fig. 5.136

Solution The branches *AE*, *DE* and *BC* consists of current sources. Here we have to apply supermesh analysis.

The combined supermesh equation is

$$10(I_1 - I_3) + I_1 - 10 + 4I_2 - 20$$
$$+ 8I_4 - 30 + 20\,(I_4 - I_3) = 0$$

or $\quad 11I_1 + 4I_2 - 30I_3 + 28I_4 = 60$

In branch *AE*, $\quad I_2 - I_1 = 5\text{ A}$

In branch *BC*, $\quad I_3 = 15\text{ A}$

In branch *DE*, $\quad I_2 - I_4 = 10\text{ A}$

Solving the above four equations, we can get the four currents I_1, I_2, I_3 and I_4 as

$$I_1 = 14.65\text{ A}$$
$$I_2 = 19.65\text{ A},\ I_3 = 15\text{ A, and } I_4 = 9.65\text{ A}$$

Example 5.53 *Determine the power delivered by the voltage source and the current in the 10 Ω resistor for the circuit shown in Fig. 5.137.*

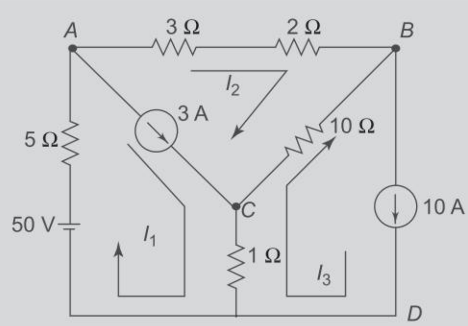


Fig. 5.137

Solution Since branches AC and BD consist of current sources, we have to use the supermesh technique.

The combined supermesh equation is

$$-50 + 5I_1 + 3I_2 + 2I_2 + 10(I_2 - I_3) + 1(I_1 - I_3) = 0$$

or $$6I_1 + 15I_2 - 11I_3 = 50$$

or $$I_1 - I_2 = 3 \text{ A and } I_3 = 10 \text{ A}$$

From the above equations we can solve for I_1, I_2 and I_3 follows

$$I_1 = 9.76 \text{ A}, I_2 = 6.76 \text{ A}, I_3 = 10 \text{ A}$$

Example 5.54 *In the circuit shown in Fig. 5.138, find the power delivered by 4 V source using mesh analysis and voltage across the 2 Ω resistor.*

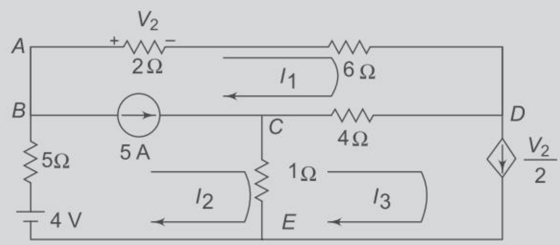


Fig. 5.138

Solution Since branches BC and DE consists of current sources, we use the supermesh technique.

The combined supermesh equation is

$$2I_1 + 6I_1 + 4(I_1 - I_3) + (I_2 - I_3) - 4 + 5I_2 = 0$$

or

$$12I_1 + 6I_2 - 5I_3 = 4$$

In branch BC, $I_2 - I_1 = 5$

In branch DE, $I_3 = \dfrac{V_2}{2}$

Solving the above equations

$$I_1 = -2 \text{ A}; I_2 = 3 \text{ A}$$

The voltage across the 2 Ω resistor $V_2 = 2I_1 = 2 \times (-2) = -4$ V

Power delivered by 4 V source $P_4 = 4I_2 = 4(3) = 12$ W

Example 5.55 *For the circuit shown in Fig. 5.139, find the current through the 10 Ω resistor by using mesh analysis.*

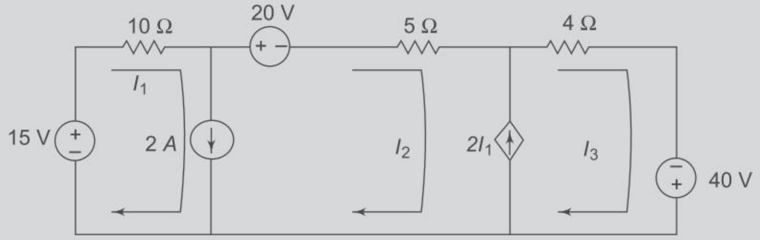


Fig. 5.139

Solution The parallel branches consist of current sources. Here we use supermesh analysis. The combined supermesh equation is.

or $\quad -15 + 10I_1 + 20 + 5I_2 + 4I_3 - 40 = 0$

and

$$10I_1 + 5I_2 + 4I_3 = 35$$

$$I_1 - I_2 = 2$$

$$I_3 - I_2 = 2I_1$$

Solving the above equations, we get

$$I_1 = 1.96 \text{ A}$$

The current in the 10 Ω resistor is $I_1 = 1.96$ A

5.4.4 Steady State AC Mesh Analysis

We have earlier discussed mesh analysis but have applied it only to resistive circuits. Some of the AC circuits presented in this chapter can also be solved by using mesh analysis. In Chapter 2, the two basic techniques for writing network equations for mesh analysis and node analysis were presented. These concepts can also be used for sinusoidal steady-state condition. In the sinusoidal steady-state analysis, we use voltage phasors, current phasors, impedances and admittances to write branch equations, KVL and KCL equations. For AC circuits, the method of writing loop equations is modified slightly. The voltages and currents in AC circuits change polarity at regular intervals. At a given time, the instantaneous voltages are driving in either the positive or negative direction. If the impedances are complex, the sum of their voltages is found by vector addition. We shall illustrate the method of writing network mesh equations with the following example.

Consider the circuit shown in Fig. 5.140, containing a voltage source and impedances.

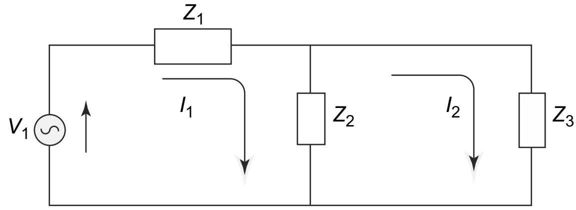


Fig. 5.140

The current in impedance Z_1 is I_1, and the current in Z_2, (assuming a positive direction downwards through the impedance) is $I_1 - I_2$. Similarly, the current in impedance Z_3 is I_2. By applying Kirchhoff's voltage law for each loop, we can get two equations. The voltage across any element is the product of the phasor current in the element and the complex impedance.

Equation for loop 1 is

$$I_1Z_1 + (I_1 - I_2)Z_2 = V_1 \qquad (5.39)$$

Equation for loop 2, which contains no source is

$$Z_2(I_2 - I_1) + Z_3 I_2 = 0 \tag{5.40}$$

By rearranging the above equations, the corresponding mesh current equations are

$$I_1(Z_1 + Z_2) - I_2 Z_2 = V_1 \tag{5.41}$$

$$-I_1 Z_2 + I_2(Z_2 + Z_3) = 0 \tag{5.42}$$

By solving the above equations, we can find out currents I_1 and I_2. In general, if we have M meshes, B branches and N nodes including the reference node, we assume M branch currents and write M independent equations; then the number of mesh currents is given by $M = B - (N - 1)$.

Example 5.56 *Write the mesh current equations in the circuit shown in Fig. 5. 141, and determine the currents.*

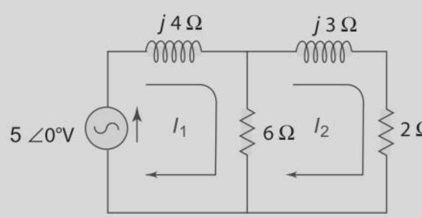


Fig. 5.141

Solution The equation for loop 1 is

$$I_1(j4) + 6(I_1 - I_2) = 5\angle 0° \tag{5.43}$$

The equation for loop 2 is

$$6(I_2 - I_1) + (j3)I_2 + (2)I_2 = 0 \tag{5.44}$$

By rearranging the above equations, the corresponding mesh current equations are

$$I_1(6 + j4) - 6I_2 = 5\angle 0° \tag{5.45}$$

$$-6I_1 + (8 + j3)I_2 = 0 \tag{5.46}$$

Solving the above equations, we have

$$I_1 = \left[\frac{(8+j3)}{6}\right] I_2$$

$$\left[\frac{(8+j3)(6+j4)}{6}\right] I_2 - 6I_2 = 5\angle 0°$$

$$I_2\left[\frac{(8+j3)(6+j4)}{6} - 6\right] = 5\angle 0°$$

$$I_2\,[10.26\angle 54.2° - 6\angle 0°] = 5\angle 0°$$

$$I_2\,[(6 + j8.32) - 6] = 5\angle 0°$$

$$I_2 = \frac{5\angle 0^\circ}{8.32\angle 90^\circ} = 0.6\angle -90^\circ$$

$$I_1 = \frac{8.54\angle 20.5^\circ}{6} \times 0.6\angle -90^\circ$$

$$I_1 = 0.855\ \angle -69.5^\circ$$

Current in loop 1, $I_1 = 0.855\underline{|-69.5^\circ}$

Current in loop 2, $I_2 = 0.6\ \angle -90^\circ$

5.4.5 Nodal Analysis

In Chapter 1, we discussed simple circuits containing only two nodes, including the reference node. In general, in a *N* node circuit, one of the nodes is choosen as reference or datum node, then it is possible to write *N* – 1 nodal equations by assuming *N* – 1 node voltages. For example, a 10 node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential. In the circuit shown in Fig. 5.142, node 3 is assumed as the reference node. The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3. Applying Kirchhoff's current law at node 1; the current entering is equal to the current leaving. (See Fig. 5.143).

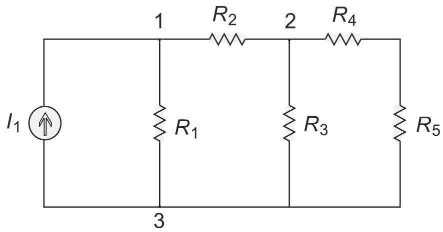


Fig. 5.142

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

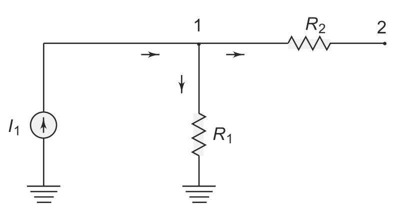


Fig. 5.143

where V_1 and V_2 are the voltages at node 1 and 2, respectively. Similarly, at node 2, the current entering is equal to the current leaving as shown in Fig. 5.144.

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0$$

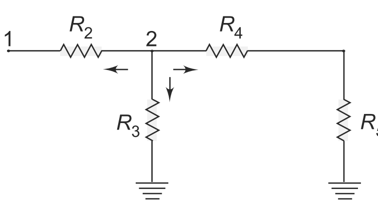


Fig. 5.144

Rearranging the above equations, we have

$$V_1\left[\frac{1}{R_1}+\frac{1}{R_2}\right]-V_2\left[\frac{1}{R_2}\right]=I_1$$

$$-V_1\left[\frac{1}{R_2}\right]+V_2\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4+R_5}\right]=0$$

From the above equations, we can find the voltages at each node.

Example 5.57 *Write the node voltage equations and determine the currents in each branch for the network shown in Fig. 5.145.*

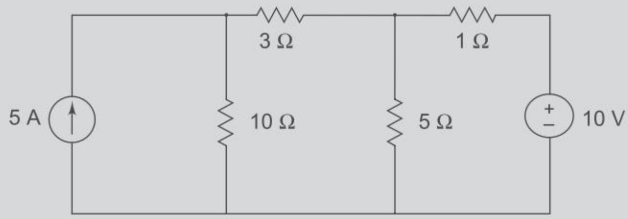


Fig. 5.145

Solution The first step is to assign voltages at each node as shown in Fig. 5.146.

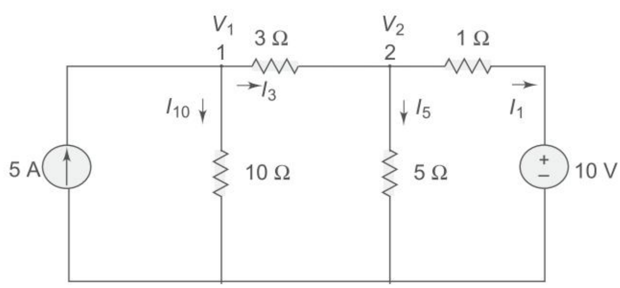


Fig. 5.146

Applying Kirchhoff's current law at node 1,

we have $5=\dfrac{V_1}{10}+\dfrac{V_1-V_2}{3}$

or $$V_1\left[\frac{1}{10}+\frac{1}{3}\right]-V_2\left[\frac{1}{3}\right]=5 \quad (5.47)$$

Applying Kirchhoff's current law at node 2,

we have $$\frac{V_2-V_1}{3}+\frac{V_2}{5}+\frac{V_2-10}{1}=0$$

or $$-V_1\left[\frac{1}{3}\right]+V_2\left[\frac{1}{3}+\frac{1}{5}+1\right]=10 \quad (5.48)$$

From Eqs 5.47 and 5.48, we can solve for V_1 and V_2 to get

$$V_1 = 19.85 \text{ V}, V_2 = 10.9 \text{ V}$$

$$I_{10} = \frac{V_1}{10} = 1.985\text{A}, \; I_3 = \frac{V_1 - V_2}{3} = \frac{19.85 - 10.9}{3} = 2.98\,\text{A}$$

$$I_5 = \frac{V_2}{5} = \frac{10.9}{5} = 2.18\,\text{A}, \; I_1 = \frac{V_2 - 10}{1} = 0.9\,\text{A}$$

Example 5.58 *Determine the voltages at each node for the circuit shown in Fig. 5.147.*

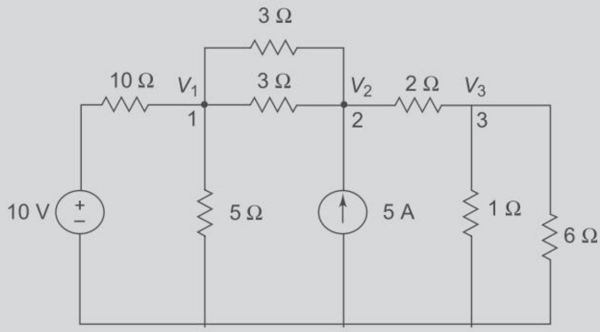


Fig. 5.147

Solution At node 1, assuming that all currents are leaving, we have

$$\frac{V_1 - 10}{10} + \frac{V_1 - V_2}{3} + \frac{V_1}{5} + \frac{V_1 - V_2}{3} = 0$$

or $$V_1\left[\frac{1}{10} + \frac{1}{3} + \frac{1}{5} + \frac{1}{3}\right] - V_2\left[\frac{1}{3} + \frac{1}{3}\right] = 1$$

$$0.96V_1 - 0.66V_2 = 1 \tag{5.49}$$

At node 2, assuming that all currents are leaving except the current from current source, we have

$$\frac{V_2 - V_1}{3} + \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{2} = 5$$

$$-V_1\left[\frac{2}{3}\right] + V_2\left[\frac{1}{3} + \frac{1}{3} + \frac{1}{2}\right] - V_3\left[\frac{1}{2}\right] = 5$$

$$-0.66\,V_1 + 1.16\,V_2 - 0.5V_3 = 5 \tag{5.50}$$

At node 3, assuming all currents are leaving, we have

$$\frac{V_3 - V_2}{2} + \frac{V_3}{1} + \frac{V_3}{6} = 0$$

$$-0.5\,V_2 + 1.66\,V_3 = 0 \tag{5.51}$$

Applying Cramer's rule, we get

$$V_1 = \frac{\begin{vmatrix} 1 & -0.66 & 0 \\ 5 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{7.154}{0.887} = 8.06\,\text{V}$$

Similarly,

$$V_2 = \frac{\begin{vmatrix} 0.96 & 1 & 0 \\ -0.66 & 5 & -0.5 \\ 0 & 0 & 1.66 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{9.06}{0.887} = 10.2\,\text{V}$$

$$V_3 = \frac{\begin{vmatrix} 0.96 & -0.66 & 1 \\ -0.66 & 1.16 & 5 \\ 0 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 0.96 & -0.66 & 0 \\ -0.66 & 1.16 & -0.5 \\ 0 & -0.5 & 1.66 \end{vmatrix}} = \frac{2.73}{0.887} = 3.07\text{ V}$$

5.4.6 Nodal Equations by Inspection Method

The nodal equations for a general planar network can also be written by inspection, without going through the detailed steps. Consider a three node resistive network, including the reference node, as shown in Fig. 5.148.

In Fig. 5.148, the points *a* and *b* are the actual nodes and *c* is the reference node. Now consider the nodes *a* and *b* separately as shown in Fig. 5.148(a) and (b).

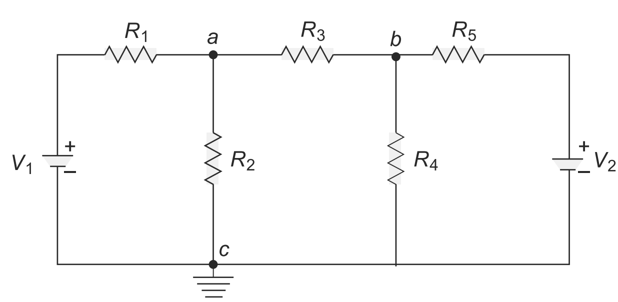


Fig. 5.148

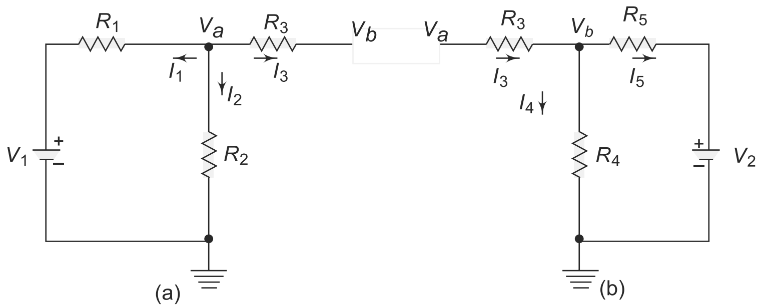


Fig. 5.149

In Fig. 5.149(a), according to Kirchhoff's current law, we have

$$I_1 + I_2 + I_3 = 0$$

$$\therefore \quad \frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0 \tag{5.52}$$

In Fig. 5.149(b), if we apply Kirchhoff's current law, we get

$$I_4 + I_5 = I_3$$

$$\therefore \quad \frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_2}{R_5} = 0 \tag{5.53}$$

Rearranging the above equations, we get

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_a - \left(\frac{1}{R_3}\right)V_b = \left(\frac{1}{R_1}\right)V_1 \tag{5.54}$$

$$\left(-\frac{1}{R_3}\right)V_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_b = \frac{V_2}{R_5} \tag{5.55}$$

In general, the above equations can be written as

$$G_{aa} V_a + G_{ab} V_b = I_1 \tag{5.56}$$

$$G_{ba} V_a + G_{bb} V_b = I_2 \tag{5.57}$$

By comparing Eqs 5.54, 5.55 and Eqs 5.56, 5.57 we have the self conductance at node a, $G_{aa} = (1/R_1 + 1/R_2 + 1/R_3)$ is the sum of the conductances connected to node a. Similarly, $G_{bb} = (1/R_3 + 1/R_4 + 1/R_5)$, is the sum of the conductances connected to node b. $G_{ab} = (-1/R_3)$, is the sum of the mutual conductances connected to node a and node b. Here all the mutual conductances have negative signs. Similarly, $G_{ba} = (-1/R_3)$ is also a mutual conductance connected between nodes b and a. I_1 and I_2 are the sum of the source currents at node a and node b, respectively. The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

Example 5.59 *For the circuit shown in Fig. 5.150, write the node equations by the inspection method.*

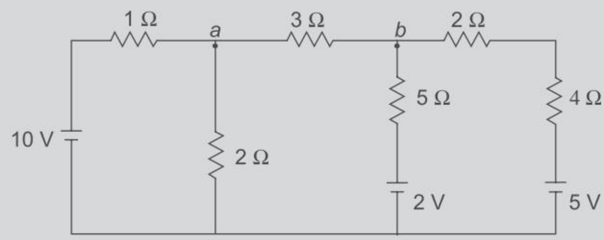


Fig. 5.150

Solution The general equations are

$$G_{aa} V_a + G_{ab} V_b = I_1 \quad (5.58)$$

$$G_{ba} V_a + G_{bb} V_b = I_2 \quad (5.59)$$

Consider Eq. 5.58

G_{aa} = (1 + 1/2 + 1/3) mho, the self-conductance at node *a* is the sum of the conductances connected to node *a*.

G_{bb} = (1/6 + 1/5 + 1/3) mho the self-conductance at node *b* is the sum of the conductances connected to node *b*.

G_{ab} = – (1/3) mho, the mutual conductance between nodes *a* and *b* is the sum of the conductances connected between nodes *a* and *b*.

Similarly, G_{ba} = – (1/3), the sum of the mutual conductances between nodes *b* and *a*.

$$I_1 = \frac{10}{1} = 10\text{ A, the source current at node } a,$$

$$I_2 = \left(\frac{2}{5} + \frac{5}{6}\right) = 1.23\text{ A, the source current at node } b.$$

Therefore, the nodal equations are

$$1.83\, V_a - 0.33\, V_b = 10 \quad (5.60)$$

$$-0.33\, V_a + 0.7\, V_b = 1.23 \quad (5.61)$$

Example 5.60 *Determine the voltage ratio V_{out} / V_{in} for the circuit shown in Fig. 5.151 by using nodal analysis.*

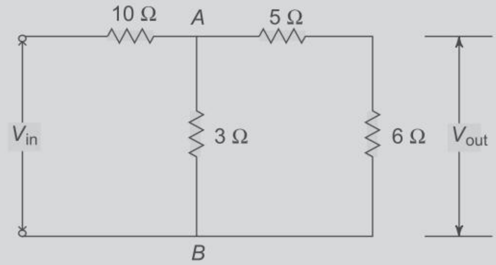


Fig. 5.151

Solution $I_{10} + I_3 + I_{11} = 0$

$$I_{10} = \frac{V_A - V_{in}}{10}$$

$$I_3 = \frac{V_A}{3}$$

$$I_{11} = \frac{V_A}{11}, \text{ or } \frac{V_{out}}{6}$$

$$\frac{V_A - V_{in}}{10} + \frac{V_A}{3} + \frac{V_A}{11} = 0$$

Also $\dfrac{V_A}{11} = \dfrac{V_{out}}{6}$

$\therefore \quad VA = V_{out} \times 1.83$

From the above equations $V_{out}/V_{in} = 1/9.53 = 0.105$

Example 5.61 *Find the voltages V in the circuit shown in Fig. 5.152 which makes the current in the 10 Ω resistor zero by using nodal analysis.*

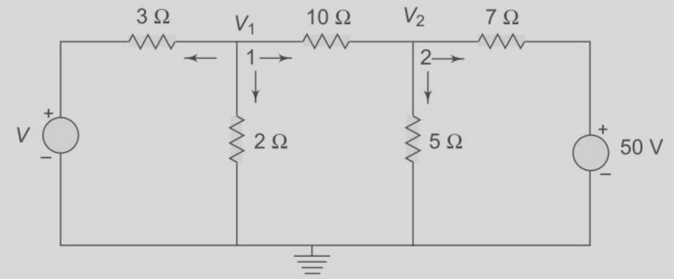


Fig. 5.152

Solution In the circuit shown, assume voltages V_1 and V_2 at nodes 1 and 2. At node 1, the current equation in Fig. 5.153 is

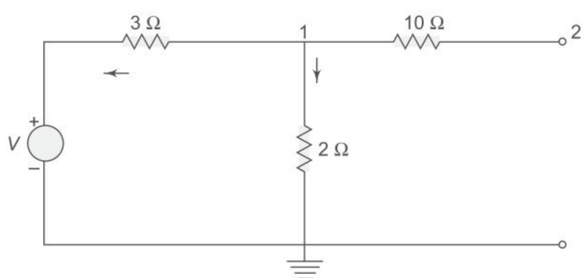


Fig. 5.153

$$\frac{V_1 - V}{3} + \frac{V_1}{2} + \frac{V_1 - V_2}{10} = 0$$

or $\quad 0.93\, V_1 - 0.1\, V_2 = V/3$

At node 2, the current equation in Fig. 5.154 is

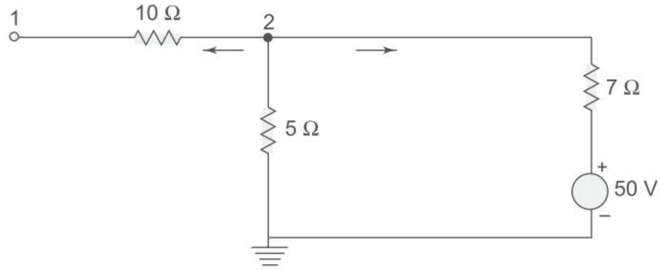


Fig. 5.154

$$\frac{V_2 - V_1}{10} + \frac{V_2}{5} + \frac{V_2 - 50}{7} = 0$$

or $\quad -0.1\, V_1 + 0.443\, V_2 = 7.143$

Since the current in 10 Ω resistor is zero, the voltage at node 1 is equal to the voltage at node 2.

$\therefore \quad V_1 - V_2 = 0$

From the above three equations, we can solve for V

$V_1 = 20.83$ Volts and $V_2 = 20.83$ volts

$\therefore \quad V = 51.87$ V

Example 5.62 *Use nodal analysis to find the power dissipated in the 6 Ω resistor for the circuit shown in Fig. 5.155.*

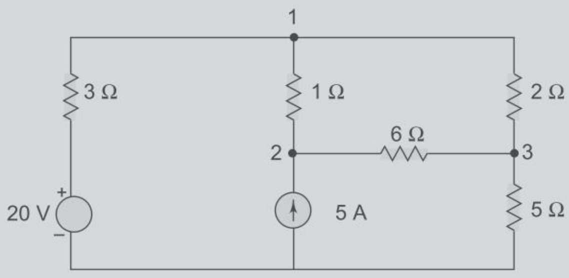


Fig. 5.155

Solution Assume voltage V_1, V_2 and V_3 at nodes 1, 2 and 3 as shown in Fig. 5.155. By applying current law at node 1, we have

$$\frac{V_1 - 20}{3} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = 0$$

or $1.83V_1 - V_2 - 0.5V_3 = 6.67$ (5.62)

At node 2

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{6} = 5\,\text{A}$$

or $-V_1 + 1.167V_2 - 0.167V_3 = 5$ (5.63)

At node 3,

$$\frac{V_3 - V_1}{2} + \frac{V_3 - V_2}{6} + \frac{V_3}{5} = 0$$

or $-0.5\,V_1 - 0.167\,V_2 + 0.867\,V_3 = 0$ (5.64)

Applying Cramer's rule to Eqs 5.62, 5.63 and 5.64, we have

$$V_2 = \frac{\Delta_2}{\Delta}$$

where $$\Delta = \begin{vmatrix} 1.83 & -1 & -0.5 \\ -1 & -1.167 & -0.167 \\ -0.5 & -0.167 & 0.867 \end{vmatrix} = -2.64$$

$$\Delta_2 = \begin{vmatrix} 1.83 & 6.67 & -0.5 \\ -1 & 5 & -0.167 \\ -0.5 & 0 & 0.867 \end{vmatrix} = 13.02$$

$$\therefore \quad V_2 = \frac{13.02}{-2.64} = -4.93\,\text{V}$$

Similarly,

$$V_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta_3 = \begin{vmatrix} 1.83 & -1 & 6.67 \\ -1 & -1.167 & 5 \\ -0.5 & -0.167 & 0 \end{vmatrix} = 1.25$$

$$\therefore \quad V_3 = \frac{1.25}{-2.64} = -0.47\,\text{V}$$

The current in the 6 Ω resistor is

$$I_6 = \frac{V_2 - V_3}{6}$$

$$= \frac{-4.93 + 0.47}{6} = -0.74\,\text{A}$$

The power absorbed or dissipated = $I_6^2 R_6$

$= (0.74)^2 \times 6$

$= 3.29$ W

Example 5.63 *For the circuit shown in Fig. 5.156 find the voltage across the 4 Ω resistor by using nodal analysis.*

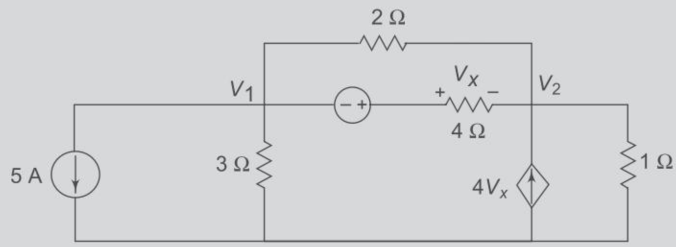


Fig. 5.156

Solution In the circuit shown, assume voltages V_1 and V_2 at nodes 1 and 2. At node 1, the current equation is

$$5+\frac{V_1}{3}+\frac{V_1+5-V_2}{4}+\frac{V_1-V_2}{2}=0$$

or $\quad 1.08\ V_1 - 0.75\ V_2 = -6.25 \qquad (5.65)$

At node 2, the current equation is

$$\frac{V_2-V_1-5}{4}+\frac{V_2-V_1}{2}-4V_x+\frac{V_2}{1}=0$$

$$V_x = V_1 + 5 - V_2$$

or $\quad -4.75\ V_1 + 5.75\ V_2 = 21.25 \qquad (5.66)$

Applying Cramer's rule to Eqs 5.65 and 5.66, we have

$$V_2=\frac{\Delta_2}{\Delta}$$

where $$\Delta=\begin{vmatrix}1.08 & -0.75\\ -4.75 & 5.75\end{vmatrix}=2.65$$

$$\Delta_2=\begin{vmatrix}1.08 & -6.25\\ -4.75 & 21.25\end{vmatrix}=-6.74$$

∴ $$V_2=\frac{\Delta_2}{\Delta}=\frac{-6.74}{2.65}=-2.54\text{ V}$$

Similarly, $$V_1=\frac{\Delta_1}{\Delta}$$

$$\Delta_1=\begin{vmatrix}-6.25 & -0.75\\ 21.25 & 5.75\end{vmatrix}=-20$$

$$V_1=\frac{\Delta_1}{\Delta}=\frac{-20}{2.65}=-7.55\text{ V}$$

The voltage across the 4 Ω resistor is

$$V_x = V_1 + 5 - V_2 = -7.55 + 5 - (-2.54)$$

$$V_x = -0.01 \text{ volts}$$

Example 5.64 *For the circuit shown in Fig. 5.157, find the current passing through the 5 Ω resistor by using the nodal method.*

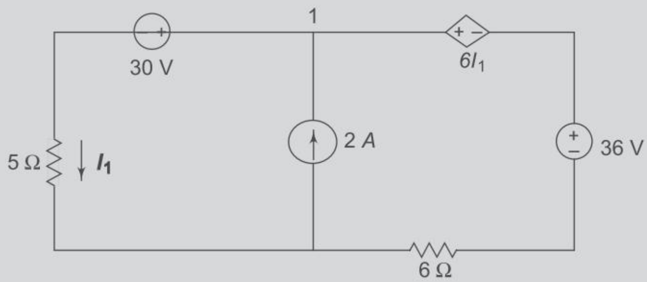


Fig. 5.157

Solution In the circuit shown, assume the voltage V at node 1.

At node 1, the current equation is

$$\frac{V-30}{5} - 2 + \frac{V-36-6I_1}{6} = 0$$

where $$I_1 = \frac{V-30}{5}$$

From the above equation

$$V = 48 \text{ V}$$

The current in 5 Ω resistor is

$$I_1 = \frac{V-30}{5} = 3.6 \text{ A}$$

Example 5.65 *Use nodal analysis, to determine the voltage V_1 and V_2 in the circuit shown.*

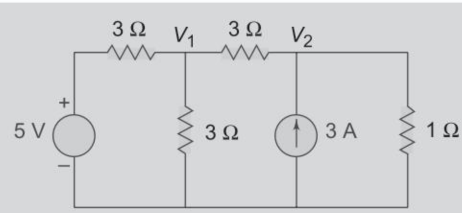


Fig. 5.158

Solution The nodal equation for the two nodes are

$$\frac{V_1 - 5}{2} + \frac{V_1}{3} + \frac{V_1 - V_2}{2} = 0 \quad (1)$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} = 3 \quad (2)$$

From 1 $\quad 1.333\, V_1 - 0.5\, V_2 = 2.5$

From 2 $\quad -0.5\, V_1 + 1.5\, V_2 = 3$

Solving the above equations for V_1 and V_2 yields

$$V_1 = 3\text{V and } V_2 = 3\text{V}.$$

Example 5.66 *For the circuit shown in Fig. 5.159, find the current through 20Ω resistor?*

[JNTU May/June 2006]

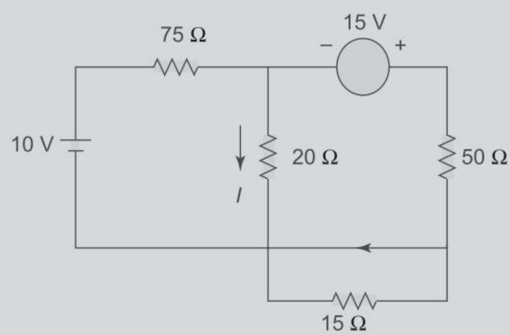


Fig. 5.159

Solution Applying nodal analysis

$$\frac{V-10}{75}+\frac{V}{20}+\frac{V+15}{50}=0$$

$$V=-2 \text{ volts } I$$

$$I=\frac{V}{20}=-0.1 \text{ A}$$

5.4.7 Supernode Analysis

Suppose any of the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis. One way to overcome this difficulty is to apply the supernode technique. In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff's current law as usual. This is explained with the help of Fig. 5.160.

It is clear from Fig. 5.160, that node 4 is the reference node. Applying Kirchhoff's current law at node 1, we get

$$I=\frac{V_1}{R_1}+\frac{V_1-V_2}{R_2}$$

Due to the presence of voltage source V_x in between nodes 2 and 3, it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

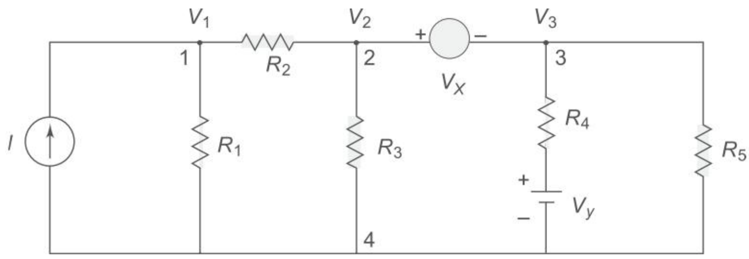


Fig. 5.160

Accordingly, we can write the combined equation for nodes 2 and 3 as under.

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_y}{R_4} + \frac{V_3}{R_5} = 0$$

The other equation

$$V_2 - V_3 = V_x$$

From the above three equations, we can find the three unknown voltages.

Example 5.67 *Determine the current in the 5 Ω resistor for the circuit shown in Fig. 5.161.*

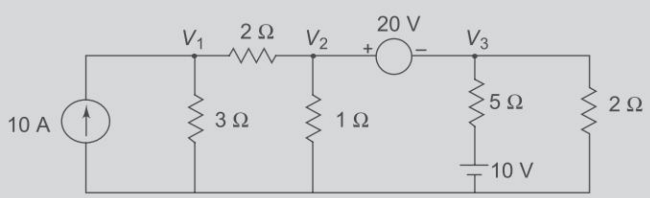


Fig. 5.161

Solution At node 1

$$10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

or $$V_1\left[\frac{1}{3} + \frac{1}{2}\right] - \frac{V_2}{2} - 10 = 0$$

$$0.83\ V_1 - 0.5\ V_2 - 10 = 0 \tag{5.67}$$

At node 2 and 3, the supernode equation is

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

or $$\frac{-V_1}{2} + V_2\left[\frac{1}{2} + 1\right] + V_3\left[\frac{1}{5} + \frac{1}{2}\right] = 2$$

$$-0.5\ V_1 + 1.5\ V_2 + 0.7\ V_3 - 2 = 0 \tag{5.68}$$

The voltage between nodes 2 and 3 is given by

$$V_2 - V_3 = 20 \tag{5.69}$$

The current in the 5 Ω resistor $I_5 = \frac{V_3 - 10}{5}$

Solving Eqs 5.67, 5.68 and 5.69, we obtain

$$V_3 = -8.42 \text{ V}$$

$\therefore$ Current $I_5 = \frac{-8.42 - 10}{5} = -3.68 \text{ A}$ (current towards node 3) i.e. the current flows towards node 3.

Example 5.68 *Determine the power dissipated by 5 Ω resistor in the circuit shown in Fig. 5.162.*

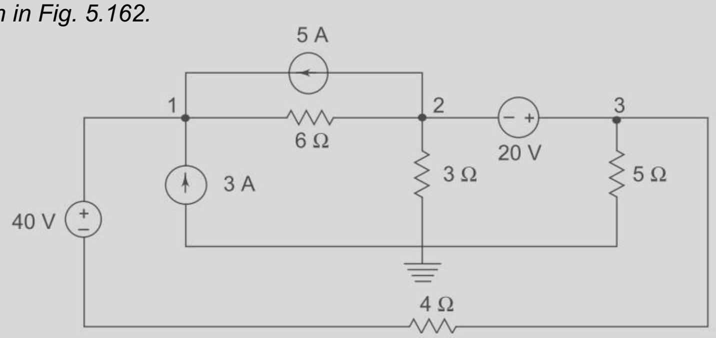


Fig. 5.162

Solution In Fig. 5.162, assume voltages V_1, V_2 and V_3 at nodes 1, 2 and 3. At node 1, the current law gives

$$\frac{V_1 - 40 - V_3}{4} + \frac{V_1 - V_2}{6} - 3 - 5 = 0$$

or $\quad 0.42\, V_1 - 0.167\, V_2 - 0.25\, V_3 = 18$

Applying the supernode technique between nodes 2 and 3, the combined equation at node 2 and 3 is

$$\frac{V_2 - V_1}{6} + 5 + \frac{V_2}{3} + \frac{V_3}{5} + \frac{V_3 + 40 - V_1}{4} = 0$$

or $\quad -0.42\, V_1 + 0.5\, V_2 + 0.45\, V_3 = -15$

Also $\quad V_3 - V_2 = 20$ V

Solving the above three equations, we get

$V_1 = 52.89$ V, $V_2 = -1.89$ V and

$V_3 = 18.11$ V

∴ The current in the 5 Ω resistor $I_5 = \frac{V_3}{5}$

$$= \frac{18.11}{5} = 3.62\text{ A}$$

The power absorbed by the 5 Ω resistor $P_5 = I_5^2 R_5$

$= (3.62)^2 \times 5$

$= 65.52$ W

Example 5.69 *Find the power delivered by the 5 A current source in the circuit shown in Fig. 5.163 by using the nodal method.*

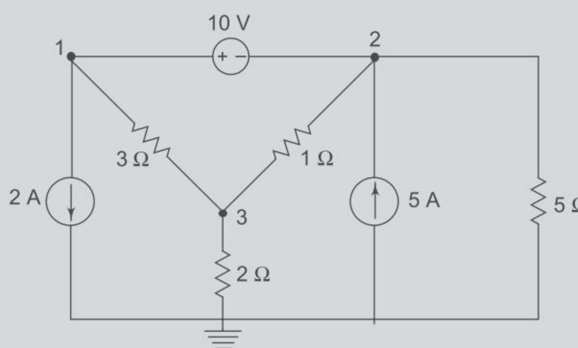


Fig. 5.163

Solution Assume the voltages V_1, V_2 and V_3 at nodes 1, 2, and 3, respectively. Here, the 10 V source is common between nodes 1 and 2. So applying the supernode technique, the combined equation at node 1 and 2 is

$$\frac{V_1 - V_3}{3} + 2 + \frac{V_2 - V_3}{1} - 5 + \frac{V_2}{5} = 0$$

or $0.34\,V_1 + 1.2\,V_2 - 1.34\,V_3 = 3$

At node 3, $\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$

or $-0.34\,V_1 - V_2 + 1.83\,V_3 = 0$

Also $V_1 - V_2 = 10$

Solving the above equations, we get

$V_1 = 13.72$ V; $V_2 = 3.72$ V

$V_3 = 4.567$ V

Hence the power delivered by the source (5 A) $= V_2 \times 5$

$= 3.72 \times 5 = 18.6$ W

Example 5.70 *Using nodal method, find the current through 5 Ω resistor, in the following circuit.*

[May/June 2006]

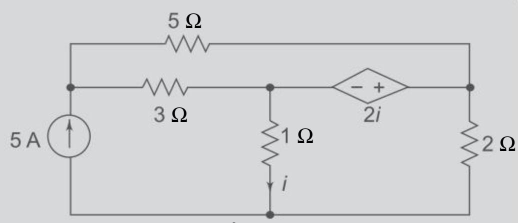


Fig. 5.164

Solution

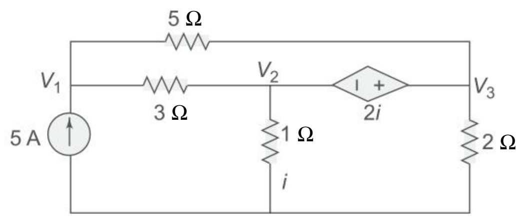


Fig. 5.165

Equation at V_1; $\frac{V_1 - V_3}{5} + \frac{V_1 - V_2}{3} = 5$

$$8V_1 - 5V_2 - 3V_3 = 75 \qquad (1)$$

Equation at supernode

$$\frac{V_2 - V_1}{3} + V_2 + \frac{V_3 - V_1}{5} + \frac{V_3}{2} = 0$$

$$-16V_1 + 40V_2 + 21V_3 = 0 \qquad (2)$$

$$V_3 - V_2 = 2i$$

$$i = V_{2/1} = V_2$$

$$V_3 - V_2 = 2V_2 \Rightarrow V_3 = 3V_2$$

Solving for V_1, V_2 and V_3

$$V_1 = 12.87;\ V_2 = 2;\ V_3 = 6 \text{ volts}$$

Current through 5 Ω from V_1 to V_3 is equal to $\frac{V_1 - V_3}{5} = 1.347$ amps.

Example 5.71 *Find the power supplied by 12 V source as shown in Fig. 5.166.*

[JNTU April/May 2006]

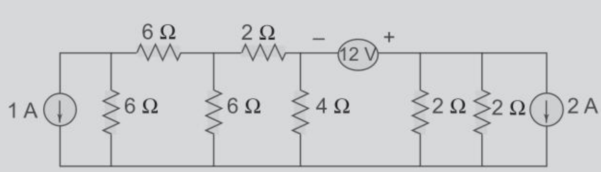


Fig. 5.166

Solution

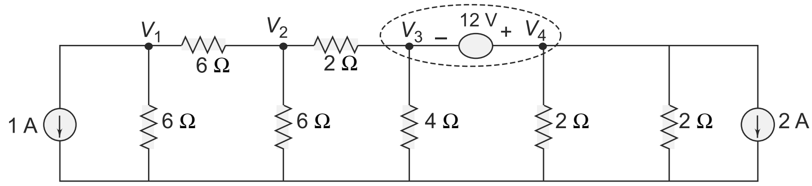


Fig. 5.167

The nodal equations are

$$1+\frac{V_1}{6}+\frac{V_1+V_2}{6}=0 \quad (1)$$

$$\frac{V_2}{6}+\frac{V_2-V_1}{6}+\frac{V_2-V_3}{2}=0 \quad (2)$$

$$\frac{V_3-V_2}{2}+\frac{V_3}{4}+\frac{V_4}{2}+\frac{V_4}{2}+2=0 \quad (3)$$

$V_4 - V_3 = 12$ is the supernode equation

(1) $\Rightarrow V_1\left[\frac{1}{3}\right]-V_2\left[\frac{1}{6}\right]+1=0$

(2) $\Rightarrow V_1\left[-\frac{1}{6}\right]+V_2\left[\frac{5}{6}\right]-V_3\left[\frac{1}{2}\right]=0$

(3) $\Rightarrow V_2\left[-\frac{1}{2}\right]+V_3\left[\frac{3}{4}\right]+V_4[1]+2=0$

$$-\frac{1}{2}V_2+\frac{7}{4}V_3+12+V_3+2=0$$

$$-\frac{1}{2}V_2+\frac{7}{4}V_3+14=0$$

$$\Rightarrow V_3=\frac{4}{7}\left[\frac{1}{2}V_2-14\right]=\frac{2}{7}V_2-8 \quad (4)$$

From (2), $-\frac{1}{6}V_1+\frac{5}{6}V_2-\frac{1}{2}\left[\frac{2}{7}V_2-8\right]=0$

$$-\frac{1}{6}V_1+\frac{5}{6}V_2-\frac{1}{7}V_2+4=0$$

$$V_1=6\left\{\frac{29}{42}V_2+4\right\}=\frac{29}{7}V_2+24 \quad (5)$$

$$\frac{1}{3}V_1-\frac{1}{6}V_2+1=0$$

Substitute for V_1

From (1), $$\frac{1}{3}\left[\frac{29}{7}V_2 + 24\right] - \frac{1}{6}V_2 + 1 = 0$$

$$\frac{29}{21}V_2 + 8 - \frac{1}{6}V_2 + 1 - 0 \Rightarrow V_2 = \frac{-126}{17}\ V$$

$$V_3 = \frac{2}{7}V_2 - 8 = \frac{-172}{17}\ V$$

$$V_4 = V_3 + 12 = \frac{32}{17}\ V$$

Current through 12 V source is

$$I = \frac{V_4}{2} + \frac{V_4}{2} + 2 = \frac{66}{17}\ \text{A}$$

Power $$V_1 = 12 \times \frac{66}{17} = \frac{792}{17}\ \text{W}$$

Example 5.72 *Find the Current I_1 and I_2 using Nodal Analysis (Fig. 5.168)* *[JNTU May/June 2006]*

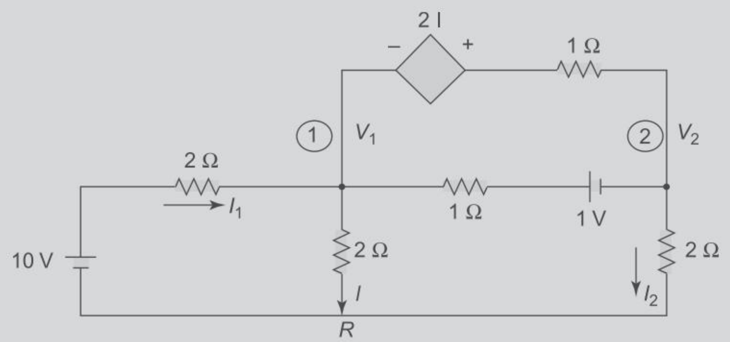


Fig. 5.168

Solution At node (1):

$$\frac{V_1 - 10}{2} + \frac{V_1}{2} + \frac{V_1 - (1 + V_2)}{1} + \frac{(2I + V_1) - V_2}{1} = 0$$

$$\Rightarrow \quad V_1\left(\frac{1}{2} + \frac{1}{2} + 1 + 1\right) + (-1 - 1)V_2 = \frac{10}{2} + 1 - 2I = 6 - 2I$$

$$\Rightarrow \quad 3V_1 - 2V_2 = 6 - 2I \qquad (1)$$

At node 2:

$$\frac{V_2}{2} + \frac{(1 + V_2) - V_1}{1} + \frac{V_2 - V_1 - 2I}{1} = 0$$

$$\Rightarrow \quad (-1 - 1)V_1 + \left(\frac{1}{2} + 1 + 1\right)V_2 = -1 + 2I$$

$$\Rightarrow \quad -2V_1 + \frac{5}{2}V_2 = 2I - 1 \tag{2}$$

But
$$I = \frac{V_1}{2} \tag{3}$$

From (3) and (1) $\Rightarrow 4V_1 - 2V_2 = 6$

From (3) and (2) $\Rightarrow 3V_1 - \frac{5}{2}V_2 = 1$

Solving,

$$V_1 = 3.25 \text{ V}$$
$$V_2 = 3.5 \text{ V}$$

$$\therefore \quad I_1 = \frac{10 - V_1}{2} = 3.375 \text{ A}; \quad I_2 = \frac{V_2}{2} = 1.75 \text{ A.}$$

Example 5.73 *For the network shown (Fig. 5.169), determine the node Voltages V_1 and V_2. Determine the power dissipated in each resistor.*

[JNTU May/June 2006]

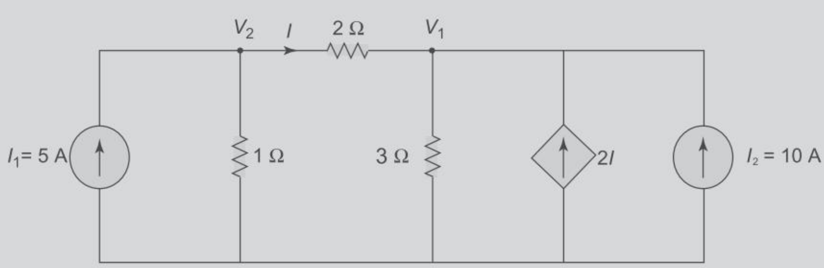


Fig. 5.169

Solution

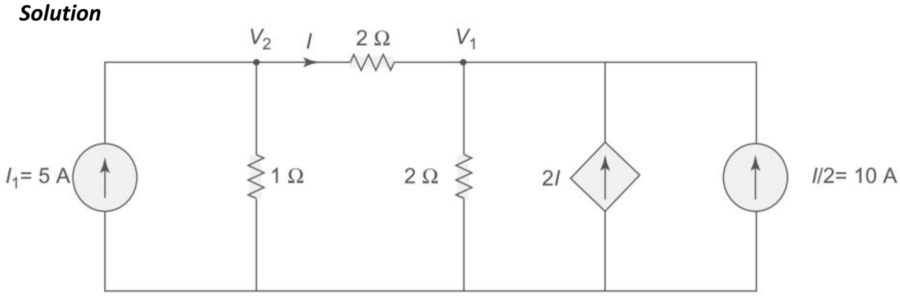


Fig. 5.170

Applying KCL

$$5 = \frac{V_2}{1} + \frac{V_2 - V_1}{2} \quad \Rightarrow \quad V_2\left(1 + \frac{1}{2}\right) - \frac{V_1}{2} = 5$$

$$3V_2 - V_1 = 10 \tag{1}$$

$$I = \frac{V_2 - V_1}{2} \tag{2}$$

$$\frac{V_1}{3} + \frac{V_1 - V_2}{2} = 10 + 2I = 10 + 2\left(\frac{V_2 - V_1}{2}\right)$$

$$\Rightarrow \quad V_1\left(\frac{1}{2} + \frac{1}{3}\right) = 10 + V_2 - V_1$$

$$\therefore \quad 11V_1 - 9V_2 = 60 \tag{3}$$

Solving (1) and (3)

$$V_1 = 11.25 \text{ volts}$$

and $\quad V_2 = 7.083$ volts

Power dissipated in 1 Ω resistor $= VI = I^2R = \frac{V^2}{R} = \frac{V_2^2}{1} = (7.083)^2$

$= 50.17$ watts

Power dissipated in 2 Ω resistor $= \frac{V^2}{R} = \frac{(V_2 - V_1)^2}{2} = 8.682$ watts

Power dissipated in 3 Ω resistor $= \frac{V_1^2}{3} = \frac{(11.25)^2}{3} = 42.19$ watts

Example 5.74 *Using nodal analysis, determine the Power supplied by 8V Voltage source. (Fig. 5.171)*

[JNTU May/June 2006]

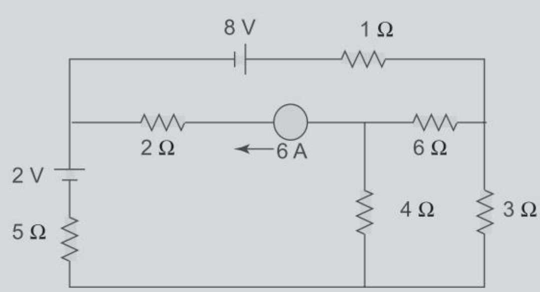


Fig. 5.171

Solution

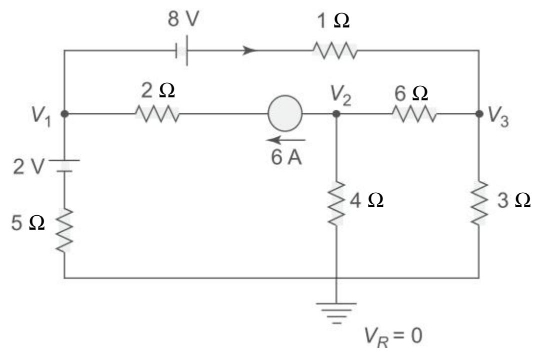


Fig. 5.172

Applying KCL at node (1);

$$\frac{V_1-2}{5}+\frac{V_1-V_3+8}{1}=6 \quad \Rightarrow \quad 5V_3-6V_1=8 \qquad (1)$$

Applying KCL at node (2);

$$6+\frac{V_2}{4}+\frac{V_2-V_3}{6}=0 \quad \Rightarrow \quad 5V_2-2V_3+72=0 \qquad (2)$$

Applying KCL at node (3);

$$\frac{V_3-V_2}{6}+\frac{V_3}{3}+\frac{V_3-V_1-8}{1}=0 \quad \Rightarrow \quad 9V_3-V_2+6V_1=48 \qquad (3)$$

Solving (1), (2) and (3), we get

$$V_1 = -4.593 \text{ volts}$$

$$V_2 = 11.56 \text{ volts}$$

$$V_3 = -7.11 \text{ volts}$$

From the circuit, $i=\dfrac{V_1+8-V_3}{1}=10.517\text{ A}$

Power supplied by 8 V source is (8×10.517)

$$= 84.136 \text{ Watts}$$

Example 5.75 *Find the current through 12 Ω resistor for the given circuit by nodal method as shown in Fig. 5.173.*

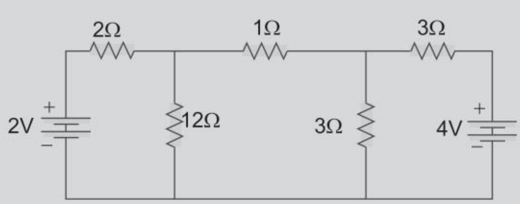


Fig. 5.173

Solution

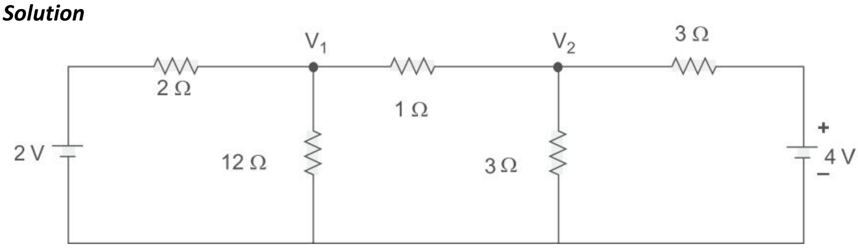


Fig. 5.174

Applying KCL at node 1

$$\frac{V_1 - 2}{2} + \frac{V_1}{12} + \frac{V_1 - V_2}{1}$$

Applying KCL at node 2

$$\therefore \quad \frac{V_2 - 4}{3} + \frac{V_2}{3} + \frac{V_2 - V_1}{1}$$

$$\therefore \quad \frac{V_1}{2} + \frac{V_1}{12} + \frac{V_1}{1} - \frac{V_2}{1} = 1 \qquad (1)$$

$$\therefore \quad \frac{V_2}{3} + \frac{V_2}{3} + \frac{V_2}{1} - \frac{V_1}{1} = \frac{4}{3} \qquad (2)$$

$$\therefore \quad V_1\left[1 + \frac{1}{2} + \frac{1}{12}\right] - V_2 = 1$$

From (1)

$$V_1\left[\frac{12+6+1}{12}\right] - V_2 = 1$$

$$\frac{19}{12}V_1 - V_2 = 1 \qquad (3)$$

From (2)

$$\therefore \quad V_2\left[\frac{1}{3} + \frac{1}{3} + 1\right] - V_1 = \frac{4}{3} \qquad (4)$$

$$\therefore \quad V_2\left[\frac{1+1+3}{3}\right] - V_1 = \frac{4}{3}$$

$$\therefore \quad \frac{5}{3}V_2 - V_1 = \frac{4}{3} \qquad (4)$$

$$\therefore \quad \frac{19}{12}V_1 - V_2 = 1 \qquad (5)$$

$$-V_1 + \frac{5}{3}V_2 = \frac{4}{3} \qquad (6)$$

Simplifying (5) & (6), we get

$$\therefore \qquad V_2 = 1.89 \text{ V}$$

$$\therefore \qquad V_1 = 1.82 \text{ V}$$

5.4.8 Steady State AC Nodal Analysis

The node voltage method can also be used with networks containing complex impedances and excited by sinusoidal voltage sources. In general, in an N node network, we can choose any node as the reference or datum node. In many circuits, this reference is most conveniently choosen as the common terminal or ground terminal. Then it is possible to write (N – 1) nodal equations using KCL. We shall illustrate nodal analysis with the following example.

