Engineering Graphics

RGPV-BE 105

Bachelor of Engineering B.E. (Common to all Branches)

About the Authors

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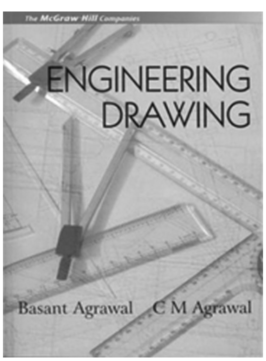
By the Same Authors



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Contents:

Drawing Instruments and Their Uses, Sheet layout, Lines Lettering and Dimensioning, Geometrical Construction, Scales, Conic Sections, Engineering Curves, Loci of Points, Orthographic Projections, Sectional and auxiliary views, Projections of Points, Projections of Straight Lines, Projections of Planes, Projections of Solids, Sections of Solids, Anti-sections, Development of Surfaces, Anti-development, Intersection of Surfaces, Isometric Projections, Oblique Projections, Perspective Projections, Computer Aided Design (CAD).

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Preface

Overview

Engineering Graphics is the 'international language of engineers'. It is a core subject for the first-year students of all branches of engineering. It is also a prerequisite subject of study for all professionals since it acts as a viable means of communicating technical ideas in a recorded form. When exact visual understanding is necessary, engineering graphics is the most appropriate way to represent information. It also develops the ability to visualize any object with all its physical and dimensional features.

Target Audience

The present book intends to meet the requirements of the first year BE students of Rajiv Gandhi Proudyogiki Vishwavidyalaya (RGPV) and other autonomous engineering colleges of Madhya Pradesh. It contains a large number of examples on drawing various shapes, and each example outlines the steps of construction. Problems from previous examination papers of RGPV are included. They have been classified from simple to advanced. The illustrations are simplified to help students grasp the basic concepts. Multiple-choice questions with answers are given at the end of each chapter for the purpose of quick revision and preparation for practical/viva-voce examination.

About the Book

One of the major strengths of the book is the excellent presentation of the subject matter in a clear, logical and concise manner. The work is an extract of the knowledge gained by the experience of successful classroom teaching of this subject with utmost devotion. No one who goes through the book can miss the enormous work that has been undertaken to prepare the text in the present form.

This book is designed to act as a comprehensive guide to cover the basic principles. It also includes every significant feature of graphics software while making use of the computer as a drawing instrument. The drawings have been prepared to scale with the help of advanced software packages as per recommendations of ISO and the latest BIS codes. Simple language, systematic introduction of concepts, variety of solved examples, questions and objective type questions are some of the major features of the text.

Salient Features

The salient features include the following:

- Excellent illustrations (2D & 3D) for effective visualization of the objects
- An exclusive chapter on application of CAD software
- Step-by-step procedures given for solved problems with classification from simple to typical ones
- Large number of solved examples from RGPV question papers
- Simplified presentation of the subject matter and illustrations
- Use of BIS code 'SP 46 : 2003' and ISO standards
- Excellent Pedagogy includes examples and practice questions:
  - Solved examples: 383
  - Practice Questions: 406
  - Review Questions: 154
  - Multiple Choice Questions: 172

Chapter Organization

This book is organized into thirteen chapters. *Chapter 1* provides the list of drawing instruments required in engineering graphics and covers the important geometrical constructions. *Chapter 2* describes the different types of engineering scales and their applications. *Chapters 3* and *4* deal with the construction of curves used in engineering practice. *Chapter 5* introduces the fundamentals of orthographic projections. *Chapters 6 through 10* present projections of points, straight lines, planes, solids and sections of solids. *Chapter 11* describes the development of surfaces as applied to sheet metal work. It is recommended for the beginners to read Chapters 5 through 11 in the same chronological order as given in this book. *Chapter 12* is devoted to isometric projections. Finally, *Chapter 13* deals with the use of computer graphics with special emphasis on AutoCAD developed by Autodesk Inc. An attempt is made to present some of the basic commands in a simple way. The readers are, however, advised to refer the manuals prepared by Autodesk for its detailed features and applications.

Acknowledgements

We are indebted to the expert faculties for their appreciation, recommendation, valuable suggestions and encouragement during preparation of this book. We would also like to thank the editorial and production staff—Ms Vibha Mahajan, Ms Shalini Jha, Ms Tina Jajoriya, Mr Manish Choudhary, Ms Sohini Mukherjee, Mr Satinder Singh, Mr Anuj kr. Shriwastava and Ms Rachna Sehgal, at McGraw-Hill Education India for their assistance and cooperation. The reviewers of this text also deserve a special note of thanks. Their names are mentioned below.

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Our acknowledgements would be incomplete if we forget to mention the love, care and patience rendered by our family members. Our wholehearted thanks go to them.

Feedback

Any further suggestion or criticism from the readers for improvement of the text will be highly appreciated. For any assistance or clarification, please contact us through our e-mail ids: *bas_agr@yahoo.co.in* and/or *cma2004@rediffmail.com*.

Basant Agrawal
C M Agrawal

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Conic Sections: Construction of ellipse, parabola, hyperbola by different methods; Normal and Tangent
Special Curves: Cycloid, Epicycloid, Hypocycloid, Involutes, Archimedean and logarithmic spirals

GO TO

***Chapter 1* – Introduction**
***Chapter 2* – Scales**
***Chapter 3* – Conic Sections**
***Chapter 4* –Special Curves**

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GO TO

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GO TO

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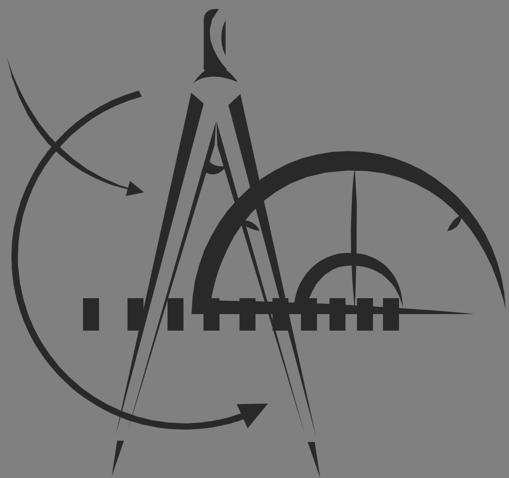
Isometric Projections: Isometric scale, Isometric axes, Isometric Projection from orthographic drawing.
Computer Aided Drafting (CAD): Introduction, benefit, software's basic commands of drafting entities like line, circle, polygon, polyhedron, cylinders; transformations and editing commands like move, rotate, mirror, array; solution of projection problems on CAD.

GO TO

***Chapter 12* – Isometric Projections**
***Chapter 13* – Introduction to Computer Aided Drafting**

1



Introduction

- Introduction
- Drawing Instruments and Accessories
- Sheet Layout
- Lines
- Lettering
- Dimensioning
- Geometrical Constructions
- Construction of Regular Polygons

1.1 INTRODUCTION

Engineering drawing is used to fully and clearly convey the ideas and information necessary for constructing engineered items. It is usually created in accordance with standard conventions for layout, nomenclature, interpretation, appearance, size, etc. The purpose of engineering drawing is to provide exact geometrical configuration for the construction or analysis of machines, structures, or systems. Today, the mechanics of the drawing task has been largely automated, and greatly accelerated, through the use of CAD systems. This chapter provides a list of drawing instruments, layout of drawing sheets, methods of dimensioning and some important methods of geometrical constructions.

1.2 DRAWING INSTRUMENTS AND ACCESSORIES

The following instruments and accessories are required for drawing:

1. Drawing board
2. Mini-drafter
3. Card sheet (A1 size)
4. Drawing sheet (A2 size)
5. Pencil (2H, H and HB)
6. Compass (big and small)
7. Divider (big and small)
8. Protractor
9. Ruler (scale)
10. French curves
11. Set-squares
12. Eraser or rubber
13. Drawing clips
14. Adhesive tape
15. Pencil cutter
16. Sandpaper pad
17. Brush or towel cloth
18. Sketchbook

A *drawing board* provides flat and smooth surface on which the drawing sheet is fixed. A *mini-drafter* is used to draw horizontal, vertical and inclined parallel lines of desired lengths anywhere on the drawing sheet with considerable ease. A *compass* is used for drawing circles. A big compass is used to draw circles, arcs and circular curves of diameter greater than 20 mm. A small bow compass is used to draw circles, arcs and circular curves of diameter less than 50 mm. A *divider* is used to divide lines or curves into a number of equal parts, to transfer measurement from one part of the drawing to another part and to step off a series of equal distances on the drawing. In engineering drawing, we need a big divider and a small bow divider. A *protractor* is used to draw and measure angles, and to divide a circle or a sector into any number of equal parts.

A flat *metric ruler* is required for measurements in centimetres and millimetres. *French curves* are used to draw smooth curves through predetermined points in short steps. *Setsquares* are used to draw lines inclined at 30º, 45º and 60º with the horizontal. Lines inclined at 15º and 75º can also be drawn with the help of a pair of set squares. An *eraser* or a rubber is used for removing unwanted pencil marks. *Drawing clips* and *adhesive tapes* are used to fix the drawing sheet on the drawing board. The pencils are prepared by a *blade-type cutter* or a *sharpener*. While sharpeners are used for preparing conical points, the blade-type cutters are suitable for removing the wood from the pencil. A *sandpaper pad* is used to sharpen the pencil lead. A *brush*, *duster* or *towel cloth* is used to keep the drawing surface clean by removing the erased crump, graphite particles or accumulated dirt.

1.3 SHEET LAYOUT

A proper sheet layout facilitates easy reading of drawings and makes it possible for essential references to be located. A standard arrangement should ensure that all necessary information for understanding the content of drawing is included and sufficient extra margin is left to facilitate easy filing and binding wherever necessary. IS 15093 : 2002 and SP46 : 2003 published by Bureau of Indian Standards specify the exact size and location for each items on the drawing sheet. A typical layout of drawing sheet containing frame, title block, space for text and other information is shown in Fig. 1.1.

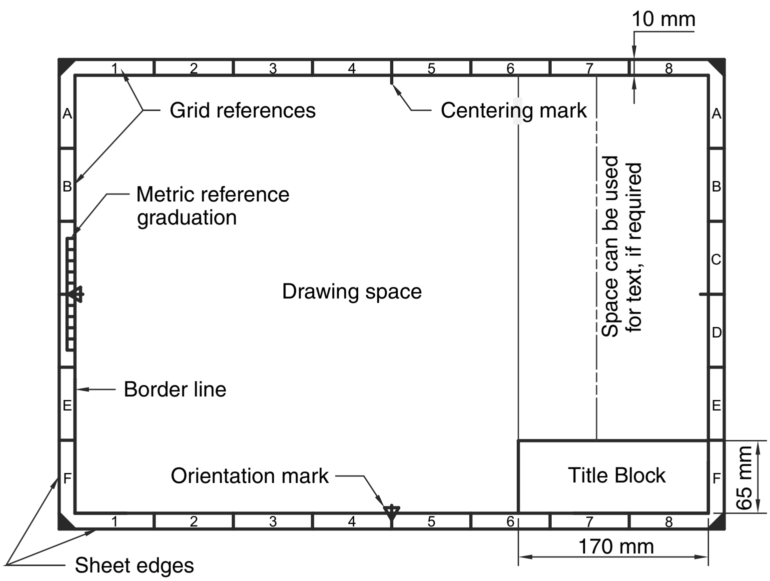


Fig. 1.1 *Typical layout of drawing sheet*

1.3.1 Title Block

A title block or a nameplate is made on the drawing sheet for identification purpose. It is normally placed in the bottom right-hand corner of the drawing frame. The size of the title block recommended by B.I.S. is 170 mm × 65 mm. Figures. 1.2(a) and 1.2(b) show sample title blocks used by draughtsmen in industries and engineering students in colleges respectively.

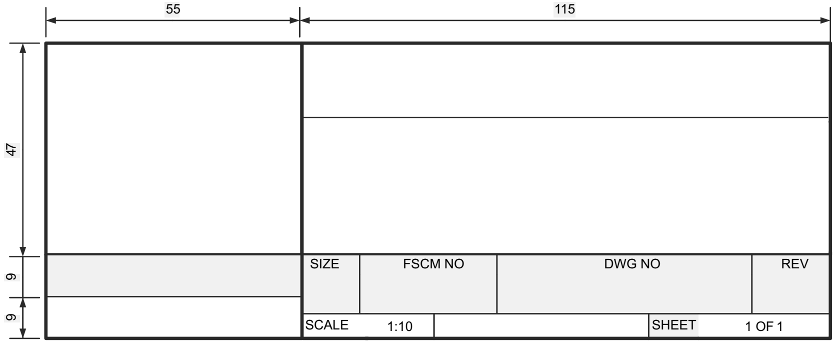


Fig. 1.2(a) *Title block used by draughtsmen in industries*

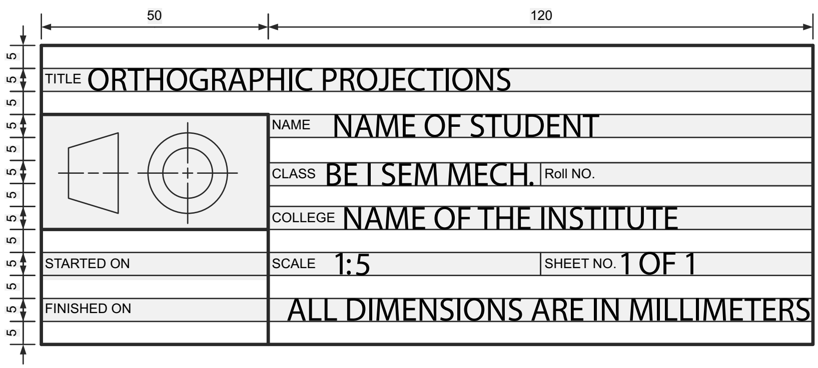


Fig. 1.2(b) *Title block used by students in engineering colleges*

A title block contains the following information:

- Name of the organization or the institute
- Title of the drawing
- Drawing sheet number
- Scale on which the drawing is prepared
- Symbol for the angle of projection used
- Initials of designing, drawing, checking, approving and issuing officer
- Any other information that may be necessary

The title block should also contain the statement "All dimensions are in millimetres unless otherwise specified".

1.3.2 Space for Text

A space for text may be used to provide (i) explanation of special symbols, designation, abbreviations and units of dimensions, (ii) material specification, surface treatment and number of units, (iii) references to supplementary drawings and other documents, (iv) location figures, usually in case of architectural and building drawings, and (v) revision tables to record all document modifications, alterations or revisions which are made from time to time.

This space is normally placed at the right-hand edge of the drawing sheet as shown in Fig. 1.1. The width of the space is equal to that of the title block. If a figure takes up the whole width of the drawing sheet, the space for the text is provided at the bottom edge of the drawing sheet. The height of the space for text is chosen as required.

1.3.3 Item References on Drawing and Item Lists

If the drawing contains a number of items, or if it is an assembly drawing, a list of items, also known as bill of materials, is attached just above the title block. The item list included in the drawing should have its sequence from bottom to top, with headings of the column immediately underneath as shown in Fig. 1.3.

5 mm

3				
2.				
1.				
S. No.	**Item Ref.**	**Item Name**	**Quantity**	**Material**

Fig. 1.3 *Item list (bill of materials)*

1.4 LINES

The Bureau of Indian Standards has recommended various types of lines to be used. The types of lines and their applications in engineering drawing given in SP 46 : 2003 are summarized in Table 1.1.

Table 1.1 *Types of lines and their applications in engineering drawing*

S. No.	*Line description and representation*	*Application*
1.	Continuous narrow lines	Imaginary lines of intersection, dimension line, extension line, leader line with reference line, hatching, short centre lines, dimension lines termination, arrowheads, diagonals for indication of flat surfaces, projection lines, grid lines
2.	Continuous narrow freehand line	[a] Preferably manually represented termination of partial or interrupted views, cuts and sections if the limit is not a line of symmetry or a centre line
3.	Continuous narrow line with zigzags	[a] Preferably mechanically represented termination of partial or interrupted views, cuts and sections if the limit is not a line of symmetry or a centre line
4.	Continuous wide line	Visible edges, visible outlines, lines of cuts and section arrows
5.	Dashed narrow line	Hidden edges, hidden outlines
6.	Dashed wide line	Indication of permissible areas of surface treatment
7.	Long-dashed dotted narrow line	Centre lines, line of symmetry, pitch circle of gears, pitch circle of holes

Table 1.1 (*Contd.*)

S. No.	Line description and representation	Application
8.	Long-dashed dotted wide line —·—·—·—·—·—·—·—·	Indication of (limited) required areas of surface treatment e.g. heat treatment, indication of cutting planes
9.	Long-dashed double-dotted narrow line —··—··—··—··—··—··—··—··	outlines of adjacent parts, extreme positions of movable parts, centroidal lines, parts situated in front of a cutting plane

[a] It is recommended to use only one type of line on one drawing.

1.4.1 Rules for Drafting Lines

The following rules should be observed while drafting lines:

1. The minimum space between parallel lines should preferably be greater than 0.7 mm.
2. Lines with dash (S.No. 5 to 9 of Table 1.1), should preferably meet at a dash.
   See Fig. 1.4.

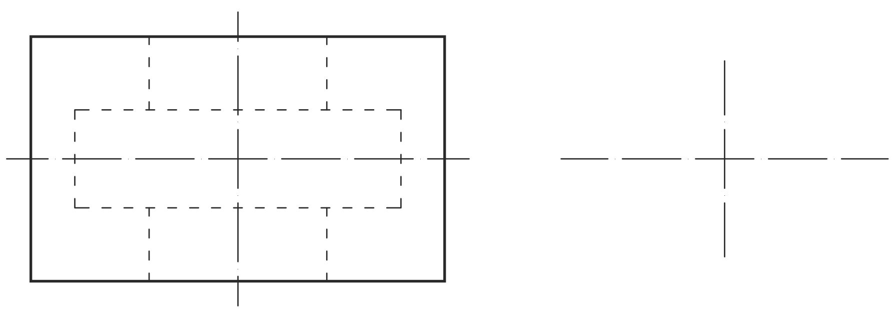


Fig.1.4 *Samples of dashed lines*

3. Lines should be drawn in black or white depending on the colour of the background.
4. In case of two or more lines of different types which may overlap or coincide, the drawing priority may be given in the following order.
   a. Visible outlines and edges
   b. Hidden outlines and edges
   c. Cutting planes
   d. Centre lines and lines of symmetry
   e. Centroidal lines
   f. Projection lines

   For example, if a visible line coincides with a hidden line then only the visible line is to be drawn, ignoring the hidden line. Similarly, if a hidden line coincides with a projection line then only the hidden line is to be drawn, ignoring the projection line.

1.5 LETTERING

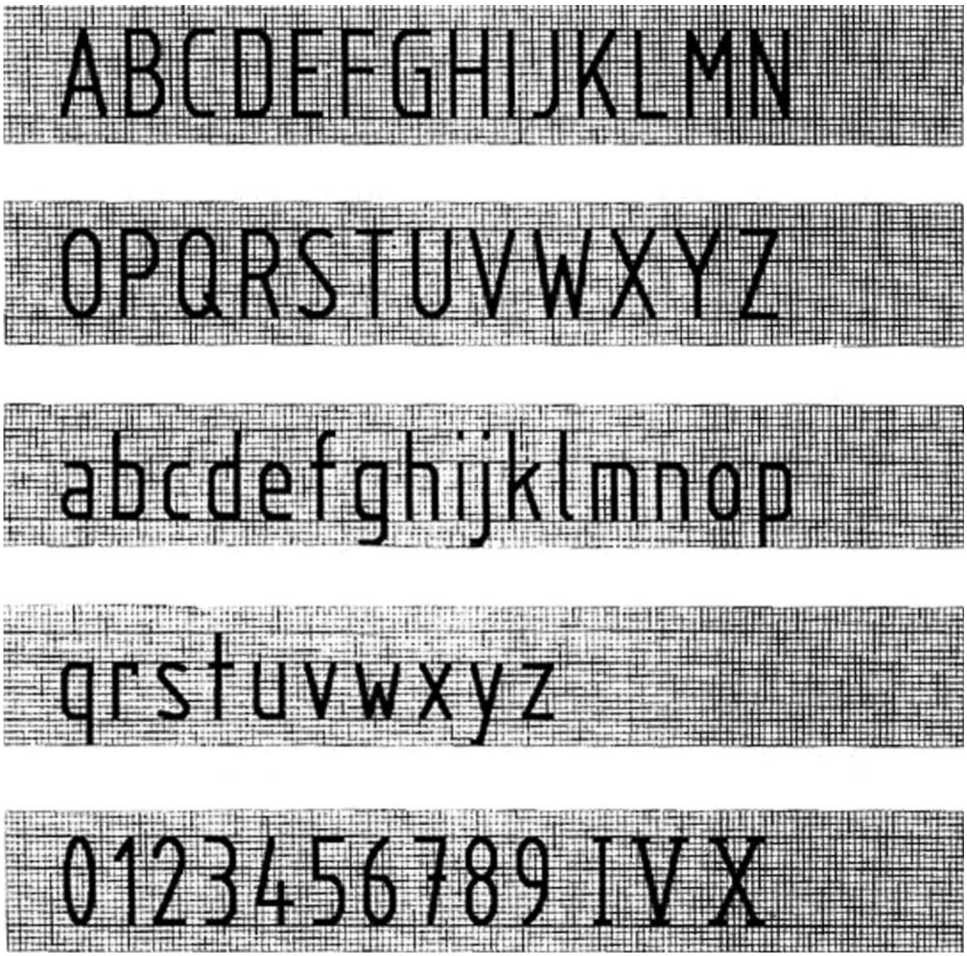


Fig. 1.5(a) *Lettering A: Single-stroke vertical letters*

Writing of titles, notes and other particulars on a drawing is called lettering. Lettering should be done on the drawing in such a manner that it may be read when the drawing is viewed from the bottom edge, except when it is used for dimensioning purposes. An HB or H pencil with conical-shaped point works best for most lettering.

The Bureau of Indian Standards recommends Latin alphabets and numerals for technical drawing in its bulletin IS 9609 (Part 1): 2006. Single-stroke vertical capital letters and numerals shown in Figure 1.5(a) are generally used in practice. Figure 1.5(a) also shows recommended single-stroke vertical lower-case letters. Inclined or italic letters, having an inclination of about 75º may also be used in special conditions. Figure 1.5(b) shows single-stroke inclined capital letters, lower-case letters and numerals.

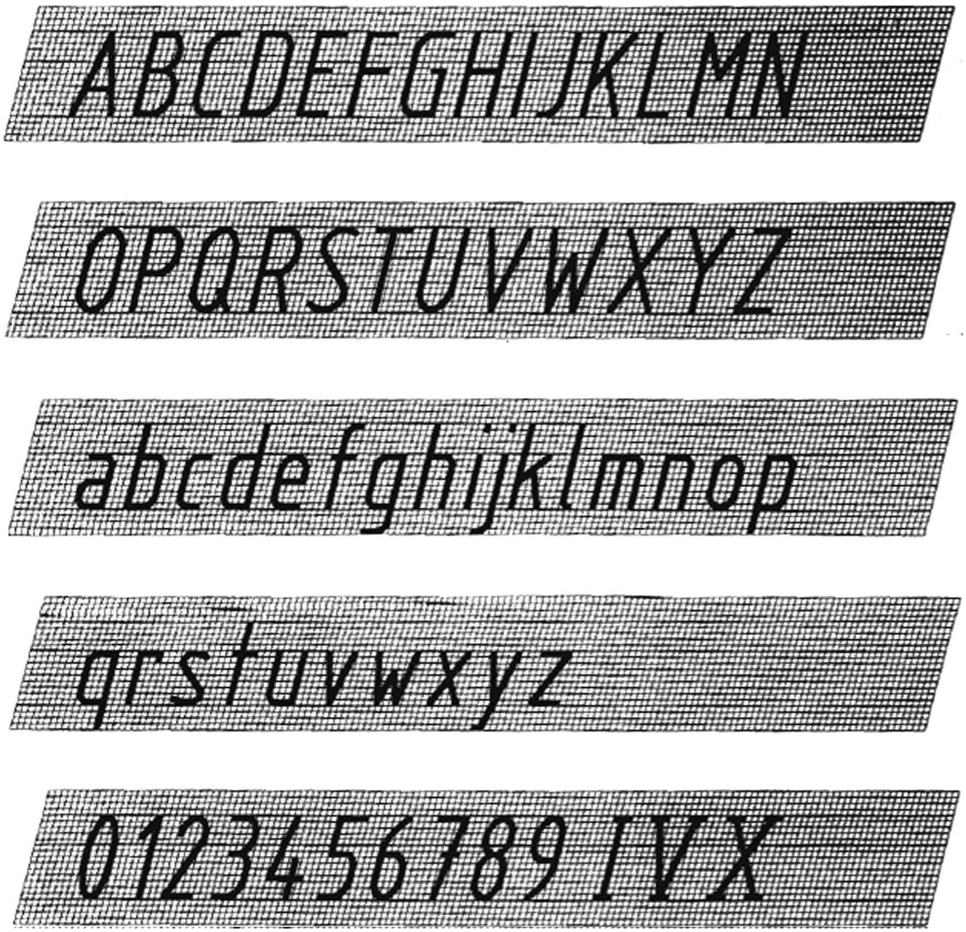


Fig. 1.5(b) *Lettering A: Single-stroke inclined letters*

1.5.1 Rules for Lettering

The following rules should be observed while lettering:

1. Guidelines ensure consistency in the size of the letter characters. If the lettering consists of capitals, draw only the cap line and base line. If the lower-case letters are included as well, draw the waist line and the drop line.
2. The thickness of the line of the letter should be such as is obtained in one stroke of pencil.
3. The width-to-height ratio should be around 1:2 for all capital alphabets [except I and W] and 1:3 for all numerals [except 1].
4. Letters should be written in capitals. Lower-case letters should be used only when they are accepted in international usage for abbreviations.
5. Letters and numerals should neither touch each other nor the lines.
6. Letters should be so spaced that the area between them appears equal. It is not necessary to keep clearances between adjacent letters equal, e.g., as in letters LA, TV or Tr.
7. Words should be spaced one letter apart.

1.6 DIMENSIONING

Dimensions are marked on the drawing to specify the size such as length, breadth, height, diameter, radius, angle and location of holes.

1.6.1 Dimensioning Terminology

Figure 1.6 shows the methodology of dimensioning a figure. Various lines and arrowheads used in it are as follows:

1. **Dimension Lines** These are thin continuous lines used to indicate the measurement. The measurement is denoted in figures and placed near the middle of the dimension lines.
2. **Projection Lines** These are thin continuous lines extended beyond outlines. Projection lines are drawn in a direction perpendicular to the feature to be dimensioned or where necessary, they may be drawn obliquely but parallel with each other.
3. **Leaders or Pointer Lines** These are the lines referring to a feature and notes. These are thin continuous lines and terminated by arrowheads or dots. Notes and figures are written above the extended dimension lines. Leaders are usually drawn at any convenient angle 30º, 45º and 60. Use of long leaders should be avoided.

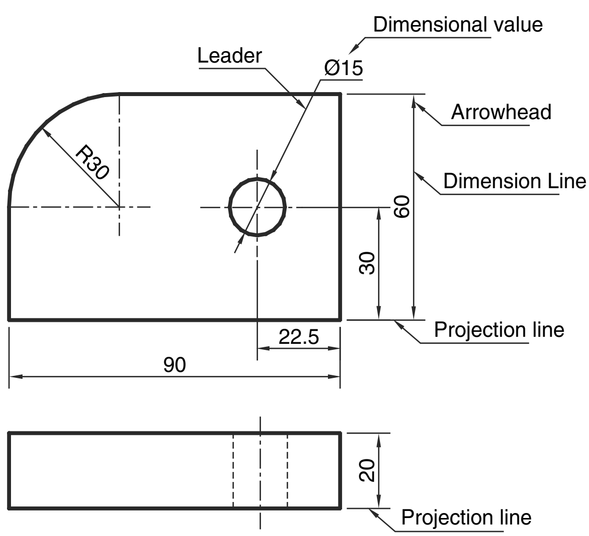


Fig. 1.6 *Dimensioning terminology*

4. **Arrowheads** The common types of arrowheads recommended by B.I.S. are shown in Fig. 1.7(a). These are used to terminate the dimension lines. It is preferable to use filled arrowheads in engineering drawings with its length about three times the depth/width as shown in Fig. 1.7(b). Normally, length of arrowheads is 3 mm for small drawings and 4 to 5 mm for large drawings.

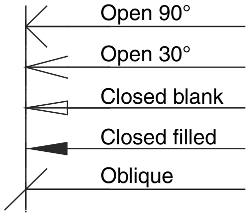


Fig. 1.7(a) *Types of arrowheads*

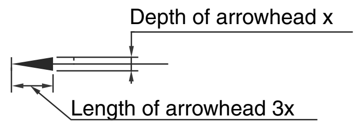


Fig. 1.7(b) *Execution of closed filled arrowhead*

1.6.2 Placing of Dimensions

Dimensions should be placed on the view which shows the relevant features more clearly. The two recommended systems of placing the dimensions are as follows:

1. **Aligned System for Linear Dimensioning** In this system, all dimensions are so placed that they may be read from the bottom or the right-hand edges of the drawing sheet. See Fig. 1.8(a). As far as possible, dimension lines should not be placed in 30º zone shown hatched in Fig. 1.8(b).

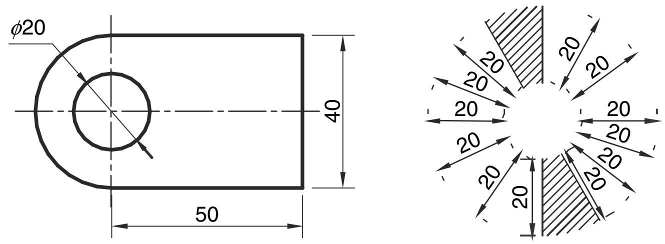


Fig. 1.8 *Aligned system for unidirectional dimensioning*

2. **Aligned System for Angular Dimensioning** Angular dimensions are represented in the same manner as the linear dimensions. See Fig 1.9(a). In certain cases, they may be written horizontally if this improves clarity. See Fig. 1.9(b).

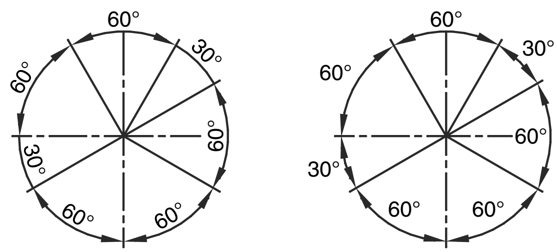


Fig. 1.9 *Aligned system for angular dimensioning*

3. **Unidirectional System for Linear Dimensioning** In this system, all dimensions are so placed that they may be read from the bottom edge of the drawing sheet. See Fig. 1.10(a). There is no restriction controlling the direction of dimension lines. See Fig. 1.10(b). This system is advantageous on large drawings, where it is inconvenient to read dimensions, from the right-hand side.

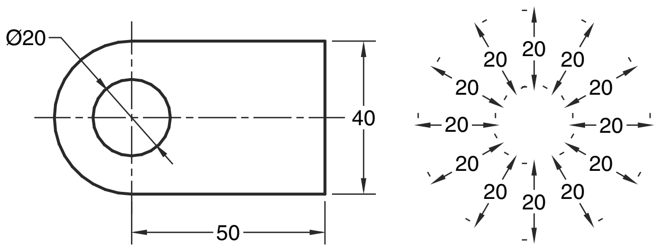


Fig. 1.10 *unidirectional system for linear dimensioning*

4. Unidirectional System for Angular Dimensioning Angular dimensions are placed in the same manner as that of dimensioning linear dimensions. See Fig 1.11.

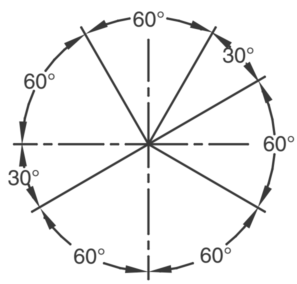


Fig. 1.11 *unidirectional system for angular dimensioning*

1.6.3 Method of Dimensioning Some Common Features

1. Circles They should be placed on the most appropriate view to ensure clarity, and should be preceded by the symbol 'ϕ'. They should be dimensioned by one of the methods depending on size. See Fig. 1.12.

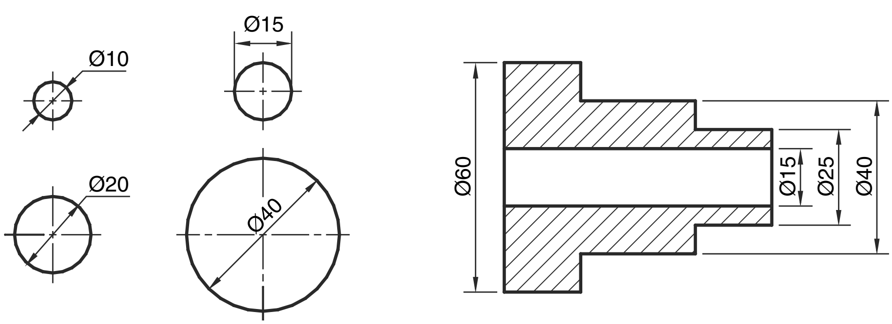


Fig. 1.12 *Dimensioning circles*

2. Radii They should be preceded by the letter 'R'. As far as possible, the dimension line of a radius shall pass through the centre of the arc. Whenever the centre is located by dimensions, the centre

shall be marked by a clear dot or small cross. While dimensioning small radii, the arrows may be reversed. See Fig. 1.13(a). Leaders which indicates radius must always be radial line.

In case the size of the radius can be derived from other dimensioning, it should be indicated with a radius arrow and symbol R without an indication of the value. See Fig. 1.13(b).

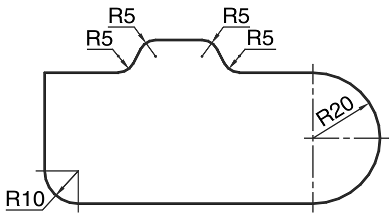


Fig. 1.13(a) *Dimensioning radii*

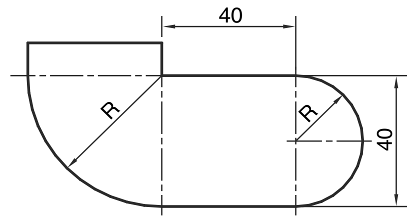


Fig. 1.13(b) *Dimensioning radii*

3. **Angles** Some typical examples are shown in Fig. 1.14 to give an idea of the methods of indicating angular dimensions.

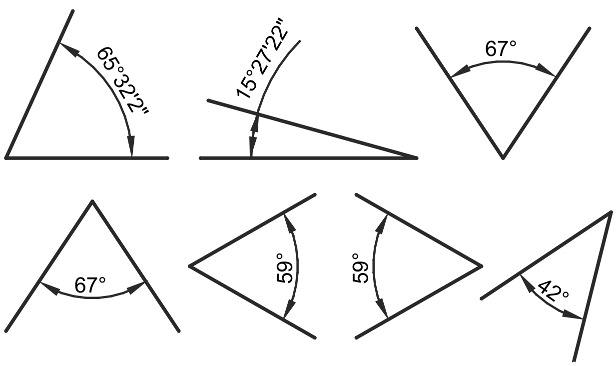


Fig. 1.14 *Dimensioning angles*

4. **Spheres** The radius or diameter dimension of a spherical surface should be preceded by the word 'S'. See Fig. 1.15.
5. **Squares and Hexagons** The dimension of a square should be preceded by the symbol '□' and the dimension of a hexagon should be preceded by the word 'HEX'.

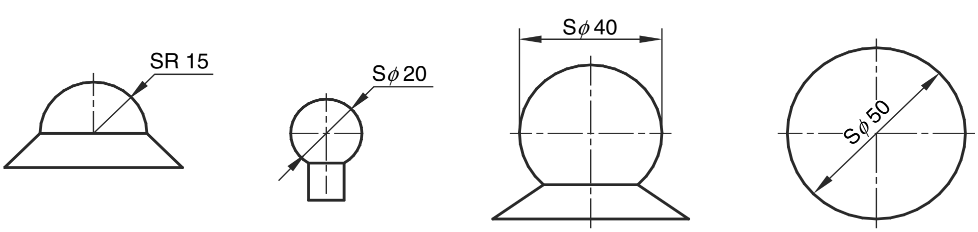


Fig. 1.15 *Dimensioning spheres*

1.6.4 Rules for Dimensioning

The following rules should be observed while dimensioning:

1. Dimensions should be clear and permit only one interpretation. Numerals and letters should be large enough to ensure easy reading.
2. Dimension values should be placed preferably near the middle. If unavoidable due to lack of space above the extended portion of the dimension line beyond the arrowheads, preferably it should be on the right-hand side.
3. As far as possible, dimensions should be placed outside the views. In case it is not possible, it may be placed within the view. However, dimensions should not be placed within a view unless the drawing becomes clear by doing so.
4. Dimensions should be placed at sufficient distance from the parts being dimensioned and also from each other.
5. Dimensions indicated in one view need not be repeated in another view, except for purpose of identification, clarity or both.
6. Dimensions should be marked with reference to the visible outlines. Dimensions should be marked from a base line or centre line of a hole or cylindrical parts or finished surfaces, etc., which may be readily established, based on design requirements and the relationship to other parts.
7. Dimensioning to a centre line should be avoided, except when the centre line passes through the centre of a hole, or a cylindrical part.
8. An axis or a contour line should never be used as a dimension line but may be used as a projection lines.
9. Intersection of projection and dimensional lines should be avoided. However, if their intersection is unavoidable, neither line should be shown with breaks.
10. Overall dimensions should be placed outside the intermediate dimensions. Where an overall dimension is shown, one of the intermediate dimensions is redundant and should not be dimensioned.
11. If the space for arrowhead termination is sufficient it should be shown within the limit of dimension lines. If the space is limited, the arrowhead termination may be shown outside the intended limits of the dimension lines that are extended for that purpose. However, where space is too small for an arrowhead, it may be replaced by an oblique stroke or a dot. See Fig. 1.16.

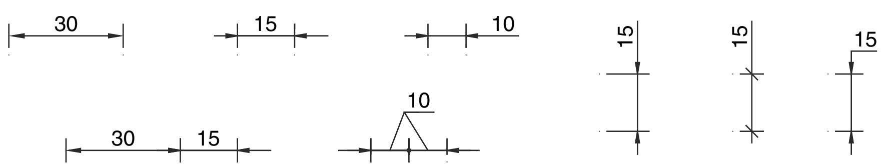


Fig. 1.16 *Arrowhead termination*

1.7 GEOMETRICAL CONSTRUCTIONS

While preparing a drawing, one or more of the following constructions may be required.

1.7.1 Divide a Line

A line may be divided into any number of equal parts as illustrated in the example below.

Example 1.1 (Fig 1.17)

Divide a 100 mm long straight line into seven equal parts.

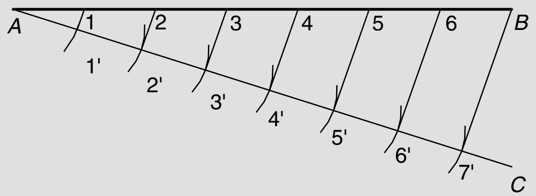


Fig. 1.17 *Dividing a line in seven segments*

Construction: Fig. 1.17

1. Draw a 100 mm long straight line *AB*.
2. Draw a line *AC* at any convenient acute angle with *AB*.
3. Set the divider to a convenient length and mark off seven spaces on *AC*. Let the points obtained be 1′, 2′, 3′, 4′, 5′, 6′, and 7′.
4. Join 7′ to the point *B*.
5. Draw lines through points 1′, 2′, 3′, 4′, 5′ and 6′ parallel to 7′*B* to meet *AB* at points 1, 2, 3, 4, 5 and 6 respectively. These points divide *AB* in equal length.

1.7.2 Bisect an Angle

An angle is bisected as explained in the following example.

Example 1.2 (Fig 1.18)

Draw an angle of 75º and bisect it.

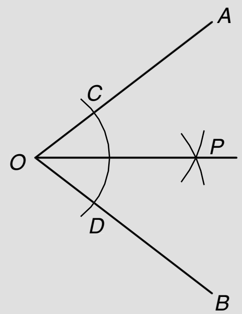


Fig. 1.18 *Bisecting an angle*

Construction: Fig. 1.18

1. Draw angle *AOB* of 75°.
2. Set the needle point of the compass at *O*, draw an arc *CD* of arbitrary radius to strike *OA* and *OB* at points *C* and *D* respectively.
3. With the ends *C* and *D* as centres, draw arcs of equal radius to intersect each other at the point *P*.
4. Draw a line from *O* through *P*. The line *OP* bisects the angle *AOB*.

1.7.3 Divide a Circle

A protractor may be used to divide a circle. However, if it is to be divided into twelve equal parts, one may use the drafter and the compass. The following example illustrates the method.

Example 1.3 (Fig 1.19)

Divide a 50 mm diameter circle into twelve equal parts.

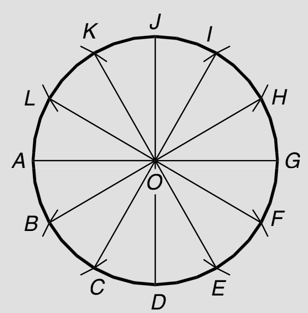


Fig. 1.19 *Dividing a circle into twelve segments*

Construction: Fig. 1.19

1. Draw a 50 mm diameter circle having *O* as the centre.
2. Using Drafter, draw diameters *AG* and *DJ* perpendicular to each other.
3. Draw arc of radius equal to the radius of the circle (= 25 mm) and *A* as the centre to meet the circumference of the circles at point *C* and *K*.
4. Similarly, draw arcs of the same radius (= 25 mm) and centres *G*, *D* and *J* respectively, to obtain points *E*, *I*, *B*, *F*, *H* and *L* on the circle. These points divide the circumference of the circle into 12 equal parts.

1.7.4 Triangles

Geometrical construction of a triangle with three given sides is illustrated in the example.

Example 1.4 (Fig 1.20)

Draw a triangle having sides of 80 mm, 60 mm and 50 mm.

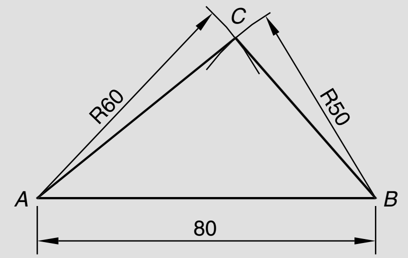


Fig. 1.20 *Triangle*

Construction: Fig. 1.20

1. Draw an 80 mm long straight line *AB*.
2. Draw an arc having *A* as the centre and radius of 60 mm.
3. Draw another arc having *B* as the centre and radius equal to 50 mm to intersect the previous drawn arc at the point *C*.
4. Draw lines connecting *A* and *B* with the point *C*, to obtain the required triangle *ABC*.

1.7.5 Rectangles and Squares with Given Sides

When the lengths of sides of the rectangle or the square are given, use of a drafter is recommended to draw them. Set the drafter along line *AB* and erect perpendiculars *AD* and *BC*, equal to the given width from the ends, as shown in Fig 1.21(a). Join points *C* and *D* to obtain the required rectangle. If all the sides are made equal the figure obtained would be a square, as shown in Fig 1.21(b).

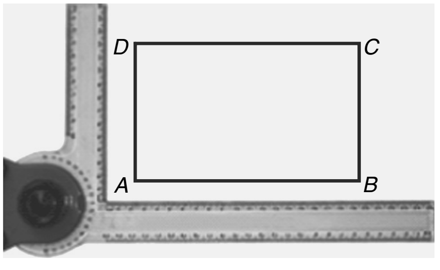


Fig. 1.21(a) *Rectangle*

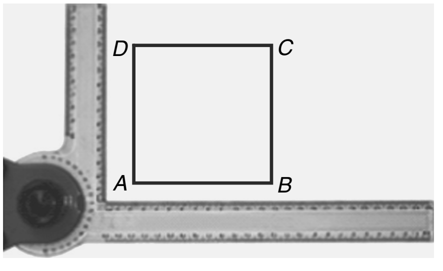


Fig. 1.21(b) *Square*

1.7.6 Squares with Given Diagonal

Diagonals of the squares are perpendicular bisector of each other. To draw a square of given diagonal, set the drafter along one of the diagonals, say *AB*, as shown in Fig 1.22. With the help of 45º-set-square, draw a 45º angle from the ends *A* and *B* on either sides of the line *AB*, and obtain points *C* and *D*. Join *ABCD* which is a square. Another method suggest to use the compass to draw the *AB* and *CD* as the perpendicular bisector of each other and making their lengths equal about the intersecting point *O*. Joining the end points will constitute the square.

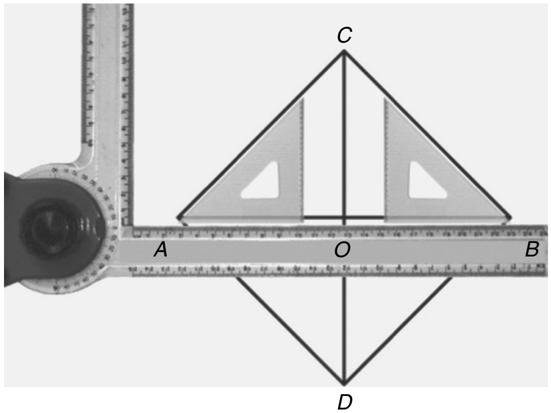


Fig. 1.22 *Square with given diagonals*

1.8 CONSTRUCTION OF REGULAR POLYGONS

Regular polygons can be drawn with the help of protractor taking internal angle of the polygon equal to $\left[\frac{(n-2)}{n}\times 180°\right]$ or external angle equal to $\left[\frac{2}{n}\times 180°\right]$, where *n* is the number of sides of the polygon.

1.8.1 General Methods for Construction of Polygons

Example 1.5 (Fig 1.23)

Draw a regular pentagon and a regular heptagon having 40 mm long sides, using general method.

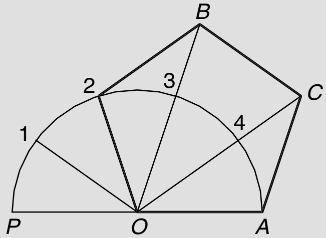


Fig. 1.23(a) *Pentagon*

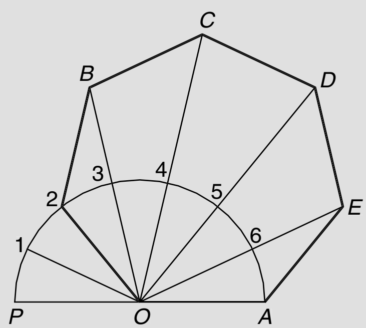


Fig. 1.23(b) *Heptagon*

Construction:

Method 1:

Fig. 1.23(a) for pentagon and Fig. 1.24(b) for heptagon

1. Draw a 40 mm long line *OA*.
2. Draw a semicircle with *O* as the centre and *OA* as radius.
3. Divide the semicircular arc *AP* into *n* equal parts (where *n* = number of sides of the polygon). Number the divisions as 1, 2, 3, etc starting from the point *P*.
4. Join *O* through the point 2.
5. Draw an arc with 2 as the centre and *OA* as the radius, to meet *O*3 produced at the point *B*.
6. Draw another arc with *B* as the centre and the same *OA* the radius, to meet *O*4 produced at the point *C*. For heptagon proceed to draw arcs of radius *OA*, from with *C* and *D* centres to meet *O*5 and *O*6 produced at points D and *E*, respectively.
7. Join the points to obtain the required polygon as shown.

Method 2:

Fig. 1.23(c) for Pentagon and Fig. 1.23(d) for heptagon

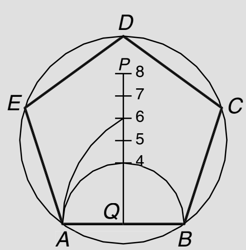


Fig. 1.23(c) *Pentagon*

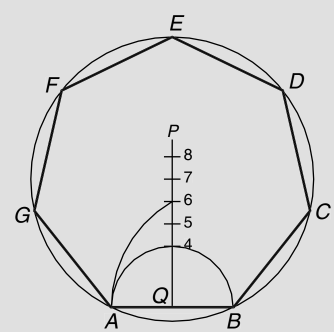


Fig. 1.23(d) *Heptagon*

1. Draw a 40 mm long line *AB*.
2. Draw a line *PQ* as the perpendicular bisector to the line *AB*.
3. Describe a semicircle having diameter *AB*, to meet the line *PQ* at the point 4.
4. With either *A* or *B* as the centre, draw an arc of radius *AB* to meet the line *PQ* at point 6.
5. Bisect line 4-6 and obtain point 5 on *PQ*.
6. Mark point 7 and 8 on *PQ* such that the lengths 4-5, 5-6, 6-7, 7-8 are equal.
7. Draw a circumcircle with centre 5 for pentagon and 7 for heptagon passing through points *A* and *B*.
8. Lay off the circle into the parts of chord length *AB* and obtain the required polygon.

1.8.2 Special Methods for the Construction of Pentagons

The example below illustrates the special geometrical constructions methods for a regular pentagon.

Example 1.6 (Fig 1.24)

Draw a regular pentagon of 40 mm long side.

Method 1: Fig. 1.24(a)

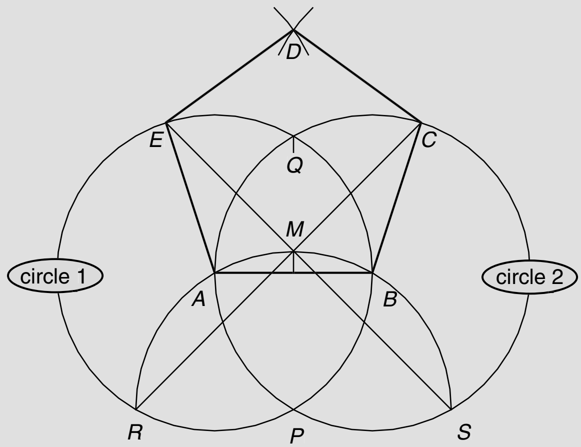


Fig. 1.24(a) *Pentagon method 1*

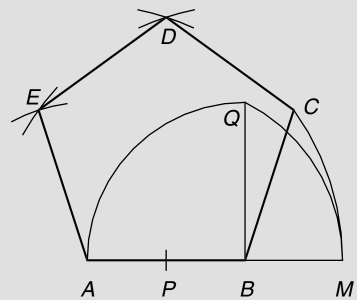


Fig. 1.24(b) *Pentagon method 2*

1. Draw a 40 mm long line *AB*.
2. Set the compass to radius *AB* and draw circle-1 and circle-2 with centres *A* and *B* respectively. Let circle-1 and circle-2 intersects each other at points *P* and *Q*.
3. With *P* as the centre and same radius *AB*, draw an arc to intersect the circle-1 and the circle-2 at points *R* and *S* respectively.
4. Join *PQ* to meet arc *RS* at point *M*.
5. Join *RM* and produce it to meet circle-2 at point *C*.
6. Join *SM* and produce it to meet circle-1 at point *E*.
7. Draw arcs having *AB* as the radius and centres *C* and *E*, respectively, to intersect each other at point *D*.
8. Join *ABCDE* to obtain the required pentagon.

Method 2: Fig. 1.24(b)

1. Draw a 40 mm long line *AB*.
2. Locate *P* as the midpoint of *AB*.
3. Erect line *BQ* perpendicular and equal to *AB*.
4. Draw an arc with *P* as the centre and length *PQ* as the radius to meet the line *AB* produced at point *M*. This line *AM* is the diagonal of the pentagon.
5. Draw an arc with *A* as the centre and *AB* as the radius to intersect the arc drawn with *B* as the centre and *AM* as the radius at the point *E*.

6. Draw another arc with *B* as the centre and *AB* as the radius to intersect the arc drawn with *A* as the centre and *AM* as the radius at the point *C*.
7. Draw arcs having *AB* as the radius and centres *C* and *E*, respectively, to intersect each other at the point *D*.
8. Join *ABCDE* to obtain the required pentagon.

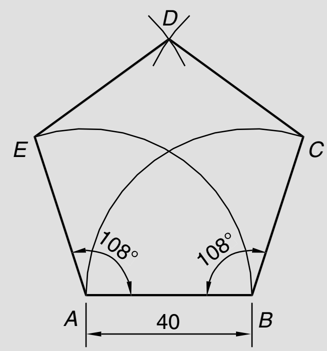


Fig. 1.24(c) *Pentagon method 3*

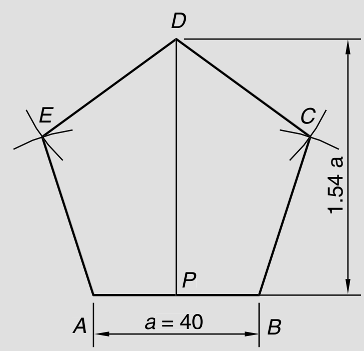


Fig. 1.24(d) *Pentagon method 4*

Method 3:

Fig. 1.24(c)

1. Draw a 40 mm long line *AB*.
2. Draw 40 mm long lines *AE* and *BC*, both inclined at 108º with line *AB*.
3. With centres *C* and *E* and radius *AB*, draw arcs intersecting each other at the point *D*.
4. Join *ABCDE* to obtain the required pentagon.

Method 4:

Fig. 1.24(d)

1. Draw a 40 mm long line *AB*.
2. Locate *P* as the midpoint of line *AB*.
3. Erect *PD* perpendicular to *AB*, with a length equal to 1.54 times of the side *AB*.
4. Draw arcs having *AB* as the radius and *B* and *D* as the centres, respectively, to intersect each other at the point *C*.
5. Draw arcs having *AB* as the radius and *A* and *D* as the centres, respectively, to intersect each other at the point *E*.
6. Join *ABCDE* to obtain the required pentagon.

1.8.3 Special Methods for the Construction of Hexagons

The example below illustrates the special geometrical constructions methods for a regular hexagon.

Example 1.7 (Fig 1.25)

Draw a hexagon of 40 mm long side.

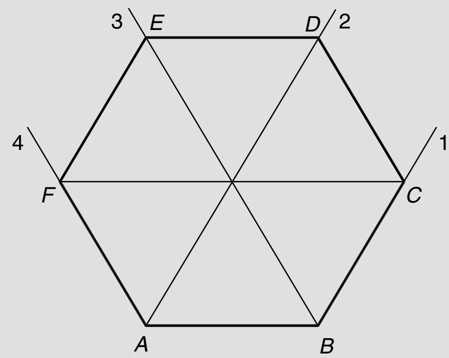


Fig. 1.25 *Hexagon*

Construction: Fig. 1.25

1. Draw a 40 mm long line *AB*.
2. Erect lines *A*2 and *A*4 at point *A*, inclined at 60º and 120º respectively with *AB*.
3. Erect lines *B*1 and *B*3 at point *B*, inclined at 60º and 120º respectively with *AB*.
4. With centres *A* and *B* and radius *AB*, draw arcs to meet lines *B*1 and *A*4 at points *C* and *F* respectively.
5. Similarly, with centres *C* and *F* and radius *AB*, draw arcs to lines meet *A*2 and *B*3 at points *D* and *E* respectively.
6. Join *ABCDEF*, which is the required hexagon.

Another simplified method for construction of hexagon is based on the principle of inscribing it in a circle. The example below illustrates this.

Example 1.8 (Fig 1.26)

Draw a regular hexagon having a 40 mm side as (a) vertical, and (b) horizontal.

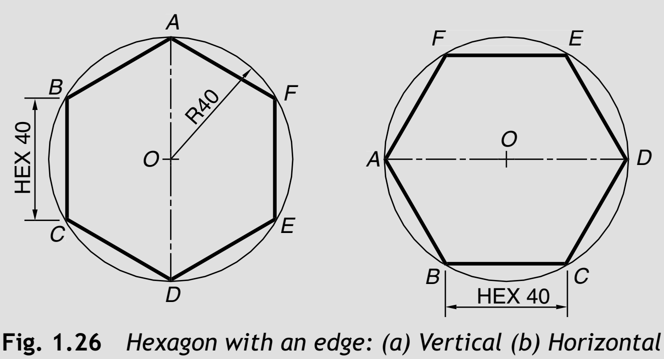


Fig. 1.26 *Hexagon with an edge: (a) Vertical (b) Horizontal*

Construction: Fig. 1.26(a) and Fig. 1.26(b)

1. Draw a circle of 40 mm radius with *O* as its centre.
2. Draw diameter of the circle *AD* at a desired inclination. [For case (a), *AD* should be vertical and for case (b), *AD* should be horizontal]
3. With radius *OA* and centres *A* and *D* respectively, draw arcs to meet the circumference of the circle on points *B*, *F*, *C* and E.
4. Join *ABCDE* to obtain the required hexagon.

EXERCISE 1

1. Divide a straight line of 90 mm length into parts that are proportional as 2:3:5.
2. Draw an angle of 45º and bisect it using compass.
3. Draw an equilateral triangle of 60 mm side with an edge inclined at 45º.
4. Draw a square of 30 mm side such that all the sides are equally inclined (45º) to a horizontal line.
5. Draw a pentagon of 30 mm side such that one of its edges is vertical.
6. Draw a heptagon of 25 mm side such that one of its edges is vertical.
7. Use a mini-drafter to draw Fig. E1.1 to E1.3 in a square of 100 mm side. Take the distance between consecutive parallel lines as 10 mm.

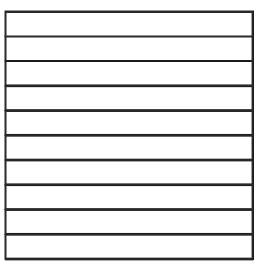


Fig. E1.1

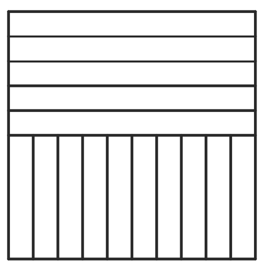


Fig. E1.2

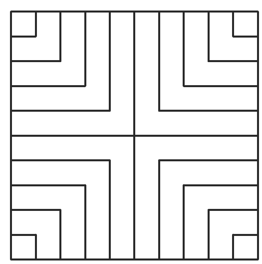


Fig. E1.3

8. Use a mini-drafter to draw Fig. E1.4 to E1.6 in a square of 90 mm side. Take the distance between consecutive parallel lines as 10 mm.

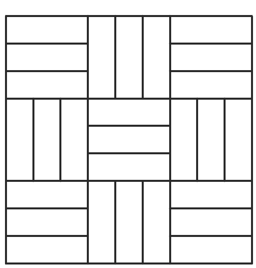


Fig. E1.4

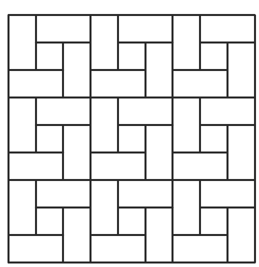


Fig. E1.5

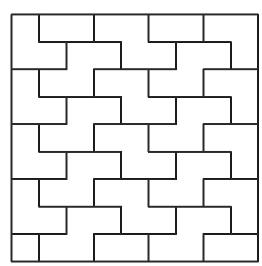


Fig. E1.6

9. Use a mini-drafter to draw Fig. E1.7 to E1.9 in a square of 100 mm side. In Fig. E1.7 and Fig. E1.8, take the distance between consecutive parallel lines of 10 mm length.

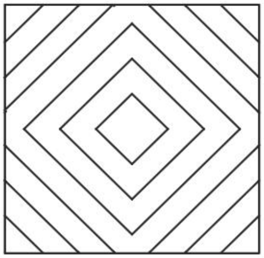


Fig. E1.7

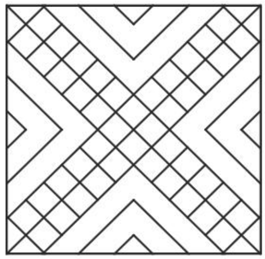


Fig. E1.8

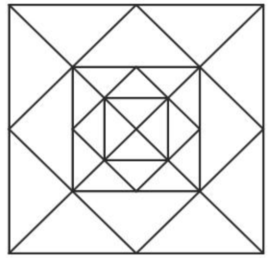


Fig. E1.9

10. Use necessary drawing instruments to draw Fig. E1.10 to E1.12 in a square of 100 mm side.

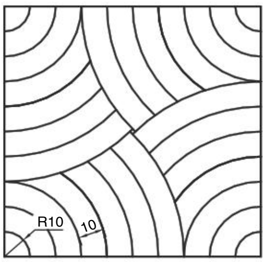


Fig. E1.10

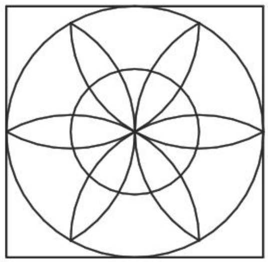


Fig. E1.11

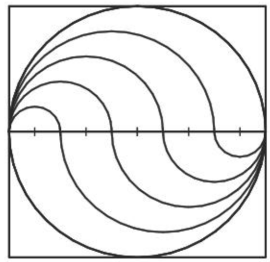


Fig. E1.12

11. Use necessary drawing instruments to draw Fig. E1.13 to E1.15 in a square of 100 mm side.

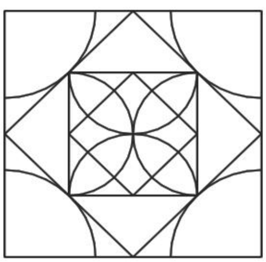


Fig. E1.13

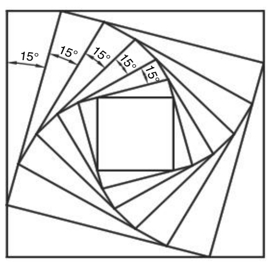


Fig. E1.14

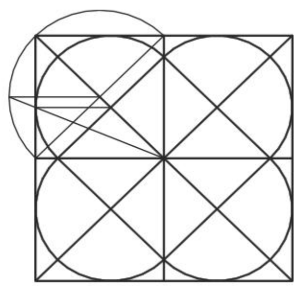


Fig. E1.15

12. Use necessary drawing instruments to draw Fig. E1.16 to E1.18 in a circle of 100 mm diameter.

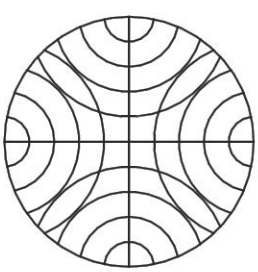


Fig. E1.16

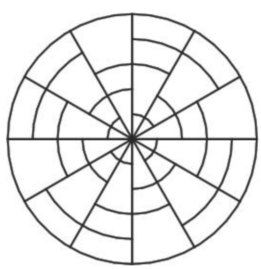


Fig. E1.17

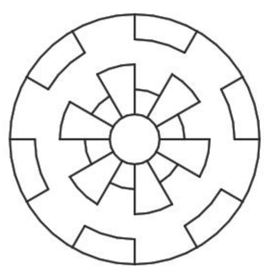


Fig. E1.18

13. Use necessary drawing instruments to draw Fig. E1.19 to E1.21 in a circle of 100 mm diameter.

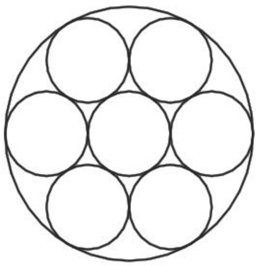


Fig. E1.19

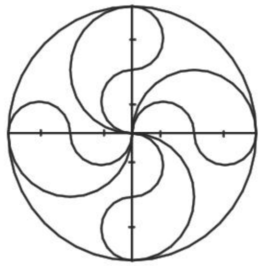


Fig. E1.20

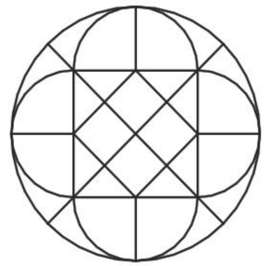


Fig. E1.21

14. Use necessary drawing instruments to draw Fig. E1.22 to E1.24 in a circle of 100 mm diameter.

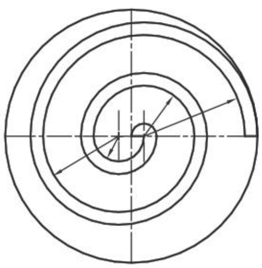


Fig. E1.22

Fig. E1.23

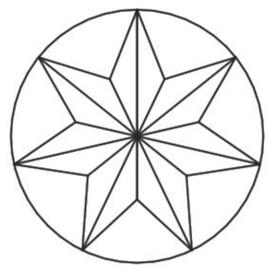


Fig. E1.24

15. Use necessary drawing instruments to reproduce Fig. E1.25 to E1.27.

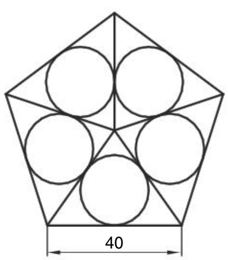


Fig. E1.25

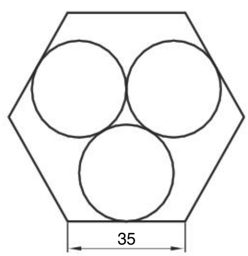


Fig. E1.26

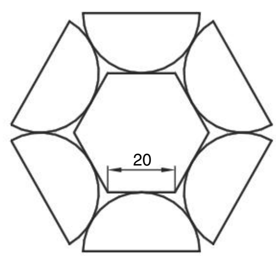


Fig. E1.27

REVIEW QUESTIONS

1. Name different types of drawing instruments.
2. What information should be contained in the title block of a drawing sheet?
3. Draw the lines recommended by B.I.S. for the following applications:
   (a) Visible outlines (b) Hidden lines (c) Centre lines (d) cutting plane line (e) projection lines

4. Write all the alphabets and numerals of 12 mm height using single stroke vertical capital letters according to latest Indian standard IS 9609 : 2006.
5. Write the name of your institute of 10 mm height using single-stroke vertical capital letters according to IS 9609 : 2006.
6. Write "the quick brown fox jumps over the lazy dog" of 12 mm height using single- stroke vertical capital letters recommended by latest Indian standards.
7. Distinguish between dimension line, projection lines and leaders with the help of a neat sketch.
8. Show various dimension line terminations (arrowheads) as recommended Bureau of Indian Standards.
9. Differentiate between aligned and unidirectional systems of linear dimensioning.
10. What do you understand by a perpendicular bisector?
11. How can you divide a line into given equal number of parts?
12. Explain the method of inscribing a hexagon in the given circle.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) A drafter helps in drawing
(a) parallel and perpendicular lines (b) concentric circles
(c) smooth curves (d) all the above

ii) Center lines are drawn as
(a) continuous narrow lines (b) dashed narrow line
(c) long-dashed dotted narrow line (d) long-dashed double dotted narrow line

iii) Long-dashed dotted narrow line is used to represent
(a) line of symmetry (b) centre lines
(c) pitch circle of gears and holes (d) all the above

iv) Which of following publications made by Bureau of Indian Standards includes standard techniques for line conventions and lettering in detail?
(a) SP 46 (b) BIS 9609
(c) ASME Y14.2M (d) ISO 9000

v) The inclination of letters as for inclined lettering as recommended by B.I.S. is
(a) 75° (b) 70° (c) 65° (d) 60°

vi) The length-to-height ratio of a closed filled arrowhead is
(a) 1:3 (b) 3:1 (c) 1:2 (d) 2:1

vii) The two recommended systems of placing the dimensions are
(a) unidirectional and aligned systems (b) upright and inclined systems
(c) linear and oblique systems (d) linear and inclined systems

viii) The dimension figure for radius of a circle should preceded by
(a) R (b) CR (c) SR (d) RAD

ix) The dimension figure for diameter of a circle should
(a) preceded by the symbol 'ϕ' (b) suffixed by the symbol 'ϕ'
(c) preceded by the symbol 'D' (d) suffixed by the symbol 'D'

x) The recommended method of dimensioning a sphere with 80 mm diameter is
(a) 80ϕS (b) ϕ80S (c) S80ϕ (d) Sϕ80

xi) When two diameters of a circle are drawn at right angles to each other, which of the following polygons inscribed in a given circle will have all of the sides at 45° to these diameters?
(a) A hexagon (b) An octagon (c) A pentagon (d) A square

xii) The included angle of a pentagon is
(a) 68º (b) 72º (c) 108º (d) 112º

Answers

(i) a (ii) c (iii) d (iv) a (v) a (vi) b (vii) a (viii) a (ix) a (x) d (xi) d (xii) c

2

Scales

2.1 INTRODUCTION

It is always convenient to represent an object to its actual size in drawing, if its size permits, e.g. a 50 mm diameter plain disc should be represented by a circle of 50 mm diameter on the drawing. When a drawing is prepared to the actual size of the object, the scale is said to be a full-size scale and the drawing is said to be a full-size drawing.

However, it is not possible to make drawings of machines, buildings, town plans, etc. to their actual size. When the objects are of very large sizes, the actual dimensions of the object have to be reduced on some regular proportion to make their drawings, e.g. a rectangular plot of size 25 m × 10 m can be represented by a rectangle of 250 mm × 100 mm. The scale selected in the present case is 1 mm = 0.10 m. In other words, 1 mm on the drawing represents 0.10 m length of the object. When a drawing is smaller than the actual size of the object, the scale is said to be a *reducing scale* and the drawing is said to be *reduced-size drawing*.

Similarly, very small objects such as gear mechanism of a wrist watch, components of an electronic instrument, atoms configuration, etc., are shown by drawing them larger than their actual size. When the drawing is larger than the actual size, the scale is said to be an *enlarging scale* and the drawing is said to be *enlarged-size drawing*.

2.2 REPRESENTATION OF SCALES

Scales can be expressed in one of the following ways.

1. Engineering scale is represented by writing the relation between the dimension on the drawing and the corresponding actual dimension of the object itself. It is expressed as
1 mm = 1 mm for full-size drawing
1 mm = 5 m, 1 mm = 8 km, etc. for reduce size drawing
1 mm = 0.2 mm, 1 mm = 5 μm etc. for enlarge size drawing
They are usually written on the drawings in numerical forms.

2. Graphical scale is represented by its representative fraction and is captioned on the drawing itself. As the drawing becomes old, the drawing sheet may shrink and the engineering scale would provide inaccurate results. However, the scale made on the drawing sheet along with drawing of object will shrink in the same relative proportion. This will always provide an accurate result. It is the basic advantage gained by graphical representation of a scale.

2.3 UNITS OF MEASUREMENTS

Table 2.1 provides the relationship of various units used for linear measurement.

Table 2.1

Metric system for linear measurement	*British system for linear measurement*
1 kilometre (km) = 10 hectometre	1 league = 3 miles
1 hectometre (Hm) = 10 decametre	1 mile (mi) = 8 furlongs

Table 2.1 (*Contd.*)

Metric system for linear measurement	*British system for linear measurement*
1 decametre (Dm or dam) = 10 metre	1 furlong (fur) = 10 chains
1 metre (m) = 10 decimetre	1 chain (ch) = 22 yards
1 decimetre (dm) = 10 centimetre	1 yard (yd) = 3 feet
1 centimetre (cm) = 10 millimetre (mm)	1 foot (ft) = 12 inches
	1 inch (in) = 8 eighth

The following linear and area conversions is also useful in construction of scales.

Linear conversion: 1 mile = 1.609 km
1 inches = 25.4 mm

Area conversion: 1 are (a) = 100 m^2
1 hectare (ha) = 100 ares = 10000 m^2
1 square mile = 640 acres
1 acre (ac) = 10 square chain = 4840 square yards

2.4 TYPES OF SCALES

Scales are classified as

a. Plain scale
b. Diagonal scale
c. Comparative scale (plain and diagonal type)
d. Vernier scale
e. Scale of chords

2.5 REPRESENTATIVE FRACTION (R.F.)

Representative fraction is defined as the ratio of the length of an element of the object in the drawing to its actual length.

$$\text{R.F.} = \frac{\text{Length of the object in the drawing}}{\text{Actual length of that object}}$$

Case 1: If 1 cm length of the drawing represents 5 m length of the object then in engineering scale it is written as 1 cm = 5 m and in graphical scale it is denoted by

$$\text{R.F.} = \frac{1\text{ cm}}{5\text{ m}} = \frac{1\text{ cm}}{500\text{ cm}} = \frac{1}{500}$$

Case 2: If a 5 cm long line in the drawing represents 3 km length of a road then in engineering scale it is written as 1 cm = 600 m and in the graphical scale, it is denoted as

$$\text{R.F.} = \frac{5\text{ cm}}{3\text{ km}} = \frac{5\text{ cm}}{3\times 1000\times 100\text{ cm}} = \frac{1}{60000}$$

Case 3: If a gear of 15 cm diameter in the drawing represents an actual gear of 6 mm diameter in graphical scale, it is expressed by

$$\text{R.F.} = \frac{15\text{ cm}}{6\text{ mm}} = \frac{150\text{ mm}}{6\text{ mm}} = \frac{25}{1}$$

Thus, the scale 1:1 represents full-size scale, scale 1:x represents reducing scale and scale x:1 represents enlarging scale, where x is greater than unity.

2.6 DATA REQUIRED FOR CONSTRUCTION OF SCALES

The data required for the construction of a plain or a diagonal scale are as follows:

1. R.F. of the scale
2. The maximum length which the scale can measure
3. Least count of the scale, i.e. minimum length which the scale can measure.
   Length of scale is determined by L_s = R.F. × Maximum length

2.7 PLAIN SCALE

A plain scale is used to represent two consecutive units, i.e. a unit and its subdivision. For example (a) metre and decimetre, (b) kilometre and hectometre, (c) feet and inches, etc. The second unit of the scale should be such that it can comfortably divide the first unit (maximum 15 subdivisions).

2.7.1 Construction of Plain Scale

The following examples illustrate the method of construction of plain scales.

Example 2.1 (Fig. 2.1)

Construct a scale of 1:60 to show metres and decimetres and long enough to measure up to 6 metres. Mark on it a distance of 4.7 m. [RGPV June 2009]

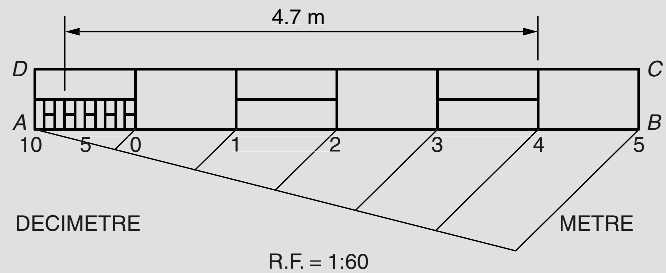


Fig. 2.1

Construction: Fig. 2.1

1. Given (a) R.F. = 1/60 (b) maximum length = 6 m (c) least count = 1 dm
2. Length of scale, L_s = R.F. × max. length = $\frac{1}{60} \times 6 \times 100 \text{ cm} = 10 \text{ cm}$
3. Draw a rectangle having length AB = 10 cm and width AD = 10 mm.
4. Here the length of scale represents 6 m. Divide it into 6 equal parts#. Each part represents 1 metre. Mark the main units.
5. Divide first division 0A of the scale into 10 sub-divisions#. Each sub-division represents 1 decimetre. Mark sub-units on the scale.
6. Write R.F. = 1:60 below the scale.
7. Mark a length of 4.7 m on the scale, i.e. 4 metre on the right side of the zero mark and 7 decimetre on the left side of the zero mark.

Example 2.2 (Fig. 2.2)

Construct a scale of 1 cm = 1 m to read metres and decimetres and long enough to measure up to 14 metres. Show on this a distance equal to 12.4 m. [RGPV Dec. 2007]

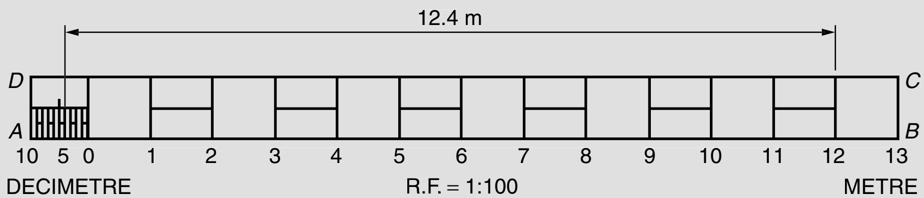


Fig. 2.2

Construction: Fig. 2.2

1. R.F. = $\frac{1 \text{ cm}}{1 \text{ m}} = \frac{1 \text{ cm}}{1 \times 100 \text{ cm}} = \frac{1}{100}$
2. Length of scale, L_s = R.F. × max. length = $\frac{1}{100} \times 14 \times 100 = 14 \text{ cm}$
3. Draw a rectangle having a 14 cm length and 10 mm width.
4. Divide the length of scale into 14 equal parts, each part representing 1 metre.
5. Divide first division of the scale into 10 equal parts, each representing 1 decimetre.
6. Mark units and sub-units on the scale and write the value of R.F.
7. Mark a length of 12.4 m on the scale, i.e. 12 m on the right side of the zero mark and 4 dm on the left side of the zero mark.

Readers are advised to refer chapter 1 for the details of methods of dividing a straight line into equal number of parts.

Example 2.3 (Fig. 2.3)

A length of 1 decametre is represented by 5 cm. Find the R.F. and construct a plain scale to measure up to 2.5 decametre and mark a distance of 19 m on it.

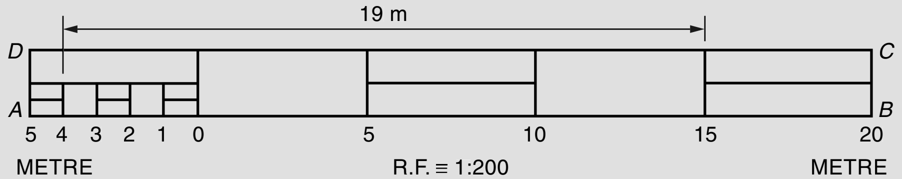


Fig. 2.3

Construction: Fig. 2.3

1. R.F. = $\dfrac{5\text{ cm}}{1\text{ Dm}} = \dfrac{5\text{ cm}}{10\times 100\text{ cm}} = \dfrac{1}{200}$
2. Length of scale, L_s = R.F. × max. length = $\dfrac{1}{200}\times 2.5\times 1000 = 12.5$ cm
3. Draw a rectangle having a 12.5 cm length and 10 mm width.
4. Divide the length of scale into 5 equal parts, each part representing 5 m.
5. Divide first division of the scale into 5 equal parts, each representing 1 m.
6. Mark units and sub-units on the scale and write the value of R.F.
7. Mark a length of 19 m on the scale, i.e. 15 metres on the right side of the zero mark and 4 metres on the left side of the zero mark.

Example 2.4 (Fig. 2.4)

Construct a scale of 1:5 to show decimetres and centimetres and long enough to measure up to 1 m. Show a distance of 6.3 dm on it.

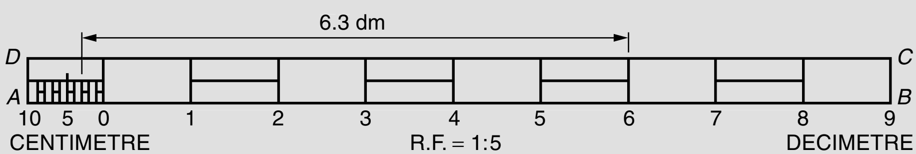


Fig. 2.4

Construction: Fig. 2.4

1. R.F. = 1:5
2. Length of scale, L_s = R.F. × max. length = $\dfrac{1}{5}\times 1\times 100 = 20$ cm
3. Draw a rectangle having a 20 cm length and 10 mm width.
4. Divide the length of scale in 10 equal parts, each part representing 1 dm.
5. Divide first division of the scale in 10 equal parts, each representing 1 cm.
6. Mark units and sub-units on the scale and write the value of R.F.
7. Mark a length 6.3 dm on the scale, i.e. 6 decimetres on the right side of the zero mark and 3 centimetres on the left side of the zero mark.

Example 2.5 (Fig. 2.5)

In a map of Bhopal, a distance of 36 km between two localities is shown by a line of 45 cm long. Calculate its R.F. and construct a plain scale to read kilometres and hectometres. Show a distance of 9.3 km on it. [RGPV April 2010]

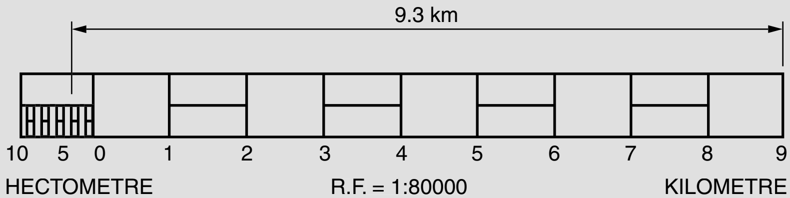


Fig. 2.5

Construction: Fig. 2.5

1. R.F. = $\dfrac{45\text{ cm}}{36\text{ km}} = \dfrac{45\text{ cm}}{36\times10^5\text{ cm}} = \dfrac{1}{80000}$
2. Here maximum length is not given. Since it is required to show a distance of 9.3 km, the maximum length should be greater than 9.3 km. Therefore, assume the max. length = 10 km.
3. Length of scale, L_s = R.F. × max. length = $\dfrac{1}{80000}\times10\times10^5 = 12.5$ cm
4. Draw a rectangle having a 12.5 cm length and 10 mm width.
5. Divide the length of scale into 10 equal parts, each part representing 1 kilometre.
6. Divide first division of the scale into 10 equal parts, each representing 1 hectometre.
7. Mark units and sub-units on the scale and write the value of R.F.
8. Mark a length 9.3 km on the scale, i.e. 9 kilometre on the right side of the zero mark and 3 hectometre on the left side of the zero mark.

Example 2.6 (Fig. 2.6)

A rectangular plot of 100 square kilometres is represented on a certain map by a rectangular area of 4 square centimetres. Draw a scale to show 50 km and mark a distance of 41 km on it. [RGPV June 2008(o)]

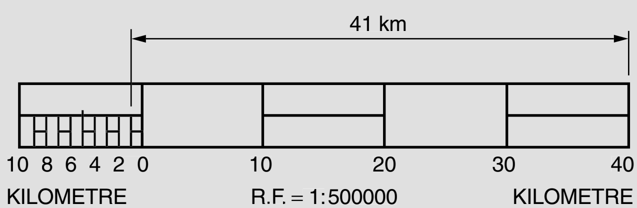


Fig. 2.6

Construction: Fig. 2.6

1. We know that R.F. is the ratio of lengths, therefore R.F. = $\sqrt{\dfrac{4\text{ cm}^2}{100\text{ km}^2}} = \dfrac{1\text{ cm}}{5\text{ km}} = \dfrac{1}{500000}$

2. Length of scale, L_s = R.F. × max. length = $\frac{1}{500000} \times 50 \times 10^5 = 10$ cm
3. Draw a rectangle having a 10 cm length and 10 mm width.
4. Divide the length of scale into 5 equal parts, each part representing 10 km.
5. Divide first division of the scale in 10 equal parts, each representing 1 km.
6. Mark units and sub-units on the scale and write the value of R.F.
7. Mark a length 41 km on the scale as shown.

Example 2.7 (Fig. 2.7)

A cube of 5 cm side represents a tank of 8000 cubic metre volume. Find the R.F. and construct a scale to measure up to 60 m and mark on it a distance of 47 m. Indicate R.F. of the scale. [RGPV Dec. 2008]

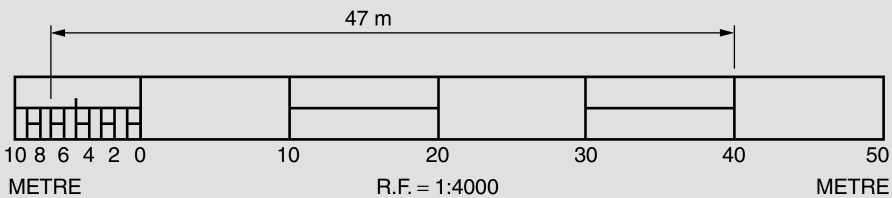


Fig. 2.7

Construction: Fig. 2.7

1. We know that R.F. is the ratio of lengths, therefore R.F. = $\frac{5 \text{ cm}}{\sqrt[3]{8000} \text{ m}} = \frac{5 \text{ cm}}{20 \times 100 \text{ cm}} = \frac{1}{4000}$
2. Length of scale, L_s = R.F. × max. length = $\frac{1}{4000} \times 60 \times 100$ cm $= 15$ cm
3. Draw a rectangle having a 15 cm length and 10 mm width.
4. Divide the length of scale into 6 equal parts, each part representing 10 metre.
5. Divide first division of the scale into 10 equal parts, each representing 1 m.
6. Mark units and sub-units on the scale and write the value of R.F.
7. Mark a distance of 47 metre on the scale as shown.

Example 2.8 (Fig. 2.8)

Construct a scale of 1:14 to read feet and inches and long enough to measure 7 feet. Show a distance of 5 feet 10 inches on it.

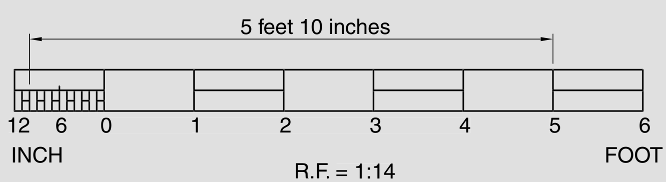


Fig. 2.8

Construction: Fig. 2.8

1. R.F. $= \frac{1}{14}$
2. Length of scale, L_s = R.F. × max. length $= \frac{1}{14} \times 7 \times 12 = 6 \text{ inches} = 15.24 \text{ cm}$
3. Draw a rectangle of length 6 inches, i.e. 15.24 cm and width 10 mm.
4. Divide the length of scale into 7 equal parts, each part representing 1 foot.
5. Divide the first division of scale into 12 equal parts, each representing 1 inch.
6. Mark units and sub-units on the scale and write the value of R.F.
7. Mark a distance of 5 feet 10 inches on the scale as shown.

Example 2.9 (Fig. 2.9)

Construct a scale of 1:54 to show yards and feet and long enough to measure 9 yards.

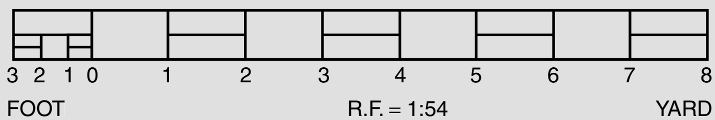


Fig. 2.9

Construction: Fig. 2.9

1. R.F. $= \frac{1}{54}$
2. Length of scale, L_s = R.F. × max. length $= \frac{1}{54} \times 9 \times 3 \times 12 = 6 \text{ inches} = 15.24 \text{ cm}$
3. Draw a rectangle of length 6 inches, i.e. 15.24 cm and width 10 mm.
4. Divide the length of scale in 9 equal parts, each part representing 1 yard.
5. Divide the first division of scale in 3 equal parts, each representing 1 foot.
6. Mark units and sub-units on the scale and write the value of R.F.

2.8 DIAGONAL SCALE

A diagonal scale is used to represent three consecutive units, i.e. main unit, its sub-unit and further subdivision of sub-unit. For example (a) metre, decimetre and centimetre (b) kilometre, hectometre and decametre (c) yards, feet and inches etc.

2.8.1 Principle of Diagonal Scale

The third unit in a diagonal scale is obtained by diagonal principle which is described as follows (see Fig. 2.10):

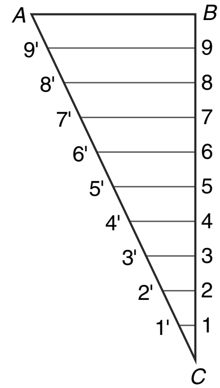


Fig. 2.10

1. Draw a small line *AB*.
2. Draw *BC* perpendicular to line *AB* taking any convenient length, say 50 mm. Join *AC*.
3. Divide *BC* into 10 equal parts and name the points as 1, 2, 3, etc.
4. Draw lines through 1, 2, 3, etc to meet line *AC* at points 1′, 2′, 3′, etc. All the lines 11′, 22′, 33′, 44′,…, 99′ are parallel to *AB*.
5. The triangles *C*11′, *C*22′, *C*33′, *C*44′, etc, are similar to triangle *CBA*. Therefore, their sides are also proportional.

   i.e. $11' = \frac{1}{10}$ of AB, $22' = \frac{2}{10}$ of AB, $33' = \frac{3}{10}$ of AB, etc.

2.8.2 Construction of Diagonal Scale

The following example illustrates the construction of the diagonal scale.

Example 2.10 (Fig. 2.11)

A map is to be drawn with R.F. 1:40. Construct a scale to read metres, decimetres and centimetres and long enough to measure up to 6 m. Show on it a distance of 3.84 m. [RGPV Dec. 2008]

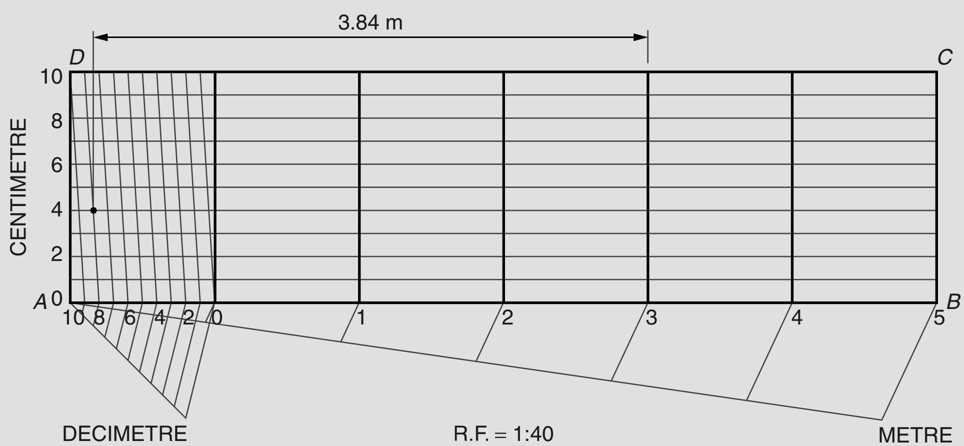


Fig. 2.11

Construction: Fig. 2.11

1. Given (a) R.F. = 1/40, (b) maximum length = 6 m and (c) least count = 1 cm.
2. Length of scale, L_s = R.F. × max. length $= \frac{1}{40} \times 6 \times 100 \text{ cm} = 15 \text{ cm}$
3. Draw a rectangle of length *AB* = 15 cm and width *AD* either 40 or 50 mm.
4. As length of the scale represents 6 m, divide line *AB* into 6 equal parts so that each part may represent 1 metre. Erect perpendicular lines through them to meet line *CD*. Mark the main units on it.
5. Divide the first part 0*A* into 10 equal subdivisions. Each subdivision represents 1 decimetre. Mark second unit on the scale as shown. Also, erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*. Mark third unit of the scale along it as shown.
7. Write the value of R.F. below the scale.
8. Mark a length of 3.84 m on the scale. i.e. 3 metre on the right side of the zero mark, 8 decimetre on the left side of zero mark and move up along the diagonal line by 4 divisions.

Example 2.11 (Fig. 2.12)

Construct a diagonal scale showing kilometre, hectometre and decametre in which a 2 cm long line represents 1 kilometre, and the scale is long enough to measure up to 7 kilometres. Find representative fraction and mark a distance of 4 kilometre 5 hectometre 3 decametre on it.

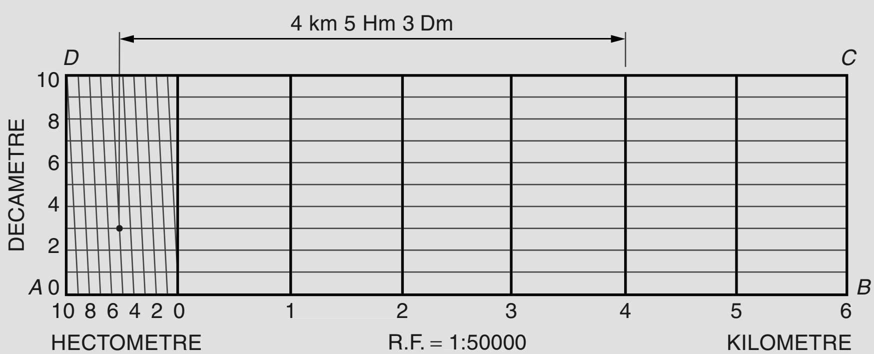


Fig. 2.12

Construction: Fig. 2.12

1. a) $\text{R.F.} = \frac{2 \text{ cm}}{1 \text{ km}} = \frac{2 \text{ cm}}{1 \times 10^5 \text{ cm}} = \frac{1}{50000}$

   b) maximum length = 7 kilometre c) least count of the scale = 1 decametre
2. Length of scale, L_s = R.F. × max. length = $\frac{1}{50000} \times 7 \times 10^5 \text{ cm} = 14 \text{ cm}$
3. Draw a rectangle *ABCD* of length *AB* = 14 cm and width *AD* either 40 or 50 mm.
4. Divide *AB* into 7 equal parts so that each part may represent 1 km. Mark the main unit as shown and erect perpendicular lines through them to meet line *CD*.

5. Divide 0*A* into 10 equal subdivisions, each representing 1 hectometre. Mark second unit on the scale as shown and erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*. Mark third unit of the scale along it as shown.
7. Write the value of R.F. below the scale.
8. Mark a length 4 km 5 hm 3 dm on the scale, i.e. 4 kilometres on the right side of the zero mark, 5 hectometres on the left side of zero mark and move up along the diagonal line by 3 divisions.

Example 2.12 (Fig. 2.13)

Draw a diagonal scale of R.F. 3:100 showing metres, decimetres and centimetres and to measure up to 5 metres. Show a length of 3.69 metres on it. [RGPV June 2009]

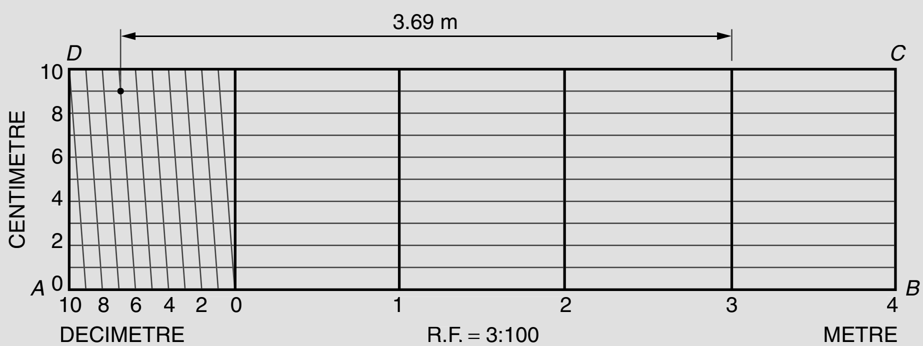


Fig. 2.13

Construction: Fig. 2.13

1. (a) R.F. = $\frac{3}{100}$ (b) maximum length = 5 m (c) least count = 0.01 m.
2. Length of the scale, L_s = R.F. × max. length = $\frac{3}{100} \times 5 \times 100 \text{ cm} = 15 \text{ cm}$
3. Draw a rectangle *ABCD* of length *AB* = 15 cm and width *AD* either 40 or 50 mm.
4. Divide *AB* into 5 equal parts so that each part may represent 1 m. Mark the main unit as shown and erect perpendicular lines through them to meet the line *CD*.
5. Divide 0*A* into 10 equal sub-divisions, each representing 1 decimetre. Mark second unit on the scale as shown and erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting line *BC*. Mark third unit of the scale along it as shown.
7. Write the value of R.F. below the scale.
8. Mark a length 3.69 m on the scale, i.e. 3 m on the right side of the zero mark, 6 decimetre on the left side of zero mark and move up along the diagonal line by 9 divisions.

Example 2.13 (Fig. 2.14)

The distance between two cities *A* and *B* is 300 km, its equivalent distance on the map measures only 6 cm. What is the R.F.? Draw a diagonal scale to show hundreds of kilometres, tens of kilometres and kilometres. Indicate on the scale, the following distances: 525 km, 313 km and 258 km. [RGPV June 2006]

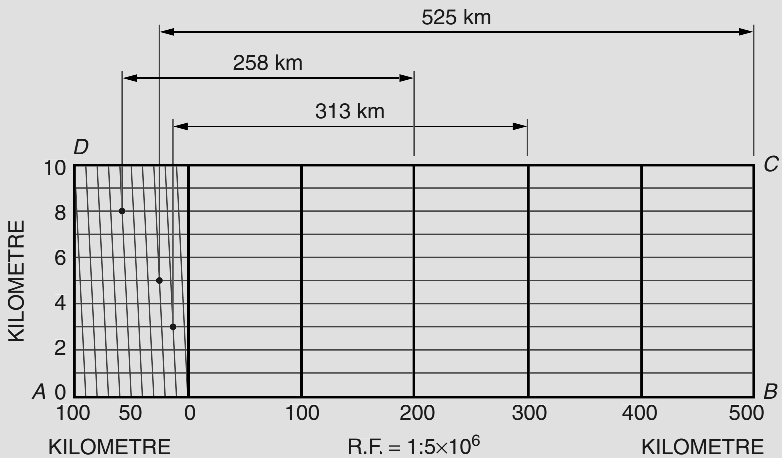


Fig. 2.14

Construction: Fig. 2.14

1. R.F. $= \dfrac{6 \text{ cm}}{300 \text{ km}} = \dfrac{6 \text{ cm}}{300 \times 10^5 \text{ cm}} = \dfrac{1}{5 \times 10^6}$
2. Since the scale has to show a distance of 525 km, the maximum length should be at least 600 km and the least count 1 km.
3. Length of scale, L_s = R.F. × max. length = $\dfrac{1}{5 \times 10^6} \times 600 \times 10^5 \text{ cm} = 12 \text{ cm}$
4. Draw a rectangle *ABCD* of 12 cm length and width either 40 or 50 mm.
5. Divide *AB* into 6 equal parts so that each part may represent 100 km.
6. Divide *A*0 into 10 equal sub-divisions, each representing 10 km. Erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*.
8. Write main unit, second unit, third unit and the value of R.F.
9. Mark the distances 525 km, 313 km and 258 km as shown.

Example 2.14 (Fig. 2.15)

Construct a scale to be used with a map; the scale of which is 1 cm = 500 m. The maximum length to be read is 5 km. Mark on the scale a distance of 3.85 km. [RGPV Aug. 2010]

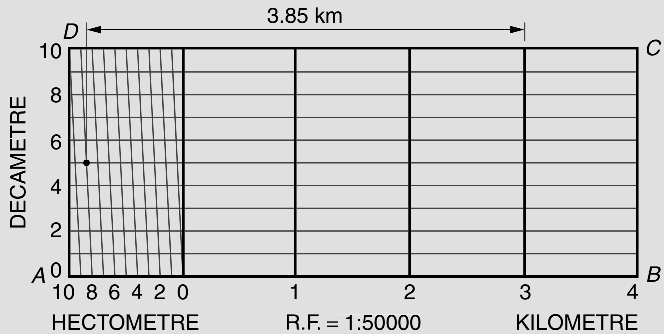


Fig. 2.15

Construction: Fig. 2.15

1. $\text{R.F.} = \dfrac{1 \text{ cm}}{500 \text{ m}} = \dfrac{1 \text{ cm}}{500 \times 10^2 \text{ cm}} = \dfrac{1}{50000}$
2. Length of scale, $L_s = \text{R.F.} \times \text{max. length} = \dfrac{1}{50000} \times 5 \times 10^5 \text{ cm} = 10 \text{ cm}$
3. Draw a rectangle *ABCD* of 10 cm length and width either 40 or 50 mm.
4. Divide *AB* into 5 equal parts, each representing 1 km.
5. Divide 0*A* into 10 equal subdivisions, each representing 1 hectometre. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*.
7. Write main unit, second unit, third unit and the value of R.F.
8. Mark a distance of 3.85 km as shown.

Example 2.15 (Fig. 2.16)

On a map, the actual distance of 5 m is represented by a line of 25 mm long. Calculate the R.F. of scale used. Construct a diagonal scale long enough to read 25 m and make distance of 19 m and 11 m. [RGPV Dec. 2007]

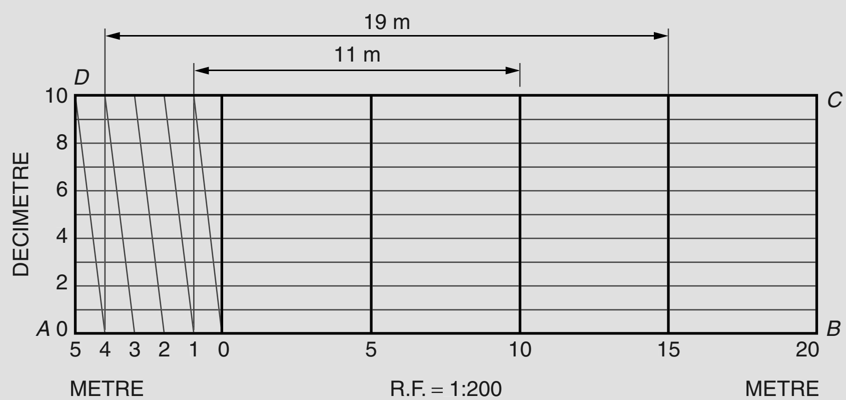


Fig. 2.16

Construction: Fig. 2.16

1. $\text{R.F.} = \dfrac{25\text{ mm}}{5\text{ m}} = \dfrac{25\text{ mm}}{5\times10^3\text{ mm}} = \dfrac{1}{200}$
2. Length of scale, L_s = R.F. × max. length = $\dfrac{1}{200}\times 25\times10^2\text{ cm} = 12.5\text{ cm}$
3. Draw a rectangle *ABCD* of 12.5 cm length and width either 40 or 50 mm.
4. Divide *AB* into 5 equal parts, each representing 5 m.
5. Divide 0*A* into 5 equal subdivisions, each representing 1 m. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*.
7. Write main unit, second unit, third unit and value of R.F.
8. Mark the distances of 19 m and 11 m as shown.

Example 2.16 (Fig. 2.17)

The distance between two stations is 100 km and on a road map it is shown by 30 cm. Draw a diagonal scale and indicate 46.8 km and 32.4 km on it.

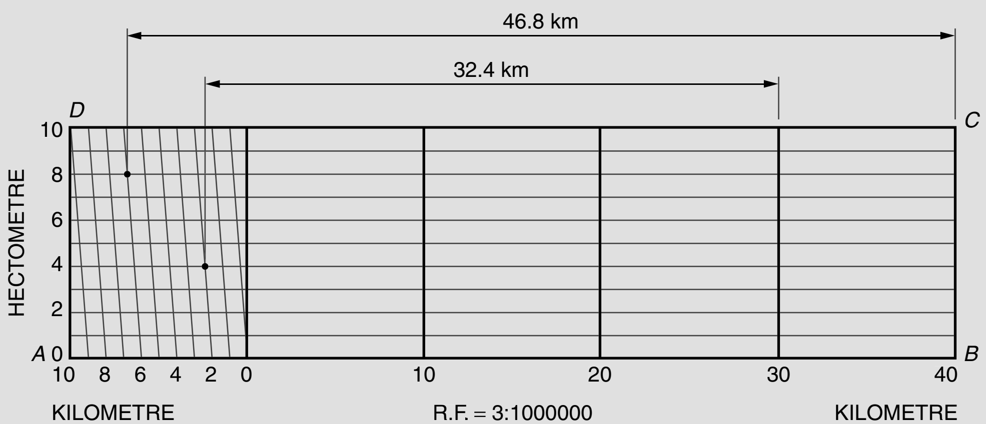


Fig. 2.17

Construction: Fig. 2.17

1. $\text{R.F.} = \dfrac{30\text{ cm}}{100\text{ km}} = \dfrac{30\text{ cm}}{100\times10^5\text{ cm}} = \dfrac{3}{10^6}$
2. Since the scale has to show a distance of 46.8 km, the maximum length should be at least 50 km and the least count 0.1 km.
3. Length of scale, L_s = R.F. × max. length = $\dfrac{3}{10^6}\times 50\times10^5\text{ cm} = 15\text{ cm}$
4. Draw a rectangle *ABCD* of 15 cm length and width either 40 or 50 mm.
5. Divide *AB* into 5 equal parts, each representing 10 km.
6. Divide 0*A* into 10 equal subdivisions, each representing 1 km. Erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*.

8. Write main unit, second unit, third unit and the value of R.F.
9. Mark 46.8 km and 32.4 km as shown.

Example 2.17 (Fig. 2.18)

Construct a scale to measure km, 1/8 km and 1/40 of a km, in which 1 km is represented by 4 cm. Mark on this scale a distance of 2.775 km. [RGPV Sep. 2009]

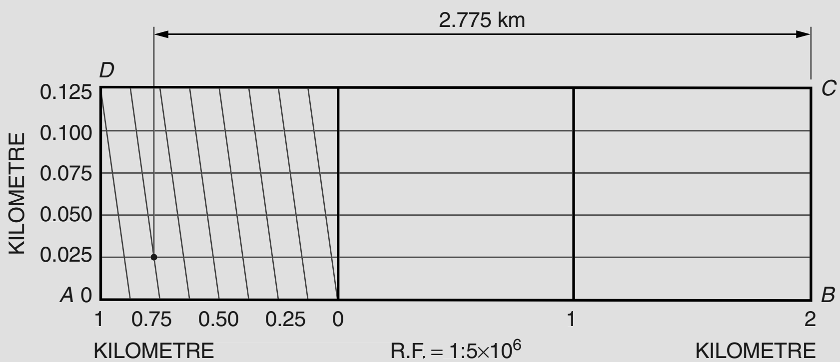


Fig. 2.18

Construction: Fig. 2.18

1. $\text{R.F.} = \frac{4\text{ cm}}{1\text{ km}} = \frac{4\text{ cm}}{1\times10^5\text{ cm}} = \frac{1}{25000}$
2. Since scale has to show a distance of 2.775 km, the maximum length should be at least 3 km.
3. Length of scale, $L_s = \text{R.F.} \times \text{max. length} = \frac{1}{25000}\times3\times10^5\text{ cm} = 12\text{ cm}$
4. Draw a rectangle *ABCD* of 12 cm length and width either 40 or 50 mm.
5. Divide *AB* into 3 equal parts, each representing 1 km.
6. Divide 0*A* into 8 equal subdivisions, each representing 1/8 km i.e. 0.125 km. Erect diagonal lines through them as shown.
7. Divide *AD* into 5 equal parts, each representing 1/40 km i.e. 0.025 km. Draw horizontal lines through each of them meeting at *BC*.
8. Write main unit, second unit, third unit and the value of R.F.
9. The breakup of 2.775 km is obtained as
   $$2.775\text{ km} = 2\text{ km} + 6\times\frac{1}{8}\text{ km} + 1\times\frac{1}{40}\text{ km}$$
10. To mark 2.775 km, take 2 km on the right side of the zero mark, 6 sub-divisions $\left(\text{i.e. } 6\times\frac{1}{8} = 0.75\text{ km}\right)$ on the left side of zero mark and move up along the diagonal line by 1 division $\left(\text{i.e. } 1\times\frac{1}{40} = 0.025\text{ km}\right)$.

Example 2.18 (Fig. 2.19)

A rectangular plot of land measuring 1.28 hectares is represented on a map by a similar rectangle of 8 sq. cm. Calculate R.F. of the scale. Draw a diagonal scale to read 1 m and long enough to measure 600 m. Show a distance of 438 m on it. [RGPV June 2008]

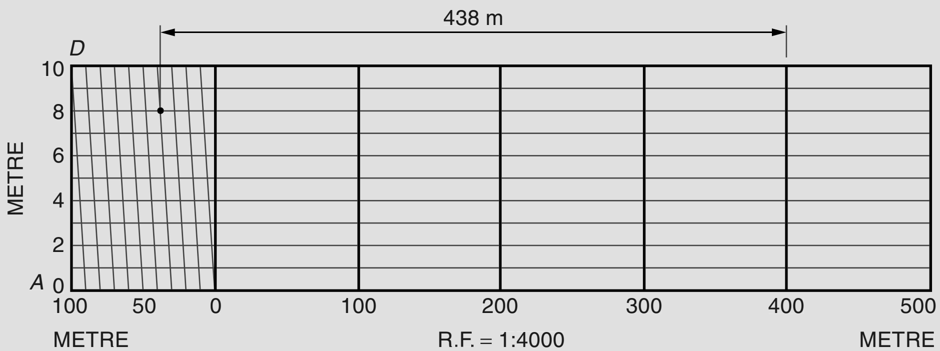


Fig. 2.19

Construction: Fig. 2.19

1. We know that R.F. is the ratio of lengths, therefore

$$\text{R.F.} = \frac{\sqrt{8\text{ cm}^2}}{\sqrt{1.28\text{ hectare}}} = \sqrt{\frac{8\text{ cm}^2}{1.28 \times 10^4\text{ m}^2}} = \frac{1\text{ cm}}{40\text{ m}} = \frac{1}{4000}$$

2. Length of scale, $L_s = \text{R.F.} \times \text{max. length} = \frac{1}{4000} \times 600 \times 10^2\text{ cm} = 15\text{ cm}$
3. Draw a rectangle *ABCD* of 15 cm length and width either 40 or 50 mm.
4. Divide *AB* into 6 equal parts, each representing 100 m.
5. Divide 0*A* into 10 equal subdivisions, each representing 10 m. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*.
7. Write main unit, second unit, third unit and the value of R.F.
8. Mark a distance of 438 m as shown.

Example 2.19 (Fig. 2.20)

The area of a field is 50000 m². The length and breadth of the field on the map is 15 cm and 8 cm respectively. Construct a diagonal scale which can read up to single metre. Mark the length of 346 m on the scale.

[RGPV Dec. 2004, Feb. 2005]

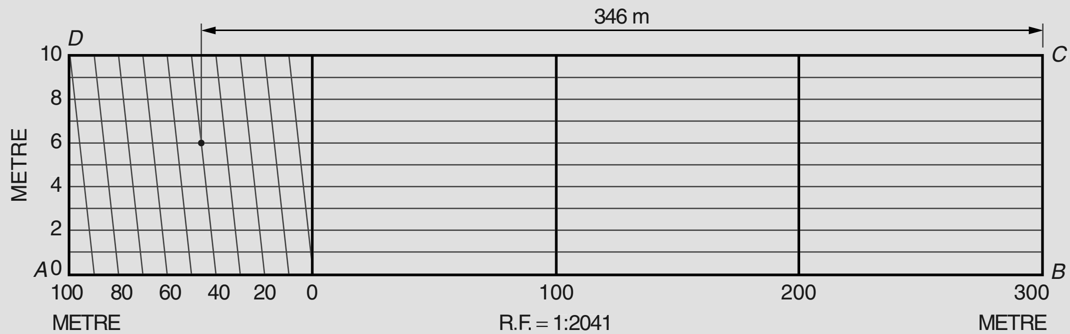


Fig. 2.20

Construction: Fig. 2.20

1. We know that R.F. is the ratio of lengths, therefore

$$\text{R.F.} = \frac{\sqrt{15 \times 8}\ \text{cm}}{\sqrt{50000}\ \text{m}} = \sqrt{\frac{120}{50000}} \times \frac{1\ \text{cm}}{100\ \text{cm}} \approx \frac{1}{2041}$$

2. Since the scale has to show a distance of 346 m, the maximum length should be at least 400 m and the least count 1 m.
3. Length of scale, L_s = R.F. × max. length = $\frac{1}{2041} \times 400 \times 100\ \text{cm} = 19.6\ \text{cm}$
4. Draw a rectangle *ABCD* of 19.6 cm length and width either 40 or 50 mm.
5. Divide *AB* into 4 equal parts, each representing 100 m.
6. Divide 0*A* into 10 equal subdivisions, each representing 10 m. Erect diagonal lines through them as shown.
7. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*.
8. Write main unit, second unit, third unit and the value of R.F.
9. Mark a distance of 346 m as shown.

Example 2.20 (Fig. 2.21)

A room of 1728 m^3 volume is shown by a cube of 4 cm side. Find the R.F. and construct a scale to measure up to 50 m. also indicate a distance of 37.6 m on the scale.

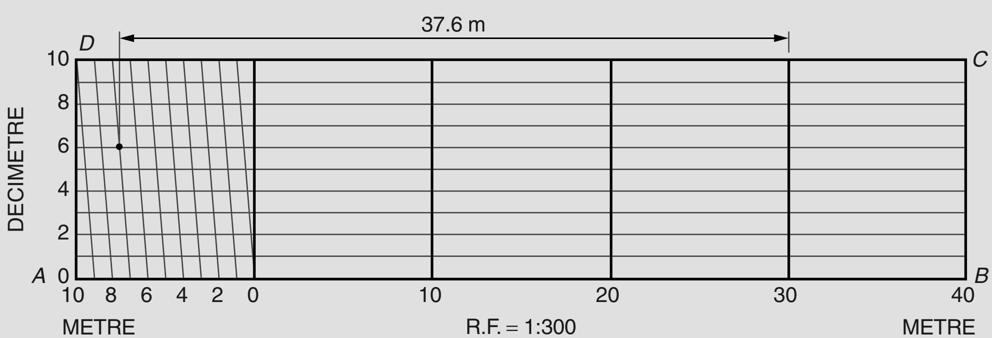


Fig. 2.21

Construction: Fig. 2.21

1. We know that R.F. is the ratio of lengths,

   therefore $\text{R.F.} = \dfrac{4 \text{ cm}}{\sqrt[3]{1728} \text{ m}} = \dfrac{4 \text{ cm}}{12 \times 100 \text{ cm}} = \dfrac{1}{300}$
2. Length of scale, L_s = R.F. × max. length = $\dfrac{1}{300} \times 50 \times 100 \text{ cm} = 16.67 \text{ cm}$
3. Draw a rectangle *ABCD* of 16.67 cm length and width either 40 or 50 mm.
4. Divide *AB* into 5 equal parts, each representing 10 m.
5. Divide 0*A* into 10 equal subdivisions, each representing 1 m. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts and draw horizontal lines through each of them meeting at *BC*.
7. Write main unit, second unit, third unit and the value of R.F.
8. Mark a distance of 37.6 km as shown.

Example 2.21 (Fig. 2.22)

Construct a diagonal scale of 1:63360 to read miles, furlongs and chains and long enough to measure up to 6 miles.

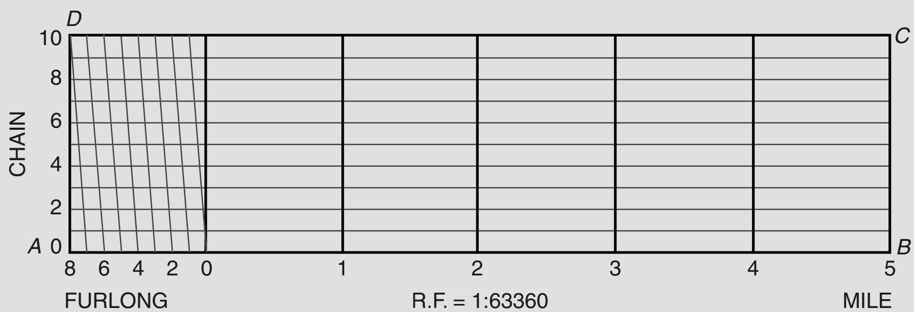


Fig. 2.22

Construction: Fig. 2.22

1. Given (a) R.F. = 1/63360 (b) maximum length = 6 miles (c) least count = 1 chain
2. Length of scale, L_s = R.F. × max. length

   $$= \frac{1}{63360} \times 6 \times 8 \times 10 \times 22 \times 3 \times 12 \text{ inches} = 6 \text{ inches} = 15.24 \text{ cm}$$
3. Draw a rectangle of length *AB* = 15.24 cm and width *AD* either 40 or 50 mm.
4. Divide *AB* into 6 equal parts, each part may represent 1 mile. Erect perpendicular lines through them to meet line *CD*. Mark the main units on it.
5. Divide 0*A* into 8 equal sub-divisions to represent 1 furlong each. Mark second unit on the scale as shown. Erect diagonal lines through them as shown.
6. Divide *AD* into 10 equal parts to represent 1 chain each. Draw horizontal lines through each of them meeting at *BC*. Mark third unit of the scale and write the value R.F. as shown.

Example 2.22 (Fig. 2.23)

Construct a diagonal scale showing yards, feet and inches in which 2 inches long line represents 1.25 yards and is long enough to measure up to 5 yards. Find R.F. and mark a distance of 4 yards 2 feet 8 inches.

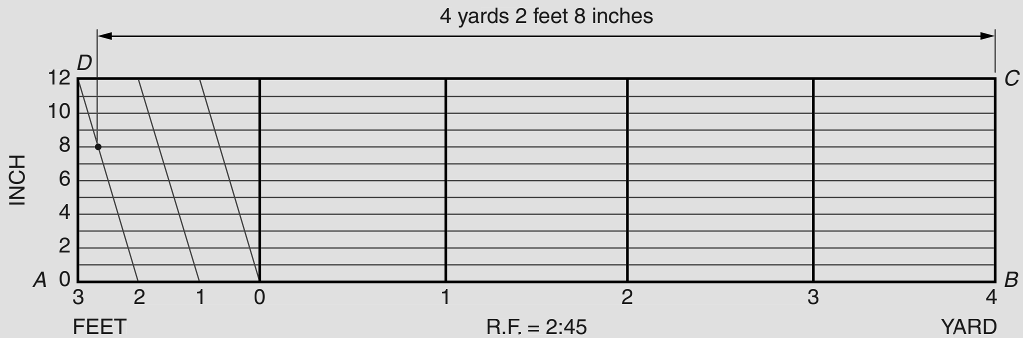


Fig. 2.23

Construction: Fig. 2.23

1. $\text{R.F.} = \dfrac{2 \text{ inches}}{1.25 \text{ yards}} = \dfrac{2 \text{ inches}}{1.25 \times 3 \times 12 \text{ inches}} = \dfrac{2}{45}$
2. Length of scale,
   $L_s = \text{R.F.} \times \text{max. length} = \dfrac{2}{45} \times 5 \times 3 \times 12 \text{ inches} = 8 \text{ inches} = 20.32 \text{ cm}$
3. Draw a rectangle of length $AB = 20.32$ cm and width AD either 50 or 60 mm.
4. Divide AB into 5 equal parts to represent 1 yard each. Erect perpendicular lines through them to meet line CD. Mark the main units on it.
5. Divide $0A$ into 3 equal subdivisions to represent 1 foot each. Mark second unit on the scale as shown. Erect diagonal lines through them as shown.
6. Divide AD into 12 equal parts to represent 1 inch each. Draw horizontal lines through each of them meeting at BC. Mark third unit of the scale and write the value of R.F.
7. Mark a distance of 4 yards 2 feet 8 inches as shown.

2.9 COMPARATIVE SCALE

Comparative scale is a pair of scales having a common representative fraction but graduated to read different units. A map drawn in miles and furlongs can be measured directly in kilometres and hectometres with the help of a comparative scale. Comparative scales may be either plain or diagonal scales depending upon the requirement.

2.9.1 Construction of Comparative Scale

The following examples illustrate the construction of the comparative scales.

Example 2.23 (Fig. 2.24)

On a railway map, an actual distance of 36 miles between two stations is represented by a 10 cm long line. Draw a plain scale to show mile and long enough to read up to 60 miles. Also draw a comparative scale attached to it to show kilometre and read up to 90 km. Take 1 mile = 1609 metres.

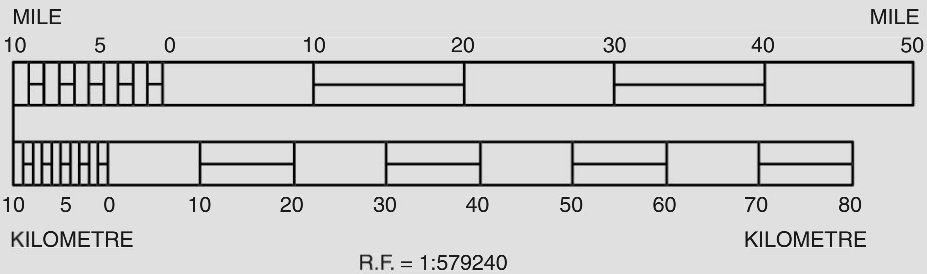


Fig. 2.24

Construction: Fig. 2.24

1. $\text{R.F.} = \dfrac{10\text{ cm}}{36\text{ miles}} = \dfrac{10\text{ cm}}{36 \times 1609 \times 10^2\text{ cm}} = \dfrac{1}{579240}$
2. Length of scales,

   $L_s\text{ (miles)} = \text{R.F.} \times \text{max. length} = \dfrac{1}{579240} \times 60 \times 1609 \times 10^2 = 16.67\text{ cm}$

   $L_s\text{ (kilometre)} = \text{R.F.} \times \text{max. length} = \dfrac{1}{579240} \times 90 \times 10^5 = 15.54\text{ cm}$
3. Draw a 16.67 cm long plain scale to represent 60 miles. Make its divisions and sub-divisions so that its least count is 1 mile.
4. Draw another 15.54 cm long plain scale to represent 90 km. Make its divisions and sub-divisions so that its least count is 1 km.
5. Write the main unit and the second unit for both the scales. Also write the value of R.F.

Example 2.24 (Fig. 2.25)

What is the R.F. of a scale which measures one-eighth inches to a mile? Draw plain comparative scales of two units to measure up to 60 miles and 100 km. Take 1 mile = 1.609 km.

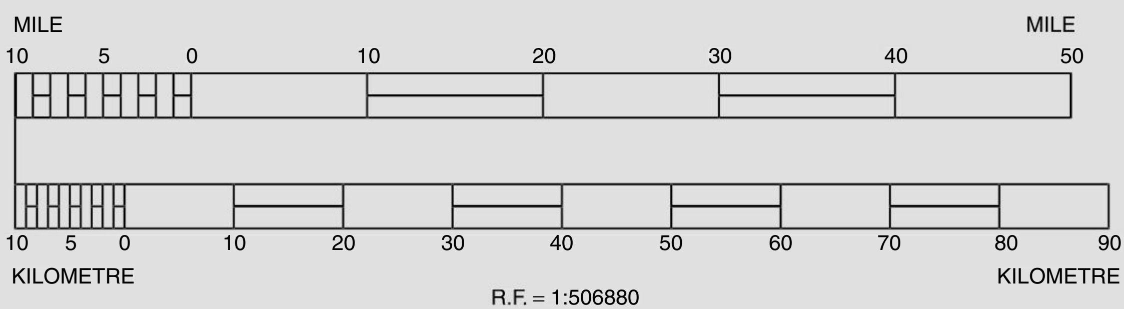


Fig. 2.25

Construction:

1. $\text{R.F.} = \dfrac{1/8 \text{ inch}}{1 \text{ mile}} = \dfrac{1/8 \text{ inch}}{1 \times 8 \times 10 \times 22 \times 3 \times 12 \text{ inch}} = \dfrac{1}{506880}$
2. Length of scales,

   $L_s \text{ (miles)} = \text{R.F.} \times \text{max. length} = \dfrac{1}{506880} \times 60 \times 1.609 \times 10^5 \text{ cm} = 19.04 \text{ cm}$

   $L_s \text{ (km)} = \text{R.F.} \times \text{max. length} = \dfrac{1}{506880} \times 100 \times 10^5 \text{ cm} = 19.72 \text{ cm}$
3. Draw a 19.04 cm plain scale long to represent 60 miles. Make its divisions and sub-divisions so that its least count is 1 mile.
4. Draw another 19.72 cm long plain scale to represent 100 km. Make its divisions and sub-divisions so that its least count is 1 km.
5. Write the main unit and the second unit for both the scales. Also write the value of R.F.

Example 2.25 (Fig. 2.26)

The distance between two towns is 120 km. A passenger train covers this distance in 4 hours. Construct a scale to measure off the distance covered by the train in a single minute and up to 1 hour. The R.F. of the scale is 1:200000. Show the distance covered by the train in 36 minutes. [RGPV Feb 2008]

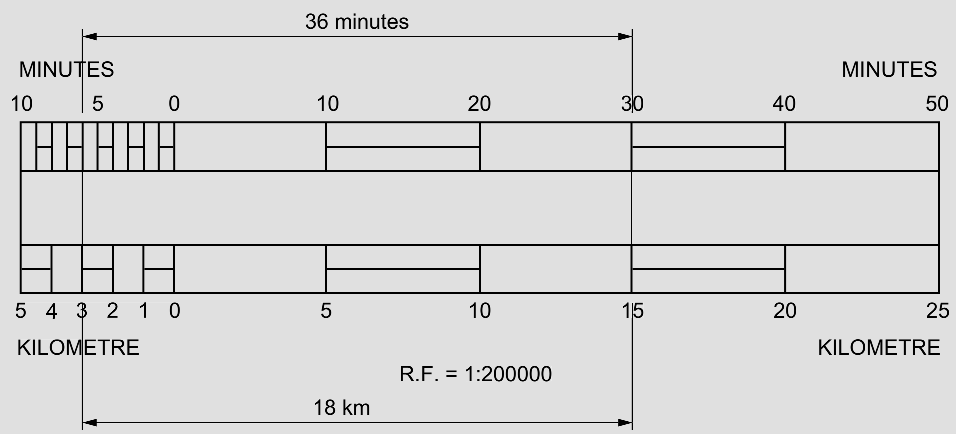


Fig. 2.26

Construction: Fig. 2.26

1. $\text{R.F.} = \dfrac{1}{200000}$
2. Length of scales,

   $L_s \text{ (minute)} = \text{R.F.} \times \text{max. length} = \dfrac{1}{200000} \times 1 \text{ hour}$

$$= \frac{1}{200000} \times \frac{120 \times 10^5}{4} \text{ cm} = 15 \text{ cm}$$

$$L_s \text{ (kilometre)} = \text{R.F.} \times \text{max. length} = \frac{1}{200000} \times 30 \times 10^5 \text{ cm} = 15 \text{ cm}$$

3. Draw a 15 cm long plain scale to represent 60 minutes km. Make its divisions and subdivisions so that its least count is 1 minute.
4. Draw another 15 cm long plain scale to represent 30 km. Make its divisions and subdivisions so that its least count is 1 km.
5. Write the main unit and the second unit for both the scales. Also write the value of R.F.
6. Mark a distance of 36 minute on the minute scale and compare it on the kilometre-scale to find an equivalent length of 18 km as shown.

Example 2.26 (Fig. 2.27)

A train is running at a speed of 40 km/h. Construct a plain scale to read up to a km and a minute. The scale should measure up to 50 km. The R.F. of the scale is 1:250000. On the scale show the distance covered by the train in 39 minutes. [RGPV Sep. 2009]

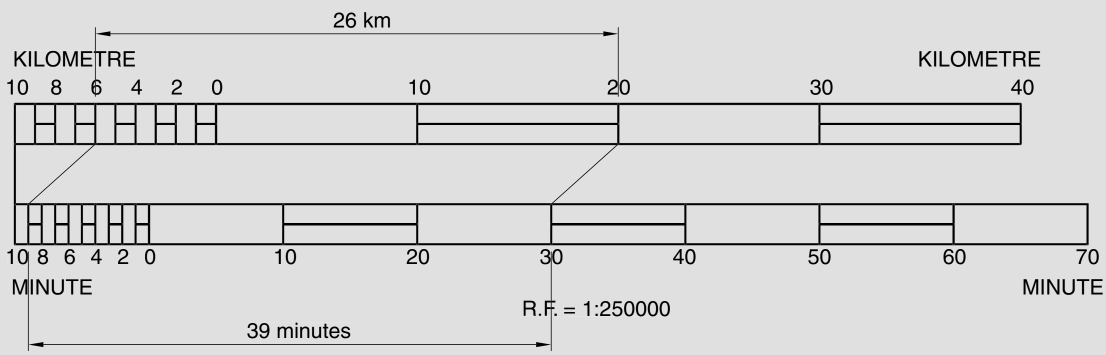


Fig. 2.27

Construction: Fig. 2.27

1. $\text{R.F.} = \frac{1}{250000}$
2. Length of scales,

$$L_s \text{ (kilometre)} = \text{R.F.} \times \text{max. length} = \frac{1}{250000} \times 50 \times 10^5 \text{ cm} = 20 \text{ cm}$$

$$L_s \text{ (minute)} = \text{R.F.} \times \text{max. length} = \frac{1}{250000} \times 80 \text{ minutes}$$

$$= \frac{80}{250000} \times \frac{40 \times 10^5}{60} \text{ cm} = 21.33 \text{ cm}$$

3. Draw a 20 cm long plain scale to represent 50 km. Make its divisions and sub-divisions so that its least count is 1 km.
4. Draw another 21.33 cm long plain scale to represent 80 minutes. Make its divisions and sub-divisions so that its least count is 1 minute.
5. Write the main unit and the second unit for both the scales. Also write the value of R.F.
6. Mark a distance of 39 minute on the minute scale and compare it on the kilometre-scale to find an equivalent length of 26 km as shown.

Example 2.27 (Fig. 2.28)

A car is running at a speed of 50 km/hr. Construct a diagonal scale to show 1 km by 3 cm and to measure up to 6 km. Mark also on the scale the distance covered by the car in 5 minutes 28 seconds. [RGPV June 2006]

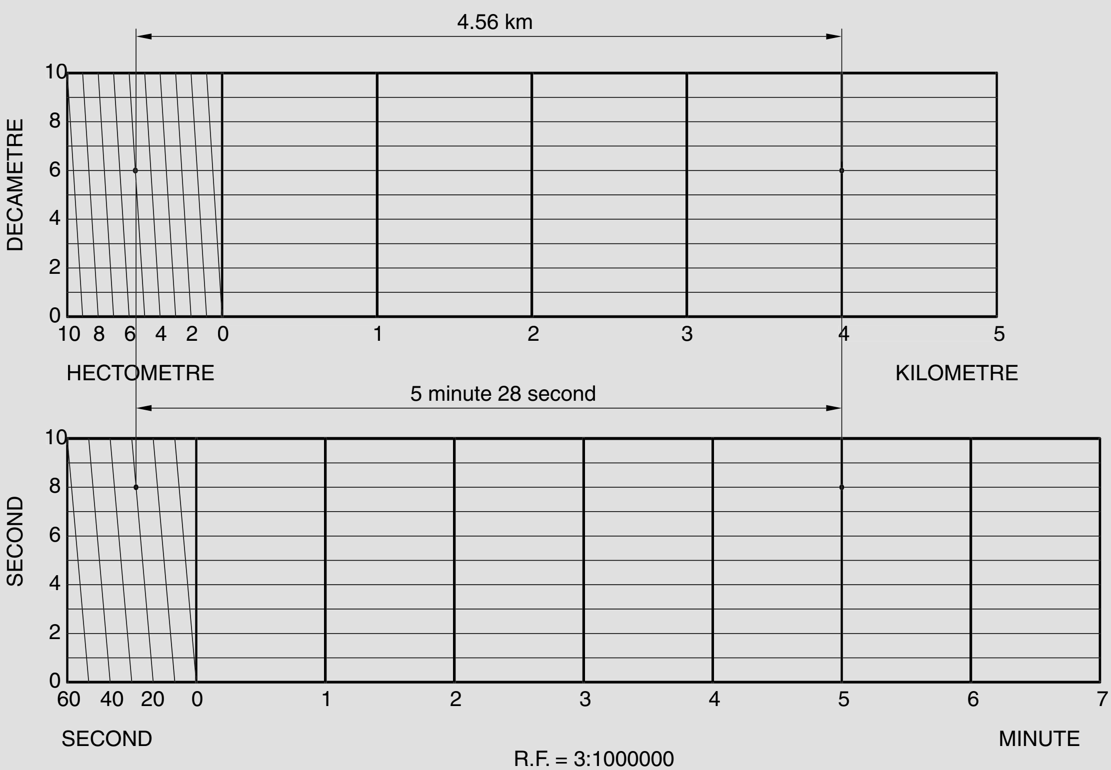


Fig. 2.28

Construction: Fig. 2.28

1. $\text{R.F.} = \dfrac{3\ \text{cm}}{1\ \text{km}} = \dfrac{3}{100000}$
2. Length of scales,

$$L_s \text{ (kilometre)} = \text{R.F.} \times \text{max. length} = \frac{3}{100000} \times 6 \times 10^5 \text{ cm} = 18 \text{ cm}$$

$$L_s \text{ (minute)} = \text{R.F.} \times \text{max. length} = \frac{3}{100000} \times 8 \text{ minutes}$$

$$= \frac{3}{100000} \times 8 \times \frac{50 \times 10^5}{60} \text{ cm} = 20 \text{ cm}$$

3. Draw an 18 cm long diagonal scale to represent 6 km. Make its divisions and sub-divisions such that least count of the scale is 1 decametre.
4. Draw another 20 cm long diagonal scale to represent 8 minutes. Make its divisions and sub-divisions such that its least count of the scale is 1 second.
5. Write the main unit, the second unit and third unit for both the scales. Also write the value of R.F.
6. Mark a distance of 5 minute 28 second on the minute-scale and compare it on the kilometre-scale to find an equivalent length of 4.56 km as shown.

2.10 SCALE OF CHORDS

In absence of a protractor, a scale of chords may be used to measure the angle or to set the required angle. The construction is based on the lengths of chords of angles measured on the same arc.

2.10.1 Construction of Scale of Chords

Fig. 2.29 shows the scale of chords. The steps of construction are as follows:

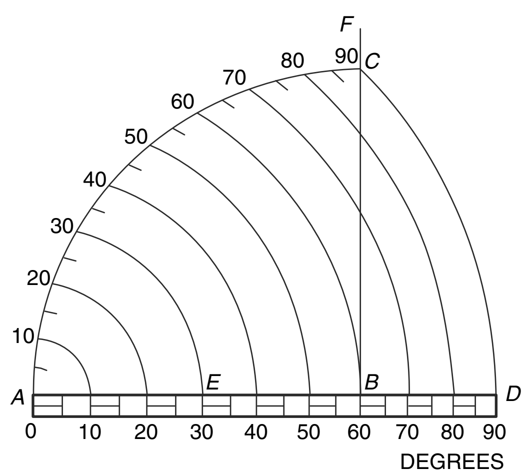


Fig. 2.29

1. Draw a line *AB* of any convenient length.
2. At point *B*, erect a line *BF* perpendicular to *AB*.
3. With centre *B* draw an arc *AC* cutting *BF* at point *C*. This arc subtends 90° angle at the centre *B*.

4. Divide this arc *AC* into 9 equal parts, each representing 10º division at *B*.
5. With centre *A* turn down the divisions to line *AB* produced. Thus, the distance *AE* on the scale represents the length of chord *A*-30, which subtends an angle 30º at the point *B*.
6. Similarly, mark 5º divisions on the line *AB*.
7. Complete the scale by drawing a rectangle below *AD* as shown. It may be noted that the divisions obtained are unequal, decreasing gradually from *A* to *D*. Scale *AD* is the required scale of chords.

2.10.2 Application of Scale of Chords

Set-off the given angle (Fig. 2.30)

Example 2.28 (Fig. 2.29–2.30)

Construct a scale of chords showing 5º divisions and with its aid set off angles 40º, 55º and 130º. [RGPV Dec. 2006]

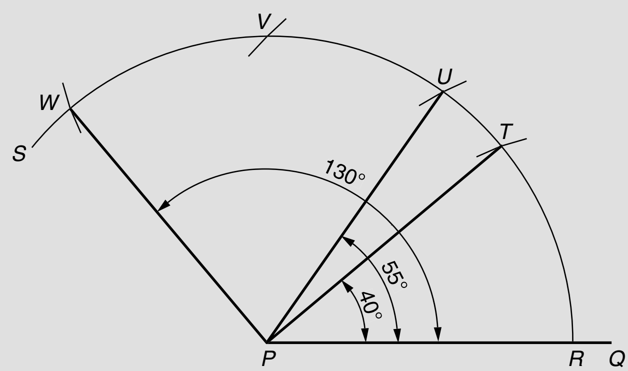


Fig. 2.30

Construction: Fig. 2.30

1. Draw scale of chords as shown in Fig. 2.29.
2. Draw a line *PQ*.
3. Draw an arc *RS* with *P* as the centre and *AB* as the radius.
4. Draw an arc with *R* as centre and radius equal to 0º–40º chord length, to intersect arc *RS* at point *T*. Join *PT*. The ∠*TPQ* represents 40º.
5. Similarly, draw an arc with *R* as centre and radius equal to 0º–55º chord length, to intersect arc *RS* at point *U*. Join *PU*. The ∠*UPQ* represents 55º.
6. Draw an arc with *R* as centre and radius equal to 0º–90º chord length, to intersect arc *RS* at point *V*. Draw another arc with *V* as centre and radius equal to 0º–40º chord length, to intersect arc *VS* at point *W*. Join *PW*. The ∠*WPQ* represents 130º (90º + 40º = 130º).

Example 2.29 (Fig. 2.31)

Construct a scale of chords showing 6º divisions and with its aid set off an angle of 54º.

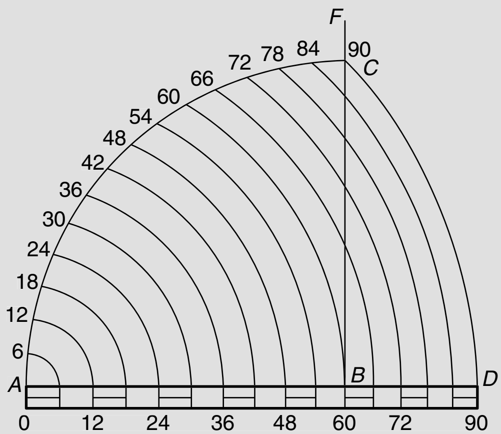


Fig. 2.31(a) *Scale of chords*

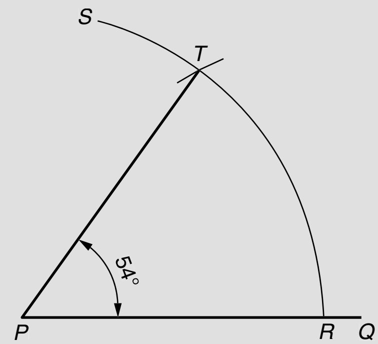


Fig. 2.31(b) *Marking an angle of 54°*

Construction: Fig. 2.31(a-b)

1. Draw a line *AB* of any convenient length.
2. At point *B*, erect a line *BF* perpendicular to *AB*.
3. With centre B draw an arc *AC* cutting *BF* at point *C*. This arc subtends 90° angle at the centre *B*.
4. Divide this arc *AC* into 15 equal parts, each representing 6° division at *B*.
5. With centre *A* turn down the divisions to line *AB* produced.
6. Complete the scale by drawing a rectangle below *AD* to represent the required scale of chords.
7. Draw a line *PQ*. Draw an arc *RS* with *P* as the centre and *AB* as the radius.
8. Draw an arc with *R* as centre and radius equal to 0°–54° chord length, to intersect arc *RS* at point *T*. Join *PT*. The ∠*TPQ* represents 54°.

Measure the Given Angle

Example 2.30 (Fig. 2.32)

Construct a scale of chords showing 5° divisions and with its aid measure angle *PQR* shown in Fig. 2.32.

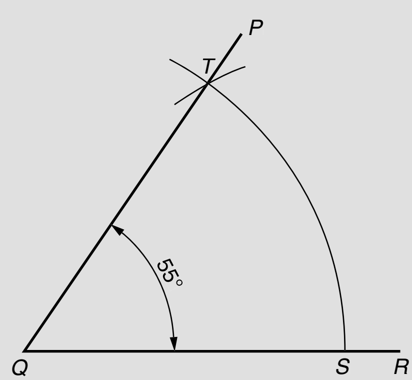


Fig. 2.32

Construction: Fig. 2.31

1. Draw scale of chord as shown in Fig. 2.29.
2. Draw an arc with *Q* as the centre and radius equal to *AB* the scale of chords, to meet line *PQ* at *T* and line *RQ* at *S*.
3. Transfer the chord length *ST* on the scale of chords and read the angle. Here it is 55º.

EXERCISE 2

Plain Scale

1. Construct a scale of 1:50 to read metres and decimetres and to measure up to 6 metre. **[RGPV Feb. 2010]**
2. Construct a scale of 1:40 to read metres and decimetres and long enough to measure up to 6 metre. Mark on it a distance of 4.7 metres.
3. Construct a scale to be used with a map, the scale of which is 1 cm = 4 m. The scale should read in metres upto 60 m. Show on it a distance of 46 m. **[RGPV Dec. 2010]**
4. Construct a scale of 1:5 to show decimetres and centimetres and long enough to measure up to 1 m. Show a distance of 6.3 dm on it.
5. Construct a scale of R.F. 1:125 to read a single metre and long enough to measure lengths up to 25 m. show the length of 16 m on this scale.
6. A 3.2 cm long line represents a distance of 4 m. Extend the line to measure up to 25 m and show on it units of metre and 5 m. Show a length of 16 m on this scale. **[RGPV Feb 2007, April 2009]**
7. Draw a plain scale to measure a maximum length of 10 km and to read in kilometre and hectometre. Mark on this scale a length of 3.6 km. Take R.F. = 1:80000. **[RGPV April 2009]**
8. Draw a scale of full size to read millimetre and long enough to measure up to 12 centimetre. Show a distance of 10.3 centimetre on it.
9. Construct a scale of 7:40000 to show hectometre and decametre and long enough to measure up to 1 kilometre. Show a distance of 0.63 kilometre on it.
10. Construct a scale of 1:6250 to read up to 1 km and to read 1 decametre on it. Show a length of 68 decametre on it.
11. A 4 cm long line on a map represents 1.5 m length. Determine the R.F. and draw a scale long enough to measure up to 6 m. Show a distance of 4.6 m on it.
12. A line of 1 cm represents a length of 4 decimetre. Draw a plain scale and mark a distance of 6.7 m on it.
13. A rectangular plot of land of area 16 square metre is represented on a map by a similar rectangle of 1 square centimetre. Calculate the R.F. of the scale of the map. Construct a plain scale to read metres and long enough to measure up to 60 m. Indicate a distance of 45 m on the scale. **[RGPV Aug. 2010, Feb. 2011]**

14. A cube of 5 cm side represents a tank of 1000 m^3 volume. Find the R.F. and construct a scale to measure up to 35 m and mark a distance of 27 m on it.
15. A container of 1000 m^3 volume is represented by a block of 125 cm^3 volume. Fine R.F. and construct a scale to measure up to 30 m. Measure a distance of 19 m on this scale. **[RGPV Dec. 2001]**
16. Construct a scale of 1:36 to show yards and feet and long enough to measure 7 yards.
17. Construct a scale of 1:5 to read feet and inches and long enough to measure 2 feet 6 inches. Show a distance of 1 foot 3 inches on it.

Diagonal Scale

18. Construct a diagonal scale of 1:32 to read metres, decimetres and centimetres and long enough to measure 4 m. Show on this scale a distance of 2.46 m. **[RGPV June 2007]**
19. Construct a diagonal scale of 1:48 showing metres, decimetres and centimetres and to measure up to 6 m length. Mark a length of 3.76 m on it. **[RGPV Jun. 2008(o), Dec. 2010]**
20. Construct a scale to be used with a map, the scale of which is 1 cm = 40 m. The scale should read in metres and maximum up to 500 m. Mark a distance of 456 m on it. **[RGPV Feb. 2011]**
21. Construct a diagonal scale of 3:200 showing metre, decimetres and centimetres and to measure up to 6 metres. **[RGPV Dec. 2006]**
22. The distance between two stations by road is 200 km and it is represented on a certain map by a 5 cm long line. Find the R.F. and construct a diagonal scale showing single kilometre and long enough to measure up to 600 km. Show a distance of 467 km on this scale.
23. On a building plan a line 15 cm long represents a distance of 300 metres. Calculate its R.F. Construct a diagonal scale to read up to 300 metres, showing single metre by diagonal division. Indicate the following distance on this scale: 245, 160, 70 and 8 metres. **[RGPV Dec. 2005]**
24. Construct a scale of R.F. = 1:250 to show decimetre and long enough to measure up to 30 m. Indicate a distance of 28.9 m on it.
25. The distance between two points on a map is 15 cm. The real distance between them is 20 km. Draw a diagonal scale to measure up to 25 km and show a distance of 13.6 km on it.
26. On a map, a 30 cm long line represents a distance of 450 m. Construct a diagonal scale for this map showing divisions of 50 m, 5 m and 0.5 m and capable of measuring 300 m. Show a distance of 175 m on this scale. **[RGPV June 2007]**
27. On a map, the distance between two points is 14 cm. The real distance between them is 20 km. Draw a diagonal scale of this map to read km hectometre and to measure up to 25 km and show a distance of 17.6 km on this scale. **[RGPV Feb. 2007]**
28. An area of 144 square cm on a map represents an area of 36 square km on the field. Find the R.F. of the scale for this map and draw a diagonal scale to show kilometre, hectometre and decametre so as to measure up to 10 km. Show a distance of 7.54 km on it. **[RGPV Feb. 2008]**
29. An area of 400 cm^2 on a map represents an area of 25 km^2 on a field. Construct a scale to measure up to 5 km and capable to show a distance of 3.56 km. Indicate this distance on the scale. **[RGPV Aug. 2010]**

30. A rectangular plot of land area 0.45 hectare is represented on a map by a similar rectangle of 5 cm^2. Calculate the R.F. of the scale of the map. Also draw the scale long enough to measure up to 400 metres and to read up to single metres from the map.
[RGPV June 2003, Dec. 2003]
31. The area of a field is 50000 m^2. The length and breadth of the field on the map is 10 cm and 8 cm respectively. Construct a diagonal scale which can read up to single metre. Mark the length of 235 m on the scale. What is the R.F. of the scale?
[RGPV June 2004]
32. A cube of 5 cm side shows a tank of 8000 m^3 volume. Find the R.F. and construct a diagonal scale to measure up to 70 m and mark a distance of 53.5 m on it.
33. Construct a diagonal scale of 1:316800 to read miles, furlongs and chains and long enough to measure 40 miles.
34. Construct a diagonal scale of 1:27 showing yards, feet and inches and long enough to measure up to 6 yards. Find R.F. and mark a distance of 4 yards 2 feet 10 inches.

Comparative Scale

35. The distance between Bhopal and Vidisha is 35 miles. On a railway map it is represented by a length of 7 cm. Find the R.F. Construct a comparative plain scale to measure up to 80 miles and read a single mile and a single kilometre. Show a distance of 67 miles and find the corresponding distances in kilometre. Take 1 mile = 1.6 km.
36. A 4 cm long line on a map represents an actual length of 200 miles. Construct a comparative diagonal scale to read up to a single mile and a single kilometre. Mark a distance of 653 miles and find the corresponding length in kilometre. Take 1 mile = 1.6 km.
37. The distance between Bhopal and Indore by train is 140 miles. It is represented by a 7 cm long line on a map. Draw a comparative diagonal scale to represent 350 miles. Mark a length of 237 miles and with the help of the scale find the corresponding distance in kilometres. Take 1 mile = 1.6 km.
38. The distance between two towns is 150 km. A superfast train covers this distance in 5 hours. Construct a scale to measure off the distance covered by the train in a single minute and up to 1 hour. The scale is drawn to $1/(2\times10^5)$. Show on it the distance covered in 38 minutes.
39. The distance between two stations *A* and *B* is 144 km and is covered by train in 4 hours. Draw a plain scale to measure the time up to single minute. R.F. of the scale is 1:240000. Calculate the distance covered by the train in 45 minutes and show minutes on the scale.
[RGPV June 2005]
40. An aeroplane is flying at a speed of 360 km/h. Draw a diagonal scale to represent 6 km by 1 cm and to show distance up to 60 km. Find the R.F. of the scale and find from the scale the distance covered by the aeroplane in (i) 3 minutes 22 seconds (ii) 7 minutes 49 seconds.
[RGPV Dec. 2002]
41. The distance between Hyderabad and Bangalore is 600 kilometres. An express train takes 12 hours to cover this distance. Construct a comparative plain scale to measure the distance covered up to 1 hour. With the help of the scale find the following: (i) the distance covered in 36 minutes, and (ii) the time taken to travel 354 km. Take R.F. as 1:312500.
42. A train is running at a speed of 40 km/h. Construct a comparative scale to read a kilometre and a minute and long enough to read up to 50 kilometres. The R.F. of the scale is 1:250000. On the scale, show the distances travelled by the train in 42 minutes.

Scale of Chords

43. Construct a scale of chords showing 10° divisions and with its aid set-off angles of 40° and 150°.
44. Construct a scale of chords showing 5° divisions and with its aid set-off angles of 35° and 110°.
45. Using the scales of chords, construct angles of 45° and 125°.
[RGPV June 2008]
46. Using the scales of chords, construct angles of 45° and 60°.
[RGPV April 2010]
47. With the help of the scale of chords and a least count of 15°, construct the following angles: (i) 75° (ii) 105° (iii) 225°.
48. The least count of a scale of chords is 18°. With the help of this scale erect angles of 72° and 108°.
49. Taking 6° least count in a scale of chords construct the following angles: (i) 42° (ii) 108° (iii) 210°.

REVIEW QUESTIONS

1. Distinguish among full size, reduced size and enlarged size drawing.
2. Explain reducing scale and give two practical applications.
3. Explain enlarging scale and give two practical applications.
4. State the advantages of graphical scale over an engineering scale?
5. What is representative fraction?
6. Enlist types of scales used in engineering practices.
7. Explain the principle of diagonal scale.
8. What are the advantages of a diagonal scale over a plain scale?
9. What is a comparative scale?
10. What are the applications of the scale of chords?

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) For drawing the components of a wrist watch, the scale used is
(a) reducing scale (b) full scale (c) enlarging scale (d) any of these

ii) The R.F. of scale is always
(a) less than 1 (b) equal to 1 (c) greater than 1 (d) any of these

iii) The unit of R.F. is
(a) cubic centimetre (b) square centimetre
(c) centimetre (d) none of these

iv) The full form of R.F. is
(a) reducing fraction (b) representative fraction
(c) rational factor (d) representative factor

v) A map of 10 cm × 8 cm represents an area of 50000 sq. metre of a field. The R.F. of the scale is
(a) 1/25 (b) 1/625 (c) 1/2500 (d) 1/6250000

vi) An area of 36 square kilometre is represented by 144 square centimetre on a map.
What is the R.F.?
(a) 1/4 (b) 1/2 (c) 1/5000 (d) 1/50000

vii) When measurements are required in three consecutive units, the appropriate scale is
(a) plain scale (b) diagonal scale (c) isometric scale (d) scales of chords

viii) The diagonal scale is most suitable for the measurement of
(a) diameter of a circle (b) diagonal of a square
(c) side of a pentagon (d) all of these

ix) The scale used for measuring in two systems of units is
(a) plain scale (b) diagonal scale (c) comparative scale (d) vernier scale

x) The diagonal of a square can be measured by
(a) plain scale (b) diagonal scale (c) vernier scale (d) all of these

xi) The R.F. of the scale on a mini-drafter is
(a) 0 (b) 1 (c) 10 (d) none of these

xii) Which of the following scale can be used for converting miles into kilometres
(a) plain scale (b) diagonal scale (c) comparative scale (d) all of these

xiii) Comparative scale is a pair of scale having a common
(a) units (b) R.F. (c) length of scale (d) least count

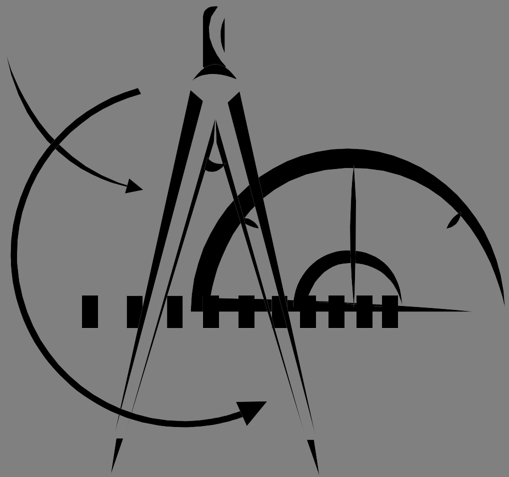
xiv) An angle can be set off and measured with the help of
(a) plane scale (b) diagonal scale
(c) comparative scale (d) scale of chords

xv) The scale of chord is used to measure
(a) length of chord (b) arc length of chord
(c) angle of chord (d) all of these

Answers

(i) c (ii) d (iii) d (iv) b (v) c (vi) d (vii) b (viii) d (ix) c (x) d (xi) b
(xii) c (xiii) b (xiv) d (xv) c

3



Conic Sections

- Introduction
- Cone, Circle, Isosceles Triangle
- Ellipse, Parabola, Hyperbola
- Construction of Ellipse
- Locate Centre, Major Axis and Minor Axis
- Tangent and Normal to the Ellipse, Empirical Relations
- Construction of Parabola
- Axis, Focus, Directrix, Tangent and Normal to the Parabola
- Construction of Hyperbola
- Asymptotes, Directrix, Tangent and Normal to the Hyperbola, Empirical Relations

3.1 INTRODUCTION

In engineering practice, we come across a number of objects containing plane curves such as ellipses, parabolas, hyperbolas, etc. The curve, which is obtained by cutting a right-circular cone with the help of a plane in different positions relative to the axis, is called a conic section. This chapter deals with a few common methods of construction of the conic sections and the field of their application.

3.2 CONE

A cone is formed if a right-angled triangle with an apex angle α is rotated about its altitude as the axis [Fig. 3.1(a)]. The height and the radius of the base of the cone are respectively equal to the altitude and base of the triangle. The apex angle of the cone is equal to 2 α [Fig. 3.1(b)]. Any imaginary line joining the apex to the circumference of the base circle is called a generator.

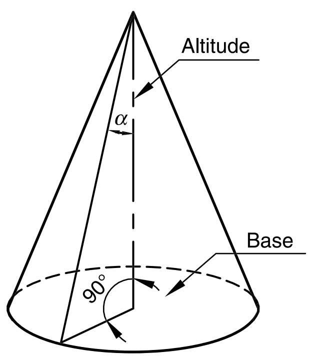


Fig. 3.1(a) *Constructing a cone*

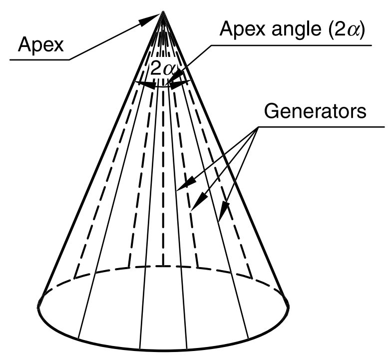


Fig. 3.1(b) *Cone formed*

3.3 CIRCLE

When the cutting plane is perpendicular to the axis of the cone ($\theta = 90°$), the curve of intersection obtained is a circle (Fig. 3.2).

