ELECTRICAL MACHINES

Fourth Edition

About the Author

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Dr Bhattacharya holds a BTech degree from Jadavpur University, an MTech degree from Calcutta University and a PhD degree from Birla Institute of Technology and Science, Pilani. He received practical training in an electricity board and a transformer factory in the Netherlands. At Huddersfield, England, he was exposed to curriculum development and technical teachers' training. He visited England again on a Commonwealth Education Fellowship to attend a six-month advance course on Educational Technology. Dr Bhattacharya has visited many universities and industries in countries like Japan, Korea, Australia, Philippines, Malaysia, Netherlands, Thailand, Hong Kong, Singapore and the UK.

As a Senior Fellow of MHRD, Government of India, he attended the Technical Teachers' Training Program at Bengal Engineering College, Shibpur. Dr Bhattacharya is a Fellow of the Institution of Engineers and a Fellow of Institution of Electronics and Telecommunication Engineers, India.

He has written a number of popular textbooks, published and presented over one hundred technical papers in journals and conference proceedings respectively, and developed a number of teaching-learning materials in the form of manuals, monographs, demonstration models, simulations, multimedia learning packages, video films, etc. Dr Bhattacharya specializes in electrical machines, power electronics and control systems.

Dr Bhattacharya, as professor of Electrical Engineering, has taught theory and design of Electrical Machines to both BTech and MTech students for two decades. He has also conducted faculty development programs on teaching of Electrical Machines to the delight of the participants. This book is the outcome of his long experience in teaching the subject.

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Director (Academics)
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Dedicated to

My Wife

Sumita

*Without whose patience and encouragement,
this work could not have been completed*

CONTENTS

PREFACE

It is indeed a matter of great satisfaction that this book on Electrical Machines has done very well in terms of catering to the needs of student's vis-à-vis the teachers' expectations for the last 20 years. The third edition of the book was brought out in 2009 which included a few new topics. In revising the third edition, a comprehensive analysis of syllabi across colleges at both degree and diploma levels was done and a detailed content outline prepared. Thus, the fourth edition of the book will satisfy the curriculum requirements for Electric Machines for students in technical universities, for students preparing for AMIE examinations, and also for students studying in polytechnic colleges.

New to this Edition

This edition has some new topics like

- Concept of energy and co-energy associated in electromagnetic circuits
- Three-phase transformer in detail
- Separation of losses in a transformer core
- Open-delta connection of transformers
- Waveform of magnetizing current of a transformer
- Three-winding transformers
- Electric braking of dc motors and four-quadrant operation
- Cooling arrangement in transformers
- Separation of hysteresis loss and eddy-current loss
- Achieving high starting torque in squirrel-cage-type motors
- Power relationships in synchronous generators
- Equivalent circuit of a single-phase induction motor
- Contactor control operation of three-phase induction motors
- Rotor slip energy
- Recovery method of speed control of slip-ring induction motors
- Induction generator

In addition, more methods of calculation of voltage regulation of synchronous generators have been added, as well as concept of short-circuit ratio, performance calculation of salient-type synchronous generators, etc.

Salient Features

The salient features of this book are the following:

- **Coverage** Covers the entire syllabus of Electrical Machines of diploma courses and a two-semester course for BTech students.
- **Approach** The book begins with a generalized approach by explaining the principles of all electrical machines, and then builds on those principles to

deal with individual machines. This provides a unique understanding that all machines, predominantly, work on the same principle.

- **Illustrations** Over 300 illustrations have been presented to clarify various concepts and principles.
- **Pedagogy** Over 150 solved numerical examples have been interspersed within the chapters. In addition, 100 multiple-choice and short-answer type questions, and over 250 numerical problems with answers have been provided for practice and feedback.

Chapter Organization

The text is organized in seven chapters.

Chapter 1, on generalized treatment of electrical machines, provides explanation about the working of all electrical machines, right from the basic concept of alignment of two magnetic fields. The expression for generalized torque and induced emf has been developed. The concept of a universal machine has been presented to give the students a feel that all electrical machines work on the same principle.

Chapter 2 deals with dc machines where the concept of commutation has been explained lucidly so that the function of brush and commutator is understood for both motoring and generating actions. All aspects of performance characteristics, starting methods, speed control, efficiency and applications, installation and maintenance, have been explained in detail.

Chapter 3 deals with transformers in a comprehensive manner with plenty of solved problems. Three-phase transformers, vector grouping of transformers for parallel operation, and three-winding transformers have been explained. Maintenance schedules of transformers have also been included.

Chapter 4, on three-phase induction machines, has comprehensively explained the basic principles, starting methods, speed-control mechanisms, and the characteristics and applications of induction motors. Methods of drawing circle diagrams for calculating the performance parameters has also been explained with solved numericals.

Chapter 5 is devoted to synchronous machines. Constructional details, operating characteristics of both non-salient-type rotors as well as salient-type rotors have been dealt with.

Chapter 6 has covered all types of single-phase motors including slipper motors and servomotors.

Chapter 7 covers, in brief, power converters including thyristor power converters.

Laboratory Experiments based on the theory have been included at the end of each chapter The laboratory instruction sheets have been prepared in such a way that after going through them, students will understand the experiments and will able to perform the experiments even if the related theory has not been taught to them.

Short-answer-type Questions and O*bjective-type Questions* have been included at the end of each chapter to enable students test their understanding. In addition, *Numerical Problems with Answers* have been provided chapterwise for practice purposes.

The development of this book has stemmed from the author's vast experience in teaching at various places and various levels—diploma, degree, and PG levels, as well as teacher training sessions. The author has conducted a series of institution-based, as well as industry-based, training programmes on the subject for the benefit of technical teachers. Interaction with the participating faculty members and industry personnel have helped the author refine this book over the years. On the basis of feedback received, a few corrections have been incorporated in this edition.

Acknowledgements

I acknowledge with thanks the permission granted by the Indian Standards Institution for making references to Indian standards. These standards are available for sale from the Indian Standards Institution (now called BIS), New Delhi, and its regional and branch offices at Ahmadabad, Bangalore, Bhopal, Bhubaneswar, Mumbai, Kolkata, Chandigarh, Hyderabad, Jaipur, Kanpur, Chennai and Thiruvananthapuram.

This book was originally reviewed by Dr L P Singh, Professor of Electrical Engineering, Indian Institute of Technology (IIT) Kanpur. He found this book well written and useful for students and teachers of polytechnics and also for undergraduate students of BTech level.

For its subsequent editions, a number of senior faculty members of various engineering colleges have reviewed the book. I am thankful to the following faculty for taking out time for this.

Praveen Kumar	*Indian Institute of Technology (IIT) Guwahati* *Guwahati, Assam*
Nitai Pal	*Indian School of Mines (ISM)* *Dhanbad, Jharkhand*
Ritesh Kumar Keshri	*Birla Institute of Technology (BIT) Mesra* *Ranchi, Jharkhand*
Yaduvir Singh	*Harcourt Butler Technological Institute (HBTI)* *Kanpur, Uttar Pradesh*
U Sreenivas	*Srinivasa Ramanujan Institute of Technology* *Chedulla, Andhra Pradesh*
Shadhik M	*Dhaanish Ahmed College of Engineering* *Chennai, Tamil Nadu*

The feedback on this latest edition has been quite encouraging as all the reviewers have appreciated the work.

I would also like to thank my students, many of whom are now teaching in various colleges, who had liked my methods of teaching and are now following this book while teaching this subject. I would also like to thank all the students who have found this book interesting to read.

Lastly, I must convey my gratitude to my wife, without whose patience and encouragement this work could not have been carried out.

Feedback

I hope the patrons of this text will continue to find this book useful and interesting to read. I will be delighted to receive further comments and suggestions on this book so that future editions are further improved to meet user requirements.

S K Bhattacharya

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Piracy-related issues may also be reported.

PART 1

GENERALISED TREATMENT, DC MACHINES AND TRANSFORMERS

- *Generalised Treatment of Electrical Machines*
- *DC Machines*
- *Transformers*

1 GENERALISED TREATMENT OF ELECTRICAL MACHINES

OBJECTIVES

After carefully studying this chapter, you should be able to

➢ Explain why electromechanical energy conversion is necessary.
➢ Explain basic principles of generating and motoring actions.
➢ Explain that torque is developed due to non-alignment of two magnetic fields.
➢ State the significance of torque angle.
➢ Explain the working principle of all common types of electrical rotating machines from the magnetic field alignment concept.
➢ Derive torque equations for rotating machines from the generalised torque equation
➢ Appreciate that the basic principle of working of all electrical rotating machines is the same.

1.1 INTRODUCTION

A conventional book on electrical machines deals with each machine separately. It also explains the principle of each type of machine independently. This gives an impression to the students that each electrical machine is a separate entity and is based on different principles. In fact, this is not true. A close look at all the rotating electrical machines will reveal that there exists two magnetic fields in each electrical machine and the operation of each electrical machine can be explained from the fact that two magnetic fields always try to align with each other. It should be possible, therefore, to evolve a common method with the help of which the working of all electrical machines can be explained. This chapter explains the working principle of electrical machines from the concept of alignment of two magnetic fields.

1.2 DEFINITION OF MOTOR AND GENERATOR

The device which converts electrical energy into mechanical energy is called a motor. The device which converts mechanical energy into electrical energy is called a generator. The process is called *electromechanical energy conversion*, and is illustrated in Fig. 1.1.

A question that can be asked is why this electromechanical energy conversion is necessary.

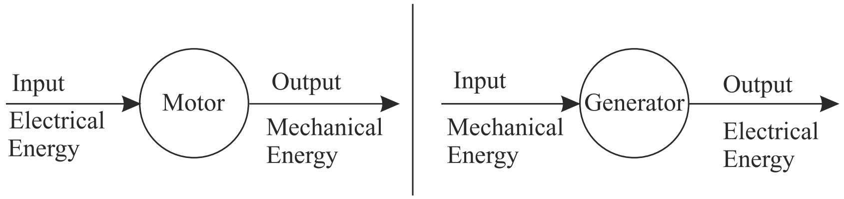


Fig. 1.1 *Schematic representation of electro-mechanical energy conversion devices*

The sources of energy available in nature are in different forms, e.g., energy can be made available in the form of water heads where a river is flowing and there is sufficient gradient. The potential energy of water can be utilised to do some work. But the amount of energy available at a place may have to be transferred to other places like cities or where there is concentration of industries. Thus it becomes necessary to transfer or transport that energy to cities and industrial areas. Electrical energy can efficiently and effectively be transmitted from the place of generation to the place of utilisation. Electrical energy can easily be converted into any required form of energy (heat, light, sound, mechanical, chemical, etc.) at the place of utilisation. Hence the natural available sources of energy (thermal, water, nuclear, etc.) are first converted into electrical energy, transmitted to the load centre and then converted into the required form of energy. That is why the potential energy of water available at a place is converted into electrical energy in a hydroelectric power station. The potential energy of water drives a turbine, which in turn drives a generator. The generator produces electricity, the electricity is transmitted through overhead lines and brought near to the load centre.

Another example may be that the source of energy is in the form of coal. Coal is available in mines. Mines are not located at every place. Electricity can be generated economically by converting thermal energy of coal at a place where there is constant availability of coal and the electrical energy so produced can conveniently be transmitted to various places.

When electrical energy is available and mechanical work is to be done by it, a device is needed which will convert electrical energy into mechanical energy. In the case of power stations, like hydroelectric power stations, or diesel-electric power stations, mechanical energy or thermal energy is converted into electrical energy with the help of generators, whereas electrical motors are used to convert electrical energy into mechanical energy to do some specific work for us. It may, however, be noted that the basic construction of a motor and a generator is the same. The process of conversion of electrical energy into mechanical energy and vice versa is a reversible one. The students will study the details of electrical machines in the chapters to follow.

1.3 BASIC PRINCIPLE OF A GENERATOR AND A MOTOR

A generator is built utilising the basic principle that emf is induced in a conductor when it cuts magnetic lines of force. A motor works on the basic principle that a current carrying conductor placed in a magnetic field experiences a force.

1.3.1 Generator

Faraday's law of electromagnetic induction states that when a conductor or a coil is rotated in a magnetic field, an emf is induced. To generate a considerable amount of emf a good number of coils connected in series must be rotated. In a generator, there is one magnetic field produced by the field magnets. The coils in which emf is generated are placed on a cylindrical rotor called an armature, which is rotated with the help of a rotating device called the primernover. Load is connected across the terminals of the armature in which the emf is induced. Current flows through the armature conductors when the load is connected across it. Thus there exist two magnetic fields in a generator, viz. the stator magnetic field, i.e., the field in which the armature rotates, and the field produced by the current flowing through conductors placed on the armature.

1.3.2 Motor

If a conductor carrying current is placed in a magnetic field, force is developed in the conductor. If a number of conductors connected in series are placed on a cylindrical rotor and current is allowed to flow through the conductors, a torque will be produced on the rotor which will cause the rotor to rotate. Thus, in any electrical motor, two magnetic fields can be found, i.e., the field produced by the magnets and the field produced by the current flowing through the armature conductors. In the following section, we will study how two magnetic fields in trying to align with each other produce a torque.

1.4 TORQUE DUE TO NON-ALIGNMENT OF TWO MAGNETIC FIELDS AND THE CONCEPT OF TORQUE ANGLE

The aim of this section is to explain how torque is produced on magnets due to non-alignment of two magnetic fields. The magnitude of the torque depends on strength of the two magnetic fields and the torque angle. The torque angle is the angle between the axis of the two magnetic fields.

In all electrical machines that will be studied here, it will be observed that there are two magnetic fields produced by two electromagnets. Figure 1.2(a) shows a bar-type permanent magnet. Figures 1.2(b), (c), (d) shows the electromagnets.

In an electromagnet, a coil is wound on a piece of magnetic material. An electromagnet is required to be energised by connecting a source of supply across the coil. When current flows through the coil, lines of force are established. In a permanent magnet, the magnetic polarities are permanently fixed but in an electromagnet the position of the North and South poles depends upon the direction of current flowing through the coil (see this difference in Figs. 1.2(c) and (d)). By applying the thumb rule or cork screw rule, the direction of flux around a current carrying coil can be found. It is to be noted that the North pole is one from where the lines of force come out of the magnet body, whereas the South pole is one where the lines of force enter the magnet body. The strength of a magnet is determined by the number of lines of force produced by it. The strength of a permanent magnet is fixed. The strength of an electromagnet is proportional to the total ampere-turns provided on the magnet

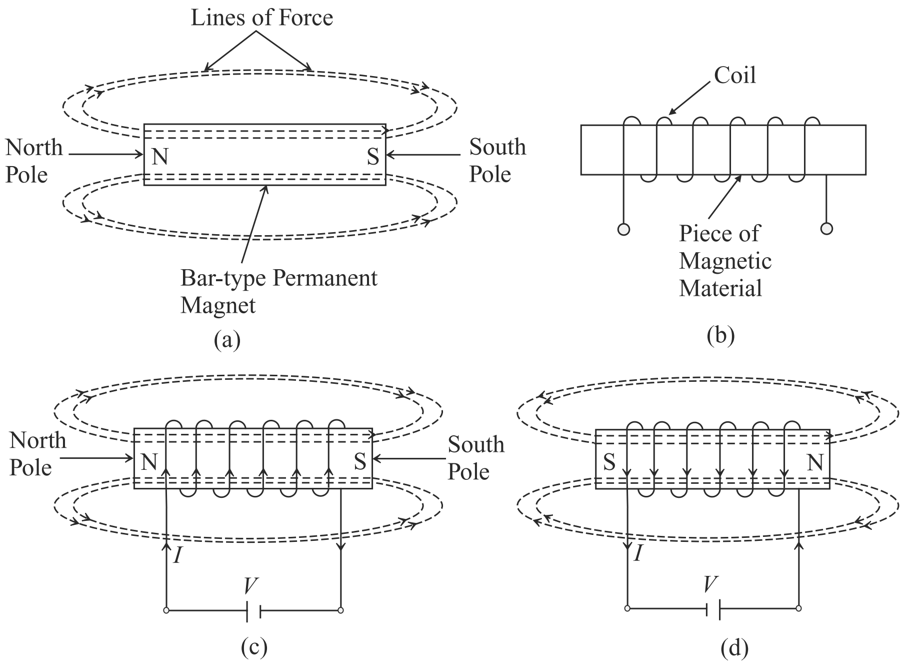


Fig. 1.2 *Magnetic fields produced by electromagnets*

(ampere-turn = NI, where N is the number of turns of the coil and I is the current flowing through the coil). Number of turns N remaining constant, the strength of an electromagnet will be proportional to the magnitude of current flowing through its coil.

The body or the core of the electromagnet is made of magnetic material because magnetic materials allow easy flow of flux through them (reluctance of a magnetic material is very low as compared to that of any nonmagnetic material). Reluctance is the property of a material by virtue of which it opposes the establishment of magnetic f1ux through it. It is the inverse of permeability. Reluctance of a magnetic circuit is analogous to resistance in an electric circuit. The material on which the coil of an electromagnet is wound is called its core. If the body or core is made of iron, then it will be called an iron core electromagnet.

The core of an electromagnet can be made of any shape and size depending on the requirement. Figure 1.3(a) shows a solid cylindrical core. The difference in the method of winding of the coil on the core may be noted in Figs. 1.3(a) and (b). A cross section of the electromagnet as shown in Fig. 1.3(b) will look like Fig. 1.3(c). Henceforth a cross-sectional view of the core will be shown as it is easier to draw it than to draw the isometric view. In Fig. 1.3(c), the winding has been shown on the surface of the core. Instead, some slots could have been made on the body of the magnet to place the coils. Such an arrangement is shown in Fig. 1.3(d).

In Fig. 1.3, a solid cylindrical electromagnet with windings placed on the surface of the core or in the slots has been shown. Figure 1.4 shows a hollow cylindrical core with current carrying coils placed in the slots. Whatever may be the shape or size of the core, when current flows through its coils, flux will be produced.

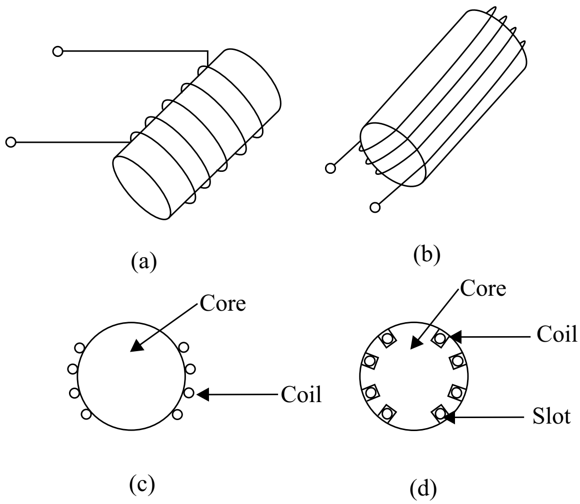


Fig. 1.3 *Various ways of placing coils on cores to make an electromagnet*

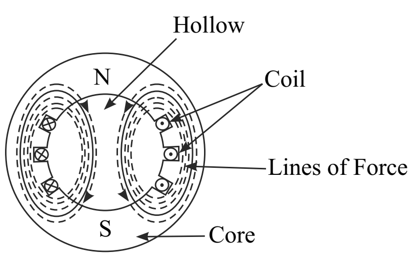


Fig. 1.4 *A hollow cylindrical electromagnet*

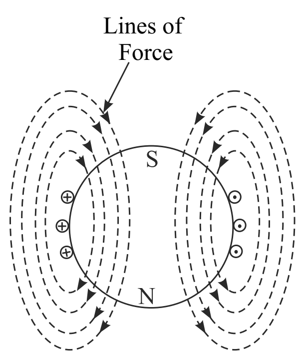


Fig. 1.5 *A solid cylindrical electromagnet*

Figure 1.5 shows a solid cylindrical electromagnet with its current-carrying coils. The polarities have been marked. The directions of flux lines have been found out by applying the cork screw rule. It is already known that the path of the current is a closed loop. Similarly, a flux line should be closed in itself. Another point to be seen from Fig. 1.5 is that the lines of force never cross each other.

The other important characteristic of magnetic lines of force is that they always pass through the path of minimum reluctance. In Fig. 1.6, it is to be noted that there is a congestion of flux in portion *a* whereas there are very few lines of force in portion *b*. This is because the lines of force find an easier path through iron than through air. Air is a non-magnetic material with very high reluctance whereas iron or steel is a good magnetic material with very low reluctance. Therefore, for the same ampere-turns, the flux produced by an iron-core electromagnet will be much more than an air-core one. In an air-core electromagnet, there is no magnetic material inside the coil [see Fig. 1.7(b)].

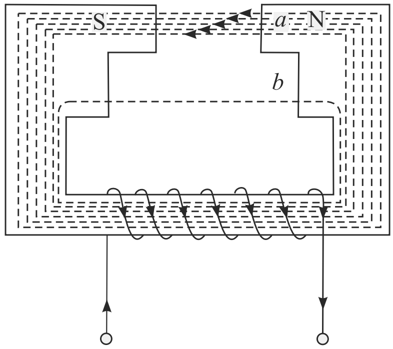


Fig. 1.6 *Non-uniform distribution of flux due to difference in reluctance to the flux path*

The flux lines around a magnet constitute a magnetic field. In fact, there will be a large number of lines of force around a magnet. But for simplicity only a few lines will be drawn. Obviously, the presence of the lines of force will have to be imagined. This will minimise the drawing work.

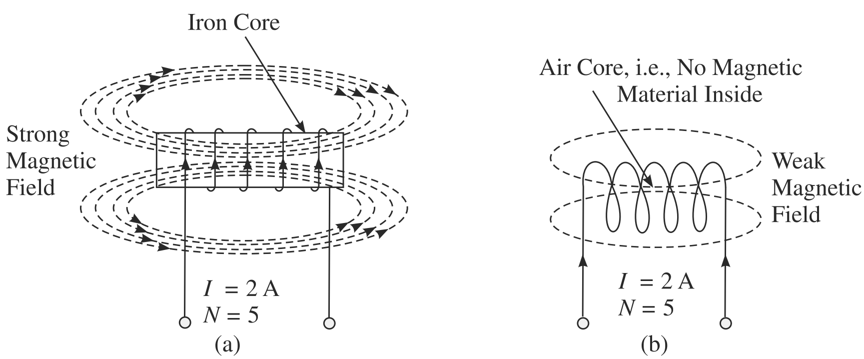


Fig. 1.7 *Difference in the amount of flux produced due to (a) iron core and (b) air core*

Figures 1.8(a) and (b) show the magnetic fields produced by an electromagnet and a permanent magnet respectively. If one magnet is brought under the influence of the magnetic field of another magnet, the two magnetic fields will try to align with each other. Under the aligned condition, the poles of one magnet will face the opposite poles of the other magnet due to the forces of attraction between opposite poles of the magnets (see Fig. 1.9).

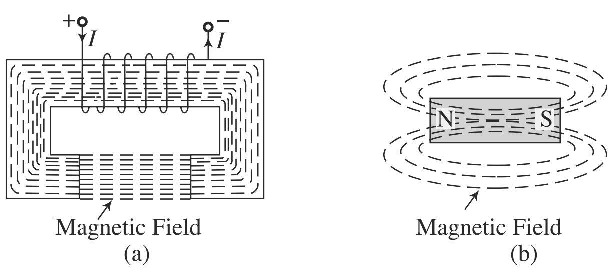


Fig. 1.8 *(a) Magnetic field produced by an electromagnet*
(b) Magnetic field produced by a permanent magnet

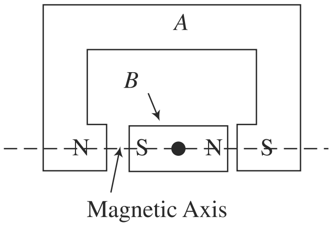


Fig. 1.9 *Two magnets are in stable equilibrium*

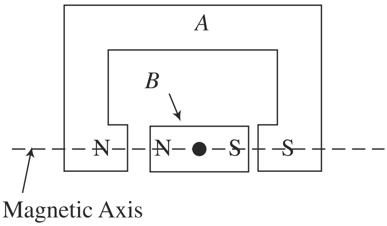


Fig. 1.10 *Two magnets are in unstable equilibrium*

Magnet B is placed in the magnetic field of magnet A such that the opposite poles face each other. The axes of the two magnets are aligned. This condition is called the *stable equilibrium condition*. Under the stable equilibrium condition there will be no tendency of magnet B to move away from its equilibrium position.

In Fig. 1.10 also, the axes of the two magnets are in alignment and the two magnets are in equilibrium. But they are unstable because a slight disturbance will cause the small magnet to rotate by 180°, to bring the two magnets into stable equilibrium.

Now, what happens when the two magnetic fields are not aligned can be examined. In Fig. 1.11(a), magnet A is stationary and magnet B is free to rotate. The position of magnet B will remain unchanged even when it is left free because this position is its stable equilibrium position.

If a small rotation is given to magnet B by an angle say α, and it is then released, the two magnetic fields will no longer be aligned [see Fig. 1.11(b)]. The force of attraction between the opposite poles will give rise to a torque. Torque is the product of force and its perpendicular distance from the centre of rotation. The torque thus produced will bring magnet B back to its stable equilibrium position, as shown in Fig. 1.11(c). There will be no torque in magnet B in the position shown in Fig. 1.11(c). Angle α between the axes of the two magnetic fields is called the torque angle.

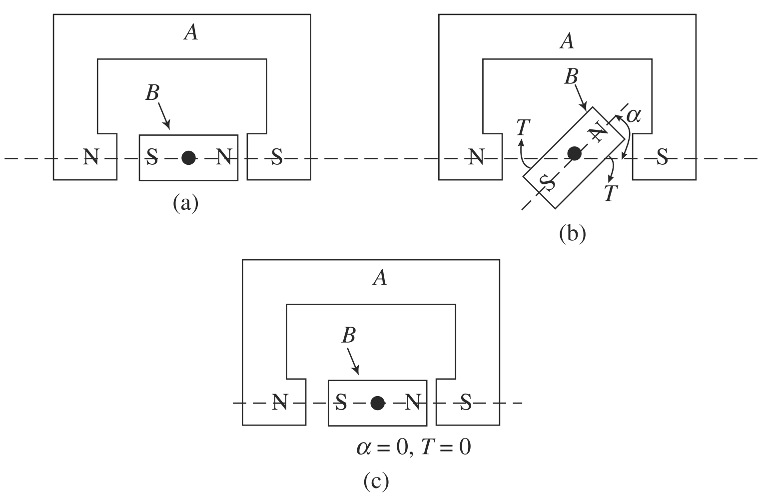


Fig. 1.11 *Torque developed due to non-alignment of two magnetic fields as shown in (b)*

It has been just seen that torque is developed when there is an angle of non-alignment between two magnetic fields. Torque is developed due to the force of attraction between the opposite poles of the two magnets.

In Fig. 1.12(a), magnets are aligned to each other. Although there exists a force of attraction between opposite poles, the torque developed is zero because the forces are co-linear with the magnetic field axis. (Torque = Force × Perpendicular distance. In this case, this perpendicular distance is zero.)

In Fig. 1.12(b), the two magnetic fields are shown non-aligned by an angle α. To calculate the magnitude of the torque, the component of force F perpendicular to the axis of the magnet is taken F_1 represents the perpendicular component of F. From Fig. 1.12(b)

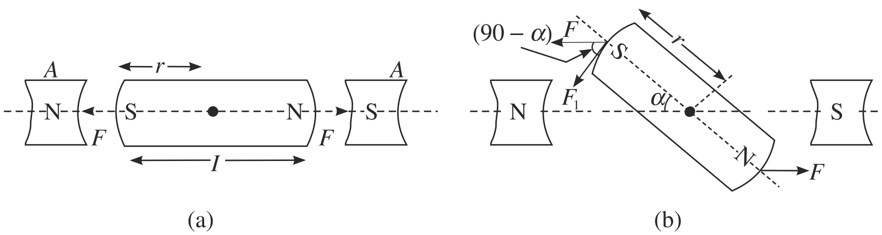


Fig. 1.12 *Torque due to non-alignment of two magnetic fields*

$$F_1 = F\cos(90° - \alpha) = F\sin\alpha$$
$$T = F_1 r = F r \sin\alpha$$

Total torque (for two pairs of poles)

$$T = 2Fr\sin\alpha$$
$$= 2rF\sin\alpha = lF\sin\alpha$$

where l represents the length of the magnet.

Therefore, $$T \propto F\sin\alpha \qquad (1.1)$$

It is seen that the torque developed due to non-alignment of two magnetic fields is proportional to $F \sin \alpha$. Torque T therefore depends upon two factors, viz. F, the force between two opposite poles which in turn depends upon the strength of the magnetic poles and $\sin \alpha$ where α is the angle of non-alignment between the axis of the two magnetic fields.

In the cases discussed so far the torque produced is not continuous. The free magnet stops turning when it becomes aligned with the stator magnetic field. Now, it is to be seen how continuous rotation of the free magnet can be achieved.

Method of Achieving Continuous Rotation of one Magnet with Respect to the Other When the two magnets are non-aligned, as shown in Fig. 1.13(a), torque will be developed on the free magnet. This magnet will rotate in the clockwise direction by an angle α, and stay in the position shown in Fig. 1.13(b).

If magnet AA is rotated in the clockwise direction by an angle α as shown in Fig. 1.13(c), again torque will be developed on the free magnet bringing it to the position as shown in Fig. 1.13(d). Magnet AA is again rotated in the clockwise direction by some more angle, as shown in Fig. 1.13(e). The free magnet will further rotate as shown in Fig. 1.13(f).

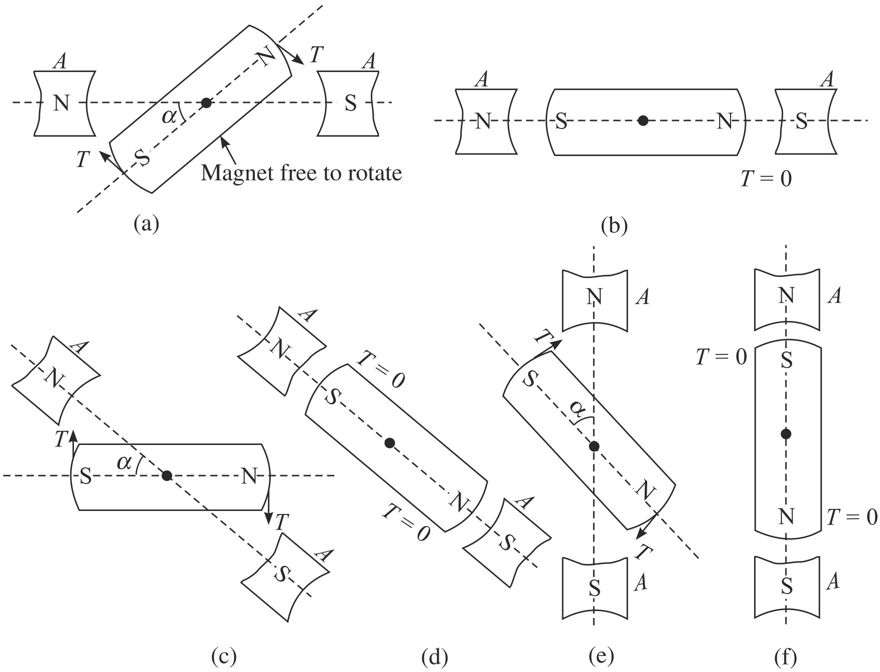


Fig. 1.13 *Gradual rotation of magnet AA causes rotation of the other magnet (rotor magnet)*

From the above facts it can be concluded that if magnet *AA* is rotated in a particular direction by some angle, the free magnet will also rotate in the same direction by the same angle.

At position 1–1 in Fig. 1.14 no torque is developed on the free magnet. But when magnet *AA* is moved to position 2–2, a torque will be developed on the free magnet causing it to move in the clockwise direction till the two fields become aligned with each other. Just before the free magnet becomes aligned, if magnet *AA* is moved to position 3–3, the free magnet will continue rotating till it again becomes aligned with magnet *AA*. If magnet *AA* is rotated by one revolution, the free magnet will also rotate by one revolution. By rotating magnet *AA* continuously, continuous rotation of the free magnet will be achieved. It is to be noted that the tendency of the two magnetic fields to remain aligned with each other will keep the free magnet rotating if magnet *AA* is rotated.

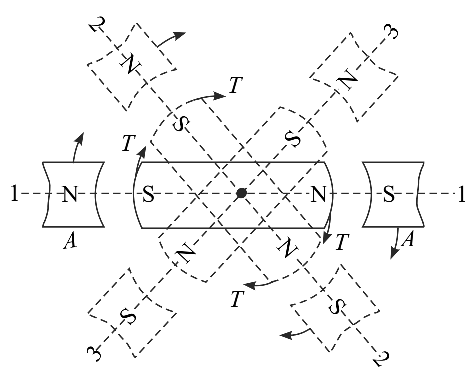


Fig. 1.14 *Rotor magnet is made rotating by physically rotating the stator magnet AA*

Now refer to the arrangement shown in Fig. 1.15. The rotor will rotate in the clockwise direction by 90° and stay in the position as shown in Fig. 1.15(b). Continuous rotation of the rotor can be obtained in the following ways:

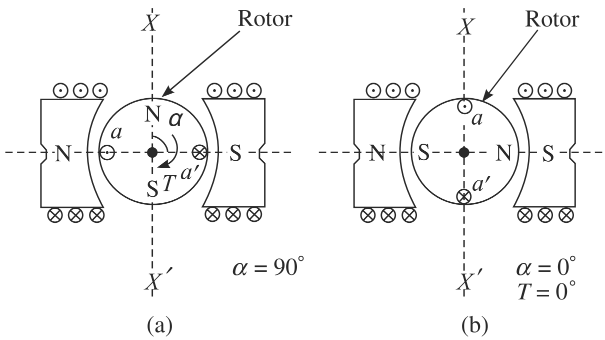


Fig. 1.15 *Illustrate the torque is developed on the rotor as long as the torque angle exists*

1. By changing the direction on currents in the rotor conductors in such a manner that the conductors facing a particular main field pole will always have the same direction of current. That is, there should be a change in the direction of current in coil *aa′* when it crosses axis *xx′*. The above is achieved through the brush and commutator arrangement in a dc machine. It will be discussed in detail in a separate chapter.
2. By rotating the main poles which will pull the rotor along because of the tendency of the rotor field to align with the main field.

The reader is advised to work out the following exercises before proceeding to the next section.

EXERCISES 1A

1.1 In Fig. 1.16, mark by cross and dot the currents in the conductors of the rotor to obtain (i) clockwise torque and (ii) anticlockwise torque.

1.2 Mark the direction of currents in the rotor conductors in Fig. 1.17 for torque to be developed in the anticlockwise direction.

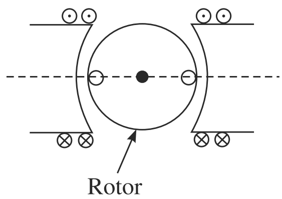


Fig. 1.16

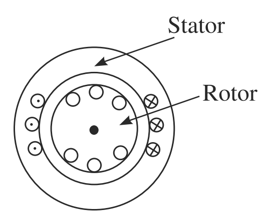


Fig. 1.17

1.3 Will there be any torque produced in the arrangements shown in Figs. 1.18(a) and (b)? If not, mention why.

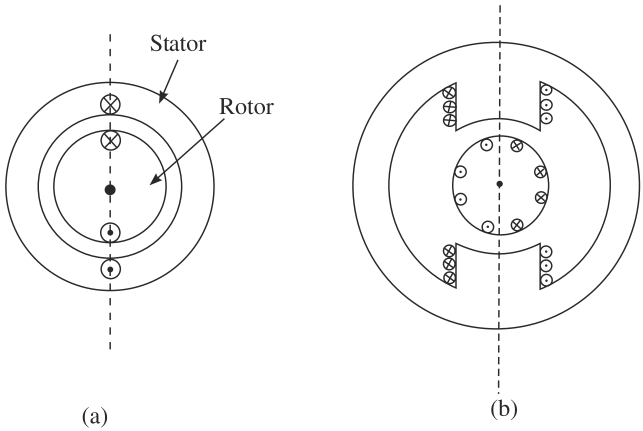


Fig. 1.18

1.4 Show the direction of torque produced and the torque angle in Fig. 1.19.

1.5 Show the direction of currents in both the pole windings and on the armature conductors in Fig. 1.20 for torque in the clockwise direction.

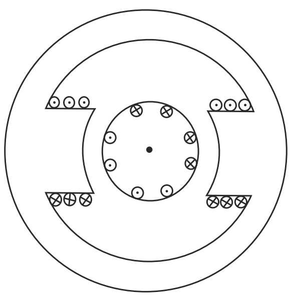


Fig. 1.19

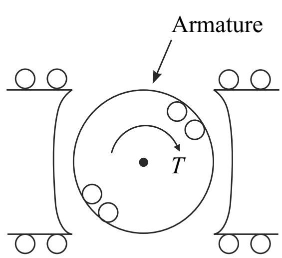


Fig. 1.20

1.6 Will the rotor in Figs. 1.21(a) and (b) rotate? Explain giving reasons.

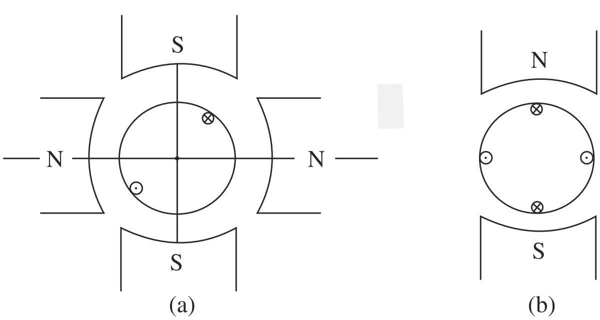


Fig. 1.21

1.5 BASIC ELECTROMAGNETIC LAWS

To understand the principle of working of electrical machines, it is necessary to know the related basic electromagnetic laws. A review of the basic electromagnetic laws has been made in this section for the benefit of the reader.

1.5.1 Magnetic Field

Whenever a current flows through a conductor, a magnetic field is established around that conductor. Figure 1.22 shows the cross-sectional views of two conductors carrying current in opposite directions. The cross sign (×) shown inside the conductor indicates that current is entering the conductor whereas the dot sign (·) shown indicates that the current is coming out in a direction perpendicular to the plane of the paper. Directions of flux lines around the conductors are also shown in the figure. The direction of the lines of force around a conductor is readily determined by the *right-hand rule* which is as follows—point the thumb of the right hand in the direction of the current in the conductor and wrap the fingers of the right hand around the conductor. The finger tips will point in the direction of the lines of force.

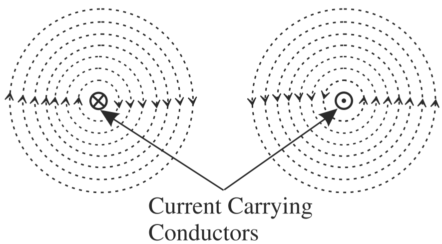


Fig. 1.22 *Flux around current-carrying conductors*

The cork screw rule can also be applied to determine the direction of the lines of force around a current-carrying conductor.

Concentration of flux can be produced by causing the current to flow through a coil instead of through a conductor. Inside the coil the flux contributed by each segment of the coil is in the same direction. Thus a strong magnetic field is produced due to current flowing through a coil as shown in Fig. 1.23. The introduction of a magnetic material as a core for the coil increases the flux. A simple right-hand rule indicates the direction of magnetic flux inside a coil: Wrap the fingers of the right hand around the coil, with the finger tips pointing in the direction of the current, the thumb points in the direction of the magnetic flux.

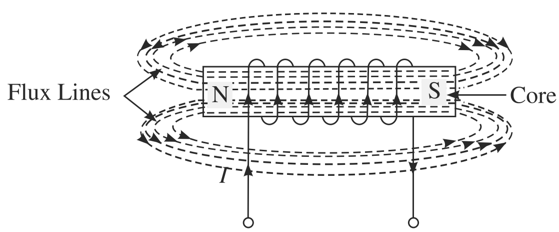


Fig. 1.23 *Flux produced by a current-carrying coil*

1.5.2 Magnetic Force and Torque

Figure 1.24 shows a current-carrying conductor placed in a magnetic field produced by a permanent magnet. Such a magnetic field, however, may be produced by an electromagnet also. There is an interaction of the two magnetic fields, i.e., the main field and the field produced by the current-carrying conductor. This interaction results in a force on the conductor which is given by

$$F = Bil \qquad (1.2)$$

where B is the flux density of the main field, i is the current in the conductor, and l the effective length of the conductor placed in the magnetic field.

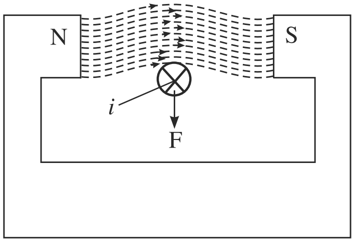


Fig. 1.24 *Force on a current-carrying conductor placed in a magnetic field*

In Fig. 1.24, the total flux above the conductor is the sum of the two fluxes, i.e., the flux produced by the magnet and the flux produced by the current. Below the conductor the flux is reduced, because the flux produced by the current opposes and cancels some of the flux of the main field. The force on the conductor is downwards, i.e., from the position of larger flux density towards that of smaller flux density. The direction of the force is perpendicular to both directions of the main flux and the direction of current i.

Now consider the case of a coil placed in a magnetic field as shown in Fig. 1.25. Equal forces act on two sides of the coil, but their directions are opposite as indicated in Fig. 1.25. The net result is a torque which causes rotation. The magnitude of torque T is given by $T = 2\ Bilr,$ where r is the effective radius of the rotor core. If the coil contains N turns, then the torque is increased by this factor. If flux density B is expressed in terms of the total flux through the coil ($\phi = BA$), then, torque T is

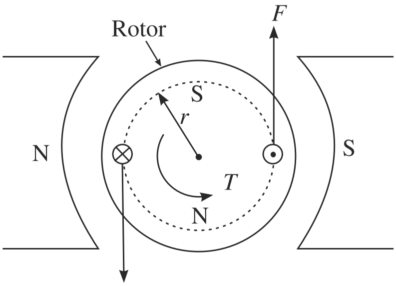


Fig. 1.25 *Force on a current carrying coil placed in a magnetic field*

$$T = 2\ Blir\ N$$

$$= \frac{2\phi\, lir\, N}{A} = \left[\frac{2\, lr\, N}{A}\right]\phi i$$

$$= K\phi i$$

Therefore, $T = K\phi i,$ where $K = \dfrac{2\, lr\, N}{A}$ = constant (1.3)

The torque produced can be used to cause mechanical rotation.

1.5.3 Electromagnetic Induction

Faraday's Law Faraday's law states that when there is change in flux linkage with a coil, emf is induced in it. The change in flux linkages can be produced either by creating a relative motion between the coil and flux or by changing the magnitude of flux linking the coil. The emf produced by former action is called *dynamically induced emf* while the latter is called *statically induced emf*.

The magnitude of induced emf e is directly proportional to the rate of change of flux linkage with the coil.

Mathematically, $$e = \frac{-d(N\phi)}{dt} = -N\frac{d\phi}{dt} \quad (1.4)$$

where $N\phi$ is called the flux linkage, N the number of turns of the coil and ϕ the average flux linking each turn. The negative sign is due to Lenz's law which is explained later. The effect of change of flux linkage by a coil is shown through an experiment described as follows.

A zero-centre galvanometer is connected between the two terminals of an air-core coil having sufficient number of turns. A bar-magnet is moved in and out of the core (see Fig. 1.26). The following will be observed:

1. The deflection of the needle of the galvanometer is proportional to the speed at which the magnet is brought in and out of the coil.
2. The direction of deflection changes with the direction of motion of the magnet.
3. The direction of deflection changes when the polarity of the magnet is reversed keeping the direction of motion of the magnet unchanged.

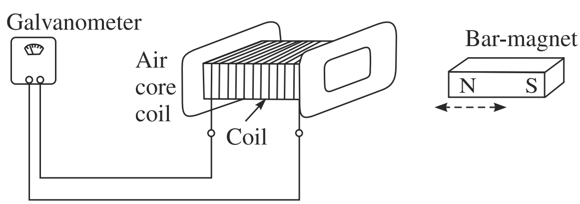


Fig. 1.26 *Effect of change of flux linkage by a coil*

The deflection of the galvanometer is due to an emf induced in the coil as a result of changes in the flux lines passing through the coil as the magnet moves to and fro. When the magnet is moved towards the coil, there is an increase in the number of lines passing through it. This passing of the lines of force through the coil is called the *linking of flux*. The total number of lines of force, ϕ, which pass through a coil multiplied by the number of turns, N, of the coil is called *flux linkage*, $N\phi$. When a magnet moves towards a coil there is an increase in the flux linkage. This increase in flux linkage induces an emf in the coil. When the magnet is moved away from the coil there is a decrease in the flux linkage. This decrease in the flux linkage also induces an emf in the coil. The direction of the deflection of the galvanometer will be opposite in the case of decrease of flux linkage as compared to that for increase of flux linkage. It may, therefore, be concluded that emf is induced in a coil when it experiences a change of flux linkage. The magnitude of the induced emf is proportional to the rate at which the flux linkage changes which is, as mentioned earlier, mathematically expressed as

$$e = -N\frac{d\phi}{dt}$$

An emf is induced in the coil irrespective of whether the magnet moves or the coil moves as long as the coil experiences a changing flux linkage.

Lenz's Law When the magnet in Fig. 1.26 is moved towards the coil, there is an increase in flux linkage. This gives rise to an induced emf in the coil which has a direction such that when the circuit is closed and a current flows, it sets up flux lines in a direction opposite to those of the main magnet, thus opposing the increase in flux linkage (see Fig. 1.27). Similarly, when the magnet is moved away from the coil, there is a decrease in flux linkage. This gives rise to an induced emf which has direction such that when the circuit is closed to allow the current to flow, it sets up flux lines in the same direction as those of the main magnet, thus opposing the decrease in flux linkage. The above phenomenon regarding the direction of the induced emf is referred to as Lenz's law.

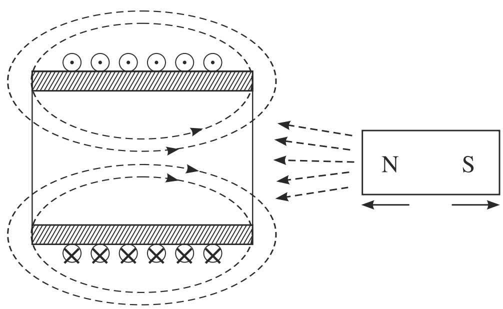


Fig. 1.27 *Direction of emf induced in a coil—application of Lenz's law*

Lenz's law states that the direction of induced emf in a coil is such that it opposes the change producing it. That is why, this effect is shown mathematically in the equation of induced emf by putting a minus sign. Thus,

$$e = -N\frac{d\phi}{dt}$$

1.6 EMF INDUCED IN A COIL ROTATING IN A MAGNETIC FIELD

Figure 1.28 is shows a coil being rotated in a magnetic field. The coil is placed on the rotor slots. As the rotor is rotated, a change of linkage of flux by the coil takes place. Thus emf is induced in the coil. The magnitude of the emf induced in the coil depends upon the rate of change of flux linkage or alternatively, on the rate of cutting of flux by the coil sides. Figure 1.29 shows coil aa' rotating in a magnetic field. The magnitude and direction of emf induced in the coil will be examined for various positions the coil occupies when rotated. Now carefully observe the positions of coil as in Fig. 1.29.

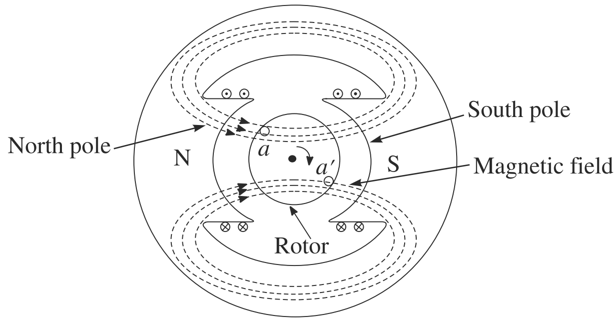


Fig. 1.28 *Emf induced in a coil rotating in a magnetic field*

1. Position 1 – 5 (i.e., coil side a in position 1 and coil side a' in position 5). There is a maximum flux linkage, i.e., maximum number of flux lines are passing through the coil. A small change in the coil position does not cause change in flux linkage. Although flux linkage is maximum, change in flux linkage is zero and hence the induced emf in the coil is zero. Further, in this position, movement of conductors does not create cutting of flux by the conductors since the conductors move parallel to the flux lines and hence induced emf is zero

2. Position 2 – 6 When the coil changes its position from 1 – 5 to 2 – 6, there is a change of flux linkage (i.e., less number of flux lines are passing through the coil). There is an induced emf in the coil. The direction of the induced emf in the coil sides are shown in Fig. 1.29. The direction of the induced emf is found by applying Fleming's right-hand rule. The flux produced by the current flowing through the coil will oppose the reduction in flux linkage by the coil caused due to its rotation in the magnetic field. The direction of induced emf in coil aa' for various position can also be found out by applying Lenz's law.

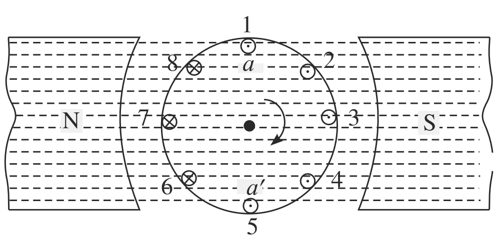


Fig. 1.29 *Emf induced in a coil rotating in a magnetic field*

3. Position 3–7 As the coil rotates in the clockwise direction from position 2–6 towards position 3–7, there will be further reduction in the flux linkage. It may be noted that as the coil advances towards position 3–7, the rate of reduction in the flux linkage is comparatively higher than when the coil moves from position 1–5 to position 2–6. Thus there will be gradual increase in induced emf in the coil. At position 3–7, the flux linkage by the coil is zero.

4. Position 3–7 to Position 5–1 When the coil passes through position 3–7, there is maximum rate of change of flux linkage, the induced emf is maximum. From position 3–7 to position 5–1, the flux linkage will again increase and the rate of change of flux linkage will gradually decrease and hence the magnitude of the induced emf will also gradually decrease.

5. Position 5 –1 to Position 1 – 5 Through Position 6 – 2, 7 – 3 and 8 – 4 The change in flux linkage and hence the induced emf in the coil will be the same as the induced emf when the coil had moved from position 1–5 to position 5–1. The direction of induced emf will however be opposite in this case. Thus the nature of the induced emf in the coil for its one revolution will be as shown in Fig. 1.30. The abscissa of the graph has been represented in time since the coil has occupied different positions at different times.

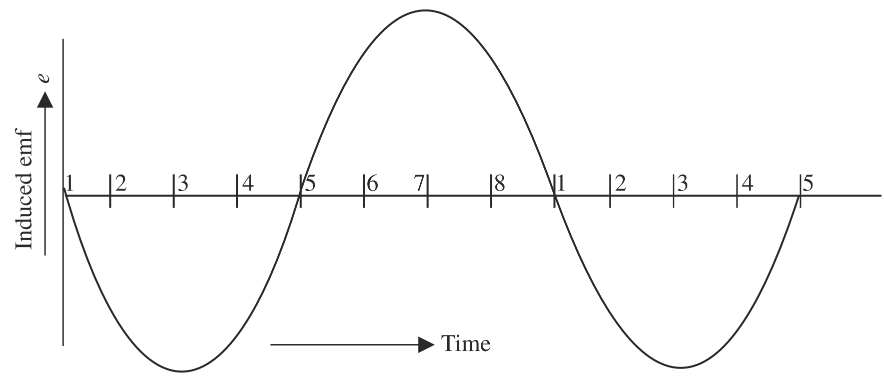


Fig. 1.30 *Graphical representation of the emf induced in coil aa′ of Fig. 1.29*

It is also possible to visualise that as the coil rotates, its coil sides cut through the lines of force produced by the main poles. Viewed in this manner, Faraday's law of electromagnetic induction may be stated as follows:

When a coil rotates in a magnetic field in such a manner that its coil sides cut the lines of force, an emf is induced in the coil sides. This induced emf is proportional to the rate at which the flux is cut.

For example, when the coil moves in Position 1–5, its movement is parallel to the lines of force and hence it does not cut any lines of force. The induced emf in coil *aa′* in Position 1–5 is therefore zero. In Position 3–7, the movement of the coil sides is perpendicular to the flux lines and hence the emf induced will be maximum in the coil.

The magnitude of the induced emf in a conductor when rotated in a magnetic field is

$$e = Blv \text{ V}$$

where B = flux density of the magnetic field in Wb/m^2
l = length of the conductor in m
v = velocity of the conductor perpendicular to the field in m/s

Since there are two coil sides in a coil, the total emf induced in the coil is equal to

$$e = 2\ Blv \text{ V}$$

If the coil has N turns,

$$e = 2\ NBlv \text{ V}$$

It may be noted that as the coil rotates in the magnetic field, the coil sides do not always cut the lines of force in a perpendicular direction. To generalise, therefore, the above expression can be rewritten as

$$e = 2\ NBlv \sin \theta \text{ V} \tag{1.5}$$

where θ = angle between the direction of coil velocity and the direction of the lines of force.

The reader is advised to work out the following exercises before proceeding to the next section.

EXERCISES 1B

1.1 Mark the direction of emf in the conductors of the rotor of Fig. 1.31, when the rotor moves: (a) in the clockwise direction and (b) in the anticlockwise direction.

1.2 Figure 1.32 shows four coils, 1–1′, 2–2′, 3–3′, 4–4′ placed in the slots of the stator. The rotor is carrying the field winding. Mark the direction of the emf in the conductors of the stator for (a) clockwise direction of rotation of the rotor and (b) anticlockwise direction of rotation of the rotor.

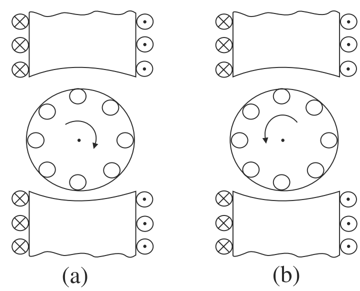


Fig. 1.31

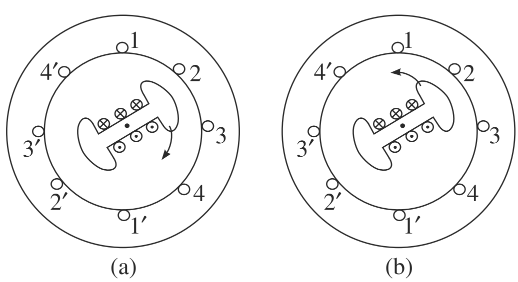


Fig. 1.32

1.3 Mark the direction of induced emf in the coils when a magnet is moved toward the coil as shown in Fig. 1.33.

1.4 Mark the direction of induced emf in the coils when a magnet is taken away from the coil as shown in Fig. 1.34.

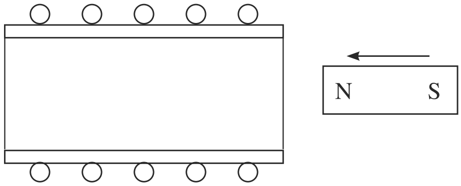


Fig. 1.33

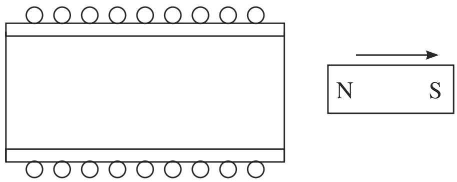


Fig. 1.34

Note: The solutions to the exercise problem have been provided at the end of the chapter.

1.7 ELEMENTARY CONCEPT OF AN ELECTRICAL MACHINE

An elementary concept of an electrical generator and a motor has been explained with a single coil placed in a magnetic field. For comparison, the directions of electromagnetic torque and mechanical torque have been shown for both motoring and generating action.

1.7.1 An Elementary Generator

Consider a coil being rotated in a magnetic field by applying a mechanical torque, T_m as shown in Fig. 1.35. An emf will be induced in the coil due to the change of flux linked by the coil. The direction of induced emf in the coil sides is shown in Fig. 1.35.

The direction of induced emf is determined by applying either Lenz's law or Fleming's right-hand rule. If the coil ends are connected to an external resistance R_L, a current i will flow in the direction shown in Fig. 1.35. The current-carrying coil will now experience a torque, T_e in the direction shown (as it is known that when a current-carrying conductor is placed in a magnetic field it experiences a force). The torque developed by the current-carrying coil is the electromagnetic torque, T_e. The electromagnetic torque developed will act in a direction opposite to the applied mechanical torque, T_m, responsible for causing rotation of the coil in the magnetic field.

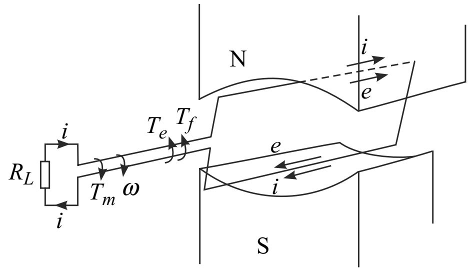


Fig. 1.35 An elementary generator

If there is no external resistance R_L connected across the coil, no current will flow through the circuit and hence no torque will he developed. The value of T_m will then be utilised in overcoming the frictional torque, T_f When the circuit of the coil is closed through an external resistance, T_e will be set up by the coil in a direction opposite to T_m. The value of T_m must therefore be increased to overcome T_f as well as T_e. Thus,

$$T_m = T_e + T_f \tag{1.6}$$

If T_f is considered negligible,

$$T_m = T_e$$

The magnitude of current through the coil is

$$i = \frac{e}{R_L + r}$$

where r = resistance of the coil
R_L = load resistance

Thus, $$e = iR_L + ir$$

Multiplying both sides by i

$$ei = i^2R_L + i^2r = vi + i^2r \tag{1.7}$$

where ei = electrical power developed
i^2R_L = electrical power used in the electrical load
$= vi$
v = terminal voltage = iR_L
and i^2r = power lost in the resistance of the coil

The electrical power developed $ei = \omega T_e$, where T_e is the portion of T_m used for conversion into electrical power.

The relation $ei = \omega T_e$ can be proved as follows:

We can write
$$\begin{aligned} ei &= 2B\,lv\,i \\ &= 2Bil\,v \\ &= 2Fr\frac{v}{r} \\ &= T_e\omega \end{aligned}$$

Thus, $$ei = \omega T_e \tag{1.8}$$

Of the total mechanical power ωT_m supplied, ωT_e is converted into electrical power, the remaining power ωT_f is wasted as frictional losses. Of the electrical power developed ei, an amount of power i^2r is wasted in the winding, the remaining power, vi, is available across the load. This is shown diagrammatically in Fig. 1.36.

In a practical generator, instead of one coil, a number of coils connected together are housed inside the slots of a cylinder called the armature so that an emf of the desired magnitude is induced and can be made available for commercial use. The details regarding the various types of generators will be discussed at a later stage.

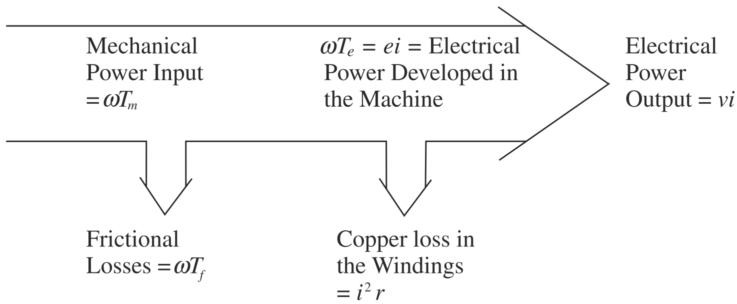


Fig. 1.36 *Power flow diagram of an elementary generator*

1.7.2 An Elementary Motor

The same arrangement of Fig. 1.35 is shown in Fig. 1.37. Here, instead of connecting a load across the coil, source of supply, i.e., a battery has been connected across it and the coil is not being rotated by a prime mover. The current flowing through the armature coil will produce a torque on the coil which will cause the coil to turn.

For the position of the coil shown in Fig. 1.37 the magnetic field produced by the coil carrying current is at 90° with the main field axis. Its North pole will be on the left-hand side and the South pole will be on the right-hand side. Thus the coil will turn in the anticlockwise direction by 90°. The two magnetic fields will then align with each other. Therefore, continuous rotation of the coil will not he obtained. Some special arrangement (brush and commutator arrangement) is made to achieve continuous rotation of the rotor which will be discussed in the sections to follow. For the timebeing, it is assumed that the coil is rotating continuously. The direction of emf induced in the coil for its anticlockwise direction is shown in Fig. 1.37. It may be noted that in the case of the motor, the directions of induced emf and current through the armature coil are opposite to each other, whereas in the case of the generator they are in the same direction. See Figs 1.37 and 1.35 for comparison.

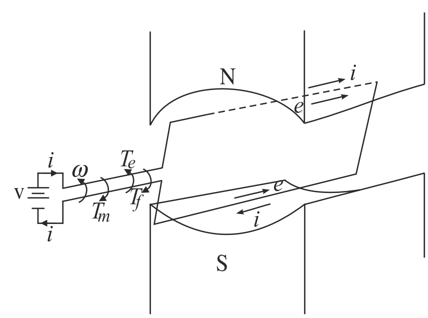


Fig. 1.37 *An elementary motor*

Considering that the coil is rotating continuously at an angular velocity ω, the following equations can he derived:

$$V - e = ir$$

where induced emf e is in opposite direction to that of applied voltage V. Multiplying both sides by i

or, $$Vi = ei + i^2 r$$

Power supplied = Electrical power being available for conversion to mechanical power + Copper loss.

Electrical power $$ei = \omega T_e$$

A portion of ωT_e is lost to overcome friction. The rest is available for supplying mechanical output, i.e., to carry the mechanical load on the motor.

$$T_e = T_m + T_f$$

or
$$\omega T_e = \omega T_m + \omega T_f \tag{1.9}$$

Thus the power flow diagram for an electrical motor can be represented as shown in Fig. 1.38.

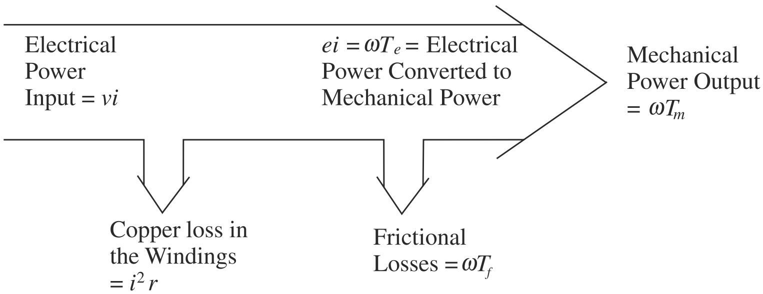


Fig. 1.38 *Power flow diagram of an elementary motor*

The following differences are found between a generator and a motor:

1. In the case of generator the directions of e and i are the same, whereas in the case of motor they are opposite. Thus it may be concluded that a circuit in which e and i are in the same direction acts as a source of electrical energy, whereas a circuit in which e and i are in opposition to each other acts as a consumer of electrical energy.
2. The direction of frictional torque T_f is opposite to the direction of rotation, both in the case of a generator and a motor.
3. For a generator, ω and T_e are in the opposite directions whereas they are in the same direction in the case of a motor.

The reader is advised to work out the following exercises before proceeding to the next section.

EXERCISES 1C

1.1 Figure 1.39 shows the cross-sectional view of a rectangular coil placed on the rotor and being rotated in a magnetic field. Show the instantaneous direction of emf induced in the coil. Also, show the direction of electromagnetic torque developed T_e, applied mechanical torque T_m and frictional torque T_f when the coil is connected to a load.

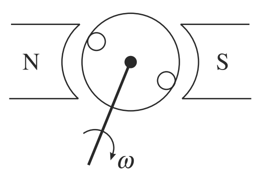


Fig. 1.39

1.2 Figure 1.40 shows the cross-sectional view of a rectangular coil rotating in a magnetic field. The direction of rotation of the coil and the direction of current in the coil sides are shown.

(a) Determine the direction of induced emf in the coil sides.

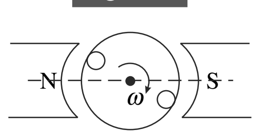


Fig. 1.40

(b) Show the direction of electromagnetic torque developed and also the torque angle.
(c) Show the direction of frictional torque.
(d) Write the relationship for mechanical, frictional and electromagnetic torques.

1.3 Figure 1.41 shows the cross-sectional view of a rectangular coil placed in a magnetic field created by electromagnets. The coil is rotated in anticlockwise direction. Show the direction of emf induced in the coil and also the direction of electromagnetic torque developed on the rotor when the coil is connected across a load resistance.

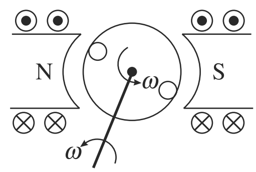


Fig. 1.41

1.8 CLASSIFICATION OF MAIN TYPES OF ELECTRICAL MACHINES

Electrical machines can be classified mainly into direct current machines and alternating current machines. Alternating current machines can further be classified into induction machines and synchronous machines. Alternating current machines are manufactured for both single-phase and three-phase operations. In this section, brief explanations of these main types of electrical machines have been included. The principle of working of the machines has been explained on the basis of alignment of magnetic fields.

1.8.1 dc Machines

Figure 1.42 shows the cross-sectional view of a dc machine. For simplicity, only the main component parts have been shown. The field windings are excited from a dc source. The polarity of the magnetic poles will depend upon the direction of field current and will be as indicated in Fig. 1.42. The armature is shown inside the field system. The armature carries conductors inside slots. Two brushes are shown placed at right angles to the main field axis. The brushes are stationary whereas the

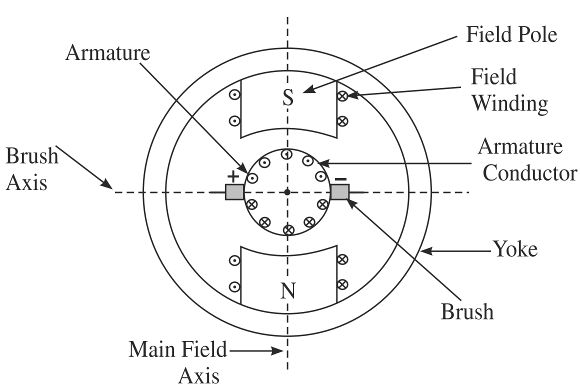


Fig. 1.42 *Cross-sectional view of a dc machine*

armature is free to rotate. When the armature is rotated in the magnetic field, emf will be induced in the armature conductors. The direction of the induced emf is found out by applying Fleming's right-hand rule and will be as shown in Fig. 1.42 through crosses and dots. The direction of induced emf will depend upon the direction of rotation if the field polarity remains unchanged. Brushes are placed at an angle of 90° with the main pole axis.

When a load (say a resistance) is connected across the armature terminals, current will flow through the armature circuit. The direction of the armature current will be the same as that of the induced emf. The armature may now be considered as an electromagnet and its polarities will be as shown in Fig. 1.43. The electromagnetic torque T_e will be developed in the anticlockwise direction as the stationary North and South poles of the field will attract the South and North poles of the armature. The magnitude of T_e will depend on the strength of the field poles and also on the strength of the armature field. The strength of the field poles and armature fields will depend upon the current flowing through the field windings and armature windings respectively. As the external load on the generator is increased, the magnitude of the armature current and hence the strength of the armature field will increase. This will in turn cause an increase in T_e. As T_m acts in a direction opposite to the applied torque T_a (which keeps the rotor rotating), with increase in load, more and more torque should be applied through the primemover to keep the armature rotating at a particular speed. For the direction of rotation shown in Fig. 1.43 the direction of current in all the conductors on the upper side of the armature will have dot currents, whereas all the conductors in the lower half of the armature will have cross currents and brush *A* will always collect cross currents as the armature continues to rotate in the clockwise direction as shown in Fig. 1.43. In the output circuit when connected across terminals *A* and *B*, current will flow in one direction.

The dc machine shown in Fig. 1.43 is working as a generator. The same machine will work as a motor if the armature is provided with electric supply. Figure 1.44 shows the cross-sectional view of a dc machine working as a motor. The armature is connected across a supply voltage *V* and the field windings are excited. The direction of current in conductors 1, 2, 3 and 4 will be inward, whereas in conductors 1′, 2′, 3′ and 4′ the direction of current will be outward. Note that 1 and 1′ are the two sides of coil 1–1′, 2 and 2′ are the two sides of coil 2–2′, and so on. The polarities developed in the armature will be as shown in Fig. 1.44. Electromagnetic torque T_e will be developed in the anticlockwise direction as the opposite poles of the armature and the stator field will attract each other. The armature will rotate in the anticlockwise direction due to the torque developed.

To reverse the direction of rotation of the armature, either the direction of current in the field winding or in the armature winding will

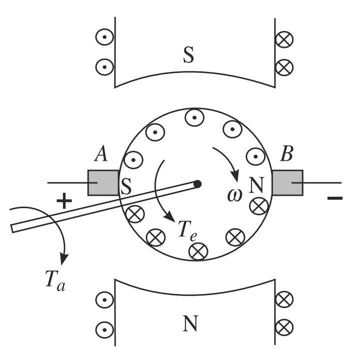


Fig. 1.43 *Electromagnetic torque, Te developed in the generator is in opposite direction of applied torque, T_a*

have to be reversed. If the directions of current in both the armature and field windings are reversed, the direction of rotation of the armature will remain unchanged.

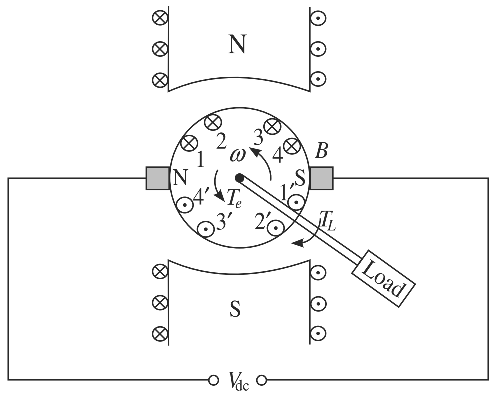


Fig. 1.44 *A dc machine working as a motor. Electromagnetic torque T_e produces rotation of the rotor and drives the load*

As the mechanical load on the rotor shaft represented by load torque T_L is increased, more and more electromagnetic torque will be developed by the armature to balance the mechanical torque requirement for which the armature will draw more current from the supply mains.

1.8.2 Synchronous Machines (Single Phase)

It was seen earlier that in a dc motor, direct current supply is provided in both field and armature windings. The polarity of the armature field is fixed at the brush axis and unidirectional torque is produced.

It is now assumed that instead of direct current, alternating current is supplied to the armature terminals of a dc machine, as shown in Fig. 1.45(a).

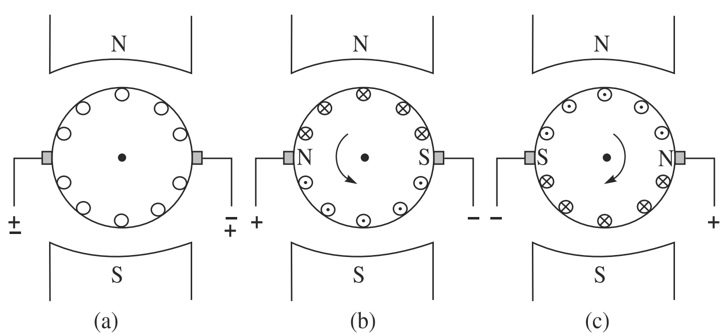


Fig. 1.45 *Effect of connecting ac supply across the armature terminals of a dc machine (a) ac supply to armature; (b) at positive half cycle of supply; (c) at negative cycle of supply*

It can be observed that as the supply polarities to the armature alternates, torque in the opposite direction is developed in every half-cycle [see Figs. 1.45 (b) and (c)].

The average torque therefore would be zero and hence the device cannot give any mechanical output. Thus in a dc machine, if ac is supplied to the armature terminals, the average torque for a complete cycle of supply will be zero.

Now consider what happens when ac is provided in the rotor circuit through slip rings and not through a brush and commutator arrangement as has been shown in Fig. 1.45. Figure 1.46 shows an armature having four coils. All the coils are connected in series and the extreme two coil-ends are connected to two slip-rings. Two brushes are fixed on the slip-ring. As the slip rings are fixed on the shaft, they also rotate when the armature rotates. The brushes, however, are stationary.

When an ac supply is connected across the brush terminals, the brushes will have alternate positive and negative polarities in every half-cycle of ac supply. When brush A is positive and brush B is negative, currents in coil sides 1, 2, 3 and 4 will have cross-currents and coil sides 1′, 2′, 3′ and 4′ will have dot currents, which are shown in Fig. 1.46 (b). The rotor will develop an anticlockwise torque and turn by only 90° when North and South poles of the rotor will respectively face the South and North poles of the main field. At that moment if the polarities of the brushes are reversed, the current in the coil sides will change their directions and therefore it can be seen that the rotor will rotate by 180°. Thus, again if the polarities of the brushes are changed, the rotor will continue rotating in the same direction by another 180°. Thus,

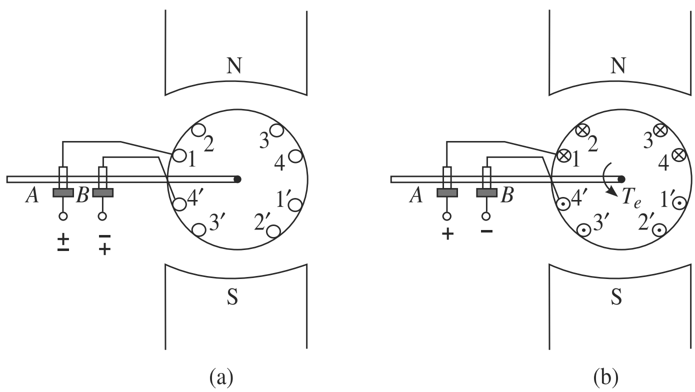


Fig. 1.46 *AC supply connected across the armature terminals through brush and slip-ring arrangement*

when ac supply is given to the rotor windings through slip rings and the current in the rotor winding reverses its direction in a time in which a coil side moves from one pole region to the next pole region, then the torque developed will be unidirectional. Since the rotor will develop unidirectional torque only when the rotor rotates at a speed N_s called the synchronous speed, the motor will have no starting torque. (It was seen above that the rotor speed is called the synchronous speed when the rotor during half-cycle of supply rotates by 180° electrical. Thus in one cycle of supply the rotor rotates by 360° electrical. For a two-pole arrangement 360° electrical is equal to 360° mechanical.)

For the benefit of the reader, a relationship between electrical degree and mechanical degree is given below.

In Figs. 1.47(a), (b) and (c) are shown two-pole, four-pole and six-pole stators. The distance between the two adjacent poles is always taken as 180° electrical.

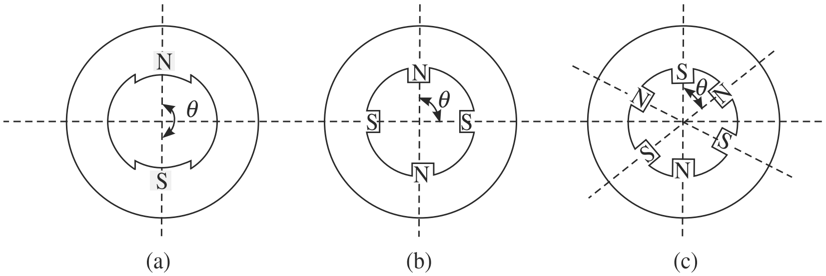


Fig. 1.47 *AC supply connected across the armature terminals through brush and slip-ring arrangement*

But the mechanical degrees covered are 180, 90 and 60 respectively in the case of a two-pole, four-pole and six-pole machine. Thus the following relation can be written for electrical and mechanical degrees which depends on the number of poles as:

$$1^\circ \text{ mechanical} = \frac{P^\circ}{2} \text{ electrical}$$

Thus, when $P = 2$

$$1^\circ \text{ mechanical} = 1^\circ \text{ electrical}$$

When $P = 4$

$$1^\circ \text{ mechanical} = 2^\circ \text{ electrical}$$

When $P = 6$

$$1^\circ \text{ mechanical} = 3^\circ \text{ electrical, and so on}$$

We know the relations

$$1^\circ \text{ mechanical} = \frac{P^\circ}{2} \text{ electrical}$$

$$1^\circ \text{ electrical} = \frac{2^\circ}{P} \text{ mechanical}$$

$$180^\circ \text{ electrical} = \frac{P}{2} \times 180^\circ \text{ mechanical}$$

In electrical machines the rotor speed is called synchronous speed when during a half-cycle of supply the rotor rotates by 180° electrical, i.e., by $\frac{2}{P} \times 180^\circ$ mechanical,

or In I cycle, the rotor rotates by $\frac{4}{P} \times 180^\circ$ mechanical,

or In f cycle, the rotor rotates by $\frac{4f}{P} \times 180^\circ$ mechanical.

As the supply frequency is f cps, the rotor speed is

$$N_s = \frac{4f}{P} \times 180/P^\circ \text{ mechanical}$$

or
$$N_s = \frac{4f \times 180 \times 60}{P \times 360} = \frac{120f}{P} \text{ rev/min}$$

(As 360° mechanical =1 revolution and 60 s = 1 min)

∴
$$N_s = \frac{120f}{P}$$

where N_s is the synchronous speed in rpm
f is the supply frequency in Hz
P is the number of poles

In the arrangement of a motor shown in Fig. 1.46, the windings have been shown rotating and the field poles as stationary. But the motor will also work if the windings are placed on the stator and the field poles on the rotor. Placing the windings on the stator has a number of advantages which will be discussed later.

1.8.3 Synchronous Machine (Three Phase)

Smaller ac machines are generally wound for single phase. High rating ac machines are usually wound for three phase due to the following reasons:

1. It is more economical to transmit and distribute three-phase power. For generation of three-phase power, three-phase generators are required.
2. For the same weight and volume, three-phase machines give more output.

Like other types of machines already discussed, torque in a three-phase synchronous machine is also produced due to the tendency of two magnets to align with each other. In a three-phase synchronous machine, the stator carries a three-phase winding and the rotor has dc poles excited through dc, supplied with the help for two slip rings (see Fig. 1.48). When the poles are rotated, emf is induced in the stator windings.

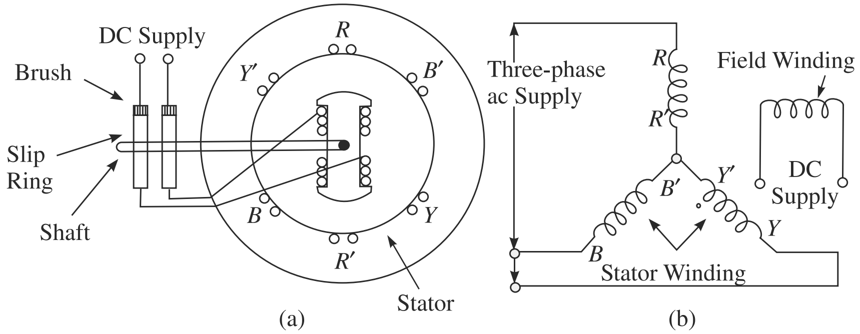


Fig. 1.48 *Simplified diagram of a three-phase synchronous machine*

In motoring operation, when a three-phase supply is applied across the stator terminals, a rotating magnetic field is produced which is rotating at synchronous speed with respect to the stator. The rotor magnet, in trying to align itself with the rotating magnetic field, will also rotate at synchronous speed in the same direction in which the stator field is rotating. At 50 Hz supply, the speed of the rotating field produced will be high and therefore the rotor due to its inertia will have no starting torque. But if the rotor is given an initial torque to bring its speed near to synchronous speed, it will speed up to rotate at synchronous speed. As mentioned earlier, the rotor field is produced by applying dc to the field winding through slip rings. How a rotating field is produced when a polyphase supply is connected across a polyphase winding is explained systematically as follows:

Production of Rotating Magnetic Field When current flows through a winding, a magnetic field is produced. Winding can be made in different ways such as:

1. Single-phase winding,
2. Two-phase winding, and so on.

Supply voltage can also be made available as:

1. Single-phase supply,
2. Two-phase supply, and
3. Three-phase supply.

In the following pages the nature of the magnetic field produced will be investigated when

1. Single-phase supply is applied to a single-phase winding,
2. Two-phase supply is applied to a two-phase winding, and
3. Three-phase supply is applied to a three-phase winding.

To start with, it is necessary to identify the various types of windings and supply systems.

Figure 1.49(a) shows a cross-sectional view of a hollow cylindrical core with a few slots on its inner side. The slots are provided to place coils inside them. Figure 1.49(c) shows a coil. The coil has two coil sides or conductors. The coil sides are shown placed inside two slots of the core in Fig. 1.49(b).

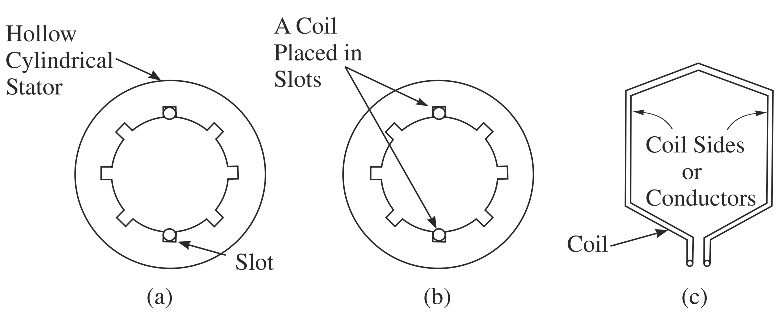


Fig. 1.49 *A coil placed in slots of a hollow cylindrical core*

Figure 1.50(a) shows three coils placed inside slots. The three coils are then connected in series as is shown in Fig. 1.50(b). Two free terminals are left open. In a winding, any number of slots and any number of coils can be used depending on the need. Whatever may be the number of coils and slots, the coils are connected in series and thus only two free end terminals are left out. Such a winding is called a single-phase winding.

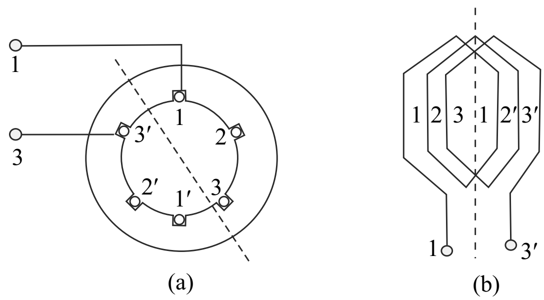


Fig. 1.50 *A single-phase winding made with three coils 1–1′, 2–2′, 3–3′, connected in series and placed in slots*

A single-phase winding can be wound on any type of core. It was seen earlier how a single-phase winding was made on a hollow cylindrical core.

Figure 1.51 shows a solid cylindrical core with a number of slots on its outer periphery. Coils are placed in the slots and are connected in series (a series connection is one where the end of one coil is connected to the beginning of the next coil). Here too, only two free terminals are left out. It is to be noted that here only three coils have been used. A single-phase winding can be made with only one coil also as shown in Fig. 1.52. But in practice a single-phase winding will have many coils.

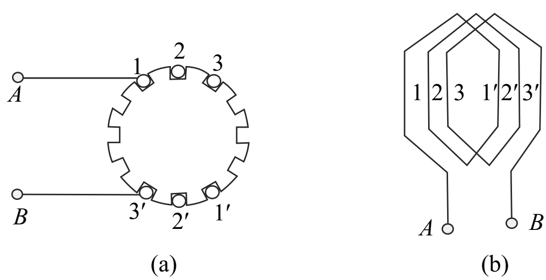


Fig. 1.51 *A single-phase winding of three coils placed in slots of a solid cylindrical core*

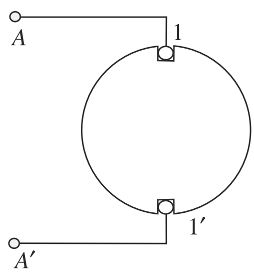


Fig. 1.52 *A single-phase winding made with only one coil, 1-1′*

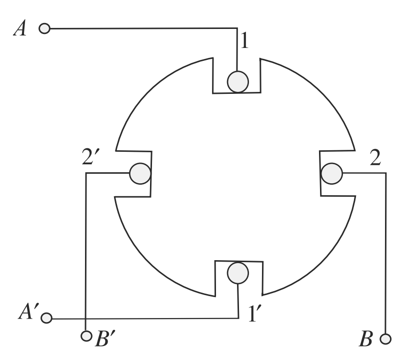


Fig. 1.53 *A simple two-phase winding with two coils placed at 90° with respect to each other*

A two-phase winding consists of two separate single-phase windings placed at an angle of 90° with each other (see Fig. 1.53). For simplicity, in each phase only one coil has been shown. Phase *A* is placed at right angles with phase *B*. For each phase two separate sets of terminals are available. It may be noted that the two-phase windings are insulated from each other. Two-phase windings are displaced in space at an angle of 90° from each other.

Figure 1.54 shows a two-phase winding with a hollow cylindrical core with two coils in each phase. It is to be noted that here also the two phases are displaced in space phase by an angle of 90°. This is evident when the angle of separation of either the beginning or the end of the two-phase windings is noticed.

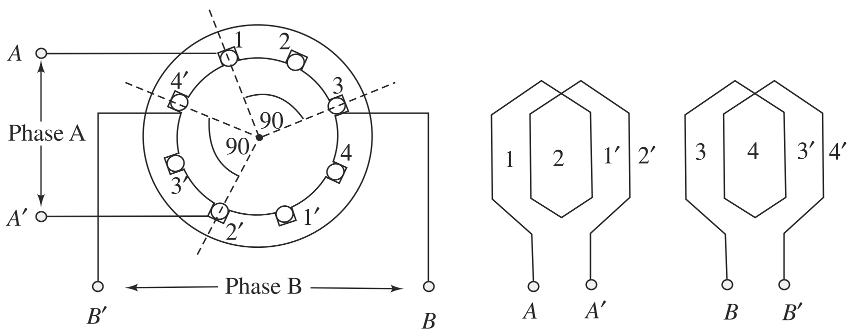


Fig. 1.54 *A two-phase winding having two coils per phase*

A three-phase winding consists of three separate single-phase windings. A simple three-phase winding is shown in Fig. 1.55. Three-phase windings are shown star-connected with three free terminals *A*, *B* and *C*.

Only one coil is shown for each phase. The three phases are displaced in space by an angle of 120° as may be seen from the beginning of each phase. Phase *B* starts after 120° from phase *A*. Phase *C* starts after another 120° from phase *B*. There will be six free terminals. The similar terminals of each phase may be connected together to form a star connection. There are thus three free terminals left for external connection.

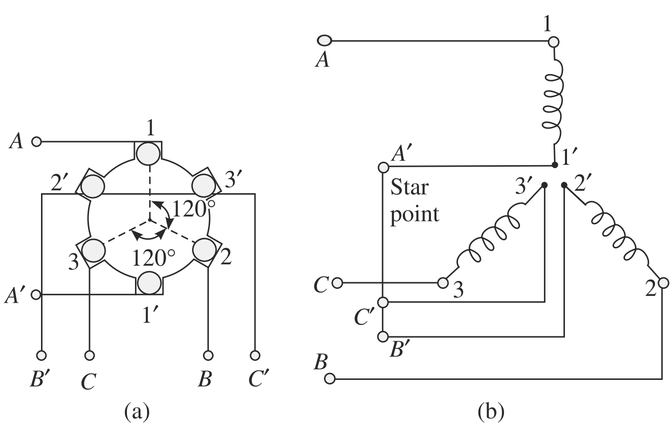
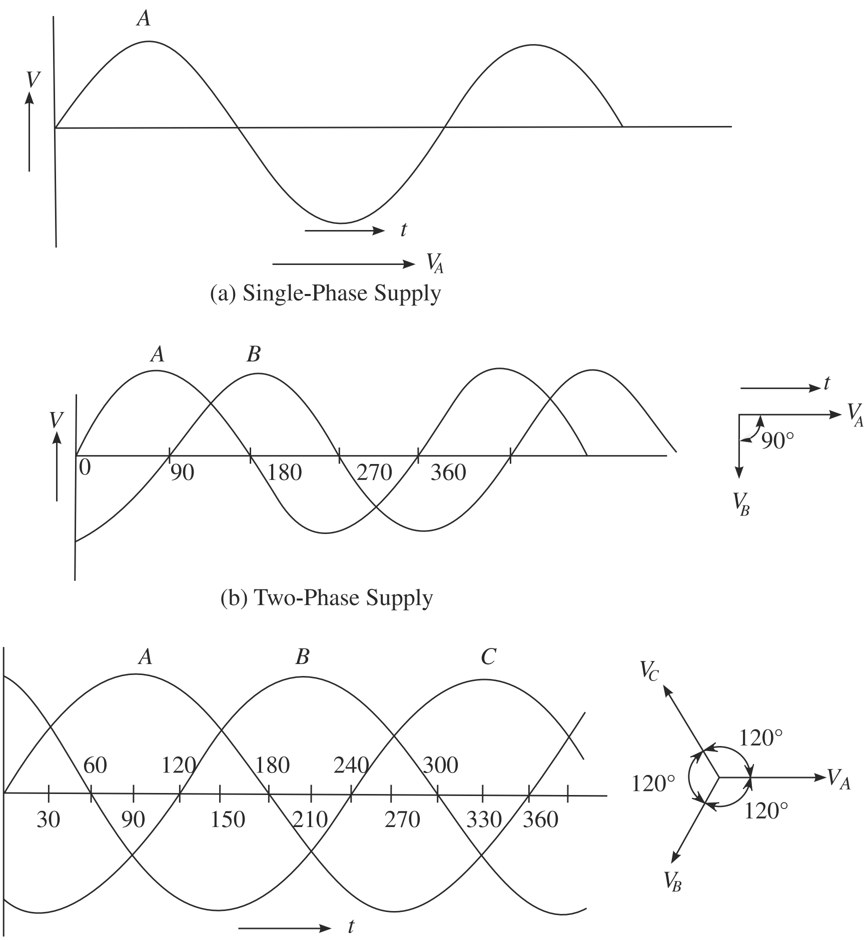


Fig. 1.55 *A simple three-phase winding with only one toil per phase*

It was seen that windings could be made for one-phase, two-phase or three-phase depending on the requirement. If the number of phases are more than one, it is called a polyphase winding.

Like windings, there are also different kinds of supply voltages, such as single-phase supply, two-phase supply and three-phase supply. In Figs. 1.56(a), (b) and (c), three types of supply voltages are shown. In a single-phase supply, only one voltage varies with time. In a two-phase supply, two voltages vary with time but have a time-phase difference of 90° between the phase voltages. In a three-phase supply, there are three time varying voltages displaced in time phase by 120° from each other. It is to be noted that in each case it was assumed that the voltages vary sinusoidally with time. In fact, the voltages supplied by the electricity authority vary sinusoidally with time. For the benefit of the readers, procedure for drawing the waves of a three-phase supply is explained as follows:



(a) Single-Phase Supply

(b) Two-Phase Supply

(c) Three-Phase Supply

Fig. 1.56 *Graphical representation of different kinds of supply voltage*

It is known that a three-phase supply consists of three voltages displaced in time-phase by 120° from each other. The voltage of phase *A* is drawn starting from the origin [see Fig. 1.57(a)]. The voltage of phase *B* will start after a time interval of 120°. So the voltage of phase *B* is drawn 120° ahead on the time axis [see Fig. 1.57(b)]. It may be observed that 90° make a quarter of cycle. Now extend the wave shape of *B* backwards to zero time limit [see Fig. 1.57 (c)]. The voltage of phase *C* will start 120° after the voltage of phase *B*. In other words, the voltage of phase *C* will start after a time interval of 240° from the voltage of phase *A* [see Fig. 1.57 (d)]. Now extend the wave shape of *C* backwards to zero time limit. The three voltages can be shown vectorially as shown in Fig. 1.57(e).

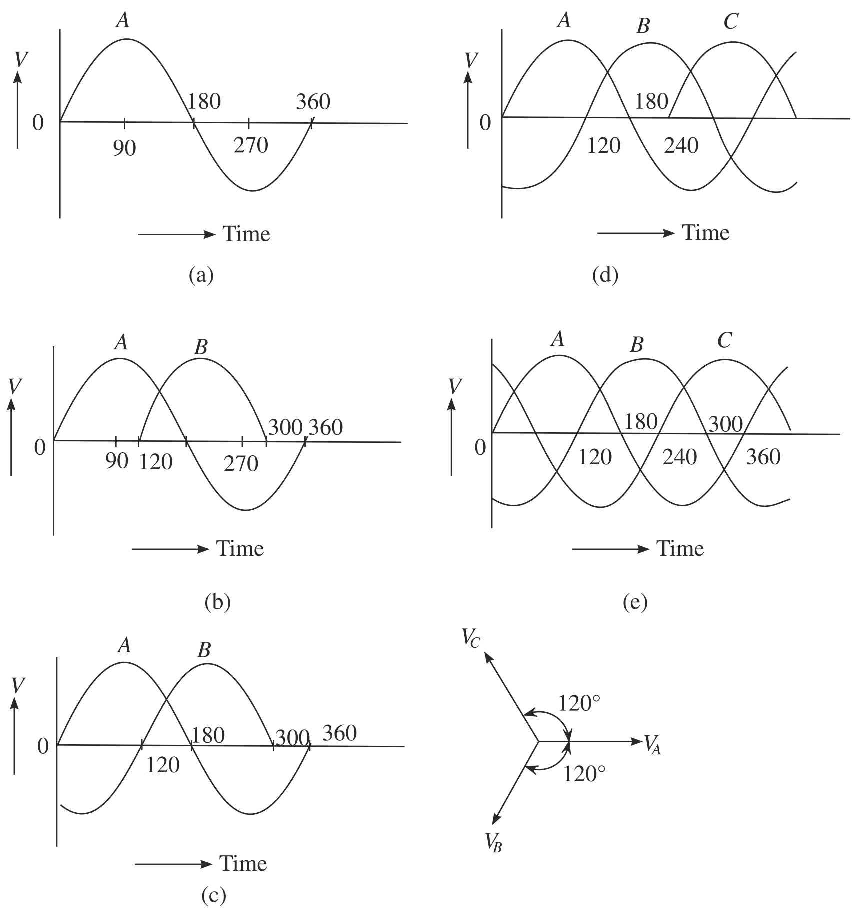


Fig. 1.57 *Method of drawing three-phase supply voltages*

The purpose of studying the different types of windings and the types of supply systems was to investigate the nature of the magnetic field produced when,

1. Single-phase supply is applied to a single-phase winding
2. Two-phase supply is applied to a two-phase winding
3. Three-phase supply is applied to a three-phase winding

It may be recalled that the aim is to produce a rotating magnetic field by some means where to mechanical work will be involved.

First, the nature of the field produced will be investigated when a single-phase supply is applied to a single-phase winding.

Figure 1.58(a) shows a single-phase supply. Figure 1.58(b) shows a single-phase supply applied across a single-phase winding. An alternating current will flow through coil 1–1′. When the voltage is positive, current will enter the coil through coil-end A and leave through end A'. When the voltage is negative the reverse will happen. As the voltage alternates, the current through the coil will also alternate.

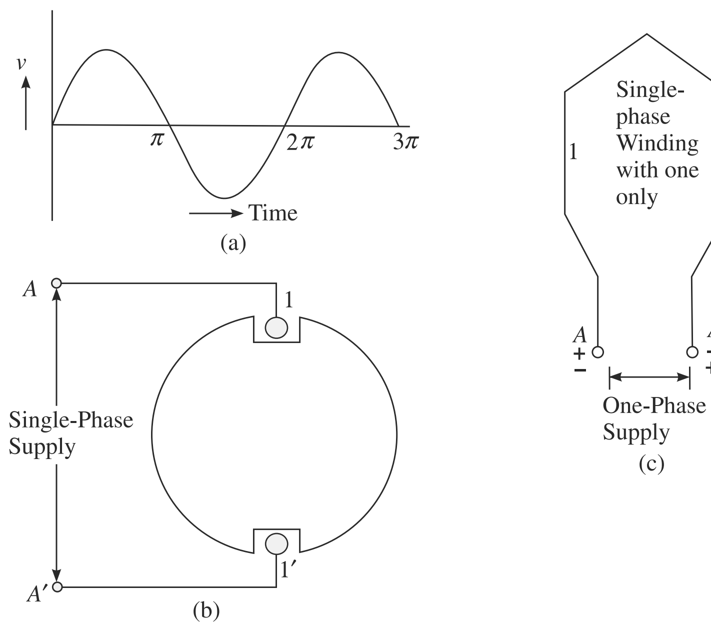


Fig. 1.58 *Single-phase supply connected across a single-phase winding (a) Single-phase supply; (b) Single-phase given to a single-phase winding placed in rotor slots (e) Single-phase winding with one coil and supply.*

Refer to Fig. 1.59, for time 0 to π, the current is positive. Current enters through coil side 1 and leaves through coil side 1′. The flux produced due to this current is also shown in the figure. The position of the magnetic polarities are also shown. In the next half-cycle from time π to 2 π the direction of current becomes opposite. The current enters through coil side 1′ and leaves through coil side 1. The flux lines and the positions of the magnetic poles are shown in Fig. 1.59(c). In the next half-cycle of current the positions of poles are again shown in Fig. 1.59(d).

It is noted that the magnetic poles just interchange their positions at intervals of one half-cycle. The strength of the magnetic poles, however, also change with time. With a single-phase supply applied across a single-phase winding, an alternating (also called pulsating) magnetic field is obtained. The aim was to produce a rotating magnetic field. But it was just seen that a single-phase supply when applied across a single-phase winding produces a pulsating magnetic field and not a rotating magnetic field.

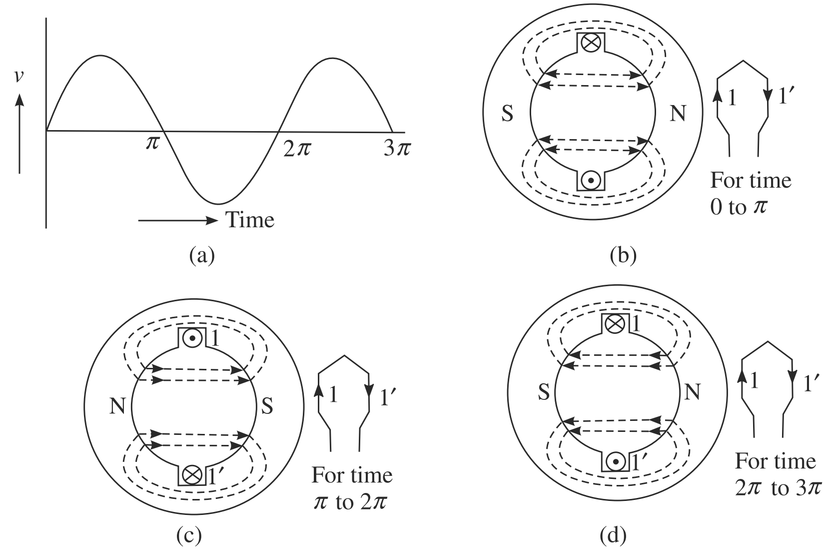


Fig. 1.59 *Nature of magnetic field produced when single phase supply is connected across a single-phase winding*

Now let the two-phase supply voltages as shown in Fig. 1.60(a) be applied across the two-phase winding as shown in Fig. 1.60(b). The voltage of phase A is connected across winding AA' and the voltage of phase B is connected across winding BB'. Two instants of time have been considered on the voltage wave. At instant of time t_1,

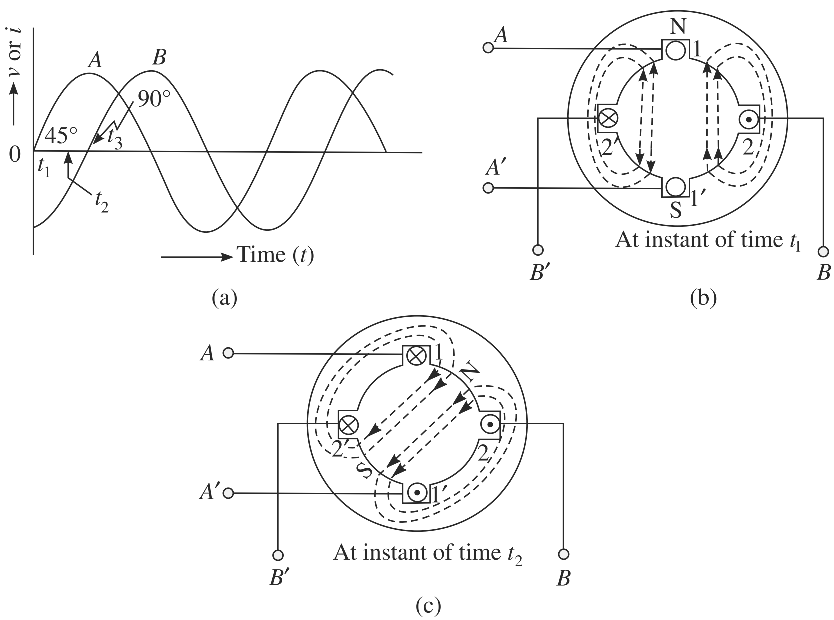


Fig. 1.60 *Resultant magnetic field produced when two-phase supply is connected across a two-phase winding*

the voltage of phase A is zero [see Fig. 1.60(a)]. There will be no current flowing through phase A at that instant. However, the voltage of phase B is maximum but negative at that instant. Negative voltage will cause the current to enter through terminal B' and leave the coil through terminal B. (If the voltage was positive the situation would have been just opposite.) Therefore, at instant of time t_1, there will be current flowing through phase B only. The flux distribution and the positions of the magnetic poles have been shown in Fig. 1.60(b).

At instant of time t_2, the voltage in phase A is positive and the voltage in phase B is negative. The current distribution, the resultant flux distribution due to the currents in the windings and the resultant magnetic field have been shown in Fig. 1.60(c). It is to be noted that the magnetic field axis has been shifted in the clockwise direction in Fig. 1.60(c) with respect to that in Fig. 1.60(b). The flux distribution and the position of the magnetic poles at different instants of time of the voltage wave have been shown in Fig. 1.61. It is seen that for one half-cycle of current flow (from time t_1 to time t_2) through the windings, the resultant magnetic field rotates by 180°. On drawing for one complete cycle it can be seen that for one complete cycle of current flow through the windings the resultant magnetic field will rotate by one revolution.

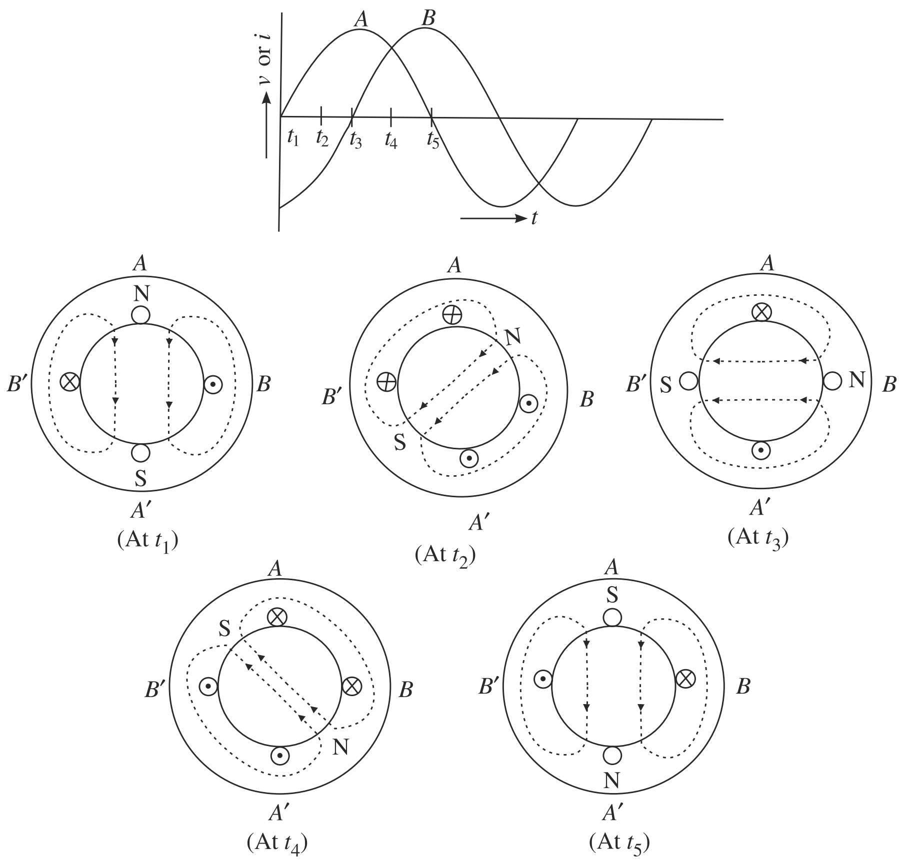


Fig. 1.61 *A rotating magnetic field produced when a two-phase supply is connected across a two-phase winding*

The flux distribution at different instants of time have been drawn separately for ease of understanding. From the above it can be concluded that: when a two-phase supply is applied across a two-phase winding a rotating magnetic field is produced.

For one cycle of current flow, the magnetic field rotates by one revolution. It is known that the supply frequency is 50 Hz. This means that the supply voltage makes 50 cps. For 50 cps of supply voltage, the resultant field will rotate by 50 rev/s. When a 50 cps supply voltage is applied, the resultant magnetic field produced will rotate by 50 × 60 = 3000 rev/min (60 s = 1 min). Therefore, the speed of rotation of the rotating magnetic field, also called synchronous speed, depends upon the supply frequency. The direction of rotation of the magnetic field will be reversed if the connections of the phase windings with the supply terminals are interchanged. The reader is advised to try this out.

It will now be seen what type of magnetic field is produced when a three-phase supply is applied across a three-phase winding. For this, different instants of time on the voltage waves will be considered and the current directions in each phase winding marked. Then the resultant flux lines will be drawn and the magnetic polarities marked. In Fig. 1.62(a) is shown a three-phase supply voltage wave.

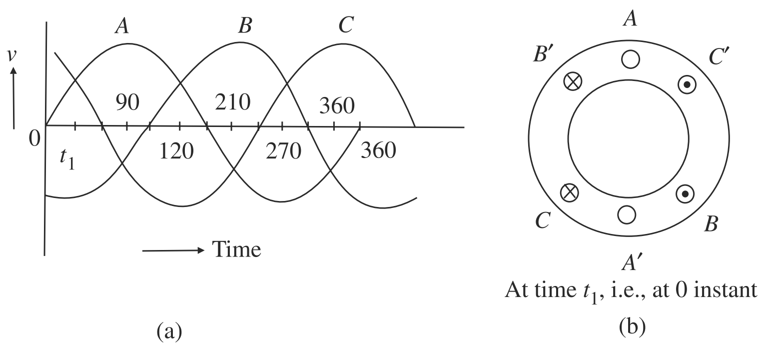


Fig. 1.62 *(a) A three-phase supply (b) A three-phase winding carrying current at time t_1*

At instant of time t_1, the voltage of phase is zero, the voltage of phase B is negative and the voltage of phase C is positive. The voltage of each phase when connected across the phase windings will cause current to flow through the three-phase windings shown in Fig. 1.62(b). As the voltages of the phases are different at different instants, the currents flowing through each phase winding will also be different at different instants of time. The current flowing through all the phases will give rise to a resultant magnetic field. The current directions in each phase winding may be represented when the three-phase voltages shown in Fig. 1.62(a) are applied across the three-phase windings, shown in Fig. 1.62(b). Consider an instant of time t_1. As the voltage of phase A is zero, there will be no current through the coil of phase A. In phase B the voltage is negative. This will cause current to flow in the reverse direction through the coil of phase B this means that the current in the coil of phase B will enter through B' and leave the coil through end B. This is shown in Fig. 1.62(b). In phase C, the voltage is positive. Therefore, current will enter through end C and leave the coil through end C'. Once the current directions are shown on the conductors, it is easy to draw the lines of force around them. It is to be noted that an independent field cannot exist around each conductor due to its current. There will be a resultant field due to the combined effect of the currents flowing through the conductors of all the phases. For finding out the direction of the flux lines, the corkscrew rule will be applied. The resultant flux will encircle all the conductors carrying current in the same direction. The resultant magnetic field produced when a three-phase supply is connected across a three-phase winding is shown in Fig. 1.63. In Figs. 1.63(b) and (c), only a few flux lines have been shown. To mark the positions of North and South poles the area from where the flux lines leave the magnet body may be observed and designated as North pole. In the South pole the flux lines will enter the magnet body. It may be seen that the flux lines do not cross each other.

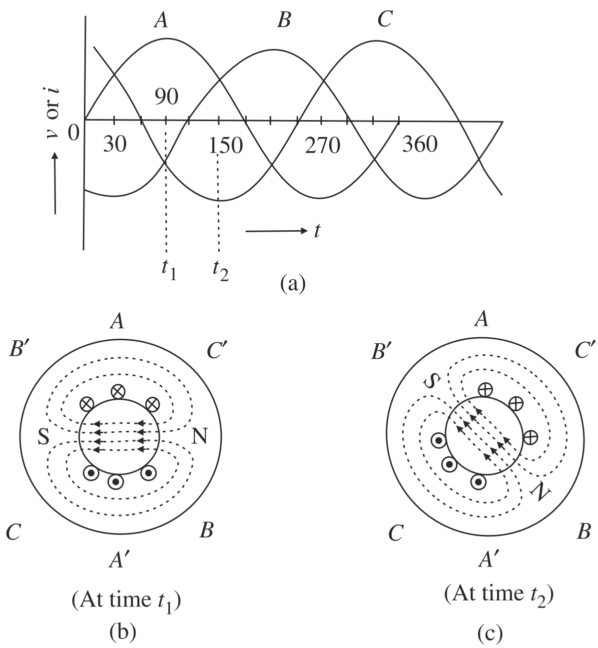


Fig. 1.63 *Nature of magnetic field produced when a three-phase supply is connected across a three-phase winding*

It is to be noted that the positions of the magnetic poles have been shifted from positions shown in Fig. 1.63(b) to that shown in Fig. 1.63(c). Figure 1.63(b) represents the positions of the poles at time t_1 and Fig. 1.63(c) at time t_2. For a lapse of time $(t_2 - t_1)$ the poles have rotated in the clockwise direction. Here, only two limits have been shown but in fact the rotation has been gradual. This can be seen by considering more intervals of time in between t_1 and t_2. At instants of time t_1, t_2, t_3, t_4, and t_5 the current distribution in the three-phase windings, the resultant lines of force and the positions of the poles have been shown in Fig. 1.64. Instants of time t_1 to t_2 make one complete cycle. For one complete cycle of current flowing through the three-phase windings, the resultant magnetic field has rotated by one revolution. It is to be noted that only five instants of time have been considered. If more instants of time in one

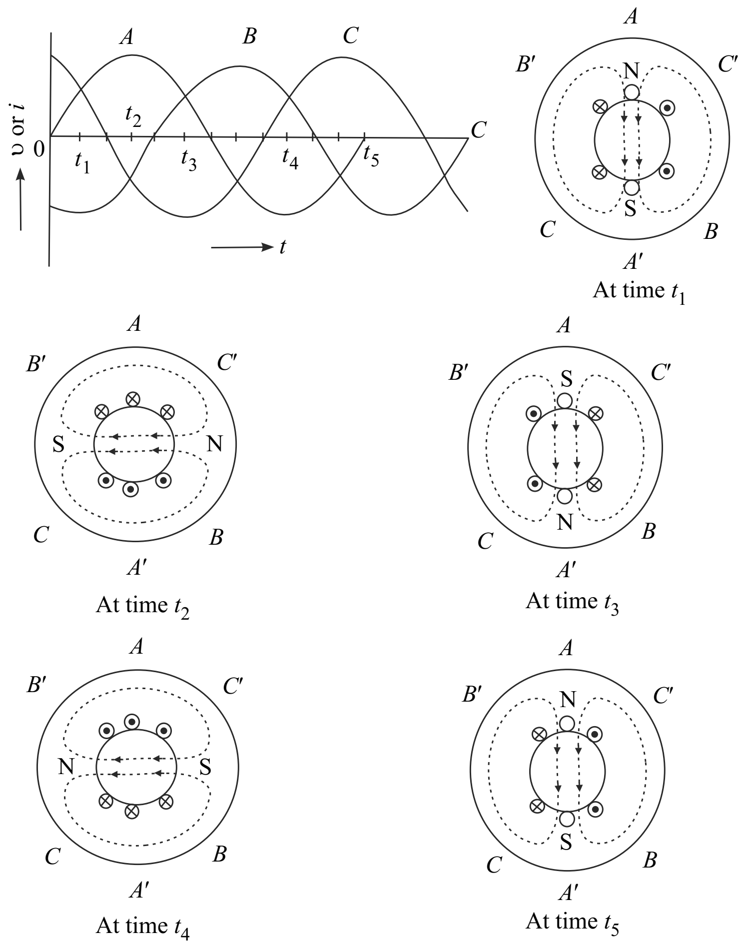


Fig. 1.64 *A rotating magnetic field produced when a three-phase supply is connected to a three-phase winding*

cycle could be considered, it would be seen that the field rotates gradually and not in steps. Thus when a three-phase supply is applied across a three-phase winding, a rotating magnetic field is produced. The rotating field will rotate at synchronous speed. To summarise

1. When a single-phase supply is applied across a single-phase winding, a pulsating or alternating magnetic field is produced.
2. When a two-phase supply is applied across a two-phase winding, a rotating magnetic field is produced.
3. When a three-phase supply is applied across a three-phase winding, a rotating magnetic field is produced.
4. In general, when a polyphase (more than one) supply is applied across a polyphase winding, a rotating magnetic field is produced.
5. The rotating magnetic field produced rotates at synchronous speed.

6. The direction of rotation of the rotating magnetic field can be reversed by interchanging the connections of any two-phase supply voltages applied across the windings.

Thus when a three-phase supply is applied across the three-phase stator windings of a synchronous machine of Fig. 1.48, a rotating magnetic field rotating at synchronous speed will be produced. The rotor, when excited, in trying to align itself with the stator rotating field, will rotate at synchronous speed. If the direction of rotation of the rotating field is reversed (this can be achieved by interchanging any two supply terminals to the motor terminals), the rotor will rotate in the reverse direction at synchronous speed. Although the rotor will rotate at synchronous speed, a torque angle will exist which is nothing but the angle between the stator and the rotor magnetic fields. When the rotor is rotating at synchronous speed, the two fields would be stationary with respect to each other, but there would exist a torque angle. For motoring action, the rotor field will lag the stator by the torque angle. When the rotor field is leading the stator field, the machine will be working as a generator.

1.8.4 Induction Machine (Three Phase)

Figure 1.65 shows the cross-sectional view of a three-phase induction machine. The stator has a three-phase winding and the rotor has a closed, i.e., short-circuited winding. When a three-phase supply is connected across the three-phase stator windings, a rotating magnetic field is produced. If the rotor rotates at synchronous speed (same speed as the rotating magnetic field produced by the stator) and in the same direction as the rotating magnetic field, there will be no emf induced in the rotor windings as there is no rate of change of cutting flux by the rotor. As there is no induced emf in the rotor winding, there will be nocurrent in that winding although the rotor winding is a closed one. Hence, there will be no torque developed on the rotor. Therefore, the machine cannot operate at synchronous speed. Now, assume that the rotor is rotated at a speed slightly lower than the synchronous speed. EMF will be induced and since the winding is closed, current will flow which will cause a torque developed by the rotor. The torque produced will increase when the difference between the rotor speed and the speed of the rotating magnetic field is increased. The torque developed by the rotor is due to the tendency of the stator field and the rotor-induced magnetic field to align with each other as shown in Fig. 1.66.

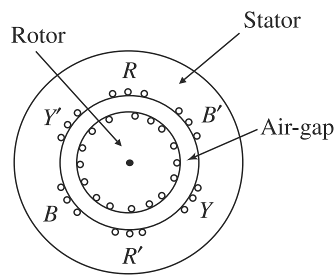


Fig. 1.65 *Cross-sectional view of a three-phase induction motor*

The direction of rotation of the rotor is shown clockwise. The rotor rotates in the same direction as the stator rotating magnetic field but at a lower speed. To understand this, assume that the stator field is stationary. Then the rotor may be assumed to be rotating in the anticlockwise direction at a speed equal to the difference in the speed of the rotating field and the rotor. The direction of the stator field at any particular instant of time is shown in Fig. 1.66(b) as F_s. Although the rotor is shown rotating in the clockwise direction, the rotor conductors on the right-hand side of the rotor may be assumed to be rotating in the anticlockwise direction with respect to a stationary stator field. In Fig. 1.66(b), only one conductor has been shown. The direction of the induced emf in this conductor can be determined by applying Fleming's right-hand rule. Thus the induced emf and current is the rotor conductors will be as shown in the figure assuming that the rotor circuit is resistive only. The direction of the rotor field will be downwards. The instantaneous polarity of both the stator and the rotor are also shown in the figure. Due to the tendency of the rotor magnetic field to be aligned with the stator field, the rotor will develop a torque in the same direction as the direction in which the rotor was rotated. As the stator field will rotate, the rotor will also continue to rotate in the same direction. As the developed torque is in the same direction as the direction of rotation of the field, the device acts as an induction motor. The rotor field is rotating at a speed of $N_s - N_r$ wrt the rotor. Thus the speed of stator field is N_s and that of rotor field is also N_s wrt the stator. These two fields are aligned with each other. However, there would exist a torque angle.

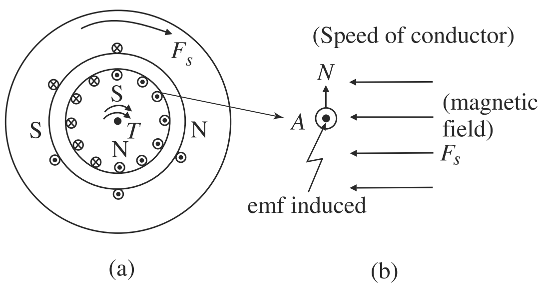


Fig. 1.66 *Torque developed by a three-phase induction motor in the direction of the rotating magnetic field*

If the rotor is rotated at speed higher than the synchronous speed, then the emf induced in the rotor conductors will be as shown in Fig. 1.67. The rotor rotates in the clockwise direction at a speed $(N - N_s)$ relative to the rotating magnetic field produced by the stator. The direction of the induced emf is determined by applying Fleming's right-hand rule. The direction of current and hence the positions of the instantaneous poles induced in the rotor are shown in the figure. The torque developed in this case is in a direction opposite to that of the direction of rotation of the rotor. Thus the device acts as an induction generator.

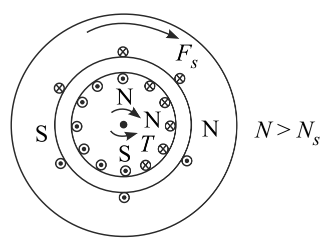


Fig. 1.67 *Induction motor working as an induction generator*

1.8.5 Induction Machine (Single Phase)

The construction of a single-phase induction machine is similar to that of a polyphase induction machine. On the stator, instead of a polyphase winding, a single-phase winding is provided. The rotor has a squirrel-cage construction.

Figure 1.68 shows a single-phase supply connected across the stator terminals. With the alternating current flowing in the stator coils, stator flux ϕ_s is stationary in space but pulsates in magnitude. Currents are induced in the rotor due to electromagnetic induction, these currents being in such a direction as to produce a flux ϕ_R which opposes the stator flux ϕ_s at every instant of time. Figure 1.68 shows the field produced by the stator current and the rotor-induced current when a single-phase supply is applied across the stator. ϕ_s is the flux produced by the stator mmf and ϕ_R is the flux produced by the rotor mmf. The directions of the two fields ϕ_s and ϕ_R for two consecutive half-cycles of stator applied voltage are shown in Figs. 1.68(a) and 1.68(b) respectively.

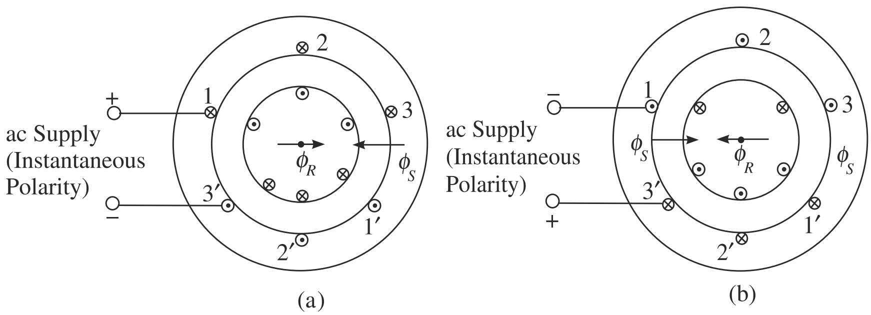


Fig. 1.68 *Emf induced in the rotor conductors by electromagnetic induction when the rotor is stationary*

It may be noted that as the axes of the two magnetic fields remain aligned all the time, no torque is developed on the rotor and hence the motor will not have any starting torque. The motor is merely a single-phase static transformer with a short-circuited secondary. When the rotor is given a rotation in any direction, an emf is induced in the rotor winding by virtue of its rotation in the stationary stator field. Figure 1.69 shows the direction of the induced emf in the rotor conductors due to its rotation in the stator field. The rotational voltage produces a component rotor current and a rotor flux whose axis is displaced in space by an angle of 90° (electrical) with the stator field axis. Thus a torque will be developed on the rotor. Detailed analysis will show that the direction of torque developed will be in the same direction as the direction of rotation given to the rotor and thus the rotor will continue to rotate in the same direction. Thus a single-phase induction motor, as such is not self-starting but would continue to rotated if an initial torque is provided.

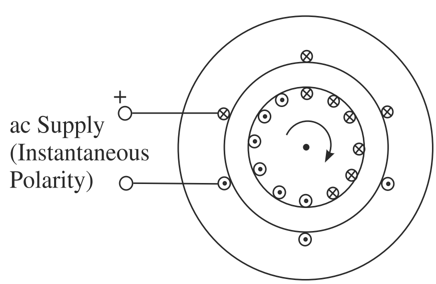


Fig. 1.69 *Shows the direction of induced emf in the rotor conductors when the rotor is given a rotation in clockwise direction*

The various types of field excitation used in the different electrical rotating machines, their voltage equations and a generalised torque equation are described in the following sections.

1.9 EXCITATION SYSTEMS IN ELECTRICAL MACHINES

A study of the performance of all rotating electrical machines includes of the study of the flux and mmf conditions due to stator and rotor ampere turns and the interaction between the two. In an electrical machine both the stator and the rotor have distributed windings or one of them has concentrated winding. The excitation system may be classified into the following categories:

1. Direct Current Excitation as in dc Machines and Synchronous Machines

In dc machines, the dc excited field poles are fixed on the stator and hence produce a stationary field where as in synchronous machines, the dc excited field system is fixed on the rotor and therefore the field is rotating in space. In dc machines, the field poles are projected type and are therefore called salient poles. In synchronous machines the field poles can be made either projected (salient) type or non-projected, i.e., cylindrical (non-salient) type. Figure 1.70 illustrates flux density waveform for salient poles with dc excitation in direct current machines and synchronous machines.

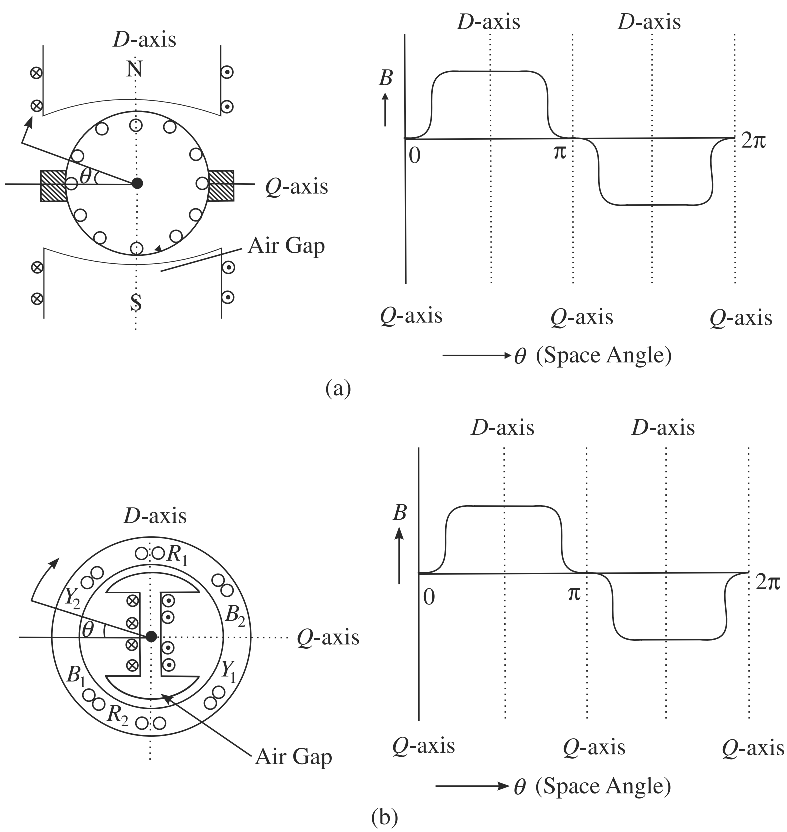


Fig. 1.70 *(a) Field form of salient poles with dc excitation (as in dc machine)*
(b) Field form of salient poles with dc excitation (as in synchronous machines)

It is seen that the air-gap reluctance varies from maximum at the Q-axis, i.e., at the inter polar region to minimum at the D-axis, i.e., at the polar region. The field form is therefore somewhat trapezoidal. By proper design of the pole shoes, the field form can be shaped and can be made more towards a sine wave. In machine analysis, the field form will be considered a sine wave, i.e., only the fundamental component will be considered neglecting the harmonic waves.

2. Non-Salient Poles with dc Excitation Figure 1.71 shows the field form of a distributed field winding excited from a dc source as in turbo alternators. The field form is a stepped one and its harmonic content is very small. The field form shown in Fig. 1.70 and Fig. 1.71 are of constant amplitude as long as the excitation current is constant. The equation for such a field can be written as $b_f = B_m \sin \theta$, where θ, is the space angle in electrical radians and is measured from the Q-axis, i.e., from the interpolar axis.

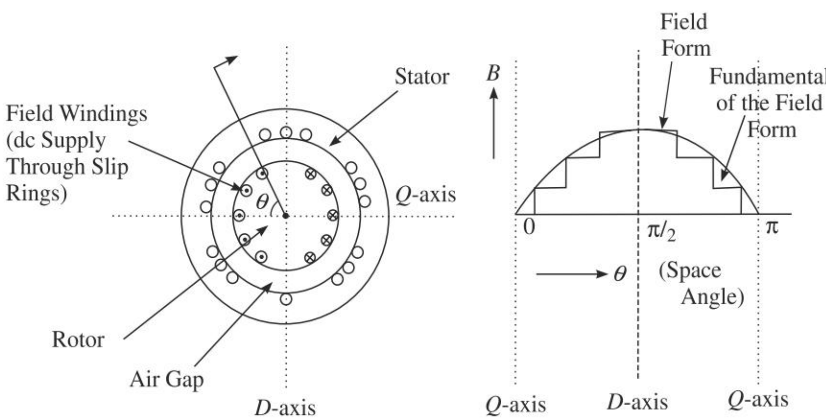


Fig. 1.71 *Field form of a distributed field winding with dc excitation (as in turbo alternators)*

3. Single-Phase ac Excitation Figure 1.72 shows the distributed stator winding excited from an ac source as in the case of single-phase induction machines. Since the winding is distributed, the field form will be a stepped one as shown in the figure.

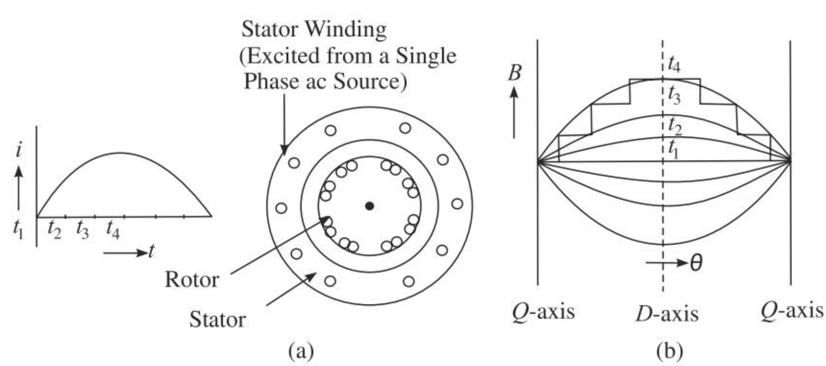


Fig. 1.72 *Field form of a distributed stator winding excited from an ac source (as in single phase induction machines)*

Further, for sinusoidally varying exciting current, the field form will change in its magnitude sinusoidally with respect to time having the wave shape more like a sine wave as shown in Fig. 1.72(b). Neglecting the space harmonic of the mmf wave, the flux density waveform can be written as

$$b_f = B_m \sin\theta \sin\omega t$$

This means that the field is sinusoidally distributed in space and is changing in magnitude sinusoidally with respect to time as the current through the exciting coil changes sinusoidally with respect to time.

4. Polyphase ac Excitation Figure 1.73 shows a stator with three-phase windings. These windings are connected across a balanced three-phase supply of constant frequency. Assuming sinusoidal excitation and neglecting the space harmonic of the field form, the field produced by the three-phase windings which are displaced in space by 120 degrees (i.e., 2 π/3 degrees) can be written as

For R phase $\quad B_R = B_m \sin\theta \sin\omega t$

For Y phase, $\quad B_y = B_m \sin\left(\theta - \frac{2\pi}{3}\right)\sin\left(\omega t - \frac{2\pi}{3}\right)$

For B phase, $\quad B_B = B_m \sin\left(\theta - \frac{4\pi}{3}\right)\sin\left(\omega t - \frac{4\pi}{3}\right)$

In the airgap, we will have the resultant flux due to all the three phase fields. Therefore, by adding, we get

Resultant flux density $= B_R + B_y + B_B$

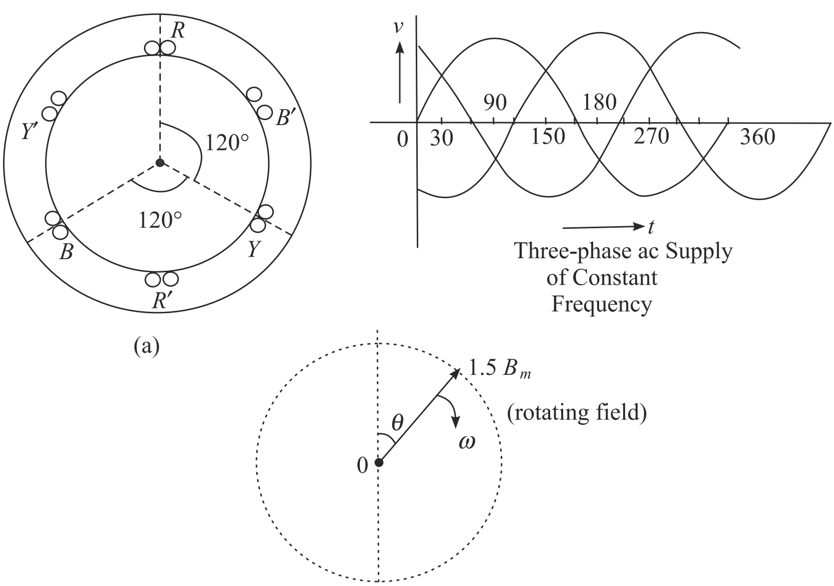


Fig. 1.73 *(a) Stator with three-phase windings connected to a balanced three-phase supply (b) Resultant magnetic field of constant magnitude 1.5 B_m but rotating in space*

$$= B_m \sin \theta \sin \omega t + B_m \sin\left(\theta - \frac{2\pi}{3}\right)\sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$+ B_m \sin\left(\theta - \frac{4\pi}{3}\right)\sin\left(\omega t - \frac{4\pi}{3}\right)$$

$$= 1.5\, B_m \cos(\omega t - \theta) \tag{1.10}$$

This shows that the resultant magnetic field is rotating in nature having a magnitude equal to 1.5 times the magnitude of the field of each phase and rotating at the same angular speed ω radians per second as that of the exciting current as shown in Fig. 1.73(b). This speed of the rotating magnetic field is, therefore, called *synchronous* speed.

5. Permanent Magnet Excitation Because of certain advantages like simple constructional arrangement, increased efficiency, compact size, etc., small dc and synchronous machines are made with permanent magnet excitation. The material used for permanent magnet poles should have high residual magnetism B_r, high coercive force H_c, and a large hysteresis loop area as shown in Figs. 1.74 (a) and (b).

In an electrical machine, the permanent magnetic field may be either on stator or on rotor. The nature of the field form due to permanent magnet is similar to the field form of a dc excited concentrated field winding as in case of dc machines explained earlier. The field form will be somewhat trapezoidal in shape as shown in Fig. 1.74(c).

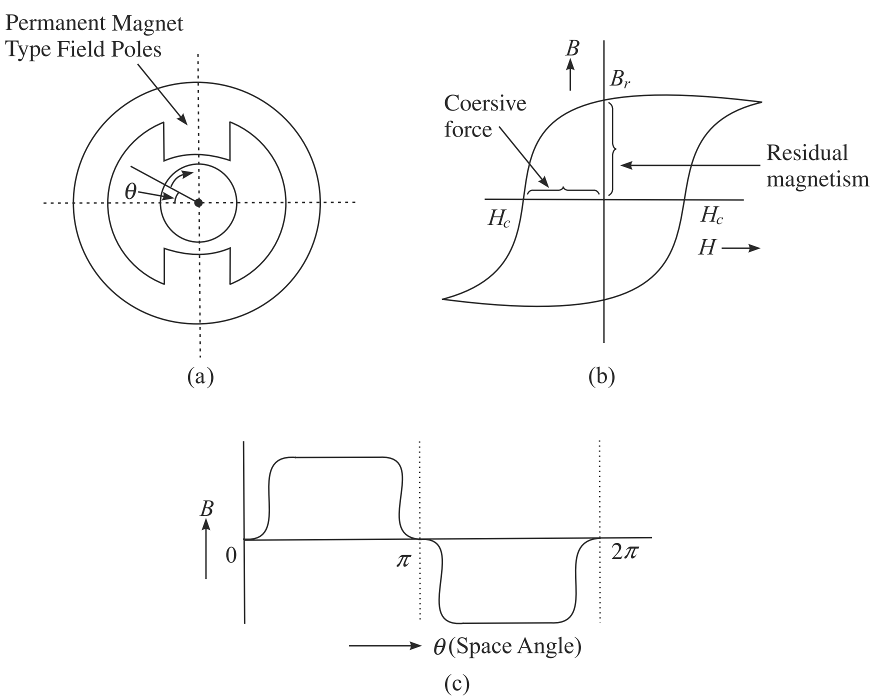


Fig. 1.74 *(a) Permanent magnet type field poles (b) Its hysteresis loop (c) The field form*

1.10 VOLTAGE GENERATED IN ELECTRICAL MACHINES

Let the space distribution of field flux density created by distributed stator winding be taken approximately as sinusoidal, i.e.,

$$b_f = B_m \sin \theta$$

For simplicity let a single coil 1-1′ of N turns be placed in the rotor slots as shown in Fig. 1.75 and rotated at a speed of ω. Let θ be the angle measured from the quadrature axis. For an elemental angle $d\theta$ of the rotor surface, the flux linkage, $d\psi$ by the coil is

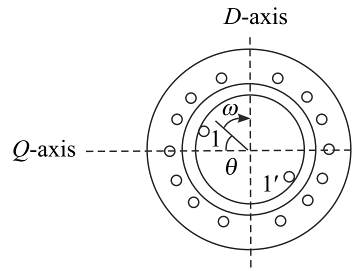


Fig. 1.75 *Stator with distributed winding and rotor with single coil 1-1′*

$$d\psi = Nd\phi$$

$$= N\, b_f\, dA \qquad [\because \text{ Flux = Flux density} \times \text{Area})$$

$$= N\, B_m \sin\theta\, lrd\theta$$

[$\because$ $rd\theta$ is the arc and l is the axial length of the armature core, r is the radius of the coil]. If P is the number of poles, angle $d\theta$ is to be converted into equivalent mechanical degrees.

We know, $\quad 1°\text{ mech} = \dfrac{P}{2}$ degree elect.

or $\quad 1°\text{elect} = \dfrac{2}{P}$ degree mech.

$$\therefore \qquad d\psi = N\, B_m \sin\,\theta\, lrd\,\theta\, \frac{2}{P} = 2N\, B_m\, lr\, \frac{1}{P} \sin\theta\, d\,\theta$$

Let the angular position of the rotor coil 1-1′ at time zero be at the Q-axis, i.e., at the origin and after a time, t let the angular position be such that the coil makes an angle α with the Q-axis.

Since a coil spans by 180°, total flux ψ linked by the coil,

$$\psi = \int_{\alpha}^{\pi+\alpha} 2NB_m\, lr \frac{1}{P} \sin\theta d\theta$$

$$= 2N\, B_m\, lr \frac{1}{P}[-\cos\theta]_{\alpha}^{\pi+\alpha}$$

$$= 4N\, B_m\, lr \frac{1}{P} \cos\,\alpha$$

$$= \frac{4}{P} NB_m\, lr \cos\,\omega t \qquad [\because\ \alpha = \omega t]$$

where $\omega = 2\pi f$, f being the frequency of alternation of induced voltage. We know that induced emf is the rate of change of flux linkage. Thus, instantaneous value of induced emf, is given as

$$e = \frac{d\psi}{dt} = -\frac{d}{dt}\left[\frac{4}{P}NB_m lr\cos\omega t\right]$$

$$= \frac{4}{P}NB_m lr\frac{d}{dt}[\cos\omega t]$$

$$= \frac{4}{P}\ NB_m\ lr\ 2\pi f\sin\ \omega t$$

$$= \frac{8}{P}\ \pi f NB_m\ lr\ \sin\ \omega t \tag{1.11}$$

We can also calculate flux per pole as

$$\phi = \text{Flux density} \times \text{Area under a pole}$$

$$= \int_0^{\pi} B_m \sin\theta lr d\theta \frac{2}{P}$$

$$= \frac{4}{P} B_m\ lr$$

Substituting the above, in Eq. (1.11), the value of included emf becomes

$$e = 2\pi f N\phi \sin\ \omega t$$

The equation is of the form

$$e = E_m \sin\ \omega t$$

where $E_m = 2\pi f N\phi$

The rms value of induced emf,

$$E = \frac{E_m}{\sqrt{2}}$$

$$E = \frac{2\pi}{\sqrt{2}}\ f N\phi = 4.44\ \phi f N$$

or $$E = 4.44\ \phi f N\ \text{ Volts} \tag{1.12}$$

The magnitude of induced emf in a distributed stator or rotor winding is effected by a factor called *distribution factor* or *breadth factor* k_d. The effect of use of short pitch coils on the magnitude of induced emf is expressed by another factor called pitch factor k_p. The expression for induced emf is to be multiplied by these two factors since they directly affect the magnitude of induced emf. The values of k_d and k_p are less than one. For a concentrated winding using full-pitch coils, values of k_d and k_p are, however, unity.

Thus, the emf equation for a distributed winding can be expressed as

$$E = 4.44\ \phi f N k_d k_p\ \text{ Volts} \tag{1.13}$$

It is interesting to note that the Eq. (1.12) is same as that of a transformer shown in Fig. 1.76. This can be verified as follows:

$$e = \frac{d\psi}{dt} = -N\frac{d\phi}{dt}$$

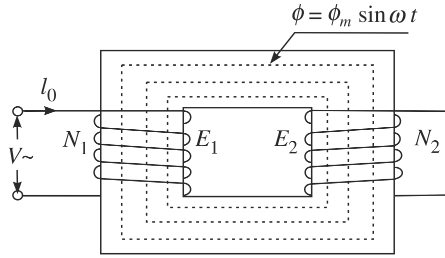


Fig. 1.76 *Emf is induced in the primary and secondary windings due to linkage of flux produced by primary current*

$$= -N\frac{d}{dt}(\phi_m \sin \omega t)$$

$$= -N\omega\phi_m \cos \omega t = 2N\pi f \phi_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

This is the form $$e = E_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $$E_m = 2\pi f \phi_m N \tag{1.14}$$

rms value of induced emf,

$$E = \frac{E_m}{\sqrt{2}}$$

$$E = \frac{2\pi}{\sqrt{2}} f\phi_m N = 4.44\ \phi_m f N$$

or $$E_1 = 4.44\ \phi_m f N_1$$

and $$E_2 = 4.44\ \phi_m f N_2$$

where E_1 and E_2 are the emfs induced in the primary and secondary windings of the transformer as shown in Fig. 1.76.

Induced emf equations in a coil rotating in a magnetic field being equal to the emf induced in the windings of a transformer indicates that the motion of a coil under a stationary flux density wave produces the same voltage as does a time varying flux density wave linking a stationary coil.

It is noted that the voltage induced in the coil 1–1′ in Fig. 1.75 is alternating in nature. In case of dc machines the alternating voltage induced in the armature coils is rectified by means of brush and commutator arrangement. The commutator, a mechanical rectifier, reverses the negative half-cycle of the alternating voltage such that the output voltage will be as shown in Fig. 1.77 (b).

In practical dc machines, however a large number of armature coils distributed in armature slots are used and as such the output voltage wave shape becomes more or less a dc wave. We can calculate the average value of the fluctuating dc of Fig. 1.77(b) as

$$E_{\text{dc}} = \frac{1}{\pi}\int_0^{\pi} E_m \sin \omega t\, d(\omega t)$$

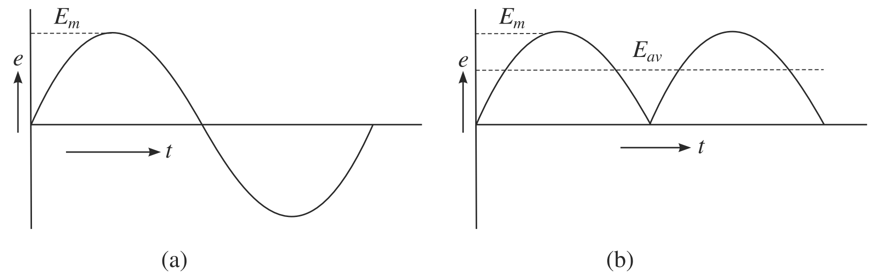


Fig. 1.77 *(a) Voltage induced in the armature coil (b) Output voltage across the brushes as in a dc machine*

Substituting value of E_m from Eq. (1.14).

$$E_{\text{dc}} = \frac{1}{\pi}\int_0^{\pi} \omega N \phi \sin \omega t \, d(\omega t)$$

$$= 4 \phi f N \text{ Volt}$$

f can be expressed in terms of speed N in rpm as $f = \dfrac{NP}{120}$

where P is the number of poles. If the armature of a dc machine has Z number of conductors, and A number of parallel paths, then the number of conductors per parallel path is Z/A. Induced emf is the emf per parallel path. In terms of turns per parallel path,

Number of turns $\quad N = \dfrac{Z}{2A}$

Thus the emf equation for a dc machine becomes

$$E_{\text{dc}} = 4\phi \frac{NP}{120}\frac{Z}{2A}$$

or

$$E_{\text{dc}} = \frac{\phi ZNP}{60A} \text{ Volts} \tag{1.15}$$

1.11 GENERALISED TORQUE EQUATION FOR ROTATING MACHINES

The variety of electrical machines available today, operate on the same basic principle but each has its own special characteristics and applications. Let us develop the torque equation for an idealised elementary machine and extend this equation for various types of basic rotating machines, i.e., synchronous machine, induction machine and dc machine.

Let us assume cylindrical rotor system as shown in Fig. 1.78.

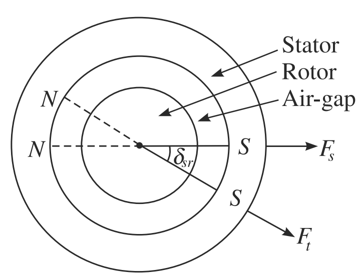


Fig. 1.78 *Cylindrical rotor system*

The machine can be regarded as having two sets of windings—one on the stator and the other on the rotor, producing magnetic fields in the air gap. The torque is produced due to the tendency of these two magnetic fields to line up in the same way as permanent magnets tend to align themselves. Voltage is generated due to the relative motion between a field and a winding as per Faraday's law of Electromagnetic induction. Let us make the following assumptions:

1. The tangential component of the magnetic field in the air gap is negligible as compared to the radial component, i.e., the flux goes straight across the gap.
2. The radial length *lg* of the air gap is small compared to the radius of the rotor or stator. With this assumption, we can say that the flux density at the rotor surface or at the stator surface or at any intermediate radial distance in the air gap is same. The resultant air gap field is then the radial field *H* or *B* whose intensity varies with angle around the periphery.

Figure 1.78 shows the axes of the stator field F_s and rotor field F_r. The angle between the two axis is δ_{sr}.The addition of the fields can be done easily by means of the phasor diagram as shown in Fig. 1.79. F_s and F_r represents the peak values of the fundamental components of the stator and rotor mmf waves and the phasor F_{sr} represent the peak value of their resultant.

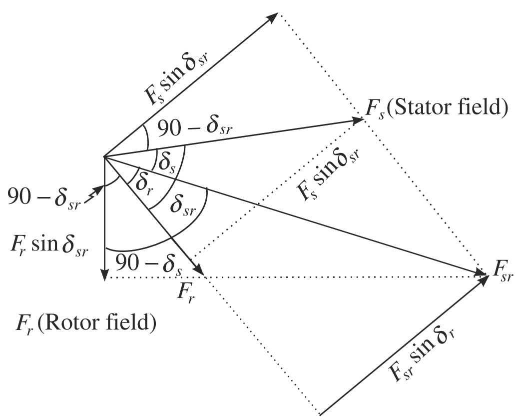


Fig. 1.79 *Phasor diagram showing relationship of stator field F_s, rotor field F_r and their resultant F_{sr}*

From the trigonometric formula for the diagonal of a parallelogram

$$F_{sr}^2 = F_s^2 + F_r^2 + 2\,F_s\,F_r \cos\,\delta_{sr}$$

Also $$F_s \sin\,\delta_{sr} = F_{sr} \sin\,\delta_r$$

and $$F_r \sin\,\delta_{sr} = F_{sr} \sin\,\delta_s$$

Now consider the magnetic field co-energy stored in the air gap. By definition, co-energy is the area under the *BH* curve and represents the energy converted into mechanical work (for details refer to Section 1.13). The co-energy density at a point where the magnetic field intensity is *H* is given by

$$\frac{1}{2}\,BH \quad \text{or} \quad \frac{1}{2}\,\mu_0\,H^2 \text{ (since } B = \mu_0\,H).$$

The average value of energy density = $\frac{\mu_0}{2}$ (Average value of H^2)

The average value of the square of the sine wave is half its square of peak value.

Since $$H = \frac{\text{Magnetising force}}{\text{Length of flux path}}$$

$$H_m = \frac{F_{sr}}{l_g} \text{ (neglecting reluctance of iron path)}$$

Average co-energy density = $\frac{\mu_0}{2}\frac{H_m^2}{2} = \frac{\mu_0}{4}\left(\frac{F_{sr}}{l_g}\right)^2$

Total co-energy = Average co-energy density × Volume of air gap

$$W_{f/d} = \frac{\mu_0}{4}\left(\frac{F_{sr}}{g}\right)^2 \pi DL.l_g$$

$$= \frac{\mu_0 \pi DL}{4l_g} F_{sr}^2$$

$$= \frac{\mu_0 \pi DL}{4l_g} (F_s^2 + F_r^2 + 2 F_s F_r \cos \delta_{sr})$$

where D is the average diameter at the airgap, l is the axial length, g is the air gap clearance and

$$\mu_0 = 4 \times 10^{-7}$$

This is the energy being converted into mechanical work. An expression for the electromagnetic torque T can not be obtained in terms of the interacting magnetic fields by taking the partial derivatives of the field co-energy with respect to the angle.

For a two pole machine.

$$T = \frac{\partial(W_{f/d})}{\partial\delta_{sr}} = \frac{\mu_0 \pi DL}{2l_g} F_s F_r \sin \delta_{sr} \tag{1.16}$$

For a P pole machine equation (1.16) gives the torque per pair of poles.

∴ the torque for a P pole machine is

$$T = \frac{P}{2}\frac{\mu_0 \pi DL}{2l_g} F_s F_r \sin \delta_{sr} \tag{1.17}$$

We know that the torque is being produced by the interaction of rotor field and the resultant air gap field. Substituting $F_s \sin \delta_{sr} = F_{sr} \sin \delta_r$ in Eq. (1.17)

We get

$$T = \frac{P}{2}\frac{\mu_0 \pi DL}{2l_g} F_s F_{sr} \sin \delta_r \tag{1.18}$$

We know $$B_{sr} = \mu_0 H_{sr} = \mu_0 \frac{F_{sr}}{l_g}$$

$$\therefore \qquad F_{sr} = \frac{B_{sr}\, l_g}{\mu_0}$$

Here B_{sr} is the maximum value of flux density.

Average value of flux density = $\frac{2}{\pi} B_{sr}$

Flux $$\phi = \frac{2}{\pi} B_{sr} \times \frac{\pi DL}{P}$$

or $$B_{sr} = \frac{\phi P \pi}{2\pi DL} = \frac{\phi P}{2DL}$$

Substituting this value in the expression of F_{sr}, we get

$$F_{sr} = \frac{\phi P}{2DL} \frac{lg}{\mu_0}$$

Substituting F_{sr} in Eq. (1.18) We get the generalized torque equation as

$$T = \left(\frac{P}{2}\right)^2 \frac{\pi}{2} \, \phi F_r \sin \delta_r \tag{1.19}$$

where ϕ is the resultant air gap flux produced by the combined effect of the stator and rotor mmfs.

Thus from Eq (1.19) it is seen that torque is proportional to the interacting fields and the sine of the electrical space angle between their magnetic axes. This is the generalised torque equation. Let us now apply this equation to different types of machines and try to explain their behaviour.

1.11.1 Torque Equation for a dc Machine

The essential features of a dc machine is shown in Fig. 1.80. The stator has salient poles and excited field coils. The air gap flux distribution created by the field windings is symmetrical about the centre line of the field poles. This axis is called the *field axis* or *direct axis*. The brushes are located so that commutation occurs when the coil sides are in the neutral zone, midway between the field poles. The axis of the armature mmf wave then is 90° electrical from the axis of the field poles, i.e., in the quadrature axis.

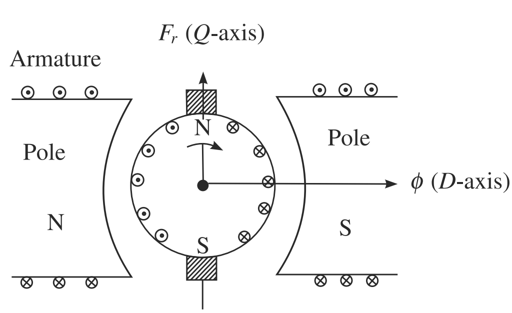


Fig. 1.80 *Essential features of a dc machine showing the field axis (D-axis) and the armature mmf axis (Q-axis)*

The torque can be expressed in terms of the interaction of the direct axis air gap flux per pole ϕ and the space fundamental component F_{r_1} of the armature mmf wave. With the brushes in the quadrature axis the angle between these fields is 90° electrical.

Thus, referring to the generalised torque equation shown in Eq. (1.19), the torque equation for dc machine becomes

$$T = \frac{\pi}{2}\left[\frac{P}{2}\right]^2 \phi F_{r_1} \sin 90°$$

$$= \frac{\pi}{2}\left[\frac{P}{2}\right]^2 \phi F_{r_1} \qquad (1.20)$$

The mmf of a distributed armature winding of a dc machine is triangular in shape as shown in Fig. 1.81.

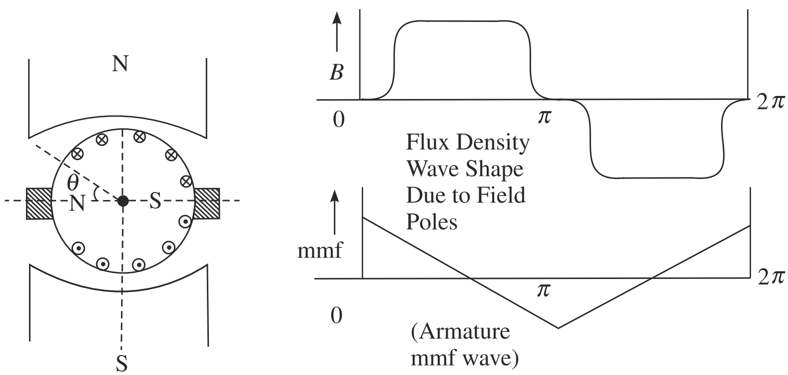


Fig. 1.81 *Flux density wave and armature mmf wave of a distributed armature winding of a dc machine*

A triangular mmf wave can be represented by a fundamental sine wave and component harmonic waves. We will consider only fundamental wave whose maximum value is $8/\pi^2$ of the maximum value of triangular wave.

If Z is the total number of conductors, A is the number of parallel paths, P is the number of poles and i is the armature current then the armature mmf per pole, F_r is calculated as,

$$F_r = \text{Amp turns/pole} = \frac{Zi}{2PA}\left[\because \text{no of turns per pole is } \frac{Z}{2}\times\frac{1}{A}\times\frac{1}{P}\right]$$

The fundamental component

$$F_{r_1} = \frac{8}{\pi^2} F_r = \frac{8}{\pi^2}\frac{Zi}{2PA}$$

Substituting this value in Eq. (1.17), we get

$$T = \frac{\pi}{2}\left(\frac{P}{2}\right)^2 \phi \frac{8}{\pi^2}\frac{Zi}{2PA} = \frac{PZ}{2\pi A}\phi i$$

or $$T = K\phi i \tag{1.21}$$

Multiply and divide RHS of Eq. (1.21) by $N/60$

$$T = \frac{\left(\frac{N}{60}\right) PZ\phi i}{\left(\frac{N}{60}\right) 2\pi A}$$

or $$\frac{2\pi NT}{60} = \frac{\phi ZNP}{60A}\cdot i = ei \qquad \left[\because\ e = \frac{\phi ZNP}{60A}\right]$$

or $$\text{P} = \omega T = ei \qquad \text{where } \omega = \frac{2\pi N}{60} \tag{1.22}$$

This is the power which is transferred through the air gap to the rotor.

1.11.2 Torque Equation for a Polyphase Synchronous Machine

A synchronous machine is one in which alternating current flows in the armature winding and dc excitation is supplied to the field winding. The armature winding is almost invariably on the stator and is having a three-phase winding. The field winding on the rotor is shown in Fig. 1.82. The dc power required for excitation is supplied through slip rings. A synchronous machine is connected to a power system. The voltage and frequency at the armature terminals are fixed by the system. A source of constant rms voltage and frequency is called an Infinite Bus. When carrying balanced polyphase currents, the armature winding will produce a rotating magnetic field rotating at synchronous speed $N_s = \frac{120f}{P}$. But the field produced by the dc rotor winding revolves with the rotor. For production of a steady unidirectional torque, the rotating field of stator and rotor must travel at the same speed. A synchronous motor connected to a constant frequency voltage source, therefore, operates at a constant steady slate speed regardless of load. Behaviour of a synchronous motor under running conditions can be visualised in terms of the torque equation,

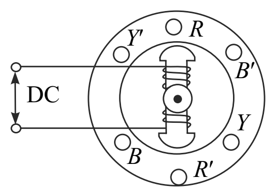


Fig. 1.82 *Rotating field and stationary armature system*

$$T = \frac{\pi}{2}\left(\frac{P}{2}\right)^2 \phi\, F_r \sin \delta_r$$

The voltage generated by the air gap flux wave is nearly equal to the terminal voltage, V_t,

$$V_t = 4.44\, f N\, \phi$$

i.e., $\phi = \frac{V_t}{4.44\, f\, N}$ = constant, if V_t and f are constants. The rotor mmf F_r is determined by the dc field current and is also constants. Therefore,

$$T = k \sin \delta_r \tag{1.23}$$

The variation in the torque requirements of the load on the machine is taken care of entirely by variation of torque angle δ_r, as shown by the torque angle curve in Fig. 1.83 in which positive value of torque represents motor action and positive value of δ_r, represents angle of lag of the rotor mmf wave w.r.t. the resultant air gap flux wave.

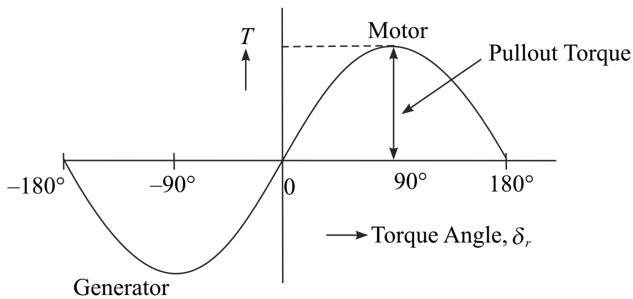


Fig. 1.83 *Torque-angle characteristics of a synchronous machine*

With a light shaft load, only a relatively small electromagnetic torque is required and δ_r is small. When more shaft load is added the rotor must fall back in space phase with respect to the rotating flux wave just enough so that δ assumes the value required to supply the necessary torque. The readjustment process is actually a dynamic one accompanied by a temporary decrease in the instantaneous mechanical speed of the rotor and a damped mechanical oscillation, called Hunting of the rotor about its new position. When δ_r becomes 90°, the maximum possible torque or power called the pull out torque or pull out power for a fixed terminal voltage and field current is reached. If the load requirement exceeds this value the motor slows down under the influence of the excess shaft torque and synchronous motor action is lost because rotor and stator fields are no longer stationary with respect to each other. This phenomenon is known as pulling out of step or losing synchronism.

1.11.3 Torque Equation for Polyphase Induction Machines

Like a synchronous motor a polyphase induction motor is supplied with a three-phase supply to its three-phase stator windings. The rotor is a closed winding and gets its excitation through electromagnetic induction. When three-phase supply is connected across the three-phase windings of the stator a rotating magnetic field ϕ_{sr} is produced whose magnitude remains constant if the supply voltage and frequency remain constant. The rotor mmf F_r is proportional to the rotor current I_r. Thus the original equation,

$$T = \frac{\pi}{2}\left(\frac{P}{2}\right)^2 \phi_{sr}\, F_r \sin \delta_r \text{ gets reduced to}$$

$$T = k\, I_r \sin \delta_r \tag{1.24}$$

The rotor rotates at a speed N_r, somewhat less than the speed of the rotating magnetic field, N_s. Because at synchronous speed of the rotor, there will be no induced current in the rotor as there will be no relative speed between the rotating field and the rotor. The difference of N_s and N_r expressed as percentage of N_s is called slip S.

$$S = \frac{N_s - N_r}{N_s} \times 100 \qquad (1.25)$$

Slip is about 3 to 5 per cent.

Various types of machines mentioned so far will be dealt with in detail separately.

1.12 COMMON FEATURES OF ROTATING ELECTRICAL MACHINES

The common features of all the rotating electrical machines just described in previous sections are listed below.

1. Each machine can either be operated as a motor or a generator.
2. Every machine consists of two main parts, namely stator and rotor.
3. Stator construction is in the form of a hollow cylinder with the inner surface either having slots in which windings are placed, or alternatively having projected poles with windings placed around them.
4. Rotor construction is in the form of either a solid cylinder having slots on its outer surface in which windings are placed or a rotor cylinder having projected poles with windings placed around them.
5. Windings of both stator and rotor carry current which produce a common magnetic flux in the space called the air-gap between the stator and the rotor.
6. Each machine has suitable devices such as fixed terminals, slip-rings, commutator, brushes, etc., for leading current in and out of its two main parts.
7. Each machine has its supporting frame, enclosure, bearings and cooling arrangements.

In the following section, we will explain the concept of energy and co-energy associated with the energy conversion process.

1.13 ELECTROMECHANICAL ENERGY CONVERSION PROCESS—CONCEPT OF ENERGY AND CO-ENERGY

The electromechanical energy conversion process is explained below.

An electromechanical energy conversion device where electrical energy is converted into mechanical energy output, like in an electrical motor, has three essential parts viz. (1) an electrical system, (2) a mechanical system, (3) a coupling magnetic field. The energy balance equation is expressed as.

Electrical input energy	=	Increase in stored energy in the coupling field	+	Energy losses ↓ I^2R loss + core loss + friction and windage loss	+	Mechanical energy output

Considering a differential time interval dt,

$$d\,W_e = d\,W_f + d\,W_m, \text{ if losses are neglected}$$

where $d\,W_e$ is the increment in electrical input energy

$d\,W_f$ is the energy supplied to the field which is either stored or lost

$d\,W_m$ is the energy converted into mechanical form

1.13.1 Field Energy

As shown in Fig. 1.84, the movable part B of the electromagnet is held in position by the spring tension as against the force of attraction by the electromagnet A when current has been increased from 0 to i. Since there is no movement of part B the mechanical output,

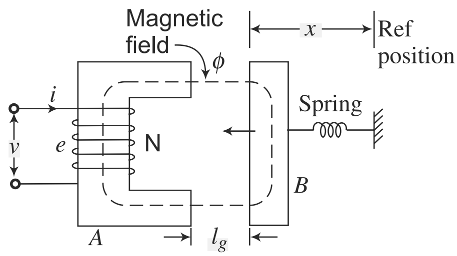


Fig. 1.84 *Shows an electromagnet with stored energy is its magnetic field*

$$dW_m = 0$$

Therefore, $$dW_e = dW_f$$

Neglecting losses, the incremental energy input, dW_e is stored as increment in field energy, dW_f.

Again, Induced emf, $e = \dfrac{Nd\phi}{dt} = \dfrac{d\lambda}{dt}$ where, $\lambda = N\phi$ (flux linkage)

Incremental input energy, $dW_e = ei\,dt = dW_f$

$$dW_f = ei\,dt = \frac{d\lambda}{dt}\,i\,dt = id\lambda$$

i.e., $$dW_f = id\lambda$$

The relationship between the flux linkage, λ and i has been shown in Fig. 1.85. The incremental field energy has been shown by cross-hatched area. When the flux linkage is increased from 0 to λ, the energy stored in given as

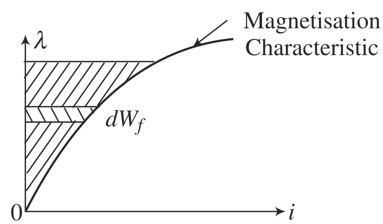


Fig. 1.85 *Energy stored in the magnetic field*

$$W_f = \int_0^{\lambda} i\,d\lambda = \text{entire hatched area shown in Fig. 1.85}$$

1.13.2 Energy and Co-energy

The flux linkage λ versus i characteristic is non-linear due to the B-H characteristic of the magnetic material, i.e., due to saturation effect. For an air-gap the λ–i relationship is linear. If the air-gap is more, the λ–i becomes less non-linear.

The characteristics becomes more non-linear if the air-gap length is reduced. In Fig. 1.86, part A is represented by energy stored in the magnetic field, W_f. The area B of $\lambda - i$ characteristic is known as the co-energy, W_i where

$$W_i = \int_0^{i} \lambda\,di \qquad (1.26)$$

and, $$W_f + W_i = \lambda\,i \qquad (1.27)$$

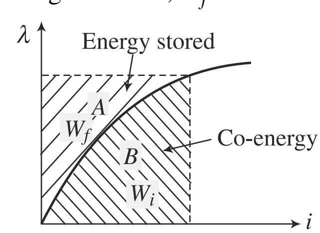


Fig. 1.86 *Representation of energy and co-energy*

Co-energy does not have any physical significance. It is used to derive force or torque equations. Accordingly Eq. (1.26) can be used to derive expression for force or torque. If $W_f = W_i$, the λ–i characteristics is linear.

1.13.3 Mechanical Force

The λ–i characteristic for different air-gap lengths has been shown in Fig. 1.87. When the air-gap is reduced, the characteristic moves upwards and become more non-linear.

Consider the electromagnetic system shown in Fig. 1.87.

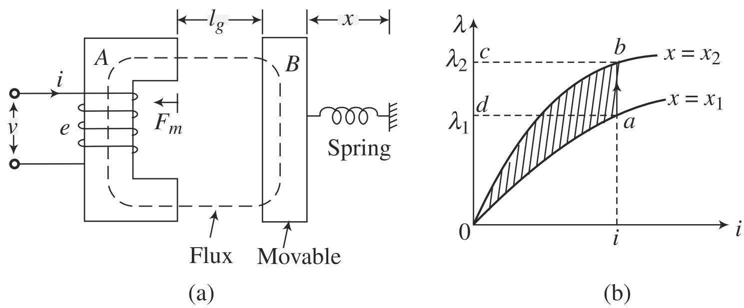


Fig. 1.87 *Mechanical Force developed on an electromagnet (a) Force developed on the movable part (b) Flux linkage λ versus current i characteristics*

As shown in Fig. 1.87(a), when a voltage v is applied, a current i will flow through the exciting coil. The flux lines will complete their path through the movable part B. The movable part B has a spring attached to it. The movable part will get attracted towards the stationary part A. Let the movement of movable part is from position $x = x_1$ to position $x = x_2$. This will reduce the air gap and hence the λ–i characteristic will move upwards. The $\lambda-i$ characteristics for the two positions of the movable part have been shown in Fig. 1.87(b).

Assuming that current i flowing through the excitation coil remains the same, the operating point 'a' on the λ–i characteristic will move up to position 'b' as has been shown. During the motion of the part B, energy input is

$$dW_e = \int e\,i\,dt = \int_{x_1}^{x_2} i\,d\lambda = \text{area } abcd.$$

Differential field energy, dW_f = area obc – area oad

Differential mechanical energy, $dW_m = dW_e - dW_f$

We had $$dW_e = \int ei\,dt = \int_{x_1}^{x_2} i\,dx = \text{area } abcd.$$

$$dW_f = \text{area } obc - \text{area } oad$$

Thus, $$dW_m = dW_e - dW_f = \text{area } abcd - (\text{area } obc - \text{area } oad)$$

or, $$dW_m = \text{area } abcd + \text{area } oad - \text{area } obc = \text{area } oab$$

= shaded area shown in Fig. 1.87 which is in between the λ–i characteristics.

A close look at this shaded are will show that this area, in fact, represents the change in co-energy of the system. Therefore,

$$dW_m = dW_i$$

The mechanical force acting on the movable part is f_m and the displacement is dx.

$$f_m\, dx = dW_m = dW_i$$

$$f_m = \left.\frac{\partial W_i\,(i, x)}{\partial x}\right|_{i\,=\,\text{constant}}$$

If we assume that during the motion of the movable part the flux linkage has remained constant, then the mechanical work done will be represented by the shaded area as shown in Fig. 1.88. The mechanical work done is represented by the area *oap*. This area, in fact, is the decrease in field energy.

Therefore,

$$f_m\, dx = dW_m = -dW_f$$

$$f_m = \left.\frac{\partial W_f\,(\lambda, x)}{\partial x}\right|_{\lambda\,=\,\text{constant}}$$

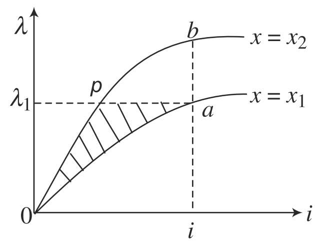


Fig. 1.88

When flux linkage is constant, the mechanical energy is supplied entirely by the field energy. For any system, torque can be calculated from the expression of force.

1.14 GENERALISED ELECTRICAL MACHINE

The essential unity underlying the principle of all electrical rotating machines leads us to the concept of a single generalised electrical machine which can be operated as any of the machines described earlier. A general view of a particular type of generalised machine is shown in Fig. 1.89. Such machines can be run as all conventional dc machines, induction machines, synchronous machines and others. In Fig. 1.89 is shown a dynamometer coupled with the generalised machine for the purpose of

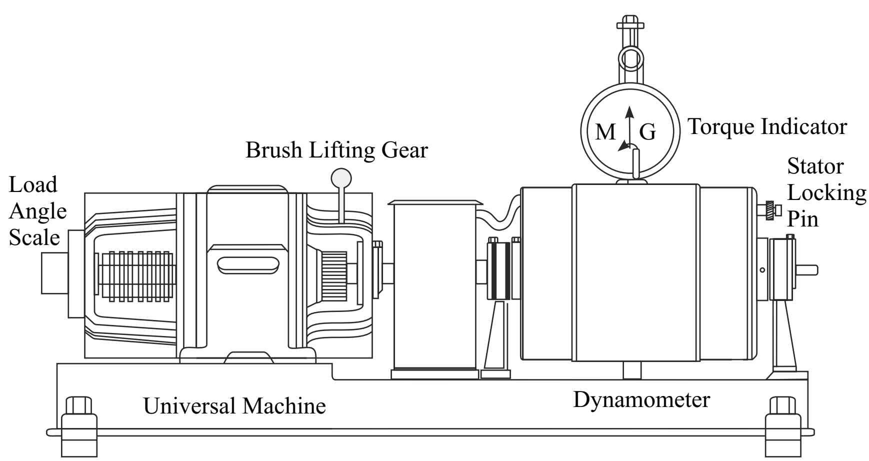


Fig. 1.89 *View of a generalised (universal) electrical machine coupled with a dynamometer*

loading and for use as prime mover. Facility for direct measurement of torque is also available. All the stator and rotor terminals are brought out on a panel. The set can be used as any machine by making suitable external connections on the panel.

MODEL QUESTIONS

Short-Answer-Type Questions

1.1 Explain the meaning of torque angle. Show that the torque developed is the maximum when the torque angle is 90°.

1.2 Mark in Fig. 1.90 the direction of current on the stator conductors for clockwise torque to be developed on the rotor. Show also the torque angle.

1.3 Directions of current through the stator and rotor conductors are shown in Fig. 1.91. Show the position of the poles, the torque angle and the direction of rotation of the rotor.

1.4 Will the rotor in Fig. 1.92 rotate? If so, in which direction?

1.5 Explain with the help of an example why in an electrical machine the number of stator poles should be equal to the number of rotor poles.

1.6 Explain Faraday's laws of electromagnetic induction and Lenz's law.

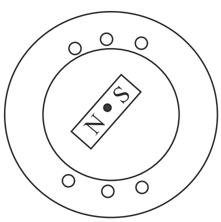


Fig. 1.90

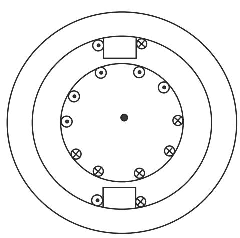


Fig. 1.91

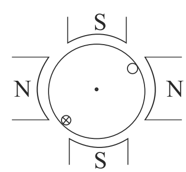


Fig. 1.92

1.7 Show the nature of the emf induced in the rotor coil when the rotor is rotating in the stator magnetic field as shown in Fig. 1.93.

1.8 Mark the direction of emf in the rotor conductors in Fig. 1.94 when the rotor rotates in the anticlockwise direction.

1.9 Mark the direction of induced emf's in the coils wound on the hollow cylinder when the cylinder is brought towards the magnet as shown in Fig. 1.95.

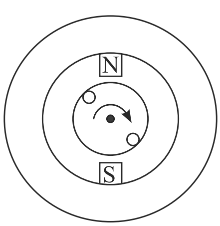


Fig. 1.93

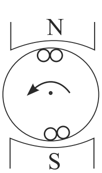


Fig. 1.94

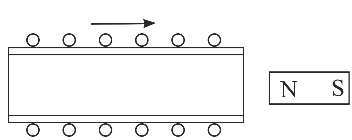


Fig. 1.95

1.10 Show the directions of electromagnetic torque, frictional torque, mechanical torque and direction of rotation (a) in a motor and (b) in a generator. Also draw power flow diagrams in each case.

1.11 Explain the principle of working of a dc machine for generating and motoring action.

1.12 Illustrate with the help of neat diagrams the nature of the magnetic field produced when

(a) a two-phase supply is connected across a two-phase winding, and

(b) a three-phase supply is connected across a three-phase winding.

1.13 A 50 Hz, three-phase supply is connected across a three-phase, two-pole winding. Show with the help of neat sketches that the resultant field is a rotating one and the speed of rotation is 3000 rpm.

1.14 Show the torque angle in Fig. 1.96. Show the position of the brushes and the torque angle for maximum torque.

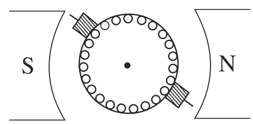


Fig. 1.96

1.15 Show, with the help of neat diagrams, the nature of the resultant magnetic field produced when a two-phase 10 Hz, supply is applied across a two-phase four-pole winding. At what speed will the resultant field rotate? Show that the magnitude of the resultant field is constant at every instant of time.

1.16 Explain from the concept of alignment of magnetic fields the principle of a three-phase induction motor. Show that the direction of torque produced on the rotor is in the same direction in which the stator field is rotating.

1.17 The rotor of four-pole synchronous motor rotates at 1000 rpm. Calculate the frequency of the stator supply voltage.

1.18 Explain why an induction motor cannot run at synchronous speed.

1.19 State the most common features of rotating electrical machines.

1.20 Derive the generalised torque equation for rotating machines.

1.21 Derive the emf equation for rotating electrical machine (a) ac machine (b) dc machine.

1.22 Show that the emf equation for a full-pitch coil concentrated winding ac machine is the same as that of a transformer.

1.23 Derive the generalised torque equation for rotating machines.

1.24 Explain the concept of field energy and co-energy with the help of an example.

1.25 Show that the torque developed in a rotating electrical machine is maximum when torque angle is 90°.

LABORATORY EXPERIMENTS

EXPERIMENT 1.1 *Measurement of the angular displacement of the rotor of a three-phase synchronous machine with respect to the stator on application of dc to the field winding and simultaneously to each phase winding in sequence.*

Objective To demonstrate the alignment of magnetic fields.

Brief Theory When one magnet is placed in the field produced by another magnet. The two fields try to align themselves with each other. This is illustrated in Fig. 1.97, where the inner magnet is free to rotate and the outer magnet is stationary.

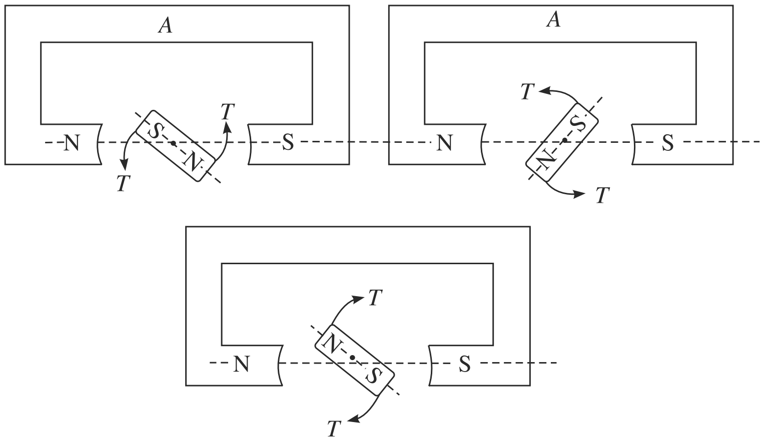


Fig. 1.97 *Principle of alignment of magnetic field*

When the two fields become aligned with each other. There is no torque developed on the field system. This is shown in Fig. 1.98(a). Figure 1.98(b) shows the North pole facing the North pole and the South pole facing the South pole. Here the two fields, though parallel, are not aligned but act in opposition to each other. The inner magnet is in unstable equilibrium and the slightest external mechanical torque applied to the inner magnet in any direction will cause a resultant torque which will make it turn by 180°, so that the two fields become aligned with each other.

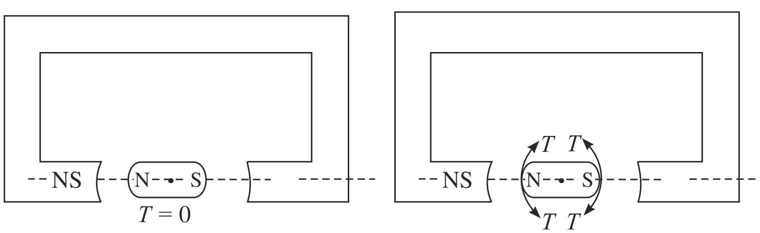


Fig. 1.98 *Shows the principle of alignment of magnetic field*

Thus it can be concluded that torque is produced due to the non-alignment of two magnetic fields in an attempt to align themselves. This is illustrated in Fig. 1.97. This principle is used to obtain torque in rotating electrical machines.

In these machines, the field system is produced in a slightly different way from that as shown in Fig. 1.97. In a three-phase synchronous machine, for example, the stator slots contain three separate windings mutually displaced by 120°. The rotor is an electromagnet which is free to rotate. A simple synchronous machine is shown in Fig. 1.99.

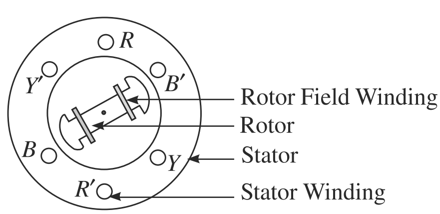


Fig. 1.99 *Shows the rotor and stator of a simple synchronous machine*

For simplicity, only one coil per phase is shown. In actual practice, there may be many coils per phase. If dc supply is given to any one stator phase, say *RR′* (Fig. 1.100(a)), a magnetic field is set up by the stator. If now the rotor is magnetised with a dc voltage applied across the rotor winding, torque will be experienced by the rotor as long as the two fields are not aligned. This is illustrated in Figs. 1.100 (a) and (b).

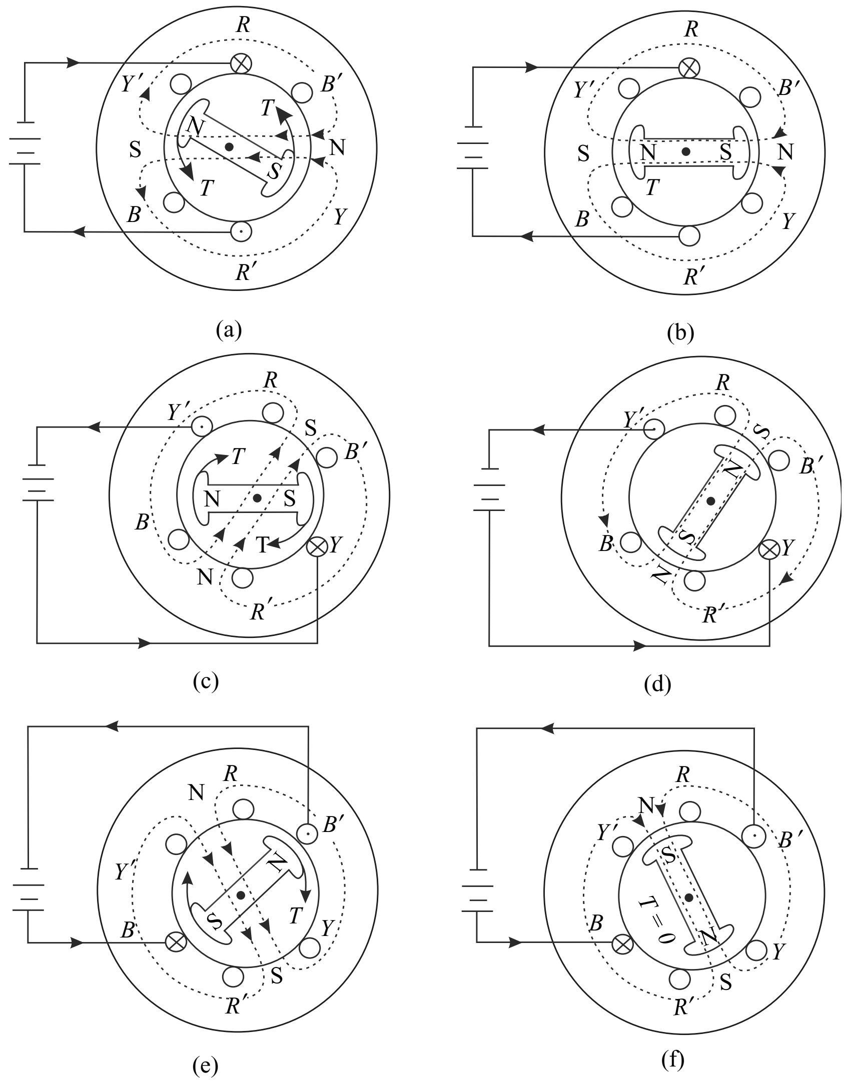


Fig. 1.100 *Shows the alignment of two magnetic fields*

In Fig. 1.100(a), a low voltage dc is applied to phase *R* (between *R* and *R′* terminals). This will induced magnetic polarity on the stator as shown. The rotor winding is energised from a 220 V or 110 V dc source (the supply connections to the rotor are not shown in the figure). The polarity of the rotor is determined by the direction of current in the rotor winding which in turn depends upon the direction of the rotor winding and the connections to the supply. Since the rotor winding is fixed the polarity of the rotor remains constant for a given direction of current through the winding. Under these conditions, the rotor will turn its North pole faces the South pole of the stator and its South pole faces the North pole of the stator. The two fields will then be aligned with each other, i.e., the axis of both stator and rotor fields will be in alignment. Now let the dc supply be removed from RR' and applied across phase Y. The connections of the rotor winding are not to be altered so that the rotor polarity remains unaltered. The magnetic polarity of the stator will no longer be as it was in Fig. 1.100(a) but will be as shown in Fig. 1.100(c). The rotor will, therefore, turn in the clockwise direction by 120°, i.e., till it comes into stable alignment. Finally, if the supply is removed from the Y phase and given to the B phase of the stator, the poles formed on the stator will be as shown in Fig. 1.100(c). The rotor will again turn by another 120° in the clockwise direction.

The above experiment shows that as dc supply is given one after another in succession to the stator phase, the poles developed by the stator change their axes in the same way as if they were rotating. The rotor field will always tend to come into alignment with the stator field. When they are not aligned, a torque is developed which aligns them.

In this experiment, the rotor is made to change its position by changing the axis of the magnetic field produced by the stator winding which in turn is caused by successively giving dc supply to each of the stator phases one after another. The principle (i.e., rotation of the rotor poles in order to keep in alignment with the rotating stator poles) is used in three-phase motors in which a three-phase winding on the stator is connected to a three-phase supply causing a rotating magnetic field to be produced. The rotating magnetic field rotates in similar way as when successively giving dc supply to the three-stator phase windings in this experiment. The reason how a three-phase supply creates a rotating magnetic field in a three-phase winding is a topic which the student will study separately. The purpose of this experiment is to demonstrate the alignment of magnetic fields. It will thus be appreciated that a rotor which develops fixed poles due to the dc excitation of its winding will continue to rotate when placed inside a rotating magnetic field.

Thus the above experiment demonstrates that if a field is produced by the stator which changes its axis continuously, i.e., rotates, a dc excited rotor will also rotate in the same direction. The rotor will rotate at the same speed with which the stator field axis rotates.

Circuit Diagram See Fig. 1.101.

Apparatus Required Three-phase synchronous machine, rheostats (two), ammeters (two), 12 V battery.

Procedure

1. Connect the synchronous machine stator terminals, in star (if not already connected).
2. Excite the rotor field winding from the dc supply source.

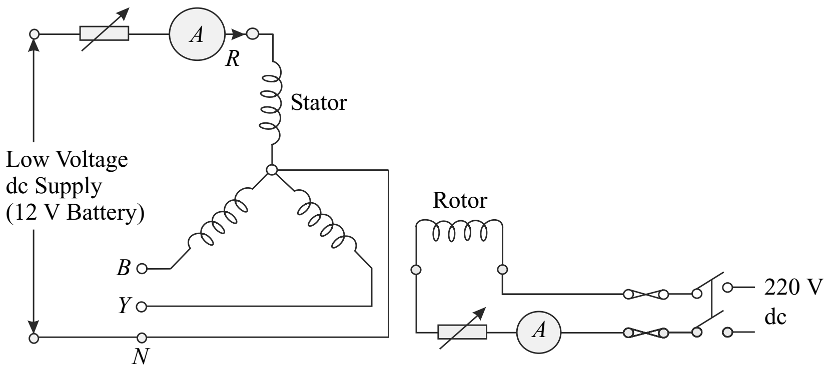


Fig. 1.101 *Shows the circuit diagram for demonstrating alignment of magnetic fields using a synchronous machine*

3. Connect one terminal of the 12 V dc source (battery) to the star point of the stator and keep this connection fixed throughout this experiment.
4. Connect the other terminal of the battery to the *R* phase of the stator winding. The rotor will move and stop at a certain position.
5. Mark the position of the rotor (when it has come to stop after Step 4 above), with respect to the stator of the synchronous machine. This may he done by putting a chalk mark on the rotor shaft and the machine frame in one line.
6. Remove the supply from the *R* phase and connect to the *B* phase of the stator. The rotor again moves and takes up another position.
7. Remove the supply from the *B* phase and connect to the *Y* phase of the stator and observe that the rotor again moves and takes up another position.
8. Having observed that the rotor rotates as the dc supply is given successively to the various stator phases, remove the supply from the *Y* phase and bring the rotor manually into a position such that the chalk marks are again in line.
9. Now give the supply to the terminals of the three stator phase successively in the sequence *RBYR*. Observe how much rotation the rotor makes with respect to the stator (the chalk mark on the stator being the reference point) when the supply to the stator phases has completed the sequence *RBYR*. Note down the direction of rotation of the rotor.
10. Next reverse the sequence of dc supply to the stator winding, i.e., give supply in the sequence *RYBR*. Observe again the direction and total angular rotation of the rotor.

Observations and Results With the rotor field excited:

(i) When supply is given to the stator phases in *RYBR* sequence, the direction of rotation of rotor is ____________.

(ii) When supply is given to the stator phases in *RYBR* sequence, the direction of rotation of the rotor is ____________.

(iii) When the three phases of the stator are excited in succession, the rotor turns by ____________ deg, when one sequence of stator excitation, either *RYBR* or *RBYR* is complete.

Questions Answer the following questions in your report:

1. If one revolution is made by the rotor when the supply to the stator is given in the sequence *RYBR*, the rotor being excited from a dc source, what is the number of poles on the synchronous machine?
2. Can you perform this experiment with ac supplied to the stator? If not, explain why?
3. If the stator field rotates at *N* rpm, at what speed will the rotor rotate?

EXPERIMENT 1.2 *Measurement of the angular displacement of the rotor of a slip-ring induction motor on application of dc to each stator phase-winding in sequence and simultaneously to one rotor phase-winding.*

Objective To demonstrate alignment of magnetic fields.

Brief Theory Same as in Experiment 1.1.

Circuit Diagram See Fig. 1.102.

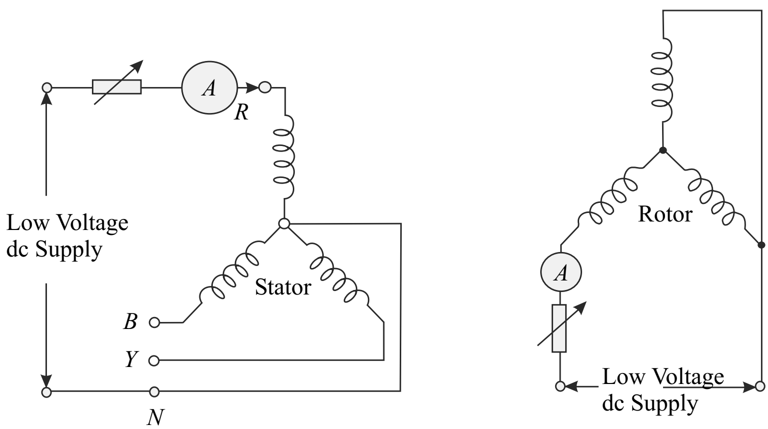


Fig. 1.102 *Circuit diagram for demonstrating alignment of magnetic fields using a slip-ring induction motor*

Apparatus Required Three-phase slip ring induction motor, ammeters (two), rheostats (two), low voltage dc supply or a 12 V battery.

Procedure

1. Make connections as per the circuit diagram.
2. Give dc supply to the *R* phase of the stator and to the rotor as shown. The rotor will take up a certain position.
3. Mark the position of the rotor with respect to the stator of the induction motor. (This may be done by putting a chalk mark on the rotor shaft and the machine frame in one line.)
4. Change the supply from the *R* to *B* phase of the stator and observe that the rotor moves and takes up another position.
5. Remove the supply from the *B* phase and connect to the *Y* phase of the stator and observe that the rotor again moves and takes up another position.

6. Having observed that the rotor rotates as the dc supply is given successively to the various stator phases, remove the supply from the *Y* phase and bring the rotor manually into a position such that the chalk marks are again in line.
7. Now connect the supply to the terminals of the three stator phases successively in the sequence *RYBR*. Observe how much rotation the rotor makes with respect to the stator (the chalk mark on the stator being the reference point) when the supply to the stator phases has completed the sequence *RYBR*. Note down the direction of rotation of the rotor.
8. Reverse the sequence of dc supply to the stator phases, i.e., give supply to the stator phases in the sequence *RBYR* and observe again the direction of rotation of the rotor and the total angle through which the rotor has rotated.

Observations and Results With the rotor circuit excited from a dc source:

(i) When supply is given to the stator phases in *RYBR* sequence the direction of rotation of the rotor is ____________.

(ii) When supply is given to the stator phases in *RBYR* sequence the direction of rotation of the rotor is ____________.

(iii) When the three phases of the stator are excited one at a time in succession, the rotor turns by ____________ deg., when one sequence of stator excitation *RYBR* or *RBYR* is incomplete.

Question Answer the following questions in your report:

1. Can you perform this experiment with ac supply to the stator and rotor? If not, explain why?
2. With the rotor excited from a dc source as shown in the circuit diagram, if the stator field is made to rotate at *N* rpm, at what speed will the rotor rotate?

EXPERIMENT 1.3 *Measurement of induced emf and frequency under open-circuit condition in one rotor phase winding of a three-phase slip-ring induction motor at different angular positions on application of ac to only one stator phase winding.*

Objectives To determine whether the induced emf in a winding which is placed within the magnetic field produced by another winding depends upon the relative position of the two windings.

Brief Theory It is known from Faraday's laws of electromagnetic induction that when ac is applied across a winding *A* and another winding *B* is placed in its vicinity, emf is induced in winding *B* (see Fig. 1.103). Application of alternating voltage V_1 to coil *A* produces alternating flux, some of which links coil. An emf E_2 is induced in coil *B* because of the change of flux linkages experienced by it.