Consider the circuit shown in Fig. 5.175.

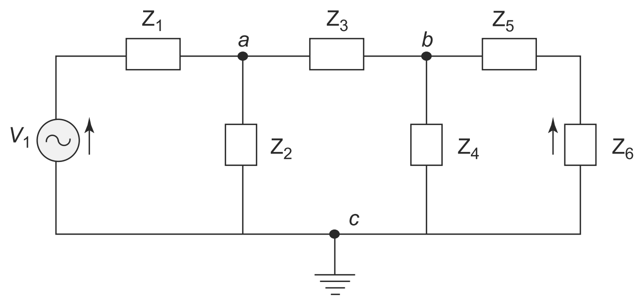


Fig. 5.175

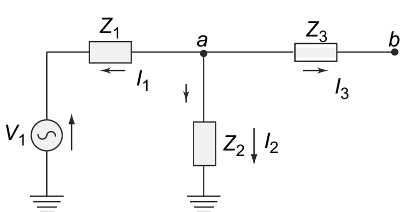


Fig. 5.176

Let us take a and b as nodes, and c as reference node. V_a is the voltage between nodes a and c. V_b is the voltage between nodes b and c. Applying Kirchhoff's current law at each node, the unknowns V_a and V_b are obtained.

In Fig. 5.176, node a is redrawn with all its branches, assuming that all currents are leaving the node a.

In Fig. 5.176, the sum of the currents leaving node a is zero.

$$\therefore \qquad I_1 + I_2 + I_3 = 0 \tag{5.70}$$

where $I_1 = \dfrac{V_a - V_1}{Z_1}, I_2 = \dfrac{V_a}{Z_2}, I_3 = \dfrac{V_a - V_b}{Z_3}$

Substituting I_1, I_2 and I_3 in Eq. 5.70, we get

$$\frac{V_a - V_1}{Z_1} + \frac{V_a}{Z_2} + \frac{V_a - V_b}{Z_3} = 0 \tag{5.71}$$

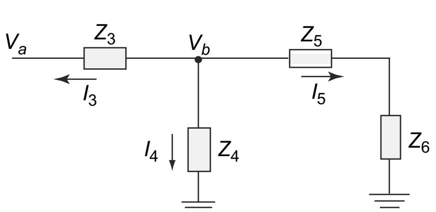


Fig. 5.177

Similarly, in Fig. 5.177, node b is redrawn with all its branches, assuming that all currents are leaving the node b.

In Fig. 5.177, the sum of the currents leaving the node b is zero.

$$\therefore \quad I_3 + I_4 + I_5 = 0 \tag{5.72}$$

where $I_3 = \dfrac{V_b - V_a}{Z_3}, I_4 = \dfrac{V_b}{Z_4}, I_5 = \dfrac{V_b}{Z_5 + Z_6}$

Substituting I_3, I_4 and I_5 in Eq. 5.72

we get
$$\frac{V_b - V_a}{Z_3} + \frac{V_b}{Z_4} + \frac{V_b}{Z_5 + Z_6} = 0 \tag{5.73}$$

Rearranging Eqs 5.71 and 5.73, we get

$$\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)V_a - \left(\frac{1}{Z_3}\right)V_b = \left(\frac{1}{Z_1}\right)V_1 \tag{5.74}$$

$$-\left(\frac{1}{Z_3}\right)V_a + \left(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5 + Z_6}\right)V_b = 0 \tag{5.75}$$

From Eqs 5.74 and 5.75, we can find the unknown voltages V_a and V_b.

Example 5.76 *In the network shown in Fig. 5.178, determine V_a and V_b.*

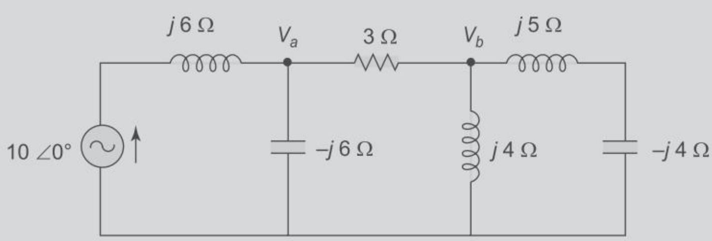


Fig. 5.178

Solution To obtain the voltage V_a at a, consider the branch currents leaving the node a as shown in Fig. 5.179.

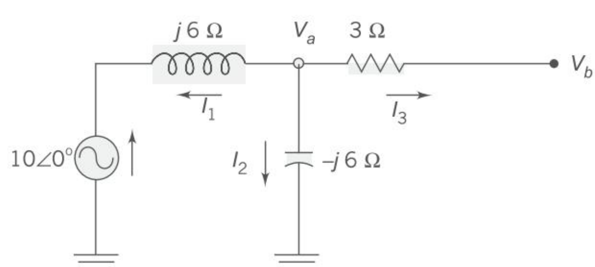


Fig. 5.179

In Fig. 5.179, $$I_1 = \frac{V_a - 10\angle 0^\circ}{j6}, I_2 = \frac{V_a}{-j6}, I_3 = \frac{V_a - V_b}{3}$$

Since the sum of the currents leaving the node a is zero,

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - 10\angle 0^\circ}{j6} + \frac{V_a}{-j6} + \frac{V_a - V_b}{3} = 0 \tag{5.76}$$

$$\left(\frac{1}{j6} - \frac{1}{j6} + \frac{1}{3}\right)V_a - \frac{1}{3}V_b = \frac{10\angle 0^\circ}{j6}$$

$$\therefore \quad \frac{1}{3}V_a - \frac{1}{3}V_b = \frac{10\angle 0^\circ}{j6} \tag{5.77}$$

To obtain the voltage V_b at b, consider the branch currents leaving node b as shown in Fig. 5.180.

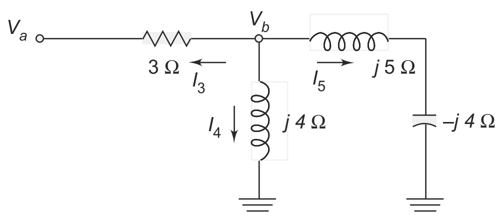


Fig. 5.180

In Fig. 5.180, $$I_3 = \frac{V_b - V_a}{3}, I_4 = \frac{V_b}{j4}, I_5 = \frac{V_b}{(j5 - j4)}$$

Since the sum of the currents leaving node b is zero

$$I_3 + I_4 + I_5 = 0$$

$$\frac{V_b - V_a}{3} + \frac{V_b}{j4} + \frac{V_b}{j1} = 0 \tag{5.78}$$

$$-\frac{1}{3}V_a + \left(\frac{1}{3} + \frac{1}{j4} + \frac{1}{j1}\right)V_b = 0 \tag{5.79}$$

From Eqs 5.78 and 5.79, we can solve for V_a and V_b.

$$0.33V_a - 0.33V_b = 1.67\angle -90^\circ \tag{5.80}$$

$$-0.33V_a + (0.33 - 0.25j - j)V_b = 0 \tag{5.81}$$

Adding Eqs 5.80 and 5.81 we get $(-1.25j)V_b = 1.67\angle -90^\circ$

$$-1.25\angle 90^\circ\, V_b = 1.67\angle -90^\circ$$

$$V_b = \frac{1.67\angle -90^\circ}{-1.25\angle 90^\circ}$$

$$= -1.34\angle -180^\circ$$

Substituting V_b in Eq. (5.80), we get

$$0.33V_a - (0.33)(-1.34\angle -180^\circ) = 1.67\angle -90^\circ$$

$$V_a = \frac{1.67\angle -90^\circ}{0.33} = -1.31\ \text{V}$$

$$V_a = 5.22\angle -104.5^\circ\ \text{V}$$

Voltages V_a and V_b are 5.22 ∠–104.5° V and –1.34 ∠–180° V respectively.

5.5 DUALITY AND DUAL NETWORKS

In an electrical circuit itself there are pairs of terms which can be interchanged to get new circuits. Such pair of dual terms are given below.

Current — Voltage
Open — Short
L — C
R — G
Series — Parallel
Voltage source — Current source
KCL — KVL

Consider a network containing R—L—C elements connected in series, and excited by a voltage source as shown in Fig. 5.181.

The integrodifferential equation for the above network is

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = \text{V}$$

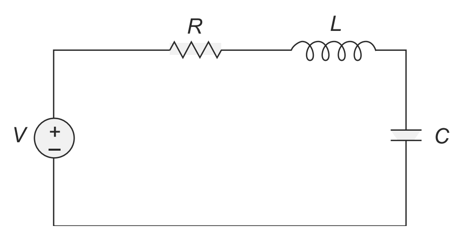


Fig. 5.181

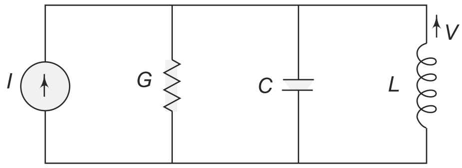


Fig. 5.182

Similarly, consider a network containing R—L—C elements connected in parallel and driven by a current source as shown in Fig. 5.182.

The integrodifferential equation for the network in Fig. 5.182 is

$$i = Gv + C\frac{dv}{dt} + \frac{1}{L}\int v\,dt$$

If we observe both the equations, the solutions of these two equations are the same. These two networks are called *duals*.

To draw the dual of any network, the following steps are to be followed.

1. In each loop of a network place a node; and place an extra node, called the *reference node*, outside the network.
2. Draw the lines connecting adjacent nodes passing through each element, and also to the reference node; by placing the dual of each element in the line passing through original elements.

For example, consider the network shown in Fig. 5.183.

Our first step is to place the nodes in each loop and a reference node outside the network.

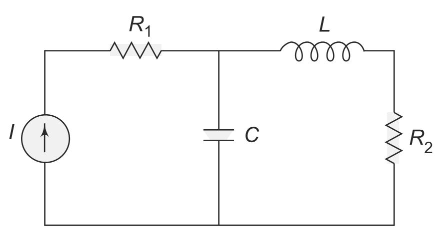


Fig. 5.183

Drawing the lines connecting the nodes passing through each element, and placing the dual of each element as shown in Fig. 5.184 (a) we get a new circuit as shown in Fig. 5.184 (b).

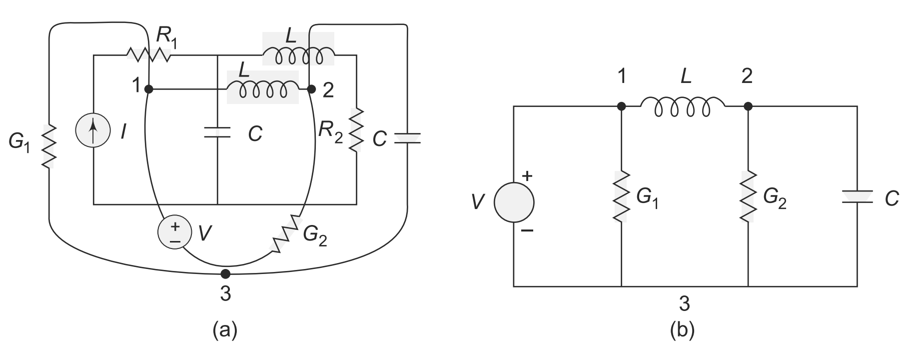


Fig. 5.184

Example 5.77 *Draw the dual network for the given network shown in Fig. 5.185.*

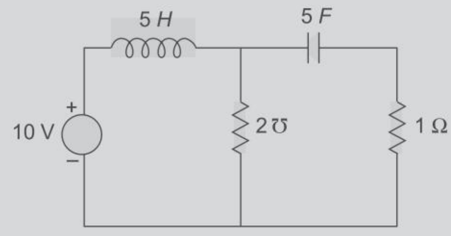


Fig. 5.185

Solution Place nodes in each loop and one reference node outside the circuit. Joining the nodes through each element, and placing the dual of each element in the line, we get the dual circuit as shown in Fig. 5.186 (a).

The dual circuit is redrawn as shown in Fig. 5.186 (b)

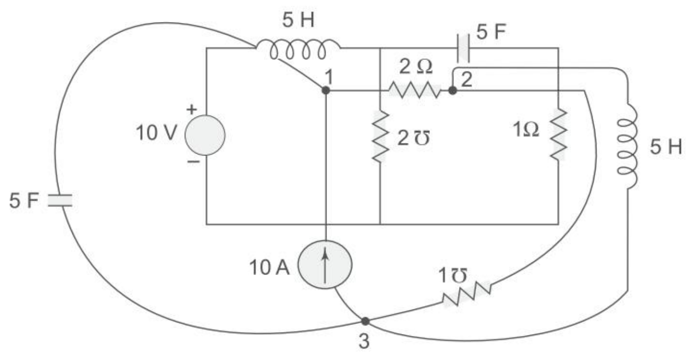


Fig. 5.186 (a)

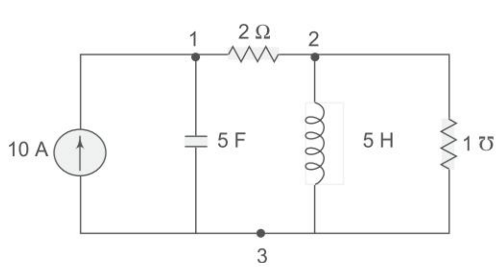


Fig. 5.186 (b)

Example 5.78 *What is duality? Explain the procedure for obtaining the dual of the given planar network shown in Fig. 5.187.*

[JNTU May/June 2004]

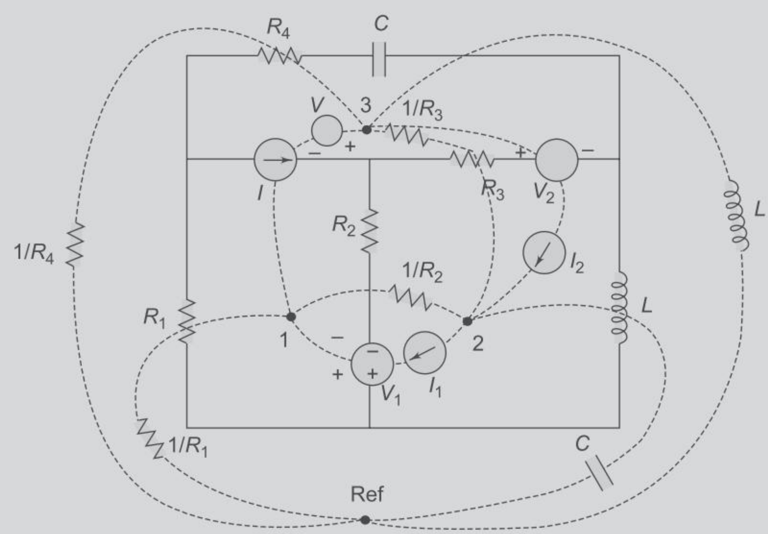


Fig. 5.187

Solution *Rule* 1: If a voltage source in the original network produces a clockwise current in the mesh, the corresponding dual element is a current source whose direction is towards the node representing the corresponding mesh.

Rule 2: If a current source in the original network produces a current in the clockwise direction in the mesh, the voltage source in the dual network will have a polarity such that the node representing the corresponding mesh is positive.

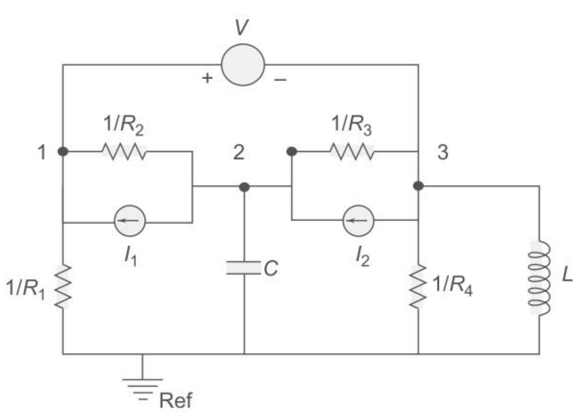


Fig. 5.188

Dual of the planar circuit given in Fig. 5.188.

Example 5.79 *Draw the dual circuit for the given circuit shown in Fig. 5.189.*

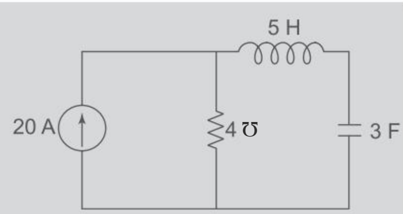


Fig. 5.189

Solution Our first step is to place nodes in each loop, and a reference node outside the circuit.

Join the nodes with lines passing through each element and connect these lines with dual of each element as shown in Fig. 5.190 (a).

The dual circuit of the given circuit is shown in Fig. 5.190 (b).

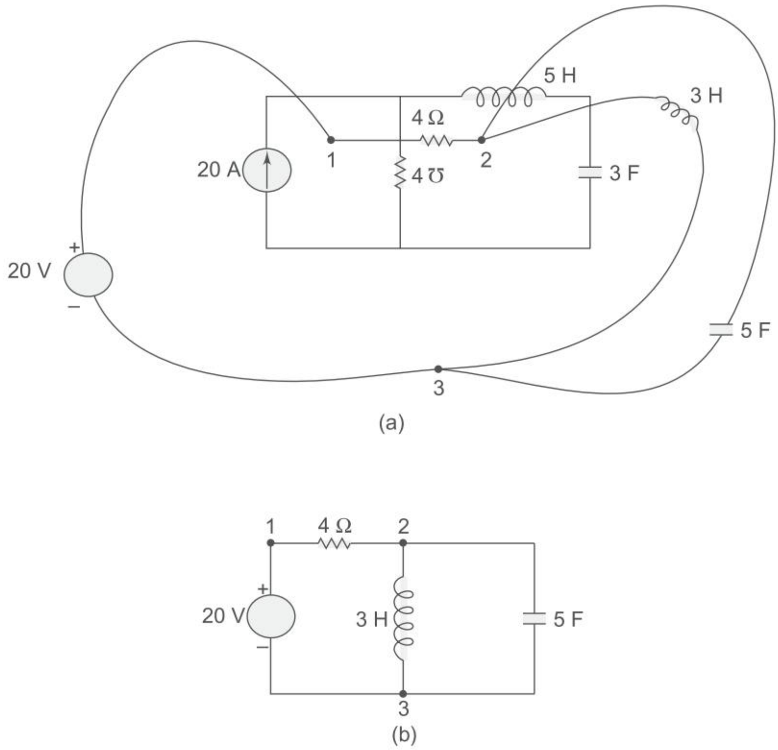


Fig. 5.190

Example 5.80 *Draw the dual circuit of the Fig. 5.191 given below.*

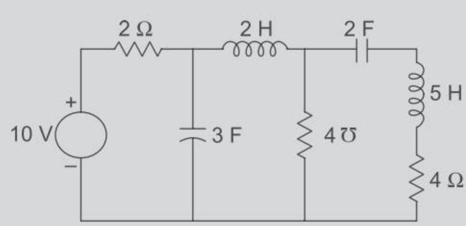
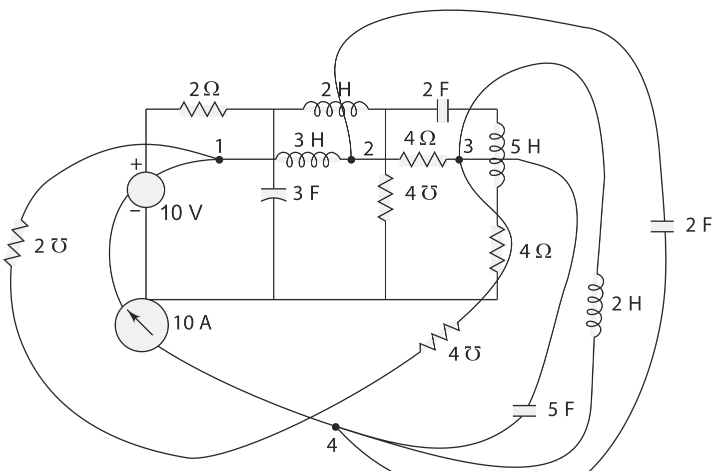


Fig. 5.191

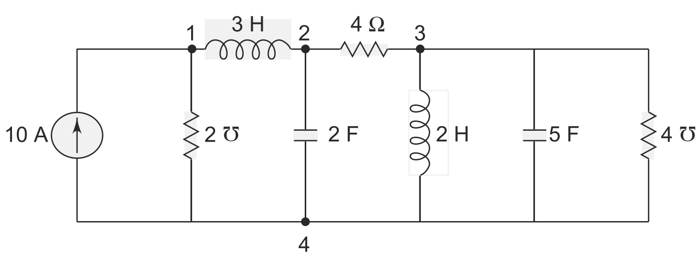
Solution Our first step is to mark nodes in each of the loop and a reference node outside the circuit.

Join the nodes with lines passing through each element and connect these lines with dual of each element as shown in Fig. 5.192 (a).

The dual circuit of given circuit is shown in Fig. 5.192 (b).



(a)



(b)

Fig. 5.192

Example 5.81 *Draw the dual network for the following circuit. Shown in Fig. 5.193.* *[JNTU June 2006]*

Solution

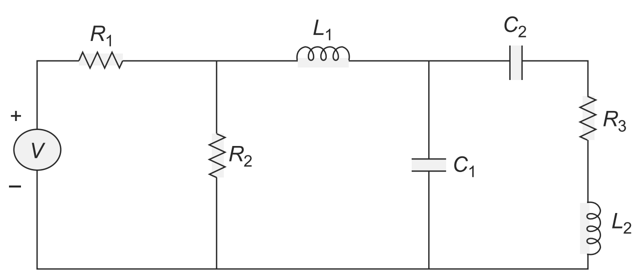


Fig. 5.193

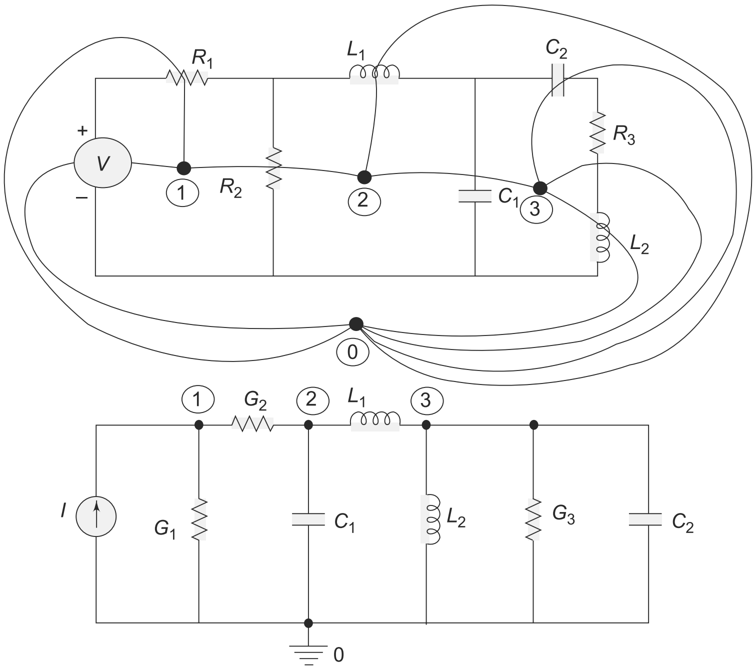


Fig. 5.194

Example 5.82 *Explain cleanly what you understand by "Duality" and "Dual network". Illustrate the procedure for drawing the dual of a given network.*
[JNTU June 2006]

Solution Two circuits are duals, if the mesh equations that characterise one of them have the same mathematical form as the nodal equations that characterise other.

Then they are said to duals (OH) satisfy duality of property i.e., if each mesh equation of one circuit is numerically identical with the corresponding nodal equation of other.

Network that satisfy duality property are called "Dual networks."

Dual pairs:

Resistance (R) → Conductance (G)
Inductance (L) → Capacitance (C)
Voltage (V) → Current (I)
Voltage Source → Current source
Node → Mesh
Series path → Parallel path
Open circuit → Short ckt
Thevenin → Norton

Steps to construct a dual circuit

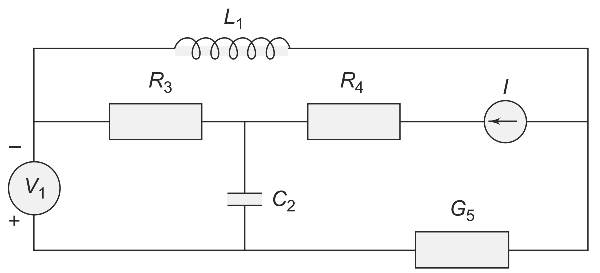
1. Place a node at the centre of each mesh of the given ckt. Place the reference node of the dual ckt outside the given ckt.
2. Draw dotted lines between the nodes such that each line crosses a network element by its dual.

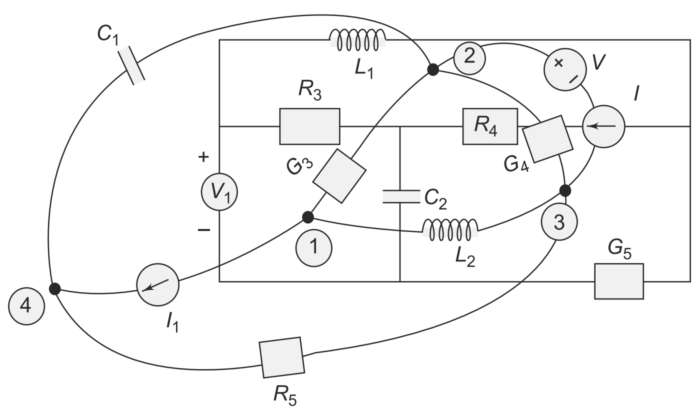
3. A voltage source that produces a positive (clockwise) mesh current has it dual or current source whose reference direction is from ground to non-reference node.

∴ Two circuits are said to be dual if they are described by the same characterising equations with dual quantities interchanged.

Example 5.83 *Draw the dual network of ckt.* *[JNTU June 2009]*

Solution





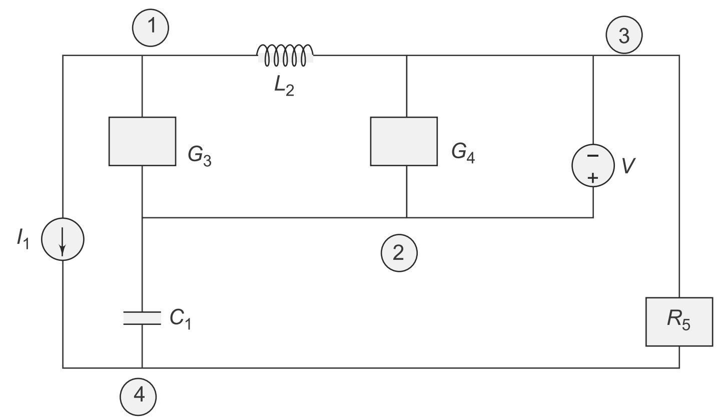


Fig. 5.195

Example 5.84 *Draw the dual network for the given network as in the following Fig. 5.196.*

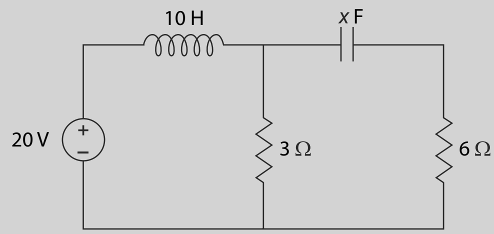


Fig. 5.196

Solution

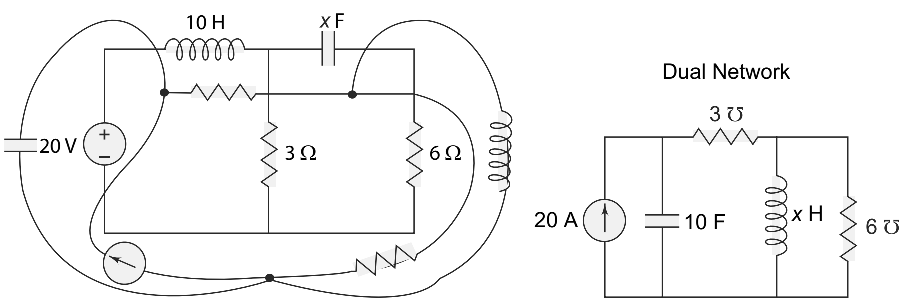


Fig. 5.197

Example 5.85 *Explain the procedure for obtaining the dual of the given planar network shown below.*

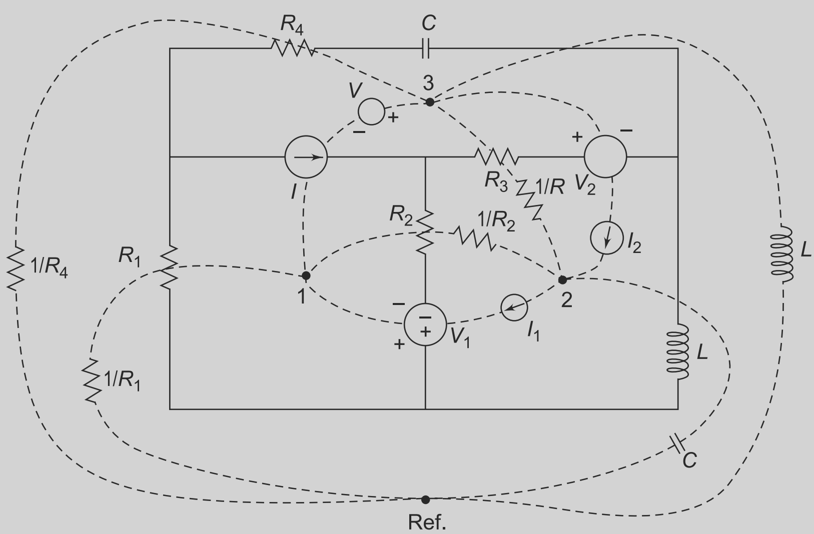


Fig. 5.198

Solution *Rule* 1: If a voltage source in the original network produces a c.w current in the mesh, the corresponding dual element is a current source whose direction is towards node representing the corresponding mesh.

Rule 2: If a current source in the original network produces a current in clockwise direction in the mesh, the voltage source in the dual network will have a polarity such that the node representing the corresponding mesh is positive.

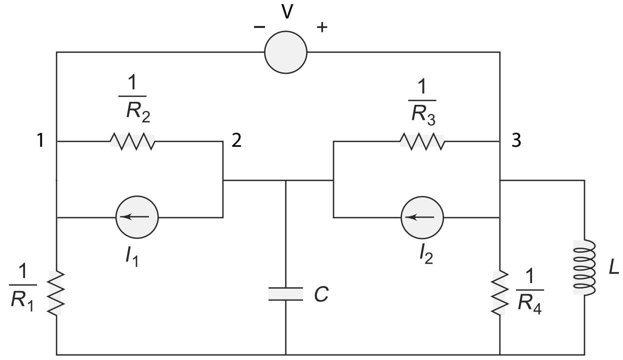


Fig. 5.199

Dual of the planar circuit given in 5.199.

Practice Problems

5.1 Determine the voltage V_{ab} and V_{bc} in the network shown in Fig. 5.200 by loop analysis, where source voltage $100\cos(314t + 45°)$.

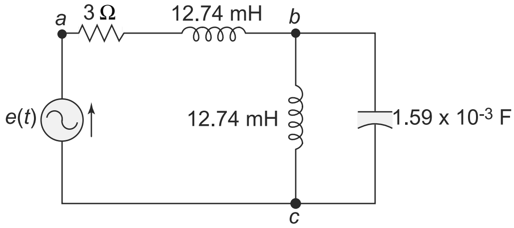


Fig. 5.200

5.2 Determine the power output of the voltage source by loop analysis for the network shown in Fig. 5.201. Also determine the power extended in the resistors.

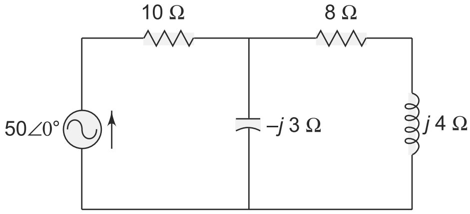


Fig. 5.201

5.3 Determine the value of source currents by loop analysis for the circuit shown in Fig. 5.202 and verify the results by using node analysis.

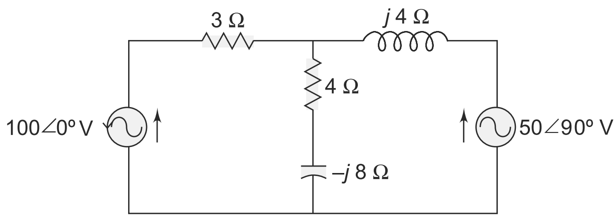


Fig. 5.202

5.4 Determine the power out of the source in the circuit shown in Fig. 5.203 by nodal analysis and verify the results by using loop analysis.

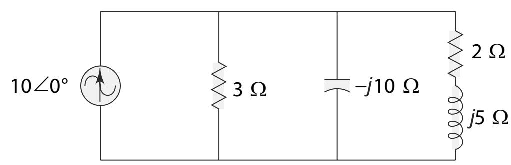


Fig. 5.203

5.5 For the circuit shown in Fig. 5.204 find the voltage across the dependent source branch by using mesh analysis.

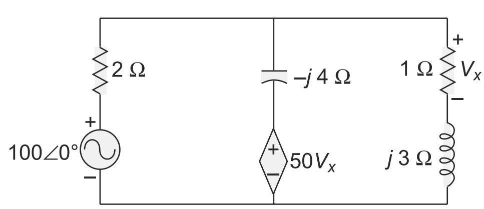


Fig. 5.204

5.6 For the circuit shown in Fig. 5.205, obtain the voltage across 500 kΩ resistor.

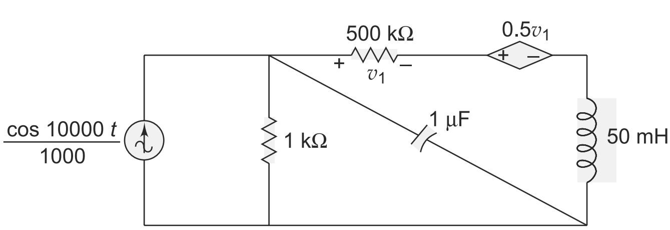


Fig. 5.205

5.7 In the circuit shown in Fig. 5.206, use mesh analysis to find out the power delivered to the 4 Ω resistor. To what voltage should the 100 V battery be changed so that no power is delivered to the 4 Ω resistor?

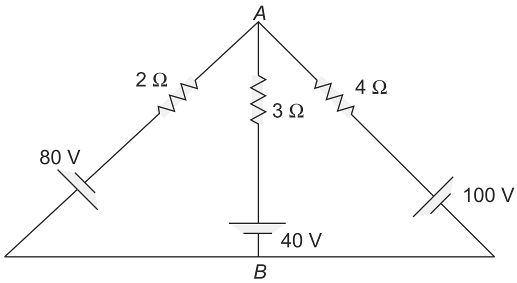


Fig. 5.206

5.8 Find the voltage between *A* and *B* of the circuit shown in Fig. 5.207 by mesh analysis.

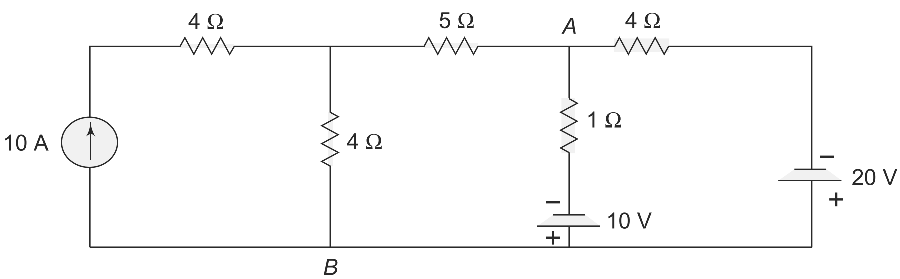


Fig. 5.207

5.9 In the circuit shown in Fig. 5.208, use nodal analysis to find out the voltage across 40 Ω and the power supplied by the 5 A source.

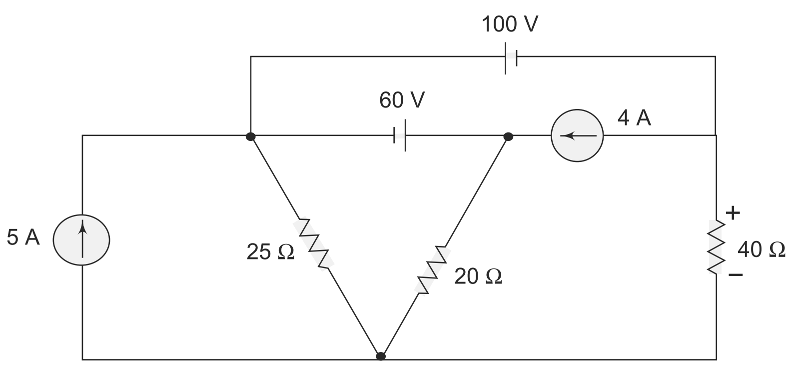


Fig. 5.208

5.10 In the network shown in Fig. 5.209, the resistance *R* is variable from zero to infinity. The current *I* through *R* can be expressed as $I = a + bV$, where *V* is the voltage across *R* as shown in the figure, and *a* and *b* are constants. Determine *a* and *b*.

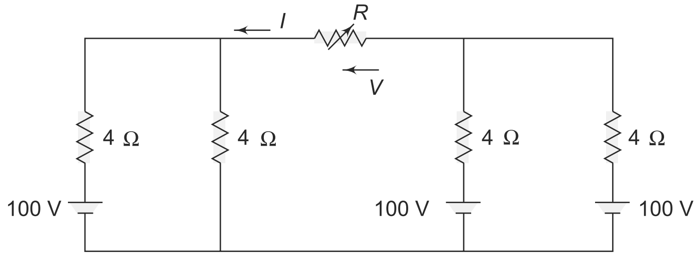


Fig. 5.209

5.11 Determine the currents in bridge circuit by using mesh analysis in Fig. 5.210.

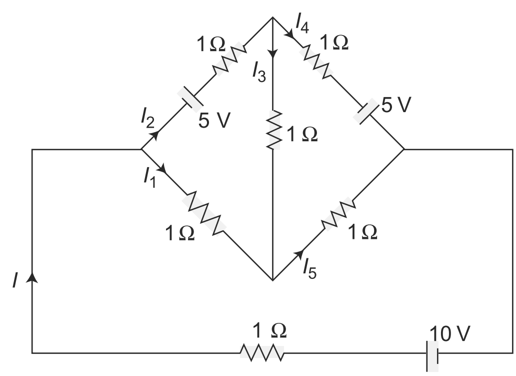


Fig. 5.210

5.12 Use nodal analysis in the circuit shown in Fig. 5.210 and determine what value of V will cause $V_{10} = 0$.

5.13 For the circuit shown in Fig. 5.212, use mesh analysis to find the values of all mesh currents.

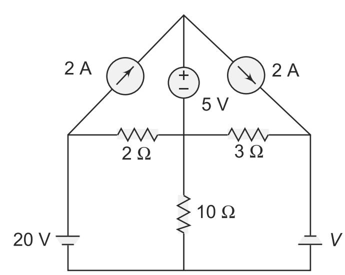


Fig. 5.211

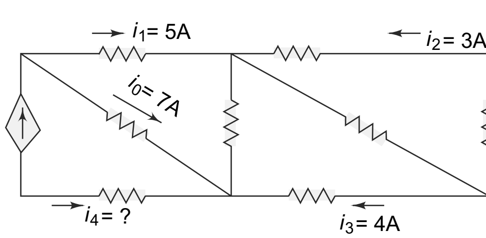


Fig. 5.212

5.14 For the circuit shown in Fig. 5.213, use node analysis to find the current delivered by the 24 V source.

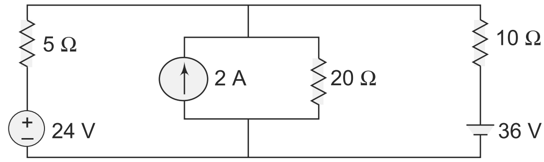


Fig. 5.213

5.15 Using mesh analysis, determine the voltage across the 10 kΩ resistor at terminals A and B of the circuit shown in Fig. 5.214.

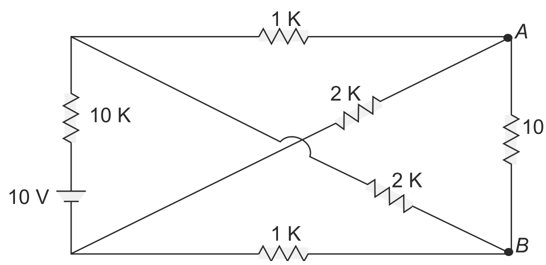


Fig. 5.214

5.16 Determine the current I in the circuit by using loop analysis in Fig. 5.215.

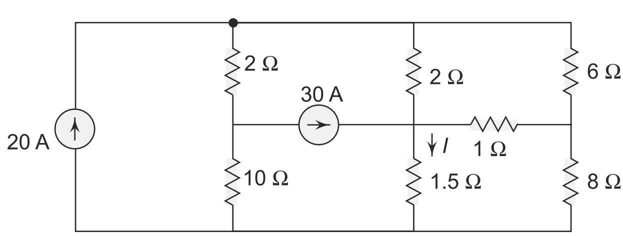


Fig. 5.215

5.17 Write nodal equations for the circuit shown in Fig. 5.216, and find the power supplied by the 10 V source.

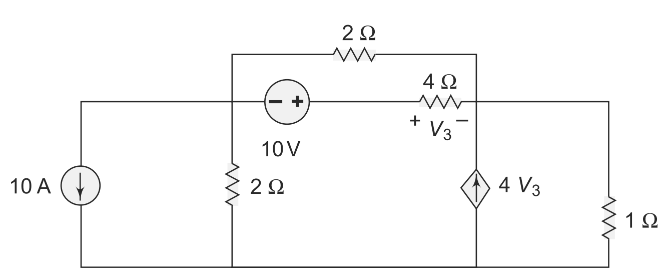


Fig. 5.216

5.18 Use nodal analysis to find V_2 in the circuit shown in Fig. 5.217.

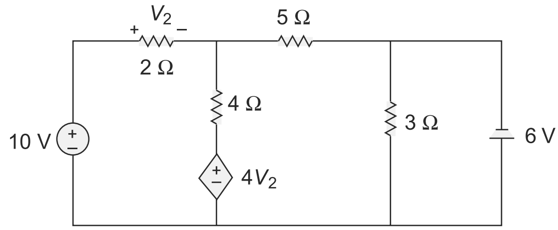


Fig. 5.217

5.19 Use mesh analysis to find V_x in the circuit shown in Fig. 5.218.

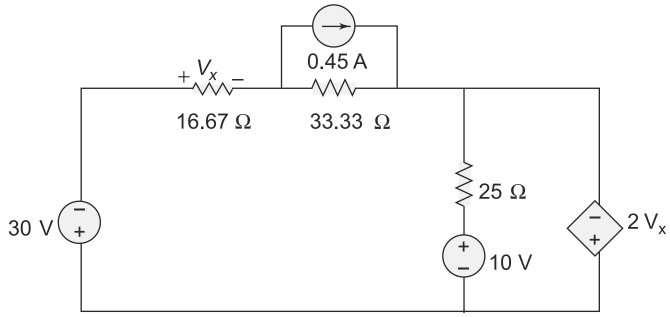


Fig. 5.218

5.20 For the circuit shown in Fig. 5.219, find the value of V_2 that will cause the voltage across 20 Ω to be zero by using mesh analysis.