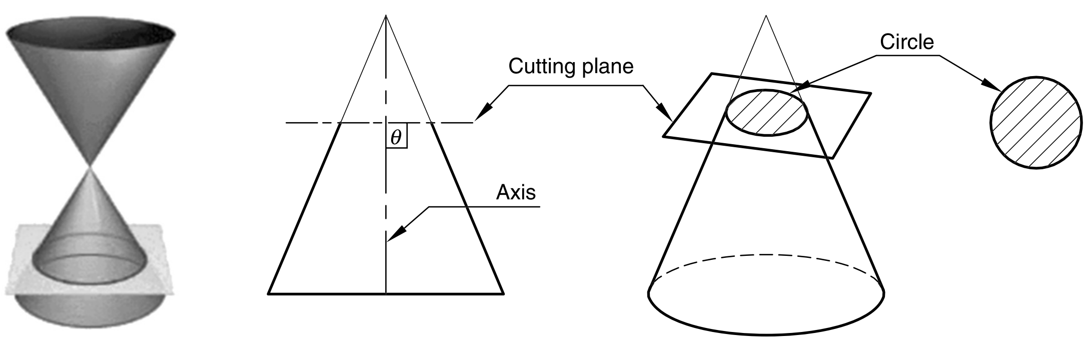


Fig. 3.2 *Formation of a circle by cutting a cone*

Applications Circles find their application in a vast number of objects such as diaphragms, discs, rings, plates, etc. A circle revolving around its axis forms a surface called a sphere.

3.4 ISOSCELES TRIANGLE

When the cutting plane passes through the apex and cuts the base of the cone, the curve of intersection is an isosceles triangle (Fig. 3.3).

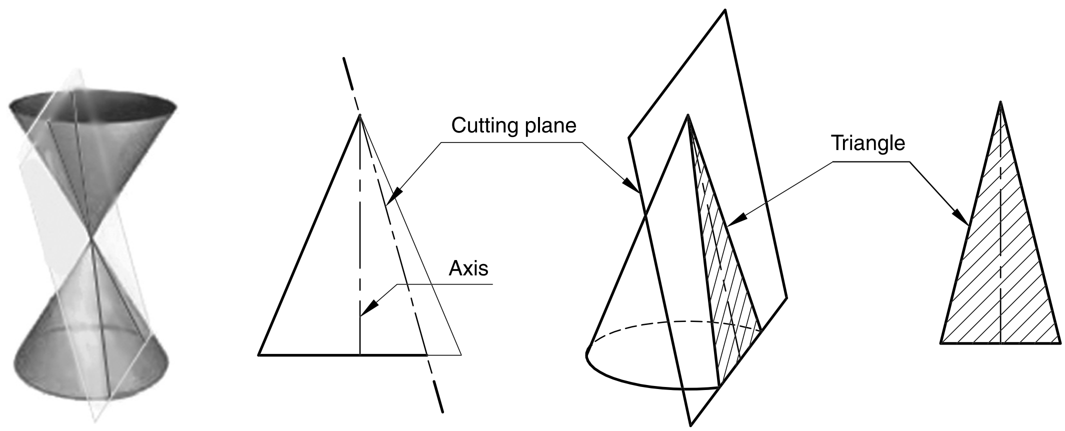


Fig. 3.3 *Formation of an isosceles triangle by cutting a cone*

3.5 ELLIPSE AND ITS APPLICATIONS

When the cutting plane is inclined to the axis and cut all the generators of the cone, the section is an ellipse. The inclination of cutting plane for an ellipse must be greater than half of the apex angle i.e. $\theta > \alpha$ (Fig. 3.4).

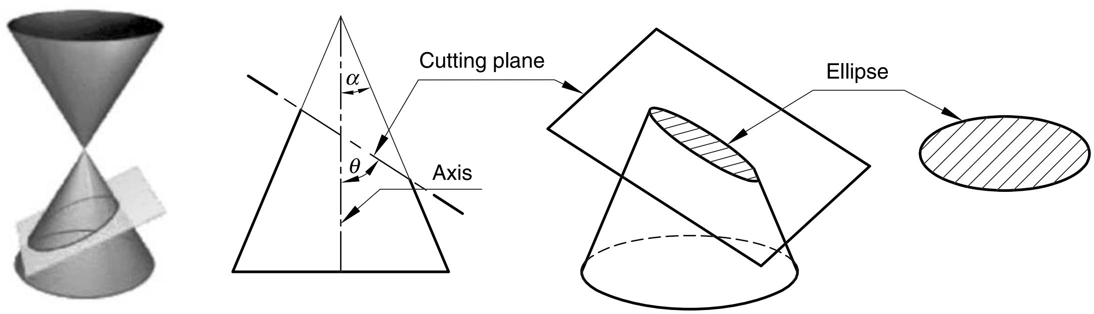


Fig. 3.4 *Formation of an ellipse by cutting a cone*

Applications Elliptical curves find their use in concrete arches [Fig. 3.5(a)], stone bridges, dams, monuments (memorial structures), man-holes, glands, stuffing boxes, etc. A planet travels around the sun in an elliptical orbit with the sun at one of its foci [Fig. 3.5(b)]. The orbits of the moon and artificial satellites of the earth are also elliptical. On a far smaller scale, the electrons of an atom move in an approximately elliptical orbit with the nucleus at one focus [Fig. 3.5(c)].

Fig. 3.5(a)

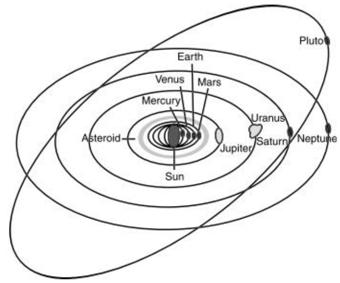


Fig. 3.5(b)

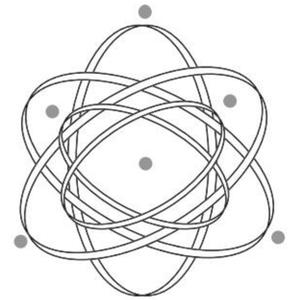


Fig. 3.5(c)

The ellipse has an important property that is used in the reflection of light and sound waves. Any light or signal that starts at one focus will be reflected to the other focus. This principle is used in lithotripsy, a medical procedure for treating kidney stones, [Fig. 3.5(d)]. The principle is also used in the construction of "whispering galleries", such as in St. Paul's Cathedral in London, in which a person, whisper near one focus is heard at the other focus though not at many places in between. [Fig. 3.5(e)]

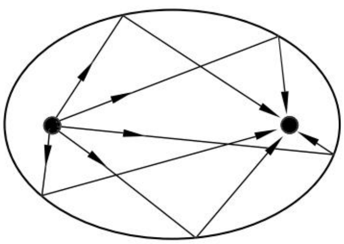


Fig. 3.5(d)

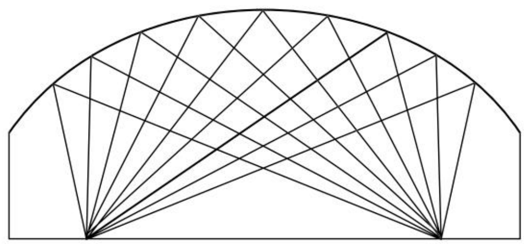


Fig. 3.5(e)

3.6 PARABOLA AND ITS APPLICATIONS

When the cutting plane is inclined to the axis and is parallel to one of the generators of the cone, the section is a parabola. The inclination of the cutting plane is equal to half of the apex angle, i.e., $\theta = \alpha$ (Fig. 3.6).

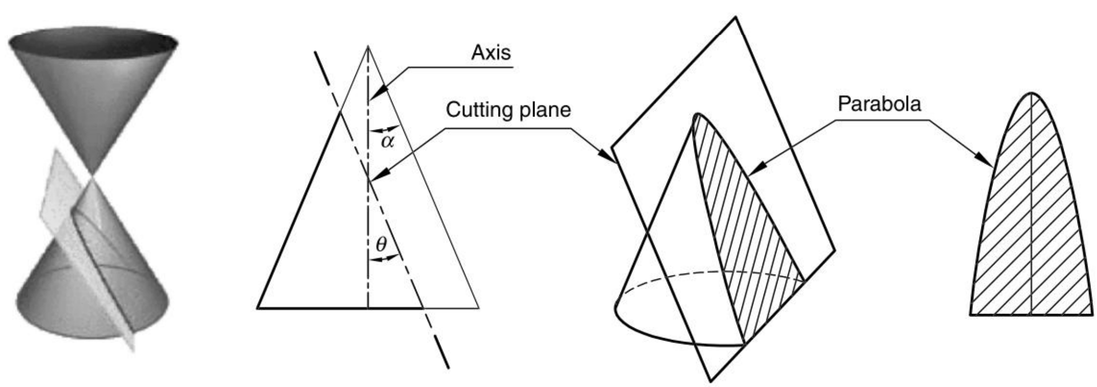


Fig. 3.6 *Formation of a parabola by cutting a cone*

Applications One of nature's best known approximations to parabolas is the path taken by a body projected upward and obliquely to the pull of gravity, as in the parabolic trajectory of a golf ball. The friction of air and the pull of gravity will change slightly the projectile's path from that of a true parabola, but in many cases the error is insignificant [Fig. 3.7(a) and Fig. 3.7(b)]. Many bridge designs use parabolic supports, eg. Golden Gate Bridge, Howrah Bridge, etc. Fig. 3.7(d).

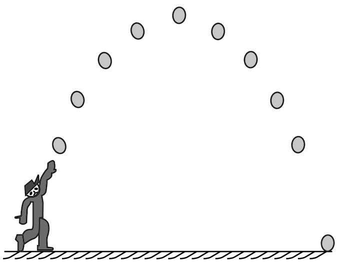


Fig. 3.7(a)

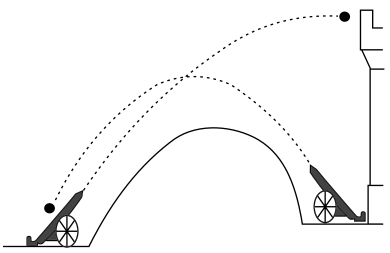


Fig. 3.7(b)

Fig. 3.7(c)

If a light is placed at the focus of a parabolic mirror, the light will be reflected in rays parallel to the axis [Fig. 3.7(d)]. This principle is used for getting a straight beam of light in headlamps, torches, etc. The opposite principle is used in giant mirrors, reflecting telescopes and antennas to collect light and radio waves from outer space and focus them at the focal point [Fig. 3.7(e)]. A solar furnace and solar cooker produces heat by focusing sunlight by means of a parabolic mirror arrangement [Fig. 3.7(f)].

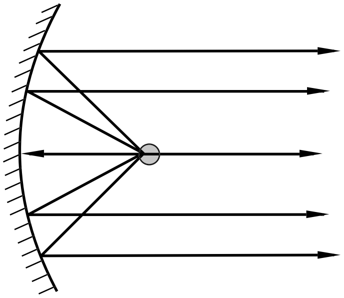


Fig. 3.7(d)

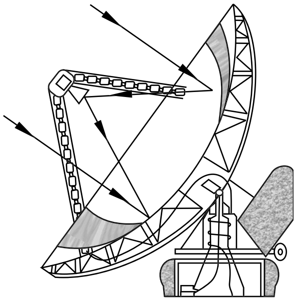


Fig. 3.7(e)

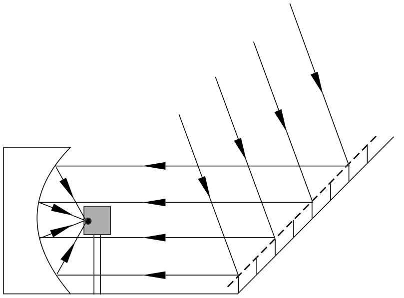


Fig. 3.7(f)

3.7 HYPERBOLA AND ITS APPLICATIONS

When the cutting plane cuts both the parts of the double cone, the section is a hyperbola. The inclination of cutting plane for the hyperbola must be less than half of the apex angle, i.e., $\theta < \alpha$ (Fig. 3.8).

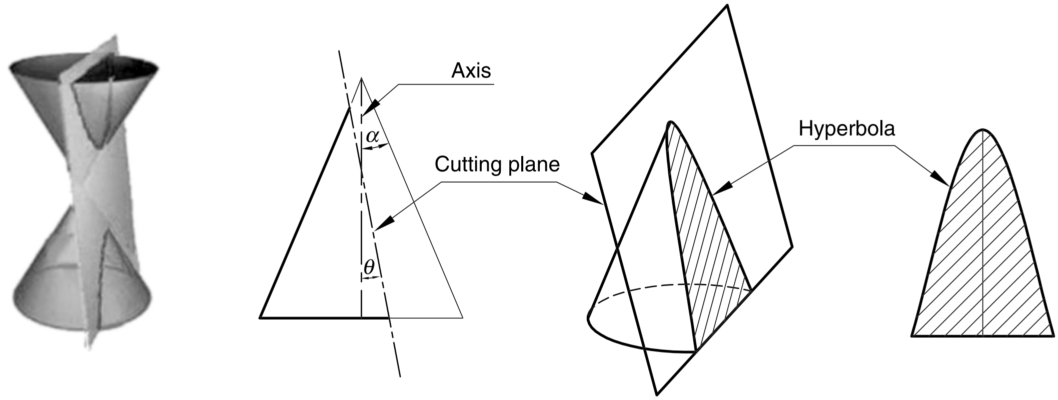


Fig. 3.8 *Formation of a hyperbola by cutting a cone*

Rectangular Hyperbola When the cutting plane is parallel to the axis of the cone, the section is a rectangular hyperbola, i.e., $\theta = 0^\circ$ (Fig. 3.9).

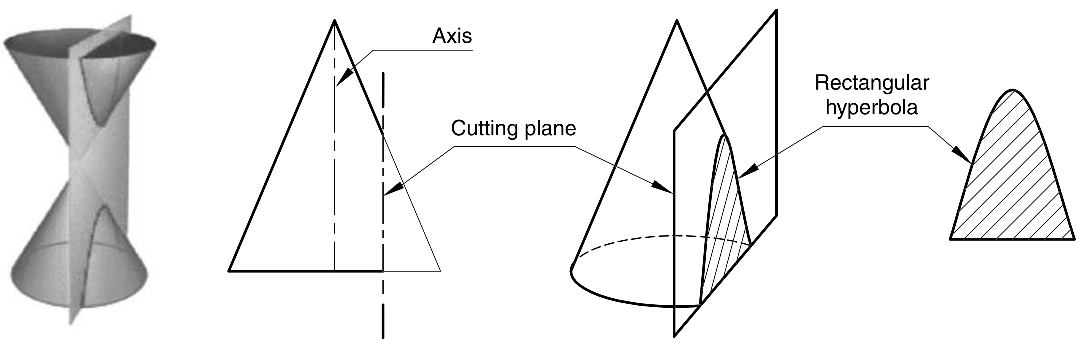


Fig. 3.9 *Formation of a rectangular hyperbola by cutting a cone*

Applications The hyperbolic curve graphically represents the Boyle's law, i.e., PV = constant. A comet that does not return to the sun follows a hyperbolic path. A household lamp casts hyperbolic shadows on a wall [Fig. 3.10(a)].

Fig. 3.10(a)

Fig. 3.10(b)

Fig. 3.10(c)

A hyperbola revolving around its axis forms a surface called a hyperboloid. Hyperboloids are useful in design of water channels, cooling towers, etc. Fig. 3.10(b) shows use of cooling tower for nuclear plant. The architecture of the James S. McDonnell Planetarium of the St. Louis Science Centre is also hyperboloid [Fig. 3.10(c)].

Two hyperboloids of revolution can provide gear transmission between two skew axes. The cogs of each gear are a set of generating straight lines [Fig. 3.10(d)]. Reflecting telescopes use hyperbolic mirrors [Fig. 3.10(e)]. If the centre of each of two sets of concentric circles is the source of a radio signal, the synchronized signals would intersect one another in associated hyperbolas. This principle forms the basis of a hyperbolic radio navigation system known as Long Range Navigation [Fig. 3.10(f)].

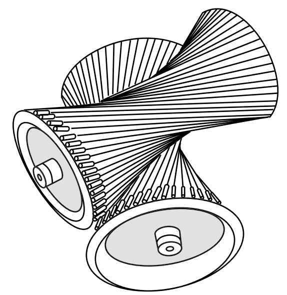


Fig. 3.10(d)

Fig. 3.10(e)

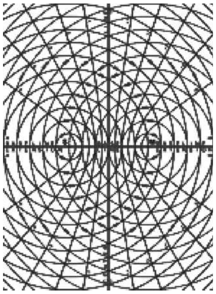


Fig. 3.10(f)

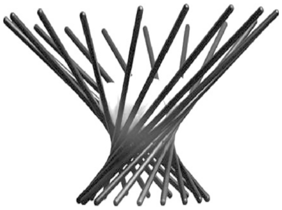


Fig. 3.10(g) *Wooden pot*

3.8 CONSTRUCTION OF ELLIPSE

An ellipse can be constructed by the following methods:

1. Eccentricity method (general method)
2. Intersecting arcs method or arcs of circles method
3. Concentric circles method
4. Oblong method
   a. Rectangle method
   b. Parallelogram method

3.8.1 Eccentricity Method

An ellipse is defined as the locus of a point P moving in a plane in such a way that the ratio of its distance from a fixed point F_1 to the fixed straight line DD' is a constant and is always less than 1 (see Fig. 3.11).

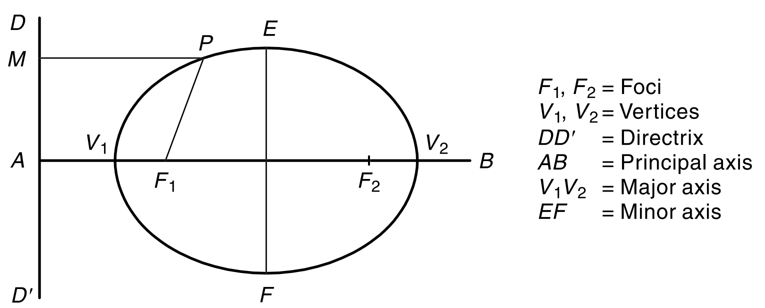


Fig. 3.11

$$\text{Eccentricity, } e = \frac{\text{distance of the point from the focus } (PF_1)}{\text{distance of the point from the directrix } (PM)}$$

$$e = \frac{PF_1}{PM} = \text{constant (and less than 1 for ellipse)}$$

The eccentricity method for construction of ellipse is based on this definition.

Example 3.1 (Fig. 3.12)

Construct an ellipse when the distance of its focus from its directrix is equal to 50 mm and eccentricity is 2/3. Also draw a tangent and a normal to the ellipse at a point 70 mm away from the directrix. [RGPV June 2007]

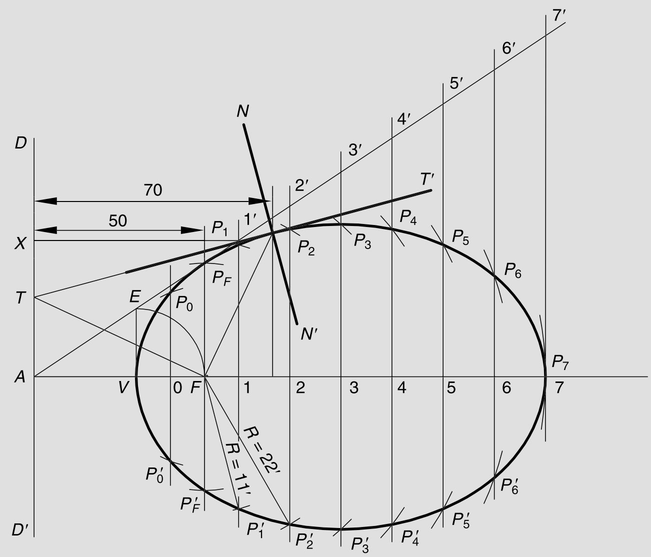


Fig. 3.12

Construction: Fig. 3.12

1. Draw a directrix DD'.
2. Draw principal axis AB perpendicular to directrix DD'.
3. Mark focus F on AB at a distance of 50 mm from the directrix DD'.
4. Divide AF into five equal parts (e = 2/3) and mark vertex V on it such that $\frac{VF}{AV} = \frac{2}{3}$. Vertex V satisfies the condition for being a point of the curve.

5. Draw a vertical line VE equal to VF. Join AE and extend it to some distance. Thus, in the triangle AVE, $\frac{VE}{AV} = \frac{VF}{AV} = \frac{2}{3}$.
6. Mark any point 1 on the axis and through it draw a perpendicular line to meet AE produced at 1′. Thus, $\frac{11'}{A1} = \frac{VE}{AV} = \frac{2}{3}$
7. With center F and radius equal to 11′, draw arcs to intersect the perpendicular line 11′ at points P_1 and P_1'. These are the points on the ellipse because ratio $\frac{FP_1}{XP_1} = \frac{11'}{A1} = \frac{2}{3}$.
8. Repeat steps 6 and 7 for points 2, 3, 4, … etc (at any convenient distances on AB which need not be equal) and obtain points P_2 and P_2', P_3 and P_3', P_4 and P_4', etc.
9. Join points P_2', P_1', V, P_1, P_2, etc. to obtain the required ellipse.

Tangent and Normal to An Ellipse

10. Mark a point P on the ellipse at the given distance 70 mm from the directrix.
11. Join PF.
12. Draw a line FT perpendicular to PF meeting directrix at T.
13. Join TP and extend it to some point T'. TT' is the required tangent.
14. At point P, draw a line NN'perpendicular to TT'. This is the required normal.

Note: Mathematically the length of the major axis = $\frac{2ed}{1-e^2}$ where, e is the eccentricity and d is the distance of focus from directrix.

3.8.2 Intersecting Arcs Method or Arcs of Circles Method

Ellipse is a curve traced by a point P moving in a way that the sum of its distance from two fixed points F_1 and F_2 is constant and is equal to the major axis (refer Fig. 3.13). Intersecting arcs method is based on this principle. The method is also known as *arcs of circles method*.

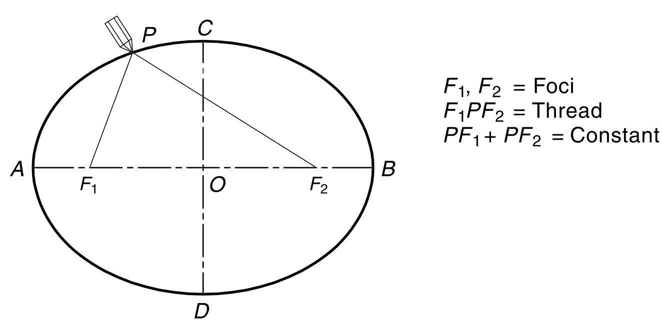


Fig. 3.13

Example 3.2 (Fig. 3.14)

A point moves in a plane in such a way that the sum of its distances from two fixed points 60 mm apart is 90 mm. Name and draw the locus of this point around the fixed points.
[RGPV Dec. 2008]

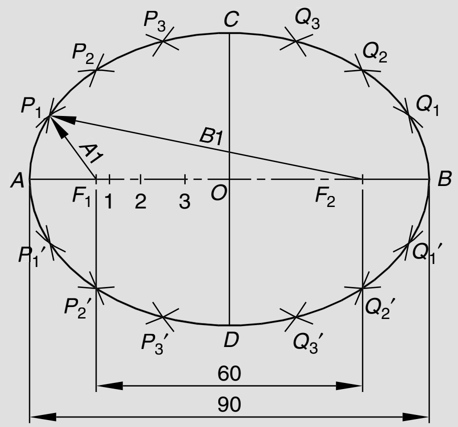


Fig. 3.14

Construction: Fig. 3.14

1. Draw major axis $AB = 90$ mm and mark its mid-point O.
2. Mark $OF_1 = OF_2 = 60/2 = 30$ mm.
3. Mark points 1, 2, 3, etc., on OF_1 at any convenient distances, which need not be equal.
4. With foci F_1 and F_2 as the centers and radius $A1$, draw arcs on both sides of AB.
5. With foci F_1 and F_2 as the centers and radius $B1$, draw arcs on both sides of AB to intersect the previous arcs at four points P_1, P_1', Q_1 and Q_1'.
6. Repeat step 3 and step 4 with radii $A2$ and $B2$, $A3$ and $B3$, etc., and obtain points P_2, P_2', Q_2, Q_2', P_3, P_3', Q_3, and Q_3'.
7. Draw a smooth curve passing through all these points. The curve obtained is ellipse.

Corollary 1: If an arc with foci F_1 and F_2 and radius AO is drawn, they will intersect each other on the perpendicular bisector through point O at points C and D. The line joining CD represents the minor axis.

Corollary 2: If lengths of major axis AB and minor axis CD are given, then draw an arc with radius AO and centre either C or D to meet the major axis at points F_1 and F_2. These points are the foci of the ellipse.

Corollary 3: If distance between the foci F_1 and F_2, and the lengths of minor axis CD are given, then draw an arc with centre O and radius CF_1 (= CF_2) to meet the line joining the foci at points A and B. The line joining AB represents the major axis.

3.8.3 Concentric Circles Method

This is a special method for construction of ellipse.

Example 3.3 (Fig. 3.15)

The major axis of an ellipse is 100 mm and minor axis is 60 mm long. Draw an ellipse by concentric circle method. [RGPV Dec. 2004, Feb. 2005, Dec. 2006]

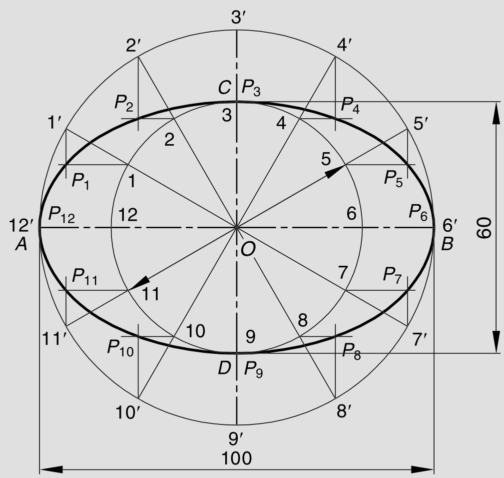


Fig. 3.15

Construction: Fig. 3.15

1. Draw major axis *AB* = 100 mm and minor axis *CD* = 60 mm. Lines *AB* and *CD* are perpendicular bisectors of each other meeting at point *O*.
2. Draw two concentric circles with *O* as the center and diameters equal to *AB* and *CD*.
3. Divide both the circles into twelve equal parts. Mark points 1′, 2′, 3′, … etc., on the circumference of the circle with diameter *AB* and points 1, 2, 3, … etc., on circumference of the circle with diameter *CD*, as shown.
4. Draw vertical line through point 1′ to meet the horizontal line drawn through point 1 at point P_1. The point P_1 represents a point on the ellipse.
5. Similarly, draw vertical lines through other points 2′, 3′, …, etc., to meet the corresponding horizontal lines drawn through points 2, 3, … etc., at points P_2, P_3, … etc. The point P_2, P_3, … etc., lie on the ellipse.
6. Draw a smooth curve passing through all these points to get the required ellipse.

3.8.4 Rectangle Method

It is basically a method of inscribing an ellipse in a rectangle.

Example 3.4 (Fig. 3.16)

Inscribe the largest possible ellipse in a rectangle with 160 mm × 100 mm sides.

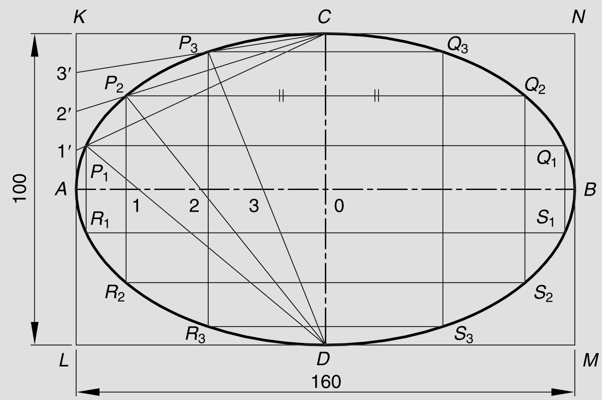


Fig. 3.16

Construction: Fig. 3.16

1. Draw a rectangle *KLMN* with sides *KL* = 100 mm and *LM* = 160 mm.
2. Mark points *A*, *B*, *C* and *D* as mid point of sides *KL*, *MN*, *NK* and *LM* respectively.
3. Mark *O* as perpendicular bisectors of *AB* and *CD*.
4. Divide lines *AO* and *AK* into same number of equal parts, say 4. Mark 1, 2, 3 on *AO* and 1′, 2′, 3′ on *AK*.
5. Join *C* with points 1′, 2′, 3′.
6. Draw lines from point *D*, passing through 1, 2 and 3, intersecting *C*1′, *C*2′ *C*3′ at points P_1, P_2, P_3 respectively.
7. Draw the curve through *A*, P_1, P_2, P_3, *C*. This is one quarter of an ellipse.
8. As the curve is symmetric about the axes, obtain points Q_1, Q_2, Q_3 of the curve in rectangle *CNBO* by drawing horizontal lines from P_1, P_2, P_3 and making them equal on either sides of *OC*.
9. Similarly, obtain points R_1, R_2, R_3 in rectangle *AODL* by drawing perpendicular lines from points P_1, P_2, P_3 and making them equal on either sides of *OA*.
10. Draw horizontal lines from R_1, R_2, R_3 and vertical lines from Q_1, Q_2, Q_3 to meet at S_1, S_2, S_3 to get points of a curve in rectangle *OBMD*.
11. Draw a curve passing through points obtained in steps 8 to 10.

3.8.5 Parallelogram Method

It is basically a method of inscribing an ellipse in a parallelogram.

Example 3.5 (Fig. 3.17)

The sides of a parallelogram are 120 mm and 80 mm. The included angle between them is 75°. Inscribe an ellipse in the given parallelogram.

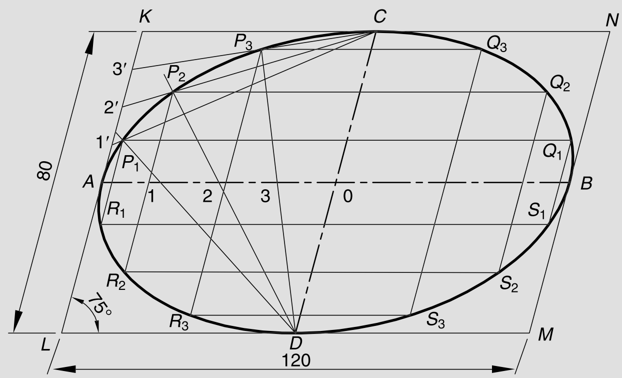


Fig. 3.17

Construction: Fig. 3.17

1. Draw a parallelogram *KLMN* with sides *KL* = 80 mm, *LM* = 120 mm and ∠*KLM* = 75°.
2. Mark *A*, *B*, *C*, *D* as mid-points of *KL*, *MN*, *NK*, *LM* respectively.
3. Mark *O* as the perpendicular bisectors of *AB* and *CD*.
4. Divide lines *OA* and *KA* into same number of equal parts, say 4. Mark 1, 2, 3 on *OA* and 1′, 2′, 3′ on *KA*.
5. Join point *C* with 1′, 2′, 3′.
6. Draw lines from point *D*, to join points 1, 2 and 3 and produce to intersect lines *C*1′, *C*2′ *C*3′ at points P_1, P_2, P_3 respectively.
7. Draw smooth curve through *A*, P_1, P_2, P_3, *C*. This is one quarter of an ellipse.
8. Draw lines parallel to *AB* through points P_1, P_2, P_3 and make each of them equal on either sides of *CD* and obtain Q_1, Q_2, Q_3.
9. Similarly, draw lines parallel to line *CD* passing through points P_1, P_2, P_3 and points Q_1, Q_2, Q_3. Make each of them equal on either sides of *AB* and obtain points R_1, R_2, R_3, S_1, S_2, S_3.
10. Join the points obtained in steps 8 and 9 with a smooth curve.

3.9 LOCATE CENTRE, MAJOR AXIS AND MINOR AXIS

Example 3.6 (Fig. 3.18)

Determine the center, major axis and minor axis of the given ellipse.

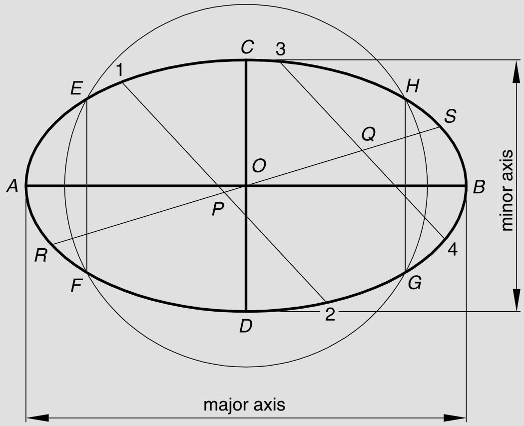


Fig. 3.18

Construction: Fig. 3.18

1. Draw any two chords 1-2 and 3-4 parallel to each other.
2. Mark *P* and *Q* as midpoints of 1-2 and 3-4 respectively.
3. Join *PQ* and extend it on both sides so that it meets the ellipse at points *R* and *S*.
4. Mark *O* as the midpoint of *RS*. Point *O* represents the centre.
5. With point *O* as the center and radius of any convenient length, draw a circle to cut the ellipse at points *E*, *F*, *G* and *H*.
6. Complete the rectangle *EFGH*.
7. Through point *O*, draw a line *AB* parallel to *EH*, and line *CD* parallel to *EF*. The lines *AB* and *CD* represent the major and the minor axes, respectively.

3.10 TANGENT AND NORMAL TO THE ELLIPSE

Example 3.7 (Fig. 3.19)

Draw tangent and normal to the ellipse through a point *P*, lying on it.

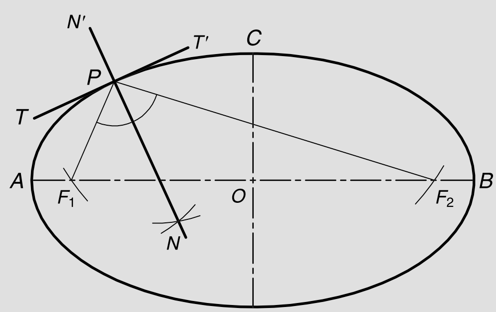


Fig. 3.19

Construction: Fig. 3.19

1. Let *ADBCA* is the given ellipse.
2. Locate its major and minor axes *AB* and *CD*. (Refer Example 3.6)
3. Let the major and the minor axes intersect at point *O*. Draw an arc with point *C* as the centre and radius *OA* to meet the major axis at points F_1, F_2. The points F_1, F_2 represents the foci.
4. Locate point *P* on the ellipse through which tangent has to be drawn.
5. Join the point *P* with foci F_1 and F_2.
6. Draw angular bisector *NN′* of the angle F_1PF_2. The line *NN′* represents the normal.
7. Through point *P*, draw a line *TT′* perpendicular to *NN′*. The line *TT′* represents the tangent.

Example 3.8 (Fig. 3.20)

Draw a pair of tangents to the ellipse through a point *P*, lying outside.

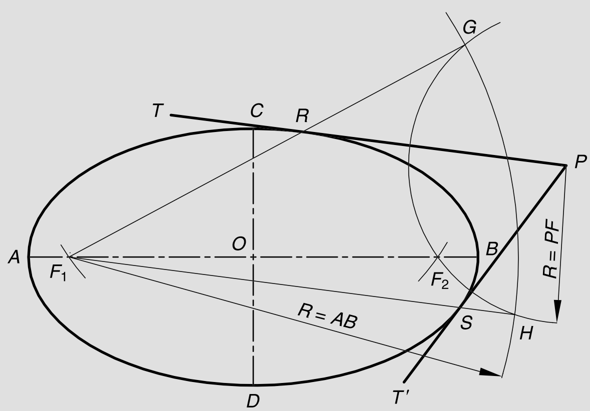


Fig. 3.20

Construction: Fig. 3.20

1. Let *ADBCA* is the given ellipse.
2. Locate its major and minor axes *AB* and *CD*. (Refer Example 3.6)
3. Let the major and the minor axes intersect at point *O*. Draw an arc with point *C* as the centre and radius *OA* to meet the major axis at points F_1, F_2. The points F_1, F_2 represents the foci.
4. Mark point *P* at the given location, through which tangent has to be drawn.
5. Draw an arc with point *P* as centre and radius PF_2 to intersect another arc drawn with F_1 as the centre and radius *AB* at points *G* and *H*.
6. Join F_1G and F_1H. Let they intersect the ellipse at points *R* and *S*, respectively.
7. Join *PR* and *PS* and extend them to points *T* and *T′* respectively. The lines *PRT* and *PST′* represents the tangents to the ellipse.

3.11 EMPIRICAL RELATIONS

The empirical relations among various parameters can be established as given below:

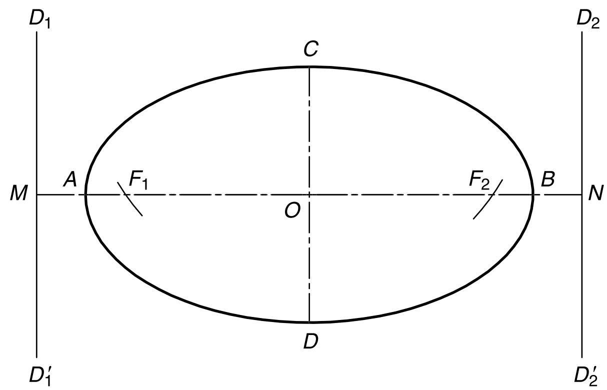


Fig. 3.21

We know that eccentricity is defined as, $e = \dfrac{\text{distance of a point from the focus}}{\text{distance of the point from the directrix}}$

In Fig. 3.21 the ends *A* and *B* of the major axis are also the points lying on the ellipse, therefore,

$$e = \frac{AF_1}{AM} \text{ and } e = \frac{BF_1}{BM}$$

1. Therefore, eccentricity may also be given by

$$e = \frac{AF_1 - BF_1}{AM - BM} = \frac{F_1F_2}{AB} = \frac{\text{distance between the foci}}{\text{length of major axis}}$$

2. Eccentricity may also be given by

$$e = \frac{AF_1 + BF_1}{AM + BM} = \frac{AB}{AM + BM} = \frac{AB}{AM + AN} \qquad (\because BM = AN)$$

$$\text{or } e = \frac{\text{length of major axis}}{\text{distance between directrices}}$$

3. Eccentricity may also be given by multiplying equation (A) and equation (B).

$$e^2 = \frac{\text{distance between foci}}{\text{length of major axis}} \times \frac{\text{length of major axis}}{\text{distance between directrices}}$$

$$\text{or } e = \sqrt{\frac{\text{distance between foci}}{\text{distance between directrices}}} = \sqrt{\frac{F_1F_2}{MN}}$$

3.12 CONSTRUCTION OF PARABOLA

The parabola can be constructed by the following methods:

1. Eccentricity method
2. Offset method
3. Tangent method
4. Oblong method
   a. Rectangle method
   b. Parallelogram method

3.12.1 Eccentricity Method

A parabola is the locus of a point P moving in a plane, such that the ratio of its distance from a fixed point F_1 to the fixed straight line DD' is a constant and is always equal to unity.

$$\text{Eccentricity, } e = \frac{\text{distance of the point from the focus } (PF)}{\text{distance of the point from the directrix } (PM)}$$

$$e = \frac{PF}{PM} = \text{constant} = 1 \text{ (unity)}$$

The eccentricity method for construction of a parabola is based on the above definition.

Example 3.9 (Fig. 3.22)

Draw a parabola when the distance between its focus and directrix is 50 mm. Also draw a tangent and a normal at a point 70 mm from the directrix.

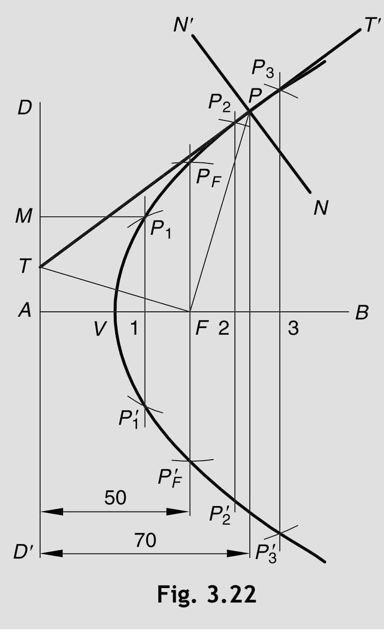


Fig. 3.22

Construction: Fig. 3.22

1. Draw principal axis *AB*.
2. Draw directrix *DD′* perpendicular to *AB*.
3. Mark focus *F* on *AB*, at a distance of 50 mm from *DD′*, i.e., *AF* = 50 mm.
4. Mark vertex *V* at the mid of *AF*. This vertex *V* satisfies the condition for being a point of the parabola. $\left[\because \dfrac{VF}{AV} = 1\right]$.
5. Mark number of points 1, 2, 3, … on *VB*. Through these points, erect lines perpendicular to *AB*.
6. With *F* as the center and radius equal to *A*1, draw arcs to intersect the perpendicular line through 1 at points P_1 and P_1'. It may be noted that $FP_1 = FP_1' = P_1M$. Thus, P_1 and P_1' are the points of the parabola.
7. Similarly, obtain P_2 and P_2', P_3 and P_3', … etc., by drawing arcs with *F* as the centre and *A*2, *A*3, … as radius, to intersect perpendicular lines through 2, 3, ….
8. Join points P_2', P_1', *V*, P_1, P_2, etc., by smooth parabolic curve.

Tangent and Normal to A Parabola

9. Mark a point P on the parabola at a given distance of 70 mm from the directrix.
10. Join *PF*.
11. Draw a line *FT* perpendicular to *PF* meeting *DD′* at point *T*.
12. Join *TP* and extend it to some point *T′*. Line *TT′* is the required tangent.
13. Through point *P*, draw line *NN′* perpendicular to *TT′*. *NN′* is the required normal.

3.12.2 Offset Method

The equation for the parabola is $x^2 = 4ay$. Thus, x^2 is directly proportional to *y*. Therefore, a curve passing through coordinates (0, 0), (1, 1), (2, 4), (3, 9), (4, 16), etc., will generate a parabolic curve. Off-set method is based on this principle.

Example 3.10 (Fig. 3.23)

Draw a parabola using 'offset method' when span and rise are 160 mm and 100 mm respectively.

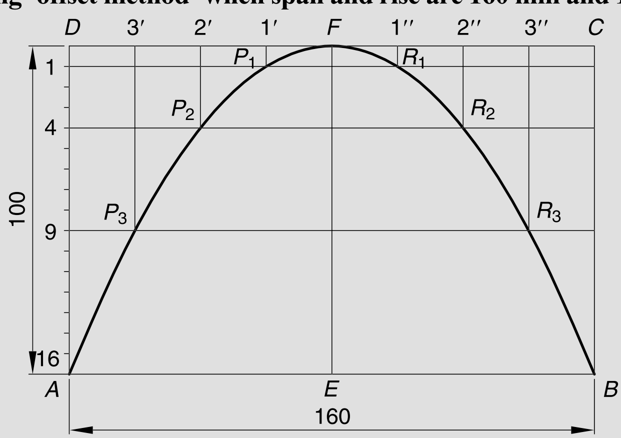


Fig. 3.23

Construction: Fig. 3.23

1. Draw a rectangle *ABCD*, with span *AB* = 160 mm and rise *AD* = 100 mm.
2. Mark *E* and *F* as the mid-points of sides *AB* and *CD* respectively. Join *EF*.
3. Divide *DF* and *CF* into number of equal parts, say 4. Divide line *DA* into square of number of parts made of line *DF,* i.e., $4^2 = 16$. Number the points as shown.
4. Draw vertical lines from points 1′, 2′, 3′ of side *DF* and from points 1″, 2″, 3″ of side *FC*.
5. Draw horizontal lines from points 1, 4, 9 of side *AD* meeting vertical lines at points P_1, P_2, P_3 and R_1, R_2, R_3 respectively.
6. Draw a smooth curve passing through all these points as shown. The curve obtained is the required parabola.

3.12.3 Tangent Method

Example 3.11 (Fig. 3.24)

Construct a parabola given the base 100 mm and height 80 mm by Tangent method.
[RGPV Dec 2005]

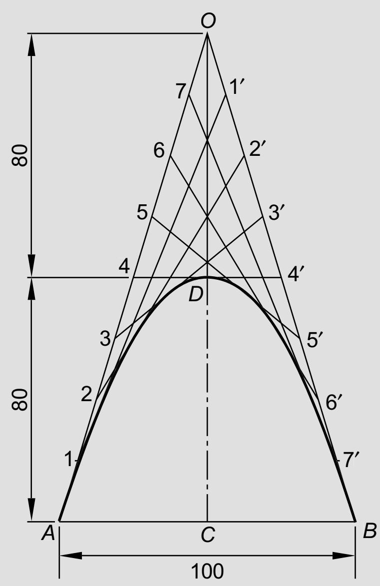


Fig. 3.24

Construction: Fig. 3.24

1. Draw *AB* = 100 mm and mark the midpoint *C*.
2. Draw *CD* = *DO* = 80 mm perpendicular *AB*.
3. Join *OA* and *OB*.
4. Divide lines *OA* and *OB* into same number of equal parts, say 8. Mark divisions of side *AO* as 1, 2, 3, …etc., and side *OB* as 1′, 2′, 3′,… etc. as shown.

5. Join 11′, 22′, 33′, … etc. These lines are tangents to the parabola.
6. Draw a smooth curve starting from point *A* touching the lines 11′, 22′, 33′, … etc tangentially ending at point *B*. The curve is the required parabola.

3.12.4 Rectangle Method

Rectangle method is basically a method of inscribing parabola in a rectangle.

Example 3.12 (Fig. 3.25)

Draw a parabola given the width and height of its enclosing rectangle as 105 mm and 75 mm respectively. [RGPV June 2007]

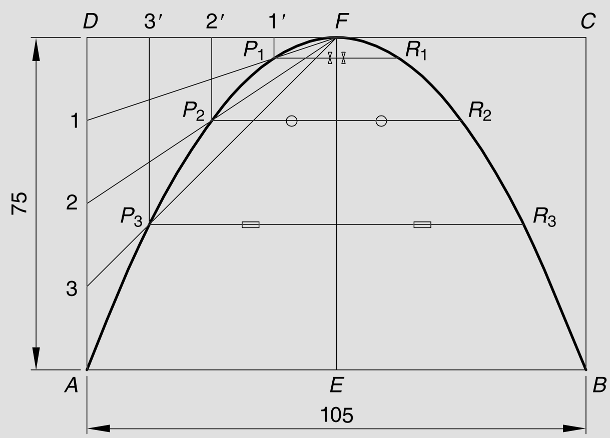


Fig. 3.25

Construction: Fig. 3.25

1. Draw a rectangle *ABCD* taking *AB* = 105 mm and *AD* = 75 mm.
2. Mark *E* and *F* as the mid-points of sides *AB* and *CD* respectively. Join *EF* to represent the axis.
3. Divide lines *FD* and *DA*, into equal number of parts, say 4. Mark division of side *DA* as 1, 2, 3 and divisions of *FD* as 1′, 2′, 3′.
4. Join *F* with points 1, 2, 3.
5. Through 1′, 2′, 3′ draw lines parallel to the axis *EF* to meeting lines *F*1, *F*2, *F*3 at points P_1, P_2, P_3 respectively.
6. As the curve is symmetric about axis, obtain points R_1, R_2, R_3 of the curve by drawing horizontal lines through points P_1, P_2, P_3 and making them equal on the other side of axis *EF*.
7. Draw a smooth curve passing through *A*, P_3, P_2, P_1, *F*, R_1, R_2, R_3 and *B*. This curve is the required parabola.

3.12.5 Parallelogram Method

Parallelogram method is basically a method of inscribing a parabola in a parallelogram.

Example 3.13 (Fig. 3.26)

Inscribe a parabola in a parallelogram of 110 mm × 80 mm sides, the included angle being 60°. Consider the longer side of the parallelogram as the base of parabola.

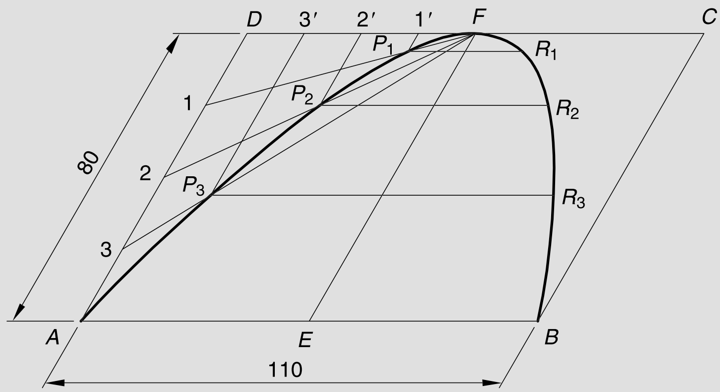


Fig. 3.26

Construction: Fig. 3.26

1. Draw a parallelogram *ABCD*, *AB* = 120 mm and *AD* = 80 mm. Let ∠*DAB* = 60°.
2. Mark *E* and *F* as the mid-points of *AB* and *CD* respectively.
3. Divide lines, *FD* and *DA*, into same number of equal parts, say 4. Mark divisions of *DA* as 1, 2, 3 and divisions of *FD* as 1′, 2′, 3′.
4. Connect point *F* with points 1, 2, 3.
5. Through 1′, 2′, 3′ draw lines parallel to axis *EF* to meet lines *F*1, *F*2, *F*3 at points P_1, P_2, P_3 respectively.
6. Draw a curve through *F*, P_1, P_2, P_3, *A*. This is one-half of the parabola.
7. Draw horizontal lines through points P_1, P_2, P_3. Make their distances equal on either side of *EF* and obtain points R_1, R_2, R_3 of the curve.
8. Draw a curve to pass through points *F*, R_1, R_2, R_3, *B*. This is other half of the parabola.

3.13 AXIS OF THE PARABOLA

Example 3.14 (Fig. 3.27)

Determine the axis of the given parabola.

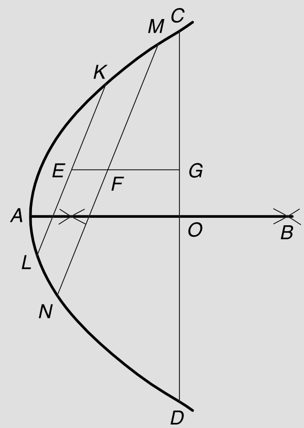


Fig. 3.27

Construction: Fig. 3.27

1. Draw two parallel chords *KL* and *MN* across the parabola, separated by some distance.
2. Locate *E* and *F* as the mid-points of the chords *KL* and *MN* respectively.
3. Join *EF* and produce it to some length.
4. Draw a line perpendicular to *EF* at any convenient distance to meet the parabola at points *C* and *D*. The line *CD* is a new chord to the parabola.
5. Locate *O* as the mid-point of chord *CD*.
6. Draw a line *AB* through *O* parallel to *EF*. This line *AB* is the required axis of the parabola.

3.14 FOCUS AND DIRECTRIX OF THE PARABOLA

Example 3.15 (Fig. 3.28)

Determine focus and directrix of the given parabola.

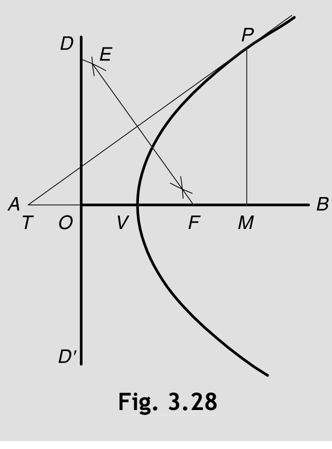


Fig. 3.28

Construction: Fig. 3.28

1. Determine axis *AB* of the given parabola (refer Example 3.14). Let *V* be the vertex.
2. Mark a point *P*, anywhere on the parabola.
3. From point *P*, draw a line *PM* perpendicular to *AB* to meet at point *M*.
4. Mark a point *T* on *AB* produced such that *TV* = *VM*.
5. Join *PT*.
6. Draw a perpendicular bisector *EF* of line *PT*, to meet axis at point *F*. The point *F* is the required focus of the parabola.
7. Mark a point *O* on the axis such that *VF* = *OV*.
8. Draw a line *DD*′ through *O* perpendicular to the axis *AB*. This line *DD*′ is the required directrix of the parabola.

3.15 TANGENT AND NORMAL TO THE PARABOLA

Example 3.16 (Fig. 3.29)

Draw a tangent and a normal to the given parabola through a point *P*, lying on it.

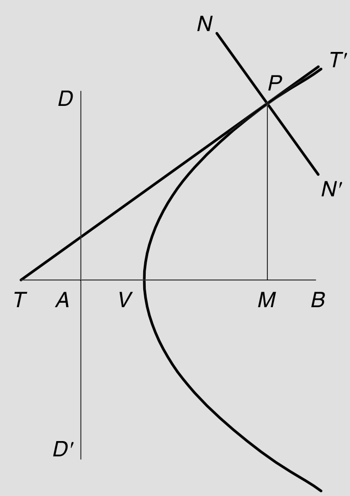


Fig. 3.29

Construction: Fig. 3.29

1. Determine axis *AB* of the parabola (refer Example 3.14). Let *V* be the vertex.
2. Mark a point *P* on the parabola where tangent is to be drawn.

3. Through point *P*, draw a line *PM* perpendicular to *AB*.
4. Mark a point *T* on the axis *AB* produced such that *TV* = *VM*.
5. Join *TP* and extend it to a distance *T′*. The line *TT′* is the tangent to the parabola.
6. Draw line *NN′* as the perpendicular bisector of *TT′*. The line *NN′* is the required normal to the parabola.

Example 3.17 (Fig. 3.30)

Draw a pair of tangents to the given parabola from an external point *P*.

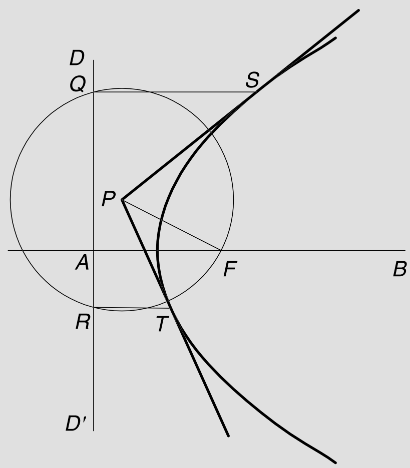


Fig. 3.30

Construction: Fig. 3.30

1. Determine axis *AB* of the parabola (refer Example 3.14).
2. Determine the focus *F* and the directrix *DD′* of the parabola (refer Example 3.15).
3. Mark point *P* through which tangents are to be drawn.
4. Draw a circle with point *P* as the center and radius *PF*. Let the circle meet the directrix *DD′* at points *Q* and *R*.
5. Draw horizontal lines through points *Q* and *R* to meet the parabola at points *S* and *T*, respectively.
6. Join *PS* and *PT* and extend them. The lines *PS* and *PT* are the required tangents of the parabola.

3.16 CONSTRUCTION OF HYPERBOLA

The hyperbola can be constructed by the following methods:

1. Eccentricity method
2. Intersecting arcs method
3. Oblong method
4. Intercept method
5. Orthogonal asymptotes method
6. Oblique asymptotes method

3.16.1 Eccentricity Method

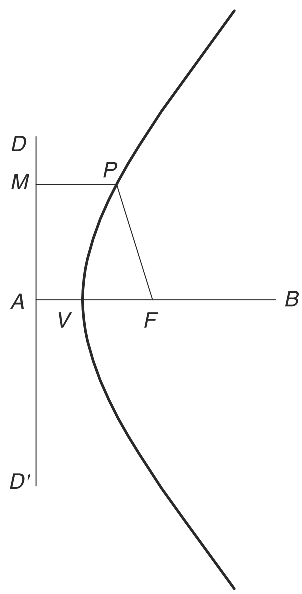


Fig. 3.31

Hyperbola is the locus of a point *P* moving in a plane, such that the ratio of its distance from a fixed point *F* to the fixed straight line *DD′* is a constant and is always greater than unity.

In Fig. 3.31,

F = Focus V = Vertex
DD' = Directrix AB = Principal axis

$$\text{Eccentricity, } e = \frac{\text{distance of the point from the focus } (PF)}{\text{distance of the point from the directrix } (PM)}$$

$$e = \frac{PF}{PM} = \text{constant and } e > 1$$

The eccentricity method for construction of hyperbola is based on the above definition.

Example 3.18 (Fig. 3.32)

Construct a hyperbola with the distance between the focus and the directrix as 50 mm and eccentricity as 3/2. Also draw normal and tangent to the curve at a point, 25 mm from the axis. [RGPV Aug. 2010]

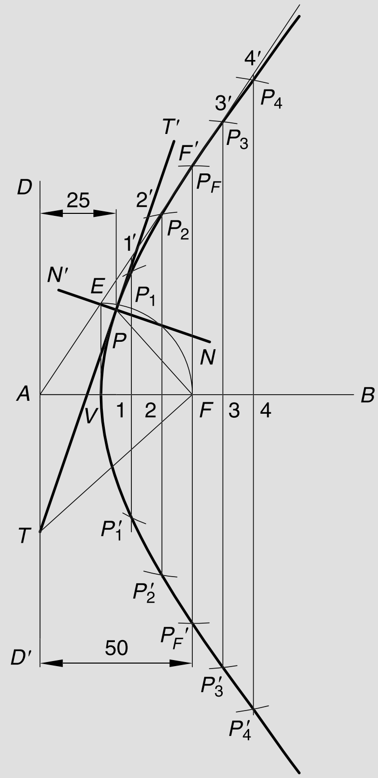


Fig. 3.32

Construction: Fig. 3.32

1. Draw directrix DD' and principal axis AB perpendicular to the DD'.
2. Mark focus F taking $AF = 50$ mm.
3. Divide the line AF into five (3 + 2) equal parts (since $e = 3/2$) and mark vertex V on it such that $\frac{VF}{AV} = \frac{3}{2}$. Thus, vertex V satisfies the condition for being a point of the curve.
4. At V, draw a vertical line VE equal to VF. Join AE and extend it to some distance. Thus, in the triangle AVE, $\frac{VE}{AV} = \frac{VF}{AV} = \frac{3}{2}$
5. Mark any point 1 on the axis and through it, draw a perpendicular line to meet AE produced at 1′. Thus, $\frac{11'}{A1} = \frac{VE}{AV} = \frac{3}{2}$

6. With center F and radius equal to 11′, draw arcs to intersect the perpendicular line 11′ at point P_1 and P_1'. These are the points of the hyperbola because ratio, $\frac{11'}{AV} = \frac{3}{2}$
7. Similarly, mark any number of points 2, 3, 4, … on VB at any convenient distances which need not be equal. Through these points erect lines 22′, 33′, 44′, …, perpendicular to principal axis AB. With F as the center and radius equal to 22′, 33′, 44′, …, draw arcs to intersect the perpendicular line 22′, 33’, 44′, … at points P_2 and P_2', P_3 and P_3’, … etc. respectively.
8. Join points P_2', P_1', V, P_1, P_2, etc., to form a smooth curve. This is the required hyperbola.

Tangent And Normal to A Hyperbola

9. Mark a point P on the hyperbola at a given distance, 30 mm from the directrix.
10. Join PF.
11. Draw a line FT perpendicular to PF meeting directrix at T.
12. Join TP and extend it to some point T'. The line TT' is the required tangent.
13. Through point P, draw a line NN' perpendicular to TT'. The line NN' is the required normal.

3.16.2 Intersecting Arcs Method

The hyperbola is a curve traced by a point P moving in such a way that the difference of its distance from two fixed points, F_1 and F_2 is always constant and equal to the distance between the vertices of the two branches of the hyperbola.

In Fig. 3.33., let F_1 and F_2 be the foci, V_1 and V_2 be the vertices. Then for hyperbola the locus of any point P should satisfy $|PF_2 - PF_1| = V_1V_2$. The distance between the vertices V_1V_2 is also known as transverse axis or major axis. Intersecting arc method is based on this definition.

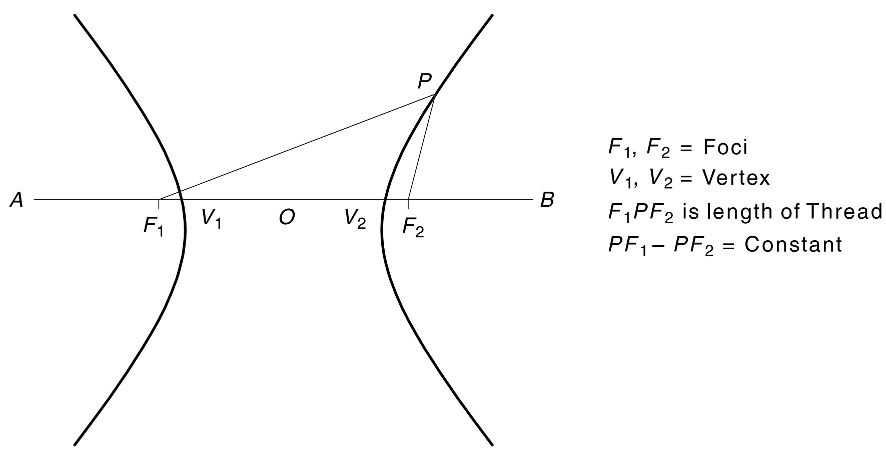


Fig. 3.33

Example 3.19 (Fig. 3.34)

Trace the locus of a point, such that the difference between the distances of the point from the two fixed points 80 mm apart is constant which is equal to 60 mm. Name the curve. [RGPV Dec 2001]

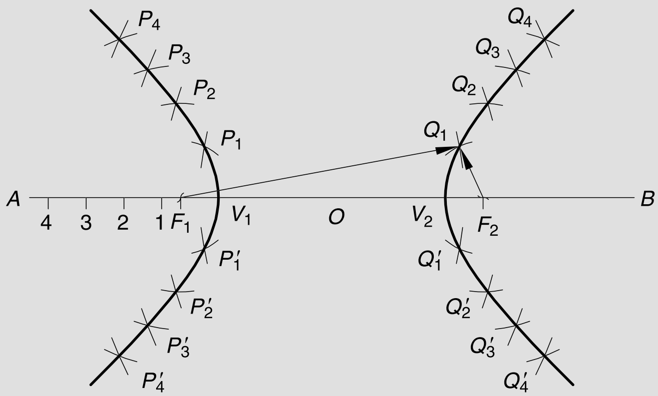


Fig. 3.34

Construction: Fig. 3.34

1. Draw the principal axis *AB* and mark a point *O* on it.
2. Mark foci F_1, F_2 and vertices V_1, V_2 on *AB* so that $F_1F_2 = 80$ mm, $V_1V_2 = 60$ mm and are symmetry about *O*.
3. Mark points 1, 2, 3, … on *AB* on one side of foci, at any convenient distances which need not be equal.
4. With centre F_1 and radius $V_1$1 draw arcs F_1P_1 and F_1P_2. Also with centre F_2 and radius $V_1$1 draw arcs F_2Q_1 and F_2Q_2.
5. Again with centre F_1 and radius $V_2$1 draw arcs F_1Q_1 and F_1Q_2. Also with centre F_2 and radius $V_2$1 draw arcs F_2P_1 and F_2P_2.
6. Repeat step 4 and step 5 with the remaining points 2, 3 and 4 and obtain P_2, P_2', Q_2, Q_2', P_3, …. etc.
7. Draw a smooth curve passing through all the points as shown. This gives required two branches of hyperbola.

3.16.3 Oblong Method

Oblong method is used to draw hyperbola when transverse axis, double ordinate and abscissa are given.

Transverse Axis A hyperbola has two vertices. The line joining the two vertices is called the major axis or the transverse axis.

Double Ordinate Any chord which is perpendicular to the axis is called the double ordinate. The chord which is perpendicular to axis and passes through the focus is called the latus rectum.

Abscissa The distance between the vertex and the double ordinate is called the abscissa. It may be noted that a parabola or a hyperbola has unlimited number of double ordinates and for each double ordinate there is an abscissa.

Asymptotes Asymptotes are the straight lines that pass through the centre of the transverse axis and tangential to the hyperbola at infinity. They approach nearer and nearer to the hyperbola while moving away from the centre and assumed to touch the hyperbola at infinity.

When the asymptotes to the hyperbola intersect at right angles, the curve is known as rectangular hyperbola and its eccentricity is $\sqrt{2}$.

Example 3.20 (Fig. 3.35)

Draw a hyperbola when half the transverse axis, double ordinate and abscissa are 50 mm, 120 mm and 40 mm long respectively.

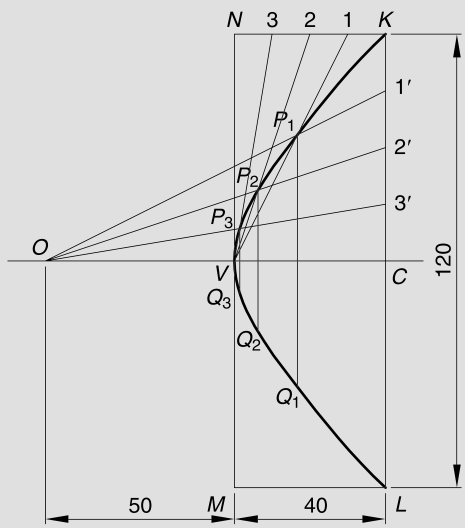


Fig. 3.35

Construction: Fig. 3.35

1. Draw a rectangle *KLMN* such that *KL* = 120 mm (double ordinate) and *LM* = 40 mm.
2. Mark *C* and *V* as the mid-points of *KL* and *NM* respectively.
3. Join *CV* and extend it to point *O* such that *VO* = 50 mm (half the transverse axis).
4. Divide *KN* and *KC* into 4 equal parts. Name the points on line *KN* as 1, 2, 3 and on line *KC* as 1′, 2′, 3′.
5. Join point *V* with points 1, 2, 3 and join point *O* with 1′, 2′, 3′.
6. Locate P_1, P_2, P_3 at the intersection of *V*1 and *O*1′, *V*2 and *O*2′, *V*3 and *O*3′, respectively.

7. Locate Q_1, Q_2, Q_3 such that their distance from VC is equal to the distance of points P_1, P_2, P_3 with VC.
8. Join K, P_1, P_2, P_3, V, Q_1, Q_2, Q_3, L and obtain the required hyperbola.

3.16.4 Intercept Method

In a rectangular hyperbola if the chord of the hyperbola is extended to intersect the axes, the intercept between the curves and the axes are equal. This principle is used to draw hyperbola using intercept method.

Example 3.21 (Fig. 3.36)

Draw a rectangular hyperbola when the position of a point *P* on the curve is at a distance of 30 mm and 50 mm from two asymptotes.

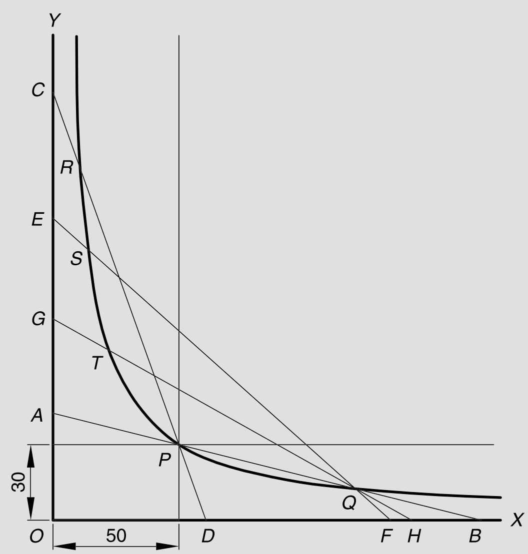


Fig. 3.36

Construction: Fig. 3.36

1. Draw asymptotes OX and OY at right angle to each other.
2. Mark a point P such that its distance from OX is 30 mm and from OY is 50 mm.
3. Draw a line passing from point P to intersect the asymptotes at points A and B. Locate point Q of the hyperbola on line AB, such that $PA = BQ$.
4. Similarly, draw another line from point P to intersect the asymptotes at C and D. Locate point R of the hyperbola on line CD, such that $PD = CR$.
5. Draw a line passing from point Q of the hyperbola to intersect the asymptotes at points E and F. Locate point S of the hyperbola on line EF, such that $QF = ES$.

6. Similarly, draw another line from point Q to intersect the asymptotes at G and H. Locate point T of the hyperbola on line GH, such that $QH = GT$.
7. Proceed to mark sufficient number of points of the hyperbola. Draw a smooth curve passing through the points P, Q, R, S, T, etc., which is the required rectangular hyperbola.

3.16.5 Orthogonal Asymptotes Method

The ‘orthogonal asymptotes method’ is used to draw rectangular hyperbolas. The included angle between asymptotes is 90º.

Example 3.22 (Fig. 3.37)

Draw a rectangular hyperbola using the ‘orthogonal asymptotes’ method when the position of a point *P* on the curve is at a distance of 35 mm and 50 mm from two asymptotes.

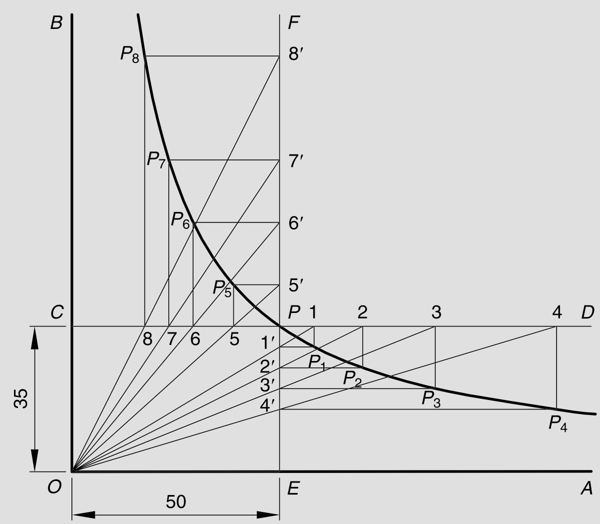


Fig. 3.37

Construction: Fig. 3.37

1. Draw asymptotes OA and OB at right angle to each other.
2. Mark a point P such that its distance from OA is 35 mm and from OB is 50 mm.
3. Through point P, draw lines CD and EF parallel to OA and OB respectively.
4. Mark points 1, 2, 3, …etc. along CD which need not be equidistant.
5. Join $O1$, $O2$, $O3$, … etc., and extend them, if necessary, to meet the line EF at points 1′, 2′, 3′, … etc.
6. Through 1, 2, 3, … etc., draw lines parallel to OB and through 1′, 2′, 3′, … etc parallel to OA. Let the lines intersect each other at points P_1, P_2, P_3, …etc., respectively.
7. Draw a smooth curve passing through points P_1, P_2, P_3, … etc. This curve is the required rectangular hyperbola.

3.16.6 Oblique Asymptotes Method

The 'oblique asymptotes' method is used to draw hyperbolas when included angle between asymptotes is either acute or obtuse.

Example 3.23 (Fig. 3.38)

Draw a hyperbola when its asymptotes are inclined at 60° to each other and it passes through a point *P* at a distance of 40 mm and 50 mm from the asymptotes.

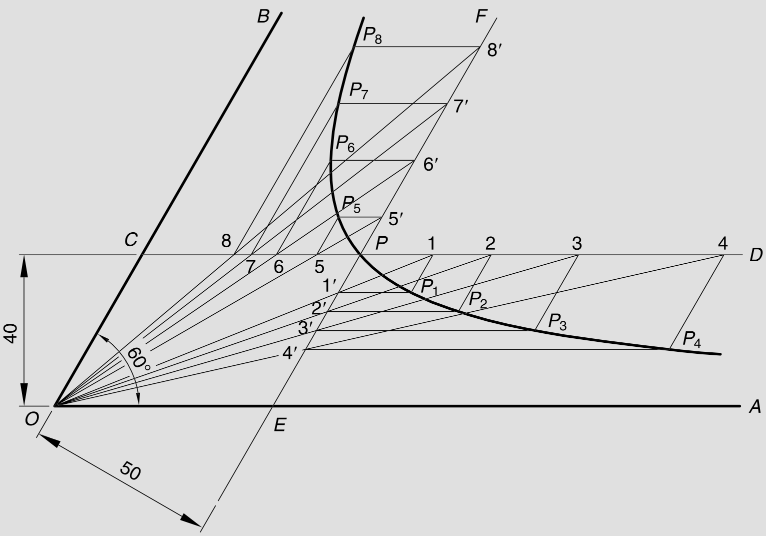


Fig. 3.38

Construction: Fig. 3.38

1. Draw asymptotes *OA* and *OB* with an included angle of 60°.
2. Mark a point *P* such that its distance from *OA* is 40 mm and from *OB* is 50 mm.
3. Through point *P*, draw lines *CD* and *EF* parallel to asymptotes *OA* and *OB* respectively.
4. Mark points 1, 2, 3, … etc along *CD* which need not be equidistant and lying on both sides of point *P*. It is advisable to mark points 1, 2, 3, at distances in increasing order.
5. Join *O*1, *O*2, *O*3, …etc and extend them, if necessary, until they meet the line *EF* at points 1′, 2′, 3′, …etc.
6. Through 1, 2, 3, …etc., draw lines parallel to *OB* and through 1′, 2′, 3′, …etc., draw lines parallel to *OA*. Let them intersect at points P_1, P_2, P_3, …etc. respectively.
7. Draw a smooth curve passing through points P_1, P_2, P_3,…etc. The obtained curve is the required hyperbola.

3.17 LOCATE ASYMPTOTES AND DIRECTRIX OF THE HYPERBOLA

Example 3.24 (Fig. 3.39)

Draw two branches of hyperbola keeping the distances between the foci as 70 mm and distances between vertices as 40 mm. Locate asymptotes and measure the angle between them. Also draw the directrix of the hyperbola.

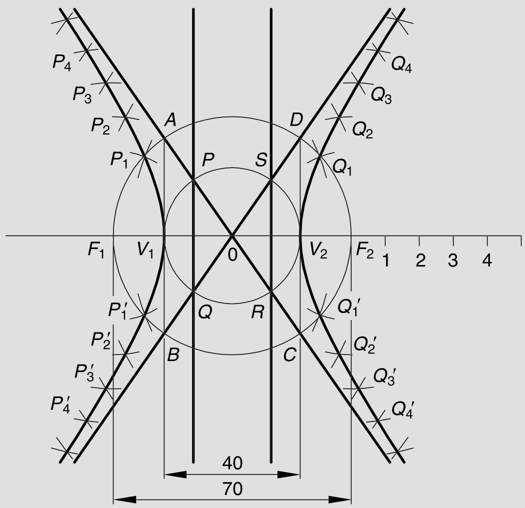


Fig. 3.39

Construction: Fig. 3.39

1. Draw hyperbola using 'Intersecting Arcs method' (refer Example 3.19).
2. With centre O and diameter $F_1 F_2$ draw a circle.
3. At V_1 and V_2 draw two vertical lines cutting the circle at points $ABCD$ as shown.
4. Join AC and BD. Extend each of them on both sides. These are the required asymptotes. Measure the angle between them.
5. With O as the center and V_1V_2 as the diameter draw a circle intersecting asymptotes at points $PQRS$.
6. Join PQ and RS. Extend each of them on both sides. These are the required directrices.

3.18 TANGENT AND NORMAL TO THE HYPERBOLA

Example 3.25 (Fig. 3.40)

Draw a tangent and a normal to the given hyperbola at a point P, when distance between foci is known.

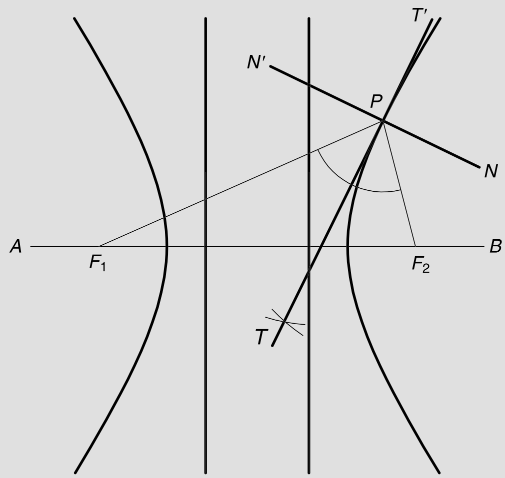


Fig. 3.40

Construction: Fig. 3.40

1. Let AB be the axis and F_1 and F_2 be the foci of the given hyperbola.
2. Mark a point P at the given location on the hyperbola.
3. Join PF_1 and PF_2.
4. Draw TT' as the bisector of $\angle F_1PF_2$. This is the required tangent.
5. Through point P, draw a line NN' perpendicular to TT'. The line NN' is the required normal.

3.19 EMPIRICAL RELATIONS

ANALYTICAL: Refer Fig. 3.33. Obtain the following analytical relations

Eccentricity is defined as, $e = \dfrac{\text{distance of a point from the focus}}{\text{distance of the point from the directrix}}$

1. Eccentricity may also be written as $e = \dfrac{\text{distance between the foci}}{\text{distance between vertices}} = \dfrac{F_1F_2}{V_1V_2}$
2. Eccentricity may also be written as $e = \dfrac{\text{distance between vertices}}{\text{distance between directrices}}$
3. Eccentricity may also be given by

$$e^2 = \frac{\text{distance between foci}}{\text{distance between vertices}} \times \frac{\text{distance between vertices}}{\text{distance between directrices}}$$

$$\text{or } e = \sqrt{\frac{\text{distance between foci}}{\text{distance between directrices}}}$$

3.20 MISCELLANEOUS EXAMPLES

Example 3.26 (Fig. 3.41)

The major and minor axes of an ellipse are 140 mm and 90 mm respectively. Find the foci and draw the ellipse using 'arcs of circle' method. Draw a tangent and a normal to the ellipse at a point 40 mm above the major axis.

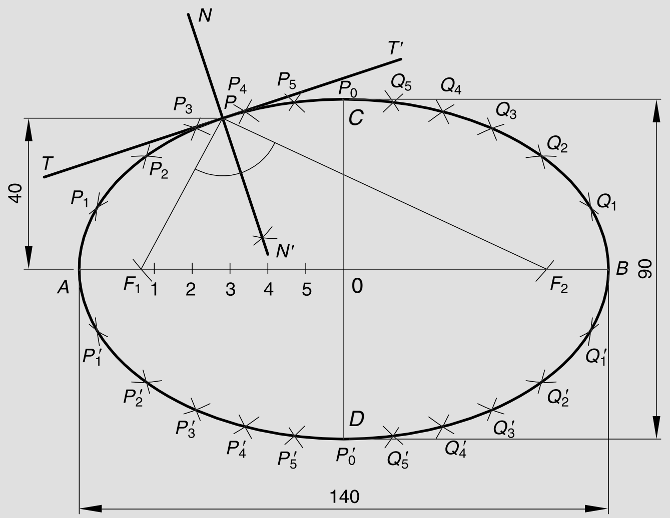


Fig. 3.41

Construction: Fig. 3.41

1. Draw the major axis $AB = 140$ mm and minor axis $CD = 90$ mm bisecting each other at O.
2. Draw an arc with centre C and radius OA to meet line AB at points F_1 and F_2. Points F_1 and F_2 are the foci.
3. Draw the ellipse using 'arcs of circle' method (refer Example 3.3).
4. Locate point P, 40 mm above AB and draw tangent and normal to the ellipse (refer Example 3.7).

Example 3.27 (Fig. 3.42)

Construct an ellipse having a major axis 100 mm and minor axis 80 mm. Locate its foci, directrices and find the eccentricity. [RGPV Sep. 2009]

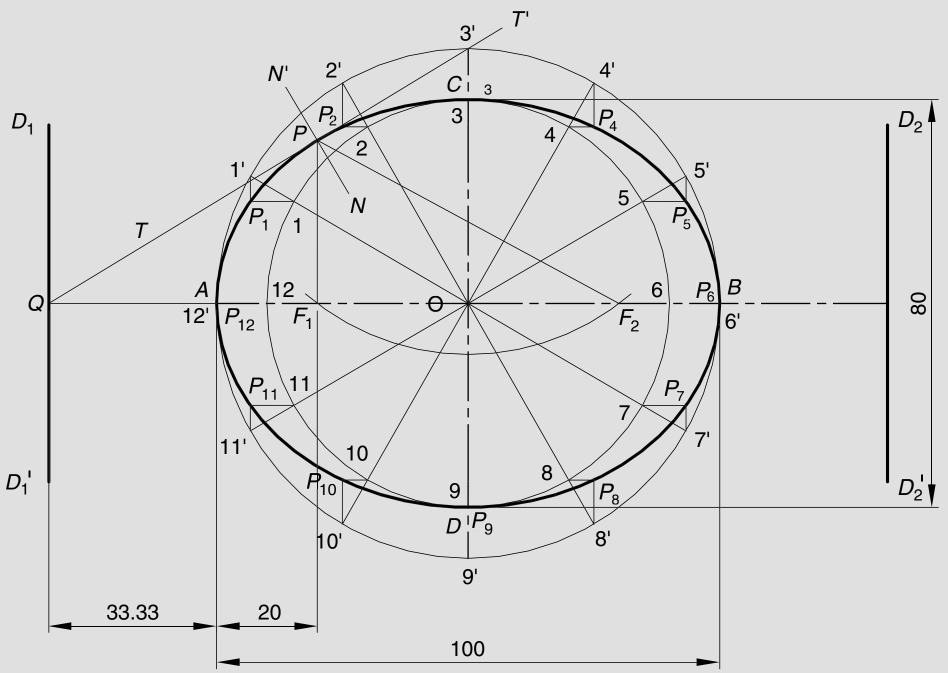


Fig. 3.42

Construction: Fig. 3.42

1. Draw the ellipse using 'concentric circles' method (refer Example 3.3).
2. Draw an arc with point C as the centre and radius OA to cut the major axis AB at points F_1 and F_2. Points F_1 and F_2 represents the foci of the ellipse.
3. Through focus F_1, draw a line F_1P perpendicular to the axis AB to meet the ellipse at a point P.
4. Draw a tangent TT' to the ellipse through point P (refer Example 3.7).
5. Let the tangent meet the axis AB produced at point Q. Through point Q, draw a line D_1D_1' perpendicular to the axis AB. The line D_1D_1' represents the directrix.
6. Draw a line D_2D_2' parallel to D_1D_1' at distance equal to OQ on the other side of ellipse. D_2D_2' is the second directrix.
7. Determine the ratio of AF_1 and AQ as eccentricity. Here, $e = \dfrac{AF_1}{AQ} = \dfrac{20}{33.33} = 0.6.$

Example 3.28 (Fig. 3.43)

The major axis of an ellipse is 110 mm long and the foci are at a distance of 15 mm from its ends. Draw the ellipse, one-half of it by 'concentric circles' method and the other half by rectangle method. Determine the eccentricity of the ellipse.

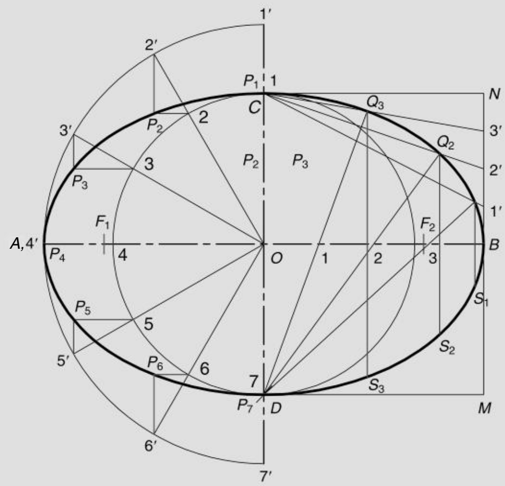


Fig. 3.43

Construction: Fig. 3.43

1. Draw a horizontal line and mark the vertices *A* and *B*, 110 mm apart. Also mark foci F_1 and F_2, 80 mm apart. Let point *O* be the midpoint of both *AB* and F_1F_2.
2. Draw an arc with focus F_1 as the centre and radius *OA* to meet the vertical line passing through *O* at points *C* and *D*. Join *CD* to represent the minor axis.
3. Draw left half of the ellipse using 'concentric circles' method (refer Example 3.3).
4. Draw a rectangle *CDMN* and inscribe other half of the ellipse using rectangle method (refer Example 3.4).
5. Determine the eccentricity using the relation

$$e = \frac{\text{distance between foci}}{\text{length of major axis}} = \frac{F_1F_2}{AB} = \frac{110 - 2\times 15}{110} = \frac{8}{11}$$

Example 3.29 (Fig. 3.44)

Draw an ellipse passing through points *A*, *B* and *C* of a triangle *ABC* having length of sides as 100 mm, 75 mm and 50 mm. Also draw a curve parallel to the ellipse and 25 mm away from it.

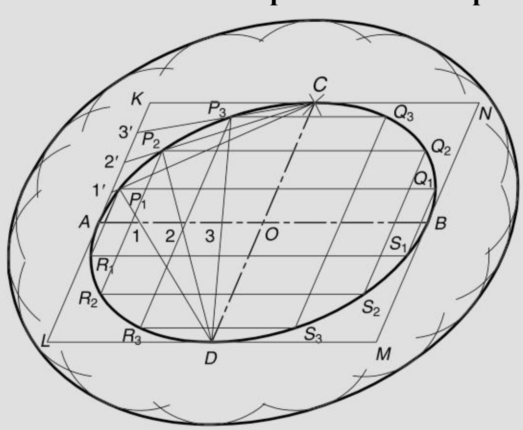


Fig. 3.44

Construction: Fig. 3.44

1. Draw a triangle *ABC* taking *AB* = 100 mm, *AC* = 75 mm and *BC* = 50 mm.
2. Mark *O* as the midpoint of *AB*. Join *CO* and extend it to *D* taking *OC* = *OD*.
3. Draw a parallelogram *KLMN* as shown and inscribe the ellipse (refer Example 3.5)
4. Through points p_1, p_2 p_3, etc., of the ellipse, draw arcs of 25 mm. Pass a curve touching the arcs, tangentially.

Example 3.30 (Fig. 3.45)

Inscribe a parabola in a rectangle having a base 80 mm and axis 60 mm. Draw a tangent to the curve at a point 30 mm from the base. Also locate the position of the focus and directrix of the parabola. [RGPV Jun. 2011]

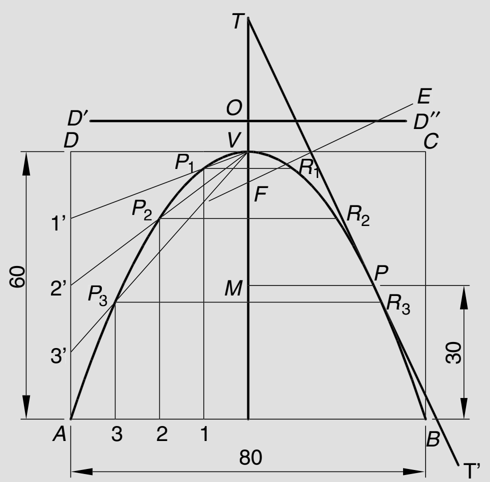


Fig. 3.45

Construction: Fig. 3.45

1. Draw a rectangle of base 80 mm and height 60 mm. Inscribe a parabola in it (refer Example 3.12).
2. Mark a point *P*, 30 mm above *AB*. Draw a tangent at point *P* (refer Example 3.16)
3. Proceed to determine the focus and the directrix of the parabola (refer Example 3.15).

Example 3.31 (Fig. 3.46)

A shot is discharged from the ground at an inclination of 45º to the ground which is horizontal. The shot returns to the ground at a point 120 m away from the point of discharge. Draw the path traced by the shot. Find the direction of the shot after it has traveled a horizontal distance of 100 m.

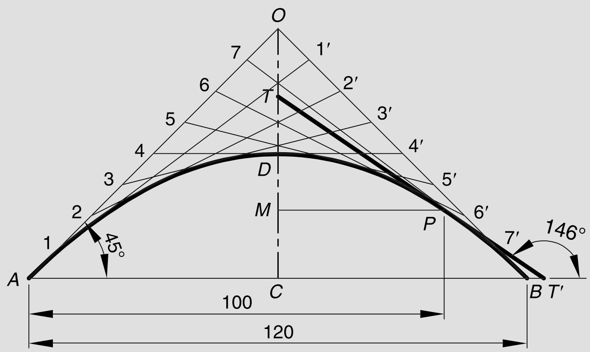


Fig. 3.46

Construction: Fig. 3.46

1. Draw an isosceles triangle *ABO* with *AB* = 120 mm and ∠*OAB* = ∠*OBA* = 45°.
2. Mark *C* as the midpoint of *AB*. Join *OC*.
3. Using tangent method to draw a parabola with *AB* as the base and half of *OC* as the altitude (refer Example 3.11).
4. Mark point *P* on the parabola at a horizontal distance of 100 m from point *A*.
5. Through *P*, draw *PM* perpendicular to *OC* to meet at *M*. Mark point *T* on the axis such that *TD* = *DM*. Join *TP* and extend it to a distance *T′*. This *TT′* is the tangent.
6. Measure angle made by *TT′* with *AB* as the direction of the shot at *P*.

Example 3.32 (Fig. 3.47)

Two points *A* and *B* are 110 mm apart. Point *C* is 90 mm and 60 mm from points *A* and *B* respectively. Draw a parabola passing through points *A*, *B* and *C*.

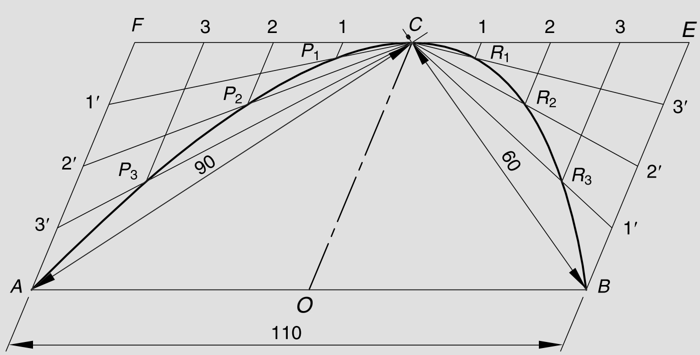


Fig. 3.47

Construction: Fig. 3.47

1. Draw a triangle *ABC*.
2. Mark *O* as its midpoint of *AB*. Join *OC*.
3. Draw a parallelogram *ABEF* and then draw a parabola by parallelogram method (refer Example 3.13).

Example 3.33 (Fig. 3.48)

A ball is thrown up in air from a 6 m high building and its highest point of flight it just crosses a 12 m high palm tree . Trace the path of the ball, if the distance between the building and the palm tree is 3 m. Choose a suitable scale. [RGPV June 2005]

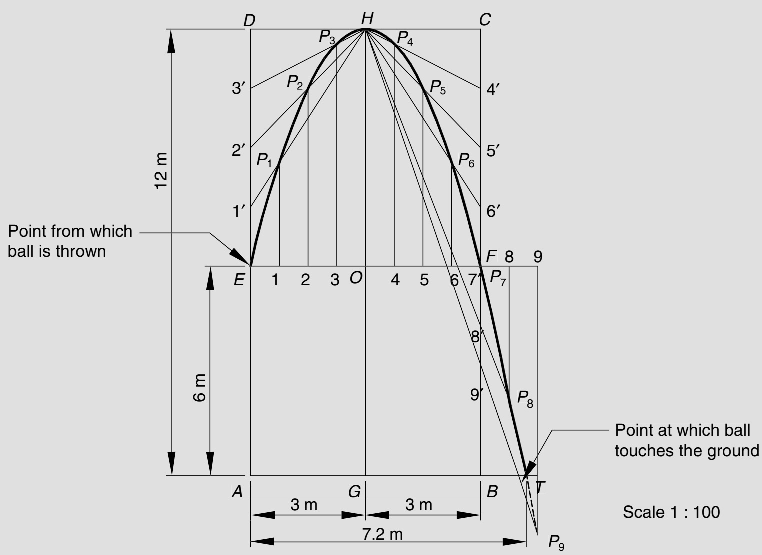


Fig. 3.48

Construction: Fig. 3.48

1. Select a scale 1 cm = 1 m.
2. Draw a rectangle *ABCD* of 6 m × 12 m. Mark point *E* at a height of 6 m above point *A*. The line *AB* represents the ground, line *AE* represents the building and point *E* represents the location through which the ball is thrown.
3. Mark points *G* and *H* as the mid-points of lines *AB* and *CD* respectively. Join *GH* to represent the palm tree.
4. Draw a parabola in rectangle *EFCD* (refer Example 3.12). It may be noted that here sides *ED*, *EO*, *FO* and *FC* are divided into four equal parts.
5. Mark points 8, 9, etc., on line *EF* produced such that divisions *F*8 and 8–9 are equal to *E*1. Similarly, mark 8′ and 9′ on *CF* produced such that 7′8′ and 8′9′ are equal to *C*4′.
6. Join *H*8′ and *H*9′ and extend them to meet vertical lines from points 8 and 9 at points as P_8 and P_9, respectively.

7. Extend the parabola to pass through points P_8 and P_9.
8. Locate point T of the parabola on AB produced. This is the point where ball will touch the ground.

Example 3.34 (Fig. 3.49)

The directrices of a hyperbola are 50 mm apart and the vertices are 70 mm apart. Locate the asymptotes and foci graphically and construct two branches of the hyperbola.

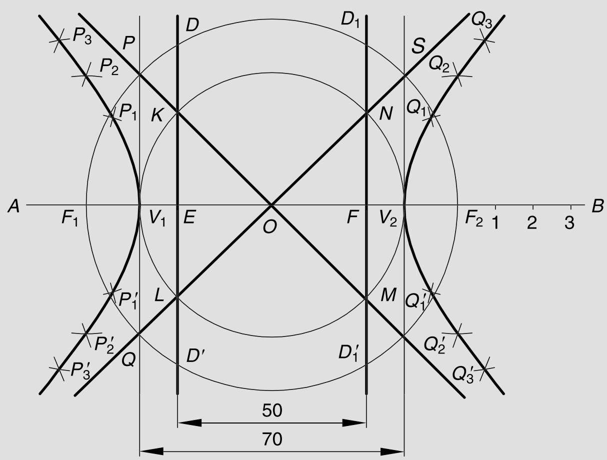


Fig. 3.49

Construction: Fig. 3.49

1. Draw principal axis AB and mark point O on it.
2. Mark points V_1 and V_2 on AB as the vertices, 70 mm apart and equidistant from O.
3. Mark points E and F on AB, 50 mm apart and equidistant from O.
4. Through E and F, erect lines DD' and D_1D_1' perpendicular to AB as the directrices.
5. With center O and diameter V_1V_2, draw a circle to cut the directrices DD' and D_1D_1' at points K, L, M and N.
6. Join KM and LN and extend them on both sides. These are the required asymptotes.
7. At points V_1 and V_2, draw vertical lines to cut the asymptotes at points P, Q, R and S.
8. With center O and radius OP, draw a circle to meet AB at points F_1 and F_2. These are the required foci.
9. Draw two branches of the hyperbola using to meet 'intersecting arcs' method (Example 3.19).

Example 3.35 (Fig. 3.50)

The foci of a rectangular hyperbola are 70 mm apart. Draw two branches of the hyperbola and mark its vertices and directrices.

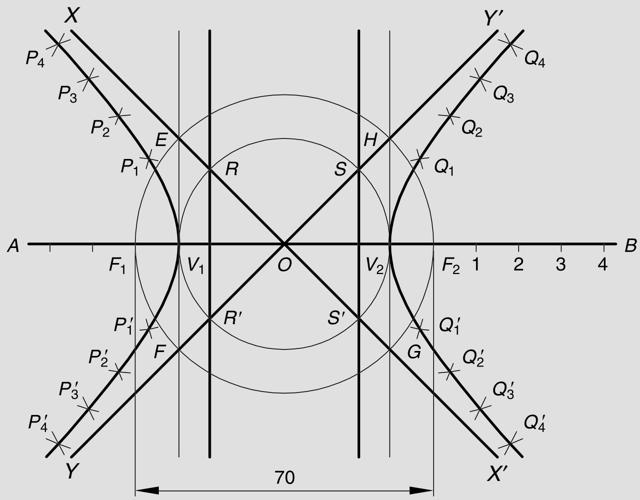


Fig. 3.50

Construction: Fig. 3.50

1. Draw principal axis *AB* and mark *O* on it.
2. Draw two asymptotes *XX′* and *YY′* at 45^0 to *AB*.
3. Mark F_1 and F_2 on *AB* taking $OF_1 = OF_2 = 35$ mm.
4. With centre *O* and diameter F_1F_2, draw a circle to cut the asymptotes at points *E*, *F*, *G* and *H*.
5. Join *EF* and *GH* to cut the principal axis at points V_1 and V_2. These are the required vertices.
6. With center *O* and diameter V_1V_2 draw a circle to cut the asymptotes at point *R*, *R′*, *S* and *S′*.
7. Join *RR′* and *SS′* and extend them. These are the required directrices.
8. Draw the hyperbola using ‘intersecting arcs method’ (refer Example 3.19).

EXERCISE 3

Ellipse

1. Draw the ellipse when the distance of its focus from its directrix is equal to 60 mm and eccentricity is 3/5. Also draw a tangent and a normal to the ellipse at a point 100 mm away from the directrix.
2. Draw an ellipse when the distance of its vertex from its directrix is 24 mm and distance of its focus from directrix is 42 mm.
3. The major axis of an ellipse is 150 mm and minor axis 100 mm. Find the foci and draw the ellipse by arcs of circles method. **[RGPV April 2009]**
4. The major axis of an ellipse is 90 mm and the minor axis is 60 mm. Find the foci and draw the ellipse by ‘arcs of circles’ method. **[RGPV June 2009]**

5. The major axis of an ellipse is 120 mm long and the minor axis is 80 mm long. Draw the ellipse by arcs of circles method.

[RGPV Feb. 2006]

6. An elliptical fish pond of largest size is to be constructed inside a rectangular plot of 150 m × 70 m. Draw the boundary of the fish pond.
7. A plot of land is in the shape of a 170 m × 100 m parallelogram with an included angle of 60º. Inscribe an elliptical flower bed on it.
8. Two points *A* and *B* are 120 mm apart. The third point *C* is 90 mm from *A* and 60 mm from *B*. Draw an ellipse passing through points *A*, *B* and *C*.
9. The major and minor axes of an ellipse are 125 mm and 100 mm long respectively. Draw the curve by any one of the standard methods and locate its foci.

[RGPV Feb. 2010]

10. The major axis of an ellipse is 150 mm long and the minor axis is 100 mm long. Draw the ellipse and then a tangent to the ellipse at a point on it 25 mm above the major axis. **[RGPV Feb. 2008, April 2010]**
11. The major and minor axes of an ellipse are 120 mm and 80 mm. Draw the ellipse and then draw a tangent and normal at a point 30 mm away from the major axis.

[RGPV June 2008]

12. Draw an ellipse with 120 mm major axis and 80 mm minor axis. Draw a normal and tangent on any point on ellipse.

[RGPV Dec. 2010]

Parabola

13. A fixed point is 75 mm from a straight line. Draw the locus of a point *P* moving such a way that its distance from the fixed straight line is equal to its distance from the fixed point. Name the curve generated.

[RGPV Feb. 2007]

14. Construct a parabola whose focus is at a distance of 40 mm from the directrix. Draw a tangent and a normal to the parabola at a point, 50 mm away from the principal axis.
15. Draw the locus of a point which moves in such a manner that its distance from a fixed point is equal to its distance from a fixed straight line. Consider the distance between the fixed point and the fixed line as 60 mm. Name the curve.
16. Construct a parabola using 'offset method' when its double ordinate is 150 mm and abscissa is 75 mm. Locate the focus and directrix to the parabola.
17. A ball is thrown up in the air where it reaches a maximum height of 45 m and travels a horizontal distance of 75 m. Trace the path of the ball assuming it to be parabolic.

[RGPV Feb. 2007]

18. A ball thrown from the ground level reaches a reaches a maximum height of 5 m and travels a horizontal distance of 11 m from the point of projection. Trace the path of the ball. **[RGPV Feb. 2011]**
19. A fountain jet discharges water from ground level at an inclination of 60º to the ground. The jet travels a horizontal distance of 14 m from the point of discharge and falls on the ground. Trace the path of the jet and name the curve.
20. Construct a parabola with 60 mm base and 40 mm length of axis. Draw a tangent to the curve at a point, 20 mm from the base. **[RGPV June 2008]**

21. Inscribe a parabola in the parallelogram of 110 mm and 70 mm long sides with longer side of it as the normal base. Consider one of the included angles between the sides as 60º.

Hyperbola

22. The focus of a hyperbola is 60 mm from its directrix. Draw the curve when eccentricity is 5/3. Draw a tangent and a normal to the curve at a point 45 mm from the directrix.
23. A fixed point is 90 mm from a fixed straight line. Draw the locus of a point *P* moving in such a way that its distance from the fixed point is twice its distance from the fixed straight line. Name the curve.
24. Two points are fixed at 100 mm apart. Draw the locus of a point moving in such a manner that the difference of its distance from the points is 75 mm. Name the curve.
25. Draw two branches of a hyperbola when the distance between its foci is 90 mm and the vertices are 15 mm from the foci. Locate the asymptotes and measure the angle between them.
26. The transverse axis of a hyperbola is 80 mm long. Its double ordinate is 90 mm long and the corresponding abscissa is 50 mm. Construct the hyperbola.
27. Draw a rectangular hyperbola when the position of a point *P* on the curve is 30 mm from the horizontal asymptote and 50 mm from the vertical asymptote. Show at least four points on either side of point *P*.
28. The asymptotes of a hyperbola are inclined at an angle of 75º. Its foci are 60 mm apart. Locate its foci graphically and construct two branches of the hyperbola. Also draw a tangent and a normal to the curve at a point 20 mm from one of the foci.

REVIEW QUESTIONS

1. What is a conic section? Enlist its various types.
2. What is the inclination of the cutting plane in order to obtain following sections from a cone: (a) parabola, (b) ellipse, (c) hyperbola, (d) rectangular hyperbola.
3. Give two practical applications for the following curves: (a) parabola, (b) ellipse, (c) hyperbola.
4. Define eccentricity.
5. State four common methods to draw (a) parabola, (b) ellipse, and (c) hyperbola.
6. State the principle of intersecting arcs method for construction of an ellipse.
7. How is a tangent drawn from an external point on the ellipse?
8. State the relationship among major axis, minor axis and distance between foci of an ellipse.
9. What do you understand by conjugate diameters?
10. State the principle of offset method for construction of a parabola.
11. How is a tangent drawn from an external point on a parabola?
12. Define ordinate, double ordinate, abscissa and latus rectum.
13. State the principle of intersecting arcs method for construction of a hyperbola.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) If a point moves in a plane in such a way that the sum of its distances from two fixed points is constant the curve so traced is called
(a) ellipse (b) parabola (c) hyperbola (d) none of these

ii) Name the curve traced out by a point moving in a plane such that the difference between its distances from two fixed points is constant.
(a) Ellipse (b) Parabola (c) Hyperbola (d) Any of these

iii) When a bullet is shot in air the path traversed by the bullet is called a
(a) cycloid (b) semicircle (c) parabola (d) hyperbola

iv) A right circular cone when cut by a plane parallel to its generator, the curve obtained is a/an
(a) ellipse (b) parabola (c) hyperbola (d) circle

v) When a right circular cone is cut by a plane passing through its apex, the section obtained is a/an
(a) ellipse (b) parabola (c) hyperbola (d) triangle

vi) When a right circular cone is cut which meets its axis at an angle greater than the semi cone angle, the curve obtained is a/an
(a) ellipse (b) parabola (c) hyperbola (d) triangle

vii) When a right circular cone is cut which meets its axis at an angle less than the semi cone angle, the curve obtained is
(a) ellipse (b) parabola (c) hyperbola (d) triangle

viii) The angle between the asymptotes of a rectangular hyperbola is
(a) 30° (b) 45° (c) 60° (d) 90°

ix) Name the curve which has zero eccentricity.
(a) Ellipse (b) Parabola (c) Hyperbola (d) Circle

x) Which of the following curves obeys Boyle's law?
(a) Ellipse (b) Parabola (c) Hyperbola (d) Circle

xi) Which of the following applications uses a hyperbolic curve ?
(a) Solar collector (b) Cooling tower (c) Lamp reflectors (d) Monuments

xii) The eccentricity of an ellipse can be determined by

(a) $\dfrac{\text{length of major axis}}{\text{distance between directrices}}$ (b) $\dfrac{\text{distance between the foci}}{\text{length of major axis}}$

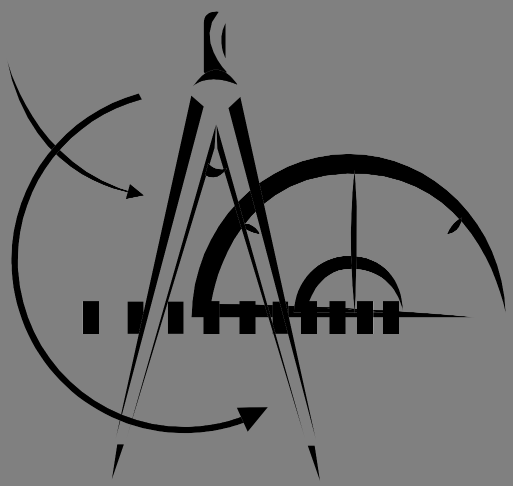
(c) $\dfrac{\text{distance of a point of ellipse from the focus}}{\text{distance of the same point from the directrix}}$. (d) all of these

xiii) The major and minor axes of an ellipse are 100 mm and 60 mm respectively. What will be the distance of its foci from the end of the minor axis?
(a) 30 mm (b) 40 mm (c) 50 mm (d) 60 mm

Answers

(i) a (ii) c (iii) c (iv) b (v) d (vi) a (vii) c (viii) d (ix) d (x) d
(xi) b (xii) d (xiii) c

4



Special Curves

4.1 INTRODUCTION

Curves generated by the rolling contact of one curve or line on another curve or line are called roulettes. There are infinite varieties of roulettes. The most common types of roulettes used in engineering applications are cycloidal curves, involutes and spirals.

4.2 CYCLOIDAL CURVES

Cycloidal curves are generated by a point lying on the circumference of a circle, when it rolls along a fixed straight or curved path without slipping. The circle which rolls is called the *rolling circle*, or *generating circle*, and the fixed straight line or the circle on which it rolls is called the *directing line* or the *directing circle*. Cycloidal curves are commonly used in kinematics (the study of motion) and in mechanisms that work with rolling contact.

4.3 CYCLOID

A cycloid is a curve traced by a point on the circumference of a circle which rolls along a fixed straight line without slipping. Consider Fig. 4.1 where a circle with centre C rolls along a straight line PQ. The path traced by any point P lying on the circumference of the circle is called cycloid.

Example 4.1 (Fig 4.1)

Construct a cycloid having a rolling circle of 50 mm diameter. Draw a normal and a tangent to the curve at a point 35 mm above the base line. [RGPV Feb 2010]

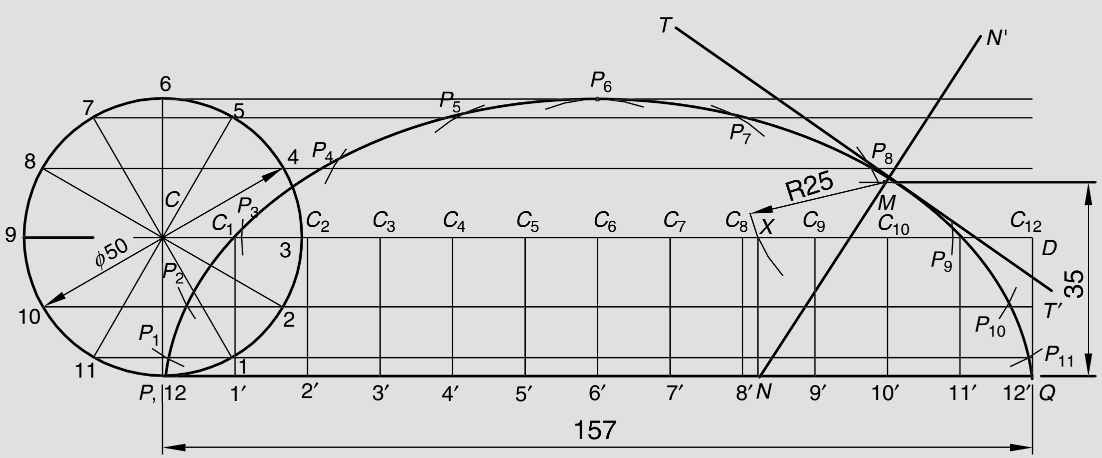


Fig. 4.1

Construction: Fig. 4.1

1. Draw a rolling circle of 50 mm diameter with C as the centre.
2. Draw the directing line $PQ = \pi D = 157$ mm, tangential to the circle at point P.
3. Divide the rolling circle into 12 equal parts and mark points 1, 2, 3, etc., as shown. Draw lines through points 1, 2, 3, etc., parallel to line PQ.
4. Divide line PQ into 12 equal parts and mark points 1′, 2′, 3′, etc., on it.
5. Draw perpendicular lines through these points to meet the centre line CB at points C_1, C_2, C_3, etc.
6. Assume that the circle rolls to the right through 1/12 rotation. Thus, point 1 of the circle will get in contacts with 1′, centre C will move to the new centre C_1, and point P will move up to the horizontal line through point 1. Take $C_1P_1 = CP = 25$ mm and draw an arc with centre C_1, to cut the horizontal line through point 1 at P_1.
7. Similarly, draw arcs with centres C_2, C_3, C_4, etc., with 25 mm radius, to meet the horizontal locus lines through points 2, 3, 4, etc. at points P_2, P_3, P_4, etc., respectively.
8. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, etc. The curve obtained is the required cycloid.

Tangent and Normal to the Cycloid

9. Mark point M on the cycloid at 35 mm from the directing line PQ.
10. With M as the centre and radius 25 mm, cut the centre line at point X.
11. Through point X, draw a line perpendicular to meet PQ at point N.
12. Join NM and produce to N'. This line NN' is the required normal.
13. Through point M draw a line TT' perpendicular to NN'. This line TT' is the required tangent.

4.4 EPICYCLOID

An epicycloid is a curve traced by a point on the circumference of a circle which rolls along another circle outside it, without slipping. Consider Fig. 4.2 where a circle with centre C rolls along the arc of circle with centre O and outside it. The path traced by a point P lying on the circumference of the rolling circle is called epicycloid. Epicycloids are often used in rotary pumps, blowers and superchargers.

Example 4.2 (Fig 4.2)

A circle of 50 mm diameter rolls on the circumference of another circle of 175 mm diameter and outside it. Trace the locus of a point on the circumference of the rolling circle for one complete revolution. Also draw a set of tangent and normal on a suitable point on the curve.
[RGPV Dec 2004, Feb 2005, Feb 2006, Apr 2009]

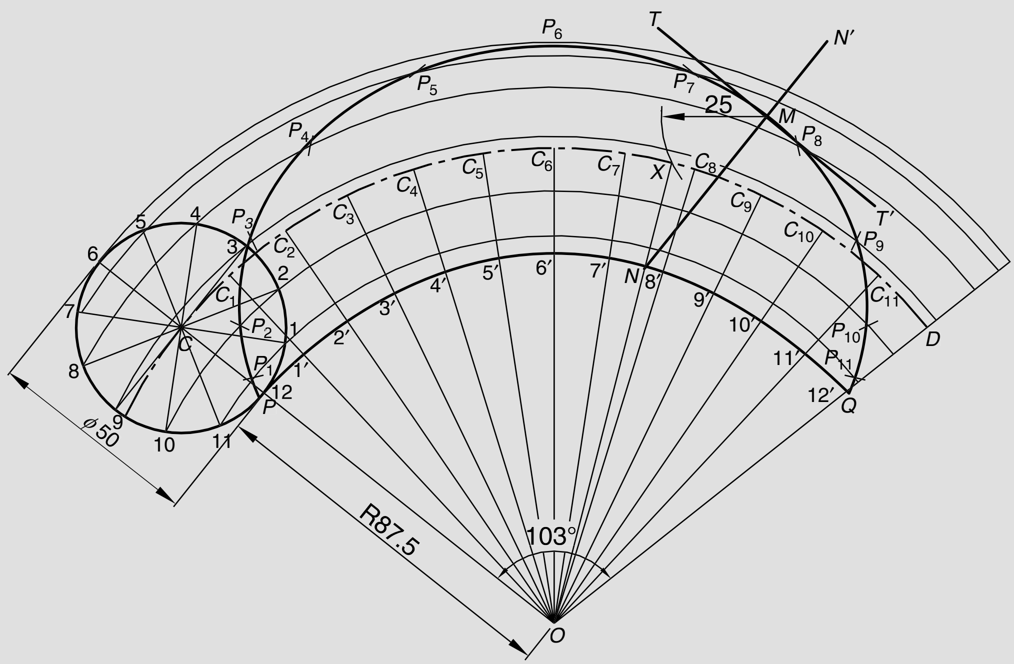


Fig. 4.2

Construction: Fig. 4.2

A generating circle will cover an arc length $PQ = 2\pi r$ in its one revolution along the directing circle. The angle subtended by the arc PQ at centre O is given by $\theta = \dfrac{d}{D} \times 360°$, where d and D are the diameters of the generating and directing circles respectively. Here $d = 50$ mm and $D = 175$ mm, therefore, $\theta = \dfrac{50}{175} \times 360° = 103°$.

1. Draw an arc PQ with O as the centre and radius (175/2) mm, to subtend angle of 103°. This represents the directing path.
2. Join OP and extend it to point C taking $CP = 25$ mm.
3. Draw a generating circle with C as the centre and radius CP. Divide the circle into 12 equal parts and name its divisions as 1, 2, 3, 4, etc.
4. Also, divide the directing arc PQ into 12 equal parts and mark its division as 1′, 2′, 3′, 4′, etc., as shown.
5. Draw arcs, with O as the centre and radii equals to $O1$, $O2$, $O3$, $O4$, etc., to meet line OQ produced.
6. Draw an arc with O as the centre and radius OC to meet OQ at point D. The arc CD is known as the centre arc.
7. Extend lines $O1'$, $O2'$, $O3'$, $O4'$, etc., to meet arc CD at points C_1, C_2, C_3, C_4, etc.
   Assume that the circle rolls to the right through 1/12 rotation. Thus, point 1 of the circle will get in contacts with 1′, centre C will move to the new centre C_1 and point P will move to arc through point 1. Also, the distance of point P will be equal to the radius of the circle, i.e., 25 mm. Therefore, you can do the following:

8. Draw an arc with centre C_1, to cut the arc through point 1 at P_1.
9. Similarly, draw arcs with centres C_2, C_3, C_4, etc., and 25 mm radius, to intersect arcs through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
10. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, etc. The curve is the required epicycloid.

Tangent and Normal to the Epicycloid

11. Mark point *M* on the epicycloid where a tangent and a normal has to be drawn.
12. Draw an arc with *M* as the centre and 25 mm radius, to meet the centre arc *CD* at point *X*.
13. Join *OX*. Let it meets the arc *PQ* at point *N*.
14. Join *NM* and produce it to *N′*. The line *NN′* is the required normal.
15. Through point *M*, draw a line *TT′* perpendicular to *NN′*. The line *TT′* is the required tangent.

4.5 HYPOCYCLOID

A hypocycloid is a curve traced by a point on the circumference of a circle which rolls along another circle and inside it, without slipping. Consider Fig. 4.3 where a circle with centre *C* rolls along the arc of circle with centre *O* and inside it. The path traced by a point *P* lying on the circumference of the rolling circle is called hypocycloid. Hypocycloids are also used in rotary pumps, blowers, and superchargers.

Example 4.3 (Fig 4.3)

A circle of 50 mm diameter rolls on the circumference of another circle of 175 mm diameter and inside it. Trace the locus of a point on the circumference of the rolling circle for one complete revolution. Also draw a set of tangent and normal on a suitable point on the curve.
[RGPV Jun 2011]

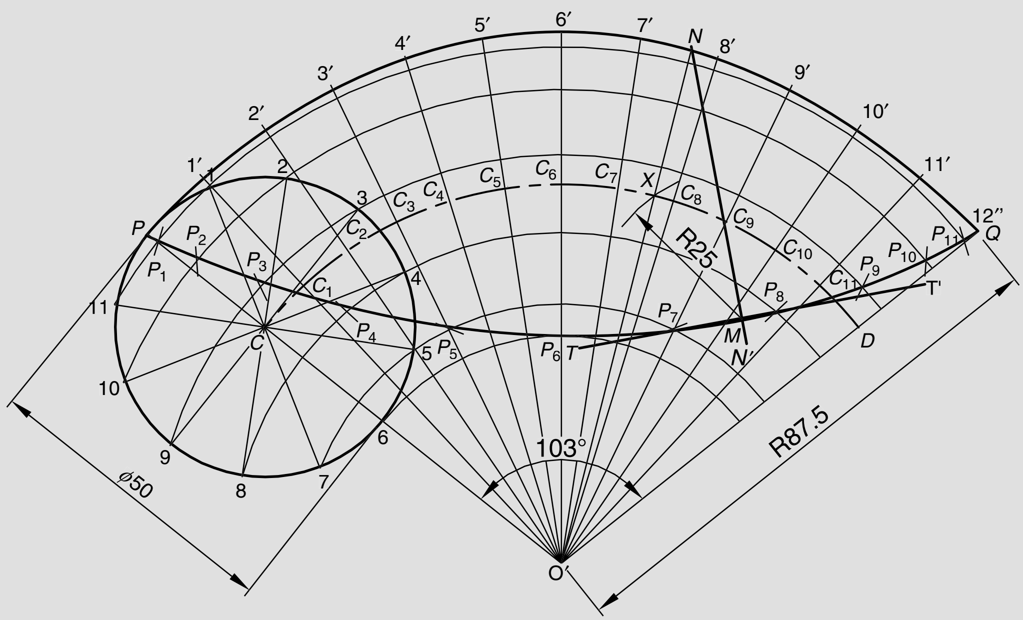


Fig. 4.3

Construction: Fig. 4.3

A generating circle will cover an arc length $PQ = 2\pi r$ in its one revolution along the directing circle. The angle subtended by the arc PQ at centre O is given by $\theta = \frac{d}{D} \times 360°$, where d and D are the directors of the generating and directing circles respectively. Here d = 50 mm and D = 175 mm, therefore, $\theta = \frac{50}{175} \times 360° = 103°$.

1. Draw an arc PQ with O as the centre and radius (175/2) mm, to subtend angle of 103°. This represents the directing path.
2. Join OP and mark point C on it taking CP = 25 mm.
3. Draw the generating circle with C as the centre and radius CP. Divide this circle into 12 equal parts and name its divisions as 1, 2, 3, 4, etc., as shown.
4. Also, divide the directing arc PQ into 12 equal parts and mark its division as 1′, 2′, 3′, 4′, etc., as shown.
5. Draw arcs, with O as the centre and radii equals to $O1$, $O2$, $O3$, etc., to meet line OQ.
6. Draw an arc with O as the centre and radius OC to meet OQ at point D. The arc CD is known as the centre arc.
7. Join $O1'$, $O2'$, $O3'$, $O4'$, etc., to meet arc CD at points C_1, C_2, C_3, C_4, etc., respectively. Assume that the circle rolls to the right through 1/12 rotation. Thus, point 1 of the circle will get in contacts with 1′, centre C will move to the new centre C_1 and point P will move to arc through point 1. Also, the distance of point P will be equal to the radius of the circle, i.e., 25 mm. Therefore,
8. Draw an arc with centre C_1, to cut arc through point 1 at P_1.
9. Similarly, draw arcs with centre C_2, C_3, C_4, etc., and 25 mm radius, to intersect arcs through points 2, 3, 4, etc., at points P_2, P_3, P_4, etc., respectively.
10. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, etc., thus obtained. The curve is the required hypocycloid.

Tangent and Normal to the Hypocycloid

11. Mark point M on the hypocycloid where a tangent and a normal has to be drawn.
12. Draw an arc with M as the centre and 25 mm radius, to meet the centre arc CD at point X.
13. Join OX. Let it meets the arc PQ at point N.
14. Join NM and produce it to N'. The line NN' is the required normal.
15. Through point M, draw a line TT' perpendicular to NN'. The line TT' is the required tangent.

4.6 INVOLUTE

An involute is a curve traced out by an end of a thread, when it is unwound from a circle or a polygon, the thread being kept tight. An involute of a circle is used as teeth profile of a large gear wheel and a gear reducer. In Fig. 4.4 the end of the thread is at point P of the circle. When the thread is unwound keeping it always tight, it is always tangential to the circle and the point P will generated an involute.

Example 4.4 (Fig 4.4)

Draw an involute of a circle of 50 mm diameter. Also, draw a normal and a tangent at a point 100 mm from the centre of the circle.

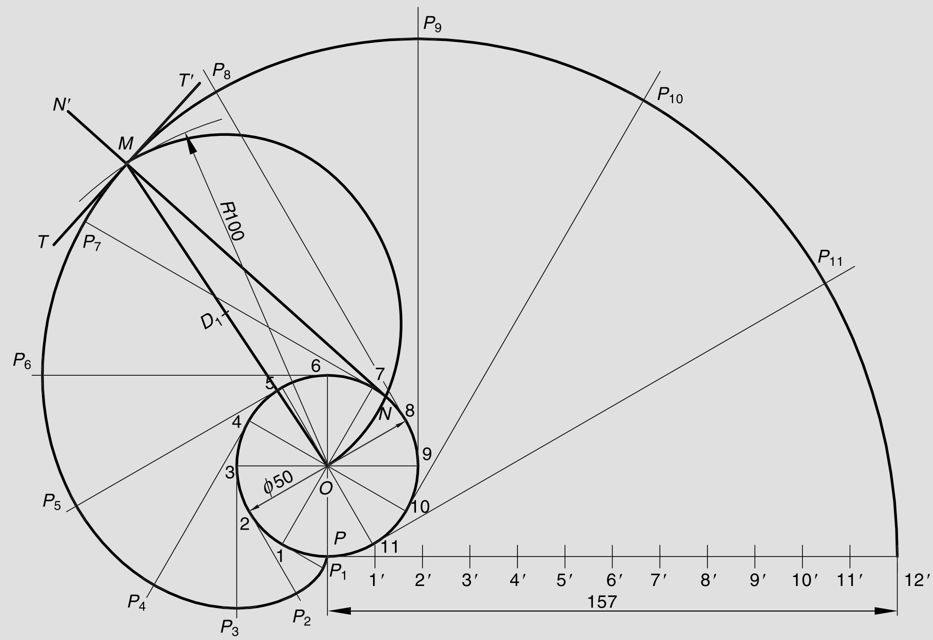


Fig. 4.4

Construction: Fig. 4.4

1. Draw a circle having a 50 mm diameter with O as the centre. Divide the circle into 12 equal parts and mark them as 1, 2, 3, 4, etc.
2. Mark a point P on the circumference of the circle which is considered to be the end of thread. Also, consider the thread is being unwound in the clockwise direction.
3. The length of thread that will get unwound in one revolution is $\pi D = 157$ mm and it will be tangential to the circle at point P. Therefore, draw a line $PQ = 157$ mm and divide it into 12 equal parts. Mark the point as 1′, 2′ 3′, 4′, etc.
4. Draw tangents to the circle though points 1, 2, 3, 4, etc., such that they represent the thread position being unwound.
5. The length of thread that is unwound by 1/12 revolution will be equal to $P1'$. Therefore, with centre 1 and radius $P1'$, mark a point $P1$ on the tangent line.

6. Similarly, with centres 2, 3, 4, etc., and radii $P2'$, $P3'$, $P4'$, etc., respectively, mark points P_2, P_3, P_4 etc., on their respective tangent lines.
7. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, etc. The curve obtained is the required involute.

Tangent and Normal to the Involute

8. Mark a point *M* on the involute at 100 mm from centre *O*.
9. Join *OM*. Mark O_1 as its mid-point.
10. With O_1 as the centre and *OM* as diameter, draw a semi-circle in clockwise direction to intersect the circle at point *N*.
11. Join *NM* and produce it to *N′*. The line *NN′* is the required normal.
12. At point *M,* draw a line *TT′* perpendicular to *NN′*. The line *TT′* is the required tangent.

Example 4.5 (Fig 4.5)

Draw an involute of a hexagon of 25 mm side.

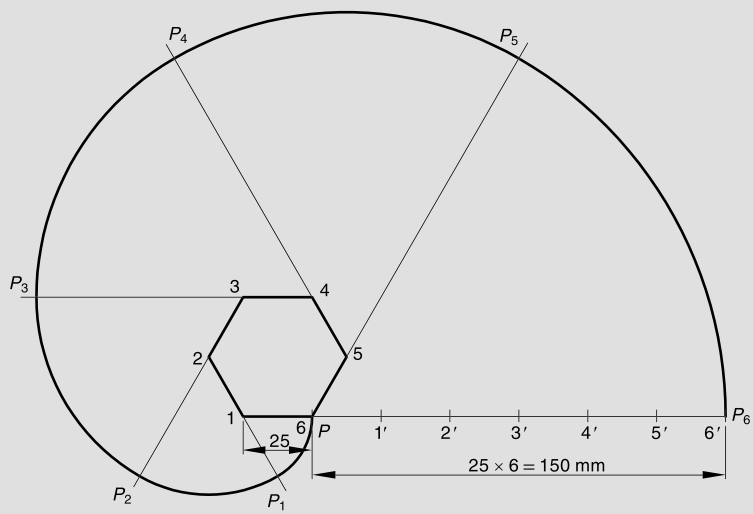


Fig. 4.5

Construction: Fig. 4.5

1. Draw a hexagon 123456 of 25 mm side. Mark point *P* on it as shown.
2. Consider the thread is being unwound in clockwise direction.
3. Produce sides 12, 23, 34, 45, 56 and 61 in unidirectionally.

4. With centres 1, 2, 3, 4, 5, 6 and radii in multiples of side length (i.e. 25 mm, 50 mm, 75 mm, 100 mm, 125 mm, 150 mm, respectively), mark points P_1, P_2, P_3, P_4, P_5, P_6 on produce sides 12, 23, 34, 45, 56, 61 respectively.
5. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, P_5, P_6. The curve is the required involute of the hexagon.

4.7 SPIRAL

*If a line rotates in a plane about one of its ends and at the same time, if a point moves along the line continuously in one direction, the curve traced out by the moving point is called a spira*l. The Following terms are used in connection with the spiral.

1. **Pole** It is the fixed end of the line about which the line rotates.
2. **Radius Vector** It is the line joining any point of the curve with the pole.
3. **Vectorial Angle** It is the angle between the initial position of the line and the instantaneous position of the line.
4. **Convolution** Rotation of the moving line through 360º is called one convolution. A spiral make any number of convolutions before it reaches the final destination.

   Two types of spirals, namely, Archimedean and logarithmic spirals are commonly used in engineering practice.

4.8 ARCHIMEDEAN SPIRAL

An Archimedean spiral is a curve traced out by a point moving uniformly along a straight line towards or away from the pole, while the line revolves about its one of the ends with uniform angular velocity. Consider Fig. 4.6 where point *P* is moving about a fixed point *O* such that for every increase in its vectorial angle of 30º, point *P* moves away through a distance of *P*1. Thus, point *P* traces an Archimedean spiral.

Example 4.6 (Fig 4.6)

Draw an Archimedean spiral of 1½ convolutions, the greatest and least radii being 115 mm and 15 mm respectively. Draw a tangent and a normal to the spiral at a point 63 mm from the pole.

[RGPV June 2003]

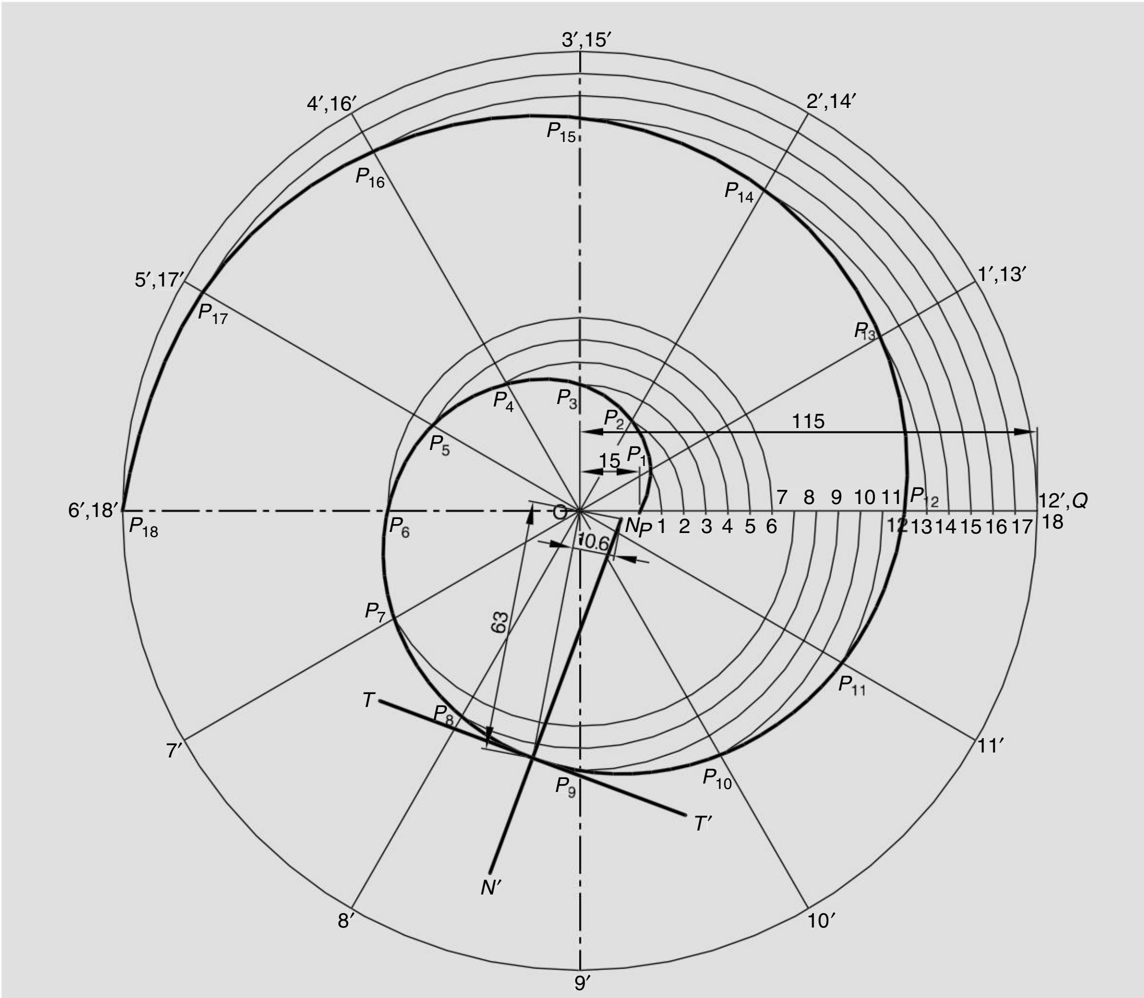


Fig. 4.6

Construction: Fig. 4.6

1. Draw a circle with O as the centre and 115 mm radius. Divide the circle into 12 equal parts. Mark points as 1′, 2′, 3′, 4′, etc.
2. On a radial line OQ mark point P at a distance of 15 mm from point O.
3. Divide PQ into 18 parts (@12 parts per convolution). Mark points as 1, 2, 3, 4, etc.
4. Draw arcs with the centre O and radii $O1$, $O2$, $O3$, $O4$, etc. to meet radial lines $O1'$, $O2'$, $O3'$, $O4'$, etc., respectively, at points P_1, P_2, P_3, P_4, etc.
5. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, etc. The curve is the required Archimedean spiral.

Tangent and normal to Archimedean Spiral

6. Mark point M on the spiral at a radial distance of 63 mm from the centre O.

7. Join OM.
8. Draw $ON = \dfrac{115-15}{1.5\times 2\pi} = 10.6$ mm perpendicular to line OM.
9. Join NM and produce it to N'. The line NN' is the required normal.
10. Through point M draw a line TT' perpendicular to NN'. The line TT' is the required tangent.

4.9 LOGARITHMIC SPIRAL

A logarithmic spiral is a curve traced by a point moving along a rotating line such that for equal angular displacement of the line, the ratio of the lengths of consecutive radius vectors is constant. Thus, in a logarithmic spiral, vectorial angles increase in arithmetic progression and the corresponding radius vectors are in geometrical progression. Consider Fig. 4.7(b) where $\dfrac{OP_1}{OP} = \dfrac{OP_2}{OP_1} = \dfrac{OP_3}{OP_2} =$ constant for uniform increase in the vectorial angle. So the curve is a logarithmic spiral.

Example 4.7 (Fig. 4.7)

Draw a logarithmic spiral of one convolution, given the shortest distance as 16 mm and ratio of the length of radius vectors enclosing an angle of 30º is 9:8. Draw a tangent and a normal to the curve at a point 50 mm from the pole.

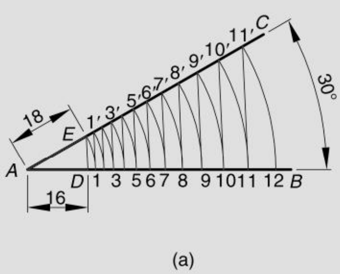


Fig. 4.7(a)

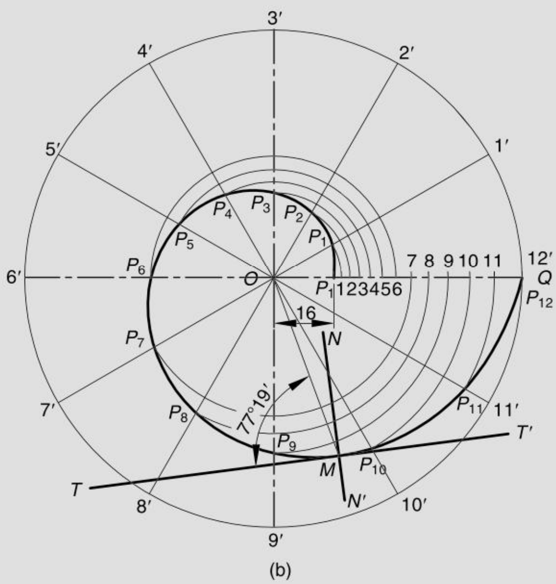


Fig. 4.7(b)

Construction: Fig. 4.7

Determine length of the radius vector graphically. Fig. 4.7(a)

1. Draw an angle *CAB* of 30º.
2. Mark a point *D* on *AB* such that *AD* = 16 mm (shortest distance).
3. Mark point *E* on *AC* such that *AE* = ratio of length of radius vectors *X* shortest distance = $\frac{9}{8}$ *X* 16 mm = 18 mm.
4. Join *DE*. With centre *A* and radius *AE*, draw an arc meet at point 1.
5. Through 1, draw a line parallel to *DE* to meet *AC* at point 1′. With centre *A* and radius *A*1′, draw an arc meet at point 2.
6. Through 2 draw a line parallel to *DE* to meet *AC* at point 2′. With centre *A* and radius *A*2′, draw an arc meet at point 3.
7. Repeat the same procedure as described in steps 5 and 6 and obtain points 4, 5, 6, 7, 8, 9, 10, 11 and 12.

Draw Logarithmic Spiral Fig. 4.7(b)

8. Draw a circle having a 50 mm, given diameter, keeping *O* as the centre. Divide this circle into 12 equal parts and number them as 1′, 2′, 3′, 4′, etc., (as shown).
9. Mark points *P*, 1, 2, 3, …, 12 on *OQ* such that *OP* = *AD*, *O*1 = *A*1, *O*2 = *A*2, *O*3 = *A*3 and, so on. Thus, *OQ* is the copy of line *AB*.
10. Draw arcs with centre *O* and radii *O*1, *O*2, *O*3, *O*4, etc., to meet radial lines *O*1′, *O*2′, *O*3′, *O*4′, etc., respectively, at point P_1, P_2, P_3, P_4, etc.
11. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, etc. The curve is the required logarithmic spiral.

Tangent and Normal to the Logarithmic Spiral

12. Mark a point *M* on the spiral at a radial distance of 50 mm from the centre *O*.
13. Determine $\alpha = \tan^{-1}\frac{\theta}{\ln r} = \tan^{-1}\frac{\pi/6}{\ln 9/8} = 77^{o}19'$.
14. Mark a point *T* such that angle *OMT* = α = 77º19′. Produce *TM* to get *T*′. The line *TT*′ is the required tangent.
15. At point *M*, draw a line *NN*′ perpendicular to *TT*′. The line *NN*′ This is the required normal.

4.10 MISCELLANEOUS EXAMPLES

Example 4.8 (Fig 4.8)

A circle of 50 mm diameter rolls on a horizontal line for a half revolution and then on a vertical line downwards for another half revolution. Draw the curve traced out by a point *P* on the circumference of the circle. Assume that the horizontal and the vertical line constitute a corner. [RGPV June 2004]

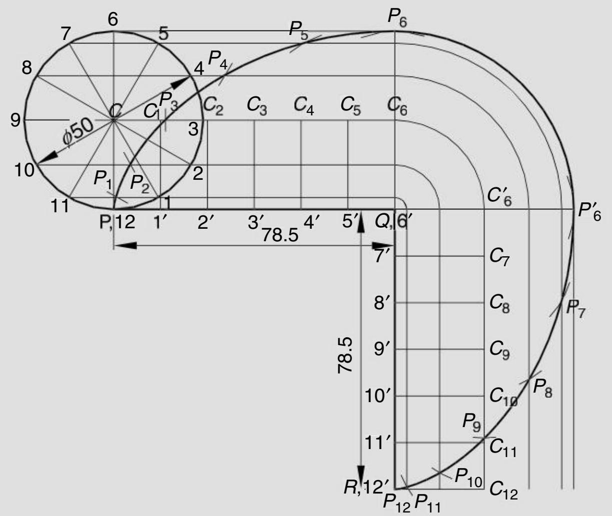


Fig. 4.8

Construction: Fig. 4.8

1. Draw a generating circle having 50 mm diameter.
2. Draw a horizontal line $PQ = \frac{\pi D}{2}$ = 78.5 mm (appox), tangential to the circle.
3. Divide the rolling circle into 12 equal parts and mark the points as 1, 2, 3, etc. Draw lines through the points 1, 2, 3, etc., parallel to *PQ*.
4. Divide *PQ* into 6 equal parts and mark 1′, 2′, 3′, 4′, 5′, 6′ on it. Draw perpendicular lines through these points to meet the centre line *CB* at points C_1, C_2, C_3, etc.
5. Draw arcs with centre C_1, C_2, C_3, C_4, C_5, C_6 of 25 mm radius, to meet the locus lines through points 1, 2, 3, 4, 5, 6 at points P_1, P_2, P_3, P_4, P_5, P_6 respectively.
6. Draw a smooth curve to pass through all the points P_1, P_2, P_3, P_4, P_5, P_6.
7. It may be noted that when the circle is at point *Q* and rolls, the point P_6 will move through one-quarter on a circular path and reach P_6'. Therefore, draw an arc with *Q* as the centre and raidus QP_6 to meet *PQ* produced at point P_6'.
8. Now draw a vertical line *QR* of 78.54 mm as the new directing line. Also, transfer the locus lines coming from points 1, 2, 3, etc., through 90º as shown and produce them vertically downwards.
9. Divide *QR* into 6 equal parts and mark 7′, 8′, 9′, 10′, 11′, 12′ on it. Draw horizontal lines through these points to meet the vertical centre line at points C_7, C_8, C_9, C_{10}, C_{11}, C_{12}.
10. Draw arcs with centres C_7, C_8, C_9, C_{10}, C_{11}, C_{12} of 25 mm radius, to meet locus lines through points 7, 8, 9, 10, 11, 12 at points P_7, P_8, P_9, P_{10}, P_{11}, P_{12} respectively.
11. Draw a smooth curve passing through the points P_7, P_8, P_9, P_{10}, P_{11} and P_{12}.

Example 4.9 (Fig 4.9)

A circle of 50 mm diameter rolls on the outside of a directing circle of the same diameter without slipping. Draw the curve traced by a point on the rolling circle for one revolution of rolling circle.
[RGPV Sep. 2009]

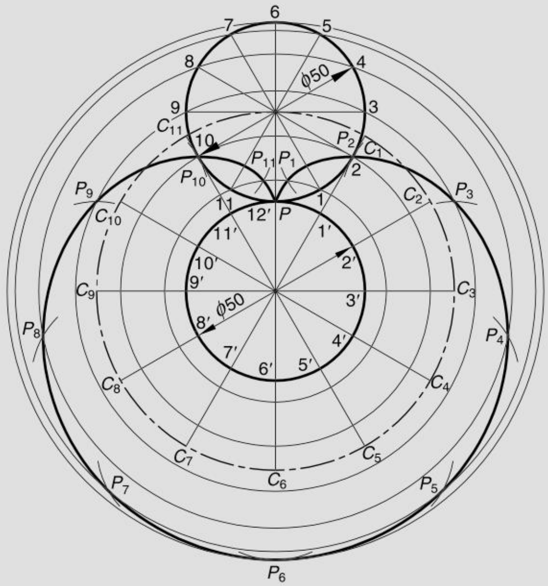


Fig. 4.9

When the diameters of the generating and directing circles are equal, the angle subtended by the generating circle at centre of the directing circle is 360° and the epicycloid traced is called a **cardioid**. Follow the steps of constructions of Example 4.2, take $r = R = 25$ mm, $\theta = 360^{\circ}$ and obtain the epicycloid as shown in Fig 4.9.

Example 4.10 (Fig 4.10)

Construct a hypocycloid, taking the diameter of the generating circle and radius of directing circle as 60 mm.

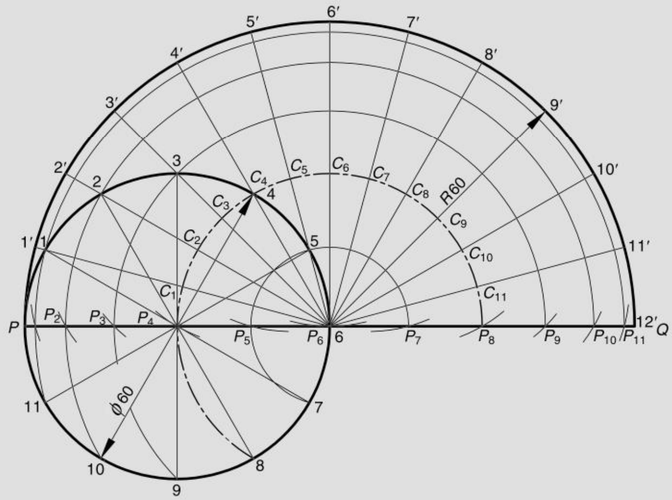


Fig. 4.10

Construction: Fig. 4.10

When the diameter of the generating (rolling) circle is half the diameter of the directing circle ($r = R/2$), the hypocycloid traced is a straight line.

For such hypocycloid, $\theta = \frac{r}{R} \times 360° = \frac{60}{2 \times 60} \times 360° = 180°$.

Follow the steps of *constructions* of Example 4.3, where $r = 30$ mm, $R = 60$ mm and $\theta = 180°$ and obtain the hypocycloid as a straight line, as shown in Fig. 4.10.

Example 4.11 (Fig 4.11)

A disc is in the form of a square of 30 mm sides surmounted by a semicircle on one of the sides of the square and a half hexagon on the opposite side. Draw the path of the end of a string which is unwound from the circumference of the disc.

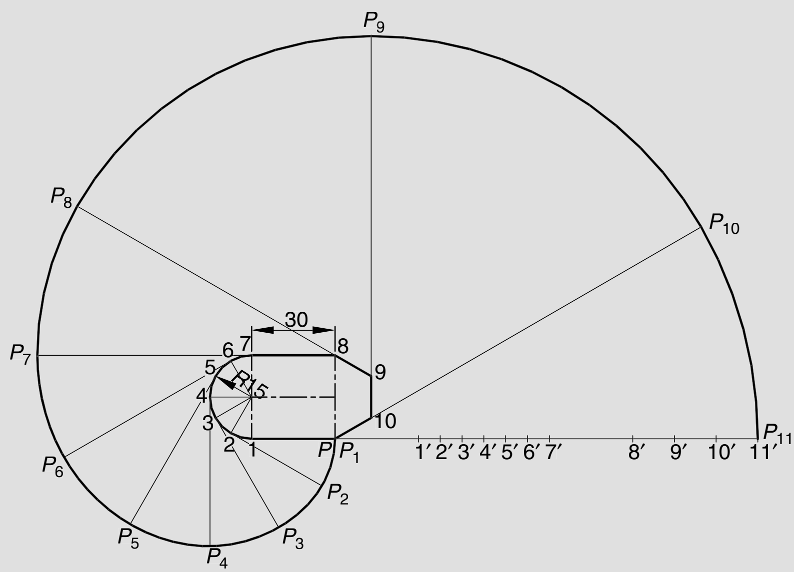


Fig. 4.11

Construction: Fig. 4.11

1. Draw the given disc.
2. Draw a line P-11′ equal to the perimeter of the disc.
3. Draw the involute. (refer Examples 4.4 and 4.5).

Example 4.12 (Fig 4.12)

Draw a path traced out by an end of a piece of thread when unwound to a length of 150 mm from a circle of 40 mm diameter, the thread being kept tight when it is being unwound. Name the curve traced.

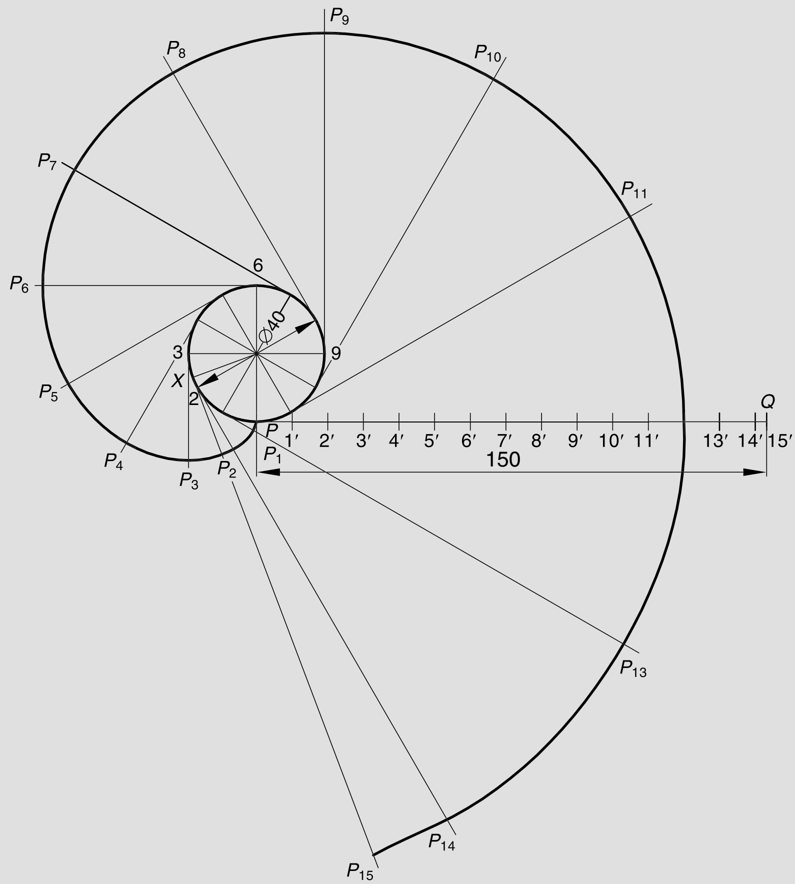


Fig. 4.12

Construction: Fig. 4.12

When the thread is wound on a circle or a polygon, the curve traced is called an involute.

As the length of thread is given as 150 mm, the angle through which the thread could be wounded can be calculated as $\theta = \dfrac{\text{Length of thread}}{\pi D} \times 360° = \dfrac{150}{\pi \times 40} \times 360° = 429.7°$. Thus, the thread could be wound through an angle of 360° + 69.7° (one revolution + 69.7°).

1. Draw a circle with a 40 mm diameter and a tangential line $PQ = 150$ mm long.
2. Mark points 1, 2, 3, 4, etc., on PQ starting from point P such that P-1, 1-2, 2-3, 3-4, etc. are of length equal to $\frac{\pi \times D}{12}$. Thus, length 14-15 will remain of shorter length.
3. Draw tangents to the circle though points 1, 2, 3, 4, etc.,
4. Draw arcs with centres 1, 2, 3, 4, etc., and radii $P1'$ $P2'$, $P3'$, $P4'$, etc., respectively, and locate points P_1, P_2, P_3, P_4, etc., on their respective tangent lines, as discussed in Example 4.4. Continue the procedure to locate points P_{13} and $P_{14.}$ point P_{15} will lie on a tangent line through a point x, lying on the circumference of the circle such that angle $oxP = 69.7°$ as shown.
5. Draw a smooth curve passing through all the points P_1, P_2, P_3, P_4, etc. The curve is the required involute.

Example 4.13 (Fig 4.13)

***PQ* is a rope 1.4 m long, tied to a peg at *Q*, as shown in Fig. 4.13(a). Keeping it always tight, the rope is wound around the pole *O*. Draw the curve traced out by the end *P*. Consider scale 1:10.** **[RGPV Dec. 2003]**

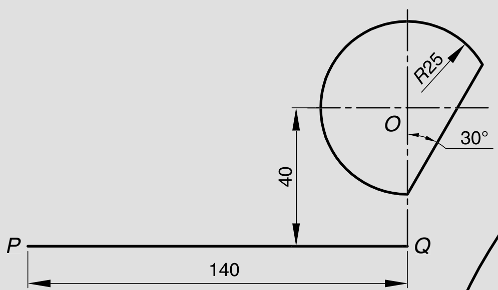


Fig. 4.13(a)

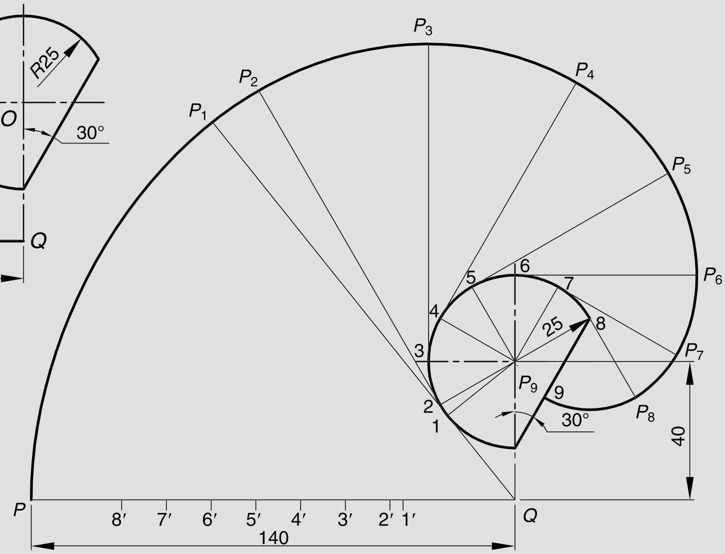


Fig. 4.13(b)

Construction: Fig. 4.13

1. Draw Fig 4.13. With point Q, draw a tangent to the circle of 140 mm meeting at 1′. Measure Q-1′.
2. Divide the circle/sector into a number of parts and number them as 2, 3, 4, etc.,
3. Mark points 1′, 2′, 3′, 4′, etc., on PQ such that Q-1 = tangent = 35 mm, 1-2 = arc 1′-2′, Thereafter 2-3 = arc 2′-3′, 3-4 = arc 3′-4′, 4-5 = arc 4′-5′, and so on.

4. Draw tangents to the circle though points 2, 3, 4, 5, etc., of lengths *P*-2, *P*-3, *P*-4, *P*-5, etc., respectively.
5. Draw a smooth curve to pass through the ends of the tangents of the circle. The curve is the required involute.

Example 4.14 (Fig 4.14)

A 150 mm long link swings on a pivot *O* from its vertical position of rest to the right, through an angle of 40º. After it swings to the left through an angle of 80º, it returns to its initial centre position. During this period, a point *P* moving at a uniform speed along the centre line of the link from a point at a distance of 22 mm from *O*, reaches the end of the link. Draw the locus of the point *P*.

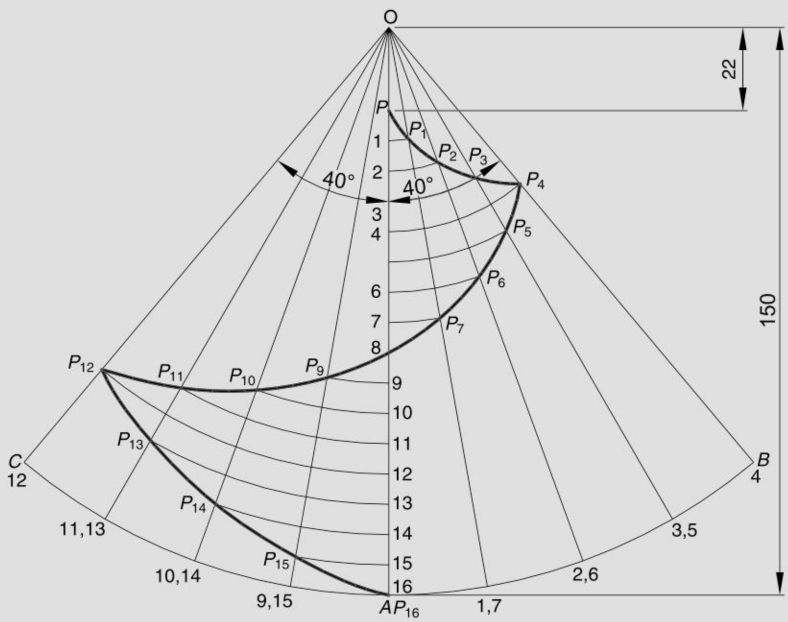


Fig. 4.14

Construction: Fig. 4.14

1. Draw a vertical line *OA* = 150 mm.
2. Draw lines *OB* and *OC* with *O* as the centre subtending angle 40º taken on the right and the left side of the vertical line respectively.
3. Mark point *P* along *OA* and at a distance of 22 mm from *O*.
4. Divide the angles *AOB* and *AOC* into 4 equal parts. As the point *P* travels through an angle of 160º (40º + 80 º+ 40º = 160º), divide the vertical line *PA* into 16 equal parts.
5. With centre *O* and radius *O*1′, draw an arc to get P_1 on line *O*1.

6. Similarly, with the same centre O and radii $O2'$, $O3'$, $O4'$…, draw arcs to get P_2, P_3, P_4,… on lines $O2$, $O3$, $O4$,… respectively.
7. Join P_1, P_2, P_3, P_4, etc., by a smooth curve. The curve is the required locus of point P.

EXERCISE 4

Cycloid

1. A circle having a 40 mm diameter rolls along a line for one revolution clockwise. Draw and name the locus of a point on the circle which is in contact with the line. **[RGPV June 2008]**
2. A circle having 50 mm diameter rolls along a straight line without slipping. Draw the curve traced out by a point P on the circumference for one complete revolution of the circle. Name the curve. **[RGPV June 2009, April 2010]**
3. A wheel of 50 mm diameter rolls on a straight road surface without slip. Trace the path of point of contact for one complete revolution of the wheel. **[RGPV Feb. 2011]**
4. Draw a cycloid given the diameter of a generating circle as 50 mm. Also draw a normal and tangent at the centre point of the cycloid. **[RGPV Dec. 2007]**
5. A circle having a 50 mm diameter rolls on a straight line without slipping. In the initial position the diameter AB of the circle is parallel to the line on which it rolls. Draw the loci of the points A and B for one revolution of the circle. **[RGPV June 2011]**
6. A circle having a 60 mm diameter rolls on a horizontal line for half revolution and then a vertical line upwards for another half. Draw the curve traced out by a point lying on the circumference of the circle. Assume that the horizontal and the vertical line constitute a corner.