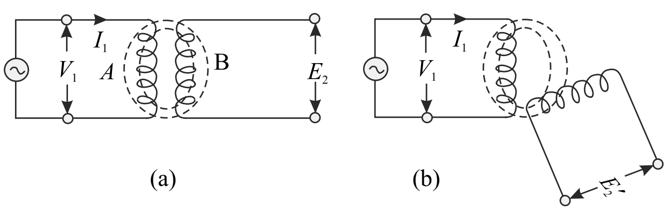


Fig. 1.103 *Dependence of induced emf in a coil upon the amount of linking of mutual flux*

Observe that the axes of the two windings in Fig. 1.103(a) are parallel to each other and in this configuration coil *B* links the maximum possible flux produced by coil *A*. The emf induced in coil, *B* in this position will be maximum.

Now if the physical position of the second coil is changed as in Fig. 1.103(b) the amount of flux linked by it will be less than it did in the configuration shown in Fig. 1.103(a). Hence the magnitude of the induced emf will be reduced. For the purpose of demonstrating this a slip-ring induction motor is taken. Coil *A* of Fig. 1.103(a) is simulated by connecting two of the three-phase windings (*R*, *Y*) of the stator in series as shown in Fig. 1.103. Supply is given as shown. Coil *B* is simulated by connecting in series two of the phase-windings of the rotor. (Say R_1 and Y_1) of a wound rotor induction motor. If the axis of coil *B* is displaced by 90 deg from coil *A*, there will be practically no flux linkage by coil *B* and the induced emf in it will be zero. Therefore, it may be concluded that the magnitude of the induced emf in coil *B* is a function of its position with respect to coil *A*. If the two axes are parallel, the induced emf is maximum. (Note that the magnitude of the induce emf is also dependent on the distance between the two coils and also on the core material.) In this experiment, it will be seen how the magnitude of the induced emf changes with change in position of the axes of the two windings.

Circuit Diagram See Fig. 1.104.

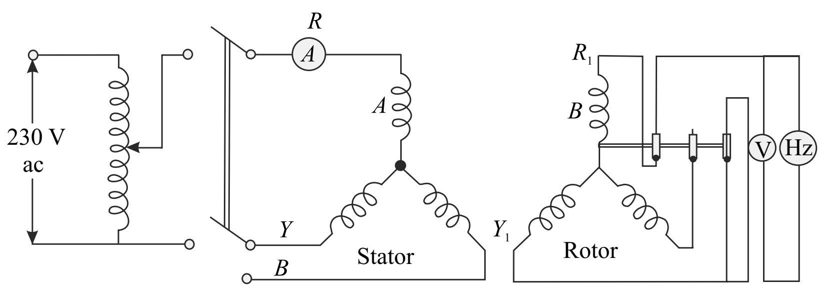


Fig. 1.104 *Circuit diagram for measurement of induced emf and frequency in the rotor winding of a slip-ring induction motor at different angular positions of the rotor on application of ac between two terminals of the stator winding*

Apparatus Required Three-phase slip ring type induction motor, single phase variac (auto-transformer), ammeter, voltmeter, frequency meter.

Procedure

1. Make connections according to the circuit diagram (keep the slip-ring terminals open circuited).
2. Apply ac voltage across the two terminals of the stator through an autotransformer.
3. Rotate the rotor slowly and find out the position of the rotor at which voltmeter *V* shows the maximum reading.
4. Mark this position of the rotor as the reference point. This can be done by putting a chalk mark on the rotor shaft and the stator frame in one line.
5. Now with the help of a chalk put marks around the whole stator frame at intervals of 30° from the reference point (measurement of angles may be approximate).

6. Rotate the rotor in steps of 30° noting down the magnitude of induced voltage and the frequency for each step. Stop taking readings when the rotor has turned through 360°.
7. Now reduce the voltage applied across the stator terminals to half its previous value with the help of the auto-transformer.
8. Repeat the same procedure as indicated under Step 6.

Observations and Results Tabulate your observations according to the table given below.

S. No.	Rotor position	Stator applied voltage	Rotor induced emf	Frequency of rotor induced emf

Take 5 or 6 readings.

From the experimental data, draw graphs showing the relation between induced emf and the rotor positions at two different stator applied voltages. Note the angular difference between the positions of maximum and minimum values of the induced emf.

Questions Answer the following questions in your report:

1. Why does the induced emf in the rotor winding change when the rotor is turned by hand?
2. On which factors does the induced emf in the rotor winding depend?
3. Can you find out the number of poles on the machine from the induced emf wave shape obtained? Explain your answer.
4. Is it possible to perform this experiment on a squirrel-cage induction motor?

EXPERIMENT 1.4 *Measurement of the induced emf and frequency under the open-circuit condition in the rotor phase winding of a three-phase slip-ring induction motor at different angular positions on application of a three-phase supply to the stator.*

Objectives

1. To verify that at standstill when a three-phase supply is given to the stator windings of a three-phase slip-ring type induction motor, the induced emf on the rotor winding is independent of the position of the rotor.
2. To verify that the magnitude of the induced emf and its frequency are dependent on the relative speed of the rotor with respect to the rotating magnetic field produced by the stator currents.

Brief Theory When a three-phase supply is given to a three-phase winding, a rotating magnetic field is produced. The effect is the same as mechanically rotating a permanent magnet. The rotating magnetic field rotates at synchronous speed. When the rotor is at a standstill, i.e., stationary, emf will be induced in the rotor winding as there exists a relative motion between the rotating field and the stationary rotor windings. The magnitude of the induced emf in the rotor at standstill remains constant irrespective of the position of the rotor windings. This is because the rotor emf depends only upon the relative motion between the rotor and the stator field and is independent of the position of the rotor. If, however, with the help of a primemover the rotor with its winding open circuited is rotated in the same direction as the rotating field, the relative motion between the rotor and the stator field is reduced. Hence, the magnitude and frequency of the induced emf in the rotor winding will be reduced. If the rotor is made to rotate at the same speed as the rotating magnetic field, the relative motion between the rotor and the rotating field will be zero and hence the induced emf will be zero.

Note: In a slip-ring-type induction motor, the terminals of the rotor winding are short circuited. In this experiment, the rotor winding terminals are kept open in order to measure the induced emf and its frequency by means of a voltmeter and a frequency meter respectively.

If the rotor is made to rotate in a direction opposite to the direction of rotation of the rotating field, the relative motion between the rotor and the rotating field increases and the magnitude of the induced emf will also increase.

Circuit Diagram See Fig. 1.105.

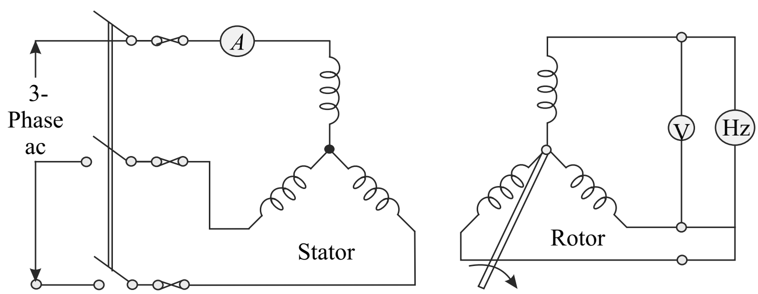


Fig. 1.105 *Connection diagram for measurement of induced emf and frequency in the rotor phase winding of a three-phase slip-ring induction motor*

Apparatus Required Slip-ring induction motor coupled with a dc machine, Voltmeters (two), Ammeter, Tachometer, Frequency meter.

Procedure

1. Make connections according to the circuit diagram.
2. Keep the slip-ring terminals of the rotor open.
3. Give a three-phases supply to the stator and measure the emf induced in the rotor between any two slip-ring terminals for different rotor positions. Record as per Table 1.1 the magnitude and frequency of the induced emf for different rotor angular positions. The rotor angular position may he determined by drawing a line on the shaft by means of a chalk and a number of lines on the end shield at an interval of say 45°. For the purpose of this experiment an approximate marking will suffice.
4. Now determine the direction of rotation of the rotating field. This is done by first short circuiting the slip-ring terminals and applying rated voltage to the stator winding. The direction in which the rotor rotates is also the direction of rotation of the rotating field. Now open circuit the slip-ring terminals and rotate the rotor in the same direction at different speeds with the help of a primemover.

5. Note down the magnitude and frequency of the induced emf of the rotor at different rotor speeds as per Table 1.2. For measurement of frequency, a CRO may also be used.

> ***Note:*** The frequency meter available in the laboratory will usually have a narrow range between 45 and 55 Hz. In this experiment the frequency of the induced emf available at the slip-ring terminals, particularly at higher speeds, will be very low. A CRO may then have to be used to measure such frequencies. However, if a CRO is not available. drive the rotor at a low speed so that the frequency of the induced emf is within the range of your frequency meter.

Observations and Results

Table 1

S. No.	Angular position of the rotor (deg)	EMF induced in rotor winding	Frequency of rotor induced emf (Hz)

Table 2

S. No.	Speed of the rotor (rpm)	EMF induced in rotor winding	Frequency of rotor induced emf (Hz)

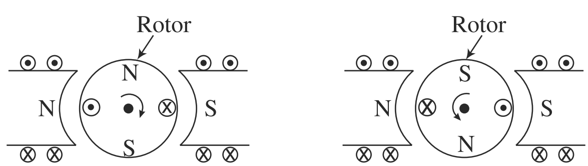
Question Answer the following questions in your report:

1. What is the magnitude of the rotor emf when the rotor is driven at synchronous speed?
2. Why does the magnitude of the rotor induced emf at standstill remain constant at different rotor angular positions?
3. Why does the magnitude of the rotor induced emf vary when the rotor is driven at different speeds?
4. In your experiment, what is the speed of the rotating magnetic field produced by the stator?
5. How can you calculate the frequency of the rotor induced emf if the speed of the rotor and that of the rotating magnetic field produced by the stator are known?

SOLUTIONS OF THE EXERCISE PROBLEMS

For Exercises 1A

Ex. 1.1



(a) For clockwise torque (b) For anticlockwise torque

Fig. 1.106

We have first marked the two poles as N and S with the given current directions. North pole is one where from flux lines come out of the magnet body and South pole is one where flux lines enter the magnet body. We have then placed the positions of North and South poles on the rotor to achieve clockwise and anticlockwise torque. We have then found the direction of current to produce the poles in (a) and (b) respectively.

Ex. 1.2 We have marked the positions of North and South poles on the stator. To develop torque in the anticlockwise direction, the position of the poles on the rotor has to be shown. For the poles shown, the direction of current in the rotor coils indicated.

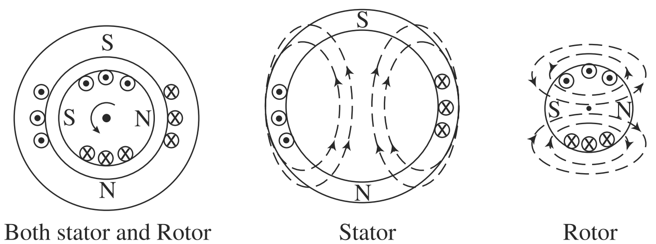


Fig. 1.107

Ex. 1.3 In Fig. 1.108(a) the positions of poles have been shown. The stator and rotor magnetic field axes are aligned and hence no torque is developed.

In Fig. 1.108(b) after showing the positions of both stator and rotor poles, it is observed that both stator and rotor are in unstable equilibrium and torque developed is zero. In this position, if a small torque is provided on the rotor in any direction, it will rotate by half revolution to attain a stable equilibrium position.

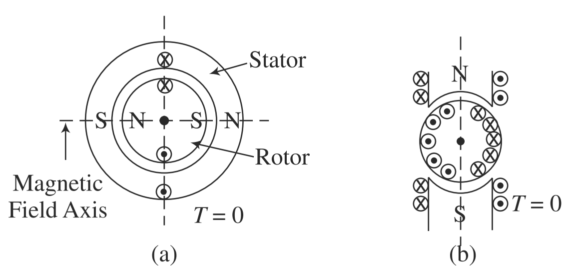


Fig. 1.108

Ex. 1.4 The axis of the stator magnetic field and the rotor magnetic field are not aligned. So, due to non-alignment of the two magnetic field torque will be developed. Torque angle is the angle between the two magnetic fields.

Ex. 1.5 The directions of currents in the pole windings have been shown so that N and S poles are formed. Then we indicate the

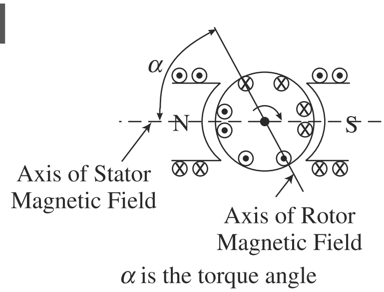


Fig. 1.109

positions of N and S poles on the rotor so as to get clockwise torque. Accordingly, the directions rotor winding currents have been shown.

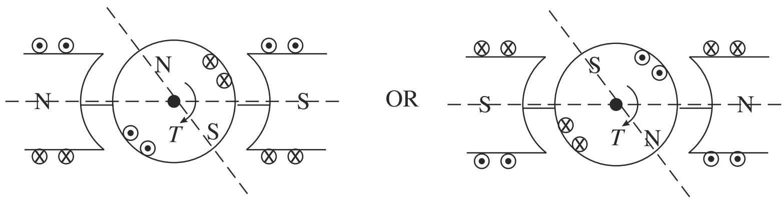


Fig. 1.110

Ex. 1.6

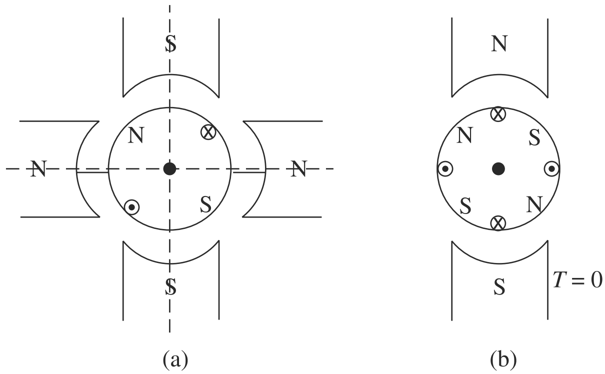


Fig. 1.111

In both the cases shown in Fig. 1.111(a) and (b), the net torque will be zero. In Fig. 1.111(a), the stator has four poles while the rotor has two poles. In Fig. 1.110(b) the stator has two poles while the rotor has two poles. For unidirectional torque, the number of stator and rotor poles should be equal.

For Exercises 1B

Ex. 1.1 The poles have been marked N and S according to the directions of the field currents. Applying Fleming's right hand rule, the directions of current in the rotor conductors determined as have been shown.

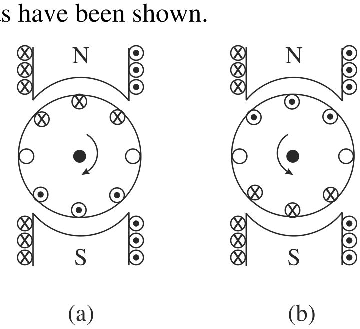
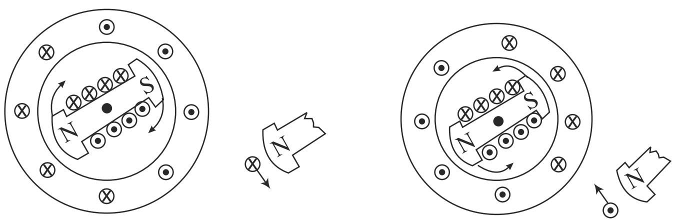


Fig. 1.112

Ex. 1.2 Here the field poles are rotating. Since there is relative velocity between the field flux and the conductors in the stator, emfs will be induced in the stator conductors. For applying Fleming's right hand rule, it will be convenient to consider the fields to be stationary but the conductors rotating in opposition direction. Accordingly, we will first mark the positions of N and S poles, consider poles as stationary and the conductors rotating in opposite direction. For example, in Fig. 1.113(a) the N-pole has been made static and the armature conductor has been assumed to have a relative motion in opposite direction. Now, by applying RFH rule, the direction of induced emf is found out as has been shown. All the conductors under N-pole will have same direction of induced emf and those under S-pole will have opposite direction of induced emf. Note that for determining the N and S pole positions of the field poles, the right hand grip rule can be applied conveniently.



(a) (b)

Fig. 1.113

Ex. 1.3 and 1.4 There will be a change in flux linkage by the coil when the magnet is either moved towards or is moved away from the coil. The change in flux linkage will cause emf induced and hence current flow in the coil, if the coil is a closed one. According to Lenz's law the flux produced by the induced current should be such that this flux will oppose the increase or decrease of flux linkage. When the magnet is moved towards the coil there is increase in flux linkage. The flux due to induced current is the coil must oppose this increase. When the magnet is moved away from the coil there will be decrease in flux linkage and the coil induced flux most oppose the decrease in flux linkage. The current directions in the coil have been accordingly indicated.

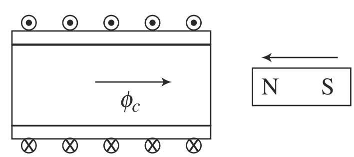


Fig. 1.114 *For Ex 1.9*

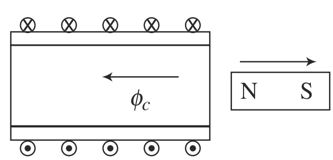


Fig. 1.115 *For Ex 1.10*

For Exercises 1C

Ex. 1.1 The applied mechanical torque T_m causes rotation of the rotor. The angular speed is ω. Thus ω and T_m are in the same direction. Here, it is clockwise direction. The direction of induced emf in the rotor coil is found by applying FRH rule. When connected to a load resistance current will flow. The direction of current will be the same as the direction of induced emf. The positions of N and S poles on the rotor have been shown. The direction of electromagnetic torque T_e, in case of generator action as in this case, is opposite to the applied mechanical torque, T_m. The more is the electrical load, the more will be T_e and hence more should be T_m. This establishes the condition of input-output balance. The frictional torque, T_f opposes the applied mechanical torque T_m.

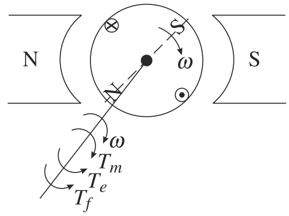


Fig. 1.116

Ex. 1.2 A current carrying coil when placed in a magnetic field develops an electromagnetic torque which causes rotation of the rotor. The mechanical load is attached to the rotor shaft. Since the electromagnetic torque T_e causes rotation, both T_e and ω are in the same direction. The load torque, i.e., the torque required to drive the load, T_L and the frictional torque, T_f will be in the opposite direction to T_e and ω. The torque angle α is the angle of nonalignment between the axis of stator field and the rotor field. The supply voltage across the rotor coil is V. The emf induced in the rotor coil when it is rotated in the clockwise direction can be found by applying FRH rule on the coil sides. The induced emf E_b will be in the opposite direction to the direction of applied voltage V. That is E_b will oppose V, by Lenz's law. Induced emf in the rotor coil, E_b is often referred to as back emf. If I_a is the current flowing through the armature coil and R_a is the resistance of the coil, then $V - E_b = I_a R_a$.

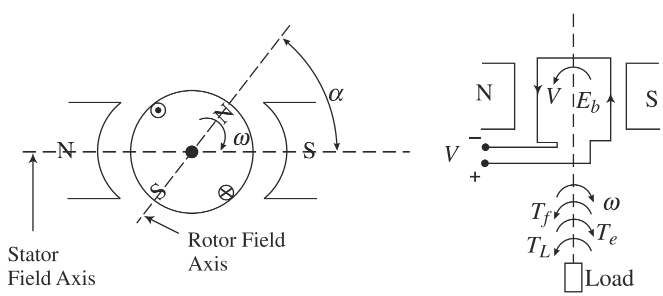


Fig. 1.117

The relationship of various torques is established by equation the developed torque with the opposing torques, as

$$T_e = T_L + T_f$$

i.e., electromagnetic torque = load torque + frictional torque

Ex 1.3 When the coil is rotated in the magnetic field, emf is induced. The direction of induced emf on the coil sides (conductors) is determined by applying FRH rule. When an electrical load, i.e., a resistance is connected across the coil, current will flow. The direction of induced emf E and of current, I will be the same. The positions of rotor poles are then indicated. It is observed that the electromagnetic torque is in clockwise direction and is in the opposite direction to the applied mechanical torque causing rotation of the rotor.

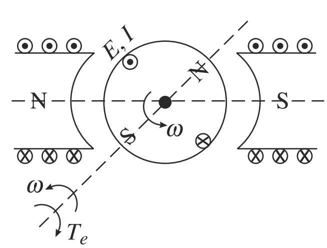


Fig. 1.118

2 DIRECT CURRENT MACHINES

OBJECTIVES

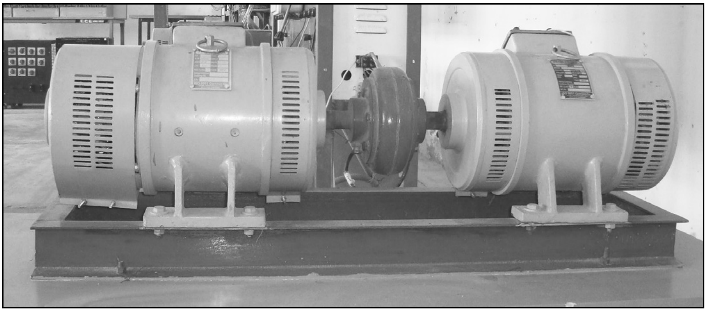
After carefully studying this chapter, you should be able to

- ➢ Explain the function of various parts of a dc machine.
- ➢ Prepare winding schemes for the armature winding.
- ➢ Explain the working of a dc machine for generating and motoring action.
- ➢ Draw the characteristics of different types of dc machines and indicate their possible fields of applications.
- ➢ Draw the starter circuits and start a dc motor using starters.
- ➢ Explain the methods of speed control of a dc motor.
- ➢ Explain the effect of armature reaction in a dc machine and suggest ways to neutralise its effect.
- ➢ Explain commutation in a dc machine.
- ➢ Describe methods of determining the efficiency of a dc machine without actually loading the machine.
- ➢ List the maintenance tasks of a dc machine.
- ➢ Perform certain basic tests on a dc machine.

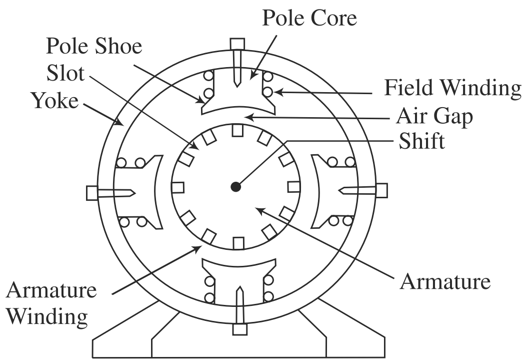
A dc machine works as a dc generator when it is driven by a primemover. The same machine works as a dc motor when electrical energy is supplied to it. Therefore, the constructional features of a dc generator and a dc motor are the same.

2.1 MAIN CONSTRUCTIONAL FEATURES

Like any other electrical rotating machine, a dc machine has two main parts, viz. the stator and the rotor. The stator is the stationary member and consists of the field system. The rotor is the rotating member which houses the armature winding. The rotor of a dc machine is also called the armature. Figure 2.1(a) shows a dc motor generator set and Fig. 2.1(b) shows the simplified cross-sectional view of such a dc machine. The pole cores are usually made of a number of steel sheets stacked and riveted together. The pole cores are then bolted to a hollow cylindrical stator frame called the yoke. The yoke may be made of cast steel or fabricated rolled steel. The field poles when excited by dc current produce the magnetic field. For a given machine the strength of the magnetic field depends upon the field current. The armature core is separated from the field poles by a small air-gap which allows the



(a)



(b)

Fig. 2.1 *(a) A dc motor generator set, (b) Cross-sectional view of the general arrangements of a dc machine*

armature of rotate freely. The air-gap is kept very small to keep the reluctance to the magnetic circuit low. The armature is a laminated cylinder and is mounted on a shaft. The armature laminations are about 0.4–0.6 mm thick and are insulated from one another. The armature is laminated to reduce the eddy-current loss in the core. Slots are stamped on the periphery of the armature laminations. The armature slots house the armature windings. The stator core, the yoke and the poles may not be laminated as they encounter dc flux.

Due to the presence of slots on the armature surface, there is flux pulsation at the stator pole-face. The stator pole shoes, therefore, should be laminated to reduce the eddy-current loss. However, for mechanical reasons, in many cases the whole of the pole core is laminated. In dc machines of high ratings slots are cut on the pole-faces to house a separate winding called the compensating winding. The compensating winding is connected in series with the armature winding and neutralises the effect of armature reaction. To neutralise the effect of armature reaction in the space in between two poles, smaller poles, called interpoles, are fixed on the yoke as shown in Fig. 2.2.

As mentioned earlier, the armature winding is placed inside the armature slots. The slots are lined with tough insulating material. This slot insulation is folded over the armature conductors. The conductors in the slots are secured in their places by hard wooden wedges or fibre glass wedges. The armature windings are first made on formers and then placed on slots.

Enamel insulated copper wires are used for the armature winding. Each armature coil end is connected with each segment of the commutator. A commutator is a cylindrical body mounted on the shaft along with the armature. In fact, the armature core and the commutator form one single unit mounted on the shaft. Brushes are placed on the commutator surface to supply or collect current to the armature coils through the commutator segments. The commutator segments are insulated from each other.

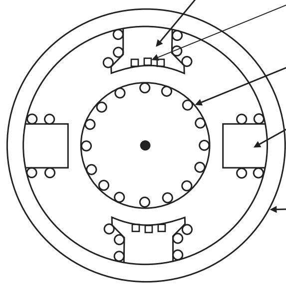


Fig. 2.2 *Cross-sectional view of a dc machine showing the positions of interpoles*

The function of the commutator is to convert alternating currents induced in the armature conductors into direct currents in the external circuit in case of generator operation. In the case of a dc motor the function of the commutator is to produce a unidirectional torque. The commutator is of cylindrical structure and is built up of a wedge-shaped segment of hard-drawn copper. Mica insulation is provided between commutator segments. Brushes are made of carbon and are housed in brush-holders. A spring in the brush-holder maintains the desirable pressure on the carbon brushes so that proper contact is maintained between the brushes and the commutator surface.

2.2 FUNCTION OF COMMUTATOR FOR MOTORING AND GENERATING ACTION

Figure 2.3 shows the cross-sectional view of a dc machine. The field system is shown excited and dc voltage is shown applied across the armature terminals. The direction of the current flowing through the armature conductors is also shown. The magnetic polarities of the armature will be as shown in the figure. Thus the axes of the two magnetic fields are at 90° with each other and hence maximum torque will be developed. The axis of the stator magnetic field is fixed. The armature magnetic field axis should maintain an angle of non-alignment of 90° all the time. This is possible if the distribution of current on both sides of the brush axis remains the same as shown in Fig. 2.3 all the time, i.e., even when the rotor starts rotating. Thus when the armature starts rotating, conductor 8 will pass under brush *A* and occupy the position of conductor 1′. Similarly, conductor 8′ will occupy the position of conductor 1. Thus to have current distribution on one side of the armature conductors the same at any time, it is necessary that there should be a change in the direction of current in these conductors while passing under the brushes. Otherwise, the armature as shown in Fig. 2.3 will turn only by 90° and thereafter stop rotating. Therefore, for continuous rotation there should be a change in the direction of current in conductors passing under the brushes. This process of current change is called commutation.

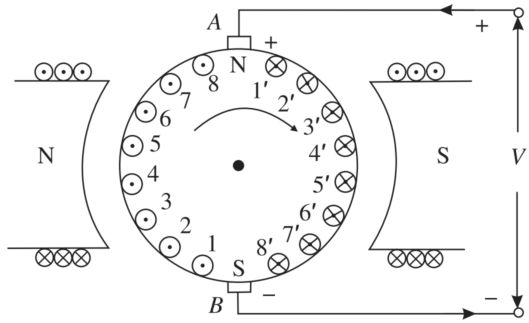


Fig. 2.3 *Cross-sectional view of a dc machine working as a motor*

In a dc motor, commutation is necessary for achieving a continuous unidirectional torque. How exactly reversal of current in a conductor takes place when it passes under a brush is illustrated in Fig. 2.4. For simplicity, only one coil has been taken in the armature. The two coil-ends are connected to two commutator segments as shown in the figure. In actual practice there will be a number of coils in the armature circuit and their ends will be connected to the commutator segments, each commutator segment being insulated from one another. A commutator in an actual dc machine is composed of a number of segments. Figure 2.4(a) shows the isometric view while Fig. 2.4(b) shows a simplified view of an armature having one coil with its ends connected to two commutator segments.

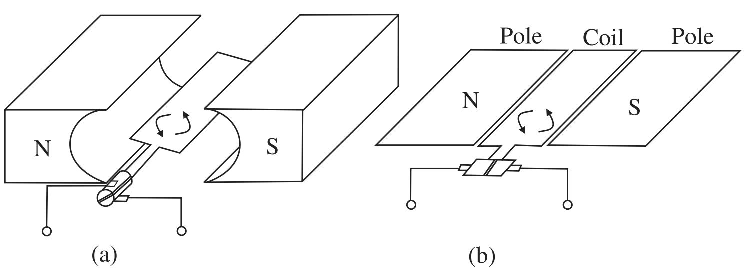


Fig. 2.4 *Armature coil of a dc machine connected with the commutator segments shown in two views*

It is now to be seen how the direction of current in the conductors is reversed when the coil is rotating. Consider the coil shown in Fig. 2.5(a). It shows the two ends a and a' of the coil connected to the two halves of conducting cylinder [see also Fig. 2.5(b)]. Coil-end a is connected to half A and coil-end a' is connected to half A' of the conducting cylinder, called the commutator. The two halves of the cylinder are separated by a layer of insulation. Two sliding contacts C and C' are made on the surface of the two parts of the cylinder by means of two brushes. Current from the source will flow from contacts of C and A to coil-sides a and a' and back through contacts of A' and C'. Current cannot flow from A to A' as the two halves are separated by insulation. Now, assume that the coil turns in an anticlockwise direction as shown in Fig. 2.5(a). When the coil turns by half a revolution the coil along with commutator segments A and A' will take the positions as shown in Fig. 2.5(c). It is to be noted that the cylinder also rotates with the coil, but the brushes are stationary. Thus, A will take the position of A' and vice-versa. Also, it is seen that the direction of current in the coil-sides has reversed.

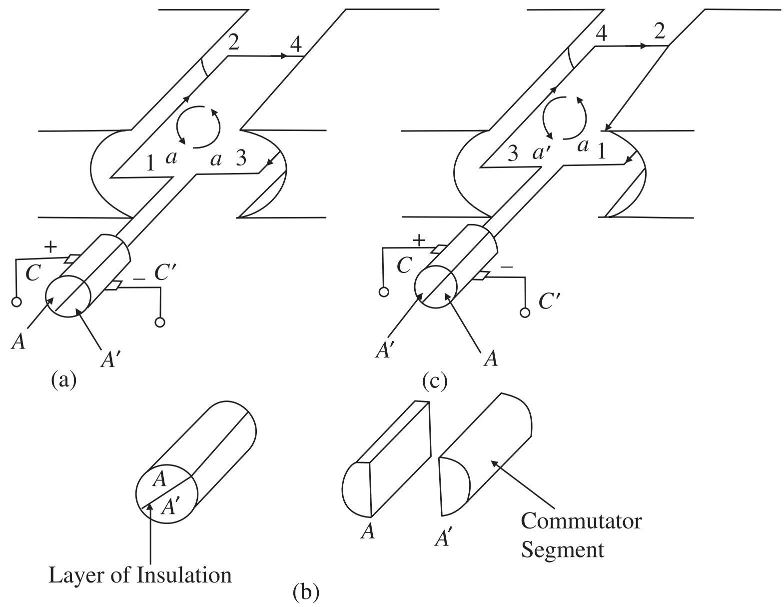


Fig. 2.5 *Directions of current in the coil-sides are reversed when a coil rotates (achieved through brush and commutator arrangement)*

For every half revolution the current in the coil-sides will reverse. Segments A and A' are called commutator segments and the brush and commutator segments together form the commutator assembly. Thus, by a brush and commutator arrangement the currents in the coil-sides can be reversed after every half revolution. The core on which the coil is housed, i.e., the armature and the commutator assembly, is mounted on a common shaft (see Fig. 2.6).

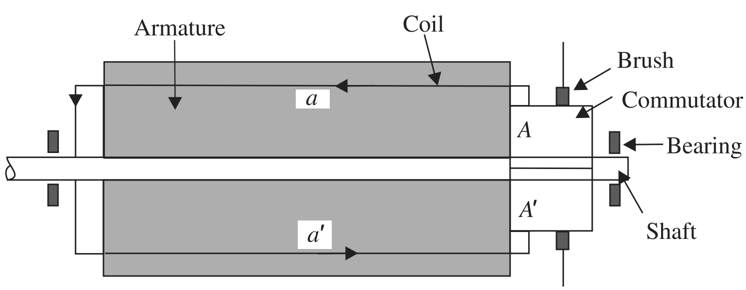


Fig. 2.6 *A coil with brush and commutator arrangement placed on armature slots. The current in the coil-sides gets reversed in every half cycle*

Consider now the function of commutator when the dc machine is working as a generator. Consider one coil rotating in a magnetic field as shown in Fig. 2.7(a). The direction of emf included in the coil-sides will be as shown in the figure. When coil-side *a* comes under *S*-pole, the direction of emf induced in it changes. If coil-end *a* and *a′* are connected through slip-rings and brushes to the external circuit, alternating voltage is obtained across the output terminals (Fig. 2.7(b)).

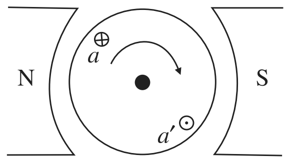


Fig. 2.7(a) *Coil rotating in a magnetic field*

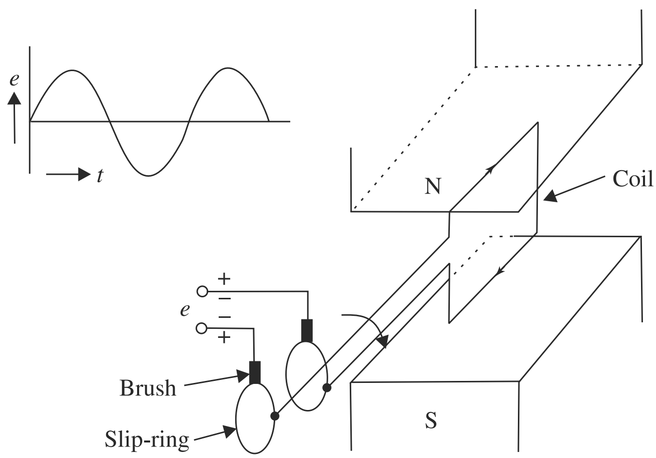


Fig. 2.7(b) *Alternating voltage available across the slip-ring terminals when a coil is rotating in a magnetic field*

From a dc generator, a dc output is expected. Through a brush and commutator arrangement the alternating emf induced in the armature coil is converted into dc voltage for the output circuit. Conversion of ac induced in the armature circuit into dc at the output circuit is called commutation. This is explained below.

In Fig. 2.8 is shown a coil rotating in a magnetic field of flux density *B*. The coil ends are connected to two commutator segments. If the emf induced in the coil is plotted for its different positions, the shape of the emf induced will be as shown in Fig. 2.8(d). Points *a*, *b* and *c* respectively show the magnitude of emf induced for coil positions shown in Fig. 2.8(a), (b) and (c). The magnitude of induced emf in the coil in positions (a) and (c) in Fig. 2.8 is the maximum as the rate of cutting of lines of force by the coil-sides is the maximum. However, the direction of the induced emf is the same due to the brush and commutator arrangement. For the position of the coil shown in Fig. 2.8(b), the induced emf in the coil-sides is zero as the rate of cutting of the lines of force by the coil-sides is zero. It may be noted that the coil-ends get short-circuited by the brushes at the position when the induced emf in them is zero.

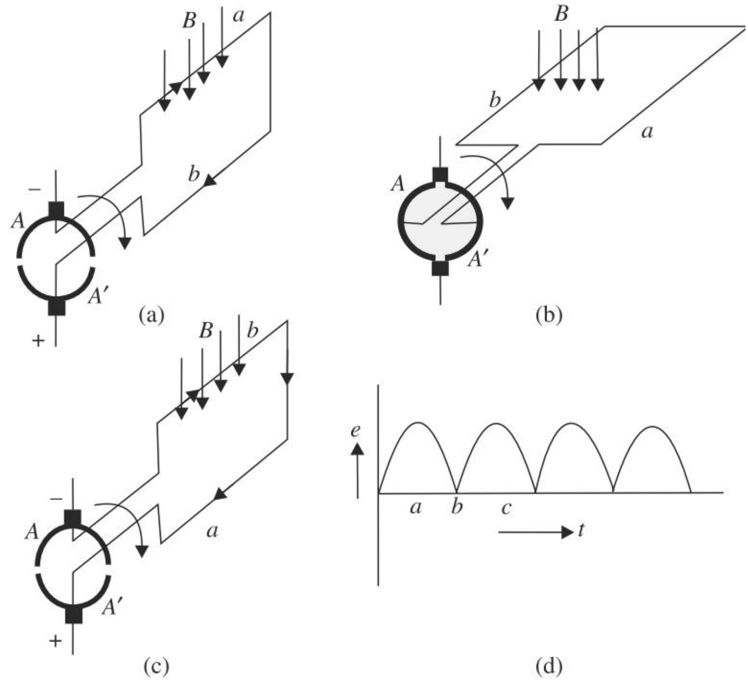


Fig. 2.8 *Direction of emf induced in a coil rotating in a magnetic field*

How ac generated in the armature coil gets converted into dc through brush and slip-ring arrangement is further explained below.

A coil *a–a′* is shown rotating in a magnetic field created by the two poles of the dc generator. The position of the coils sides *a* and *a′* gets interchanged in every half cycle of rotation of the coil. The direction of current in the coil in Fig. 2.9(a) is from *a* to *a′*. After half a revolution the coil-sides change their positions. The current flow is now from *a′* to *a*. As the coil continues to rotate in the clockwise direction, the direction of current in the armature coil changes continuously from *a* to *a′*, *a′* to *a*, *a* to *a′*, *a′* to *a*, and so on. This is called alternating current. The current through the load resistance remains unidirectional as has been shown in Fig. 2.9(b).

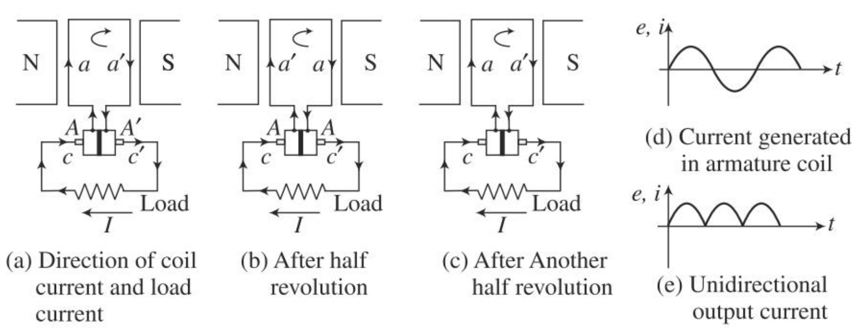


Fig. 2.9 *Function of brush and commutator in a dc generator. Current in the armature coil is ac but current through the load is dc*

2.3 ARMATURE WINDING

The armature winding is a vital part of a dc machine. This is where emf is induced and force is developed that results in the turning of the rotor. The design of the armature winding is more critical than the design of other parts of a dc machine.

The armature winding is housed in slots made on the armature surface. Formed coils are placed on slots. The ends of the coils are joined with commutator segments.

2.3.1 Materials Required for Armature Winding

Coils for the armature winding are made from insulated copper conductors. Hard-drawn annealed higher conductivity copper is used. Aluminium wires are not used because of the restriction on winding space in slots and a number of other limitations, such as the formation of oxide coating and jointing problems. For large industrial machines, however, aluminium may be used for field windings as an alternative to copper.

Insulating materials used should have the following characteristics:

(i) high dielectric strength at increased temperature,

(ii) good thermal conductivity to transfer heat generated due to conductor I^2R loss to the surrounding structure and coolant,

(iii) ease of working having sufficient mechanical strength, and

(iv) resistance to failure by moisture, vibration, abrasion and bending.

Unfortunately, none of the available materials has all the above-mentioned desirable properties. Superior synthetic materials are now available which are gradually replacing the natural insulating materials. Available insulating materials are classified according to their thermal limits as shown in Table 2.1.

Table 2.1 *Classification of insulating materials*

Insulation class	Materials	Maximum permissible operating temperature °C
Class Y	Cotton, silk, paper, press board, wood, etc., not impregnated nor oil immersed, PVC with or without plasticizer, vulcanized natural rubber, etc.	90
Class A	Cotton, silk, paper, etc., when impregnated or immersed in a liquid dielectric such as oil, (in class Y material impregnated with natural resins, cellulose esters, insulating oils, etc.), also laminated wood, varnished paper, cellulose, acetate film, etc.	105
Class E	Synthetic resin enamels, cotton and paper laminates with formaldehyde bonding, etc.	120
Class B	Mica, glass fibre, asbestos, etc., with suitable bonding substances, built-up mica, glass-fibre and asbestos laminates	130
Class F	Class B materials with thermally resistant bonding materials.	155
Class H	Glass fibre, asbestos, built-up mica, etc., with silicon resin binder.	180
Class C	Mica, ceramic, glass, quartz and asbestos without or with an inorganic binder.	above 180

Conductor Insulation For small-size machines double-cotton-covered copper wires are used. For medium-size machines the conductors are rectangular in shape. Each conductor is machine-taped with superfine cotton tape, whereas for large-size machines each conductor is machine-taped with one layer of 0.2 mm thick impregnated cotton tape with half overlap.

Slot Insulation For slot insulation, leatheroid, manila paper or mica folium of appropriate thicknesses are used. Overhangs, i.e., the back portions of the coils not lying in slots, are insulated with varnished and impregnated cotton tape.

Commutator The commutator is made up of a number of commutator segments. Coil-ends are connected to each commutator segment. The segments of the commutator are made of hard-drawn copper and are separated by thin sheets of mica or micanite.

The induced emf per conductor in a dc machine is small. The problem is how these conductors are to be connected together so as to form a complete winding. Figure 2.10(a) shows the cross-sectional view of the armature of a four-pole machine.

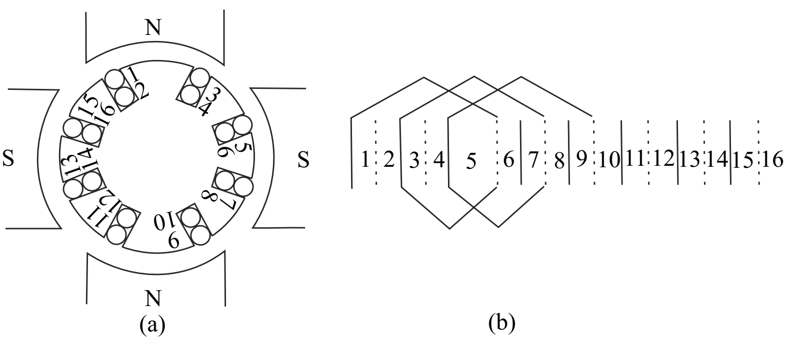


Fig. 2.10 *(a) Cross-sectional view of the armature of a 4-pole dc machine (b) Incomplete developed diagram of the armature winding*

For ease of understanding, a developed diagram of armature of Fig. 2.10(a) is drawn as shown in Fig. 2.10(b). Conductors should be so connected that the total emf is maximum. Therefore, conductor 1 should be connected to conductor 6 shown by dotted line as conductor 6 is placed below conductor 5 so that they occupy identical positions under two adjacent poles. Similarly conductor 3 should be connected with conductor 8 and so on. Figure 2.11 shows the developed winding diagram of the 16 armature conductors of Fig. 2.10(a). The average pitch Y_a back pitch Y_b, and the front pitch Y_f are calculated as:

$$Y_a = \frac{16}{4} = 4$$

$$Y_a = \frac{Y_b + Y_f}{2}$$

$$Y_b - Y_f = \pm 2$$

For progressive lap winding

$$Y_b - Y_f = 2$$

$$\therefore \quad Y_b = 5, Y_f = 3$$

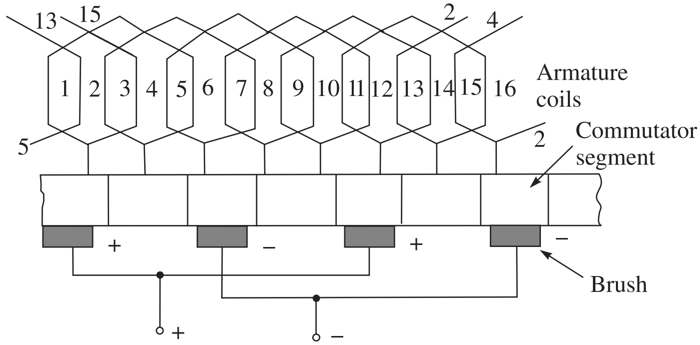


Fig. 2.11 *Armature winding of a dc machine*

Figure 2.11 gives the details of end connections of the conductors, connection of coils with commutator segments, and the position of brushes on the commutator surface with their polarities. This type of winding is called lap winding. In the winding shown in Fig. 2.11, single-turn conductors are used. As many as 16 conductors make eight coils. The coils are 1-6, 3-8, 5-10, 7-12, 9-14, 11-16, 13-2 and 15-4. The design of a lap winding of the type shown in Fig. 2.11 is described as follows.

2.3.2 Lap Winding

In a lap winding, the finishing end of one coil is connected via the commutator segment to the starting end of the adjacent coil situated under the same pole. In this way all the coils are connected. The winding is known as lap winding because the sides of successive coils overlap each other (see Fig. 2.11). A coil may consist of any number of turns. The number of slots required on the armature is equal to the number of coil-sides if two coil-sides are placed in each slot. With two coil-sides in each slot, a two-layer winding is obtained. While making a winding diagram in a two-layer winding, all top coil-sides are numbered odd whereas the bottom coil-sides are numbered even (shown by dotted lines) as shown in Fig. 2.12. For an eight-coil armature, therefore, eight slots are required on the armature surface. The following terminologies are required to be understood for preparing an armature winding diagram.

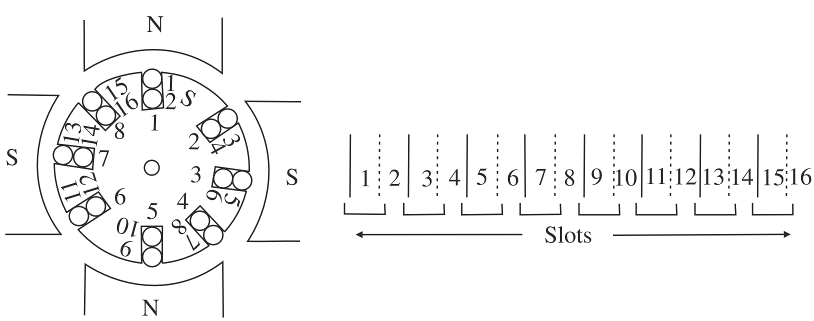


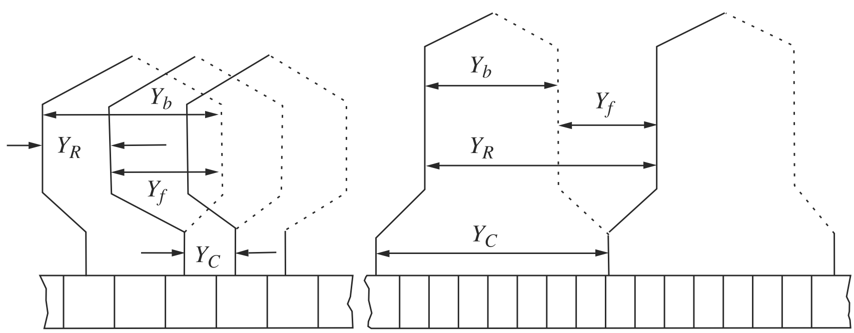
Fig. 2.12 *Position of coil-sides in slots of a two-layer armature winding*

Pole Pitch It is equal to the number of coil-sides per pole. For a single turn, eight-coil, four-pole armature pole pitch is calculated as:

$$\text{Pole pitch} = \frac{\text{No. of coils} \times 2}{\text{No. of poles}} = \frac{8 \times 2}{4} = 4$$

Coils and Coil-sides The dc armature windings are double-layer type having at least two coil-sides per slot. Each coil consists of an upper coil-side at the top of one slot and a lower coil-side situated at the bottom of another slot. The distance between the two coil-sides of a coil is approximately equal to the pole pitch. A coil may be of single turn or of many turns. If two coil-sides are placed in one slot, then the number of slots required on the armature of housing the coils is equal to the number of coils of the winding. For low-speed high-voltage winding, however, the number of coil-sides per slot is more than two. This is because the winding will have a large number of coils and it may not be possible to have an equal number of slots on the armature.

Back Pitch The distance measured in terms of the number of armature conductors (coil sides) between the two coil-sides of a coil measured around the back of the armature, i.e., away from the commutator end of the armature is called the back pitch, Y_b (see Fig. 2.13).



(a) Lap Winding (b) Wave Winding

Fig. 2.13 *Shows back pitch Y_b, front pitch y_f resultant pitch Y_R, and commutator pitch Y_C in (a) lap winding (b) wave winding*

Front Pitch The distance between two coil-sides connected to the same commutator segment is called the front pitch, Y_f.

Resultant Pitch It is defined as the distance in terms of the number of coil-sides between the start of one coil and the start of the next coil to which it is connected.

Commutator Pitch It is defined as the distance measured in terms of commutator segments between the segments to which the two ends of a coil are connected.

For calculating back pitch Y_b and front pitch Y_f for a lap winding, the following relations are used:

(i) $Y_b - Y_f = \pm 2m$ Also, $Y_b = \frac{Z}{P} \pm 1$

where m = 1 for simplex winding
= 2 for duplex winding

When Y_b is greater than Y_f, the winding is a progressive one, i.e., it progresses from left to right. If Y_b is less than Y_f, the winding is called a retrogressive one, i.e., it progresses from right to left.

(ii) The back pitch and front pitch must be odd.

(iii) The average pitch, $Y_a = \frac{Y_b + Y_f}{2}$, should be equal to the pole pitch, i.e., equal to Z/P, where z is the number of coil sides.

(iv) The commutator pitch is equal to m, i.e., equal to 1, 2, etc. for simplex, duplex etc. type of winding.

(v) The number of parallel paths in the armature winding for a simplex lap winding is equal to the number of poles, P.

(vi) The resultant pitch is always even, being the difference of two odd numbers.

EXAMPLE 2.1 *Prepare a layout winding diagram for a simplex lap-type dc armature winding. The winding is for four poles. The armature has 16 slots and 16 commutator segments.*

Solution Number of armature coils = Number of commutator segments = 16

Number of coil-sides (conductors)

$$Z = 16 \times 2 = 32$$

$$\text{Back pitch } Y_b = \frac{Z}{P} \pm 1 = \frac{32}{4} \pm 1 = 9 \text{ or } 7$$

$$Y_b - Y_f = 2$$

$$\therefore \quad Y_f = Y_b - 2 = 9 - 2 \text{ (using } Y_b = 9)$$
$$= 7$$

$$Y_b = 9$$
$$Y_f = 7$$

Since $Y_b > Y_f$, the winding is a progressive one.

As there are 32 coil-sides and 16 slots, the number of coil-sides per slot is 2. The connection scheme of the coil-sides is shown in Fig. 2.14.

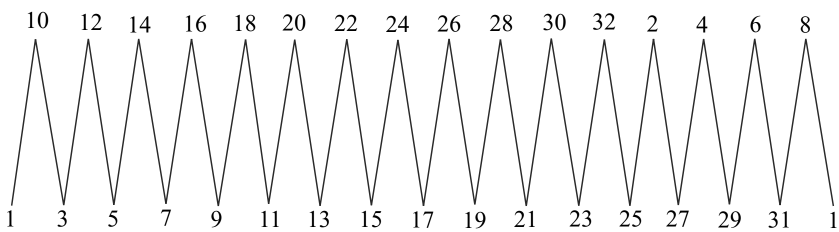


Fig. 2.14 *Scheme for connections of the coil-sides of a dc armature windings*

Coil-side 1 is connected to coil-side 10 on the other side of the commutator (since Y_b is 9, coil-side 1 is connected to coil-side 1 + 9, i.e., 10). Coil-side 10 is connected to coil-side 3 on the commutator end (Since Y_f is 7, coil-side 10 is connected to coil-side 10–7, i.e., 3). The winding progresses according to the above scheme. It may be noted that each coil is used once and the winding is a closed one.

The layout diagram of the winding along with commutator connections and brush positions is shown in Fig. 2.15.

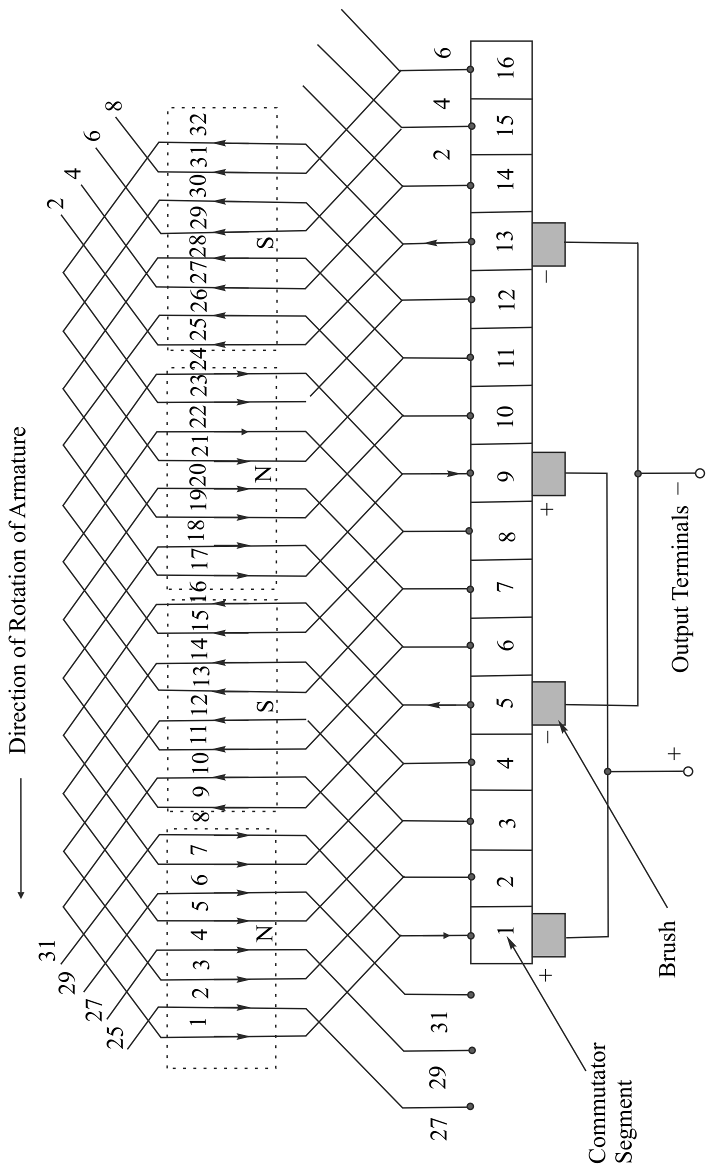


Fig. 2.15 *Layout diagram for a lap winding*

Connections of the coil-sides are made as follows: for connections at the back end of the armature, add the back pitch with the coil-side which is to be connected. Thus coil-side 1 is to be connected with coil-side $1 + Y_b$, i.e., $1 + 9 = 10$. On the commutator end side, coil-side 10 is connected to coil-side 3. This is achieved by subtracting Y_f, i.e., 7 from coil-side number 10 ($10 - 7 = 3$). Coil-side 3 is now connected to $3 + Y_b = 3 + 9 = 12$. In this way the winding is completed.

The positions of the four poles are also shown in Fig. 2.15. Eight coil-sides placed in four slots are under each pole. Assuming a direction of rotation of the armature, say anticlockwise in Fig. 2.15, the direction of the induced emf in the armature conductors is determined by applying Fleming's right-hand rule. The direction of the current in the coil-sides under north poles will be downward and under south poles upward as shown in Fig. 2.15.

The position of brushes can be determined by tracing the directions of current in various coil-sides. From Fig. 2.15, it can be observed that directions of current in coil-sides 1 and 8 are downward and they are connected to commutator segment 1. A brush placed on commutator segment 1 will have positive polarity. Similarly in coil-sides 9 and 16, the current is upwards. The two coil-sides are connected to commutator segment 5. The brush placed on commutator segment 5 will have negative polarity. Similarly the positions of the other two brushes are fixed. Two positive brushes and two negative brushes are joined together to output terminals *A* and *B* respectively.

The number of parallel paths of the armature winding across the output terminals is four (equal-to the number of poles) which can be examined as follows: Redraw the armature winding of Fig. 2.15 in a simplified manner as shown in Fig. 2.16. Between terminals *A* and *B* there are four parallel paths shown as *M*, *N*, *O* and *P*. The total emf generated in the machine is equal to the emf generated in one parallel path.

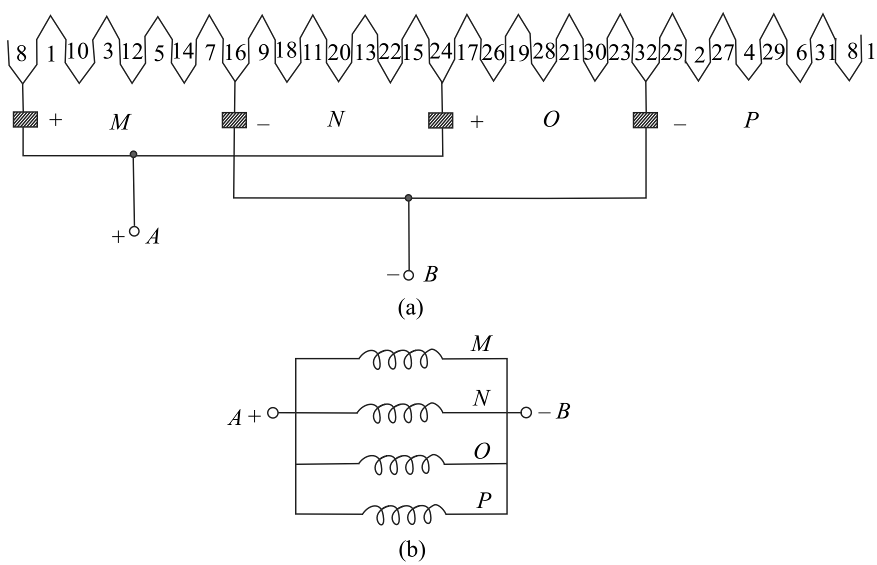


Fig. 2.16 *(a) Armature winding of a dc machine shown in a simplified manner (b) Shows the number of parallel paths in the armature*

Equaliser Connections in Lap Winding As mentioned earlier, a simplex lap winding has as many number of parallel paths as there are poles. The emf induced in each parallel path may not be exactly equal due to a number of reasons, such as the difference in the lengths of the air-gap under each pole, the difference in the field strength due to some error in putting field windings, etc.

Unequal values of emf generated in the parallel paths will circulate a considerable amount of current in the armature circuit without doing any useful work. This circulating current will be large as the armature circuit resistance is generally very low. This circulating current will generate heat and while circulating through the brush contacts will cause commutation difficulties (like sparking on the commutator surface).

To overcome this problem arising from the circulating current, equaliser connections are made in lap wound armatures. These equaliser connections or equalisers are low-resistance copper conductors which connect those points in the winding which under ideal conditions should be at equal potential. The difference in potential between these points created due to reasons mentioned earlier will be equalised as a result of flow of current through these low resistance conductors which will bypass the current from flowing through the brushes.

2.3.3 Wave Winding

In a wave winding a coil-side under one pole is connected to a second coil-side which occupies approximately the same position under the next pole through back connection. The second coil-side is then connected forward to another coil-side under the next pole (in the case of lap winding the second coil is connected back through the commutator segment to a coil-side under the original pole). The difference in lap and wave winding connections has been illustrated in Fig. 2.13(a) and (b).

The characteristics of a wave winding are:

(i) Average pitch, $$Y_a = \frac{Y_b \pm Y_f}{2} = \frac{Z \pm 2}{P}$$

If Y_a is taken equal to Z/P, as is the case in a lap winding the winding after one round will close itself without including all the coils which is not desirable. Hence the product of the average pitch and the number of pairs of poles must be two greater or less than the number of coil-sides.

Average pitch should be a whole number.

(ii) Both back pitch and front pitch should he odd numbers.

(iii) To make the average pitch a whole number, wave winding is not possible with any number of coil-sides. For example if $Z = 32$ and $P = 4$,

$$Y_a = \frac{Z \pm 2}{P} = \frac{32 \pm 2}{4} = 8\tfrac{1}{2} \text{ or } 7\tfrac{1}{2}$$

Thus wave winding is not possible with 32 coil-sides. In this case the number of effective coil-sides needs to be 30.

EXAMPLE 2.2 *Prepare a winding diagram for a four-pole wave-connected armature of a dc generator having 22 coil sides.*

$$Y_a = \frac{Z \pm 2}{P} = \frac{22 \pm 2}{4} = 6 \text{ or } 5$$

If Y_a is taken to be odd, i.e., 5, then the front pitch and back pitch will be equal. Thus, $Y_a = Y_b = Y_f = 5$.

Connections of the coil sides will be as shown in Fig. 2.17.

The connection diagram is achieved by adding Y_b and Y_f with the coil numbers progressing in the forward direction. Coil-side 1 is connected at the back with coil-side 6 ($1 + Y_b = 6$). Coil side 6 is connected at the front with coil-side 11 ($6 + Y_f = 11$) and so on.

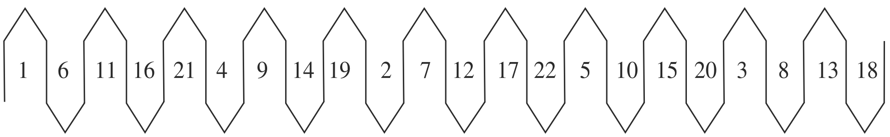


Fig. 2.17 *Connection diagram of the coil-sides for a dc wave winding*

In Fig. 2.17 it is to be noted that coil-side 19 is connected with coil-side 2. This is obtained by adding Y_b to 19 which gives 24. Coil-side 24 does not exist as there are in all 22 coil-sides. Therefore after 22 count two more numbers starting from 1. This gives coil-side 2. Similarly it can be seen that coil-side 20 is connected in the front with coil-side 3. By adding Y_f (= 5) to 20, the number 25 is obtained. After 20 five numbers are counted as 21, 22, 1, 2, and 3. Thus coil-side 20 should be connected to coil-side 3. In this way, the whole winding is completed by connecting all the coil-sides with one another. The actual layout diagram of the winding along with the position of the poles and the direction of induced emf in the coil-sides for a particular direction of rotation of the armature are shown in Fig. 2.18. The positions of the four brushes are also shown in the figure.

The positions of brushes are fixed as follows: for ease in understanding, the connection diagram of Fig. 2.17 is reproduced in Fig. 2.19. The directions of current in the coil-sides are also shown by observing the directions from Fig. 2.18. By carefully examining the directions of current in the coil-sides it is seen that between points *P* and *Q* current gets divided in two parallel paths. From point *P* the current flows to *Q* via two paths, viz. through 11-16-21- ... 6-11-18-13-

The point *P* in Fig. 2.19 is the separating point of the emf in the two sections of the winding and therefore corresponds to the position of one of the brushes, viz. the negative brush. For placing of the positive brush, it is seen from Fig. 2.19 that at point Q current is coming out from both the coil-sides. Therefore, point *Q* corresponds to the position of the positive brush.

It may be noted from Fig. 2.18 that coil-sides 6 and 17 lie in the interpolar region. The direction of current in these coil-sides will depend upon the direction of current in the other coil side of the respective coils, viz. coils 1-6 and 17-22.

Dummy Coils As mentioned earlier wave winding is possible with a particular number of coil-sides. But if standard stampings with a definite number of slots are to be used, the number of coil-sides needed to be placed in all the slots may be more than the required number. In such a case, the extra coils are left unconnected. These coils are called dummy coils. Dummy coils are used so as to make the armature dynamically balanced. They, otherwise, do not contribute to the induced emf or developed torque.

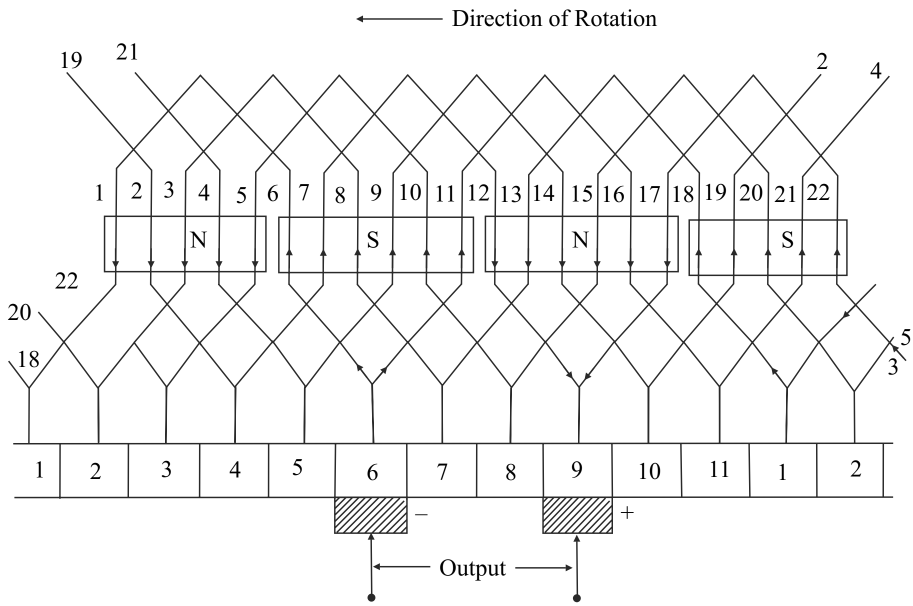


Fig. 2.18 *Layout diagram for a wave winding*

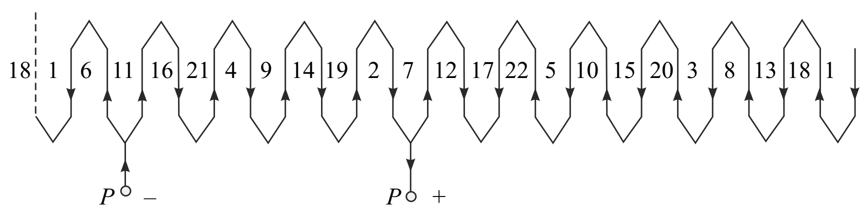


Fig. 2.19 *Connection diagram for armature winding of Fig. 2.18*

EXAMPLE 2.3 *Calculate the winding pitches and draw developed and sequence diagrams of the winding for a four-pole wave connected armature winding of a dc generator having seven coils. In the diagram, show the position of poles and the position and polarity of brushes.*

Solution

Number of coil-sides = 7 × 2 = 14

$$Y_a = \frac{Z \pm 2}{P} = \frac{14 \pm 2}{4} = 3 \text{ or } 4$$

Y_a should be an integer, Y_b and Y_f should be odd numbers.

Therefore, we choose

$$Y_a = Y_b = Y_f = 3$$

The sequence and layout diagrams of the winding are shown in Fig. 2.20.

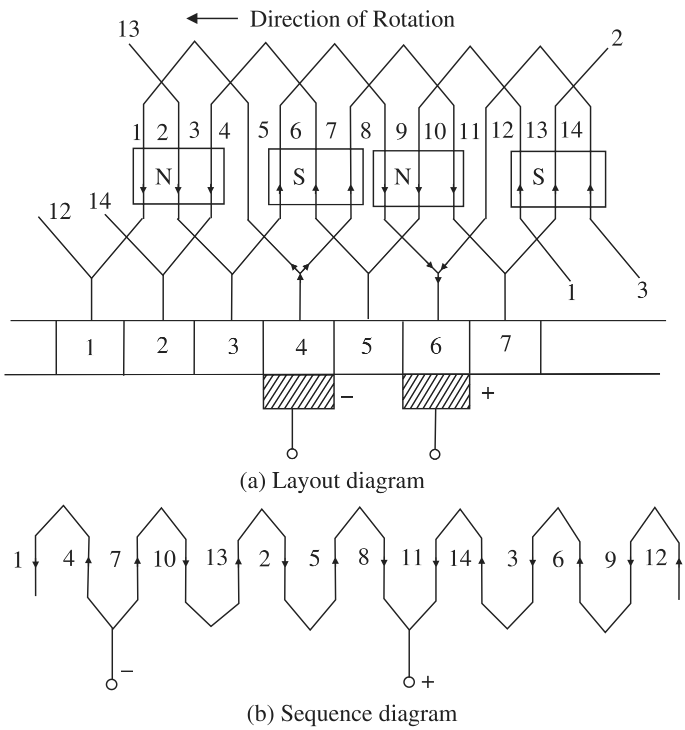


Fig. 2.20 *Layout and sequence diagram for a wave wound dc armature*

2.4 FACTORS DETERMINING INDUCED EMF

Whether a dc machine is working as a generator or a motor, its armature rotates in the magnetic field which exists in the air-gap. The conductors of the armature winding cut the magnetic flux and therefore the emf is induced in them. In the case of a generator this emf is responsible for supplying power to the load. In the case of a motor this emf opposes the applied voltage and thereby limits the current drawn by the armature conductors. The expression for induced emf is the same for a particular dc machine irrespective of its mode of operation. Figure 2.21 shows a portion of a dc machine. The armature conductors are seen rotating in the magnetic field produced by the field pole. If B is the average flux density over the pole pitch, then the average emf induced in one conductor is given by

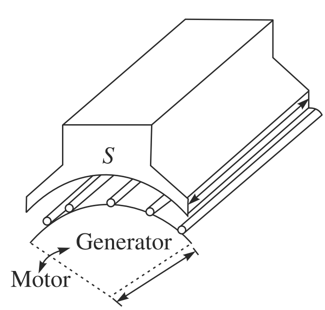


Fig. 2.21 *A portion of a dc machine*

$$e/\text{conductor} = \frac{d\phi}{dt}$$

$$\phi = BA$$

$$A = \text{arc} \times \text{length} = r\theta l$$

$$e = \frac{d}{dt} Br\theta l$$

or $$e = Brl\frac{d\theta}{dt} = Bl\omega r \text{ Volts.}$$

where, ω is the angular velocity of the armature; r is the radius of armature; and l is the length of the pole.

If Z is the total number of conductors in the armature and A the number of parallel paths in the armature, then the conductors per parallel path are Z/A.

The induced emf in the armature is equal to the total emf induced in all the conductors connected in parallel. Thus the total induced emf

$$E = Bl\omega r \frac{Z}{A}$$

Again $$B = \frac{\phi}{a}$$

where a is the area of the pole and is equal to $2\pi rl/P$ and ϕ is flux in Wb/pole.

Therefore, $$B = \frac{P\phi}{2\pi rl}$$

Substituting the value of B in the equation for E,

$$E = \frac{P\phi l\omega r}{2\pi rl}\frac{Z}{A}$$

or $$E = \frac{P\phi l 2\pi Nr}{2\pi rl \times 60}\frac{Z}{A} \text{ V} \qquad \left[\because \ \omega = \frac{2\pi N}{60}\right]$$

or $$E = \frac{\phi ZNP}{60A} \text{ V} \tag{2.1}$$

We may drive the emf equation in the following manner also.

Let ϕ be the flux per pole of the dc machine and P be the number of poles.

Total flux = $P\phi$ Webers

When a conductor rotates, it cuts $P\phi$ Wbs per revolution. The conductor rotates in N revolutions per minute (rpm), i.e., the conductor rotates in $N/60$ revolution per second (rps).

The conductor makes $N/60$ in 1 second.

Time taken by the conductor in making 1 revolution is, therefore, $60/N$ seconds. Flux cut by the conductor in 1 revolution is $P\phi$ Wbs.

$$\text{Induced emf, } E = \text{flux cut per second} = \frac{\text{Flux cut}}{\text{Time taken}}$$

or, $$E = \frac{P\phi}{60/N} \text{ volts} = \frac{P\phi N}{60} \text{ volts}$$

Since Z is the total number of armature conductors and A is the number of parallel paths in the armature winding, the number of conductors per parallel path is Z/A.

Total induced emf in the dc machine is the same as the induced emf in each parallel path. Therefore,

$$E = \frac{P\phi N}{60} \times \frac{Z}{A} = \frac{\phi ZNP}{60A} \text{ volts}$$

For a particular machine, Z the number of armature conductors, P, the number of poles, and the number of parallel paths of the armature, A are constants. Thus,

Induced emf, $E \propto$ Flux, $\phi \times$ Speed, N

From the above equation it is evident that the magnitude of induced emf will increase if the field strength as well as the speed of rotation of the armature are increased. This is a very important relation and will often be referred to in later sections.

EXAMPLE 2.4 *The wave connected armature of a 2-pole, 200 V generator has 400 conductors and runs at 300 rpm. Calculate the useful flux per pole. If the number of turns in each field coil is 1200, what is the average value of the emf induced in each coil on breaking the field if the flux dies away completely in 0.15 second?*

Solution Given $P = 2$, $Z = 400$, $N = 300$ rpm, $E = 200$ V, and number of parallel paths, $A = 2$ (for wave connected armature winding)

We know, $$E = \frac{\phi ZNP}{60A} \text{ Volts}$$

Substituting the given data,

$$200 = \frac{\phi \times 400 \times 300 \times 2}{60 \times 2}$$

or $$\phi = 0.1 \text{ Wb}$$

Induced emf in the field coil,

$$e = N\frac{d\phi}{dt} = 1200\left[\frac{0.1}{0.15}\right] = 800 \text{ Volts}$$

EXAMPLE 2.5 *A 6-pole, wave connected armature has 250 conductors and runs at 1200 rpm. The emf generated on open-circuit is 600 V. Calculate the useful flux per pole.*

Solution Given $P = 6$, $A = 2$, $Z = 250$, $N = 1200$ rpm, and $E = 600$ V

We know, $$E = \frac{\phi ZNP}{60\text{A}}$$

Substituting the given data,

$$600 = \frac{\phi \times 250 \times 1200 \times 6}{60 \times 2}$$

or, $$\phi = 0.04 \text{ Wb}$$

EXAMPLE 2.6 *An 8-pole, lap connected armature has 960 conductors, a flux of 40×10^{-3} Wb per pole, and a speed of 400 rpm. Calculate the emf generated on open circuit.*

Solution Given $P = 8$, $A = 8$, $Z = 960$, $\phi = 40 \times 10^{-3}$ Wb, and $N = 400$ rpm

We know,
$$E = \frac{\phi ZNP}{60\ A}$$

Substituting the given data,
$$E = \frac{40 \times 10^{-3} \times 960 \times 400 \times 8}{60 \times 2} = 256 \text{ Volts}$$

2.5 FACTORS DETERMINING ELECTROMAGNETIC TORQUE

In rotating electrical machines the electromagnetic torque works in the opposite direction to the applied mechanical torque or the load torque. It is necessary to know the factors on which electromagnetic torque depends.

2.5.1 Generating Action

Assume that a single-turn coil a–a' placed on the rotor is rotated by an externally applied torque T (Nm) at a constant velocity ω (rad/s) in a radial magnetic field of constant flux density B (tesla).

The mechanical power, P_m, supplied to rotate the coil is equal to the product of T and ω. Thus,

$$P_m = T\omega \text{ Nm/s}$$

As the coil rotates, both the coil-sides (conductors) cut the magnetic flux and emf is generated in them due to their rotation in the magnetic field. The total emf in a coil of two conductors,

$$e = 2Bl\omega r \text{ V} \qquad \text{(i)}$$

where, l is the length of conductor and r the radial distance of the conductors from the centre of rotation.

When the coil ends are connected across a resistive load of value R, a current i will flow which is given by

$$i = \frac{e}{R} \text{ A}$$

Since the velocity of rotation is constant the principle of ‘equal action and reaction’ requires that the forward torque T must be counter-balanced by an equal torque say T' as shown in Fig. 2.22.

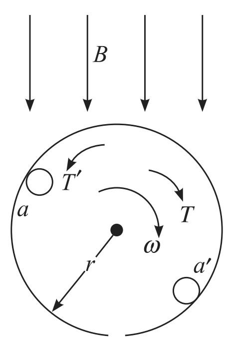


Fig. 2.22 *Electromagnetic torque developed in a dc machine*

The opposing torque T' is due to a force exerted on the two sides of the coil by the current flowing in the coil while it rotates through the magnetic field.

$$T' = 2Fr$$
$$= 2Bilr \text{ (since } F = Bil)$$

and since $T' = T$

therefore $T = 2Bilr$

$$P_m = T\omega = 2\,Bl\omega ri$$
$$= ei \text{ [as } e = \text{induced emf per coil} = 2\,Bl\omega r]$$

Therefore, $T\omega$ (Nm/s) = ei Watts or $T = ei/\omega$

Thus in a generator, mechanical power is converted into electrical power through the magnetic field.

2.5.2 Motoring Action

If the coil a–a' of Fig. 2.22 is now connected across a voltage source V capable of supplying a current i A, which reverses its direction at intervals equal to half the revolution of the rotor, then the electrical power supplied to the machine is

$$P_e = vi = ei \text{ Watts}$$

(Since $v = e$, Kirchhoff's voltage law)

If the resulting speed of the rotor is constant and equal to ω (rad/s) and the torque is T (Nm) then,

$$P_e = ei = 2\,Bl\omega r\, i = 2\,(Bli)r\omega$$
$$= 2\,Fr\omega = T\omega = P_m$$

i.e. $$P_e = P_m$$

or $$ei\text{(W)} = T\omega \text{ (Nm/s)}$$

Thus, in motoring action, electrical energy is converted into mechanical energy.

It is observed that the equation $T\omega = ei$ holds good for both motoring and generating action.

The equation for torque can therefore be written as

$$T = \frac{ei}{\omega}$$
$$= \frac{\phi Z N P}{60\,\text{A}}\frac{i}{\omega} = \frac{\phi Z N P i}{60\,\text{A}\dfrac{2\pi N}{60}}$$
$$T = \frac{ZP}{2\pi\,\text{A}}\phi\, i \text{ Nm}$$
$$= \left[\frac{ZP}{2\pi A}\right]\phi\, i \text{ Nm}$$

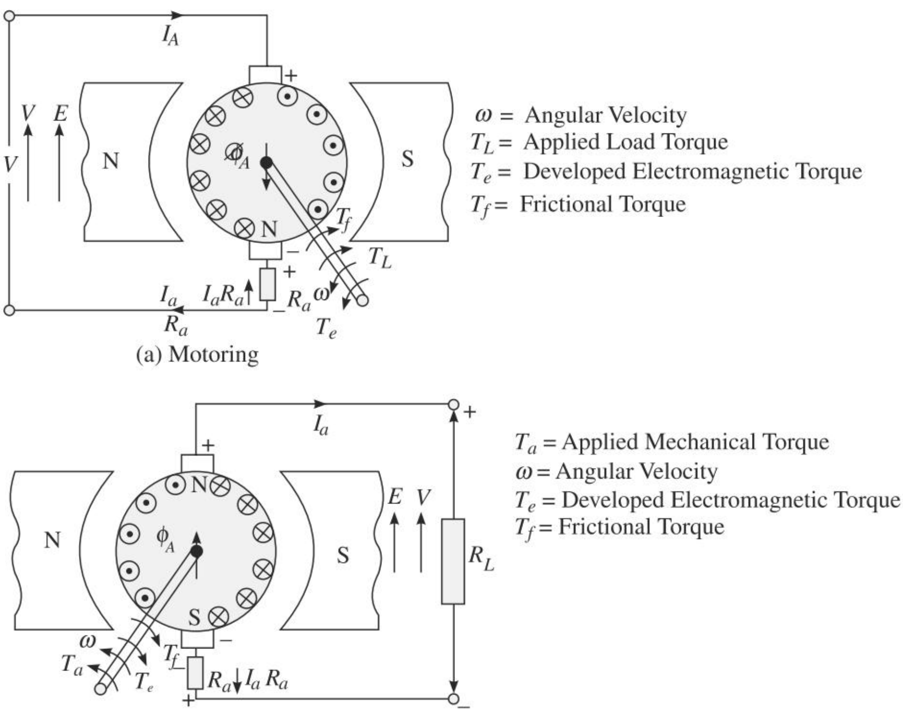
∴ Torque ∝ flux × current

or $$T = K_t\,\phi\, i \text{ Nm} \qquad (2.2)$$

where $$K_t = \frac{ZP}{2\pi A}$$

2.6 PRINCIPLES OF GENERATING AND MOTORING ACTION AND RELATIONSHIP BETWEEN TERMINAL VOLTAGE AND INDUCED EMF

As mentioned in an earlier section, in a dc generator mechanical energy is converted into electrical energy. The electrical energy thus made available is supplied to the electrical load. In a dc motor electrical energy is converted into mechanical energy. This mechanical energy is utilised to drive some mechanical load connected to the shaft of the motor. Thus, electrical power flows out of the machine in a generator and flows into the machine in a motor. It therefore depends on the direction of the flow of power whether a particular machine is generating or motoring. The directions of current and torque are opposite in these two cases of generating and motoring action. This is illustrated in Fig. 2.23.



(b) Generating

Fig. 2.23 *Illustrates motoring and generating action of a dc machine*

From Fig. 2.23(b) considering the generator circuit where induced emf E causes current I_a to flow to the load, the equation for the induced emf in terms of terminal voltage and armature resistance drop can be written as

$$E - I_a R_a = V$$ (This is because $E > V$ and the difference $E - V = I_a R_a$ drop)

or $$E = V + I_a R_a \tag{2.3}$$

i.e., induced emf in the armature = Terminal voltage available across the load
+ Armature resistance drop

Similarly from Fig. 2.23(a) considering the motor circuit, where $V > E$ and $V - E = I_a R_a$ drop, the equation for the induced emf in terms of the terminal voltage and the armature resistance drop can be written as

$$V - E = I_a R_a$$

or $$E = V - I_a R_a \quad (2.4)$$

Thus in the case of generating, $E > V$ and in the case of motoring, $E < V$. In both the cases, however, $E \propto \phi N$ and $T \propto \phi I_a$.

Multiplying Eq. (2.3) for a generator by I_a,

$$EI_a = V I_a + I^2{}_a R_a$$

or $$\omega T_e = V I_a + I^2{}_a R_a \quad (\because \omega T_e = EI_a) \quad (2.5)$$

In a generator,

Mechanical power input = Electrical power developed
+ Electrical power lost in the armature circuit

i.e., $$\omega T_a = \omega T_e + \omega T_f$$

If we neglect ωT_f, i.e., the power lost due to friction, then for simplicity we can write.

$$\omega T_a = \omega T_e$$

If the electrical load on the generator increases, I_a increases and hence ωT_e increases. To balance this, ωT_a should also increase. This means that as the load on the generator increases the applied mechanical torque should also increase.

Observe the directions of T_a, ω, T_e and T_f in case of generating and motoring actions as shown in Fig. 2.23. In the case of a generator, the directions of ω and T_e are opposite to each other, whereas in the case of a motor the directions of ω and T_e are the same.

For motoring action,

$$E I_a = VI_a - I_a^2 R_a \text{ [multiplying Eqn. (2.4) by } I_a]$$

or $$V I_a = E I_a + I_a^2 R_a$$

or $$V I_a = \omega T_e + I_a^2 R_a \quad (2.6)$$

i.e., Electrical power input = Mechanical power developed
+ Loss in the armature resistance

and $$\omega T_e = \omega T_L + \omega T_f$$

Mechanical power developed = Mechanical power output at the shaft
+ Frictional losses

If we neglect ωT_f $\quad \omega T_e = \omega T_L$

Therefore from Eq. (2.6),

$$V I_a = \omega T_L + I_a^2 R_a$$

If load torque T_L on the motor shaft is increased, $V I_a$ should increase, V being constant, the motor will draw more current from the supply mains when load is increased.

EXAMPLE 2.7 *The induced emf in a dc machine while running at 500 rpm is 180 V. Calculate the induced emf when the machine is running at 600 rpm. Assume constant flux.*

We know, $E = K \phi N$

If flux ϕ is assumed constant,

$$E = K_1 N$$

Thus from the data given,

$$180 = K_1 \times 500$$

or $$K_1 = \frac{180}{500}$$

Therefore induced emf at 600 rpm is

$$E = \frac{180}{500} \times 600 = 216 \text{ V}$$

EXAMPLE 2.8 *The induced emf in a dc machine while running at 750 rpm is 220 V. Calculate (a) assuming constant flux the speed at which the induced emf will be 250 V; and (b) the percentage increase in the field flux for an induced emf of 250 V and speed of 700 rpm.*

Solution

(a) $$E_1 = K_1 N_1$$

or $$220 = K_1 \times 750$$

or $$K_1 = \frac{220}{750}$$

$$E_2 = K_1 N_2$$

$$N_2 = \frac{E_2}{K_1} = 250 \times \frac{750}{220} = 852 \text{ rpm}$$

(b) $$E_1 = K \phi_1 N_1$$

or $$220 = K \phi_1 \times 750 \quad (1)$$

and $$E_2 = K \phi_2 N_2$$

or $$250 = K \phi_2 \times 700 \quad (2)$$

Dividing Eqn. (2) by Eqn. (1)

$$\frac{250}{220} = \frac{K\phi_2}{K\phi_1} \times \frac{700}{750}$$

$$\frac{\phi_1}{\phi_2} = \frac{250}{220} \times \frac{750}{700} = 1.217$$

Therefore the percentage increase in flux is 21.7%.

EXAMPLE 2.9 *The induced emf in a dc machine is 200 V at a speed of 1200 rpm. Calculate the electromagnetic torque developed at an armature current of 15 A.*

Solution

We know, $EI = T\omega$

or $$T = \frac{EI}{\omega} = \frac{200 \times 15}{\frac{2\pi \times 1200}{60}} = \frac{200 \times 15}{6.28 \times 20} = 23.9 \text{ Nm}$$

EXAMPLE 2.10 *On the armature of a dc machine is placed one coil having 10 turns. The length and diameter of the armature are both 0.2 m. The armature is rotating at a uniform magnetic field density of 1 Wb/m^2 at 1500 rpm. If the coil is connected to the load, the total resistance of the circuit becomes 4 Ω. Calculate the torque developed on the armature coil.*

Solution The induced emf in one conductor is given by

$$e = Bl\omega r$$

Each turn of the armature coil has two coil-sides (i.e., two conductors). The coil has 10 such turns. Therefore the total induced emf

$$E = Bl\omega r\, 2 \times 10$$

$$= 1 \times 0.2 \times \frac{2\pi \times 1500}{60} \times 0.1 \times 2 \times 10$$

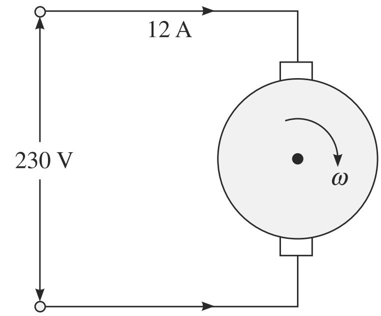
$$= 62.8 \text{ V}$$

The current through the armature coil when connected to the load,

$$I = \frac{62.8}{4} = 15.7 \text{ A}$$

Torque, $$T = \frac{EI_a}{\omega} = \frac{62.8 \times 15.7}{\frac{2\pi \times 1500}{60}} = 6.27 \text{ Nm}$$

EXAMPLE 2.11 *The armature supply voltage of a dc motor is 230 V. The armature current is 12 A, the armature resistance is 0.8 Ω, and the speed is 100 rad/s (Fig. 2.24). Calculate (a) the induced emf, (b) the electromagnetic torque, (c) the electrical power input to the armature, (d) the mechanical power developed by the armature, and (e) the armature copper-losses.*



R_a = 0.8 Ω
ω = 100 Radians/Second

Fig. 2.24

Solution

(a) $$E = V - I_a R_a$$

$$= 230 - 12 \times 0.8$$
$$= 220.4 \text{ V}$$

(b) $$E I_a = T_e \omega$$

$\therefore$ $$T_e = \frac{EI_a}{\omega}$$
$$= \frac{220.4 \times 12}{100} = 26.448 \text{ Nm}$$

(c) $$P_{\text{input}} = V I_a$$
$$= 230 \times 12 = 2760 \text{ W}$$

(d) $$P_d = T \omega$$
$$= E I_a = 220.4 \times 12 = 2644.8 \text{ W}$$

(e) $$I_a^2 R_a = (12)^2 \times 0.8 = 115.2 \text{ W}$$

EXAMPLE 2.12 *A dc generator is connected to a 220 V dc mains (Fig. 2.25). The current delivered by the generator to the mains is 100 A. The armature resistance is 0.1 Ω. The generator is driven at a speed of 400 rpm. Calculate (a) the induced emf, (b) the electromagnetic torque, (c) the mechanical power input to the armature neglecting iron, windage and friction losses, (d) the power input and output of the armature when the speed drops to 350 rpm. State whether the machine is generating or motoring. Assume constant flux.*

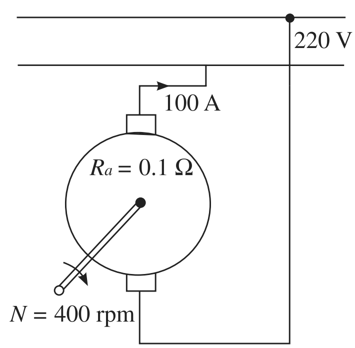


Fig. 2.25

Solution

(a) $$E = V + I_a R_a$$
$$= 220 + 100 \times 0.1 = 230 \text{ V}$$

(b) $$T_e = \frac{E I_a}{\omega} = \frac{230 \times 100}{2\pi \times \frac{400}{60}} = 549.08 \text{ Nm}$$

(c) Mechanical power input = Electromagnetic power developed
+ Iron-loss in the armature
+ Windage and friction-losses

Neglecting iron loss, windage-loss and friction-loss,

Mechanical power input = $\omega T_e = EI_a$
$= 230 \times 100 = 23000$ W

(d) When the machine runs at 400 rpm

$$E = K \phi N$$

$$230 = K\phi \times 400$$

or $$K\phi = \frac{230}{400}$$

$K\phi$ remains constant when the machine runs at 350 rpm also.

Therefore, $$E = K\phi N$$

or $$E = \frac{230}{400} \times 350$$
$$= 201.25 \text{ V}$$

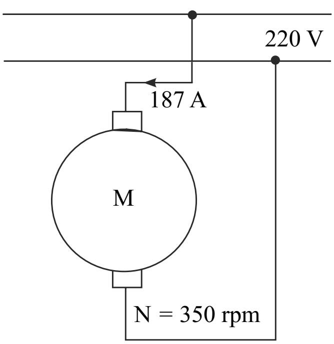


Fig. 2.26

As induced emf E is less than terminal voltage V (V remains constant as the machine is connected across a 220 V mains), the machine will now work as a motor (Fig. 2.26). Thus it will draw current from the mains.

Current drawn from the mains can be calculated as

$$E = V - I_a R_a$$

$$I_a = \frac{V - E}{R_a} = \frac{220 - 201.25}{0.1} = 187.5 \text{ A}$$

$$P_{\text{input}} = 220 \times 187.5 = 41.25 \text{ kW}$$

P_{output} (neglecting iron friction and windage-losses)

$$= E I_a$$
$$= 201.25 \times 187.5 = 37.73 \text{ kW}$$

EXAMPLE 2.13 *A shunt generator delivers 50 kW at 250 V and 400 rpm. The armature resistance is 0.02 Ω and field resistance is 50 Ω. Calculate the speed of the machine when running as a shunt motor and taking 50 kW input at 250 V.*

Solution

$$I_f = \frac{250}{50} = 5\text{A}$$

$$I_L = \frac{50000}{250} = 200 \text{ A}$$

$$I_a = 200 + 5 = 205 \text{ A}$$

$$E_G = V + I_a R_a = 250 + 205 \times 0.02 = 254.1 \text{ V}$$

As a motor, $$E_M = V - I_a R_a$$

$$I_L = \frac{50000}{250} = 200 \text{ A}$$

$$I_a = I_L - I_f = 200 - 5 = 195 \text{ A}$$

$$E_M = 250 - 195 \times 0.02 = 246.1 \text{ V}$$

We know $$\frac{N_M}{N_G} = \frac{E_M}{E_G}$$

Therefore speed of the motor,

$$N_M = N_G \frac{E_M}{E_G}$$

$$N_M = 400 \times \frac{246.1}{254.1} = 387 \text{ rpm}$$

EXAMPLE 2.14 *A dc shunt machine, connected to 250 mains, has an armature resistance of 0.12 Ω, and the resistance of the field circuit is 100 Ω. Calculate the ratio of the speed as a generator to the speed as a motor, the line current in each case being 80 A.*

Solution Given $V = 250$ Volts, $R_a = 0.12\ \Omega$, $R_f = 100\ \Omega$, $I_L = 80$ A

Field current, $$I_f = \frac{V}{R_f} = \frac{250}{100} = 2.5 \text{ A}$$

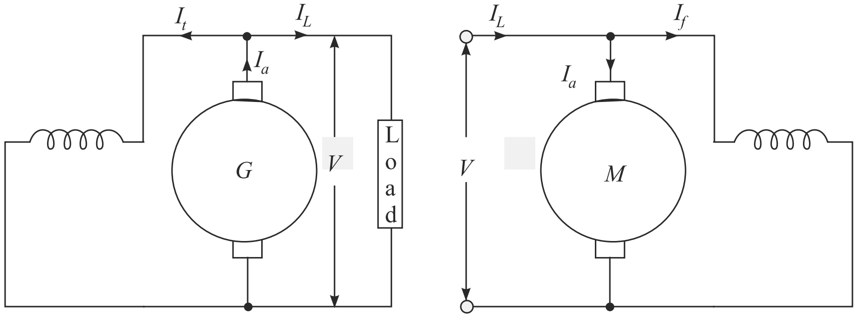
When the machine is generating,

$$I_a = I_L + I_f$$
$$= 0 + 2.5 = 82.5 \text{ A}$$
$$E_G = V + I_a R_a$$
$$= 250 + 82.5 \times 0.12 = 259.9 \text{ Volts}$$

When the machine is motoring,

$$I_L = I_a + I_f$$

or $$I_a = I_L - I_f$$
$$= 80 - 2.5 = 77.5 \text{ A}$$



(a) Generating (b) Motoring

Fig. 2.26(a)

$$E_M = V - I_a R_a$$
$$= 250 - 77.5 \times 0.12 = 240.7 \text{ Volts}$$

Induced emf, $E \propto N$ (other terms remaining constant)

$$\frac{E_G}{E_M} = \frac{N_G}{N_M}$$

Ratio of speeds, $$\frac{N_G}{N_M} = \frac{E_G}{E_M}$$
$$= \frac{259.9}{240.3} = 1.081$$

EXAMPLE 2.15 *A 1500 kW, 550 V, 16 pole, generator runs at 150 rpm. What must be the useful flux per pole if there are 2500 conductors in the armature and the winding is lap connected and full-load armature copper loss is 25 kW? Calculate the area of the pole shoe if the air gap flux density has a uniform value of 0.9 Wb/m^2. Also find the no-load terminal voltage. Neglect change in speed.*

Solution Given $V = 550$ Volts, $P = 16$, $N = 150$ rpm, $Z = 2500$, $A = 16$.

Power = 1500 kW

Full-load copper loss = 25 kW

Flux density in the pole $B = 0.9$ Wb/m^2

Full load current (I_a) is calculated as $= \dfrac{1500 \times 1000}{550} = 2727.3$ A

We have $$I_a^2 R_a = 25 \times 1000$$

or, $$R_a = \frac{25 \times 1000}{(2727.3)^2} = 3.36 \times 10^{-3}\ \Omega$$

Induced emf, $$E = V + I_a R_a$$
$$= 550 + 2727.3 \times 3.36 \times 10^{-3} = 559.2 \text{ Volts}$$

We have, $$E = \frac{\phi Z N P}{60 A}$$

Substituting the values,

$$559.2 = \frac{\phi \times 2500 \times 150 \times 16}{60 \times 16}$$

or, $$\phi = 0.0895 \text{ Wb}$$

To calculate the area of the pole shoe, we take,

Flux = Flux density × Area of the pole shoe

Thus, Area of pole shoe $= \dfrac{\text{flux}}{\text{flux density}}$

$$= \frac{0.0895}{0.9} \text{ m}^2 = 994.4 \text{ cm}^2$$

EXAMPLE 2.16 *A commutator having a diameter of 76 cm rotates at 600 rpm. Calculate the approximate time of commutation if the width of brush is 1.5 cm.*

Solution Given commutator diameter = 76 cm = 0.76 m.

Commutator radius = 0.38 m, speed $N = 600$ rpm, speed in rps, $n = 10$

Brush width = 1.5 cm = 1.5×10^{-2} m and peripheral speed of commutator,

$$V_p = r \, . \, \omega = r \times 2\,\pi\, n \text{ m/sec}$$

Substituting the data, $V_p = 0.38 \times 2 \times 3.14 \times 10$

$= 23.86$ m/sec

$$\text{Time of Commutation, } t_c = \frac{\text{brush width}}{\text{peripheral speed, } V_p}$$

$$\text{Substituting the data,} \quad t_c = \frac{15 \times 10^{-2}}{23.86} \text{ seconds}$$

$= 0.628 \times 10^{-3}$ seconds

2.7 FACTORS DETERMINING THE SPEED OF A dc MOTOR

The emf equation for a dc machine is

$$E = K\,\phi\, N$$

For a dc motor $\quad E = V - I_a R_a$

From the above two equations

$$N = \frac{V - I_a R_a}{K\phi} \tag{2.7}$$

From Eq. (2.7), it can be concluded that the speed of a dc motor depends on

(i) the applied voltage V

(ii) the field flux ϕ and

(iii) drop in the armature circuit resistance $I_a R_a$

Equation (2.7) also suggests the different methods of speed control of a dc motor. They are:

(i) control of speed by varying the applied voltage.

(ii) control of speed by varying the field flux.

(iii) control of speed by adding extra resistance in the armature circuit.

i.e., by increasing the voltage drop in the armature circuit as $I_a\,(R_a + R)$, where R is the extra resistance connected in the armature circuit.

The various methods of speed control of dc motors will be discussed in a separate section.

2.8 DIFFERENT TYPES OF EXCITATION

In this section we will be discussing the various methods by which the dc machine is excited, i.e., its field flux is created. Field flux can be created by using either permanent magnets or electromagnets. Permanent magnets are used in machines of small sizes and in special motors used in toys, battery-operated cassette-recorders, techogenerators, dynamos in motorcycles, etc. The advantages of using permanent magnets are that they are: (i) compact (ii) require no field windings and (iii) have no field copper-losses. The disadvantages are: (i) they have low flux and (ii) there is flux-density deterioration of the magnets with time. Electromagnets are therefore used in large dc machines. The dc machines are named depending upon how the field winding along with the armature winding is connected across the mains.

The simplest dc machine consists of a minimum of two windings, viz. a concentrated field winding and the armature winding. In some machines a separate winding is added to the poles so that the poles carry two windings. The extra winding is termed as the series field winding. Based on the methods of interconnection of windings the following types of dc machines are available:

(i) Separately excited dc machines; and
(ii) Self-excited dc machines with
  (a) shunt-field connection,
  (b) series-field connection, and
  (c) compound-field connection.

2.8.1 Separately Excited dc Machines

Here the field winding, i.e., the excitation winding is fed from a dc supply which is not connected to the armature winding.

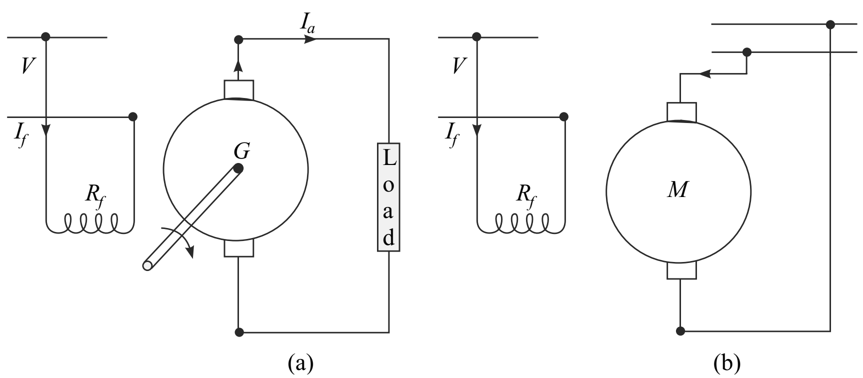


Fig. 2.27 *(a) Separately excited dc generator (b) Separately excited dc motor*

Figures 2.27 (a) and (b) show respectively the connection diagrams for a separately excited dc generator and a dc motor. It may be noted that the current flowing through the field winding is independent of the load and is equal to V/R_f, where R_f is the field circuit resistance. The flux produced is proportional to the field current, i.e., $\phi \times I_f$.

2.8.2 dc Machines with Shunt Field Connection

The field winding in the case of a dc shunt machine is connected in parallel with the armature as shown in Fig. 2.28.

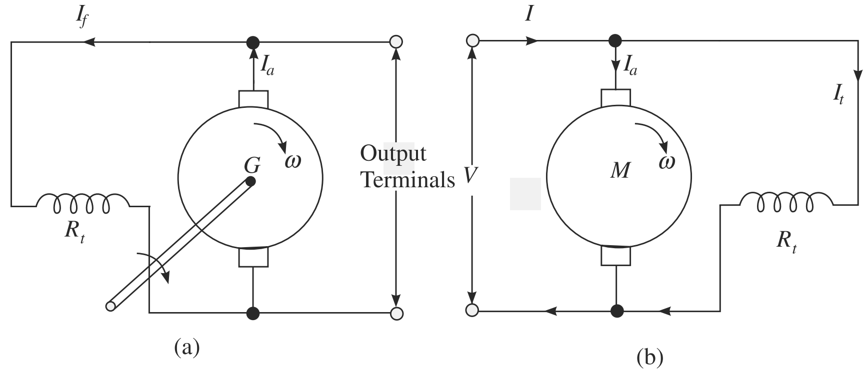


Fig. 2.28 *(a) dc shunt generator (b) dc shunt motor*

In the case of a motor the armature and the field windings are connected across a constant source of dc supply. Field current I_f is drawn from the source. In the case of a dc shunt generator, the field winding is connected across the armature terminals.

Field current flows due to the emf induced in the armature because of the rotation of the armature by the primemover. When the armature just starts rotating a small amount of emf is induced in the armature due to the residual magnetism of the field. This emf causes some current to flow in the field winding which causes a rise in the magnitude of the emf induced. This in turn increases the field current which causes more flux and hence more induced emf. A shunt generator will excite only if the poles have some residual magnetism and the resistance of the field circuit winding has some value less than a critical value.

Process of Induced emf in a dc Shunt Generator How exactly emf is induced in a dc shunt generator is explained as follows. Assume that the curve *RB* in Fig. 2.29 represents the open-circuit characteristics of the machine. The open-circuit characteristic is the relationship between the field current and the armature induced emf.

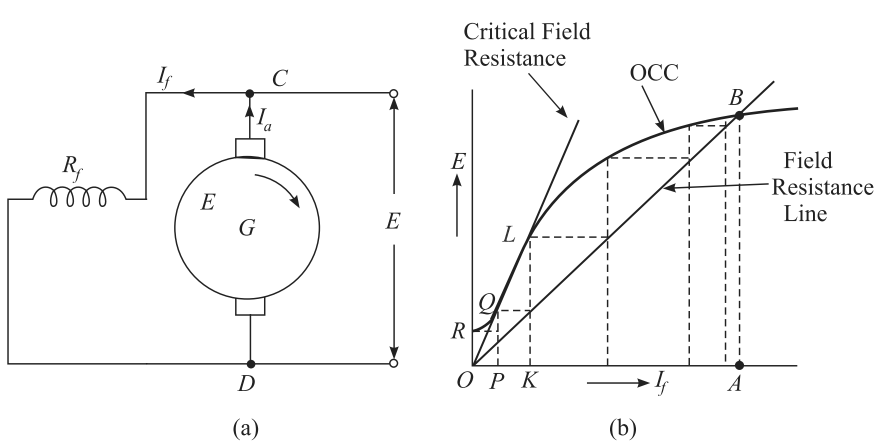


Fig. 2.29 *Shows how emf is induced in a dc shunt generator*

When the generator is driven at rated speed with the shunt-field circuit open, an emf *OR* (see Fig. 2.29(b)) is induced across the terminal *CD* which will, when the field circuit is closed, circulate a current *OP* through the field winding such that $OP = OR/R_f$. This field current *OP* will induce an emf *PQ* in the armature which will circulate a current *OK* in the field-winding. Thus the emf will go on increasing up to *AB*, where the field resistance line cuts the open-circuit characteristics. If the field-circuit resistance is too high (higher than the critical resistance), emf will not build up.

In a shunt generator the field current is not independent of the load on the generator. As the generator is loaded, the field current will decrease due to the fall of the armature-induced emf which will be explained later.

The reasons for failure of a dc shunt generator to build up voltage across its terminals are mentioned below.

1. There may not be any residual magnetism in the field system. In such a case, there will be no initial voltage induced when the armature is rotated and hence voltage will not build up.
2. The field winding connection may be such that it may not assist voltage to get built-up. If the field winding connection with the armature winding is made in such a way that the induced emf due to residual magnetism gets destroyed, then voltage will not build up.
3. The total field-circuit resistance, i.e., the sum to field-winding resistance and any extra resistance connected in the field circuit is more than the critical field resistance. In such cases, the slope of field resistance line will be high and no voltage will build up.

2.8.3 dc Series Machines

Figure 2.30(a) and (b) show the connection diagram for a dc series motor and generator respectively as the field winding is connected in series with the armature, $I_f = I_a = I_L$. In the case of a dc series machine, therefore, the field winding is made up of thick winding wires such that the armature current can flow through wires of the field winding without overheating. In the case of a series connection, flux $\phi \propto I_a$ or $\phi = K_2 I_a$.

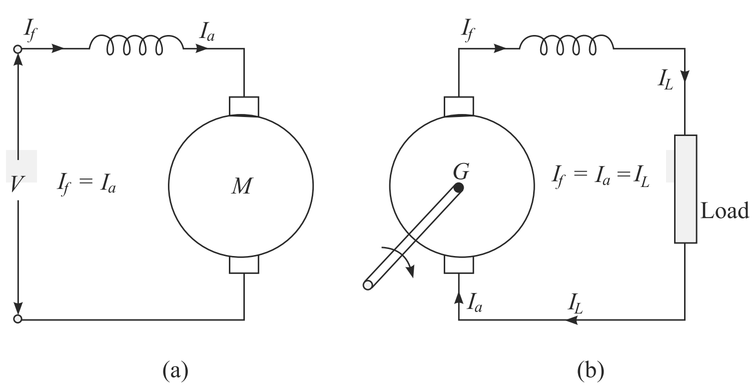


Fig. 2.30 *Connection diagrams for dc series motor and generator*

2.8.4 dc Compound Machines

In a compound machine there are two field windings, namely a shunt-field winding and a series-field winding. The shunt-field winding is connected in parallel with the armature and the series-field winding is connected in series with the combinations as shown in Fig. 2.31.

The series winding will carry a large armature current I_a or I_L and therefore it is made of wires of large cross-section and has a few turns only. The resistance of a series winding is very small. The shunt-winding is made up of wires of small cross-sections and has high resistance. Since the resistance of the shunt-field winding is high, the current flowing through it is small in comparison with the machine's armature current. The main flux is created by the shunt winding but it is modified by the flux of the series winding. A compound machine therefore combines the best features of shunt machines and series machines. In a compound machine the shunt winding can be connected in two ways, namely

(i) across the armature winding only, or
(ii) across both the armature winding and the series winding.

The difference is shown in Fig. 2.31(a) and (c) and also in Fig. 2.31(b) and (d). Connection (i) mentioned above is called a short shunt [Fig. 2.31(a) and (b)] and connection (ii) as mentioned above is called a long shunt [Fig. 2.31(c) and (d)].

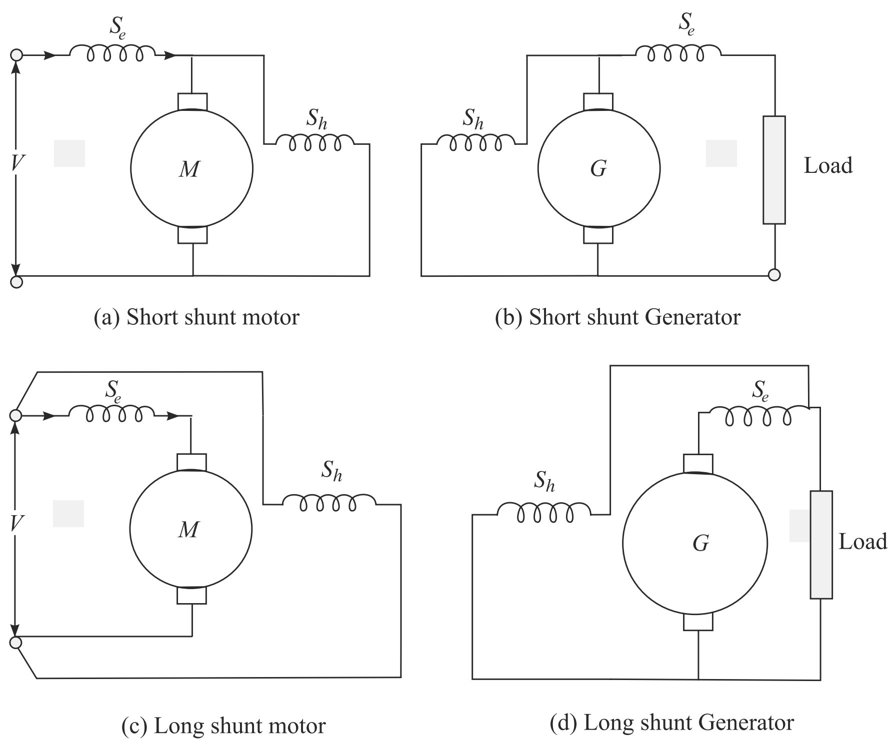


Fig. 2.31 *Connection diagrams of dc compound motor as in Fig. (a) and (c) and compound generator as in Fig. (b) and (d)*

The series-field winding flux can act either in the same direction as the shunt field flux or in opposition to it, depending on the direction in which the series field is wound with respect to the shunt field. When the series-field flux helps the shunt-field flux, the connection is called cumulative compound and when the series-field flux opposes the shunt-field flux the connection is called differential compound. In practice the cumulative compound connection is more commonly used. Compound excitation, which is a combination of shunt and series excitation, has the resultant field flux, $\phi = K_1 I_f \pm K_2 I_a$. A plus or minus sign is applicable depending upon whether the windings are cumulatively compounded or differentially compounded.

2.9 PERFORMANCE AND CHARACTERISTICS OF DIFFERENT TYPES OF dc MACHINES

How a dc machine, whether operating as a generator or a motor, behaves when it is loaded will now be discussed. This knowledge will enable one to suitably select a particular type of dc machine for a specific use.

2.9.1 Load Characteristic of dc Generators

A dc machine working as a generator must supply a power VI_L to the load circuit at a voltage V and a current I_L. When a dc generator is loaded, its terminal voltage varies. How V varies with I_L is an important information and can be experimentally obtained by driving the dc machine at a constant speed and loading the generator with the help of a variable loading resistance. The terminal voltage V and load current I_L can be measured and plotted graphically. The values of terminal voltage V and load current I_L when plotted give a characteristic called the load characteristic or external characteristic.

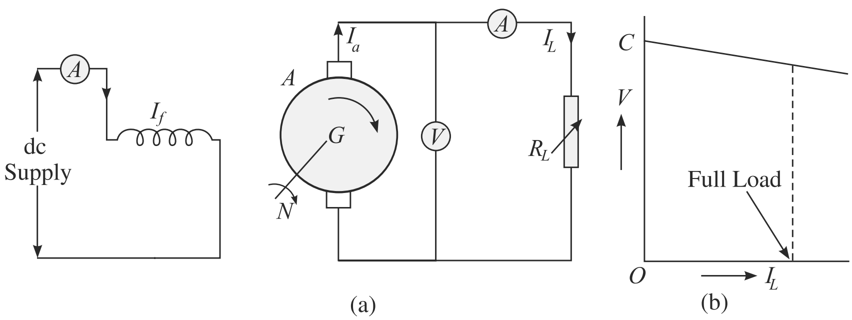


Fig. 2.32 *Load characteristic of a separately excited dc generator*

In Fig. 2.32(a) is shown a separately excited dc generator driven by a primemover at a constant speed and loaded with the help of a variable resistance. Assume OC (see Fig. 2.32(b)) as the voltage generated across the armature terminals when there is no load connected across the terminals. This terminal voltage at no load is determined by the speed of rotation of the armature, N and the field current I_f. Now let the generator be loaded with the help of a loading resistance R_L. As the load resistance gets decreased (due to more load resistances getting connected in parallel), load current I_L will increase. The increase in I_L will cause a drop in the terminal voltage as shown in Fig. 2.32(b). The drop in voltage is due to two reasons, viz.

(i) $I_a R_a$ drop in the armature circuit, and

(ii) Reduction in the air-gap flux due to the effect of armature reaction and hence reduction in the no-load terminal voltage. (Armature reaction is the effect of flux produced by current flowing through the armature conductors on the flux created by the field winding current.)

Thus as load on the generator increases, i.e., the value of the load resistance decreases (in actual practice an increase of load is characterised by a decrease in load resistance as electrical loads are connected in parallel), there will be a fall in the terminal voltage.

To keep the terminal voltage constant either the field current or the speed of the primemover will have to be increased.

The load characteristic of a shunt generator [Fig. 2.33 (a)] will have a relatively more drooping characteristic as compared to a separately excited one as illustrated in Fig. 2.33 (b). This is because in addition to the voltage drop due to $I_a R_a$ and the reduction of the air-gap flux due to the effect of the armature reaction, there will be a voltage drop due to the reduction of field current I_f. As $I_f = V/R_f$, if V is reduced, I_f will be reduced.

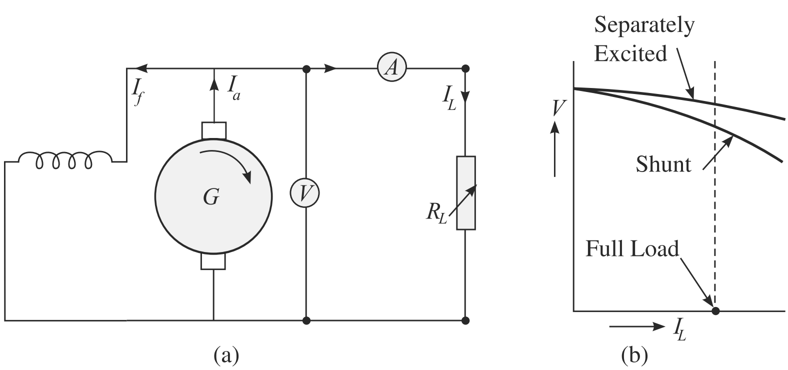


Fig. 2.33 *Load characteristics of a dc shunt generator*

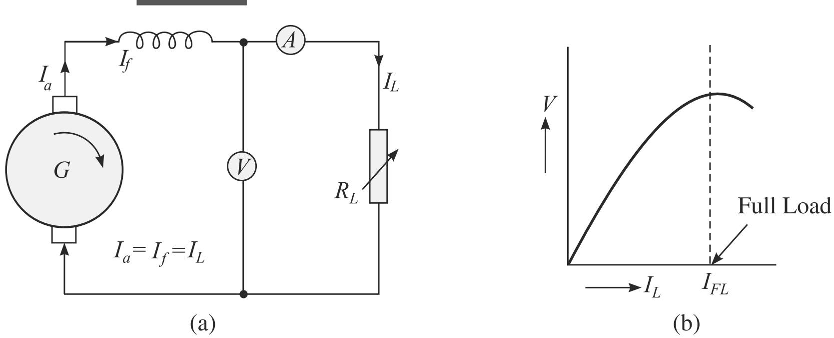


Fig. 2.34 *Load characteristics of a dc series generator*

The load characteristic of a dc series generator is shown in Fig. 2.34(b). At no load the induced emf across the generator terminals is practically zero. As the generator is loaded, I_L increases and hence field current I_f increases (as $I_L = I_a = I_f$). The terminal voltage will continue to rise till the effect of the armature reaction and $I_a R_a$ drop increases to a great extent (beyond the full-load current) due to loading where the curve starts falling as shown in Fig. 2.34(b).

2.9.2 dc Motor Characteristics

The important characteristics of dc motors are
(i) speed-armature current (load) characteristics,
(ii) torque-armature current (load) characteristic, and
(iii) speed-torque characteristic.

It is essential to know the above characteristics of different types of dc motors as this enables the selection of specific type for a specific purpose.

Characteristic of dc Shunt Motor

(i) Speed-armature Current (Load) Characteristic For a dc motor the following equation is known

$$N = \frac{V - I_a R_a}{K\phi}$$

A shunt motor is connected across the mains having voltage V. The mains voltage V is assumed constant. The field winding is connected across the armature terminals as shown in Fig. 2.35.

Flux ϕ produced by field current I_f will be constant as V remains constant. But in actual practice the air-gap flux is slightly reduced due to the effect of the armature reaction. From the expression of speed it is evident that as the armature current I_a increases, speed will decreases by a small amount due to an increase in the $I_a R_a$ drop ($I_a R_a$ drop is very small as compared to V).

The speed versus armature current characteristic is shown in Fig. 2.36. A slight variation of speed of the shunt motor from no load to full load can be made up by inserting an extra resistance in the field circuit thereby reducing the field flux.

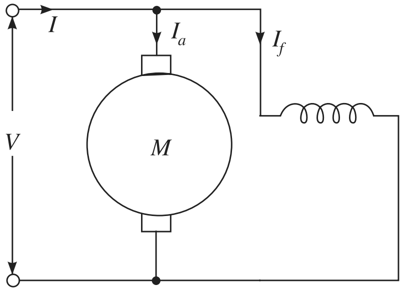


Fig. 2.35 *A dc shunt motor connected across mains voltage*

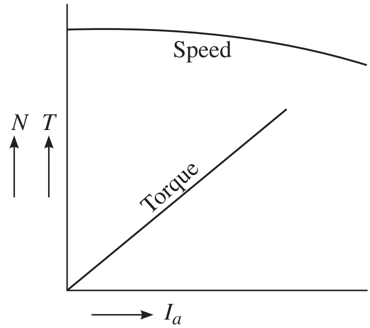


Fig. 2.36 *Speed versus armature current (load) characteristic of a dc shunt motor*

The shunt motor, being thus more or less a constant speed motor, can be used in applications such as driving of line shafts, lathes, conveyors, etc.

(ii) Torque-armature Current (Load) Characteristic The equation for torque can be written as

$$T = K_t \phi I_a$$

If flux ϕ is taken as constant, torque T becomes directly proportional to I_a as shown graphically in Fig. 2.36. It is a straight line passing through the origin.

(iii) Speed-torque Characteristic Knowing the relation between T and I_a and N and I_a, as shown in Fig. 2.36, the relationship between speed and torque can be drawn. The speed-torque characteristic for a dc shunt motor is shown in Fig. 2.37.

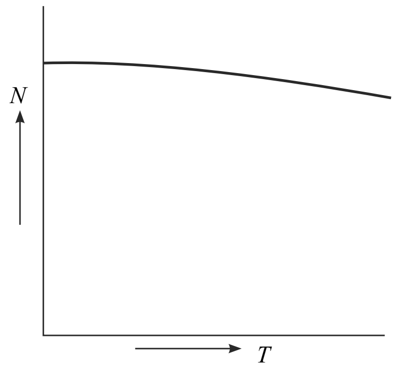


Fig. 2.37 *Speed torque characteristic of a dc shunt motor*

Characteristic of dc Series Motor

(i) Speed-armature Current (Load) Characteristic From the expression of speed $N = \dfrac{V - I_a R_a}{K\phi}$, it is seen that speed N is inversely proportional to flux ϕ.

For a dc series motor, as seen from Fig. 2.38(a) flux is proportional to I_a. Thus if V is constant, N is inversely proportional to I_a. The N versus I_a characteristic is therefore a rectangular hyperbola as shown in Fig. 2.38 (b). It is seen from the characteristic that the speed decreases as the load on the motor increases. At a very low load, the speed is dangerously high. Thus if a series motor is allowed to run at a very light load or at no load, its speed will become much higher than its normal speed which may cause damage to the motor. For this reason series motors are never started on no-load and are not used in applications where there is a chance of the load being completely removed when the motor remains connected to the supply. The load on the series motors is to be connected through gears and not through a belt-pulley arrangement.

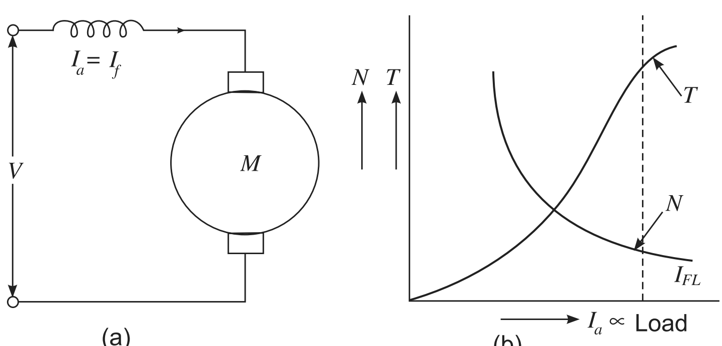


Fig. 2.38 *Speed load and torque load characteristics of a dc series motor*

This is because in the case of failure of the belt, the load will get removed from the motor and thereby the motor will attain a dangerously high speed. In the case of loads connected through gears, however, in the event of an accidental release of load, gears provide some load on account of the frictional resistance of the gear teeth.

(ii) Torque-armature Current (Load) Characteristic The equation for torque for a dc machine is given by

$$T = K_t \phi I_a$$

Flux ϕ for a series motor is proportional to armature current I_a. Thus $T \propto I_a^2$. The relationship between torque and armature current is, therefore, of the form of a parabola. With increase in I_a, the field flux increases linearly but due to saturation of the magnetic core, beyond a certain magnitude of I_a, the increase in flux is very negligible. Thus T is proportional to the square of I_a up to the saturation point beyond which T varies linearly with I_a. This has been shown in Fig. 2.38. From the torque load characteristic it is seen that a series motor when started with load develops a very high starting torque. Hence series motors are used in applications where high starting torque is required such as in electric trains, hoists, trolleys, etc.

(iii) Speed-torque Characteristic From the characteristics shown in Fig. 2.38 the speed-torque characteristic can be drawn as shown in Fig. 2.39.

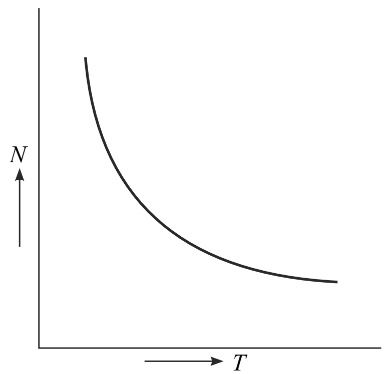


Fig. 2.39 *Speed torque characteristic of a dc series motor*

Characteristics of dc Compound Motors As mentioned in the earlier section, compound motors can be of two types, namely, the cumulative compound motor and differential compound motor. In a cumulative compound-wound motor the series-field winding is connected in such a way that the flux produced by it helps the flux produced by the main field winding, whereas in differential compound-wound motor the series-field flux opposes the main field flux. The characteristics of compound-wound motors are combinations of shunt and series wound motors.

The speed-armature current characteristic and torque-armature current characteristic of the two types of compound-wound motors are shown in Fig. 2.40. In the case of a cumulative compound motor the N versus I_a characteristic is slightly more drooping than that of a shunt motor as there is an increase in flux with load (the series-field flux adds with the main-field flux). As speed is inversely proportional to flux, increase in flux causes decrease in speed. Applying the same argument, the other characteristics shown in Fig. 2.40 can be understood.

The torque developed by a cumulative compound motor increases with sudden increase in load, and at no-load it has a definite speed (does not attain excessive speed at no-load). Cumulative compound motors are therefore suitable in situations where there is sudden application of heavy loads like in shears, punches, rolling mills, etc.

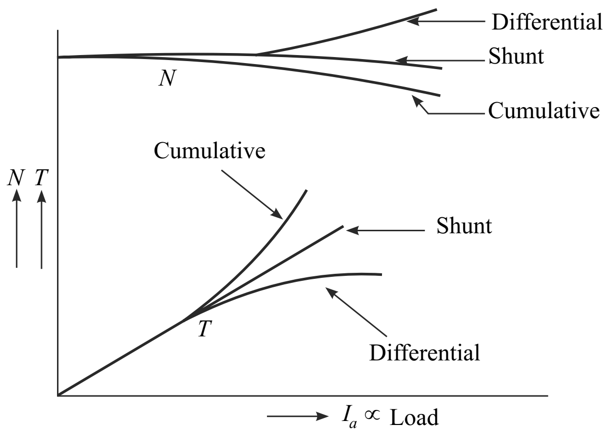


Fig. 2.40 *Speed-load and torque-load characteristics of a dc compound motor*

The speed of a differential compound motor remains more or less constant with increase in load but its torque decreases with load. Since the shunt motor develops a good torque and its speed does not vary appreciably with increase in load, differential compound motors are not preferred over shunt motors and hence are rarely used.

The speed-torque characteristic of a compound-wound motor is shown in Fig. 2.41.

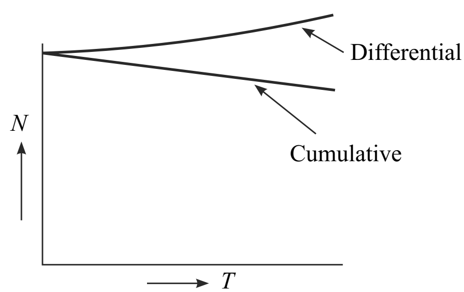


Fig. 2.41 *Speed torque characteristic of dc compound motor*

2.10 STARTING OF dc MOTORS AND MOTOR STARTERS

When a dc motor is started with full-voltage applied across its armature terminals it will, during the starting period, draw much more current than its rated current. This excessive current will overheat the armature winding and may even damage the winding. During the starting period a variable resistance called a starter is connected in series with the armature circuit to limit the starting current.

2.10.1 Starting of dc Motors

A dc motor, when switched onto the supply mains directly, draws a very heavy current which is many times more than its rated current. This can be understood from the expression for current for a dc motor which is

$$I_a = \frac{V - E}{R_a} \tag{2.8}$$

At the moment of start, induced emf E (which is also called back emf, E_b)in the dc armature is zero since $N = 0$ (as $E = K\,\phi\,N$). If E is zero, armature current I_a is the ratio of V and R_a. The armature resistance, R_a for the dc machine is very small, say of the order of a fraction of an ohm. For a 220 V dc motor, therefore, the starting current drawn by the motor armature will be very high, of the order of hundreds of amperes. This current which is many times more than the full-load rated current of the motor if continues to flow through the armature winding will cause damage to the winding due to overheating. However, the magnitude of this initial high current goes on reducing as the motor begins to pick up speed. As speed increases, induced emf E increases and thus I_a decreases and gradually comes down to its rated value when the motor has reached its normal speed. It is therefore necessary that the current drawn by the motor during starting should be brought down to a reasonably low value for the safety of the motor winding.

To limit the initial heavy current drawn by the motor, a resistance is connected in series with the armature circuit as shown in Fig. 2.42. This extra starting resistance is gradually cut out of the circuit as the motor starts rotating. By introducing an extra resistance R in the armature circuit, the expression for I_a is modified as

$$I_a = \frac{V - E}{R_a + R}$$

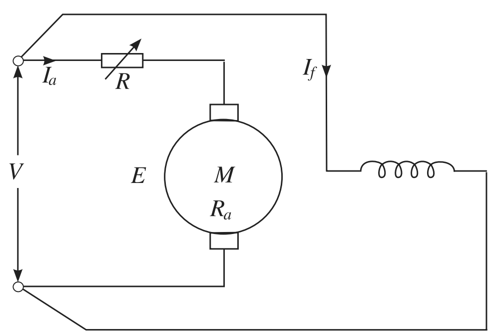


Fig. 2.42 *Starting of a dc motor by connecting a resistance in the armature circuit*

Armature current I_a is reduced to a safe value by introduction of R which comes in the denominator of the above expression. With the increase of speed of the motor, E increases and hence the numerator is reduced. Thus the need for keeping R in the circuit gradually decreases. It is therefore cut out of the circuit. Keeping the extra resistance in the armature circuit, even when it is not necessary, will cause wastage of energy in the form of $I_a^2 R$ loss which is undesirable.

As seen from Fig. 2.42, the field winding is connected across the supply terminals and not across the armature terminals. If the field winding is connected across the armature terminals, current drawn by the field winding will be reduced (due to heavy $I_a R$ drop in resistance R, the voltage available across the armature terminals will be small and hence I_f will be small) and hence sufficient torque may not get developed to rotate the rotor.

In actual practice the starting resistance is placed inside an enclosure with its terminals brought out. Tappings from the resistor are connected to brass studs. The resistance is gradually cut out of the circuit by moving a handle (starter arm) which makes sliding contacts with the brass studs. This device is called a starter. It is very essential that the starter is provided with protective devices to enable the starter arm to return to the off position in the event of failure of supply and overload on the motor. If the supply fails and the starter arm does not return to the off position, the armature will get directly connected across the mains when the supply is restored. When the motor becomes overloaded or develops some fault, the armature will take excessive current. The arm of the starter should therefore immediately return to the off position. A starter should provide for off-load protection as well as over-load protection.

2.10.2 dc Motor Starters

Various types of manual face plate dc motor starters commonly known as two-point, three-point and four-point starters are available. A degree of similarity exists among these starters. All have a face plate rotary switch with a connected group of current-limiting resistors. The differences lie in the form of protection they contain.

Two-Point Starter A two-point starter is used for starting a dc series motor which has the problem of overspeeding due to the loss of load from its shaft. Such a starter is shown in Fig. 2.43.

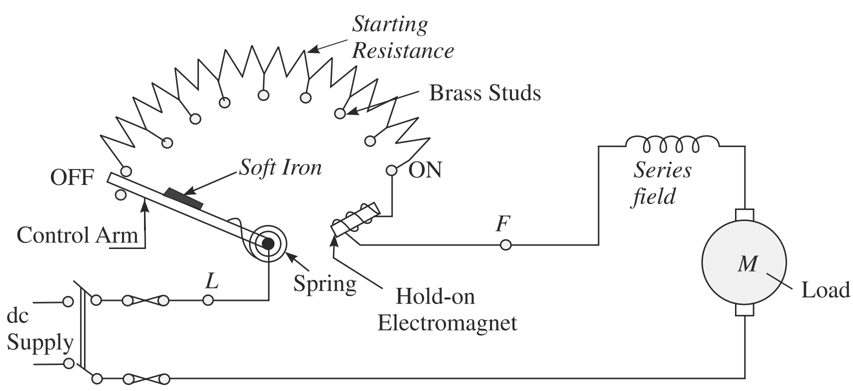


Fig. 2.43 *Two-point-starter for a dc series motor*

Here for starting the motor, the control arm is moved clockwise from its OFF position to the ON position against the spring tension. The control arm is held in the ON position by the hold-on electromagnet. The hold-on electromagnet is connected in series with the armature circuit. If the motor loses its load, current decreases and hence the strength of the electromagnet also decreases. The control arm returns to the OFF position due to spring tension, thus preventing the motor from overspeeding. The starter arm also returns to the OFF position when the supply voltage decreases appreciably. *L* and *F* are the two points of the starter which are connected with the supply and motor terminals.

Three-Point Starter A three-point starter is used for starting a dc shunt or a compound motor. The coil of the hold-on electromagnet is connected in series with the shunt-field coil. Such a starter is shown in Fig. 2.44. When the starter arm is moved towards ON position from its OFF position, supply will be available to the field winding as also to the armature but through the starter resistance.

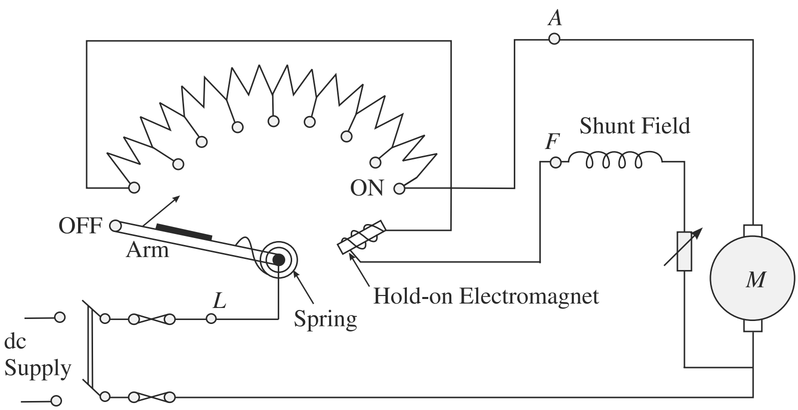


Fig. 2.44 *Three-point starter for a dc shunt or compound motor*

In the case of disconnection in the field circuit due to internal failure or field-rheostat failure, the control arm will return to its OFF position due to spring tension. This is necessary because the shunt motor will overspeed in the same manner as the series motor if it loses its field excitation. The starter also returns to the OFF position in case of low supply voltage or complete failure of supply voltage. This protection is therefore called no volt release (NVR). Overload protection for the motor can be incorporated by connecting another electromagnetic coil in series with the armature. When the motor is overloaded it draws a heavy current. This heavy current also flows through the electromagnetic coil. The electromagnet then pulls an iron piece upwards which short-circuits the coils of the hold-on electromagnet (NVR coil, see Fig. 2.45). The hold-on electromagnet gets de-energised and therefore the starter arm returns to

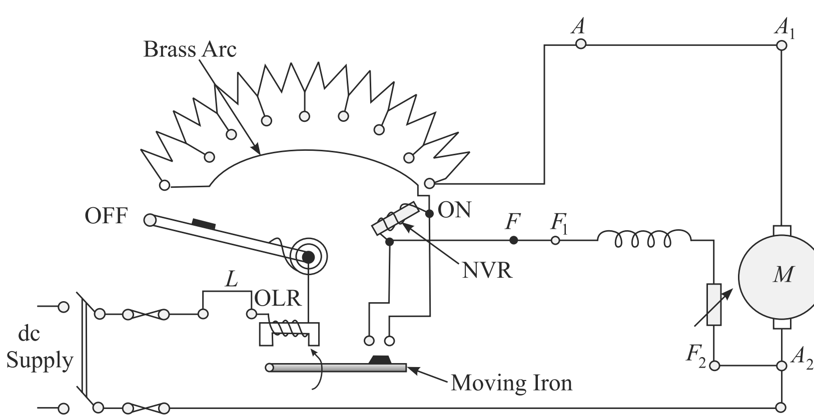


Fig. 2.45 *A three-point starter with NO VOLT RELEASE (NVR) and OVERLOAD RELEASE (OLR) arrangement*

the OFF position, thus protecting the motor against overload. The complete circuit connection for a three-point starter with NVR and overload release OLR is shown in Fig. 2.45. It is to be noted that points *L*, *A* and *F* are the three terminals of a three-point starter. Use of a brass arc as shown in the figure enables connection of the field circuit directly with the supply instead of via the starter resistance.

A three-point starter may not be suitable where a large field current adjustment by using a field regulator is needed. This may cause weakening of the field current to such an extent that the hold-on electromagnet may not be able to keep the starter arm in the ON position. This may therefore disconnect the motor from the supply when it is not desired. Such a problem is overcome by using a four-point starter.

Four-Point Starter The disadvantage of a three-point starter as mentioned above is overcome in a four-point starter by connecting the hold-on coil across the line instead of in series with the shunt-field circuit when the arm is moved in clockwise direction from its OFF position. This makes a wide range of field adjustments possible. The connection diagram for a four-point starter is shown in Fig. 2.46. In a four-point starter, therefore, when the starter arm touches the starting resistance, current from the supply is divided into three paths. One through the starting resistance and armature, one through the field circuit, and one through the NVR coil. A protective resistance is connected in series with the NVR coil. Since in a four-point starter the NVR coil is independent of the field-circuit connection, the dc motor may overspeed if for some reason there should be a break in the field circuit. It is therefore recommended that before starting a dc motor the field circuit be checked against an open circuit.

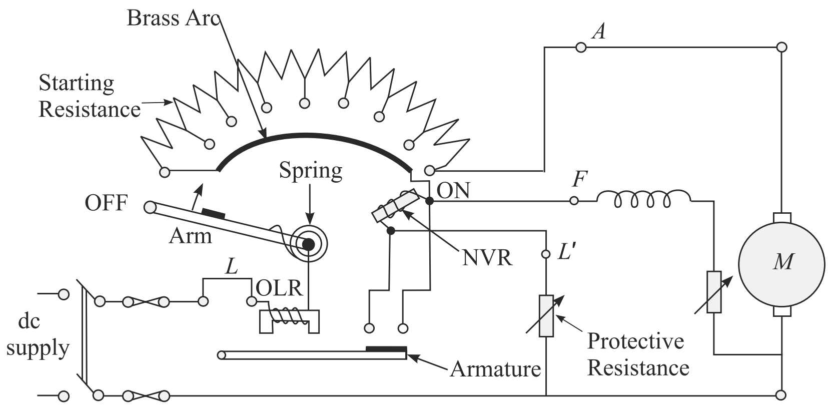


Fig. 2.46 *A four-point starter with NVR and OLR arrangement*

A dc motor should be stopped by opening the main switch rather than by throwing back manually the starting arm to the OFF position. If the starting arm is thrown back manually, the field circuit suddenly gets opened. The electromagnetic energy stored in the field gets discharged through the OFF contact button of the starter in the form of an arc, thereby gradually burning the contact. If the motor is stopped by opening the main switch, the electromagnetic energy of the field circuit gets slowly discharged through the armature. The starting resistance used in various types of starters is cut out of the circuit in steps while starting a motor. The steps of the starting resistance are so designed that the armature current will remain within certain limits and will not change the torque developed by the motor to a great extent.

So far face plate type starters have been discussed. The same type of control as achieved with a face-plate starter can be obtained with a drum-rotary-switch starter. The drum switch has the advantage of being able to handle a large current and therefore is suitable for motors with high ratings. Current-limiting starting resistors are not part of the drum switch and they may be made as large as possible which are rated to carry current continuously. (In a conventional starter, the starting resistors are rated to carry current for small durations of time. This reduces the size of the resistances.) As a result, the drum switch is able to control the motor continuously at intermediate or even at the lowest speed. Drum-rotary switch starters are suitable for tram-car-motor controls and for large crane-motor controls.

Nowadays automatic motor-control starters are also available which can be operated by pressing push-buttons. The working of such automatic starters is as follows. Upon pressing an ON push-button, current-limiting starting resistors get connected in series with the armature circuit. Then some form of automatic control progressively disconnects these resistors until full line-voltage is available to the armature circuit. ON pressing an OFF push-button the system should get back to its original position. Protective devices, such as OLR, NVR, etc. are usually the same as that of a manual starter. Automatic starter circuits are designed using electromagnetic contactors and time-delay relays.

2.11 SPEED CONTROL OF dc MOTORS

Speed control of a motor means the intentional variation of speed according to the requirement of the work-load connected with the motor. This can be done by mechanical means, such as by using stepped pulleys, a set of change gears, a friction clutch mechanism, etc. However, control of speed of a motor by electrical means has greater advantages over mechanical speed control The dc motors offer easy speed control and that is why dc motors are preferred over other types of motors in many applications. Various methods of speed control of dc motors can be understood by examining the expression for speed of a dc motor. The expression for speed can be written from the two basic equation, viz., $E = K\phi N$ and $E = V - I_a R_a$ as

$$N = \frac{V - I_a R_a}{K\phi}$$

Speed N can be varied by varying the field flux ϕ, by adding some extra resistance in the armature circuit and by varying the supply voltage V. These methods are discussed below.

2.11.1 Speed Control by Varying the Field Flux

A variable resistor called a field regulator is connected in series with the shunt-field winding as shown in Fig. 2.47(a). The overall resistance of the field circuit can be changed by varying the field-regulator resistance. When the field-circuit resistance is increased (made more than the resistance of the winding), the field current and therefore the flux produced by the field is decreased. The induced emf is also decreased causing more current to flow through the armature circuit. This will increase the torque which will accelerate the motor until the induced emf is again nearly equal to the supplied voltage. By introducing the field regulator, the field-circuit resistance can only be increased, i.e., the field flux can only be decreased, and thereby the speed of the motor can be increased. It is not possible to decrease the speed by the use of a field regulator. This method of speed control is applicable in shunt and compound motors.

For series motors, a resistor called the diverter is connected in parallel with the series-field winding. The diverter consists of one fixed resistor r and variable resistor R [see Fig. 2.47(b)]. By varying R, the current flowing through the series field winding can be controlled. The fixed resistance r prevents the series winding being short-circuited when R is made zero. Due to introduction of the diverter-circuit resistance, the field current can be reduced and there by the speed of the series motor can be increased. Reduction of speed is not possible by using a diverter because field current cannot be increased from its original value, i.e., when the diverter was not introduced.

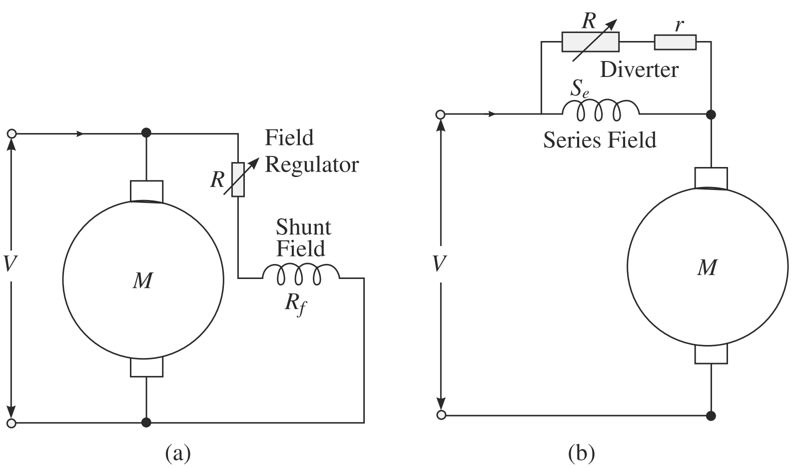


Fig. 2.47 *Speed control of dc motors by varying field flux (a) dc shunt motor (b) dc series motor*

2.11.2 Speed Control by Connecting a Resistance in Series with the Armature

A resistor, called a controller is connected in series with the armature as shown in Fig. 2.48. The field winding is connected directly across the supply terminals. The connections are similar to that of a starter. However, the resistance element of the controller is designed to carry currents continuously and not for a short duration of time as required in the case of a starter.

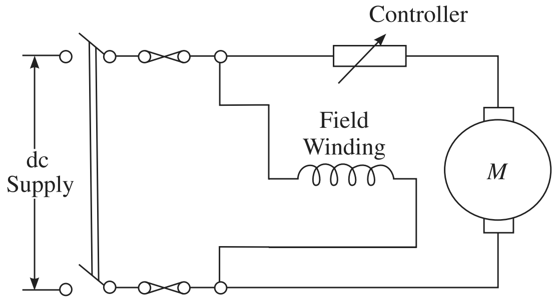


Fig. 2.48 *Speed control by using a controller in the armature circuit*

In this system the speed of the motor can be reduced as desired. Using a high-value resistance for the controller the potential drop across it can be increased to a great extent, causing a very small voltage to appear across the motor armature. Thus the speed of the motor can be reduced to a large extent. The field winding should be connected across the supply terminals, otherwise the flux produced will be badly affected and sufficient torque may not be produced to rotate the motor.

This armature resistance control method has a number of disadvantages:

(i) The overall efficiency of the system is low as much of the input energy is dissipated in the controller as heat.
(ii) The controller has relatively high cost.
(iii) The speed may vary largely with variation of load.

The advantage of this method is that the speed can be smoothly varied from zero up to the rated speed. This method of speed control is used in controlling the speed of tram-cars, cranes, hoists, etc., where smooth control is the main consideration rather than efficiency.

2.11.3 Speed Control by Controlling the Voltage Applied Across the Armature Terminals

In this method of speed control the armature is supplied with a variable voltage with the help of a motor-generator set since the supply voltage available from the electricity authority cannot be varied at will. The field winding is connected across the supply terminals since otherwise the torque produced may be insufficient to start the motor. Figure 2.49 shows the connection diagram for such a system.

This system of speed control is also known as the Ward-Leonard system. The speed of the motor can be conveniently varied from a very low value to above the rated value. If a reversing switch is incorporated, by changing the polarity of the armature supply terminals, speed can be controlled in the opposite direction also. The variable voltage across the motor armature terminals is obtained by varying the generator field circuit regulator resistance *R*. This system is advantageous over the other systems because

(i) smooth control of speed over a wide range in both directions is possible; and
(ii) the system is more efficient at low speeds as there are no resistor connected in series with the armature circuit.

The only disadvantage of this system is that a separate motor-generator set is required which may be a costly affair.

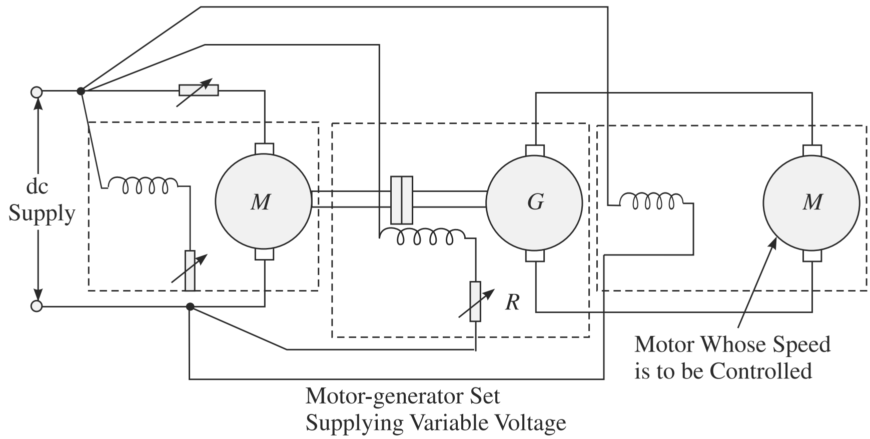


Fig. 2.49 *Speed control of a dc motor by varying armature applied voltage*

2.11.4 Speed Control by Using Triac

The motor-generator set, for obtaining variable dc voltage can be replaced by various forms of grid-controlled mercury-arc rectifiers or by thyristors or triac circuits Fig. 2.50 shows one such scheme which utilises triacs. Triac is a semiconductor device which operates as a static switch. When a signal is applied to its gate *G*, the triac

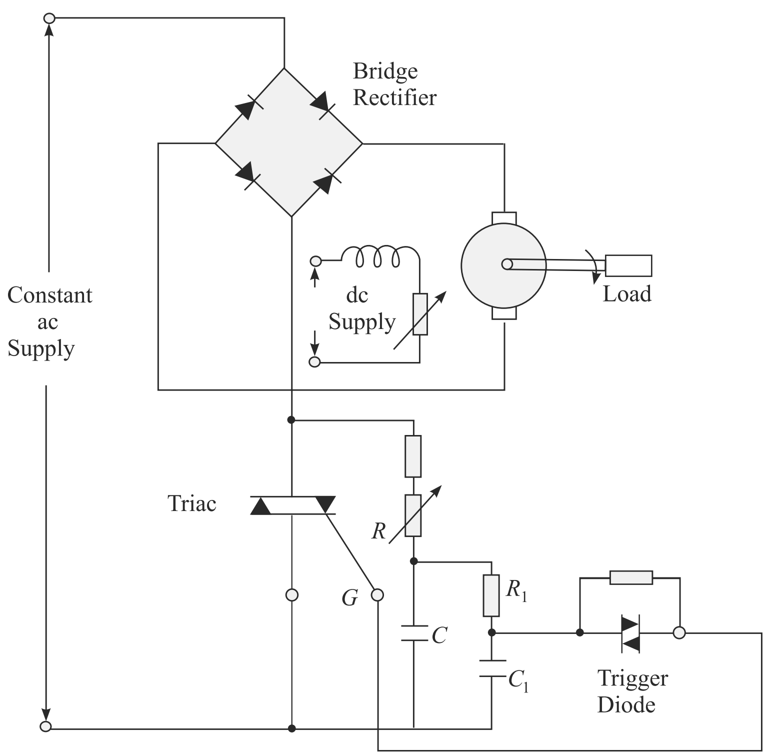


Fig. 2.50 *Speed control of a small dc motor using a triac*

behaves as a switch, the contacts of which are closed. When the alternating current flowing through it in any direction falls to zero, the triac automatically opens the circuit. The circuit can be closed again by application of an impulse to its gate. The signal applied to the gate is generated by a $R - C$ network via an auxiliary $R_1 - C_1$ network and a trigger diode (DIAC). The closing of the triac at each half-cycle can be controlled by adjusting the variable resistor R. The dc voltage appearing across the motor armature can be controlled by adjusting the ac voltage applied to the bridge circuit during each half-cycle of the ac input at which the triac closes the circuit.

EXAMPLE 2.17 *A 240 V dc shunt motor runs on no-load at 800 rpm with no extra resistance in the field or armature circuit, the armature current being 2 A. Calculate the resistance required in series with the shunt winding so that the motor may run at 950 rpm when taking a line current of 30 A. Shunt winding resistance is 160 Ω, armature resistance is 0.4 Ω, Assume that flux is proportional to field current.*

Solution At no-load

$$E = V - I_a R_a$$
$$= 240 - 2 \times 0.4 = 239.2 \text{ V}$$
$$E = K \phi N$$
$$= K_1 I_f N$$
$$I_f = \frac{V}{R_f} = \frac{240}{160} \text{ A} = 1.5 \text{ A}$$

Substituting

$$239.2 = K_1 \frac{240}{160} \times 800$$

$$\therefore \quad K_1 = \frac{239.2 \times 160}{240 \times 800} = 0.199$$

At a load of 30 A

$$I_{a_1} = I_{L_1} - I_f = 30 - 2 = 28 \text{ A}$$
$$E_1 = V - I_{a_1} R_a$$
$$= 240 - 28 \times 0.4 = 228.8 \text{ V}$$
$$E_1 = K_1 I_{f_1} N_1$$

Substituting

$$228.8 = 0.199 I_{f_1} \times 950$$
$$I_{f_1} = \frac{228.8}{950 \times 0.199} \text{ A}$$
$$R_f + R = \frac{240 \times 950 \times 0.199}{228.8} = 198.3\ \Omega$$

$\therefore$ Extra resistance required in the field circuit

$$R = 198.3 - R_f = 198.3 - 160 = 38.3\ \Omega$$

EXAMPLE 2.18 *A 230 V dc shunt motor takes an armature current of 20 A on a particular load. The armature circuit resistance is 0.5 Ω. Find the resistance required in series with the armature to reduce the speed by 50% if (a) the load torque is constant and (b) the load torque is proportional to the square of the speed.*

Solution

(a)

$$E = V - I_a R_a$$
$$= 230 - 20 \times 0.5 = 220 \text{ V}$$
$$E = K\phi N_1$$
$$N_1 = \frac{220}{K\phi}$$

When an extra resistance is introduced in the armature circuit, the speed is halved, i.e., the new speed

$$N_2 = \frac{N_1}{2}$$
$$E_2 = K\phi N_2 = K\phi \frac{220}{K\phi \times 2} = 110 \text{ V}$$

Here, $E_2 = V - I_a (R_a + R)$

where R is the extra resistance in the armature circuit.

Substituting $110 = 230 - 20 (0.5 + R)$

or, $R = 5.5\ \Omega$

(b)

$$T = K_t \phi I_a$$
$$T \propto I_a \text{ and } T \propto N^2$$

∴ $I_a \propto N^2$

If N is halved, I_a is one-fourthed.

∴ $E_2 = V - I_{a1} (0.5 + \text{R})$

Substituting $110 = 230 - 5 (0.5 + R)$

∴ $R = 23.5\ \Omega$

EXAMPLE 2.19 *The resistance of the armature circuit of a 250 V dc shunt motor is 0.3 Ω and its full-load speed is 1000 rpm. Calculate the resistance required in series with the armature to reduce the speed with the full-load torque to 800 rpm, the full-load armature current being 50 A. If the load torque is then halved, at what speed will the motor run? The armature reaction effect is to be neglected.*

Solution

$$E = V - I_a R_a$$
$$= 250 - 50 \times 0.3 = 235 \text{ V}$$
$$E = K\phi N$$
$$235 = K\phi \times 1000$$

or $$K\phi = \frac{235}{1000} = 0.235$$

To calculate E at 800 rpm,

$$E = K\phi \times 800 = 0.235 \times 800 = 188 \text{ V}$$

Let an extra resistance R be put in series with the armature circuit. Then,

$$E = V - I_a(R_a + R)$$

$$188 = 250 - 50\,(0.3 + R)$$

or $$R = 0.94\ \Omega$$

If the load torque is halved, I_a is halved since there is a linear relationship between torque and I_a. Now

$$I_a = 25 \text{ A}$$

$$E_1 = V - I_a(R_a + R)$$

$$= 250 - 25(0.3 + 0.94)$$

$$= 219 \text{ V}$$

$$\frac{E}{E_1} = \frac{N}{N_1}$$

$$N_1 = N\frac{E_1}{E} = 800 \times \frac{219}{188} = 932 \text{ rpm}$$

EXAMPLE 2.20 *A 250 shunt motor on no-load runs at 1000 rpm and draws 5 A from the lines. The armature resistance is 0.2 Ω and the field circuit resistance is 250 Ω. Calculate the speed of the motor when it is loaded and draws a current of 50 A. The armature reaction weakens the field by 3 per cent.*

Solution

$$I_L = 5 \text{ A at no-load}$$

$$I_f = \frac{250}{250} = 1 \text{ A}$$

∴ $$I_a = I_L - I_f = 5 - 1 = 4 \text{ A}$$

As at a load current of 50 A, the armature reaction weakens the field by 3 per cent,

$$I_f = 0.97 \text{ A} \qquad (\because\ \phi \propto I_f)$$

and $$I_a = 50 - 0.97 = 49.03 \text{ A}$$

$$E = K\phi N \text{ and } E = V - I_a R_a$$

or $$\frac{E_1}{E_2} = \frac{K\phi_1 N_1}{K\phi_2 N_2} = \frac{\phi \times 1000}{0.97\phi \times N_2}$$

$$N_2 = \frac{1000 \times E_2}{0.97 \times E_1} = \frac{1000(250 - 49.03 \times 0.2)}{0.97(250 - 4 \times 0.2)}$$

∴ $$N_2 = 994 \text{ rpm}$$

EXAMPLE 2.21 *A 4 pole, 500 V, shunt motor has a total of 720 armature conductors which are wave connected. The full-load armature current is 60 A, and the flux per pole is 0.03 Wb. The armature resistance is 0.2 Ω. The voltage drop across a brush is 1 volt. Calculate the full-load speed of the motor.*

Solution Given, $P = 4$, $V = 500$ V, $Z = 720$, $A = 2$, $I_a = 60$ A, $\phi = 0.03$ Wb, $R_a = 0.2\ \Omega$

We know, $E = V - I_a R_a$

Since, voltage drop across each brush = 1 V,

$$E = V - I_a R_a - 2 \quad \text{(voltage drop for a pair of brushes)}$$
$$= 500 - 60 \times 0.2 - 2 = 486 \text{ Volts}$$

Again, $$E = \frac{\phi ZNP}{60\,A}$$

or $$N = \frac{E \times 60\,A}{\phi ZP} = \frac{486 \times 60 \times 2}{0.03 \times 720 \times 4} = 675 \text{ rpm}$$

EXAMPLE 2.22 *A dc series motor, connected to a 440 V supply, runs at 600 rpm when taking a current of 50 A. Calculate the value of resistance which, when inserted in series with the motor, will reduce the speed to 400 rpm, the gross torque being then half its previous value. The resistance of the motor is 0.2 Ω. Assume flux to be proportional to field current.*

Solution Before an extra resistance is introduced,

$$E = V - I_a R_a$$
$$= 440 - 50 \times 0.2 = 430 \text{ V}$$
$$E = K\,\phi\,N = K_1\,I_a N \text{ (for a series motor } \phi \propto I_a)$$

$\therefore$ $$K_1 = \frac{E}{I_a N} = \frac{430}{50 \times 600} = 0.0143$$

$$T = K_t\,\phi\,I_a = K_{t_1} I^2_a$$

When torque is half, say, T_1,

$$\frac{T}{T_1} = \frac{K_{t_1} I_a^2}{K_{t_1} I_{a_1}^2}$$

or $$\frac{T}{T/2} = \frac{50 \times 50}{(I_{a_1})^2}$$

$$I_{a_1} = \sqrt{\frac{50 \times 50}{2}} = 35.35 \text{ A}$$

At this armature current I_{a_1} and with a resistance R introduced in the circuit, the induced emf E_1 is given by

$$E_1 = V - I_{a_1}(R_a + R)$$

But $$E_1 = K_1 I_{a_1} N_1 = 0.0143 \times 35.35 \times 400 = 202 \text{ V}$$

∴ $$202 = 440 - 35.35\,(0.2 + R)$$

or, $$R = 6.53\ \Omega$$

EXAMPLE 2.23 *A dc series motor (Fig. 2.51) runs at 1000 rpm when taking 20 A at 200 V. The resistance of the armature circuit is 0.5 Ω and that of the field winding is 0.2 Ω. Find the speed for a total current of 20 A, 200 V, when a 0.2 Ω resistor is joined in parallel with the field winding. The flux for a field current of 10 A is 70% of that for 20 A.*

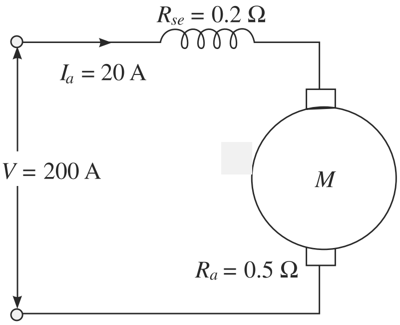


Fig. 2.51

Solution In the first case

$$E = V - I_a(R_a + R_{se})$$

$$= 200 - 20\,(0.5 + 0.2) = 186 \text{ V}$$

$$E = K\phi N$$

or $$186 = K\phi \times 1000$$

$$K\phi = 0.186$$

When a resistance R of value 0.2 Ω is connected in parallel with the series field, 20 A current will be equally divided between the series field winding and the parallel resistance called the diverter. In this case flux will be produced due to 10 A current flowing through the series field.

Induced emf,

$$E_1 = K\phi_1 N_1$$

$$= 0.7 \times K\phi \times N_1 \quad (\because K\phi_1 = 0.7\,K\phi)$$

$$E_1 = 0.7 \times 0.186 \times N_1$$

But $$E_1 = V - (I_a R_a + I_{se} R_{se})$$

$$= 200 - (20 \times 0.5 + 10 \times 0.2) = 188 \text{ V}$$

Thus, $$188 = 0.7 \times 0.186 \times N_1$$

$$N_1 = \frac{188}{0.7 \times 0.186}$$

$$N_1 = 1444 \text{ rpm}$$

EXAMPLE 2.24 *A series motor has a resistance of 1 ohm between its terminals. The motor runs at 800 rpm at 200 V taking a current of 15 A. Calculate the speed at which the motor will run when connected in series with a 5 Ω resistance and taking the same current at the same supply voltage.*

Solution In the first case,

$$E_1 = V - I_a R_a = 200 - 15 \times 1 = 185 \text{ V}$$

In the second case when 5 Ω resistance is connected in series,

$$E_2 = V - I_a (R_a + R)$$
$$= 200 - 15 (1 + 5) = 110 \text{ V}$$

We have, $$\frac{N_2}{N_1} = \frac{E_2}{E_2}$$

Therefore, $$N_2 = N_1 \frac{E_2}{E_2} = 800 \times \frac{110}{185} = 476 \text{ rpm}$$

2.12 ARMATURE REACTION AND COMMUTATION

When a dc generator is loaded, the armature mmf flux reacts with the field flux and thereby distorts the distribution of flux in the air-gap. The magnetic neutral axis shifts from the geometrical neutral axis, i.e., from the interpolar region. This adversely affects the reversal of current in a conductor passing under the brush.

2.12.1 Armature Reaction

The distribution of flux in the air-gap of a dc machine due to its field excitation is shown in Fig. 2.52.

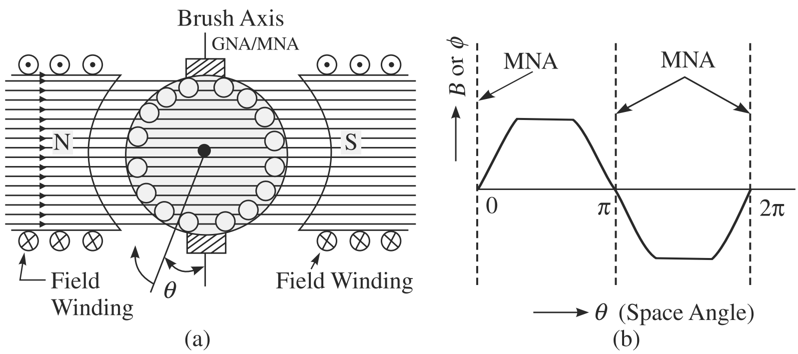


Fig. 2.52 *Flux distribution in the air-gap due to field excitation only*

It is seen from Fig. 2.52(a) that the geometrical neutral axis (GNA) and magnetic neutral axis (MNA) are the same in this case (GNA is the line passing through midway between any two magnetic poles, while MNA is a line passing through the region of zero flux existing between poles).

When the machine is loaded, considerable amount of current will flow through the armature conductors. The flux produced by the current flowing through the armature conductors will distort the air-gap distribution of flux produced by field winding current. This effect of armature flux on the field winding flux is called the armature reaction. The effect of armature flux on the field flux is shown graphically in Fig. 2.53.

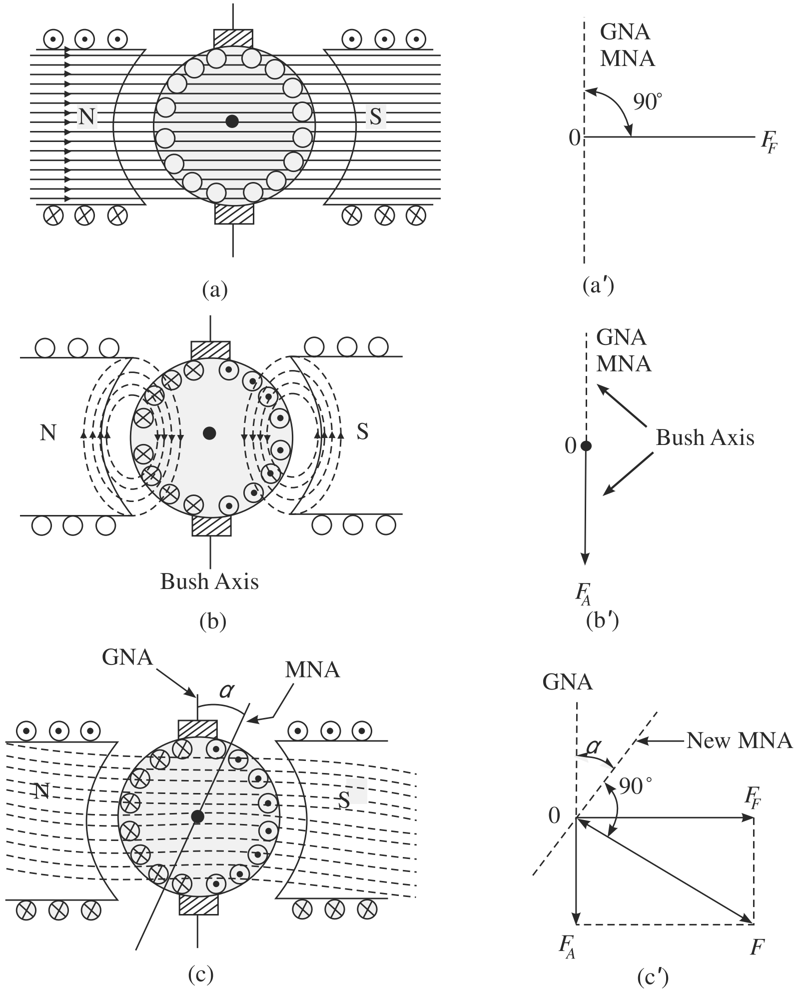


Fig. 2.53 *(a) Flux distribution in air-gap due to field current only (b) flux distribution due to armature current only (c) Flux distribution due to the combined effect of field current and armature current. (a′), (b′) and (c′) are the vectorial representations of flux density distribution due to field current, armature current, and combined effect of field and armature current respectively*

It is seen from the figure that due to the armature flux, the distribution of the air-gap flux is distorted as shown in Fig. 2.53(c). The MNA is now shifted by an angle α as shown in Fig. 2.53 (c and c′). The brush axis is therefore to be shifted by an angle α in the forward direction if the brushes are to be in the MNA (a requirement for sparkless commutation, details of which will be discussed in the following section). The angle of shift of the MNA will depend upon the magnitude of armature current, i.e., on the load on the machine. Therefore, the angle of shift of the brush axis to the MNA will depend upon the magnitude of load on the machine. If the brushes are not shifted to the MNA, emf will be induced in the coil undergoing commutation (coil in which current change takes place when passing under the brushes), which will be a cause of sparking on the commutator surface. Continuous shifting of the brush axis, depending on the magnitude of load on the machine, is an impractical solution to the problem. Alternative arrangements are made to get rid of the problem of sparking. This is being discussed in the section to follow.

Assume, for example, that the brushes are shifted to MNA (to avoid sparking during commutation) as shown in Fig. 2.54(b). The brush axis has been shown shifted by angle α in the clockwise direction to the MNA. The distribution of current in armature conductors is changed. Some conductors which were earlier under the influence of the south pole have come under the influence of the north pole and vice versa. The armature mmf will now lie along the new position of the brush axis as shown in Fig. 2.54(c). This is shown by vector F_a, which can be resolved into two components, viz. F_{ac} along the GNA and F_{ad} at perpendicular to GNA. The component F_{ac} is in quadrature with the main field mmf, i.e., F_m [see Fig. 2.54(b)]. This component will have a cross-magnetising effect on the main field. Its effect will be to distort the flux distribution in the air-gap. The component F_{ad} will have a demagnetising effect since it works in the opposite direction to the main field. This will reduce the air-gap field strength and hence the induced emf.

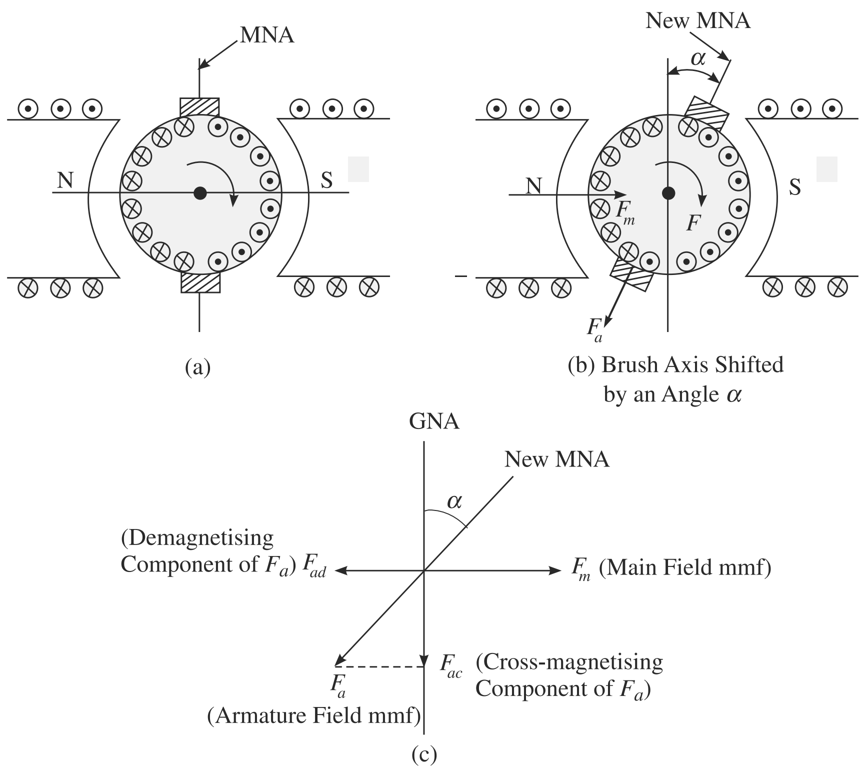


Fig. 2.54 *Brush axis shifted to new magnetic neutral axis created due to armature reaction*

2.12.2 Commutation

It has been explained earlier that when an armature coil passes under the brushes, the direction of current in the coil changes. The reversal of current takes place when the conductors are along the brush axis, i.e., when the brushes short-circuit the coil. The process of commutation is shown in Fig. 2.55. In Fig. 2.55(a) the coil *AA′* is seen touching the brushes *BB′* and in Fig. 2.55(c), the coil is seen leaving the brushes as the armature continues to rotate. Reverse of current takes place during this period.

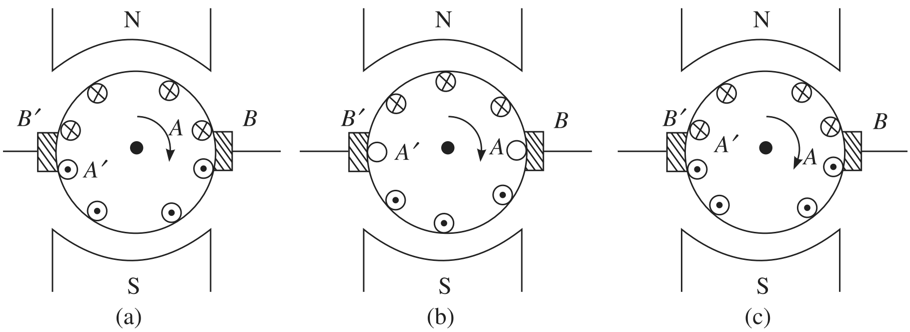


Fig. 2.55 *Reversal of current takes place in an armature coil AA′ when the armature conductors pass under the brushes BB′*

The process by which current in the coil is reversed during the period in which it passes under the brushes is called commutation. The period during which a coil undergoing commutation remains short-circuited is known as the commutation period. The commutation period is shown by the period which is taken for coil *AA′* to change its position from that shown in Fig. 2.55(a) to that in Fig. 2.55(c).

During the period of short circuit, the current in the coil undergoing commutation should be reversed to its full value as shown graphically in Fig. 2.56 (from $+I$ to $-I$ as shown by line a). If the current does not fully reverse to its full value as shown by line b in Fig. 2.56, the difference has to jump from the commutator bar to the brush causing sparking. Thus the cause of sparking in a dc machine is the failure of the current in the short-circuited coil to reach to its full value in the reverse direction by the end of the short-circuited period, i.e., during the period of commutation.

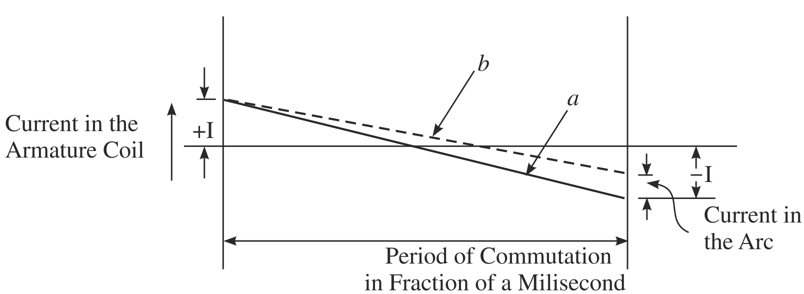


Fig. 2.56 *Period of commutation i.e., the time taken by the armature coil AA′ to pass under the brushes while rotating*

It is required to investigate the reason for the failure of a coil to get the current reversed to its full value during commutation. This is due to the production of self- induced emf in the coil undergoing commutation. This induced emf is called *reactance voltage* which opposes the reversal of current. Reactance voltage is induced due to the presence of any flux in the air-gap in the interpolar region, i.e., in the brush axis. As mentioned earlier, due to the armature reaction, the pattern of the air-gap flux distribution changes, thereby changing the MNA. Due to this, flux axists in the air-gap at the GNA. The strength of this flux will depend upon the magnitude of armature current, i.e., on the load. Brushes are to be shifted to the MNA to avoid reactance voltage to be induced in the coils undergoing commutation. The angle of shift will depend on the magnitude of load. Shifting of brushes to the MNA for every change in load is not practicable. The brushes are therefore fixed at the GNA and the effect of armature reaction on the GNA is neutralised by some suitable method which are explained in the following section.

2.12.3 Methods of Neutralising Effect of Armature Reaction for Better Commutation

Use of High Brush-Contact Resistance The process of commutation is shown with the help of a series of diagrams in Fig. 2.57. The way in which the current direction in the coil a reverses has been illustrated in Fig. 2.57. When commutator segment 2 approaches the brush [Fig. 2.57(b)] a part x of current I still flows through coil a. The current in coil a becomes zero in the position shown in Fig. 2.57(c). In position (d), the current in coil a starts flowing in the reverse direction. In position (e), there is complete reversal of current in coil a, if the brush-contact resistance with the commutator is made higher. By using say carbon brushes, this reversal of current in the coil will be easier and there will be less sparking on the commutator surface. (Sparking on the commutator surface is undesirable as it reduces the life of the commutator which is a very important part of the dc machine).

Now it is to be seen how high brush-contact resistance helps the reversal of current in a coil undergoing commutation. When the commutator segment is leaving brush 1 (see Fig. 2.57(d)) more and more current should flow through coil a in the direction shown so that complete reversal takes place by the time commutator segment 1 completely leaves its contact with the brush (as in Fig. 2.57(e)). The contact resistance will depend on the resistance of the carbon brush and also on the area of contact between the brush and the commutator segment. Lower the contact area, higher is the contact resistance. For the position shown in Fig. 2.57(d). Let r be the contact resistance between the brush and commutator segment 2. The contact resistance between the brush and commutator segment I will be higher, let it be equal to $10r$ (say). Let R be the resistance of the coil a. Then,

Potential drop between commutator segment 1 and brush Q = Potential drop in coil a + Potential drop between commutator segment 2 and brush Q

Thus, $$(I - X)\ 10r = xR + (I + X)r$$

or $$10\ Ir - 10\ rX = xR + Ir + Xr$$

or $$X\ (11r + R) = 9\ Ir$$

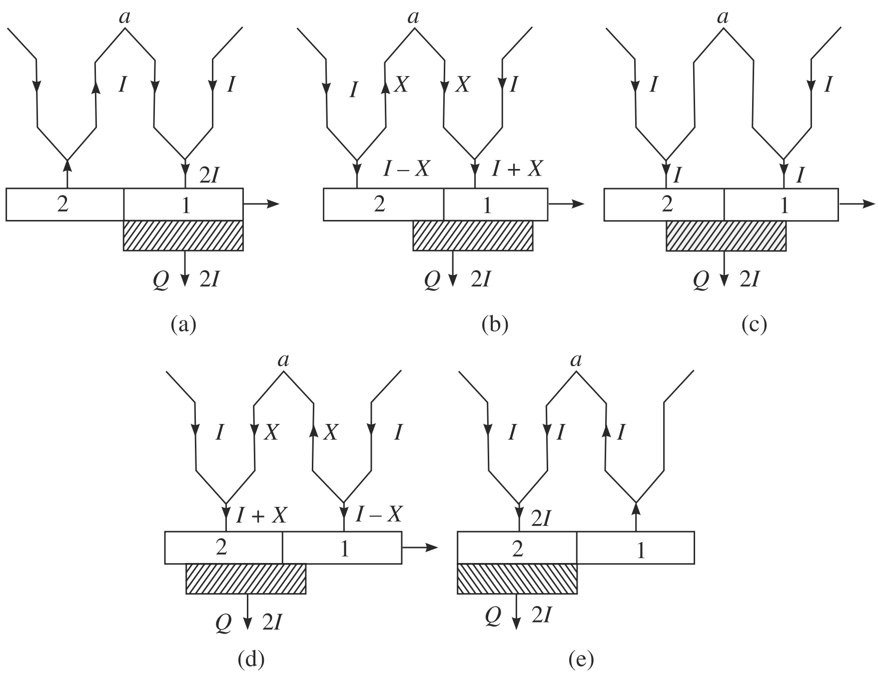


Fig. 2.57 *Process of commutation and effect of high brush contact resistance*

$$\therefore \qquad X = I\,\frac{9r}{11r + R}$$

The value of r should be such that x is high.
Now the effect of varying r relative to R is to be studied.

Let $\qquad r = 0.01\,R$

Then $\qquad X = \frac{9 \times 0.01R}{11 \times 0.01R + R}\,I = 0.081\,I$

Let $\qquad r = 0.1\,R$

$$X = \frac{9 \times 0.01R}{11 \times 0.01R + R}\,I = 0.43\,I$$

Let $\qquad r = R$

$$X = \frac{9R}{11R + R}I = \frac{9}{12}\,I = 0.75\,I$$

From the above it is obvious that if the value of brush-contact resistance r is high, reverse current x through the coil increases very fast which is highly desirable for sparkless commutation. For this reason high-resistance carbon brushes are used in dc machines. The carbon brushes also lubricate and polish the commutator surface as it rotates.

Shifting of Brushes By shifting the brushes to the new MNA, sparkless commutation can be achieved. This has been explained earlier. The brushes are to be shifted by a certain angle in the forward direction (in the direction of rotation) in a generator, and backward in a motor. The disadvantage with this method is that the angle of shift will depend upon the load on the machine and therefore is practically difficult to arrange. This method, therefore, is not used.

Use of Commutating Poles (Interpoles) The use of a high brush-contact resistance aids commutation but this is not a complete solution to the problem. The other way is to create a flux in the interpolar region (GNA) in a direction opposite to that produced due to the armature reaction so that there is no reactance voltage induced in the coil undergoing commutation. In fact a flux slightly higher in magnitude but in a direction reverse to that produced due to the armature reaction effect is produced with the help of small extra poles fixed in the interpolar region. Such poles are called *compoles* or *interpoles*. This extra reverse flux will induce a small amount of reactance voltage in a direction such that it aids reversal of the current in the coil. The polarity of a compole in a dc generator must be the same as that of the main pole immediately ahead as shown in Fig. 2.58.

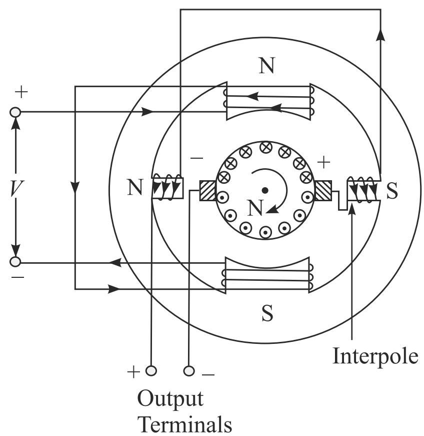


Fig. 2.58 *Connection of commutating poles in a dc generator*

Since the purpose of introducing compoles is to neutralise the effect of armature reaction caused due to armature current, the compole field windings are connected in series with the armature winding. This enables the compole ampere turns to be in proportion with the armature ampere-turns.

Use of Compensating Winding To neutralise the demagnetising effect of the armature ampere-turns (component F_{ad} of F_a as shown in Fig. 2.54(c)), an auxiliary winding called the compensating winding is provided in large dc machines. The compensating winding is placed in slots made on the main pole faces as shown in Fig. 2.59. Compensating windings are connected in series with the armature winding. They produce mmf in the opposite direction to that produced by the armature current.

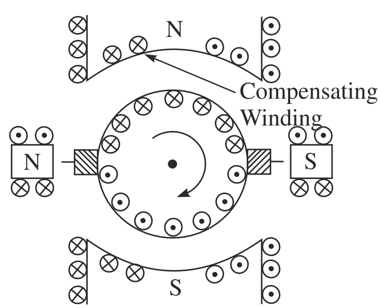


Fig. 2.59 *Compensating windings used to neutralise the effect of armature reaction*

The magnitude of mmf produced by this compensating winding per pole face is equal to the mmf produced by the number of armature conductors lying under the pole.

EXAMPLE 2.25 *Calculate the ampere-turns for each commutating pole of an 8-pole generator with 107 slots, each containing 1000 ampere conductors. The interpole air-gap is 1.2 cm. The gap flux density is to be 0.32 Wb/m². Neglect the effect of iron parts of the circuit and of leakage.*

Solution Ampere-turns for each commutating pole is the sum of ampere-turns of armature winding per pole and the ampere-turns required for the air-gap (neglecting the effect of iron parts of the circuit and of leakage).

Given, $P = 8$, $I_a Z = 107 \times 1000$, $l_g = 0.012$ m, $B_{ag} = 0.32$ Wb/m^2

We know, $$H = \frac{AT(ie\,NI)}{l_g}$$

AT required for the air gap $= H \times l_g$

and $$B_{ag} = \mu H$$

$\therefore$ AT required for the air gap $= \dfrac{B_{ag} \times l_g}{\mu}$

Number of armature winding ampere-turns per pole $= \dfrac{I_a Z}{2 \times P}$

Thus total interpole ampere turns = AT of armature winding per pole + AT required for the air-gap

$$= \frac{I_a Z}{2 \times P} + \frac{B_{ag} \times l_g}{\mu}$$

Substituting, the values,

Total $$AT = \frac{107 \times 1000}{2 \times 8} + \frac{0.32 \times 0.012}{4\pi \times 10^{-7}} = 9743$$

EXAMPLE 2.26 *Estimate the number of turns needed on each commutating pole of a 6-pole generator delivering 200 kW at 200 V, given that the number of armature conductors is 540 and the winding is lap connected, interpole air-gap is 1.0 cm, and the flux density in the interpole air-gap is 0.3 Wb/m². Neglect the effect of iron parts of the circuit and of leakage.*

Solution Given $P = 6$, $A = 6$ (as the winding is lap connected), $Z = 540$, interpole air-gap, $l_g = 0.01$ m

Interpole flux density, $B_{ip} = 0.3$ Wb/m^2

Power delivered, $P = 200$ kW at $V = 200$ Volts

Armature current, $$I_a = \frac{200{,}000}{200} = 1000 \text{ A}$$

Ampere-turns (mmf) to be produced by an interpole

$$= N_{ip} \times I_a = N_{ip} \times 1000$$

(as interpole winding is connected in series with the armature windings)

This interpole mmf should be equal to the sum of armature mmf per pole and the mmf required to produce a flux density of 0.3 Wb/m^2 in the interpole air-gap. (neglecting the effect of iron-parts of the circuit and of leakage).

Number of armature conductors, $Z = 540$

Number of armature winding turns = $\dfrac{540}{2} = 270$

As there are 6 parallel paths in the armature, the number of armature winding turns

per parallel path = $\dfrac{270}{6} = 45$

Number of armature ampere turns/pole = $\dfrac{1000 \times 45}{6} = 7500$

For the air-gap,

$$B_{ag} = \mu H$$

or $$H = \frac{B_{ag}}{\mu}$$

Again, $$H = \frac{N_{ipg} \times I_a}{l_g}$$

$\therefore$ $$B_{ag} = \frac{\mu \times N_{ipg} \times I_a}{l_g}$$

or $$N_{ipg} \times I_a = \frac{B_{ag} \times l_g}{\mu} = \frac{0.3 \times 0.01}{4\pi \times 10^{-7}} = 2387$$

Total interpole ampere turns = Ampere turns of the armature winding per pole + Ampere turns for the air-gap

Therefore, $$N_{ip} \times I_a = 7500 + 2387$$

or, $$N_{ip} = \frac{9887}{1000} \simeq 10$$

2.13 LOSSES AND EFFICIENCY

The whole amount of input energy to a dc machine is not available at the output side because of various losses taking place in the machine during the process of electromechanical energy conversion. The various losses in a dc machine are mentioned below.

2.13.1 Losses in a dc Machine

I^2R Loss in the Armature Winding Due to the current flow in the armature winding an appreciable amount of electrical energy is lost and can be expressed as $I_a^2 R_a$. This loss varies with the variation of load on the machine and is called variable loss. It varies as the square of the load current. If load current is halved, the $I_a^2 R_a$ loss will be one-fourth and so on. The resistance of the armature winding can be measured by the ammeter-voltmeter method. By knowing the value of R_a at room temperature, its value at operating temperature can be calculated.

Iron Loss in the Armature Core Iron-loss consists of two parts, namely hysteresis loss and eddy-current loss. These losses occur in any part of the machine that is constructed out of magnetic material and is subjected to variations of magnetic flux. The armature of a dc machine rotates in a magnetic field. Due to rotation, the magnetic material of the rotor comes under the north and south poles alternately. The hysteresis loss is due to alternate magnetisation of the atoms, forming domains in the magnetic material of the core. Each domain behaves as a tiny magnet which aligns itself with the magnetic flux in which it is placed. As the flux changes its direction (due to the changing of position of the rotor when the rotor rotates), these tiny magnets rotate clockwise and counter-clockwise. Power is wasted due to this movement and this process develops heat in the armature core. Hysteresis loss depends upon flux density and frequency of variation of flux and can be expressed as

$$\text{Hysteresis loss} = K_h B_m^{1.6} f V_c \text{ Watts}$$

where K_h is a constant whose value depends upon the core material, B_m is the maximum flux density of the magnetic field in which the core is placed, f is the frequency of variation of flux and V_c is the volume of the core material in m^3.

When the armature core rotates in the magnetic field, emf is induced in the core in the same way as emf is induced in the windings. The emf induced in the core produces a circulating current in the core material. These are called eddy currents and they produce power loss in the resistance of all the magnetic parts. To reduce eddy-current loss, the magnetic core is made up of thin sheets called laminations which are insulated from one another. This causes increase in the resistance of the current path and thereby reduces the eddy-current loss. The expression for eddy-current loss is given as

$$\text{Eddy-current loss} = K_e B_m^2 f^2 t^2 V_c \text{ Watts}$$

where K_e is another constant which depends upon the core material and t is the thickness of the core laminations.

If the flux density and speed of a dc machine remain constant, the core losses as described above will also remain constant.

Eddy current loss also takes place in the pole cores clue to flux pulsation as a result of armature reaction.

Losses in the Field Windings Losses take place in the field windings due to the current flow through them. Loss in the shunt field is expressed as the product of the field current and terminal voltage. Losses in the series field, interpoles and compensating windings are proportional to the square of the armature current.

Frictional Losses Frictional losses take place due to rotation of the armature. These losses are bearing-friction loss, brush-friction loss and windage-friction loss. Windage-friction loss takes place due to rotation of the armature in air. Windage-friction loss is proportional to the cube of the speed. Brush-friction loss and bearing-friction loss are proportional to the speed.

The total losses in a dc machine can be expressed as

$$\text{Total losses} = I_a^2 R_a + \text{Field copper loss} + C$$

$$C = \text{sum of iron, friction and windage losses}$$

where, the right-hand quantity is obviously positive.

The input, output and losses in a dc machine can he diagrammatically expressed as shown in Fig. 2.60.

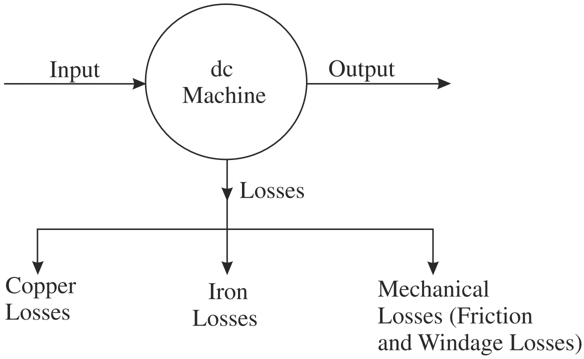


Fig. 2.60 *Input, output and losses in a dc machine*

2.13.2 Efficiency of a dc Machine

The efficiency of a dc machine is the ratio of the useful output and the input. Efficiency can be expressed as

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$= \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

In the case of separately excited dc generator, knowing the output and losses, the efficiency can be expressed as

$$\eta_G = \frac{V I_a}{V I_a + I_a^2 R_a + V I_f + C} \tag{2.9}$$

In case of a dc motor, knowing the input and losses, the efficiency can be expressed as

$$\eta_M = \frac{V I_a - I_a^2 R_a - V I_f - C}{V I_a} \tag{2.10}$$

Efficiency is generally expressed in percentage.

Condition for Maximum Efficiency The expression for the efficiency of a dc generator is written as

$$\eta_G = \frac{V I_a}{V I_a + I_a^2 R_a + V I_f + C}$$

Efficiency will be maximum when the denominator of the above expression is minimum. To determine the condition for which the denominator is minimum, differentiate it with respect to the variable quantity, i.e., I_a and equate it to zero.

$$\eta_G = \frac{V}{V + I_a R_a + \dfrac{(V I_f + C)}{I_a}}$$

(dividing the expression for η_G by I_a)

For η_G to be maximum,

$$\frac{d}{d I_a}\left[V + I_a R_a + \frac{V I_f + C}{I_a}\right] = 0$$

or $$R_a - \frac{V I_f + C}{I_a^2} = 0$$

or $$I_a^2 R_a = V I_f + C$$

or $$\text{Variable loss} = \text{Constant loss}$$

Again, for η_G to be maximum $\dfrac{d^2 \eta_G}{d I_a^2}$ should be positive.

Therefore, $\dfrac{d}{d I_a}\left[R_a - \dfrac{V I_f + C}{I_a^2}\right]$ should be positive

$$\frac{d}{d I_a}\left[R_a - \frac{V I_f + C}{I_a^2}\right] = \frac{2(V I_f + C)}{I_a^3}$$

The right-hand quantity is obviously positive.