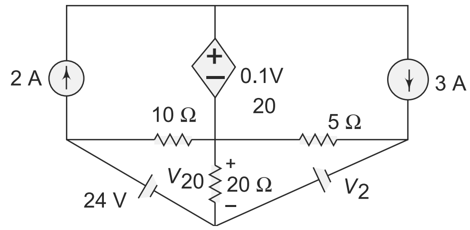


Fig. 5.219

Objective Type Questions

5.1 A tree has
(a) a closed path
(b) no closed paths
(c) none

5.2 The number of branches in a tree is_______the number of branches in a graph.
(a) less than
(b) more than
(c) equal to

5.3 The tie-set schedule gives the relation between
(a) branch currents and link currents
(b) branch voltages and link currents
(c) branch currents and link voltages
(d) none of the above

5.4 The cut-set schedule gives the relation between
(a) branch currents and link currents
(b) branch voltages and tree branch voltages
(c) branch voltages and link voltages
(d) branch current and tree currents

5.5 Mesh analysis is based on
(a) Kirchhoff's current law (b) Kirchhoff's voltage law
(c) Both (d) None

5.6 If a network contains B branches, and N nodes, then the number of mesh current equations would be
(a) $B - (N - 1)$ (b) $N - (B - 1)$
(c) $B - N - 1$ (d) $(B + N) - 1$

5.7 A network has 10 nodes and 17 branches. The number of different node pair voltages would be
(a) 7 (b) 9 (c) 45 (d) 10

5.8 A practical voltage source consists of
(a) an ideal voltage source in series with an internal resistance
(b) an ideal voltage source in parallel with an internal resistance
(c) both (a) and (b) are correct
(d) none of the above

5.9 A practical current source consists of
(a) a ideal current source in series with an resistance
(b) a ideal current source in parallel with an resistance
(c) both are correct
(d) none of the above

5.10 A network has seven nodes and five independent loops. The number of branches in the network is
(a) 13 (b) 12 (c) 11 (d) 10

5.11 The nodal method of circuit analysis is based on
(a) KVL and Ohm's law (b) KCL and Ohm's law
(c) KCL and KVL (d) KCL, KVL and Ohm's law

5.12 The number of independent loops for a network with n nodes and b branches is
(a) $n - 1$
(b) $b - n$
(c) $b - n + 1$
(d) independent of the number of nodes

5.13 Relative to a given fixed tree of a network
(a) link currents form an independent set
(b) branch currents form an independent set
(c) link voltages form an independent set
(d) branch voltages form an independent set

5.14 The number of independent loops for a network with 3 nodes and 6 branches is
(a) 2 (b) 1 (c) 4 (d) 6

5.15 A circuit consists of two resistances, 4 Ω and 4 Ω in parallel. The total current passing through the circuit is 10 A. The current passing through R_1 is
(a) 5 A (b) 10 A (c) 4 A (d) 2 A

5.16 A network has eight nodes and five independent loops. The number of branches in the network is
(a) 13 (b) 11 (c) 12 (d) 15

5.17 Mesh analysis is based on
(a) Kirchhoff's current law (b) Kirchhoff's voltage law
(c) Both (d) None

5.18 If a network contains B branches, and N nodes, then the number of mesh current equations would be
(a) $B - (N - 1)$ (b) $N - (B - 1)$
(c) $B - N - 1$ (d) $(B + N) - 1$

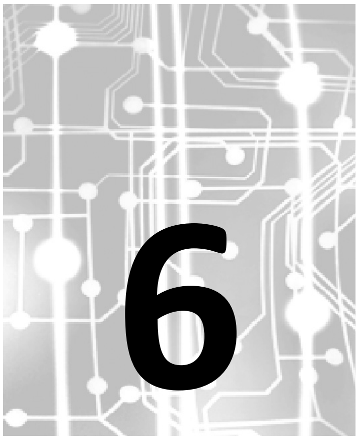
5.19 A network has 10 nodes and 17 branches. The number of different node pair voltages would be
(a) 7 (b) 9 (c) 45 (d) 10

5.20 A circuit consists of two resistances, R_1 and R_2, in parallel. The total current passing through the circuit is I_T. The current passing through R_1 is
(a) $\frac{I_T R_1}{R_1 + R_2}$ (b) $\frac{I_T (R_1 + R_2)}{R_1}$
(c) $\frac{I_T R_2}{R_1 + R_2}$ (d) $\frac{I_T R_1 + R_2}{R_2}$

5. 21 A network has seven nodes and five independent loops. The number of branches in the network is
(a) 13 (b) 12 (c) 11 (d) 10

5. 22 The nodel method of circuit anaysis is based on
(a) KVL and Ohm's law (b) KCL and Omp's law
(c) KCL and KVL (d) KCL, KVL and Omp's law

5. 23 The number of independent loops for a network with n nodes and b branches is
(a) $n - 1$ (b) $b - n$
(c) $b - n + 1$ (d) independent of the number of nodes



6 Network Theorems

6.1 SUPERPOSITION THEOREM

6.1.1 Superposition Theorem (dc Excitation) *[JNTU Nov. 2011]*

The superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative; that is, while considering the effect of individual sources, other ideal voltage sources and ideal current sources in the network are replaced by short circuit and open circuit across their terminals. This theorem is valid only for linear systems. This theorem can be better understood with a numerical example.

Consider the circuit which contains two sources as shown in Fig. 6.1.

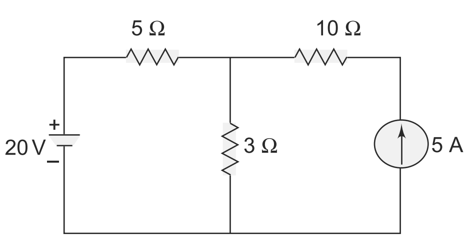


Fig. 6.1

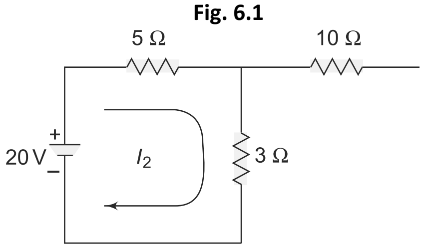


Fig. 6.2

Now let us find the current passing through the 3 Ω resistor in the circuit. According to superposition theorem, the current I_2 due to the 20 V voltage source with 5 A source open circuited = 20/(5 + 3) = 2.5 A. (See Fig. 6.2)

The current I_5 due to 5 A source with 20 V source short circuited is

$$I_5 = 5 \times \frac{5}{(3+5)} = 3.125 \text{ A}$$

The total current passing through the 3 Ω resistor is

$$(2.5 + 3.125) = 5.625 \text{ A}$$

Let us verify the above result by applying nodal analysis.

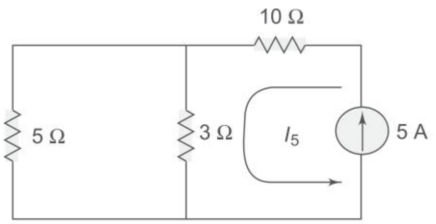


Fig. 6.3

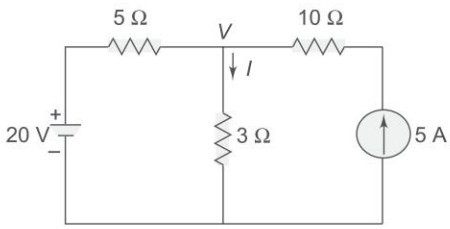


Fig. 6.4

The current passing in the 3 Ω resistor due to both sources should be 5.625 A.

Applying nodal analysis to Fig. 8.4, we have

$$\frac{V-20}{5}+\frac{V}{3}=5$$

$$V\left[\frac{1}{5}+\frac{1}{3}\right]=5+4$$

$$V=9\times\frac{15}{8}=16.875 \text{ V}$$

The current passing through the 3 Ω resistor is equal to $V/3$

$$\text{i.e.} \quad I=\frac{16.875}{3}=5.625 \text{ A}$$

So the superposition theorem is verified.

Let us now examine the power responses.

Power dissipated in the 3 Ω resistor due to voltage source acting alone

$$P_{20} = (I_2)^2R = (2.5)^2\, 3 = 18.75 \text{ W}$$

Power dissipated in the 3 Ω resistor due to current source acting alone

$$P_5 = (I_5)^2R = (3.125)^2\, 3 = 29.29 \text{ W}$$

Power dissipated in the 3 Ω resistor when both the sources are acting simultaneously is given by

$$P = (5.625)^2 \times 3 = 94.92 \text{ W}$$

From the above results, the superposition of P_{20} and P_5 gives

$$P_{20} + P_5 = 48.04 \text{ W}$$

which is not equal to $P = 94.92$ W

We can, therefore, state that the superposition theorem is not valid for power responses. It is applicable only for computing voltage and current responses.

Example 6.1 *Find the voltage across the 2 Ω resistor in Fig. 6.5 by using the super-position theorem.*

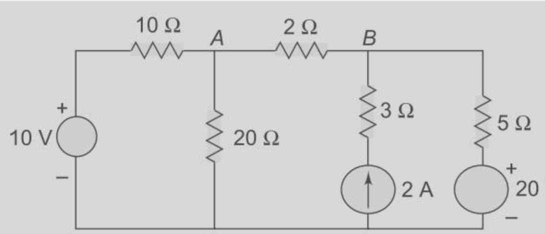


Fig. 6.5

Solution Let us find the voltage across the 2 Ω resistor due to individual sources. The algebraic sum of these voltages gives the total voltage across the 2 Ω resistor.

Our first step is to find the voltage across the 2 Ω resistor due to the 10 V source, while other sources are set equal to zero.

The circuit is redrawn as shown in Fig. 6.6(a).

Assuming a voltage V at node 'A' as shown in Fig. 6.6(a), the current equation is

$$\frac{V-10}{10}+\frac{V}{20}+\frac{V}{7}=0$$

$$V\,[0.1 + 0.05 + 0.143] = 1$$

or $$V = 3.41\text{ V}$$

The voltage across the 2 Ω resistor due to the 10 V source is

$$V_2=\frac{V}{7}\times 2=0.97\text{ V}$$

Our second step is to find out the voltage across the 2 Ω resistor due to the 20 V source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 6.6(b).

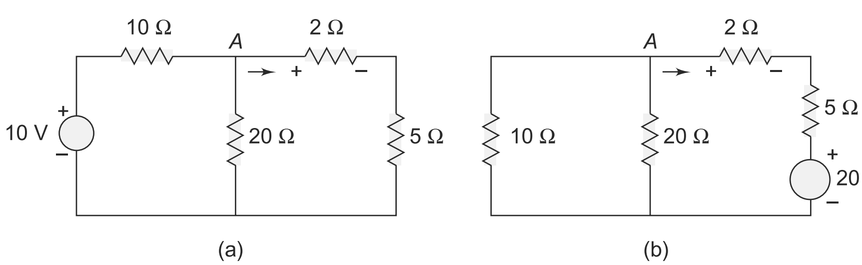


Fig. 6.6

Assuming voltage V at node A as shown in Fig. 6.6(b), the current equation is

$$\frac{V-20}{7}+\frac{V}{20}+\frac{V}{10}=0$$

$$V\,[0.143 + 0.05 + 0.1] = 2.86$$

or $$V=\frac{2.86}{0.293}=9.76\text{ V}$$

The voltage across the 2 Ω resistor due to the 20 V source is

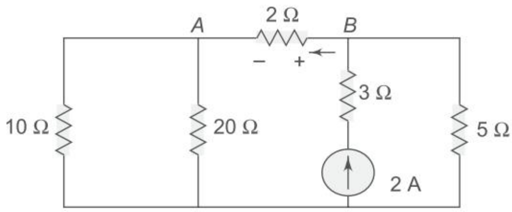
$$V_2=\left(\frac{V-20}{7}\right)\times 2=-2.92\text{ V}$$

The last step is to find the voltage across the 2 Ω resistor due to the 2 A current source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 6.6(c).

$$\text{The current in the } 2\ \Omega \text{ resistor} = 2 \times \frac{5}{5+8.67}$$

$$= \frac{10}{13.67} = 0.73\ \text{A}$$

The voltage across the 2 Ω resistor = 0.73 × 2 = 1.46 V



(c)

Fig. 6.6

The algebraic sum of these voltages gives the total voltage across the 2 Ω resistor in the network

$V = 0.97 - 2.92 - 1.46$

$= -3.41$ V

The negative sign of the voltage indicates that the voltage at '*A*' is negative.

Example 6.2 *For the resistive network shown in Fig. 6.7, find the current in each resistor, using the superposition principle.*

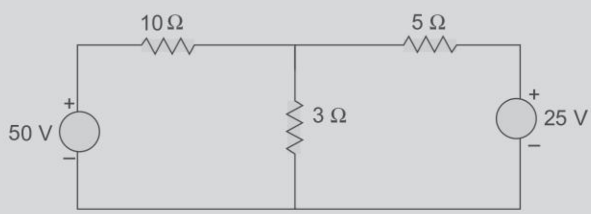


Fig. 6.7

Solution The current due to the 50 V source can be found in the circuit shown in Fig. 6.8(a).

Total resistance $R_T = 10 + \dfrac{5 \times 3}{8} = 11.9\ \Omega$

Current in the 10 Ω resistor $I_{10} = \dfrac{50}{11.9} = 4.2\ \text{A}$

Current in the 3 Ω resistor $I_3 = 4.2 \times \dfrac{5}{8} = 2.63\ \text{A}$

Current in the 5 Ω resistor $I_5 = 4.2 \times \dfrac{3}{8} = 1.58\ \text{A}$

The current due to the 25 V source can be found from the circuit shown in Fig. 6.8(b).

Total resistance

$$R_T = 5 + \frac{10 \times 3}{13} = 7.31\ \Omega$$

Current in the 5 Ω resistor

$$I_5' = \frac{25}{7.31} = 3.42\ \text{A}$$

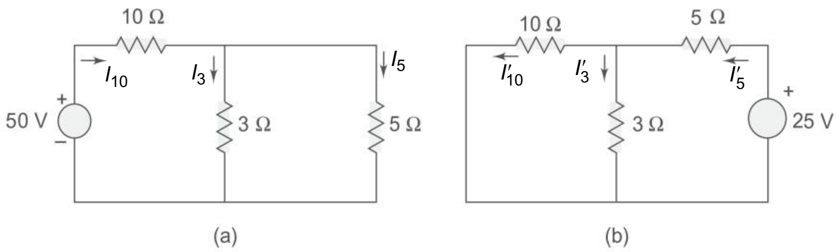


Fig. 6.8

Current in the 3 Ω resistor $\quad I'_3 = 3.42 \times \dfrac{10}{13} = 2.63\,\text{A}$

Current in the 10 Ω resistor $\quad I'_{10} = 3.42 \times \dfrac{3}{13} = 0.79\,\text{A}$

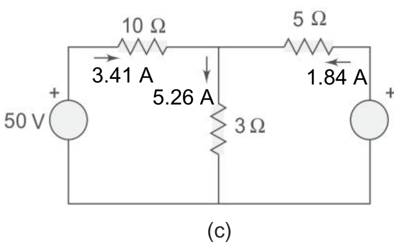


Fig. 6.8

According to superposition principle
Current in the 10 Ω resistor

$= I_{10} - I'_{10} = 4.2 - 0.79 = 3.41$ A

Current in the 3 Ω resistor

$= I_3 + I'_3 = 2.63 + 2.63 = 5.26$ A

Current in the 5 Ω resistor

$= I'_5 - I_5 = 3.42 - 1.58 = 1.84$ A

When both sources are operative, the directions of the currents are shown in Fig. 8.8(c).

Example 6.3 *Determine the voltage across the terminals AB in the circuit shown in Fig. 6.9.*

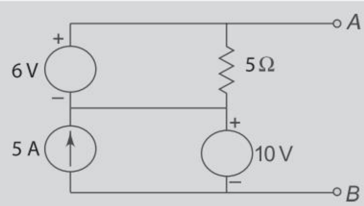


Fig. 6.9

Solution Voltage across AB is $V_{AB} = V_{10} + V_5$.

To find the voltage across the 5 Ω resistor, we have to use the superposition theorem.

Voltage across the 5 Ω resistor V_5 due to the 6 V source, when other sources are set equal to zero, is calculated using Fig. 6.10(a).

$$V_5 = 6\text{ V}$$

Voltage across the 5 Ω resistor V'_5 due to the 10 V sources, when other sources are set equal to zero, is calculated using Fig. 6.10(b).

$$V'_5 = 0$$

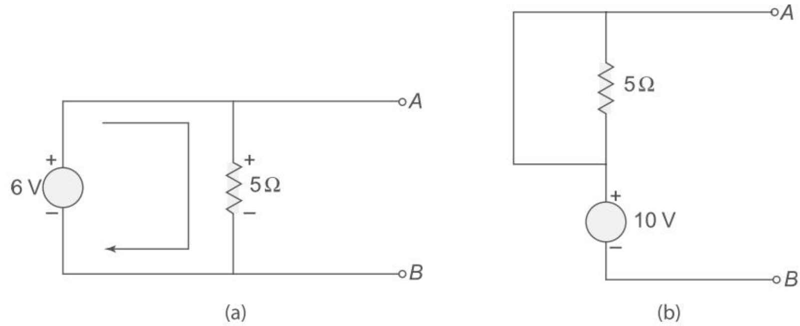


Fig. 6.10

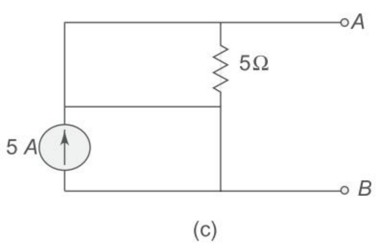


Fig. 6.10

Voltage across the 5 Ω resistor V'_5 due to the 5 A source only, is calculated using Fig. 6.10(c)

$V''_5 = 0$

According to the superposition theorem, Total voltage across the 5 Ω resistor $= 6 + 0 + 0 = 6\text{V}$.

So the voltage across terminals AB is $V_{AB} = 10 + 6 = 16$ V.

Example 6.4 *For the circuit shown Fig. 6.11, find the current i_4 using the superposition principle.*

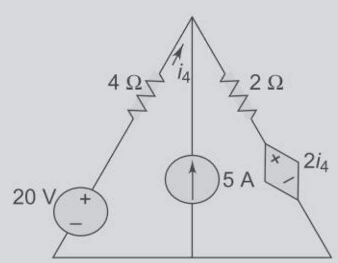


Fig. 6.11

Solution The circuit can be redrawn as shown in Fig. 6.12(a).

The current i'_4 due to the 20 V source can be found using the circuit shown in Fig. 6.12(b).

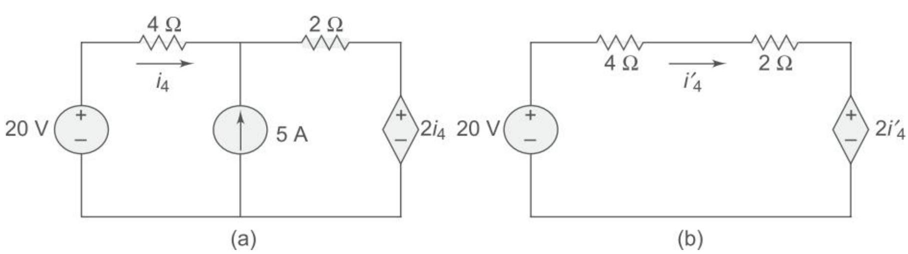


Fig. 6.12

Applying Kirchhoff's voltage law

$$-20 + 4i'_4 + 2i'_4 + 2i'_4 = 0$$

$$i'_4 = 2.5 \text{ A}$$

The current i'_4 due to the 5 A source can be found using the circuit shown in Fig. 6.12(c).

By assuming V' at node shown in Fig. 6.12(c) and applying Kirchhoff's current law

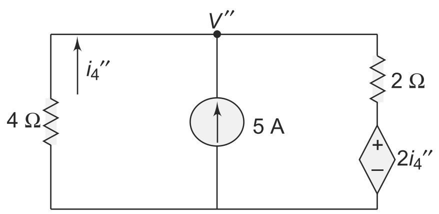


Fig. 6.12(c)

$$\frac{V''}{4} - 5 + \frac{V'' - 2i''_4}{2} = 0$$

$$i''_4 = \frac{-V''}{4}$$

From the above equations

$$i'_4 = -1.25 \text{ A}$$

$$\therefore \text{Total current } i_4 = i'_4 + i'_4 = 1.25 \text{ A}$$

Example 6.5 *Determine the current through the 2 Ω resistor as shown in the Fig. 6.13 by using the superposition theorem.*

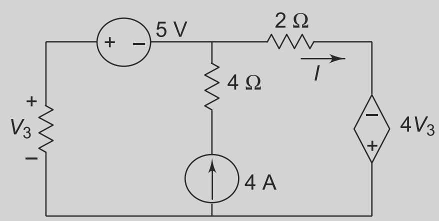


Fig. 6.13

Solution The current I' due to the 5 V source can be found using the circuit shown in Fig. 6.14(a).

By applying Kirchhoff's voltage law, we have

$$3I' + 5 + 2I' - 4V'_3 = 0$$

we know $\quad V'_3 = -3I'$

From the above equations

$$I' = -0.294 \text{ A}$$

The current I' due to the 4 A source can be found using the circuit shown in Fig. 8.14(b).

By assuming node voltage V'_3, we find

$$I'' = \frac{V''_3 + 4V''_3}{2}$$

By applying Kirchhoff's current law at node we have

$$\frac{V''_3}{3} - 4 + \frac{V''_3 + 4V''_3}{2} = 0$$

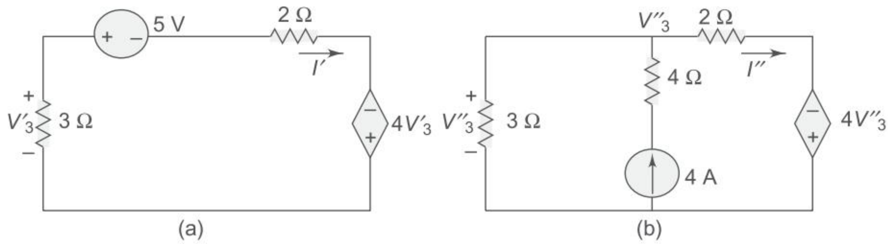


Fig. 6.14

$$V_3' = 1.55 \text{ V}$$

$$\therefore \qquad I'' = \frac{V_3'' + 4V_3''}{2} = 3.875 \text{ A}$$

Total current in the 2 Ω resistor $I = I' + I'' = -0.294 + 3.875$

$$\therefore \qquad I = 3.581 \text{ A}$$

Example 6.6 *Find the current I the circuit shown in Fig. 6.15.*

[JNTU May/June 2006]

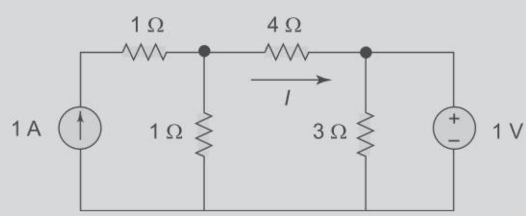


Fig. 6.15

Solution Applying superposition

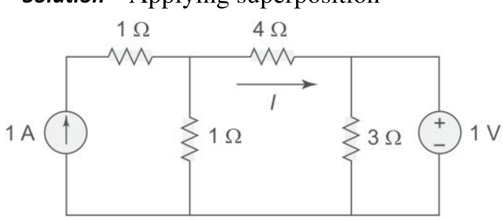


Fig. 6.16

Open circuit the current source

$$I_1 = \frac{1}{5}$$

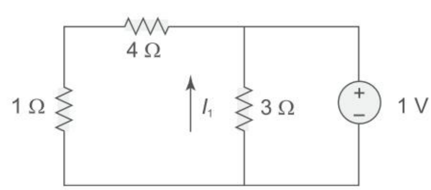


Fig. 6.17

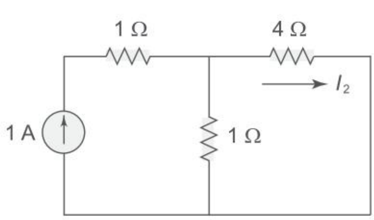


Fig. 6.18

Short the voltage source

$$I_2 = 1 \times \frac{1}{5} = \frac{1}{5}$$

Total current through $4\ \Omega = \frac{1}{5} - \frac{1}{5} = 0$

Example 6.7 *Find the current i in the circuit shown in Fig. 6.19 using superposition theorem.* *[May/June 2006 Network Analysis]*

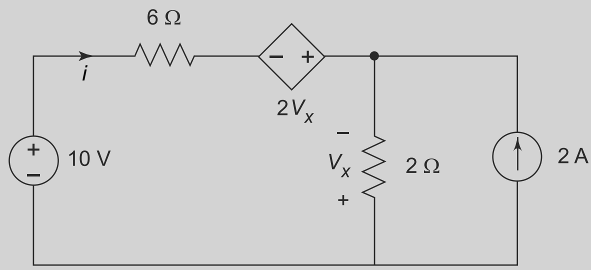


Fig. 6.19

Solution Consider 2 A current source acting alone by short circuiting voltage source 10 V as shown in Fig. 6.19(a) $6i_1 - 2V_x - V_x = 0$

$$i_2 = -2\text{ A}$$

$$V_x = -2(i_1 - i_2)$$

$$= -2i_1 - 4$$

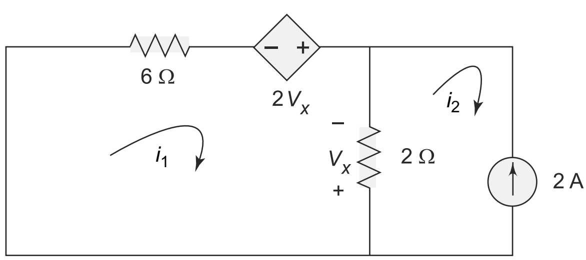


Fig. 6.19(a)

$$6i_1 - 3V_x = 0 \Rightarrow 6i_1 - 3(-2i_1 - 4) = 0$$

$$6i_1 + 6i_1 + 12 = 0 \Rightarrow i_1 = -1\text{ A}$$

Consider 10V voltage source acting alone by opening 2A current source in Fig. 6.19(b)

$$-10 + 6i_1' - 2V_x - V_x = 0$$

$$V_x = -2i_1'$$

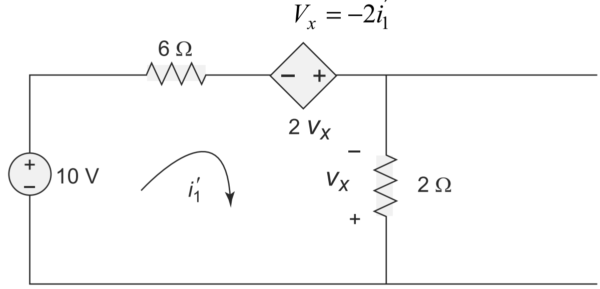


Fig. 6.19(b)

$$-10+6i_1'-3V_x=0 \Rightarrow -10+6i_1'+6i_1'=0 \Rightarrow i_1'=5/6$$
$$i=i_1+i_1'=-1+5/6=-1/6A$$

Example 6.8 *Is superposition valid for power? Explain.* *[JNTU May/June 2004]*

Solution Superposition theorem is valid only for linear systems.

Superposition cannot be applied for power because the equation for power is nonlinear.

Let us consider a network with a voltage source and current source as shown below and find the power consumed in 9 Ω resistor by super position.

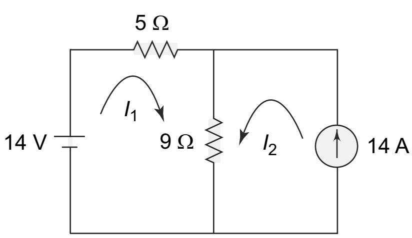


Fig. 6.20

When 14 V source is acting, the current in 9 Ω is 1 A

The power = $i^2 \times 9$ = 9 watts

When 14 A source is acting, the current in 9 Ω is 5 A

The power = $i^2 \times 9$ = 225 watts

Total power = 225 + 9 = 234 watts

When both are acting the KVL for loop 1 and 2

are $14 = 5i_1 + 9(i_1 + i_2)$

$$14i_1 = -112$$

$$i_1 = -8\text{ A};\ i_2 = 14\text{ A}$$

Current in 9 Ω resistor is $i_1 + i_2 = 6$ A

Power = $(6)^2 \times 9$ = 324 watts

Since power is not the same in both the cases, the superposition theorem does not hold true.

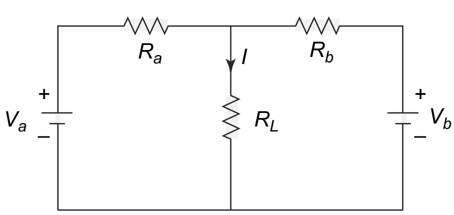


Fig. 6.21

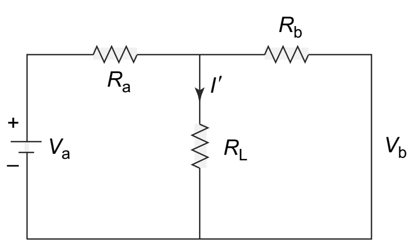


Fig. 6.22

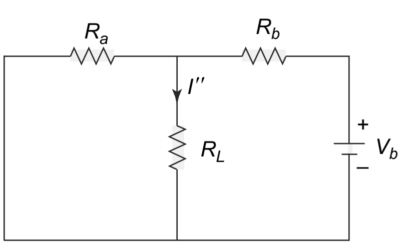


Fig. 6.23

Consider the circuit shown below.

When V_a is acting.

I^1 be the current through R_L: and Power = $(I')^2 R_L$

When V_b is acting I'' be the current through R_L and Power = $(I'')^2 R_L$

Total current: Through R_L by superposition

$I = I' + I''$ and power $= I^2 R_L$

$(I')^2 R_L + (I'')^2, R_L \neq I^2 R_L$

because $I^2 = (I' + I'')^2 = (I')^2 + (I'')^2 + 2I'I''$

Hence $(I')^2 + (I'')^2 \neq I^2$ and therefore superposition theorem is not valid for power.

Example 6.9 *Using superposition theorem, find V_{AB}.* *[JNTU May/June 2004]*

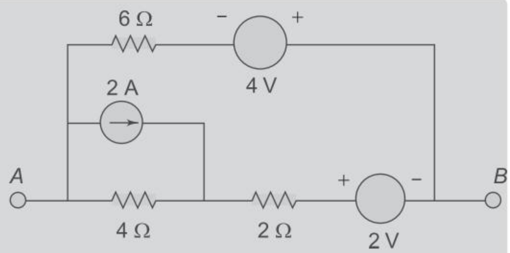


Fig. 6.24

Solution When 4 V source is acting alone, the circuit becomes

Current through the circuit

$$i' = \frac{-1}{3}\text{A}$$

$$\therefore \quad V_{AB1} = i' \times 6 = -2\text{V}$$

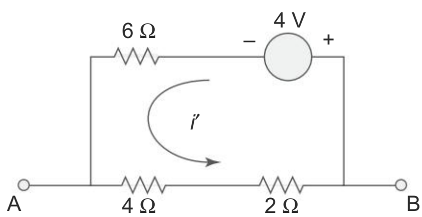


Fig. 6.25

When 2V source is acting alone, the circuit becomes

Current through the circuit

$$i' = \frac{2}{12} = \frac{1}{6}\text{A}$$

$$\therefore \quad V_{AB2} = -i'' \times 6 + 2$$

$$= -1 + 2 = 1\text{ V}$$

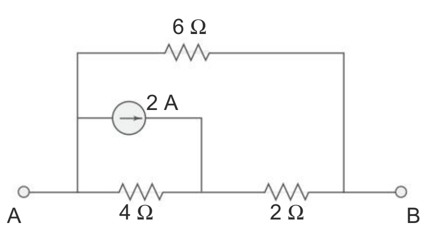


Fig. 6.26

When 2 A source is acting alone, the circuit becomes

Current in 4 Ω resistor $= 2 \times \frac{8}{12} = \frac{4}{3}\text{A}$

Voltage across 4 Ω resistor $= \frac{16}{3}\text{ V}$

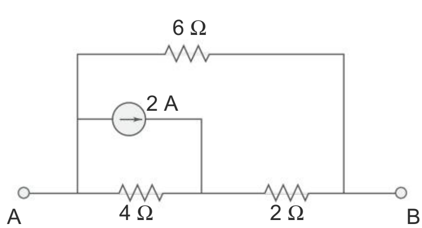


Fig. 6.27

$$\text{Current in } 2\ \Omega \text{ resistor} = 2 \times \frac{4}{12} = \frac{2}{3}\text{ A}$$

$$\text{Voltage across } 2\ \Omega \text{ resistor} = \frac{4}{3}\text{ V}$$

$\therefore$

$$V_{AB3} = -V_4 + V_2 = \frac{-16}{3} + \frac{4}{3} = -4\text{ V}$$

Voltage across *AB*

$$V_{AB} = V_{AB1} + V_{AB2} + V_{AB3}$$

$$= -2 + 1 - 4 = -5 \text{ volts.}$$

Example 6.10 *Solve for current in 5 ohms resistor by principle of super position theorem shown in Fig. 6.28.* *[JNTU June 2009]*

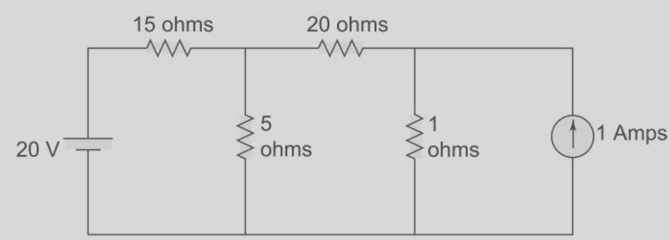


Fig. 6.28

Solution Open circuiting current source

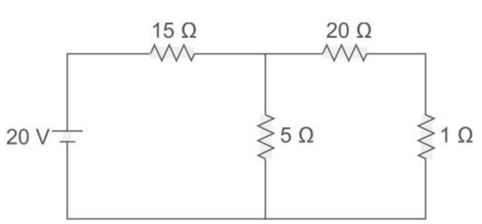


Fig. 6.29

Replacing series combination of 20 Ω and 1 Ω by (20 + 1) Ω = 21 Ω and 20 V voltage source with series resistance of 15 Ω by current source of $\left(\frac{20}{15}\right)$ amp with parallel resistance of 15 Ω.

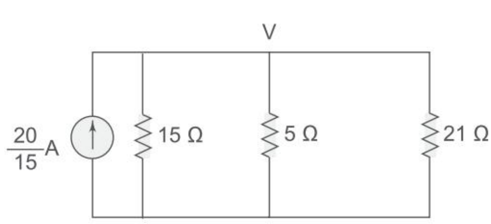


Fig. 6.30

$$\therefore \quad \frac{20}{15} = \frac{V}{15} + \frac{V}{5} + \frac{V}{21}$$

or, $V = 4.232$ volt

$\therefore$ Current in 5 Ω

$$= \frac{4.232}{5}\text{ amp}$$

$$= 0.846 \text{ amp}$$

Short circuiting voltage source

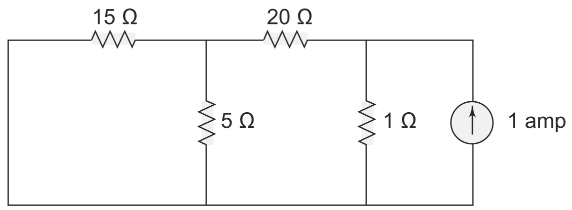


Fig. 6.31

Replacing 1 amp current source with parallel resistance of 1 Ω by a voltage source of 1 V with series resistance of 1 Ω

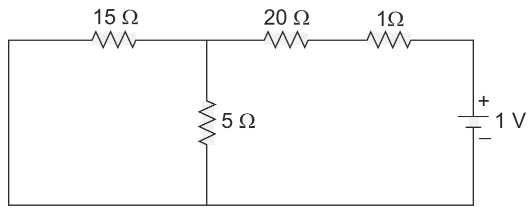


Fig. 6.32

Replacing series combination of 20 Ω and 1 Ω by (20 + 1) Ω = 21 Ω

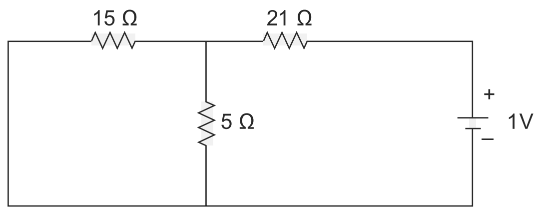


Fig. 6.33

Replacing voltage source of 1V with series resistance of 21 Ω by a current source of (1/21) amp with a parallel resistance of 21 Ω

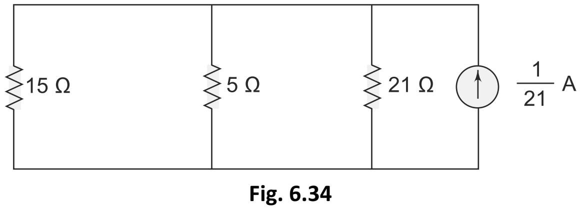


Fig. 6.34

$$\therefore \qquad \frac{1}{21} = \frac{V}{15} + \frac{V}{5} + \frac{V}{21}$$

$\therefore \quad V = 0.151 \text{ volt}$

$\therefore$ Current through $5\,\Omega = \dfrac{0.151}{5}$ amp

$= 0.03$ amp

$\therefore$ Total current in $5\ \Omega = (0.846 + 0.03)$ amp

$= 0.876$ amp.

6.1.2 Superposition Theorem (ac Excitation)

[JNTU Jan 2010]

The superposition theorem can be used to analyse ac circuits containing more than one source. The superposition theorem states that the response in any element in a circuit is the vector sum of the responses that can be expected to flow if each source acts independently of other sources. As each source is considered, all of the other sources are replaced by their internal impedances, which are mostly short circuits in the case of a voltage source, and open circuits in the case of a current source. This theorem is valid only for linear systems. In a network containing complex impedance, all quantities must be treated as complex numbers.

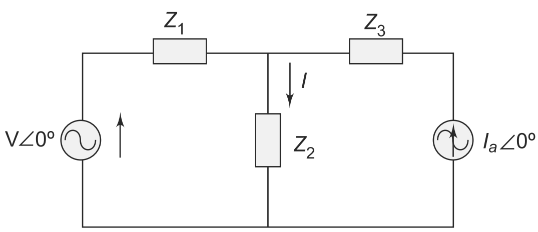


Fig. 6.35

Consider a circuit which contains two sources as shown in Fig. 6.35.

Now let us find the current I passing through the impedance Z_2 in the circuit. According to the superposition theorem, the current due to voltage source $V\ \angle 0°$ V is I_1 with current source $I_a\ \angle 0°$ A open circuited.

$$I_1 = \frac{V\ \angle 0°}{Z_1 + Z_2}$$

The current due to $I_a\ \angle 0°$ A is I_2 with voltage source $V\angle 0°$ short circuited.

$$\therefore \quad I_2 = I_a\ \angle 0° \times \frac{Z_1}{Z_1 + Z_2}$$

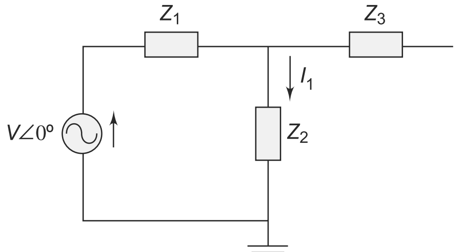


Fig. 6.36

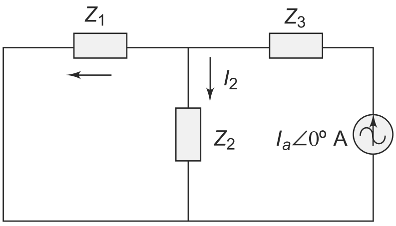


Fig. 6.37

The total current passing through the impedance Z_2 is

$$I = I_1 + I_2$$

The superposition theorem finds use in the study of AC circuits, amplifier circuits, where sometimes AC is often superimposed with DC. This theorem defines the behaviour of a linear circuit. Within the context of linear circuit analysis, this theorem provides the basis for all other theorems. Given a linear circuit, it is easy to see how mesh analysis and nodal analysis make use of the principle of superposition.

It is not possible to apply superposition theorem directly to determine power associated with an element. In addition, application of superposition theorem does not normally lead to simplification of analysis. It is not best technique to determine all currents and voltages in a circuit, driven by multiple sources. Superposition theorem works only for circuits that are reducible to series/parallel combinations for each of the sources at a time. This theorem is useless for analyzing an unbalanced bridge circuit. Networks containing components like lamps or varistors could not be analyzed.

Example 6.11 *Determine the voltage across (2 + j5) Ω impedance as shown in Fig. 6.38 by using the superposition theorem.*

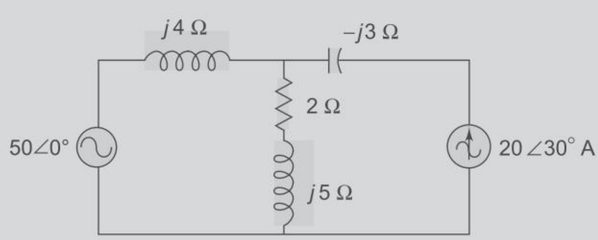


Fig. 6.38

Solution According to the superposition theorem, the current due to the $50\angle 0°$ V voltage source is I_1 as shown in Fig. 6.39 with current source $20\angle 30°$ A open circuited.

Current $$I_1 = \frac{50\angle 0°}{2 + j4 + j5} = \frac{50\angle 0°}{(2 + j9)}$$

$$= \frac{50\angle 0°}{9.22\angle 77.47°} = 5.42\angle -77.47° \text{ A}$$

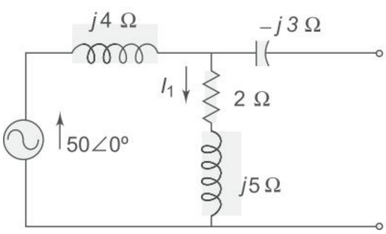


Fig. 6.39

Voltage across $(2 + j5)$ Ω due to current I_1 is

$$V_1 = 55.42 \angle -77.47° \,(2 + j5)$$

$$= (5.38)\,(5.42) \angle -77.47° + 68.19°$$

$$= 29.16 \angle -9.28°$$

The current due to $20\angle 30°$ A current source is I_2 as shown in Fig. 6.40, with voltage source $50\angle 0°$ V short circuited.

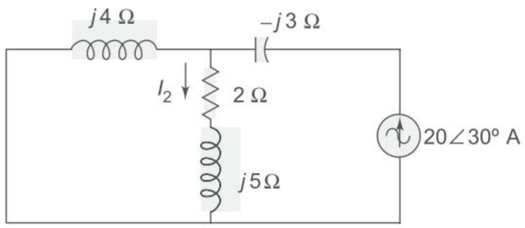


Fig. 6.40

Current $$I_2 = 20\angle 30° \times \frac{(j4)\,\Omega}{(2+j9)\,\Omega}$$

$$= \frac{20\angle 30° \times 4\angle 90°}{9.22\angle 77.47°}$$

$\therefore$ $$I_2 = 8.68\angle 120° - 77.47° = 8.68\angle 42.53°$$

Voltage across $(2 + j5)$ Ω due to current I_2 is

$$V_2 = 8.68\angle 42.53°\ (2 + j5)$$

$$= (8.68)\ (5.38)\ \angle 42.53° + 68.19°$$

$$= 46.69\ \angle 110.72°$$

Voltage across $(2 + j5)$ Ω due to both sources is

$$V = V_1 + V_2$$

$$= 29.16\ \angle -9.28° + 46.69\angle 110.72°$$

$$= 28.78 - j4.7 - 16.52 + j43.67$$

$$= (12.26 + j38.97)\ \text{V}$$

Voltage across $(2 + j5)$ Ω is $V = 40.85\ \angle 72.53°$.

Example 6.12 *For the circuit shown in Fig. 6.41, determine the voltage V_{AB} using the superposition theorem.*

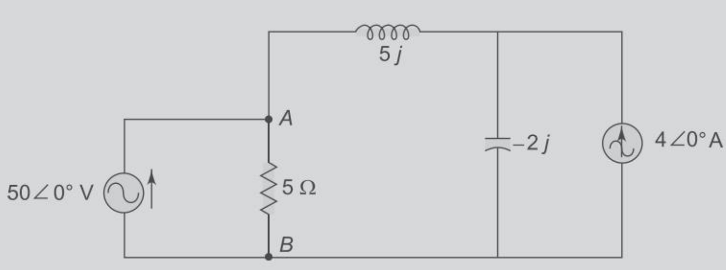


Fig. 6.41

Solution Let source $50\angle 0°$ V act on the circuit and set the source $4\angle 0°$ A equal to zero. If the current source is zero, it becomes open-circuited. Then the voltage across AB is $V_{AB} = 50\angle 0°$.

Now set the voltage source $50\angle 0°$ V is zero, and is short circuited, or the voltage drop across AB is zero.

The total voltage is the sum of the two voltages.

$$\therefore \quad V_T = 50\angle 0°$$

Example 6.13 *For the circuit shown in Fig. 6.42, determine the current in (2+j3) Ω by using the superposition theorem.*

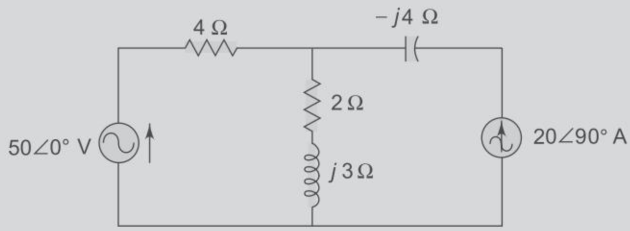


Fig. 6.42

Solution The current in (2 + j3) Ω, when the voltage source $50\angle 0°$ acting alone is

$$I_1 = \frac{50\angle 0°}{(6+j3)} = \frac{50\angle 0°}{6.7\angle 26.56°}$$

$$\therefore \quad I_1 = 7.46\ \angle -26.56°\ \text{A}$$

Current in (2 + j3) Ω, when the current source $20\angle 90°$ A acting alone is

$$I_2 = 20\angle 90° \times \frac{4}{(6+j3)}$$

$$= \frac{80\angle 90°}{6.7\angle 26.56°} = 11.94\angle 63.44°\ \text{A}$$

Total current in (2 + j3) Ω due to both sources is

$$I = I_1 + I_2$$
$$= 7.46\angle -26.56° + 11.94\angle 63.44°$$
$$= 6.67 - j3.33 + 5.34 + j10.68$$
$$= 12.01 + j7.35 = 14.08\angle 31.46°$$

Total current in (2 + j3) Ω is $I = 14.08\angle 31.46°$

Example 6.14 *Find the current in the 6 Ω resistor using superposition theorem as shown in Fig. 6.43.* *[JNTU May/June 2006]*

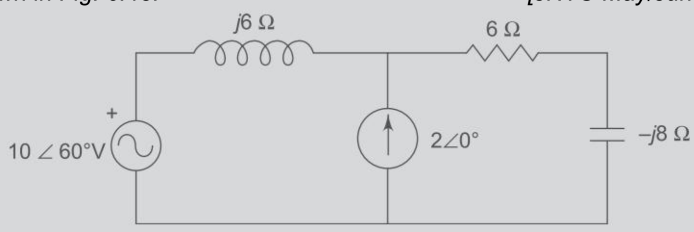


Fig. 6.43

Solution

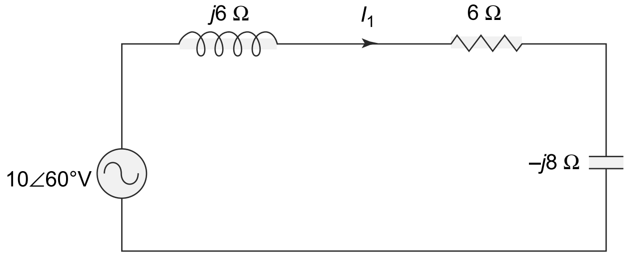


Fig. 6.44 (a)

$$I_1 = \frac{10\angle 60^\circ}{6+j6-j8} = \frac{10\angle 60^\circ}{6-j2} = \frac{10\angle 60^\circ}{6.32\angle -18.43^\circ} = 1.58\angle 78.43^\circ \text{ A}$$

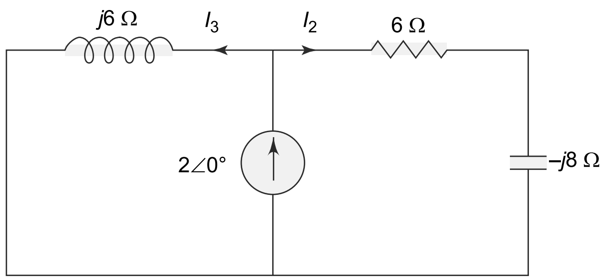


Fig. 6.44 (b)

$$I_2 = 2\angle 0^\circ \times \frac{j6}{6+j6-j8} = 2\angle 0^\circ \times \frac{j6}{6-j2}$$

$$= 1\angle 0^\circ \times \frac{j6}{3-j1} = \frac{1\angle 0^\circ \times 6\angle 90^\circ}{3.16\angle -18.43^\circ} = 1.899\angle 108.43^\circ \text{ A}$$

By superposition theorem, current through

$$6\ \Omega = I_1 + I_2$$
$$= 1.58\angle 78.43^\circ + 1.899\angle 108.43^\circ$$
$$= 0.317 + j1.548 + [-0.6 + j1.8]$$
$$= -0.283 + j3.348 = 3.36\angle 94.83^\circ\text{A}$$

Example 6.15 *Determine the current I in the circuit shown in Fig. 6.45 using superposition theorem:* *[JNTU May/June 2002]*

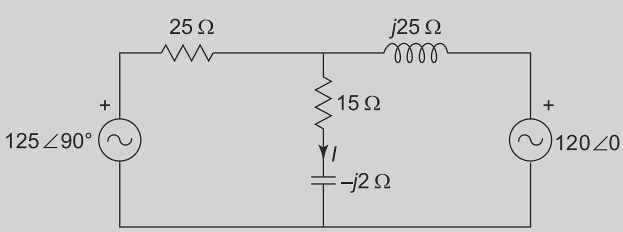


Fig. 6.45

Solution Consider $125\angle 90^\circ$ volt voltage source and short circuiting the other voltage source.