Epicycloid

7. Draw an epicycloid generated by a rolling circle having a 60 mm diameter for one complete revolution. The radius of the directing circle is 100 mm. Draw a tangent and a normal to the epicycloid at 150 mm from the centre of the directing circle.
8. A cycle wheel having a 50 cm diameter rolls over a culvert with 175 cm diameter. Draw the path traced out a point on the circumference of the cycle wheel for one complete revolution. **[RGPV Dec. 2006]**
9. Draw the locus of a point lying on the circumference of a wheel having a 60 mm diameter for its one complete revolution, when it passes over a segmental arched culvert of 0.15 m radius.
10. A rolling circle having a 40 mm diameter AB, rolls on a fixed disc having a 60 mm diameter with external contact. Draw the loci of path traced by the points A and B of the rolling circle for one complete revolution, when one of the end points of diameter AB is in contact to the disc at the starting position.
11. Construct a cardioid taking the diameter of the rolling and directing circles as 60 mm.

12. A circle of 40 mm diameter rolls on the outside of a base circle of the same diameter. Draw the curve traced by a point of the same diameter. Draw the curve traced by a point on the rolling circle for one complete revolution of the rolling circle.
[RGPV June 2003]

Hypocycloid

13. Draw a hypocycloid generated by a rolling circle of 60 mm diameter for its one complete revolution. The radius of the directing circle is 100 mm. Draw a tangent and a normal to the hypocycloid at 50 mm from the centre of the directing circle.
14. A circle of 50 mm diameter rolls within a circle of 150 mm diameter with internal contact. Draw the locus of a point lying on the circumference of the rolling circle for its complete turn. Name the curve. Also draw a tangent and a normal to the curve, at a point 40 mm from the centre of the bigger circle.
15. A circus man rides on a motorcycle inside a globe of 6 m diameter. The motor cycle wheel is 1 m in diameter. Draw the locus of a point lying on the circumference of the wheel of motor cycle for its one complete turn.
16. Draw the locus of a point lying on the circumference of a circle of 70 mm diameter, which rolls on a circle of 140 mm diameter with internal contact for one complete rotation.

Involute

17. A coir is unwound from a drum of 30 mm diameter. Draw the locus of the free end of the coir for unwinding through an angle of 360º. Draw also a normal and tangent at any point on the curve.
[RGPV Aug. 2010]
18. Draw the involute of an equilateral triangle of 25 mm side.
[RGPV Dec. 2010]
19. Draw an involute of a given square of 25 mm side. **[RGPV June 2006]**
20. A line rolls over a square of 30 mm side without slipping. Draw the curve traced out by a point on the line. Name the curve. **[RGPV Dec. 2005]**
21. An elastic string of 150 mm length has its one end attached to the circumference of a circular disc of 40 mm diameter. Draw the curve traced out by the other end of the string when it is completely wound around the disc, keeping the string always tight.

Archimedean Spiral

22. Construct an Archimedean spiral of 1-1/2 convolution given, the greatest diameter 120 mm and minimum 30 mm diameter. **[RGPV June 2006]**
23. Construct an Archimedean spiral of two convolutions given the greatest and shortest radii as 84 mm and 12 mm respectively. **[RGPV Dec. 2007]**
24. A point *P* moves towards another point *O*, 60 mm from it and reaches it while moving around it once, its movement towards *O* being uniform with its movement around it. Draw the curve traced out by the point *P*. **[RGPV June 2004]**

25. Draw an Archimedean spiral of 1.5 convolutions, for the shortest and the greatest radii as 20 mm and 90 mm respectively. Also, draw a tangent and a normal to the spiral at a point 65 mm from the pole.
26. Draw an Archimedean spiral for 2 convolutions, which starts at the pole and has a radial movement of 48 mm during each convolution.
27. Draw an Archimedean spiral for one convolution with a shortest distance of 125 mm and radial increment of 5 mm for each 30º.
28. A link *OA*, 80 mm long rotates about *O* in anticlockwise direction. A point *M* on the link, 20 mm away from *O* moves and reaches the end *A*, while the link has rotated through 2/5 of a revolution. Assuming the movement of the link and the point to be uniform, trace the path of the point *M*. **[RGPV Dec. 2003]**
29. A point moves along a bar at an uniform speed. The bar rotates about its end *O* at an uniform speed. Name and construct the path of a point *P* starting from a position 20 mm away and move up to 60 mm away from the fixed end of bar during its one revolution. Draw a tangent at a point 45 mm away from *O*. **[RGPV Dec. 2008]**

Logarithmic Spiral

30. Draw a logarithmic spiral of one convolution, given the shortest distance as 10 mm and the ratio of the length of adjacent radii enclosing 30º as 10:9. Draw a tangent and a normal to the curve at a point 50 mm from the pole.
31. Draw a logarithmic spiral for 1.25 convolutions such that the angle between the consecutive radii is 30º, the ratio of succeeding radii is 7:6 and the greatest radius is 120 mm. Also draw a tangent and normal at a point 70 mm from the pole.
32. A circular disc having 120 mm diameter *AB* rotates with uniform angular velocity. The point *P* which is at *A* moves with uniform linear velocity and reaches the point *B*, when the disk completes one revolution. Trace the locus of the point *P* moving from *A* to *B*.

REVIEW QUESTIONS

1. Differentiate between epicycloid and hypocycloid.
2. Define a cycloid? How a tangent is drawn at a point on a cycloid?
3. What is an epicycloid? Give its practical applications.
4. What is a hypocycloid? Give its practical applications.
5. Define an involute of a polygon.
6. What is an Archimedean spiral? Define the term convolution.
7. Differentiate between an Archimedean and a logarithmic spiral.
8. What is the nature of hypocycloid when radius of the directing circle is (a) equal to the diameter of the rolling circle, and (b) twice the diameter of the rolling circle?

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) The locus of a point lying on the circumference of the circle which rolls on a straight line is known as
(a) cycloid (b) hypocycloid (c) epicycloid (d) circle

ii) Name the curve traced out by a point on the circumference of a circle, which rolls outside another circle of same diameter.
(a) Cycloid (b) Hypocycloid (c) Cardioid (d) None of these

iii) Name the curve traced out by a point on the circumference of a circle, which rolls on another circle of larger diameter.
(a) Epicycloid (b) Involute (c) Spiral (d) None of these

iv) When a circle rolls inside another circle of twice its diameter, the curve traced out by a point on the circumference of the rolling circle will be a/an
(a) straight line (b) epicycloid (c) spiral (d) none of these

v) The curve traced by a point on a straight line which rolls on a circle, without slipping is called a/an
(a) cycloid (b) epicycloid (c) hypocycloid (d) involute

vi) When a straight line rolls on the circumference of a semicircle, the locus of its end point is called a/an
(a) cycloid (b) epicycloid (c) hypocycloid (d) involute

vii) Involute curve is used in
(a) chains (b) gears (c) cams (d) pulleys

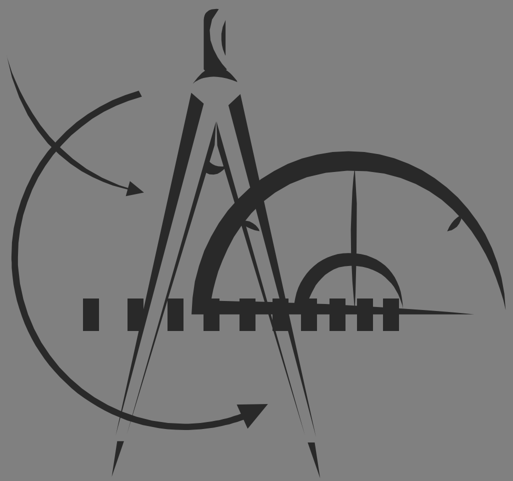
viii) When a pendulum oscillates, name the locus of a point moving along its string at a constant speed.
(a) Cycloid (b) Involute (c) Spiral (d) Helix

ix) The geometrical name of the curvature of the coil used in spiral binding is
(a) archimedean spiral (b) logarithmic spiral
(c) involute (d) none of these

Answers

(i) a (ii) c (iii) a (iv) a (v) d (vi) d (vii) b (viii) c (ix) d

5



Orthographic Projections

- Introduction
- Multi-View Projection
- First-Angle Projection
- Third-Angle Projection
- Second-Angle and Fourth-Angle Projections
- Symbols for Orthographic Projection
- Assumptions
- General Preparation for Multi-View Drawings

5.1 INTRODUCTION

An orthographic projection is a parallel projection in which the projectors are parallel to each other and perpendicular to the plane of projection. See Fig. 5.1(a) and (b). An observer viewing a surface, a block or an object, is considered to be at an infinite distance so that the rays of sight from the eyes are parallel to each other and perpendicular to the plane of projection. The view of the surface, block or object on the plane of projection will be exactly similar to its front surface. The lines from the planes to the block (i.e., rays of sight) are called projectors.

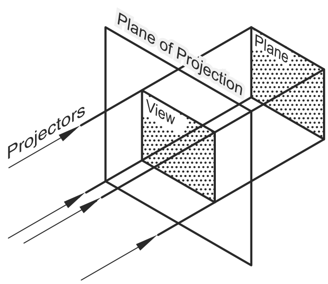


Fig. 5.1(a)

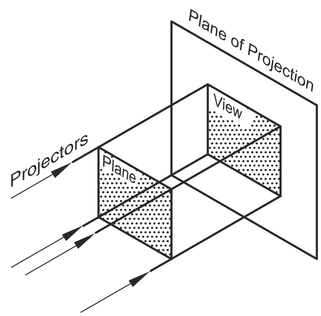


Fig. 5.1(b)

The orthographic projection technique can produce either pictorial drawings that show all the three dimensions in one view, or multi-views that show only two dimensions of the object in each view.

5.2 MULTI-VIEW PROJECTION

The multi-view projection is an orthographic projection in which the object is preferred to be kept in a manner that one of its faces is perpendicular to the projectors and parallel to the plane of projection. In such a case, the image of a three-dimensional object gives the true dimension of its front face.

The orthographic projections are obtained on two principal planes (also known as reference planes) having negligible thickness, namely, vertical plane (V.P.) and horizontal plane (H.P.), as shown in Fig. 5.2(a). The principal planes are perpendicular to each other and they divide the space into four quadrants. They are usually called angles, and the following terms are used to define them.

1. First-angle projection: It is the space in front of V.P. and above H.P.
2. Second-angle projection: It is the space behind V.P. and above H.P.
3. Third-angle projection: It is the space behind V.P. and below H.P.
4. Fourth-angle projection: It is the space in front of V.P. and below H.P.

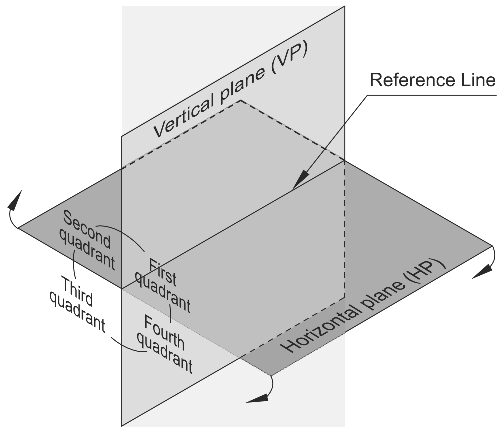


Fig. 5.2(a) *Principal planes*

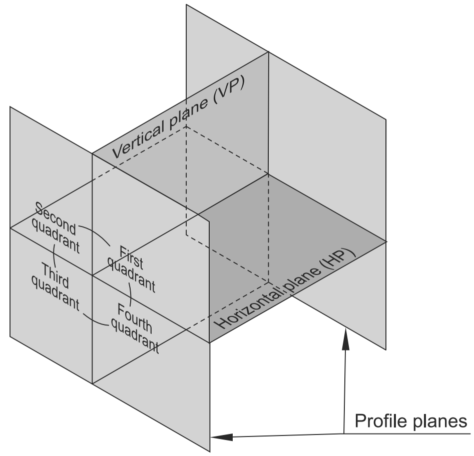


Fig. 5.2(b) *Profile planes*

5.3 TERMINOLOGY

For multi-views, the following terms are used frequently.

Vertical plane A vertical plane, also known as 'Front Reference Plane', is assumed to be placed vertically and is denoted by V.P. See Fig. 5.2(a) and (b).

Horizontal plane A horizontal plane, also known as 'Horizontal Reference Plane', is assumed to be placed horizontally and is denoted by H.P. It is perpendicular to the V.P.

Profile plane The plane perpendicular to both the principal planes, as shown in Fig. 5.2(b), is known as a profile plane. The plane on the right end of the principal planes is known as the right profile plane, while the plane on the left end of the principal planes is known as the left profile plane.

Reference line The line of intersection between the principal planes is the known as reference line. It is also popularly called an *XY* line.

Front view The view of an object obtained by observing it from the front and drawn on the V.P. is known as the front view (FV) or elevation.

Top view The view of an object obtained by observing it from the top and drawn on the H.P. is known as the top view (TV) or plan.

Side view The view of an object obtained by observing it from the left-hand side or right-hand side drawn on a profile plane, is known the side view.

5.4 FIRST-ANGLE PROJECTION

In first-angle projection, a three-dimensional object is considered, to lie in the first-angle, i.e., in front of the V.P. and above the H.P. The observer, who is theoretically at infinite distance, looks at the object from the front. The rays of sight are parallel to each other and perpendicular to the V.P. See Fig. 5.3(a). The view obtained on the V.P. is similar to the front surface of the object and is known as front view. It may be noted that the front view shows only the length and height of the object. It does not indicate the width.

Again, the observer looks at the object from the top, such that the rays of sight are parallel to each other and perpendicular to the H.P. See Fig. 5.3(b). The view obtained on the H.P. is similar to the top surface of the object and is known as top view. It may be noted that the top view shows only the length and width of the object. It does not indicate the height.

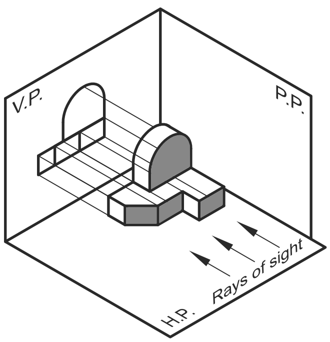


Fig. 5.3(a) *Rays of sight are perpendicular to V.P. to obtain the front view*

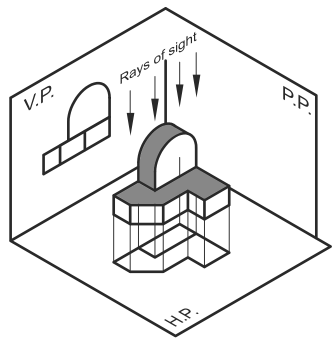


Fig. 5.3(b) *Rays of sight are perpendicular to H.P. to obtain the top view*

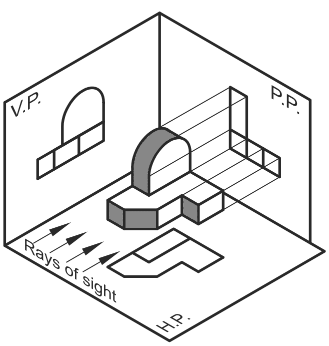


Fig. 5.3(c) *Rays of sight are perpendicular to P.P. to obtain the left-hand side view*

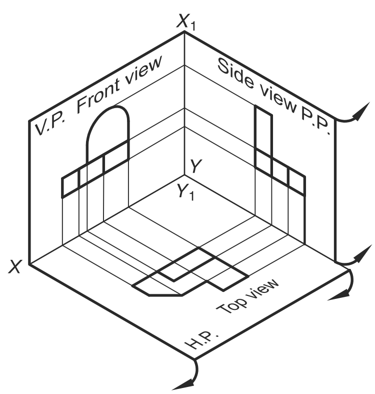


Fig. 5.3(d) *Shows co-relation between front, top and side views*

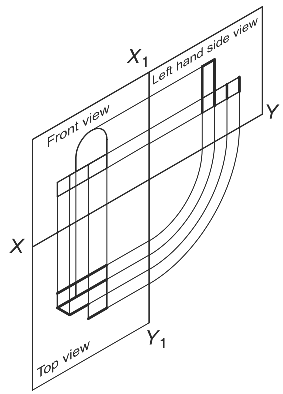


Fig. 5.3(e) *Rays of sight are perpendicular to P.P. to obtain the left-hand side view*

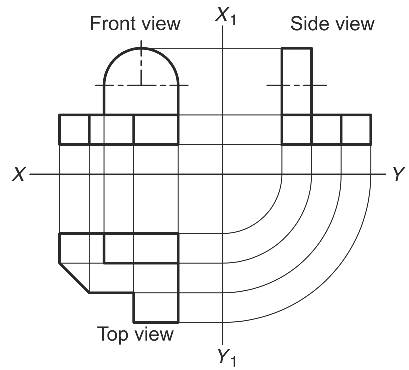


Fig. 5.3(f) *Final orthographic projections (front, top and left-hand side views)*

Now consider that the observer looks at the object from the left-hand side, such that the rays of the sight are parallel to each other and perpendicular to the profile plane (P.P.). See Fig. 5.3(c). The view obtained on the P.P. is similar to the side surface of the object and is known as side view. It may be noted that the side view shows only the width and height of the object. It does not indicate the length.

The co-relation among the front, top and side views are shown in Fig. 5.3(d). The front, top and side views formed on the planes, V.P., H.P. and P.P., respectively, are at right angle to each other and cannot be drawn on a plain drawing sheet. Therefore, it is customary to rotate the H.P. in a clockwise direction about the reference line *XY* through 90º, such that it becomes co-planer with V.P. Similarly, the profile plane (P.P.) is rotated about X_1Y_1 line. Thus, top and side views become co-planer with the front view, as shown in Fig. 5.3(e). This gives the final projection and is produced on the drawing sheet, as shown in Fig. 5.3(f).

Figure 5.4 shows another example of first-angle projection. An attempt has been made to draw the front, top, left-hand side, right-hand side, bottom and rear views.

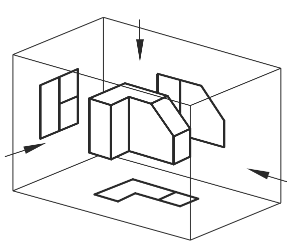


Fig. 5.4(a) *Object placed in mutually perpendicular planes*

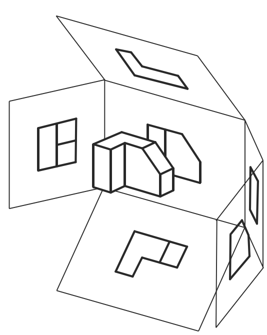


Fig. 5.4(b) *Box is opened for multi-view drawing*

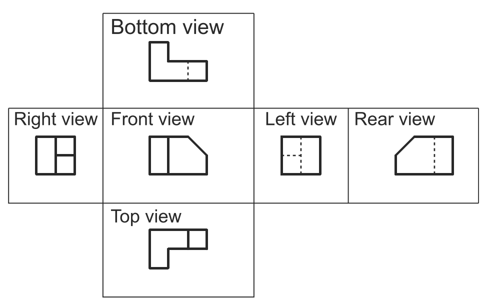


Fig. 5.4(c) *Multi-view drawing*

5.4.1 Features of First-Angle Projection

The features of a first-angle projection are given below.

1. The object lies in the first-angle, i.e., in front of the V.P. and above the H.P.
2. The object lies between the observer and the plane of projection.
3. The top view is drawn below the front view.
4. The left-hand side view is drawn to the right side of the front view.
5. The right hand side view is drawn to the left side of the front view.

5.5 THIRD-ANGLE PROJECTION

In a third-angle projection, a three-dimensional object is considered to lie in the third-angle, i.e., behind the V.P. and below the H.P. The observer who is theoretically at infinite distance looks at the object from the front. The rays of sight are parallel to each other and perpendicular to the V.P. See Fig. 5.5(a). The view obtained on the V.P. is similar to the front surface of the object and is known as front view. It may be noted that the front view shows only the length and height of the object. It does not indicate the width.

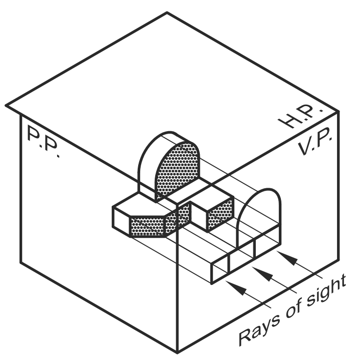


Fig. 5.5(a) *Rays of sight are perpendicular to V.P. to obtain the front view*

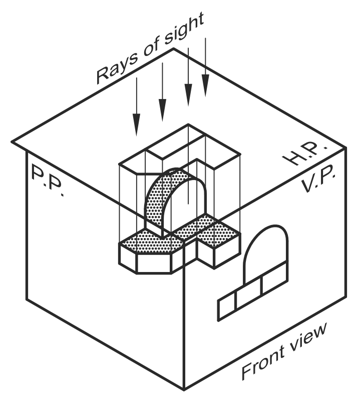


Fig. 5.5(b) *Rays of sight are perpendicular to H.P. to obtain the top view*

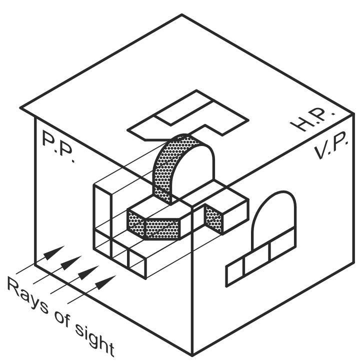


Fig. 5.5(c) *Rays of sight are perpendicular to P.P. to obtain the left-hand side view*

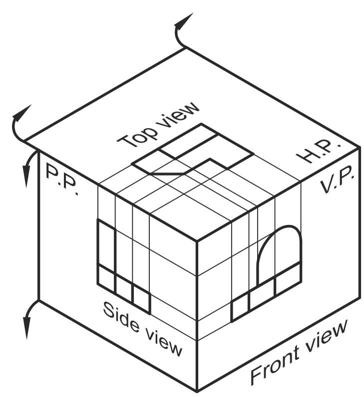


Fig. 5.5(d) *The figure shows co-relation between front, top and side views*

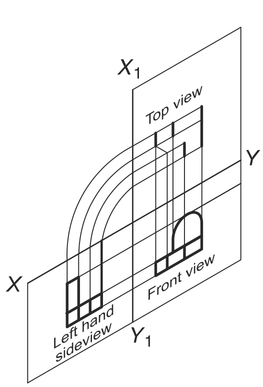


Fig. 5.5(e) *Rays of sight are perpendicular to P.P. to obtain the left-hand side view*

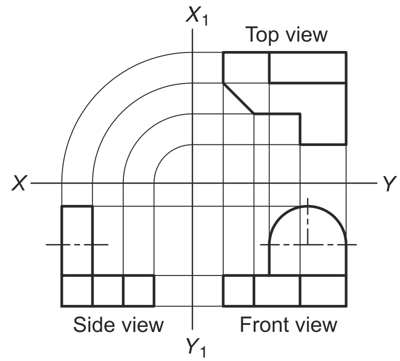


Fig. 5.5(f) *Final orthographic projections (front, top and left-hand side views)*

Again, the observer looks at the object from the top, such that the rays of sight are parallel to each other and perpendicular to the H.P. See Fig. 5.5(b). The view obtained on the H.P. is similar to the top surface of the object and is known as top view. It may be noted that the top view shows only the length and width of the object. It does not indicate the height.

Now consider that the observer looks at the object from the left-hand side such that the rays of the sight are parallel to each other and perpendicular to the profile plane (P.P.). See Fig. 5.5(c). The view obtained on the P.P. is similar to the side surface of the object and is known as side view. It may be noted that the side view shows only the width and height of the object. It does not indicate the length.

The co-relation among the front, top and side views are shown in Fig. 5.5(d). The front, top and side views formed on the planes, V.P., H.P. and P.P. respectively, are at right angle to each other and can not be drawn on a plain drawing sheet. Therefore, it is customary to rotate the H.P. in a clockwise direction about the reference line *XY* through 90º such that it becomes co-planer with V.P. Similarly the profile plane (P.P.) is rotated about X_1Y_1 line. Thus, the top and side views become co-planer with the front view, as shown in Fig. 5.5(e). This gives the final projection and is produced on the drawing sheet, as shown in Fig. 5.5(f).

Figure 5.6 shows another example of third-angle projection. An attempt has been made to draw the front, top, left-hand side, right-hand side, bottom and rear views.

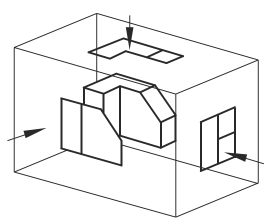


Fig. 5.6(a) *Object is placed in mutual perpendicular planes*

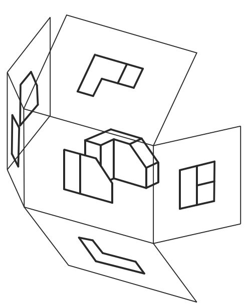


Fig. 5.6(b) *Box is opened for multi-view drawing*

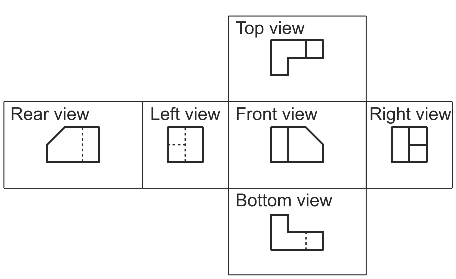


Fig. 5.6(c) *Multi-view drawing*

5.5.1 Features of Third-Angle Projection

The features of a third-angle projection are given below.

1. The object lies in the third-angle, i.e., behind the V.P. and below the H.P.
2. The plane of projection lies between the object and the observer.
3. The top view is drawn above the front view.
4. The left-hand side view is drawn to the left side of the front view.
5. The right-hand side view is drawn to the right side of the front view.

5.6 SECOND-ANGLE AND FOURTH-ANGLE PROJECTIONS

In the second angle projection, the object is considered to lie in the second-Angle, i.e., behind the V.P. and above the H.P., as shown in Fig. 5.7(a). In the fourth-Angle projection, the object is considered to lie in the fourth-angle, i.e., in front of V.P. and below the H.P., as shown in Fig. 5.7(b). In both of these cases, there is a possibility of overlapping of the front and top views after rotation of the H.P. in the clockwise direction about the reference line *XY* and making co-planer with V.P. Thus, these methods of projection are not considered useful.

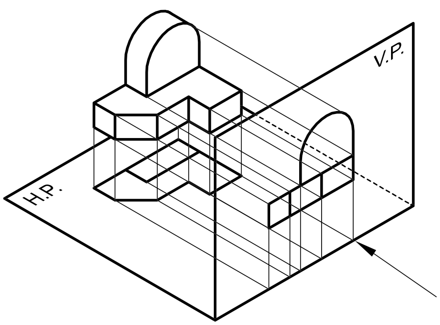


Fig. 5.7(a) *Object placed in second-angle*

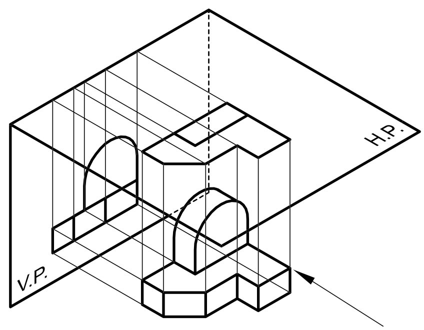


Fig. 5.7(b) *Object placed in fourth-angle*

5.7 SYMBOLS FOR ORTHOGRAPHIC PROJECTION#

It has been seen that the front view and top view will give the clear picture and will not overlap if the object is placed in the first or the third-angle. Thus, internationally, only two methods are adopted for drawing multi-view projections, i.e., the first-angle projection and the third-angle projection.

The symbol recommended by BIS to draw two views of the frustum of a cone is shown in Fig. 5.8(a). The diameters of the frustum of the cone may be taken in the ratio of 1:2, while the length may be taken equal to the diameter at the bigger end. Figure 5.8(b) shows the symbol for the first-angle projection and Fig. 5.8(c)

Readers are advised to refer chapter 1, Fig. 1.2 for the method of inserting symbol in the title block.

shows symbol for the third-angle projection. The method of projection must be indicated in the space provided for the purpose in the title block of the drawing sheet.

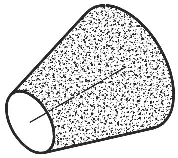


Fig. 5.8(a) *Frustum of a cone*

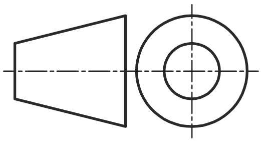


Fig. 5.8(b) *Symbol for first-angle projection*

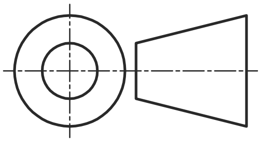


Fig. 5.8(c) *Symbol for third-angle projection*

5.8 ASSUMPTIONS

1. The direction for the principal view is generally indicated on a pictorial view by an arrow. If it is not given, the view of the object showing the important features, which may be chosen from the point of design, assembly, sales, service or maintenance is considered as the front view. It is the most informative view.
2. The hidden part of a symmetrical object is to be treated similar to the visible part.
3. The holes, grooves, etc., are assumed to be drilled or cut through, unless otherwise specified.
4. When the radii for small curves of the fillet are not specified, the lengths of the radii should be assumed.

5.9 GENERAL PREPARATION FOR MULTI-VIEW DRAWINGS

1. Observe the shape and dimensions of the given object carefully and determine the overall dimension for each view. Take a convenient scale for drawing to accommodate the views on the drawing sheet.
2. Decide the direction of the side view and fix the relative positions of the front, top and side views according to the method of projection used. In the first-angle projection, the top view must be placed below the front view, the left-hand side view must lie on the right side of the front view, and right-hand side view must lie on the left side of the front view.
3. There should be sufficient space between the views (front, top and side views).
4. Preferably start drawing the views in which the circular parts of the object are seen as circles or part of them. It becomes simpler to project the points of the circle in other views.
5. The front and top views always lie between the same vertical projectors.
6. The front and side views always lie between the same horizontal projectors.
7. The surface parallel to the reference plane will be seen as true shape of the surface.
8. The surface perpendicular to the reference plane will be seen as a straight line.
9. The invisible edges of the object are represented by dotted lines.
10. The method of projection should be indicated on the drawing.
11. All the views should be dimensioned as discussed in dimensioning.
12. All views should be properly labeled.

5.10 MISCELLANEOUS EXAMPLES

Example 5.1 Fig. 5.9(b)

Pictorial view of an object is shown in Fig. 5.9(a). Using first angle projection, draw its (i) front view, and (ii) top view. Use the direction *X* for the front view.

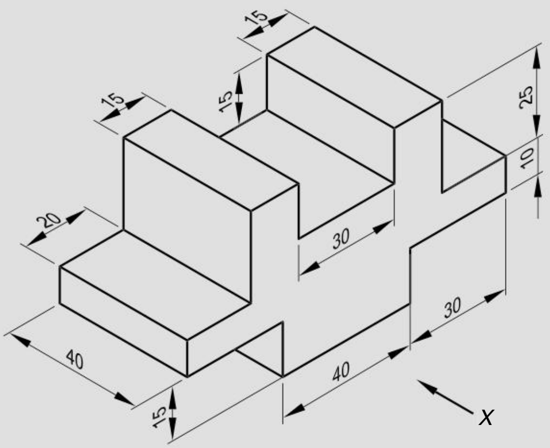


Fig. 5.9(a) *Pictorial view*

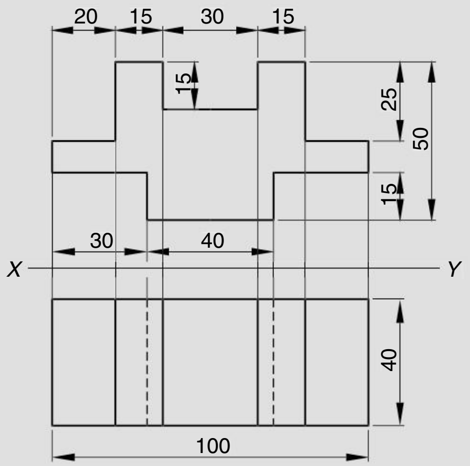


Fig. 5.9(b) *Orthographic views*

Example 5.2 Fig. 5.10(b)

Draw the front view, side view from the left and top view of the block given in Fig. 5.10(a).
[RGPV June 2007]

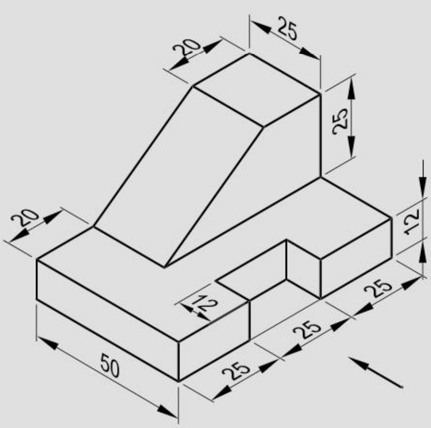


Fig. 5.10(a) *Pictorial view*

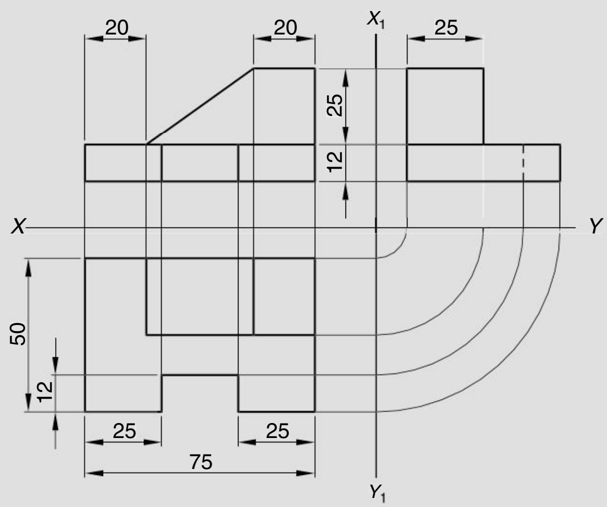


Fig. 5.10(b) *Orthographic views*

Example 5.3 Fig. 5.11(b)

The pictorial view of a block is given in Fig. 5.11(a). Draw the front view, the top view and the side view looking in the direction *A*, *B* and *C* in first-angle projection. [RGPV June 2008]

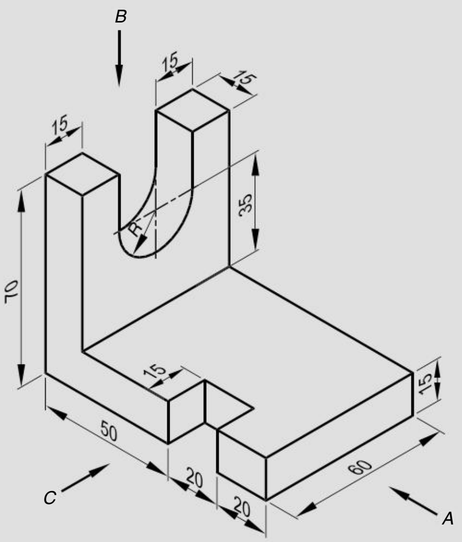


Fig. 5.11(a) *Pictorial view*

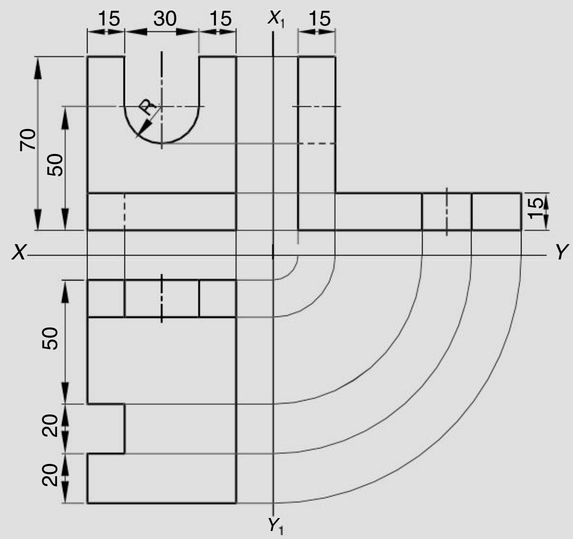


Fig. 5.11(b) *Orthographic views*

Example 5.4 Fig. 5.12(b)

The isometric view of an object is shown in Fig. 5.12(a). Draw its three views looking from the directions shown. [RGPV Dec. 2008]

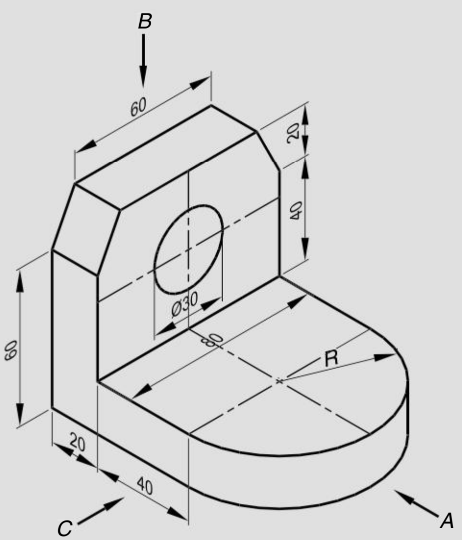


Fig. 5.12(a) *Pictorial view*

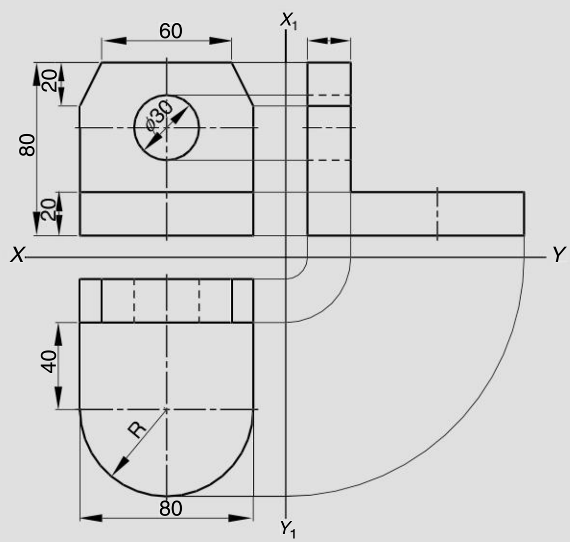


Fig. 5.12(b) *Orthographic views*

Example 5.5 Fig. 5.13(b)

Draw the front view, top view and right-hand side view of the object shown in Fig. 5.13(a).
[RGPV Dec. 2004, Feb. 2005]

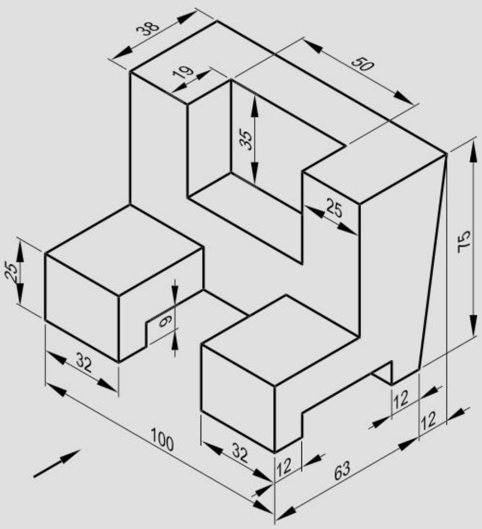


Fig. 5.13(a) *Pictorial view*

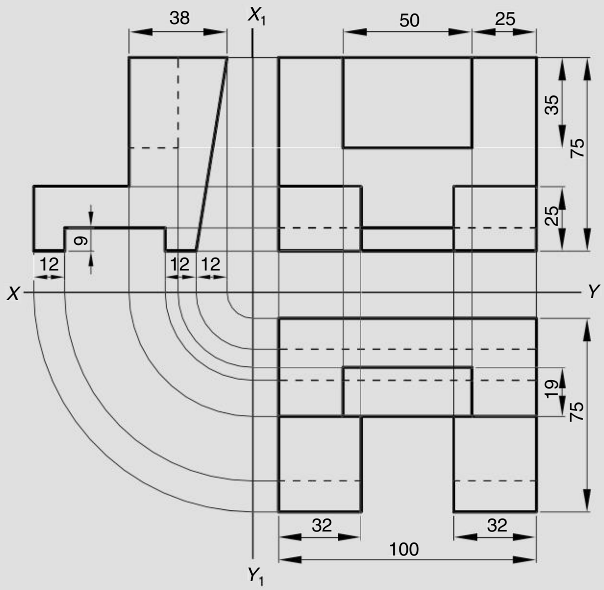


Fig. 5.13(b) *Orthographic views*

Example 5.6 Fig. 5.14(b)

Pictorial view of an object is shown in Fig. 5.14(a). Using first-angle projection, draw its (i) front view looking from the direction *X*, (ii) top view and (iii) end view.

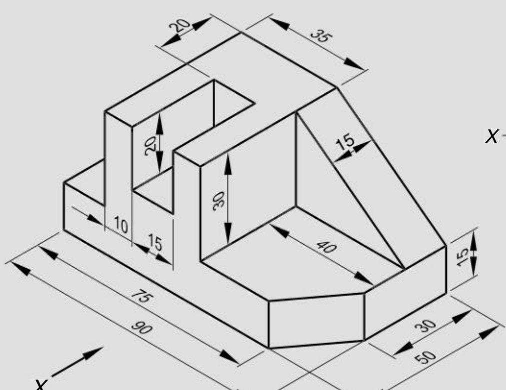


Fig. 5.14(a) *Pictorial view*

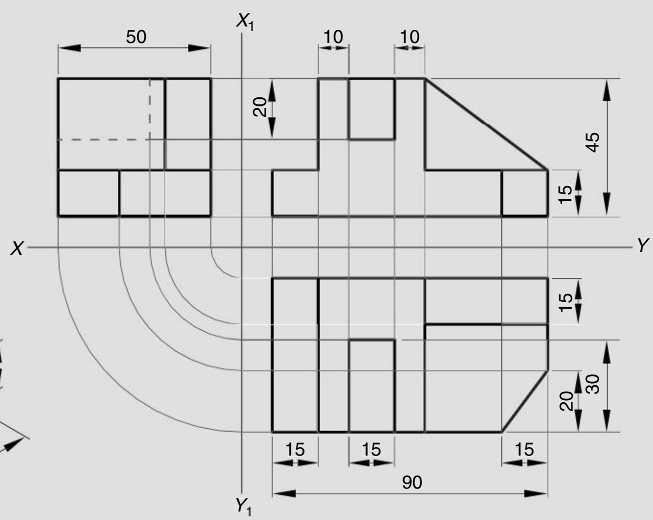


Fig. 5.14(b) *Orthographic views*

Example 5.7 Fig. 5.15(b)

Pictorial view of an object is shown in Fig. 5.15(a). Using first-angle projection, draw its (i) front view in the direction *X*, (ii) top view, and (iii) side view.

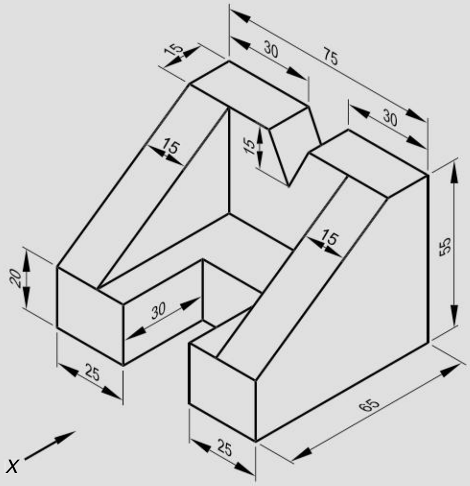


Fig. 5.15(a) *Pictorial view*

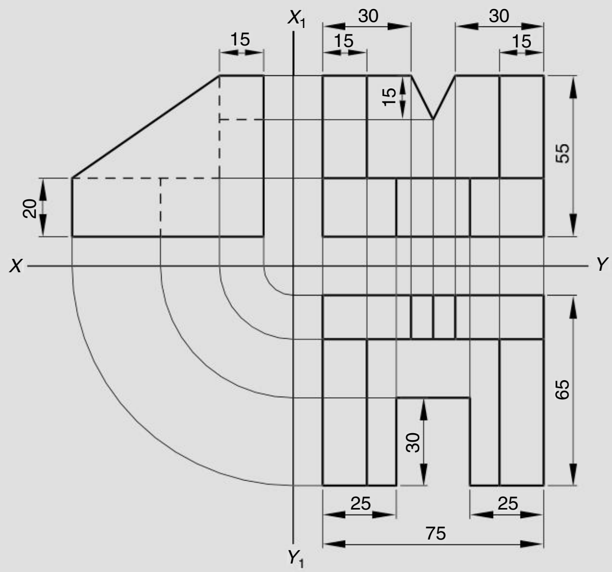


Fig. 5.15(b) *Orthographic views*

Example 5.8 Fig. 5.16(b)

Pictorial view of an object is shown in Fig. 5.16(a). Using first-angle projection, draw its (i) front view, (ii) top view, and (iii) side view. Assume suitable direction for the views. [RGPV Dec. 2003]

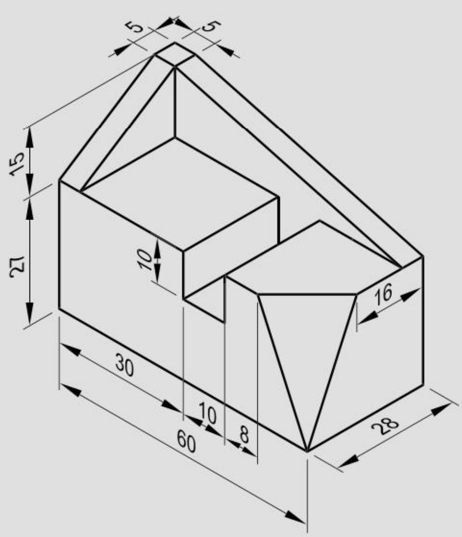


Fig. 5.16(a) *Pictorial view*

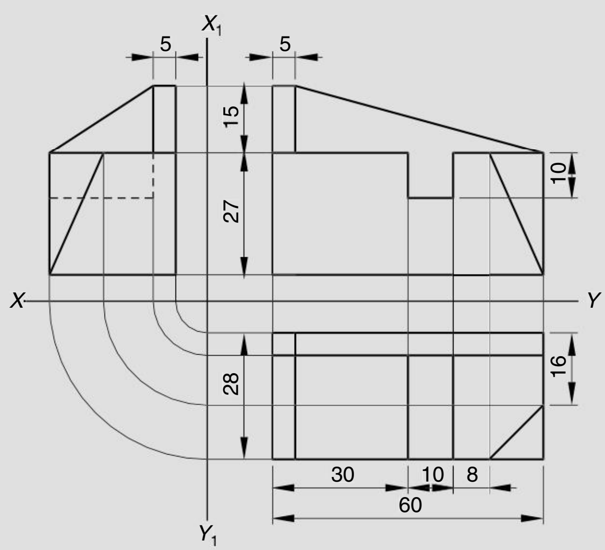


Fig. 5.16(b) *Orthographic views*

Example 5.9 Fig. 5.17(b)

The pictorial view of a block is shown in Fig. 5.17(a). Draw the front view and top view in first angle method of projection. Use the direction *X* for front view. [RGPV Sep. 2009]

Also, draw the end view of the block.

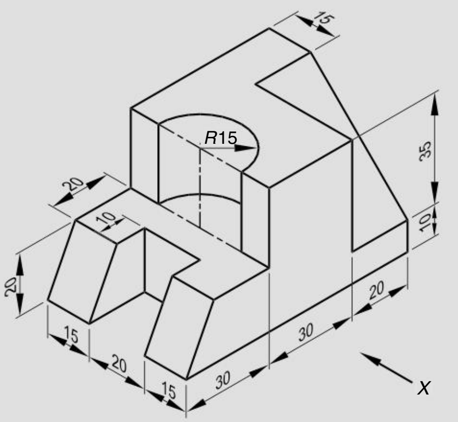


Fig. 5.17(a) *Pictorial view*

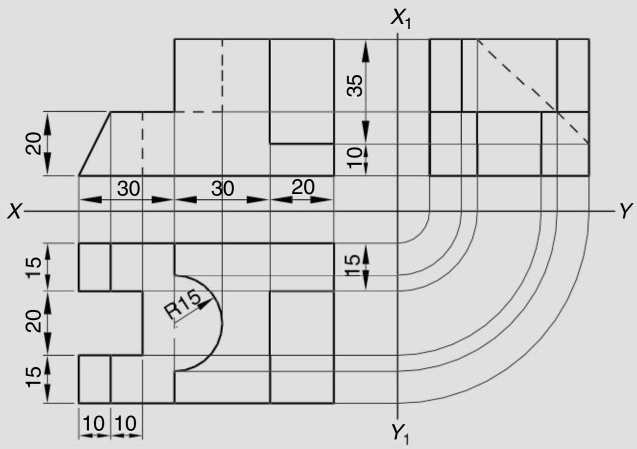


Fig. 5.17(b) *Orthographic views*

Example 5.10 Fig. 5.18(b)

Pictorial view of an object is shown in Fig. 5.18(a). Using first-angle projection, draw its (i) front view in the direction of arrow, (ii) top view and (iii) left-hand side view.

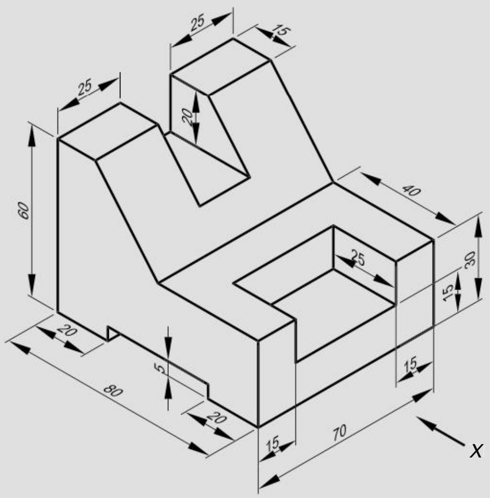


Fig. 5.18(a) *Pictorial view*

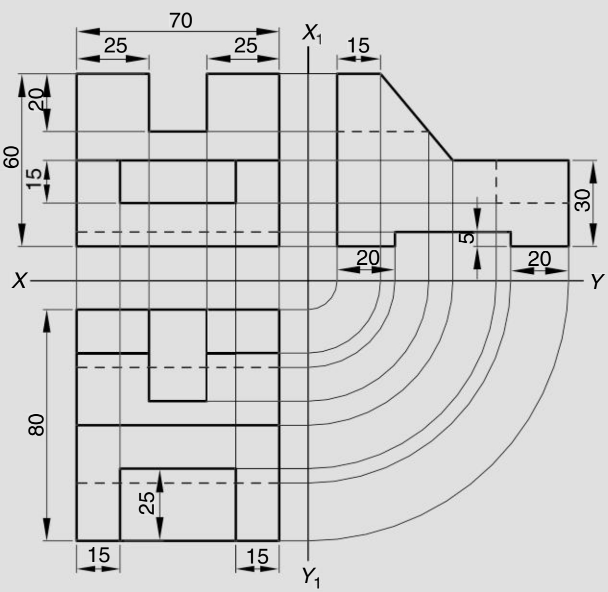


Fig. 5.18(b) *Orthographic views*

Example 5.11 Fig. 5.19(b)

Pictorial view of an object is shown in Fig. 5.19(a). Using first-angle projection, draw its (i) front view in the direction of arrow, (ii) top view and (iii) side view.

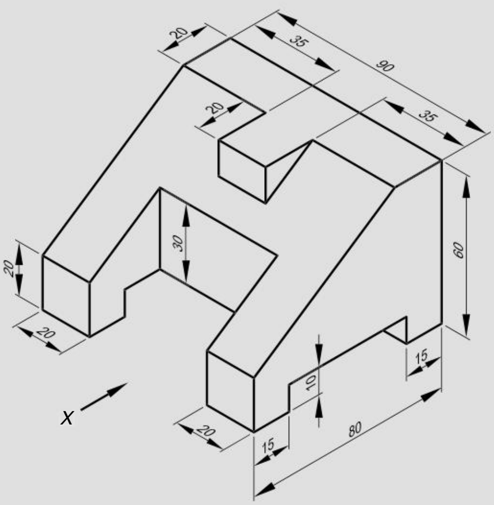


Fig. 5.19(a) *Pictorial view*

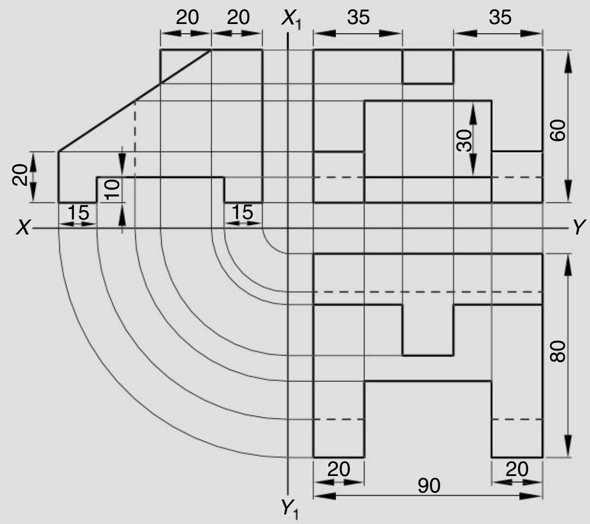


Fig. 5.19(b) *Orthographic views*

Example 5.12 Fig. 5.20(b)

Draw the following views of the drawing shown in the Fig. 5.20(a). (i) front view in the direction of arrow, and (ii) top view. [RGPV June 2006]

Also, draw the right-hand side view of the object.

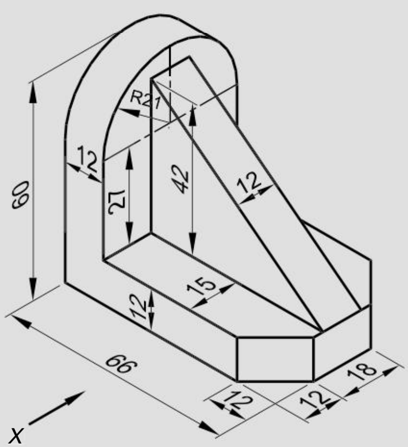


Fig. 5.20(a) *Pictorial view*

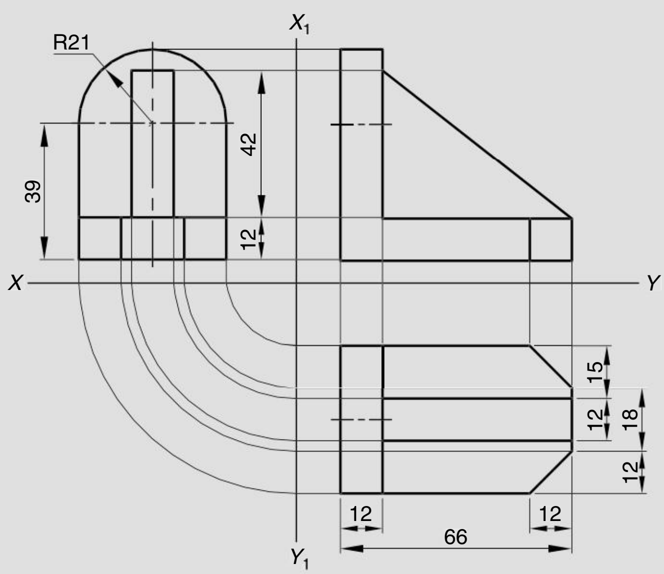


Fig. 5.20(b) *Orthographic views*

Example 5.13 Fig. 5.21(b)

Pictorial view of an object is shown in Fig. 5.21(a). Using first-angle projection, draw its (i) front view in the direction *X*, (ii) top view, and (iii) side view.

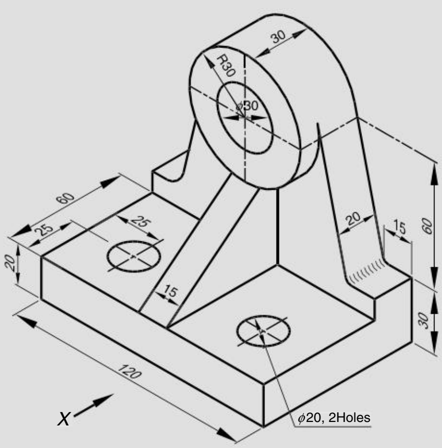


Fig. 5.21(a) *Pictorial view*

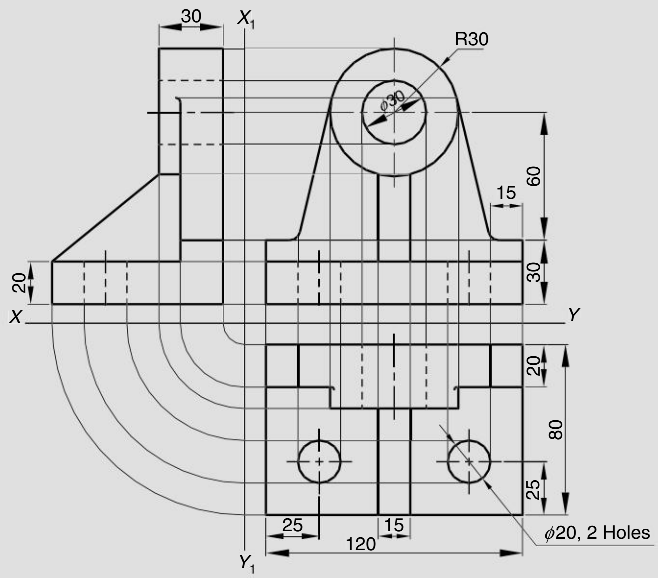


Fig. 5.21(b) *Orthographic views*

Example 5.14 Fig. 5.22(b)

Pictorial view of an object is shown in Fig. 5.22(a). Using first-angle projection, draw its (i) front view in the direction of arrow, (ii) top view and (iii) right-hand side view.

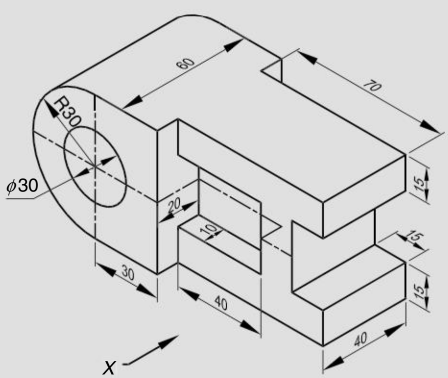


Fig. 5.22(a) *Pictorial view*

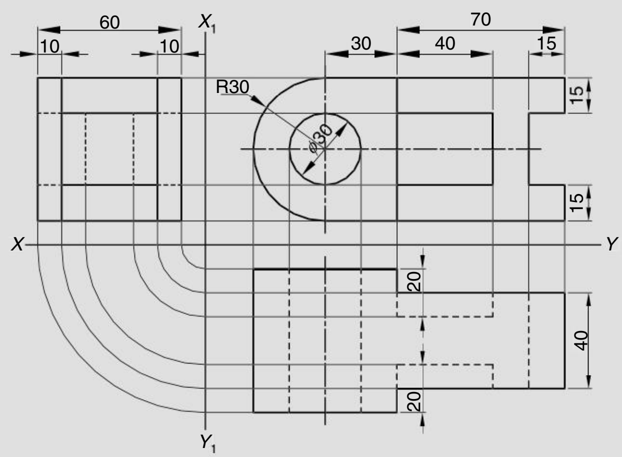


Fig. 5.22(b) *Orthographic views*

Example 5.15 Fig. 5.23(b)

Draw the following views of the drawing shown in Fig. 5.23(a). (i) Front view in the direction of arrow (ii) Side view from the right (iii) Top view. [RGPV Dec. 2006]

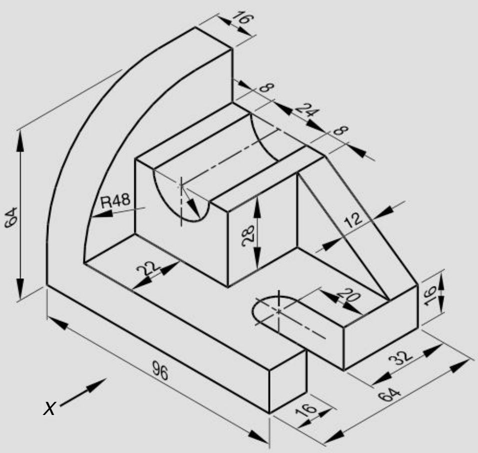


Fig. 5.23(a) *Pictorial view*

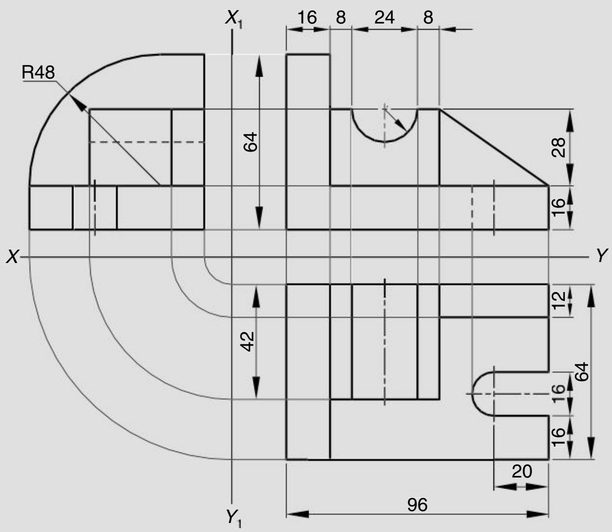


Fig. 5.23(b) *Orthographic views*

Example 5.16 Fig. 5.24(b)

Draw the three views of the bracket shown in Fig. 5.24(a) taking direction *X* as the view for front view. [RGPV Dec. 2007]

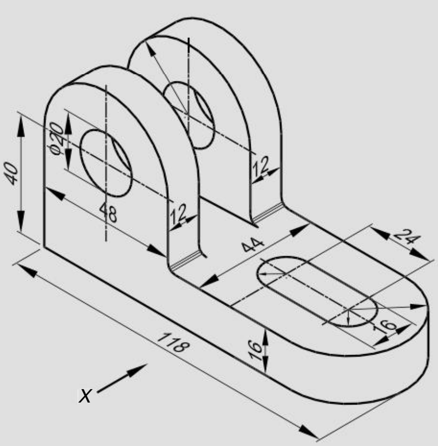


Fig. 5.24(a) *Pictorial view*

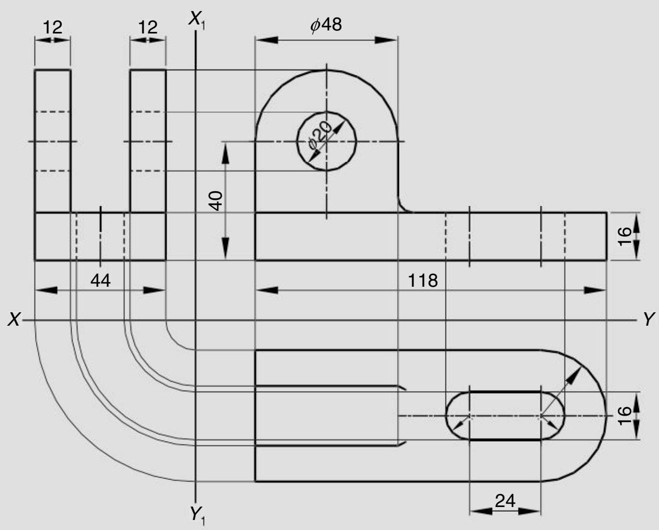


Fig. 5.24(b) *Orthographic views*

Example 5.17 Fig. 5.25(b)

Draw the three views of the bracket shown in Fig. 5.25(a) taking direction *X* as the view for front view. [RGPV Dec. 2007]

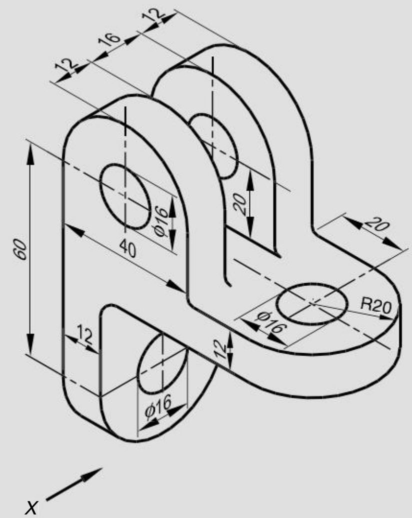


Fig. 5.25(a) *Pictorial view*

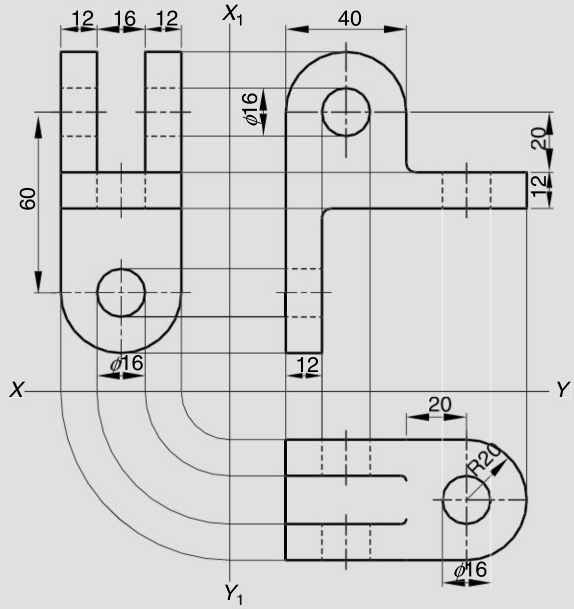


Fig. 5.25(b) *Orthographic views*

EXERCISE 5

Draw the three views of the object shown in Figs. E5.1 to E5.30 taking direction of arrow for the front view. Use first-angle projections. Assume missing data suitably.

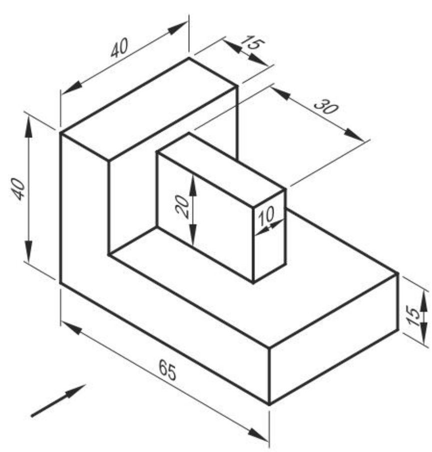


Fig. E5.1

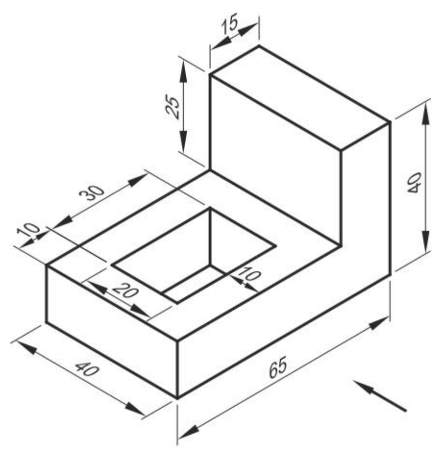


Fig. E5.2

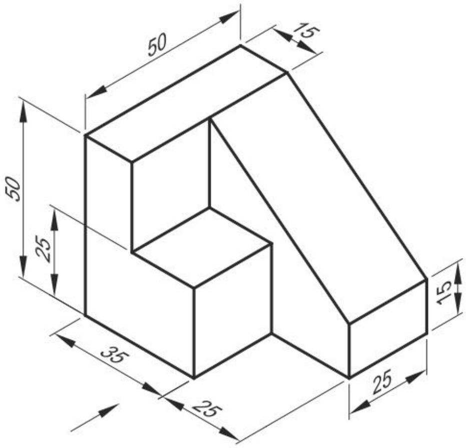


Fig. E5.3

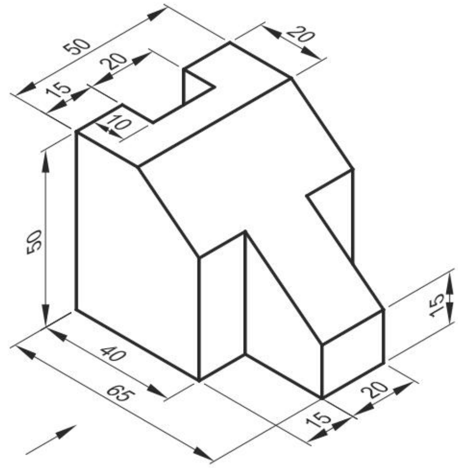


Fig. E5.4

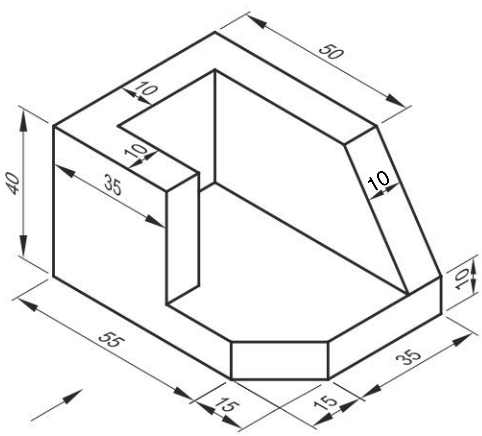


Fig. E5.5

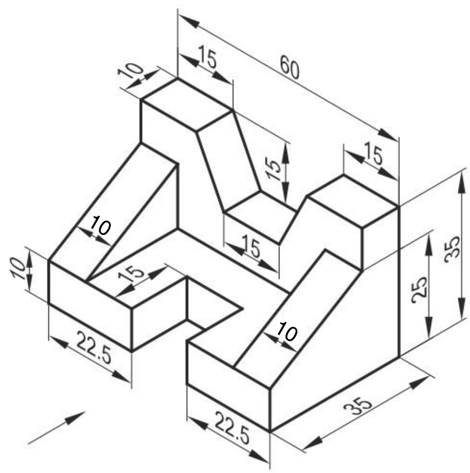


Fig. E5.6

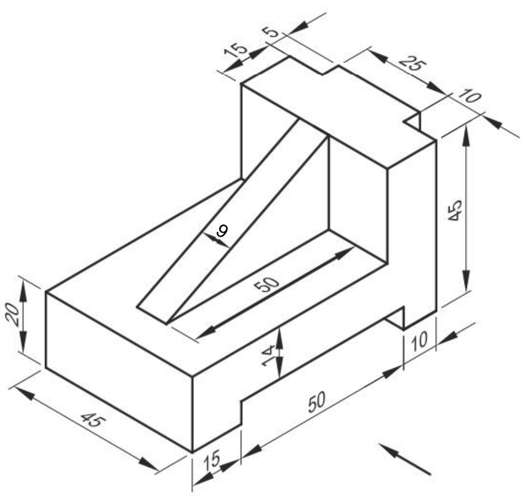


Fig. E5.7

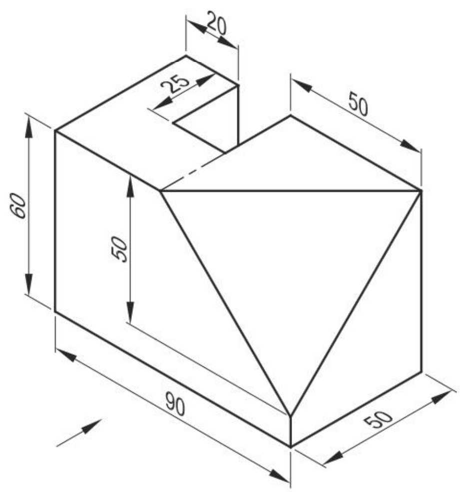


Fig. E5.8

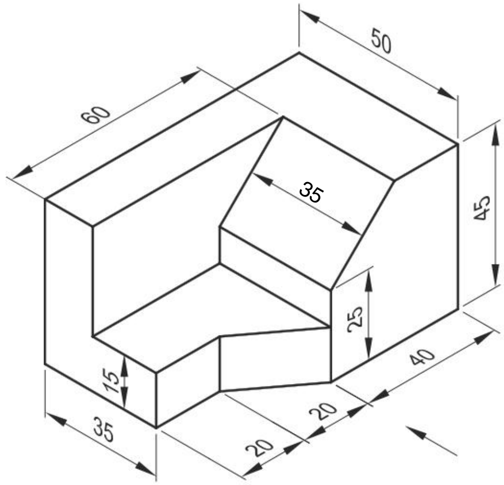


Fig. E5.9

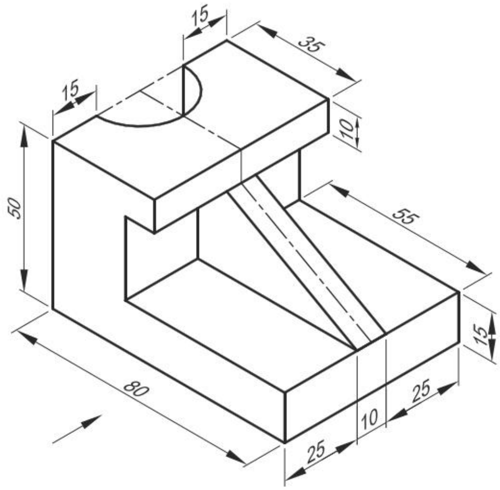


Fig. E5.10

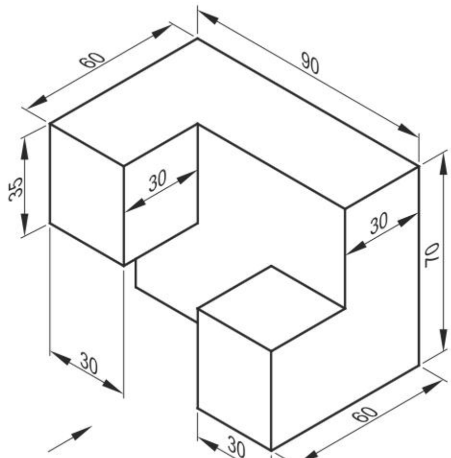


Fig. E5.11

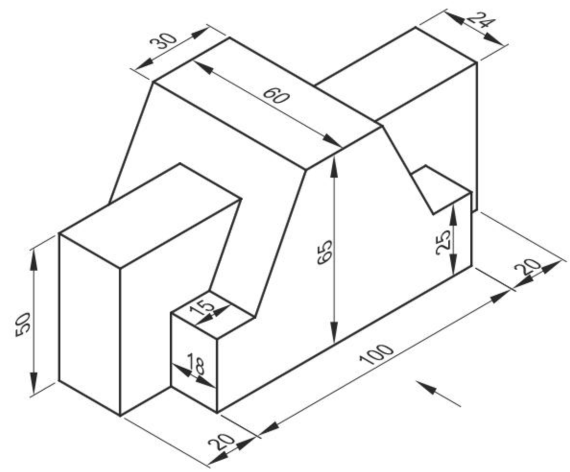


Fig. E5.12

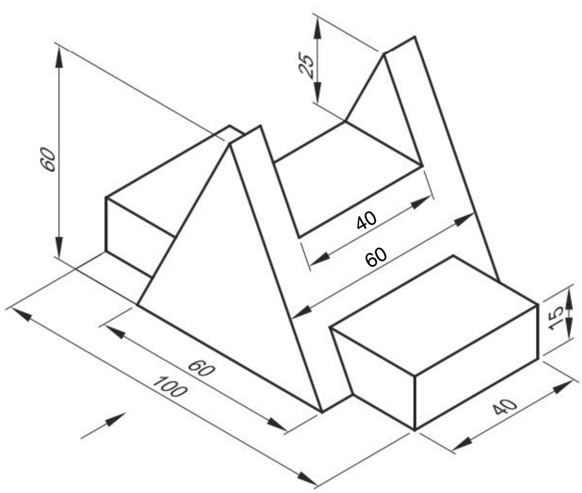


Fig. E5.13

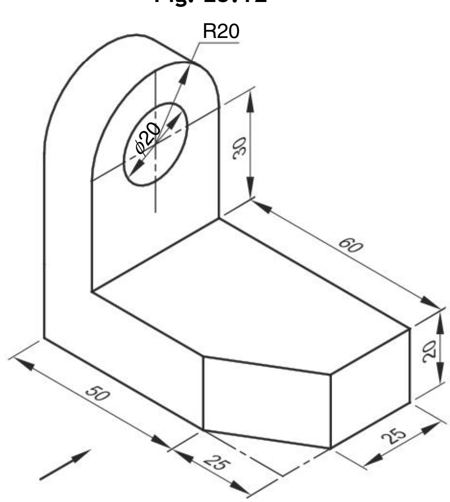


Fig. E5.14

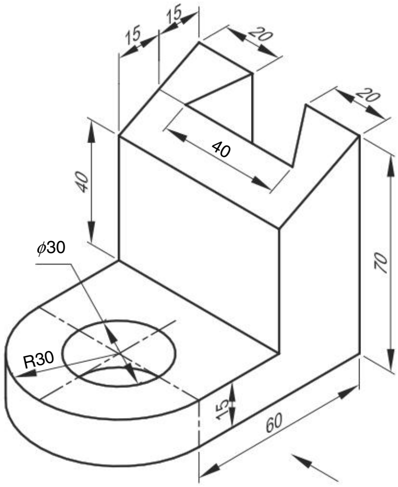


Fig. E5.15

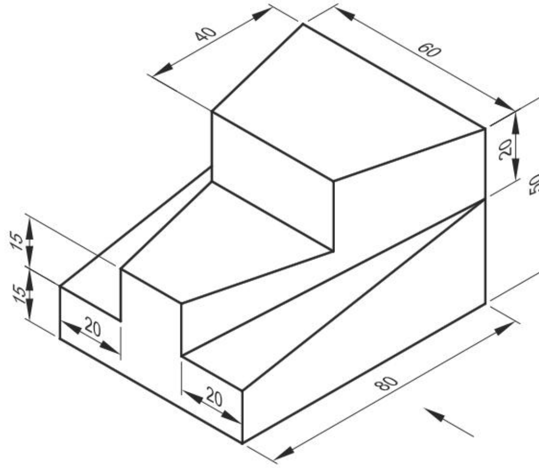


Fig. E5.16

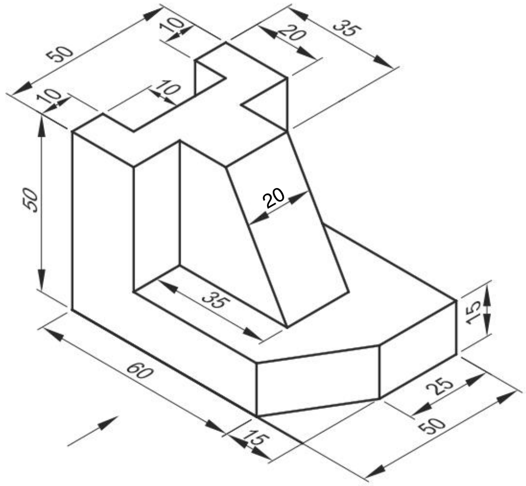


Fig. E5.17

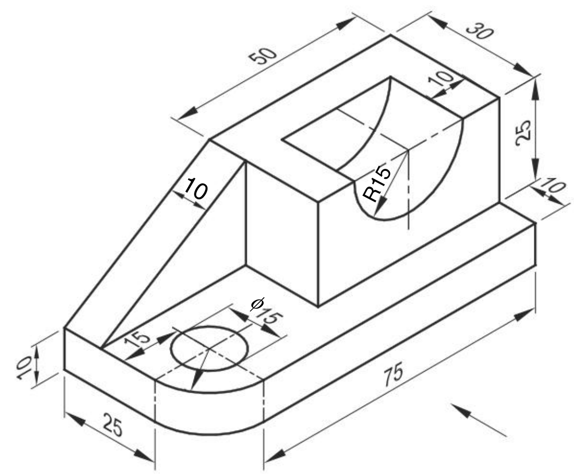


Fig. E5.18

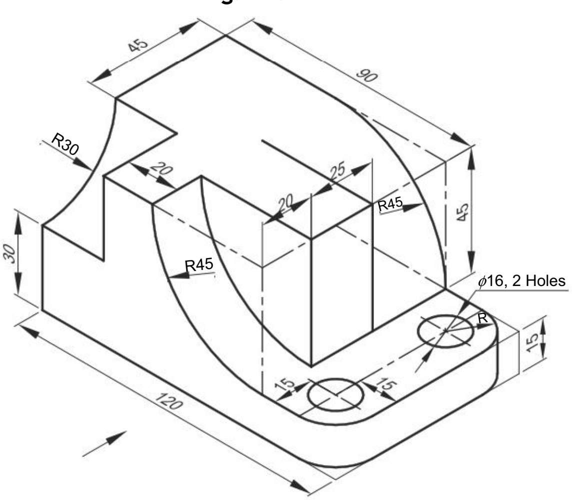


Fig. E5.19

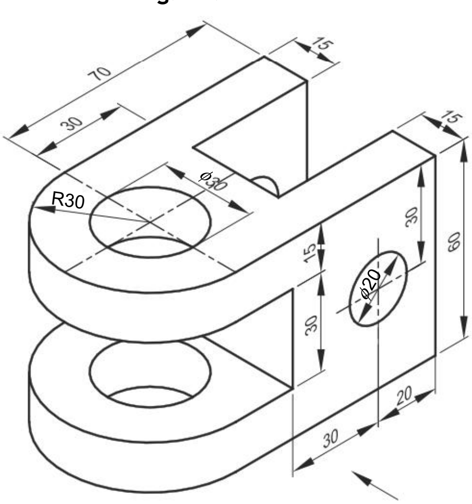


Fig. E5.20

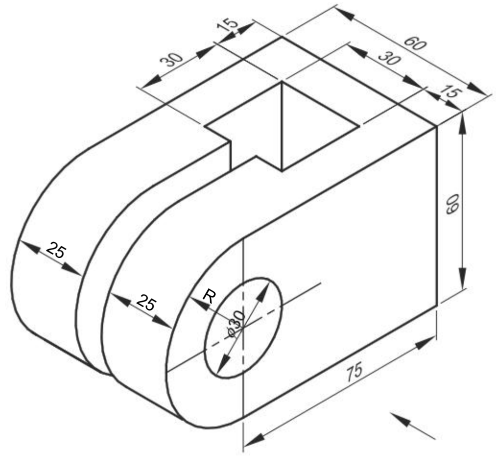


Fig. E5.21

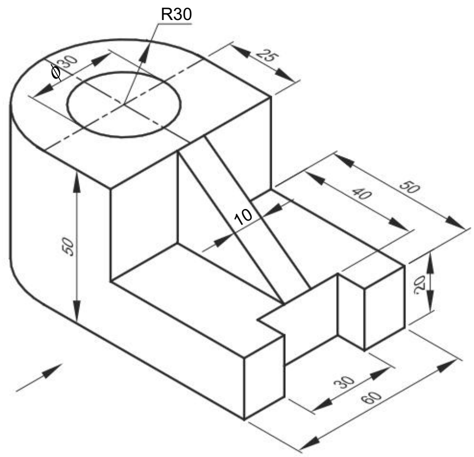


Fig. E5.22

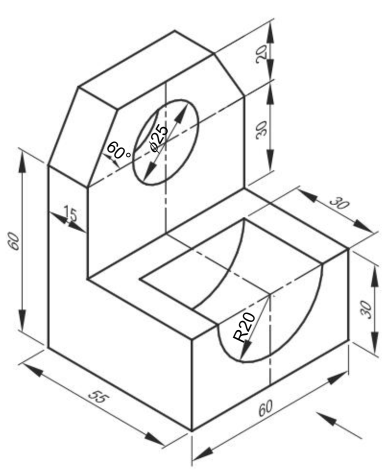


Fig. E5.23

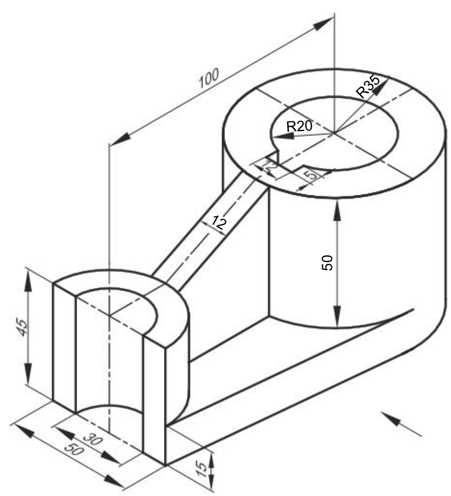


Fig. E5.24

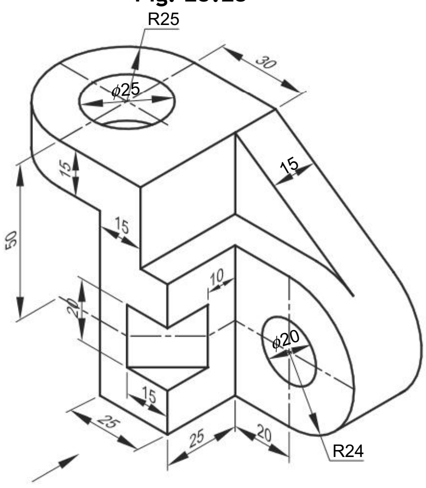


Fig. E5.25

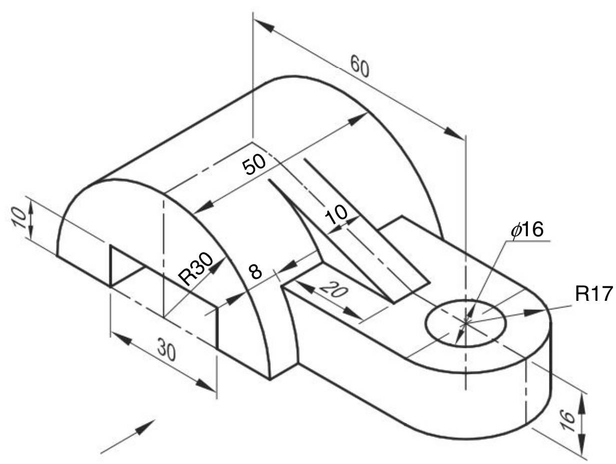


Fig. E5.26

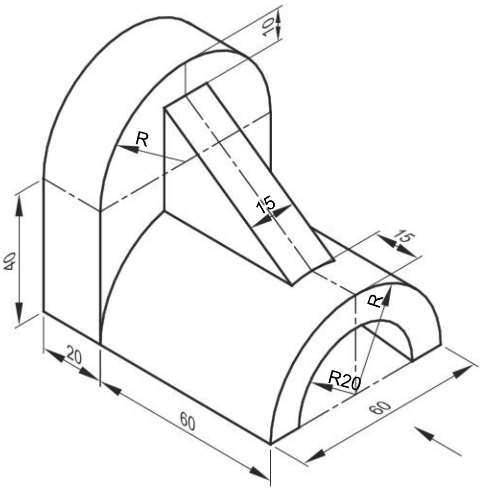


Fig. E5.27

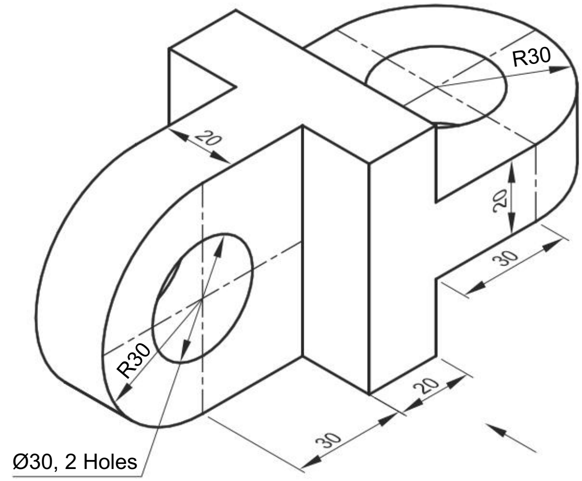


Fig. E5.28

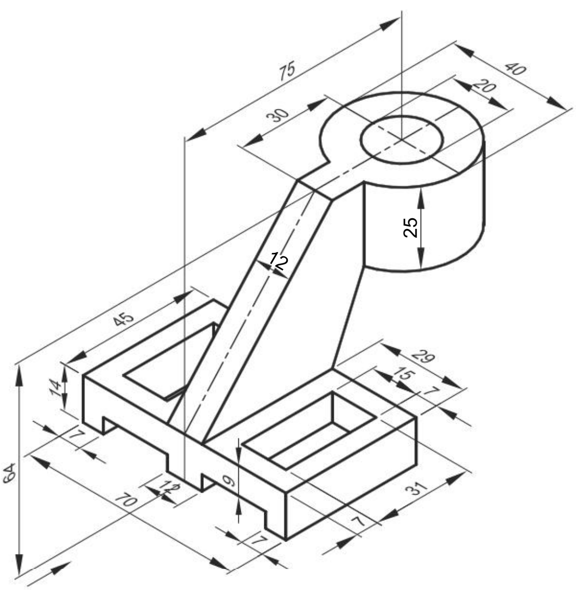


Fig. E5.29

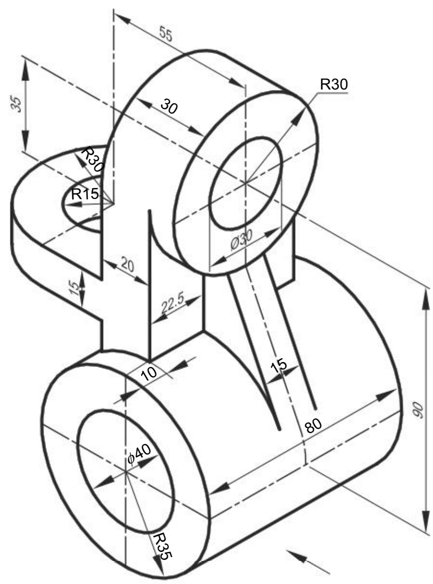


Fig. E5.30

REVIEW QUESTIONS

1. What do you mean by projection? Give its classification.
2. Differentiate between a pictorial view and multi-view.
3. What is an orthographic projection?

4. What is a multi-view projection? How does it differ from axonometric projection?
5. How a solid or an object should be placed on the planes to obtain multi-views. Explain it with the help of necessary sketches.
6. Define vertical, horizontal and profile planes.
7. Define elevation, plan and end view.
8. Differentiate between first-angle and third-angle projection.
9. Give the symbolic representation of first- and third-angle projection.
10. What is the criterion for selection of the face of an object suitable for front view, while drawing multi-views?

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) Projection of an object shown by three views is known as
(a) perspective (b) isometric (c) oblique (d) orthographic

ii) Which of the following describes the theory of orthographic projection?
(a) Projectors parallel to each other and perpendicular to the plane of projection
(b) Projectors parallel to each other and parallel to the plane of projection
(c) Projectors parallel to each other and oblique to the plane of projection
(d) Projectors perpendicular to each other and parallel to the plane of projection

iii) In orthographic projection, the elevation is obtained on a plane called
(a) horizontal (b) vertical (c) profile (d) auxiliary

iv) In multi-view projections, the *XY* line is also known as
(a) horizontal line (b) horizontal trace (c) reference line (d) all of these

v) In first-angle projection method, the relative positions of the object, plane and observers are the following:
(a) Object is placed in between (b) Plane is placed in between
(c) Observer is placed in between (d) May be placed in any order

vi) In first-angle projection system, the right-hand side view of an object is drawn
(a) above of the elevation (b) below of the elevation
(c) left of the elevation (d) right of the elevation

vii) If the front view of an object exhibits width and height, then what dimensions of an object are exhibited by a right side view?
(a) Length and width (b) Length and height
(c) Height and width (d) Length and breadth

viii) For orthographic projections, BIS recommends the following:
(a) First-angle projection (b) Third-angle projection
(c) Second-angle projection (d) Fourth-angle projection

ix) The recommended symbol for indicating the angle of projection shows two views of the frustum of a
(a) square pyramid (b) triangular pyramid
(c) cone (d) any of these

x) For the object shown in Fig. E5.31, select the correct front view:

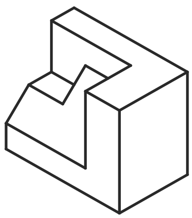
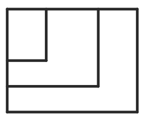
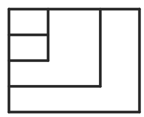


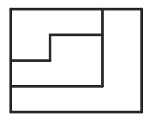
Fig. E5.31



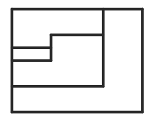
(a)



(b)



(c)



(d)

xi) For the object shown in Fig. E5.32, select the correct front view:

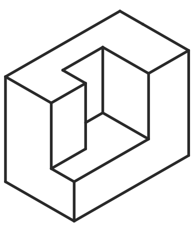
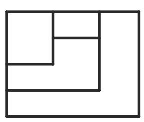
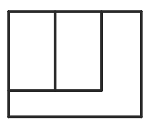


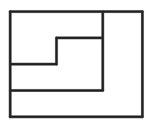
Fig. E5.32



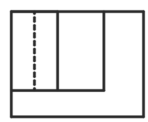
(a)



(b)



(c)

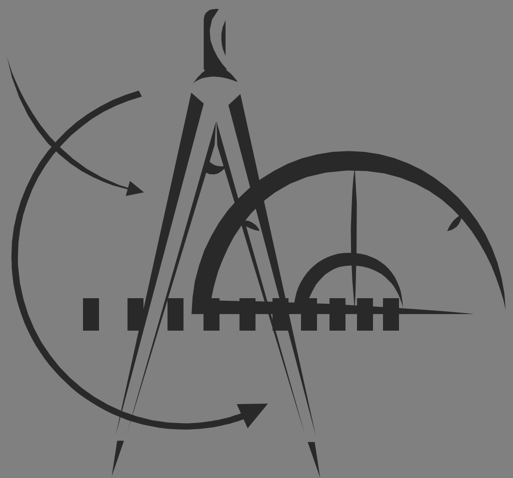


(d)

Answers

(i) d (ii) a (iii) b (iv) c (v) a (vi) c (vii) b (viii) a (ix) c (x) c (xi) b

6



Projections of Points

- Introduction
- Location of a Point, Conventional Representation
- Point Above the H.P. and in Front of the V.P.
- Point Above the H.P. and Behind the V.P.
- Point Below the H.P. and Behind the V.P
- Point Below the H.P. and in Front of the V.P.
- Point on the H.P. and in Front of the V.P.
- Point Above the H.P. and on the V.P.
- Point on the H.P. and Behind the V.P.
- Point Below the H.P. and on V.P.
- Point on Both H.P. and V.P.

6.1 INTRODUCTION

A point is defined as a geometrical element that has no dimensions. In engineering drawing or graphics, the point is represented as a dot. This chapter deals with the projections of points.

6.2 LOCATION OF A POINT

We know that the reference planes divide the space in four quadrants. A point lying in the space may be situated in the following positions with respect to principle planes of projections.

1. Point above the H.P and in front of the V.P.
2. Point above the H.P and behind the V.P.
3. Point below the H.P and behind the V.P.
4. Point below the H.P and in front of the V.P.
5. Point on the H.P and in front of the V.P.
6. Point above the H.P and on the V.P.
7. Point on the H.P and behind the V.P.
8. Point below the H.P and on the V.P.
9. Point on the H.P and V.P both

6.3 CONVENTIONAL REPRESENTATION

1. The actual position of a point is designated by the capital letters. i.e., A, B, C, P, Q, R, … etc.
2. The front view of a point is conventionally represented by small letters with dashes. i.e. a', b', c', p', q', r', …etc.
3. The top view of a point is conventionally represented by small letters. i.e., a, b, c, p, q, r, … etc.
4. The side view of a point is conventionally represented by small letters with double dashes. i.e., a'', b'', c'', p'', q'', r'', …etc.

The intersection of the reference planes is a line known as the reference line. It is denoted as xy. The reference line is drawn by a thin line.

The line which connects the front view and the top view is called the projection line. It is drawn by a thin line. The projection line is always perpendicular to the principal axis (XY).

6.4 POINT ABOVE THE H.P. AND IN FRONT OF THE V.P.

A point situated above the H.P. and in front of the V.P. would lie in the first angle.

Example 6.1 (Fig 6.1)

A point *P* is 50 mm above the H.P. and 70 mm in front of the V.P. Draw its projections.

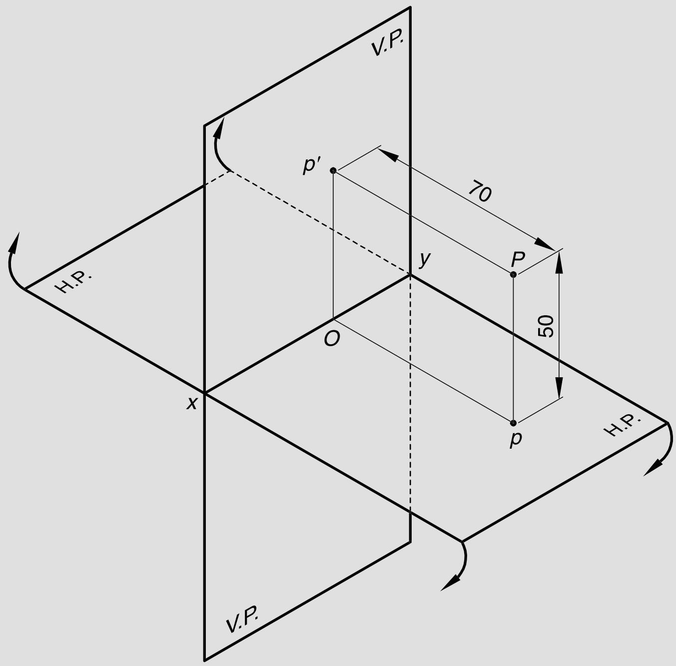


Fig. 6.1(a)

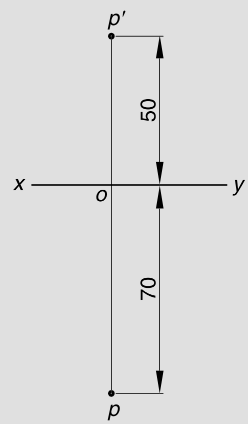


Fig. 6.1(b)

Visualization: Fig. 6.1(a) shows a point *P* situated in the first angle such that its distance above the H.P. is 50 mm and in front of the V.P. is 70 mm. The front view of the point is obtained by drawing a horizontal line through *P* to intersect V.P. at point *p*′. Top view is obtained by drawing a vertical line through *P* to intersect the H.P. at point *p*.

After projecting the point *P* on the V.P. and the H.P., the H.P. is rotated about the *XY* line in a clockwise direction till it lies in the plane with V.P. The projections of the point after rotation of the H.P. is shown in Fig. 6.1(b). The front view *p*′ is 50 mm above the *XY* line and the top view *p* is 70 mm below the *XY*.

Construction: Fig. 6.1(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark front view *p*′ on the projector 50 mm above the *XY*.
4. Mark top view *p* on the projector 70 mm the below *XY*.

Conclusion

1. If a point is situated above the H.P., then its front view (F.V.) is above the *XY* and the distance of front view (F.V.) from the *XY* is equal to the distance of the given point from the H.P.
2. If a point is situated in front of the V.P., then its top view (T.V.) is below the *XY* and the distance of top view (T.V.) from the *XY* is equal to the given distance of point from the V.P.

6.5 POINT ABOVE THE H.P. AND BEHIND THE V.P.

A point situated above the H.P. and behind the V.P. would lie in the second-angle.

Example 6.2 (Fig 6.2)

A point *Q* is 40 mm above the H.P. and 60 mm behind the V.P. Draw its projections.

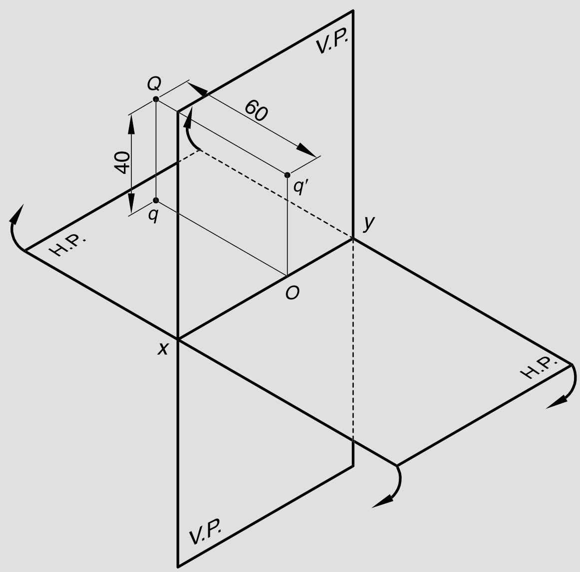


Fig. 6.2(a)

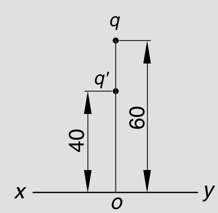


Fig. 6.2(b)

Visualization: Fig. 6.2(a) shows a point *Q* situated in the second angle such that its distance above the H.P. is 40 mm and behind the V.P. is 60 mm. The front view of the point is obtained by drawing a horizontal line through *Q* to intersect the V.P. at point *q′*. The top view is obtained by drawing a vertical line through *Q* to intersect the H.P. at point *q*.

After projecting the point *Q* on the V.P. and H.P., the H.P. is rotated about *XY* in a clockwise direction till it lies in the plane with the V.P. The projections of the point after rotation of the H.P. is shown in Fig. 6.2(b). The front view *q′* is 40 mm above the *XY* and the top view *q* is 60 mm above the *XY*.

Construction: Fig. 6.2(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark the front view *q′* on the projector 40 mm above the *XY*.
4. Mark the top view *q* on the projector 60 mm above the *XY*.

Conclusion

1. If a point is situated above the H.P., then its front view (F.V.) is above the *XY* and the distance of front view (F.V.) from the *XY* is equal to the distance of the point from the H.P.
2. If a point is situated behind the V.P., then its top view (T.V.) is above the *XY* and the distance of the top view (T.V.) from the *XY* is equal to the distance of the given point from the V.P.

6.6 POINT BELOW THE H.P. AND BEHIND THE V.P.

A point situated below the H.P. and behind the V.P. would lie in the third-angle.

Example 6.3 (Fig 6.3)

A point *R* is 80 mm below the H.P. and 50 mm behind the V.P. Draw its projections.

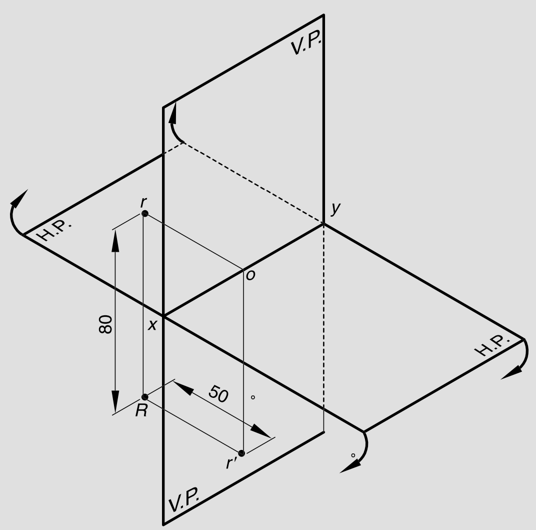


Fig. 6.3(a)

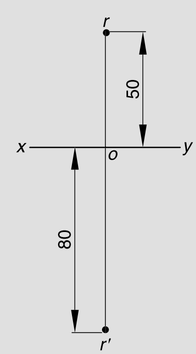


Fig. 6.3(b)

Visualization: Fig. 6.3(a) shows a point *R* situated in the third angle such that its distance below the H.P. is 80 mm and behind the V.P. is 50 mm. The front view of the point is obtained by drawing a horizontal line through *R* to intersect V.P. at point *r′*. The top view is obtained by drawing a vertical line through *R* to intersect the H.P. at point *r*.

After projecting the point *R* on the V.P. and the H.P., the H.P. is rotated about the *XY* in clockwise direction till it lies in the plane with the V.P. The projections of the point after the rotation of the H.P. is shown in Fig. 6.3(b). The front view *r′* is 80 mm below the *XY* and the top view *r* is 50 mm above the *XY*.

Construction: Fig. 6.3(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark the front view *r′* on the projector 80 mm below the *XY*.
4. Mark the top view *r* on the projector 60 mm above the *XY*.

Conclusion

1. If a point is situated below the H.P., then its front view (F.V.) is below the *XY* and the distance of front view (F.V.) from the *XY* is equal to distance of the point from the H.P.
2. If a point is situated behind the V.P., then its top view (T.V.) is above the *XY* and the distance of top view (T.V.) from the *XY* is equal to distance of the point from the V.P.

6.7 POINT BELOW THE H.P. AND IN FRONT OF THE V.P.

A point situated below the H.P. and in front of the V.P. would lie in the fourth-angle.

Example 6.4 (Fig 6.4)

A point *S* is 80 mm below the H.P. and 50 mm in front of the V.P. Draw its projections.

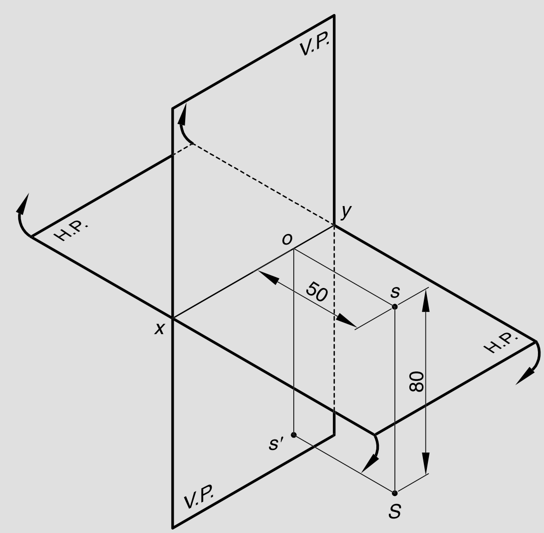


Fig. 6.4(a)

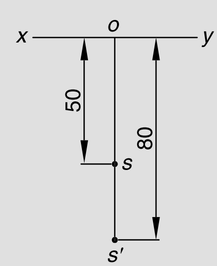


Fig. 6.4(b)

Visualization: Fig. 6.4(a) shows a point *S* situated in the fourth-angle such that its distance below the H.P. is 80 mm and in front of the V.P. is 50 mm. The front view of the point is obtained by drawing horizontal line through *S* to intersect V.P. at point s′. The top view is obtained by drawing a vertical line through *S* to intersect the H.P. at point *s*.

After projecting the point *S* on the V.P. and the H.P., the H.P. is rotated about the *XY* in a clockwise direction till it lies in the plane with V.P. The projection of the point after rotation of the H.P. is shown in Fig. 6.4(b). The front view *s*′ is 80 mm below the *XY* and the top view *s* is 50 mm below the *XY*.

Construction: Fig. 6.4(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark the front view *s*′ on the projector 80 mm below the *XY*.
4. Mark the top view *s* on the projector 60 mm below the *XY*.

Conclusion

1. If a point is situated below the H.P., then its front view (F.V.) is below the *XY* and the distance of front view (F.V.) from the *XY* is equal to distance of point from the H.P.
2. If a point is situated in front of the V.P., then its top view (T.V.) is below the *XY* and the distance of top view (T.V.) from the *XY* is equal to the distance of point from the V.P.

6.8 POINT ON THE H.P. AND IN FRONT OF THE V.P.

Example 6.5 (Fig 6.5)

A point *P* lies in the H.P. and 70 mm in front of the V.P. Draw its projections.

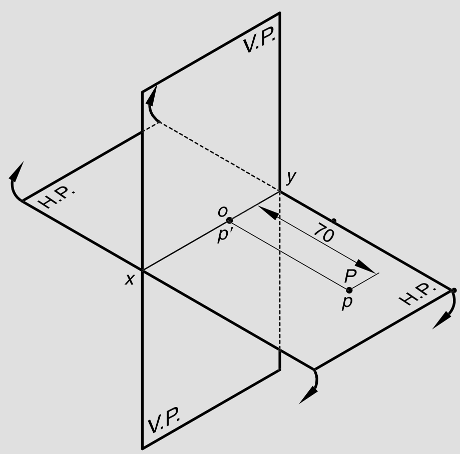


Fig. 6.5(a)

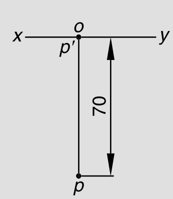


Fig. 6.5(b)

Visualization: Fig. 6.5(a) shows a point *P* which lies on the H.P. and is 70 mm in front of the V.P. The front view of the point is obtained by drawing the horizontal line through *P* to intersect the V.P. on the reference line at point *p*′. The top view *p* is obtained on point *P* as the point *P* lies on the H.P.

After projecting the point *P*, the H.P. is rotated about the *XY* in a clockwise direction till it lies in the plane with V.P. The projections of the point after rotation of the H.P is shown in Fig. 6.5(b). The front view *p*′ is on *XY* and the top view *p* is 70 mm below the *XY*.

Construction: Fig. 6.5(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark the front view *p*′ on the *XY*.
4. Mark the top view *p* on the projector 70 mm below the *XY*.

Conclusion

If a point is situated on the H.P., then its front view (F.V.) is on the *XY*.

6.9 POINT ABOVE THE H.P. AND ON THE V.P.

Example 6.6 (Fig 6.6)

A point *Q* is 70 mm above the H.P. and on the V.P. Draw its projections.

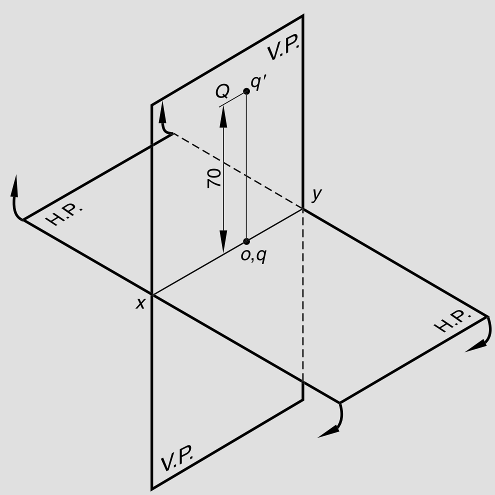


Fig. 6.6(a)

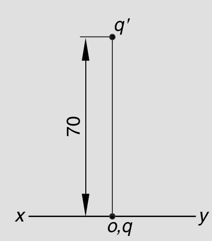


Fig. 6.6(b)

Visualization: Fig. 6.6(a) shows a point *Q* which is situated 70 mm above the H.P. and on the V.P. The front view *q′* of the point is obtained on point *Q*, as point *Q* is on the V.P. The top view is obtained by drawing a vertical line through *Q* to intersect the H.P. on the reference line at point *q*.

After projecting the point *Q*, the H.P. is rotated about the *XY* in a clockwise direction till it lies in the plane with the V.P. The projections of the point after rotation of the H.P. is shown in Fig. 6.6(b). The front view *q′* is 70 mm above the *XY* and the top view *q* is on the *XY*.

Construction: Fig. 6.6(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark the front view *q′* on the projector 70 mm above the *XY*.
4. Mark the top view *q* on the projector on *XY*.

Conclusion

If a point is situated on the V.P., then its top view (T.V.) is on the *XY*.

6.10 POINT ON THE H.P. AND BEHIND THE V.P.

Example 6.7 (Fig 6.7)

A point *R* is on the H.P. and 60 mm behind the V.P. Draw its projections.

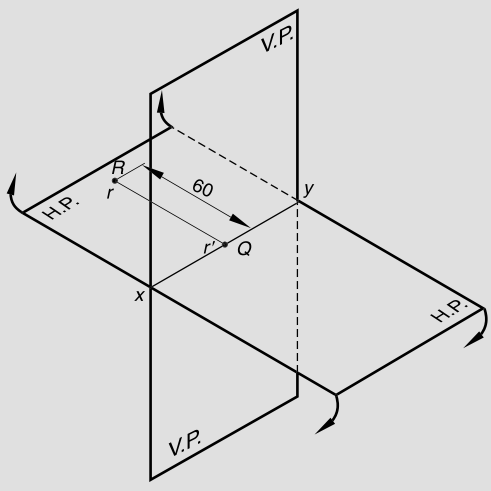


Fig. 6.7(a)

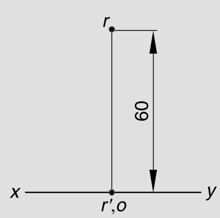


Fig. 6.7(b)

Visualization: Fig. 6.7(a) shows a point *R* situated on the H.P. and 60 mm behind the V.P. The front view of the point is obtained by drawing a horizontal line through *R* to intersect the V.P. on the reference line at point *r*′. The top view *r* coincides with point *R*, as the point *R* lies on the H.P.

After projecting the point *R*, the H.P. is rotated about the *XY* in a clockwise direction till it lies in the plane with the V.P. The projections of the point after rotation of the H.P. is shown in Fig. 6.7(b). The front view *r*′ is on *XY* and the top view *r* is 60 mm above the *XY*.

Construction: Fig. 6.7(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark the front view *r*′ on the *XY*.
4. Mark the top view *r* on the projector 60 mm above the *XY*.

Conclusion

If a point is situated on the H.P., then its front view (F.V.) is on the *XY*.

6.11 POINT BELOW THE H.P. AND ON V.P.

Example 6.8 (Fig 6.8)

A point *S* is 70 mm below the H.P. and on the V.P. Draw its projections.

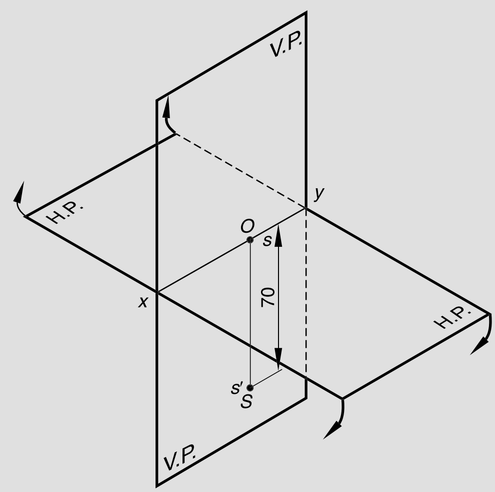


Fig. 6.8(a)

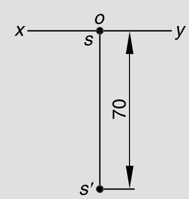


Fig. 6.8(b)

Visualization: Fig. 6.8(a) shows a point *S* which is situated 70 mm below the H.P. and on the V.P. The front view *s*′ of the point is obtained on point *S*, as the point *S* lies on the V.P. The top view is obtained by drawing a vertical line through *S* to intersect the H.P. on the reference line at point *s*.

After projecting the point *S*, the H.P. is rotated about the *XY* in a clockwise direction till it lies in the plane with the V.P. The projections of the point after rotation of the H.P. is shown in Fig. 6.8(b). The front view *s′* is 70 mm below the *XY* and the top view s is on the *XY*.

Construction: Fig. 6.8(b)

1. Draw a reference line *XY*.
2. Draw a projector perpendicular to the *XY*.
3. Mark the front view *s′* 70 mm below the *XY*.
4. Mark the top view *s* on the projector on the *XY*.

Conclusion

If a point is situated on the V.P., then its top view (T.V.) is on the *XY*.

6.12 POINT ON BOTH H.P. AND V.P.

Example 6.9 (Fig 6.9)

A point *T* is on the H.P. and the V.P. both. Draw its projections.

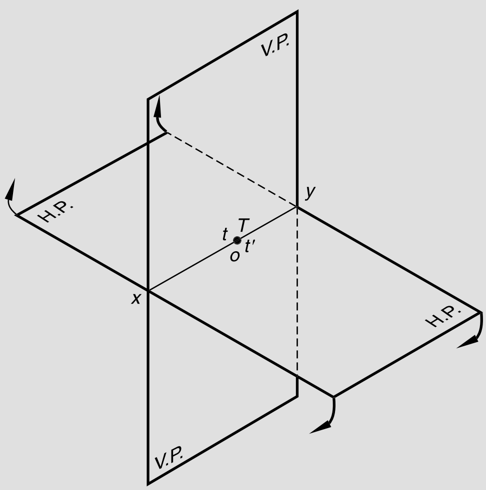


Fig. 6.9(a)

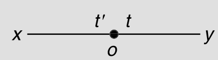


Fig. 6.9(b)

Visualization: Fig. 6.9(a) shows a point *T* situated both on the H.P. and the V.P. The Point lies on the reference line. The front and top views of the point are obtained on the reference line itself.

After projecting the point *R*, the H.P. is rotated about the *XY* in a clockwise direction till it lies in the plane with the V.P. The projections of the point after rotation of the H.P. is shown in Fig. 6.9(b). The front view *r′* and the top view *r* are on the *XY*.

Construction: Fig. 6.9(b)

1. Draw a reference line *XY*.
2. Mark front view *r′* and top view *r* coinciding on the *XY*.

Conclusion

If a point is situated both on the H.P. and the V.P., then its front and top views coincide on the *XY*.

6.13 MISCELLANEOUS EXAMPLES

Example 6.10 (Fig. 6.10)

Draw the projections of the following points on the same ground lines, keeping the projectors 15 mm apart:

(a) *A* in the H.P. and 20 mm behind V.P.
(b) *B* 25 mm below the H.P. and 25 mm behind V.P.
(c) *C* 15 mm above the H.P. and 20 mm in front of V.P.
(d) *D* 40 mm below H.P. and 25 mm in front of V.P. [RGPV April 2010]

Solution

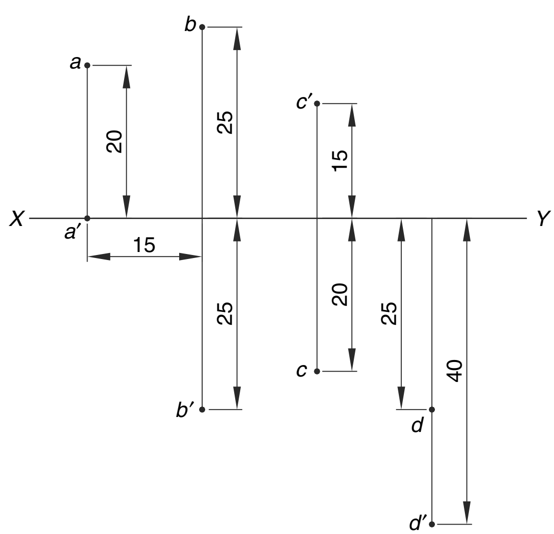


Fig. 6.10

Example 6.11 (Fig. 6.11)

Draw the projections of the following points on a common reference line, keeping the distance between their projectors 30 mm apart.

(a) Point *A* is 20 mm below the H.P. and 50 mm in front of the V.P.
(b) Point *B* is in the H.P. and 40 mm behind the V.P.
(c) Point *C* is 30 mm in front of the V.P. and in the H.P.
(d) Point *D* is 50 mm above the H.P. and 30 mm behind the V.P.
(e) Point *E* is 20 mm below the H.P. and 50 mm behind the V.P.
(f) Point *F* is in the V.P. and 50 mm below the H.P.

Solution

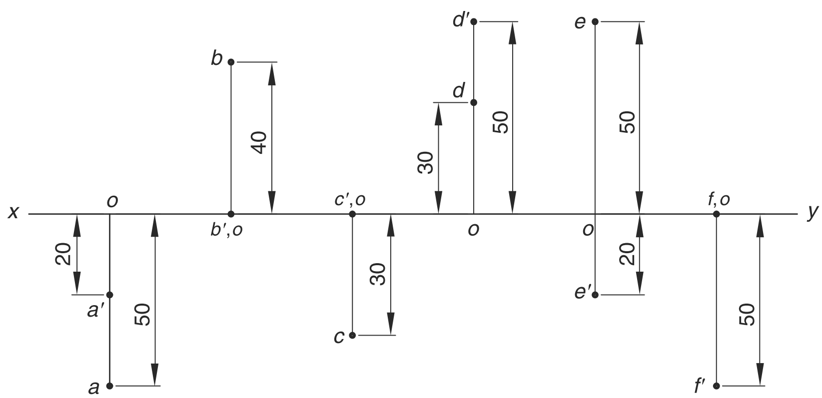


Fig. 6.11

Example 6.12 (Fig. 6.12)

Projection of various points are given in Fig. 6.12. State the position of each point with respect to the planes of projection.

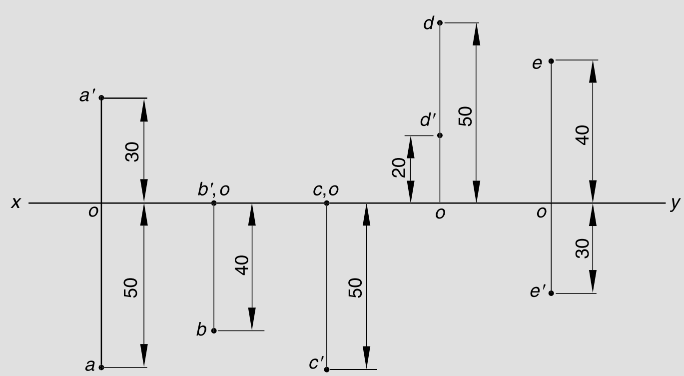


Fig. 6.12

Solution

(a) Point *A* is 30 mm above the H.P. and 50 mm in front of the V.P.
(b) Point *B* is in the H.P and 40 mm in front of the V.P.
(c) Point *C* is 50 mm below the H.P and in the V.P.
(d) Point *D* is 20 mm above the H.P and 50 mm behind the V.P.
(e) Point *E* is 30 mm below the H.P and 40 mm behind the V.P.

EXERCISE 6

1. A point is 30 mm from the H.P. and 50 mm from the V.P. Draw its projections keeping it in all possible positions.
2. Draw the projections of the following points on a common reference line keeping the distance between their projectors 25 mm apart.
   (a) Point *A* is 40 mm above the H.P and 25 mm in front of the V.P.
   (b) Point *B* is 40 mm above the H.P and on the V.P.
   (c) Point *C* is 25 mm in front of the V.P. and on the H.P.
   (d) Point *D* is 25 mm above the H.P and 30 mm behind the V.P.
   (e) Point *E* is on the H.P and 30 mm behind the V.P.
   (f) Point *F* is 40 mm below the H.P and 30 mm behind the V.P.
   (g) Point *G* is 25 mm below the H.P and 40 mm in front of the V.P.
   (h) Point *H* is on the V.P and 30 mm below the H.P.
3. Draw the projections of the following points on a common reference line keeping the distance between their projectors 30 mm apart.
   (a) Point *P* is 35 mm below the H.P. and on the V.P.
   (b) Point *Q* is 40 mm in front of the V.P. and 25 mm below the H.P.
   (c) Point *R* is 45 mm above the H.P. and 20 mm behind the V.P.
   (d) Point *S* is 30 mm below the H.P. and 45 mm behind the V.P.
   (e) Point *T* is both on the H.P. and the V.P.

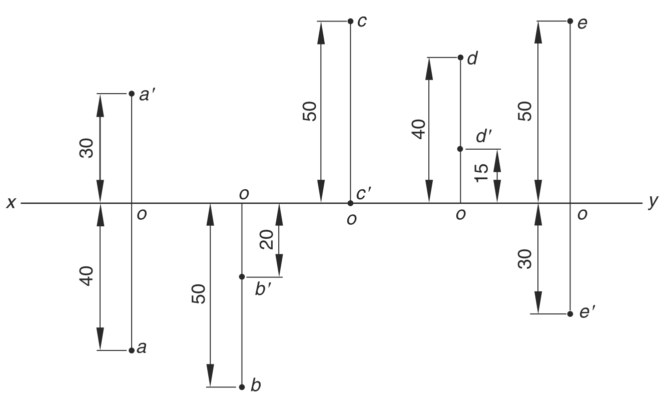


Fig. E 6.1

4. Projection of various points is given in Fig. E 6.1. State the position of each point with respect to the planes of projection.
5. State the quadrants in which the following points are located:
   (i) A - front view and top view are above *XY*.
   (ii) B-front view below *XY* and top view above *XY*.
   (iii) C-front view and top view below *XY*.
   (iv) front view above *XY* and top view below *XY*. [RGPV Aug. 2010]

REVIEW QUESTIONS

1. If both the views of a point coincide with each other and lie below the reference line, state the angle in which the point lies.
2. State the similarities and dissimilarities in the projections of points which lie in the second-angle and the fourth-angle.
3. State the position of the point, the front view of which lies 50 mm below the reference line and the top view 30 mm above the front view.
4. State the position of the point, the top view of which lies 50 mm above the reference line and the front view 30 mm below the top view.
5. If the front view of a point lies above the reference line, state the possible angles in which the point may lie.
6. If the top view of a point lies above the reference line, state the possible angles in which the point may lie.
7. If the front view of a point lies below the reference line, state the possible angles in which the point may lie.
8. If the top view of a point lies below the reference line, state the possible angles in which the point may lie.
9. State the relationship between front view and top view of a point.
10. State the position of the point if its both views lie on the reference line.
11. State the position of the point, the top view of which lies on the reference line and the front view 50 mm below it.
12. State the position of the point, the front view of which lies on the reference line and the top view 50 mm below it.
13. State the position of the point, the top view of which lies on the reference line and the front view 45 mm above it.
14. State the position of the point, the front view of which lies on the reference line and the top view 35 mm above it.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives.

(i) The line joining the front and top views of a point is called
 (a) reference line (b) projector
 (c) connector (d) locus

(ii) A point lying in the H.P. has its top view above the *XY*. Its front view will be
 (a) on *XY* (b) above *XY*
 (c) below *XY* (d) any of these

(iii) A point whose elevation and plan are above *XY* is situated in
 (a) first-angle (b) second-angle
 (c) third-angle (d) fourth-angle

(iv) A point whose elevation is above *XY* may be situated in
 (a) first-angle (b) second-angle
 (c) vertical-plane (d) any of these

(v) A point is 20 mm below the H.P. and 30 mm behind the V.P. Its top view will be
 (a) 20 mm below *XY* (b) 30 mm below *XY*
 (c) 20 mm above *XY* (d) 30 mm above *XY*

(vi) If the front view of a point is 50 mm above the *XY* and the top view is 20 mm below the front view, the point lies in
 (a) First-angle (b) Second-angle
 (c) Third-angle (d) Fourth-angle

(vii) If both the front and the top views of a point lie on opposite sides of the reference line, the point may be situated in the following angles:
 (a) First or second (b) First or third
 (c) Second or fourth (d) Third or fourth

(viii) If both the front and the top views of a point lie on the same side of the reference line, the point may be situated in following angles:
 (a) First or second (b) First or third
 (c) Second or fourth (d) Third or fourth

(ix) If the top view of a point is situated 60 mm below the reference line and its front view is 20 mm above the top view, the point lies in
 (a) first-angle (b) second-angle
 (c) third-angle (d) fourth-angle

(x) If the front view of a point is 40 mm above the *XY* and the top view is 50 mm below the *xy*, the position of point is

(a) 40 mm above H.P. (b) 40 mm below the H.P.
(c) 50 mm above H.P. (d) 50 mm below the H.P.

(xi) State the position of a point the front view of which lies on the reference line and the top view is 40 mm above it.

(a) 40 mm above the H.P. and on the V.P. (b) 40 mm behind the V.P. and on the H.P.
(c) 40 mm below the H.P. and on the V.P. (d) 40 mm in front of the V.P. and on the H.P.

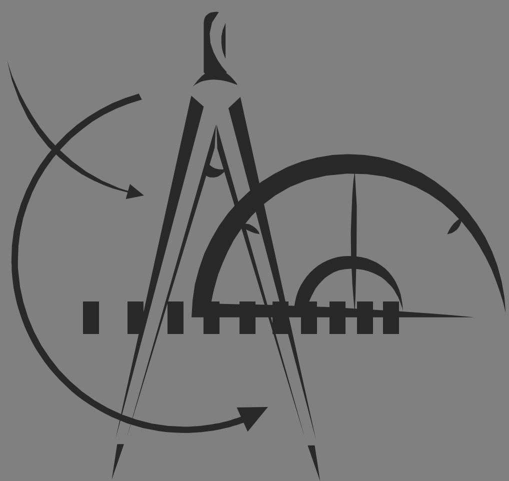
(xii) State the position of a point the top view of which lies on the reference line and the front view is 30 mm below it.

(a) 30 mm above the H.P. and on the V.P. (b) 30 mm behind the V.P. and on the H.P.
(c) 30 mm below the H.P. and on the V.P. (d) 30 mm in front of the V.P. and on the H.P.

Answers

(i) (b) (ii) (a) (iii) (b) (iv) (d) (v) (d) (vi) (b) (vii) (b) (viii) (c) (ix) (d) (x) (a)
(xi) (b) (xii) (c)

7



Projections of Straight Lines

- Introduction
- Orientation of a Straight Line
- Trace of a Straight Line
- Line Parallel to Both H.P. and V.P.
- Line Perpendicular to H.P. or V.P.
- Line Inclined to H.P. or V.P.
- Line Situated in the H.P., V.P. or both
- Line in the First Angle Inclined to Both the Reference Planes
- True Length and True Inclination of Given Line
- Trapezoid Method
- Line Inclined to Both the Reference Planes the Ends of Which Lie in Different Angles

7.1 INTRODUCTION

A straight line is the shortest distance between two points. The projections of straight lines can be drawn by joining the respective projections of its end points. We have used the word 'line' for straight lines for the sake of simplicity. The actual length of the line is commonly called true length and is denoted by T.L.

7.2 ORIENTATION OF A STRAIGHT LINE

A straight line may be in one of the following positions.

1. Line parallel to both H.P. and V.P.
2. Line perpendicular to H.P. (and parallel to V.P.)
3. Line perpendicular to V.P. (and parallel to H.P.)
4. Line inclined to H.P. and parallel to V.P.
5. Line inclined to V.P. and parallel to H.P.
6. Line situated in H.P.
7. Line situated in V.P.
8. Line situated in both H.P. and V.P. (i.e. on the reference line, *XY*)
9. Line inclined to both the reference planes
    a. Line inclined to both H.P. and V.P. such that $\theta + \phi < 90^\circ$.
    b. Line inclined to both H.P. and V.P. such that $\theta + \phi = 90^\circ$.

Let us consider the line in the first angle. Projections of a straight line lying in the first angle will have its front view above *XY* and the top view below *XY*. A clear concept of orthographic projections and projections of points is required to understand the projections of straight lines.

7.3 TRACE OF A STRAIGHT LINE

The point of intersection of a given straight line (extended if necessary) with the reference plane is known as traces of that lines.

The point at which the line (extended if necessary) intersects the H.P. is known as horizontal trace and is denoted by either H.T. or letter h. The front view of the horizontal trace lies on *XY* and is denoted by h'.

The point at which the line (extended if necessary) intersects the V.P. is known as vertical trace and is denoted by either V.T. or letter v'. The top view of the vertical trace lies on *XY* and is denoted by v.

7.4 LINE PARALLEL TO BOTH H.P. AND V.P.

This is the basic position of any line. Both the front and top views shall be the true lengths.

Example 7.1 (Fig. 7.1)

A 50 mm long line *AB* is parallel to both the H.P. and the V.P. It is 25 mm in front of the V.P. and 60 mm above the H. P. Draw its projections and determine the trace.

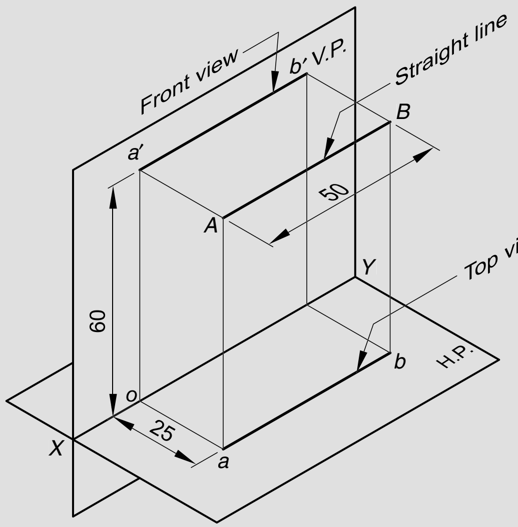


Fig. 7.1(a)

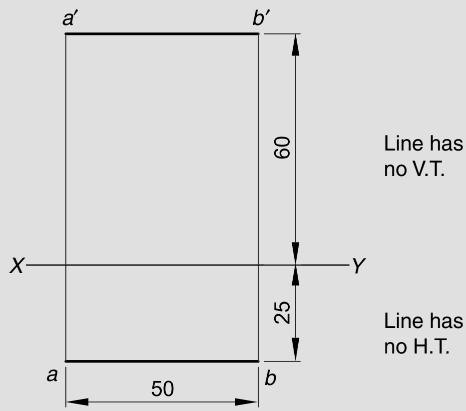


Fig. 7.1(b)

Visualisation Fig. 7.1(a) shows the straight line *AB* which is 25 mm in front of the V.P. and 60 mm above the H.P. The front view *a′b′* of the line is obtained by joining the front view of end points of line *AB*. The line *a′b′* shows the true length and is parallel to the *XY*. The top view *ab* of the line is obtained by joining the top view of end points of line *AB*. The line *ab* also shows the true length and is parallel to the *XY*.

Construction: Fig. 7.1(b)

1. Draw a reference line *XY*.
2. Mark point *a′* 60 mm above *XY* and point a 25 mm below *XY*.
3. Draw a 50 mm long line *a′b′* parallel to *XY*, to represent the front view.
4. Draw another 50 mm long line *ab* parallel to *XY*, to represent the top view.

Locate H.T. and V.T.

5. As the line is parallel to H.P., it will never intersect H.P., even if it is extended. So it has no H.T.
6. As the line is parallel to V.P., it will never intersect the V.P., even if it is extended. So it has no V.T.

7.5 LINE PERPENDICULAR TO H.P.

A line perpendicular to H.P. is always parallel to V.P. The front view shall be true length perpendicular to *XY* whereas the top view shall be only a point.

Example 7.2 (Fig. 7.2)

A 60 mm long line *AB* has its end *A* 20 mm above the H.P. The line is perpendicular to H.P. and 40 mm in front of the V.P. Draw its projections and locate the traces.

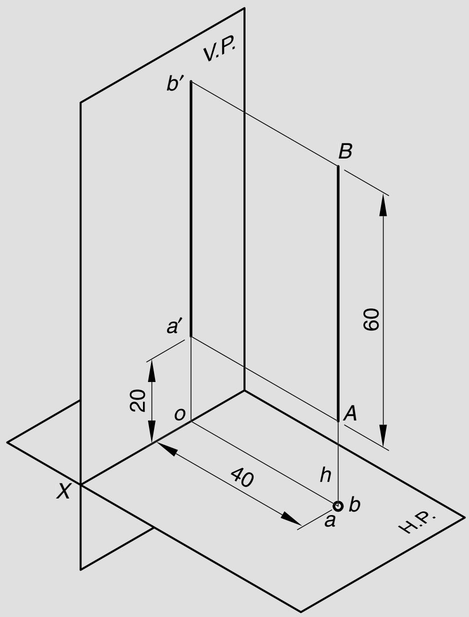


Fig. 7.2(a)

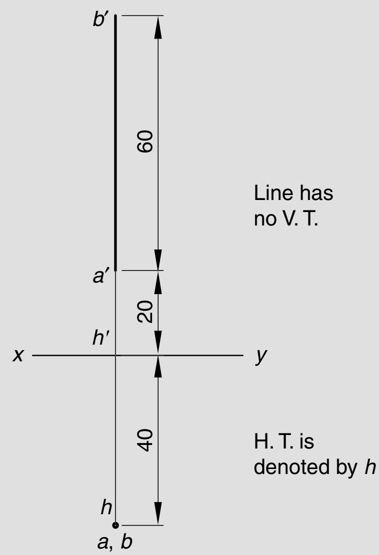


Fig. 7.2(b)

Visualisation Fig. 7.2(a) shows the straight line *AB* whose end *A* is 20 mm above the H.P. and 40 mm in front of the V.P. As the line is perpendicular to the H.P. the end *b* is 80 mm (20 + 60) above the H.P. and 40 mm in front of the V.P. Obtain front and top views of *A* and *B*. Join them to obtain *a′b′* as the front view and *AB* as the top view.

Construction: Fig. 7.2(b)

1. Draw a reference line *XY*.
2. Mark point *a′* 20 mm above *XY* and point a 40 mm below *XY*.
3. Draw a 60 mm long line *a′b* perpendicular to *XY*, to represent the front view.
4. Mark point *b* to overlap point *a*. This point represents the top view of the line.

Locate H.T. and V.T.

5. The line *AB* when produced, meets the H.P. at point *h*, 40 mm in front of V.P. Thus, the H.T. coincides with the top view. It is denoted by *h*.
6. As the line *AB* is parallel to V.P., it doesnot intersect the V.P. anywhere. Therefore it has no V.T.

7.6 LINE PERPENDICULAR TO V.P.

A line perpendicular to V.P. is always parallel to H.P. The top view shall be true length perpendicular to *XY* whereas the front view shall be only a point.

Example 7.3 (Fig. 7.3)

A 60 mm long line *AB* has its end *A* at a distance of 20 mm in front of the V.P. The line is perpendicular to the V.P. and 40 mm above the H.P. Draw the projections of the line and determine its traces.

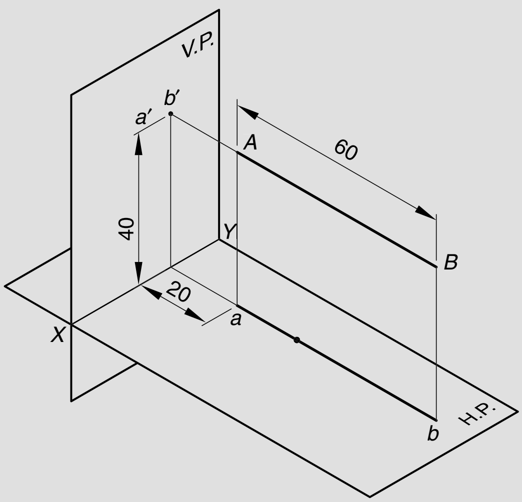


Fig. 7.3(a)

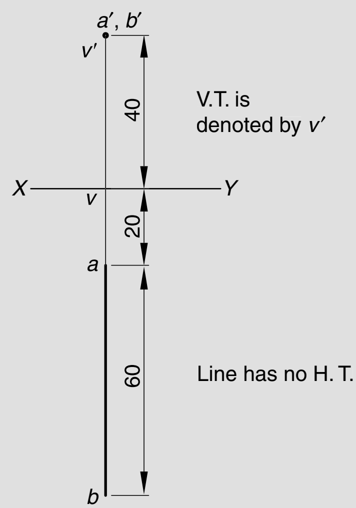


Fig. 7.3(b)

Visualisation Fig. 7.3(a) shows the straight line *AB* whose end *A* is 40 mm above the H.P. and 20 mm in front of the V.P. As the line is perpendicular to the V.P. the end *B* is 80 mm (20 + 60) in front of the V.P. and 40 mm above the H.P. Obtain front and top views of *A* and *B*. Join them to obtain *ab* as the top view and *a′b′* as the front view.

Construction: Fig. 7.3(b)

1. Draw a reference line *XY*.
2. Mark point *a′* 40 mm above *XY* and point a 20 mm below *XY*.

3. Mark point b' overlap point a'. This point represents the front view of the line.
4. Draw a 60 mm long line *ab* perpendicular to *XY*, to represent the top view.

Locate H.T. and V.T.

5. As the line *AB* is parallel to H.P., it doesnot intersect the H.P. Therefore it has no H.T.
6. The line *AB* when produced, meets the V.P. at point v', 40 mm above H.P. Thus, the V.T. coincides with the front view. It is denoted by v'.

7.7 LINE INCLINED TO H.P. AND PARALLEL TO V.P.

When a line is inclined to H.P. and parallel to the V.P. its front view will be true length inclined to *XY*. The top view will be projected length smaller than the true length parallel to *XY*.

Example 7.4 (Fig. 7.4)

An 80 mm long line *AB* has end *A* at a distance of 20 mm above the H.P. and 40 mm in front of the V.P. The line is inclined at 30° to the H.P. and is parallel to the V.P. Draw the projections of the line and determine its traces.

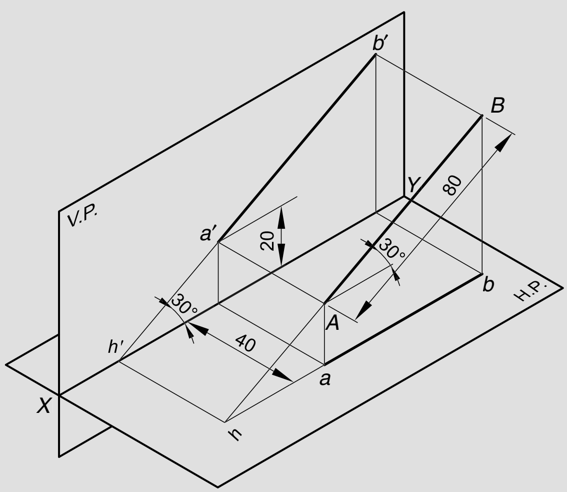


Fig. 7.4(a)

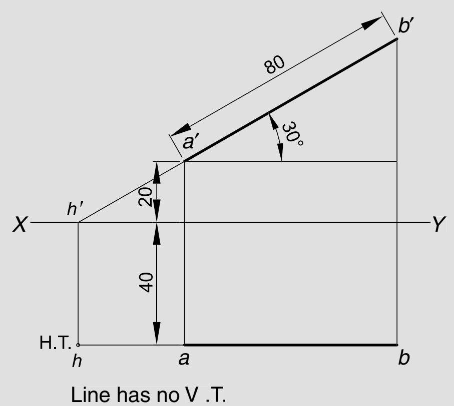


Fig. 7.4(b)

Visualisation Fig. 7.4(a) shows the straight line *AB* whose end *A* is 20 mm above the H.P. and 40 mm in front of the V.P. The line is inclined at 30° to the H.P. and parallel to the V.P. As the line is parallel to V.P., its front view is true length (80 mm) making an angle 30^0 with *XY*. The top view is projected length parallel to *XY*.

Construction: Fig. 7.4(b)

1. Draw a reference line *XY*.
2. Mark point *a′* 20 mm above *XY* and point a 40 mm below *XY*.
3. Through point *a′*, draw an 80 mm long line *a′b′* making 30º with *XY*. This represents the front view.
4. Through point *a*, draw a line parallel to *XY* to meet the vertical projector of *b′* at *b*. The line *ab* represents the top view.

Locate H.T. and V.T.

5. Extend line *a′b′* to meet *XY* at point *h′*. Project point *h′* to meet *ab* produced at point *h*. The point *h* denotes the H.T.
6. As the line is parallel to V.P., it doesnot intersect the V.P. Therefore it has no V.T.

7.8 LINE INCLINED TO V.P. AND PARALLEL TO H.P.

When a line is inclined to V.P. and parallel to H.P. its top view will be true length inclined to *XY*. The front view will be projected length smaller than the true length parallel to *XY*.

Example 7.5 (Fig. 7.5)

An 80 mm long line *AB* is inclined at 30º to the V.P. and is parallel to the H.P. The end *A* of the line is 20 mm above the H.P. and 40 mm in front of the V.P. Draw the projections of the line and determine its traces.

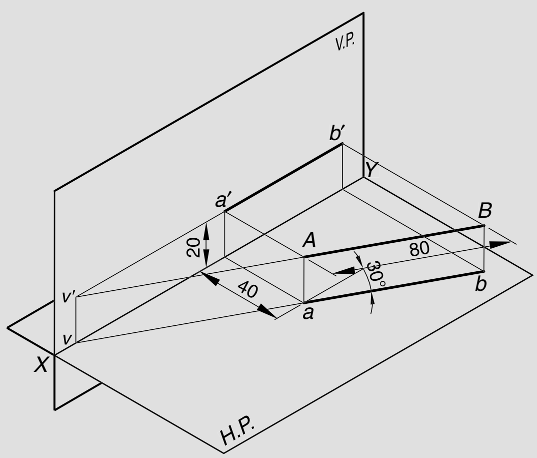


Fig. 7.5(a)

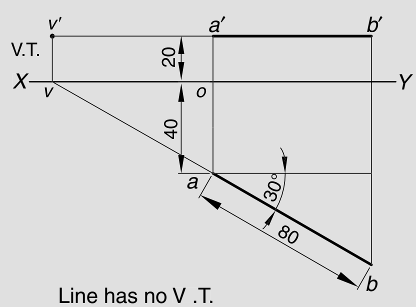


Fig. 7.5(b)

Visualisation Fig. 7.5(a) shows the straight line *AB* whose end *A* is 20 mm above the H.P. and 40 mm in front of the V.P. The line is inclined at 30º to the V.P. and parallel to the H.P. As *AB* is parallel to H.P., its top view is true length (80 mm) making true angle 30º with *XY*. The front view is projected length parallel to *XY*.

Construction: Fig. 7.5(b)

1. Draw a reference line *XY*.
2. Mark point *a′* 20 mm above *XY* and point a 40 mm below *XY*.
3. Through point *a*, draw an 80 mm long line *ab* making 30º with *XY*. This represents the top view.
4. Through point *a′*, draw a line parallel to *XY* to meet the vertical projector of *b* at *b′*. The line *a′b′* represents the front view.

Locate H.T. and V.T.:

5. Extend line *AB* to meet *XY* at point *v*. Project *v* to meet *a′b′* produced at point *v′*. The point *v′* denotes the V.T.
6. As the line is parallel to H.P., it doesnot intersect the H.P. Therefore it has no H.T.

7.9 LINE SITUATED IN THE H.P.

When a line is situated in H.P., its top view will be true length. The front view will be projected length on the reference line.

Example 7.6 (Fig. 7.6)

A 60 mm long line *PQ* is situated in the H.P. and is inclined at 30º to the V.P. The end *P* of the line is situated 20 mm in front of the V.P. Draw the projections of the line and determine its traces.

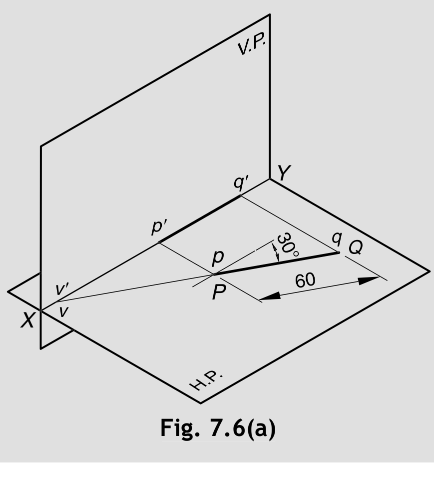


Fig. 7.6(a)

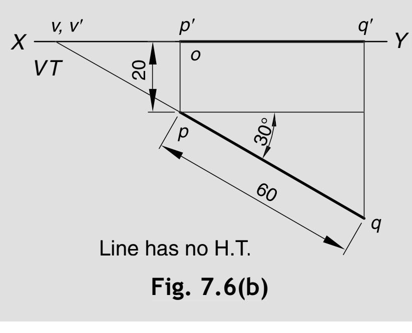


Fig. 7.6(b)

Visualisation Fig. 7.6(a) shows the straight line *PQ* whose end *P* is on the H.P. and 20 mm in front of the V.P. The line is inclined at 30º to the V.P. As the line *PQ* is on the H.P., the top view is true length (60 mm) making 30º with *XY*. The front view is projected length and lies on *XY*.

Construction: Fig. 7.6(b)

1. Draw a reference line *XY*.
2. Mark point *p′* on *XY* and point *p* 20 mm below *XY*.
3. Through point *p*, draw a 60 mm long line *pq* making 30º with *XY*. This represents the top view.
4. Through point *p′*, draw a line along *XY* to meet the projector of *q* at *q′*. The line *p′q′* represents the front view.

Locate H.T. and V.T.:

5. As the line is parallel to H.P., it doesnot intersect the H.P. Therefore it has no H.T.
6. Extend line *pq* to meet *XY* at *v*. This coincides with extension of *p′q′* at *v′*. The point *v′* denotes V.T.

7.10 LINE SITUATED IN THE V.P.

When a line is situated in the V.P. its front view will be true length. The top view will be of projected length on the reference line.

Example 7.7 (Fig. 7.7)

Draw the projections of a 70 mm long line *PQ*, situated in V.P. and inclined at 30º to the H.P. The end *P* of the line is 25 mm above the H.P. Also, determine the traces of the line.

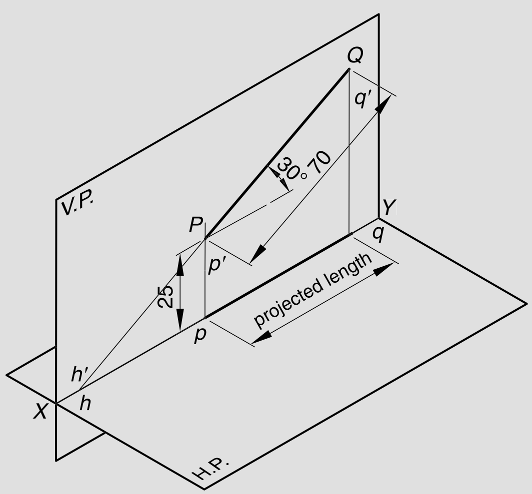


Fig. 7.7(a)

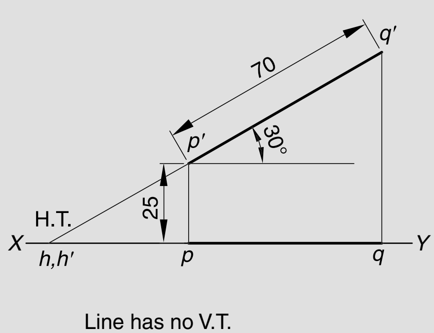


Fig. 7.7(b)

Visualisation Fig. 7.7(a) shows the straight line *PQ* whose end *P* is 25 mm above the H.P. and on the V.P. It is inclined at 30º to the H.P. As the line is on the V.P., its front view is true length (70 mm) making true angle 30º with *XY*. Its top view is of projeted length and lies on *XY*.

Construction: Fig. 7.7(b)

1. Draw a reference line *XY*.
2. Mark point *p′* 25 mm above *XY* and point *p* on the *XY*.
3. Through point *p′*, draw a 70 mm long line *p′q′* making 30º with *XY*. This represents the front view.
4. Through point *p*, draw a line along *XY* to meet the vertical projector of *q′* at *q*. The line *pq* represents the top view.

Locate H.T. and V.T.:

5. Extend line *p′q′* to meet *XY* at point *h′*. This coincides with extension of *pq* at *h*. The point *h* denotes H.T.
6. As the line is parallel to V.P., it doesnot intersect the H.P. Therefore it has no V.T.

7.11 LINE SITUATED BOTH IN H.P. AND V.P.

When a line is situated both in H.P. and V.P., it is on the reference line. Both the front and the top views will coincide on the *XY*.

Example 7.8 (Fig. 7.8)

Draw the projections of a 60 mm long line *PQ*, which is situated in the H.P. and the V.P. both. Also, determine the traces of the line.

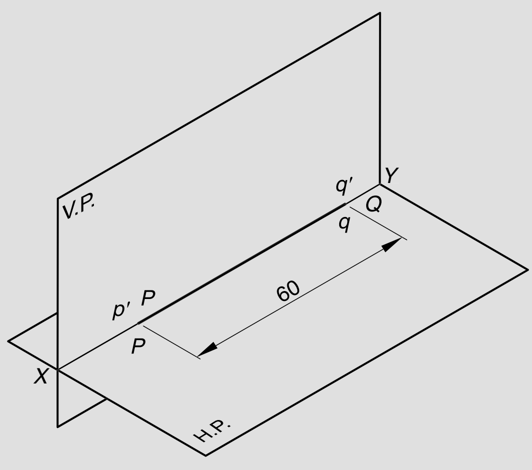


Fig. 7.8(a)

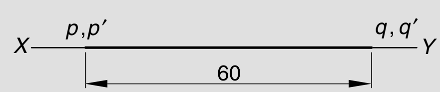


Fig. 7.8(b)

Visualisation Fig. 7.8(a) shows the straight line *PQ* situated on the reference line. Hence it is situated on the H.P. and the V.P. both. The front and top views of the line are obtained on *XY* and show the true length.

Construction: Fig. 7.8(b)

1. Draw a reference line *XY*.
2. Mark points p' and p coinciding on *XY*.
3. Mark points q' and q on *XY* at 60 mm from pp'.
4. As the line is parallel to both H.P. and V.P., it has neither H.T. nor V.T.

7.12 SUMMARY

Table 7.1

S.No.	*Orientation / Position of line*	*Front view or elevation*	*Top view or plan*	*Horizontal Trace*	*Vertical Trace*
1.	Line parallel to both H.P. and V.P.	True length, parallel to *XY*	True length, parallel to *XY*	Does not exist	Does not exist
2.	Line perpendicular to H.P.	True length, perpendicular to *XY*	Point	Coincides with top view	Does not exist
3.	Line perpendicular to V.P.	Point	True length, perpendicular to *XY*	Does not exist	Coincides with front view
4.	Line inclined at θ to H.P. and parallel to V.P.	True length, inclined at θ to *XY*	Shorter than the true length, parallel to *XY*	Exists	Does not exist
5.	Line inclined at ϕ to V.P. and parallel to H.P.	Shorter than the true length, parallel to *XY*	True length, inclined at ϕ to *XY*	Does not exist	Exists
6.	Line situated in H.P. and inclined at ϕ to V.P.	Shorter than the true length, lying on *XY*	True length, inclined at ϕ to *XY*	Does not exist	Exists on *XY*
7.	Line situated in V.P. and inclined at θ to H.P.	True length, inclined at θ to *XY*	Shorter than the true length, lying on *XY*	Exists on *XY*	Does not exist
8.	Line situated both in H.P. and V.P.	Both front and top views are true length and coincide on *XY*		Does not exist	Does not exist

7.14 MISCELLANEOUS EXAMPLES

Example 7.9 (Fig. 7.9)

A 50 mm long line *AB* has its end *A* 30 mm above H.P. and 20 mm in front of V.P. The front view of the line is a point. Draw its projections.

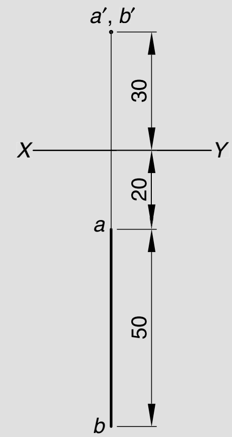


Fig. 7.9

Interpretation: As the front view of the line is a point, the line is perpendicular to V.P.

Construction: Fig. 7.9

1. Draw a reference line *XY*.
2. Mark point *a′* 30 mm above *XY* and point *a* 20 mm below *XY*.
3. As front view of the line is a point, point *b′* will overlap point *a′*.
4 Draw a 50 mm long line *ab*, perpendicular to *XY*. This represents the top view.