Therefore efficiency of the generator is maximum when the load is such that the variable loss is equal to the constant loss.

The condition for maximum efficiency of a dc motor can also be determined by using the expression for efficiency of a dc motor and is found as the same as that of a dc generator. Efficiency of a dc machine can be determined by using any one of the following methods.

Determination of Efficiency by Actually Loading the Machine In this method the machine is actually loaded by applying some brake in the case of motor operation or by loading electrically in the case of generator operation. The output power of a dc motor can be measured by using some sort of a mechanical brake arrangement as shown in Fig. 2.61.

A belt is placed on the pulley attached to the motor shaft. One end of the pulley is attached to the floor or to a base via spring balance *S*. A known weight is attached to the other end. If the balance reading is *S* kgf and the suspended load has weight of *W* kgf, then

Net pull on the belt = $(W - S)$ kgf

$= 9.81\,(W - S)$ Newtons

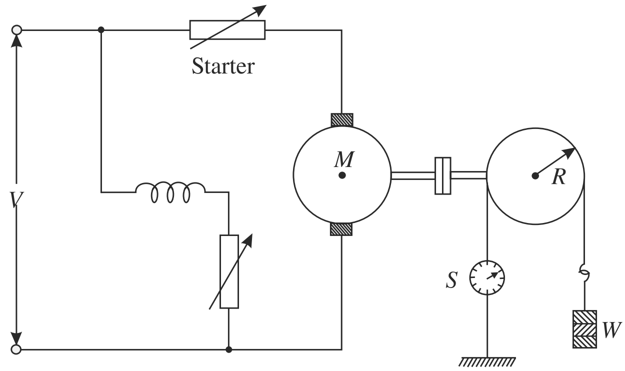


Fig. 2.61 *Determination of efficiency of a dc motor by directly loading the motor mechanically*

If N_r is the speed of the pulley in rpm and R is its radius in metres,

$$\text{Output power} = \frac{2\pi N_r\, 9.81(W - S)}{60} R \text{ Watts} \quad \text{(i)}$$

$$= \frac{2\pi N_r\, 9.81(W - S)R}{60 \times 746} \text{ Horse power}$$

$$\text{Input power} = VI \text{ Watts} \quad \text{(ii)}$$

$$\therefore \quad \text{Efficiency} = \frac{2\pi N_r\,(W - S) \times 9.81 \times R}{60 \times V I}$$

It should be understood here that the whole of the output power is wasted as heat in this method. For machines of very high ratings, it is a problem to get this heat dissipated. That is why this direct-loading method of determination of efficiency is restricted to only small machines.

Determination of Efficiency by Measurement of Losses (Swinberne's Method) This is an indirect method where the efficiency is calculated by measuring the losses. This method is known as Swinberne's method. The machine is run as a motor on no-load. The supply voltage and motor speed are adjusted at their rated values. The connection diagram for performing Swinberne's test on a dc shunt motor is shown in Fig. 2.62.

When the motor is running on no-load,

Input power to armature = $V I_{ao}$

and Shunt field-loss = $V I_f$

The input power at no-load to the motor is wasted as (i) iron-loss in the core, (ii) friction and windage-loss and (iii) a small amount of $I_{ao}^2 R_a$ loss in the armature winding. It may be noted that the output at no-load is zero and hence the input power is spent to supply the above losses.

The small amount of $I_{ao}^2 R_a$ loss taking place at no-load can be calculated by knowing the armature circuit resistance and no-load armature current.

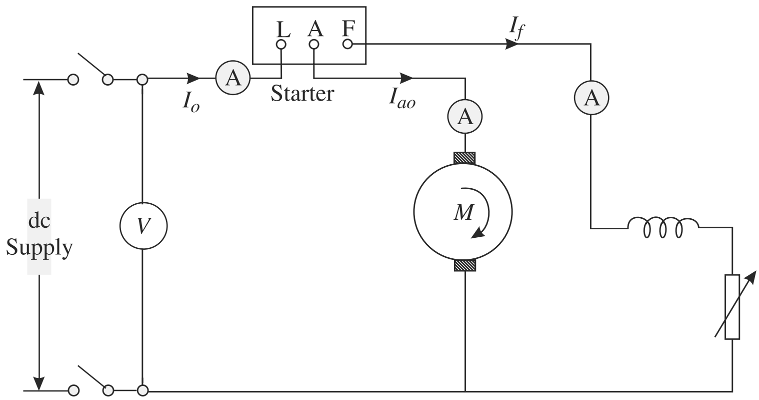


Fig. 2.62 *Connections for Swinberne's test for determination of efficiency of a dc motor*

Iron-loss and friction and windage-loss can be calculated by subtracting the no-load armature copper loss from the no-load armature input.

Iron-loss and friction and windage-loss are constant losses and are the same at any load. They are dependent upon the supply voltage and the speed, and not on the load on the machine.

By knowing the constant losses, the efficiency at a load current (line current), of say I, can be calculated.

Efficiency of the machine as a shunt motor,

$$\eta_M = \frac{VI - I_a^2 R_a - (I_{ao} + I_f)V}{VI}$$

where, VI is input to the motor at a load I, I_a is the current through the armature when load current is I, and $(I_{ao} + I_f)\,V$ is the input to the motor at no-load and rated speed.

Similarly, the efficiency of the machine as a shunt generator is given by

$$\eta_G = \frac{VI}{VI - I_a^2 R_a - (I_{ao} + I_f)V}$$

Armature resistance R_a is measured by the ammeter-voltmeter method. If the windings are at room temperature when this measurement is made, the resistance at normal working temperature should be calculated.

The advantages of Swinberne's method are:

(i) It is an economical method of determining efficiency since the power required is very small. Large machines can be tested by spending a small amount of energy.

(ii) Efficiency of the machine can be calculated for any load.

The disadvantages of this method are the following:

(i) The effect of armature reaction on flux distribution and on any possible change in iron-loss is not considered.

(ii) It is not possible to know whether at full-load commutation would be satisfactory and the temperature rise would be the same as specified.

(iii) A series motor cannot be tested by this method since it is not advisable to run series motors on no-load.

Determination of Efficiency by Regenerative or Hopkinson's Method

This requires two identical machines mechanically coupled to each other. The electrical connections are shown in Fig. 2.63. One of the machines will work as a motor and drive the other machine which will work as a generator. The generator will feed back power to the motor. The power to be drawn from the supply is only for supplying the losses in the machines. Large machines can therefore be tested under load conditions without spending much energy from the supply. The main disadvantage of this method is that two identical machines are required.

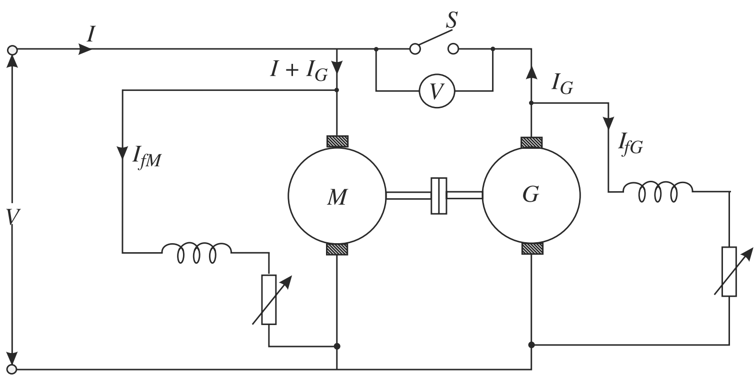


Fig. 2.63 *Regenerative or Hopkinson's test*

Keeping switch S open machine M is started as a motor with the help of starter. Excitation I_{fG} of the other machine is adjusted till voltmeter V gives zero reading. Switch S is then closed. Current I_G can be adjusted to any desired value by increasing field current I_{fG} or by reducing the excitation amount I_{fM}. Note that ammeters of appropriate ratings are to be connected in the circuit.

If V is the supply voltage, then

Output power of $G = V I_G$ (i)

Input power to $M = (I + I_G) V$

Output power of $M = \eta(I + I_G) V$

$= \text{Input power of } G$

Output power of $G = \eta \times \text{Input power of } G$

$= \eta^2 (I + I_G) V$ (ii)

Therefore, from Eqs (i) and (ii)

$$V I_G = \eta^2 (I + I_G) V$$

or

$$\eta = \sqrt{\frac{I_G}{I + I_G}}$$

The above expression gives the efficiency of a dc machine assuming that losses in the two machines while on test are equal. But for accurate measurement some corrections are to be applied since the losses in the two machines are not equal.

The armature current of machine M is higher than that of machine G. The armature copper-losses in the two machines are therefore different. Excitation current of G is higher than that of M which makes the field copper-loss and iron-losses in machine G higher than that in machine M.

Most errors due to difference in losses can be eliminated by calculating efficiency in the following way. Let R_a be the resistance of the armature winding of each machine.

Winding loss in the armature in $G = (I_G + I_{fG})^2 R_a$

Winding loss in the armature in $M = (I + I_G - I_{fM})^2 R_a$

Loss in the shunt-field circuit of $G = V I_{fG}$

Loss in the shunt-field circuit of $M = V I_{fM}$

Total losses in G and M supplied by the mains = VI

Iron-loss in the two machines = (Power supplied by the mains) – (Armature and field losses in the two machines)

$$= VI - \{(I_G + I_{fG})^2 R_a + (I + I_G - I_{fM})^2 R_a + V(I_{fG} + I_{fM})\} = C \text{ (say)}$$

Assume iron, friction and windage losses in the two machines to be equal.

Iron, friction and windage loss per machine = $\dfrac{C}{2}$

$$\text{Efficiency of } G = \frac{V I_G}{V I_G + (I_G + I_{fG})^2 R_a + V I_{fG} + \dfrac{C}{2}}$$

$$\text{Efficiency of } M = \frac{V(I + I_G) - \left[(I + I_G - I_{fM})^2 R_a + I_{fM} V + \dfrac{C}{2}\right]}{V(I + I_G)}$$

EXAMPLE 2.27 *In a brake test on a dc motor, the effective load on the brake drum was 23 kgf, the effective diameter of the drum 45 cm and the speed 960 rpm. The input to the motor was 28 A at 230 V. Calculate the efficiency of the motor.*

Solution

$$\text{Output power} = \frac{2\pi\ NFr\ 9.81}{60}\ \text{W}$$

$$= \frac{2 \times 3.14 \times 960 \times 23 \times 45 \times 9.81}{60 \times 100 \times 2} = 5100\ \text{W}$$

Input power = 230 × 28 = 6440 W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{5100}{6440} \times 100\% = 79.2\%$$

EXAMPLE 2.28 *A 100 kW, 500 V shunt generator was run as a motor (Fig. 2.64) on no load at its rated voltage and speed. The total current taken was 9.5 A, including a shunt-field current of 2.5 A. The resistance of the armature circuit at normal working temperature is 0.1 Ω. Calculate the efficiency of the generator at full-load and at half-load.*

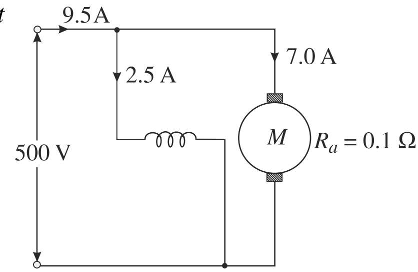


Fig. 2.64

Solution

Input at no-load = 9.5 × 500 W
= 4750 W

This includes iron, friction and windage losses, armature copper-loss at no-load and field copper-loss. Therefore,

Iron, friction and windage losses = $4750 - (7)^2 \times 0.1 - 500 \times 2.5 = 3495$ W

Full-load output current of the generator = $\frac{100 \times 1000}{500} = 200$ A

∴ Armature current at full-load = 200 + 2.5 = 202.5 A

Full-load armature copper-loss = $(202.5)^2 \times 0.1 = 4100$ W

Shunt-field copper-loss = 500 × 2.5 = 1250 W

Efficiency of the generator at full-load

$$= \frac{100 \times 1000 \times 100}{100 \times 1000 + 3495 + 4100 + 1250}\% = 91.9\%$$

For calculating efficiency at half-load, the variable loss, viz. the armature copper-loss at half-load is to be calculated.

Half-load output = 100 A

Armature current at half-load = 100 + 2.5 = 102.5 A

Armature copper-loss at half-load = $(102.5)^2 \times 0.1 = 1050$ W

Efficiency of the generator at half-load = $\frac{50 \times 1000 \times 100}{50 \times 1000 + 1050 + 1250 + 3495}\%$

= 89.6%

EXAMPLE 2.29 *A dc shunt machine when run as a motor (Fig. 2.65) on no-load takes 440 W at 220 V and runs at 1000 rpm. The field current and armature resistance are 1.0 A and 0.5 Ω respectively. Calculate the efficiency of the machine when running (a) as a generator delivering 40 A at 220 V and (b) as a motor taking 40 A from a 220 V supply.*

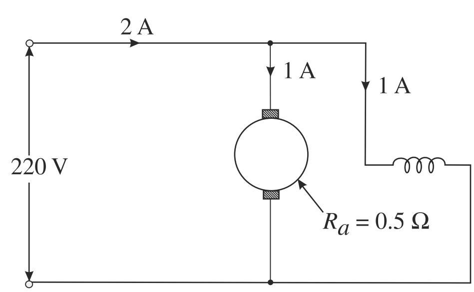


Fig. 2.65

Solution

Input at no load = 440 W

$$\text{Input current at no-load} = \frac{440}{220} = 2\text{ A}$$

$$\text{Armature current at no-load} = 2 - 1 = 1\text{ A}$$

$$\text{Iron, friction and windage losses} = \text{Input} - \text{Armature copper-loss at no-load} - \text{field copper-loss}$$

$$= 440 - (1)^2 \times 0.5 - 220 \times 1 = 219.5\text{ W}$$

Efficiency as a generator when delivering 40 A at 220 V,

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} \times 100\%$$

$$= \frac{220 \times 40 \times 100}{220 \times 40 + (40 + 1)^2 \times 0.5 + 220 \times 1 + 219.5}\%$$

$$= 87.3\%$$

Efficiency is as motor when taking 40 A from a 220 V supply,

$$\text{Efficiency} = \frac{\text{Input} - \text{Losses}}{\text{Input}} \times 100\%$$

$$= \frac{\{220 \times 40 - (40 - 1)^2 \times 0.5 - 220 \times 1 - 219.5\}}{220 \times 40} \times 100\%$$

$$= 86.4\%$$

EXAMPLE 2.30 *A 400 V dc motor (Fig. 2.66) takes 5 A at no-load. Its armature and field resistances are 0.5 Ω and 200 Ω respectively. Calculate the efficiency when the motor takes 50 A on full-load. Also calculate the percentage change in speed from no-load to full-load.*

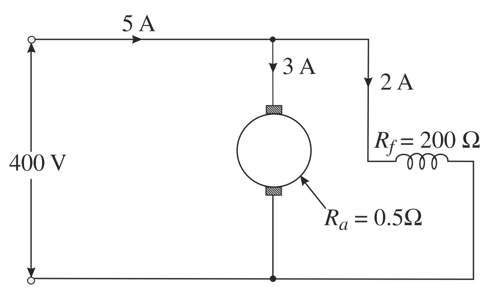


Fig. 2.66

Solution

$$I_f = \frac{V}{R_f} = \frac{400}{200} = 2\text{ A}$$

Motor input at no-load = 400 × 5 = 2000 W

Iron, friction and windage losses = 2000 – (3^2 × 0.5) – (400 × 2) = 1195.5 W

$$\text{Efficiency at full-load} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$= \frac{\{400 \times 50 - (48)^2 \times 0.5 - 400 \times 2 - 1195.5\}}{400 \times 50} \times 100\%$$

$$= 84.2\%$$

$$\text{Induced emf } E = K\,\phi\,N$$

Let E_1 and E_2 be respectively the induced emf in the armature at no-load and full-load and N_1 and N_2 the speeds. Then

$$E_1 = K\,\phi\,N_1 \text{ and}$$

$$E_2 = K\,\phi\,N_2$$

Field flux ϕ is constant at no-load and at full-load.

$$E_1 = V - I_a R_a$$

$$= 400 - 3 \times 0.5 = 398.5 \text{ V}$$

$$E_2 = 400 - 48 \times 0.5 = 376 \text{ V}$$

Therefore, $$\frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{398.5}{376}$$

Percentage change in speed from no-load to full-load

$$= \frac{N_1 - N_2}{N_1} \times 100\% = \left[1 - \frac{N_2}{N_1}\right] \times 100\% = \left[1 - \frac{E_2}{E_1}\right] \times 100\%$$

Substituting values, percentage change $= \left[1 - \dfrac{376}{398.5}\right] \times 100\% = 5.64\%$

EXAMPLE 2.31 *A dc shunt machine (Fig. 2.67) has an armature resistance of 0.5 Ω and a field-circuit resistance of 750 Ω. When run as a motor on no-load at 500 V applied terminal voltage, the line current was 3 A. Estimate the efficiency of the machine when it operates as a generator with an output of 2 kW at 500 V, the field-circuit resistance remaining unchanged.*

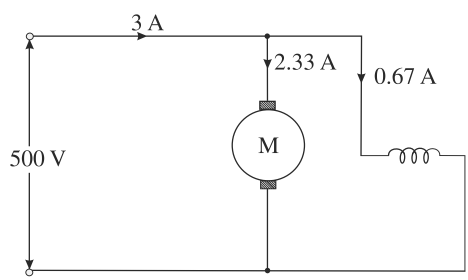


Fig. 2.67

Solution

$R_a = 0.5\ \Omega$; $R_f = 750\ \Omega$;

$$I_f = \frac{500}{750} = \frac{2}{3} \text{ A}$$

Iron, friction and windage loss

$$= 500 \times 3 - (2.33)^2 \times 0.5 - 500 \times 0.67 = 1163 \text{ W}$$

The output current of the generator under loaded condition

$$I = \frac{20 \times 1000}{500} = 40 \text{ A}$$

$$I_a = 40 + 0.67 = 40.67 \text{ A}$$

$$\text{Efficiency of the machine} = \frac{20 \times 1000 \times 100}{20 \times 1000 + (40.67)^2 \times 0.5 + 500 \times 0.67 + 1163}$$

$$= 89.6\%$$

EXAMPLE 2.32 *In Hopkinson's test on two similar dc machines (Fig. 2.68), the readings obtained are as follows: Line voltage = 100 V, motor current = 30 A, generator current = 25 A and armature resistance of each machine = 0.25 Ω. Calculate the approximate efficiency of each machine ignoring the field currents and assuming that their iron and mechanical-losses are the same.*

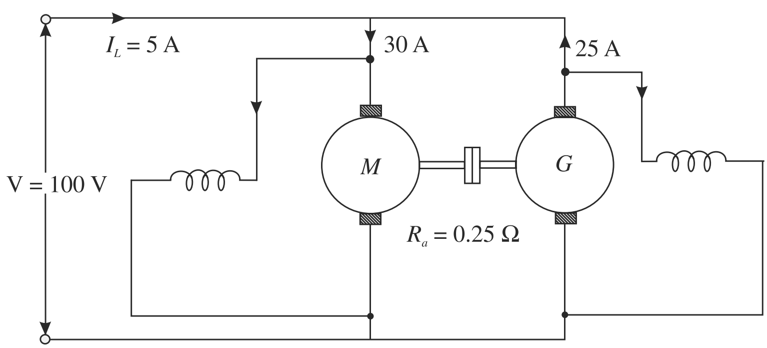


Fig. 2.68

Solution

Here $I_G = 25$ A

$$I_L = 30 - 25 = 5 \text{ A}$$

$$I_a^2 R_a \text{ loss in } G = (25)^2 \times 0.25 = 156 \text{ W}$$

$$I_a^2 R_a \text{ loss in } M = (30)^2 \times 0.25 = 225 \text{ W}$$

Total loss in G and M = 381 W (neglecting other losses)

Power supplied from mains = 100 × 5 = 500 W

∴ Iron, friction and windage loss in the two machines

$$= 500 - 381 = 119 \ \Omega$$

Iron, friction and windage-loss in each machine

$$= \frac{119}{2} = 59.5 \text{ W}$$

$$\text{Efficiency of the motor} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$= \frac{(30 \times 100 - 30^2 \times 0.25 - 9.5)100}{30 \times 100}\%$$

$$= 90.5\%$$

$$\text{Efficiency of the generator} = \frac{\text{Output}}{\text{Output} + \text{Losses}}\%$$

$$= \frac{100 \times 25 \times 100}{100 \times 25 + (25)^2 \times 0.25 + 59.5}\% = 92\%$$

EXAMPLE 2.33 *In a test on two similar 440 V, 220 kW generators, the circulating current is equal to the full-load current, and in addition, 90 A are taken from the supply. Calculate the approximate efficiency of each machine.*

Solution

$$\text{Rated current of each machine} = \frac{200 \times 1000}{440}\text{A}$$

$$= 454 \text{ A}$$

Assuming losses in the machines to be equal

$$\text{Approximate efficiency } \eta = \sqrt{\left[\frac{I_G}{I_G + I}\right]} = \sqrt{\frac{454}{454 + 90}} = 0.91 = 91\%$$

EXAMPLE 2.34 *Two similar dc machines, each rated at 500 V, 1000 kW, were tested by Hopkinson's method (Fig. 2.69). The test data obtained were as follows.*

Output current of generator = 2000 A
Input current from supply mains = 380 A
Shunt-field current of generator = 22 A
Shunt-field current of motor = 17 A
Resistance of the armature circuit of each machine = 0.01 Ω

Calculate the efficiency of the generator at full-load, assuming (a) equal efficiencies and (b) equal iron, friction and windage losses.

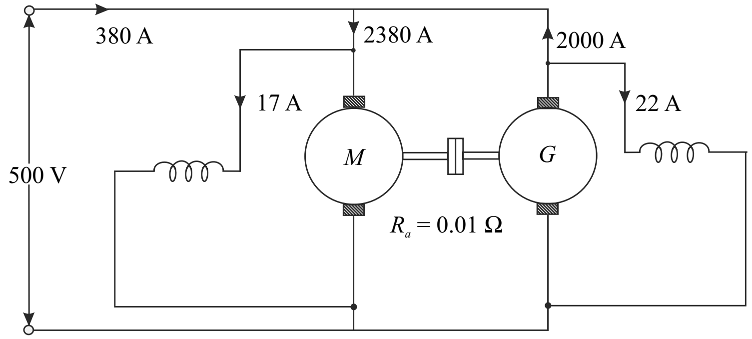


Fig. 2.69

Solution

(a) Efficiency of the generator assuming equal efficiencies of the two machines

$$\text{Efficiency} = \sqrt{\frac{I_G}{I_G + I}} \times 100\%$$

$$= \sqrt{\frac{2000}{2000 \times 380}} \times 100\% = 91.6\%$$

(b) Armature current of G = 2000 + 22 = 2022 A

Copper-loss in armature circuit of G = $(2022)^2 \times 0.01$ = 40.9 kW

Loss in the field circuit of G = 500 × 22 = 11 kW

Armature current in motor = 2380 – 17 = 2363 A

Loss in the armature circuit of motor = $(2363)^2 \times 0.01$ = 55.84 kW

Loss in the shunt field circuit of motor = 500 × 17 = 8.5 kW

Total input to M and G = 500 × 380 = 190 kW

Iron, friction and windage loss in both the machines

= 190 – (40.9 + 11 + 55.84 + 8.5)

= 73.76 kW

Iron, friction and windage loss in each machine

$$= \frac{73.76}{2} = 36.88 \text{ kW}$$

$$\text{Full-load output of the generator} = \frac{2000 \times 500}{1000} = 1000 \text{ kW}$$

$$\text{Efficiency of the generator at full-load} = \frac{1000 \times 100}{1000 + 40.9 + 11 + 36.88}\%$$

= 91.84%

2.14 ELECTRIC BRAKING OF DC MOTORS AND FOUR QUADRANT OPERATION

Braking of motors is required to quickly and accurately stop the motor driving a mechanical load. For example, suburban electrical trains driven by motors may require frequent and quick stops. The metro rails also require frequent starting and stopping. Again when a loaded train goes down a steep gradient, breaking is required to hold the speed of the train to a safe limit. In all such applications, electric braking system is employed.

Braking may be done using mechanical brakes also. Mechanical brakes have the disadvantage of wear and tear requiring regular maintenance and replacement of brake shoes. Also the energy of the moving system is wasted in the brakes as heat.

There are three methods of electric braking, namely

1. Dynamic or rheostatic braking
2. Plugging or reverse current braking
3. Regenerative braking

These methods are discussed in the following sections.

2.14.1 Dynamic Braking of dc Motors

In this method of braking, the armature terminals of the motor are disconnected from the supply terminals and immediately connected across a resistor (rheostat) while the field winding is kept energised.

The quick stop of the motor due to dynamic braking is explained below:

For Separately Excited dc Motors When the motor is running a counter emf is induced in it. This emf is also called back emf. When the armature is disconnected from the supply and is connected across a resistor the counter emf causes a reverse current to flow through the armature as shown in Fig. 2.70.

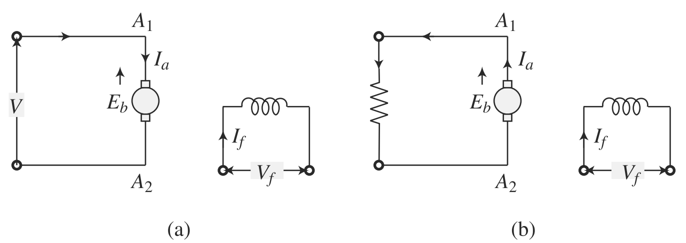


Fig. 2.70 *(a) Separately excited dc motors in running condition (b) For braking supply is disconnected and a resistance is connected across the armature terminals*

The reverse current flow through the armature, as shown in Fig. 2.70(b) develops a reverse torque and causes the motor to slow down. This reverse torque is developed due to the interaction of field flux and the reverse armature current.

For dc Shunt Motors When the motor is disconnected from the supply and is connected across a resistor the counter emf causes a current to flow through the resistor resulting in I^2R heat loss in the resistor. The mechanical energy of the rotating armature is dissipated, i.e., wasted as heat energy in the resistance. The rapidity with which this energy can be dissipated determines how quickly the motor can be stopped. If a low value of resistor is used the motor comes to stop quickly.

For dynamic braking of shunt motors the connections of the resistors is done across the motor terminals as shown in Fig. 2.71.

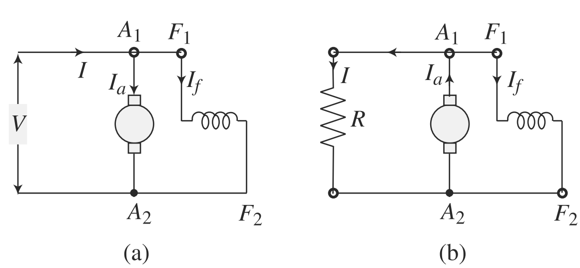


Fig. 2.71 *(a) A dc shunt motor in running condition (b) Dynamic braking applied to the dc shunt motor where a reverse armature current causes a reverse torque developed*

For dc Series Motors For dynamic braking of dc series motors, the motor is disconnected from the supply, the field connections are reversed and then the motor is connected across the resistor as shown in Fig. 2.72.

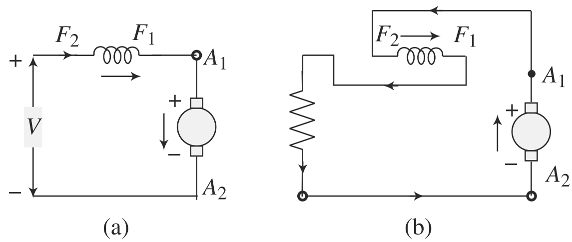


Fig. 2.72 *(a) a dc series motor (b) dynamic braking*

By reversing the field connections we make sure that current through the series field winding flows in the same direction, i.e., from F_2 to F_1 in running condition and also during braking operation as has been shown. If this is not done, the field current will destroy the field flux thereby not aiding braking operation. Once the field connections are reversed and the motor terminals connected across a resistor, the motor works as a self excited generator dissipating its energy in the resistor.

2.14.2 Plugging or Reverse Current Braking

Plugging or plug stopping of a dc shunt or a separately excited motor means bringing the motor to standstill quickly by creating a reverse torque on the armature. Plugging of a dc motor is done by reversing the supply to the armature while the motor is still running in the forward direction. When the supply connection to the armature is reversed the motor develops torque in the opposite direction creating a braking action. The counter torque makes the motor slow down quickly. When the motor reaches zero speed and tends to rotate in the reverse direction, supply to the motor is disconnected. A special device is used to cut off the supply exactly at the instant when the motor stops. The special device, called zero speed plugging switch, which opens at zero speed, is used.

During plugging, the impressed reverse voltage and the back emf of the armature adds up. A plugging resistance is required to be connected in the armature circuit to avoid excessive current flow.

Plugging is not only used to bring the motor to stop but also used in drives where quick reversal of the drive is required, such as, in cranes and in rolling mills. Fig. 2.73 shows the connections for plugging in dc shunt motors.

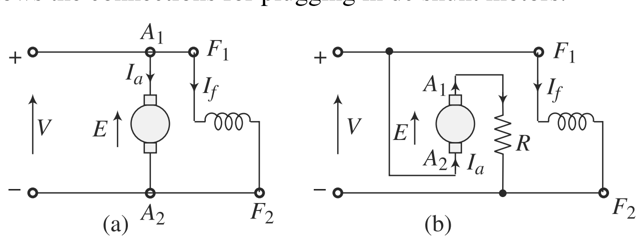


Fig. 2.73 *(a) dc shunt motor in normal running condition*
(b) dc shunt motor during plugging

The connections for a dc series motor during normal running condition and during plugging (braking) have been shown in Fig. 2.74. It is ensured that the direction of field current remains the same during running condition and during plugging operation.

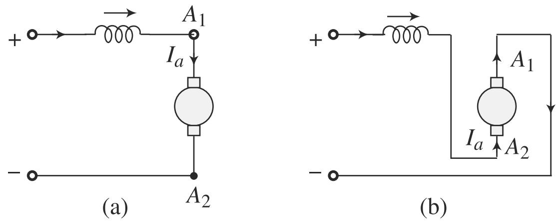


Fig. 2.74 *(a) A dc series motor in running operation (b) A dc series motor during plugging*

In the plugging condition, the back emf and the supply voltage becomes additive causing almost twice the supply voltage appearing across the armature circuit causing a heavy current flow. To limit this heavy current a resistor is connected in the circuit. In plugging period extra energy is drawn from the supply mains.

2.14.3 Regenerative Braking

In both rheostatic braking and plugging, the energy stored in the rotating system is wasted. In regenerative braking mechanical energy of the rotating system is converted into electrical energy and is returned to the supply source. Some examples of possibility of applying regenerative braking are mentioned below.

Suppose a dc motor driven car is moving down a load. The speed of the drive motors may become more than its no-load speed due to gravitational force. Under such a condition, the back emf will be more than the supply voltage and hence the dc motor will start working as a generator. The direction of current will reverse because $E_b > V$ and hence the machine will return energy to the supply mains. The direction of electromagnetic torque will reverse and act as a brake to the speeding motor. Regenerative braking is applicable in holding a descending load with high potential energy, e.g., a metro-train going down the slope; dc motors driving loads such as elevators, cranes, hoists, etc. In all these cases the motor speeds up, E_b becomes greater than V and hence the motor, instead of drawing current from the supply mains returns power to the mains.

The necessary condition for regenerative braking is that the induced emf also called the back emf must be higher than the supply voltage so that the direction of current flow in the motor gets reversed.

In case of dc shunt motors if the speed increases to greater than no-load speed, the back emf E_b becomes greater than the supply voltage V. The armature current becomes negative as is evident from the relation.

$$I_a = \frac{V - E_b}{R_a}$$

If E_b is greater than V, I_a becomes negative the motor now starts working as a generator. The direction of electromagnetic torque changes and works as a brake. Thus in dc shunt motors regenerative braking is possible.

However, in case of dc series motors there is problem with regenerative braking. The speed load characteristic of a dc series motor is redrawn for reference as in Fig. 2.75.

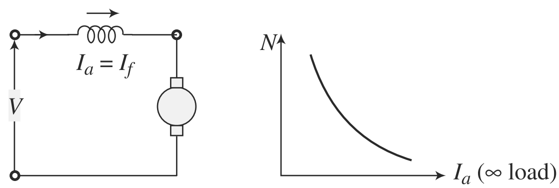


Fig. 2.75 *Speed-armature current characteristic of a dc series motor*

It is observed from the N versus I_a characteristics that if speed of the motor increases, the armature current decreases. Again the field current I_f is the same as I_a. So, if N increases I_f decreases and hence field flux ϕ also decreases. Back emf

$$E_b = \frac{\phi ZNP}{60 \text{ A}}$$

As N increases, ϕ decreases and therefore with increase of speed, E_b will not increase. As a consequence regeneration is not possible with a simple dc series motor. However, since dc series motors are used extensively as traction motors in electric trains, in tram cars, in elevators, etc., regenerative braking is desirable for dc series motors also. One of the methods used in achieving regenerative braking in dc series motors is to run the series motor during regeneration period as a dc shunt motor by reconnecting the field winding accordingly. Since the series winding has low resistance, an extra resistance of high value should be connected in series with the series field winding before connecting the series winding as a shunt winding, i.e., in parallel with the armature.

2.14.4 Four-Quadrant Operation of dc Drives

A motor may run in either direction, i.e., in forward direction and in reverse direction. In forward motoring both speed and torque are positive. In reverse motoring both speed and torque are negative as shown as 1st and 3rd quadrant operation in Fig. 2.76. The 2nd quadrant represents operation of the motor as a brake because direction of speed remaining positive, torque becomes negative. Negative torque means braking operation. The 4^{th} quadrant represents operation of the motor in the reverse direction. Since torque is positive and direction of rotation is negative. This is called reverse braking.

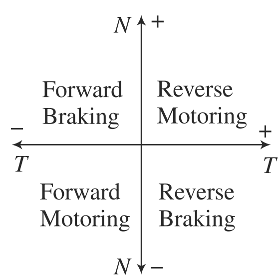


Fig. 2.76 *Four quadrant operation of a dc motor*

2.15 PERMANENT MAGNET dc MOTORS

In permanent magnet dc (PMDC) motors we use strong permanent magnets as field poles. No excitation current is required. The field magnets are made of materials having high residual magnetism and high coercive force. The *B-H* characteristics of materials required for making strong field magnets is shown in Fig. 2.77.

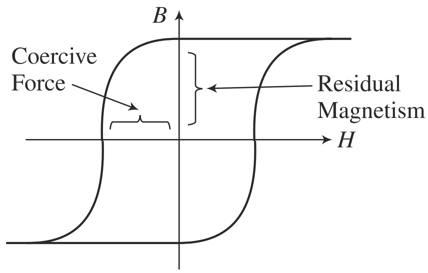


Fig. 2.77 *Typical B–H loop of material for permanent magnets*

Alnico (an alloy of Aluminum, Nickel, and Cobalt) and rare earth magnetic materials are suitable for making strong magnets. When field magnets are made of permanent magnets, no field winding and no field current is required.

Figure 2.78 shows the constructional details of a PMDC motor. Permanent magnets in the form of poles are mounted on the inner periphery of a hollow cylindrical stator frame made of steel. The rotor construction is similar to the armature of a typical dc motor having rotor core, rotor slots, armature winding, and the commutator and brush arrangement.

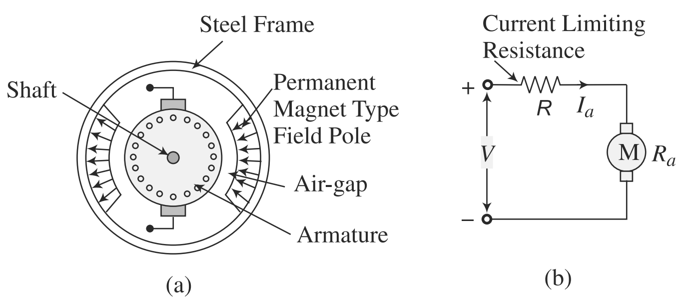


Fig. 2.78 *(a) Cross-sectional view of a permanent magnet dc motor (b) Circuit model*

The field magnets are shaped such that flux lines emanate radially as has been shown through arrows. The supply voltage is applied across the armature only. A low voltage dc from a battery source is normally applied. As shown in figure, a voltage of V volts has been applied to the armature terminals through a current limiting resistance. The back emf E_b is expressed as

$$E_b = \frac{\phi ZNP}{60\ A} = \text{kN (as } \phi \text{ is also constant)} \tag{2.11}$$

where $K = \dfrac{\phi ZP}{60\ A}$

The voltage equation for a dc motor is

$$E_b = V - I_a R_a \tag{2.12}$$

From Eqs. (2.11) and (2.12),

$$E_b = kN = V - I_a R_a$$

or

$$N = \frac{V - I_a R_a}{K} \tag{2.13}$$

The equation of torque is given as

$$T = K_t \, \phi \, I_a$$

or

$$T = K_1 \, I_a \tag{2.14}$$

where $K_1 = k_t \, \phi$

From Eqs. (2.13) and (2.14) it is seen that both speed N and torque T of a PMDC motor can be controlled by varying the armature voltage. Since voltage V can be reduced by using either a rheostat or through chopper control, in this method the speed can only be reduced from its normal value. The motor cannot be run at higher speeds than its rated speed.

The characteristics of PMDC motors are shown in Fig. 2.79. The following equations have been used in drawing the characteristics.

$$N = \frac{V - I_a(R_a + R)}{K} \text{ and } T = K_t \, I_a$$

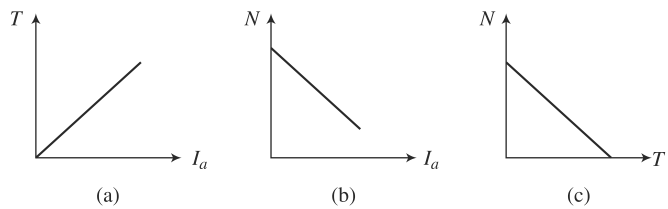


Fig. 2.79 *(a) Torque-load characteristics (b) Speed-load characteristics (c) Speed-torque characteristics*

As can been observed from Fig. 2.79 (c), the torque produced at starting, i.e., at $N = 0$, is quite high in PMDC motors.

Some of the application areas of permanent magnet dc motors are mentioned below.

Small PMDC motors are used in automobiles as drives for wind shield wipers, lowering and raising of window glasses; in computer drives, in battery operated toys, in portable electric tools, etc.

2.16 APPLICATIONS OF dc MACHINES

Applications of electrical machines depends on factors like cost, nature of power supply required, maintenance requirement, ease of speed control, etc. Direct current generators as a source of power supply have been replaced by ac generators because of many reasons. Direct current motors, however, are still preferred in many applications because their speed can be controlled smoothly.

2.16.1 dc Generators

The use of dc machines as generators of electricity for commercial use is very limited since the bulk of the energy is generated in the form of alternating current. This is because generation and transmission of electricity as alternating current have many advantages. The use of dc generators is confined to supplying excitation of medium-capacity ac generators or to converting ac to dc (ac motor driving a dc generator) for industrial applications.

Direct current is widely used for electrolytic-processes, for variable-speed motor drives and to some extent in welding processes. In these cases, the use of dc generators is gradually being replaced by the use of static rectifiers. Semi-conductor rectifiers are slowly replacing dc generators as a source of supply for even steel mill applications. It may be mentioned here that the quality of dc obtained from a dc generator is superior to that obtained by using rectifiers in terms of freedom from ripple in the wave shape. A dc generator with a rotating commutator requires constant maintenance and replacement of parts which are responsible for its limited use.

2.16.2 dc Motors

In applications like variable-speed devices and where severe torque variations occur, dc motors are widely used. In recent years variable frequency ac drives are slowly replacing dc drives for some applications but for many applications dc motors are still more suitable. The dc motors are often provided with a semiconductor rectifier as the supply voltage available is ac. The characteristics of various types of motors suggest their field of applications. Applications of shunt, series and compound motors are discussed separately below.

dc Shunt Motors (i) Shunt motors are used in situations, such as driving a line shafting, etc., where the speed has to be maintained approximately constant between no-load and full-load.

(ii) In situations where a variable load is to be driven at different speed but at each load the speed is to be kept constant, such as driving a lathe. The speed change is obtained by the use of a shunt-field regulator.

dc Series Motors Series motors are used in applications such as driving hoists, cranes, trains, etc., as in these cases a large starting torque is required. They are also used where the motor can be permanently coupled to the load, such as a fan, whose torque increases with speed.

Where constancy of speed is not essential, the decrease of speed with increase of load has the advantage that the power absorbed by the motor does not increase as rapidly as the torque.

Series motors acquire very high speed at no-load or at very light load. That is why they should not be used for a belt drive where there is a possibility of the load decreasing to a very small value.

dc Compound Motors Direct current compound motors are used in applications where large starting torques are required but where the load may fall to such a small value that a series motor would reach a dangerously high speed. Where the supply voltage may fluctuate, for instance on a traction system, the series winding reduces the fluctuation of armature current, partly by its inductance and partly by its influence on the value of the flux and therefore on the induced emf.

When the load is of fluctuating nature, e.g., for driving stamping processes, etc., the shunt excitation prevents the speed becoming excessive on light load, and the decrease of speed with increase of load enables the flywheel, usually fitted to such a machine, to assist the motor in dealing with the peak load by giving up some of its kinetic energy.

2.17 MAINTENANCE OF dc MACHINES

While installing a dc machine care should be taken to see that it is not inconvenient to inspect, maintain, repair or replace the machine. Proper maintenance improves the performance and life of a machine. In a dc machine the commutator and brushes should be regularly checked for arcing and mechanical wear. Finely powdered carbon particles appearing on the commutator surface should be regularly cleaned. Otherwise, these dust particles mixed up with the shaft-bearing oil or grease stick to the commutator surface. This greased layer of carbon dust is conductive and may lead to flashover, thereby damaging the commutator surface.

Bearings need periodic checking. Periodic greasing will increase the life of a bearing and also give trouble-free service. Failure of bearings may cause failure of motor windings. The bearing life is highly affected due to non-alignment during installation or by too light belt tension.

Accumulation of dust on the machine reduces the heat dissipation and causes overheating of the machine. Over-lubrication of machine bearings should be avoided as lubricating oil or grease mixed up with dust and dirt depositing in various parts of the machine over a long period may ultimately cause electrical failure of the machine.

A machine, before failing due to improper maintenance, gives indications or shows some symptoms, such as abnormal sounds, vibrations, excess surface temperature, etc. The operator should be aware of these abnormalities and take immediate preventive measures. If such abnormalities are ignored these may ultimately result in failure of the machine. 'Prevention is better than cure' is true for human beings as also for machines.

MODEL QUESTIONS

Short-Answer-Type Questions

2.1 Draw a neat sketch of a dc machine and label the component parts. Name the material used for each component part.

2.2 Explain the function of a commutator in a dc machine for motoring and generating action.

2.3 With the help of simple examples explain the difference between lap-winding and wave-winding.

2.4 Draw a developed winding diagram for a dc armature having 24 slots with two coil-sides per slot. The winding is for six poles.

2.5 Draw a developed winding diagram in wave connection for a four-pole dc armature with 30 coil-sides housed in 15 slots.

2.6 Explain the meaning of the following terms used in designing dc armature windings front pitch, back pitch, commutator pitch, pole pitch and average pitch.

2.7 Explain why equaliser connections are used in lap-winding and dummy coils are sometimes used in wave-windings.

2.8 Deduce the equation for the emf induced in a dc machine.

2.9 Explain why the armature winding is made distributed on the whole armature surface and not kept concentrated in only two slots.

2.10 State the factors on which electromagnetic torque developed in a dc machine depends. Deduce the expression for torque developed in a dc motor.

2.11 Deduce the equation for speed of a dc motor and hence suggest various methods of speed control.

2.12 Draw and explain torque-speed characteristics for the following types of dc motors: (i) shunt motor (ii) series motor and (iii) compound motor.

2.13 Draw the characteristics of a dc series motor and from the nature of the curve explain the applications of dc series motors.

2.14 Explain why a dc motor should not be started direct-on-line.

2.15 Draw and explain a three-point started for a dc motor.

2.16 Explain the various methods of speed control of dc motors. Mention the limitations of each method.

2.17 Explain the Ward-Leonard method of speed control of a dc motor.

2.18 Compare the various methods of speed control of dc motors.

2.19 Explain armature reaction in a dc machine.

2.20 Explain how the effect of armature reaction can be neutralised by using commutating poles and compensating windings.

2.21 Explain clearly how commutation takes place in a dc machine.

2.22 State and explain the various losses which take place in a dc machine.

2.23 Explain hysteresis loss and eddy-current loss that occur in a dc machine. State the factors on which these losses depend.

2.24 Explain how efficiency of a dc machine can be found out without actually loading the machine.

2.25 Explain the regenerative method of determination of efficiency of a dc machine.

2.26(a) Describe in brief the three methods of braking of dc motors.

2.26(b) Explain the principle of working and characteristics of permanent magnet dc motors.

2.27(a) State with reasons applications of various types of dc motors.

2.27(b) Describe how a dc machine is to be maintained for a long satisfactory performance.

Multiple-Choice Questions

2.28 The armature of a dc machine is made up of laminated sheets to
(a) reduce hysteresis loss
(b) reduce eddy-current loss

(c) reduce armature copper-loss
(d) increase dissipation of heat from the armature surface.

2.29 In a dc machine, interpoles are used to
(a) neutralise the effect of armature reaction in the interpolar region
(b) generate more induced emf in the armature
(c) avoid interference of the armature flux with the main-field flux
(d) reduce the demagnetising effect of armature reaction.

2.30 The function of a brush and commutator arrangement in a dc motor is
(a) to produce unidirectional torque
(b) to produce undirectional current in the armature
(c) to help in changing the direction of rotation of the armature
(d) to reduce sparking.

2.31 The emf induced in a coil rotating in a magnetic field is maximum when
(a) the rate of change of flux linkage by the coil is maximum
(b) the rate of change of flux linkage by the coil is minimum
(c) the rate of change of cutting of flux by the coil-sides is minimum
(d) the flux linkage by the coil is maximum.

2.32 The armature winding of a dc machine is made up of a number of coils distributed in a large number of armature slots instead of placing all the coils in two slots to
(a) get sinusoidal emf at the output terminals
(b) have minimum heat dissipation from the armature
(c) make the armature dynamically balanced
(d) get maximum generated emf in the armature.

2.33 Aluminium is not used as winding wire for the armature of a dc machine because
(a) aluminium has low resistivity
(b) a large winding space is taken by aluminium conductors and creates jointing problems
(c) the thermal conductivity of aluminium is low
(d) aluminium has low conductivity as compared to copper.

2.34 The maximum permissible operating temperature of class-E insulating material is
(a) 130 °C (b) 180 °C (c) 120 °C (d) 105 °C

2.35 The commutator segments of a dc machine are made up of
(a) stainless steel (b) hard drawn copper
(c) brass (d) bronze

2.36 The number of parallel paths in the armature winding of a four-pole wave connected dc machine having 22 coil-sides is
(a) 4 (b) 2 (c) 22 (d) 1

2.37 The induced emf in the armature of a lap-wound four-pole dc machine having 100 armature conductors rotating at 600 rpm and with 1 Wb flux per pole is
(a) 1000 V (b) 100 V (c) 600 V (d) 10,000 V

2.38 The direction of rotation of a dc motor can be reversed
(a) by reversing the connections of both armature and the field windings with the supply

(b) by reversing the connections of either the armature or the field-winding connection with the supply
(c) by reducing the field flux
(d) by introducing an extra resistance in the armature circuit.

2.39 A dc series motor should always be started with load because
(a) at no-load it will rotate at a dangerously high speed
(b) at no-load it will not develop high starting torque
(c) it cannot start without load
(d) it draws a small amount of current at no-load.

2.40 To have sparkless commutation the armature reaction effect in a dc machine is neutralised by
(a) using compensating windings and commutating poles
(b) shifting the brush axis from the geometrical neutral axis to the magnetic neutral axis
(c) fixing the brush axis in alignment with the main pole axis
(d) increasing the field excitation.

2.41 Swinberne's method cannot be used for determining the efficiency of a dc series motor because
(a) it is not advisable to run a series motor on no-load
(b) a series motor takes excessive current at no-load
(c) a series motor develops very high starting torque
(d) it is not possible to load a series motor in steps.

Numerical Problems

2.42 A four-pole wave-wound dc armature has 50 slots with 10 conductors per slot and is rotated at 1000 rpm. If the useful flux per pole is 30 m Wb, calculate the value of the generated emf.

2.43 A six-pole armature is wound with 600 conductors. The magnetic flux and the speed are such that the average emf generated in each conductor is 2.3 V. Calculate the terminal voltage on no-load when the armature is (a) lap-connected and (b) wave-connected.

2.44 An eight-pole lap-connected armature has 96 slots with six conductors per slot and is driven at 600 rpm. The useful flux per pole is 0.1 Wb. Calculate the generated emf.

2.45 A four-pole armature has 624 conductors and is lap-connected. The armature is driven at 1500 rpm, calculate the useful flux per pole required to generate on emf of 220 V.

2.46 A six-pole armature has 410 wave-connected conductors. The useful flux per pole is 0.02 Wb. Calculate the speed at which the armature must be driven for the generated emf to be 400 V.

2.47 The emf generated by a four-pole dc generator is 400 V when the armature is driven at 1200 rpm. Calculate the flux per pole if the wave-wound armature has 39 slots having 16 conductors per slot.

2.48 A 50 kW, 250 V dc shunt generator has a field-circuit resistance of 60 Ω and an armature resistance of 0.02 Ω. Calculate: (a) the load current, field current, and armature current, and (b) the generated armature voltage, when delivering rated current at rated speed and voltage.

2.49 A separately excited dc generator has a no-load voltage of 120 V at a field current of 2 A when driven at 1500 rpm. Assuming that it is operating on the straight-line portion of its saturation curve, calculate: (a) the generated voltage when the field current is increased to 2.5 A, and (b) the generated voltage when the speed is reduced to 1400 rpm and the field current is increased to 2.8 A.

2.50 A 250 V dc shunt generator has a voltage regulation of 8%. Calculate the voltage at no-load.

2.51 A 200 V dc shunt motor having an armature circuit resistance of 0.2 Ω and a field-circuit resistance of 100 Ω draws a line current of 50 A at full-load at a speed of 1500 rpm. Calculate its speed at half-load.

2.52 A 240 V dc shunt motor has an armature resistance of 0.2 Ω. The rated full-load current is 80 A. Calculate the current at the instant of starting and express it in terms of the full-load current. Calculate the value of the starting resistance to limit the current in the motor at 150% of the rated load at the instant of starting.

2.53 The input to a 220 V dc shunt motor is 11 kW. Calculate (a) the torque developed, (b) the efficiency, and (c) the speed at this load when the following particulars of the motor are given: no-load current = 5 A; no-load speed = 1150 rpm; armature resistance = 0.5 Ω; shunt-field resistance = 110 Ω.

2.54 A shunt motor has an armature resistance of 0.25 Ω. The motor takes 125 A from a 400 V supply and runs at 1000 rpm. If the total torque developed remains unchanged, calculate the speed and armature current if the magnetic field is reduced to 80% of the initial value.

2.25 A 500 V dc series motor has a resistance of 0.25 ohm and runs at 500 rpm when taking a current of 50 A. Calculate the speed when taking a current of 30 A if the flux is reduced to 60% of that at 50 A.

2.56 A 220 V shunt motor has an armature resistance of 0.5 Ω and takes an armature current of 40 A on full-load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant?

2.57 In a brake test the effective load on the pulley was 36.5 kg, the diameter of the pulley is 66 cm and the speed 900 rpm. The motor took 60 A at 220 V. Calculate the brake horsepower and the efficiency at this load.

2.58 A 500 V shunt motor takes a current of 5 A on no-load. The resistances of the armature and field circuits are 0.5 Ω and 250 Ω respectively. Calculate the efficiency when the motor takes a current of 100 A.

Answers

The answers to Multiple-choice Questions and Numerical Problems are given below.

2.28 (b)	2.29 (a)	2.30 (a)	2.31 (a)
2.32 (b)	2.33 (b)	2.34 (c)	2.35 (b)
2.36 (b)	2.37 (a)	2.38 (b)	2.39 (a)
2.40 (a)	2.41 (a)		

2.42	500 V	2.43	230 V, 690 V
2.44	576 V	2.45	14.1 mWb
2.46	975 rpm	2.47	16 mWb
2.48	200 A, 4.16 A, 204. 16 A, 254.08 V		
2.49	150 V, 156.8 V	2.50	270 V
2.51	1539 rpm	2.52	15 times, 1.8 Ω
2.53	87 Nm, 79.5%, 1030 rpm	2.54	1224 rpm, 156.25 A
2.55	842 rpm	2.56	37.3%
2.57	14.93 hp, 84.36%	2.58	85.4%

LABORATORY EXPERIMENTS

EXPERIMENT 2.1 *Determination of the brush positions of a dc generator for maximum induced emf.*

Objective To observe that for a dc generator there is only one particular position for which the induced emf across the brushes is maximum.

Brief Theory Let us consider a two-pole dc generator having eight coils (16 coil-sides) on the armature. When the armature is rotated in the magnetic field, emf is induced in the armature conductors whose directions can be found out by applying Fleming's right hand rule. Figure 2.80(a) shows a two-pole dc generator having eight coils on its armature. The armature is rotated in the clockwise direction. Figure 2.80(b) shows the exploded view of the armature where the direction of induced emf in each coil-side is shown by arrows.

Let the induced emf is the coils 1, 2, 3, 4 etc. be e_1, e_2, e_3, e_4, etc., respectively. As the coils are identical, $e_1 = e_2 = e_3 = e_4$ (say). Between brushes P and Q the group of coils 1, 2, 3, and 4 are connected in series and the group of coils 5, 6, 7 and 8 are also connected in series, the two groups being in parallel. The induced emf in coils 1, 2, 3 and 4 have the same direction for the brush position shown in Fig. 2.80(a). Similarly the induced emf in coils 5, 6, 7 and 8 have the same direction. Therefore, between brushes P and Q there are two parallel circuits, each circuit having four coils in series. This is represented in Fig. 2.81.

As the induced emf in coils 1, 2, 3 and 4 connected in series between brushes P and Q are in the same direction, voltage E available across the brushes P and Q is

$$E = e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8 = 4e$$

$$(\text{as } e_1 = e_2 = e_3 = e_4, \text{ etc.} = e)$$

Thus the maximum value of the induced emf which is possible to obtain for this type of connection of armature coils is 4e. This magnitude of the voltage can be available across the brushes if the brushes are placed in the position shown in Fig. 2.82(a) and (b) i.e., when the brush axis makes 90° with the main-pole axis.

Note: The brushes in Fig. 2.80(a) are shown in the inter-polar region. This representation means that the brushes are short-circuiting the coils passing through the inter-polar region. As a matter of fact the brushes are physically placed along the axis of the main poles as shown in Fig. 2.80(b) but they short circuit the coils that are 90° displaced from this axis.

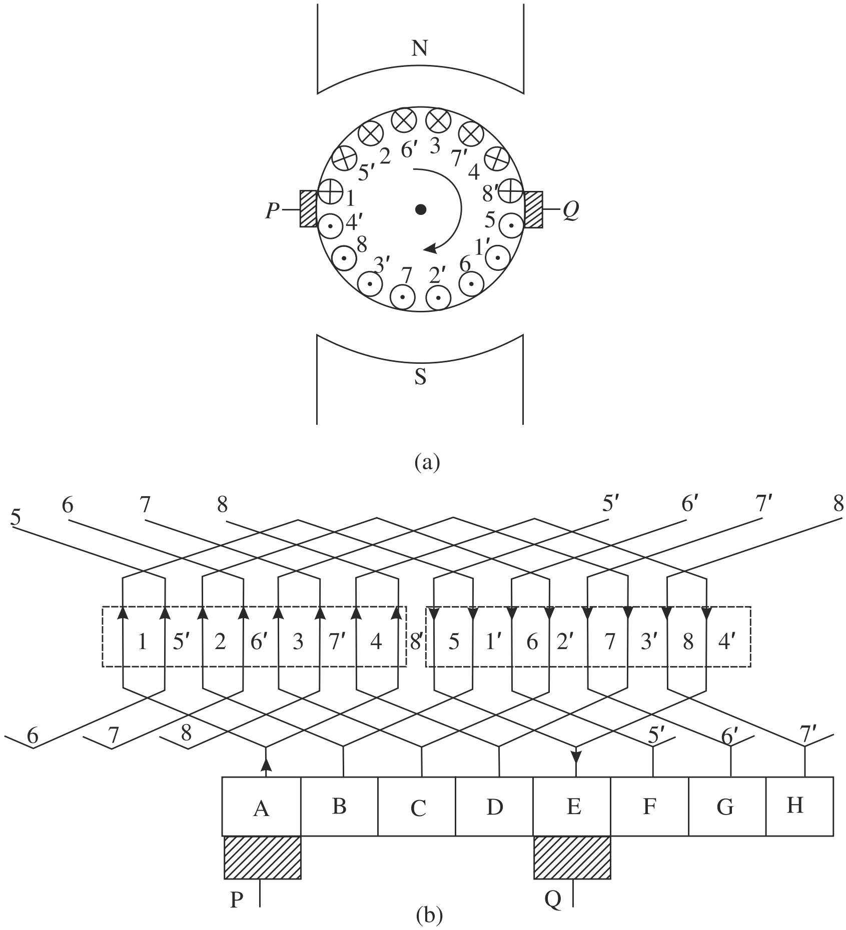


Fig. 2.80 *(a) Shows a 2-pole dc generator having 8 coils on its armature and brushes P and Q on the neutral axis (b) Shows the exploded view of the armature of figure (a)*

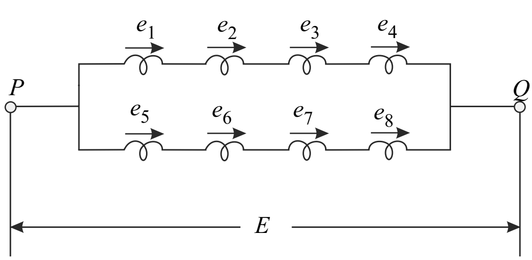


Fig. 2.81 *Shows two parallel circuits between brushes P and Q with the direction of induced emf in each coil for the brush positions shown in Fig. 2.80(a)*

Now observe the magnitude of the voltage available across the brushes *P* and *Q* when they are shifted to the new position as shown in Fig. 2.82(a). Across brushes *P* and *Q* are connected the group of coils 2, 3, 4 and 5 in series and also the group of coils 6, 7, 8 and 1 in series, the two groups are in parallel. The directions of the emf are as shown in Fig. 2.83. From Fig. 2.83 it is seen that the resultant emf E available across the brushes is

$$\begin{aligned} E &= e_2 + e_3 + e_4 - e_5 = e_6 + e_7 + e_8 - e_1 \\ &= 2e \text{ (as } e_2 = e_3 = e_4, \text{ etc.} = e) \end{aligned}$$

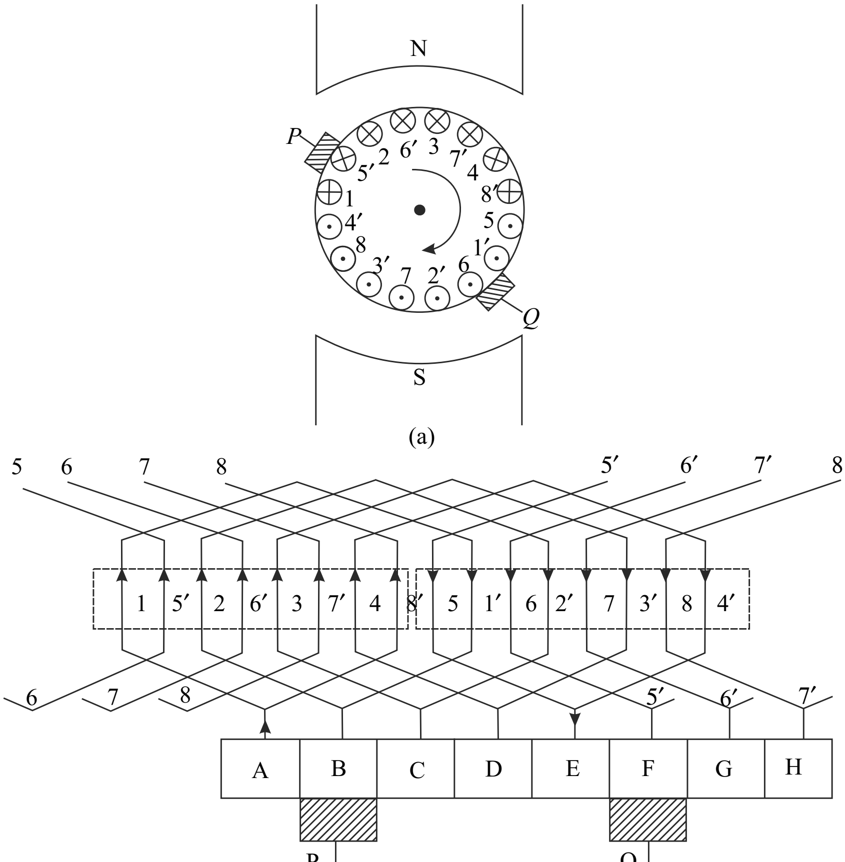


Fig. 2.82 *(a) Shows a 2-pole dc generator having 8 coils on its armature and brushes P and Q shifted from the neutral axis (b) Shows the exploded view of the armature of Fig. 2.82(a)*

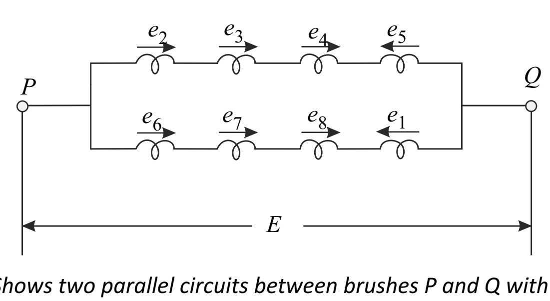
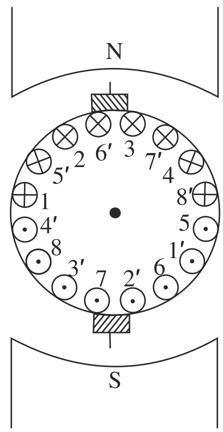
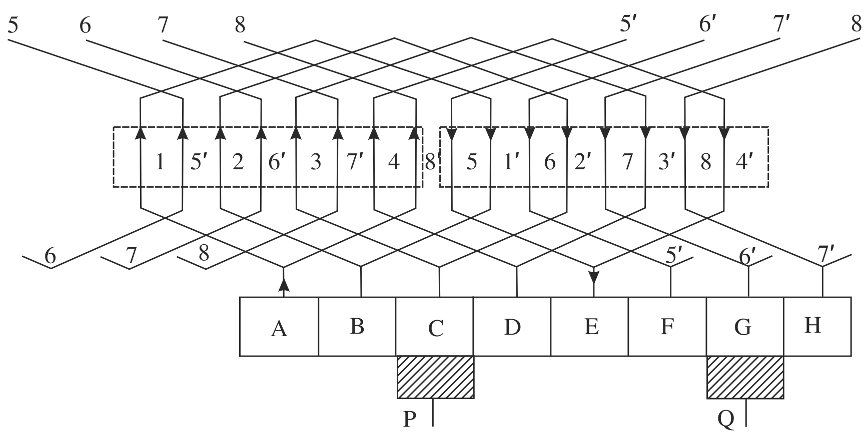


Fig. 2.83 *Shows two parallel circuits between brushes P and Q with the direction of induced emf in each coil for the brush position shown in Fig. 2.82 (a)*

Thus, it is seen that the voltage available across the brushes is reduced for this new position of the brushes. Now shift the brush position further and see how the magnitude of induced emf across the brushes change (see Fig. 2.84). In this new position of brushes, the connection of coils between the brushes is shown in Fig. 2.85.



(a)



(b)

Fig. 2.84 *(a) Shows the two-pole dc generator of Fig. 2.80, with its brushes shifted by 90 degrees from the neutral axis (b) shows the exploded view of the armature of Fig. 2.84(a)*

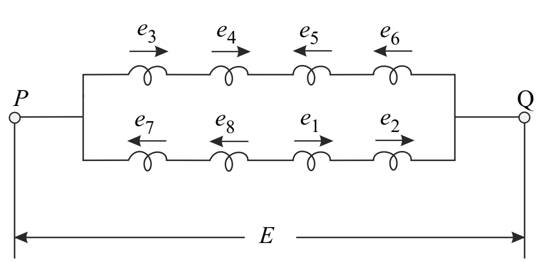


Fig. 2.85 *Shows two parallel circuits between brushes P and Q with the direction of induced emf in each coil for the brush position shown in Fig. 2.84*

The voltage available across the brushes:

$$E = e_3 + e_4 - e_5 - e_6 = e_1 + e_2 - e_8 = 0$$

This is the position of zero induced emf across the brushes. Hence, when the brush axis coincides with the main field axis, the voltage available across the brushes is zero.

Apparatus Required The dc motor-generator set, voltmeter, starting rheostat/starter insulated plier/hand gloves (insulated).

Procedure

1. Make connections as per the circuit diagram (Fig. 2.86).

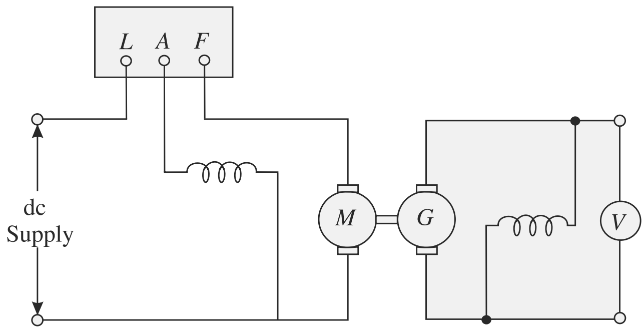


Fig. 2.86 *Shows the connection diagram for determination of brush position of a dc generator for maximum induced emf*

2. Loosen the brush-ring of the dc generator.
3. Start the dc motor with the help of the starter or starting rheostat.
4. With the help of an insulated plier or hand gloves (insulated) rotate the brush-ring in one direction slowly and observe the magnitude of the induced emf.
5. Mark the position of brushes for maximum induced emf in the generator (chalk marking can be made on the generator frame).
6. Rotate the brush-ring further and mark the position of the zero induced emf.
7. Rotate the brush-ring in the opposite direction and observe the position of brushes for maximum and zero induced emf.

Observations and Results Shown diagrammatically, the position of the brushes for maximum and zero induced emf in the dc generator.

Questions Answer the following questions in your report:

1. At what position should the brushes be fixed to get maximum induced emf in a dc machine?
2. At what position of the brushes is the induced emf in a dc machine minimum?

EXPERIMENT 2.2 *Measurement of the induced emf of a separately excited dc machine as a function of the field current.*

Objective To plot the complete magnetisation curve.

Brief Theory To develop the equation for induced emf in a dc generator, consider a two-pole dc generator having only one conductor on its armature as shown in Fig. 2.87. When the conductor is rotating in the magnetic field produced by the two poles, the flux cut by the conductor in one revolution is 2ϕ Wb. If ϕ is the flux per pole. For P number of poles, the flux cut per revolution is $P\ \phi$ Wb. If the rotor is rotating at n rps (revolutions per second), the flux cut per second will be $P\ \phi_n$ Wbs. It is already known that the induced emf in a conductor is given by the flux cut per second ($e = d\phi/dt$). Therefore, the induced emf $e = P\ \phi_n$ V. If the speed of the rotor is expressed in rpm (N) then

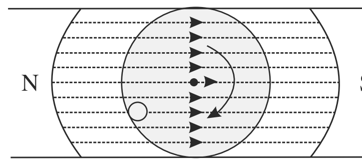


Fig. 2.87 *Shows a simple two-pole dc generator with only one conductor on its armature*

$$E = \frac{\phi ZNP}{60} \text{ Volts}$$

i.e., induced emf E in a dc generator can be expressed as

$$E = K\ \phi\ N \text{ Volts}$$

The field flux ϕ in a dc generator is proportional to the field current I_f. Thus, the above equation can be rewritten as

$$E = K_1\ I_f N$$

By increasing or decreasing the magnitude of current in the field circuit, E can be increased or decreased.

In this experiment the change in induced emf E in a dc generator will be recorded with change in the field current I_f, keeping N, the speed of rotation of the rotor, constant.

Circuit Diagram See Fig. 2.88.

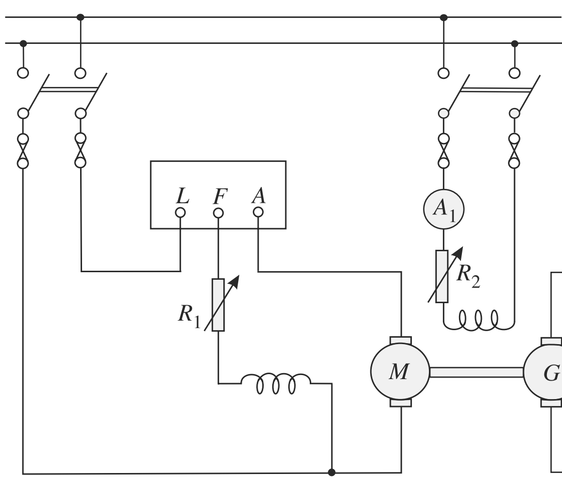


Fig. 2.88 *Shows the connection diagram for measurement of induced emf of a separately excited dc generator at different field currents*

Apparatus Required DC motor-generator set, starter for the dc motor, rheostats, ammeter, voltmeter.

Procedure

1. Make connections as per the circuit diagram (Note: the primemover connection will depend upon the type of motor-generator set available in the laboratory).
2. Run the generator at its rated speed with the help of the motor. Rheostat R_1 may be adjusted for bringing the speed to the rated value.
3. Note down the voltmeter reading. The generator field circuit is open, but the voltmeter will read a few volts.
4. With full resistant R_2 in the generator field circuit, close the switch. (The additional field circuit resistance R_2 should have a high value, may be of the order of a hundred ohms.) Note the values of the field current and the corresponding values of the induced emf from ammeter A_1 and voltmeter V respectively.
5. Now increase the value of generator field current I_f by reducing resistance R_2 in the field circuit in steps and record at least 10 values of the field current and induced emf. Go up to 120% of the rated voltage of the generator.
6. Now reduce the field current with the help of rheostat R_2 in steps and record the induced emf for decreasing values of the field current. (While taking readings for increasing values of the field current, it should be continuously increased. At no time should it be momentarily decreased and then increased. Similarly when taking readings for decreasing values of the field current, it should be continuously decreased and at no time should it be momentarily increased. Otherwise, minor hysteresis loops will be introduced and the curve will not be a smooth one.)
7. When the field current is zero, reverse the supply to the field and increase the field current in steps in the reverse direction. Record the generated emf up to 120% of its rated value and also the corresponding values of the field current.
8. Again decrease the field current from its negative maximum value to zero in steps and record the magnitude of the corresponding induced emf. Note that all the above readings should be taken at constant speed.

Observations and Results Tabulate your data as follows:

Speed of the motor-generator set = (Keep constant)

S. No.	Increasing from zero to positive		Decreasing from positive maximum to zero		Increasing from zero to negative maximum		Decreasing from negative maximum to zero	
	Field current (A)	Induced emf (V)	Field current (A)	Induced emf (V)	Field current (A)	Induced emf (V)	Field current (A)	Induced emf (V)
Take 9-10 readings								

Plot the magnetisation characteristic for a dc generator. Remember that the independent variable should be taken along the abscissa and the dependent variable along the ordinate.

Questions Answer the following questions in your report:

1. The curve for decreasing values of field current lies above the curve for increasing values of field current. Explain why.
2. Describe the type of curve you obtain. Explain its shape
3. If the above experiment is repeated for (a) higher and (b) lower values of speed, in what way will the new graph plotted differ from the one you have plotted for your experiment?

EXPERIMENT 2.3 *Measurement of the terminal voltage of a separately-excited dc generator as a function of the load current.*

Objective To determine the external or load characteristic of a separately excited dc generator by actually loading the machine.

Brief Theory For a generator, the induced emf is equal to the terminal voltage when the generator is not loaded. When, however, the generator is loaded, there is a fall in the terminal voltage. The fall (or the drop) in the terminal voltage of a separately excited dc generator is due to:

(a) $I_a R_a$ drop in the armature winding,
(b) the effect of armature reaction, and
(c) brush contact resistance.

When the current flows through the armature winding it sets up an armature flux which opposes the main-field flux. The main-field flux thus available for inducing emf in the armature winding is reduced. Hence the induced emf is decreased thereby causing a reduction in terminal voltage. As the load on the generator is increased, these drops also increase as they are proportional to the load current.

Circuit Diagram See Fig. 2.89.

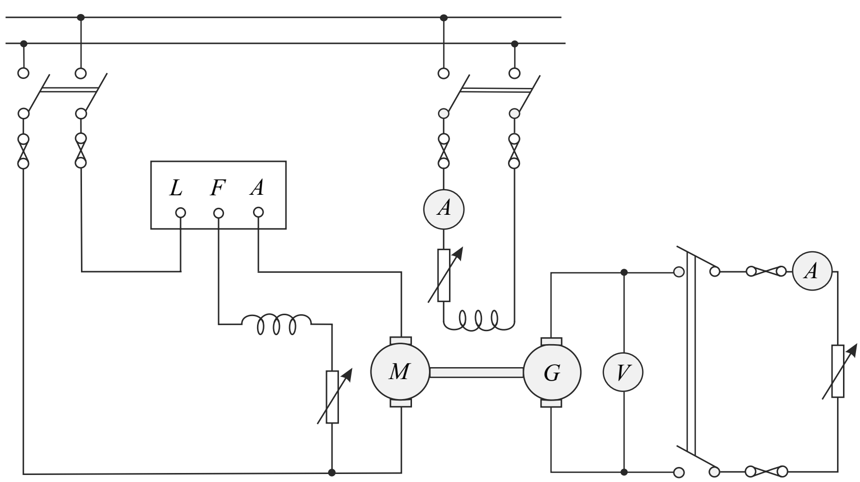


Fig. 2.89 *Shows the connection diagram for measurement of terminal voltage of a separately excited dc generator as a function of load current*

Apparatus Required Ammeters (2), voltmeter, MG set, field regulating rheostats (2), dc motor starter, loading resistance, tachometer.

Procedure

1. Make connections as per your circuit diagram (see Fig. 2.89). (The connections of the primemover will depend upon the type of motor-generator set available in your laboratory. In the given circuit diagram, the primemover is a dc motor).
2. Start the dc motor with the help of a starter and bring the speed of the set-up to the rated value by adjusting the field-regulating rheostat.
3. Give supply to the field winding of the dc generator from a dc source. Adjust the magnitude of current through the field-winding of the dc generator with the help of the field regulating rheostat to obtain the rated value of the induced emf on the generator terminals.
4. Load the dc generator step-by-step with the help of lamps or loading resistances. For each step keep the speed and field current of the dc generator constant. Take at least six readings.
5. Measure the armature resistance of the dc generator by the ammeter-voltmeter method.

Observations and Results

Armature resistance R_a =

Tabulate your observations as per the table shown below.

S. No.	*Speed in rpm*	*Generator field current (A)*	*Load current (A)*	*Generator terminal voltage (V)*
Take 7-8 readings				

From the experimental results, plot a graph showing the relation between the terminal voltage and the load current for the separately excited dc generator. Also plot a graph showing the relation between voltage drop due to armature resistance and the load current. Subtract the values of the voltage due to armature resistance from the corresponding values of the terminal voltage from the results obtained. The resultant graph will show how much voltage is dropped due to armature reaction and brush contact resistance.

Questions Answer the following questions in your report:

1. From the graph plotted by you what is the effect of load on the terminal voltage? What are the reasons for the effect you have mentioned?
2. In this experiment why is it necessary to keep the excitation and speed of the dc generator constant?

Experiment 2.4 *Measurement of the terminal voltage of a dc shunt generator as a function of the load current.*

Objective To determine the external or load characteristic of a dc shunt generator by actually loading the machine.

Brief Theory For a generator the induced emf is equal to the terminal voltage when the generator is not loaded. However, when the generator is loaded there is a fall in terminal voltage. The fall in terminal voltage of dc shunt generator is due to

(a) $I_a R_a$ drop in the armature
(b) the effect of armature reaction
(c) brush-contact resistance.

When the generator is loaded, the voltage across the generator terminals falls due to reasons given in (a), (b) and (c). Now these causes reduce the voltage available across the field terminals of the dc generator. Thus the field current and hence the induced emf is reduced which results in further reduction of the terminal voltage. The external characteristic of a dc shunt generator is more drooping than that of a separately excited generator.

Circuit Diagram See Fig. 2.90.

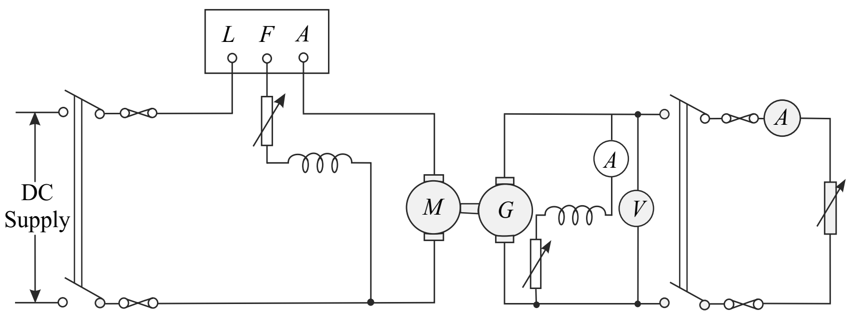


Fig. 2.90 *Shows the connection diagram for measurement of terminal voltage of a dc shunt generator as a function of load current*

Apparatus Required Ammeters (2), voltmeter, MG set, field regulating rheostats (2), dc motor starter, loading resistance, tachometer.

Procedure

1. Make connections as per your circuit diagram (see Fig. 2.80). (The connection of the primemover will depend upon the type of motor-generator set available in your laboratory. In the circuit diagram shown, the primemover is a dc motor).
2. Start the dc motor with the help of the starter and make the speed of the set equal to the rated value by adjusting the field regulating rheostat.
3. Adjust the magnitude of current through the field winding of the dc generator with the help of the field regulating rheostat to obtain rated value of induced emf on the generator terminals.
4. Load the de generator step-by-step with the help of loading resistances. For each step keep the speed of the dc generator constant. Take at least six readings.
5. Measure the armature resistance of the dc generator by the ammeter-voltmeter method.

Note: It is advisable to perform this experiment on the same motor-generator set on which Experiment 2.3 was performed.

Observations and Results Tabulate your observations as per the table shown below Armature resistance $R_a =$

S. No.	Speed in rpm	Generator field current (A)	Load current (A)	Generator terminal voltage (V)
Take 6-7 readings				

From the experimental results plot a graph showing the relation between terminal voltage and load current for the shunt generator. If the generator is the same as used for Experiment 2.3, then plot the curve of terminal voltage versus load current on the same graph paper using the same scale as was used for Experiment 2.3. This will enable a ready comparison. Also draw a graph showing how voltage drop due to armature resistance varies with load current. Subtract this graph from the external characteristic graph drawn.

Questions Answer the following questions in your report:

1. In the graph plotted by you what is the effect of load on the terminal voltage? What are the reasons for the effect mentioned by you?
2. In this experiment, why is it necessary to keep the speed of the dc generator constant?
3. Compare the results of the experiment with those obtained in Experiment 2.3. Comment on the difference, if any.

EXPERIMENT 2.5 *Measurement of the speed of a separately excited dc motor as a function of the load-torque*

(a) *at rated armature voltage and with maximum field current,*
(b) *at 50% of the rated armature voltage and with maximum field current, and*
(c) *at rated armature voltage and with 85% of the maximum field current.*

Objective

(i) To plot the torque-speed characteristics of a separately excited dc motor and to determine how it is influenced by changes in the armature voltage and field current.
(ii) To determine the efficiency of a dc motor under the conditions listed in (a) to (c) above.

Brief Theory We know the following equations for a dc motor

$$E = V - I_a R_a \qquad \text{[from Eq. (2.4)]}$$

and

$$E = K \phi N \qquad \text{[from Eq. (2.1)]}$$

From the above two equations the expression for speed can be written as follows

$$N = \frac{V - I_a R_a}{K \phi} \qquad \text{[from Eq. (2.7)]}$$

The expression for torque can be written as

$$T = K_t \phi I_a \qquad \text{[from Eq. (2.2)]}$$

Keeping the expressions of speed and torque in view it is possible to predict the influence of variable terms like terminal voltage and field current. In this experiment it is required to determine how the speed and torque of a separately excited dc motor are affected under the conditions listed in (a) to (c) of the experiment. Now theoretically consider the effects under the given conditions one after the other.

Conditions *(a) At rated armature voltage and with maximum field current* Referring to Eq. (2.1) it is seen that as V and ϕ are constant quantities and $I_a R_a$ drop increases as the load current increases, the numerator of Eq. (2.7) decreases which means that speed N decreases as the load current is increased. This gives a drooping speed characteristic [see Fig. 2.91(a)].

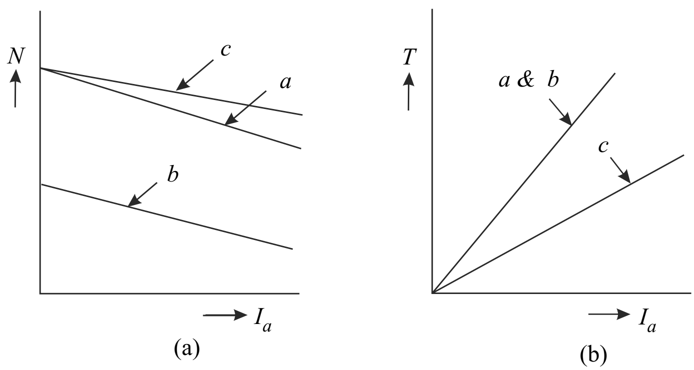


Fig. 2.91 *Shows the speed-load and torque-load characteristics of a dc separately excited motor. Curves a, b, c refer to conditions a, b, c mentioned under the theory*

From Eq. (2.2) it is seen that torque T will increase linearly with I_a, since ϕ is constant [see Fig. 2.81 (b)].

Since $T \propto \phi\, I_a \propto I_a$ (for a separately excited machine), the $N - T$ characteristic shown in Fig. 2.92 can be arrived at. The $N - T$ characteristic of Fig. 2.92 can be arrived at in the following manner also.

Equation (2.2) may be rewritten as

$$I_a = \frac{1}{K_t} \times \frac{T}{\phi}$$

Substituting this value of I_a in Eq. (2.7),

$$N = \frac{V - \left[\frac{1}{K_t} \times \frac{T}{\phi}\right] R_a}{K\phi}$$

or $$K\,\phi\, N = V - \left[\frac{1}{K_t} \times \frac{T}{\phi}\right] R_a$$

or $$K_1 N = V - K_2 T$$

or $$N = \frac{1}{K_1} V - \frac{K_2}{K_1} T$$

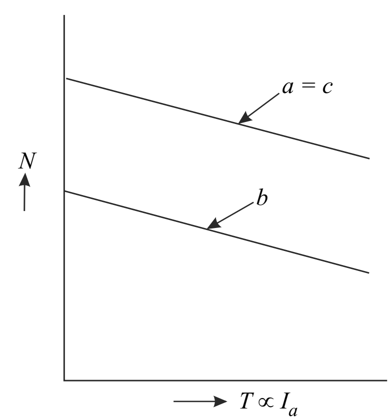


Fig. 2.92 *Shows speed-torque characteristics of a separately excited dc motor*

or $$N = C_1 V - C_2 T$$

or $$N = -C_2 T + C_1 V$$

This equation is of the form

$$y = -mx + C$$

where $y = N$, $m = C_2$, $C = C_1$, V and $x = T$.

A look at Fig. 2.92 will show that the curves obtained are consistent with the above expression.

(b) At 50% of the Rated Armature Voltage and with Maximum Field Current In this case the armature voltage is reduced to half. The flux remains constant at its maximum value. From Eq. (2.7) the speed-load characteristic will be a drooping one but will lie below the characteristic curve obtained for condition (a). The torque-load curve will be the same as obtained under condition (a) because torque is independent of the armature voltage ($T \propto \phi I_a$). The speed-torque characteristic will be the same in nature as that obtained for condition (a) but will lie below it (see Fig. 2.92).

(c) At Rated Armature Voltage and with 85% of the Maximum Field Current As flux is reduced, the speed-load characteristic curve will be less drooping than as obtained for condition (a). The torque-load characteristic will be linear but will have slightly less slope than that obtained for conditions (a) and (b) (see Fig. 2.91). The nature of the speed-torque characteristic will be almost the same as that for condition (a).

Note: The dc motor may be loaded with a brake or with the help of a dc generator.

Circuit Diagram See Fig. 2.93.

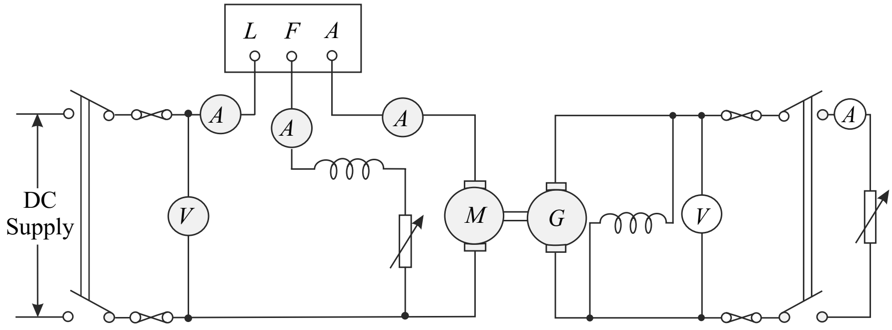


Fig. 2.93 *Shows the connection for measurement of speed of a separately excited dc motor as a function of load-torque*

Apparatus Required DC motor generator set, ammeters (three), voltmeters (two), field regulating resistance for the dc motor, loading resistance for the generator, tachometer, potential divider.

Procedure

1. Make connections as per Fig. 2.93 in which the motor has been shown loaded through a dc generator.

2. (a) Start the dc motor with the help of a starter (or a starting resistance). Adjust the field-regulating rheostat to its minimum value. Apply rated voltage to the motor. Load the dc motor in steps through the dc generator coupled with it. Record for each step the current drawn by the motor and its speed and the output voltage and current of the generator. Keep the field current and applied voltage constant for all the steps and note their values.
   (b) Now apply 50% of the rated voltage to the motor with the help of a potential divider or otherwise. Do not alter the field current. Starting from no-load, increase the load on the motor step. Note down all the readings as mentioned in 2(a) keeping the voltage and field current constant.
   (c) Adjust the applied voltage back to its rated value. Adjust the field regulating rheostat so that the field current is 85% of its maximum value. Starting from no-load, increase the load on the motor step-by-step. Note down all the readings as mentioned in 2 (a) keeping the voltage and field current constant.

   De-couple the dc generator from the dc motor. Run the dc generator as a motor at no-load and record the no-load input voltage and current. From this calculate the input power. This no-load input gives the iron, friction and windage-losses and the small amount of armature copper-loss which occurs at the load.
3. Measure the armature resistance of the dc generator by the ammeter-voltmeter method.

Observations and Results For the three conditions (a) to (c), tabulate the results as follows:

Armature resistance of the dc generator =

After decoupling when the dc generator was run as a dc motor at no-load

Input power =

S. No.	Motor field current (A)	Motor applied voltage (V)	Motor armature current (A)	Speed of the set (rpm)	Generator terminal voltage (A)	Load current (A)
Take 6-7 readings						

As the dc motor is loaded through a generator, the torque can be calculated thus:

$$T = \frac{\text{Input power to generator}}{\omega}$$

where ω = speed of motor (= speed of generator) in rad/s

and Input to generator = (Output of generator) + (Copper, windage, friction and iron losses of generator)

Windage, friction and iron losses of generator

= (Input to the generator when run as a motor at no-load) – (Armature $I_a^2 R_a$ loss at no-load)

The input to the generator is the output of the motor. From this calculate the value of the output torque of the motor for different speeds. Further, calculate the input to the dc motor (i.e., supply voltage × current) and hence find out its efficiency at different loads. Show sample calculations for at least one reading for each condition. Tabulate the calculated results for each reading. Plot curves for conditions (a) to (c) showing the relation between

(i) load-torque and speed
(ii) load-torque and armature current
(iii) speed and armature current
(iv) efficiency and armature current.

Note: It is suggested that graphs (i), (ii), (iii) and (iv) mentioned above should be drawn on separate graph sheets. However, each sheet should contain the graphs for conditions (a) to (c). This will enable ready comparison which is an important purpose of this experiment.

Questions Answer the following questions from the results obtained by you in your report:

1. What is the effect on the speed of a separately excited dc motor when it is loaded at constant supply voltage?
2. What is the effect of the variation of field current on the speed-torque characteristic of a separately excited dc generator, the supply voltage remaining constant?
3. How does the efficiency of a separately excited dc motor vary with load? At what percentage of full load did the efficiency become maximum for the different conditions under which you performed the experiment? What conclusions do you draw from this comparison?

EXPERIMENT 2.6 *Measurement of the speed of a dc series motor as a function of the load-torque.*

(a) At rated armature voltage
(b) At 50% of the rated armature voltage

Objective

(i) To plot the torque-speed characteristic of a dc series-motor and to determine how it is influenced by change in armature voltage.
(ii) To determine the efficiency of a dc series motor under the conditions listed in (a) and (b) above.

Brief Theory The following equations for a dc motor are known:

$$N = \frac{V - I_a R_a}{K\phi}$$

and

$$T = K_t \, \phi \, I_a$$

For a series motor flux ϕ is proportional to armature current I_a. Therefore, the above expressions can be rewritten as:

$$N = \frac{V - I_a R_a}{K_1 I_a} \tag{i}$$

$$T = K_2 \, I_a^2 \tag{ii}$$

In this experiment, how the speed and torque of a dc series motor are affected under conditions listed in (a) and (b) are to be determined. Now theoretically consider the effects under the given conditions one after the other.

Conditions *(a) At Rated Armature Voltage* Referring to Eq. (i) it is seen that speed N is approximately inversely proportional to armature current I_a, (= load current). Equation (i) can be rewritten as

$$N I_a = \frac{1}{K_1}(V - I_a R_a)$$

With increase in armature current I_a, the term $(V - I_a R_a)$ does not change very much as the $I_a R_a$ drop is small compared to V. So the above equation is of the from

$$xy = \text{constant}$$

which is the equation for a rectangular hyperbola. The speed versus armature current characteristic will be approximately a rectangular hyperbola as shown in Fig. 2.94.

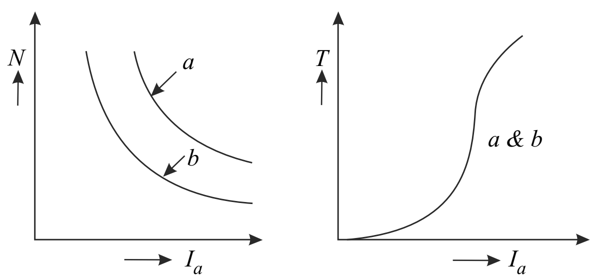


Fig. 2.94 *Shows the speed-load and torque-load characteristics of a series motor respectively*

From Eq. (ii) it is seen that torque T is proportional to the square of the armature current. Hence the torque versus armature current characteristic will be a parabola. It is to be noted that after saturation, flux ϕ will be constant and torque T will then be directly proportional to the armature current only. From the graphs shown in Fig. 2.94 (a) and (b) the speed-torque characteristic shown in Fig. 2.95 can be arrived at. It is to be noted that if $T - I_a$ characteristic was linear then the shape of $N - T$ characteristic would be the same as $N - I_a$ characteristic. But as the $T - I_a$ characteristic is parabolic, the $N - T$ characteristic will be much more curved than the $N - I_a$ curve.

(b) At 50% of the Rated Armature Voltage In this case the armature voltage is reduced to half. From the speed equation $N I_a = (I/k_1)(V - I_a R_a)$ it is seen that the magnitude of the right-hand-side term will be approximately halved and

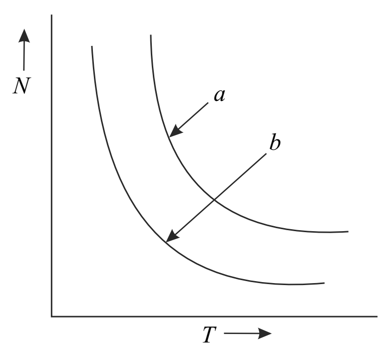


Fig. 2.95 *Shows the speed-torque characteristic of a dc series motor*

will not change very much with a change in the armature current. The shape of the $N - I_a$ characteristic will be as shown in Fig. 2.94 (see curve *b*).

From Eq. (ii) it is seen that torque is independent of the armature voltage. Therefore, the shape of the $N - I_a$ characteristic will be exactly the same as obtained under condition (a).

As in this case $N - I_a$ characteristic lies below the curve obtained under condition (a), the $N - T$ characteristic will also be below the $N - T$ characteristic obtained for condition (a) The nature of the curve is shown in Fig. 2.95 (see curve *b*).

Note: The dc series motor may be loaded with a brake or a dc generator coupled with its shaft

Circuit Diagram See Fig. 2.96.

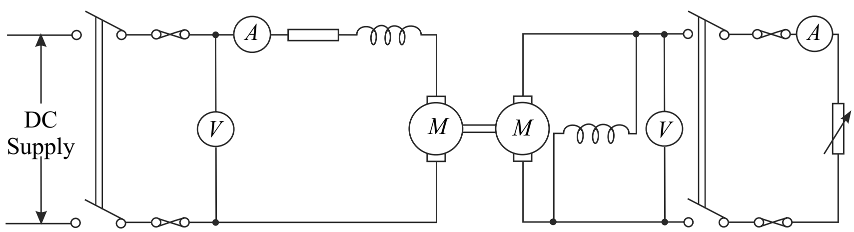


Fig. 2.96 *Shows the connection diagram for measurement of speed of a dc series motor as a function of load-torque*

Apparatus Required The dc series motor coupled with a dc generator, ammeters (two), voltmeters (two), starting resistance for the dc motor, loading for the resistance for the generator, tachometer, potential divider.

Procedure

1. Make connections according to the circuit diagram shown in Fig. 2.96.
2. Switch on the loading resistances on the generator terminals. This is important because a series motor should not be run on no-load.
3. (a) Start the series motor with the help of a starting resistance. The generator will act as a load on the series motor. Apply rated voltage to the motor. Reduce the load on the dc generator step-by-step and record the speed, current taken by the motor and the output voltage and current of the generator for each step. Keep the applied voltage constant for all the steps.
   (b) Now apply 50% of the rated voltage to the motor with the help of a potential divider. Starting from full load, decrease the load on the motor in steps. Note down all the readings as mentioned in 3(a) keeping the applied voltage constant at 50%.
4. De-couple the generator from the dc motor. Run the dc generator as a motor at no-load and record the no-load input voltage and current. From this calculate the input power. This no-load input gives the iron, friction and windage losses and the small amount of armature copper-loss which occurs at no-load.
5. Measure the armature resistance of the de generator by the ammeter-voltmeter method.

Observations and Results For conditions (a) and (b), tabulate the results as follows

S. No.	Motor applied voltage (V)	Motor armature current (A)	Speed of the set (rpm)	Generator terminal voltage (A)	Load current (A)

Armature resistance of the dc generator =

After decoupling when the dc generator was run as de motor at no-load:

Input power =

As the dc motor is loaded through a generator, the torque can be calculated thus:

$$T = \frac{\text{Input power to generator}}{\omega}$$

where ω = speed of motor (= speed of generator) in rad/s, and input to generator T = (output of generator) + (copper, windage, friction and iron losses of generator).

Windage, friction and iron losses of generator
= (input to generator when run as motor at no load)
– (armature $I_a^2 R_a$ loss at no-load)

Input to the generator is the output of the motor. From this calculate the value of the output torque for different speeds. Further, calculate the input to the dc motor (i.e., supply voltage × current) and hence find out its efficiency at different loads. Show sample calculations for at least one reading for each condition. Tabulate the calculated results for each reading. Plot curves for conditions (a) and (b) showing the relation between

(i) load-torque and speed,
(ii) load-torque and armature current,
(iii) speed and armature current, and
(iv) efficiency and armature current.

Note: It is suggested that the characteristics (i), (ii), (iii) and (iv) just mentioned should be drawn on separate graph sheets. However, each sheet should contain characteristic corresponding to conditions (a) and (b). This will enable a ready comparison.

Questions Answer the following questions in your report:

1. In this experiment load is provided to the dc series motor by means a dc generator. Why is it necessary to switch the load on to the generator before switching on supply to the motor?
2. Study the $T – N$ curve for the dc series motor obtained by you. From it explain what can be the practical applications for which such a motor can be used.

Experiment 2.7 *To perform Swinberne's test on a dc shunt motor.*

Objective To determine the efficiency of a dc shunt motor by measurement of losses (without actually loading the motor).

Brief Theory The most obvious method of determining the efficiency of any machine at any load would be to measure the output and input when the machine is running at that load. This requires application of load on the machine. For machines of higher rating, loads of the required size may not be available. Moreover, even if it was possible to provide such loads, large power would be consumed, making it an expensive method. All big electrical machines are, therefore, tested indirectly by measuring the losses as discussed below. The machine is run as a motor on no-load at its rated voltage and speed and the input power measured. The armature and field resistances are also measured. Since these values are measured when the machine is cold, they must be converted to values corresponding to the temperature at which the machine would work on full-load. This is done because the resistance of the winding wire increases with temperature. The admissible working temperature depends upon the design and class of insulations used. For instance, machines designed with class *A* insulation can work up to a temperature rise of 65 °C above an ambient of 40 °C. If *R* is the measured winding resistance when cold (i.e., at ambient temperature), the hot resistance when the machine will be running at full-load can be calculated thus

$$R_2 = R_1[1 + \alpha_1(t_2 - t_1)] \qquad \text{(i)}$$

Let the ambient temperature be 40 °C and the let the rise in winding temperature be assumed to be 65 °C. Using Eq. (i) resistance is to be calculated at 105 °C. Therefore

$$R_{105} = R_{40}\,(1 + \alpha_{40} \times 65)$$

where $\alpha_{40} = \dfrac{\alpha_0}{1 + \alpha_0 \times 40}$

The value of α_0 for copper may be taken as 0.00427. The hot resistance, both of the armature and of the field windings, may be calculated by the above method. From the cold resistance (resistance measured at room temperature) of the armature, the copper-loss ($I_a^2 R_a$) can be calculated. From the cold resistance of the shunt field, the field copper-loss (V^2/R_{sh}) can be calculated. The sum of these two losses deducted from the input at no-load (i.e., $V I_0$) gives the friction and iron-losses. If now the calculated field copper-loss (V^2/R_{sh}), using the hot resistance, is added to the friction and iron-losses, the result will be constant losses. Let this be denoted by P_c. Then for any armature current I_a, the armature copper-loss will be $I_a^2 R_a$, where R_a is the hot armature resistance. For calculating full-load armature copper-loss, take I_a as the full-load armature current. Hence the total loss

$$P_t = I_a^2 R_a + P_c$$

Let $V I_1$ be the full-load input of the dc motor ($I_1 = I_a + I_{sh}$), I_{sh} being the shunt field current.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$= \frac{VI_1 - P_t}{VI_1}$$

Similarly, efficiencies at any other load can be calculated.

For dc machines the simplest of indirect tests is Swinberne's test. It can be applied to shunt and compound machines only. The dc series machines cannot be tested by this method as it requires the machine to run as a motor on no-load. A dc series motor should not be run at no-load because at no-load the speed of a dc series motor is extremely high. An extremely high speed may cause damage to the mechanical parts of the machine and also of the foundation. Also, in a series machine iron and friction losses are not constant with load. Therefore, the losses measured at no-load for a series machine will not be the constant losses.

Circuit Diagram See Fig. 2.97.

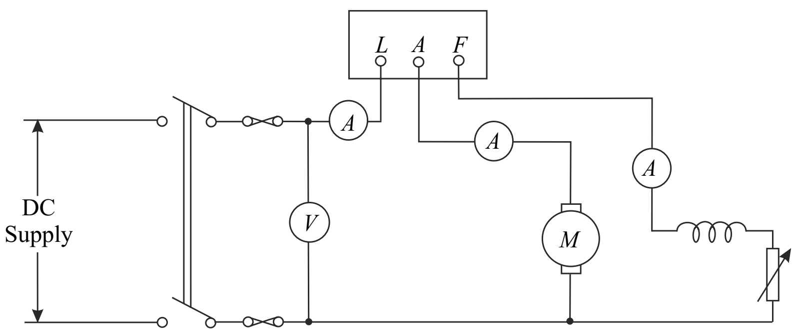


Fig. 2.97 *Shows the connection diagram for performing Swinberne's test on a dc machine*

Apparatus Required dc machine, voltmeter (one), ammeters (three), tachometer, voltmeter, (low range), rheostat.

Procedure

1. Make connections as per the circuit diagram shown in Fig. 2.87.
2. Start the dc motor with the help of a starter
3. Bring the motor to its rated speed by adjusting the field regulating rheostat.
4. Note the applied voltage the armature and field currents.
5. Disconnect the circuit and measure the annature and field circuit resistances at room temperature by the ammeter-voltmeter method. Very low voltage should be applied to the armature for measurement of armature resistance by this method because the armature resistance is very low.

Observations and Results

S. No	Applied voltage (V)	Field current (A)	Armature current (A)	Speed (rmp)

Measurement of armature resistance and field resistance

S. No	Applied Voltage (V)		Current (A)	
	Armature circuit	Field circuit	Armature circuit	Field circuit

Cold resistance of armature =
Hot resistance of armature =
Cold resistance of field =
Hot resistance of field =

Show at least one sample calculation for calculating the hot resistance and also for calculating the efficiency of the dc motor (use the method as given under *brief-theory*).

Questions Answer the following questions in your report:

1. Why is Swinberne's method of finding out the efficiency not suitable for dc series motor?
2. Why do we use indirect method of finding out efficiency for electrical machines instead of direct methods?
3. Are there any assumptions made when using Swinberne's method for determining the efficiency of dc shunt and compound machines?

3 TRANSFORMERS

OBJECTIVES

After carefully studying this chapter, you should be able to

- ➢ Explain the function of various parts of a transformer.
- ➢ Explain the principle of working of a transformer.
- ➢ Draw the equivalent circuit and phasor diagram of a transformer.
- ➢ Calculate voltage regulation of a transformer.
- ➢ Calculate the efficiency of a transformer from test results.
- ➢ Explain the use of instrument transformers.
- ➢ Explain in brief the various tests that are to be performed on transformers before certification.
- ➢ Prepare a maintenance schedule for a transformer.
- ➢ Select transformers for parallel operation.
- ➢ Perform certain basic tests on a transformer.

3.1 INTRODUCTION

A transformer is an electrical device, having no moving parts, which by electromagnetic induction transfers electric energy from one circuit to another at the same frequency, usually with changed values of voltage and current. In its simplest form it consists of two windings insulated from each other and wound on a common core made up of magnetic material. Alternating voltage is connected across one of the windings called the primary winding. In both the windings emf is induced by electromagnetic induction. The second winding is called the secondary winding (see Fig. 3.1).

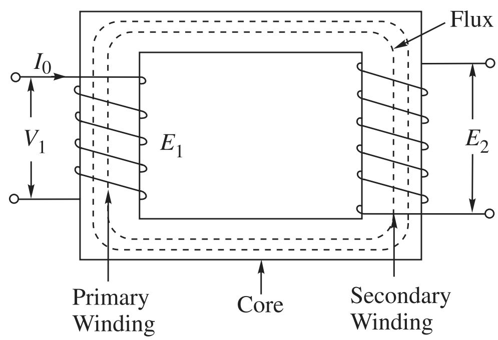


Fig. 3.1 *Illustration of transformer principle*

When the primary winding is connected to an ac source an exciting current I_0 flows through the winding. As the current is alternating, it will produce an alternating flux in the core which will be linked by both the primary and secondary windings. The induced emf in the primary winding is almost equal to the applied voltage V_1 and will oppose the applied voltage. The emf induced in the secondary winding can be utilised to deliver power to any load connected across the secondary. Thus power is transferred from the primary to the secondary circuit by electromagnetic induction. The flux in the core will alternate at the same frequency as the frequency of the supply voltage. The induced emf in the windings will also have the same frequency as that of the supply voltage. The magnitude of the emf induced in the secondary winding will depend upon its number of turns. Thus by providing a higher number of turns in the secondary winding with respect to the primary winding, a higher voltage can be obtained in the secondary winding and vice versa. Thus by having different ratios of the number of turns between the primary and secondary windings, power at lower or at higher voltages can be obtained through a transformer. When, in a transformer, the number of turns in the secondary winding is less than those in the primary winding, it is called a step-down transformer; when the number of turns in the secondary winding is higher, it is called a step-up transformer.

3.2 APPLICATIONS OF TRANSFORMERS

Electricity is generated at places which are often away from the places of its consumption. This is because the location of natural sources of energy such as water head, coal, etc., needed for the generation of electricity is not necessarily near the load centre. Electricity generated at far-away places is to be brought to the places of consumption. For example, electricity generated at Bhakra should be brought to places like Chandigarh where it will be used in residences, offices and in industries. Electricity from Bhakra is brought to Chandigarh through overhead transmission lines as shown through a single line diagram in Fig. 3.2.

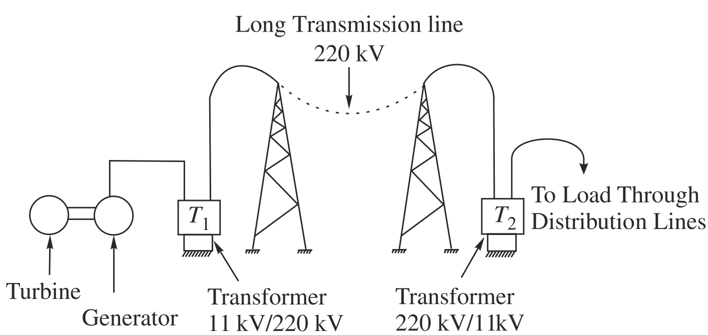


Fig. 3.2 *Electricity generated at the power station is brought to the places of consumption through transmission lines*

Normally electricity at the power stations is generated at a voltage as high as 11 kV. (Generations at a higher voltage than 11 kV give rise to insulation problems within the generator.) The generated power is then stepped up to a much higher voltage such as 220 kV, through a step-up transformer (T_1 in Fig. 3.2 is a step-up 11 kV/220 kV transformer). Electricity is then transmitted through a transmission line at 220 kV and at the receiving end stepped down to a lower voltage through one step-down transformer T_2. The secondary side of transformer T_2 is connected to the various loads through distribution lines.

It is economical to transmit electricity at a very high voltage. The higher the voltage in the transmission line the lower is the current which will flow through the transmission line for a given amount of power to be transmitted. (Power is the product of voltage and current. For the same power to be transmitted, if the voltage is increased, current will be reduced.) Thus a higher transmission voltage will result in a lower cross-sectional area of the transmission wire and hence lower cost. Thus, the transmission of electric energy over long distances can be achieved economically when the transmission voltage is high. However. as mentioned earlier, it is difficult to generate electricity at a voltage as high as 220 kV. Moreover, the use of electricity at a very high voltage is difficult and may also be dangerous. Thus it becomes necessary to transform electricity from a low voltage—high current form to a high voltage—low current one. This, in the case of an alternating current system, is made possible very easily by the use of transformers.

Transformers of lower capacity are used in electronics circuits and equipment. For example, almost all electronic equipment require a low voltage dc supply which is obtained by using a step-down transformer and a bridge rectifier. In an inverter circuit low voltage dc voltage of 12 V is first converted into ac and then stepped up to 230 V ac using a step-up transformer. Thus, transformers find wide applications in electrical and electronics engineering field.

3.3 CONSTRUCTIONAL DETAILS

A transformer is a static device and its construction is simple as there are no moving parts. Fig. 3.3(a) shows a transformer installed outdoors and connected to supply lines. The main components of a transformer as listed below are shown in Fig. 3.3(b).

(i) the magnetic core,
(ii) primary and secondary windings,
(iii) insulation of windings,
(iv) lead and tappings for coils with their supports, terminals and terminal insulators,
(v) tank, oil, cooling arrangement, conservators, dryers, etc.

3.3.1 Magnetic Core

The magnetic circuit of the transformer can be of core-type or shell-type as shown in Fig. 3.4. A core-type transformer is one in which there is only one iron path, usually in rectangular form, and the windings are wound on two opposite limbs. The core-type is not often used in small-size transformers as its shape makes mounting difficult, but it offers advantages, considering the insulation requirement, when very high voltages have to be made available through small transformers. In shell-type construction there are two parallel magnetic paths into which the flux from the central limb gets divided. The primary and secondary windings are wound on the central limb one around the other. This provides a better magnetic coupling between the primary and secondary windings.

In both the types, the magnetic circuit is made up of a laminated iron core. Thin silicon sheet-steel cut into particular sizes and shapes are stacked together to form the core. The core is laminated to reduce eddy-current losses. Eddy currents are induced in the core because the core is experienced by alternating flux produced in the core by the alternating current flowing through the windings. These eddy currents circulate in the core and give rise to I^2R loss in the core. To reduce this eddy-current loss, the core is built up of laminations. Laminated sheets are insulated from each other by applying a thick layer of varnish insulation on the laminations. The thickness of laminations

Fig. 3.3 (a) *Photographic view of a transformer installed in the premises of an engineering college*

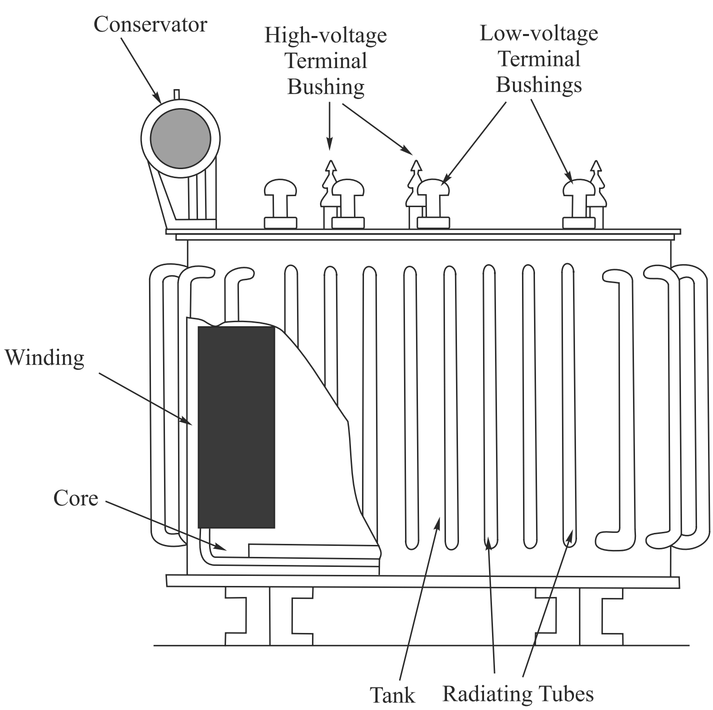


Fig. 3.3(b) *Cut view of a transformer showing various important parts*

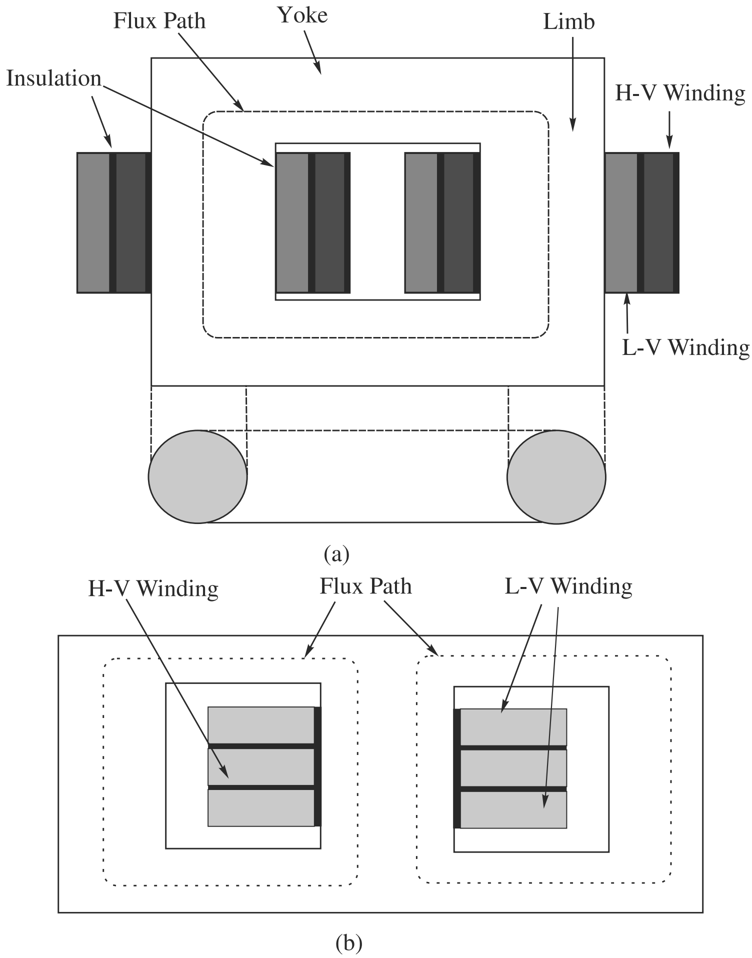


Fig. 3.4 *Constructional details of single-phase transformers (a) Core-type (b) Shell-type*

used is 0.4 mm or less. In order to reduce the eddy-current loss the thickness of the laminations should be very small. But the thickness cannot be reduced too much because the mechanical strength of the laminations will be greatly reduced (they may buckle while handling).

In addition to eddy-current loss, hysteresis loss occurs in the core as it is subjected to magnetisation in opposite directions in every half cycles. Hysteresis loss depends upon the area of the hysteresis loop of the core material. Special silicon steel (silicon content 4 to 5%) is used for laminations. This has a small hysteresis loop area but a high value of saturation flux density and high resistivity. Thus the hysteresis loss and eddy-current loss will be minimised (high resistivity will cause a low value of eddy-current flowing in the core). The sum of hysteresis loss and eddy-current loss is called iron-loss or core-loss of a transformer. Manufacturers of electrical sheet-steel supply curves showing the total iron loss in their material under specified conditions of frequency, flux density, etc. (see Fig. 3.5).

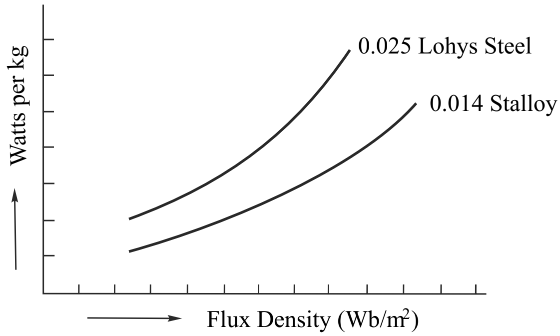


Fig. 3.5 *Shapes of iron-loss curves for different grades of silicon steel in the form of small laminations*

By reference to such graphs it is possible to quickly calculate what iron-loss may be expected at the particular frequency and flux density for the type of lamination used.

Nowadays cold-rolled grain-oriented (CRGO) silicon-steel laminations are used for the construction of transformer core. This reduces the amount of magnetising current taken by the primary winding. The effect of use of CRGO sheet on the magnetising current is explained as follows.

The phenomenon of magnetisation of ferromagnetic material can be explained by assuming that magnetic substances such as iron are composed of particles which themselves are tiny magnets. In the demagnetised state these tiny magnets, also called grains (magnetic dipoles), are distributed in different directions in random fashion as shown in Fig. 3.6(a). The resultant magnetism is zero as the magnetism of one dipole is cancelled by that of the other. When a magnetising force is applied, the dipoles orient themselves in the direction of magnetisation as shown in Fig. 3.6(b). The higher the magnetising force the more is the orientation of the dipoles (grains). When almost all the dipoles (grains) become oriented in a particular direction, the core is said to have become saturated.

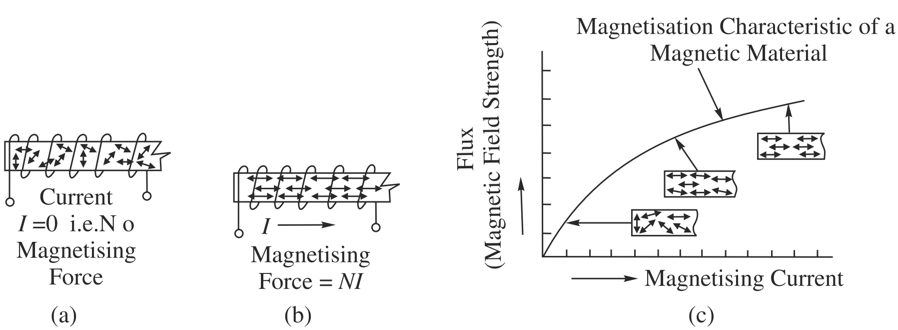


Fig. 3.6 *Orientation of the grains (magnetic dipoles) due to application of magnetising force (a) A piece of magnetic material with no magnetising force applied (NI = 0) (b) A piece of magnetic material being magnetised by applying a magnetising force (NI) (c) Magnetisation characteristic of a magnetic material showing at three different stages the orientation of the grains in the material being magnetised*

By the cold-rolling of the laminated sheets the grains are made to orient to some extent in the direction of rolling. Thus the magnetising current required to fully orient the grains in that particular direction will be comparatively smaller than in the case of nonoriented steels.

While using CRGO sheet steel to build transformer cores, care must be taken to assemble the core in such a manner that the orientation of the grains is parallel to the flux path. Otherwise, the core will offer a higher reluctance in certain portions and the advantage of cold-rolling will not be fully utilised. To understand this consider a piece of grain-oriented laminated sheet as shown in Fig. 3.7(a). Pieces like *ABCD*, as shown in Fig. 3.7(b), can be cut out of this sheet.

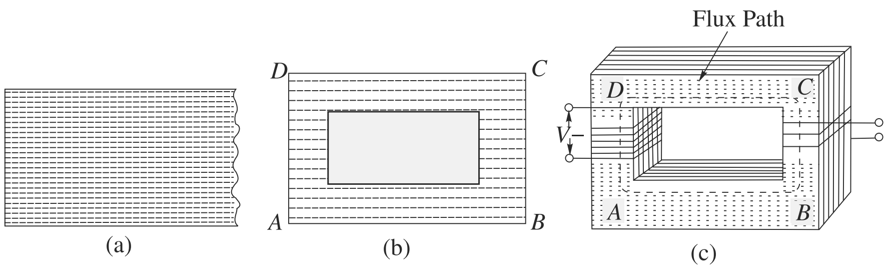


Fig. 3.7 *(a) A piece of grain-oriented sheet steel (b) A rectangular sheet ABCD cut out of the steel sheet of figure (a) (c) A rectangular core built with laminations as shown in figure (b)*

Figure 3.7(c) shows a rectangular-core transformer built by assembling cut laminations as shown in Fig. 3.7(b). It will be seen from Fig. 3.7(c) that in limbs *AD* and *BC* the grains are oriented at right angles to the direction of the flux path. Thus to produce flux in the core in the direction shown, a large magnetising force will be required to orient the grains in limbs *AD* and *BC*. If however, rectangular sheets AA', BB', CC', and DD' are cut from the sheet shown in Fig. 3.7(a) and assembled as shown in Fig. 3.8 to build the required thickness of the core, it will be obvious that in such an arrangement the grain orientation will always be parallel to the flux path.

Thus the exciting current needed to produce a particular amount of flux will be small. It may however be noted that the grains in the four corners are not oriented in the direction of the flux path. This can be avoided by using strips of the shapes shown in Fig. 3.9. The strips are cut at angles of 45° at the two ends. For providing mechanical support between the laminations and also to distribute the air-gap in the corner region the lamination sheets are cut at angles of, say 47° and 43°, as shown in Fig. 3.10.

Small core-type transformers are made of rectangular-section core limbs. But in large core-type transformers, economical use of core material requires that the cross-section of the core should be circular. A circular cross-section gives minimum peripheral length for a particular area; this reduces the length of the mean turn of winding and thereby reduces the cost of copper wire required for the winding. To make the limb cross-sectional area circular, the width of each lamination should be made variable which will be very uneconomical. As a compromise the core section is arranged in steps as shown in Fig. 3.11(b). In the figure laminations of three different widths have been used.

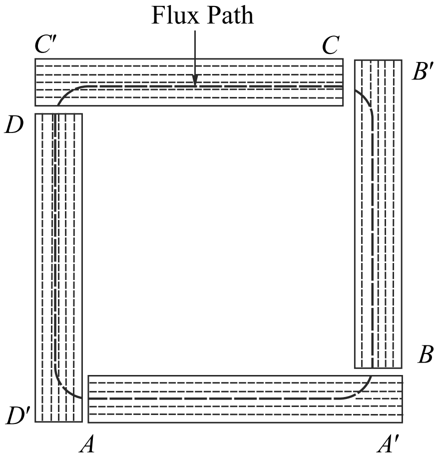


Fig. 3.8 *Grain-oriented sheet steel strips cut into rectangular shapes and arranged to build a transformer core such that the grains are oriented in the direction of the magnetic flux paths in all the limbs*

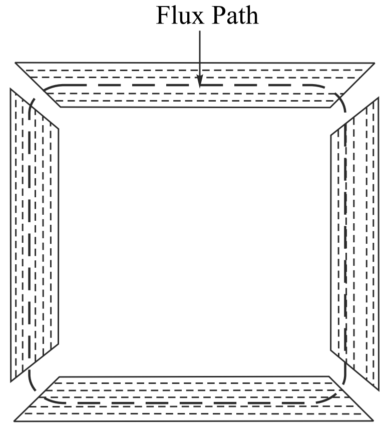


Fig. 3.9 *Grain oriented sheet steel laminations cut at angle of 45° and assembled to build transformer core enabling grains at the four corners also to be oriented in the direction of the flux path*

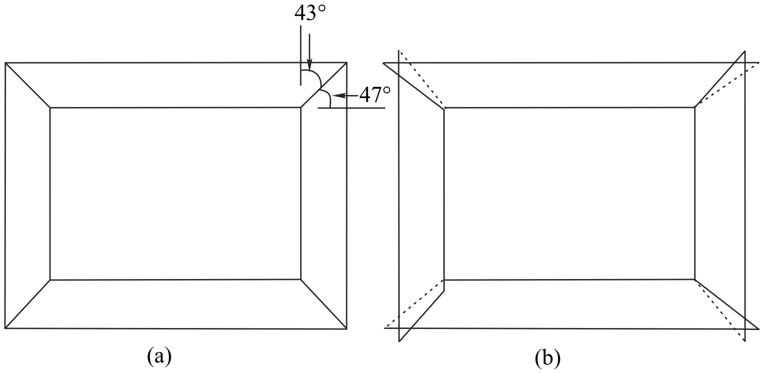


Fig. 3.10 *(a) Laminated sheets cut at angles of say 47° and 43° on two ends and assembled to form a layer of core laminations (b) Layer of tore laminations placed one above the other in a manner as shown to form a complete core*

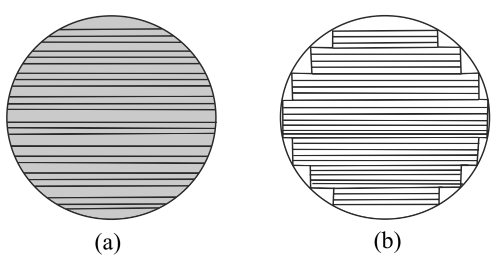


Fig. 3.11 *(a) Cross-section of the core limb using laminations of variable widths (b) Cross-section of the core limb using laminations of three different widths*

3.3.2 Three-phase Transformer Core

The core construction of a three-phase transformer is similar to a single-phase one as explained earlier. A three-limbed core as shown in Fig. 3.12 is used. Three-phase windings are placed on the three core limbs as shown. For a three-phase circuit, however, three single-phase transformers could also be used with their primaries and secondaries connected in star or delta as required.

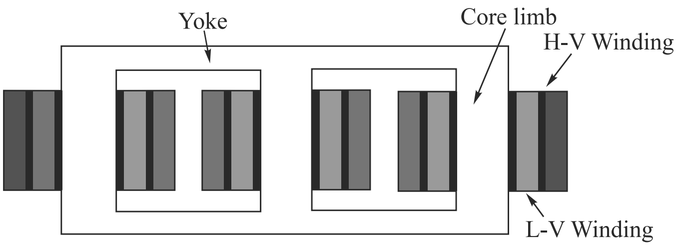


Fig. 3.12 *Three-phase transformer construction*

The advantages of three-phase transformers over the banks of three single-phase transformer are that they occupy less space, weigh less, cost less and there is only one unit to instal and operate. Their disadvantages are that a trouble in one phase may be transferred to the other two-phases and in the case of a fault the whole system has to be shut off for repairs, whereas in the case of three single-phase transformers only the faulty unit need be disconnected. Like single-phase transformers, three-phase transformers can also be built either core-type or shell-type (A core-type is one in which the iron core is surrounded by windings whereas a shell-type is one in which the windings are surrounded by iron cores).

3.3.3 Primary and Secondary Windings

The primary and secondary windings basically consist of a series of turns, called coils, wound round the core. The coils of transformer windings are generally of two main types, viz.

(i) Cylindrical Concentric coils, and (ii) Sandwich coils.

Core-type transformers usually have cylindrical coils taking up the whole length of the transformer-core limb with the high-voltage winding placed concentric with the low-voltage winding. The low-voltage winding is placed closer to the core than the high-voltage winding as this will involve less insulation problem between the core and the windings. Shell-type transformers usually have a series of flat coils with primary and secondary-winding sections placed one above the other on the core limb. Two low-voltage winding sections are placed at the ends as this will require less insulation material to insulate the winding sections from the core yoke at the two ends. The two types of winding arrangements are shown in Fig. 3.13. The end low-voltage sections have half the turns of the normal low-voltage sections.

For transformers of high ratings a large area of cross-section of winding wires has to be provided. Conductors of large cross-section give rise to eddy-current losses within the winding wires and also they are difficult to handle. The conductor section is therefore subdivided to reduce the eddy-current loss in the winding wires. In such

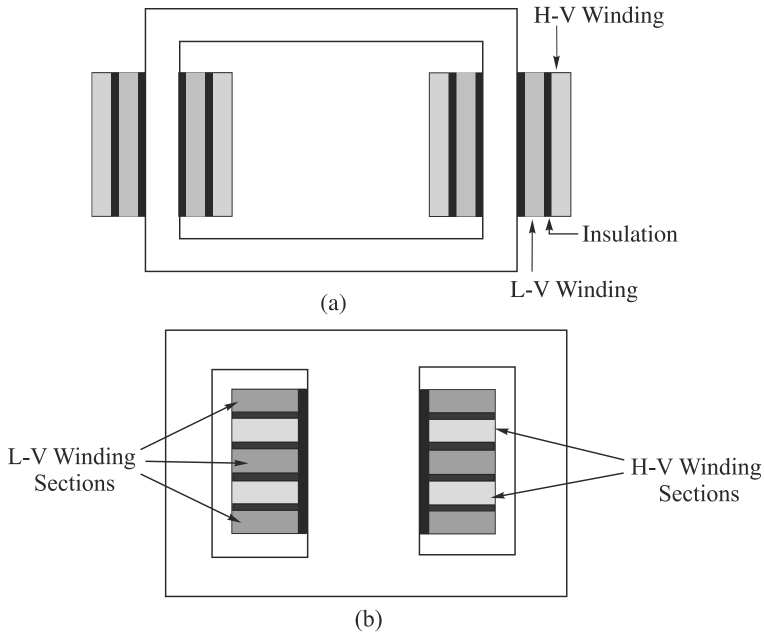


Fig. 3.13 *(a) Concentric coils in core-type transformers*
(b) Sandwich coils used in shell-type transformers

cases instead of round conductors, flat conductor sections are used. Figure 3.13(b) shows high and low-voltage windings sectionalised. Both the primary and secondary windings are divided into sections and the sections are interconnected. The use of such sectionalised windings makes handling easy and improves the cooling of windings. Oil can flow easily through the space available between the winding sections. It also facilitates the control of leakage reactance as it provides better coupling between the primary and secondary windings. In making windings of large transformers, instead of using a single conductor with large cross-section, a number of flat conductor sections are used. The conductor sections are transposed while making the coil winding so that each section conductor occupies all the positions.

3.3.4 Insulation of Windings and Terminals

Enamel insulation is used as the inter-turn insulation of low-voltage transformers. For power transformers enamelled copper with paper insulation is also used. Narrow strips of insulating paper are used for insulating winding wire, whereas wide rolls of paper form the inter-layer insulation. Paper in the form of tape may be utilised for taping winding leads and other parts. Press board in the form of collars, cylinders, sheets and boards is used as insulation between windings and the core. Press board is also used to make spacer blocks which are used to maintain radial and axial cooling ducts in transformer windings. Cloth untreated form is used as a tape over paper to prevent it from unwrapping. Cotton tape impregnated with varnish is used for reinforcement of insulation between turns and coils. Synthetic resin bonded materials are used in transformers in the form of laminated cylinders, boards and tubes.

Connections from the transformer windings are brought out by means of bushing. Ordinary porcelain insulators can be used as bushings up to a voltage of 33 kV. Above 33 kV, capacitor and oil-filled type of bushings are used. Bushings are fixed on the transformer tank. In oil-filled bushings the conductor is passed through the hollow porcelain insulator which is filled with oil. Due to the electric field existing around the conductor, the impurities in oil will try to align themselves in the radial direction, thus creating a possible path for the breakdown of insulation. To avoid this happening a number of hollow bakelite tubes (called baffle tubes) are placed concentrically around the conductor inside the bushing (see Fig. 3.14).

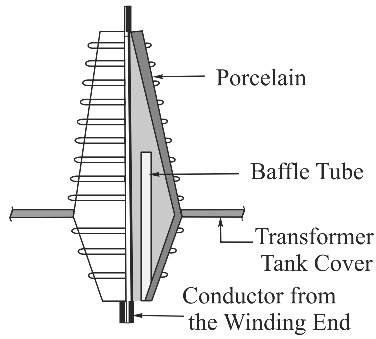


Fig. 3.14 *Cross-sectional view of an oil-filled bushing used in extra high voltage transformers*

3.3.5 Cooling Arrangement in Transformers

A transformer is a static device and hence its losses are less than that of rotating machines. In transformers, losses take place in the windings and in the core. To keep down the temperature of the windings, when the transformer is on load, the heat generated due to losses should be radiated to the atmosphere. The various methods of cooling employed in transformers are:

(i) air natural, (ii) air blast,
(iii) oil natural, (iv) oil blast,
(v) forced circulation of oil, (vi) oil and water-cooled, and
(vii) forced oil and water-cooled.

The air-natural method of cooling is used for cooling small low-voltage transformers where the natural circulation of the surrounding air is utilised to carry away the heat from the transformer core and windings.

In the air-blast method the transformer is cooled by a continuous blast of cool air produced by a fan and forced through the windings. The air supplied is filtered to avoid dust entering the ventilating ducts.

Transformers with high ratings are oil-cooled. Oil, in addition to cooling, provides insulation to the windings. The transformer is placed inside the tank filled with transformer oil and the tank is sealed. Heat is radiated to the atmosphere through the outside surface of the transformer tank. To increase the radiating area, hollow tubes or fins are provided with the transformer tank. Oil circulates through these radiating tubes or fins and thereby accelerates heat dissipation. Figure 3.15 shows a transformer tank with radiating tubes.

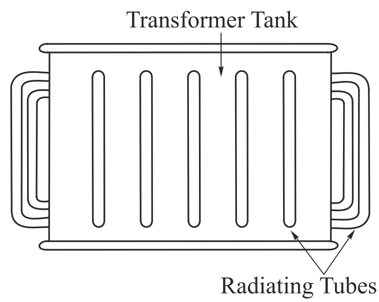


Fig. 3.15 *Transformer tank with radiating tubes*

Heat generated due to losses in the core and in the windings inturn heats the oil. Hot oil will flow in the upward direction and its place will be occupied by cool oil from the top. The heat will be radiated out to the atmosphere through the radiating duct and hence it will flow in the downward direction. Thus there will be a continuous flow of oil inside the transformer tank.

The efficiency of heat dissipation can further be increased by creating a blast of air directed over the outer surface of the radiating tubes as is done in the case of oil-blast type of cooling.

The oil used for the cooling of transformers is obtained by refining crude petroleum. This mineral oil has a good insulating property. Vegetable and other animal oils cannot be used in transformers as they react with the fibrous insulating material used in the transformer. Contact of oil with the atmosphere should be avoided as this will help in the formation of sludge in the transformer oil due to its heating and oxidation. Sludge formation is very harmful as it may in due course of time block the ducts through which oil is circulated inside the transformer and thereby cause improper cooling. In very large transformers more effective cooling is achieved through forced circulation of oil by the use of pumps or allowing cooling water to circulate through coils or pipes placed near the top of the tank under the oil surface.

3.3.6 Use of Conservators and Breathers

Inside the transformer tank some space should be kept for allowing the expansion of transformer oil. (Transformer oil will expand due to the heat generated because of losses taking place in the transformer.) If some free space above the oil level is kept by not filling the transformer tank completely, as the tank is completely sealed, when oil expands the air above the oil level will be under pressure and stresses will develop on the tank. Therefore some ventilating arrangement is to be made for free expansion of oil. This may be done by connecting a hollow pipe on the top of the tank and giving a downward bend to the other end of the pipe. Thus, as the transformer oil expands, air will come out of the tank and as the oil cools, fresh air will be drawn in. Atmospheric air contains moisture and the oil in the transformer is now exposed to the outside moist air. This is avoided by keeping some drying agent, such as calcium chloride or silica gel, in a container filled at the end of the tube connected with the tank. Silica gel or calcium chloride absorbs moisture and allows dry air to enter the transformer tank. The drying agent is replaced periodically as routine maintenance. To avoid the whole surface of the transformer oil to be exposed to the atmosphere, a separate small cylindrical tank called a conservator is fixed on the top of the transformer tank (see Fig. 3.16).

The whole of the transformer tank and a portion of the conservator are filled with oil. The breather is connected on one side of the conservator. Thus a small surface area of transformer oil is exposed to the atmosphere through the breather. Figure 3.17 shows the various important parts of a transformer so far discussed as viewed from different angles.

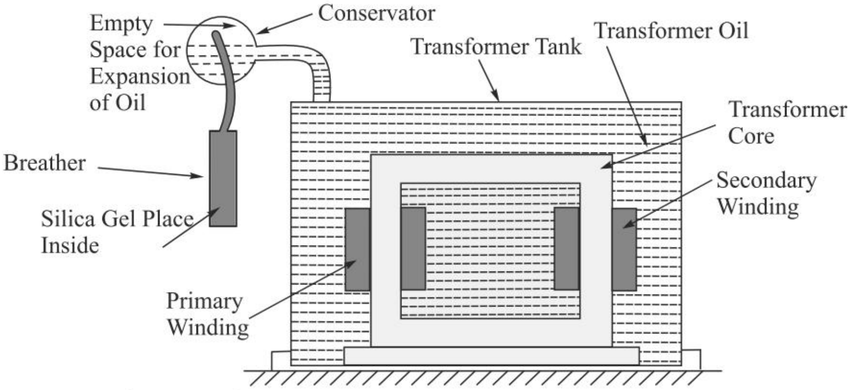


Fig. 3.16 *Transformer tank with conservator and breather*

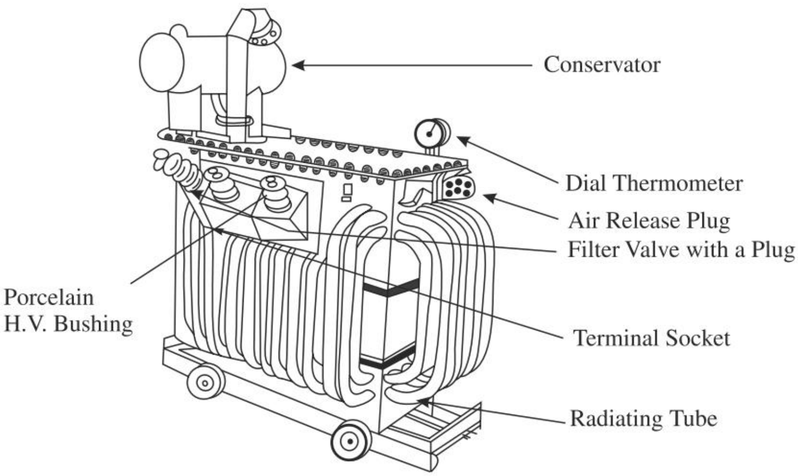


Fig. 3.17(a) *Outside view of a transformer*

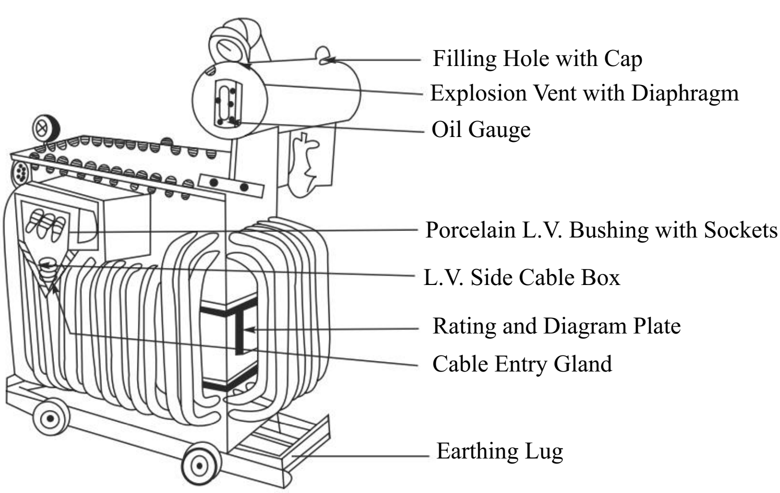


Fig. 3.17(b) *A transformer fitted with various accessories*

3.4 EMF EQUATION OF A TRANSFORMER

Figure 3.18 shows a transformer having N_1 and N_2 number of turns in the primary and secondary windings respectively. When alternating voltage V_1 of frequency f Hz is applied across the primary winding, current I_m will flow and magnetise the core. This will produce an alternating flux which while completing its path through the core will link both primary and secondary windings. As the current drawn by the primary winding is alternating at a frequency of f, the flux produced will also have the same frequency. This alternating flux will produce induced emf in the primary and secondary windings. The magnitude of the induced emf in the windings can be determined in the following way: the flux produced by the magnetising current will be sinusoidal as the supply voltage is sinusoidal. Thus the equation for core flux is $\phi = \phi_m \sin \omega t$, where ϕ_m is the maximum value of the core flux produced.

or $$\phi = \phi_m \sin 2\pi f t \qquad (\therefore\ \omega = 2\pi f)$$

Instantaneous value of induced emf,

$$e = -N\frac{d\phi}{dt}$$ (where N is the number of turns of the winding)

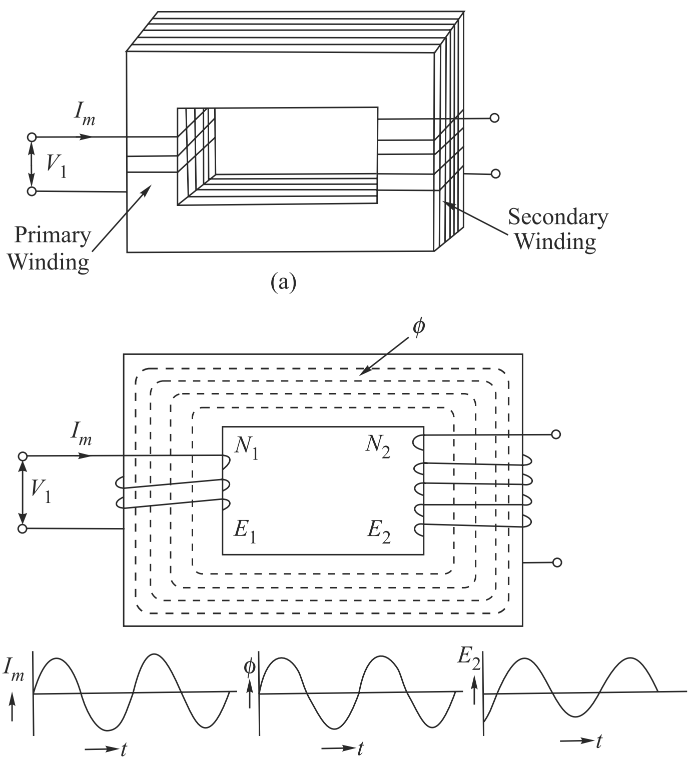


Fig. 3.18 *(a) A transformer having different number of turns in the primary and secondary windings (b) Emf is induced in the secondary winding due to linkage of changing flux produced by the primary current*

$$= -N \frac{d}{dt} \phi_m \sin 2\pi f t$$
$$= -N\, 2\pi f \phi_m \cos 2\pi f t$$
$$= 2\pi f \phi_m N \sin(2\pi f t - \pi/2) \text{ V}$$
$$= (2\pi f N \phi_m) \sin(\omega t - \pi/2) \text{ V}$$

This equation is of the form

$$e = E_m \sin(\omega t - \pi/2) \text{ V} \tag{3.1}$$

Thus the maximum value of the induced emf is

$$E_m = 2\pi f N \phi_m \text{ V}$$

Therefore, the rms value of the induced emf in a coil is

$$E = \frac{E_m}{\sqrt{2}} = \frac{2\pi f N \phi_m}{\sqrt{2}}$$
$$= 4.44\, \phi_m f N \text{ Volts}$$

The primary and secondary windings of the transformer have respectively N_1 and N_2 number of turns. Emf induced in the two windings would therefore be

$$E_1 = 4.44\, \phi_m f N_1 \text{ Volts} \tag{3.2}$$
$$E_2 = 4.44\, \phi_m f N_2 \text{ Volts} \tag{3.3}$$

Dividing Eq. (3.2) by Eq. (3.3)

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{3.4}$$

It can also be observed that the induced emf, $e = E_m \sin(\omega t - \pi/2)$ lags the flux $\phi = \phi_m \sin \omega t$ by 90°.

The emf equation can alternatively be deduced as follows. From Fig. 3.19, it is seen that the value of flux changes from $+\phi_m$ Wb to $-\phi_m$ Wb. The total change is $2\,\phi_m$ Wb in $T/2$ s, i.e., in the $1/(2f)$ seconds. Thus, average rate of change of flux linkage

$$= 2\,\phi_m / \left(\frac{1}{2f}\right) = 4 f \phi_m \text{Wb/s}$$

Rate of change of flux linkage causes induced emf.

Therefore, the average value of induced emf in a coil of N turns

$$E_{av} = 4 f \phi_m N \text{ Volts}$$

The ratio of the rms value and average value of a sinusoidal wave is 1.11. The rms value of induced emf is

$$E = 4 \times 1.11\, \phi_m f N \text{ Volts} = 4.44\, \phi_m f N \text{ Volts}$$

The rms values of the emf induced in the primary and secondary windings respectively are

$$E_1 = 4.44\, \phi_m f N_1 \text{ Volts}$$
$$E_2 = 4.44\, \phi_m f N_2 \text{ Volts}$$

From the above,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

When the secondary winding is open-circuited, the induced emf E_2 of the secondary is the same as its terminal voltage V_2 (see Fig. 3.19). The current drawn by the primary winding is small and is only about 3 to 5% of the current drawn under full-load condition. Therefore the induced voltage E_1 is approximately equal and opposite to the primary applied voltage V_1. ($V_1 = E_1 + I_1 Z_1$, $I_1 Z_1$ at no-load is very small and hence $V_1 \approx E_1$)

$$\frac{E_1}{E_2} \approx \frac{V_1}{V_2}$$

$$\therefore \qquad \frac{V_1}{V_2} \approx \frac{N_1}{N_2} \qquad (3.5)$$

The efficiency at full-load of a transformer is nearly equal to 100% (may be as high as 98%).So, we may assume input an approximately equal to output

In that case,

Input = Output (approximately)

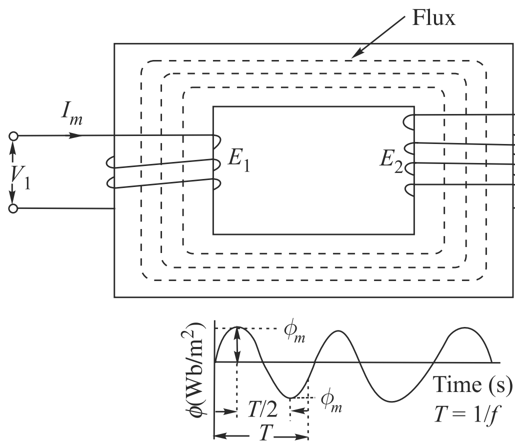


Fig. 3.19 *Emf is induced in the windings due to linkage of alternating flux*

Assuming losses to be negligibly small,

$$V_1 I_1 \cos \phi_1 = V_2 I_2 \cos \phi_2$$

At full-load $\qquad \cos \phi_1 = \cos \phi_2$

$$V_1 I_1 \approx V_2 I_2$$

or $$\frac{V_1}{V_2} \approx \frac{I_1}{I_2} \qquad (3.6)$$

From Eqs (3.4) and (3.5),

$$\frac{N_1}{N_2} \approx \frac{I_2}{I_1}$$

or $$N_1 I_1 \approx N_2 I_2 \qquad (3.7)$$

Thus the primary ampere turns of a transformer are approximately equal to its secondary ampere turns.

EXAMPLE 3.1 *A 500 kVA, 11000 V/400 V, 50 Hz, single-phase transformer has 100 turns on the secondary winding. Calculate (a) the approximate number of turns in the primary winding, (b) the approximate value of the primary and secondary currents, and (c) the maximum value of flux in the core.*

Solution From Eq. (3.5),

$$\frac{V_1}{V_2} \approx \frac{N_1}{N_2}$$

or

$$N_1 \approx \frac{V_1}{V_2} N_2 = \frac{11000 \times 100}{400}$$

$$= 2750 \text{ turns}$$

$$V_1 I_1 \approx V_2 I_2 = 500 \times 1000$$

$$I_1 \approx \frac{500 \times 1000}{11000} = 45.54 \text{ A}$$

$$I_2 = \frac{500 \times 400}{1000} = 1250 \text{ A}$$

Also,

$$E_1 = 4.44\, \phi_m f N_1 \text{ V}$$

But

$$E_1 = V_1$$

$\therefore$

$$11{,}000 = 4.44 \times \phi_m \times 50 \times 2750$$

or

$$\phi_m = \frac{11000}{4.44 \times 50 \times 2750} = \frac{22}{4.44 \times 275}$$

$$= 0.018 \text{ Wb}$$

EXAMPLE 3.2 *A 6600 V/230 V, 50 Hz single-phase core-type transformer has a core section 25 cm × 25 cm (Fig. 3.20). Calculate the approximate number of primary and secondary turns if the flux density in the core should not exceed 1.1 Wb/m^2.*

Solution

$$A = \frac{25}{100} \times \frac{25}{100} \text{ m}^2 = 6.25 \times 10^{-2} \text{ m}^2$$

$$\phi_m = B_m \times \text{A}$$

$$= 1.1 \times 6.25 \times 10^{-2} = 6.875 \times 10^{-2} \text{ Wb}$$

$$V_1 \approx E_1 \text{ and } V_2 \approx E_2$$

and

$$E = 4.44\, \phi_m f N$$

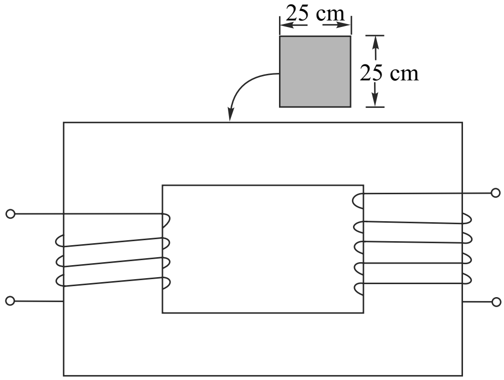


Fig. 3.20 *Core cross-section of the given transformer*

$$\therefore \quad 6600 = 4.44 \times 6.875 \times 10^{-2} \times 50 \times N_1$$

$$\therefore \quad N_1 = \frac{6600}{15.262} = 432 \text{ (approx)}$$

Similarly $$230 = 4.44 \times 6.875 \times 10^{-2} \times 50 \times N_2$$

$$\therefore \quad N_2 = \frac{230}{15.262} = 15 \text{ (approx)}$$

EXAMPLE 3.3 *A single-phase 50 Hz transformer has 100 turns on the primary winding and 400 turns on the secondary winding. The net cross-sectional area of the core is 250 cm². If the primary winding is connected to a 230 V, 50 Hz supply, determine (a) the emf induced in the secondary winding, and (b) the maximum value of the flux density in the core.*

Solution

$$\frac{V_1}{V_2} \approx \frac{N_1}{N_2}$$

$$V_2 \approx V_1 \left[\frac{N_2}{N_1}\right]$$

$$E_2 \approx V_1 \left[\frac{N_2}{N_1}\right] \text{ (since at no-load } E_2 = V_2\text{)}$$

$$= 230 \times \frac{400}{100} = 920 \text{ V}$$

The emf induced in the secondary winding is 920 V (approx.)

$$E_2 = 4.44\, \phi_m f N_2 \text{ V}$$

or $$920 = 4.44 \times \phi_m \times 50 \times 400$$

or $$\phi_m = 10.36 \times 10^{-3} \text{ Wb}$$

$$\therefore \qquad B_m = \frac{\phi_m}{A} = \frac{10.36 \times 10^{-3}}{250 \times 10^{-4}} \text{ Wb/m}^2 = 0.414 \text{ Wb/m}^2$$

Maximum flux density in the core

$$B_m = 0.414 \text{ Wb/m}^2$$

3.5 TRANSIENT IN-RUSH CURRENT IN A TRANSFORMER

The magnetizing current of a transformer is about three to five percent of its rated full-load current. This is the steady-state current. However, when a transformer is switched on to supply, an initial transient current (very short duration current) of high magnitude may flow through the primary winding. This transient *inrush current* may be as high as 10 to 20 times the rated current.

The magnitude of inrush current will depend upon the instant of switching on the supply voltage. The inrush current will be maximum when the transformer is switched on at the instant of zero input voltage. The inrush current will be minimized if switching is done at the pick of the input voltage. However, it may not be possible to select a particular instant of time on the input voltage for switching on a transformer.

This inrush current causes maximum mechanical stress on the transformer winding. The windings of the transformer are required to be firmly placed to be able to withstand the strong transient mechanical forces due to high inrush current.

When a transformer is switched on to supply, in the worst case, the transient inrush current many cause high value of core flux. The core flux may be two times its normal value. This is called *doubling effect*.

Further, when a transformer is switched off, it may not be possible to do it at zero flux crossing. Because of this, the core of the transformer will retain some residual flux, ϕ_r.

If the instant of switching happens to be at the instant of zero input voltage, and if the residual core flux is ϕ_r, the maximum value of core flux may become equal to $\phi = 2\phi_m + \phi_r$.

As mentioned earlier, the magnitude of inrush current depends on the instant of the voltage wave at which the transformer is connected to the supply.

We will consider two extreme cases, namely (a) the transformer is switched on when the voltage is maximum; and (b) the transformer is switched on when the input voltage passes through zero. We will assume in both the cases that residual flux in the core, i.e., $\phi_r = 0$.

Transformer is Switched on to the Supply Mains when the Voltage Wave Reaches its Maximum Let the supply voltage be, $v = V_m \sin \omega t$

$$\text{or,} \qquad v = \sqrt{2}\, V \sin \omega t \tag{3.8}$$

where V is the rms value.

Neglecting core losses and resistance of the primary winding,

$$v = -e = N \frac{d\phi}{dt}$$

or, $$d\phi = \frac{1}{N} v\, dt$$

or $$\phi = \frac{1}{N} \int v\, dt \tag{3.9}$$

From Eqs (3.8) and (3.9),

$$\phi = \frac{1}{N} \int \sqrt{2}\, V \sin \omega t\, dt$$

$$\phi = \frac{\sqrt{2}V}{N} \int \sin \omega t\, dt \tag{3.10}$$

$$\phi = \frac{\sqrt{2}V}{N\omega} \sin(\omega t - 90°) = \phi_m \sin(\omega t - 90°) \tag{3.11}$$

for $\omega t > 90°$

where $$\phi_m = \frac{\sqrt{2}V}{N\omega} \tag{3.12}$$

There is no transient in flux and there is no inrush current as has been shown in Fig. 3.21.

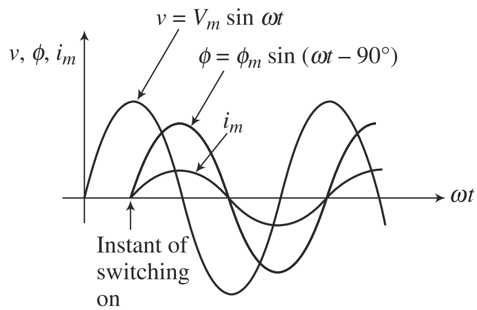


Fig. 3.21 *Transformer switch on to supply when voltage is at maximum*

It can be seen that no heavy inrush current will flow and the quantities are at their steady state values. The magnetizing current, i_m will contain third harmonic components as created due to non-linear characteristics of the *B-H* loop for the magnetic material of the transformer core.

Transformer is Switched on to the Supply when the Voltage Wave Passes Through Zero From Eq. (3.10),

$$\phi = \frac{\sqrt{2}V}{N} \int_0^t \sin \omega t\, dt$$

$$= \frac{\sqrt{2}V}{N\omega}(1 - \cos \omega t)$$

$$= \phi_m - \phi_m \cos \omega t \qquad \text{[from Eq. (3.11)]} \tag{3.13}$$

When $\omega t = \pi$,

$$\phi = 2\,\phi_m$$

If there is residual flux in the core as ϕ_r, then total flux, $\phi = 2\,\phi_m + \phi_r$.

Figure 3.22 shows the variation of input voltage, core flux, and the magnetizing current. It is observed that the peak value of flux has doubled and the corresponding magnetizing current has reached a very high peak value. This large inrush current will quickly decay because of the winding resistance as has been shown in Fig. 3.22 (b).

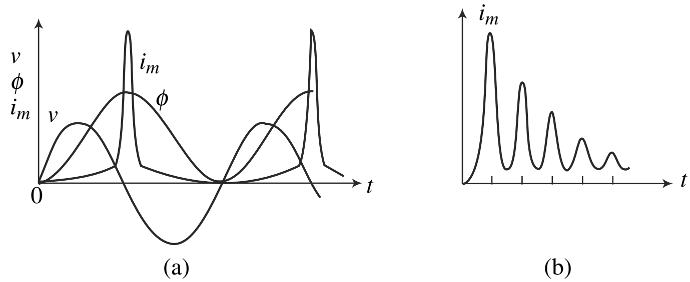


Fig. 3.22 *(a) Inrush current of a transformer (b) Decay of inrush current*

In a three-phase transformer the inrush current will always be there irrespective of the time of switching. This is because voltages of all the three phases are never equal to zero or equal to maximum simultaneously.

3.6 PHASOR DIAGRAM AND EQUIVALENT CIRCUIT OF A TRANSFORMER

The phase relationship between the various quantities like applied voltage, core flux, induced emf and current in the windings, and voltage drop in the windings can be expressed in the form of a phasor diagram. Based on the phasor diagram an equivalent electrical circuit representing a transformer can be drawn. Performance characteristics like voltage regulation and efficiency of a transformer for different power factor loads can be calculated using the equivalent circuit.

3.6.1 Phasor Diagram of a Transformer on No-load

Under the no-load condition the secondary winding is open and therefore no current flows through it. When an alternating voltage V_1 is applied across the primary winding, current I_0 will flow which will set up a flux in the core (Fig. 3.23a). A part of the current I_0 will however be spent in supplying iron-losses (hysteresis loss and eddy-current loss) in the core and a small amount of $I_0^2\,R_t$ loss in the primary winding. Alternating flux ϕ, which is common to both the primary and secondary windings, will induce emf in both the windings. The magnitude of induced emf will depend upon their number of turns. If the number of primary and secondary turns are equal, then E_1 will be equal to E_2. The induced emfs due to the linking of changing flux will lag the flux by 90° [see Eq. (3.1)].

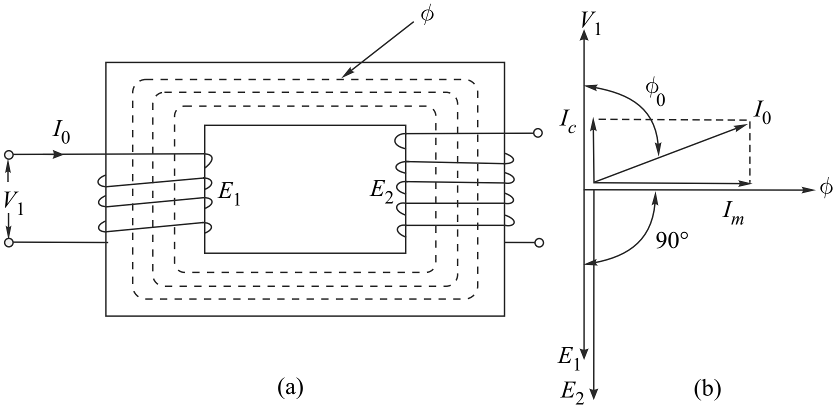


Fig. 3.23 *(a) Transformer on no-load (b) Phasor diagram of a transformer on no-load*

The magnitude of the induced emf in the primary winding, i.e., E_1 will be approximately equal (slightly less) but opposite to the applied voltage V_1. Therefore phasor E_1 should be drawn at 180° to phasor V_1 [see Fig. 3.23(b)]. The no-load current I_0 is divided up into two components I_m and I_c. I_m will lag voltage V_1 by 90° since in a loss-less inductor coil the current lags the applied voltage by 90°. However, the loss component of current I_0, i.e., I_c will be in phase with V_1, $V_1 I_c$ is equal to the core-loss. In a transformer I_c is kept small by using laminated core and silicon-steel material as mentioned in the earlier section. From the phasor diagram,

$$I_0 \cos \phi_0 = I_c \qquad \therefore \cos \phi_0 = \frac{I_c}{I_0}$$

$$I_0 \sin \phi_0 = I_m$$

$$I_0^2 = I_m^2 + I_0^2$$

$$I_0 = \sqrt{I_m^2 + I_c^2}$$

To sum up, referring to the phasor diagram, the primary current I_0 has two components I_m and I_c. I_m lags behind V_1 by 90°. I_m produces flux in the core which links both the windings. Flux ϕ which is common to both primary and secondary windings is taken as the reference axis. This flux induces emf E_1 in the primary winding and E_2 in the secondary winding. The two emf E_1 and E_2 lag the flux by 90°, their magnitudes depending on the number of turns of the respective windings. The induced emf E_1 will be approximately equal but opposite to applied voltage V_1. Further, a component of current I_0, i.e., I_c is in phase with V_1 such that $V_1 I_c$ = core-loss.

3.6.2 Phasor Diagram of a Transformer on Load Assuming no Voltage Drop in the Windings

For simplicity it is assumed for the time being that there is no voltage drop in the windings of the transformer. It will be included in the next section. The no-load phasor diagram is redrawn here in Fig. 3.24(a).

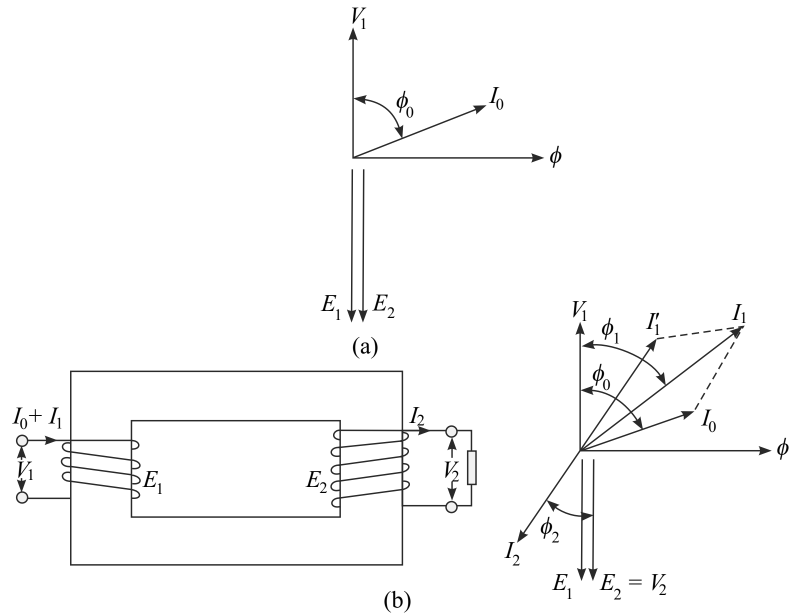


Fig. 3.24 *(a) Phasor diagram of a transformer on no-load assuming equal numbers of primary and secondary turns; (b) Phasor diagram on load assuming no voltage drops in the windings*

The secondary terminal voltage V_2 is the same as the secondary induced emf E_2 as voltage drop in the winding has not been considered. The load current flowing through the secondary winding is I_2 and the load power factor is cos ϕ_2. I_2 is drawn lagging V_2 by an angle ϕ_2 (assuming a lagging power-factor load). The primary winding, in adding to no-load current I_0, will draw a current I_1' which will neutralise the demagnetising effect of the secondary current I_2. Current I_1', therefore, has been drawn equal and opposite to I_2 [see Fig. 3.24(b)]. If the number of primary and secondary turns are different, then $I_1 = I_2 N_2 / N_1$ (as $I_1' N_1 = I_2 N_2$). The phasor sum of $\underline{I_1}'$ and I_0 is equal to the total current I_1 drawn by the primary winding.

3.6.3 Mutual and Leakage Flux, Leakage Reactance, and Equivalent Circuit of a Transformer

Mutual and Leakage Flux It was assumed earlier that all the flux produced by the primary magnetising current will pass through the core and they will link both the windings. But in actual practice a small percentage of the flux will complete their path through the air. Their percentage will be very small as the reluctance of air is much higher than that of iron. When the secondary is connected to the load some flux produced by the secondary current will also complete their path through the air. In other words, there will be some flux that will link only the primary winding and some that will link only the secondary winding. The major amount of the flux will however pass through the magnetic core and will link both the windings. The core flux common to both the windings are called useful flux or mutual flux or main flux whereas flux linking individual windings only are called leakage flux as shown in

Fig. 3.25. The main flux induces emf E_1 and E_2 in the two windings. The leakage flux will also induce some emf in the two windings. The two emfs induced due to the main flux and leakage flux can be thought to be equivalent to voltage drop due to two reactances, namely the voltage drop due to magnetising reactance and that due to leakage reactance respectively. The leakage fluxes are proportional to the currents flowing through the respective windings as more is coil current more is leakage flux.

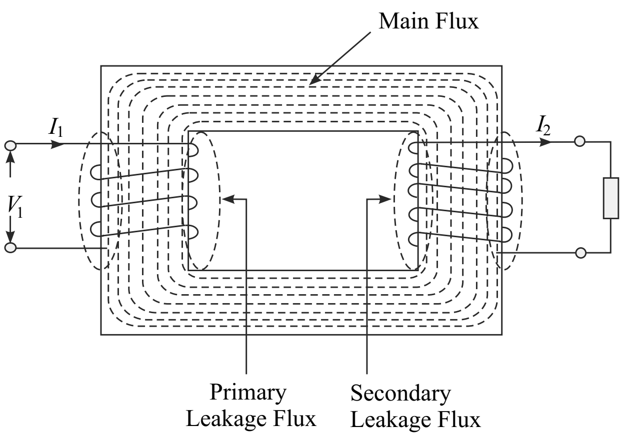


Fig. 3.25 *Main flux and leakage flux in a transformer*

Leakage Reactance The effect of leakage flux of the two windings can be considered as equivalent to two reactors X_1 and X_2 connected respectively in series with primary and secondary windings causing voltage drops. In addition, there will be voltage drop in the primary and secondary windings due to their winding resistances.

The magnitude of leakage flux and hence of leakage reactance can be reduced by increasing the magnetic coupling between the primary and secondary windings.

3.6.4 Equivalent Circuit of a Transformer

Primary current I_1 in a transformer has two components I_0 and I_1' ((Fig. 3.26). Current I_1' will counter balance the secondary current I_2. I_0 will magnetise the core and in the process produce losses in the core. E_1 is the emf induced in the primary due to core flux ϕ. E_1 will oppose the applied voltage V_1, its magnitude being slightly less than V_1. The difference of voltage V_1 and E_1 will be the drop across the winding resistance R_1 and its leakage reactance X_1. Magnetising current I_m produces flux which induces an emf E_1. The reactance due to flux ϕ of the primary is X_m such that $E_1 = I_m X_m$. Therefore, I_m is shown flowing through a magnetising reactance X_m. The losses in the core, i.e., hysteresis loss and eddy-current loss are proportional to core flux and can be represented by a resistance R_m connected parallel to the magnetising reactance X_m. The component current I_c of the no-load current I_0 flows through R_m such that

$$I_c^2 R_m = \frac{E_1^2}{R_m} = \text{Core loss}$$

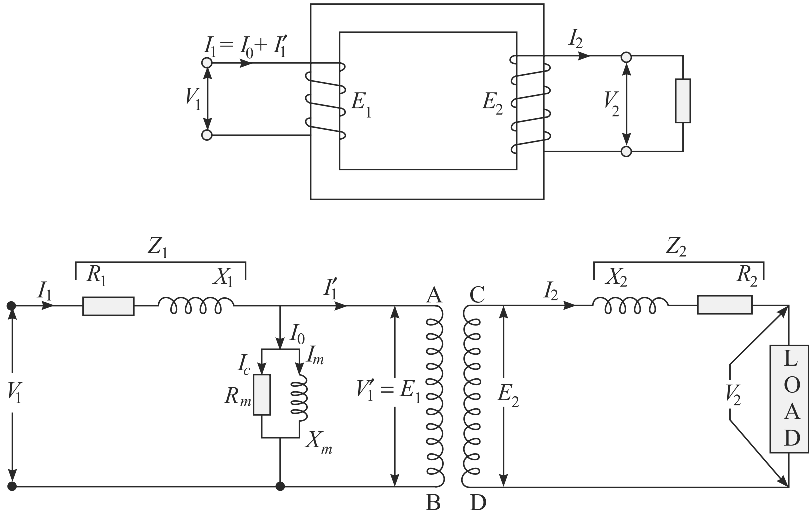


Fig. 3.26 *Equivalent circuit of a transformer*

As shown in Fig. 3.26, current I_1', shown flowing through the primary winding with resistance and reactance shown separately counterbalances current I_2 of the secondary winding. Similarly, the secondary-winding resistance and reactance have been shown separately in the secondary winding.

If N_1 and N_2 are equal, then E_1 will be equal to E_2 (as $N_1/N_2 = E_1/E_2$). When $E_1 = E_2$, terminals *A* and *B* of the primary winding in Fig. 3.26 can be connected directly to terminals *C* and *D* respectively of the secondary, without disturbing the circuit conditions. However, if E_1 and E_2 are unequal, then the secondary circuit quantities X_2 and R_2 should be converted to the primary voltage level, i.e., to the level of E_1 so that the two circuits can be connected together which will facilitate simplified calculations. Similarly, the primary circuit quantities also can be transferred to the secondary voltage levels and the primary circuit can be connected to the secondary circuit at points *AB* and *CD*.

Approximate Equivalent Circuit of a Transformer The no-load current I_0 drawn by a transformer is about 3 to 5% of the rated full-load current. As such, the parallel branch of the primary circuit of Fig. 3.26 can be neglected, for the purpose of simplification, without introducing much error. The idea is to arrive at a simple circuit which will make calculation very easy. Thus the equivalent circuit of Fig. 3.26 can be drawn as shown in Fig. 3.27. The transformer is assumed to have no core-loss. Also, for magnetisation it is assumed that no current is drawn by the transformer. The winding resistances and leakage reactances have also been shown separately. Thus the transformer indicated as an ideal transformer in Fig. 3.27 shown inside a dotted rectangle can be considered as a transformer having no core-loss and having no winding resistance and reactance.

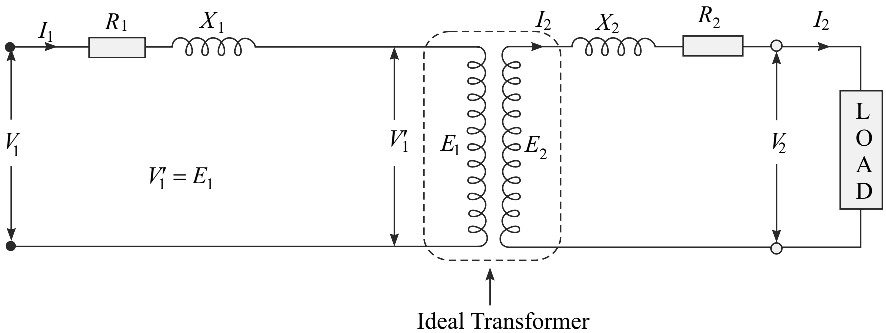


Fig. 3.27 *Approximate equivalent circuit of a transformer*

The equivalent circuit shown in Fig. 3.27 can be further simplified by transferring the secondary-circuit parameters to the primary circuit or vice versa. The secondary-circuit parameters R_2 and X_2 can be transferred to the primary circuit without disturbing the circuit conditions as follows: R_2 of the secondary can be transferred to the primary circuit by introducing an additional resistance R_2' in the primary such that the power absorbed in R_2' when carrying primary current I_1 is equal to the power absorbed by R_2 when carrying secondary current I_2. Therefore,

$$I_1^2 R_2' = I_2^2 R_2$$

from which
$$R_2' = \left[\frac{I_2}{I_1}\right]^2 R_2$$

It is already known that

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{V_1}{V_2}$$

$$R_2' \approx \left[\frac{N_1}{N_2}\right]^2 R_2$$

$$\approx \left[\frac{V_1}{V_2}\right]^2 R_2 \tag{3.14}$$

Similarly to transfer X_2, in the primary circuit it may be realised that the inductance and hence the inductive reactance of a coil is proportional to the square of the number of turns.

Inductance,
$$L = N\frac{d\phi}{di}$$

Assuming a linear relationship between ϕ and i, L can be written as:

$$L = N\frac{\phi}{I}$$

$$= N\frac{BA}{I}$$

(where, Flux, ϕ = Flux density $B \times$ Area of cross-section of the core A)

$$= N\frac{\mu HA}{I}$$

$$= N\mu\left[\frac{NI}{l}\right]\frac{A}{I} \qquad \left(\because H = \frac{NI}{l}\right)$$

$$\therefore \qquad L = \mu\frac{N^2A}{l}$$

where μ = permeability of the core material of the inductor,
N = number of turns of the coil

and $$X = \omega L = 2\,\pi f L$$

where A = core cross-sectional area
l = length of the coil

$$X \propto N^2$$

Therefore, X_2 referred to primary, i.e., $X_2' = \dfrac{X_2}{N_2^2} \times N_1^2$

$$X_2' = \left[\frac{N_1}{N_2}\right]^2 X_2 \approx \left[\frac{V_1}{V_2}\right]^2 X_2 \qquad (3.15)$$

Thus the approximate equivalent circuit with secondary quantities referred to the primary can be represented as Fig. 3.28.

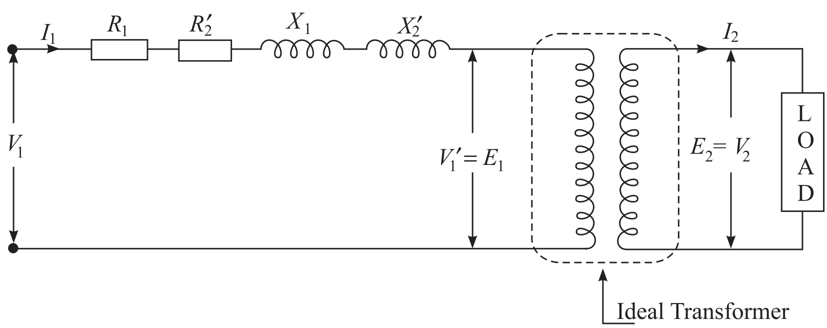


Fig. 3.28 *Approximate equivalent circuit of a transformer with secondary quantities referred to the primary side*

Equivalent impedance, Z_e' of the transformer,

$$Z_e' = \sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}$$

$$= \sqrt{R_e'^2 + X_e'^2}$$

where $$R_e' = R_1 + R_2'$$

$$= R_1 + \left[\frac{N_1}{N_2}\right]^2 R_2$$

and $$X'_e = X_1 + X'_2$$

$$= X_1 + \left[\frac{N_1}{N_2}\right]^2 X_2$$

From this simplified equivalent circuit it is easy to calculate voltage regulation of a transformer which is being dealt with in the following section.

3.6.5 Equivalent Circuit and Phasor Diagram of a Transformer on Load

For reference purpose the equivalent circuit of a transformer is redrawn in Fig. 3.29. The phasor diagram of a transformer on load is shown in Fig. 3.29(b). Here the voltage drops in the winding due to winding resistance and reactances have also been included. For reference the reader may refer to the phasor diagram of a loaded transformer assuming no voltage drop in the windings drawn in Fig. 3.24(b).

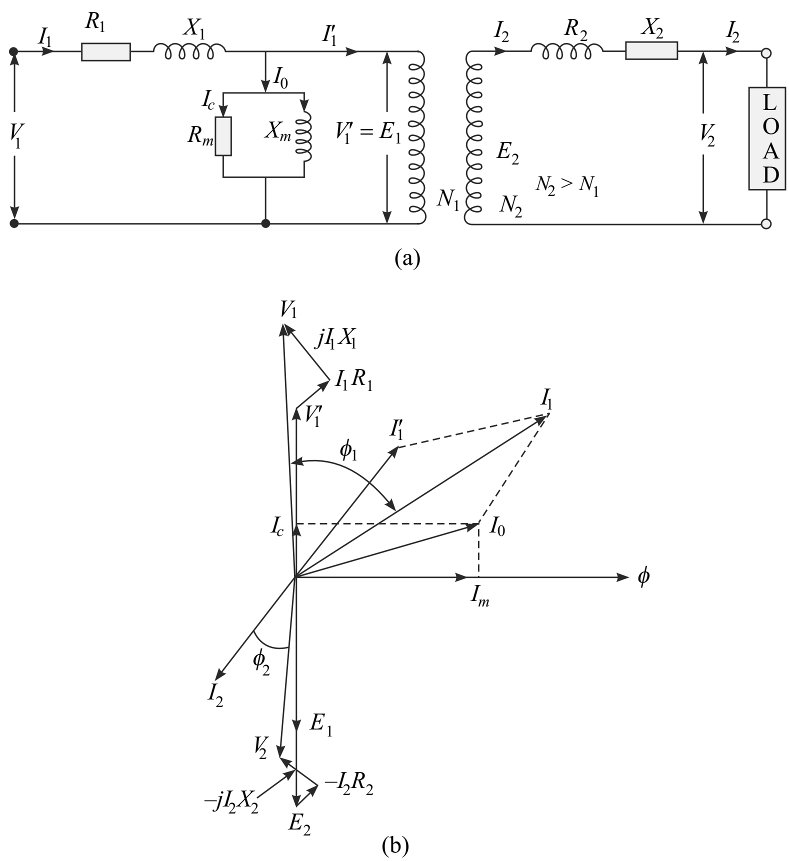


Fig. 3.29 *(a) Equivalent circuit of a transformer on load*
(b) Phasor diagram of a transformer on load

In Fig. 3.29(b) the voltage drops in the windings due to the resistances are shown in phase with the currents flowing through them and the drops in the reactances are shown at right angles to the current according to the following relations

$$V_1 = V_1' + I_1 R_1 + j I_1 X_1 \tag{3.16}$$

$$V_2 = E_2 - I_2 R_2 - j I_2 X_2 \tag{3.17}$$

where phasor E_2 has been shown longer than E_1 since N_2 is greater than N_1.

EXAMPLE 3.4 *A 40 kVA, 2000/250 V transformer has a primary resistance of 1.15 Ω and a secondary resistance of 0.0155 Ω. Calculate (a) the total resistance in terms of the secondary winding, (b) the total resistance drop on full-load, and (c) the total copper loss on full-load.*

Solution Let the primary resistance R_1 when referred to secondary side be R_1''. Considering equal power loss,

$$I_1^2 R_1 = I_2^2 R_1''$$

or $$R_1'' = \left[\frac{I_1}{I_2}\right]^2 R_1$$

Here, $$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

(a) The total resistance of the transformer in terms of the secondary winding,

$$R_e'' = R_2 + R_1''$$

$$= R_2 + \left[\frac{V_2}{V_1}\right]^2 R_2$$

$$= 0.0155 + \left[\frac{250}{2000}\right]^2 \times 1.15 = 0.03346\ \Omega$$

(b) Full load secondary current,

$$I_2 = \frac{40 \times 1000}{250} = 160\text{ A}$$

Total resistance drop on full load $= I_2 R_e''$

$$= 160 \times 0.0334 = 5.35\text{ Volts}$$

(c) Total copper loss on full load $= I_2^2 R_e''$

$$= (160)^2 \times 0.0334 = 855\text{ Watts}$$

EXAMPLE 3.5 *A single-phase transformer has 1200 turns on the primary and 300 turns on the secondary. The no-load current is 2.5 A and the no-load power factor is 0.2 lagging. Calculate the current and power factor of the primary circuit when the secondary draws a current of 300 A at a power factor of 0.8 lagging. The voltage may be neglected.*

Solution

$$I_1' N_1 = I_2 N_2$$

$$I_1' = \frac{300 \times 300}{1200} = 75 \text{ A}$$

$$I_0 = 2.5 \text{ A}$$

$$\cos \phi_0 = 0.2$$

$$\sin \phi_0 = 0.98$$

$$\cos \phi_2 = 0.8$$

$$\sin \phi_2 = 0.6$$

From Fig. 3.30, resolving the vertical and horizontal components,

$$I_1 \cos \phi_1 = I_1' \cos \phi_2 + I_0 \cos \phi_0$$

$$= 75 \times 0.8 + 2.5 \times 0.2 = 60.5 \text{ A}$$

$$I_1 \sin \phi_1 = I_1' \sin \phi_2 + I_0 \sin \phi_0$$

$$= 75 \times 0.6 + 2.5 \times 0.98$$

$$= 45 + 2.45 = 47.45 \text{ A}$$

$$I_1 = \sqrt{(I_1 \cos\phi_1)^2 + (I_1 \sin\phi_1)^2}$$

$$= \sqrt{(60.5)^2 + (47.45)^2} = 76.88 \text{ A}$$

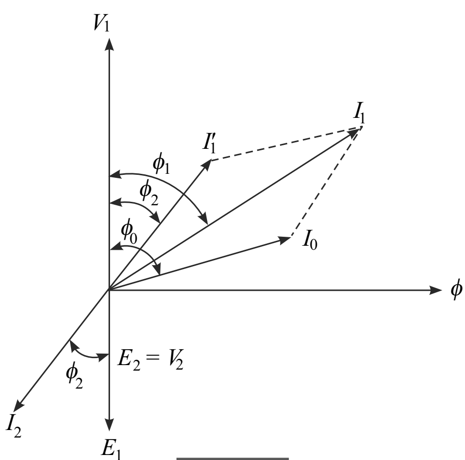


Fig. 3.30

Primary current = 76.88 A

$$\tan \phi_1 = \frac{I_1 \sin\phi_1}{I_1 \cos\phi_1} = \frac{47.45}{60.5} = 0.78$$

$$\phi_1 = 38°$$

∴ Primary power factor,

$$\cos \phi_1 = \cos 38° = 0.788$$

EXAMPLE 3.6 *A transformer on no-load takes 1.5 A at power factor 0.2 lagging when connected across a 50 Hz, 230 V supply. The ratio between the primary and secondary number of turns is 3. Calculate the value of the primary current when the secondary is supplying a current of 40 A at a power factor of 0.8 lagging. Neglect the voltage drop in the windings. Draw the relevant phasor diagram.*

Solution Given

$$I_0 = 1.5 \text{ A}$$
$$\cos \phi_0 = 0.2$$
$$\phi_0 = 78°$$
$$I_2 = 40$$
$$\cos \phi_2 = 0.8$$
$$\phi_2 = 37°$$

It is known that,

$$I_1' N_1 = I_2 N_2$$

$$I_1' = \frac{N_2}{N_1} I_2 = \frac{1}{3} \times 40 = 13.33 \text{ A}$$

Figure 3.31 shows the transformer and its phasor diagram. From the phasor diagram, resolving the vertical and horizontal components of I_1', I_1 and I_0

$$I_1 \cos \phi_1 = I_1' \cos \phi_2 + I_0 \cos \phi_0$$
$$= 13.33 \times 0.8 + 1.5 \times 0.2$$
$$= 10.964 \text{ A} \quad \text{(i)}$$

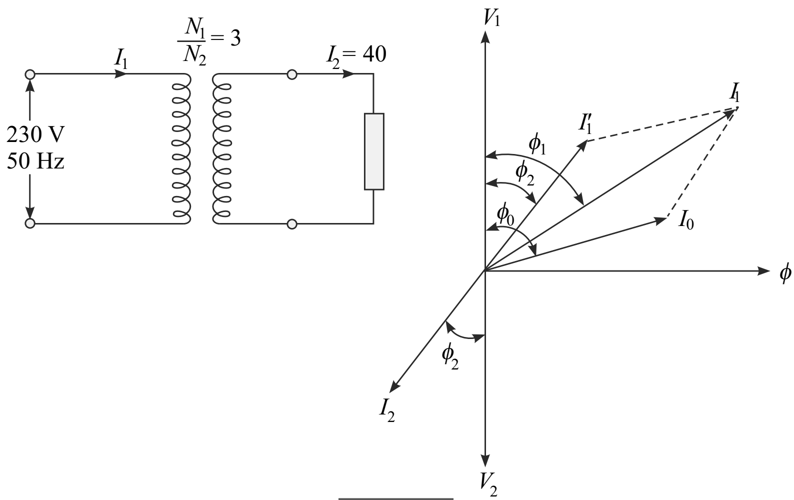


Fig. 3.31

and $$I_1 \sin \phi_1 = I_1' \sin \phi_2 + I_0 \sin \phi_0$$

$$= 13.33 \times 0.6 + 1.5 \times 0.978 = 9.47 \text{ A} \quad \text{(ii)}$$

From Eqs (i) and (ii)

$$I_1^2 \sin^2 \phi_1 + I_1^2 \cos^2 \phi_1 = (10.964)^2 + (9.47)^2$$

$$I_1 = \sqrt{[10.964]^2 + [9.47]^2} = 14.5 \text{ A (approx.)}$$

EXAMPLE 3.7 *A transformer on no-load takes 4.5 A at a power factor of 0.25 lagging when connected to a 230 V, 50 Hz supply. The number of turns of the primary winding is 250. Calculate (a) the magnetising current, (b) the core loss, and (c) the maximum value of the flux in the core.*

Solution Given

$$V_1 = 230 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$N_1 = 250$$

$$I_0 = 4.5 \text{ A}$$

$$\cos \phi_0 = 0.25$$

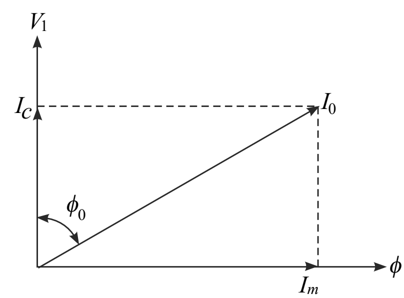


Fig. 3.32

The phasor diagram is shown in Fig. 3.32.

Magnetising current

$$I_m = I_0 \sin \phi_0$$

$$= 4.5 \times 0.968$$

$$= 4.35 \text{ A}$$

Core-loss $$P_c = V_1 I_0 \cos \phi_0$$

$$= 230 \times 4.5 \times 0.25 \text{ W} = 259 \text{ W}$$

(Neglecting a small amount of $I^2 R$ loss in the primary winding at no-load)

It can be assumed that

$$E_1 \approx V_1$$

$$V_1 = 4.44 \phi_m f N_1$$

or $$230 = 4.44 \phi_m \times 50 \times 250$$

$$\phi_m = \frac{230}{4.44 \times 50250} \text{ Wb} = 4.14 \times 10^{-3} \text{ Wb}$$

EXAMPLE 3.8 *A 660 V/220 V single-phase transformer takes a no-load current of 2 A at a power factor of 0.225 lagging. The transformer supplies a load of 30 A at a power factor of 0.9 lagging. Calculate the current drawn by the primary from the mains and the primary power-factor. Neglect the winding resistance and reactance.*

Solution Given

$$I_0 = 2 \text{ A}$$
$$\cos \phi_0 = 0.225, \phi_0 = 77°$$
$$I_2 = 30 \text{ A}$$
$$\cos\phi_2 = 0.9, \phi_2 = 26°$$
$$\sin \phi_2 = \sin 26° = 0.438$$
$$N_1 I_1' = N_2 I_2$$
$$I_1' = \frac{N_2}{N_1} \times I_2 \approx \frac{V_2}{V_1} \times I_2$$
$$= \frac{220}{660} \times 30 = 10 \text{ A}$$

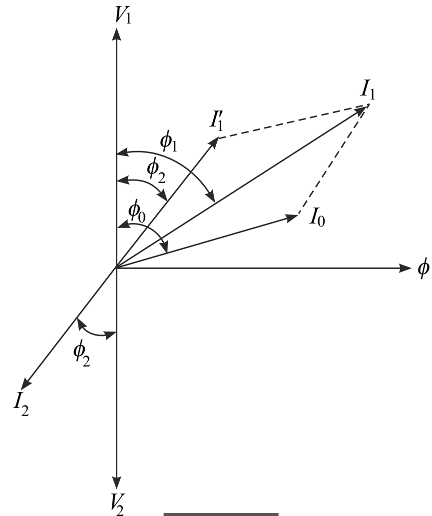


Fig. 3.33

The phasor diagram for the transformer is shown in Fig. 3.33.

$$I_1 \cos \phi_1 = I_1' \cos \phi_2 + I_0 \cos \phi_0$$
$$= 10 \times 0.9 + 2 \times 0.225$$
$$= 9.45 \text{ A} \quad \text{(i)}$$
$$I_1 \sin \phi_1 = I_1' \sin \phi_2 + I_0 \sin \phi_0$$
$$= 10 \times 0.438 + 2 \times 0.974 = 6.33 \text{ A} \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i)

$$\tan \phi_1 = \frac{6.33}{9.45} = 0.672, \phi_1 = 34°$$

Primary power-factor,

$$\cos \phi_1 = 0.829$$
$$I_1 = \sqrt{[9.45]^2 + [6.33]^2} = 11.38 \text{ A}$$

EXAMPLE 3.9 *A single-phase transformer has a turns ratio of 144/432 and operates at a maximum flux of 7.5 × 10^{-3} Wb, at 50 Hz. When on no-load, the transformer takes 0.24 kVA, at a power factor of 0.26 lagging from the supply. If the transformer supplied a load of 1.2 kVA at a power factor of 0.8 lagging, determine (a) the magnetising current, (b) the primary current, and (c) the primary power-factor.*

Solution

$$V_1 \approx E_1$$
$$= 4.44\, \phi_m f N_1$$
$$= 4.44 \times 7.5 \times 10^{-3} \times 50 \times 144 = 240 \text{ V}$$
$$I_0 = \frac{0.24 \times 1000}{V_1}$$

$$= \frac{0.24 \times 1000}{240} = 1 \text{ A}$$

$$\cos \phi_0 = 0.26$$

$$\phi_0 = 75°$$

$$I_m = I_0 \sin \phi_0 = 1 \sin 75° = 0.97 \text{ A}$$

$$V_2 \approx E_2$$

$$= E_1 \times \frac{N_2}{N_1} = 240 \times \frac{432}{144} = 720 \text{ V}$$

$$I_2 = \frac{1.2 \times 1000}{720} = 1.67 \text{ A}$$

$$I_1' = 1.67 \times \frac{432}{144} = 5.0\text{A}$$

$$I_1 = \sqrt{(I_1' \sin\phi_2 + I_0 \sin\phi_0)^2 + (I_1' \cos\phi_2 + I_0 \cos\phi_0)^2}$$

$$= \sqrt{(5 \times 0.6 + 0.97)^2 + (5 \times 0.8 + 1 \times 0.26)^2}$$

$$= \sqrt{33.9} = 5.82 \text{ A}$$

and
$$\cos \phi_1 = \frac{I_1' \cos\phi_2 + I_0 \cos\phi_0}{I_1} = \frac{5 \times 0.8 + 1 \times 0.26}{5.28}$$

$$= \frac{4.26}{5.28} = 0.73 \text{ lagging}$$

Example 3.10 *A small substation has a single-phase 6600/240 V transformer supplying four feeders which take the following loads:*

(i) 10 kW at 0.8 p.f. lag
(ii) 50 A at 0.7 p. f. lag
(iii) 5 kW at unity p.f.
(iv) 8 kVA at 0.6 p.f. lead

Determine the primary current and the power factor which the transformer takes from the 6600 V system. Neglect losses in the transformer.