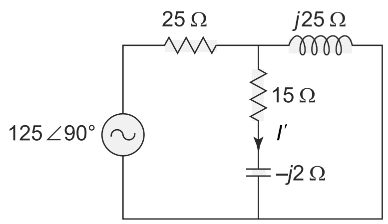


Fig. 6.46 (a)

$$R_{eq} = \frac{j25(15-j2)}{j25+15-j2} + 25 = 37.9\underline{|9.01}\ \Omega$$

$$I_s = \frac{125\underline{|90^\circ}}{37.9\underline{|9.01}}$$

$$= 3.29\underline{|80.99}\ \text{A}$$

$$I' = I_s \times \frac{j25}{15-j2+j25} = 3.29\underline{|80.99} \times \frac{25\underline{|90^\circ}}{27.45\underline{|56.88^\circ}}$$

$$I' = 2.99\underline{|114.11}\text{A}$$

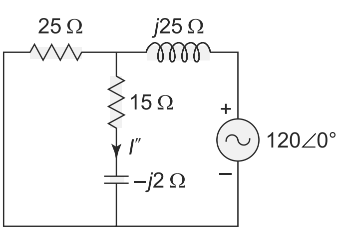


Fig. 6.46 (b)

Now consider $120\angle 0^\circ$ V voltage source and short circuit the other voltage source.

$$R_{eq} = \frac{25(15-j2)}{25+15-j2} + j25$$

$$R_{eq} = 25.99\underline{|68.75^\circ}$$

$$I_s = \frac{120\underline{|0^\circ}}{25.99\underline{|68.75^\circ}} = 4.617\underline{|-68.75^\circ}\ \text{A}$$

$$I'' = I_s \times \frac{25}{15-j2+25} = I_s \times \frac{25}{40-j2}$$

$$I'' = 4.617\angle{-68.75} \times \frac{25}{40 - j2}$$

$$I'' = 2.88\angle{-65.89^\circ}$$

$$I = I' + I''$$

$$= 2.99\angle{114.11^\circ} + 2.88\angle{-65.89^\circ}$$

$$= -1.22 + j2.729 + 1.176 - j2.62$$

$$= -0.044 + j0.1 = 0.1\angle{113.74^\circ}\ \text{A}$$

6.2 THEVENIN'S THEOREM *[JNTU May/June 2008, Nov. 2011]*

6.2.1 Thevenin's Theorem (dc Excitation)

In many practical applications, it is always not necessary to analyse the complete circuit; it requires that the voltage, current, or power in only one resistance of a circuit be found. The use of this theorem provides a simple, equivalent circuit which can be substituted for the original network. Thevenin's theorem states that any two terminal linear network having a number of voltage current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance, where the value of the voltage source is equal to the open circuit voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their internal resistances. According to Thevenin's theorem, an equivalent circuit can be found to replace the circuit in Fig. 6.47.

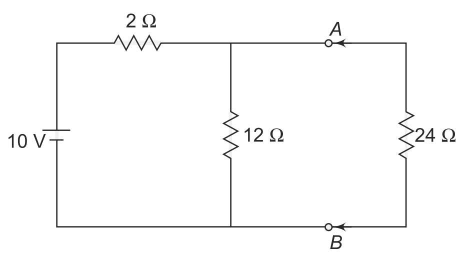


Fig. 6.47

In the circuit, if the load resistance 24 Ω is connected to Thevenin's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experienced in the original circuit. To verify this, let us find the current passing through the 24 Ω resistance due to the original circuit.

$$I_{24} = I_T \times \frac{12}{12 + 24}$$

where $$I_T = \frac{10}{2+(12\|24)} = \frac{10}{10} = 1\,\text{A}$$

∴ $$I_{24} = 1 \times \frac{12}{12+24} = 0.33\,\text{A}$$

The voltage across the 24 Ω resistor = 0.33 × 24 = 7.92 V. Now let us find Thevenin's equivalent circuit.

The Thevenin voltage is equal to the open circuit voltage across the terminals 'AB', i.e. the voltage across the 12 Ω resistor. When the load resistance is disconnected from the circuit, the Thevenin voltage

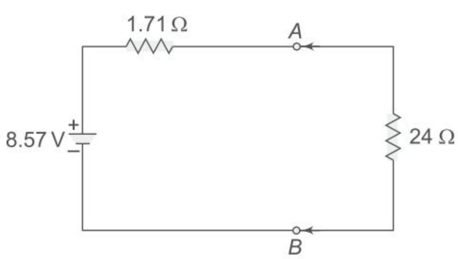


Fig. 6.48

$$V_{\text{Th}} = 10 \times \frac{12}{14} = 8.57\,\text{V}$$

The resistance into the open circuit terminals is equal to the Thevenin resistance

$$R_{\text{Th}} = \frac{12 \times 2}{14} = 1.71\,\Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.48.

Now let us find the current passing through the 24 Ω resistance and voltage across it due to Thevenin's equivalent circuit.

$$I_{24} = \frac{8.57}{24+1.71} = 0.33\,\text{A}$$

The voltage across the 24 Ω resistance is equal to 7.92 V. Thus, it is proved that R_L (= 24 Ω) has the same values of current and voltage in both the original circuit and Thevenin's equivalent circuit.

Example 6.16 *Determine the Thevenin's equivalent circuit across 'AB' for the given circuit shown in Fig. 6.49.*

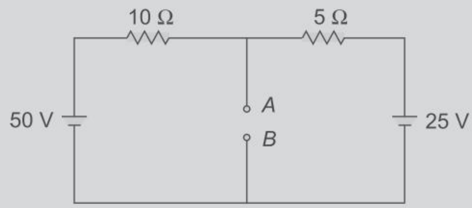


Fig. 6.49

Solution The complete circuit can be replaced by a voltage source in series with a resistance as shown in Fig. 6.50(a)

where V_{Th} is the voltage across terminals ***AB*** and

R_{Th} is the resistance seen into the terminals ***AB***.

To solve for V_{Th}, we have to find the voltage drops around the closed path as shown in Fig. 6.50(b).

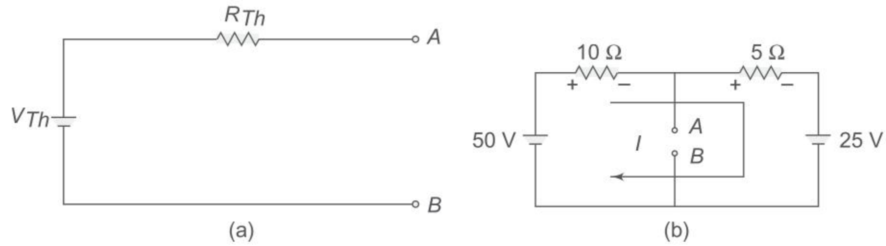


Fig. 6.50

We have $50 - 25 = 10I + 5I$

or $15I = 25$

$\therefore$ $I = \dfrac{25}{15} = 1.67\text{ A}$

Voltage across $10\ \Omega = 16.7$ V

Voltage drop across $5\ \Omega = 8.35$ V

or $V_{Th} = V_{AB} = 50 - V_{10}$

$= 50 - 16.7 = 33.3$ V

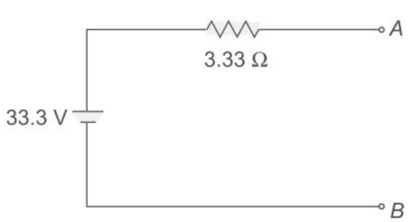


Fig. 6.50(c)

To find R_{Th}, the two voltage sources are removed and replaced with short circuit. The resistance at terminals AB then is the parallel combination of the $10\ \Omega$ resistor and $5\ \Omega$ resistor; or

$$R_{Th} = \frac{10 \times 5}{15} = 3.33\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.50(c).

Example 6.17 *Use Thevenin's theorem to find the current in 3 Ω resistor in Fig. 6.51.*

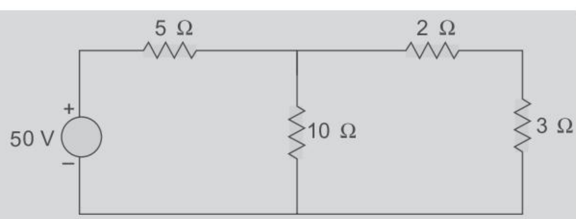


Fig. 6.51

Solution Current in the 3 Ω resistor can be found by using Thevenin's theorem.

In circuit shown in Fig. 6.52(a) can be replaced by a single voltage source in series with a resistor as shown in Fig. 6.52(b).

$$V_{Th} = V_{AB} = \frac{50}{15} \times 10 = 33.3\text{ V}$$

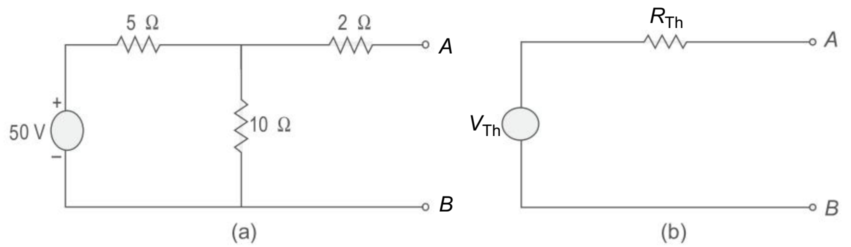


Fig. 6.52

$R_{Th} = R_{AB}$, the resistance seen into the terminals AB

$$R_{AB} = 2 + \frac{5 \times 10}{15} = 5.33\ \Omega$$

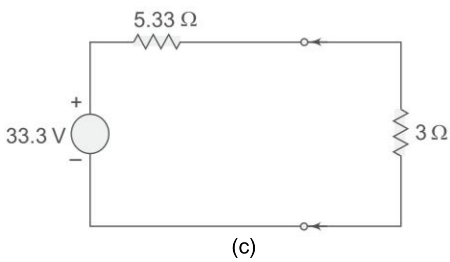


Fig. 6.53

The 3 Ω resistor is connected to the Thevenin equivalent circuit as shown in Fig. 6.53.

Current passing through the 3 Ω resistor

$$I_3 = \frac{33.3}{5.33 + 3} = 4.00\ \text{A}$$

Example 6.18 *Use Thevenin's theorem to find the current through the 5 Ω resistor in Fig. 6.54.*

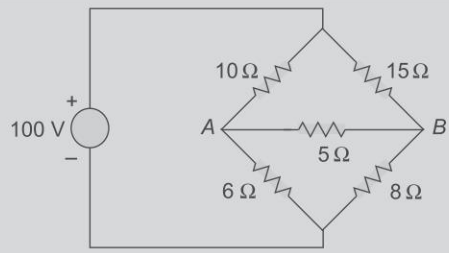


Fig. 6.54

Solution Thevenin's equivalent circuit can be formed by obtaining the voltage across terminals AB as shown in Fig. 6.55(a).

Current int the 6 Ω resistor $\quad I_6 = \frac{100}{16} = 6.25\ \text{A}$

Voltage across the 6 Ω resistor $V_6 = 6 \times 6.25 = 37.5\ \text{V}$

Current in the 8 Ω resistor $I_8 = \frac{100}{23} = 4.35\ \text{A}$

Voltage across the 8 Ω resistor is $V_8 = 4.35 \times 8 = 34.8\ \text{V}$

Voltage across the terminals AB is $V_{AB} = 37.5 - 34.8 = 2.7\ \text{V}$

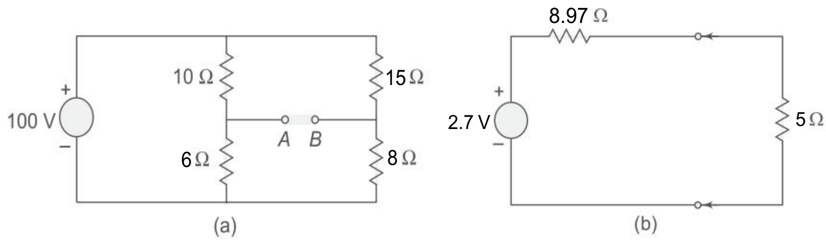


Fig. 6.55

The resistance as seen into the terminals R_{AB}

$$= \frac{6\times10}{6+10} + \frac{8\times15}{8+15}$$

$$= 3.75 + 5.22 = 8.97\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.55(b).

Current in the 5 Ω resistor $I_5 = \dfrac{2.7}{5+8.97} = 0.193$ A

Example 6.19 *Find Thevenin's equivalent circuit for the circuit shown in Fig. 6.56.*

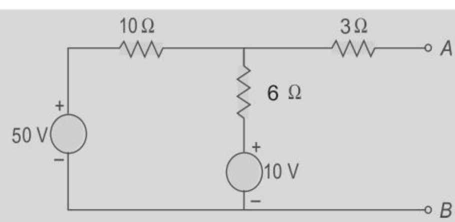


Fig. 6.56

Solution Thevenin's voltage is equal to the voltage across the terminals AB.

$\therefore$ $$V_{AB} = V_3 + V_6 + 10$$

Here the current passing through the 3 Ω resistor is zero.
Hence $V_3 = 0$
By applying Kirchhoff's law we have

$$50 - 10 = 10I + 6I$$

$$I = \frac{40}{16} = 2.5\,\text{A}$$

The voltage across 6 Ω is V_6 with polarity as shown in Fig. 6.57(a), and is given by

$$V_6 = 6 \times 2.5 = 15\ \text{V}$$

The voltage across terminals AB is $V_{AB} = 0 + 15 + 10 = 25$ V.
The resistance as seen into the terminals AB

$$R_{AB} = 3 + \frac{10\times6}{10+6} = 6.75\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.57(b).

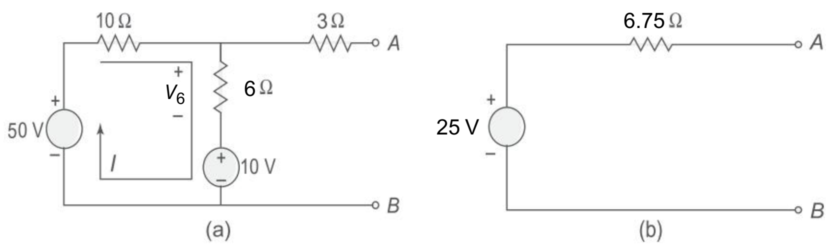


Fig. 6.57

Example 6.20 *Determine the Thevenin's equivalent circuit across terminals AB for the circuit in Fig. 6.58.*

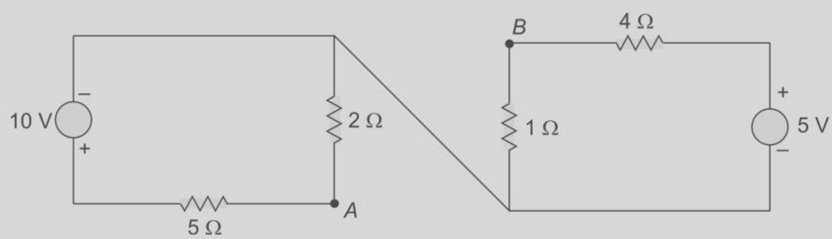


Fig. 6.58

Solution The given circuit is redrawn as shown in Fig. 6.59(a).

Voltage $V_{AB} = V_2 + V_1$

Applying Kirchhoff's voltage law to loop 1 and loop 2, we have the following

Voltage across the 2 Ω resistor $\quad V_2 = 2 \times \frac{10}{7} = 2.85\,\text{V}$

Voltage across the 1 Ω resistor $\quad V_1 = 1 \times \frac{5}{5} = 1\,\text{V}$

$$\therefore \quad V_{AB} = V_2 + V_1$$

$$= 2.85 - 151.85\ \text{V}$$

The resistance seen into the terminals AB

$$R_{AB} = (5 \,||\, 2) + (4 \,||\, 1)$$

$$= \frac{5 \times 2}{5+2} + \frac{4 \times 1}{4+1}$$

$$= 1.43 + 0.8 = 2.23\ \Omega$$

Thevenins's equivalent circuit is shown in Fig. 6.59(b).

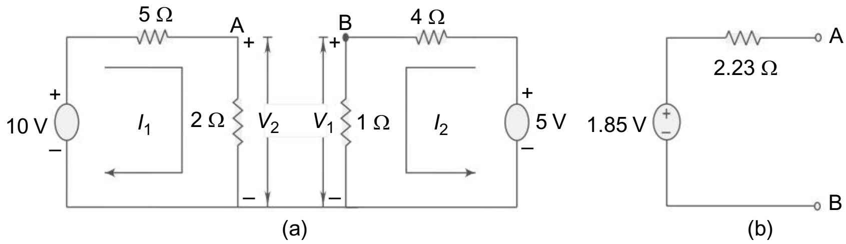


Fig. 6.59

Example 6.21 *For the circuit shown in Fig.6.60, obtain Thevenin's equivalent circuit.*

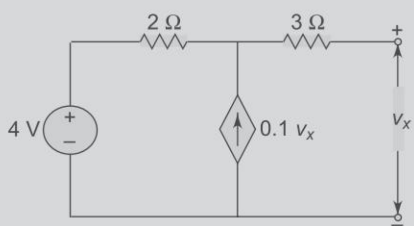


Fig. 6.60

Solution The circuit consists of a dependent source. In the presence of dependent source R_{Th} can be determined by finding v_{OC} and i_{SC}

$$\therefore \qquad R_{Th} = \frac{v_{OC}}{i_{SC}}$$

Open circuit voltage can be found from the circuit shown in Fig. 6.61(a).
Since the output terminals are open, current passes through the 2 Ω branch only.

$$v_x = 2 \times 0.1\, v_x + 4$$

$$v_x = \frac{4}{0.8} = 5\,\text{V}$$

Short circuit current can be calculated from the circuit shown in Fig. 6.61(b).
Since $v_x = 0$, dependent current source is opened.

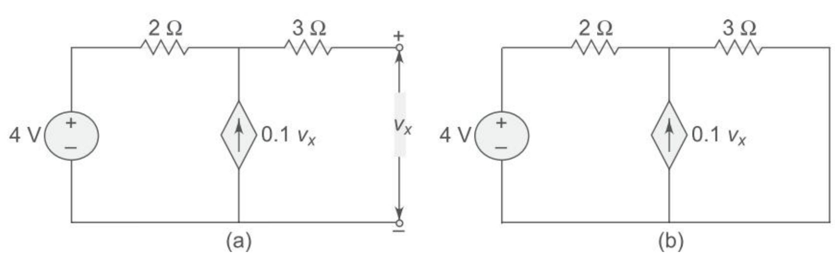
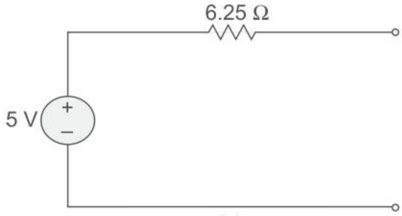


Fig. 6.61



(c)

Fig. 6.61

The current $i_{SC} = \frac{4}{2+3} = 0.8\ \text{A}$

$$\therefore \quad R_{\text{Th}} = \frac{v_{OC}}{i_{SC}} = \frac{5}{0.8} = 6.25\ \Omega$$

The Thevenin's equivalent circuit is shown in Fig. 6.61(c).

Example 6.22 *For the circuit shown in Fig. 6.62, find the current i_2 in the $2\ \Omega$ resistor by using Thevenin's theorem.*

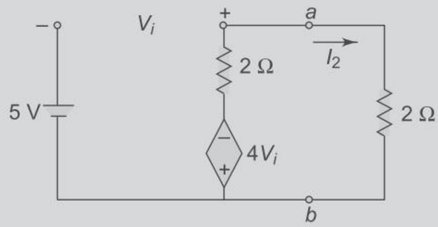


Fig. 6.62

Solution From the circuit, there is open voltage at terminals ***AB*** which is

$$V_{OC} = -4V_i$$

where $\quad V_i = -4V_i - 5$

$\therefore \quad V_i = -1$

Thevenin's voltage $V_{OC} = 4\ \text{V}$

From the circuit, short circuit current is determined by shorting terminals ***a*** and ***b***. Applying Kirchhoff's voltage law, we have

$$4V_i + 2i_{SC} = 0$$

We know $\quad V_i = -5$

Substituting V_i in the above equation, we get

$$i_{SC} = 10\ \text{A}$$

$$\therefore \quad R_{\text{Th}} = \frac{V_{OC}}{i_{SC}} = \frac{4}{10} = 0.4\ \Omega$$

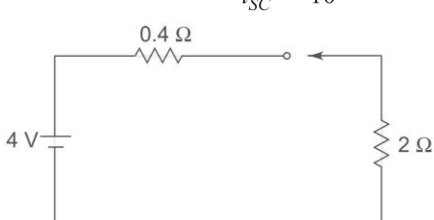


Fig. 6.63

The Thevenin's equivalent circuit is as shown in Fig. 6.63.

The current in the 2 V resistor

$$i_2 = \frac{4}{2.4} = 1.67\ \text{A}$$

Example 6.23 *Find Thevenin's equivalent circuit at terminal AB for the network shown in Fig. 6.64.* *[JNTU April/May 2002]*

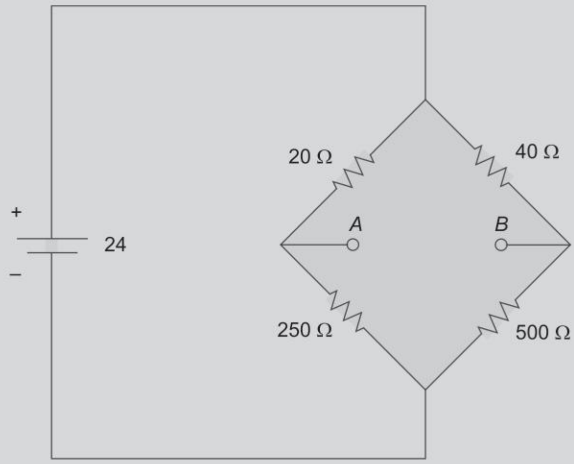


Fig. 6.64

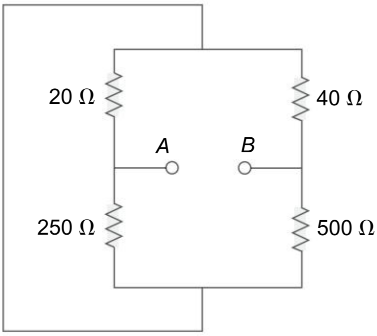


Fig. 6.65

Solution For finding R_{Th}

Remove voltage source and short the terminals

$$R_{AB} = 20 \,||\, 250 + 40 \,||\, 500$$

$$= \frac{20 \times 250}{20 + 250} + \frac{40 \times 500}{40 + 500} = 55.54\ \Omega$$

For finding $V_{OC}\ (V_{AB})$

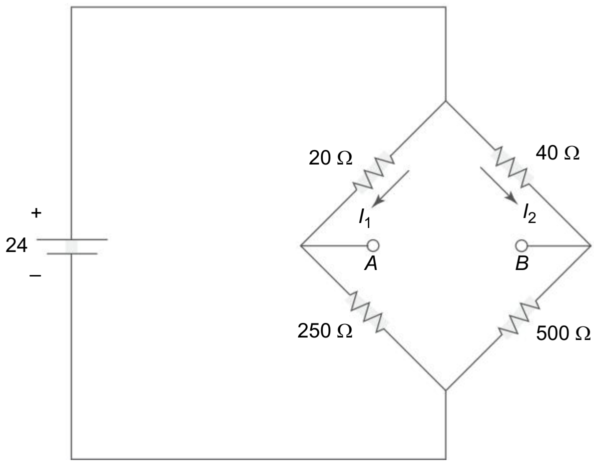


Fig. 6.66

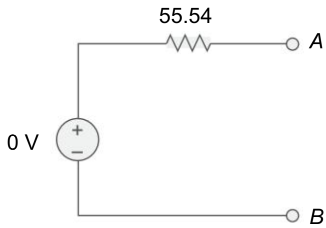


Fig. 6.67

$$I_1 = \frac{24}{20+250} = 0.0888 \text{ A}$$

$$I_2 = \frac{24}{40+500} = 0.0444 \text{ A}$$

For loop OAB apply KVL

$$V_{AB} = 20 \times 0.0888 - 40 \times 0.0444 = 0$$

$$V_{AB} = 0$$

Thevenin's equivalent circuit is given in Fig. 6.67.

Example 6.24 *Find the Thevenin's equivalent for the circuit in Fig. 6.68.*
[JNTU April/May 2003]

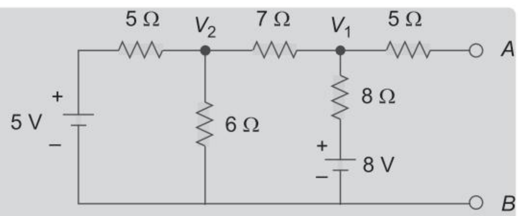


Fig. 6.68

Solution The Thevenin's equivalent resistance is calculated assuming all voltage sources shorted and as seen from AB, the circuit will be as shown below:

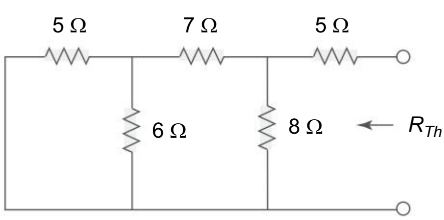


Fig. 6.69

$$R_{Th} = \left[\left\{\left(5 \,//\, 6 + 7\right)\right\} //\, 8\right] + 5$$

$$\left[\left\{\frac{30}{11} + 7\right\} //\, 8\right] + 5 = \left[\frac{\frac{107}{11} \times 8}{\frac{107}{11} + 8}\right] + 5$$

$$= 4.389 + 5$$

$$= 9.389 \ \Omega$$

Let us assume voltages at nodes (1) and (2) be V_1 and V_2.
Now writing node equations.

$$\frac{V_1 - 8}{8} + \frac{V_1 - V_2}{7} = 0$$

$$7V_1 - 56 + 8V_1 - 8V_2 = 0 \Rightarrow 15\, V_1 - 8V_2 = 56 \qquad (1)$$

$$\frac{V_2}{6} + \frac{V_2 - V_1}{7} + \frac{V_2 - 5}{5} = 0 \Rightarrow -30V_1 + 107V_2 = 210 \qquad (2)$$

on solving equations (1) and (2) we get

$V_1 = 5.6 \qquad \Rightarrow \qquad V_{OC} = 5.6$

∴ Thevenins equivalent circuit is

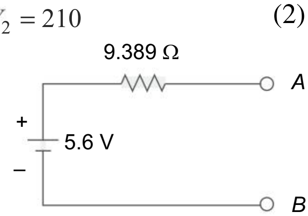


Fig. 6.70

Example 6.25 *Find the current through 10 Ω resistor using Thevenin's theorem.* *[JNTU May/June 2004]*

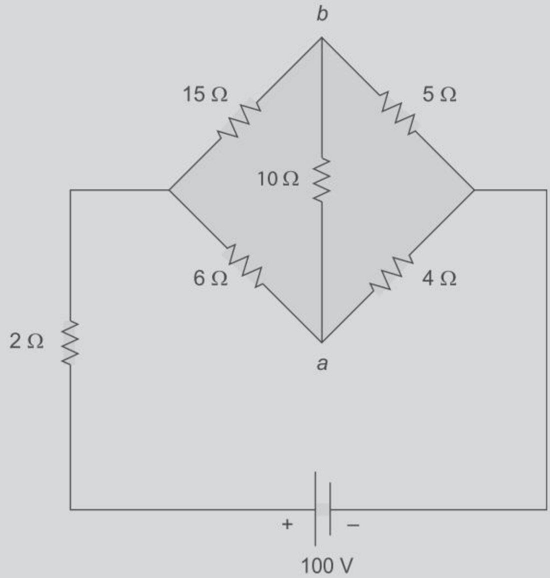


Fig. 6.71

Solution Let us redraw the circuit by removing 10 Ω

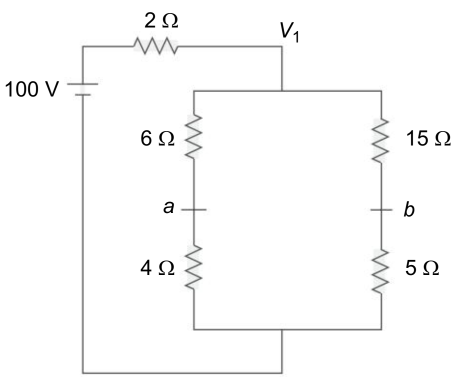


Fig. 6.72

Applying KCL at V_1

$$\frac{V_1 - 100}{2} + \frac{V_1}{10} + \frac{V_1}{20} = 0$$

from which

$$V_1 = 76.92 \text{ V}$$

$$V_{Th} = V_a - V_b$$

$$= \frac{V_1}{6+4} \times 4 - \frac{V_1}{15+5} \times 5$$

$$= 11.538 \text{ V}$$

$$R_1 = \frac{6 \times 15 + 15 \times 2 + 2 \times 6}{2} = 66$$

$$R_2 = \frac{132}{15} = 8.8$$

$$R_3 = \frac{132}{6} = 22$$

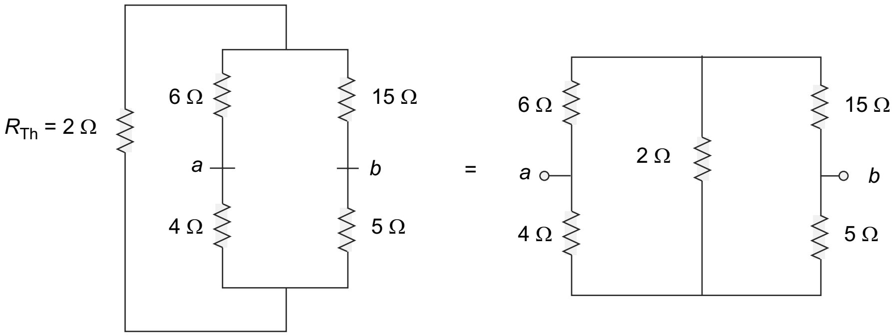


Fig. 6.73

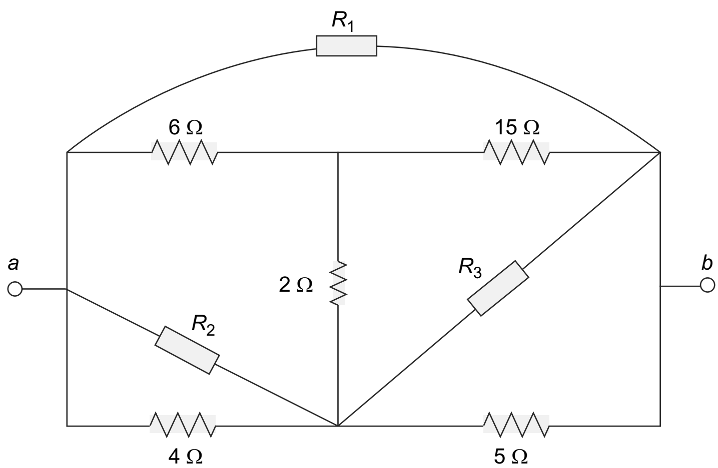


Fig. 6.74

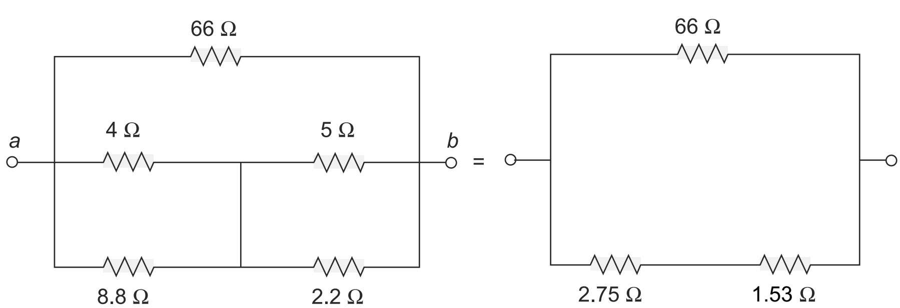


Fig. 6.75

$$R_{ab} = R_{th} = \frac{66 \times 4.28}{70.28} = 4.02\ \Omega$$

Thevenin’ s equivalent circuit is given by

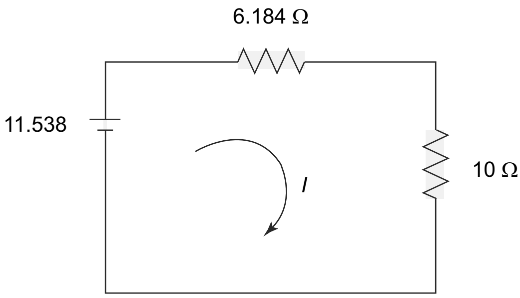


Fig. 6.76

where

$$I = \frac{11.538}{14.02} = 0.823 \text{ A}$$

Example 6.26 *What are the limitations of Thevenin's Theorem?* *[JNTU May/June 2008]*

Solution ***Limitations of Thevenin's theorem***:

If there are two sub-networks which are connected between the terminals ***AB***, at which we have to replace the Thevenin's network then the independent sources on one network do not depend on the voltages and currents in the other network.

Example 6.27 *Explain the steps to apply Thevenin's Theorem and draw the Thevenin's equivalent circuit.* *[JNTU May/June 2008]*

Solution ***Steps to apply Thevenin's theorem***:

Let us consider the given circuit.

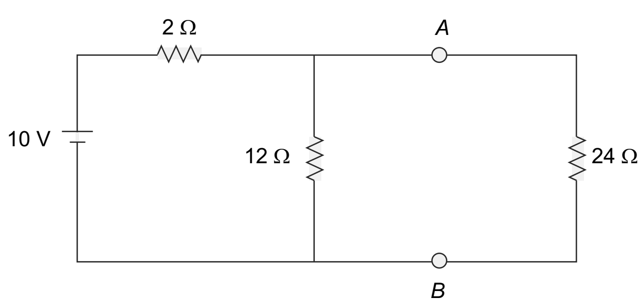


Fig. 6.77

An equivalent Circuits should be replaced across ***AB***.

In the circuit, if the load resistance of 24 Ω is connected to Thevenin's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experienced in the original circuit. To verify this, let us find the current passing through the 24 Ω resistance due to the original circuit.

$$I_{24} = I_T \times \frac{12}{12+24}$$

$$I_T = \frac{10}{2+(12\|24)} = \frac{10}{10} = 1\text{ A}$$

$$I_{24} = 1 \times \frac{12}{12+24} = 0.33\text{ A}$$

The voltage across the 24 Ω resistor = 0.33 × 24 = 7.92 V.

The Thevenin's voltage is equal to the open circuit voltage across the terminals '***A***' i.e., the voltage across the 12 Ω resistor. When the load resistance is disconnected from the circuit, the Thevenin's voltage

$$V_{th} = 10 \times \frac{12}{14} = 8.57\text{ V}$$

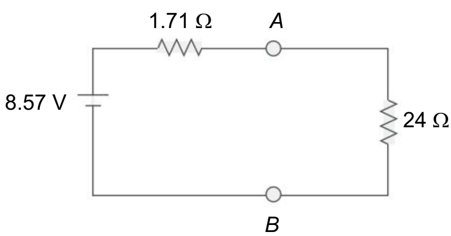


Fig. 6.78

The resistance into the open circuit terminals is equal to the Thevenin resistance

$$R_{\text{Th}} = \frac{12 \times 2}{14} = 1.71\ \Omega$$

Thevenin's equivalent circuits is shown above.

The current passing through the 24 Ω resistance and voltage across it due to Thevenin's equivalent circuit

$$I_{24} = \frac{8.57}{24+1.71} = 0.33\text{ A}$$

The voltage across the 24 Ω resistance is equal to 7.92 V. Thus, it is proved that R_L (=24 Ω) has the same values of current and voltage in both the original circuit and Thevenin equivalent circuit.

Example 6.28 *Using Thevenin's theorem, find the current through 1 Ω resistor in the circuit shown in Fig. 6.79.* *[JNTU May/June 2006]*

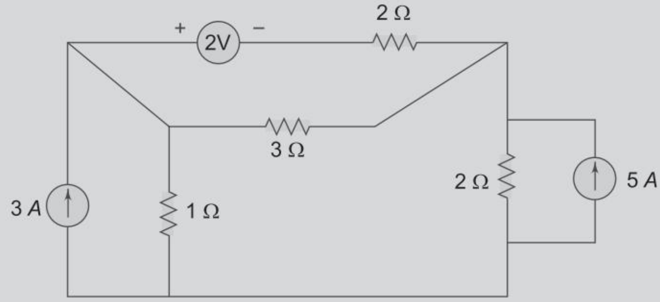


Fig. 6.79

Solution The given circuit is

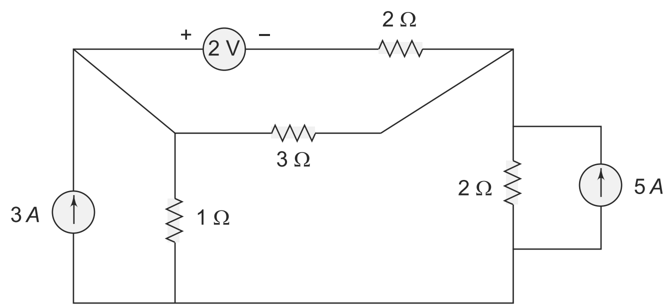


Fig. 6.80

To find $\boldsymbol{R_{Th}}$

By keeping all the sources to zero, the circuit reduces to

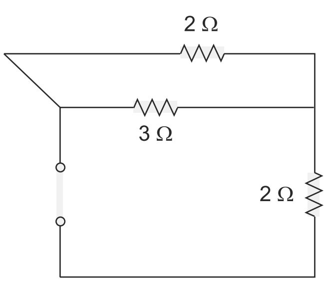


Fig. 6.81

$$R_{Th} = 2 \| 3 + 2$$

$$R_{Th} = \frac{6}{5} + 2$$

$$R_{Th} = \frac{16}{5}$$

To find $\boldsymbol{V_{Th}}$

Transforming current source of 5 A to voltage source the circuit reduces to

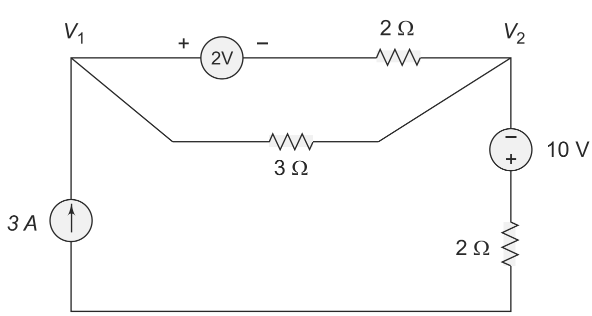


Fig. 6.82

Applying Nodal analysis

$$\frac{V_1 - V_2 - 2}{2} + \frac{V_1 - V_2}{3} = 3$$

$$V_1 - V_2 = \frac{24}{5} \qquad (1)$$

$$\frac{V_2 - V_1 + 2}{2} + \frac{V_2 - V_1}{3} + \frac{V_2 + 10}{2} = 0$$

$$-10\,V_1 + 16\,V_2 = -72 \qquad (2)$$

From (1) and (2)

$$V_1 = 0.8\text{ V}$$

$$V_2 = -4\text{ V}$$

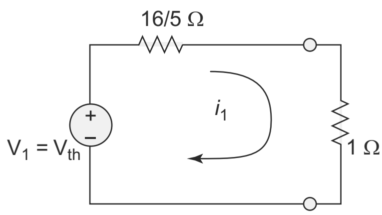


Fig. 6.83

The Thevenin's circuit with 1 Ω resistance is shown in figure

∴ The current through 1 Ω resistor

$$i_1 = \frac{0.8}{\frac{16}{5} + 1} = 0.19\text{ A}$$

Example 6.29 *Determine the Thevenin's equivalent across the terminals A and B as shown in Fig. 6.84.* *[JNTU June 2009]*

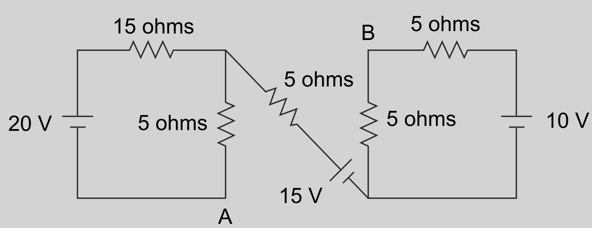


Fig. 6.84

Solution

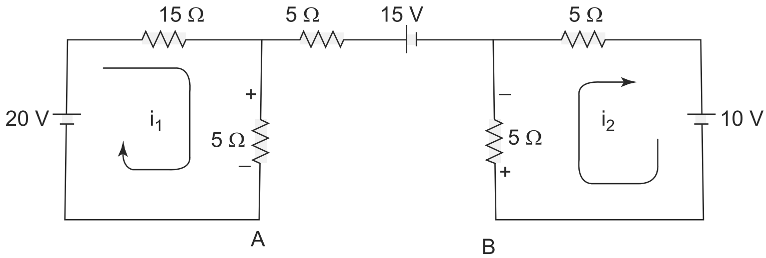


Fig. 6.85

$$i_1 = \frac{20\text{ V}}{20\ \Omega} = 1\text{A}, \qquad i_2 = 1\text{ A}$$

$$\therefore \quad V_{AB} = -5\text{ V} + 15\text{ V} - 5\text{ V} = +5\text{ V} = V_{Th}$$

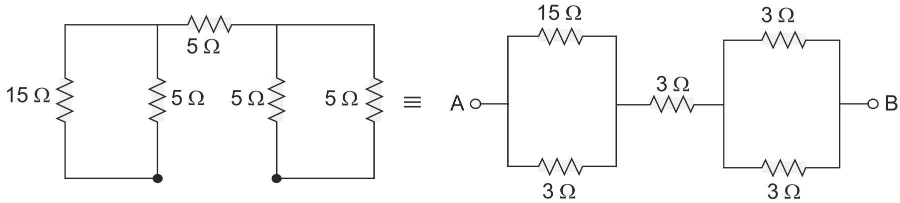


Fig. 6.86

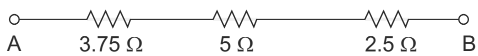


Fig. 6.87

$\therefore \quad R_{AB} = 11.25\,\Omega = R_{\text{Th}}$

The Thevenin's equivalent circuit is

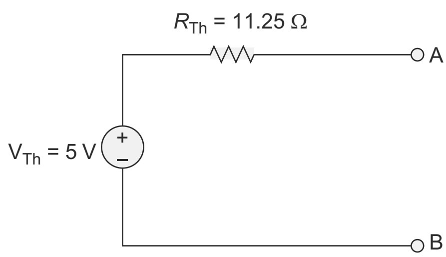


Fig. 6.88

6.2.2 Thevenin's Theorem (ac Excitation) *[JNTU Jan 2010]*

Thevenin's theorem gives us a method for simplifying a given circuit. The Thevenin equivalent form of any complex impedance circuit consists of an equivalent voltage source V_{Th}, and an equivalent impedance Z_{Th}, arranged as shown in Fig. 6.89. The values of equivalent voltage and impedance depend on the values in the original circuit.

Though the Thevenin equivalent circuit is not the same as its original circuit, the output voltage and output current are the same in both cases. Here, the Thevenin voltage is equal to the open circuit voltage across the output terminals, and impedance is equal to the impedance seen into the network across the output terminals.

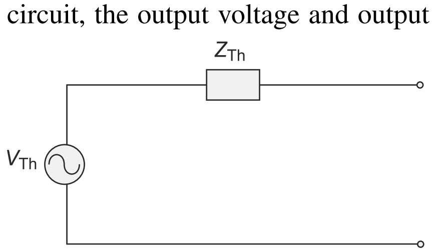


Fig. 6.89

Consider the circuit shown in Fig. 6.90.

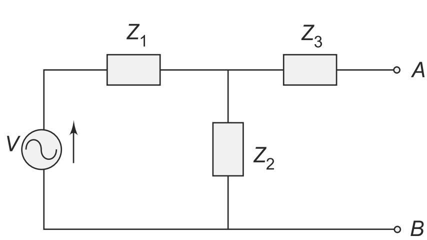


Fig. 6.90

Thevenin equivalent for the circuit shown in Fig. 6.90 between points A and B is found as follows.

The voltage across points A and B is the Thevenin equivalent voltage. In the circuit shown in Fig. 6.90, the voltage across A and B is the same as the voltage across Z_2 because there is no current through Z_3.

$$\therefore \quad V_{Th} = V\left(\frac{Z_2}{Z_1 + Z_2}\right)$$

The impedance between points ***A*** and ***B*** with the source replaced by short circuit is the Thevenin equivalent impedance. In Fig. 6.90, the impedance from ***A*** to ***B*** is $\boldsymbol{Z_3}$ in series with the parallel combination of $\boldsymbol{Z_1}$ and $\boldsymbol{Z_2}$.

$$\therefore \quad Z_{Th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

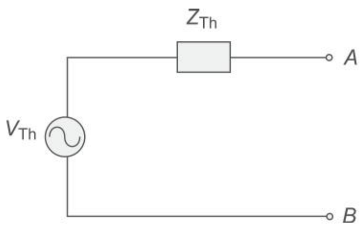


Fig. 6.91

The Thevenin equivalent circuit is shown in Fig. 6.91.

Thevenin's theorem is especially useful in analyzing power systems and other circuits where load resistance/impedance is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it. Many circuits are only linear over a certain range of values, thus Thevenin's equivalent is valid only within this linear range and may not be valid outside the range. The Thevenin's equivalent has an equivalent I-V characteristic only from the point of view of the load. Since power is not linearly dependent on voltage or current, the power dissipation of the Thevenin's equivalent is not identical to the power dissipation of the real system.

Example 6.30 *For the circuit shown in Fig. 6.92, determine Thevenin's equivalent between the output terminals.*

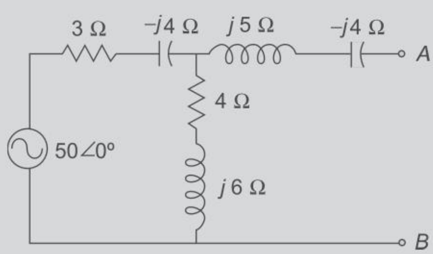


Fig. 6.92

Solution The Thevenin voltage, $\boldsymbol{V_{Th}}$, is equal to the voltage across the $(4 + j6)\ \Omega$ impedance. The voltage across $(4 + j6)\ \Omega$ is

$$V = 50\angle 0° \times \frac{(4 + j6)}{(4 + j6) + (3 - j4)}$$

$$= 50\angle 0° \times \frac{4 + j6}{7 + j2}$$

$$= 50\angle 0° \times \frac{7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= 50\angle 0° \times 0.99\angle 40.35°$$

$$= 49.5\angle 40.35°\ \text{V}$$

The impedance seen from terminals ***A*** and ***B*** is

$$Z_{Th} = (j5 - j4) + \frac{(3 - j4)(4 + j6)}{3 - j4 + 4 + j6}$$

$$= j1 + \frac{5\angle 53.13° \times 7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= j1 + 4.95\angle -12.78° = j1 + 4.83 - j1.095$$

$$= 4.83 - j0.095$$

$$\therefore \quad Z_{Th} = 4.83\angle -1.13°\ \Omega$$

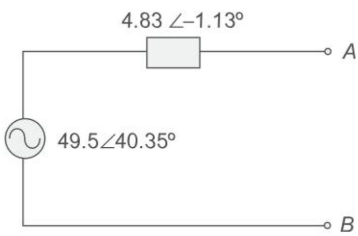


Fig. 6.93

The Thevenin equivalent circuit is shown in Fig. 6.93.

Example 6.31 *For the circuit shown in Fig. 6.85, determine the load current by applying Thevenin's theorem.*

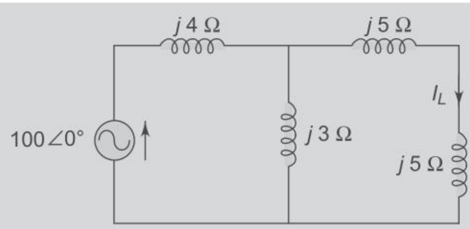


Fig. 6.94

Solution Let us find the Thevenin equivalent circuit for the circuit shown in Fig. 6.95 (a).

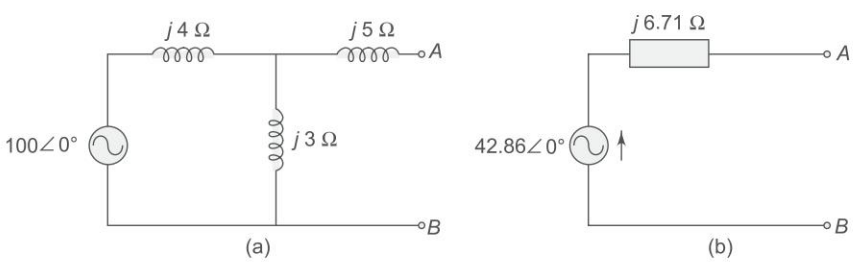


Fig. 6.95

Voltage across ***AB*** is the voltage across (*j*3) Ω

$$\therefore \qquad V_{AB} = 100\ \angle 0° \times \frac{(j3)}{(j3) + (j4)}$$

$$= 100\ \angle 0° \frac{(j3)}{j7} = 42.86\angle 0°$$

Impedance seen from terminals ***AB***

$$Z_{AB} = (j5) + \frac{(j4)(j3)}{j7}$$

$$= j5 + j1.71 = j6.71\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.95 (b).