Example 7.10 (Fig. 7.10)

The line *AB* of 100 mm length is inclined at an angle 30º to the H.P. and parallel to V.P. The point *A* is 15 mm above H.P. and 20 mm in front of V.P. Draw the front view and top view of the line. [RGPV Feb. 2010]

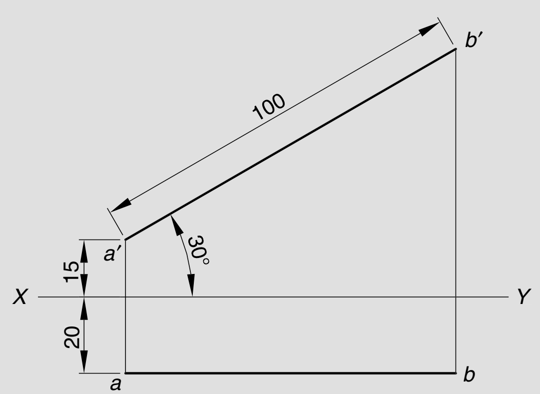


Fig. 7.10

Construction: Fig. 7.10

1. Draw a reference line *XY*.
2. Mark point *a′* 15 mm above *XY* and point a 20 mm below *XY*.
3. Draw a 100 mm long line *a′b′* at 30º to *XY*. This represents the front view.
4. Draw a line *ab* parallel to *XY* to meet the projector from *b′* at point *b*. Line *ab* represents the top view.

Example 7.11 (Fig. 7.11)

A 60 mm long line *AB* is parallel to and 20 mm in front of the V.P. The ends *A* and *B* are 10 mm and 50 mm above the H.P. respectively. Draw the projections of the line and determine its inclination with the H.P. Also, locate the traces.

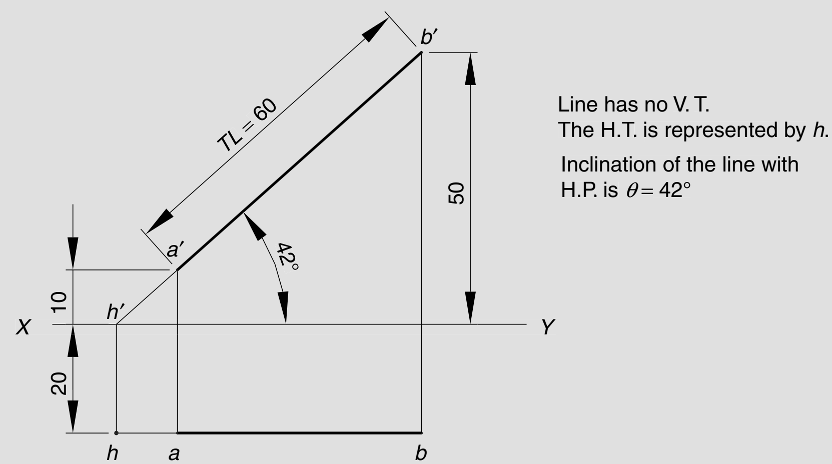


Fig. 7.11

Interpretation: As the line is parallel to the V.P., the front view is of true length and top view is parallel to *XY*.

Construction: Fig. 7.11

1. Draw a reference line *XY*.
2. Mark point *a′* 10 mm above *XY* and point a 20 mm below *XY*.
3. Draw a horizontal line 50 mm above *XY*. It is the locus of point *b′*.
4. Draw an arc of 60 mm radius with *a′* as centre to intersect the locus line of point *b′* at point *b′*. Join *a′b′* to represent the front view.

5. Draw a line from point a parallel to *XY* to meet the vertical projector from point *b′* at point *b*. The line *ab* represents the top view.
6. Determine inclination of *a′b′* with *XY* as $\theta = 42°$.
7. Produce *a′b′* to meet *XY* at point *h′*. Produced line *ab* to meet the vertical projector of point *h′* at point *h*. Point *h* represents the H.T. of the line.
8. As the line is parallel to V.P. so it has no V.T.

Example 7.12 (Fig. 7.12)

The length of the top view of a line parallel to the V.P. and inclined at 45º to the H.P. is 50 mm. One end of the line is 12 mm above the H.P. and 25 mm in front of the V.P. Draw the projections of the line and determine its true length. [RGPV June 2008]

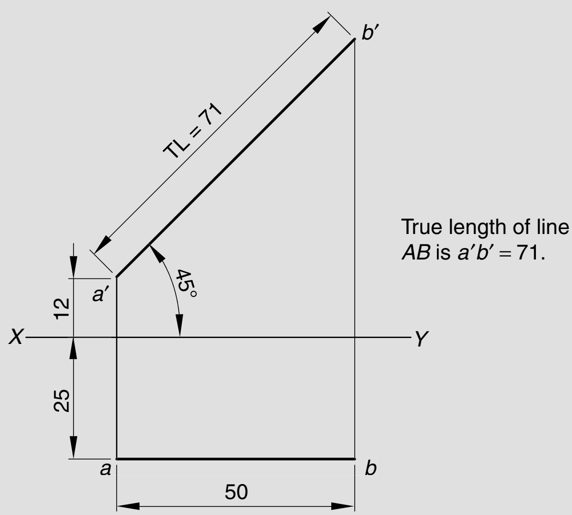


Fig. 7.12

Interpretation: When a line is parallel to V.P. its front view is of true length and top view is parallel to *XY*.

Construction: Fig. 7.12

1. Draw a reference line *XY*.
2. Mark point *a′* 12 mm above *XY* and point a 25 mm below *XY*.
3. Draw a 50 mm long line *ab* parallel to *XY*. This represents the top view.
4. Draw a line from point *a′*, inclined at 45º to *XY* to meet the vertical projectors from point *b* as point *b′*. Line *a′b′* represents the front view.
5. Measure length *a′b′*, 71 mm as true length of the line.

Example 7.13 (Fig. 7.13)

The front view of a line inclined at 30º to the V.P. is 65 mm long. Draw the projections and true length of the line when it is parallel to and 40 mm above the H.P. and its one end being 30 mm in front of the V.P. [RGPV April 2010]

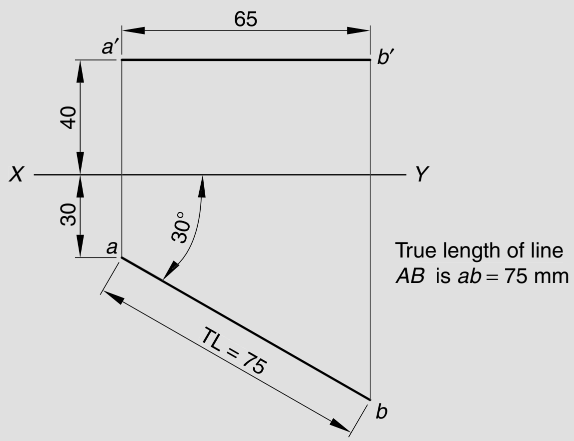


Fig. 7.13

Construction: Fig. 7.13

1. Draw a reference line *XY*.
2. Mark point *a′* 40 mm above *XY* and point a 30 mm below *XY*.
3. Draw a 65 mm long line *a′b′* parallel to *XY*. This represents the front view.
4. Draw a line from point *a*, inclined at 30º to *XY* to meet the projector from *b′* at point *b*. Line *ab* represents the top view.
5. Measure length *a′b′*, 75 mm as true length of the line.

Example 7.14 (Fig. 7.14)

A 60 mm long line is inclined at 45º to the V.P. The V.T. is 25 mm above *XY* and the H.T. does not exist. Draw the projections of the line when an end is 10 mm in front of the V.P.

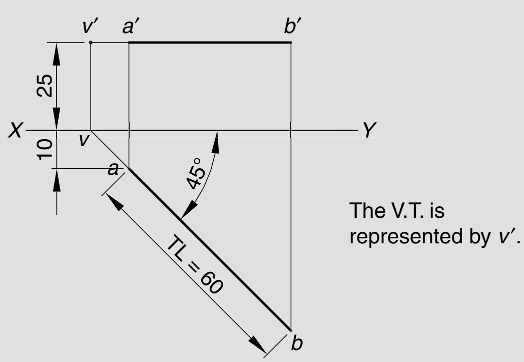


Fig. 7.14

Interpretation: As the line has no H.T., it is parallel to H.P. The front view of the line and the V.T. will be equidistant from *XY*.

Construction: Fig. 7.14

1. Draw a reference line *XY*.
2. Mark point a' 25 mm above *XY* and point a 10 mm below *XY*.
3. Draw a line *ab*, 60 mm long, making 45° with *XY*. Line *ab* represents the top view.
4. Draw a line from point a' parallel to *XY* to meet the vertical projector from point *b* at point b'. The line $a'b'$ represents the front view.
5. Produce *ab* to meet *XY* at point *v*. Produce line $a'b'$ to meet the vertical projector of *v* at v'. Point v' represents the V.T.

Example 7.15 (Fig. 7.15)

An electric switch and a bulb fixed on a wall are 5 m apart. The distance between them measured parallel to the floor is 4 metres. If the switch is 1.5 m above the floor, find the height of the bulb and inclination of line joining the two with the floor. [RGPV Dec. 2007]

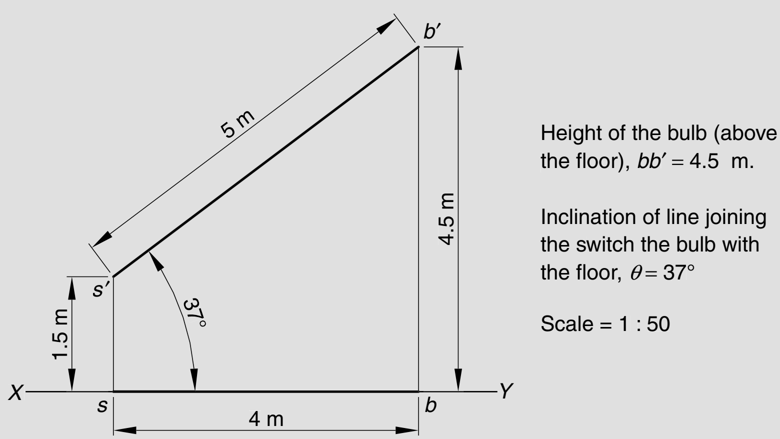


Fig. 7.15

Construction: Fig. 7.15

1. Draw a reference line *XY*.
2. Let *S* represents switch and *B* represents the bulb. Consider scale 1:50.
3. Mark point s' 1.5 m above *XY* and point *s* on *XY*.
4. Draw a 4 m long line *sb* along *XY*. This represents the top view of the line joining the switch and the bulb.
5. Draw an arc with s' as the centre and 5 m radius to meet the vertical projector from point *b* at point b'. Join $s'b'$ to represents the front view.

6. Measure, $bb' = 4.5$ m, as height of the bulb above the floor. Also, measure the inclination of the line $s'b'$ with XY, $\theta = 37^\circ$, as the inclination of line joining the switch the bulb with the floor.

EXERCISE 7A

1. A line 70 mm long parallel to H.P. and V.P. lies 20 mm above H.P. and 50 mm in front of V.P. Draw its projections.
2. The end A of a 65 mm long line AB is 20 mm above the H.P. and 15 mm in front of the V.P. Draw the projections of the line when it is perpendicular to H.P.
3. A 50 mm long line is perpendicular to the V.P. and 40 mm above the H.P. An end of the line is 10 mm in front of the V.P. Draw its projections and the traces.
4. A 70 mm long line AB is inclined at 45° to the H.P. and parallel to the V.P. The end A is 15 mm above the H.P. and 25 mm in front of the V.P. Draw its projections and locate the traces.
5. An 80 mm long line is inclined at 60° to the V.P. and parallel to the H.P. One end of the line is 30 mm above the H.P. and 10 mm in front of the V.P. Draw its projections and locate the traces.
6. An 80 mm long line is parallel to and 20 mm above the H.P. Its one end is in the V.P. while the other end lies 40 mm in front of the V.P. Draw its projections and determine the true inclination of the line with the V.P. [Ans: 30°]
7. A 60 mm long line AB is parallel to and 20 mm in front of V.P. The ends A and B of the line are 10 mm and 50 mm above the H.P., respectively. Draw the projections of the line and determine its inclination with the H.P. Also, locate the traces. [Ans: 42°]
8. A 70 mm long line AB is parallel to the V.P. and inclined to the H.P. The end A is 20 mm above the H.P. and 30 mm in front of the V.P. The top view of the line measures 45 mm. Draw its projections. [Ans: 50°]
9. The front view of 80 mm long line measures 50 mm. The whole line is in the H.P. and its one end is 30 mm in front of the V.P. Draw its projections and find inclination of the line with the V.P. [Ans: 40°]
10. The top view of an 80 mm long line measures 55 mm. The line is in the V.P. and its one end lies 20 mm above the H.P. Draw its projections and find the inclination with the H.P. [Ans: 43°]
11. The top view of a line is 60 mm long. The line is parallel to the V.P. and inclined at 45° to the H.P. One end of the line is 25 mm in front of the V.P. and in the H.P. Draw the projections and determine its true length. [Ans: 85 mm]
12. The front view of a line, parallel to the V.P. and inclined 60° to the H.P., is 50 mm long. One end of the line is 20 mm in front of the V.P. and 25 mm above the H.P. Draw its projections and determine the true length of the line. [Ans: 80 mm]
13. A 70 mm long line PQ has no H.T. and V.T. An end of the line is 30 mm in front of the V.P. and 20 mm above the H.P. Draw its projections.
14. A 70 mm long line is making 30° with the H.P. The H.T. of the line lies 15 mm below the reference line and the V.T. of the line does not exist. Draw its projections when an end of the line is 25 mm above the H.P.

15. A 75 mm long line is inclined at 45º to the V.P. The V.T. is 30 mm above the reference plane and H.T. does not exist. One end of the line is 20 mm in front of the V.P. Draw its projections.
16. A line inclined at 60º to the V.P. has a 60 mm long front view. One end of the line is 20 mm in front of the V.P. The V.T. is 15 mm above the reference line and H.T. does not exist. Draw its projections.

7.15 LINE IN THE FIRST ANGLE INCLINED TO BOTH THE REFERENCE PLANES

Here we will understand the projections of lines inclined to both the reference planes. The line lies in the first angle and is inclined θ to H.P. and ϕ to V.P.

7.16 PROJECTIONS OF A LINE INCLINED TO BOTH THE REFERENCE PLANES

When a line is inclined at θ with H.P. and ϕ with V.P., then both front and top views will be of projected (reduced) length and will appear to be inclined at apparent angles, α with H.P. and β with V.P.

Example 7.16 (Fig. 7.16)

A 70 mm long line *PQ* has its end *P* 20 mm above the H.P. and 30 mm in front of the V.P. The line is inclined at 45º to the H.P. and 30º to the V.P. Draw its projections.

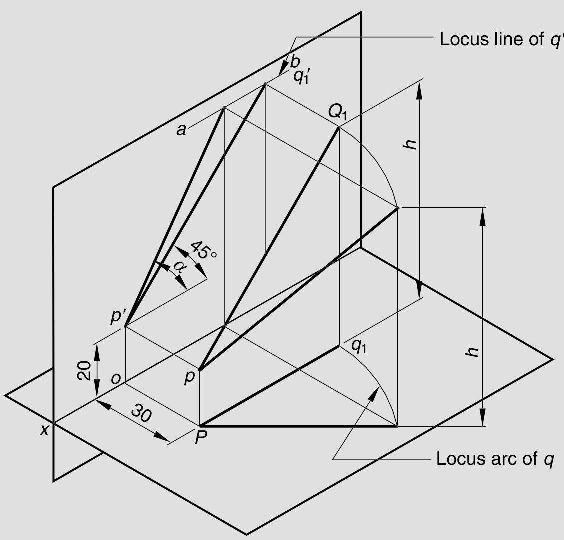


Fig. 7.16(a)

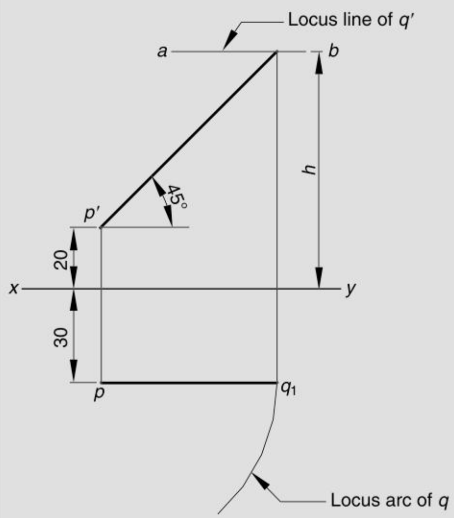


Fig. 7.16(b)

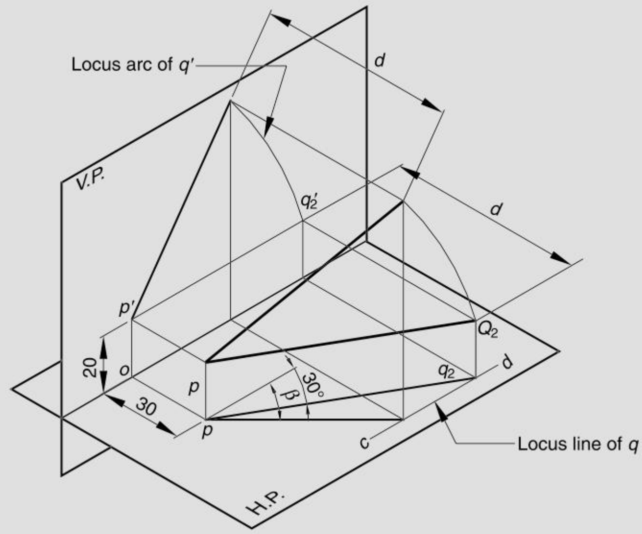


Fig. 7.16(c)

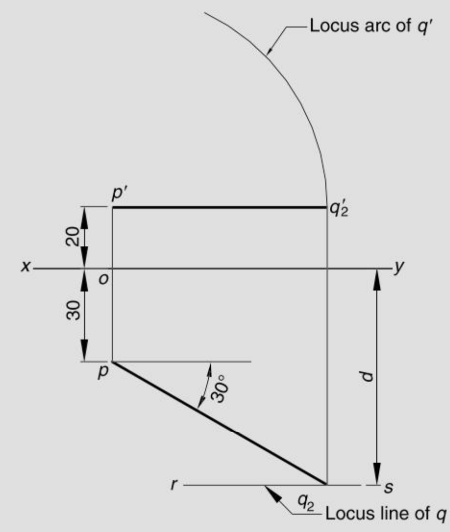


Fig. 7.16(d)

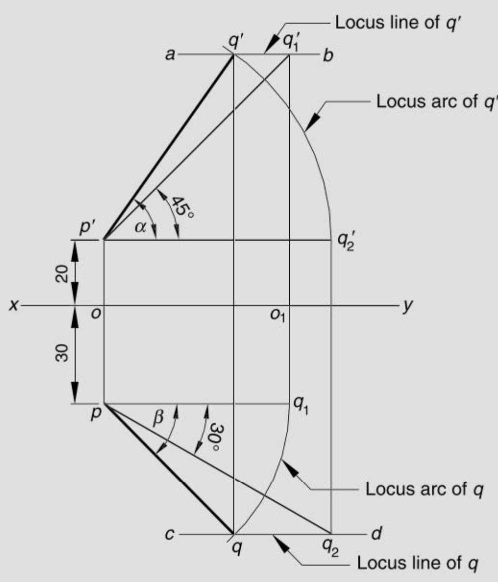


Fig. 7.16(e)

Visualisation:

1. *Fig. 7.16(a) and Fig. 7.16(b)*: Consider a line PQ_1, 70 mm long has its end P 20 mm above the H.P. and 30 mm in front of the V.P. The line is inclined $\theta = 45^\circ$ to the H.P. and parallel to the V.P.
   a. Draw the front view $p'q_1'$ equal to the true length making $\theta = 45^\circ$ with the XY.

b. Draw the top view pq_1 of projected length and parallel to *XY*.

Now keeping the position of the end *P* fixed and the inclination $\theta = 45°$ with H.P. as constant, let the line be turned to make it inclined ϕ to V.P. While turning the line following observations were made.

a. Since the inclination ($\theta = 45°$) with H.P. is constant, the length of the top view will remain same. Therefore, the point q_1 will move to the new position q along an arc (known as locus of q) drawn with p as the centre and pq_1 as the radius.

b. The distance of the end Q_1 from H.P. is constant (say h). In the front view q_1' will move along the line *ab* parallel to *XY*. The line *ab* is called the locus of q'.

2. *Fig. 7.16(c) and Fig. 7.16(d)*: Consider a line PQ_2, 70 mm long has its end *P* 20 mm above the H.P. and 30 mm in front of the V.P. The line is inclined $\phi = 30°$ to the V.P. and parallel to the H.P.

   a. Draw the top view pq_2 equal to the true length making $\phi = 30°$ with the *XY*.

   b. Draw the front view $p'q_1'$ of projected length and parallel to *XY*.

   Now keeping the position of the end *P* fixed and the inclination $\phi = 30°$ with V.P. as constant, let the line be turned to make it inclined θ to H.P. While turning the line following observations were made.

   a. Since the inclination ($\phi = 30°$) with V.P. is constant, the length of the front view will remain same. Therefore, the point q_2' will move to the new position q' along an arc (known as locus of q') drawn with p' as the centre and pq_1 as the radius.

   b. The distance of the end Q_2 from V.P. is constant (say d). In the top view q_2 will move along the line *rs* parallel to *XY*. The line *rs* is called the locus of point q.

3. Step 1 states point q' lies on the locus line *mn* and point q lies on the locus of arc. Step 2 states point q' lies on the locus of arc and point q lies on the locus line *rs*.

   Thus, on combining steps, i.e., Fig. 7.16(b) and Fig. 7.16(d) we can locate point q' and q, as shown in Fig. 7.16(e).

Note: The projector joining qq' is perpendicular to *XY*.

Construction: Fig. 7.16(e)

1. Draw a reference line *XY*.
2. Mark point p' 20 mm above *XY* and point p 30 mm below *XY*.
3. Draw a 70 mm long $p'q_1'$ inclined at $\theta = 45°$ to *XY*.
4. Draw another 70 mm long pq_2, inclined at $\phi = 30°$ to *XY*.
5. Project point q_1' to meet the horizontal line from point p at point q_1.
6. Draw an arc with p as the centre and radius pq_1 to meet the horizontal line from point q_2 at point q.
7. Join pq to represent the top view.
8. Project point q_2 to meet the horizontal line from point p' at point q_2'.
9. Draw an arc with p' as the centre and radius $p'q_2'$ to meet the horizontal line from point q_1' at point q'.
10. Join $p'q'$ to represent the front view.
11. Ensure that the line qq' is perpendicular to *XY*, i.e. representing a vertical projector.

Note: α is the inclination of the front view $p'q'$ with XY whereas β is the inclination of the top view pq with XY. They are called apparent angles. It may be observed that $\alpha > \theta$ and $\beta > \phi$. Both the front and the top views are smaller than the true length.

Notation used:

In this example the following notations have been used

PQ	Actual line in space. It is equal to the true length of the line $PQ = TL$
θ	True inclination of line with the H.P.
ϕ	True inclination of line with the V.P.
PQ_1	Line assumed parallel to the V.P. and inclined θ to the H.P.
$p'q_1'$	Front view of the line PQ_1
pq_1	Top view of the line PQ_1
PQ_2	Line assumed parallel to the H.P. and inclined ϕ to the V.P.
$p'q_2'$	Front view of the line PQ_1
pq_2	Top view of the line PQ_1
$p'q'$	Final / Actual front view of the line PQ
pq	Final / Actual top view of the line PQ
α	Apparent angle made by the front view $p'q'$ with XY
β	Apparent angle made by the top view pq with XY

7.17 TRUE LENGTH AND TRUE INCLINATION OF THE GIVEN LINE

To obtain the true length and true inclination of a line, each view has to be turned to make it parallel to the reference line XY. This is the reverse of Example 7.16.

Example 7.17 (Fig. 7.17)

A straight line *PQ* has its end *P* 20 mm above the H.P. and 30 mm in front of the V.P. and the end *Q* is 80 mm above the H.P. and 70 mm in front of the V.P. If the end projectors are 60 mm apart, draw the projections of the line. Determine its true length and true inclinations with the reference planes.

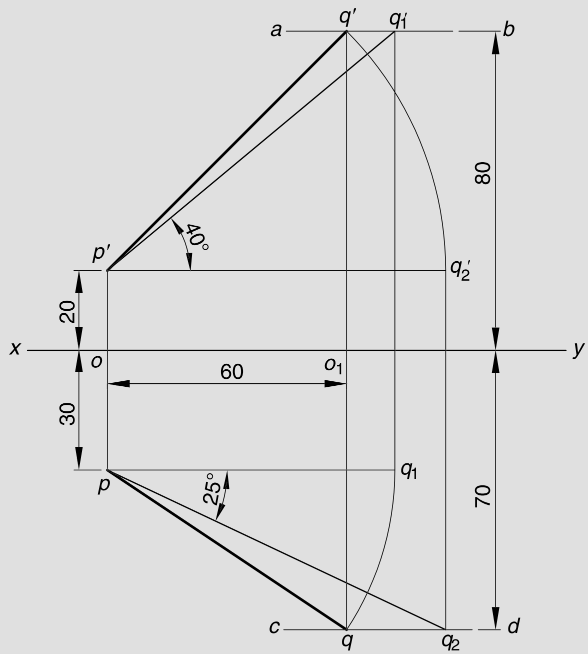


Fig. 7.17

Construction: Fig. 7.17

Draw the projections of the line.

1. Draw a reference line *XY*. Mark points o and o_1 on it such that they are 60 mm apart.
2. On the vertical projector through o, mark point p' 20 mm above *XY* and point p 30 mm below *XY*.
3. On the vertical projector through o_1, mark point q' 80 mm above *XY* and point q 70 mm below *XY*.
4. Join $p'q'$ and pq to represent the front and the top views of the line, respectively.

Find true inclination of line with H.P. (θ) and true length of the line

5. Draw an arc with p as centre and radius pq to meet the horizontal line through point p at point q_1.
6. Project point q_1 to meet horizontal line ab through point q' at point q_1'.
7. Join $p'q_1'$. The length $p'q_1'$ represents the true length of *PQ*. The inclination of $p'q_1'$ with *XY* represents true inclination of *PQ* with H.P. Here, T.L. = 94 mm and $\theta = 40°$.

Find true inclination of line with V.P. (ϕ) and true length of the line.

8. Draw an arc with p' as centre and radius $p'q'$ to meet the horizontal line through point p' at point q_2'.
9. Project point q_2' to meet horizontal line cd through point q at point q_2.
10. Join pq_2. The length pq_2 represents the true length of *PQ*. The inclination of pq_2 with *XY* represents true inclination of *PQ* with V.P. Here, $\phi = 25°$. Ensure length pq_2 is equal to length $p'q_1'$.

7.18 TRAPEZOID METHOD

The true length, true inclination and traces of a line can also be determined by trapezoid method as described below.

Example 7.18 (Fig. 7.18)

A straight line *PQ* has its end *P* 20 mm above the H.P. and 30 mm in front of the V.P. and the end *Q* is 80 mm above the H.P. and 70 mm in front of the V.P. If the end projectors are 60 mm apart, draw the projections of the line. Determine its true length and true inclinations with the reference planes by trapezoid method.

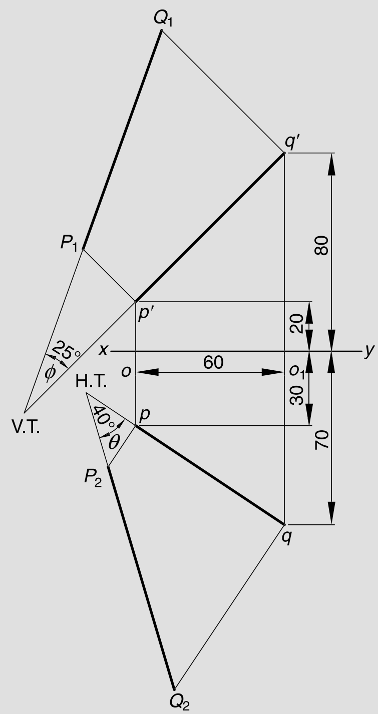


Fig. 7.18

Construction: Fig. 7.18

1. Draw a reference line *XY*. Mark points *o* and o_1 on *XY* 60 mm apart.
2. On the vertical projector through point *o*, mark point *p′* 20 mm above the *XY* and point *p* 30 mm below the *XY*.
3. On the vertical projector through point o_1, mark point *q′* 80 mm above the *XY* and point *q* 70 mm below the *XY*.

4. Join $p'q'$ and pq to represent the front and top views of line PQ, respectively.
5. Draw perpendiculars $p'P_1$ and $q'Q_1$ to the line $p'q'$ such that $p'P_1 = op$ and $q'Q_1 = oq$.
6. Join P_1Q_1. Measure its length to represent the true length and its inclination with $p'q'$ to represent true inclination with V.P. (ϕ).
7. Draw perpendiculars pP_2 and qQ_2 to the line Pq such that $pP_2 = op$ and $qQ_2 = oq$.
8. Join P_2Q_2. Measure its length to represent the true length and its inclination with pq to represent true inclination with H.P. (θ).
9. Ensure that true length obtained in step 7 and step 9 are same.
10. The point at which line P_1Q_1 produced meets with front view $p'q'$ produced represents the V.T.
11. The point at which line P_2Q_2 produced meets with front view pq produced represents the H.T.

7.19 TRACES OF A LINE INCLINED TO BOTH THE REFERENCE PLANES

When a line is inclined to both the reference planes, it will meet both the H.P. and the V.P. Therefore, the line will have both the H.T. and the V.T. Consider the following example.

Example 7.19 (Fig. 7.19)

Locate the traces of a straight line *PQ*, kept in first angle for the following cases.

A. End *P* is 20 mm above the H.P. and 30 mm in front of the V.P. and the end *Q* is 80 mm above the H.P. and 60 mm in front of the V.P. The end projectors are 60 mm apart.

B. End *P* is 30 mm above the H.P. and 20 mm in front of the V.P. and the end *Q* is 60 mm above the H.P. and 80 mm in front of the V.P. The end projectors are 60 mm apart.

C. End *P* is 20 mm above the H.P. and 30 mm in front of the V.P. and the end *Q* is 60 mm above the H.P. and 10 mm in front of the V.P. The end projectors are 60 mm apart.

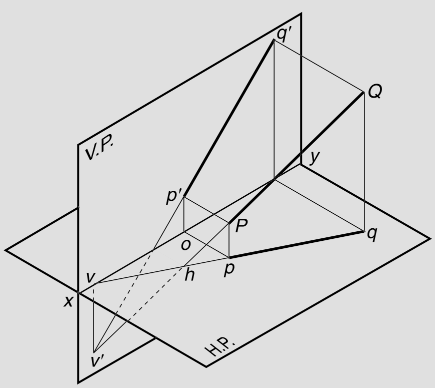


Fig. 7.19(a)

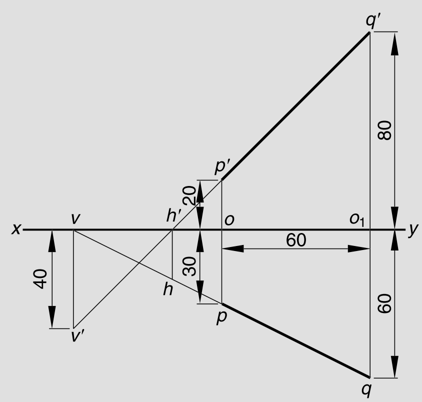


Fig. 7.19(b)

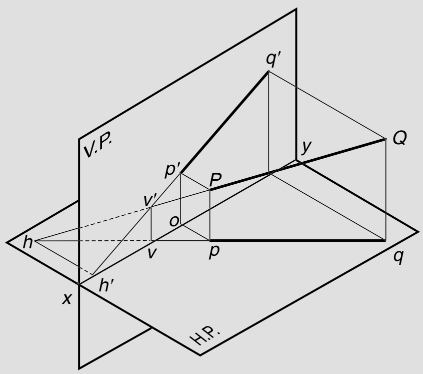


Fig. 7.19(c)

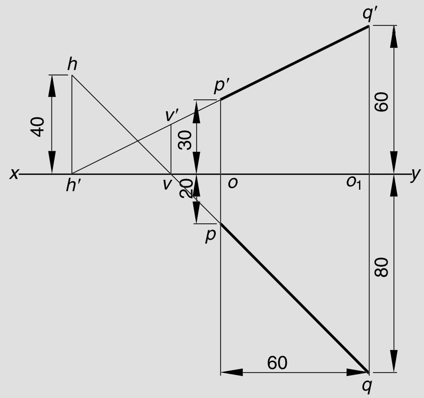


Fig. 7.19(d)

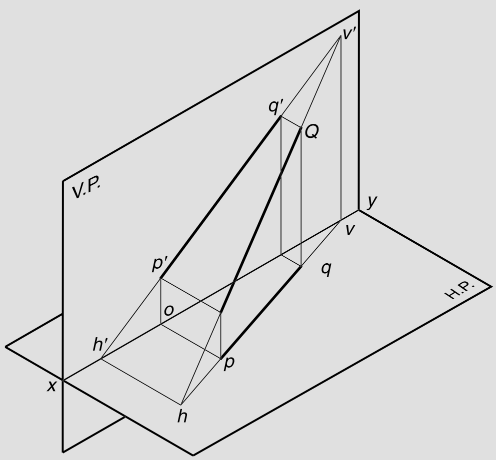


Fig. 7.19(e)

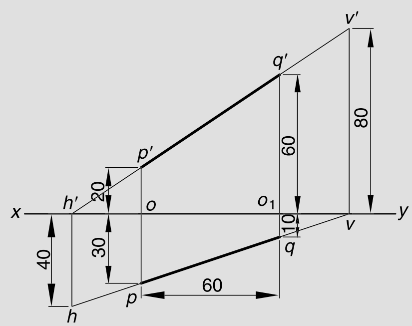


Fig. 7.19(f)

Visualisation: *Case A*: Fig. 7.19(a) shows the line situated in the space with respect to the reference plane and Fig. 7.19(b) shows the corresponding projections of the given line.

Case B: Fig. 7.19(c) shows the line situated in the space with respect to the reference plane and Fig. 7.19(d) shows the corresponding projections of the given line.

Case B: Fig. 7.19(e) shows the line situated in the space with respect to the reference plane and Fig. 7.19(f) shows the corresponding projections of the given line.

Construction: *Case A* — Fig. 7.19(b), *Case B* — Fig. 7.19(d) and *Case C* — Fig. 7.19(f)

1. Draw a reference line *XY*. Mark points *o* and o_1 on *XY* such that they are 60 mm apart.
2. On the vertical projector through point *o*, mark points *p*′ and *p* as the front and the top views of point *P*.

3. Similarly, on the vertical projector through point o_1, mark points q' and q as the front and the top views of point Q.
4. Join $p'q'$ and pq to represent the front and the top views of the line PQ.
5. Produce the front view $p'q'$ to meet XY at a point h'. Draw a vertical projector through point h' to meet the top view pq, produced if required, at point h. The point h represents the H.T.
6. Produce the top view pq to meet XY at a point v. Draw a vertical projector through point v to meet the front view $p'q'$, produced if necessary, at point v'. The point v' represents the V.T.
7. Measure the distance of h and v' from XY.

Conclusion: The H.T. and V.T. may lie either on the same side or on the opposite side of XY.

7.20 PROJECTIONS OF A LINE CONTAINED BY A PROFILE PLANE (i.e. $\theta + \phi = 90°$)

A profile plane is a plane perpendicular to both the reference planes (H.P. and V.P.). When a line is contained by a profile plane, the sum of its inclination with the H.P. and V.P. is 90º. (i.e. $\theta + \phi = 90°$). In such a case, both the front and the top views will be perpendicular to XY and have their lengths shorter than the true length. In other words, the apparent angles, α and β both will be 90º.

Example 7.20 (Fig. 7.20)

A 100 mm long line *PQ* has its end *P* 10 mm above the H.P. and 70 mm in front of the V.P. The line is inclined at 60º to the H.P. and 30º to the V.P. Draw its projections.

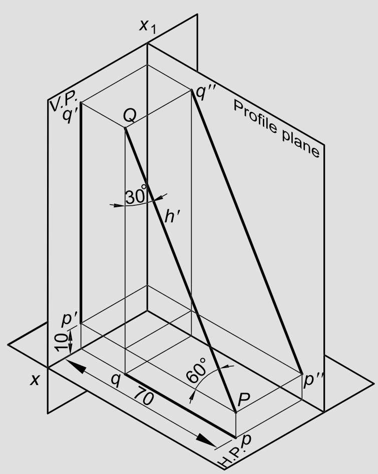


Fig. 7.20(a) *Position of line*

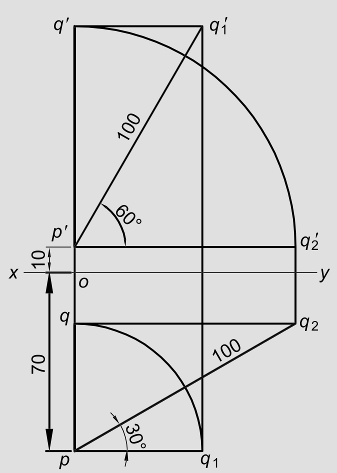


Fig. 7.20(b) *Method 1*

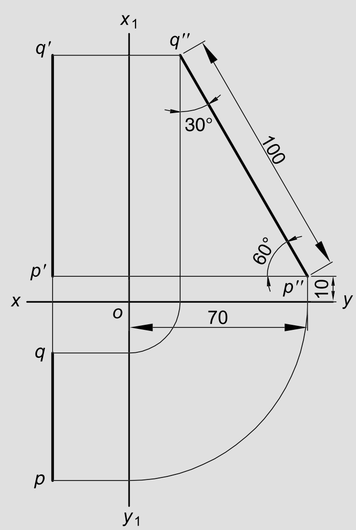


Fig. 7.20(c) *Method 2*

Visualisation: Fig. 7.20(a) shows the position of the line *PQ* kept on the reference planes. The example can be solved by two methods as follows:

Method 1: Change of position of line. (Similar to Example 7.16):

Construction: Fig. 7.20(b)

1. Draw a reference line *XY*.
2. Mark point *o* on the reference line and draw a vertical projector through it.
3. On the vertical projector mark p' 10 mm above *XY* and *p* 70 mm below *XY*.
4. Draw 100 mm long $p'q_1'$ inclined at $\theta = 60°$ to *XY*.
5. Draw another 100 mm long pq_2 inclined at $\phi = 30°$ to *XY*.
6. Project point q_2 to meet the horizontal line through point p' at point q_2'. Draw an arc with p' as the centre and radius $p'q_2'$ to meet the horizontal line through point q_1' at point q'. Join $p'q'$ to represent the front view.
7. Project point q_1' to meet the horizontal line through point *p* at point q_1. Draw an arc with *p* as the centre and radius pq_1 to meet the horizontal line through point q_2 at point *q*. Join *pq* to represent the top view.

Method 2: Use of profile plane / auxiliary plane.

Visualisation: Consider the line is kept in the reference planes as shown in Fig. 7.20(a). Let there be a profile plane on the right side of the line. The line is parallel to the profile plane such that it is inclined 60° to the H.P. and 30° with the V.P. It can be clearly seen that for such line, the front and the top views will appear as a line perpendicular to *XY*.

Construction: Fig. 7.20(c)

1. Draw a reference line *XY*.
2. Draw another reference line X_1Y_1, perpendicular to the *XY* to represent the profile plane.
3. Mark point p'' such that it is 10 mm above *XY* and 70 mm right to X_1Y_1.
4. Draw $p''q''$ inclined at 60° to *XY*. The line would also be inclined at 30° to X_1Y_1.
5. Draw horizontal lines from points p'' and q'', and locate points p' and q' such that they lay on a line perpendicular to *XY*. Join $p'q'$ to represent the front view of the line.
6. Draw vertical projectors from p'' and q'' to meet *XY*. Rotate the points obtained with *o* as the centre to meet X_1Y_1. Draw horizontal lines from them, known as locus lines for the top view.
7. Draw vertical projectors from p' and q' to meet the locus lines of the top view at points *p* and *q*. Join *pq* to represent the top view.

Conclusion: When a line is inclined to the reference planes such that $\theta + \phi = 90°$ then apparent angles $\alpha = \beta = 90°$. i.e. final front and top views are perpendicular to *XY*.

7.21 TRACES OF A LINE CONTAINED BY A PROFILE PLANE

When a line is inclined at θ to H.P. and ϕ to V.P. such that $\theta + \phi = 90°$, then the line is said to be contained by a profile plane. To determine the H.T. and V.T. we can not follow the procedure of the Example 7.19. To locate H.T. and V.T. in such case consider the following example.

Example 7.21 (Fig. 7.21)

The ends of a straight line *PQ* are on the same projectors. Locate its traces when

A. End *P* is 20 mm above the H.P. and 30 mm in front of the V.P. and the end *Q* is 80 mm above the H.P. and 70 mm in front of the V.P.
B. End *P* is 80 mm above the H.P. and 20 mm in front of the V.P. and the end *Q* is 30 mm above the H.P. and 70 mm in front of the V.P.

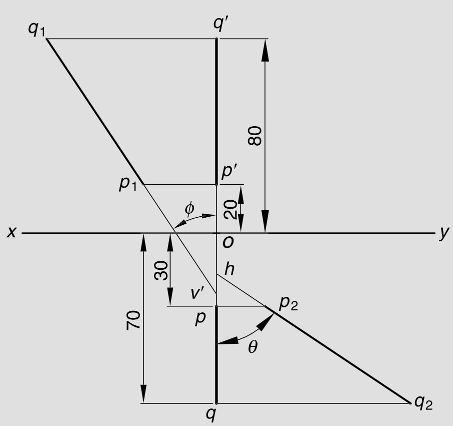


Fig. 7.21(a)

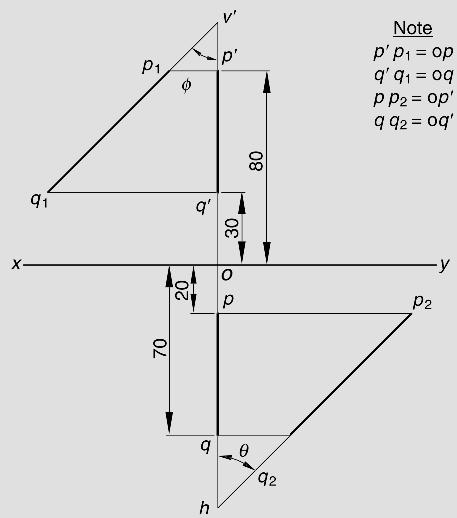


Fig. 7.21(b)

Visualisation: As ends of the line lies on the common projector the line is parallel to the profile plane, i.e. $\theta + \phi = 90$. First draw the projections of the line and then determine the traces of the line in each of the cases. Fig. 7.21(a) and Fig. 7.21(b) shows the final result. Follow steps 1 to 7 to locate H.T. and V.T. for both the cases.

Construction: Fig. 7.21(a) and Fig. 7.21(b)

1. Draw a reference line *XY*.
2. Mark points p', p, q' and q for each cases as given in the example. Join $p'q'$ and p-q to represent the front and the top views of line *PQ*, respectively.
3. From points p' and q', draw lines $p'p_1$ and $q'q_1$ such that $p'p_1 = op$, $q'q_1 = oq$ and they are perpendicular to $p'q'$. Points p_1 and q_1 must lie on the same side of $p'q'$.
4. Join p_1q_1 and produce it to meet front view $p'q'$ produced at point v'. Point v' denotes the V.T. of the line. The angle made by p_1q_1 with $p'q'$ represents the true inclination (ϕ) of the line *PQ* with V.P.
5. From points p and q, draw lines pp_2 and qq_2 such that $pp_2 = op'$, $qq_2 = oq'$ and they are perpendicular to pq. Points p_2 and q_2 must lie on the same side of pq.

6. Join p_2q_2 and produce it to meet top view pq produced at point h. Point h denotes the H.T. of the line. The angle made by p_2q_2 with pq represents the true inclination (θ) of line PQ with H.P.

Conclusion: When $\theta + \phi = 90°$ then the H.T. and the V.T. lies on the common projector that of the line PQ itself.

7.22 MISCELLANEOUS EXAMPLES

Example 7.22 (Fig. 7.22)

A line *CD*, 80 mm long is inclined at 45º to H.P. and 30º to the V.P., its end *C* is in H.P. and 40 mm in front of V.P. Draw the projections. [RGPV Dec. 2004, Feb 2005]

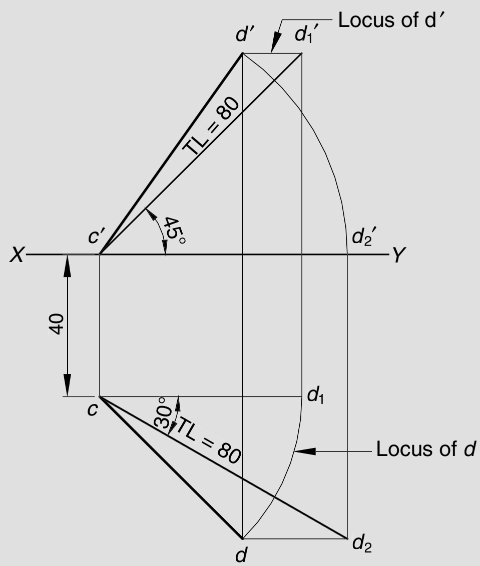


Fig. 7.22

Given data	*Interpretation*
CD = 80 mm	$c'd_1' = cd_2$ = 80 mm
CD is inclined at 45º to H.P. (θ= 45º)	$c'd_1'$ is inclined at 45º to XY
CD is inclined at 30º to V.P. (ϕ= 30º)	cd_2 is inclined at 30º to XY
C is in H.P.	Point c' is on XY
C is 40 mm in front of V.P.	Point c is 40 mm below XY

Construction: Fig. 7.22

1. Draw the reference line *XY*.
2. Mark point c' on *XY* and point *c* 40 mm blow *XY*.
3. Draw an 80 mm long line $c'd_1'$ making 45º with *XY*.
4. Draw another 80 mm long line cd_2 making 30º with *XY*.
5. Project d_1' to meet horizontal line from *c* at point d_1. Draw an arc with *c* as the centre and radius cd_1 to meet the horizontal line through d_2 at point *d*. Join *cd* to represent the top view.
6. Project d_2 to meet horizontal line through c' at point d_2'. Draw an arc with c' as the centre and $c'd_2'$ as the radius to meet the horizontal line through d_1' at point d'. Join $c'd'$ to represent the front view.
7. Join $d'd$. If $d'd$ is exactly perpendicular to *XY*, it indicates that the drawing is accurate.

Example 7.23 (Fig. 7.23)

A 100 mm long line *PQ* is inclined at 30º to H.P. and 45º to the V.P. Its mid-point is 35 mm above the H.P. and 50 mm in front of V.P. Draw its projections.

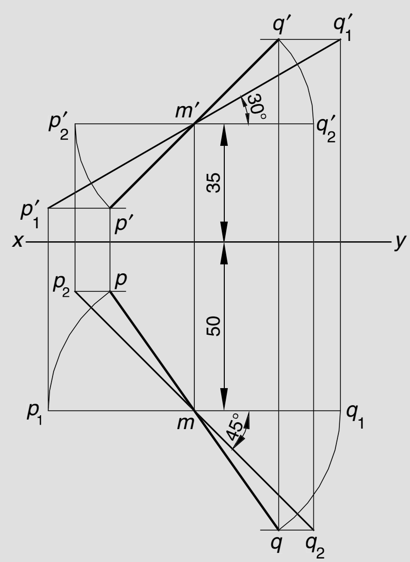


Fig. 7.23

Given data	*Interpretation*
PQ = 100 mm and *M* is the mid-point	$p_1'm' = m'q_1' = 50$ mm, $p_2m = mq_2 = 50$ mm
Mid-point *M* is 35 mm above the H.P.	m' is 35 mm above *XY*
Mid-point *M* is 50 mm in front of V.P.	*m* is 50 mm below *XY*
Line is inclined at 30º to H.P. ($\theta = 30º$)	$p'm'q_1'$ is making 30º with *XY*
Line is inclined at 45º to V.P. ($\phi = 45º$)	p_2mq_2 is making 45º with *XY*

Construction: Fig. 7.23

1. Draw a reference line *XY*. Mark point *m′* 35 mm above *XY* and point *m* 50 mm below *XY*.
2. Draw a 50 mm long line $m'q_1'$ inclined at 30º to *XY*. Produce it such that $p_1'q_1' = 100$ mm.
3. Draw another 50 mm line mq_2 inclined at 45º to *XY*. Produce it such that $p_2q_2 = 100$ mm.
4. Project points p_1' and q_1' to meet the horizontal line through point *m* at points p_1 and q_1 respectively. Draw an arc with *m* as the centre and radius mp_1 (or mq_1) to meet the horizontal lines through points p_1' and q_1' at points *p* and *q*, respectively. Join *pmq* to represent the top view.
5. Draw vertical projectors from points p_2 and q_2 to meet the horizontal line through point *m′* at points p_2' and q_2' respectively. Draw an arc with *m′* as the centre and radius $m'p_2'$ (or $m'q_2'$) to meet the horizontal lines through points p_2 and q_2 at points *p′* and *q′*, respectively. Join *p′m′q′* to represent front view.
6. Join *p′p* and *q′q* and ensure that these are perpendicular to *XY*.

Example 7.24 (Fig. 7.24)

An 80 mm long line *PQ* has its end *P* 15 mm from both H.P. and V.P. The other end *Q* is 40 mm above H.P. and 50 mm in front of V.P. Draw the projections of the line and determine the inclinations with H.P. and V.P.

[RGPV June 2008, Feb. 2010, Aug. 2010]

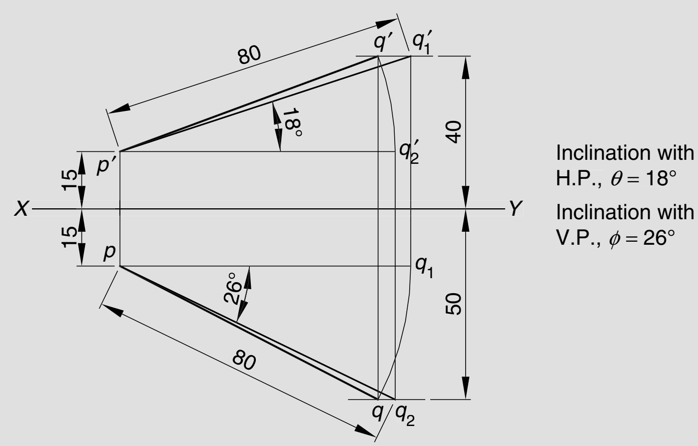


Fig. 7.24

Given data	*Interpretation*
PQ = 80 mm	$p'q_1' = pq_2 = 80$ mm
P is 15 mm above H.P.	Point *p′* is 15 mm above *XY*
P is 15 mm in front of V.P.	Point *p* is 15 mm below *XY*
Q is 40 mm above H.P.	Points *q′* and q_1' are 40 mm above *XY*
Q is 50 mm in front of V.P.	Points *q* and q_2 are 50 mm below *XY*

Construction: Fig. 7.24

1. Draw a reference line *XY*. Mark point p' 15 mm above *XY* and point p 15 mm below *XY*.
2. Draw a horizontal line 40 mm above *XY* as locus line of point q' and q_1'.
3. Draw another horizontal line 50 mm below *XY* as locus line of point q and q_2.
4. Draw an arc with p' as the centre and radius 80 mm to intersect locus of q_1' at point q_1'. Measure the inclination of line $p'q_1'$ with *XY*. It is the required inclination θ of the line with H.P. Here $\theta = 18^\circ$.
5. Draw an arc with p as the centre and 80 mm radius to intersect locus of q_2 at point q_2. Measure the inclination of line pq_2 with *XY*. It is the required inclination ϕ of the line with V.P. Here $\phi = 26^\circ$.
6. Project q_1' to meet horizontal line through p at point q_1. Draw an arc with p as the centre and radius pq_1 to intersect the locus of q at point q. Join pq to represent the top view.
7. Project q_2 to meet horizontal line through p' at point q_2'. Draw an arc with p' as the centre and radius $p'q_2'$ to intersect locus of q' at point q'. Join $p'q'$ to represent the front view.
8. Join $q'q$ and ensure that it is perpendicular to *XY*.

Example 7.25 (Fig. 7.25)

Draw projections and find out the true length of a line *AB*, with end *B* on H.P. and 40 mm in front of V.P. *AB* is inclined at 30º to H.P. and 45º to V.P. and its plan measures 50 mm.
[RGPV April 2009]

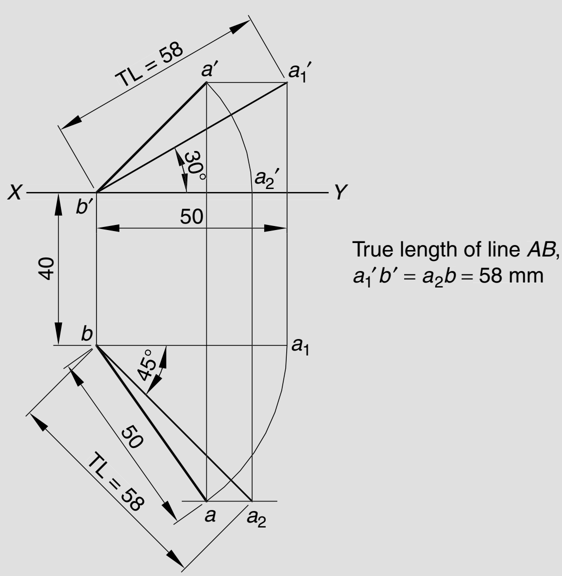


Fig. 7.25

Given data	*Interpretation*
End *B* is on H.P.	Point b' is on *XY*
End *B* is 40 mm in front of V.P.	Point *b* is 40 mm below *XY*
AB is inclined at 30º to H.P. (θ= 30º)	$b'a_1'$ is inclined at 30º to *XY*
AB is inclined at 45º to V.P. (ϕ= 30º)	ba_2 is inclined at 45º to *XY*
Plan measures 50 mm	$ba = ba_1 = 50$ mm

Construction: Fig. 7.25

1. Draw a reference line *XY*. Mark point b' on *XY* and point *b* 40 mm below *XY*.
2. Draw a 50 mm long line ba_1 parallel to *XY*.
3. From point b', draw a line inclined at 30º to meet the vertical projector through point a_1 at point a_1'. Measure length $b'a_1' = 58$ mm as true length of the line.
4. From point *b*, draw a line ba_2 equal to $b'a_1'$, inclined at 45º to *XY*.
5. Draw an arc with *b* as the centre and radius ba_1 to meet the horizontal line through point a_2 at point *a*. Join *ba* to represent the top view.
6. Project a_2 to meet horizontal line through b' at point a_2'. Draw an arc with b' as the centre and radius $b'a_2'$ to meet the horizontal line through point a_1' at point a'. Join $b'a'$ to represent the front view.
7. Join $a'a$ and ensure that it is perpendicular to *XY*.

Example 7.26 (Fig. 7.26)

The top view of a 80 mm long line *AB* measures 65 mm, while the length of its front view is 55 mm. Its one end *A* is in the H.P. and 12 mm in front of the V.P. Draw the projections of *AB* and determine its inclination with the H.P. and V.P. [RGPV June 2009]

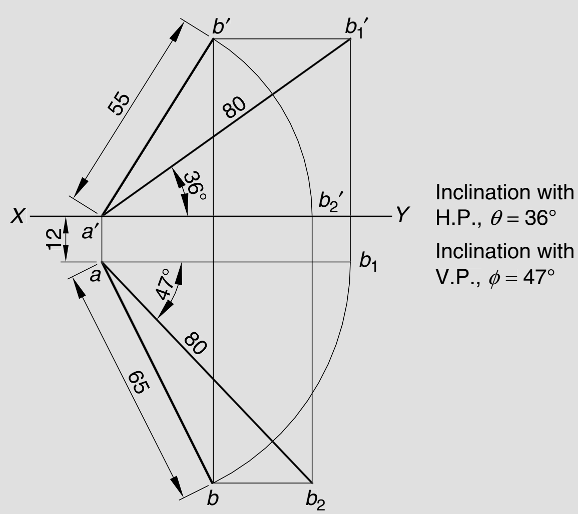


Fig. 7.26

Given data	*Interpretation*
AB = 80 mm	$a'b_1' = ab_2$ = 80 mm
Top view measures 65 mm	$ab = ab_1$ = 65 mm
Length of front view is 55 mm	$a'b' = a'b_2'$ = 55 mm
End A is in the H.P.	a' is on XY
End A is 12 mm in front of the V.P.	a is 12 mm below XY

Construction: Fig. 7.26

1. Draw a reference line XY. Mark point a' on XY and point a 12 mm below XY.
2. Draw a 65 mm long line ab_1 parallel to XY.
3. Draw another 55 mm long line $a'b_2'$ parallel to XY.
4. Draw an arc with a' as the centre and 80 mm radius to meet the vertical projector from point b_1 at point b_1'. Join $a'b_1'$. Measure the inclination of the line $a'b_1'$ with XY. Here θ= 36º which is the required inclination of line with H.P.
5. Draw an arc with a as the centre and 80 mm radius to meet the vertical projector from b_2' at point b_2. Join ab_2. Measure the inclination of the line ab_2 with XY. Here ϕ = 47º which is the required inclination of line with V.P.
6. Draw an arc with a' as centre and radius $a'b_2'$ to meet the horizontal line through point b_1' at point b'. Join $a'b'$ to represent the front view.
7. Draw an arc with a as centre and radius ab_1 to meet the horizontal line through point b_2 at point b. Join ab to represents the top view.
8. Join $b'b$ and ensure that it is perpendicular to XY.

Example 7.27(Fig. 7.27)

The front view and top view of a straight line *PQ* measures 50 mm and 65 mm respectively. Point *p* is in the H.P. and 20 mm in front of the V.P. and the front view of the line is inclined at 45º to the reference line. Determine the true length of *PQ*, true angles of inclination with the reference planes and the trace.

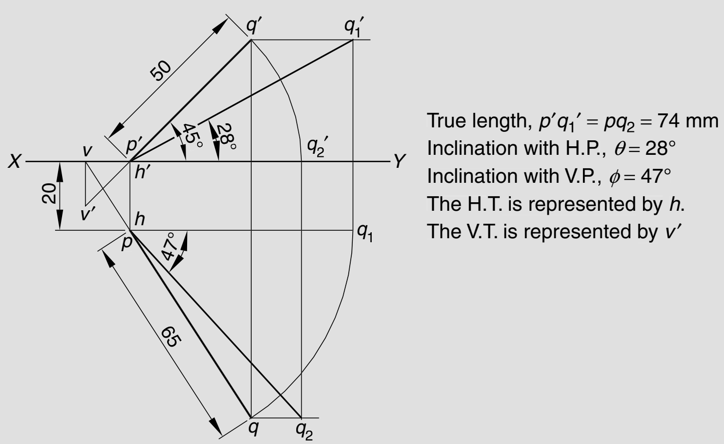


Fig. 7.27

Given data	*Interpretation*
Front view = 50 mm	$p'q' = p'q_2' = 50$ mm
Top view = 65 mm	$pq = pq_1 = 65$ mm
End P is in the H.P.	Point p' is on the XY
End P is 20 mm in front of the V.P.	Point p is 20 mm below the XY
Front view is inclined 45º to XY	$\alpha = 45º$ (i.e. $p'q'$ makes 45º with XY)

Construction: Fig. 7.27

1. Draw a reference line XY. Mark point p' on XY and point p 20 mm below XY.
2. From point p', draw a 50 mm long line $p'q'$ such that it is inclined at $\alpha = 45º$ with XY. Line $p'q'$ represents the front view.
3. Draw an arc with p as the centre and radius 65 mm to meet the vertical projector from point q' at point q. Join pq to represent the top view.
4. Draw an arc with p' as the centre and radius $p'q'$ to meet the horizontal line through p' at point q_2'. Project q_2' to meet the horizontal line through point q at point q_2. Join pq_2 to represent the true length. The inclination of pq_2 with XY represents the inclination of line with V.P. Here $\phi = 47º$.
5. Draw another arc with p as the centre and radius PQ to meet the horizontal line through p at point q_1. Project q_1 to meet the horizontal line through point q' at point q_1'. Join $p'q_1'$ to represent the true length. The inclination of $p'q_1'$ with XY represents the inclination of line with H.P. Here $\theta = 28º$.
6. Ensure that true lengths of the line $p'q_1' = pq_2 = 74$ mm.
7. Line $p'q'$ meet XY at point h'. Project point h' to meet line pq at point h. Point h represents the H.T. Here, point h coincides with point p.
8. Produce line pq to meet XY at point v. Project point v to meet line $p'q'$ produced at point v'. Point v' represents the V.T.

Example 7.28 (Fig. 7.28)

A 70 mm long line PQ is inclined at 45º to the V.P. Its end P lies in the H.P. and 15 mm in front of the V.P. The top view of the line measures 60 mm. Draw its projections and determine true inclination with H.P.

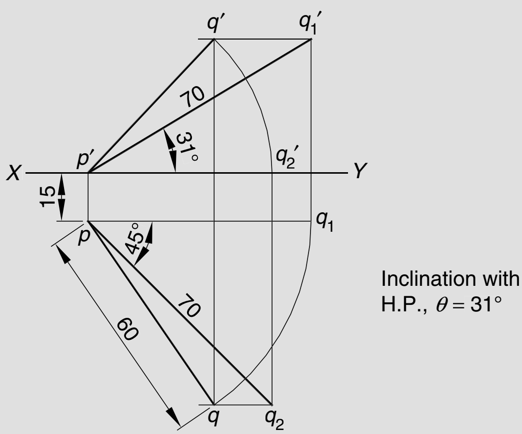


Fig. 7.28

Given data	*Interpretation*
$PQ = 70$ mm	$p'q_1' = pq_2 = 70$ mm
Line is inclined at 45º to V.P. ($\phi = 45º$)	pq_2 is inclined at 45º to XY
End P is in the H.P.	Point p' is on XY
End P is 15 mm in front of the V.P.	Point p is 15 mm below XY
Top view measures 60 mm	$pq = pq_1 = 60$ mm

Construction: Fig. 7.28

1. Draw a reference line XY. Mark point p' on XY and point p 15 mm below XY.
2. From point p, draw a 70 mm long line pq_2 such that it is inclined at $\phi = 45º$ with XY.
3. Draw an arc with p as the centre and 60 mm radius to meet the horizontal line through point q_2 at point q. Join pq to represent the top view.
4. Draw an arc with p as the centre and radius pq to meet the horizontal line through p at point q_1. Draw another arc with p' as the centre and 70 mm radius long to meet the vertical projector from point q_1 at point q_1'. Join $p'q_1'$ to represents the true inclination of line with H.P. Here, $\theta = 31º$
5. Draw a vertical line from point q_2 to meet the horizontal line through p' at point q_2'. Draw an arc with p' as the centre and radius $p'q_2'$ to meet the horizontal line through point q_1' at point q'. Join $p'q'$ to represent the front view.
6. Join $q'q$ and ensure that it is perpendicular to XY.

Example 7.29 (Fig. 7.29)

A line *AB*, 90 mm long is inclined at 30º to the H.P. Its end *A* is 12 mm above H.P. and 20 mm in front of the V.P. Its front view measures 65 mm. Draw the top view of *AB* and determine its inclination with the V.P. Also, locate the V.T. and H.T. of the line. [RGPV Feb. 2007]

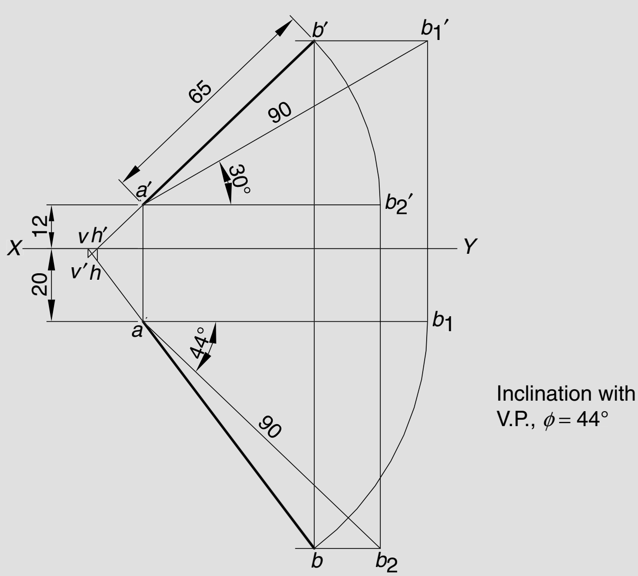


Fig. 7.29

Given data	***Interpretation***
AB is 90 mm long	$a'b_1' = ab_2 = 90$ mm
Line is inclined at 30º to H.P. ($\theta = 30º$)	$a'b_1'$ is making 30º with *XY*
End *A* is 12 mm above H.P.	a' is 12 mm above *XY*
End *A* is 20 mm in front of the V.P.	*a* is 20 mm below XY
Front view measures 65 mm	$a'b' = a'b_2' = 65$ mm

Construction: Fig. 7.29

1. Draw *XY*. Mark point a' 12 mm above *XY* and point *a* 20 mm below *XY*.
2. From point a', draw a 90 mm long line $a'b_1'$ at $\theta = 30º$ with *XY*.
3. Draw an arc with a' as the centre and 65 mm radius to meet the horizontal line through point b_1' at point b'. Join $a'b'$ to represent the front view.
4. Draw an arc with a' as the centre and radius $a'b'$ to meet the horizontal line through a' at point b_2'. Draw another arc with *a* as the centre and 90 mm radius to meet the vertical projector from point b_2' at point b_2. Join ab_2 and measure its inclination with V.P. as $\phi = 44º$.
5. Draw a vertical line from point b_1' to meet the horizontal line through point *a* at point b_1. Draw an arc with a as the centre and radius ab_1 to meet the horizontal line through point b_2 at point *b*. Join *ab* to represent the top view.
6. Join $b'b$ and ensure that it is perpendicular to *XY*.
7. Produce $a'b'$ to meet *XY* at point h'. Project point h' to meet extended line *ab* at point *h*. Point *h* represents the H.T.
8. Produce line *ab* to meet *XY* at point *v*. Project point *v* to meet extended line $a'b'$ at point v'. Point v' represents the V.T.

Example 7.30 (Fig. 7.30)

The front view and top view of a 80 mm long line *PQ* measures 70 mm and 60 mm respectively. End *P* is in the H.P. and *Q* in the V.P. Draw the projections and determine true inclinations with H.P. and V.P. Also, locate the traces.

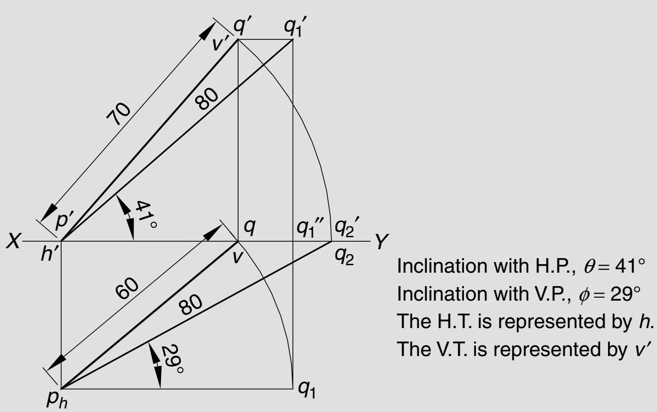


Fig. 7.30

Given data	*Interpretation*
$PQ = 80$ mm	$p'q_1' = pq_2 = 80$ mm
Front view measures 70 mm	$p'q' = p'q_2' = 70$ mm
Top view measures 60 mm	$pq = pq_1 = 60$ mm
End P is in the H.P.	Point p' is on XY
End Q is in the V.P.	Point q is on XY

Construction: Fig. 7.30

1. Draw a reference line XY. Mark point p' on XY.
2. From point p', draw a 60 mm long line $p'q_1''$ long. Draw an arc with p' as the centre and 80 mm radius to meet the vertical projector from point q_1'' at point q_1'. Join $p'q_1'$. Measure the inclination of line $p'q_1'$ with XY as inclination with H.P. Here $\theta = 41°$.
3. Draw an arc with p' as the centre and 70 mm radius to meet the horizontal line through point q_1' at point q'. Join $p'q'$ to represent the front view.
4. Project point q' to meet XY at point q (because q lies on XY).
5. From point q, draw an arc of 60 mm radius to meet the vertical projector from point p' at point p. Join pq to represents the top view.
6. Draw an arc with p' as the centre and radius $p'q'$ to meet the horizontal line through p' at point q_2. Join pq_2. Measure the inclination of pq_2 with XY as inclination of line with V.P. Here $\phi = 29°$.
7. Ensure that length pq_2 is equal to 80 mm.
8. Line $p'q'$ meets XY at point h'. Project point h' to meet line pq at point h. Point h represents the H.T. Here, point h coincides with point p.
9. Similarly, line pq meets XY at point v. Project point v to meet line $p'q'$ at point v'. Point v' represents the V.T. Here, point v' coincides with point q'.

Example 7.31 (Fig. 7.31)

A line *AB* has its end *A* 12 mm above H.P. and 10 mm in front of V.P. The end *B* is 50 mm above the H.P. and the line is inclined at 30° to the H.P. The distance between the end projectors of the line is 50 mm. Draw the projections of the line, find its inclination with V.P. and locate its traces. [RGPV June 2009]

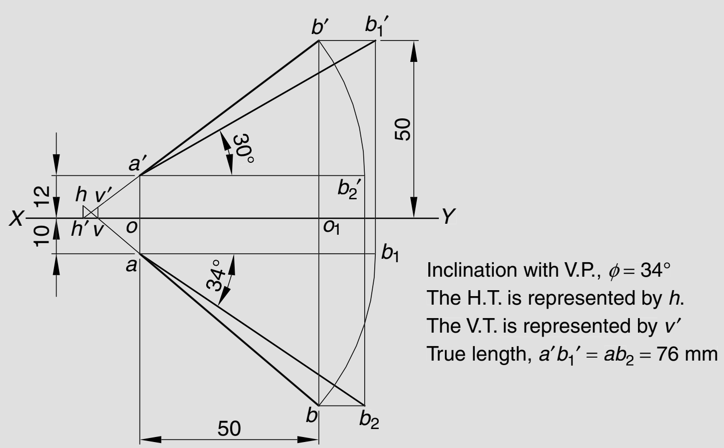


Fig. 7.31

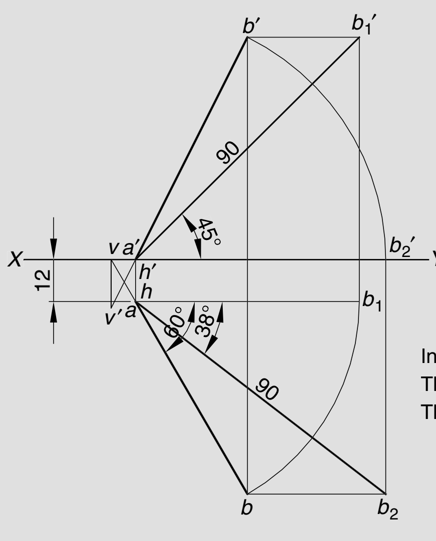
Given data	*Interpretation*
A is 12 mm above H.P.	a' is 12 mm above XY
A is 10 mm in front of V.P.	a is 10 mm below XY
B is 50 mm above the H.P.	b' is 50 mm above XY
Line is inclined at 30º to H.P. (θ = 30º)	$a'b_1'$ is inclined at 30º to XY
distance between end projectors = 50 mm	oo_1 = 50 mm,

Construction: Fig. 7.31

1. Draw a reference line XY. Mark points o and o_1 on XY such that oo_1 = 50 mm.
2. On the vertical projector through point o, mark point a' 12 mm above XY and point a 10 mm below XY.
3. On the vertical projector through point o_1, mark point b' 50 mm above XY.
4. Join $a'b'$ to represent the front view.
5. Draw a line from point a' inclined at 30º with XY to meet horizontal line through point b' at point b_1'.
6. Project point b_1' to meet the horizontal line through point a at point b_1. Draw an arc with point a as the centre and radius ab_1 to meet the vertical projector from point b' at point b. Join ab to represent the top view.
7. Draw an arc with a' as centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Project point b_2' to meet the horizontal line through point b at point b_2. Join ab_2. Measure the inclination of ab_2 with XY as the inclination of line with V.P. Here ϕ = 34º.
8. Ensure that true length represented by line $a'b_1'$ and line ab_2 are equal. Here, T.L. = 76 mm.
9. Produce line $a'b'$ to meet XY at point h'. Project point h' to meet line ab produced at point h. Point h represents the H.T.
10. Produce line ab to meet XY at point v. Project point v to meet line $a'b'$ produced at point v'. Point v' represents the V.T.

Example 7.32 (Fig. 7.32)

A line *AB*, 90 mm long is inclined at 45º to H.P. and its top view makes an angle of 60º with V.P. The end *A* is in the H.P. and 12 mm in front of V.P. Draw its front view and find its true inclination with V.P. Also, locate its traces. [RGPV Dec. 2005, June 2007]



Inclination with V.P., ϕ = 38°
The H.T. is represented by *h*.
The V.T. is represented by *v′*

Fig. 7.32

Given data	***Interpretation***
AB = 90 mm	$a'b_1' = ab_2$ = 90 mm,
Line is inclined at 45º to H.P. (θ = 45º)	$a'b_1'$ is inclined at 45º to XY
Top view makes an angle of 60º with V.P. (β = 60º)	ab is inclined at 60º to XY
A is in the H.P.	Point a' is on XY
A is 12 mm in front of the V.P.	Point a is 12 mm below XY

Construction: Fig. 7.32

1. Draw a reference line XY. Mark point a' on XY and point a 12 mm below XY.
2. Draw a 90 mm long line $a'b_1'$, inclined at 45º with XY.
3. Project point b_1' to meet horizontal line through point a at point b_1. Draw an arc with point a as the centre and radius ab_1.
4. Draw a line from point a inclined at 60º with XY to intersecting the arc from ab_1 at point b. Join AB to represent the top view.
5. Draw a horizontal line from point b as the locus of b_2. Cut an arc with a as the centre and 90 mm radius to meet locus of b_2 at point b_2. Join ab_2. Measure the inclination of line ab_2 with XY. This is the inclination ϕ of line with V.P. Here ϕ = 38º.
6. Project point b_2 to meet the horizontal line through a' at point b_2'. Draw an arc with a' as the centre and radius $a'b_2'$ to meet the horizontal line through point b_1' at point b'. Join $a'b'$ to represent the front view.
7. Join $b'b$ and ensure that it is perpendicular to XY.
8. Line $a'b'$ meet XY at point h'. Project point h' to meet line ab at point h. Point h represents the H.T. Here, point h coincides with point a.
9. Produce line ab to meet XY at point v. Project point v to meet line $a'b'$ produced at point v'. Point v' represents the V.T.

Example 7.33 (Fig. 7.33)

A line *AB* inclined at 40º to H.P. has its front view 60 mm long and inclined at 60º to the reference line. One end is 20 mm away from both the reference planes in first angle. Locate the position of end *B*. Find true length and true inclination of the line with V.P. Also, show its traces.

[RGPV Dec. 2008]

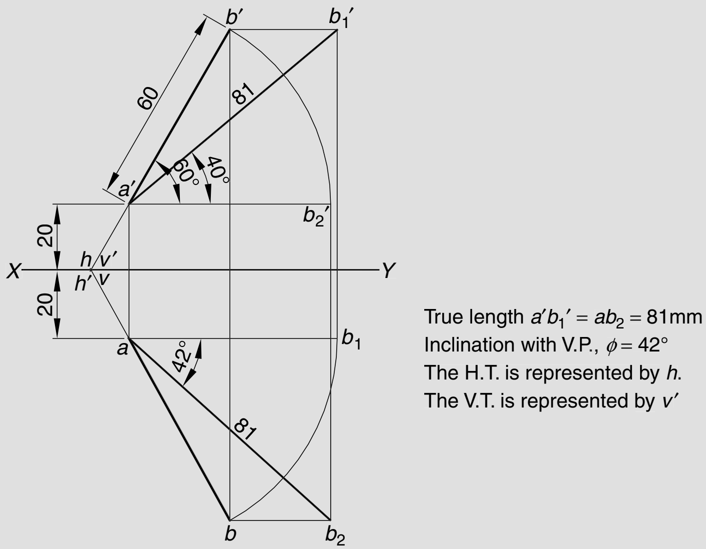


Fig. 7.33

Given data	***Interpretation***
Line is inclined at 40º to H.P. ($\theta = 40º$)	$a'b_1'$ is inclined at 40º to *XY*
Front view is 60 mm long	$a'b' = a'b_2' = 60$ mm
Front view is inclined at 60º with V.P. ($\alpha = 60º$)	$a'b'$ is inclined at 60º to *XY*
End *A* is 20 mm from H.P.	a' is 20 mm above *XY*
End *A* is 20 mm from V.P.	*a* is 20 mm below *XY*

Construction: Fig. 7.33

1. Draw a reference line *XY*. Mark point a' 20 mm above *XY* and point *a* 20 mm below *XY*.
2. Draw a 60 mm long line $a'b'$, inclined at 60º to *XY* to represent the front view.
3. From point a', draw a line inclined at 40º to *XY* to meet the horizontal line through point b' at point b_1'. Measure $a'b_1'$ as true length of the line. Here T.L. = 81 mm.
4. Draw an arc with a' as the centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Draw another arc with *a* as the centre of 81 mm radius to meet the vertical projector from point b_2' at point b_2. Join ab_2. Measure its inclination with *XY*. This is the inclination ϕ of line with V.P. Here, $\phi = 42º$.
5. Project point b_1' to meet the horizontal line through *a* at point b_1. Draw an arc with *a* as the centre and radius ab_1 to meet the horizontal line through point b_2 at point *b*. Join *ab* to represent the top view.

6. Join $b'b$ and ensure that it perpendicular to XY.
7. Produce line $a'b'$ to meet XY at point h'. Project point h' to meet line ab produced at point h. Point h represents the H.T.
8. Produce line ab to meet XY at point v. Project point v to meet line $a'b'$ produced at point v'. Point v' represents the V.T.

Example 7.34 (Fig. 7.34)

A 80 mm long line *PQ* has its end *P* 10 mm above the H.P. and 25 mm in front of the V.P. The line is inclined at 30º to the H.P. and 60º to the V.P. Draw its projections.

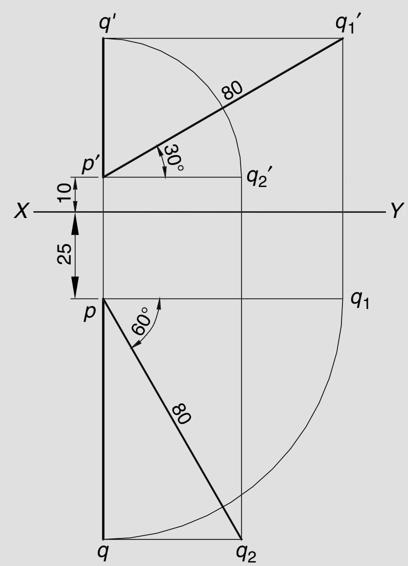


Fig. 7.34

Given data	*Interpretation*
$PQ = 80$ mm	$p'q_1' = pq_2 = 80$ mm,
P is 10 mm above the H.P.	p' is 10 above XY
P is 25 mm in front of the V.P.	p is 25 mm below XY
Line is inclined at 30º to H.P. ($\theta = 30º$)	$p'q_1'$ is inclined at 30º to XY
Line is inclined at 60º to V.P. ($\phi = 60º$)	pq_2 is inclined at 60º to XY

Construction: Fig. 7.34

1. Draw a reference line XY. Mark point p' on XY and point p 25 mm below XY.
2. Draw 80 mm long line $p'q_1'$ inclined at 30º to XY.
3. Draw another 80 mm long line pq_2 inclined at 60º to XY.

4. Project q_1' to meet horizontal line through p at point q_1. Draw an arc with p as the centre and radius pq_1 to meet the horizontal line through q_2 at point q. Join pq to represent the top view.
5. Project q_2 to meet horizontal line through p' at point q_2'. Draw an arc with p' as the centre and $p'q_2'$ as the radius to meet the horizontal line through q_1' at point q'. Join $p'q'$ to represent the front view.
6. Join $q'q$ and ensure that it is perpendicular to XY. Here, it may be noted that here $\theta + \phi = 90°$, hence both the front and the top views are perpendicular to XY.

Example 7.35 (Fig. 7.35)

A 75 mm long line *PQ* has its end *P* 15 mm above the H.P. and 20 mm in front of the V.P. The front and top views are 45 mm and 60 mm long, respectively. Draw its projections and determine the true inclinations with the reference planes.

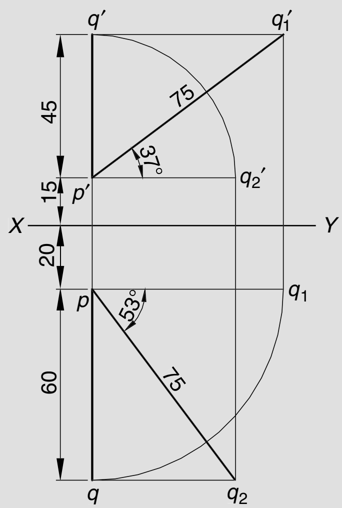


Fig. 7.35

Given data	*Interpretation*
$PQ = 75$ mm	$p'q_1' = pq_2 = 75$ mm,
P is 15 mm above the H.P.	Point p' is 15 above XY
P is 20 mm in front of the V.P.	Point p is 20 mm below XY
Front view is 45 mm long	$p'q' = p'q_2' = 45$ mm
Top view is 60 mm long	$pq = pq_1 = 60$ mm

Construction: Fig. 7.35

1. Draw a reference line XY. Mark point p' 15 mm above XY and point p 20 mm below XY.
2. Draw a 45 mm long line $p'q_2'$ and 60 mm long line pq_1, parallel to XY.

3. Draw an arc with p' as the centre and 75 mm radius to meet the vertical projector from point q_1 at point q_1'. Join $p'q_1'$. Measure the inclination of line $p'q_1'$ with XY as true inclination of line with H.P. Here $\theta = 37^\circ$.
4. Draw another arc with p as the centre and 75 mm radius to meet the vertical projector from point q_2' at point q_2. Join pq_2. Measure the inclination of line pq_2 with XY as true inclination of line with V.P. Here, $\phi = 53^\circ$.
5. Draw an arc with p' as the centre and radius $p'q_2'$ to meet the horizontal line through q_1' at point q'. Join $p'q'$ to represent the front view.
6. Draw an arc with p as the centre and radius pq_1 to meet the horizontal line through q_2 at point q. Join PQ to represent the top view.
7. Join $q'q$ and ensure that it is perpendicular to XY. Here, it may be noted that $45^2+60^2 = 75^2$, hence both the front and the top views are perpendicular to XY.

Example 7.36 (Fig. 7.36)

A 80 mm long line *PQ* has its end *P* 15 mm above the H.P. and 50 mm in front of the V.P. while the end *Q* in the V.P. Draw the projections of the line when the sum of its inclination with the H.P. and V.P. is 90º. Determine the true inclination with the reference planes and its traces.

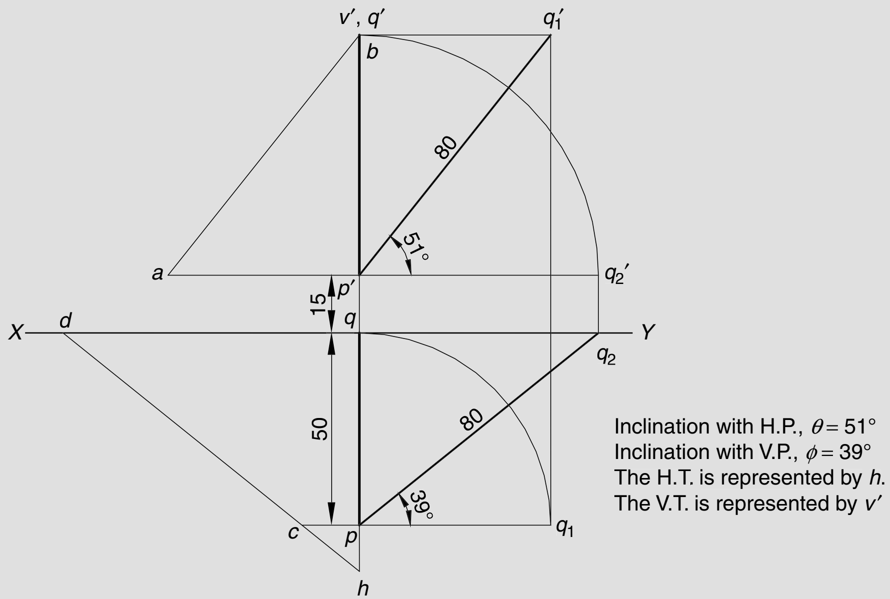


Fig. 7.36

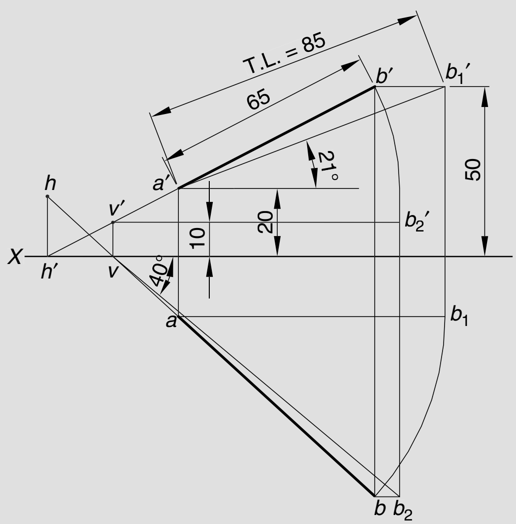
Given data	*Interpretation*
$PQ = 80$ mm	$p'q_1' = pq_2 = 80$ mm,
End p is 15 mm above the H.P.	p' is 15 above XY
End p is 50 mm in front of the V.P.	p is 50 mm below XY
End q is in the V.P.	q is on XY
$\theta + \phi = 90^\circ$	$\alpha = \beta = 90^\circ$, $p'q'$ and PQ are perpendicular to XY

Construction: Fig. 7.36

1. Draw a reference line *XY*. On a vertical projector, mark point *p′* 15 mm above *XY*, point *p* 50 mm below *XY* and point *q* on *XY*. Join *pq* to represent the top view.
2. Draw an arc with *p* as the centre and 80 mm radius to meet the horizontal line through *q* at point q_2. Join pq_2. Measure the inclination of line pq_2 with *XY* as true inclination with V.P. Here, $\phi = 39°$.
3. Draw an arc with *p* as the centre and radius *pq* to meet the horizontal line through *p* at point q_1. Draw another arc with *p′* as the centre and 80 mm radius to meet the vertical projector from point q_1 at point q_1'. Join $p'q_1'$. Measure the inclination of line $p'q_1'$ with *XY* as the true inclination with H.P. Here, $\theta = 51°$.
4. Draw a vertical line from q_2 to meet the horizontal line through *p′* at point q_2'. Draw an arc with *p′* as the centre and radius $p'q_2'$ to meet the horizontal line through q_1' at point *q*. Join *p′q′* to represent front view.
5. Draw lines *p′a* and *q′b* perpendicular to *p′q′* on the same side of *p′q′* such that $p'a = op$, $q'b = oq$. Join *ab* and produce it to meet line *p′q′* produced at point *v′*. Point *v′* represents the V.T.
6. Draw lines *pc* and *qd* perpendicular to *pq* on the same side of *pq* such that $pc = op'$, $qd = oq'$. Join *cd* and produce it to meet line *pq* produced at point *h*. Point *h* represents the H.T.

Example 7.37 (Fig. 7.37)

A line *AB*, inclined at 40º to the V.P. has its ends 50 mm and 20 mm above the H.P. The length of its front view is 65 mm and its V.T. is 10 mm above the H.P. Determine the true length of *AB*, its inclination with the H.P. and its H.T. [RGPV Feb. 2006]



True length $a'b_1' = 85$ mm
Inclination with H.P., $\theta = 21°$
The H.T. is represented by *h*.

Fig. 7.37

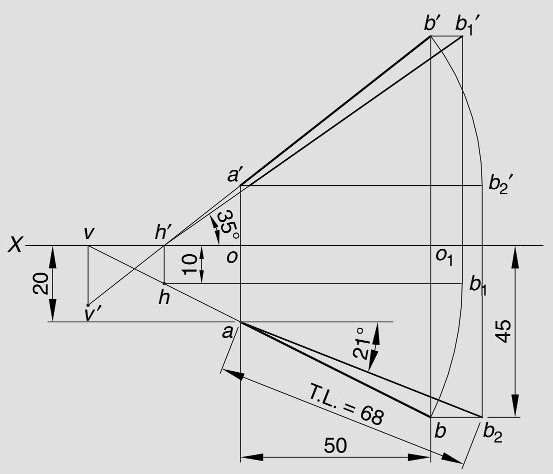
Given data	*Interpretation*
AB inclined at 40º to the V.P. ($\phi = 40°$)	$a'b_1'$ is inclined at 40º to *XY*
B is 50 mm above the H.P.	*b′* is 50 mm above *XY*
A is 20 mm above the H.P.	*a′* is 20 mm above *XY*
Length of front view is 65 mm	$a'b' = p'q_2' = 65$ mm
V.T. is 10 mm above the H.P.	*v′* is 10 mm above *XY*, $vv' = 10$ mm

Construction: Fig. 7.37

1. Draw a reference line *XY*. Mark point a' 20 mm above *XY*.
2. Draw a horizontal line 50 mm above *XY* as the loci of b' and b_1'. Draw an arc with a' as the centre and 65 mm radius long to intersect the locus of b' at point b'. Join $a'b'$ to represent the front view.
3. Draw a line 10 mm above *XY* as the locus of v'. Produce $a'b'$ to meet the locus of v' at point v'. Point v' represents the V.T. Project point v' to meet *XY* at point v.
4. Draw an arc with v' as the centre and radius $v'b'$ to meet the horizontal line through v' at point b_2'. Draw a line from point v inclined at 40º to *XY* to meet the vertical projector from b_2' at point b_2. Join vb_2.
5. Project point b' to meet the horizontal line through b_2 at point b. Join vb. Project point a' to meet line vb at point a. Join ab to represent the top view.
6. Draw an arc with a as the centre and radius ab to meet the horizontal line through a at point b_1. Project point b_1 to meet the horizontal line through point b_1' at point b_1'. Join $a'b_1'$. Measure $a'b_1'$ as true length. Here T.L. = 85 mm.
7. Produce $a'b'$ to meet *XY* at point h'. Project h' to meet ab produced at point h. Point h represents the H.T.

Example 7.38 (Fig. 7.38)

A line *AB* has its ends *A* and *B* 20 mm and 45 mm in front of V.P. respectively. The end projectors of the line are 50 mm apart. The H.T. of the line is 10 mm in front of the V.P. The line is inclined at 35º to H.P. Draw the projections of the line and determine the true length of line and locate its V.T. of the line from H.P. and inclination of the line with V.P. [RGPV June 2008]



True length $ab_2 = 68$ mm
Inclination with V.P., $\phi = 21°$
The V.T. is represented by v'.

Fig. 7.38

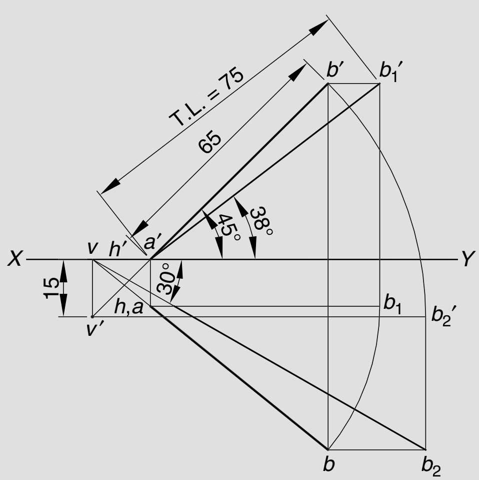
Given data	*Interpretation*
A is 20 mm in front of V.P.	a is 20 mm below *XY*
B is 45 mm in front of V.P.	b is 45 mm below *XY*
End projectors are 50 mm apart	oo_1 = 50 mm
H.T. is 10 mm in front of V.P.	h is 10 mm below *XY*, hh' = 10 mm
Line is inclined at 35º to H.P. (θ= 35º)	$a'b_1'$ is inclined at 35º to *XY*

Construction: Fig. 7.38

1. Draw a reference line *XY*. Mark points *o* and o_1 on *XY* such that oo_1 = 50 mm.
2. On the vertical projector of point *o*, mark point *a* 20 mm below *XY*. Similarly, on the vertical projector of point o_1, mark point *b* 45 mm below *XY*. Join *ab* to represent the top view.
3. Draw a horizontal line 10 mm below *XY* as the locus of *h*. Produce *ab* to meet the locus of *h* at point *h*. Point *h* represents the H.T. Project point *h* to meet *XY* at point *h*′.
4. Draw an arc with *h* as the centre and radius *hb* to meet the horizontal line through *h* at point b_1. Draw a line from point *h*′ inclined at 35º to *XY* to meet the vertical projector from b_1 at point b_1'. Join $h'b_1'$.
5. Project point *b* to meet the horizontal line through b_1' at point *b*′. Join *h*′*b*′. Project point *a* to meet line *h*′*b*′ at point *a*′. Join *a*′*b*′ to represent the front view.
6. Draw an arc with *a*′ as the centre and radius *a*′*b*′ to meet the horizontal line through *a*′ at point b_2'. Project point b_2' to meet the horizontal line through *b* at point b_2. Join ab_2. Measure ab_2 as true length and its inclination with *XY* as true inclination with V.P. Here, T.L. = 68 mm and $\phi = 21º$.
7. Produce *ab* to meet *XY* at point *v*. Project *v* to meet *a*′*b*′ produced at point *v*′. Point *v*′ represents the V.T.

Example 7.39 (Fig. 7.39)

The front view of a line *AB* measures 65 mm and makes an angle of 45º with *XY*. *A* is in the H.P. and the V.T. of the line is 15 mm below the H.P. The line is inclined at 30º to the V.P. Draw the projections of *AB* and find its true length and inclination with the H.P. Also, locate its H.T. [RGPV Feb. 2006]



True length, $a'b_1'$ = 75 mm
Inclination with H.P., θ = 38°
The H.T. is represented by *h*.

Fig. 7.39

Given data	*Interpretation*
Front view measures 65 mm	*a*′*b*′ = 65 mm
Front view makes 45º with *XY* ($\alpha = 45º$)	*a*′*b*′ is inclined at 45º to XY
A is in the H.P.	*a*′ is on *XY*
V.T. is 15 mm below the H.P.	*v*′ is 15 mm below *XY*, *vv*′ = 15 mm
Line is inclined at 30º to the V.P. ($\phi = 30º$)	ab_2 is inclined at 30º to *XY*

Construction: Fig. 7.39

1. Draw a reference line *XY*. Mark point *a′* on H.P.
2. Draw a 65 mm long line *a′b′* inclined at 45º to *XY*. This represents the front view.
3. Produce *a′b′* to meet at a point *v′*, 15 mm below *XY*. Point *v′* denotes V.T. Project *v′* to meet *XY* at point *v*.
4. Draw an arc with *v′* as the centre and radius *v′b′* to meet the horizontal line through *v′* at point b_2'. From point *v*, draw a line inclined at 30º to *XY* to meet the vertical projector from point b_2' at point b_2. Join vb_2.
5. Project point *b′* to meet the horizontal line from point b_2 at point *b*. Join *vb*. Project *a′* to meet *vb* at point *a*. Join *ab* to represent the top view.
6. Draw an arc with *a* as the centre and radius *ab* to meet the horizontal line through point *a* at point b_1. Project point b_1 to meet the horizontal line from point *b′* at point b_1'. Join $a'b_1'$. Measure $a'b_1'$ as true length and its inclination with *XY* as true inclination with H.P. Here, T.L. = 75 mm and θ= 38º.
7. Front view *a′b′* meets *XY* at *h′* which coincides with *a′*. Project *h′* to meet *ab* at point *h* which also coincides with *a*. Point *h* denotes the H.T.

Example 7.40 (Fig. 7.40)

A line *AB* measures 100 mm. The projectors through its V.T. and the end *A* are 40 mm apart. The end *A* is 30 mm above H.P. and 20 mm in front of V.P. The V.T. is 10 mm below the H.P. Draw the projections of the line and determine H.T. and inclination of line with H.P. and V.P.

[RGPV June 2004]

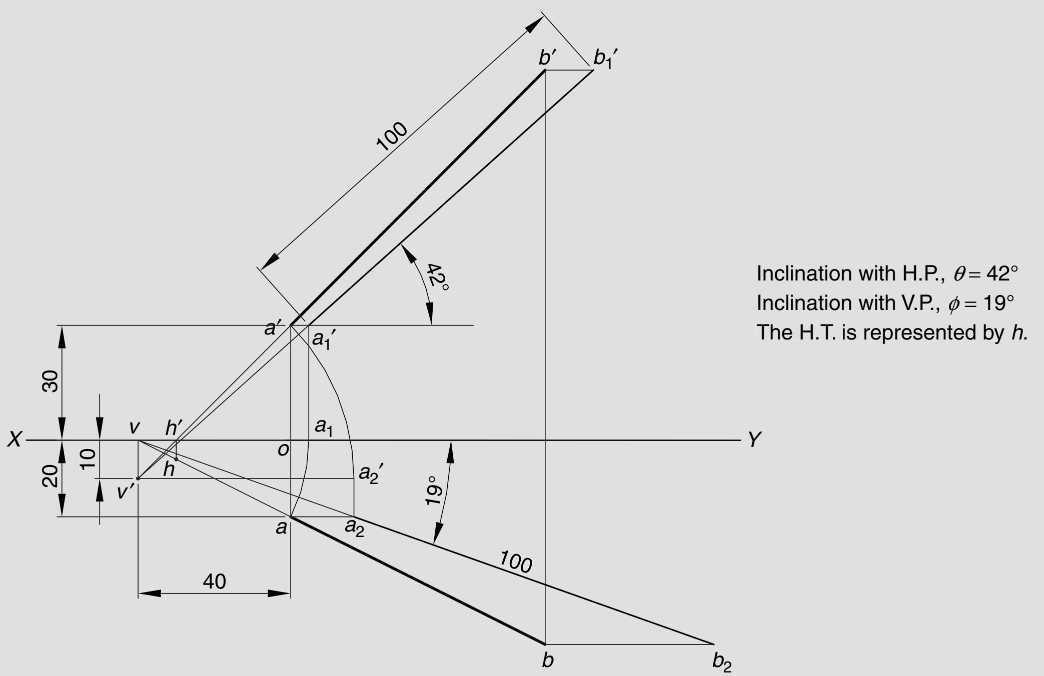


Fig. 7.40

Given data	***Interpretation***
AB measures 100 mm	$a'b_1' = a_2b_2 = 100$ mm
Projector through V.T. and end *A* are 40 mm apart	*ov* = 40 mm
A is 30 mm above the H.P.	*a*′ is 30 mm above *XY*
A is 20 mm in front of the V.P.	*a* is 20 mm below *XY*
V.T. is 10 mm below the H.P.	*v*′ is 10 mm below *XY*

Construction: Fig. 7.40

1. Draw a reference line *XY*. Mark points *o* and *v* on *XY* such that *ov* = 40 mm.
2. On the projector through *o* mark point *a*′ 30 mm above *XY* and *a* 20 mm below *XY*. On the projector through *v* mark point *v*′ 10 mm below *XY*. Join *v*′*a*′ and *va*.
3. Consider lines *v*′*a*′ and *va* as the front and top views of line *VA* and determine its true length.
4. Draw an arc with *v* as the centre and radius *va* to meet the horizontal line through *v* at point a_1. Draw a vertical projector from a_1 to meet the horizontal line from *a*′ at point a_1'. Join $v'a_1'$ and produce it to b_1' such that $a_1'b_1' = 100$ mm. Measure the inclination of the line $a_1'b_1'$ with *XY*. This is true inclination θ of *AB* with H.P. Here $\theta = 42°$.
5. Draw an arc with *v*′ as the centre and radius *v*′*a*′ to meet the horizontal line through *v*′ at point a_2'. Project a_2' to meet the horizontal line from *a* at point a_2. Join va_2 and produce it to b_2 such that $a_2b_2 = 100$ mm. Measure the inclination of the line a_2b_2 with *XY*. This is true inclination ϕ of *AB* with V.P. Here $\phi = 19°$.
6. Draw a horizontal line from b_1' to meet *v*′*a*′ produced at point *b*′. Line *a*′*b*′ represents the front view.
7. Draw another horizontal line from b_2 to meet *va* produced at point *b*. Line *ab* represents the top view.
8. Join *b*′*b* to ensure that it is perpendicular to *XY*.
9. Produce *a*′*b*′ to meet *XY* at point *h*′. Project *h*′ to meet *ab* produced at point *h*. Point *h* represents the H.T.

Example 7.41 (Fig. 7.41)

The front view of a line *AB* makes an angle of 30º with *XY* line. The H.T. of the line is 45 mm in front of V.P., while its V.T. is 30 mm below the H.P. End *A* is 12 mm above H.P. and end *B* is 105 mm in front of V.P. Draw the projections of the line and find its true length, inclinations with H.P. and V.P.

[RGPV Dec. 2003]

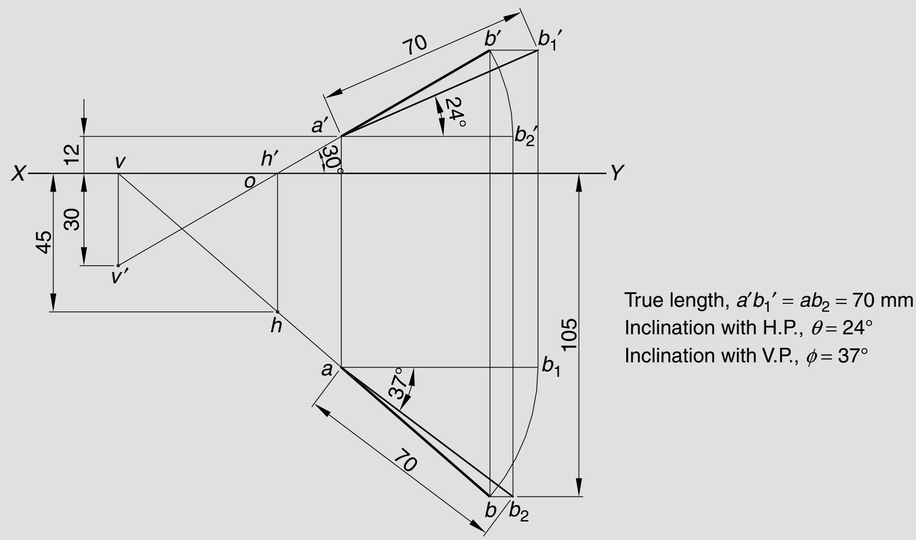


Fig. 7.41

Given data	*Interpretation*
Front view makes 30º with *XY* (α = 30º)	$a'b'$ is inclined at 30º to *XY*
H.T. is 45 mm in front of V.P.	*h* is 45 mm below *XY*, $h'h$ = 45 mm
V.T. is 30 mm below the H.P.	v' is 30 mm below *XY*, vv' = 30 mm
A is 12 mm above H.P.	a' is 12 mm above *XY*
B is 105 mm in front of V.P.	*b* is 105 mm below *XY*

Construction: Fig. 7.41

1. Draw a reference line *XY*.
2. Draw a line inclined at 30º to *XY* and mark on it; point a' 12 mm above *XY*, point h' on *XY* and point v' 30 mm below *XY*.
3. Project v' to meet *XY* at point *v*.
4. Project h' to meet point *h* 45 mm below *XY*.
5. Join *vh* and produce it to meet a point *b*, 105 mm below *XY*.
6. Project point a' to meet line *vhb* at point *a*. Line *ab* represents the top view.
7. Project point *b* to meet line $v'h'a'$ produced at point b'. Line $a'b'$ represents the front view.
8. Draw an arc with a' as the centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Project point b_2' to meet the horizontal lines through point *b* at point b_2. Join ab_2. Measure the inclination of line ab_2 with *XY* which is the true inclination ϕ with V.P. Here $\phi = 37º$.
9. Draw an arc with *a* as the centre and radius *ab* to meet the horizontal line through point *a* at point b_1. Project b_1 to meet the horizontal lines through point b' at point b_1'. Join $a'b_1'$. Measure the inclination θ of line $a'b_1'$ with *XY*. Here $\theta = 24º$.
10. Measure $a'b_1' = ab_2 = 70$ mm as true lengths.

Example 7.42 (Fig. 7.42)

The end projectors of a line *AB* are 50 mm apart, while those drawn for its H.T. and V.T. are 90 mm apart. The H.T. is 40 mm in front of the V.P. and the V.T. is 80 mm above the H.P. Draw the projections of *AB*, if its end *A* is 10 mm above the H.P. Also, determine its true length and inclinations with the reference planes.

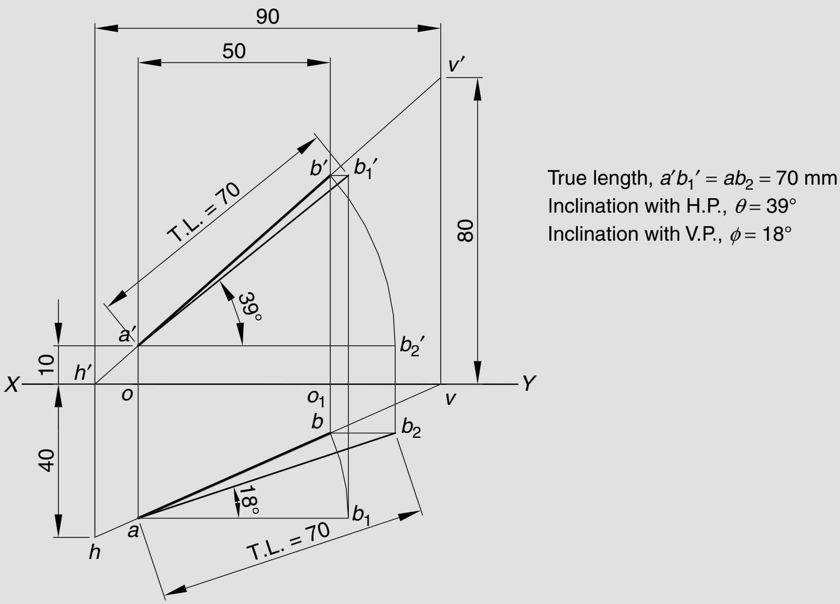


Fig. 7.42

Given data	*Interpretation*
End projectors are 50 mm apart	oo_1 = 50 mm
Projectors of H.T. and V.T. are 90 mm apart	$h'v$ = 90 mm
H.T. is 40 mm in front of the V.P.	h is 40 mm below XY, $h'h$ = 40 mm
V.T. is 80 mm above the H.P.	v' is 80 mm above XY, vv' = 80 mm
End A is 10 mm above the H.P.	a' is 10 mm above XY

Construction: Fig. 7.42

1. Draw a reference line XY. Mark points h' and v on XY such that $h'v$ = 90 mm.
2. On the vertical projector through h', mark point h 40 mm below XY. Similarly, on the the vertical projector through v, mark point v' 80 mm above XY.
3. Join $h'v'$ and hv, to show inclinations of front and top views, respectively.
4. On line $h'v'$ mark point a′ such that it is 10 mm above XY. Draw a vertical projector through a' to meet XY at point o and line hv at point a.
5. Mark point o_1 on XY such that oo_1 = 50 mm. Draw a vertical projector through o_1 to meet line $h'v'$ at point b' and line hv at point b. Lines $a'b'$ and ab represent the front and the top views, respectively.

6. Draw an arc with a as centre and radius ab to meet the horizontal line through point a at point b_1. Project point b_1 to meet the horizontal lines through point b' at point b_1'. Join $a'b_1'$. Measure the inclination of line $a'b_1'$ with XY as true inclination with H.P. Here, $\theta = 39°$.
7. Draw an arc with a' as centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Project point b_2' to meet the horizontal lines through point b at point b_2. Join ab_2. Measure the inclination of line ab_2 with XY true inclination with V.P. Here $\phi = 18°$.
8. Measure lengths $a'b_1' = ab_2 = 70$ mm as true lengths.

Example 7.43 (Fig. 7.43)

A 100 mm long line *PQ*, has its end *P* at 50 mm in front of the V.P. The H.T. is 60 mm in front of the V.P. and V.T. is 80 mm above the H.P. The distance between the H.T. and V.T. is 130 mm. Draw the projections of the line *PQ*, and determine its inclinations with the H.P. and the V.P.

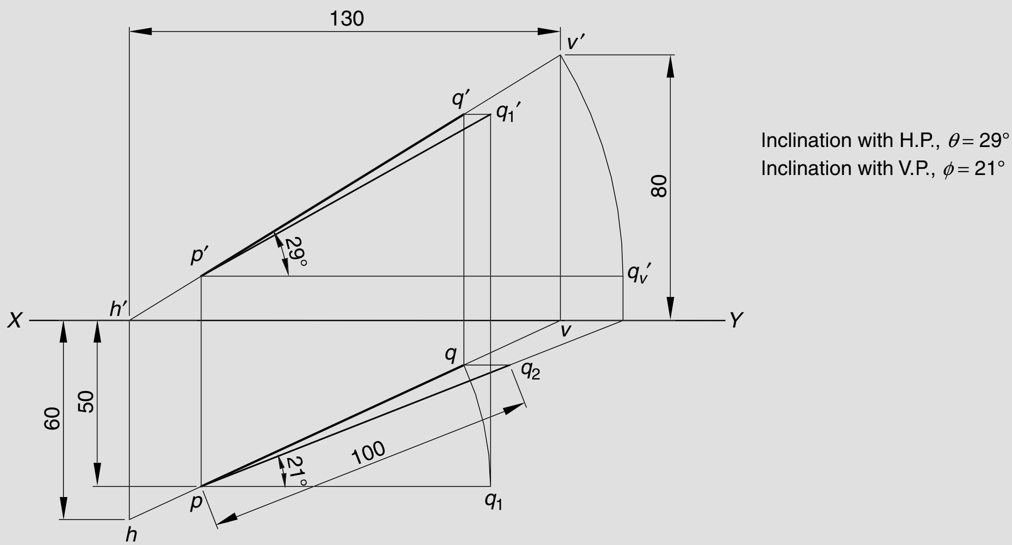


Fig. 7.43

Given data	*Interpretation*
$PQ = 100$ mm	$p'q_1' = pq_2 = 100$ mm
P is 50 mm in front of the V.P.	p' is 50 mm below XY
H.T. is 60 mm in front of the V.P.	h is 60 mm below XY, $h'h = 60$ mm
V.T. is 80 mm above the H.P.	v' is 80 mm above XY, $vv' = 80$ mm
Distance between the H.T. and V.T. is 130 mm	$h'v = 130$ mm

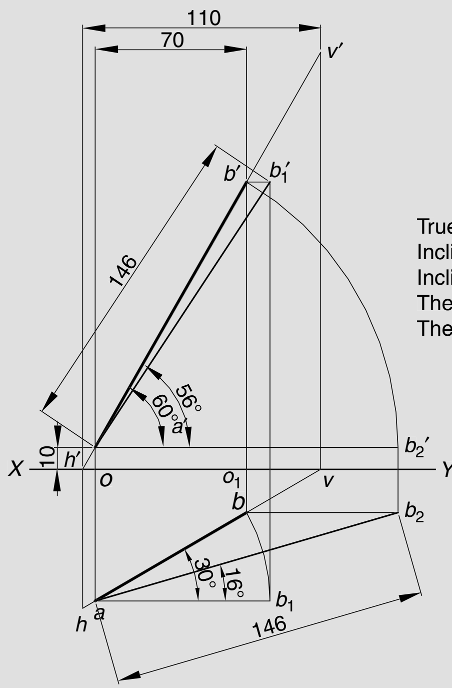
Construction: Fig. 7.43

1. Draw a reference line XY. Mark points h' and v on XY such that $h'v = 130$ mm.

2. On the vertical projector through h', mark point h at 60 mm below XY. Similarly, on the vertical projector through v, mark point v' at 80 mm above XY.
3. Join $h'v'$ and hv to show inclinations of front and top views, respectively.
4. On line hv, mark point p such that it is 50 mm below XY. Draw a vertical projector through point p to meet line $h'v'$ at point p'.
5. Draw an arc with p' as centre and radius $p'v'$ to meet the horizontal line through p' at a point q_v'. Project q_v' to meet the horizontal line from v at point q_v. Join pq_v. Measure the inclination of pq_v with XY as true inclination with V.P. Here, $\phi = 21°$.
6. Mark point q_2 on line pq_v such that $pq_2 = 100$ mm. Draw a horizontal line from q_2 to meet line pv at point q. Join pq to represent the top view.
7. Project point q to meet line $p'v'$ at point q'. Join $p'q'$ to represent the front view.
8. Draw an arc with p as the centre and radius pq to meet the horizontal line through point p at point q_1. Project q_1 to meet the horizontal line from point q' at a point q_1'. Join $p'q_1'$. Measure the inclination of line $p'q_1'$ with XY as true inclination with H.P. Here $\theta = 29°$. Also, ensure $p'q_1'$ is 100 mm long.

Example 7.44 (Fig. 7.44)

The distance between the end projectors of a line *AB* is 70 mm and the projectors through the traces are 110 mm apart. The end *A* of the line is 10 mm above H.P. If the top and the front views of the line make 30º and 60º with *XY* line respectively, draw the projections of the line and determine (i) the traces, (ii) the angles with the H.P. and the V.P., and (iii) the true length of the line. [RGPV Dec. 2002, June 2004]



True length, $a'b_1' = ab_2 = 146$ mm
Inclination with H.P., $\theta = 56°$
Inclination with V.P., $\phi = 16°$
The H.T. is represented by h.
The V.T. is represented by v'.

Fig. 7.44

Given data	*Interpretation*
Distance between End projectors is 70 mm	oo_1 = 70 mm
Projectors through traces are 110 mm apart	$h'v$ = 110 mm
A is 10 mm above the H.P.	a' is 10 mm above *XY*
Top view makes 30º with *XY* (β= 30º)	*ab* is inclined at 30º to *XY*
Front view makes 60º with *XY* (α= 60º)	$a'b'$ is inclined at 60º to *XY*

Construction: Fig. 7.44

1. Draw a reference line *XY*. Mark points *o* and o_1 on *XY* such that oo_1 = 70 mm.
2. On the vertical projector through point *o*, mark point a' at 10 mm above *XY*.
3. Through point a', draw a line inclined at 60º to *XY*. Let it meets *XY* at point h' and vertical projector from point o_1 at point b'.
4. Mark point *v* on *XY* such that $h'v$ = 110 mm. Project point *v* to meet the line $h'a'b'$ produced at point v'.
5. Through point *v*, draw a line inclined at 30º to *XY*. Let it meets vertical projectors through points h', a' and b′ at points *h*, *a* and *b* respectively. Lines $a'b'$ and *ab* represent the front and the top views.
6. Draw an arc with *a* as centre and radius *ab* to meet the horizontal line through point *a* at point b_1. Project point b_1 to meet the horizontal lines through point b' at point b_1'. Join $a'b_1'$. Measure the inclination of line $a'b_1'$ with *XY*. This is true inclination θ with H.P. Here, θ= 56º.
7. Draw an arc with a' as centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Project point b_2' to meet the horizontal lines through point *b* at point b_2. Join ab_2. Measure the inclination of line ab_2 with *XY*. This is true inclination ϕ with V.P. Here ϕ= 16º.
8. Measure lengths $a'b_1' = ab_2$ = 146 mm as true lengths.

Example 7.45 (Fig. 7.45)

Two straight lines *PQ* and *QR* make an angle of 120° between them in their front view and top view. *PQ* is 60 mm long and is parallel to and 15 mm from both H.P. and V.P. Determine the true angle between *PQ* and *QR*, if point *R* is 50 mm above H.P.

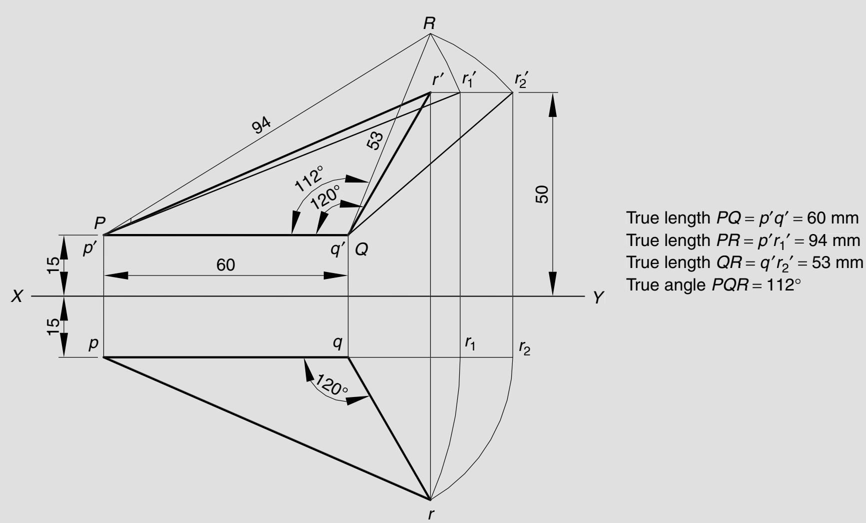


Fig. 7.45

Given data	*Interpretation*
Angle between *PQ* and *QR* in front and top views is 120º.	Angle $p'q'r'$ = angle pqr = 120º
PQ is 60 mm long and parallel to both H.P. and V.P.	$p'q' = PQ = 60$ mm
PQ is and 15 mm from both H.P. and V.P.	$p'q'$ and pq are 15 mm from *XY*
R is 50 mm above H.P.	r' is 50 mm above *XY*

Construction: Fig. 7.45

1. Draw a reference line *XY*. Mark point p' at 15 mm above *XY* and point *p* at 15 mm below *XY*.
2. Draw 60 mm long lines $p'q'$ and pq, parallel to *XY*.
3. Through point q', draw a line inclined at 120º to *XY* such that it meets the horizontal line at 50 mm above *XY* at point r'. Join $q'r'$ and $p'r'$.
4. From point *q* draw a line inclined at 120º to *XY* such that it meets the projector from point r' at a point *r*. Join *qr* and *pr*.
5. As lines *pq* and $p'q'$ are parallel to *XY*, they represent the true length of side *PQ*. Here, *PQ* = 60 mm.
6. Draw an arc with *p* as the centre and radius *pr* to meet the horizontal line through point *p* at point r_1. Project point r_1 to meet horizontal lines from point r' at point r_1'. Join $p'r_1'$. This is true length of line *PR*. Here, $PR = p'r_1' = 94$ mm.
7. Draw an arc with *q* as the centre and radius *qr*, to meet the horizontal line through point *q* at point r_2. Project point r_2 to meet horizontal lines from point r' at point r_2'. Join $q'r_2'$. This is true length of line *QR*. Here, $QR = q'r_2' = 53$ mm.
8. Draw actual triangle *PQR* taking true lengths i.e. 60 mm, 94 mm and 53 mm. Measure the included angle *PQR* as the actual angle between sides *PQ* and *QR*. Here, it is 112º.

Example 7.46 (Fig. 7.46)

A room is 4.8 m × 4.2 m × 3.6 m high. Determine graphically the distance between a top corner and the bottom corner diagonally opposite to it. [RGPV Feb. 2006]

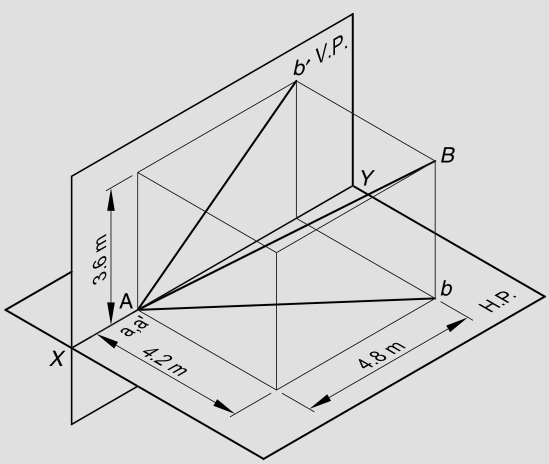
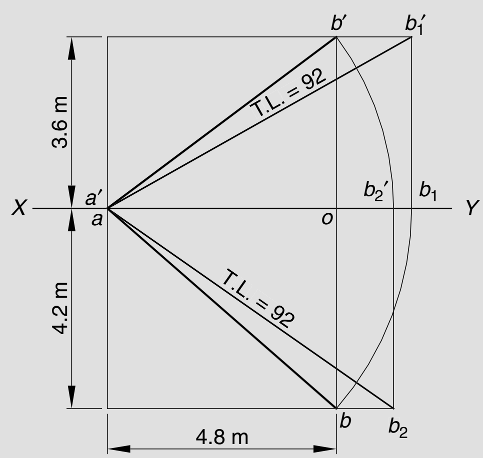


Fig. 7.46(a)



Scale 1:50
Distance between the diagonally opposite corners = $AB = a'b_1' = ab_2 = 92$ mm

Fig. 7.46(b)

Visualization: Fig. 7.46(a) shows the 3-dimensional view of the given room. We need to determine true length of body diagonal *AB*.

Construction: Fig. 7.46(b)

1. Take scale 1:50. Draw a reference line *XY*. Mark points *a′* and *a* coinciding on *XY*.
2. Also mark point *o* on *XY* at a distance 4.8 m from point *a′*.
3. Project point *o* and mark on it point *b′* at 3.6 m above *XY* and point *b* 4.2 m below *XY*. Join *a′b′* and *ab* to represent the front and the top views respectively.
4. Draw an arc with *a* as the centre and radius *ab*, to meet the horizontal line through *a* at point b_1. Project point b_1 to meet horizontal lines from *b′* at b_1'. Join $a'b_1'$ to represent the true length. Here, T.L. = 92 mm.
5. Draw an arc with *a′* as the centre and radius *a′b′*, to meet the horizontal line through point *a′* at point b_2'. Project b_2' to meet horizontal lines from *b* at b_2. Join ab_2 and ensure that it is equal to $a'b_2'$ i.e. 92 mm.

Example 7.47 (Fig. 7.47)

A room is 6 m × 5 m × 3.5 m high. An electric bulb *B* is above the centre of the longer wall and 1 m below the ceiling. The bulb *B* is 3.5 m away from the longer wall. The switch *S* for the light is 1.25 m above the floor on the centre of the adjacent wall. Determine graphically, the shortest distance between the bulb *B* and the switch *S*. [RGPV Dec. 2003]

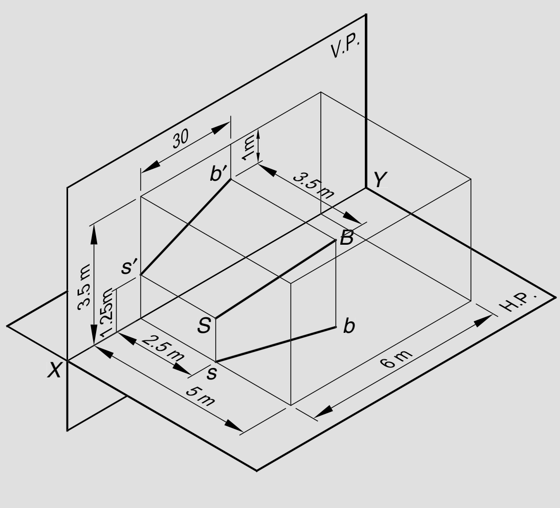
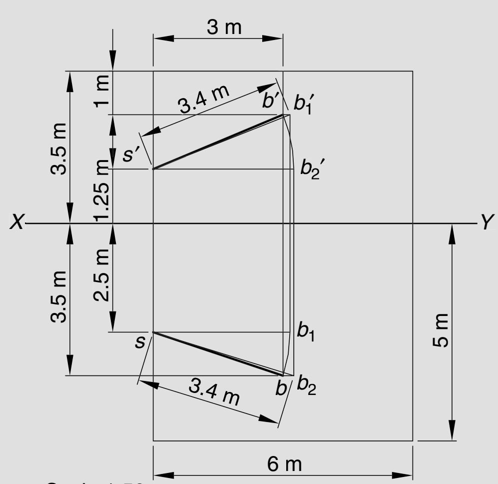


Fig. 7.47(a)



Scale 1:50

Distance between the bulb and the switch is 3.4 m

Fig. 7.47(b)

Visualization: Consider a room of the given measurement in Fig. 7.47(a). Points *S* and *B* are switch and bulb respectively.

Construction: Fig. 7.47(b)

1. Take scale 1:50. Draw a reference line *XY*.
2. Mark point *s′* 1.25 m above *XY* and point *s* 2.5 m below *XY*.
3. Draw a vertical projector 3 m away from *s′s*. Mark on it, point *b′* 2.5 m above *XY* and point *b* 3.5 m below *XY*. Join *s′b′* and *sb* to represent the front and the top views of the line joining the switch and the bulb.
4. Draw an arc with *s* as the centre and radius *sb*, to meet the horizontal line through *s* at point b_1. Project b_1 to meet horizontal lines from *b′* at point b_1'. Join $s'b_1'$ to represent the true length. Here, T.L. = 3.4 m.
5. Draw an arc with *s′* as the centre and radius *s′b′*, to meet the horizontal line through *s′* at point b_2'. Project b_2' to meet horizontal lines from *b* at point b_2. Join sb_2 and ensure it is equal to length $a'b_2'$, i.e. 3.4 m.

Example 7.48 (Fig. 7.48)

A wireless aerial tower 20 m high is tied at top by two guy ropes having angles of depression of 30º and 45º. The other end of guy ropes are connected to two poles at heights of 5 m are 2.5 m respectively. The two poles are 12 m apart. Draw projections of the arrangement and determine lengths of guy ropes. [RGPV June 2003]

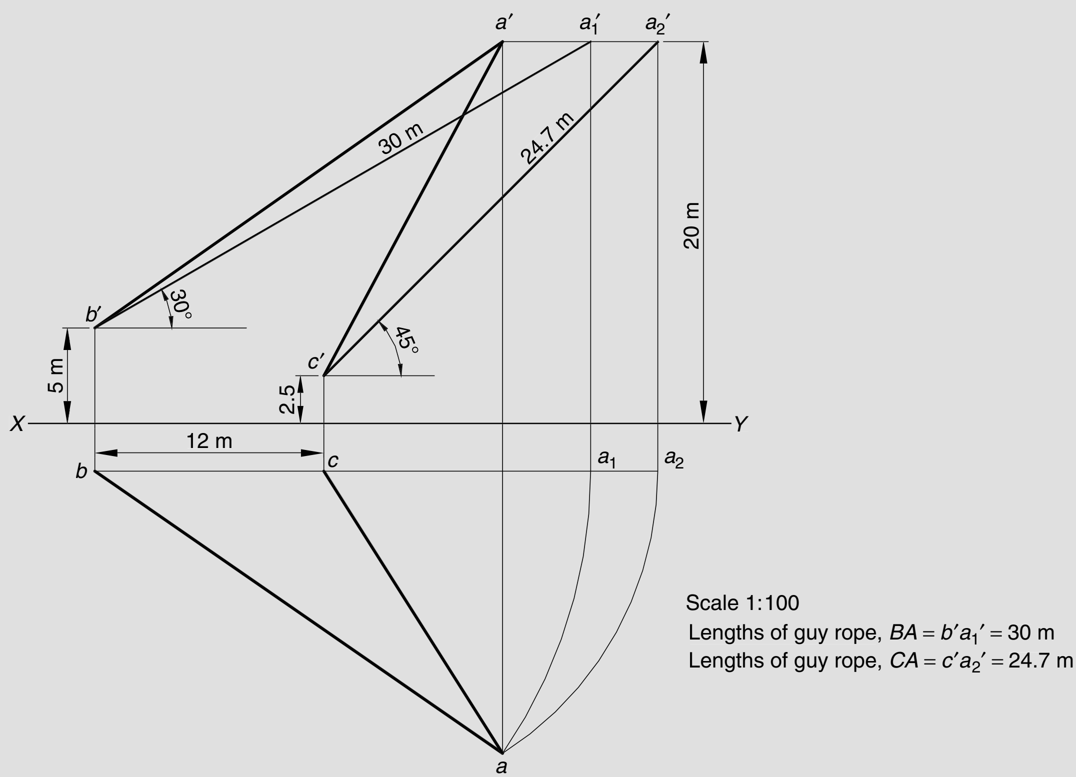


Fig. 7.48

Given data	*Interpretation*
Tower is 20 m high.	a' is 20 mm above *XY*.
Angles of depression are 30º and 45º (θ_1 = 30º, θ_2 = 45º).	$b'a_1'$ and $c'a_2'$ are at 30º and 45º with *XY*, respectively.
Height of pole are 5 m are 2.5 m.	b' and c' are 5 m are 2.5 m above *XY*.
Poles are 12 m apart.	*bc* = 12 m.

Construction: Fig. 7.48

1. Take scale 1:100. Draw a reference line *XY*.
2. Draw line *bc* = 12 m parallel to *XY* in the top view.
3. On the vertical projector through point *b*, mark point b' 5 m above *XY*. Similarly, on the vertical projector through point *c*, mark point c' 2.5 m above *XY*.
4. Draw a line from point b', inclined at 30º to *XY* such that it meets a point a_1' 20 mm above *XY*.
5. Draw another line from point c', inclined at 45º to *XY* such that it meets a point a_2' 20 mm above *XY*.
6. It may be noted that $b'a_1'$ and $c'a_2'$, represent true length of the ropes *BA* and *CA*, respectively. Here, $b'a_1'$ = 30 m and $c'a_2'$ = 24.7 m.
7. Project points a_1' and a_2' to meet the horizontal line through line *bc* at points a_1 and a_2 respectively.
8. Draw arc with *b* as the centre and radius ba_1' to meet another arc drawn with *c* as the centre and radius ca_2' at point a. Join *b*-*a* and *c*-*a* to represent the top views of the ropes.
9. Project point *a* to meet the horizontal line through points a_1' and a_2' at point a'. Join b'-a' and c'-a' to represent the front views of the ropes.

EXERCISE 7B

1. A line *AB*, 50 mm long has its end *A* in both H.P. and V.P. It is inclined at 30º to the H.P. and 45º to the V.P. Draw its projections. [RGPV June 2005]
2. A straight line *AB*, 55 mm long makes an angle of 30º with H.P. and 45º with the V.P. The end *A* is 12 mm in front of V.P. and 15 mm above H.P. Draw the projections. [RGPV June 2005]
3. End *A* of a line *AB* is 30 mm above H.P. and 5 mm in front of V.P. and end *B* is 10 mm above H.P. and 25 mm in front of V.P. The distance between the end projectors is 40 mm. Draw the projections of the line and locate its traces. [RGPV Dec. 2007]
4. The distance between the projectors of two ends of a straight line is 40 mm. One end is 15 mm above H.P. and 10 mm in front of the V.P. The other end is 40 mm above H.P. and 40 in front of the V.P. Find the true length and true inclination of the line.
   [Ans: 56 mm, 27º, 32º] [RGPV Feb. 2011]
5. The end *A* of a line *AB* is in the H.P. and 25 mm in front of V.P. The end *B* is in the V.P. and 50 mm above H.P. Distance between the end projectors is 75 mm. Draw the projections of the line *AB* and determine its true length, traces and inclinations with the planes.
   [Ans: 94 mm, 32º, 16º] [RGPV June 2007]

6. Draw projections and find out the true length of a line *AB* with end *B* on H.P. and 40 mm in front of V.P. *AB* is inclined at 45º to H.P. and 30º to V.P. and its plan measures 50 mm.
[Ans: 71 mm] [RGPV June 2008]
7. The top view of a 75 mm long line *AB* measures 65 mm, while the length of its front view is 50 mm. Its one end *A* is in the H.P. and 12 mm in front of the V.P. Draw the projections of the line *AB* and determine its inclination with H.P. and V.P.
[Ans: 30º, 48º] [RGPV Dec. 2006]
8. A line *AB*, 75 mm long has one of its ends 50 mm in front of V.P. and 15 mm above H.P. The top view of the line is 50 mm long. The other end is 15 mm in front of V.P and is above H.P. Draw the views and determine its true inclinations.
[Ans: 48º, 28º] [RGPV Dec. 2001]
9. A line *AB*, 65 mm long has its end *A* 20 mm above the H.P. and 25 mm in front of the V.P. The end *B* is 40 mm above H.P. and 65 mm in front of the V.P. Draw the projections of *AB*. Show its inclination to H.P. and V.P. Locate its traces.
[Ans: 18º, 38º] [RGPV June 2006]
10. The front view of a 75 mm long line *PQ* measures 50 mm, while its top view measures 60 mm. If end *P* of the line is 35 mm above the H.P. and 15 in front of the V.P., draw its projections and the traces. Determine the true inclinations of the line *PQ* with H.P. and V.P.
[Ans: 37º, 48º]
11. The top view of line *AB*, 70 mm long measures 55 mm and front view measures 45 mm. Its end *A* is 10 mm from H.P. and 15 mm from V.P. Draw the projections of the line and determine its inclination with the H.P. and V.P. Also draw its traces if the line is in first quadrant.
[RGPV Dec. 2010]
12. A 70 mm long line *PQ* is inclined at 30º to the H.P. Its end *P* is 15 mm in front of the V.P. and 25 mm above the H.P. The front view of the line measures 45 mm. Draw its projections and determine true angle of inclination with V.P. [Ans: 50º]
13. A line *AB* is inclined at 40º to H.P. Its one end *A* is 25 mm above H.P. and 30 mm in front of V.P. The top view of the line is 70 mm and is inclined at 30º to *XY*. Draw the projections of the line and determine its true length and inclination with V.P.
[Ans: 91 mm, 23º] [RGPV Feb. 2011]
14. The distance between the end projectors of a line *PQ* is 50 mm. The end *P* is 50 mm in front of the V.P. and 25 mm above the H.P. The end *Q* is 10 mm in front of the V.P. and above the H.P. The line is inclined at 30º to the V.P. Draw its projections. Determine its true length and true angle of inclination with the H.P.
[Ans: 80 mm, 37º]
15. A 90 mm long line *AB* has the end *A* at 15 mm above the H.P. and 25 mm in front of the V.P. The line is inclined at 60º to the H.P. and 30º to the V.P. Draw its projections.
16. A 75 mm long straight line *PQ* lying in the first angle has its end *P* in the H.P. and end *Q* in the V.P. The line is inclined at 45º to the H.P. and 30º to the V.P. Draw its projections.
17. A line *PQ* 75 mm long has its end *P* in the V.P. and the end *Q* in the H.P. The line is inclined at 30º to the H.P. and at 60º to the V.P. Draw its projections.
[RGPV June 2008]
18. A 100 mm long line *AB* has the end *A* at 15 mm above the H.P. and 20 mm in front of the V.P. The front and top views are 80 mm and 60 mm long, respectively. Draw its projections and determine the true inclinations with the reference planes. [Ans: 53º, 37º]

19. A 90 mm long line *AB* has its end *A* is in the H.P. and 70 mm in front of the V.P. while the end *B* is 10 mm in front of the V.P. Draw the projections of the line when the sum of its inclination with the H.P. and V.P. is 90º. Determine the true inclination with the reference planes and locate its traces. [Ans: 48º, 42º]
20. A line *PQ* is in first angle. Its ends *P* and *Q* are 15 mm and 45 mm in front of the V.P. respectively. The distance between the end projectors is 55 mm. The line is inclined at 30º to the H.P. and its H.T. is 8 mm above *XY* line. Draw the projections of the line *PQ* and find its true length and locate its V.T.
[Ans: 72 mm] [RGPV June 2007]
21. A line *PQ* inclined at 30º to the V.P. has its end *P* at 15 mm above the H.P. Its front view measures 70 mm and makes an angle of 45º with the reference line. The V.T. of the line is 25 mm below the H.P. Draw the projections of the line and determine its true length and the H.T.
[Ans: 80 mm]
22. A line *AB* inclined at 40º to the V.P. has its ends 50 mm and 20 mm above the H.P. The length of its front view is 65 mm and its V.T. is 10 mm above H.P. Determine the true length of *AB*, its inclination with H.P. and its H.T.
[RGPV Dec. 2006]
23. A line *PQ* measures 70 mm. The projectors through its V.T. and the end *P* are 40 mm apart. The point *P* is 30 mm above the H.P. and 40 mm in front of the V.P. The V.T. is 10 mm above the H.P. Draw the projections of the line and determine its H.T. and inclinations with the H.P. and the V.P.
[Ans: 19º, 42º]
24. The front view of a line makes an angle of 30º with the reference line. The H.T. of the line is 30 mm in front of the V.P. while the V.T. is 20 mm below the H.P. One end of the line is 15 mm above the H.P. and the other end of the line is 100 mm in front of the V.P. Draw the projections of the line and determine its true length and true angles of inclination with the reference planes.
[Ans: 80 mm, 24º, 37º]
25. Find the length of a solid diagonal of a cube of side 30 mm graphically and measure it.
[Ans: 52 mm] [RGPV June 2002]
26. Find graphically the length of a largest rod that can be kept inside a hollow cuboid of 60 mm × 40 mm × 30 mm.
[Ans: 78 mm]
27. An object is placed 2.4 m above the ground and in the centre of the hall of 8.4 m × 7.2 m × 7.2 m. Determine its distance from one of the corners between the roof and two adjacent walls by graphical method.
[Ans: 7.3 m] [RGPV June 2002]
28. Three wires *AB*, *CD* and *EF* are tied at points *A*, *C*, *E* on a 14 m long vertical pole at height 12 m, 10 m and 8 m from the ground, respectively. The lower ends of the wires are tied to hooks at points *B*, *D* and *F* on the ground level, all of which lie at the corners of an equilateral triangle of 7.5 m side. If the pole is situated at the centre of the triangle, determine the length of each rope and its inclination with the ground.
[Ans: 128 mm *AB* at 70º, 109 mm *CD* at 67º, 91 mm *EF* at 62º]

7.23 LINE INCLINED TO BOTH THE REFERENCE PLANES THE ENDS OF WHICH LIE IN DIFFERENT ANGLES

This part deals with the projections of straight lines inclined to one or both the reference planes and the ends of the line lie in different angles.

Example 7.49 (Fig. 7.49)

A line *PQ*, 75 mm long is inclined at 30º to the V.P. and parallel to the H.P. Draw its projections when whole line lies in the same angle and end *P* is (a) 25 mm in front of the V.P. and 40 mm above the H.P., (b) 25 mm behind the V.P. and 40 mm above the H.P., (c) 25 mm behind the V.P. and 40 mm below the H.P., and (d) 25 mm in front of the V.P. and 40 mm below the H.P.

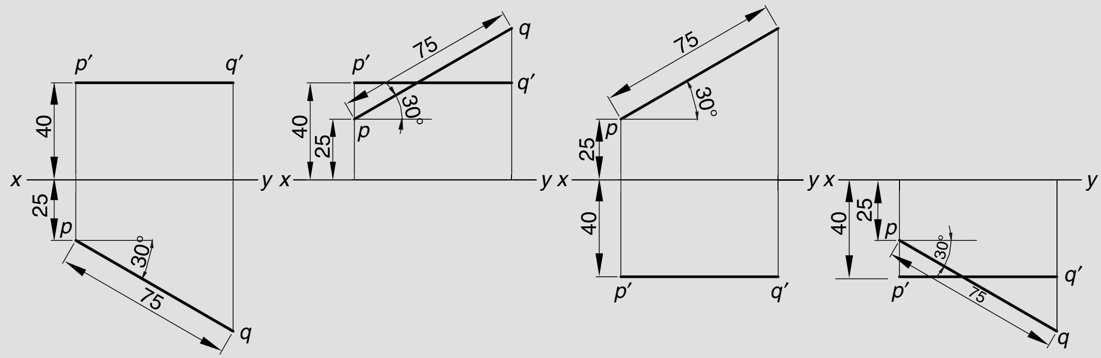


Fig. 7.49(a) **Fig. 7.49(b)** **Fig. 7.49(c)** **Fig. 7.49(d)**

Construction:

Follow the following steps of construction for each of the cases.

1. Draw a reference line *XY*.
2. Locate front and top views of point *p* as points *p′* and *p*, respectively.
3. Draw the top view *PQ* such that *PQ* = 75 mm, inclined 30º to *XY* and lie on one side of *XY*.
4. Project point *q* to meet the horizontal line through point *p′* at point *q′*. Join *p′*-*q′* which represents the front view of the line.

Example 7.50 (Fig. 7.50)

A line *PQ*, 90 mm long is inclined at 30º to the H.P. and 45º to the V.P. Its end *P* is 15 mm above the H.P. and 25 mm in front of the V.P. Draw its projections. Assume end *Q* in the (a) first angle, (b) second angle, (c) third angle, and (d) fourth angle.

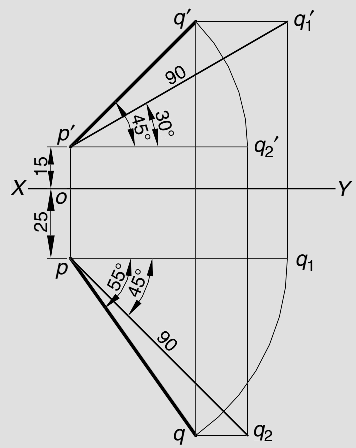


Fig. 7.50(a)

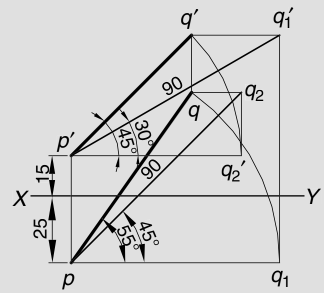


Fig. 7.50(b)

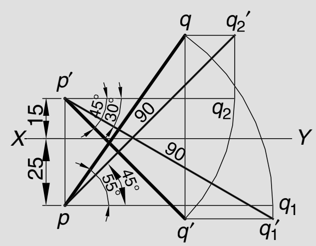


Fig. 7.50(c)

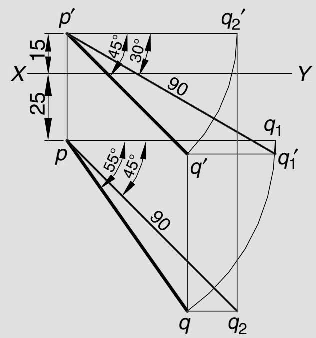


Fig. 7.50(d)

Construction:

1. Mark p', 15 mm above XY and p, 25 mm below XY.
2. Through p', draw line $p'q_1'$ equal to 90 mm, inclined at 30º to XY. Draw a line through point q_1', parallel to XY as the locus of q'.
   As q_1' represents the locus of q', it may be noted that (i) For the end Q to lie above H.P., q_1' should lie above XY, and (ii) For the end Q to lie below H.P., q_1' should lie below XY.
3. Through p, draw pq_2 equal to 90 mm, inclined at 45º to XY. Draw a line through point q_2, parallel to XY as the locus of q.
   As q_2 represents the locus of q, it may be noted that (i) For the End Q to lie in front of the V.P., q_2 should lie below XY, and (ii) For the end Q to lie behind the V.P., q_2 should lie above XY.
4. Project q_1' to meet the horizontal line through p at point q_1. Draw an arc with p as the centre and radius pq_1 to intersect the locus of q at point q. Join pq representing the top view.
5. Project qq_2 to meet the horizontal line from p' at point q_2'. Draw an arc with p' as the centre and radius $p'q_2'$ to meet the locus of q' at point q'. Join $p'q'$ representing the front view.

The following may be noted:

1. When Q lies in the first angle, point q_1' is marked above XY and point q_2 is marked below XY. Fig. 7.50(a)
2. When Q lies in the second angle, points q_1' and q_2 are marked above XY. Fig. 7.50(b)
3. When Q lies in the third angle, point q_1' is marked below XY and point q_2 is marked above XY. Fig. 7.50(c)
4. When Q lies in the fourth angle, points q_1' and q_2 are marked below XY. Fig. 7.50(d)

7.24 MISCELLANEOUS EXAMPLES

Example 7.51 (Fig. 7.51)

The front view of a 75 mm long line measures 55 mm. The line is parallel to H.P. and one of its ends is in the V.P. and 25 mm below the H.P. Draw the projections of line and determine its inclination with the V.P. [RGPV June 2005]

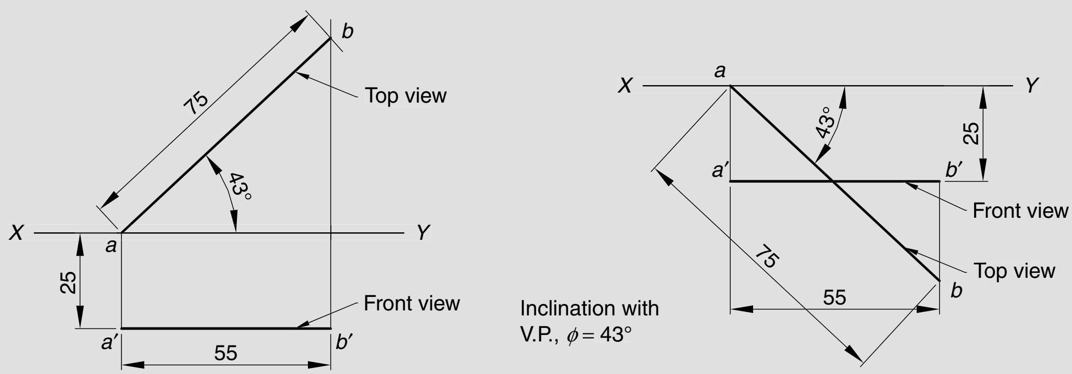


Fig. 7.51 *When the line is situated in the (a) third angle (b) fourth angle*

Interpretation: When a line is parallel to H.P. and 25 mm below H.P., the top view will represent the true length. The line may be situated either in the third or the fourth angle. Readers need to draw any one of these figures.

Construction: Fig. 7.51(a) or (b)

1. Draw a reference line XY.
2. Draw a 55 mm long line $a'b'$, parallel to and 25 mm below XY.
3. Project point a′ to meet XY at point a.
4. Draw an arc of 75 mm radius with point a' as the centre to meet the vertical projector from point b' at point b.
5. Join ab to represent the top view. Measure its inclination with XY as $\phi = 43°$.

Example 7.52 (Fig. 7.52)

The front view of a line is 70 mm long. The line is parallel to H.P. and inclined at 45º to the V.P. One end of the line is 20 mm behind the V.P. and 30 mm below the H.P. while the other end is in the fourth angle. Draw its projections and determine the traces.

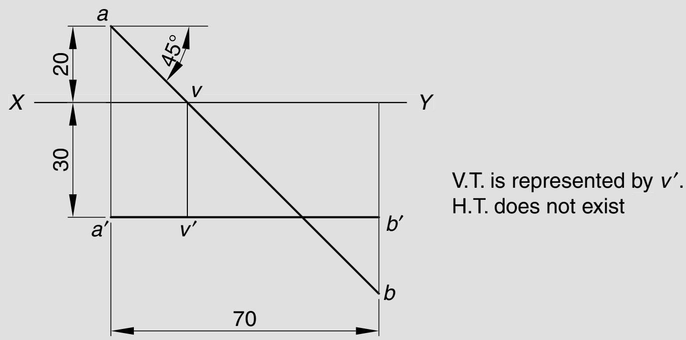


Fig. 7.52

Construction: Fig. 7.52

1. Draw a reference line *XY*. Mark point *a′* 30 mm below *XY* and point *a* 20 mm above *XY*.
2. Draw 70 mm long line *a′b′* parallel to *XY*.
3. Draw line *pq* inclined at 45º to *XY* such that it meets the vertical projector from point *b′* at a point *b*, below *XY*. Line *ab* represents the top view.
4. Mark *v* at the intersection of line *pq* with *XY*. Project *v* to meet *p′q′* at point *v′*. It represents the V.T. As the line is parallel to H.P., the H.T. does not exist.

Example 7.53 (Fig. 7.53)

A 90 mm long line *AB* has 60 mm long front view parallel to *XY*. The end *A* is 25 mm from the V.P, 40 mm from the H.P. and lies in the second angle. The other end *B* lies in the first angle. Draw its projections and determine the true inclination with V.P. and the traces.

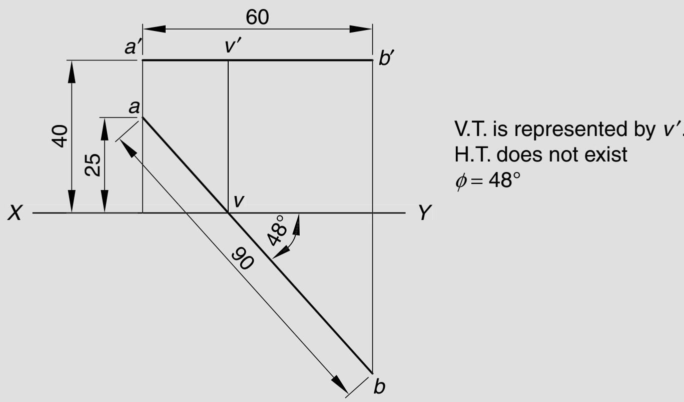


Fig. 7.53

Construction: Fig. 7.53

1. Draw a reference line *XY*. As point *A* lies in second angle, mark point *a′* 40 mm above *XY* and point *a* 25 mm above *XY*.
2. Draw 60 mm long line *a′b′*, parallel to *XY*, to represent the front view.
3. Draw an arc with *a* as the centre and 90 mm radius to meet the vertical projector from point *b′* at point *b*, below *XY*. Join *ab* to represent the top view.
4. Mark point *v* at the intersection of line *ab* with *XY*. Project point *v* to meet line *p′q′* at point *v′*. Point *v′* represents the V.T. As the front view line is parallel to *XY*, it does not have H.T.

Example 7.54 (Fig. 7.54)

The end *A* of a 100 mm long line *AB* is in the V.P. and 30 mm above the H.P. The end *B* is below the H.P. and behind the V.P. The line is inclined at 45º to the H.P. and 30º to the V.P. Draw its projections and locate its H.T. and V.T.

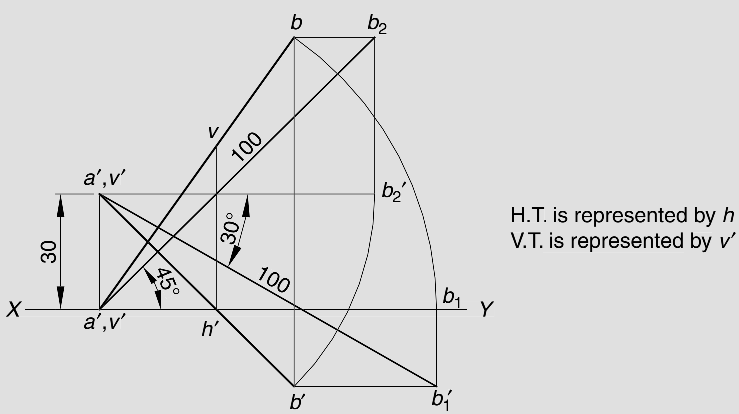


Fig. 7.54

Given data	*Interpretation*
Line *AB* is 100 mm long	$a'b_1' = ab_2 = 100$ mm
End *A* is in the V.P.	*a* on *XY*
End *A* is 30 mm above the H.P.	*a′* is 30 mm above *XY*
The line is inclined at 45º to the H.P. ($\theta = 45º$)	$a'b_1'$ is inclined at 45º to *XY*
The line is inclined at 30º to the V.P. ($\phi = 30º$)	ab_2 is inclined at 30º to *XY*

Construction: Fig. 7.54

1. Draw the reference line *XY*. Mark *p′* 30 mm above *XY* and *p* on *XY*.
2. Draw $p'q_1'$ 100 mm long making 30º with *XY* such that q_1' is below *XY*. Draw a horizontal line through q_1' as the locus of *q′*.
3. Draw pq_2 100 mm long making 60º with *XY* such that q_2 is above *XY*. Draw a horizontal line through q_2 as the locus of *q*.

4. Projector q_1' to meet horizontal line through p at point q_1. Draw an arc with p as the centre and radius pq_1 to meet the locus of q at point q. Join pq.
5. Project q_2 to meet horizontal line through p' at point q_2'. Draw an arc with p' as the centre and $p'q_2'$ as the radius to meet the locus of q' at point q'. Join $p'q'$.
6. Join q'-q to check that it represent a projector perpendicular to XY.
7. Line $p'q'$ meets XY at point h'. Project h' to meet pq at point h. Point h is the H.T.
8. Line pq meets XY at point v. Project v to meet $p'q'$ at point v'. Point v' is the V.T.

Example 7.55 (Fig. 7.55)

A 100 mm long line *PQ*, is inclined at 30º to the H.P. and at 45º to the V.P. Its mid-point is in the V.P. and 20 mm above H.P. Draw its projections if its end *P* is in the third angle and *Q* in the first angle. [RGPV Feb. 2007]

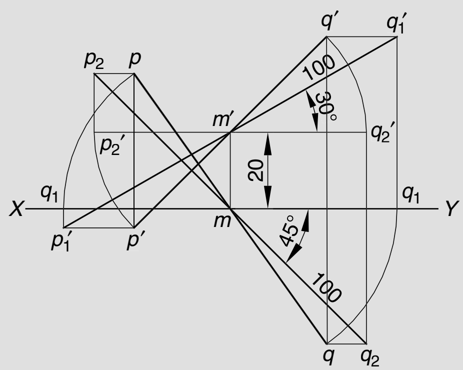


Fig. 7.55

Given data	*Interpretation*
Line *PQ* is 100 mm long, *M* is the mid-point	$p_1'm' = m'q_1' = p_2m = mq_2 = 50$ mm
The line is inclined at 30º to the H.P. ($\theta = 30º$)	$p_1'm'q_1'$ is inclined at 30º to XY
The line is inclined at 45º to the V.P. ($\phi = 45º$)	p_2mq_2 is inclined at 45º to XY
Mid-point *M* is in the V.P.	m is on XY
Mid-point *M* is 20 mm above H.P.	m' is 20 mm above XY

Construction: Fig. 7.55

1. Draw a reference line XY. Mark point m' 20 mm above XY and point m on XY.
2. Draw a 50 mm long line $m'q_1'$ inclined at 30º to XY. Produce it such that $p_1'q_1' = 100$ mm.
3. Draw another 50 mm line mq_2 inclined at 45º to XY. Produce it such that $p_2q_2 = 100$ mm.
4. Project points p_1' and q_1' to meet the horizontal line through point m at points p_1 and q_1 respectively. Draw an arc with m as the centre and radius mp_1 (or mq_1) to meet the horizontal lines through points p_1' and q_1' at points p and q, respectively. Join pmq to represent the top view.

5. Draw vertical projectors from points p_2 and q_2 to meet the horizontal line through point m' at points p_2' and q_2' respectively. Draw an arc with m' as the centre and radius $m'p_2'$ (or $m'q_2'$) to meet the horizontal lines through points p_2 and q_2 at points p' and q', respectively. Join $p'm'q'$ to represent front view.
6. Join $p'p$ and $q'q$ and ensure that they are perpendicular to XY.