Solution First calculate the total current delivered by the secondary to the four loads. Let I_1, I_2, I_3 and I_4 be the currents supplied to the four loads at power factors $\cos \phi_1$, $\cos \phi_2$, $\cos \phi_3$ and $\cos \phi_4$.

$$I_1 = \frac{10 \times 1000}{0.8 \times 240} \text{ at 0.8 lagging p.f.}$$

$$I_2 = 50 \text{ A at 0.7 lagging p.f.}$$

$$I_3 = \frac{5 \times 1000}{1 \times 240} \text{ at 1.0 p.f.}$$

$$I_4 = \frac{8 \times 1000}{240} \text{ at 0.6 leading p.f.}$$

Let I_5 be the total current supplied by the secondary at a power factor cos ϕ. Resolving all the currents in the horizontal and vertical axes

$$I_H = I_1 \cos \phi_1 + I_2 \cos \phi_2 + I_3 + I_4 \cos \phi_4$$

$$I_V = I_1 \sin \phi_1 + I_2 \sin \phi_2 + 0 - I_4 \sin \phi_4$$

$$I_5 = \sqrt{I_H^2 + I_V^2}$$

$$= \sqrt{(4166 + 35 + 20.83 + 20)^2 + (312 + 35 - 26.6)^2}$$

$$= 124 \text{ A}$$

Therefore the current drawn by the primary from 6600 V mains is equal to

$$I_P = I_s \frac{V_s}{V_P} = 124 \times \frac{240}{6600} = 4.51 \text{ A}$$

$$\cos \phi = \cos \tan^{-1} \frac{I_v}{I_H} = \cos \tan^{-1} \frac{39.6}{117.5}$$

$$= \cos 19° = 0.94 \text{ lagging}$$

EXAMPLE 3.11 *A 100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 and 0.01 Ω respectively, and the corresponding leakage reactances are 1.1 and 0.035 Ω respectively. Calculate the equivalent impedance referred to the primary circuit.*

Solution Equivalent impedance of the transformer referred to the primary side is given by

$$Z_e' = \sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}$$

$$R_2' = \left[\frac{N_1}{N_2}\right]^2 R_2 = \left[\frac{400}{80}\right]^2 \times 0.01 = 0.25 \ \Omega$$

$$X_2' = \left[\frac{N_1}{N_2}\right]^2 X_2 = \left[\frac{400}{80}\right]^2 \times 0.035 = 0.875 \ \Omega$$

$$Z_e' = \sqrt{(0.3 + 0.25)^2 (1.1 + 0.875)^2} = 2.05 \ \Omega$$

3.6.6 Concept of an Ideal Transformer

In a transformer there is core loss and I^2R loss in the windings which is called the copper loss. Even when on no-load, i.e., when output is zero, there is some input power which is mainly due to core loss. When the transformers is loaded, there is, in addition to core loss, there will be $I_1^2R_1$ loss in the primary winding and $I_2^2R_2$ loss in the secondary winding. Also, there is voltage drop in the windings due to current flow through them. In the approximate equivalent circuit of the transformer we neglect the parallel branch through which no-load current flows. The resistances and reactances of the windings are shown separately so that the windings can be assumed having no resistance and leakage reactance. If all losses and voltage drops are taken out of a transformer, it becomes an ideal transformer.

So, an ideal transformer is one which has no core loss, no winding loss, the windings have no resistance and reactance and hence no voltage drops in them when current flows through them. The efficiency of such an ideal transformer is 100 percent. Since there is no voltage drop in the windings, the voltage regulation (which is the change in voltage from no-load to full-load) is zero. Such ideal conditions, however, is not possible to achieve, although desirable.

3.7 VOLTAGE REGULATION OF A TRANSFORMER

Voltage regulation of a transformer is defined as the change in secondary terminal voltage from no-load to full-load and is expressed as a percentage of either no-load or the full-load value. (Voltage regulation can also be calculated as a percentage of no-load voltage).

$$\text{Voltage regulation} = \frac{\text{No load voltage} - \text{Full load voltage}}{\text{Full load voltage}}$$

The expression for voltage regulation can be derived from the approximate equivalent circuit of the transformer and the associated phasor diagram.

The approximate equivalent circuit of a transformer referred to the secondary side and the associated phasor diagrams are shown in Fig. 3.34 (a), (b) and (c). In the figure the primary resistance and reactance have been shown referred to the secondary. Primary and secondary resistances are shown together as R_e''. Primary and secondary reactance are shown together as X_e'', where,

$$R_e'' = R_2 + R_1'' \text{ and } X_e'' = X_2 + X_1''$$

To calculate R_1'' we equate power-loss in the two equivalent circuits as:

$$I_2^2 R_1'' = I_1^2 R_1$$

$$R_1'' = \left[\frac{I_1}{I_2}\right]^2 R_1 = \left[\frac{N_2}{N_1}\right]^2 R_1$$

$$\therefore \qquad R_e'' = R_2 + \left[\frac{I_1}{I_2}\right]^2 R_1 = R_2 + \left[\frac{N_2}{N_1}\right]^2 R_1$$

Similarly, $X_e'' = X_2 + X_1''$

$$X_e'' = X_2 + \left[\frac{N_2}{N_1}\right]^2 X_1$$

(Since reactances are proportional to the square of the number of winding turns) From the triangle OCB of Fig. 3.34 (b) (for lagging power factor)

$$OB^2 = OC^2 + CB^2$$

or $$OB^2 = (OA + AC)^2 + CB^2$$

$$E_2^2 = [V_2 + I_2 Z_e'' \cos(\phi_e - \phi_2)]^2 + [I_2 Z_e'' \sin(\phi_e - \phi_2)]^2$$

In actual practical $I_2 Z_e'' \sin(\phi_e - \phi_2)$ is very small compared to E_2 and therefore E_2 can be written as

$$E_2 = V_2 + I_2 Z_e'' \cos(\phi_e - \phi_2)$$

$$E_2 - V_2 = I_2 Z_e'' \cos(\phi_e - \phi_2)$$

Referring Fig. 3.34(c) for leading power factor,

$$OB^2 = (OA - AC)^2 + CB^2$$

$$E_2^2 = [V_2 - I_2 Z_e'' \cos(180 - (\phi_e + \phi_2))]^2 + [I_2 Z_e'' \sin(180 - (\phi_e + \phi_2))]^2$$

We know

$$\sin(180 - \theta) = \sin\theta$$

$$\cos(180 - \theta) = -\cos\theta$$

Neglecting $I_2 Z_e'' \sin(\phi_e + \phi_2)$ and using the above two relationships, we get

$$E_2 = V_2 + I_2 Z_e'' \cos(\phi_e + \phi_2)$$

$$E_2 - V_2 = I_2 Z_e'' \cos(\phi_e + \phi_2)$$

in general for lagging and leading power factor load,

$$E_2 - V_2 = I_2 Ze'' \cos(\phi_e \mp \phi_2)$$

where –ve is for lagging and +ve is for leading power factor load.

$$\therefore \quad \text{Regulation} = \frac{E_2 - V_2}{V_2} = \frac{I_2 Z_e'' \cos(\phi_e \mp \phi_2)}{V_2}$$

$$I_2 Z_e'' \cos(\phi_e \mp \phi_2) = I_2 Z_e'' (\cos\phi_e \cos\phi_2 \pm \sin\phi_e \sin\phi_2)$$

$$= I_2 \cos\phi_2 (Z_e'' \cos\phi_e) \pm I_2 \sin\phi_2 (Z_e'' \sin\phi_e)$$

$$\therefore \quad \text{Regulation} = \frac{I_2(R_e'' \cos\phi_2 \pm X_e'' \sin\phi_2)}{V_2}$$

since $I_2 Z_e'' \cos\phi_e = I_2 R_e''$

and $I_2 Z_e'' \sin\phi_e = I_2 X_e''$

where +ve is for lagging and –ve is for loading power factor load.

∴ % voltage regulation of a transformer

$$V_{reg} = \frac{I_2(R_e'' \cos\phi_2 \pm X_e'' \sin\phi_2)}{V_2} \times 100\% \tag{3.18}$$

The smaller the value of voltage regulation the better suited is the transformer for supplying power to the load. The voltage regulation varies with power factor and has a maximum value, assuming lagging power factor load, when

$$\frac{dV_{reg}}{d\phi_2} = 0 = -R_e'' \sin\phi_2 + X_e'' \cos\phi_2$$

(i.e., by differentiating expression for voltage regulation with respect to load power factor angle and equating to zero)

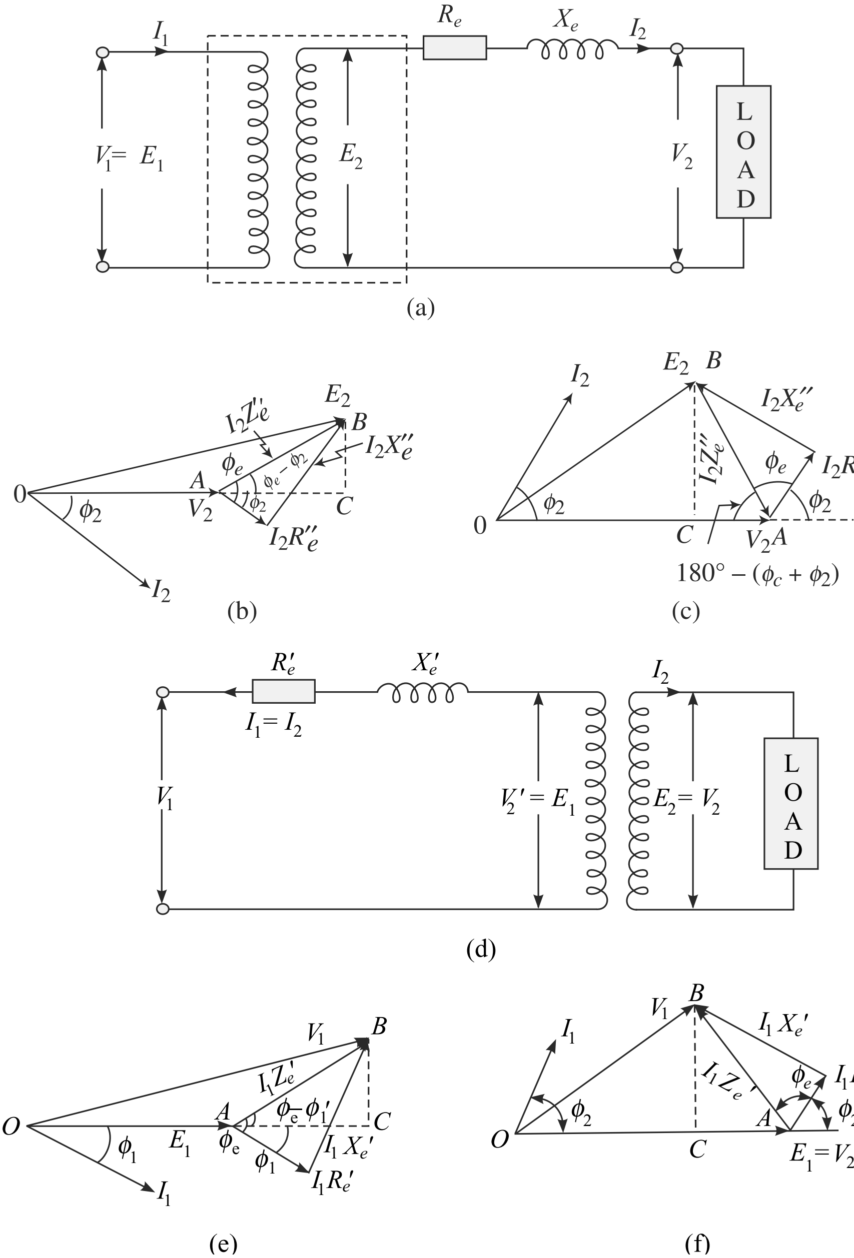


Fig. 3.34 *(a) Approximate equivalent circuit referred to secondary side*
(b) phasor diagram for lagging power factor
(c) corresponding phasor diagram for leading power factor
(d) approximate equivalent circuit referred to primary side
(e) corresponding phasor diagram for lagging power factor
(f) corresponding phasor diagram for leading power factor

or $$\tan \phi_2 = \frac{X_e''}{R_e''} = \tan \phi_e$$

It means that voltage regulation is the maximum when the load power factor (lagging) angle has the same value as the angle of equivalent impedance.

The voltage regulation is zero when

$$R_e'' \cos \phi_2 - X_e'' \sin \phi_2 = 0$$

or $$\tan \phi_2 = \frac{R_e''}{X_e''}$$

or power factor $$\cos \phi_2 = \frac{X_e''}{\sqrt{R_e''^2 + X_e''^2}}, \text{ leading}$$

The expression for voltage regulation can also be derived by referring the secondary quantities to the primary side and drawing the relevant phasor diagram as shown in Fig. 3.34 (d), (e) and (f).

Proceeding as before we get

$$V_1^2 = [V_2' + I_2' Z_e' \cos(\phi_e \mp \phi_2)]^2 + [I_2' Z_e' \sin(\phi_e \pm \phi_2)]^2$$

Neglecting the second term on RHS,

$$V_1^2 = [V_2' + I_2' Z_e' \cos(\phi_e \mp \phi_2)]$$

$$V_1 - V_2' = I_2' Z_e' \cos(\phi_e \mp \phi_2)$$

where –ve is for lagging power factor and +ve is for leading power factor load. The expression for voltage regulation becomes:

$$\text{Regulation} = \frac{V_1 - V_2}{V_2} = \frac{I_2 Z_e' \cos(\phi_e \mp \phi_2)}{V_2}$$

Expanding $\cos(\phi_e \mp \phi_2)$ as before, % voltage regulation of a transformer

$$= \frac{I_2'(R_e' \cos\phi_2 \pm X_e' \sin\phi_2)}{V_2} \times 100 \qquad (3.19)$$

where +ve is for lagging power factor and –ve is for leading power factor load.

EXAMPLE 3.12 *A single-phase, 50 Hz transformer has a turn ratio of 6. The resistances are 0.90 Ω and 0.03 ohm, and reactances are 5 Ω and 0.13 Ω for high voltage and low voltage windings respectively. Calculate (a) the voltage to be applied to the high voltage side to obtain full-load current of 200 A in the low-voltage winding on short-circuit, (b) the power factor on short-circuit.*

Solution Referring the secondary quantities to the primary (high voltage) side,

$$R_e' = R_1 + R_2 \left[\frac{N_1}{N_2}\right]^2$$

$$= 0.9 + 0.03 \times (6)^2 = 1.98\ \Omega$$

$$X_e' = X_1 + X_2 \left[\frac{N_1}{N_2}\right]^2$$

$$= 5 + 0.13 \times (6)^2 = 9.68\ \Omega$$

$$Z_e' = \sqrt{R_e'^2 + X_e'^2} = \sqrt{(1.98)^2 + (9.68)^2}$$
$$= 9.88\ \Omega$$

Secondary current referred to primary side

$$I_2' = \frac{N_1}{N_2}\ I_2 = \frac{1}{6} \times 200 = 33.33\ \text{A}$$

(a) Voltage to be applied to the high voltage side

$$I_2'\ Z_e' = 33.33 \times 9.88 = 330\ \text{Volts}$$

(b) Power factor $\cos\phi = \dfrac{R_e'}{Z_e'} = \dfrac{1.98}{9.68} = 0.2$

EXAMPLE 3.13 *The primary winding of a 6600/250 V, 50 Hz, single-phase transformer has resistance and reactance of 0.21 Ω and 1.0 Ω respectively. The corresponding values of the secondary winding are 2.72×10^{-4} Ω and 1.3×10^{-4} Ω. Calculate the current and power input when the high voltage winding is connected to a 400 Volt, 50 Hz supply, the secondary winding being short circuited.*

Solution Given,

$$R_1 = 0.21\ \Omega,\ X_1 = 1.0\ \Omega$$
$$R_2 = 2.72 \times 10^{-4}\ \Omega,\ X_2 = 1.3 \times 10^{-3}\ \Omega$$

Equivalent resistance referred to primary side,

$$R_e' = R_1 + \left[\frac{N_1}{N_2}\right]^2 R_2$$

$$= 0.21 + \left[\frac{6600}{250}\right]^2 \times 2.72 \times 10^{-4}$$

$$= 0.3995\ \Omega \qquad \left[\text{Taking } \frac{N_1}{N_2} = \frac{V_1}{V_2}\right]$$

Similarly,

$$X_e' = X_1 + \left[\frac{N_1}{N_2}\right]^2 X_2$$

$$= 1.0 + \left[\frac{6600}{250}\right]^2 \times 1.3 \times 10^{-3} = 1.906\ \Omega$$

$$Z_e' = \sqrt{R_e'^2 + X_e'^2}$$

$$= \sqrt{(0.3995)^2 + (1.906)^2} = 1.9474\ \Omega$$

$$I_1 = \frac{400}{1.9474} = 205.4 \text{ A}$$

$$\text{Power input} = I_1^2 R_e' = (205.4)^2 \times 0.3995 = 16.85 \text{ kW}$$

EXAMPLE 3.14 *A single-phase transformer has 90 primary turns and 180 secondary turns. The primary and secondary winding resistances are 0.067 Ω and 0.233Ω respectively. Calculate the equivalent resistance of (a) the primary winding in terms of secondary winding, (b) the secondary winding in terms of primary, winding and (c) the total resistance of the transformer referred to the primary winding.*

Solution

(a) Primary winding resistance referred to secondary side,

$$R_1'' = R_1 \left[\frac{N_2}{N_1}\right]^2$$

$$= 0.067 \left[\frac{180}{90}\right]^2 = 0.268 \ \Omega$$

(b) Secondary winding resistance in terms of primary side,

$$R_2' = R_2 \left[\frac{N_1}{N_2}\right]^2$$

$$= 0.233 \left[\frac{90}{180}\right]^2 = 0.0582 \ \Omega$$

(c) Total resistance of the transformer referred to primary side,

$$R_e' = R_1 + R_2' = 0.067 + 0.0582 = 0.1252 \ \Omega$$

EXAMPLE 3.15 *The primary and secondary windings of a 30 kVA, 6000/230 V transformer have resistances of 10 and 0.016 Ω respectively. The total reactance of the transformer referred to the primary is 23 Ω. Calculate the percentage regulation of the transformer when supplying full-load current at a power factor of 0.8 lagging.*

Solution Given

$$V_1 = 6000 \text{ V}$$

$$V_2 = 230 \text{ V, Rating} = 30 \text{ kVA}$$

$$R_1 = 10 \ \Omega$$

$$R_2 = 0.016 \ \Omega$$

$$X_e' = 23 \ \Omega, \ \cos\phi_2 = 0.8 \text{ lagging}$$

% regulation at full-load and 0.8 p.f. lagging

$$= \frac{I_2'(R_e' \cos\phi_2 + X_e' \sin\phi_2)}{V_2} \times 100$$

$$R_e' = R_1 + R_2'$$

$$= R_1 + \left[\frac{N_1}{N_2}\right]^2 R_2$$

$$= R_1 + \left[\frac{V_1}{V_2}\right]^2 R_2$$

$$= 10 + \left[\frac{6000}{230}\right]^2 \times 0.016 = 20.9\ \Omega$$

$$\sin \phi_2 = 0.6$$

$$I_2' = \frac{30 \times 1000}{230} \times \frac{230}{6000} = 5\text{ A}$$

$$V_2' = 6000 - 5(0.8 - j0.6)\,(20.9 + j23) = 5847\text{ V}$$

Putting the values

$$\%\text{ regulation} = \frac{5(20.9 \times 0.8 + 23 \times 0.6)}{5847} \times 100 = 2.6\%$$

EXAMPLE 3.16 *A 10 kVA single-phase transformer, rated for 2000/400 V has resistances and leakage reactance as follows. Primary winding: $R_1 = 5.5\ \Omega$, $X_1 = 12\ \Omega$. Secondary winding: $R_2 = 0.2\ \Omega$, $X_2 = 0.45\ \Omega$. Determine the approximate value of the secondary voltage at full-load 0.8 power-factor lagging when the primary voltage is 2000 V and also calculate the voltage regulation at this load.*

Solution Assume

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Referring the primary quantities to the secondary side (Fig. 3.35),

$$R_e'' = R_2 + R_1 \left[\frac{N_2}{N_1}\right]^2 = R_2 + \left[\frac{V_2}{V_1}\right]^2 R_1$$

$$= 0.2 + \left[\frac{400}{2000}\right]^2 \times 5.5 = 0.42\ \Omega$$

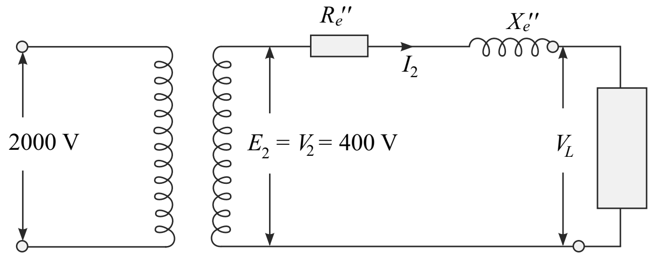


Fig. 3.35

$$X_e'' = X_2 + \left[\frac{N_2}{N_1}\right]^2 X_1$$

$$= 0.45 + \left[\frac{400}{2000}\right]^2 \times 12 = 0.93\ \Omega$$

$$I_2 = \frac{10 \times 1000}{400} = 25\ \text{A}$$

$$Z_e'' = \sqrt{(R_e'')^2 + (X_e'')^2} = 1.02\ \Omega$$

Voltage across the load at full-load and 0.8 p.f. lagging

$$V_L = 400 - I_2\,(0.8 - j\,0.6)\,(R_e'' + j\,X_e'')$$

$$= 400 - 25\,(0.8 - j\,0.6)\,(0.42 + j\,0.93)$$

$$= 374.5\ \text{V}$$

$$\%\ \text{voltage regulation} = \frac{400 - 374.5}{374.5} \times 100 = 6.8\ \%$$

EXAMPLE 3.17 *A 80 kVA, 2000/200 V, 50 Hz single-phase transformer has impedance drop of 8% and resistance drop of 4%. Calculate the regulation of the transformer at full-load 0.8 p.f. lagging. Also determine at what power factor the regulation will be zero.*

Solution

Percentage impedance drop = $\frac{I_2 Z_e''}{V_2} \times 100$

Substituting given data,

$$8 = \frac{I_2 Z_e''}{V_2} \times 100$$

or $I_2 Z_e'' = 16\ \text{V}$

Similarly $\frac{I_2 Z_e''}{V_2} \times 100 = 4$

or $I_2 R_e'' = \frac{4 \times 200}{100} = 8\ \text{V}$

$$I_2 X_e'' = \sqrt{(I_2\,Z_e'')^2 - (I_2\,R_e'')^2}$$

$$= \sqrt{(16)^2 - (8)^2} = 13.86\ \text{V}$$

$$\%\ \text{regulation} = \frac{I_2\,R_e''\cos\phi_2 + I_2\,X_e''\sin\phi_2}{V_2} \times 100$$

$$= \frac{8 \times 0.8 + 13.86 \times 0.6}{200} \times 100 = 7.35\%$$

Equating the expression for voltage regulation at leading power factor load to zero,

$$\text{Power factor for zero regulation, } \cos\phi = \frac{X_e''}{\sqrt{(R_e'')^2 + (X_e'')^2}}$$

$$= \frac{13.86}{\sqrt{(8)^2 + (13.86)^2}} = 0.86 \text{ (leading)}$$

3.8 LOSSES IN A TRANSFORMER

With the help of a transformer electrical energy is transferred from one circuit to the other. The whole of the input energy cannot be transferred to the output circuit as certain amount of it is lost in the core and the windings of the transformer as heat. By proper design of a transformer the losses are kept as minimum as possible.

It may be mentioned that total losses in a transformer are less than that in an equivalent electrical rotating machine as there is no rotational loss in a transformer.

3.8.1 Types of Losses

In a transformer power is lost in the resistance of the windings and in the magnetic core. The losses in the windings are $I_1^2 R_1$ and $I_2^2 R_2$ respectively in the primary and secondary. The loss in the magnetic core is called core-loss or iron-loss. The core-loss is composed of two losses, namely (i) hysteresis loss, and (ii) eddy-current loss. Hysteresis loss is due to the alternate magnetisation of the atoms forming domains in the magnetic material of the core. Each domain behaves as a very small magnet oriented in different directions as shown in Fig. 3.36(a).

Due to the application of the magnetising force, these tiny magnets orient themselves in the direction of magnetisation. If the magnetising force is alternating, the small magnets will orient themselves alternately in opposite directions. Power is expended in this process which is called hysteresis loss. The power lost is dissipated as heat from the core.

Figure 3.36 (c) shows two thick laminated sheets forming a part of the transformer-core limb. However, the laminated sheets are actually very thin and insulated from each other. In these sheets emf is induced due to the presence of an alternating flux in the core (similar to the way in which emf is induced in the windings). The emf induced in laminated sheets will produce circulating currents as has been shown. These are called eddy-currents and they produce power loss in the resistance of all iron paths. To reduce the eddy-current loss in the core, the core is made up of thin laminated sheets (instead of a solid mass) so that the resistance to the eddy-current path is increased and hence the value of eddy current is reduced.

Thus the losses in a transformer are:

(i) $I^2 R$ loss in the windings, and

(ii) Core-loss which is composed of hysteresis loss and eddy-current loss.

Core loss or iron loss is also called constant loss as this loss does not depend upon the amount of electrical load connected to the transformer. This means that the core loss remains the same as on no-load and on full-load or on any other load.

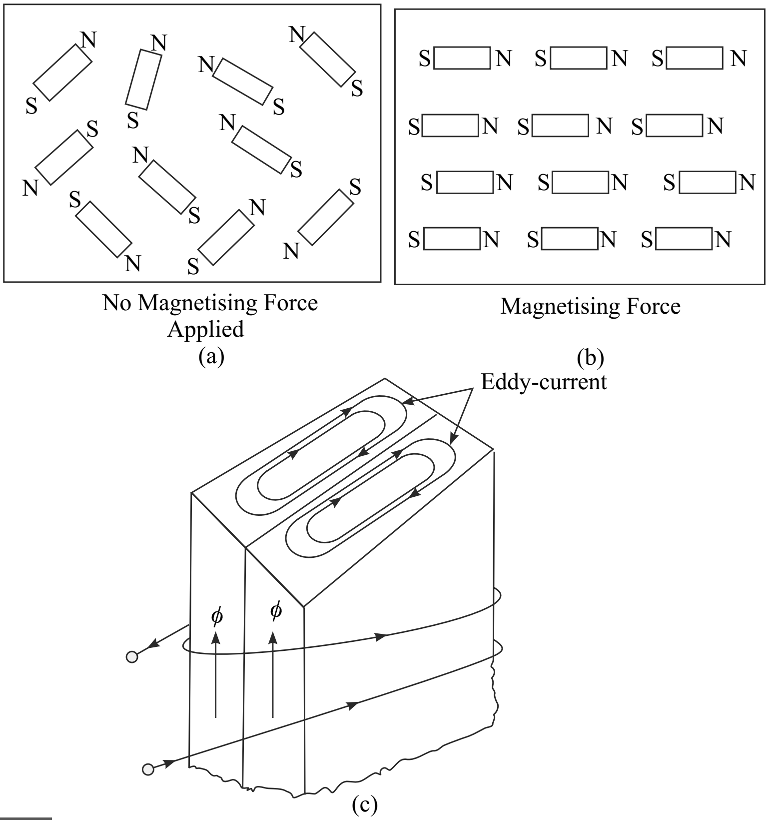


Fig. 3.36 *(a) Orientation of gains in different directions in an unmagnetised magnetic material (b) Orientation of grains on application of a magnetic force (c) Eddy-currents flowing in laminated sheets of a transformer core when current flows through the winding*

Copper loss or winding loss or I^2R loss in the windings is a variable loss as this loss depends upon the value of load current, I. It may be noted that sometimes the windings of transformers are made of aluminium wires to reduce the cost of production. So, the winding loss is often mentioned as I^2R loss instead of copper loss. I^2R loss varies as the square of the load current. If I^2R loss on full-load is w watts, at half load its value will be $w/4$ watts; at one-third of full load its value will be $w/9$ watts; and so on.

As variation of losses are concerned, I^2R loss is independent of variation of flux density, applied voltage, and frequency. Core loss components, i.e, hysteresis and eddy current losses vary with flux density, frequency, and applied voltage as

$$\text{Hysteresis loss, } W_h \propto B_m^{1.6} f \text{ watts}$$

$$\text{Eddy current loss, } W_e \propto B_m^2 t^2 f^2 \text{ watts}$$

where, B_m is the maximum flux density of the core, t is the thickness of laminations used to make the core, and f is the frequency of transformer primary supply voltage.

3.8.2 Separation of Hysteresis Loss and Eddy Current Loss

The core of a transformer is subjected to alternating magnetization at the frequency of the supply connected to its primary winding. Due to this an amount of power is wasted in the core material.

The hysteresis loss, W_h depends on the area of the hysteresis loop and is expressed as

$$W_h = \eta f B_m^{\,x} \tag{3.20}$$

where η is Steinmetz constant which depends on the material, f is the supply frequency, B_m is the maximum flux density in the core, and value of x varies from 1.5 to 2.0 for steel.

As mentioned earlier when a certain small percentage of silicon is added to steel, the hysteresis loop area becomes smaller and hence the hysteresis loss gets reduced.

Changing magnetic flux causes eddy currents to flow in the core material causing eddy current loss. The eddy current loss depends upon the square of the thickness of the laminated sheets used for making the core.

The eddy current loss, W_e is expressed as

$$W_e = K f^2 t^2 B_m^{\,2} \tag{3.21}$$

where t is the thickness of the laminations used and K is a constant.

The sum of two losses, i.e., W_h and W_e is called the core loss or iron loss, W_c or W_i.

The emf equation of the transformer is expressed as

$$E \simeq V = 4.44\, \phi_m f N = 4.44\, A_i B_m f N \tag{3.22}$$

Where A_i is the area of the core and B_m is the flux density.

From the above, we can write,

$$K B_m f = V \qquad \text{where } k = 4.44\, A_i \text{ N}$$

or,

$$B_m \propto \frac{V}{f}$$

The maximum flux density, B_m in the core will remain constant if the ratio of applied voltage and frequency is kept constant.

The core loss or iron loss is then expressed as

$$W_i = W_h + W_e$$

$$= \eta f B_m^{\,x} + K f^2 t^2 B_m^{\,2}$$

If B_m is kept constant by keeping the ratio V/f constant, then W_i can be expressed as

$$W_i = Af + Bf^2$$

where A and B are constants.

If the measurement of W_i is taken as two different frequencies by keeping the ratio of V/f constant then we will have

$$W_{i1} = A f_1 + B f_1^{\,2}$$

$$W_{i2} = A f_2 + B f_2^{\,2}$$

The core loss or iron loss can be found out by performing the no-load test on the transformer. The value of W_i can be calculated from the no-load wattmeter reading W_0 as

$$W_i = W_0 - I_0^2 R_1$$

If the measurement is taken at two different frequencies f_1 and f_2 by adjusting the voltage V accordingly so that B_m is constant, then the two component losses, i.e., hysteresis loss and eddy current loss can be separated out from the core loss.

One or two examples will clarify the procedure of separation of hysteresis and eddy current losses.

EXAMPLE 3.18 *The core loss of a transformer at 50 Hz is found to be equal to 550 Watts. To separate out the hysteresis loss and eddy current loss, the core loss was determined by conducting no-load test at another frequency of 40 Hz by the maintaining the ratio of V/f constant. The core loss at 40 Hz is found to be 400 Watts. Calculate the hysteresis loss and eddy current loss at normal frequency of 50 Hz.*

Solution Given

$$W_{i_1} = 550 \text{ W at } f_1 = 50 \text{ Hz}$$

and $$W_{i_2} = 400 \text{ W at } f_2 = 40 \text{ Hz.}$$

$$W_{h_1} = A f_1 \text{ and } W_{e_1} = B f_1^2$$

We formulate the two equations from the above values as

$$W_i = Af + B f^2$$

Substituting two sets of values,

$$W_{i_1} = Af_1 + B f_1^2$$

or, $$550 = A \times 50 + B \times (50)^2 \quad \text{(i)}$$

and $$W_{i_2} = Af_2 + B f_2^2$$

or, $$400 = A \times 40 + B \times (40)^2 \quad \text{(ii)}$$

From (i) and (ii),

$$50\,A + 2500\,B = 550$$

and $$40\,A + 1600\,B = 400$$

or, $$A + 50\,B = 11$$

and $$A + 40\,B = 10$$

Solving, $$B = 0.1$$

And $$A = 6$$

Using these values,

Wh_1 i.e., W_h at $f = 50$ Hz is calculated as

$$Wh_1 = A f_1 = 6 \times 50 = 300 \text{ W}$$

and $$W_{e_1} = B f_1^2 = 0.1 \times (50)^2 = 0.1 \times 2500 = 250 \text{ W}$$

thus, $$W_{i_1} = W_{h_1} + W_{e_1} = 300 + 250 = 550 \text{ W}$$

EXAMPLE 3.19 *No-load test conducted on a transformer gave the following readings*

250 V	*50 Hz*	*55 W*
200 V	*40 Hz*	*40 W*

Calculate the hysteresis and eddy current losses at 250 V, 50 Hz.

Solution $\frac{V_1}{f_1}$ must be equal to $\frac{V_2}{f_2}$ so that B_m remains constant.

Here, $\frac{V_1}{f_1} = \frac{250}{50} = 5$ and $\frac{V_2}{f_2} = \frac{200}{40} = 5$

Hence, the core flux density B_m is constant. So we can write, $W_i = W_h + W_e$

or, $$W_i = A\,f + B\,f^2$$

Substituting the two sets of values, we have

$$55 = A \times 50 + B \times (50)^2$$

or $$55 = 50A + 2500\,B \quad \text{(i)}$$

And $$40 = 40\,A + 1600\,B \quad \text{(ii)}$$

From (i) and (ii), $B = 0.01$ and $A = 0.6$ at 250 V, 50 Hz,

$$W_h = A\,f = 0.6 \times 50 = 30 \text{ W}$$

And $$W_e = Bf^2 = 0.01 \times 2500 = 25 \text{ W}$$

To check, $$W_i = W_h + W_e = 30 + 25 = 55 \text{ W}$$

EXAMPLE 3.20 *A 1000V/2000V transformer has 750 W hysteresis loss and 250 W eddy current loss. When the applied voltage is doubled and the frequency of supply voltage is halved, what will be the values of hysteresis is loss and eddy-current loss ?*

Solution We know,

$$W_h \propto B_m^{1.6} f \text{ and } W_e \propto B_m^2 f^2$$

and $$E \approx V = 4.44\,\phi f t$$

and $$\phi = B_m A$$

Thus, $$V \propto B_m f$$

or, $$B_m \propto \frac{v}{f}$$

Substituting, $w_h = \left(\frac{V}{f}\right)^{1.6} f$ ...(1) and $W_e \propto \left(\frac{V}{f}\right)^2 f^2 \propto V^2$ ...(2) Now let hysteresis and eddy current losses at the new voltage and frequency be w_h^1 and w_e^1 respectively.

As new $V = 2V$ and new $f = \frac{f}{2}$

$$w_h^1 \propto (2V)^{1.6}\,(0.5f)^{-0.6} \propto 2^{1.6} \times (0.5)^{-0.6} \times V^{1.6} f^{-0.6} \quad (3)$$

Dividing (3) by (2),

$$\frac{W_h^1}{W_h} = \frac{2^{1.6} \times (0.5)^{-0.6}\, V^{1.6} f^{-0.6}}{V^{1.6} f^{-0.6}}$$

$$= 2^{1.6} \times (0.5)^{-0.6} = 4.59$$

$$w_h^1 = 4.59 \times w_h = 4.59 \times 750 \text{ watts} = 3446 \text{ watts.}$$

$$w_e^1 \propto (2v)^2 \propto 4v^2 \quad (4)$$

Dividing (4) by (2), $w_e^1 = 4\,w_e = 4 \times 250$ W = 1000 watts

3.8.3 Estimation of Losses from Practical Tests

The losses in a transformer can be estimated by performing two tests, namely:

(i) open-circuit test, and

(ii) short-circuit test

From the open-circuit test an estimate of the power wasted in the core is obtained, whereas the short-circuit test gives an account of the power lost in the winding resistances, i.e., $I^2 R$ loss in the windings. With the help of these test results it is possible to calculate the efficiency and regulation of the transformer without actually loading it.

Open-circuit Test The transformer is connected to a rated voltage supply as shown in Fig. 3.37. The secondary is kept open. The voltmeter V_1 reads the rated voltage applied to the primary whereas V_2 reads the secondary induced voltage. Ammeter *A* reads the no-load current, wattmeter *W* reads the power consumed by the transformer.

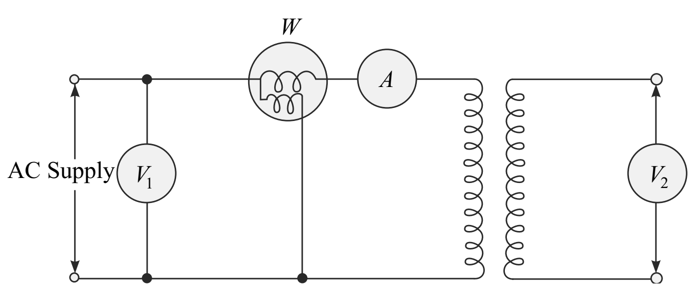


Fig. 3.37 *Circuit diagram for open circuit test on a transformer*

At no-load the current taken by the transformer is about 3 to 5% of the full-load current. Thus the $I^2 R$ loss at no-load will be about 0.09% to 0.25% of the full-load copper-loss. Thus compared to the full-load value, this copper-loss at no-load is very negligible. Therefore, the power consumed by the transformer on no-load can be taken as approximately equal to the core-loss of the transformer. Core-loss depends upon the applied voltage. As the no-load test is carried out at rated voltage, the wattmeter reading will give the value of core-loss at full-load.

The ratio of the voltmeter readings, i.e., V_1/V_2 gives the ratio of number of turns of the primary and secondary windings. Thus

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

and $$V_1 I_0 \cos \phi_0 = W$$

$$\cos \phi_0 = \frac{W}{V_1 I_0}$$

Short-circuit Test In this test, one winding, usually the low-voltage winding, is short-circuited through an ammeter as shown in Fig. 3.38. A low voltage is applied across the other winding through a variable voltage supply, i.e., an auto transformer.

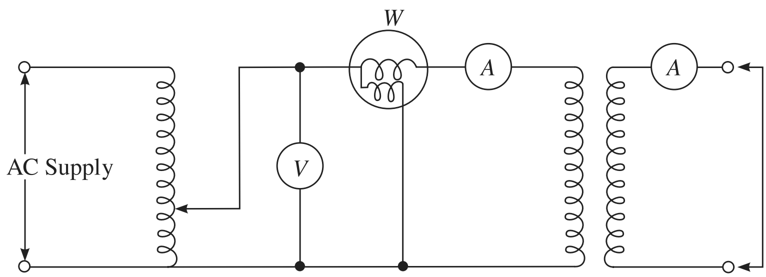


Fig 3.38 *Circuit diagram for short-circuit test on a transformer*

The applied voltage is adjusted (raised from 0 upwards) such that the full-load current flows through the primary and secondary windings. The amount of voltage needed to circulate the full-load current will be very small. The wattmeter reading will approximately give an account of the full-load copper-loss in both the windings as the core-loss at this reduced voltage is very small.

Thus the wattmeter reading when the short-circuit test is conducted at rated current will be approximately equal to full-load copper-losses. The equivalent circuit of Fig. 3.38 with the primary circuit parameters (resistance and reactance) referred to the secondary can be drawn as shown in Fig. 3.39.

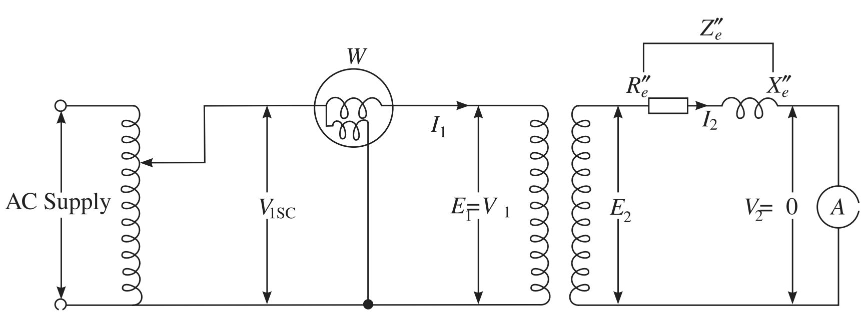


Fig. 3.39 *Equivalent circuit of Fig. 3.38 with primary quantities referred to the secondary*

From the short-circuit test data the resistance and reactance of the transformer with primary quantities referred to the secondary can be calculated thus:

$$W_{(sc)} = I_{2\,(sc)}^2 R_e''$$

$$R_e'' = \frac{W_{(sc)}}{I_{2(sc)}^2}\ \Omega$$

$$Z_e'' = \sqrt{(R_e'')^2 + (X_e'')^2}$$

$$Z_e'' = \frac{E_{2(sc)}}{I_{2(sc)}}\ \Omega$$

where
$$E_{2(sc)} = \frac{N_2}{N_1} \times E_{1(sc)}$$

$$= \frac{N_2}{N_1} \times V_{1(sc)}$$

and
$$X_e'' = \sqrt{Z_e''^2 - R_e''^2}$$

Thus, knowing the losses in a transformer, the efficiency for a particular output can be calculated as

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + losses}}$$

Percentage regulation can be calculated using the Eq. (3.18) as follows.

$$\%\ \text{regulation} = \frac{I_2 Z_e'' \cos(\phi_e - \phi_2) \times 100}{V_2}$$

$I_2 Z_e''$ is equal to the voltage induced in the secondary under the short-circuit condition as the whole of the voltage drops across the winding impedance. Wattmeter reading, $W_{(sc)} = I_{(sc)} E_{(sc)} \cos \phi_e$

$$\cos \phi_e = \frac{W_{(sc)}}{I_{2(sc)} E_{2(sc)}}$$

$$E_{2(sc)} = \frac{N_2}{N_1} \times V_{1(sc)}$$

Thus the regulation can be calculated at full-load at a particular power-factor.

Alternatively, if the secondary quantities referred to the primary side are considered, then the calculations would be as follows:

$$W_{(sc)} = I_1 V_{1(sc)} \cos \phi_e$$

∴
$$\cos \phi_e = \frac{W_{(sc)}}{I_1 V_{1(sc)}}$$

$$\%\ \text{regulation} = \frac{I_2 Z_e'' \cos(\phi_e - \phi_2)}{V_2} \times 100 \text{ [from Eq. (3.19)]}$$

and
$$I_1 Z_e'' = V_{1(sc)}$$

where, V_1 is the rated primary voltage.

Thus voltage regulation at full-load and at a particular power-factor can be calculated.

EXAMPLE 3.21 *The open-circuit and short-circuit tests conducted on a 50 kVA transformer gave the following results:*

OC test: Primary voltage = 3300 V
Secondary voltage = 400 V
Primary power-input = 460 W
SC test: Primary voltage = 124 V
Primary current = 15.4 A
Primary power = 540 W

Calculate (a) the efficiency at full-load and 0.8 power-factor lagging and (b) the voltage regulation at 0.8 power-factor lagging.

Solution

From the open-circuit test, core-loss = 460 W

From the short-circuit test, copper-loss = 540 W

Thus the efficiency at full-load and 0.8 lagging power-factor

$$= \frac{50 \times 1000 \times 0.8}{50 \times 1000 \times 0.8 + 460 + 540} \times 100\%$$

$$= 97.5\%$$

$$\text{Voltage regulation} = \frac{V_{1(sc)} \cos(\phi_e - \phi_2)}{V_2'} \times 100$$

$$Z_e'' = \frac{124}{15.4} = 8.05\ \Omega$$

$$R_e'' = \frac{540}{(15.4)^2} = 2.27\ \Omega$$

$$X_e'' = 7.72$$

$$V_2' = 3300 - 15.15\,(0.8 - j\,0.6)\,(2.27 + j\,7.72)$$

$$= 3203$$

$$V_{1(sc)} = 124\ \text{V}$$

$$V_1 = 3300\ \text{V}$$

$$\cos\phi_2 = 0.8,\ \phi_2 = 37°$$

$$V_{1(sc)}\, I_{1(sc)} \cos\phi_e = 540$$

$$\cos\phi_e = \frac{540}{124 \times 15.4} = 0.28 \quad \phi_e = 74°$$

$$\therefore \quad \text{Voltage regulation} = \frac{124\cos(74° - 37°)}{3300} \times 100\% = 3\%$$

EXAMPLE 3.22 *A 100 kVA, 6600/330 V, 50 Hz, single-phase transformer took 10 A and 436 W at 100 V in a short-circuit test, the figures referring to the high voltage side. Calculate the voltage to be applied to the high voltage side on full load at power factor 0.8 lagging when the secondary terminal voltage is 330 volts.*

Solution

Total resistance $R_e' = \frac{W}{I^2} = \frac{436}{(10)^2} = 4.36\ \Omega$

Total impedance $Z_e' = \frac{100}{10} = 10\ \Omega$

$$X_e' = \sqrt{(10)^2 - (4.36)^2} = 8.999\ \Omega$$

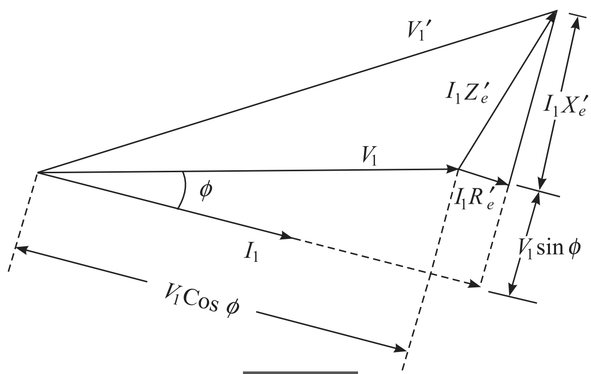


Fig. 3.40

Full-load current $I_1 = \frac{10000}{6600} = 15.15$

$$I_1 R_e' = 66.06\ \text{V}$$
$$I_1 X_e' = 136.35\ \text{V}$$
$$\cos\phi = 0.8$$
$$\sin\phi = 0.6$$
$$V_1' = \sqrt{(V_1 \cos\phi + IR_e')^2 + (V_1 \sin\phi + I_1 X_e')^2}$$
$$= \sqrt{(6600 \times 0.8 + 66.06)^2 + (6600 \times 0.6 + 136.35)^2}$$
$$V_1' = 6735\ \text{V}$$

EXAMPLE 3.23 *A 250/500 V transformers gave the following test results:*

Circuit test: 250 V, 1 A, 80 W on low voltage side

Short circuit test: with low voltage winding

Short circuited – 20 V, 12 A, 100 W.

Calculate the circuit constants and insert them on the equivalent circuit. Also, calculate the efficiency when the output is 10A at 500 volts and power factor is 0.8 lagging.

Solution From open circuit test data,

$$V_1 I_0 \cos\theta_0 = 80 \text{ W}$$

$$\cos\theta_0 = \frac{80}{250 \times 1} = 0.32$$

Loss component of no-load current, $I_c = I_0 \cos\phi_0$

$$= 1 \times 0.32 = 0.32 \text{ A}$$

Magnetizing current, $I_m = \sqrt{I_0^{\,2} - I_c^{\,2}} = \sqrt{1^2 - (0.32)^2} = 0.95 \text{ A}$

$$R_m = \frac{V_1}{I_c} = \frac{250}{0.32} = 781.25\ \Omega$$

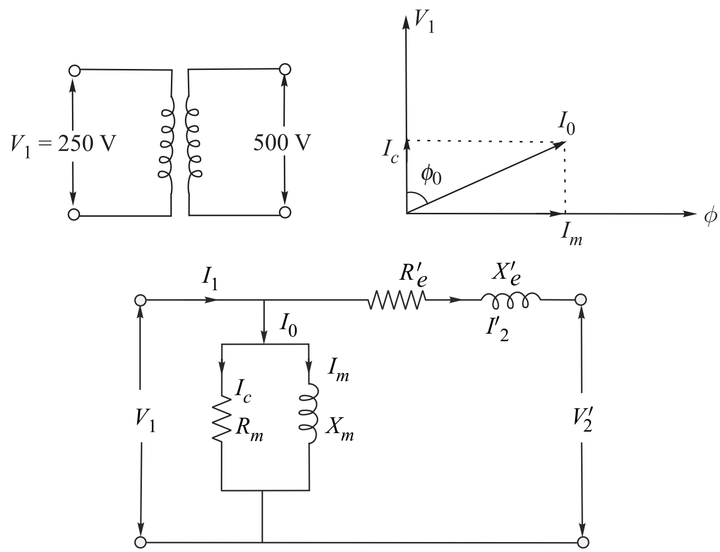


Fig. 3.41

$$X_m = \frac{V_1}{I_m} = \frac{250}{0.95} = 263.15\ \Omega$$

This is a step-up transformer. The short-circuit test has been conducted on high voltage (secondary) side.

Therefore,

$$I_{sc}^2 R_e'' = 100 \text{ W}$$

where R_e'' is the equivalent resistance reformed to secondary (H.V.) side.

or, $$R_e'' = \frac{100}{(12)^2} = 0.694\ \Omega$$

$$Z_e'' = \frac{V_{sc}}{I_{sc}} = \frac{20}{12} = 1.67\ \Omega$$

$$Z_e'' = \sqrt{(Z_e'')^2 - (R_e'')^2} = \sqrt{(1.67)^2 - (0.694)^2} = 1.518\ \Omega$$

These values are referred to secondary side, i.e., to the high voltage side. They have to be converted to low-voltage side as the open-circuit test has been conducted on low-voltage side. All the parameters will then be referred *ab* low-voltage side, i.e., primary side. The turn ratio, $K = \frac{V_2}{V_1} = \frac{500}{250} = 2$

The referred values are

$$R_e' = \frac{R_e''}{K^2} = \frac{0.694}{4} = 0.173\ \Omega$$

$$X_e' = \frac{1.518}{K^2} = \frac{1.518}{4} = 0.379\ \Omega$$

$$Z_e' = \frac{1.67}{4} = 0.416\ \Omega$$

$$\text{Efficiency } \eta = \frac{\text{Output} \times 100}{\text{Output + losses}} = \frac{VI\cos\phi \times 100}{VI\cos\phi + \text{core loss} + \text{Copper loss}}$$

$$= \frac{500 \times 10 \times 0.8 \times 100}{500 \times 10 \times 0.8 + 80 + 10^2(0.694)}$$

$$= 96.4 \text{ per cent}$$

3.8.4 Determination of Efficiency and Temperature Rise of Transformers by Back-to-Back Test

Small transformers can be tested for efficiency by directly loading them. By measuring the output and input the efficiency can be calculated. The temperature rise of a transformer can be determined by running the transformer on load for a long time and measuring the temperature of the oil at periodic intervals of time using an alcohol thermometer. For big transformers, however, it may be difficult to load the transformer as such heavy loads may not be available in the test laboratory. It will also be expensive as a large amount of energy will be wasted in the load during the duration of the test.

Large transformers can be tested for determining the temperature rise by the Sumpner Test which is also called the Back-to-Back Test. The efficiency can also be calculated by noting down the losses. In the back-to-back test two identical transformers are needed. In this test the two primary windings are connected across the supply having the normal voltage rating of the transformers. A wattmeter, an ammeter and a voltmeter are connected to the input side as shown in Fig. 3.42. The wattmeter reading W_1 when the secondary windings are open will give the core-losses of the two transformers. The two secondary windings are connected in phase opposition such that the voltage measured across terminals B_2 and D_2 is zero. This can be done by first connecting any two terminals, say A_2 and C_2, together and connecting a double-range voltmeter across the other terminals, in this case A_1 and D_2. For correct back-to-back connections, the voltmeter should give zero reading. If the voltmeter gives a reading twice the voltage rating of the secondary, the terminal connections should be reversed. With the secondary windings connected back-to-back, if the free terminals are shorted, no current will flow in the secondary windings. Current can be allowed to flow in the secondary windings by injecting some voltage through a regulating transformer as shown in Fig. 3.42. The output voltage of the regulating transformer can be adjusted to circulate full-load current to flow through the windings. The secondary current will then induce full-load current in the primary windings also. This current will circulate through the primary windings only and will not affect the wattmeter reading, connected in the primary circuit. Wattmeter W_2 connected in the secondary circuit will, therefore, give the copper-losses in the windings of the two transformers. The core and copper-losses in one transformer will be $(W_1 + W_2)/2$.

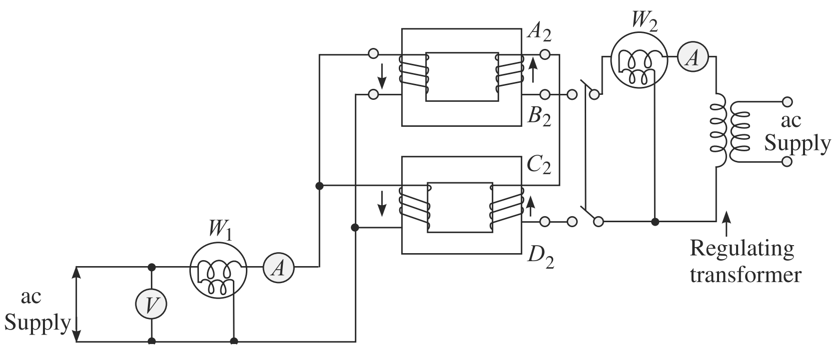


Fig. 3.42 *Back-to-back test on two similar transformers*

The efficiency of the transformer can be determined by running these trans formers back-to-back for a long time, say 48 hours, and measuring the temperature of the oil at periodic intervals of time, say every one hour. The main advantage of this test is that the transformers are tested under full-load conditions without much expenditure of energy, i.e., by simply supplying energy equal to the losses of the two transformers only. As mentioned earlier, two identical transformers are necessary for this test.

A single transformer can, however, be tested for temperature rise by short-circuiting one winding and applying a voltage of such a value to the other that the input power recorded by the wattmeter is equal to the sum of its core-loss and full-load copper-loss. As only a small percentage of voltage is required to be applied, the core-loss will be small whereas the copper-loss will be much more than the normal full-load copper-loss. The temperature rise of the windings will be more and the temperature rise of iron will be less than what would happen under actual loading conditions. The temperature rise of oil, however, will be the same as would happen under actual loading conditions and hence can be used for checking the temperature rise of a transformer when two identical transformers are not available for back-to-back connections.

EXAMPLE 3.24 *Power input to two identical 200 kVA, single-phase transformers when connected back-to-back was 3.4 kW. Power supplied through a regulating* *transformer to the secondary circuit in passing full-load current was 5.2 kW. Calculate the efficiency of each transformer at 0.8 power factor lagging.*

Solution

Core loss of the two transformers = 3.4 kW

Therefore, core loss of each transformer = $\frac{3.4}{2}$ = 1.7 kW

Full-load copper loss of the two transformer = 5.21 W

Therefore, full-load copper loss of each transformer = $\frac{5.2}{2}$ = 2.6 kW

$$\text{Full-load \% efficiency at 0.8 p.f. lagging} = \frac{\text{Output} \times 100}{\text{Output + losses}}$$

$$= \frac{200 \times 0.8 \times 100}{200 \times 0.8 + 1.7 + 2.6} = 97.3\%$$

3.9 EFFICIENCY—CONDITION FOR MAXIMUM EFFICIENCY AND ALL-DAY EFFICIENCY

The efficiency of any device is the ratio of its output and input. Input can be expressed as the sum of the output and losses. Since losses are different at different loads and are not directly proportional to the output, efficiency will be different at different loads. *Efficiency is maximum at a particular load.* It is possible to determine the load at which the efficiency of a transformer will be maximum:

The all-day efficiency of a transformer is different from that of commercial efficiency (ratio of output and input). The all-day efficiency gives an idea of how effectively a distribution transformer is used throughout the day.

3.9.1 Efficiency

The efficiency of a transformer is given by the expression,

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Output power} + \text{losses}}$$

$$= \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + \text{core loss} + \text{copper loss}}$$

$$= \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_c + I_1^2 R_1 + I_2^2 R_2} \quad (3.23)$$

The core loss, P_c in a transformer is constant if the input voltage is constant. The copper-losses vary as the square of the load current. Equation (3.23) can be rewritten as

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_c + I_2^2 R_e''}$$

where $R_e'' = R_2 + R_1 \left[\frac{N_2}{N_1}\right]^2$

and $I_2^2 R_e''$ = total copper-loss in the windings
Dividing both numerator and denominator by I_2

$$\eta = \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + \dfrac{P_c}{I_2} + I_2 R_e''} \tag{3.24}$$

V_2 can be taken as approximately constant for a transformer. Thus, for a particular load power factor, efficiency is maximum if the denominator is the minimum.

3.9.2 Condition for Maximum Efficiency

To determine at what load the efficiency will be maximum, we need to differentiate the denominator of Eq. (3.24) with respect to load current I_2 and equate to zero, thus

$$\frac{d}{dI_2}\left[V_2 \cos \phi_2 + \frac{P_c}{I_2} + I_2 R_e''\right] = 0$$

or
$$\frac{P_c}{I_2^2} + R_e'' = 0$$

or
$$I_2^2 R_e'' = P_c \tag{3.25}$$

Thus efficiency will be maximum at a load in which the total copper-loss in the windings is equal to the core-loss. A transformer is designed such that its efficiency is generally maximum at a load slightly lower than the full-load. This is because the transformer generally works at a load lower than the full-load rating (when a transformer is installed, its rating is chosen higher than the estimated load). Thus by design the transformer is put to work at near to maximum efficiency.

3.9.3 All-Day Efficiency

The efficiency of a transformer is given by

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

This is called ordinary or commercial efficiency. The efficiency of a distribution transformer is also expressed as the ratio of the kilowatthour output and kilowatthour input in a day. The utilisation of a transformer can be judged from the all-day efficiency.

A distribution transformer is energised all the 24 hours, although it may or may not supply any energy to the consumer or may supply very little load. Loading on a transformer will vary at different hours of the day. For example, a distribution transformer supplying mainly light and fan loads may be loaded lightly in the morning hours and late in the evening hours. But the transformer is to be kept energised throughout the 24 hours. Thus, in the transformer, core-loss will occur throughout the day whereas copper-loss will occur only when the transformer is loaded and will depend on the magnitude of the load. Therefore, while designing such distribution transformers, care is taken to keep the core-losses to a minimum.

To calculate the all-day efficiency, one must know the load-cycle of the trans former, i.e., how the transformer is loaded during 24 hours in a day. The method of calculation of all-day efficiency is illustrated through an example.

EXAMPLE 3.25 *A 100 kVA distribution transformer supplying light and fan. loads has full-load copper-loss and core-loss of 1.5 kW and 2 kW respectively. During 24 h in a day the transformer is loaded as follows:*

6 AM	*to*	*10 AM*	*(4 h)*	*Half-load*
10 AM	*to*	*6 PM*	*(8 h)*	*One-fourth load*
6 PM	*to*	*10 Pm*	*(4 h)*	*Full-load*
10 PM	*to*	*6 AM*	*(8 h)*	*Negligible load*

Calculate the all-day efficiency of the transformer.

Solution

Full-load core-loss = 2.0 kW

This core-loss is constant for any load.

Total core-loss for 24 h = 2.0 × 24 = 48 kWh

Full-load copper-loss = 1.5 kW

Copper-loss is proportional to the square of the load, i.e., if copper-loss at full-load is x, copper-loss at half-load will be 1/4 x. Thus,

$$\text{Copper-loss from 6 AM to 10 AM} = \frac{1.5}{4} \times 4 = 1.5 \text{ kWh}$$

$$\text{Copper-loss from 10 AM to 6 PM} = \frac{1.5}{16} \times 8 = 0.75 \text{ kWh}$$

Copper-loss from 6 PM to 10 PM = 1.5 × 4 = 6.0 kWh

Copper-loss from 10 PM to 6 AM = Negligible

Total copper-loss for 24 h = 1.5 + 0.75 + 6.0 = 8.25 kWh

Output for 24 h = 50 × 4 + 25 × 8 + 100 × 4 + 0 = 800 kWh

(load is considered of unity power-factor)

Thus,

$$\text{All-day efficiency} = \frac{\text{Output in kWh for 24 h}}{\text{Input in kWh for 24 h}} \times 100$$

$$= \frac{\text{Output in kWh for 24 h} \times 100}{\text{Output in kWh for 24 h + losses in 24 h}}$$

$$= \frac{800}{800 + 48 + 8.25} \times 100 = 93.4\%$$

EXAMPLE 3.26 *A 50 kVA, 6360/240 V transformer gave the following test results:*

Open-circuit test :Primary voltage, 6360 V; primary current, 1 A; power input, 2 kW.

Short-circuit test :Primary voltage, 1325 V; secondary winding current, 175 A; power input, 2 kW.

Calculate the efficiency of the transformer when supplying full-load current at a power factor of 0.8 lagging.

Solution From the open-circuit test,

Core-loss = 2 kW

From the short-circuit test,
Copper-loss at secondary current of 175 A = 2 kW

$$\text{Full-load secondary current } I_2 = \frac{50 \times 1000}{240} = 208.3 \text{ A}$$

$$\text{Efficiency} = \frac{50 \times 0.8}{50 \times 0.8 + 2.2\left[\frac{208.3}{175}\right]^2} \times 100$$

$$= \frac{40}{40 + 2 + 2.83} \times 100 = 89.2\%$$

EXAMPLE 3.27 *The primary and secondary windings of a 500 KVA transformer have resistances of 0.4 ohm and 0.001 ohm respectively. The primary and secondary voltages are 6000 volts and 400 volts respectively. The iron loss is 3 kW. Calculate the efficiency on full load at 0.8 power factor lagging.*

Solution We know, efficiency at 0.8 p.f. can be calculated as,

$$\eta = \frac{\text{Output}}{\text{Output + losses}} \times 100$$

$$= \frac{\text{KVA}\cos\theta \times 100}{\text{KVA}\cos\theta + \text{core loss} + \text{Copper loss}}$$

We have to calculate the Copper loss in the primary and secondary windings on full-load.

$$\text{Primary winding current,} \quad I_1 = \frac{\text{Primary Volt amp}}{\text{Primary Voltage}} = \frac{500 \times 1000}{6600}$$

$$= 75.7 \text{ Amps.}$$

$$\text{Secondary winding current, } I_2 = 75.7 \times \frac{V_1}{V_2} = 75.7 \times \frac{6600}{400}$$

$$= 75.7 \times 16.5 \text{ amps. (Since } V_1 I_1 = V_2 R_2)$$

$$\text{Copper losses in the two winding } = I_1^2 R_2 + I_2^2 R_2$$

$$= (75.7)^2 \times 0.4 + (75.7 \times 16.5)^2 \times 0.001$$

$$= 3845 \text{ Watts} = 3.845 \text{ kW}$$

$$\text{Efficiency at 0.8 p.f.,} \quad \eta = \frac{500 \times 0.8 \times 100}{500 \times 0.8 + 3 + 3.845} = 98.3 \text{ per cent}$$

EXAMPLE 3.28 *A 400 kVA transformer has an iron-loss of 2 kW and the maximum efficiency at 0.8 power factor occurs when the load is 240 kW. Calculate (a) the maximum efficiency at unity power-factor, and (b) the efficiency on full-load at 0.71 power-factor lagging.*

Solution

$$\text{kVA}\cos\phi = \text{kW}$$

$$\therefore \qquad \text{kVA} = \frac{240}{0.8} = 300$$

Efficiency is maximum when, core-loss = copper-loss

= 2 kW

Maximum efficiency occurs at 240 kW, 0.8 power-factor, i.e., at 300 kVA load

∴ Copper-loss at 300 kVA load = 2 kW

$$\text{Copper-loss at 400 kVA load (full-load)} = 2 \times \left[\frac{400}{300}\right]^2 \text{kW} = 3.55 \text{ kW}$$

Efficiency at full-load and 0.71 power factor

$$= \frac{400 \times 0.71}{400 \times 0.71 + 2 + 3.55} = 98\%$$

Maximum efficiency occurs at 300 kVA load.

∴ Maximum efficiency at 300 kVA and unity power factor,

$$= \frac{300 \times 1}{300 \times 1 + 2 + 2} \times 100 = 98.6\%$$

EXAMPLE 3.29 *A 40 kVA transformer has a core-loss of 450 W and a full-load copper-loss of 800 W. If the power factor of the load is 0.8, calculate (a) the full-load efficiency, (b) the maximum efficiency, and (c) the load at which maximum efficiency occurs.*

Solution

$$\text{Full-load efficiency} = \frac{40 \times 0.8}{40 \times 0.8 + 0.450 + 0.800} \times 100 = 96.24\%$$

For maximum efficiency,

$$\text{Core-loss} = \text{copper-loss} = 450 \text{ W}$$

To calculate the load at which efficiency is maximum assume that n is the fraction of the full load at which efficiency is maximum. Copper-loss at the load is $n_2 \times 800$ W. For maximum efficiency

$$n^2 \times 800 = 450$$

$$\therefore \qquad n^2 = \frac{450}{800}$$

$$\text{or} \qquad n = \frac{3}{4}$$

Therefore at a load 3/4 × 40, i.e., 30 kVA, the efficiency will be maximum.

$$\text{Value of maximum efficiency} = \frac{30 \times 0.9}{30 \times 0.8 + 0.450 + 0.450} = 96.3\%$$

EXAMPLE 3.30 *Two transformers A and B each rated for 40 kVA have core-losses of 500 and 250 W respectively and full-load copper-losses of 500 and 750 W respectively. Compare the all-day efficiencies of the two transformers if they are to be used to supply a lighting load 'with outputs varying as follows: Output–four hours at full-load, eight hours at half-load and the remaining 12 hours at no-load.*

Solution

Transformer A

$$\text{Rated output} = 40 \text{ kVA, core-loss} = 500 \text{ W} = 0.5 \text{ kW}$$

$$\text{Full-load copper-loss} = 500 \text{ W} = 0.5 \text{ kW}$$

$$\text{Copper-loss at half-load} = \frac{500}{4} = 125 \text{ W} = 0.125 \text{ kW}$$

$$\text{Power factor for lighting load} = 1.0$$

All-day efficiency of transformer A

$$= \frac{\text{Output energy in kWh in 24 h}}{\text{Input engery in kWh in 24 h}}$$

$$= \frac{(40 \times 1 \times 4 + 20 \times 1 \times 8) \times 100}{40 \times 1 \times 4 \times 20 \times 1 \times 8 + 0.500 \times 24 + 0.500 \times 4 + 0.125 \times 8}$$

$$= 95.5\%$$

Similarly the all-day efficiency of transformer *B* is calculated as:

All-day efficiency of transformer *B*

$$= \frac{(40 \times 1 \times 4 + 20 \times 1 \times 8) \times 100}{40 \times 1 \times 4 \times 20 \times 1 \times 8 + 0.250 \times 24 + 0.750 \times 4 + 0.1875 \times 8}$$

$$= 96.8\%$$

Thus all-day efficiency of transformer *B*, which has less core-loss as compared to transformer *A*, is higher than that of transformer *A*.

EXAMPLE 3.31 *A 50 kVA single-phase transformer draws a primary current of 250 A on full-load. The total resistance referred to primary side is 0.006 Ω. If the iron loss of the transformer is 200 W, calculate the efficiency on full-load and on half-load at 0.8 power factor lagging.*

Solution

$$\text{Full-load copper loss} = I_1^2 R_e'$$

$$= (250)^2 \times 0.006 = 375 \text{ W}$$

$$\text{Total loss on full-load} = \text{Copper-loss} + \text{Core-loss}$$

$$= 375 + 200$$

$$= 575 \text{ W} = 0.575 \text{ kW}$$

$$\text{Total loss on half load} = [(1/2)^2 \times 375] + 200$$

$$= 93.75 + 200$$

= 293.75 W (Copper loss being proportional to the square of the load)

Efficiency at full-load, 0.8 power factor lagging

$$\eta = \frac{\text{Output} \times 100}{\text{Output + losses}} = \frac{50 \times 0.8 \times 100}{50 \times 0.8 + 0.575}$$

$$= 98.6\%$$

Efficiency at half load, 0.8 power factor lagging

$$\eta = \frac{25 \times 0.8 \times 100}{25 \times 0.8 + 0.29375} = 98.5\%$$

EXAMPLE 3.32 *A 10 kVA, 400/200 V, single-phase, 50 Hz transformer has a maximum efficiency of 96% at 75% of full load at unity power factor. Calculate the efficiency at full-load 0.8 power factor lagging.*

Solution

Output at 75% of full load at unity power factor = 10 × 0.75 × 1 = 7.5 kW

Input at 75% of full-load unity power factor

$$= \frac{\text{Output}}{\text{Efficiency}} = \frac{7.5}{0.96} = 7.812 \text{ kW}$$

$$\text{Total losses} = \text{Input} - \text{Output}$$

$$= 7.812 - 7.5 = 0.312 \text{ kW}$$

At maximum efficiency, core loss is equal to copper loss. Here, maximum efficiency occurs at 3/4 th of full-load

∴ Iron loss, $P_i = (3/4)^2 \times$ full load copper loss, P_c

or $P_i = 0.5625\, P_c$

$$\text{Iron loss, } P_i = \frac{\text{Total loss}}{2} = \frac{0.312}{2} \text{ kW} = 156 \text{ W}$$

$$P_i = 0.5625\, P_c$$

or

$$P_c = \frac{156}{0.5625} = 277 \text{ W}$$

Thus, total losses on full-load

$$= P_i + P_c$$

$$= 156 + 277 = 433 \text{ W}$$

Efficiency on full-load, 0.8 power factor lagging,

$$\eta = \frac{\text{Output} \times 100}{\text{Output} + \text{Losses}}$$

$$= \frac{10 \times 0.8 \times 100}{10 \times 0.8 + 0.433} = 94.8\%$$

EXAMPLE 3.33 *The maximum efficiency of a 500 kVA, 3300/500 V, 50 Hz, single phase transformer is 97% and occurs at 75% of full-load, unity power factor. If the impedance is 10%, calculate the regulation at full-load power factor 0.8 lagging.*

Solution Output at maximum efficiency

$$= \text{kVA rating} \times 0.75 \times \text{p.f.}$$

$$= 500 \times 1000 \times 0.75 \times 1$$

$$= 375000 \text{ Watts}$$

Efficiency $\eta = \dfrac{\text{Output}}{\text{Output + Losses}}$

or $\text{Output} = \eta\,(\text{Output + Losses})$

$\eta \times \text{Losses} = \text{Output}\,(1 - \eta)$

or $\text{Losses} = \left[\dfrac{1}{\eta} - 1\right] \times \text{Output}$

Thus, at maximum efficiency,

$$\text{Losses} = \left[\frac{1}{0.97} - 1\right] \times 375000 = 11598 \text{ W}$$

At maximum efficiency,

$$\text{Core-losses} = \text{Copper-losses}$$

Therefore, $\text{Copper-losses} = \dfrac{\text{Total loss}}{2} = \dfrac{11598}{2} = 5799 \text{ W}$

These copper losses occur at 75% of full-load.
Thus, copper losses at full-load

$$= 5799(4/3)^2 = 10309 \text{ W}$$

(Copper losses are proportional to the square of the load current)
Equivalent resistance,

$$R_e \text{ in per unit} = \frac{I_1 R_e}{V_1}$$

$$= \frac{I_1^2 R_e}{V_1 I_1}$$

∴ $R_e(\text{p.u.}) = \dfrac{\text{Ohmic losses}}{\text{kVA rating}}$

$$= \frac{10309}{500 \times 1000} = 0.02\ \Omega$$

Given, $Z_e\ (\text{p.u.}) = 0.1$

Thus $X_e\ (\text{p.u.}) = \sqrt{Z_e^2\,(\text{pu}) - R_e^2\,(\text{pu})}$

$$= \sqrt{(0.1)^2 - (0.02)^2} = 0.098$$

$$\%\text{ voltage regulation} = \left[\frac{I R_e}{V_1}\cos\phi_2 + \frac{I_1 X_e}{V_1}\sin\phi_2\right] \times 100$$

$$= [0.02 \times 0.8 + 0.098 \times 0.6] \times 100 = 7.5\%$$

3.10 AUTOTRANSFORMERS

An autotransformer is a one-winding transformer. The same winding acts as the primary and a part of it as the secondary. The winding is tapped at a suitable point to obtain the desired output voltage across the secondary (see Fig. 3.43).

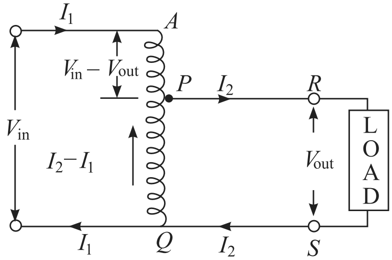


Fig. 3.43 *Connection diagram of an autotransformer*

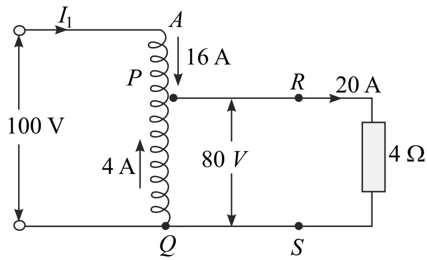


Fig. 3.44 *An autotransformer supplying a specific load*

The current flowing through the common portion *PQ* of the winding is the difference of the primary and secondary currents. Let us consider a specific example where the input voltage is 100 V and the output voltage 80 V as shown in Fig. 3.44.

A resistive load of say 4 Ω is connected across the output terminals. Current through the load from *R* to *S* will be 20 A and the power will be $(20)^2 \times 4$, i.e., 1600 W. If the losses are neglected (for simplification), the input to the autotransformer will be 1600 W. Current I_1 entering the winding *AP* is 1600/100 = 16 A. By applying Kirchhoff's current law at *P*, current flowing from *Q* to *P* will be 4 A.

The current 16 A drops 20 V in potential in flowing from *A* to *P* representing 320 W. This power must either be dissipated or appear elsewhere. Actually, this power is transferred to the flux in the core. This flux, by transformer action, raises the potential of the current of 4 A flowing from *Q* to *P* by 80 V. Thus 320 W of the total input of 1600.W are transformed and delivered to the load. The remaining 1280 W flow conductively to the load through *AP* and *PR* (see Fig. 3.44).

From the above example it is evident that all the input power in an autotransformer is not transformed but a certain portion of it flows conductively to the load. Therefore the size of an autotransformer will be smaller than a conventional two winding transformer. The ratio of the size of an autotransformer to the size of a two-winding transformer is given by $(n–1)/n$ where n is the primary to secondary turn or voltage ratio. For example, if an autotransformer has primary to secondary voltage ratio of 2 : 1, its size will be only (2 – 1)/2, i.e., one-half of the size of a two-winding transformer having the same voltage ratio.

In an autotransformer, the smaller the ratio of the primary to secondary voltage, the smaller is the amount of power transformed and hence larger is the amount of power conducted from the primary to the load. With a 100/75 V (i.e., 4 : 3) voltage ratio, only one-fourth of the total power is transformed whereas three-fourth is conducted. With a 100/50 V (i.e., 2 : 1) voltage ratio only half of the total power is transformed whereas half is conducted. The amount of power conducted directly from the primary to the load increases if the voltage ratio is decreased. Hence autotransformers become more and more economical than two-winding transformer if the ratio of transformation, i.e., the primary to secondary voltage ratio, is small. With a large ratio of transformation there is practically no advantage of an autotransformer over a two-winding transformer. Moreover, as the primary and secondary windings are not insulated from one another, in the case of any fault the secondary winding may be energised to the high potential of the primary causing damage to the instruments, etc. connected to the secondary side, unless special precautions are taken.

3.10.1 Comparison between Copper Used in an Autotransformer and a Two-winding Transformer of the Same Rating

It is known that the length of copper required in a winding is proportional to the number of turns, and the area of cross section of the winding wire is proportional to the current rating. Thus the weight of copper required in a conventional two-winding transformer is proportional to $I_p \times N_p + I_s \times N_s$, where N_p and N_s are respectively the number of primary and secondary turns.

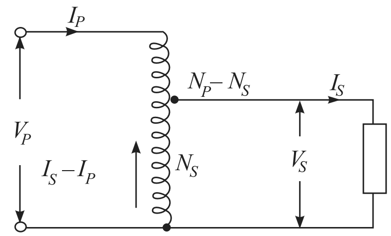


Fig. 3.45 *Connection diagram of an autotransformer*

For an autotransformer, the weight of copper required as can he seen from Fig. 3.45 is proportional to $I_p(N_p - N_s) + (I_s - I_p)N_s$.

$$\text{Thus, } \frac{\text{Weight of copper required in an auotransformer}}{\text{Weight of copper required in a two-winding transformer}} = \frac{I_p(N_p - N_s) + (I_s - I_p)N_s}{I_pN_p + I_sN_s}$$

$$= \frac{I_pN_p - I_pN_s + I_sN_s - I_p N_s}{I_pN_p + I_sN_s}$$

$$= \frac{I_pN_p + I_sN_s - 2I_pN_s}{I_pN_p + I_sN_s}$$

$$= 1 - \frac{2I_pN_s}{2I_pN_p} \qquad (\text{as, } I_p N_p = I_s N_s)$$

$$= 1 - \frac{N_s}{N_p} = 1 - \frac{1}{n}$$

$$= \frac{n-1}{n} \tag{3.26}$$

If, for the example, the ratio of the primary to secondary voltage of an autotransformer is 100: 50, then the above ratio would be (2–1)/2 = 1/2. Thus the saving in copper by using an autotransformer would be 50%.

EXAMPLE 3.34 *The primary and secondary voltages of an autotransformer are 230 V and 75 V respectively. Calculate the currents in different parts of the winding when the load current is 200 A. Also calculate the saving in the use of copper.*

Solution

$$I_2 N_s = I_1 N_p$$

$$I_1 = I_2 \frac{N_s}{N_p} = I_2 \frac{V_s}{V_p} = 200 \times \left[\frac{75}{230}\right]$$

Primary current, $I_1 = 65.2$ A

Load current, $I_2 = 200$ A

Current flowing through the common portion of the

$$\text{winding} = I_2 - I_1 = 200 - 65.2 = 134.8 \text{ A}$$

Let weight of copper required in a two-winding transformer = W_{TW}, and weight of copper required in an auto transformer = W_a

$$\text{Economy in saving in copper in percentage} = \frac{W_{TW} - W_a}{W_{TW}}$$

$$= \left[1 - \frac{W_{TW}}{W_a}\right] \times 100$$

$$= \left[1 - \left(1 - \frac{N_s}{N_p}\right)\right] \times 100$$

$$= \frac{N_s}{N_p} \times 100$$

$$= \frac{V_s}{V_p} \times 100 = \frac{75 \times 100}{230} = 32.6\%$$

3.10.2 Applications of Autotransformer

One of the most common application of autotransformers is the variac used in laboratories. A variac has a winding wound on a toroidal core. A thick carbon brush makes contact with the winding in the desired position. The output voltage can be varied from zero up to say 120% of the input voltage. Figure 3.46 shows photographic view and connection diagram of a single phase variac. Variacs are used for making available ac voltage from fixed supply voltage.

Single-phase autotransformers are also used for supplying improved voltage to domestic appliances. Three-phase auto-transformers are used in the interconnection of grids, say a 132 kV grid with a 220 kV grid.

Three-phase induction motors are often started with three-phase autotransformers where a reduced voltage is applied across the motor-terminals at starting.

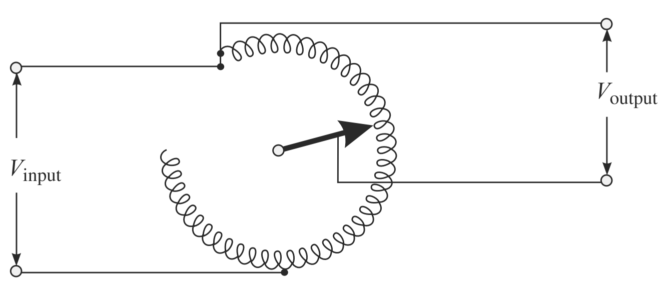


Fig. 3.46 *Connection diagram of a single-phase autotransformer (varier)*

3.11 INSTRUMENT TRANSFORMERS

Instrument transformers are small transformers intended to supply low values of current or voltage to measuring instruments, protective relays and other similar devices. It may become dangerous to the operator if measuring instruments are connected directly to the high voltage system. By means of instrument transformers, viz. current transformer (CT) and potential transformer (PT), instruments can be completely insulated from the high voltage system and yet measure accurately the current and voltage of a high-voltage circuit. For measurement of power of a high-voltage circuit a wattmeter of low rating can be used where the current coil and potential coil of the wattmeter of low rating can be used where the current coil and potential coil of the wattmeter are connected to the circuit through a CT and a PT respectively.

3.11.1 Current Transformers (CT)

As mentioned earlier, connecting ac ammeters and current coils of wattmeters directly on a high voltage system can be avoided through the use of current transformers. Current transformers, in addition to insulating the instruments from high voltage, step down the current to a known ratio, making it possible to use ammeters of a lower range.

The primary winding of a current transformer consists of a few turns and is connected in series with the line whose current is to be measured. The secondary has comparatively more number of turns. A low-range ammeter, generally of rating 0–5 A is connected in the secondary circuit as shown in Fig. 3.47.

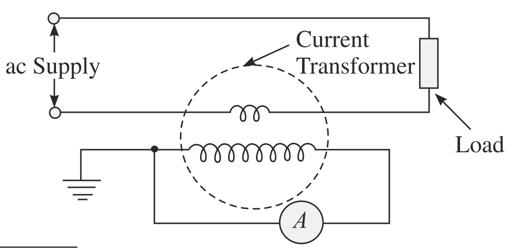


Fig. 3.47 *Connection diagram of a current transformer*

The ratio of the number of secondary to primary current is approximately the inverse ratio of their turns. The secondaries of all current transformers are rated at 5 A irrespective of the primary current rating.

A 1000 A current transformer has a ratio of 200 : 1. The ratio of primary and secondary turns will be 1 : 200. The primary of a CT may be of the form of a straight conductor as shown in Fig. 3.48. The secondary is insulated from the primary and is assembled on a core.

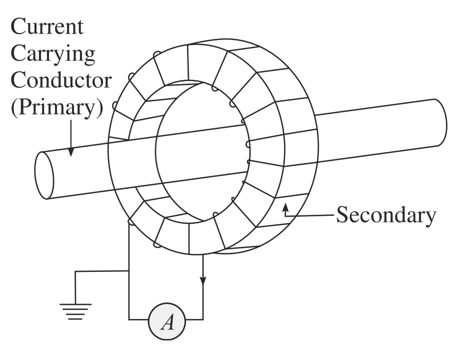


Fig. 3.48 *A type of current transformer*

The main difference between a current transformer and a two-winding power transformer is that the primary current of a CT is determined by the load on the system and not by the load on its secondary. If by any chance the secondary is open-circuited, the whole of the primary current (load current of the system) will work as a magnetising current. A dangerously high voltage will be induced in the secondary winding because the secondary has many number of turns as compared to the primary. This exceedingly high voltage may give fatal shock to the operator or may cause damage to the insulation. For this reason the secondary of a current transformer must always be earthed.

3.11.2 Potential Transformer (PT)

Basically, potential transformers are similar to power transformers except that their power rating is considerably low. The primary of a potential transformer has many number of turns as compared to its secondary and is connected across the high-voltage system whose voltage is to be measured. The secondary winding is usually rated for 110 V irrespective of the primary voltage rating. With the secondary winding wound for 110 V, the primary is wound for the voltage which is to be measured. For example, in an electric sub-station, if a voltage of 132 kV is to be measured, the PT will have a turn ratio of 132,000/110, i.e., 1200 : 1. The connection of PT is shown in Fig. 3.49.

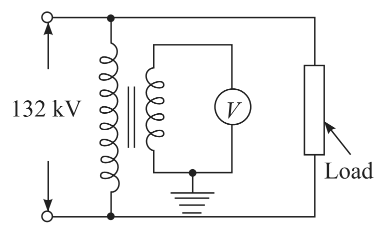


Fig. 3.49 *Connection diagram of a potential transformer to 132 kV circuits*

To ensure safety to the operator, the secondary of a PT should be earthed.

3.12 TESTING OF TRANSFORMERS

Transformers should be tested in the factory by the manufacturer before despatching them to the customers. Tests should be carried out at room temperature after fitting HT and LT bushings, other fittings in the tank and filling filtered transformer oil in the tank.

Indian standard IS: 2026 (specification for power transformers) has laid down for power transformers (including autotransformers) the type of tests to be carried out. The standard does not cover small and special purpose transformers like single-phase transformers rated at less than 1 kVA and poly-phase transformers rated at less than 5 kVA; outdoor type three-phase distribution transformers up to and including 100 kVA, 11 kV (covered by IS: 1180–1964); instrument transformers; transformers for static converters; starting transformers; testing transformers; traction transformers mounted on rolling stock; welding transformers mining transformers; earthing transformers; X-ray transformers; reactors; and furnace-type transformers.

Three types of tests, namely Routine Tests, Type Tests and some Special Tests have been specified in IS: 2026 (Part I)–1977.

3.12.1 Routine Tests

The routine tests specified are as follows:
(a) Measurement of winding resistance
(b) Measurement of voltage ratio and check of voltage vector relationship
(c) Measurement of impedance voltage/short-circuit impedance (principal tapping) and load loss test
(d) Measurement of no-load loss and current
(e) Measurement of insulation resistance
(f) Dielectric tests
(g) Tests on-load tap-changers, where appropriate.

3.12.2 Type Tests

The following shall constitute the type tests:
(a) Measurement of winding resistance
(b) Measurement of voltage ratio and check of voltage vector relationship
(c) Measurement of impedance voltage/short-circuit impedance (principal tapping) and load loss test
(d) Measurement of no-load loss and current
(e) Measurement of insulation resistance
(f) Dielectric tests
(g) Temperature-rise tests
(h) Tests on no-load tap-changers, where appropriate.

3.12.3 Special Tests

The following tests are to be carried out by mutual agreement between the purchaser and the supplier:
(a) Dielectric tests
(b) Measurement of zero-sequence impedance of three-phase transformers
(c) Short-circuit tests
(d) Measurement of acoustic noise level
(e) Measurement of harmonics of no-load current
(f) Measurement of the power taken by fans and oil pumps

The test procedure for the testing of transformers is given in IS: 2026 (Part-I)—1977 to IS : 2026 (Part-IV)—1977. However, a brief idea of some of the above tests is given below.

Measurement of Winding Resistance Measurement of winding resistance can be made accurately either by a Wheatstone bridge or by a Kelvin bridge. The ammeter-voltmeter method can also be used for the measurement of low resistance. A heavy current, of say 50 A, and sensitive meters would give accurate result. The test may be made when the transformer is either cold or hot. It should be ensured that the winding and oil temperature have been equalised when measurement is made. Temperature should also be recorded when the winding resistance is measured.

Measurement of Voltage Ratio and Check of Voltage-Vector Relationship

The voltage ratio should be measured on each tapping. The ratio test can be carried out by using a precisely calibrated voltmeter. A better method is to use a ratio-testing set called a ratiometer. The test set consists of a transformer with a fixed primary winding and a secondary winding with a large number of tappings at equal intervals connected to two selector switches—one for coarse adjustment and another for fine adjustment so that any desired voltage can be obtained and read off directly. The high-voltage side of the transformer under test is connected directly to the 230 V or 400 V mains. The voltage induced on the low-voltage side is compared with the output voltage of the ratiometer by connecting the two windings in such a way that the two voltages are in phase opposition. An ammeter is connected between the two windings to ensure that for accurate reading no circulating current flows.

The polarity of single-phase transformers should be checked. It is necessary to know the relative polarities of the terminals of a transformer in order to make proper connections when transformers are to be connected in parallel for a distribution system and also when transformers are interconnected to supply three-phase power.

The polarity of the primary-winding terminals of a transformer is determined by the supply voltage which changes every half-cycle. The polarity of secondary winding terminals is determined by the direction of winding of the secondary. This is illustrated in Fig. 3.50.

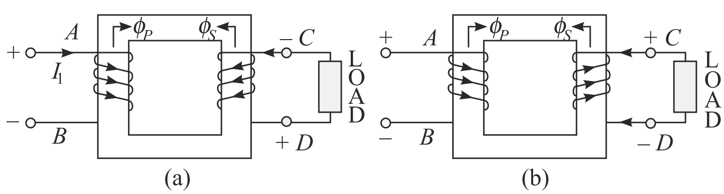


Fig. 3.50 *Determination of polarity of a transformer*

Induced emf in the secondary winding will set up current in such a direction that the current coil sets up a flux to oppose the flux produced by the primary winding in accordance with Lenz's law. Terminals *A* and *D* are terminals of identical polarity as are terminals *B* and *C* as shown in Fig. 3.50(a). If the secondary winding is wound in opposite directions as shown in Fig. 3.50(b) the polarities of the secondary terminals will be opposite to those shown in Fig. 3.50(a).

The voltage vector of three-phase transformers should be checked. This can be checked with the help of an ordinary voltmeter. The winding connections should be correctly made as per the vector diagram given on the name plate.

Measurement of Impedance Voltage/Short-circuit Impedance (Principal Tapping) and Load-Loss Test

Impedance Voltage/Short-Circuit Impedance Test Impedance voltage is expressed as a percentage of the rated voltage of a transformer. An impedance voltage of 10% for a 1100 V/400 V transformer means that when voltage of

110 V (10% of 110 V) is applied across the high-voltage terminals, full-load current will flow through the 400 V windings if it is kept short-circuited. This test can be performed by using a variable voltage source and ordinary voltmeter and ammeter. A tolerance of ±10% over the specified impedance voltage is permissible for a two-winding transformer.

Load-Loss Test This loss mainly comprises copper-loss. The load-loss can be determined by measuring the power input when impedance voltage is applied across one winding and the other winding is short circuited allowing full-load current through the windings.

Load-loss can be determined by performing the back-to-back test on two identical transformers, if available. In the back-to-back test the primary windings of the two transformers are connected across the supply mains. The secondaries are connected in opposition (back-to-back) through an ammeter. By adjusting the tappings on the transformer windings a circulating current equal to the full-load current can be allowed to flow. The wattmeter reading under this condition will approximately give the full-load copper-loss. The back-to-back test can also be performed to determine the temperature rise of a transformer. When performed for this purpose such a test is also called the heat-run test.

Measurement of No-Load Loss and Current No-load loss, i.e., the core-loss or the iron-loss is measured by performing the no-load test at normal voltage. Normal voltage is applied across one winding keeping the other windings open. The power input measured with the help of a wattmeter gives approximately the iron-loss. The ammeter reading records the no-load current. The no-load current will be about 3 to 5% of the full-load current depending upon the size and design of the transformer. The no-load power factor is very poor. The wave shape of the no-load current contains a number of harmonics of appreciable magnitude.

Measurement of Insulation Resistance Insulation resistance can be measured with the help of a megger. It may be noted that insulation resistance depends on temperature. For example, the insulation resistance for class *A* insulators becomes one-half for every 10 to 15° rise in temperature. It is therefore necessary to record the temperature while measuring insulation resistance. There is no hard-and-fast rule regarding the minimum permissible insulation resistance. The following values of insulation resistance are considered safe with the transformer working at 45 °C.

Rated voltage	400 V	11 kV	33 kV	132 kV
Insulation resistance	2 MΩ	50 MΩ	150 MΩ	500 MΩ

Dielectric Test The transformer should pass the appropriate dielectric tests specified in IS: 2026 (Part III) – 1977.

Temperature-Rise Test (Also called Heat-Run Test) This test is conducted to check the temperature rise of a transformer when operating at full-load. Test is performed either by loading the transformer indirectly, i.e., through a back-to-back arrangement or by applying impedance voltage. Temperature is measured at regular intervals of time till no further appreciable temperature rise is recorded (may be 48 hours or so). The temperature rise should be within the permissible limit as specified.

3.12.4 Back-to Back Test or Heat Run Test

This test is also called Sumpner's test or Regenerative test. This test is also called heat-run test. This test is conducted to draw the temperature rise characteristic of a transformer. When a transformer is loaded i.e., when it is supplying power to a load, temperature of the windings and the core rises gradually and reaches a final temperature after a period of time. Normally, the temperature rise stabilizes after about 48 hours. The simple way to draw the temperature rise characteristic is to load the temperature and record its temperature after each interval of time, say after one hour for 48 hours. The temperature rise will be exponential as shown in Fig. 3.51.

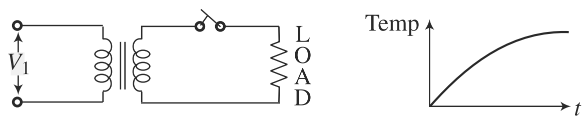


Fig. 3.51 *Temperature rise of a transformer when it is supplying a load*

For high capacity transformers, however, it may not be possible to arrange for the load in the laboratory. Moreover, since the experiment is to be conducted for a long period of time, there will be huge amount of power loss in the load resistance and therefore the experimentation will be expensive.

For these two reasons the experiment is conducted by fictitiously loading the transformer. This type of loading is also called phantom loading.

When a transformer is supplying a load, there is iron-loss in the core and I^2R loss in the windings. In fictitious loading system, a condition is created where core loss at rated voltage and I^2R loss in the windings at rated current takes place in the transformer but without actually loading the transformer. To conduct heat run test on one transformer, another identical transformer is required.

The connection diagram is shown in Fig. 3.52. Two identical transformers T_1 and T_2 have been connected back to back. The primary windings of the two transformers have been connected across the supply of rated voltage and rated frequency. The secondary windings have been connected in series opposition so that the output voltage of the secondary windings in series is zero. A voltmeter V_1 will measure the primary input voltage, an ammeter will measure the total primary current and a wattmeter, W_1 will measure the total input power to the two transformers. After connecting the transformers back to back, supply is given to the transformer primaries by closing the switch S_1. If the transformer secondaries have been connected back to back, the reading of the voltmeter, V_2 will be zero. Otherwise, the voltmeter reading will be twice the secondary voltage of the two windings. If the voltmeter reads twice the secondary voltage of a transformer, then connections of the secondary winding terminals have to be changed so that voltmeter reading, V_2 is zero.

The voltmeter V_2 should be chosen such that it can read twice the secondary voltage of each transformer. When the voltmeter reading V_2 is zero, there will be no current flowing through the secondary windings. This is equivalent to no-load condition of the transformers. Remember that on no-load, the input power to a

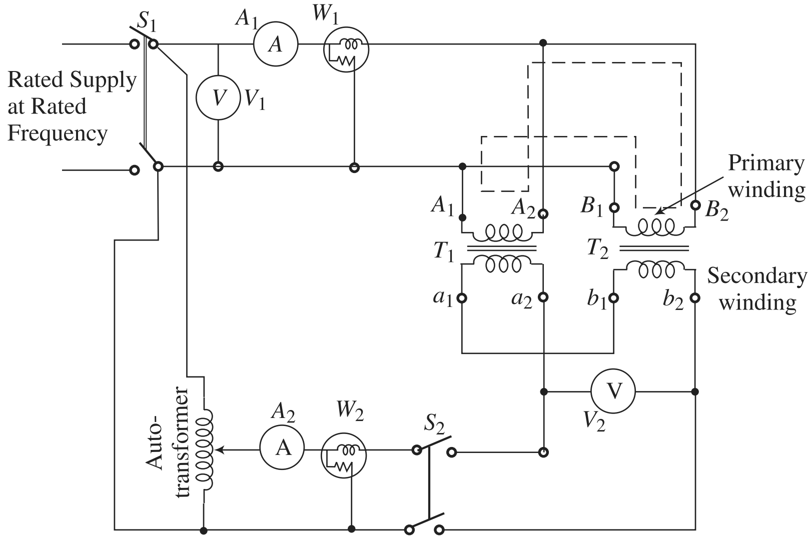


Fig. 3.52 *Back-to-back test of transformers*

transformers is approximately equal to the core loss of the transformer. Here the wattmeter reading will be equal to the core losses of the two transformers. Keeping the supply to the primary windings on, a small voltage with the help of an auto-transformer is applied to the secondary windings. This voltage is adjusted such that full load current flows through the secondary windings which can be checked from the reading of armature A_2. This secondary current will cause primary to draw more current from the supply mains such that full-load current flows through the primary windings also. This primary current will circulate through the secondary windings as shown by dotted lines in the figure. The wattmeter W_2 will record the I^2R losses in the primary and secondary windings of both the transformers.

In this arrangement we have created a full-load condition of the two transformers without actually using any load.

The power drawn from the supply is $W_1 + W_2$ which is the sum of core loss and copper loss in the two transformers. The ammeter A_1 records the no-load current drawn by the two transformers.

By connecting the two identical transformers back to back and by supplying a small voltage in the secondary winding so as to circulate the full-load current, the temperature rise of the transformers is recorded on hourly or two-hourly basis for a period of say 48 hours and the data is plotted. The temperature of the transformer is measured by measuring the temperature of the transformer oil in the tank with the help of an alcohol thermometer (mercury thermometer will give higher reading due to eddy current loss and hence heat generated in the mercury).

3.13 INSTALLATION AND MAINTENANCE OF TRANSFORMERS

A transformer is despatched from a factory with its tank filled with oil or dry air of inert gas under pressure. On receipt of the transformer the following may be checked.

Dielectric Strength of Oil Samples of oil should be taken from near the top and bottom of the tank and the dielectric strength tested. If the dielectric strength is below the acceptable value (lower than 30 kV for a 4 mm gap), the oil should be filtered and dried.

Insulation Resistance of Windings The insulation resistance of each winding should be measured with the help of an insulation tester. The insulation resistance should not be less than a certain minimum value at a particular temperature.

In addition to the above two tests checks should be made to detect any possible unwanted connections by removing the tank top cover.

Location of Transformers Transformers may be mounted either indoors or outdoors. When mounted indoors, the room for housing a transformer should be large enough to allow free access on all sides with adequate ventilation. Outdoor installation of transformers can be made in any of the following ways depending on the rating of the transformer:

(i) Directly clamped to a pole (Pole mounted)
(ii) Mounted on a cross-arm fixed between two poles
(iii) Platform mounting
(iv) Floor mounting

When transformers are located close to one another, fireproof barrier walls should be provided to restrict the damage caused due to fire from one to the others. Transformers installed outdoors should be protected through enclosure allowing free circulation of air from all sides.

Maintenance of Transformers For a long trouble-free service, transformers should be given due attention regularly. The programme of maintenance depends on the size of the transformers, operating conditions and its location. Maintenance schedule for transformers installed in attended indoor and outdoor substations and also in unattended outdoor substations is given in Table 3.1.

Table 3.1 *Maintenance Schedule of Transformers*

Inspection Frequency	*Items to be inspected in attended indoor or outdoor transformer and actions to be taken*	*Items to be inspected in unattended outdoor transformer and action to be taken*
Hourly	—Check winding temperature, oil temperature, ambient temperature, load and voltage. Adjust load to keep the temperature rise within a permissible limit.	
Daily	(a) Check oil level. If low, fill in dry oil. (b) Check the colour of the silica gel in the breather. Colour should be blue. If the colour of the silica gel becomes pink replace them.	
Quarterly	Check for proper working of cooling fans, circulating pump, etc.	Clean bushings. Examine conditions of switches and tighten connections. Check condition of silica gel. Check oil level.

(Contd.)

Half-yearly	Check dielectric strength of oil. Check bushes and insulators. Check cable boxes/filter/replace Oil	
Yearly	Check oil for acidity, sludge formation, etc. Check alarms, relays, and contacts. Check earth resistance Check lightning arrestors.	Check dielectric strength of oil. Check earth resistance. Check lightning arrestors.
Five yearly	Carry out overall inspection of the transformer including lifting of core and coils. Clean the transformer with dry transformer oil.	Open tank for cleaning and re-assembling. Washdown with transformer oil

In large substations and grid substations a standby transformer should be kept to be switched on at a moment's notice in case of need. Such a standby transformer should be maintained at the same standard as the other installed transformers.

Proper maintenance will ensure trouble free operation of transformers for many years and thereby ensure uninterrupted power supply to consumers.

3.14 TERMINAL MARKINGS, POSITION OF TERMINALS, RATING AND RATING PLATES

3.14.1 Terminal Markings

Winding terminals of a transformer should be designated by letters. The higher voltage winding is designated by a capital letter and the lower voltage winding by the corresponding small letter. Additional windings are designated by a capital letter appropriate to its phase preceded by the numerals 3, 4, etc. Thus the designation of winding terminals should be:

(i) For a single-phase transformer
   A—higher-voltage winding
   a—lower-voltage winding
   3*A*—third winding (if any)

(ii) For a three-phase transformer
   A, *B*, *C*—higher-voltage windings
   a, *b*, *c*—lower-voltage windings
   3*A*, 3*B*, 3*C*—third windings (if any)

An example of the above is given in Fig. 3.53.

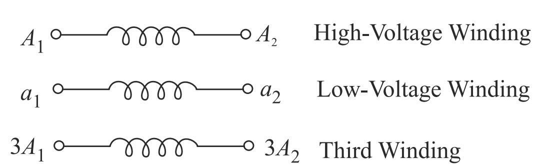


Fig. 3.53 *Example of designating terminal markings for a single-phase winding without tappings*

In Figure 3.53 the markings are such that if in a higher voltage winding the direction of the induced voltage is at a given instant from A_1 to A_2, the direction of the induced voltage in the lower voltage winding at the same instant will also be from a_1 to a_2.

The arrangement of terminals is decided by considering the case of a two-winding transformer where the higher-voltage and lower-voltage winding terminals are mounted on opposite sides of the tank. For three-phase transformers, the arrangement of terminals viewed from the higher-voltage side should be from left to right. The neutral terminal is fitted to the extreme left. For a single-phase transformer the terminal subscript numbers should be arranged in descending order from left to right. These are illustrated in Fig. 3.54.

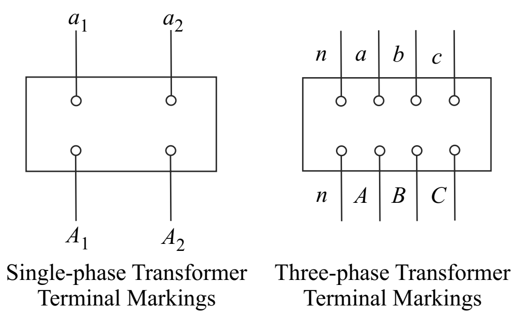


Fig. 3.54 *Markings of terminals of transformers*

3.14.2 Rating Plate

All transformers should be fitted with rating plates giving the following informations:

(i) Number of Indian Standard, viz. IS: 2026
(ii) Manufacturer's name, country of manufacture
(iii) Manufacturer's serial number
(iv) Rated kVA/MVA
(v) Number of phases
(vi) Frequency
(vii) Rated voltage at no-load
(viii) Rated current
(ix) Percentage impedance voltage at 75°C
(x) Winding connections and phase displacement symbols of vector diagrams (vector group)
(xi) Type of cooling
(xii) Weight of core and windings in kg
(xiii) Total quantity and weight of insulating liquids in litres and kilograms respectively
(xiv) Temperature-rise in oil for which transformer is designed
(xv) Year of manufacture

3.14.3 Transformer Ratings

As mentioned the nameplate of a transformer, in addition to other information, it provides information on rated power (Apparent power) in kVA or in MVA, and the voltages of each winding.

For example, let us say that the data taken from the name plate of a transformer are 10 kVA, 200/100 V. The following information are derived from the above:

1. The transformer full-load power rating of 10 kVA. It can provide supply of 10 kVA on a continuous basis.
2. It is a step down transformer with

$$V_1 = 200 \text{ V and } V_2 = 100 \text{ V}.$$

3. The magnitudes of primary and secondary currents are

$$\frac{V_1 I_1}{1000} = 10 \quad \text{or} \quad I_1 = \frac{10 \times 1000}{200} = 50 \text{ A}$$

and $$\frac{V_2 I_2}{1000} = 10 \quad \text{or} \quad I_2 = \frac{10 \times 1000}{100} = 100 \text{ A}$$

4. The ratio of transformation is

$$N_1 I_1 = N_2 I_2$$

or, $$\frac{N_1}{N_2} = \frac{I_2}{I_1} = \frac{100}{50} = 2$$

Also $$\frac{N_1}{N_2} = \frac{E_1}{E_2} \simeq \frac{V_1}{V_2} = \frac{200}{100} = 2$$

3.15 PARALLEL OPERATION OF TRANSFORMERS

To supply a load in excess of the existing load on a transformer, there are two alternatives, viz. replacing the existing transformer by another of a higher rating or by connecting another transformer in parallel with the existing one. In practice, to supply excess load we connect transformers in parallel. In a substation, instead of installing a very large capacity transformer, a number of lower capacity transfomers are seen in parallel. In case of fault in one transformer, supply of electricity can be continued and the faulty transformer taken out for repair. There are certain conditions which need to be fulfilled for satisfactory parallel operation of transformers. The parallel operation of both single-phase and three-phase transformers is discussed below.

3.15.1 Parallel Operation of Single-phase Transformers

Figure 3.55 shows two single-phase transformers connected in parallel with their primary windings connected to the same supply and their secondary windings supplying a common load. In the same manner more transformers can be connected in parallel depending upon the increase in magnitude of the load to be supplied.

From Fig. 3.55, by applying Kirchhoff's law, the following equations can be written for the primary side and secondary side respectively.

$$V_p = E_{A_1A_2} = E_{B_1B_2} \tag{3.27}$$

and $$V_S = E_{a_1a_2} = E_{B_1B_2} \tag{3.28}$$

The two secondary voltages $E_{a_1a_2}$ and $E_{b_1b_2}$ must be in phase because primary supply voltage V_p is common and is equal to $E_{A_1A_2}$ and $E_{B_1B_2}$.

For satisfactory parallel operation, transformer terminals must be correctly marked and like terminals joined together. Wrong connections will result in a short-circuit in the windings.

The primary and secondary windings of both the transformers are assumed to have N_1 and N_2 turns respectively. Now, dividing Eq. (3.27) by Eq. (3.28),

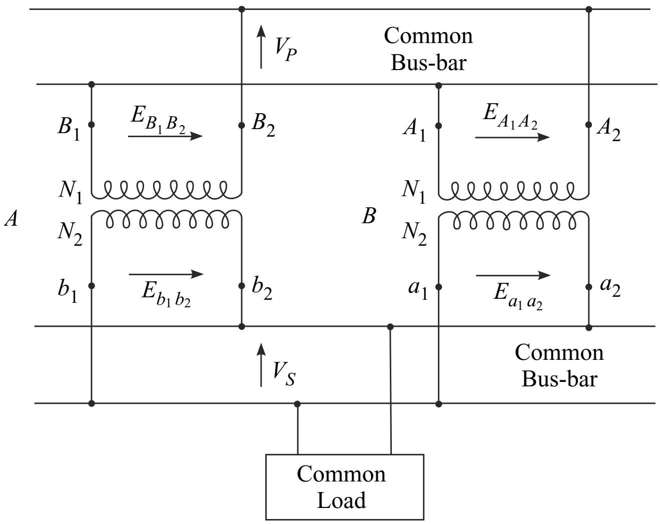


Fig. 3.55 *Two single-phase transformers connected in parallel*

$$\frac{V_p}{V_s} = \frac{E_{A_1A_2}}{E_{B_1B_2}} = \frac{E_{a_1a_2}}{E_{b_1b_2}} = \frac{N_1}{N_2} \tag{3.29}$$

If the turn ratios of the two transformers as assumed are not equal, circulating current will flow between the two transformers to equalize the voltage in each. This will reduce the capacity of the transformers supplying a common load, which is not desirable. It may, however, be realised that in practice it is difficult to manufacture two exactly identical transformers. As long as the difference is very small, the transformers will run satisfactorily in parallel. There are two more conditions that are necessary to be satisfied for efficient parallel operation of transformers.

The potential differences at full-load across the transformer internal impedances should be equal. In other words, per-unit or percentage impedance of the two transformers should be equal. If this condition is fulfilled, load sharing of the total connected load between the two transformers will be proportional to their kVA ratings. The reader is advised to study the concept of per-unit representation of values given at the end of this section.

The other desirable condition is that the ratio of the winding resistance to reactance for the transformers should be equal. This condition, if satisfied, will ensure that both the transformers will operate at the same power factor supplying a common load.

To summarise, *the conditions for satisfactory parallel operation of single-phase transformers are*

(i) Voltage ratings of the primary windings should be suitable for supply system voltage and frequency. The turn ratios of the transformers must be equal.

(ii) Transformers should be properly connected with regard to their polarity.

(iii) The per-unit or percentage impedance of the two transformers should be equal.

Note: The concept of per-unit value is given at the end of this section.

(iv) Ratios of winding resistances to reactance for the transformers should be equal.

Assuming that all the above-mentioned conditions are satisfied and that the terminals are correctly labelled, the two transformers can be correctly paralleled as shown in Fig. 3.55 by first connecting a voltmeter between terminal b_1 and common secondary busbar 2. If the connections have been made with like polarities joined together, then the voltmeter reading will be zero. If the connections are made for reversed polarities, the voltmeter reading will be twice the secondary voltage. When all the four conditions of parallel operation are satisfied the two transformers running parallel will have the same voltage ratio and identical impedance voltage triangles. The phasor diagram of the various quantities of transformers running in parallel is shown in Fig. 3.56(b).

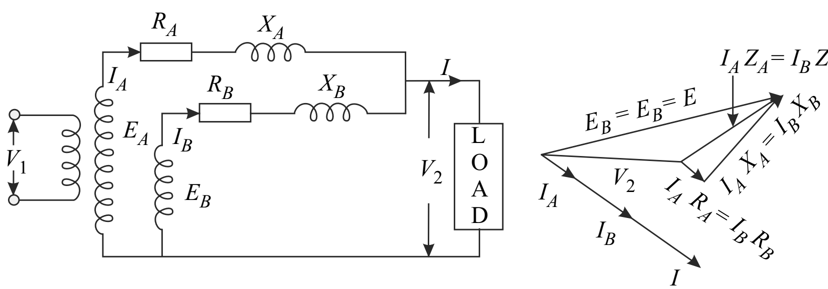


Fig. 3.56 *(a) Equivalent circuit of two transformers supplying a common load (b) Phasor diagram of two transformers running in parallel*

From Fig. 3.56 $\quad I_A + I_B = I$

$$I_A Z_A = I_B Z_B$$

$$I_A = I_B \frac{Z_B}{Z_A}$$

$$I_A + I_B = I_B + I_B \frac{Z_B}{Z_A} \quad I_B = \left[1 + \frac{Z_B}{Z_A}\right]$$

or
$$I = I_B \frac{Z_A + Z_B}{Z_A}$$

∴
$$I_B = I \frac{Z_A}{Z_A + Z_B} \tag{3.30}$$

and
$$I_A = I \frac{Z_B}{Z_A + Z_B} \tag{3.31}$$

Multiplying both sides of the above two expressions by V_2,

$$V_2 I_B = V_2 I \frac{Z_A}{Z_A + Z_B}$$

and $$V_2 I_A = V_2 I \frac{Z_B}{Z_A + Z_B}$$

$$V_2 I_A \times 10^{-3} = \text{kVA of the total load} = Q \text{ (say)}$$

Thus, kVA supplied by transformer *A*,

$$Q_A = Q \frac{Z_B}{Z_A + Z_B} \tag{3.32}$$

Similarly, kVA supplied by transformer *B*,

$$Q_B = Q \frac{Z_A}{Z_A + Z_B} \tag{3.33}$$

If the ratios of resistance of reactance of the two transformers are not equal (i.e., if the fourth condition for parallel operation is not satisfied), the power factor of the load supplied by the two individual transformers will be different. The expressions for kVA supplied by each transformer will however be the same as above.

EXAMPLE 3.35 *Two transformers are connected in parallel to supply a common load of 125 kVA at 0.8 power-factor lagging. Rating of transformer A is 100 kVA and has resistance and reactance of 0.9% and 10% respectively. Rating of transformer B is 50 kVA and has resistance and reactance of 1.0% and 5% respectively. How will the two transformers share the common load?*

Solution The percentage impedances of the two transformers should be brought to the same kVA level say 100 kVA.

Thus percentage impedance of transformer *B* at 100 kVA level,

$$Z_B = \frac{100}{50}(1 + j\,5) = 2 + j\,10$$

$$Z_A = 0.9 + j\,10$$

$$\cos \phi = 8.0,\ \phi = 37°$$

Total load $$Q = 125 \angle -37° \text{ kVA}$$

$$\frac{Z_A}{Z_A + Z_B} = \frac{0.9 + j10}{0.9 + j10 + 2j10} = 0.497 \angle 3°$$

and $$\frac{Z_B}{Z_A + Z_B} = \frac{2 + j10}{0.9 + j10 + 2 + j10} = 0.5 \angle -3°$$

Load shared by transformer *A*,

$$Q_A = Q \frac{Z_B}{Z_A + Z_B} = 125 \angle -37° \times 0.5 \angle -3°$$

$$= 62.5 \angle -40° \text{ kVA}$$

Load shared by transformer *B*,

$$Q_B = Q \frac{Z_A}{Z_A + Z_B} = 125 \angle -37° \times 0.497 \angle +3°$$

$$= 62.12 \angle -34° \text{ kVA}$$

Since the percentage impedances of the transformers are not equal, the transformers are not loaded in proportion to their ratings.

3.16 PER-UNIT REPRESENTATION OF VALUES

When it is expressed that the amount of current supplied by one transformers is say 50 A and by another transformer is say 80 A it does not give us an idea as to which of the two transformers is of higher capacity and how the transformers are loaded with respect to their capacity. If the full-load current rating of the transformer is 200 A and that of the second transformer is 100 A, then their loading can be compared as:

$$\text{Loading of first transformer} = \frac{50}{200} = 0.25 \text{ per unit or pu}$$

$$\text{Loading of second transformer} = \frac{80}{100} = 0.8 \text{ per unit or pu}$$

In machine problems it is often convenient to compare quantities with their pu values corresponding to full-load conditions. Values of current, voltage, resistance, reactance, impedance, power, etc., can usefully be expressed as per-unit values by dividing the actual values of amperes, volts, ohms, kilowatts, etc. by the corresponding values at full-load according to the rating of the machine. Consider the following example.

Consider a 5 MVA, 11 kV, three-phase, star-connected alternator having an impedance of $0.242 + j\ 12.1\ \Omega$ delivering 150 A at 10.5 kV at a pf of 0.8.

$$\text{Full-load current of the alternator} = \frac{5 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 262.5 \text{ A}$$

$$\text{Voltage per phase} = \frac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$$

In pu representation

$$1 \text{ pu voltage} = 6351 \text{ V}$$

$$1 \text{ pu current} = 262.5 \text{ A}$$

$$1 \text{ pu impedance} = \frac{V}{I} = \frac{6351}{262.5}\ 24.2 \text{ pu}$$

$$\text{Generator voltage} = \frac{10.5}{11} = 0.9545 \text{ pu}$$

$$\text{Generator current} = \frac{150}{262.5} = 0.571 \text{ pu}$$

$$\text{Generator impedance} = \frac{1}{24.2}\ (0.242 + j\ 12.1) \text{ pu} = (0.01 + j\ 0.5) \text{ pu}$$

The pu resistance of 0.01 and pu reactance of 0.5 are actually the resistance and reactance drop in pu at full-load but they are in short called pu resistance and pu reactance.

$$\text{Base pu resistance} = \frac{V}{I_a}$$

Resistance in pu of an armature having an actual resistance R_a

$$= \frac{\text{Actual resistance}}{\text{Base pu resistance}} = \frac{R_a}{V/I_a} = \frac{I_a R_a}{V}$$

3.17 THREE-PHASE TRANSFORMERS

Nearly all alternating current electrical energy is generated by three-phase alternating current generators. Similarly, three-phase systems are used for transmission and distribution of electrical energy. There are several reasons why a three-phase system is preferred over a single-phase system. Some of the important reasons are:

(i) kVA ratings of three-phase generators and horsepower ratings of three-phase motors, for a given physical size, are higher than those of similar single-phase units.

(ii) Operating characteristics of three-phase motors and other appliances are superior to those of similar single-phase units.

(iii) The efficiency of transmission and distribution of power in a three-phase system more than the efficiency in a single-phase system.

Alternating current generated through a three-phase generator has to be transmitted at a high-voltage level for economic reasons. At the receiving end of the transmission line it is necessary to transform the energy to suitable lower voltage level for distribution. It is therefore often necessary to transform the three-phase voltage system to a higher or lower value.

Electrical energy may be transferred from one three-phase circuit to another three-phase circuit with a change in voltage by means of a three-phase transformer. Power transmission on a three-phase system may also be accomplished by using three separate single phase transformers with the windings of the transformers connected in star or delta.

3.17.1 Advantages of Three-phase Transformer over a Bank of Three Single-phase Transformers

Recently, three-phase transformers are increasingly being used for both step up and step-down applications for the following reasons:

(i) The cost of one three-phase transformer is less than the cost of three single-phase transformers required to supply the same kVA or MVA output.

(ii) A three-phase transformer weighs less and occupies less space than three single-phase transformers.

(iii) The bus-bar structure, switchgear and other wiring for a three-phase transformer installation are simpler than those for three single-phase transformers.

It may, however, be mentioned that there is one major advantage in using three-phase transformers rather than a three-phase transformer. If a single-phase transformer in a three-phase bank becomes defective, it can be disconnected and partial power supply restored until replacement/repair is possible. In a three-phase transformer, however, if one of the phase windings becomes defective, the entire transformer must be taken out of service for repair work, thereby completely disturbing the power supply.

3.17.2 Three-phase Transformer Construction

Three single phase transformers can be connected together to supply a three-phase load. However, a single unit of three-phase transformer can also be created with three primary and three secondary windings wound on the three limbs of a common core.

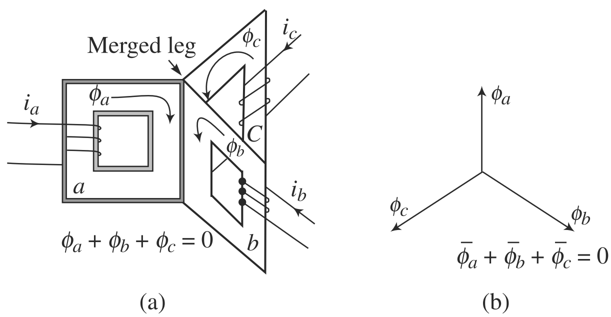


Fig. 3.57 *Development of a common core for a three-phase transformer*

To understand how a three-phase transformer can be built on a common core, let us consider the three single-phase transformers placed at 120° apart as shown in Fig. 3.57(a). In Fig. 3.57(a), the three primary windings constituting the three phases have been shown. When a balanced three-phase supply in applied across these three phase windings current i_a, i_b, i_c will flow through these windings as has been shown. These currents will produce respectively flux ϕ_a, ϕ_b, and ϕ_c in the three limbs. These fluxes will be 120° apart. The other three limbs of the cores are merged together forming one merged leg. Fluxes ϕ_a, ϕ_b, ϕ_c will pass through the merged leg. The phasor sum of these three fluxes is however equal to zero. So, in reality, no flux will flow through the merged leg. This merged leg, therefore, serves no purpose and hence can be ignored. But the core structure as shown in Fig. 3.57(a), and without the merged leg is not simple to construct.

This structure is made simple by pushing the section b in between section a and section c and removing its yokes. This modification makes the core of a three-phase transformer as has been shown in Fig. 3.58. The core is made of a large number of laminated sheets to reduce the eddy current loss. The primary and secondary windings of each phase are placed on the three legs. In fact, the primary and secondary windings, instead of placing side by side, are placed concentrically one below the other.

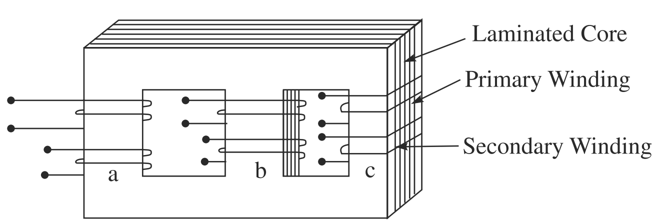


Fig. 3.58 *Development of core of a three-phase transformer*

Figure 3.59 shows the core of a three-phase transformer with low-voltage and high-voltage windings.

Each limb carries both high-voltage and low-voltage windings. The windings are similar to those of a single-phase transformer. The low-voltage windings are placed nearer the core limb as it is easier to insulate the low-voltage winding from the core than the high-voltage winding. The flux produced by the primary ampere turns will be linked by the secondary windings.

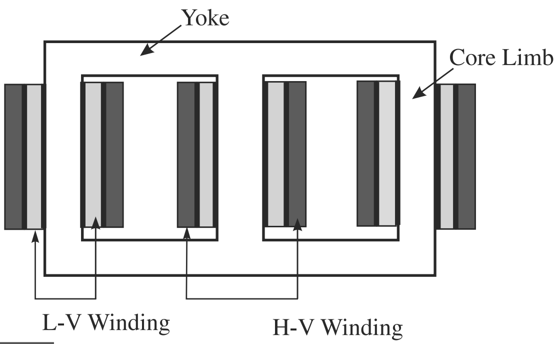


Fig. 3.59 *Placement of high-voltage and low-voltage windings on the core limbs of a three-phase transformer*

The flux in each leg will be out of phase by 120° and therefore will reach their maximum at different instants of time. At any instant of time at least one of the core legs will act as a return path for the flux of the other phases.

A shell-type core, less commonly used in three-phase construction, will have five limbs. Windings are made on the three limbs. The two outer limbs provide a return path for the fluxes.

The core structure along with the windings on the limbs is placed in a tank and immersed in transformer oil. The connections between the coil ends are made inside the tank of the transformer. The winding connections of the primary and secondary may be in delta-delta, star-star, delta-star, or star-delta. The terminals are brought out through high-voltage and low-voltage bushings for external connections.

3.17.3 Transformer Winding Connections

The high-voltage and low-voltage winding terminals of a three-phase transformer are connected either in star or in delta for connections to a three-phase system.

A bank of three single-phase transformers can also be connected similarly. When the primary high-voltage winding terminals are connected in, say, star and the secondary low-voltage winding terminals are connected in, say, delta, it is said that transformer windings are connected in star-delta (or Y–Δ). Similarly, star-star (Y–Y), delta-delta (Δ–Δ), and delta-star (Δ–Y) connections can be used. There is a definite time-phase relationship between the terminal voltages of the high-voltage side and low-voltage side for these connections. The time-phase relationship between the voltages of high-voltage and low-voltage sides will depend upon the manner in which the windings are connected. If the high-voltage side and low-voltage side windings are connected in star-star or in delta-delta, as shown in Fig. 3.60(a) and (c), the phase displacement will be zero. If, however, the low-voltage winding connections are reversed, the time-phase displacement in induced voltages between the high-voltage and low-voltage windings will be 180° as shown in Fig. 3.60(b) and (d).

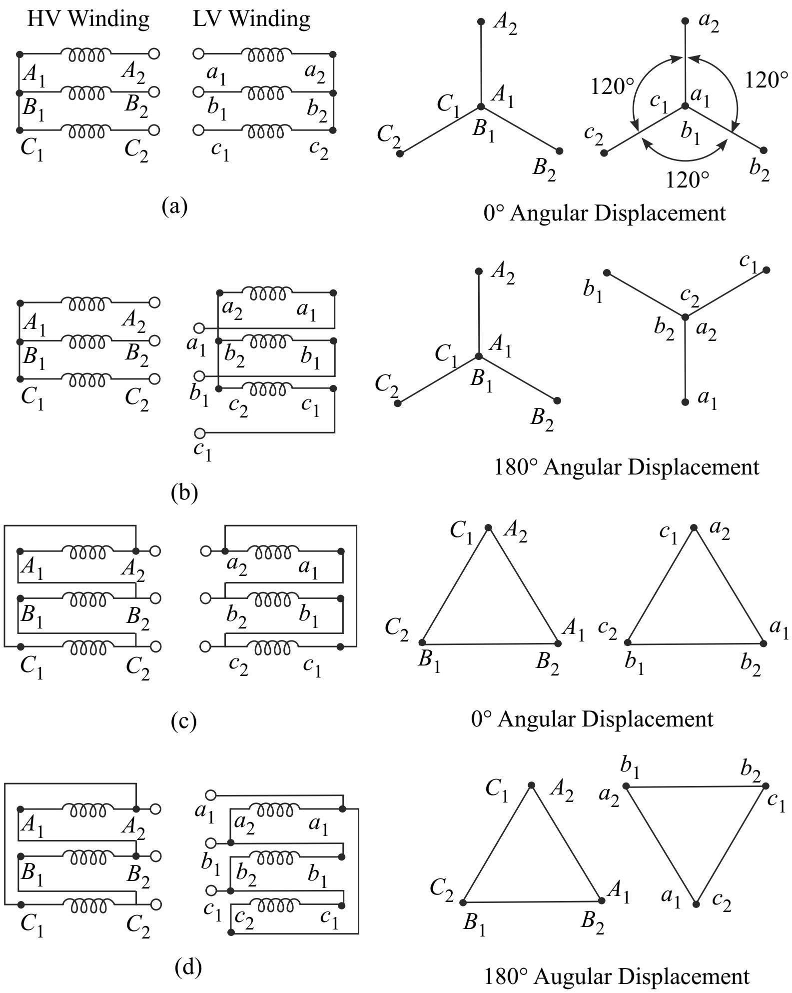


Fig. 3.60 *Transformer winding connection (a) Yy 0 (b) Yy 6 (c) Dd 0 (d) Dd 6*

If the primary high-voltage windings are connected in star and the secondary low-voltage windings are connected in delta the phase displacement will be –30°. If the winding connections of the low-voltage side are reversed the phase displacement will be +30° as shown in Fig. 3.61(a) and (b) respectively. If the high-voltage windings are now delta-connected and the low-voltage windings are star-connected, the phase displacement of the terminal voltages will be –30°. The phase displacement will be +30° if the winding connections of the high-voltage side are changed in Fig. 3.61(c) and (d).

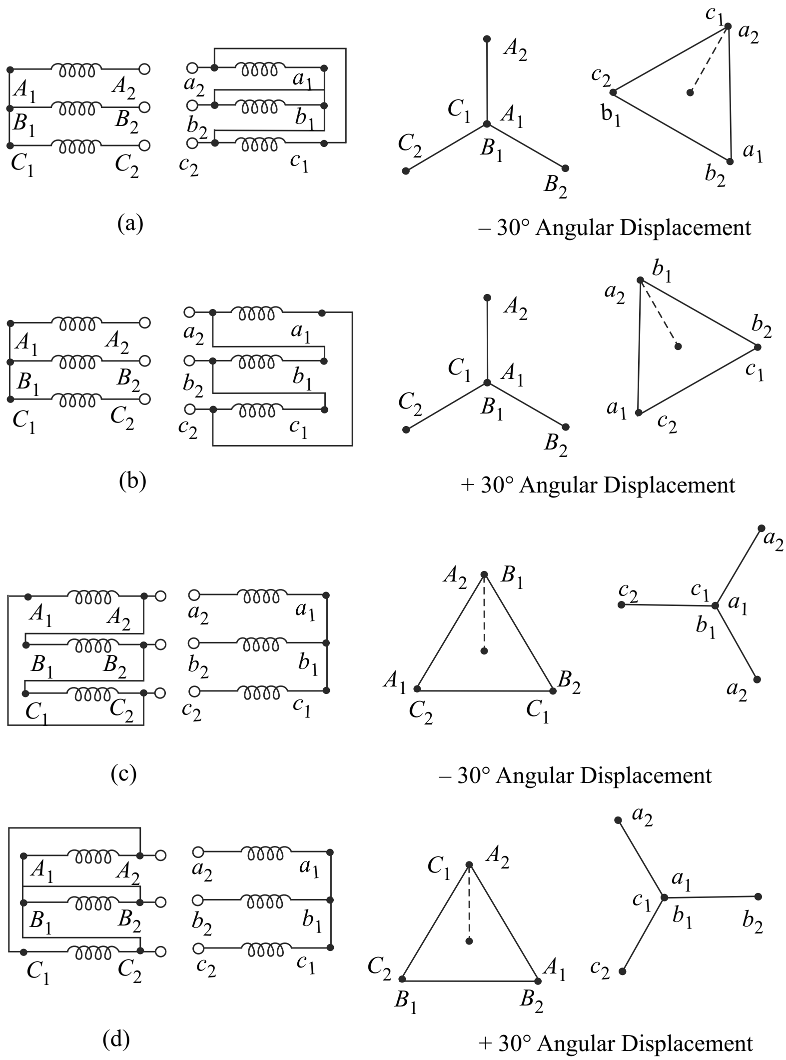


Fig. 3.61 *Various types of connections of transformer windings (a) Yd I (b) YdI I, (c) DY I (d) DY II*

The phase difference between the high-voltage and low-voltage windings for different types of connections can be represented by comparing it with the hour hand of a clock. When the hour hand of the clock is at the 12 o'clock position it is considered zero displacement. When it is the 11 o'clock position the displacement is +30° (anticlockwise is positive). When the hand is at the 1 o'clock position the displacement is –30° and at the 6 o'clock position it is 180°, as shown in Fig. 3.62.

Thus the connections of Fig. 3.60(a), (b), (c) and (d) can respectively be represented as *Yy* 0, *Yy* 6, *Dd* 0 and *Dd* 6. The connections at Fig. 3.61(a), (b), (c) and (d) are often represented as *Yd*1, *Yd*11, *Dy*1 and *DY*11 respectively.

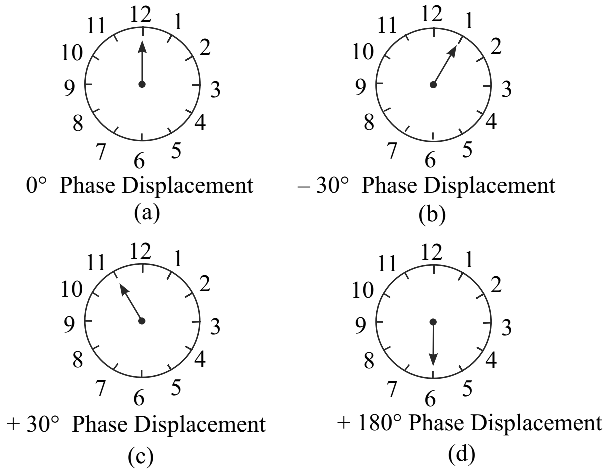


Fig. 3.62 *Positions of the hour-hand of the clock used to represent the phase displacement between hv and lv side voltages*

Depending on the phase displacement of the voltages of high-voltage and low-voltage sides, transformers are classified into groups called 'vector groups'. Transformers with the same phase displacement between the high-voltage and low voltage sides are classified into one group. For satisfactory parallel operation of transformers, they should belong to the same vector group. For example, a star-star connected three-phase transformer (or bank of three single-phase transformers) can be paralleled with another three-phase transformer (or bank of three other single-phase transformers) whose windings are either star-star connected or delta-delta connected. A star-star connected transformer cannot be paralleled with another star-delta connected transformer as this may result in short-circuiting of the secondary side.

3.17.4 Open-Delta or *V–V* connection of Transformers

Three single-phase transformers can be connected in delta-delta formation to supply a three-phase load. If one of the transformers, of the three transformers connected in Δ–Δ, supplying a three-phase load, is disconnected due to some fault in it, the remaining two transformers make an open-delta or *V–V* connection. The three-phase load can still be carried by the two single-phase transformers in open-delta connection. However, the current in the existing two transformers supplying the three-phase load will immediately increase. Each transformer will be overloaded by 73.2 percent. To keep the current in the transformer windings within their rated values, the load has to be decreased. The volt-ampere (*VA*) supplied by each transformer in *V–V* connection is only 57.7 percent of their rated capacity. As shown in Fig. 3.63, the third transformer is removed, and yet supply to the three-phase load is being maintained at reduced load.

When three single-phase transformers are carrying a three-phase load, and if one transformer is taken out, it may appear that one-third of the three-phase load should be reduced, i.e., the existing two transformers must be able to carry two-third of the load. But this is not true. Let us examine the *VA* rating of Δ–Δ system and *V–V* system as shown in Fig. 3.64 (only secondary windings shown).

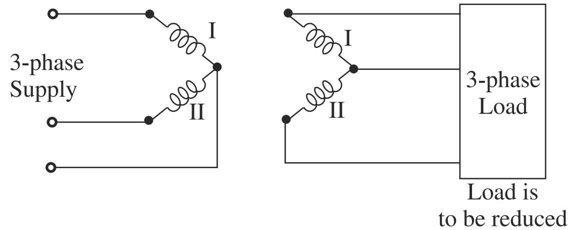


Fig. 3.63 *Open-delta connection by two single-phase transformers*

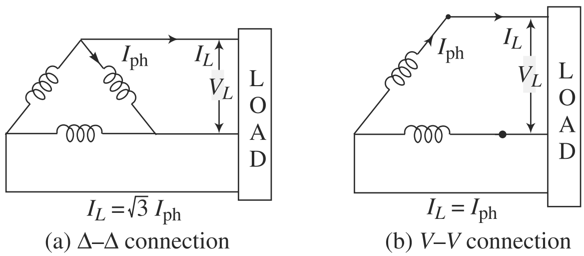


Fig. 3.64 *Delta-delta connection and V–V connection compared (only secondary shown)*

For closed delta connection as in Fig. 3.64 (a).

$$(VA)_{\Delta\text{–}\Delta} = \sqrt{3}\, V_L\ I_L = \sqrt{3}\, V_L\, \sqrt{3}\, I_{ph} = 3\, V_L\ I_{ph}$$

$$(VA)_{V\text{–}V} = \sqrt{3}\, V_L\ I_L = \sqrt{3}\, V_L\ I_{ph} \qquad (\because I_{ph} = I_L \text{ as in Fig. 3.64(b)})$$

The ratio $\dfrac{(VA)_{V-V}}{(VA)_{\Delta-\Delta}} = \dfrac{\sqrt{3}\, V_L\ I_{ph}}{3\, V_L\ I_{ph}} = \dfrac{1}{\sqrt{3}} = \dfrac{1}{1.732} = 0.577$

This shows that the load that can be carried by open delta system without being overloaded is only 57.7 percent of the original load where three single phase transformers were supplying the load.

Therefore when one of the transformers of a delta connected system becomes defective and has to be disconnected, the load should be reduced to $1/\sqrt{3}$, i.e., 57.7 percent times of original load.

The loading of the two transformers connected in *V–V* mode and supplying the three-phase load will be equal only when the load power factor is unity. At any other power factor the power supplied by transformers will be $V_L I_L$ cos (30 + ϕ) and $V_L I_L$ cos (30-ϕ) respectively.

Open delta connection of two transformers find applications when one of the transformers develop some fault and has to be disconnected for repair. The supply can continue but the total load has to be reduced.

Open-delta connected system is also used in areas where, load may increase at a later date, particularly in new areas being developed. Whenever the load requirement increases a third single-phase transformers is connected to convert the open-delta connection into a closed-delta connection thereby saving in the capital investment.

EXAMPLE 3.36 *Three 300 kVA 1100/230 V, 50 Hz transformers are connected in delta-delta formation to supply a 600 kVA load at 0.8 power factor lagging. One of these single-phase transformers is to be removed for emergency repair, thus making an open delta connection of two single-phase transformers supplying the three-phase load. Calculate the percent increase in load on each transformer when the defective transformer is removed. Also calculate the ratio of rating of open-delta and full-delta connections.*

Solution

(i) The original load on each of the three single-phase transformers connected in Δ–Δ.

$$= \frac{1}{3} \times \text{total load} = \frac{1}{3} \times 600 = 200 \ \text{kVA}$$

(ii) The load on each transformer when connection in open-delta,

$$= \frac{1}{\sqrt{3}} \times \text{original load}$$

$$= \frac{1}{\sqrt{3}} \times 600 = 346.41 \text{ kVA}$$

(iii) Percent increase in load,

$$= \frac{(346.41 - 200)}{200} \times 100$$
$$= 73.2 \text{ percent}$$

(iv) Total kVA rating of V–V connected bank of transformers

$$= \sqrt{3} \times \text{kVA rating of each transformer}$$

$$= \sqrt{3} \times 300 \text{ kVA} = 519.6 \text{ kVA}$$

(v) Total kVA rating of Δ–Δ connected bank of transformer

$$= 3 \times \text{kVA rating of each transformer}$$

$$= 3 \times 300 = 900 \text{ kVA}$$

(vi) Ratio of rating of open-delta to full-delta

$$= \frac{519.6}{900} \times 100 = 57.7 \ \text{percent}$$

3.17.4 Waveform of the Magnetizing Current of a Transformer

The relationship between the flux and magnetizing current for the core of a transformer is represented by the hysteresis loop. Let us assume that flux is sinusoidal. If the flux is sinusoidal, the induced emfs will be sinusoidal and will lag the flux by quarter of a cycle. When the induced emf is sinusoidal, the applied voltage to the primary winding is to be sinusoidal.

When the flux in the core is sinusoidal, the magnetizing current is not sinusoidal because of the non-linear relationship between ϕ and i or B and H as has been shown in Fig. 3.65.

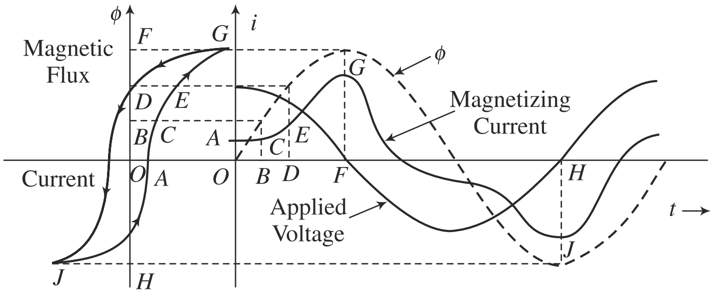


Fig. 3.65 *Non-linear wave shape of the magnetizing current of a transformer*

3.17.5 Harmonics in Three-phase Transformer Banks

For economical reason, a transformer is operated at a higher flux density. This can be understood easily from the fact that induced emf in the windings depends on the amount of flux linking the windings, i.e., the core flux. Core flux, ϕ is the product of flux density, B and core area, A. For a particular value of flux in the core, if B is increased, A will be reduced. Less area of cross section of the core means less weight of core material as also of winding and insulating material required. All these will lead to saving in cost of the transformer.

Flux in the core needs to be sinusoidal. Flux produces induced emf and if flux is sinusoidal, emfs induced will be sinusoidal.

Because of the non-linear nature of the *B-H* characteristics, the magnetizing current of a transformer is non-sinusoidal. Thus flux to remain sinusoidal, the magnetizing current has to be non-sinusoidal.

An analysis of the non-sinusoidal magnetizing current shows that it will contain the fundamental and all odd harmonics. However, the presence of third harmonic is predominant in the magnetizing current. This third harmonic component can be 5 to 10 percent of the fundamental wave.

The non-sinusoidal nature of transformer magnetizing current is necessary to produce sinusoidal flux in the core of a transformer. This non-sinusoidal magnetizing current may cause some undesirable effect when a bank of three single phase transformer windings are connected to supply three-phase loads. Let us assume that the primary and secondary windings are connected in Y–Y formation with no neutral connection as shown in Fig. 3.66.

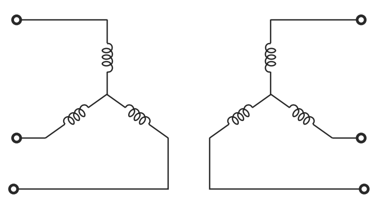


Fig. 3.66 *Star-star connection of three single phase transformer windings*

Let us assume that the exciting currents contain only fundamental and third harmonic currents expressed as

$$i_R = I_{m1} \sin \omega t + I_{m3} \sin 3\,\omega t$$
$$i_Y = I_{m1} \sin (\omega t - 120°) + I_{m3} \sin 3(\omega t - 120°)$$
$$i_B = I_{m1} \sin (\omega t - 240°) + I_{m3} \sin 3(\omega t - 240°)$$

The phasor sum of the currents of the three phases will pass through the neutral wire if it is grounded. In the absence of neutral wire, These third harmonic currents cannot flow and hence will not exist.

The current in the neutral wire is calculated as

$$i_N = i_R = i_Y + i_B$$

$$= [I_{m1} \sin \omega t + I_{m1} \sin (\omega t - 120°) + I_{m3} \sin (\omega t - 240°) + I_{m3} \sin 3\omega t + I_{m3} \sin 3\ \omega t + I_{m3} \sin 3\omega t$$

$$= 0 + 3\ I_{m3} \sin 3\omega t$$

$$= 3\ I_{m3} \sin 3\omega t$$

The neutral wire, if present will carry only the third harmonic current in a star-star connection. With neutral grounded, third harmonic current will flow through the neutral ground wire. This presence of third harmonic current of the magnetizing current is essential as it produces a sinusoidal core flux and a sinusoidal induced voltage in the windings. If we represent the fundamental and third-harmonic component of the magnetizing current through phasor as shown in Fig. 3.67, it will be observed that the third harmonic components add up and are co-phasal whereas the fundamentals add upto zero.

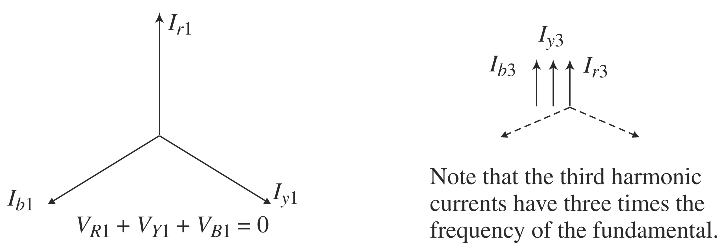


Fig. 3.67 *Harmonic currents in three phase transformer connections.*

When Windings Connected in Delta Let us assume that third harmonic magnetizing currents are present in the three phases of a delta connected three phase transformer. The line current is found by subtracting the phase currents as shown in Fig. 3.68. In the line current, the third harmonic current of the phases cancel with each other since these are co-phasal. Hence, there will be no third harmonic present in the line currents. The third harmonic magnetizing currents flow round the closed loop of the delta connected windings.

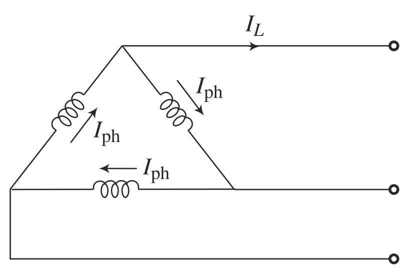


Fig. 3.68 *Phase and line current*

Three-phase transformer will generally have one delta connected set of windings so as to allow flow of third harmonic currents in the windings keeping the flux wave sinusoidal and thereby maintaining sinusoidal line voltages. If there is no path allowed for the flow of third harmonic magnetizing current, the core flux becomes non-sinusoidal and hence the line voltages and currents also become non-sinusoidal which is not desirable.

For this reason three-phase transformers are generally connected in star-delta or delta-star.

Star Connection When three-phase transformers windings are connected in star-star with no neutral wire, the flow of third harmonic magnetizing currents is suppressed, i.e., they cannot flow. The fundamental components of the magnetizing current are displaced 120° apart and their phasor sum is zero. But the phasor sum of the third harmonic magnetizing currents is not zero. Their phasor sum is three times the phase current. Since these currents are all in phase (co-phasal) they do not cancel. There must be a path available for their flow which is not provided in a purely star-star connection.

For this reason, when transformer windings are to be connected in star-star manner, a *delta connected tertiary winding* is provided. This third winding is called tertiary winding making the transformer a three winding transformer as shown in Fig. 3.69. The delta connected tertiary winding provides a path for the third harmonic magnetizing current to flow. Because of this, there will be sinusoidal flux in the core limbs of the three-phase transformer, which is desirable. The tertiary windings can also supply local auxiliary loads for running of fans, lights, pumps, etc.

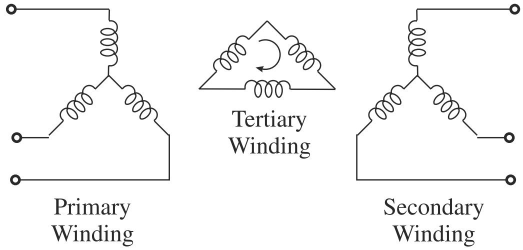


Fig. 3.69 *Three windings transformer—the tertiary winding providing a path for the flow of third harmonic current*

For a star-star connection a four-wire star connected system with neutral wire is to be formed. The in-phase third harmonic currents forming the neutral current will flow through the neutral wire which is essential.

3.17.6 Three Winding Transformers

A transformer may be provided with a third winding called tertiary winding. Thus a three winding transformer will have its primary, secondary, and tertiary windings wound on its limbs as shown in Fig. 3.70. The third windings, i.e., the tertiary windings are low voltage windings connected in delta and is used to supply power to the local loads of the substation where the transformer is installed. The substation loads

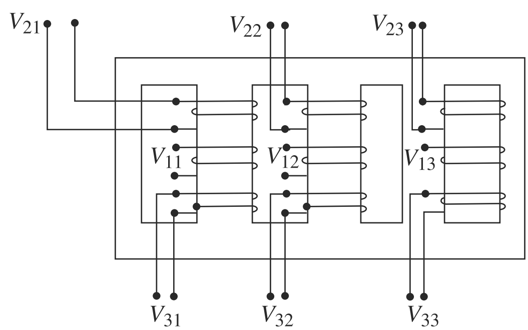


Fig. 3.70 *Three winding transformer*

may include lights, fans, pumps, etc. The voltage rating of the tertiary windings will be different from the voltage ratings of the primary and secondary windings. The primary windings represented by voltages V_{11}, V_{12}, V_{13}. Voltages V_{21}, V_{22}, V_{23} are the voltages of secondary winding. Voltages V_{31}, V_{32}, V_{33} are low voltage windings voltages of the tertiary and are connected in delta to supply the substation loads. The primary and secondary windings may be rated at 33 kV and 11 kV respectively while the tertiary windings may be rated at 400 Volts only.

The kVA rating of the primary winding and the secondary winding is the same for the transformer. However, the kVA rating of the tertiary winding is much lower. Since the tertiary windings are always connected in delta, the three winding transformer windings may be connected in Y–Y–Δ.

The other important reason of using delta connected tertiary windings is that they suppress the harmonic voltages that may be generated in primary and secondary star-connected windings. Tertiary windings also reduces the secondary load imbalance.

The unbalance and third harmonic problems do not arise when either the primary windings or the secondary windings are connected in delta.

Equivalent Circuit The equivalent circuit of a three-winding transformer is shown in Fig. 3.71. R_1, X_1; R_2, X_2; and R_3, X_3 are respectively the resistance and leakage reactance of the primary, secondary, and tertiary windings. The no-load current I_0 is shown flowing through the parallel circuit consisting of R_m and X_m. The equivalent circuit has been drawn for one phase only. The secondary and tertiary quantities have been shown referred to the primary side.

Across the terminals 2 and 3 loads are connected. I_1, I_2 and I_3 are respectively the currents in the primary, secondary, and tertiary windings. The equivalent circuit neglecting the parallel branch through which no-load current, I_0 flows is shown as an approximate equivalent circuit in Fig. 3.72.

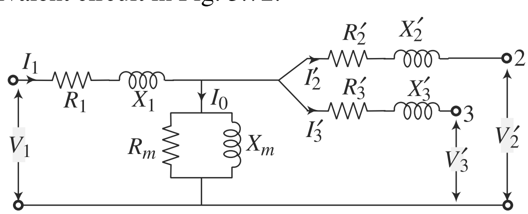


Fig. 3.71 *Single-phase equivalent circuit of a three-winding transformer*

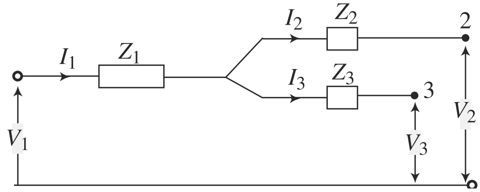


Fig. 3.72 *Approximate equivalent circuit of a three-winding transformer*

Determination of Equivalent Circuit Parameters Three short-circuit tests and one open circuit test are required to be conducted on the three winding transformer.

The circuit diagrams and their equivalent circuits have been shown in Fig. 3.73.

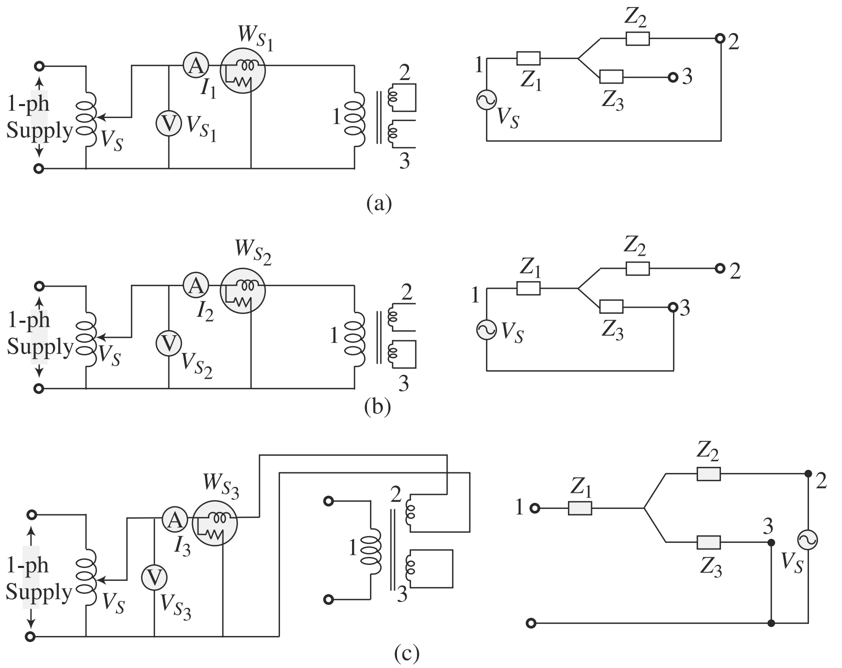


Fig. 3.73 *Short-circuit tests on a three-winding transformer.*

From the test conducted according to Fig. 3.73(a),

$$Z_{12} = \frac{V_{S_1}}{I_1}; \; Z_{12} = Z_1 + Z_2; \; R_{12} = \frac{W_{S_1}}{I_1^2} \tag{3.34}$$

From the test as in Fig. 3.73(b),

$$Z_{13} = \frac{V_{S_2}}{I_1}; \; Z_{13} = Z_1 + Z_3; \; R_{13} = \frac{V_{S_2}}{I_1^2} \tag{3.35}$$

From the test in accordance to Fig. 3.73(c),

$$Z_{23} = \frac{V_{S_3}}{I_1}; \; Z_{23} = Z_2 + Z_3; \; R_{23} = \frac{V_{S_3}}{I_1^2} \tag{3.36}$$

From Eqs (3.34), (3.35), and (3.36),

$$Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23})$$

$$Z_2 = \frac{1}{2}(Z_{23} + Z_{12} - Z_{13})$$

$$Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12})$$

The open circuit test is conducted by keeping the secondary and tertiary windings open. Rated voltage is applied to the primary winding. The readings of Ammeter, Voltmeter and Wattmeter are recorded. From the no-load test data, the magnetizing impedance, R_m and X_m and the ratio of turns can be calculated as.

$$W_0 = V_0 I_0 \cos\phi_0;\ \cos\phi_0 = \frac{W_0}{V_0 I_0}$$

$$I_m = I_0 \sin\phi_0;\ I_C = I_0 \cos\phi_0$$

$$X_m = \frac{V_0}{I_m}\ \textit{and}\ R_m = \frac{V_0}{I_C}$$

3.17.7 Three-phase Transformer Connections—Advantages and Drawbacks

We have seen that three-phase transformers can be connected in four ways, namely star-star, star-delta, delta-star, and delta-delta. Some of the advantages and drawbacks of each of these connections are mentioned below.

Star-Star Connection The main advantage of a Y/Y connection is that a neutral point is available on both the sides. The neutral can be grounded and an unbalanced load can be supplied. The unbalanced current can flow through the neutral wire. It is to be noted that a Y/Y connection will work satisfactorily without grounding of the neutral only if the three-phase load on the transformer is balanced.

If the neutral is not grounded, the wave shape of the induced emfs will get distorted (becomes non-sinusoidal). This is because, the third harmonic components of the non-sinusoidal magnetizing current cannot flow. In fact they are forced to add up to zero. This affects the waveform of the induced voltage.

Delta-Delta Connection In delta connection the line voltage is the same as the phase voltage. Therefore, in Δ–Δ connection the line-to-line voltage on either side of the transformer is equal to the corresponding phase voltages. Since the phase voltage is stressed to line voltage, delta connected windings require more insulation than a star connected winding of the same rating.

The advantage of this connection is that even on unbalanced loads the three phase load voltages remain more or less constant. The disadvantage is that no neutral connection is available on either side.

Star-Delta Connection Here, line-to-neutral voltage is applied to the primary phase windings whereas line-to-line voltage appears across the secondary windings. In star-delta connection, the secondary winding current (phase current) is 57.7 percent of the load current. Star-delta connection is, therefore, suitable for step-down applications.

In a star-star connected transformer without neutral grounding with no path for the third harmonic magnetizing current to flow, there is distortion of the waveform of the induced voltage. However, in star-delta connection, even if no neutral connection is available, the third harmonic current in the primary causes a circulating current in the secondary and tries to oppose any distortion of the sinusoidal flux distribution in the core.

Delta-Star Connection This type of connection is suitable for step-up application. Both three-phase and single-phase supply is possible with the help of three line wires and the neutral wire. For balanced operation the single-phase loads on the three phases are distributed equally so that loading is balanced.

3.18 PARALLEL OPERATION OF THREE-PHASE TRANSFORMERS

The need for parallel operation of transformers has been explained in an earlier section. The conditions for parallel operation of single-phase transformers have also been dealt with. In addition to the conditions of parallel operation of single-phase transformers, for three-phase transformers another essential condition is that the phase sequence and vector grouping should be the same for the transformers to be connected in parallel.

The phase sequence, i.e., the order in which the phases reach their maximum positive voltage, must be identical for the paralleled transformers, otherwise during the cycle each pair of phases will be short-circuited.

The paralleled transformers should belong to the same vector group, i.e., there should not be any phase difference between the secondary voltages.

3.19 TRANSFORMATION FROM THREE-PHASE TO TWO-PHASE AND SIX-PHASE

It may often be necessary to transform or convert a three-phase supply to two-phase, six-phase or more number of phases for specific applications. This can be achieved by using transformers having tappings on their windings.

3.19.1 Transformation from Three-phase to Two-phase

Conversion from three-phase supply to two-phase supply is achieved through Scott or Tee connections of two-phase transformers. This three-phase to two-phase conversion is commonly used in electric furnace installations where it is intended to run two single-phase furnaces together and draw a balanced currents from a three-phase supply system. Two specially tapped transformers are required for this purpose. One is called the Main Transformer which has a centre-tapped primary. The other is called the Teaser Transformer and has a primary voltage rating of 0.866 of the voltage rating of the main transformer primary. The secondaries of both transformers have equal voltage ratings. The connection diagram is shown in Fig. 3.74.

A two-phase three-wire system is produced by connecting the ends of transformer secondaries and bringing out the neutral wire as has been shown.

3.19.2 Scott Connection

The secondary windings can also be used to supply two single-phase loads independently as shown in Fig. 3.75.

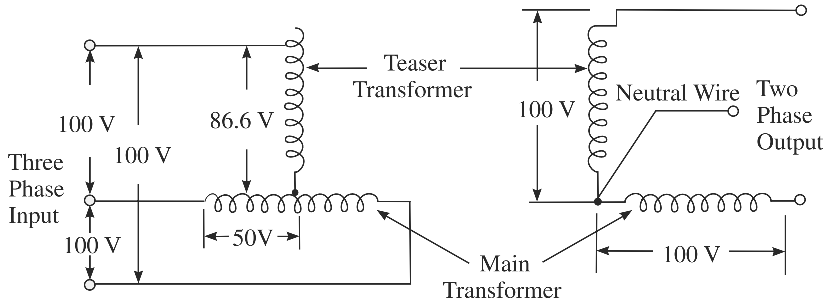


Fig. 3.74 *Scott connection, of two single-phase transformers for three-phase to two phase conversion*

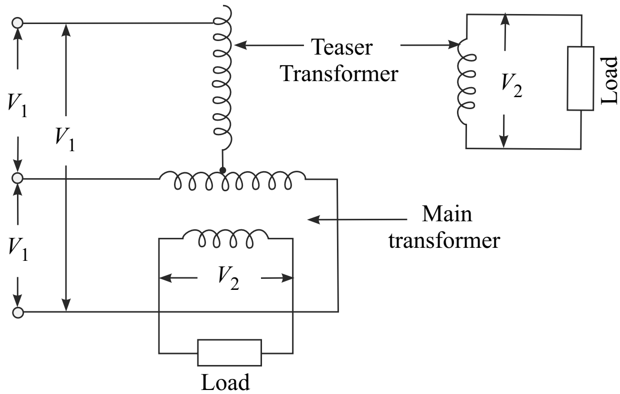


Fig. 3.75 *Scott connection of two single-phase transformers for conversion of three-phase supply system to two single-phase loads*

3.19.3 Transformation from Three-Phase to Six-Phase

A six-phase supply or more phases of supply are often required for supplying power rectifiers. The three primary windings of the three transformers are star connected, the secondary windings are centre-tapped. If the centre points of the three secondary windings are joined together as a star point, it gives two three phase systems which are displaced by 180° to produce a six-phase supply system.

If there are two separate windings, they can be connected in two-delta formations with a phase displacement of 180° to obtain an equivalent six-phase supply.

EXAMPLE 3.37 *Two transformers are supplying two single-phase furnace loads at 200 V each through Scott connection. The supply voltage is 440 volts. One load of each of the furnaces is 75 kVA. Calculate the turn ratio of the main and the teaser transformers and also the primary and secondary currents.*

Solution Voltage ratio of main transformer and winding currents:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{440}{200} = \frac{2.2}{1}$$

KVA = 75, VA = 75 × 1000

Therefore, $I_2 = \dfrac{75 \times 1000}{200} = 375$ Amps.

Primary current, $I_1 = 375 \times \dfrac{200}{440} = 170$ Amps.

Voltage ratio of the teaser transformer:

Primary winding voltage = Voltage across *AD*

= 0.866 × 440 = 381 volts.

$$\text{Voltage ratio} = \frac{V_1}{V_2} = \frac{381}{200} = 1.9$$

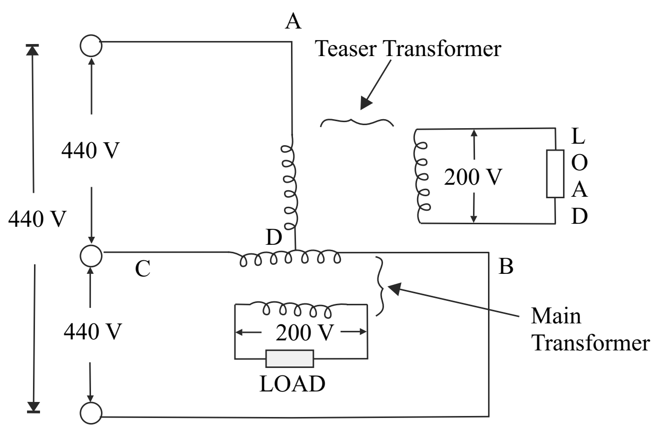


Fig. 3.76

EXAMPLE 3.38 *A 440 V, 3-phase supply is available to feed two single-phase furnaces at 200 V. The supply is provided through Scott connection of two single-phase transformers. The load of each of the furnaces is 250 KW and the power factor of the load is unity. Neglecting the no-load currents of the transformers calculate the line currents on the supply side.*

Solution Supply voltage is *V*, voltage across *AD* is 0.866 V. The neutral point is not at *D* but at a point *N* such that $V_{AN} = V_{BN} = V_{CN}$. Thus point *N* is faced on *AD* such that *AN* : *ND*= 2 : 1. The line voltage and phase voltage are *V* and $\dfrac{V}{\sqrt{V_3}}$ respectively as shown (this has to be for a balanced three-phase supply system).

The secondary voltages of teaser transformer and the main transformer are V_{2T} and V_{2M} respectively as has been shown.

$$I_{2T} = I_{2M} = \frac{250 \times 1000}{200} = 1250$$

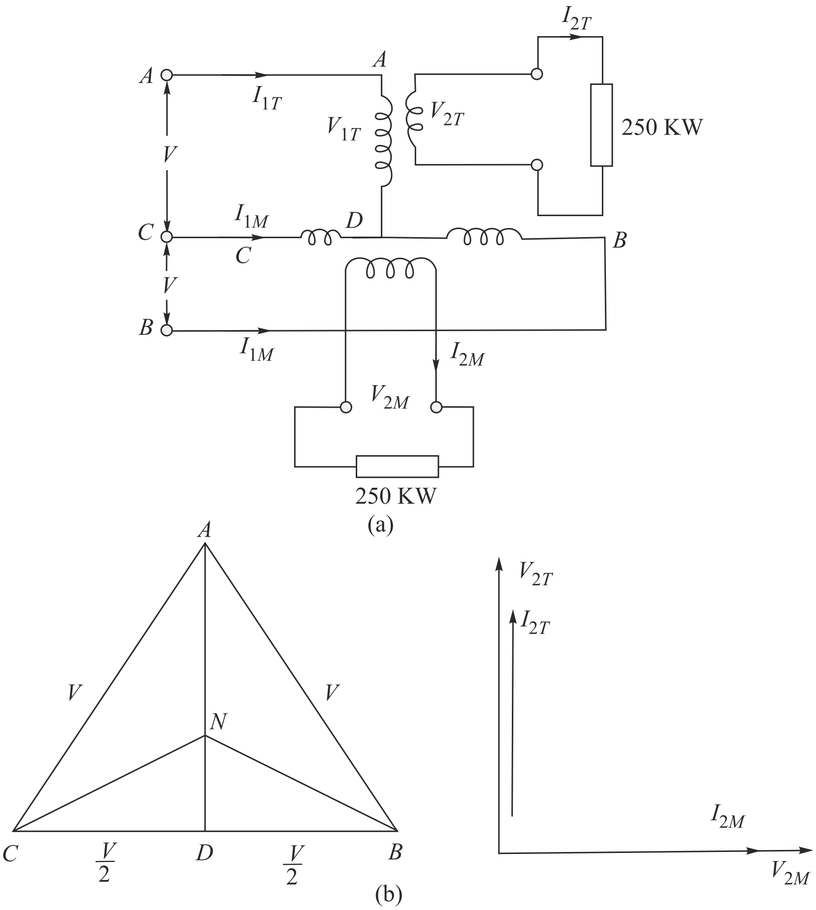


Fig. 3.77

Primary phase current of the main transformer = $I_{2M} \times \dfrac{V_2}{V_1}$

$$= 1250 \times \frac{200}{440} = 568$$

Primary phase current of the teaser transformer = $\dfrac{I_{2T} \times V_2}{0.866\, V_1}$

$$= \frac{1}{0.866} \frac{1250 \times 200}{440}$$

$$= 1.15 \times 568 = 654 \text{ A}$$

These currents have been shown vectorically as below.

$$I_{1M} = \sqrt{(568)^2 + (327)^2} = 654 \text{ A}$$

I_{1M} is the line current which is the same as the phase current as the primary windings are star-connected.

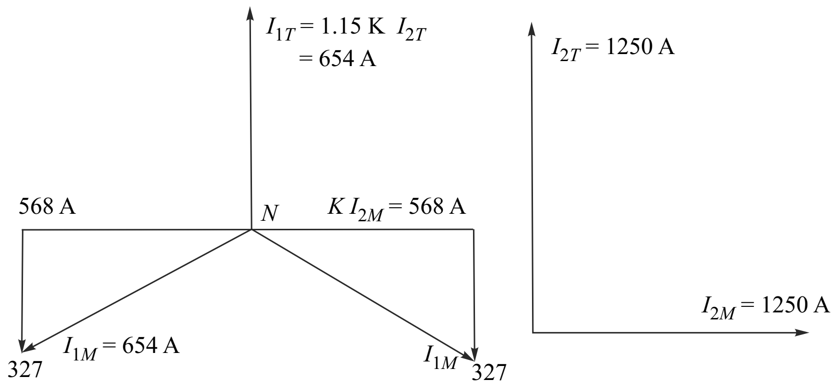


Fig. 3.78

As can be seen from above diagram, I_{1M} has two components. One component $K I_{2M}$ is balancing the secondary current I_{2M} and the second part is equal to one half of the teaser current providing return path of the teaser current through the two halves of the primary of the main transformer the resultant is calculated by adding these two component currents vectorically.

Thus the three line currents are equal showing that the balanced three-phase currents are being drawn from the supply source. The three phase currents are in phase with phase voltage V_{AN}, V_{BN} and V_{CN} as the power factor is unity. For lagging power factor loads, the phase currents will lag the voltages by the power factor angle, ϕ.

EXAMPLE 3.39 *A 3300 V, three-phase supply system provides connections to two single-phase furnace loads through two scott-connected transformers. The power taken by each furnace is 300 KW at 200 V and the load power factor is 0.8 lagging. Calculate the values of the line currents on the supply side.*

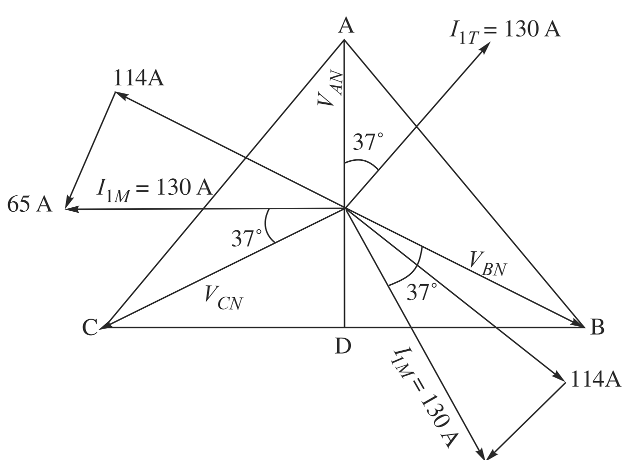


Fig. 3.79

Solution

$$\text{Power} = VI\cos\phi,\ \text{Given, } \cos\phi = 0.8 \text{ lagging}$$

$$\phi = \cos^{-1} 0.8 = 37°$$

$$I_{2T} = \frac{300 \times 1000}{200 \times 0.8} = 1875 \text{ A}$$

$$\text{Teaser primary current,} \quad I_{1T} = \frac{1875 \times V_2}{0.866\, V_1}$$

$$= \frac{1875 \times 200}{0.866 \times 3300} = 130 \text{ A}$$

Primary winding balancing current of the

$$\text{secondary current} = \frac{1875 \times 200}{3300} = 114 \text{ A}$$

$$\text{Line current, } I_{1M} = \sqrt{(114)^2 + (65)^2} = 130 \text{ A.}$$

A balanced three-phase currents of 130 A is supplied from the system.

3.20 TRANSFORMER TAP CHANGING

Due to the vast distribution network, voltage variation is a normal phenomenon in electrical systems. The voltage is to be controlled to:

(i) adjust the customer's terminal voltage within a specific tolerance.

(ii) control the active and reactive power flow in the network.

This adjustment is done by changing the effective turns ratio of the system transformer by proper selection of tappings on the windings as shown in Fig. 3.80. Occasional adjustment is made by off circuit tap changing and frequent adjustment is generally made by means of on-load tap changing gear. For off-load tap changing, as the name implies it is essential to switch off the transformer before changing the tap. On load tap changers are employed to regulate voltage while the transformer is delivering normal load. Since the low voltage winding is placed next to the core, the tappings are generally provided on the outer winding, i.e., the *hv* side. Moreover the h*v* side has more number of turns. The circuit continuity is maintained throughout the tap changing operation. During transition, two adjacent taps are momentarily connected and the short-circuit current is limited by automatic insertion of impedance in between the corresponding tappings.

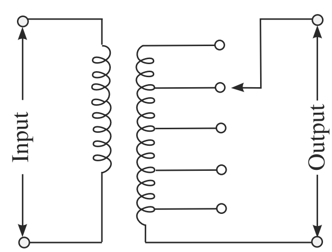


Fig. 3.80 *Voltage regulation by adjustment of tappings*

Figure 3.81 shows an arrangement for on load tap changing (shown for one phase only) on neutral end of the star-connected winding of a transformer. While changing from tapping 6 to tapping 5, contact A_1 is opened first, transferring the load current via contact B_1 through resistor R_1. Then B_2 closes connecting R_1 and R_2 in series across tappings 5 and 6. Since there is voltage difference between tapping 5 and 6, circulating current flows through these resistors. The load current divides and passes through each resistor to tappings. B_1 then opens interrupting the circulating current and transferring the load current to tapping 5 through resistor R_2. Finally A_2 closes, takes the load current and completes the tap change. The whole process takes place in 40 to 80 m secs.

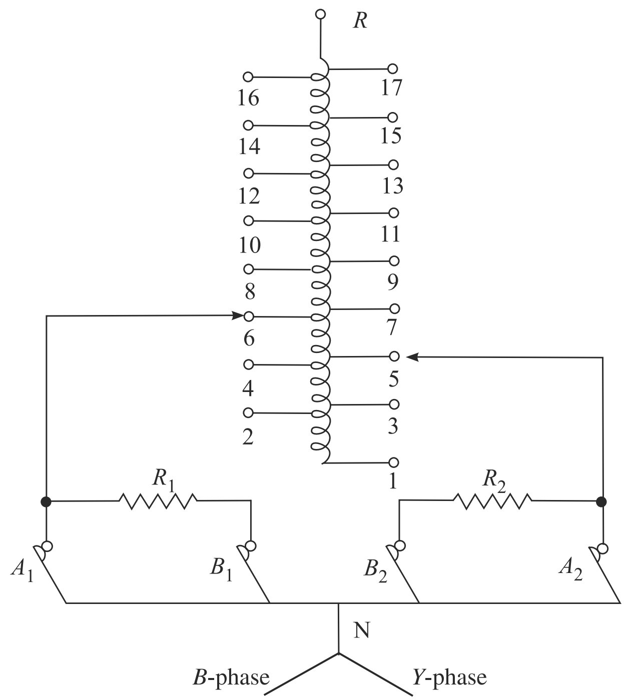


Fig. 3.81 *Illustration of on-load tap changing operation (for one phase only)*

3.21 POWER TRANSFORMERS AND DISTRIBUTION TRANSFORMERS

Transformers of large size used in generating stations at the sending end of the transmission line to step-up the voltage and at the receiving end of the transmission line to step-down the voltage are known as Power Transformers. A number of such transformers are connected in parallel. They are operated up to full-load capacity by connecting or disconnecting transformers depending upon the load condition. Power transformers are designed to have maximum efficiency near full-load.

Distribution transformers are comparatively smaller transformers of rating of the order of hundreds of KVA and are connected directly to supply the load at 400/230 volts. Thus, the secondary side of a distribution transformer is directly connected to the load. They are to be kept in operation for all the time for all the days irrespective of whether the consumer is utilizing the power or not. Thus, the load on a distribution transformer varies throughout the day depending upon how much the consumers utilize electricity. The average load on a distribution transformer is quite less than its rated capacity. That is why they are rated to have maximum efficiency at a load lower than their full-load capacity. They are designed to have good all-day-efficiency rather than highest efficiency at or near full-load.

MODEL QUESTIONS

Short-Answer-Type Questions

3.1 Explain the principle of working of a transformer.

3.2 Show with the help of a diagram how electricity generated at the power station is brought to the load centre by the use of transformers at the sending and receiving ends of transmission line.

3.3 Give the constructional differences between a core-type and a shell-type transformers.

3.4 State why the core of a transformer should be made of magnetic material.

3.5 Explain why the magnetising current of a transformer is less than that of an equivalent rotating electrical machine.

3.6 Explain how the magnetising current of a transformer is reduced by the use or grain-oriented sheet-steel laminations.

3.7 Illustrate with the help of sketches how full advantage of grain orientation is taken by assembling steel-sheet strips to form a core for the transformer.

3.8 State the two functions of using transformer oil inside the transformer tank.

3.9 The use of radiating tubes in the transformer tank increases the cooling effect. Explain.

3.10 Explain the purpose of using (i) conservator, and (ii) breather in a transformer.

3.11 Deduce the emf equation of a transformer.

3.12 Prove that the emf induced in the windings of a transformer will lag the alternating flux producing that emf.

3.13 Draw the no-load phasor diagram of a transformer. Express the magnetising current and loss component of the no-load current in terms of the no-load current and no-load power factor.

3.14 Explain mutual flux, leakage flux, magnetising reactance and leakage reactance of a transformer.

3.15 Draw the complete phasor diagram of a transformer on-load.

3.16 Draw the equivalent circuit of a transformer. Show how the equivalent circuit can be simplified without introducing much error. State the practical usefulness of such equivalent circuits.

3.17 Draw the equivalent circuit of a transformer with (i) primary quantities referred to the secondary side, and (ii) secondary quantities referred to the primary side.

3.18 Define voltage regulation of a transformer. Deduce the expression for voltage regulation.

3.19 Explain why hysteresis loss and eddy-current loss occur in a transformer.

3.20 Explain how hysteresis loss and eddy-current loss in a transformer core can be reduced by properly selecting the core material and suitably designing the core.

3.21 Explain how various losses of a transformer can be found out from practical tests without actually loading the transformer.

3.22 Write the expression for efficiency of a transformer and hence establish the condition for maximum efficiency.

3.23 Mention the important point of difference in the design between a power transformer and a distribution transformer.

3.25 Make a comparison in the weight of copper required in an autotransformer and a two-winding transformer of the same rating.

3.26 State two specific uses of autotransformers in the field of electrical engineering.

3.27 Explain how with the use of current transformer and potential transformer, high-current and high-voltage measurements are possible through low-range instruments.

3.28 Explain why one terminal of a CT or PT should be earthed.

3.29 Mention the various type tests and routine tests that are to be performed on transformers as per Indian Standards.

3.30 Mention the different tests that are to be performed on a transformer after it is received from the supplier.

3.31 Prepare a maintenance chart for an outdoor-installed high-capacity transformer.

3.32 Mention the various information printed on the name-plate of a transformer.

3.33 Explain vector grouping of transformers. Mention its usefulness.

3.34 State the need for parallel operation of transformers.

3.35 State and explain the various conditions of parallel operation of single phase and three-phase transformers.

Multiple-Choice Questions

3.36 The core of a transformer is made of
(a) silicon steel (b) annealed copper
(c) seasoned wood (d) aluminium

3.37 The core of a transformer is assembled with laminated sheets to reduce
(a) hysteresis loss (b) eddy-current loss
(c) magnetic noise (d) magnetising current

3.38 The use of grain-oriented laminated sheets in building a transformer core
(a) reduces magnetising current
(b) reduces eddy-current loss in the core
(c) reduces hysteresis loss
(d) increases the no-load power factor angle

3.39 Cooling of transformers is necessary to
(a) increase the efficiency
(b) dissipate the heat generated in the windings

(c) reduce the losses
(d) reduce humming

3.40 The emf induced in the windings of a transformer will
(a) lag the core flux by 90°
(b) be in-phase with the core flux
(c) be out-of-phase with the core flux
(d) be independent of the core flux

3.41 A100 kVA, 1100/400 V, 50 Hz single-phase transformer has 100 turns on the secondary winding. The number of turns on its primary will be
(a) 550 (b) 275 (c) 2750 (d) 5500

3.42 The emf induced in the secondary winding of a 50 Hz single-phase transformer having 1000 turns on its secondary is 222 V. The maximum flux density in the core is 0.1 Wb/m^2. The cross-sectional area of the core is
(a) 0.1 m^2 (b) 0.01 m^2 (c) 1 m^2 (d) 0.001 m^2

3.43 The copper-loss and core-loss of a transformer at various loads are as shown below:

	Load	*Core-loss*	*Copper-loss*
(a)	50 kVA	320 W	500 W
(b)	40 kVA	320 W	320 W
(c)	30 kVA	320 W	180 W
(d)	20 kVA	320 W	80 W

At what load will the efficiency of the transformer be maximum?

3.44 A transformer when supplying a load maintained 11 kV across load terminals. When the load was switched off, the terminal voltage became 11550 V. What is the voltage regulation at this load?
(a) 11.55% (b) 5.5% (c) 5% (d) 55%

3.45 Power lost in the open-circuit and short-circuit tests on a transformer gives approximately an account of the following losses
(a) core-losses and copper-losses respectively
(b) copper-losses and core-losses respectively
(c) eddy-current loss and hysteresis loss respectively
(d) hysteresis loss and eddy-current loss respectively

3.46 Maximum efficiency of a transformer occurs when
(a) hysteresis loss and eddy-current loss are minimum
(b) the sum of hysteresis loss and eddy-current loss is equal to copper-loss in the windings
(c) Power factor of the load is leading
(d) hysteresis loss is equal to eddy-current loss

3.47 The all-day efficiency of a transformer is the ratio of
(a) kWh output and kWh input per day
(b) kWh output and kWh input in a day
(c) output power and input power
(d) input power and output power

3.48 The ratio of the primary to secondary voltage of a transformer is 2:1. The saving in terms of weight of copper required if an autotransformers is used instead of a two-winding transformer will be

(a) 50% (b) 33.33% (c) 66.67% (d) 97%

3.49 For satisfactory parallel operation of two single-phase transformers a number of conditions are to be filled. A number of conditions are written below. Indicate which of these is not required to be filled:

(a) kVA ratings of the two transformers should be equal
(b) Transformers should be properly connected with regard to their polarity
(c) Voltage ratings of the primary windings should be suitable for supply system voltage and frequency. The turn ratio of the transformers must be equal
(d) The percentage impedance of the two transformers should be equal

3.50 An additional condition for parallel operation of three-phase transformers over single-phase transformers is that

(a) the transformers should belong to the same vector group
(b) ratios of the winding resistances to resistances for the transformers should be equal
(c) the transformers should have the same kVA ratings
(d) the transformers should not be belong to the same vector group

Numerical Problems

3.51 A single-phase transformer is required to step down voltage from 1100 V to 400 V at 50 Hz. The cross-sectional area of the core is 25 cm^2 and the maximum value of the flux density is 5 Wb/m^2. Determine the number of turns of the primary and secondary windings.

3.52 The primary winding of a 500 kVA, 3300/400 V transformer has 800 turns. Determine (a) the secondary turns, (b) the emf per turn and (c) the secondary current at unity power factor.

3.53 A single-phase 40 kVA transformer has a primary voltage of 6600 V, a secondary voltage of 230 V and has 30 turns on the secondary winding. Calculate the number of primary turns. Also calculate the primary and secondary currents.

3.54 A 50 Hz, 230/115 V transformer has 500 turns on its low-voltage side. Calculate (a) the number of turns on its high voltage side, (b) volts/turn ratio of high-voltage side, and (c) volts/turn-ratio of low-voltage side.

3.55 The number of turns on the high-voltage and low-voltage sides of a transformer are 750 and 50 respectively. When the high-voltage side is connected to a rated voltage of 230 V, 50 Hz and a rated load of 40 A is connected to the low voltage side, calculate (a) the secondary voltage assuming no internal transformer voltage drops and (b) the volt-ampere rating of the transformer.

3.56 A 100 kVA, 1100/230 V, 50 Hz single-phase transformer has 60 turns on the secondary. Calculate (a) the approximate values of primary and secondary currents, (b) the approximate number of primary turns and (c) the maximum value of the core flux.

3.57 A single-phase transformer has 500 turn on the primary and 100 turns on the secondary. The no-load current is 2 A at a power factor of 0.2 lagging. Calculate the primary current and power factor when the secondary current is 200 A at a power factor of 0.8 lagging. Assume the voltage drop in the windings to be negligible.

3.58 A 5 kVA, 200/400 V, 50 Hz, single-phase transformer gave the following test data:

O.C. test (low voltage side): 200 V, 0.7 A, 60 W

S.C. test (high voltage side): 22 V, 16 A, 120 W

If the transformer operates on full-load, determine the regulation at 0.9 power factor lagging.

3.59 A 100 kVA transformer has 300 turns on the primary and 60 turns on the secondary. The primary and secondary resistances are 0.4 and 0.01 Ω respectively, and the corresponding leakage reactance are 1 and 0.03 Ω respectively. The supply voltage is 2200 V. Calculate (a) the equivalent impedance and voltage regulation at full-load, 0.8 power factor lagging and (b) voltage regulation at full-load, 0.8 power factor leading.

3.60 The primary and secondary windings of a 500 kVA transformer have resistances of 0.4 and 0.001 Ω respectively. The primary and secondary voltages are 6600 and 400 V respectively. The iron-loss is 3.0 kW. Calculate the efficiency on full load, the load power factor being 0.8 lagging.

3.61 The efficiency of a 200 kVA 1100/230 V transformer is a maximum of 98.0% at 50% of rated load. Calculate (a) core-loss, (b) efficiency at rated load and (c) efficiency at a load of 75%.

3.62 A 20 kVA, 1100/230 V transformer which is continuously excited, is loaded at unity power factor for 24 h as follows: 6 h at half-load, 6 h at quarter-load, remaining 6 h on no-load. The maximum efficiency of 97% occurs at full load. Calculate the all-day efficiency of the transformer.

3.63 A single-phase transformer working at unity power factor has an efficiency of 90% at both half-load and full-load of 1 kW. Calculate the efficiency of the transformer at 70% of full-load.

3.64 A 50 kVA transformer has a core-loss of 500 W and a full-load copper-loss of 900 W. If the power factor of the load is 0.8 lagging, calculate (a) the full load efficiency (b) the maximum efficiency and (c) the load at which efficiency becomes maximum.

3.65 A 100 kVA distribution transformers has a full-load loss of 4 kW, the losses being equally divided between iron and copper. During 24 h in a day the transformer operates on full-load for 4 h, on half-load for 6 h, the output being negligible for the remainder of the day. Calculate the all-day efficiency of the transformer.

Answers

The answers to Multiple-choice Questions and Numerical Problems are given below.

3.36 (a)	3.37 (b)	3.38 (a)	3.39 (b)
3.40 (a)	3.41 (b)	3.42 (b)	3.43 (b)

3.44 (c)	3.45 (a)	3.46 (b)	3.47 (b)
3.48 (a)	3.49 (a)	3.50 (a)	
3.51 (396, 144)		3.52 (97, 4.12, 1250 A)	
3.53 (860, 6.06 A, 173.9 A)		3.54 (1000, 0.23 V, 0.23 V)	
3.55 (15.33 V, 613.33 VA)			
3.56 (90-9A, 434-78 A, 287, 17.26 mWb)			
3.57 (41.52 A, 0.78)		3.58 (3.09%)	
3.59 (1.866 Q, 3.24%, –1.1%)		3.60 (98.3%)	
3.61 (1.02 kW, 97.5%, 97.8%)		3.62 (95.5%)	
3.63 (90.5%)	3.64 (96.6%, 97.5%, 37.26 kVA)		3.65 (92.2%)

LABORATORY EXPERIMENTS

EXPERIMENT 3.1 *No load test on a single-phase transformer*

Objective To find how the no-load current and no-load losses of a single-phase transformer vary with the applied voltage.

Brief Theory The no-load current I_c, taken by the primary consists of two components: (a) a reactive or magnetising component, I_m, responsible for producing flux in the core, and (b) an active or energy component, I_w, which supplies the hysteresis and eddy current losses taking place in the core and $I^2{}_0 R_1$ loss taking place in the primary winding. The $I^2{}_0 R_1$ loss in the primary winding at no-load is very small and can be neglected for all practical purposes.

The no-load current can thus be broken up into its two components, i.e., (a) the magnetising current, I_m and (b) the energy current, I_w. How magnetising current varies with the applied voltage can be explained as follows: At no-load the induced emf in the primary winding is approximately equal and opposite to the applied voltage. The equation for the induced emf in any winding of a transformer is given by the expression:

$$E = 4.44\ \phi_m f N \text{ V}$$

If the frequency of supply ϕ and the number of turns N are constant then induced emf E is proportional to flux ϕ. Now flux ϕ is produced by the magnetising current I_m (a component of the no-load current). The curve giving the relation between B and I_m for a magnetic material is known as the magnetising characteristic.

Therefore, in this case the relation between the induced emf (or terminal voltage) and the magnetising current will be similar to the magnetising characteristic of the core material.

Second, as mentioned earlier, the losses at no-load are: (a) hysteresis loss (W_h) and (b) eddy-current loss (W_e). The two losses can be expressed mathematically thus:

$$W_h \propto B_m^{1.6} f \text{ W}$$

$$W_e \propto B_m{}^2 f^2 \text{ W}$$

Thus, Total iron-loss $W_{\text{iron}} = W_h + W_e \propto B_m{}^{1.6} f + B_m{}^2 f^2$

If f is constant, then W_{iron} is approximately proportional to the square of flux density B_m. Again, the applied voltage is directly proportional to the flux density. Therefore, the iron-losses of a transformer are proportional to the square of the applied voltage. From the following experiment the above relationship can be verified

Circuit Diagram See Fig. 3.82.

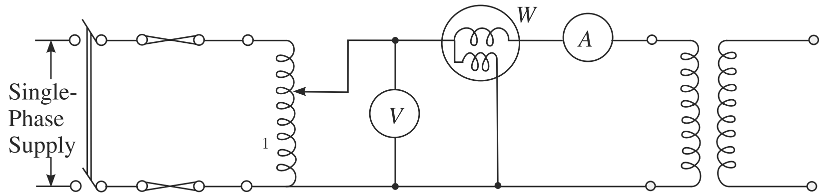


Fig. 3.82 *Connection diagram for no-load test on a single-phase transformer*

Apparatus Required Autotransformer, transformer (single-phase), low power factor wattmeter, ammeter, voltmeter.

Procedure

1. Make connections according to Fig. 3.82 by properly choosing the current and voltage ranges of the wattmeter.
2. By varying the applied voltage in step from zero upwards up to about 120% of the rated voltage, take for each value of the applied voltage readings of the wattmeter and the ammeter. Note down the multiplying factor of the wattmeter and convert the wattmeter reading into actual values.
3. Enter the results in a tabular form according to the table shown below.

Observation and Results

Take 8.9 *Readings*

No. of obs.	Applied voltage	No-load current	Power input	Energy current	Magnetising current
	V	A	W	W/V	$\sqrt{I^2 - (W/V)^2}$

Plot graphs showing the following relationship

(a) Applied voltage versus no-load current.
(b) Applied voltage versus magnetising current.
(c) Applied voltage versus energy current.
(d) Applied voltage versus no-load power.

Questions Answer the following questions in your report

1. Why is it advisable to use a low power wattmeter in this experiment?
2. Is it possible to calculate the efficiency of the transformer from the above experimental data if the values of winding resistances are given? If so, how?
3. How would you calculate the multiplying factor of a wattmeter?
4. In this experiment what percentage of the full-load current is drawn by the transformer on no-load at rated voltage?

5. Show with the help of an example how the magnetising and energy components of the no-load current can be calculated from the test data.

EXPERIMENT 3.2 *Short-circuit test on a single-phase transformer.*

Objective (a) To determine the internal impedance of a transformer and to Verify that it is constant: (b) to determine the copper-losses of a transformer and to observe how these vary with load.

Brief Theory When the secondary winding of a transformer is short-circuited a very low voltage (say about 5 to 10% of normal voltage) applied across the primary terminals will cause full-load current to flow through the windings. As the secondary winding is short-circuited, the secondary-terminal voltage will be zero. A voltage V_{sc1} applied to the primary winding causes a currently I_1 to be drawn. Therefore, the impedance of the device = $V_{sc1}1/I_1$ (see Fig. 3.83). This is the impedance of the transformer as seen from the supply. When the supply is given to the primary, the impedance so obtain is called the equivalent impedance of the transformer with reference to the primary. As mentioned above, only a very low voltage applied to the primary causes full-load current to flow when the secondary is short-circuited. Under these conditions, therefore, the flux density is very low. Since the iron-losses (i.e., hysteresis and eddy-current losses) are proportional approximately to the square of the voltage, these losses will be very low and may be neglected. If a wattmeter is connected on the primary side, the reading will give the power loss in the resistance of the windings or what are commonly called copper-losses.

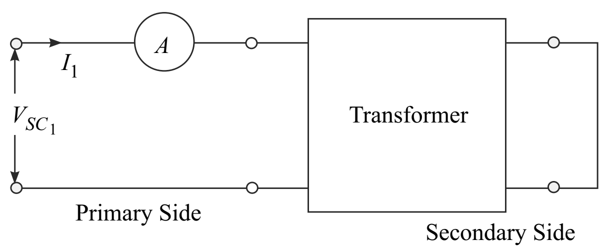


Fig. 3.83 *Measurement of equivalent impedance of a transformer*

When the full-load current is flowing with the secondary short-circuited,

$$\text{Wattmeter reading} = \text{Copper-loss at full-load} = W_c$$

Equivalent impedance referred to primary, $Z_e' = V_{sc1}/I_1$

$\therefore$ Equivalent resistance referred to primary $R_e' = \dfrac{W_c}{I_1^2}$

X_e' can be calculated as

$$X_e' = \sqrt{\left[Z_e'^2 - R_e'^2\right]}$$

If the test was performed on the secondary side, i.e., low voltage was applied to the secondary and the primary was short-circuited, then the voltage applied to the secondary, V_{sc2}, and the current drawn by the secondary I_2 would give the equivalent impedance, $Z_e' = V_{sc2}/I_2$.This is the equivalent impedance referred to the secondary. A wattmeter placed on the secondary side will give the total input power W_c, which in this case is all copper-loss. From this, the equivalent resistance as seen from the secondary side R_e' can be found as $R_e'\ W_c/I^2{}_2$.

However, it is possible to obtain equivalent values referred to the secondary if the equivalent values referred to the primary are known and vice versa thus:

$$Z_e'' = Z_e'\ (N_1/N_1)^2\ = Z_e'\ (V_2/V_1)^2$$

$$R_e'' = R_e'\ (N_1/N_1)^2\ = R_e'\ (V_2/V_1)^2$$

$$E_x'' = X_e'\ (N_1/N_1)^2\ = X_e'\ (V_2/V_1)^2$$

Similarly,

$$Z_e' = Z_e''\ (N_1/N_1)^2 = Z_e''\ (V_1/V_2)^2$$

$$R_e' = R_e''\ (N_1/N_2)^2 = R_e''\ (V_1/V_2)^2$$

$$X_e' = X_e''\ (N_1/N_2)^2 = X_e''\ (N_1/N_2)^2$$

Circuit Diagram See Fig. 3.84.

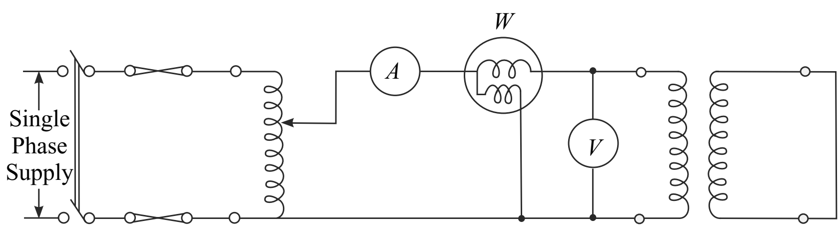


Fig. 3.84 *Connection diagram for short-circuit test on a transformer*

Apparatus Required Autotransformer-single phase, ammeter, voltmeter, wattmeter (low range), transformer single phase.