If we connect a load to Fig. 6.95(b), the current passing through ($j5$) Ω impedance is

$$I_L = \frac{42.86\angle 0^\circ}{(j6.71 + j5)} = \frac{42.86\angle 0^\circ}{11.71\angle 90^\circ} = 3.66\angle -90^\circ$$

Example 6.32 *For the circuit shown in Fig. 6.96, determine Thevenin's equivalent circuit.*

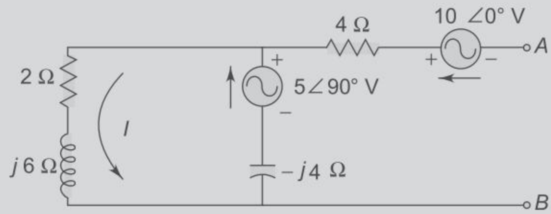


Fig. 6.96

Solution Voltage across ($-j4$) Ω is

$$V_{-j4} = \frac{5\angle 90^\circ}{(2 + j2)}(-j4)$$

$$= \frac{20\angle 0^\circ}{2.83\angle 45^\circ} = 7.07\angle -45^\circ$$

Voltage across ***AB*** is $V_{AB} = -V_{10} + V_5 - V_{-j4}$

$$= -10\angle 0^\circ + 5\angle 90^\circ - 7.07\angle -45^\circ$$

$$= j5 - 10 - 4.99 + j4.99$$

$$= -14.99 + j9.99$$

$$V_{AB} = 18\angle 146.31^\circ$$

The impedance seen from terminals ***AB***, when all voltage sources are short circuited is

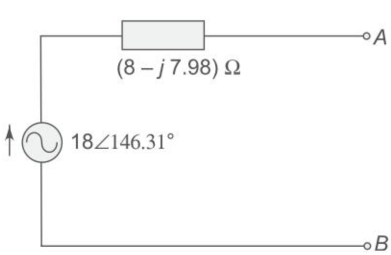


Fig. 6.97

$$Z_{AB} = 4 + \frac{(2 + j6)(-j4)}{2 + j2}$$

$$= 4 + \frac{6.32\angle 71.56^\circ \times 4\angle -90^\circ}{2.83\angle 45^\circ}$$

$$= 4 + 8.93\angle -63.44^\circ$$

$$= 4 + 4 - j\,7.98 = (8 - j7.98)\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.97.

Example 6.33 *Convert the active network shown in Fig. 6.98 by a single voltage source in series with impedance.*

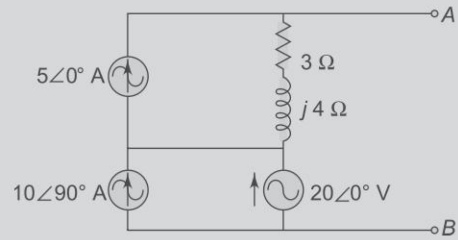


Fig. 6.98

Solution Using the superposition theorem, we can find Thevenin's equivalent circuit. The voltage across AB, with $20\angle 0°$ V source acting alone, is V'_{AB}, and can be calculated from Fig. 6.99 (a).

Since no current is passing through the $(3 + j4)\ \Omega$ impedance, the voltage

$$V'_{AB} = 20\angle 0°$$

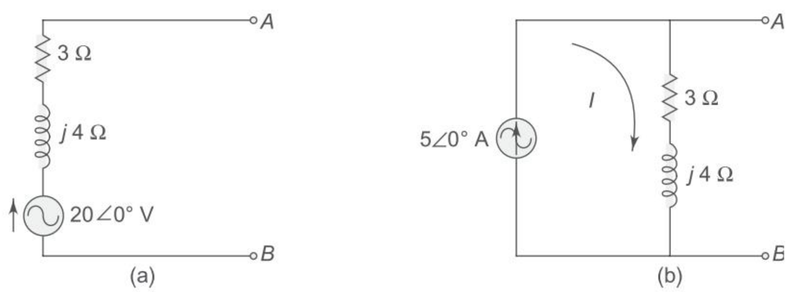


Fig. 6.99

The voltage across AB, with $5\angle 0°$ A source acting alone, is V'_{AB}, and can be calculated from Fig. 6.90 (b).

$$V''_{AB} = 5\angle 0°\ (3 + j4) = 5\angle 0° \times 5\angle 53.13° = 25\angle 53.13°\ \text{V}$$

The voltage across AB, with $10\angle 90°$ A source acting alone, is V'''_{AB}, and can be calculated from Fig. 6.100 (a).

$$V'''_{AB} = 0$$

According to the superposition theorem, the voltage across AB due to all sources is

$$V_{AB} = V'_{AB} + V''_{AB} + V'''_{AB}$$

$$\therefore \quad V_{AB} = 20\angle 0° + 25\angle 53.13° = 20 + 15 + j19.99$$

$$= (35 + j19.99)\ \text{V} = 40.3\angle 29.73°\ \text{V}$$

The impedance seen from terminals AB

$$Z_{\text{Th}} = Z_{AB} = (3 + j4)\ \Omega$$

∴ The required Thevenin circuit is shown in Fig. 6.100 (b).

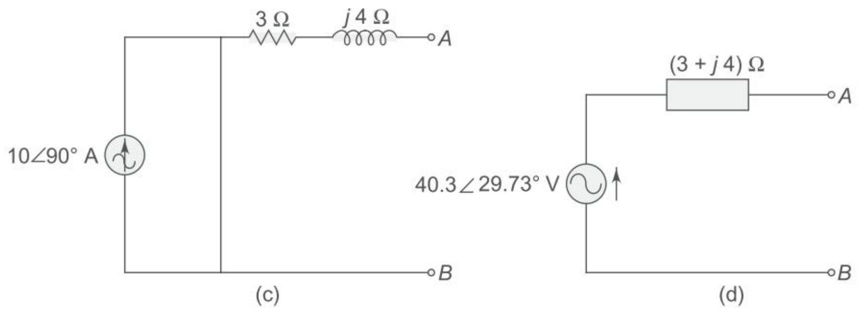


Fig. 6.100

Example 6.34 *For the circuit shown in Fig. 6.101, find the current in the j5 Ω inductance by using Thevenin's theorem.*

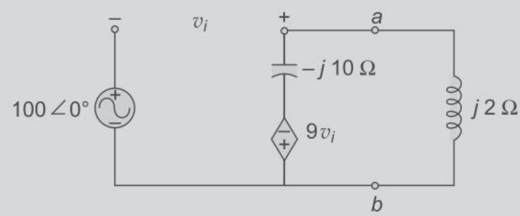


Fig. 6.101

Solution From the circuit shown in Fig. 6.101 the open circuit voltage at terminals $\boldsymbol{a}$ and $\boldsymbol{b}$ is

$$V_{oc} = -9\,V_i$$

where

$$V_i = -9V_i - 100\angle 0^\circ$$

$$10V_i = -100\angle 0^\circ$$

$$V_i = -10\angle 0^\circ$$

Thevenin's voltage $V_{oc} = 90\angle 0^\circ$

From the circuit, short circuit current is determined by shorting terminals $\boldsymbol{a}$ and $\boldsymbol{b}$. Applying Kirchhoff's voltage law, we have

$$9V_i - j10\,i_{sc} = 0$$

$$i_{sc} = 9\angle 90^\circ$$

$$\therefore \quad Z_{\text{Th}} = \frac{V_{oc}}{I_{sc}} = \frac{90\angle 0^\circ}{9\angle 90^\circ} = 10\angle -90^\circ$$

$$Z_{\text{Th}} = -j10\ \Omega$$

The Thevenin's equivalent circuit is shown in Fig. 6.102.

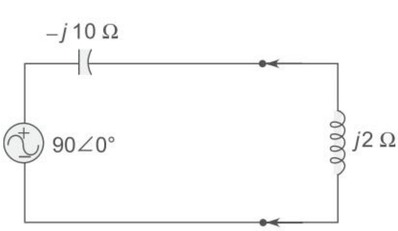


Fig. 6.102

The current in the $j2\ \Omega$ inductor is $= \dfrac{90\angle 0^\circ}{j8}$

$$= 11.25\angle 90^\circ$$

Example 6.35 *Use Thevenin's Theorem and find the current through (5 + j4) ohms impedance, for the network as shown in Fig. 6.103.*

[JNTU May/June 2008]

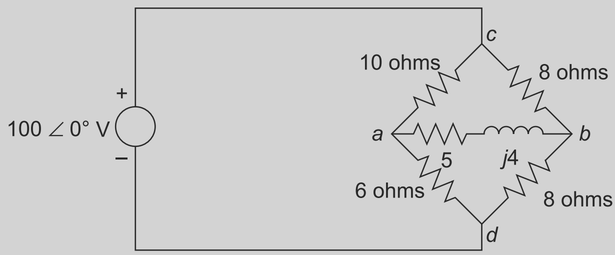


Fig. 6.103

Solution The given circuit is Thevenin's equivalent circuit can be obtained across the terminals ***ab***.

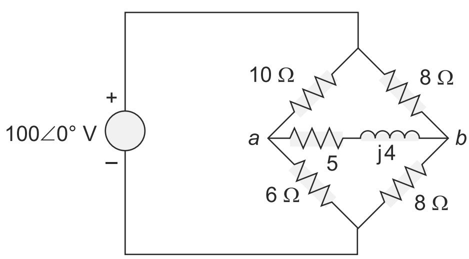


Fig. 6.104(a)

Current in the 6 Ω resistor

$$I_6 = \frac{100}{16} = 6.25\,\text{A}$$

Voltage across the 6 Ω resistor

$$V_6 = 6 \times 6.25 = 37.5\,\text{V}$$

Current in the 8 Ω resistor $\quad I_8 = \frac{100}{16} = 6.25\,\text{A}$

Voltage across the 8 Ω resistor $\quad V_8 = 8 \times 6.25 = 50\,\text{V}$

Voltage across the terminals ***AB*** $\quad V_{AB} = 37.5 - 50$

$$= -12.5\,\text{V}$$

The resistance as seen through the terminals

$$R_{AB} = \frac{6\times10}{6+10} + \frac{8\times8}{8+8} = \frac{60}{16} + 4$$

$$3.75 + 4 = 7.75\,\Omega$$

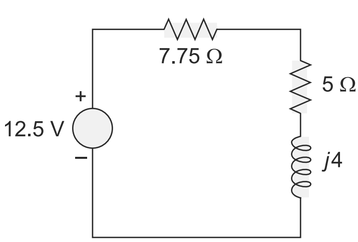


Fig. 6.104(b)

Equivalent circuit is

The current flowing in (5 + *j*4) Ω is

$$= \frac{12.5}{7.75+5+j4} = \frac{12.5}{12.75+j4}$$

$$= \frac{12.5}{13.362\angle 17.41} = 0.935\angle -17.41\,\text{A}$$

Example 6.36 *Find the current in load impedance Z_L of the network shown in Fig. 6.105, by applying Thevenin's theorem.*

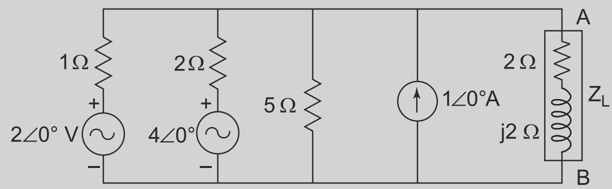


Fig. 6.105

Solution The current source has been replaced by the voltage source and the load impedance is removed from the network.

Then the network becomes as shown as Fig. 6.106(a)

The mesh equations are

$$\begin{bmatrix} 3 & -2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -2\angle 0^\circ \\ -1\angle 0^\circ \end{bmatrix}$$

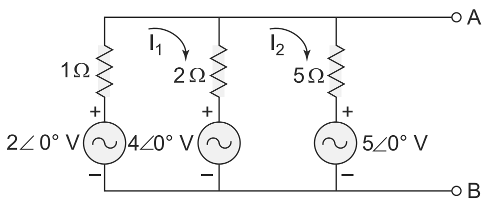


Fig. 6.106(a)

$$\therefore \quad I_2 = \frac{\begin{vmatrix} 3 & -2 \\ -2 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -2 & 7 \end{vmatrix}} = \frac{-7}{17}$$

Then $V_{AB} = 5I_2 + 5\ \angle 0^\circ$

$$= 5 \times \frac{-7}{17} + 5$$

$$= \frac{50}{17} \text{ volts}$$

$$\mathrm{R}_{Th} = (1) \,||\, (2) \,||\, (5) = \frac{10}{17}\ \Omega$$

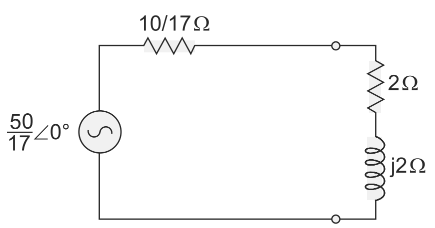


Fig. 6.106(b)

The Thevenin's circuit is

Hence the current ***i*** through the load impedance is

$$i = \frac{50/17}{\frac{10}{17} + 2 + j2}$$

$$= 0.899\underline{|-37.69^\circ}\ \text{A}$$

Example 6.37 *Find the current through the branch A-B of the network shown in Fig. 6.107 using Thevenin's theorem.* *[JNTU Jan 2010]*

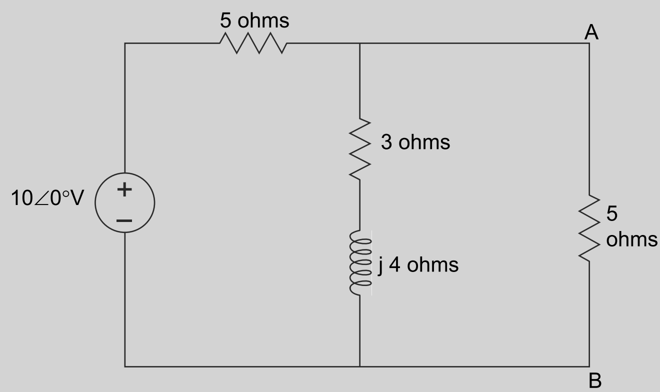


Fig. 6.107

Solution

V_{Th} is calculated by open circuiting AB terminal

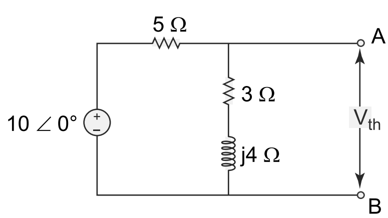


Fig. 6.108(a)

$$\therefore V_{Th} = \frac{10}{5+3+j4} \times (3+j4)\text{ volt}$$

$$= \frac{10\times 5}{8.94} \angle 26.56° \text{ volt}$$

$$= 5.6\angle 26.56° \text{ volt.}$$

$\boldsymbol{Z_{int}}$ is determined by open circuiting A-B terminal and short circuiting voltage source

$$Z_{int} = \frac{(3+4j)5}{3+4j+5} \text{ ohm}$$

$$= 2.8\angle 26.56° \text{ ohm}$$

$$\therefore \text{ Current through } AB = \frac{V_{th}}{Z_{int}+Z_L}$$

$$= \frac{5.6\angle 26.56°}{2.8\angle 26.56° + 5} \text{amp}$$

$$= \frac{5.6\angle 26.56°}{7.5+1.252j} \text{ amp}$$

$$= 0.74\angle 17.083° \text{ amp}$$

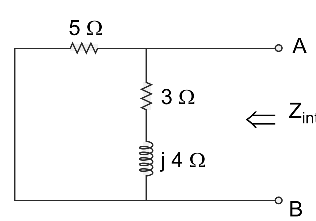


Fig. 6.108(b)

6.3 NORTON'S THEOREM

6.3.1 Norton's Theorem (dc Excitation)

[JNTU Jan 2010, Nov 2011]

Another method of analysing the circuit is given by ***Norton's theorem***, which states that any two terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance. The value of the current source is the short circuit current between the two terminals of the network and the resistance is the equivalent resistance measured between the terminals of the network with all the energy sources are replaced by their internal resistance.

According to Norton's theorem, an equivalent circuit can be found to replace the circuit in Fig. 6.109.

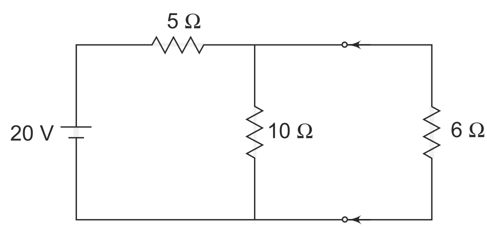


Fig. 6.109

In the circuit if the load resistance 6 Ω is connected to Norton's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experiences in the original circuit. To verify this, let us find the current passing through the 6 Ω resistor due to the original circuit.

$$I_6 = I_T \times \frac{10}{10+6}$$

where $$I_T = \frac{20}{5+(10 \parallel 6)} = 2.285 \text{ A}$$

∴ $$I_6 = 2.285 \times \frac{10}{16} = 1.43 \text{ A}$$

i.e. the voltage across the 6 Ω resistor is 8.58 V. Now let us find Norton's equivalent circuit. The magnitude of the current in the Norton's equivalent circuit is equal to the current passing through short circuited terminals as shown in Fig. 6.110.

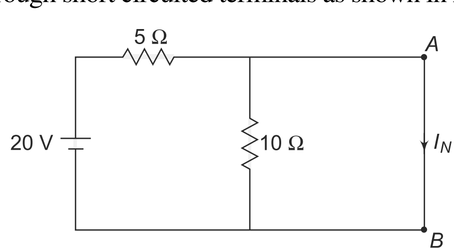


Fig. 6.110

Here $$I_N = \frac{20}{5} = 4\text{ A}$$

Norton's resistance is equal to the parallel combination of both the 5 Ω and 10 Ω resistors

$$R_N = \frac{5\times 10}{15} = 3.33\ \Omega$$

The Norton's equivalent source is shown in Fig. 6.44.

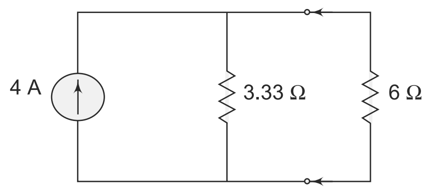


Fig. 6.111

Now let us find the current passing through the 6 Ω resistor and the voltage across it due to Norton's equivalent circuit.

$$I_6 = 4 \times \frac{3.33}{6+3.33} = 1.43\text{ A}$$

The voltage across the 6 Ω resistor = 1.43 × 6 = 8.58 V

Thus, it is proved that $\boldsymbol{R_L}$ (=6 Ω) has the same values of current and voltage in both the original circuit and Norton's equivalent circuit.

Example 6.38 *Determine Norton's equivalent circuit at terminals AB for the circuit shown in Fig. 6.112.*

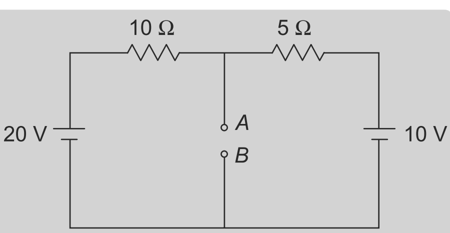
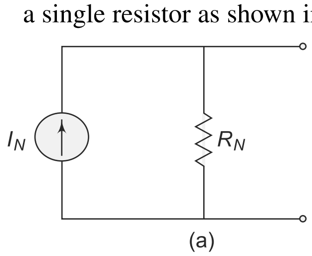


Fig. 6.112

Solution The complete circuit can be replaced by a current source in parallel with a single resistor as shown in Fig. 6.113(a), where $\boldsymbol{I_N}$ is the current passing through the short circuited output terminals ***AB*** and $\boldsymbol{R_N}$ is the resistance as seen into the output terminals.



(a)

Fig. 6.113

To solve for $\boldsymbol{I_N}$, we have to find the current passing through the terminals ***AB*** as shown in Fig. 7.113(b).

From Fig. 6.113(b), the current passing through the terminals ***AB*** is 4 A. The resistance at terminals ***AB*** is the parallel combination of the 10 Ω resistor and the 5 Ω resistor,

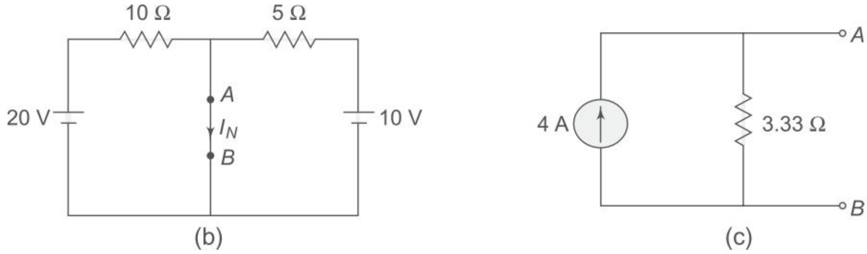


Fig. 6.113

or $$R_N = \frac{10 \times 5}{10+5} = 3.33\ \Omega$$

Norton's equivalent circuit is shown in Fig. 6.113(c).

Example 6.39 *Determine Norton's equivalent circuit for the circuit shown in Fig. 6.114.*

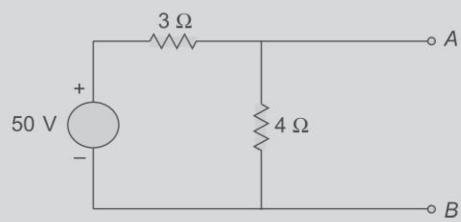


Fig. 6.114

Solution Norton's equivalent circuit is given by Fig. 6.115(a).

where I_N = Short circuit current at terminals AB

R_N = Open circuit resistance at terminals AB

The current I_N can be found as shown in Fig. 6.115(b).

$$I_N = \frac{50}{3} = 16.7\,\text{A}$$

Norton's resistance can be found from Fig. 6.115(c).

$$R_N = R_{AB} = \frac{3 \times 4}{3+4} = 1.71\Omega$$

Norton's equivalent circuit for the given circuit is shown in Fig. 6.115(d).

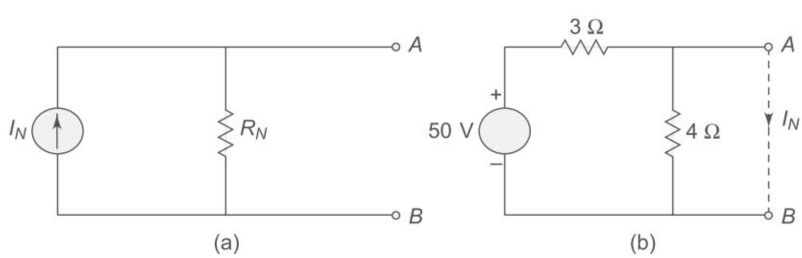


Fig. 6.115

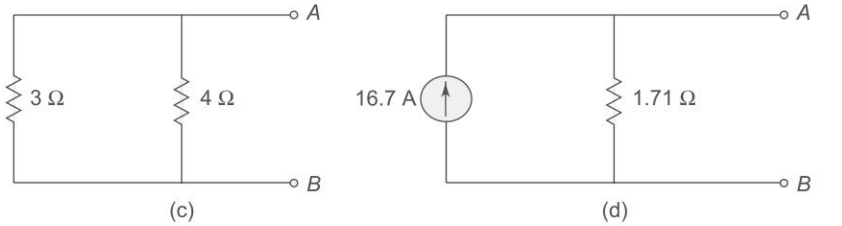


Fig. 6.115

Example 6.40 *Determine Norton's equivalent circuit for the given circuit shown in Fig. 6.116.*

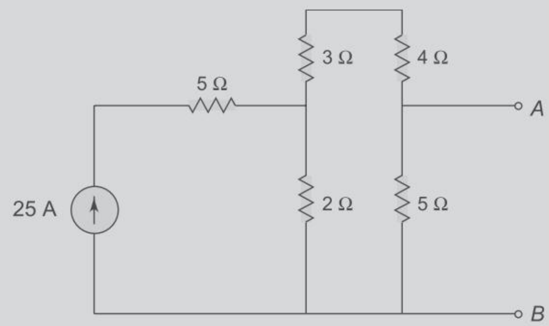


Fig. 6.116

Solution The short circuit current at terminals ***AB*** can be found from Fig. 6.117(a) and Norton's resistance can be found from Fig. 6.117(b).

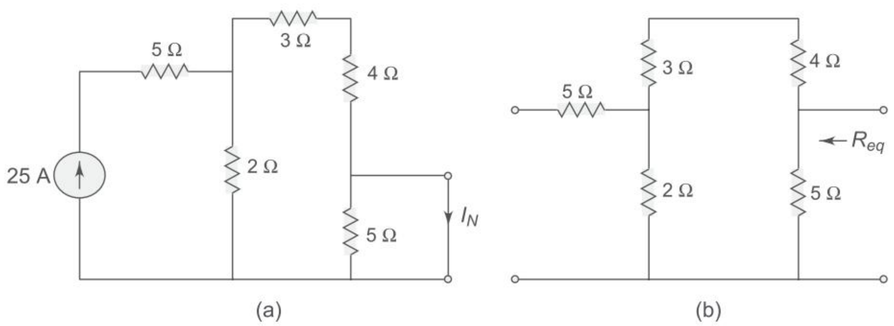


Fig. 6.117

The current I_N is same as the current in the 3 Ω resistor or 4 Ω resistor.

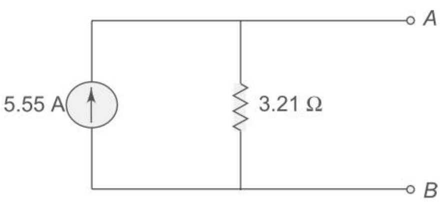


Fig. 6.117(c)

$$I_N = I_3 = 25 \times \frac{2}{7+2} = 5.55\,\text{A}$$

The resistance as seen into the terminals ***AB*** is

$$R_{AB} = 5 \parallel (4 + 3 + 2)$$

$$= \frac{5 \times 9}{5+9} = 3.21\,\Omega$$

Norton's equivalent circuit is shown in Fig. 6.117(c).

Example 6.41 *Determine the current flowing through the 5 Ω resistor in the circuit shown in Fig. 6.118 by using Norton's theorem.*

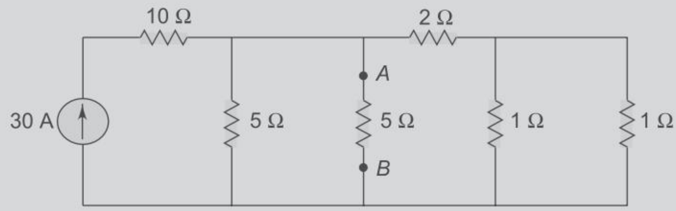


Fig. 6.118

Solution The short circuit current at terminals ***AB*** can be found from the circuit as shown in Fig. 6.119(a). Norton's resistance can be found from Fig. 6.119(b).

In Fig. 6.119(a), the current I_N = 30 A.

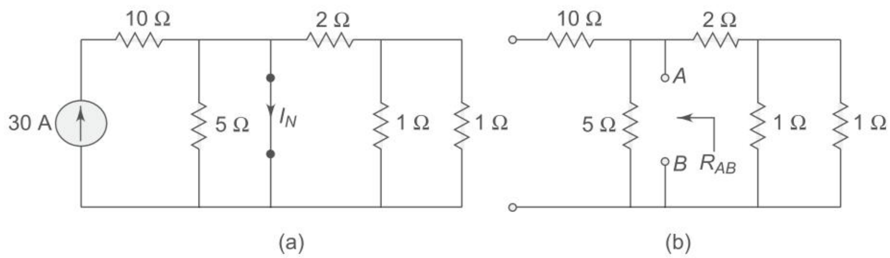


Fig. 6.119

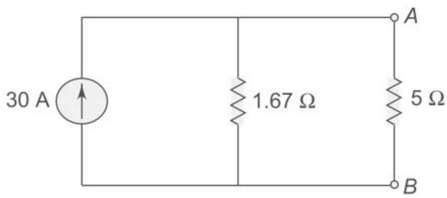


Fig. 6.119(c)

The resistance in Fig. 6.119(b)

$$R_{AB} = 5 \,||\left(2 + \frac{1 \times 1}{2}\right)$$

$$= 5 \,||\, (2.5) = \frac{5 \times 2.5}{7.5} = 1.67\,\Omega$$

Norton's equivalent circuit is shown in Fig. 6.119(c).

∴ The current in the 5 Ω resistor

$$I_5 = 30 \times \frac{1.67}{6.67} = 7.51 \text{ A}$$

Example 6.42 *Replace the given network shown in Fig. 6.120 by a single current source in parallel with a resistance.*

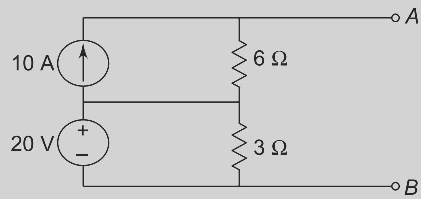


Fig. 6.120

Solution Here, using superposition technique and Norton's theorem, we can convert the given network.

We have to find a short circuit current at terminals ***AB*** in Fig. 6.121(a) as shown

The current I'_N is due to the 10 A source $I'_N = 10$ A

The current I''_N is due to the 20 V source (See Figs 7.121(b) and (c))

$$I''_N = \frac{20}{6} = 3.33 \text{ A}$$

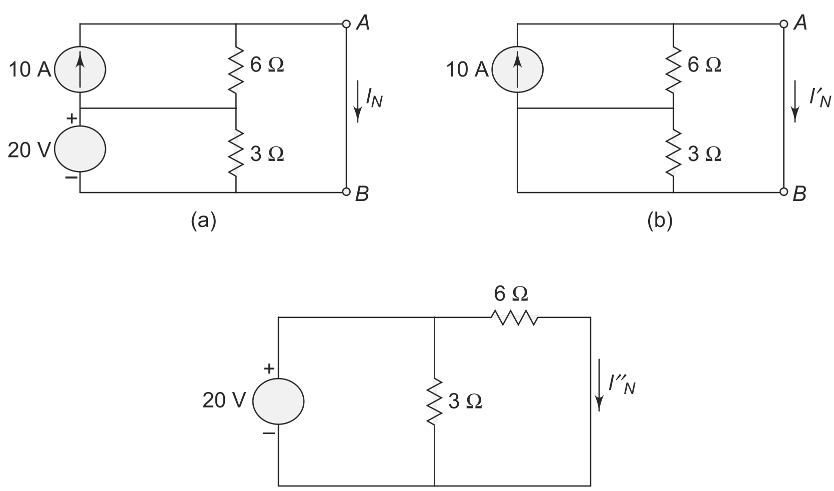


Fig. 6.121

The current I_N is due to both the sources

$$I_N = I'_N + I'_N$$

$$= 10 + 3.33 = 13.33 \text{ A}$$

The resistance as seen from terminals ***AB***

$$R_{AB} = 6\ \Omega \text{ (from the Fig. 6.121(d))}$$

Hence, the required circuit is as shown in Fig. 6.121(e).

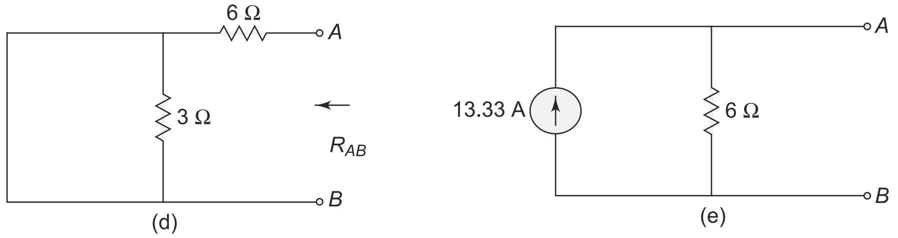


Fig. 6.121

Example 6.43 *For the circuit shown in Fig. 6.122, find Norton's equivalent circuit.*

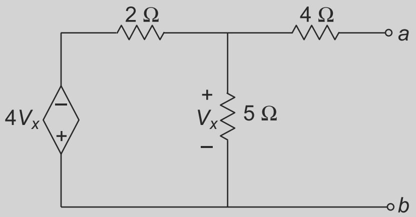


Fig. 6.122

Solution In the case of circuit having only dependent sources (without independent sources), both V_{OC} and i_{SC} are zero. We apply a 1 A source externally and determine the resultant voltage across it, and then find $R_{Th} = \frac{V}{1}$ or we can also apply the 1 V source externally and determine the current through it and then we find $R_{Th} = 1/i$.

By applying the 1 A source externally as shown in Fig. 6.123(a).

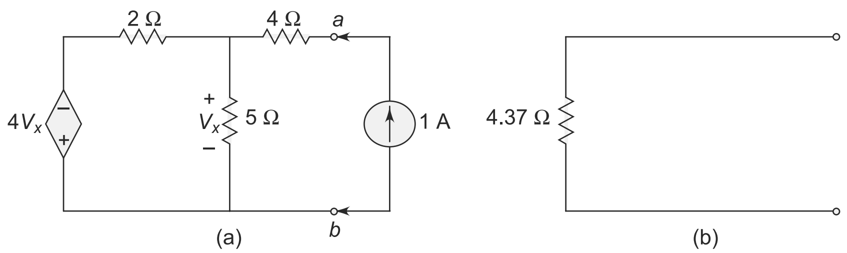


Fig. 6.123

and application of Kirchhoff's current law, we have

$$\frac{V_x}{5} + \frac{V_x + 4V_x}{2} = 1$$

$$V_x = 0.37 \text{ V}$$

The current in the 4 Ω branch is

$$\frac{V_x - V}{4} = -1$$

Substituting V_x in the above equation, we get

$$V = 4.37 \text{ V}$$

$$\therefore \qquad R_{Th} = \frac{V}{1} = 4.37\ \Omega$$

If we short circuit the terminals ***a*** and ***b*** we have

$$\frac{V_x - 4V_x}{2} = 0$$

$$V_x = 0$$

$$I_{SC} = \frac{V_x}{4} = 0$$

Therefore, Norton's equivalent circuit is as shown in Fig. 6.123(b).

Example 6.44 *Obtain the Norton's equivalent circuit at the terminals A, B for the following Fig. 6.124.*

[JNTU April/May 2003]

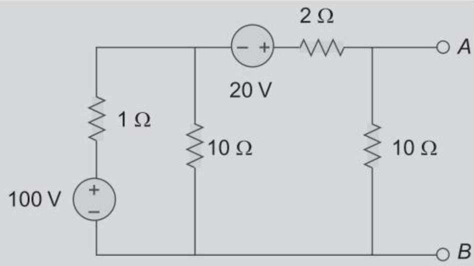


Fig. 6.124

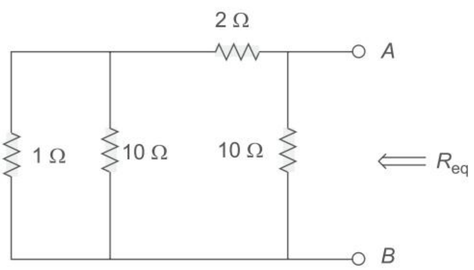


Fig. 6.125

Solution For finding the Nortons resistance, replace the voltage sources by the short circuit.

$$R_{eq} = \{[(1||10)+2]||10\}$$

$$= 2.253\ \Omega$$

For finding the I_N short the terminals ***A*** and ***B*** find current I_N. Apply superposition

(i) With 100 V source

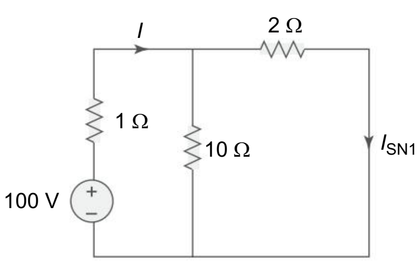


Fig. 6.126

$$Z = [(2||10) + 1] = 2.67$$

$$I_{SN1} = \frac{100}{Z}$$

$$= \frac{100}{2.67} = 37.45 \text{ A}$$

(ii) With 20 V source

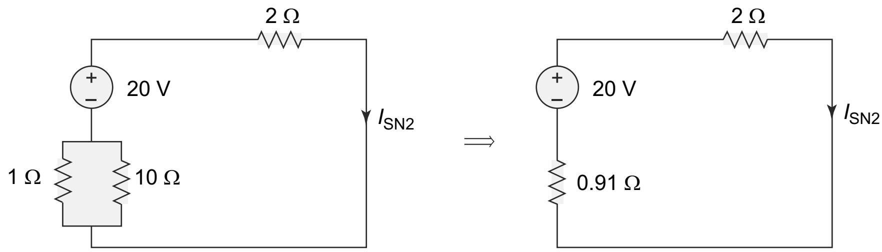


Fig. 6.127

$$I_{SN2} = \frac{20}{2.91} = 6.872 \text{ A}$$

$$\therefore \quad I_{SN} = I_{SN1} + I_{SN2}$$

$$= 37.45 + 6.872$$

$$= 44.322 \text{ A}$$

∴ Nortons equivalent circuit is shown in Fig. 6.128.

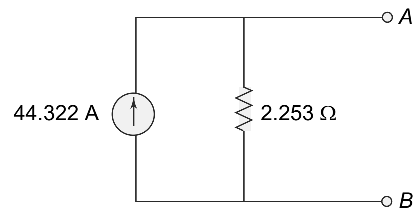


Fig. 6.128

Example 6.45 *Find the Norton's equivalent across the terminals ab as shown in Fig. 7.129. Hence find current through 10 ohms.* *[JNTU June 2009]*

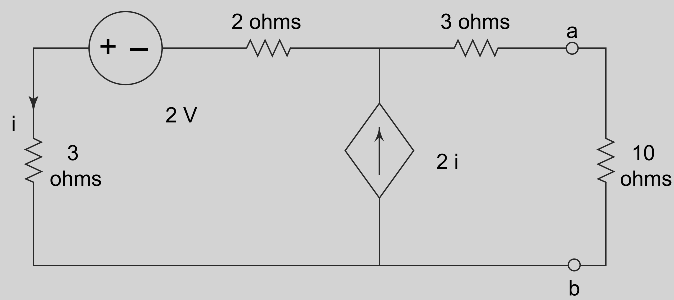


Fig. 6.129

Solution Short circuiting a-b terminal–

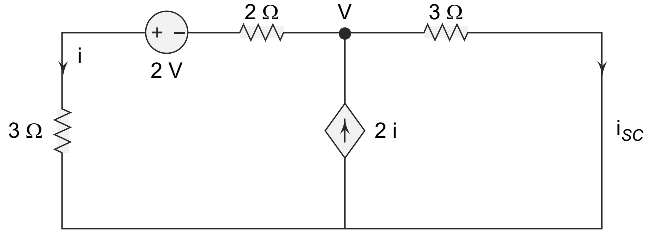


Fig. 6.130

$$2i = i + i_{SC} \Rightarrow i_{SC} = i$$

$$i = \frac{2+V}{5} = \frac{V}{3}$$

or, $6 + 3V = 5V$

or, $6 = 2V$

or, $V = 3$

$$\therefore \quad i = \frac{2+3}{5} = 1 \text{ amp}$$

$$\therefore \quad i_{SC} = 1 \text{ amp}$$

Open circuiting a-b terminal and deactivating independent voltage source–

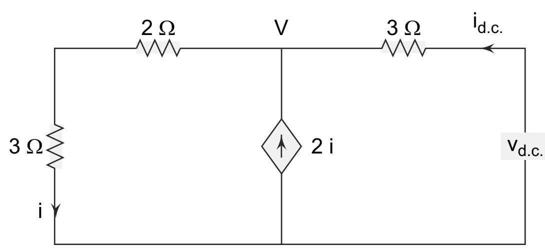


Fig. 6.131

$$2i + i_{d.c.} = i$$

$$\therefore \quad i_{d.c.} = -i$$

Now, $\frac{V}{5} = i$ or, $V = 5i$

Now, $\frac{V_{d.c.} - V}{3} = i_{d.c.}$

or, $\frac{V_{d.c.} - 5i}{3} = -i$

or, $V_{d.c.} - 5i = -3i$

or, $V_{d.c.} = 2i = -2i_{d.c.}$

$$\therefore \quad R_{int} = 2 \text{ ohm}$$

6.3.2 Norton's Theorem (ac Excitation) *[JNTU Jan 2010]*

Another method of analysing a complex impedance circuit is given by Norton's theorem. The Norton equivalent form of any complex impedance circuit consists of an equivalent current source I_N and an equivalent impedance Z_N, arranged as shown in Fig. 6.132. The values of equivalent current and impedance depend on the values in the original circuit.

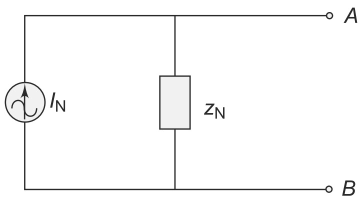


Fig. 6.132

Though Norton's equivalent circuit is not the same as its original circuit, the output voltage and current are the same in both cases; Norton's current is equal to the current passing through the short circuited output terminals and the value of impedance is equal to the impedance seen into the network across the output terminals.

Consider the circuit shown in Fig. 7.133.

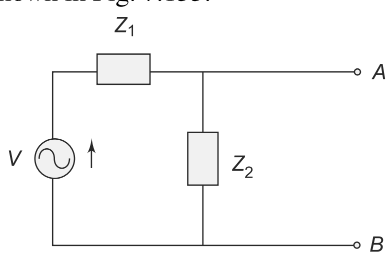


Fig. 6.133

Norton's equivalent for the circuit shown in Fig. 6.133 between points ***A*** and ***B*** is found as follows. The current passing through points ***A*** and ***B*** when it is short-circuited is the Norton's equivalent current, as shown in Fig. 6.134.

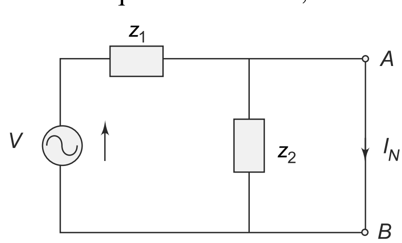


Fig. 6.134

Norton's current $\boldsymbol{I_N = V/Z_1}$

The impedance between points ***A*** and ***B***, with the source replaced by a short circuit, is Norton's equivalent impedance. In Fig. 6.100, the impedance from ***A*** to ***B***, $\boldsymbol{Z_2}$ is in parallel with $\boldsymbol{Z_1}$.

$$\therefore \qquad Z_N = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Norton's equivalent circuit is shown in Fig. 6.135.

The advantages seen with Thevenin's theorem apply to Norton's theorem. If we wish to analyze load resistor voltage and current over several different values of

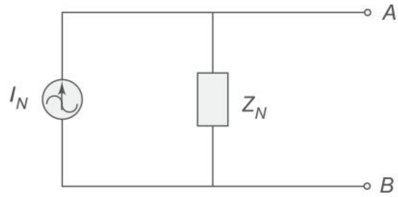


Fig. 6.135

load resistance, we can use the Norton's equivalent circuit again and again, applying nothing more complex than simple parallel circuit analysis to determine what's happening with each trial load. This theorem is not applicable to circuits consisting of nonlinear elements and not valid to unilateral circuits. This theorem is not valid where the magnetic coupling exists between load and the circuit.

Example 6.46 *For the circuit shown in Fig. 7.136, determine Norton's equivalent circuit between the output terminals, AB.*

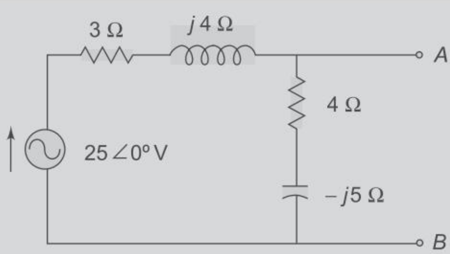


Fig. 6.136

Solution Norton's current $\boldsymbol{I_N}$ is equal to the current passing through the short circuited terminals ***AB*** as shown in Fig. 6.137.

The current through terminals ***AB*** is

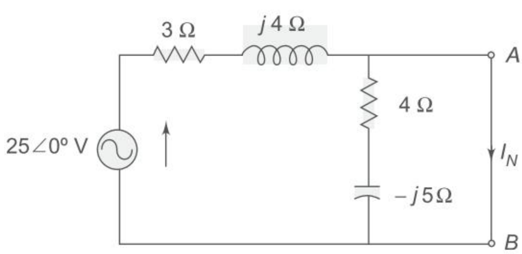


Fig. 6.137

$$I_N = \frac{25\angle 0^\circ}{3 + j4} = \frac{25\angle 0^\circ}{5\angle 53.13^\circ}$$

$$= 5\angle -53.13^\circ$$

The impedance seen from terminals ***AB*** is

$$Z_N = \frac{(3 + j4)(4 - j5)}{(3 + j4) + (4 - j5)}$$

$$= \frac{5\angle 53.13^\circ \times 6.4\angle -51.34^\circ}{7.07\angle -8.13^\circ}$$

$$= 4.53\angle 9.92^\circ$$

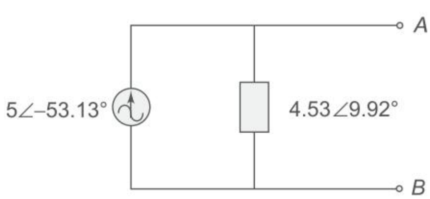


Fig. 6.138

Norton's equivalent circuit is shown in Fig. 6.138.

Example 6.47 *For the circuit shown in Fig. 6.139, determine the load current I_L by using Norton's theorem.*

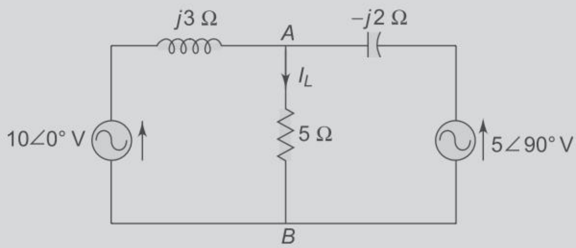


Fig. 6.139

Solution Norton's impedance seen from terminals ***AB*** is

$$Z_{AB} = \frac{(j3)(-j2)}{(j3)-(j2)} = \frac{6}{j1}$$

$\therefore \quad \boldsymbol{Z_{AB}} = 6\angle -90°$

Current passing through ***AB***, when it is shorted

$$I_N = \frac{10\angle 0°}{3\angle 90°} + \frac{5\angle 90°}{2\angle -90°}$$

$\therefore \quad \boldsymbol{I_N} = 3.33\ \angle -90° + 2.5\ \angle 180°$

$= -j3.33 - 2.5$

$\boldsymbol{I_N} = 4.16\ \angle -126.8°$

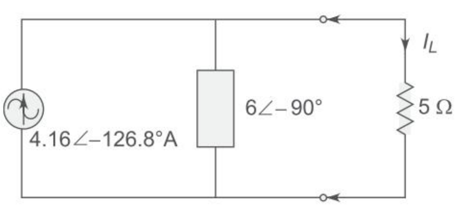


Fig. 6.140

Norton's equivalent circuit is shown in Fig. 6.140.

$$\text{Load current is } I_L = I_N \times \frac{6\angle -90°}{5+6\angle -90°}$$

$$= 4.16\angle -126.8° \times \frac{6\angle -90°}{5-j6}$$

$$= \frac{4.16 \times 6\angle -216.8°}{7.81\angle -50.19°}$$

$$= 3.19\ \angle -166.61°$$

Example 6.48 *For the circuit shown in Fig. 6.141, determine Norton's equivalent circuit.*

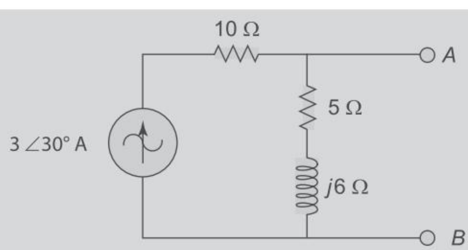


Fig. 6.141

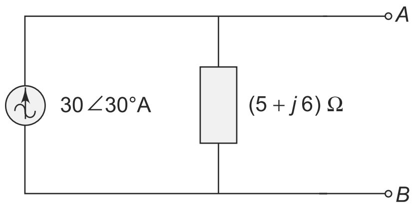


Fig. 6.142

Solution The impedance seen from the terminals when the source is reduced to zero

$$Z_{AB} = (5 + j6)\ \Omega$$

Current passing through the short circuited terminals, ***A*** and ***B***, is

$$I_N = 30\ \angle 30°\ \text{A}$$

Norton's equivalent circuit is shown in Fig. 6.142.

Example 6.49 *Determine the current through the load impedance $Z_L = (8 + j6)\ \Omega$ connected across AB in the network shown in Fig. 6.143 by applying Norton's theorem.* *[JNTU April/May 2002]*

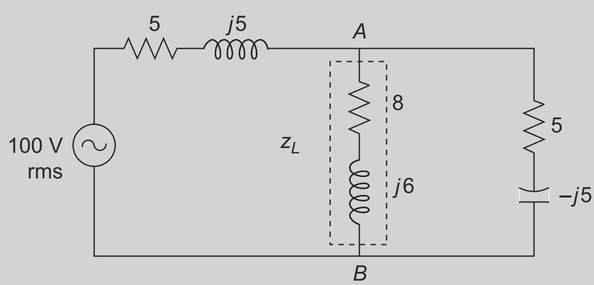


Fig. 6.143

Solution

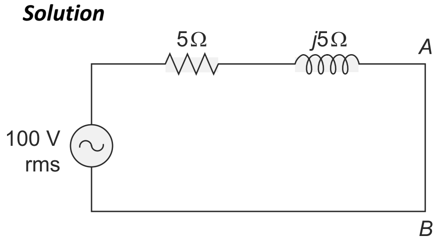


Fig. 6.144

(i) ***To find the Norton's current***

Short the load terminals as shown in Fig. 6.144.

$$I_N = \frac{100}{5 + j5} = 14.142\angle -45^\circ\ \text{A}$$

(ii) ***To find R_N***

Open the load terminals and replace the source with short circuit as shown in Fig. 6.112.