Example 7.56 (Fig. 7.56)

The projectors of the ends of a line *AB* are 60 mm apart. The end *A* is 20 mm above H.P. and 30 mm in front of the V.P. The end *B* is 10 mm below H.P. and 40 mm behind the V.P. Determine the true length and traces of *AB* and its inclinations with the two planes.
[RGPV April 2010]

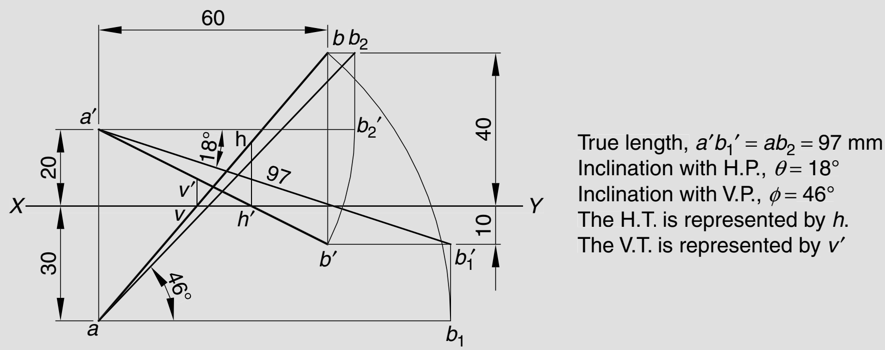


Fig. 7.56

Given data	***Interpretation***
The projectors of the ends are 60 mm apart	Distance between $a'a$ and bb' is 60 mm
The end *A* is 20 mm above H.P.	a' is 20 mm above *XY*
The end *A* is 30 mm in front of the V.P.	a is 30 mm below *XY*
The end *B* is 10 mm below H.P.	b' is 10 mm below *XY*
The end *B* is 40 mm behind the V.P.	b is 40 mm above *XY*

Construction: Fig. 7.56

1. Draw a reference line XY. Mark point a' 20 mm above XY and point a 30 mm below XY.
2. Draw a vertical projector on XY at a distance 50 mm from the projector aa'. On it, mark point b' 10 mm below XY and point b 40 mm above XY.
3. Join $a'b'$ and ab to represent the front and the top views, respectively.
4. Draw an arc with a as centre and radius AB to meet the horizontal line through point a at point b_1. Project point b_1 to meet the horizontal line through point b' at point b_1'. Join $a'b_1'$ which is true length and its inclination with XY is true inclination with H.P. Here, T.L. = 97 mm, $\theta = 18^{\circ}$.

5. Draw an arc with a' as centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Project point b_2' to meet the horizontal line through point b at point b_2. Join ab_2 which is true length and its inclination with XY is true inclination with V.P. Here, $\phi = 46°$.
6. Mark point h' at the intersection of line $a'b'$ with XY. Project point h' to meet line ab at point h. Point h represents the H.T.
7. Mark point v at the intersection of line ab with XY. Project point v to meet line $a'b'$ at point v'. Point v' represents the V.T.

Example 7.57 (Fig. 7.57)

A straight line *AB*, equally inclined to H.P. and V.P. has its end *A* in the front of V.P. and 20 mm above the H.P. End *B* is behind the V.P. and 40 mm below H.P. A point on this line is in V.P. and 10 mm below H.P. Draw the projections and find true length and inclination of the line with H.P., if distance between projectors of the ends is 60 mm.

[RGPV June 2003, Sep. 2009]

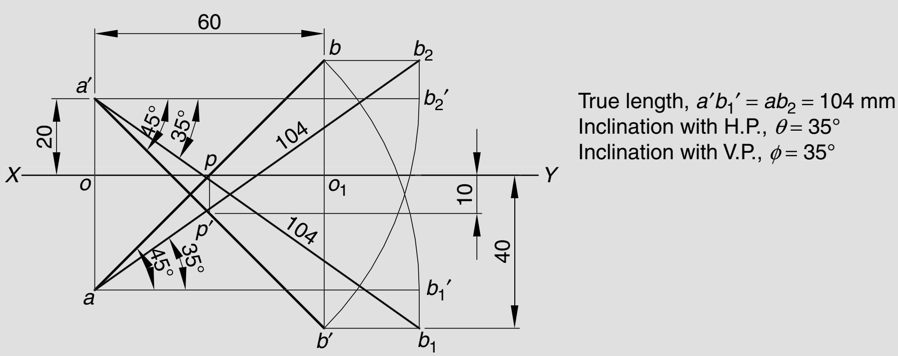


Fig. 7.57

Given data	*Interpretation*
Line *AB* is equally inclined to H.P. and V.P.	$\theta = \phi$ and $\alpha = \beta$
End *A* in the front of V.P. is 20 mm above H.P.	a is below XY and a' is 20 mm above XY
End *B* is behind the V.P. and 40 mm below H.P.	b is above XY and b' is 40 mm below XY
Point *P* on the line is in V.P. and 10 mm below H.P.	p is above XY and p' is 10 mm below XY
Distance between projectors of the ends is 60 mm.	$oo_1 = 60$ mm

Construction: Fig. 7.57

1. Draw a reference line XY. Mark point o on XY and on its projector mark point a' at 20 mm above XY.
2. Mark point o_1 on XY such that $oo_1 = 60$ mm. On the vertical projector from point o_1, mark point b' 40 mm below XY. Join $a'b'$ to represent the front view. Measure its inclination with XY as $\alpha = 45°$.
3. Mark point p' on line $a'b'$, 10 mm from XY. Project it to meet XY at point p.
   When a line is equally inclined to H.P. and V.P. ($\theta = \phi$), then the apparent angles will be also equally inclined to XY ($\alpha = \beta$).

4. Through point *p*, draw a line inclined at $\alpha(= 45°)$ angle to *XY* to meet the vertical projectors from points *a*′ and *b*′ at points *a* and *b*. Line *apb* represents the top view.
5. Draw an arc with *a* as centre and radius *ab* to meet the horizontal line through point *a* at point b_1. Project point b_1 to meet the horizontal line through point *b*′ at point b_1'. Join $a'b_1'$. Measure its length as true length and inclination with *XY* as true inclination with H.P. Here, T.L. = 104 mm, $\theta = 35°$.
6. Draw an arc with *a*′ as centre and radius *a*′*b*′ to meet the horizontal line through point *a*′ at point b_2'. Project point b_2' to meet the horizontal line through point *b* at point b_2. Join ab_2. Measure its length as true length and inclination with *XY* as true inclination with V.P. Here, $\phi = 46°$.

Example 7.58 (Fig. 7.58)

The top view of a line *PQ* 75 mm long measures 54 mm. The mid-point of the line is 50 mm from V.P. and 15 mm from H.P. The point *Q* is 24 mm from the V.P. Draw its projections and find inclinations with the H.P. and V.P. Also, locate its traces. [RGPV Dec. 2007]

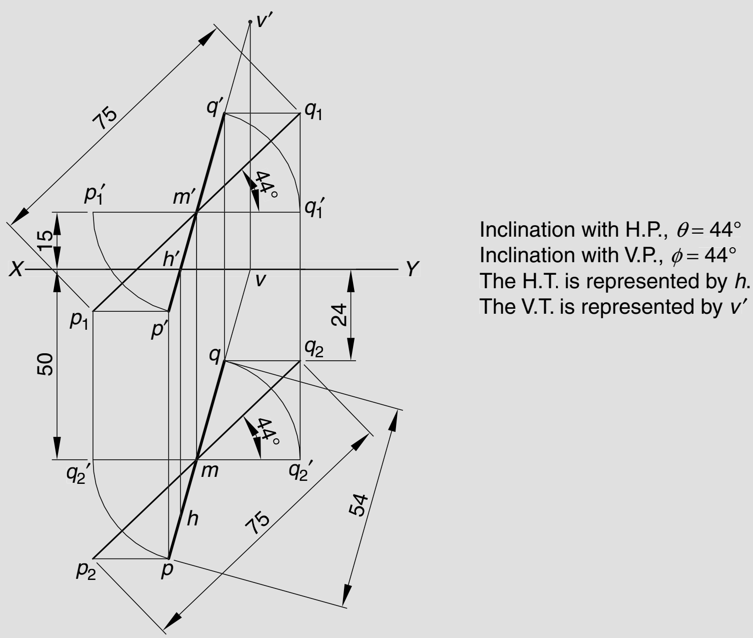


Fig. 7.58

Given data	*Interpretation*
Line *PQ* is 75 mm long, *M* is the mid-point	$p_1'm' = m'q_1' = p_2m = mq_2 = 75/2$ mm
The top view measures 54 mm.	$p'm' = m'q' = 54/2$ mm
The mid-point *M* is 50 mm from V.P.	*m* is 50 mm below *XY*
The mid-point *M* is 15 mm from H.P.	*m*′ is 15 mm above *XY*
The end *Q* is 24 mm from the V.P.	*q* is 24 mm below *XY*

Construction: Fig. 7.58

1. Draw a reference line *XY*. Mark point *m′* 15 mm above *XY* and point *m* 50 mm below *XY*. This represents the front and top view of the mid-point of the line.
2. Draw a line parallel to 24 mm from *XY* as loci of points *q* and q_2. Draw an arc with *p* as centre and radius 27 mm (=54/2) to meet the locus of point *q* at point *q*. Join *qm* and produce it to point *p* such that *pmq* = 54 mm. Line *pq* represent the top view.
3. Similarly, draw another arc with *p* as centre and 37.5 mm radius (=75/2) to meet the locus of point q_2 at point q_2. Join q_2m and produce it to mark point p_2 such that p_2q_2 = 75 mm. Measure its inclination with *XY* as true inclination with V.P. Here, $\phi = 44°$.
4. Draw an arc with *m* as the centre and radius *mq* (=*mp*) to meet horizontal line through point *m* at points p_2' and q_2'. Draw another arc with point m′ as centre and 37.5 mm (=75/2) radius to meet the vertical projectors from points p_2' and q_2' at points p_1 and q_1. Join p_1q_1 and measure its inclination with *XY* as inclination with H.P. Here, $\theta = 44°$.
5. Project points p_2 and q_2 to meet horizontal line from point *m′* at points p_1' and q_1'. Draw an arc with *m′* as centre and radius $m'q_1'$ (=$m'p_1'$) to meet horizontal line from points p_1 and q_1 at points *p′* and *q′*, respectively. Join p′q′ to represent the front view.
6. Line *p′q′* to meet *XY* at point *h′*. Project *h′* to meet line *pq* at point *h*. Point *h* represents H.T.
7. Produce line *pq* to meet *XY* at point *v*. Project point *v* to meet line *p′q′* at point *v′*. Point *v′* represents V.T.

Example 7.59 (Fig. 7.59)

The front view of a line *AB* is 50 mm long and it makes an angle of 35° with *XY*. The point *A* lies 10 mm above H.P. and 25 mm behind V.P. The difference between the distances of *A* and *B* from V.P. is 25 mm. The line *AB* is in second angle. Draw the projections of the line and determine its true length and inclinations with the H.P. and V.P. [RGPV Aug. 2010]

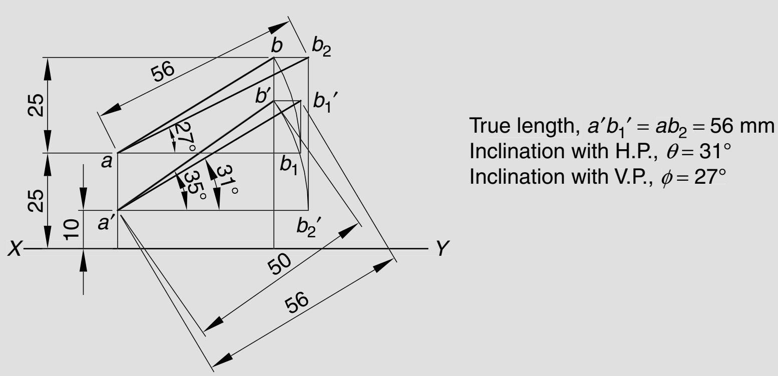


Fig. 7.59

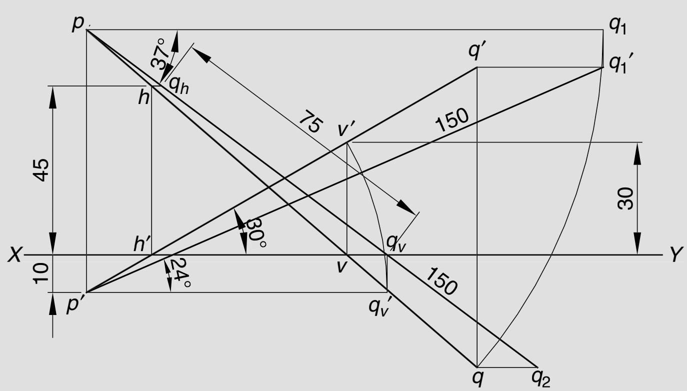
Given data	*Interpretation*
Front view of a line *AB* is 50 mm long	$a'b'$ = 50 mm
Front view makes an angle of 35º with *XY* (α = 35º)	$a'b'$ is inclined at 35º to *XY*
Point *A* lies 10 mm above H.P.	a' is 10 mm above *XY*
Point *A* lies 25 mm behind V.P.	*a* is 25 mm above *XY*
The difference between the distances of *A* and *B* from V.P. is 25 mm	*b* is 25 mm above *a*, (i.e. *b* is 50 mm above *XY)*

Construction: Fig. 7.59

1. Draw a reference line *XY*. Mark point a' 10 mm above *XY* and point *a* 25 mm above *XY*.
2. Draw a 50 mm long line $a'b'$ inclined at 35º to *XY*. This represents the front view.
3. Project point b' to meet a horizontal line parallel to and 50 mm from *XY* at point *b*. Join *ab* to represent the top view.
4. Draw an arc with *a* as centre and radius *ab* to meet the horizontal line through point *a* at point b_1. Project point b_1 to meet the horizontal line through point b' at point b_1'. Join $a'b_1'$. Measure its length as true length and inclination with *XY* as true inclination with H.P. Here, T.L. = 56 mm, θ = 31º.
5. Draw an arc with a' as centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Project point b_2' to meet the horizontal line through point *b* at point b_2. Join ab_2. Measure its length as true length and inclination with *XY* as true inclination with V.P. Here, ϕ = 27º.

Example 7.60 (Fig. 7.60)

The front view of a line *PQ* makes an angle 30º with *XY* line. The H.T. of the line is 45 mm behind V.P. while its V.T. is 30 mm above H.P. The end *P* of the line is 10 mm below H.P. and the end *Q* is in the first angle. The line is 150 mm long. Draw the projections of the line and determine the true length of the portion of the line, which is in second angle. Also, find the angle of the line with the H.P. and V.P. [RGPV June 2006]



True length of the portion of the line, which is in second angle, q_hq_2 = 75 mm

Inclination with H.P., θ = 24°

Inclination with V.P., ϕ = 37°

Fig. 7.60

Given data	*Interpretation*
Front view makes an angle 30º with *XY*	$p'q'$ is inclined at 30º to *XY*
H.T. is 45 mm behind V.P.	hh' = 45 mm, *h* lies above *XY*
V.T. is 30 mm above H.P.	vv' = 30 mm, v' lies above *XY*
End *P* is 10 mm below H.P.	p' is 10 mm below *XY*
The line is 150 mm long	$p'q_1' = pq_2 = 150$ mm

Construction: Fig. 7.60

1. Draw a reference line *XY*. Mark point h' on *XY* and *h* 40 mm above *XY*.
2. Draw a line $h'v'$ inclined at 30º to meet a point v' 30 mm above *XY*.
3. Produce line $h'v'$ to meet a point p' 10 mm below *XY*.
4. Project point v' to meet *XY* at point *v*. Join *vh* and produce it to meet the vertical projector from point p' at point *p*.
5. Draw an arc with p' as centre and radius $p'v'$ to meet the horizontal line through point p' at point q_v'. Project point q_v' to meet the horizontal line through point *v* at point q_v. Join pq_v and produce it to mark point q_2, 150 mm from point *p*. Measure the inclination of line *pq* with *XY* as true inclination with V.P. Here, $\phi = 37º$.
6. Produce line *phv* to meet the horizontal line from point q_2 at point *q*. Line *pq* represents the top view.
7. Draw an arc with *p* as centre and radius *pq* to meet the horizontal line through point *p* at point q_1. Draw an arc with p' as the centre and radius 150 mm to meet the vertical projector from point q_1 at point q_1'. Join $p'q_1'$. Measure its inclination with *XY* as true inclination with H.P. Here, $\theta = 24º$.
8. Produce line $p'h'v'$ to meet the horizontal line from point q_1' at point q'. Join qq' and ensure that it is perpendicular to *XY*.
9. Portion of the line lying between its H.T. and V.T. lies in the second angle. Draw a horizontal line from point *h* to meet line *PQ* at point q_h. Measure the length q_hq_v which is 75 mm long.

Example 7.61 (Fig. 7.61)

Two apples on a tree are respectively 1.8 m and 3 m above the ground and 1.2 m and 2.1 m from a 0.3 m thick wall but on the opposite side of it. The distance between the apples along the wall is 2.7 m. Determine the true distance between the apples. [RGPV April 2009]

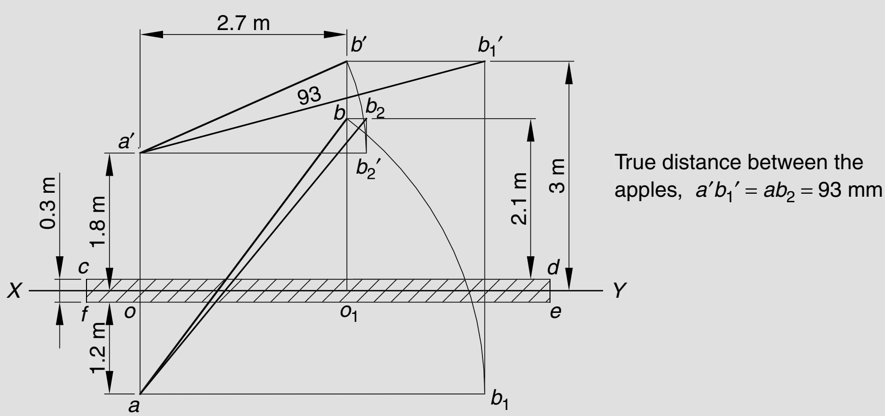


Fig. 7.61

Given data	*Interpretation*
Apple *A* is 1.8 m above the ground	a' is 1.8 m above *XY*
Apple *B* is 3 m above the ground	b' is 3 m above *XY*
Apple *A* is 1.2 m from a 0.3 m thick wall	*a* is 1.2 m below *fe*
Apple *B* is 2.1 m from a 0.3 m thick wall (on opposite side)	*b* is 2.1 m above *cd*
The distance between the apples along the wall is 2.7 m	oo_1 = 2.7 m

Construction: Fig. 7.61

1. Take scale 1:50. Draw a reference line *XY*. Mark points *o* and o_1 on *XY* such that oo_1 = 2.7 m.
2. Draw lines *cd* and *ef* parallel to and 0.15 m from *XY*, to represent 0.3 m thick wall.
3. On the vertical projector through point *o* mark point a' 1.8 m above *XY* and point *a* 1.2 m from *ef*.
4. On the vertical projector through point o_1 mark point b' 3 m above *XY* and point *b* 2.1 m from *cd*.
5. Join $a'b'$ and *ab* to represent the front and the top views of the line joining two apples.
6. Draw an arc with *a* as centre and radius *ab* to meet the horizontal line through point *a* at point b_1. Project point b_1 to meet the horizontal line through point b' at point b_1'. Join $a'b_1'$. Measure its length as true distance between the apples. Here, T.L. = 93 mm.
7. Draw an arc with a' as centre and radius $a'b'$ to meet the horizontal line through point a' at point b_2'. Project point b_2' to meet the horizontal line through point *b* at point b_2. Join ab_2. Ensure that its length is equal to that of $a'b_1'$.

EXERCISE 7C

1. A 90 mm long line *PQ* is inclined at 45º to the H.P. and 30º to the V.P. Its end *P* is in the H.P. and 40 mm in front of the V.P. Draw its projections keeping end *Q* in the fourth angle.
2. End point *C* of a 80 mm long line *CD* is 15 mm above the H.P. and 10 mm in front of the V.P. The line is inclined at 30º to the H.P. and 45º to the V.P., and the other end point *D* lies in the second angle. Draw its projections and determine its traces.
3. Find the distance between the two points *P* and *Q* when point *P* is 35 mm above the H.P. and 50 mm behind the V.P. The point *Q* is 40 mm above the H.P. and 25 mm in front of the V.P. The distance between the projectors being 50 mm. **[Ans: 91 mm] [RGPV Sep. 2009]**
4. The projectors of the ends of a line *AB* are 60 mm apart. The end *A* is 25 mm above H.P. and 30 mm in front of the V.P. The end *B* is 20 mm below H.P. and 40 mm behind the V.P. Determine the true length and traces of *AB* and inclinations with the two planes.
[Ans: 103 mm, 26º, 43º] [RGPV Feb. 2008]
5. The projectors of the ends of a line are 50 mm apart. The end *A* is 20 mm above H.P. and 30 mm in front of the V.P. The end *B* is 10 cm below H.P. and 40 mm behind V.P. Determine the true length and traces of *AB* and its inclination with the two planes.
[Ans: 91 mm, 19º, 50º] [RGPV Feb. 2007]

6. The distance from the end projections of a line *PQ* is 50 mm. A point *P* is 29 mm above H.P. and 22 mm behind V.P., while the point *Q* is 40 mm below H.P. and 30 mm in front of V.P. Determine the projections of the line and determine the true length and true inclinations with H.P. and V.P.

[Ans: 100 mm, 44º, 31º] [RGPV June 2006]

7. The end *A* of a line *AB* is in H.P. and 25 mm behind V.P. The end *B* is in the V.P. and 50 mm above H.P. The distance between the end projectors is 75 mm. Draw the projections of *AB* and determine its true length.

[Ans: 94 mm] [RGPV Dec. 2004, Feb. 2005]

8. A 150 mm long line *AB* is inclined at 30º to the H.P. and 45º to the V.P. A point *M* lies on the line at a distance of 90 mm from end *A* and has its front view 20 mm above the *XY* and top view at 30 mm below its front view. Draw its projections and determine the traces.
9. An 80 mm long line *AB* has its end *A* 20 mm from the reference planes and lies in the third angle. The end *B* is 40 mm from both the reference planes and lies in the second angle. Draw its projections and find the true inclination with the reference planes.

[Ans: 49º, 14º]

10. A 120 mm long line *AB* has its projectors 50 mm apart. The ends *A* and *B* are 10 mm and 60 mm below the H.P., respectively. The mid-point of *PQ* lay in the V.P. Draw the projections and determine inclination of the line with the reference planes. Consider end *A* is in the fourth angle.

[Ans: 25º, 54º]

11. The end *A* of a 150 mm long line *AB* is 50 mm behind the V.P. and 35 mm below the H.P. The other end *B* is in the first angle. The line is inclined at 30º to the H.P. and has a point *C* in both H.P. and V.P. Draw the projections of the line and find its inclination with the V.P.

[Ans: 46º]

12. A 100 mm long line *PQ* has its end point *A* 30 mm below H.P. and 20 mm behind V.P. Its V.T. is 10 mm above the H.P. The projectors drawn through its V.T. and the end *A* are 40 mm apart. Draw the projections of the line and determine its H.T. and inclination with the reference planes.

[Ans: 42º, 19º]

13. A 75 mm long line *AB* has its end *A*, 50 mm below H.P. and 20 mm behind V.P. Its front and top views are 60 mm and 45 mm long, respectively. Draw its projections when the end *B* is in the first angle. Determine the true inclination of the line with H.P. and V.P.

[Ans: 53º, 37º]

14. The front view of a 75 mm long straight line *AB* measures 45 mm, while its top view measures 60 mm. Its end *A* lies 15 mm below HP and 20 mm behind VP, while the other end lies in first angle. Draw the projections of *AB* and obtain true inclinations of *AB* with the reference planes.

[Ans: 37º, 53º] [RGPV Dec. 2008]

15. Two bulbs on two poles are respectively 2 m and 3 m above the ground and 1 m and 2 m from the wall, 0.3 m thick but on opposite side of it. The distance between the bulbs measured along the ground and parallel to the wall is 3 m. Determine the real distance between the bulbs.

[Ans: 4.9 m] [RGPV Dec. 2010]

REVIEW QUESTIONS

1. A straight line is parallel to and 25 mm in front of the V.P. and inclined at 30º to the H.P. What is the position of its H.T. and V.T.?
2. A straight line is parallel to and 40 mm above the H.P. and inclined at 45º to the V.P. What is the position of its H.T. and V.T.?
3. The front view of a line is parallel to *XY* and measures 30 mm. What is its true length if the top view measures 65 mm?
4. The top view of a line is parallel to *XY* and measures 40 mm. What is its true length if the front view measures 75 mm?
5. A line is inclined at 30º to the H.P. and 60º to the V.P. Which orthographic view of this line will show its true length?
6. The distance between end projectors of a line is zero. Which orthographic view of this line will show its true length?
7. A line is inclined to both the reference planes. State the positions of the front and top views of its H.T.
8. A line is inclined to both the reference planes. State the positions of the front and top views of its V.T.
9. The top view of a line is represented by a point on the reference line. State the position of the line.
10. A point on *XY* represents the front view of a straight line. What is the position of the line?
11. The top view of a line is 30 mm long. If the length of the line is extended by one third of its original length, what will be the measure of the new top view?
12. The front view of a line is 40 mm long. If the length of the line is reduced by one fourth of its original length, what will be the measure of the new front view?
13. One end of a line lies in the first angle and the other in the second angle. Which of the two views of the line will intersect the reference line?
14. One end of a line lies in the second angle and the other in the third angle. Which of the two views of the line will cross the reference line?
15. A line is inclined at an angle of 30º with H.P. What will be its inclination with V.P. if the distance between its end projectors is zero?
16. If the front view of a line lies in the reference line, state all the possible positions of the line.
17. If the top view of a line lies in the reference line, state all the possible positions of the line.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) If a line is parallel to both H.P. and V.P., its true length will be seen in
   (a) front view (b) top view (c) side view (d) both front and top views

ii) If the apparent and the true inclinations of a line with H.P. are equal, the line is
(a) parallel to horizontal plane (b) parallel to vertical plane
(c) parallel to profile plane (d) inclined to both reference planes

iii) The point at which the line intersects the V.P., extended if necessary, is known as
(a) profile trace (b) horizontal trace (c) vertical trace (d) auxiliary trace

iv) If the front view of a line is parallel to the *XY* its true length is shown in
(a) front view (b) top view (c) side view (d) both front and top views

v) If top view of a line is a point, its front view is
(a) parallel to *XY* and of true length
(b) parallel to *XY* and of apparent length
(c) perpendicular to *XY* and of true length
(d) perpendicular to *XY* and of apparent length

vi) Horizontal trace of a line exits when the line is
(a) parallel to horizontal plane (b) inclined to horizontal plane
(c) perpendicular to vertical plane (d) perpendicular to profile plane

vii) If a line is inclined at 45º to the H.P. and 30º to the V.P., its front view is inclined at
(a) 30º to *xy* (b) 45º to *xy* (c) between 30º and 45º (d) greater than 45º

viii) If a line is inclined at 30º to the H.P. and 60º to the V.P., its front and top views are inclined at an angle of
(a) 30º and 60º to *XY* respectively (b) 60º and 30º to *XY* respectively
(c) both at 90º to *XY* (d) both greater than 30º but less than 90º

ix) For a line situated in the first angle which of the following is not correct?
(a) H.T. and V.T. may lie below *XY* (b) H.T. lies below *XY* and V.T. lies above *XY*
(c) H.T. and V.T. may lie above *XY* (d) H.T. lies above *XY* and V.T. lies below *XY*

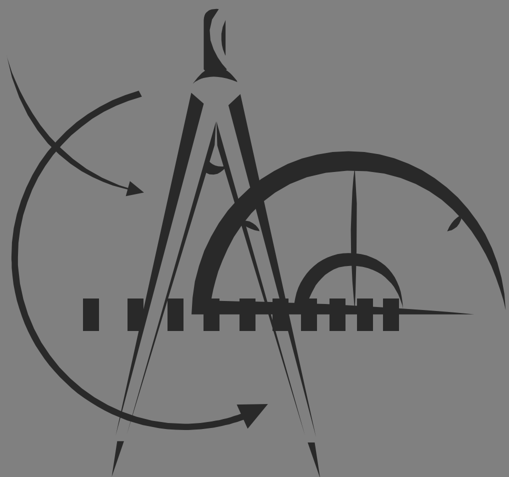
x) A 90 mm long line *PQ*, inclined at 30º to the H.P. and 45º to the V.P. has end *P* 15 mm above H.P. and 25 mm in front of V.P. The other end *Q* will lie in
(a) first angle (b) third angle (c) second or fourth angle (d) any of these

xi) If the front and top views of a line are inclined at 30º and 45º to the reference line, the true inclination of the line with H.P. will be
(a) 30º (b) 45º (c) less than 30º (d) greater than 45º

xii) If both the front and top views of a line are perpendicular to the reference line, the true inclination of the line with H.P. and V.P. may be respectively
(a) 15º and 75º (b) 30º and 60º (c) both 45º (d) any of these

(i) d (ii) b (iii) c (iv) b (v) c (vi) b (vii) d (viii) c (ix) d (x) d (xi) c (xii) d

8



Projections of Planes

- Introduction
- Orientation of Planes
- Plane Parallel to H.P. or V.P.
- Plane Perpendicular to Both H.P. and V.P.
- Plane Inclined to H.P. and Perpendicular to V.P.
- Plane Inclined to V.P. and Perpendicular to H.P.
- Plane Inclined to Both the Reference Planes
- Plane Inclined θ to H.P. and ϕ to V.P. such that $\theta + \phi = 90^\circ$

8.1 INTRODUCTION

In this chapter we deal with two-dimensional objects called planes. Planes have length, breadth and negligible thickness. Only those planes are considered in the chapter whose shape can be defined geometrically[#] and are regular in nature. Some of these are shown in Fig. 8.1.

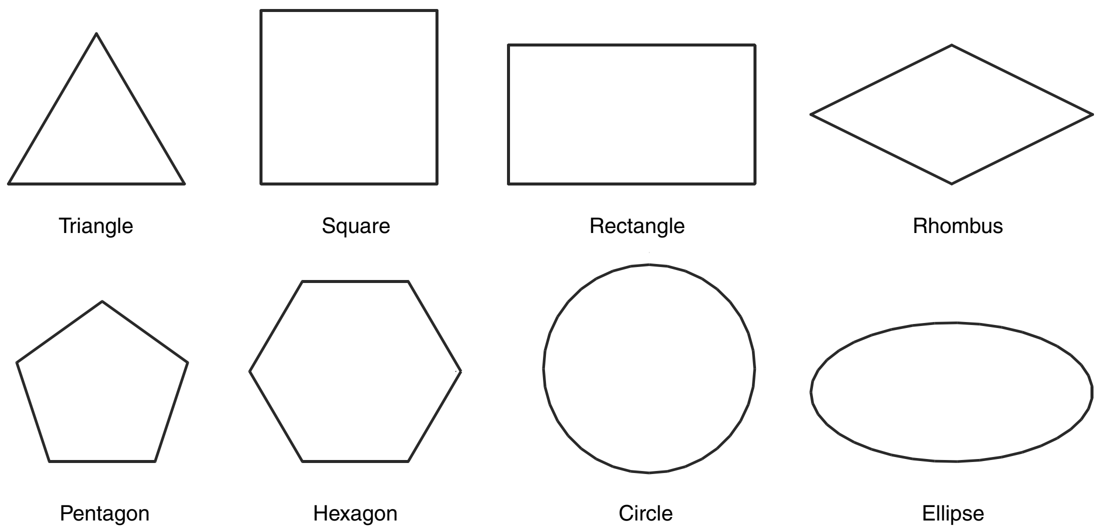


Fig. 8.1 *Planes*

8.2 ORIENTATION OF PLANES

The surface of a plane may be

1. Parallel to H.P. (and perpendicular to V.P.)
2. Parallel to V.P. (and perpendicular to H.P.)
3. Perpendicular to both H.P. and V.P. (i.e. parallel to profile plane)
4. Inclined to H.P. and perpendicular to V.P.
5. Inclined to V.P. and perpendicular to H.P.
6. Inclined to both H.P. and V.P.

8.3 PLANE PARALLEL TO H.P.

This is one of the basic positions of a plane. A plane parallel to H.P. is always perpendicular to the V.P. The true shape and size of the plane is seen in its top view. Hence, top view of the plane is drawn first and then it is projected to get a straight line as its front view.

[#] Readers are advised to refer chapter 1 for the details of methods of construction of regular polygons.

Example 8.1 (Fig. 8.2b)

A square plane *ABCD* of 30 mm side has its surface parallel to H.P. and 20 mm away from it. Draw its projections of the plane when two of its sides are (i) parallel to V.P., (ii) inclined at 30º to V.P., and (iii) all sides are equally inclined to V.P. [RGPV June 2008(O)]

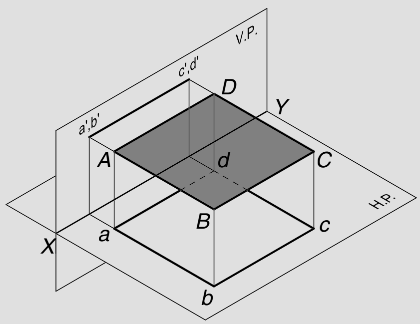


Fig. 8.2(a) *3-D view*

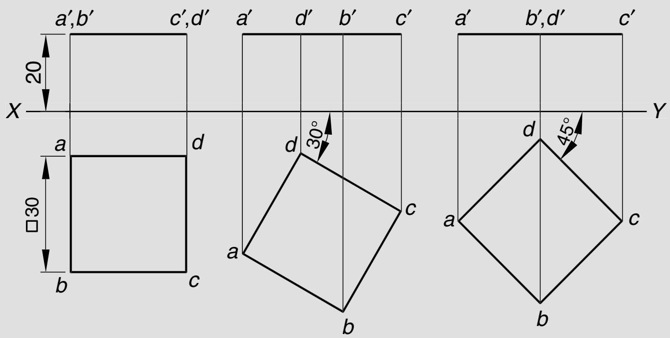


Fig. 8.2(b) *Projections in cases (i), (ii) and (iii)*

Visualization: Fig. 8.2(a) shows a square plane *ABCD* with the surface parallel to H.P. and above it. Side *AD* is parallel to the V.P. as desired in case (i). Since all the sides of the square are parallel to the H.P., the top view is a square *abcd* in which *ad* is parallel to *XY.* As plane is perpendicular to V.P., its front view is a line parallel to *XY.*

Construction: Fig. 8.2(b)

Case (i) Draw a square *abcd* in the top view keeping *ad* parallel to *XY.* Project the corners from the top view and obtain points *a′*, *b′*, *c′* and *d′* 20 mm above *XY*. Join *a′b′c′d′* to get the required front view.

Case (ii) Draw a square *abcd* in the top view keeping *ab* at 30º to *XY.* Project the corners from the top view and obtain points *a′*, *b′*, *c′* and *d′* 20 mm above *XY*. Join *a′b′c′d′* to get the required front view.

Case (iii) Draw the square *abcd* in the top view keeping *ab* at 45º to *XY.* Project the corners from the top view and obtain points *a′*, *b′*, *c′* and *d′* 20 mm above *XY*. Join *a′b′c′d′* to get the required front view.

8.4 PLANE PARALLEL TO V.P.

This is another basic position of a plane. A plane parallel to V.P. is always perpendicular to the H.P. The true shape and size of the plane is seen in its front view. Hence, front view of the plane is drawn first and then it is projected to get a straight line as its top view.

Example 8.2 (Fig. 8.3b)

A hexagonal plane of 25 mm side has its surface parallel to and 20 mm in front of V.P. Draw its projections, when a side is (a) parallel to H.P., (b) perpendicular to H.P., (c) inclined at 45º to H.P.

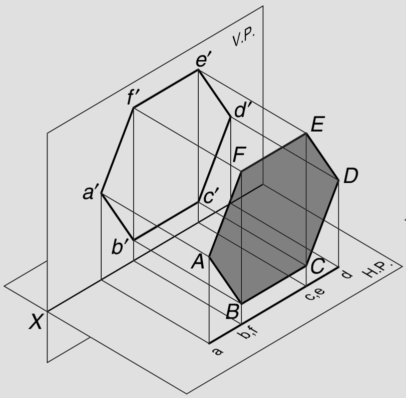


Fig. 8.3(a) *3-D view*

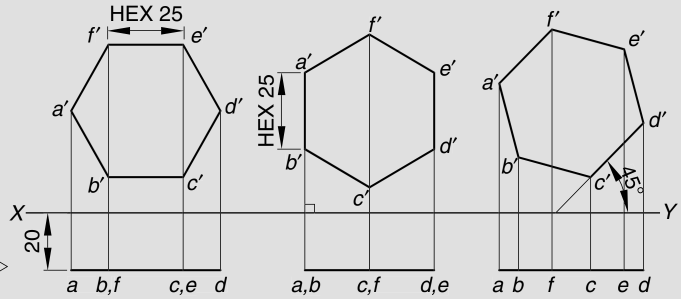


Fig. 8.3(b) *Projections in cases (i), (ii) and (iii)*

Visualization: Fig. 8.3(a) shows a hexagonal plane *ABCDEF* with its surface parallel to and in front of V.P. Side *BC* is parallel to the H.P. as desired in case (i). Its front view is a regular hexagon *abcdef* with edge *bc* parallel to *XY.* Since the plane is perpendicular to H.P., its top view is a line parallel to *XY.*

Construction: Fig. 8.3(b)

Case (i) Draw a hexagon *a′b′c′d′e′f′* as the front view keeping a side *b′c′* parallel to *XY.* Project the corners and obtain points *a*, *b*, *c*, *d*, *e* and *f* 20 mm below *XY.* Join *abcdef* to get the required top view.

Case (ii) Draw a hexagon *a′b′c′d′e′f′* as the front view keeping a side *a′b′* perpendicular to *XY.* Project the corners and obtain points *a*, *b*, *c*, *d*, *e* and *f* 20 mm below *XY.* Join *abcdef* to get the required top view.

Case (iii) Draw a hexagon *a′b′c′d′e′f′* as the front view keeping a side *c′d′* inclined at 45º to *XY.* Project the corners and obtain points *a*, *b*, *c*, *d*, *e* and *f* 20 mm below *XY.* Join *abcdef* to get the required top view.

8.5 PLANE PERPENDICULAR TO BOTH H.P. AND V.P.

A plane perpendicular to both H.P. and V.P. has its surface parallel to the profile plane. The true shape and size of the plane can be obtained only on the profile plane, popularly known as *end view* or *side view*. The front and top views are projected from the side view.

Example 8.3 (Fig. 8.4)

A triangular plane is in the form of an isosceles triangle of 30 mm side base and 40 mm long altitude. It is kept in the first quadrant such that the surface is perpendicular to both H.P. and V.P. Draw its projections when the base is parallel to the V.P.

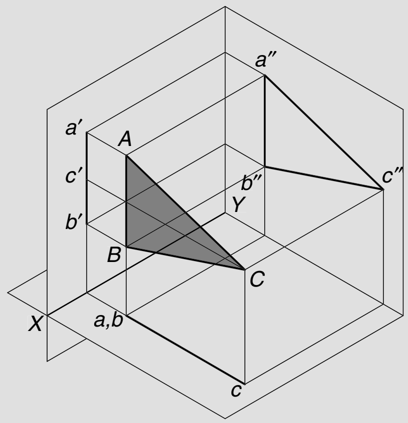


Fig. 8.4(a) *3-D view*

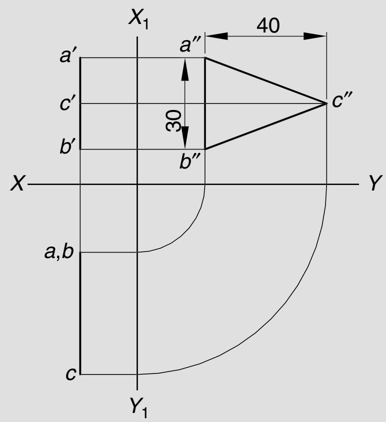


Fig. 8.4(b) *Projections*

Visualization: Let the plane *ABC* be perpendicular to both H.P. and V.P. keeping the side *AB* is parallel to the V.P., as shown in Fig. 8.4(a). Thus, the plane is parallel to the profile plane.

Construction: Fig. 8.4(b)

1. Draw a triangle *a″b″c″* as the side view keeping *a″b″* perpendicular to *XY*.
2. Project the corners and obtain points *a′*, *b′* and *c′* at some distance from X_1Y_1 line.
3. Project *a′*, *b′* and *c′* on *XY* line and extend them to meet projectors coming from the side view to intersect at points *a*, *b* and *c*. Join *abc*.

8.6 PLANE INCLINED TO H.P. AND PERPENDICULAR TO V.P.

When the surface of the plane is perpendicular to the V.P. and inclined at θ to the H.P., the projections are drawn in two stages. In the first stage, it is assumed that the plane is lying in the H.P. Here true shape of the plane is seen in the top view and the front view is projected from the top view. In the second stage, the surface of the plane is tilted at θ to the H.P. Therefore, reproduce the front view at an inclination of θ to *XY*. Obtain the top view by joining the points of intersection of the vertical projectors of the corners from the front view with the horizontal projectors from the top view of the preceding stage.

Note 1. If the plane has a side in the H.P. (or parallel to the H.P. or on the ground) then in the top view of the first stage, an edge should be perpendicular to the *XY* line.

Note 2. If the plane has a corner in the H.P. (or on the ground) then the line joining the top view of one of the corners and the centre of the plane should be parallel to *XY*.

Example 8.4 (Fig. 8.5)

A hexagonal plane of 25 mm side has one side on the ground. The surface of the plane is inclined at 45º to the H.P. and perpendicular to the V.P. Draw its projections.

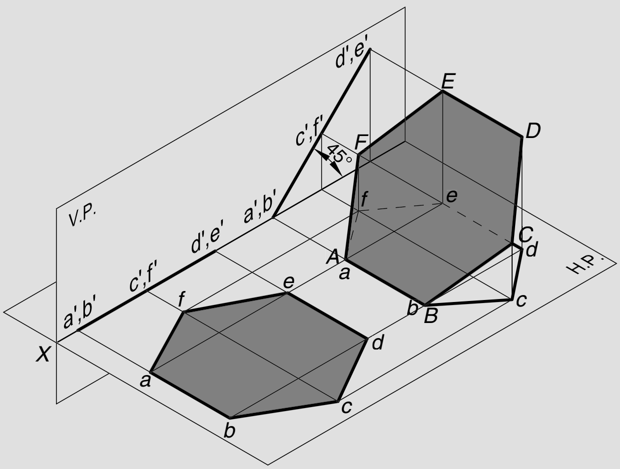


Fig. 8.5(a) *3-D view*

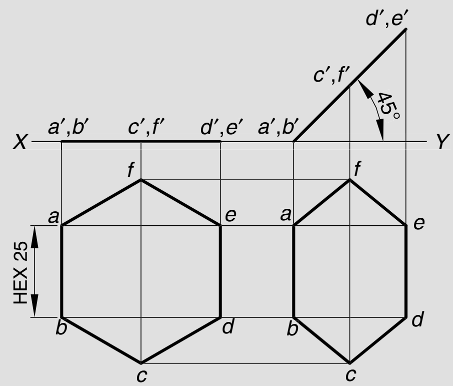


Fig. 8.5(b) *Projections*

Construction: Fig. 8.5(b)

1. *First stage*: The plane has an edge in the H.P., so assume that the plane *ABCDEF* is placed on the H.P. with side *AB* perpendicular to the V.P. Draw the hexagon *abcdef* as the top view with *ab* perpendicular to *XY* line. Project the corners to *XY* and obtain the front view *b′d′*.
2. *Second stage*: Reproduce the front view of first stage, such that *b′d′* is inclined at 45º to *XY* line. Obtain new points *a*, *b*, *c*, *d*, *e* and *f* of the top view by joining the points of intersection of the projectors (vertical lines) drawn from the front view of the second stage (i.e. vertical lines from *a′*, *b′*, *c′*, *d′*, *e′* and *f′*) with the corresponding horizontal locus lines drawn from the top view of the first stage (i.e. from *a*, *b*, *c*, *d*, *e* and *f*). Join new *abcdef* to represent the final top view.

Example 8.5 (Fig. 8.6)

A hexagonal plate of 30 mm side is resting on one of its corner on H.P. The plate is perpendicular to V.P. and inclined at 45º to the H.P. Draw its projections. [RGPV Dec 2010]

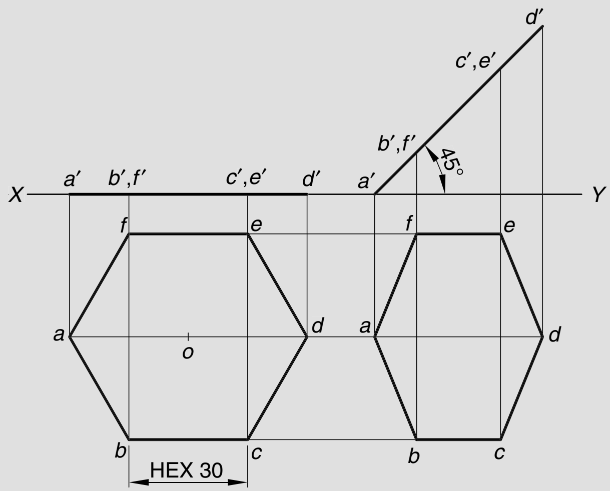


Fig. 8.6

Construction: Fig. 8.6

1. *First stage*: The plate resting on a corner in the H.P., so assume that the plane ABCDEF is placed on the H.P., such that the line joining the corner and the centre of the plate i.e. *OA* is parallel to V.P. Draw a hexagon *abcdef* as the top view such that *oa* is parallel to the *XY*. Project the corners to *XY* and obtain *a′d′* as the front view.
2. *Second stage*: Reproduce the front view *a′d′* of first stage such that it is inclined at 45º to *XY*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* of the new top view by joining the points of intersection of the vertical projectors drawn from the front view of the second stage (i.e. from *a′*, *b′*, *c′*, *d′*, *e′* and *f′*) with the horizontal locus lines drawn from the top view of the first stage (i.e. from *a*, *b*, *c*, *d*, *e* and *f*). Join *abcdef* to represent the final top view.

Example 8.6 (Fig. 8.7)

Draw the projections of a circle of 40 mm diameter, resting on H.P. on a point on the circumference. Its plane is inclined at 30° to the H.P. and perpendicular to the V.P. Its centre is 35 mm in front of the V.P.

[RGPV Dec. 2007]

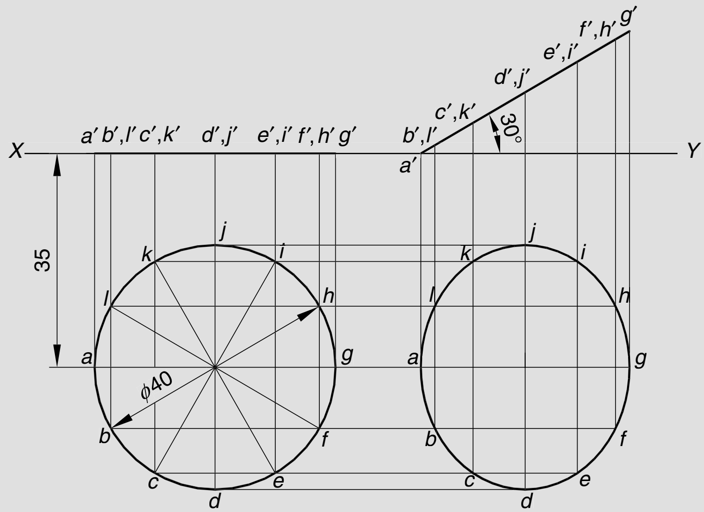


Fig. 8.7

Construction: Fig. 8.7

1. *First stage*: As circle has a point on the H.P., so assume that initially the circle is placed on the H.P. Draw the circle *abcdefghijkl* having centre 35 mm below *XY* line as the top view. Divide the circle in 12 equal parts and project all the points to *XY* and obtain the front view $a'g'$.
2. *Second stage*: Reproduce the front view of first stage such that $a'g'$ is inclined at 30º to the *XY* line. Obtain point *a*, *b*, *c*, etc. of the new top view by joining the points of intersection of the vertical projectors drawn from front view of the second stage (i.e. from a', b', c', d', etc.) with the horizontal locus lines drawn from the top view of the first stage (i.e. from *a*, *b*, *c*, *d*, etc.). Join new *abcdefghijkl* to get the final top view.

8.7 PLANE INCLINED TO V.P. AND PERPENDICULAR TO H.P.

When the surface of the plane is perpendicular to the H.P. and inclined at ϕ to the V.P., then the projections are drawn in two stages. In the first stage, it is assumed that the plane is parallel to V.P. or on the V.P. True shape of the plane is seen in the front view and a straight line in the top view. In the second stage, the surface of the plane is inclined at ϕ to V.P.

If the plane has a side parallel to the V.P. or in the V.P. then in the first stage the front view should have an edge perpendicular to the *XY* line. If the plane has a corner in the V.P., then in the first stage the line joining the front view of one of the corners with the front view of the centre of the plane should be parallel to the *XY* line.

Example 8.7 (Fig. 8.8)

A hexagonal plate of 25 mm side and negligible thickness has one of its edges in the V.P. The surface of the plate is perpendicular to the H.P. and inclined at 45º to the V.P. Draw its projections.

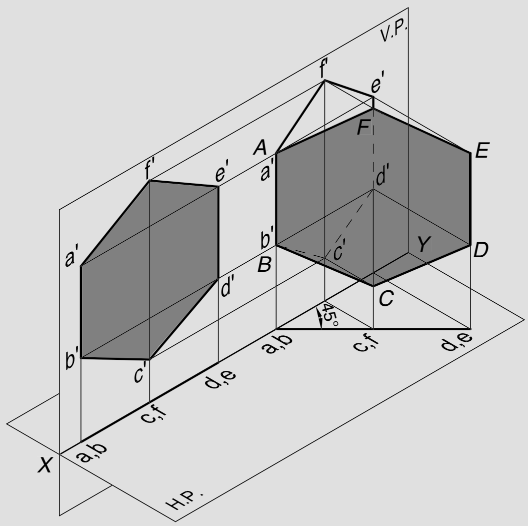


Fig. 8.8 (a)

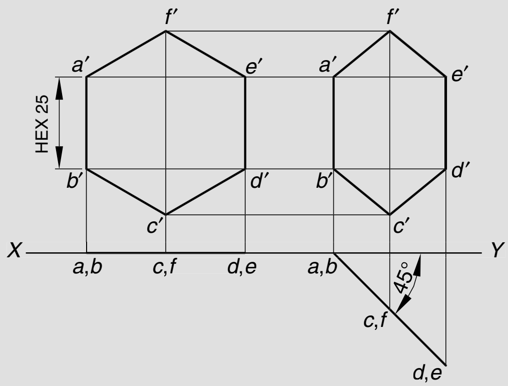


Fig. 8.8 (b)

Construction: Fig. 8.8(b)

1. *First stage*: The plane has one of the sides in the V.P., so assume that the plane *ABCDEF* is placed in the V.P. keeping *AB* perpendicular to H.P. Draw the hexagon *a′b′c′d′e′f′* as the front view with *a′b′* perpendicular to *XY*. Project all the corners to *XY* and obtain the top view *ad*.
2. *Second stage*: Reproduce the top view of first stage such that *ad* makes 45º to *XY*. Obtain point *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the front view by joining the points of intersection of the vertical projectors drawn from the top view of the second stage (i.e. from *a*, *b*, *c*, *d*, *e* and *f*) with the corresponding horizontal locus lines drawn from the front view of the first stage (i.e. from *a′*, *b′*, *c′*, *d′*, *e′* and *f′*). Join *a′b′c′d′e′f′* to get the final front view.

Example 8.8 (Fig. 8.9)

A circular plate of 50 mm diameter is held such that its plane is perpendicular to H.P. and inclined at 30º to V.P. with its centre 30 mm above the H.P. and 20 mm in front of V.P. Draw its projections.

[RGPV Feb. 2007]

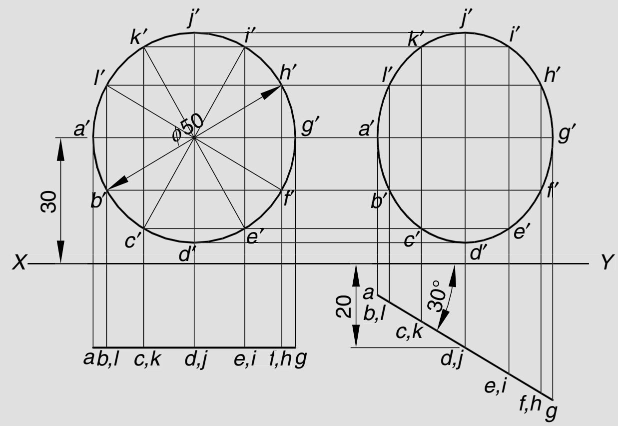


Fig. 8.9

Construction: Fig. 8.9

1. *First stage*: As the circle has a point on the V.P., so assume that initially the circle is placed in the V.P. Draw the circle *a′b′c′d′e′f′g′h′i′j′k′l′* having centre 30 mm above *XY* as the front view. Divide the circle in 12 equal parts and project all the points to *XY* to obtain the top view *ag*.
2. *Second stage*: Reproduce the top view of first stage such that *ag* is inclined at 30º to the *XY* line and its centre is 20 mm below *XY* line. Obtain point *a′*, *b′*, *c′*, etc., of the new front view by joining the points of intersection of the vertical projectors drawn from top view of the second stage (i.e. from *a*, *b*, *c*, *d*, etc.) with the horizontal locus lines drawn from the front view of the first stage (i.e. from *a′*, *b′*, *c′*, *d′*, etc.). Join new *a′b′c′d′e′f′g′h′i′j′k′l′* to get the final front view.

8.8 TRACE OF A PLANE

A plane which is not parallel to the reference plane will meet with it in a line (extended if necessary). This line is called trace of the plane. When the plane meets the H.P. then that line is called the *horizontal trace* or H.T. Similarly, when the plane meets the V.P. then that line is called the *vertical trace* or V.T.

8.8.1 Plane Parallel to H.P. and Perpendicular to V.P.

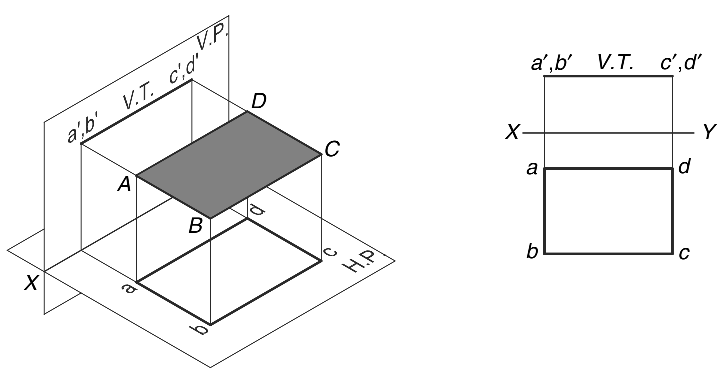


Fig. 8.10

In Fig. 8.10, the plane *ABCD* is parallel to H.P.

1. On extending the plane it does not intersect the H.P., therefore it has no H.T.
2. On extending the plane it intersects the V.P. on line *a′d′* (front view). Therefore, the line *a′d′* represents the V.T. of the plane.

8.8.2 Plane Parallel to V.P. and Perpendicular to H.P.

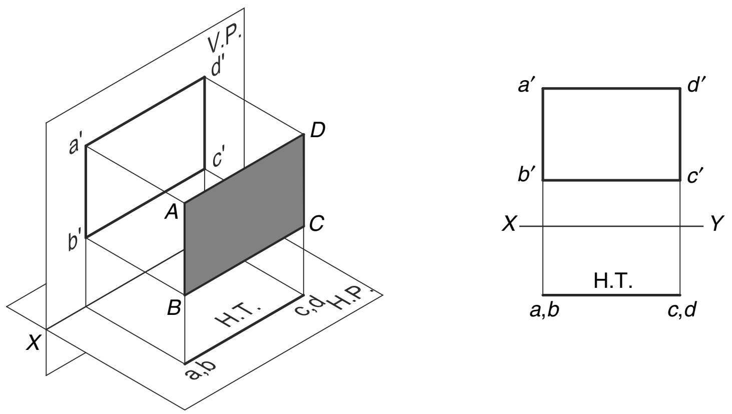


Fig. 8.11

In Fig. 8.11, the plane *ABCD* is parallel to V.P.

1. On extending the plane, it does not intersect the V.P., therefore it has no V.T.
2. On extending the plane, it intersects the H.P. on the line *ad* (top view). Therefore, the line *ad* represents the H.T. of the plane.

8.8.3 Plane Perpendicular to Both H.P. and V.P.

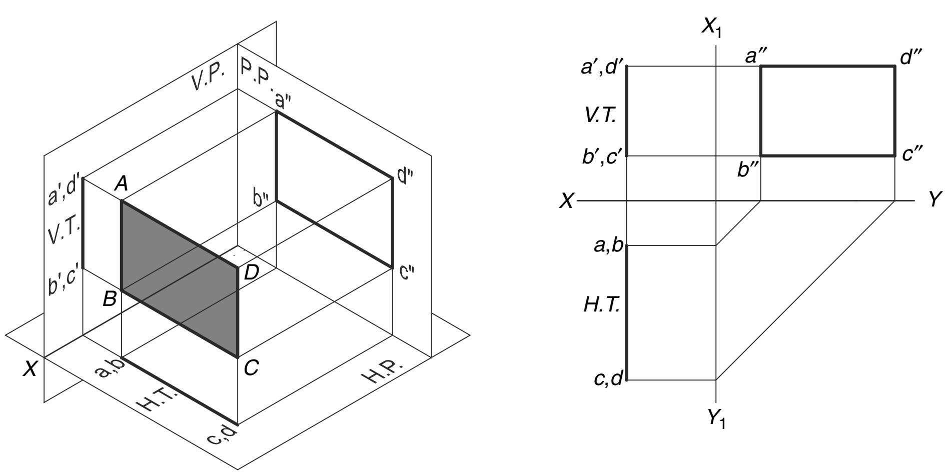


Fig. 8.12

In Fig. 8.12, the plane *ABCD* is parallel to the profile plane.

1. On extending the plane, it intersects the V.P. on the line $a'b'$. Therefore, the line $a'b'$ represents the V.T. of the plane.
2. On extending the plane it intersect the H.P. on the line *ad*. Therefore, the line *ad* represents the H.T. of the plane.

8.8.4 Plane Inclined to H.P. and Perpendicular to V.P.

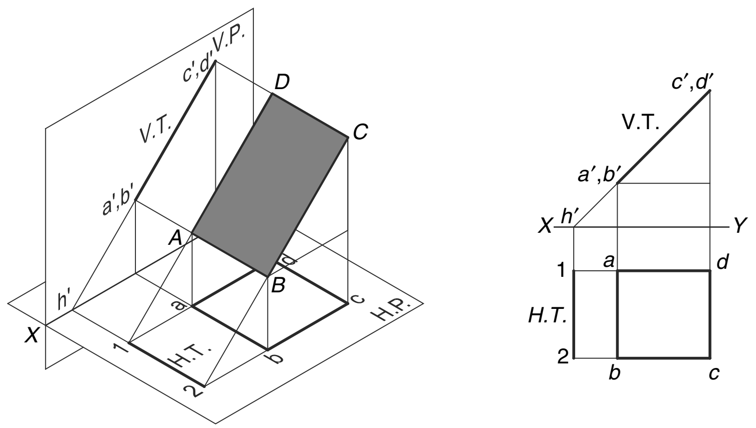


Fig. 8.13

In Fig. 8.13, the plane *ABCD* is perpendicular to the V.P. and inclined to the H.P.

1. On extending the plane it intersect the V.P. on line $a'd'$. Therefore, the line $a'd'$ represents the V.T. of the plane.
2. On extending the plane it intersect the H.P. on line 1-2. Therefore, the line 1-2 represents the H.T. of the plane. To draw the H.T, (a) extend the front view to meet *XY* line at point h' and (b) draw the projector from h' to meet horizontal lines from the front view at 1-2.

8.8.5 Plane Inclined to V.P. and Perpendicular to H.P.

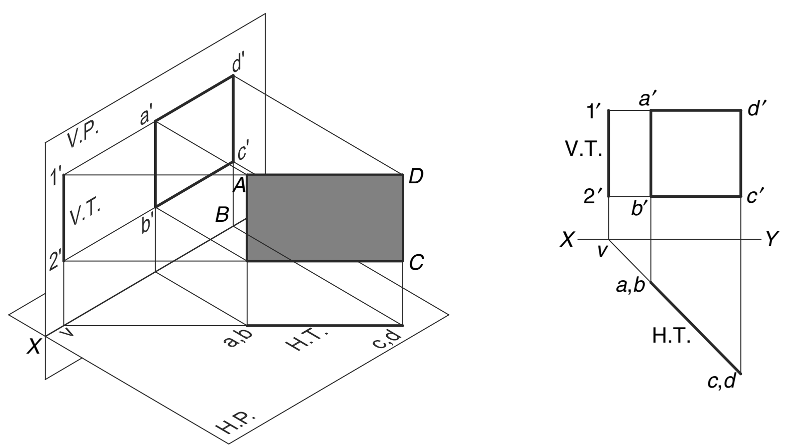


Fig. 8.14

In Fig. 8.14, the plane *ABCD* is perpendicular to the H.P. and inclined to the V.P.

1. On extending the plane it intersect the H.P. on line *ad*. Therefore, the line *ad* represents the H.T. of the plane.
2. On extending the plane it intersect the V.P. on line 1′-2′. Therefore, the line 1′-2′ represents the V.T. of the plane. To draw the V.T., (a) extend the top view to meet *XY* line at point *v*, and (b) draw the projector from *v* to meet horizontal lines from the front view at 1′-2′.

8.9 MISCELLANEOUS EXAMPLES

Example 8.9 (Fig. 8.15)

A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45º to the H.P. and perpendicular to the V.P. Draw its projections and show its traces.

[RGPV Dec. 2006, June 2009]

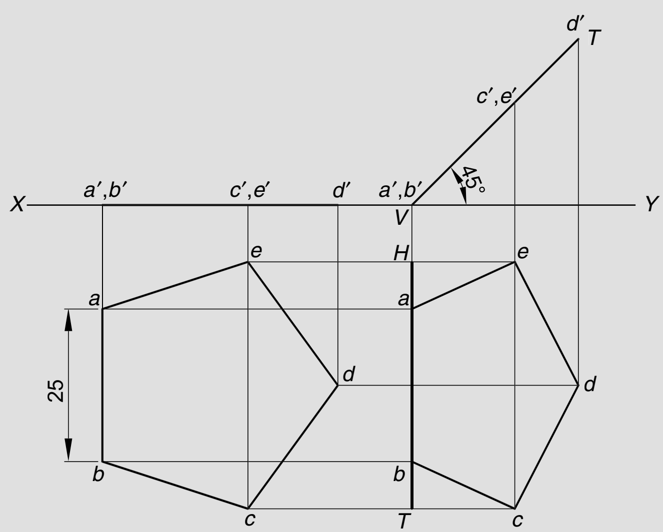


Fig. 8.15

Construction: Fig. 8.15

1. *First Stage*: Draw a pentagon *abcde* as the top view keeping *ab* perpendicular to *XY*. Project all the corners and obtain line *a′d′* on *XY* as the front view.
2. *Second stage*: Reproduce the front view of first stage, such that *a′d′* is inclined at 45º to *XY*. Obtain new points *a*, *b*, *c*, *d* and *e* of the top view by joining the points of intersection of the vertical projectors drawn from the front view of the second stage with the corresponding horizontal locus lines drawn from the top view of the first stage. Join new *abcde* to represent the final top view.
3. The H.T. and V.T. are marked on the final views.

Example 8.10 (Fig. 8.16)

A regular hexagonal lamina of 40 mm sides has a square hole of 25 mm side centrally cut through it. Draw the projections when it is resting on one of its sides on the H.P. with its surfaces inclined at 60º to the V.P. and its corner nearest to the V.P. is 24 mm from the V.P.

[RGPV June 2008]

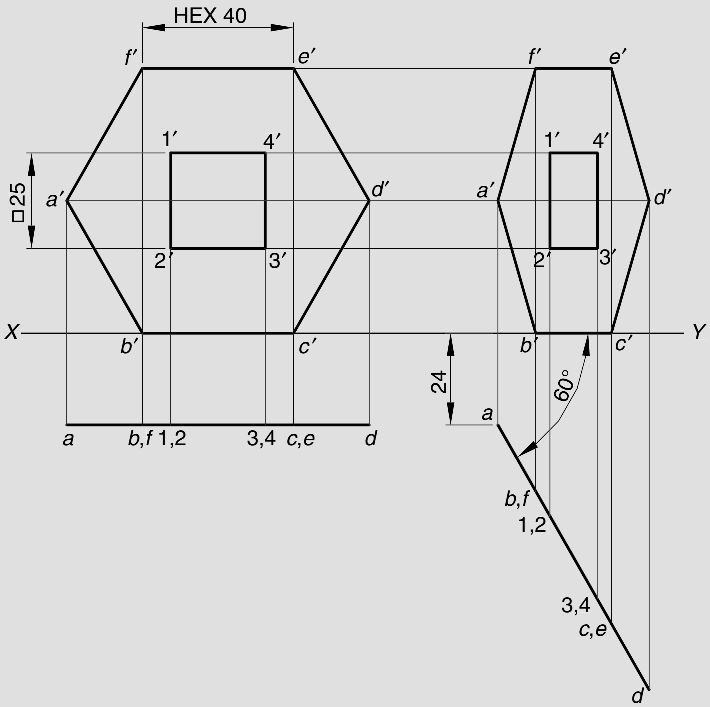


Fig. 8.16

Construction: Fig. 8.16

1. *First stage*: Draw a hexagon *a′b′c′d′e′f′* keeping *b′c′* on *XY*. Draw a rectangle 1′2′3′4′ at the centre of the hexagon. Project all the corners of the hexagon and the square to get the top view *ad* 24 mm below *XY*.
2. *Second stage*: Reproduce this top view *ad* inclined at 60º to the *XY* keeping its end a 24 mm below *XY*. Obtain point *a′*, *b′*, *c′*, *d′*, *e′*, *f′*, 1′, 2′, 3′ and 4′ of the front view by joining the points of intersection of the vertical projectors drawn from the top view of the second stage with the corresponding horizontal locus lines drawn from the front view of the first stage. Join *a′b′c′d′e′f′* and 1′2′3′4′, to represent the final front view.

Example 8.11 (Fig. 8.17)

A pentagon of 30 mm side has one corner on H.P. Its plane is inclined at 60º to V.P. and perpendicular to H.P. Draw the projection of the pentagon. [RGPV Aug. 2010]

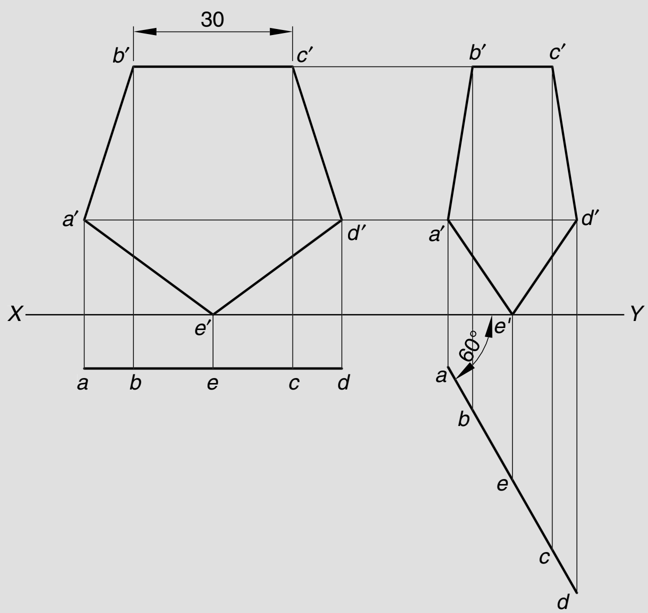


Fig. 8.17

Construction: Fig. 8.17

1. *First stage*: Draw a pentagon *a′b′c′d′e′* keeping the corner *e′* is on the *XY* line. Project all the corners to obtain the top view *ad*.
2. *Second stage*: Reproduce this top view *ad* at 60º to the *XY*. Obtain point *a′*, *b′*, *c′*, *d′* and *e′* of the front view by joining the points of intersection of the vertical projectors drawn from the top view of the second stage with the corresponding horizontal locus lines drawn from the front view of the first stage. Join *a′b′c′d′e′* to get the final front view.

Example 8.12 (Fig. 8.18)

The top view of a lamina whose surface is perpendicular to V.P. and inclined at an angle of 45º to H.P. appears as a regular hexagon of 30 mm side, having a side parallel to the reference line. Draw the projections of plane and obtain its true shape. [RGPV Dec. 2008]

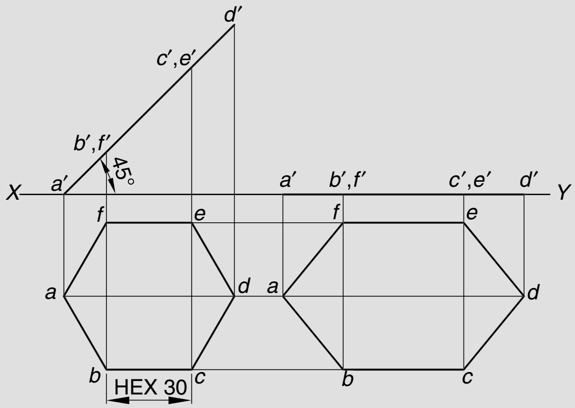


Fig. 8.18

Construction: Fig. 8.18

1. *First Stage*: Draw a hexagon *abcdef* as the top view keeping *bc* parallel to *XY*.
2. Project point *a* to meet *XY* line at point *a*′. Draw a line from point *a*′ inclined at 45º to the *XY* line. Project point *d* to meet the inclined line at point *d*′. The line *a*′*d*′ represents the front view.
3. Project all other points of the top view to meet *a*′*d*′.
4. *Second stage*: Reproduce the front view *a*′*d*′ on the *XY* line. Obtain new points *a*, *b*, *c*, *d*, *e* and *f* of the top view by joining the points of intersection of the vertical projectors drawn from the front view of the second stage with the corresponding horizontal locus lines drawn from the top view of the first stage. Join new *abcdef* to obtain the required true shape.

Example 8.13 (Fig. 8.19)

Determine the true shape of figures, the top view of which is a regular pentagon of 35 mm side, having one side inclined at 30º to the V.P. and whose front view is a straight line making an angle of 45º to the H.P. [RGPV Sep. 2009]

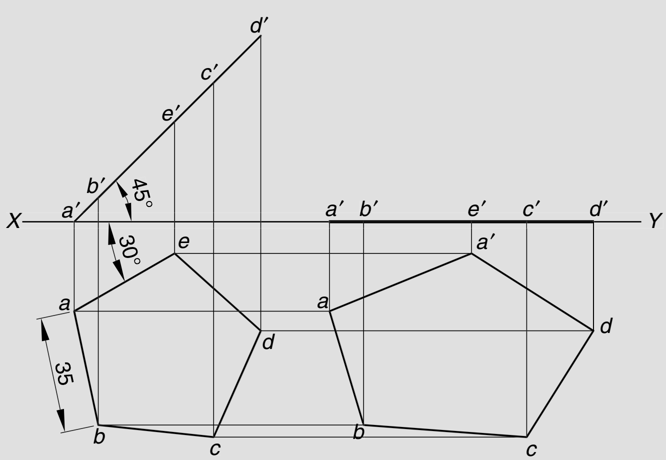


Fig. 8.19

Construction: Fig. 8.19

1. *First Stage*: Draw a pentagon *abcde* as the top view, such that side *ae* is inclined at 30º to the *XY* line.
2. Project point *a* to meet *XY* line at point *a*′. Draw a line from point *a*′ inclined at 45º to *XY*. Project point *d* to meet this inclined line to get *d*′. The line *a*′*d*′ represents the front view.
3. Project all other points of the top view to meet *a*′*d*′.
4. *Second stage*: Reproduce *a*′*d*′ on *XY*. Obtain new points *a*, *b*, *c*, *d* and *e* of the top view by joining the points of intersection of the vertical projectors drawn from the front view of the second stage with the corresponding horizontal locus lines drawn from the top view of the first stage. Join new *abcde* to represent the true shape of the plane.

EXERCISE 8A

Surface Parallel or Perpendicular to the Reference Planes

1. A hexagonal lamina with 30 mm sides has one of the sides perpendicular to V.P. The surface of lamina is parallel to and 15 mm above H.P. Draw its projections.
2. A thin hexagonal plate with 30 mm sides has one of the sides inclined at 45º to the V.P. Its V.T. is parallel to and 25 mm above *XY* and the H.T. does not exist. Draw its projections.
3. A circular plane with 60 mm diameter has its centre 20 mm above the H.P. and 30 mm in front of the V.P. The surface of the plane is parallel to the H.P. Draw its projections.
4. A composite plate with negligible thickness is made up of a rectangle (60 mm and 40 mm long sides) and a semicircle on its longer side. The plate lies in the H.P. with one of its shorter sides parallel to V.P. Draw its projections.
5. A square lamina with 40 mm sides has its surface parallel to and 30 mm in front of the V.P. Draw the projections when one of its sides is inclined at 30º to the H.P.
6. A square plane with 40 mm sides is situated in the V.P. with all the sides equally inclined to H.P. Draw its projections.
7. A pentagonal plane with 35 mm sides has its corner on the H.P. and the side opposite to this corner is parallel to the H.P. The plane is parallel to and 20 mm in front of the V.P. Draw its projections and locate its traces.
8. A rectangular plane with 50 mm and 30 mm sides is perpendicular to both H.P. and V.P. The longer edges are parallel to the H.P. and nearest one is 20 mm above it. The shorter edge nearer to V.P. is 15 mm from it. Draw its projections.
9. A square plate with 40 mm sides has one of its sides inclined at 30º to the H.P. The surface of the plate is perpendicular to both H.P. and V.P. Draw its projections and locate its traces.

Surface Inclined to One of the Reference Planes

10. A square lamina with 50 mm sides rests on the H.P. on one of its corners, such that the diagonal through that corner is parallel to V.P. and inclined at 30º to the H.P. Draw its projections when the lamina is perpendicular to the V.P. Measure the distance of the topmost corner from the H.P. [Ans: 35 mm]

11. A pentagonal plane with 30 mm sides is resting on one of its edges in the H.P. with its surface perpendicular to the V.P. The corner opposite to that edge is 40 mm above the H.P. Draw the projections of the plane and determine its inclination with the H.P.

[Ans: 60º]

12. A rhombus of 60 mm and 40 mm long major and minor diagonals respectively, is placed on one of the end points of the major diagonal on H.P. such that the minor diagonal is perpendicular to V.P. and the surface is inclined at 60º to the H.P. Draw its projections.
13. A regular hexagonal 30 mm lamina side rests on one of its sides on H.P. such that it is perpendicular to V.P. and inclined to the H.P. at 45º. Its nearest corner to V.P. is 15 mm away from V.P. Draw its projections.

[RGPV June 2007]

14. A pentagonal plane with 30 mm sides has a centrally punched circular hole of 24 mm diameter. The plane is placed on a side in the H.P. such that the surface is perpendicular to the V.P. and inclined at 45º to the H.P. Draw its projections when the centre of the plane is 35 mm in front of the V.P. Locate the traces of the plane.
15. A square plate of 40 mm side is perpendicular to H.P. and inclined to V.P. at 40º. One of its edge is on V.P. Draw the projections when one of the corners is 12 m from the H.P.

[RGPV Dec. 2010]

16. A regular hexagonal lamina side 20 mm rests on H.P. on one of its sides such that it is perpendicular to the H.P. and inclined to V.P. at 30º. Draw its projections when the corner nearest to the V.P. is 15 mm away from it.

[RGPV Feb. 2007]

17. Draw the projections of a circle 60 mm diameter resting on V.P. on a point on the circumference. The plane is inclined at 45º to V.P. and perpendicular to H.P. The centre of the plane is 30 mm above H.P.

[RGPV Feb. 2011]

18. Draw the projections of a circle of 60 mm diameter resting on V.P. on a point on the circumference. The plane is inclined at 45º to V.P. and perpendicular to the H.P. The centre of the plane is 40 mm above H.P. Mark H.T. of the plane also.

[RGPV June 2008(o), Feb. 2010]

19. A regular hexagonal thin plate of 45 mm side has a circular hole of 45 mm diameter in its centre. It is resting on one of its sides on H.P. Draw its projections when the plate surface is vertical and inclined at 30º to the V.P.

[RGPV April 2009]

Determine Inclination of the Plane

20. A rectangular plate with 60 mm and 40 mm long sides, rests on a shorter edge on the H.P. with its surface perpendicular to the V.P. such that centre of the plate lies 20 mm above the H.P. and 30 mm in front of the V.P. Draw the projections of the plate and determine angle made by it with the H.P.
21. A rhombus with 60 mm and 40 mm long diagonals has a corner in the V.P. The surface of the plane is perpendicular to H.P. and the front view appears as a square. Draw its projections and determine the inclination of the rhombus with the V.P.

22. A square plane with 100 mm long diagonals is placed perpendicular to the V.P., such that a corner touches the H.P. In its top view, the plane appears as a rhombus with 100 mm and 60 mm long diagonals. Draw the front view and determine the inclination of the plane with the H.P.
23. A square plane has one of its corners in the H.P. and its surface is perpendicular to the V.P. The top view of the plane appears as a rhombus with 70 mm and 40 mm long diagonals. Draw the projections of the plane and determine its inclination with the H.P.
24. A circular plate with a 50 mm diameter is resting on a point of the rim in the V.P. The plane is perpendicular to H.P. and the front view appears as an ellipse of the major and minor axes of 50 mm and 30 mm length respectively. Draw the front and top views of the plate and determine its inclination with the V.P.
25. A circular plane is placed perpendicular to the V.P. and it appears as an ellipse in the top view having 80 mm and 50 mm long major and minor axes respectively. Draw three views of the plane and determine its inclination with the H.P.

Determine True Shape of the Plane

26. The top view of a triangular plane appears as an equilateral triangle with 50 mm sides placed on a side perpendicular to the reference line. The front view of the plane appears as a 65 mm long line. Determine the true shape of the plane.
27. The front view of a plane whose surface is perpendicular to H.P. and inclined at 30º to the V.P. appears as a regular pentagon with 30 mm sides having a side parallel to the reference line. Draw the projections of the plane and determine its true shape.
28. The top view of a plane whose surface is perpendicular to the V.P. and inclined at 45º to the H.P. is a circle with a 60 mm diameter. Draw the projections of the plane and determine its true shape.
29. The front view of a plane is a straight line inclined at 30º with the reference line. The top view of the plane is a regular hexagon with 30 mm sides having an edge inclined at 45º with the reference line. Draw its projections and determine its true shape.

8.10 PLANE INCLINED TO BOTH THE REFERENCE PLANES

When the surface of a plane is inclined to both the reference planes, its projections are drawn in three stages. It is the extension of the examples discussed earlier in this chapter on projections of planes inclined to one of the reference planes.

8.10.1 Plane Placed on an Edge Parallel to the H.P. such that the Surface is Inclined to H.P. and that Edge is Inclined to the V.P.

Example 8.14 (Fig. 8.20)

Draw the projections of a regular hexagon of 25 mm side having one of its sides in the H.P. and inclined at 60º to the V.P. and its surface making an angle of 45º with the H.P.

[RGPV April 2010]

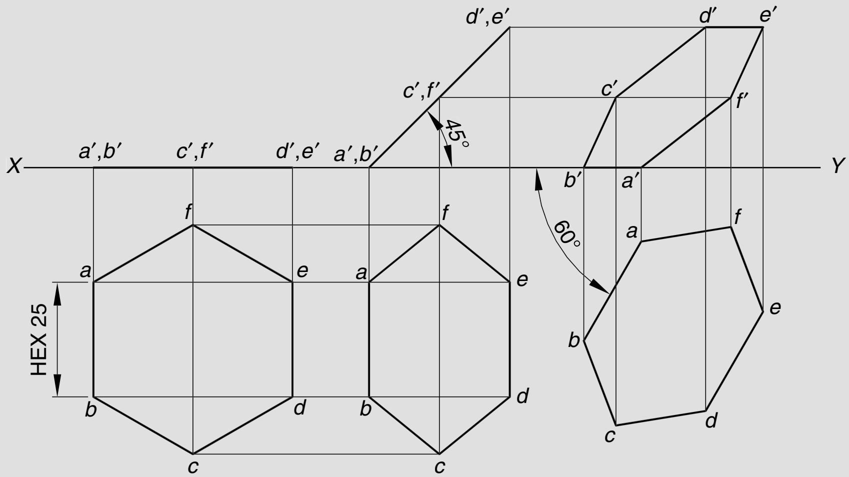


Fig. 8.20

Construction: Fig. 8.20

1. *First stage*: Draw a hexagon *abcdef* as the top view with *ab* perpendicular to *XY*. Project all the corners to *XY* and obtain *a′d′* as the front view.
2. *Second stage*: Reproduce the front view of first stage such that *a′* lies on the *XY* line and *a′d′* is inclined at 45º to *XY*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* of the top view by joining the points of intersection of the vertical projectors drawn from points *a′*, *b′*, *c′*, *d′*, e′ and *f′* of the second stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, *e* and *f* of first stage. Join *abcdef*.
3. *Third stage*: Reproduce the top view of the second stage such that the side *ab* is inclined at 30º to *XY*. Obtain point *a′*, *b′*, *c′*, *d′*, *e′* and *f′* for the front view by joining the points of intersection of vertical projectors drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the third stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage. Join *a′b′c′d′e′f′*. This stage represents the final projections.

8.10.2 Plane Rests on an Edge on the V.P. such that the Surface is Inclined to the V.P. and that Edge is Inclined to the H.P.

Example 8.15 (Fig. 8.21)

A hexagonal plane of 30 mm side, rests on the V.P. on an edge such that the surface is inclined at 45º to the V.P. and the edge on which it rests is inclined at 30º to the H.P. Draw its projections.

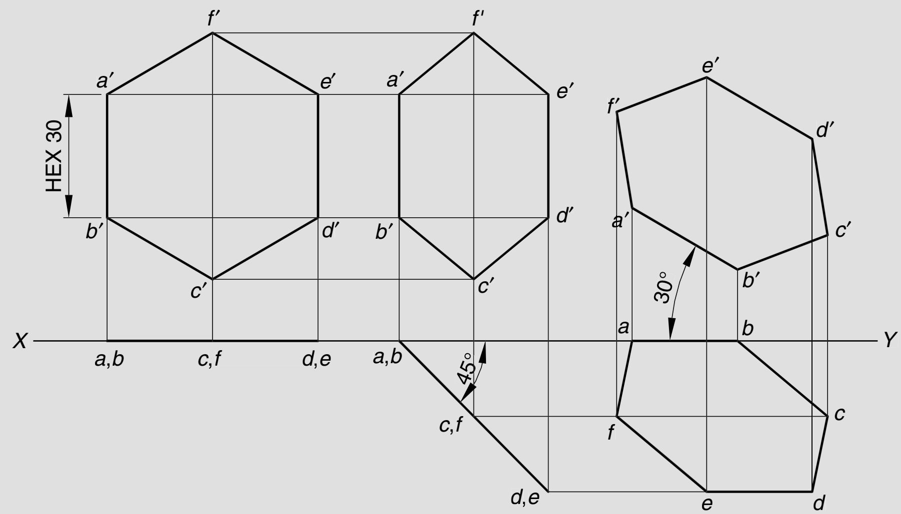


Fig. 8.21

Construction: Fig. 8.21

1. *First stage*: Draw a hexagon $a'b'c'd'e'f'$ as the front view with $a'b'$ perpendicular to *XY*. Project all the corners to *XY* and obtain *ad* as the top view.
2. *Second stage*: Reproduce the top view of first stage such that point a lies on the *XY* line and *ad* is inclined at 45° to *XY*. Obtain points a', b', c', d', e' and f' of the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage with the horizontal locus lines drawn from points a', b', c', d', e' and f' of first stage. Join $a'b'c'd'e'f'$.
3. *Third stage*: Reproduce the front view of the second stage such that the side $a'b'$ is inclined at 30° to *XY*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* for the top view by joining the points of intersection of vertical projectors drawn from points a', b', c', d', e' and f' of the third stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage. Join *abcdef*. This stage represents the final projections.

Example 8.16 (Fig. 8.22)

A semicircular plate of 80 mm diameter has its straight edge on the V.P. and inclined at 30° to the H.P., while the surface of the plate is inclined at 45° to the V.P. Draw the projection of the plate.

[RGPV Aug. 2010]

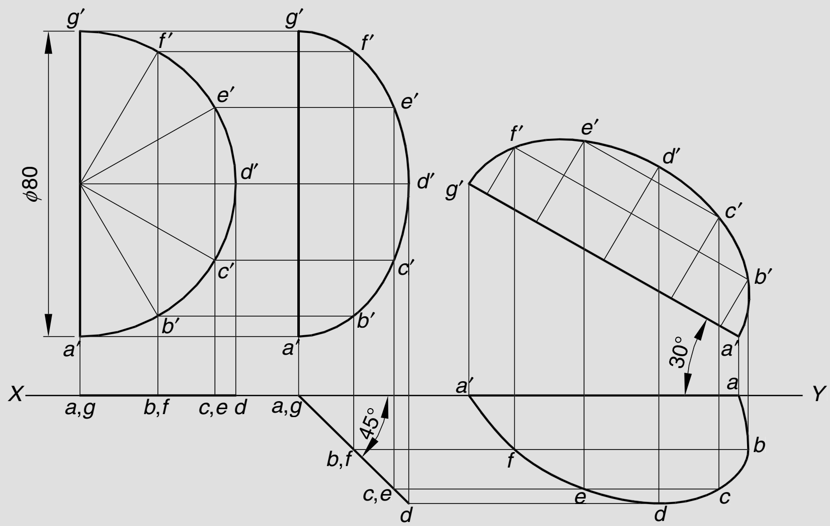


Fig. 8.22

Construction: Fig. 8.22

1. *First stage*: Draw a semicircle *a′b′c′d′e′f′g′* as the front view with *a′g′* perpendicular to the *XY* line. Divide the semicircle in 6 equal parts and project all these points to *XY* and obtain *ad* as the top view.
2. *Second stage*: Reproduce the top view of first stage such that point *a* lies on *XY* and *ad* is inclined at 45º to *XY*. Obtain points *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *g′* of the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c*, *d*, *e*, *f* and *g* of the second stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *g′* of first stage. Join *a′b′c′d′e′f′g′*.
3. *Third stage*: Reproduce the front view of the second stage such that the side *a′g′* is inclined at 30º to *XY*. Obtain points *a*, *b*, *c*, *d*, *e*, *f* and *g* for the top view by joining the points of intersection of vertical projectors drawn from points *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *g′* of the third stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, *e*, *f* and *g* of the second stage. Join *abcdefg*. This stage represents the final projections.

Example 8.17 (Fig. 8.23)

A thin 30–60 degree set-square has its longest edge in the V.P. and inclined at 30º to the H.P. Its surface makes an angle of 45º with the V.P. Draw its projections.

[RGPV June 2006, June 2007, June 2008]

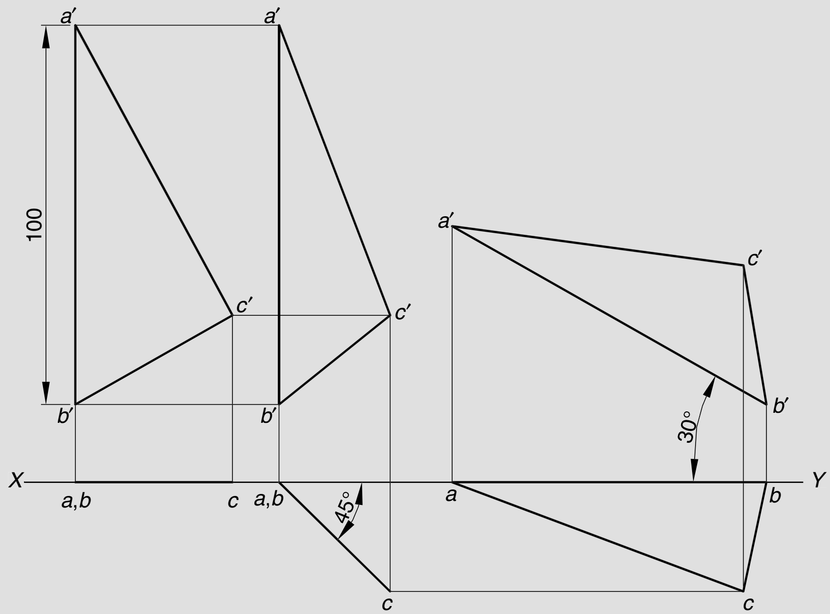


Fig. 8.23

Construction: Fig. 8.23

1. *Assumption*: Length of the sides of the set-square is not given, so assume a suitable length for the longest edge (say *AB* = 100 mm).
2. *First stage*: Draw a right-angled triangle *a′b′c′* in which angle *a′b′c′* = 60º and angle *b′a′c′* = 30º keeping *a′b′* and perpendicular to *XY,* to represent the front view of the set-square. Project all the corners to *XY* and obtain *ac* as the top view.
3. *Second stage*: Reproduce the top view of first stage such that point *a* lies on the *XY* line and *ac* is inclined at 45º to *XY.* Obtain points *a′*, *b′* and *c′* of the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b* and *c* of the second stage with the horizontal locus lines drawn from points *a′*, *b′* and *c′* of first stage. Join *a′b′c′*.
4. *Third stage*: Reproduce the front view of the second stage such that the side *a′b′* is inclined at 30º to *XY.* Obtain points *a*, *b* and *c* for the top view by joining the points of intersection of vertical projectors drawn from points *a′*, *b′* and *c′* of the third stage with the horizontal locus lines drawn from points *a*, *b* and *c* of the second stage. Join *abc*. This stage represents the final projections.

8.10.3 Plane Rests on a Corner (or an End of Diameter or Diagonal) on the H.P. such that its Surface is Inclined to H.P. and an Edge or Line which is Parallel to H.P. is Inclined to V.P.

Example 8.18 (Fig. 8.24)

A regular hexagon of 30 mm side has a corner in the H.P. Its surface is inclined at 45º to the H.P. and the top view of the diagonal through the corner which is in the H.P. makes an angle of 60º with the V.P. Draw its projections. [RGPV Feb. 2006]

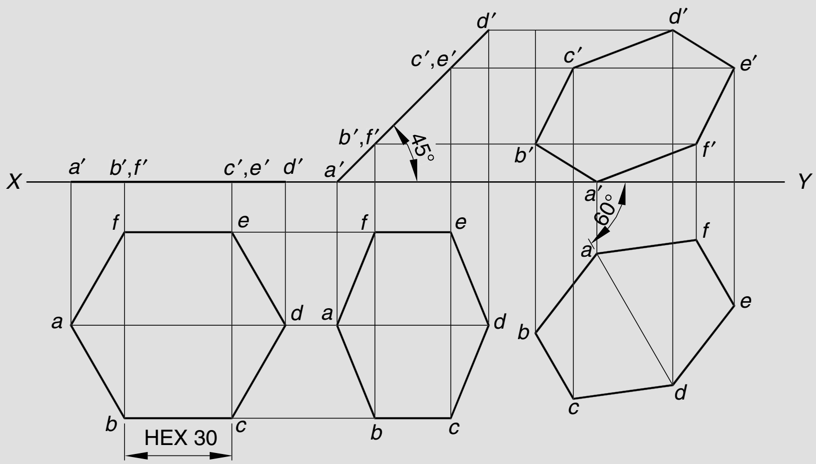


Fig. 8.24

Construction: Fig. 8.24

1. *First stage*: Draw a hexagon *abcdef* as the top view with *ad* parallel to *XY*. Project all the corners to *XY* and obtain *a′d′* as the front view.
2. *Second stage*: Reproduce the front view of first stage such that *a′* lies on *XY* and *a′d′* is inclined at 45º to *XY*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* of the top view by joining the points of intersection of the vertical projectors drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, *e* and *f* of first stage. Join *abcdef*.
3. *Third stage*: Reproduce the top view of the second stage such that the line *ad* is inclined at 30º to *XY*. Obtain point *a′*, *b′*, *c′*, *d′*, *e′* and *f′* for the front view by joining the points of intersection of vertical projectors drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the third stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage. Join *a′b′c′d′e′f′*. This stage represents the final projections.

Example 8.19 (Fig. 8.25)

A pentagonal lamina of 30 mm side rests on the H.P. on one of its corners with its surface inclined at 30º to the H.P. Draw its projections when the side opposite to the corner in the H.P. is parallel to the V.P. [RGPV Dec. 2008]

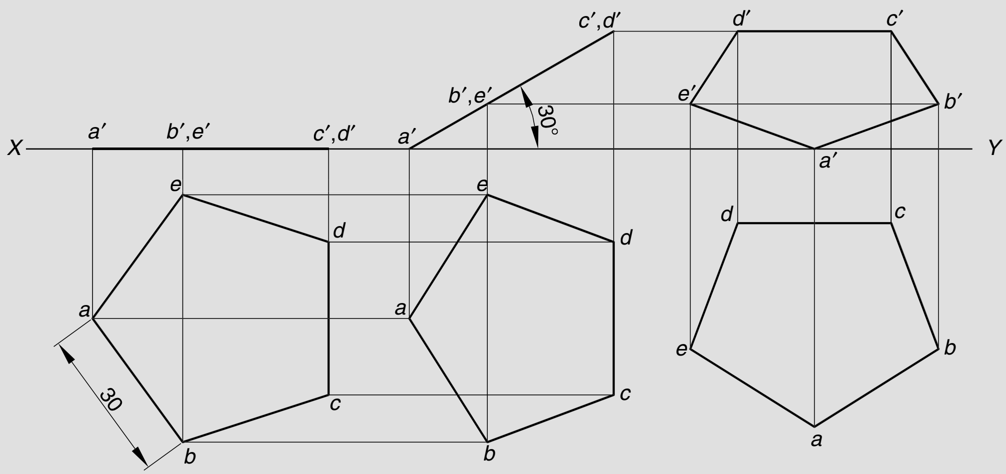


Fig. 8.25

Construction: Fig. 8.25

1. *First stage*: Draw a pentagon *abcde* such that *cd* is perpendicular to *XY* as the top view. Project all the corners to *XY* and obtain *a′c′* as the front view.
2. *Second stage*: Reproduce the front view of first stage such that *a′* lies on *XY* and *a′c′* is inclined at 30º to *XY*. Obtain points *a*, *b*, *c*, *d* and *e* of the top view by joining the points of intersection of the vertical projectors drawn from points *a′*, *b′*, *c′*, *d′* and *e′* of the second stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d* and *e* of first stage. Join *abcde*.
3. *Third stage*: Reproduce the top view of the second stage such that the side *dc* is parallel to *XY*. Obtain points *a′*, *b′*, *c′*, *d′* and *e′* for the front view by joining the points of intersection of vertical projectors drawn from points *a*, *b*, *c*, *d* and *e* of the third stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′* and *e′* of the second stage. Join *a′b′c′d′e′*. This stage represents the final projections.

8.10.4 Plane Rests on a Corner (or an end of Diameter or Diagonal) on the V.P. such that its Surface is Inclined to V.P. and an Edge or a Line which is Parallel to H.P. is Inclined to H.P.

Example 8.20 (Fig. 8.26)

A hexagonal plane figure of 30 mm side is resting on a corner in the V.P. with its surface making an angle of 30º with the V.P. The view from the front of the diagonal passing through that corner is inclined at 35º to the H.P. Draw the three principal views.

[RGPV Sep. 2009]

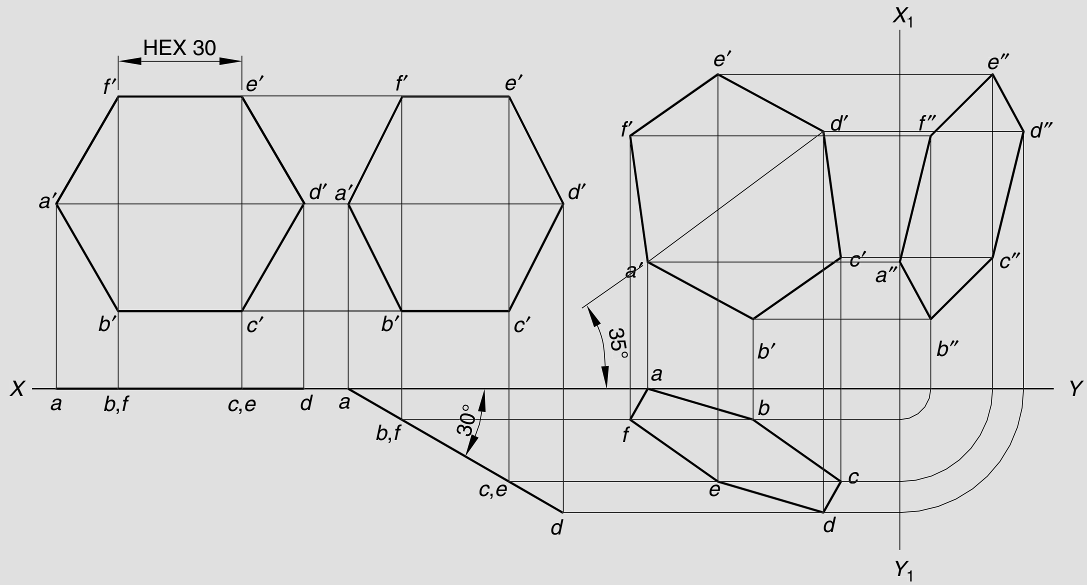


Fig. 8.26

Construction: Fig. 8.26

1. *First stage*: Draw a hexagon *a′b′c′d′e′f′* as the front view with diagonal *a′d′* parallel to *XY*. Project all the corners to *XY* and obtain *ad* as the top view.
2. *Second stage*: Reproduce the top view of first stage such that point *a* lies on *XY* and *ad* is inclined at 30º to *XY*. Obtain points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of first stage. Join *a′b′c′d′e′f′*.
3. *Third stage*: Reproduce the front view of the second stage such that *a′d′* is inclined at 35º to *XY*. Obtain point *a*, *b*, *c*, *d*, *e* and *f* for the top view by joining the points of intersection of vertical projectors drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the third stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the second stage. Join *abcdef*. This stage represents the final projections.
4. *Side view*: Draw a reference line X_1Y_1 perpendicular to *XY*. Project *a*, *b*, *c*, *d*, *e* and *f* of the top view of the third stage up to the line X_1Y_1 and then rotate them through 90º. Now project them vertically to meet the corresponding locus lines drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the front view of the third stage at points *a″*, *b″*, *c″*, *d″*, *e″* and *f″*. Join *a″b″c″d″e″f″*.

8.10.5 Plane Inclined to H.P. and an Edge or a Diagonal Already Inclined to H.P. is also Inclined to the V.P.

Example 8.21 (Fig. 8.27)

A hexagonal plane of 30 mm side has its corner *A* in the H.P. The surface of the plane is inclined at 45º to the H.P. and the diagonal containing corner *A* is inclined at 30º to the V.P. Draw its projections.

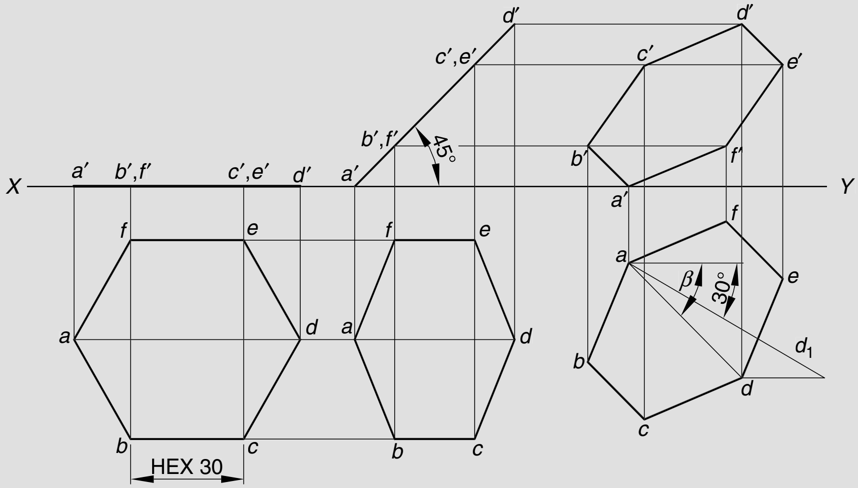


Fig. 8.27

Construction: Fig. 8.27

1. *First stage*: Draw a hexagon *abcdef* such that line *ad* is parallel to *XY* as the top view. Project all the corners to *XY* and obtain *a′d′* as the front view.
2. *Second stage*: Reproduce the front view of first stage such that a′ lies on *XY* and *a′d′* is inclined at 45º to *XY*. Obtain points *a*, *b*, *c*, *d*, *e* and *f* of the top view by joining the points of intersection of the vertical projectors drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, *e* and *f* of first stage. Join *abcdef*.
3. *Third stage*: As the diagonal *ad* is already inclined to H.P. and we have to be turn it further at 30º to the V.P., we have to determine the apparent angle β. Therefore, Draw a line ad_1 equal to true length of the diagonal (*ad* of first stage) at 30º to *XY* line. Draw an arc with *a* as the centre and radius equal to projected length of the diagonal (*ad* of second stage), to meet horizontal locus line from d_1 at point *d*. Join *ad*. The line *ad* is inclined at β angle with *XY*.
4. Reproduce the top view of the second stage such that the line *ad* is inclined at β angle to *XY*. Obtain point *a′*, *b′*, *c′*, *d′*, *e′* and *f′* for the front view by joining the points of intersection of vertical projectors drawn from points *a*, *b*, *c*, *d*, *e* and *f* of the third stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′*, *e′* and *f′* of the second stage. Join *a′b′c′d′e′f′*. This stage represents the final projections.

8.10.6 Plane Inclined (at ϕ) to V.P. and an Edge or a Diagonal Already Inclined to V.P. is also Inclined (at θ) to the H.P.

Example 8.22 (Fig. 8.28)

A rectangular plate of size 70 mm × 40 mm rests on its shorter side in the V.P. and the surface makes 45º with the V.P. The longer side of the plane is inclined at 30º to the H.P. Draw its projections.

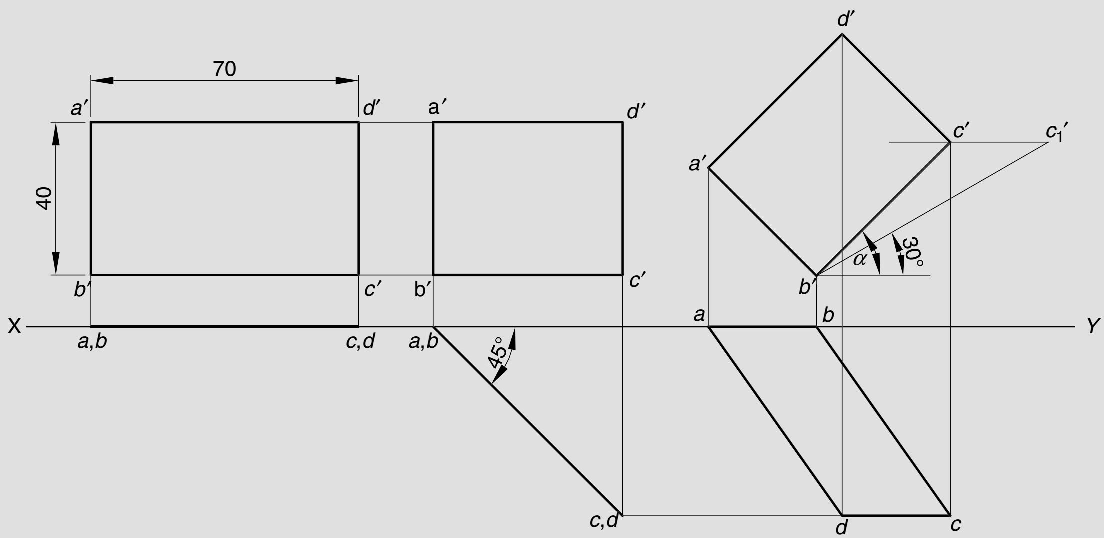


Fig. 8.28

Construction: Fig. 8.28

1. *First stage*: Draw a rectangle $a'b'c'd'$ such that shorter edge $a'b'$ is perpendicular to *XY* line as the front view. Project all the corners to *XY* and obtain *ac* as the top view.
2. *Second stage*: Reproduce the top view of first stage such that a lies on *XY* and *ac* is inclined at 45º to *XY*. Obtain point a', b', c' and d' of the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c* and *d* of the second stage with the horizontal locus lines drawn from points a', b', c' and d' of the first stage. Join $a'b'c'd'$.
3. *Third stage*: As the longer side *BC* is already inclined to V.P. and we have to turn it further at 30º to the H.P., we have to determine the apparent angle α. Therefore, Draw a line $b'c_1'$ equal to true length (70 mm) and inclined at 30º to *XY*. Draw an arc with centre b' and radius equal to projected length of longer edge ($b'c'$ of second stage), to intersect the horizontal line from c_1' at point c'. The line $b'c'$ is now inclined at α angle to *XY*.
4. Reproduce the front view of the second stage such that the side $b'c'$ is inclined at α angle to *XY*. Obtain point *a*, *b*, *c* and *d* for the top view by joining the points of intersection of vertical projectors drawn from points a', b', c' and d' of the third stage with the horizontal locus lines drawn from points *a*, *b*, *c* and *d* of the second stage. Join *abcd*. This stage represents the final projections.

8.11 PLANE INCLINED θ TO H.P. AND ϕ TO V.P. SUCH THAT $\theta + \phi = 90°$

Example 8.23 (Fig. 8.29 and Fig. 8.30)

A circular plane of 80 mm diameter has one of the ends of the diameter in the H.P. while the other end is in the V.P. The plane is inclined at 30° to the H.P. and 60° to the V.P. Draw its projections.

8.11.1 Using Change of Position Method

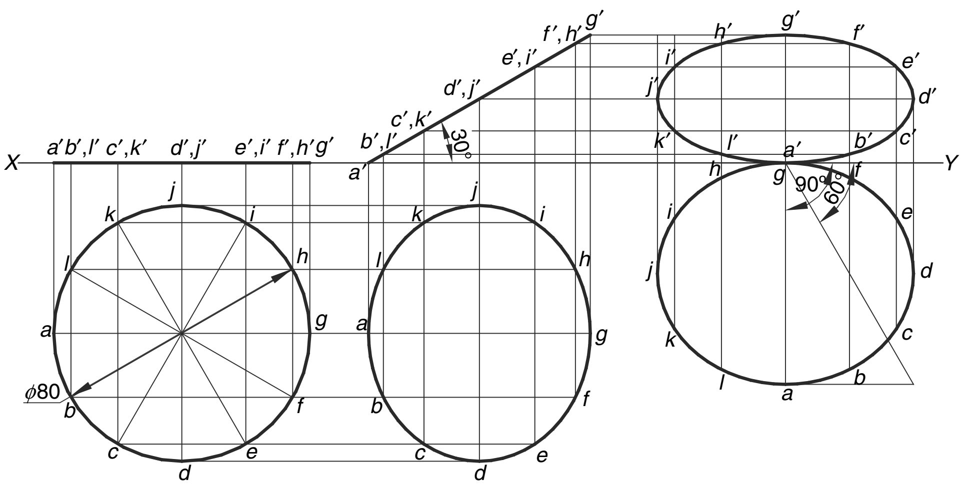


Fig. 8.29

Construction: Fig. 8.29.

1. *First stage*: Draw a circle *adgj* as the top view. Divide the circumference of the circle into 12 equal parts. Project all the points to *XY* and obtain *a′g′* as the front view.
2. *Second stage*: Reproduce the front view of first stage such that point *a′* lies on *XY* and line *a′g′* is inclined at 30° to *XY*. Obtain points *a*, *b*, *c*, *d*, etc. of the top view by joining the points of intersection of the vertical projectors from points *a′*, *b′*, *c′*, *d′* etc. from the second stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, etc. drawn from the first stage. Join points *a*, *b*, *c*, *d*, etc. using a smooth curve.
3. *Third stage*: We know that if a line / diameter is inclined to HP and VP both such that sum of their inclination is 90° ($\theta + \phi = 90°$) then the apparent angle $\beta = 90°$. Therefore, reproduce the top view of the second stage such that point *g* touches the *XY* and *ag* is inclined at 90° to the *XY*. Obtain points *a′*, *b′*, *c′*, *d′*, etc. for the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c*, *d*, etc., of the third stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′*, etc., of the second stage. Join *a′b′c′d′e′f′g′h′j′k′l′* using a smooth curve.

8.11.2 Using Auxiliary Plane Method

Note: This procedure is valid only when $\theta + \varnothing = 90^\circ$.

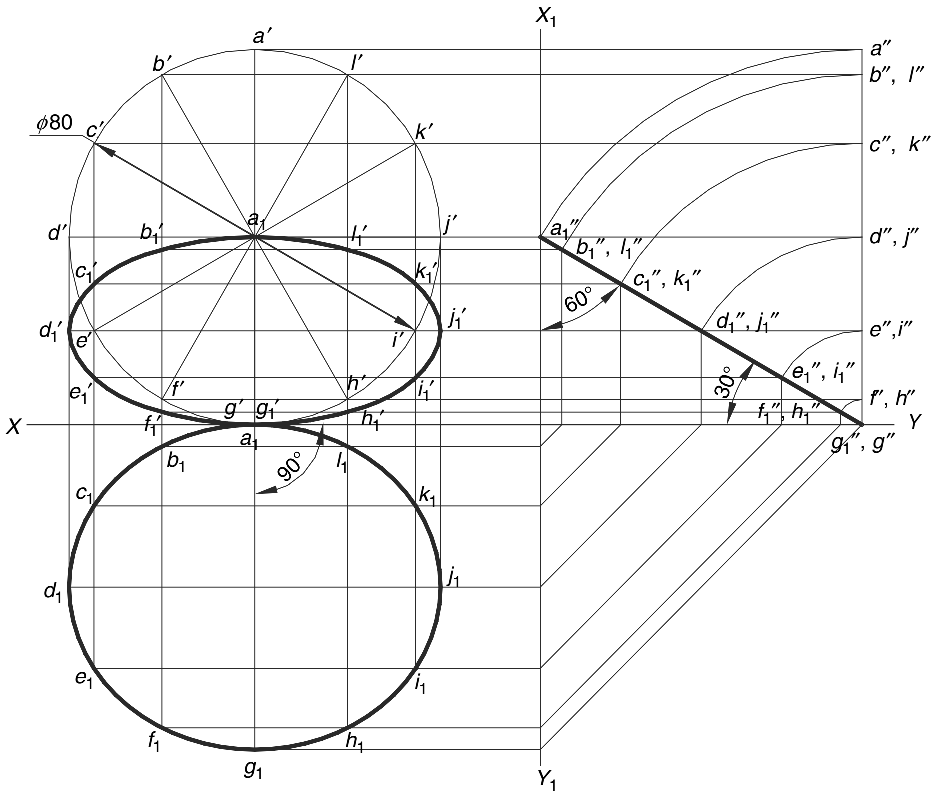


Fig. 8.30

Construction: Fig. 8.30.

1. Draw a reference lines *XY*. Consider the circular plane is parallel to VP.
2. Draw a circle $a'd'g'j'$ as the front view. Divide the circle into 12 equal parts. Divide the circumference of the circle into 12 equal parts.
3. Project all the points horizontally to an arbitrary distance to obtain vertical line $a''g''$ as the side view.
4. Now the circular plane is turned such that it is inclined at 30º to the H.P. and 60º to the V.P. Therefore, reproduce the side view such with that g_1'' is on *XY* and line $a_1''g_1''$ is inclined at 30º to the *XY*. This is the final side view.
5. Draw another reference line X_1Y_1 passing through a_1'' and perpendicular to *XY*.
6. Obtain points a_1', b_1', c_1', d_1', etc., as the final front view by joining the points of intersection of the vertical projectors from points a', b', c', d', etc., of the front view with the horizontal locus lines drawn from points a_1'', b_1'', c_1'', d_1'', etc ,of the final side view. Join a_1', b_1', c_1', d_1' etc., with a smooth curve. This is the final front view.

7. Obtain points a_1, b_1, c_1, d_1, etc., of the top view by joining the points of intersection of the vertical projectors from points a_1', b_1', c_1', d_1', etc., with the locus lines from points a_1'', b_1'', c_1'', d_1'', etc., of the side views.
8. Join points $a_1b_1c_1d_1e_1f_1g_1h_1i_1j_1k_1l_1$ by a smooth curve representing top view.

8.12 MISCELLANEOUS EXAMPLES

Example 8.24 (Fig. 8.31)

Draw the projections of a circle of 50 mm diameter resting in the H.P. on a point *A* on the circumference, its plane is inclined at 45º to the H.P. and (a) the top view of the diameter *AG* making 30º angle with the V.P., and(b) the diameter *AG* making 30º angle with the V.P.
[RGPV June 2003]

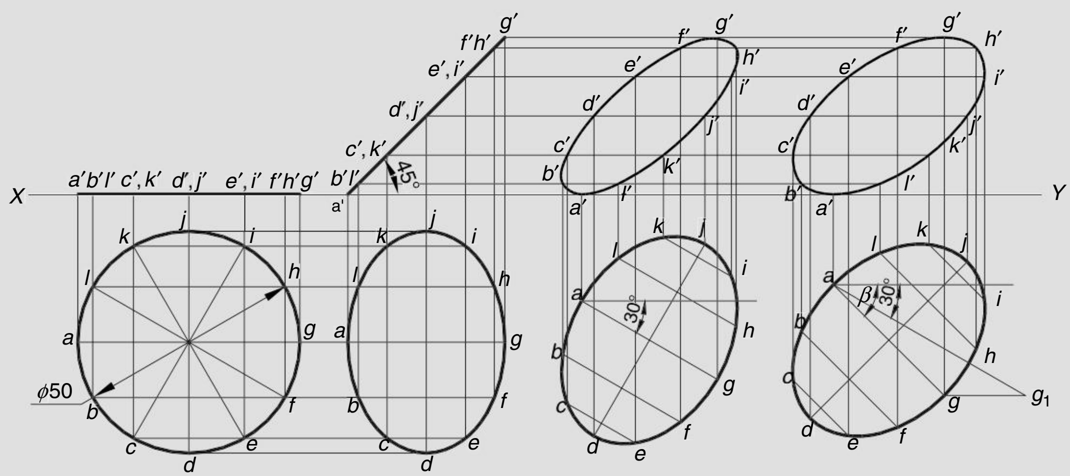


Fig. 8.31

Construction: Fig. 8.31.

1. *First stage*: Draw a circle *adgj* as the top view. Divide the circumference of the circle into 12 equal parts. Project all the points to *XY* and obtain *a′g′* as the front view.
2. *Second stage*: Reproduce the front view of the first stage such that *a′g′* is inclined at 45º to *XY.* Obtain points *a*, *b*, *c*, *d*, etc., of the top view by joining the points of intersection of the vertical projectors from points *a′*, *b′*, *c′*, *d′*, etc., of the second stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d*, etc of the first stage. Join *a*, *b*, *c*, *d*, etc., with a smooth curve.
3. *Third stage* (when top view of the diameter through point *A* is inclined at 30º to the V.P.): Reproduce the top view of the second stage such that diameter *ag* is inclined at 30º to *XY.* Obtain points *a′*, *b′*, *c′*, *d′*, etc., for the front view by joining the points of intersection of the vertical projectors

drawn from points *a*, *b*, *c*, *d*, etc., of the third stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′*, etc., of the second stage. Join *a′*, *b′*, *c′*, *d′*, etc. using a smooth curve.

4. *Fourth stage* (when the diameter through point *A* is inclined at 30º to the V.P.): The diagonal *AG* is already inclined to H.P. and it has to be turned to make 30º with the V.P. we have to determine the apparent angle β. Therefore, Draw a line ag_1 of length equal true length of the diameter (50 mm) inclined at 30º to *XY.* Draw an arc with *a* as the centre and radius equal to *ag* of second stage, to meet horizontal line through point g_1 at point *g*. Join *ag*. Thus line *ag* makes β angle with *XY* line.
5. Reproduce the top view of the second stage such that line *ag* is inclined at β to *XY.* Obtain points *a′*, *b′*, *c′*, *d′*, etc., for the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c*, *d*, etc. of the fourth stage with the horizontal lines drawn from points *a′*, *b′*, *c′*, *d′*, etc of the second stage. Join *a′*, *b′*, *c′*, *d′*, etc., using a smooth curve.

Example 8.25 (Fig. 8.32)

Draw the projections of a rhombus having 100 mm and 40 mm long diagonals. The bigger diagonal is inclined at 30º to H.P. with one of the end point in H.P. and the smaller diagonal is parallel to both the planes. [RGPV Dec 2003, Feb 2012]

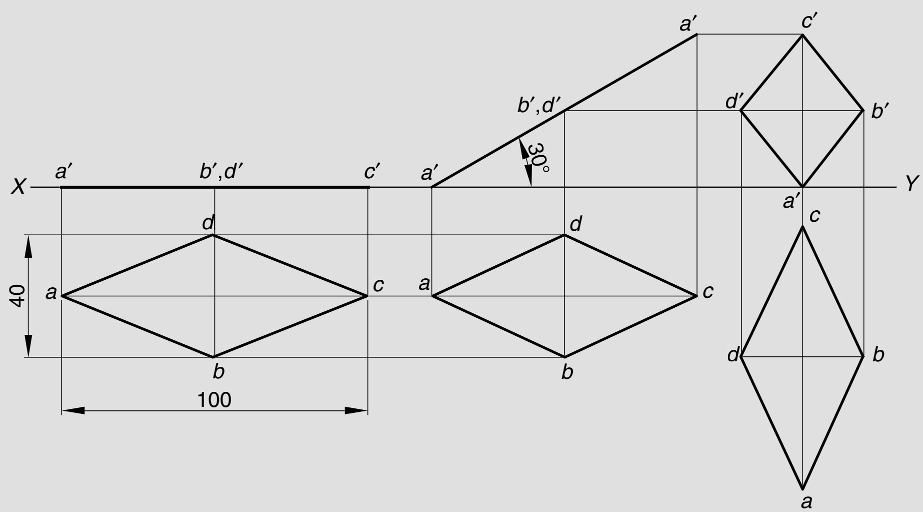


Fig. 8.32

Construction: Fig. 8.32

1. *First stage*: Draw a rhombus *abcd* such that diagonal *ac* is parallel to *XY* and diagonal *bd* is perpendicular to *XY,* as the top view. Project all the corners to *XY* and obtain *a′c′* as its front view.
2. *Second stage*: Reproduce the front view with *a′c′* inclined at 30º to *XY* line. Obtain points *a*, *b*, *c* and *d* of the top view by joining the points of intersection of the vertical projectors drawn from

points *a′*, *b′*, *c′* and *d′* of the second stage with the horizontal locus lines drawn from points *a*, *b*, *c* and *d* of the first stage. Join *abcd*.

3. *Third stage*: Reproduce the top view of the second stage such that *bd* is parallel to *XY*. Obtain points *a′*, *b′*, *c′* and *d′* for the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c* and *d* of the third stage with the horizontal lines from points *a′*, *b′*, *c′* and *d′* of the second stage. Join *a′b′c′d′*.

Example 8.26 (Fig. 8.33)

A pentagon *ABCDE* of 30 mm side has its side *AB* in the V.P. and inclined at 30° to the H.P. and the corner *B* is 15 mm above the H.P. and the corner *D* is 30 mm in front of the V.P. Draw the projections of the plane and find its inclination with the V.P.

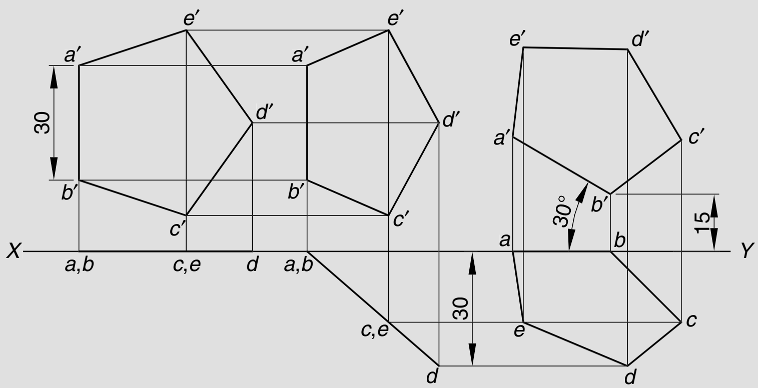


Fig. 8.33

Construction: Fig. 8.33

1. *First stage*: Draw a pentagon *a′b′c′d′e′* as the front view with side *a′b′* perpendicular to *XY*. Project all the corners to *XY* and obtain *ad* as the top view.
2. *Second stage*: Reproduce the top view of first stage such that *ab* lies on *XY* and point *d* is 30 mm below *XY*. Measure its inclination with *XY* which is found to be 41°.
3. Obtain points *a′*, *b′*, *c′*, *d′* and *e′* of the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c*, *d* and *e* of the second stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′*, *d′* and *e′* of the first stage. Join *a′b′c′d′e′*.
4. *Third stage*: Reproduce the front view of the second stage such that point *b′* is 15 mm above *XY* and side *a′b′* is inclined at 30° to *XY*. Obtain points *a*, *b*, *c*, *d* and *e* for the top view by joining the points of intersection of vertical projectors drawn from points *a′*, *b′*, *c′*, *d′* and *e′* of the third stage with the horizontal locus lines drawn from points *a*, *b*, *c*, *d* and *e* of the second stage. Join *abcde*.

Example 8.27 (Fig. 8.34)

A plate having shape of an isosceles triangle has 50 mm long base and 70 mm altitude. It is so placed that in the front view it is seen as an equilateral triangle of 50 mm side and one side inclined at 45º to *XY*. Draw its top view. [RGPV Feb. 2006]

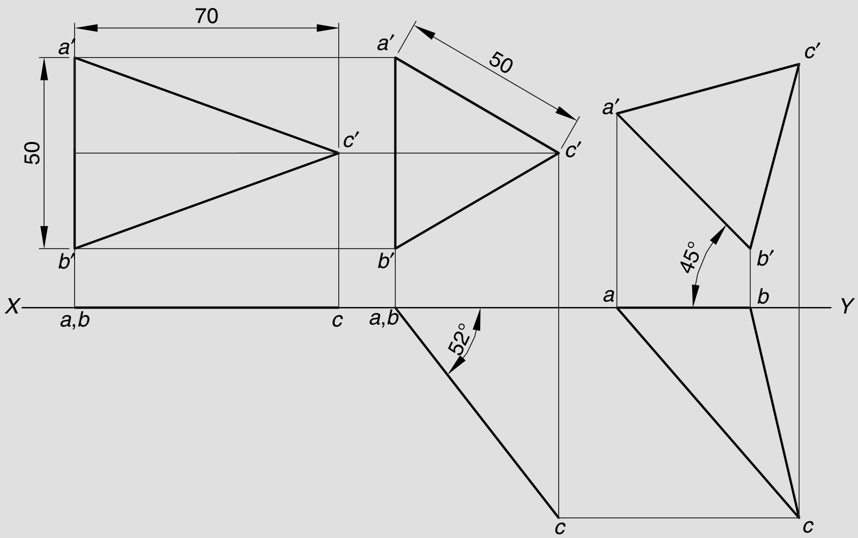


Fig. 8.34

Construction: Fig. 8.34

1. *First stage*: Draw an isosceles triangle *a′b′c′* such that side *a′b′* is 50 mm long and is perpendicular to *XY*. Project all the corners to *XY* and obtain *ac* as the top view.
2. *Second stage*: Draw an equilateral triangle *a′b′c′* on the horizontal locus lines drawn through points *a′*, *b′* and *c′*. Obviously, the length of sides of the triangle would is 50 mm.
3. Project points *a′* and *b′* vertical to meet *XY* at points *a* and *b* respectively. Project point *c′* vertically downwards as the locus of point *c*. Draw an arc with *a* as the centre and radius *ac* to meet the locus of point *c* at point *c*. Join *ac*.
4. *Third stage*: Reproduce the front view of the second stage such that side *a′b′* is inclined at 30º to *XY*. Obtain points *a*, *b* and *c* for the top view by joining the points of intersection with the vertical projectors drawn from points *a′*, *b′* and *c′* of the third stage with the horizontal locus lines drawn from points *a*, *b* and *c* of the second stage. Join *abc*.

Example 8.28 (Fig. 8.35)

A thin rectangular plate of 50 mm and 30 mm sides has its shorter side in the V.P. and inclined at 30º to the H.P. Project its top view, if its front view is a square of 30 mm long sides. [RGPV Feb. 2008]

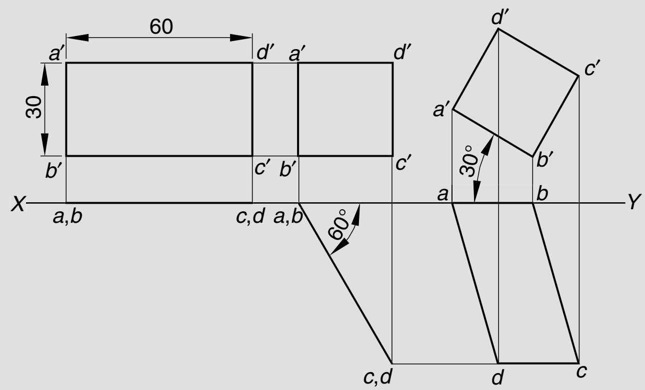


Fig. 8.35

Construction: Fig. 8.35

1. *First stage*: Draw a rectangle *a′b′c′d′* such that side *a′b′* is perpendicular to *XY* as the front view. Project all the corners to *XY* and obtain *ac* as the top view.
2. *Second stage*: Draw a square *abcd* on the horizontal locus lines drawn through points *a′*, *b′*, *c′* and *d′* as shown.
3. Project point *a′* to meet *XY* at point *a*. Project point *c′* vertically downwards as the locus of *c*. Draw an arc with *a* as the centre and radius equal to length *ac* of the first stage to meet the locus of *c* at point *c*. Join *ac*.
4. *Third stage*: Reproduce the front view of the second stage such that *a′b′* is inclined at 30º to *XY*. Obtain points *a*, *b*, *c* and *d* for the top view by joining the points of intersection of the vertical projectors drawn from points *a′*, *b′*, *c′* and *d′* of the third stage with the horizontal locus lines drawn from points *a*, *b*, *c* and *d* of the second stage. Join *abcd*.

Example 8.29 (Fig. 8.36)

A elevation of a rectangular lamina *ABCD* of 25 mm × 50 mm sides is a square of 25 mm side when its side *AB* is in V.P. and the side *AD* is making an angle of 20º to the H.P. Draw its projections. [RGPV Dec. 2001]

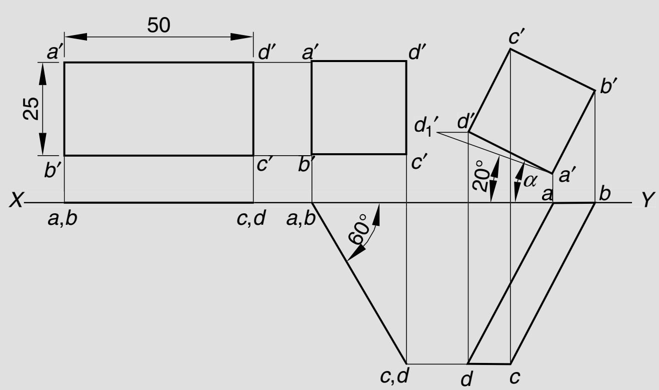


Fig. 8.36

Construction: Fig. 8.36

1. *First stage*: Draw a rectangle *a′b′c′d′* such that side *a′b′* is perpendicular to *XY* as the front view. Project all the corners to *XY* and obtain *ac* as the top view.
2. *Second stage*: Draw a square *abcd* on the horizontal locus lines drawn through points *a′*, *b′*, *c′* and *d′* as shown.
3. Project point *a′* to meet *XY* at point *a*. Project point *c′* vertically downwards as the locus of *c*. Draw an arc with a as the centre and radius equal to length *ac* of the first stage to meet the locus of *c* at point *c*. Join *ac*.
4. *Third stage*: Here the side *AD* is inclined at 60º to V.P. To make is further 20º to the H.P., we need to determine the apparent angle α. Therefore, draw a line $a'd_1'$ equal to true length (50 mm) and inclined at 20º to *XY*. Draw an arc with centre *a′* and radius equal to *a′d′* of the second stage (25 mm), to intersect the horizontal locus line from d_1' at point *d′*. The line *a′d′* is now inclined at α angle to *XY*.
5. Reproduce the front view of the second stage such that the side *a′d′* is inclined at α angle to *XY*. Obtain point *a*, *b*, *c* and *d* for the top view by joining the points of intersection of vertical projectors drawn from points *a′*, *b′*, *c′* and *d′* of the third stage with the horizontal locus lines drawn from points *a*, *b*, *c* and *d* of the second stage. Join *abcd*.

Example 8.30 (Fig. 8.37)

A thin square plate of 50 mm side stands on one of its corners in the HP and the opposite corner is raised so that one of the diagonals is twice that of the others. If, one of the diagonal is parallel to both the planes, draw its projection and determine an inclination of the plane of the plate with the HP. [RGPV Dec. 2005]

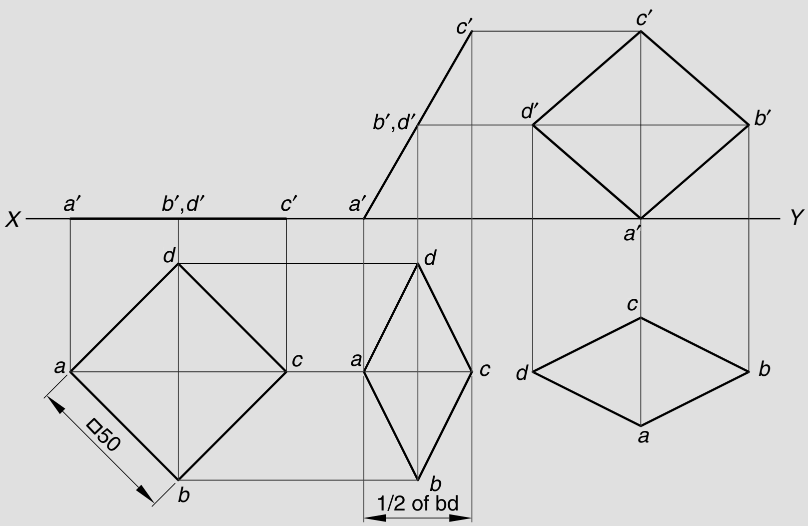


Fig. 8.37

Construction: Fig. 8.37

1. *First stage*: Draw a square *abcd* such that line *ac* is parallel to *XY,* as the top view. Project all the corners and obtain *a′c′* as the front view.
2. *Second stage*: Draw a rhombus *abcd* on the horizontal locus lines drawn through points *a*, *b*, *c* and *d* such that diagonal *bd* is equal to the diagonal *bd* of the first stage and diagonal *ac* is equal to half of the diagonal *bd*.
3. Project point *a* to meet *XY* line at point *a′*. Project point *c* vertically upward as the locus line of point *c′*. Draw an arc with *a′* as the centre and radius *a′c′* of the first stage, to meet locus line of *c′* at point *c′*. Join *a′c′*. Mark points *b′* and *d′* on line *a′c′*. Determine inclination of line *a′c′* with *XY* as 60º, inclination of the surface with H.P.
4. *Third stage*: Reproduce the top view of the second stage such that line *bd* is parallel to *XY.* Obtain points *a′*, *b′*, *c′* and *d′* for the front view by joining the points of intersection of the vertical projectors drawn from points *a*, *b*, *c* and *d* of the third stage with the horizontal locus lines drawn from points *a′*, *b′*, *c′* and *d′* of the second stage. Join *a′b′c′d′*.

Example 8.31 (Fig. 8.38)

A circular plate of negligible thickness and 50 mm diameter appears as an ellipse in the front view, having major axis 50 mm and minor axis 30 mm long. Draw its top view when the major axis of the ellipse is horizontal. [RGPV Dec. 2002]

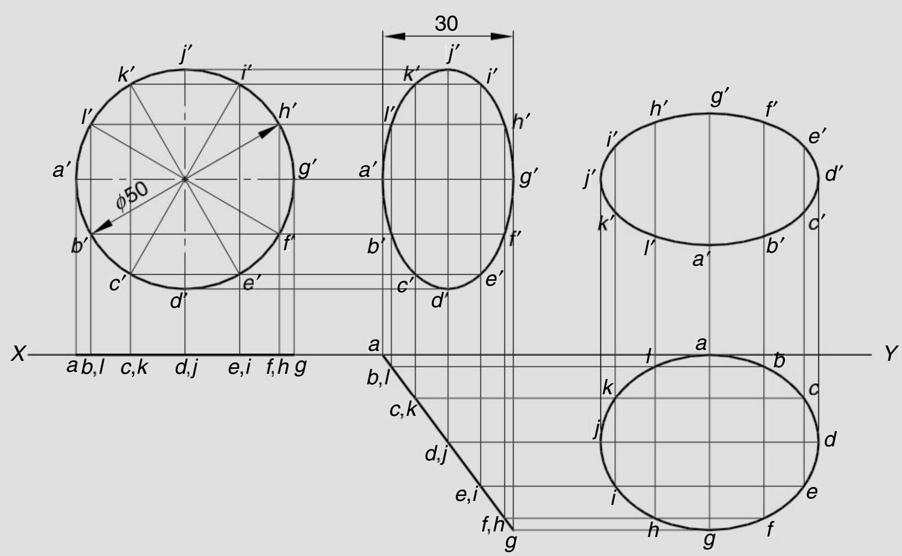


Fig. 8.38

Construction: Fig. 8.38

1. *First stage*: Draw a circle *a′d′g′j′* as the front view. Divide it into 12 equal parts. Project all the points to *XY* and obtain *ag* as the top view.

2. *Second stage*: Draw horizontal locus line from points a' and g' of the first stage. Mark on it points a' and g' such that they are 30 mm apart (equal to the length of the minor axis).
3. Project a' to meet *XY* line at point *a*. Project g' vertical as the locus of point *g*. Draw an arc with *a* as the centre and radius equal to *ag* of the first stage (50 mm) to meet the locus of point *g* at point *g*. Join *ag*. Mark other points on line *ag*.
4. Obtain points a', b', c', d', etc., of the front view by joining the points of intersection of the vertical projectors from points *a*, *b*, *c*, *d*, etc., of the second stage with the horizontal lines drawn from points a', b', c', d', etc., of the first stage. Join $a'b'c'd'e'f'g'h'i'j'k'l'$.
5. *Third stage*: Reproduce the front view of the second stage such that line *dj* parallel to *XY*. Obtain points *a*, *b*, *c*, *d*, etc., for the top view by joining the points of intersection of the projectors drawn from points a', b', c', d' of the third stage with the horizontal lines from points *a*, *b*, *c*, *d*, etc., of the second stage. Join *abcdefghijkl* using a smooth curve.

EXERCISE 8B

Plane Inclined to Both the Reference Planes

1. Draw the projections of a regular hexagon of 30 mm side, having one of its sides in H.P. and inclined at 60º to V.P. and its surface makes an angle of 45º to the H.P. **[RGPV Dec. 2005]**
2. A regular pentagon of 30 mm side is resting on one of its edges on the H.P. which is inclined at 45º to the V.P. Its surface is inclined at 30º to the H.P. Draw its projections. **[RGPV Dec. 2007]**
3. A rectangular plate of sides 60 mm × 30 mm is resting on its shorter side on H.P. and inclined at 30º to V.P. Its surface is inclined at 60º to H.P. Draw its projections. **[RGPV June 2002]**
4. An equilateral triangle of 40 mm long side has an edge on the ground and inclined at 60º to the V.P. Its plane makes an angle of 45º with the H.P. Draw its projections. **[RGPV June 2004]**
5. A rectangular lamina *ABCD* of sides 60 mm × 30 mm has its shorter side in H.P. and inclined at 60º to V.P. and the plane of the lamina is inclined at 60º to the H.P. Draw its projections. **[RGPV April 2009]**
6. A square *ABCD* with 60 mm long diagonals has a corner *A* on the H.P. The diagonal *AC* is inclined 30º to the H.P. while the diagonal *BD* is inclined at 45º to the V.P. and is parallel to the H.P. Draw its projections.
7. A pentagonal lamina of sides 40 mm resting with one of its corners on the V.P. and an edge opposite this corner makes an angle of 30º to the ground. The surface of the lamina itself is inclined at 45º to the V.P. Draw the projections. **[RGPV June 2011]**
8. A pentagonal lamina with 40 mm sides is resting on one of its sides on the H.P. having that side parallel to and 25 mm in front of the V.P. It is tilted about that side so that its highest corner rests in the V.P. Draw the projections of the lamina.

9. A circular disc of 40 mm diameter and negligible thickness rests on H.P. on its rim and makes an angle of 45º to it. One of its diameters is inclined to the V.P. at 30º. Draw its projections.
[RGPV June 2009]
10. A circle of 40 mm diameter is resting on H.P. on a point with its surface inclined at 30º to H.P. Draw the projections of the circle when the top view of the diameter through the resting point makes an angle of 45º with *XY*.
[RGPV Feb. 2008, Dec. 2004, Feb. 2005, April 2010]
11. A 30–60 set-square is kept on its 80 mm long hypotenuse in the H.P. inclined at 30º to the V.P. Draw its projections when the set-square itself is inclined at 45º to the H.P.
12. A 30–60 set square with longest side of 75 mm is kept such that the longest side is in the V.P. inclined at 45º to the H.P. The set-square is inclined at 45º to the V.P. Draw its projections.
13. A semicircular plate of 80 mm diameter has its straight edge in the H.P. and inclined at 30º to the V.P. The surface of the plate is inclined at 45º to the H.P. Draw its projections.
14. Draw the projections of a rhombus having diagonals 125 mm and 50 mm long, the smaller diagonal of which is parallel to both the planes (H.P. and V.P.), while the other is inclined at 30º to H.P.
[RGPV Dec. 2003, Dec. 2006]
15. A thin hexagonal piece of metal sheet with 40 mm sides has a hole with a 40 mm diameter punched centrally. It is placed on a corner in the H.P. Its surface is inclined at 30º to the H.P. and the top view of the diagonal through the corner in the H.P. is inclined at 45º to the V.P. Draw its projections.

To Determine Apparent Angle

16. A hexagonal plane with 30 mm sides has one of the corners in the H.P. and the corner opposite to it is in the V.P. The plane is inclined at 60º to the H.P. and 30º to the V.P. Draw its projections.
17. A square plate with 50 mm long sides is resting on one of its corners on the H.P. Its surface is inclined at 45º to the H.P. and the diagonal passing through that corner is inclined at 30º to the V.P. Draw the projections of the plate.
18. A regular hexagonal plate of 30 mm side has a corner on H.P. and its surface is inclined at 45º to H.P. Draw the projections when the diagonal through the corner which is on H.P. makes 30º with the V.P. **[RGPV Feb. 2011]**
19. A circular plate with a 60 mm diameter is kept on a point of its rim on the V.P. The surface of the circular plate is inclined at 30º to the V.P. Draw the projections of the plate when diameter passing through that point is inclined at 30º to the H.P.
20. A rectangular plate with 40 mm and 60 mm long sides is resting on one of its corners on the V.P. such that the diagonal passing through that corner is inclined at 30º to the H.P. and 45º to the V.P. Draw its projections.

Shape of One View Is Given

21. A thin rectangular plate of sides 60 mm × 30 mm has its shorter side in the V.P. and inclined at 30º to the H.P. Project its top view, if its front view is a square of 30 mm long sides.
[RGPV June 2006]

22. A thin rectangular plate of size 60 mm × 40 mm has its shorter edge on the H.P. It is kept in such a way that its top view appears as a square of 40 mm sides. Draw its projections when the edge resting on the H.P. is inclined at 30º to the V.P.
23. The top view of a square plane with 80 mm long diagonals appears as a rhombus of 80 mm and 50 mm long diagonals. One of the corners of the plane is in the H.P. Draw its projections when one of the diagonals is parallel to both the principal planes.
24. A rhombus has 100 mm and 60 mm long diagonals. It is placed such that its top view appears to be a square. Draw its projections when vertical plane containing the longer diagonal is inclined at 30º to the V.P.
25. A circular plate with a 60 mm diameter has a point *A* on its circumference in the H.P. Its surface is inclined to H.P. such that top view appears as an ellipse with minor axis 40 mm long. Draw its projections when top view of the diameter through point *A* makes 45º with the V.P. Neglect the thickness of the plate.
26. A trapezium *ABCD* (*AB* = 70 mm, *CD* = 40 mm) having parallel sides 60 mm apart is kept on its side *AB* in the V.P. such that its front view appears as another trapezium of same parallel sides but 30 mm apart. Draw the projections of the trapezium when the side in the V.P. makes an angle of 45º with the H.P.
27. A rectangular plate with 45 mm and 75 mm long sides is resting on one of its smaller sides in the H.P. Its top view appears as another rectangle with 45 mm and 60 mm long sides. Draw its projections when one of its longer sides makes an angle of 45º with the V.P.

    (*Hint*: determine apparent angle β)
28. A rectangular plane *ABCD* with 40 mm and 60 mm long sides has its shorter side in the V.P. and longer side is inclined at 30º to the H.P. Draw the projections of the rectangle if its front view appears as a square.

    (*Hint*: determine apparent angle α)

REVIEW QUESTIONS

1. If the top view of a plane is a straight line, will its front view always be the true shape?
2. If the front view of a plane lies in the reference line, will its top view always be the true shape?
3. The projections of a plane lying in the H.P. are drawn. What will be the change in the shape, size and position of the front view if the surface of the plane is inclined at 30º to the H.P.?
4. The projections of a plane parallel to V.P. are drawn. What will be the change in the shape, size and position of the top view if the surface of the plane is inclined at 45º to the V.P.?
5. A rectangular plane 60 mm long and 30 mm wide is parallel to and 20 mm above the H.P. What will be the shape and position of its front view if the longer side is inclined at 30º to the V.P.?
6. The top view of a plane is a circle and the front view is a line inclined at 60º to *XY*. What is the true shape of the plane?
7. The surface of a hexagonal plane is perpendicular to both H.P. and V.P. Which orthographic view will show the true shape?

8. The true shape of a pentagonal plane is seen in the side view. What will be the shapes of its front and top views?
9. Define the position of a plane rhombus such that its top view appears as a square.
10. Define the position of an elliptical plane such that its front view appears as a circle.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) If a thin set-square is kept perpendicular to both the horizontal and vertical planes, its true shape is seen in
   (a) horizontal plane (b) vertical plane
   (c) auxiliary inclined plane (d) profile plane

ii) Planes which are inclined to both the horizontal and vertical planes are called
   (a) oblique planes (b) profile planes (c) auxiliary planes (d) none of these

iii) If a thin rectangular plate of 60 mm $\times$ 30 mm is inclined at an angle of 60° to H.P. its top view may be
   (a) square of 60 mm side (b) square of 30 mm side
   (c) rectangle of 60 mm $\times$ 45 mm (d) rectangle of 45 mm $\times$ 30 mm

iv) In multiview orthographic projection, the front view of a circular plane may be
   (a) a circle (b) an ellipse (c) a straight line (d) any one of these

v) If both front and top views of a plane are straight lines the true shape will lie on
   (a) profile plane (b) horizontal plane (c) vertical plane (d) any of these

vi) If a circular plane is inclined at 30° with the H.P. and 60° with the V.P. its side view will be
   (a) an ellipse (b) a straight line (c) a circle (d) true shape

vii) The front view of an elliptical plane may be
   (a) an ellipse (b) a circle (c) a straight line (d) any of these

viii) If the top view of a plane is a rhombus the object may be
   (a) a square (b) a rhombus (c) either (a) or (b) (d) neither (a) nor (b)

ix) The trace of a hexagonal plane may be
   (a) a straight line (b) a point (c) a hexagon (d) an equilateral triangle

x) A 60° set-square has its shortest edge in the V.P. The surface is perpendicular to the H.P. and inclined to the V.P. Its front view may appear as
   (a) an equilateral triangle (b) an isosceles triangle
   (c) an obtuse-angled triangle (d) an acute-angled triangle

xi) A 60º set-square has its shortest edge in the H.P. and the surface is perpendicular to the V.P. Its top view may appear as

(a) an isosceles triangle (b) a right-angled triangle
(c) a straight line (d) any of these

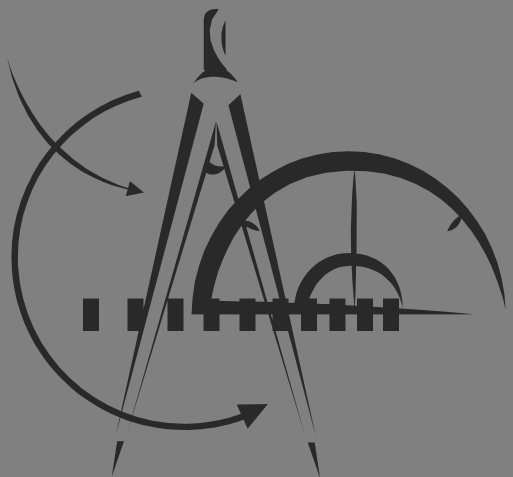
xii) If both the principal views of a plane object are ellipse of the same size, the side view will be

(a) a horizontal line (b) a vertical line (c) an inclined line (d) an ellipse

Answers

(i) d (ii) a (iii) b (iv) d (v) d (vi) b (vii) d (viii) c (ix) d (x) b (xi) d (xii) c

9



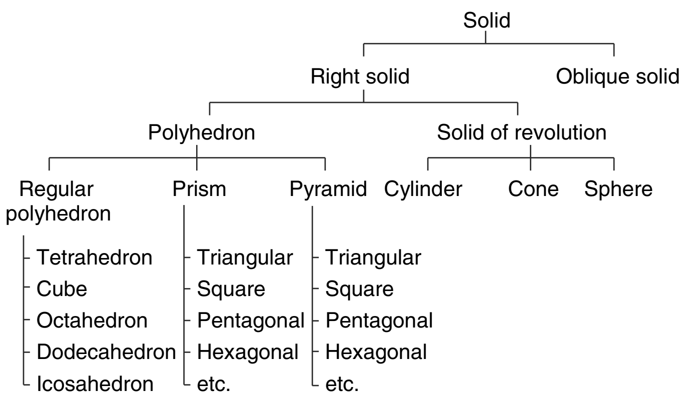
Projections of Solids

9.1 INTRODUCTION

This chapter deals with the orthographic projections of three-dimensional objects called solids. However, only those solids are considered, the shape of which can be defined geometrically and are regular in nature. The basic concepts of orthographic projections discussed in earlier chapters shall also apply here.

9.2 CLASSIFICATION OF SOLIDS

Solids are usually classified as follows:



Polyhedron: A polyhedron is defined as a solid bounded by planes called faces which meet in straight lines called edges.

Regular polyhedron: It is a polyhedron having all the faces equal and regular, see Fig. 9.1.

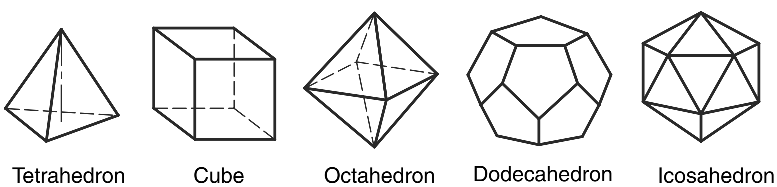


Fig. 9.1 *Regular polyhedron*

Tetrahedron: It is a solid having four equal equilateral triangular faces.

Cube: It is a solid having six equal square faces.

Octahedron: It is a solid having eight equal equilateral triangular faces.

Dodecahedron: It is a solid having twelve equal pentagonal faces.

Icosahedron: It is a solid having twenty equal equilateral triangular faces.

Prism: This is a polyhedron having two equal and similar regular polygons called its ends or bases parallel to each other and joined by other faces which are rectangles, see Fig. 9.2. The imaginary line joining the centres of the bases is called the axis.

A right and regular prism has its axis perpendicular to the base. All the faces are equal rectangles.

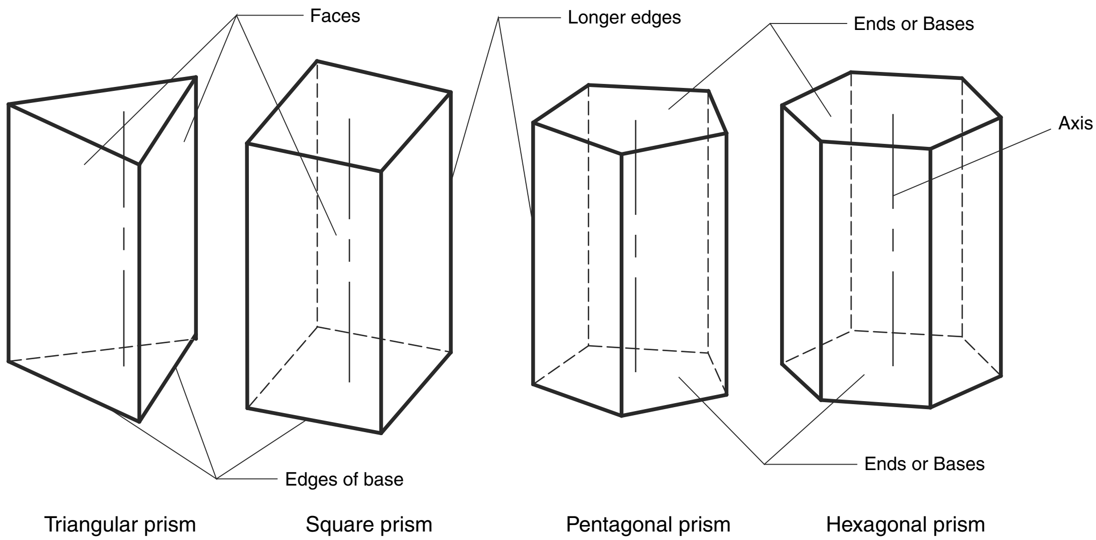


Fig. 9.2 *Prisms*

Pyramid: This is a polyhedron having a plane figure as a base and a number of triangular faces meeting at a point called the vertex or apex, see Fig. 9.3. The imaginary line joining the apex with the centre of the base is known as the axis.

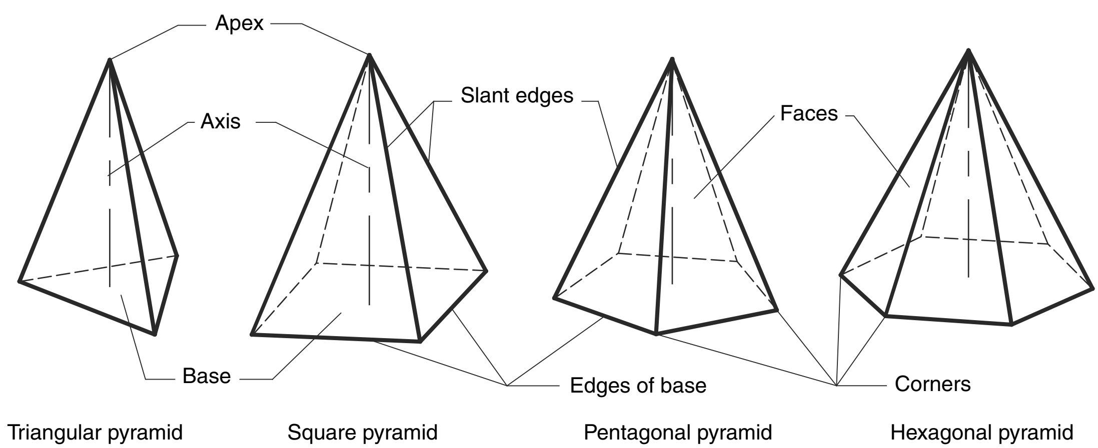


Fig. 9.3 *Pyramids*

A right and regular pyramid has its axis perpendicular to the base which is a regular plane figure. All the faces are isosceles triangles.

Solids of revolution: These solids are obtained by revolving a plane figure like rectangle, triangle or a semicircle about a fixed line, see Fig. 9.4.

Cylinder: A cylinder is obtained by revolving a rectangle about a fixed line called axis.

Cone: A cone is obtained by revolving a right-angled triangle about its perpendicular side which remains fixed.

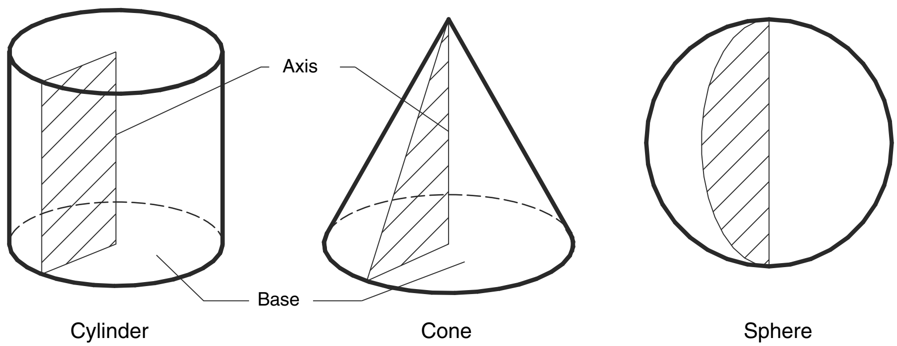


Fig. 9.4 *Solids of revolution*

Sphere: A sphere is obtained by revolving a semicircle about its diameter.

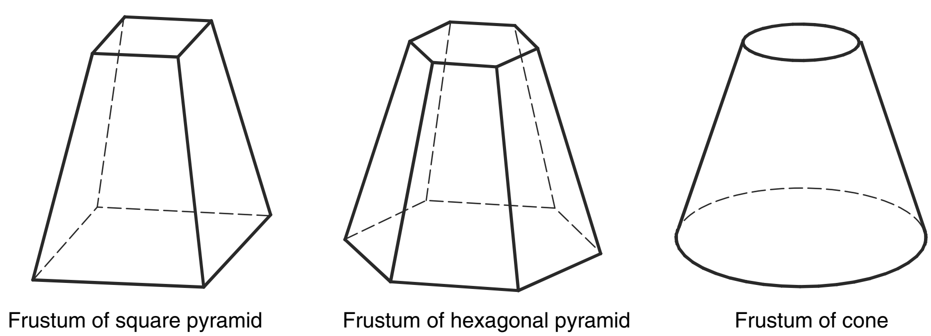


Fig. 9.5 *Frustums*

Frustum of pyramid or cone: When a regular pyramid or cone is cut by a section plane parallel to its base and on removing the upper portion of the solid, the remaining portion is called the frustum of that pyramid or cone, see Fig. 9.5.