Procedure

1. Make connections according to Fig. 3.84. Set the autotransformer at zero output voltage. Ensure that a proper range is selected on the wattmeter.
2. Increase the voltage input to the transformer starting from zero in very small steps, with the help of the autotransformer. Record for each step the readings of the ammeter, voltmeter and the wattmeter. Increase the voltage till the ammeter indicates a value of 120% of the full-load current of the transformer. (The full-load current value can be calculated from the name-plate data).
3. Note down the multiplying factor of the wattmeter.
4. Plot on a graph paper the relation between (a) applied voltage versus short-circuit current and (b) short-circuit losses versus short-circuit current.

Questions Answer the following questions in your report

1. Why is iron-loss under the short-circuit test considered as negligible?
2. By measuring the copper-losses at full load, is it possible to calculate copper-losses at any other load?

3. Will there be any difference in the wattmeter reading if the short-circuit test is performed (a) by short-circuiting the primary when voltage is applied to the secondary and (b) by short-circuit the secondary when voltage is applied to the primary?
4. Explain with reasons the shape of the curves (a) applied voltage versus short-circuit current and (b) short-circuit losses versus short-circuit current.

EXPERIMENT 3.3 *Wave shape of the no-load current of a transformer*

Objective (a) To observe the wave shape of the no-load current of a transformer with the help of a cathode ray oscilloscope (CRO) (b) To be conversant with the use of a CRO.

Brief Theory When a transformer primary winding is supplied with a sinusoidal alternating voltage with the secondary open-circuited, the current flowing through the primary winding produces an alternating magnetic field which in turn induces an emf in this winding approximately equal and opposite to that of the applied voltage. For this emf of be sinusoidal, the flux must vary sinusolidally with time. The magnetic flux is produced by the magnetising current flowing through the primary. The curve showing the relation between magnetic flux and magnetising current is called the magnetising characteristic. The wave shape of the magnetising current would be sinusoidal if the magnetisation curve for the core material was linear. However, due to the nonlinear characteristic of the magnetisation curve the wave shape of the magnetising current is nonsinusoidal. A typical oscillogram (wave shape as seen on a CRO screen) of the magnetising current wave-shape is shown in Fig. 3.85.

Apparatus Required Transformer-single-phase, ammeter, variable resistor, autotransformer (single-phase), cathode oscilloscope.

Circuit Diagram See Fig. 3.86.

Procedures

1. Make connections according to Fig. 3.86(a). Make sure that the neutral of the autotransformer is connected to the earth terminal of the CRO and that the variable point of the resistor is connected to the other input of the CRO.
2. Observe and trace the shape of the magnetising characteristics on the CRO. Note that the shape of the no-load current is non-sinusoidal.

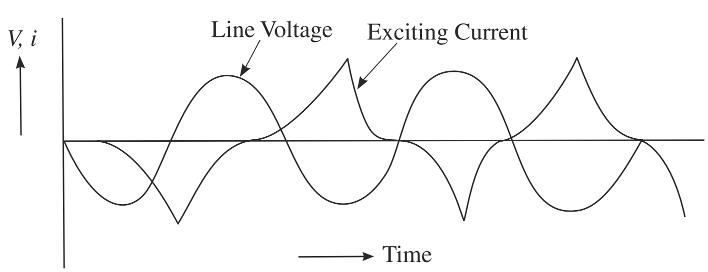


Fig. 3.85 *Typical exciting current wave shape of a transformer*

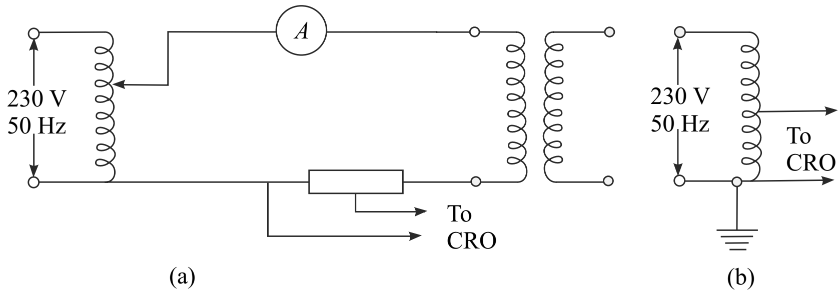


Fig. 3.86 *Connection diagram for observing wove shape of the no-load current of a transformer*

3. Also observe and trace the wave shape of the input supply voltage by giving a small voltage to CRO through an autotransformer [see Fig. 3.87(b)]. It is to be noted that the wave shape is sinusoidal. Tracing of wave shape should preferably be done on tracing paper. Label the axes of the waves so traced and from the wave shape observed on the CRO screen and calculate the frequency by noting down the time period *T*.

Note:

(i) Frequency = $\frac{1}{\text{Time period}}$

(ii) While giving input to the CRO ensure that the *X*-amplification scale is at its maximum. This may be reduced afterwards according to the requirement. Otherwise, the wave may go out of the screen.

Observation and Results Show the wave shapes of the no-load current and voltage (tracing from CRO screen) in your report.

Questions Answer the following in your report:

1. Why is the no-load current wave shape of a transformer nonsinusoidal?
2. You have been instructed to make sure that the neutral of the autotransformer is connected to the earth terminal of the CRO and that the variable point of the resistor should be connected to the other input terminal of the CRO. Why is it advisable to follow this instruction?
3. Suppose you were to measure the magnitude of current in a circuit by means of a CRO. How would you do this?
4. Mention the various uses that a CRO can be put to form what you have learnt so far.

EXPERIMENT 3.4 *Determination of efficiency and regulation of a transformer*

Objective To determine the efficiency and regulation of a transformer by the indirect method, i.e., by means of open-circuit and short-circuit tests.

Brief Theory (For open-circuit and short-circuit tests read the brief theories given under Experiments 3.1 and 3.2).

From the open-circuit or no-load test, the wattmeter reading gives the iron-losses in the transformer core. The copper-loss at no-load is considered negligible because then the primary current is very small. In this test, the secondary being open, the primary draws a current only to magnetise the core and to supply the iron-losses. Therefore, the wattmeter reading at no-load gives practically the iron-losses.

The short-circuit test is carried out at very low voltage and at the rated full load current. Therefore, the iron-loss at that voltage is very low. The wattmeter reading thus gives the copper-loss in the windings for full-load. After having known the iron and copper-losses from open-circuit and short-circuit tests respectively, the efficiency of the transformer can be calculated by using the formula

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$= \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + \text{Iron-loss} + \text{Copper-loss}}$$

Note: The short-circuit test may be carried out on either side, i.e., the voltage may be applied to the LV winding and the HV winding short-circuited or vice versa.

Calculation of Regulation Calculate R_e' and X_e' from the test data. Then for various values of load current I_1 (say 50%. 75%, 100% and 125%) and for various values of power factor (say 0, 0.7, 0.8 and unity), calculate the regulation using the following approximate standard formula

$$\text{Regulation} = \frac{I_1 (R_e' \cos \phi_2 + X_e' \sin \phi_2)}{V_2'}$$

Apparatus Required Same as for Experiments 3.1 and 3.2.

Circuit Diagram Same as that of Experiments 3.1 and 3.2.

Procedure Same as that of Experiments 3.1 and 3.2.

Observation and Results Record experimental observations in tabular form for both the open and short-circuit tests. Show at least one sample calculation for efficiency and regulation from the test data. Calculate the efficiency of the transformer at one-eight load, one-fourth load, one-half load, three-fourths load, full-load and at 1.25 times the full-load. Draw a graph showing how efficiency varies with load.

Questions *Answer the following questions in your report:*

1. By knowing the losses at full-load, how can you calculate the efficiency of a transformer at any other load?
2. Why is it preferred to determine the efficiency of a transformer indirectly rather than by directly loading it?
3. What is the significance of poor regulation of a transformer?
4. From the efficiency graph drawn by you, indicate at what load, in terms of the full-load, is the efficiency of the transformer maximum? Verify whether the copper-loss at that load is equal to the core-loss.

PART 2

INDUCTION MACHINES, SYNCHRONOUS MACHINES, FRACTIONAL KILOWATT MOTORS AND POWER CONVERTERS

- *Three-Phase Induction Machines*
- *Three-Phase Synchronous Machines*
- *Single-Phase Motors*
- *Power Converters*

4 THREE-PHASE INDUCTION MACHINES

OBJECTIVES

After carefully studying this chapter, you should be able to

- ➢ Explain the constructional features of three-phase induction motors
- ➢ Explain how a rotating magnetic field is produced when a polyphase supply is connected across a polyphase winding and its use in a polyphase induction motor
- ➢ Explain from the concept of alignment of two magnetic fields, the principle of working of a polyphase induction motor
- ➢ Develop an expression for torque of a polyphase induction motor and draw torque-speed characteristic
- ➢ Explain the effect of variation of applied voltage and rotor resistance on the torque-speed characteristic
- ➢ Explain how high starting torque can be achieved by increasing the rotor circuit resistance
- ➢ Describe he various methods of starting polyphase induction motors
- ➢ Explain the different methods of speed control of polyphase induction motors
- ➢ State the various losses in an induction motor and their effects on the efficiency
- ➢ Draw an electrical equivalent circuit of an induction motor
- ➢ Draw circle diagram for determining the various performance data, like starting torque, maximum torque, torque developed at full-load, full-load efficiency, full-load current and power factor, etc., of the motor
- ➢ Make calculations about torque developed, current drawn, power factor, motor speed, etc., using standard equations and test data
- ➢ Perform some basic tests on the motor to determine its performance characteristics
- ➢ Compare the performance of a three-phase induction motor with other motors
- ➢ Suggest suitability of three-phase induction motors for specific applications
- ➢ Supervise installation of induction motors by consulting relevant Indian Standards Specifications
- ➢ Supervise routine maintenance work of induction motors.

4.1 INTRODUCTION

The most common type of ac motor being used throughout the world today is the induction motor. Induction motors are more rugged, require less maintenance, and are less expensive than dc machines of equal kilowatt and speed ratings. Induction motors are constructed both for single-phase and three-phase operations. Three-phase induction motors are widely used for industrial applications such as in lifts, cranes, pumps, exhaust fans, lathes, etc., where as single-phase induction motors are used mainly for domestic-electrical appliances such as fans, refrigerators, washing machines, exhaust pumps, etc. Three-phase induction motors are of two types namely, squirrel-cage type and slip-ring type.

In 1891, Tesla exhibited a crude type of a three-phase motor at an exhibition. Subsequently an improved construction with distributed stator windings and a cage rotor was built. The slip-ring rotor was developed after a decade or so. Since then a lot of improvement has taken place on the design of these two types of induction motors. Lot of research work has been carried out to improve its power factor and to achieve suitable methods of speed control. As a result of research work, the basic induction motor has been modified and variations like synchronous induction motor for the improvement of power factor, pole changing windings for speed control, introduction of commutator for adjustment of speed and power factor (Schrage motor) came up. However, these special machines are only a very small fraction of the total number of induction motors manufactured. Today perhaps 80 per cent of the motors manufactured in the world are plain polyphase induction motors. Figure 4.1 shows photographic view of a three-phase induction motor.

Fig. 4.1 Three-phase induction motor

Like any other electrical rotating machine, an induction machine can also be used as an induction generator, if the rotor is rotated by a primemover. However, induction generators have restricted applications as a source of power supply.

This chapter deals with three-phase induction motors in detail.

4.2 CONSTRUCTION

An induction motor essentially consists of two parts, namely a stationary part called the stator, and a rotating part called the rotor. The rotor is placed inside the stator and is supported on both sides by two end-shields which house the bearings. Figure 4.2 shows the parts of a three-phase induction motor.

Energy is supplied to the windings placed in the stator slots. Energy is transferred to the rotor windings through electromagnetic induction and hence such machines are called induction motors. Figure 4.3 shows the essential parts of a three-phase induction motor in cut sections. The stator, as shown in Fig. 4.3(b) consists of three-phase windings which are placed in slots of a laminated stator core. The stator-core

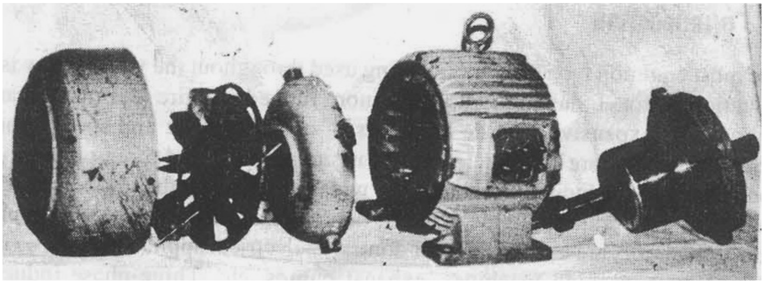
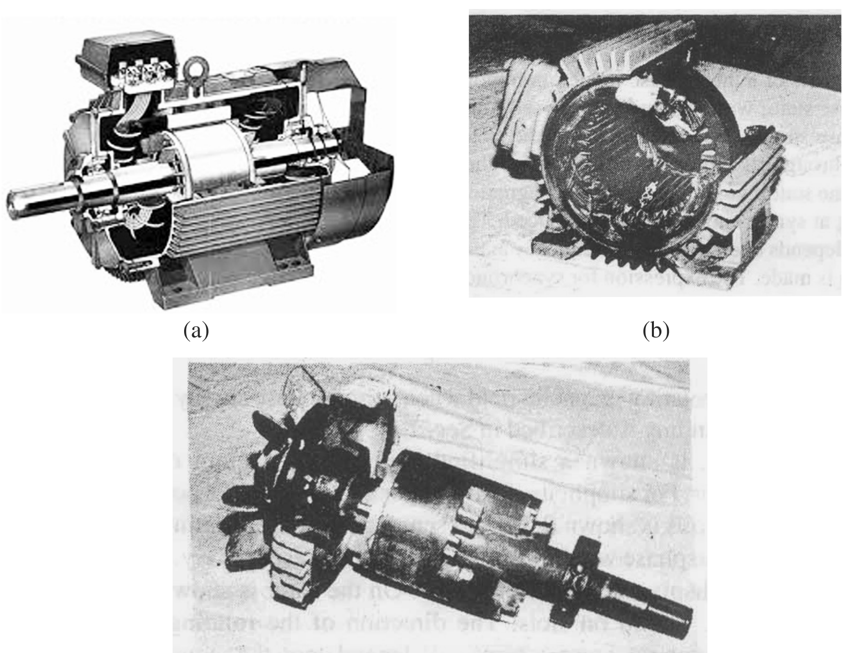


Fig. 4.2 *Parts of a three-phase induction motor*

is supported by the stator frame. The windings are made up of formed coils and are placed in insulated stator slots. Three separate single-phase windings insulated from each other and displaced in space by 120° electrical are made. These windings are either connected permanently in star or in delta internally or all the six terminals of the three-phase windings are brought out to the terminal box so that the operator can connect the machine in star or delta as per requirement. The rotor core is a laminated steel cylinder having slots in which aluminium conductors are die-cast or copper conductors are wound approximately parallel to the shaft. The rotor aluminium conductors in a squirrel-cage induction motor need not be insulated from the rotor-core, since the current will flow through the path of minimum resistance,



(a) (b)

(c)

Fig. 4.3 *Essential parts of a three-phase induction motor in cut sections: (a) Assembled view in cut sections (b) Stator (c) Rotor* i.e., through the rotor conductors an not through the core. The rotor bars are shorted at both the rotor ends by end-rings.

The rotor slots are not made parallel to the rotor shaft but are *skewed* at a certain angle with the shaft (i) to reduce magnetic noise during motor operation, (ii) to produce a more uniform torque, and (iii) to prevent possible magnetic locking (also called cogging) of the rotor with the stator.

In case of slip-ring induction motor rotors, the rotor winding is made on the insulated rotor slots with copper conductors similar to the stator winding. The windings are connected in star. The three free ends are brought to three slip-rings mounted on the shaft. Slip-rings are also insulated from the rotor shaft. External resistance can be connected with the rotor winding through brush and slip-ring connection. Such an arrangement is shown diagrammatically in Fig. 4.4.

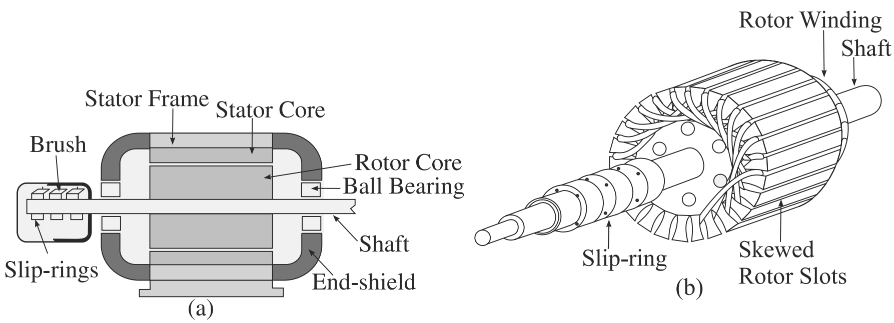


Fig. 4.4 *(a) Simplified diagrammatic representation (in sectional front view) of a slip-ring type induction motor (b) Rotor of a slip-ring type induction motor*

4.3 WORKING PRINCIPLE OF A THREE-PHASE INDUCTION MOTOR

The two essential parts of a three-phase induction motor, are (a) a three-phase stator winding, and (b) a closed rotor winding. To start the motor, a three-phase supply is connected across the stator terminals. The rotor gets its excitation through electromagnetic induction. When three-phase supply is connected across the stator windings, a rotating magnetic field, constant in magnitude but rotating at synchronous speed is produced. The speed of the rotating field so produced depends upon the supply frequency and the number of poles for which the winding is made. The expression for synchronous speed is given by

$$N_S = 120f/P \tag{4.1}$$

Production of rotating magnetic field when a three-phase supply is connected to a three-phase winding has already been described in Sec. 1.4.

In Fig. 4.5, is shown a simplified connection diagram of a three-phase induction motor. For simplicity, on the stator a three-phase, 2 pole winding made only with six coils has been shown coil *R*-*R*′ represents the *R*-phase winding. Similarly *Y*-*Y*′ and *B*-*B*′ are the phase windings of *Y* and *B* phases respectively. The three-phase windings are displaced in space by 120°. On the rotor is shown six coils, 1-1′, 2-2′, 3-3′, etc., placed on slots. The direction of the rotating magnetic field produced by the stator ampere-turns will depend upon the sequence in which the supply terminals R_S Y_S and B_s are connected across the stator winding terminals R, Y and B. In Fig. 4.5, the stator has been shown wound for two poles. The rotor will also have two poles induced in it. Let us assume that the stator field is rotating in the anticlockwise direction. The position of stator poles at a particular instant of time has been shown in the figure. The rotating magnetic field produced by the stator will induce emf in the rotor conductors. The direction of induced emf in the rotor conductors can be determined thus:

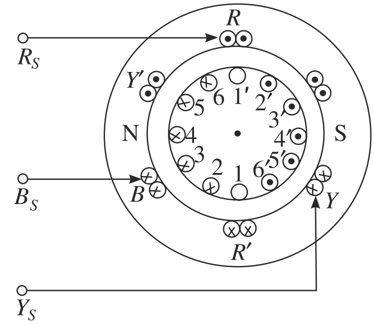


Fig. 4.5 *Three-phase supply connected across the three phase stator windings of an induction motor*

Assume that the stator field which is rotating in the anti-clockwise direction is made stationary. The rotor conductors then can be assumed to be rotating in clock wise direction with respect to the stator field. Applying Fleming's Right Hand Rule, the direction of induced emf can be determined and will be as shown by crosses and dots in Fig. 4.5. Emf induced in the coil 4.4′ will be maximum at this position. Emf induced by the coil 1-1′ will be zero. Emf induced in coils 3-3′ and 5-5′ will be somewhat less than that induced in coil 4-4′. Emf induced in coils 2-2′ and coil 6-6′ will still be less than that induced in coil 4-4′. The magnitude and direction of induced emf in various coils at the particular instant under consideration are shown graphically in Fig. 4.6. Since the rotor winding is a closed one, the rotor induced emf will cause flow of current in the rotor conductors. Assume, for the time being, the rotor to be a purely resistive circuit. The distribution of current flowing through the rotor conductors will be the same as the induced emf, since in a resistive circuit current is in time-phase with the voltage. Due to the current flow in the rotor conductors the rotor will be magnetised. The position of the rotor poles and the direction of the torque developed on the rotor are shown in Fig. 4.6(a).

It may be seen that the rotor will rotate in the same direction as the stator rotating magnetic field. If the direction of rotation of the rotating magnetic field is reversed, the rotor will rotate in the opposite direction. The torque angle in this case (assuming the rotor circuit to be a purely resistive one) is maximum, i.e., 90°. In actual practice, the rotor circuit is not purely resistive but will have inductive reactance in addition to resistance. The rotor current therefore will lag and rotor induced emf by some angle. The current distribution in various armature conductors for lagging current has been shown in Fig. 4.6(c and d). From Fig. 4.6(d), it can be seen that when induced emf in, say, conductor 4 is maximum, current is maximum in conductor 6. In Fig. 4.6(c) the position of the rotor poles due to lagging current flowing through the rotor is shown. The rotor-field axis is now making an angle somewhat less than that in Fig. 4.6(a) with the stator-field axis. The torque angle and hence the torque produced is therefore less when the rotor is an inductive circuit. The torque angle and torque are maximum when the rotor is a purely resistive circuit. To achieve higher torque at starting, therefore, in slip-ring type induction motors, extra resistance is connected in the rotor circuit so that the rotor becomes more resistive and torque angle is increased.

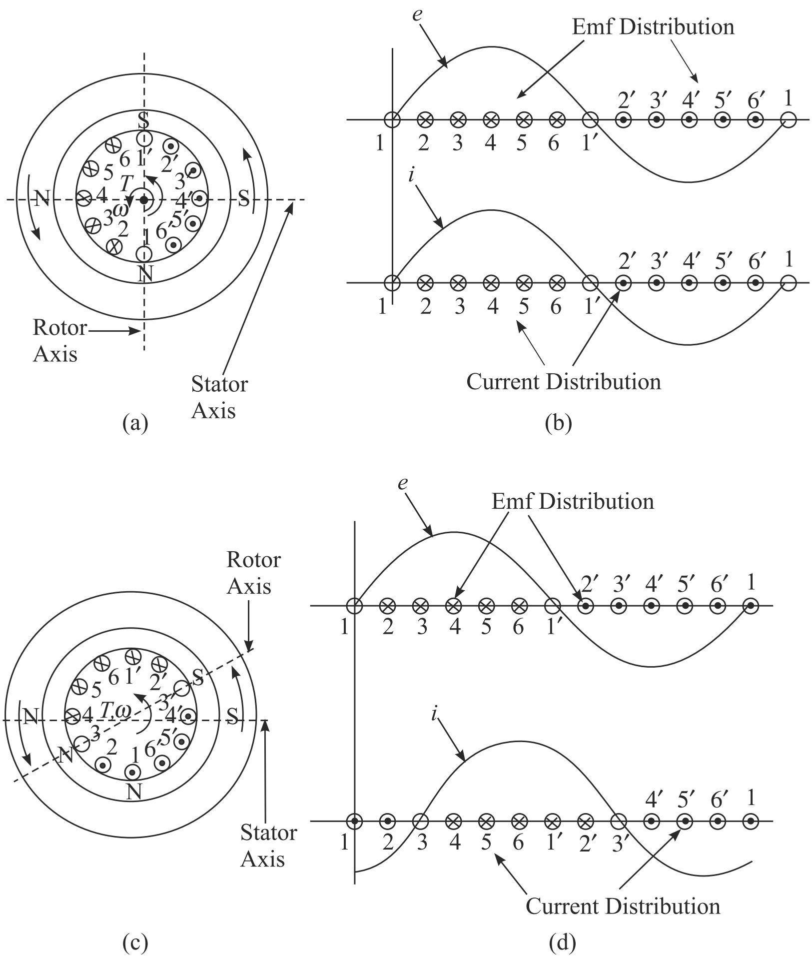


Fig. 4.6 *(a) Direction of current in rotor conductors due to emf induced by stator rotating magnetic field assuming rotor circuit to be purely resistive (b) Direction of induced emf and current in rotor conductors shown separately assuming rotor circuit to be purely resistive (c) Direction of current in rotor conductors considering rotor circuit inductance (d) Direction of induced emf and current in rotor conductors shown separately considering rotor circuit inductance*

If the sequence of supply to the stator winding terminals is changed, the direction of rotation of the stator magnetic field is reversed. The rotor will, then rotate in the opposite direction. The speed of the rotor will, however, be always less than the synchronous speed. At synchronous speed, induced emf in the rotor will be zero and hence there will be no rotor current and rotor torque. The speed of the rotor relative to that of the stator rotating field is called *slip*. If the load on the rotor shaft is increased the slip increases, i.e., the rotor speed decreases. The induced emf in the rotor circuit will increase. The rotor develops more torque to balance the increased load torque requirement.

It is a usual practice to express the slip either as per unit or as a percentage of synchronous speed.

$$\text{Per unit slip} = \frac{\text{Synchronous speed } (N_s) - \text{Rotor speed } (N_r)}{\text{Synchronous speed } (N_s)}$$

and $$\text{Percentage slip} = \frac{N_s - N_r}{N_s} \times 100 \qquad (4.2)$$

The value of slip at full-load varies from about 6 per cent for small motors to about 2 per cent for all large motors.

It may be noted that in an induction motor the stator field rotates at N_s speed with respect to the stator. The rotor rotates at N_r speed with respect to the stator. The magnetic field produced by the rotor mmf rotates at a speed $N_s - N_r$ with respect to the rotor. In other words, the rotor field rotates at $N_r + (N_s - N_r)$, i.e., N_s speed with respect to the stator.

Thus the two magnetic fields are stationary with respect to each other and therefore a steady torque is developed. The torque magnitude is dependent upon the space angle between the two fields, i.e., on the torque angle. This is the condition for development of torque in all rotating electrical machines.

EXAMPLE 4.1 *A three-phase, 6-pole induction motor is supplied from a 50 Hz, 400 V supply. Calculate (a) the synchronous speed, and (b) the speed of the rotor when the slip is 4 per cent.*

Solution

$$\text{Synchronous speed, } N_s = \frac{120f}{p} = \frac{120 \times 50}{6} = 1000$$

$$\text{Percentage slip, } S = \frac{N_s - N_r}{N_s} \times 100$$

In this case, $4 = \frac{1000 - N_r}{1000} \times 100$. Therefore, $N_r = 960$ rpm

EXAMPLE 4.2 *Two three-phase induction motors when connected across a 400 V, 50 Hz supply are running at 1440 and 940 rpm respectively. Determine which of the two motors is running at higher slip.*

Solution Synchronous speeds for motors fed from 50 Hz supply system are

when $P = 2$, $$N_s = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

when $P = 4$, $$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

when $P = 6$, $$N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

when $P = 8$, $$N_s = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

A three-phase induction motor runs at a speed slightly lower than the synchronous speed.

By observing the synchronous speeds calculated above, it is concluded that the number of poles of the motor running at 1440 rpm should be 4 and synchronous speed of the stator rotating field should be 1500 rpm.

$$\text{Slip of the motor, } S_1 = \frac{N_s - N_r}{N} \times 100$$

$$= \frac{1500 - 1440}{1500} \times 100 = 4 \text{ per cent}$$

Number of poles of the motor running at 940 rpm should be 6 and synchronous speed of the stator rotating field should be 1000 rpm.

$$\text{The slip of the motor, } S_2 = \frac{1000 \times 940}{1000} \times 100 = 6 \text{ per cent}$$

Thus the slip of the motor running at 940 rpm is higher than the slip of the motor running at 1440 rpm.

EXAMPLE 4.3 *A 10-pole, 3-phase alternator is coupled to an engine running at 600 rpm. It supplies an induction motor which has a full-load speed of 1440 revolutions per minute. Calculate the percentage slip and the number of poles of the motor.*

Solution Frequency of voltage generated by the alternator,

$$f = \frac{P \times N}{120} = \frac{10 \times 600}{120} = 50 \text{ Hz}$$

$$\text{Synchronous speed, } N_s = \frac{120 \times f}{P}$$

when $P = 2$, $$N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

when $P = 4$, $$N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Induction motor speed is somewhat less than the synchronous speed. In this case induction motor speed, N_r is 1440 rpm.

$$\text{Percentage slip, } S = \frac{N_s - N_r}{N_s} \times 100$$

$$= \frac{1500 - 1440}{1500} = 4\%$$

Thus, slip is 4% and number of poles of the motor is 4.

4.4 PRODUCTION OF ROTATING MAGNETIC FIELD WITH THREE-PHASE WINDING AND A THREE-PHASE SUPPLY

Figure 4.7(a) shows a stator with three-phase winding. One end of each of the phases are connected together. Other ends are kept free to be connected to the supply terminals. Each phase winding is shown to have been made of two coils. In practice, however, there will be more coils per phase and the windings will be distributed throughout the stator slots. When a three-phase supply, as shown in Fig. 4.7(b) is applied across the stator winding terminals, magnetic fields will be produced by the current flowing through the phase windings. The direction of field produced by each

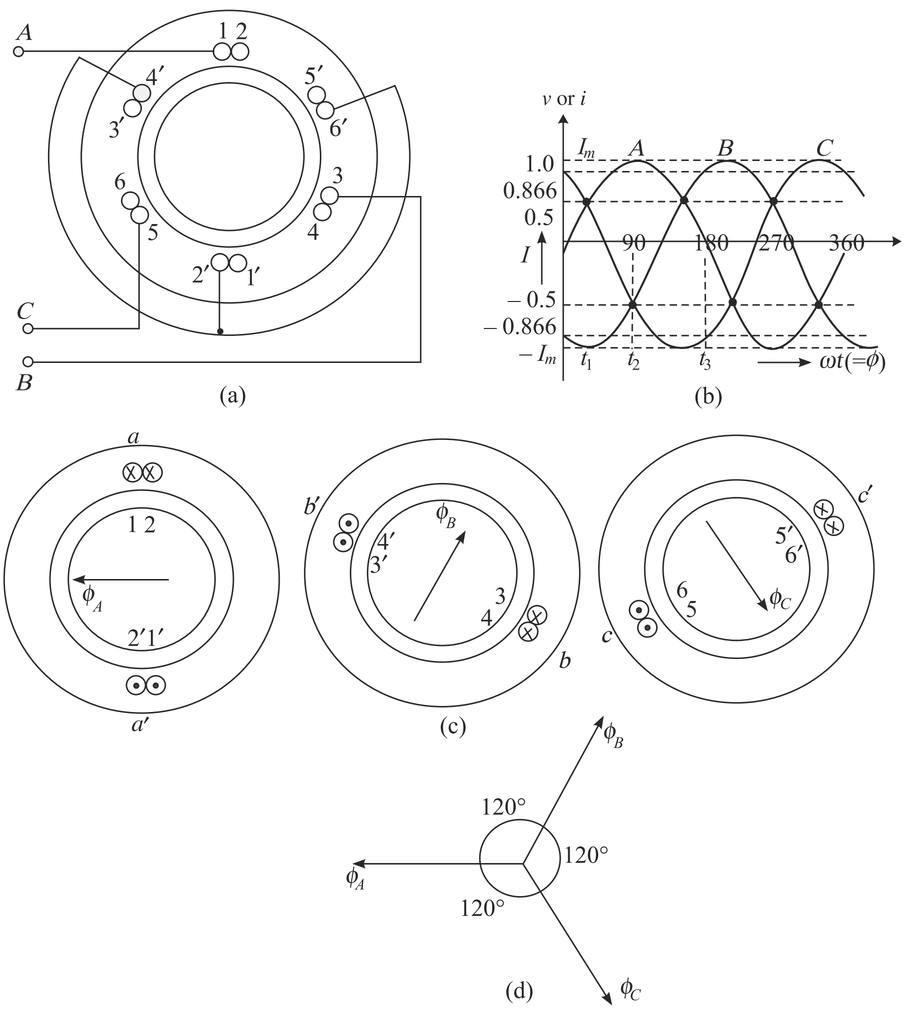


Fig. 4.7 *Magnetic fields produced due to current flowing in each phase of a three-phase stator winding (a) Three-phase winding with two coils per phase wound on stator; (b) Three-phase supply; (c) Flux produced by the individual phase currents; (d) Phasor representation of the fields produced by the three phase winding mmfs*

phase at any particular instant of time will be different as shown in Fig. 4.7(c). These three magnetic fields will give rise to a resultant magnetic field. We will study the nature of the resultant magnetic field at different instants of time when the three-phase currents will continue to flow through the three-phase stator windings.

The directions of flux produced by the currents flowing through the three-phase windings are shown vectorially in Fig. 4.7(d). The magnitude and direction of the three-phase currents flowing through the windings will change with time. In Fig. 4.7(d), while representing the flux produced by each phase through phasors, we have assumed positive current of equal magnitude flowing through the windings. To determine the magnitude and direction of the resultant magnetic field we will take into account the actual magnitude and direction of current flowing through the three windings. We have assumed that supply current represented by wave-shapes *A*, *B* and *C* as shown in Fig. 4.7(b) are connected respectively to the phases *A*, *B* and *C* of Fig. 4.7(a). Let us consider three instants of time t_1, t_2, t_3 of the current waves. At the instant of time t_1, current in phase *A* is zero, current in phase *B* is – 0.866 I_m and current in phase *C* is + 0.866 I_m as can be seen from Fig. 4.7(b).

The direction of fields produced by the three phases at the instant of time t_1 and the resultant magnetic field is shown in Fig. 4.8(a). Since there is no current flowing through phase *A* coils, no flux will be produced, hence ϕ_A is shown as zero. In phase *B*, current is 0.866 I_m and is flowing in the negative direction. Current flowing in the negative direction is shown by a cross in conductors 3′, 4′ and by dots in conductors 3 and 4. The magnitude of the field will be 0.866 ϕ_m and will be in the direction shown by phasor ϕ_B. In phase *C*, current is 0.866 I_m and is flowing in the positive direction. The magnitude of field will be 0.866 ϕ_m and the direction will be as shown by phasor ϕ_c. The sum of these phasors at the instant of time t_1 of the current waves is shown as ϕ_R and is equal to,

$$\phi_R = 1.5\ \phi_m$$

Referring to Fig. 4.7(b) we observe that at the time t_2 current in phase *A* is positive maximum, i.e., I_m. The flux produced by phase *A* ampere turns is therefore $\phi_A = \phi_m$, its direction is shown in Fig. 4.8(b). Currents in phase *B* and phase *C* are negative and their magnitudes are 0.5 fluxes, $\phi_B = \phi_C = -0.5\ \phi_m$ are shown in Fig. 4.8(b). The resultant flux, ϕ_R of ϕ_A, ϕ_B and ϕ_C is also shown. The magnitude of ϕ_R is 1.5 ϕ_m, but its direction is now changed. The resultant field has rotated in the clockwise direction by 90°.

Similarly flux produced by the individual phase windings and the magnitude and direction of the resultant field for the instant of time t_3 have been shown in Fig. 4.8(c).

From Fig, 4.8 it is observed that for the time t_1 to t_2, i.e., by the time the current has flown for one-half cycle through the windings, the resultant magnetic field has rotated by 180° mechanical, i.e., by half a revolution in the clockwise direction. the magnitude of the resultant field has been 1.5 ϕ_m all the time. It could be shown that if current had flown for one cycle through the windings, the resultant field would have rotated by one complete rotation.

The supply frequency is generally 50 cps. In one second therefore the resultant field will rotate by 50 revolutions. In one minute the resultant field will rotate by

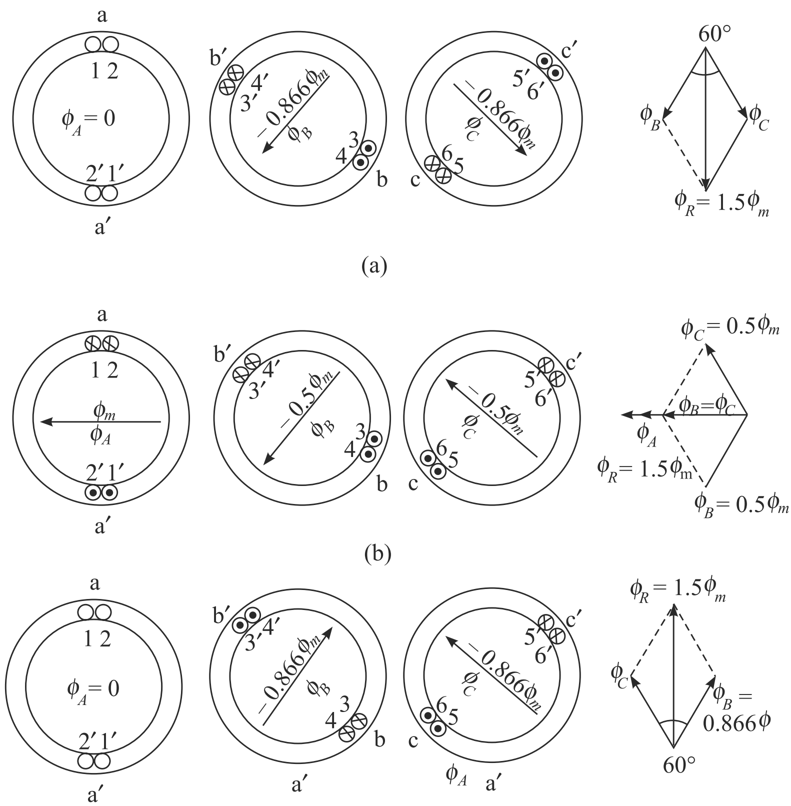


Fig. 4.8 *Directions of magnetic field produced by the ampere-turn of each phase and the resultant field produced by the three phases mmfs*

50 × 60 = 3000 revolutions. This speed is called synchronous speed. Synchronous speed, in addition to the supply frequency, depends on the number of poles for which the winding is made. In this case the windings have been made for two poles. The relation between frequency, number of poles, and the synchronous speed, as mentioned earlier, is given by

$$N_s = \frac{120\,f}{P}$$

If the sequence of supply to the different phase windings is changed, i.e., if we connect *A* phase supply to *B* phase winding and *B* phase supply to *A* phase winding, keeping *C* phase winding supply connection unchanged, the resultant magnetic field will rotate in the opposite direction, in this case in the anticlockwise direction. The magnitude of the resultant magnetic field and its speed will however remain unchanged. To summarise:

(a) When a three-phase supply is connected across a three-phase winding, a rotating field of constant mangitude rotating at synchronous speed is produced.

(b) The direction of rotation of the rotating field so produced depends on the sequence of supply to the phase windings.

4.5 ROTOR FREQUENCY, ROTOR INDUCED EMF, ROTOR CURRENT AND POWER FACTOR

4.5.1 Rotor Frequency

The relationship between synchronous speed, stator supply frequency and the stator number of poles is given by

$$f = \frac{PN_s}{120} \tag{4.3}$$

When the rotor rotates at a speed N_r the rotor conductors cut the rotating field at a speed, $N_s - N_r$. The frequency of the rotor induced emf f_r can be expressed as

$$f_r = P\frac{(N_s - N_r)}{120} \tag{4.4}$$

Slip is defined as the difference between the synchronous speed N_s and the rotor speed N_r. Value of slip is expressed in per unit or in percentage of N_s.

Per unit slip is the ratio of slip in rpm and the synchronous speed in rpm.

Thus per unit slip,

$$S = \frac{N_s - N_r}{N_s} \quad \text{or,} \quad (N_s - N_r) = SN_s \tag{4.5}$$

From Eqs (4.4) and (4.6),

$$f_r = \frac{PSN_s}{120} = S\frac{(PN_s)}{120} \tag{4.6}$$

Substituting the value of from Eq. (4.3),

$$f_r = Sf \tag{4.7}$$

Alternatively, when the rotor is at standstill condition, the emf induced in the rotor due to stator rotating field will have the same frequency as the frequency of stator supply voltage. When the rotor rotates at N_r speed, emf is induced in the rotor due to relative velocity of $N_s - N_r$. Thus,

when N_s is the relative velocity rotor frequency, $f_r = f$.

when $N_s - N_r$ is the relative velocity rotor frequency, $f_r = \frac{f}{N_s}(N_s - N_r) = sf$

$$\therefore \quad f_r = sf$$

Thus, frequency of the rotor induced emf (i.e. rotor frequency) when the rotor rotates at a speed N_r, is equal to the product of rotor slip and stator supply frequency. At standstill, i.e., when the rotor is not rotating, the rotor frequency is the same as the stator frequency, because the slip at rotor standstill is unity.

The magnetic field produced by the polyphase stator currents rotates at a speed N_s with respect to the stator. The rotor rotates at a speed N_r with respect to the stator. The rotor currents due to rotor emf produced a rotating magnetic field which rotates at a speed $N_s - N_r$ with respect to the rotor. Therefore, the speed of the rotor field relative to the stator is

$$N_r + N_s - N_r = N_s \text{ rpm}$$

Thus, the rotor magnetic field and the stator magnetic field rotates at a speed N_s with respect to the stator and hence they are stationary with respect to each other.

A polyphase induction motor can, therefore, the assumed as a polyphase transformer having the primary and secondary windings separated by an air-gap. Because of the presence of the air-gap, an induction motor takes higher magnetising current as compared to a transformer of the same rating. Due to the occurrence of rotational losses, the efficiency of an induction motor is less than that of an equivalent transformer.

EXAMPLE 4.4 *The rotor of a three-phase induction motor rotates at 1440 rpm when a 50 Hz supply is connected, across the stator terminals. Calculate the frequency of the rotor induced emf.*

Solution

$$N_r = 1440 \text{ rpm}$$

We know, $$N_s = \frac{120 f}{P}$$

The number of poles can be 2, 4, 6, 8, etc.

when $P = 2$, $$N_s = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

when $P = 4$, $$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

when $P = 6$, $$N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

when $P = 8$, $$N_s = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

Thus at 50 Hz supply, the possible synchronous speeds of the rotating field are 3000, 1500, 1000, 750, etc.

The rotor of an induction motor rotates at a speed slightly less than the synchronous speed. For the rotor speed of 1440 rpm, the synchronous speed should be 1500 rpm and the motor must have been wound for four poles.

$$N_s = 1500 \text{ rpm}, N_r = 1440 \text{ rpm}$$

$$f_r = s.f = 50 \times \frac{N_s - N_r}{N_s}$$

or $$f_r = 50 \times \frac{1500 - 1440}{1500}$$

or $$f_r = \frac{50 \times 60}{1500} = 2 \text{ Hz}$$

Rotor frequency $$f_r = 2 \text{ Hz}$$

EXAMPLE 4.5 *If the electromotive force in the stator of an 8-pole induction motor has a frequency of 50 Hz, and that in the rotor 1½ Hz, at what speed is the motor running and what is the slip?*

Solution

Stator frequency, $$f = 50 \text{ Hz}$$

Rotor frequency, $f_r = 1.5$ Hz

$$f_r = S \times f$$

Slip, $$S = \frac{f_r}{f} = \frac{1.5}{50} = 0.03$$

Synchronous speed, $$N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

Again, $$S = \frac{N_s - N_r}{N_s}$$

or $$0.03 = \frac{750 - N_r}{750}$$

or $$N_r = 750 - 0.03 \times 750 = 728 \text{ rpm}$$

Thus, the motor is running at 728 rpm and the slip is 3 per cent.

EXAMPLE 4.6 *A 6 pole, 3-phase induction motor is connected across a 400 V, 50 Hz supply source. Calculate the speed of the rotating magnetic field produced. What would be the speed of the rotor when slip is 0.04. Also calculate the frequency of rotor current at standstill and at a slip of 0.03.*

Solution

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$S = \frac{N_s - N_r}{N_s}$$

or, $$S\,N_s = N_s - N_r$$

or, $$N_r = (1 - s)\,N_s$$

Substituting values,

Rotor speed, $N_r = (1 - 0.04)\,1000 = 960$ rpm

Rotor frequency, $f_r = sf$

Substituting values, $f_r = 0.03 \times 50 = 1.5$ Hz.

f_r at standstill will be calculated by putting $s = 1$.

Thus, $f_r = sf = 1 \times 50 = 50$ Hz

4.5.2 Rotor Induced Emf

The rotating magnetic field produced by the stator ampere-turns will induce emf in both stator and the rotor windings. The induced emf will depend upon the magnitude of the rotating flux and the speed at which this flux cuts the stator and rotor conductors. When the rotor is stationary (i.e., at standstill) the stator flux cuts the rotor conductors at a speed N_s. Let E_{20} be the induced emf in the rotor winding when the rotor is not rotating. When the rotor starts rotating at a speed N_r, the rotating field cuts the rotor conductors at a speed $(N_s - N_r)$ rpm, i.e., at SN_s rpm. Since at N_s speed of flux cutting, induced emf in the rotor is E_{20}, at SN_s speed of flux cutting induced emf in the rotor will be SE_{20}. Let E_2 be the induced emf in the rotor winding when the rotor is rotating.

Rotor induced emf, $E_2 = SE_{20}$ (4.8)

At the instant of starting, slip is equal to one. Therefore at start, $E_2 = E_{20}$ (maximum emf is induced in the rotor). As the motor picks up speed, its slip decreases, and, therefore, the rotor induced emf decreases. When the rotor approaches synchronous speed, its slip reduces to a very small value and hence rotor induced emf becomes very small. The rotor cannot attain synchronous speed because at synchronous speed no emf will be induced in the rotor and no torque will be produced. Rotor will, therefore, always rotate at a lower speed than synchronous speed.

4.5.3 Magnitude of Stator and Rotor EMF, Rotor Current and Power Factor

Let V_1 = the stator applied voltage per phase
T_1 = number of stator winding turns in series per phase
T_2 = number of rotor winding turns in series per phase
ϕ = flux per pole produced by the stator ampere-turns
E_{20} = emf induced per phase when the rotor is at standstill
E_2 = emf induced in the rotor per phase when the rotor is rotating at a slip S
R_2 = resistance of rotor winding per phase
X_{20} = leakage reactance or the rotor winding per phase when the rotor is not rotating (i.e., rotor is at standstill)
L_{20} = rotor inductance per phase at stand still due to leakage flux
X_2 = reactance of rotor winding per phase when the rotor is rotating
K_d = distribution factor of winding
K_p = pitch factor of winding

Expression for emf induced in the stator and rotor windings can be found out as follows.

When the rotor is at standstill, emf will be induced in both stator and rotor windings due to the cutting of flux by them at a speed of N_s, since the rotating magnetic field produced by the stator ampere-turns will cut the stator and rotor conductors at synchronous speed. Flux cut by the stator or rotor conductors per revolution of the rotating magnetic field = $P\phi$ webers. The revolving field makes $N_s/60$ revolutions in one second. The time taken by the revolving field to make 1 revolution is $60/N_s$ secs.

$$\text{Flux cut per second} = \frac{P\phi}{60/N_s} = \frac{P\phi N_s}{60} \text{ Wb}$$

= average induced emf in each conductor

Rms value of induced emf per conductor

$$= 1.11\ \frac{P\phi N_s}{60} \text{ V}$$

We know, $$f = \frac{PN_s}{120}$$

Therefore, $E_{\text{rms}} = 2.22 f\phi$ V

In the stator there are T_1 turns per phase, i.e., there are 2 T_1 conductors per phase in series.

Stator induced emf, $E_1 = 4.44\, f\phi T_1$ V

Considering distribution factor and pitch factor,

$$E_1 = 4.44\, f\phi T_1\, K_p K_d \text{ V}$$

Stator supply voltage V_1 is approximately equal to stator induced emf, E_1.

Therefore, $V_1 \approx E_1 = 4.44\, f\phi T_1 K_p K_d$ V per phase (4.9)

Similarly, induced emf in the rotor when the rotor is at standstill,

$$E_{20} = 4.44\, f\phi T_2\, K_{\text{p}}\, K_{\text{d}} \text{ V per phase}$$

When the rotor is rotating at a slip S, the rotor induced emf E_2 is equal to SE_{20}

Therefore, $E_2 = SE_{20} = 4.44\ \text{S}\, f\phi\ T_2\ K_p\ K_d$ V per phase

Since rotor frequency $f_r = Sf$,

$$E_2 = 4.44\, f_r\ \phi\ T_2\ K_p\ K_d \text{ V per phase} \tag{4.10}$$

Rotor reactance when the rotor is rotating,

$$X_2 = 2\pi f_r L_{20}$$

$$X_2 = 2\pi s\, f L_{20}$$

$$X_2 = S\, 2\pi f L_{20} = SX_{20}$$

Rotor impedance per phase,

$$Z_2 = \sqrt{R_2^2 + (SX_{20})^2}$$

If I_{20} is the rotor current per phase at standstill and I_2 is the rotor current per phase at a slip, S,

$$I_{20} = \frac{E_{20}}{\sqrt{R_2^2 + X_{20}^2}}$$

and

$$I_2 = \frac{SE_{20}}{\sqrt{R_2^2 + (SX_{20})^2}} \tag{4.11}$$

If ϕ_2 is the phase difference between rotor voltage, E_2 and rotor current, I_2 then rotor power factor,

$$\cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + (SX_{20})^2}} \tag{4.12}$$

EXAMPLE 4.7 *The induced emf between the slip-ring terminals of a three-phase induction motor, when the rotor is at standstill is 100 V. The rotor winding is star connected and has resistance and standstill reactance of 0.05 Ω and 0.1 Ω per phase respectively. Calculate the rotor current and phase difference between rotor voltage and rotor current at (a) 4 per cent slip, and (b) 100 per cent slip.*

Solution

Given $E_{20} = 100$ V, $R_2 = 0.05\ \Omega$, $X_{20} = 0.1\ \Omega$

$$E_{20} \text{ per phase} = \frac{100}{\sqrt{3}} = 57.7 \text{ V}$$

(a) When $S = 0.04$

$$I_2 = \frac{E_2}{Z_2} = \frac{SE_{20}}{\sqrt{R_2^2 + (SX_{20})^2}}$$

$$= \frac{0.40 \times 57.7}{\sqrt{0.05^2 + (0.40 \times 0.1)^2}}$$

or $$I_2 = 46 \text{ A}$$

$$\cos \phi_2 = \frac{R_2}{\sqrt{R_2^2 + (SX_{20})^2}}$$

$$= \frac{0.05}{\sqrt{0.05^2 + (0.40 \times 0.1)^2}}$$

or $$\cos \phi_2 = 0.9968$$

Phase angle between rotor voltage and rotor current $\phi_2 = 4.57°$

(b) When $S = 1.0$

$$I_2 = \frac{1.0 \times 57.7}{\sqrt{(0.05)^2 + (0.1)^2}}$$

or $$I_2 = 516 \text{ A}$$

$$\cos \phi_2 = \frac{0.05}{\sqrt{0.05^2 + 0.1^2}} = 0.449$$

$$\phi_2 = 63.3°$$

From Example 4.4, it is seen that the magnitudes of rotor current and rotor power factor depend on the rotor slip. Therefore, the torque developed, which is a function of rotor current and rotor power factor, will depend on rotor slip.

EXAMPLE 4.8 *A 4-pole induction motor is energised from a 50 Hz supply system. If the machine runs on full-load at 3 per cent slip, determine the running speed and the frequency of the rotor currents.*

Solution Synchronous speed

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{Percentage slip } S = \frac{N_s - N_r}{N_s} \times 100$$

or $$3 = \frac{1500 - N_r}{1500} \times 100$$

or $N_r = 1455$ rpm

Rotor frequency $f_r = S\,f = 0.03 \times 50 = 1.5$ Hz

EXAMPLE 4.9 *A 6-pole induction motor is supplied from a 400 V, three-phase, 50-Hz supply system. The frequency of the rotor induced emf is 2 Hz. Calculate (a) the percentage slip, (b) the rotor speed.*

Solution Rotor frequency

$$f_r = S\,f$$

or $$2 = S \times 50$$

or $$S = \frac{2}{50} = 0.04 = 4 \text{ per cent}$$

$$S = \frac{N_s - N_r}{N_s} \times 100$$

or $$4 = \frac{1000 - N_r}{1000} \times 100$$

or $$N_r = 960 \text{ rpm}$$

EXAMPLE 4.10 *A 12-pole, 50 Hz three-phase induction motor runs at 485 rpm. What will be the frequency of the rotor current?*

Solution Synchronous speed

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{12} = 500 \text{ rpm}$$

Slip $$S = \frac{N_s - N_r}{N_s} = \frac{500 - 485}{500} = 0.03$$

Rotor frequency $$f_r = Sf = 0.03 \times 50 = 1.5 \text{ Hz}$$

EXAMPLE 4.11 *The induced emf between the slip-ring terminals of an induction motor at standstill is 100 V. The rotor windings are stator connected and has a resistance of 0.4 Ω per phase and standstill reactance of 2.25 Ω per phase. Calculate, the rotor current when the slip-ring terminals are short-circuited and the rotor is rotating at a slip of 4 per cent.*

Solution Induced emf in the rotor at standstill,

$$E_{20} = 100 \text{ V}$$

$$E_{20} \text{ per phase} = \frac{100}{\sqrt{3}} \text{ V}$$

Rotor induced emf at a slip S,

$$E_2 = SE_{20} = \frac{0.40 \times 100}{\sqrt{3}} = 2.31 \text{ V}$$

Rotor impedance at a slip S,

$$Z_2 = \sqrt{R_2^2 + (SX_{20})^2} = \sqrt{(0.4)^2 + (0.40 \times 2.25)^2}$$

$$= \sqrt{0.16 + 0.0081} = 0.41\ \Omega$$

Rotor current $$I_2 = \frac{E_2}{Z_2} = \frac{2.31}{0.41} = 5.63\ \text{A}$$

4.6 POWER FLOW DIAGRAM FOR AN INDUCTION MOTOR

Figure 4.9 illustrates how the electrical power supplied to the stator winding of an induction motor is converted into mechanical power output at the shaft.

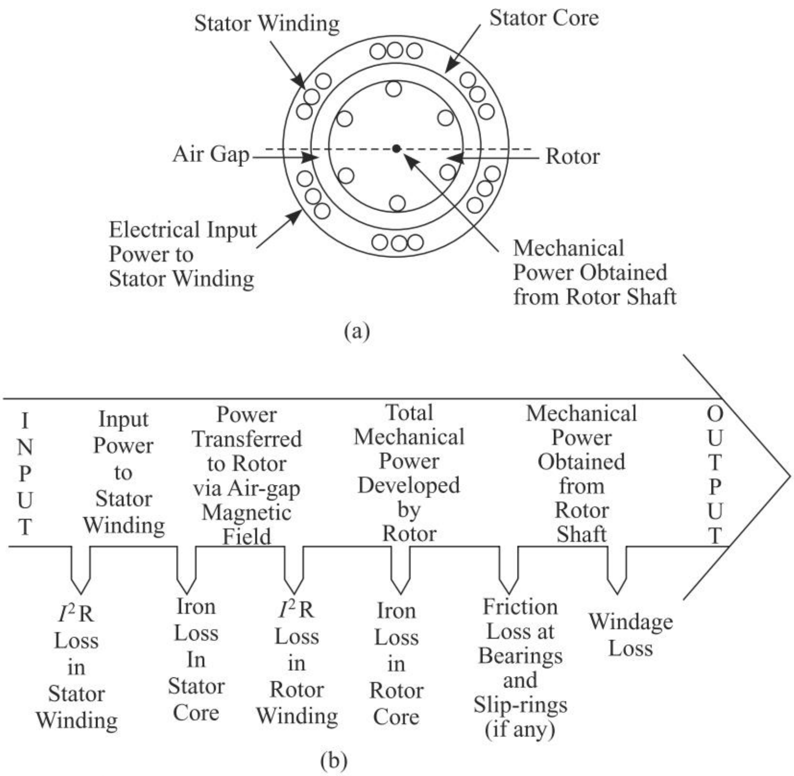


Fig. 4.9 *Power flow diagram of an induction motor (a) Cross-sectional view of the motor (b) Flow of power showing electrical input power to mechanical power output*

At rated speed, iron-loss in the rotor core is negligible because iron-loss is a function of rotor frequency and the frequency of the rotor current is very small ($f_r = Sf$). Let T be the torque in Newton-metres exerted on the rotor by the rotating magnetic field rotating at synchronous speed N_s rpm or $N_s/60$ rps.

Power transferred from stator to rotor (i.e., rotor input)

$$= \frac{2\pi T N_s}{60} \text{ W} \tag{4.13}$$

This input power to the rotor is termed torque in synchronous watts.

When the rotor is rotating at a speed of N_r rpm, total mechanical power developed by the rotor is $2\pi T N_r/60$ W. From Fig. 4.9 we observe that the difference between power transferred to the rotor via the magnetic field of airgap and the mechanical power developed by the rotor is equal to I^2R-loss in the rotor winding (assuming iron-loss in the rotor core as negligible). This is expressed as:

(Power transferred from stator to rotor) – (Mechanical power developed by the rotor) = Rotor I^2R-loss

or $$\frac{2\pi T N_s}{60} - \frac{2\pi T N_r}{60} = \text{Rotor } I^2R\text{-loss} \tag{4.14}$$

or $$\frac{2\pi T N_s}{60} \frac{(N_s - N_r)}{N_s} = \text{Rotor } I^2R\text{-loss}$$

[dividing and multiplying L.H. S. by N_s]

or $$\text{Rotor Input} \times S = \text{Rotor } I^2R\text{-loss}$$

or $$\text{Rotor } I^2R\text{-loss} = \text{S} \times \text{Rotor input} \tag{4.15}$$

This is an important relation which will be used while deriving the expression for torque.

4.7 FACTORS DETERMINING TORQUE AND TORQUE-SLIP CHARACTERISTIC

Electrical power generated in the rotor is equal to $mE_2I_2 \cos \phi_2$, where m is the number of phases. This electrical power is lost as I^2R-loss in the rotor circuit.

$$\text{Rotor } I^2R\text{-loss} = mE_2I_2 \cos \phi_2$$

$$= \frac{mSE_{20}SE_{20}}{\sqrt{R_2^2 + (SX_{20})^2}} \frac{R_2}{\sqrt{R_2^2 + (SX_{20})^2}}$$

$$= \frac{mS^2E_{20}^2R_2}{R_2^2 + S^2X_{20}^2} \tag{4.16}$$

From Eq. (4.13),

$$\text{Rotor input} = \frac{2\pi T N_s}{60}$$

From Eq. (4.15)

$$\text{Rotor } I^2R\text{-loss} = \text{Slip} \times \text{Rotor input}$$

or $$\frac{mS^2E_{20}^2R_2}{R_2^2 + S^2 + X_{20}^2} = S2\pi T\frac{N_s}{60}$$

$$T = \frac{60}{2\pi N_s} \times \frac{mS\,E_{20}^2R_2}{R_2^2 + S^2X_{20}^2} \tag{4.17}$$

Therefore, $$T \propto \frac{SE_{20}^2 R_2}{R_2^2 + S^2 X_{20}^2} \quad (4.18)$$

Since rotor induced emf at standstill, is proportional to air-gap flux ϕ,

$$T \propto \frac{S\phi^2 R_2}{R_2^2 + S^2 X_{20}^2} \quad (4.19)$$

From the expression of torque, it is seen that for given values of rotor resistance, rotor reactance, and rotor slip, torque is proportional to the square of the flux.

Since the flux produced in the air-gap is approximately proportional to the supply voltage to the stator, from expression (4.19) it is seen that the torque on rotor, T is proportional to the square of the stator applied voltage, V_1.

Therefore, $$T \propto V_1^2 \quad (4.20)$$

From the expression (4.18) of torque, the shape of the torque-slip characteristic can be predicted as follows:

Assuming constant flux produced by the stator applied voltage, the expression for torque can be written as

$$T \propto \frac{SR_2}{R_2^2 + S^2 X_{20}^2}$$

$$T \propto \frac{KSR_2}{R_2^2 + S^2 X_{20}^2} \quad (4.21)$$

The value of rotor standstill reactance X_{20} is greater than rotor resistance R_2. Assuming certain values of R_2, X_{20} and constant K the value of T can be calculated for different values of slip (value of slip varies from 0 to 1) and the torque-slip characteristic can be drawn. It is, however, possible to predict the nature of the torque-slip characteristic without making actual calculations. In Eq. (4.21), at very small values of slip, the term $S^2 X_{20}^2$ can be neglected as compared to R_2^2. In such a case, $T \propto S/R_2$; or $T \propto S$, i.e., torque is directly proportional to slip. For large values of slip, say from 0.1 to 1.0, R_2^2 is small as compared to $S^2X_{20}^2$. Therefore, the torque expression can be written as

$$T \propto \frac{SR_2}{S^2 X_{20}^2} \propto \frac{1}{S} \text{ (since, } R_2 \text{ and } X_{20} \text{ are constant for a motor)}$$

i.e., torque is inversely proportional to slip. Thus the torque-slip characteristic is drawn as shown in Fig. 4.10(a). For lower values of slip, the torque-slip characteristic is represented as a straight line whereas, for higher values of slip the torque-slip characteristic is represented as a rectangular hyperbola.

Knowing that zero slip means synchronous speed, N_s, and slip = 1 means zero speed, the torque as a function of rotor speed can also be drawn as shown in Fig. 4.10 (b).

The direction of increasing slip is opposite to the direction of increasing rotor speed. The magnitude of torque at zero rotor speed is called starting torque T_{st}. The maximum torque T_m, developed by the motor is always higher than the full-load torque. The motor is never operated on maximum torque developed condition.

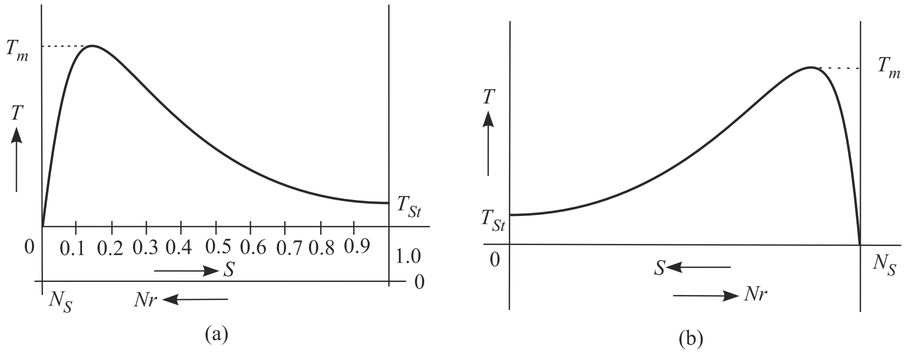


Fig. 4.10 *Torque-slip or torque-speed characteristic of a three-phase induction motor*

Starting torque developed by the rotor should be more than the torque required by the load connected to the motor shaft to enable the motor to start with the load. The magnitude of the starting torque and the slip at which maximum torque is developed depend upon the rotor resistance. The effect of variation of rotor resistance on the torque-speed characteristic of an induction motor is explained in the following section.

Let us now calculate the ratio of starting torque, T_s and full-load torque T_f when the induction motor is started direct-on-line, i.e., on full-voltage. It will be seen that although the starting current on full-voltage is 6 to 7 times the full-load current, the starting torque is not so high. This can be seen through the following calculations.

We have seen earlier that

Rotor Copper loss = slip × Rotor input

Rotor input is provided by the rotating magnetic filed which is rotating at synchronous speed, N_s. Thus,

$$I^2{}_2 R_2 = S \times \frac{2\pi T N_s}{60}$$

or,
$$T \propto \frac{I_2^2}{S}$$

If torque at full load is T_f and torque at starting is T_s, and S_f is the slip at full-load, then

$$T_f \propto \frac{I_2^2}{s} \propto \frac{I_f^2}{s_f}$$

$$T_s \propto \frac{I_2^2}{s} \propto \frac{I_{sc}^2}{1}$$ [Since $S = 1$ at starting;

$I_2 = I_f$ on full-load; and $I_2 = I_{sc}$ at starting because the motor is on blocked rotor condition with full voltage applied]

As mentioned earlier, an induction motor when started on line, at the moment start, it is similar to a short-circuited transformer. That is why the starting current is taken as short-circuit current which is about 6 to 7 times the full-load current.

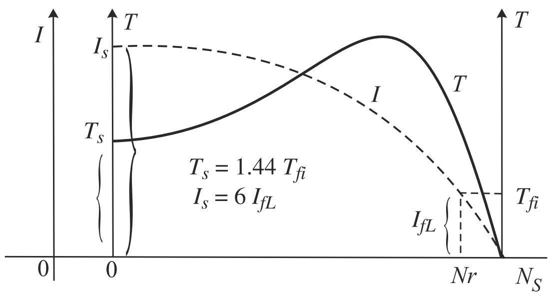
Dividing T_s by T_f we get

$$\frac{T_s}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 S_f \tag{4.22}$$

Now, let us assume that an induction motor is running at 1440 rpm. Its slip will be 4 per cent. If the motor draws 6 times its full-load current at starting, then

$$\frac{T_s}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 S_f = \left(\frac{6I_f}{I_f}\right)^2 \times 0.04 = 1.44$$

Thus, it is seen that starting torque is 1.44 times the full-load torque with a starting current equal to 6 times the full-load current as shown in Fig. 4.10(c).



(c)

Fig. 4.10 *Showing the starting current starting torque as against full-load current and full-load torque*

4.8 EFFECT OF VARIATION OF ROTOR RESISTANCE ON THE TORQUE-SLIP CHARACTERISTIC

The torque equation for an induction motor at constant input voltage is rewritten as

$$T = \frac{KSR_2}{R_2^2 + S^2 X_{20}^2}$$

Let the values of R_2 and X_{20} for a particular induction motor be 1 Ω and 10 Ω respectively. Let us also assume that it is possible to increase the rotor circuit resistance by some external means. (In slip-ring induction motors, external resistance can be connected across the rotor terminals with the help of brush and slip-ring arrangement). Let us assume that the total rotor resistance is made 1, 2, 6 and 10 Ω. respectively. The rotor standstill reactance X_{20} = 10 Ω will remain constant since X_{20} is fixed by the design of the rotor. If we calculate the value of torque, T, for different values of slip, S, we shall get a number of points on the torque-slip characteristic. Table 4.1 gives the magnitudes of torque at different values of slip for $R_2 = 1$, $X_{20} = 10$; $R_2 = 2$, $X_2 = 10$; $R_2 = 6$, $X_{20} = 10$ and $R_2 = 10$, $X_{20} = 10$.

For studying the nature of torque-slip characteristics, a suitable value of K can be taken. Let us take $K = 100$.

Thus

$$T = \frac{100\,SR_2}{R_2^2 + S^2 X_{20}^2}$$

The values of torque calculated at different values of slip for various combinations of R_2 and X_{20} are shown in Table 4.1.

Torque-slip characteristics for different values of rotor circuit resistance as per Table 4.1 have been drawn as shown in Fig. 4.11. Torque-speed characteristics for four different values of rotor circuit resistance have been drawn together for the sake of comparison.

Table 4.1 *Calculated Values of Torque at Different Slips Having Variable Rotor Resistance*

SLIP	TORQUE FOR $R_2 = 1, X_{20} = 10$	TORQUE FOR $R_2 = 2, X_{20} = 10$	TORQUE FOR $R_2 = 6, X_{20} = 10$	TORQUE FOR $R_2 = 10, X_{20} = 10$
0.01	0.99	0.49	0.16	0.09
0.02	1.92	0.99	0.33	0.199
0.05	4.0	2.35	0.82	0.49
0.1	5.0	4.0	1.62	0.99
0.15	4.6	4.8	2.35	1.46
0.2	4.0	5.0	3.0	1.92
0.3	3.0	4.61	4.0	2.75
0.4	2.35	4.0	4.61	3.45
0.5	1.92	3.45	4.92	4.0
0.6	1.62	3.0	5.0	4.41
0.9	1.09	2.11	4.61	4.97
1.0	0.99	1.92	4.41	5.0
0.7	1.23	2.35	4.8	4.87

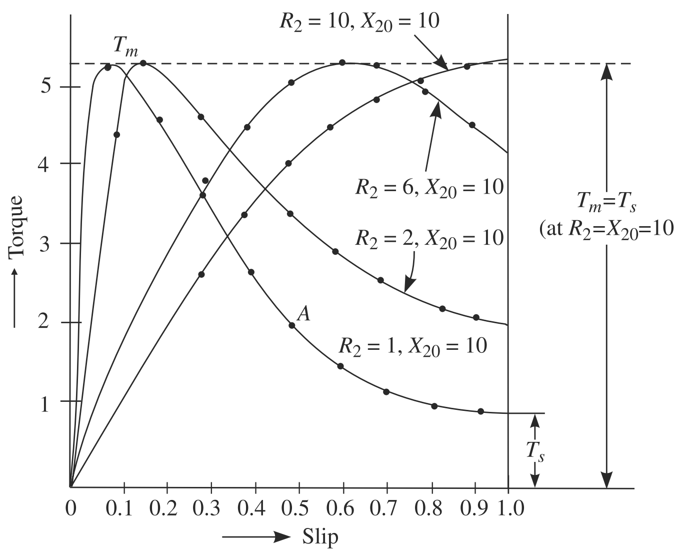


Fig. 4.11 *Effect of variation of rotor circuit resistance on torque slip characteristic of an induction motor*

From Fig. 4.11, the following observations can be made:

(a) Starting torque increases with increase in value of rotor resistance.

(b) Maximum torque remains constant and is independent of the value of rotor resistance.

(c) The slip at which maximum torque occurs varies with the variation of rotor resistance.

(d) Maximum torque is developed at starting when rotor resistance is equal to the standstill rotor reactance, i.e., when R_2 is equal to X_{20}.

(e) Torque is maximum when the rotor reactance $X_2 = (SX_{20})$ is equal to the rotor resistance R_2 (in graph A, for example, maximum torque occurs when $S = 0.1$. Thus the relationship $R_2 = SX_{20}$ holds good. For $R_2 = 10$, and $X_2 = 10$, maximum torque occurs when $S = 1$).

Let us now determine the ratio of starting torque and the maximum torque. We rewrite the torque equation as

$$T = \frac{KSR_2}{R_2^2 + S^2X_{20}^2}$$

The starting torque T_s is determined by taking the value of $S = 1$

$$T_s = \frac{KR_2}{R_2^2 + X_{20}^2}$$

The maximum torque T_m is determined by putting

$$S = \frac{R_2}{X_{20}} \text{ in the torque equation.}$$

$$T_m = \frac{K}{2X_{20}}$$

The ratio of T_s and T_m is found as

$$\frac{T_s}{T_m} = \frac{K\,R_2}{R_2^2 + X_{20}^2} \cdot \frac{2X_{20}}{K} = \frac{2R_2X_{20}}{R_2^2 + X_{20}^2} = \frac{2(R_2/X_{20})}{1 + (R_2/X_{20})^2}$$

Putting the ratio $\dfrac{R_2}{X_{20}} = a$

$$\frac{T_s}{T_m} = \frac{2a}{1 + a^2} \tag{4.23}$$

Let us again draw the complete torque-slip curve of an induction motor and mark the operating region, starting torque and pull-out torque.

The equation for torque of an induction motor is

$$T = \frac{KSE_{20}^2R_2}{R_2^2 + S^2X_{20}^2}$$

The value of X_{20} is higher than R_2. The value of S varies from a very low value to 1. At low value of slip, i.e., when slip is say 0.01 or 0.02, the term $S^2X_{20}^2$ becomes too small as compared to R_2^2 and hence can be neglected. Thus, torque becomes directly proportional to slip. At higher values of slip, $S^2X R_{20}^2$ is higher that R_2^2. In that case R_2^2 can be neglected while comparing with the term $S^2X_{20}^2$. Thus, torque becomes inversely proportional to slip. Thus,

$$T \propto S \text{ for low values of } S$$

and $$T \propto 1/S \text{ for high values of } S$$

The nature of torque slip characteristic can be drawn as shown in Fig. 4.12.

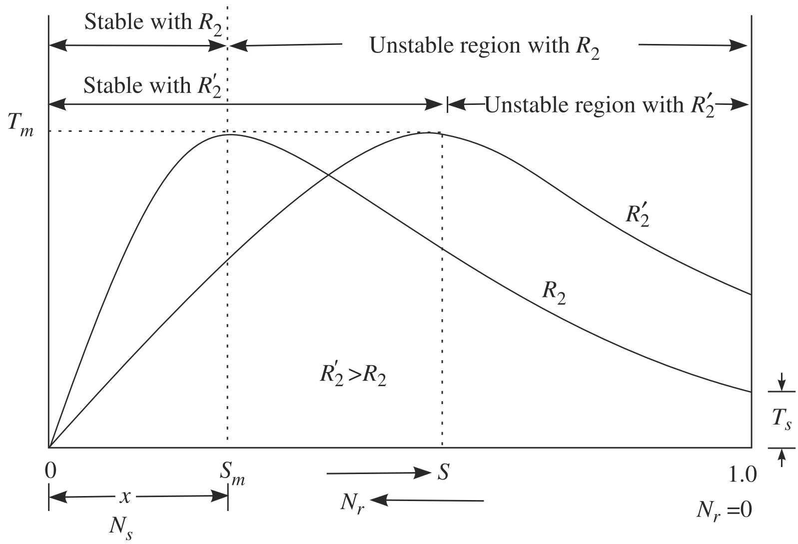


Fig. 4.12 *Torque-slip characteristics showing the stable and unstable region of operation of an induction motor*

For slip, $S = 1$, speed $N_r = 0$, torque corresponding to $S = 1$ is called the starting torque. Starting torque depends on the rotor resistance. It increases with the increase in rotor circuit resistance. The value of maximum torque T_m is equal to $k/2X_{20}$ and is constant. The slip S_m at which maximum torque occurs for different rotor circuit resistance, changes. Maximum torque is developed at starting when rotor circuit resistance equals X_{20}, i.e., stand-still rotor reactance.

The stable operating region of the motor has been indicated as a speed range denote by 'x' in the figure. The rotor will settle for any speed in this region depending upon the load on its shaft. The torque required by the load on the motor should be lower than the starting torque of the motor, otherwise the rotor will not accelerate. Once the motor starts rotating it will accelerate to higher speeds as the torque developed is higher than the torque required.

The motor torque developed and load torque requirement becomes equal at a speed N_r at which the motor will continue to rotate with the load on its shaft. If the load is reduced, the load characteristic line will come down and the crossing point of motor torque and load torque will change as shown in Fig. 4.13. The motor speed will somewhat increase. If the load on the motor is increased, the load characteristic line will move upwards, the crossing point will change, and the speed will decrease. If we continue to load the motor further, a time will come when load torque requirement will reach the maximum torque that the motor is able to develop. Any increase in load beyond this value will cause, sliding of the motor speed to zero. This torque, T_m is therefore called pull-out torque.

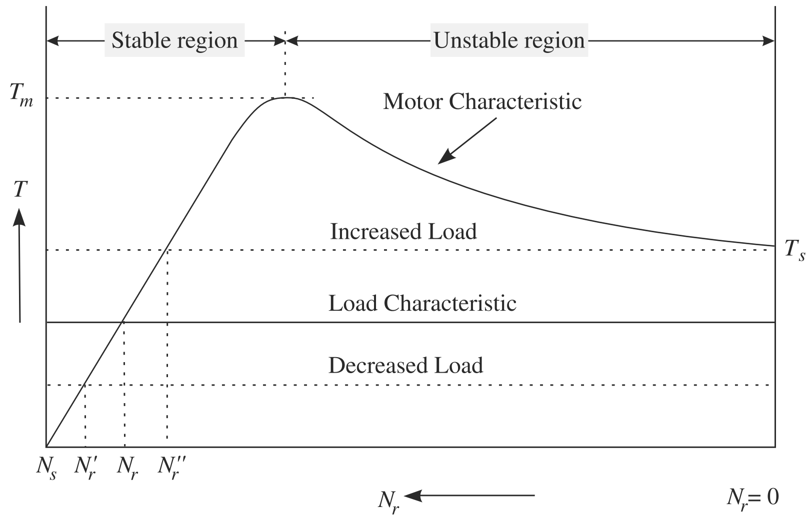


Fig. 4.13 *Effect of change of load on an induction motor*

4.9 CONDITION FOR MAXIMUM TORQUE AND THE VALUE OF MAXIMUM TORQUE

The condition for maximum torque can be derived mathematically by differentiating the expression for torque with respect to slip and equating to zero we get.

$$\frac{d}{ds}\frac{SR_2}{R_2^2 + S^2X_{20}^2} = \frac{(R_2^2 + S^2X_{20}^2)R_2 - SR_2 \times 2 \times SX_{20}^2}{(R_2^2 + S^2X_{20}^2)^2}$$

$$= \frac{R_2[R_2^2 + S^2X_{20}^2 - 2\,SX_{20}^2]}{(R_2^2 + S^2X_{20}^2)^2}$$

Equating this to zero we have,

$$R_2^2 - S^2X_{20}^2 = 0$$

or $$R_2 = SX_{20}$$

Thus, the condition for maximum torque is given by

$$R_2 = SX_{20} \tag{4.24}$$

which means torque is maximum when rotor resistance is equal to slip times rotor reactance under running condition.

The value of maximum torque is found out by substituting R_2 for SX_{20} in the torque equation as

Maximum torque $$T_m = \frac{KSSX_{20}}{S^2X_{20}^2 + S^2X_{20}^2} = \frac{K}{2X_{20}}$$

Since X_{20} is constant, the maximum torque is the same for any value of rotor resistance, which is evident from the torque-slip characteristics shown in Fig. 4.11.

EXAMPLE 4.12 *The power input to a three-phase induction motor is 50 kW and the corresponding stator losses are 2 kW. Calculate (a) the total mechanical power developed and the rotor 1^2R-loss when the slip is 3 per cent, (b) the output horse power of the motor if the friction and windage losses are 1.0 kW, and (c) efficiency of the motor.*

Solution

Input to rotor = Input to stator – Stator losses
= 50 – 2 = 48 kW

Power developed by the rotor = Rotor input – Rotor I^2 R-loss – Rotor core-loss.

Rotor I^2R-loss = Slip × Rotor input
= 0.03 × 48 = 1.44 kW

Rotor core-loss at 3 per cent slip is very small and can be neglected.

Thus,

Power developed by the rotor = 48 –1.44 = 46.56 kW

$$= \frac{46.56}{0.7355} \text{ hp} = 63.3 \text{ hp}$$

Output power = Power developed by the rotor – Friction and windage loss

= 46.56 – 1.0 = 45.56 kW

$$\text{Efficiency of the motor} = \frac{\text{Output power}}{\text{Input power}} \times 100$$

$$= \frac{45.56}{50} \times 100 = 91.1 \text{ per cent}$$

EXAMPLE 4.13 *A 20 hp, three-phase, 50 Hz, 4 pole induction motor has a full-load slip of 4 per cent. The friction and windage losses are 500 W. Calculate the rotor I^2R-loss and rotor speed.*

Solution

Output of the motor = 20 hp
= 20 × 735.5 W = 14710 W

Power developed by the rotor = Output of the motor + Friction & windage loss
= 14710 + 500 = 15210 W

Rotor input = Rotor I^2R-loss + Power developed by the rotor
= Slip × Rotor input + Power developed by the rotor

or (1 – S) Rotor input = Power developed by the rotor

Again,

Rotor I^2R-loss = S × Rotor input

or $$\text{Rotor input} = \frac{\text{Rotor } I^2R\text{-loss}}{S}$$

Substituting,

$$(1 - S)\ \frac{\text{Rotor } I^2R\text{-loss}}{S} = \text{Power developed by the rotor}$$

or $$\text{Rotor } I^2R\text{-loss} = \frac{S}{(1 - S)} \times \text{Power developed by the rotor}$$

In this case,

$$\text{Rotor } I^2R\text{-loss} = \frac{0.04}{(1 - 0.04)} \times 15210 = 633.75 \text{ W}$$

$$\text{Slip} = \frac{N_s - N_r}{N_s}$$

and $$N_s = \frac{120\,f}{P}$$

In this case,

$$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

and $$\text{Slip, } S = 0.04 = \frac{1500 - N_r}{1500}$$

or $$N_r = 1440 \text{ rpm}$$

EXAMPLE 4.14 *A three-phase, 6-pole, 50 Hz induction motor develops maximum torque at 940 rpm. The rotor resistance per phase is 0.1 Ω. What is the standstill rotor reactance?*

Solution Synchronous speed

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Slip, $$S = \frac{N_s - N_r}{N_s} = \frac{1000 - 940}{1000} = 0.06$$

Maximum torque occurs at a slip, S such that

$$R_2 = SX_{20}$$

or $$X_{20} = \frac{R_2}{S} = \frac{0.1}{0.06} = 1.66\ \Omega$$

EXAMPLE 4.15 *A 4 pole, three-phase, 50 Hz slip-ring type induction motor rotates at 1440 rpm with the slip-ring terminals short circuited. The rotor resistance and standstill reactance are 0.1 and 0.6 Ω respectively. If an extra resistance of 0.1 Ω per phase is added to the rotor circuit, what will be the new full-load speed?*

Solution Since the load on the motor is not changed, full-load torque of the motor will remain unchanged but the slip at which full-load torque will be developed with extra resistance in the rotor circuit will change. This is shown in Fig. 4.14. We are to calculate Nr' as shown in the figure.

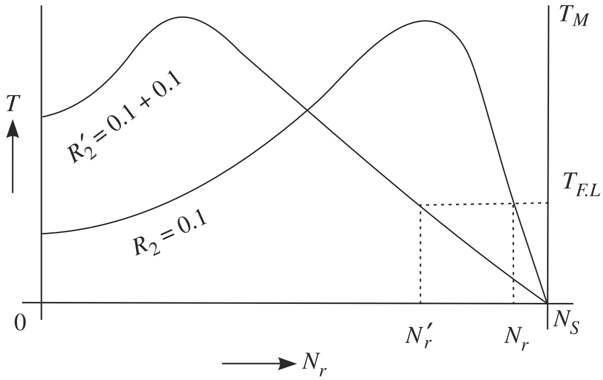


Fig. 4.14 *Refers to Example 4.15*

Synchronous speed, $N_S = \dfrac{120 f}{P}$

$$= \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Full-road slip with rotor resistance R_2, i.e.,

$$S_1 = \frac{1500 - 1440}{1500} = 0.04$$

Torque equations for two different values of rotor circuit resistance, R_2 and R_2' can respectively be expressed as:

$$T_1 = \frac{K S_1 R_2}{R_2^2 + S_1^2 X_{20}^2}$$

and

$$T_2 = \frac{K S_1 R_2'}{(R_2')^2 + S_2^2 X_{20}^2}$$

where S_1 and S_2 are the slip at which the motor will be running on full-load and with rotor circuit resistance, R_2 and R_2' respectively.

Since $T_1 = T_2 = T_{FL}$

we can write

$$\frac{S_1 R_2}{R_2^2 + S_1^2 X_{20}^2} = \frac{S_1 R_2'}{R_2'^2 + S_2^2 X_{20}^2}$$

Substituting the values we have,

$$\frac{0.04 \times 0.1}{0.1^2 + (0.04 \times 0.6)^2} = \frac{S_2 (0.1 + 0.1)}{0.2^2 + S_2^2 (0.6)^2}$$

or $$0.378 = \frac{0.2 \times S_2}{0.04 + 0.36S_2^2}$$

or $$S_2 = 0.08$$

To calculate N_r'

$$S_2 = \frac{N_s - N_r'}{N_s}$$

or $$0.08 = \frac{1500 - N_r'}{1500}$$

or $$N_r' = 1380 \text{ rpm}$$

EXAMPLE 4.16 *A particular three-phase, 4 pole, induction motor has rotor resistance of 0.04 Ω per phase. The maximum torque occurs at a speed of 1200 rpm. Calculate the starting torque as a percentage of maximum torque.*

Solution

$$\text{Synchronous speed} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Slip at maximum torque

$$S = \frac{1500 - 1200}{1500} = 0.2$$

From Eq. (4.22), the condition for maximum torque is

$$R_2 = SX_{20}$$

$$X_{20} = \frac{R_2}{S} = \frac{0.04}{0.2} = 0.2\ \Omega$$

Starting torque is the torque developed when $S = 1$,

$$T_{st} = \frac{KR_2}{R_2^2 + X_{20}^2} = \frac{K \times 0.04}{0.04^2 + 0.2^2} = 0.96 \text{ K}$$

Maximum torque, $$T_m = \frac{K}{2X_{20}} = \frac{K}{2 \times 0.2} = \frac{K}{0.4}$$

Thus, T_{st}, in terms of T_m can be expressed as

$$\frac{T_{st}}{T_m} = 0.96K \times \frac{0.4}{K}$$

or $$T_{st} = 0.384\, T_m$$

Therefore, starting torque is 38.4 per cent of the maximum torque.

EXAMPLE 4.17 *An 8-pole, 50 Hz induction motor has a full-load slip of 2.5 per cent and a maximum torque of twice the full-load torque. At what value of slip does the maximum torque occur?*

Solution

$$\text{Slip} = 2.5\ \% = 0.025$$

Condition for maximum torque is

$$R_2 = SX_{20}$$

or $$S = \frac{R_2}{X_{20}}$$

For the given condition,

$$T_m = 2T_{FL}$$

We know, $$T_m = \frac{K}{2X_{20}} \text{ and } T_{FL} = \frac{KSR_2}{R_2^2 + S^2X_{20}^2}$$

Substituting in (i),

$$\frac{K}{2X_{20}} = \frac{KSR_2}{R_2^2 + S^2X_{20}^2}$$

or $$4\ SR_2\ X_{20} = R_2^2 + S_2\ X_{20}^2$$

Dividing both sides by X_{20}^2,

$$\frac{4\,SR_2}{X_{20}} = \frac{R_2^2}{X_{20}^2} + S^2$$

or $$\left[\frac{R_2}{X_{20}}\right]^2 - 4S\left[\frac{R_2}{X_{20}}\right] + S^2 = 0$$

Putting the value of $S = 0.025$

$$\left[\frac{R_2}{X_{20}}\right] - 4 \times 0.025\left[\frac{R_2}{X_{20}}\right] + (0.025)^2 = 0$$

or $$\left[\frac{R_2}{X_{20}}\right]^2 - 0.1\left[\frac{R_2}{X_{20}}\right] + (6.25) \times 10^{-4} = 0$$

$$\frac{R_2}{X_{20}} = 0.093$$

Thus the value of slip at which the maximum torque occurs is 0.093.

EXAMPLE 4.18 *The rotor of a 4-pole, 50 Hz, 3-phase, slip-ring induction motor has a resistance of 0.25 Ω per phase and runs at 1440 rpm on full load. Calculate the external resistance per phase which must be added to lower the speed to 1200 rpm, the torque remaining constant in both the cases.*

Solution

$$N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{6} = 1500 \text{ rpm}$$

$$N_r = 1440 \text{ rpm}$$

$$S_1 = \frac{N_s - N_r}{N_s} = \frac{1500 - 1440}{1500} = 0.04$$

Rotor circuit resistance is 0.25 Ω

In the second case,

$$S_2 = \frac{1500 - 1200}{1500} = 0.2$$

Now, let the rotor circuit resistance is R_2'

The expression for torque is

$$T = \frac{KSR_2}{R_2^2 + S^2 X_{20}^2}$$

For low value of slip, S^2 is small and, therefore,

$$T \approx \frac{KSR_2}{R_2^2} = \frac{KS}{R_2}$$

In the first case torque, $T_1 = \dfrac{K S_1}{R_2}$

In the second case torque, $T_2 = \dfrac{K S_2}{R_2'}$

Since torque remains constant, $T_1 = T_2$

Therefore, $R_2' = R_2 \times \dfrac{S_2}{S_1}$

Substituting $R_2' = 0.25 \times \dfrac{0.2}{0.04} = 1.25\ \Omega$

Extra resistance to be connected in the motor circuit $= R_2' - R_2$

$= 1.25 - 0.25 = 1.0\ \Omega$

EXAMPLE 4.19 *A 6-pole, 50 Hz, 3-phase induction motor has rotor resistance and reactance per phase of 0.02 Ω and 0.1 Ω respectively. At what speed is the torque maximum? What must be the value of the external rotor resistance per phase to produce two-third of the maximum torque at starting?*

Solution From the condition for maximum torque we have,

$$R_2 = SX_{20}$$

or $$S = \frac{R_2}{X_{20}} = \frac{0.02}{0.1} = 0.2$$

And $$N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Substituting the values in $S = \dfrac{N_s - N_r}{N_s}$,

$$0.2 = \frac{1000 - N_r}{1000}$$

$$N_r = 1000 - 200 = 800 \text{ rpm}$$

Torque, $$T = \frac{KSR_2}{R_2^2 + S^2 X_{20}^2}$$

At starting, $S = 1$

$\therefore$ Starting torque, $$T_{st} = \frac{KR_2}{R_2^2 + X_{20}^2}$$

Maximum torque, $$T_m = \frac{K}{2X_{20}}$$

Here, $$T_{st} = \frac{2}{3} T_m$$

or $$\frac{KR_2}{R_2^2 + X_{20}^2} = \frac{K}{3X_{20}}$$

Substituting the value of $X_{20} = 0.1$, we have

$$\frac{R_2}{R_2^2 + (0.1)^2} = \frac{1}{3 \times 0.1}$$

or $$0.3\,R_2 = R_2^2 + 0.01$$

or $$R_2^2 - 0.3\,R_2 + 0.01 = 0$$

or $$100\,R_2^2 - 30R_2 + 1 = 0$$

Calculating $R_2 = 0.262$ or 0.038

This R_2 is the value of the total rotor circuit resistance.

Rotor winding resistance/phase = 0.02

Therefore, extra resistance = 0.262 – 0.02 = 0.242 Ω

or 0.038 – 0.02 = 0.018 Ω

EXAMPLE 4.20 *A 20 hp. 4-pole, three-phase induction motor has friction and windage losses of 2 per cent of the output. The full-load slip is 3 per cent. Calculate for full-load (a) the rotor I^2R-loss; (b) the rotor input; and (c) the output torque.*

Solution Motor shaft output = 20 hp = 20 × 735.5 W = 14710 W

Friction and windage loss = 0.02 × 14710 W = 294.2 W

Power developed by the rotor = 14710 + 294.2 W = 15004.2 W

$$\text{Rotor } I^2R\text{-loss} = \frac{S}{(1-S)} \times \text{Power developed by the rotor}$$

$$= \frac{0.03}{1 - 0.03} \times 15004.2 \text{ W} = 464 \text{ W}$$

Rotor input = Power developed by the rotor + Rotor I^2R-loss + Rotor iron-loss

Rotor iron-loss at 3 per cent slip is very small and can be neglected.

Therefore, rotor input = 15004.2 + 464 W = 15468.2 W

$$\text{Output torque,} = \frac{\text{Shaft output in watts}}{2\pi N_r}$$

$$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S = 0.03 = \frac{1500 - N_r}{1500}$$

or $N_r = 1455$ rpm

$$= \frac{1455}{60} \text{ rpm} = 24.25 \text{ rpm}$$

Output in Watts = $2\pi T N_r$, (in rpm), where T = Torque in Nm

Substituting the values,

$$T = \frac{14710}{2 \times 3.14 \times 24.25} = 96.59 \text{ Nm}$$

EXAMPLE 4.21 *The power input to a 500 V, 50 Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW and friction and windage losses total 2 kW. Calculate (a) the slip, (b) the rotor copper loss; (c) the output horse-power; (d) the efficiency.*

Solution

Synchronous speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000$ rpm

Rotor speed, $N_r = 975$ rpm

Slip, $S = \frac{N_s - N_r}{N_s} = \frac{1000 - 975}{1000} = 0.025$

Power input to stator = 40 kW

Stator losses = 1 kW

Output from stator = Input to rotor

Therefore, Rotor input = Stator input – Stator losses

= 40 – 1 = 39 kW

Rotor copper losses = Slip × Rotor input

= 0.025 × 39 = 0.975 kW

Rotor output, i.e., power obtained from the rotor shaft, P = Rotor input – Rotor copper loss – frictional windage losses (assuming iron loss in the rotor core as negligible)

or $P = 39 - 0.975 - 2 = 36.025$ kW

$$\text{Output horse-power} = \frac{36025}{735.5} = 48.98$$

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}} \times 100$$

$$= \frac{36025 \times 100}{40000} = 90\%$$

EXAMPLE 4.22 *The power input to the rotor of a 440 V, 50 Hz, 6-pole, 3-phase induction motor is 80 kW. The rotor electromotive force is observed to make 120 alternations per minute. Calculate (a) the slip; (b) the rotor speed; (c) the mechanical power developed; (d) the rotor copper loss per phase; (e) the rotor resistance per phase if the rotor current is 60 A.*

Solution

Synchronous speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$

Rotor frequency, $f_r = \frac{120}{60} = 2 \text{ Hz}$

$$f_r = \text{slip} \times f$$

or $\text{slip} = \frac{f_r}{f} = \frac{2}{50} = 0.04$

Again, $S = \frac{N_s - N_r}{N_s}$

Therefore, $0.04 = \frac{1000 - N_r}{1000}$

$$N_r = 960 \text{ rpm}$$

$$\text{Rotor input} = 80 \text{ kW}$$

$$\text{Rotor copper loss} = \text{Slip} \times \text{Rotor input}$$

$$= 0.04 \times 80 = 3.2 \text{ kW}$$

$$\text{Rotor copper loss per phase} = \frac{3.2}{3} = 1.067 \text{ kW} = 1067 \text{ W}$$

Mechanical power developed = Rotor input – Copper loss in rotor winding
– Iron loss in rotor core
(see Fig. 4.9)

Therefore, mechanical power developed = 80000 – 3200

$$= 76800 \text{ W} = \frac{76800}{735.5} \text{ hp}$$

$$= 104.4 \text{ hp}$$

Rotor current, $I_r = 60 \text{ A}$

Let rotor resistance per phase is, R_2

Rotor copper loss per phase = $I_r^2 R_2$

$$R_2 = \frac{\text{Rotor copper loss per phase}}{I_r^2}$$

$$= \frac{1067}{(60)^2} = 0.296\ \Omega$$

4.10 STARTING TORQUE

From the torque-speed characteristics (Fig. 4.11), it is observed that with a motor having low rotor-circuit resistance, the starting torque is small as compared to the maximum torque developed. Starting torque is increased if the rotor circuit resistance is increased. Although increased rotor resistance increases the starting torque, it reduces the efficiency due to high I^2R-loss. For achieving both high starting torque and high efficiency, the motor should have high rotor-circuit resistance at starting and low rotor-circuit resistance under running condition. In slip-ring motors this is achieved by inserting extra resistance in the rotor circuit at starting and by gradually reducing this resistance as the motor accelerates. In squirrel cage type motors, high starting torque is achieved by using double-cage rotors.

4.10.1 Achieving High Starting Torque in Slip-ring Type Motors

For achieving high starting torque, an extra starting resistance is connected across the slip-ring terminals of the rotor while starting. The connection diagram is shown in Fig. 4.15.

The starting procedure is to switch-on the supply to the stator and gradually move the rotor starting resistance arms in the clockwise direction from position of START to position of RUN. At START position, full extra resistance gets connected to the rotor circuit and at RUN position the full resistance is cut out of the rotor circuit and thereby the rotor circuit gets closed in itself. The starting resistance is required to be used for a short time only and, therefore, can be of short-time rating. If this resistance is also to be used for control of speed by keeping it in the circuit under running condition, the resistance should be of full-time rating.

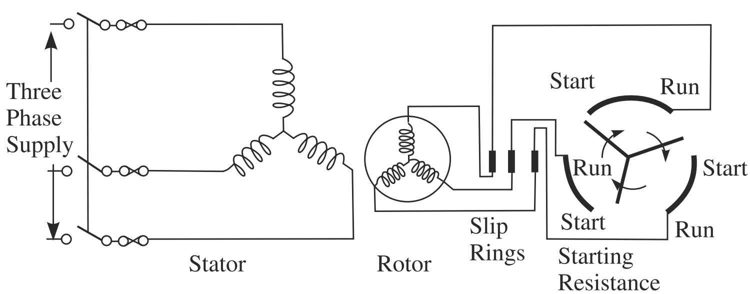


Fig. 4.15 *Starting resistance used in slip-ring type induction motor rotor circuit for obtaining high starting torque*

4.10.2 Achieving High Starting Torque in Squirrel-cage Type Motors

High starting torque in squirrel-cage motors can be achieved by using double-cage rotors. In a double-cage rotor there are two separate squirrel cages on the rotor as shown in Fig. 4.14(a). The outer cage can be of high resistance alloy, such as brass and the inner one can be of copper. The inner cage conductors are situated deep inside the core so that they are enclosed by more iron. At the moment of start, the frequency of the rotor induced emf and hence that of the current is the same as the stator supply frequency. The reactance of the lower cage will be much higher than the outer cage.

This can be understood from the following explanation: As the lower cage conductors are placed deep inside the rotor, there will be more magnetic flux around them as compared to the upper cage conductors. This has been shown in Fig. 4.14(b). Inductance, L ($L \propto d\phi di$) and hence the inductive reactance ($2\,\pi f L$) of the lower cage will be higher than that of the upper cage. Thus the impedance offered by the lower cage to the current flow will be very high as compared to the upper cage. Most of the current will, therefore, flow through the upper cage conductors. Since upper cage conductors are of high resistance material, the effective rotor circuit resistance at start will be high. Starting torque developed, therefore, will be high. As the motor picks up speed and attains nearly its rated speed, the frequency of the rotor induced emf will be very small, may be one or two Hz. The reactance will have very little effect on the current flow. Current flow in the rotor cage windings will depend mainly on the rotor circuit resistance.

As the lower cage conductors have low resistance, a shift of current from the upper cage to the lower cage will occur once the motor starts picking up speed. Under running condition, most of the rotor current will flow through the lower cage conductors. Thus, the I^2R-loss in the rotor circuit under running condition will not be high and, therefore, will not adversely affect the motor efficiency.

The design of the two cages can be made as shown in Figs 4.16(b)) and 4.16(c) respectively. If the number of slots for the upper cage and the lower cage is made the same, the width of the iron at the roots would be very small. It is, therefore, a common practice to have half as many number of slots for the inner cage than the outer cage as shown in Fig. 4.16(c).

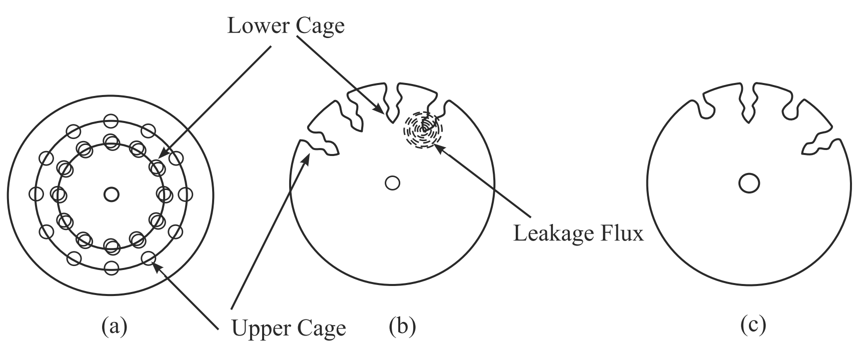


Fig. 4.16 *Laminations of double-cage rotor of different designs*

Torque-speed characteristic of the two cages if considered separately will be different. The shape of torque-speed characteristic of a double-cage induction motor will be similar to the curve 3 of Fig. 4.17. The shape of the resultant characteristic can be modified by proper design of the two cages. The starting current drawn by the motor will be comparatively small because of the high rotor impedance at starting.

The advantages of a double-cage induction motor are:

(a) high starting torque and comparatively low I^2R-loss under running condition, and

(b) possibility of starting the motor direct-on-line, because of comparatively small starting current.

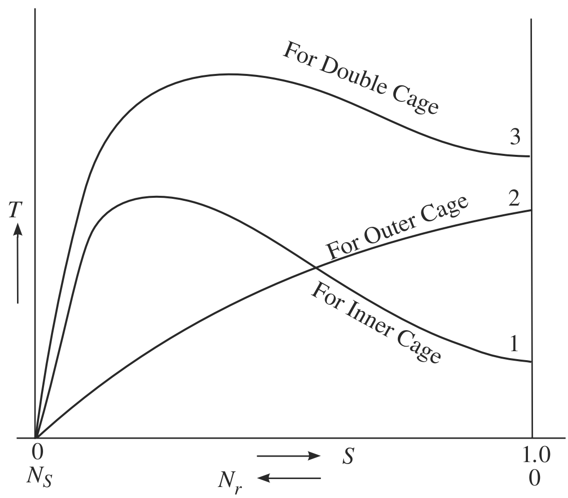


Fig. 4.17 *Torque-speed characteristics of double-cage induction motors*

EXAMPLE 4.23 *A three-phase wound rotor induction motor has rotor circuit resistance and standstill reactance per phase of 0.05 ohm and 0.4 ohm respectively. If the rotor circuit resistance is varied, the slip at which maximum torque occurs changes. You are required to determine the value of additional resistance to be introduced in the rotor circuit so that maximum torque occurs at starting.*

Solution

We have from (4.23), $\dfrac{T_s}{T_m} = \dfrac{2a}{1+a^2}$ where $a = \dfrac{R_2}{X_{20}}$

Since T_m occurs at starting, we can put

$$T_m = T_s$$

Therefore, $$\frac{T_s}{T_m} = 1 = \frac{2a}{1+a^2}$$

or, $$a_2 + 1 - 2a = 0$$

or, $$a = 1$$

when we put an extra resistance, r in the rotor circuit, total rotor circuit resistance becomes equal to $R_2 + r$.

Thus,

$$a = 1 = \frac{R_2 + r}{X_2}$$

Putting values, $$1 = \frac{0.05 + r}{0.4}$$

or, $$r = 0.4 - 0.05 = 0.35\ \Omega$$

EXAMPLE 4.24 *A 400 V, 50 Hz, 6 pole, 3-phase induction motor has rotor resistance of 0.03 ohm and standstill rotor reactance per phase of 0.4 ohm. Calculate the speed of the motor when developing maximum torque and also calculate the ratio of maximum torque to full-load torque. The full-load speed is 960 rpm.*

Solution

Synchronous speed, $$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Corresponding ship, $$S = \frac{N_s - N_r}{N_s} = \frac{1000 - 960}{1000} = 0.04$$

$$T = \frac{K S R_2}{R_2{}^2 + S^2 X_{20}^2}$$

$$T_m \text{ occurs at } S = \frac{R_2}{X_{20}}$$

Therefore, $$T_m = \frac{K \dfrac{R_2}{X_{20}} \cdot R_2}{R_2^2 + \dfrac{R_2^2\, X_{20}^2}{X_{20}^2}} = \frac{K \dfrac{R_2^2}{X_{20}}}{R_2^2 + R_2^2} = \frac{K}{2X_{20}}$$

$$\frac{T_m}{T} = \frac{K}{2X_{20}} \times \frac{R_2^2 + S^2 X_{20}^2}{KSR_2} = \frac{R_2^2 + S^2 X_{20}^2}{2SR_2 X_{20}} = \frac{\dfrac{R_2^2}{X_{20}^2} + S^2}{2\dfrac{R_2}{X_{20}} S}$$

Putting $\dfrac{R_2}{X_{20}} = a$,

$$\frac{T_m}{T} = \frac{a^2 + s^2}{2as}$$

$$a = \frac{R_2}{X_{20}} = \frac{0.03}{0.4} = 0.075$$

$$\frac{T_m}{T} = \frac{a^2 + s^2}{2as} = \frac{(0.075)^2 + (0.04)^2}{2 \times 0.075 \times 0.04} = 1.2$$

Slip at maximum torque,

$$S_m = \frac{R_2}{X_{20}} = \frac{0.03}{0.4} = 0.075$$

$$\begin{aligned} \text{Corresponding speed} &= (1 - S_m)\, N_s \\ &= (1 - 0.075)\, 1000 \\ &= 925 \text{ rpm} \end{aligned}$$

EXAMPLE 4.25 *A 400 V, 50Hz, 4 pole, three-phase induction motor takes 6 times its full-load current of 30 A at starting and has a full-load slip of 4 percent. What must be the voltage applied at starting so that starting torque equals full-load torque? What will then be the starting current?*

Solution

From Eq. (4.24), $$\frac{T_s}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 s_f = \left(\frac{6I_f}{I_f}\right)^2 \times 0.04 = 1.44$$

or, $T_s = 1.44\, T_f$.

1.44 times of full-load torque is developed at starting when a full-voltage of 400 V is applied. Since torque is proportional to square of the voltage, we can write, 1.44 T_f is produced when input voltage is 400 V. $T_s = T_f$ will be produced when input voltage is,

$$V_{in} = \sqrt{\frac{400^2}{1.44}} = \sqrt{\left(\frac{400}{1.2}\right)^2} = 333.3 \text{ volts}$$

Full-load current at 333.3 volts is

$$I_s = 6I_f\left(\frac{333.3}{400}\right) = 150.5 \text{ A}$$

4.11 STARTING OF INDUCTION MOTORS

Induction motors if started with full voltage take about six to eight times its rated full-load current at the time of starting, This heavy current although may not be dangerous for the motor because of the short duration of time during which such a large current flows through the motor windings, will cause a large drop in the line-voltage supplying the motor. Other machines connected to the supply system will be adversely affected. It is therefore recommended that large three-phase induction motors be started with reduced voltage applied across the stator terminals at starting. Small motors below 5 hp ratings may however be started direct-on-line (DOL). Various methods of starting three-phase squirrel-cage induction motor are described as follows.

4.11.1 Full Voltage or Direct-on-line Starting

When full voltage is connected across the stator terminals of an induction motor, large current is drawn by the windings. This is because, at starting (i.e., before the rotor starts rotating) the induction motor behaves as a short circuited transformer. The short circuited, i.e., secondary, i.e., the rotor is separated from the primary, i.e., the stator by a small air-gap.

At starting, when the rotor is at standstill, emf is induced in the rotor circuit exactly similar to the emf induced in the secondary winding of a transformer. This induced emf of the rotor will circulate a very large current through its windings. The primary will draw very large current from the supply mains to balance the rotor ampere-turns. To limit the stator and rotor currents at starting to a safe value, it may be necessary to reduce the stator supply voltage to a lower value.

With full voltage applied across stator terminals, the starting current will be very high. This current will however be gradually decreasing as the motor will pick up speed. The current drawn by the motor during the time of starting with full-voltage connected across the stator terminals is shown in Fig. 4.18.

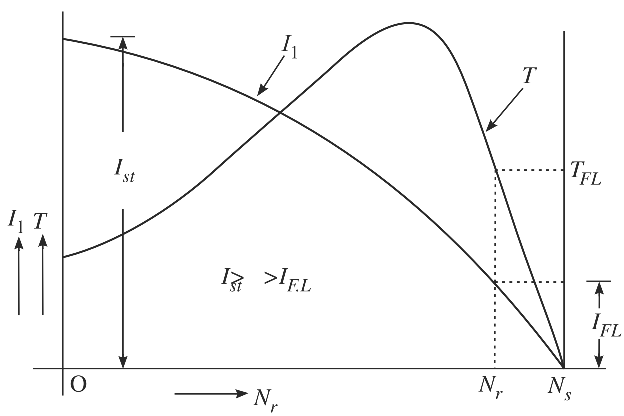


Fig. 4.18 *Current drawn by an induction motor when started direct-on-line*

If induction motors are started direct-on-line, such a heavy starting current of short duration may not cause harm to the motor since the construction of induction motors are rugged, i.e., not so delicate. Moreover, it takes time for intolerable temperature rise to endanger the insulation of the motor windings. But this heavy inrush of current will cause a large voltage drop in the lines leading to the motor. Other motors and equipment connected to the supply lines will receive reduced voltage. The reader must have observed that the lights in his home dim momentarily at the instant the refrigerator motor starts. The situation in this case may not be so objectionable as the motor used in the refrigerator is of a small size. In industrial installations, however, if a number of large motors are started direct-on-line, the voltage drop will be very high and may be really objectionable for the other types of loads connected to the system. The amount of voltage drop will not only be dependent on the size of the motor but also on factors like the capacity of the power supply system, the size and length of the line leading to the motors, etc. Indian Electricity Rule restricts direct-on-line starting of three-phase induction motors upto a maximum of 5 hp rating. For motors of higher ratings, methods are to be found out to apply reduced voltage at starting across the windings, such that the motor starting current is reduced considerably.

Direct-on-line method of starting of induction motors, applicable up to a rating of 5 hp, is shown in Fig. 4.19. In the circuit in addition to fuses, thermal overload relay has been used to protect the motor windings against over-load. When the ON push-button is pressed, the contactor coil *A* becomes energised and its open contacts are closed. The motor gets connected across the supply mains through the main contacts of the contactor. The motor continues to get supply even when the pressure on the push-button is released, since the contactor coil will then get supply through the contact *a* of the contactor. Contact *a* of the contactor *A* is therefore called the hold-on contact. When the OFF push-button is pressed the coil gets de-energised; the main contacts of the contactor opens and the motor stops. In case of over-load on the motor, the contact *e* of the overload relay (OLR) will open, and subsequently the motor will stop. Fuses are provided for short-circuit protection. For the sake of understanding, the constructional details of an electromagnetic type contactor used in motor control circuits has been shown in Fig. 4.20. When the contactor coil is energised the armature is pulled against the spring pressure. The contacts of the contactor closes. When deenergised, the contacts return to their original position.

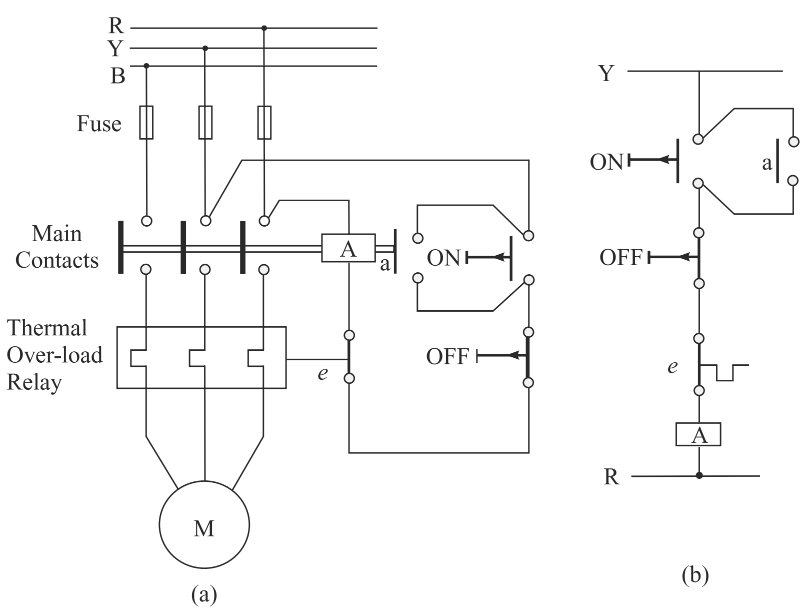


Fig. 4.19 *Push button operated direct-on-line starting of a three-phase induction motor (a) Complete wiring diagram (b) Schematic diagram of the control circuit*

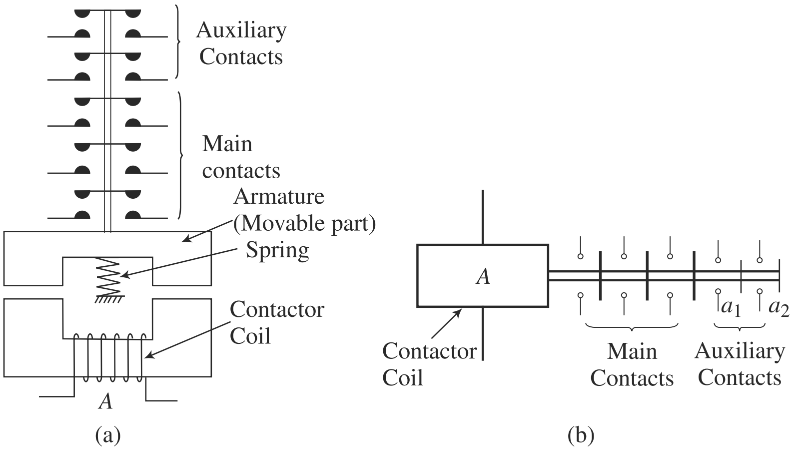


Fig. 4.20 *An electromagnetic type contactor, (a) Constructional details; (b) Symbolic representation*

4.11.2 Reduced Voltage Starting

Reduced voltage can be applied across the stator circuit by any of the following ways: (i) by use of an auto-transformer or (ii) by connecting resistors or inductors in series with the stator winding or (iii) by connecting the stator windings at the time of starting in star and then in delta (star-delta method of starting). These methods are described as follows:

Auto-transformer Starting An auto-transformer starter consists of an auto-transformer and a switch as shown in Fig. 4.21. When the switch *S* is put on START position, a reduced voltage is applied across the motor terminals. When the motor picks up speed, say to 80 per cent of its normal speed, the switch is put to RUN position. Then the auto-transformer is cut out of the circuit and full rated voltage gets applied across the motor terminals. The circuit diagram in Fig. 4.21 is for a manual auto-transformer starter. This can be made push-button operated automatic controlled starter so that the contacts switch over from start to run position when the motor speed picks up to 80% of its speed. Over-load protection relay has not been shown in the figure. The switch *S* is air-break type for small motors and oil-break type for large motors. Auto-transformer may have more than one tapping to enable the user to select any suitable starting voltage depending upon the conditions.

Series Resistors and Reactors Series resistors or reactors can be used to cause voltage drop in them and thereby allow low voltage to be applied across the motor terminals at starting. These are cut out of the circuit as the motor picks up speed.

Star-delta Method of Starting The stator phase windings are first connected in star and full voltage is connected across its free terminals. As the motor picks up speed, the windings are disconnected through a switch and they are re-connected in delta across the supply terminals. The current drawn by the motor from the lines is reduced to 1/3 as compared to the current it would have drawn if connected in delta.

From Fig. 4.22 it is seen that for star-connection of windings, phase current is equal to line current, i.e.,

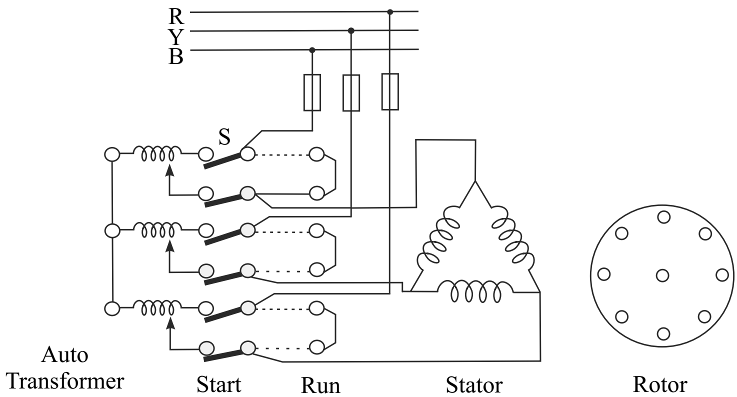


Fig. 4.21 *A manual auto-transformer starter for an induction motor*

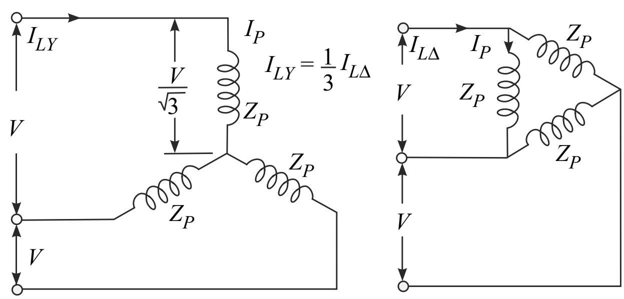


Fig. 4.22 *Comparison of current drawn from the lines when windings are star connected and delta connected*

$$I_p = I_{LY} = \frac{V_p}{Z_p} = \frac{V}{\sqrt{3}Z_p}$$

where I_p is the phase current, I_{LY} is the line current when windings are star-connected and Z_p is the winding impedance per phase, V_p is the phase voltage and V is the line voltage.

For delta-connection of windings, $V_p = V$. So,

$$I_P = \frac{V}{Z_p} \text{ and } I_{L\Delta} = \sqrt{3}\, I_P = \sqrt{3}\frac{V}{Z_p}$$

The ratio of line currents drawn in star and delta-connection is therefore,

$$\frac{I_{LY}}{I_{L\Delta}} = \frac{V}{\sqrt{3}\,Z_p}\frac{Z_p}{\sqrt{3}\,V} = \frac{1}{3}$$

or
$$I_{LY} = \frac{1}{3} I_{L\Delta} \tag{4.25}$$

Thus by connecting the motor windings, first in-star and then in-delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting with the windings delta connected.

From expression (4.18), it is known that torque developed by an induction motor is proportional to the square of the applied voltage. In star connection of windings, the phase voltage is $1/\sqrt{3}$ times the line voltage. So, the starting torque will be reduced to one-third. A simple manual star-delta starter is shown in Fig. 4.23(a).

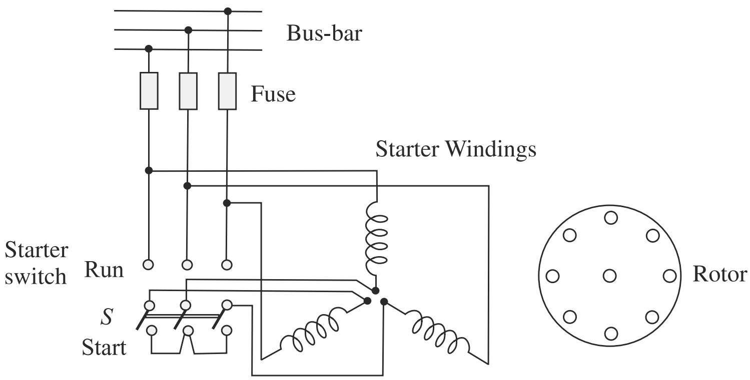


Fig 4.23 (a) *A manual star-delta starter for an induction motor*

In making connections for star delta starting, care should be taken such that sequence of supply connections to the winding terminals does not change while changing from star-connection to delta-connection. Otherwise the motor will start rotating in the opposite direction, when connections are changed from star to delta. Start-delta starters are available for manual operation using push-button control. An automatic star-delta starter uses time-delay relays (TDR) through which star to delta connections take place automatically with some pre-fixed time delay. The delay time of the TDR is set keeping in view the starting time of the motor.

Manual Star-delta Starter (Push Button Type) Figure 4.23(b) shows the connection diagram of a manual star-delta starter (push-button type). When push button *P* is pressed and the handle of starter switch *S* is brought to *Y* position the contactor coil A gets energised. The stator windings terminals get star connected and receive three-phase supply. The motor starts rotating with the stator windings star connected.

The starting current drawn from the lines will be only one-third of the current which the motor would have drawn if the windings were delta connected during starting. After the rotor has picked up speed the current drawn by the motor automatically gets reduced and, therefore, the windings are connected in delta configuration by turning the switch *S* to Δ position with the help of its handle. The pressure on the push-botton *P* can now be released. The contactor coil *A* remains energised by getting supply through the hold-on contact 'a' of the contactor *A*.

The thermal overload relay contact, *e* remains connected in series with the contactor coil for all the time of motor operation. In the event of any persistent overload on the motor the thermal over-load relay will operate, its contact *e* will open and the contactor will get de-energised thereby switching off the motor. For stopping the motor, the stop push-button Q has to be pressed.

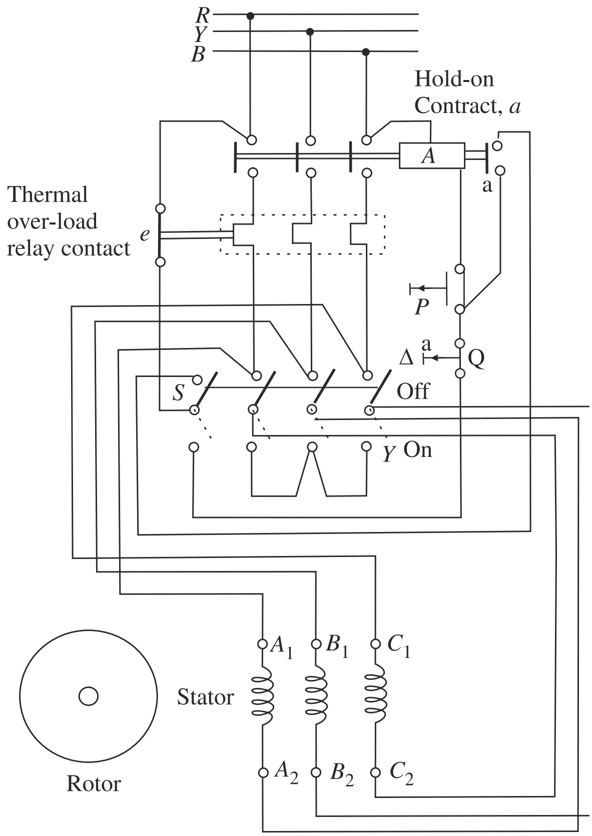


Fig. 4.23 (b) *Wiring diagram of a manual star delta starter*

Semi-automatic Star-delta Starter for a Three-phase Induction Motor In the semi-automatic star-delta starter the time delay from star to delta connection of the stator windings is achieved by keeping the start push-button pressed. The change-over from star to delta takes place at the instant the start push-button is released. Pressing the start button again will not have any effect as long as the starter is on.

The schematic diagram for the control circuit has been shown in Fig. 4.23(c) (1). The power circuit diagram has been shown in Fig. 4.23(c) (2). In all, three contactors have been used. When start push button is pressed contactor coil A gets energised. The motor winding terminals get connected in star.

Since contact a_1 of push-button A is now closed, contactor B will be energised and three phase supply will be available across the motor terminals (see Fig. 4.23 (c-1)). The motor will continue to run with the stator terminals star connected. The start push-button is kept pressed for the motor to pick up sufficient speed. When the start push button pressure is released contact A will get deenergised. Contactor B will remain energised as its coil will continue to receive supply through its own contact b_1

which is now closed. Contactor *C* will also become energised and connect the stator terminals in delta. When the motor is running with its stator terminals connected in delta it will not be possible to energise the contactor *A* by pressing the start push-button since contact *c* of contactor *C* is now open. Therefore, pressing of the start push-button again will have no effect once the motor is in operation.

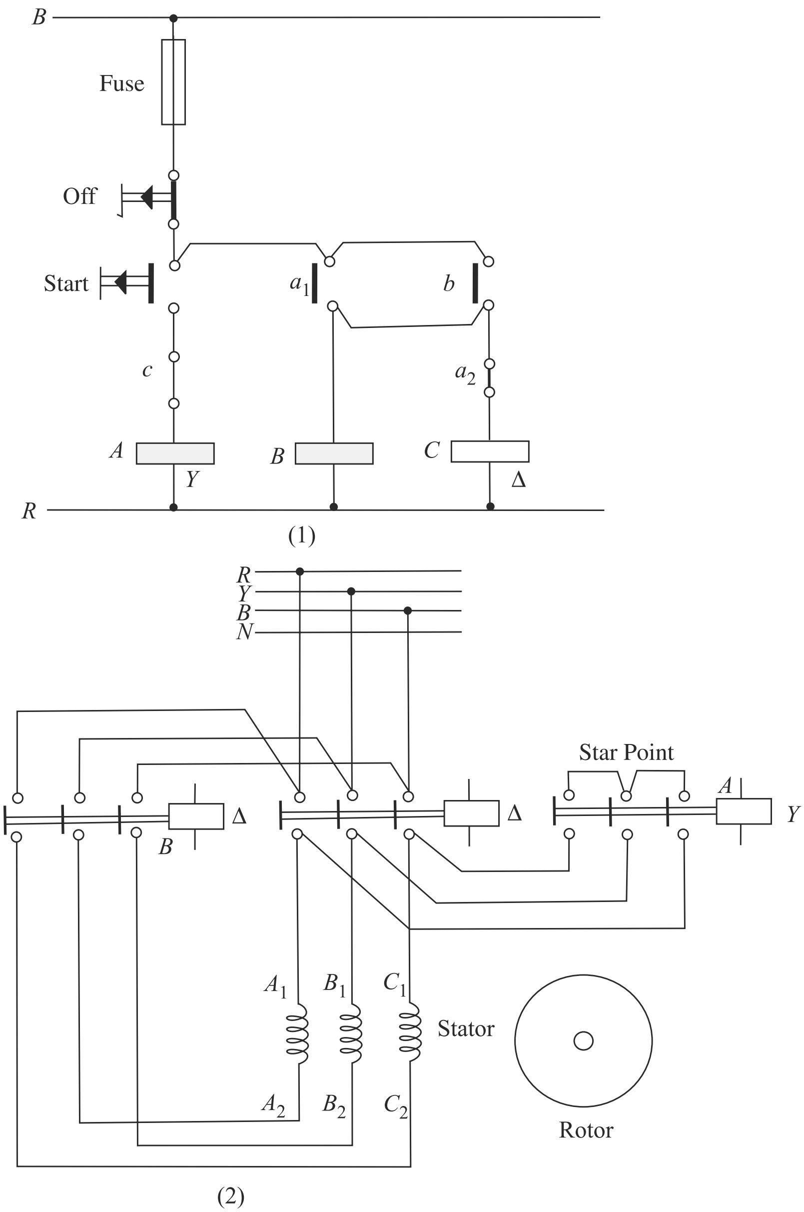


Fig. 4.23 (c) *Semi-automatic star-delta starter (1) Control circuit diagram (2) Power circuit diagram*

Automatic Star-delta Starter for a Three-phase Induction Motor The time delay in any automatic star-delta starter, before changing over from the star to delta connections should be sufficient to allow the motor to come up to its normal running speed. This period may be taken as 10 secs, but could be less for a lightly loaded motor and greater for a slow starting or heavily loaded motor.

In an automatic star-delta starter, this delay is obtained by using a timer. The time delay can be adjusted by rotating the dial screw on the timer, clockwise or anticlockwise for decreasing or increasing the time delay.

The power circuit diagram is similar to the circuit shown in Fig. 4.23(c) (2).The schematic diagram for the control circuit has been shown in Fig. 4.23(d).

When the START push button is pressed contactor S gets energised connecting the stator terminals in star. The windings get three phase supply through the contactor M which is now energised due to the closing of contact s_1 of contactor S. The motor will start rotating with its stator windings star connected. When the motor picks up normal speed (say in 10 secs) the time delay contact t_1 of the TDR will open and de-energise the contactor S. Contact S_2 of contactor S will now come to close condition and, therefore, contactor D will get energised connecting the winding terminals in delta across the supply. For interlocking between the two contactors S and D, one *NC* contact of S, i.e., s_2 has been connected in series with contactor D and one *NC* contact of D, i.e., d_1 has been connected in series with contactor S.

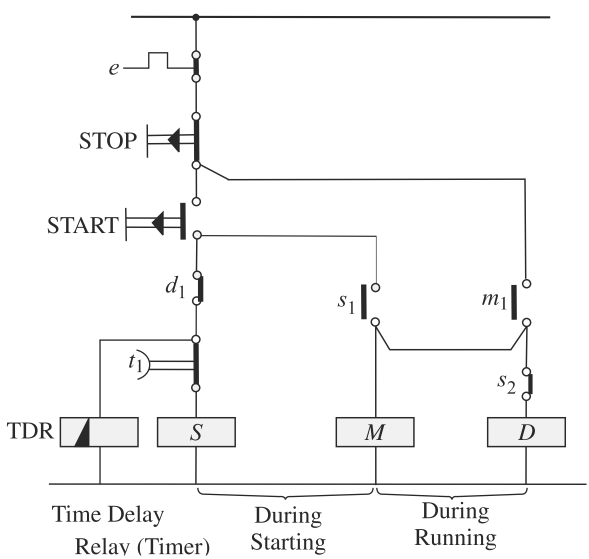


Fig. 4.23 (d) *Automatic star-delta starter—schematic diagram of the control circuit*

While designing the circuit for a star-delta starter care should be taken that the direction of rotation of the motor remains the same while the change in connections of the stator windings occur from star to delta.

EXAMPLE 4.26 *A 400 V, 50 Hz, three-phase induction motor takes a starting current of 75 A and develops a starting torque of 1.5 times its full-load torque when* *full voltage is supplied across the winding terminals connected in delta. Calculate the starting line current and starting torque if the motor windings are connected in star and full voltage applied across its terminals.*

Solution

Current taken when delta-connected = 75 A

Current taken when star-connected $= \dfrac{75}{3} = 25$ A

Torque T is proportional to square of voltage. Voltage applied per phase when star-connected is $1/\sqrt{3}$ times the voltage applied when windings are delta-connected. Since torque is proportional to square of the applied voltage, starting torque with windings star-connected will be one-third of the starting torque with windings delta-connected.

Starting torque with windings delta-connected = $1.5 \times T_{FL}$

Starting torque with windings star-connected $= \dfrac{1.5 \times T_{FL}}{3} = 0.5 \times T_{FL}$

EXAMPLE 4.27 *The ratio of maximum torque to full-load torque in a 3-phase squirrel cage induction motor is 2.5. Calculate the ratio of starting torque to full-load torque for (a) direct-on-line starting; (b) star-delta starting; and (c) auto-transformer starting with tapping at 75 per cent.*

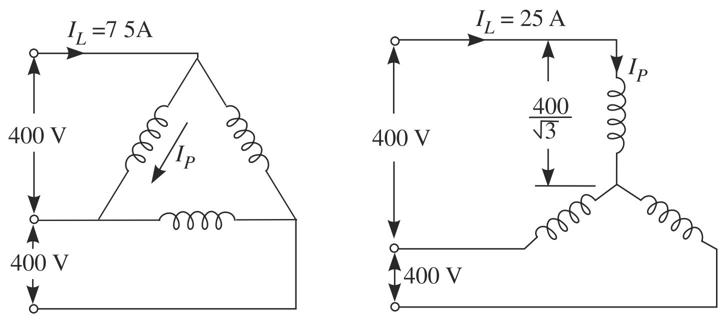


Fig. 4.24 *Refers to Example 4.26*

Solution The rotor resistance is 0.5 Ω and rotor standstill reactance is 5 Ω

$$T = \frac{KSE_{20}^2 R_2}{R_2^2 + S^2 X_{20}^2}$$

At starting, $S = 1$

$$\therefore \qquad T_{st} = \frac{KE_{20}^2 R_2}{R_2^2 + X_{20}^2}$$

For maximum torque, $S = \dfrac{R_2}{X_{20}}$

$$T_m = \frac{K\left[\dfrac{R_2}{X_{20}}\right]E_{20}^2 R_2}{R_2^2 + \left[\dfrac{R_2}{X_{20}}\right]^2 X_{20}^2} = \frac{K\,E_{20}^2}{2\,X_{20}}$$

Given $$\frac{T_m}{T_{FL}} = 2.5$$

$$T_{FL} = \frac{T_m}{2.5} = \frac{KE_{20}^2}{2.5 \times 2X_{20}} = \frac{KE_{20}^2}{2.5 \times 2 \times 5} = \frac{KE_{20}^2}{25}$$

(a) For direct-on-line starting,

$$\frac{T_{st}}{T_{FL}} = \frac{K\,E_{20}^2\,R_2}{R_2 + X_{20}^2} \times \frac{25}{K\,E_{20}^2}$$

$$= \frac{25\,R_2}{R_2^2 + X_{20}^2} = \frac{25 \times 0.5}{[0.5]^2 + [5]^2} = \frac{12.5}{0.25 + 25}$$

$$= \frac{12.5}{25.25} = 0.49$$

(b) For star-delta starting,

Starting rotor voltage per phase = $\dfrac{E_{20}}{\sqrt{3}}$

Therefore, $$T_{st} = \frac{K\left[\dfrac{E_{20}}{\sqrt{3}}\right]^2 R_2}{\left[R_2^2 + X_{20}^2\right]}$$

T_{FL} remains unchanged as the motor operates in delta connection under running condition

$$\frac{T_{st}}{T_{FL}} = \frac{K\,E_{20}^2\,R_2}{3\left[R_2^2 + X_{20}^2\right]} \times \frac{25}{K\,E_{20}^2}$$

$$= \frac{1}{3} \times \frac{25R_2}{R_2^2 + X_{20}^2} = \frac{1}{3} \times 0.49 = 0.16$$

(c) For auto-transformer starting with a tapping at 75%, stator voltage per phase is 0.75 times, the voltage on direct switching. Therefore, rotor voltage at starting is 0.75 E_{20}

$$\frac{T_{st}}{T_{FL}} = \frac{K[0.75E_{20}]^2\,R_2}{R_2^2 + X_{20}^2} \times \frac{25}{KE\dfrac{2}{20}}$$

$$= 0.56 \times \frac{25R_2}{R_2^2 + X_{20}^2}$$

$$= 0.056 \times 0.49 = 0.27$$

Therefore, $$T_{st} = 0.27\, T_{FL}$$

EXAMPLE 4.28 *A 3-phase, squirrel cage induction motor takes a starting current of 6 times the full-load current. Find the starting torque as a percentage of full-load torque if the motor is started (a) Direct-on-line, (b) through a star-delta starter; the full-load slip of the motor being 4 per cent.*

Solution We know, Rotor copper loss = Slip × Rotor input

i.e., $$I_2^2 R_2 = S \times \frac{2\pi T N_s}{60}$$

or $$T \propto \frac{I_2^2}{S}$$

Again $$I_2 \propto I_1$$

Therefore, $$T \propto \frac{I_1^2}{S}$$

At start, $S = 1$ and hence $T_{st} = kI_{st}^2$

and at full-load, $$T_{FL} = \frac{kI_{FL}^2}{S}$$

$$\frac{T_{st}}{T_{FL}} = \left[\frac{I_{st}}{I_{FL}}\right]^2$$

For direct-on-line starting, I_{st} is 6 times of I_{FL}

Therefore, $$\frac{T_{st}}{T_{FL}} = (6)^2 \times 0.04 = 1.44$$

In terms of percentage, starting torque is 144 per cent of the full-load torque.

In star-delta starting, the phase current in start is $1/\sqrt{3}$ times the phase current in delta.

Thus, $$\frac{T_{st}}{T_{FL}} = \left[\frac{I_{st}}{\sqrt{3}\, I_{FL}}\right]^2 \times S$$

$$= \frac{1}{3}\left[\frac{I_{st}}{I_{FL}}\right]^2 \times S = \frac{1}{3}\,(6)^2 \times 0.04 = 0.48$$

In terms of percentage, starting torque is 48 per cent of the full-load torque.

This also shows that starting torque in case of star-delta starting is one-third the starting torque in direct-on-line starting. The above is also verified from the fact that torque is proportional to the square of the voltage and voltage per phase in star connection is $1/\sqrt{3}$ times the voltage in delta connection.

EXAMPLE 4.29 *A 4-pole, 3-phase, 400 V, 50 Hz induction motor develops 1.6 times its full-load torque at stating. The ratio of maximum torque to full-load torque* *is 2. Calculate the speed of the motor when it is developing maximum torque. Also, calculate its full-load speed.*

Solution

Given $\dfrac{T_s}{T_{fL}} = 1.6$ and $\dfrac{T_m}{T_{fL}} = 2$

Dividing, $$\frac{\dfrac{T_s}{T_{fL}}}{\dfrac{T_m}{T_{fL}}} = \frac{T_s}{T_m} = \frac{1.6}{2} = 0.8$$

Again, $$\frac{T_s}{T_m} = \frac{2a}{1+a^2} \qquad \text{where } a = \frac{R_2}{X_{20}}$$

Hence, $$\frac{2a}{1+a^2} = 0.8$$

or, $$a = 0.04 \quad \frac{R_2}{X_{20}} = 0.04$$

$$R_2 = 0.04\, X_{20}$$

Slip at maximum torque, $S_m = 0.04$

$$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$N_r = (1 - s)N_s = (1 - 0.04) \times 1500 = 1440 \text{ rpm}$$

$$\frac{T_m}{T_{fL}} = \frac{a^2 + S_{fL}^2}{2a\, S_{fL}} = 2$$

Substituting value of a,

$$\frac{(0.40)^2 + S_{fL}^2}{2 \times 0.04 \times S_{fL}} = 2$$

or, $s_{fL} = 0.01$

Full load speed, $N_r = (1 - S)N_s$

$$= (1 - 0.01) \times 1500 = 1485 \text{ rpm}$$

4.11.3 Starting of Wound-rotor Motors

The easiest method of starting wound-rotor (slip-ring) induction motors is to connect some extra resistance in the rotor circuit as shown in Fig. 4.15. Connection of extra resistance in the rotor circuit decreases the starting current and at the same time increases the starting torque. As the motor starts rotating the extra resistance is gradually cut out. When the motor attains rated speed the resistance is fully cut out and the slip-ring terminals are short circuited. The motor now operates on its own characteristic which provides the required torque at a low slip.

4.12 CURRENT AND POWER FACTOR

Current drawn by an induction motor at no-load consists of magnetising current which produces flux and a component current which is spent in supplying no-load losses.

In a transformer, no-load current is about 2 to 5 per cent of the full-load current. In an induction motor the no-load current may be as high as 30 to 40 per cent of the full-load current. Because of the presence of air-gap between the stator and the rotor, the magnetising current in an induction motor is much higher than that of a transformer which is required to produce rotating flux. In a transformer, only hysteresis and eddy-current losses take place in the transformer core. In an induction motor, in addition to these two losses, there will be friction and windage losses due to rotation of the rotor. Therefore, both the magnetising component and the power-loss component of the no-load current of an induction motor will be higher than that of an equivalent transformer. There is also some current in the rotor circuit at no-load. But its effect on the stator circuit may be neglected.

The no-load power factor of an induction motor is less than 0.4 lagging. This is because the magnetising component of no-load current, I_m is much higher than loss component of no-load current, I_c as shown in Fig. 4.25. As the motor is loaded, power factor of the motor increases. This can be understood from the following explanation.

As the motor is loaded its speed decreases, i.e., its slip increases. The rotor induced emf and hence rotor current also increases. The rotating field produced by the rotor mmf rotates at slip frequency with respect to the rotor and at synchronous speed with respect to the stator. At the same time the rotating field produced by the stator mmf rotates also at synchronous speed with respect to the stator. Thus both the stator and the rotor fields rotate at synchronous speed with respect to the stator. In other words, both the fields are stationary with respect to each other. The rotor flux will produce a demangetising effect on the stator field. The stator will draw more current from the lines to compensate for this. This maintains a flux which is more or less constant over the operating range of the motor. At low value of slip, the rotor power factor is quite high since the rotor reactance SX_{20} is very small as compared to R_2. The total current drawn by the stator, I_1 will be the phasor sum of the no-load current, I_0 and a current, I_2' drawn by stator due to loading effect on the rotor. This is shown in Fig. 4.25. Power factor angle on load, ϕ_L is less than power factor angle, ϕ_0 on no-load.

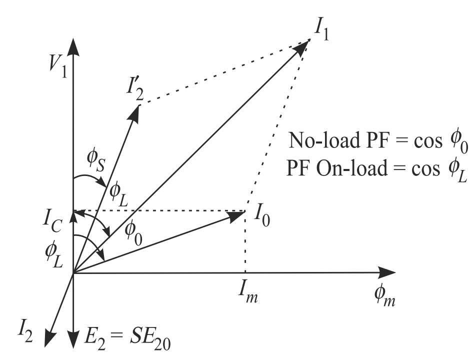


Fig. 4.25 *Power factor on no-load and on-load*

It is seen that the power factor of the motor is increased as the motor is loaded. With further increase in load, its slip increases and the rotor circuit reactance can no longer be taken as negligibly small. The power factor of the motor will gradually decrease if the motor is continued to be loaded. The power factor of a fully loaded induction motor may be as high as 0.85 to 0.9. To maintain a high power factor, the air-gap of the motor should be kept minimum. The necessity to keep a small air-gap, sometimes, puts a limitation to the size of the motor to be built, due to mechanical considerations.

4.13 MOTORING GENERATING AND PLUGGING MODES OF OPERATION

We have studied the operation of an induction machine as a motor where the rotor rotates in the same direction as the rotating magnetic field at a speed slightly lower than the synchronous speed. Three-phase supply is applied to the stator windings and the rotor tries to catch up with the speed of the rotating magnetic field but slips behind so that N_r is less than N_s. This is the natural mode of operation of the machine and is called Motoring Mode of Operation.

The same machine will work in Generating Mode if the machine is rotated by a prime-mover, say a dc machine at a speed higher than the synchronous speed in the same direction as the rotating field. Thus N_r is more than N_s in generating mode of operation. When the rotor is rotated at supersynchronous speed by the prime-mover and three-phase supply being applied to the stator terminals of the induction machine, the torque developed will be acting in the opposite direction to the rotation of the rotor.

In generating mode, the induction machine will cause power flow in the negative direction ie power will be fed back to the supply.

If now the dc motor direction of rotation is so adjusted that the dc motor rotates the induction motor rotor in the opposite direction to the stator rotating field, the torque developed by the motor will be in the same direction as the rotating field but will oppose the rotation of the rotor. This torque will act as breaking torque. This mode of operation is called Plugging. An induction machine rotating as a motor in a particular direction can be stopped quickly by first switching off the supply and immediately changing the sequence of supply. This will cause the stator field rotate in opposite direction to the rotation of the rotor producing a plugging or braking action. The three modes of operation of an induction machine has been shown in Fig. 4.26.

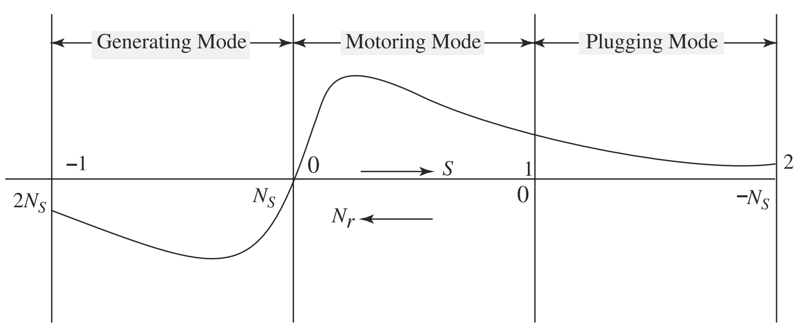


Fig. 4.26 *Three modes of operation of an induction machine*

4.14 SPEED CONTROL OF INDUCTION MOTOR

The relationship between synchronous speed, rotor speed and slip is given by,

$$S = \frac{N_s - N_r}{N_s}$$

or $$N_r = N_s (1 - S)$$

Therefore, rotor speed, $$N_r = \frac{120f}{P} (1 - S) \quad (4.26)$$

This equation shows that speed of an induction motor depends on (a) slip S; (b) frequency of the stator supply f, and (c) number of poles for which the windings are made, i.e., P.

The ability of varying any one of the above three quantities will provide methods of speed control of an induction motor.

4.14.1 Control of Speed by Changing Slip

In slip-ring type motors, slip at a particular load can be changed by changing the rotor circuit resistance. In squirrel-cage motors, rotor circuit resistance cannot be varied. Therefore, speed of squirrel-cage type motors cannot be varied by changing of slip. The effect of change of rotor circuit resistance on slip when the motor is connected to a mechanical load is shown in Fig. 4.27. Torque-slip characteristics corresponding to rotor resistances R_1 and R_2 are shown respectively by the curves A and B. Curve L shows the load characteristic. The motor runs at a slip, S_1 with R_1 as its rotor circuit resistance.

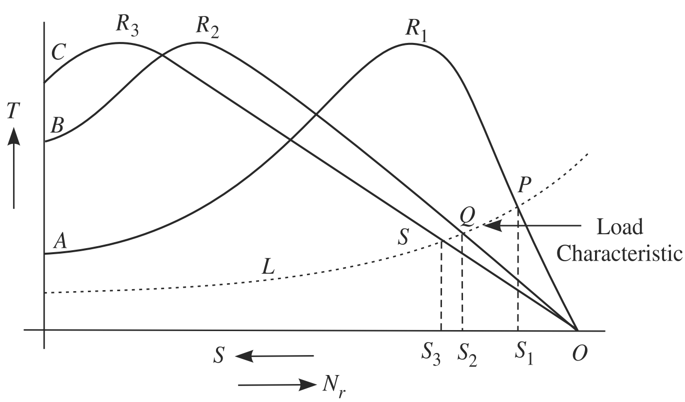


Fig. 4.27 *Shows the change of rotor slip with change of rotor circuit resistance when the motor is running with a particular load*

With rotor circuit resistance R_2 which is greater than R_1, the rotor slip increases from S_1 to S_2. Thus the speed of the rotor decreases. If we further change the rotor circuit resistance, the speed of the motor will change to a new value as indicated by slip S_3.

Thus, the speed of the motor can be varied by changing the rotor circuit resistance.

The disadvantages of this method of speed control are as follows.

Decreased Efficiency Due to Power Lost in the External Rotor Circuit Resistance At higher values of slip, the rotor current and hence rotor I^2R-loss are increased considerably. This can be better understood by referring back to expression,

$$\text{Rotor } I^2R\text{-loss} = \text{Slip} \times \text{Rotor input}$$

If slip is increased, rotor I^2R-losses increase. At 50 per cent slip, for example, half of the rotor input will be wasted as rotor I^2R-loss.

Poor Speed Regulation when the Motor is Working with Higher Rotor Circuit Resistance Figure 4.27 shows the torque-speed characteristics of an induction motor for two values of rotor-circuit resistance R_1 and R_2. Speed regulation is expressed as the variation of speed as the load on the motor varies. From Fig. 4.28, it can be seen that for change of load from half-load to full-load, the variation of motor speed is from N_1 to N_2 with R_1 as the rotor-circuit resistance. The variation of speed is N_3 to N_4 for R_2 as the rotor-circuit resistance which is much higher in this case as compared to the variation with R_1 as rotor-circuit resistance, i.e., $(N_3 - N_4) > (N_1 - N_2)$.

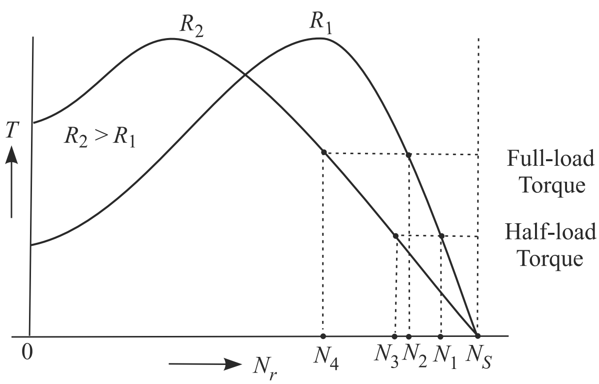


Fig 4.28 *Speed regulation with different values of rotor circuit resistance*

Although by changing the rotor-circuit resistance from R_1 to R_2 at a particular load, say at half-load, the speed can be changed from N_1 to N_3 but the speed regulation, i.e., the variation of speed with variation of load becomes higher with higher rotor circuit resistance.

4.14.2 Control of Speed by Controlling Supply Voltage

Speed of an induction motor can also be varied by changing the applied stator voltage. If the voltage is reduced, torque is reduced as the square of the voltage. For example, if the applied voltage is reduced from V to 0.9 V, the torque will be reduced from T to 0.81 T. The torque-speed characteristic at reduced stator voltage say 0.9 V is shown in Fig. 4.29.

Since the torque is reduced to 81 per cent, the rotor cannot continue to rotate at speed N_1, its speed will be reduced, i.e., its slip will increase until the increased rotor current will make up for the reduced stator voltage and produce the required load torque at a lower speed N_2. This method of speed control is rarely used for industrial

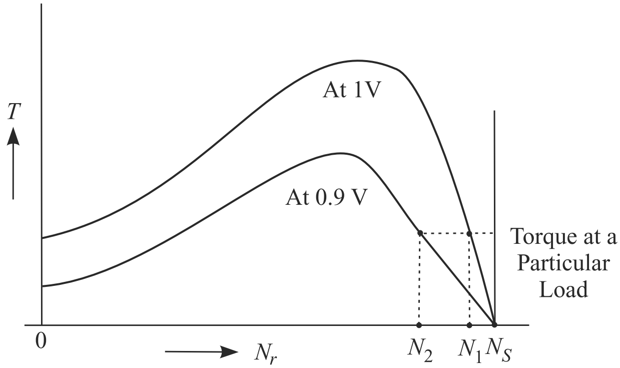


Fig. 4.29 *Variation of motor speed due to variation of stator applied voltage*

three-phase motors because of the requirement of additional costly voltage changing auxiliary equipment. For small induction motors used in home appliance, voltage control method of speed changing is often used.

4.14.3 Injected Voltage Method of Speed Control

The speed of an induction motor can be changed by injecting a voltage in the rotor circuit. The emf induced in the rotor is of slip frequency. Therefore, the emf to be injected in the rotor circuit should also be of slip frequency. The rotor equivalent circuit with an injected voltage $\bar{E}_i$ has been shown in Fig. 4.30.

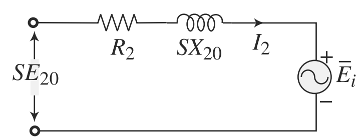


Fig. 4.30 *Control of speed by injecting a voltage in the rotor circuit*

The injected voltage may be in phase or in phase opposition to the rotor induced emf. The magnitude of the injected emf can also be varied. If the injected emf is in phase with the induced emf of the rotor, i.e., if E_i and SE_{20} are in phase the rotor current will increase because

$$\bar{I}_2 = \frac{S\bar{E}_{20} + \bar{E}_i}{\bar{Z}_2}$$

Increase in rotor current is equivalent to decrease in rotor circuit resistance. When $\bar{E}_i$ is in phase opposition to $S\bar{E}_{20}$ the rotor current will decrease. Decrease in rotor current is equivalent to increase in rotor circuit resistance. This shows that changing the phase of the injected voltage becomes equivalent to changing the resistance of the rotor circuit. When rotor circuit resistance changes the operating speed also changes. Control of speed can also be achieved by changing the magnitude of the injected voltage.

A successful system developed by K.H. Schrage of Sweden was applied in a motor named after him as *Schrage motor*. If the injected emf has a component in direct opposition the rotor emf, there will be decrease in I_2 which means increase in R_2 and hence the motor speed will decrease. If the injected emf has a component in phase with the rotor induced emf, the speed will be above original speed.

These days Schrage motors are rarely used because of the bulky construction and prohibitive cost.

4.14.4 Control of Speed by Changing Supply Frequency

The speed of an induction motor is directly proportional to supply frequency. By gradually changing supply frequency, speed can be increased or decreased smoothly. There are, however, several drawbacks in this system. Electricity supply authorities supply power at a fixed frequency of 50 Hz. Provision for supply at variable frequency can be made by the consumers by having separate arrangements. Frequency conversion equipment are, therefore, to be installed by the industries at additional cost. Variable frequency supply can be obtained from solid-state equipment, or rotary converters, i.e., motor-generator set.

It is very important to note that if speed control is to be achieved by changing frequency, the supply voltage should also simultaneously be changed. This is because if the supply frequency is reduced keeping the applied voltage constant, the flux is increased ($E = 4.44\ \phi_m fT$). If flux is increased, core-losses will increase and cause reduction in the efficiency. On the other hand, if the frequency is increased, flux will decrease, thereby reducing the torque developed. It is important, therefore, that the frequency changing device should change frequency and voltage simultaneously as a direct ratio. That is, if frequency is increased, the supply voltage must also be increased and if the frequency is decreased the supply voltage must also be decreased proportionately.

This method had limited applications because of cost involved in arranging a variable frequency power supply. With the recent developments in semiconductor devices, frequency control method of speed variation of induction motors is becoming popular. A fundamental block diagram showing the scheme of speed control of an induction motor using thyristors has been shown in Fig. 4.31. Three-phase supply at the input is first converted into controlled dc. This dc is applied to inverter circuit, whose frequency is controlled by pulses from voltage to frequency converter units. A large smoothing reactor, L is connected in the circuit to filter the controlled dc.

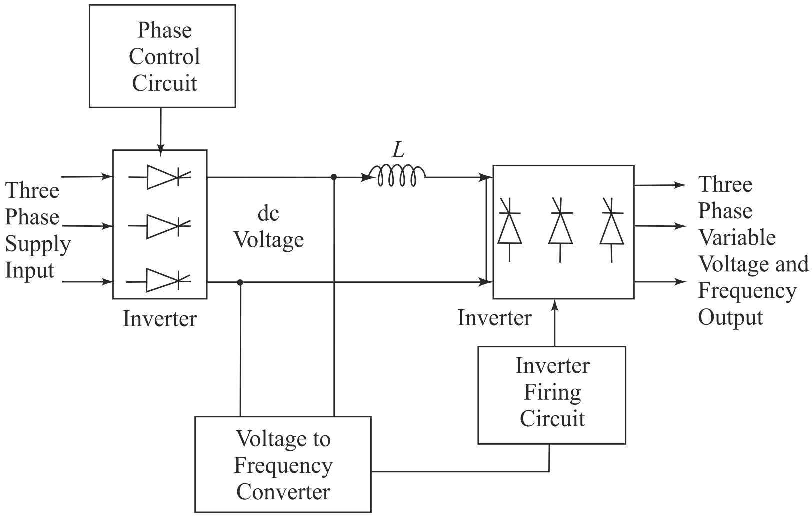


Fig. 4.31 *Block diagram representation of speed control of induction motor using static semiconductor devices*

A simple voltage-to-frequency converter as indicated in Fig. 4.31 can be made by using one UJT (unijunction-transistor) as shown in Fig. 4.32. The UJT is biased properly by using R_2, R_3 and V_{BB}. Capacitor C_1 is charged through R_1 due to the input voltage. The charging rate of the capacitor C_1 increases as the input voltage increases. The output pulses are thus produced at shorter intervals, which means an increase in output frequency. The circuit, therefore, works as a voltage to frequency converter.

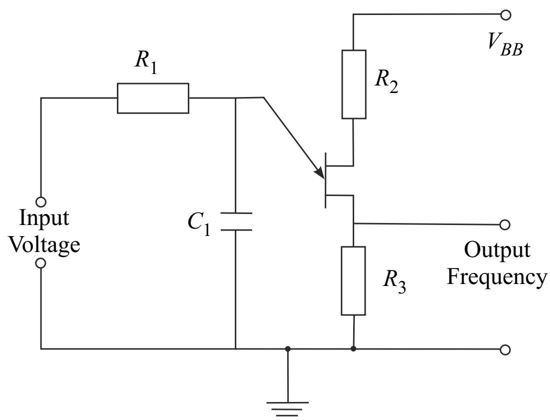


Fig. 4.32 *Voltage to frequency converter circuit*

The inverter used in the circuit of Fig. 4.31 has been used for converting dc into ac.

While converting ac to dc through a converter, the thyristor is fired at a desired point in positive half-cycle of ac to make it conduct. It shuts off automatically when ac cycle goes through zero to negative half-cycle. This type of switching off of the thyristor is called natural commutation.

While converting dc to ac through an inverter, once a thyristor is fired, it will never be switched off, since the applied voltage is unidirectional. To obtain a continuous ac, the thyristor is to be switched off. External means are to be employed to force the switching off of the thyristor. Such a commutation is called forced commutation. Based on different forced commutation schemes, various inverter circuits are designed but these are beyond the scope of this book.

4.14.5 Control of Speed by Pole Changing

Speed Control Using Two Separate Windings on the Stator An induction motor stator is wound for a definite number of poles. The speed of an induction motor depends upon the number of poles for which the stator is wound. If instead of one stator winding, two independent windings, for different number of poles say for four poles and for eight poles are made on the stator, two definite rotor speeds can be obtained. The two windings are to be insulated from one another. When any of the windings is used, the other winding should be kept open circuited by the switch or at least left in star-connection. In the rotor, poles are induced in accordance with the number of poles in the stator circuit. The limitation of this method is that, only two definite speeds can be obtained. Smooth control of speed over a wide range is not possible.

Speed Control Using Consequent-pole Technique An alternative method is to use only one winding wound for a particular number of poles, but the end connections of the coils with the supply is changed such that different number of poles are formed. This is explained in a simplified manner in Fig. 4.33. Only one stator phase winding of a balanced three-phase, squirrel-cage motor has been considered. In Fig. 4.33(a), a simplified 2-pole stator winding has been shown. It is seen from Fig. 4.33(b) that due to change in supply connections to the windings, two instantaneous south poles are formed. The development of these two south poles gives a 4-pole windings.

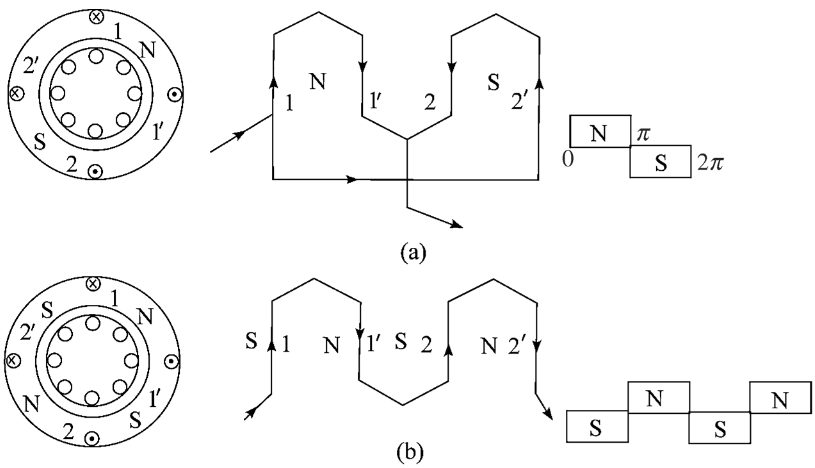


Fig. 4.33 *Pole changing method of speed control (a) Parallel connection of coils (b) Series connection of coils*

With the combination of having two separate windings on the stator and by series-parallel connections of the coils across the supply, four distinct speeds of the rotor can he obtained.

Speed Control Using Pole-Amplitude-Modulation Technique In this method, by reversing the connections of one-half of the windings it is possible to get pole changing of different ratios other than 1 : 2, as was shown in Fig. 4.33. Let us assume a stator winding made of 10 poles. Reversing half of the coil mmf can effectively be considered equivalent to multiplying the total coil mmf by a modulating space mmf wave of unit amplitude and of period equal to the length of the stator periphery. This is illustrated in Fig. 4.34.

Figure 4.34(a) shows a 10 pole stator mmf wave. Reversing of one half of the stator mmf, i.e., from point B to C will result in mmf distribution as shown in Fig. 4.34(c). This can be viewed as if mmf wave of Fig. 4.34(a) has been multiplied by mmf wave of Fig. 4.34(b) to achieve mmf wave of Fig 4.34(c). Thus a 10-pole mmf wave is converted into an 8-pole mmf wave by reversing the mmf of half of the winding. Note that in Fig. 4.34(c), the extreme two North poles combine to make one North pole.

In other words this conversion from a 10-pole to an 8-pole is achieved as a result of multiplication of the original 10-pole mmf wave by a 2-pole modulating wave. Conversion to different number of poles can also be achieved by omitting a section of winding from each half of the stator winding and then reversing the second-half of the winding with respect to the first-half. It is observed from Fig. 4.34(c) that there is nonuniformity in the modulated wave achieved through simple reversal of one-half of the winding mmf. Improvement over this coil reversal method of speed control is necessary to so as to achieve good performance of the motor at other speeds.

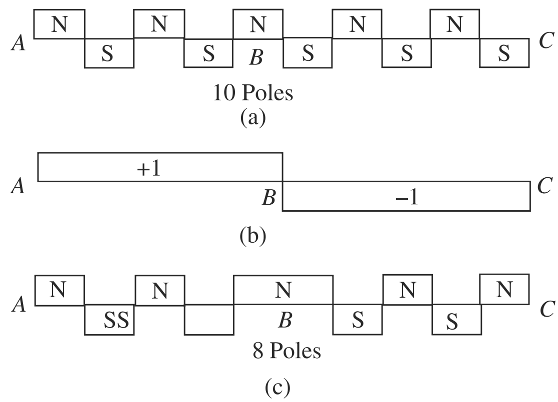


Fig. 4.34 *Illustration of pole amplitude modulation (a) 10-pole stator mmf wave (b) 2-pole modulating wave (c) 8-pole modulated wave*

Pole changing methods of speed control as mentioned earlier is mostly used for squirrel-cage motors since the rotor poles are induced in accordance with the number of poles on the stator. In a wound rotor induction motor, rotor is to be wound for the same number of stator poles. Therefore, if any change of poles is made on the stator, the same must be done for the rotor also. To achieve this, additional slip-rings must be provided for the rotor circuit to change the rotor circuit connections.

4.14.6 Rotor Slip Energy Recovery (Static Scherbius Scheme and Static Kramer Drive System) Method of Speed Control

We have seen that speed of slip-ring induction motors can be controlled by adding extra resistance in the rotor circuit. However, power is lost in the rotor circuit as I^2R loss. The efficiency of operation gets drastically reduced, particularly during low speed operation when rotor current is high. The speed control is achieved by virtue of additional I^2R loss in the rotor circuit.

If the slip power lost in the resistance could be returned to the source, the efficiency of the motor operation would be increased to a large extent. A method of recovering the slip power from the rotor is shown in Fig. 4.35.

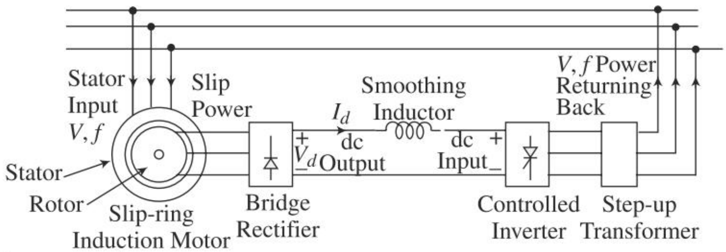


Fig. 4.35 *Basic scheme of speed control by rotor slip energy recovery (also called static Scherbius scheme of speed control)*

The rotor power is first rectified using a diode bridge rectifier. The rectified output wave shape is smoothened by using a smoothing inductor as has been shown. The output of the rectifier is then connected to the input terminals of controlled inverter which inverts the dc power to ac power of supply frequency. The output of the inverter is then connected through a step-up transformer to the supply mains. This way, instead of wasting the slip power in the rotor circuit resistance, it is returned back to the supply mains. This scheme of speed control where rotor slip energy is recovered and returned to the supply mains is called *static scherbius scheme of control.*

This method of speed control is advantageous in applications where variation of speed over a wide range involves a considerable amount of slip power.

There is another scheme of control, called *static kramer drive system* where slip power from the rotor of the induction motor is fed to the armature of a dc motor after rectification as shown in Fig. 4.36. Both the induction motor and the dc motor together drive a common load. The speed of the dc motor is controlled by varying the field current of the dc motor.

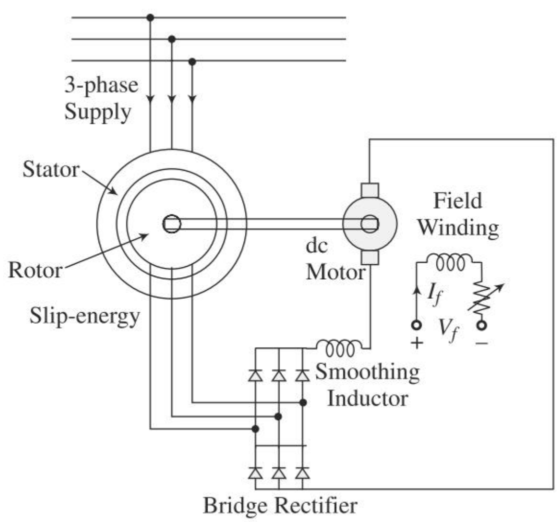


Fig. 4.36 *Slip-power recovery using static Kramer drive system*

The torque supplied to the load, which is connected on the common shaft, is the sum of torque developed by the slip-ring induction motor and the dc motor.

4.15 LOSSES AND EFFICIENCY

4.15.1 Relationship between Output, Losses and Efficiency

An induction motor stator is supplied with electrical power. Power is transferred to the rotor through the air-gap and is made available at the shaft to do mechanical work. The whole of the power input to the stator winding is not available at the shaft as output. Some power is lost in the stator and rotor windings and on the core as heat. In addition, there is friction and windage loss due to the rotation of the rotor in air. The various losses in an induction motor are listed as:

(a) I^2R-loss in the stator winding.
(b) Core loss (hysteresis and eddy-current loss) in the stator core.
(c) I^2R-loss in the rotor winding.
(d) Core loss in the rotor core.
(e) Frictional losses in the bearings, brush and slip-rings (if any).
(f) Windage loss caused due to the rotation of the rotor in air.

The relationship between input power, output power and the various losses in an induction motor can be represented as

Electrical power input to stator winding – [I^2R-loss in the stator winding + Core loss in the stator core]
= Power transferred through the air-gap to the rotor circuit
= Power input to the rotor circuit

Power input to the rotor circuit – [I^2R-loss in the rotor + Core loss in the rotor]
= Mechanical power developed.

Mechanical power developed – Friction and windage losses
= Mechanical output at the shaft

Efficiency of an induction motor like any other machine is the ratio of output and input. Thus

$$\text{Efficiency,} \qquad \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Ouput} + \text{Losses}}$$

4.15.2 Determination of Efficiency

Efficiency of an induction motor can be determined by directly loading the motor and by measuring its input and output power. Small motors can be conveniently tested by this direct method. For motors of high ratings, however, it may be difficult to arrange loads for the motor in the testing laboratory.

Even if load is arranged, there will be heavy power loss in the testing process. In order to avoid the wastage of power indirect methods are used for determining efficiency.

In an indirect method of determining efficiency, the various losses of the motor are determined. For determining the various losses in an induction motor, two tests, similar to those conducted on transformers, are performed on the motor. They are, (a) no-load test and (b) blocked-rotor test.

No-load Test In no-load test, without connecting any load on the motor shaft, full voltage is applied across the winding terminals. Since output of the motor at no-load is zero, the whole of the input power is wasted as various losses. At no-load, the speed of the rotor is very nearly equal to synchronous speed. The emf induced in the rotor and the rotor current is negligibly small. The rotor can, therefore, be approximately considered as an open circuit. No-load test of an induction motor is therefore, similar to no-load test on a transformer. The losses at no-load are:

(a) I^2R-loss in the stator winding;
(b) Core losses in the stator and rotor;
(c) Friction and windage losses.

No-load current drawn by an induction motor is much higher than that of a transformer and, therefore, cannot be assumed as negligible. From the total input at no-load, the I^2R-loss in the stator winding can be subtracted to get core loss plus friction and windage losses. These losses at no-load are nearly the same as would occur under full-load condition. This is because core loss depends on applied voltage whereas friction and windage losses depend upon speed of rotation of the rotor. Applied voltage is assumed to be constant and the variation of speed of an induction motor from no-load to full-load is negligibly small.

Connection diagram for no-load test on an induction motor is shown in Fig. 4.37.

The sum of the wattmeter readings gives the no-load power input to the motor. Subtracting the stator I^2R loss from the input power we get the core loss plus the friction and windage loss.

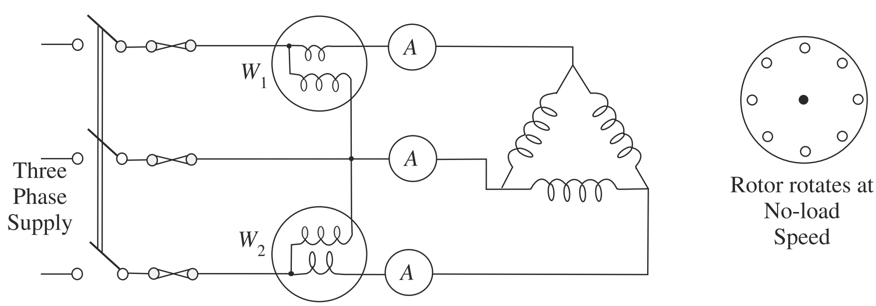


Fig. 4.37 *No-load test on an induction motor*

Blocked Rotor Test In this test the rotor of the motor is blocked, i.e., the rotor is not allowed to rotate. Low voltage is applied across the stator terminals through a three-phase autotransformer. Voltage is gradually increased to a value so that full rated current flows through the windings. Since the rotor circuit is closed and is not rotating, this test is similar to short-circuit test on a transformer. The voltage needed to circulate full-load current under blocked rotor condition is very low. The power input to the stator is mainly wasted as I^2R-loss in the stator and the rotor windings. The core-loss at reduced voltage can be neglected. The circuit diagram for blocked rotor test is given in Fig. 4.38.

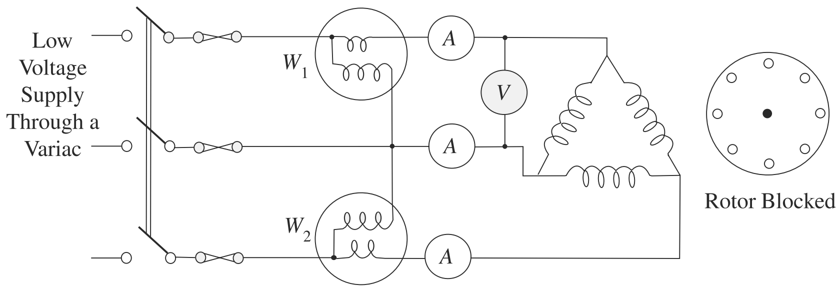


Fig. 4.38 *Blocked-rotor test on an induction motor*

The sum of the two wattmeter readings gives the total input power. Since full-load current is allowed to flow through the stator and rotor windings, the input power can be considered approximately equal to full-load I^2R-losses. From the input power it is possible to calculate equivalent resistance of the motor referred to the stator side. By knowing the value of this resistance, we can calculate the value of I^2R-loss at no-load. As mentioned earlier, if we subtract I^2R-loss at no-load from the no-load power input of no-load test, we get the constant losses of the induction motor.

From the data of the no-load and blocked-rotor tests, therefore, it is possible to calculate the efficiency of the induction motor. The procedure for calculation of efficiency from no-load and blocked rotor test data has been given in example 4.31.

EXAMPLE 4.30 *A 20 hp, 4 pole, 50 Hz, three-phase induction motor has friction and windage losses of 2 per cent of the output. The full-load slip is 4 per cent. calculate for full-load:*

(a) the rotor I^2R-loss, (b) the rotor input, and (c) the output torque.

Solution Synchronous speed,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S = \frac{N_s - N_r}{N_s}$$

or

$$N_r = N_s (I - S)$$

$$= 1500\,(1 - 0.04) = 1440 \text{ rpm}$$

$$\text{Output power} = 20 \times 735.5 \text{ W} = 14710 \text{ W}$$

$$\text{Output torque} = \frac{\text{Output power}}{2\pi \dfrac{N_r}{60}} = \frac{14710}{6.28 \times 1440/60} \text{ Nm} = 97.6 \text{ Nm}$$

Friction and windage losses = 2 per cent of output

$$= 0.02 \times 14710 = 294.2 \text{ W}$$

Power developed by the rotor = Shaft output + Friction and windage losses

$$= 14710 + 294.2 = 15004.2 \text{ W}$$

From Eq. (4.15),

$$\text{Rotor } I^2R\text{-loss} = S \times \text{Rotor input}$$

or $$\text{Rotor input} = \frac{\text{Rotor } I^2R\text{-loss}}{S}$$

Again, $$\text{Rotor input} = \text{Rotor } I^2R\text{-loss} + \text{Power developed by the rotor}$$

or $$\frac{\text{Rotor } I^2R\text{-loss}}{S} = \text{Rotor } I^2R\text{-loss} + \text{Power developed by the rotor}$$

or $$\text{Rotor } I^2R\text{-loss}\left[\frac{1}{S} - 1\right] = \text{Power developed by the rotor}$$

or $$\text{Rotor } I^2R\text{-loss} = \frac{S}{(1-S)} \times \text{Power developed by the rotor}$$

or $$\text{Rotor } I^2R\text{-loss} = \frac{0.04}{0.96} \times 15004.2 \text{ W} = 625.2 \text{ W}$$

$$\text{Rotor input} = \frac{\text{Rotor } I^2R\text{-loss}}{S} = \frac{625.2}{0.04} \text{ W} = 15630 \text{ W}$$

EXAMPLE 4.31 *A 4-pole, 50 Hz, 230 V, 5 hp squirrel-cage induction motor gave the following test data:*

No-load test: Power input	*= 275 W*
No-load current	*= 6.3 A*
No-load input voltage	*= 230 V*
Blocked-rotor test: Power input	*= 735 W*
Blocked rotor full-load current	*= 15 A*
Blocked rotor input voltage	*= 40 V*

Determine the full-load efficiency of the motor from the above test data.

Solution Neglecting the small amount of core-loss under blocked-rotor condition, the input to the motor in block rotor test gives the full-load I^2R-losses.

$$\text{Full-load } I^2R\text{-losses} = 735 \text{ W}$$

Input at no-load gives the core-loss plus friction and windage loss in addition to no-load I^2R-loss. I^2R-loss at no-load can be calculated by knowing the value of the resistance of the windings.

Winding resistance can be calculated from the blocked-rotor test data thus:

$$3I_1{}^2R_e' = \text{Power input under blocked rotor test.}$$

where, R_e' is the per phase equivalent resistance of the windings referred to stator circuit.

Thus, $$3 \times (15)^2 \times R_e' = 735$$

$$R_e' = 1.09\ \Omega$$

$$I^2R\text{-loss at no-load} = 3I_0^2 R_e' = 3 \times (6.3)^2 \times 1.09 = 130 \text{ W}$$

Core loss plus friction and windage loss

$$= 275 - 130 = 145 \text{ W}$$

Thus, Total losses = 735 + 145 = 880 W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} \times 100$$

$$= \frac{5 \times 735.5}{5 \times 735.5 + 880} \times 100$$

= 80.7 per cent

EXAMPLE 4.32 *Calculate the full-load efficiency of a 415 V, three-phase, 50 Hz delta connected induction motor from the following test data:*

On no load, power intake is 1500 W at rated input voltage.

On full load, line current is 50 A, power factor is 0.85 and slip is 0.04. Resistance of the stator winding per phase is 0.5 Ω.

Assume the ratio of stator core loss to friction and windage loss as 3 : 2.

Solution Input to the motor on full-load

$$= \sqrt{3} \times V_L I_L \cos \phi$$

= 1.732 × 415 × 50 × 0.85 = 30548 W

Stator I^2R-loss on full-load

$$= 3 \times I_1^2 \times R_1$$

$$= 3\left[\frac{50}{\sqrt{3}}\right]^2 \times 0.5$$

= 1250 W

No load input power of 1500 W is spent as stator core loss plus friction and windage loss (neglecting a small amount of stator I^2R-loss). Since the ratio of stator core loss to friction and windage loss is 3 : 2,

$$\text{Stator core loss} = \frac{3}{5} \times 1500 = 900 \text{ W}$$

$$\text{Friction and windage loss} = \frac{2}{5} \times 1500 = 600 \text{ W}$$

Stator losses = 1250 + 900 = 2150 W

Stator input = 30548 W

Power transferred through the air-gap

= 30548 – 2150 = 28398 W

= Input to the rotor

Rotor I^2R-loss = Slip × Rotor input

= 0.04 × 28398 W = 1136 W

Rotor losses = Friction and windage losses + I^2R-loss
(Rotor iron loss is negligible)

= 600 + 1136 = 1736 W

Output power at the shaft = 28398 – 1736 = 26,662 W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100$$

$$= \frac{26662}{30548} \times 100 = 87.28 \text{ per cent}$$

4.16 EQUIVALENT CIRCUIT OF AN INDUCTION MOTOR AND ITS USE

4.16.1 Equivalent Circuit

Similar to a transformer, an indiction motor can be represented by an equivalent electrical circuit. Equivalent circuit enables has to evaluate the performance characteristics of the induction machine for steady-state conditions by ordinary processes of ac network solution. The equivalent circuit of an induction motor drawn for only one phase has been shown in Fig. 4.39. In Fig. 4.39(a) the stator resistance, R_1 and leakage reactance, X_1 are separated from the common magnetic circuit and the magnetising and loss currents are considered as flowing through the parallel branches R_m and X_m.

The rotor current is given by

$$I_2 = \frac{SE_{20}}{\sqrt{R_2^2 + S^2 X_{20}^2}}$$

$$= \frac{E_{20}}{\sqrt{\left[\frac{R_2}{S}\right]^2 + X_{20}^2}}$$

The rotor circuit can, therefore, be represented by a resistance R_2/S and a reactance X_{20} connected in series across a voltage source E_{20} which causes a current I_2 flowing through the circuit.

To show the mechanical power conversion in the rotor circuit, the resistance $\frac{R_2}{S}$ of the rotor circuit can be represented as two separate resistances, viz.,

R_2 and $R_2 \left[\frac{1-S}{S}\right]$. This is verified from the following:

$$\frac{R_2}{S} = \frac{R_2}{S} - R_2 + R_2 \text{ [by subtracting and adding } R_2]$$

or

$$\frac{R_2}{S} = R_2 \left[\frac{1-S}{S}\right] + R_2$$

This has been shown in Fig. 4.39(b).

By transferring the rotor quantities to the stator side, i.e., by referring all the rotor quantities from voltage level E_{20} to the voltage level of E_1 the two circuits can be joined at AA' and BB' and will be as shown in Fig. 4.39(c). When the rotor circuit elements are referred to the stator voltage level, their values will change as represented by R_2', X_{20}', etc., in the figure. For simplicity, the two parallel branches representing R_m and X_m can be shifted to the left as shown in Fig. 4.39(d) without introducing much error. This simplification will enable easy calculations.

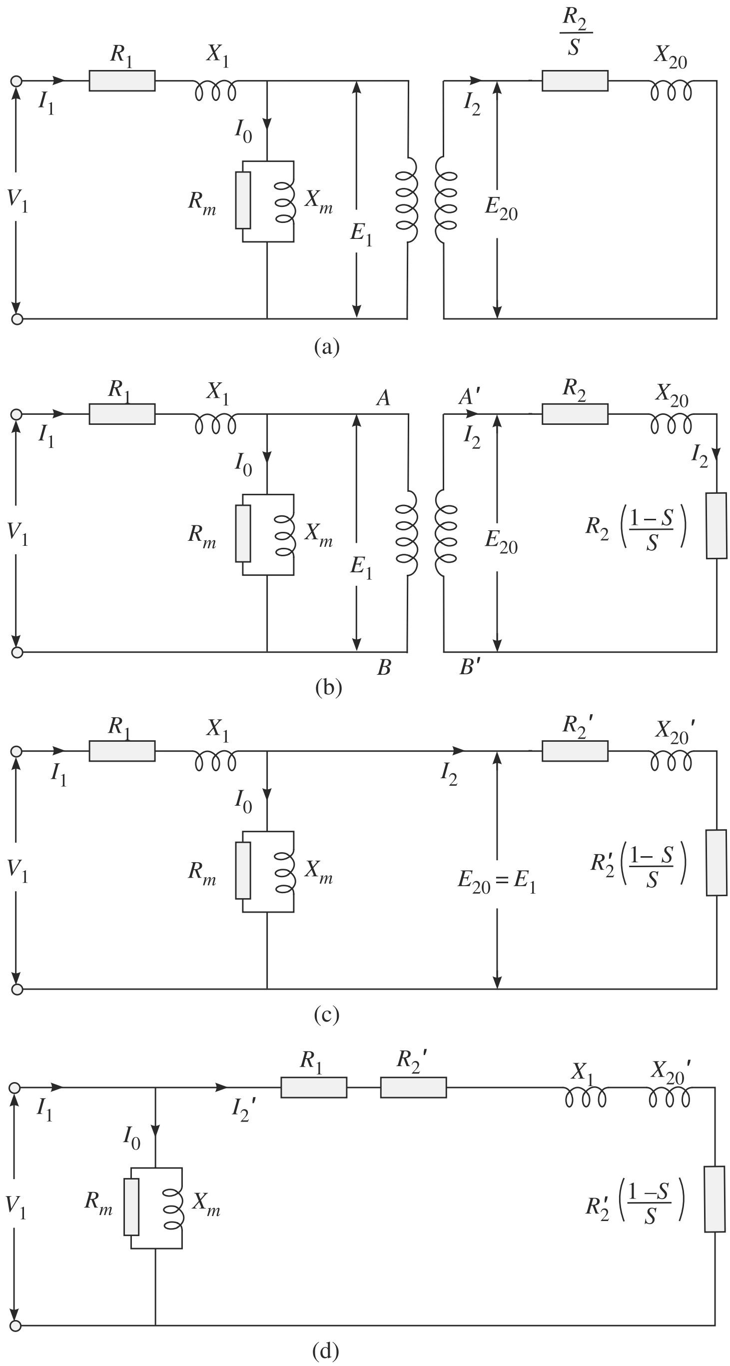


Fig. 4.39 *Equivalent circuit of an induction motor developed step by step as in (a) to (d)*

4.16.2 Calculation of Rotor Output and Torque Using the Equivalent Circuit

From Fig. 4.39(b), expression for output power and torque can be derived as shown below.

Rotor output power, $P_r = I_2^2\, R_2\left[\dfrac{1-S}{S}\right]$

Rotor input power, $P_I = I_2^2\, R_2\left[\dfrac{1-S}{S}\right] + I_2^2 R_2 = \dfrac{I_2^2 R_2}{S} - I_2^2 R_2 + I_2^2 R_2$

That is, $$P_I = \frac{I_2^2 R_2}{S} \quad (4.27a)$$

or, $SP_I = I^2{}_2\, R_2$

We can write,

$P_i = SP_i + P_i - SP_i$ (by adding and subtracting SP_i)

or $$P_i = SP_i + (1 - S)P_i \quad (4.27b)$$

This shows that rotor input, P_i is utilised in the rotor resistance as SP_i and the equivalent resistance representing mechanical load $(1 - S)\, P_i$

Rotor power output, $P_r = (1 - S)\, P_i$

$$= (1 - S)\ \frac{I_2^2 R_2}{S}$$

$$P_r = I_2^2\, R_2\left[\frac{1-S}{S}\right] \quad (4.28)$$

Rotor current $$I_2 = \frac{S\, E_{20}}{\sqrt{R_2^2 + (SX_{20})^2}}$$

Substituting the value of I_2 in Eq. (4.28),

$$P_r = \frac{S^2 E_{20}^2}{R_2^2 + S^2 X_{20}^2} R_2\, \frac{(1-S)}{S}$$

$$P_r = \frac{S\, E_{20}^2\, R_2 (1-S)}{R_2^2 + S^2 X_{20}^2} \quad (4.29)$$

Again $$P_r = \frac{2\pi T N_r}{60} \quad (4.30)$$

Equating Eqs (4.29) and (4.30),

$$\frac{2\pi T N_r}{60} = \frac{S\, E_{20}^2 R_2\, (1-S)}{R_2^2 + S^2 X_{20}^2}$$

or $$T = \frac{60}{2\pi N_r} \times \frac{SE_{20}^2 R_2 (1-S)}{R_2^2 + S^2 X_{20}^2}$$

Substituting N_r as N_s (1 – S),

$$T = \frac{60}{2\pi N_s(1-S)} \times \frac{S E_{20}^2 R_2 (1-S)}{R_2^2 + S^2 X_{20}^2}$$

or
$$T = \frac{60}{2\pi N_s} \times \frac{S E_{20}^2 R_2}{R_2^2 + S^2 X_{20}^2} \qquad (4.31)$$

This expression of torque is the same as developed earlier in Sec. 4.7.

Using the equivalent circuit concept and the expressions for power and torque developed above, calculations for the performance of an induction motor can be made.

EXAMPLE 4.33 *A three-phase induction motor at standstill has 100 V induced between its slip-ring terminals. The rotor winding is star-connected and has resistance and standstill reactance of 0.2 and 1 Ω per phase respectively. Calculate (a) the rotor current when the slip is 3 per cent and the rings are short circuited; and (b) the slip and rotor current when the rotor develops maximum torque.*

Solution Inducted emf between the slip-rings when the rotor is at standstill is E_{20} = 100 V.

$$E_{20} \text{ per phase} = \frac{100}{\sqrt{3}} = 57.6 \text{ V}$$

Rotor current I_2 at a slip of 0.03,

$$I_2 = \frac{S E_{20}}{\sqrt{R_2^2 + (SX_{20})^2}}$$

$$= \frac{0.03 \times 57.6}{\sqrt{(0.2)^2 + (0.03 \times 1)^2}} = 8.5 \text{ A per phase}$$

We know when torque becomes maximum, $R_2 = SX_{20}$.

Therefore
$$S = \frac{R_2}{X_{20}} = \frac{0.2}{1} = 0.2$$

Rotor current I_2 at a slip of 0.2,

$$I_2 = \frac{0.2 \times 57.6}{\sqrt{(0.2)^2 + (0.2 \times 1)^2}} = 40.7 \text{ A per phase}$$

EXAMPLE 4.34 *A pole, three-phase, 50 Hz induction motor at standstill has 120 V induced across its star-connected rotor terminals. The rotor resistance and standstill reactance per phase are 0.2 and 1 Ω respectively. Calculate the rotor speed when the rotor is drawing a current of 16 A at a particular load. Also calculate the speed at which the torque is maximum and the corresponding value of rotor input.*

Solution Rotor induced emf at standstill,

$$E_{20} = 120 \text{ V}$$

$$E_{20} \text{ per phase} = \frac{120}{\sqrt{3}} = 69.3 \text{ V}$$

Rotor current,

$$I_2 = \frac{S E_{20}}{\sqrt{R_2^2 + S^2 X_{20}^2}}$$

or $$16 = \frac{S \times 69.3}{\sqrt{(0.2)^2 + S^2 X_{20}^2}}$$

or $$S = 0.048$$

$$S = \frac{N_s - N_r}{N_s}$$

and $$N_s = \frac{120 f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Therefore $$N_r = (I - S)\, N_s = (1 - 0.048) \times 1500 = 1428 \text{ rpm}$$

From condition for maximum torque, we have

$$R_2 = S X_{20}$$

$$S = \frac{R_2}{X_{20}} = \frac{0.2}{1} = 0.2$$

$$S = \frac{N_s - N_r}{N_s}$$

$$0.2 = \frac{1500 - N_r}{1500}$$

or $$N_r = 1200 \text{ rpm}$$

Rotor current at $S = 0.2$,

$$I_2 = \frac{S E_{20}}{\sqrt{R_2^2 + S^2 X_{20}^2}}$$

$$I_2 = \frac{0.2 \times 69.3}{\sqrt{(0.2)^2 + (0.2 \times 1)^2}} = 49 \text{ A per phase}$$

Rotor input for the three-phase (from expression 4.26),

$$P_i = \frac{3 I_2^2 R_2}{S}$$

$$= \frac{3 \times (49)^2 \times 0.2}{0.2} = 7203 \text{ W} = 7.203 \text{ kW}$$

4.16.3 Determination of Circuit Elements of the Equivalent Circuit

The various elements of the equivalent circuit can be determined from the readings of two tests conducted on the induction motor, namely, the *no load test* and the *blocked rotor test*.

When the motor is running on no load it has to develop only a small fraction of the full-load torque in order to overcome friction, windage and iron-losses in addition to the small amount of I^2R-loss in the stator winding. The slip at no load is very small and therefore the quantity $R_2(1 - S)/S$ of the equivalent circuit of Fig. 4.39 is very high. The rotor circuit can, therefore, be assumed to carry negligible current. The rotor for practical purposes may be considered as an open circuit and the equivalent circuit of the motor at no-load will be shown in Fig. 4.40.

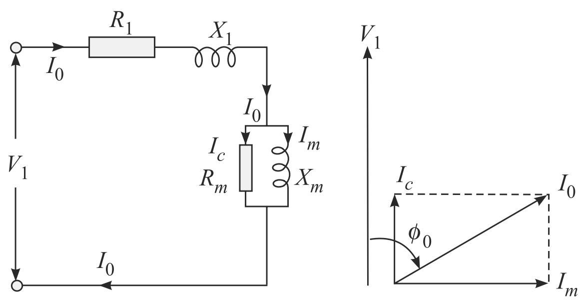


Fig. 4.40 *Equivalent circuit of an induction motor at no-load*

Input power at no load W_0 is spent as friction and windage loss $W_{F\&W}$, core loss in the stator W_i and I^2R-loss in the stator winding W_c due to no-load current I_0. Therefore,

$$W_0 = W_{F\&W} + W_i + W_c$$

If a number of readings of W_0 at no-load are taken at different stator applied voltage and W_0 is plotted against applied voltage, V, we will get a characteristic as shown in Fig. 4.41. If we extend the curve of W_0 to the left, it cuts the vertical axis at A.

OA represents the losses due to friction and windage since this is the loss which would occur if the applied voltage was zero, i.e., if there were no iron loss and I^2R-loss in the windings. If we know the stator resistance, I^2R loss at no-load at different applied voltages can be calculated and plotted as shown by the curve W_{cu}. If no-load I^2R-loss is subtracted from the curve of W_0, we will get the core loss, also called iron loss W_i of the motor.

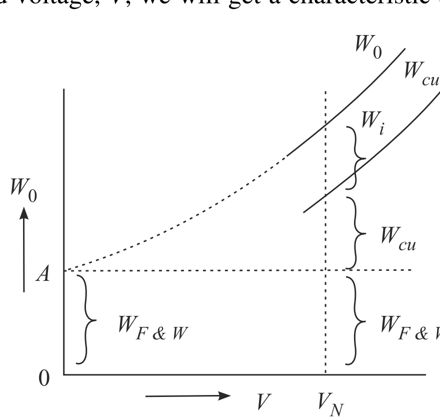


Fig. 4.41 *Separation of iron, friction and windage loss of an induction motor*

Thus, the various components of the no load loss are separated out. If V_1, I_0 and cos ϕ_0 are respectively the stator applied voltage per phase, no-load stator current per phase, and no-load power factor, then,

$$V_1 I_0 \cos \phi_0 = W_0$$

$$\cos \phi_0 = \frac{W_0}{V_0\, I_0}$$

I_0 divides itself into two components namely I_c and I_m as shown in Fig. 4.40. I_c is in phase with V_1 and I_m is lagging V_1 by 90°.

$$I_m = I_0 \sin \phi_0$$

and $$I_m = I_0 \cos \phi_0$$

The values of R_m and X_m can be calculated as

$$R_m = \frac{V_1}{I_c}$$

and $$X_m = \frac{V_1}{I_m}$$

In the above calculations, voltage drop in the stator winding has been neglected.

Blocked rotor test on an induction motor is analogous to short-circuit test on a transformer. Under blocked rotor condition the rotor is at rest and, therefore, the slip is unity. The load resistance $R'_2\left(\frac{1-S}{S}\right)$ is zero. Thus the equivalent circuit of blocked rotor condition will be as shown in Fig. 4.42.