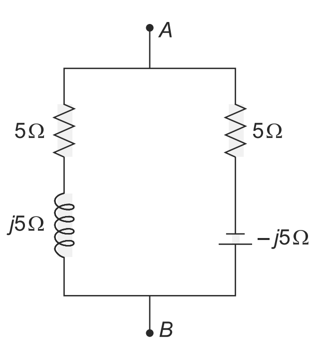


Fig. 6.145

$$R_N = \frac{(5 + j5)(5 - j5)}{10} = \frac{25 + 25}{10} = 5\ \Omega$$

$$I_L = \frac{(14.14\angle -45^\circ)5}{13 + j6} = \frac{70.7\angle -45^\circ}{14.317\angle 24.77^\circ}$$

$$I_L = 4.93\angle -69.77^\circ\ \text{A}$$

$$I_L = 1.704 - j\,4.625\ \text{A}$$

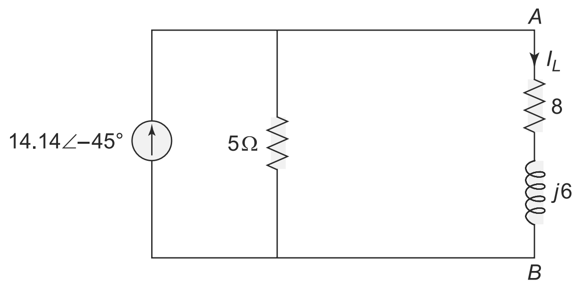


Fig. 6.146

Example 6.50 *Using Norton's theorem, find the current through the load impedance Z_L, for the network as shown in Fig. 6.147. [JNTU May/June 2008]*

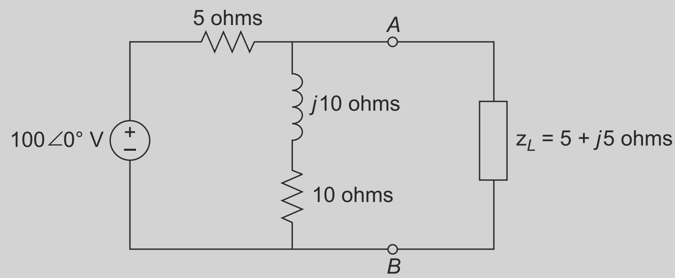


Fig. 6.147

Solution The given network is

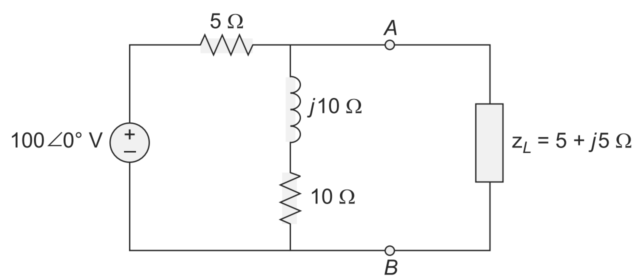


Fig. 6.148(a)

First replace with Norton's equivalent across the terminals ***AB***. Norton's current I_N is equal to the current passing through the short-circuited terminals ***AB***.

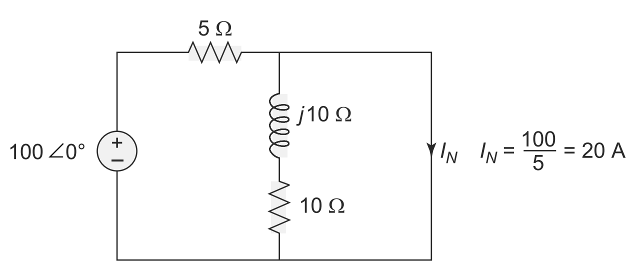


Fig. 6.148(b)

The impedance across the terminals ***AB***

$$Z_n = 5 \,||\, (10 + j10)$$

$$= \frac{5 \times (10 + j10)}{15 + j10} = \frac{5 \times 10 \times (1 + j)}{3 + j2} = 3.92\underline{|11.31^\circ}$$

The circuit when replaced is shown below.

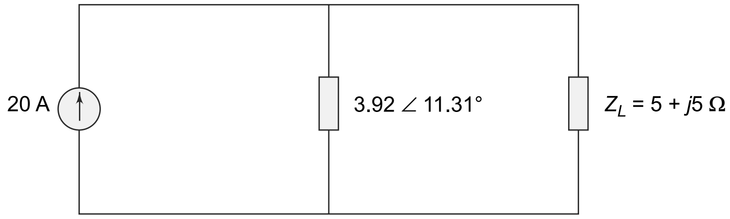


Fig. 6.148(c)

The current flowing through $\boldsymbol{Z_L}$

$$I_1 = \frac{20 \times 3.92\underline{|11.31^\circ}}{3.92\underline{|11.31^\circ} + 7.071\underline{|45^\circ}}$$

$$= \frac{20 \times 3.92\underline{|11.31^\circ}}{3.843 + j0.768 + 5 + j5}$$

$$= \frac{20 \times 3.92\underline{|11.31^\circ}}{8.843 + j5.768}$$

$$= \frac{20 \times 3.92\underline{|11.31^\circ}}{10.557\underline{|33.11^\circ}} = 7.426\underline{|-21.8^\circ}$$

Example 6.51 *Using Norton's theorem, find the current through the load impedance Z_L as shown in Fig. 6.149.* *[JNTU June 2009]*

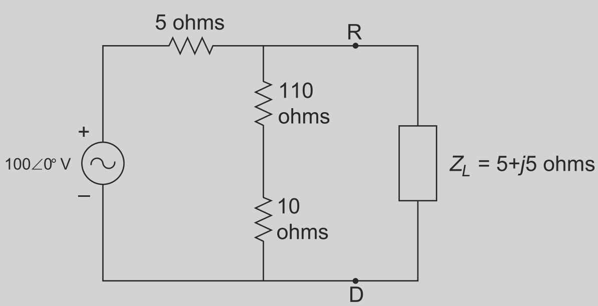


Fig. 6.149

Solution

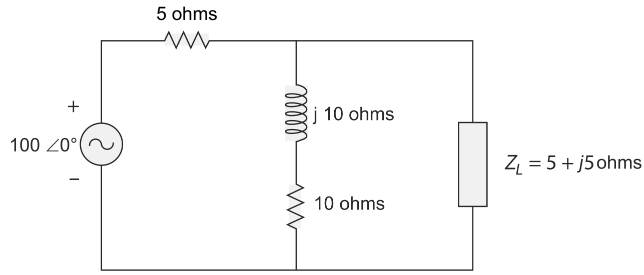


Fig. 6.150(a)

Short circuiting load terminal

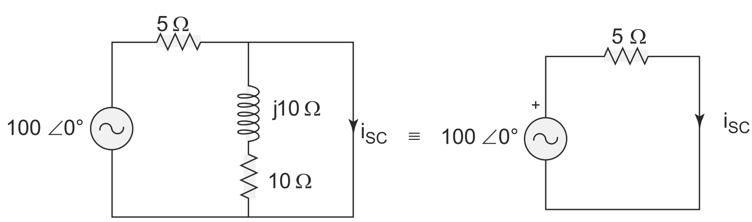


Fig. 6.150(b)

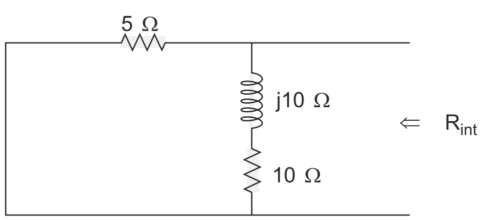


Fig. 6.150(c)

$$\therefore \quad i_{sc} = \frac{100}{5} \text{ amp} = 20 \text{ amp.}$$

To determine the equivalent resistance of the circuit looking through load terminal, the constant source is deactivated as shown

$$\therefore \quad R_{int} = \frac{(10+10j)5}{10+10j+5} \text{ ohm} = \frac{50(1+j)}{5(3+2j)} \text{ ohm} = \frac{10(1+j)}{(3+2j)} \text{ ohms.}$$

So, Norton's equivalent circuit is given as

$$\therefore \quad \text{Current through load } = I_L = i_{sc} \times \frac{R_{int}}{R_{int} + z_L}$$

$$= 20 \times \frac{10(1+j)/(3+2j)}{10(1+j)/(3+2j) + 5(1+j)}$$

$$= 20 \times \frac{10(1+j)}{10(1+j) + 5(1+j)(3+2j)}$$

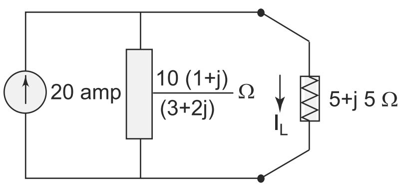


Fig. 6.150(d)

$$= \frac{20 \times 10}{5(1+j)} \times \frac{(1+j)}{2+3+2j}$$

$$= \frac{40}{5+2j} \text{ amp} = 7.428\angle -21.8° \text{ amp}$$

$$= 6.897 - 2.758j$$

Example 6.52 *Obtain Norton's equivalent across terminals A and B for network shown in Fig. 6.151.* *[JNTU June 2009]*

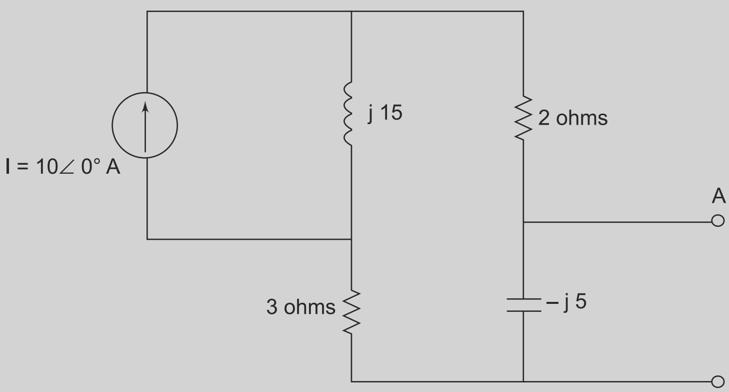


Fig. 6.151

Solution

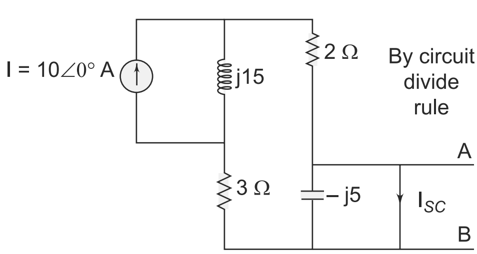


Fig. 6.152(a)

$$I_{SC} = \frac{(j15)10\angle 0°}{(5+j15)} \text{A}$$

$$= 9 + 3j$$

$$= 9.486\angle 18.435°$$

Now deactivating the source

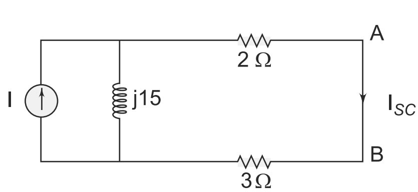


Fig. 6.152(b)

= (−*j*5)||(5+15*j*)

= 1−*j*7.

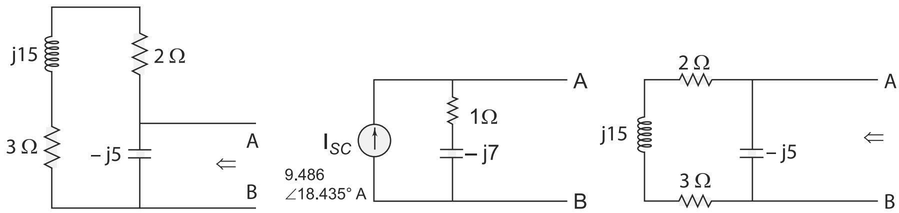


Fig. 6.152(c)

Example 6.53 *Using Norton's theorem, find the current through the load impedance Z_L, for the network as shown in Fig. 6.153.* *[JNTU Jan 2010]*

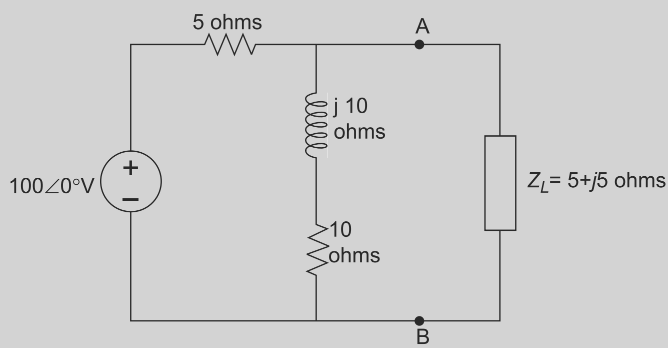


Fig. 6.153

Solution

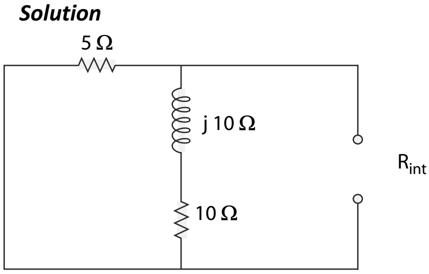


Fig. 6.154

To measure internal resistance Z_L is removed and voltage source is short circuited giving

$$\therefore \qquad R_{\text{int}} = \frac{(10+10j)5}{10+10j+5}\,\Omega$$

$$= \frac{\cancel{5}\times 10(1+j)}{\cancel{5}(3+2j)} = \frac{10\sqrt{1^2+1^2}}{\sqrt{3^2+2^2}} \angle \tan^{-1}\left(\frac{1}{1}\right) - \tan^{-1}\left(\frac{2}{3}\right)$$

$$= 3.92\angle 11.31°\Omega$$

$$\therefore \qquad Y_{11} = \frac{\Delta 11}{\Delta} = \frac{2+\frac{4}{s}+\frac{1}{s^2}}{\frac{4}{s}+\frac{2}{s^2}} = \frac{2s^2+4s+1}{2s(1+2s)}$$

$$Y_{12} = -\frac{\Delta 21}{\Delta} = \frac{1+2s+2s^2}{2s(1+2s)}$$

$$Y_{21} = -\frac{\Delta 12}{\Delta} = \frac{1+2s+2s^2}{2s(1+2s)}$$

$$Y_{22} = \frac{\Delta 22}{\Delta} = \frac{2s^2+4s+1}{2s(1+2s)}$$

In this problem, $\Delta_{11} = \Delta_{22}, \Delta_{12} = \Delta_{21}$

$Y_{11} = Y_{22}, Y_{12} = Y_{21}$

∴ The network is symmetrical and reciprocal.

6.4 MAXIMUM POWER TRANSFER THEOREM

6.4.1 Maximum Power Transfer Theorem (dc Excitation) *[JNTU Jan 2010]*

Many circuits basically consist of sources, supplying voltage, current, or power to the load; for example, a radio speaker system, or a microphone supplying the input signals to voltage pre-amplifiers. Sometimes it is necessary to transfer maximum voltage, current or power from the source to the load. In the simple resistive circuit shown in Fig. 6.65, R_S is the source resistance. Our aim is to find the necessary conditions so that the power delivered by the source to the load is maximum.

It is a fact that more voltage is delivered to the load when the load resistance is high as compared to the resistance of the source. On the other hand, maximum current is transferred to the load when the load resistance is small compared to the source resistance.

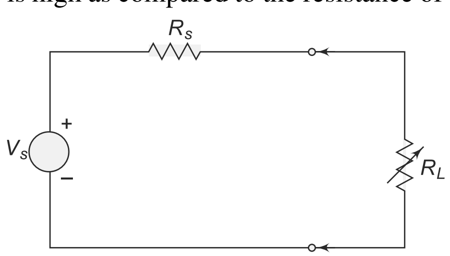


Fig. 6.155

For many applications, an important consideration is the maximum power transfer to the load; for example, maximum power transfer is desirable from the output amplifier to the speaker of an audio sound system. The maximum Power Transfer Theorem states that maximum power is delivered from a source to a load when the load resistance is equal to the source resistance. In Fig. 6.155, assume that the load resistance is variable.

Current in the circuit is $I = V_S/(R_S + R_L)$

Power delivered to the load R_L is $P = I^2R_L = V^2_S R_L/(R_S + R_L)^2$

To determine the value of R_L for maximum power to be transferred to the load, we have to set the first derivative of the above equation with respect to R_L, i.e. when $\frac{dP}{dR_L}$ equals zero.

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_S^2}{(R_S + R_L)^2} R_L\right]$$

$$= \frac{V_S^2 \left\{(R_S + R_L)^2 - (2R_L)(R_S + R_L)\right\}}{(R_S + R_L)^4}$$

$\therefore$ $$(R_S + R_L)^2 - 2R_L(R_S + R_L) = 0$$

$$R^2_S + R^2_L + 2R_S R_L - 2R^2_L - 2R_S R_L = 0$$

$\therefore$ $$R_S = R_L$$

So, maximum power will be transferred to the load when load resistance is equal to the source resistance.

Example 6.54 *In the circuit shown in Fig. 6.156 determine the value of load resistance when the load resistance draws maximum power. Also find the value of the maximum power.*

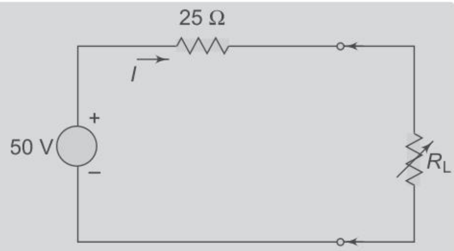


Fig. 6.156

Solution In Fig. 6.156, the source delivers the maximum power when load resistance is equal to the source resistance.

$$R_L = 25\ \Omega$$

The current $I = 50/(25 + R_L) = 50/50 = 1$ A

The maximum power delivered to the load $P = I^2R_L$

$$= 1 \times 25 = 25 \text{ W}$$

Example 6.55 *Determine the maximum power delivered to the load in the circuit shown in Fig. 6.157.*

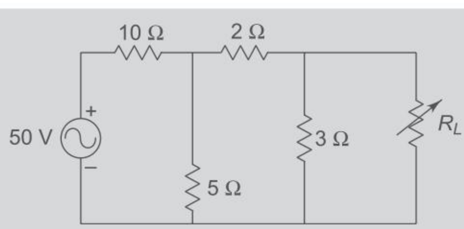


Fig. 6.157

Solution For the given circuit, let us find out the Thevenin's equivalent circuit across AB as shown in Fig. 6.158(a).

The total resistance is

$$R_T = [\{(3 + 2) \parallel 5\} + 10]$$

$$= [2.5 + 10] = 12.5\ \Omega$$

Total current drawn by the circuit is

$$I_T = \frac{50}{12.5} = 4\ \text{A}$$

The current in the 3 V resistor is

$$I_3 = I_T \times \frac{5}{5+5} = \frac{4\times 5}{10} = 2\ \text{A}$$

Thevenin's voltage $V_{AB} = V_3 = 3 \times 2 = 6$ V

Thevenin's resistance $R_{\text{Th}} = R_{AB} = [((10 \parallel 5) + 2) \parallel 3]\ \Omega = 1.92\ \Omega$

Thevenin's equivalent circuit is shown in Fig. 6.158(b).

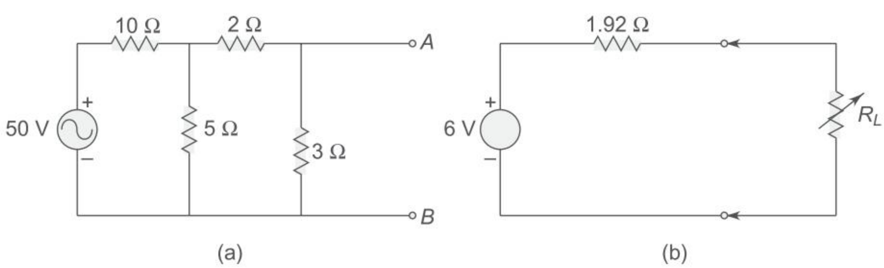


Fig. 6.158

From Fig. 6.158(b), and maximum power transfer theorem

$$R_L = 1.92\ \Omega$$

∴ Current drawn by load resistance R_L

$$I_L = \frac{6}{1.92+1.92} = 1.56\ \text{A}$$

Power delivered to the load $= I^2_L R_L$

$$= (1.56)^2 \times 1.92 = 4.67\ \text{W}$$

Example 6.56 *Determine the load resistance to receive maximum power from the source; also find the maximum power delivered to the load in the circuit shown in Fig. 6.159.*

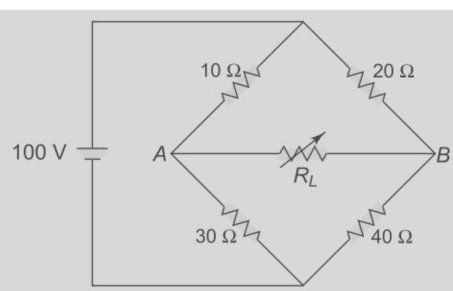
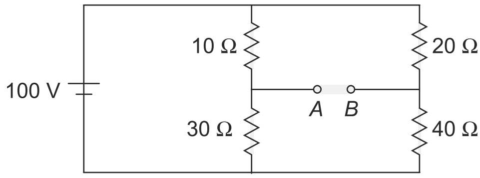


Fig. 6.159



(a)

Fig. 6.160

Solution For the given circuit, we find out the Thevenin's equivalent circuit.

Thevenin's voltage across terminals A and B

$$V_{AB} = V_A - V_B$$

Voltage at point A is

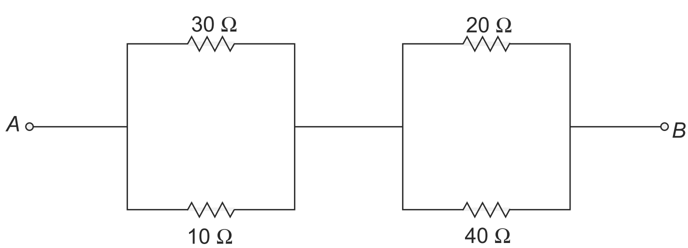
$$V_A = 100 \times \frac{30}{30+10} = 75 \text{ V}$$

Voltage at point B is

$$V_B = 100 \times \frac{40}{40+20} = 66.67 \text{ V}$$

$$\therefore V_{AB} = 75 - 66.67 = 8.33 \text{ V}$$

To find Thevenin's resistance the circuit in Fig. 6.161 (a) can be redrawn as shown in Fig. 6.161(b).



(b)

Fig. 6.161

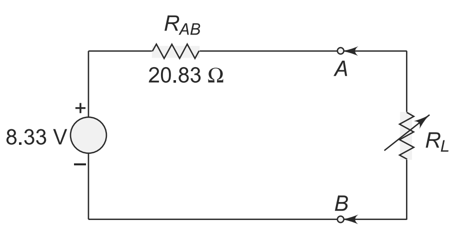
From Fig. 6.161(b), Thevenin's resistance

$$R_{AB} = [(30 \parallel 10) + (20 \parallel 40)]$$

$$= [7.5 + 13.33] = 20.83\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.161(c).

According to maximum power transfer theorem



(c)

Fig. 6.161

$$R_L = 20.83\ \Omega$$

Current drawn by the load resistance

$$I_L = \frac{8.33}{20.83 + 20.83} = 0.2 \text{ A}$$

$\therefore$ Maximum power delivered to

load = $I^2_L R_L$

$$= (0.2)^2 (20.83) = 0.833 \text{ W}$$

Example 6.57 *The circuit shown in the Fig. 6.162 below has resistance R which absorbs maximum power. Compute the value of R and maximum power.* *[JNTU April/May 2003]*

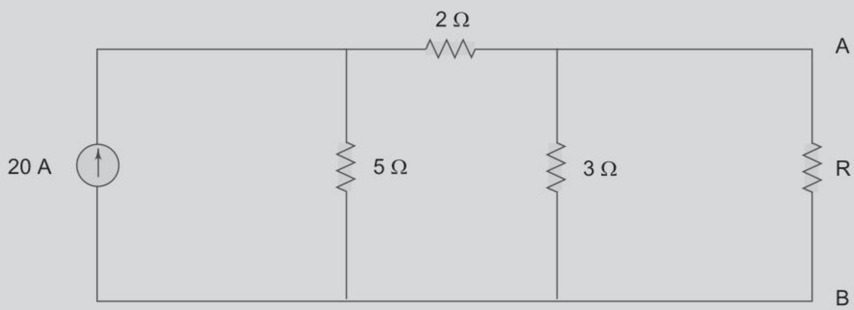


Fig. 6.162

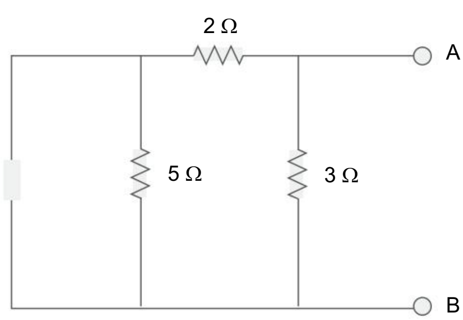


Fig. 6.163

Solution According to maximum power transfer theorem, maximum power can be transferred when load resistance is equal to the interval resistance of the source which can be calculated as the resistance seen from AB with source open.

$$\therefore \quad R_{Th} = (5 + 2)//3$$

$$\frac{21}{10} = 2.1\ \Omega$$

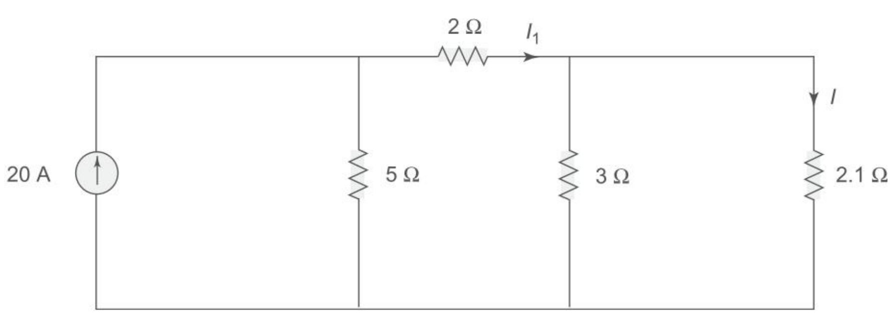


Fig. 6.164

Now the circuit can be drawn as
According to current dividing rule

$$I_1 = \frac{20 \times 5}{(5 + 3.235)} = 12.14 \text{ A}$$

$$I_2 = \frac{I_1 \times 3}{5.1} = \frac{12.14 \times 3}{5} = 7.14 \text{ A}$$

So the maximum power that can be delivered to resistor ***R*** is

$$\boldsymbol{I^2 R} = (7.14)^2 \times 2.1 = 107 \text{ watts.}$$

6.4.2 Maximum Power Transfer Theorem (ac Excitation) *[JNTU Jan 2010, Nov 2011]*

The maximum power transfer theorem has been discussed for resistive loads. The maximum power transfer theorem states that the maximum power is delivered from a source to its load when the load resistance is equal to the source resistance. It is for this reason that the ability to obtain impedance matching between circuits is so important. For example, the audio output transformer must match the high impedance of the audio power amplifier output to the low input impedance of the speaker. Maximum power transfer is not always desirable, since the transfer occurs at a 50 per cent efficiency. In many situations, a maximum voltage transfer is desired which means that unmatched impedances are necessary. If maximum power transfer is required, the load resistance should equal the given source resistance. The maximum power transfer theorem can be applied to complex impedance circuits. If the source impedance is complex, then the maximum power transfer occurs when the load impedance is the complex conjugate of the source impedance.

Consider the circuit shown in Fig. 6.165, consisting of a source impedance delivering power to a complex load.

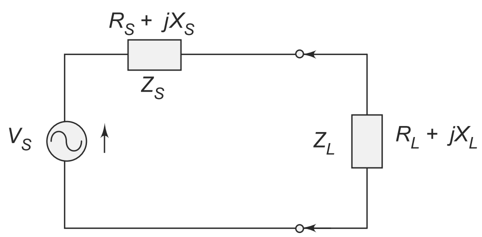


Fig. 6.165

Current passing through the circuit shown

$$I = \frac{V_s}{(R_s + j X_s) + (R_L + j X_L)}$$

Magnitude of current $I = |I| = \dfrac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$

Power delivered to the circuit is

$$P = I^2 R_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

In the above equation, if R_L is fixed, the value of P is maximum when

$$X_s = -X_L$$

Then the power
$$P = \frac{V_s^2 R_L}{(R_s + R_L)^2}$$

Let us assume that R_L is variable. In this case, the maximum power is transferred when the load resistance is equal to the source resistance (already discussed in Chapter 3). If $R_L = R_s$ and $X_L = -X_s$, then $Z_L = Z^*_s$. This means that the maximum power transfer occurs when the load impedance is equal to the complex conjugate of source impedance Z_s.

Maximum power transfer does not coincide with maximum efficiency. Application of the maximum power transfer theorem to AC power distribution will not result in max or even high efficiency. The goal of high efficiency is more important for AC power distribution, which dictates a relatively low generator impedance compared to load impedance. Maximum power transfer does not coincide with the goal of lowest noise. The low level radio frequency amplifier between the antenna and a radio receiver is often designed for lowest possible noise. This often requires a mismatch of the amplifier input impedance to the antenna as compared with that dictated by the maximum power transfer theorem.

Example 6.58 *For the circuit shown in Fig. 6.166, find the value of load impedance for which the source delivers maximum power. Calculate the value of the maximum power.*

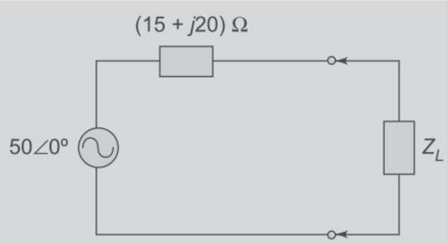


Fig. 6.166

Solution In the circuit shown in Fig. 6.166, the maximum power transfer occurs when the load impedance is complex conjugate of the source impedance

$$\therefore \quad Z_L = Z^*_s = 15 - j20$$

When $Z_L = 15 - j20$, the current passing through circuit is

$$I = \frac{V_s}{R_s + R_L} = \frac{50\angle 0^\circ}{15 + j20 + 15 - j20} = \frac{50\angle 0^\circ}{30} = 1.66\angle 0^\circ$$

The maximum power delivered to the load is

$$P = I^2 R_L = (1.66)^2 \times 15 = 41.33 \text{ W}$$

Example 6.59 *For the circuit shown in Fig. 6.167, find the value Z that will receive maximum power, also determine this power.*

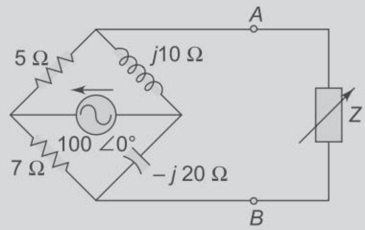


Fig. 6.167

Solution The equivalent impedance at terminals AB with the source set equal to zero is

$$Z_{AB} = \frac{5(j10)}{5+j10} + \frac{7(-j20)}{(7-j20)}$$

$$= \frac{50\angle 90^\circ}{11.18\angle 63.43^\circ} + \frac{140\angle -90^\circ}{21.19\angle -70.7^\circ}$$

$$= 4.47\ \angle 26.57^\circ + 6.6\ \angle -19.3^\circ$$

$$= 3.99 + j1.99 + 6.23 - j2.18$$

$$= 10.22 - j0.19$$

The Thevenin equivalent circuit is shown in Fig. 6.168(a).
The circuit in Fig. 6.168(a) is redrawn as shown in Fig. 6.168(b).

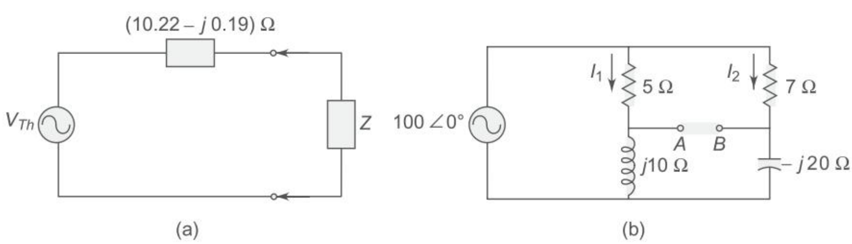


Fig. 6.168

$$\text{Current} \quad I_1 = \frac{100\angle 0^\circ}{5+j10}$$

$$= \frac{100\angle 0^\circ}{11.18\angle 63.43^\circ} = 8.94\angle -63.43^\circ\text{A}$$

$$\text{Current} \quad I_2 = \frac{100\angle 0^\circ}{7-j20} = \frac{100\angle 0^\circ}{21.19\angle -70.7^\circ} = 4.72\angle 70.7^\circ$$

Voltage at A, $V_A = 8.94\ \angle -63.43^\circ \times j10 = 89.4\ \angle -26.57^\circ$

Voltage at B, $V_B = 4.72\ \angle 70.7^\circ \times -j20 = 94.4\ \angle -19.3^\circ$

Voltage across terminals AB

$$V_{AB} = V_A - V_B$$
$$= 89.4\angle 26.57° - 94.4\angle -19.3°$$
$$= 79.96 + j39.98 - 89.09 + j31.2$$
$$= -9.13 + j71.18$$
$$V_{Th} = V_{AB} = 71.76\angle 97.3° \text{ V}$$

To get maximum power, the load must be the complex conjugate of the source impedance.

$\therefore$ Load $Z = 10.22 + j0.19$

Current passing through the load Z

$$I = \frac{V_{Th}}{Z_{Th} + Z} = \frac{71.76\angle 97.3°}{20.44} = 3.51\angle 97.3°$$

Maximum power delivered to the load is

$$= (3.51)^2 \times 10.22 = 125.91 \text{ W}$$

Example 6.60 *For the circuit shown in Fig. 6.169, the resistance R_s is variable from 2 Ω to 50 Ω. What value of R_s results in maximum power transfer across the terminals AB?*

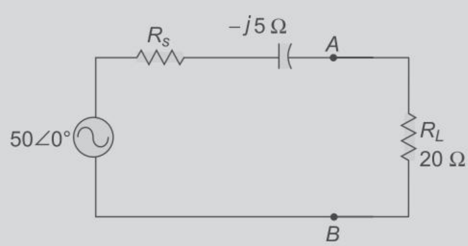


Fig. 6.169

Solution In the circuit shown the resistance R_L is fixed. Here, the maximum power transfer theorem does not apply. Maximum current flows in the circuit when R_s is minimum. For the maximum current

$$R_s = 2$$

But $Z_T = R_s - j5 + R_L = 2 - j5 + 20 = (22 - j5)$

$$= 22.56\angle -12.8°$$

$$\therefore \quad I = \frac{V_s}{Z_T} = -\frac{50\angle 0°}{22.56\angle -12.8°} = 2.22\angle 12.8°$$

Maximum power $P = I^2R = (2.22)^2 \times 20 = 98.6$ W

Example 6.61 *For the circuit shown in Fig. 6.170, find the value of Z that will receive maximum power; also determine this power.*

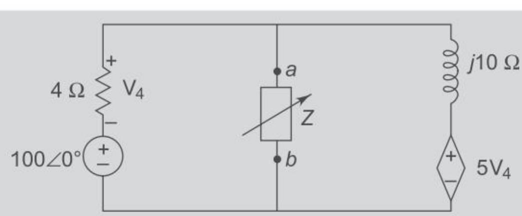


Fig. 6.170

Solution The equivalent impedance can be obtained by finding $\boldsymbol{V_{oc}}$ and $\boldsymbol{i_{sc}}$ at terminals ***a b***. Assume that current ***i*** is passing in the circuit.

$$i = \frac{100\angle 0° - 5V_4}{4 + j10}$$

$$= \frac{100\angle 0°}{4 + j10} - \frac{5 \times 4i}{4 + j10}$$

$$\boldsymbol{i} = 3.85 \angle -22.62°$$

$$\boldsymbol{V_{oc}} = 100 \angle 0° - 4 \times 3.85 \angle -22.62°$$

$$= 86 \angle 3.94°$$

$$\boldsymbol{i_{sc}} = 25 + \boldsymbol{j}50 = 56 \angle 63.44°$$

Thevenin's equivalent impedance

$$Z_{\text{Th}} = \frac{V_{oc}}{i_{sc}} = 1.54\angle -59.5°$$

$$= 0.78 - \boldsymbol{j}1.33$$

The circuit is drawn as shown in Fig. 6.171.

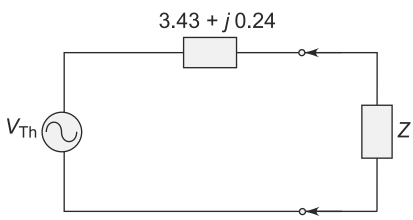


Fig. 6.171

To get maximum power, the load must be the complex conjugate of the source impedance.

∴ Load $\boldsymbol{Z} = 0.78 + \boldsymbol{j}1.33$

Current passing through load $\boldsymbol{Z}$

$$I = \frac{V_{\text{Th}}}{Z_{\text{Th}} + Z} = \frac{86\angle 3.94°}{1.56} = 55.13\angle 3.94°$$

Maximum power delivered to the load is $(55.13)^2 \times (0.78) = 2370.7$ W.

Example 6.62 *In the network shown in Fig. 6.172, find the value of Z_L so that the power transfer from the source is maximum. Also find P_{max}.*

[JNTU May/June 2006]

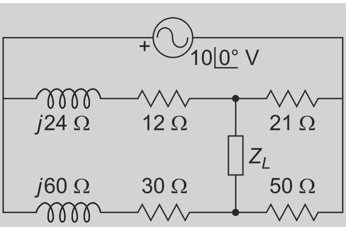


Fig. 6.172

Solution Let us remove 'z_L'. The Internal impedence of the circuit looking through $\boldsymbol{x} - \boldsymbol{y}$ is given by

$$z_{in} = \frac{(21)(12 + j24)}{21 + 12 + j24} + \frac{50(30 + j60)}{50 + 30 + j60}$$

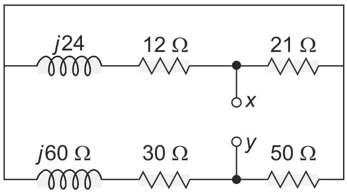


Fig. 6.173(a)

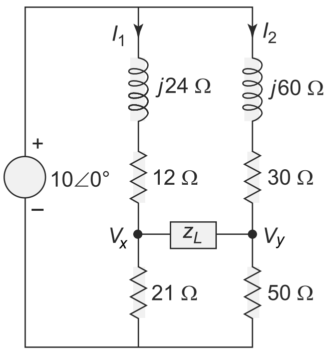


Fig. 6.173(b)

$$= \frac{563.44\angle 63.43^\circ}{40.8\angle 36^\circ} + \frac{3354.10\angle 63.43^\circ}{100\angle 36.87^\circ}$$

$$= 13.81\angle 27.43^\circ + 33.54\angle 26.56^\circ$$

$$z_{in} = 42.19 + j21.49\ \Omega$$

As per maximum power transfer theorem, $\boldsymbol{Z_L}$ should be the complex of $\boldsymbol{z_{in}}$

$$Z_L = z_{in}^* = (42.19 - j21.49)\ \Omega$$

$$\boldsymbol{V_{OC} = V_{xy}}$$

$$V_x = \frac{12 + j24}{12 + j24 + 21} \times 10\angle 0^\circ$$

$$= 6.577\angle 27.43^\circ\ \text{V}$$

$$V_y = \frac{30 + j60}{30 + j60 + 50} \times 10\angle 0^\circ$$

$$= 6.71\angle 26.56^\circ$$

$$V_{OC} = V_x - V_y = 6.577\angle 27.43^\circ - 6.71\angle 26.56^\circ$$

$$= -0.163 + j0.029$$

$$V_{OC} = 0.1657\angle 170^\circ\ \text{V}$$

$$P_{max} = \frac{V_{OC}^2}{4R_L} = \frac{(0.1657)^2}{4 \times 42.19} = 0.1627\ \text{mW}$$

$$\boldsymbol{P_{max}} = 0.1627\ \text{mW}$$

Example 6.63 *In the circuit shown in the given Fig. 6.174, find the value of R_L which results in max power transfer. Calculate the value of the maximum power.*

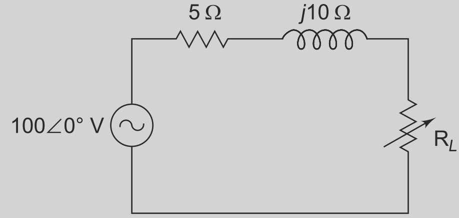


Fig. 6.174

Solution The value of $\boldsymbol{R_L}$ for which the maximum power transfer

$$R_L = |5 + j10| = \sqrt{5^2 + 10^2} = 11.18\ \Omega$$

Then the circuit current is

$$I = \frac{100\angle 0^\circ}{11.18+5+j10} = \frac{100\angle 0^\circ}{19.02\angle 31.78^\circ}$$

$$= 5.26\angle -31.718^\circ \text{ A}$$

The maximum power across R_L is

$$P_{max} = I^2 R = (5.26)^2\, 11.18 = 309 \text{ watts}$$

6.5 RECIPROCITY THEOREM

6.5.1 Reciprocity Theorem (dc Excitation)

[JNTU June 2009, Nov 2011]

In any linear bilateral network, if a single voltage source V_a in branch '*a*' produces a current I_b in branch '*b*', then if the voltage source V_a is removed and inserted in branch '*b*' will produce a current I_b in branch '*a*'. The ratio of response to excitation is same for the two conditions mentioned above. This is called the *reciprocity theorem.*

Consider the network shown in Fig. 6.175. AA' denotes input terminals and BB' denotes output terminals.

The application of voltage V across AA' produces current I at BB'. Now if the positions of the source and responses are interchanged, by connecting the voltage source across BB', the resultant current I will be at terminals AA'. According to the reciprocity theorem, the ratio of response to excitation is the same in both cases.

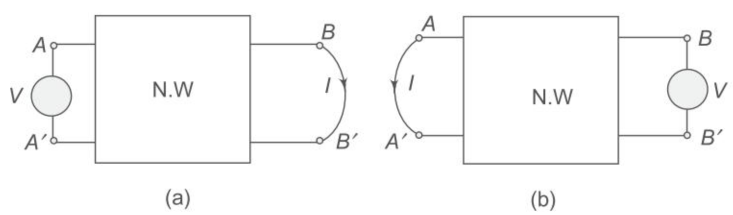


Fig. 6.175

Example 6.64 *Verify the reciprocity theorem for the network shown in Fig. 6.176.*

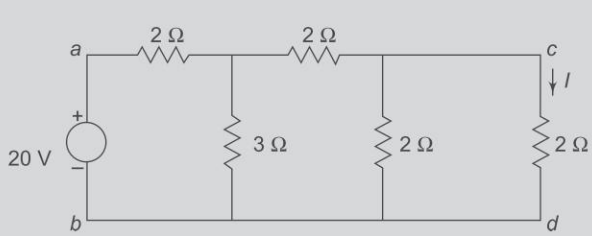


Fig. 6.176

Solution Total resistance in the circuit = 2 + [3 || (2 + 2 || 2)] = 3.5 Ω.

The current drawn by the circuit (See Fig. 6.177(a)).

$$I_T = \frac{20}{3.5} = 5.71\ \Omega$$

The current in the 2 Ω branch *cd* is I = 1.43 A.

Applying the reciprocity theorem, by interchanging the source and response we get (See Fig. 6.177(b)).

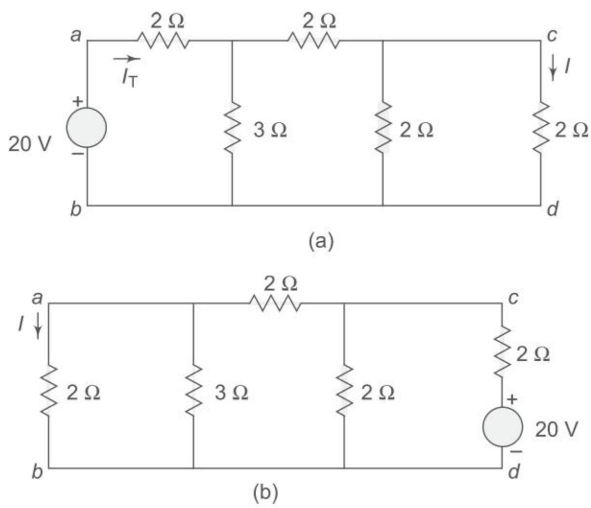


Fig. 6.177

Total resistance in the circuit = 3.23 Ω.

$$\text{Total current drawn by the circuit} = \frac{20}{3.23} = 6.19\ \text{A}$$

The current in the branch *AB* is I = 1.43 A

If we compare the results in both cases, the ratio of input to response is the same, i.e. (20/1.43) = 13.99.

Example 6.65 *Verify the reciprocity theorem for the given circuit shown in Fig. 6.178.*

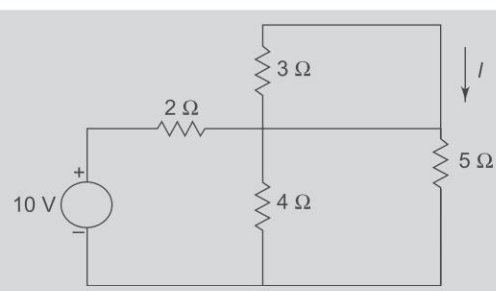


Fig. 6.178

Solution In Fig. 6.38, the current in the 5 Ω resistor is

$$I_5 = I_2 \times \frac{4}{8+4} = 2.14 \times \frac{4}{12} = 0.71\ \text{A}$$

where $$I_2 = \frac{10}{R_T}$$

and $$R_T = 4.67$$

$\therefore$ $$I_2 = \frac{10}{4.67} = 2.14 \text{ A}$$

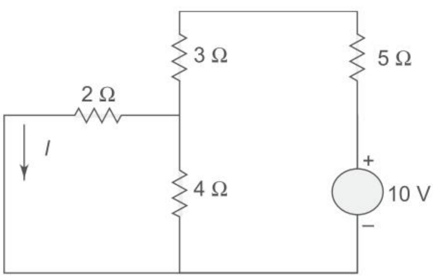


Fig. 6.179

We interchange the source and response as shown in Fig. 6.179.

In Fig. 6.179, the current in 2 Ω resistor is

$$I_2 = I_3 \times \frac{4}{4+2}$$

where $I_3 = \frac{10}{R_T}$ and $R_T = 9.33\ \Omega$

$\therefore$ $$I_3 = \frac{10}{9.33} = 1.07 \text{ A}$$

$$I_2 = 1.07 \times \frac{4}{6} = 0.71 \text{ A}$$

In both cases, the ratio of voltage to current is $\frac{10}{0.71} = 14.08$.

Hence the reciprocity theorem is verified.

Example 6.66 *Verify the reciprocity theorem in the circuit shown in Fig. 6.180.*

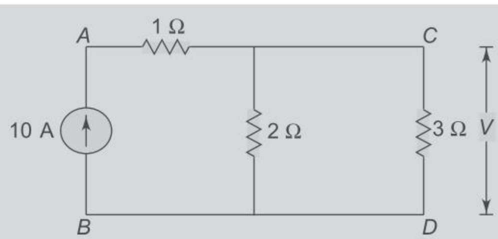


Fig. 6.180

Solution The voltage V across the 3 Ω resistor is

$$V = I_3 \times R$$

where $$I_3 = 10 \times \frac{2}{2+3} = 4 \text{ A}$$

$\therefore$ $$V = 4 \times 3 = 12 \text{ V}$$

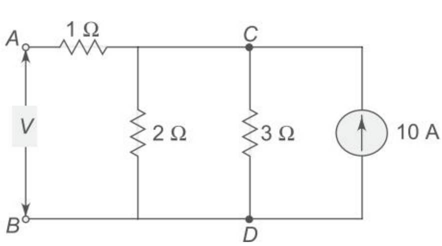


Fig. 6.181

We interchange the current source and response as shown in Fig. 6.181.

To find the response, we have to find the voltage across the 2 Ω resistor

$$V = I_2 \times R$$

where $I_2 = 10 \times \frac{3}{5} = 6 \text{ A}$

∴ $V = 6 \times 2 = 12 \text{ V}$

In both cases, the ratio of current to voltage is the same, i.e. it is equal to 0.833. Hence the reciprocity theorem is verified.

Example 6.67 *Verify reciprocity theorem in circuit shown in the following Fig. 6.182.*

[JNTU April/May 2003]

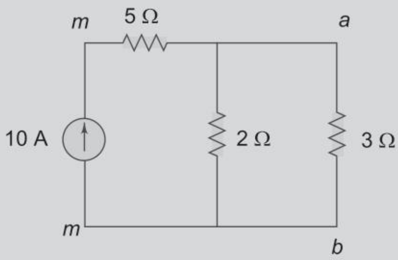


Fig. 6.182

Solution Let us find current in 3 V resistor.

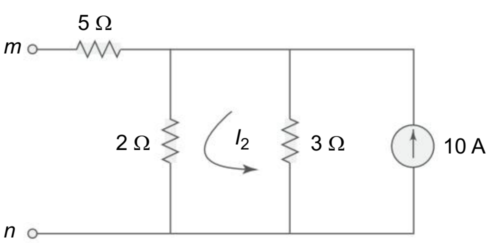


Fig. 6.183

$$I_3 = 10 \times \frac{2}{2+3} = 4 \text{ A}$$

$$V_{ab} = 3 \times 4 = 12$$

According to reciprocity theorem the voltage across *AB* $V_{ab} = 12$

Now connect the current source across *AB* and find the voltage across *m* and *n*.

$$I_2 = 10 \times \frac{3}{5} = 6 \text{ A}$$

The voltage across *mn* = 2 × 6 = 12 volts, same as V_{ah}. Hence, the reciprocity theorem is proved.