9.3 RECOMMENDED METHOD FOR NAMING CORNERS OF THE SOLIDS

It is recommended to label the corners of the solids in a manner shown in Fig. 9.6.

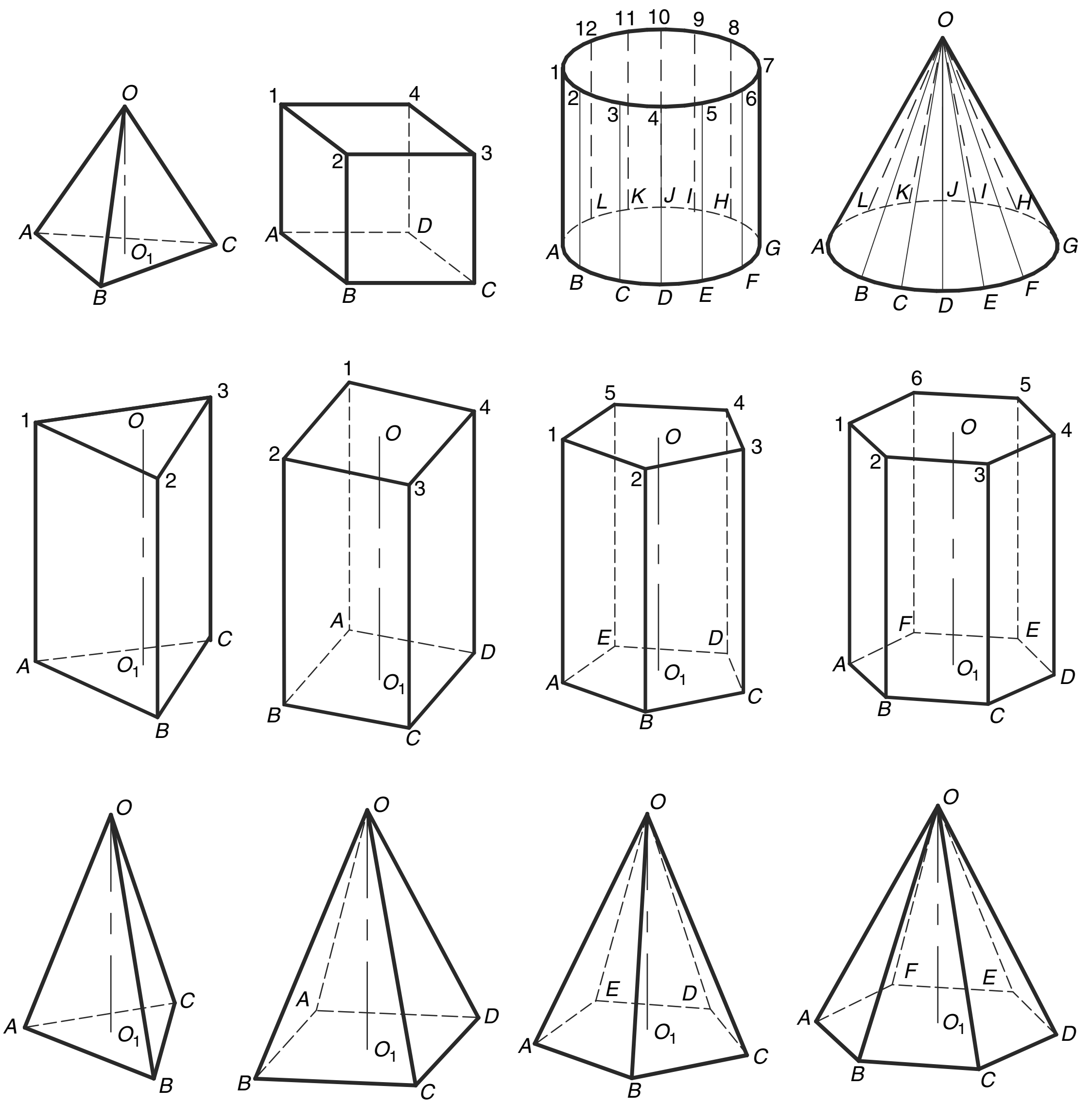


Fig. 9.6 *Suggested method of labelling*

9.4 ORIENTATION OF SOLIDS

The possible orientations of solids with respect to the principal planes are as given below.

1. Axis perpendicular to H.P.
2. Axis perpendicular to V.P.
3. Axis parallel to both H.P. and V.P. (i.e. axis perpendicular to the profile plane)
4. Axis inclined to H.P. and parallel to V.P.

5. Axis inclined to V.P. and parallel to H.P.
6. Axis inclined to both H.P. and V.P.

9.5 DECIDING INITIAL POSITION OF THE SOLID

General rule adopted to decide the initial position of the solids when it is inclined to one or both the axes are as follows:

S.No	*Required final position of the solid*	*Assume initial position of the solid*
1(a)	Axis inclined to H.P. and parallel to the V.P. with an edge of the base in the H.P.	Base in H.P. with an edge of the base perpendicular to the reference line.
1(b)	Axis inclined to H.P. and parallel to the V.P. with a corner of the base in the H.P.	Base in H.P. keeping the line joining the corner and centre of the base parallel to the reference line.
2(a)	Axis inclined to V.P. and parallel to the H.P. with an edge of the base in the V.P.	Base in V.P. with an edge of the base perpendicular to the reference line.
2(b)	Axis inclined to V.P. and parallel to the H.P. with a corner of the base in V.P.	Base in V.P. keeping the line joining the corner and centre of the base parallel to the reference line.
3	Pyramid having one of its triangular faces in H.P. or inclined to the H.P. and axis parallel to V.P.	Base in H.P. with an edge of the base perpendicular to the reference line.
4	Pyramid having one of its triangular faces in V.P. or inclined to the V.P. and axis parallel to H.P.	Base of the pyramid in V.P. with an edge of the base perpendicular to the reference line.
5	Pyramid lying on one of its slant edges in H.P. or inclined to the H.P. and axis parallel to V.P.	Base in H.P. keeping the line joining the corner and centre of the base parallel to the reference line.
6	Pyramid lying on one of its slant edges in V.P. or inclined to the V.P. and axis parallel to H.P.	Base in V.P. keeping the line joining the corner and centre of the base parallel to the reference line.
7	Cone lying on one of its generators in H.P. or inclined to the H.P. and parallel to V.P.	Base of the cone in the H.P.
8	Cone lying on one of its generators in V.P. or inclined to the V.P. and parallel to H.P.	Base of the cone in the V.P.

9.6 RULES TO IDENTIFY VISIBLE AND HIDDEN LINES

General rule adopted to identify and distinguish the visible and the hidden edges in the orthographic views of the solid are as follows:

1. Outlines of an object are always visible, the outer edges of any view should be shown with continuous lines (i.e. boundary lines of any view are never dotted).
2. All the edges or faces which are towards the observer are visible. Therefore, their projections in the other view are shown by full lines.
3. All the edges or faces which are away from the observer, i.e. towards the reference line are not visible. Therefore, their projections in the other view are shown by dotted lines.

4. When two lines representing the edges cross each other, one of them must be dotted.
5. Two continuous lines never cross each other inside. Similarly, two hidden lines never cross each other.

These rules are applicable for only single solid. They do not apply for the solids with holes or combination of solids.

9.7 AXIS PERPENDICULAR TO H.P.

This is one of the basic positions of the solid. It is evident that if the axis of a solid is perpendicular to H.P., its base will be parallel to the H.P. The true shape and size of the base will be seen in the top view. Hence top view of the solid should be drawn first and then it is projected to draw the front view.

Example 9.1 (Fig 9.7)

A square pyramid, side of base 40 mm and axis 60 mm is resting on its base on H.P. Draw its projections when (a) a side of the base is parallel to V.P., (b) a side of the base is inclined at 30º to V.P., and (c) all the sides of the base are equally inclined to V.P.

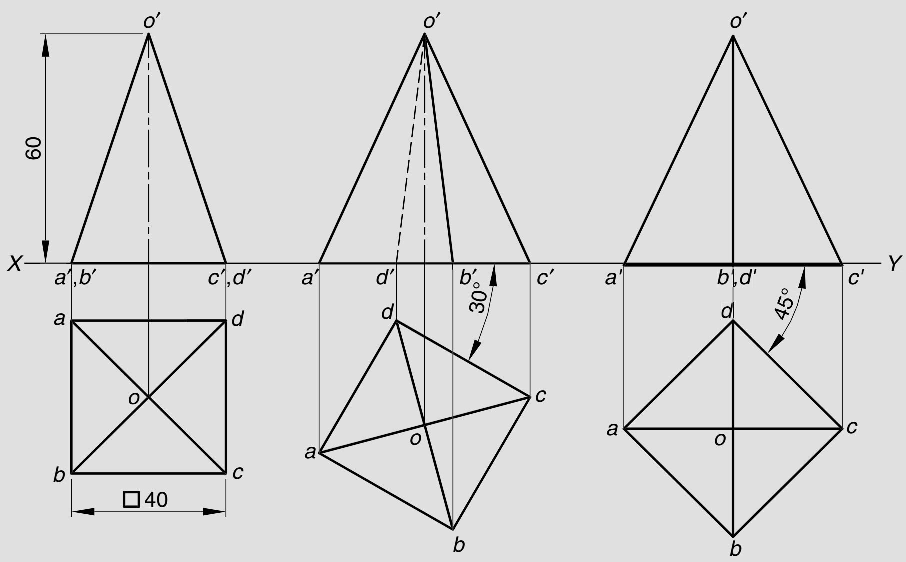


Fig. 9.7(a) Fig. 9.7(b) Fig. 9.7(c)

Construction:

1. *A side of the base parallel to V.P. [Fig 9.7 (a)]:* Draw a square *abcd* with side *ad* parallel to the *XY*. Locate centre *o* and join it with all the corners *a*, *b*, *c*, *d*. This represents the top view. Project points *a*, *b*, *c* and *d* on *XY* and mark points *a*′, *b*′, *c*′ and *d*′. Project point *o*, 60 mm above the *XY* and mark it as *o*′. Join *o*′*a*′*b*′ and *o*′*c*′*d*′. This is the required front view.

2. *A side of the base inclined at 30º to V.P. [Fig 9.7 (b)]:* Draw a square *abcd* with *dc* inclined at 30º to the *XY*. Locate centre *o* and join it with the corners *a*, *b*, *c*, *d*. This represents the top view.
   Project points *a*, *b*, *c* and *d* on *XY* and mark points *a*′, *b*′, *c*′ and *d*′. Project point *o*, 60 mm above the *XY* and mark it as *o*′. Join *a*′*o*′, *b*′*o*′, *c*′*o*′ and *d*′*o*′. This is the required front view. It may be noted that *d*′*o*′ is not visible and should be shown as dotted line.
3. *All the sides of the base equally inclined to V.P. [Fig 9.7 (c)]:* Draw a square *abcd* with sides inclined at 45º with the *XY*. Locate centre *o* and join it with the corners *a*, *b*, *c*, *d*. This is the required top view.
   Project points *a*, *b*, *c* and *d* on *XY* and mark points *a*′, *b*′, *c*′ and *d*′. Project point *o*, 60 mm above the *XY* and mark it as *o*′. Join *a*′*o*′, *b*′*d*′*o*′ and *c*′*o*′. This is the required front view.

Example 9.2 (Fig 9.8)

A square prism of 40 mm base edges and 60 mm long axis is resting on its base on ground. Draw its projections when (a) a face is perpendicular to the V.P., (b) a face is inclined at 30º to the V.P., and (c) all the faces are equally inclined to the V.P.

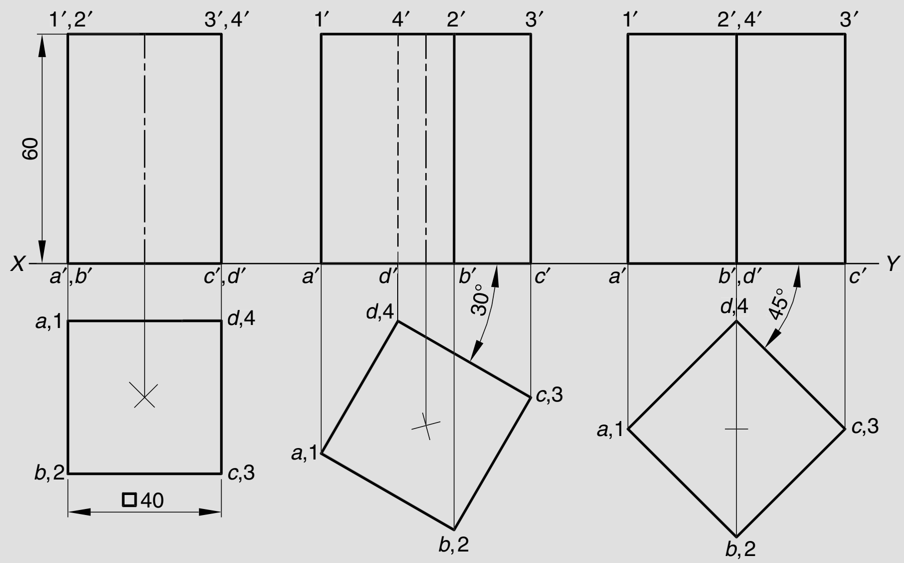


Fig. 9.8(a) Fig. 9.8(b) Fig. 9.8(c)

Construction:

1. *A face perpendicular to the V.P. [Fig 9.8 (a)]:* Draw a square *abcd* such that side *ab* is perpendicular to *XY*. This represents the top view.
   Project all the corners of the top view on *XY* and mark *a*′, *b*′, *c*′ and *d*′. Mark points 1′, 2′, 3′ and 4′, 60 mm above *XY*. Join all the edges and obtain the required front view.
2. *A face inclined at 30º to the V.P. [Fig 9.8 (b)]:* Draw a square *abcd* such that side *cd* is inclined at 30º to *XY*. This represents the top view.

Project all the corners of the top view on *XY* and mark *a*′, *b*′, *c*′ and *d*′. Mark points 1′, 2′, 3′ and 4′, 60 mm above *XY*. Join all the edges and obtain the required front view. Edge *D*4 is not visible in the front view so *d*′4′ should be drawn as dotted line.

3. *All the faces equally inclined to the V.P. [Fig 9.8 (c)]:* Draw a square *abcd* such that sides are inclined at 45° to *XY*. This represents the top view.
   Project all the corners of the top view on *XY* and mark *a*′, *b*′, *c*′ and *d*′. Mark points 1′, 2′, 3′ and 4′, 60 mm above *XY*. Join all the edges and obtain the required front view.

9.8 AXIS PERPENDICULAR TO V.P.

This is another basic position of the solid. It is evident that if the axis of a right solid is perpendicular to V.P., its base will be parallel to the V.P. The true shape and size of the base will be seen in the front view. Hence front view of the solid should be drawn first and from its projectors top view must be drawn.

Example 9.3 (Fig 9.9)

A pentagonal prism of 30 mm base edges and 60 mm long axis, has one of its bases in the V.P. Draw its projections when (a) a rectangular face is parallel to and 15 mm above H.P., (b) a face is perpendicular to the H.P., and (c) a face is inclined at 45° to the H.P.

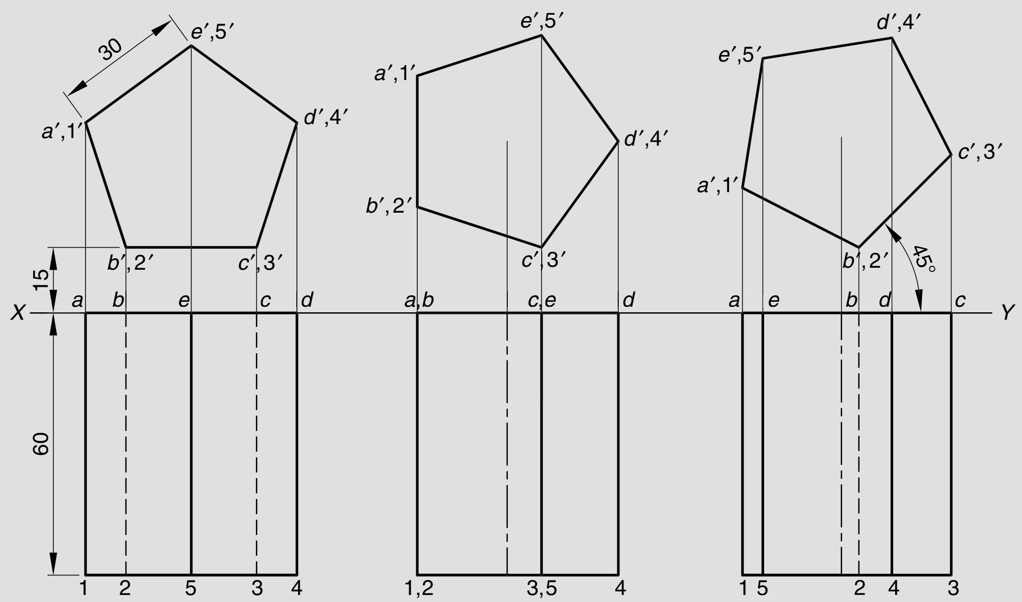


Fig. 9.9(a) **Fig. 9.9(b)** **Fig. 9.9(c)**

Construction:

1. *A rectangular face parallel to and 15 mm above H.P. [Fig 9.9 (a)]*: Draw a pentagon *a*′*b*′*c*′*d*′*e*′ such that side *b*′*c*′ is parallel to and 15 mm above *XY*, to represent the front view.

Project all corners from the front view on the *XY* and mark them as *a*, *b*, *c*, *d* and *e*. Also project points 60 mm below *XY* and mark them as 1, 2, 3, 4 and 5. Join the points and obtain the required top view. Edges *b*2 and *c*3 are not visible and should be drawn as dotted lines.

2. *A face perpendicular to the H.P. [Fig 9.9 (b)]:* Draw a pentagon *a′b′c′d′e′* such that side *a′b′* is perpendicular to *XY*, to represent the front view.
   Project all corners from the front view on the *XY* and mark them as *a*, *b*, *c*, *d* and *e*. Also project points 60 mm below *XY* and mark them as 1, 2, 3, 4 and 5. Join the points and obtain the required top view.
3. *A face inclined at 45° to the H.P. [Fig 9.9 (c)]:* Draw a pentagon *a′b′c′d′e′* such that side *b′c′* is inclined at 45° to the *XY*, to represent the front view.
   Project all corners from the front view on the *XY* and mark them as *a*, *b*, *c*, *d* and *e*. Also project points 60 mm below *XY* and mark them as 1, 2, 3, 4 and 5. Join the points and obtain the required top view. Edge *b*2 is not visible and should be drawn as a dotted line.

9.9 AXIS PARALLEL TO BOTH H.P. AND V.P.

It is evident that if the axis of right solids is parallel to both H.P. and V.P., the base of the solid will be perpendicular to the reference planes and parallel to the profile plane. The true shape and size of the base will be seen in the side view. Hence side view of the solid should be drawn first and from its projectors front and top views must be drawn.

Example 9.4 (Fig 9.10)

A triangular prism of base 30 mm and axis 55 mm long lies on its rectangular face in H.P. with its axis parallel to the V.P. Draw the three views of the prism. [RGPV Feb. 2007]

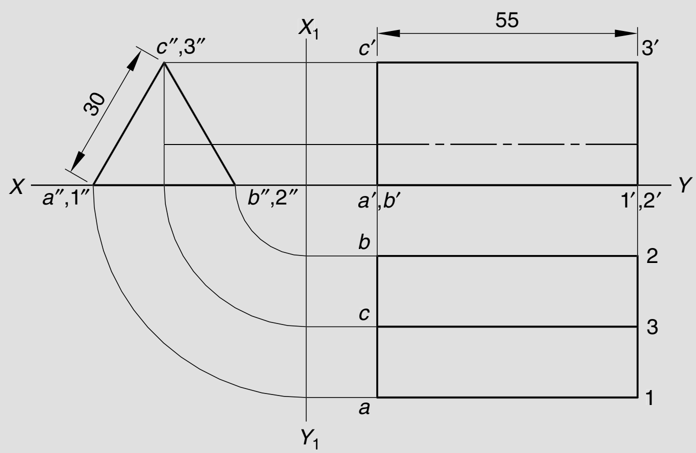


Fig. 9.10

Construction: Fig. 9.10

1. As the axis is parallel to both the planes, the triangle would be seen in the side view. Thus, begin with the side view. Draw a triangle $a''b''c''$ such that side $a''b''$ is on the XY line.
2. Project the corners from the side view parallel to XY and obtain the front view $a'1'3'c'$.
3. Project the corners from the side view and the front view to meet each other and obtain the top view $a12b$.

9.10 AXIS INCLINED TO H.P. AND PARALLEL TO V.P.

When the axis of a right solid is inclined (at θ) to the H.P. and parallel to the V.P., then the projections are drawn in two stages. Consider the following solved example.

Example 9.5 (Fig 9.11)

A right regular pentagonal prism, 25 mm edge of base and 55 mm height rests on an edge of its base in H.P. such that its axis is parallel to V.P. and inclined to the H.P. at 45°. Draw the projections of the solid. [RGPV Dec. 2007]

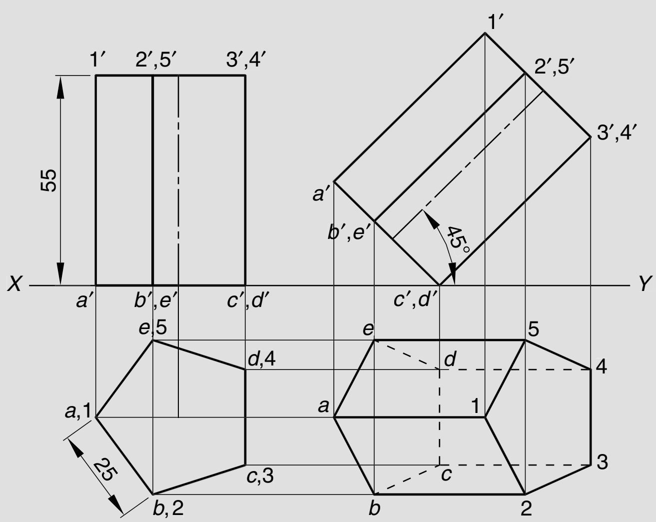


Fig. 9.11

Construction: Fig. 9.11

1. *First stage*: Draw a pentagon *abcde* such that side *cd* is perpendicular to *XY*, to represent the top view.
2. Project all the corners and obtain $a'd'4'1'$ to represent the front view.

3. *Second stage*: Reproduce the front view of the first stage keeping $c'd'$ is on XY and $c'3'$ is inclined at 45º to it. Thus, the axis is inclined at 45º to XY.
4. Obtain points *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.6 (Fig 9.12)

A hexagonal pyramid, base 25 mm side and axis 50 mm long has an edge of its base on the ground. Its axis is inclined at 40º to the ground and parallel to the V.P. Draw its projections.

[RGPV June 2009]

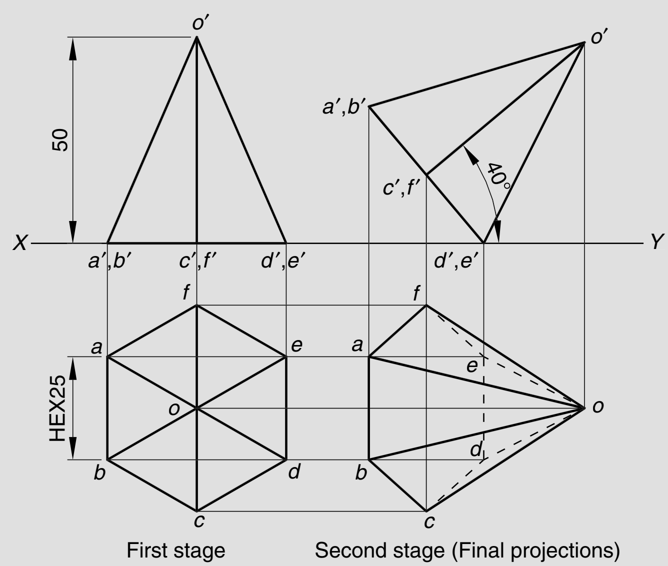


Fig. 9.12

Construction: Fig. 9.12

1. *First stage*: Consider edge *ED* of the hexagonal prism lie on the H.P. in the final stage. Draw a hexagon *abcdef* such that side *de* is perpendicular to *XY*. Join the centre *o* of the hexagon to meet the corners. This represents the top view.
2. Project the corners and obtain $a'd'4'1'$ to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping $d'e'$ on XY and $a'd'$ inclined at 50º to it. Thus, the axis becomes inclined at 40º to XY.

4. Obtain points *a*, *b*, *c*, *d*, *e*, *f*, 1, 2, 3, 4, 5 and 6 in the top view as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.7 (Fig 9.13)

A hexagonal pyramid, with 30 mm base edges and 70 mm long axis, has a triangular face on the ground and the axis parallel to the V.P. Draw its projections.

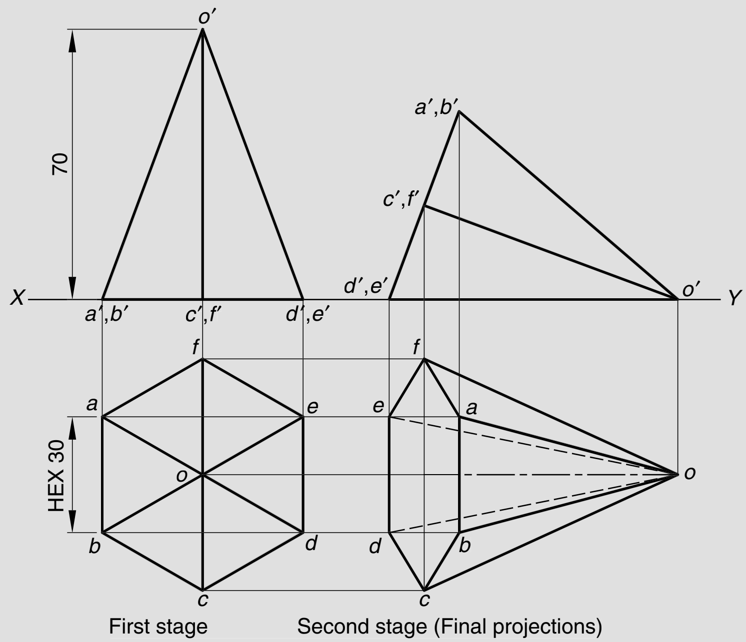


Fig. 9.13

Construction: Fig. 9.13

1. *First stage*: Draw a hexagon *abcdef* keeping side *de* perpendicular to *XY*. Locate the mid-point *o* and join it with the corners of the hexagon. This represents the top view.
2. Project all the corners and obtain *a′d′o′* to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage such that line *e′d′o′* representing the triangular face is on *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all visible and hidden lines as shown.

Example 9.8 (Fig 9.14)

A pentagonal prism of 25 mm base side and 50 mm axis length is resting on the H.P. on one of its base corners with its axis inclined at 40º to the H.P. and parallel to the V.P. Draw its projection when the base sides containing the resting corner are equally inclined to the H.P.

[RGPV June 2002]

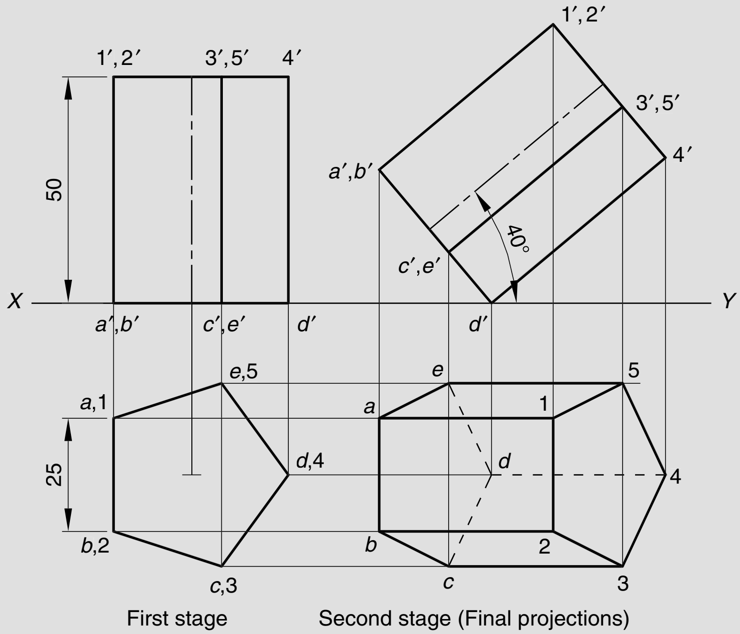


Fig. 9.14

Construction: Fig. 9.14

1. *First stage*: Draw a pentagon *abcde* keeping side *ab* perpendicular to *XY*. This represents the top view.
2. Project the corners and obtain $a'd'4'1'$ to represent the front view.
3. *Second stage*: Reproduce the front view of the first keeping that corner d' on *XY* and the axis inclined at 40º to the *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 in the top view as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.9 (Fig 9.15)

Draw the projections of a cylinder of 40 mm diameter and 60 mm long axis when it is lying on H.P. with axis inclined at 45º to H.P. and parallel to V.P. Follow the change of position method.
[RGPV June 2008(o)]

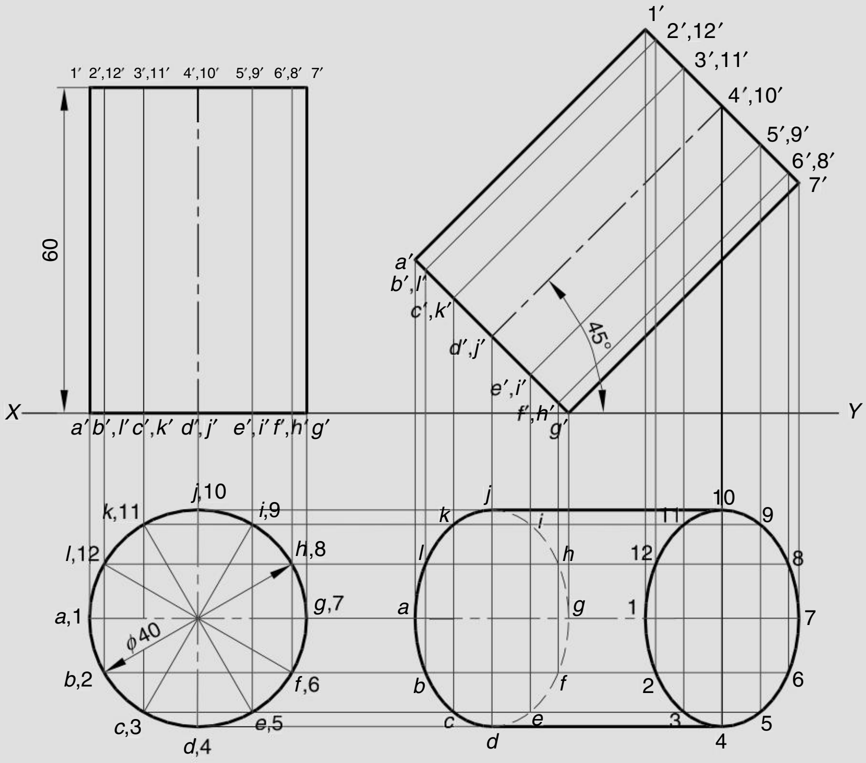


Fig. 9.15

Construction: Fig. 9.15

1. *First stage*: Draw a circle *adgj* to represent the top view. Divide this circle into 12 equal parts using radial lines.
2. Project the ends of the radial lines and obtain $a'g'7'1'$ to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping point g' on the *XY* and the base $g'a'$ inclined at 45º to *XY*. Thus, the axis is inclined at 45º to *XY*.
4. Obtain points *a*, *b*, *c*, *d*, etc., and 1, 2, 3, 4, etc., as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible points with continuous curve and hidden points with dotted curve, following the rules of visibility.

Example 9.10 (Fig 9.16)

A pentagonal pyramid of base edge 30 mm and height 50 mm is resting on one of its corners in H.P. Draw the projection when the axis is inclined at 45º to H.P. [RGPV Dec. 2010]

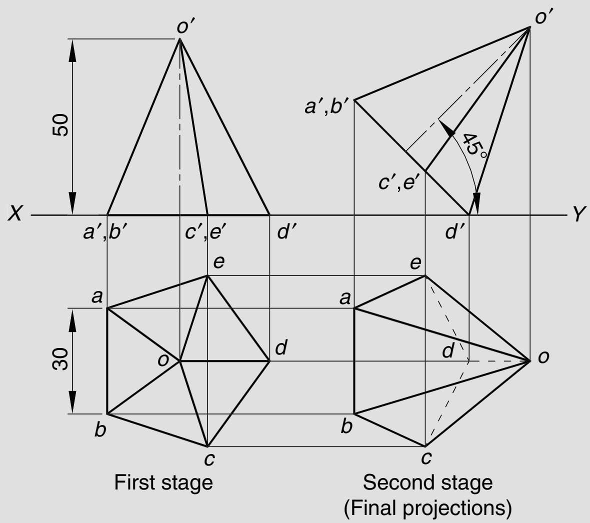


Fig. 9.16

Construction: Fig. 9.16

1. *First stage*: Draw a pentagon *abcde* keeping *ab* perpendicular to *XY*. Locate the centroid *o* and join it with all the corners of the pentagon. This represents the top view.
2. Project the corners *abcde* upto *XY* and *o* 50 mm above it. Join *a′d′o′* to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping corner *d′* on *XY* and the base *a′d′* inclined at 45º to it. Thus, the axis is inclined at 45º to *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.11 (Fig 9.17)

A hexagonal pyramid with 30 mm base sides and 70 mm long axis is lying on a slant edge on the ground such that the axis is parallel to the V.P. Draw its projections.

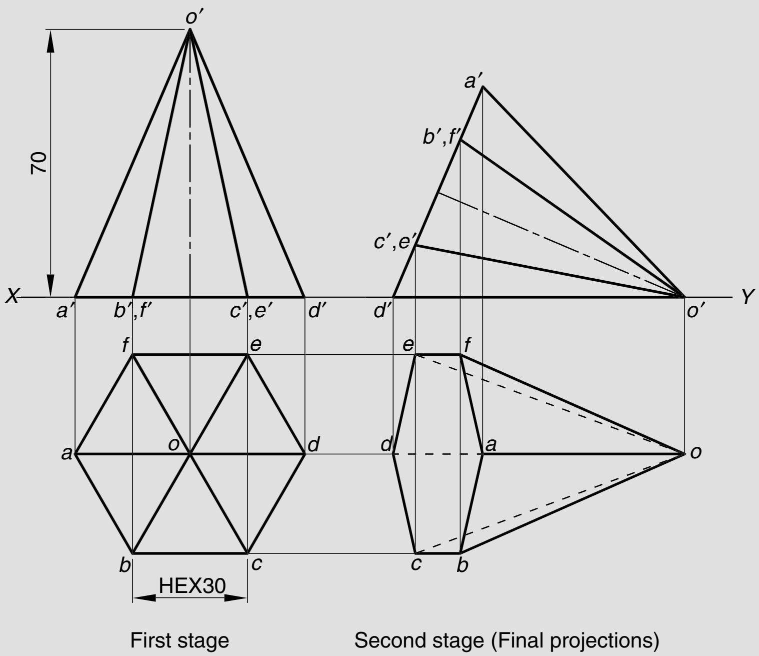


Fig. 9.17

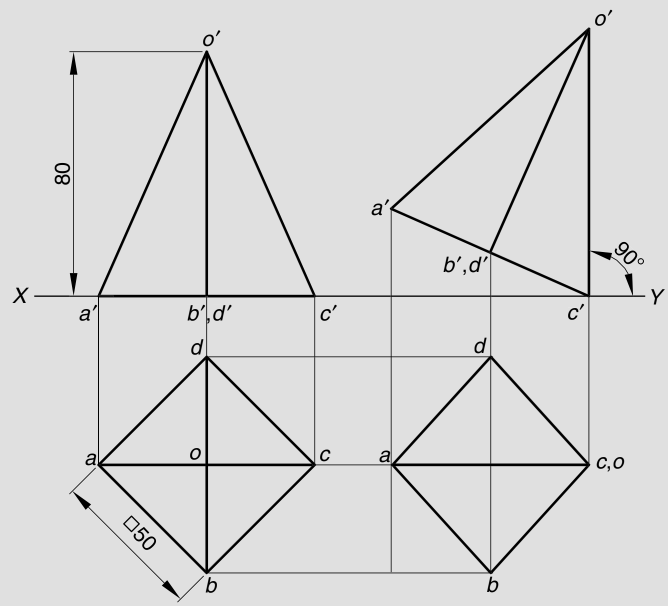
Construction: Fig. 9.17

1. *First stage*: Draw a hexagon *abcdef* keeping diagonal *ad* parallel to *XY*. Locate the mid-point *o* and join it with the corners of the hexagon. This represents the top view.
2. Project all the corners and obtain $a'd'o'$ to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage such that slant edge $o'd'$ is on the *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible and hidden lines as shown.

Example 9.12 (Fig 9.18)

A square pyramid of 50 mm base side and 80 mm height is held in such a way that one of the edges connecting one of the corners of the base and the apex is perpendicular to the ground and parallel to V.P. Draw the projections.

[RGPV Dec. 2001]



First stage Second stage (Final projections)

Fig. 9.18

Construction: Fig. 9.18

1. *First stage*: Draw a square *abcd* keeping *ac* parallel to *XY*. Locate the centre *o* and join it with all the corners of the square. This represents the top view.
2. Project the corners and obtain *a′c′o′* to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping *c′o′* perpendicular to *XY*.
4. Obtain points *a*, *b*, *c*, *d* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all edges as shown.

Example 9.13 (Fig 9.19)

A right circular cone with 50 mm diameter of base and 65 mm long axis rests on its base rim on H.P. with its axis parallel to V.P. and one of the generators perpendicular to H.P. Draw the projections of the cone.

[RGPV June 2009]

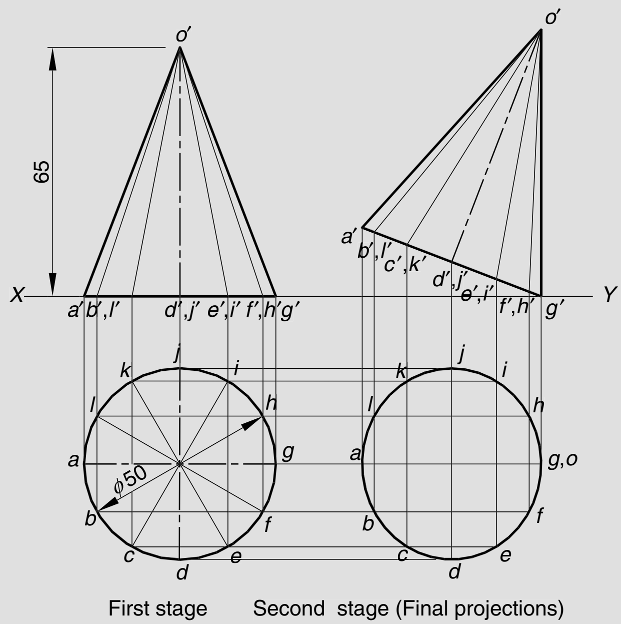


Fig. 9.19

Construction: Fig. 9.19

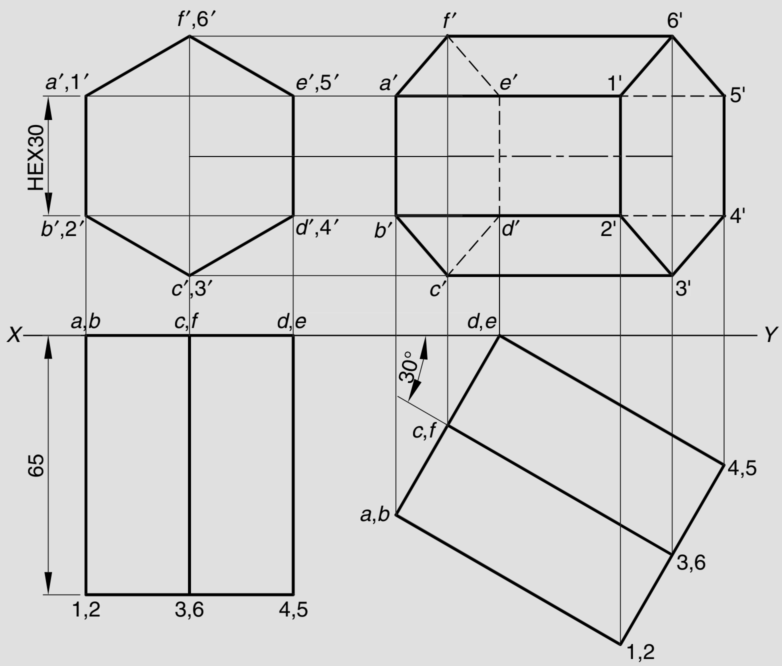
1. *First stage*: Draw a circle *adgj* to represent the top view. Divide this circle into 12 equal parts using radial lines.
2. Project the ends of the radial lines and obtain *a′g′o′* to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping point *g′* on the *XY* and the generator *g′o′* perpendicular to it.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible points with continuous curve as shown.

9.11 AXIS INCLINED TO V.P. AND PARALLEL TO H.P.

When the axis of a right solid is inclined (at an angle) to the V.P. and parallel to the H.P., then the projections are drawn in two stages. Consider the following solved Example.

Example 9.14 (Fig 9.20)

A hexagonal prism of 30 mm base edges and axis 65 mm long, has an edge of its base in the V.P. such that the axis is inclined at 30º to the V.P. and parallel to the H.P. Draw its projections.



First stage   Second stage (Final projections)

Fig. 9.20

Construction: Fig. 9.20

1. *First stage*: Consider edge *DE* of the hexagonal prism lies in the V.P. in the final position. Draw a hexagon $a'b'c'd'e'f'$ keeping side $d'e'$ perpendicular to *XY*, to represent the front view.
2. Project the corners and obtain *ad*41 to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage keeping point *ed* on *XY* and *d*4 inclined at 30º to it. This will make the axis at 30º to the V.P.
4. Obtain points a', b', c', d', e', f', 1′, 2′, 3′, 4′, 5′ and 6′ in the front view as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.15 (Fig 9.21)

A pentagonal pyramid of base 30 mm sides and axis 55 mm long, has a triangular face in the V.P. and the axis parallel to the H.P. Draw its projections.

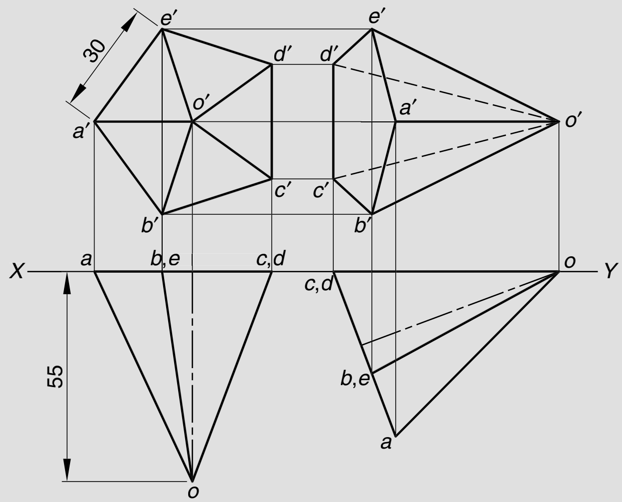


Fig. 9.21

Construction: Fig. 9.21

1. *First stage*: Draw a pentagon *a′b′c′d′e′* keeping side *c′d′* perpendicular to *XY*. Locate the mid-point *o′* and join it with the corners of the pentagon. This represents the front view.
2. Project all the corners and obtain *ado* to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage such that line *cdo* representing the triangular face is on *XY*.
4. Obtain points *a′*, *b′*, *c′*, *d′*, *e′* and *o′* as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.16 (Fig 9.22)

A cone with a 50 mm base diameter and 55 mm long axis, has a generator in the V.P. and the axis parallel to the H.P. Draw its projections.

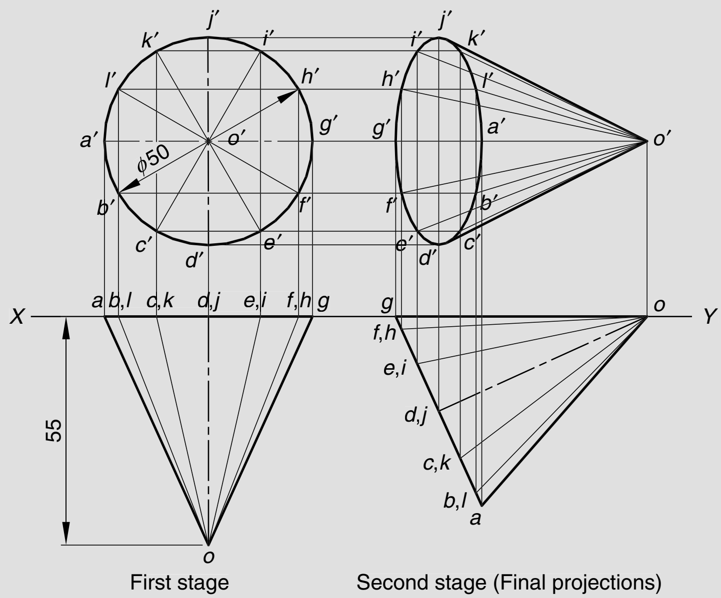


Fig. 9.22

Construction: Fig. 9.22

1. *First stage*: Draw a circle $a'd'g'j'$ to represent the front view. Divide this circle into 12 equal parts using radial lines.
2. Project the ends of the radial lines and obtain *aog* to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage such that slant edge *og* is on the *XY*.
4. Obtain points a', b', c', d', e', f', g', h', i', j', k', l' and o' as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join visible points with continuous curve to obtain ellipse and then join it with the tangents from o'. This represents the final front view.

9.12 MISCELLANEOUS EXAMPLES

Example 9.17 (Fig 9.23)

Draw the projections of a tetrahedron with 65 mm long edges lying on a face in the H.P. and an edge of that face is perpendicular to the V.P.

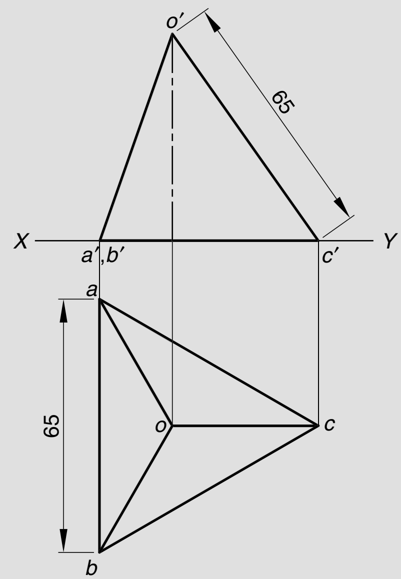


Fig. 9.23

Construction: Fig 9.23

1. Draw an equilateral triangle *abc* keeping *ab* perpendicular to *XY*. Locate *o* as its centroid and join it with all the corners *a*, *b* and *c*. This represents the top view.
2. Project points *a*, *b* and *c* on *XY* and obtain *a*′, *b*′ and *c*′.
3. As *oc* is parallel to *XY*, its front view *o*′*c*′ will be of true length. Therefore, draw an arc with point *c*′ as the centre and 65 mm radius to meet the projector of point *o* at point *o*′. Join *o*′*a*′, *o*′*b*′ and *o*′*c*′. This represents the required front view.

Example 9.18 (Fig 9.24)

A cylinder of 50 mm base diameter and 75 mm long axis, has a square hole of 25 mm side cut through it so that the axis of the hole coincides with that of the cylinder. The cylinder is lying on the ground with the axis perpendicular to the V.P. and the faces of the hole equally inclined to the H.P. Draw its projections.

[RGPV Dec. 2002]

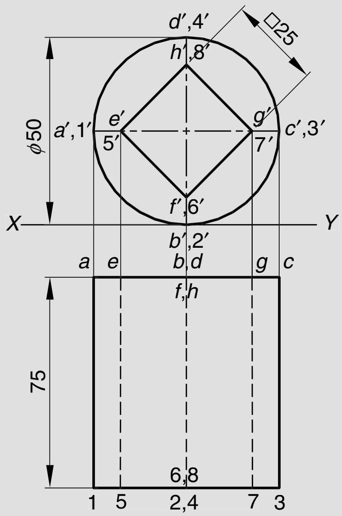


Fig. 9.24

Construction: Fig 9.24

1. Draw a circle $a'b'c'd'$ touching XY. Also draw a square $e'f'g'h'$ concentric to the circle keeping the sides at 45º to XY. This represents the front view.
2. Project all the points and obtain $ac31$ to represent the top view.

Example 9.19 (Fig 9.25)

Draw the projections of a pentagonal prism of 25 mm base side and 50 mm long axis, resting on one of its rectangular faces on the H.P. with the axis inclined at 45º to the V.P. [RGPV June 2008]

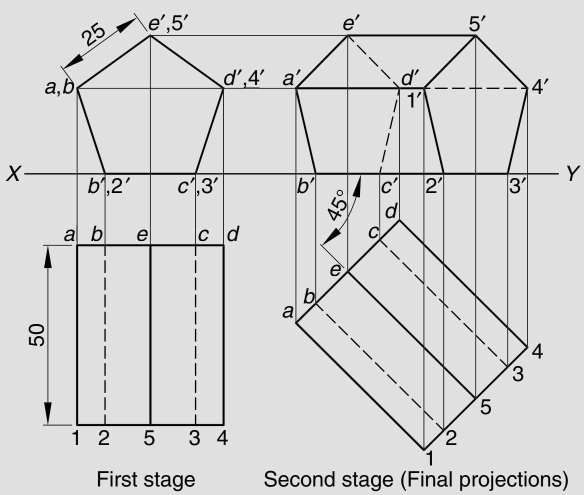


Fig. 9.25

Construction: Fig. 9.25

1. *First stage*: Draw a pentagon $a'b'c'd'e'$ keeping $b'c'$ on XY, to represent the front view.
2. Project the corners and obtain $ad41$ to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage such that $e5$ is inclined at 45º to XY. This will make the axis inclined at 45º to the V.P.
4. Obtain points a', b', c', d', e', $1'$, $2'$, $3'$, $4'$ and $5'$ in the front view as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.20 (Fig 9.26)

A hexagonal pyramid of 25 mm side of base and 60 mm long axis, is resting on an edge of the base on H.P. Draw the projections of the solid when the axis makes an angle of 45º with V.P. and the base of the solid is nearer to V.P. [RGPV Aug. 2010]

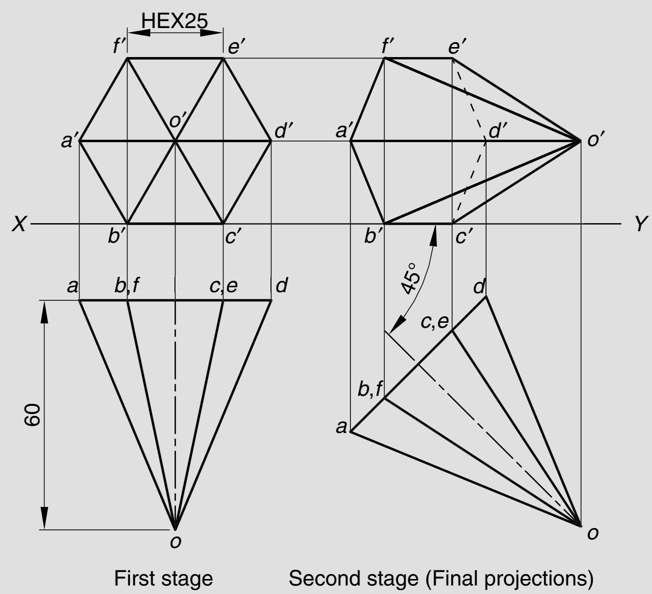


Fig. 9.26

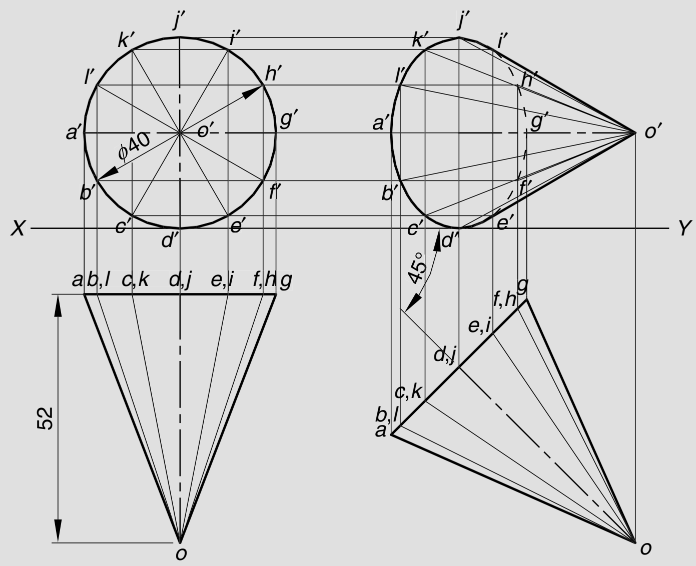
Construction: Fig. 9.26

1. *First stage*: Draw a hexagon $a'b'c'd'e'f'$ keeping $b'c'$ on XY. Locate the centre point o of the hexagon and join it with all the corners. This represents the front view.
2. Project the corners and obtain ado to represent the top view.

3. *Second stage*: Reproduce the top view of the first stage such that *ad* is inclined at 45º to *XY*. This will make the axis inclined at 45º to the V.P. It should be noted that base *ad* should be nearer to the *XY* than the apex *o*.
4. Obtain points *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *o′* in the front view as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.21 (Fig 9.27)

A right circular cone with 40 mm diameter of base and 52 mm height is held on ground such that its axis is inclined at 45º to V.P. and is parallel to the H.P. Draw its projections when its apex is away from the V.P. [RGPV April 2009]



First stage Second stage (Final projections)

Fig. 9.27

Construction: Fig. 9.27

1. *First stage*: Consider the cone held with corner *D* on the ground in the final position. Draw a circle *a′d′g′j′* to represent the front view. Divide this circle into 12 equal parts using radial lines.
2. Project the ends of the radial lines and obtain ago to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage keeping *g* on the *XY* and *do* inclined at 45º to *XY*. It should be noted that the apex *o* should be away to the *XY* than base *ag*.

4. Obtain points a', b', c', d', e', f', g', h', i', j', k', l' and o' as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join visible points with continuous curve and hidden points with dotted curve as shown.

Example 9.22 (Fig 9.28)

A pentagonal pyramid of 30 mm base sides and 60 mm long axis has an edge of base parallel to H.P. Its axis is parallel to V.P. and inclined at 45º to the H.P. Draw its projections when the apex lies in the H.P.

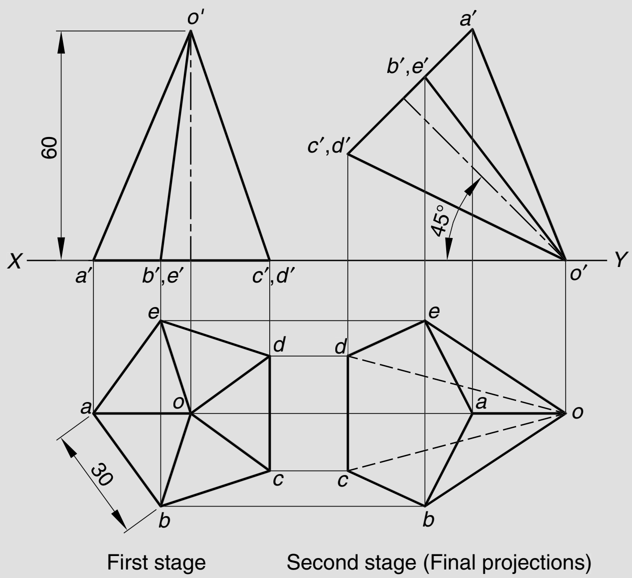


Fig. 9.28

Construction: Fig. 9.28

1. *First stage*: Draw a pentagon *abcde* keeping *cd* perpendicular to *XY*. Locate the centroid *o* and join it with all the corners of the pentagon. This represents the top view.
2. Project the corners and obtain $a'd'o'$ to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage such that apex o' is on *XY* and the axis is inclined at 45º to *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dotted lines, following the rules of visibility.

Example 9.23 (Fig 9.29)

A tetrahedron of 70 mm long edge has an edge on the ground and the faces containing that edge are equally inclined to the H.P. Draw its projections when the edge lying on the ground is perpendicular to the V.P.

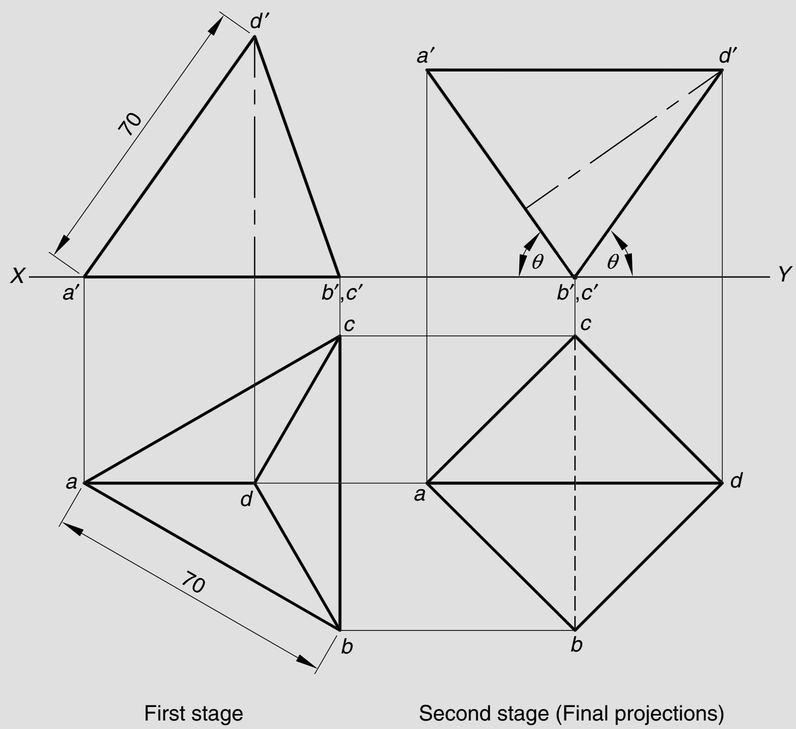


Fig. 9.29

Construction: Fig. 9.29

1. *First stage*: Draw an equilateral triangle *abc* keeping *bc* perpendicular to *XY*. Locate the centroid *d* and join it with all the corners of the tetrahedron. This represents the top view.
2. Project points *a*, *b* and *c* on *XY* and obtain *a′*, *b′* and *c′*. As *ad* is parallel to *XY*, its front view *a′d′* will be of true length. Therefore, draw an arc with point *d′* as the centre and 70 mm radius to meet the projector of point *d* at point *d′*. Join *d′a′*, *d′b′* and *d′c′*.
3. *Second stage*: Reproduce the front view of the first stage such that *b′c′* remains on *XY* and the inclination of faces *a′b′c′* and *b′c′d′* with *XY* are equal.
4. Obtain points *a*, *b*, *c* and *d* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible edges with continuous lines and hidden edges with dotted lines.

Example 9.24 (Fig 9.30)

A hexagonal prism of side of base 25 mm and 60 mm long axis is freely suspended from a corner of the top base. Draw the projections of the prism in suspended position.

[RGPV Feb. 2008]

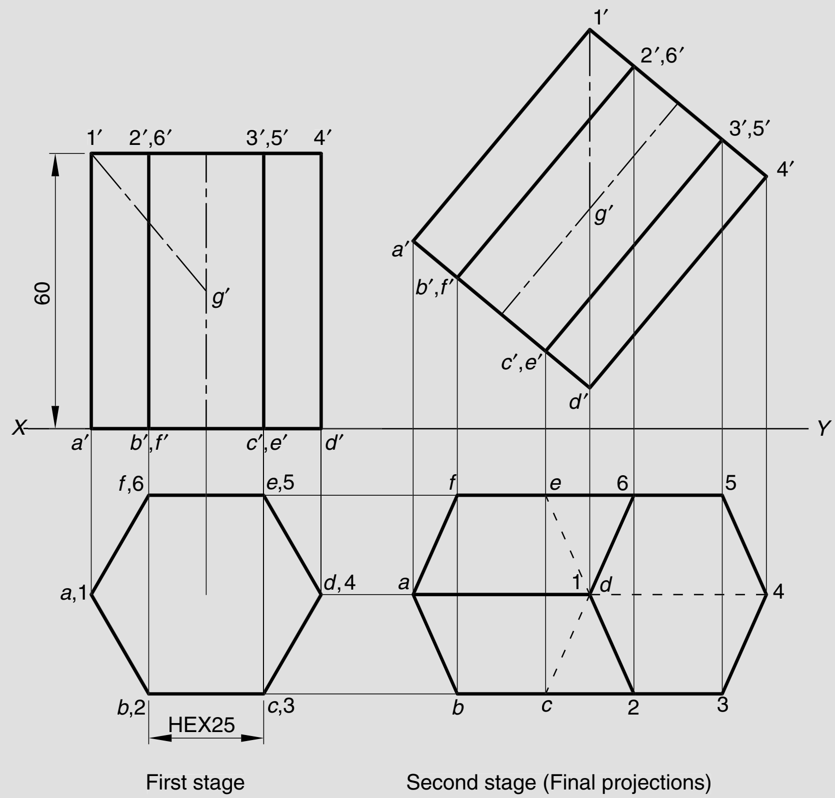


Fig. 9.30

Construction: Fig. 9.30

1. *First stage*: Draw a hexagon *abcdef* keeping *ad* parallel to *XY*, to represent the top view.
2. Project all the corners and obtain *a′d′*4′1′ to represent the front view. Mark *g′* as the mid-point of the axis to represent the centre of gravity (C.G.) of the prism.
3. *Second stage*: Reproduce the front view of the first stage such that line 1′*g′* is perpendicular to *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f*, 1, 2, 3, 4, 5 and 6 in the top view as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines.

Note: Centre of gravity (g') of prisms and cylinders lie on the mid-point of the axis while that of the pyramids and cones lie on its axis at a distance ¼ of the axis from the base of the solid. On suspending the solid, line joining the point of suspension and centre of gravity (g') becomes perpendicular to the ground.

Example 9.25 (Fig 9.31)

A hexagonal prism of 40 mm base sides and 40 mm long axis has a centrally drilled circular hole of 40 mm diameter. Draw its projections when the prism is resting on an edge of its base on the H.P. and the axis inclined at 60º to the H.P. and parallel to the V.P.

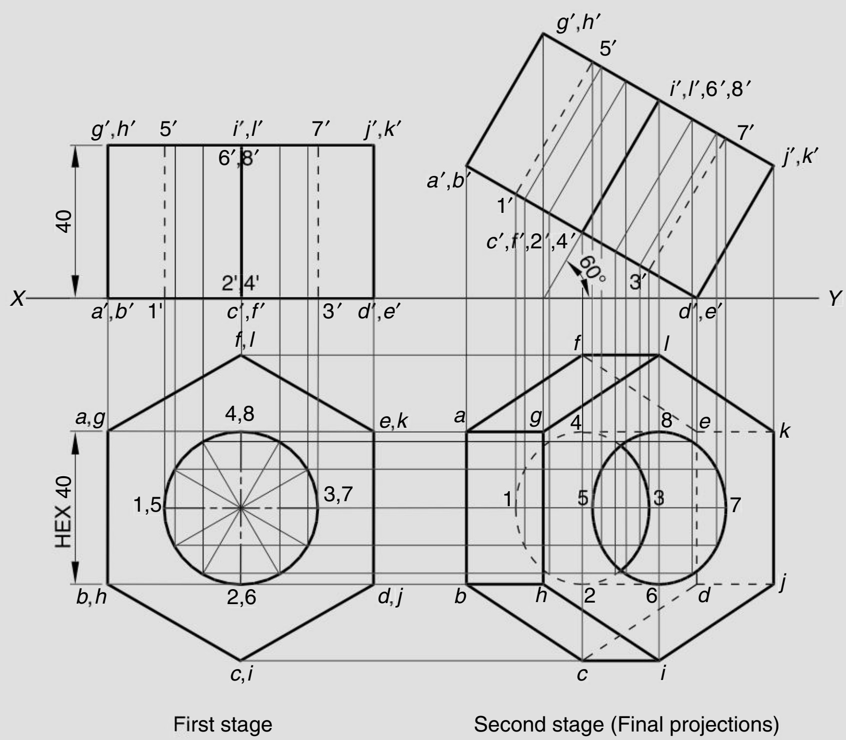


Fig. 9.31

Construction: Fig. 9.31

1. *First stage*: Draw a hexagon *abcdef* keeping *de* perpendicular to *XY*. Draw a circle 1234 concentric to the hexagon to represent the hole. The figure represents the top view.
2. Project all the corners and obtain $a'd'j'g'$ to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping $d'e'$ on *XY* and line $d'j'$ is inclined at 60º to *XY*. This will make axis at 60º to H.P.

4. Obtain points *a*, *b*, *c*, etc. and 1, 2, 3, etc., in the top view as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines.

Example 9.26 (Fig 9.32)

Draw three views of the frustum of a hexagonal pyramid, with 35 mm edges of base, 20 mm edges of top and 45 mm long axis, has one of its faces on the H.P. and axis parallel to V.P.

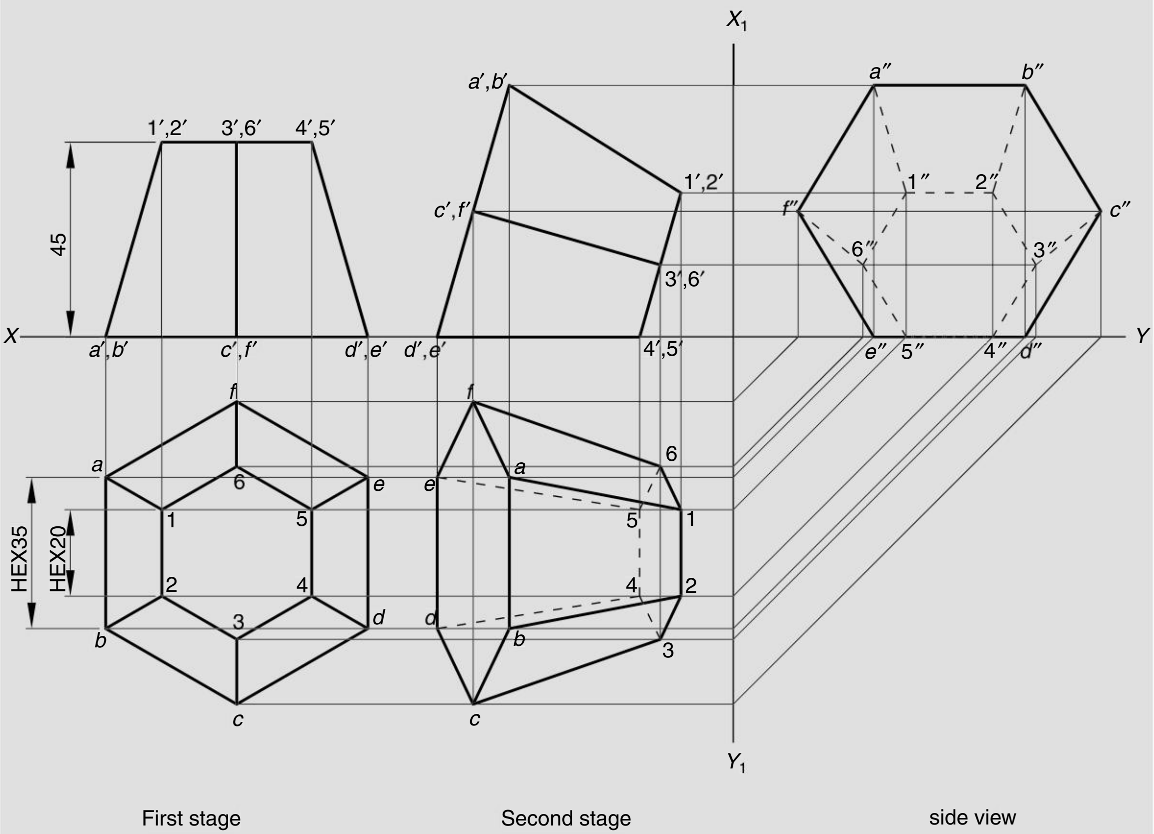


Fig. 9.32

Construction: Fig. 9.32

1. *First stage*: Consider that frustum would lie on its face *DE*54 on the H.P. in the final stage. Draw two concentric hexagons *abcdef* and 123456 with sides *de* and 45 perpendicular to *XY*. Join 1*a*, 2*b*, 3*c*, 4*d*, 5*e* and 6*f*. This represents the top view.

2. Project all the corners and obtain $a'd'4'1'$ to represent the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping $d'e'5'4'$ on *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f*, 1, 2, 3, 4, 5 and 6 in the top view as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join all the visible edges with continuous lines and hidden edges with dotted lines.
5. *Side view*: Draw lines parallel to *xy* line from the top view to meet X_1Y_1. Thereafter rotate them through 90º and produce to meet the horizontal line from the front view at points a'', b'', c'', d'', e'', $1''$, $2''$, $3''$, $4''$ and $5''$. Join the points as shown. This is the required side view.

EXERCISE 9A

Simple Position

1. Draw the projections of a pentagonal prism, with 30 mm base edges and 50 mm long axis, resting on one of its bases on the H.P. with a vertical face parallel to and 20 mm in front of V.P.
2. Draw the projections of a triangular pyramid, with 45 mm base edges and 50 mm long axis, having its base on the H.P. and an edge of the base perpendicular to the V.P. Also draw its side view.
3. A right circular cylinder having 40 mm diameter of base and 60 mm height is resting on H.P. on its base rim such that its axis is parallel to V.P. Draw its projections.
[RGPV Dec. 2007]
4. A right circular cone with 50 mm diameter of base and 62 mm height lies on the H.P. on one of its elements with axis parallel to V.P. Draw the projections of the cone.
[RGPV Feb. 2007]
5. A cube of 40 mm side is resting with a face on H.P. such that the vertical faces are equally inclined to V.P. Draw its projections. **[RGPV Feb. 2010]**
6. A tetrahedron with 50 mm long edges is resting on the H.P. on one of its faces such that an edge of that face is parallel to the V.P. Draw its projections and measure the distance of its apex from the H.P.
[Ans: 41 mm]
7. Draw the projections of a pentagonal prism, 30 mm base sides and 60 mm long axis, lying on one of its rectangular faces on the H.P. with axis perpendicular to the V.P.
8. A hexagonal pyramid with 25 mm base sides and 50 mm long axis, has its base in the V.P. and an edge of the base inclined at 45º to the H.P. The axis is 35 mm above the H.P. Draw its projections.
9. A hexagonal prism, with 25 mm base sides and 60 mm long axis, is resting on one of its rectangular faces on the H.P. with the axis perpendicular to the V.P. A right circular cone with 40 mm base diameter and 45 mm long axis is placed centrally on the top of the prism. Draw the projections of the composite solid.

Axis Inclined to One of the Reference Planes

10. A hexagonal prism with 30 mm base sides and 70 mm long axis rests on an edge of the base on the H.P. The axis is parallel to the V.P. and inclined at 45º to the H.P. Draw its projections.
11. A square prism with 30 mm base sides and 50 mm long axis, is resting on one of its sides of the base on the H.P. while the plane of the base is inclined at 60º to the H.P. and perpendicular to the V.P. Draw its projections.
12. A pentagonal prism with 30 mm base sides and 75 mm long axis, has a corner of its base on the ground and axis is inclined at 60º to the H.P. Draw its projections, if the plane containing that corner and the axis is parallel to the V.P.
13. A pentagonal pyramid with 30 mm base sides and 70 mm long axis, has an edge of its base on the H.P. Draw its projections when the axis is inclined at 60º to the H.P. and parallel to the V.P.
14. A right regular pentagonal pyramid side of base 20 mm and height 45 mm, rests on a corner of its base on H.P. such that its axis is inclined at 45º to the H.P. and is parallel to the V.P. Draw its projections. **[RGPV Jun. 2007]**
15. Draw three views of a square pyramid of 40 mm base side and 70 mm long axis, which is resting on a corner of its base on the H.P. and the slant edge containing that corner is vertical.
16. A cone with a 50 mm base diameter and 70 mm height is resting on a point of its base rim on the H.P. while the axis is parallel to the V.P. and inclined at 45º to the H.P. Draw its projections.
17. A hexagonal pyramid with 30 mm base sides and 70 mm long axis lies on one of its triangular faces in the H.P. and axis parallel to the V.P. Draw its three views.
18. A hexagonal pyramid with 30 mm base sides and 75 mm long axis, rests on an edge of its base on the V.P. with a triangular face containing that edge inclined at 45º to the V.P. and perpendicular to the H.P. Draw its projections.
19. A square pyramid with 40 mm base edges and 70 mm long axis is resting on a triangular face on the V.P. Draw its projections when the axis is parallel to and 30 mm above the H.P.
20. A cone with 50 mm base diameter and 65 mm long axis, lies on one of its generators in the V.P. and axis parallel to the H.P. Draw its projections.
21. A hexagonal prism with 30 mm base sides and 70 mm long axis, rests on a rectangular face on the H.P. such that the longer edges of the face are inclined at 60º with the V.P. Draw its projections.
22. Draw the projections of a cylinder with 75 mm diameter and 100 mm length lying on the ground with its axis inclined at 30º to the V.P. and parallel to the ground. **[RGPV June 2007, April 2009]**
23. A square pyramid of side of base 30 mm and axis 50 mm long is freely suspended from a corner of its base. Draw its projections. **[RGPV Feb. 2011]**
24. A pentagonal prism of 30 mm base side and 70 mm long axis is standing on a corner of the base on the ground with longer edge containing that corner inclined at 45º to the H.P. Draw its projections if the face opposite to that longer edge is perpendicular to the V.P.
25. A cylindrical block of 75 mm base diameter and 25 mm thickness, has an axially drilled hexagonal hole of 25 mm side. Draw the projections of the block when it has its flat faces vertical and inclined at 45º to the V.P. and two faces of the hole are parallel to the H.P.

9.13 AXIS INCLINED TO BOTH THE REFERENCE PLANES

When the axis of a solid is inclined to both the reference planes, the projections are drawn in three stages. It is an extension of the examples on projections of solids inclined to one of the reference planes.

9.13.1 Solid Rests on Its Edge in the H.P. with Its Axis Inclined (at θ) to H.P. and the Resting Edge is Inclined (at ϕ) to V.P.

Example 9.27 (Fig 9.33)

A square prism, 25 mm edge of base and 45 mm long axis, has its axis inclined at 45º to the H.P. and an edge of its base on which the prism rests is inclined at 30º to the V.P. Draw its projections. [RGPV June 2005]

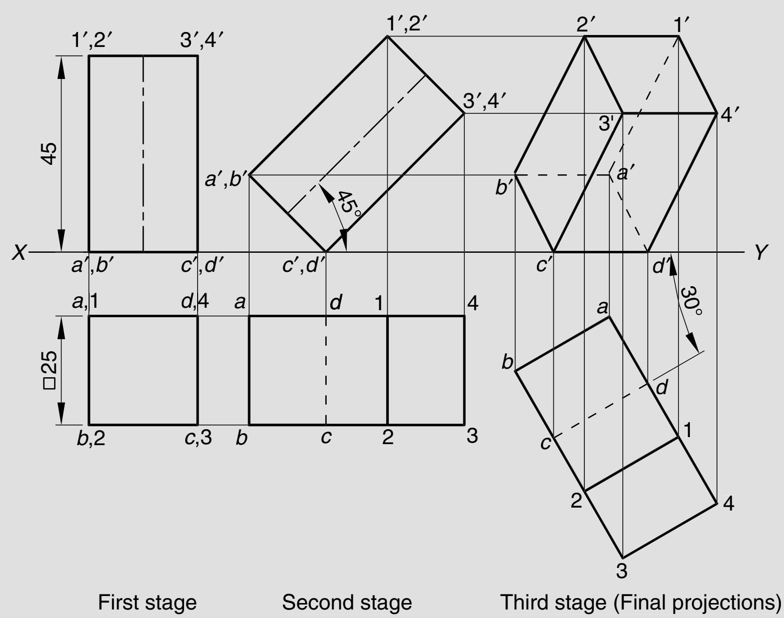


Fig. 9.33

Construction: Fig. 9.33

1. *First stage*: Draw a square *abcd* keeping *cd* perpendicular to *XY*, to represent the top view.
2. Project the corners and obtain $b'c'3'2'$ as the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping $c'd'$ on *XY* and $c'd'3'4'$ inclined at 45º to *XY*. This will make the axis incline at 45º to *XY*.

4. Obtain points *a*, *b*, *c*, *d*, 1, 2, 3 and 4 as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *ab*34 as the top view.
5. *Third stage*: Reproduce the top view of the second stage keeping *cd* is inclined at 30º with *XY*.
6. Obtain points *a′*, *b′*, *c′*, *d′*, 1′, 2′, 3′ and 4′ as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain *b′c′d′*4′1′2′ as the required front view.

Example 9.28 (Fig 9.34)

A right pentagonal prism, 60 mm high with each side of the base 30 mm is resting on one of the edges on the horizontal plane and inclined at 30º to V.P. and the face containing that edge is inclined at 45º to H.P. Draw the projections of the pentagonal prism. [RGPV June 2004]

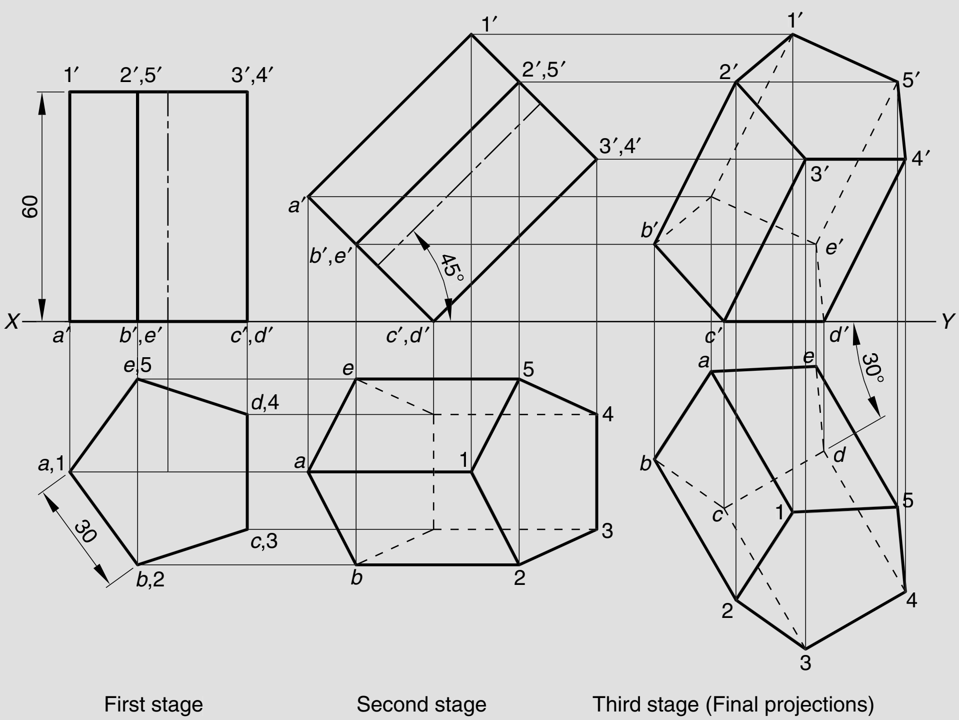


Fig. 9.34

Construction: Fig. 9.34

1. *First stage*: Draw a pentagon *abcde* keeping *cd* perpendicular to *XY*, to represent the top view.
2. Project the corners and obtain *a′c′*3′1′ as the front view.

3. *Second stage*: Reproduce the front view of the first stage keeping $c'd'$ on XY and $c'd'3'4'$ inclined at 45º to XY. This will make axis at 45º to XY.
4. Obtain points a, b, c, d, e, 1, 2, 3, 4 and 5 as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain $ab2345e$ as the top view.
5. *Third stage*: Reproduce the top view of the second stage keeping cd is inclined at 30º with XY.
6. Obtain points a', b', c', d', e', $1'$, $2'$, $3'$, $4'$ and $5'$ as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain $b'c'd'4'5'1'2'$ as the required front view.

Example 9.29 (Fig 9.35)

A hexagonal pyramid of 30 mm base side and 60 mm long axis has an edge of its base on the ground and the axis inclined at 30º to the H.P. The edge of the base on which it rests is inclined at 45º to the V.P. Draw its projections.

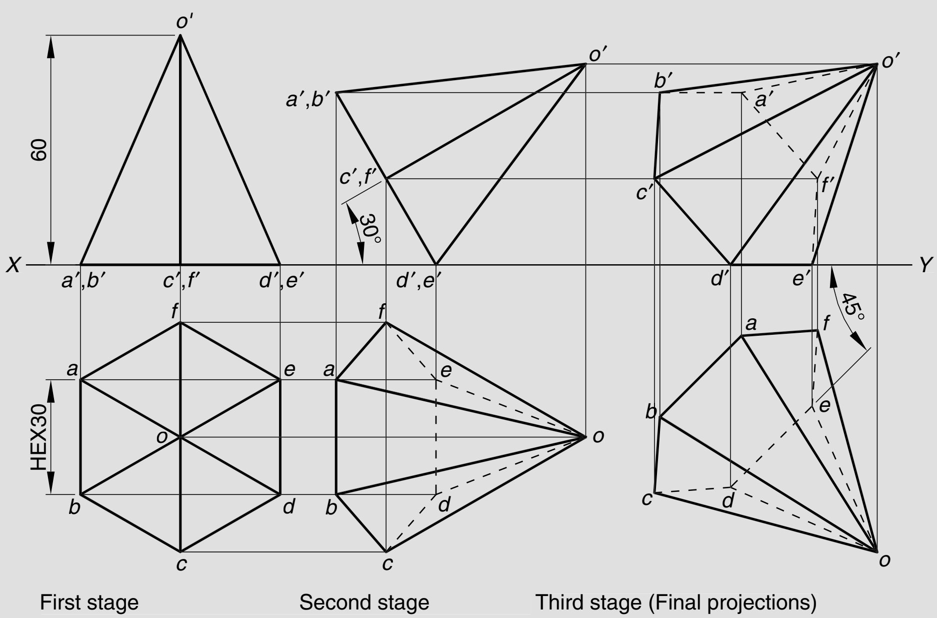


Fig. 9.35

Construction: Fig. 9.35

1. *First stage*: Draw a hexagon *abcdef* keeping *de* perpendicular to XY. Also join the diagonal lines. This represents the top view.
2. Project the corners and obtain $b'd'o'$ as the front view.

3. *Second stage*: Reproduce the front view of the first stage keeping *d′e′* on *XY* and *b′d′* at 60º to it. This will make the axis incline at 30º to *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *abcof* as the top view.
5. *Third stage*: Reproduce the top view of the second stage keeping *de* is inclined at 45º with *XY*.
6. Obtain points *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *o′* as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain *b′c′d′e′o′* as the required front view.

Example 9.30 (Fig 9.36)

A pentagonal pyramid of 30 mm base side and 60 mm long axis rests on an edge of its base on the ground so that the highest point of the base is 20 mm above the ground. Draw its projections, if the vertical plane containing the axis is inclined at 30º to the V.P.

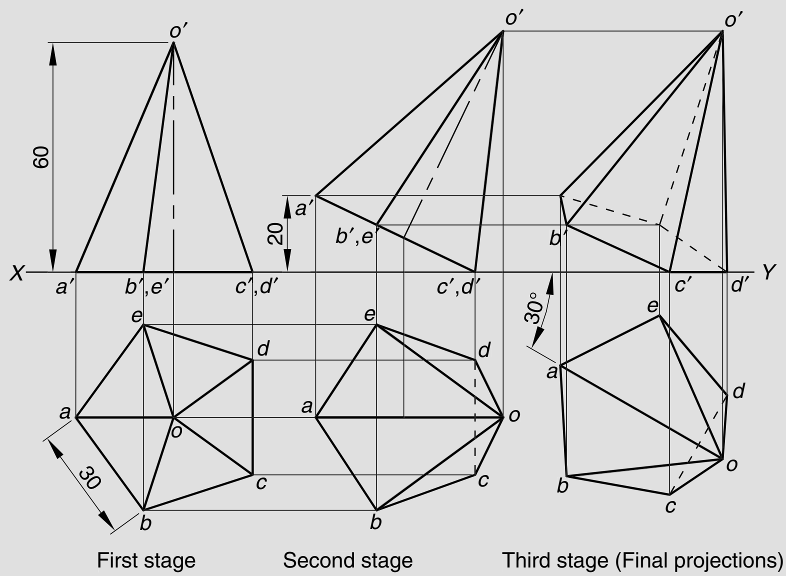


Fig. 9.36

Construction: Fig. 9.36

1. *First stage*: Draw a pentagon *abcde* keeping *cd* perpendicular to *XY*. Locate centroid *o* and join it with all the corners of the pentagon. This represents the top view.
2. Project the corners and centroid to obtain *a′c′o′* as the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping *c′d′* on *XY* and *a′* 20 mm above it. (For this, draw a horizontal line 20 mm above *XY*. Draw an arc with *c′d′* as the centre and radius equal to *a′c′* of the first stage to meet the horizontal line at *a′*.)

4. Obtain points *a*, *b*, *c*, *d*, *e* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *abcode* as the top view.
5. *Third stage*: Reproduce the top view of the second stage such that *ao* is inclined at 30º with *XY*.
6. Obtain points *a′*, *b′*, *c′*, *d′*, *e′* and *o′* as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain *a′b′c′d′o′* as the required front view.

Example 9.31 (Fig 9.37)

A pentagonal pyramid, with 25 mm side of base and 60 mm height has one of its slant faces on the horizontal plane and the edge of the base contained by that slant face makes an angle of 30º to the V.P. Draw its projections. [RGPV Dec. 2003]

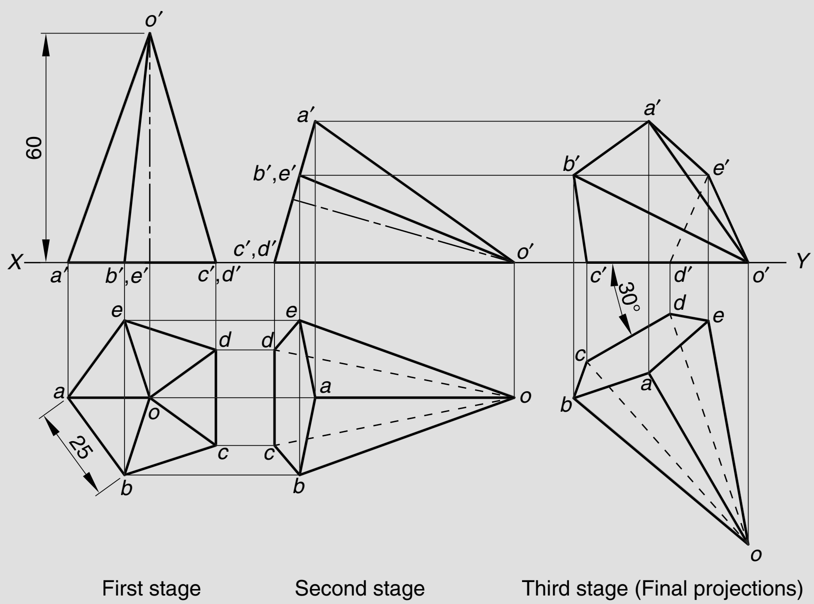


Fig. 9.37

Construction: Fig. 9.37

1. *First stage*: Draw a pentagon *abcde* keeping *cd* perpendicular to *XY*. Locate centroid *o* and join it with all the corners of the pentagon. This represents the top view.
2. Project the corners and centroid to obtain *a′c′o′* as the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping *c′d′o′* on *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *bcdeo* as the top view.

5. *Third stage*: Reproduce the top view of the second stage such that *cd* is inclined at 30º with *XY*.
6. Obtain points *a′*, *b′*, *c′*, *d′*, *e′* and *o′* as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain *a′b′c′d′o′e′* as the required front view.

Example 9.32 (Fig 9.38)

A hexagonal pyramid of 30 mm base side and 60 mm axis length is kept with a side of base parallel to the V.P. and the triangular face containing that side being vertical. Draw the projections of the solid. [RGPV Sep. 2009]

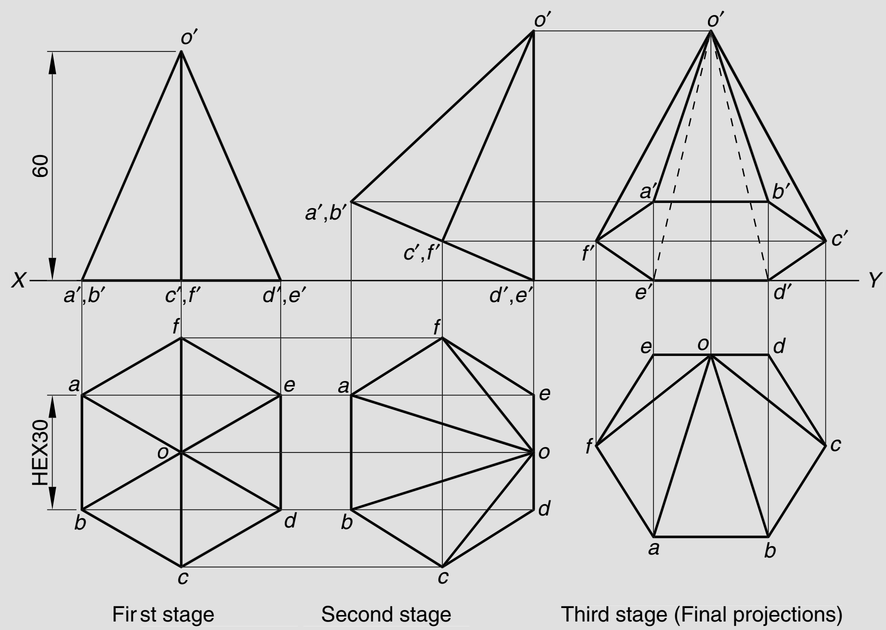


Fig. 9.38

Construction: Fig. 9.38

1. *First stage*: Draw a bexagon *abcdef* keeping *de* perpendicular to *XY*. Locate centroid *o* and join it with all the corners of the hexagon. This represents the top view.
2. Project the corners and centroid to obtain *b′d′o′* as the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping *d′e′* on *XY* and *d′e′o′* is perpendicular to *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f* and *o* as the intersecting points of the vertical projectors drawn from the corners of the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *abcdoef* as the top view.

5. *Third stage*: Reproduce the top view of the second stage such that *de* is parallel to *XY*.
6. Obtain points a', b', c', d', e', f' and o' as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain $c'd'e'f'o'$ as the required front view.

9.13.2 Solid Rests on Its Corner in the H.P. (or Ground) with Its Axis Inclined (at θ) to H.P. and Vertical Plane Containing the Axis and that Corner is Inclined (at $\varnothing$) to V.P.

Example 9.33 (Fig 9.39)

A hexagonal pyramid, having a base of 30 mm side and a 50 mm long axis, rests on one of its base corners on the ground with axis inclined at 45º to the H.P. Draw its projections when the vertical plane containing the axis and the corner that lies in the H.P. is inclined at 30º to the V.P.

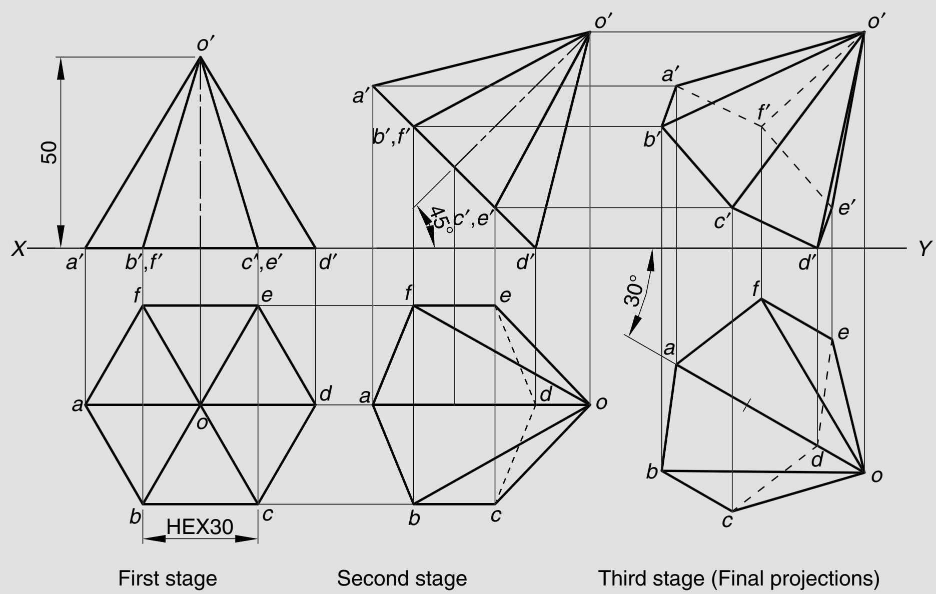


Fig. 9.39

Construction: Fig. 9.39

1. *First stage*: Draw a hexagon *abcdef* keeping *ad* parallel to *XY*. Locate centroid *o* and join it with all the corners of the pentagon. This represents the top view.
2. Project the corners and centroid to obtain $a'd'o'$ as the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping d' on *XY* and $a'd'$ inclined at 45º to *XY*. This will make the axis incline at 45º to *XY*.

4. Obtain points *a*, *b*, *c*, *d*, *e*, *f* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *abcoef* as the top view.
5. *Third stage*: Reproduce the top view of the second stage such that *ao* is inclined at 30º to *XY*.
6. Obtain points *a′*, *b′*, *c′*, *d′*, *e′*, *f′* and *o′* as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain *a′b′c′d′e′o′* as the required front view.

Example 9.34 (Fig 9.40)

A hexagonal pyramid of 25 mm base side and 55 mm long axis, has one of its slant edges on the ground. A plane containing that edge and the axis is perpendicular to H.P. and inclined at 45º to V.P. Draw its projections, when the apex is nearer to V.P. then the base.

[RGPV Dec. 2004, Feb. 2005, June 2008]

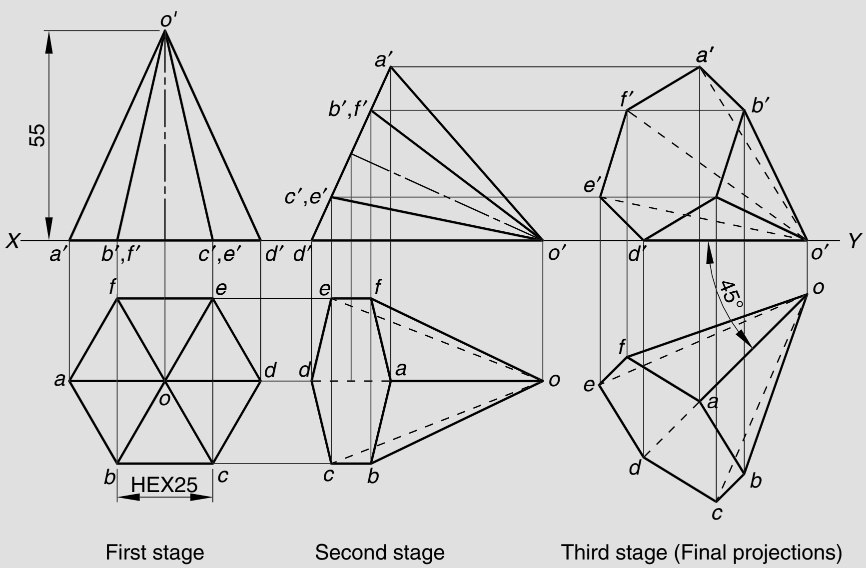


Fig. 9.40

Construction: Fig. 9.40

1. *First stage*: Draw a hexagon *abcdef* keeping *ad* parallel to *XY*. Locate centroid *o* and join it with all the corners of the hexagon. This represents the top view.
2. Project the corners and centroid to obtain *a′d′o′* as the front view.
3. *Second stage*: Reproduce the front view of the first stage keeping *d′o′* on *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e*, *f* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *abcoef* as the top view.

5. *Third stage*: Reproduce the top view of the second stage such that *oad* is inclined at 45º to *XY*.
6. Obtain points *a*′, *b*′, *c*′, *d*′, *e*′, *f*′ and *o*′ as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain *a*′*b*′*c*′*d*′*e*′*o*′ as the required front view.

Example 9.35 (Fig 9.41)

A cylinder of 50 mm base diameter and 65 mm long axis rests on a point of its base circle on the H.P. Draw its projections when the axis is making an angle of 30º with H.P. and top view of the axis is perpendicular to V.P.

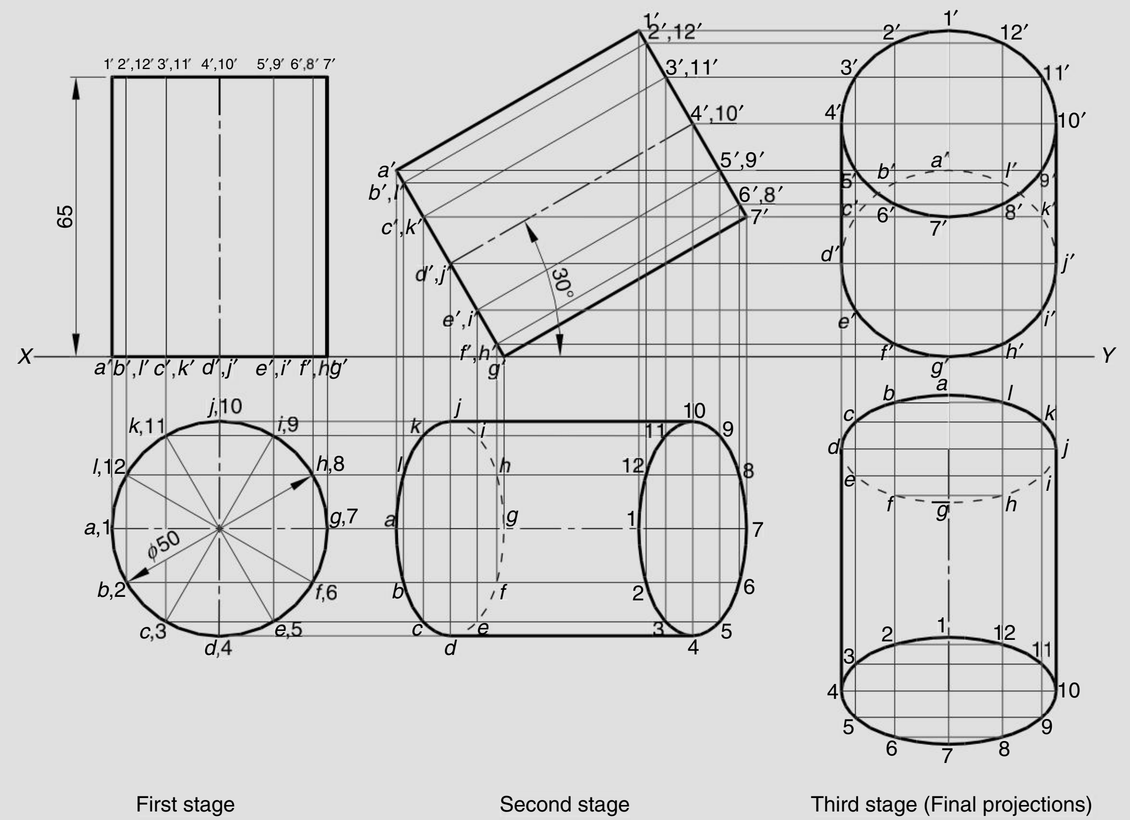


Fig. 9.41

Construction: Fig. 9.41

1. *First stage*: Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts using radial lines.
2. Project all the points and join *a*′*g*′7′1′ to represent the front view.

3. *Second stage*: Reproduce the front view of the first stage keeping g' on XY and $d'7'$ is inclined at 30º to XY. This will make axis to incline at 30º to XY.
4. Obtain points a, b, c, etc., and 1, 2, 3, etc., as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible points with continuous curve/lines as shown.
5. *Third stage*: Reproduce the top view of the second keeping $ag17$ perpendicular to XY.
6. Obtain points a', b', c', etc., and $1'$, $2'$, $3'$, etc., as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain the required front view.

9.13.3 Solid Rests on an Edge of Base in V.P. (or Parallel to V.P.) with Axis Inclined (at ϕ) to V.P. and the Resting Edge Inclined (at θ) to H.P.

Example 9.36 (Fig 9.42)

A pentagonal prism of 35 mm base side and 70 mm long axis has its axis inclined at 30º to the V.P. An edge of its base is in the V.P. and inclined at 45º to the H.P. Draw its projections.

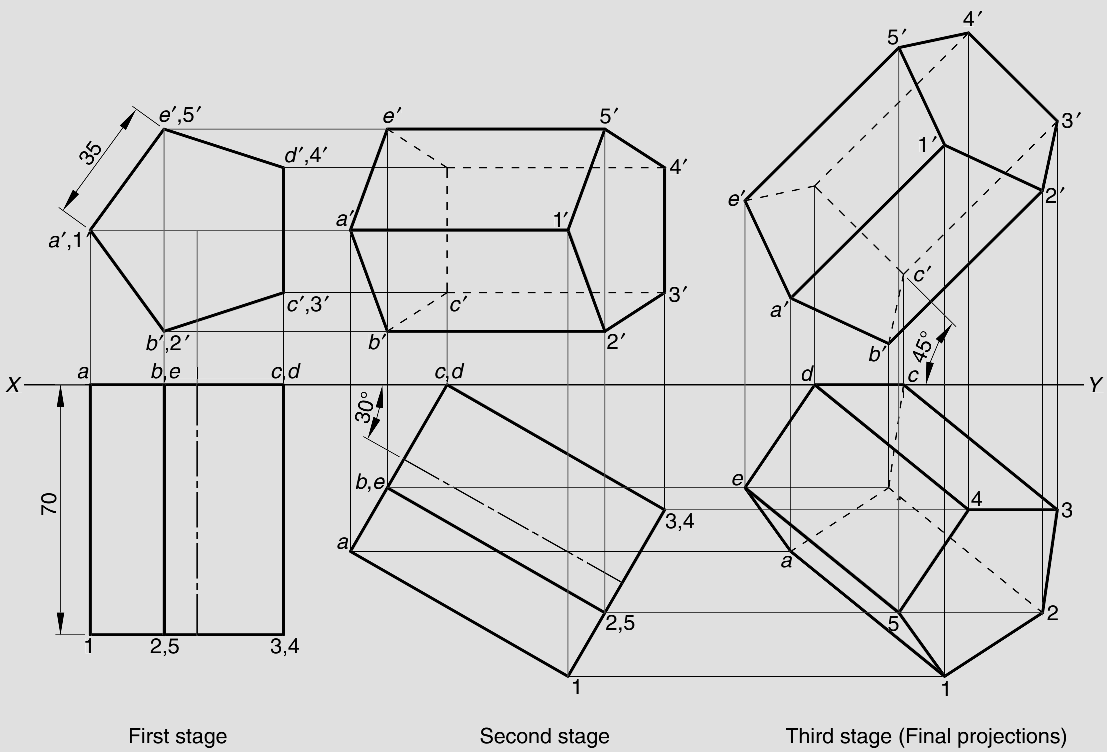


Fig. 9.42

Construction: Fig. 9.42

1. *First stage*: Draw a pentagon *a′b′c′d′e′* keeping *c′d′* perpendicular to *XY*, to represent the front view.
2. Project the corners and obtain *ac*31 to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage keeping *cd* on *XY* and *cd*34 at 30º to it. This will also make the axis at 30º to *XY*.
4. Obtain points *a′*, *b′*, *c′*, *d′*, *e′*, 1′, 2′, 3′, 4′ and 5′ as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join these points and obtain *a′b′*2′3′4′5′*e′* as the front view.
5. *Third stage*: Reproduce the front view of the second stage keeping *c′d′* at 30º with *XY*.
6. Obtain points *a*, *b*, *c*, *d*, *e*, 1, 2, 3, 4 and 5 as the intersecting points of the vertical projectors drawn from the front view of the third stage with the horizontal projectors drawn from the top view of the second stage. Join these points and obtain a123*cde* as the required top view.

Example 9.37 (Fig 9.43)

A hexagonal prism of 25 mm base edge and 60 mm long axis has an edge of its base in the V.P. and inclined at 60º to the H.P. Draw its projections, when the edge of the other base farthest away from V.P. is at a distance of 70 mm from the V.P.

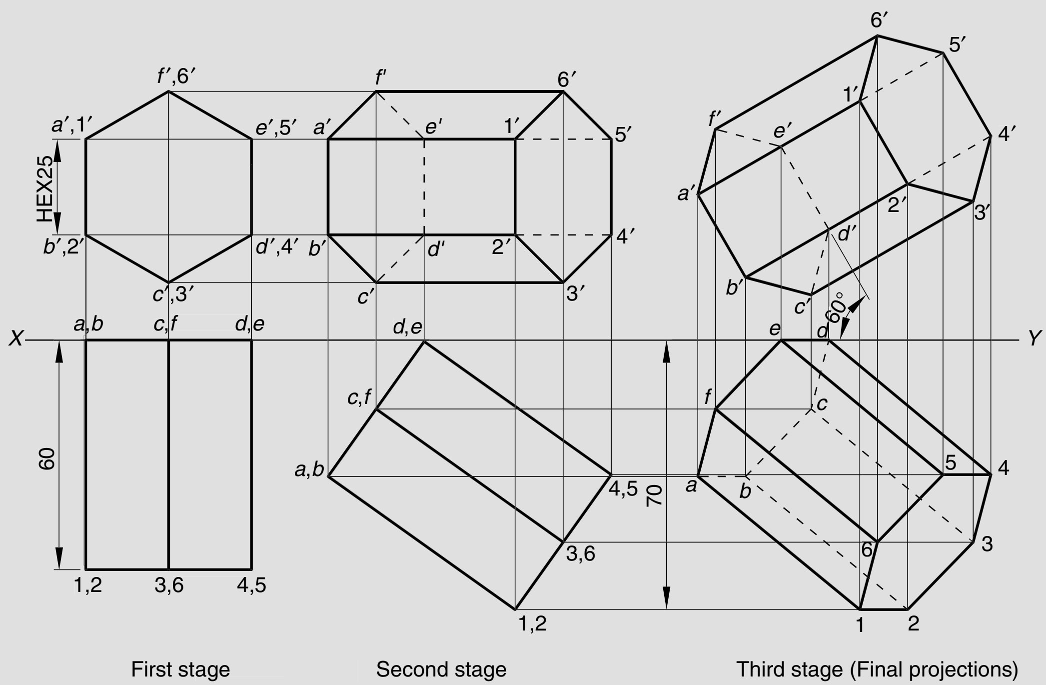


Fig. 9.43

Construction: Fig. 9.43

1. *First stage*: Draw a hexagon *a′b′c′d′e′f′* keeping *d′e′* perpendicular to *XY*, to represent the front view.

2. Project the corners and obtain *ad*41 to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage keeping *de* on *XY* and 12 at a distance of 70 mm from *XY*. (For this, draw a horizontal line at a distance of 70 mm. Now draw an arc with 12 as the centre and radius *d*1 of the first stage to meet the horizontal line at point 1.)
4. Obtain points *a′*, *b′*, *c′*, *d′*, *e′*, *f′*, 1′, 2′, 3′, 4′, 5′ and 6′ as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join the points and obtain *a′b′*2′3′4′5′6′*f′* as the front view.
5. *Third stage*: Reproduce the front view of the second stage keeping *d′e′* at 60º with *XY*.
6. Obtain points *a*, *b*, *c*, *d*, *e*, *f*, 1, 2, 3, 4, 5 and 6 as the intersecting points of the vertical projectors drawn from the front view of the third stage with the horizontal projectors drawn from the top view of the second stage. Join the points and obtain *a*1234*def* as the required top view.

9.13.4 Solid Rests on a Corner in the V.P. with Its Axis Inclined (at ϕ) to V.P. and the Plane Containing the Axis and that Corner is Inclined (at θ) to H.P.

Example 9.38 (Fig 9.44)

A square pyramid of 40 mm base side and 75 mm long axis has a corner of its base on the V.P. A slant edge contained by that corner is inclined at 45º to the V.P. and the plane containing the slant edge and the axis is inclined at 60º to the H.P. Draw its projections.

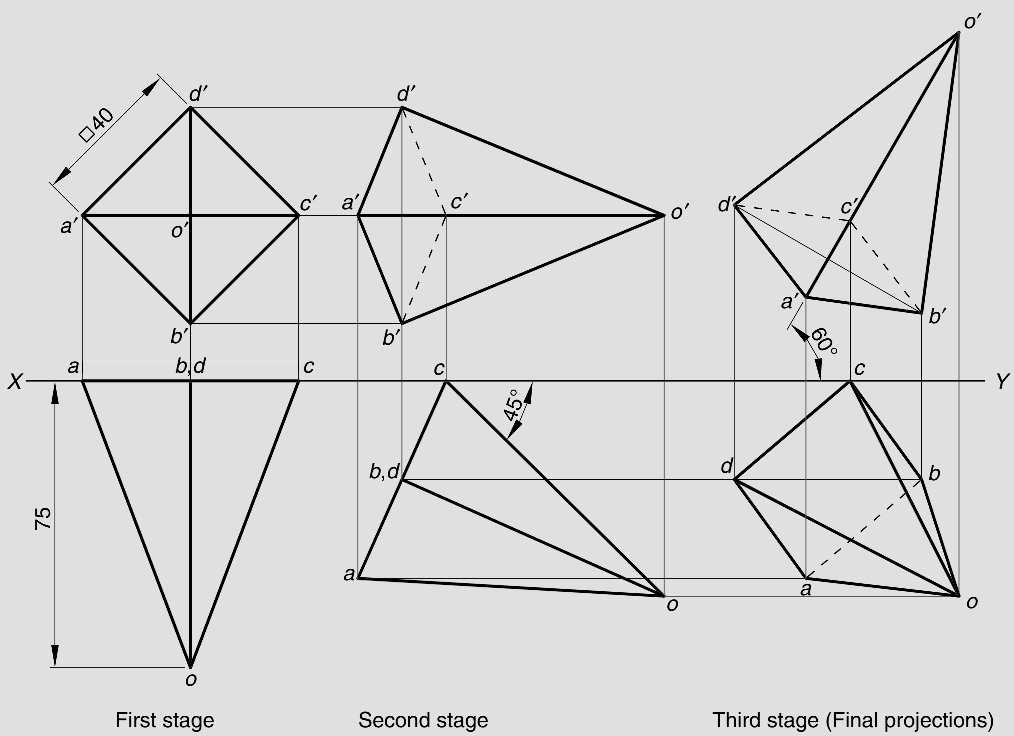


Fig. 9.44

Construction: Fig. 9.44

1. *First stage*: Draw a square *a′b′c′d′* keeping *a′c′* parallel to *XY*. Locate its centre *o′* and join it with the corners of the square. The figure represents the front view.
2. Project all the corners and obtain *aco* as its top view.
3. *Second stage*: Reproduce the top view of the first stage keeping *c* on *XY* and the slant edge *oc* at 45º to it.
4. Obtain points *a′*, *b′*, *c′*, *d′* and *o′* as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join these points and obtain *a′b′o′d′* as the front view.
5. *Third stage*: Reproduce the front view of the second stage keeping *o′c′a′* at 60º to *XY*. This is the final front view.
6. Obtain points *a*, *b*, *c*, *d* and *o* as the intersecting points of the vertical projectors drawn from the front view of the third stage with the horizontal projectors drawn from the top view of the second stage. Join these points and obtain *aobcd* as the final top view.

Example 9.39 (Fig 9.45)

Draw the three views of the cone, 50 mm base diameter and 65 mm long axis, having one of its generators in the V.P. and inclined at 30º to the H.P., the apex being in H.P.

[RGPV Feb. 2006]

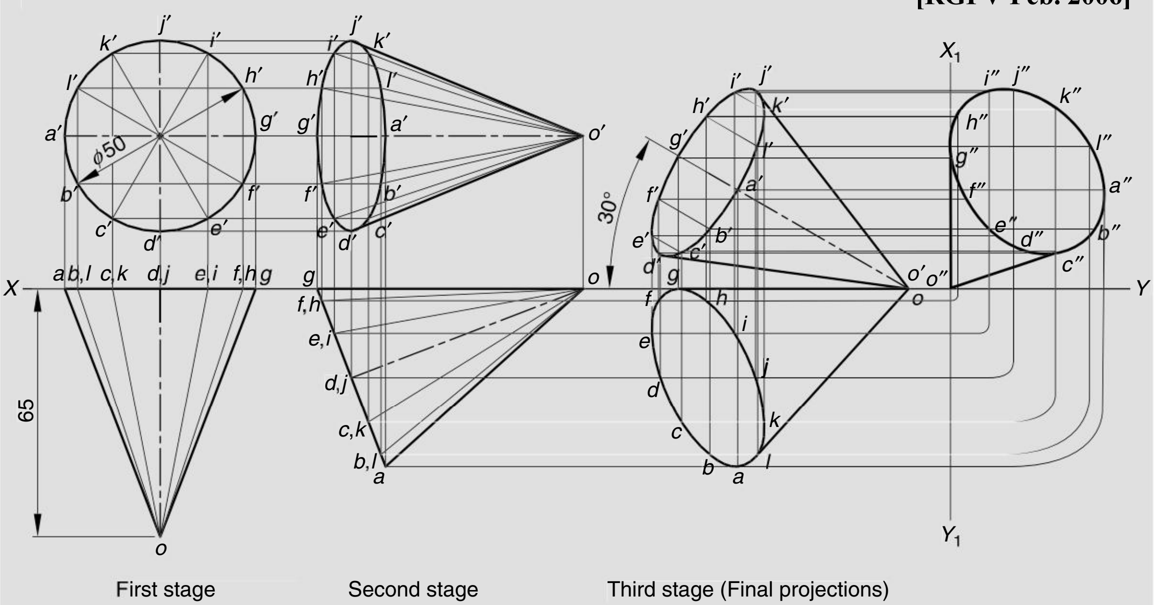


Fig. 9.45

Construction: Fig. 9.45

1. *First stage*: Draw circle *a′d′g′j′* to represent the front view. Divide the circle into 12 equal parts using radial lines.

2. Project these points of the generators and obtain *ago* to represent the top view.
3. *Second stage*: Reproduce the top view of the first stage keeping *og* on *XY*.
4. Obtain points *a′*, *b′*, *c′*, *d′*, *e′*, *f′*, *g′*, *h′*, *i′*, *j′*, *k′*, *l′* and *o′* as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join these points and obtain the front view.
5. *Third stage*: Reproduce the front view of the second stage keeping *o′* on *XY* and *a′g′o′* at 30º to it.
6. Obtain points *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l* and *o* as the intersecting points of the vertical projectors drawn from the front view of the third stage with the horizontal projectors drawn from the top view of the second stage. Join these points and obtain the final top view.

9.13.5 Solid Rests on Its Element (Corner or Edge) in the H.P. with Its Axis Inclined (at θ) to H.P. and (at ϕ) to V.P.

Example 9.40 (Fig 9.46)

A hexagonal prism of base 25 mm side and axis 45 mm long is positioned with one of its base edges on H.P. such that the axis is inclined at 30º to H.P. and 45º to the V.P. Draw its projections of the prism by change of position method. [RGPV Feb. 2010]

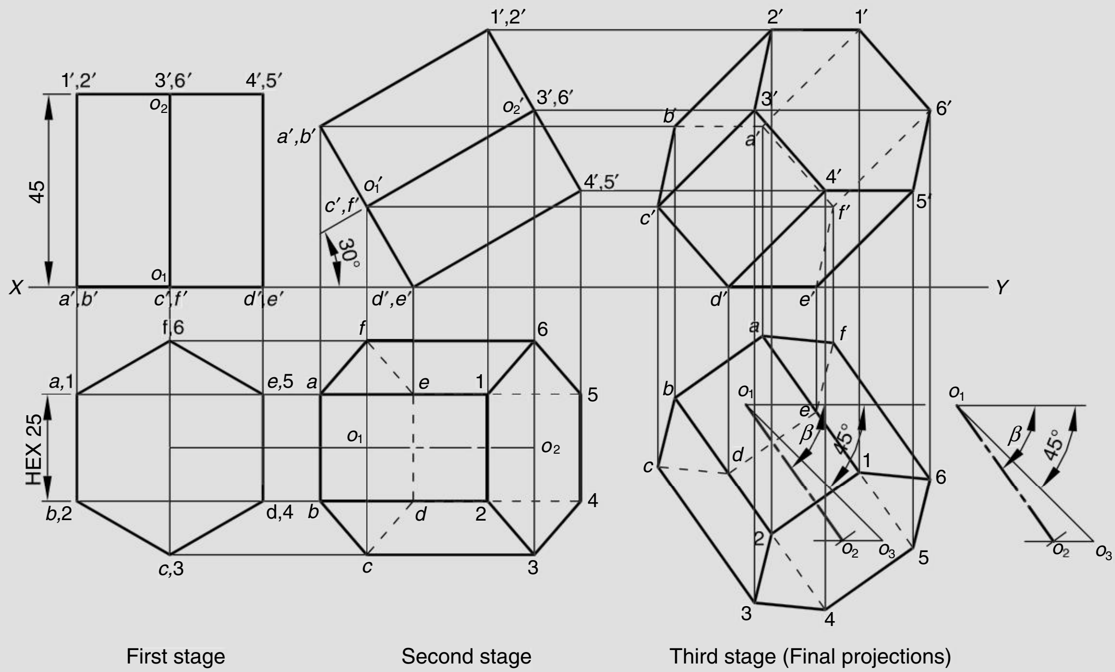


Fig. 9.46

Construction: Fig. 9.46

1. *First stage*: Draw a pentagon *abcdef* keeping *de* perpendicular to *XY*, to represent the top view.
2. Project the corners and obtain *b′d′*4′2′ as the front view.

3. *Second stage*: Reproduce the front view of the first stage keeping $d'e'$ on XY and $d'4'$ is inclined at 30º to XY. This will also make the axis incline at 30º to XY.
4. Obtain points a, b, c, d, e, f, 1, 2, 3, 4, 5 and 6 as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain $abc3456f$ as the top view.
5. *Third stage*: As we know that when an axis or a line is inclined at θ to the H.P. and ϕ to the V.P., its top view is inclined at β to XY. Therefore, draw a line o_1o_3, 45 mm (true length of the axis) inclined at 45º to XY. Draw an arc with o_1 as the centre and radius equal to the length of the axis o_1o_2 in the top view of the second stage to meet horizontal line from o_3 at point o_2.
6. Reproduce the top view of the second stage keeping o_1o_2 inclined at β with XY.
7. Obtain points a', b', c', d', e', f', $1'$, $2'$, $3'$, $4'$, $5'$ and $6'$ as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain $b'c'd'e'5'6'1'2'$ as the required front view.

Example 9.41 (Fig 9.47)

A pentagonal pyramid of 25 mm edge of base and 60 mm height is resting on the corner of its base on H.P. and the slant edge containing that corner is inclined at 45º with H.P. Draw the projections of the solid, when its axis makes an angle of 30º with V.P. [RGPV April 2010]

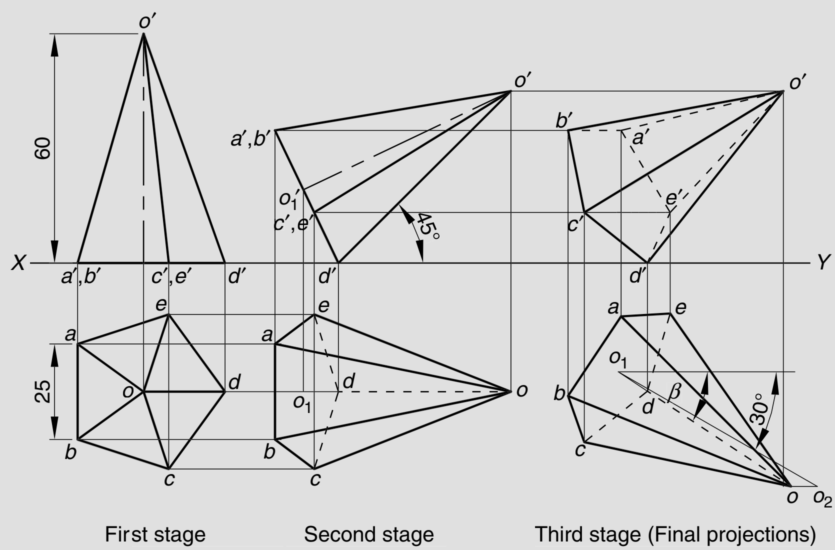


Fig. 9.47

Construction: Fig. 9.47

1. *First stage*: Draw a pentagon $abcde$ keeping ab perpendicular to XY. Locate centroid o and join it with all the corners of the pentagon. This represents the top view.
2. Project the corners and centroid to obtain $a'd'o'$ as the front view.

3. *Second stage*: Reproduce the front view of the first stage keeping d' on *XY* and $d'o'$ is inclined at 45º to *XY*.
4. Obtain points *a*, *b*, *c*, *d*, *e* and *o* as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join the points and obtain *abcoe* as the top view.
5. *Third stage*: As we know that when an axis or a line is inclined at θ to the H.P. and ϕ to the V.P., its top view is inclined at β to *XY*. Therefore, draw a line o_1o_2, 60 mm (true length of the axis) inclined at 30º to *XY*. Draw an arc with o_1 as the centre and radius equal to the length of the axis o_1o in the top view of the second stage to meet horizontal line from o_2 at point *o*.
6. Reproduce the top view of the second stage such that axis o_1o is inclined at β with *XY*.
7. Obtain points a', b', c', d', e' and o' as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain $b'c'd'o'$ as the required front view.

Example 9.42 (Fig 9.48)

Draw the projections of a cone, having a base with a 50 mm diameter and a 60 mm axis, when it is resting on the ground on a point of its base circle with the axis inclined at 30º to the H.P. and (i) the top view of the axis is inclined at 45º with the V.P., and (ii) the axis is inclined at 45º with the V.P.

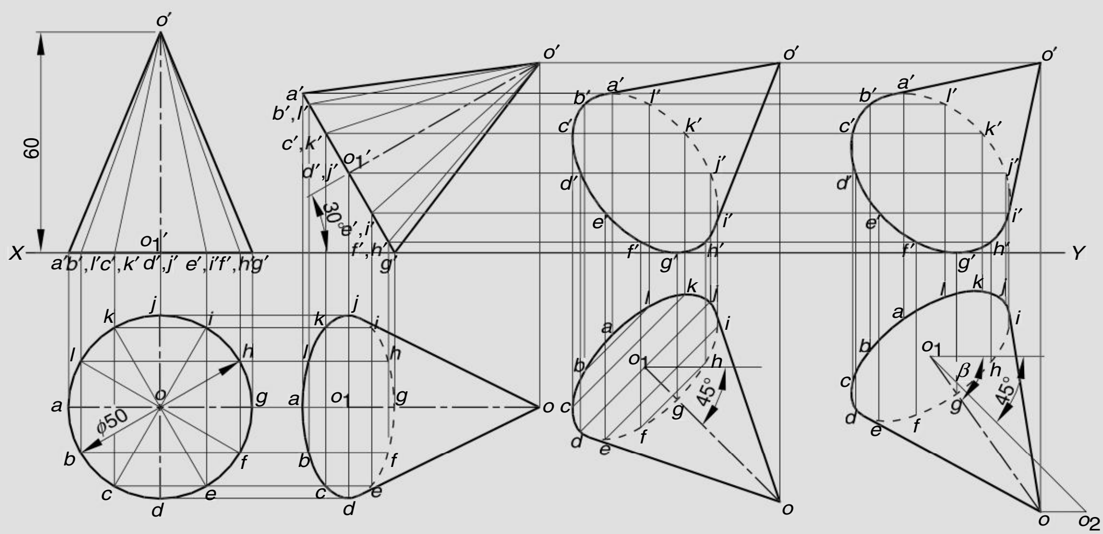


Fig. 9.48

Construction: Fig. 9.48

1. *First stage*: Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts using radial lines.
2. Project all the points and obtain $a'g'o'$ to represent the front view.

3. *Second stage*: Reproduce the front view of the first stage keeping g' on XY and $g'a'$ at 60º to it. This will make axis at 30º to XY.
4. Obtain points a, b, c, etc., as the intersecting points of the vertical projectors drawn from the front view of the second stage with the horizontal projectors drawn from the top view of the first stage. Join visible points with continuous curve/lines as shown.
5. *Third stage [case (i), the top view of the axis is inclined at 45º with the V.P.]*: Reproduce the top view of the second stage such that ago is inclined at 45º to XY.
6. Obtain points a', b', c', etc., as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain the required front view.
7. *Third stage [case (ii), the axis is inclined at 45º with the V.P.]:* Determine the apparent angle β. For this, draw a line o_1o_2, 60 mm long (true length of the axis) inclined at 45º to XY. Draw an arc with o_1 as the centre and radius equal to the top view of the axis to meet horizontal line from o_2 at point o.
8. Reproduce the top view of the second stage such that oo_1 is inclined at β angle to XY.
9. Obtain points a', b', c', etc., as the intersecting points of the vertical projectors drawn from the top view of the fourth stage with the horizontal projectors drawn from the front view of the second stage. Join the points and obtain the required front view.

9.13.6 Solid Rests on Its Element (Corner or Edge) on the V.P. with Its Axis Inclined (at θ) to H.P. and (at ϕ) to V.P.

Example 9.43 (Fig 9.48)

A cylinder of 50 mm base diameter and 70 mm long axis has a point of its base circle in the V.P. Its axis is inclined at 30º to V.P. and 45º to the H.P. Draw its projections.

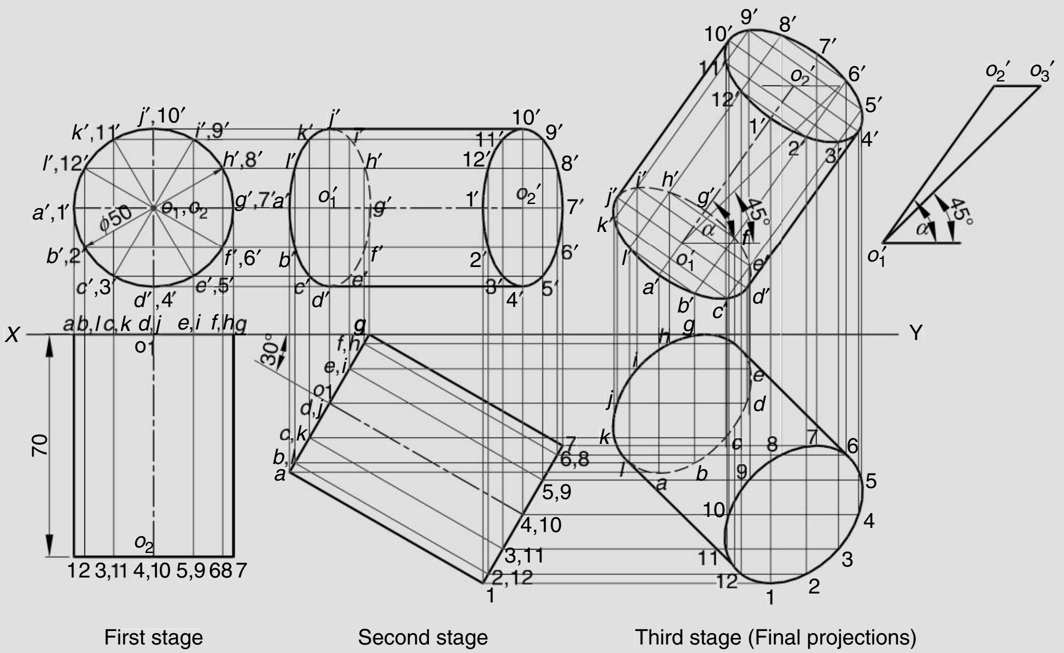


Fig. 9.49

Construction: Fig. 9.49

1. *First stage*: Draw a circle $a'd'g'j'$ as the front view. Divide the circle into 12 equal parts using radial lines.
2. Project ends of the radial lines and obtain $a17g$ as its top view.
3. *Second stage*: Reproduce the top view of the first stage keeping g on XY and $g7$ at 30º to it. This will make the axis at 30º to XY.
4. Obtain points a', b', c', d', etc., and $1'$, $2'$, $3'$, $4'$, etc, as the intersecting points of the vertical projectors drawn from the top view of the second stage with the horizontal projectors drawn from the front view of the first stage. Join these points and obtain $a'd'4'7'10'j'$ as the top view.
5. *Third stage*: As we know that when an axis or a line is inclined at θ to the H.P. and ϕ to the V.P., its front view is inclined at α to XY. Therefore, draw a line $o_1'o_3'$, 70 mm (True length of the axis) inclined at 45º to XY. Draw an arc with o_1' as the centre and radius equal to the front view of the axis from the second stage to meet horizontal line from o_3' at point o_2'.
6. Reproduce the front view of the second stage such that $o_1'o_2'$ is the axis of the figure. This is the final front view.
7. Obtain points a, b, c, d, etc., and 1, 2, 3, 4, etc. as the intersecting points of the vertical projectors drawn from the top view of the third stage with the horizontal projectors drawn from the front view of the second stage. Join the points following rules of visibility and obtain the required front view as shown.

9.14 MISCELLANEOUS EXAMPLES

Example 9.44 (Fig 9.50)

A regular pentagonal pyramid with 30 mm side of base and 60 mm height rests on an edge of the base on the H.P. The base is tilted until its apex is 50 mm above the level of the edge of the base on which it rests. Draw the projections of the pyramid when the edge on which it rests, is parallel to the V.P. and apex of the pyramid points towards V.P. [RGPV Dec. 2002]

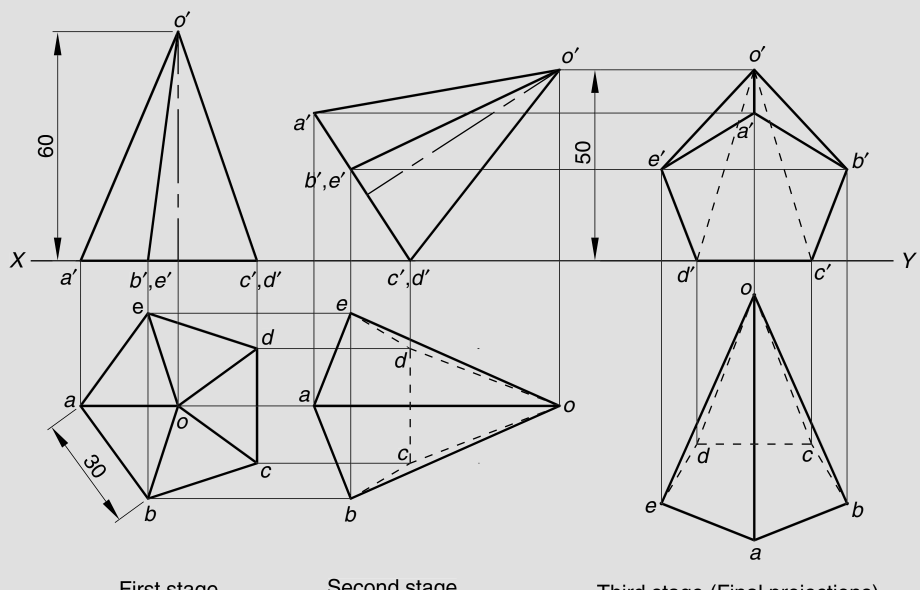


Fig. 9.50

Construction: Fig. 9.50

1. *First stage*: Draw pentagon *abcde* keeping *cd* perpendicular to *XY*. Locate its centroid *o* and join it with the corners of the pentagon. The figure represents the top view. Project all the points and obtain *a′o′c′* to represent the front view.
2. *Second stage*: Reproduce the front view of the first stage keeping *c′d′* on *XY* and apex *o′* 50 mm above *XY*. Project the front view points to meet locus lines from the top view of the first stage and obtain *aboe* as the new top view.
3. *Third stage*: Reproduce the top view of the second stage keeping *cd* parallel to *XY*. This is the final top view. Project this top view to meet the locus lines from the front view of the second stage and obtain *b′c′d′e′o′* as the final front view.

Example 9.45 (Fig 9.51)

A pentagonal pyramid has an edge of the base in the V.P. and inclined at 30º to H.P., while the triangular face containing that edge makes an angle of 45º to the V.P. Draw the three views of the pyramid. Length of the side of base is 30 mm, while that of the axis is 60 mm long.

[RGPV Dec. 2005]

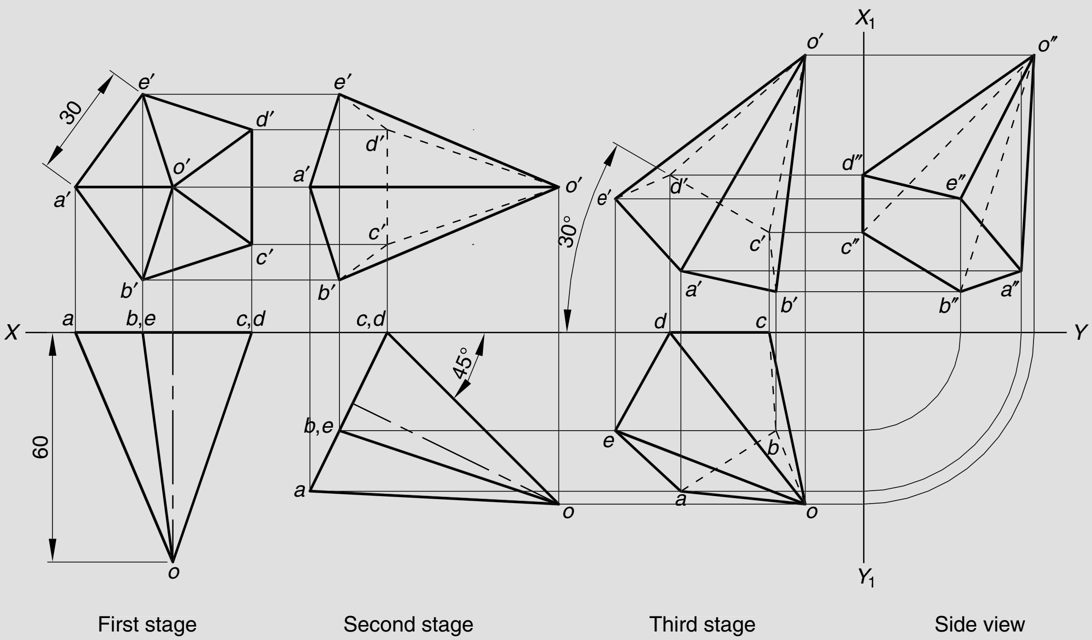


Fig. 9.51

Construction: Fig. 9.51

1. *First stage*: Draw pentagon *a′b′c′d′e′* keeping side *c′d′* perpendicular to *XY*. Locate its centroid *o′* and join it with the corners of the pentagon. The figure represents the front view. Project all the points and obtain *aoc* to represent the top view.

2. *Second stage*: Reproduce the top view of the first stage keeping *cd* on *XY* and face *cdo* inclined at 45º to *XY*. Project the top view points to meet locus lines from the front view of the first stage and obtain *a′b′o′e′* as the new front view.
3. *Third stage*: Reproduce the front view of the second stage keeping *c′d′* inclined at 30º to *XY*. This is the final front view. Project this front view to meet the locus lines from the top view of the second stage and obtain *aocde* as the final top view.
4. *Side view*: Draw lines parallel to *XY* from the top view to meet X_1Y_1. Thereafter rotate them through 90º and produce them vertically to meet the horizontal line from the front view at points *a″*, *b″*, *c″*, *d″*, *e″* and *o″*. Join the points as shown. This is the required third view.

Example 9.46 (Fig 9.52)

A pentagonal pyramid of 30 mm base side and 50 mm long axis is held on a corner of its base on the ground and the triangular face opposite to it is horizontal. Draw the projections of the pyramid when the edge of the base contained by this triangular face makes an angle of 60º to the V.P. and the apex is towards the observer.

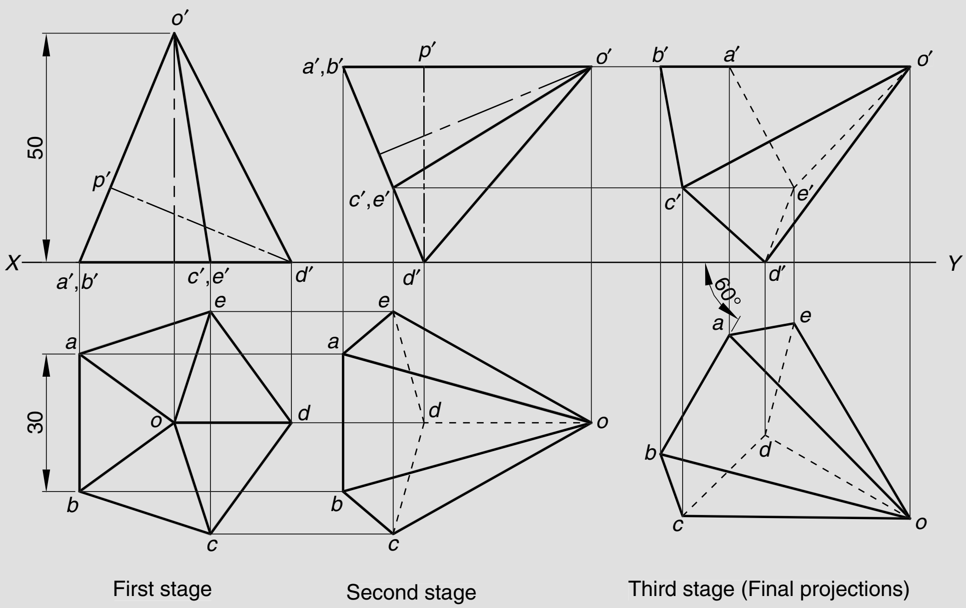


Fig. 9.52

Construction: Fig. 9.52

1. *First stage*: Draw pentagon *abcde* keeping *ab* perpendicular to *XY*. Locate its centroid *o* and join it with the corners of the pentagon. The figure represents the top view.
2. Project all the points and obtain *a′o′d′* to represent the front view. Draw a line *d′p′* perpendicular to line *a′o′*.

3. *Second stage*: Draw a line $d'p'$ perpendicular to *XY* keeping d' on *XY*. Draw line $a'b'p'o'$ parallel to *XY* and reproduce the front view of the first stage. Project this front view points to meet locus lines from the top view of the first stage and obtain *abcoe* as the new top view.
4. *Third stage*: Reproduce the top view of the second stage keeping *ab* at 60º to *XY*. This is the final top view. Project this top view to meet the locus lines from the front view of the second stage and obtain $a'b'c'd'o'$ as the final front view.

Example 9.47 (Fig 9.53)

A tetrahedron of 75 mm long edges has one edge parallel to the H.P. and inclined at 45º to the V.P. while a face containing that edge is vertical. Draw its projections. [RGPV Feb. 2006]

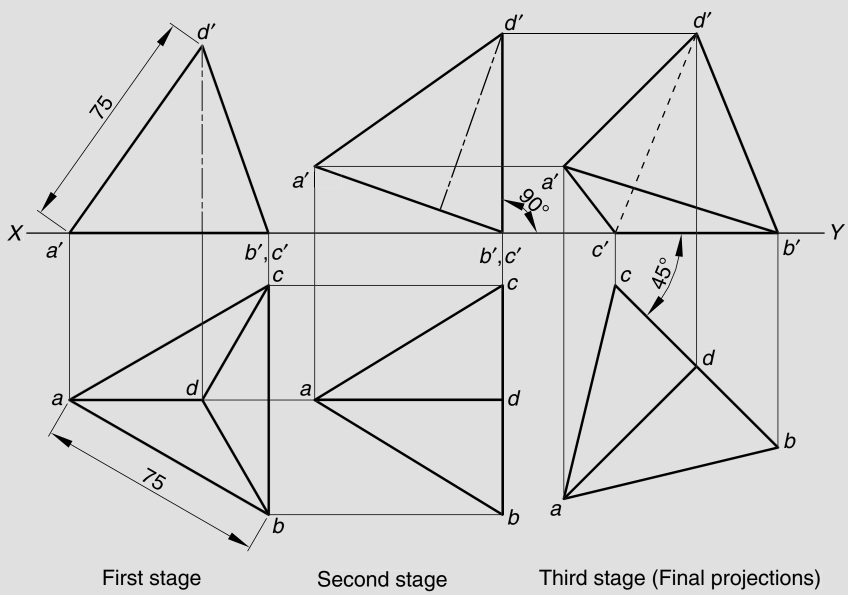


Fig. 9.53

Construction: Fig. 9.53

1. *First stage*: Draw a triangle *abc* keeping *bc* perpendicular to *XY*. Locate its centroid *o* and join it with the corners of the triangle. The figure represents the top view. Project all the points and obtain $a'b'd'$ to represent the front view.
2. *Second stage*: Reproduce the front view of the first stage keeping $b'c'$ on *XY* and $b'c'd'$ perpendicular to *XY*. Project the front view points to meet locus lines from the top view of the first stage and obtain *abc* as the new top view.

3. *Third stage*: Reproduce the top view of the second stage keeping *bc* at 45º to *XY*. This is the final top view. Project this top view to meet the locus lines from the front view of the second stage and obtain *a′c′b′d′* as the final front view.

Example 9.48 (Fig 9.54)

Draw the projections of a cube with 40 mm long edges resting on the H.P. on one of its corners with a solid diagonal perpendicular to the V.P. [RGPV April 2010]

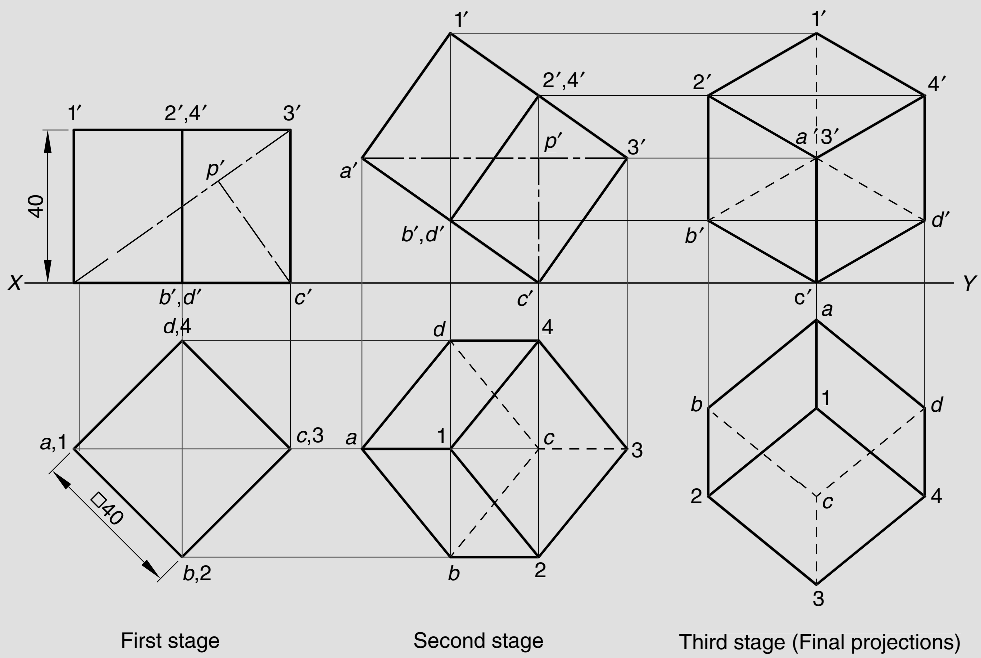


Fig. 9.54

Construction: Fig. 9.54

1. *First stage*: Draw a square *abcd* keeping *ab* at 45º to *XY*. This is the top view. Project the corners and obtain the front view.
2. Mark a diagonal *a′*3′. Draw a perpendicular line *c′p′* from *c′* on *a′*3′.
3. *Second stage*: Draw line *c′p′* perpendicular to *XY* and copy line *a′*3′ parallel to *XY*. Now, reproduce the front view of the first stage. Project these front view to meet locus lines from the top view of the first stage and obtain the new top view.
4. *Third stage*: Reproduce the top view of the second stage keeping *a*1*c*3 perpendicular to *XY*. Project this top view to meet locus lines from the front view of the second stage and obtain the final front view.

Example 9.49 (Fig 9.55)

A square pyramid, 40 mm base side and 60 mm long axis, is freely suspended from one of its corners of its base. Draw its projections, when the axis as a vertical plane is inclined at 45º to the V.P.

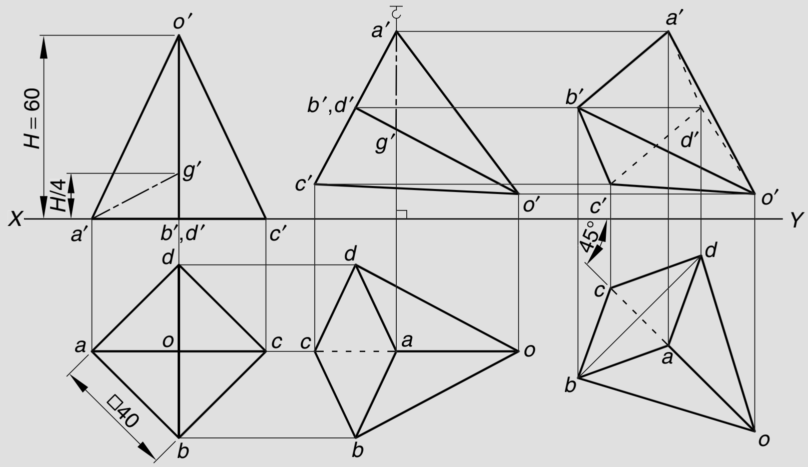


Fig. 9.55

Construction: Fig. 9.55

1. *First stage*: Draw a square *abcd* keeping *ac* parallel to *XY*. Locate its centroid *o′* and join it with the corners of the square. The figure represents the top view.
2. Project all the points and obtain the front view. Mark centre of gravity (*g′*) on the axis in the front view at a height *H*/4 = 15 mm from *XY*.
3. *Second stage*: Reproduce the front view of the first stage keeping *a′g′* perpendicular to *XY*. Project the front view to meet locus lines from the top view of the first stage and obtain the new top view.
4. *Third stage*: Reproduce the top view of the second stage keeping *cao* at 45º to *XY*. Project the top view to meet locus lines from the front view of the second stage and obtain the final front view.

Example 9.50 (Fig 9.56)

A regular pentagonal prism lies with its axis inclined at 60º to the H.P. and 30º to the V.P. The prism is 60 mm long and has a face width of 25 mm. The nearest corner is 10 mm away from the V.P. and the farthest shorter edge is 100 mm from the H.P. Draw the projections of the solid.

[RGPV June 2003]

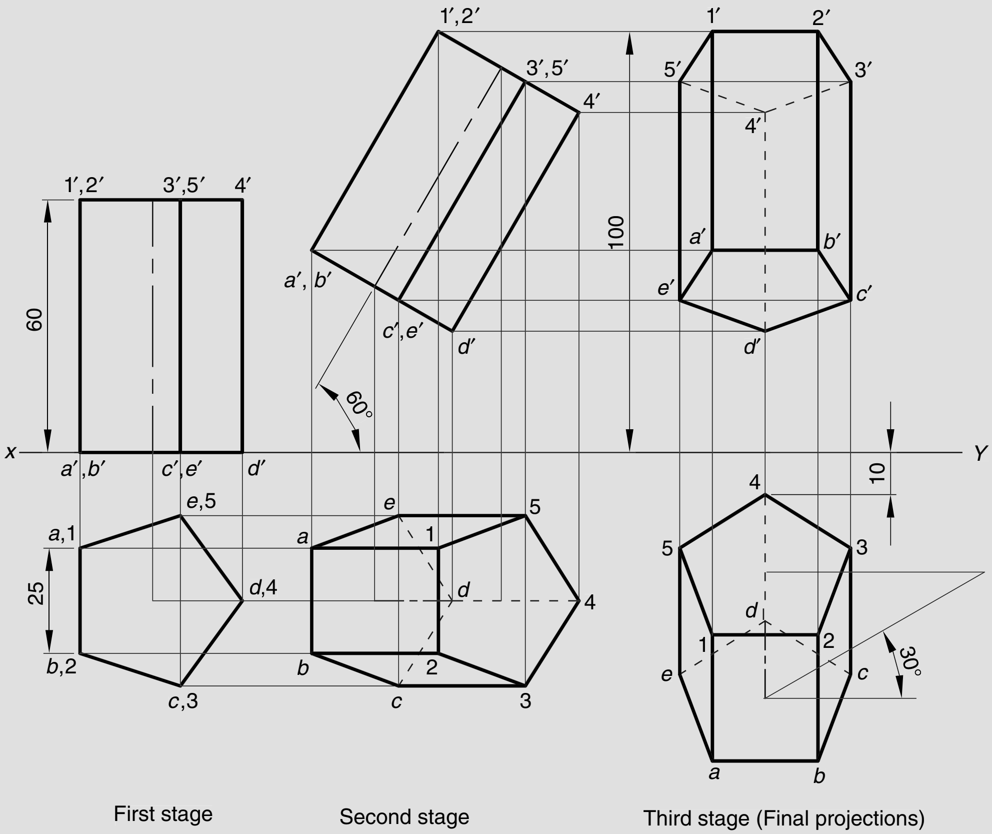


Fig. 9.56

Construction: Fig. 9.56

1. *First stage*: Draw a pentagon *abcde* keeping *ab* perpendicular to *XY*. Project all the points and obtain the front view.
2. *Second stage*: Mark a point 1′2′ 100 mm above *XY*. Reproduce the front view of the first stage keeping 1′2′*a*′*b*′ at 60º to *XY*. This will also make axis inclined at 60º to *XY*. Project this front view to meet locus lines from the top view of the first stage and obtain the new top view.
3. *Third stage*: Here axis is inclined 60º to H.P. and 30º to V.P. Therefore, β angle shall be equal to 90º. Reproduce the top view of the second stage such that point 4 is 10 mm below *XY* and axis is perpendicular to *XY*. Project this top view to meet locus lines from the front view of the second stage and obtain the final front view.

EXERCISE 9B

An Element of the Solid in the H.P.

1. A hexagonal prism with 30 mm base sides and 75 mm long axis is kept on one of its base edges on the H.P. in such a way that the axis is inclined at 30º to the H.P. and the base edge resting on the H.P. is inclined at 45º with the V.P. Draw its projections.
2. A square prism with 40 mm base sides and 60 mm long axis has one of its base edges parallel to the H.P. and inclined at 45º to the V.P. Its axis makes 60º with the H.P. Draw its projections.
3. A square pyramid with 40 mm base sides and 70 mm long axis has a triangular face on the H.P. and the vertical plane containing the axis is inclined at 45º to the V.P. Draw its projections.
4. A hexagonal pyramid, with 30 mm base sides and 65 mm long axis, has a triangular face on the H.P. and the edge of the base containing that face makes an angle of 30º with the V.P. Draw its projections.
5. A hexagonal pyramid with 30 mm base sides and 70 mm long axis has one of its triangular faces perpendicular to the H.P. and inclined at 45º to the V.P. Draw its projections when the base is visible.
6. A pentagonal pyramid, 30 mm base sides and 60 mm long axis, has one of the edges of the base in the H.P. The solid is tilted in such a manner that the highest point of the base is 40 mm above H.P. and the edge of the base on which it is resting is parallel to the V.P. Draw its projections.
7. A pentagonal prism with 30 mm base sides and 70 mm long axis is resting on a corner of its base on the ground with a longer edge containing that corner inclined at 45º to the H.P. A vertical plane containing that longer edge and the axis is inclined at 30º to the V.P. Draw its projections.
8. A hexagonal pyramid with 30 mm base sides and 75 mm long axis has one of its slant edges on the H.P. and vertical plane containing this edge and the axis is inclined at 30º to the V.P. Draw its projections when apex is 15 mm in front of the V.P.
9. Draw the projections of a cone of 45 mm base diameter and 50 mm long axis, when it is resting on the ground on a point on its base circle with the axis making 30º with the H.P. and its top view making 45º with the V.P. **[RGPV Feb. 2008]**
10. A right circular cone of diameter 70 mm and axis height 80 mm is resting on one of its generators in H.P. The top view of the axis is inclined at 45º to V.P. Draw the projections of the cone. **[RGPV Dec. 2010]**
11. Draw the projections of a cone of base 50 mm diameter and altitude 60 mm long lying on one of its generators on H.P. when the top view of the axis makes an angle of 30º with XY. **[RGPV Feb. 2011]**
12. A right circular cone of 50 mm base diameter and 70 mm long axis is resting on its circular rim in such a way that one of its extreme generators is normal to H.P. and then the plan of the axis makes an angle of 45º to the V.P. Draw its projections. **[RGPV Dec. 2005]**
13. A square pyramid with 50 mm base sides and 60 mm long axis has one of its triangular faces on the H.P. and a slant edge containing that face is parallel to the V.P. Draw its projections.
14. A frustum of a pentagonal pyramid, with 40 mm base edges, 20 mm top edges and 60 mm long axis, is resting on its face on the H.P. Draw its projections when an edge of its base is parallel to the V.P. and the small base is towards the observer.

15. Draw the projections of a cube of 25 mm long edges resting on the H.P. on one of its corners with the solid diagonal perpendicular to the V.P. **[RGPV Dec. 2006]**
16. A cube with 40 mm side has a corner in the H.P. Draw its projections when a solid diagonal is parallel to the H.P. and inclined at 30º to the V.P.
17. A hexagonal pyramid with 30 mm base sides and 70 mm long axis is suspended freely from one of its base corners such that its axis is in a vertical plane which makes an angle of 45º with the V.P. Draw its projections.
18. A cone having a base with 50 mm diameter and 65 mm long axis is freely suspended from a point of its rim such that the top view of the axis is perpendicular to V.P. and the apex is towards the observer. Draw its projections.

An Element of the Solid in the V.P.

19. A hexagonal prism, with 30 mm base sides and 75 mm long axis, has an edge of the base on the V.P. and inclined at 30º to the H.P. The rectangular face containing that edge makes an angle of 45º with the V.P. Draw its projections.
20. A hexagonal pyramid with 30 mm base sides and 70 mm long axis has an edge of its base in the V.P. and inclined at 30º to the H.P. The triangular face containing that edge is inclined at 45º to the V.P. Draw its projections.
21. Draw the projections of a pentagonal pyramid having side of base 40 mm and length of axis 90 mm when it is resting with a triangular face in the V.P. and the base edge of that face inclined at 60º to the H.P. **[RGPV June 2011]**
22. A hexagonal pyramid of 30 mm base side and 70 mm long axis has a corner of its base on the V.P. The axis is inclined at 45º to the V.P. and the plane containing the slant edge and the axis is inclined at 60º to the H.P. Draw its projections.
23. A square pyramid, of 40 mm base side and 60 mm long axis, has one of its slant edges on the V.P. A plane containing that edge and the axis is perpendicular to the V.P. and inclined at 45º to the H.P. Draw its projections, when the apex is nearer to the H.P. then the base.