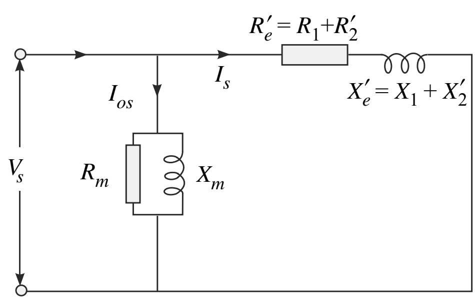


Fig. 4.42 *Equivalent circuit of an induction motor under blocked rotor condition*

In an induction motor, if full voltage is applied across the stator terminals with the rotor not allowed to rotate, about six to eight times its full-load current will flow in the circuit. That is why blocked-rotor test is conducted at a reduced voltage V_s such that current, I_s equal to only the full-load current flow through the circuit. The applied voltage V_s under blocked rotor condition is, therefore, about one-sixth to one-eight of the normal voltage. At this reduced voltage the current I_{0s} flowing through the parallel branch is very small, and can be neglected. As shown in Fig. 4.43, R_e' and X_e' are stationary resistance and reactance in series connected across the applied voltage, V_s.

If W_s is the per phase input power under blocked-rotor condition, I_s is the current and cos ϕ_s is the power factor, then

$$V_s I_s \cos \phi_s = W_s$$

or $$\cos \phi_s = \frac{V_s}{V_s I_s}$$

The impedance

$$Z_e' = \frac{V_s}{I_s}$$

$$R_e' = Z_e' \cos \phi_s$$

and $$X_e' = Z_e' \sin \phi_s$$

R_1 can be measured by ammeter-voltmeter method by applying dc to the stator winding. To get ac resistance at 50 Hz, the value of R_1 so obtained is to be multiplied by a factor, say, 1.5. Thus R_1 and R_2' from R_e' can be separated as $R_2' = R_e' - R_1$. There is no simple way of separating X_1 and X_2' from X_e' except arbitrarily. X_1 may be taken equal to X_2' so that $X_1 = X_2' = X_e'/2$.

Thus the equivalent circuit elements as envisaged in Fig. 4.39(d) are found out.

As mentioned earlier, all calculations regarding the performance of an induction motor can be done using the equivalent circuit with some approximations.

EXAMPLE 4.35 *A 400, V, 40 hp, 50 Hz, three-phase induction motor gave the following test data:*

No-load test: 400 V, 20 A, 1200 W

Blocked-rotor test: 100 V, 45 A, 2750 W

Stator dc resistance per phase is 0.01 Ω

The ratio of ac to dc resistance is 1.5. The friction and windage loss is 300 W. Calculate the circuit elements of the approximate equivalent circuit of the motor.

Solution

$$R_1(\text{ac}) = 1.5 \times R_1(\text{dc})$$

$$= 1.5 \times 0.01 = 0.015\ \Omega$$

At no-load,

I^2R-loss in the stator phases $= 3I^2{}_0R_1$

$$= 3 \times 20^2 \times 0.015 = 18\ \text{W}$$

$$\text{Core-loss} = 1200 - \text{stator } I^2R\text{-loss} - \text{friction and windage loss}$$

$$= 1200 - 18 - 300 = 882\ \text{W}$$

$$\text{Core-loss per phase} = \frac{882}{3} = 294\ \text{W}$$

$$\text{Applied voltage per phase} = \frac{400}{\sqrt{3}}\ \text{V} = 231\ \text{V}$$

To calculate R_m,

$$\frac{V^2}{R_m} = \text{Core-loss}$$

or $$\frac{400 \times 400}{3 \times R_m} = 294$$

$$R_m = 181\ \Omega$$

or $$\cos \phi_0 = \frac{294}{231 \times 20}$$

or $\cos \phi_0 = 0.064$

$$\cos \phi_0 = 86°$$

$$\sin \phi_0 = 0.997$$

$$X_m = \frac{V}{I_0 \sin \phi_0} = \frac{400}{\sqrt{3} \times 20 \times 0.997} = 11.6\ \Omega$$

Under blocked rotor condition,

$$V/\text{phase} = \frac{100}{\sqrt{3}} V$$

Motor impedance referred to stator side

$$Z_e' = \frac{100}{\sqrt{3} \times 45} = 1.28\ \Omega$$

Neglecting core-loss under blocked rotor condition.

$$3\, I_{sc}^2 R_e' = 2750$$

$$R_e' = \frac{2750}{3 \times 45^2} = 0.45\ \Omega$$

Stator resistance, $R_1 = 0.015\ \Omega$

Rotor resistance referred to stator side

$$R_2' = R_e' - R_1 = 0.45 - 0.015 = 0.435\ \Omega$$

Equivalent reactance of the motor referred to stator side

$$X_e' = \sqrt{Z_2'^2 - R_2'^2}$$

$$= \sqrt{(1.28)^2 - (0.45)^2} = 1.2\ \Omega$$

Assuming $X_1 = X_2'$

$$X_2' = \frac{X_e'}{2} = 0.6\ \Omega$$

and $X_1 = 0.6\ \Omega$

Thus the element of the approximate equivalent circuit are:

$$R_m = 181\ \Omega;\ X_m = 11.6\ \Omega$$

$$R_1 = 0.015\ \Omega;\ R_2' = 0.435\ \Omega;\ R_e' = 0.45\ \Omega$$

$$X_1 = 0.6\ \Omega;\ X_2' = 0.6\ \Omega\ X_e' = 1.2\ \Omega$$

EXAMPLE 4.36 *A 400 V, 50 Hz, three-phase star-connected, 4-pole induction motor has stator impedance, $Z_1 = (0.07 + j\,0.3)\ \Omega$ and rotor impedance referred to stator side, $Z_2' = (0.08 + j\,0.3)\ \Omega$. The magnetising reactance is 10 Ω and resistance representing core-loss is 50 Ω.. Calculate (a) stator current and power factor, (b) rotor current, (c) developed torque and (d) gross efficiency by using approximate equivalent circuit. Assume a slip of 4 per cent.*

Solution The equivalent circuit of the induction motor is shown in Fig. 4.42. The values of the circuit elements have been found out from the given data.

Let Z is the total impedance of the circuit *CEFD* of Fig. 4.43.

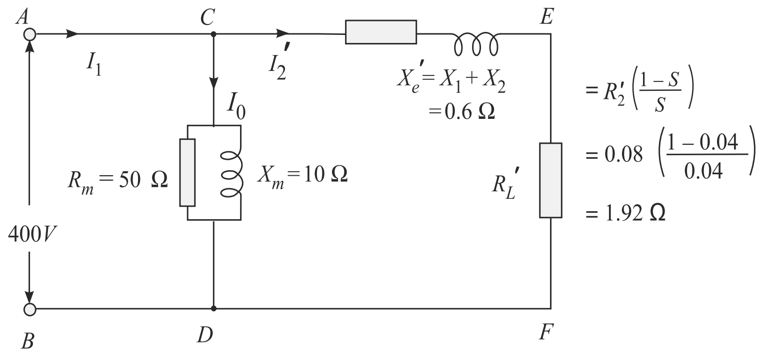


Fig. 4.43 *Refer to Example 4.22*

Rotor current, $$I_2' = \frac{V}{Z} = \frac{400}{\sqrt{3}(1.92 + 0.15 + j0.6)}$$

$$= \frac{400}{\sqrt{3}(2.07 + j0.6)}$$

or $$I_2' = 107 \angle -16.5^\circ\ A$$

$$I_0 = I_c + I_m$$

$$R_e' = R_1 + R_2'\ X_e' = X_1 + X_2'$$

$$I_0 = I_c + I_m = \frac{V}{R_m} + \frac{V}{jX_m}$$

$$= \frac{400}{\sqrt{3} \times 50} - \frac{j400}{\sqrt{3} \times 10} = 4.6 - j\,23$$

$$I_0 = 23.4 \angle -79^\circ\ A$$

$$I_1 = I_0 + I_2'$$

$$= 23.4 \angle -79^\circ + 107 \angle -16.5^\circ$$

$$= 119.7 \angle -26.5^\circ$$

Primary current $I_1 = 119.7$ A

Primary power factor

$$\cos \phi_1 = \cos 26^\circ$$

$$= 0.89 \text{ lagging}$$

Synchronous speed $$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Rotor speed $$N_r = N_s(1 - S)$$

$$= 1500\,(1 - 0.04) = 1440 \text{ rpm}$$

$$\text{Rotor output} = 3I_2'^2 R_L'$$
$$= 3 \times (107)^2 \times 1.92 = 65946 \text{ W}$$

Again $$\text{Rotor output} = 2\pi T_d \frac{N_r}{60} = 65946$$

or $$T_d = \frac{65946 \times 60}{2\pi \times 1440} \text{ Nm}$$

or $$T_d = \text{Torque developed} = 437.3 \text{ Nm}$$

$$\text{Input power to stator} = \sqrt{3}\, V_1 I_1 \cos \phi_1$$
$$= \sqrt{3} \times 400 \times 119.7 \times 0.89 = 073808 \text{ W}$$

Percentage gross efficiency

$$= \frac{\text{Gross output}}{\text{Input}}$$
$$= \frac{65946 \times 100}{73808} = 89.4\%$$

4.17 CIRCLE DIAGRAM OF AN INDUCTION MOTOR

Although it is possible to compute the operating characteristics of an induction motor by using the equivalent circuit, it is simpler and more convenient to use a circle diagram for this purpose. If we neglect the parallel branch of the equivalent circuit of an induction motor, it reduces into a simple series circuit having a constant voltage supply. The locus of current drawn by an induction motor is referred to as a circle diagram. The circle diagram can be drawn by using no-load and blocked-rotor test data. From the circle diagram it is possible to obtain graphically a considerable range of information like full-load current and power factor, maximum power output, pull-out torque, full-load efficiency, etc.

4.17.1 Locus of Current Drawn by an Induction Motor

The locus of current in a series circuit with a constant reactance and a variable resistance connected across a fixed voltage is a circle. This can be seen in Fig. 4.44.

From Fig. 4.44,

$$I = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{X}\frac{X}{\sqrt{R^2 + X^2}} = \frac{V}{X} \sin \phi$$

$I = (V/X) \sin \phi$ is the equation of a circle in polar from. If the values of I for different values of R are represented, the locus of current phasors will be on a semicircle as shown in Fig. 4.44(b). When the value of R is zero, $\sin \phi = 1$, i.e., $\phi = 90°$ and $I = V/X$. The magnitude of current I is maximum and it lags the voltage by 90°. The position of current vector is shown as OA in the figure. When the value of R is infinitely large, the value of $\sin \phi$ is zero and hence current I is also infinitely small which is represented by point. O in the figure. Thus the two extreme points, i.e., O and A of the current locus are determined. Now, assume that $R = X$. Then $\sin \phi = 1/\sqrt{2}$, i.e.,

$\phi = 45°$. The magnitude of current $I = (V/X)\ (1/\sqrt{2})$ is lagging V by 45° which is represented by vector OB. If the variable values of I are shown vectorially, the locus of the tip of the vectors will lie on a semicircle as shown.

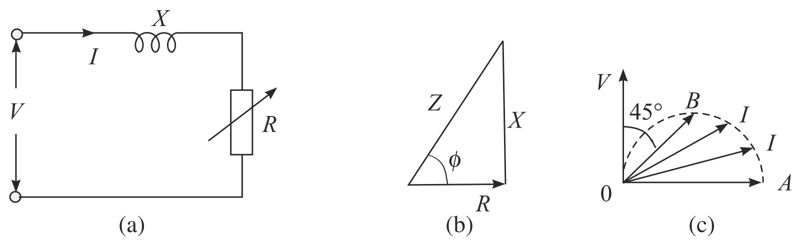


Fig. 4.44 *(a) A variable resistance in series with a fixed value reactance connected across a supply voltage; (b) the impedance triangle; (c) the locus of current in the series circuit as resistance is varied from zero to infinity*

The approximate equivalent circuit of the induction motor is redrawn as shown in Fig. 4.45.

The fixed resistance R_e' and the variable resistance R_L' can together be considered as a variable resistance. Thus, across the terminals AB we have a fixed reactance X_e' and a variable resistance $R_e' + R_L'$ connected across a fixed voltage V_1.

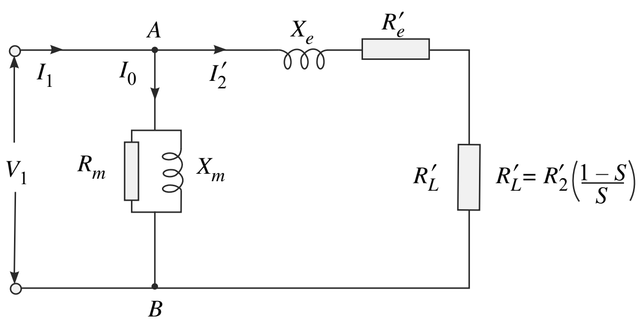


Fig. 4.45 *Approximate equivalent circuit of an induction motor*

The locus of current I_2' will, therefore, be a semicircle. The diameter of the semi circle representing the maximum value of current I_2' will be V_1/X_e' as shown in Fig. 4.46. It may be remembered that I_2' is the rotor current referred to the stator.

I_0 is the no-load current and I_1 is the stator current. The vector sum of I_2' and I_0 is I_1. The no-load current I_0 is shown lagging V_1 by a large angle as the no-load power factor is low. If R_m and X_m are assumed to be constant, then both I_0

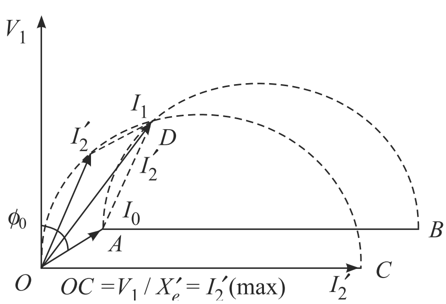


Fig. 4.46 *Locus of current drawn by an induction motor*

and cos ϕ_0 will be constant. Then the locus of current I_1 will also lie on a semicircle shown as *ADB*.

4.17.2 Construction of Circle Diagram

To construct the circle diagram three tests are to be performed, namely (a) no-load test, (b) blocked-rotor test and (c) stator resistance test. From no-load test I_0 is known and ϕ_0 is found out as (taking all quantities on per phase values).

$$V_1 I_0 \cos \phi_0 = W_0$$

or
$$\phi_0 = \cos^{-1} \frac{W_0}{V_1 I_0}$$

By choosing a proper current scale, I_0 can be drawn as *OC* shown in Fig. 4.47, with V_1 as vertical reference axis.

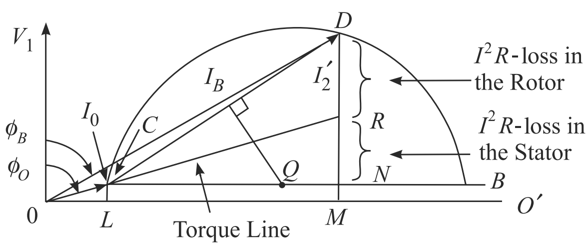


Fig. 4.47 *Circle diagram of an induction motor*

From blocked rotor test I_2' is known. Convert this I_2' at normal voltage level as:

$$I_2' \text{ (at normal voltage)} = I_2' \frac{V_1}{V_s}$$

Power factor under blocked-rotor condition with all quantities on per phase values is given as:

$$\cos \phi_s = \frac{W_1}{V_s I_s}$$

With the same current scale, I_2' corresponding to normal voltage can be drawn at an angle ϕ_s with the vertical axis shown by vector *CD*. Thus *OD* represents the total current drawn by the stator. The locus of the current I_1 will pass through the two extreme points, namely through *C* at no-load and through *D* which corresponds to blocked-rotor condition. The centre of semicircle may be found out thus: draw a perpendicular bisector from the line *CD*. This will cut the horizontal line *CB* drawn from *C* at the point *Q*. With *Q* as centre and *QC* as radius draw a semicircle *CDB*. Draw also a horizontal line *OO*′, parallel to the line *CB*. Draw perpendicular lines from the points *C* and *D* on the horizontal line as *CL* and *DM*. *DM* cuts the horizontal line *CB* at *N* such that *NM* = *CL*. As applied voltage V_1 is shown as vertical line, *CL* which is equal to $I_0 \cos \phi_0$ is proportional to the no-load input. Thus the length *CL* can be equated to the no-load input power W_0 which supplies core-loss, friction and windage loss and a small amount of I^2R loss.

The power scale can be determined by considering the vertical distance CL in cm as equal to W_0. The power scale is calculated thus

$$\text{Power scale: } 1 \text{ cm} = \frac{W_0}{CL \text{ in cm}}$$

For better accuracy power scale should be calculated considering length *DM* as equivalent to the input under blocked rotor condition.

The vertical distance *DM* represents the input power under blocked-rotor condition with rated voltage applied across the stator. Distance *NM* has been assumed to be equal to *CL* representing core-loss and friction and windage losses. This is an approximation as under blocked-rotor condition, there is no friction and windage loss. The remaining part, *DN* of the input at blocked rotor condition is wasted as I^2R-loss in the stator and rotor, the output under blocked rotor condition being zero. If we assume, for the time being, stator I^2R-loss as equal to rotor I^2R-loss. then we may divide the line *DN* at *R*. *NR* represents the stator I^2R-loss and *RD* which is equal to the rotor input is wasted as I^2R-loss in the rotor.

The line *CD* represents the output line. The vertical distance above this line up to the periphery of the circle expressed in power scale will represent the output power. The line *CR* separating the stator and rotor I^2R-loss is the torque line. Vertical distance above this line up to the periphery of the circle is the developed torque. We have assumed stator and rotor I^2R-loss to be equal and divided the line *DN* at *R*. The exact position of the point *R* can be located as follows:

Input power, W_s under blocked-rotor condition is wasted as I^2R-loss in the stator and rotor. The stator circuit resistance can be measured as follows:

$$\text{Stator Cu-loss} = 3I_1^2 R_1$$

$$\text{Rotor } I^2R\text{-loss} = W_S - 3I_1^2 R_1$$

Therefore, for squirrel-cage motors,

$$\frac{RN}{RD} = \frac{3I_1^2 R_1}{W_s - 3I_1^2 R_1}$$

For slip-ring motors, resistances R_1 and R_2 can be determined and the ratio

$$\frac{RN}{RD} = \frac{I_1^2 R_1}{I_2^2 R_2} = \frac{R_1}{R_2}\left[\frac{I_1}{I_2}\right]^2$$

From the circle diagram as drawn in Fig. 4.47 we can calculate the various quantities which will illustrate the performance of the motor at any particular load. Let us redraw the circle diagram of Fig. 4.47 and let us assume that the motor is taking a current represented by *OP* in Fig. 4.48.

The perpendicular line *PG* represents power input. *GH* represents fixed losses. *HK* and *KJ* represents respectively the stator and rotor I^2R-loss. *JP* represents output power and *KP* represent output torque.

To determine the maximum power developed by the motor, draw a line parallel to the output line tangent to the circle at point *S*. Draw a vertical line *ST* from the point *S* to the output line. The length *ST* represents the maximum output.

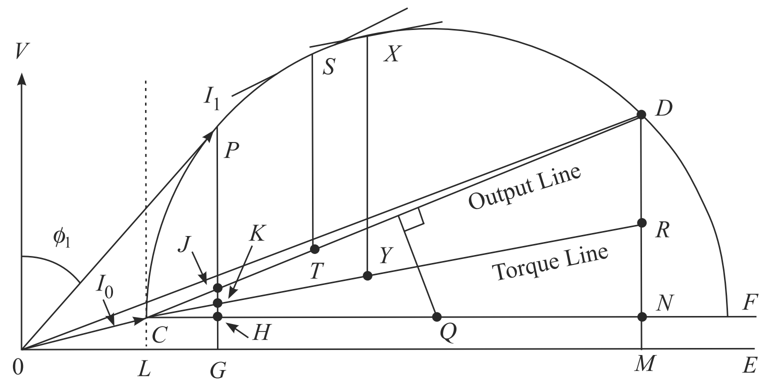


Fig. 4.48 *Method of construction of circle diagram of an induction motor*

Similarly, to determine maximum torque developed by the motor, draw a line parallel to the torque line tangent to the circle at point X. Draw a vertical line XY from the point X to the torque line. The length XY represents the maximum torque.

EXAMPLE 4.37 *A 400 V, 40 hp, 50 Hz, 4 pole delta-connected induction motor gave the following test data.*

No-load test: 400 V, 20 A, 1200 W

Blocked-rotor test: 100 V, 45 A, 2800 W

Draw the circle diagram and determine

(a) the line current and power factor at rated output;

(b) the maximum output;

(c) the maximum torque;

(d) the full-load efficiency;

(e) the full-load rotor speed.

Assume stator and rotor I^2R*-losses to be equal at standstill.*

Solution The test data given are assumed to be of line values.

$$\text{No-load power factor, } \cos\phi_0 = \frac{W_0}{\sqrt{3}\,V_1 I_1} = \frac{1200}{1.732 \times 400 \times 200}$$

$$\cos\phi_0 = 0.0866$$

$$\phi_0 = 85°$$

Similarly, power factor under blocked-rotor condition,

$$\cos\phi_s = \frac{2800}{1.732 \times 100 \times 45} = 0.359$$

$$\phi_s = 69°$$

Input current and input power at blocked rotor condition are at a reduced voltage of 100 V. These quantities are to be converted into rated voltage of 400 V.

Thus, Input current at blocked-rotor condition at 400 V

$$= 45 \times \frac{400}{100} = 180 \text{ A}$$

Input power at blocked-rotor condition at 400 V

$$= 2800 \left[\frac{400}{100}\right]^2 = 44800 \text{ W}$$

Input power at no-load at 400 V = 1200 W

Let us now choose a convenient current scale of

$$1 \text{ cm} = 10 \text{ A}$$

The circle diagram shown in Fig. 4.49 is constructed through the following steps:

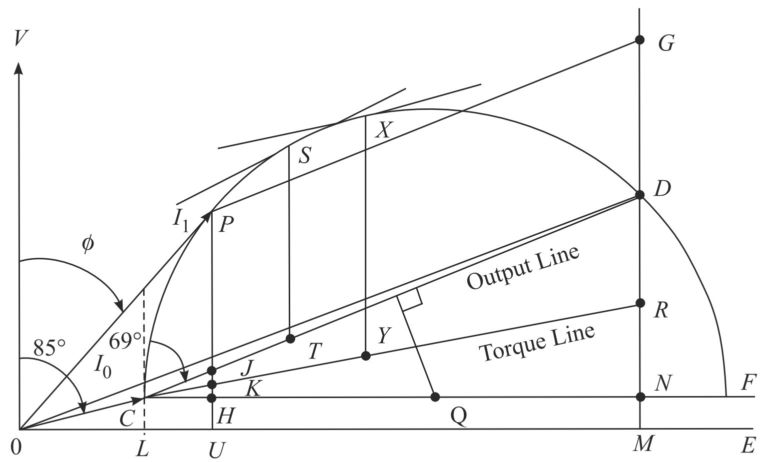


Fig. 4.49 *Circle diagram of an induction motor*

Represent voltage *V* on the vertical axis. Draw a horizontal line *OE* from *O*. Since current scale is 1 cm = 10 A, no-load current of 20 A is equivalent to 2 cm. No-load power factor angle is 85°. Thus, represent no-load current by the vector *OC* whose length is 2 cm and is lagging the vertical voltage axis by 85°.

Similarly, represent the input current of 180 A under blocked rotor condition by a vector *CD* of 18 cm length and lagging voltage vector by 69°.

Draw a horizontal line *CF* from *C* parallel to the line *OE*. Join *OD*. Draw a perpendicular bisector from *CD* to cut the horizontal line *CF* at *Q*. With *Q* as centre and *QC* as radius draw a semicircle. From *D* on the semicircle draw a vertical line *DM* on the horizontal axis. *DM* cuts *CF* at *N* such that *NM* = *CL*. Since stator and rotor I^2R-losses are assumed to be equal, divide *DN* at *R* and join *CR*. Now *CD* represents the output line and *CR* represents the torque line.

In the circle diagram, length *DM* represents input at blocked-rotor condition, such that

$$DM = 44800 \text{ W}$$
$$DM = 6.8 \text{ cm}$$

From this, $$1 \text{ cm} = \frac{44800}{6.8} = 6588 \text{ W}$$

which is the power scale for the circle diagram.

Now, full-load output of the motor

$$= 40 \text{ hp}$$

$$= 40 \times 735.5 = 29420 \text{ W}$$

On the power scale 29420 W represent

$$\frac{29420}{6588} = 4.46 \text{ cm}$$

JP represents 4.46 cm above the output line. To locate the position of *JP* we may raise the vertical line *MD* and cut 4.46 cm from it. *DG* is 4.46 cm in the figure. Now draw a line from *G* parallel to the output line *CD* to cut the circle at *P*. From *P* drop a vertical line *PJ* onto the output line. *OP* represents the full-load input current.

Full-load current, $OP = 6.1 \text{ cm} \times 10 \text{ A} = 61 \text{ A}$

Input line current at full-load = 61 A

Power factor all full-load

$$\cos \phi = \cos 34° = 0.829$$

$$\text{Input} = PU \text{ in cm} \times \text{power scale}$$

$$= 5.1 \times 6488 \text{ W}$$

$$\text{Output} = PJ \text{ in cm} \times \text{power scale}$$

$$= 4.46 \times 6488 \text{ W}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{PJ}{PU} = \frac{4.46}{5.1}$$

$$= 0.874 = 87.4\%$$

To determine the maximum output, draw a line parallel to the output line, tangent to the semicircle at point *S*. The vertical distance *ST* represents the maximum output.

In this case *ST* measures 6.7 cm

Using power scale of

$$1 \text{ cm} = 6588 \text{ W}$$

$$\text{Maximum output} = 6.7 \times 6588 \text{ W} = 60 \text{ hp}$$

To determine the maximum torque, draw a line parallel to the torque line, tangent to the circle at point *X*. The vertical distance *XY* represents the maximum torque. In this case *XY* is 8.1 cm. Using power scale, maximum torque = 6588 × 8.1 Syn. *W* = 53363 Syn. W.

To determine full-load rotor speed we can use the relation

$$\text{Rotor } I^2\text{-loss} = \text{Slip} \times \text{Rotor input}$$

At full-load rotor I^2R-loss is represented by the distance *JK* which is equal to 0.3 cm. Rotor input is represented by *PK* and is equal to 4.65 cm.

Thus, $$\text{slip } S = \frac{0.3}{4.65} = 0.0646$$

Synchronous speed,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Slip $$S = \frac{N_s - N_r}{N_s}$$

or $$0.0646 = \frac{1500 - N_r}{1500}$$

Therefore, rotor speed at full-load

$$N_r = 1403 \text{ rpm}$$

4.18 OPERATION OF INDUCTION MACHINE AS INDUCTION GENERATOR

Here the stator windings of an induction motor are connected across a three-phase supply of constant voltage and frequency and the rotor is mechanically coupled to a prime-mover. If the prime-mover is capable of driving the induction machine at a speed higher than the synchronous speed, it will be observed that energy is returned to the ac mains. The machine will then work as an induction generator. Since the speed of the rotor is different from the synchronous speed, such generators will be called asynchronous generators. As the primemover rotates the rotor at a speed higher than the synchronous speed, the value of slip becomes negative.

In the equivalent circuit of an induction motor of Fig. 4.39(d), the induction motor has been shown delivering power to an equivalent load resistance of $R_2(1 - S)/S$, which represents the power converted to mechanical work. In an induction generator, the slip is negative and, therefore, the load resistance is $-R_2(1+S)/S$ which is negative. This means, the load resistance no longer absorbs power, but acts as a source of power. In other words, in an induction generator, there is a transfer of energy from mechanical form to electrical form.

When, for negative slip torque is calculated using Eq. (4.21), the torque speed characteristics is extended as shown in Fig. 4.50.

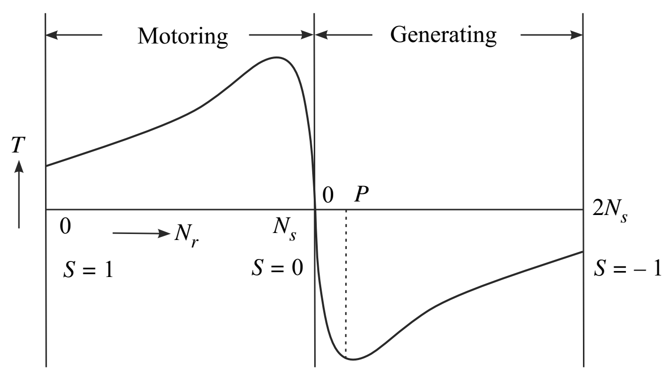


Fig. 4.50 *Torque-speed characteristic of an induction machine*

The operating range of the machine as a generator is limited to the maximum value of torque corresponding to a slip of *OP* as indicated in Fig. 4.50. It should be understood that an induction generator is capable of supplying power to only an existing supply system and not to a simple resistive load connected across its terminals.

4.18.1 Difference between a Synchronous Generator and an Induction Generator

The main points of difference between a synchronous generator (conventional ac generator) and an induction generator are as follows

(a) Induction generator needs no direct current excitation.
(b) Induction generator will generate only when it is connected to the lines.
(c) For an induction generator, no synchronising is required since the machine will generate only when it is connected to the lines.

Asynchronous induction generator as explained above have very limited applications Some uses are mentioned below:

(a) When the load is being lowered by an ac driven hoist, gravity can act as primemover to the motor driving the hoist. If the speed of the motor is allowed to exceed synchronous speed, a braking action will be created to the falling load, as the driving motor will now work as a generator. For speed lower than the synchronous speed, however, braking will have to be done by some other means.
(b) Small and variable amount of water power which do not call for the setting up of a normal station, can be utilised by arranging for a water wheel to drive squirrel-cage induction machine.

When there is adequate water, the machine gets automatically switched on to overhead line energised from another normal power station. In case of insufficient water supply to drive the rotor above synchronous speed, the machine gets switched off automatically.

4.18.2 Isolated Induction Generator

We have noted earlier that an induction generator will generate only when it is connected to the supply lines. However, an induction machine can be made to work as an induction generator even without an external power supply in isolated places like in hilly areas, or desert areas. Delta connected capacitor bank is connected across the terminals of the induction machine as shown in Fig. 4.51. These delta connected capacitors across the generator terminals provide the excitation current required for the machine to work as a generator. When the prime-mover rotates the motor, a small amount of emf is induced in the stator because of the residual magnetism present in the rotor. The frequency of the emf induced will depend upon the rotor speed. This way, the machine works as a self excited induction generator. The induction generator does not require three-phase supply for excitation. Due to residual magnetism of the rotor, a small amount of voltage is induced in the stator winding when the rotor is rotated by a prime-mover, which may be a windmill. The current flowing through the capacitor, $I_C = \dfrac{V}{X_C} = \dfrac{V}{1/\omega C} = V\omega C.$ This current is equal to the magnetizing current, I_m so that

$$I_m = \frac{V}{X_m} = I_C = \frac{V}{X_C} = V\omega C$$

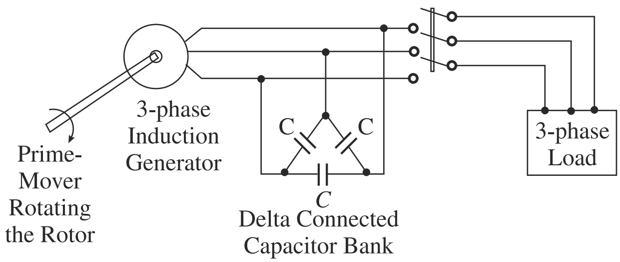


Fig. 4.51 *An isolated induction generator*

The induced voltage, V as a function of I_m has been shown in Fig. 4.52. Like a self-excited dc generator, the voltage across the stator terminals will build up. The ultimate voltage induced will be decided by the point of intersection of V versus I_m curve and the capacitive reactance line, X_C as has been shown.

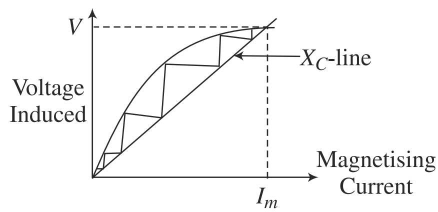


Fig. 4.52 *Voltage build up in an induction generator*

Induction machine working as induction generator are used for power generation using renewable energy sources like wind mill generators, in isolated places. Such a generating unit is simple, costs less and requires less maintenance as compared to a synchronous generator.

Induction generators can be used as stand-alone power supply unit to loads in remote places where power supply from the electricity grid is not available.

4.19 CRAWLING OF INDUCTION MOTORS

Squirrel-cage induction motors may sometimes show a tendency to run at a very low speed usually one-seventh of its normal speed. This is because of the presence of harmonics in the nonsinusoidal flux wave shape produced by the stator mmf. The harmonic flux will also give rise to torque-speed characteristic similar to the one produced by the fundamental sine wave. The synchronous speed of the rotating magnetic field produced by the harmonic flux will be different from the one due to the fundamental flux. The effect of harmonic waves of the air-gap flux on torque-speed characteristic of a three phase squirrel cage induction motor is explained as follows. The effect of time harmonics of the supply voltage and the effect of space harmonics of the air-gap flux on the torque-speed characteristic will be considered separately.

4.19.1 Effect of Time Harmonics of the Supply Voltage on the Torque-Speed Characteristic

If the voltage supplied to a polyphase induction motor contains time harmonics, the air-gap flux will have components that will rotate at speeds other than the speed related to the fundamental frequency. For example, a fifth time-harmonic flux would tend to rotate the motor at five times the speed due to the fundamental. An analysis of the phenomena will show that such harmonics are of little significance because the torque that these time harmonics would produce are usually very small throughout the operating range of the motor. The time-harmonic voltage is usually small in proportion to the fundamental voltage. The reactance of the motor to the time-harmonics is high ($X = 2\pi f_n L$; where f_n is the frequency of the harmonic flux and is multiple of the fundamental). Hence harmonic current flowing through the winding will be small and therefore, the torque produced by these time-harmonics are negligible in the operating range of the motor.

4.19.2 Effect of Space-harmonic of the Air-gap Flux on the Torque-Speed Characteristic

Space-harmonics may be produced by the sinsusoidal currents received from the supply when the stator windings do not possess perfect distribution; or, there is a variation of permeance of the air-gap due to the slots.

Figure 4.53 shows a stator coil *a-a′* carrying current *i* which is varying sinusoidally with time. The space flux produced by this current is shown rectangular in shape but the amplitude of this rectangular wave varies sinusoidally with time. The space distribution is such that the flux wave contains, in addition to its fundamental, an infinite number of space-harmonics, the time variation of all the harmonic waves of the air-gap flux is the same as for the fundamental, since they all are product by the same Current.

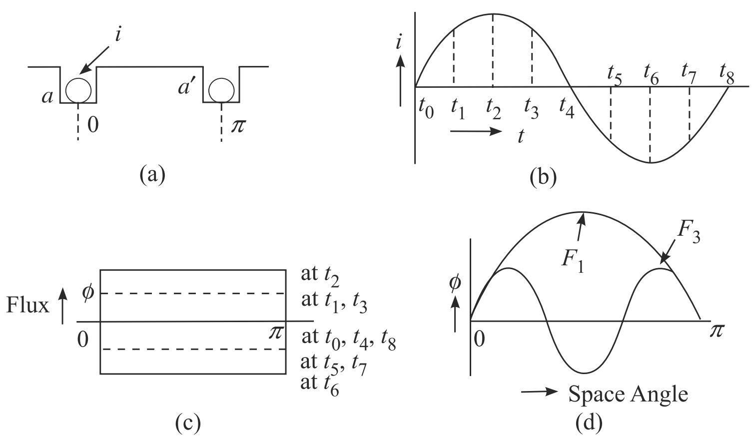


Fig. 4.53 *(a) Current carrying coil placed in stator slots (b) Variation of stator current with time (c) Variation of space distribution of flux with time (d) Fundamental and third harmonic component of the space flux*

In Fig. 4.53(d), for simplicity, is shown only the fundamental and the third-harmonic flux. From figure it is evident that the third-harmonic flux will produce three times as many magnetic poles as the fundamental component. Likewise, the fifth harmonic would produce five times the number of poles as the fundamental, and so on. Therefore, if the space-harmonics are present in the air-gap flux, they will produce rotating fields that would travel around the air-gap at a speed equal to the synchronous speed divided by the order of the harmonic, Harmonic flux may rotate in either direction depending upon the order of the harmonic. Each harmonic rotating flux will produce torque on the rotor; their magnitude, direction, etc., will depend upon the order of the harmonic. Since there is induction motor action corresponding to each space-harmonic of the stator mmf, a polyphase induction motor can be considered as equivalent to a number of mechanically coupled motors, with different number of poles. For example, let us consider induction motor action due to the fundamental, the fifth and the seventh harmonics in the flux distribution. The motor is equivalent to three mechanically coupled motors with number of poles P_1, 5 P_1 and 7 P_1. Torque-speed characteristics due to the fundamental and the seventh-harmonic flux are shown in Fig. 4.54.

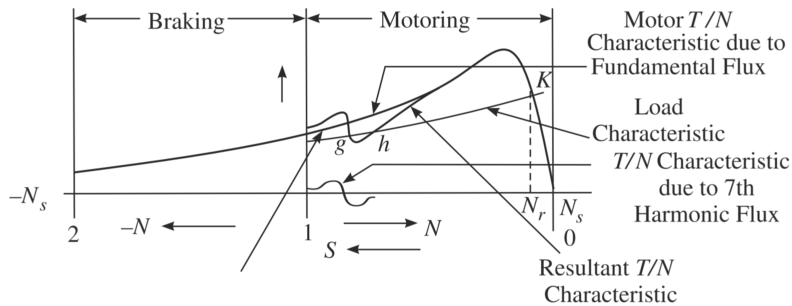


Fig. 4.54 *Effect of harmonic flux torque on the torque-speed characteristic of an induction motor*

The seventh harmonic produces a dip in the torque-speed curve at one-seventh of the synchronous speed. There will be a dip in the torque-speed curve due to the fifth harmonic but because the phase order of the fifth harmonic is opposite to that of the fundamental, this dip occurs at a slip which is greater than unity, i.e., in the negative direction of motor rotation (not shown in the figure).

If the motor torque developed is due to the fundamental flux alone, the motor will accelerate to the point K as a final operating speed for the given load. On the other hand, due to the presence of seventh-harmonic flux the resultant torque-speed characteristic will be as shown in the figure. Torque due to harmonic flux causes dip in the torque-speed characteristic. The motor will hang or *crawl* at the reduced speed corresponding to g in the figure. A momentary reduction in load may permit the motor to accelerate to h, at which developed torque by the motor is equal to the load torque. Such operation, however, is unstable, and the motor, once it has reached the speed of h, may accelerate to the point K.

By proper choice of coil pitch and distribution of coils while designing the winding, the possibility of the presence of harmonics in the air-gap flux wave is either made zero or reduced to a very low value to eliminate the crawling effect.

4.20 COGGING

If the number of stator and rotor slots of an induction motor are either equal or have an integral ratio, the rotor may fail to start. With the number of stator slots equal to or an integral multiple of rotor slots, there would exist a strong alignment force between the stator and the rotor at standstill condition because of the pronounced variation of air-gap reluctance. The alignment force at the instant of start may become stronger than the starting torque resulting in failure of the motor to start. This phenomenon of motor refusing to start is known as cogging. To avoid cogging the number of stator and rotor slots are never made to be equal or have an integral ratio.

4.21 INDUCTION REGULATOR

Due to the vast distribution network, voltage variation is a normal phenomenon in electrical systems. Without the use of auxiliary apparatus, it is not possible to maintain constant voltage over the whole length of feeder. The variation of voltage at the end of the feeder depends on the length of the feeder as also on the loading of the feeder. An induction regulator is used to maintain the voltage at the feeder end, i.e., at the distribution point nearly constant.

An induction regulator enables a smooth variation of output voltage, whereas a tap changer transformer enables voltage control only in steps. In induction voltage regulators, the output voltage is controlled by varying the angle between the magnetic axes of primary (Rotor) and secondary (Stator) windings. An induction regulator may be single-phase or three-phase. In both the types the rotor is moved usually by a maximum of 180 degrees.

4.21.1 Single-phase Induction Regulator

The general construction of an induction regulator is similar to an induction motor. In this case, however, the rotor carries the primary winding and the stator houses the secondary winding. A compensating winding is also placed in the rotor slots whose axis is perpendicular to the axis of the primary winding as shown in Fig. 4.55(a). The compensating winding is kept short circuited. The primary winding of the rotor is fed with input voltage through flexible leads (no slip-ring is required as the rotor is not expected to rotate continuously). Maximum emf will be induced in the secondary winding when the axes of the two windings are parallel. When the rotor is rotated by 90 degrees, no emf will be induced in the secondary winding. However, when the rotor is rotated by 180 degrees the emf induced will be negative maximum.

The connection diagram for the various windings has been shown in Fig. 4.55(b). Since the axes of primary winding and the secondary winding are in phase, the voltage V_1 and induced emf E_2 get added up vectorially to give the output voltage V_2 as shown in Fig. 4.55(c). If the rotor is rotated by 180 degrees, the two voltages V_1 and E_2 will be in phase opposition and the resultant output voltage will be equal to $V_1 - E_2$. Thus in one of the two extreme cases output voltage will be boosted upto $V_1 + E_2$, and in case of the other output voltage will be bucked to $V_1 - E_2$. At any intermediate position, when the angle between the axes of the two windings becomes θ, the resultant output voltage will be $V_1 + E_2 \cos \theta$. Note that the angle between the axes of the primary winding and the compensating winding is always 90 degrees as they are both placed on the rotor When the rotor winding makes an angle of 90 degrees with the stator winding the relative positions of the three windings will be as shown in Fig. 4.56. The emf induced in the secondary winding in this position will be zero because it is at right angle with the primary winding. The relative angle

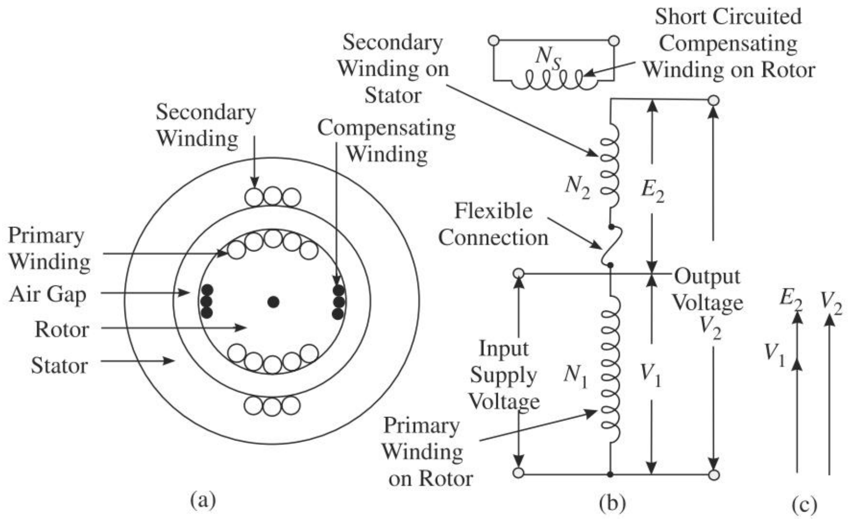


Fig. 4.55 *(a) Constructional features of a single phase induction regulator (b) Connection diagram (c) Voltage phases diagram when the axis of primary and secondary winding are parallel*

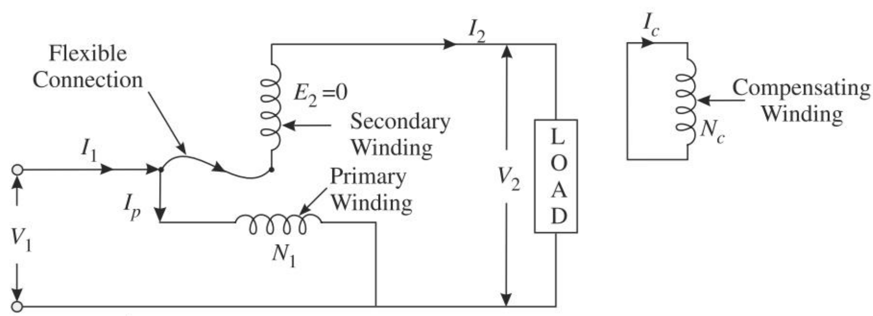


Fig. 4.56 *Relative positions of three windings of a single-phase induction regulator when the rotor winding is in quadrature with stator winding*

between the primary winding and the compensating is always 90 degrees as shown. The output voltage V_2 will be equal to input voltage V_1 provided there is no voltage drop across the secondary winding. However, in the absence of the compensating winding there will be large voltage drop across the secondary winding due to the flow of load current through this winding as shown in Fig. 4.56. The mmf I_2N_2 is neutralise by mmf of the compensating winding I_C N_C making the reactance of secondary winding almost equal to zero. The rating of a single phase induction regulator is the product of E_2 and I_2, where, E_2 is the maximum value of the emf induced in the secondary winding and I_2 is the full-load output current.

4.21.2 Three-phase Induction Regulator

A three-phase induction regulator is similar in construction to that of a single-phase induction regulator except that here both the rotor and stator houses three-phase windings. Three-phase supply is applied across the three-phase primary windings placed in the rotor slots through flexible wire connections. A rotating magnetic field of constant magnitude is produced which cuts the secondary windings placed in the stator slots. Emf's of constant magnitude and having a phase difference of 120 degrees are induced in the three-phase secondary windings. So the phasor sum of the primary voltage and the secondary induced emf gives rise to increased or decreased output voltage depending on the rotor position. The constructional features and the connection diagram of a three-phase induction regulator have been shown in Fig. 4.57(a) and (b) respectively. It may be noted that there is no requirement of a compensating winding in a three phase induction regulator.

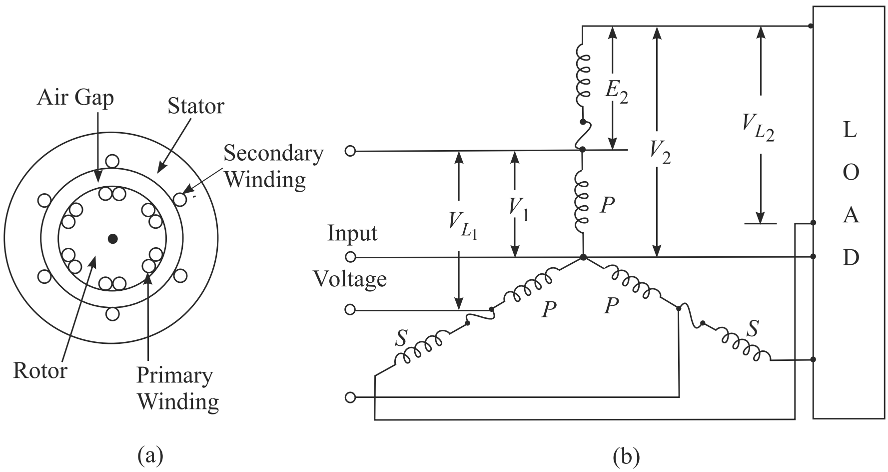


Fig. 4.57 *(a) Constructional features of a three-phase induction regulator (b) Connection diagram*

In Fig. 4.58 is shown the phasor diagram representing the primary supply voltage V_1, the secondary induced emf E_2 and the output voltage V_2 for all the three phases for a particular rotor position. Voltage across *AB*, *BC*, *CA* represent the input line voltage while voltage across *PQ*, *QR*, *RS* represent the output line voltages. Depending on the rotor position the tips of the secondary voltages of the three phases will lie on circles as shown by dotted lines and hence the phasor sum of V_1 and E_2 of all the three phases will change accordingly. Induction regulators

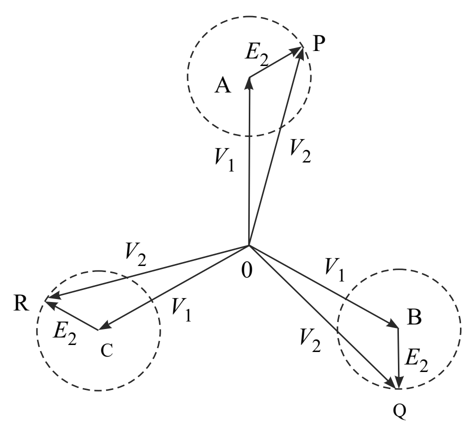


Fig. 4.58 *Phasor diagram representing the primary supply voltage V_1, the secondary induced E_2 and the output voltage V_2 for all the three phases for a particular rotor position*

have the advantage that they provide a constant voltage adjustment depending upon the loading of the lines. However, their use involves more investment as compared to tap changing transformers.

4.22 LINEAR INDUCTION MOTOR

If we cut axially an ordinary induction motor and is able to lay it flat, a linear induction motor (LIM) will be produced. Such a linear induction motor will work on the same principle as rotating induction motor. The linear version of the induction motor will produce linear or translational motion.

The stator of a rotating three-phase induction motor forms the primary of the LIM. The rotor of the induction motor forms the secondary of the LIM. Instead of terms stator and rotor, for LIM we use primary and secondary respectively. The secondary of the LIM may be made of a simple metal sheet instead of a winding made on a linear core. If we compare an ordinary induction motor with a linear induction motor, the angular velocity of an induction motor becomes linear velocity of a LIM and torque becomes thrust. In an induction motor continuous rotation of the rotor is produced while in a LIM a constant thrust or force is produced.

In a LIM, the primary winding is made shorter than the secondary. In certain applications, the secondary may be made shorter than the primary. The secondary of the LIM which is of any metal sheet is backed by a sheet of ferromagnetic material, say iron, so as to reduce the reluctance to flux path as has been shown in Fig. 4.59.

A LIM may also have primary winding on two sides having the secondary in between with some air-gap.

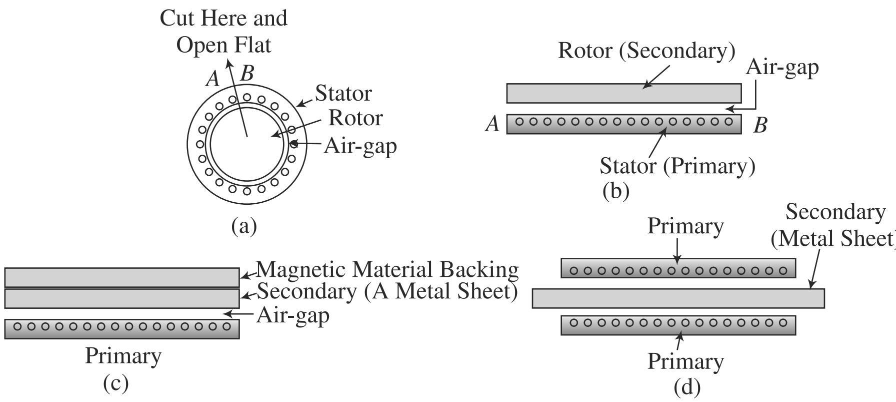


Fig. 4.59 *(a) An induction motor being cut and open flat to make a LIM; (b) A LIM with one primary winding and a secondary of metal sheet; (c) A LIM with a secondary metal sheet backed by a sheet of ferromagnetic material; (d) A double-sided secondary type linear induction motor*

As shown in Fig. 4.59, a LIM may have either a single-sided primary or a double-sided primary. The secondary is generally a sheet of aluminum which is a non-magnetic material. Often, an iron plate is used as backing material for the secondary. Some practical LIMS have double-sided primary as has been shown.

When a three-phase supply is applied across the three-phase primary windings a *moving linear magnetic field* (similar to rotating magnetic field in case of ordinary induction motor) *is produced*. The secondary metal plate is usually made of aluminum with a backing material of ferromagnetic material. The secondary metal plate will have eddy currents induced in it thus creating an opposing magnetic field, in accordance with Lenz's law.

The two opposing fields will repel each other and will create motion as the magnetic field sweeps through the metallic sheet. The interaction of the magnetic field with the induced currents in the secondary exerts a thrust (force) on the secondary to move in the same direction if the primary is held stationary. If the secondary side is kept stationary and the primary is made free to move, the primary will move in the direction opposite to that of the direction of the magnetic field.

Important applications of LIM is in transportation system. A double-sided LIM as a transportation test vehicle has been shown in Fig. 4.60. The two primaries are placed on the vehicle while the secondary is placed on the track on which the vehicle moves. Transportation test vehicles use LIM as drive and are in operation in countries like Canada, Japan, etc.

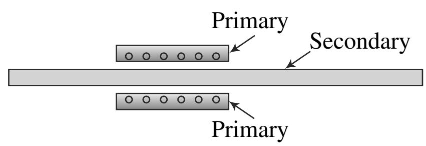


Fig. 4.60 *A LIM used as transportation system*

There are other applications of LIM as in a sliding door closer, opening and closing of stage curtain, material transportation, etc.

Performance of a LIM We know that when a three-phase supply is applied to the stator windings of an induction motor a rotating magnetic field, rotating at synchronous speed in produced. In a LIM when a three-phase supply is applied to the primary windings a travelling flux density wave is produced which travels along the length of the primary as shown in Fig. 4.61. Emf will be induced in the secondary conductor and hence a current will flow through the secondary. The induced current and travelling flux wave will interact with each other to produce force.

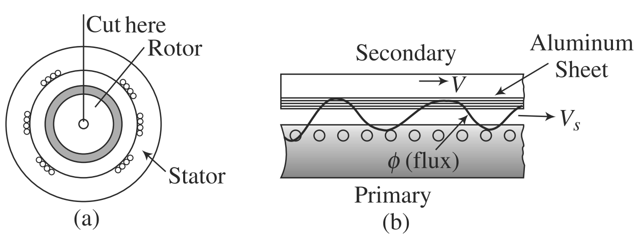


Fig. 4.61 *Linear induction motor as shown in (b) derived from a rotary induction motor as shown in (a)*

The force developed will be a translational force or a thrust. If the primary is kept stationary the secondary will be free to move in the same direction as the travelling wave. If the travelling wave moves with a velocity V_s, the secondary member will move at a velocity V. The translational slip is expressed as

$$S = \frac{V_s - V}{V_s}$$

The synchronous velocity V_s of the travelling wave is

$$V_s = 2\ T_P f \text{ m/sec.}$$

where $T_p = \frac{2\pi}{P}$, is the pole pitch.

[We have for an induction motor, $Ns = \frac{120\ f}{P}$ rpm

In terms of rps, $N_S = \frac{2f}{P}$

Velocity, $V_s = 2\pi n_s = \frac{2\pi}{P} \times 2f = 2f\, T_P$

Similar to torque-speed characteristic of an induction motor, the thrust/force versus speed characteristic of a linear induction motor will be as shown in Fig. 4.62.

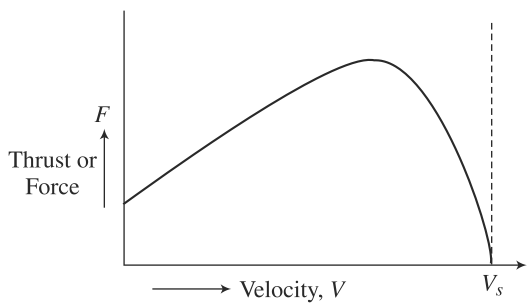


Fig. 4.62 *Thrust-speed characteristic of a linear induction motor*

The air-gap required for a linear induction motor is much more than that of a rotary induction motor. The air-gap varies from 15 mm to 30 mm for a LIM whereas for rotary induction motor the air-gap is kept to a minimum of about 1 mm or so.

Since the air-gap is large, the magnetizing current required to produce the magnetic field is large. Since the no load current is large, the power factor is low. The slip of a LIM is large and hence losses in the rotor is high resulting in lower operating efficiency as compared to rotary induction motor.

4.23 TESTING OF INDUCTION MOTORS

Instructions for conducting and reporting the generally acceptable applicable tests to determine the performance characteristics of three-phase induction motors have been included in the Indian Standard IS: 4029–1967 (Note, now all IS codes are called BIS codes). A brief idea of the various tests recommended are mentioned as follows. For details, the reader is advised to consult the Indian Standard IS: 4029–1967 and IS: 325–1961.

Insulation Resistance Test Insulation resistance is measured between winding and frame (earth) with the help of an insulator tester of suitable voltage.

High Voltage Test A test voltage in accordance with Table 4.2 is to be applied between windings and frame of the motor, with the core connected to the frame and the windings not under test. The test voltage is to be applied once and only once to the new and complete motor in normal working condition with all its parts in place, and the test should be carried out along with the insulation resistance test at the works of the manufacturer.

Table 4.2 *High Voltage Test*

	Part of Motor	Test Voltage
1	Primary (stator) windings	1000 V plus twice the rated voltage with a minimum of 2000 V
2	Secondary (rotor) windings not permanently short-circuited: (a) For non reversing motors or motors reversible from standstill only.	4000 V plus twice the open circuit standstill voltage as measured between slip-ring or secondary terminals with rated voltage applied to the primary windings, with a minimum of 2000 V.
	(b) For motors to be reversed or braked by reversing the primary supply while the motor is running	1000 V plus four times the open circuit stand-still secondary voltage as defined in (a) with a minimum of 2000 V.

Resistance Measurement Resistance of the windings are measured either by (a) voltmeter-ammeter method or by (b) bridge method.

If R_1 is the resistance measured at temperature t_1 °C, its resistance R_2 at any other temperature t_2°C can be calculated by using the relation

$$R_2 = \frac{(235 + t_2)}{(235 + t_1)} \times R_1 \tag{4.32}$$

4.23.1 Performance Characteristics

No-load Test The motor is run on no-load at rated voltage and frequency until the watts input is constant. Readings of voltage, current, frequency and power input are taken. From these readings, no-load current, core-loss and friction and windage losses are found out.

Open-circuit Test With rated or reduced voltage applied across the stator terminals, the voltage induced across the slip-ring terminals of the motor is measured. If any rotor unbalance is detected, number of readings with several rotor positions should be taken and average readings calculated.

Blocked Rotor Test This is performed to determine the soundness of rotor in the case of squirrel-cage motors and to measure the starting current, power factor, starting torque and impedance. This test also enables circle diagram to be drawn, which has been discussed in detail in Sec. 4.17.

Test for Speed-Torque and Speed-Current Curves Speed-torque and speed-current test may be carried out by the following methods:

(i) Dynamometer,

(ii) Pony brake,
(iii) Rope and pulley, and
(iv) Calibrated machine.

These tests are conducted at rated voltage or as near to it as practical. It is necessary to avoid temperatures exceeding the limits of temperature rise for a given class of insulation as specified. Measurement of voltage, current, speed and torque is to be made. For wound-rotor motors, speed-torque and speed-current tests may be conducted between synchronous speed and the speed corresponding to maximum torque.

Load Test This test is performed at 125, 100, 75, 50 and 25 per cent of the full-load values for the determination of efficiency, power-factor, speed and temperature rise. Loading of the motor can be done with the help of brake-pulley arrangement, dynamometer method or with the help of a calibrated/uncalibrated generator.

Temperature-rise Test This test is performed to determine the temperature rise on different parts of the motor, while running at rated conditions. The duration of temperature-rise test is dependent on the type of rating of the motor. For motors with continuous rating, the temperature rise test should be continued till thermal equilibrium has been reached. Whenever possible, the temperature should be measured, both while running and after switching off the supply. Measurement of temperature rise of windings and other parts can be done by any one of the following methods:

Embedded Temperature Detector Method Temperature detectors such as resistance thermometers or thermocouples are embedded inside the machine during manufacture, in intimate contact with the surface whose temperature is to be measured.

Resistance Method Temperature of a winding is determined by measuring the increase in resistance of the winding, usually the stator winding. Temperature rise $(t_2 - t_1)$ can be calculated using the Eq. (4.32) mentioned earlier.

Thermometer Method This method is applicable is cases where neither the embedded temperature detector method nor the resistance method is applicable.

In this method, temperature is determined by thermometers applied to the accessible surface of the motor. The term thermometer includes mercury or alcohol-bulb thermometers, provided the latter are applied to the points accessible to the usual bulb thermometer.

When bulb thermometers are used, alcohol thermometers are to be preferred over mercury thermometers in places where there is any varying or moving magnetic field present.

Measurement of Shaft Current and Voltages For details the reader may refer to IS: 4029-1967.

Measurements of Noise For details the reader may refer to IS: 4029-1967.

Test on motor are normally carried out at the manufacturer's work. Two types of tests viz., *type tests*, and *routine tests* are carried out.

4.23.2 Type Tests

These tests are carried out on a *type motor* (the motor which is representative of others in essential details) to verify conformity to the performance requirements of Indian Standard Specifications. The following are the type tests:

(a) Measurement of stator resistance and rotor resistance on slip-ring motors;
(b) No-load running of motor and reading of voltage, current, power input and speed;
(c) Open-circuit voltage ratio on slip-ring motors;
(d) Reduced voltage running-up test at no-load to check the ability of the motor to run up to no-load, in both directions of rotation with $1/\sqrt{3}$ of the rated line voltage applied to the motor;
(e) Locked rotor reading of voltage, current, power input and torque of squirrel-cage motor (Note: This test may be made at a reduced voltage);
(f) Full-load reading of voltage, current, power input and slip;
(g) Temperature-rise test;
(h) Momentary overload-in-torque tests;
(i) Insulation resistance test (both before and after the high-voltage test); and
(j) High-voltage test.

It is recommended that reports of type tests be made in the form indicated:

Form for Type Test Report of Three-Phase Induction Motor

(Squirrel-cage/Slip-ring)

Name and Address of the Manufacturer

As per IS:______________

Purchaser______________ Certificate No.______________

Purchaser's Order No.______________ Order Acceptance No.____________

Name-Plate Data

kW output	syn speed rev/min	Full load rev/min	Phase conn	C_y	Volts	Amp full load	Encl	Ratings	Frame	Temp. rise	Class of insulation	Rotor volts	Rotor amps

Test Characteristics

						Power Factor		Efficiency	
Loading Conditions	Volts	Amps	Watts	Precent	Load	Guar-anteed	Test	Guaranteed	Test
No-load									
Load						1/2			
						3/4			
						1			

Temperature Test Run

Condition						Temperature-Rise in °C for Stator and Rotor
Hours Run	Line Volts	Line Amps	Watts Input	Calculated	Cooling Air °C	Windings by Resistance or Thermometer
Torque and Starting Current						Insulation and High-Voltage Test
Locked Rotor Torque in kgm with Volts Applied		Stg. Current (looked rotor) with Volts Applied		Insulation		High Voltage
				Stator		
				Rotor		
Open Circuit Rotor Volts		Resistance per Phase				
		Stator		Rotor		
		Ω at ... °C		Ω at ... °C		

Momentary overload for 15 s... Test O.K. Approved by ...
Tests conducted on Machine No..... Date ......
Tested by....... on ......

4.23.3 Routine Tests

These tests are carried out on each motor to ascertain that it is electrically and mechanically sound. The following are the routine tests:

(a) Insulation resistance test (before the high-voltage test only);
(b) High-voltage test;
(c) No-load running of motor and reading of current in the three-phases and of voltage;
(d) Locked rotor test at a suitable voltage (for squirrel-cage motors only);
(e) Reduced voltage running-up test at no-load, to check the ability of the motor to run to full speed on no-load in both directions of rotation with $1/\sqrt{3}$ of the rated line voltage applied to stator terminals; and
(f) Open circuit voltage ratio (for slip-ring motors only).

4.23.4 Test Certificates

Unless otherwise specified, when inviting tenders the purchasers, if so desired by the manufacturer, shall accept the manufacturer's certificates as evidence of the compliance of the motor with the requirements of type test, on a motor identical in essential details with the one purchased, together with routine tests on each individual motor. In the case when a batch of 20 or more similar motors is supplied to one order, type tests, as specified, shall be made on one of these motors, if the purchaser so requires.

Certificates of routine tests shall show that the motor purchased has been run and has been found to be electrically and mechanically sound and in working order in all particular.

4.24 INSTALLATION AND PREVENTIVE MAINTENANCE OF INDUCTION MOTORS

An electrical engineer is expected to instal, undertake preventive maintenance work and use logical methods to identify and localise faults in an electric motor. Informations about these are available in sources like Indian Standards (refer IS: 900-1965), manufacturers manual and reference books. In this section, the relevant information has been given in a precise form for the benefit of the students, teachers and working technicians in the field.

4.24.1 Installation of Induction Motors

Installation of an induction motor involves inspection of the motor on arrival and its storage, deciding a location for the motor and control gear, preparing foundation and arrange for levelling, checking for proper alignment, fitting of pulleys and couplings, earthing, etc.

Inspection on Arrival and Storage The wooden crates containing motors should be unloaded by using cranes. Sliding the motor down an inclined plank using pipes or bars should be avoided. However, where use of inclined plank becomes unavoidable, care should be taken to reinforce plank by placing boxes or tresses under them. Ropes and wedges should be employed in bringing the crates down in a steady manner to avoid injury to the personnel.

Handling of Motors Motors have a hook on the frame, which should be used for lifting it. Cranes, chain pulley block and ropes, etc., used for this purpose should have adequate lifting capacity.

Inspection The opening of a crate should be done in an upright position by using proper tools for removing nails, screws, and steel-strip fasteners. On opening the crate, the materials with the packing list should be checked. Loss or damage during transit, if any, should be informed to the supplier and also to the insurance company. In case of visible sign of damage on the crate, open delivery from carrier should be taken and loss and damage certificate got signed.

Storage The motor should be stored in a clean and dry place, using space heaters if provided specially during rainy seasons. Motors provided with ball and roller bearings, if stored at locations susceptible to vibration, should have resilient (rubber) pads underneath to avoid stationary indentation. Rotor should be rotated through 90° and relocked periodically, say every fortnight.

Location of Motors and Control Apparatus The following points are to be considered while deciding a location for the motor to be installed:

(a) Place of installation of motor should have sufficient space all around the motor to facilitate movement of operating personnel and for maintenance purposes.
(b) Motor should be accessible for carrying out repair and maintenance. If required, lifting arrangement should be provided with the structure of the building.
(c) The motor and control apparatus should not be located where they are liable to exposure to water, corrosive liquid, oil, steam, carbon, metallic dust, dirt, etc., unless they are suitably enclosed to withstand such conditions.

Foundation and Levelling The main requirement of motor foundation is that it should be strong and rigid. Cement concrete foundation is advised where vibrations are present and load is fluctuating. Foundation block should be at least 20 cm longer and 20 cm wider than the motor feet, side rails, or bed plates. Depth of the foundation should not be less than that given in Table 4.3.

Table 4.3 *Depth of Foundation for Installation of Motors*

HP OF THE MOTOR	DEPTH OF FOUNDATION IN CM
Up to 10 hp	10 to 15
10 to 25	15 to 20
25 to 50	20 to 25
50 to 75	25 to 38
75 to 100	38 to 61

Where the anchor or rag bolts, which hold down the motor are to be embedded in concrete, their location should be determined with great accuracy. Where the motor rests directly on the foundation, care should be taken in the levelling of the foundation.

It should be ensured that undue amount of vibration from other machinery is not transmitted to the motor. Though motors are generally required to be rigidly fixed, resilient mounting using springs, rubber or other similar material may be necessary, where transmission of vibration is not desirable.

Where the motor rests directly on foundation, great care is needed in the levelling of foundation, so that the motor is not strained when the mounting bolts are tightened. The motor with base plates is to be placed in position having the shaft lined up approximately with the driven apparatus and providing packing of steel plates evenly spaced. Care is to be taken to provide packing close to the foundation bolts, otherwise distortion may occur on tightening the foundation bolts. The bed plate is then to be levelled up with the help of spirit level by adding or removing packing sheets. After levelling, the motor is carefully aligned with the driven equipment.

Alignment One of the considerations in alignment is that the motor shaft should be perfectly levelled. If this is not done, undue loading may occur on the bearings of the motor under the influence of the drive forces. Proper alignment for directly coupled drives is achieved by ensuring three operations, namely axial positioning of shaft, paralleling of shaft and centering of shaft.

1. Axial Positioning of Shaft It is ensured that the specified gap between the faces of the two couplings is kept while the motor shaft is fully extended towards the driven shaft. If this is not allowed, it will cause a thrust in the bearing under running condition.

2. Paralleling of Shaft in Vertical and Horizontal Plane Coupling gap reading should be taken at 12 o'clock and 6 o'clock positions and the two half-couplings rotated together through 180°, when a second set of reading at 12 o'clock and 6 o'clock should be taken. For correct alignment in the vertical plane, the algebraic difference between the readings should be equal. Similar set of readings should be taken at 3 o'clock and 9 o'clock positions. Equal algebraic difference between the readings will indicate correct alignment in the horizontal plane.

3. Centering of Shafts In addition to correct alignment in the vertical and horizontal plane, the axis of both shafts should be in the same line and not make an angle with each other. This can be checked by measuring the gap between the flange faces at four points, i.e., top, bottom, front and back (in other words at 12 o'clock, 6 o'clock, 9 o'clock and 3 o'clock positions). This can be done by laying the edge of a steel ruler against the sides of the two flanges and checking whether the steel edge rests fully against the sides of the two flanges or if there is any gap. Any variation in levels is corrected by using steel shims (packings).

Non-direct coupled drives include flat belt, V-belt, rope, chain and gear drive. For such drives, requirement of alignment, in addition to the paralleling of the shafts is, that the middle of the pulleys, gears, etc., should be in one and the same plane, i.e., perpendicular to the shaft axis. For chain drives, the driving and driven shafts should be parallel to each other and the chains running centrally on their sprockets at right angles to the shaft.

Checks before Commissioning Certain mechanical checks, like checks for correct alignment, air-gap between stator and rotor, proper greasing of the bearings if required, etc., should be carried out.

The following electrical checks are also to be done before commissioning of a motor:

(a) All fixed connections should be checked for tightness.. Before connecting to the mains all connections should be checked with the wiring diagram.
(b) Rating of fuses should be checked.
(c) Earth connections should be checked for tightness and earth resistance should be measured.
(d) Where relays are used in conjunction with current transformers, tests should be performed for checking of the relay by simulating loading conditions.

Commissioning of Motor Before switching on the supply, tests should be carried out for continuity and insulation resistance. Protective devices should be set at their minimum current settings and minimum operating time in order to avoid any undesirable consequence of any fault condition arising at the time of starting. The motor should be started slowly. If the direction of rotation, by any chance is opposite, the same can be reversed by interchanging connections of two supply terminals to the motor terminals. When the motor is running at full-speed, observe that there is no excessive vibration and noise. If the motor fails to start before attempting to restart, check whether:

(a) The connections are correct.
(b) Correct voltage is available at the starter terminals.
(c) All the terminals are tight.
(d) The brush gear handle (in case of a slip-ring motor) is in the start position.

After checking the above, attempt should be made to restart the motor. If the motor still fails to start or fails to accelerate, the load should be uncoupled and fresh attempts be made to start the motor. If the direction of rotation is correct and the motor runs to normal speed without load, but trips when load is applied, then the settings of the overload relays should be checked. If the motor still refuses to carry the load, it will be advisable to consult the manufacturer of both the motor and the driven load.

4.24.2 Preventive Maintenance of Three-Phase Induction Machines

After the motor is installed properly, periodic attention will give a trouble free service for a long time. It is always better to prevent fault to occur rather than repairing afterwards. For this purpose a motor is kept dry and clean and is properly lubricated.

A few important aspects of maintenance of a motor are:

(a) Protective paint should be kept in good condition and repainted, if needed.
(b) Motor should be cleaned by blowing air at regular intervals.
(c) Motor should be kept dry and clean. The stator and rotor windings should be kept free from oil, grease, dampness, dirt, etc.
(d) Air-gap between the stator and rotor should be kept free from any accumulation of dirt.
(e) Terminal connection should be kept clean and tight to maintain a good contact.
(f) Insulation resistance of the windings should be tested periodically. If its value falls below 1 MΩ, the motor should be dried out and then put into service If low insulation resistance is indicated for a number of times, a coat of insulating varnish in the machine winding is to be provided after drying.
(g) Ball and roller bearings should be greased in case they make noise. Required quantities of grease should be put. Every three years, old grease should be replaced by new grease.
(h) Dust, oil, moisture should not be allowed to accumulate on the surface of slip-rings and brush gear. Tension of the springs should be checked for correct brush pressure on the rings.
(i) The covers of the controllers, starters and rheostats should be removed periodically for inspection. The contacts and insulating parts should be cleaned.
(j) Resistance of the earth continuity wire should be measured periodically to check the effectiveness of the system.
(k) Control circuit schemes if used for remote control, interlocks, etc., should be checked for proper functioning.

Maintenance schedule of induction motors, as recommended by ISI, is reproduced for the benefit of the reader.

4.25 RECOMMENDED MAINTENANCE SCHEDULE

(Reproduced from IS: 900-1965 by permission of BIS)

1. Daily Maintenance

1.1 Examine visually earth connections and motor leads.

1.2 Check motor windings for overheating (the permissible maximum temperature is about that which can be comfortably felt by hand)

1.3 Examine control equipment.

1.4 In the case of oil-ring lubricated motors:

(a) Examine bearings to see that oil-rings are working.

(b) Note temperature of bearings.

(c) Add oil, if necessary.

(d) Check end-play.

2. Weekly Maintenance

2.1 Check belt tension. In case where this is excessive, it should immediately be reduced and in the case of sleeve bearing machines, the air gap between rotor and stator should be checked.

2.2 Blow out windings of protected type motors situated in dust locations.

2.3 Examine starting equipment for burnt contacts where motor is started and stopped frequently.

2.4 Examine oil in the case of oil-ring lubricated bearing for contamination by dust, grit, etc. (This can be roughly judged by the colour of the oil.)

3. Monthly Maintenance

3.1 Overhaul controllers.

3.2 Inspect and clean oil-circuit breakers.

3.3 Renew oil in high-speed bearings in damp and dusty locations.

3.4 Wipe brush holders and check bedding of brushes of slip-ring motors.

4. Half-Yearly Maintenance

4.1 Clean windings of motors subjected to corrosive or other elements; also bake and varnish, if necessary.

4.2 In the case of slip-ring motors, check slip-rings for grooving on unusual wear.

4.3 Check grease in ball and roller bearings and make it up where necessary, taking care of avoiding overfilling.

4.4 Drain all oil from bearings, wash with petrol to which a few drops of oil have been added; flush with lubricating oil and refill with clean oil.

5. Annual Maintenance

5.1 Check all high-speed bearings and renew, if necessary.

5.2 Blow out all motor windings thoroughly with clean dry air. Make sure that the pressure is not so high as to damage the insulation.

5.3 Clean and varnish dirty and oily windings.

5.4 Overhaul motors which have been subjected to severe operating conditions.
5.5 Renew switch and fuse contacts, if damaged.
5.6 Check oil.
5.7 Renew oil in starters subjected to damp or corrosive elements.
5.8 Check insulation resistance to earth and between phases of motor windings, control gear and wiring.
5.9 Check resistance of earth connections.
5.10 Check air-gaps.

4.26 APPLICATIONS OF INDUCTION MACHINES

The majority of the induction machines used today are operated as induction motors. Use of induction machines as induction generators is insignificant. An induction motor as a nearly constant speed motor has hardly any competitor. Other types of motors are considered only when a variable speed drive is needed.

Applications of three-phase induction motors of size varying from half a kilowatt to thousands of kilowatts are numerous. Induction motors are found everywhere from a small workshop to a large manufacturing industry.

Before it is possible to specify an appropriate motor for a given application, it is necessary to know the load characteristic. Load characteristic will include items like horse power requirement, starting torque, speed variation, duty cycle, environmental condition, etc.

Application of various types (with respect to starting torque and starting current) of squirrel-cage motors and wound-rotor motors are mentioned below:

Squirrel-cage Motor

(i) General purpose with normal torque and normal starting current.
Applications Fans, blowers, centrifugal pumps, line shafting, etc.

(ii) High torque, low starting current.
Applications Conveyors, compressors, crushers, agitators, reciprocating pumps, etc.

(iii) High torque, medium and high slip.
Applications Used for high inertia loads, such as sheers, punch presses, die stamping, etc.

(iv) Low starting torque, normal starting current.
Applications For loads of low inertia and low starting torque requirement, such as fans and centrifugal pumps.

Wound-rotor Motor These motors are used in situations where high starting torque is required such as in hoists, compressors, lifts, crushers, large ventilating fans, cranes etc.

MODEL QUESTIONS

Short-Answer-Type Questions

4.1 Explain the principle of working of a three-phase induction motor on the basis of the concept of alignment of magnetic fields.

4.2 Describe the differences in construction between a synchronous motor, slip-ring type induction motor and a squirrel-cage induction motor.

4.3 Describe the two types of rotor construction of induction motors.

4.4 Explain why the stator and rotor cores of an induction motor are made of magnetic material .

4.5 Explain why the number of poles of the stator and the rotor of an electrical motor be equal.

4.6 State the conditions under which a polyphase induction motor will have:
(a) rotor frequency equal to stator frequency,
(b) rotor frequency higher than the stator frequency,
(c) maximum voltage and current induced in the rotor,
(d) minimum voltage and current induced in the rotor.

4.7 Explain the nature of magnetic field produced when a single-phase supply is applied across a single-phase stator winding.

4.8 Write the equation for torque of a polyphase induction motor at any value of slip defining all the terms.

4.9 Using the torque equation, draw the nature of the torque-slip characteristic of a three-phase induction motor. Show on the curve, starting torque, maximum torque and slip at which torque is maximum.

4.10 Derive the condition for maximum torque and hence obtain the value of maximum torque of a polyphase induction motor.

4.11 State under what condition the maximum torque will be developed by an induction motor at (a) starting, (b) 50 per cent of the synchronous speed, and (c) 75 per cent of the synchronous speed.

4.12 Show how increase in rotor circuit resistance will increase the starting torque of an induction motor. What should be the maximum limit of such increase of rotor circuit resistance?

4.13 Explain how high starting torque is achieved in double-cage type induction motors. State two specific applications of such motors.

4.14 Explain why an induction motor draws heavy current as compared to its full-load current at starting.

4.15 Explain why the no-load current of an induction motor is much higher than that of an equivalent transformer.

4.16 Define the term slip of an induction motor. What is the value of slip when the rotor is stationary? What is normally the value of slip when properly designed induction motor runs on full-load?

4.17 Explain how a rotating magnetic field is produced when a three-phase supply is connected across the three-phase stator windings of an induction motor.

4.18 Prove that maximum torque developed by an induction motor is independent of the rotor circuit resistance.

4.19 Prove that the frequency of the rotor induced emf in an induction motor is slip times its stator supply frequency.

4.20 Explain why an induction motor cannot run at synchronous speed.

4.21 A three-phase, 50 Hz, 4-pole induction motor runs at a slip of 4 per cent at full-load. Calculate the rotor speed. (Ans. 1440 rpm)

4.22 Write the expression for rotor current of an induction motor and hence show that current is maximum at starting and zero if the rotor is rotated at synchronous speed.

4.23 Show through a power flow diagram, how electrical input is converted into mechanical power output in an induction motor.

4.24 By using power flow diagram or otherwise, derive the torque equation of a polyphase induction motor.

4.25 Establish the relation, 'Rotor copper-loss = Slip × Rotor input' for a polyphase induction motor.

4.26 Explain with the help of characteristic curves, the effect of variation of rotor circuit resistance on the torque-slip characteristic of an induction motor.

4.27 Explain how higher starting torque in a polyphase squirrel-cage type induction motor is achieved by use of double-cage rotors.

4.28 Show with the help of a circuit diagram, the arrangement for achieving higher starting torque in a slip-ring type induction motor.

4.29 State the advantages of a double-cage induction motor. Draw the torque-speed characteristics of a double squirrel-type induction motor.

4.30 Explain why it is recommended that large induction motors be started with reduced voltage applied across its terminals at starting.

4.31 Explain why a starter is needed for starting a large capacity induction motor.

4.32 Draw and explain a push-button operated direct-on-line starter, for a three-phase induction motor.

4.33 Draw and explain a manual auto-transformer starter for a three-phase induction motor.

4.34 Compare the value of current drawn from the lines by an induction motor when it is started with its stator winding star-connected, as against delta-connected.

4.35 Draw the connection diagram of a manual star-delta starter for a three-phase induction motor. Explain the circuit. How do you make sure that the direction of rotation is not reversed while changing the connections from star to delta?

4.36 Name the different methods of speed control of a polyphase squirrel-cage type induction motor. What are the limitations and disadvantages of this method?

4.37 Explain how the speed of a slip-ring type induction motor can be changed by changing the rotor circuit resistance. What are the limitations and disadvantages of this method?

4.38 Explain why the frequency changing device should change frequency and voltage simultaneously while controlling speed of a polyphase induction motor.

4.39 List the various losses that take place in an induction motor, starting the factors on which such losses depend.

4. 40 Describe how efficiency of a polyphase induction motor can be determined from the data obtained in no-load and blocked-rotor tests.

4.41 Show how an induction motor can be represented by an equivalent electrical circuit. State the practical utility of such an equivalent circuit.

4.42 Show how the circuit elements of the equivalent circuit of an induction motor can be determined from the readings of no-load and blocked-rotor tests.

4.43 Show that the locus of the rotor of an induction motor is semicircle.

4.44 What tests are to be performed on an induction motor to be able to draw its circle diagram? What are information one can get about the performance of the motor from the circle diagram? What assumptions or approximations are made in drawing the circle diagram?

4.45 What is an induction generator? State the main points of difference between a synchronous generator and an induction generator. Explain why an induction generator has very limited applications.

4.46 Name the different tests which are to be conducted on induction motor under the *type tests* and *routine tests*.

4.47 Explain the construction and principle of working of linear induction motor. Mention applications of such motors.

Numerical Problems

4.48 A 5 hp, three-phase, 400 V, 50 Hz, 4-pole induction motor runs at a speed of 1440 rpm on full-load. Calculate the slip. What is the frequency of the rotor induced emf ? (Ans. 4 per cent, 2 Hz)

4.49 The frequency of the rotor induced emf of a 400 V, three-phase 50 Hz, 8-pole induction motor is 2 Hz. Calculate the percentage slip and the rotor speed. (Ans. 4 per cent, 720 rpm)

4.50 A three-phase 400 V, 50 Hz, 4-pole, start-connected induction motor has rotor resistance and reactance of 1 and 2 Ω respectively. The ratio of stator to rotor turns is 3. Calculate the power developed by the rotor at a slip and the rotor speed. (Ans. 678.7 W)

4.51 A 20 hp, 4-pole, 50 Hz, three-phase induction motor has friction and windage loss of 3 per cent of the output. For full-load slip of 4 per cent, calculate for full-load, (a) rotor I^2R-loss, (b) rotor input, and (c) output torque. (Ans. 631.3 W, 15782 W, 97.6 Nm)

4.52 A 400 V, 40 Hz, three-phase slip-ring type induction motor has rotor resistance of 0.1 Ω and standstill reactance of 0.5 Ω per phase with the rotor circuit star-connected. The full-load slip is 4 per cent. Calculate the extra resistance to be connected in the rotor circuit to achieve maximum torque at 50 per cent slip. (Ans. R = 0.15 Ω)

4.53 A 400 V, 50 Hz slip-ring type three-phase induction motor rotor is star connected and has per phase rotor reactance and standstill reactance of 0.5 and 1.5 Ω respectively. Calculate the resistance to be added per phase to achieve starting torque equal to maximum torque. (Ans. R = 1.0 Ω)

4.54 The rotor of a 4-pole, 50 Hz, slip-ring type induction motor has a resistance of 0.2 Ω per phase and runs at 1440 rpm at full load. Calculate the external resistance per phase which must be added to reduce the speed to 1200 rpm, the torque remaining same in both the cases. (Ans. R = 0.8 Ω)

4.55 A 400 V, 30 hp, 50 Hz, 4-pole, delta-connected induction motor give the following test data:

No-load test: 400-V, 12-A, 1.2-kW.
Short circuit test: 100-V, 40-A, 3-kW.
Draw the circle diagram and find:
(a) stator current and power factor at full-load;
(b) starting torque;
(c) full-load torque;
(d) maximum torque;
(e) efficiency at full-load.

(Ans. 43 A, 0.88 lagging, 23997 Syn. W, 23644 Syn. W, 46582 Syn. W, 82.9 per cent)

Multiple-Choice Questions

4.56 Three-phase induction motors are widely used for industrial applications because
(a) They are rugged in construction, requires less maintenance and are less expensive than other motors
(b) their speed can be controlled very smoothly over a wide range
(c) their operating characteristics are superior over other electrical motors
(d) they can be manufactured easily for any hp rating.

4.57 In a three-phase induction motor.
(a) three-phase supply is to be given to stator winding and dc supply to the rotor winding
(b) only three-phase supply is to be given to stator winding
(c) three-phase supply is to be given to both stator and rotor windings
(d) three-phase supply is to be given to rotor winding.

4.58 The stator and rotor cores of an induction motor are made up of laminated sheets.
(a) to reduce the hysteresis loss in the core
(b) to reduce the eddy-current loss in the core
(c) to make the rotor and stator mechanically strong
(d) to enable the stator and rotor cores dissipate heat more effectively.

4.59 The stator and rotor cores of an induction motor are made up of magnetic material
(a) to keep the cost of construction low
(b) to reduce the magnetising current
(c) to make the parts strong
(d) because the reluctance of the magnetic material is high.

4.60 In wound-rotor type induction motors the rotor terminals are brought out through slip rings
(a) to enable extra resistance to be connected across them during starting
(b) to enable closing the rotor circuit externally
(c) to enable three-phase supply to be applied across the rotor winding through brush and slip-ring arrangement
(d) to enable connecting the rotor windings either in star or in delta depending upon the need.

4.61 A pulsating (alternating) magnetic field will be produced when
(a) two-phase supply is applied across a two-phase stator winding
(b) three-phase supply is applied across a three-phase stator winding
(c) a polyphase supply is applied across a polyphase stator winding
(d) a single-phase supply is applied across a single-phase stator winding.

4.62 To make the simplest 6-pole single-phase winding, at least
(a) two coils are needed
(b) one coil is needed
(c) three coils are needed
(d) six coil are needed

4.63 The direction of the rotating magnetic field produced by the stator ampere-turns of a three-phase induction motor changes, if
(a) the sequence of supply to the stator terminals is changed
(b) a variable frequency voltage is applied across the stator terminals
(c) the supply voltage is changed
(d) supply to any one phase is disconnected.

4.64 A 400 V, 50 Hz, 4-pole, three-phase induction motor cannot run at 1500 rpm because
(a) at 1500 rpm there will be no emf induced in the rotor circuit and hence no torque will be developed
(b) an induction motor can run only at a speed higher than its synchronous speed
(c) at 1500 rpm, torque developed by the rotor may not be sufficient to rotate the rotor
(d) at 1500 rpm, the rotor will draw excessive current and may be harmful to the motor.

4.65 A 400 V, 50 Hz three-phase induction motor rotates at 1440 rpm on full-load. The motor is wound for
(a) 2 poles (b) 4 poles (c) 6 poles (d) 8 poles.

4.66 The slip of 400 V, three phase, 4-pole induction motor when rotating at 1440 rpm is
(a) 2 per cent (b) 3 per cent (c) 4 per cent (d) 5 per cent.

4.67 The relation between synchronous speed, stator supply frequency and stator number of poles of a three-phase induction motor is given by
(a) $N_S = \dfrac{P}{120f}$ (b) $f = \dfrac{120N_s}{P}$
(c) $f = \dfrac{P\,N_s}{120}$ (d) $N_S = \dfrac{120P}{f}$

4.68 When a 400 V, 50 1-1z, 6-pole induction motor is rotating at 960 rpm on no-load, its slip is
(a) 1 per cent (b) 2 per cent (c) 3 per cent (d) 4 per cent.

4.69 The torque-slip characteristic for a three-phase induction motor is such that
(a) for lower values of slip, torque is directly proportional to slip and for higher values of slip, torque in inversely proportional to slip

(b) for lower values of slip, torque is inversely proportional to slip and for higher values of slip, torque in directly proportional to slip
(c) for lower values of slip, torque is directly proportional to the square of the slip and for higher values of slip, torque is inversely proportional to slip
(d) for lower values of slip, torque is directly proportional to slip and for higher values of slip, torque is inversely proportional to square of slip.

4.70 Torque developed by a three-phase, 400 V, induction motor is 100 N-m. If the applied voltage is reduced to 200-V, the developed torque will be
(a) 50 N-m (b) 25 N-m
(c) 200 N-m (d) 62.5 N-m

4.71 If the rotor circuit resistance of a three-phase induction motor is increased
(a) its starting torque will increase and the maximum torque developed will also increase
(b) both the starting torque and maximum torque developed will decrease
(c) its starting torque will increase but the maximum torque developed will decrease
(d) its starting torque will increase but the maximum torque will remain unchanged.

4.72 To achieve higher starting torque in a three-phase slip-ring type induction motor
(a) extra resistance should be connected across the slip-ring terminals
(b) the phase sequence of the supply to the motor should be reversed
(c) the supply voltage should be increased
(d) the windings should first be connected in star and then in delta.

4.73 For a three-phase induction motor having rotor circuit resistance of 6 Ω, maximum torque occurs at a slip of 0.6. The value of standstill rotor circuit reactance is
(a) 4.44 Ω (b) 0.36 Ω (c) 1 Ω (d) 10 Ω

4.74 While starting a three-phase induction motor, a star-delta starter is used to
(a) reduce the starting current of a safe value
(b) achieve higher starting torque
(c) enable the motor to start in the right direction
(d) be able to reverse the direction of rotation of the rotor as and when necessary.

4.75 A delta-connected 400 V, 50 Hz, three-phase induction motor when started direct-on-line takes a starting current of 30 A. When the motor is started through a star-delta starter, the starting current will be
(a) 3 A (b) 10 A (c) 15 A (d) 30 A

4.76 The magnetising current of a three-phase induction motor is much higher than an equivalent transformer because
(a) the size of an induction motor is higher than an equivalent transformer
(b) of the presence of air-gap between stator and rotor in an induction motor
(c) gain oriented magnetic material is used for the core of an induction motor
(d) inferior magnetic material is used for the core of an induction motor.

4.77 Smooth speed control of a three-phase induction motor over a wide range is possible, by
(a) pole changing method
(b) frequency control method
(c) using consequent pole technique
(d) by voltage control method.

4.78 The power input in blocked-rotor test performed on a three-phase induction motor is approximately equal to
(a) hysteresis loss in the core
(b) I^2R-loss in the windings
(c) eddy-current loss in the core
(d) iron-loss in the core.

4.79 The power input in no-load test performed on a three-phase induction motor is approximately equal to
(a) hysteresis loss in the core
(b) I^2R-loss in the winding
(c) eddy-current loss in the core
(d) iron-loss in the core.

4.80 In the equivalent circuit of a three-phase induction motor the mechanical load on the motor can be represented by a resistance of value
(a) R_2 (b) R_2/S
(c) $R_2 \dfrac{(I - S)}{S}$ (d) $\dfrac{R^2}{S} + 1$

4.81 The phenomenon of squirrel-cage motors sometime showing a tendency to run at a very low speed is known as
(a) cogging (b) crawling (c) damping (d) skewing.

4.82 The speed of revolving field for a 50 Hz, 8-pole machine will be
(a) 1500 rpm (b) 1440 rpm (c) 1000 rpm (d) 750 rpm.

4.83 The torque developed by an induction motor is
(a) directly proportional to the square of the rotor resistance
(b) directly proportional to the square of the supply voltage
(c) inversely proportional to the supply voltage
(d) inversely proportional to the slip.

4.84 The torque power factor of an induction motor will be high when
(a) running at no-load (b) running at full-load
(c) rotor is blocked (d) the rotor is crawling.

4.85 A 5 hp, three-phase, 400 V star-connected squirrel-cage induction motor meant to drive a milling machine, at starting takes about
(a) 40 A (b) 100 A (c) 150 A (d) 200 A.

4.86 The starting torque of an induction motor can be increased by
(a) increasing the rotor reactance
(b) increasing the rotor resistance
(c) increasing the supply frequency
(d) giving supply through a star-delta starter.

4.87 Developed at starting if the rotor parameters are as follows
(a) $R_2 = 2\ \Omega, X_2 = 8\ \Omega$ (b) $R_2 = 4\ \Omega, X_2 = 8\ \Omega$
(c) $R_2 = 8\ \Omega, X_2 = 8\ \Omega$ (d) $R_2 = 16\ \Omega, X_2 = 8\ \Omega$

4.88 The speed of a three-phase induction motor will increase, if the
(a) number of poles of the stator winding is increased
(b) number of poles of the stator winding is decreased
(c) frequency of the stator supply is decreased
(d) resistance of the rotor circuit is increased.

4.89 Induction motors now-a-days use die-cast aluminium rotor because
(a) aluminium is lighter than copper
(b) aluminium is cheaper than copper
(c) aluminium is easy to cast because of low melting point and is easily available
(d) aluminium has less resistivity than copper.

4.90 When the rotor circuit resistance of a polyphase induction motor is increased
(a) the starting torque increases
(b) the maximum value of torque decreases
(c) the slip at which maximum torque occurs remains unchanged
(d) maximum torque is developed at starting.

Fill in the Blanks

4.91 A number of statements are given below with missing words. You are required to fill in the blanks with appropriate words:

(i) A single phase supply fed to a single-phase winding produces _______ field.

(ii) A revolving field can be produced when _______ phase supply is fed to a _______ phase winding.

(iii) In a three-phase 2-pole winding, the space angle between two consecutive phases is _______ deg. electrical.

(iv) In a three-phase 2-pole winding, the space angle between two consecutive phases is _______ deg. mechanical.

(v) In a three-phase, 6-pole winding, the space angle between two consecutive phases is _______ deg. electrical.

(vi) In a three-phase, 6-pole winding, the space angle between two consecutive phases is _______ deg. mechanical.

(vii) The rotating magnetic field produced by a stator is in clockwise direction. The motor will rotate in _______ direction.

(viii) If the voltage applied to an induction motor is half the rated voltage, the starting torque will be _______ times the starting torque at the rated voltage.

(ix) Induction motor on the basis of their rotor construction are classified as _______ type motor and _______ type motor.

(x) A 400 V, 50 Hz, three-phase supply is connected across a three 6-pole stator winding. The rotating magnetic field will rotate at _______ revolutions per minute.

(xi) To reverse the direction of rotation of the magnetic field produced through a three-phase winding, the _______ of the supply should be reversed.

(xii) A 400 V, 50 Hz, three-phase, 4-pole induction motor rotates at 1440 rpm. The frequency of the rotor induced emf is _______ Hz.

(xiii) The frequency of the rotor induced emf of a, 400 V, 50 Hz, three-phase 6-pole induction motor is 2 Hz, the rotor speed is _______

(xiv) An induction motor takes more magnetising current than that of an equivalent transformer because of the presence of _______ in its magnetic circuit.

(xv) Blocked-rotor test on an induction motor is equivalent to _______ test on a transformer.

(xvi) Iron-loss in the rotor of an induction motor rotating at its normal speed is negligibly small because the _______ of the rotor induced emf is very small.

(xvii) The relationship between rotor input and rotor I^2R-loss in an induction motor is given as Rotor I^2R-loss _______ Rotor input.

(xviii) The expression for torque developed by a polyphase induction motor is given as: T = (_______)

(xix) Torque developed by a polyphase induction motor is directly proportional to slip for very small value of _______.

(xx) The rotor circuit resistance of a slip-ring type induction motor is increased. The starting torque will _______ the maximum torque will _______.

(xxi) The condition of maximum torque in terms of rotor resistance R_2, rotor reactance X_2 and slip S, can be expressed as: S =

(xxii) To achieve high _______ torque in a slip an induction motor, double-cage r otors are used.

(xxiii) To achieve high _______ torque in a slip-ring type induction motor, _______ is connected across the slip-ring terminals.

(xxiv) As per Electricity Rules, three-phase induction motors above _______ hp rating should not be started direct-on-line.

(xxv) Speed of a three-phase type induction motor can be controlled by using any of the following two methods:
(a) _______________. (b) _______________.

(xxvi) Efficiency of an induction motor can be calculated by performing the following tests on the motor _______.

(xxvii) The phenomenon of polyphase induction motors often running at a very low speed due to the presence of space harmonics in the air-gap flux is called _______.

Answers

4.56 (a)	4.57 (b)	4.58 (b)	4.59 (b)
4.60 (a)	4.61 (d)	4.62 (c)	4.63 (a)
4.64 (a)	4.65 (b)	4.66 (c)	4.67 (c)
4.68 (d)	4.69 (a)	4.70 (b)	4.71 (d)
4.72 (a)	4.73 (d)	4.74 (a)	4.75 (b)
4.76 (b)	4.77 (b)	4.78 (b)	4.79 (d)
4.80 (c)	4.81 (b)	4.82 (d)	4.83 (b)
4.84 (b)	4.85 (a)	4.86 (b)	4.87 (c)

4.88 (b) 4.89 (c) 4.90 (a).

4.91 (i) Alternative or pulsating; (ii) poly, poly;
(iii) 120 ; (iv) 120;
(v) 120; (vi) 40;
(vii) Clockwise; (viii) One-fourth;
(ix) Squirrel-cage, slip-ring; (x) 1000;
(xi) Phase-sequence; (xii) 2;
(xiii) 960 rpm; (xiv) Air-gap;
(xv) Short-circuit; (xvi) Frequency;
(xvii) Slip; (xviii) $T = \dfrac{60_m SE_{20}^2 R_2}{2\pi N_s (R_2^2 + S^2 X_{20}^2)}$
(xix) Slip; (xx) Increase remain unchanged;
(xxi) $S = R_2/X_2$; (xxii) Starting;
(xxiii) Starting, resistance; (xxiv) 5;
(xxv) Frequency control method, pole changing method;
(xxvi) No-load test and blocked rotor test;
(xxvii) Crawling.

LABORATORY EXPERIMENTS

EXPERIMENT 4.1 *No-load test on an induction motor.*

Objective To determine for an induction motor on no-load, relationship between

(a) applied voltage and speed,
(b) applied voltage and rotor current,
(c) applied voltage and stator current,
(d) applied voltage and power factor,
(e) applied voltage and power input.

Brief Theory In this experiment it is intended to study the effect of variation of applied voltage on the speed, power input, power factor, stator and rotor currents of an induction motor running on no-load. The effect of change of applied voltage on the above-mentioned quantities are explained as follows:

(a) *Effect on Speed* Speed remains practically constant until very low voltages are reached. Unless heavily loaded, the speed of an induction motor is affected very little by fluctuations of voltage.

(b) *Effect on Rotor Current* Torque is proportional to air-gap flux and rotor current. Air-gap flux is produced by applied stator voltage. Therefore, torque is proportional to applied voltage and rotor current
Torque being nearly constant, if applied voltage is reduced, rotor current will increase. Thus, the rotor current varies in inverse proportion to the applied stator voltage.

(c) *Effect on Stator Current* As applied voltage is increased, stator current rises gradually on account of the increase in magnetising current required to produce the stator flux. The component of the stator current which provides the ampere-turns balancing the rotor ampere-turns will steadily diminish as the rotor current decreases with the increase in rotor speed. The increase in the magnetising component is however, more than sufficient to balance this decrease. At very low voltages the induction is so low that almost the whole of the stator current is employed in balancing the rotor current. At normal voltages the rotor current requires only a small proportion of the stator currents to balance them. The higher saturation of the magnetic circuit requires a much stronger magnetising current to maintain the air-gap flux.

(d) *Effect on Power Factor* As explained above, the magnetising component of the stator current becomes larger as the voltages increases. Thus, there is a continuous increase in the power factor angle and hence a fall in power factor.
Frictional losses of the motor are practically constant as the speed does not change with voltage. The loss component of the stator current, I_w is due to frictional losses and iron-losses. As voltage is increased, iron-loss component and magnetising component of stator current will increase. The increase in magnetising current will be more than the increase in iron-loss component of stator current. Thus there will be a fall in power factor as the voltage is increased.

(e) *Effect on Power Input* No-load power input is spent in overcoming both iron laid frictional losses. As stated earlier, frictional losses are nearly constant at all voltages (until the motor speed falls rapidly), while the iron-losses continue to increase with the increase in the applied voltage.
In Fig. 4.63, by extrapolating the power input curve to the left until it cuts the ordinate of zero voltage, when there can be no iron-loss, it is possible to make a rough estimate of the power spent in friction and windage.
The effect of change of stator input voltage on the above-mentioned quantities are shown graphically in Fig. 4.63.

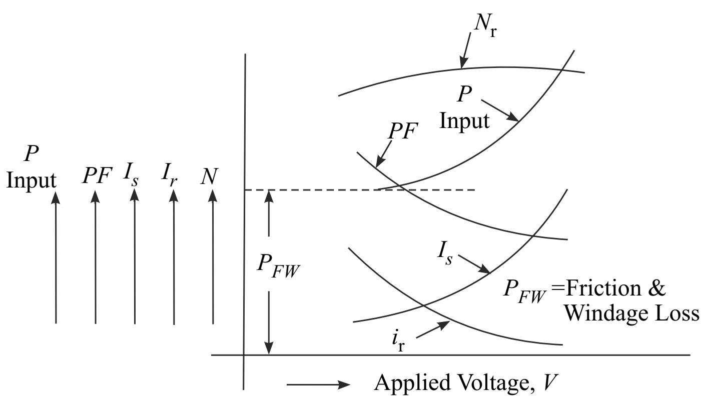


Fig. 4.63 *Effect of change of applied voltage on speed, rotor current, stator current, power factor and power input of an induction motor running on no-load*

Circuit Diagram See Fig. 4.64.

Apparatus Required Three phase slip-ring type induction motor, three phase variac, wattmeters (two), ammeter moving-iron type (two), voltmeter-moving iron type, tachometer.

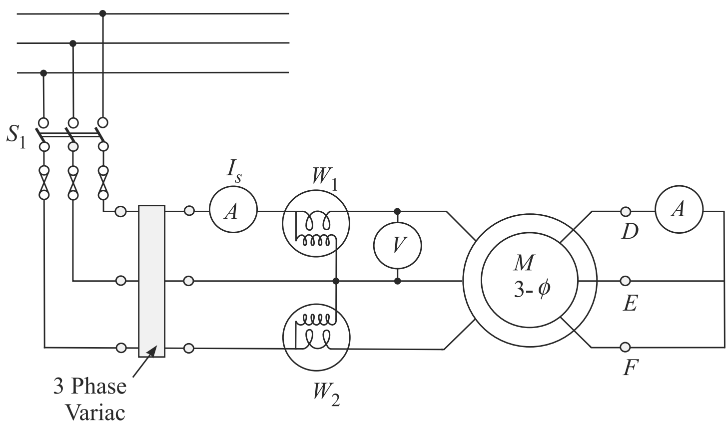


Fig. 4.64 *Connection diagram for no-load test on a three-phase induction motor*

Procedure

1. Make connections according to Fig. 4.64. If the experiment is performed on a squirrel-cage induction motor, no ammeter can be connected in the rotor circuit as has been shown in Fig. 4.63. As such, part (b) of the objectives of the experiment cannot be achieved.
2. Close the main switch St and apply voltage in the stator with the help of the auto-transformer. At about 20 per cent of the rated voltage, the rotor will start rotating and will pick up speed near to its rated speed current, speed and power inputs. Increase the voltage in steps and record the above readings for each step. Increase voltage up to rated value.
3. Tabulate observations and calculate power input and power factor for each reading. Plot characteristics of quantities as indicated in Fig. 4.63.

Observation and Results

S. No.	Stator Voltage	Stator Current	Rotor Current	Speed	Wattmeter Readings		Power Input	Power Factor
					W_1	W_2		
Take 6-7 Readings								

Questions Answer the following questions in your report:

1. From the power input curve determine the value of friction and windage loss. State why friction and windage loss for an induction motor remains approximately constant with change of stator voltage.
2. Express the no-load stator current corresponding to rated stator voltage as a percentage of rated full-load current.
3. Explain why the no-load current of an induction motor is much higher than that of a transformer of the same rating.
4. The power factor of an induction motor decreases as the applied voltage is increased. Explain this with the help of a phasor diagram.

5. An induction motor picks up its rated speed at about 25 to 30 per cent of rated input voltage. Explain why it is necessary to apply more voltage than 25 to 30 per cent of the rated voltage.

EXPERIMENT 4.2 *Blocked-rotor test pm an induction motor.*

Objectives To determine the relationship between the following for a three-phase induction motor under blocked-rotor condition:

(a) applied voltage and input power,
(b) applied voltage and rotor current,
(c) applied voltage and stator current.

Brief Theory This test is similar to short-circuit test on a transformer. This experiment is performed on a three-phase slip-ring type motor when the rotor is not allowed to rotate (if performed on a squirrel-cage type induction motor, part (b) listed under 'objective' cannot be achieved).

The effects of variation of stator voltage on input power, rotor current and stator current are explained as follows:

(a) *Effect on Input Power* When the rotor is blocked, only a small amount of voltage can be applied across stator terminals to allow up to normal full-load current to flow through the windings. The iron-losses will be very small as at that low voltage magnetisation will be low. The power taken by the motor when the rotor is blocked is, therefore, almost entirely due to copper losses. With the increase in stator applied voltage, the losses will increase as the square of the current.

(b) *Effect on Stator Current.* The stator current will increase in proportion to the rotor current, as in a transformer, in order to balance the rotor currents.

Figure 4.65 shows the effect of change of stator voltage on the quantities.

Circuit Diagram See Fig. 4.66.

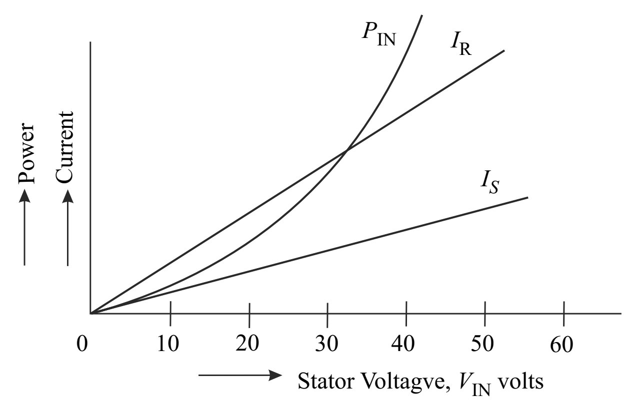


Fig. 4.65 *Effect of exchange of stator applied voltage on power input, rotor and stator current*

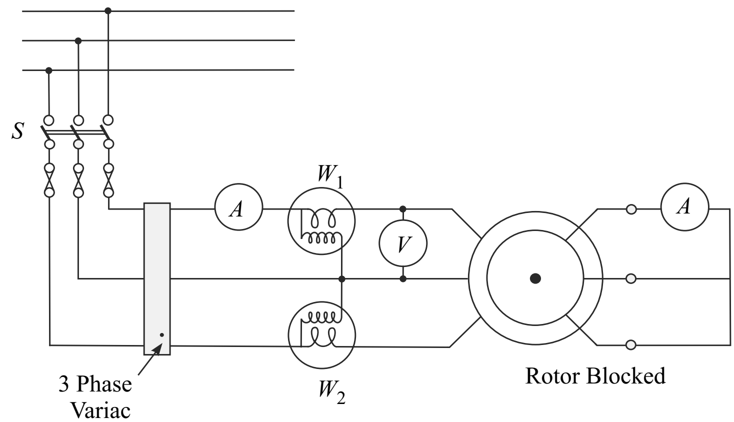


Fig. 4.66 *Connection diagram for the study of the effect on variation of stator voltage on power input, stator and rotor currents of an induction motor under blocked rotor condition*

Apparatus Required Three phase induction motor slip ring type, three-phase auto-transformer, wattmeter (two), ammeters-moving iron type (two), voltmeter-moving iron type, blocking arrangement of the rotor.

Procedure

1. Make connections according to Fig. 4.66. Tie or clamp the rotor in such a way that it cannot rotate. Apply very low voltage and then gradually step by step increase the voltage to the stator windings until a current of about 30 per cent above the normal full-load current of the motor is reached. Record at each step the readings of all the meters.
2. Tabulate your readings as per table shown. Draw characteristics of the quantities similar to as shown in Fig. 4.65.

Observation and Results

S. No.	Stator Voltage	Stator Current	Rotor Current	Wattmeter Readings		Power Input
	V	I_s	I_r	W_1	W_2	P_{in}
Take 6-7 Readings						

Precautions

1. Blocking of the rotor should be done properly.
2. Make sure to apply only a small voltage across the stator terminals with the rotor blocked.

Questions Answer the following questions in your report:

1. State the frequency of the rotor induced emf when the rotor is (a) rotating at a slip of *S* and (b) blocked. The stator supply frequency is 50 Hz.
2. Explain why the power input to the stator with the rotor blocked is nearly equal to copper-losses in the windings.

3. Show how you can calculate the efficiency of an induction motor from the results of no-load test and blocked rotor test.
(Brief theory of the no-load test has been given in experiment No. 4.1)

EXPERIMENT 4.3 *Load test on an induction motor.*

Objectives To determine how speed, efficiency, power factor, stator current, torque, and slip of an induction motor vary with load.

Brief Theory The motor is loaded either by applying brake through belt-pulley arrangement or by loading a dc generator of known efficiency. The effect of applying load on the above mentioned quantities are discussed as follows.

(a) *Effect on Speed* When the induction motor is on no-load, the speed is slightly below the synchronous speed. The current due to induced emf in the rotor winding is responsible for production of torque required at no-load. As the load is increased, the rotor speed is slightly reduced. The emf induced in the rotor and hence the current increases to produce higher torque required, until the torque developed is equal to the torque required by the load on the motor.

(b) *Effect on Slip* Slip is expressed as the difference in the speed of the rotor relative to that of the rotating magnetic field which rotates at synchronous speed. Slip is expressed as a percentage of the synchronous speed thus

$$S = \frac{N_s - N_r}{N_s} \times 100$$

where N_s = synchronous speed, i.e., the speed of the rotating magnetic field. N_r = rotor speed.

Synchronous speed N_s depends upon frequency of stator supply voltage and number of poles for which the motor winding is made:

$$N_s = \frac{120f}{P}$$

Therefore, if f and P are constant, then N_z is constant for a particular motor. Thus with increase in load on the motor if N_r decreases, S will increase.

(c) *Effect on Stator Current* Current drawn by the stator is determined by two factors. Its one component is the magnetising current required to maintain the rotating field. The second component produces a field which is equal and opposite to that formed by the rotor currents. The rotor current increases with load. The stator current will, therefore, increase with load.

(d) *Effect on Power Factor* Power factor of an induction motor on no-load is very low because of the high value of magnetising current. With load the power factor increases because the power component of the current is increased. Low power factor operation is one of the disadvantages of an induction motor. An induction motor draws a heavy amount of magnetising current due to presence of air-gap between the stator and the rotor (unlike a transformer). The reduce the magnetising current in an induction motor, the air-gap is kept as small as possible. It is, therefore, usual to find the air-gap of induction motors smaller than any other type of electrical machine.

(e) *Effect on Efficiency* To study the effect of load on efficiency, we should study the effect of load on the various losses taking place in an induction motor as

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output + losses}}$$

The losses occurring in a motor are of three kinds, viz., (i) losses in the stator and rotor windings, (ii) iron-losses in stator and rotor Core, and (iii) friction and windage losses.

Iron-losses in the stator is proportional to stator flux density and frequency of supply. The strength of the stator field is constant at all loads and hence the stator iron-loss will not change with load.

The iron-loss in the rotor is very small as the frequency of the rotor current is small and therefore, iron-loss in the rotor may be neglected as compared to the stator iron-loss.

Thus, the iron-loss is independent of load on the motor.

As the speed of the motor does not vary very much with load, friction and windage losses also can be assumed as constant.

If the I^2R-losses would have been constant, the efficiency of the motor would increase with load.

But I^2R-loss in both stator and rotor increase as square of the load. Therefore, with load, efficiency will increase but the curve would be dropping at very high loads.

(f) *Effect on Torque*

$$\text{Output} = \text{Torque} \times \text{Speed}$$

As the speed of the motor does not vary appreciably with load, torque will increase with increase in load, i.e., with increase in output.

The variations of the above-mentioned quantities with load are shown in Fig. 4.67.

Circuit Diagram See Fig. 4.68.

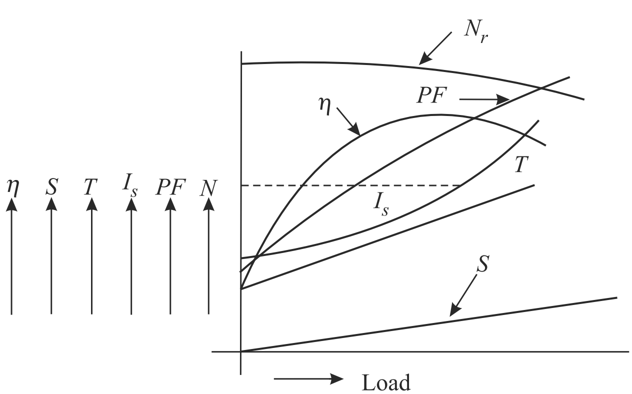


Fig. 4.67 *Graphical representation of the effect of load on rotor speed, efficiency power factor, output torque, stator current and slip of an induction motor*

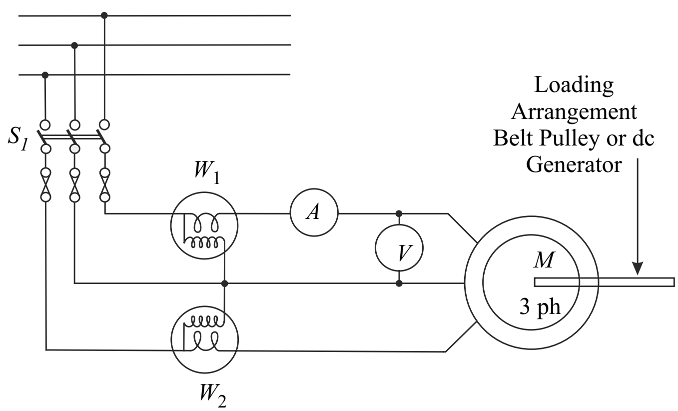


Fig. 4.68 *Connection diagram for loud test on an induction motor*

Apparatus Required Induction motor with loading arrangement, Wattmeter (two), Ammeter and Voltmeter-moving iron type, Tachometer.

Procedure

1. Male connections as per Fig. 4.68. Start the induction motor direct-on-line at no-load by closing switch S. Record no-load speed, power input, stator current and applied voltage.
2. Load the motor step-by-step (either by applying brake or by loading a generator coupled with the motor) and record the quantities mentioned above. Also note the output power/torque in each case. Load the motor till rated current flows through the stator winding.
3. Tabulate observations/results as per table shown.
4. If the efficiency of the dc generator is not known, decouple the generator from the motor and run it as dc motor at no-load. Note the power input to the motor. Also measure armature resistance of the motor.
5. For each reading, calculate power input and power factor from the wattmeter readings. If the motor is loaded through a dc generator, the output torque is to be calculated thus.

Input torque to the generator is equal to the output of the induction motor (as they are coupled together).

Output torque, T of the induction motor is

$$T = \frac{\text{Input power to generator}}{\omega}$$

where ω = angular speed in rad/s of the induction

$$= \frac{2\pi N_r}{60} \quad (N_r \text{ is the rotor speed in rpm})$$

Input to generator = Output of generator + copper, windage, friction and iron-losses of generator.

Windage, friction and iron-losses of generator = Input to the generator when run as motor at no-load minus the armature copper loss ($I_a^2 R_a$) at no-load.

Thus for each speed, T can be calculated For each value of input to the induction motor, its efficiency can be calculated thus:

$$\text{Efficiency} = \frac{\text{Output of the motor (is equal to the input to the dc generator}}{\text{Input to the motor}}$$

Observation and Results

S. No.	Rotor Speed N_r	Applied Voltage V	Stator Current I_s	Wattmeter Readings		Power Input P_{in}	Output Power Torque	Efficiency
				W_1	W_2			
Take 6-7 Readings								

Precautions

1. While loading the induction motor by brakes, check whether cooling water is circulated in the drum. Before starting the motor, loosen the strap and then tighten it gradually when the motor has picked up speed.
2. In case, induction motor is coupled to a dc generator, keep the load switch of the generator in off position at the time of starting the induction motor.

Questions Answer the following in your report:

1. Show one sample calculation for finding output torque, efficiency and slip of the induction motor.
2. Mention at what value of load (expressed in terms of full load) the efficiency of the induction motor under test is maximum. Explain the dropping characteristic of the efficiency curve near full-load.
3. Explain why the power factor of an induction motor on no-load is very poor.

EXPERIMENT 4.4 *Determination of the effect of rotor resistance on the torque-speed curves of an induction motor.*

Objectives To determine how starting torque and torque slip curve change with change in rotor resistance of slip-ring type induction motor.

Brief Theory The equation for torque of an induction motor is

$$T \propto \frac{SE_{20}^2 R_2}{R_2^2 + (SX_{20}^2)}$$

where E_{20} is the induced emf in the rotor winding at rotor stand-still
R_2 is the rotor resistance,
S is the slip,
X_{20} is the rotor reactance at standstill (i.e., when the rotor is not rotating). If the supply voltage is constant, E_{20} will be constant. Thus torque is

$$T \propto \frac{SR_2}{R_2^2 + (SX_{20})^2}$$

The value of X_{20} is usually far greater than the resistance R_2 of the rotor winding. For simplicity assume $R_2 = 1\ \Omega$ and $X_{20} = 8\ \Omega$ and calculate $SR_2/(R_2^2 + S^2X_{20}^2)$ for various values of slip between 1 and 0. The results are plotted by curve *A* in Fig. 4.68. It will be seen that for small values of slip, torque is directly proportional to the slip, whereas for slips between 0.15 and 1, torque is almost inversely proportional to the slip. To study the effect of variation of rotor resistance, the simplest method will be to repeat the calculation of $SR_2/(R_2^2 + S^2X_{20}^2)$ with $R_2 = 2$, $R_2 = 4$ and $R_2 = 8\ \Omega$. The results are represented graphically in Fig. 4.69.

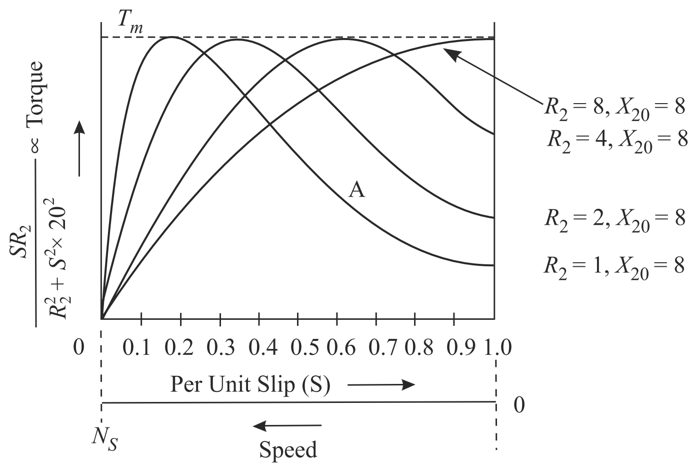


Fig. 4.69 *Torque-slip/speed characteristics of an induction motor at various rotor resistance*

It will be seen that for a particular slip, say, 0.05 the effect of doubling the rotor resistance is to reduce the torque by about 40 per cent, whereas for a slip of 1, i.e., at starting, the torque is nearly doubled when the rotor resistance is increased from 1 to 2 Ω. Hence if a large starting torque is required, the rotor must have a relatively high resistance. It will also be noticed that the maximum value of the torque, T_m is the same for the four values of R_2 and that the larger the resistance, the greater is the starting torque. When the rotor circuit resistance is equal to the rotor reactance at standstill, i.e., when $R_2\ X_{20}$, maximum torque is developed at starting.

Induction motor when started at full voltage takes about six to eight times its rated current. The variation of current with speed is shown in Fig. 4.70. It will be difficult to determine experimentally the complete torque-speed characteristic as the current drawn by the motor at low speed will be very high. The difficulty will be more at very low speed when the current drawn by the motor will be much beyond the permissible limit. It is, therefore, recommended to perform this experiment at a low voltage say 40 per cent of the rated voltage. The shape of the curve will remain undisturbed. The induction motor can be loaded gradually through brake-pulley arrangement or by loading a dc generator coupled to it. In case the motor is loaded through a generator, output torque of the motor can be assumed to be proportional to the output current of the dc generator, the generator being separately excited.

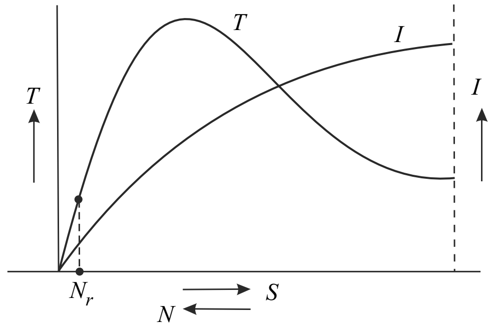


Fig. 4.70 *Variation of current drawn by an induction motor at various speeds*

Apparatus Required Three phase slip ring induction motor having loading arrangement and additional resistance to bc inserted in the rotor circuit, Three phase autotransformer, ammeter and voltmeter (moving iron type), ammeter moving coil type (two), tachometer.

Procedure

1. Make connections as per Fig. 4 71, the figure the loading of the induction motor is shown to be through a de generator. The generator output current I_G will be proportional to the output torque of the induction motor. If loading is done through a brake-puley arrangement, magnitude of output torque can be directly read.

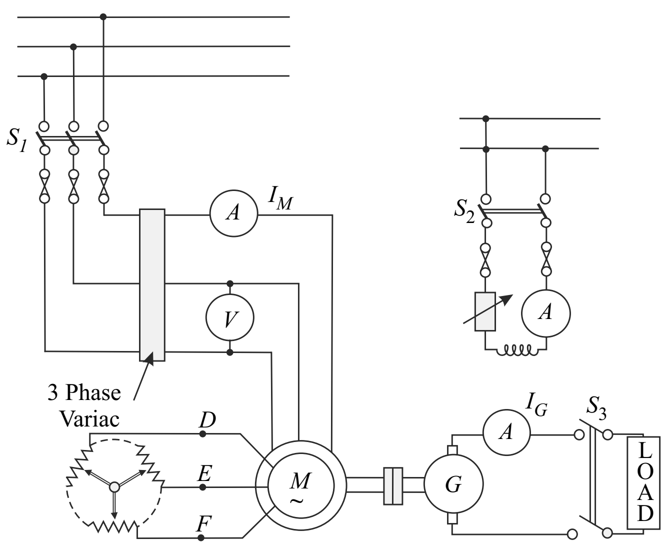


Fig. 4.71 *Connection diagram for determining the effect of rotor resistance on the torque-speed characteristic of an induction motor*

2. Adjust the variac output voltage to about 40 per cent of the rated input voltage of the motor and then start the induction motor on no-load by closing switch S_1 with no extra rotor resistance. (The terminals D, E and F are shorted). Excite the dc generator separately and starting from no load take readings of I_G and speed. Continue loading the dc generator till the speed of the set falls to a very low value. At each step record I_G (or torque) and speed. At low speed the readings should be taken very quickly as the current drawn by the motor, I_M will be about two to three times normal current.
3. Switch off the load on the motor. Insert some resistance in the rotor circuit (of the order of 2 to 3 Ω) and repeat steps mentioned under 2 above. Tabulate observation as follows:
4. Plot on graph paper torque-speed/slip characteristics with (i) no additional resistance in the rotor circuit, and (ii) additional resistance in the rotor circuit.

Precautions While taking observations at low speed, do it quickly as at this stage the rotor current is two to three times its normal current.

Questions Answer the following in your report:

1. Explain in brief the effect of introducing additional rotor resistance in an induction motor as evident from your experimental results.

Observation and Results

S. No.	Stator Input Voltage	Stator Current	Speed	Motor Output Torque	Slip
Run I Without extra resistance in the rotor circuit					
Run II With additional resistance in the rotor circuit					

2. Mention if it is possible to control the speed of an induction motor at a particular load by varying the rotor circuit resistance.
3. State why it is advisable to perform this experiment at a reduced voltage.
4. If by introducing additional resistance in the rotor circuit, starting torque of an induction motor can be increased, then why do we not make rotor windings highly resistive? Explain.
5. If the rotor circuit resistance is increased, the starting torque also increases. When rotor circuit resistance is equal t theory). Mention what would happen to the value of starting torque if rotor circuit resistance, i.e., R_2 is made more than X_{20}.

5 THREE-PHASE SYNCHRONOUS MACHINES

OBJECTIVES

After carefully studying this chapter, you should be able to

- ➢ Explain the constructional details of a synchronous machine.
- ➢ Explain how polyphase emfs are induced in a synchronous generator.
- ➢ Appreciate the advantages of a rotating field system in a synchronous machine over a rotating armature.
- ➢ Explain the basic concepts of making a three-phase armature winding.
- ➢ Derive emf equation and explain the need of distribution of armature winding and use of short pitch coils.
- ➢ Explain armature reaction effect at different power factor loads.
- ➢ Determine voltage regulation by synchronous impedance and other methods.
- ➢ Synchronise synchronous generators for load sharing.
- ➢ Start a synchronous motor.
- ➢ Explain the effect of change of excitation of a synchronous motor on its armature current.
- ➢ Mention applications of synchronous machines giving reasons.
- ➢ Perform basic tests on a synchronous machine to determine performance characteristics.

5.1 INTRODUCTION

The most commonly used machine for generation of electrical power for commercial purpose is the *synchronous generator*. Such a synchronous generator is also called an *alternator* since it generates alternating voltage. A synchronous generator like any other electrical rotating machine has two main parts, viz. the stator and the rotor. The part of the machine in which voltage is induced is called armature. In a synchronous generator the armature winding is placed on the stator slots. The rotor carries the field poles which produce the required magnetic lines of force.

A synchronous machine works as a generator when its rotor carrying the field system is rotated by a primemover. The same machine will work as a synchronous motor when three-phase voltage is applied across the armature winding placed on the stator slots.

The construction of a synchronous generator depends upon the type of primemover used to rotate the rotor. Three types of primemovers are generally used.

In a thermal or nuclear power station a steam turbine is used to drive the alternator. Steam turbines are designed to rotate at a high speed (3000 rpm) as at high speeds the efficiency of the steam turbine is high. In all power stations electricity is generated at a constant frequency of 50 Hz. This is necessary because a large number of alternators are to be connected in parallel to supply the load system. An alternator driven by a steam turbine which is required to generate voltage at 50 Hz will have only two rotor poles. The relationship between rotor speed N_s, frequency of generated emf f and the rotor poles P is given by $N_s = \frac{120f}{P}$. A steam turbine generator set is mounted horizontally.

In hydel power stations hydraulic turbines of different types are used. Hydraulic turbines are of three types, viz. Pelton wheels, Francis turbines and Kaplan turbines. The type of turbine to be used depends upon the water head available in the power station. For water heads up to 50 m, Kaplan turbines are used. Francis turbines are used up to a water head of about 380 m. For higher heads Pelton wheels are used. Since the water heads generally available are not very high, Francis, or Kaplan turbines are normally used. The water turbine and the alternator are coupled together and mounted vertically. The speed of such primemover vary from 50 to 500 rpm. The rotor of the alternator will, therefore, have a large number of poles. For example, in Bhakra hydroelectric power house (situated in Himachal Pradesh, India), the alternators are driven at 166.7 rpm and have 36 poles on their rotors.

Diesel engines are used as primemovers for low rating synchronous generators. Diesel engines are of low speeds as compared to steam turbines. The generators have more than two poles and are always mounted horizontally.

A synchronous machine when working as a motor is called a synchronous motor. Synchronous motors are synchronous machines most often with projected poles which run at constant speed called synchronous speed (N_s = 120 f/P). Change of excitation at any particular load changes the power factor of the motor. An overexcited synchronous motor is often used for power factor correction of a system.

5.2 GENERATION OF SINGLE-PHASE AND POLYPHASE EMFs

According to Faraday's law of electromagnetic induction, when there is cutting of magnetic flux by a conductor or when their is a change of flux linkage by a coil, emf is induced in the conductor or the coil. Figure 5.1 shows a simple arrangement of a coil A_1 A_2 placed on the rotor slots and are free to rotate in a magnetic field created by the stator poles. When the coil is rotated by some means in the magnetic field, there will be a change in the flux linkage by the coil. By flux linkage we mean the amount of the flux passing through the coil. The magnitude of induced emf in the coil will depend upon the rate of change of flux linkage whereas the direction of induced emf will depend upon the direction of flux and the direction of rotation of the coil. Let us examine the nature of induced emf in the coil A_1 A_2. In Fig. 5.2(a), maximum flux passes through the coil A_1 A_2. A slight change in position of the coil from this position does not change the flux linkage. Therefore, we say that at this position the rate of change of flux linkage is minimum (rate of change of flux linkage is determined by the change in flux linkage caused due to a little change in position of the coil). Induced emf in the coil is zero at this position.

As the coil rotates to occupy the position as shown in Fig. 5.1(b), the flux linkage is reduced but the rate of change of flux linkage is increased. Therefore, the induced emf in the coil increases. Rate of change of flux linkage and induced emf is maximum in position shown in Fig. 5.1(c).

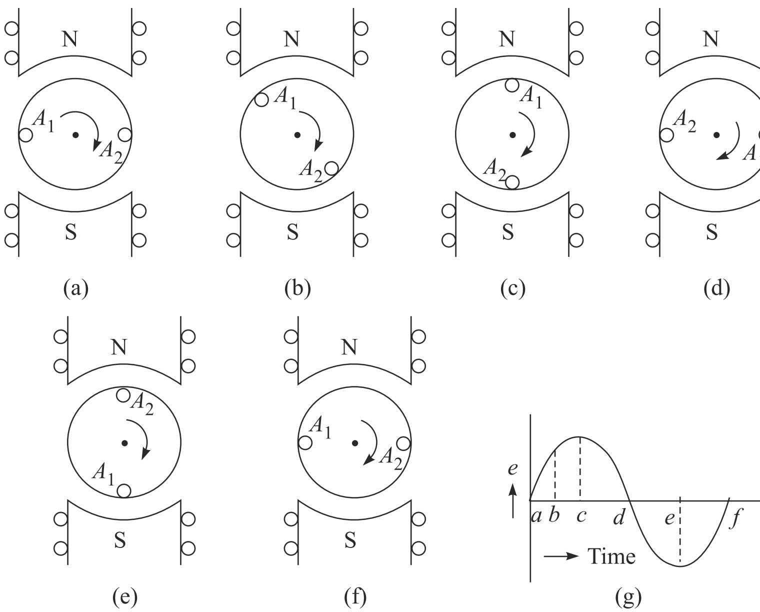


Fig. 5.1 *Induced emf in a coil depends upon the rate of change of flux-linkage by the coil. Figures (a) to (f) shows the coil positions at different instants of time. The corresponding induced emf has been shown in Fig. (b) marked as a, b, c, d, e and f*

Induced emf in the coil goes on reducing as the coil approaches to occupy the position shown in Fig. 5.1(d). Beyond this position the direction of emf changes as the coil A_1 A_2 is now facing the south pole. Emf induced in the coil as the coil makes one revolution has been shown in Fig. 5.1(g). The emf induced in the rotating coil can be collected through brush and slip-ring arrangement for further connection across a load as shown in Fig. 5.2. To generate emf at 50 Hz, the rotor carrying the coil A_1A_2 is to be rotated by a primemover at synchronous speed. This is the simplest form of a single-phase ac generator.

To generate voltage for commercial purposes, a large number of armature coils connected in series are to be used. For generation of three-phase emfs three separate windings insulated from each other should be placed on the rotor. The windings should be displaced by an angle of 120° with each other as shown in Fig. 5.3. When the rotor is rotated, emfs will be induced in the three coils (phases). Here, for simplicity only one coil per phase has been shown. Voltage induced in the three phases R_1–R_1, Y_1–Y_2 and B_1–B_2 will have a time-phase difference of 120°. This can be understood by observing that similar rate of change of flux linkage as that of the coil R_1–R_2 will take place in the coil Y_1–Y_2 after the rotor has rotated by 120° electrical. Time taken by the rotor to rotate by 120° electrical in this case is $\frac{60}{N_s} \times \frac{120}{360}$ s, i.e., $\frac{20}{N_s}$ s.

(This is because the rotor makes N_s revolutions in 60 s and one revolutions means rotation by 360°).

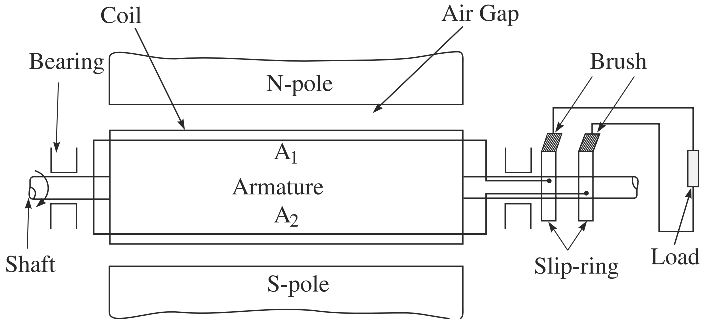


Fig. 5.2 *Single-phase generator with only one coil on the armature*

The three-phase windings can be connected either in star or in delta inside the rotor. Three slip-rings are necessary for making connections with external circuit. For generation of emf at 50 Hz, the rotor is to be rotated at a constant speed called synchronous speed. Synchronous speed depends on the generation frequency and the number of poles for which a machine is made. To generate emfs at 50 Hz, a two-pole generator is to be rotated at 3000 rpm.

A generator being rotated at synchronous speed and generating three-phase emfs is called a three-phase synchronous generator. In Fig. 5.3 the armature windings have been placed on the rotor and the field poles have been shown stationary. Instead, the armature windings could be placed on stator slots and the field system could be made rotating as is done in a practical synchronous generator. The details of construction of a practical three-phase synchronous machine are discussed in the following section.

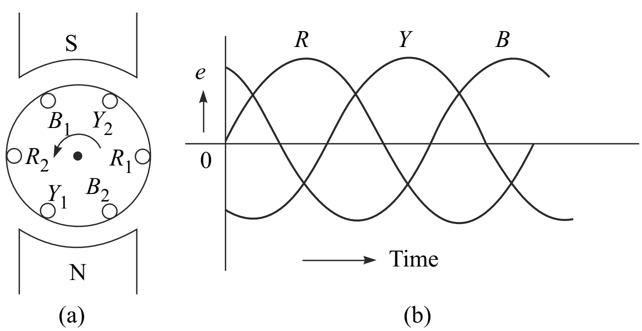


Fig. 5.3 *(a) A simple three-phase winding with three coils R_1–R_2, Y_1–Y_2, and B_1–B_2 placed in rotor slots. (b) Three-phase voltage induced in the three-phases of the rotor windings when the rotor is being rotated*

5.3 CONSTRUCTIONAL DETAILS OF A THREE-PHASE SYNCHRONOUS MACHINE

A synchronous machine works as a generator when the rotor is rotated and as a motor when a three-phase supply is connected across its armature. The basic construction of a synchronous generator and a synchronous motor is the same. As mentioned earlier a synchronous machine while working as a generator is often called an alternator as it generates alternating emf.

In a dc machine, the field system is stationary and the-armature winding is placed on the rotor. The same arrangement can be done in a synchronous machine also. But in a synchronous machine, due to a number of advantages, the field system is made rotating and the armature winding is placed in stator slots. The two possible arrangements of armature and field system are shown in Fig. 5.4.

In synchronous machine construction, the arrangement shown in Fig. 5.4(a) in which the field is stationary and the armature is rotating has limited applications. In almost all commercial synchronous machines, rotating field stationary armature system as shown in Fig. 5.4(b) is used.

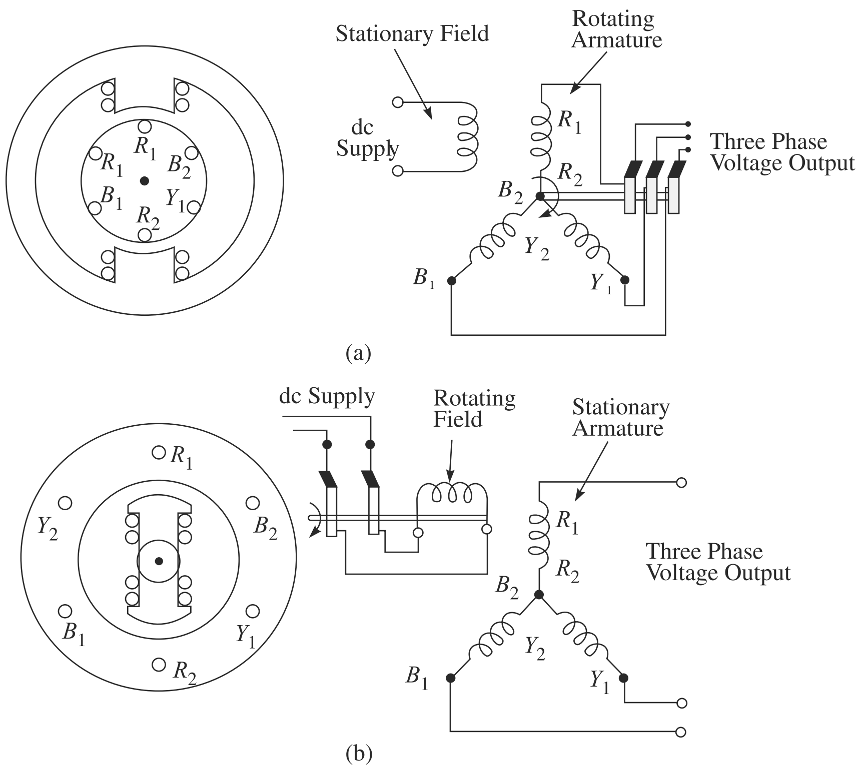


Fig. 5.4 *(a) Stationary field and rotating armature system as in a dc machine, (b) Rotating field and stationary armature system as in an alternator*

5.3.1 Advantages of Rotating Field and Stationary Armature System

The following are the main reasons which favour a stationary armature and rotating field construction for three-phase synchronous machines used in commercial applications.

Ease of Construction For large three-phase synchronous machines, the armature winding is more complex than the field winding. The coil and phase connections including bracing of the windings can be done more easily and securely on a stationary structure, i.e., on the stator than on the rotor.

Number of Slip-rings Required Referring to Fig. 5.4 it is seen that when armature winding is made rotating, at least three slip-rings are needed to receive the generated power for the output circuit from the synchronous generator. For large synchronous machines rated in MVAs and voltage ratings in kilo volts (generally 11 kV) transferring power through brush and slip-ring arrangement may cause some problems. It is also difficult to insulate the slip-rings from the rotating shaft for high voltage. The distance between the slip-rings is to be kept sufficiently large so that flash-over does not take place.

With the stationary armature and rotating field arrangement, none of these problems occur. Only two slip-rings of much smaller size are required to supply excitation current to the rotating windings, as power required for excitation is much less and is supplied at a low voltage.

Better Insulation to Armature Large size commercial synchronous machine armature coils carry heavy currents at high voltage. It is easier to insulate the armature coils from the core if the windings are placed on the stator instead of on the rotor. It is comparatively easier to insulate the low voltage dc winding placed on the rotor.

Reduced Rotor Weight and Rotor Inertia The weight of the field system placed on the rotor is comparatively much lower than the armature winding placed on the stator. This is because the field windings are made with thinner wires and are required to be insulated for a lower voltage. The inertia of the rotor is, therefore, reduced. With rotating field system, the rotor will take comparatively less time to come up to the rated speed.

Improved Ventilation Arrangement Arrangement for forced air-cooling or hydrogen cooling for large machine can easily be made on a stationary armature by enlarging the stator core and providing radial air-ducts and ventilation holes.

Due to the reasons mentioned, the idea of rotating armature for commercial synchronous machines has been unpopular altogether. All the large synchronous machines built today have stationary armature and rotating field structure as shown in Fig. 5.4(b).

5.3.2 Types of Rotor Construction

Three-phase windings are made on the stator of a synchronous machine. The field construction of the rotor may be made *nonsalient type* (cylindrical type) or salient type (projected type) as shown in Fig. 5.5. Type of rotor construction depends upon the type of primemover used to drive the synchronous generator.

5.3.3 Salient Type Rotor for Alternators Driven at Low Speeds

Alternators driven at low speeds by primemovers like water turbines will have salient pole rotors. This is because, to generate electricity at 50 Hz with the rotor rotated at slow speeds, the number of rotor poles required becomes large. It is convenient to build a rotor having large number of poles in projected pole, i.e., salient pole construction. The diameters of such rotors become bigger than their lengths.

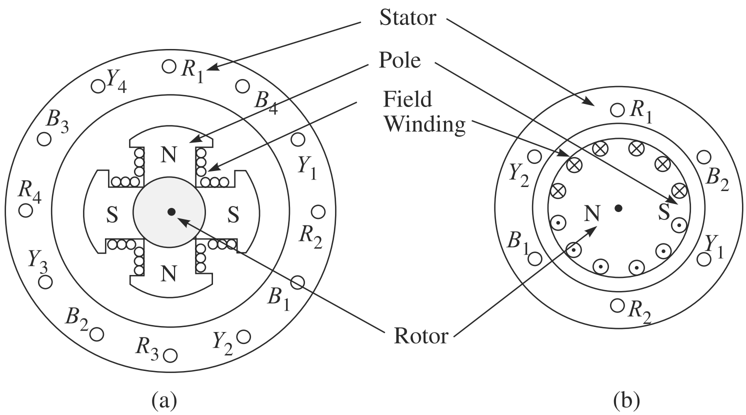


Fig. 5.5 *(a) Salient type rotor; (b) Non-salient or cylindrical type rotor*

5.3.4 Nonsalient Type Rotor for Alternators Driven at High Speeds

For alternators using high-speed turbines (3000 rpm) like steam turbines as primemovers, the number of rotor poles required to generate electricity at 50 Hz is only two. To reduce the centrifugal force developed on the rotor winding at high speed, the rotor diameter is to be kept small. Nonsalient, i.e., cylindrical type rotor construction is made for such synchronous generators. The length of such generators are more than their diameters. For alternators using medium-speed primemover, like diesel engines, the number of rotor poles is more than two and the rotor is made salient type.

5.3.5 Excitation for Rotating Field System

The field windings of an alternator are excited by direct current supply which may be obtained in any of the following ways:

(a) From a dc generator called *exciter*, mounted on the shaft extension of the alternator. For moderately rated alternators, exciters are dc shunt generators. Exciters for large alternators may be separately excited type, whose field windings are fed from another shunt generator called *pilot exciter*. The pilot exciter is also mounted on the same shaft as that of the alternator.

(b) Using a separate three-phase synchronous generator as exciter, mounted on the same shaft as the main synchronous generator. The output of the exciter is rectified through a bank of rectifiers and then fed to the field windings of the main generator.

Figure 5.6 shows the connection diagram of the excitation system of a large alternator using main exciter and pilot exciter.

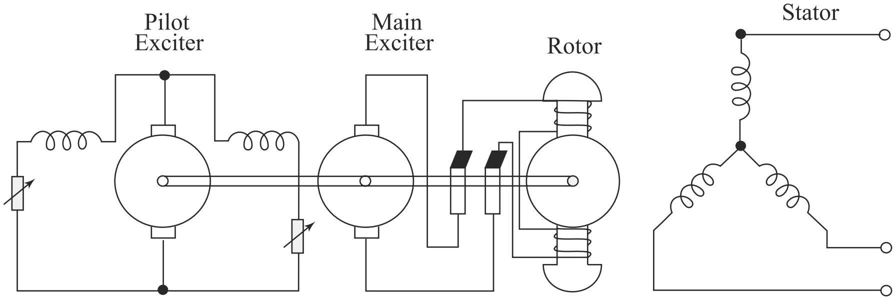


Fig. 5.6 *Excitation system of a synchronous generator*

5.3.6 Use of Damper Windings

When the synchronous machine is also to work as a motor, the rotor is provided with a *damper winding* to enable the motor develop starting torque, since as such, a synchronous motor does not have self-starting ability. The damper windings, are placed in slots made on the pole faces as shown in Fig. 5.7.

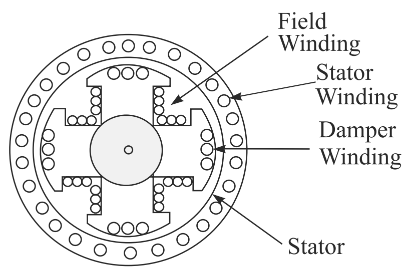


Fig. 5.7 *Damper winding placed in slots of pole faces*

The damper winding is a short-circuited winding similar to the squirrel-cage winding of an induction motor rotor. When three-phase supply is connected across the three-phase stator windings, the rotor starts rotating as a squirrel-cage motor and attains near synchronous speed. When dc supply is applied to the rotor field windings, the rotor is pulled up into synchronous speed.

5.4 THREE-PHASE WINDINGS

It is known that the emfs induced in the armature coils of a dc machine are alternating in nature. By making brush and commutator arrangement, a constant unidirectional emf is obtained across the brush terminals. If slip-rings are used instead of the commutator, single-phase alternating voltage can be obtained from the dc armature. Armature windings of small single-phase ac generators are made exactly similar to the windings of a dc armature winding, and the output can be obtained from the rotating armature through brush and slip-ring arrangement. For large synchronous machine, three-phase windings are wound for a particular number of poles, depending upon the type of primemover to be used to drive the rotor. The three windings of the three phases are insulated from one another and are displaced in space at an angle of 120° electrical. A simple three-phase 2-pole stator winding with only one coil per phase is shown in Fig. 5.8.

Assuming three-phase current flowing through the windings the direction of current flowing through the conductors at a particular instant of time has been shown in the developed winding diagram. The instantaneous positions of the two poles formed on the stator have also been shown in the figure.

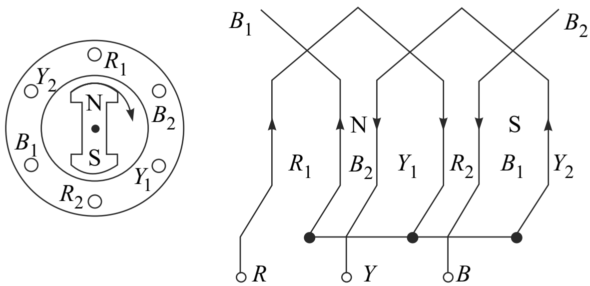


Fig. 5.8 *A simple three-phase two-pole stator winding with only one coil per phase*

Since the winding is made for two poles, the number of electrical degrees is equal to the number of mechanical degrees. Coil pitch is 180° for the full-pitch coils used in this case. In full-pitch coils, the two coil-sides of a coil occupy identical position under opposite poles. Three phase windings are displaced at an angle of 120°. Referring to Fig. 5.8 it is seen that Y phase starts after 120° from the start of R phase. Similarly, B phase starts after 120° from the start of Y phase. When the rotor rotates, three separate alternating emfs will be induced in the three-phase windings. These emfs will have time-phase displacement of 120°. Note that in this winding, only one coil per phase has been used and the winding has been made for two poles. In actual winding, the number of coils per phase will be more for various reasons.

Figure 5.9 shows a three-phase 4-pole winding. The direction of current in the phase windings and the position of the four poles shown in the figure are for a particular instant of time. Here R_1–R_2–R_3–R_4 constitute R phase winding. Similarly, Y_1–Y_2–Y_3–Y_4 and B_1–B_2–B_3–B_4 constitute respectively Y phase and B phase windings. Here also full-pitch coils have been used. The coil pitch is 180° electrical (i.e., 90° mechanical). The phases are displaced physically by 120° electrical (i.e., 60° mechanical). The total number of coils per phase has been only two. Each coil should have a large number of turns to get considerable amount of induced emf per phase. Such a winding is called concentrated winding. In actual practice, distributed windings are used in machine winding instead of a concentrated winding.

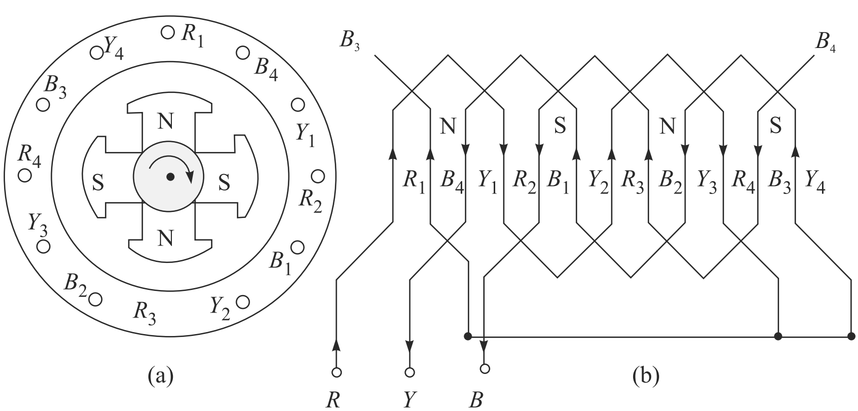


Fig. 5.9 *A three-phase four-pole winding (a) cross-sectional view, (b) developed winding diagram*

The advantages of a distributed winding are: Better dissipation of heat produced due to I^2R-losses in the winding wires and better emf waveform (generated emf is more towards a sine wave in a distributed winding than in a concentrated winding). However, the magnitude of the total emf generated per phase is somewhat less in the case of a distributed winding as compared to a concentrated winding. This is because, when the coils are distributed, the emf generated in the coil-sides occupying adjacent slots are displaced from each other in time-phase and their total vectorial sum is less than their arithmetic sum. The distributed windings for each phase can be made in several ways as shown in Fig. 5.10.

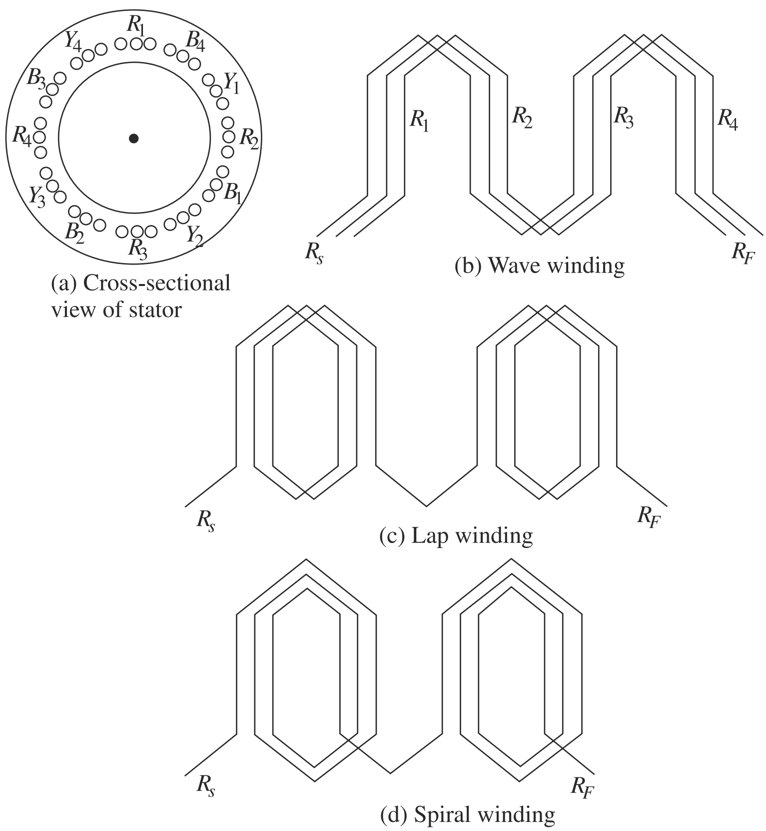


Fig. 5.10 *(a) A three-phase four pole distributed stator winding with coils of the phases connected in wave, lap, and spiral as shown respectively in (b), (c) and (d) above*

The windings of a phase may be connected in wave, in lap or in spiral as shown in the figure. The windings shown in Fig. 5.10(a) are of single layer type. It is also possible to arrange the windings in two layers. The coils used for windings so far have been full-pitch coils. When a winding is made with coil span less than full-pitch, the winding is called *fractional pitch winding.* The coil-sides of a fractional pitch coil will not occupy identical positions under opposite poles. In practice, fractional pitch windings are used, so that waveform of the induced emf is more towards a sinewave. Harmonics present in a nonsinusoidal wave can be eliminated through a proper choice of fractional pitch coils for the winding. An added advantage of short-pitch coil winding is that there is saving of copper required in the end connections of coils. However, the magnitude of emf induced in a coil will be reduced. The emfs induced in the two coil-sides will not he added arithmetically since now they do not occupy exactly identical positions under opposite poles. Fractional pitch windings are ordinarily of the two-layer lap-wound type.

5.5 INDUCED EMF IN A SYNCHRONOUS MACHINE

Emf is induced in the armature windings due to cutting of flux by the rotation of the rotor poles. Distribution of coils in the stator slots and the use of short-pitch coils affect the magnitude and the waveshape of the induced emf.

5.5.1 Emf Equation

Figure 5.11 shows the cross-sectional view of a synchronous generator. The stator has a simple 2-pole three-phase winding. A salient pole type rotor is shown rotating in clockwise direction. When the rotor rotates, magnetic lines of force will cut the three-phase winding and emf will be induced in them. For simplicity, only one coil per phase has been shown.

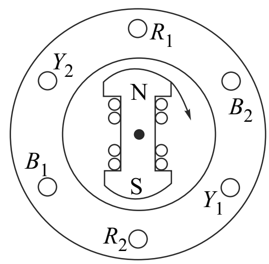


Fig. 5.11 *Cross-sectional view of a synchronous generator*

In actual practice there will be more coils per phase of the stator winding and each coil will have many number of turns.

Let,

T be the number of turns in the coils connected in series in each phase,
ϕ is the flux per pole in webers,
P is the number of poles, and
N is the rpm of the rotor.

Magnetic flux cut by a conductor in one revolution of the rotor poles = $P \times \phi$ webers. The rotor poles make N revolutions per minute (i.e., per 60 s).

Time taken by the rotor poles to make one revolution

$$= \frac{60}{N}\ s$$

Therefore, flux cut per second by a conductor of the stator.

$$= \left(P\phi \div \frac{60}{N}\right) \text{Wb/s}$$

$$= \frac{P\phi N}{60}\ \text{Wb/s}$$

Average induced emf in a conductor = Flux cut per second

$$= \frac{P\phi N}{60}\text{Wb/s} = \frac{P\phi N}{60}\text{Volts}$$

Since T is the total number of turns connected in series per phase, total number of conductors per phase = $2T = Z$ (say).

Average emf induced per phase

$$= \frac{P\phi N 2T}{60} \text{Volts}$$

or $$E_{av} = \frac{P\phi N Z}{60} \text{Volts}$$

For a sinusoidally distributed flux, the waveshape of the induced emf will be sinusoidal.

If the generated emf waveshape is assumed to be sinusoidal, then using the relation

$$\frac{E_{\text{rms}}}{E_{av}} = 1.11 \text{ (for a sine wave)}$$

The rms value of induced emf,

$$E = \frac{2.22 P\phi NT}{60} \text{V} = \frac{2.22\ \phi T PN}{60} \text{V}$$

The relationship between f, P and N is given by

$$\text{N} = \frac{120f}{P}$$

or $$\frac{PN}{60} = 2f$$

Substituting this value in the emf equation

$$E = 2.22\ \phi T \times 2f \text{ Volts}$$

or $$E = 4.44\ \phi f T \text{ Volts} \tag{5.1}$$

It may be noted that the above equation is the same as that of a transformer. In case of a synchronous machine the relative motion of steady rotor flux and stator coils produce the voltage which is the same as that produced in a transformer by a time varying flux in association with stationary coils.

Three independent emfs as expressed by Eq. (5.1) will be induced in the three phases R, Y and B. Since the phases are displaced physically by 120°, the induced emfs in them will have a time-phase difference of 120°.

In Fig. 5.12 only six slots (only two slots for one phase shown in Fig. 5.12 (a)) have been used for a three-phase 2-Pole winding. The number of slots per pole per phase has been $\frac{6}{2 \times 3} = 1$.

This means that all the phase winding coils have been put together. The induced emf in each coil of a phase has, therefore, been added arithmetically. The coil pitch used in the winding has also been chosen to be of full-pitch. Such concentrated windings by having only one slot per pole per phase wound with full-pitch coils are not used in practice. Instead, distributed windings are made using short-pitch coils. Proper distribution of windings on the stator and the use of short pitch coils for the windings improves the waveshape of the induced emf, i.e., the waveshape becomes more sinusoidal. The magnitude of induced emf per phase, however, gets reduced due to distribution of coils and use of short-pitch coils. The effect of distribution of the windings on the magnitude of induced emf is expressed by a factor called *distribution factor* or *breadth factor,* K_d. The effect of use of short-pitch coils on the magnitude of induced emf is expressed by a factor called *pitch factor*, K_p. The expression for induced emf is to be multiplied by these two factors since they directly affect the magnitude of induced emf. The values of K_d and K_p are less than one. For a concentrated winding using full-pitch coils, values of K_d and K_p are, however, unity.

Thus the emf equation for a synchronous generator can be expressed as

$$E = 4.44\, \phi f T K_d K_p \text{ V} \tag{5.2}$$

Winding Factor The distribution factor, K_d and the pitch factor, K_p of an armature winding are often combined into a single factor, called winding factor, K_ω which is the product of K_p and K_d.

Thus, winding factor, $K_\omega = K_p K_d$

The effect of distribution of the phase winding on the stator slots is explained in detail as follows:

5.5.2 Effect of Distributing the Winding on Induced EMF

Effect of Distribution on the Magnitude of Total Induced EMF In Fig. 5.12(a) is shown the coil-sides of three coils of a synchronous generator stator winding placed together, whereas in Fig. 5.12(b) they are shown placed in different adjacent slots. Since all the coil-sides in Fig. 5.12(a) are placed in one slot, the emfs induced in all the coils will be in time-phase. If the coils are connected in series, the total emf available across them will be the arithmetic sum of the individual emfs as has been shown in Fig. 5.12(a).

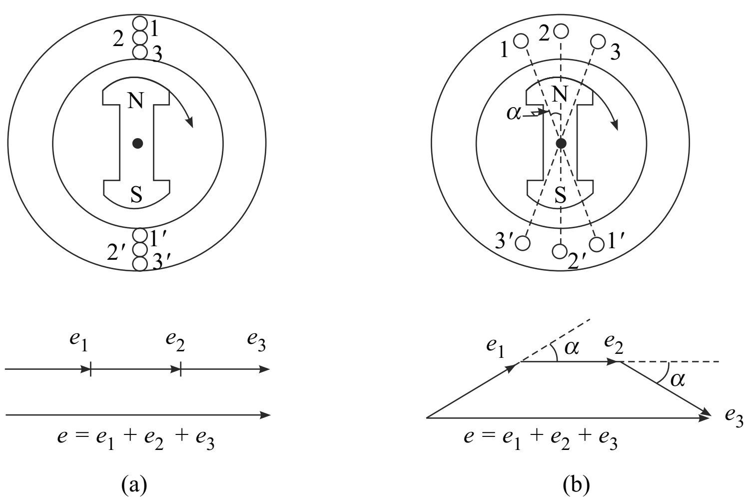


Fig. 5.12 *(a) Induced emf in a concentrated winding (arithmetic sum of coil voltages), (b) Induced emf in a distributed winding (Vector sum of coil voltages)*

In Fig. 5.12(b) are shown the same coils distributed in adjacent slots and are connected in series. The emfs induced in the conductors will have a time-phase difference of $\alpha°$. This is because, similar rate of cutting of flux in coils 2-2′ and 3-3′ will occur after a time lag of α and 2α respectively, from that in coil 1-1′ for the direction of rotation of the rotor shown in figure. For determining the total emf, the individual emfs should be added considering their instantaneous directions. If we represent the individual emfs by phasors, the resultant emf will be as shown in Fig. 5.12(b). It can be seen that in a distributed winding, the magnitude of the total emf induced in the winding is reduced as compared to the emf induced in a concentrated winding. Angle α is called the slot-angle which is the angular distance between the two adjacent slots.

Distribution Factor Assume that the slot angle, α in Fig. 5.12 be equal to say, 20°. The induced emf in the three coils, forming one phase winding, will differ in time phase by 20° and can be represented by phasors as shown in Fig. 5.13(a). The magnitude of total emf is reduced when the winding is distributed. In this case the total emf, e is less than arithmetic sum of the three emfs e_1, e_2, e_3.

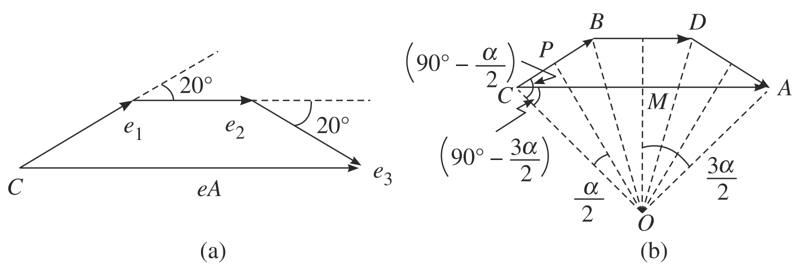


Fig. 5.13 *(a) Emf induced in the coils has a time phase difference due to their placement in different slots, (b) calculation of resultant voltage in terms of emf induced in the individual coils*

Distribution Factor K_d is defined as the ratio of emf induced per phase with a distributed winding to the emf induced with a concentrated winding.

In this case,

$$K_d = \frac{\text{phasor sum of } e_1, e_2, e_3, \text{ i.e., } e}{\text{arithmetic sum of } e_1, e_2, e_3}$$

For deriving a general expression for distribution factor in terms of slot angle α, and the number of slots per pole per phase m, let us refer to Fig. 5.13(b). The magnitude and direction of induced emfs in the three coils are represented by phasors *CB, BD* and *DA*. The resultant emf is represented by the phasor *CA*.

The magnitude of induced emf in the coils are equal

Therefore,

$$CB = BD = DA = e_c \text{ (say)}$$

$$e_c = CB = CP + PB$$

$$CP = OC \cos (90 - \alpha/2)$$

$$PB = OC \cos (90 - \alpha/2)$$

Therefore, $$e_c = 2\,OC \cos(90 - \alpha/2)$$
$$= 2\,OC \sin \alpha/2$$

Arithmetic sum of the three voltages $= 3e_c = 3 \times 2\,OC \sin \alpha/2$.

If there are m number of slots per pole per phase and α is the slot angle, the arithmetic sum of the voltage induced in the coils will be:

Arithmetic sum (emf with concentrated winding)

$$= m \times 2\,OC \sin \alpha/2 \tag{5.3a}$$

Magnitude of the resultant voltage is represented as the phasor sum of the voltage in all the three coils. Referring to Fig. 5.13(b),

Phasor sum, $$CA = CM + MA$$

$$CM = MA = OC \cos\left[90 - \frac{3\alpha}{2}\right]$$

$$CA = 2\,OC \cos\left[90 - \frac{3\alpha}{2}\right]$$

$$= 2\,OC \sin 3\,\alpha/2$$

For m number of slots per pole phase an α as slot angle, phasor sum (emf with distributed winding)

$$= 2\,OC \sin \frac{m\alpha}{2} \tag{5.3b}$$

Distribution factor, $$K_d = \frac{\text{emf with distributed winding}}{\text{emf with concentrated winding}}$$

$$= \frac{\text{phasor sum of emf}}{\text{arithmetic sum of emf}}$$

Therefore, distribution factor is found by dividing Eq. (5.3b) by Eq. (5.3a) as

$$K_d = \frac{2OC \sin \frac{m\alpha}{2}}{m\,2OC \sin \alpha/2} \quad \text{or,} \quad K_d = \frac{\sin \frac{m\alpha}{2}}{m \sin \alpha/2} \tag{5.4a}$$

The value of K_d for a distributed winding is always less than unity.

Effect of Distribution on the Waveshape of the Total Induced EMF Let us now examine how the distribution of coils in slots affect the waveshape of the total induced emf. For the sake of comparison, the graphical summation of the emfs in the coils for a concentrated winding and a distributed winding have been shown in Fig. 5.14. The resultant emf waveshape has been drawn by adding the magnitude of individual emfs at every instant of time. In Fig. 5.14(a), it is observed that the waveshape of the total emf is similar to the waveshape of the induced emf in each coil. For a distributed winding, however, the emfs e_1, e_2, e_3 are displaced from each other and the waveshape of the resultant emf has changed to a stepped rectangular one as shown in Fig. 5.14(b).

It is observed that the waveshape is more like a sine wave than a rectangular wave. By proper distribution of the winding coils, it is possible to improve the waveshape further. In addition to distribution of windings, by proper design of the magnetic circuit, the air-gap flux distribution can also be modified such that it contributes ultimately to the production of a nearly sinusoidally varying induced emf.

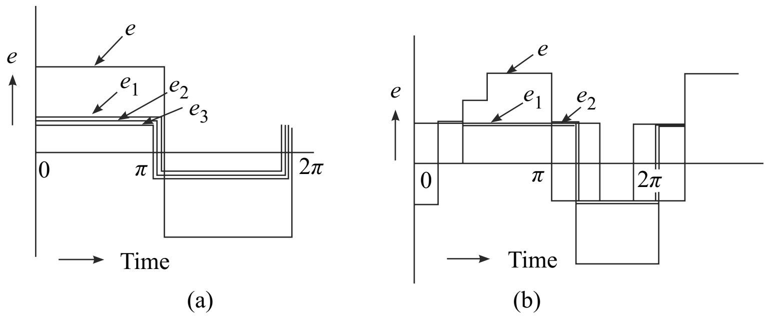


Fig. 5.14 *(a) Wave shape of the total emf in a concentrated winding*
(b) Wave shape of the total emf in a distributed winding

Generation of a sinusoidally varying emf is necessary for satisfactory working of the various machines connected to the supply system. A nonsinusoidal emf will contain a number of harmonic waves (frequencies of harmonic waves are multiples of the main wave) and will result in complications in performance of machines supplied with such a nonsinusoidal voltage.

5.5.3 Effect of Using Short-Pitch Coil on Induced EMF

Effect of Using Short-Pitch Coils on the Magnitude of Induced EMF

A full-pitch coil is one in which the two coil-sides of a coil when placed in slots occupy identical positions under adjacent opposite poles. The distance between the two coil-sides of a coil expressed in degrees is called coil span. For a full-pitch coil, the coil span is 180°. In a short-pitch coil, the coil span is reduced by some degrees from the full-pitch span. In Fig. 5.15 are shown a full-pitch coil, $A_1 A_2$ and a short-pitch coil $A_1 A_2'$. The angle of short-pitching used is $\beta°$.

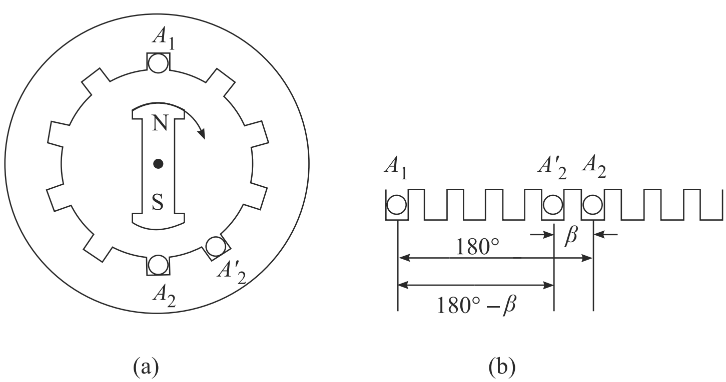


Fig. 5.15 *Short-pitch coils for armature winding*

The magnitude of induced emf in a coil is the sum of the emfs induced in the two coil-sides. For a full-pitch coil the emfs of the two coil-sides are in phase as shown in Fig. 5.16. In a short-pitch coil since the two coil-sides of a coil do not occupy identical positions under opposite poles, there will be some time-phase difference between the emfs induced in the coil-sides. Due to this phase difference, the phasor sum of the voltages of the two coilsides will be less than their arithmetic sum as shown in the figure.

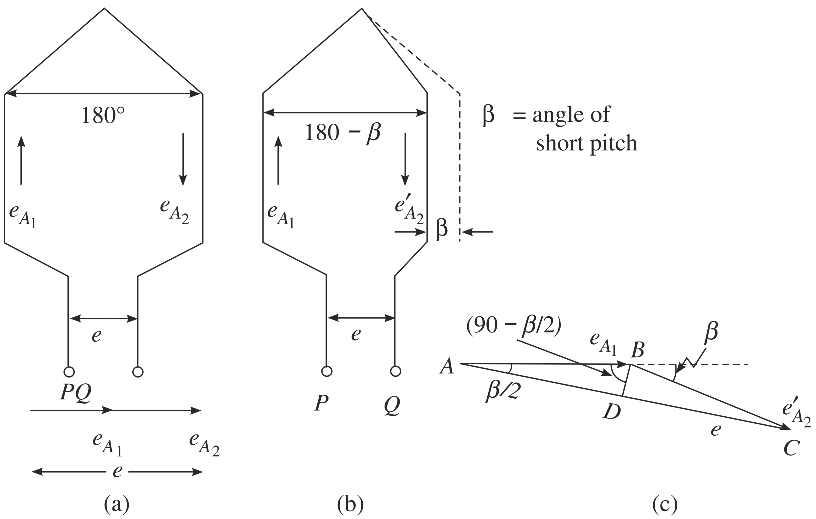


Fig. 5.16 *(a) Induced emf in a full-pitch coil (b) Induced emf in a short-pitch coil (c) Calculation of pitch factor*

5.5.4 Pitch Factor

The total emf induced in a phase, therefore, will be less in a winding made of short-pitch coils as compared to the emf induced in a phase winding made with full-pitch coils. The factor involved is called pitch factor, K_p which is defined as

$$K_p = \frac{\text{Emf induced in short-pitch coil}}{\text{Emf induced in a full-pitch coil}}$$

Referring to Fig. 5.16(c)

$$K_p = \frac{AC}{AB + BC} = \frac{AD + DC}{AB + BC}$$

$$= \frac{AB\cos\beta/2 + BC\cos\beta/2}{AB + BC}$$

$$= \frac{2\,AB\cos\beta/2}{2\,AB} \qquad \text{[since } AB = BC\text{]}$$

$$= \cos\beta/2$$

Therefore, pitch factor,

$$K_p = \cos\beta/2 \qquad (5.4b)$$

If, for example, the coil span is 150°, i.e., if angle of short-pitch β is

$$\beta = 30°$$

$$K_p = \cos\frac{30°}{2} = 0.96$$

The value of pitch factor of a winding using fractional pitch coils (coil pitch less than 180°) is always less than unity. For a winding using full-pitch coils, this factor is 1.

Effect of using Short-pitch Coils on the Waveshape of the Induced EMF

As mentioned earlier, using short-pitch coils, the waveshape of the emfs induced in the coils and hence across the phase windings can be improved. Let us examine how this is achieved. The waveshape of the induced emf in a coil will contain harmonic waves if the air-gap flux wave contains harmonics. By proper short-pitching of the coils the predominant harmonic waves other than the fundamental can be filtered out in the emf induced in a coil. Let e_{11} and e_{21} be the emfs induced in the coil-sides due to fundamental frequency air-gap flux and let e_{13} and e_{23} be the emfs induced in the coil-sides due to presence of a third harmonics air-gap flux. If short-pitch coils of coil span of 120° ($\beta = 60°$) is chosen, the time-phase difference between the emfs of the two coil-sides for the fundamental frequency wave will be 60° whereas the time-phase difference between the emfs of the two coil-sides for the third harmonic wave will be 60 × 3 = 180°. The emfs induced in the coil-sides for fundamental frequency wave and third harmonic wave and their phase relationship will be as shown in Fig. 5.17.

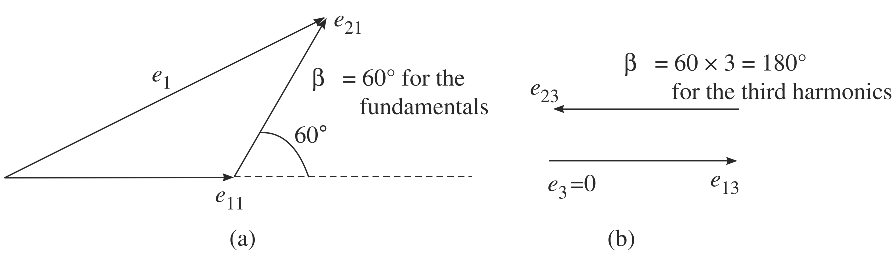


Fig. 5.17 *(a) EMF induced in the coilsides and the resultant emf in a coil due to fundamental frequency flux wave (b) EMF induced in the coilsides and the resultant emf in a coil due to third harmonic flux wave*

The sum of the third harmonic emfs induced in the two coil-sides of a coil is zero as the component emfs are in opposition (see Fig. 5.17b). Thus, using short pitching by 60° the third harmonic, if present, can be eliminated. By Fourier analysis of the air-gap flux wave it is possible to identify the pre-dominant harmonic present in the flux wave. Accordingly, appropriate short-pitching may be used to eliminate the emf induced in the coil due to that harmonic flux. In this process of eliminating the harmonic emf, the magnitude of the fundamental frequency emf in the coil is reduced. The magnitude of induced emf is to be multiplied by pitch factor when a winding is made with short-pitch coils.

EXAMPLE 5.1 *Calculate the distribution factor for a single layer 18 slot 2-pole three-phase stator winding.*

Solution There are 18 slots on the stator. Since one coil will occupy two slots, nine coils will be used in this winding. The number of coils per phase will be three. The coils of each phase are connected in series. Since the three coils of a phase are placed in different slots (distributed winding), the emfs induced in them due to rotation of the rotor poles will have a time-phase displacement. The 18 slots of the stator subtend an angle of 360° which means the slot angle (angle between two adjacent slots) α = 360/18 = 20° mechanical. The placement of coils in the stator slots are shown in Fig. 5.18. The number of slots/pole/phase,

$$m = \frac{18}{2 \times 3} = 3$$

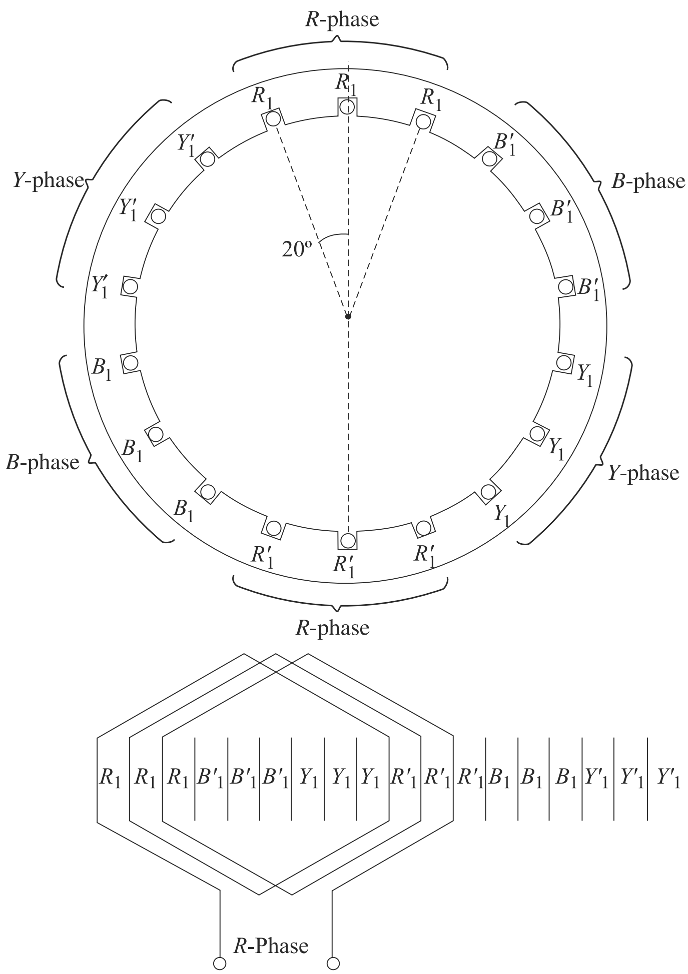


Fig. 5.18 *Refers to Example 5.1. Coils $R_1R_1R_1$–$R_1'R_1'R_1'$ constitute R-phase; coils $Y_1Y_1Y_1$–$Y_1'Y_1'Y_1'$ constitute Y-phase; and coils $B_1B_1B_1$–$B_1'B_1'B_1'$ constitute B-phase*

The emfs of the coils of each phase will have a time-phase difference of 20°. Their phasor sum will be less than their arithmetic sum.

The distribution factor,

$$K_p = \frac{\sin\frac{m\alpha}{2}}{m\sin\alpha/2}$$

Here, $m = 3$

and $\alpha = 20°$ mechanical

$= 20°$ electrical [since $P = 2$ and $1°\text{mech} = \frac{P}{2}$ degree elec.]

Therefore, $$K_d = \frac{\sin\frac{3\times 20}{2}}{3\sin\frac{20}{2}} = \frac{\sin 30°}{3\times\sin 10°} = \frac{0.5}{3\times 0.1736} = 0.96$$

EXAMPLE 5.2 *Calculate the distribution factor for a 36 slot, 4-pole, single layer three-phase winding.*

Solution Number of slots used per phase = $\frac{36}{3} = 12$

Since it is a 4-pole winding, number of slots per pole per phase, is calculated as

$$m = \frac{12}{4} = 3$$

Slot angle, $$\alpha = \frac{360}{36} = 10° \text{ mechanical}$$

$= 20°$ electrical [since $P = 4$]

Note: 1° mechanical = $P/2°$ electrical

$$K_d = \frac{\sin\frac{3\times 20}{2}}{3\sin\frac{20}{2}} = \frac{\sin 30°}{3\sin 10°} = \frac{0.5}{3\times 0.1736} = 0.96$$

EXAMPLE 5.3 *The stator winding of a synchronous machine has 48 slots. A 4-pole, three-phase winding is made on the stator. Each coil span 11 slot pitches. Calculate the pitch factor.*

Solution

Total number of slots = 48

These are spread over 360° mechanical.

Slot angle $$\alpha = \frac{360}{48} = 7\frac{1°}{2}$$

$= 15°$ electrical [since $P = 4$]

For a 4-pole winding with 48 slots, a full-pitch coil will subtend 12 slots which is equivalent to 180°, electrical. For short-pitch coils used in this winding the coil span is 11 slots, i.e., the coils are short pitched by one slot. This is shown in Fig. 5.19. Here β is equal to 15°.

Pitch factor, $K_p = \cos \beta/2 \cos = 15/2 = \cos 7.5 = 0.99$.

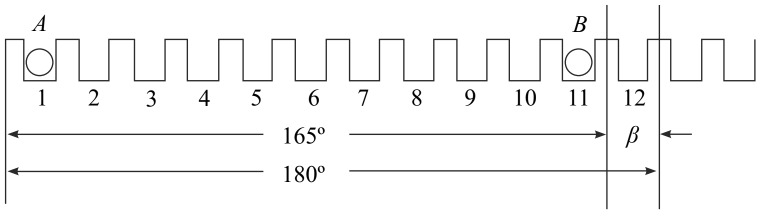


Fig. 5.19

EXAMPLE 5.4 *The stator of a three-phase, 8-pole synchronous generator driven at 750 rpm has 72 slots. The winding has been made with 36 coils having 10 turns per coil. Calculate the rms value of the induced emf per phase if the flux per pole is 0.15 Wb, sinusoidally distributed. Assume that full-pitch coils have been used.*

Solution The emf equation for a synchronous generator is

$$E = 4.44\, \phi f T K_d K_p \text{ V}$$

Given $\phi = 0.15$ Wb

$$f = \frac{P N_s}{120} = \frac{8 \times 750}{120} = 50 \text{ Hz}$$

Number of coils per phase = 36/3 = 12

Number of turns per phase, $T = 12 \times 10 = 120$.

Since full-pitch coils are used, $K_p = 1$

To calculate K_d we will use the expression

$$K_d = \frac{\sin m\alpha/2}{m \sin \alpha/2}$$

Number of slots per pole per phase,

$$m = \frac{72}{8 \times 3} = 3$$

Slot angle,

$$\alpha = \frac{360}{72} = 5° \text{ mechanical}$$

$$= 20 \text{ electrical} \quad [\text{since } P = 8]$$

Substituting these values,

$$K_d = \frac{\sin \frac{3 \times 20}{2}}{3 \sin \frac{20}{2}} = 0.96$$

Rms value of emf induced per phase,

$$E = 4.44 \times 0.15 \times 50 \times 120 \times 0.96 \times 1 \text{ V} = 3836 \text{ V}$$

EXAMPLE 5.5 *A three-phase, star-connected synchronous generator driven at 750 rpm is required to generate a line-to-line voltage of 440 V at 50 Hz on open circuit. The stator is wound with 2 slots per pole per phase and each coil has 4 turns. Calculate the useful flux per pole.*

Solution

$$E \text{ (line-to-line)} = 440 \text{ V}$$

E (per phase) $\quad E = \dfrac{440}{\sqrt{3}} = 254 \text{ V}$

$$P = \frac{120\, f}{N_s} = \frac{120 \times 50}{750} = 8$$

Number of slots per pole per phase, $m = 2$

Total number of stator slots = 2 × 8 × 3 = 48

Slot angle, $\quad \alpha = \dfrac{360}{48} = 7\dfrac{1}{2}$ deg mechanical

[since 1° mechanical = P/2° electrical]

$$= 30° \text{ electrical}$$

$$K_d = \frac{\sin \dfrac{2 \times 30}{2}}{2 \sin \dfrac{30}{2}} = \frac{\sin 30}{2 \sin 15} = 0.966$$

$K_p = 1$ (assuming full-pitch coils)

Number of slots per phase = 2 × 8 = 16
Number of turns per coil = 4
Number of turns per phase, $T = 8 \times 4 = 32$
Rms value of emf induced per phase is,

$$E = 4.44\, \phi f T K_d K_p \text{ V}$$

Substituting the values,

$$254 = 4.44 \times \phi \times 50 \times 32 \times 0.966 \times 1$$

or $\quad$ Flux per pole $\phi = 36.9 \times 10^{-3}$ Wb

EXAMPLE 5.6 *A 3-phase, 16-pole synchronous generator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, sinusoidally distributed, and the speed is 375 revolutions per minute. Calculate the frequency, and line induced emf.*

Solution

Synchronous speed, $\quad N_s = \dfrac{120 f}{P}$

$$N_s = \frac{N_s \times P}{120} = \frac{375 \times 16}{120}$$

Assuming full-pitch coils being used, $K_p = 1$

$$K_d = \frac{\sin m\alpha}{m \sin \frac{\alpha}{2}}$$

Here, solt angle, $\alpha = \frac{360}{144}$ degree mechanical

$$= \frac{360}{144} \times \frac{p}{2} \text{ degree electrical}$$

$$= \frac{360 \times 16}{144 \times 2}$$

$$= 20° \text{ electrical}$$

Number of slots per pole per phase, $m = \frac{144}{16 \times 3} = 3$

Substituting values,

$$K_d = \frac{\sin \frac{3 \times 20}{2}}{3 \sin \frac{20}{2}} = \frac{\sin 30}{3 \sin 10} = 0.96$$

Number of turns per phase, $T = \frac{144 \times 10}{2 \times 3} = 240$

Rms value of induced emf per phase,

$$E = 4.44\, \phi f T K_d K_p \text{ V}$$

$$= 4.44 \times 0.03 \times 50 \times 240 \times 0.96 \times 1 \text{ V} = 1534 \text{ V}$$

Induced emf across the lines = $\sqrt{3} \times 1534 = 2657$ V

EXAMPLE 5.7 *Find the number of armature conductors is series per phase required for the armature of a 3-phase, 10-pole, 50 Hz, synchronous generator with 90 slots. The winding is to be star-connected so as to have line voltage of 11 kV. The flux per pole is 0.16 Wb.*

Solution

Slot angle, $\alpha = \frac{360}{90} \times \frac{P}{2}$ degree electrical

$$= \frac{360}{90} \times \frac{10}{2} \text{ degree electrical}$$

$$= 20° \text{ electrical}$$

Number of slots per pole per phase, $m = \frac{90}{10 \times 3} = 3$

$$K_d = \frac{\sin\frac{m\alpha}{2}}{m\sin\frac{\alpha}{2}} = \frac{\sin\frac{3\times 20}{2}}{3\sin\frac{20}{2}}$$

$$= \frac{\sin 30}{3\sin 10} = 0.96$$

Line voltage, $E_L = \sqrt{3}\ E_{ph}$

or $11000 = \sqrt{3}\ \times 4.44 \times f \times \phi \times T \times K_p \times K_d$

or $11000 = 1.732 \times 4.44 \times 50 \times 0.16 \times \frac{Z}{2} \times 1 \times 0.96$

Thus, $Z = 372$ (as 1 turn has 2 conductors)

EXAMPLE 5.8 *A 6 pole, 3 phase, 50 Hz alternator has 12 slots per pole and 4 conductors per slot. The winding is five-sixth pitch and the flux per pole is 1.5 wb. The armature coils are all connected in series with star connection. Calculate the induced emf per phase.*

Solution

$$p = 6, f = 50$$

Total number of slots = 12 × 6 = 72

Total number of slots per phase $= \frac{72}{3} = 24$

Total number of conductors per phase = 24 × 4 = 96

Total number of turns per phase $= \frac{96}{2} = 48$

Number of slots per pole per phase $= \frac{72}{6\times 3} = 4$ m

Slot angle $= \frac{\text{Total mach. angle}}{\text{Total number of slots}} = \frac{360}{72} = 5°$ mechanical

Since 1° mechanical $= \frac{P°}{2}$ electrical

Slot angle, a = 5° mech. $= 5 \times \frac{P°}{2}$ electrical $= 5 \times \frac{6}{2} = 15°$ electrical.

Distribution factor, $K_d = \frac{\sin\frac{m\alpha}{2}}{m\sin\alpha/2}$

Substituting the values of m and α,

$$K_d = \frac{\sin\frac{4\times 15}{2}}{4\sin\frac{15}{2}} = \frac{\sin 30°}{4\sin 7.5°} = \frac{0.5}{4\times 0.13} = 0.96$$

Coil pitch is 5/6 of full-pitch.

A full-pitch coil has 180° electrical between the coil sides A 5/6 pitched coil will have an electrical angle of $\frac{5}{6}\times 180°$, i.e., 150° between the coil sides.

Thus, short pitch angle, $\beta = 180 - 150 = 30°$.

Pitch factor, $K_p = \cos\frac{\beta}{2}$

Substituting the value β, $K_p = \cos\frac{\beta}{2} = \cos 15° = 0.96$

Induced emf, $E = 4.44\,\phi f_T K_p K_d$ volts.

Substituting the values we get,

$$E = 4.44 \times 1.5 \times 50 \times 48 \times 0.96 \times 0.96$$

$$= 14730 \text{ volts} = 14.73 \text{ kV}$$

5.6 SYNCHRONOUS GENERATOR ON NO-LOAD

When a synchronous generator is driven at constant speed the terminal voltage on open circuit can be expressed as:

$$E = K\phi$$

where, $K = 4.44\, fT\, K_p K_d$

This means that the no-load terminal voltage depends upon flux per pole. Flux per pole is produced by field ampere-turns. Number of field turns being constant, flux per pole is proportional to field current, I_f.

Therefore, induced emf with the rotor driven at constant speed can be expressed as:

$$E = K_1\, I_f \qquad (5.5)$$

The relationship between the field current and the induced emf on no-load is referred to as open circuit characteristic (OCC). If the reluctance of the magnetic flux path through the iron is neglected and only the reluctance of air-gap is considered, the OCC will be a straight line.

However, if the reluctance of both air-gap and iron path are considered, the open circuit characteristic of a synchronous generator will be similar to the one shown in Fig. 5.20. The deviation of the OCC from a straight line relationship is due to the saturation effect of the iron, i.e., at higher values of excitation current, I_f, the rate of rise of induced emf gets reduced. For cylindrical type rotors, the saturation effect is identical along the whole of the air-gap. For salient pole type rotors, the saturation effect along the pole axis (also called direct axis) is different from that

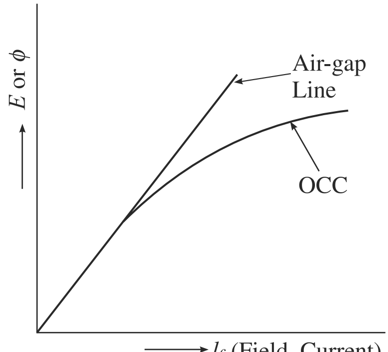


Fig. 5.20 *Open circuit characteristic of a synchronous generator*

along the interpolar axis (also called *quadrature axis*) due to the difference of iron along the two axis.

5.7 SYNCHRONOUS GENERATOR ON LOAD

When a synchronous generator is running on no-load, there will be no current flowing through the armature windings. The flux produced in the air-gap will be due to the field ampere-turns only. When load is connected across the armature terminals, current will flow through the armature windings. These three-phase currents will produce a rotating magnetic field in the air-gap. The effect of the armature flux on the flux produced by the field ampere-turns is called *armature reaction*. The armature flux will distort, oppose or help the field flux causing reduction or increase in the air-gap flux depending upon the power factor of the load. The armature reaction effect at various power-factor load is discussed as follows.

5.7.1 Synchronous Generator Loaded with Unity Power Factor Load

In Fig. 5.21(a) is shown a synchronous generator supplying power to a resistive load. The flux produced by the field ampere-turns, when the alternator is not connected across the load alone is shown in Fig. 5.21(b). The direction of emf induced in the armature conductors is shown by crosses and dots in Fig. 5.21(c). Since the generator is loaded with resistive load, the instantaneous direction of current in the armature conductors will be the same as the direction of emfs induced in them.

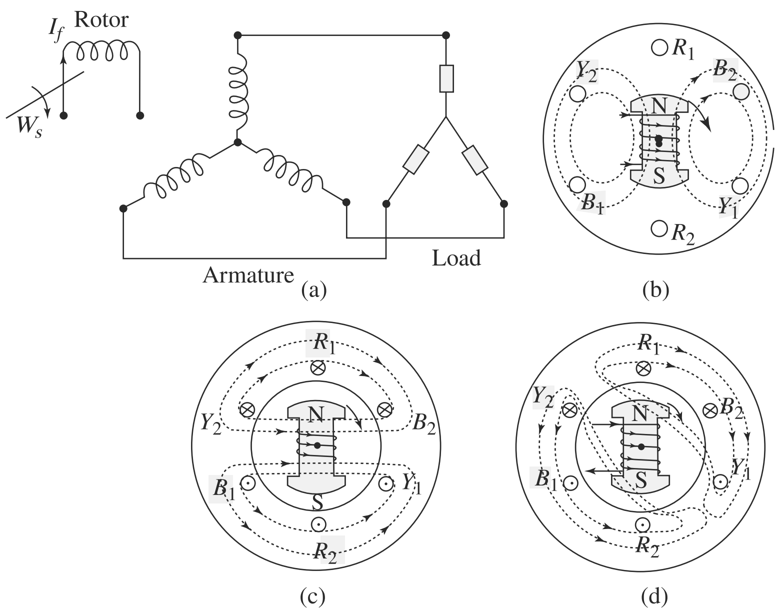


Fig. 5.21 *(a) Synchronous generator supplying power to a resistive load, (b) Flux distribution due to field ampere-turns alone, (c) Direction of armature induced emf and armature current on resistive load, and flux distribution due to armature ampere turns alone, (d) Armature reaction effect on flux distribution*

Flux produced by the armature ampere-turns alone have been shown in the figure. This armature flux will be rotating at synchronous speed. The flux produced by the rotor ampere-turns is also rotating at synchronous speed. Thus these two fluxes will be stationary with respect to each other. The two fluxes will give rise to a resultant air-gap flux distribution similar to the one shown in Fig. 5.21(d).