Example 6.68 *Verify reciprocity theorem for the network shown in Fig. 6.184. [JNTU May/June 2006]*

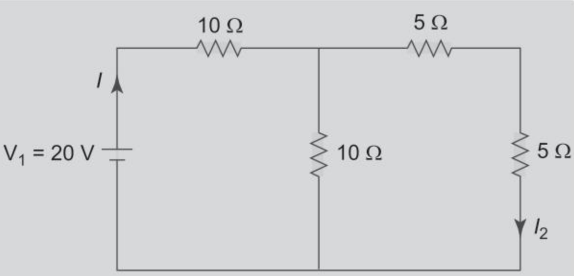


Fig. 6.184

Solution Reciprocity theorem states that in any passive linear bilateral single source network interchanging the positions of `ideal voltage source and an ammeter does not change the ammeter reading (current) and interchanging the positions of current source and voltmeter does not change voltmeter reading (voltmeter)

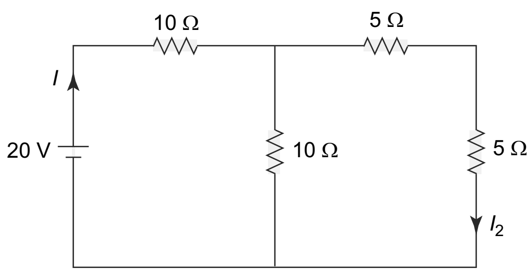


Fig. 6.185

Verifying theorem for the above ckt

$$I = \frac{20}{10+5} = \frac{4}{3} \qquad \therefore\ I_2 = \frac{2}{3}$$

(Current divider rule)

Interchanging the voltage source,

$$I = \frac{20}{15} = \frac{4}{3} \quad \Rightarrow \quad I_1 = \frac{2}{3}$$

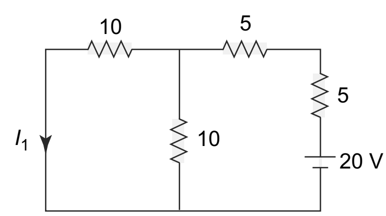


Fig. 6.186

⇒

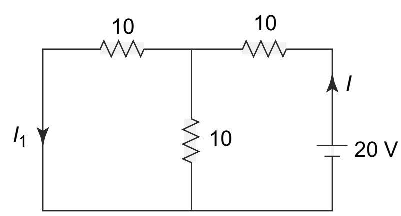


Fig. 6.187

∴ The ratio of excitation to response when only one excitation is applied is constant when positions of excitation and response are interchanged. Hence reciprocity theorem is verified.

Example 6.69 *Verify reciprocity theorem for the voltage V and current I in the network shown in Fig. 6.188.*

[JNTU Jan 2010]

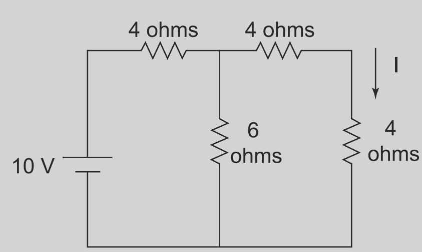


Fig. 6.188

Solution

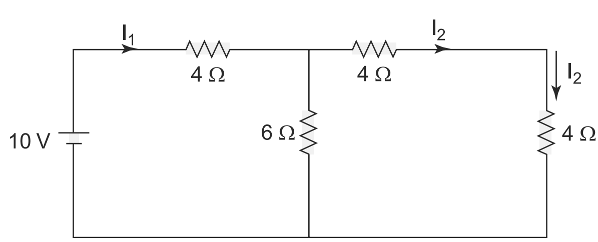


Fig. 6.189

$$\therefore \quad R_{eq} = [(4+4)\|6]+4$$

$$\therefore \quad R_{eq} = \frac{8\times 6}{14}+4$$

$$R_{eq} = \frac{48}{14}+4$$

$$R_{eq} = \frac{24}{7}+4$$

$$R_{eq} = \frac{52}{7}\,\Omega$$

$$I_1 = \frac{10\times 7}{52} = \frac{70}{52} = \frac{35}{76}$$

$$= 1.346\,\text{A}$$

$$\therefore \quad I_2 = 1.346\times\frac{6}{6+4+4}$$

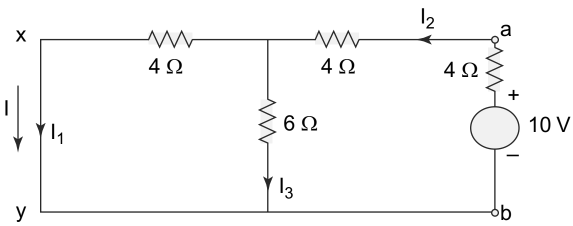


Fig. 6.190

$$I_2 = 1.346\times\frac{6}{14}$$

$$= 0.576\,\text{A}$$

$$I_2 = I = 0.576\text{ A}$$

$$R_{eq} = (4\|6) + 4 + 4$$

$$\therefore \quad R_{eq} = \frac{24}{10}+8$$

$$\therefore \quad R_{eq} = 2.4 + 8 = 10.4\ \Omega$$

$$I_2 = \frac{10}{10.4} = 0.961\,\text{A}$$

$$\therefore \quad I_1 = 0.961\times\frac{6}{6+4}$$

$$= 0.961\times\frac{3}{5}$$

$$I_1 = 0.576\text{ A}$$

$$\therefore \quad I_1 = I = 0.576\text{ A}$$

6.5.2 Reciprocity Theorem (ac Excitation) *[JNTU Jan 2009, Nov 2011]*

In a linear bilateral single source network, if a single voltage (current) source in one branch 'a' of the network produces a current (voltage) in branch 'b', then if the voltage (current) source is shifted to branch 'b' will produce a current (voltage) in branch 'a'. The ratio of excitation and response is same in both the cases. This theorem is valid for networks comprising of linear, bilateral, passive elements energised by a single voltage or current source.

The above theorem can be verified by a simple example.

Example 6.70 *Verify the reciprocity theorem for the network shown in Fig. 6.191.*

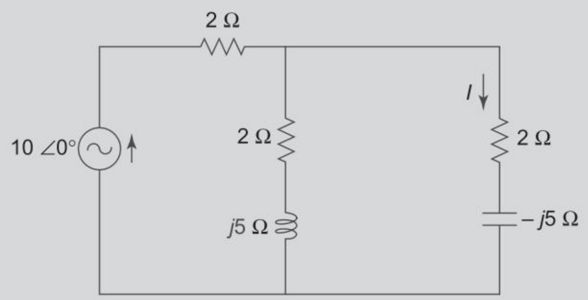


Fig. 6.191

Solution Total impedance in the circuit = $2+[(2+j5) \parallel (2-j5)] = 9.25\ \Omega$

The current drawn by the circuit

$$I_T = \frac{10\angle 0^\circ}{9.25} = 1.08\angle 0^\circ \text{ A}$$

The current in the $(2 - j5)\ \Omega$ branch

$$I = I_T \times \frac{2+j5}{2+j5+2-j5}$$

$$= 1.08\angle 0^\circ \times \frac{2+j5}{4} = 1.45\angle 68.2^\circ$$

Applying the reciprocity theorem, by interchanging the source and response, we get

Total Impedance in the circuit = $[2 \parallel (2 + j5)] + 2 - j5$

$$= \frac{2(2+j5)}{2+2+j5} + 2 - j5$$

$$= 5.8\angle -50.1^\circ\ \Omega$$

Total current drawn by the circuit $= \dfrac{10\angle 0^\circ}{5.8\angle -50.1^\circ}$

$$= 1.72\angle 50.1^\circ \text{ A}$$

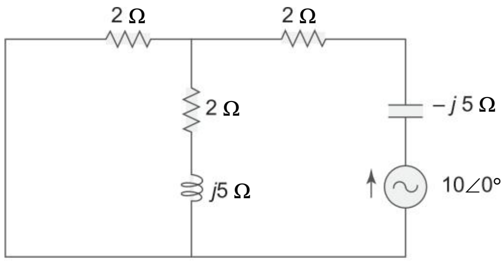


Fig. 6.192

The current in the 2 Ω branch is $= 1.72\angle 50.1^\circ \times \dfrac{2+j5}{4+j5}$

$$= 1.45\angle 67^\circ \text{ A}$$

If we compare the results in both cases, the ratio of input to response is same.

Example 6.71 *Verify reciprocity theorem for the voltage source and the current I in the circuit shown in Fig. 6.193.*

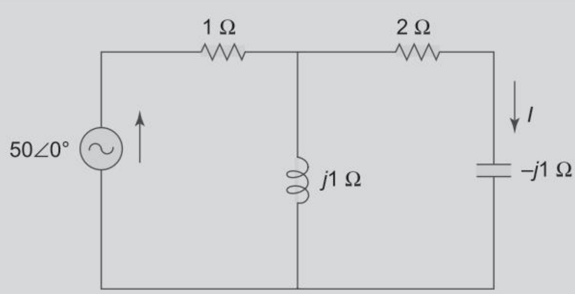


Fig. 6.193

Solution Total impedance in the circuit $Z_T = [1 + [(j1) \parallel (2 - j1)]]$

$$Z_T = 1.81\angle 33.69^\circ\ \Omega$$

Total current drawn by the circuit

$$I_T = \frac{5\angle 0^\circ}{1.81\angle 33.69^\circ}$$

$$= 2.76\angle -33.69^\circ \text{ A}$$

The current in the (2 − j1) Ω branch

$$I = I_T \times \frac{j1}{2 - j1 + j1}$$

$$= 2.76\angle -33.69^\circ \times \frac{1\angle 90^\circ}{2}$$

$$= 1.38\angle 56.31^\circ \text{ A}$$

Applying the reciprocity theorem, by interchanging the source and response, we get
Total impedance in the circuit $Z_T = [1 \| (j1) + (2 - j1)]$

$$Z_T = 2.55\angle{-11.30^\circ}\ \Omega$$

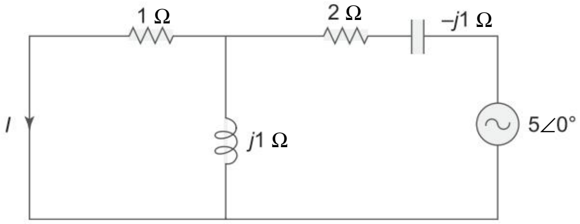


Fig. 6.194

Total current drawn by the circuit

$$I_T = \frac{5\angle{0^\circ}}{2.55\angle{-11.30^\circ}}$$

$$= 1.96\angle{11.30^\circ}\ \text{A}$$

The current I in $1\ \Omega$ branch

$$I = \frac{1.96\angle{11.30^\circ} \times 1\angle{90^\circ}}{1.414\angle{45^\circ}}$$

$$= 1.38\angle{56.36^\circ}\ \text{A}$$

The voltage to current ratio is same in both the circuits.

Example 6.72 *In a single current source circuit shown in Fig. 6.195, find the voltage V. Verify the reciprocity theorem for the circuit.*

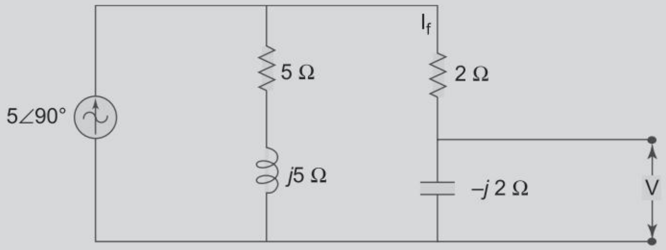


Fig. 6.195

Solution The voltage across $(-j2)\ \Omega$ impedance

$$V = I(-j2) \text{ volts}$$

where the current passing through $(-j2)\ \Omega$ is

$$I = 5\angle{90^\circ} \times \frac{5 + j5}{5 + j5 + 2 - j2}$$

$$= 4.65\angle{111.8^\circ}\ \text{A}$$

$\therefore$ The voltage $V = 4.65\angle 111.8^\circ \times 2\angle -90^\circ$

$= 9.24\angle 21.8^\circ$ Volts

Applying reciprocity theorem by interchanging source and response as shown in the circuit of Fig. 6.196.

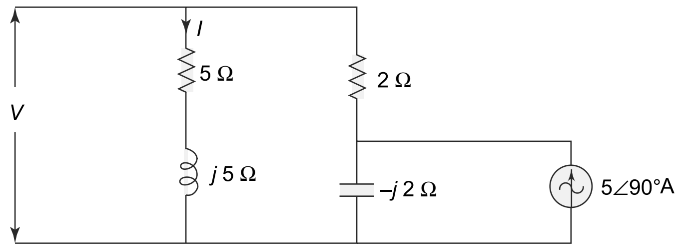


Fig. 6.196

The current passing through (5 + *j*5) Ω impedance

$$I = 5\angle 90^\circ \times \frac{-j2}{7 + j5 - j2}$$

$$I = 1.31\angle -23.2^\circ \text{ A}$$

The voltage across (5 + *j*5) Ω impedance

$$V = (5 + j5) \times 1.31\angle -23.2^\circ$$

$$= 9.25\angle 21.8^\circ \text{ voltage}$$

The response to excitation ratio is same in both the circuits.

6.6 MILLMAN'S THEOREM

6.6.1 Millman's Theorem (dc Excitation) *[JNTU June 2009, Nov 2011]*

Millman's theorem states that in any network, if the voltage sources V_1, V_2, … V_n in series with internal resistances R_1, R_2, … R_n, respectively, are in parallel, then these sources may be replaced by a single voltage source V' in series with R' as shown in Fig. 6.197.

where $$V' = \frac{V_1G_1 + V_2G_2 + \cdots V_nG_n}{G_1 + G_2 + \ldots + G_n}$$

Here G_n is the conductance of the *n*th branch,

and $$R' = \frac{1}{G_1 + G_2 + \cdots + G_n}$$

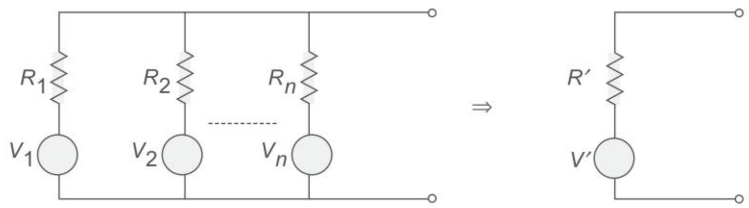


Fig. 6.197

A similar theorem can be stated for n current sources having internal conductances which can be replaced by a single current source I' in parallel with an equivalent conductance.

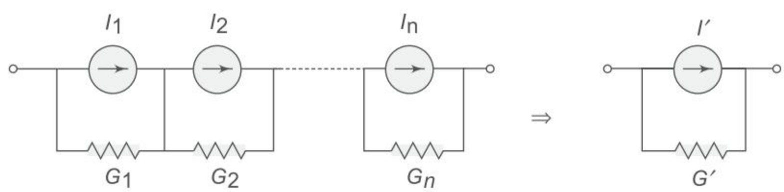


Fig. 6.198

where $$I' = \frac{I_1R_1 + I_2R_2 + \cdots I_nR_n}{R_1 + R_2 + \cdots + R_n}$$

and $$G' = \frac{1}{R_1 + R_2 + \cdots + R_n}$$

Example 6.73 *Calculate the current I shown in Fig. 6.199 using Millman's Theorem.*

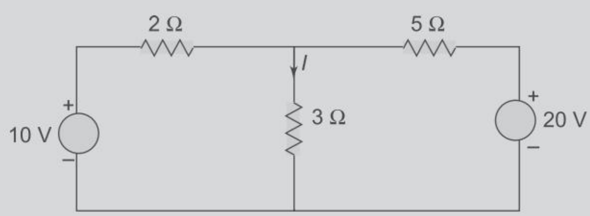


Fig. 6.199

Solution According to Millman's theorem, the two voltage sources can be replaced by a single voltage source in series with resistance as shown in Fig. 6.200.

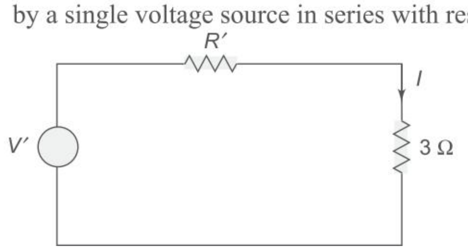


Fig. 6.200

we have $$V' = \frac{V_1G_1 + V_2G_2}{G_1 + G_2}$$

$$= \frac{[10(1/2) + 20(1/5)]}{1/2 + 1/5} = 12.86 \text{ V}$$

and $R' = \dfrac{1}{G_1 + G_2} = \dfrac{1}{1/2 + 1/5} = 1.43\ \Omega$

Therefore, the current passing through the 3 Ω resistor is

$$I = \frac{12.86}{3 + 1.43} = 2.9\ \text{A}$$

Example 6.74 *Find the current I_L. Use Millman's theorem.*
[JNTU May/June 2004]

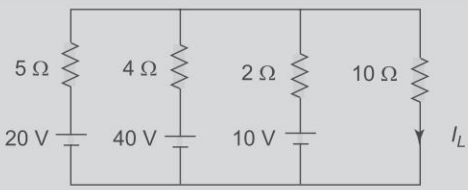


Fig. 6.201

Solution From Millman's theorem,

$$V' = \frac{V_1 G_1 + V_2 G_2 + \ldots + V_n G_n}{G_1 + G_2 + \ldots + G_n}$$

$$R' = \frac{1}{G_1 + G_2 + \ldots + G_n}$$

$$\therefore\ V' = \frac{20 \times \frac{1}{5} + 40 \times \frac{1}{4} + \left(-10 \times \frac{1}{2}\right)}{\frac{1}{5} + \frac{1}{4} + \frac{1}{2}} = 9.47736$$

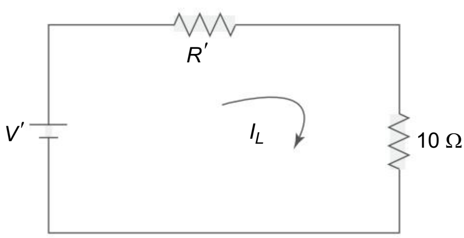


Fig. 6.202

$$R' = \frac{1}{\frac{1}{5} + \frac{1}{4} + \frac{1}{2}} = 1.0526$$

$$I_L = \frac{9.4736}{1.0526 + 10} = 0.857\text{A}$$

6.6.2 Millman's Theorem (ac Excitation) *[JNTU June 2009, Jan 2010]*

Millman's Theorem states that in any network, if the voltage sources V_1, V_2, . . .V_n in series with internal impedances Z_1, Z_2, . . . Z_n, respectively, are in parallel, then these sources may be replaced by single voltage source V' in series with an impedance Z' as shown in Fig. 6.203.

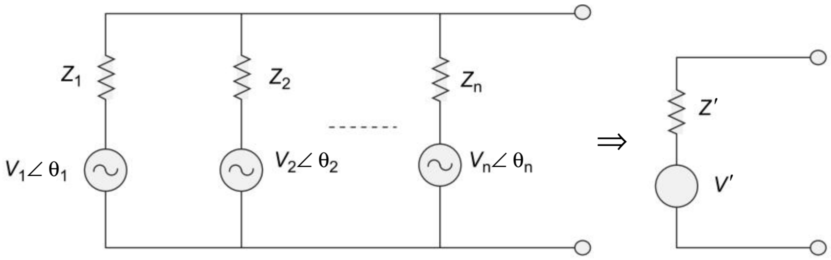


Fig. 6.203

where $V' = \dfrac{\sum_{i=1}^{n} V_i Y_i}{\sum_{i=1}^{n} Y_i}$ and $Z' = \dfrac{1}{\sum_{i=1}^{n} Y_i}$

A similar theorem can be stated for ***n*** current sources having internal admittances which can be replaced by a current source ***I***[1] in parallel with an equivalent admittance.

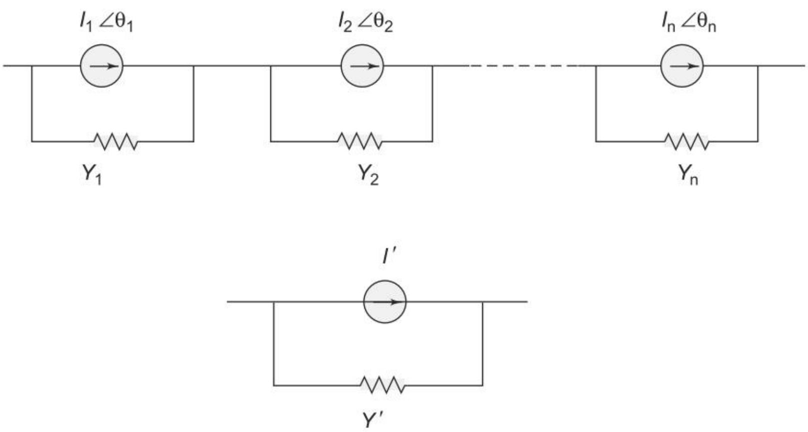


Fig. 6.204

where $I' = \dfrac{\sum_{i=1}^{n} I_i Z_i}{\sum_{i=1}^{n} Z_i}$ and $Y' = \dfrac{1}{\sum_{i=1}^{n} Z_i}$

$$Z_i = \frac{1}{Y_i}$$

Millman's theorem is very convenient for determining the voltage across a set of parallel branches, where there are enough voltage sources present to preclude solution via regular series – parallel reduction method. It doesn't require the use of simultaneous equations. However, it is limited in that it only applied to circuits which can be redrawn to fit this form. It can not be used to solve an unbalanced bridge circuit.

Example 6.75 *Calculate the current flowing in 2 Ω resistance shown in Fig. 6.205.*

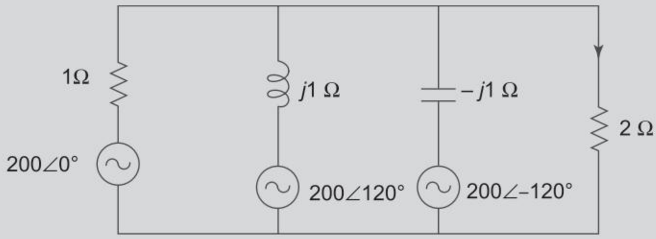


Fig. 6.205

Solution The above circuit can be redrawn as shown in Fig. 6.206.

Fig. 6.206

∴ From Millman's theorem, the equivalent impedance is given by

$$Z' = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{1 - \frac{1}{j1} + \frac{1}{j1}} = 1\,\Omega$$

$$V' = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y}$$

$$= 200 + 200\angle{-120^\circ}(-j1) + 200\angle{120^\circ}(j1)$$

$$= 200 - 200\angle{-30^\circ} + 200\angle{210^\circ}$$

$$= -146.5\angle{0^\circ}\text{ V}$$

The current in 2 Ω resistance

$$I = \frac{V^1}{Z^1 + 2} = \frac{-146.5\angle{0^\circ}}{3}$$

$$= -48.67\angle{0^\circ}\text{ A}$$

Example 6.76 *Use Millman's theorem to find the current in the load Z_L in the circuit shown in Fig. 6.207.*

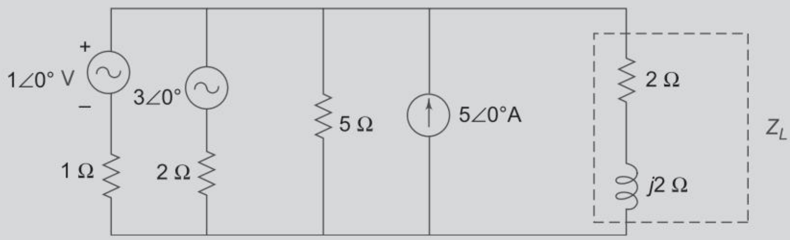


Fig. 6.207

Solution First converting the current source $5\angle 0°$ A in parallel with 5 Ω resistance is connected into the voltage source in series with resistance.

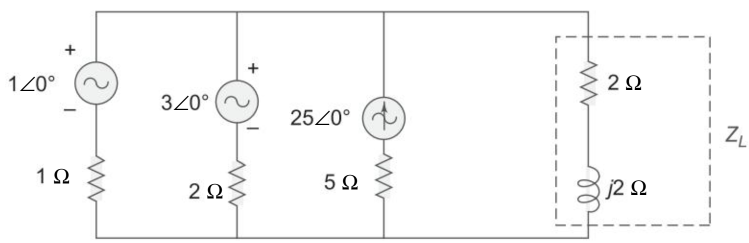


Fig. 6.208

The above circuit can be redrawn as shown in Fig. 6.209.

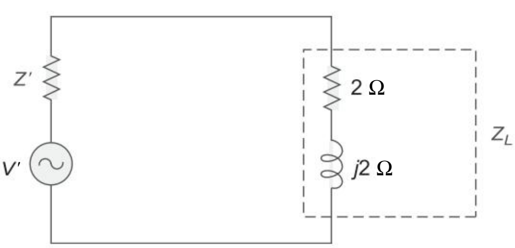


Fig. 6.209

∴ From Millman's theorem, the equivalent impedance is given be

$$Z' = \frac{1}{Y_1 + Y_2 + Y_3}$$

$$= \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{5}} = 0.59\ \Omega$$

Voltage source $$V' = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y^1}$$

where $$Y' = \frac{1}{Z^1} = \frac{1}{0.59} = 1.695\ \mho$$

$$\therefore \quad V' = \frac{1\angle 0^\circ \times 1 + 3\angle 0^\circ \times \frac{1}{2} + 25\angle 0^\circ \times \frac{1}{5}}{1.695}$$

$$V' = \frac{7.5\angle 0^\circ}{1.695}$$

$$= 4.42\angle 0^\circ \text{ Volts}$$

The load current $$I_L = \frac{V'}{Z' + Z_L}$$

$$= \frac{4.42\angle 0^\circ}{0.59 + 2 + j2 \ \Omega}$$

$$I_L = \frac{4.42\angle 0^\circ}{3.27\angle 37.67^\circ} = 1.35\angle -37.67^\circ \text{ A}$$

6.7 COMPENSATION THEOREM

6.7.1 Compensation Theorem (dc Excitation)

The *compensation theorem* states that any element in the linear, bilateral network, may be replaced by a voltage source of magnitude equal to the current passing through the element multiplied by the value of the element, provided the currents and voltages in other parts of the circuit remain unaltered. Consider the circuit shown in Fig. 6.210(a). The element R can be replaced by voltage source V, which is equal to the current I passing through R multiplied by R as shown in Fig. 6.210(b).

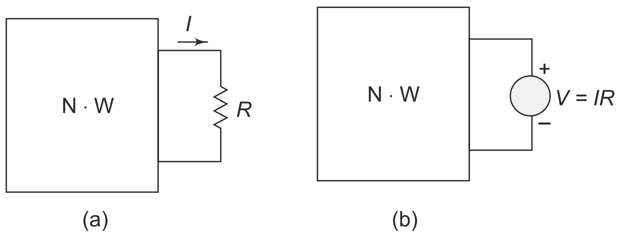


Fig. 6.210

This theorem is useful in finding the changes in current or voltage when the value of resistance is changed in the circuit. Consider the network containing a resistance R shown in Fig. 6.211(a). A small change in resistance R, that is $(R + \Delta R)$, as shown in Fig. 6.211(b) causes a change in current in all branches. This current increment in other branches is equal to the current produced by the voltage source of voltage $I.\ \Delta R$ which is placed in series with altered resistance as shown in Fig. 6.211(c).

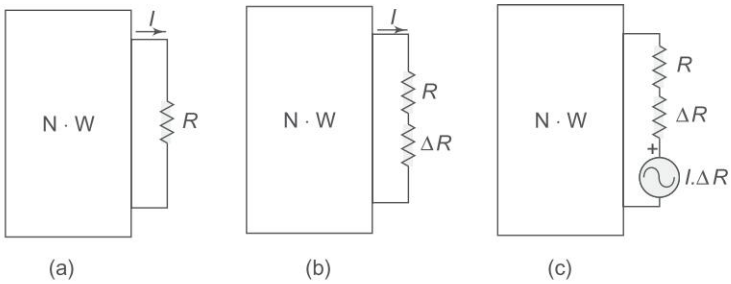


Fig. 6.211

Example 6.77 *Determine the current flowing in the ammeter having 1 Ω internal resistance connected in series with a 3 Ω resistor as shown in Fig. 6.212.*

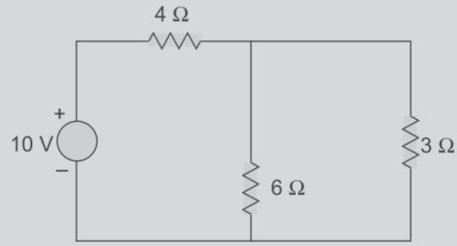


Fig. 6.212

Solution The current flowing through the 3 Ω branch is $I_3 = 1.11$ A. If we connect the ammeter having 1 Ω resistance to the 3 Ω branch, there is a change in resistance. The changes in currents in other branches then result as if a voltage source of voltage $I_3\,\Delta R = 1.11 \times 1 = 1.11$ V is inserted in the 3 Ω branch as shown in Fig. 6.213.

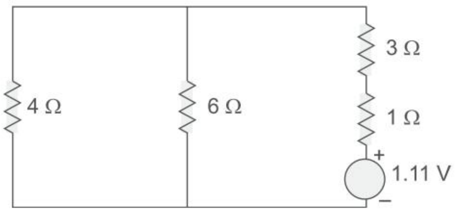


Fig. 6.213

Current due to this 1.11 V source is calculated as follows.

Current $I'_3 = 0.17$ A

This current is opposite to the current I_3 in the 3 Ω branch.

Hence the ammeter reading $= (1.11 - 0.17) = 0.94$ A.

Example 6.78 *Using the compensation theorem, determine the ammeter reading where it is connected to the 6 Ω resistor as shown in Fig. 6.214. The internal resistance of the ammeter is 2 Ω.*

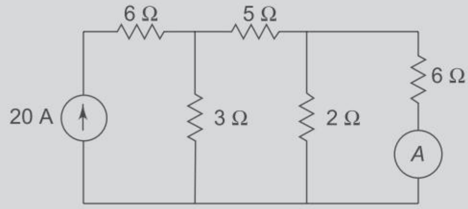


Fig. 6.214

Solution The current flowing through the 5 Ω branch

$$I_5 = 20 \times \frac{3}{3+6.5} = 6.315 \text{ A}$$

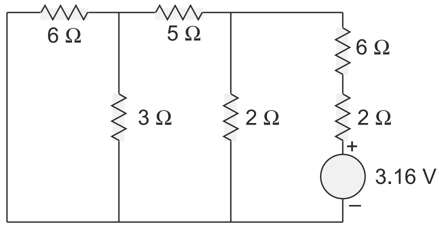


Fig. 6.215

So the current in the 6 Ω branch

$$I_6 = 6.315 \times \frac{2}{6+2} = 1.58 \text{ A}$$

If we connect the ammeter having 2 Ω internal resistance to the 6 Ω branch, there is a change in resistance. The changes in currents in other branches results if a voltage source of voltage $I_6 \Delta R = 1.58 \times 2 = 3.16$ V is inserted in the 6 Ω branch as shown in Fig. 6.215.

The current due to this 3.16 V source is calculated.

The total impedance in the circuit

$$R_T = \{[(6 \parallel 3) + 5] \parallel [2]\} + \{6 + 2\}$$
$$= 9.56\ \Omega$$

The current due to 3.16 V source

$$I_6' = \frac{3.16}{9.56} = 0.33 \text{ A}$$

This current is opposite to the current I_6 in the 6 Ω branch.

Hence, the ammeter reading = (1.58 − 0.33)

= 1.25 A

6.7.2 Compensation Theorem (ac Excitation)

The compensation theorem states that any impedance having voltage across its terminal in the linear, bilateral network, may be replaced by a voltage source of zero internal impedance equal to the current passing through the impedance multiplied by the value of the impedance, provided the currents and voltages in other part of the network remain unaltered.

Let a branch of a network contain impedance Z_1 and Z_2. If the current in this branch is I, the voltage drop across Z_1 is IZ_1 with polarity as shown in Fig. 6.216(a). Fig. 6.216(b) shows the compensation source $V_C = IZ_1$ which replace Z_1. However V_C must have polarity as shown in Fig. 6.216(b). If any chance which should effect I occurs in the network then the compensation source must be changed accordingly. The compensation is often referred as substitution theorem. This theorem is of use, when it is required to evaluate the changes in magnitudes of currents and voltages in the different branches of a network, due to a small change in the impedance of one of the branches.

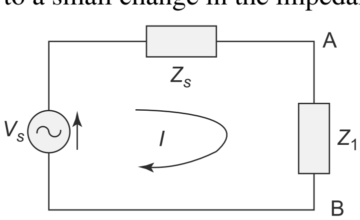


Fig. 6.216(a)

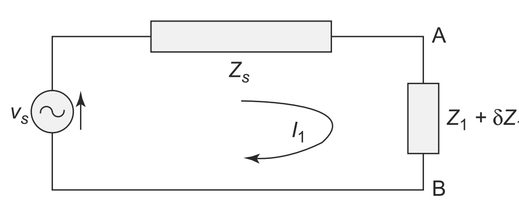


Fig. 6.216(b)

Consider a network shown in Fig. 6.216(a).

The current in the circuit is $I = \dfrac{V_s}{Z_s + Z_1}$

Let the impedance of branch AB change from Z_1 to $(Z_1 + \delta Z_1)$. Let I_1 be the new current.

The current $I_1 = \dfrac{V_s}{Z_s + Z_1 + \delta Z_1}$

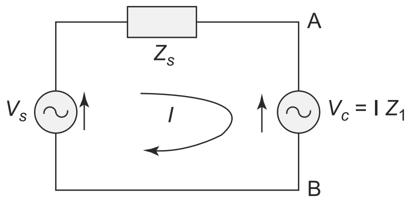


Fig. 6.216(c)

The impedance Z_1 of the network shown in Fig. 6.216(a) may be replaced by a voltage source, V_C. By substitution theorem $V_C = IZ_1$ with polarity as shown in Fig. 6.216(c).

Similarly, the network shown in Fig. 8.216(b) can be replaced by the network shown in Fig. 6.216(d).

Let δI_1 denote the small change in current, due to the small change in the impedance value by δZ_1.

$$\therefore \qquad \delta I_1 = I - I_1 = \frac{V_s}{Z_s + Z_1} - \frac{V_s}{Z_s + Z_1 + \delta Z_1}$$

$$= \frac{V_s \cdot \delta Z_1}{(Z_s + Z_1)(Z_s + Z_1 + \delta Z_1)}$$

$$\delta I_1 = I \cdot \frac{\delta Z_1}{Z_s + Z_1 + \delta Z_1}$$

$$\text{since} \qquad I = \frac{V_s}{Z_s + Z_1}$$

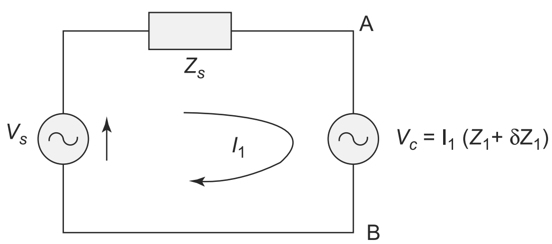


Fig. 6.216(d)

The network for which the above relationship holds good is as shown in Fig. 8.216(e).

By compensation theorem the small change in the magnitude of current due to a small change in a branch impedance is given by

$$\delta I_1 = \frac{I\,\delta Z_1}{Z_s + Z_1 + \delta Z_1}$$

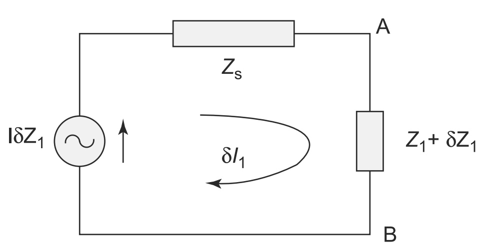


Fig. 6.216(e)

Therefore, the original voltage source should be set equal to zero and a new voltage source $I\delta Z_1$ must be introduced with correct polarity.

Example 6.79 *For the circuit shown in Fig. 6.217 find the change in the current by using compensation theorem when the reactance has changed to j5 Ω.*

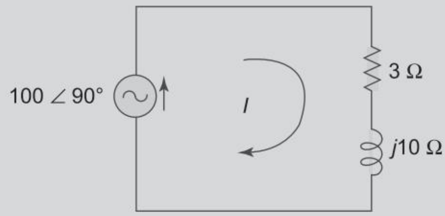


Fig. 6.217

Solution The current in the circuit shown is $I = \dfrac{100\angle 90^\circ}{3+j10} = 9.58\angle 16.7^\circ$ A

The inductive reactance is changed from $j10\ \Omega$ to $j5\ \Omega$

$\therefore$ Change in impedance $\delta Z = j5\,\Omega$.

The new circuit is shown in Fig. 8.82.

The change in current due to change in impedance

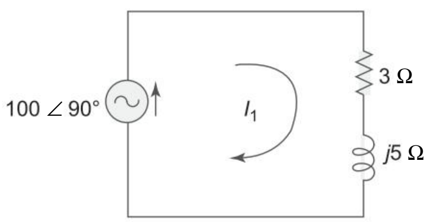


Fig. 6.218

$$\delta I = \frac{I \cdot \delta Z}{Z_{\text{total}}} = \frac{9.58\angle 16.7^\circ \times j5}{3+j5}$$

$$\delta I = 8.22\angle 47.7^\circ \text{ A.}$$

Example 6.80 *In the network shown in Fig. 6.219, the 2 Ω resistor is changed to 4 Ω. Determine the resulting change in current Δ I through the load impedance, using compensation theorem.*

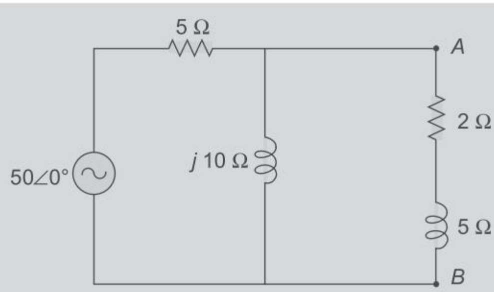


Fig. 6.219

Solution The thevenin's equivalent circuit of a given network with open circuit terminals shown in Fig. 6.220.

Open circuit voltage across terminals AB

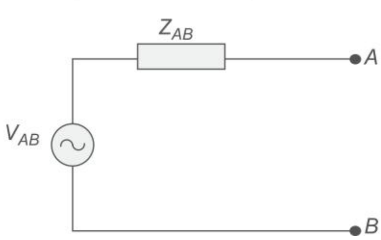


Fig. 6.220

$$V_{AB} = (j10)\frac{50\angle 0^\circ}{5+j10} = \frac{10\angle 90^\circ\ 50\angle 0^\circ}{11.18\angle 63.43^\circ}$$

$$V_{AB} = 44.72\angle 26.57^\circ \text{ Volts}$$

The impedance seen into the terminals AB

$$Z_{AB} = 5 \parallel (j10) = \frac{5(10\underline{|90^\circ})}{5 + j10}$$

$$Z_{AB} = 4.472\underline{|26.57^\circ} = (4 + j2)\ \Omega$$

The Thevenin's equivalent circuit is shown in Fig. 6.221

Current $$I = \frac{44.72\underline{|26.57^\circ}}{6 + j7} = 4.86\underline{|-22.93^\circ}\ \text{A}$$

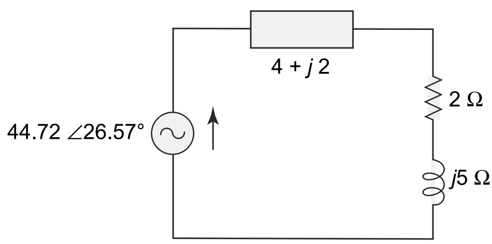


Fig. 6.221

when the impedance of 2Ω is change to 4Ω, the Thevenin's equivalent circuit with new load impedance is shown in Fig. 6.222.

Change in impedance $\delta Z = 4 - 2 = 2\ \Omega$

Total impedance $= (8 + j7)\ \Omega$ $= 10.63\underline{|41.18^\circ}\ \Omega$

By compensation theorem, we have
Change in current

$$\delta I = \frac{I \cdot \delta Z}{Z_{\text{total}}} = \frac{4.86\underline{|-22.93^\circ} \times 2}{10.63\underline{|41.18^\circ}}$$

$$\therefore \quad \delta I = 0.914\underline{|-64.11^\circ}\ \text{A}$$

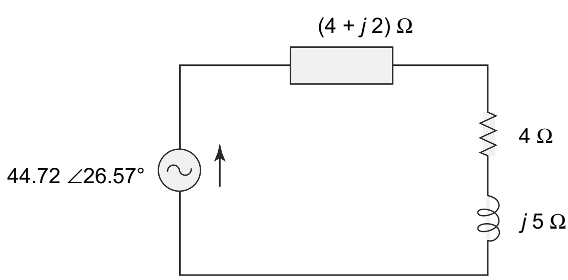


Fig. 6.222

6.8 TELLEGEN'S THEOREM

6.8.1 Tellegen's Theorem (dc Exitation) *[JNTU Jan 2010, Nov 2011]*

Tellegen's theorem is valid for any lumped network which may be linear or non linear, passive or active, time-varying or time-invarient. This theorem states that in an arbitrary lumped network, the algebraic sum of the powers in all branches at any instant is zero. All branch currents and voltages in that network must satisfy Kirchhoff's laws. Otherwise, in a given network, the algebraic sum of the powers delivered by all sources is equal to the algebraic sum of the powers absorbed by all elements. This theorem is based on Kirchhoff's two laws, but not on the type of circuit elements.

Consider two networks N_1 and N_2, having the same graph with different types of elements between the corresponding nodes.

Then $$\sum_{K=1}^{b} v_{1K} i_{2K} = 0$$

and $$\sum_{K=1}^{b} v_{2K} i_{1K} = 0$$

To verify Tellegen's theorem, consider two circuits having same graphs as shown in Fig. 6.223.

In Fig. 6.223(a)

$$i_1 = i_2 = 2\text{ A};\ i_3 = 2\text{ A}$$

and $$v_1 = -2\text{ V},\ v_2 = -8\text{ V},\ v_3 = 10\text{ V}$$

In Fig. 6.223(b),

$$i_1^1 = i_2^1 = 4\text{ A};\ i_3^1 = 4\text{ A}$$

and $$v_1^1 = -20\text{ V};\ v_2^1 = 0\text{ V};\ v_3^1 = 20\text{ V}$$

Now $$\sum_{K=1}^{3} v_K i_K^1 = v_1 i_1^1 + v_2 i_2^1 + v_3 i_3^1$$

$$= (-2)(4) + (-8)(4) + (10)(4) = 0$$

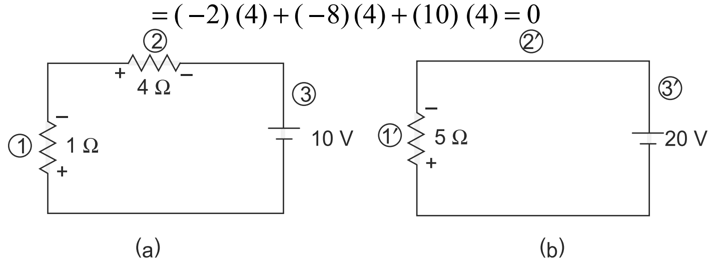


Fig. 6.223

and $$\sum_{K=1}^{3} v_K^1 i_K = v_1^1 i_1 + v_2^1 i_2 + v_3^1 i_3$$

$$= (-20)(2) + (0)(2) + (20)(2) = 0$$

Similarly,

$$\sum_{K=1}^{3} v_K i_K = v_1 i_1 + v_2 i_2 + v_3 i_3$$

$$= (-2)(2) + (-8)(2) + (10)(2) = 0$$

and $$\sum_{K=1}^{3} v_K^1 i_K^1 = (-20)(4) + (0)(4) + (20)(4) = 0$$

This verifies Tellegen's theorem.

Example 6.81 *Verify Tellegen's theorem in the network shown in the Fig. 6.224.* *[JNTU May/June 2006]*

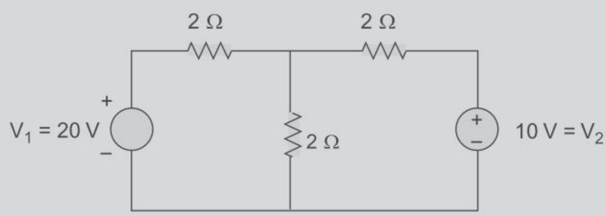


Fig. 6.224

Solution Tellegen's theorem states that in any arbitrary lumped network, the algebraic sum of the powers in all the branches at any instant is zero and all the branch currents and voltages must satisfy Kirchhoff's law.

Verifying Tellegen's theorem for the above circuit.

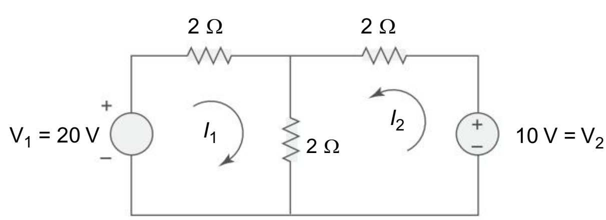


Fig. 6.225

There are 5 elements in the above circuit.

Applying mesh equations.

$$4i_1 + 2i_2 = 20$$

$$\Rightarrow 2i_1 + i_2 = 10$$

$$2i_1 + 4i_2 = 10 \quad (1)$$

$$i_1 + 2i_2 = 5 \quad (2)$$

Solving (1) and (2)

$$i_1 = 5, i_2 = 0$$

$\sum_{k=1}^{5} V_k I_k$ for this circuit is

$$-100 + 50 + 50 + (0)^2 (2) - (0)(10) = 0$$

Hence, verified.

Example 6.82 *Verify Tellegen's Theorem for the network shown in Fig. 6.226.*
[JNTU Jan 2010]

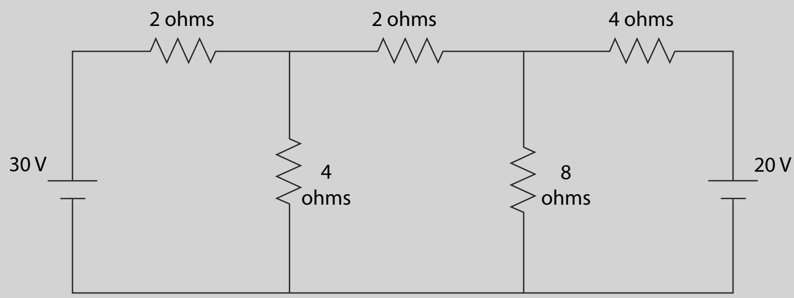


Fig. 6.226

Solution

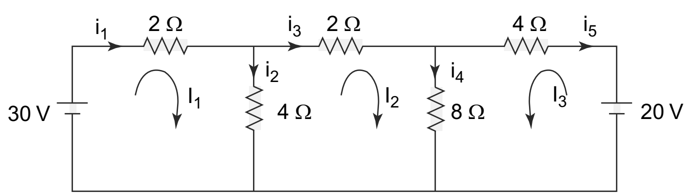


Fig. 6.227

$$30 = 6I_1 - 4I_2$$
$$0 = -4I_1 + 14I_2 + 8I_3$$
$$20 = 8I_2 + 12I_3$$

$$\Delta = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 14 & 8 \\ 0 & 8 & 12 \end{bmatrix} = 624 - 192 = 432$$

$$\Delta_1 = \begin{bmatrix} 30 & -4 & 0 \\ 0 & 14 & 8 \\ 20 & 8 & 12 \end{bmatrix} = 3120 - 640 = 2480$$

$$\Delta_2 = \begin{bmatrix} 6 & 30 & 0 \\ -4 & 0 & 8 \\ 0 & 20 & 12 \end{bmatrix} = -960 + 1440 = 480$$

$$\Delta_3 = \begin{bmatrix} 6 & -4 & 30 \\ -4 & 14 & 0 \\ 0 & 8 & 20 \end{bmatrix} = 1680 - 320 - 960 = 400$$

$\therefore \qquad I_1 = 5.74 \text{ amp}, I_2 = 1.11 \text{ amp}, I_3 = 0.93 \text{ amp}$

$$\therefore \quad i_1 = I_1 = 5.74 \text{ amp}$$
$$i_2 = I_1 - I_2 = 4.63 \text{ amp}$$
$$i_3 = I_2 = 1.11 \text{ amp}$$
$$i_4 = I_2 + I_3 = 2.04 \text{ amp}$$
$$i_5 = -I_3 = -0.93 \text{ amp}$$

$\therefore$ Total power supplied $= (30 \times 5.74) + (20 \times 0.93)$ watt

$\therefore$ Total power dissipated $= (5.74^2 \times 2) + (4.63^2 \times 4) + (1.11^2 \times 2) + (2.04^2 \times 8) + (0.93^2 \times 4)$ watt

$\therefore$ Total power in all branches = Power supplied – Power dissipated
$= 0$

$\therefore$ Tellegani's theorem is verified.

6.8.2 Tellegen's Theorem (ac Excitation)

[JNTU Jan 2010, Nov 2011]

The Tellegen's theorem states that the summation of instantaneous power or summation of complex power of sinusoidal sources in a network is zero. The network power may be linear or non linear, passive or active and time invariant or variant.

The Tellegen's theorem is used to design filters in signal processing applications. The assumptions for electrical circuits are generalized for dynamic systems obeying the laws of irreversible thermodynamics. Topology and structure of reaction networks can be analyzed using the Tellegen's theorem. Another application of Tellegen's theorem is to determine stability and optimality of complex process systems.

Consider a network shown in Fig. 6.228.

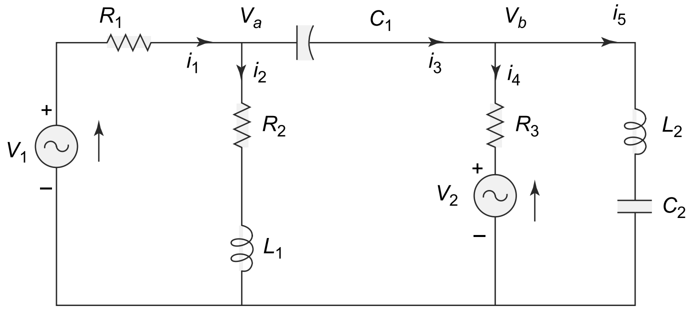


Fig. 6.228

Applying Kirchoff's current law at nodes, we get

At node a,

$$i_1 - i_2 - i_3 = 0$$

At node b,

$$i_3 - i_4 - i_5 = 0$$

Total instantaneous powers delivered by the voltage sources

$$= V_1 i_1 - V_2 i_4 \quad (1)$$

Total instantaneous power absorbed by all the passive element

$$= i_1 (-V_1 - V_a) + V_a i_2 + (V_a - V_b) i_3 + (V_b + V_2) i_4 + V_b i_5 \quad (2)$$

∴ Summation of all instantaneous powers = (1) + (2)

$$V_1 i_1 - V_2 i_4 - V_1 i_1 - V_a i_1 + V_a i_2 + V_a i_3 - V_b i_3 + V_b i_4 + V_2 i_4 + V_b i_5$$

$$V_1 (i_1 - i_1) + V_2(-i_4 + i_4) + V_a(-i_1 + i_2 + i_2) + V_b(-i_3 + i_4 + i_5) = 0$$

Since the algebraic sum of the currents at each of the nodes is zero.