Apparent Angle α or β

24. A square prism, with 40 mm base sides and 65 mm long axis, is resting on an edge of its base on the H.P. with axis inclined at 45º to the H.P. and 30º to the V.P. Draw its projections.
25. Draw the projections of a cylinder of 75 mm diameter and 100 mm length lying on the ground with axis inclined at 30º to the V.P. and 45º to the H.P. **[RGPV Dec. 2006]**
26. A square pyramid, with 35 mm base sides and 50 mm long axis, has a triangular face on the ground and the axis is inclined at 45º to the V.P. Draw its projections.
27. A cone having 50 mm base diameter and 60 mm long axis is lying on one of its generators on the H.P. Draw its projections when the axis is making an angle of 45° with the V.P.

REVIEW QUESTIONS

1. Differentiate between a triangular pyramid and a tetrahedron.
2. State the shape and number of faces in a dodecahedron and icosahedron.

3. Define cylinder and cone in terms of surface of revolution.
4. What do you understand by a right regular solid?
5. Differentiate between frustum of a pyramid and a truncated pyramid.
6. A cube is resting on one of its corners in the H.P. with a solid diagonal vertical. What will be the outer shape of its top view?
7. A cube is resting on one of its corners in the H.P. with a solid diagonal perpendicular to the V.P. What will be the outer shape of its front view?
8. What is the difference between the top view of a hexagonal prism and that of a hexagonal pyramid when both solids rest on their bases on the H.P. with similar orientation?
9. State the position of a tetrahedron so as to get a square as the outer shape in its top view.
10. State the location of the centroid of a square pyramid.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) Among the following solids, a regular polyhedron is
(a) square prism (b) square pyramid (c) cube (d) sphere

ii) A solid having minimum number of faces is
(a) tetrahedron (b) triangular prism (c) square pyramid (d) cube

iii) A pyramid is cut by a plane parallel to its base removing the apex, the remaining part is known as
(a) truncated (b) frustum (c) sectioned (d) prism

iv) Number of faces in a dodecahedron are
(a) 4 (b) 8 (c) 12 (d) 20

v) If three orthographic views of a sphere containing a circular hole are drawn, the maximum number of circles that may appear altogether
(a) 1 (b) 3 (c) 4 (d) 6

vi) An orthographic view of a hemisphere may appear as
(a) circle (b) ellipse (c) parabola (d) hyperbola

vii) The number of stages that are necessary to get the orthographic views of a solid having its axis inclined to both the reference planes are
(a) one (b) two (c) three (d) four

viii) A tetrahedron is resting on its face on the H.P. with a side perpendicular to the V.P. Its front view will be an
(a) equilateral triangle (b) isosceles triangle
(c) scalene triangle (d) right-angled triangle

ix) A square pyramid is resting on a face in the V.P. The number of dotted lines appearing in the front view will be

(a) one (b) two (c) three (d) four

x) The solid will have two dotted lines in the top view when it is resting on its face in the H.P.

(a) square pyramid (b) pentagonal pyramid
(c) hexagonal pyramid (d) all of these

xi) A cube is resting on H.P. with a solid diagonal perpendicular to it. The top view will appear as

(a) square (b) rectangle (c) irregular hexagon (d) regular hexagon

xii) A right circular cone resting on a point of its base circle in the H.P. having the axis inclined at 30º to the H.P. and 45º to the V.P. The angle between the reference line and top view of the axis will be

(a) 30º (b) between 30º and 45º (c) 45º (d) more than 45º

xiii) A right circular cone resting on a generator in the H.P. and axis inclined at 45º to the V.P. The angle between the reference line and top view of the axis will be

(a) less than 45º (b) 45º (c) more than 45º (d) any of these

xiv) A cylinder rests on a point of its base circle in the H.P. having the axis inclined at 30º to the H.P. and 60º to the V.P. The inclination of the top view of the axis with the reference line will be

(a) 30º (b) 60º (c) 90º (d) none of these

Answers

(i) c (ii) b (iii) b (iv) c (v) c (vi) a (vii) c (viii) b (ix) b (x) d (xi) d (xii) d (xiii) c (xiv) c

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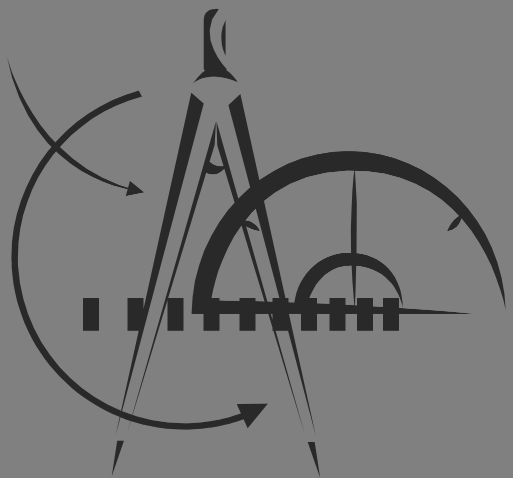


Sections of Solids

- Introduction
- Terminology for Sections of Solids
- Types of Section Planes
- Sections of Solids by Horizontal Plane
- Sections of Solids by Plane Parallel to V.P
- Sections of Solids by Auxiliary Inclined Plane
- Sections of Solids by Auxiliary Vertical Plane
- Sections of Solids by a Profile Plane

10.1 INTRODUCTION

It is observed that the orthographic views of a solid may contain a number of dotted lines. These lines indicate the presence of hidden details which may lie behind or somewhere in the middle of the object. The interpretation of the object's shape becomes difficult with increasing number of such lines. As a remedy, it becomes obligatory to draw sectional views for a better and easier interpretation of the internal details. The present chapter describes the methods of obtaining sectional views and other related drawing.

The object considered to be cut by a plane is called a section or a cutting plane. The portion of the object, which falls between the cutting plane and the observer, is assumed to be removed. Thus, the exposed internal details become visible. The projections of the remaining object are termed as sectional views.

10.2 TERMINOLOGY

The terms used very frequently for sections of solids are given follow.

1. ***Section plane*** Section planes are the imaginary planes which cut the objects completely or partially to show their invisible and interior details clearly. These planes are represented by their traces.
2. ***Cut surface*** Section or cut surface is the surface created due to cutting the object by section planes. It is shown by hatching lines.
3. ***Hatching lines*** Hatching lines are the continuous thin lines drawn at an angle of 45° to the reference planes, parallel to each other at a uniform space of 2 to 3 mm in between.
4. ***Apparent section*** The projection of the section on the principle plane to which the section plane is inclined is known as apparent section.
5. ***True shape of section*** The projection of the section on a plane parallel to the section plane is known as true shape of section. It shows actual shape and size of cut surface.

10.3 TYPES OF SECTION PLANES

The shape of the section obtained will depend upon the orientation of the solid and the section plane, with respect to the principle planes of projection. Some of the positions of the section planes are given below.

1. Horizontal plane
2. Auxiliary Inclined Plane (A.I.P.)
3. Plane parallel to V.P.
4. Auxiliary Vertical Plane (A.V.P.)
5. Profile plane
6. Oblique plane (inclined to both H.P. and V.P.) not considered in current study

10.3.1 Horizontal Plane

A section plane parallel to H.P. is known as horizontal section plane [Fig. 10.1(a)]. Its V.T. is a straight line parallel to *XY* and has no H.T.[Fig. 10.1(b)].

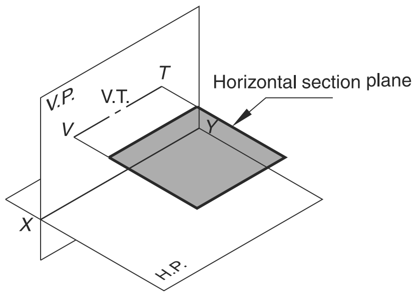


Fig. 10.1(a)

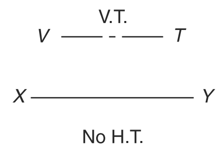


Fig. 10.1(b)

If a solid is cut by a horizontal section plane, the front view of the solid is superimposed by the V.T. of the section plane. The portion of the solid lying between the V.T. and the observer is considered to be removed. The remaining cut solid is then projected to get its sectional top view.

10.3.2 Plane Parallel to V.P.

A section plane parallel to V.P. is known as vertical section plane [Fig. 10.2(a)]. Its H.T. is a straight line parallel to *XY* and has no V.T. [Fig. 10.2(b)].

If a solid is cut by a vertical section plane, the top view of the solid is superimposed by the H.T. of the section plane. The portion of the solid lying between the H.T. and the observer is considered to be removed. The remaining cut solid is then projected to get its sectional front view.

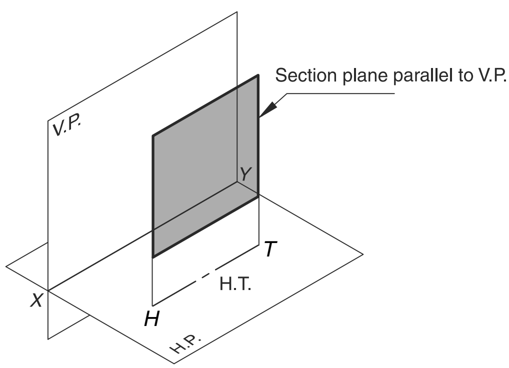


Fig. 10.2(a)

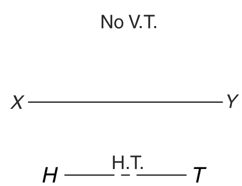


Fig. 10.2(b)

10.3.3 Auxiliary Inclined Plane (A.I.P.)

A plane perpendicular to V.P. and inclined at an angle θ to H.P. is known as auxiliary inclined plane (A.I.P.) [Fig. 10.3(a)]. Its V.T. is a straight line inclined at θ to *XY*. The H.T. of this plane is a line perpendicular to *XY* [Fig. 10.3(b)].

If a solid is cut by an auxiliary inclined plane, the front view of the solid is superimposed by the V.T. of the section plane. The portion of the solid lying between the V.T. and the observer is considered to be removed. The remaining cut solid is then projected to get its sectional top view.

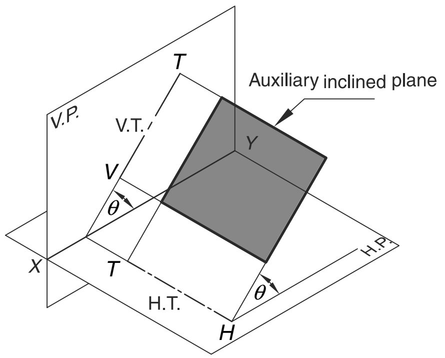


Fig. 10.3(a)

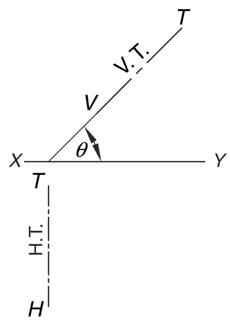


Fig. 10.3(b)

10.3.4 Auxiliary Vertical Plane (A.V.P.)

A plane perpendicular to H.P. and inclined at an angle Ø to V.P. is known as an auxiliary vertical plane (A.V.P.) [Fig. 10.4(a)]. Its H.T. is a straight line inclined at Ø to *XY*. The V.T. of this plane is a line perpendicular to *XY* [Fig. 10.4(b)].

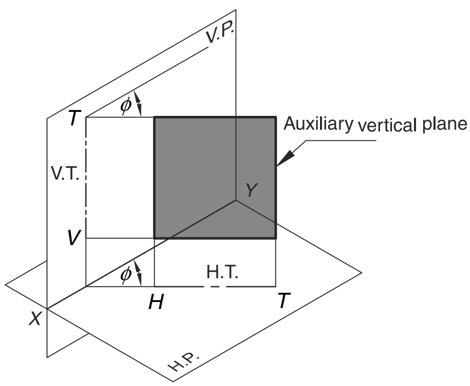


Fig. 10.4(a)

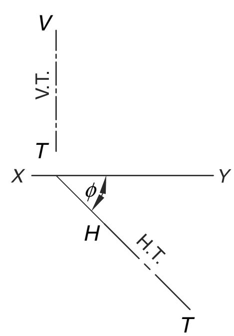


Fig. 10.4(b)

If a solid is cut by an auxiliary vertical plane, the top view of the solid is superimposed by the H.T. of the section plane. The portion of the solid lying between the H.T. and the observer is considered to be removed. The remaining cut solid is then projected to get its sectional front view.

10.3.5 Profile Plane

A plane perpendicular to both H.P. and V.P. is known as profile plane [Fig. 10.5(a)]. Both of its V.T. and H.T. are straight lines perpendicular to *XY* [Fig. 10.5(b)].

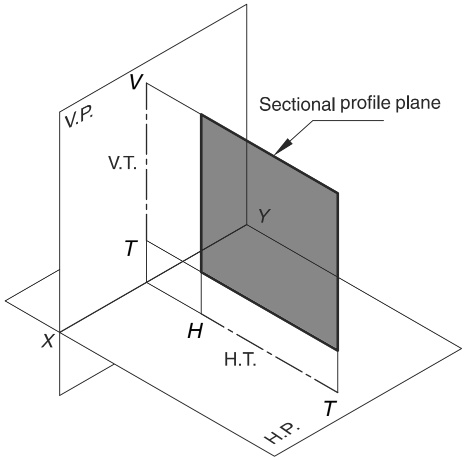


Fig. 10.5(a)

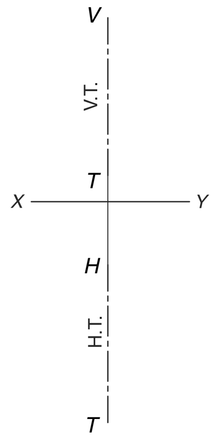


Fig. 10.5(b)

If a solid is cut by a profile plane, both the front view and the top view are superimposed by the V.T. and H.T. of the section plane respectively. The true shape of section is seen in the side view.

Note

1. In case where two or more positions of the section plane is possible, which satisfy the given conditions, then as far as possible, the section plane through which minimum portion of the solid is cut away is selected.
2. The part / portion of the solid in between the observer and the section plane are assumed to be removed. The retained part of the solid is drawn with thick continuous lines and the removed part with thin lines.
3. The hatching lines (section lines) are used to indicate cut surface in the sectional views. Usually, the hatching lines are inclined at 45º to the principal planes. If the boundary of the object itself makes 45º to the reference planes, hatching lines are drawn at 30º or 60º to the principal axis.

10.4 SECTIONS OF SOLIDS BY HORIZONTAL PLANE

Example 10.1 (Fig. 10.6)

A cube of 50 mm side rests with one of its edges on H.P. such that the square faces containing that edge are making equal inclinations with H.P. A horizontal section plane cuts the cube at a distance 18 mm below the horizontal edge nearer to the observer. Draw the sectional top view and front view of the cube.

[RGPV June 2008]

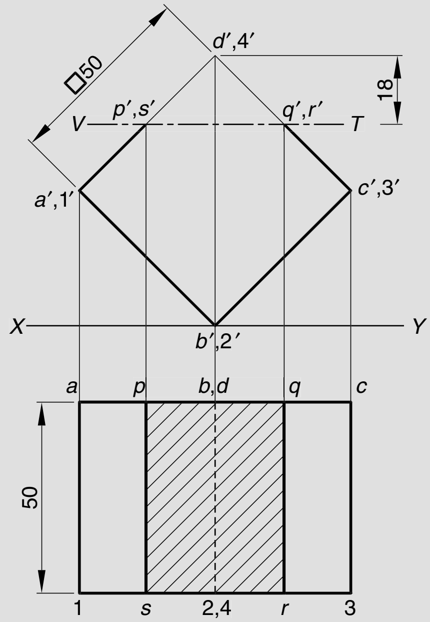


Fig. 10.6

Construction: Fig. 10.6

1. Draw a square $a'b'b'd'$ keeping edges at 45º with *XY*, to represent the front view. Project all the corners and obtain *ac*31 to get the top view.
2. Draw section plane V.T. parallel to *XY* and 18 mm below edge $d'4'$.
3. Let the section plane cut the edges of the prism $a'd'$ at p' and s', $c'd'$ at q' and r'. Project points p', q', r' and s' on edges *ad*, *cd*, 3-4 and 1-2 to obtain points *p*, *q*, *r* and *s*. Join *pqrs* and hatch the enclosed portion with lines inclined at 45º to *XY*.

Note: When a solid is cut by a horizontal section plane the true shape of the section is seen in the top view.

Example 10.2 (Fig. 10.7)

A triangular prism of 30 mm base side and 50 mm long axis is lying on the H.P. on one of its rectangular faces with its axis inclined at 30º to the V.P. It is cut by a horizontal section plane at a distance of 12 mm above the ground. Draw its front view and sectional top view.

[RGPV Dec. 2007]

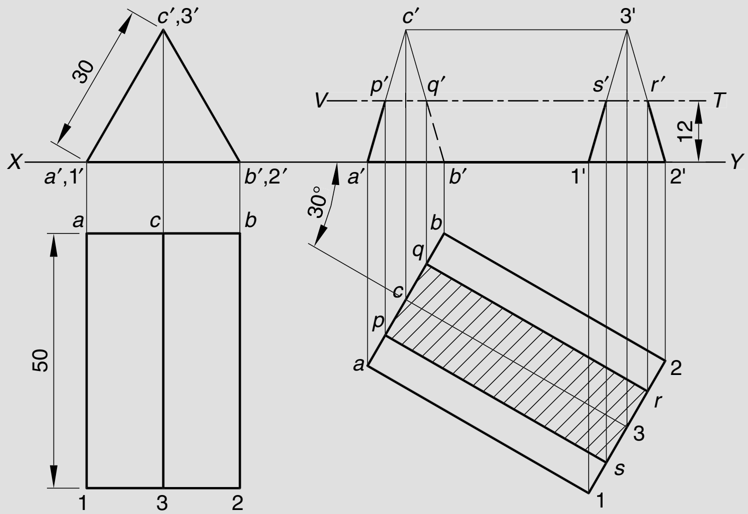


Fig. 10.7

Construction: Fig. 10.7

1. Draw a triangle *a′b′c′* keeping *a′b′* on *XY* to represent the front view. Project all the corners to obtain *ab*21 to represent the top view.
2. Reproduce this top view keeping *c*3 at 30º with *XY*. Project it to meet locus lines from the front view of the first stage and obtain the new front view.
3. Draw section plane V.T. parallel to and 12 mm above *XY*.
4. The section plane cuts *a′c′* at *p′*, *b′c′* at *q′*, 2′3′ at *r′*, and 1′3′ at *s′*. Project points *p′*, *q′*, *r′* and *s′* to meet *ac*, *bc*, 2-3 and 1-3 at points *p*, *q*, *r* and *s*. Join *pqrs* and hatch the enclosed portion.

Example 10.3 (Fig. 10.8)

A hexagonal prism of 20 mm base and 60 mm height is resting on one of its corners on the ground with the base making 60º with the ground. The axis is parallel to V.P. A section plane parallel to H.P. and perpendicular to V.P. cuts the object such that it is 15 mm from the base as measured along the axis. Draw its sectional view from the above and the view from the front.

[RGPV Dec. 2001]

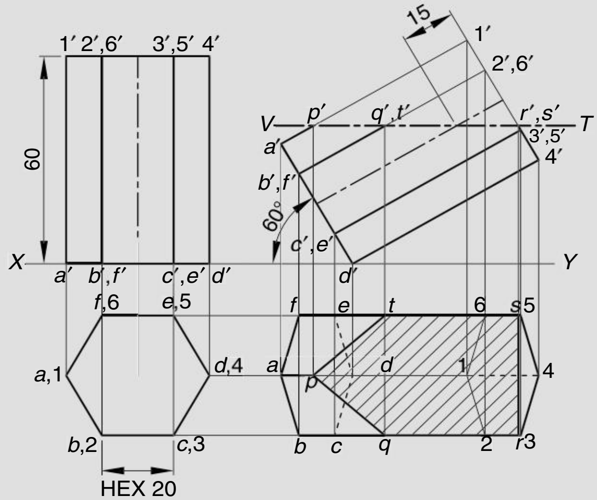


Fig. 10.8

Construction: Fig. 10.8

1. Draw a hexagon *abcdef* keeping diagonal *ad* parallel to *XY* to represent the top view. Project all the corners and obtain *a′d′*4′1′ to represent the front view.
2. Reproduce the front view of the first stage keeping *d′* on *XY* and *d′a′* at 60º to *XY*. Project the top view to meet locus lines from the front view of the first stage and obtain the new front view.
3. Draw section plane V.T. parallel to *XY* passing through a point 15 mm away from the base along the axis.
4. The section plane cuts *a′*1′ at *p′*, *b′*2′ at *q′*, 2′3′ at *r′*, 5′6′ at *s′* and *f′*6′ at *t′*. Project *p′*, *q′*, *r′*, *s′* and *t′* to meet edges *a*1, *b*2, 2-3, 5-6 and *f*6 at points *p*, *q*, *r*, *s* and *t*. Join *pqrst* and hatch the enclosed portion.

Example 10.4 (Fig. 10.9)

A hexagonal pyramid of 30 mm side of base and 60 mm long axis rests with its base on H.P. and one of the edges of the base is parallel to V.P. It is cut by a horizontal section plane at a distance 30 mm above the base. Draw the front view and sectional top view. [RGPV Feb. 2011]

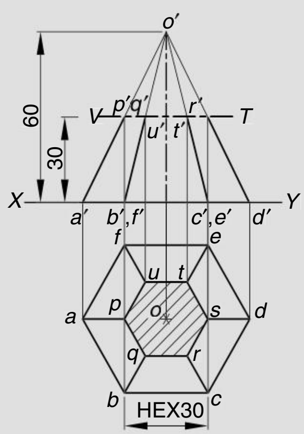


Fig. 10.9

Construction: Fig. 10.9

1. Draw a hexagon *abcdef* keeping *ef* parallel to *XY*. Locate the centroid *o* and join it with all the corners of the hexagon. The figure represents the top view. Project all the corners to obtain $a'd'o'$ as the front view.
2. Draw section plane V.T. parallel to and 30 mm above *XY*.
3. The section plane cuts $o'a'$ at p', $o'b'$ at q', $o'c'$ at r', $o'd'$ at s', $o'e'$ at t' and $o'f'$ at u'. Project p', q', r', s', t' and u' to meet edges *oa*, *ob*, *oc*, *od*, *oe* and *of* at points *p*, *q*, *r*, *s*, *t* and *u*. Join *pqrstu* and hatch the enclosed portion.

Example 10.5 (Fig. 10.10)

A right circular cone of the 45 mm base diameter and 55 mm axis long is lying on one of its generators on the H.P. It is cut by a horizontal section plane passing through the mid-point of the axis. Draw the projections of the cone and its true section.

[RGPV Feb. 2008, April 2010]

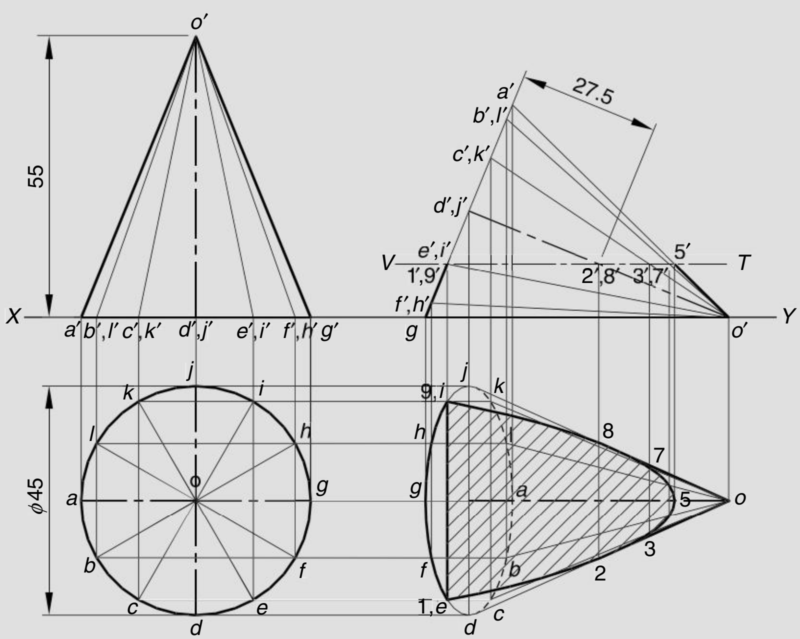


Fig. 10.10

Construction: Fig. 10.10

1. Draw a circle *adgj* to represent the top view. Divide the circle into 12 equal parts to represent generators in the top view. Project ends of the generators and obtain $a'g'o'$ as the front view.
2. Reproduce the front view of the first stage keeping generator $g'o'$ on *XY*. Project the front view to meet locus lines from the top view of the first stage and obtain the new top view.

3. Draw section plane V.T. parallel to *XY* bisecting the axis.
4. The section plane cuts the base at points 1′ and 9′ while the generators *d′o′*, *e′o′*, *f′o′*, *g′o′*, *h′o′*, *i′o′*, *j′o′* at points 2′, 3′, 4′, 5′, 6′, 7, 8′. Project points 1′, 2′, 3′, 4′, 5′, 6′, 7, 8′ and 9′ to meet the corresponding generators in the top view at points 1, 2, 3, 4, 5, 6, 7, 8 and 9. Join 123456789 and hatch the enclosed portion.

10.5 SECTIONS OF SOLIDS BY PLANE PARALLEL TO V.P.

Example 10.6 (Fig. 10.11)

A cube of 30 mm long edges is resting on the H.P. on one of its faces with a vertical face inclined at 30º to the V.P. It is cut by a section plane parallel to the V.P. and 10 mm away from the axis and further away from the V.P. Draw the sectional front view and top view of the cube. [RGPV June 2009]

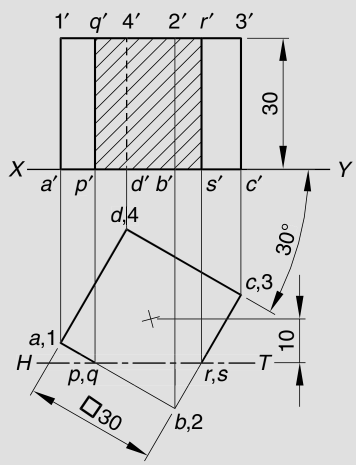


Fig. 10.11

Construction: Fig. 10.11

1. Draw a square *abcd* keeping *cd* at 30º to *XY* in the top view. Project all the corners to obtain *a′b′*3′1′ as front view.
2. Locate centre *o* of the square *abcd*. Draw section plane H.T. parallel to *XY* and 10 mm from the centre *o*.
3. The section plane cuts the edges *ab*, 1-2, 2-3 and *bc* at points *p*, *q*, *r* and *s*. Project points *p*, *q*, *r* and *s* to meet the edges *a′b′*, 1′2′, 2′3′ and *b′c′* at points *p′*, *q′*, *r′* and *s′*. Join *p′q′r′s′* and hatch the enclosed portion.

Note: When a solid is cut by a vertical section plane, the true shape of the section is seen in the front view.

Example 10.7 (Fig. 10.12)

A pentagonal prism of 30 mm base side and 60 mm long axis rests on one of its rectangular faces on H.P. with its axis inclined at 30º to V.P. A section plane parallel to V.P. cuts the solid through the centre of the axis into two halves. Draw the projections of the solid.

[RGPV June 2008(o)]

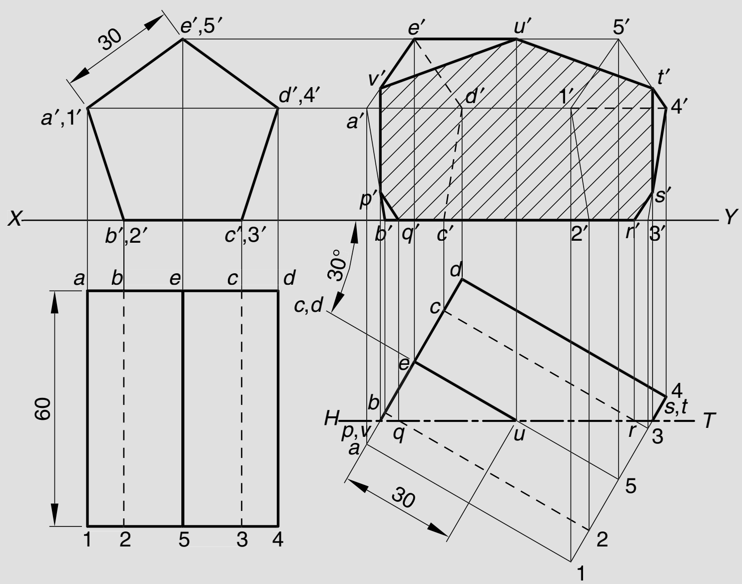


Fig. 10.12

Construction: Fig. 10.12

1. Draw a pentagon *a′b′c′d′e′* keeping *b′c′* on *XY*. This represents the front view. Project the corners and obtain *ad*41 as its top view.
2. Reproduce this top view keeping the axis at 30º to *XY*. Project it to meet locus lines from the front view of the first stage and obtain the new front view.
3. Draw section plane H.T. parallel to *XY* bisecting the axis.
4. The section plane cut the edges *ab* at *p*, *b*2 at *q*, *c*3 at *r*, 3-4 at *s*, 4-5 at *t* and *e*5 at *u*. Project points *p*, *q*, *r*, *s*, *t* and *u* to meet their corresponding edges in the front view at points *p′*, *q′*, *r′*, *s′*, *t′* and *u′*. Join *p′q′r′s′t′u′* and hatch the enclosed portion.

Example 10.8 (Fig. 10.13)

A cylinder of 50 mm base diameter and 60 mm long axis is resting on its base on H.P. A section plane parallel to V.P. cuts the cylinder at a distance 17 mm from the axis. Draw its sectional front view and top view.

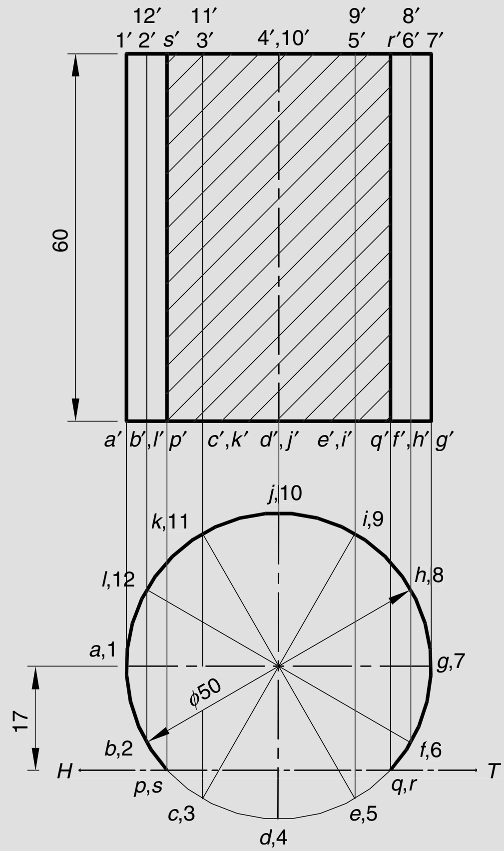


Fig. 10.13

Construction: Fig. 10.13

1. Draw a circle *adgj* of 50 mm diameter as the top view. Divide it into 12 equal parts. Project the circle and obtain *a*′*g*′7′1′ as the front view.
2. Draw section plane H.T. parallel to *XY* and passes through a point 17 mm from the axis *o* towards the observer.
3. The section plane cuts the base circle at *p*, *q*, *r* and *s*. Project *p*, *q*, *r* and *s* and obtain *p*′, *q*′, *r*′ and *s*′ in the front view. Join *p*′*q*′*r*′*s*′ and hatch the enclosed portion.

Example 10.9 (Fig. 10.14)

A pentagonal pyramid side of 35 mm base and 60 mm long axis rests with its base on H.P. such that one of the edges of the base is perpendicular to V.P. A section plane perpendicular to H.P. and parallel to V.P. cuts the pyramid at a distance of 20 mm from the corner of the base nearer to the observer. Draw its top and sectional front views.

[RGPV June 2008, Aug. 2010]

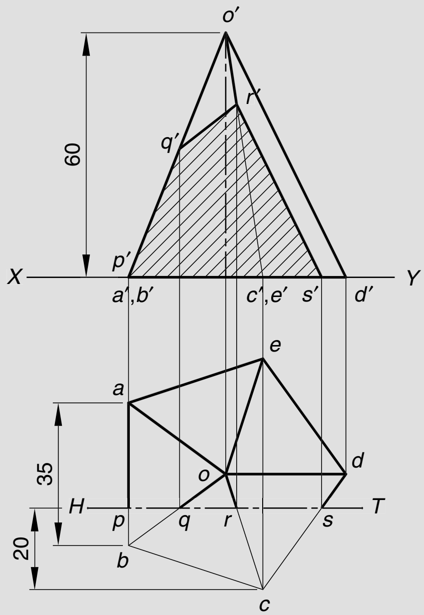


Fig. 10.14

Construction: Fig. 10.14

1. Draw a pentagon *abcde* keeping *ab* perpendicular to *XY*. Locate the centroid *o* and join it with all the corners of the pentagon. This figure represents the top view. Project the corners and obtain *a′d′o′* as the front view.
2. Draw section plane H.T. parallel to *XY* and passing through a point 20 mm from point *c*.
3. The section plane cuts the edges *ab* at *p*, *ob* at *q*, *oc* at *r* and *cd* at *s*. Project points *p*, *q*, *r* and *s* to meet their respective edges *a′b′*, *o′b′*, *o′c′* and *b′d′* at points *p′*, *q′*, *r′* and *s′*. Join *p′q′r′s′* and hatch the portion enclosed by it.

Example 10.10 (Fig. 10.15)

A cone of 50 mm base diameter and 60 mm long axis is resting on its base on the H.P. It is cut by a section plane parallel to the V.P. and passing through a point 15 mm away from the axis towards the observer. Draw its sectional front view.

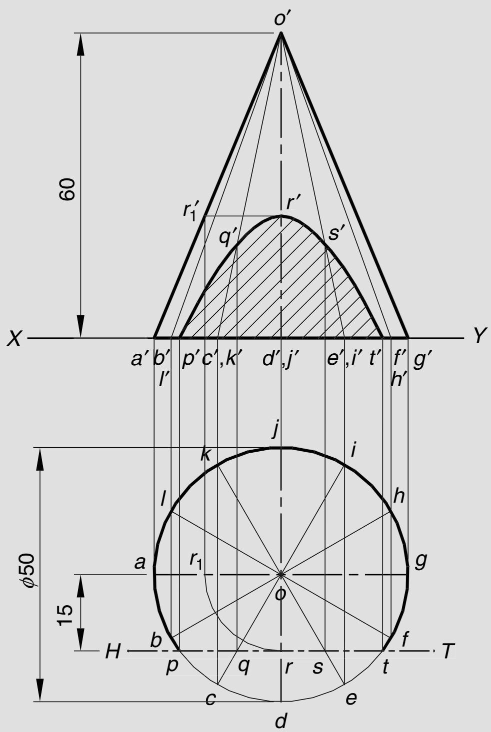


Fig. 10.15

Construction: Fig. 10.15

1. Draw a circle *adgj* in top view. Divide it into 12 equal parts and project to get $a'g'o'$ as the front view.
2. Draw section plane H.T. parallel to *XY* and 15 mm away from *o*.
3. The section plane cuts the base circle and generators at *p*, *q*, *r*, *s* and *t*. Project *p*, *q*, *s* and *t* to meet $b'c'$, $o'b'$, $o'e'$ and $e'f'$ at points p', q', s' and t' respectively.
4. Point *r* cannot be directly projected to meet $o'd'$. For this draw an arc with centre *o* and radius *ro* to meet *oa* at point r_1. Project points r_1 to meet $o'a'$ at points r_1'. Draw horizontal line from r_1' to meet $o'd'$ at r'. Join $p'q'r's't'$ and hatch the portion enclosed by it.

10.6 SECTIONS OF SOLIDS BY AUXILIARY INCLINED PLANE

Example 10.11 (Fig. 10.16)

A square prism (25 mm base side × 60 mm height) is kept on H.P. with its axis vertical and two adjacent base sides equally inclined to V.P. It is cut by a section plane whose V.T. makes an angle of 30º with the reference line and bisects the axis. Draw sectional top view and true shape of section. [RGPV Dec. 2008]

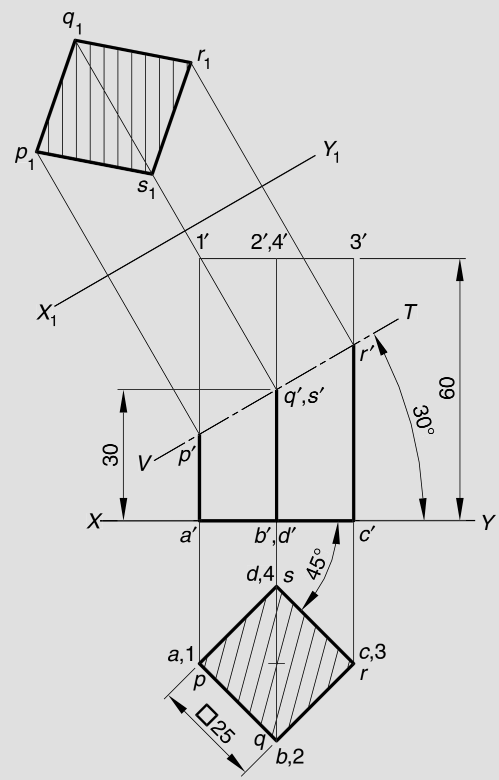


Fig. 10.16

Construction: Fig. 10.16

1. Draw a square *abcd* keeping edges at 45º to *XY* in the top view. Project all the corners and obtain *a*′*c*′3′1′ as the front view.
2. Draw section plane V.T. inclined at 30º to *XY* bisecting the axis.
3. The section plane cuts the edges *a*′1′ at point *p*′, *b*′2′ at point *q*′, *c*′3′ at point *r*′ and *d*′4′ at point *s*′. Project *p*′, *q*′, *r*′ and *s*′ to meet the edges *a*1, *b*2, *c*3 and *d*4 at point *p*, *q*, *r* and *s*. Join *pqrs* and hatch

the portion enclosed by it. Since the boundary lines of *pqrs* are inclined at 45º to *XY* the hatching lines should be drawn at 30º or 60º to *XY*.

4. Draw another reference plane X_1Y_1 parallel to the V.T. and project points p', q', r' and s' on X_1Y_1. Locate points p_1, q_1, r_1 and s_1 on these projectors such that their distances from X_1Y_1 are equal to distances of *p*, *q*, *r* and *s* from *XY* respectively. Join $p_1q_1r_1s_1$ and hatch the portion enclosed by it.

Note 1: When the boundaries of the sectional surface are inclined at 45º to the reference line, the hatching lines may be drawn at 30º or 60º to *XY*.

Note 2: When a solid is cut by an A.I.P. the true shape of the section is seen on the plane parallel to the A.I.P.

Example 10.12 (Fig. 10.17)

A cylinder of 45 mm diameter and 70 mm long is resting on one of its bases on H.P. It is cut by a section plane inclined at 60º with H.P. and passing through a point on the axis at 15 mm from one end. Draw the two views of the solid. Also obtain the true shape of the section.

[RGPV Dec. 2010]

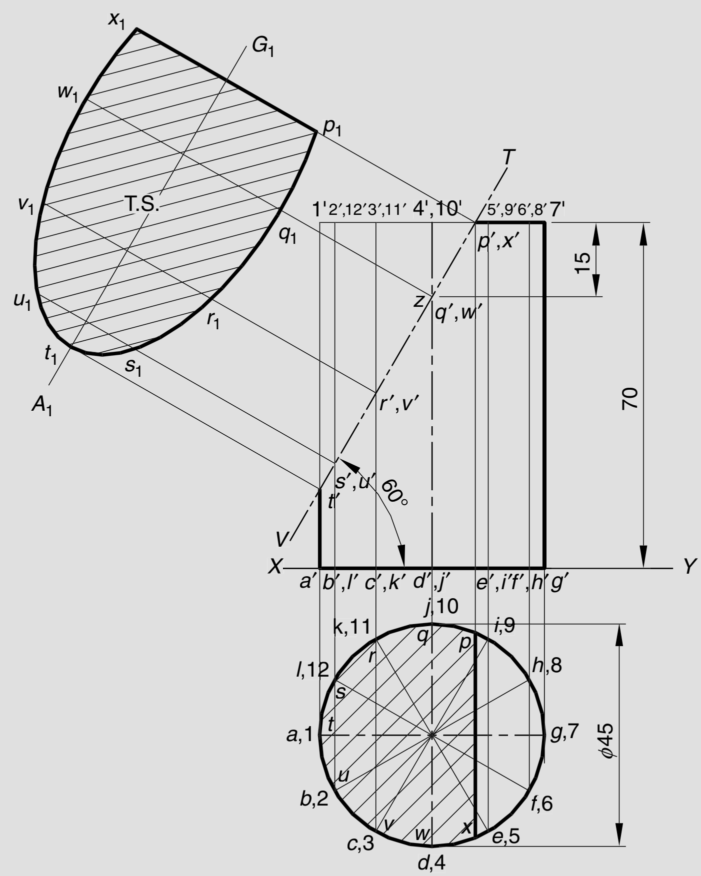


Fig. 10.17

Construction: Fig. 10.17

1. *Projections*: Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts. Project the circle and obtain $a'g'7'1'$ as its front view.
2. *Sectional plane*: Mark a point *z* on the axis, 15 mm below top end, i.e. 55 mm above *XY*. Draw a V.T. in the front view inclined at 60º to *XY* and passes through point *z*.
3. *Sectional top view*: Let the cutting plane cut the generators and the rim of the cylinder in the front view $9'10'$ at p', $j'10'$ at q', $k'11'$ at r' and $l'12'$ at s', $a'1'$ at t', $b'2'$ at u', $c'3'$ at v', $d'4'$ at w', and $4'5'$ at x'. Project points p', q', r', s', t', u', v', w' and x' to meet their corresponding edges at points *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w* and *x*. Join *pqrstuvwx* and hatch the enclosed portion.
4. *True shape*: Draw X_1Y_1 parallel to V.T. Project points p', q', r', s', t', u', v', w' and x' perpendicular on X_1Y_1 and extend them. Locate points p_1, q_1, r_1, s_1, t_1, u_1, v_1, w_1 and x_1 on the projectors such that their distance from X_1Y_1 is equal to distance of points *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w* and *x* from *XY*. Join $p_1q_1r_1s_1t_1u_1v_1w_1x_1$ and hatch the enclosed portion.

Example 10.13 (Fig. 10.18)

A hexagonal pyramid of 30 mm base side and axis 65 mm long is resting on its base on the H.P. with two edges parallel to the V.P. It is cut by a section plane perpendicular to the V.P. inclined at 45º to the H.P. and intersecting the axis at a point 25 mm above the base. Draw the front view, sectional top view and true shape of the section.

[RGPV June 2004, Dec. 2007, April 2010]

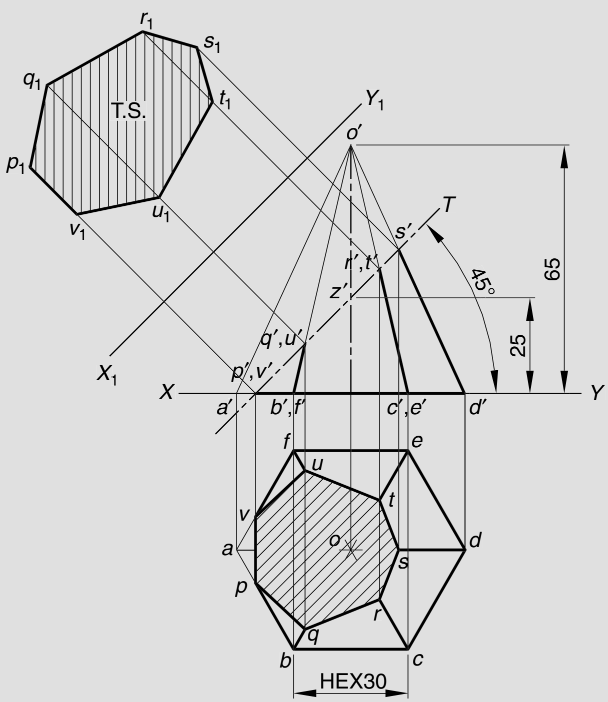


Fig. 10.18

Construction: Fig. 10.18

1. Draw a hexagon *abcdef* keeping *ef* parallel to *XY*. Join the corners with centroid *o*. This represents the top view. Project the corners and obtain *a′d′o′* as the front view.
2. Mark *z′* on the axis 25 mm above the base. Draw V.T. at 45º to *XY* passing through *z′*.
3. Let V.T. cut the edges *a′b′* at *p′*, *o′b′* at *q′*, *o′c′* at *r′*, *o′d′* at *s′*, *o′e′* at *t′*, *o′f′* at *u′* and *e′a′* at *v′*. Project *p′*, *r′*, *s′*, *t′*, *u′* and *v′* to meet their respective edges *ab*, *oc*, *od*, *oe*, of and *ea* in the top view at points *p*, *r*, *s*, *t*, *u* and *v*.
4. Draw X_1Y_1 parallel to V.T. Project *p′*, *q′*, *r′*, *s′*, *t′*, *u′* and *v′* on X_1Y_1. Locate points p_1, q_1, r_1, s_1, t_1, u_1 and v_1 on these projectors such that their distances from X_1Y_1 are equal to distances of *p*, *q*, *r*, *s*, *t*, *u* and *v* from *XY* respectively. Join $p_1q_1r_1s_1t_1u_1v_1$ and hatch to represent the true shape of the section.

Example 10.14 (Fig. 10.19)

A square pyramid of 40 mm base side and 60 mm long axis is resting with its base on H.P. with a side of base parallel to the V.P. Draw its sectional views and true shape of the section, if it is cut by a plane perpendicular to the V.P. inclined at 60º to the H.P. bisecting the axis.

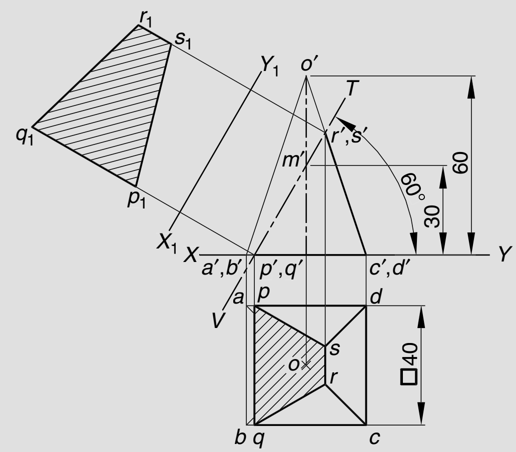


Fig. 10.19

Construction: Fig. 10.19

1. Draw a square *abcd* keeping side *ad* parallel to *XY* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain *a′b′o′* as the front view.
2. Draw section plane V.T. inclined at 60º to *XY* bisecting the axis.
3. The section plane cuts the edges *a′d′* at *p′*, *b′c′* at *q′*, *o′b′* at *r′* and *o′d′* at *s′*. Project *p′*, *q′*, *r′* and *s′* to meet their respective edges *ad*, *bc*, *oc* and *od* in the top view at points *p*, *q*, *r* and *s*.
4. Draw X_1Y_1 parallel to the V.T. Project points *p′*, *q′*, *r′* and *s′* on X_1Y_1. Locate points p_1, q_1, r_1 and s_1 on these projectors such that their distances from X_1Y_1 are equal to distances of *p*, *q*, *r* and *s* from *XY* respectively. Join $p_1q_1r_1s_1$ and hatch to represent the true shape of the section.

Example 10.15 (Fig. 10.20)

A square pyramid of 40 mm base side and 65 mm long axis has its base on the H.P. and all the edges of the base equally inclined to the V.P. It is cut by a section plane perpendicular to the V.P. and inclined at 45º to H.P. and bisecting the axis. Draw the sectional top view and true shape of the section.

[RGPV April 2009]

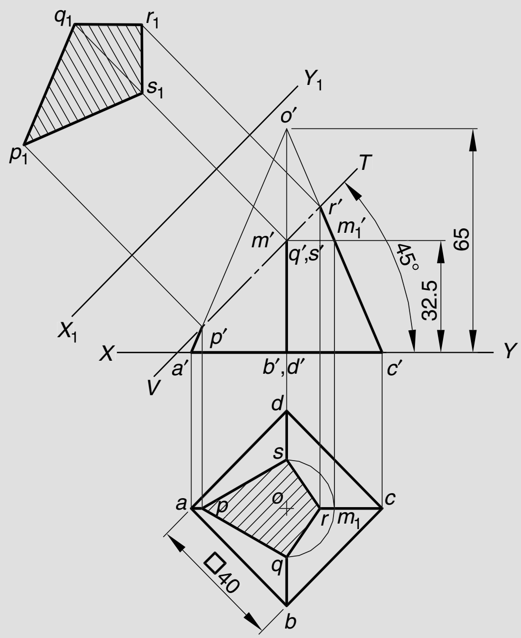


Fig. 10.20

Construction: Fig. 10.20

1. Draw a square *abcd* keeping sides at 45º to *XY*. Join the corners with centroid *o*. This represents the top view. Project the corners and obtain $a'c'o'$ as the front view.
2. Draw section plane V.T. inclined at 45º to *XY* bisecting the axis.
3. Let V.T. cut the edges $o'a'$ at p', $o'b'$ at q', $o'c'$ at r' and $o'd'$ at s'. Project p' and r' to meet *oa* and *oc* at *p* and *r* respectively.
4. Points q' and s' cannot be projected directly on *ob* and *od*. For this, draw a horizontal line from q' and s' to meet $o'c'$ at m_1'. Project m_1' to meet oc at m_1. Draw an arc with centre *o* and radius om_1 to meet *ob* and *od* at points *q* and s respectively. Join *pqrs* and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to V.T. Project points p', q', r' and s′ on X_1Y_1. Locate points p_1, q_1, r_1 and s_1 on these projectors such that their distances from X_1Y_1 are equal to distances of *p*, *q*, *r* and *s* from *XY* respectively. Join $p_1q_1r_1s_1$ and hatch to represent the true shape of the section.

Example 10.16 (Fig. 10.21)

A square pyramid of 40 mm base side and 70 mm long axis rests with its base on H.P. with all the edges of the base equally inclined to the V.P. It is cut by a section plane perpendicular to the V.P. and inclined at 60º to H.P. and passing through a point on the axis at 30 mm from the base. Draw the sectional top view and front view of the pyramid.

[RGPV June 2008(o)]

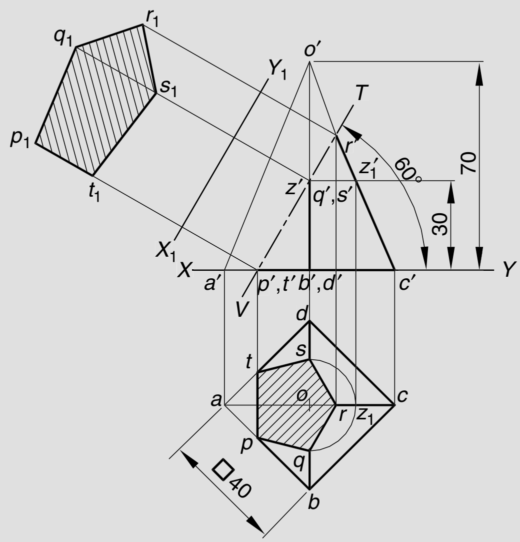


Fig. 10.21

Construction: Fig. 10.21

1. Draw a square *abcd* keeping sides at 45º to *XY*. Join the corners with centroid *o*. This represents the top view. Project the corners and obtain *a′b′o′* as the front view.
2. Draw section plane V.T. inclined at 60º to *XY*, 30 mm on the axis above the base.
3. Let V.T. cut the edges *a′b′* at *p′*, *o′b′* at *q′*, *o′c′* at *r′*, *o′d′* at *s′* and *d′a′* at *t′*. Project *p′*, *r′* and *t′* to meet *ab*, *oc* and *da* at *p*, *r*, and *t* respectively.
4. Points *q′* and *s′* cannot be projected directly on *ob* and *od*. For this, draw a horizontal line from *q′* and *s′* to meet *o′c′* at z_1'. Project z_1' to meet *oc* at z_1. Draw an arc with centre *o* and radius oz_1 to meet *ob* and *od* at points *q* and *s* respectively. Join *pqrst* and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to the V.T.. Project points *p′*, *q′*, *r′*, *s′* and *t′* on X_1Y_1. Locate points p_1, q_1, r_1, s_1 and t_1 on these projectors such that their distances from X_1Y_1 are equal to distances of points *p*, *q*, *r*, *s* and *t* from *XY* respectively. Join $p_1q_1r_1s_1t_1$ and hatch to represent the true shape of the section.

Example 10.17 (Fig. 10.22)

A right circular cone with 50 mm diameter of base and 65 mm height rests on its base on H.P. A section plane perpendicular to V.P. and inclined to H.P. at 45º cuts the cone meeting its axis at a distance of 36 mm from its base. Draw its front view, sectional top view and true shape of the section. [RGPV June 2007]

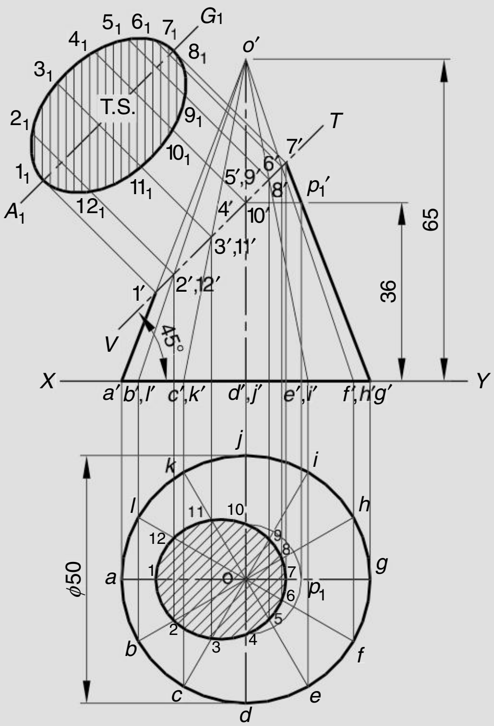


Fig. 10.22

Construction: Fig. 10.22

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. Draw section plane V.T. inclined at 45º to *XY* 36 mm on the axis from the base.
3. Let V.T. cut the generators $o'a'$ at 1′, $o'b'$ at 2′, $o'c'$ at 3′, $o'd'$ at 4′, $o'e'$ at 5′, $o'f'$ at 6′, $o'g'$ at 7′, $o'h'$ at 8′, $o'i'$ at 9′, $o'j'$ at 10′, $o'k'$ at 11′ and $o'l'$ at 12′. Project points 1′, 2′, 3′, 5′, 6′, 7′, 8′, 9′, 11′ and 12′ to meet in the top view at points 1, 2, 3, 5, 6, 7, 8, 9, 11 and 12.
4. Points 4′ and 10′ cannot be projected directly on *od* and *oj*. For this draw a horizontal line from 4′ to meet $o'g'$ at p_1'. Project p_1' to meet *og* at p_1. Draw an arc with centre *o* and radius op_1 to meet *od* and *oj* at points 4 and 10 respectively.

5. Join 1-2-3-4-5-6-7-8-9-10-11-12 and hatch the enclosed portion.
6. Draw A_1G_1 parallel to the V.T. Project 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, 9′, 10′, 11′ and 12′ on A_1G_1. Locate points 1_1, 2_1, 3_1, 4_1, 5_1, 6_1, 7_1, 8_1, 9_1, 10_1, 11_1 and 12_1 on the projectors such that their distances from A_1G_1 are equal to distances of points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 from line *ag*, respectively. Join $1_12_13_14_15_16_17_18_19_110_111_112_1$ and hatch to represent the true shape of the section.

Example 10.18 (Fig. 10.23)

A cone of 50 mm base diameter and 65 mm long axis is resting on its base on the H.P. It is cut by an A.I.P. passing through a point on the axis 25 mm below the apex parallel to one of the extreme generators. Draw its sectional top view and obtain true shape of the section.

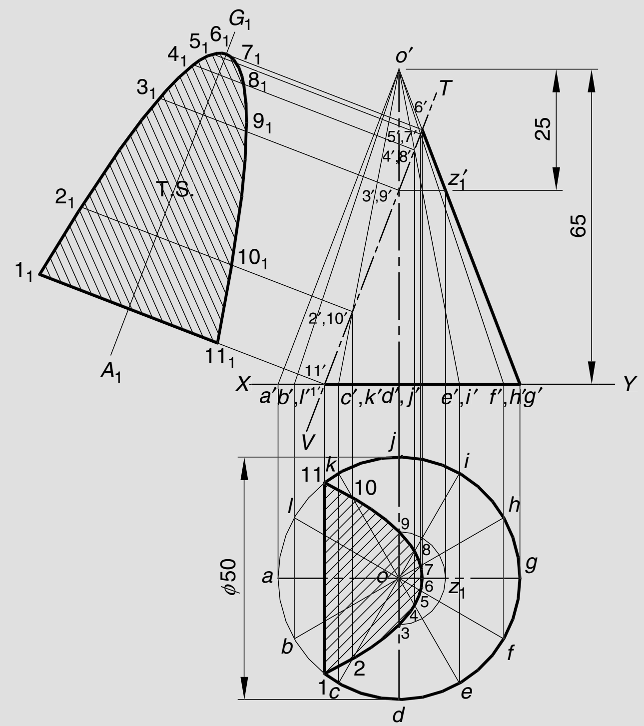


Fig. 10.23

Construction: Fig. 10.23

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts and project to obtain *a′g′o′* as the front view.
2. Draw section plane V.T. parallel to *o′a′* 25 mm on the axis below the apex.
3. Let V.T. cut the base and generators *b′c′* at 1′, *o′c′* at 2′, *o′d′* at 3′, *o′e′* at 4′, *o′f′* at 5′, *o′g′* at 6′, *o′h′* at 7′, *o′i′* at 8′, *o′j′* at 9′, *o′k′* at 10′ and *k′l′* at 11′. Project points 1′, 2′, 4′, 5′, 6′, 7′, 8′, 10′ and 11′ to meet the top view at points 1, 2, 4, 5, 6, 7, 8, 10 and 11.

4. Points 3′ and 9′ cannot be projected directly on *od* and *oj*. For this draw a horizontal line from 4′ to meet *o′g′* at z_1'. Project z_1' to meet *og* at z_1. Draw an arc with centre *o* and radius oz_1 to meet *od* and *oj* at points 3 and 9 respectively.
5. Join 1-2-3-4-5-6-7-8-9-10-11 and hatch the enclosed portion.
6. Draw A_1G_1 parallel to the V.T. Project points 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, 9′, 10′ and 11′ on A_1G_1. Locate points 1_1, 2_1, 3_1, 4_1, 5_1, 6_1, 7_1, 8_1, 9_1, 10_1 and 11_1 on the projectors such that their distances from A_1G_1 line are equal to distances of points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11 from *ag* respectively. Join $1_12_13_14_15_16_17_18_19_110_111_1$ and hatch to represent the true shape of the section.

10.7 SECTIONS OF SOLIDS BY AUXILIARY VERTICAL PLANE

Example 10.19 (Fig. 10.24)

A square prism, having a 40 mm base side and an 60 mm long axis, rests on its base on H.P. such that one of its rectangular faces makes an angle of 30º with V.P. It is cut by a section plane perpendicular to H.P. and inclined at 60º to V.P. passing through the prism such that the face which makes 60º with V.P. is bisected. Draw its sectional front view, top view and true shape of section.

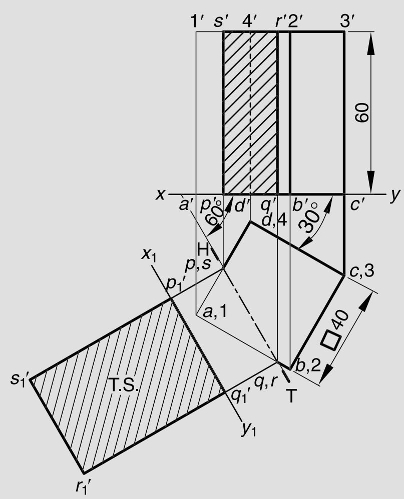


Fig. 10.24

Construction: Fig. 10.24

1. Draw a square *abcd* of 40 mm side keeping *cd* at 30º to *XY* to represent the top view. Project the corners and obtain *a′c′*3′1′ as the front view.
2. Draw section plane H.T. at 60º to *XY* bisecting *ad*.

3. The H.T. cuts *ad* at *p*, *ab* at *q*, 2-1 at *r* and 1-4 at *s*. Project *p*, *q*, *r*, and *s* to meet the front view at *p*′, *q*′, *r*′ and *s*′. Join *p*′*q*′*r*′*s*′ and hatch the enclosed portion.
4. Draw X_1Y_1 parallel to H.T. Project *p*, *q*, *r* and *s* on X_1Y_1. Locate points p_1', q_1', r_1' and s_1' on these projectors such that their distances from X_1Y_1 line are equal to distances of *p*, *q*, *r* and *s* from *XY*. Join $p_1q_1r_1s_1$ and hatch the enclosed portion to represent the true shape of the section.

Note: When a solid is cut by an A.V.P. the true shape of the section is seen on any plane parallel to A.V.P.

Example 10.20 (Fig. 10.25)

A cylinder with 50 mm base diameter and 60 mm long axis, is resting on its base on the H.P. It is cut by an A.V.P. inclined at 60º to the reference line at a distance 17 mm from the axis. Draw its sectional front view, top view and true shape of the section.

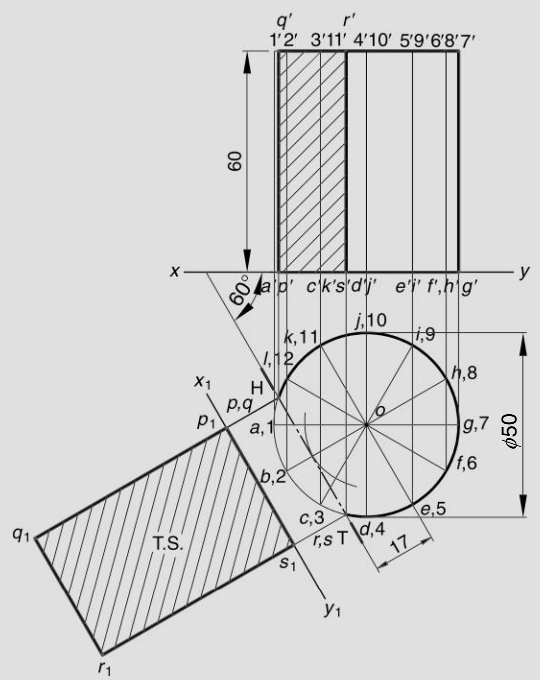


Fig. 10.25

Construction: Fig. 10.25

1. Draw a circle with a 50 mm diameter in the top view. Divide it into 12 equal parts and project it to obtain *a*′*g*′7′1′ as its front view.
2. Draw section plane H.T. at 60º to *XY* tangent to the arc with centre *o* and 17 mm radius.

3. Let H.T. cut the top view *p*, *q*, *r* and *s*. Project these points to meet the front view at *p′*, *q′*, *r′* and *s′* respectively. Join *p′q′r′s′* and hatch the enclosed portion.
4. Draw X_1Y_1 parallel to H.T. Project points *p*, *q*, *r* and s on X_1Y_1. Locate points p_1', q_1', r_1' and s_1' on the projectors such that their distances from X_1Y_1 line are equal to distances of points *p*, *q*, *r* and *s* from *XY*. Join $p_1q_1r_1s_1$ and hatch the enclosed portion to represent the true shape of the section.

Example 10.21 (Fig. 10.26)

A pentagonal pyramid having a base with 30 mm sides and 70 mm long axis, is resting on its base in the H.P. such that an edge of the base nearer to the V.P. is parallel to it. A vertical section plane inclined at 45º to the V.P. cut the pyramid at a distance of 8 mm from the axis. Draw its top view, sectional front view and true shape of the section.

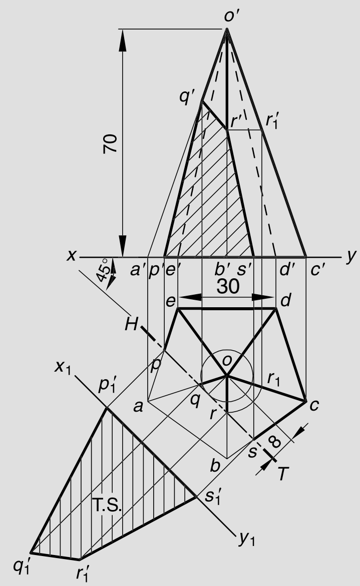


Fig. 10.26

Construction: Fig. 10.26

1. Draw a pentagon *abcde* keeping *de* parallel to *XY* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain *a′c′o′* as the front view.
2. Draw H.T. at 45º to *XY* and tangent to an arc with centre *o* and radius 8 mm.
3. Let H.T. cut the edges *ae* at *p*, *oa* at *q*, *ob* at *r* and *bc* at *s*. Project *p*, *q* and *s* to meet their corresponding edges *a′e′*, *o′a′* and *b′c′* at points *p′*, *q′* and *s′*.
4. Point *r* cannot be directly projected to *o′b′*. For this draw an arc with centre *o* and radius *oq* to meet *oc* at point z_1. Project z_1 to meet *o′c′* at z_1'. Draw horizontal line from z_1' to meet *o′b′* at point *r′*. Join *p′q′r′s′* and hatch the enclosed portion.

5. Draw X_1Y_1 parallel to H.T. and project *p*, *q*, *r* and *s* on X_1Y_1. Locate points p_1', q_1', r_1' and s_1' on these projectors such that their distances from X_1Y_1 are equal to the distances of points *p′*, *q′*, *r′* and *s′* from *XY* respectively. Join $p_1'q_1'r_1's_1'$ and hatch the enclosed portion to represent the true shape of the section.

Example 10.22 (Fig. 10.27)

A cone with 60 mm base diameter and 70 mm long axis is resting on its base on the H.P. It is cut by a section plane whose H.T. is inclined at 60º to the reference line and passes through a point that is 20 mm away from the axis. Draw its sectional front view and obtain true shape of the section.

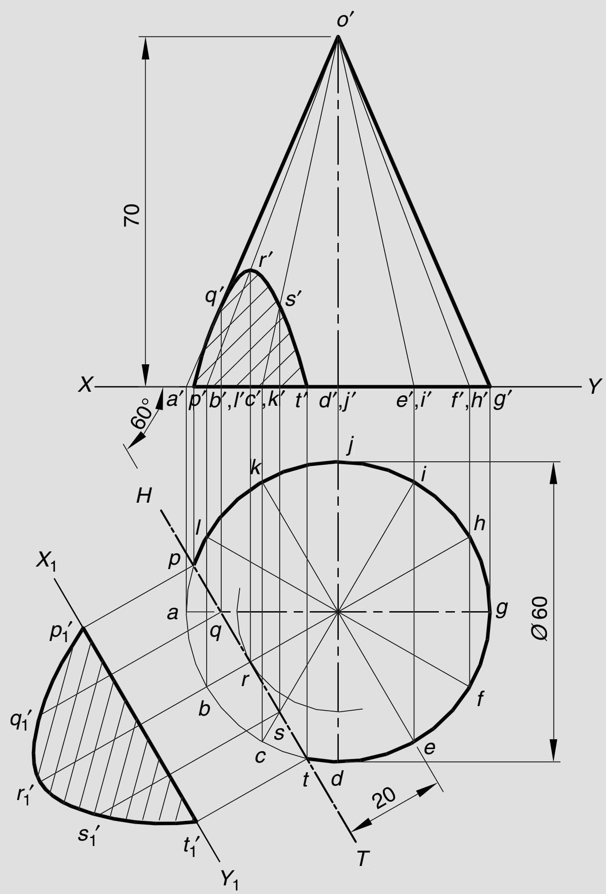


Fig. 10.27

Construction: Fig. 10.27

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts and project to obtain *a′g′o′* as the front view.

2. Draw H.T. at 60º to *XY* and tangent to an arc with centre *o* and 20 mm radius.
3. Let H.T. cut the top view at *p*, *q*, *r*, *s* and *t*. Project these points to get *p*′, *q*′, *r*′, s′ and *t*′. Join *p*′*q*′*r*′*s*′*t*′ and hatch the enclosed portion.
4. Draw X_1Y_1 parallel to H.T. Project points *m*, *n*, *p*, *q* and *r* on X_1Y_1. Locate points p_1', q_1', r_1', s_1' and t_1' on the projectors such that their distances from X_1Y_1 are equal to distances of points *p*′, *q*′, *r*′, *s*′ and *t*′ from *XY*. Join $p_1'q_1'r_1's_1't_1'$ and hatch the enclosed portion to represent the true shape of the section.

10.8 SECTIONS OF SOLIDS BY A PROFILE PLANE

Example 10.23 (Fig. 10.28)

A square prism having base with 40 mm sides and 60 mm long axis, rests on its base on the H.P. such that one of its rectangular faces is inclined at 30º to the V.P. It is cut by a section plane perpendicular to both the H.P. and the V.P., and passes through one of the vertical edges. Draw its front view, top view and sectional side view.

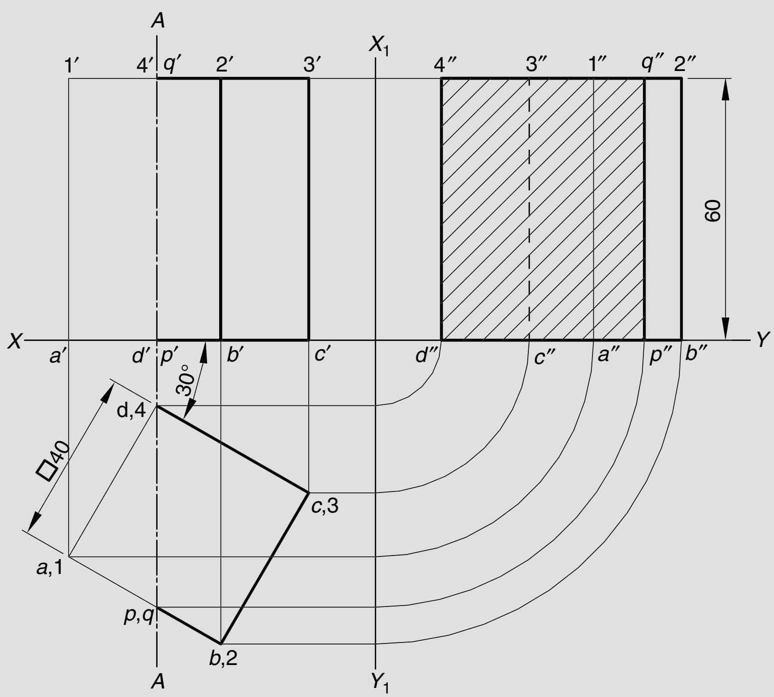


Fig. 10.28

Construction: Fig. 10.28

1. Draw a square *abcd* keeping *cd* at 30º to *XY*. This represents the top view. Project the corners and obtain *a*′*c*′3′1′ as the front view. Also, obtain *d*″*b*″2″4" as the side view.

2. Draw line *AA* perpendicular to *XY* and passing through *d*-4. This represents both the V.T. and the H.T. of the cutting plane.
3. Let *AA* cut the edges in the top view at *p* and *q*. Transfer points *p* and *q* in the side view to meet the corresponding edges *a″b″* and 1″2″ at points *p″* and *q″*. Join *d″p″q″*1″ and hatch the enclosed portion. As the section plane is parallel to the profile plane, *d″p″q″*1″ represents the true shape of the section.

Note: When a solid is cut by a profile plane the true shape of the section is seen in the side view.

Example 10.24 (Fig. 10.29)

A pentagonal pyramid having a base with 40 mm sides and 70 mm long axis is resting on the H.P. on an edge of its base such that axis is inclined at 45º to the H.P. and parallel to the V.P. It is cut in such manner that H.T. and V.T. of the section plane are perpendicular to the *XY* and pass through the edge on which the pyramid is resting. Draw the front view, top view and sectional side view.

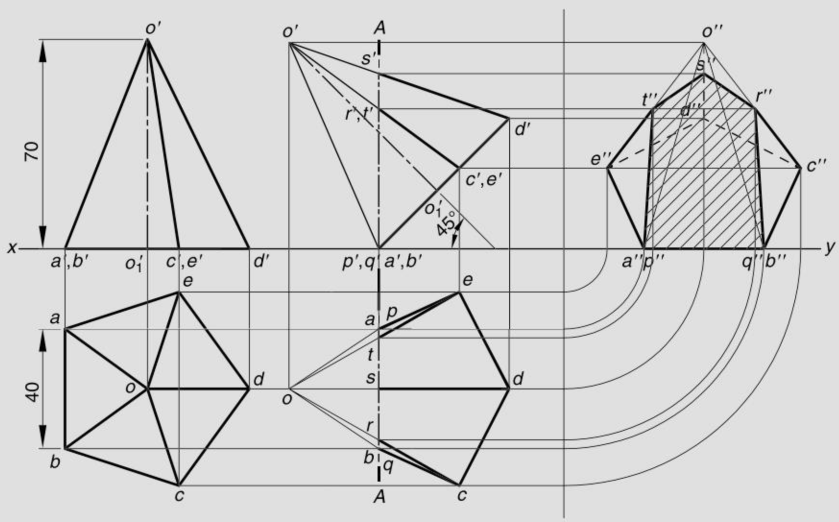


Fig. 10.29

Construction: Fig. 10.29

1. Draw a pentagon *abcde* keeping *ab* perpendicular to *XY* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain *a′d′o′* as the front view.
2. Reproduce the front view such that *a′b′* is on *XY* and axis $o'o_1'$ is inclined at 45º to *XY*. Project the front view to meet horizontal lines from the top view of the first stage and obtain *obcdea* as the new top view.
3. Also, obtain the side view by taking projectors from the front and top views.
4. Draw *AA* perpendicular to the *XY* passing through *a′b′* and *ab* to represent V.T. and H.T. in the front and top views respectively.

5. Let H.T. cut the edges *oc* at *r*, *od* at *s* and *oe* at *t*. Project *r*, *s* and *t* to mark *r′*, *s′* and *t′* in the front view. Also project *r*, *s* and *t* to meet their respective edges in the side view at points *r″*, *s″* and *t″*. Join *a″b″r″s″t″* and hatch the enclosed portion.

10.9 MISCELLANEOUS EXAMPLES

Example 10.25 (Fig. 10.30)

A rectangular block of 66 × 48 × 22 mm dimensions has a rectangular hole of 40 × 20 mm cut centrally through it. It rests on its base on H.P. A section plane perpendicular to V.P. and inclined at 45º to H.P. passes through the top end of the axis in front view. Draw its front view, sectional top view and true shape of the section. [RGPV Feb. 2007]

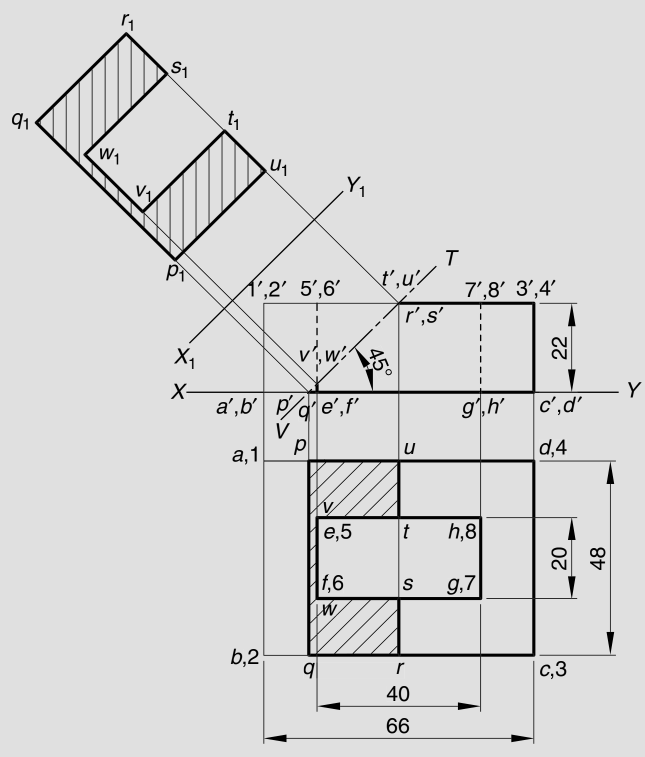


Fig. 10.30

Construction: Fig. 10.30

Assumption: Solid rests on the H.P. with a side of the base parallel to the V.P.

1. Draw two concentric rectangles *abcd* and *efgh* keeping longer side parallel to *XY*. This represents the top view. Project the corners and obtain *b′c′*3′1′ as the front view.

2. Draw V.T. at 45º to XY bisecting the top edge.
3. Let V.T. cut the edges $a'd'$ at p', $b'c'$ at q', $2'3'$ at r', $6'7'$ at s', $5'8'$ at t', $1'4'$ at u', $e'5'$ at v' and $f'6'$ at w'. Project points p', q', r', s', t', u', v' and w' to meet their respective edges at points p, q, r, s, t, u, v and w. Join *pqrswvtu* and hatch the enclosed portion.
4. Draw X_1Y_1 parallel to the V.T. Project points p', q', r', s', t', u', v' and w' on X_1Y_1. Locate points p_1, q_1, r_1, s_1, t_1, u_1, v_1 and w_1 on the projectors such that their distances from X_1Y_1 is equal to distance of points p, q, r, s, t, u, v and w from XY respectively. Join $p_1q_1r_1s_1w_1v_1t_1u_1$ and hatch to represent the true shape of the section.

Example 10.26 (Fig. 10.31)

A cylinder of 60 mm base diameter and 70 mm long axis is resting on its base in the H.P. It is cut by two auxiliary inclined planes which make angles of 60º and 45º with the H.P. passing through the top end of the axis. Draw its sectional top view and true shape of the section.

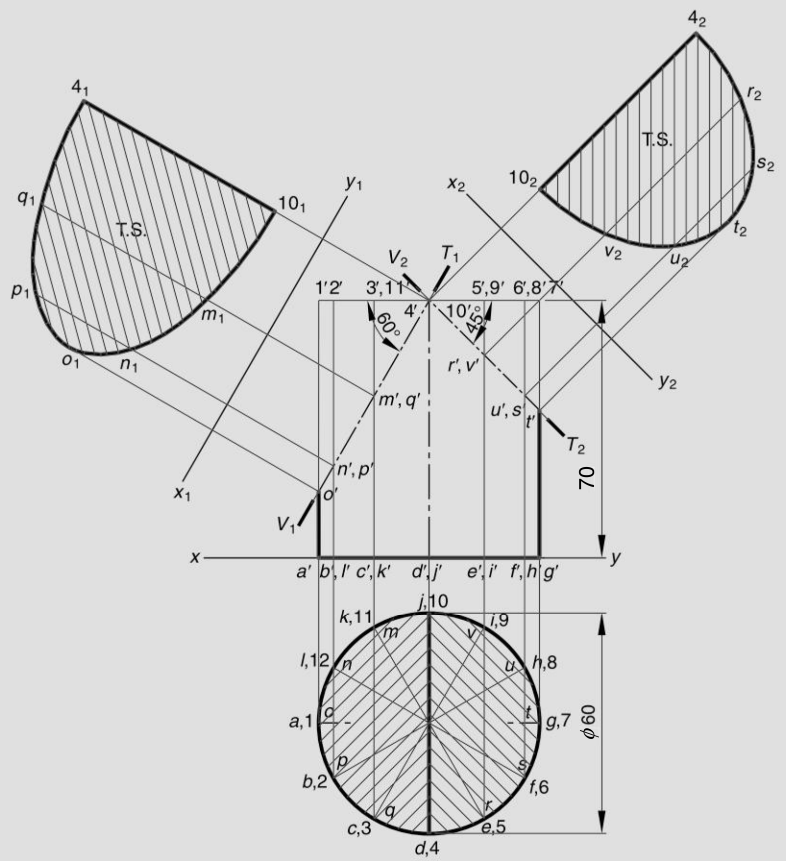


Fig. 10.31

Construction: Fig. 10.31

1. Draw a circle *adgj* as the top view. Project and obtain $a'g'7'1'$ as the front view.

2. Draw V_1T_1 and V_2T_2 inclined at 60º and 45º with *XY* respectively and passing through the top end of the axis.
3. As the cutting planes cut all the generators. Hatch the circle in the top view to represent the sectional top view. It may be noted that a line 4-10 will be visible in the top view.
4. Draw X_1Y_1 parallel to V_1T_1 and project points 10′, *m*′, *n*′, *o*′, *p*′, *q*′ and 4′ on X_1Y_1. Locate 10_1, m_1, n_1, o_1, p_1, q_1 and 4_1 on the projectors such that their distances from X_1Y_1 are equal to distances of points 4, *m*, *n*, *o*, *p*, *q* and 10 from *XY*. Join $10_1m_1n_1o_1p_1q_14_1$.
5. Draw X_2Y_2 parallel to V_2T_2. Project points 4′, *r*′, *s*′, *t*′, *u*′, *v*′ and 10′ on X_1Y_1 . Locate 4_2, r_2, s_2, t_2, u_2, v_2 and 10_2 on these projectors such that their distances from X_2Y_2 are equal to distances of points 4, *r*, *s*, *t*, *u*, *v* and 10 respectively from *XY*. Join $4_2r_2s_2t_2u_2v_210_2$.

Example 10.27 (Fig. 10.32)

A pentagonal prism of 25 mm base and 60 mm height has an edge of its base on the H.P. and the axis parallel to the V.P. and inclined at 60º to the H.P. A section plane having its H.T. perpendicular to *XY* and the V.T. inclined at 60º to *XY* and passing through the highest corner cuts the prism. Draw the sectional top view and true shape of the section. [RGPV Feb. 2008]

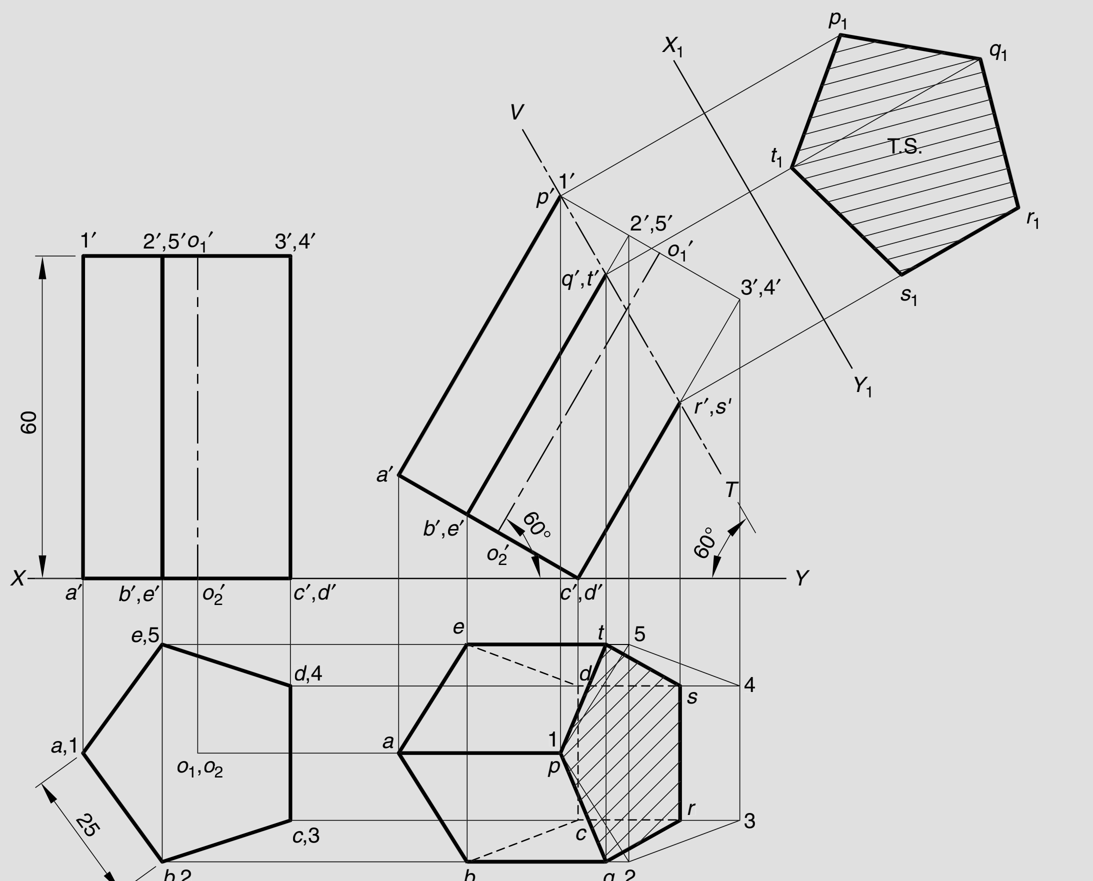


Fig. 10.32

Construction: Fig. 10.32

1. Draw a pentagon *abcde* keeping *cd* perpendicular to *XY*. This represents the top view. Project the corners and obtain $a'd'4'1'$ as its front view.
2. Reproduce the front view of the first stage keeping $c'd'$ on *XY* and $o_1'o_2'$ inclined at 60º to it. Project this front view to meet the horizontal lines from the top view of the first stage and obtain the new top view.
3. Draw V.T. inclined at 60º to *XY* passing through the highest corner $1'$.
4. V.T. cuts the edges $a'1'$ at p', $b'2'$ at q', $c'3'$ at r', $d'4'$ at s' and $e'5'$ at t'. Project points p', q', r', s' and t' to meet their corresponding edges *a*1, *b*2, *c*3, *d*4 and *e*5 at points *p*, *q*, *r*, *s* and *t*. Join *pqrst* and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to V.T. Project points p', q', r', s' and t' on X_1Y_1. Locate points p_1, q_1, r_1, s_1 and t_1 on these projectors such that their distance from X_1Y_1 line are equal to distance of points *p*, *q*, *r*, *s* and *t* from *XY*. Join $p_1q_1r_1s_1t_1$ and hatch the enclosed portion.

Example 10.28 (Fig. 10.33)

A hexagonal prism having a base with 30 mm sides and 70 mm long axis, is resting on a face on the ground with axis parallel to the V.P. It is cut by an A.V.P. which makes an angle of 45º with the V.P. and passes through a point 25 mm on the axis from one of its ends. Draw its sectional front view and obtain true shape of the section.

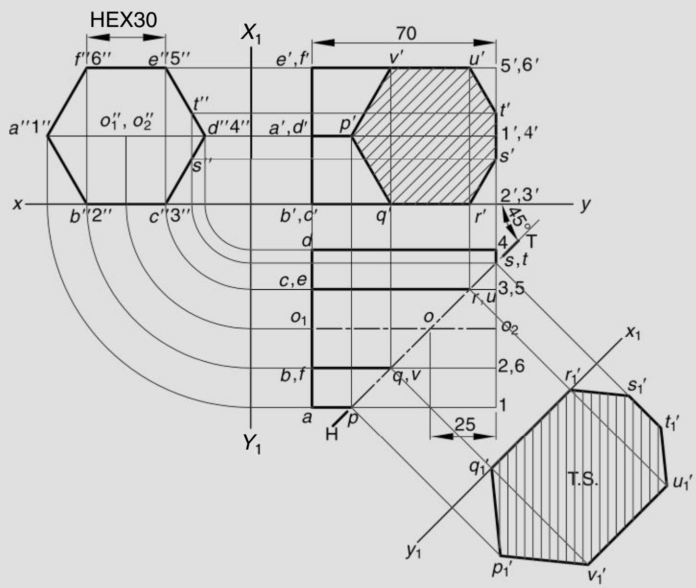


Fig. 10.33

Construction: Fig. 10.33

1. Draw a hexagon $a''b''c''d''e''f''$ keeping $b''c''$ on *XY* to represent the side view. Project the corners to get $b'2'5'e'$ as the front view and *a*14*d* as the top view.
2. Mark point *o* on the axis o_1o_2 25 mm from o_2. Draw H.T. inclined at 45º to *XY* passing through a point *o*.

3. H.T. cuts the edges *a*1 at *p*, *b*2 at *q*, *c*3 at *r*, 3-4 at *s*, 4-5 at *t*, *e*5 at *u* and *f*6 at *v*. Project *p*, *q*, *r*, *u* and *v* to points *p*′, *q*′, *r*′, *u*′ and *v*′ respectively in the front view.
4. Points *s*′ and *t*′ cannot be obtained directly by projecting from the top view. Project *s* and *t* in the side view as *s*″ and *t*″. Draw horizontal lines from *s*″ and *t*″ to meet 3′4′ at *s*′ and 4′5′ at *t*′. Join *p*′*q*′*r*′*s*′*t*′*u*′*v*′ and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to H.T. Project *p*, *q*, *r*, *s*, *t*, *u* and *v* on X_1Y_1. Locate p_1', q_1', r_1', s_1', t_1', u_1' and v_1', etc., on the projectors such that their distances from X_1Y_1 are equal to distance of points *p*′, *q*′, *r*′, *s*′, *t*′, *u*′ and *v*′ from *XY*. Join $p_1'q_1'r_1's_1't_1'u_1'v_1'$ and hatch the enclosed portion.

Example 10.29 (Fig. 10.34)

A cylinder with 50 mm base diameter and 80 mm long axis, is lying on a generator on the H.P. with its axis parallel to the V.P. It is cut by an A.I.P. inclined at 30º to the H.P. and passes through a point on the axis 30 mm from one of its ends. Draw its sectional top view and obtain true shape of the section.

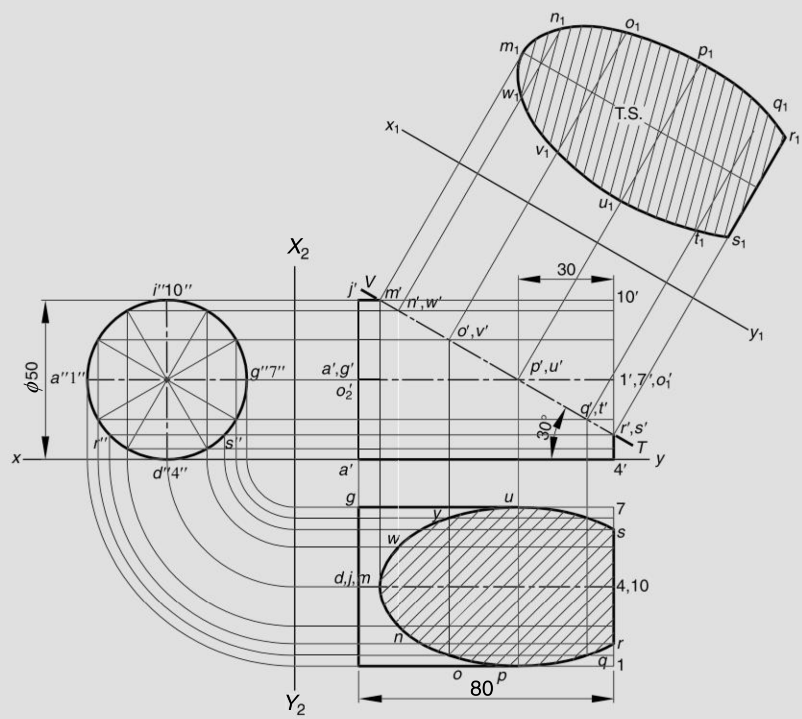


Fig. 10.34

Construction: Fig. 10.34

1. Draw a circle *a*″*d*″*g*″*j*″ touching the reference line to represent the side view. Project the side view and obtain *d*′4′10′*j*′ as its front view and *a*-1-7-*g* as the top view.
2. Mark a point on the axis $o_1'o_2'$ 30 mm from o_1'. Draw V.T. inclined at 45º to *XY* passing through point o_1'.
3. *Sectional top view*: Let V.T. cut the edges in the front view as *j*′10′ at *m*′, *a*′1′ at *p*′, 1′4′ at *r*′, 4′7′ at *s*′, *g*′7′ at *u*′. Project *m*′, *p*′ and *u*′ to meet their corresponding projectors at points *m*, *p* and *u*.

4. Points r' and s' cannot be projected directly. For this project r' and s' to meet at points r'' and s'' in the side view. Project r'' and s'' perpendicular on XY and then rotate them through 90º and then draw horizontal lines from them such that they meet 1-4 at r and 4-7 at s. Join *mprsu* and hatch the enclosed portion.
5. Draw a line X_1Y_1 parallel to V.T. Project points m', p', r', s' and u' on X_1Y_1. Locate points m_1, p_1, r_1, s_1, u_1 on the projectors such that their distances from X_1Y_1 are equal to the distances of m, p, r, s, u from XY. Join $m_1p_1r_1s_1u_1$ and hatch the enclosed portion by it.

Example 10.30 (Fig. 10.35)

A cylinder of 50 mm diameter and 70 mm long is resting on H.P. with its axis inclined at 30º to H.P. and parallel to V.P. A section plane inclined at 45º to V.P. passes through the axis at 25 mm from one end of it. Draw the projections of the cut solid. Also obtain the true shape of the section. [RGPV Feb. 2011]

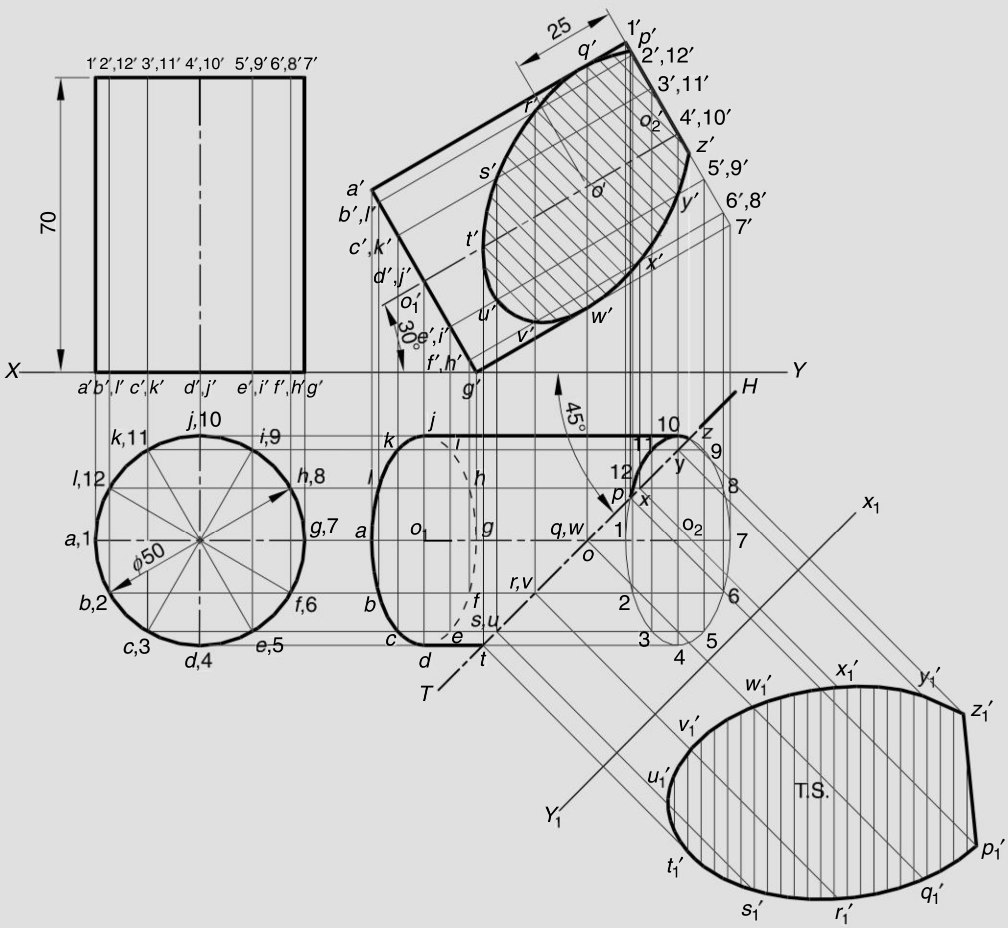


Fig. 10.35

Construction: Fig. 10.35

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts and project to obtain $a'g'7'1'$ as the front view.
2. Reproduce the front view of the first stage keeping g' on *XY* with axis inclined at 45º to it. Project the front view to meet the horizontal lines from the top view of the first stage and obtain the new top view.
3. Mark point o' on the axis $o_1'o_2'$ 25 mm from o_2'. Project it to meet o_1o_2 at point *o*. Draw H.T. inclined at 45º to *XY* passing through point *o*.
4. H.T. cuts 1-2 at *p*, *a*1 at *q*, *b*2 at *r*, *c*3 at *s*, *d*4 at *t*, *e*5 at *u*, *f*6 at *v*, *g*7 at *w*, *h*8 at *x*, *i*9 at *y* and 9-10 at *z*. Project *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y* and *z* to meet at points p', q', r', s', t', u', v', w', x', y' and z' and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to H.T. Project points *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x*, *y* and *z* to X_1Y_1. Locate points p_1', q_1', r_1', s_1', t_1', u_1', v_1', w_1', x_1', y_1' and z_1' on the projectors such that their distances from X_1Y_1 are equal to distances of p', q', r', s', t', u', v', w', x', y' and z' from *XY*. Join $p_1'q_1'r_1's_1't_1'u_1'v_1'w_1'x_1'y_1'z_1'$ and hatch the enclosed portion.

Example 10.31 (Fig. 10.36)

A cone of 60 mm base diameter and 70 mm long axis is resting on its base in the H.P. It is cut by a horizontal plane and an A.I.P. which makes 45º with the H.P. Both the planes meet at a point on the axis 40 mm above the base. Draw its sectional top view and obtain true shape of the section.

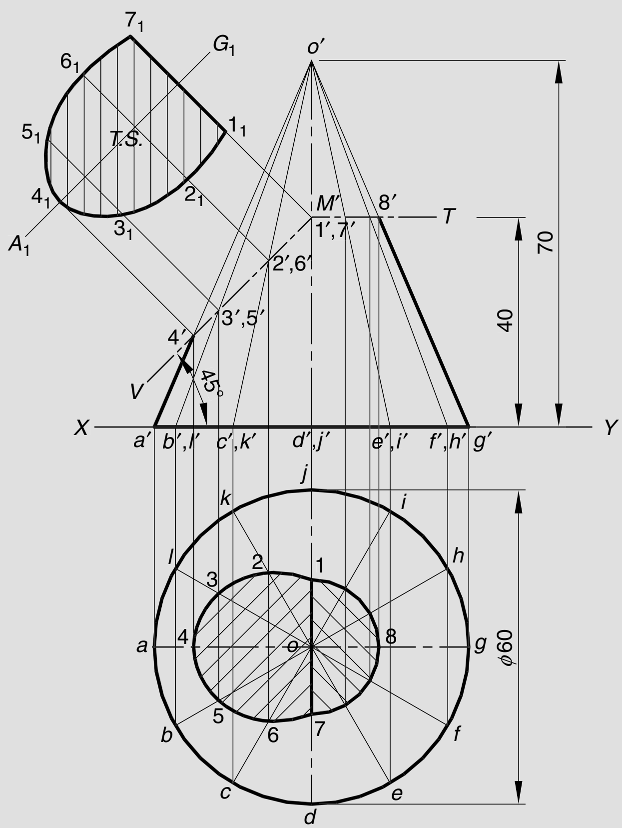


Fig. 10.36

Construction: Fig. 10.36

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts project to obtain $a'g'o'$ as the front view.
2. Mark point M' on the axis 40 mm above the base. Draw VM' inclined at 45º to *XY* and $M'T$ parallel to *XY*.
3. V.T. cuts the generators $o'j'$ at 1′, $o'k'$ at 2′, $o'l'$ at 3′, $o'a'$ at 4′, $o'b'$ at 5′, $o'c'$ at 6′, $o'd'$ at 7′ and $o'g'$ at 8′. Project point 8′ to meet at point 8. Draw a semicircle to meet *oj* and *od* at points 1 and 7 respectively. Project points 2′, 3′, 4′, 5′ and 6′ to meet at points 2, 3, 4, 5, 6 and 7. Join 1-2-3-4-5-6-7-8 and hatch the enclosed portion.
4. *True shape*: Draw A_1G_1 parallel to the V.T. Project points 1′, 2′, 3′, 4′, 5′, 6′ and 7′ perpendicular on A_1G_1 and extend them. Locate points 1_1, 2_1, 3_1, 4_1, 5_1, 6_1 and 7_1 on the projectors such that their distance from A_1G_1 line is equal to distance of points 1, 2, 3, 4, 5, 6 and 7 from line *ag*. Join $1_12_13_14_15_16_17_1$ and hatch. $1_12_13_14_15_16_17_1$ and 1-8-7 represent the true shape.

Example 10.32 (Fig. 10.37)

A pentagonal pyramid of 30 mm base side and 60 mm long axis is on a triangular face in the H.P. with its axis parallel to the V.P. It is cut by an A.I.P. making an angle of 60º to the H.P. and passing through the highest point of the base. Draw its sectional top view and true shape of the section.

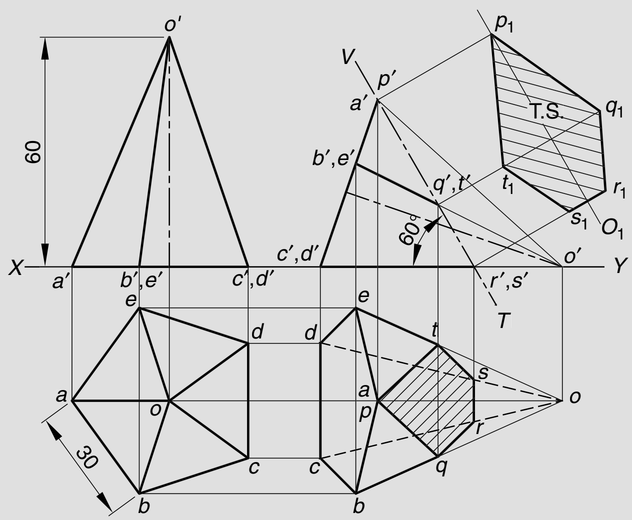


Fig. 10.37

Construction: Fig. 10.37

1. Draw a pentagon *abcde* keeping *cd* perpendicular to *XY* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain $a'd'o'$ as its front view.
2. Reproduce the front view keeping $c'd'o'$ on *XY*. Project it to meet the horizontal lines from the top view of the first stage and obtain the new top view.

3. Draw V.T. inclined at 60º to XY passing through a'.
4. Let V.T. cut the edges $b'o'$ at q', $c'o'$ at r', $d'o'$ at s' and $e'o'$ at t'. Project q', r', s' and t' to meet bo, co, do and eo at points q, r, s and t. Join $pqrst$ and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to V.T. Project points p', q', r', s' and t' on X_1Y_1. Locate p_1, q_1, r_1, s_1 and t_1 on the projectors such that their distances from X_1Y_1 line are equal to distances of points p, q, r, s and t from XY. Join $p_1q_1r_1s_1t_1$ and hatch the enclosed portion.

Example 10.33 (Fig. 10.38)

A cone base of 60 mm diameter and 60 mm long axis is lying on the ground on one of its generators with axis parallel to the V.P. A vertical section plane parallel to the generator which is tangent to the ellipse in the top view cuts the cone bisecting the axis and removing a portion containing the apex. Draw its sectional front view and true shape of the section. [RGPV June 2003]

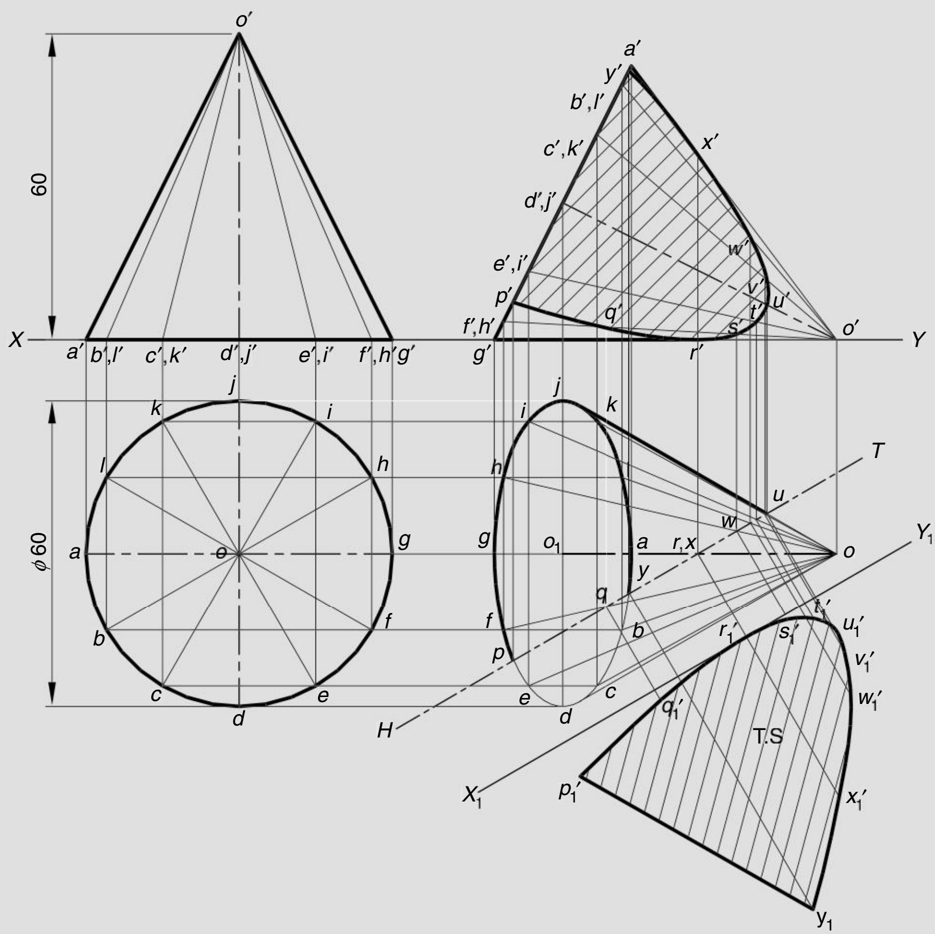


Fig. 10.38

Construction: Fig. 10.38

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts and project to obtain $a'g'o'$ as the front view.
2. Reproduce the front view keeping the generator $g'o'$ on *XY*. Project the front view to meet horizontal lines from the top view of the first stage and obtain the new top view.
3. Draw the section plane H.T. parallel to $o'd'$ bisecting the axis oo_1.
4. Let the section plane cut the base at points *p* and *y* while the generators *of*, *og*, *oh*, *oi*, *oj*, *ok*, *ol*, *oa* at points *q*, *r*, *s*, *t*, *u*, *v*, *w* and *x*. Project points *q*, *r*, *s*, *t*, *u*, *v*, *w* and *x* to meet at points p', q', r', s', t', u', v', w', x' and y'. Join $p'q'r's't'u'v'w'x'y'$ and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to H.T. Project points *p*, *q*, *r*, *s*, *t*, *u*, *v*, *w*, *x* and *y* to X_1Y_1. Locate points p_1', q_1', r_1', s_1', t_1', u_1', v_1', w_1', x_1' and y_1' on the projectors such that their distances from X_1Y_1 are equal to distances of p', q', r', s', t', u', v', w', x' and y' from *XY*. Join $p_1'q_1'r_1's_1't_1'u_1'v_1'w_1'x_1'y_1'$ and hatch the enclosed portion.

Example 10.34 (Fig. 10.39)

A pentagonal pyramid of 30 mm base side and 60 mm long axis is on a triangular face in the H.P. with its axis parallel to the V.P. It is cut by a vertical section plane making an angle of 60º to the V.P. and passing through the centre of the base. Draw its sectional front view and obtain true shape of the section retaining the apex.

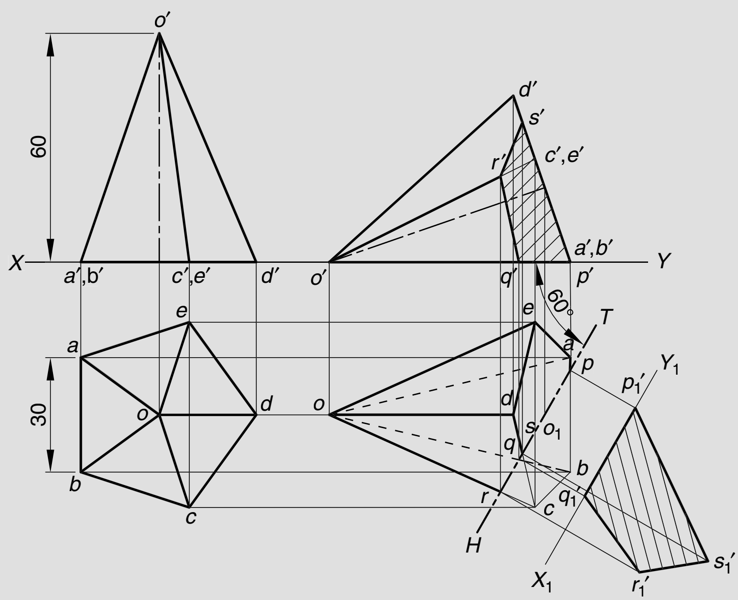


Fig. 10.39

Construction: Fig. 10.39

1. Draw a pentagon *abcde* keeping *ab* perpendicular to *XY* and join the corners with centroid *o*. This represents the top view. Project the corners and obtain $a'd'o'$ as its front view.

2. Reproduce the front view keepng $o'a'b'$ on XY. Project it to meet the horizontal lines from the top view of the first stage and obtain the new top view.
3. Draw H.T. inclined at 60º to XY passing through the centre of the base o_1.
4. Let H.T. cut the edges ab at p, ob at q, oc at r and cd at s. Project points p, q, r and s to meet their corresponding edges $a'b'$, $o'b'$, $o'c'$ and $c'd'$ at points p', q', r' and s'. Join $p'q'r's'$ and hatch the enclosed portion.
5. Draw X_1Y_1 parallel to H.T. Project points p, q, r and s on X_1Y_1. Locate points p_1', q_1', r_1' and s_1' on the projectors such that their distances from X_1Y_1 are equal to distances of points p', q', r' and s' from XY. Join $p_1'q_1'r_1's_1'$ and hatch the enclosed portion.

Example 10.35 (Fig. 10.40)

A right regular hexagonal pyramid (30 mm base edge × 75 mm long axis) has one of its slant edges in H.P. and the vertical plane containing this edge and axis is inclined at 30º to the V.P. Draw the projection when apex is 20 mm in front of V.P. It is now cut by a section plane whose H.T. makes an angle of 60º with the reference line. Draw sectional view and true shape of section when the section plane bisects the axis. [RGPV Dec. 2008]

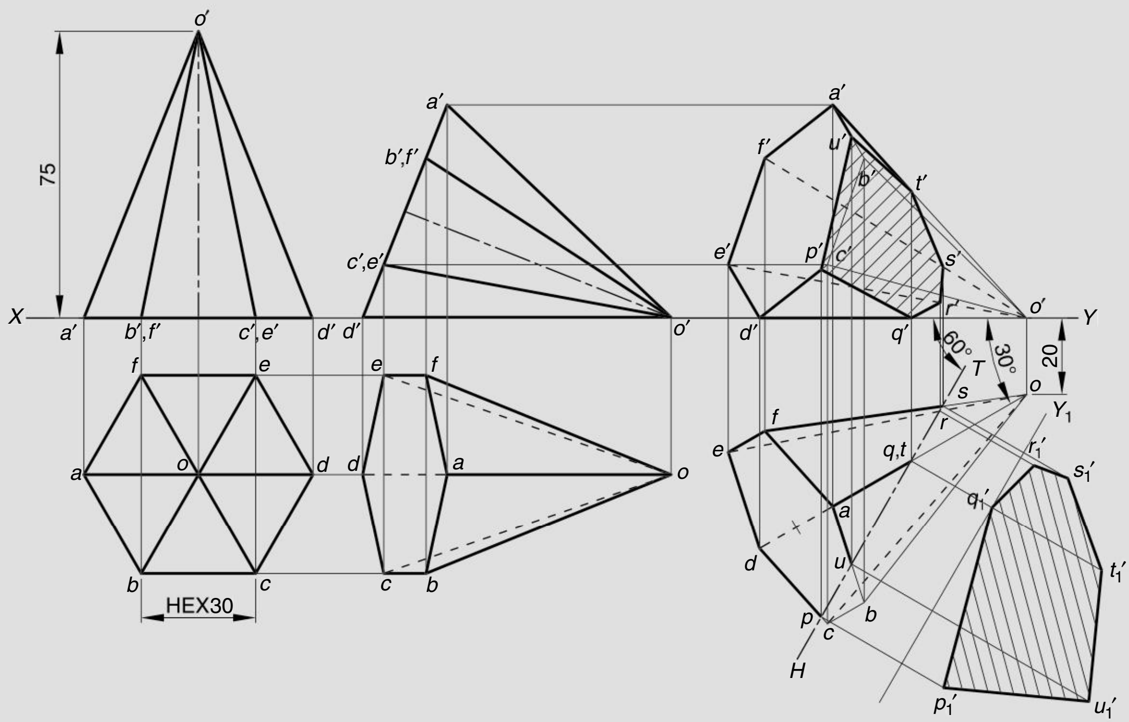


Fig. 10.40

Construction: Fig. 10.40

1. Draw a hexagon $abcdef$ keeping ef parallel to XY and join the corners with centroid o. This represents the top view. Project the corners and obtain $a'd'o'$ as its front view.

2. Reproduce the front view keepng $o'd'$ on XY. Project the front view to meet the horizontal lines from the top view of the first stage and obtain the new top view.
3. Reproduce the top view of the second stage keeping od at 30º to XY. Project this top view to meet the horizontal lines from the front view of the second stage and obtain the new front view.
4. Draw H.T. inclined at 60º to XY bisecting the axis oo_1. (It may be noted that there are two possible ways of drawing H.T. One of them is shown here.)
5. Let H.T. cut the edges cd at p, od at q, oe at r, of at s, oa at t and ab at u. Project points p, q, r, s, t and u to meet their corresponding edges at points p', q', r', s', t' and u'. Join $p'q'r's't'u'$ and hatch the enclosed portion.
6. Draw X_1Y_1 parallel to H.T. Project points p, q, r, s, t and u on X_1Y_1. Locate points p_1', q_1', r_1', s_1', t_1' and u_1' on the projectors such that their distances from X_1Y_1 are equal to distances of points p', q', r', s', t' and u' from XY. Join $p_1'q_1'r_1's_1't_1'u_1'$ and hatch the enclosed portion.

EXERCISE 10

1. A triangular prism having a base of 50 mm sides and 80 mm long axis, is lying on one of its rectangular faces in the H.P. with its axis perpendicular to the V.P. It is cut by a section plane parallel to and 20 mm above the H.P. Draw its front view and sectional top view.
2. A pentagonal pyramid having a base of 30 mm sides and 60 mm long axis, is resting on a triangular face in the H.P. with its axis parallel to the V.P. It is cut by a horizontal section plane passing through the centroid of the pyramid. Draw its projections.
3. A cube of 35 mm long edge is resting on H.P. on one of its faces with a vertical face inclined at 30º to the V.P. It is cut by a section plane parallel to V.P. and 9 mm away from the axis. Draw its sectional front view and top view. **[RGPV April 2009]**
4. A pentagonal prism having 30 mm base sides and 60 mm long axis, lies on one of its rectangular faces on the H.P. with its axis inclined at 45º to the V.P. A vertical section plane parallel to the V.P. cuts the prism at a distance of 20 mm from one of the end faces. Draw its sectional front view and top view.
5. A pentagonal pyramid having 30 mm base side and 70 mm long axis, is resting on its base on the H.P. with a side of base parallel to the V.P. and nearer to it. It is cut by a section plane parallel to the V.P. and 12 mm in front of the axis of the pyramid. Draw its sectional front view and top view.
6. A square prism having 40 mm base sides and 60 mm long axis, rests on its base on the H.P. such that one of the vertical faces makes an angle of 30º with the V.P. A section plane perpendicular to the V.P., inclined at 45º to the H.P. and passing through the axis at a point 20 mm from its top end, cuts the prism. Draw its front view, sectional top view and true shape of section.
7. A cylinder of 40 mm diameter and 60 mm height and having its axis vertical, is cut by a section plane perpendicular to V.P. and inclined at 45º to the H.P. and intersecting the axis 32 mm above the base. Draw its front view, sectional top view and true shape of the section. **[RGPV June 2007]**

8. A hexagonal pyramid of 30 mm side of base and 60 mm long axis is resting on its base on H.P. with an edge of the base perpendicular to the V.P. It is cut by a section plane inclined at 30º to the H.P. and passing through the axis at 20 mm from the base. Draw the sectional view, top view and true shape of the section. **[RGPV Feb. 2010]**
9. A pentagonal pyramid having 30 mm base sides and 70 mm long axis, is resting on its base in the H.P. with an edge of the base parallel to the V.P. It is cut by a section plane perpendicular to the V.P, inclined at 60º to the H.P. and bisecting the axis. Draw its front view and sectional top view and true shape of the section.
10. A right regular square pyramid, 35 mm edge of base and 50 mm height rests on its base on H.P. with all the base edges equally inclined to the V.P. A section plane perpendicular to the V.P. and inclined to the H.P. at 35º, cuts the pyramid bisecting its axis. Draw the front view, sectional top view and true shape of the section of the truncated pyramid. **[RGPV Sep. 2009]**
11. A cone base of 75 mm diameter and 80 mm long axis is resting on its base on the H.P. It is cut by a section plane perpendicular to V.P. and inclined at 45º to the H.P. and cutting axis at a point 35 mm from the apex. Draw its front view, sectional top view and true shape of the section. **[RGPV Feb. 2007]**
12. A cone having a 60 mm base diameter and 70 mm long axis, is resting on its base on the H.P. It is cut by an A.I.P. inclined at 45º to the H.P. and passes through a point on the axis, 20 mm above the base. Draw its sectional top view and obtain true shape of the section.
13. A hexagonal prism having a 30 mm base sides and 70 mm long axis has an edge of its base on the H.P. The axis parallel is to the V.P. and inclined at 45º to the H.P. It is cut by an A.V.P. inclined at 45º to the V.P. and passing through a point on the axis, 25 mm from the top end. Draw its sectional front view and obtain true shape of the section.
14. A pentagonal pyramid having 30 mm base sides and 70 mm long axis, is on a triangular face in the H.P. with its axis parallel to the V.P. It is cut by a plane whose H.T. makes an angle of 30º with the reference line and bisects the axis such that the apex is removed. Draw its sectional front view and obtain true shape of the section.
15. A square prism with 45 mm edge of base and 90 mm long axis has its axis parallel to both H.P. and V.P. The lateral surfaces are equally inclined to H.P. It is cut by a vertical section plane inclined at 60º to the V.P. and passing through the axis at 65 mm from one end. Draw the projections of the solid. Also draw true shape of section. **[RGPV Dec. 2010]**
16. A hexagonal pyramid having 30 mm base sides and 72 mm long axis, is resting on an edge in the H.P. The axis is parallel to both H.P. and V.P. A section plane whose V.T. bisects the axis, is inclined at 30º to the reference line. Draw the sectional front view and true shape of the section, retaining the portion containing the apex.
17. A hexagonal pyramid having 30 mm base sides and 72 mm long axis, is resting on an edge in the H.P. The axis is parallel to both H.P. and V.P. A vertical section plane whose H.T. bisects the axis, is inclined at 30º to the V.P. Draw the sectional front view and true shape of the section, removing the portion containing the apex.
18. A thin cylindrical glass vessel having 50 mm base diameter and 75 mm height, is resting on the H.P. contains water up to 45 mm from its base. The vessel is then tilted so that water is just at the point of tricking out. Draw the projections of the glass in its tilted position, showing clearly the water surface.

19. A square pyramid of 50 mm base side and 75 mm long axis, rests on one of its triangular faces on the ground. The top view of the axis makes an angle of 30º with the V.P. It is cut by a horizontal section plane, the V.T. of which intersects the axis at a point 20 mm from the base. Draw its front view and sectional top view.

REVIEW QUESTIONS

1. State the relationship of an auxiliary vertical plane with the reference planes.
2. Define an auxiliary inclined plane, auxiliary vertical plane and a profile plane.
3. How can the true shape of section be obtained when a solid is cut by an AIP?
4. How can the true shape of section be obtained when a solid is cut by AVP?
5. A solid is cut by a profile plane. Which orthographic view will show the true shape of section?
6. How would you locate the section plane which cuts a cone to get an isosceles triangle as true shape of section?
7. How would you locate the section plane which cuts a square pyramid to get a trapezium as true shape of section?
8. How would you locate the section plane which cuts a cube to get an equilateral triangle of largest possible side as true shape of section?

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) Name the view that provides the internal features of an object?
(a) sectional view (b) oblique view (c) auxiliary view (d) pictorial view

ii) A cube is resting on a face in the H.P. with vertical faces equally inclined to the V.P. It is cut by an A.I.P. The true shape of section view is
(a) triangle (b) rhombus (c) hexagon (d) any of these

iii) A cone is cut by a section plane parallel to the profile plane. Its true shape of section is seen in
(a) front view (b) top view (c) side view (d) auxiliary view

iv) A square pyramid resting on its base in the H.P. and a side of base parallel to V.P. It is cut by an A.I.P. Its true shape will be
(a) square (b) rectangle (c) trapezium (d) parallelogram

v) A square pyramid of 50 mm side resting on its base in the H.P. is cut by a horizontal section plane bisecting its axis. Its true shape of section is
(a) square of 25 mm side (b) trapezium with parallel sides 25 mm & 50 mm
(c) square of 50 mm side (d) triangle of base 50 mm side

vi) A square pyramid of 45 mm side and 60 mm long axis, resting on its base in the H.P. is cut by a horizontal section plane passing through a point on the axis 20 mm below the apex. Its true shape of section is a square of side
(a) 15 mm (b) 30 mm (c) 40 mm (d) 45 mm

vii) A triangular prism is resting on a rectangular face in the H.P. It is cut by a horizontal plane. Its sectional top view is a/an
(a) Equilateral triangle (b) Isosceles triangle
(c) Rectangle (d) None of these

viii) A cone resting on its base on the H.P. cut by a section plane parallel to V.P. has its sectional front view as
(a) ellipse (b) parabola (c) hyperbola (d) semicircle

ix) A cube is resting on a face in the H.P. with vertical faces equally inclined to the V.P. It is cut by an A.I.P. passing through the solid diagonal. The true shape of the section view is
(a) square (b) rectangle (c) hexagon (d) rhombus

x) A cylinder of 50 mm diameter and 120 mm long axis is lying on its generator in H.P. It is cut by a vertical section plane to get largest ellipse as the true shape of section. The major axis of this ellipse will be
(a) 50 mm (b) between 50 mm and 120 mm
(c) 120 mm (d) 130 mm

xi) A cylinder of 60 mm diameter and 80 mm long axis is lying on its generator in H.P. It is cut by a section plane to get an ellipse as the true shape of section. The minor axis of this ellipse will be
(a) 60 mm (b) 80 mm (c) 100 mm (d) none of these

xii) If a polyhedron is cut by any section plane, the true shape of section is a closed figure made up of
(a) straight lines (b) curves
(c) combination of lines and curves (d) any of these

Answers

(i) a (ii) d (iii) c (iv) c (v) a (vi) a (vii) c (viii) c (ix) d (x) d (xi) a (xii) a

11

Development of Surfaces

- Introduction
- Classification of Surfaces
- Methods of Development
- Development of Prism
- Development of Cylinder
- Development of Cone
- Development of Pyramid
- Anti-development

11.1 INTRODUCTION

In engineering practice, a large number of objects like funnel, bucket, hopper, chimney, duct of air conditioner, boiler shell, storage tank and tray, etc., are made of metal sheets. The fabrication of these objects can be planned in an economic way if the accurate shape and size of metal sheet is known. This chapter deals with proper layout planning of the surface of the object on a single plane called the development of surfaces.

11.2 CLASSIFICATION OF SURFACES

Surfaces of various geometrical objects may be classified as:

1. *Plane surfaces*: Surfaces of objects like prisms, pyramids, cubes and polyhedra are plane surfaces.
2. *Singly curved surfaces*: Surfaces of object like cylinders and cones are singly curved surfaces.
3. *Doubly curved surfaces*: Surfaces of spheres, paraboloids, ellipsoids, hyperboloids are doubly curved surfaces.

11.3 METHODS OF DEVELOPMENT

The methods of development may be classified as:

1. *Parallel-line method*: This method is adopted in the development of prisms and cylinders.
2. *Radial-line method*: This method is adopted in the development of pyramids and cones.
3. *Triangulation method*: This method is generally applied for the development of transition pieces and oblique solids.
4. *Approximation method*: Spherical and other doubly curved surfaces are developed by this method.

Note 1: Since the development is the true representation of the surface of an object, it is obtained by taking true length only.

Note 2: Since it is not possible to spread the doubly curved surface on any plane, one has to be contented by its approximate development only.

11.4 DEVELOPMENT OF PRISM

Prisms are developed by parallel-line method. In this method, first of all, the front view and top view of the prism are drawn. Two parallel lines called stretch-out lines are drawn from the ends of the prism in a direction perpendicular to the axis. The length of these lines is same as the perimeter of the base of the prism. The faces of the prism are marked between the stretched outlines, which represent the development of the lateral surface.

Example 11.1 (Fig. 11.1)

A square prism of 40 mm side of base and 80 mm long axis is resting on its base on H.P. such that a rectangular face of it is parallel to V.P. Draw the development of the prism.

[RGPV Feb. 2010]

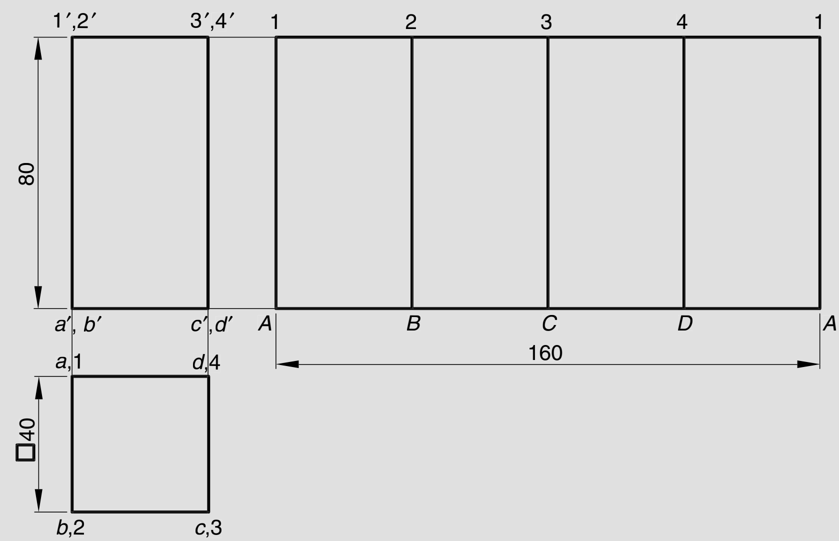


Fig. 11.1

Visualisation: The lateral surface of the given prism comprises of four rectangles of 80 mm × 40 mm sides.

Construction: Fig. 11.1

1. Draw a square *abcd* keeping *ad* parallel to *XY* to represent the top view. Project all the points to obtain rectangle $a'd'4'1'$ as the front view.
2. Stretch out lines 1-1 and *A*-*A* from the front view, equal to the perimeter of the base.
3. Divide 1-1 and *A*-*A* in four equal parts and name their intermediate points as 2, 3, 4 and *B*, *C*, *D* respectively. Join vertical edges 1*A*, 2*B*, 3*C* and 4*D* in the development.

Note 1: In development of the lateral surfaces of the closed objects, the first and the last edge have same name.

Note 2: Usually, development of the lateral surfaces of the object is drawn and the ends are omitted from the development. They can easily be added whenever required.

Example 11.2 (Fig. 11.2)

Draw the development of the surfaces of the part *P* of the cube whose front view is shown in Fig. 11.2(a). [RGPV June 2007, April 2009]

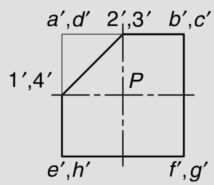


Fig. 11.2(a)

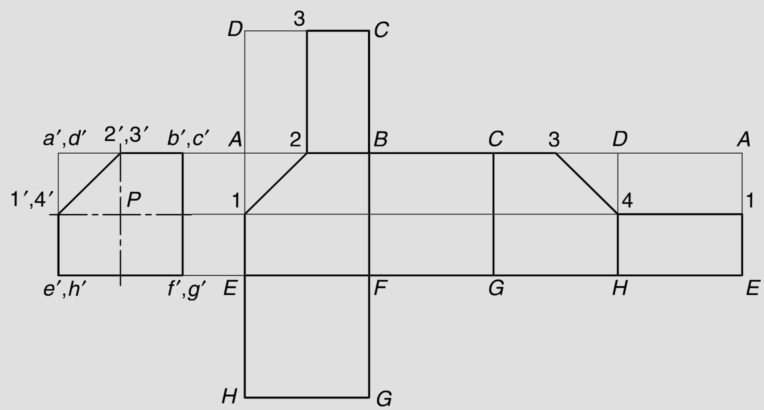


Fig. 11.2(b)

Construction: Fig. 11.2(b)

1. Redraw Fig. 11.2(a) considering arbitrary length for the sides of the cube (say 40 mm), as the front view.
2. Stretch out lines *A*-*A* and *E*-*E* from the front view, equal to 4 times the length of the side.
3. Divide *A*-*A* and *E*-*E* in four equal parts and name their intermediate points as *B*, *C*, *D* and *F*, *G*, *H* respectively. Join *AE*, *BF*, *CG* and *DH*. Attach square *EFGH* and rectangle *BC*32 to add the lower and upper bases.
4. Draw horizontal lines from points 1′ to meet edges *AE* and *DH* at points 1 and 4 respectively.
5. Also mark point 2 on *AB* and 3 on *CD* such that $A2 = a'2'$ and $C3 = c'3'$. Also, mark point 3 on *CD* in the top square.
6. Join lines 1-2, 2-3, 3-4 and 4-1 and complete the development as shown.
7. Darken the portion of the development which remains, after truncating the prism.

Example 11.3 (Fig. 11.3)

A pentagonal prism, having a base with a 30 mm side and a 70 mm long axis, is resting on its base on the H.P. such that one of the rectangular faces is parallel to the V.P. It is cut by an auxiliary inclined plane whose V.T. is inclined at 45º with the reference line and passes through the mid-point of the axis. Draw the development of the lateral surface of the truncated prism.

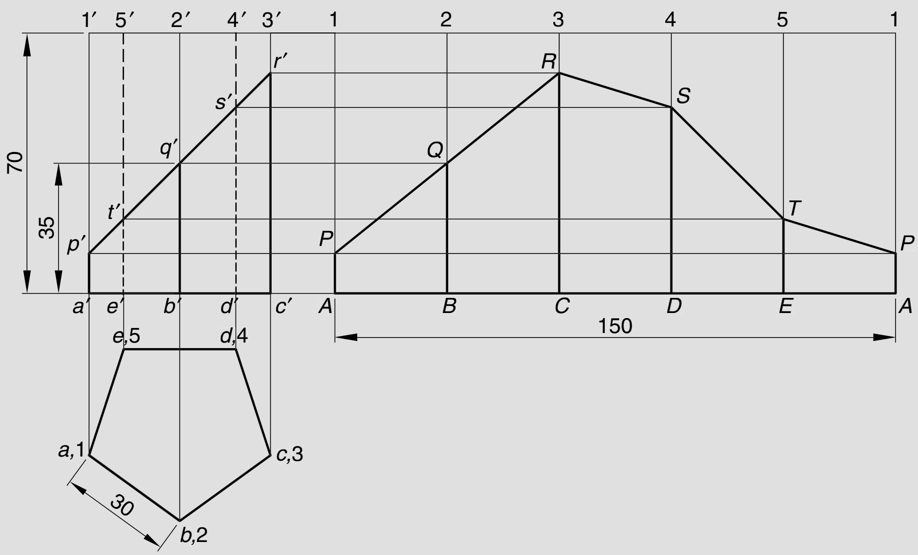


Fig. 11.3

Construction: Fig. 11.3

1. Draw a pentagon *abcde* keeping *de* parallel to *XY* to represent the top view. Project all points to obtain *a*′*c*′3′1′ as the front view.
2. In the front view, draw *p*′*r*′ as V.T. of the cutting plane at 45° to *XY* passing through the mid-point of the axis (point *q*′). Line *p*′*r*′ cuts the edges *a*′1′ at point *p*′, *b*′2′ at point *q*′, *c*′3′ at point *r*′, *d*′4′ at point *s*′ and *e*′5′ at point *t*′.
3. Consider the seam along 1-*a*. Stretch out lines 1-1 and *A*-*A* equal to five times the side of the base. Divide 1-1 and *A*-*A* in five equal parts and name their intermediate points as 2, 3, 4, 5 and *B*, *C*, *D*, *E* respectively. Join 1*A*, 2*B*, 3*C*, 4*D* and 5*E*.
4. Draw horizontal lines from points *p*′, *q*′, *r*′, *s*′ and *t*′ to meet corresponding edges *A*1, *B*2, *C*3, *D*4 and *E*5 at points *P*, *Q*, *R*, S and *T* respectively. Join each of *PQ*, *QR*, *RS*, *ST*, *TP* with straight lines.
5. Darken the portion of the development which remains after truncating the prism.

Example 11.4 (Fig. 11.4)

A hexagonal prism having base with a 30 mm side and a 70 mm long axis, is resting on its base on the ground with a side of the base inclined at 45° to the V.P. It is cut by an auxiliary inclined plane making an angle of 45° with the H.P. and passes through a point 15 mm below the top end of the axis. Obtain the development of the lateral surface of the truncated prism.

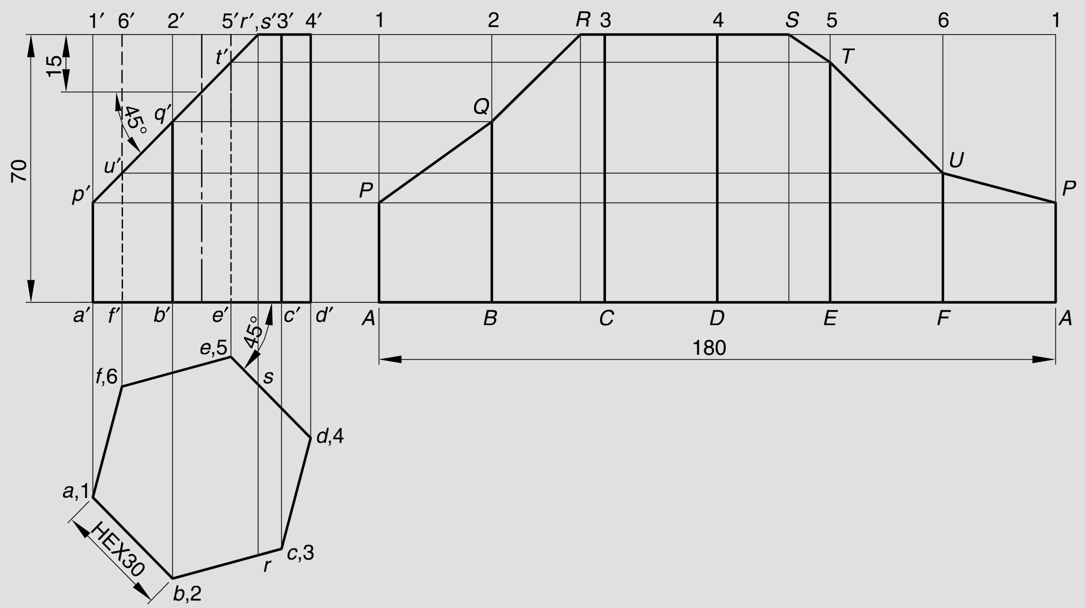


Fig. 11.4

Construction: Fig. 11.4

1. Draw a hexagon *abcdef* keeping *ed* at 45º with *XY* to represent the top view. Project all points to obtain *a′d′*4′1′ as the front view.
2. Draw line *p′r′* as the cutting plane inclined at 45º to *XY*, 15 mm below the top end of the axis. Line *p′r′* cuts *a′*1′ at *p′*, *b′*2′ at *q′*, 2′3′ at, *e′*5′ at *t′*, *f′*6′ at *u′*, 2′3′ at *r′* and 4′5′ at *s′*.
3. Consider seam at 1-*a*. Stretch out lines 1-1 and *A*-*A* from the front view equal to the perimeter of the base. Divide 1-1 and *A*-*A* in six equal parts and name their intermediate points as 2, 3, 4, 5, 6 and *B*, *C*, *D*, *E*, *F* respectively. Join vertical edges 1*A*, 2*B*, 3*C*, 4*D*, 5*E* and 6*F* in the development.
4. Draw horizontal lines from points *p′*, *q′*, *t′* and *u′* to meet their corresponding edges *A*1, *B*2, *E*5, *F*6 at points *P*, *Q*, *T*, *U*, respectively.
5. To locate *r′* and *s′* on the development project *r′s′* to meet the top view at *r* and *s* respectively. Mark points *R* and *S* such that 2*R* (in the development) = 2*r* (in the top view), and distance 3*S* (in the development) = 3*s* (in the top view).
6. Join all the points with straight lines as shown. Dark the portion of the development which remains after truncating the prism.

Example 11.5 (Fig. 11.5)

Develop the lateral surface of a right regular hexagonal prism of 35 mm base side and 75 mm height, kept vertically with a base side perpendicular to V.P. and having a cylindrical hole of 40 mm diameter drilled centrally with the axis of hole being perpendicular to V.P.

[RGPV Dec. 2008]

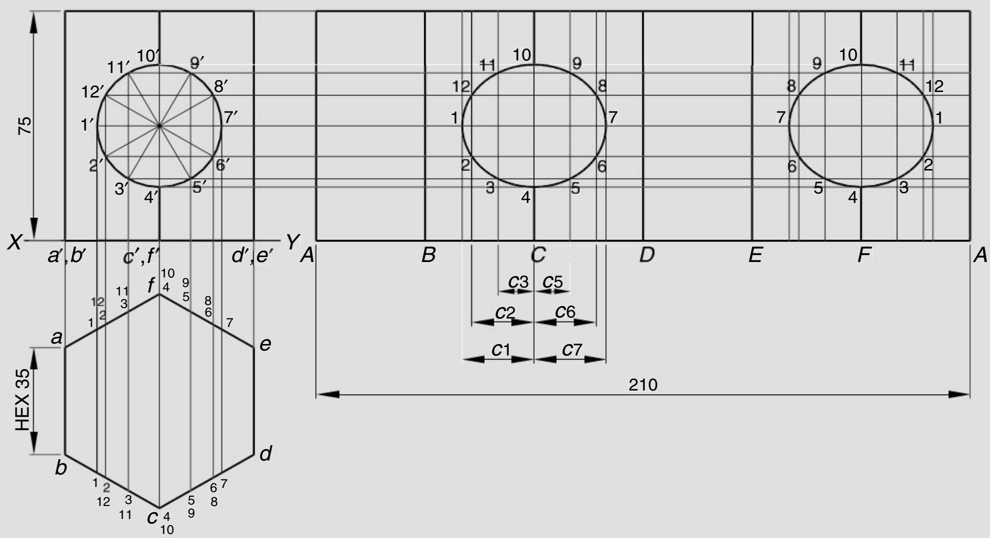


Fig. 11.5

Construction: Fig. 11.5

1. Draw a hexagon *abcdef* keeping *ab* perpendicular to *XY* as the top view. Project all the points to obtain the front view.
2. Draw a circle in the front view and divide it into 12 equal parts. Project all the points in the top view.
3. Consider seam at 1-*a*. Stretch out lines 1-1 and *A*-*A* equal to the perimeter of the hexagon. Divide 1-1 and *A*-*A* in six equal parts and name their intermediate points as 2, 3, 4, 5, 6 and *B*, *C*, *D*, *E*, *F* respectively. Join 1*A*, 2*B*, 3*C*, 4*D*, 5*E* and 6*F*.
4. Mark locus lines of 1, 2, 3,…, etc in the development such that their distances from points *C* and *F* is equal to distances of points 1, 2, 3,…, etc., from points *c* and *f* of the top view.
5. Project 1′, 2′, 3′,…, etc., to meet their respective locus lines in the development at points 1, 2, 3,…, etc. Join them to obtain the required development as shown.

Example 11.6 (Fig. 11.6)

Figure 11.6(a) shows the front view of a truncated hexagonal prism with a 30 mm base edge and 90 mm long axis resting on the H.P. such that an edge of the base is parallel to the V.P. Draw the development of its lateral surface.

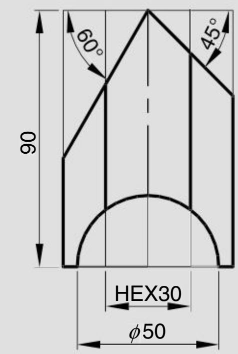


Fig. 11.6(a)

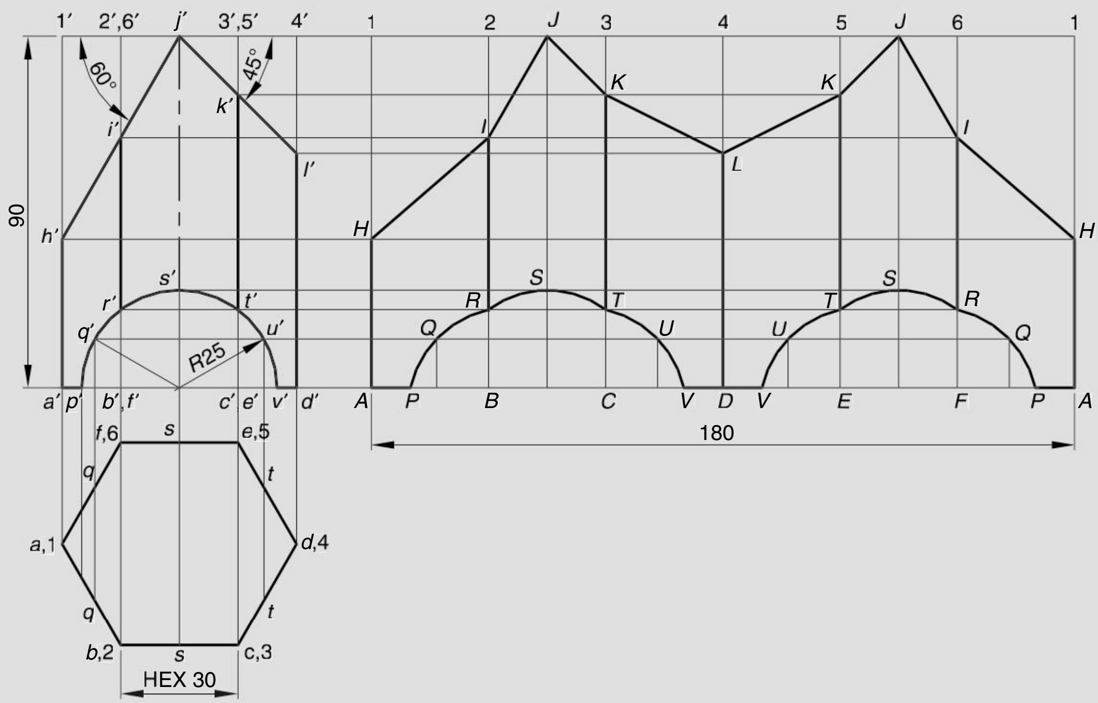


Fig. 11.6(b)

Construction: Fig. 11.6(b)

1. Draw a hexagon *abcdef* keeping *ef* parallel to *XY* to represent the top view. Project all points to obtain $a'd'4'1'$ as the front view.
2. Draw the cutting planes as given in Fig. 11.6(a). Name the point of intersection as h', i', j', k', l', p', r', s', t' and v'. For semicircle consider some more points q' and u'.

3. Stretch out lines 1-1 and *A-A* and draw 1*A*, 2*B*, 3*C*, 4*D*, 5*E* and 6*F* to complete the development of the uncut prism.
4. Project points *q′* and *u′* vertically downwards and obtain points *q* and *u* in the top view. In the development draw locus at a distance *aq* and *dt* from points *A* and *D* respectively.
5. Draw horizontal lines through points *h′*, *i′*, *j′*, *k′*, *l′*, and *p′*, *q′*, *r′*, *s′*, *t′*, *u′*, *v′* to meet their corresponding locus lines or generators and obtain *H*, *I*, *J*, *K*, *L* and *P*, *Q*, *R*, *S*, *T*, *U*, *V*. Join them to obtain the development as shown.

11.5 DEVELOPMENT OF CYLINDER

Cylinders are also developed by parallel-line method in a way similar to the prisms. Here, the length of stretch line is equal to the circumference of the base circle of the cylinder.

Example 11.7 (Fig. 11.7)

A cylinder of 40 mm diameter of base and 55 mm long axis is resting on its base on H.P. It is cut by a section plane perpendicular to V.P. and inclined at 45º to H.P. The section plane is passing through the top end of an extreme generator of the cylinder. Draw the development of the lateral surface of the cut cylinder. [RGPV June 2008(o), Aug. 2010]

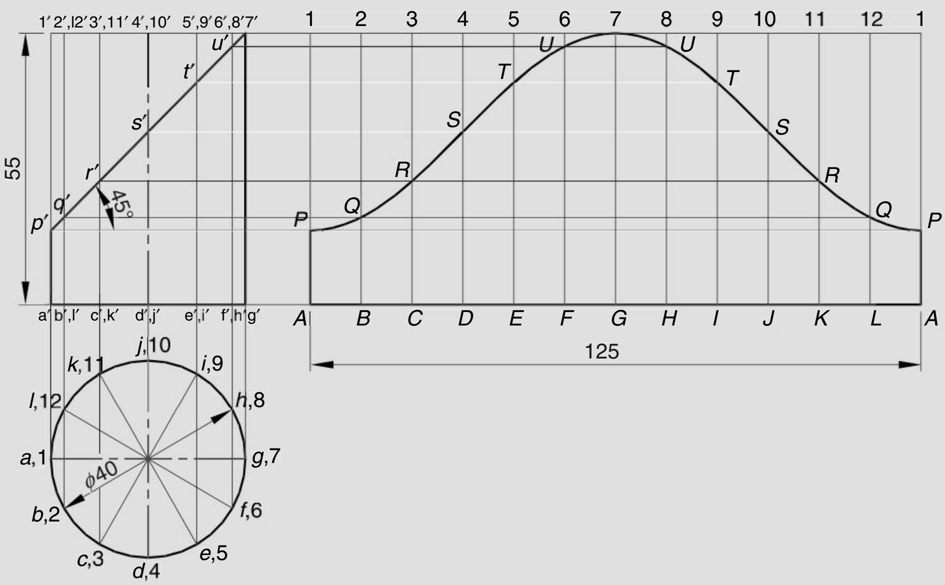


Fig. 11.7

Construction: Fig. 11.7

1. Draw a circle *adgj* to represent the top view and divide it into 12 equal parts. Project all the points to obtain *a′g′*7′1′ as the front view.

2. In the front view, draw a line $p'7'$ at 45º to XY passing through 7′. This represents the V.T. of the cutting plane. Let the line $p'7'$ cut generators $a'1'$ at p', $b'2'$ at q', $c'3'$ at r', $d'4'$ at s',…, etc.
3. Stretch out lines 1-1 and A-A through the front view equal to the perimeter of the cylinder (i.e. $\pi \times 40$ = 125 mm). Divide 1-1 and A-A into 12 equal parts and join generators A-1, B-2, C-3, D-4,..., A-1.
4. Draw horizontal lines from points p', q', r', s',…, etc., of line to meet their corresponding generators A1, B2, C3, D4,…, etc., at points P, Q, R, S,…, etc., respectively.
5. Join all the points with a continuous smooth curve.
6. Dark the portion of the development which remains after truncation of the cylinder.

Example 11.8 (Fig. 11.8)

A right circular cylinder with 40 mm diameter of base 60 mm height is truncated at its two ends by two different section planes as shown in Fig. 11.8(a). Develop the lateral surface of the truncated cylinder.

[RGPV June 2009]

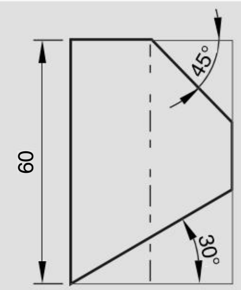


Fig. 11.8(a)

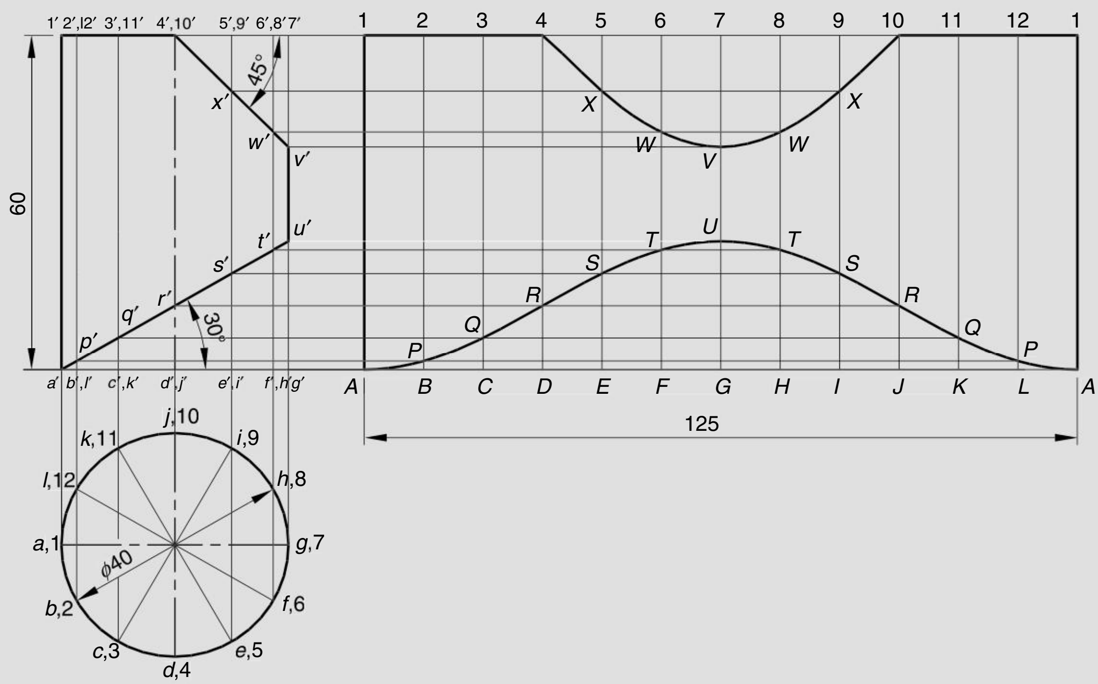


Fig. 11.8(b)

Construction: Fig. 11.8(b)

1. Draw a circle *adgj* to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. Draw V.T. of the cutting planes as given in Fig. 11.8(a). Name the point of intersection of the V.T. with generators as p', q', r', s', t', u', v', w' and x'.
3. Stretch out lines 1-1 and *A-A* of length equal to the perimeter of the cylinder. Divide 1-1 and *A-A* into 12 equal parts and join them.
4. Draw horizontal lines from points p', q', r', s', t', u', v', w' and x' to meet their corresponding generators in the development at points *P*, *Q*, *R*, *S*, *T*, *U*, *V*, *W* and *X* respectively.
5. Join all the points with smooth curves. Dark the portion of the development which remains after truncation of the cylinder.

Example 11.9 (Fig. 11.9)

Draw the development of truncated cylinder shown in Fig. 11.9(a). [RGPV Dec. 2005]

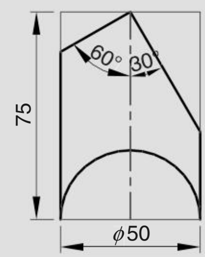


Fig. 11.9(a)

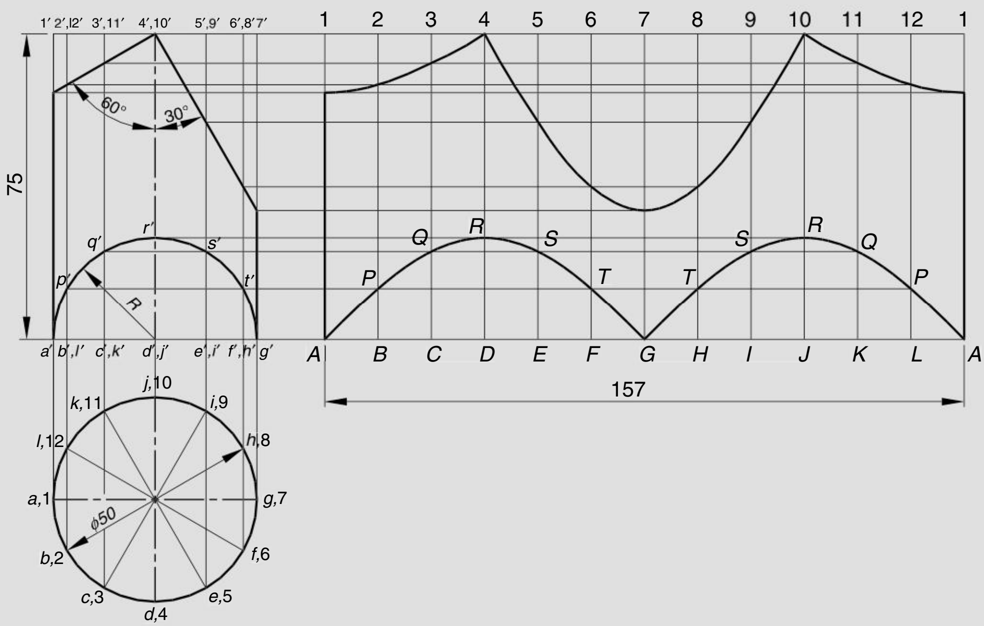


Fig. 11.9(b)

Construction: Fig. 11.9(b)

1. Draw a circle *adgj* to represent the top view and divide it into 12 equal parts. Project all the points to obtain $a'g'7'1'$ as the front view.
2. Draw the cutting planes as given in Fig. 11.9(a). Name the point of intersection of the planes with generators as p', q', r', etc.
3. Stretch out lines 1-1 and *A-A* of length equal to the perimeter of the cylinder. Divide 1-1 and *A-A* into 12 equal parts and join them.
4. Draw horizontal lines from points p', q', r', etc., to meet their corresponding generators in the development at points *P*, *Q*, *R*, etc., respectively.
5. Join all the points with smooth curves. Darken the portion of the development which remains after truncation of the cylinder.