It can be observed that flux distribution is now distorted. The flux lines along the air-gap have been lengthened. The flux lines in trying to shorten their path through the air-gap will exert a backward pull on the rotor. The extent of flux distortion and hence the magnitude of backward pull will depend upon the magnitude of armature current, i.e., on the load. The primemover driving the generator should, therefore, develop more torque to enable to rotor the continue to rotate at synchronous speed.

5.7.2 Synchronous Generator with Lagging Power Factor load

With a purely inductive load connected across the armature terminals the armature current will lag the induced emf by 90°. Figure 5.22(a) shows the direction of induced emf and current in the armature conductors when a resistive load is connected across the armature terminals. Since the load is resistive, emf and current are in time phase. When induced emf in *R* phase is maximum, current is also maximum in the same phase.

For a purely inductive load, however, current in *R* phase will be maximum only after 90° elapse of time. By this time, the rotor would have moved forward by quarter of a revolution and the rotor would occupy the position shown in Fig. 5.22(b).

It is seen that the armature flux is in direct opposition to the rotor field flux. Thus the effect of armature reaction with a purely inductive load connected across the armature terminals is to reduce the air-gap flux created by the rotor field poles. The distribution of flux will remain symmetrical and hence no additional torque has to be developed by the primemover to enable the rotor rotate at synchronous speed. The reduction of air-gap flux will cause a reduction of induced emf and therefore a drop in terminal voltage. The amount of voltage drop will depend upon the magnitude of the load.

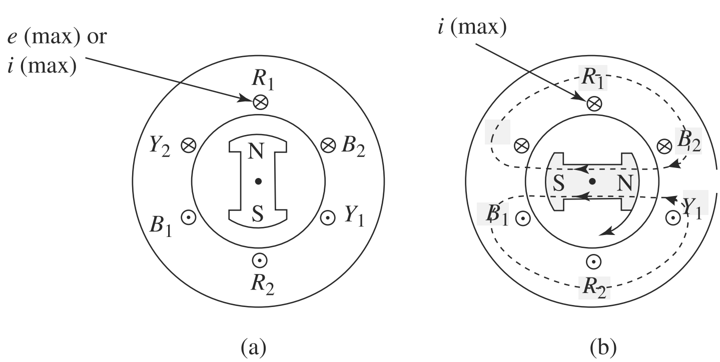


Fig. 5.22 *(a) Maximum value of induced emf and current in R phase of the armature on resistive load and the corresponding positions of the rotor poles, (b) Maximum current in R-phase on purely inductive load and the corresponding positions of the rotor poles*

5.7.3 Synchronous Generator with Leading Power Factor Load

When a purely capacitive load is connected across the armature terminals, the current flowing through the windings will lead the induced emf by 90 degrees. Maximum current in R phase, in this case, will occur 90° before the occurrence of maximum induced emf in that phase. Therefore, under purely capacitive load when maximum current in *R* phase occurs, the position of the rotor would remain 90° behind as compared to its position under resistive load. The direction of armature flux and the corresponding position of the rotor are shown in Fig. 5.23. It is seen from the figure that the armature flux is in the same direction as the rotor field flux. The two fluxes will help each other and thereby strengthen the air-gap flux.

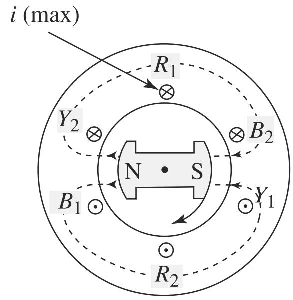


Fig. 5.23 *Maximum current in R-phase on purely capacitive load and the corresponding positions of the rotor poles*

The effect of armature reaction under capacitive load is, therefore, to help the main field flux and thereby increasing the emf induced in the armature. Therefore, when a synchronous generator is capacitively loaded, its terminal voltage will increase.

Effect of armature reaction on terminal voltage at various power-factor loads is shown in Fig. 5.24. At unity power-factor load, the change in terminal voltage with load is somewhat less as compared to inductive and capacitive load.

At zero lagging or leading power-factor load, the change in terminal voltage with load is large due to the demagnetising and magnetising effect of the armature reaction respectively. Normally the load on a synchronous generator will be of resistive-inductive type. Capacitive loading if occurs, due to say switching off of the load at the receiving end of a transmission line connected to the generator, may create a serious problem of rise in the terminal voltage unless preventive measures are incorporated.

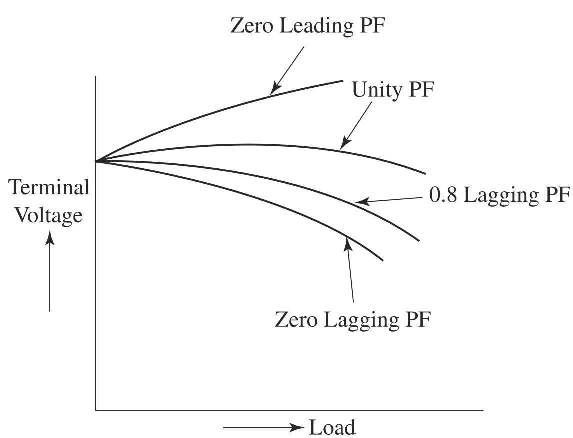


Fig. 5.24 *Effect of armature reaction on terminal voltage of a synchronous generator at various power factor loads*

In addition to armature reaction effect, terminal voltage will change with load due to voltage drop in the winding resistance and leakage reactance. Thus, when a synchronous generator is loaded, its terminal voltage changes due to a voltage drop in the winding resistance, a voltage drop in the winding leakage reactance and armature reaction effect. The load characteristics of an alternator at various power-factor loads will be similar to the characteristics shown in Fig. 5.24.

5.8 SYNCHRONOUS IMPEDANCE AND PHASOR DIAGRAM OF A SYNCHRONOUS GENERATOR

The resistance of each phase winding of a synchronous generator is designated as R_a. Some of the flux lines, produced by the armature ampere-turns, which do not cross the air-gap are called leakage flux. The reactance due to these leakage fluxes is called leakage reactance, X_L. When a synchronous generator is loaded, there will be a change in the terminal voltage due to a voltage drop in armature resistance and armature leakage reactance.

The change in terminal voltage due to armature reaction effect can also be viewed as a reactance voltage drop. This can be understood from the following explanation:

The rotor field flux, ϕ_f produces induced emf, E in the armature winding. When loaded, this emf causes an armature current, I_a to flow through the winding and the load. The armature ampere-turns produces a flux, ϕ_a in the air gap. This flux, ϕ_a produces another emf E_a in the armature windings.

The phase relationship between the field flux, ϕ_f; armature induced emf due to field flux E; the armature current, I_a; the flux produced by armature current, ϕ_a, and the emf induced E_a; in the armature due to armature flux at different power-factor loads, are shown in Fig. 5.25. Induced emf, E will lag the field flux, ϕ_f as shown in the figure.

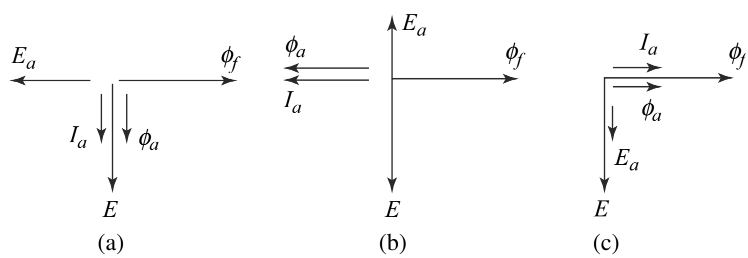


Fig. 5.25 *Phase relationship between the various quantities on (a) Resistive load (unity power factor), (b) Inductive load (zero power factor lagging) (c) Capacitive load (zero power factor leading)*

The phase relationship between the induced emf, E and the current flowing through the armature winding, I_a will depend upon the power factor of the load. At unity power-factor load, I_a will be in phase with E. At zero lagging power-factor load, I_a will lag E by 90° whereas at zero leading power-factor load, I_a will lead E by 90°. Flux, ϕ_a produced by armature current I_a will be in time-phase. Emf induced E_a in the armature windings due to ϕ_a will lag ϕ_a by 90°. A component of the generated voltage that would be necessary to overcome this armature reaction voltage must act in the opposite direction.

Since the armature reaction induced voltage always lags the armature current and the flux producing it by 90°, the component of the voltage drop necessary to overcome this generated voltage will always lead the armature current by 90°. This voltage drop is similar to the component of applied voltage needed to overcome leakage reactance drop due to emf of self-induction. Thus the voltage induced due to armature reaction effect can be considered as a reactance drop in the armature winding of the synchronous generator. This fictitious reactance due to armature flux, ϕ_a is called X_a. Reactance due to armature leakage flux, as mentioned earlier, is called leakage reactance, X_l. The sum of X_a and X_l is called synchronous reactance X_s.

When an alternator is loaded, there will be voltage drop due to $I_a R_a$ which is in phase with I_a and due to $I_a X_s$, which is leading I_a by 90°. The difference between the terminal voltage V and induced emf E is due to voltage drops in the resistance and reactance, $I_a R_a$ and $I_a X_s$. The relationship between induced emf E and the terminal voltage V can be represented as

$$E = V + I_a R_a + jI_a (X_l + X_a)$$

or $$E = V + I_a R_a + jI_a X_s$$

or $$E = V + I_a(R_a + jX_s)$$

Therefore $$E = V + I_a Z_s$$

The vector sum of R_a and X_s is called synchronous impedance, Z.

Phasor diagram representing the various quantities of a synchronous generator at different power-factor loads are shown in Fig. 5.26.

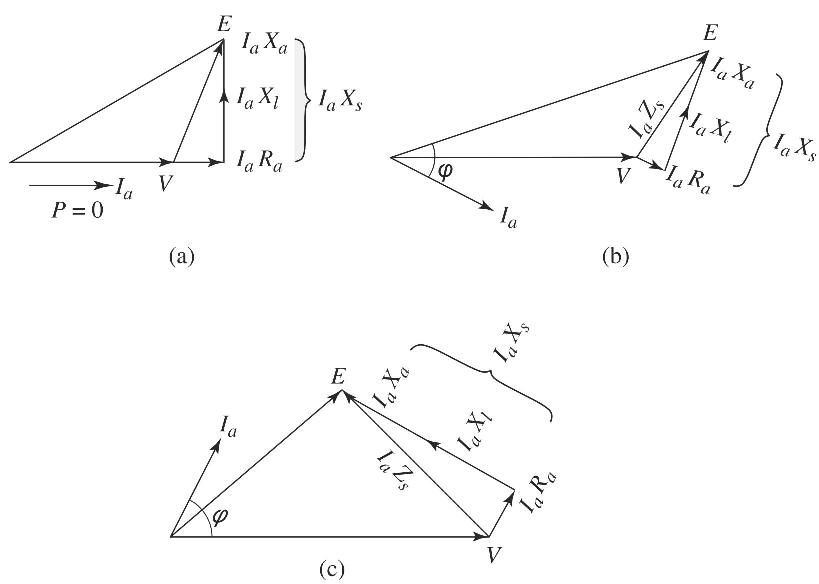


Fig. 5.26 *Phasor diagrams at (a) Unity pf load, (b) Lagging pf load, (c) Leading pf load*

5.9 POWER RELATIONSHIPS

The output power, P_o of a three-phase synchronous generator is expressed as

$$P_o = 3\,VI_a \cos\phi$$

where V is the terminal voltage, I_a is the armature current and $\cos\phi$ is the power factor. We know that in a synchronous generator, mechanical power is the input. This input power P_i is expresses as

$$P_i = T_s\,\omega_s$$

where T_s is the torque exerted by the primemover to rotate the rotor poles and ω_s is the angular velocity of the rotor. In addition an electrical power input of VI_f is provided to the field winding for excitation.

The total input, P_{in} is given as

$$P_{\text{in}} = T_s\,\omega_s + VI_f$$

The following are the losses in the synchronous machine

Losses = Copper loss in the three-phase armature windings $(3I_a^2R_a)$
+ core loss + Field winding loss (VI_f) + Rotational loss (P_r)

We know,

Input – Losses = Output

or P_{in} – Losses = P_o

Again losses = Constant losses + Variable losses

$= P_c + 3I_a^2R_a$

P_C = constant losses = $VI_f + P_r$

Efficiency of the generator, $\eta = \dfrac{\text{Output}}{\text{Output + Losses}} = \dfrac{3\,VI_a\cos\phi}{3VI_a\cos\phi + P_c + 3\,I_a^2R_a}$

Condition for maximum efficiency is determined by taking derivative of η with respect to I_a and equating to zero.

$$\frac{d\eta}{dI_a} = \frac{d}{dI_a}\,\frac{3VI_a\cos\phi}{3VI_a\cos\phi + P_C + 3\,I_a^2R_a} = 0$$

from which we get,

$$P_C = 3I_a^2R_a$$

That is, constant losses = Variable loss (copper loss)

5.10 POWER ANGLE CHARACTERISTICS OF CYLINDRICAL ROTOR SYNCHRONOUS GENERATOR

We have seen earlier that two types of rotor construction are made for synchronous machines. In non-salient, i.e., cylindrical rotor type, the air-gap between the stator and the rotor is uniform. The synchronous reactance, x_s which is the sum of leakage reactance, x_l and a fictitious reactance which replaces the effect of armature reaction, x_a is the same throughout the entire air-gap between the stator and the rotor. The armature winding resistance, R_a is very small as compared to synchronous reactance, x_s. The equivalent circuit of the non-salient pole synchronous generator and its phasor diagram are shown in Fig. 5.27(a). Figure 5.27(b) shows the approximate equivalent circuit and the corresponding phasor diagram where the armature resistance has been neglected.

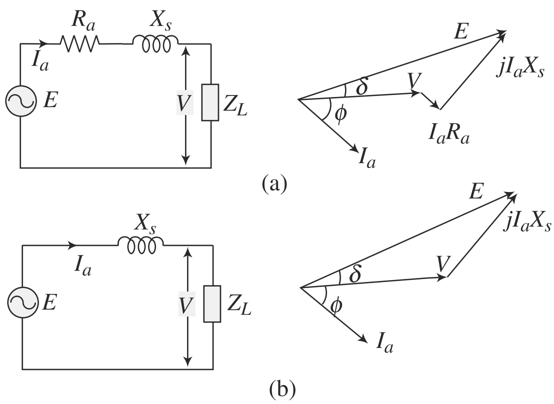


Fig. 5.27 *(a) Equivalent circuit and phasor diagram; (b) Equivalent circuit and phasor diagram of a synchronous generator neglecting R_a*

The terminal voltage across the load is V. The induced emf is E. Current, I_a flows from the generator to the load. The equation relating the induced emf and the terminal voltage is represented as

$$E = V + I_a R_a + jI_a X_s$$

If we neglect R_a,

$$E = V + j I_a X_s$$

or,

$$I_a = \frac{E - V}{j X_s} \tag{5.6a}$$

$$\bar{I}_a = \frac{\bar{E} - \bar{V}}{j X_s} \tag{5.6b}$$

From the phasor diagram,

$$\bar{E} = E \cos \delta + j E \sin \delta$$

$$\bar{I}_a = I_a \cos \phi - j I_a \sin \phi$$

Substituting the values of $\bar{E}$ and $\bar{I}_a$ in Eq. (5.5),

$$\bar{I}_a = \frac{E \cos \delta + j E \sin \delta - V}{j X_s}$$

$$= \frac{E \sin \delta}{X_s} + \frac{E \cos \delta - V}{j X_s}$$

$$= \frac{E \sin \delta}{X_s} - j^2 \frac{(E \cos \delta - V)}{j X_s}$$

or,

$$\bar{I}_a = \frac{E \sin \delta}{X_s} - j \frac{(E \cos \delta - V)}{X_s} \tag{5.7}$$

Again,

$$\bar{I}_a = I_a \cos \phi - j I_a \sin \phi \tag{5.8}$$

From (5.7) and (5.8)

$$I_a \cos \phi = \frac{E \sin \delta}{X_s}$$

Output active or real power, $P_o = 3VI_a \cos \phi = \dfrac{3V E \sin \delta}{X_s}$

or, $$P_o = P_{\max} \sin \delta \qquad (5.9)$$

where V, E, and X_s are constants, the output power varies as sin δ where δ is the angle between E and V. The angle δ is called the power angle.

From the expression, $P_o = \left(\dfrac{3\,VE}{X_S}\right) \sin \delta$

The power and torque-angle characteristics is drawn as shown in Fig. 5.28.

The maximum value of output power is at δ = 90°, so that

$$P_{o(\max)} = \frac{3\,VE}{X_s}$$

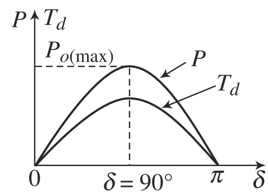


Fig. 5.28 *Power/torque-angle characteristic of a cylindrical rotor type synchronous generator*

The torque developed is calculated as

$$T_d = \frac{P_o}{\omega_s} = \frac{3\,VE \sin \delta}{X_s\,\omega_s}$$

where $\omega_s = \dfrac{2\pi\,N_s}{60}$, N_s is the synchronous speed is rpm.

The reactive power, is calculated using Eqs. (5.7) and (5.8) waveform we get,

$$I_a \sin \phi = \frac{E \cos \delta - V}{x_s}$$

So, reactive power, $Q_0 = 3\,VI_a \sin \phi$

or, $$Q_0 = \left[\frac{3VE}{X_s} \cos \delta - \frac{3V^2}{X_s}\right] \text{VAR} \qquad (5.10)$$

5.11 VOLTAGE REGULATION OF A SYNCHRONOUS GENERATOR

It has been explained that the terminal voltage of a synchronous generator changes on application of load across its output terminals. The change is due to voltage drops in the windings and armature reaction effect. The change in terminal voltage due to armature reaction effect depends upon the magnitude and power factor of the load. At lagging power-factor load the armature reaction effect is just opposite to that of leading power-factor load. Figure 5.29 shows the relationship between terminal voltage and load current of a synchronous generator at different power-factor loads.

Let *OL* be the rated load on the generator. At this load, *OP* is the terminal voltage. If this load of unity power factor is removed, keeping speed and excitation of the alternator constant, the terminal voltage will rise to *OB*, whereas if the load is of lagging power-factor, the terminal voltage will rise to *OC*. For leading power-factor load terminal voltage, however, will fall to *OA*. It can be noticed that the change of terminal voltage from full-load to no-load is more in case of lagging or leading power-factor load as compared to unity power-factor load. This is because of the demagnetising or magnetising effect of armature reaction on the main field flux.

The variation of terminal voltage from no-load to full-load expressed per unit or percentage of full-load voltage is called *regulation* of a synchronous generator. The per unit regulation of the generator having load characteristics as shown in Fig. 5.29 can be expressed as:

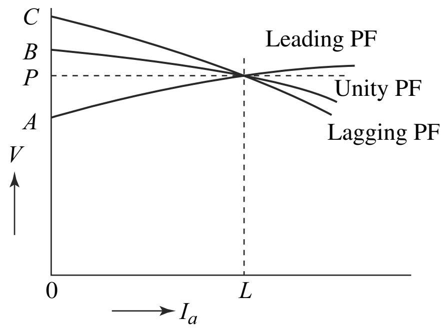


Fig. 5.29 *Variation of terminal voltage of a synchronous generator at different power factor loads*

Per unit regulation

$$= \frac{\text{Change of terminal voltage from no-load to full-load}}{\text{Full-load terminal voltage}}$$

$$= \frac{OB - OP}{OP} = \frac{BP}{OP} \text{ at unity pf load} = \frac{OC - OP}{OP} = \frac{CP}{OP} \text{ at unity pf load}$$

$$= \frac{OA - OP}{OP} = \frac{AP}{OP} \text{ at unity pf load}$$

It is noticed that at leading power-factor load, the regulation is negative. Since regulation of an alternator depends on the load and the load power-factor, it is, therefore, necessary to mention power factor also while expressing regulation at a particular load. Synchronous generators are designed to perform at a desired regulation limit while supplying full-load, usually at 0.8 lagging power-factor. This is because most of the loads connected across the supply generally have power factor near 0.8 lagging.

Since a generator does not always run on full-load and also the voltage drops due to armature reaction, winding resistance and leakage reactance vary with the load, it may become difficult to maintain a constant terminal voltage across the load.

It may be remembered that in a dc generator the armature reaction effect was neutralised by using compensating winding. In alternator, also, efforts are made to produce compounding action using feedback circuits to neutralise the armature reaction effect at different power-factor loads.

In alternators, a constant output voltage at different power-factor loads is maintained by using an automatic voltage regulator which automatically increases or decreases the field excitation, depending upon the magnitude of load power-factor. To determine an expression for voltage regulation, the phasor diagram of a loaded synchronous generator at a lagging power-factor load is given in Fig. 5.30.

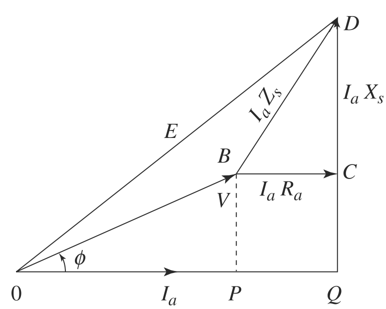


Fig. 5.30 *Phasor diagram of a synchronous generator at lagging power factor load*

From the triangle *OQD* of Fig. 5.30,

$$OD^2 = (OQ)^2 + (QD)^2$$
$$= (OP + PQ)^2 + (QC + CD)^2 = (OP + PQ)^2 + (BP + CD)^2$$

or $$E^2 = (V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2$$

or $$E = \sqrt{(V \cos\phi + I_a R_a)^2 + (V \sin\phi + I_a X_s)^2} \quad (5.11)$$

No-load voltage, E corresponding to a particular load, I_a can be calculated if the values of terminal voltage on load, load power-factor angle, armature resistance and synchronous reactance are known.

For a leading power-factor load, the expression for *E* can be similarly derived and can be expressed as

$$E = \sqrt{(V \cos\phi + I_a R_a)^2 + (V \sin\phi - I_a X_s)^2} \quad (5.12)$$

Thus, in general, the expression of no-load voltage can be written as,

$$E = \sqrt{(V \cos\phi + I_a R_a)^2 + (V \sin\phi \pm I_a X_s)^2} \quad (5.13)$$

where, + sign is for lagging power-factor load and, – sign is for leading power-factor load. To determine the regulation of an alternator, therefore in addition to other machine parameters, value of synchronous reactance should be known.

5.12 DETERMINATION OF REGULATION OF A SYNCHRONOUS GENERATOR

Commercial generators are manufactured in ratings as high as 500 MVA. To determine voltage regulation directly, such high capacity generators are to be loaded. Loading of such generators to determine their regulation or efficiency in the test laboratory will be a difficult task. Moreover, the primemover required for driving such a generator may not be available in the test laboratory. It is, therefore, a common practice to test such large machines indirectly by simulating the load conditions. Such indirect methods will consume only a small amount of power as compared to the power consumed in direct loading method.

There are several methods of determining regulation of an alternator. The simplest of the methods is the synchronous impedance method.

5.12.1 Determination of Voltage Regulation by Synchronous Impedance Method

In this method of determination of regulation, two tests are required to be performed on the machine, namely the open-circuit test and the short-circuit test. Open-circuit test is performed by running the alternator on no-load and at rated speed. The terminal voltage on no-load is measured at different values of excitation current. The relationship between no-load voltage and excitation current gives the open-circuit characteristics (OCC). For circuit diagram and procedure for determining open-circuit characteristic of an alternator the reader may refer to experiment 5.1.

Short-circuit test is performed by running the alternator at rated speed. Keeping the output terminals short-circuited through an ammeter, reduced excitation current is allowed to flow through the field winding. The relationship between armature current, I_a and the field current I_f gives the short-circuit characteristics (SCC). The OCC and SCC of an alternator are shown in Fig. 5.31.

At any particular value of I_f the ratio of open-circuit voltage and short-circuit armature current gives the synchronous impedance. Referring to Fig. 5.31 at a field current of say OA, the induced emf is is AB. With this excitation, if the armature terminals are short-circuited, a current AC will flow through the armature windings. The emf induced, AB on open circuit is regarded as being responsible for circulating a short-circuit current through the synchronous impedance of the winding. Thus the value of synchronous impedance, Z_s, at this excitation is given by

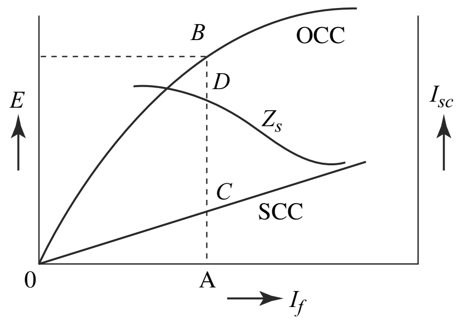


Fig. 5.31 *Open circuit and short-circuit characteristics of a synchronous generator*

$$Z_s = \frac{\text{OC Voltage}}{\text{SC Current}} \text{ at the same excitation}$$

$$= \frac{AB(V)}{AC(A)}\ \Omega$$

To calculate per-phase value of Z_s, the values of emf and current should be taken as their per-phase values. Because of the non-linear nature of the OCC, the ratio of open-circuit voltage and short-circuit current at various values of excitation currents are different. If the values of Z_s at different excitation are calculated and plotted, we shall get a curve for Z_s, as shown in Fig. 5.31. It is seen that at lower values of excitation current, the value of Z_s, is more than its value at higher excitations.

Under short-circuit test, small amount of field current is necessary to circulate full-load current through the winding. The induced emf corresponding to this excitation is small. The value of synchronous impedance calculated from open-circuit and short-circuit test data is, therefore, more than its value under actual loading condition. The regulation calculated using this value of synchronous impedance will, therefore, be more than the actual value of regulation. The method of calculation of voltage regulation by synchronous impedance method is illustrated through the following examples.

EXAMPLE 5.9 *A 500 kVA, three-phase, star-connected alternator has a rated line-to-line terminal voltage of 3300 V. The resistance and synchronous reactance per phase are 0.3 and 4.0 Ω respectively. Calculate the voltage regulation at full-load, 0.8 power-factor lagging.*

Solution

Output power in $\text{VA} = \sqrt{3}\ V_L I_L = 500 \times 1000$

or $$I_L = \frac{500 \times 1000}{1.732 \times 3300} = 87.5 \text{ A}$$

For a star-connected alternator, line current is equal to phase current.

Therefore, $I_a = 87.5$ A

$$\cos\phi = 0.8;\ \sin\phi = 0.6$$

$$R_a = 0.3,\ X_s = 4.0\ \Omega$$

$$\text{V/phase} = \frac{3300}{\sqrt{3}} = 1905 \text{ V}$$

Induced emf, $$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 87.5 \times 0.3)^2 + (1905 \times 0.6 + 87.5 \times 4)^2}$$

$$= 2152 \text{ V/phase}$$

$$\text{Percentage regulation} = \frac{E - V}{V} \times 100 = \frac{2152 - 1905}{1905} \times 100 = 12.96 \text{ percent}$$

EXAMPLE 5.10 *In a 2000 V. single-phase synchronous generator, a full-load current of 100 A is produced on short-circuit by a field excitation of 2.5 A; an emf of 500 V is produced on open-circuit by the same excitation. The armature resistance is 0.8 Ω. Determine the voltage regulation when the generator is delivering a current of 100 A at (a) unity power factor, (b) 0.71 power factor lagging; and (c) 0.8 power factor leading.*

Solution

Synchronous impedance, $$Z_s = \frac{\text{OC Voltage}}{\text{SC Current}} = \frac{500}{100} = 5\ \Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{5^2 - 0.8^2} = 4.935\ \Omega$$

Induced emf $$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

(a) At unity pf $$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

$$= \sqrt{(2000 \times 1 + 100 \times 0.8)^2 + (2000 \times 0 + 100 \times 4.935)^2}$$

$$= \sqrt{(2080)^2 + (4.935)^2} = 2138 \text{ V}$$

$$\text{Regulation} = \frac{E - V}{V} \times 100 = \frac{(2138 - 2000)}{2000} \times 100 = 6.9\%$$

(b) At 0.71 pf lagging, ($\cos\phi = 0.71$, $\sin\phi = 0.704$)

$$E = \sqrt{(2000 \times 0.71 + 100 \times 0.8)^2 + (2000 \times 0.704 + 100 \times 4.935)^2}$$

$$= \sqrt{(1420 + 80)^2 + (1408 + 493.5)^2} = 2422\text{ V}$$

$$\text{Regulation} = \frac{E - V}{V} \times 100$$

$$= \frac{2422 - 2000}{2000} \times 100 = 21.1\%$$

(c) At 0.8 pf leading, ($\cos\phi = 0.8$, $\sin\phi = 0.6$)

$$E = \sqrt{(2000 \times 0.8 + 100 \times 0.8)^2 + (2000 \times 0.6 - 100 \times 4.935)^2}$$

$$= \sqrt{(1680)^2 + (706.5)^2} = 1822.5\text{ V}$$

$$\text{Regulation} = \frac{E - V}{V} \times 100$$

$$= \frac{(1822.5 - 2000)}{2000} \times 100 = -8.87\%$$

Note that the regulation is negative at this leading power factor load.

EXAMPLE 5.11 *If a field excitation of 10 A in a synchronous generator gives a current of 150 A on short-circuit and a terminal voltage of 900 V on open-circuit, what will be the internal voltage drop with a load current of 60 A?*

Solution Induced emf of 900 V on open-circuit can be regarded as being responsible for circulating short-circuit current of 150 A, through the synchronous impedance of the winding when the excitation current is 10 A.

The value of synchronous impedance at this excitation,

$$Z_s = \frac{\text{OC Voltage}}{\text{SC Current}} = \frac{900}{150} = 6\ \Omega$$

Internal voltage drop when the load current is 60 $A = I_a X_s = 60 \times 6 = 360$ V.

EXAMPLE 5.12 *A 2000-KVA, 6600 V, three-phase, star-connected synchronous generator has a resistance of 0.4 Ω per phase and a synchronous reactance of 4.5 Ω. per phase. Calculate the percentage change in terminal voltage when the rated output of 2000 KVA at a power factor of 0.8 lagging is switched off. The speed and exciting current remain unchanged.*

Solution

$$\text{V/phase} = \frac{6600}{\sqrt{3}} = 3810\text{ V}$$

$$I_L = I_a = \frac{200 \times 1000}{\sqrt{3} \times 6600} = 175\text{ A}$$

$$R_a = 0.4\ \Omega;\ X_s = 4.5\ \Omega$$

$$\cos\phi = 0.8,\ \sin\phi = 0.6$$

$$E = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi + I_aX_s)^2}$$

$$= \sqrt{(3810 \times 0.8 + 175 \times 0.4)^2 + (3810 \times 0.6 + 175 \times 4.5)^2}$$

= 4378 V per phase

Percentage change in terminal voltage

$$= \frac{(4378 - 3810)}{3810} \times 100 = 14.9 \text{ percent}$$

EXAMPLE 5.13 *A 1200 KVA,0 V, 50 Hz, three-phase, star-connected alternator has armature resistance of 0.25Ω per phase. A field current of 40 A produces a short-circuit current of 200 A and an open-circuit emf of 1100 V line-to-line. Calculate the regulation on (a) full-load 0.8 power factor lagging, and (b) full-load 0.8 leading power factor.*

Solution

$$\text{Output power} = \sqrt{3}\ V_L I_L = 1200 \times 1000$$

$$I_L = \frac{1200 \times 1000}{\sqrt{3} \times 3300} = 210 \text{ A}$$

For star-connection, per phase,

$$I_a = I_L = 210 \text{ A}$$

$$\text{V per phase} = \frac{3300}{\sqrt{3}} = 1905 \text{ V}$$

Synchronous impedance,

$$Z_s = \frac{1100}{\sqrt{3} \times 200} = 3.175\ \Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2}$$

$$= \sqrt{(3.175)^2 - (0.25)^2} = 3.165\ \Omega$$

(a) For lagging pf load

$$E = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi + I_aX_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 210 \times 0.25)^2 + (1905 \times 0.6 + 210 \times 3.165)^2}$$

= 2398 V

$$\text{Regulation} = \frac{E - V}{V} \times 100$$

$$= \frac{2398 - 1905}{1905} \times 100 = 25.9\%$$

(b) For leading power factor load,

$$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi - I_a X_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 210 \times 0.25)^2 + (1905 \times 0.6 - 210 \times 3.165)^2}$$

or $$E = 1647 \text{ V}$$

$$\text{Regulation} = \frac{E - V}{V} \times 100$$

$$= \frac{1647 - 1905}{1905} \times 100 = -13.54 \text{ per cent}$$

It can be noticed that for leading power factor load, the regulation is negative.

EXAMPLE 5.14 *A 1500 KVA, 6600 V, 3-phase, star connected alternator with a resistance of 0.4 Ω and reactance of 6 Ω per phase, delivers full-load current at 0.8 power factor lagging, and at normal rated voltage. Calculate the terminal voltage for the same excitation and load current at 0.8 power factor leading.*

Solution

Full-load current, $$I_a = \frac{1500 \times 100}{\sqrt{3} \times 6600} = 131 \text{ A}$$

Voltage per phase, $$V = \frac{6600}{\sqrt{3}} = 3810 \text{ V}$$

Induced emf, $$E = \sqrt{(3810 \times 0.8 + 131 \times 0.4)^2 + (3810 \times 0.6 + 131 \times 6)^2}$$

$$= \sqrt{(3100)^2 + (3072)^2} = 4364$$

As excitation remains constant, E at 4364 V remains constant.

Let the terminal voltage for the same excitation and load current at 0.8 power factor leading be V'.

Then at 0.8 leading power factor,

$$4364 = \sqrt{(V'\cos\phi + I_a R_a)^2 + (V'\sin\phi - I_a X_s)^2}$$

or, $$4364 = \sqrt{(V' \times 0.8 + 131 \times 0.4)^2 + (V' \times 0.6 - 131 \times 6)^2}$$

or, $$V' = 4743 \text{ V}$$

Terminal Voltage, line-to-line = $\sqrt{3} \times 4743 = 8215$ V

5.12.2 Short Circuit Ratio (SCR) and its Significance

It is the ratio of field currents required to generate rated voltage on no-load to the field current required to circulate rated armature current on short circuit. As shown in Fig. 5.32, I_{f_1} is the field current required to generate rated voltage V on no-load

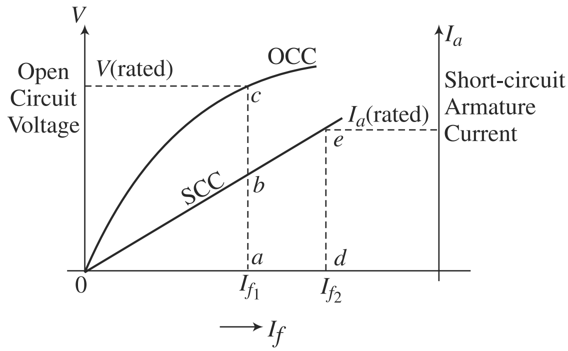


Fig. 5.32 *Determination of short-circuit ratio from OCC and SCC*

as can be found from *OCC*, and I_{f_2} is the field current necessary to circulate rated armature current on short circuit. Thus, SCR is given as

$$\text{SCR} = \frac{I_{f_1}\text{, for rated voltage on no-load}}{I_{f_2}\text{, for rated current on short circuit}}$$

$$= \frac{oa}{od} \text{ (See Fig. 5.32)}$$

We can establish a relationship between synchronous impedance, Z_s and SCR.

Synchronous impedance is defined as the ratio of open-circuit voltage to the short-circuit current for the same field current.

From OCC and SCC, synchronous impedance, Z_s is calculated as

$$Z_s = \frac{ac}{ab};\ Z_s\text{(per unit)} = \frac{Z_s}{\text{base impedance}} = \frac{Z_s}{\dfrac{V\text{(rated)}}{I\text{ (rated)}}} = \frac{Z_S}{ac/de}$$

and $$SCR = \frac{oa}{od}$$

From triangle *ode*, $$\frac{oa}{od} = \frac{ab}{de}$$

$$SCR = \frac{oa}{od} = \frac{ab}{de} = \frac{1}{de/ab}$$

$$Z_S\text{ (pu)} = \frac{Z_S}{ac/dc} = \frac{ac}{ab}\frac{de}{ac} = \frac{de}{ab}$$

Therefore, $$\text{SCR} = \frac{1}{de/ab} = \frac{1}{Z_s\text{(pu)}}$$

If R_a is neglected, $$\text{SCR} = \frac{1}{X_s\text{ (pu)}}$$

Thus, short circuit ratio is the reciprocal of per unit value of synchronous impedance.

Significance of SCR We have seen that SCR is inversely proportional to X_s (pu). If *SCR* is low, synchronous impedance is high. High value of synchronous impedance will cause poor voltage regulation of the synchronous generator. This means that there will be large variation of terminal voltage with changes in load. To maintain a reasonably constant terminal voltage, there will be a requirement of wide range of variation of the field current.

Again, synchronizing power is inversely proportional to X_s. So, if SCR is small, X_s is more and synchronizing power is small. Synchronizing power is responsible for keeping stable, the synchronous generators running in parallel. Low synchronous power will reduce the stability limit of synchronous generators running in parallel.

High value of SCR will lead to better stability of operation and also better voltage regulation. Since high value of SCR leads to low value of X_s, the short circuit fault current will be high. Typical value of SCR for a cylindrical rotor machine varies from 0.5 to 0.9 and for a salient pole rotor varies from 1.0 to 1.5.

We will now describe the two other methods of determining voltage regulation.

5.12.3 Voltage Regulation by MMF Method

Two other methods namely the magneto motive force (mmf) method and zero power-factor (potier triangle) method may be used to determine the voltage regulation of an alternator.

In mmf method, two tests, i.e., OCC test and SC test similar to synchronous impedance methods are to be performed on the alternator. $I_a R_a$ drop is added vectorially with the terminal voltage. An mmf in terms of field current corresponding to this voltage is found out from the open circuit characteristic. From the short-circuit characteristic, the field current necessary to send rated armature current is determined. The mmf representing this field current is assumed to be necessary to send rated current through the armature leakage reactance and at the same time overcome armature reaction. The phasor sum of these two mmfs are found out. The value of the emf on the open-circuit characteristic corresponding to this resultant field current is assumed to be the no-load emf of the alternator and accordingly the value of regulation calculated. The value of regulation found out by mmf method is lower than the actual regulation of the alternator.

Details of mmf method are given below.

In this method of finding voltage regulation, the following three tests are required to be conducted.

(i) Open-circuit test (OC test)
(ii) Short-circuit test (SC test)
(iii) Test for measurement of armature resistance.

In mmf (magneto motive force) method, the mmf required to produce an emf of, $E' = V + I_aR_a$ and an mmf required to circulate rated current on short-circuit are determined. These mmfs or ampere-turns are produced by field currents say I_{f_1} and I_{f_2} respectively. I_{f_1} is found out from OCC and I_{f_2} is found out from SCC as has been shown in Fig. 5.33(a).

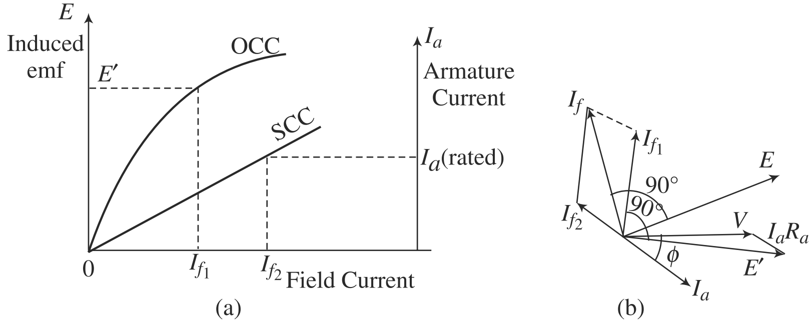


Fig. 5.33 *(a) OCC and SCC of a synchronous generator (b) Phasor diagram for determining resultant field current*

I_{f_1} produces E' and leading E' by 90°. $E' = V + I_a R_a$. I_{f_2} produces an emf which will be able to circulate the rated current through the armature on short circuit. This emf is equal to the voltage drop in the armature due to synchronous impedance.

The phasor sum of I_{f_1} and I_{f_2} gives the total field current required to induce an emf E. The value of E corresponding to I_f is found from the OCC and voltage regulation is calculated as

$$\text{Voltage regulation in percentage} = \frac{(E - V)}{V} \times 100$$

The step by step procedure for calculation of voltage regulation by mmf method is presented below.

1. Perform open-circuit test and draw OCC; perform short-circuit test and draw SCC; calculate armature resistance by ammetere-voltmeter method by applying a low voltage dc. Consider ac resistance as equal to 1.5 times the dc resistance.
2. Draw to the scale the armature rated voltage V as reference phasor. Calculate armature current, I_a and power factor angle ϕ. Draw phasor I_a making an angle of lag of ϕ with the V-axis. Add $I_a R_a$ with V to get E'. The $I_a R_a$ drop is in phase with I_a. For a voltage, E' find the corresponding field current I_{f_1} from the OCC. Draw to the scale I_{f_1} such that the induced emf E' lags I_{f_1} by 90° (see figure 5.33 b).
3. From the SCC determine the field current, I_{f_2} required to circulate the rated current through the armature on short-circuit. This field current is required to induce an emf which will balance the synchronous reactance voltage drop, $I_a X_s$. Draw to scale I_{f_2} in phase opposition to I_a. Draw the resultant of I_{f_1} and I_{f_2} to give I_f.
4. Convert I_f to the scale to get I_f in amperes; from the OCC determine the value of induced emf E corresponding to a field current of I_f and then calculate the voltage regulation.

MMF Method Neglecting Armature Resistance A more simplified method to find regulation is to neglect the armature resistance. The phasor diagram and the method of calculation of total field current are described below.

Draw the phasor diagram with the rated voltage, V as the reference axis. I_{f_1} is the field current required to induce an emf V on no-load as determined from the OCC. Draw I_{f_1} leading V by 90° as shown in Fig. 5.34.

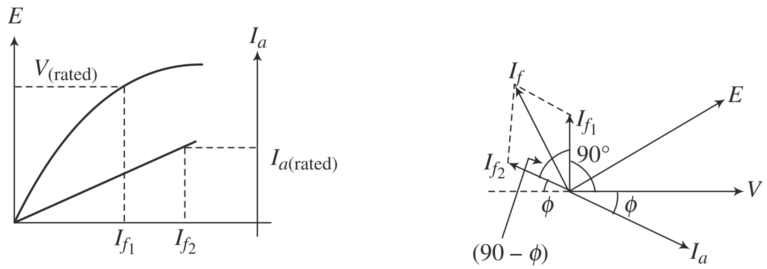


Fig. 5.34 *Simplified mmf method*

I_{f_2} is the field current required to calculate full-load current at short-circuit, determined from the SCC, and is drawn in phase opposition to I_a. The angle between I_{f_1} and I_{f_2} is $(90 - \phi)$ degrees. The resultant of I_{f_1} and I_{f_2} which is I_f is calculate as

$$I_f = \sqrt{I_{f_1}^2 + I_{f_2}^2 + 2I_{f_1} I_{f_2} \cos(90 - \phi)}$$

After calculating I_f, from OCC determine E corresponding to I_f and then calculate voltage regulation.

5.12.4 Voltage Regulation Using Zero Power Factor (ZPF) Characteristic and Potier Triangle Method

In Potier triangle method of determining voltage regulation of a synchronous generator, two characteristics are to be drawn.

One is the open-circuit or no-load characteristic discussed earlier. The no-load characteristic gives the relationship between induced emf, E and the field current, I_f on no-load at rated speed.

The other, the zero power factor (ZPF) characteristic is obtained by loading the synchronous generator with a balanced and variable three-phase purely inductive load. The generator is run at synchronous speed. The load on the generator is purely inductive. The load is adjusted such that rated current flows through the armature while the field current is adjusted and values of terminal voltage against each value of field current is recorded.

Figure 5.35 shows the OCC and ZPF characteristics. The short-circuit characteristic has also been drawn. Distance *OA* represents a field current required to cause flow of rated armature current when the terminal voltage is zero with an inductive load. Point *b* on ZPFC corresponds to the terminal voltage at a field current, I_f when the alternator is supplying rated current at zero power factor inductive load.

The distance *OA* is equal to distance *ab*. Line *ac* is drawn parallel to the air-gap line to touch the OCC at point *c*. Join point *c* and *b*. Triangle *abc* is called the Potier triangle. The vertical distance *cd* of the Potier triangle represents the leakage reactance drop, $I_a X_l$. Distance *db* represents the field current due to armature reaction mmf. Distance a*d* represents the field current required to overcome leakage reactance voltage drop.

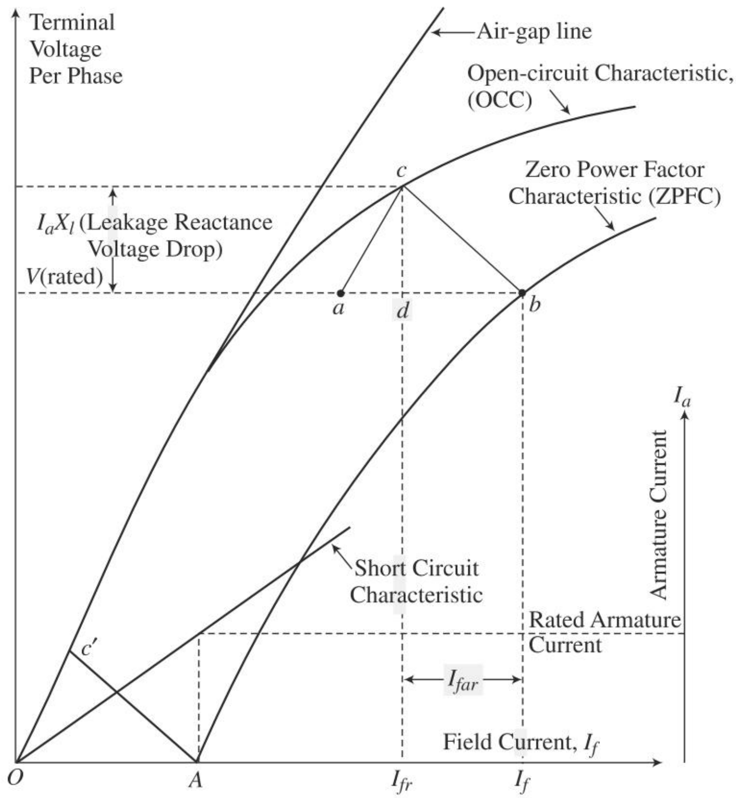


Fig. 5.35 *Open circuit and zero power factor load characteristics*

From the Potier triangle, therefore leakage reactance of the armature is calculated as

$$X_l = \frac{\text{Voltage drop per phase (distance } cd\text{)}}{\text{Rated armature current per phase}}$$

It is not necessary to draw the complete ZPF characteristic by taking readings. If we slide the Potier triangle downwards such that the point c of the Potier triangle always rests on the OCC, then the locus of point b becomes the ZPF characteristic.

The phasor diagram of an alternator at zero power factor load and by neglecting armature resistance, R_a has been shown in Fig. 5.36.

$$V + I_a X_l = E'$$

F_f the main field excitation mmf

F_{ar} is the armature reaction mmf.

F_r is the resultant mmf

F_{ar} is in phase with I_a.

$$F_f = F_r + F_{ar}$$

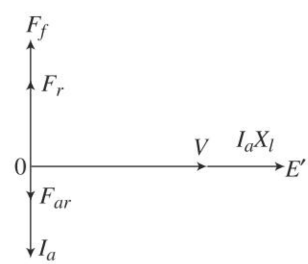


Fig. 5.36 *Phasor diagram at zero power factor lagging load with R_a neglected*

In terms of field current,

$$I_f = I_{fr} + I_{far} \text{ (refer to Fig. 5.35)}$$

After determining the armature circuit leakage reactance and the magnitude of armature reaction mmf, it is now possible to calculate the voltage regulation at any power factor load.

The step by step method of calculation of voltage regulation using Potier triangle method are stated below.

1. Draw to the scale the open circuit characteristic by choosing a suitable voltage scale. The voltage is to be the per phase values.
2. By choosing a suitable current scale draw OA representing the field current required on short circuit corresponding to rated armature current.
3. Shift point A to point b. The point b is located by taking the field current I_f required to produce rated terminal voltage on full-load zero power factor lagging (found from ZPF test).
4. Take OA equal to ab. From point a draw a line parallel to the air-gap line to touch the OCC at c.
5. Join c and b. Triangle abc is the Potier triangle.
6. Drop a perpendicular from c to d on line ab.
7. Measure the distance cd and calculate the leakage reactance X_l by considering the voltage scale. Determine the value of R_a.
8. Calculate $E' = V + I_a R_a + j\, I_a\, X_l$
9. Corresponding to voltage E' find the field current from the OCC, (which is I_{fr} in Fig. 5.37).
10. Now draw the phasor diagram as shown in Fig. 5.37. In the phasor diagram I_{fr} is leading E' by 90°. Draw I_{far} in phase opposition to I_a. I_a is lagging voltage V by the power factor angle, ϕ. Determine the total field current, I_f, by vectorially adding I_{fr} and I_{far}.
11. From OCC find E corresponding to I_f.
12. Calculate percentage voltage regulation as

$$\% \text{ Voltage regulation} = \frac{E - V}{V} \times 100$$

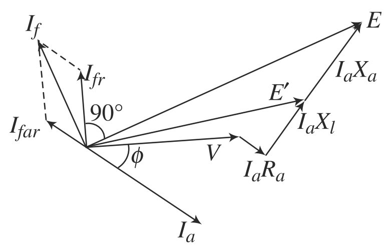


Fig. 5.37 *Phasor diagram of a synchronous generator-determining voltage regulation*

EXAMPLE 5.15 *The no-load test performed on a 1000 kVA, 3000 V, 50 Hz, three-phase star connected alternator gave the following readings:*

If (A)	15	30	50	75	90	120	150
V/ph (V)	345	690	1200	1675	1900	2130	2200

The effective armature resistance is 0.25 ohms.

When short-circuit test was conducted, a field current of 50 A was required to circulate the full-load current.

Determine the percentage voltage regulation of the alternator on full-load at 0.8 lagging power factor by mmf method.

Solution On the basis of data provided, the OCC is drawn as in Fig. 5.39. As given, $V_L = 3000$ V

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{3000}{1.732} = 1732 \text{ V}, \text{ kVA} = 1000$$

So, $$\frac{\sqrt{3}\, V_L\, I_L}{1000} = 1000$$

$\therefore$ $$I_L = \frac{1000 \times 1000}{1.732 \times 3000} = 192 \text{ A}$$

Since winding are star connected, $I_L = I_{\text{ph}} = I_a = 192$ A

Taking per phase values,

$$\bar{E}' = \bar{V} + \bar{I}_a R_a$$

The power factor = cos ϕ = 0.8 lagging

$$\phi = 37°$$

Taking $\bar{V}$ as the reference phasor,

$$\bar{E}' = 1732\angle 0° + 192\angle{-37°} \times$$

$$= 1732 + 192(\cos 37° - j\sin 37°) \times 0.25$$

$$= 1732 + 38.4 - j\,28.8$$

$$E' = 1732 + 38.4 - j\,28.8$$

$$= 1770.4 - j\,28.8$$

$$= 1770.63\ \angle{-1}$$

The phasor diagram is shown in Fig. 5.38. For $E' = 1770.63$, the field current I_{f_1} is found from the OCC is found as 81A. I_{f_2} is 50 A and is drawn in phase opposition to I_a.

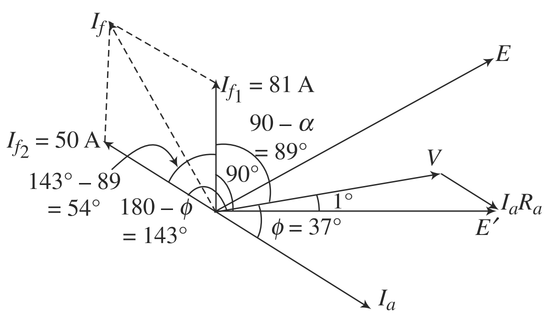


Fig. 5.38 *Phasor diagram to determine resultant mmf and the corresponding E from the OCC*

The angle between I_{f_1} and I_{f_2} is calculated as 54°

The resultant of I_{f_1} and I_{f_2} is I_f and is calculated as

$$I_{f_2} = I_{f_1}^2 + I_{f_2}^2 + 2I_{f_1} I_{f_2} \cos 54°$$

$$= 81^2 + 50^2 + 2 \times 81 \times 50 \times 0.58$$
$$= 13759$$
$$\therefore \quad I_f = 117.3 \text{ A}$$

From OCC, emf corresponding to $I_f = 117.3$ A is found as 2100 volts. Therefore, $E = 2100$

$$\text{Percentage voltage regulation} = \frac{E - V}{V} \times 100 = \frac{2100 - 1732}{1732} \times 100 = 21.2\ \%$$

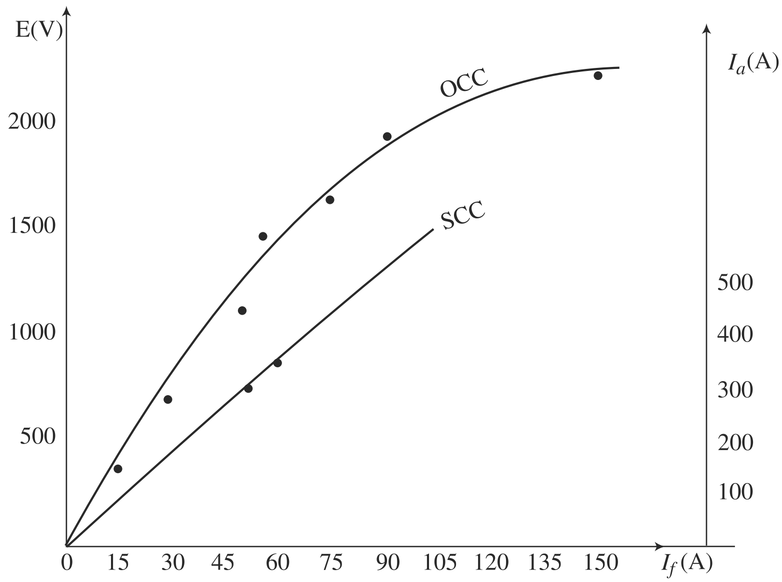


Fig. 5.39 *OCC and SCC on the basis of data provided in Example 5.15*

EXAMPLE 5.16 *A 4-pole 25 kVA, 400 V, 50 Hz, three-phase star connected synchronous generator gave the following test data.*

Field current, I_f(A)	2	4	6	8	10	12	14	16
No-load terminal voltage, (V)	138	277	355	415	468	502	533	554
Zero power factor load terminal voltage (V)			0	108	218	295	346	415

Determine the voltage regulation at full-load 0.8 power factor lagging by Potier triangle method. The armature resistance is 0.2 ohms.

Solution The OCC and ZPFC are drawn as in Fig. 5.40. The line voltage have been converted into phase voltages.

The rated line voltage is 400 V

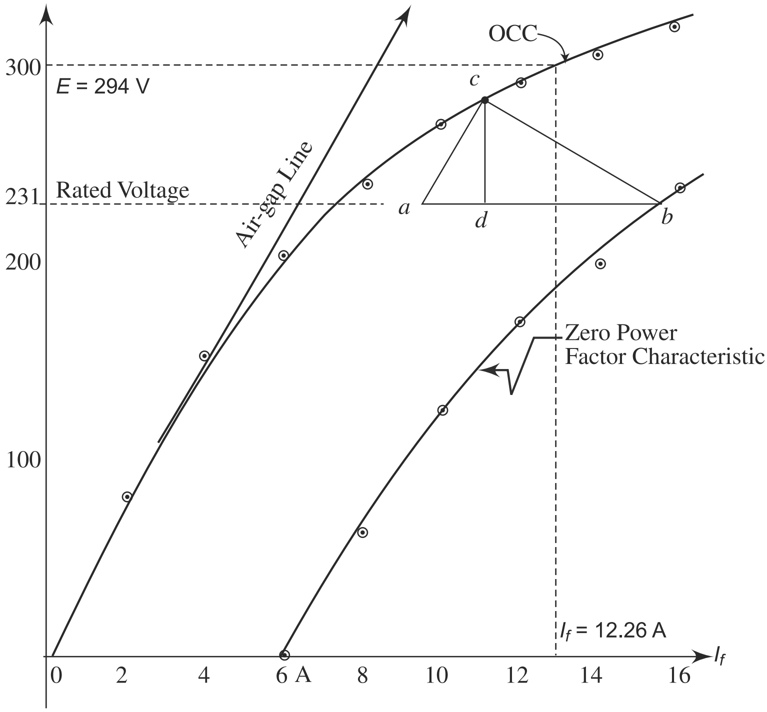


Fig. 5.40 *Development of Potier triangle for the example 5.16*

$$V_{\text{phase}} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$\text{kVA rating} = 25$$

$$\sqrt{3}\, V_L I_L = 25 \times 1000$$

$$I_L = \frac{25 \times 1000}{\sqrt{3} \times 400} = 36 \text{ A}$$

Since the windings are star connected, $I_L = I_a$

As shown in Fig. 5.40, the Potier triangle *abc* has been drawn following the set procedure. At rated voltage of 231 volts, a line is drawn to meet the *ZPFC* at *b*. *ab* is cut out from the horizontal line making it equal to *OA*. Form '*a*' a line is drawn parallel to the air-gap line to meet OCC at *c*. Now we join *cb* to get the Potier triangle *abc*. From Point *c*, a perpendicular line is drawn upto *d*. The distance *cd* when multiplied by the voltage scale provides the voltage drop due to leakage reactance.

$$cd = 53 \text{ V}$$

$$I_a X_l = 53 \text{ V}$$

$$X_l = \frac{53}{36} = 1.47 \text{ ohms}$$

The field current required to overcome armature reaction on load = *db* = 4.4 Amps. = I_{f_2}

$$E' = V + I_a R_a + jI_a X_l$$
$$= 231 + 36 \times 0.2 + j\,36 \times 1.47$$
$$= 249 + j\,53$$

or, $$E' = 254.6 \angle 12° \text{ Volt}$$

The field current required to induce of voltage of E' = 254.6 V is found from the OCC as equal to 8.6 A = I_{f_1}

Now the phasor diagram showing the field current I_{f_1} and I_{f_2} and their resultant has been drawn as in Fig. 5.41.

Power factor = cos ϕ = 0.8

$$\phi = 37°$$

V is drawn as the reference phasor. I_a is drawn lagging *V* by 37°. We add I_aR_a in phase with I_a and I_aX_l perpendicular to I_a to get E'.

$$E' = V + I_aR_a + j'I_aX_l$$

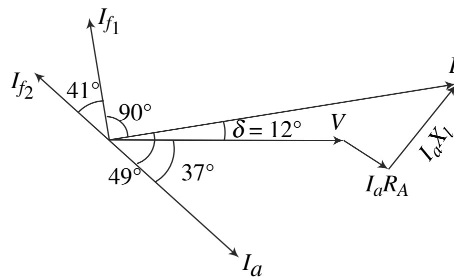


Fig. 5.41

I_{f_1} = 8.6 A is drawn perpendicular to E' and leading E'. I_{f_2} = 4.4 A is drawn in phase opposition to I_a. The angle between I_{f_1} and I_{f_2} is 41°.

The resultant of I_{f_1} and I_{f_2} is I_f where

$$I_f = \sqrt{I_{f_1}^2 + I_{f_2}^2 + 2I_{f_1}I_{f_2}\cos 41°}$$
$$= (8.6)^2 + (4.4)^2 + 2 \times 8.6 \times 4.4 \times 0.7547$$
$$I_f = 12.26 \text{ A}$$

The emf, *E* corresponding to I_f = 12.26 A is found from OCC as equal to 294 volts.

$$\text{Voltage regulation in percentage} = \frac{E - V}{V} \times 100$$
$$= \frac{294 - 231}{231} \times 100 = 27.27 \text{ percent}$$

5.13 A SYNCHRONOUS GENERATOR INDEPENDENTLY SUPPLYING A LOAD

It has been observed that the change of terminal voltage of a synchronous generator depends on load and load power-factor. A series of load characteristics as shown in Fig. 5.42 can be drawn for different power factor loads. When a synchronous generator is independently supplying a load, increasing its excitation will increase the no-load voltage. This will raise the level of the whole load characteristic and increase the value of the terminal voltage.

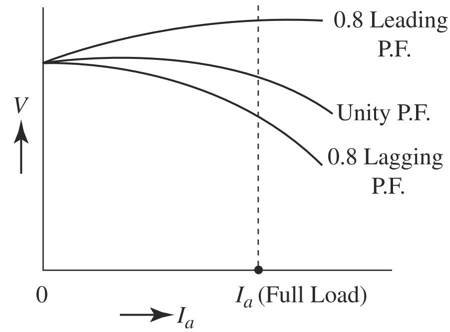


Fig. 5.42 *Load characteristics of a synchronous generator independently supplying loads of different power factors*

The power input to an alternator is applied by a primemover, which in most cases is a steam turbine. This input is directly proportional to the output. If output is increased, more power must be developed by the primemover, otherwise, speed will drop. If speed drops, there will be a drop in output voltage and frequency. On the other hand, if by increasing the primemover steam supply, input power is increased without increasing the electrical output, the speed of the set will increase. Increase in speed will cause increase in terminal voltage and frequency.

It is rare to find a synchronous generator supplying its own load independently. For emergency power supply requirement, small synchronous generators driven by diesel engines are used.

5.14 SYNCHRONOUS GENERATORS CONNECTED IN PARALLEL TO SUPPLY A COMMON LOAD

It is an established practice these days to connect a number of synchronous generators in parallel to supply a common load. In power stations, instead of having one large capacity generator, a number of smaller units are installed and their output terminals connected in parallel. Moreover, for a variety of reasons, large number of stations in a country are interconnected through transmission and distribution lines. All the synchronous generators of the system, therefore, work in parallel which is equivalent to a very large synchronous machine. Similarly all the electrical loads of the consumers are connected in parallel and form a very large variable load.

A supply system bus-bar with a large number of synchronous generators connected in parallel is referred to as *infinite bus-bar*. Any additional machine, whether to work as a generator or as a motor is connected in parallel with the system. The characteristics of an infinite bus-bar system are constant terminal voltage, constant bus-bar frequency and very small synchronous impedance (since a large number of generators are in parallel). There are a number of advantages of connecting alternators in parallel to such an infinite bus-bar system.

5.14.1 Advantage of Parallel Operation of Synchronous Generators

The following are the advantages of connecting a large number of synchronous generators in parallel to supply a common load:

(a) Repair and maintenance of individual generating units can be done keeping the continuity of supply by properly scheduling maintenance of generators one after the other. If only one large generator is installed, supply is to be cut off for maintenance work.

(b) For operating an alternator on maximum efficiency it is to be run near to its full-load capacity. It is uneconomic to operate large alternators on low loads. If several small units are used, units can be added or put off depending upon the load requirement and thus the units can be operated at near to their rated capacity.
(c) Additional sets can be connected in parallel to meet the increasing demand, thereby reducing the initial capital cost of buying larger units in anticipation of increasing demands.
(d) There is physical and economic limit to the possible capacity of alternators that can be built. The demand of a single power station may be as high as 1200 MVA. It may not be feasible to build a single alternator of such a high rating due to physical and economic considerations.

5.14.2 Parallel Connection or Synchronising of Alternators

Before a synchronous generator can be put to share the load, it should be properly connected in parallel with the common bus-bar. Interconnection of the terminals of a generator with the terminals of another or a bus-bar, to which a large number of synchronous generators are already connected is called synchronising of generator.

Conditions for Parallel Connection or Synchronisation For satisfactory parallel connection of alternators, the following three conditions must be fulfilled:
(a) The generated voltage of the incoming alternator to be connected in parallel with a bus-bar should be equal to the bus-bar voltage.
(b) Frequency of the generated voltage of the incoming alternator should be equal to the bus-bar frequency.
(c) Phase sequence of the voltage of the incoming alternator should be the same as that of the bus-bar.

Generated voltage of the incoming alternator can be adjusted by adjusting the field excitation. Frequency of the incoming alternator can be controlled and made equal to bus-bar frequency by controlling the speed of the primemover driving the alternator. Phase sequence of the alternator and the bus-bar can be checked by a phase sequence indicator. Alternatively, three lamps as shown in Fig. 5.43 can be used for checking of phase sequence. Three lamps L_1, L_2 and L_3 are to be connected as shown in the figure. With the synchronous generator driven at rated speed if all the lamps glow together and become dark together then the phase sequence of the incoming alternator is the same as that of the

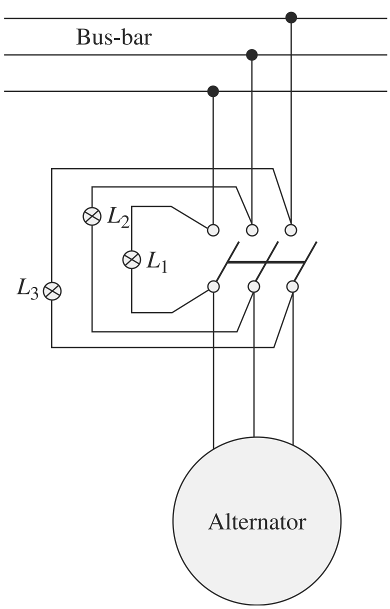


Fig. 5.43 *Three lamp method of checking phase sequence of an alternator*

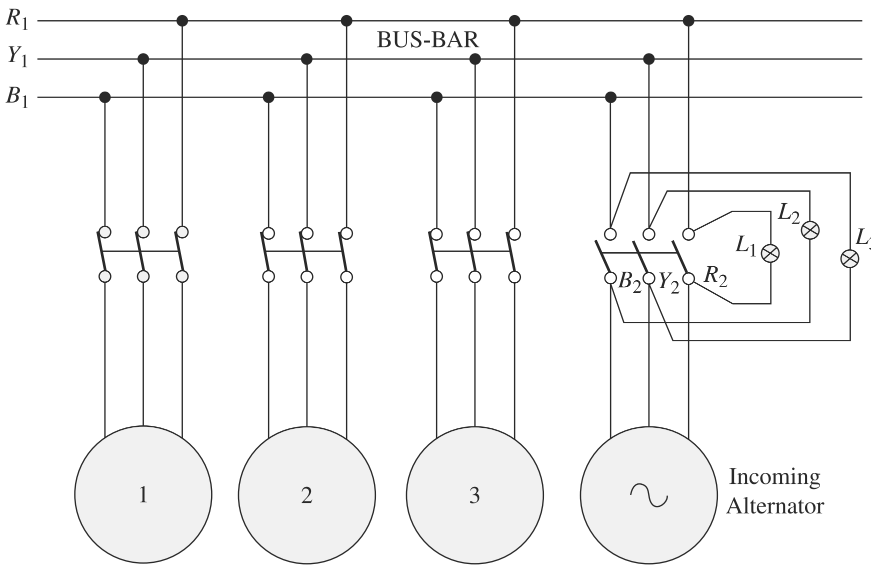
bus-bar. Once the three conditions of parallel connection, i.e., synchronisation mentioned earlier are satisfied, the incoming alternator can be switched on to the bus-bar, provided the instant when the voltages of the incoming generator and the bus-bar are in exact phase is known. For this purpose the two commonly used methods are described as follows.

Methods of Synchronisation Synchronisation or parallel connection of alternators can be achieved by any one of the following two methods:

(a) By three-lamp (one dark two bright) method.

(b) By using a synchroscope.

(a) Synchronising by Three-Lamp Method In this method of synchronising an alternator, three lamps are connected as shown in Fig. 5.44. Two lamps are cross connected with the bus-bars. In this method the brightness of the lamps will vary in sequence. A particular sequence will indicate if the incoming alternator is running



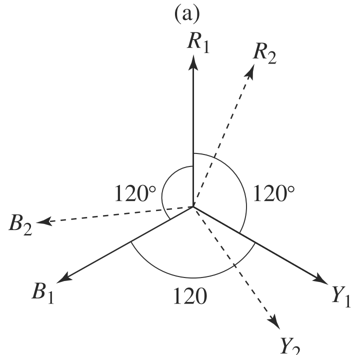


Fig. 5.44 *Synchronisation of an alternator with the bus-bar*

too fast or too slow. Perfect synchronising will occur when lamp L_1 is dark while lamps L_2 and L_3 are equally bright.

When the speed and voltage have been adjusted, the switch of the incoming synchronous machine can be closed only when lamp L_1 is dark while lamps L_2 and L_3 are equally bright. If the frequency of the incoming alternator is higher than the bus-bar frequency, the phasors R_2–Y_2–B_2 representing the alternator voltages will be rotating faster than the phasors R_1–Y_1–B_1 representing the bus-bar voltages. At the instant when R_1 is in phase with R_2, lamp L_1 will be dark and the other two lamps will be equally bright. After one-third of the cycle, B_2 will be in phase with Y_2. Since the lamp L_2 is connected across B_2, and Y_2, it will be dark. After another one-third of a cycle, lamp L_3 will be dark. Thus if the frequency of the incoming alternator is higher, the lamps will become dark in the sequence L_1–L_2–L_3.

Similarly, if the frequency of the incoming alternator is lower, the lamps will become dark in the sequence L_1–L_3–L_2. The speed of the alternator will, therefore have to be slowly adjusted so that the lamp L_1 is dark and lamps L_2 and L_3 are bright. At this instant, the switch can be closed. The incoming machine thus gets connected in parallel with the bus-bar.

In this three-lamp method, in addition to knowing the exact instant of closing of synchronising switch, it is also known whether the incoming alternator's frequency is less or more than the bus-bar frequency.

(b) Synchronisation by Using a Synchroscope A synchroscope determines the instant of synchronism more accurately than the three-lamp method. A synchroscope consists of a rotor (moving coil) and a stator (fixed coil); one of which is connected to the incoming alternator and the other to the bus-bar as shown in Fig. 5.45.

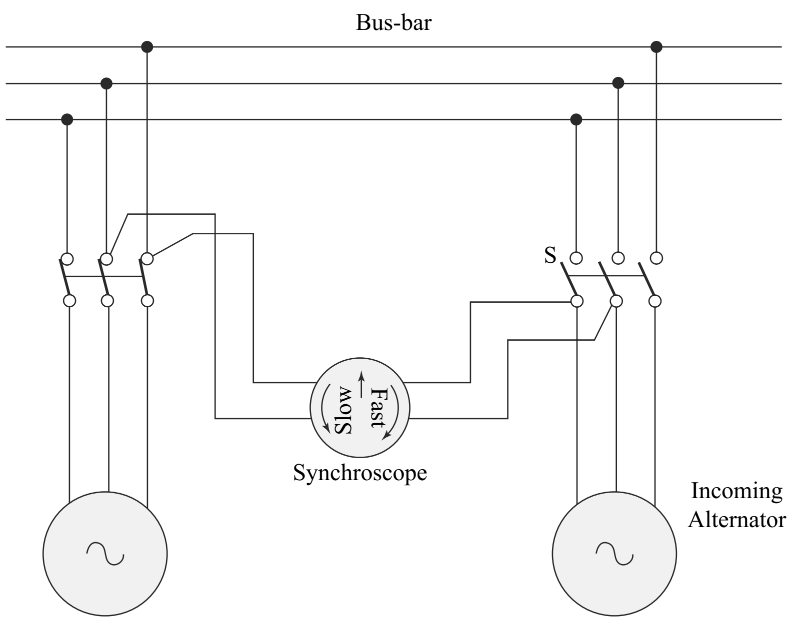


Fig. 5.45 *Synchronisation of alternators using synchroscope*

A pointer connected to the rotor will rotate if there is a difference in frequencies of the incoming alternator and bus-bar. Anticlockwise rotation of the rotor pointer indicates that the frequency of the incoming alternator is slower, whereas clockwise rotation of the pointer indicates that the frequency is higher than the bus-bar frequency. The speed of the primemover driving the alternator will, therefore, have to be adjusted such that, the frequencies are equal and the pointer is stationary. The alternator can be switched on to the bus-bar by closing the switch, S at this instant.

Concept of Infinite Bus When a large number of synchronous generators operate in parallel to supply a common load, the machines are connected to the same bus but may be physically located at different places. The capability of such a system of power supply is such that the supply voltage and frequency remain more or less constant. Connection or disconnection of a small load on such a supply system will not disturb the system voltage and frequency. Large number of generators connected in parallel may be considered as one very big generator with very small internal impedance. The voltage and frequency of an infinite bus therefore, does not change with load.

Synchronising Power A synchronous machine when synchronised to an infinite bus will tend to remain in synchronism for limited electrical and mechanical disturbances.

The power angle characteristic of a cylindrical rotor synchronous generator is governed by the expression,

$$P = \left(\frac{E\,V}{X_s}\right)\sin\delta$$

Assume that the generator is supplying a load of P_0 with a load angle δ_0. Any sudden change of load on the generator will cause a momentary retardation of the rotor thereby increasing the load angle by $\Delta\delta$. This increase in δ will cause increase in power output by ΔP which intern will cause the rotor to oppose the increase in δ. The reverse will happen when δ will tend to decrease. The rotor will settle for its original load angle δ in an oscillatory manner. Therefore, ΔP caused by $\Delta\delta$ is the power that brings the machine in its stable mode of operation.

The ratio of $dP/\Delta\delta$ is called the synchronising power coefficient or stiffness of the electromagnetic coupling and is an indicator of the capability of the synchronous machine to stay in synchronism.

Synchronising power,

$$P_{\text{syn}} = \frac{dP}{d\delta} = \left(\frac{EV}{X_S}\right)\cos\delta \quad \text{Watt/elec. radian}$$

Synchronising power gives rise to synchronising torque which is

$$T_{\text{syn}} = \frac{1}{\omega_S}\frac{dP}{d\delta} \quad \text{Nm/elec. radian}$$

where $$\omega_s = \frac{2\pi\,N_s}{60}$$

From the expression for synchronising power, it is observed that P_{syn} is directly proportional to E and inversely proportional to synchronous reactance X_s. Machines with over excitation and small value of X_s will have high value of synchronizing power. Further, when value of δ is zero, P_{syn} is maximum and when δ is nearly 90° there is hardly any synchronising power or restoring action to counter the disturbances.

Machine Floats on the Bus-Bar When synchronised, the generated emf of the incoming machine is just equal to the bus-bar voltage. The synchronous machine will be just *Floating* on the bus-bar, i.e., it will neither deliver nor receive any power. The primemover driving the machine will be supplying the no-load losses only.

Once a synchronous machine is synchronised, it will tend to remain in step with the other alternators. Any tendency to depart from the above condition is opposed by a synchronising torque developed due to circulating current flowing through the alternators. The alternator, which due to some disturbances tends to speed up will develop a circulating current and power will flow from this alternator to the others, thereby having a loading effect on this advancing alternator. This will bring retarding action on its rotor and thus put it back in step with the other alternators. On the other hand, if any alternator tends to retard, power will flow from the other alternators to this alternator and the synchronising torque will tend to keep this machine in synchronism with the others.

5.14.3 Active and Reactive Load Sharing

A synchronous machine after synchronisation just floats on the bus-bar. It neither delivers power nor receives power. When a generator is connected in parallel, it should share a portion of the total load depending upon its kVA rating. We shall examine how load sharing of alternators running in parallel can be achieved. We will study the effect of change of excitation and that of prime-mover input.

Effect of Change of Excitation For dc generators, load sharing between a number of machines running in parallel can be achieved by adjusting their excitations. For synchronous generators, change of excitation, i.e., change of field current does not change the active power shared by them. Change of excitation only changes the reactive power supplied by each machine. This is explained with the help of a phasor diagram shown in Fig. 5.46. In figure, V_B represents the bus-bar voltage, E_{in} is the voltage induced in the incoming machine. Since the incoming machine is connected in parallel, these two voltages are opposing each other as shown. Let excitation of the incoming machine

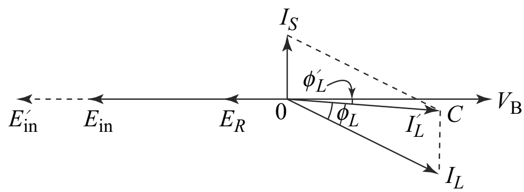


Fig. 5.46 *Effect of change of excitation of a synchronous machine connected in parallel with the bus-bar*

is changed to E'_{in}. This will cause a resultant voltage, E_R to appear which will cause a current I_s to flow from the machine to the bus-bar, i.e., to the load. Current. I_s will lag E_R by about 90°, because the synchronous reactance of the machine is much higher than its resistance. I_L is the current supplied to the load from the bus-bar and the total per phase power supplied was $V_B I_L \cos \phi_L$. Now the current supplied from the bus-bar is changed to I_L' since the incoming machine is supplying a reactive current, I_s. Since V_B is constant, active load power is proportional to the length OC. The active power supplied by the existing machines connected to the bus-bar has not changed, i.e., the $I_L \cos \phi_L$ has remained equal to $I_L' \cos \phi_L'$. Change of excitation of the incoming machine has only changed the reactive power delivered by the existing machines.

Effect of Change of Primemover Input If the input to the primemover of the incoming generator is increased, it will start sharing load while remaining in synchronism with the existing alternators connected to the bus-bar. Control of active power shared between the alternators is achieved by changing the input to their primemovers. Change in the input to primemovers in a thermal power station is achieved by a change of throttle opening thus by allowing more or less steam into the turbine, whereas in a hydel power station primemover input is controlled by controlling the water inlet into the water turbine.

Let the primemover input to the incoming alternator be increased. This will move the generated emf phasor E_{in} forward as shown in Fig. 5.47.

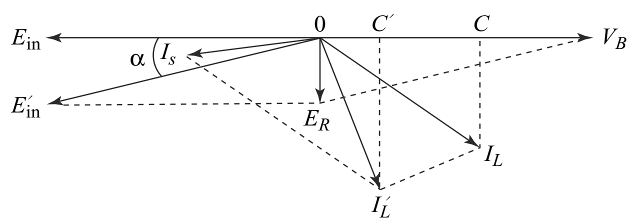


Fig. 5.47 *Effect of change of primemover input of synchronous machine connected in parallel with the bus-bar*

Let E'_{in} be the new position of the generated emf of the incoming alternator. The resultant voltage E_R will now cause a current I_s which has a strong in-phase component with the voltage. Thus the incoming machine will supply active power to the load. The I_L' will be the new load current supplied by the existing alternators, which has an active component represented by OC'. Thus, there is a reduction of active power load on the existing generators due to the sharing of active load by the incoming generator achieved by changing of primemover input.

5.14.4 Operating Characteristics of a Synchronous Generator

The circuit model and phasor diagram of a generator are shown in Fig. 5.48.

From the phasor diagram,

$$E = V + I_a R_a + j I_a X_s$$

Neglecting R_a,

$$E = V + j I_a X_s$$

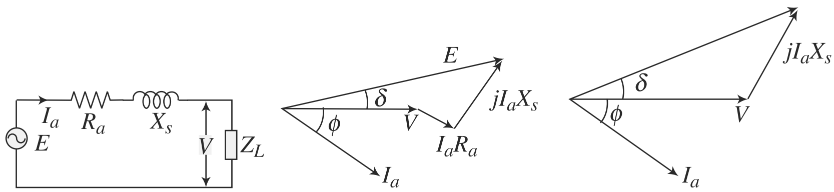


Fig. 5.48 *Circuit model and phasor diagram of a synchronous generator*

or,
$$\bar{I}_a = \frac{\bar{E} - \bar{V}}{j X_s} \tag{5.14}$$

With V as the reference axis,

$$\bar{E} = E \cos\delta + jE \sin\delta \tag{5.15}$$

and
$$\bar{I}_a = I_a \cos\phi - jI_a \sin\phi \tag{5.16}$$

From (5.14) and (5.15),

$$\bar{I}_a = \frac{E \cos\delta + jE \sin\delta - V}{jX_s}$$

or,
$$\bar{I}_a = \frac{E \sin \delta}{X_S} + \frac{E \cos \delta - V}{jX_S}$$

or,
$$\bar{I}_a = \frac{E \sin \delta}{X_s} - \frac{j(E \cos\delta - V)}{X_s} \tag{5.17}$$

Comparing (5.16) and (5.17),

$$I_a \cos \phi = \frac{E \sin \delta}{X_s}$$

Output real power, $P_o = 3\, VI_a \cos \phi$

or,
$$P_o = \frac{3\, V E}{X_s} \sin \delta$$

The angle between V and E is called the power angle δ.

$$P_{o\,(\max)} = \frac{3\, V E}{X_s}$$

The maximum power occurs at $\delta = 90°$ beyond which the machine loses its synchronism. Thus power angle $\delta = 90°$ is the steady state stability limit of the synchronous machine. The machine is normally operated at $\delta < 90°$.

We will now examine the operation of the synchronous generator under (a) constant excitation and variable load condition; and (b) constant load and variable excitation condition.

Constant Excitation, Variable Load Operation With constant excitation, the magnitude of induced emf E will remain constant but angle δ will be variable. The tip of E will lie on a E-constant circle as shown in Fig 5.49.

The phasor diagram has been drawn for the machine developing maximum power with $\delta = 90°$. The new position of I_a has been shown. With $V + jI_{a_1}X_s = E$, the magnitude of I_aX_s is large and hence the current delivered I_{a_1} will be larger than the rated current I_a.

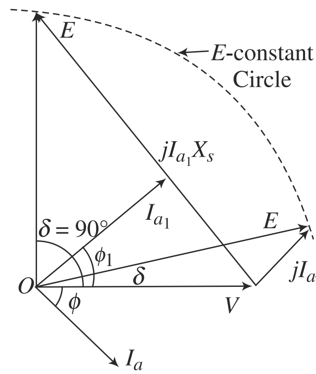


Fig. 5.49 *Phasor diagram of a synchronous generator at steady state stability limit*

Constant Load Variable Excitation Operation

Output, $P_o = V I_a \cos \phi$ and

$$P_o = \frac{VE \sin \delta}{X_s}$$

As V and X_s are taken as constants, with constant load, $I_a \cos \phi$ and $E \sin \delta$ will remain constant. With load remaining constant, as excitation is varied (I_f is varied), the magnitude of E will vary in such a way that the tip of phasor E will touch the constant load line with changing values of δ. With changing E and constant V, the I_aX_s and hence I_a will change. The tip of phasor I_a which is at right angles with I_aX_s will lie on the constant $I_a \cos \phi$ line.

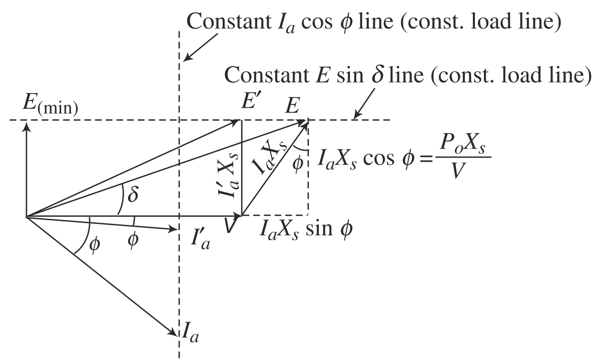


Fig. 5.50 *Constant load but variable excitation operation*

The excitation emf at which I_a is minimum is called normal excitation emf or normal excitation. The current I_a is in phase with V and hence power factor of the generator is unity. Excitation more than normal excitation is called over excitation and excitation less than normal excitation is called under excitation.

An over excited generator will supply lagging power factor current while an under excited generator will supply leading power factor current.

It is observed that change in excitation at constant load reactive power supplied by the generator changes as $I_a \sin\phi$ changes with change of excitation. $V I_a \sin\phi$ is the reactive power supplied.

Load Sharing by Changing Primemover Input Alternators are driven by primemovers. The primemovers are generally turbines. The active or real power generated by an alternator depends upon the input to the primemover. Generators are run in parallel to share a common load. The speed-load or frequency-load characteristics of the primemovers are somewhat drooping in nature. That is, with increase in load there will be some decrease in speed (or frequency of voltage generated).

Frequency versus power characteristic of a primemover driven generator has been shown in Fig. 5.51. If the prime-mover input is slightly increased, the characteristic move upward as has been shown.

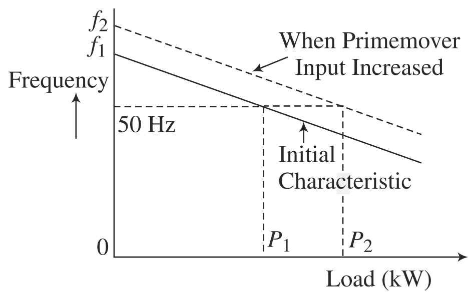


Fig. 5.51 *Speed or frequency versus load characteristic of a generator driven by a primemover*

Now consider that two alternators A and B are running in parallel. The frequency-load characteristics of the two machines have been shown in Fig. 5.52. W_1 and W_2 are the full-load power rating of alternators A and B respectively. Let P be the actual load connected to the machines running in parallel, P_1 is the load shared by machine A and P_2 is the load shared by machine B.

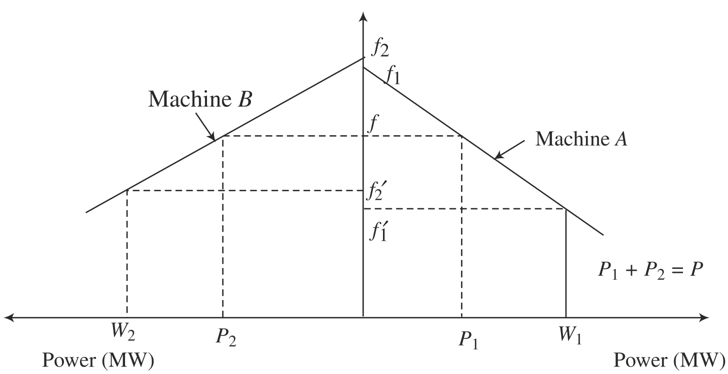


Fig. 5.52 *Frequency-load characteristics of two alternators running in parallel and sharing a common load*

f_1 and f_1' are the frequencies on no-load and on full-load of machine A. Similarly, f_2 and f_2' are the frequencies on no-load and on full-load of machine B. When the two alternators are running in parallel sharing a common load P, the frequency is f as has been shown.

Total load on the two generators is P. Load shared by generator A is P_1 and by B is P_2.

$$P = P_1 + P_2 \tag{5.18}$$

For generator A,

$$\text{Drop in frequency per unit load} = \frac{f_1 - f_1'}{W_1}$$

$$\text{Drop in frequency for a load} \quad P_1 = \frac{f_1 - f_1'}{W_1} \times P_1$$

No load frequency is f_1

$$\text{Operating frequency, } f = f_1 - \frac{f_1 - f_1'}{W_1} \times P_1 \tag{5.19}$$

For generator B, no-load frequency $= f_2$

$$\text{Operating frequency, } f = f_2 - \frac{f_2 - f_2'}{W_2} \times P_2 \tag{5.20}$$

Equating Eqs (5.18) and (5.19),

$$f_1 - \frac{f_1 - f_1'}{W_1} \times P_1 = f_2 - \frac{f_2 - f_2'}{W_1} \times P_2 \tag{5.21}$$

From Eq. (5.18) and Eq. (5.21), load shared by the two generators, i.e., P_1 and P_2, and the operating frequency f can be calculated.

EXAMPLE 5.17 *Two generators A and B rated at 25 MW and 15 MW respectively are supplying power to a total load of 28 MW. The no-load frequency of both the generators is 50 Hz. The full-load speed regulation of generators A and B respectively are 3 percent and 2 percent respectively. Calculate how the load of 28 MW is shared by the two generators.*

Solution For machine A, the drop in frequency from no-load to full-load is 3 percent of 50 Hz = 1.5 Hz

$$\text{Drop in frequency per unit load} = \frac{1.5}{25}$$

$$\text{Drop in frequency for load, } P_1 = \frac{1.5}{25} \times P_1$$

$$\text{Operating frequency, } f = 50 - \frac{1.5}{25} P_1 \tag{1}$$

For machine B, the drop in frequency from no-load to full-load is 2 percent of 50 Hz = 1.0 Hz

$$\text{Drop in frequency for a load, } P_2 = \frac{1.0}{15} \times P_2$$

$$\text{Operating frequency, } f = 50 - \frac{1}{15} P_2 \tag{2}$$

Equating Eqs (1) and (2),

$$50 - \frac{1.5}{25} P_1 = 50\ 0 \frac{1}{15} P_2$$

or, $$\frac{1.5}{25} P_1 = \frac{1}{15} P_2$$

or, $$P_1 = \frac{25}{15 \times 1.5} P_2 = \frac{5}{4.5} P_2$$

Again, $$P_1 + P_2 = 28$$

Substituting value of P_1, $\frac{5}{4.5} P_2 + P_2 = 28$

or, $$5P_2 + 4.5\ P_2 = 28 \times 4.5$$

or, $$P_2 = \frac{28 \times 4.5}{9.5} = 13.263 \text{ MW}$$

$$P_1 + P_2 = 28$$

or, $$P_1 = 28 - 13.263$$

$$= 14.737 \text{ MW}$$

EXAMPLE 5.18 *Two generators A and B rated respectively at 40 MW and 60 MW are running in parallel. Both the generators have a full-load speed regulation of 3 percent. Calculate how a load of 50 MW will be shared by them.*

Solution Let the no-load frequency be f_0.

For machine *A*,

Operating frequency, $f = f_0 - \frac{0.04}{40} f_0 \times P_1$ (1)

For machine *B*,

Operating frequency, $f = f_0 - \frac{0.04}{60} f_0 \times P_2$ (2)

Equating Eqs (1) and (2),

$$f_0 - \frac{0.04}{40} f_0 \times P_1 = f_0 - \frac{0.04}{60} f_0 \times P_2$$

$$P_1 = \frac{2}{3} P_2$$

Again, $$P_1 + P_2 = 50$$

or, $$\frac{3}{2} P_2 + P_2 = 50$$

or, $$P_2 = 20 \text{ MW}$$

So, $$P_1 = 50 - 20 = 30 \text{ MW}$$

EXAMPLE 5.19 *Two alternators A and B operate in parallel and supply a load of 8 MW at 0.8 pf lagging. The power output of A is adjusted to 5000 kW by changing its steam supply and its pf is adjusted to 0.9 lagging by changing its excitation. Find the pf of alternator B.*

Solution

$$\cos\phi = 0.8$$
$$\phi = \cos^{-1} 0.8 = 36.9°$$
$$\tan\phi = 0.75$$

For alternator A

$$\cos\phi_A = 0.9$$
$$\phi_A = \cos^{-1} 0.9 = 5.84°$$
$$\tan\phi_A = 0.484$$
$$\text{Active load, kW} = \text{VI}\cos\phi$$
$$\text{Reactive load, KVAr,} = \text{VI}\sin\phi$$

$$\therefore \quad \frac{\text{kVAr}}{\text{kW}} = \frac{\text{VI}\sin\phi}{\text{VI}\cos\phi}$$

$$\therefore \quad \text{Reactive load} = \text{Active load} \times \tan\phi$$
$$\text{Active load} = 8000\text{ kW}$$
$$\text{Reactive load} = 8000\tan\phi$$
$$= 8000 \times 0.75 = 6000\text{ kVAr}$$
$$\text{Active load } A = 5000\text{ kW}$$
$$\text{Reactive load } A = 5000\tan\phi$$
$$= 5000 \times 0.484 = 2420\text{ kVAr}$$
$$\text{Active load of } B = 8000 - 5000 = 3000\text{ kW}$$
$$\text{Reactive load of } B = 6000 - 2420 = 3580\text{ kVAr}$$
$$\tan\phi_B = \frac{\text{Reactive load of } B}{\text{Active load of } B}$$
$$\tan\phi_B = \frac{3580}{3000} = 1.19$$
$$\phi_B = \tan^{-1}(1.19)$$
$$\phi_B = 50.03°$$
$$\text{Power factor of } B = \cos\phi_B = \cos 50.03° = 0.64$$

EXAMPLE 5.20 *Two similar 6600 V, 3-phase alternators are running in parallel sharing equally a total load of 10,000 kW at a lagging power factor of 0.8, the two machines being similarly excited. If the excitation of one machine is adjusted so that its armature current is 438 A, find the armature current and power factor of the second machine if the steam supply is not changed.*

Solution

$$\text{Load current} = \frac{10{,}000 \times 1000}{\sqrt{3} \times 6600 \times 0.8} = 1093.5 \text{ A}$$

$$\cos \phi = 0.8$$

$$\phi = \cos^{-1} 0.8 = 36.87°$$

$$\sin \phi = 0.6$$

$$\text{Active component of current} = I_L \cos \phi = 1093.5 \times 0.8 = 874.8 \text{ A}$$

$$\text{Reactive component of current} = I_L \sin \phi = 1093.5 \times 0.6 = 656.1 \text{ A}$$

$$\text{Current supplied by each alternator} = \frac{1093.5}{2} = 546.75 \text{ A}$$

Active component of current supplied by each alternator

$$= \frac{874.8}{2} = 437.4 \text{ A}$$

Reactive component of current supplied by each alternator

$$= \frac{656.1}{2} = 328 \text{ A}$$

Since steam supply is same, the active component remain the same ie at 437.4A

$$\text{Reactive component of } I_L = \sqrt{(438)^2 - (437.4)^2} = 23 \text{ A}$$

Hence reactive component of $I_2 = 656.1 - 23 = 633.1$ A

Hence $$I_2 = \sqrt{(437.4)^2 + (633.1)^2} = 769.5 \text{ A}$$

$$\tan \phi_2 = \frac{633.1}{437.4} = 1.447$$

$$\phi_2 = \tan^{-1} 1.447 = 55.36°$$

Hence $$\cos \phi_2 = 0.568 \text{ (lag)}$$

5.15 SALIENT POLE SYNCHRONOUS MACHINE—TWO-REACTION MODEL

A salient pole synchronous machine as shown in Fig. 5.53 has non-uniform air-gap. The flux has to cross large air-gaps in the interpolar region than in the polar region. Accordingly the reluctance to flux path in the inter-polar region is more than in polar region. We know that reactance is more where reluctance is low, i.e., permeability is high. The polar region offers low reluctance or high permeability to flux path. Hence synchronous reactance will be more along the polar axis than in the interpolar axis.

In order to take into consideration the difference in reluctance and hence reactance in the polar and inter-polar region, synchronous reactance is split into two reactances. The synchronous reactance X_s along the pole-axis, which is also called the direct-axis or d-axis, is designated as direct-axis synchronous reactance, x_d. See Fig. 5.53.

The synchronous reactance along the inter-polar axis, which is also referred to as the quadrature axis or q-axis, is referred to as the quadrature axis synchronous reactance, x_q.

As can be seen from the figure, along the d-axis air-gap is low and hence permeability of flux path is high. Along the q-axis air-gap is high and hence permeability of flux path is low. Reactance depends on permeability of flux path. Higher permeability will cause higher value of reactance. Therefore, the reactance along the d-axis will be higher than the reactance along the q-axis, i.e., $x_d > x_q$.

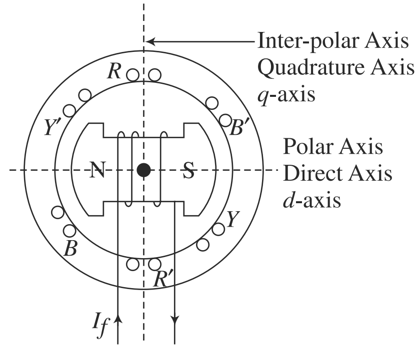


Fig. 5.53 *A salient pole type synchronous machine showing the d-axis and q-axis*

The rotor is excited by the field current I_f. When the rotor is rotated, emf is induced in the stator windings. Let E be the emf induced per-phase under no-load condition. When the machine is loaded an armature current I_a will be flowing. For a lagging power factor load, I_a will lag the induced emf E by an angle ψ. Current I_a will lag the terminal voltage V by an angle ϕ.

The armature current I_a is resolved into two components namely the direct axis component I_d and the quadrature axis component I_q as has been shown in Fig. 5.54.

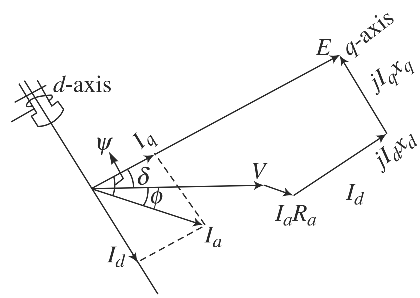


Fig. 5.54 *Phasor diagram of a salient-pole synchronous generator*

The direct component, I_d of the armature current, I_a produces armature flux along the d-axis while the quadrature component, I_q of armature current, I_a produces flux in the q-axis. The rotor flux which is along the d-axis produces induced emf E which is at right angles with the d-axis as has been shown. Therefore, E lies along the q-axis and hence, E-axis and q-axis is the same. I_d is along the field-pole axis and I_q is along the q-axis. The flux produced by I_d and I_q induces emfs E_d and E_q respectively in the armature winding. We can express these induced emfs in terms of x_d and x_q as

$$E_d = -j\, I_d\, x_d \text{ and } E_q = -j\, I_q\, x_q$$

These voltages are directly proportional to the stator current and lags the current by 90°. Considering the voltage drops due to $I_a\, R_a$ and $I_d\, x_d$ and $I_q x_q$, the emf E can be expressed as

$$E - I_aR_a - j\, I_dx_d - j\, I_q\, x_q = V$$

where V is the terminal voltage on load

or, $$E = V + I_aR_a + j\, I_dx_d + j\, I_qx_q \tag{5.22}$$

The phasor diagram has been drawn using the above relation as shown in Fig. 5.54. I_aR_a has been drawn in parallel to I_a. $j\, I_d\, x_d$ has been drawn perpendicular to I_d and $j\,I_q\,x_q$ has been draw perpendicular to I_q. The angle between V and E is δ, called the power angle.

We can express $j\,I_d x_d$ as

$$j\,I_d\,x_d = j\,I_d\,x_q + j\,I_d\,x_d - j\,I_d\,x_q$$

$$= j\,I_d\,x_q + j\,I_d(x_d - x_q)$$

The phasor diagram is redrawn with these values as shown in Fig. 5.55(a). If we neglect armature resistance R_a, the phasor diagram will be as shown in Fig. 5.55(b).

The part of phasor diagram of Fig. 5.55(a) is redrawn as in Fig. 5.56, for the purpose of calculating the value of tan δ.

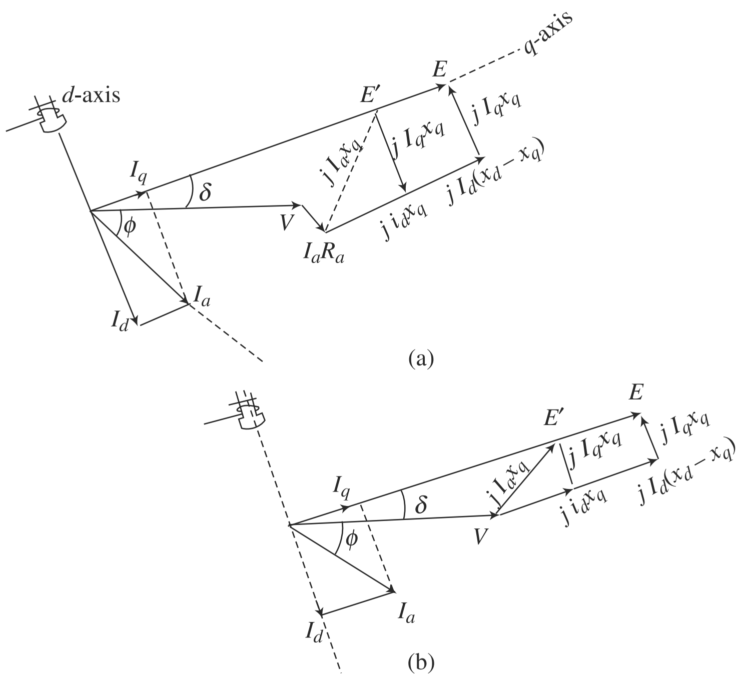


Fig. 5.55 *(a) Phasor diagram of a salient pole synchronous generator (b) Phasor diagram neglecting R_a*

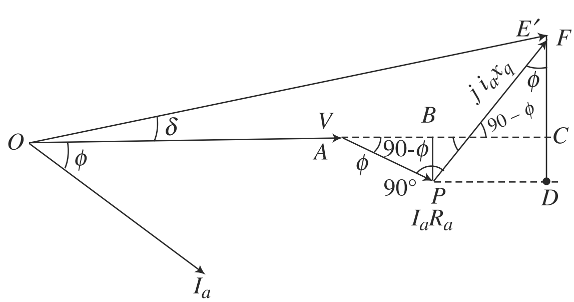


Fig. 5.56 *Part of phasor diagram of Fig. 5.55 (a) redrawn*

From triangle OCF,

$$\tan\delta = \frac{FC}{OC} = \frac{FD - CD}{OA + AB + BC} = \frac{FD - PB}{OA + AB + PD}$$

$$\tan\delta = \frac{I_a x_q \cos\phi - I_a R_a \sin\phi}{V + I_a R_a \cos\phi + I_a X_q \sin\phi} \qquad (5.23)$$

If we neglect R_a,

$$\tan\delta = \frac{I_a\, x_q \cos\phi}{V + I_a\, X_q \sin\phi} \qquad (5.24)$$

Using Eqs (5.22) or (5.24), the value of torque angle δ can be calculated.

Expression for Output Power We will now develop the expression for output power of the salient pole synchronous generator from the expression,

$$P_o = 3\, V I_a \cos\phi$$

We will neglect the armature resistance R_a and draw the phasor diagram as in Fig. 5.57.

From Fig. 5.57,

$$OB = V\cos\delta$$

$$BC = I_d x_d$$

$$OC = E$$

$$BP = V\sin\delta = QC = I_q x_q$$

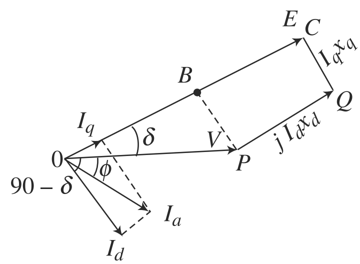


Fig. 5.57

So, $$V\sin\delta = I_q x_q$$

or, $$I_q = \frac{V\sin\delta}{x_q} \qquad (5.25)$$

and $$E - I_d x_d = V\cos\delta$$

or, $$I_d = \frac{E - V\cos\delta}{x_d} \qquad (5.26)$$

Again, resolving I_a and its components I_d and I_q along V-axis,

$$I_a\cos\phi = I_d\cos(90 - \delta) + I_q\cos\delta$$

or, $$I_a\cos\phi = I_d\sin\delta + I_q\cos\delta \qquad (5.27)$$

Substituting the values from Eqs (5.25), (5.26) and (5.27) in the expression for output power, we have

$$P_o = 3V I_a\cos\phi$$

$$= 3V(I_d\sin\delta + I_q\cos\delta)$$

$$= 3V\left[\frac{(E - V\cos\delta)}{x_d}\sin\delta + \frac{V\sin\delta}{x_q}\cos\delta\right]$$

$$= \frac{3\,VE}{x_d}\sin\delta - \frac{3V^2\sin\delta\cos\delta}{x_d} + \frac{3V^2\sin\delta\cos\delta}{x_q}$$

or, $$P_o = \frac{3\,VE}{x_d}\sin\delta - \frac{3V^2\sin 2\delta}{2x_d} + \frac{3V^2\sin 2\delta}{2x_q}$$

or, $$P_o = \frac{3\,VE}{x_d}\sin\delta + \frac{3V^2\sin 2\delta\,(x_d - x_q)}{2x_d x_q}$$

or, $$P_o = \frac{3\,VE}{x_d}\sin\delta + \frac{3V^2(x_d - x_q)}{2x_d x_q}\sin 2\delta \tag{5.28}$$

The above expression for output power contains two terms. The first term is the same as the one developed for cylindrical rotor type synchronous generator. The second term is due to the variation of reluctance in the air-gap in salient type synchronous generators. This part of the power developed is called the reluctance power. In case x_d is equal to x_q (no saliency) then the second term becomes zero.

The reluctance power or torque developed for a salient pole machine as a function of power angle or torque angle δ is shown in Fig. 5.58.

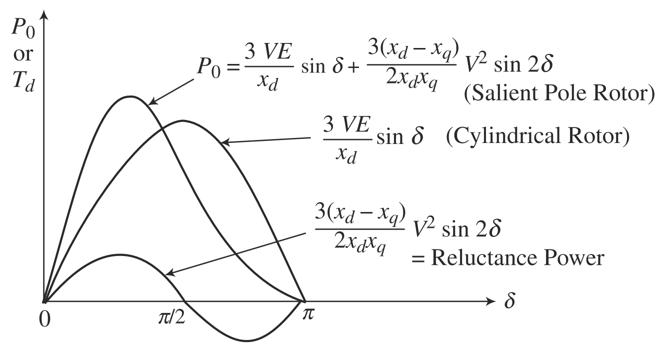


Fig. 5.58 *Power-angle characteristic of a salient pole synchronous generator*

This shows that a salient pole synchronous generator can generate more power than a cylindrical pole synchronous generator. The maximum torque is developed when the value of power angle δ is somewhat lass than 90 degrees. It is interesting to note that power is developed even when the excitation is reduced to zero, i.e., when E is zero.

Reluctance power, $$P_r = \frac{3(x_d - x_q)}{2\,x_d\,x_q}V^2\sin 2\delta$$

$$\text{Reluctance torque} = \frac{P_r}{\omega_s} = \frac{1}{2\pi n_s}\,\frac{3(x_d - x_q)}{2x_d\,x_q}V^2\sin 2\delta$$

where n_s is rps.

It is seen that reluctance power or reluctance torque is independent of excitation or excitation voltage, E. Even when the excitation is reduced to zero, reluctance torque is developed due to the salient type of poles.

EXAMPLE 5.21 *A 3-phase 50 MVA, 13 kV, star connected salient pole type alternator has x_d = 1.8 Ω and x_q = 1.2 Ω. The armature resistance is negligible. Determine at rated load and 0.8 p.f. lagging the following:*
(a) the voltage regulation; (b) power angle; (c) power developed

Solution

$$V \text{ per phase} = \frac{V_L}{\sqrt{3}} = \frac{13000}{1,732} = 7505 \text{ V}$$

$$I_a = \frac{50 \times 10^6}{\sqrt{3} \times 13000} = 2220 \text{ A}$$

p.f. = cos θ = 0.8, θ = 37°, sin θ = sin 37° = 0.6

If armature resistance is neglected,

$$\tan \delta = \frac{I_a\, x_q \cos\theta}{V + I_a\, x_q \sin\theta} = \frac{2220 \times 1.2 \times 0.8}{7505 + 2220 \times 1.2 \times 0.6} = \frac{2131.2}{9103} = 0.234$$

$$\delta = \tan^{-1} 0.234 = 13.5°$$

Taking V as the reference axis,

$$\bar{V} = V\angle 0° = 7505\ \angle 0°$$

$$\bar{I}_a = 2220\ \angle{-37°}$$

From the phasor diagram of Fig. 5.59.

$$I_q = I_a \cos(\theta + \delta)$$
$$= I_a \cos(37° + 13.5°)$$
$$= 2220 \cos 50.5°$$
$$= 1412 \text{ A}$$
$$I_d = I_a \sin(\theta + \delta)$$
$$= 2220 \sin 50.5°$$
$$= 1712 \text{ A}$$

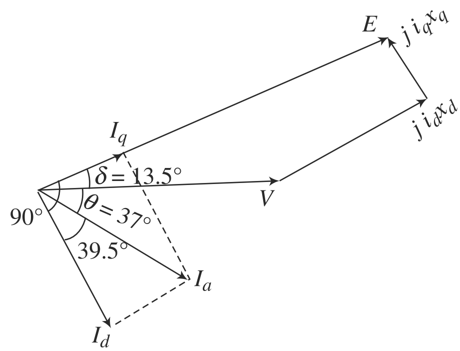


Fig. 5.59

With V as the reference phasor,

$$\bar{I}_q = I_q\ \angle{+\delta} = 1412\angle 13$$

$$\bar{I}_d = I_d\ \angle \theta + 39.5° = 1712\angle 76.$$

The generated induced voltage, E is expressed as

$$\bar{E} = V\underline{|0°} + jI_d x_d + jI_q x_q$$

$$= 7505\,\underline{|0°} + j\,1712 \times 1.8 + j\,1412 \times 1.2$$

$$= 7505 + j\,4775 = \sqrt{(7505)^2 + (4775)^2} = 8895 \text{ V}$$

(a) Voltage Regulation $= \dfrac{E - V}{V} \times 100 = \dfrac{(8895 - 7505)}{7505} \times 100 = 15.85$ percent

(b) Power angle, $\delta = 13.5°$

(c) Power developed, $P_d = P_o = 3\, V_{ph}\, I_{ph} \cos\phi$

$$P_d = \frac{3 \times 7505 \times 2220 \times 0.8}{10^6} \text{ MW}$$
$$= 39.98 \text{ MW}$$

Alternatively,

$$P_d = \frac{3\,V\,E \sin\delta}{x_d} + \frac{3(x_d - x_q)}{2\,x_d\,x_q} V^2 \sin 2\delta$$
$$= \frac{3 \times 7505 \times 8895 \times 0.2334}{1.8} + \frac{3(1.8 - 1.2)}{2 \times 1.8 \times 1.2}(7505)^2 \times 0.454$$
$$= 37.88 \text{ MW}$$

The difference is due to the value of δ which was calculated neglecting armature circuit resistance.

EXAMPLE 5.22 *A 400 V, 50 Hz alternator has x_d = 0.1 Ω and x_q = 0.08 Ω respectively. The armature resistance is small and can be neglected. The armature windings are delta connected. The line current supplied by alternator is 800 A at 0.8 power factor lagging. (a) calculate the voltage regulation. (b) Also calculate voltage regulation when x_d is considered equal to x_s, i.e., when the effect of saliency is neglected.*

Solution Since the windings are delta connected, the phase voltage is the same as line voltage. The phase current is $\frac{1}{\sqrt{3}}$ times the line current.

$$V_{ph} = 400 \text{ V}$$
$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{800}{1.732} = 462 \text{ A}$$

The powers factor $\cos\phi = 0.8$

$$\phi = \cos^{-1} 0.8 = 37°$$

We will first calculate the voltage regulation when $x_s = x_d = x_q$, i.e., saliency of the rotor is neglected.

We know, $E = V + I_a R_a + j\,I_a X_s$

When R_a is neglected

$$E = V + j\,I_a X_s$$

Taking V as the reference axis,

$$E = V + j\,I_a X_s$$
$$= 400 + j\,462\ \angle -37° \times 0.1$$
$$= 400 + j\,46.2\ \angle 37°$$
$$= 400 + j\,46.2\,(\cos 37° - j \sin 37°)$$
$$= 400 + j\,46.2 \times 0.8 - j^2\,46.2 \times 0.6$$
$$= 400 + 27.72 + j\,36.96$$

$$= \sqrt{(427.72)^2 + (36.96)} \angle \tan^{-1} \frac{36.96}{427.72}$$

or, $E = 429.3 \angle 5°$

$$\text{Regulation} = \frac{E - V}{V} \times 100$$

$$= \frac{(429.3 - 400)}{400} \times 100 = 7.32 \text{ percent}$$

Now we will calculate E for salient pole where $x_d = 0.1\ \Omega$ and $x_q = 0.08\ \Omega$

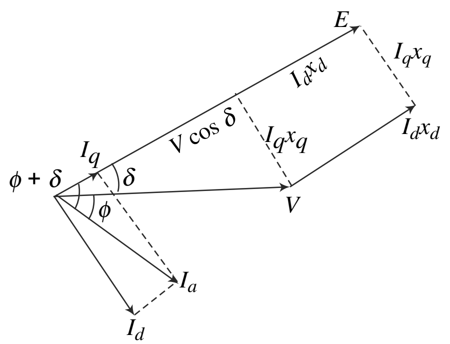


Fig. 5.60

From Fig. 5.60,

$$E = V \cos \delta + I_d x_d$$

$$I_d = I_a \sin (\phi + \delta) = 462 \sin (37° + 4.1°) = 303.7$$

$$\tan \delta = \frac{I_a x_q \cos \phi}{V + I_a x_q \sin \phi}$$

$$= \frac{462 \times 0.08 \times 0.8}{400 + 462 \times 0.08 \times 0.6}$$

$$= \frac{29.57}{422.2} = 0.07$$

$$\delta = 4.1°$$

$$E = V \cos \delta + I_d x_d = 400 \times 0.9974 + 303.7 \times 0.1 = 429.4$$

$$\text{Regulation} = \frac{E - V}{V} \times 100 = \frac{(429.4 - 400)}{400} \times 100 = 7.35 \text{ percent}$$

5.16 DETERMINATION OF x_d AND x_q FROM SLIP-TEST

For a cylindrical rotor type synchronous machine the synchronous reactance is the same along d-axis and q-axis, i.e., $x_d = x_q$.

For a salient pole type machine the value of x_d is greater than x_q due to variation in the air-gap in the polar-axis and inter-polar axis.

The direct and quadrature-axis synchronous reactances can be determined by conducting *slip-test*. The arrangement for slip-test is shown in Fig. 5.61. A reduced three-phase supply is given to the stator windings. The field winding on the rotor is kept open. The rotor is rotated by a primemover at a speed close to synchronous speed.

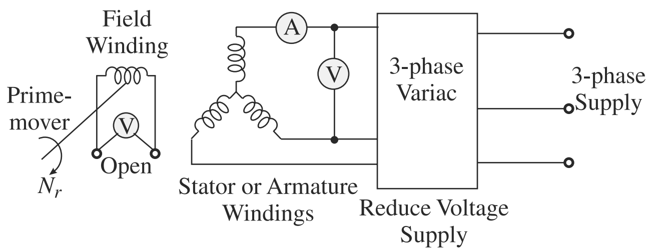


Fig. 5.61 *Arrangement for slip-test to determine x_d and x_q*

Because of giving a three-phase supply to the three-phase armature windings, a rotating magnetic field rotating at synchronous speed will be produced. The rotor is rotated at a speed slightly lower than synchronous speed. The relative velocity between the armature field and the field poles is ($N_s - N_r$).

When the stator field is aligned with the *d*-axis field poles, the effective reactance of the armature is x_d. When the stator field aligns with the quadrature axis, the effective armature circuit reactance is x_q. The current drawn by the armature therefore will vary cyclically being minimum on *d*-axis and maximum at *q*-axis. Along *d*-axis the reluctance offered to flux path is minimum and hence synchronous reactance is of higher value. This will result in minimum of current drawn by the armature winding as will be indicated by the ammeter connected on the line.

Along *d*-axis the flux linkage is maximum but the rate of change of flux linkage by the field winding is almost zero. Hence along d-axis, the emf induced in the field winding, as indicated by a voltmeter, will be zero. The emf induced in the field winding will be maximum along the quadrature axis. The emf induced *Ef* in the field winding will be at slip frequency, i.e., at *Sf* where *S* is the slip.

The low frequency emf induced in the field winding has been shown in Fig. 5.62(a). The position of *d*-axis and *q*-axis have also been indicated. The oscillogram of armature current has been shown in Fig. 5.62(b).

Because of the cyclic variation of armature current and the consequent voltage drop in the supply lines, the voltage, V across the armature terminals will also vary cyclically as has been shown in Fig. 5.62(c).

The values of x_d and x_q are calculated as

$$x_d = \frac{\text{Armature terminal voltage at minimum of armature current}}{\text{Minimum value of armature current}} = \frac{V_{\max}}{I_{\min}}$$

$$x_q = \frac{\text{Armature terminal voltage at maximum of armature current}}{\text{Maximum value of armature current}} = \frac{V_{\min}}{I_{\max}}$$

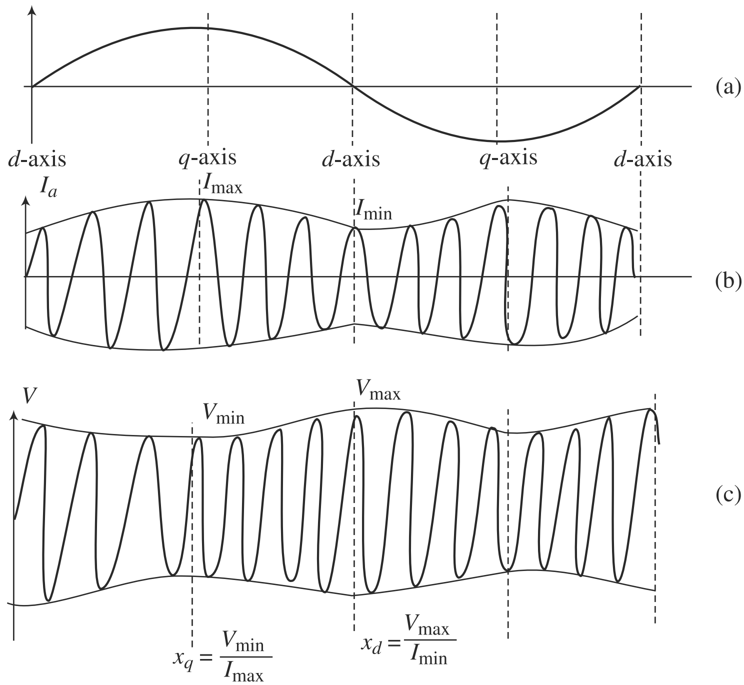


Fig. 5.62 *Oscillograms of (a) Voltage induced in the field winding at slip-frequency; (b) Armature current drawn by the stator windings; (c) Applied voltage*

The maximum and minimum values of current and voltage are to be recorded from the oscillogram taken the help of a double beam CRO fitted with a camera for recording of oscillograms. Alternately, the minimum and maximum values of ammeter and voltmeter readings be taken. In this experiment, slip has to be kept to a minimum so that maximum and minimum values of instrument readings can be taken conveniently. Otherwise, there will be too much of oscillations of the pointers of the meters and taking of correct reading may not be possible.

5.17 SYNCHRONOUS MACHINE WORKING AS A MOTOR

When a synchronous machine is just synchronised with the bus-bar, it is neither receiving nor delivering any power. The primemover driving the machine supplies only the no-load losses. Under this condition the induced emf in the synchronous machine will be exactly equal and opposite in direction to the bus-bar voltage. No current will be flowing through the armature winding.

Now if the primemover is removed, the rotor of the synchronous machine will fall back by some angle though rotating at synchronous speed. The phasor representing induced emf, E will lag v by an angle say α as shown in Fig. 5.63. A current I will be drawn by the stator windings due to the resultant voltage E_R. This current flowing through the synchronous impedance of the armature winding will lag the resultant voltage E_R by approximately 90°.

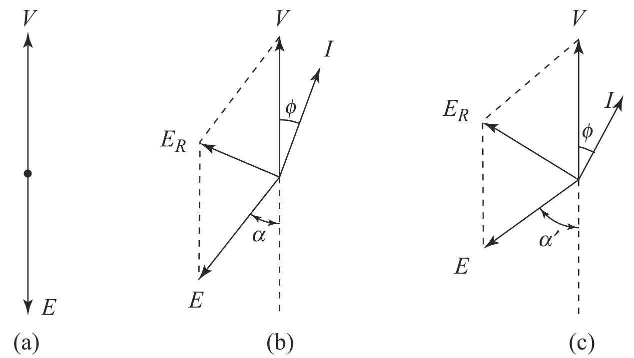


Fig. 5.63 *(a) Synchronous machine on bus-bar, (b) Synchronous machine receiving power from the bus -bar and working as a motor on no-load, (c) Synchronous machine working as a motor with some mechanical load on its shaft*

Now the synchronous machine is working as a synchronous motor on no-load, i.e., it is receiving power from the bus-bar. The magnitude of power supplied by the bus-bar per phase is $VI \cos \phi$ which is equal to the no-load losses. If now some mechanical load is applied on the synchronous motor, its rotor axis will lag by some more angle such that the power taken from the bus-bar is proportional to the mechanical output of the motor. Depending upon the magnitude of load, the rotor while rotating at synchronous speed will adjust its rotor axis such that the power drawn balances the power output.

5.18 HUNTING

A synchronous motor may be subjected to oscillations in speed when it is suddenly loaded or unloaded. The rotor speed changes momentarily until the torque angle adjusts itself to the new output requirement. If the load increases, the rotor slips backwards to an increased torque angle, while a load reduction causes the rotor to advance to a smaller torque angle position. But because of the moment of inertia, the rotor overshoots the final position, slowing down or speeding up more than the required value. Thus the rotor is subjected to periodic but momentary speed change while the rotor is attempting to adjust to a correct torque angle, the average speed of the motor remaining constant. This quick forward and backward motion of the rotor as it revolves at the average constant speed is called "hunting". The rotor is said to be hunting (i.e. in search of) for the correct torque angle in response to the changed loading condition.

Such an oscillation in speed produces undesirable current and torque pulsation. However, the squirrel cage winding made on the pole faces, that provides the motor with its starting torque, also dampens the oscillation in speed. Since, the damper winding is short-circuited in itself, there results a rotating mmf which in conjunction with the rotating field develops a damping torque, thus minimising oscillation. The damper winding remains ineffective as long as the speed is constant at the synchronous speed.

5.19 EFFECT OF CHANGE OF EXCITATION OF A SYNCHRONOUS MOTOR DRIVING A CONSTANT LOAD

When the load on a synchronous motor is constant, the input power $VI \cos \phi$ drawn from the bus-bar will remain constant. As the bus-bar voltage V is constant, $I \cos \phi$ will remain constant. Under this condition, effect of change of field excitation on the armature current, I drawn by the motor will be as follows:

When excitation is changed, the magnitude of induced emf E changes. The torque-angle α, i.e., the angle of lag of E from the axis of V remains constant as long as the load on the motor is constant.

Figure 5.64 shows the effect of increase and decrease of excitation on the magnitude and power factor of the current drawn by the motor. It is seen from Fig. 5.64(b) that when the excitation is increased the motor draws a leading current. Note that in the figure, $I \cos \phi$ which is equal to OD has been assumed constant. If the excitation is reduced, the motor will draw a more lagging current as shown in Fig. 5.64(c). From the above it can be said that at some value of excitation the motor will draw current at unity power factor which is shown in Fig. 5.64(d). The excitation corresponding to unity power factor current drawn by the motor is called normal excitation. Excitation lower than the normal is called under excitation. It is seen from Fig. 5.64 that over excitation causes leading power-factor current and under excitation causes lagging power-factor current drawn by the motor. It can also

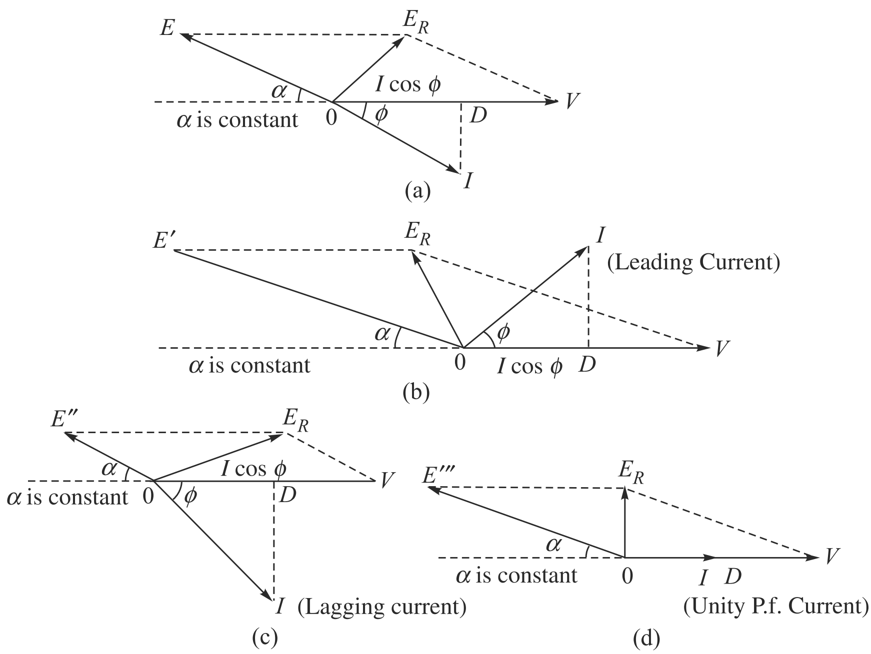


Fig. 5.64 *(a) Synchronous motor driving a constant load with an excitation emf E (b) Load remaining constant, excitation emf increased to E′ (c) Load remaining constant, excitation emf reduced to E″ (d) Load remaining constant, excitation emf increased to E‴ such that* current drawn is at unity power factor.

be observed that the magnitude of current at normal excitation is the minimum. The relation between excitation current and armature current at a particular load on a synchronous motor has been shown in Fig. 5.65.

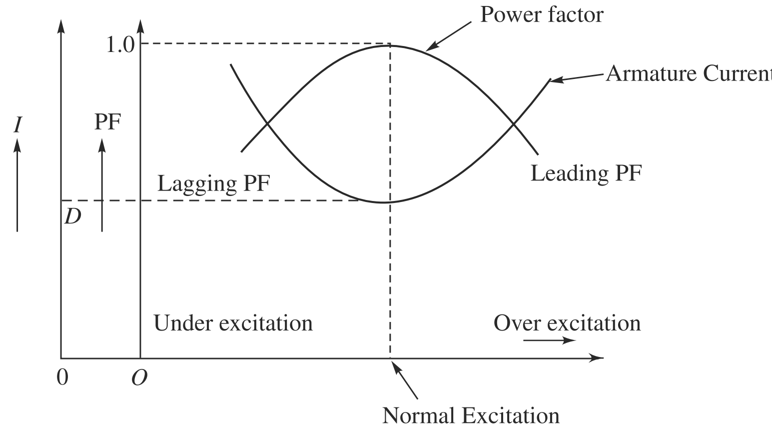


Fig. 5.65 *Effect of change of excitation on armature current and power factor of a synchronous motor*

In Fig. 5.65, at normal excitation, power factor has been shown as unity. The magnitude of armature current at this excitation is the minimum and is equal to *OD*. For excitation higher than the normal excitation, the magnitude of armature current will increase and the power factor will be leading. At excitation lower than the normal excitation, the magnitude of armature current will again increase but the power factor will be lagging as has been shown in the figure. The shape of the I versus I_f characteristic is similar to the letter *V* of the English alphabet and that is why this characteristic of synchronous motor is often referred to as synchronous motor *V*-curve. A series of *V*-curves will be obtained if the load on the motor is changed to say half-load, three-fourth-load, full-load, etc., and keeping the load constant at a particular value, excitation of the motor is varied.

Since an overexcited synchronous motor, also called a synchronous condenser, draws leading power factor current, an over excited synchronous motor can be used for power factor improvement.

5.20 SYNCHRONOUS CONDENSER

Most of the motors used in industries generally are three-phase induction motors. Induction motors have a full-load power factor of about 0.8 lagging. On lower loads, their power factor gets further reduced. In an industry since all the motors do not necessarily run on full-load, the overall power-factor of the load may be as low as 0.6 lagging. According to Electric Supply Authority regulation, the power factor of any industrial load is to be maintained to a certain minimum value, say, 0.85 lagging to avoid penalty in the rate of electricity charges. This is because for a given system voltage and power, the magnitude of current will reduce with increase in power factor. When the current is less, the $I^2 R$ losses would be less and hence the system will be more efficient. Moreover for a system with higher value of power factor the required rating of system equipment will decrease. It will also decrease the operating cost and increase the system capacity. A synchronous motor without a shaft extension, i.e., without any load on it's shaft, used exclusively for power factor improvement in a system, is called a synchronous condenser. Since it drives no load, it develops little torque and hence has a very light frame. Installation of overexcited synchronous motors in parallel with the other loads will improve the power factor of the line. Generally, the motor is run under load and by overexcitation can provide some power factor improvement of the system as also drive a mechanical load.

EXAMPLE 5.23 *An industrial plant has a load of 800 kW at of power factor of 0.8 lagging. It is desired to instal a synchronous motor to deliver a load of 200 kW and also serve as a synchronous condenser to improve the overall power factor of the plant to 0.92. Determine the kVA rating of the synchronous motor and its power factor. Assume that the synchronous motor has an efficiency of 90 per cent.*

Solution Power factor, cos ϕ of the existing load is 0.8 lagging.

$$\therefore \quad \cos\phi = 0.8$$

$$\phi = 37°$$

As in Fig. 5.66(a), $\tan 37° = \dfrac{\text{kVAr}_1}{\text{kW}_1}$

or, $\text{kVAr}_1 = 800 \times 0.75 = 600$

A synchronous motor delivering a load of 200 kW is connected to the system as shown in Fig. 5.66(b).

$$\text{Input to the synchronous motor} = \frac{\text{Output}}{\text{Efficiency}}$$

$$= \frac{200}{0.9} = 222.2 \text{ kW}$$

Total load on the system = 800 kW + 222.2 kW = 1022.2 kW

Now overall power factor of the load is to be raised to 0.92 lagging

i.e. $\cos\phi = 0.92$

or $\phi = 23°$

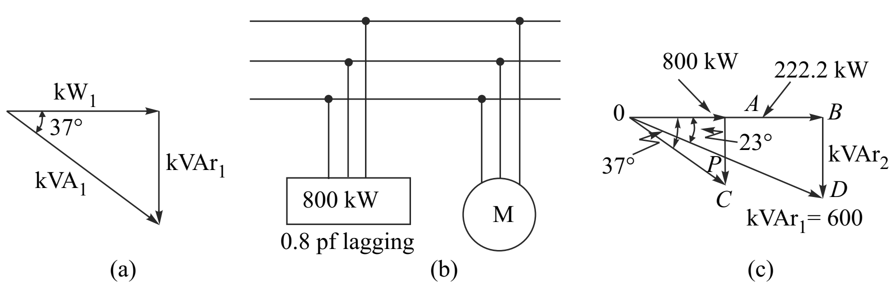


Fig. 5.66

As in Fig. 5.66(c)

$$\tan 23° = \frac{kVAr_2}{1022.22}$$

or $$kVAr_2 = 1022.2 \times 0.424 = 433.4$$

The lagging kVAr neutralised by the synchronous condenser are given by

$$= kVAr_1 - kVAr_2$$
$$= 600 - 433.4 = 166.6$$

Thus, for the synchronous condenser,

$$\text{kW input} = 222.2 \text{ kW}$$

Leading kVAr supplied by the motor = 166.6

$$\tan \phi = \frac{kVAr}{kW} = \frac{166.6}{222.2} = 0.75$$

$$\phi = 37°$$

Power factor of the synchronous motor,

$$pf = \cos 37° = 0.8 \text{ leading}$$

For kVA rating of the synchronous motor,

$$k\,V\,A \cos \phi = kW$$

or, $$kVA = \frac{kW}{\cos\phi} = \frac{222.2}{0.8} = 277.75$$

5.21 STARTING OF SYNCHRONOUS MOTORS

We have so far discussed the behaviour of a synchronous machine as a motor when already connected to the bus-bar. The machine was driven by a primemover to synchronous speed and then synchronised. If three-phase supply is given to the stator phases of a stationary synchronous machine with the rotor excited, no steady starting torque will be developed. Instead a sinusoidally time-varying torque is developed, the average value of which is zero. That is why a synchronous motor as such is not self-starting. The starting of a synchronous motor from its stationary condition can be achieved by the following methods:

Starting with the Help of Damper Winding To enable the synchronous machine to start independently as a motor, a damper winding is made on pole face slots. Bars of copper, aluminium, bronze or similar alloys are inserted in slots made on pole shoes as shown in Fig. 5.67. These bars are short-circuited by end-rings on each side of the poles. Thus these short-circuited bars form a squirrel-cage winding. On application of three-phase supply to the stator, a synchronous motor

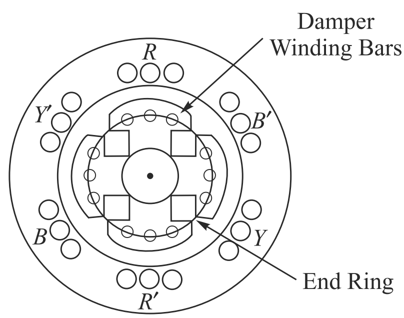


Fig. 5.67 *Damper winding made on pole faces of a synchronous machine*

with damper winding will start as a three-phase induction motor and rotate at a speed near to synchronous speed. Now with the application of dc excitation to the field windings, the rotor will be pulled into synchronous speed since the rotor poles are now rotating at only slip-speed with respect to the stator rotating magnetic field. To limit the starting current drawn by the motor, a reduced voltage may be necessary, to apply for high capacity synchronous motors. Reduced voltage can be applied through an auto-transformer or through a star-delta starter. During starting period before the application of dc excitation, the field windings are kept closed through a resistor. DC is supplied from an independent source or through the armature of the dc exciter, namely the dc generator carried on the shaft extension of the synchronous motor. If this is not done, a high voltage induced in the dc winding during starting period will strain the insulation of the field winding.

Since starting of the motor is done as an induction motor, the starting torque developed is rather low and, therefore, large capacity motors may not be able to start on full load.

Starting with the Help of a Separate Small Induction Motor In this method, the synchronous motor is brought to synchronous speed with the help of a separate induction motor. The number of poles of the induction motor should be less than the number of poles of the synchronous motor to enable it to rotate at the synchronous speed of the synchronous motor. In this method, however, the motor will have to be synchronised with the bus-bar.

Starting by Using a dc Motor Coupled to the Synchronous Motor The dc motor drives the synchronous motor and brings it to synchronous speed. The synchronous machine is then synchronised with the bus-bar. Once in parallel, the synchronous machine will work as a motor, driving the dc machine which may be used to work as a generator. The field current of the dc generator is to be increased so that its generated emf is more than the dc bus-bar voltage.

Out of the three methods mentioned, starting by using damper winding is the most commonly used method since no auxiliary machine is required.

5.22 CAPABILITY CURVE OF A SYNCHRONOUS GENERATOR

Capability curve of a synchronous generator shows the bounds or limits within which it can be operated safely.

The bounds or limits are defined by the MVA rating of the generator; the capacity of the prime-mover driving the generator which is related to the MW loading of the generator; the limit of increase of field current without overheating the field winding; and the limit of increase of load angle δ.

Capability curves are also called operating chart or capability chart.

To draw the capability curve, the phasor diagram of the generator on load is drawn as has been shown in Fig. 5.68. For simplicity the armature resistance is neglected.

As in Fig. 5.68(b),

$$BD = I_a X_s \,;\; BC = I_a X_s (\cos (90 - \phi) = I_a X_s \sin \phi$$

$$CD = I_a X_s \sin (90 - \phi) = I_a X_s \cos \phi$$

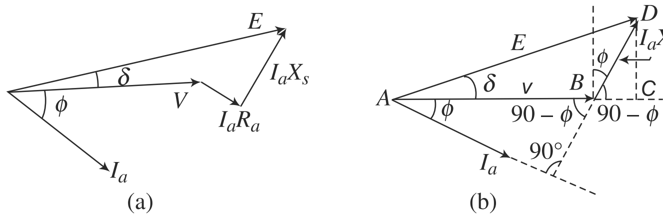


Fig. 5.68 *Phasor diagram of a synchronous generator on load (a) Considering armature resistance; (b) Neglecting armature resistance*

Now, by multiplying all sides of the phasor diagram by $\frac{3V}{X_S}$, we redraw the phasor diagram as show in Fig. 5.69.

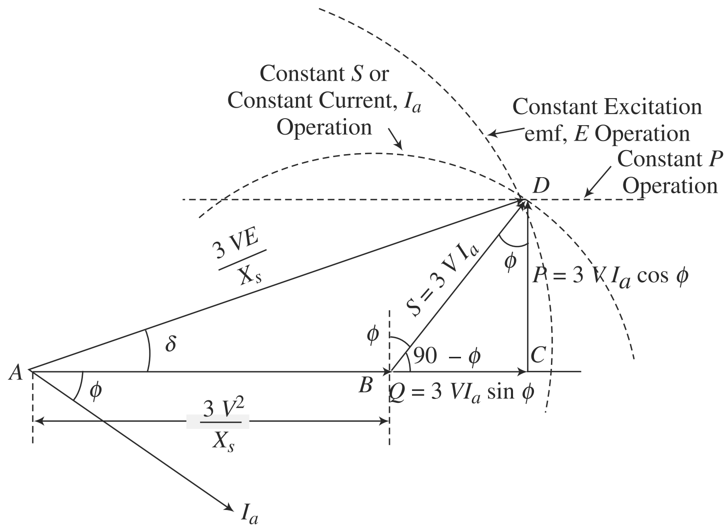


Fig. 5.69 *Phasor diagram of a synchronous generator on load showing the apparent power, S; active power, P, and reactive power, Q in the phasor diagram*

As shown in Fig. 5.69,

$BD = S = 3\ VI_a$, equal to 3-phase kVA or MVA rating;

$CD = P = 3\ VI_a \cos\phi$, equal to 3-phase kW or MW rating;

$BC = Q = 3\ VI_a \sin\phi$, equal to 3-phase kVAR or MVAR rating

$$BD = \sqrt{BC^2 + CD^2}$$

i.e., $$S = \sqrt{P^2 + Q^2}$$

Q is positive for lagging power factor load and is negative for leading power factor load.

By referring to Figure 5.69 we now examine the limits of operation for (i) constant value of S, i.e., constant value of load current I_a (since $S = 3\ VI_a$, with V constant, I_a is constant); (ii) constant value of output power, P; and (iii) constant value of excitation emf E.

(i) Constant *S* Operation For constant armature current, the value of apparent power $S = 3\ V\ I_a$ will remain constant and hence the locus of S will be a circle of radius $S = BD$ with the centre at B.

(ii) Constant *P* Operation Constant real power $P = 3\ VI_a \cos\phi = CD$ will give rise to locus of P parallel to the line BC touching the point D.

(iii) Constant *E* Operation Constant excitation means constant value of excitation emf, E. Constant excitation operation will mean constant value of AD = $\dfrac{3\ VE}{X_S}$. This will give rise to a locus of a circle with radius AD with centre at A.

These three limits of constant operation have been shown by dotted curves in Fig. 5.69.

The maximum limit of power angle δ is 90 degrees. Keeping some safety limit the maximum operating limit of δ is kept somewhat less than 90 degree.

Now, by considering the maximum operating limits of armature current I_a; the real power P which is also related to the maximum power of the prime-mover that is the driving turbine; the maximum limit of excitation current inducing emf, E; and the safe limit of operation of power angle, δ which is usually 10% less that 90 degree, the capability curve can be drawn as shown in Fig. 5.70.

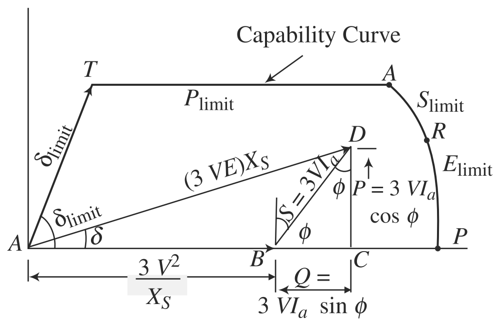


Fig. 5.70 *Capability curve PRATAP, of a synchronous generator*

Between A and T of the capability curve the operation is limited by the maximum permissible limit of power angle, δ operation. Between T and A the operation is limited by the maximum permissible limit of the primemover operation. Between A and R the operation is limited by the maximum limit of apparent power in kVA or MVA operation which is dependent upon the maximum allowable armature current without the generator getting overheated. Between R and P or the capability curve, the operation of the generator is limited by the maximum permissible value of field current.

Thus the capability curve PRATAP represents the operating zone or area of the synchronous generator. Within this limit, the operation is safe with respect to overheating of the generator and it's stability.

Any operating point on this operating area can be located to find the corresponding values of S, P, Q, E, δ, I_a, and power factor.

Similarly, capability curve for synchronous machine working as a motor can also be drawn.

5.23 APPLICATIONS OF SYNCHRONOUS MACHINES

Synchronous generators are almost universally used to generate electricity for commercial purpose. Large capacity generators are being designed due to increased demand for electric power. The design of large capacity generators has been possible due to the availability of better insulating materials and the provision of better cooling system. For emergency power supply, however, small generators driven by diesel engines are made available. These are popularly called Gen-sets.

The advantages of synchronous motors over other types of motors are:

(a) Easy Control of Power Factor An over excited synchronous motor drawing a leading current from the bus-bar may be used to raise the overall power factor of the energy supplied by the system to the load.

Synchronous motors sometimes are run on no-load with over excitation for improving the voltage regulation of a transmission line. Such over excited synchronous motors are referred to as *synchronous capacitors* or *synchronous condensers.*

(b) Constant Speed The speed of a synchronous motor is constant and is independent of load. The motor can be used to drive another alternator to generate electricity at a different frequency than the supply frequency.

The disadvantages of synchronous motors are higher cost, necessity for a dc excitation source and greater initial and maintenance cost.

MODEL QUESTIONS

Short-Answer-Type Questions

5.1 Draw a neat sketch showing the various parts of a synchronous machine. State the type of synchronous generator used in a hydroelectric power station.

5.2 Explain the constructional details of a synchronous machine, giving reasons for making two different types of rotors.

5.3 Explain why salient type rotors are not used in alternators driven by steam turbines.

5.4 Write the expression, showing the relationship between speed, frequency and number of poles of a synchronous machine. The speed of rotation of the turbine driving an alternator is 166.7 rpm. What should be the number of poles of the alternator if it is to generate voltage at 50 Hz? (Ans. 36 poles)

5.5 Explain the advantages of having a rotating field system rather than a rotating armature system in a synchronous machine.

5.6 Show with the help of a diagram a simple three-phase, 4-pole winding made on stator slots.

5.7 Explain the different excitation systems of large synchronous machines.

5.8 Explain the need for using damper winding in a synchronous machine.

5.9 Explain why distributed windings are preferred over concentrated windings in making armature winding of synchronous machines.

5.10 Drive emf equation for an alternator. Also derive expressions for distribution factor and pitch factor.

5.11 Explain why efforts are made to generate sinusoidal induced emf in an alternator.

5.12 Explain distribution factor and pitch factor used in the emf equation of a synchronous machine.

5.13 Explain how the waveshape of the induced emf in an alternator can be made more towards a sinewave by using distributed winding and short-pitch coils.

5.14 Explain the effect of distribution of winding and use of short-pitch coil on the magnitude and waveshape of the induced emf of an alternator.

5.15 Draw the open-circuit and short-circuit characteristics of a synchronous generator. Explain the shape of the characteristics.

5.16 Explain the effect of armature flux on the main field flux of a synchronous generator at (a) unity power-factor load; (b) zero lagging power-factor load; and (c) zero leading power-factor load.

5.17 Explain the term *armature reaction*. Explain armature reaction at lagging power-factor load in a synchronous generator.

5.18 Draw the phasor diagram of a synchronous generator on load. Explain the meaning of synchronous reactance.

5.19 Define voltage regulation of an alternator. Explain synchronous impedance method of determining regulation of an alternator.

5.20 Explain why the regulation of large alternators are determined by indirect methods. Explain why the value of regulation calculated by synchronous impedance method is more than the actual value.

5.21 Explain why the value of regulation of an alternator is negative under capacitive loading.

5.22 Explain why synchronous machines are designed to have a high ratio of armature reactance to resistance.

5.23 State the need for parallel operation of alternators. What are the conditions for parallel operation of three-phase alternators?

5.24 State under what conditions an alternator is said to be floating on the bus-bar. What should be done to cause an alternator to share load?

5.25 Explain the effect of change of excitation of a synchronous generator connected to an infinite bus-bar.

5.26 Explain three-lamp method of synchronising an alternator with the bus-bar. What should be done to cause an alternator to share load after synchronisation?

5.27 Explain why a synchronous motor does not have starting torque. Explain one method of starting a synchronous motor.

5.28 Explain the effect of change of excitation of a synchronous motor on its armature current.

5.29 An overexcited synchronous motor is called a synchronous condenser, explain.

5.30 Explain the operation of a synchronous generator under constant load and variable excitation condition.

5.31 Using two reaction model, develop an expression for output power of a salient pole synchronous generator.

5.32 Draw and explain the power angle characteristic of a synchronous generator.

5.33 Explain how slip test is conducted to find x_d and x_q.

5.34 Draw and explain the capability curve of a synchronous generator.

5.35 Explain the effect of change of excitation of a synchronous motor on its power factor.

5.36 State two important applications of synchronous motor giving reasons.

Numerical Problems

5.37 A 2-pole, three-phase alternator stator has a total of 270 armature conductors on it. The total flux per pole is 1.8×10^6 lines. If the frequency of the generated voltage is 50 Hz, find the induced emf per phase. (Ans. 179.8 V)

5.38 The stator of a three-phase synchronous machine is wound for four poles and has a double layer winding wound in 48 slots. Calculate the distribution factor. (Ans. 0.96)

5.39 A three-phase 4-pole synchronous generator has a double layer winding having four turns per coil placed in a total of 48 slots. If the flux per pole of the generator is 2×10^6 lines and the speed of the rotor is 1500 rpm, calculate the magnitude of generated voltage per phase. (Ans. 272.8 V)

5.40 Calculate the pitch factor of an alternator, whose armature is wound with coils of 9/10 coil pitch. (Ans. 0.98)

5.41 A 6-pole, three-phase, 50 Hz alternator has 12 slots per pole and four conductors per slot. The winding is of five-sixth pitch. The flux per pole is 1.5 Wb. The armature coils are all connected in series. The winding is star connected. Calculate the induced emf per phase. (Ans. 14792 V)

5.42 A three phase, 200 kVA, 400 V, 50 Hz alternator has per phase armature resistance and synchronous reactance of 0.1 and 1.2 Ω respectively. Determine induced emf when the machine is delivering rated current at a load power-factor of unity. Draw the phasor diagram also. (Ans. 748 V)

5.43 A three-phase, 10 kVA, 50 Hz alternator has per phase armature resistance and synchronous reactance of 0.5 and 2.5 Ω respectively. Calculate the generated voltage corresponding to (a) full-load unity power factor; (b) full-load 0.8 lagging power factor. (Ans. 417 V, 449.3 V)

5.44 A 500 kVA, 1.1 kV, 50 Hz, three-phase star-connected alternator has an effective armature resistance of 0.2 Ω per phase. The synchronous reactance per phase is 1.5 Ω. Find the full-load voltage regulation at (a) 0.8 lagging power-factor, and (b) 0.8 leading power-factor. Draw phasor diagrams in each case. (Ans. 50%, – 11%)

Multiple-Choice Questions

5.45 A 4-pole, 1200 rpm alternator win generate emf at

(a) 50 Hz (b) 40 Hz (c) 60 Hz (d) 25 Hz

5.46 The span for a full-pitch coil wound for six poles is

(a) 180° mechanical (b) 90° mechanical

(c) 60° mechanical (d) 45° mechanical

5.47 The pitch factor for a two-thirds short-pitch coil is
(a) 0.5 (b) 0.66 (c) 0.866 (d) 0.707

5.48 The armature flux opposes the main field flux when the load power factor is
(a) unity (b) zero-lagging (c) 0.8 lagging (d) zero leading

5.49 The armature flux helps the main field flux when the load power factor is
(a) unity (b) zero-lagging (c) 0.8 lagging (d) zero leading

5.50 A commercial alternator has
(a) rotating armature and stationary field
(b) stationary armature and rotating field
(c) both armature and field rotating
(d) both armature and field stationary

5.51 The stator core of a synchronous machine is made up of laminated sheet to
(a) increase the magnitude of flux produced
(b) make the machine lighter in weight
(c) minimise the eddy current loss
(d) minimise the hysteresis loss

5.52 In alternators damper windings are used to
(a) reduce eddy current loss
(b) prevent hunting
(c) make the rotor dynamically balanced
(d) reduce armature reaction

5.53 A 4-pole, 50 Hz synchronous machine runs at
(a) 750 rpm (b) 1500 rpm (c) 3000 rpm (d) 1440 rpm

5.54 The magnitude of the resultant magnetic field produced by a three-phase current flowing through a three-phase winding is equal to
(a) the maximum value of flux due to any one phase
(b) 1.5 times the maximum value of flux due to any one phase
(c) half the value of maximum flux due to any one phase
(d) twice the maximum value of flux due to any one phase

5.55 Voltage regulation of an alternator may be negative when
(a) the load power factor is lagging
(b) the load power factor is leading
(c) it is loaded beyond its full-load capacity
(d) the machine is run at very low loads

5.56 Pitch factor for 5/6 short pitch coil is
(a) 0.966 (b) 0.833 (c) 1.0 (d) 3.454

5.57 Distribution factor for a winding having 3 slots/pole phase and as lot angle of 20° is
(a) 0.96 (b) 1.0 (c) 0.5 (d) 0.707

5.58 Armature reaction in an electrical machine is the effect of
(a) armature flux on the main field flux
(b) heat produced on the armature windings
(c) armature current on the output
(d) armature flux on the output

5.59 An infinite bus-bar should maintain
(a) infinite frequency and infinite voltage
(b) constant frequency and constant voltage
(c) constant frequency but variable voltage
(d) variable frequency and variable voltage.

5.60 The speed regulation of a synchronous motor is
(a) unity (b) zero
(c) infinity (d) always less than one

5.61 Synchronous motors are to be used in situation where
(a) the load is constant
(b) the load is required to be driven at very high speeds
(c) the load is to be driven at constant speed
(d) the starting torque requirement of the load is very high

True or False

5.62 State whether for a synchronous machine the following statements are true or false:

(a) Salient pole rotor construction is used in alternators driven by steam turbines
(b) Cylindrical type rotor construction is used in alternators used in thermal power stations
(c) The synchronous speed for an eight-pole 50-Hz alternator is 1500 rpm
(d) The magnitude of induced emf is increased because of using distributed winding
(e) Use of short-pitch coils in the armature winding reduces the magnitude of the induced emf
(f) Armature reaction flux opposes the main field flux under inductive loading of an alternator
(g) The voltage regulation of an alternator can be negative under capacitive loading
(h) An underexcited synchronous motor is often called a synchronous condenser
(i) For starting a synchronous motor a star-delta starter is used
(j) To cause an alternator share more load while running in parallel with other alternators, its excitation should be increased
(k) Voltage regulation of an alternator will always be positive
(l) Frequency of the emf induced in the armature conductors of an alternator is inversely proportional to its number of poles
(m) Frequency of the emf induced in the armature conductors of an alternator is directly proportional to the speed of the rotor
(n) In an alternator the load power factor does not affect armature reaction
(o) Armature reaction at zero leading power factor loads causes an increase in the resultant air-gap flux
(p) If one phase of a three-phase alternator is synchronised with the bus-bar, the other two phases are automatically synchronised
(q) Load sharing by an alternator running in parallel with other alternators is affected by change of its excitation
(r) Power factor of a synchronous motor can be varied by change of its excitation

(s) The speed of synchronous motor does not vary with load
(t) The rating of an alternator is generally expressed in kW
(u) Synchronous motors can be used for power factor improvement of a system.

Answers

5.45 (b)	5.46 (c)	5.47 (c)	5.48 (b)
5.49 (d)	5.50 (b)	5.51 (c)	5.52 (b)
5.53 (b)	5.54 (b)	5.55 (b)	5.56 (a)
5.57 (a)	5.58 (a)	5.59 (b)	5.60 (b)
5.61 (c)			

5.62

(a) False;	(b) True;	(c) False;	(d) False;
(e) True;	(f) True;	(g) True;	(h) False;
(i) False;	(j) False;	(k) False;	(l) False;
(m) True;	(n) False;	(o) True;	(p) False;
(q) False;	(r) True;	(s) True;	(t) False;
(u) True;			

LABORATORY EXPERIMENTS

EXPERIMENT 5.1 *Determination of the magnetisation characteristics of an alternator, (a) at no-load rated speed, (b) at no-load half-rated speed, and (c) at full-load (non-inductive load) rated speed.*

Objectives

(a) To verify that the generated voltage in an alternator is proportional to the speed and excitation current

(b) To verify that the terminal voltage of an alternator or resistive load is less than its no-load voltage.

Brief Theory (a) The magnitude of induced emf per phase of an alternator is

$$E = 4.44\, f\, \phi\, TK_p K_d \text{ V}$$

In the above formula

(i) The frequency of the induced emf can be expressed as

$$f = \frac{PN}{120}$$

where, P is the number of poles, and N is the speed of the rotor in rpm.

(ii) Flux, ϕ depends on the magnitude of the magnetising current, i.e., the current flowing through the field winding. Thus,

$$\phi \propto I_f \quad \text{or} \quad \phi = kI_f$$

The above relation is true to the stage of saturation of the magnetic circuit of the machine. After saturation stage any further increase in I_f does not give rise to appreciable increase in the flux produced.

Substituting the values of f and ϕ in the emf equation,

$$E = 4.44\, \frac{PN}{120}\, k\, I_f\, TK_p\, k_d \text{ V}$$

In the above expression, all terms except I_f and N are constant for a particular machine. Therefore,

$$E = K_1 I_f N \text{ V}$$

Therefore, the induced emf of an alternator varies in direct proportion to the field current and the speed. Further, if speed is kept constant, emf will vary in direct proportion to the field current and if field current is kept constant, emf will vary in direct proportion to the speed.

The relation between the field current and the induced emf when plotted on a graph paper gives the magnetisation characteristic or no-load characteristic of an alternator. A typical magnetisation characteristic of an alternator is shown by the curve *A* in Fig. 5.71(a). Similar magnetisation characteristic can be drawn at different rotor speeds. For example, if the speed is reduced by 50 per cent and kept constant at that value and the readings of induced emf are taken for various value of field current, another characteristic as shown by curve *B* of Fig. 5.71(a) can be plotted.

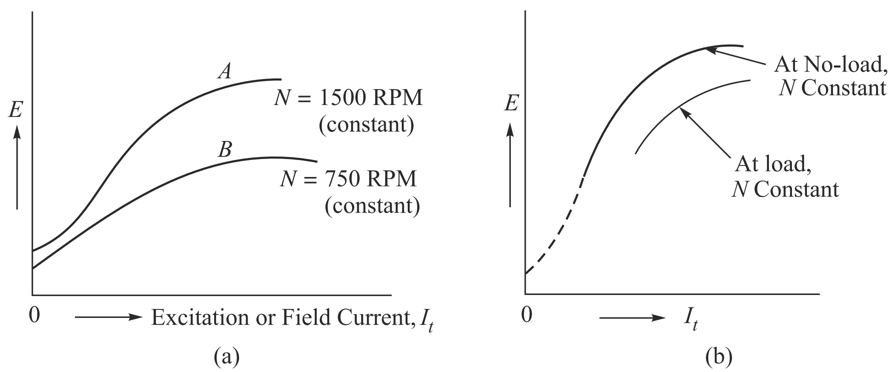


Fig. 5.71 *(a) Magnetisation characteristics of an alternator (b) Effect of applying load on the terminal voltage of an alternator*

In Fig. 5.71(a) it is seen that the characteristics do not start from the origin but a little above it. This shows that even when $I_f = 0$, the machines develops some induced emf. This is due to residual magnetism of the field poles.

To determine the effect of load on the terminal voltage, two sets of reading of terminal voltage corresponding to a particular value of field current are to be taken, one when there is no load and-one when there is load connected across the output terminals, the speed of the rotor remaining constant in both the cases. For each set of reading the magnitude of load current is to be kept constant. Such a characteristic is shown in Fig. 5.71(b).

The reasons for drop in terminal voltage of an alternator when loaded are voltage drop in the armature winding and armature reaction effect.

Circuit Diagram Make connections as per connection diagram shown in Fig. 5.72.

Apparatus Required Synchronous generator coupled with a dc motor, starter for de motor, field regulating resistance for dc motor, ammeter-moving coil type, tachometer, loading rheostat, ammeter and voltmeter-moving iron type.

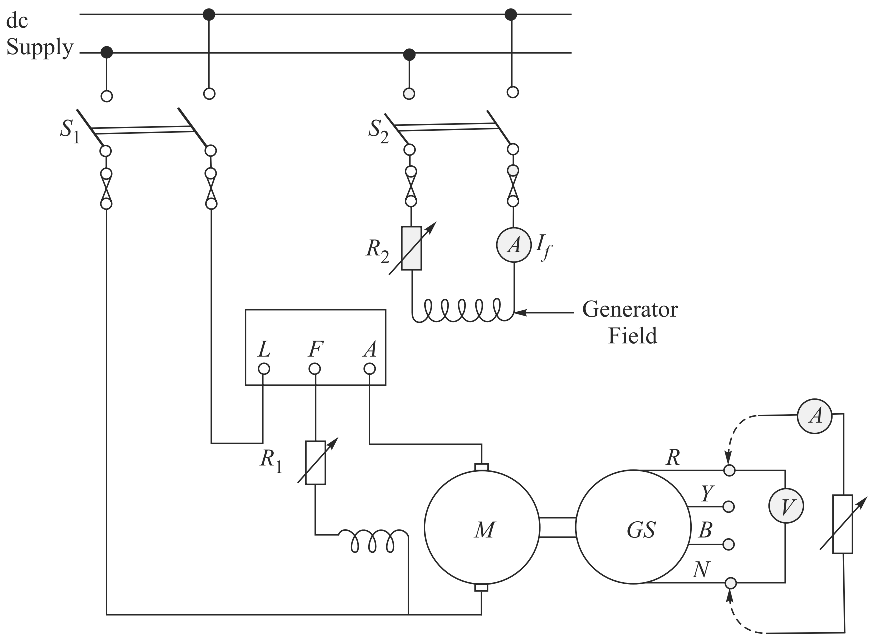


Fig. 5.72 *Connection diagram for determining the magnetisation characteristics of an alternator*

Procedure

1. After making proper connections, start the dc motor with the help of starter and bring the speed of the set-up to the rated speed. A little adjustment with the help of field regulating rheostat R_1 may be necessary. The switch S_2 of the alternator field circuit should remain open. Measure the value of induced emf when $I_f = 0$. The lowest range of the voltmeter should be used as the value of induced emf will be a few volts only.
2. Close the field circuit of the alternator with the help of S_2 with full value of R_2 (the value of R_2 should be high, of the order of say 500 Ω). Note the value of I_f and corresponding induced emf E. Take a good number of readings up to 120 per cent of the rated voltage of the alternator (i.e., if the rated voltage is 400 V, take readings up to say 480 V). In each case the speed should be kept constant. Take readings of E for increasing values of I_f only. Do not increase and then decrease I_f.
3. Starting from zero excitation, take another set of readings of I_f and E but at a lower speed. To reduce the speed of the set you may need to connect a resistor in the armature circuit of the dc motor.
4. Now keeping the speed of the set constant at rated value, connect a load on the alternator and take readings of terminal voltage against excitation current. At each stage the load current is to be kept constant.
5. Plot characteristics as follows
   (a) E versus I_f at rated N with no-load;
   (b) E versus l_f at less than rated speed with no-load;
   (c) E versus I_f at rated N with a constant load.

Observations and Results The readings should be recorded in a tabular form as shown below:

Run I

Sr. No.	N = N Rated	I_f	E	
			at no-Load	on Load
Take 5–6 readings				

Run II

Sr. No.	N = Less than Rated Speed	I_f	E at no-Load
Take 5–6 readings			

Questions Answer the following questions in your report:

1. Explain the shape of the magnetization characteristic of the alternator as drawn by you.
2. Explain why the magnitude of induced emf in an alternator is dependent upon
   (a) speed of the rotor,
   (b) the magnitude of field current.
3. State the reasons for drop in terminal voltage of an alternator when loaded.

Experiment 5.2 *Determination of excitation required to maintain constant voltage in an alternator.*

Objective To determine the relationship between field current in an alternator for constant terminal voltage and speed.

Brief Theory When an alternator is loaded, its terminal voltage changes due to: (a) voltage drop in the armature winding, and (b) due to armature reaction effect. The effect of armature reaction on terminal voltage is also dependent upon the nature of the load. To keep the terminal voltage/constant when load is connected across the terminals, the field current will have to be changed. The nature of graph showing the amount of excitation required to maintain constant voltage in an alternator at different load currents is shown in Fig. 5.73.

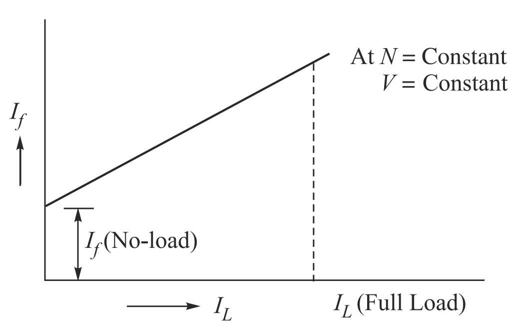


Fig. 5.73 *Excitation required to maintain constant terminal voltage in an alternator when loaded*

From the characteristic shown in figure, by knowing the value of I_f at no-load and at full-load, the rating of the field regulating resistance can be calculated thus:

Let V_f is the field applied voltage,

r_f is the field winding resistance,

I_{f_1} is the field current at no-load,

I_{f_2} is the field current at full-load.

At no-load and at full-load, field circuit resistances (field winding resistance plus the resistance of the regulator) are respectively

$$\frac{V}{I_{f_1}}\ \Omega \text{ and } \frac{V}{I_{f_2}}\ \Omega$$

By subtracting the field winding resistance from each of the above values, the rating of the field regulating resistance can be found out. The two extreme ranges of field regulating resistance are:

$$\left[\frac{V}{I_{f_1}} - r_f\right]\Omega,\ I_{f_2}A \qquad \left[\frac{V}{I_{f_2}} - r_f\right]\Omega,\ I_{f_2}A$$

Note that $\left[\frac{V}{I_{f_1}} - r_f\right]$ *is greater than* $\left[\frac{V}{I_{f_2}} - r_f\right]$ *as* I_{f_1} *is less than* I_{f_2}.

The values when calculated may be like: 150 Ω, 0.5 A and 50 Ω., 5 A.

From the above it can be concluded that we need a single value regulating resistance of 150 Ω, 5 A rating or a graded resistance of 150 Ω having 5 A rating at the lower side and 0.5 A at the higher side.

Circuit Diagram See Fig. 5.74.

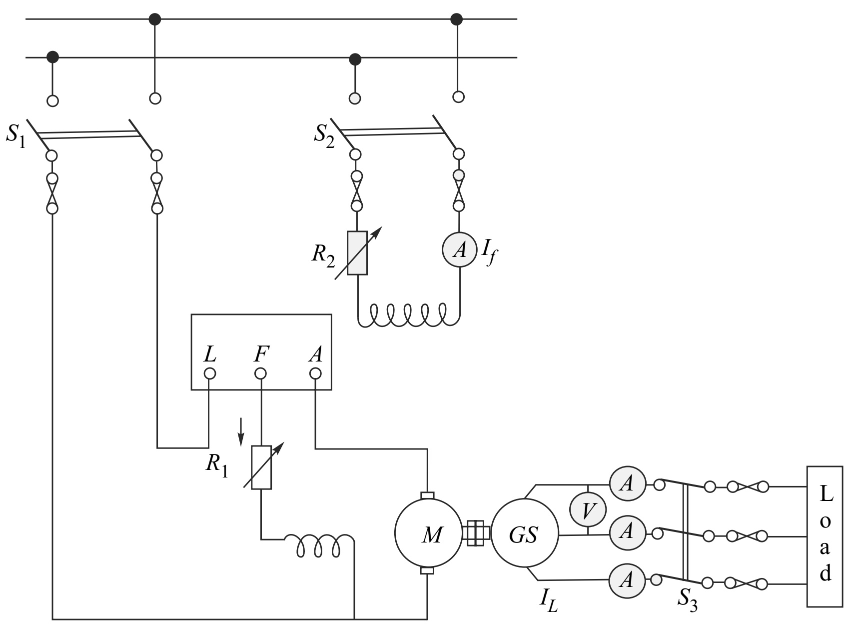


Fig. 5.74 *Connection diagram for determining excitation required to maintain constant terminal voltage in an alternator*

Apparatus Required Synchronous generator coupled with a dc motor, starter for dc motor, field regulating rheostat (two), Tachometer, Ammeter-moving coil type, Loading resistance-three phase, Voltmeter and three ammeters-moving iron type.

Procedure

1. Make connections as shown in Fig. 5.74. Start the dc motor with the help of the starter. Adjust the speed of the set to the rated speed. Close the alternator field circuit switch S_2, keeping the load switch S_3 open. Adjust value of R_2 such that rated voltage is induced across the alternator terminals. Record the value of I_f and V. This reading corresponds to $I_L = 0$.
2. Now close the load switch and load the alternator step by step. At each step of loading, keep the value of terminal voltage and speed constant. Readings should be taken up to rated load of the alternator which can be read from the name plate of the machine. Record your observations in a tabular form as shown.
3. Plot graph of I_f versus I_L for constant V and N.

Observations and Results

Sr. No.	*Load Current* I_L	*Terminal Voltage* V	*Speed* N	*Field Current* I_f
Take 6-7 Readings				

Questions Answer the following questions in your report:

1. From your experimental data calculate the rating of the field regulating resistance which must be connected in the field circuit for maintaining constant terminal voltage on variation of load up to full-load.
2. Explain why is it necessary to increase the excitation of the alternator to keep the terminal voltage unchanged when load is applied.
3. State any situation when it may be necessary to reduce the excitation to keep the terminal, voltage unchanged when an alternator is loaded.

Experiment 5.3 *Determination of the relationship between terminal voltage and load current of an alternator, keeping excitation and speed constant*

Objectives (a) To determine that the terminal voltage of an alternator drops with increase in load,

(b) To calculate the regulation of an alternator by direct loading method.

Brief Theory Like a dc generator, the terminal voltage of an alternator falls when loaded (except for capacitive load). In other words, it can be said that the terminal voltage will rise when the load is switched off. For instance, suppose *OB* in Fig. 5.75 represents the full-load current and *OA* the rated terminal voltage of an alternator. When the load is removed with the field current and speed unaltered, the terminal voltage rises to *OC*. This variation of the terminal voltage between full-load and no-load, expressed as a percentage of the full-load voltage, is termed as the percentage voltage regulation of the alternator. The amount of rise in terminal voltage from full-load to no-load will depend upon the power factor of the load.

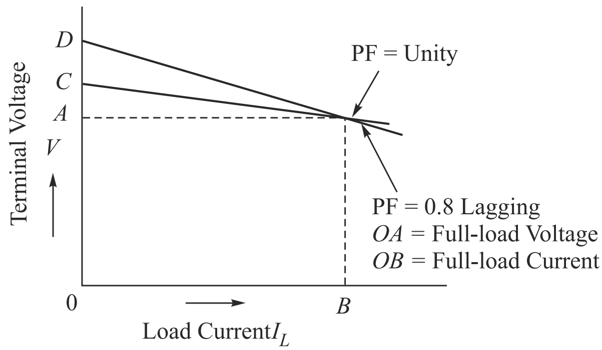


Fig. 5.75 *Variation of terminal voltage with load*

Thus, voltage regulation

$$= \frac{\text{Change in terminal voltage when full-load is removed}}{\text{Full-load terminal voltage}}$$

The voltage regulation for a power factor of 0.8 lagging is normally far greater than that at unity power-factor, and it is, therefore, important to include the power factor of the load when stating the voltage regulation of an alternator.

Circuit Diagram Same as Fig. 5.74 of previous experiment.

Apparatus Required Synchronous generator coupled with a dc motor, starter for the dc motor, Field regulating rheostats (two), tachometer, ammeter-moving coil type, ammeter (three) and voltmeter-moving iron type, variable loading resistance.

Procedure

1. Make connections as shown in Fig. 5.74. Start the dc motor with the help of the starter. Adjust the speed to the rated speed of the alternator.
2. Close the switch S_2 of the alternator field. Adjust the field current to get rated output voltage across the terminals of the alternator.
3. Now close the load switch and switch on a few loads (resistive load). Note that the terminal voltage falls as load is increased.
4. Now adjust field current, such that at rated load and speed the terminal voltage of the alternator is at rated value (rated value has been indicated on the nameplate of the machine).
5. Keep the excitation fixed at the above value. Reduce the load step-by-step and record terminal voltage, V and field current, I_f. Keep the speed of the set constant throughout.
6. Switch off the load completely and record the terminal voltage at rated speed. Tabulate the data.
7. Draw graph showing relationship between I_L and V.

Questions Answer the following questions in your report:

1. Calculate the value of regulation from the characteristics drawn by you.
2. Mention any disadvantage of determining the regulation of an alternator by direct loading method.

3. Regulations of two alternators are 10 per cent and 70 per cent respectively. Out of the above, which one do you think is more suitable for supplying electricity to a residential area? Give reasons for your answer.

EXPERIMENT 5.4 *Determination of regulation and efficiency of an alternator from open-circuit and short-circuit tests.*

Objectives (a) To determine the regulation of an alternator by synchronous impedance method.

(b) To determine the efficiency of an alternator by measuring its losses through open-circuit and short-circuit tests.

Brief Theory The variation of the terminal voltage of an alternator between full-load and no-load, expressed as percentage of full-load voltage is called the percentage voltage regulation of the alternator.

Regulation of an alternator can be determined by measuring the terminal voltage of the alternator, i.e., V when loaded and E when the load is taken off. In actual practice it will be difficult to load a big alternator in the testing laboratory as the laboratory may not have such heavy loads. Moreover, during the testing period a considerable amount of electrical energy will be wasted as losses in the machine and in the load. This is why regulation of large alternators are not generally determined by direct loading method.

Regulation of an alternator can alternatively be determined from the results of the following two tests:

(a) Open-circuit test,

(b) Short-circuit test.

Open-Circuit Test This test is carried out with the alternator running on no-load and at rated speed. The field current and corresponding-terminal voltage is recorded up to about 120 per cent of rated terminal voltage. The characteristic showing the relationship between field current and the terminal voltage on no-load is called the open-circuit characteristic (OCC).

Short-Circuit Test Short-circuit test is performed when the alternator is running at rated rpm. The armature terminals are short-circuited with a very low excitation current. Armature current up to rated value is recorded for various values of field current. A plot of field current versus armature current is called short-circuit characteristic (SCC). Since the emf generated on open circuit may be regarded as being responsible for circulating short-circuit current through the synchronous impedance, the value of synchronous impedance is taken as the ratio of the open-circuit voltage per phase to the short-circuit current per phase for a particular field current.

It may be noted that the value of the synchronous impedance calculated here is the unsaturated value (higher than the actual value), since the excitation under short-circuit condition is much lower than the normal value. The dc resistance of the stator winding can be calculated by ammeter-voltmeter method. The ac resistance is higher than the de resistance. The value of dc resistance calculated may be multiplied by a factor of 1.3 to calculate the ac resistance. The synchronous reactance can be calculated as $X_s = \sqrt{Z_s^2 - R_a^2}$

Calculation of Voltage Regulation The relationship between terminal voltage and induced emf for a lagging/leading power-factor load is given by the following expression

$$E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi \pm I_a X_s)^2}$$

Value of regulation can be calculated by calculating E.

Calculation of Efficiency In open-circuit, test the input to the alternator, i.e., the power required to drive the alternator is spent as, friction and windage loss and iron-loss. If the field of the alternator is kept unexcited, the input to the alternator will be equal to the friction and windage loss of the alternator. Input to the alternator can be calculated by measuring the input to the motor driving the alternator and by knowing the efficiency of the driving motor thus:

$$\text{Input to the alternator} = \frac{\text{Input of the driving motor}}{\text{Efficiency of the driving motor}}$$

(Since output of the driving motor is equal to the input to the alternator.)

Note: The efficiency of the driving motor should be known or may be determined For the purpose of calculation. The efficiency of the driving motor may be assumed suitably. Thus by measuring the input to the driving motor the friction and windage loss at rated speed and iron-loss at rated excitation voltage and speed can be calculated (two readings, one with no field excitation and one with rated excitation will have to be taken).

In short-circuit test the input to the alternator is spent as: $I_a^2 R_a$-loss in the armature windings, friction and windage loss and a small amount of iron-loss. If the input power is recorded when full-load current is flowing through the short-circuited armature, neglecting the small amount of iron-loss, $I_a^2 R_a$-loss can be calculated thus:

$I_a^2 R_a$-loss at full-load armature current = Input to the alternator at full-load short circuit condition – Friction and windage loss at rated speed

Thus from the open-circuit and short-circuit test data the following can be calculated:

(a) Full-load $I_a^2 R_a$-loss (X)
(b) Friction and windage loss at rated speed (Y).
(c) Iron-loss at rated speed and rated voltage (Z).

The full-load efficiency of the alternator can be calculated thus:

$$\text{Full-load efficiency} = \frac{\text{output at full load}}{\text{output at full load} + \text{losses (i.e, } X + Y + Z)} \times 100$$

Circuit Diagram See Fig. 5.76.

Note: For short-circuit test the circuit connection will be the same except that the alternator terminals will be short-circuited through an ammeter of appropriate rating.

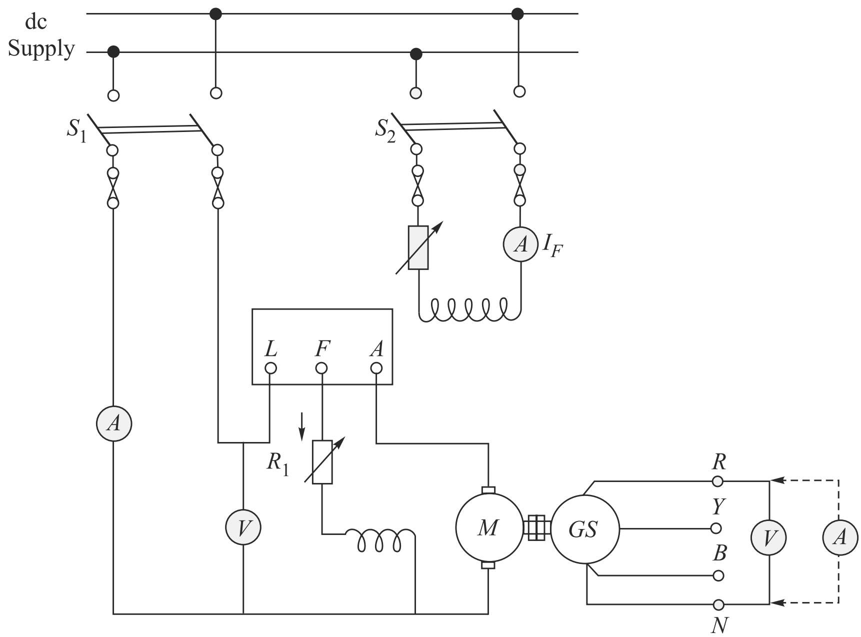


Fig. 5.76 *Circuit diagram for open-circuit/short-circuit test on an alternator*

Apparatus Required dc motor-three phase alternator set, ammeters (two) and voltmeter-dc, ammeter and voltmeter-ac, tachometer, starter, rheostat (two).

Procedure

1. Make connections as per circuit diagram shown in Fig. 5.76. Start the set with the help of a dc motor and bring the speed of the set-up to rated speed of the alternator.
2. Excite the alternator with maximum resistance in its field circuit. Increase the field current in steps by reducing the field circuit resistance. Record the induced emf of the alternator for various values of field current at constant speed up at 120 per cent of the rated voltage of the alternator. Take at least ten readings at approximately equal increasing values of field current. While taking readings, at no time decrease the field current of the alternator. Note down the readings of all the meters with rated excitation and also with no excitation.
3. With the field excitation of the alternator switched off, adjust the speed of the alternator to its rated speed.
4. Short circuit the armature terminals of the alternator with no field excitation.
5. Switch on the field excitation with maximum resistance in the field circuit. Increase the field current till rated full-load current flows through the armature windings of the alternator. Record the data as per tables indicated and then switch off the set. Note the specifications of all the instruments and the set.
6. Measure the armature resistance of the alternator by ammeter-voltmeter method by applying very low dc voltage to its armature.

Observations and Results Open-circuit Test

Speed of the set = rated speed = _____ rpm

Alternator Excitation Current, I_t							
Alternator Induced *emf*, *E*/phase							

Input to dc motor with alternator at rated speed and		*V*	*I*	*P* = *VI*
1.	at rated excitation			
2.	no excitation			
3.	full load current through armature (short circuit)			

Short-Circuit Test

Alternator Excitation Current, I_t							
Alternator Armature Current, I_a							

Note: Assuming a particular value of efficiency say 80 per cent for the dc motor, make calculations for various losses of the alternator and hence find its efficiency.

Questions Answers the following questions in your report:

1. Draw the open-circuit and short-circuit characteristics of the alternator at rated speed on a common scale of field current on *X*-axis, and calculate the synchronous impedance and synchronous reactance. Explain why the value of synchronous reactance thus calculated is called the unsaturated value. is the unsaturated value smaller than the saturated value?
2. Calculate the value of regulation of the alternator at full-load and at (a) unity power-factor, (b) 0.8 lagging power-factor and (c) 0.8 leading power-factor
3. Write down the value of the following losses of the alternator under test:
   (a) full-load armature $I_a^2 R_a$-loss;
   (b) friction and windage losses;
   (c) iron loss.
4. Calculate the full-load efficiency of the alternator.
5. Mention the advantages and disadvantages (if any) of finding regulation of an alternator by synchronous impedance method (indirect method) than by direct loading method.
6. Mention why the speed of rotation of the alternator should remain constant at rated speed while performing the open-circuit test.
7. Explain the shape of the open-circuit characteristic of the alternator drawn by you.
8. Explain why the regulation of an alternator is more at 0.8 lagging power-factor than at unity power-factor.

EXPERIMENT 5.5 *Determination of the relationship between terminal voltage and load current of an alternator for varying power-factor load, the speed and excitation remaining constant.*

Objectives To verify that the change in terminal voltage of an alternator depends on load and also on load power-factor and to find out the regulation at different power-factor loads.

Brief Theory The change in terminal voltage of an alternator with the change in load connected across its terminals depends upon the nature of the load. For example, if the nature of the load is either resistive or inductive, there will be a fall in terminal voltage with increase of load. The fall will be comparatively more in case of inductive loads than in the case of resistive loads. In case of capacitive loads the terminal voltage will rise. In case of inductive load the fall in terminal voltage is more because, the armature flux due to current flowing in the armature winding opposes the main field flux. Thus when an alternator is loaded inductively, there is a considerable reduction in air-gap flux and hence there is a fall in terminal voltage. In case of capacitive loads however, the armature flux helps the main field flux so that there is an overall increase in air-gap flux.

In this experiment, the alternator may be loaded with the help of a synchronous motor. By varying the excitation of the synchronous motor the power factor of the load on the alternator may be changed (the power factor of a synchronous motor changes with change of its field current). An over-excited synchronous motor behaves like a capacitive load whereas an underexcited synchronous motor behaves like an inductive load. A synchronous motor with normal excitation behaves like a resistive load.

Circuit Diagram See Fig. 5.77.

Apparatus Required Synchronous generator coupled with a dc motor, starter for the dc motor, synchronous motor for loading the alternator, dc generator coupled with the synchronous motor, Field regulating rheostats (three), Ammeter-moving coil type (three), Ammeter (three) and voltmeter-moving iron type, wattmeters-single phase (two), tachometer.

Procedure

1. Make connections as shown in Fig. 5.77. If variable resistive, capacitive and inductive loads are available, then loading of the alternator may be done through them otherwise, a synchronous motor coupled with a dc generator may be used.
2. Start the dc motor and bring the speed of the set to rated speed of the alternator. Close S_2 and adjust R_2 and R_1 to get rated induced emf across the terminals of synchronous generator at rated speed.
3. Start the synchronous motor as an induction motor by closing the switch S_5. Closing the field circuit of the synchronous motor by the switch S_3. The synchronous motor will now run at synchronous speed. Increase the excitation to its highest permissible value.

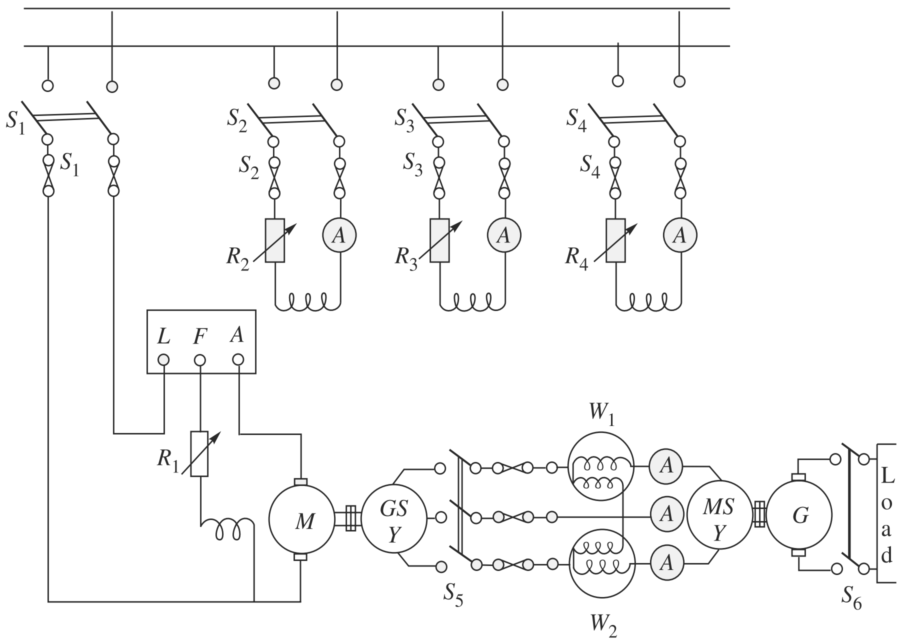


Fig. 5.77 *Connection diagram for determining the relationship between terminal voltage and load current of an alternator for varying power factor load*

4. Excite the dc generator by closing the switch S_4. Load the dc generator by closing the switch S_6 till the reading, I_L of the ammeters (current delivered by the alternator) becomes its full-load rated value at rated terminal voltage, V and at rated speed. Record readings of alternator speed, terminal voltage, load current and power output (reading of W_1 and W_2).
5. Now, switch off, S_5, S_3 and S_4. At this time the alternator is running on no-load. Adjust the speed of the alternator to its rated value. Do not change the excitation. Record the value of alternator terminal voltage.
6. Repeat step 2, 3 and 4 for two more values of synchronous motor excitation less than in the first case. While reducing the excitation of the synchronous motor, it will be observed that the armature current goes on decreasing and comes to a minimum value. This minimum current corresponds to unity power-factor loading. Further decrease in excitation will again cause increase in armature current which corresponds to lagging power-factor condition. The three sets of readings that should be taken are:
   (a) at leading pf full-load condition,
   (b) at unity pf full-load condition,
   (c) at lagging pf full-load condition.
7. Tabulate results as per the table shown. Draw graphs showing the variation of terminal voltage from no-load to full-load in the three cases. For each set of readings, calculate the power-factor of the load from the wattmeter readings by using the formula:

$$\text{Power factor, } \cos\phi = \cos\tan^{-1}\sqrt{3}\,\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

8. Calculate the value of regulation at different power-factor loads.

Observations and Results

	Wattmeter Readings W_1	W_2	Load *pf* $\cos\phi$ by Calculation	Full-load Voltage V	No-load Voltage E
Full-load Current, I_L at Leading Power Factor					
Full-load Current, I_L at Unity Power Factor.					
Full-load Current, I_L at Lagging Power Factor					

Questions Answer the following questions in your report:

1. Define the term regulation of an alternator.
2. Comment on the values of full-load regulations obtained for different power-factor loads.
3. State why the regulation of an alternator is negative at leading power-factor loads.

EXPERIMENT 5.6 *Parallel operation of three-phase alternators and load sharing.*

Objective To synchronize a three-phase alternator with another alternator by using three-lamp method and to study load sharing.

Brief Theory In power stations a number of alternators run in parallel. All the alternators are synchronized with a common bus-bar. The load is connected across the bus-bar. The alternators running in parallel share the total load proportionate to their ratings. In the event of increased demand of load on a power station, additional generating sets are required to be connected in parallel with the bus-bar. Before an alternator can be connected in parallel with another alternator it should be synchronized. Synchronization can be done by using a synchroscope or by three-lamp method. The following conditions are to be fulfilled before connecting an alternator in Parallel with others:

(a) The voltage of the incoming alternator should be the same as the running alternator voltage.

(b) The frequency of voltage of the incoming alternator should be the same as the running alternator voltage frequency.

(c) The phase sequence of the voltages of the incoming alternator to be connected in parallel should be the same as that of the running alternator.

(d) At the instant when the paralleling switch is closed, the voltages of the incoming alternator should be in time-phase with the running alternator voltage.

Three-lamp Method of Synchronization Three pairs of lamps are connected between the incoming alternator and running alternator as shown in Fig. 5.78. Note that one pair of lamps are connected between the R phase of the alternator, and the R phase of the running alternator. Other two pairs of lamps are connected between the Y phase of the alternator and the B phase of the running alternator and vice versa. Here two lamps are connected in series as the maximum voltage which will appear across the lamps will be the line voltage, i.e., 400 V (for a 400 V machine). As lamps are generally rated for 230 V, for safety reasons, we need to connect two lamps in series.

When the incoming alternator is brought to rated, speed and provided with excitation, voltage will be induced across the alternator terminals which can be

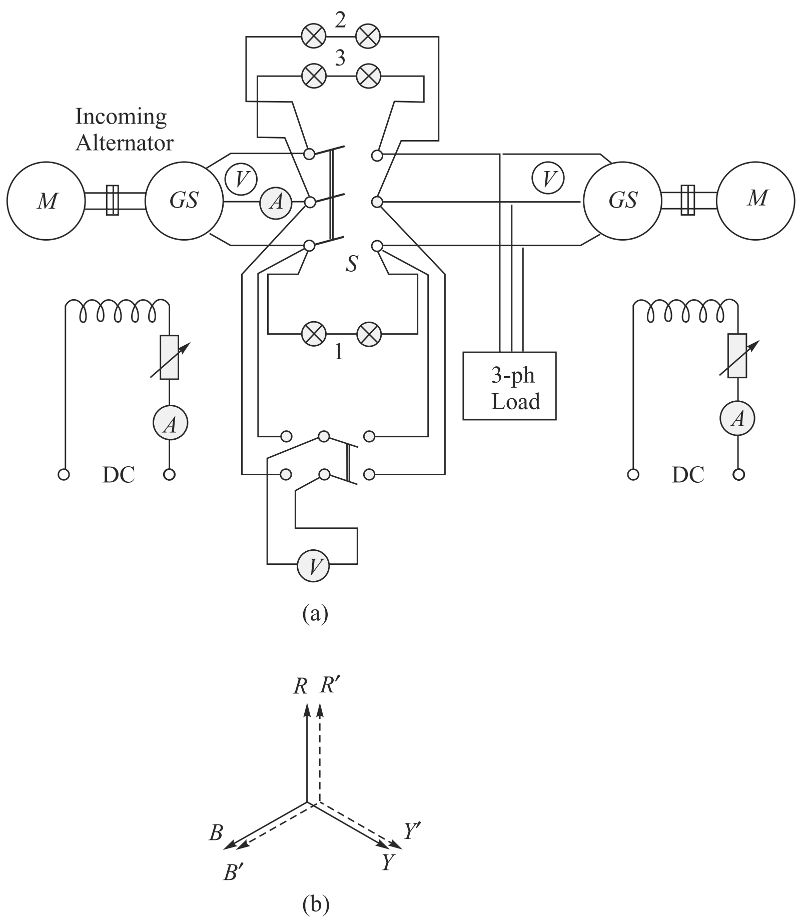


Fig. 5.78 *(a) Connection diagram for synchronization of a three-phase alternator (b) Three voltages of the running alternator and the incoming alternator are shown in time phase*

made equal to the running alternator voltage by adjusting the field excitation of the incoming alternator. The three voltages of the running alternator and the incoming alternator are shown in Fig. 5.78(b). Phasors drawn in thick lines indicate the three-phase voltages of the running alternator whereas the dotted ones indicate the three voltages of the incoming alternator. Hall the conditions of synchronization are fulfilled, then the two sets of phasors at a particular instant of time will be in position as shown in Fig. 5.78(b). At that instant the switch S can be closed and the alternator will run in parallel with the running alternator.

To enable the incoming alternator to share load, the input to the incoming alternator must be increased. In the three-lamp method of synchronization, at the moment of synchronization the lamps connected in phases *RR′* will be dark whereas other two pairs of lamps will be bright as full-line voltages will be appearing across them. If the frequency of the incoming alternator is more than the running alternator frequency, the lamps will darken in the sequence 132132... This will indicate that the speed of the alternator should be reduced. On the other hand, if the frequency of the incoming alternator is less than the running voltage frequency then the lamps will darken in the sequence 123123... This will indicate that the speed of the incoming alternator should be increased. The above is explained through series of phasor diagrams as shown in Fig. 5.79.

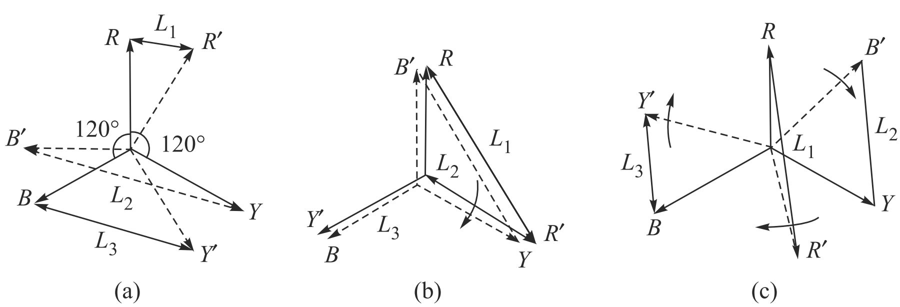


Fig. 5.79 *Phasor diagrams showing synchronization of a 3-phase alternator by three-lamp method*

In Fig. 5.79(a), as voltage *R′Y′B′* are revolving faster than voltages *RYB*, the voltages across lamps L_1 and L_2 are increasing, whereas voltage across lamp L_3 is decreasing. Therefore, lamp L_3 will be becoming dark as shown in Fig. 5.79(b). In Fig. 5.79(b), voltage across lamp L_3 is zero and hence lamp L_3 will be dark, other two lamps will he bright. In Fig. 5.79(c) voltages across lamp L_2 is decreasing. Thus after lamp L_3, lamp L_2 will be dark.