Example 6.83 *Verify Tellegen's theorem for the network shown in Fig. 6.229*

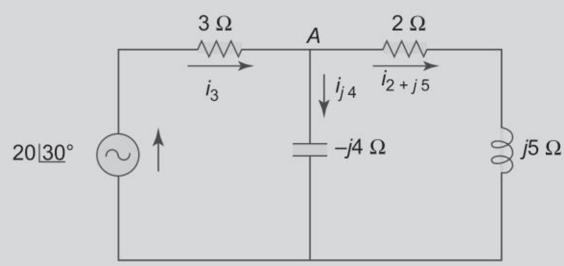


Fig. 6.229

Solution Assume that the voltage at node a is V_A. By applying modal analysis, we have

$$\frac{20\angle 30° - V_A}{3} = \frac{V_A}{-j4} + \frac{V_A}{2+j5}$$

$$V_A\left[\frac{1}{3} + \frac{1}{2+j5} - \frac{1}{j4}\right] = \frac{20\angle 30°}{3}$$

$$V_A = \frac{6.67\angle 30°}{0.41\angle 11.09°} = 16.27\angle 18.91° \text{ V}$$

Current in 3 Ω branch

$$I_3 = \frac{20\angle 30° - V_A}{3} = \frac{20\angle 30° - 16.27\angle 18.91°}{3}$$

$$I_3 = 1.7\angle 67.8° \text{ A}$$

Current in $-j4$ Ω branch

$$I_{-j4} = \frac{16.27\angle 18.91°}{4\angle -90°} = 4.067\angle 108.91° \text{ A}$$

Current in (2 + j5) Ω branch

$$I_{2+j5} = \frac{16.27\angle 18.91^\circ}{5.385\angle 68.198^\circ} = 3.021\angle -49.3^\circ \text{ A}$$

Power in 3 Ω branch $P_3 = V_3 \times I_3$

where voltage $V_3 = I_3 \times 3 = 1.7\angle 67.8^\circ \times 3 = 5.1\angle 67.8^\circ$ V

$$\therefore \quad P_3 = 5.1\angle 67.8^\circ \times 1.7\angle 67.8^\circ = 8.67\angle 135.6^\circ \text{ W}$$

$$= -6.2 + j6.066 \text{ W}$$

Power in ($-j$4) Ω branch

$$P_{-j4} = 16.27\angle 18.91^\circ \times 4.067\angle 108.91^\circ$$

$$= 66.161\angle 127.82^\circ$$

$$= -40.6 + j52.26 \text{ W}$$

Power in (2 + j5) Ω branch

$$P_{2+j5} = 16.27\angle 18.91^\circ \times 3.02\angle -49.3^\circ$$

$$= 49.135\angle -30.39^\circ$$

$$= 42.2 - j24.86 \text{ W}$$

Power delivered by the source

$$P_{20} = 20\angle 30^\circ \times 1.7\angle 67.8^\circ$$

$$= 34\angle 97.8^\circ = -4.61 + j33.68 \text{ W}$$

Sum of the powers in the circuit is zero, which proves Tellegen's theorem.

Practice Problems

6.1 Find the Thevenin's and Norton's equivalents for the circuit shown in Fig. 6.230 with respect to terminals ***ab***.

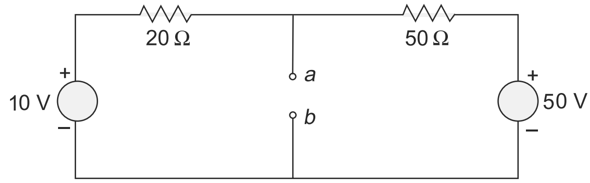


Fig. 6.230

6.2 Determine the Thevenin and Norton's equivalent circuits with respect to terminals ***AB*** for the circuit shown in Fig. 6.231.

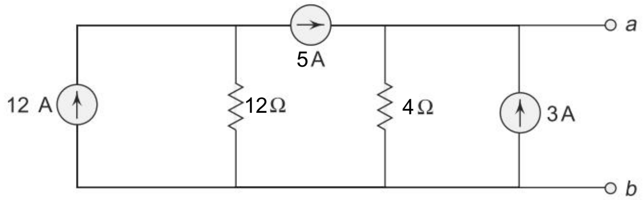


Fig. 6.231

6.3 By using source transformation or any other technique, replace the circuit shown in Fig. 6.232 between terminals ***AB*** with the voltage source in series with a single resistor.

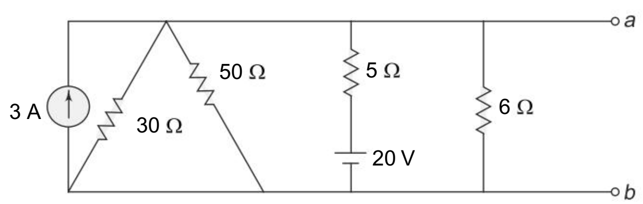


Fig. 6.232

6.4 For the circuit shown in Fig. 6.233, what will be the value of R_L to get the maximum power? What is the maximum power delivered to the load? What is the maximum voltage across the load? What is the maximum current in it?

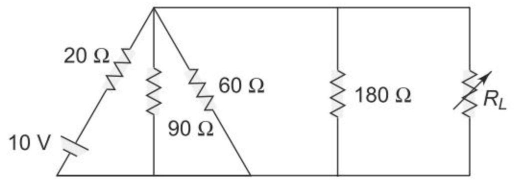


Fig. 6.233

6.5 For the circuit shown in Fig. 6.234 determine the value of R_L to get the maximum power. Also find the maximum power transferred to the load.

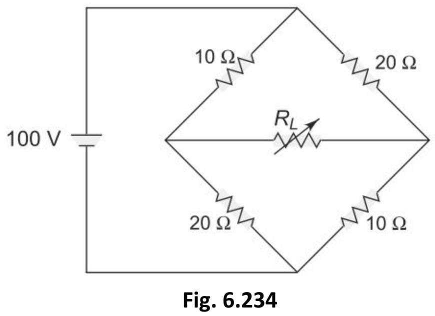


Fig. 6.234

6.6 Determine the current passing through 2 Ω resistor by using Thevenin's theorem in the circuit shown in Fig. 6.235.

6.7 Find Thevenin's equivalent circuit for the network shown in Fig. 6.236 and hence find the current passing through the 10 Ω resistor.

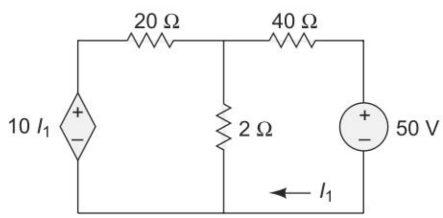


Fig. 6.235

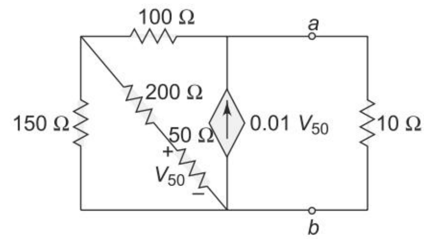


Fig. 6.236

6.8 Obtain Norton's equivalent circuit of the network shown in Fig. 6.237

6.9 For the circuit shown in Fig. 6.238, determine the value of current $\boldsymbol{I_x}$ in the impedance $\boldsymbol{Z} = 4 + j5$ between nodes ***a*** and ***b***.

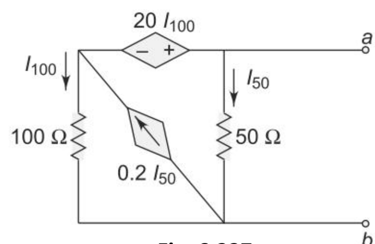


Fig. 6.237

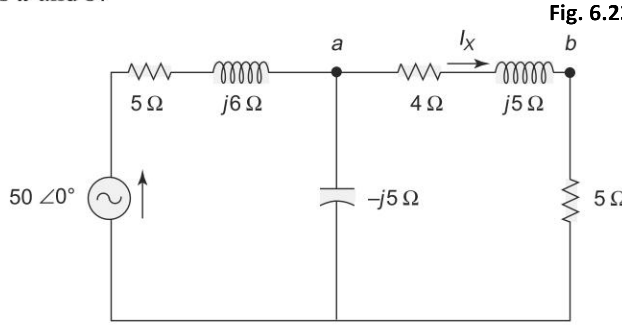


Fig. 6.238

6.10 Determine (i) the equivalent voltage generator and (ii) the equivalent current generator which may be used to represent the given network in Fig. 6.239 at the terminals ***AB***.

6.11 For the circuit shown in Fig. 6.240, find the value of ***Z*** that will receive the maximum power. Also determine this power.

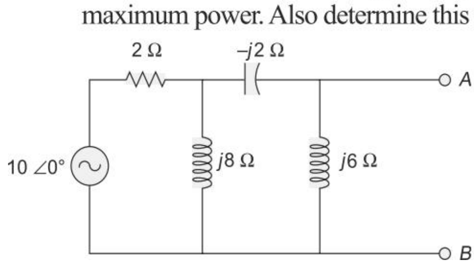


Fig. 6.239

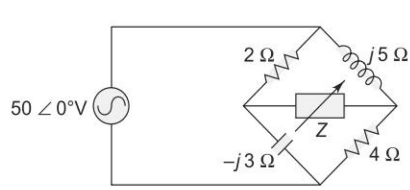


Fig. 6.240

6.12 Determine the voltage V_{ab} and V_{bc} in the network shown in Fig. 6.241 by Thevenin's theorem, where source voltage

$$e(t) = \sqrt{2} \times 100 \cos(314t + 45°).$$

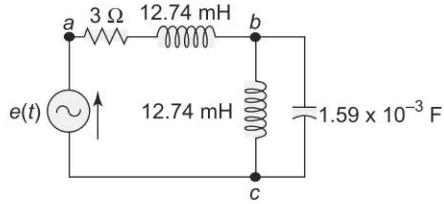


Fig. 6.241

6.13 Find the current in the 15 Ω resistor in the network shown in Fig. 6.242 by Thevenin's theorem.

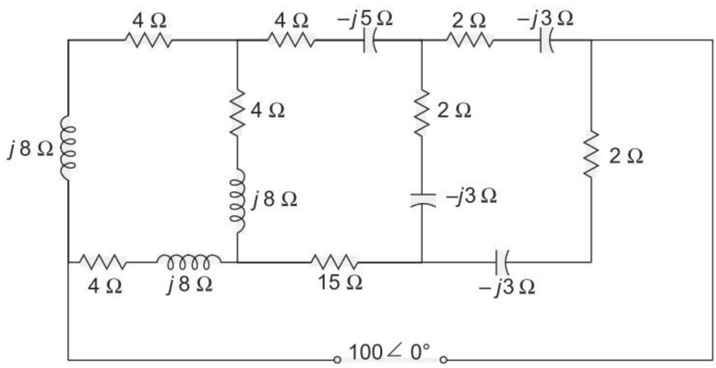


Fig. 6.242

6.14 Determine the power output of the voltage source by loop analysis for the network shown in Fig. 6.243. Also determine the power extended in the resistors.

6.15 In the circuit shown in Fig. 6.244, determine the power in the impedence $(2+j5)\,\Omega$ connected between ***A*** and ***B*** using Norton's theorem.

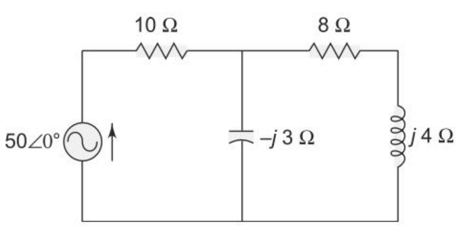


Fig. 6.243

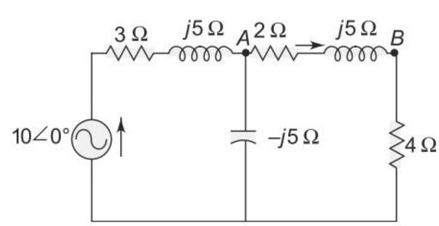


Fig. 6.244

6.16 Convert the active network shown in Fig. 6.245 by a single voltage source in series with an impedance, and also by a single current source in parallel with the impedance.

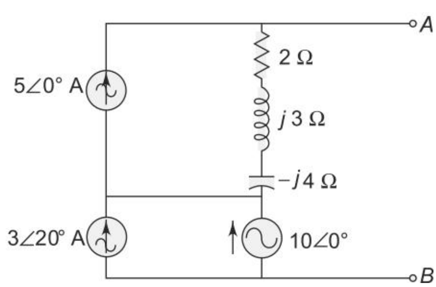


Fig. 6.245

6.17 Determine the power out of the source in the circuit shown in Fig. 6.246 by Thevenin's theorem and verify the results by using Norton's theorem.

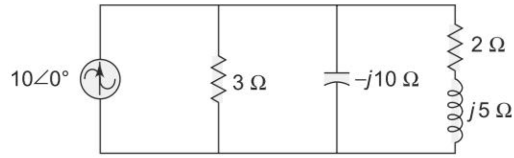


Fig. 6.246

6.18 Use Thevenin's theorem to find the current through the $(5 + j4)\ \Omega$ impedance in Fig. 6.247. Verify the results using Norton's theorem.

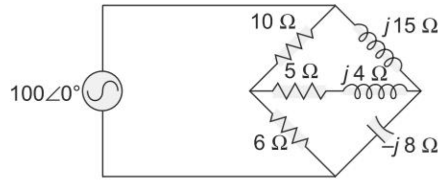


Fig. 6.247

6.19 Determine Thevenin's and Norton's equivalent circuits across terminals ***AB***, in Fig. 6.248.

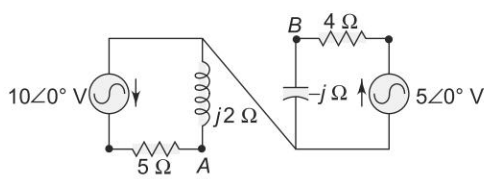


Fig. 6.248

6.20 Determine Norton's and Thevenin's equivalent circuits for the circuit shown in Fig. 6.249.

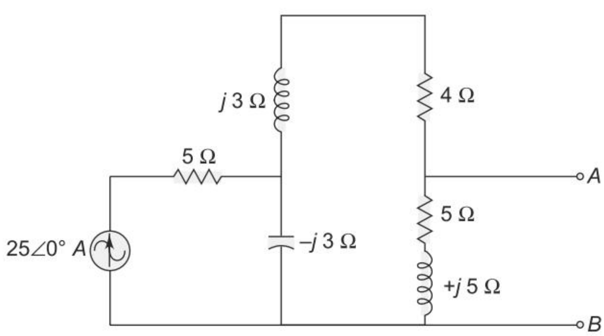


Fig. 6.249

6.21 Determine the maximum power delivered to the load in the circuit shown in Fig. 6.250.

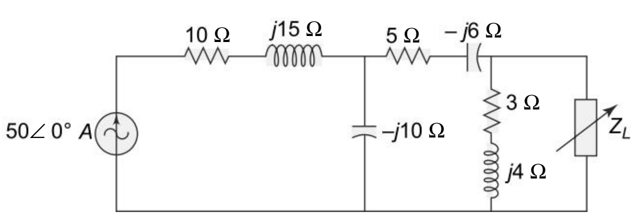


Fig. 6.250

6.22 For the circuit shown in Fig. 6.251, find the voltage across the dependent source branch by using Norton's theorem.

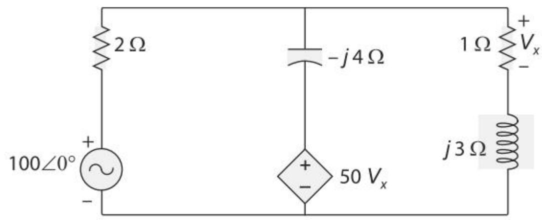


Fig. 6.251

6.23 Find Thevenin's equivalent for the network shown in Fig. 6.252.

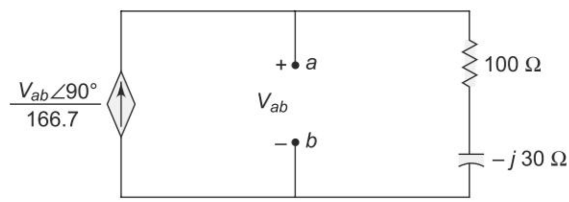


Fig. 6.252

6.24 For the circuit shown in Fig. 6.253, obtain the voltage across 500 kΩ resistor.

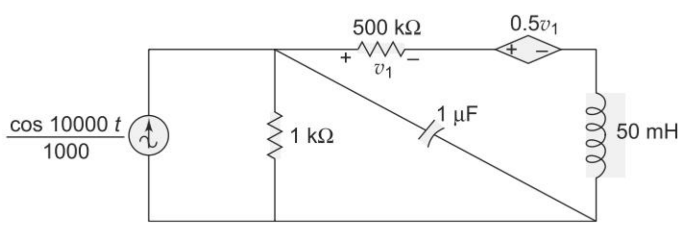


Fig. 6.253

6.25 For the circuit shown in Fig. 6.254, obtain the Thevenin's equivalent circuit at terminals ***ab***.

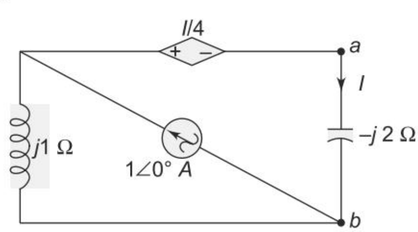


Fig. 6.254

6.26 Find the current *I* in the circuit shown in Fig. 6.255 by using the superposition theorem.

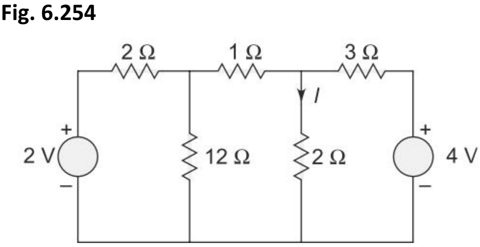


Fig. 6.255

6.27 Determine the current I in the circuit shown in Fig. 6.256 by using the superposition theorem.

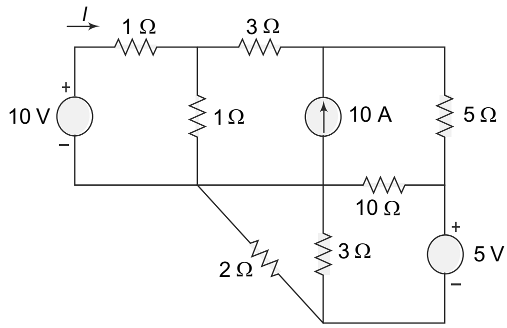


Fig. 6.256

6.28 Calculate the new current in the circuit shown in Fig. 6.257 when the resistor R_3 is increased by 30%.

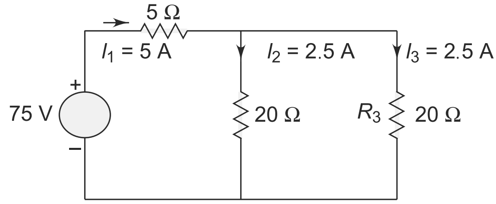


Fig. 6.257

6.29 The circuit shown in Fig. 6.258 consists of dependent source Use the superposition theorem to find the current I in the 3 Ω resistor.

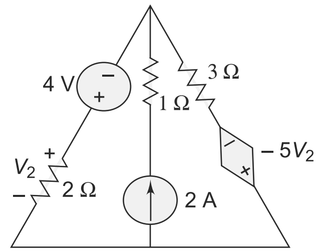


Fig. 6.258

6.30 Obtain the current passing through 2 Ω resistor in the circuit shown in Fig. 6.259 by using the superposition theorem.

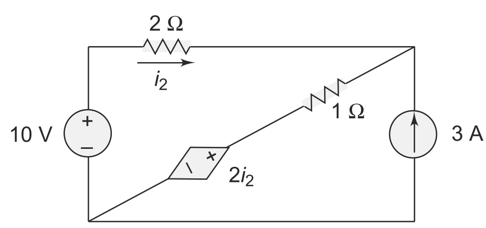


Fig. 6.259

6.31 Determine the value of source currents by superposition theorem for the circuit shown in Fig. 6.260 and verify the results by using nodal analysis.

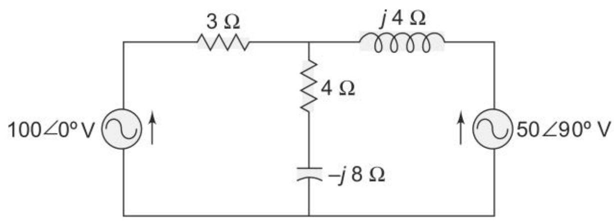


Fig. 6.260

6.32 For the circuit shown in Fig. 6.261, find the current in each resistor using the superposition theorem.

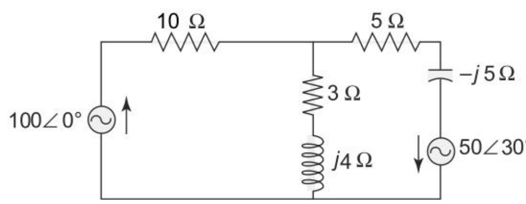


Fig. 6.261

Objective Type Questions

6.1 Reduce the circuit shown in Fig. 6.262 to its Thevenin equivalent circuit as viewed from terminal ***A*** and ***B***.

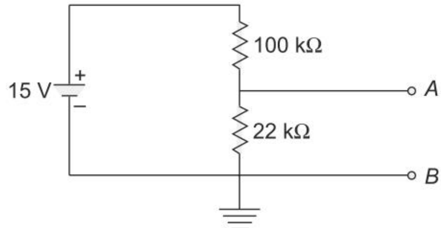


Fig. 6.262

(a) The circuit consists of 15 V battery in series with 100 kΩ
(b) The circuit consists of 15 V battery in series with 22 kΩ
(c) The circuit consists of 15 V battery in series with parallel combination of 100 kV and 22 kΩ
(d) None of the above

6.2 Norton's equivalent circuit consists of
(a) voltage source in parallel with resistance
(b) voltage source in series with resistance
(c) current source in series with resistance
(d) current source in parallel with resistance

6.3 Maximum power is transferred when load impedance is

(a) equal to source resistance
(b) equal to half of the source resistance
(c) equal to zero
(d) none of the above

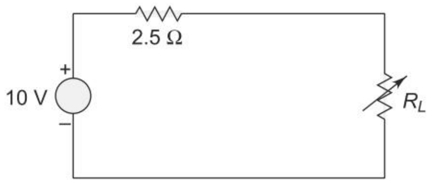


Fig. 6.263

6.4 In the circuit shown in Fig. 6.263, what is the maximum power transferred to the load

(a) 5 W (b) 2.5 W
(c) 10 W (d) 25 W

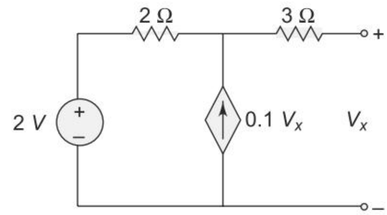


Fig. 6.264

6.5 Thevenins voltage in the circuit shown in Fig. 6.264 is

(a) 3 V (b) 2.5 V
(c) 2 V (d) 0.1 V

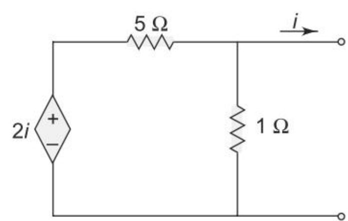


Fig. 6.265

6.6 Norton's current in the circuit shown in Fig. 6.265 is

(a) $\frac{2i}{5}$ (b) zero
(c) infinite (d) None

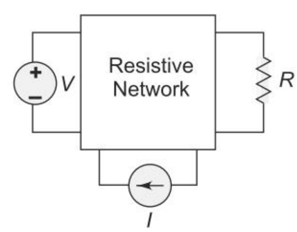


Fig. 6.266

6.7 A dc circuit shown in Fig. 6.266 has a voltage ***V***, a current source ***I*** and several resistors. A particular resistor ***R*** dissipates a power of 4 W when ***V*** alone is active. The same resistor dissipates a power of 9 W when ***I*** alone is active. The power dissipated by ***R*** when both sources are active will be

(a) 1 W (b) 5 W
(c) 13 W (d) 25 W

6.8 While applying Thevenin's theorem, the Thevenin's voltage is equal to

(a) short circuit voltage at the terminals
(b) open circuit voltage at the terminals
(c) voltage of the source
(d) total voltage available in the circuit

6.9 Thevenin impedance Z_{Th} is found

(a) by short-circuiting the given two terminals
(b) between any two open terminals
(c) by removing voltage sources along with the internal resistances
(d) between same open terminals as for V_{Th}

6.10 Thevenin impedance of the circuit at its terminals ***A*** and ***B*** in Fig. 6.267 is

(a) 5 H
(b) 2 Ω
(c) 1.4 Ω
(d) 7 H

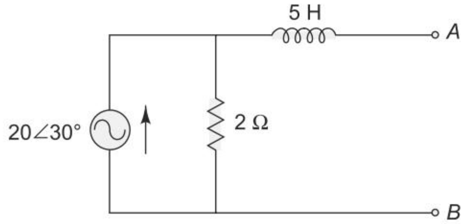


Fig. 6.267

6.11 Norton's equivalent form in any complex impedance circuit consists of

(a) an equivalent current source in parallel with an equivalent resistance.
(b) an equivalent voltage source in series with an equivalent conductance.
(c) an equivalent current source in parallel with an equivalent impedance.
(d) None of the above.

6.12 The maximum power transfer theorem can be applied

(a) only to dc circuits (b) only to ac circuits
(c) to both dc and ac circuits (d) neither of the two

6.13 Maximum power transfer occurs at a

(a) 100% efficiency (b) 50% efficiency
(c) 25% efficiency (d) 75% efficiency

6.14 In the circuit shown in Fig. 6.268, the power supplied by the 10 V source is

(a) 6.6 W
(b) 21.7 W
(c) 30 W
(d) 36.7 W

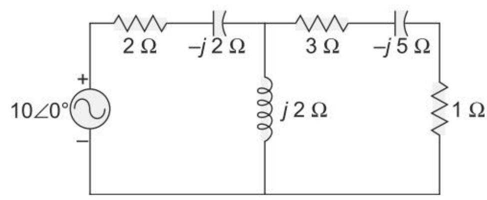


Fig. 6.268

6.15 A source has an emf of 10 V and an impedance of 500 + ***j***100 Ω. The amount of maximum power transferred to the load will be

(a) 0.5 mW (b) 0.05 mW
(c) 0.05 W (d) 0.5 W

6.16 For the circuit shown in Fig. 6.269, find the voltage across the dependent source.

(a) 8 ∠0°
(b) 4 ∠0°
(c) 4 ∠90°
(d) 8 ∠−90°

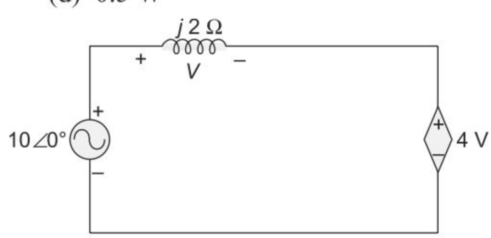


Fig. 6.269

6.17 Superposition theorem is valid only for

(a) linear circuits (b) non-linear circuits
(c) both linear and non-linear (d) neither of the two

6.18 Superposition theorem is not valid for

(a) voltage responses (b) current responses
(c) power responses (d) all the three

6.19 Determine the current I in the circuit shown in Fig. 6.270. It is

(a) 2.5 A (b) 1 A
(c) 3.5 A (d) 4.5 A

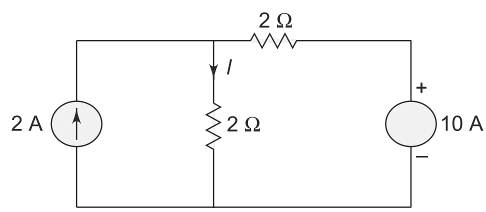


Fig. 6.270

6.20 The reciprocity theorem is applicable to

(a) linear networks only
(b) bilateral networks only
(c) linear/bilateral networks
(d) neither of the two

6.21 Compensation theorem is applicable to

(a) linear networks only
(b) non-linear networks only
(c) linear and non-linear networks
(d) neither of the two

6.22 When the superposition theorem is applied to any circuit, the dependent voltage source in that circuit is always

(a) opened (b) shorted (c) active (d) none of the above

6.23 Superposition theorem is not applicable to networks containing.

(a) non-linear elements
(b) dependent voltage sources
(c) dependent current sources
(d) transformers

6.24 The superposition theorem is valid

(a) only for ac circuits
(b) only for dc circuits
(c) For both, ac and dc circuits
(d) neither of the two

6.25 When applying the superposition theorem to any circuit

(a) the voltage source is shorted, the current source is opened
(b) the voltage source is opened, the current source is shorted

(c) both are opened
(d) both are shorted

6.26 In a complex impedance circuit, the maximum power transfer occurs when the load impedance is equal to

(a) complex conjugate of source impedance
(b) source impedance
(c) source resistance
(d) none of the above

6.27 The Thevenin equivalent impedance of the circuit in Fig. 6.271 is

(a) $(1 + j5)\ \Omega$
(b) $(2.5 + j25)\ \Omega$
(c) $(6.25 + j6.25)\ \Omega$
(d) $(2.5 + j6.25)\ \Omega$

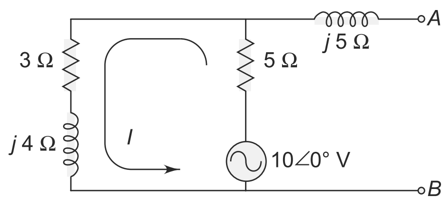


Fig. 6.271

Subject Code: R13212/R13 **Set No-1**

I B. Tech II Semester Regular Examinations August – 2014

ELECTRICAL CIRCUITS ANALYSIS-I

(Electrical And Electronics Engineering)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
There Questions should be answered from **Part-B**

****

PART-A

Q1. (i) What are the differences between dependent and independent sources?

Ans. (i) Refer Section 1.4.

(ii) Write the volt-ampere relations of *R, L, C* parameters.

Ans. (ii) Refer Sections 1.3.1, 1.3.2, 1.2.3.

(iii) Define the average and root mean square value of an alternating quantity.

Ans. (iii) Refer Sections 2.1.7 and 2.1.8.

(iv) Draw the impedance triangle of series *R-L* and *R-C* circuits.

Ans. (iv) Refer Section 2.5.5.

(v) Define the quality factor. What is the significance?

Ans. (v) Refer Sections 3.6 and 3.11.

(vi) Define reluctance and magnetic flux.

Ans. (vi) Refer Section 4.6.

(vii) List the properties of an incidence matrix.

Ans. (vii) Refer Section 5.1.6.

(viii) State the maximum power transfer theorem.

Ans. (viii) Refer Section 6.4.

PART-B

Q2. (a) Obtain the expressions for star-delta and delta-star equivalence of a resistive network.

Ans. (a) Refer Section 1.6.

Q2. (b) Find the value of resistance *R*, if the current is *I* = 11 A and source voltage is 66 V as shown in Figure 1.

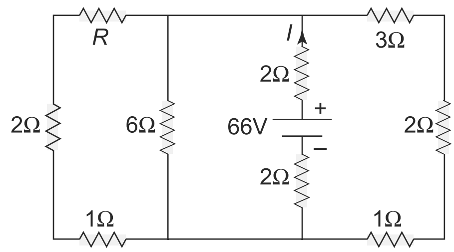


Fig. 1

Ans. (b)

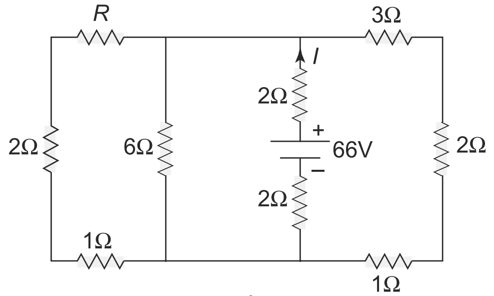


Fig. 2

Given source voltage = 66 V and

Current $I = 11$ A

From the above circuit, the resistances 3 Ω, 2 Ω, and 1 Ω are in series and also R Ω and 2 Ω, 1 Ω are in series. The equivalent circuit can be as follows:

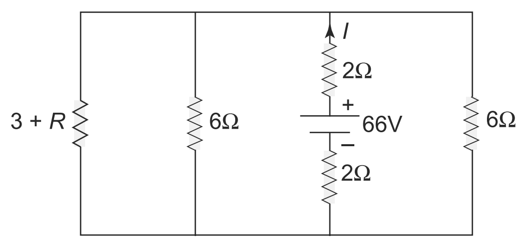


Fig. 3

The resistances 6 Ω, 6 Ω and the resistance (3 + R) are in parallel

The equivalent parallel resistance is

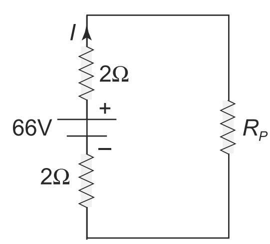


Fig. 4

$$R_p = \frac{{}^{3}\cancel{36}(3+R)}{{}_{3}\cancel{36}+\cancel{12}(3+R)}$$

$$R_P = \frac{3(3+R)}{6+R}$$

From Ohm's Law, we know that

$$V = IR$$

$$66 = I\,(R_p + 4)$$

Given $\qquad I = 11$ A

$$\overset{6}{\cancel{66}} = \cancel{11}\left(\frac{3(3+R)}{6+R}+4\right)$$

$$6(6 + R) = 9 + 3R + 4(6 + R)$$

$$36 + 6R = 33 + 7R$$

$$R = 3\Omega$$

Q2. (c) Use the nodal analysis to determine voltage at Node 1 and the power supplied by the dependent current source in the network shown in Figure 5.

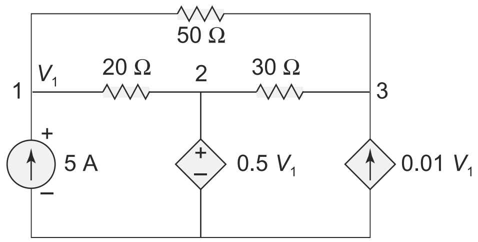


Fig. 5

Ans. (c)

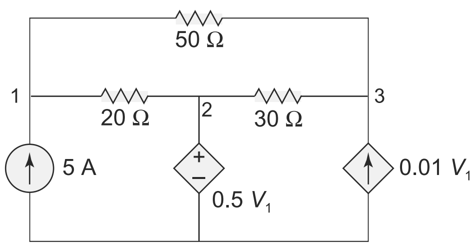


Fig. 6

Consider the node voltages at nodes 1, 2, 3 as V_1, V_2 and V_3

At Node 1,

$$-5 + \frac{V_1 - V_2}{20} + \frac{V_1 - V_3}{50} = 0$$

$$V_1\left(\frac{1}{20} + \frac{1}{50}\right) - \frac{V_2}{20} - \frac{V_3}{50} - 5 = 0 \quad (1)$$

At Node 3,

$$\frac{V_3 - V_2}{30} - 0.01V_1 + \frac{V_3 - V_1}{50} = 0 \quad (2)$$

At Node 2,

$$V_2 = 0.5\ V_1 \quad (3)$$

Put Eq. (3), $V_2 = 0.5\ V_1$ in Eq. (2)

$$\Rightarrow \frac{V_3 - 0.5V_1}{30} - 0.01V_1 + \frac{V_3 - V_1}{50} = 0$$

$$\Rightarrow -V_1\left(\frac{0.5}{30} + 0.01 + \frac{1}{50}\right) + V_3\left(\frac{1}{30} + \frac{1}{50}\right) = 0$$

$$\Rightarrow -0.046\ V_1 + V_3\ (0.05) = 0 \quad (4)$$

Put Eq. (3), $V_2 = 0.5\ V_1$ in Eq. (1)

$V_1\ (0.07) - 0.025\ V_1 - 0.02\ V_3 = 5$

$\Rightarrow V_1\ (0.045) - 0.02\ V_3 = 5$ (5)

By Solving equations (4) and (5), we get

$V_1 = 187.96$ and $V_3 = 172.93$

Power supplied by the dependent current source

$P = 0.01\ (187.96) \times 172.93$

$= 325.04$ W

Q3. (a) Explain the procedure to draw the locus diagram of *R*-*L* series circuit when *L* is varying.

Ans. (a) Refer Section 3.1.1.

Q3. (b) A coil of inductance 0.0805 H takes a current of 5A when connected in series with a 50 μF loss-free capacitor across a 240 V, 50 Hz supply. Calculate (i) resistance of the coil, (ii) power factor of the coil, and (iii) the overall power factor. Sketch the Phasor diagram. [7+9]

Ans. (b) Given inductance of coil (L) = 0.0805 H

Current (I) = 5A

Capacitance (C) = 50 μF

The source is of 240 V, 50 Hz. The equivalent circuit of inductance and capacitance connected in series with source is given as

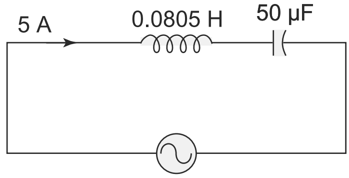


Fig. 7

$$Z = \frac{240}{5} + 48$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 48$$

$$X_L = 2\pi fL = 2\pi\ (50)\ (0.0805)$$

$$= 25.29$$

$$X_C = \frac{1}{2\pi(50)(50)\times 10^{-6}} = 63.6$$

$$R^2 + (X_L - X_C)^2 = (48)^2$$

$$R = \sqrt{(48)^2 - (25.29 - 63.6)^2} = 28.83\Omega$$

Power factor of coil = cos (ϕ_{coil})

$$\phi_{coil} = \tan^{-1}\left(\frac{X_L}{R}\right) = 41.2576°$$

Power factor of coil = cos (41.2576) = 0.75

Power factor of overall circuit = cos ϕ

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \phi = -53.036°$$

$$\cos\phi = 0.601$$

Power factor of overall circuit = 0.601

Q4. (a) Show that average power consumed by a pure inductor and a pure capacitor is zero.

Ans. (a) Refer Section 2.5.1.

Q4. (b) A coil of inductance L and resistance R in series with a capacitor is supplied at a constant voltage from a variable frequency source. If the frequency is to, ω_r find in terms of L, R and ω_r the values of those frequency at which the circuit current would be half as much as that at resonance. Hence or otherwise determine the bandwidth and selectivity of the circuit.

Ans. (b) Refer 3.4 of Chapter 3.

Q5. (a) Explain the procedure for obtaining fundamental tie-set matrix of a given network.

A5. (a) Refer Section 5.3.1.

Q5. (b) A ring has a mean diameter of 21 cm and cross-sectional area of 10 cm^2. The ring is made up of semicircular sections of cast iron and cast steel with each joint having reluctance equal to an air gap of 0.2 mm. Find the ampere turns required to produce a flux of 0.8 milli-WB. The relative permeability of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects. **[7+9]**

Ans. (b) Given mean diameter (D) = $\dfrac{Da+Db}{2}$ = 21 cm

$Da = Db = 21$ cm

We have $NI = \phi\, S$ = No. of required Ampere turns

Given flux (ϕ) = 0.8×10^{-3} Wb

We know that $S = \left(\dfrac{l}{\mu a}\right)_{\text{steel}} + \left(\dfrac{l}{\mu a}\right)_{\text{castiron}} + \left(\dfrac{l}{\mu a}\right)_{\text{airgap}}$

$$= \frac{1}{a}\left(\frac{\pi \times \frac{21}{2}}{\mu_0 \mu_s} + \frac{\pi \times \frac{21}{2}}{\mu_0 \mu_i} + \frac{0.4 \times 10^{-2}}{\mu_0}\right)$$

Given relative permeability of steel (μs) = 800

Relative permeability of iron (μi) = 166

Cross-sectional area (a) = 10 cm^2

Substituting all the values, we get

$$S = \frac{1}{10\mu_0}\left[\frac{\pi \times \frac{21}{2}}{800} + \frac{\pi \times \frac{21}{2}}{166} + \frac{0.4 \times 10^{-2}}{1}\right] / \text{cm}$$

$$= \frac{1}{10\mu_0} \times 0.243 \times 10^{-2} / cm = \frac{0.243 \times 10^{-2} / \text{cm}}{10 \times 4\pi \times 10^{-7}\,\text{wb/m}^2}$$

$$= \frac{0.243 \times \cancel{10^{-2}} / \cancel{10^{-2}}\,\text{cm}}{10 \times 4\pi \times 10^{-7}\,\text{wb/m}^2}$$

$\boxed{S = 19, 337.326}$

Ampere turns required $= NI = \phi_S = 0.8 \times 10^{-3} \times 19337.326$

$= \mathbf{15.47}$ Ampere turns

Q6. (a) Two identical coupled coils have an equivalent inductance of 80 mH when connected series aiding and 35 mH in series opposing. Find L_1, L_2, M and K.

Ans. (a) Let the inductances of coupled coils be L_1, L_2

Given both are identical $L_1 = L_2$

When connected in series, aiding equivalent inductance

$\Rightarrow L_1 + L_2 + 2M = 80$ mH

$\Rightarrow 2L_1 + 2M = 80$ mH $\quad (\because L_1 = L_2)$

$\Rightarrow L_1 + M = 40$ mH (1)

When connected in opposing equivalent inductance is

$L_1 + L_2 - 2M = 35$ mH

$\Rightarrow 2L_1 - 2M = 35$ mH

$\Rightarrow L_1 - M = 17.5$ mH (2)

(1) + (2) $\Rightarrow 2L_1 = 57.5$ mH $\Rightarrow L_1 = 28.75$ mH $= L_2$

Form (1), $M = 40$ mH $- L_1$

$= 40 - 28.75 = 11.25$ mH

We know that $M = K\sqrt{L_1 L_2}$

As $L_1 = L_2$ $\quad \Rightarrow M = K\sqrt{L_1^2} = KL_1$

$$\Rightarrow K = \frac{M}{L_1} = \frac{11.25}{28.75} = 0.391304.$$

$\boxed{K = 0.391304}$

Q6. (b) Draw the oriented graph of a network with fundamental cut-set matrix as shown below:

Twigs				Links		
1	2	3	4	5	6	7
1	0	0	0	–1	0	0
0	1	0	0	1	0	1
0	0	1	0	0	1	1
0	0	0	1	0	1	0

Also find number of cut-sets and draw them. **[7+9]**

Ans. (b) Given

	1	2	3	4	5	6	7
A	1	0	0	0	–1	0	0
B	0	1	0	0	1	0	1
C	0	0	1	0	0	1	1
D	0	0	0	1	0	1	0

The Graph for the fundamental cut-set matrix is

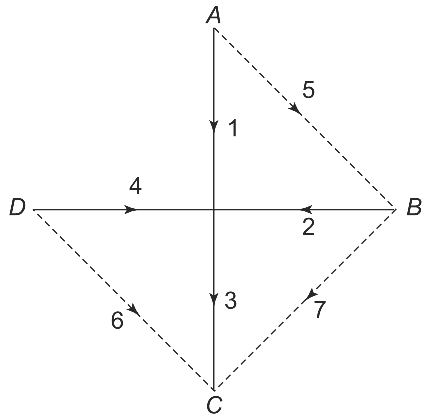


Fig. 8

The number of cut-sets are

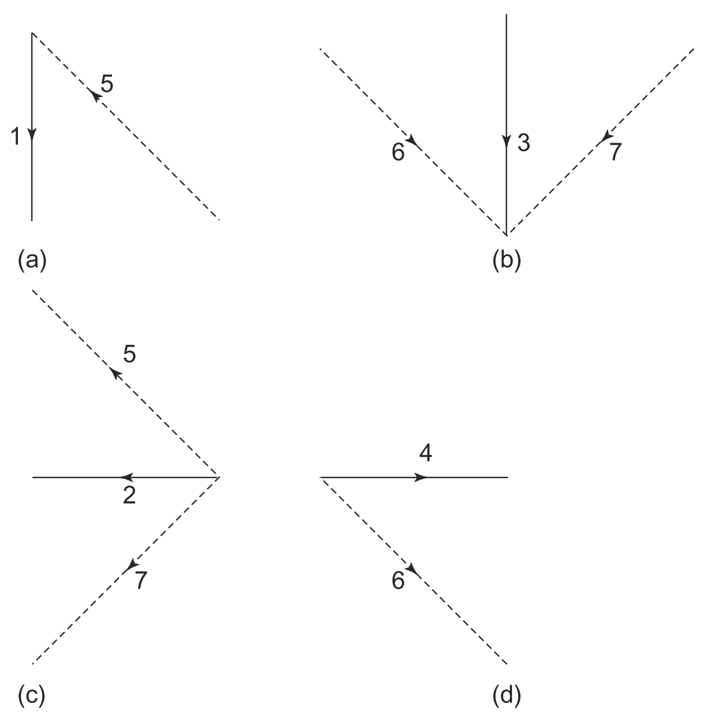


Fig. 9

Q7. (a) State and explain Norton's theorem.

Ans. (a) Refer Section 6.3.

Q7. (b) For the network shown in Figure 10, (i) determine the current through R = 10 ohms resistor using Thevenin's theorem, (ii) verify the result using Norton's theorem, and (iii) calculate the maximum power

transfer through *R* and find the value fo *R*. **[6+10]**

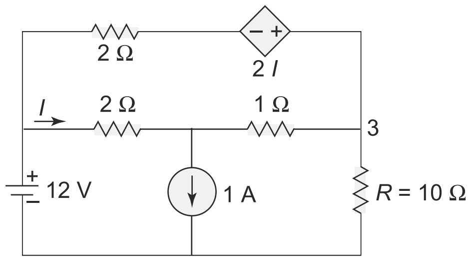


Fig. 10

Ans. (b) Given network is

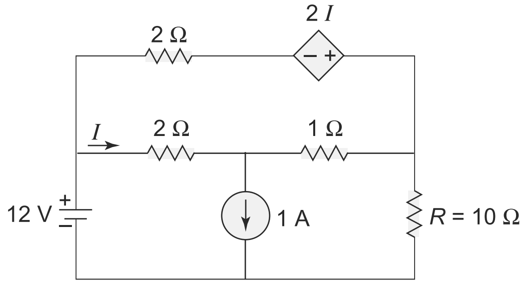


Fig. 11

To find Thevenin's voltage, open resistance $R = 10\ \Omega$

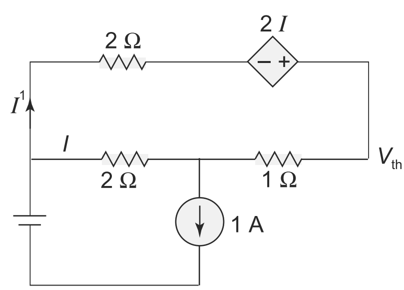


Fig. 12

$$I = \frac{12 - V}{2} \qquad (1)$$

At V_{Th}, $I' = 0 + \dfrac{V_{Th} - V}{1}$

$$I' = V_{\text{Th}} - V$$

$$I = \frac{12 - V}{2} + \frac{V_{Th} - V}{1}$$

$$\boxed{I + I' = 1}$$

$$I' = \frac{12 + 2I - V_{Th}}{2}$$

$$I' = \frac{12 + 2(1 - I') - V_{Th}}{2}$$

$$I' = 6 + 1 - I' - V_{Th}$$

$$\boxed{2I' = 7 - V_{Th}} \qquad (2)$$

At V

$$1 - I' = 6 - \left(\frac{V_{Th} - I'}{2}\right)$$

$$= 6 - \frac{V_{Th}}{2} + \frac{I'}{2}$$

$$\frac{V_{Th}}{2} + \frac{I'}{2} - I' = 5$$

$$\frac{V_{Th}}{2} - \frac{I'}{2} = 5$$

$$\boxed{V_{Th} - I' = 10} \qquad (3)$$

Solving equations (2) and (3), we get

$$V_{Th} - \left(\frac{7 - V_{Th}}{2}\right) = 10$$

$$2V_{Th} - 7 + V_{Th} = 20$$

$$3V_{Th} = 27 \Rightarrow \boxed{V_{Th} - 9\text{ V}}$$

To find I_{SC}

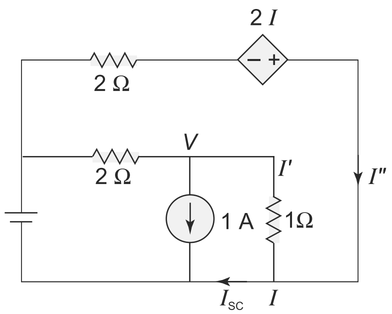


Fig. 13

$$I_{SC} = I' + I''$$

Using nodal equation at v,

$$I = \frac{12 - V}{2}$$

$$\text{I} = 6 - \frac{V}{2} \qquad (1)$$

Nodal at V

$$I = 1 + \frac{V}{1}$$

$$I = 1 + V \qquad (2)$$

Solving equations (1) and (2), $1 + V = 6 - \frac{V}{2}$

$$\Rightarrow -5 = -\frac{3V}{2} \Rightarrow \boxed{V = \frac{10}{3}}$$

$$I = 1 + V = 1 + \frac{10}{3} = \boxed{\frac{13}{3} A}$$

$$I'' = \frac{12+2I}{2} = \frac{12+2\left(\frac{13}{3}\right)}{2} = \frac{12+\frac{26}{3}}{2} = \frac{62}{2\times 3} = \frac{31}{3}$$

$$I_{sc} = I' + I'' = \frac{10}{3} + \frac{31}{3} = \frac{41}{3} = 13.66\,\text{A}$$

$$R_{\text{Th}} = \frac{V_{Th}}{I_{sc}} = \frac{9}{13.66} = 0.65\,\Omega$$

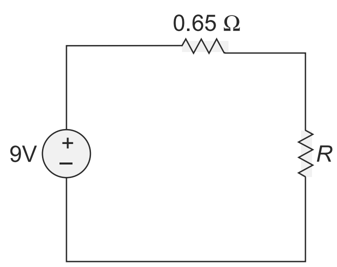


Fig. 14

Using power transfer technique, by maximum power transfer theorem maximum power is delived to resistor if and only if

$$\boxed{R = 0.65\ \Omega}$$