From the above it is seen that if the frequency of incoming alternator is higher than the running frequency, the lamps will darken in the sequence 132132.....

Circuit Diagram See Fig. 5.78(a).

Note: Primemover connection is not shown in Fig. 5.78. If it is a dc motor, make connections as shown in Fig. 5.76.

Apparatus Required 400-V alternators driven by a primemover, Six 230 V filament lamps, voltmeter (ac), double pole 2-way switch, ammeter (one dc, one ac), Field regulating rheostats, tachometer, phase sequence indicator.

Procedure

1. Check the phase sequence of the running alternator and incoming alternator voltages with the help of the phase sequence indicator. Make connections as per circuit diagram.
2. Run the incoming alternator at rated speed. Adjust the field excitation of the incoming alternator such that the induced emf is equal to the running alternator voltage. This can be checked by connecting the voltmeter across the running alternator and across the incoming alternator terminals by turn, with the help of a double-pole-double-throw switch.
3. By adjusting the speed of the primemover, bring the incoming alternator to a speed such that the pair of lamps connected to the identical phases will be dark and the other two pairs of lamps will be bright. At that time check the running alternator and incoming alternator voltages also. If the two voltages are same, connect the incoming alternator across the running alternator with the help of the switch *S*. The incoming alternator is now synchronized and is floating on the load, i.e., it is neither receiving from nor delivering to the load any power.
4. For the incoming alternator to share any load, the input to the primemover will have to be increased. In case the primemover is a dc machine, its excitation may be increased slowly. Note the ammeter readings connected in one phase of the alternators which will indicate the current supplied by each to the load.

Questions Answer the following questions in your report:

1. Explain the need for parallel operation of alternators.
2. Explain the effect of change of (a) excitation of an alternator, (b) primemover input to an alternator synchronized with another alternator.
3. Explain why an alternator should be synchronized before if can share load while running in parallel with other alternators connected to the bus-bar.

EXPERIMENT 5.7 *Determination of the effect of variation of excitation of a synchronous motor.*

Objective To determine how armature current and power factor of a synchronous motor varies with change of excitation at different loads, supply voltage remaining constant.

Brief Theory Similar to a dc motor, in a synchronous motor also applied voltage *V* is opposed by an induced emf *E*. The resultant voltage causes a current to flow through the armature winding which has an impedance of Z_s. When a synchronous motor is loaded, it speed does not change. To supply the additional output power requirement, the induced emf falls back by a certain angle 'α' which is called the load angle. See Fig. 5.80(a). (In a synchronous motor the rotor field rotates at the same speed as the rotating magnetic field produced by the polyphase stator current. When load on the rotor increases, its field axis makes a comparatively bigger angle with the axis of the rotating field and thereby supply the additional load. In case of a dc motor, however, the torque or load angle is kept fixed at 90 deg., i.e., to its maximum value and does not change.) The effect of change of excitation of a synchronous motor at a particular load is shown in Fig. 5.80. Load angle α will remain constant. Angle between E_R and I_a will also remain constant at above 90°. Since it is fixed by the ratio of reactance to resistance of the armature circuit (as the value of reactance is much higher than the value of armature resistance, the value of ϕ is usually about 90°).

Input power to the motor

$$= \sqrt{3}\, VI_a \cos\phi$$

where cos ϕ is the power factor. If V is constant and load is constant, input should remain constant and hence $I_a \cos\phi$ should remain constant, ($I_a \cos\phi$ is component of I_a on the voltage axis. In Fig. 5.80 OX represents $I_a \cos\phi$).

Assuming the supply voltage and load on the motor to remain constant, any change of excitation, I_f will simply increase or decrease the magnitude of E.

From the phasor diagram in Fig. 5.80, it is seen that increase in field current causes leading power-factor current, while decrease in field current causes lagging power-factor current drawn by the motor. There will be some excitation which will cause unity power-factor current drawn by the motor. The magnitude of armature current at that excitation would be the minimum. Excitation corresponding to unity power-factor current drawn by the motor is called normal excitation. Excitation more than the normal excitation (also called over-excitation) causes leading power-factor current and excitation less than the normal excitation (also called underexcitation) causes lagging power-factor current. A typical characteristic showing the relationship between field current I_f and armature current I_a of a synchronous motor for a particular load is shown in Fig. 5.81. Normal excitation (i.e., minimum armature current) corresponds to unity power-factor. Excitation below normal gives rise to lagging power-factor current whereas excitation more than normal gives rise to leading power-factor current drawn by the motor.

Therefore, by varying the excitation of a synchronous motor, it can be made to behave either like an inductive load (when unuerexcited) or a capacitive load (when overexcited). An

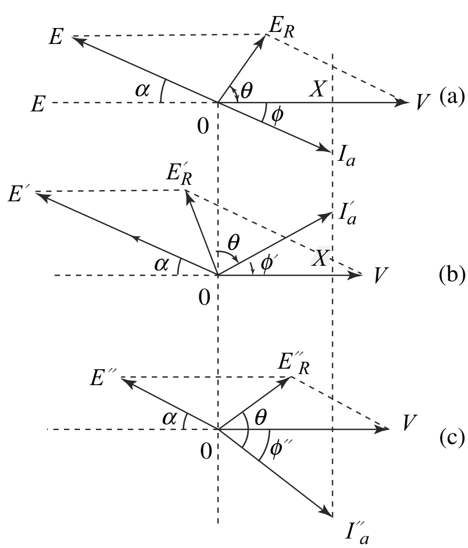


Fig. 5.80 *Phasor representation of the effect of change of excitation of a synchronous motor (a) With initial excitation (b) When excitation is increased (c) When excitation is decreased*

overexcited synchronous motor is, therefore, also called a synchronous condenser. The shape of the I_f versus I_a characteristics shown in Fig. 5.80 are often referred to as synchronous motor *V* curves. I_f by changing loads on the synchronous motor effects of If on la is studied, a series at *V* curves will be obtained as shown in the figure.

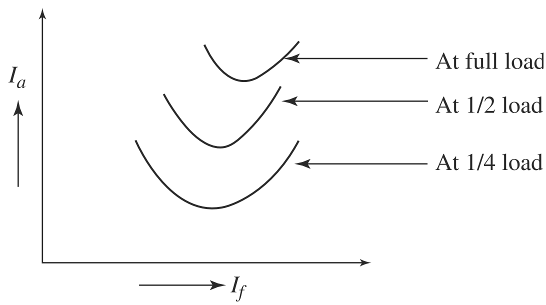


Fig. 5.81 *Graphic representation of the effect of change of excitation on armature current of a synchronous motor at various shaft loads*

Circuit Diagram See Fig. 5.82.

Apparatus Required Synchronous motor coupled with a dc machine or having any other loading arrangement, voltmeter and three ammeters-moving coil type, wattmeters (two), ammeter and voltmeter-moving iron type, loading rheostat, field regulating rheostats (two).

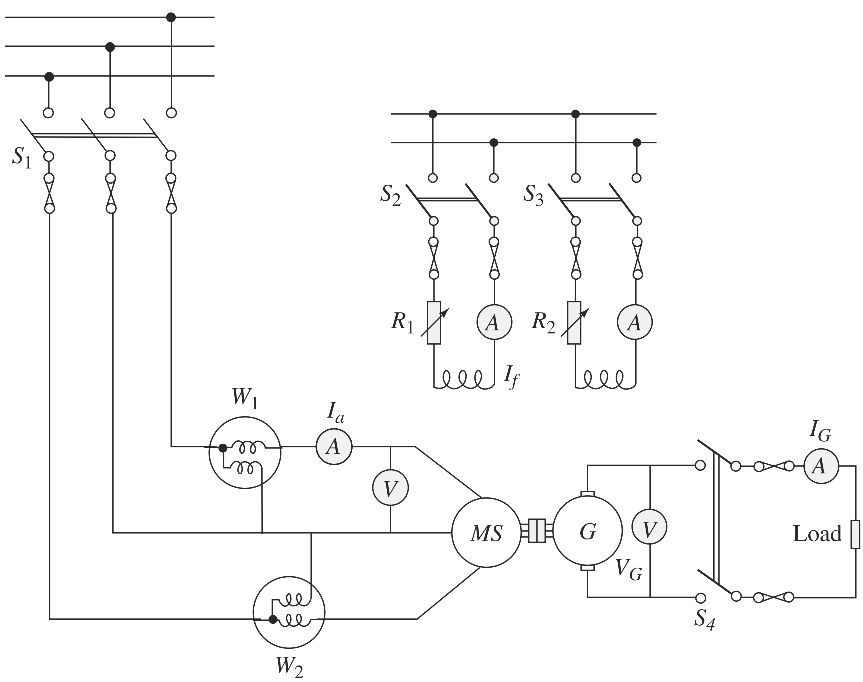


Fig. 5.82 *Connection diagram for studying the effect to variation of excitation on armature current and power factor of a synchronous motor*

Procedure

1. Make connections as per circuit diagram shown in Fig. 5.82. Start the synchronous motor as an induction motor by closing switch (S_1) and keeping the field circuit closed through a high resistance. The rotor will run at a speed very near to its synchronous speed. Close the field circuit of the synchronous motor through motor switch S_2. The motor will now be running at synchronous speed. Increase the field current to its maximum permissible value by varying the rheostat, R_1 of the field circuit.
2. Run I Reduce the field current, I_f step by step and record values of I_f, I_d, V, W_1 and W_2 in a tabular from. While reducing I_f from its maximum value, it would be observed that the value of armature current I_a will be decreasing to a minimum value and then again increasing. The range of variation of excitation current should be such that it does not cause excessive current (more than rated full-load current) to flow through the armature circuit of the motor.
3. Run II Increase the excitation of the field to its permissible maximum value again. Close switch S_3 of the dc generator and load the synchronous motor by loading the de generator by closing switch, Sr. Reduce excitation and record at each step values of I_f, I_a, V, W_1, W_2, V_G and I_G. Both in RUN I and RUN II take at least ten readings. Record data in a tabular form.

Observations

Sr. No.	Synchronous motor						DC generator	
	I_r	I_a'	V	W_1	W_2	I_f'	V_G	I_o
RUN I								
RUN II								

Sample Calculation and Result Calculate the values of power factor corresponding to each reading. Show one sample calculation.

Draw graph showing the effect of I_f, on I_f, and power factor (a) at no-load, (b) at a particular load.

6 SINGLE-PHASE MOTORS

OBJECTIVES

After carefully studying this chapter, you should be able to

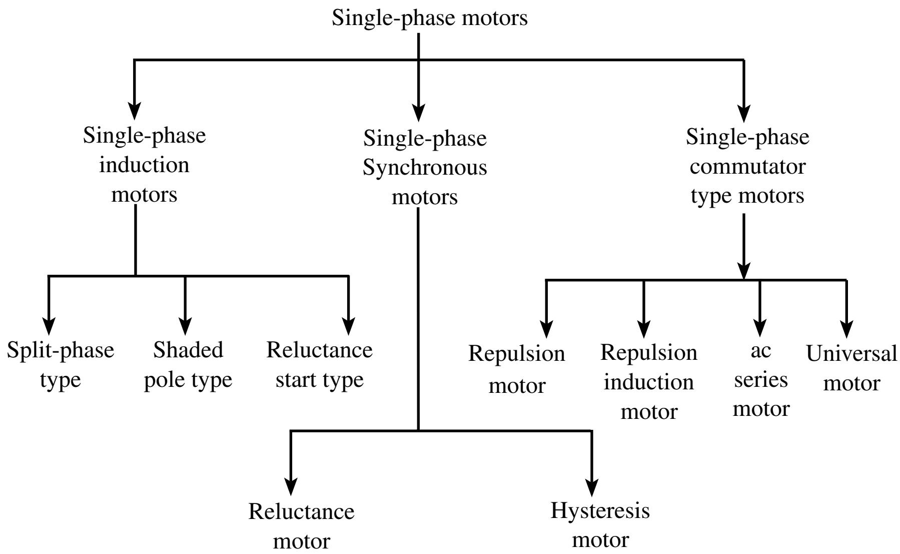
- ➢ Name the various types of single-phase motors.
- ➢ Explain the construction and principle of working of single-phase induction motors.
- ➢ Explain the construction and principle of working of single-phase synchronous motors.
- ➢ Explain the construction and principle of working of single-phase commutator type motors .
- ➢ Draw performance characteristics of different types of single-phase motors.
- ➢ Select a specific type of single-phase motor for a particular purpose.

6.1 INTRODUCTION

Single-phase motors are mostly manufactured in fractional kilowatt range to be operated on single-phase supply and for use in numerous applications like ceiling fans, refrigerators, food mixers, hair driers, portable drills, vacuum cleaners, washing machines, sewing machines, electric shavers, office machinery, etc. Single-phase motors are manufactured in different types to meet the requirements of various applications. Single-phase motors are classified on the basis of their construction and starting methods employed. The main types of single-phase motors are: (a) induction motors; (b) synchronous motors; and (c) commutator motors. The various types of motors under each class are shown in a diagrammatic form as follows.

Most single-phase motors as mentioned above are fractional kilowatt motors. But single-phase motors are also manufactured in standard integral kilowatt sizes. At this point it may be useful to define a fractional kilowatt (FKW) motor. According to American Standard Association (ASA) and National Engineering Manufacturers Association (NEMA) of USA, fractional *kilowatt* motor *is a motor built in a frame smaller than that having a continuous rating of 1 kW, open type, at 1700 rpm to 1800 rpm.*

According to the definition, since determination of FKW is based on frame size, a 3/4 kW, 900 rpm motor may require a bigger frame size than a 1 kW 1700–1800 rpm one and therefore cannot be called a fractional *kilowatt* motor.



6.2 SINGLE-PHASE INDUCTION MOTORS

Single-phase induction motors are similar to those of three-phase induction motors except for the fact that the stator has a single-phase winding instead of a three-phase winding. Performance characteristics of single-phase induction motors are less satisfactory than three-phase induction motors. However, single-phase induction motors have found wide range of applications where only single-phase supply is available. Gradual improvements in design has made these motors quite satisfactory in fractional kilowatt ratings.

6.2.1 Construction and Principle of Working

Construction A single-phase induction motor physically looks similar to that of a three-phase induction motor except that its stator is provided with a single-phase winding. The rotor construction is identical to that of a polyphase squirrel-cage type induction motor. In fact the rotor of any single-phase induction motor is interchangeable with that of a polyphase induction motor. There is no physical connection between the rotor and the stator and there is uniform air-gap between the stator and the rotor. The stator slots are distributed uniformly, and usually a single-phase double-layer winding is used. A simple single-phase winding would produce no rotating magnetic field and no starting torque. It is, therefore, necessary to modify or split the stator winding into two parts, each part winding displaced in space on the stator to make the motor self-starting. Single-phase motors are classified into split-phase type, capacitor-type, and shaded-pole type depending upon the starting devices employed. Fig. 6.1(a) shows a capacitor-type single-phase induction motor. Figure 6.1(b) shows the parts of a capacitor-type single-phase induction motor.

Fig. 6.1(a) *A capacitor-type single phase induction motor*

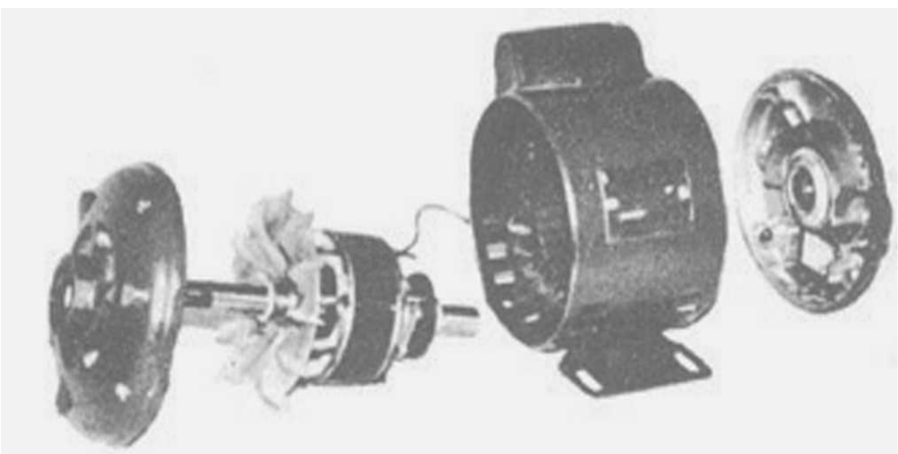


Fig. 6.1 (b) *Parts of a small capacitor type single-phase induction motor*

Principle of Working A single-phase induction motor with a distributed stator winding and a squirrel-cage rotor is shown in Fig. 6.2. When single-phase supply is applied across the single-phase stator winding, an alternating field is produced. The axis of this field is stationary in the horizontal direction as shown in the figure. The alternating field will induce an emf in the rotor conductors by transformer action. Since the rotor has a closed circuit, current will flow through the rotor conductors.

The direction of induced emf and current in the rotor conductors is shown in Fig. 6.2. For the direction of stator field as shown, the force experienced by the upper conductors of the rotor will be downward and the force experienced by the lower conductors of the rotor will be directed upward, which have been shown in Fig. 6.2(b). The two sets of forces will cancel each other and the rotor will experience no torque. The axis of the stator and rotor magnetic fields are aligned and the torque angle is

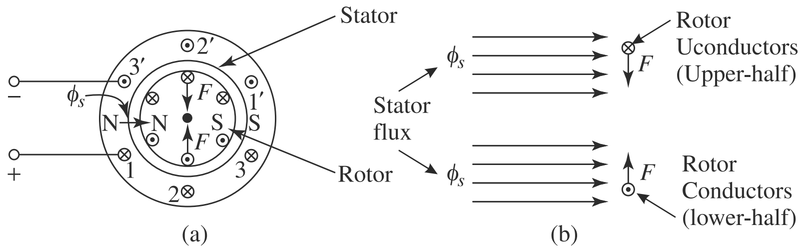


Fig. 6.2 *(a) When the rotor is stationary the rotor field produced by transformer emf is in alignment with the stator field, (b) Directions of force developed on the rotor current carrying upper and lower conductors*

zero. Therefore, when a single-phase supply is applied across the stator winding, the rotor does not rotate.

It has, however, been experienced that when the rotor is given an initial rotation in any direction, it continues to pick up speed in that particular direction. A starting torque, therefore, is to be provided to enable the rotor to pick up speed in any direction.

A motor should be designed such that it is able to start on its own. Several methods have been developed to make the motor self-starting. The starting device is needed during the starting period only. Once the motor starts rotating, the starting device becomes redundant. It is interesting to understand how the motor continues to rotate with one single-phase winding on the stator once it is given an initial rotation.

How the rotor develops torque when it is given an initial rotation can be explained with the help of any one of the two theories namely: (a) cross-field theory, and (b) double revolving field theory. These two theories are explained in a simplified manner as follows:

(a) Cross-Field Theory Consider a single-phase induction motor having a single-phase winding on the stator as shown in Fig. 6.3. The rotor is squirrel cage type. When a single-phase supply is applied across the stator winding an alternating field ϕ_s is produced along the horizontal axis. With the rotor at standstill, this field will induce a transformer emf in the rotor winding. As the axes of both the stator and rotor field lie in the horizontal direction no torque will be developed.

Assume that the rotor is now given an initial rotation, say in the clockwise direction. An emf, called rotational emf, will be induced in the rotor winding by virtue of its rotation in the stationary stator field. The direction of emf induced in the rotor conductors are shown in Fig. 6.3. Emf induced in the rotor conductors is in one direction on one side of the vertical axis and in the other direction on the other side of the vertical axis.

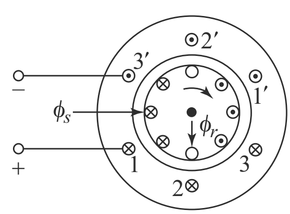


Fig. 6.3 *Rotor field produced by speed emf current is at 90° with the stator field when the rotor is given an initial rotation*

As the rotor circuit is closed the rotational voltage so induced will produce a component rotor current and a rotor mmf wave whose axis is displaced 90° electrical from the stator axis. Unlike a poly phase induction motor, the frequency of the rotor-induced emf is high, and therefore, the rotor reactance is high ($X \propto f$). The rotor current will lag the rotor induced emf by about 90°. The field produced by rotor current, ϕ_r known as crossfield will have a time phase difference of about 90° with the stator field ϕ_s. Thus the stator flux, ϕ_s and rotor flux ϕ_r are in the space and time quadrature. These two fields will produce a revolving field which will rotate in the direction in which the rotor was given an initial rotation. Thus the torque produced will be in the same direction as that of given rotation.

It can be seen that if the initial rotation is given in the anti-clockwise direction, the direction of the rotating field produced will also be in the anti-clockwise direction. Once rotating field is produced the rotor will continue to rotate in the direction of the rotating field in the same way as a polyphase induction motor. The magnitude of the component rotor field depends upon speed. At synchronous speed, the magnitude of rotor field is the same as the stator field. At lower speed the magnitude of the rotor field is less than the stator field. When the rotor speed becomes zero, the rotor cross-field is zero and, therefore, the resultant field is reduced to an alternating field acting along the horizontal axis.

The behaviour of a single-phase induction motor with a single winding on the stator can be summarised as follows:

(i) At rotor standstill, only transformer emf is induced in the rotor. The rotor current produces a flux along the same axis as the stator flux axis. The two magnetic fields act along the same axis. No torque is developed as the magnetic fields are aligned.

(ii) When the rotor is given an initial rotation in any particular direction, speed emf is induced in the rotor. This emf acts along an axis which is 90° with the stator field axis. The rotor flux produced by this speed emf and the stator flux are in time and space quadrature. These two component fluxes produce a resultant rotating magnetic field in the air-gap, in the direction in which the rotor is given an initial rotation. Hence torque is exerted on the rotor and the motor continues to rotate.

(iii) The magnitude of the rotating field is a variable one.

(b) Double Revolving Field Theory This theory is based on the fact that the alternating field produced by the stator winding can be represented as the sum of two oppositely rotating fields of identical strength. The magnitude of each of these fields will be equal to one-half of the maximum field strength of the stator alternating field. If these fields are represented by vectors that rotate in opposite directions as shown in Fig. 6.4, the summation of the vectors is a stationary vector that changes in length along the horizontal axis. In other words, as the alternating field ϕ_s oscillates between $+\phi_{sm}$ and $-\phi_{sm}$ the two component fields ϕ_f and ϕ_b rotate in the opposite directions but at the same speed. This is explained in detail as follows.

When single-phase supply is connected across the stator winding, an alternating field along the horizontal axis as shown in Fig. 6.4 (a) will be produced. This field flux will change sinusoidally, because the stator current is sinusoidal. Stator field flux, ϕ_s can be represented by a vector whose value changes with time from $+\phi_{sm}$ to $-\phi_{sm}$ for each cycle of current flowing through the stator winding. This alternating field has been broken into two component fields ϕ_f and ϕ_b whose magnitudes are one-half of ϕ_{sm} but are shown rotating in opposite directions as the magnitude and direction of stator field changes with time.

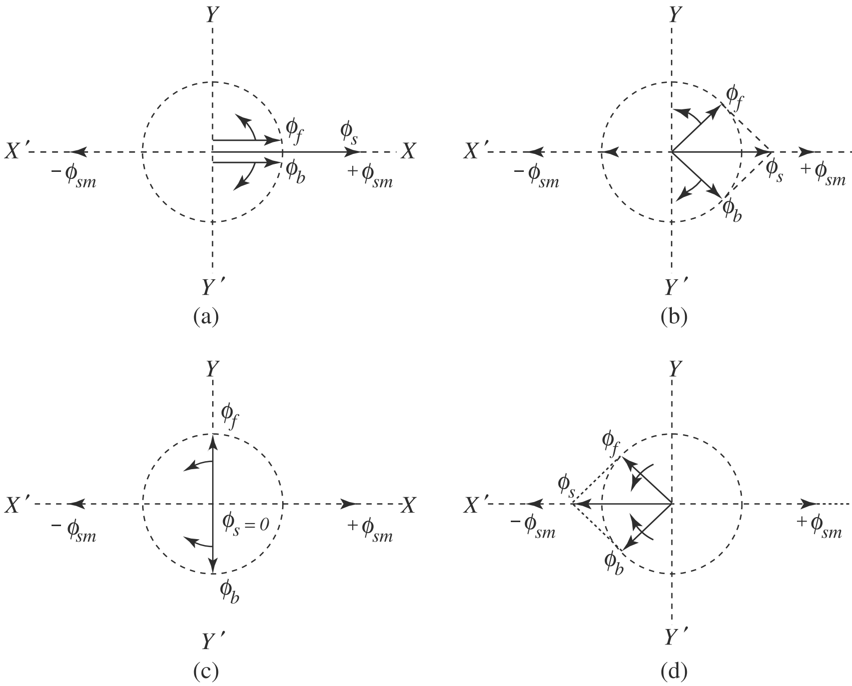


Fig. 6.4 *An alternating field is shown equivalent to two component revolving fields*

From Fig. 6.5(a), it can be seen that the magnitudes of ϕ_s at intervals of time t_1, t_2, t_3, t_4 and t_5 are respectively equal to 0, 0.707 ϕ_{sm}, ϕ_{sm}, 0.707 ϕ_{sm} and 0. It may be noted that time intervals between t_1, t_2, t_3, etc., is 45°. The stator field, ϕ_s and two component field ϕ_f and ϕ_b whose magnitudes are 0.5 ϕ_{sm} are shown in Fig. 6.5(b) at five intervals of time. At time t_1, $\phi_s = 0$. The two component fluxes ϕ_f and ϕ_b are shown in opposite directions. After a time interval of 45°, i.e., at time, t_2 ϕ_s increases to 0.707 ϕ_{sm}; the two component fields have rotated by 45°, one in clockwise direction and the other in anti-clockwise direction. At every instant of time the vector sum of the component fields is equal to the stator flux, ϕ_s.

At the interval of time representing one-fourth of a cycle, i.e., at time t_3, ϕ_s is maximum and is equal to + ϕ_{sm}. The component fields now have rotated by 90° as shown in Fig. 6.5(b). From the figure it is seen that for the time interval t_1 to t_5, i.e., for a half-cycle when the flux ϕ_s has changed from zero to its maximum value and then to zero, the two component field ϕ_f and ϕ_b have rotated in opposite directions by 180°, i.e., by half a revolution.

If the component vectors are drawn for one cycle, it will be observed that each of the component flux vectors will rotate by one revolution. For a 50 Hz supply the component fields will rotate by 50 revolutions per second. In other words, it can be concluded that the component fields would rotate at synchronous speed but in opposite directions. The field, ϕ_f which rotates in clockwise direction can be termed as forward field and the field, ϕ_b which rotates in anticlockwise direction can be termed as backward field.

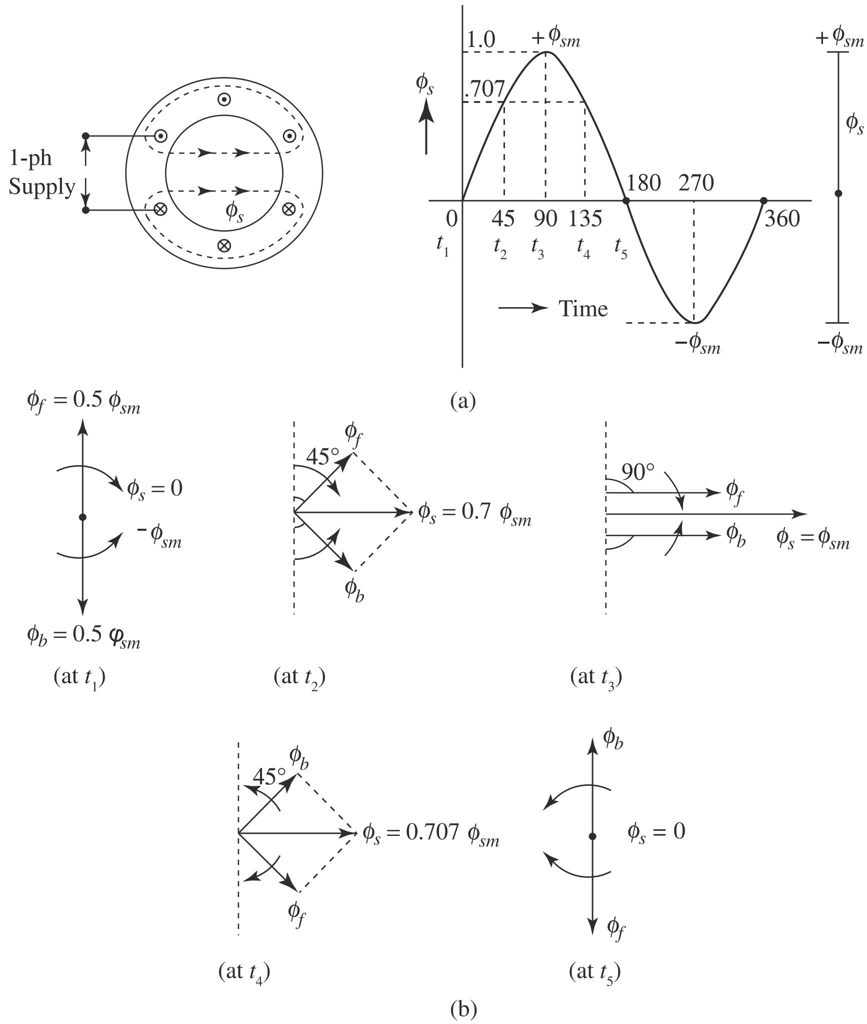


Fig. 6.5 *(a) An alternating flux ϕ_s whose value changes from $+\phi_{sm}$, to $-\phi_{sm}$ is produced by the stator mmf (b) As the stator flux changes with time, the components of the stator flux, ϕ_f and ϕ_b rotate in opposite directions at synchronous speed*

The complete torque-speed curves corresponding to each of the component fields considered independently are shown in Fig. 6.6. Note that for backward field the torque-speed curve is to be drawn in the reverse direction.

Torque developed by the two rotating fields are acting in opposite directions, each field develops a torque that tends to rotate the rotor in the direction in which the field rotates. The resultant torque developed on the rotor is the summation of the torques produced by the two rotating fields. It may be noted that torque-speed curves have been drawn for a speed range of $-N_s$ to $+ N_s$ The resultant torque-speed curve

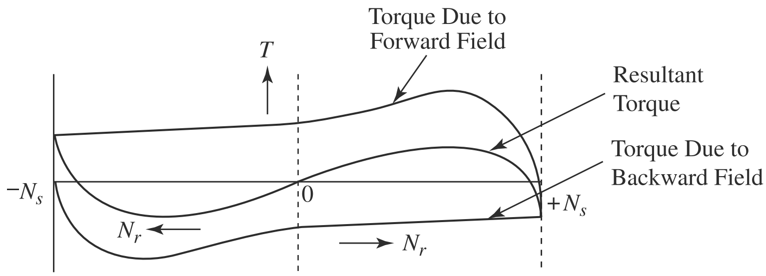


Fig. 6.6 *Torque-speed characteristic of a single-phase induction motor as derived from double revolving field theory*

is also shown in the figure. From the resultant torque-speed curve the following can be observed:

(a) Average torque at standstill is zero and, therefore, the motor is not self-starting (at zero speed, torque developed by the forward and backward fields cancel each other).

(b) When the rotor is given an initial rotation in any direction, the average torque developed causes the rotor to continue to rotate in the direction in which it is given an initial rotation.

(c) The average torque becomes zero at some value of speed below the synchronous speed (whereas in a three-phase induction motor, torque is zero at synchronous speed). This indicates that a single-phase motor operates with a greater percentage of slip at full-load than a corresponding three-phase induction motor.

Brief descriptions of two theories have been given to explain why a single-phase induction motor will continue to rotate in a direction in which the rotor is given some initial rotation. It is seen that the two theories supplement rather than contradict each other.

To make the motor self-starting, some starting device or method will have to be employed. Single-phase induction motors are named according to the starting methods employed.

6.2.2 Methods of Making Single-phase Induction Motors Self-starting

As single-phase induction motors are not self-starting, some special method must be employed to provide the initial torque. The various methods employed are described as follows:

Split-phase Method The principle of split-phase method is to create at starting a condition similar to a two-phase stator winding carrying two-phase currents so that a rotating magnetic field is produced. This is achieved by providing, in addition to main single-phase winding, a starting winding which is displaced in space by 90° with the main winding on the stator slots. The resistance and reactance (inductive or capacitive) of the two winding circuits are made such that, when connected across a single-phase supply the currents flowing through the windings will have a considerable time-phase difference. Thus, although the supply is a single-phase one, currents flowing through the two windings are to some extent similar to two-phase currents. A rotating magnetic field will, therefore, be produced which will develop starting torque on the rotor. Once the motor has started, this extra starting winding may be cut out from the supply by some device.

There are different methods of creating a time-phase difference between the currents flowing through the two windings fed from a single-phase supply. Motors are named according to the method applied for achieving this time-phase difference. These are explained as follows:

(a) Split Phase Resistance-Start Motor An auxiliary winding, also called starting winding is placed at 90° with the stator main winding as shown in Fig. 6.7. The ratio of resistance to reactance of this auxiliary winding is made higher than the main winding. This is achieved by using, for the auxiliary winding, thinner wires of higher resistivity as compared to the material used for the main winding.

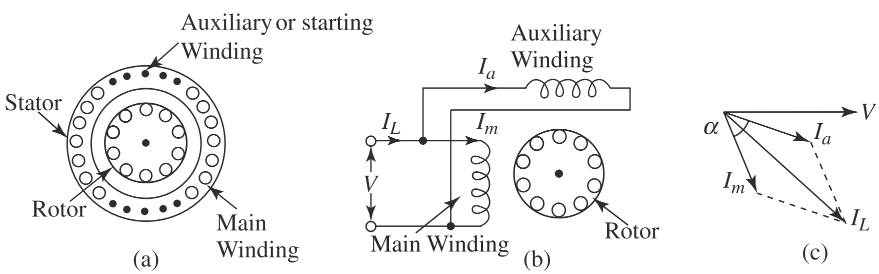


Fig. 6.7 *(a) Cross-sectional view of a split-phase type single-phase induction motor (b) Winding connections (c) Time-phase relationship between main and auxiliary winding currents*

The auxiliary winding is connected in parallel with the main winding across a single-phase supply. Since the ratio of resistance to reactance, of the auxiliary winding is more than the main winding, the angle of lag of current I_a drawn by the auxiliary winding will be less than the main winding current, I_m as shown in Fig. 6.7(c). The angle between the currents I_a and I_m is less than 90°. It can be seen from the figure that the axes of the main and auxiliary windings have a space-phase displacement of 90°. The current drawn by them have a time-phase displacement of less than 90°. The resultant flux created by the two currents, which have a time-phase displacement and are flowing through two windings having space-phase displacement, will be rotating in nature, This rotating field will produce torque on the rotor which will cause the rotor to rotate. It may be understood here that once the rotor starts rotating, it will continue to rotate and, therefore, there would be no need of the auxiliary winding to remain connected across the supply. After the motor has reached approximately 75 per cent of synchronous speed, the main winding can develop nearly as much torque as the two windings. The auxiliary winding is cut out of the circuit once the motor has picked up 70 to 80 per cent of synchronous speed with the help of a centrifugal switch or an over-current relay.

Working of a Centrifugal Switch Figure 6.8 illustrates the action of a centrifugal switch. With the rotor at standstill, the pressure of the spring on the bakelite piece C keeps the contacts PP' closed. The starting winding circuit which is connected across the supply terminals through the centrifugal switch terminals *PQ* remains closed. The auxiliary winding flux helps the motor to start. As the motor picks up speed, centrifugal force acts on the parts *a* and *b*.(see Fig. 6.8(b)). These two parts are pushed backwards. They in turn push against the spring pressure, the cup-shaped bakelite piece *C* forward. The spring pressure is such that at about 75 per cent of the synchronous speed the centrifugal switch contact points *PP′* open, thereby cutting out the auxiliary winding from the supply.

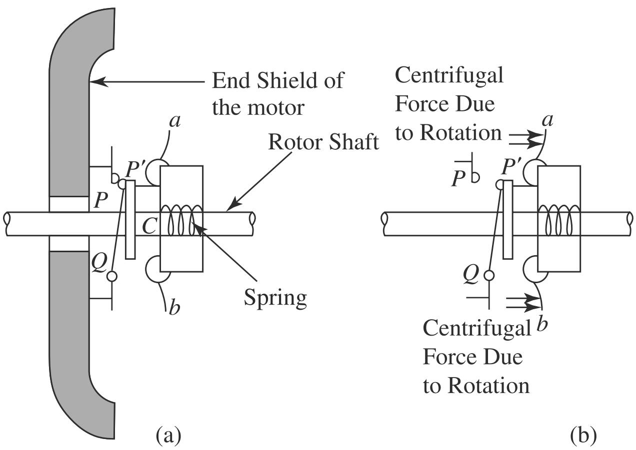


Fig. 6.8 *(a) A simplified diagram of a centrifugal switch used in the auxiliary winding circuit (b) Contacts of the centrifugal switch opens out due to centrifugal force acting on the bakelite movable cup when the rotor picks up about 75 percent speed*

Working of an Over-Current Relay A single phase induction motor, like a polyphase induction motor takes heavy current from the line during starting when started direct-on-line. Advantage is taken of the high starting current to operate an electromagnetic type over-current relay which performs the same function as the centrifugal device. Connection diagram for such an over-current relay is shown in Fig. 6.9.

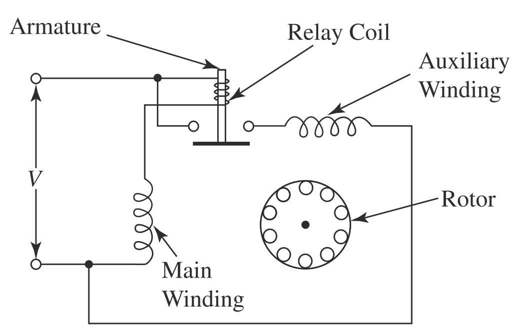


Fig. 6.9 *Wiring diagram for an electromagnetic relay used in a split-phase motor*

The relay has a coil which is connected in series with the main winding. The auxiliary winding is connected across the supply through a normally open contact of the relay. Since split-phase motors are usually started direct-on-line, the initial current inrush may be as high as five to six times the rated current. During the starting period, when the main winding current is high, the armature of the relay will be drawn upwards, thereby closing the relay contacts. The auxiliary winding will, therefore, get connected across the supply thus helping the motor to start rotating. As the rotor starts rotating, the line current gradually goes on decreasing. After the motor reaches proper speed, the main winding current drops to a low value and causes the armature of the relay to fall downwards and open the contacts, thereby cutting out the auxiliary winding from the supply. Such relays are located outside the motor so that they can be easily serviced or replaced. As centrifugal switches are mounted internally, their serving or replacement is not as simple as an externally mounted over-current relay.

Torque-Speed Characteristic Torque speed characteristic of a split phase type single-phase induction motor has been shown in Fig. 6.10. In figure, N_r represents the full-load speed whereas N'_r represents the switch operating speed. Characteristic *A* shows the torque developed when both the windings are connected to the supply. Curve *B* is the torque develop due to main winding only.

The torque-speed characteristic of the motor will follow curve *A* up to the speed N_r'. At this speed the centrifugal switch or the electromagnetic relay disconnects the starting winding. The torque developed by the main winding alone is sufficient to carry the load forward up to the rated speed N_r. Torque-speed curve beyond speed N_r' will follow curve *B* as shown in Fig. 6.10. A well designed split-phase motor will have a starting torque as much as the normal running torque. Starting torque as high as 150 per cent of normal running torque is also very common.

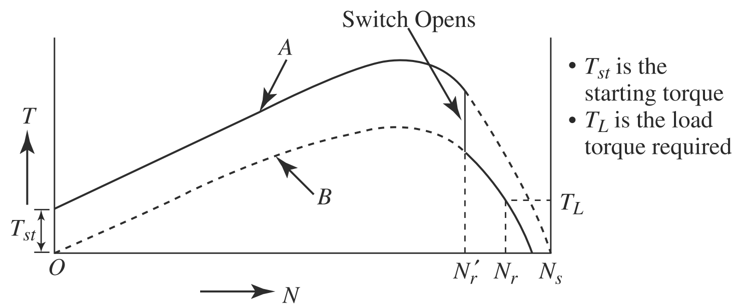


Fig. 6.10 *Torque-speed characteristic of a standard split-phase motor*

Method of Changing Direction of Rotation The direction of rotation of a split phase motor is determined by the way the main and auxiliary windings are connected to the supply with respect to each other. Since the ratio of *R:X* of the auxiliary winding is higher than the main winding, its current and flux produced will be leading the main winding current and flux. This leads to the fact that the field will rotate in a direction from a given auxiliary winding pole to an adjacent main winding pole of the same polarity. The direction of rotation of split phase motors can be reversed by reversing the terminal connections of either the main or the auxiliary winding. For motors using centrifugal switches, the reversal of connections of the auxiliary winding should be done when the rotor is at rest or at a slow speed, since otherwise the reversal will not be effective, as the centrifugal switch will be open at normal speed.

Application Split phase induction motors can be designed to develop starting torques in the range 150–200 per cent of full-load torque. Such motors are, therefore, used where starting torque requirement is moderate. Constant speed and moderate starting torque make this type of motor suitable for applications like washing machines, drill, press, oil burners, etc. Because of the varying magnitude of the cross-field, the torque developed under load is pulsating in nature. This makes the motor somewhat noisy.

(b) Split-phase Capacitor Motor In resistance split-phase method, high value of auxiliary winding circuit resistance creates phase angle between the currents of the main and auxiliary windings. Time-phase difference between the currents of the main winding and auxiliary winding can also be produced by connecting a capacitor in the auxiliary winding circuit.

Three different arrangements are possible which are described as follows:

(i) *Permanent-split Single-Value Capacitor Type* In this type, one capacitor is connected in series with the auxiliary winding as shown in Fig. 6.11. There is no centrifugal switch or over-current relay connected in the auxiliary winding circuit. Thus, this winding along with the capacitor remains energised for both starting and running conditions. Since the same capacitor is used in series with the auxiliary winding for both starting and running condition, the motor is called a permanent split capacitor motor. By using suitable capacitance in the auxiliary winding circuit, it is possible to make a time-phase difference of practically 90° between the currents, so that the motor becomes like a two-phase motor. The starting torque achieved therefore is higher than the resistance split-phase motor of the same rating. As the auxiliary winding together with the capacitor remain in the circuit during all the period of motor operation, the power factor of the motor is improved considerably. Since the auxiliary winding is to remain energised for all time, it should be designed accordingly. The value of the capacitor can be so chosen that time-phase difference between the currents I_m and I_a is exactly 90°. The rotating field produced will be a uniform one and the performance of the motor will be less noisy. Due to the leading current drawn by the auxiliary winding, the magnitude of the resultant current I_L which is drawn from the line is reduced as compared to a resistance split-phase motor explained earlier. The full-load efficiency of this type of motor will also be higher.

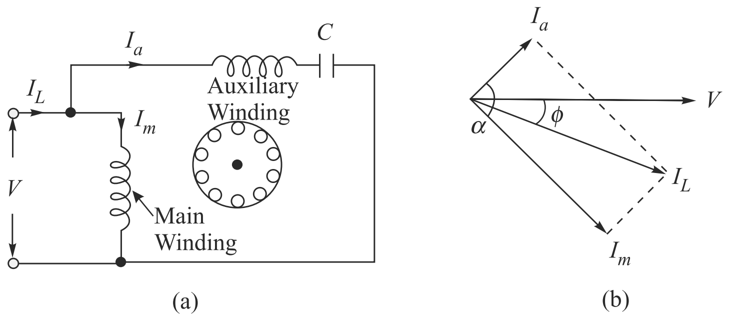


Fig. 6.11 *(a) Permanent-split single value capacitor type induction motor,*
(b) Time-phase relationship between main and auxiliary winding currents

The capacitor to be used should be of full-time rating, since it will always be in the circuit. The use of an electrolytic type capacitor is therefore prohibited. Paper-spaced oil filled type capacitors are used. The size of such capacitors, however, is much larger than electrolytic ones and are more expensive. The auxiliary winding is to be designed to have continuous duty rating. A permanent split capacitor motor is therefore more expensive than the equivalent resistance split-phase motor.

(*ii*) *Two-Value Capacitor Type* If a large starting torque is required to be developed, two capacitors as shown in Fig. 6.12 are used. During starting the large value capacitor C_1 is connected in circuit and under running condition the small value capacitor C_2 remains connected in the circuit through the operation of a switch *S* disconnecting C_1. Switch S is a centrifugal switch which remains normally closed. As the motor picks up about 75 per cent of its rated speed, the switch opens up.

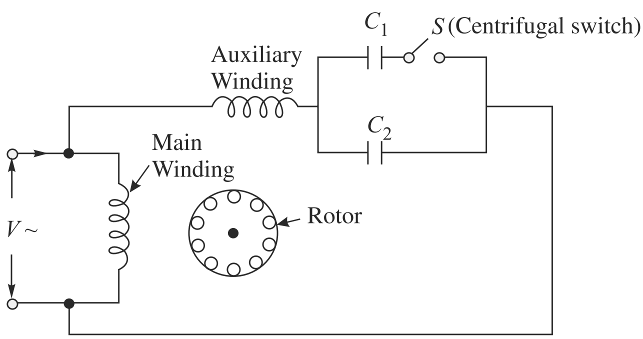


Fig. 6.12 *Two value capacitor type single-phase induction motor*

Electrolytic capacitor C_1 is used to develop high starting torque and the oil capacitor C_2 is used to help the motor work with improved power factor. Since the auxiliary winding is designed to be in the circuit for all the time, failure of the centrifugal switch to open up at a particular speed will not cause any harm to the winding. The electrolytic capacitor C_1 will however break down if the switch *S* does not open up. If the switch remains open all the time, the motor may be able to start if the load-torque requirement is low.

(iii) *Single-Value Capacitor-start Type* In this type, one electrolytic capacitor along with the centrifugal switch is connected in series with the auxiliary winding as shown in Fig. 6.13. Once the motor has picked up speed, the centrifugal switch operates and opens up the auxiliary winding circuit. By choosing a correct value of the capacitor, it is possible to create phase-split angle of nearly 90°. In a resistance split-phase motor the number of winding turns of the auxiliary winding is to be kept low to keep its *R*:*X* ratio high (less number of turns means lower value of *X*). In a capacitor motor however, this condition is unnecessary as the capacitance of the capacitor can more than neutralise the inductive reactance of the auxiliary winding. There are thus more number of turns in the auxiliary winding in a capacitor motor than a resistance split-phase motor. This leads to larger number of ampere-turns and hence a larger rotating flux and starting torque. The starting torque of a capacitor-start motor is of the range of three to four times its full-load torque whereas for a resistance split-phase motor it is about 1.5 times only.

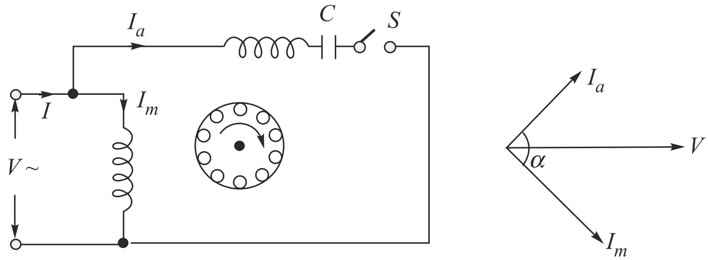


Fig. 6.13 *Single value capacitor type induction motor*

The capacitor used is a dry-type electrolytic one, designed for alternating current use only. It is different from the dc electrolytic type capacitor used as a filter in radios, amplifiers and television sets. It is because of the development of inexpensive type capacitors that the capacitor-start motors have become very popular and found wide applications, where high starting torque is a requirement. Starting capacitors used are designed for a definite duty cycle and not for continuous use. A faulty switch may keep the auxiliary winding together with the capacitor energised for a long period and thereby shorten their life spans. Reversal of direction of rotation can be obtained by changing the terminal connections of one of the windings. Speed control, if required, can be achieved by controlling the voltage applied across the winding terminals through a voltage regulator.

Capacitor-start motors are suitable for applications in pumps and compressors. Such motors are therefore widely used in refrigerators.

(c) Shaded Pole Method Single-phase induction motors using shaded poles called shaded pole motors are manufactured for very small ratings and are used in applications where the starting torque requirement of the load is very low. Some of the applications of shaded-pole type induction motors are in fans, blowers, fans used in air-heaters, slide and film projectors, advertising display devices, etc. The outstanding features are the low initial cost, small size and ruggedness. An advantageous feature in such small motors is that the starting current (blocked-rotor current) is only slightly higher than the full-load current, so that stalled motor (blocked rotor) condition is not harmful to the motor windings.

The cross-sectional view of a shaded-pole type motor is shown in Fig. 6.14. The stator poles shown are projected type. Short-circuited coils known as shading coils (rings) are fixed on one portion of each pole. A pole on which a shading ring is fixed is called a shaded pole. Shading coils can be of thick single turn in the form of a ring or have a number of short-circuited turns. The rotor is of conventional squirrel-cage type. The construction of the stator may be any one of the types shown in Figs 6.14(a) and (b).

How starting torque is produced due to the introduction of the shading coils is explained as follows:

When single-phase supply is applied across the stator winding, an alternating field is created. Flux distribution in the pole area will be nonuniform due to the shading coils on the poles.

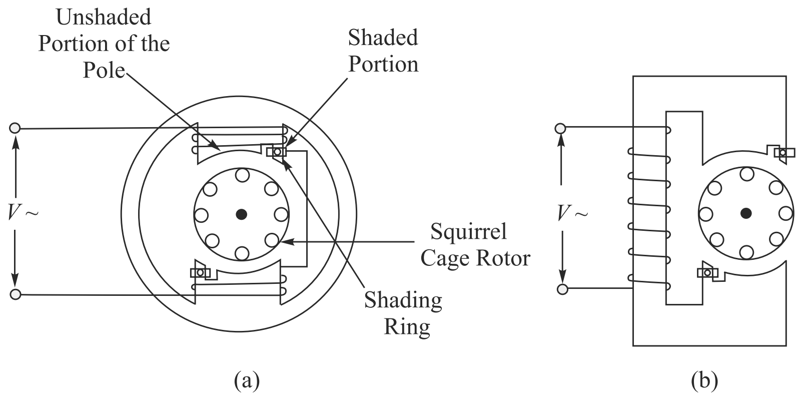


Fig. 6.14 *Shaded pole type induction motors of different designs*

Three instants of time to t_1, t_2, t_3 of the flux wave will be considered to examine the effect of shading coil. In Fig. 6.15(a) is shown the wave shape of sinusoidally varying stator current and stator flux. To understand the distribution of flux in the poles, as the stator current varies, three instants of time t_1, t_2, and t_3 have been considered. The flux distribution at these instants of time in the poles have been shown in Fig. 6.15(b).

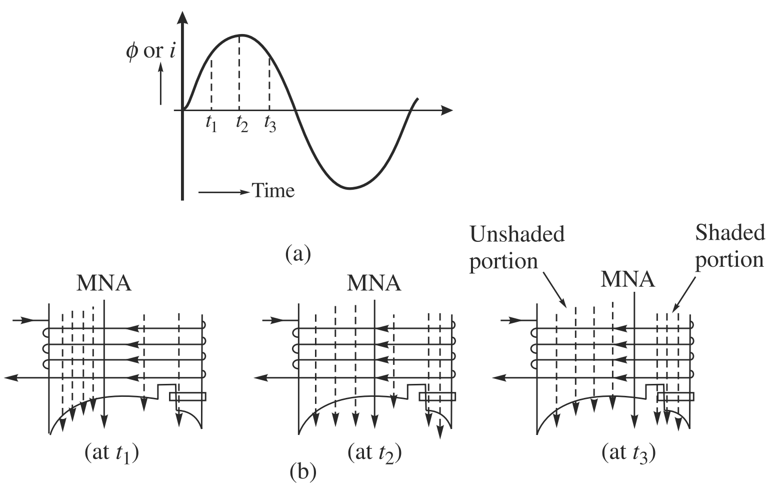


Fig. 6.15 *(a) Alternating flux produced by alternating current flowing through the stator pole winding, (b) Distribution of flux in the pole and the position of the magnetic neutral axis (MNA) at three instants of time of the current wave*

At time t_1, the rate of rise of current in the coil and therefore the rate of rise of flux in the pole is high. The rate of change of flux will induce an emf in the shading coil, the shading coil being short-circuited, current will flow in it. This current will produce a flux which by Lenz's law will oppose the pole flux created by stator current in the shaded area of pole. As a result, the distribution of flux will be nonuniform in the pole. There will be more flux in the non-shaded portion of the pole than in the shaded portion. The position of the magnetic neutral axis (MNA) will be slightly to the left instead of being at the centre of the pole. (Physical axis of a pole always lies along the centre of the pole, whereas magnetic neutral axis is along a line with equal number of lies on both sides. When flux distribution is uniform, physical neutral axis and magnetic neutral axis lie together).

At the instant of time t_2, current is at its maximum and therefore the rate of change of flux is minimum. So, the induced emf in the shading coil will be negligible and hence practically no opposing flux will be created by the shading coil. The distribution of flux in the pole will therefore be uniform and the position of the MNA will be shifted to the centre of the pole.

At the instant of time t_3, current is gradually falling in the stator coil and therefore, the flux is reducing. Since there is a change of flux, emf will be induced in the shading coil. The flux produced by a current in the shading coil will now oppose the reduction of flux in the pole in the shaded area. As a result, in the unshaded area of the pole there will be more reduction of flux than in the shaded area. The position of the MNA will now be shifted to the right from the centre of the pole. Thus for the duration of current flow from t_1 to t_3 it is observed that the axis of the flux shifts gradually from left to right . In the negative half-cycle, the direction of the flux will reverse but the shift of MNA will be from left to right. Thus in every half-cycle there will be a shift of the MNA from left to right, i.e., from the non-shaded area of the pole to the shaded area. This gives, to some extent, a rotating field effect which may be sufficient to provide starting torque to the squirrel-cage rotor. The starting torque so produced will, however, be very small. Shaded pole motors are therefore suitable for applications where starting torque requirement is low.

Since there will be considerable amount of I^2R-loss in the shading ring, the efficiency of such motors will be very low. For tiny motors, however, such low efficiency may not be objectionable, as the total quantity of power loss is also very small. The direction of rotation of the rotor is fixed, i.e., the direction is from non-shaded side to shaded side of the stator poles. If reversal of direction of rotation is desired, then two sets of shading rings on the two sides of the poles are to be provided. A suitable switching arrangement will close one set of shading rings and keep open the other set of rings for a particular direction of rotation.

(d) Reluctance Start Method In this method, starting torque is achieved by creating nonuniform air-gap of the salient poles as shown in Fig. 6.16. Due to the variation of reluctance, the flux in the portion where there is greater air-gap will be more in phase with the current. There will be a greater lag between the flux and the current producing that flux, in the portion of the pole where reluctance is low, i.e., the air-gap is smaller.

Since both the fluxes are produced by the same current, the flux across the larger air-gap will lead the flux across the smaller air-gap. To explain this further, two coils, one air-cored and one iron-cored are shown in Fig. 6.17(a). Assume that there is no I^2R loss in the winding.

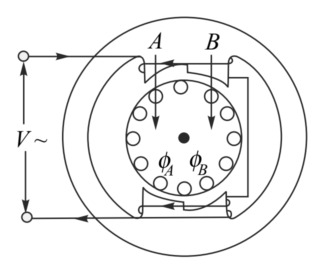


Fig. 6.16 *Reluctance-start single-phase induction motor*

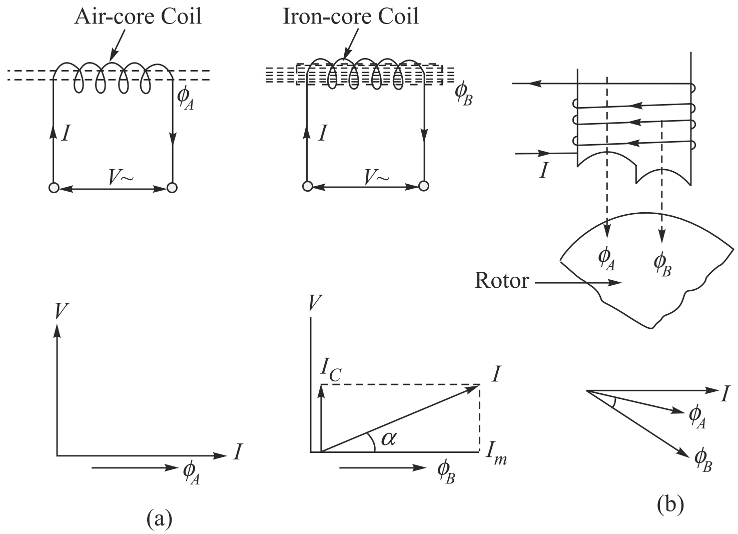


Fig. 6.17 *(a) Phase relationship of current and flux in an air-core and iron-core coil, (b) Phase relationship between the flux produced in the two portions of the pole of a reluctance-start motor*

Since in the air-core, there is no core-loss, I will lag V by 90°. The whole of I is spent in creating the flux and is called the magnetising current, the flux produced, ϕ_A and I are in time phase. In iron-core case there is core-loss. A component, I_c of I is spent to supply the core-loss. Another component, I_m produces the flux ϕ_B. The resultant of I_c and I_m is I as shown in the figure. There is a time-phase difference between I and ϕ_B, because of the presence of iron-core.

In a reluctance-start induction motor, the two portions of a pole have different amounts of iron. Hence the fluxes created in these portions will lag the current by different angles, ϕ_A being more in phase with I than ϕ_B .The two fluxes, ϕ_A and ϕ_B are displaced in time. The magnetic axis will shift across the poles from the longer air-gap region to the shorter air-gap region. This will enable the rotor to start rotating in the same direction. Once started, the rotor will continue to rotate like other types of single-phase induction motors. It is evident that the direction of rotation of such motors is fixed by the construction and cannot be reversed.

For most of the small power applications, the shaded pole motors are preferred. Reluctance-start motors have limited use in applications where the starting torque requirement is very low.

EXAMPLE 6.1 *The impedance of the main and auxiliary winding of a single phase induction motor are 3 + j3 ohms and 6 + j3 ohms respectively. What will be the value of the capacitor to be connected in series with the auxiliary winding to achieve a phase difference of 90° between the currents of the two windings?*

Solution Let applied voltage, V be the reference vector as shown in Fig. 6.18(b). Current through the main winding,

$$I_m = \frac{V\angle 0}{3+j3} = \frac{V\angle 0}{4.24\angle 45°} = \frac{V\angle -45°}{4.24}$$

Current through the auxiliary winding

$$I_a = \frac{V\angle 0}{6+j3} = \frac{V\angle 0}{6.7\angle 26.5°} = \frac{V\angle -26.5°}{6.7}$$

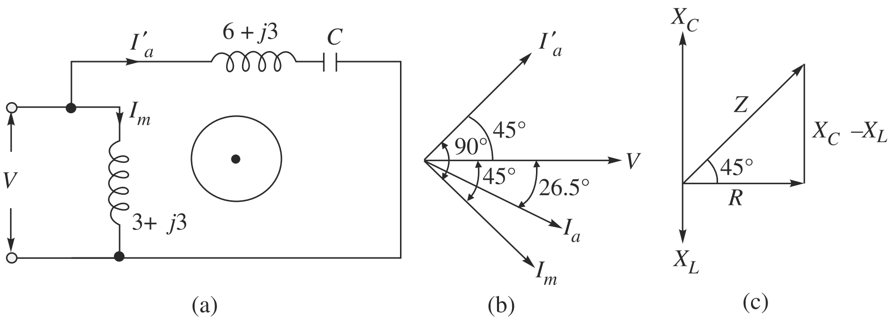


Fig. 6.18 *(a) Circuit diagram (b) Phasor diagram (c) Impedance triangle*

I_a', the current flowing through the auxiliary winding after connecting a capacitor C in series, should make an angle of 90° with I_m or make an angle of 90° – 45° = 45° with the applied voltage V.

Since current I_a' needs to be leading the voltage V by an angle of 45° (as against lagging by 26.5° earlier), the capacitive reactance of the auxiliary circuit is greater than the inductive reactance.

From Fig. 6.18(c),

$$\tan 45° = \frac{X_C - X_L}{R}$$

$$1 = \frac{\frac{1}{\omega C} - 3}{6}$$

$$C = 353.6 \text{ micro farad}$$

EXAMPLE 6.2 *A.50 Hz split phase induction motor has a resistance 5 Ω and an inductive reactance of 20 Ω in both main and auxiliary windings. Determine the value of resistance and capacitance to be added in series with auxiliary winding to send the same current in each winding with a phase difference of 90°.*

Solution Since impedance of both the main and auxiliary windings is the same, their currents will make equal phase angle with the applied voltage, V. However, when an additional resistance and a capacitance are connected in series with the auxiliary winding, auxiliary winding current, I_a' will make a phase angle of 90° with the main winding current, I_m as shown in Fig. 6.19 (b).

$$Z = 5 + j\,20 = 20.6\angle 76°$$

$$I_m = I_a = \frac{V}{Z} = \frac{V\angle 0°}{5+j20} = \frac{V\angle 0°}{20.6\angle 76°} = \frac{V\angle -76°}{20.6}$$

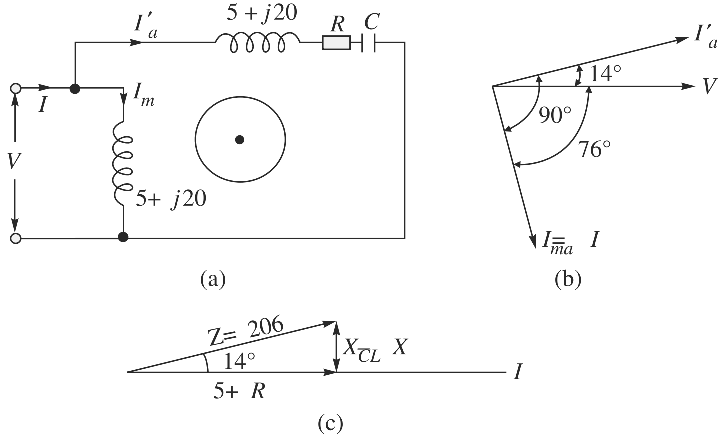


Fig. 6.19 *(a) Circuit diagram (b) Phasor diagram (c) Impedance triangle*

As shown in Fig. 6.19(b), auxiliary winding current I_a' will now make an angle of 14° with V.

Note that introduction of C in the auxiliary winding will reduce the total reactance of the circuit to $X_C – X_L$ from its initial value of X_L. Since current has to be kept constant in both the windings, a resistance R is introduced. The total impedance of the auxiliary winding circuit will remain as that of the main winding, i.e., equal to 20.6 Ω. The purpose of C has been to create a phase difference between I_a and I_m. The purpose of adding an additional resistance R in the circuit has been to keep I_a equal to I_m even after inclusion of C in the auxiliary winding circuit.

After the introduction of an additional resistance, R and a capacitor C, in the auxiliary winding circuit we get,

$$\cos 14° = \frac{5 + R}{Z}$$

$$5 + R = Z \cos 14°$$

$$R = Z \cos 14° – 5$$

$$= 20.6 \cos 14° – 5 = 19.99 – 5$$

$$R = 14.99\ \Omega$$

Again,

$$\sin 14° = \frac{X_C - X_L}{Z}$$

$$X_c = X_L + Z \sin 14°$$

$$X_c = 20 + 20.6 \times \sin 14° = 24.98\ \Omega$$

$$C = 127 \times 10^{-6}\ \text{F}$$

6.2.3 Speed Control of Single-phase Induction Motors

Single-phase induction motors are used in many domestic electrical appliances like in ceiling fans, pumps, washing machines, etc. It may be necessary to change the speed of the motors used. By varying the voltage applied to the motor, the speed can be changed. In earlier days a variable resistance was connected in series with the motor in which a portion of the input voltage was dropped and the remaining voltage was made available across the motor. The speed of the motor and also the torque developed would change with reduced voltage applied across it. This method of speed control was convenient but a good amount of power was lost in the variable resistance and hence the method was not efficient.

Resistance controllers (fan speed regulators), as for example, used in controlling speed of fans, are gradually being replaced by solid state controllers. The method of providing reduced voltage across the motor electronically; and a simple electronic controller for a fan motor are explained as follows.

A triac can be used to control voltage in both positive and negative half-cycles. This can be done by changing the firing angle. Figure 6.20 shows the circuit diagram for speed control of an ac series motor using a triac. If the firing angle is increased, the voltage available across the motor will decrease changing the speed of the motor. However, too high value of firing angle will cause non-sinusoidal voltage to appear across the load, i.e., the motor. This will lead to overheating of the motor due to harmonics and also create noise in the motor operation.

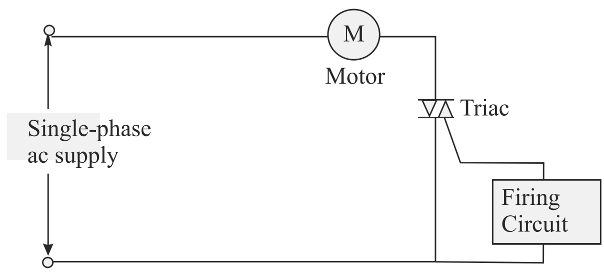


Fig. 6.20 *Basic circuit diagram for speed control of single-phase series motor using a triac*

Figure 6.21 shows a solid state fan regulator where the speed of a fan motor (single-phase induction motor) is controlled. The conduction time of the triac has been controlled by a bidirectional diode, called a diac.

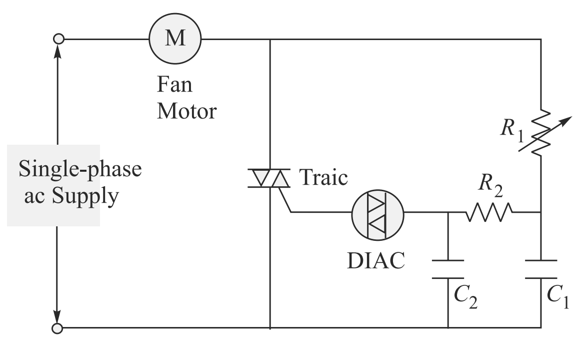


Fig. 6.21 *Circuit for solid state speed control method for a fan motor*

The circuit shown is a double *R–C* phase shift circuit. The potentiometer R_1 controls the gate current to the diac by controlling the voltage across C_2. At a particular voltage the diac operates and controls the firing angle of the triac in each half-cycle, thus controlling the voltage available across the fan motor. Thus, the diac works as the triggering device for the triac providing symmetrical triggering in both positive and negative half cycles of the supply voltage wave.

6.3 EQUIVALENT CIRCUIT OF A SINGLE-PHASE INDUCTION MOTOR

According to double revolving field theory, discussed earlier, a single-phase induction motor can be considered equivalent to two motors with a common stator winding and two rotors rotating in opposite directions at synchronous speed.

At standstill condition, these two equivalent rotors develop equal and opposite torques and hence the resultant torque i.e., net torque is zero.

Let the supply voltage to the stator winding be V_1 volts. The resistance and leakage reactance of he stator winding are R_1 and X_1 respectively.

Since an induction motor at standstill is exactly like a short circuited transformer, it will be helpful to refer to the equivalent circuit of a transformer, which have been dealt with in the chapter on transformers.

Figure 6.22(a) shows the equivalent circuit of a transformer with the secondary quantities referred to the primary side. An induction motor has a short circuited rotor winding. When the rotor is at standstill, the motor is the same as that of a short-circuited transformer. Hence the equivalent circuit of an induction motor is as shown in Fig. 6.22(b).

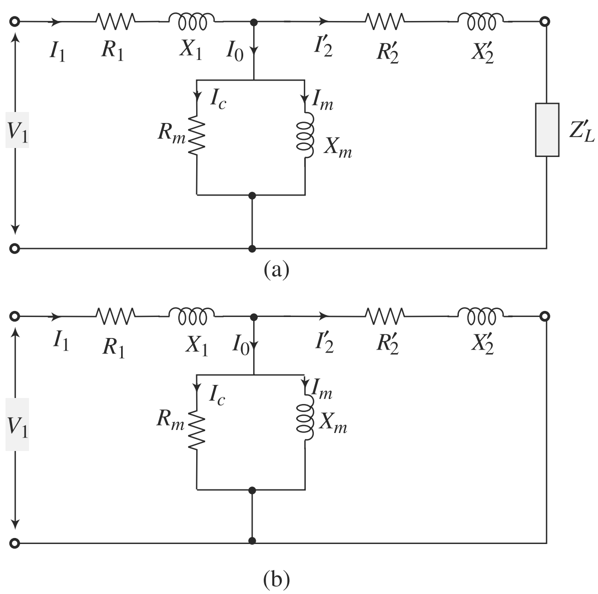


Fig. 6.22 *(a) Equivalent circuit of a single-phase transformer; (b) Equivalent circuit of a short-circuited transformer which is also the equivalent circuit of a single-phase induction motor*

In the equivalent circuit $I_c^2 R_m$ represents the core loss. For simplicity if we consider the core loss as part of the rotational loss, the resistance, R_m representing the core-loss can be removed from the equivalent circuit.

By neglecting the branch representing the core loss, the equivalent circuit of a single-phase induction motor with the main winding on the stator is represented as in Fig. 6.23.

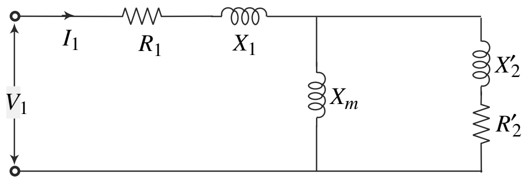


Fig. 6.23 *Equivalent circuit of a single-phase induction motor at standstill*

In the equivalent circuit, R_1 is the resistance of the main winding, X_1 is the leakage reactance of the main winding; X_m is the magnetizing reactance; R_2' is the resistance of the rotor circuit referred to the stator winding and X_2' is the standstill rotor circuit leakage reactance referred to the stator winding. The supply voltage to the main winding is V_1 and I_1 is the current drawn by the main winding from the supply.

According to double revolving field theory, the alternating magnetic field produced by the single-phase supply applied across the stator main winding, can be resolved into two revolving fields which are equal to half the magnitude of the alternating field but are rotating in opposite directions at synchronous speed.

We can assume that the two revolving magnetic fields or fluxes are acting on two separate imaginary rotors. Accordingly, the equivalent circuit of a single-phase induction motor with the main winding and two equivalent rotors at standstill condition is shown in Fig. 6.24. The standstill impedance of each of the imaginary rotor has been shown as $\frac{R_2'}{2} + j\frac{X_2'}{2}$. Impedance of the forward moving rotor is Z_f. Impedance of the backward moving rotor is Z_b. At standstill condition the two circuits representing Z_f and Z_b are identical. Let E_f and E_b be the voltage induced in the rotor winding by forward field flux and backward field flux respectively. The resultant induced voltage in the rotor is E.

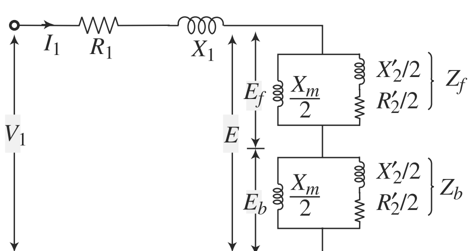


Fig. 6.24 *Equivalent circuit of a single-phase induction motor at standstill*

Now let the motor be started with the help of its auxiliary winding (starting winding) and the auxiliary winding is disconnected after the rotor has picked up speed. The rotor will rotate at a speed somewhat less than the synchronous speed. The slip of the rotor with respect to the forward field is, S. The effective rotor circuit resistance becomes $\frac{R_2'}{2S}$. To understand this, let us assume that the rotor induced emf at standstill is E_f and at a slip s is SE_f. The rotor leakage reactance at stand still is X_2' and at a slip s is SX_2'.

$$\text{Rotor current, } I_2 = \frac{SE_f}{\sqrt{\left(\frac{R_2'}{2}\right)^2 + \left(\frac{SX_2'}{2}\right)^2}} = \frac{E_f}{\sqrt{\left(\frac{R_2'}{2S}\right)^2 + \left(\frac{X_2'}{2}\right)^2}}$$

[Dividing both numerator and denominator by S].

The effective rotor circuit resistance is $\frac{R_2'}{2S}$.

Now, slip of the rotor with respect to the backward field is $(2 - S)$. This is because for the forward field,

$$S_f = \frac{N_s - N_r}{N_s}$$

The backward field is rotating at $-N_s$ rpm.

$$\text{So for backward field, } S_b = \frac{-N_s - N_r}{-N_s}$$

$$= \frac{-2N_s - N_s - N_r + 2N_s}{-N_s}$$

$$= \frac{-2N_s}{-N_s} + \frac{N_s - N_r}{-N_s}$$

or, $$S_b = 2 - \frac{N_s - N_r}{N_s} = 2 - S$$

Therefore, the effective rotor circuit resistance with the backward field is $R_2'/2(2 - S)$.

By taking into consideration the effect of forward and backward fields on the two rotor circuit resistances, the equivalent circuit of a single-phase induction motor under running condition is drawn as shown in Fig. 6.25.

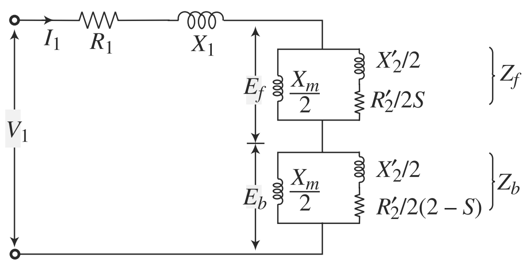


Fig. 6.25 *Equivalent circuit of a single-phase induction motor under running condition*

The equivalent circuit has two parallel branches. The equivalent circuit can be simplified by calculating Z_f and Z_b as

$$Z_f = \frac{\left(\frac{R_2'}{2S} + j\frac{X_2'}{2}\right)\left(j\frac{X_m}{2}\right)}{\frac{R_2'}{2S} + j\frac{X_2'}{2} + j\frac{X_m}{2}} = R_f + jX_f$$

and

$$Z_b = \frac{\left(\frac{R_2'}{2(2-S)} + j\frac{X_2'}{2}\right)\left(j\frac{X_m}{2}\right)}{\frac{R_2'}{2(2-S)} + j\frac{X_2'}{2} + j\frac{X_m}{2}} = R_b + jX_b$$

The simplified equivalent circuit is shown in Fig. 6.26.

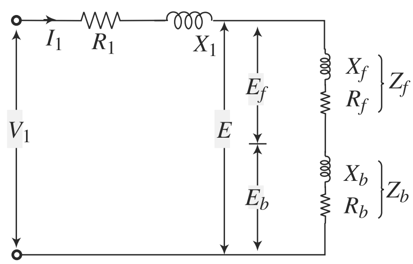


Fig. 6.26 *Simplified equivalent circuit of a single-phase induction motor*

6.4 TESTING OF SINGLE-PHASE INDUCTION MOTORS

The equivalent circuit parameters can be determined by conducting two tests on the motor. They are blocked rotor test and no-load test. These tests are similar to those conducted on three-phase induction motors described earlier. For single-phase induction motors these tests are conducted by exciting one winding at a time keeping the other winding open. However, except for capacitor run motors, these tests are conducted with the main winding only. Procedures for conducting blocked rotor test and no-load test are explained below.

6.4.1 Blocked Rotor Test

The circuit diagram and the simplified equivalent circuit have been shown in Fig. 6.27. This test is conducted by adjusting the input voltage with the help of an auto-transformer until the main winding carries the rated current.

Under blocked rotor condition, motor speed is zero, so slip, $s = 1$.

The magnetizing reactance X_m is usually higher than the rotor circuit impedance. That is, X_m is much higher than R_2' and X_2'. In the equivalent circuit under blocked rotor condition, the high impedance path can be considered as open circuit. For the blocked rotor test, the approximate equivalent circuit will be as shown in Fig. 6.27(b). In this circuit, the magnetizing reactance X_m has been omitted. The stator impedance of the main winding in series with the rotor circuit impedance in two equal sections have been shown. Let the readings of the blocked rotor test with the main winding excited be:

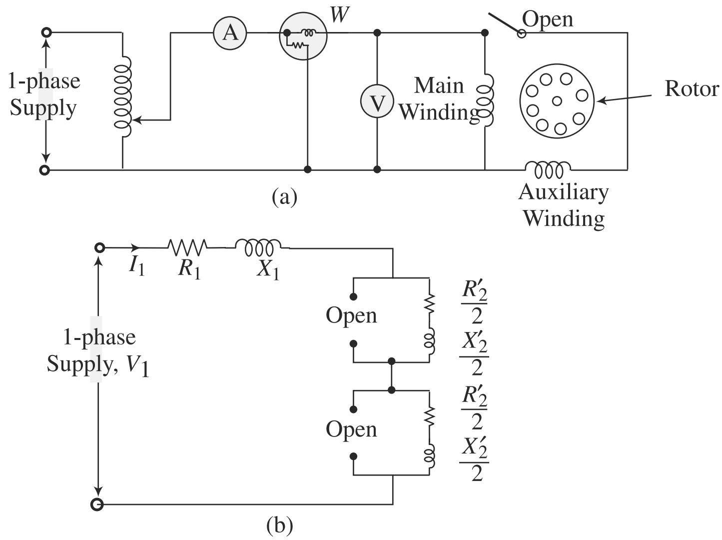


Fig. 6.27 *(a) Connection diagram for blocked rotor test; (b) Equivalent circuit under blocked rotor condition*

Applied voltage across main winding on blocked rotor condition = V_{mb}
Current through the main winding = I_{mb}
Power consumed = W_{mb}
Total resistance in the circuit = $R_1 + R_2'/2 + R_2'/2 = R_1 + R_2'/2$

$$\text{Total impedance of the circuit, } Z_{mb} = \frac{V_{mb}}{I_{mb}}$$

Again, $I_{mb}^2\ (R_1 + R_2') = W_{mb}$

$$R_1 + R_2' = R = \frac{W_{mb}}{I_{mb}^2}$$

$$X_{mb} = \sqrt{Z_{mb}^2 - R^2} = X_1 + X_2'$$

We will assume primary leakage reactance X_1 as equal to rotor circuit reactance X_2'. Therefore, $X_1 = X_2' = \dfrac{X_{mb}}{2}$. Knowing R_1, R_2' can be calculated.

No-load Test No-load test is conducted with rated voltage applied to the main winding of the stator. The auxiliary winding is kept open. The slip of the rotor is very small, may be 0.03 or 0.04. The rotor circuit resistances $R_2/2S$ is very high as compared to the magnetizing reactance, $X_m/2$ for the forward field equivalent circuit. So, the resistance branch circuit can be shown open. For the backward field circuit, $X_m/2$ is higher than $\dfrac{R_2'}{2(2-s)}$. So, the magnetizing reactance branch can be shown open. These assumptions are done only to simplify the circuit and make calculations easy. The equivalent circuit under no-load condition is shown in Fig. 6.28.

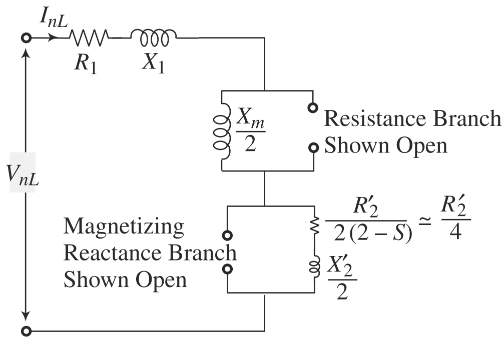


Fig. 6.28 *Approximate equivalent circuit under no-load condition*

Resistance of the circuit at no-load, $R_{nl} = R_1 + \dfrac{R_2'}{4}$

Reactance of the circuit at no-load, $X_{nl} = X_1 + 0.5\,X_m + 0.5\,X_2'$

From the data of no-load test, i.e., voltmeter, ammeter, and wattmeter readings V_{nL}, I_{nL} and W_{nL},

$$Z_{nL} = \frac{V_{nL}}{I_{nL}} \quad \text{and} \quad R_{nL} = \frac{W_{nL}}{I_{nL}^2}$$

$$X_{nL} = \sqrt{Z_{nL}^2 - R_{nL}^2}$$

$$X_{nL} = X_1 + 0.5\,X_m + 0.5\,X_2'$$

From blocked rotor test,

$$X_1 = X_2' = \frac{X_{mb}}{2}$$

Therefore,
$$X_{nL} = 0.5\,X_m + X_1 + 0.5\,X_2'$$

$$= 0.5\,X_m + \frac{X_{mb}}{2} + 0.5 \times \frac{X_{mb}}{2}$$

or,
$$X_{nL} = 0.5\,X_m + 0.75\,X_{mb}$$

or,
$$2X_{nL} = X_m + 1.5\,X_{mb}$$

or,
$$X_m = 2\,X_{nL} - 1.5\,X_{mb}$$

$$R_{nL} = R_1 + \frac{R_2'}{4} = R_1 + 0.25\,R_2'$$

No-load power consumed = W_{nL}

This power is lost as $I_{nL}^2\,R_{nL}$, as core loss and as rotational loss. As core loss component has been added to rotational losses,

Rotational losses, $W_r = W_{nL} - I_{nL}^2\,(R_1 + 0.25\,R_2')$

This shows that from block rotor test and no-load test data, all the parameters of the equivalent circuit can be determined. The performance of the motor in terms of torque developed, power output, line current, power factor, efficiency, etc. can be calculated from the equivalent circuit parameters.

6.5 PERFORMANCE CALCULATIONS OF SINGLE-PHASE INDUCTION MOTORS

We redraw the equivalent circuit as shown in Fig. 6.29

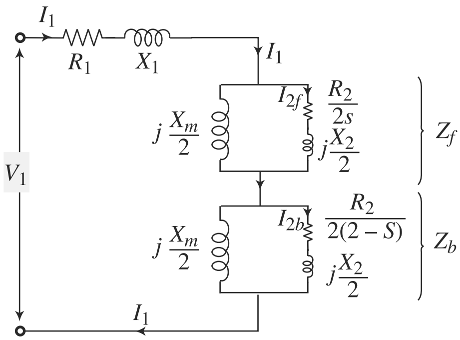


Fig. 6.29 *Equivalent circuit of a single-phase induction motor*

Note that for simplicity we will refer R_2' and X_2' as R_2 and X_2 respectively, assuming that R_2 and X_2 are the referred values.

$$Z_f = R_f + j\,X_f = \frac{j\dfrac{X_m}{2}\left(\dfrac{R_2}{2S} + j\dfrac{X_2}{2}\right)}{j\dfrac{X_m}{2} + \dfrac{R_2}{2S} + j\dfrac{X_2}{2}}$$

$$\text{or, } Z_f = R_f + j\,X_f = \frac{\dfrac{1}{2}\,jX_m\,\dfrac{1}{2}\left(\dfrac{R_2}{S} + j\dfrac{X_2}{2}\right)}{\dfrac{1}{2}\left[\dfrac{R_2}{S} + j(X_m + X_2)\right]} = \frac{0.5\,jX_m\left(\dfrac{R_1}{S} + jX_2\right)}{\dfrac{R_2}{S} + j(X_m + X_2)} \tag{6.1}$$

$$Z_b = R_b + j\,X_b = \frac{j\dfrac{X_m}{2}\left(\dfrac{R_2}{2(2-S)} + j\dfrac{X_2}{2}\right)}{j\dfrac{X_m}{2} + \dfrac{R_2}{2(2-S)} + j\dfrac{X_2}{2}} = \frac{0.5\,jX_m\left(\dfrac{R_2}{(2-S)} + jX_2\right)}{\dfrac{R_2}{(2-S)} + j(X_2 + X_m)} \tag{6.2}$$

The equivalent circuit is drawn in a simplified way as in Fig. 6.30.

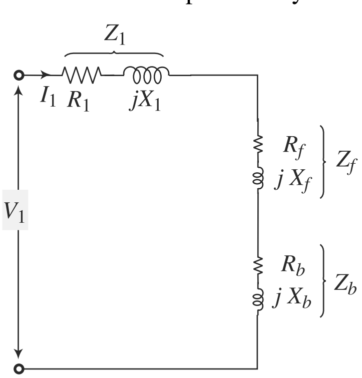


Fig. 6.30 *Simplified equivalent circuit*

$$Z = Z_1 + Z_f + Z_b$$

$$I_1 = \frac{V_1}{Z_1 + Z_f + Z_b} = \frac{V_1}{(R_1 + R_f + R_b) + j(X_1 + X_f + X_b)}$$

Input power, $P_{in} = V_1 I_1 \cos\phi$

where $\cos\phi$ is the power factor. Angle ϕ is the angle of lag of I_1 with respect to the applied voltage, V_1.

Power Flow Diagram: We will refer to the power flow diagram shown in Fig. 6.31 to develop expressions for performance indicators..

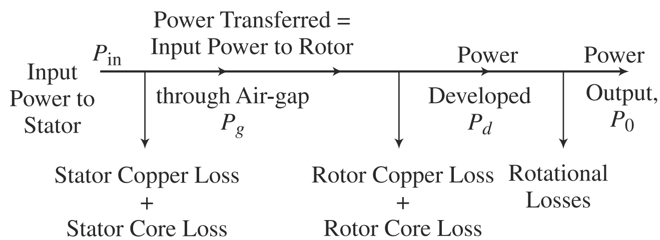


Fig. 6.31 *Power flow diagram of a single-phase induction motor*

Power transferred to rotor through air-gap, P_g = input power – (stator copper loss + stator core loss)

Neglecting stator core loss $P_g = V_1 I_1 \cos\phi - I_1^2 R_1$

Assuming iron loss in the rotor as negligible, we can write

Power transferred from stator to rotor – Power developed by rotor = Rotor copper loss.

$$\frac{2\pi T N_s}{60} - \frac{2\pi T N_r}{60} = \text{Rotor copper loss}$$

or, $$\frac{2\pi T}{60}(N_s - N_r) = \text{Rotor copper loss}$$

or, $$\frac{2\pi T N_s}{60}\left(\frac{N_s - N_r}{N_s}\right) = \text{Rotor copper loss}$$ [multiplying and dividing by N_s]

or, $$\text{Rotor input} \times S = \text{Rotor copper loss} \quad (6.3)$$

From power flow diagram, Rotor input – Rotor copper loss = Power developed

∴ Power developed P_d = Rotor input – (S × Rotor input)

or, P_d = Rotor input $(1 - S) = P_g(1 - S)$

$$T_d = \frac{p_d}{\omega_r}$$

$$P_d = P_g(1 - S) = T_d\,\omega_r = T_d\,(1 - S)\,\omega_s$$

$$\omega_r = \frac{2\pi N_r}{60}, \quad \omega_s = \frac{2\pi N_s}{60}, \quad S = \frac{N_s - N_r}{N_s}$$

or, $SN_s = N_s - N_r$

or, $N_r = (1 - S)\, N_s$. In terms of angular velocity, $\omega_r = (1 - S)\, \omega_s$

Output power, $P_o = P_d$ – rotational loses, P_r

Rotational losses, P_r includes frictional and windage losses plus core loss (in this case).

The torque developed, T_d on the rotor is due to the effect of forward field and the backward field. If torque developed by forward field is T_{df} and by the backward field is T_{db} respectively, the resultant torque, T_d is

$$T_d = T_{df} - T_{db}$$

This is shown in the torque-slip characteristics in Fig 6.32.

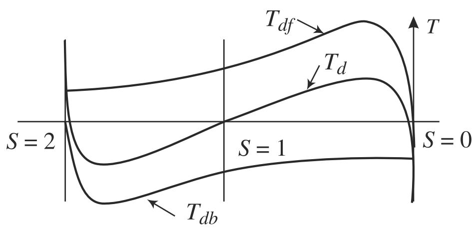


Fig. 6.32 *Torque-slip characteristics due to forward field, backward field and the resultant*

I_{2f} and I_{2b} can be calculated from the equivalent circuit shown in Fig. 6.29, in terms of I_1 using current divider rule as

$$I_{2f} = I_1 \frac{j X_m}{\frac{R_2}{S} + j(X_2 + X_m)} \tag{6.4}$$

and

$$I_{2b} = I_1 \frac{j X_m}{\frac{R_2}{(2 - S)} + j(X_2 + X_m)} \tag{6.5}$$

From the simplified equivalent circuit shown in Fig 6.30, considering R_f and R_b as the equivalent resistances in the forward and backward branches of the rotor circuit.

$$p_{gf} = I_1^2 R_f \text{ and } P_{gb} = I_1^2 R_b$$

$$\text{Efficiency} = \frac{\text{Output power, } P_o}{\text{Input power, } P_{in}}$$

Thus, the performance of a single-phase induction motor in terms of output power, output torque, line current, power factor, efficiency, etc., can be calculated.

Example 6.3 *A 230 V, 50 Hz, 8-pole, 1/2 hp, single-phase induction motor has the following parameters;*

$$R_1 = 2\ \Omega,\ X_1 = 1\ \Omega,\ R_2' = 4\ \Omega,\ X_2' = 1.5\ \Omega,\ X_m = 180\ \Omega.$$

The motor runs at 720 rpm on load. Calculate the output power, output torque and efficiency. The friction and windage loss including core loss is 55 Watts.

Solution

$$P = 8, f = 50 \text{ Hz}$$

$$N_S = \frac{120\,f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

$$N_r = 720 \text{ rpm}$$

$$S = \frac{N_S - N_r}{N_S} = \frac{750 - 720}{750} = 0.04$$

From the equivalent circuit using Eq. (6.1) and Eq. (6.2),

$$Z_f = \frac{jX_m \left(\frac{R_2'}{S} + j\,X_2'\right) \times 0.5}{\frac{R_2'}{S} + j(X_2' + X_m)} = \frac{j180\left(\frac{4}{0.04} + j1.5\right) \times 0.5}{\frac{4}{0.04} + j(1.5 + 180)}$$

$$= \frac{j180\,(100\,\_\,j1.5) \times 0.5}{100 + j181.5} = \frac{j1.8\,(100 + j1.5) \times 0.5}{1 + j1.815}$$

$$= \frac{(-2{,}7 + j\,180) \times 0.5}{1 + j\,1.815} = \frac{-1.35 + j90}{1 + j1.815} = \frac{90.01\,\angle 90.9^\circ}{2.07\,\angle 61.1^\circ}$$

$$= 43.48\angle 30.8^\circ = 43.48 \cos 30.8^\circ + j\,43.48 \sin 30.8^\circ$$

$$= 37.35 + j\,22.26$$

$$= R_f + jX_f$$

$$Z_b = \frac{jX_m \left[\frac{R_2'}{2 - S} + jX_2'\right] 0.5}{\frac{R_2'}{2 - S} + j\,(X_2' + X_m)}$$

$$= \frac{j180\left[\frac{4}{(2 - 0.04)} + j1.5\right] \times 0.5}{\frac{4}{(2 - 0.04)} + j(1.5 + 180)}$$

$$= \frac{j18\left[\frac{4}{1.96} + j1.5\right] \times 0.5}{\frac{4}{1.96} + j181.5}$$

$$= \frac{j\,9\,(2.004 + j\,1.5)}{2.004 + j181.5} = \frac{-13.5 + j\,18.036}{2.004 + j181.5}$$

$$= \frac{22.5\,\angle 126.9^\circ}{181.51\,\angle 89.3^\circ} = 0.1239\,\angle 37.6^\circ = 0.098 + j\,0.0954$$

or, $$Z_b = R_b + j\,X$$

$$Z_{\text{in}} = R_1 + j\,X_1 + Z_f + Z_b$$

$$= (2 + j_1) + (37.35 + j\,22.26) + (0.098 + j\,0.0954)$$

$$= 39.448 + j\,23.355$$

$$I_1 = \frac{V_1}{Z_{in}} = \frac{230\angle 0°}{39.448 + j\,23.355} = \frac{230\,\angle 0°}{45.83\,\angle 30.8°} = 5.02\angle -30.8°$$

$$P_{gf} = I_1^2\,R_f = (5.02)^2 \times 37.35 = 941.2 \text{ W}$$

$$P_{gb} = I_1^2\,R_b = (5.02)2 \times 0.098 = 2.47 \text{ W}$$

$$P_g = P_{gf} - P_{gb} = 941.2 - 2.47 = 938.73 \text{ W}$$

$$P_d = (1 - S)\,P_g = (1 - 0.04) \times 938.73 = 901.18 \text{ W}$$

$$P_r = 55 \text{ W}$$

$$P_o = p_g - P_r = 938.73 - 55 = 883.73 \text{ W}$$

$$T_o\,\omega_r = P_o$$

$$\omega_r = \frac{2\pi\,N_r}{60} = \frac{2 \times 3.14 \times 720}{60} = 75.36 \text{ rad/sec}$$

$$T_o = \frac{P_o}{\omega_r} = \frac{883.73}{75.36} = 11.72 \text{ N-m}$$

$$P_{in} = V\,I_1 \cos\phi = 230 \times 5.02 \times \cos 30.8° = 999.80 \text{ W}$$

$$\%\eta = \frac{P_o \times 100}{P_{in}} = \frac{883.73 \times 100}{999.80} = 88.39 \text{ percent}$$

EXAMPLE 6.4 *A 230 V, 50 Hz, 6-pole, single-phase induction motor has the following equivalent circuit parameters:*

$R_1 = 2\ \Omega$, $X_1 = 3\ \Omega$, $R_2' = 5\ \Omega$, $X_2' = 3\ \Omega$, $X_m = 100\ \Omega$

Friction and windage loss including core loss = 60 W

The motor is running at 960 rpm on load.

Calculate (a) line current; (b) input power; (c) power factor; (d) developed power; (e) shaft output power; (f) efficiency.

Solution

$$N_s = \frac{120\,f}{P} = \frac{129 \times 50}{6} = 1000 \text{ rpm}$$

$$S = \frac{N_s - N_r}{N_s} = \frac{1000 - 960}{1000} = 0.04$$

$$Z_f = R_f + j\,X_f = \frac{jX_m\left(\frac{R_2'}{S} + j\,X_2'\right) \times 0.5}{\frac{R_2'}{S} + j(X_2' + X_m)} = \frac{j100\left(\frac{5}{0.04} + j3\right) \times 0.5}{\frac{5}{0.04} + j(3 + 100)}$$

$$= \frac{j\,50\,(125 + j\,3)}{125 + j\,103} = \frac{-150 + j\,6250}{125 + j103} = \frac{6251.7\,\angle 91.4°}{161.96\,\angle 39.5°}$$

$$= 38.6 \angle 51.9° = 23.8 + j\,30.38$$

$$= R_f + j\,X_f$$

$$Z_b = R_b + j\,X_b = \frac{\left[\dfrac{R_2'}{2(2-S)} + j\dfrac{X_2'}{2}\right] j\dfrac{X_m}{2}}{\dfrac{R_2'}{2(2-S)} + j\dfrac{X_2'}{2} + j\dfrac{X_m}{2}}$$

$$= \frac{\left[\dfrac{5}{2(2-0.04)} + j\dfrac{3}{2}\right] j\dfrac{100}{2}}{\dfrac{5}{2(2-0.04)} + j\dfrac{3}{2} + j\dfrac{100}{2}} = \frac{-75 + j63.77}{1.175 + j\,51.5}$$

$$Z_b = \frac{98.4 \angle 139.5°}{51.51 \angle 88.5°} = 1.91 \angle 51° = 1.2 + j1.48 = R_b + jX_b$$

$$Z = Z_1 + Z_f + Z_b$$

$$= R_1 + jX_1 + R_f + jX_f + R_b + jX_b$$

$$= 2 + j\,3 + 23.8 + j\,30.38 + 1.2 + j\,1.48$$

$$= 27 + j\,34.86 = 44.09 \angle 52.2°$$

(a) Input line current, $I_1 = \dfrac{V_1}{Z} = \dfrac{230 \angle 0°}{44.09 \angle 52.2°} = 5.21 \angle -52.2°$

(b) Power factor = $\cos\phi = \cos 52.2° = 0.613$ lagging

(c) Power input = $V_1 I_a \cos\phi = 230 \times 5.21 \times 0.613 = 734.5$ W

$$P_{gf} = I_1^2 R_f = (5.21)^2 \times 23.8 = 646 \text{ W}$$

$$P_{gb} = I_1^2 R_b = (5.21)^2 \times 1.2 = 32.6 \text{ W}$$

$$P_g = P_{gf} - P_{gb} = 646 - 32.6 = 613.4 \text{ W}$$

(d) $$P_d = P_g(1 - s) = 613.4(1 - 0.04) = 588.86 \text{ W}$$

$$P_r = 60 \text{ W}$$

(e) $$P_0 = P_g - P_r = 613.4 - 60 = 553.4 \text{ W}$$

(f) $$\eta = \frac{\text{Output}}{\text{Input}} = \frac{553.4}{734.5} = 0.7534 = 75.34\ \%$$

6.6 SINGLE-PHASE SYNCHRONOUS MOTORS

Single-phase synchronous motors are small, constant speed motors which do not need dc excitation and are self-starting. These motors are built for a wider range of output and speed than fractional kilowatt single-phase induction motors. Synchronous motors built in miniature ratings, as low as 0.001 kW, are used for clocks, control apparatus, timing devices, etc. Single-phase synchronous motors are of two types, namely, reluctance type and hysteresis type.

6.6.1 Reluctance Motor

Motors both single-phase and three-phase can be built with no dc excitation on the rotor and having nonuniform air-gap reluctance. The stator of such motors will have windings similar to induction motor winding but the magnetic reluctance along the air-gap will be made variable. Such induction motors when built as three-phase motors are called *synchronous induction motors*. Single-phase induction motors built with a variable air-gap reluctance and with having no dc supply on the rotor are called reluctance motors. Such motors start as an induction motor but are pulled into synchronous speed because of the variation in air-gap reluctance. This pull-in force on such motors is based on the reluctance principle and is explained as follows:

Let us consider a piece of magnetic material, free to rotate, placed in a magnetic field as shown in Fig. 6.33. A torque will act on the material shown in Fig. 6.33(a) to bring it to the position shown in Fig. 6.33(b), so as to produce minimum magnetic reluctance to the flux path.

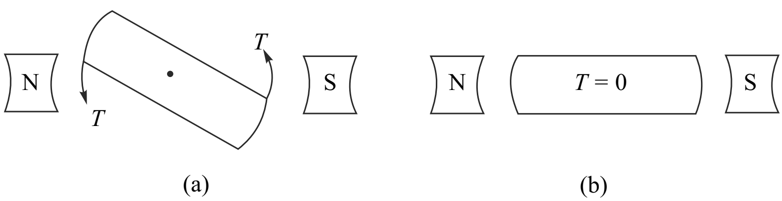


Fig. 6.33 *Reluctance torque developed on a magnetic material placed in a magnetic field*

Similarly, a reluctance motor is pulled into synchronous speed due to the variation of air-gap reluctance, intentionally created by design. The stator of such motors can be of any type of single-phase induction motor. For starting torque, the stator, in addition to the main winding, may be provided with an auxiliary winding. The rotor is a modified squirrel-cage one, with bars on the rotor slots. Some of the rotor teeth are removed to create variation in air-gap reluctance. Variation of air-gap and hence variation in reluctance to flux path between the stator and rotor can also be produced by shaping the rotor laminations. Two different types of rotor laminations are shown in Fig. 6.34. Such motors coming up to speed near synchronous speed as an induction motor, will be pulled into synchronous speed with the stator field by the reluctance torque developed at the salient poles which have lower air-gap reluctance. Torque-speed characteristic of a reluctance motor is shown in Fig 6.35.

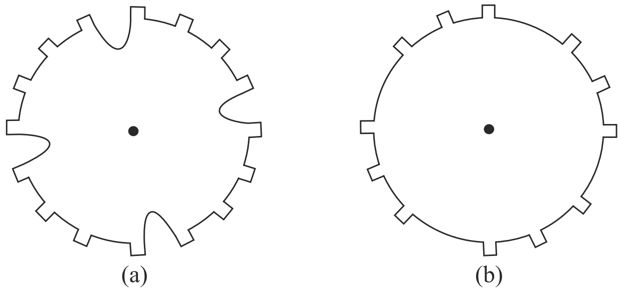


Fig. 6.34 *Different types of rotor laminations of reluctance motor*

The motor starts as a single phase induction motor. The starting winding gets disconnected at a speed of about 75 percent of the synchronous speed, with the help of a centrifugal switch or over-current relay. The motor continues to develop torque through its main winding. As the rotor speed approaches synchronous speed, the reluctance-torque developed is sufficient to pull the rotor to synchronous speed. The motor works as a non excited synchronous motor up to about 200 per cent of the full-load torque. At a load beyond 200 percent of the full-load, the motor will continue to work as a single-phase induction motor. The direction of rotation of such motors can be reversed in the same manner as a single-phase induction motor.

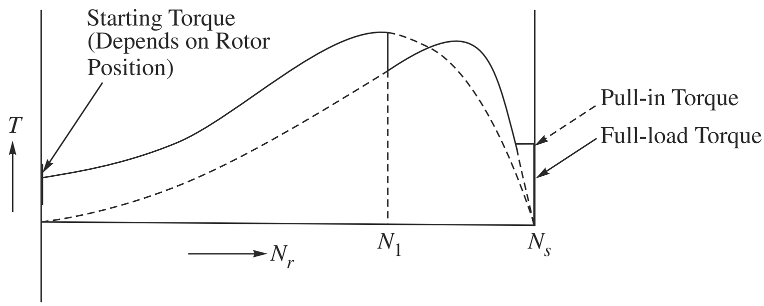


Fig. 6.35 *Torque-speed characteristic of a reluctance motor*

The advantages of constant speed, ruggedness in construction, non-requirement of dc supply and the minimum maintenance required have made such motors widely applicable in recording instruments, timing devices, control apparatus, automatic regulators, etc. However, the reluctance motors have larger size and are heavier than an ordinary induction motor. The efficiency and power factor are also poor.

6.6.2 Hysteresis Motor

A hysteresis motor is a single-phase synchronous motor without any projected poles and without dc excitation. Such motors start by virtue of the hysteresis losses induced in the hardened steel rotor by the rotating magnetic field produced by the stator windings, and operate at synchronous speed due to the retentivity property of the rotor core material.

The rotor of hysteresis motors are made with magnetic material of high hysteresis losses, i.e., whose hysteresis loop area is very large. A ring of cobalt or chrome steel is mounted on a nonmagnetic arbor made with, say, aluminium as shown in Fig. 6.36. No winding is provided on this rotor. The stator construction is either split-phase type or shaded-pole type.

The motor starts rotating due to eddy-current and hysteresis torque developed on the rotor. At synchronous speed there is no induced emf in the rotor as the stator synchronously rotating field and the rotor are stationary with respect to each other. In the absence of induced eddy current the torque due to eddy current is zero. At synchronous speed the rotor torque is only due to hysteresis effect.

When the rotor is rotating at synchronous speed, the stator revolving field flux induces poles on the rotor. Due to the hysteresis effect, the rotor polarities linger an instant after the stator poles pass on. Hence the rotor is attracted towards the moving poles as shown in Fig. 6.37.

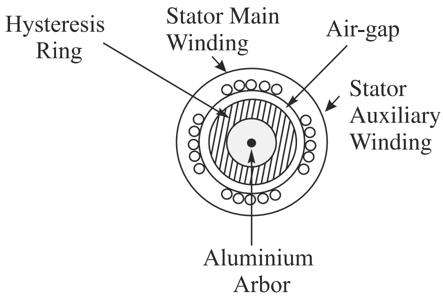


Fig. 6.36 *Cross-sectional view of a hysteresis motor*

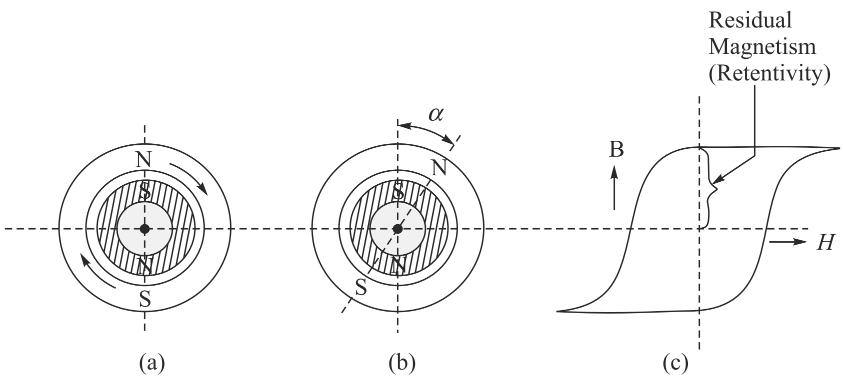


Fig. 6.37 *(a) Stator poles induce poles on the rotor, (b) Torque developed on the rotor due to residual magnetism of the rotor, (c) Hysteresis loop of the rotor material*

Since some magnetism is left on the rotor poles even when the stator poles have moved in the clockwise direction as shown in Fig. 6.37(b), the rotor will develop torque in the same direction. The magnetic strength of the rotor poles after the stator poles have moved forward will depend upon the residual magnetism (i.e., retentivity) of the magnetic material used. Higher the retentivity, the greater is the torque developed. On the contrary, if there is no residual magnetism in the rotor magnetic material, the magnetisation of the rotor will be in phase with the stator mmf and the magnetic axis of the stator and rotor will always remain aligned and hence no torque will be developed. Hysteresis-torque depends on the residual magnetism of the rotor-hysteresis ring material and is a constant. The torque is independent of the rotor speed. Because of the steady hysteresis-torque, such motors are most quiet in operation and are very popular as drives in high quantity record players, tape recorders and clocks.

6.7 COMMUTATOR-TYPE SINGLE-PHASE MOTORS

This type of motors have a wound rotor with brush and commutator arrangement like a dc armature. Commutator motors consist of two classes, namely, those operating on the principle of repulsion and those operating on the principle of series motor.

6.7.1 Repulsion Motor

The repulsion motor has a wound stator and a wound rotor. The stator winding is similar to that of the main winding of a single-phase induction motor. The rotor construction is similar to that of a dc machine. But the brushes are short-circuited and are movable on the commutator surface. Single-phase supply is connected across the stator winding. The rotor receives power through electromagnetic induction. Since the brushes are movable on the commutator surface, they can be placed at right angles to the stator pole axis, along the stator pole axis, or at an angle with the stator pole axis as shown in Fig. 6.38(a), (b) and (c).

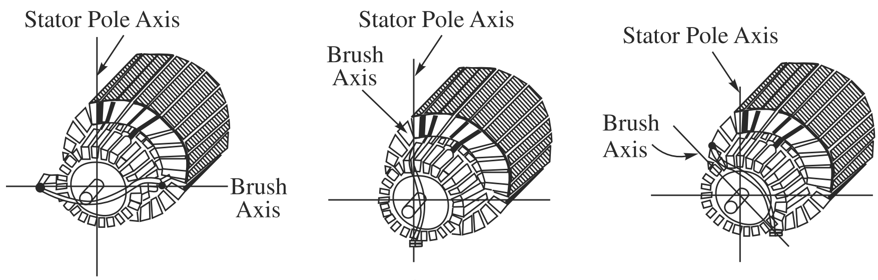


Fig. 6.38 *Brush-axis fixed at various positions with respect to the stator field axis as (a) Brush-axis at right angles to stator pole axis; (b) Brush-axis along the stator pole-axis; (c) Brush-axis at an angle with stator pole-axis*

Figure 6.39(a) shows diagrammatically the brush axis, in line with the stator field axis. The direction of induced emf and current in the rotor conductors when the field flux is increasing are shown in Fig. 6.39(a). The direction of induced emf and current in the upper-half conductors in the rotor is such that they will experience a force in the downward direction. The lower-half conductors will experience force in the upward direction. The resultant torque, therefore, will be zero and the rotor will not rotate. The magnitude of armature current will be very high as the rotor is like short-circuited secondary of a transformer. This position of the brushes, i.e., when the brush-axis lie along the stator field axis is called *hard neutral* position.

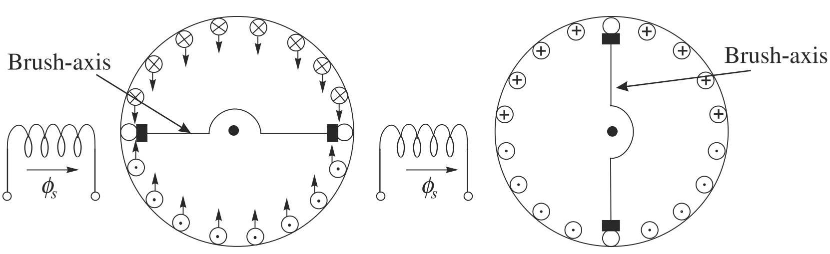


Fig. 6.39 *(a) Brushes at hard neutral position, (b) Brushes at soft neutral position*

Let us now consider the brushes at 90° with the stator field axis as shown in Fig. 6.39(b). The directions of induced emf in the rotor conductors will be the same as in the previous case. But half the conductors will have positive voltage and the other half will have negative voltage on both sides of the brush axis.

The resultant voltage across the brushes is now zero, and therefore no current will flow through the rotor conductors. Hence, no torque will be developed and the rotor will not rotate. This position of the brushes, i.e., when at 90° with the stator field axis is called neutral or *soft neutral* position.

With the brushes in hard neutral and soft neutral positions, no torque is developed on the rotor.

Let the brush axis now be shifted so that it makes small angle with the stator field axis as shown in Fig. 6.40. When the brush axis is shifted in the clockwise direction as shown in Fig. 6.40(a), induced emf in most of the upper right-half of the armature conductors will be the direction as shown by the crosses. Since all the conductors in this half are in series, the current in these conductors should flow in one direction only, i.e., in the direction of the resultant induced emf in the upper half of the brush axis (in this case current will be in inward direction). The reverse condition will be there for the current in the other half of the conductors. The magnetic polarity of the rotor and that of the stator are shown. The rotor will develop torque in the clockwise direction, due to the repulsive force of the similar poles of the stator and rotor. Similarly, if the brush axis is shifted in the anti-clockwise direction as shown in Fig. 6.40(b), the rotor will develop torque in the anti-clockwise direction due to the repulsive force of the like poles of the stator and the rotor. A repulsion motor, therefore, rotates in the direction in which the brush axis is shifted from the hard neutral position, i.e., from the stator pole axis.

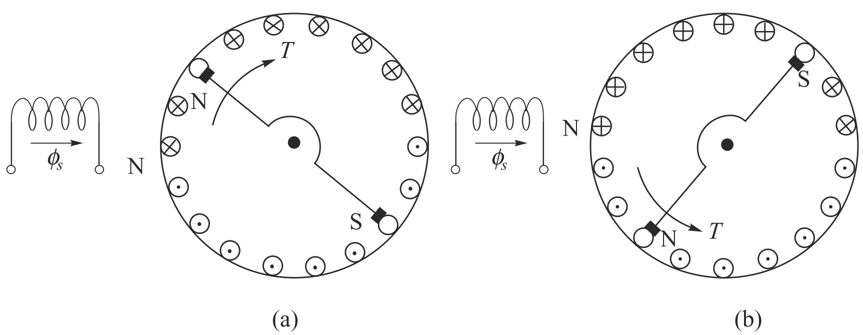


Fig. 6.40 *(a) When brushes are moved in the clockwise direction, torque is developed in the same direction, (b) When brushes are moved in the anti-clockwise direction, torque is developed in the anti-clockwise direction*

Torque developed is proportional to the product of stator flux and rotor current. Rotor current depends on stator flux and stator flux is proportional to stator current. Therefore, torque developed by a repulsion motor is proportional to the square of the stator current. Magnitude of starting torque of a repulsion motor is high. Magnitude of starting torque depends on the position of the brush axis. Its speed regulation, like a series motor, is poor.

The advantages of a repulsion motor are: excellent starting torque, low starting current and wide range of speed control with smooth variation of speed. The disadvantages are: noisy performance, poor speed regulation and periodic commutator maintenance requirement.

Because of the disadvantages mentioned above, repulsion motors have largely been replaced by capacitor-type motors and therefore, very few repulsion motors are manufactured now a days.

There is another type of single-phase motor called *repulsion-start induction motor* which, exactly like a repulsion motor, is capable of developing high starting torque. At about 75 per cent of synchronous speed, a centrifugally operated device short-circuits the entire commutator. From this speed onwards, the motor behaves like an induction motor. In some motors the brushes are lifted from the commutator surface before short-circuiting the commutator.

Repulsion-start induction motors have largely been replaced by capacitor motors because repulsion-start motors requires more maintenance (because of commutator and brushes and the centrifugal device), are more expensive and are noisy.

6.7.2 Repulsion-Induction Motor

The construction of the stator of this type of motor is similar to that of a repulsion motor, i.e., similar to the main stator winding of a single-phase induction motor. In the rotor there are two separate windings. One winding is similar to the rotor winding of a repulsion motor. The other winding is of squirrel-cage type, placed below the repulsion-motor type winding. The behaviour of the repulsion induction motor is therefore the combination of the behaviour of repulsion motor and an induction motor. Under starting condition very little current will flow through the inner squirrel-cage winding since the reactance of the squirrel-cage winding which is placed deep into the rotor slot is very high. As the rotor picks up speed, the frequency of the rotor induced emf and hence the rotor reactance will decrease. More current will flow in the squirrel-cage rotor winding. The motor will work as a combination of repulsion and induction motor. The torque-speed characteristics will be as shown in Fig. 6.41.

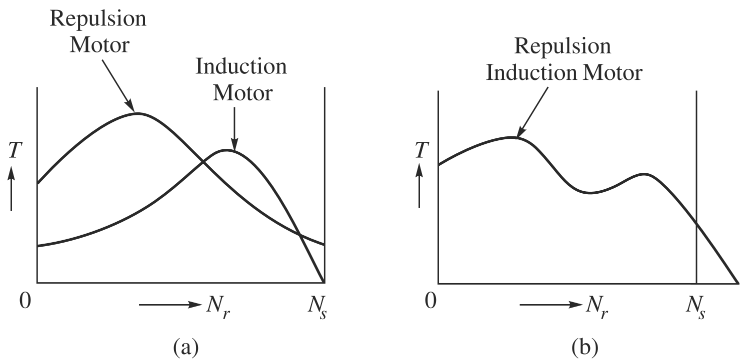


Fig. 6.41 *(a) Torque-speed characteristics of repulsion motor and induction motor shown separately, (b) Torque-speed characteristic of a repulsion induction motor*

As shown in Fig. 6.41(b), the single-phase repulsion-induction motor has the advantages of high starting torque and good speed regulation. It has the ability to continue to develop torque under sudden heavy applied loads. It is also manufactured in integral kilowatt rating and is used to drive reciprocating pumps and compressors, where only single-phase power is available. Single-phase repulsion induction motors are also used in stokers, conveyors, and deep-well pumps.

6.7.3 AC Series Motor

Operation of a dc Series Motor on ac Supply If the polarities of the line terminals supplying a dc motor is reversed, the direction of field flux and the direction of armature current reverse simultaneously. The direction of torque developed therefore remains unchanged as shown in Fig. 6.42. The rotor therefore continues to rotate in the same direction.

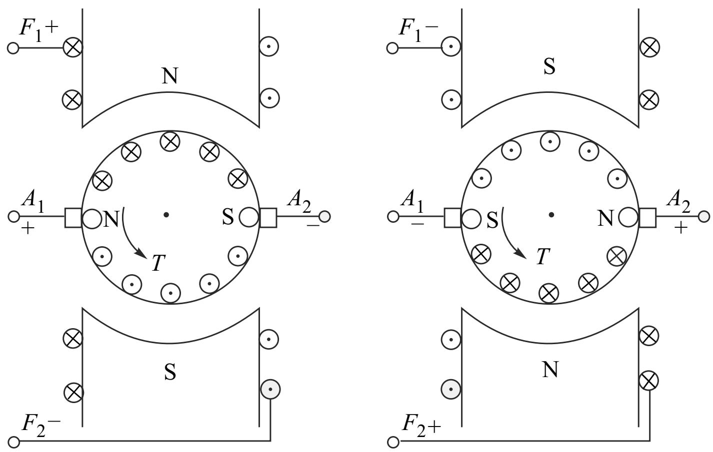


Fig. 6.42 *No change in the direction of rotation takes place when the polarities of both armature and field terminals of a dc motor are reversed*

In a dc series motor, the armature and the field terminals are connected in series. When a dc series motor is connected across an ac supply, the polarities of both the armature and the field changes at every half cycle. The direction of torque developed and the direction of rotation of the rotor, therefore, remains unaltered. From this it may appear that a dc series motor should work satisfactorily on ac supply also. But in actual practice, following operating problems arise when a dc series motor is allowed to work on ac supply.

(a) Because of the increase in hysteresis and eddy-current losses due to the alternating flux created by ac supply, efficiency of the motor will be poorer.
(b) Power factor of the motor will be less. This is because of the inductive reactance of the field and armature winding.
(c) Considerable sparking at the brushes will occur. This is because in addition to the causes of sparking that occur in a dc motor, transformer action on coil undergoing commutation further intensifies commutation difficulties. The coil, short-circuited by the brushes, links part of the constantly changing main field flux, and hence a voltage is induced in it.

Design Considerations for an ac Series Motor To enable a dc series motor operate satisfactorily on alternating current supply, the following modifications in the design are to be incorporated.

(a) The yoke and the poles should be completely laminated to minimise the eddy-current losses.

(b) The field is to be wound with fewer turns than a dc motor. The field pole area may be increased so that the field is operated at a comparatively low flux density. This will reduce the iron-loss and reactive voltage drop. In order to obtain the required torque with this low field flux, the number of armature coils should be increased.

(c) Voltage induced by transformer action in a coil undergoing commutation may be minimised, somewhat by constructing the armature coils with fewer turns. Sometimes even a single turn armature coil may be used.

(d) Increased number of armature coils will increase the armature reaction and may cause more commutation problems. More number of armature coils will increase the armature reactance also. To reduce the effect of armature reaction and thereby improving commutation and to reduce armature reactance a compensating winding may be used. The compensating winding as shown in Fig. 6.43 is placed on the stator slots at 90° electrical with the main field axis. It may be connected in series with the armature and field winding or may be short-circuited in itself. Since the axis of the compensating winding coincides with the brush axis, the alternating flux of the armature induces an emf in the short-circuited winding. The current in the winding due to this induced emf opposes the flux causing it and hence it opposes the armature reaction.

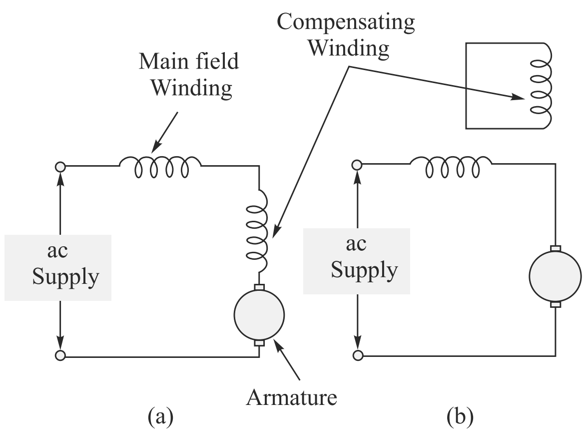


Fig. 6.43 *Single-phase series motor with compensating winding*

Torque-speed Characteristics and Applications The torque-speed characteristics of an ac series motor is similar to that of a dc series motor as shown in Fig. 6.44. Because of high starting torque developed, ac series motors are used in railway systems for electric locomotives.

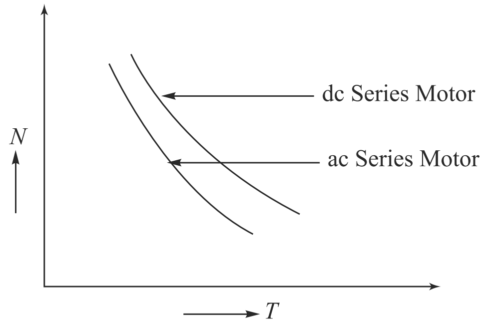


Fig. 6.44 *Torque speed characteristics of series motors*

6.7.4 Universal Motor

On the demand of developing small motors for appliances like portable drills, electric shavers, food mixers, hair driers, sewing machines, office machinery and many other similar applications to be operated either on ac or on dc, universal motors of fractional kilowatt ratings have been developed.

A dc series motor for operation on ac supply will need certain modifications in its design. These have already been discussed.

Small series motors of rating less than 1 kW rating called universal motors are designed to operate on both dc and ac, are built with some modifications over the design of dc series motors. The field of such motors are operated on low flux density and the number of armature turns is increased. Figure 6.45 shows a photographic view of a dissembled universal motor.

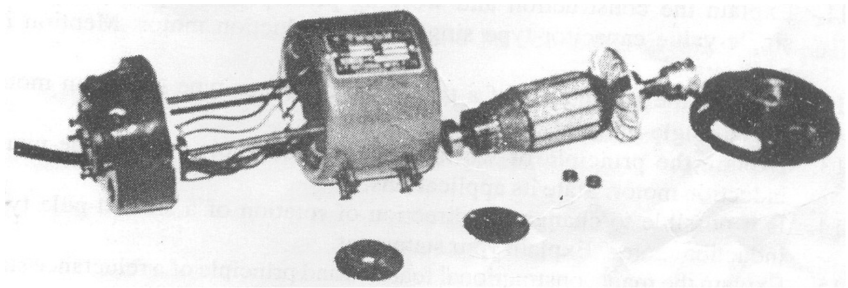


Fig. 6.45 *Photographic view of a dissembled universal motor*

Full-load speed of such motors may be as high as 10,000 rpm. Many applications do not need such high speeds. Therefore, gear trains are provided as an integral part of the motor. Since power is the product of torque and speed, very high torque at low speeds can, therefore, be achieved. No-load speeds of small universal motors may be much higher than their full-load speed. For applications such as vacuum cleaners, hair driers, electric shavers, etc., the motor is always loaded to some extent, even if it is not doing any useful work. This enables avoiding a situation of the motor attaining a dangerously high speed.

As an extension of the chapter we shall be discussing some *special types of electrical motors* used in applications like in servo mechanism, robotics, computer printers, photocopiers, etc. These motors are not necessarily single phase motors.

6.8 STEPPER MOTORS

A stepper motor is one whose rotor rotates in steps, i.e., by certain angles when its stator windings are energised sequentially by a train of pulses. In other words, a stepper motor is a brushless dc motor whose stator windings are supplied with dc pulses sequentially. The rotor is generally of permanent magnet type. Torque is produced due to the magnetic interaction between the stator and the rotor poles. The rotor does not have any winding but has projected poles.

Stepper motors are of two types. In one case, neither the stator nor the rotor is made of permanent magnets. The rotor is of variable reluctance type. While the stator has windings in which sequential dc pulses are applied. The other type of stepper motor is known as permanent magnet hybrid stepper motor. Here the rotor is of permanent-magnet type and the stator has windings in which dc voltage pulses are applied.

6.8.1 Variable Reluctance Stepper Motor

In a variable reluctance stepper motor there is no permanent magnet either on rotor or on stator. The rotor is made of soft iron stampings of variable reluctance, i.e., having non-uniform air gap between the stator and the rotor as shown in Fig. 6.46. The stator has been shown as made up of twelve poles while the rotor has eight poles. The stator has three-phase windings, i.e., phase *A*, phase *B* and phase *C*. In figure only the phase *A* winding involving four poles has been shown. Other phase windings are made on poles marked *B* and *C* respectively.

It may be noted that if bidirectional rotation is required the minimum number of stator phase windings must be three. The number of poles of the stator is an even multiple of the number of phases of the stator windings. In this case, the number of phases is three while the number of poles is four times the number of phases, i.e., twelve.

The stator is also made of soft iron stampings and is of salient (projected) pole type carrying stator windings. The number of poles of the stator is an even multiple of the number of phases for which the stator is wound. It may again be noted that the minimum number of phases of stator winding must be three if bidirectional rotation of the rotor is desirable. As shown in Fig. 6.46 the stator has three-phase winding and has twelve poles, the rotor is of eight pole construction.

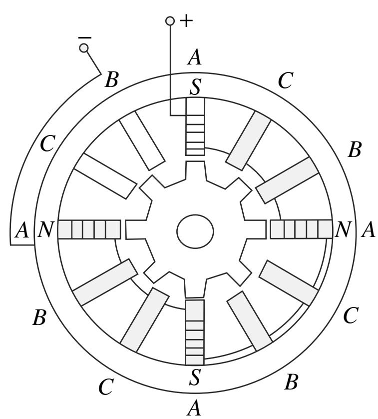


Fig. 6.46 *Schematic diagram of a three phase variable reluctance Stepper Motor, only the 'A' phase winding has been shown for clarity*

As shown in Fig. 6.46 when phase *A* is energised by giving supply at *A*, the rotor moves to the position in which the rotor teeth align themselves with the teeth of phase *A*. In this position, the reluctance of the magnetic circuit is the minimum. After this, if phase *A* is de-energised and phase *B* is energised by giving supply to its winding (not shown in figure), the rotor will rotate through an angle of 15 degrees in clockwise direction so as to align its teeth with those of phase *B*. This is the angle between any rotor pole axis and stator pole axis. By movement of 15 degrees, the rotor pole will face the stator poles which are formed due to excitation of stator pole windings. After this, de-energising phase *B* and energising phase *C* will make the rotor rotate by another 15 degrees in the clockwise direction. Thus by sequencing power supply to the phases the rotor could be made to rotate by a step of 15 degrees each time. The direction of rotation could be reversed by changing the sequence of supply to the phases, i.e., for anticlockwise rotation supply should be given in the sequence of *A*-*C*-*B*

6.8.2 Permanent Magnet Type Stepper Motors

The most popular type of stepper motor is known as permanent magnet hybrid (PMH) stepper motor.

The simplest type of PMH motor is shown in Fig. 6.47. The stator has four poles, each pole carrying its own winding. Windings of two alternate poles are connected in series to form phase *A* winding. Similarly, windings of other two alternate poles are connected in series to form phase *B* winding.

The axes of the two windings are perpendicular to each other in space. The rotor is a bar magnet having North and South poles. See Fig. 6.47.

To understand the working of the motor, assume that a voltage of +V is applied to A-phase winding. Due to this, *N* and *S* poles in the stator will be formed. This will produce a stator magnetic field shown by vector F_A. The rotor will align itself to the direction of F_A so that its *N* pole faces the *S*-pole and vice-versa. Keeping the supply to phase *A* unchanged, if supply to phase *B* is also given with +V at *B*, a resultant magnetic field in the direction *F*, as shown in Fig. 6.47(b), will be produced. The rotor will now rotate by 45 degrees to align it self to the stator field so produced. Next, the supply to phase *A* is disconnected and supply to *B* is kept unchanged as shown in Fig. 6.47(c). The direction of the stator field will shift by another 45 degrees and the rotor will rotate by another 45 degrees to keep itself aligned with the stator field. Thus, it can be seen that by changing the supply to the two stator phases as also changing the direction of supply to the phases, the rotor can be made to rotate in steps.

Electronic drive circuits for hybrid stepper motors such as logic sequence generators, pulse generators and current suppression circuits are used to feed the stator windings with certain pattern of power supply.

In Fig. 6.47 steps *a, c, e, g* constitutes single phase energisation (1-1) so that only one phase is energised at a time. Steps *b, d, f, h* on the other hand, form two phase energisation (2-2) sequence, as both *A* and *B* phases are energised either with +V or –V at any time.

Both the above sequences are four step sequences, because the rotor rotates by 90 degrees in each step.

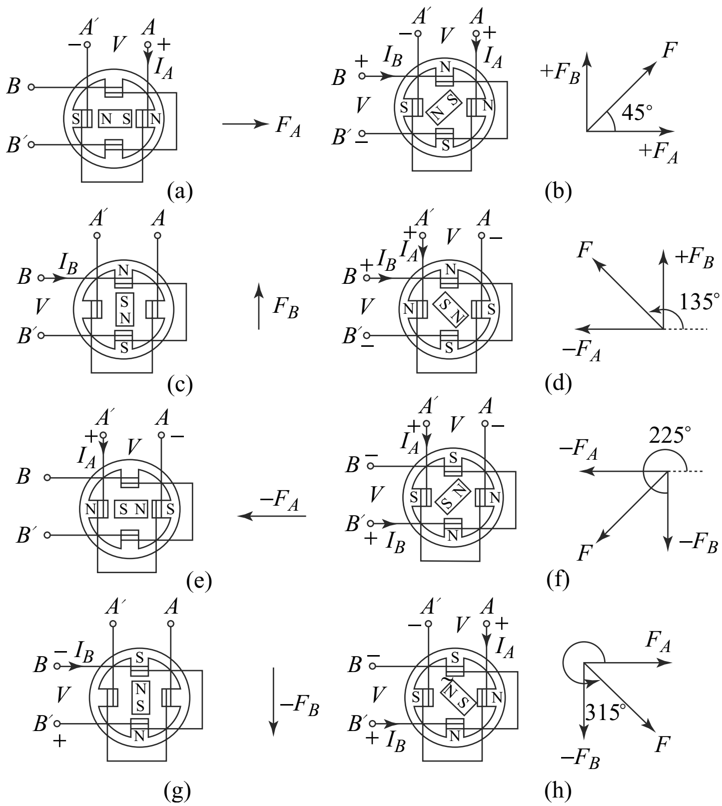


Fig. 6.47 *Principle of operation of hybrid stepper motor*

Figure 6.47 (a to h) constitutes an eight step sequence, because the rotor rotates by 45 degrees in each step. The direction of rotation of the rotor could be changed by changing the sequence of supply to the stator phase windings. Steppers motors will require a drive circuit through which command signals are obtained from a digital integrated logic circuit.

6.8.3 Applications of Stepper Motors

Various types of stepper motors find applications such as paper feed motor of a printer or head drive motor of a floppy and hard disk drive, CNC systems for machine tools, X–Y plotter, milling machines, Lathes, X-ray table positioning system, quartz clock, cameras, robotics etc.

6.9 SWITCHED RELUCTANCE MOTORS (SRM)

Figure 6.48 shows the cross-sectional view of a switched reluctance motor. The stator has eight poles while the rotor has six poles as shown. This is because SRMs have one pair of poles less in their rotors than in the stator. Both stator and rotor poles are projected type. The stator poles are wound with excitation windings while the rotor poles have no windings. The diagonally opposite stator poles are excited simultaneously in a sequence. Torque is produced due to the tendency of the rotor pole to align with stator pole so as to achieve minimum reluctance of the flux path created due to the excitation current. In the figure excitation coils for only one pair of poles in the stator have been shown. When the stator poles are excited in a particular sequence in clockwise direction, the rotor will rotate in clockwise direction in synchronism with the rotation of the stator field axis.

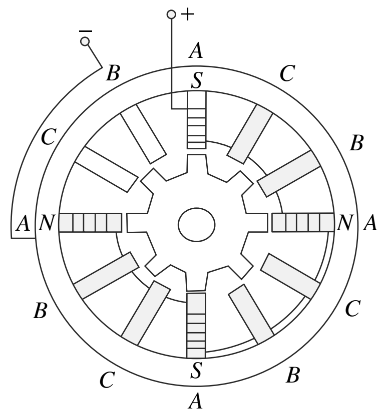


Fig. 6.48 *Cross-sectional view of a SRM*

6.10 TACHOMETERS

Tachometers are also called tachogenerators which are mounted on the shaft of a rotating machine to produce a voltage signal which is proportional to the speed. Thus the purpose of tacho-generator is to convert mechanical angular speed into a directly dependent voltage signal. Tacho-generators are used in feedback control systems for providing correcting feedback signal for constant speed servomechanism, for providing compensatory damping to stabilize a relatively unstable system and for instrumentation in a control process. The tachometer is generally an integral part of the motor either geared to or connected through V-belt or pulley arrangement. Tacho-generators are of two types, viz, dc tacho-generators and ac tacho-generators.

6.10.1 dc Tachometers

A dc tachometer is a small dc machine of permanent magnet type whose magnetic field flux is constant. The induced emf due to rotation is therefore proportional to the angular speed. The disadvantage with a dc tachometer is that due to ageing the permanent field magnets may become weak and hence produce some error in the induced emf, i.e., not being exactly proportional to speed. However, if the poles are made of good industrial magnetic material like Alnico, Magnico, etc., excellent resistance to ageing can be offered. The armature of a dc tachometer is similar in design to that of a dc machine. The induced emf, E can be expressed as

$$E \propto \phi \cdot \omega \quad \text{where, } \phi \text{ is the flux and } \omega \text{ is the angular speed;}$$

or $$E = k\,\omega \quad \omega = d\theta/dt \text{ and } k \text{ is a constant.}$$

Taking Laplace transform,

$$E(s) = K\omega(s)$$

The block diagram is shown in Fig. 6.49.

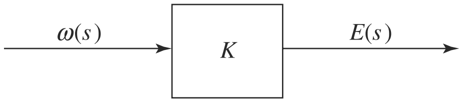


Fig. 6.49 *Block diagram of a dc tachometer*

6.10.2 ac Tachometers

An ac tachometer is similar to a two-phase induction motor whose one phase is supplied with the reference system voltage. The other winding is left open to be connected to an amplifier as shown in Fig. 6.50.

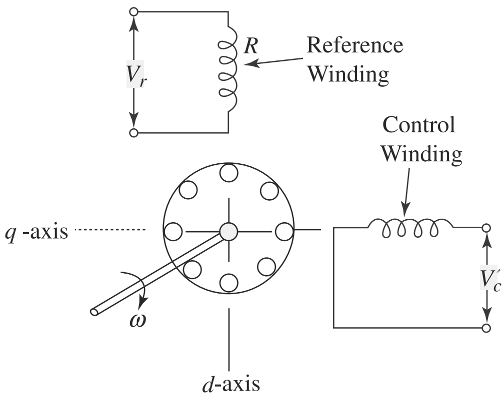


Fig. 6.50 *ac tachometer*

The stator has two windings displaced at 90 degrees. The rotor is either a squirrel-cage type or a drag-cup type. An ac tachometer converts rotational speed into proportional voltage. The voltage applied across the reference winding is fixed in magnitude at a fixed frequency, known as the carrier frequency. The axis of the reference winding can be called the direct axis, whereas, the axis of the control winding can be called the quadrature axis. The principle of working of a two-phase tachometer is explained as follows.

When a reference ac voltage is applied to the stator winding *R*, a pulsating field will be produced along the *d*-axis. When the rotor is stationary a transformer emf will be induced in it. No emf will be induced in the control winding due to the pulsating field of the reference winding because the two windings are placed at right angles to each other. Due to emf induced in the rotor windings a current will flow. A flux will be produced in the rotor which will oppose the stator pulsating field flux due to Lenzs' law. If the rotor is made of an aluminium disc having no windings, then also transformer emf will be produced in the rotor disc which will cause eddy current (circulating current) to flow. Due to this an opposing field will be developed along the direct axis. A resultant pulsating flux will thus be developed along the direct axis.

If the rotor now rotates, it would cut the pulsating flux of the direct-axis and a rotational emf will be induced in it. This rotational emf induced in the rotor will cause a current flow. A flux will be produced which will act at the quadrature axis.

This emf will be proportional to the speed of the rotor. This flux, in turn, will induce an emf in the control winding which will be proportional to the speed.

The emf available at the terminals of the quadrature axis winding can be expressed as

$$E \propto \omega \qquad \text{where } \omega \text{ is the angular speed}$$

or

$$E = k\omega \qquad \text{where } k \text{ is a constant}$$

Taking Laplace transform

$$E(s) = k\omega(s)$$

The block diagram is similar to the one shown in Fig. 6.49.

6.11 SERVO MOTORS

In control systems, a servo motor is used to convert the final control element into mechanical displacement, velocity, torque etc., as the desired output. Servo motors can be either dc or ac. They are constructed for ratings varying from a watt to several kilowatts. The commonly used servo motors are separately excited dc motors and squirrel-cage type and drag-cup type induction motors. Such servo motors should meet the starting and regulating characteristics required for the control operation. Some of the important requirements of servo motors are low moment of inertia of the rotor, linear speed-torque curve having negative slope, and capacity to withstand frequent starting and stopping.

6.11.1 dc Servo Motors

These are dc separately excited servo motors and are similar to conventional dc motors. Their control modes are either through field control with constant armature current or through armature control with constant field supply.

6.11.2 ac Servo Motors

The ac servo motors are two-phase induction motors with some modifications and are normally used from 50 Hertz mains or from 400 Hertz supply. AC servo motors are available in the range varying from a fraction of a watt to one kilowatt. Attempts are being made to manufacture servo motors of higher ratings. The schematic diagram of a two-phase servo motor is shown in Fig. 6.51.

The main field, also called the reference field, is provided with fixed voltage from the supply source. The control field voltage is connected across the control winding. This control field voltage controls the torque and speed of the motor. The control field voltage is usually supplied from an ac servo amplifier. If the phase of the control signal voltage reverses, the direction of rotation of the motor will reverse.

The voltage of the control circuit winding is made 90 degrees out of phase with respect to the voltage of the reference field. Thus we will have two phase windings carrying current which are out of phase so that a revolving field is produced that is required for the production of torque.

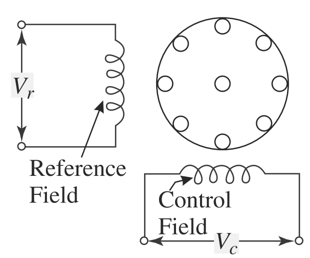


Fig. 6.51 *Schematic diagram of a two-phase ac servo motor*

The two stator windings can be excited by two-phase power supply. In the absence of two-phase power supply, the reference winding may be connected to a single-phase supply through a capacitor to provide a 90 degree phase shift. The amplifier to which the control winding is connected may be supplied from the same single-phase supply.

The torque speed characteristics of a conventional poly-phase induction motor have been shown in Fig. 6.52(a).

It can be seen from Fig. 6.52(a) that increasing rotor circuit resistance to R_3 makes the torque-speed characteristics somewhat linear. By proper design of rotor circuit, the torque-speed characteristics can be properly modified to approach an idealised characteristic. However, by trying to make T-N characteristic more linear, the starting torque gradually gets reduced which means a smaller initial acceleration and sluggish response of the servo motor. In designing a servo motor, a compromise is made between linearity of T-N characteristic and the starting torque.

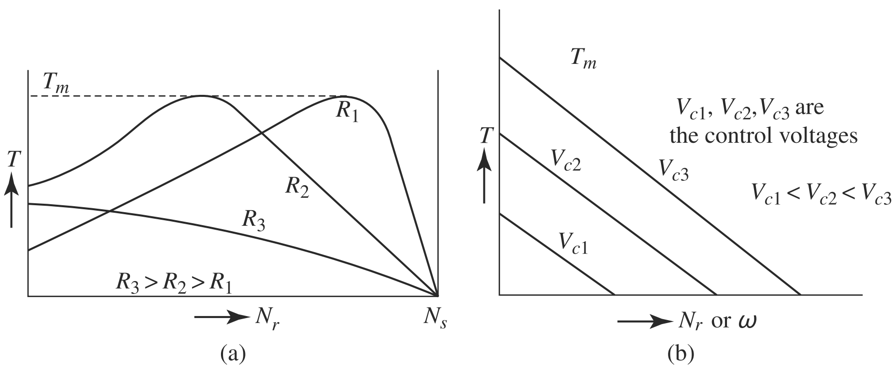


Fig. 6.52 *(a) Torque-speed characteristics of a two-phase induction motor (b) Torque-speed characteristics of a two-phase servo motor*

The torque-speed characteristic curves, when a rated voltage is applied across the reference winding and various voltages are applied across the control winding are redrawn as in Fig. 6.53(a). These give the steady-state characteristics of the servomotor.

The transfer function of the two-phase servo motor can be determined from the linear torque-speed characteristics.

Figure 6.53(a) shows a set of torque-speed curves for various values of control voltages. The slope of these curves are negative. If the control phase voltage is zero, the servo motor develops no torque. The servomotor develops a variable torque at zero speed from varying control voltages. Rapid acceleration requires high starting torque. From Fig. 6.53(a), we can notice that the torque-speed curves are straight lines with negative slopes and have an intersection in the Y-axis. The equation of such a lines is of the form;

$$y = -m\,x + c$$

In this case we can write,

$$T = -k_1\dot{\theta} + k_2 V_c$$

where, $\omega = d\theta/dt = \dot{\theta}$ and k_1 and k_2 are constants.

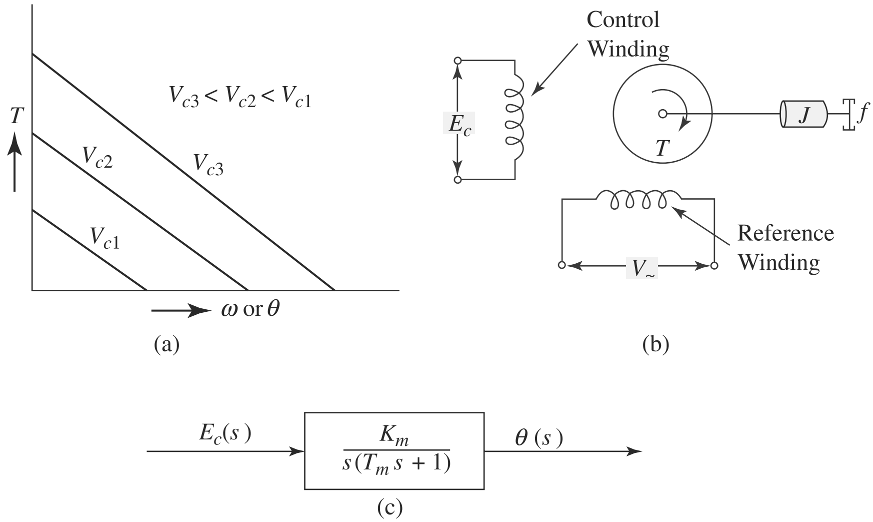


Fig. 6.53 *(a) T-N characteristic; (b) Schematic representation of a servomotor; (c) Block diagram of a servomotor*

From Fig. 6.53(b), torque-balance equation is written as

$$T = J\ddot{\theta} + f\dot{\theta}$$

where $\dot{J}$ is the moment of inertia of the motor and load and f is the coefficient of viscous friction.

Equating the above two equations we get,

$$-k_1\dot{\theta} + k_2V_c = J\ddot{\theta} + f\dot{\theta}$$

or $$J\ddot{\theta} + (f + k_1)\dot{\theta} = k_2V_c$$

Here, it may be noted that the control voltage V_c is the input and the displacement of the servo motor shaft, θ is the output. The transfer function of the system, in Laplace transform form is

$$\frac{\theta(s)}{V_c(s)} = \frac{k_2}{JS^2 + (f + k_1)S} = \frac{k_m}{S(T_mS + 1)}$$

where, $K_m = \frac{k_2}{f + k_1}$ = motor gain constant

and $T_m = \frac{J}{f + k_1}$ = motor time constant

The block diagram representation of the transfer function has been shown in Fig. 6.53(c).

AC servo motors are extensively used in position control of feedback control systems.

MODEL QUESTIONS

Short-Answer-Type Questions

6.1 Name the various type of single-phase induction motors manufactured in fractional kilowatt ratings.

6.2 Explain why a single-phase induction motor should be provided with an auxiliary winding on the stator.

6.3 Explain the principle of split-phasing used in a single-phase induction motor. Name the different methods employed. Explain each method.

6.4 Explain the principle of working of a single-phase induction motor with the help of cross-field theory.

6.5 Explain the working of principle of a single-phase induction motor with the help of double revolving field theory.

6.6 Explain why a single-phase induction motor with only one single-phase winding on the stator and connected across a single-phase supply does not develop any starting torque but continues to rotate in a direction in which the rotor is given an initial rotation.

6.7 Explain how the direction of rotation of split-phase type single-phase induction motor be reversed.

6.8 Draw the complete torque-speed characteristic of a single-phase induction motor when no auxiliary winding is provided on the stator. Show how the torque-speed characteristic is modified when an auxiliary winding is provided.

6.9 Mention the conditions necessary for production of a rotating magnetic field with the help of stationary windings

6.10 With the help of a diagram, explain how an electromagnetic relay can be used to disconnect the auxiliary winding, once the induction motor has attained a certain speed.

6.11 Explain the construction and working principle of a permanent-split single-value capacitor-type single-phase induction motor. Mention its applications.

6.12 Mention the advantages of a two-value capacitor-type induction motor over a single-value one.

6.13 Explain the principle of working of a shaded-pole type single-phase induction motor. State its applications.

6.14 Is it possible to change the direction of rotation of a shaded-pole type induction motor? Explain your statement.

6.15 Explain the main constructional features and principle of a reluctance start single-phase induction motor.

6.16 Explain why the starting torque- of a capacitor-start induction motor is better than that of a resistance-start induction motor.

6.17 The centrifugal switch contacts fail to close when a resistance-start induction motor is switched off from the supply. Explain what will happen when the motor is switched on to supply for re-starting.

6.18 A resistance-start induction motor due to heavy overload is unable to accelerate and fails to disconnect the auxiliary winding through the opening of the centrifugal switch contacts. Explain what may happen?

6.19 Explain why no rotational torque is developed by a repulsion motor when the brushes are placed along the axis of the stator poles? Will the motor develop torque when the brushes are shifted at an angle of 90° electrical with the stator poles axis?

6.20 Explain the construction and working principle of a repulsion motor. How is a repulsion-induction motor different from a repulsion motor?

6.21 Will a dc shunt motor work satisfactorily when connected across an ac supply? Explain your answer with reasons.

6.22 How can the direction of rotation of a repulsion motor be reversed?

6.23 Mention the modifications in the design to be incorporated to enable a dc series motor work satisfactorily on ac supply also.

6.24 Explain the constructional details and principle of working of an ac series motor. Draw its torque-speed characteristic and mention its applications.

6.25 Explain why a dc shunt motor will not work satisfactorily on ac supply?

6.26 Explain the constructional difference between an ac and a dc series motor.

6.27 State why small fractional kilowatt ac series motors are called universal motors.

6.28 What is an universal motor? How is it different from a dc series motor? Mention its applications.

6.29 Explain a switched reluctance motor

6.30 What is a tacho-generator? Where do we use them?

6.31 Derive the transfer function of an ac servo motor.

Multiple-Choice Questions

6.32 For production of a rotating magnetic field

(a) a single-phase supply is to be connected across a single-phase winding
(b) a two-phase supply should be connected across a two-phase winding
(c) a dc supply is to be connected across a single-phase winding
(d) the polarities of the dc supply across a single phase winding should be continuously reversed through a suitable switching device.

6.33 When a single phase supply is connected across a single-phase winding, the nature of the magnetic field produced is

(a) pulsating in nature
(b) rotating in nature
(c) constant in magnitude but rotating at synchronous speed
(d) constant in magnitude and direction.

6.34 In a resistance split-phase type single-phase induction motor, a time-phase difference between the currents in the main and auxiliary winding is achieved by

(a) placing the two windings at an angle of 90° electrical in the stator slots
(b) applying two-phase supply across the two windings

(c) having different ratio of resistance to inductive reactance for the two windings supplied from a single-phase supply system
(d) connecting the two windings in series across a single-phase supply.

6.35 In a split-phase capacitor-start induction motor, a time-phase difference between the currents in the main and auxiliary winding is achieved by
(a) placing the two windings at an angle of 90° electrical in the stator slots
(b) applying two-phase supply across the two windings
(c) introducing capacitive reactance in the auxiliary winding circuit
(d) connecting the two windings in series across a single-phase supply.

6.36 The direction of rotation of an ordinary shaded pole single-phase induction motor
(a) can be reversed by reversing the supply terminal connections to the stator winding
(b) cannot be reversed
(c) can be reversed by open-circuiting the shading rings
(d) can be reversed by short-circuiting the shading rings.

6.37 Direction of rotation of split-phase type single-phase induction motor can be reversed by
(a) reversing the supply terminal connections
(b) reversing the connections of only the auxiliary winding across the supply terminals
(c) reversing the connections of either the main winding or the auxiliary winding terminals
(d) reversing the connections of only the main winding across the supply terminals.

6.38 In a single-phase repulsion motor, torque is developed on the rotor when the brush axis is fixed
(a) at 90° electrical with the stator field axis
(b) in alignment with the stator field axis
(c) at an acute angle with the stator field axis
(d) at 90° mechanical with the stator field axis.

6.39 If the centrifugal switch of a resistance split-phase induction motor fails to close when the motor is de-energised, then
(a) no starting torque will be developed when supply is connected again across the motor terminals
(b) a dangerously high current will flow through the main winding when supply is connected again across the motor terminals
(c) starting torque developed may not be sufficient to enable the motor to restart
(d) the motor will develop high starting torque when an attempt is made to restart.

6.40 A dc series motor when connected across an ac supply will
(a) develop torque in the same direction
(b) not develop any torque
(c) draw dangerously high current

(d) develop a pulsating torque.

6.41 To enable a dc series motor work satisfactorily with an ac supply, the following modifications should be done

(a) the yoke and the poles should be completely laminated
(b) only the poles should be made of laminated sheets
(c) the air-gap between the stator and the rotor be reduced
(d) compensating poles should be introduced.

True or False

6.42 State whether the following statements are True or False:

(a) The stator of a split-phase type single-phase induction motor is provided with two windings.

(b) The rotor construction of a single-phase induction motor is similar to that of a three-phase squirrel cage induction motor.

(c) The auxiliary winding of a single-phase induction motor is placed at 180° electrical with the main winding.

(d) The currents in the main winding and auxiliary winding of a single-phase induction motor are in time-phase with each other.

(e) An alternating field may be considered equivalent to two component fields rotating in the same direction at synchronous speed.

(f) In a split-phase induction motor the ratio of resistance to reactance is made different for main and auxiliary winding.

(g) The purpose of auxiliary winding in single-phase induction motor is to make the motor self-starting.

(h) There is no need of centrifugal switch in a permanent split single capacitor type single-phase induction motor.

(i) Shaded pole motors are used in applications where the starting torque requirement is low.

(j) The direction of rotation of a shaded pole motor cannot be reversed.

(k) AC series motors are used in situations where the load is required to be driven at constant speed.

(l) AC series motors are use das a drive in electric traction system.

(m) No winding is provided on the rotor of a hysteresis motor.

Answers

6.32 (b)	6.33 (a)	6.34 (c)	6.35 (c)
6.36 (b)	6.37 (c)	6.38 (c)	6.39 (a)
6.40 (a)	6.41 (a)		

6.42 (a) True; (b) True; (c) False; (d) False;
(e) False; (f) True; (g) True; (h) True;
(i) True; (j) True; (k) False; (l) True;
(m) True.

LABORATORY EXPERIMENTS

EXPERIMENT 6.1 *Experiment to study the effect of capacitor on the starting and running of a single-phase induction motor and the method of reversing the direction of rotation.*

Objective To investigate the need of the capacitor in the auxiliary winding circuit of a single-phase induction motor, both in starting and running condition and to determine the method of reversing the direction of rotation.

Brief Theory When single-phase supply is applied across one single-phase winding on the stator of a single-phase induction motor, the nature of the field produced is alternating and as such the rotor will not develop any starting torque. It has however been observed that once the motor is given an initial rotation it continues to rotate.

In a single-phase motor, to provide starting torque, an additional winding is provided, which is called the auxiliary winding. The main and the auxiliary windings are connected in parallel across a single phase supply. The impedance of the two windings are made different so that currents flowing through these windings will have a time phase difference as shown in Fig. 6.54(a).

(a) *Need of a Capacitor in the Auxiliary Winding Circuit* A single-phase motor having a main winding and an auxiliary winding fed from a single-phase supply can be considered as equivalent to a two-phase motor having a single phase supply.

Since the two windings are not identical, the two currents I_m and I_a will have a time-phase displacement. Now if by any means the time-phase displacement between the two currents, I_m and I_a flowing through the two windings can be made 90°, a single phase motor will behave exactly like a two-phase motor. The time-phase displacement between 1 and la can be increased by using a capacitor in the auxiliary winding as shown in Fig. 6.54(b). The capacitor will also improve the overall power factor of the motor. From the phasor diagrams of Figs 6.54(a) and 6.54(b) it will be observed that the power factor of the motor is improved when a capacitor is introduced in the auxiliary winding circuit. If a capacitor is to be used only for achieving high starting torque, then the auxiliary winding can be switched off when the motor picks up speed.

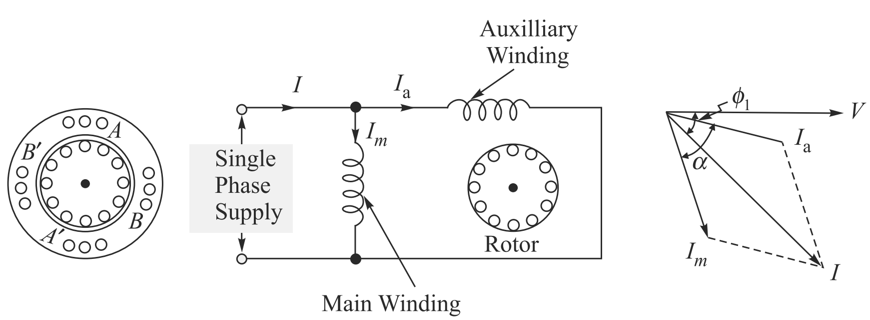


Fig. 6.54(a) *Single-phase induction motor winding carrying currents which have a time-phase difference of α degrees*

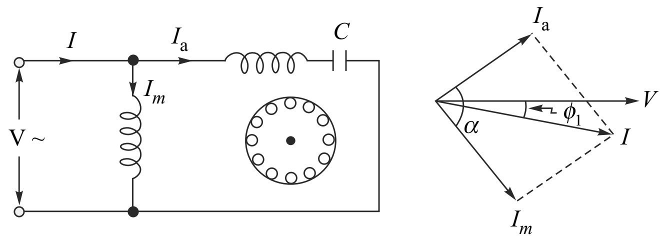


Fig. 6.54(b) *Time-phase difference of nearly 90° between the main and auxiliary winding current is achieved by using a capacitor in the auxiliary winding circuit*

(b) *Method of Reversal of Direction of Rotation* The direction of rotation of a split phase type induction motor having main and auxiliary winding, gets reversed if the current direction in any one of its windings is reversed. This is done by reversing the two terminal connections of the auxiliary or main winding across the supply. The leads of the main and auxiliary windings can be differentiated from each other (if lead marks are not labelled) by measuring resistances of the two windings.

The resistances of auxiliary winding for motors of 1/16 kW and more are generally greater than the resistance of the main winding.

Circuit Diagram See Fig. 6.55.

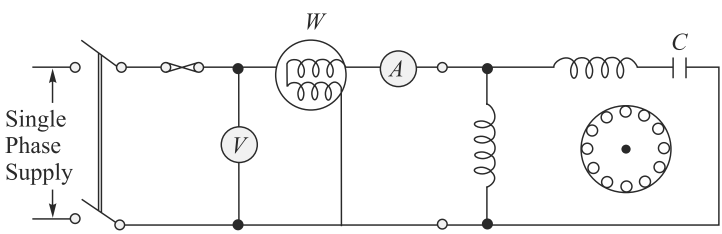


Fig. 6.55 *Connection diagram for determining the effect of capacitor on the performance of a single-phase induction motor*

Apparatus Required Single-phase induction motor-split phase type with capacitor in the auxiliary winding, single-phase wattmeter (one), voltmeter and ammeter (moving iron type).

Procedure

1. Make connections as per Fig. 6.55. Switch on the supply. Note the direction of rotation of the rotor. Remove the auxiliary winding connections after switching off the supply. Switch on supply and note that the rotor does not rotate. Give a slight rotation to the rotor in a particular direction and note that the rotor picks up speed in that direction.
2. Reconnect the auxiliary winding across the supply but without the capacitor in the circuit (short the two terminals across which the capacitor was connected). Switch on the supply and observe if the rotor starts rotating. In case the rotor rotates, feel the magnitude of starting torque by holding the shaft by hand. Allow the rotor to rotate and then record voltmeter, ammeter and wattmeter readings.

3. Run the rotor with auxiliary winding connected across the supply with the capacitor in the circuit. At starting feel the magnitude of starting torque by holding the rotor by hand. Then release the rotor and record the meter readings. Note the direction of rotation of the rotor.
4. Interchange the terminal connections of the auxiliary Winding across the supply. Switch on supply and note the direction of rotation of the rotor. Repeat this for the main winding also.

Observations

No. Sr.	Input Power (W)	Input Voltage (V)	Input Current (I)	Power Factor Circuit Condition cos ϕ
1 2				Without capacitor in the auxiliary winding With capacitor in the auxiliary winding

II Normal direction of rotation: Clockwise/Anti-clockwise.
Direction of rotating when the auxiliary winding terminal connections are interchanged: Clockwise/Anti-clockwise
Direction of rotation when the main winding terminal connections are interchanged: Clockwise/Anti-clockwise.

Questions Answer the following questions in your report.

1. Mention the purpose of providing Capacitor in the auxiliary winding circuit of a split phase type single-phase induction motor.
2. How can you reverse the direction of rotation of a single-phase induction motor?
3. What difference have you observed in the magnitude of starting torque with and without the capacitor in the auxiliary winding circuit?

7 POWER CONVERTERS

OBJECTIVES

After carefully studying this chapter, you should be able to

- ➢ Name the various types of power converters.
- ➢ Explain the principle and working of mercury arc rectifier.
- ➢ Explain the principle and working of synchronous converter.
- ➢ Explain the principle and working of selenium rectifier.
- ➢ Explain the principle and working of thyristor.
- ➢ Draw the rectified waveforms for all the four types of converters.

Supply from the electricity authority is available in the form of single-phase and three-phase ac. Direct current supply is necessary in various applications. Alternating current supply is, therefore, converted into direct current supply at the place of use. For low power ac to dc conversion, as required in electronic circuits, silicon diodes can be used. For high power conversion, devices like rotary convertors, mercury arc rectifiers, selenium rectifiers, and thyristors are used. These days thyristors have almost replaced the other converting devices.

Brief descriptions of all the types of conversion devices are given in the following sections.

7.1 MERCURY ARC RECTIFIERS

Similar to a common diode, a mercury arc rectifier consists of a cathode and an anode. Mercury is used as cathode and the anode is made of iron. Figure 7.1 shows a single-phase mercury arc rectifier with two anodes *A* and *B*.

When the rectifier is in operation, mercury forms an arc between the mercury surface and the iron anode. This arc contains ionized mercury vapour consisting of electrons and positive ions. The area on the surface of the mercury where the arc concentrates is known as the cathode spot. This is a region of high temperature and most of the ionization (formation of ions by loss of electrons), takes place here. In the arc the electrons are attracted by the positive anode whereas positive ions are attracted by the negative cathode. Since the positive ions are heavy, they carry a lot of kinetic energy which is released in the form of heat energy when they strike the cathode. This heat maintains the arc throughout the operation, i.e., when the anode is positive and the cathode negative. It has been observed that ionization temperature for mercury is 2087° C. The anodes are kept cool so that they do not emit electrons freely. As the electrons travel through the arc, this arc forms the conducting path for current and there is a voltage drop in the arc known as arc voltage. Normally the arc voltage drop is of the order of 12 to 18 V. In well-designed mercury-arc rectifiers, the arc drop is independent of the load current. Initially, as there is no current, no arc is formed. Therefore, the arc is initiated by an ignition rod or starting electrode. At this moment, let anode *A* be positive and anode *B* negative. Because of the high temperature at the mercury surface, it emits electrons and therefore ionizes the mercury vapour. Electrons are attracted to *A* and positive ions strike the cathode. When the anode is negative, it repels all the free electrons reaching it, and these are absorbed by the cathode. Now there are no free electrons available for ionization of vapour. Hence current drops to zero in anode *A*. But now anode *B* becomes positive, and hence current flows from the cathode to anode *B*. The above circuit gives out a full-wave rectified output. Usually, mercury arc rectifiers are not used for half-wave rectification, because during the negative half-cycle the arc is extinguished and it will not restrike in the next positive half-cycle by itself. External means will be needed to restrike the arc after every negative half-cycle.

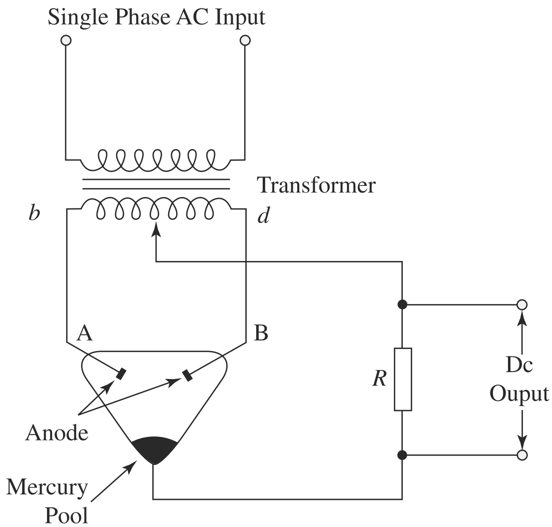


Fig. 7.1 *Single-phase mercury arc rectifier*

Figure 7.2 shows the ac supply voltage with a peak value of *E*. Let the voltage drop across arc be V_{arc}. From Fig. 7.1, according to Kirchhoff's voltage law, the voltage drop across *R* (output voltage) will be equal to the difference between the supply voltage and arc voltage. There will be current in the circuit only when supply voltage *E* exceeds the arc voltage V_{arc}. Therefore the current does not start at the instant zero, of the supply voltage but at a time t_1 where the supply voltage crosses the arc voltage. Figure 7.2 shows full-wave rectified output voltage and current waveforms. As the output is taken across a pure resistance, the circuit is resistive, and hence the voltage V_0 and current *I* are in time phase. From Fig. 7.2, it can be seen that the greater the arc voltage drop, the greater will be the time delay O_{t_1}, and hence t_2 to t_1'. It is observed that during the time interval $t_2\ t_1'$, there is no current and if this time is too large, then the arc may be extinguished because of the deficiency of electrons which is necessary to maintain the arc. Then the starting electrode will have to be employed to restart the arc. Therefore, in the design of mercury arc rectifiers, the arc voltage is kept to a minimum.

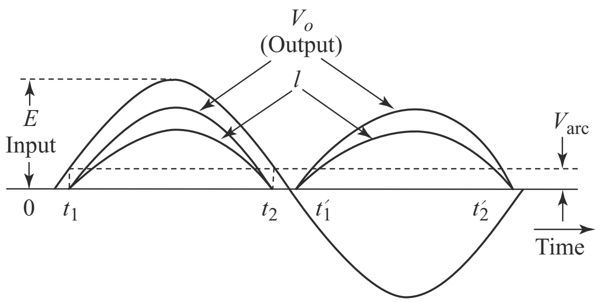


Fig. 7.2 *Output voltage and current of a mercury arc rectifier*

A three-phase mercury arc rectifier is shown in Fig. 7.3(a). The primary windings of the transformer are connected in star with neutral at O'. Secondaries are connected to three anodes A, B and C and the neutral is connected to the cathode through a

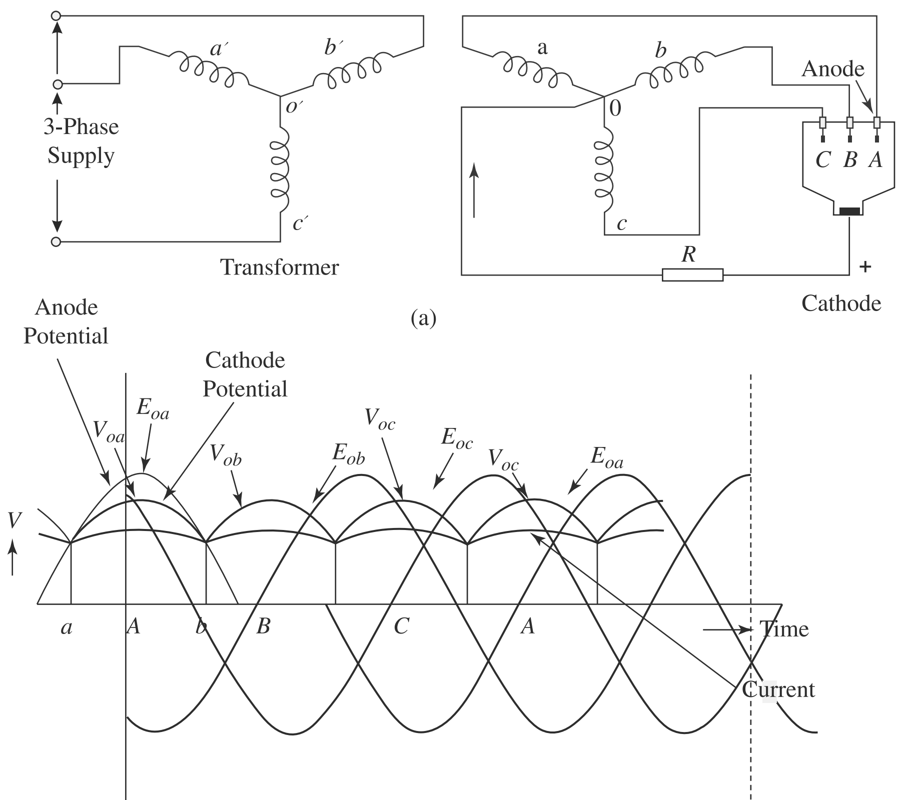


Fig. 7.3 *(a) Three-phase mercury arc rectifier, (b) Supply and output voltage waveform of three-phase rectifier*

resistance R. The three secondary emfs E_{oa}, E_{ob} and E_{oc} are shown in Fig. 7.3(b). V_{oa}, V_{ob} and V_{oc} are the cathode potentials above the neutral point O and are determined by subtracting the arc voltage drop from E_{oa}, E_{ob} and E_{oc}, respectively. During interval of time *ab*, it is observed that the potential E_{oa} is greater than E_{ob} or E_{oc}. Therefore, anode *A* will attract electrons whereas anodes *B* and *C* will not attract any. Hence during time *ab*, current flow is due to anode *A*. At time *b*, we can see that E_{oa} and E_{ob} intersect and are equal. Hence at this point of time arc will be divided between the two anodes *A* and *B* for a moment but then E_{ob} becomes greater than E_{oa} and E_{oc}, and therefore current flows due to anode *B*. In this manner the arc is transferred from one anode to other in each successive cycle. As shown in Fig. 7.3(b), voltages, namely V_{oa}, V_{ob} and V_{oc} represent the dc output voltage waveform.

7.2 SYNCHRONOUS CONVERTERS

To convert a large amount of ac power to dc power, motor generator sets (ac motor coupled with dc generator) could also be used. The disadvantage of this is that it requires comparatively more space, and the efficiency is also poor (as the overall efficiency is the product of the efficiencies of the motor and the generator). A single machine is also available to convert ac to dc. Such a machine is known as the synchronous converter or rotary convertor. A synchronous converter combines the functions of a synchronous motor and a dc generator.

7.2.1 Principle of Synchronous Converter

When a coil is rotated in a magnetic field, an alternating emf is induced in the coil. This emf can be taken out through brush and slip-ring arrangement. However, if the output is taken through the brush and commutator arrangement, dc will be available in the external circuit. A synchronous converter as shown in Fig. 7.4 has fixed poles on the stator, as in a dc generator, and has a rotating armature with both commutator and slip-ring arrangements.

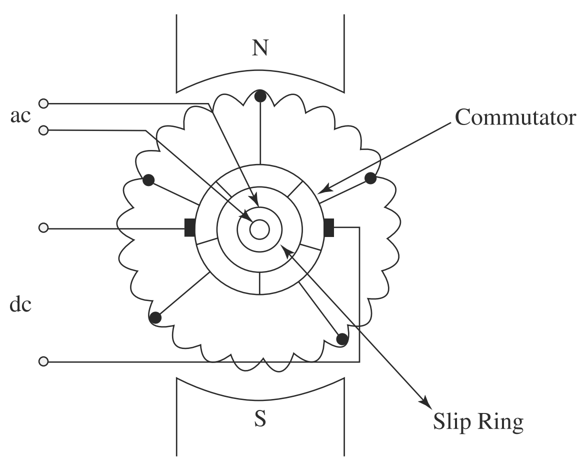


Fig. 7.4 *Single-phase synchronous converter*

In the synchronous converter, alternating current is supplied to the slip-rings, and direct current is taken from the commutator. If no output is taken from the brushes, it only means that alternating current is being fed to the machine via slip-rings, making the motor rotate. That makes it a synchronous motor. On the other hand, if dc is fed through the brushes and commutator, it works as a dc motor. If the rotor is driven mechanically, then it is known as a double current generator.

In the synchronous converter, alternating current is supplied to the slip-rings, so that the machine rotates as a synchronous motor. Simultaneously, direct current is taken out from the commutator, as in a dc generator.

Therefore, a synchronous converter is a combination of a synchronous motor and a dc generator.

7.2.2 Polyphase Converters

The output of a converter increases with the number of phases of ac supply. Therefore, three phase converters are generally in use. The connection diagram of a polyphase converter has been shown in Fig. 7.5. In the case of a two-pole converter, for a 3-ϕ of supply, three equidistant taps are taken and three slip rings used.

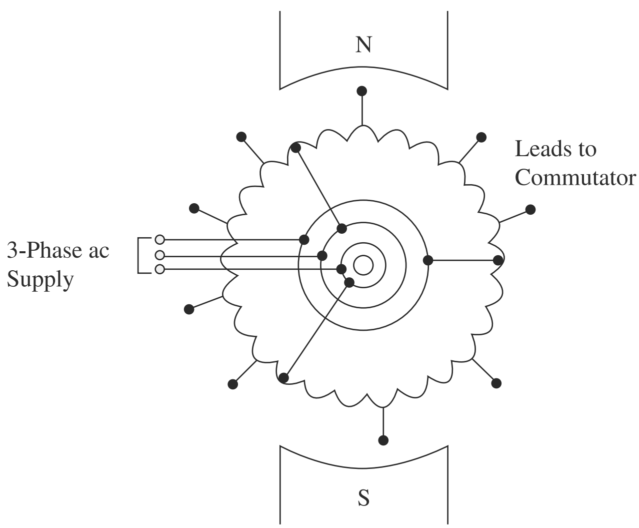


Fig. 7.5 *Three-phase, two-pole and four-pole power converter*

In the case of a four-pole converter, for three-phase supply, each ring (there are three slip-rings for three-phase) will have two taps.

If there are six poles, the number of taps to each slip-ring will be three. Thus we can formulate a simple rule stating that the number of taps to a ring will be equal to that of pole-pairs, and that the number of slip-rings will be equal to that of number of phases.

The number of dc brushes in a single-phase converter is equal to that of poles, and these brushes are placed a pole pitch apart. Therefore, the same number of conductors lie between the dc brushes and ac slip-rings. These conductors cut the same field produced by main poles but produce ac and dc emf together. The emf between the brushes is the sum of those induced in each of the individual conductors connected in series. Consider an armature with two coils displaced by 90° from each other. The ac emfs in the two will be displaced in phase by 90°. The dc emf is equal to the sum of rectified ac waves as shown in Fig. 7.6. The maximum dc emf is equal to the peak value of the ac emf. As the peak value of the ac emf is equal to $\sqrt{2}$ times the rms value, the dc emf is equal to $\sqrt{2}$ times the rms value of the ac emf.

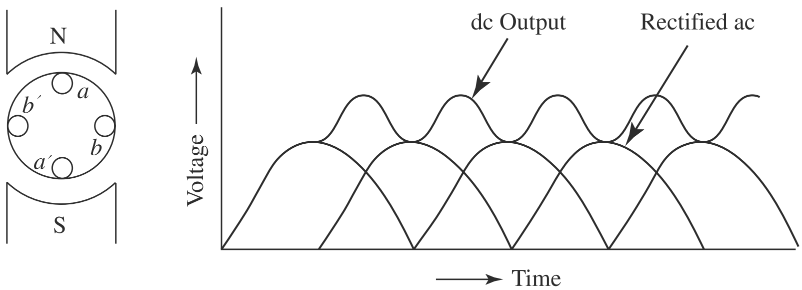


Fig. 7.6 *Output waveform far a two-phase converter*

The net current in the armature conductors of synchronous converter is the difference between the alternating and direct current which exist in the conductors separately. This is because the alternating current enters the conductors through slip-rings. It drives the machines as a motor, and in a motor, the driving voltage is in opposition to the induced emf. The dc current is a generator current and aids the induced emf. Therefore, the two currents oppose each other. The net current in each conductor is, therefore, the difference between the two currents. Figure 7.7 shows dc and ac and their difference, i.e., the resultant current.

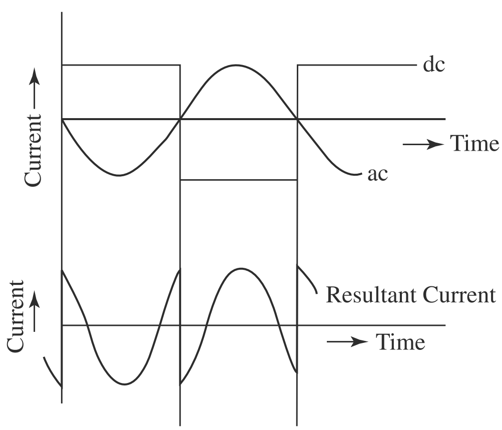


Fig. 7.7 *Waveform of different currents in a power converter*

7.3 SELENIUM RECTIFIER

The working of selenium rectifier is based on the unilateral conduction property of selenium. Unilateral conduction means that the device passes current in one direction, and blocks it in the reverse direction. This property is exhibited by selenium when placed in intimate contact with two metallic electrodes. Figure 7.8 shows the basic arrangement of a selenium rectifier unit. Selenium is deposited as a film about 0.05 mm thick on a plate of iron or aluminium. Then a low melting point alloy is sprayed on the selenium surface, forming the counter electrode. By means of chemical treatment a film 'blocking' or 'barrier' layer is formed between the selenium and the counter-electrode.

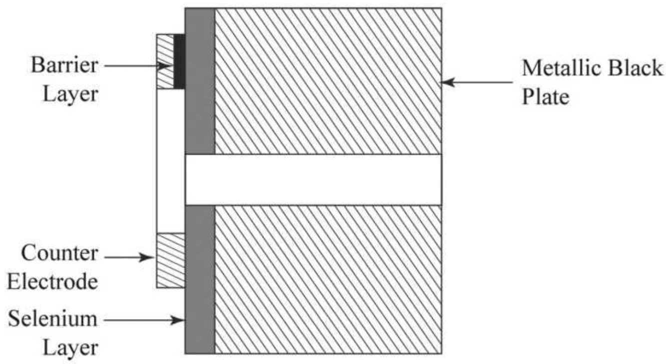


Fig. 7.8 *Selenium rectifier unit*

The rectification is in the direction from black plate to selenium, i.e., the rectifier unit passes current from the metallic black plate to selenium but blocks it from selenium to the black plate.

The above unit is capable of blocking a reverse voltage of 18 V, without breakdown. These units can be combined in series and parallel and assembled in stacks. This type of rectifier is widely used for battery charging, telephony and telegraphy, control circuits, regulated power supply units, etc.

A practical circuit which employs the above-mentioned rectifier unit is discussed below. This converts three phase 400 V ac to 220 V dc. The maximum swing of the output voltage can be ± 5%. The circuit diagram is shown in Fig. 7.9. The output voltage can be varied manually as well as automatically. Such rectifiers are often used in laboratories and workshops for dc supply.

AC supply is given through a three-phase variac (dimmerstat). The sliders of the dimmerstat (shown arrowed) can be moved up or down by a motor. As the three sliders are interconnected, by moving them up or down the ac input to the rectifier can be varied which can further vary the dc output voltage. From the dimmerstat, the supply is fed to the main transformer through a series (buck-boost) transformer. The secondaries of the series transformer are connected to the line voltage of the supply. The primaries are connected to the sliders of the dimmerstat. Hence the voltage induced in the secondaries of the series transformer will add to or subtract from the reference supply voltages. Therefore, even in the minimum position of the slider, the output of the dimmerstat will not be zero.

The main transformer is star-delta connected. It's output is fixed to six selenium rectifier units connected as shown in Fig. 7.9. For the positive half of the supply waveform, the rectifiers 1, 2 and 3 conduct. When the supply reverse it's polarity, rectifier 4, 5, 6 conduct maintaining the same direction of current in the output circuit.

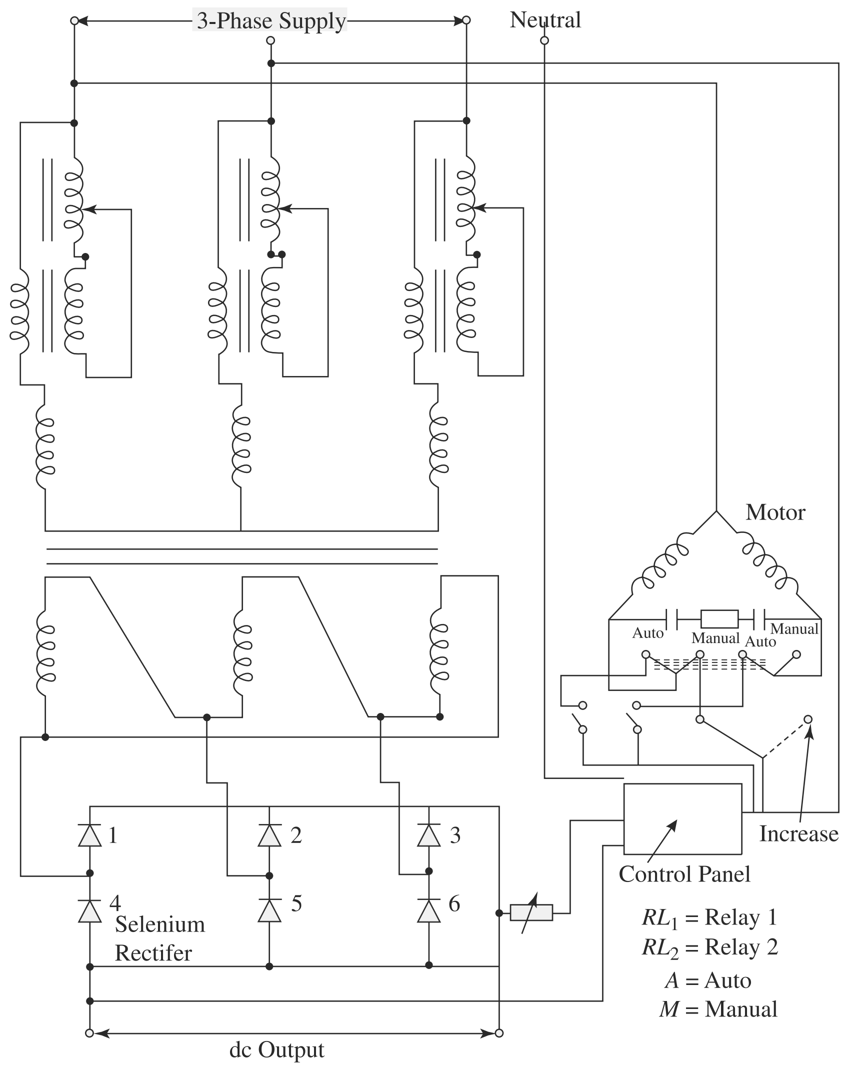


Fig. 7.9 *A practical dc power supply unit using selenium rectifiers*

To control the output a selector switch is provided to make the control either manual or automatic. The control panel (shown as a box in Fig. 7.9) is energised by a 20 V supply. There are two level detectors (relay type switches) RL_2 and RL_1 with control unit. These two detectors are locked at slightly different voltage levels. If this level difference is exceeded by a variation in the output dc voltage, one of the two relays operates and activates the motor. The motor moves the slider on the dimmerstat up or down depending on whether the output voltage is high or low.

When the output voltage is restored to its set value, the switch comes back to the normal position. Thus, the dc output is controlled.

7.4 THYRISTOR POWER CONVERTERS

In the area of conversion of power from ac to dc the use of semiconductor diodes was limited to a low power level and therefore bulky mercury arc rectifiers were in use for higher power levels. But now, thyristors, popularly known as silicon controlled rectifiers (SCR) have replaced almost all the rectifying devices used so far. These days thyristorised power converters are available with a current carrying capacity of more than 1000 A.

As seen from Fig. 7.10(a), a thyristor is composed of four alternate layers of *p* and *n* type material. It has three terminals, namely, anode, cathode and gate. Usually the gate is used to turn the thyristor on.

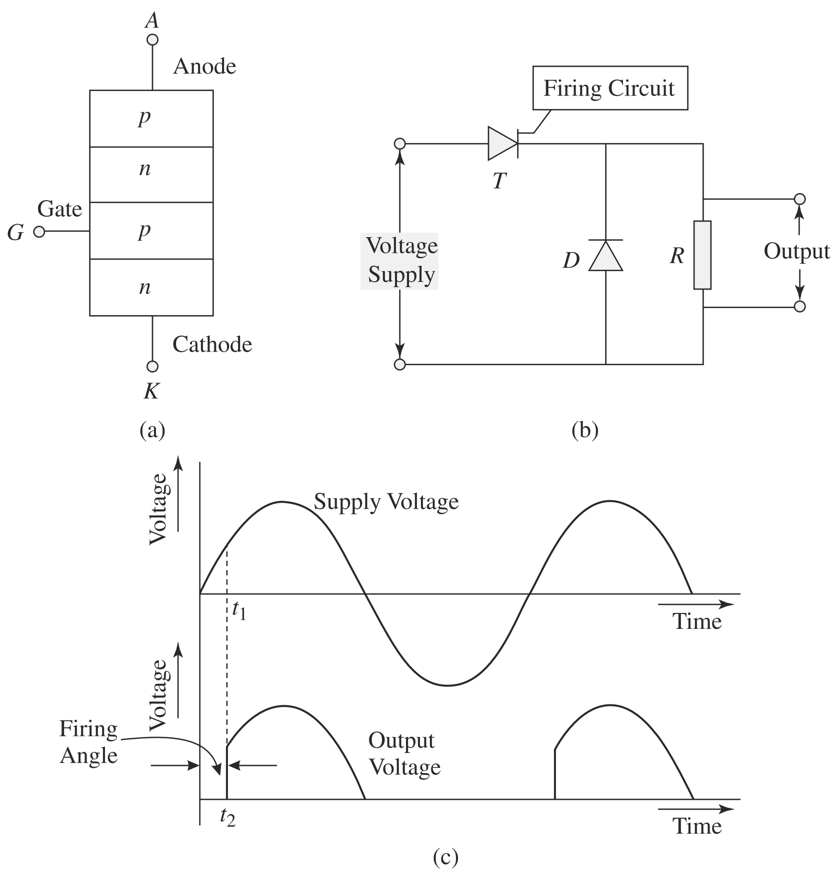


Fig. 7.10 *Half-wave thyristor converter circuit and corresponding output waveform*

A thyristor can be used to act as a switch which when in the conducting state, i.e., when its anode is more positive with respect to the cathode, allows flow of current in only one direction. A thyristor can be switched on or off by giving a low pulse to its gate. Thus a very small current can control the conduction of very high currents. Once the thyristor, T starts conducting, the gate loses its control over it. To bring the thyristor back to the off state, current has to be brought to a value lower than the holding current. A simple circuit has been taken up to illustrate the working of a thyristor as a rectifier. Figure 7.10(b) shows the simplest circuit of a half-wave rectifier. The gate is supplied a starting pulse from a firing circuit. Throughout the cycle of supply voltage, the thyristor does not conduct if the forward block over voltage is not exceeded. Hence no current flows up to t_1 and there is zero output across the resistor R. Now, if the gate is fired at an instant t_1, the thyristor starts conducting, i.e., it is switched on. The whole of the supply voltage falls across resistor R and the voltage waveform is as shown in Fig. 7.10(c). The instant at which the gate is fired can be changed by the firing circuit. The angle between instant zero and instant t_1 is known as the *firing angle*. It can be observed that by varying the firing angle, the shape of the output waveform, and hence the average value of the output, can be changed over a wide range.

The diode, D across the load known as free wheeling diode is used to provide closed path to any inductive current during the non-conducting mode of the thyristor. We can see that we are getting positive output only during the positive half cycle of the ac supply. To get positive output during both halves of the ac supply, *a*, full wave converter is to be used which is shown in Fig. 7.11.

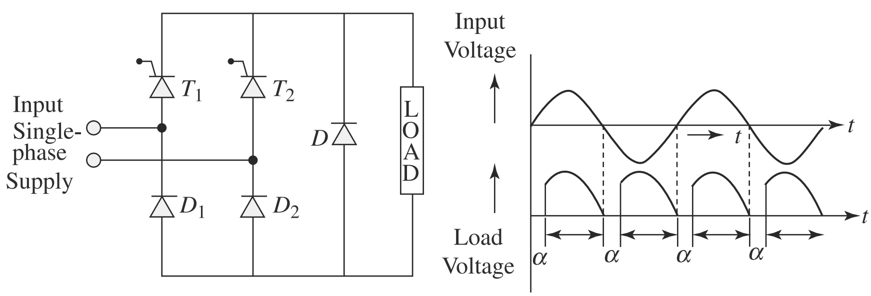


Fig. 7.11 *Full wave converter*

The thyristors T_1 and T_2 have similar characteristics. They are fired alternately at a fixed interval. The firing instant can be varied to get desirable output. T_1 and D_2 are in conducting mode during positive half of input cycle; and T_2 and D_1 are in conducting mode during the negative half of the input cycle. D is the free wheeling diode.

Figure 7.12 shows a simple circuit for the conversion of three-phase supply to dc output. It consists of three thyristors connected in such a way that no two thyristors conduct simultaneously. The firing angles and waveforms for the three thyristors are shown in Fig. 7.13.

V_{an}, V_{bn} and V_{cn}, are respectively the phase voltages of *a*, *b*, and *c*. It is assumed that firing-angle is zero for each rectifier. This means that each thyristor conducts when the voltage across it is positive at its anode with respect to its cathode. From Fig. 7.13, it is clear that between points *p* and *q*, voltage V_{an} across thyristor 1, is more positive than voltages V_{bn} and V_{cn} which are the voltages across thyristors 2 and 3 respectively. Hence this voltage switches on thyristor 1, but switches off the other two thyristors. Hence between *p* and *q* thyristor 1 conducts, and a portion of voltage V_{bn} between *p* and *q* is available at the output. After *q* (between *q* and *r*), V_{bn} becomes greater than V_{an} and V_{cn}. Hence only thyristor 2 conducts while thyristors 1 and 3

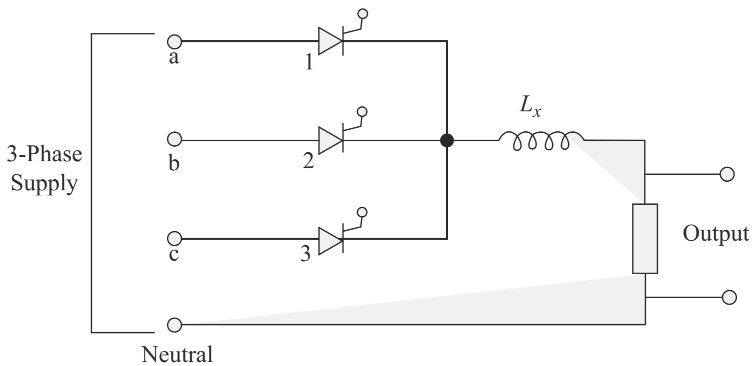


Fig. 7.12 *A three-phase thyristor converter circuit*

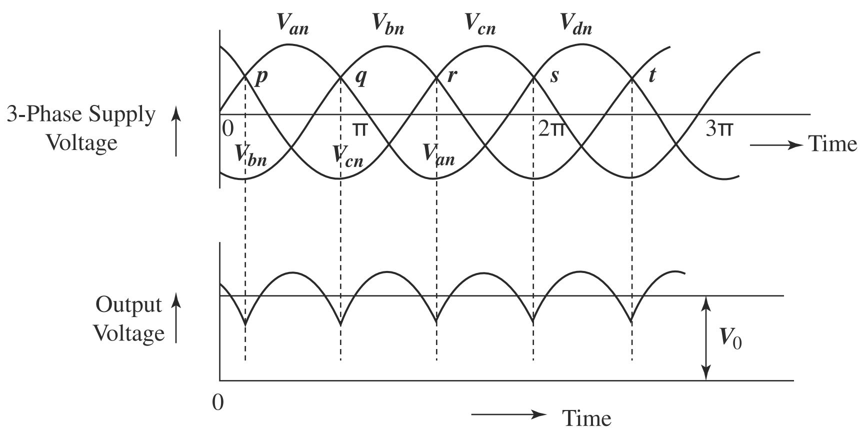


Fig. 7.13 *Three-phase supply and dc output waveforms of a thyristor converter*

are in the off state. Similarly between *r* and *s*, only thyristor 3 conducts. Thus, a dc pulsating waveform as shown in Fig. 7.13 is available at the output. It is seen that this waveform has ripples. Therefore, to reduce these ripples an inductance L_x is connected in the circuit (because inductance offers high reactance (ωL) to the alternating part and zero reactance to the direct current part of the pulsating wave shape).

MODEL QUESTIONS

Short-Answer-Type Questions

7.1 Name various types of converters available for the conversion of high voltage.

7.2 Explain the principle of the mercury arc rectifier. Also, clarify how the arc is maintained, throughout the operation.

7.3 Draw the circuit output diagram of mercury arc rectifier.

7.4 Draw the output diagram and waveform of a three-phase mercury arc rectifier.

7.5 Explain the principle of the synchronous converters.

7.6 Draw a neat sketch of a three-phase synchronous converter clearly showing the connections of the slip rings.

7.7 Draw the waveform of the current that flows in the armature of a single-phase synchronous converter.

7.8 Explain the principle of a selenium rectifier.

7.9 Draw and explain the working of a three-phase rectifier using a selenium rectifier.

7.10 Explain the principle of silicon controlled rectifier.

7.11 Explain how a thyristor can be switched on by its gate.

7.12 What is firing angle and explain how it varies the output.

7.13 Explain the constructional different between a mercury arc rectifier and thyristor.

7.14 Draw a circuit diagram of a three-phase converter using only thyristors.

7.15 Draw the output voltage waveform of a thyristorised converter.

7.16 State how the ripple in the output voltage can be eliminated in a thyristorised converter.

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