Example 11.10 (Fig. 11.10)

A square hole of 25 mm side is cut in a cylindrical drum of 50 mm diameter and 70 mm height. The faces of the hole are inclined at 45º to the H.P. and axis intersects with that of the drum at right angles. Draw the development of its lateral surface.

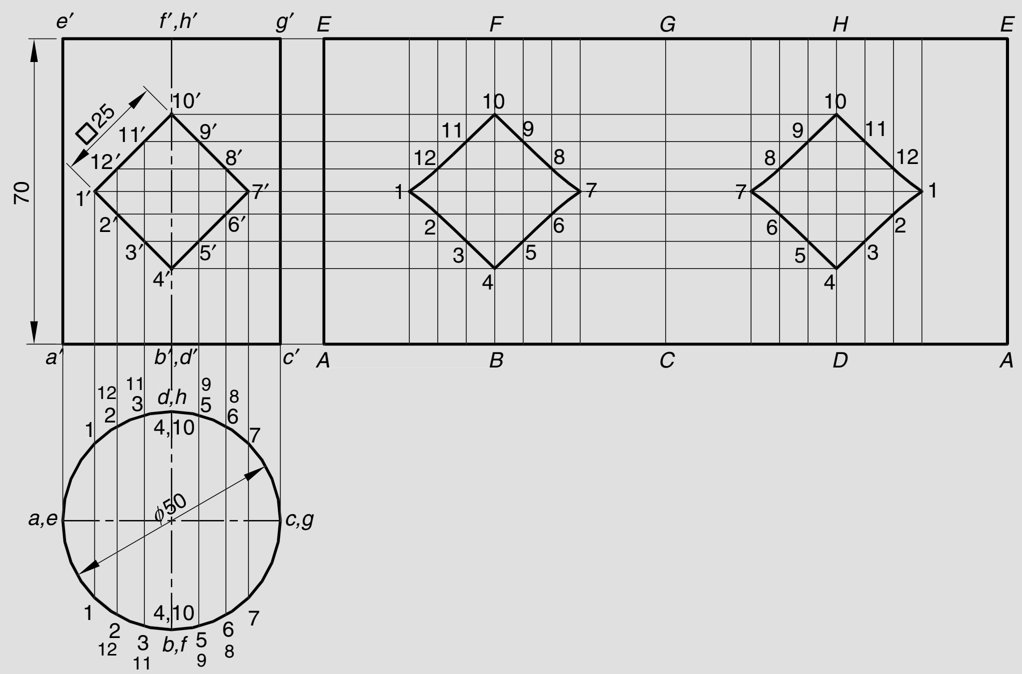


Fig. 11.10

Construction: Fig. 11.10

1. Draw a circle *abcd* to represent the top view. Project all the points to obtain $a'b'g'e'$ as the front view.
2. Draw a square $1'4'7'10'$ such that all the edges are inclined at 45º to *XY* keeping the centre 35 mm above the *XY*. On the edges of the square consider some more points as 2′, 3′, 5′, 6′, 8′, 9′, 11′ and 12′.
3. Stretch out lines *A*-*A* and *E*-*E* equal to the perimeter of the cylinder. Divide 1-1 and *A*-*A* into 4 equal parts and join all the generators.
4. Project all the points of the square vertically downwards and obtain points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 in the top view.
5. In the development, draw locus corresponding to 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 such that $A1$ = acr $a1$, $A2$ = arc $a2$, $A3$ = arc $a3$ and so on.
6. Draw horizontal lines from points 1′, 2′, 3′, 4′, 5′, 6′, 7′, 8′, 9′, 10′, 11′ and 12′ to meet their corresponding locus lines or generators in the development at points 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 respectively.
7. Join all the points with smooth curves. Darken the portion of the development which remains after truncation of the cylinder.

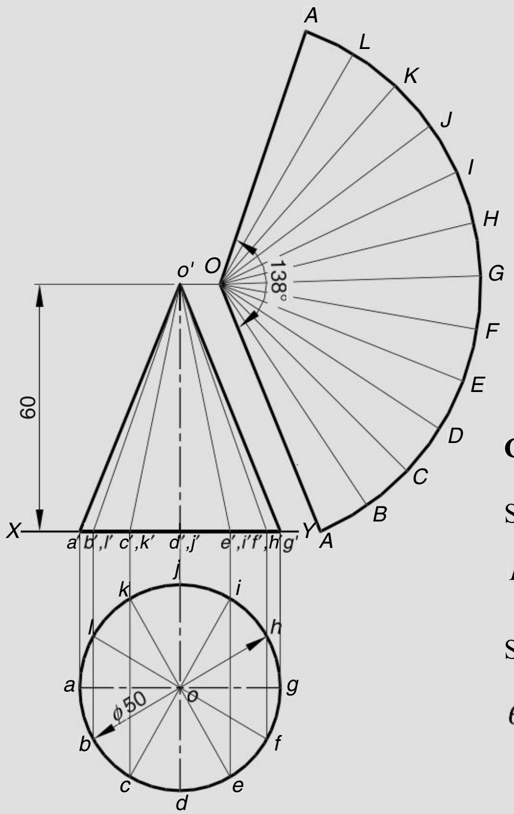
Note: In the development of cylinder the cutting lines converges to form arc. Therefore, in the development 1-4, 4-7, 7-10 and 10-1 are arcs of circles.

11.6 DEVELOPMENT OF A CONE

Development of lateral surface of a cone is obtained by radial-line method. In this method, the development is in the form of sector of a circle, the radius of which is equal to the slant height of the cone. The subtended angle 6 of this sector is calculated by $\theta = \frac{r}{R} \times 360°$ where r = the radius of the base circle, and R = the slant height of the cone. In an approximate method, subtended angle 6 can be determined by transferring arc of length, $\frac{1}{12}$th of the base circle in the top view, twelve times over the sector of the circle in the development.

Example 11.11 (Fig. 11.11)

Draw the development of lateral surface of the cone whose base diameter is 50 mm and axis is 60 mm long. The cone is resting on H.P. on its base.



Calculation of θ

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360° = \frac{25}{65} \times 360° = 138° \text{ (approx.)}$$

Fig. 11.11

Construction: Fig. 11.11

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.
2. The end generators $o'a'$ and o′g′ gives the true length of the generators because their top views are parallel to *XY*. Therefore, mark *OA* parallel to $o'g'$.
3. Determine the subtended angle θ of the development.
4. Draw a sector *A*-*O*-*A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc. This is the required development of the cone.

Example 11.12 (Fig. 11.12)

A cone base of 50 mm diameter and 60 mm long axis rests with its base on H.P. A section plane perpendicular to V.P. and inclined at 45º to H.P. bisects the axis of the cone. Draw the development of the lateral surface of the remaining portion of the cone. [RGPV Aug. 2010]

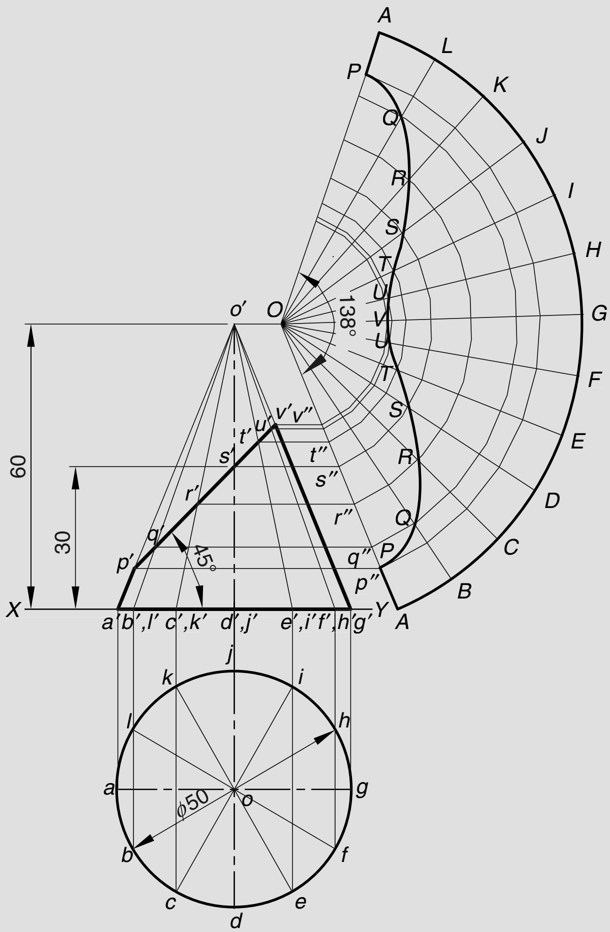


Fig. 11.12

Calculation of θ

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65 \text{ mm}$$

Subtended angle

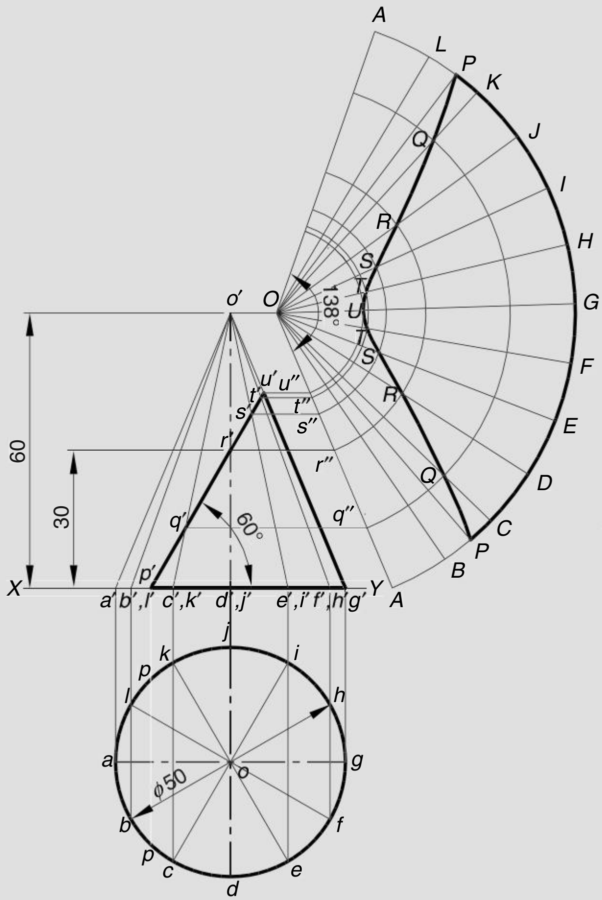
$$\theta = \frac{r}{R} \times 360° = \frac{25}{65} \times 360° = 138° \text{ (approx.)}$$

Construction: Fig. 11.12

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain *a′o′g′* as the front view.
2. Draw *p′v′* as V.T. of the cutting plane in the front view such that it is inclined at 45º to *XY* and passes through the mid-point of the axis. Line *p′v′* cut the generators *a′o′* at point *p′*, *b′o′* at point *q′*, *c′o′* at point *r′*, *d′o′* at point *s′*,…, etc.
3. Determine the subtended angle *6* as 138º. Draw a sector *A*-*O*-*A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
4. Draw the horizontal lines from points *p′*, *q′*, *r′*,…, etc., to meet *OA* in the development at points *p″*, *q″*, *r″*,…, etc. Draw arcs with *O* as the centre and radii *Op″*, *Oq″*, *Or″*, …, etc., to meet the corresponding generators at points *P*, *Q*, *R*,…, etc.
5. Join all the points obtained in the development with smooth curves. Darken the portion of the development which remains after truncation of the cone.

Example 11.13 (Fig. 11.13)

A cone with a 50 mm base diameter and 60 mm long axis, rests with its base on the H.P. Draw the development of its lateral surface when it is cut by an auxiliary inclined plane bisecting the axis and inclined at 60º to the H.P.



Calculation of θ

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 60^2} = 65 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360° = \frac{25}{65} \times 360° = 138° \text{ (approx.)}$$

Fig. 11.13

Construction: Fig. 11.13

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.
2. Draw $p'u'$ as V.T. of the cutting plane in the front view such that it is inclined at 60º to *XY* and passes through the mid-point of the axis. Line $p'u'$ cut the generators $c'o'$ at point q', $d'o'$ at point r', $e'o'$ at point s', $f'o'$ at point t', $g'o'$ at point u' and base circle at point p'.
3. Determine the subtended angle θ as 138º. Draw a sector *A-O-A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
4. Draw the horizontal lines from points of intersection, i.e. q', r', s', t' and u′ which meets line *OA* in the development at points q'', r'', s'', t'' and u'', respectively. Draw arcs with *O* as the centre and

radii Oq'', Or'', Os'', Ot'' and Ou'' to meet the corresponding generators at points Q, R, S, T and U, respectively.

5. Project point p' vertically downwards to meet the circle in the top view at point p. Locate point P in the development such that $BP = LP = bp$ and obtain point P.
6. Join all the points obtained in the development with smooth curves. Darken the portion of the development which remains after truncation of the cone.

Example 11.14 (Fig. 11.14)

The frustum of a cone of 60 mm base diameter 20 mm top diameter and 50 mm height is resting on its base in the H.P. It is cut by an A.I.P. inclined at 30º to the H.P., the H.T. of which is tangential to the base circle. Draw the development of the lateral surface of the retained frustum.

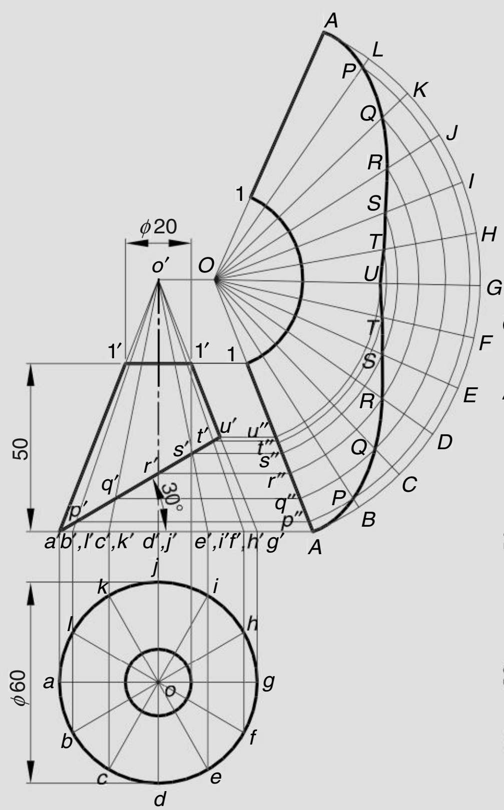


Fig. 11.14

Calculation of θ

$\Delta a'g'o'$ and $\Delta 1'1'o'$ are similar

$$\therefore \frac{60}{o'd'} = \frac{20}{o'd' - 50} \Rightarrow o'd' = 75 \text{ mm}$$

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{30^2 + 75^2} = 80.8 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360° = \frac{30}{80.8} \times 360° = 134° \text{ (approx.)}$$

Construction: Fig. 11.14

1. Draw two concentric circles of 60 mm and 20 mm diameter as the top view and divide it into 12 equal parts. Project all the points and obtain $a'g'1'1'$ as the front view.
2. Draw $a'u'$ as V.T. of the cutting plane in the front view such that it is inclined at 30º to XY. Line $a'u'$ cut the generators $b'o'$ at point p', $c'o'$ at point q', $d'o'$ at point r', $e'o'$ at point s', $f'o'$ at point t' and $g'o'$ at point u'.

3. Determine the subtended angle θ as 134º. Draw a sector *A-O-A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
4. Draw the horizontal lines from 1′ to meet *OA* at point 1. Draw an arc 1-1 with *O* as the centre and radius *O*-1.
5. Draw the horizontal lines from p', q', r', s', t' and u' to meet line *OA* at points p'', q'', r'', s'', t'' and u'', respectively. Draw arcs with *O* as the centre and radii Op'', Oq'', Or'', Os'', Ot'' and Ou'' to meet the corresponding generators at points *P*, *Q*, *R*, *S*, *T* and *U*, respectively. Draw a smooth curve to join *APQRSTUTSRQPA*.

Example 11.15 (Fig. 11.15)

Draw the development of the lateral surface of the truncated cone shown in fig. 11.15(a) with a base of diameter 40 mm. [RGPV June 2009]

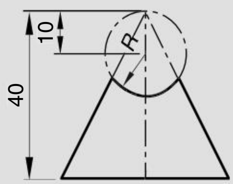


Fig. 11.15(a)

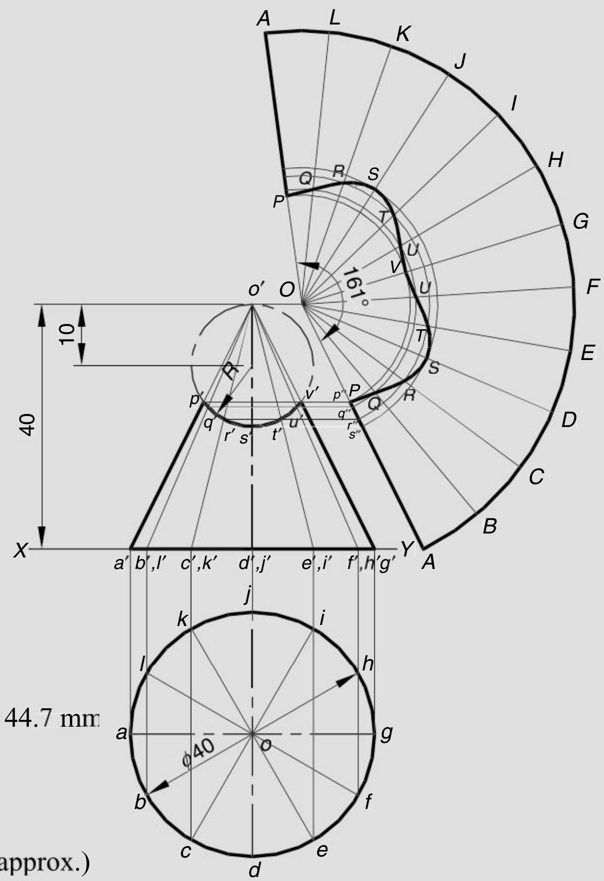


Fig. 11.15(b)

Calculation of θ

Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{20^2 + 40^2} = 44.7 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360° = \frac{20}{44.7} \times 360° = 161° \text{ (approx.)}$$

Construction: Fig. 11.15(b)

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.

2. Draw arc $p'v'$ as V.T. of the cutting plane in the front view as shown. Line $p'v'$ cut the generators $a'o'$ at point p', $b'o'$ at point q', $c'o'$ at point r', $d'o'$ at point s',…, etc.
3. Determine the subtended angle θ as 161º. Draw a sector *A-O-A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
4. Draw the horizontal lines from points p', q', r',…, etc., to meet *OA* in the development at points p'', q'', r'',…, etc. Draw arcs with *O* as the centre and radii Op'', Oq'', Or'', …, etc. to meet the corresponding generators at points *P*, *Q*, *R*,…, etc.
5. Join all the points obtained in the development with smooth curves. Darken the portion of the development which remains after truncation of the cone.

Example 11.16 (Fig. 11.16)

A cone of 60 mm base diameter and 75 mm long axis is resting on its base on the H.P. A square hole of 20 mm side is made in it such that axis of the hole intersect the axis of the cone at a height of 25 mm from the base and the faces of the hole are equally inclined to the H.P. Draw the development of its lateral surface.

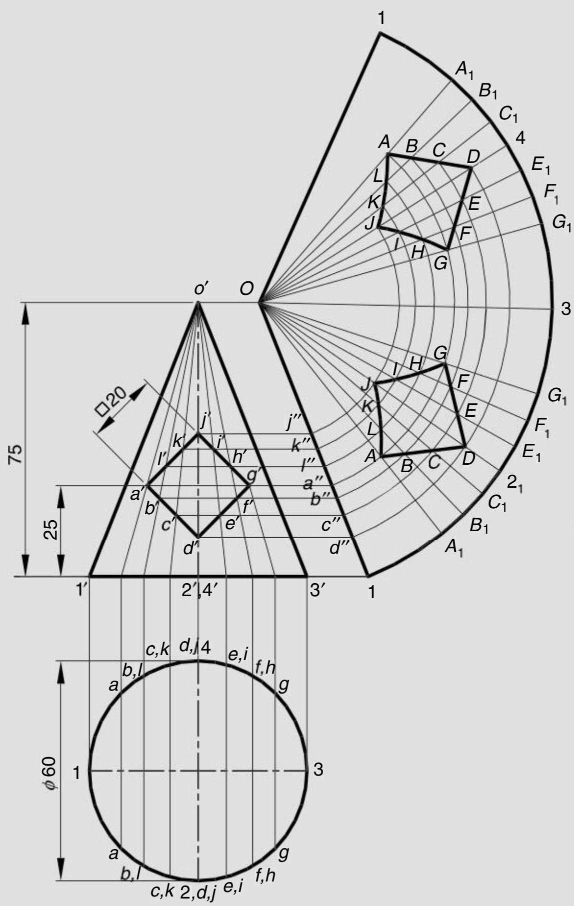


Fig. 11.16

Calculation of θ

Slant height of cone

$$R = o'3' = \sqrt{r^2 + h^2} = \sqrt{30^2 + 75^2} = 80.8 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360° = \frac{30}{80.8} \times 360° = 134° \text{ (approx.)}$$

Construction: Fig. 11.16

1. Draw a circle 1-2-3-4 as the top view. Project all the points and obtain $1'o'3'$ as the front view.
2. Draw a square $a'd'g'j'$ such that all the edges are inclined at 45º to *XY* and it centre lies at a distant of 25 mm above the *XY*. On the edges of the square mark some more points as b', c', e', f', h', i', k' and l', which may not be equidistant.
3. Determine the subtended angle θ as 134º. Draw a sector 1-*O*-1 with included angle θ.
4. Draw generator through the critical points a' and g'. Also draw generators through points b', c', e', f', h', i', k' and l'. Project them to the top view as *a*, *b*, *c*,… etc.
5. Mark the generators in the development as OA_1, OB_1, OC_1,… etc., such that $1A_1$= arc 1*a*, $1B_1$ = arc 1*b*, $1C_1$ = arc 1*c*, etc. They represent the locus line for points *A*, *B*, *C*,… etc.
6. Draw horizontal lines from the points a', b', c', e', f', g', h', i', k' and l' to meet *OA* at points a'', b'', c'',… etc. Draw arcs with *O* as the centre and radii Oa'', Ob'', Oc'',… etc to meet the corresponding generators at points *A*, *B*, *C*,… etc.
7. Join all the points to obtain the required development as shown. It may be noted that the cutting edges of the square converges in the development to form arc. Therefore, *AD*, *DG*, *GJ* and *JA* are arcs of circles.

11.7 DEVELOPMENT OF PYRAMID

Development of lateral surface of pyramids consists of a series of isosceles triangles. It can be drawn using radial line method, similar to that of the cone. The following examples illustrate the development of the lateral surface of the pyramids.

Example 11.17 (Fig. 11.17)

Draw the development of lateral surface of a square pyramid with a 40 mm base side and a 60 mm long axis, resting on its base in the H.P., such that all the sides of the base are equally inclined to the V.P.

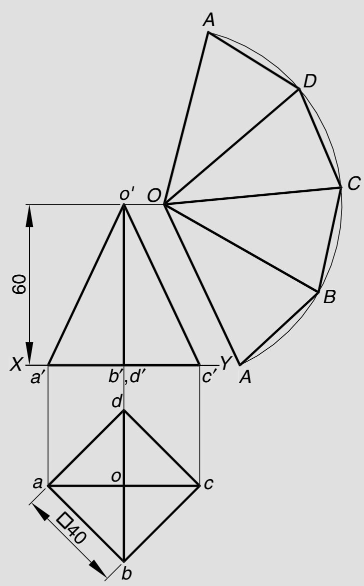


Fig. 11.17

Construction: Fig. 11.17

1. Draw a square *abcd* with side *ab* inclined at 45º to *XY*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain triangle *a′o′c′* as the front view. Consider seam at *o′a′*.
2. Slant edges *o′a′* and *o′c′* in the front view represents the true length because their top views are parallel to *XY*. Therefore, draw a line *OA* parallel to *o′c′*.
3. Draw an arc with *O* as the centre and radius *OA*. Step off a distance of 40 mm on the arc to obtain *B*, *C*, *D* and *A*. Thus, *AB* = *BC* = *CD* = *DA* = 40 mm.
4. Join the base sides *AB*, *BC*, *CD*, *DA* and slant edges *OA*, *OB*, *OC*, *OD*, *OA*. This is the required development of the pyramid.

Example 11.18 (Fig. 11.18)

Draw the development of lateral surface of a square pyramid with a 40 mm base side and a 60 mm long axis, resting on its base in the H.P. such that a side of the base is parallel to the V.P.

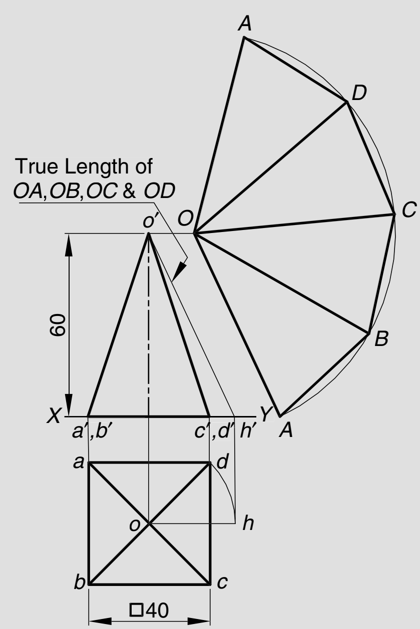


Fig. 11.18

Construction: Fig. 11.18

1. Draw a square *abcd* with side *ad* parallel to *XY*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain triangle *a′o′c′* as the front view. Consider seam at *o′a′*.

2. As slant edges *oa*, *ob*, *oc* and *od*, in the top view of are inclined to *XY* therefore, slant edges *o′a′*, *o′b′*, *o′*c′ and *o′d′* in the front view do not represent the true lengths. So first determine the true length of the slant edges.
   a. Draw an arc *dh* with *o* as the centre and radius *od* to meet the horizontal line through centre *o* at point *h*.
   b. Project point *h* to meet *XY* at point *h′*. Join *o′h′*. Line *o′h′* represents the true length of slant edges.
3. Draw line *OA* parallel to and equal to *o′h′*. Draw an arc with *O* as the centre and radius *OA*. Step off a distance of 40 mm on the arc to obtain *B*, *C*, *D* and *A*. Thus, *AB* = *BC* = *CD* = *DA* = 40 mm.
4. Darken the base sides *AB*, *BC*, *CD*, *DA* and slant edges *OA*, *OB*, *OC*, *OD*, *OA*. This is the required development of the pyramid.

Example 11.19 (Fig. 11.19)

A square pyramid with 30 mm side of base and 50 mm long axis is resting on its base with a side of the base parallel to V.P. It is cut by a section plane perpendicular to V.P. and inclined at 45º to H.P. The section plane is passing through the mid-point of the axis. Draw the development of the cut pyramid. [RGPV Feb. 2008]

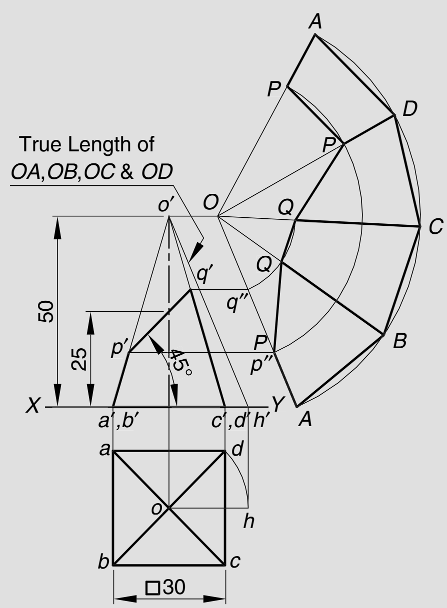


Fig. 11.19

Construction: Fig. 11.19

1. Draw a square *abcd* with side *ad* parallel to *XY*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain triangle *a′o′c′* as the front view. Consider seam at *o′a′*.

2. Determine true length of the slant edges: Draw an arc *dh* with *o* as the centre and radius *od* to meet the horizontal line through centre *o* at point *h*. Project point *h* to meet *XY* at point *h*′. Join *o*′*h*′.
3. Draw *p*′*q*′ as V.T. of the cutting plane in the front view such that it is inclined at 45° to *XY* and passes through the mid-point of the axis. Line *p*′*q*′ cut the generators *o*′*a*′ and *o*′*b*′ at point *p*′, *o*′*c*′ and *o*′*d*′ at point *r*′.
4. Draw line *OA* parallel to and equal to *o*′*h*′. Draw an arc with *O* as the centre and radius *OA*. Step off a distance of 30 mm on the arc to obtain *B*, *C*, *D* and *A*. Thus, *AB* = *BC* = *CD* = *DA* = 30 mm.
5. Draw the horizontal lines from *p*′ and *q*′ to meet line *OA* at points *p*″ and *q*″, respectively. Draw arcs with *O* as the centre and radii *Op*″ and *Oq*″ to meet *OA* and *OB* at point *P*; *OC* and *OD* at point *Q*.
6. Join all the points to obtain the required development as shown. Darken the portion of the development which remains after truncation of the pyramid.

Example 11.20 (Fig. 11.20)

A square pyramid with 30 mm side of base and 50 mm long axis is resting on its base such that all the sides of the base are equally inclined to the V.P. It is cut by a section plane perpendicular to V.P. and inclined at 60° to H.P. The section plane is passing through the mid-point of the axis. Draw the development of the cut pyramid. [RGPV Feb. 2008]

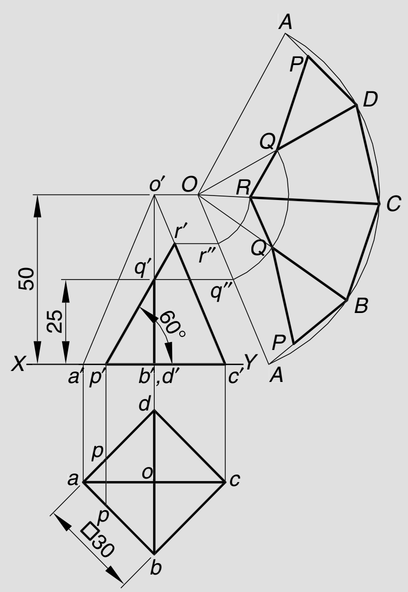


Fig. 11.20

Construction: Fig. 11.20

1. Draw a square *abcd* with side *ab* inclined at 45° to *XY*. Also, draw the diagonal lines of the square. This represents the top view. Project all the corners to obtain triangle *a*′*o*′*c*′ as the front view.

2. Draw $p'r'$ as V.T. of the cutting plane in the front view such that it is inclined at 60º to XY and passes through the mid-point of the axis. Line $p'r'$ cuts the base edges $a'b'$ and $a'd'$ at point p', generators $o'b'$ and $o'd'$ at point q', $o'c'$ at point r'.
3. Line $o'c'$ represents the true length of the slant edge. Therefore, draw line OA parallel to and equal to $o'c'$. Draw an arc with O as the centre and radius OA. Step off a distance of 30 mm on the arc to obtain B, C, D and A.
4. Draw horizontal lines from points q' and r' to meet line OA at points q'' and r'', respectively. Draw arcs with O as the centre and radii Oq'' and Or'' to meet OB and OD at point Q; OC at point R.
5. Project point p' vertically downwards to obtain point p in the top view. Mark point P in the development such that $AP = ap$.
6. Join all the points to obtain the required development as shown.

Example 11.21 (Fig. 11.21)

A pentagonal pyramid of 30 mm base side and 60 mm axis, rests on its base in the H.P. It is cut by two section plane which meet at a height of 20 mm from the base. One of the section planes is horizontal, while the other is an auxiliary inclined plane whose V.T. makes 45º with H.P. Draw the development of the lateral surface of the solid when apex is removed.

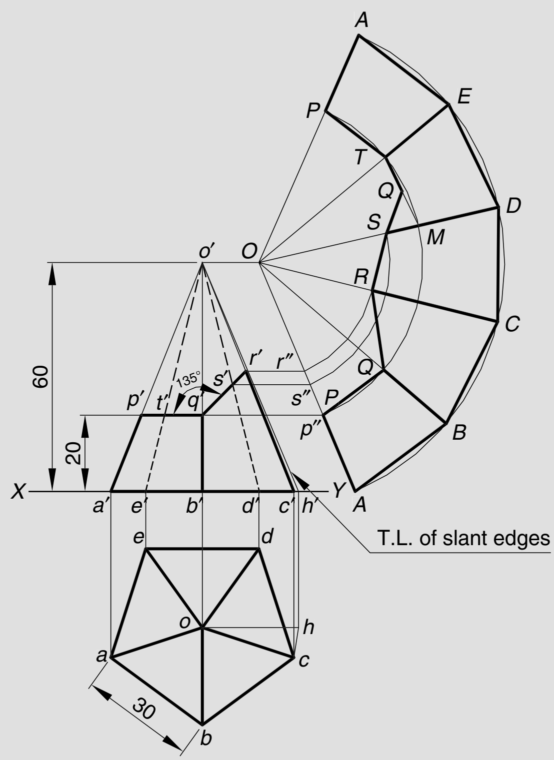


Fig. 11.21

Construction: Fig. 11.21

1. Draw a pentagon *abcde* with side *de* parallel to *XY*. Also, draw the radial lines. This represents the top view. Project all the corners to obtain triangle $a'o'c'$ as the front view.
2. Draw $p'q'$ parallel to *XY* and $q'r'$ inclined at 45° to *XY* to represent the V.T. of the cutting plane in the front view.
3. With center *o* and radius *oc* draw an arc to meet horizontal line through *o* at point *h*. Project *h* to obtain h'. Join $o'h'$. Line $o'h'$ represents true length of the slant edges.
4. Draw line *OA* parallel to and equal to $o'h'$. Draw an arc with *O* as the centre and radius *OA*. Step off a distance of 30 mm on the arc to obtain *B*, *C*, *D*, *E* and *A*.
5. Draw horizontal lines from p', q' and r' to meet *OA* at points p'', q'' and r'', respectively. Draw arcs with *O* as the centre and radii Op'', Oq'' and Or'' to meet *OA* at point *P*, *OB* at point *Q*, *OC* at point *R*, *OD* at point *S*, *OE* at point *T*. Also, mark point *Q* as the mid-point of chord *TM*.
6. Join all the points to obtain the required development as shown.

Example 11.22 (Fig. 11.22)

A hexagonal pyramid, 30 mm base side and 60 mm long axis rests on the H.P. with a side of base parallel to V.P. It is cut by planes perpendicular to V.P., to obtain the front view as shown in Fig. 11.22(a). Draw the development of the lateral surface of the retained solid.

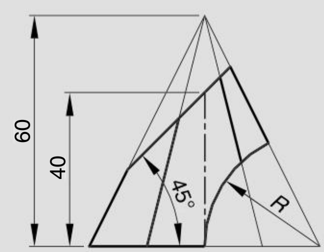


Fig. 11.22(a)

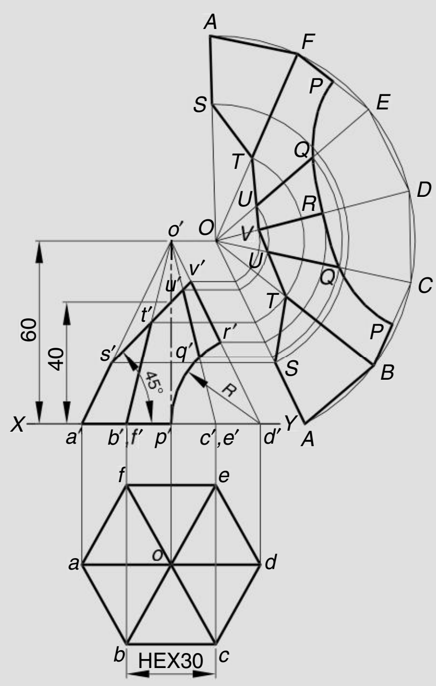


Fig. 11.22(b)

Construction: Fig. 11.22(b)

1. Draw a hexagon *abcdef* with side *ef* parallel to *XY*. Also, draw the radial lines. This represents the top view. Project all the corners to obtain triangle *a′o′d′* as the front view.
2. Draw a curve *p′r′* and line *s′v′* in the front view to represent the V.T. of the cutting planes.
3. Line *o′d′* represents true length of the slant edges. Therefore, draw line *OA* parallel to and equal to *o′d′*. Draw an arc with *O* as the centre and radius *OA*. Step off a distance of 30 mm on the arc to obtain *B*, *C*, *D*, *E*, *F* and *A*.
4. Draw horizontal lines from points *q′*, *r′*, *s′*, *t′*, *u′* and *v′* to meet *OA*, and thereafter rotate them to meet the corresponding generators at points *Q*, *R*, *S*, *T*, *U* and *V*. Point *p′* lies at the mid of base edges *bc* and *ef*. Therefore, in the development, mark point *P* at the mid of *BC* and *EF*.
5. Join all the points to obtain the required development as shown.

Example 11.23 (Fig. 11.23)

A frustum of a square pyramid of 40 mm base side, 15 mm top side and 40 mm height, rests on its base on the H.P. with all the sides of the base equally inclined to the V.P. A rectangular slot of 30 mm × 15 mm is cut through it. Figure 11.23(a) shows the front view of the solid. Draw the development of its lateral surface.

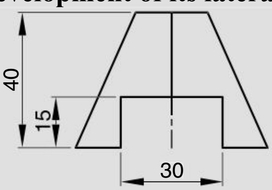


Fig. 11.23(a)

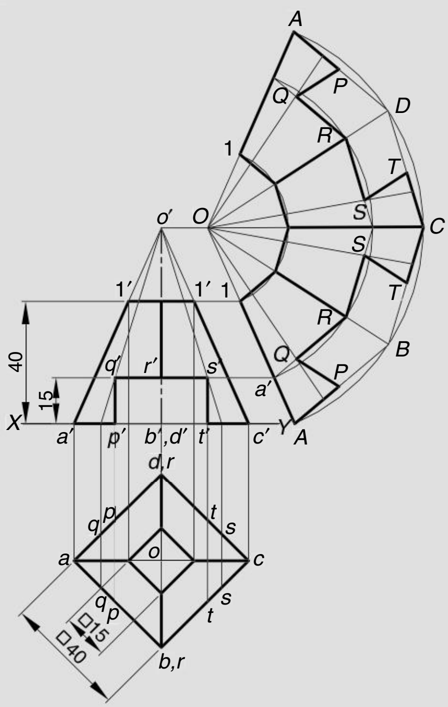


Fig. 11.23(b)

Construction: Fig. 11.23(b)

1. Draw two concentric squares to represent the top view. Project all the corners to obtain triangle *a′*1′1′*c′* as the front view.

2. Slant edge $o'c'$ in the front view represents the true length. Therefore, draw a line *OA* parallel to $o'c'$.
3. Draw arcs with *O* as the centre and radii *OA* and *O*-1. Step off a distance of 40 mm on the arc *AA* to obtain *B*, *C*, *D*. Similarly, step off a distance of 15 mm on the arc 1-1.
4. Draw the V.T. of the cutting plane to obtain the front view as given. Name the intersecting points as p', q', r', s' and t'.
5. Project points p', q', r', s' and t' on the top view to obtain points *p*, *q*, *r*, *s* and *t*. Transfer these points on the edges *AB*, *BC*, *CD* and *DA*.
6. Draw horizontal lines from q', r' and s' up to *OA*, and thereafter rotate them to obtain *Q*, *R* and *S* on their corresponding generators. It may be noted that points *P*, *Q*, *S* and *T* lies on the chords. Join all the points to obtain the required development as shown.

11.8 ANTI-DEVELOPMENT

In the previous section we have discussed the methods of obtaining the development of surfaces of any solid. Any point on the object could be transferred to its corresponding position in the development. Now we shall do the other way. Any point or a set of points from the development shall be transferred to its corresponding position in the front view / top view. This is the reverse of the development commonly know as anti-development. The following examples illustrate anti-development of surfaces.

Example 11.24 (Fig. 11.24)

A pentagonal prism, having a base with a 25 mm side and a 60 mm long axis, stands on the ground on its base such that one of its rectangular faces is parallel to and nearer the V.P. A thread is wound around the prism, starting from the corner of the lower base farthest away from the V.P. to the corresponding corner of the upper base. Find the minimum length of the thread and show it on the front view of the prism.

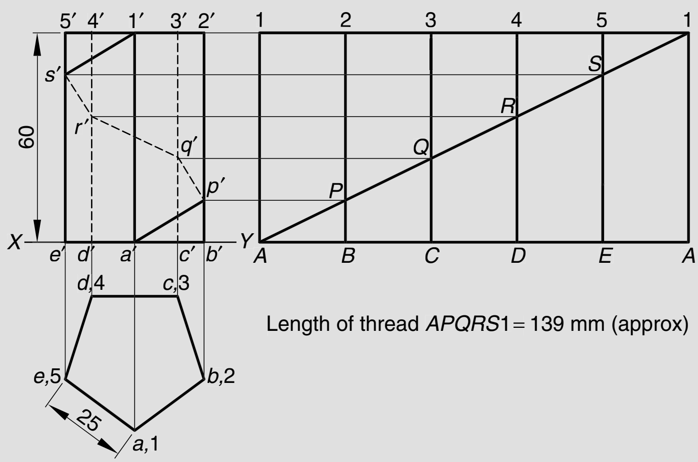


Fig. 11.24

Construction: Fig. 11.24

1. Draw a pentagon *abcde* keeping *cd* parallel to *XY* to represent the top view. Project all points to obtain $a'c'3'1'$ as the front view.
2. Consider the seam is along 1-*a*. Stretch out lines 1-1 and *A*-*A* from the front view. Locate intermediate points 2, 3, 4, 5 and *B*, *C*, *D*, *E*. Join vertical edges 1*A*, 2*B*, 3*C*, 4*D* and 5*E*.
3. Draw a diagonal line *APQRS*1 to represent the thread. The line meets the vertical edges *B*2, *C*3, *D*4 and *E*5 at points *P*, *Q*, *R* and *S*.
4. Draw horizontal lines from *P*, *Q*, *R* and *S* to meet $b'2'$, $c'3'$, $d'4'$ and $e'5'$ at points p', q', r' and s', respectively.
5. Join points $a'p'q'r's'1'$ as shown. It may be noted that the line joining $a'p'$ and $s'1'$ will be visible because they lies on the faces of the prism which is towards the observer.

Example 11.25 (Fig. 11.25)

Draw the projections of a cone resting on the ground on its base and show on them the shortest path by which a point *P*, on the circumference of the base moving around the cone will return to the same point. Base of cone is 65 mm diameter and axis is 75 mm long. [RGPV Dec. 2003, Dec. 2004, Feb. 2005]

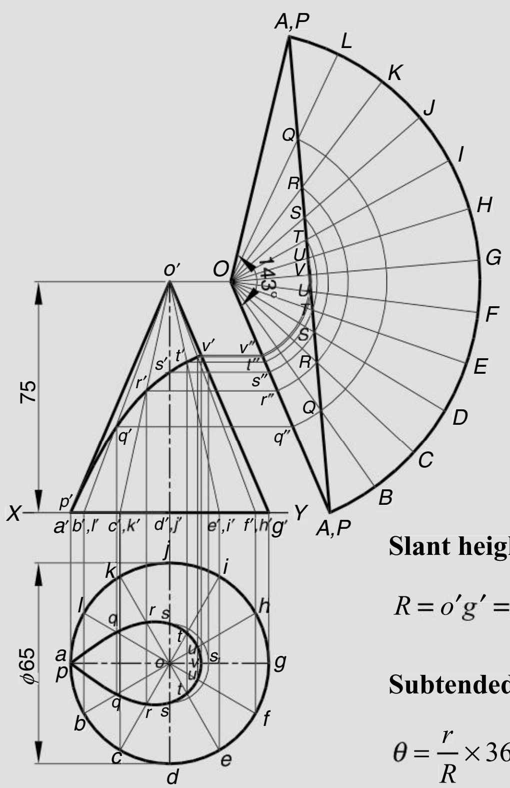


Fig. 11.25

Slant height of the cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{32.5^2 + 75^2} = 81.74 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360° = \frac{32.5}{81.75} \times 360° = 143° \quad \text{(approx.)}$$

Construction: Fig. 11.25

1. Draw a circle *adgj* to represent the top view. Divide it into 12 equal parts. Project all the points and obtain $a'o'g'$ as the front view.
2. Determine the subtended angle θ as 138º. Draw a sector *A-O-A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
3. Assume that *P* starts from the point *A*. Join *A-A* on the development to represent the shortest path. Let line *P-P* meets the generators at points *Q*, *R*, *S*, *T*, *U* and *V* in the development as shown.
4. Draw arcs with *O* as the centre and radii *OQ*, *OR*, *OS*, *OT*, *OU* and *OV* to meet line *OA* at points q'', r'', s'', t'', u'' and v''. Draw horizontal lines from q'', r'', s'', t'', u'' and v'' to meet their corresponding generators at points q', r', s', t', u' and v' in the front view. Join the points and obtain the front view of the shortest path.
5. Transfer points q', r', t', u' and v' to the top view on the respective generators and mark as *q*, *r*, *t*, *u* and *v*. To transfer points s', draw horizontal line from point p' to meet $o'g'$ at s_1'. Project it vertically downwards *t* meet *og* at point s_1. With centre *o* and radius os_1, draw arcs to meet *od* and *oj* at points *s*.
6. Join the points *pqrstuvutsrqp* by a smooth curve which represents the required shortest path in the top view.

Example 11.26 (Fig. 11.26)

A semicircle of 100 mm diameter represents the development of lateral surface of a right circular cone. Inscribe the largest possible circle in the development and draw the projections of the cone resting on its base in H.P. and showing the projection of the circle in them.
[RGPV Sep. 2009]

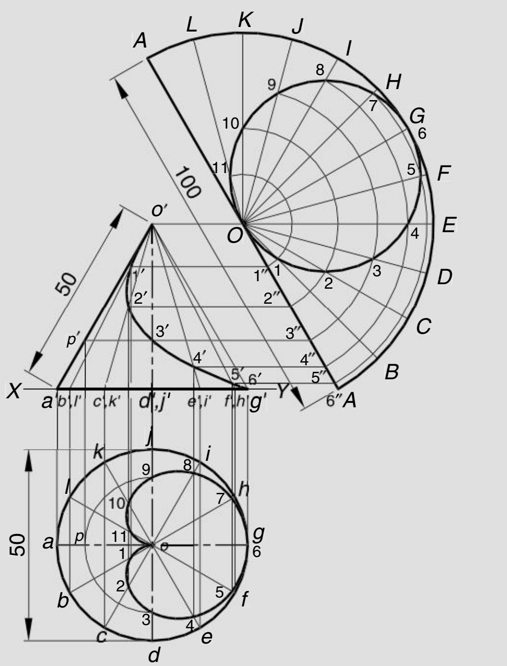


Fig. 11.26

Radius of base circle of the cone

$$r = \text{Slant height} \times \frac{\theta}{360} = 50 \times \frac{180°}{360°} = 25 \text{ mm}$$

Construction: Fig. 11.26

1. As semicircle in the development has a 100 mm diameter, the slant height of the cone is 50 mm. Calculate the radius of the base circle of the cone as 25 mm.
2. Draw a circle *adgj* of 25 mm radius to represent the top view. Divide it into 12 equal parts. Project all the points to obtain *a′o′g′* as the front view.
3. Draw a semicircle *A*-*O*-*A* to represent the development of the cone. Divide it into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
4. Draw a circle of 50 mm diameter on the development to represent the required circular hole. Mark the points of intersection of the generators with the circular hole as 1, 2, 3,… etc.
5. Draw arcs with *O* as the centre and radii *O*1, *O*2, *O*3,… etc., to meet *OA* at points 1″, 2″, 3″,… etc. Draw horizontal lines from 1″, 2″, 3″,… etc. to meet their corresponding generators in the front view at points 1′, 2′, 3′, etc. Join the points to obtain the mark of the circular hole in the front view.
6. Project points 1′, 2′, 3′, 5′, 6′, 7′, 8′, 9′ and 11′ vertically downwards to meet the respective generators in the top view and obtain points 1, 2, 3, 5, 6, 7, 8, 9 and 11.
7. Draw horizontal line points 4′ and 10′ to meet to *o′a′* at point *p′*. Project *p′* to meet *oa* at point *p*. Draw arc with *o* as the centre and *op* as the radius to meet *od* and *oj* at points 4 and 10 respectively.
8. Join all the points in the top view as shown to represent mark of the circular hole in the top view.

11.9 MISCELLANEOUS EXAMPLES

Example 11.27 (Fig. 11.27)

The projection of a solid composed of a truncated half cylinder and a half prism are shown in Fig 11.27(a). Draw the development of its lateral surface. [RGPV June 2004]

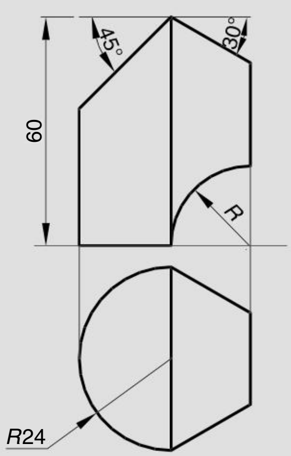


Fig. 11.27(a)

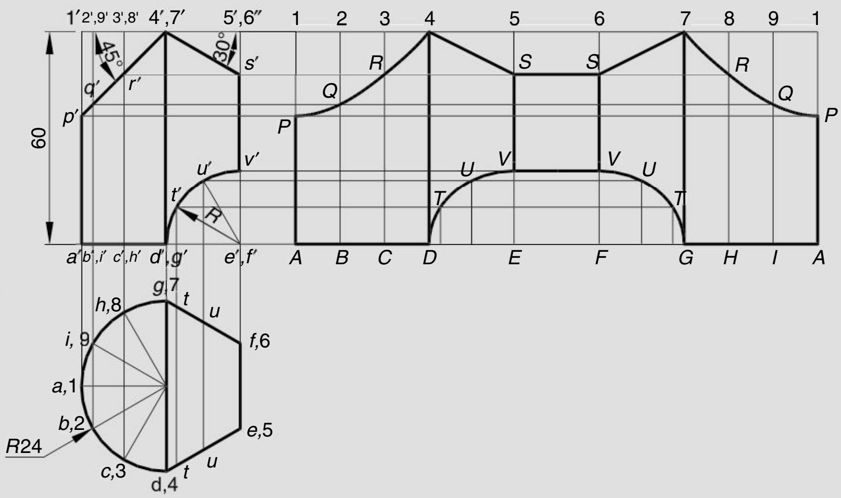


Fig. 11.27(b)

Construction: Fig. 11.27(b)

1. Draw front and top views of the solid as given. Draw generator of the half-cylinder.
2. Name the points of intersection of V.T. in the front view as *p′*, *q′*, *r′*, *s′*, *t′*, *u′* and *v′*.
3. Consider seam at 1-*a*. Stretch out 1-1 and *A*-*A* from the front view and step off *AD* = arc *ad*, *DE* = *de*, *EF* = *ef*, *FG* = *fg*, *GA* = arc *ga*. Draw generators *B*2, *C*3, *H*8 and *I*9. Also, draw the locus lines for points *T* and *U* on the faces *DE*54 and *FG*76 respectively.
4. Draw horizontal lines from points *p′*, *q′*, *r′*, *s′*, *t′*, *u′* and *v′* to meet the corresponding generators in the development at points *P*, *Q*, *R*, *S*, *T*, *U* and *V*.
5. Join all the points in the development as shown to represent the development of the lateral surface of the solid.

Example 11.28 (Fig. 11.28)

A solid is composed of half-pyramid and half-cone. It is cut by an auxiliary inclined plane bisecting the axis and making 45º with the H.P., as shown in Fig. 11.28(a). Draw the development of the lateral surface of the retained solid.

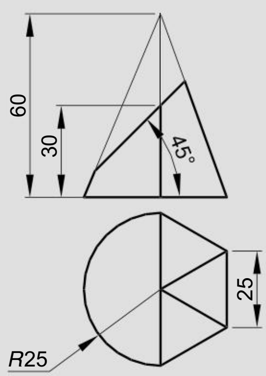


Fig. 11.28(a)

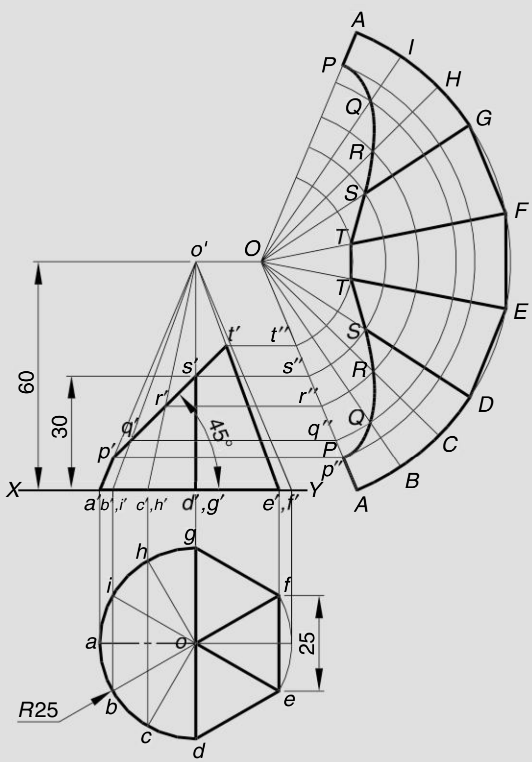


Fig. 11.28(b)

Construction: Fig. 11.28(b)

1. Draw front and top views of the solid as given. Draw generator of the half-cone.
2. Name the points of intersection of V.T. in the front view as p', q', r', s' and t'.
3. Draw a line OA of length equal to the true length. Draw an arc with O as the centre and radius OA. On the arc AA. Step off arcs of ab, bc, cd, de, ef, fg, gh, hi and ia and obtain points B, C, B, E, F, G, H and I. Join OB, OC, OD,…, etc.
4. Draw the horizontal lines from points p', q', r', s' and t' to meet OA at points p'', q'', r'', s'' and t''. Draw arcs with O as the centre and radii Op'', Oq'', Or'', Os'' and Ot'' to meet the corresponding generators at points P, Q, R, S and T.
5. Join all the points obtained in the development as shown.

Example 11.29 (Fig. 11.29)

A cone with a 60 mm base diameter and 75 mm long axis stands on its base on the H.P. An auxiliary vertical plane having H.T. inclined at 45º to the V.P. cuts the cone, at a distance of 12 mm away from the axis. Draw the sectional front view and develop the lateral surface of the retained cone.

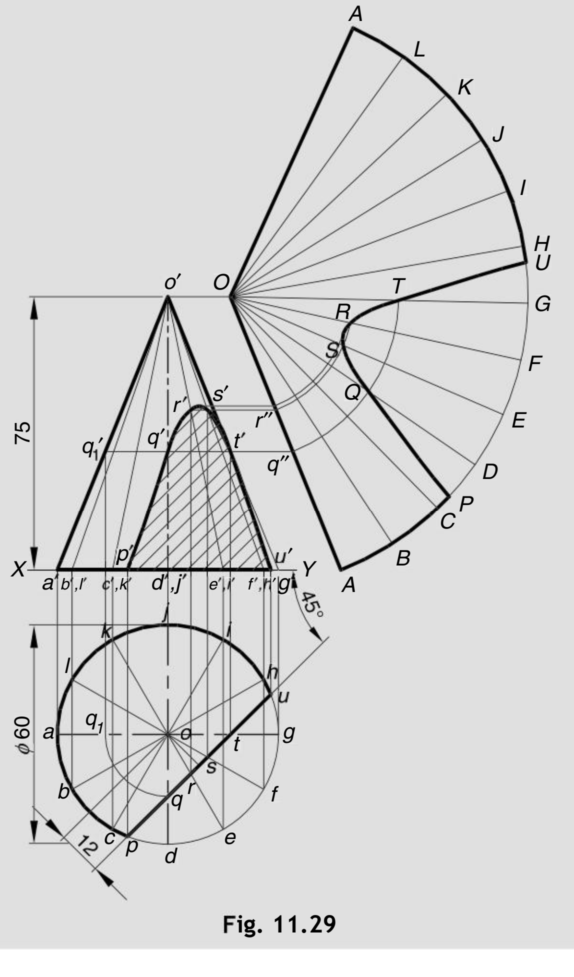


Fig. 11.29

Construction: Fig. 11.29

1. Draw a circle *adgj* as the top view and divide it into 12 equal parts. Project all the points and obtain *a′o′g′* as the front view.
2. Draw *pu* as H.T. of the cutting plane in the top view such that it is inclined at 45º to *XY* and meets tangentially an arc of 12 mm radius. The line *pu* cuts the generator *od* at point *q*, *oe* at point *r*, *of* at point *s*, *og* at point *t* and base circle at points *p* and *u*.
3. Project *p*, *q*, *r*, *s*, *t* and *u* vertical to meet their respective generators in the front view at points *p′*, *q′*, *r′*, *s′*, *t′* and *u′*.
4. Determine the subtended angle θ as 134º. Draw a sector *A-O-A* with included angle θ. Divide sector into 12 equal parts and mark the generators as *OB*, *OC*, *OD*,…, etc.
5. Locate points *P* and *Q* in the development such that *CP* = *cP* and *GU* = *gu*.
6. Draw horizontal lines from points *q′*, *r′*, *s′* and *t′* up to *OA* and thereafter rotate them to obtain points *Q*, *R*, *S* and *T*.
7. Join all the points obtained in the development as shown.

Example 11.30 (Fig. 11.30)

Develop the lateral surface of the funnel as shown in Fig 11.30(a). **[RGPV June 2003]**

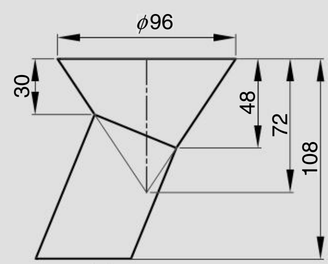


Fig. 11.30(a)

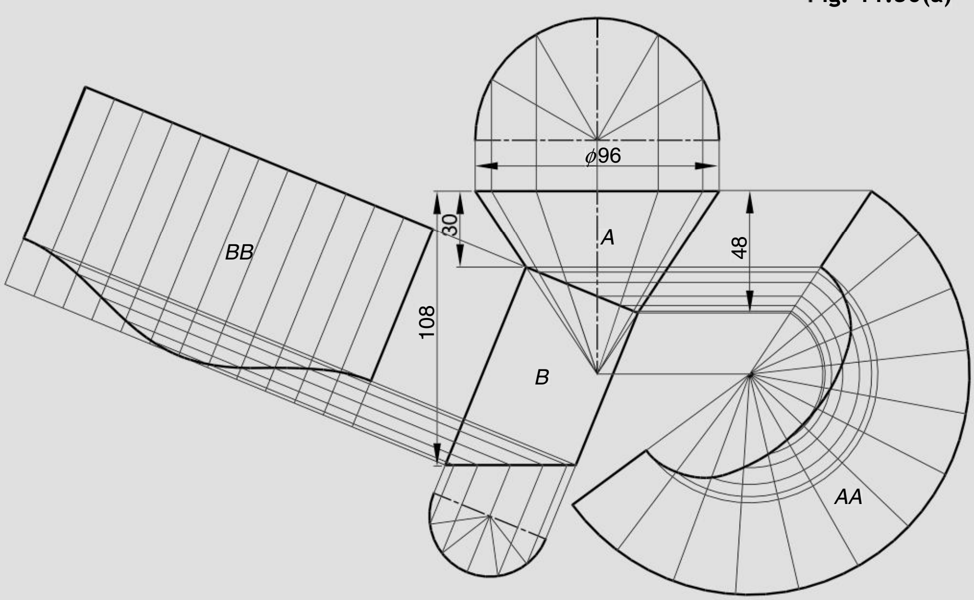


Fig. 11.30(b)

Construction: Fig. 11.30(b)

1. Draw the front view of the funnel as shown in Fig. 11.30(a).
2. Divide the front view in two parts *A* and *B* as shown.
3. Part *A* is a truncated cone. Follow the steps of Example 11.12 to obtain its development as *AA*.
4. Part *B* is a truncated cylinder. Follow the steps of Example 11.7 to obtain its development as *BB*.
5. These two developed pieces *AA* and *BB* are required to prepare the funnel.

Example 11.31 (Fig. 11.31)

Draw the shape of metal sheet required to prepare a pipe joint whose front view is shown in Fig. 11.31(a).

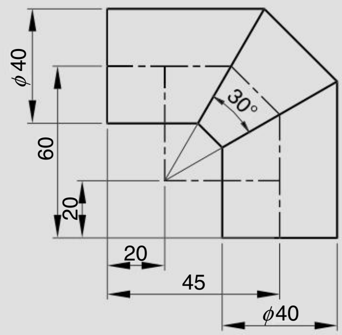


Fig. 11.31(a)

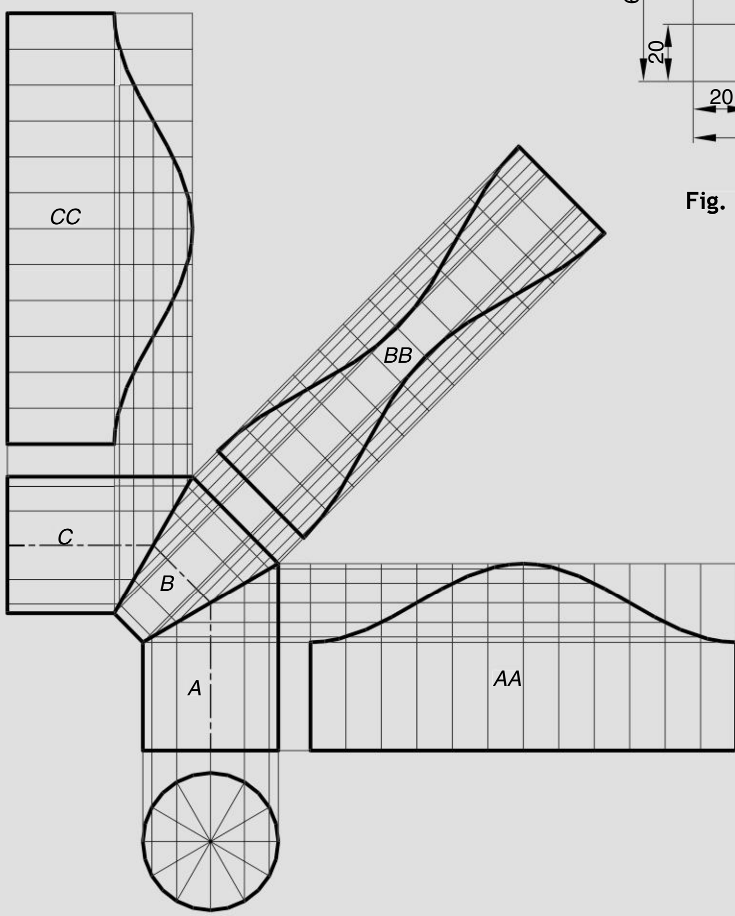


Fig. 11.31(b)

Construction: Fig. 11.31(b)

1. Draw the front view of the pipe joint as shown in Fig. 11.31(a).
2. Divide the front view in three parts *A*, *B* and *C*.
3. Part *A* and *C* are truncated cones from one end only. Follow the steps of Example 11.7 to obtain its development as *AA* and *CC*.
4. Part *B* is a truncated cone from both ends. Follow the steps of Example 11.7 for both ends of the cylinder to obtain its development as *BB*.
5. Thus, the three pieces *AA*, *BB* and *CC* are required to prepare the pipe joint.

EXERCISE 11

Prisms

1. A pentagonal prism of 30 mm base side and axis 65 mm long is resting on its base in the H.P. with a rectangular face parallel to V.P. It is cut by a section plane perpendicular to V.P., making 30º with the H.P. and passing through a point on the axis, 25 mm from one of the bases. Draw the development of its lateral surface.
2. A hexagonal prism of 25 mm base side and 50 mm axis is resting on H.P. on its base with two of its vertical faces perpendicular to V.P. It is cut by a plane inclined at 50º to H.P. and perpendicular to V.P. and meets the axis of the prism at a distance 10 mm from the top end. Draw the development of the lateral surface of the prism.
   [RGPV Dec. 2001]
3. A square prism of 50 mm edges and 65 mm height stands on its base in H.P. with its vertical faces inclined at 45º to the V.P. A horizontal hole of 25 mm diameter is drilled centrally with the axis normal to the V.P. Draw the development of the surface of the prism with the hole.
   [RGPV June 2006]

Cylinders

4. A cylinder of 45 mm base diameter and 55 mm long axis rests with its base on H.P. It is cut by a plane perpendicular to V.P. inclined at 60º to H.P. and passing through a point on the axis 12 mm from its top. Draw the top view and development of lateral surface of the truncated cylinder.
   [RGPV Feb. 2011]
5. A vertical chimney of 60 cm diameter joins a roof slopping at an angle of 30º with the horizontal. The shortest portion over the roof is 30 cm. Determine the shape of the sheet metal from which the chimney can be fabricated. **[RGPV June 2002]**
6. A cylinder of 50 mm base diameter and 70 mm long axis is resting on its base in the H.P. It is cut by two section planes inclined at 30º and 60º with the H.P. respectively. Both the planes are perpendicular to the V.P. and pass through top end of axis. Draw development of the lateral surface of the retained solid.

7. A cylinder has been truncated by a circular surface as shown in the Fig. E11.1. Draw the development of the surface of the cylinder. **[RGPV Apr. 2010]**
8. A cylinder of 60 mm base diameter and 80 mm long axis is resting on its base on the H.P. A circular hole of 44 mm diameter is drilled through the cylinder such that the axis of the hole is perpendicular bisector of the axis of the cylinder. Draw the development of its lateral surface.

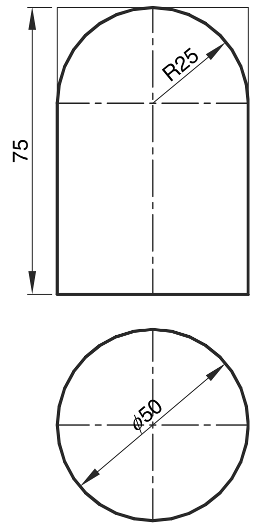


Fig. E11.1

Cone

9. A right circular cone diameter of base 40 mm and height 50 mm rests on its base on H.P. A section plane perpendicular to V.P. and inclined to H.P. at 45º cuts the cone bisecting the axis. Draw the projections of the truncated cone and develop its lateral surface. **[RGPV Dec. 2007]**
10. A cone of 50 mm base diameter and 65 mm height rests with its base in H.P. A section plane perpendicular to V.P. and inclined at 30º to H.P. bisects the cone. Draw the development of the lateral surface of the truncated cone. **[RGPV June 2008]**
11. A right cone of 50 mm base diameter and 60 mm long axis is resting on its base in the HP. It is cut by an auxiliary inclined plane parallel to and 8 mm away from the extreme generator. Draw the development of the lateral surface of the remaining solid.
12. A cone of 60 mm base diameter and 75 mm long axis is resting on its base on H.P. A profile section plane cut the cone, 10 mm away from the axis. Draw the development of the cone retained.
13. An isosceles triangle having a 60 mm base and 75 mm altitude, has a circular hole of 30 mm diameter at a height of 25 mm from the base. The figure is the front view of a truncated cone. Draw the development of its lateral surface.

Pyramid

14. A pentagonal pyramid of side of base 30 mm and axis 60 mm long is resting on its base on H.P. with an edge of the base parallel to V.P. Draw the development of the lateral surface of the pyramid. **[RGPV Dec. 2010]**
15. The frustum of a square pyramid has a 60 mm base side, 25 mm top side and 70 mm height is resting in H.P. with one of its edges parallel to V.P. Draw the lateral surface development. **[RGPV Dec. 2006]**
16. A frustum of a square pyramid has its 50 mm base side, 25 mm top side and 75 mm height. Draw the development of its lateral surface. [RGPV Feb. 2006]
17. Draw the development of the frustum of a hexagonal pyramid of side of base 35 mm at the bottom and 15 mm at the top, the height of the frustum being 50 mm. **[RGPV Dec. 2010]**
18. A pentagonal pyramid of side of base 30 mm and height 52 mm stands with its base on H.P. and an edge of the base parallel to V.P. It is cut by a plane perpendicular to V.P. and inclined at 40º to H.P. and passing through a point 30 mm above the base. Draw the development of the lateral surface of the truncated pyramid. **[RGPV Feb 2011]**
19. A square pyramid, 40 mm base side and 70 mm long axis, is resting on the ground with a side of base inclined at 30º to V.P. It is cut by a section plane perpendicular to V.P., making 45º to H.P. and bisects the axis. Draw the development of the lateral surface of truncated pyramid.

20. A hexagonal pyramid, 30 mm base side and 70 mm long axis rests on the ground with a side of base parallel to V.P. A circular hole of 30 mm diameter is cut through the faces of the pyramid such that axes of the hole and the pyramid intersect at right angles, 25 mm above the base. Draw the development of its lateral surface.

Anti-development

21. A hexagonal prism, 30 mm base side and 75 mm long axis stand on ground on its base. A thread is wound around the prism, starting from the corner of the lower base to the corresponding corner of the upper base. Find the minimum length of the thread and show it on the elevation of the prism.
22. The development of a truncated cylinder of 80 mm height is an isosceles triangle of 175 mm base and 80 mm altitude. Draw the projections of the truncated cylinder.
23. In a semi-circular plate of 120 mm diameter, a square hole is made such that center line of the semi-circle is one of the diagonals of the square. The plate is then folded to form a cone. Draw the two views of the cone.
24. In a semicircular plate of 120 mm diameter, an equilateral triangle of largest size is made. The plate is folded to form a truncated cone. Draw the two views of the truncated cone.
25. The frustum of a square pyramid has its 40 mm base side, 20 mm top side and height 60 mm, is placed on its base on the H.P. with an edge of the base perpendicular to V.P. A wire connects the mid-point of bottom edge of the front face to the mid-point of the top edge of the opposite face by passing over the surfaces of the frustum by the shortest distance. Draw the projections of the frustum and show path of the wire in the front view, top view and the development.

REVIEW QUESTIONS

1. Differentiate between singly curved surface and doubly curved surface.
2. Name the method used for drawing the development of prism and cylinder.
3. Name the method used for drawing the development of pyramid and cone.
4. What precaution should be taken while drawing the development of pyramid?
5. What are the dimensions of the cone whose development is a semicircle of 120 mm diameter?
6. State a few practical applications of development of surfaces.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) Methods for the development can be
   (a) parallel line method (b) radical line method
   (c) triangulation method (d) all of them

ii) The nature of lateral surface of a cylinder is
   (a) plane surface
   (b) singly curved surface
   (c) doubly curved surface
   (d) singly or doubly curved surface

iii) If a semicircular thin sheet is folded to form a cone then the front view looks like
   (a) equilateral triangle
   (b) isosceles triangle
   (c) rectangle
   (d) semicircle

iv) Sector of a circle of 60 mm radius and 120º represents development of the lateral surface of a cone. The top view of the cone is a circle of diameter
   (a) 20 mm (b) 40 mm (c) 60 mm (d) 80 mm

v) If the front view of a cone is represented by an equilateral triangle of 60 mm side. The area of its lateral surface is
   (a) 1200π (b) 1800π (c) 3600π (d) 120π

vi) The development of surface of a tetrahedron of 60 mm edge can be represented by an equilateral triangle of side
   (a) 60 mm (b) 90 mm (c) 120 mm (d) None of these

vii) The development of suface of a tetrahedron of 60 mm edge can be represented by a parallelogram of adjacent sides
   (a) 60 mm and 90 mm
   (b) 60 mm and 120 mm
   (c) 90 mm and 120 mm
   (d) None of these

viii) A rectangle of 120 mm × 60 mm represents the development of the lateral surface of
   (a) a square prism of side 30 mm
   (b) a hexagonal prism of side 20 mm
   (c) a cylinder of diameter $120/\pi$
   (d) all of these

ix) A string is wound around a hexagonal prism of 20 mm base side and 50 mm long axis, to connect opposite ends of the same longer edge. The minimum length of string required is
   (a) 110 mm (b) 120 mm (c) 130 mm (d) 140 mm

x) When a semicircular plate with diameter D is folded to for a cone, the cone formed will have
   (a) diameter and height equal to *D*
   (b) diameter and generator equal to *D*
   (c) diameter and height equal to *D*/2
   (d) diameter and generator equal to *D*/2

Answers

(i) d (ii) b (iii) a (iv) b (v) b (vi) c (vii) b (viii) d (ix) c (x) d

12

Isometric Projections

12.1 INTRODUCTION

Isometric projection is a type of single-view projection in which a pictorial view is obtained by keeping the object in such a way that all the three mutually perpendicular geometrical axes are equally inclined to the plane of projection. The projectors follow the rules of multi-view projections, i.e. projectors are parallel to each other and perpendicular to the plane of projection.

In multi-view orthographic projections, each view provides information of two axes (length and breadth, length and height, or breadth and height). For a complete understanding, there is always a need of more than one view of the object. These views can only be correctly interpreted and visualized by those persons who have a good knowledge of principles used for these projections. Whereas in isometric projection, a single view is drawn in such a manner that it gives an overall view of the object at the first sight. Thus, it is necessary to draw a pictorial view of one kind or the other so as to enable a common man to understand.

12.2 PRINCIPLE OF ISOMETRIC PROJECTION

Consider a cube which rests on one of its corners on the H.P. with a solid diagonal perpendicular to the V.P. See Fig. 12.1. The final front view is the isometric projection of the cube.

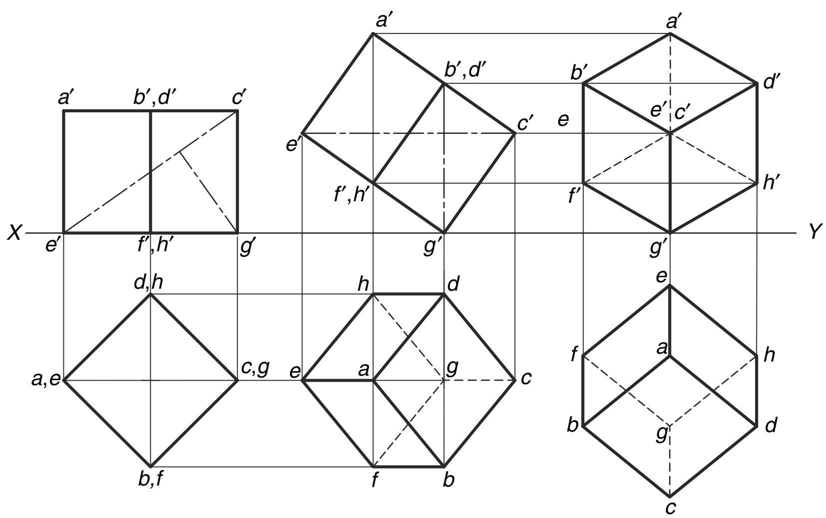


Fig. 12.1

The front view obtained in the final stage of Fig. 12.1 is redrawn for the analysis in Fig. 12.2(a), where hidden lines are removed. The corners are renamed in capital letters. A close observation of this view reveals the following information.

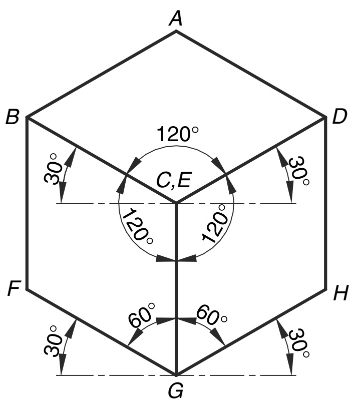


Fig. 12.2(a)

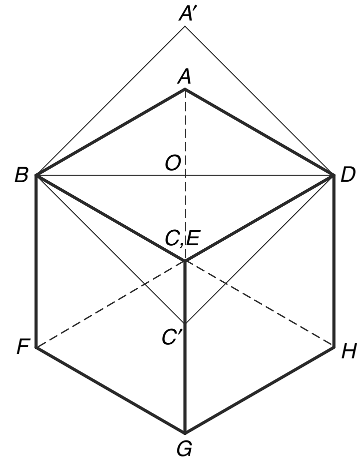


Fig. 12.2(b)

1. The outer boundary *ABFGHDA* is a regular hexagon.
2. All the faces of the cube which are actually square in shape appear as rhombus.
3. The three lines *CB*, *CD* and *CG* meeting at *C*, represent three edges of the cube
   a. They make equal angles of 120º with each other.
   b. They are equal in length but smaller than the true length of the edge of the cube.
   c. The line *CG* is vertical, and the other lines *CB* and *CD* make 30º with the horizontal.
4. All other lines representing the edges of the cube are parallel to one or the other of the above three lines, i.e. *CB*, *CD* and *CG*, and are equally foreshortened.
5. The diagonal *BD* of the top face *ABCD* is parallel to V.P., and hence shows its true length.

A comparison of the rhombus *ABCD* of the front view with the square face of the cube (represented by *A′BC′D* in the figure) is shown in Fig. 12.2(b).

12.3 TERMINOLOGY

1. *Isometric axes*: The three lines *CB*, *CD* and *CG*, meeting at point *C* and inclined at an angle of 120° with each other are called isometric axes.
2. *Isometric lines*: The lines parallel to the isometric axes are called isometric lines. Here lines *AB*, *BF*, *FG*, *GH*, *DH* and *AD* are isometric lines.
3. *Non-isometric lines*: The lines which are not parallel to isometric axes are known as non-isometric lines. For example, diagonals *BD*, *AC*, *CF*, *BG*, etc., are non-isometric lines.
4. *Isometric plane*: A plane representing any face of the cube as well as other plane parallel to it is called an isometric plane. For example, *ABCD*, *BCGF*, *CGHD*, etc., are isometric planes.
5. *Isometric scale*: It is the scale which is used to convert the true length into isometric length. Mathematically, Isometric length = 0.816 × True length

12.4 CONSTRUCTION OF AN ISOMETRIC SCALE

Fig. 12.2(b) shows that all the edges of the cube are equally foreshortened. Therefore, the square faces are seen as rhombuses in the isometric projection. The foreshortening of the edge can be calculated as follows:

In triangle *ABO*, $\dfrac{BA}{BO} = \dfrac{1}{\cos 30^0} = \dfrac{2}{\sqrt{3}}$

In triangle *A′BO*, $\dfrac{BA'}{BO} = \dfrac{1}{\cos 45^0} = \dfrac{\sqrt{2}}{1}$

Therefore, $\dfrac{\text{Isometric length}}{\text{True length}} = \dfrac{BA}{BA'} = \dfrac{2}{\sqrt{3}} \times \dfrac{\sqrt{2}}{1} = \dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{9}{11}$ (approx) or 0.816 (approx)

This reduction of the true length can be obtained either by multiplying it by a factor 0.816 or by taking the measurement with the help of an isometric scale. Fig. 12.3(a) shows the isometric scale. The steps of construction are as follows:

1. Draw a horizontal line *bo*.
2. Draw lines *ba′* and *ba* inclined at 45º and 30º with line *bo*, respectively.
3. Mark off the true scale on the line *ba′* as 0′, 10′, 20′, 30′, etc.
4. Draw vertical lines from points 0′, 10′, 20′, 30′, etc. to meet line *ba* at points 0, 10, 20, 30, etc. The mark-off divisions of *ba* represents the isometric length.

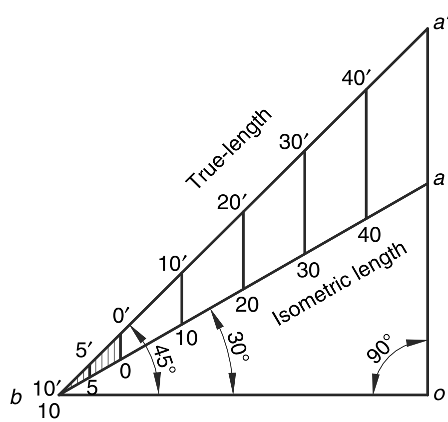


Fig. 12.3(a)

Simplified form of isometric scale

The isometric scale can also be constructed in a simplified way by using the principles of plane or diagonal scales taking $\text{R.F.} = \sqrt{2} : \sqrt{3}$. Figs. 12.3(b) and (c) show plano and diagono-isometric scales for a maximum length of 11 cm. The length of scale can be calculated as $L_s = \dfrac{\sqrt{2}}{\sqrt{3}} \times 11 = 8.98 \text{ cm} \approx 9 \text{ cm}$ (approx). The steps of construction are self explanatory.

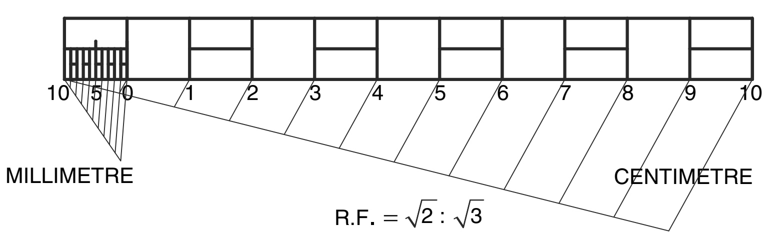


Fig. 12.3(b) *Plano-isometric scale*

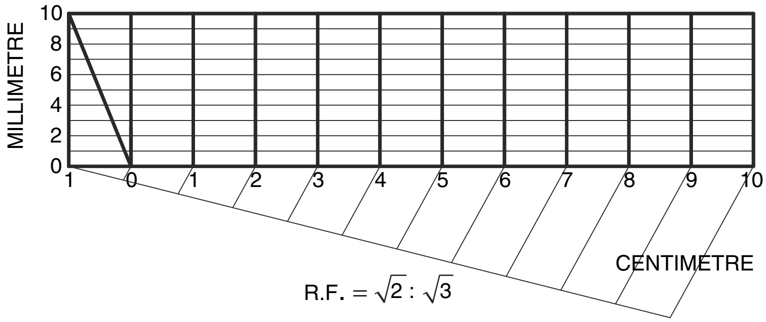


Fig. 12.3(c) *Diagono-isometric scale*

12.5 CHARACTERISTICS OF THE PRINCIPAL LINES IN ISOMETRIC PROJECTION

The following are the characteristics of the principal lines in an isometric projection.

1. All lines that are parallel on the object are parallel on the isometric projection.
2. Vertical line on the object remains vertical in the isometric projection.
3. The horizontal lines on the object are drawn at an angle of 30º with the horizontal.
4. The lines parallel to the principal lines known as isometric lines are equally foreshortened.
5. The lines which are not parallel to principal lines known as non-isometric lines are not equally foreshortened. For example, diagonals *BD* and *AC* are of equal lengths in front view but are of different lengths in the isometric projection. The non-isometric lines are drawn by locating positions of their ends on isometric planes.

12.6 ISOMETRIC PROJECTION AND ISOMETRIC VIEW

In an isometric projection, a scale factor of 0.816 is used to prepare the drawing whereas in an isometric view the true length is used. Thus, the isometric view of an object is larger than the isometric projection. Because of ease of construction and advantage of measuring the dimensions directly from the drawing, it has become a general practice to use the true lengths instead of isometric lengths.

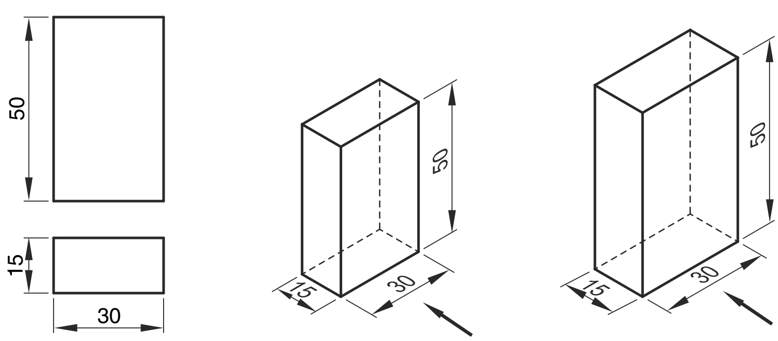


Fig. 12.4 *(a) Orthographic projection (b) Isometric projection (c) Isometric view*

Fig. 12.4(a) shows the orthographic views of a cuboid. Fig. 12.4 (b) shows its isometric projection whereas Fig. 12.4 (c) shows its isometric view. Thus, isometric projection looks smaller in size than the isometric view.

12.7 DIMENSIONING ON ISOMETRIC PROJECTION

1. While dimensioning isometric projection or isometric view, the true lengths are written as dimension values.
2. As far as possible, all extension lines and dimension lines must be isometric lines, lying in isometric planes.
3. It is usual practice to avoid the hidden lines unless they are essential to make the drawing clear.

12.8 FOUR-CENTRE METHOD TO DRAW ELLIPSE

In an isometric projection, a circle on an isometric plane appears as an ellipse as shown in Fig. 12.5(a)-(c). The ellipse can be drawn with the help of a compass by the four-centre method. Steps of construction are described below.

1. Draw rhombus *ABCD* with sides equal to the diameter of the circle to represent an isometric plane.
2. Mark 1, 2, 3 and 4 as the mid-points of the sides *AB*, *BC*, *CD* and *DA* respectively.
3. Join the ends of the minor diagonal *B* to meet mid-points 3 and 4 and *D* to meet mid-points 1 and 2. Let lines *B*4 and *D*1 meet at point *E* whereas lines *B*3 and *D*2 meet at point *F*. Then *B*, *E*, *D* and *F* are the four-centres for construction of the ellipse.

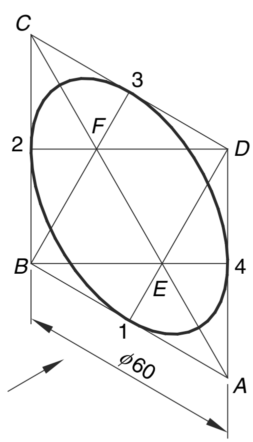


Fig. 12.5(a)

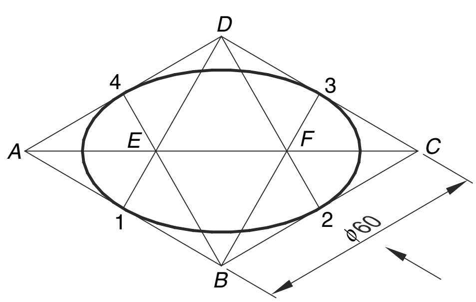


Fig. 12.5(b)

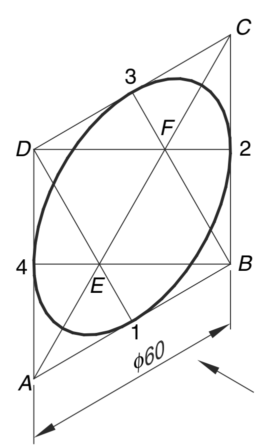


Fig. 12.5(c)

4. With centre *B* and radius *B*3 draw arc 3-4. With centre *D* and radius *D*1, draw arc 1-2. With centre *E* and radius *E*1 draw arc 1-4. With centre *F* and radius *F*2 draw arc 2-3. The ellipse thus constructed represents the circle of diameter *AB* in the isometric view.

12.9 ISOMETRIC VIEW OF RIGHT SOLIDS

For drawing the isometric view of a square prism, rectangular prism, etc., the edges are drawn parallel to the three isometric axes. The following points should be kept in mind.

1. The isometric view should be drawn such that maximum possible details are visible.
2. For every outer corner of the solid, at least three lines for the edges must converge. Of these, at least two must be for the visible edges.
3. It is usual practice to avoid the hidden lines unless they are essential to make the drawing clear. However, it is advisable to check every corner so that no line for a visible edge is left out.
4. Two lines showing visible edge will never intersect each other.

Example 12.1 (Fig 12.6)

Draw an isometric view of a square prism of base 40 mm side and axis 60 mm long resting on the H.P. (a) on its base with axis perpendicular to the H.P., (b) on its rectangular face with axis perpendicular to the V.P., and (c) on its rectangular face with axis parallel to the V.P.

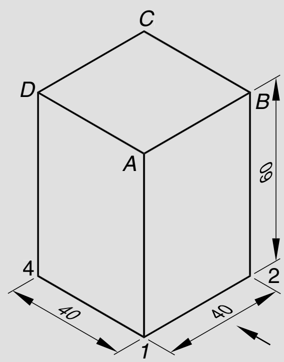


Fig. 12.6(a)

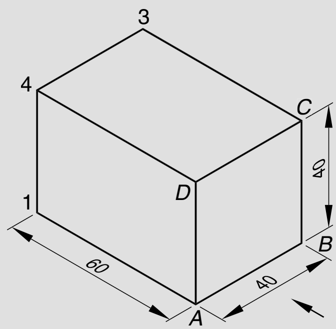


Fig. 12.6(b)

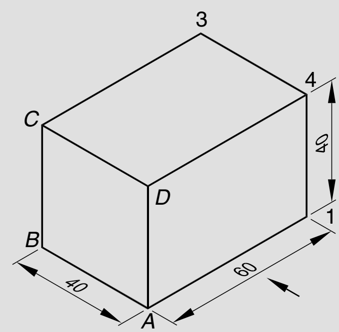


Fig. 12.6(c)

Construction:

Case a [Fig. 12.6(a)]: Draw rhombus *ABCD* such that *AB* and *AD* are inclined at 30º to the horizontal. Draw 60 mm long vertical lines *A*1, *B*2 and *D*4. Join 412.

Cases b and c [Fig. 12.6(b and c)]: Draw rhombus *ABCD* such that *AB* is inclined at 30º to the horizontal and *AD* is vertical. Draw 60 mm long lines *A*1, *C*3 and *D*4 inclined at 30º to the horizontal. Join 143. If the object is viewed along the face *ABCD* the axis would be perpendicular to the V.P. and if the object is viewed along the rectangular face *AD*41 the axis would be parallel to the V.P.

12.10 ISOMETRIC VIEW OF OBJECTS CONTAINING NON-ISOMETRIC LINES

The inclined lines of an object are represented by non-isometric lines. These are drawn by one of the following methods.

1. **Box method:** In the box method, the object is assumed to be enclosed in a rectangular box and both the isometric and non-isometric lines are drawn by locating the corresponding points of contact with the surfaces and edge of the box.
2. **Offset method:** In the offset method, the lines parallel to the isometric axes are drawn from every corner or the reference point of an end to obtain the corner or the reference point at the other end.

Example 12.2 (Fig 12.7)

Draw isometric view of a hexagonal prism with side of base 25 and 60 mm long axis. The prism is resting on its base on the H.P. with an edge of the base parallel to the V.P. [RGPV Dec. 2010]

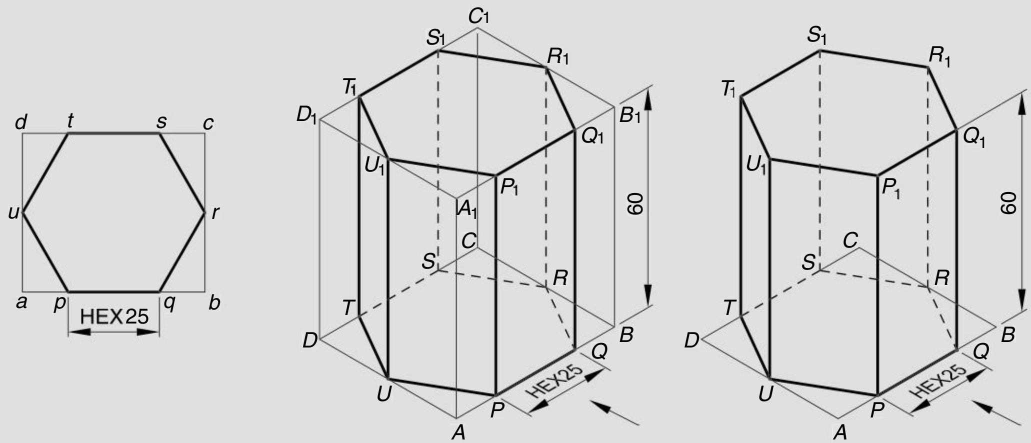


Fig. 12.7(a) Fig. 12.7(b) Fig. 12.7(c)

Construction:

1. Draw a hexagon *pqrstu* with 25 mm side to represent the top view of the prism. Enclose the hexagon into a rectangle *abcd* as shown in Fig 12.7(a).

Box method: Fig. 12.7(b)

2. Draw a parallelogram *ABCD* of sides *ab* and *ad* keeping *AB* and *AD* at 30º to the horizontal.
3. Draw 60 mm long vertical lines AA_1, BB_1, CC_1 and DD_1. Join $A_1B_1C_1D_1$.
4. Mark points *P*, *Q*, *R*, *S*, *T* and *U* in the isometric view such that $AP = ap$, $AQ = aq$, $BR = br$, $DT = dt$, $DS = ds$ and $DU = du$. Also Mark points P_1, Q_1, R_1, S_1, T_1 & U_1 such that $A_1P_1 = ap$, $A_1Q_1 = aq$, $B_1R_1 = br$, $D_1T_1 = dt$, $D_1S_1 = ds$ and $D_1U_1 = du$.
5. Join all the corners as shown to get the required isometric view.

Offset method: Fig. 12.7(c)

2. Draw a parallelogram *ABCD* of sides *ab* and *ad* keeping *AB* and *AD* at 30º to the horizontal.
3. Mark points *P*, *Q*, *R*, *S*, *T* and *U* in the isometric view such that $AP = ap$, $AQ = aq$, $BR = br$, $DT = dt$, $DS = ds$ and $DU = du$.
4. Draw 60 mm long vertical lines PP_1, QQ_1, RR_1, SS_1, TT_1 and UU_1. Join $P_1Q_1R_1S_1T_1U_1$ to get the required isometric view.

Example 12.3 (Fig 12.8)

Draw an isometric projection of a pentagonal prism of 40 mm base side and 70 mm long axis resting on its base on the H.P. with an edge of the base parallel to the V.P.

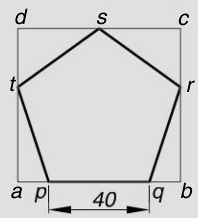


Fig. 12.8(a)

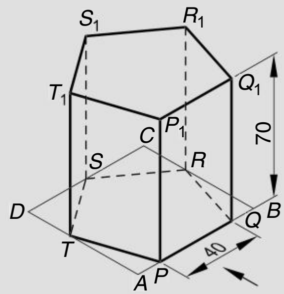


Fig. 12.8(b)

Construction: Fig. 12.8(b)

1. Use isometric scale for all measurements because we have to draw isometric projection.
2. Draw a pentagon *pqrst* taking 40 mm side on isometric scale. Enclose it in a rectangle *abcd* as shown in Fig 12.8(a).
3. Draw a parallelogram *ABCD* of side *ab* and *ad* keeping *AB* and *AD* at 30º to the horizontal.
4. Mark points *P*, *Q*, *R*, *S* and *T* in the isometric view such that *AP* = *ap*, *AT* = *at*, *BQ* = *bq*, *BR* = *br* and *DS* = *ds*.
5. Draw (70 × 0.816) mm long vertical lines PP_1, QQ_1, RR_1, SS_1 and TT_1. Join $P_1Q_1R_1S_1T_1$ to obtain the required isometric view. Dotted lines may be drawn for clarity.
6. Dimension the figure indicating true lengths.

Note: In case of isometric projection, although the measurements are taken on isometric scale, while dimensioning, only the original lengths are marked.

Example 12.4 (Fig 12.9)

Draw isometric view of a cylinder of 50 mm base diameter and 70 mm long axis when the axis is perpendicular to the (a) H.P., (b) V.P.

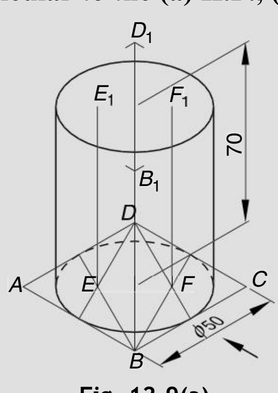


Fig. 12.9(a)

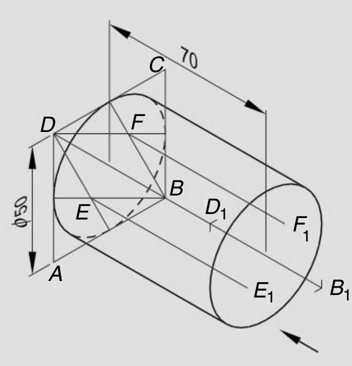


Fig. 12.9(b)

Construction: Offset method Fig. 12.9(a) and Fig. 12.9(b)

1. Draw a rhombus *ABCD* with 50 mm long sides. In case (a), *AB* and *BC* are inclined at 30º to the horizontal while in case (b), *AB* is inclined at 30º to the horizontal and *AD* is vertical.
2. Inscribe ellipse in *ABCD* using four-centre method, as explained in Article 12.8.
3. Draw 70 mm long lines BB_1, EE_1, DD_1 and FF_1 along the third isometric axis. Draw ellipse with the help of the centre-points B_1, E_1, D_1 and F_1, using four-centre method.
4. Draw common tangents to the above two ellipse.
5. Erase the inner half of the ellipse [lower half in case (a), rear half in case (b)] to obtain the required isometric view.

Example 12.5 (Fig 12.10)

Draw isometric view of a pentagonal pyramid of 30 mm base side and 50 mm long axis (a) when its axis is vertical, and (b) when its axis is horizontal.

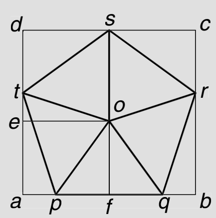


Fig. 12.10(a)

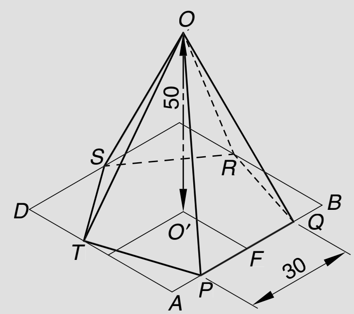


Fig. 12.10(b)

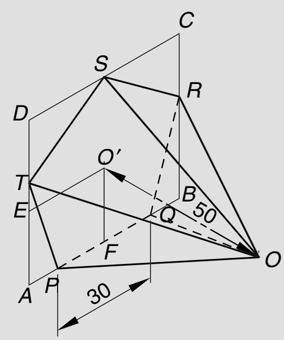


Fig. 12.10(c)

Construction: Offset method Fig. 12.10(b) and Fig. 12.10(c)

1. Draw a pentagon *pqrst* to represent the top view of the prism, as shown in Fig. 12.10(a). Enclose this pentagon into a rectangle *abcd*.
2. Draw a parallelogram *ABCD* of side lengths *ab* and *ad*. In case (a) *AB* and *AD* are inclined at 30º to the horizontal while in case (b) *AB* is inclined at 30º to the horizontal and *AD* is vertical.
3. Mark points *P*, *Q*, *R*, *S* and *T* on the edges of the rhombus such that *AP* = *ap*, *BQ* = *bq*, *AT* = *at*, *BR* = *br* and *DS* = *ds*.
4. Mark point *O′* such that its distance from *AB* = *of* and from *AD* = *eo*. Draw a 50 mm long line *OO′* along the isometric axis. [In case (a) *OO′* is vertical and in case (b), *OO′* is inclined at 30º to the horizontal].
5. Join points *P*, *Q*, *R*, *S* and *T* with apex *O* to obtain the required isometric view.

Example 12.6 (Fig 12.11)

Draw isometric projection of a cone of 50 mm base diameter and 70 mm long axis when the base is (a) on the H.P. (b) in the V.P.

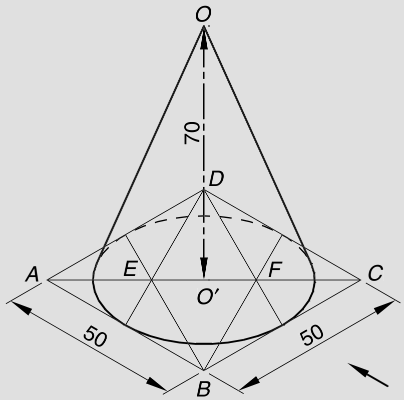


Fig. 12.11(a)

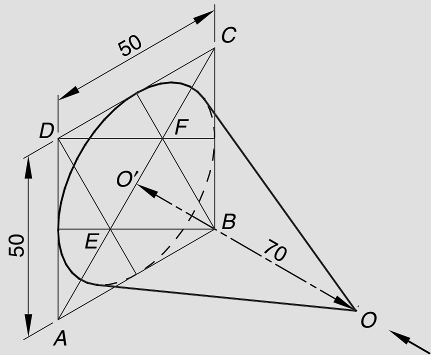


Fig. 12.11(b)

Construction: Offset method Fig. 12.11(a) and (b)

1. Draw a (50 × 0.816) mm side rhombus *ABCD*. In case (a), *AB* and *BC* are inclined at 30º to the horizontal while in case (b), *AB* is inclined at 30º to the horizontal and *AD* is vertical.
2. Inscribe ellipse in the rhombus *ABCD* using four-centre method, as explained in Article 12.8.
3. Mark point *O*′ as the mid-point of the rhombus *ABCD*. Draw a (70 × 0.816) mm long line *OO*′ along the isometric axis. [In case (a) *OO*′ is vertical and in case (b) *OO*′ is inclined at 30º to the horizontal].
4. Draw two tangents from point *O* to the ellipse.
5. Erase the inner half of the ellipse and obtain the required isometric view.

12.11 ISOMETRIC VIEW OF TRUNCATED SOLIDS

Example 12.7 (Fig 12.12)

Draw an isometric projection of the frustum of a hexagonal pyramid having 40 mm base side, 25 mm long top side and 60 mm height.

[RGPV June 2006]

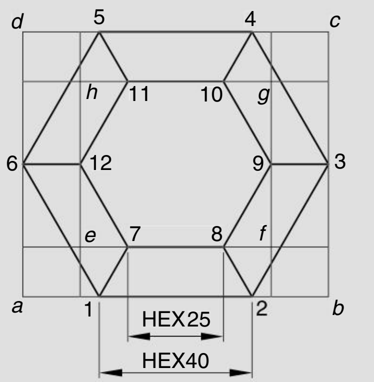


Fig. 12.12(a) *Orthographic view*

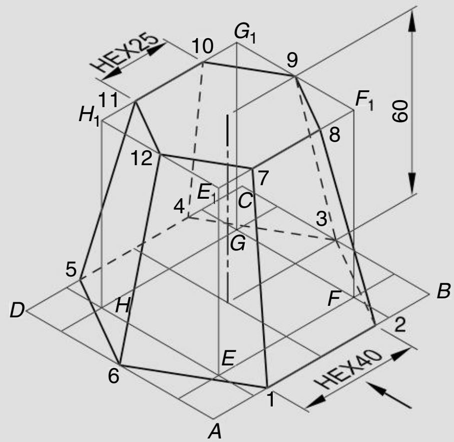


Fig. 12.12(b) *Isometric view*

Construction: Fig. 12.12(b)

1. Draw two concentric hexagons 1-2-3-4-5-6 and 7-8-9-10-11-12 to represent the top view of the frustum of the hexagonal pyramid. Enclose hexagons into rectangles *abcd* and *efgh* as shown in Fig. 12.12(a).
2. On a scale factor of 0.816, draw two concentric parallelograms *ABCD* and *EFGH* with their sides inclined at 30º to the horizontal.
3. Mark points 1, 2, 3, 4, 5 and 6 on the edges of the rhombus *ABCD* such that $A1 = a1$, $B2 = b2$, $B3 = b3$, $C4 = c4$, $D5 = d5$ and $A6 = a6$. Join 1-2-3-4-5-6 to represent the hexagon of the lower base.
4. Draw (60×0.816) mm long vertical lines EE_1, FF_1, GG_1 and HH_1. Join $E_1F_1G_1H_1$ to represent the rhombus for the upper face of the frustum.
5. Mark points 7, 8, 9, 10, 11 and 12 on the edges of the rhombus $E_1F_1G_1H_1$ such that $E_17 = e7$, $F_18 = f8$, $F_19 = f9$, $G_110 = g10$, $H_111 = h11$ and $E_112 = e12$. Join 7-8-9-10-11-12 to represent the hexagon for the upper base.
6. Join 1-7, 2-8, 3-9, 4-10, 5-11, 6-12 to represent the slant edges. Show visible edges of the frustum with dark lines and dimension the figure as shown.

Example 12.8 (Fig 12.13)

Draw the isometric projection of the frustum of a cone of 50 mm base diameter, 25 mm top diameter and 60 mm height.

[RGPV June 2006]

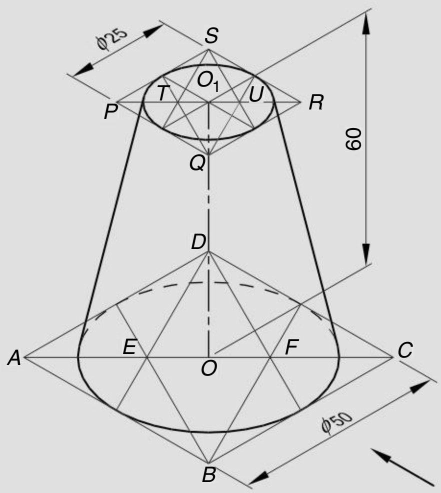


Fig. 12.13

Construction: Offset method Fig. 12.13

1. Draw a (50 × 0.816) mm side rhombus *ABCD*. Locate centre *O*.
2. Inscribe an ellipse in the rhombus *ABCD* using four-centre method, refer Article 12.8.
3. Locate centre O_1 at a height of (60 × 0.816) mm from the centre *O*. Describe another rhombus *EFGH* about centre O_1, having a (25 × 0.816) mm side.
4. Inscribe an ellipse in the rhombus *ABCD* using four centre method, refer Article 12.8.
5. Darken the visible edges to obtain the required isometric view.

Example 12.9 (Fig 12.14)

Draw isometric view of a sphere of 60 mm diameter truncated by a horizontal plane at a distance of 20 mm from the centre.

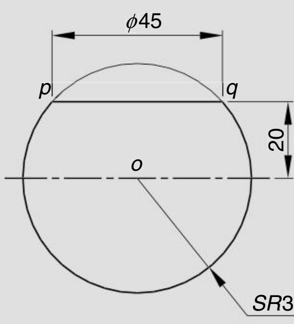


Fig. 12.14(a) *Orthographic view*

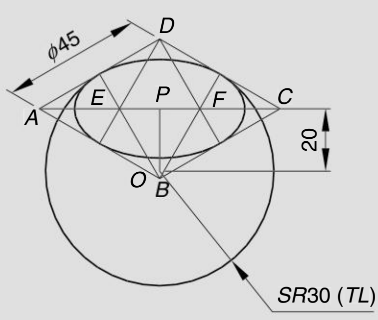


Fig. 12.14(b) *Isometric view*

Construction: Fig. 12.14(b)

1. Draw a 60 mm diameter circle. Also, draw a horizontal line *pq* such that it passes through a point 20 mm above the centre, shown in Fig. 12.14(a). The figure represents the front view of the solid. Measure the length *pq* as 45 mm, the diameter of circle at the cut surface.
2. Draw a circle of 30 mm true radius with *O* as the centre, to represent the isometric projection of the whole sphere.
3. Mark a point *P* at a height of (20×0.816) mm from centre *O*, to represent centre of the cut surface. About point *P*, describe a rhombus *ABCD* having $(pq \times 0.816)$ mm side.
4. Inscribe ellipse in the rhombus *ABCD* using four centre method, refer Article 12.8. The ellipse touches the sphere tangentially at two points.
5. Erase the part of the sphere lying above the tangent point to obtain the required isometric view or projection.

Note: Isometric view and projection of a sphere are drawn on the isometric scale, using scale factor of 0.816.

12.12 ISOMETRIC VIEW OF COMPOSITE SOLIDS

Example 12.10 (Fig 12.15)

A cube of 25 mm edge is placed centrally on the top of another square block of 40 mm edge and 15 mm thickness. Draw the isometric drawing of two solids. [RGPV June 2009]

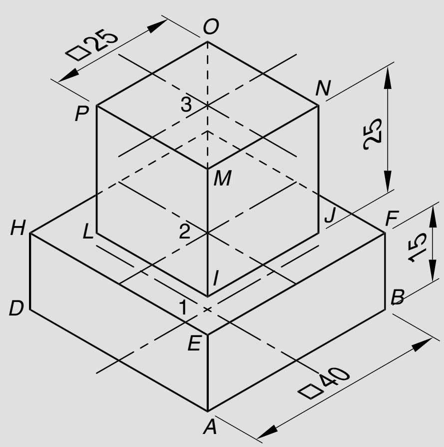


Fig. 12.15 *Isometric view*

Construction: Fig. 12.15

1. Draw a centre line 1-2-3 such that distance between points 1-2 = 15 mm and 2-3 = 25 mm.

2. Draw a rhombus *ABCD* with 40 mm side, keeping point 1 as its centre.
3. Draw another rhombus *EFGH* with 40 mm side, keeping point 2 as its centre.
4. Join *AE*, *BF*, *CG* and *DH* to represent the vertical edges of the square block.
5. Draw rhombuses *IJKL* and *MNOP* with 25 mm side each, keeping point 2 and poin 3 as their centres respectively. Join *IM*, *JN*, *KO* and *LP* to represent the vertical edges of the cube.
6. Darken the visible portion of the solid and dimension the figure.

Example 12.11 (Fig 12.16)

A square pyramid rests centrally over a cylindrical block. Draw the isometric projection of the arrangement. Consider the pyramid has a base with 25 mm side and 40 mm long axis whereas the cylindrical block has a base with 50 mm diameter and 20 mm thickness.

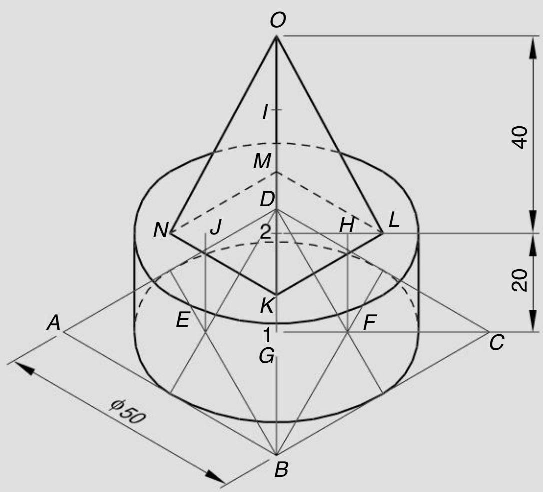


Fig. 12.16 *Isometric view*

Construction: Fig. 12.16

1. Draw a line 1-2-*O* such that distance between points 1-2 = (20 × 0.816) mm and 2-*O* = (40 × 0.816) mm.
2. Draw a (50 × 0.816) mm side rhombus *ABCD*, keeping point 1 as its centre. Determine points *E* and *F* and inscribe an ellipse inside rhombus *ABCD* using four-centre method, refer Article 12.8.
3. Draw (20 × 0.816) mm long vertical lines *BG*, *FH*, *DI* and *EJ* to transfer the centre points for the upper face of the cylinder. Draw another ellipse using four centre method where *G*, *H*, *I* and *J* are the centre-points. [Alternatively, draw a (50 × 0.816) mm side rhombus keeping point 2 as its centre. Inscribe an ellipse inside this rhombus using four centre method.]
4. Draw rhombus *KLMN* with 25 mm side keeping point 2 as its centre to represent the base of the pyramid. Join *OK*, *OL*, *OM* and *ON* to represent the slant edges of the pyramid.
5. Darken the visible portion of the solid and dimension the figure.

Example 12.12 (Fig 12.17)

Orthographic projections of a square funnel made of very thin sheet are shown in Fig. 12.17(a). Draw the isometric projection of the funnel. [RGPV Feb. 2008]

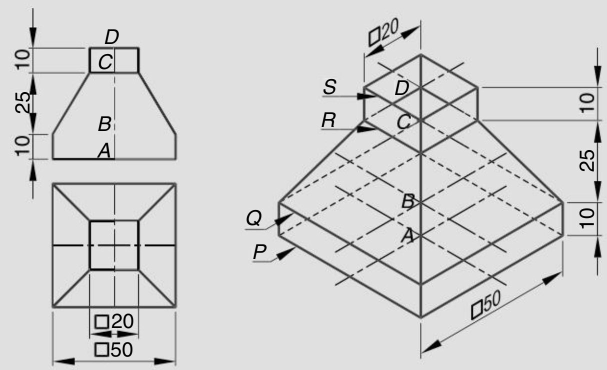


Fig. 12.17(a) *Orthographic view* **Fig. 12.17(b)** *isometric projection*

Construction: Fig. 12.17(b)

1. Draw a centre line *A-B-C-D* such that *AB* = (10 × 0.816) mm, *BC* = (25 × 0.816) mm and *CD* = (10 × 0.816) mm.
2. Draw a (50 × 0.816) mm side rhombus *P*, keeping point *A* as its centre.
3. Draw another (50 × 0.816) mm side rhombus *Q*, keeping point *B* as its centre. Join the corners of rhombus *P* with the corresponding corners of rhombus *Q* to represent the vertical edges.
4. Draw a (20 × 0.816) mm side rhombus *R*, keeping point *C* as its centre. Join the corners of rhombus *Q* with the corresponding corners of rhombus *R* to represent the slant edges.
5. Draw another (20 × 0.816) mm side rhombus *S*, keeping point *D* as its centre. Join the corners of rhombus *R* with the corresponding corners of rhombus *S* to represent the vertical edges.
6. Darken the visible portion of the solid and dimension the figure.

Example 12.13 (Fig 12.18)

Draw the isometric projection of a sphere of 25 mm radius which rests centrally on the top of a square prism of base edge 60 mm and height 30 mm. [RGPV Dec. 2004, Feb. 2005, June 2008]

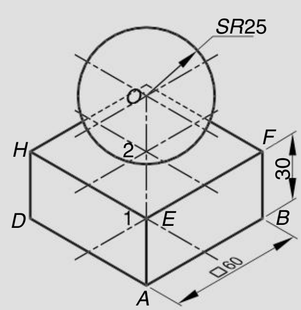


Fig. 12.18 *Isometric projection*

Construction: Fig. 12.18

1. Draw a centre line 1-2-*O* such that distance between points 1-2 = (30×0.816) mm and 2-*O* = (25 × 0.816) mm.
2. Draw a rhombus *ABCD* of (30 × 0.816) mm keeping point 1 as its centre.
3. Draw another rhombus *EFGH* of (30 × 0.816) mm keeping point 2 as its centre.
4. Join *AE*, *BF*, *CG* and *DH* to represent the vertical edges of the square prism.
5. Draw a circle with *O* as the centre and radius 25 mm to represent the sphere.
6. Darken the visible portion of the solid and dimension the figure.

Example 12.14 (Fig 12.19)

A sphere of 60 mm diameter is placed centrally on the top of a frustum of square pyramid. The base of the frustum is 60 mm square, the top is 40 mm square and its height is 50 mm. Draw the isometric projection of the arrangement.

[RGPV Dec. 2007]

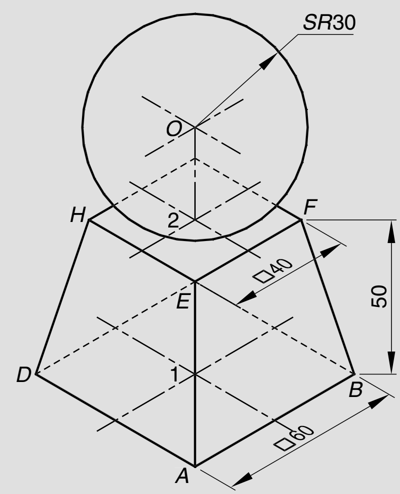


Fig. 12.19 *Isometric projection*

Construction: Fig. 12.19

1. Draw a centre line 1-2-*O* such that distance between points 1-2 = (50 × 0.816) mm and 2-*O* = (30 × 0.816) mm.
2. Draw a rhombus *ABCD* of (60 × 0.816) mm side keeping point 1 as its centre.
3. Draw another rhombus *EFGH* of (40 × 0.816) mm side with point 2 as its centre.
4. Join *AE*, *BF*, *CG* and *DH* to represent the slant edges of the frustum of the square pyramid.
5. Draw a circle with *O* as the centre and radius 30 mm to represent the sphere.
6. Darken the visible portion of the solid and dimension the figure.

Example 12.15 (Fig 12.20)

A hemisphere of 50 mm diameter rests centrally with its flat surface at the top, over a frustum of a cone of 80 mm base diameter, 60 mm top diameter and 50 mm height. Draw isometric projection of the arrangement.

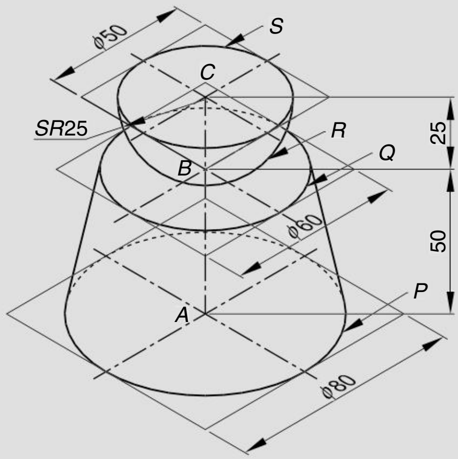


Fig. 12.20

Construction: Fig. 12.20

1. Draw a centre line *ABC* such that $AB = 50 \times 0.816$ mm and $BC = 25 \times 0.816$ mm.
2. Draw a rhombus of (80×0.816) mm side keeping point *A* as its centre. Inscribe an ellipse *P* inside this rhombus using four-centre method.
3. Draw another rhombus of (60×0.816) mm side keeping point *B* as its centre. Inscribe another ellipse *Q* inside this rhombus using four-centre method.
4. Connect the ellipses drawn in step 2 and 3 by tangent lines.
5. Draw a semicircle *R* with *B* as the centre and 25 mm radius to represent the sphere.
6. Draw another rhombus of (25×0.816) mm side keeping point *C* as its centre. Inscribe an ellipse *S* inside the rhombus using four-centre method.
7. Darken the visible portion of the solid and dimension the figure.

Example 12.16 (Fig 12.21)

The front view of an object is shown in Fig. 12.21(a). Draw the isometric projection of the object.

[RGPV Dec. 2002]

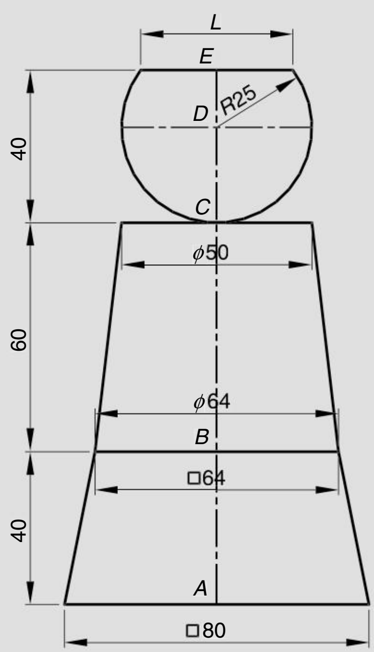


Fig. 12.21(a) *Front view*

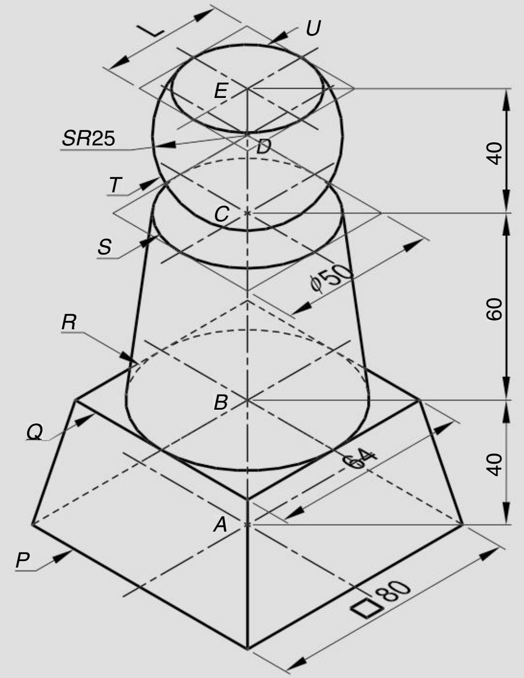


Fig. 12.21(b) *Isometric projection*

Construction: Fig. 12.21(b)

1. Draw the front view of the sphere and determine length *L* for the top circle.
2. Draw a centre line *ABCDE* such that $AB = 40 \times 0.816$ mm; $BC = 60 \times 0.816$ mm; $CD = 25 \times 0.816$ mm and $CD = 40 \times 0.816$ mm.
3. Draw a rhombus *P* of (80×0.816) mm side keeping point *A* as its centre.
4. Draw another rhombus *Q* of (64×0.816) mm side keeping point *B* as its centre. Also inscribe an ellipse *R* inside the rhombus *Q* using four-centre method.
5. Draw lines connecting the corners of the rhombuses *P* and *Q* to represent the slant edges of the frustum of the pyramid.
6. Draw another rhombus of (50×0.816) mm side keeping point *C* as its centre. Inscribe an ellipse *S* inside the rhombus using four-centre method.
7. Connect the ellipses *R* and *S* by tangent lines.
8. Draw a circle *T* with *D* as the centre and 25 mm radius to represent the sphere.
9. Draw another rhombus of (40×0.816) mm side keeping point *E* as its centre. Inscribe an ellipse *U* inside the rhombus using four-centre method.
10. Darken the visible portion of the solid and dimension the figure.

Example 12.17 (Fig 12.22)

Draw the isomeric projection of a hexagonal prism with a hemispherical top touching all the sides. The sides of the hexagonal prism are 60 mm and height 100 mm.

[RGPV Dec. 2001]

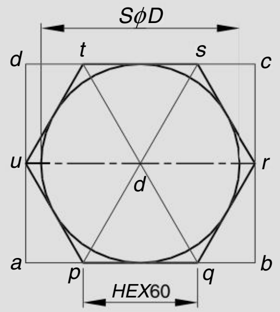


Fig. 12.22(a) *Top view*

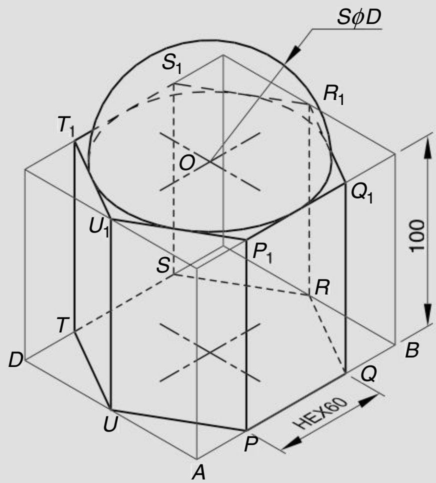


Fig. 12.22(b) *Isometric projection*

Construction: Fig. 12.22(b)

1. Draw a 60 mm side hexagon *pqrstu* and inscribe a circle inside it. This represents the top view of the solid.
2. Enclose the hexagon in a rectangle *abcd* as shown in Fig. 12.22(a). Also measure the diameter of the circle *SØD* as 104 mm to represent the diameter of the sphere.
3. Draw a parallelogram *ABCD* of ($ab \times 0.816$) mm and ($ab \times 0.816$) mm sides.
4. Locate points *P*, *Q*, *R*, *S*, *T* and *U* on the edges of the parallelogram such that $AP = ap \times 0.816$; $BQ = bq \times 0.816$; $BR = br \times 0.816$, $CS = cs \times 0.816$; $DT = dt \times 0.816$ and $AU = au \times 0.816$.
5. Draw (100×0.816) mm vertical lines PP_1, QQ_1, RR_1, SS_1, TT_1 and UU_1 to represent the vertical edges of the prism. Join $P_1Q_1R_1S_1T_1U_1$ to represent the hexagon for the top surface.
6. Join mid-points of the edges P_1Q_1, Q_1R_1, R_1S_1, S_1T_1 and T_1P_1 to get a smooth ellipse.
7. Locate the centre of the ellipse as point *O* and describe a semicircle of *SØD* as shown.
8. Darken the visible portion of the solid and dimension the figure.

12.13 ISOMETRIC VIEWS OF OBJECTS FROM ORTHOGRAPHIC VIEWS

12.13.1 Objects Extruded in One Direction Only

Plane figures with holes or cut of certain shape, called features, when extruded give a solid object. The orthographic projections of such objects show features on either front, top or side view and the remaining two views show rectangular boxes with straight lines running across. The isometric views of such objects are prepared by drawing the features on one of the isometric plane and thereafter extruding it to a mentioned depth. Consider the following examples.

Example 12.18 (Fig 12.23)

Front and top views of a casting are shown in Fig. 12.23(a). Draw its isometric view.

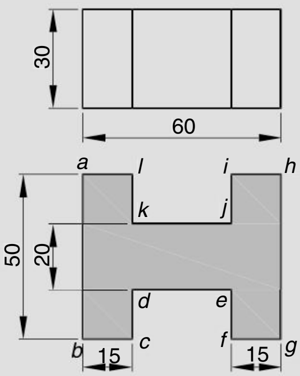


Fig. 12.23(a)

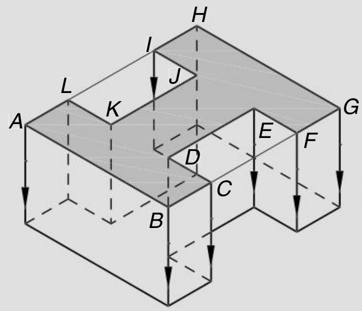


Fig. 12.23(b)

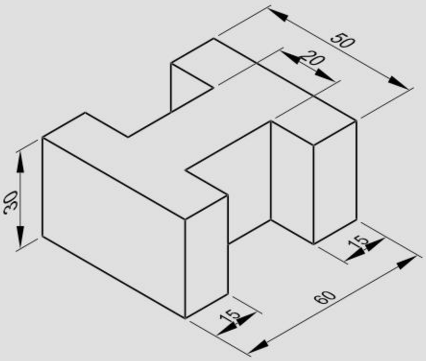


Fig. 12.23(c)

Visualization: The basic feature of the casting is seen in the top view while the front view shows the extruded thickness of 30 mm. As the object is symmetrical, length with 60 mm side can be plotted on either *x* or *y*-direction.

Construction: Fig. 12.23(c)

1. Draw *ABCDEFGHIJKL* on the *x-y* plane to represent the top view, shown by shading in Fig. 12.23(b).
2. Extrude all the corners 30 mm in *z*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the figure to obtain Fig. 12.23(c).

Example 12.19 (Fig 12.24)

Front and top views of a casting are shown in Fig. 12.24(a). Draw its isometric view.

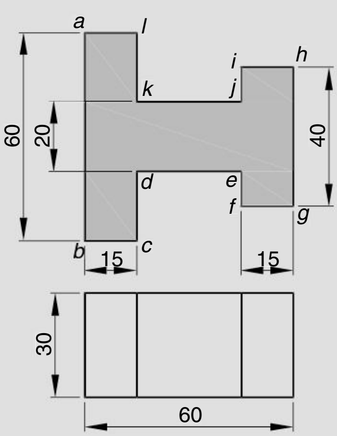


Fig. 12.24(a)

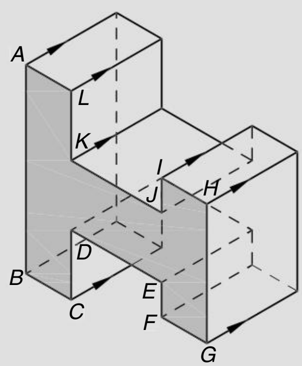


Fig. 12.24(b)

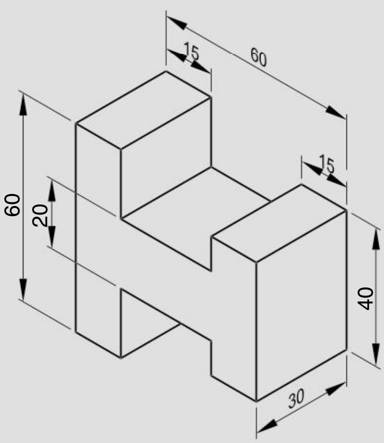


Fig. 12.24(c)

Visualization: The basic feature of the casting is seen in the front view while the top view shows the extruded thickness of 30 mm. To view major portion of the object in the isometric, the length with 60 mm side should be plotted along y-axis.

Construction: Fig. 12.24(c)

1. Draw *ABCDEFGHIJKL* on the *y-z* plane to represent the front view, shown by shade in Fig. 12.24(b).
2. Extrude all the corners 30 mm in *x*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the figure to obtain Fig. 12.24(c).

Example 12.20 (Fig 12.25)

The front and side views of an I-beam are shown in Fig. 12.25(a). Draw the isometric view of the beam.

[RGPV June 2008]

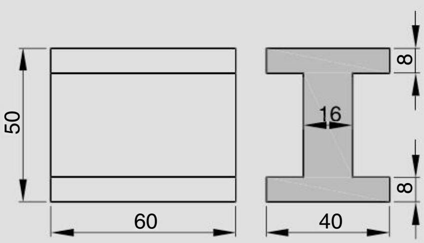


Fig. 12.25(a)

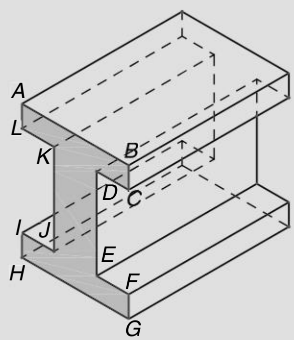


Fig. 12.25(b)

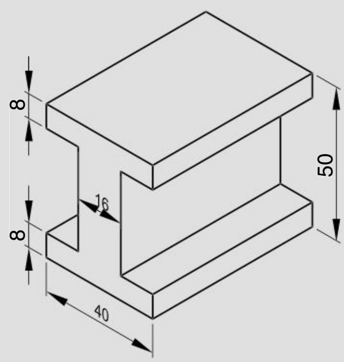


Fig. 12.25(c)

Visualization: The basic feature of the casting is seen in the front view while the right hand side view shows the extruded thickness of 60 mm. To obtain the right hand side view visible in the isometric, the 40 mm side should be plotted along the *y*-axis.

Construction: Fig. 12.25(c)

1. Draw *ABCDEFGHIJL* on the *y-z* plane to represent the front view, shown by shade in Fig. 12.25(b).
2. Extrude all the corners 60 mm in *x*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the figure to obtain Fig. 12.25(c).

Example 12.21 (Fig 12.26)

Front and left-hand side views of a casting are shown in Fig. 12.35(a). Draw its isometric view.

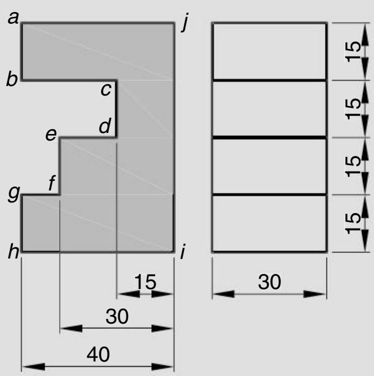


Fig. 12.26(a)

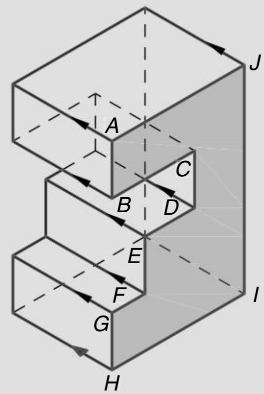


Fig. 12.26(b)

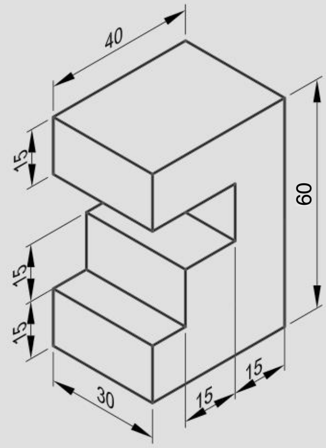


Fig. 12.26(c)

Visualization: The basic feature of the casting is seen in the front view while the left-hand side view shows the extruded thickness of 30 mm. To obtain the left hand side view the length with 40 mm side should be plotted along the *x*-axis.

Construction: Fig. 12.26(c)

1. Draw *ABCDEFGHIJ* on the *x-z* plane to represent the front view, shown by shade in Fig. 12.26(b).
2. Extrude all the corners 30 mm in *y*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the figure to obtain Fig. 12.26(c).

12.13.2 Isometric Projections of Extruded Object with Holes, Slots, Ribs and/or Webs

The objects extruded may have holes, slots, ribs and/or web in the perpendicular direction. The isometric projections of such objects can be drawn by carefully extruding the plane to obtain the basic solid and then placing the web or cutting the holes/slots at the specified location. Consider the following examples.

Example 12.22 (Fig 12.27)

Front and top views of a casting are shown in Fig. 12.27(a). Draw its isometric view.

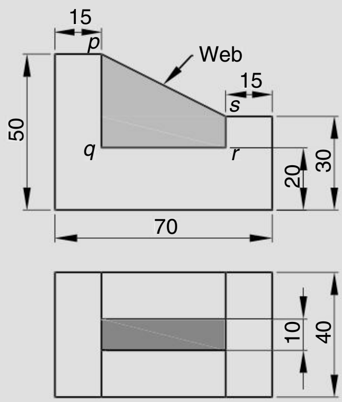


Fig. 12.27(a)

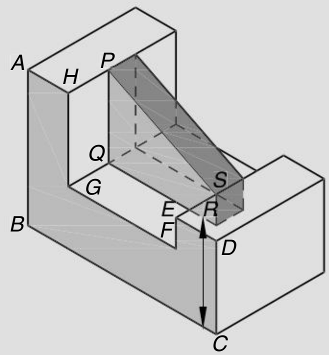


Fig. 12.27(b)

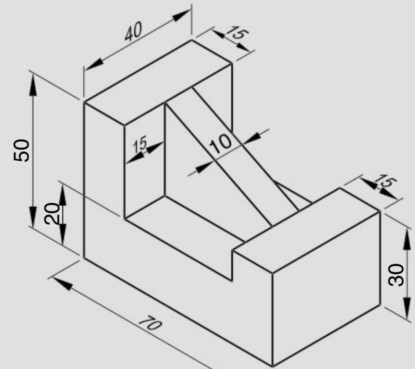


Fig. 12.27(c)

Visualization: The basic feature of the casting is seen in the front view while the top view shows the extruded thickness of 40 mm. The shaded portion in the front and the top view represents the web. To view major portion of the object in the isometric, the length with 70 mm side should be plotted along the *y*-axis.

Construction: Fig. 12.27(c)

1. Draw *ABCDDEFGH* on the isometric *y-z* plane corresponding to the front view excluding web. Extrude all the points by 40 mm along the *x*-direction and join their end points.
2. Mark points *P*, *Q*, *R* and *S* on the extruded surface, 15 mm away towards *x*-direction from points *H*, *G*, *F* and *E* respectively. Join *PQRS*.
3. Extrude points *P*, *Q*, *R* and *S*, 10 mm in *x*-direction and join their end points.
4. Darken the visible edges of the object to obtain Fig. 12.27(c) and dimension it.

Example 12.23 (Fig 12.28)

Front and top views of a casting are shown in Fig. 12.28(a). Draw its isometric view.

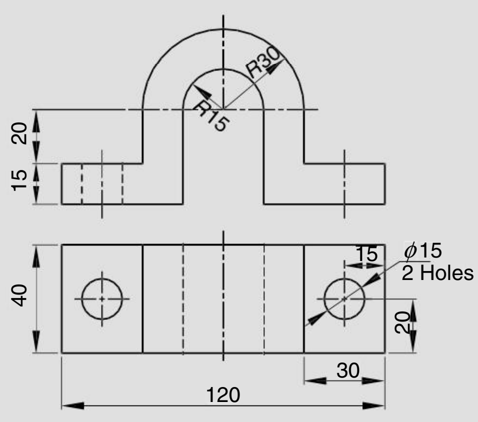


Fig. 12.28(a)

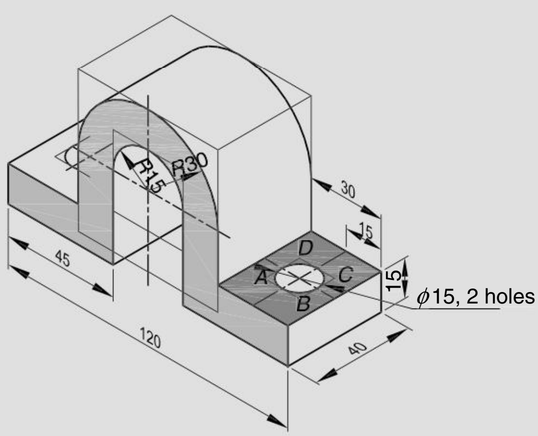


Fig. 12.28(b)

Visualization: The basic feature of the casting is seen in the front view while the top view shows the extruded thickness of 40 mm. The front and the top views also show that there are two holes of 15 mm diameter on the base. As the object is symmetrical, the length of 120 mm can be taken along either *x* or *y*-axis.

Construction: Fig. 12.28(b)

1. Draw the front view without hole on the isometric *y-z* plane. Draw semi-ellipse by four-centre method. Extrude all the points 40 mm towards the *x*-axis and join them.
2. On the extruded surfaces, draw rhombus *ABCD* of side equal to 15 mm side (hole diameter). Inscribe an ellipse in the rhombus using four-centre method. Transfer all the centres 25 mm downward and ensure that the ellipse corresponding to the lower edge of the hole is not visible through the upper ellipse.
3. Precede to the other side of the object and the draw the visible part of the edges of the drilled hole.
4. Darken the visible edges of the object and dimension the figure.

12.13.3 Isometric Projections of Angle Plates

Angle plate is composed of two plates of any arbitrary shape placed at an angle (usually at right angles) to each other. A careful observation may help in separating both the plates. The isometric projections of angle plates can also be drawn by extruding. Consider the following examples.

Example 12.24 (Fig. 12.29)

Front and top views of an angle plate are shown in Fig. 12.29(a). Draw its isometric view.

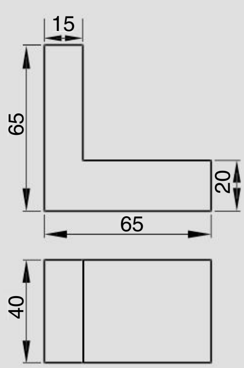


Fig. 12.29(a)

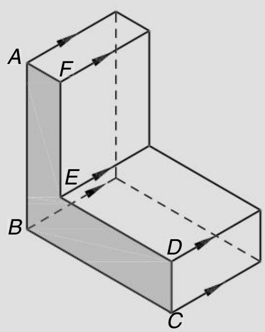


Fig. 12.29(b)

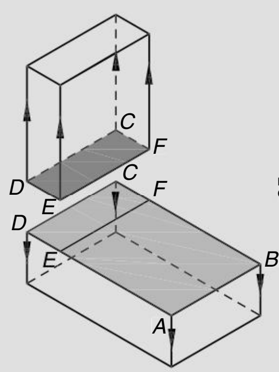


Fig. 12.29(c)

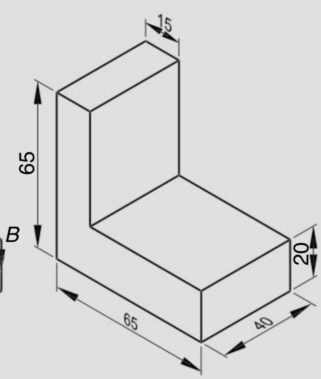


Fig. 12.29(d)

Visualization: The front view shows that angle plates are at right angles to each other and the top view shows that they are of rectangular cross-sectional area. To view major portion of the object in the isometric, the length with 65 mm side should be plotted along the *y*-axis.

Construction: Method 1, Fig. 12.29(d)

1. Draw *ABCDEF* on the *y-z* plane to represent the front view, shown by shade in Fig. 12.29(b).
2. Extrude all the corners, 40 mm in *x*-axis direction and join their end points.
3. Darken the visible edges of the object and dimension the figure to obtain Fig. 12.29(d).

Construction: Method 2, Fig. 12.29(d)

1. Draw the parallelogram *ABCD* on the *x-y* plane. Extrude *ABCD* 20 mm along the *z*-axis in downward direction.
2. On the surface of *ABCD* mark points *E* and *F*, as shown in Fig. 12.29(c). Extrude *CDEF* 20 mm along the *z*-axis in upward direction.
3. Finally darken the visible edges and dimension the figure to obtain Fig. 12.29(d).

Example 12.25 (Fig 12.30)

Front and right-hand side views of an angle plate are shown in Fig. 12.30(a). Draw its isometric view.

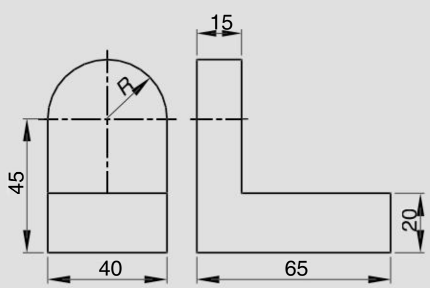


Fig. 12.30(a)

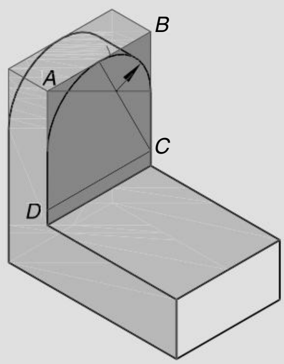


Fig. 12.30(b)

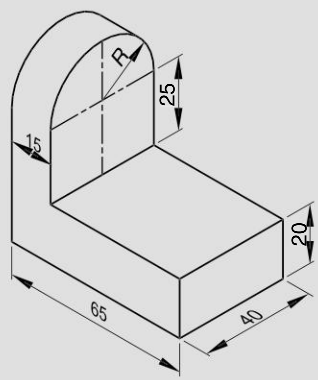


Fig. 12.30(c)

Visualization: The front view shows that angle plates are at right angles to each other and the side view shows that one of the plates has a semicircular shape on the upper end. To view major portion of the object in the isometric, the length with 65 mm side should be plotted along the *y*-axis.

Construction: Method 1, Fig. 12.30(c)

1. Draw the angle plate by any one of the methods explained in Example 12.24, and obtain the figure similar to Fig. 12.29(d).
2. On the vertical surface of the angle plate, draw the rhombus ABCD of 40 mm side as shown in Fig. 12.30(b). Draw a semi-ellipse using four-centre method inside the ellipse to represent the semicircle seen in the side view.

3. Extrude the points of the semi-ellipse 15 mm in the *y*-axis direction and obtain another semi-ellipse.
4. Darken the visible edges of the object and dimension to obtain Fig. 12.30(c).

Example 12.26 (Fig. 12.31)

Front and top views of a casting are shown in Fig. 12.31(a). Draw its isometric view.

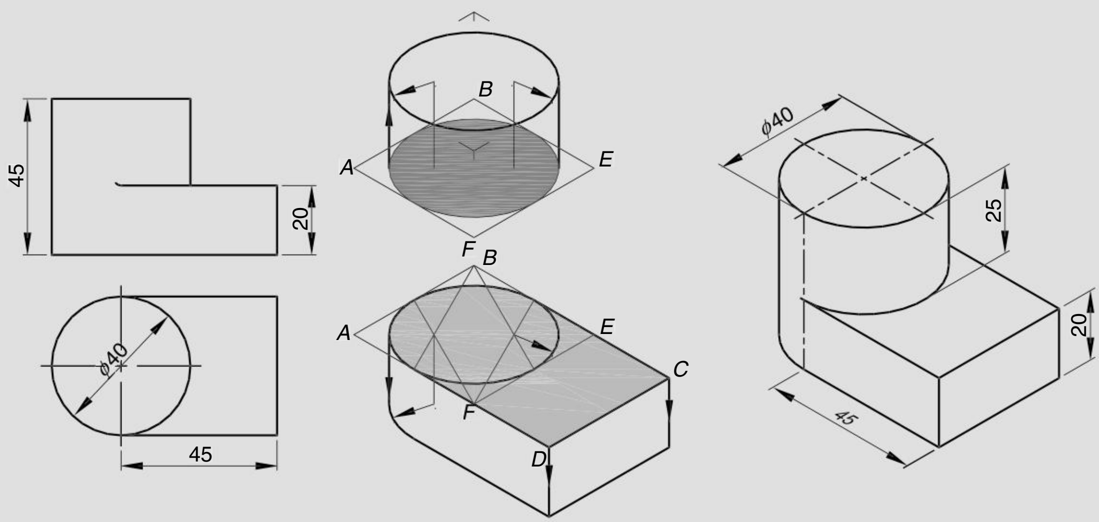


Fig. 12.31(a) **Fig. 12.31(b)** **Fig. 12.31(c)**

Visualization: The front view shows that angle plates are at right angles to each other and the top view shows that one of the plate is of cylindrical cross-sectional area. To view major portion of the object in the isometric, the length should be plotted along the *y*-axis.

Construction: Fig. 12.31(c)

1. Draw the parallelogram *ABCD* on the *x*-*y* plane. Also mark a rhombus *ABEF* and inscribe an ellipse, as shown in Fig. 12.31(b), to represent the cylinder at its end.
2. Extrude all the points 20 mm along the *z*-axis in downward direction and complete the bottom plate.
3. Extrude the points of the ellipse 25 mm along the *z*-axis in upward direction.
4. Join all the ellipses by tangent lines. Darken the visible edges and dimension the figure to obtain Fig. 12.31(c).

12.13.4 Isometric Projections of Angle Plates with Holes, Slots and/or Ribs

The orthographic view of angle plate gets complicated when slots are cut or ribs are extended. The isometric projections of such objects can be drawn by first drawing the angle plate and then drawing slots or ribs in the given position. So there is a need first to observe the basic shape of angle plates and then to observe the holes drilled and/or ribs attached. Consider the following examples.

Example 12.27 (Fig. 12.32)

Front and top views of an angle plate are shown in Fig. 12.41(a). Draw its isometric view.

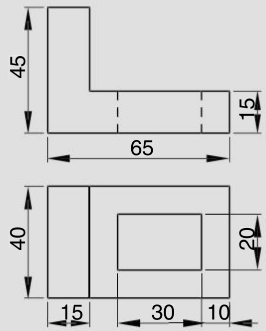


Fig. 12.32(a)

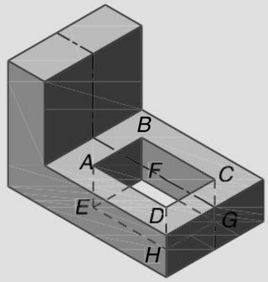


Fig. 12.32(b)

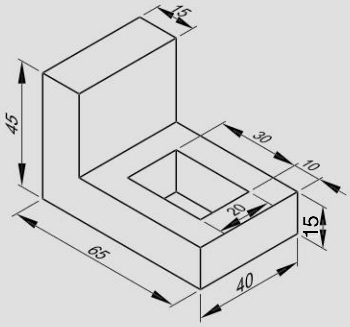


Fig. 12.32(c)

Visualization: The front view shows that the angle plates are at right angles to each other and the top view shows that they are of rectangular cross-sectional area. The top view also shows that a rectangular hole has been created on the base plate. To enable major portion of the object visible, length with 65 mm side should be plotted along the *y*-axis.

Construction: Fig. 12.32(c)

1. Draw the isometric view of the angle plate using one of the methods explained in Example 12.24.
2. Draw a parallelogram *ABCD* of 30 mm × 20 mm on the top surface of the horizontal plate to represent the rectangle of the slot, as shown in Fig. 12.32(b).
3. Extrude the points of the parallelogram *ABCD* 15 mm along the *z*-axis in downward direction to obtain *EFGH*.
4. Darken the portion of the parallelogram *EFGH* falls within parallelogram *ABCD* and dimension to obtain Fig. 12.32(c).

Example 12.28 (Fig. 12.33)

Front and top views of an angle plate are shown in Fig. 12.33(a). Draw its isometric view.

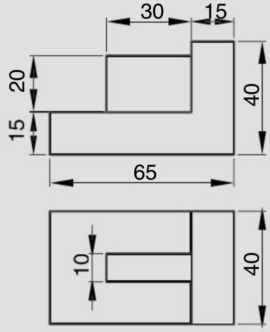


Fig. 12.33(a)

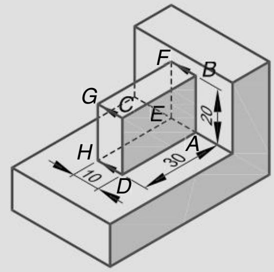


Fig. 12.33(b)

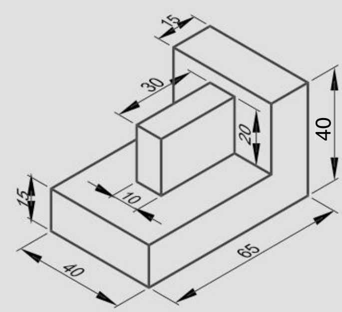


Fig. 12.33(c)

Visualization: The front view shows that the angle plates are at right angles to each other and the top view shows that they are of rectangular cross-sectional area. The views also show that there is a rectangular block of 30 mm × 20 mm × 10 mm placed in contact with both the plates of angle plate. To enable major portion of the object visible, length with 65 mm side should be plotted along the *x*-axis.

Construction: Fig. 12.33(c)

1. Draw the isometric view of the angle plate using a method explained in Example 12.24.
2. Locate a point *A* at a distance of 15 mm from point *O* in the *y*-axis direction. Draw a parallelogram *ABCD* of 30 mm × 10 mm to represent the rectangle, as shown in Fig. 12.33(b). Extrude all the points 10 mm in *y*-axis direction and obtain *EFGH*.
3. Darken the visible edges of the rectangular block and dimension to obtain Fig. 12.33(c).

Example 12.29 (Fig 12.34)

Front and side views of an angle plate are shown in Fig. 12.34(a). Draw its isometric view.

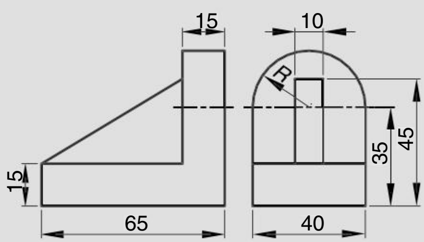


Fig. 12.34(a)

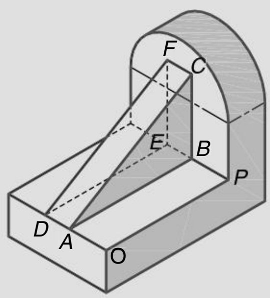


Fig. 12.34(b)

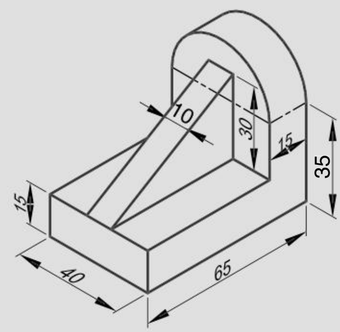


Fig. 12.34(c)

Visualization: The front view shows that the angle plates are at right angles to each other and the left-hand side view shows that they are of rectangular cross-sectional area. The views also show that one of the plates has a semicircular shape on the upper end. In addition to this the view show that there is a rib strengthening the angle plates. To enable major portion of the object visible, there is a need to plot length with 65 mm side on the *x*-axis.

Construction: Fig. 12.34(c)

1. Draw the isometric view of the angle plate using a method explained in Example 12.25.
2. Locate a points *A* and *B* on the top surface of the horizontal plate at a distance of 15 mm from points *O* and *P*.
3. Draw a triangle *ABC* to represent the right angled triangle of the rib, as shown in Fig. 12.34(b). Extrude all the points 10 mm in *y*-axis direction and obtain *DEF*.
4. Darken the visible edges of the rib and dimension to obtain Fig. 12.34(c).

12.14 MISCELLANEOUS EXAMPLES

Example 12.30 (Fig 12.35)

Draw the isometric view of the block shown in Fig. 12.35(a).

[RGPV Apr. 2010]

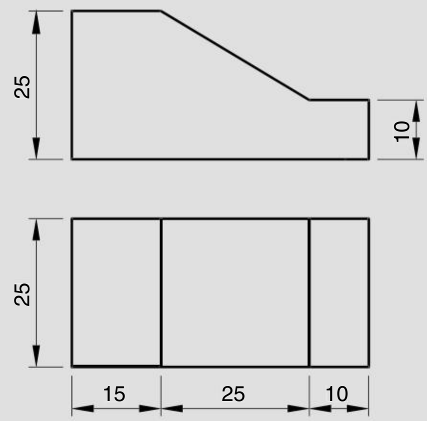


Fig. 12.35(a)

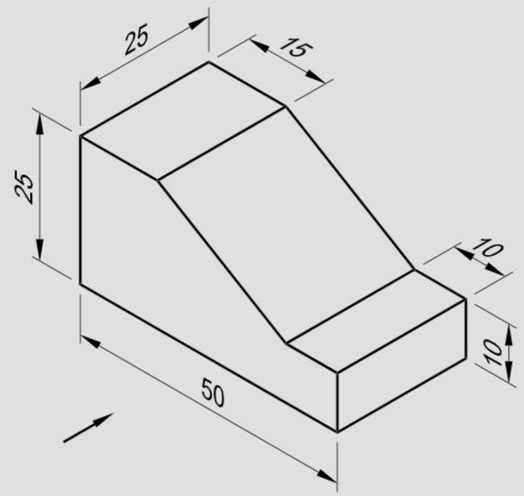


Fig. 12.35(b)

Fig. 12.35(b) shows the required isometric view.

Example 12.31 (Fig 12.36)

Front and top views of a casting are shown in Fig. 12.36(a). Draw its isometric view.

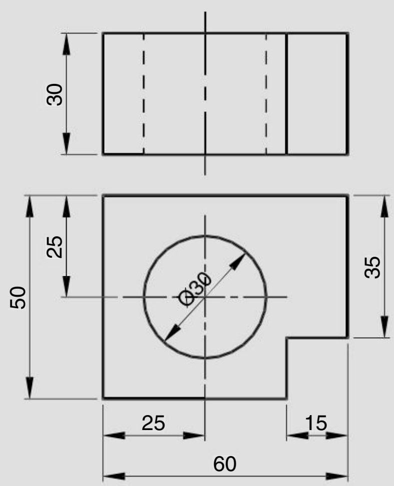


Fig. 12.36(a)

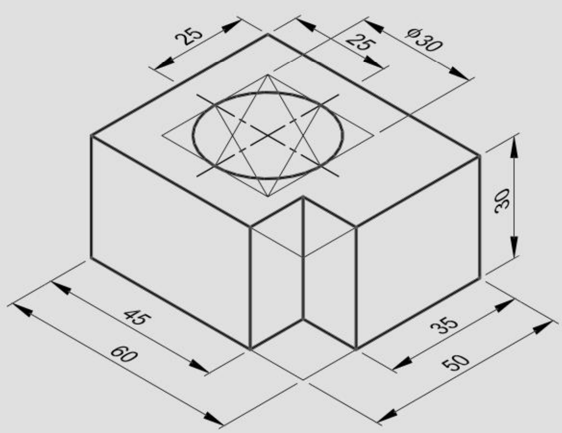


Fig. 12.36(b)

Fig. 12.36(b) shows the required isometric view. Construction lines are left intact for guidance.

Example 12.37 (Fig 12.37)

Front and left-hand side views of a casting are shown in Fig. 12.37(a). Draw its isometric view.

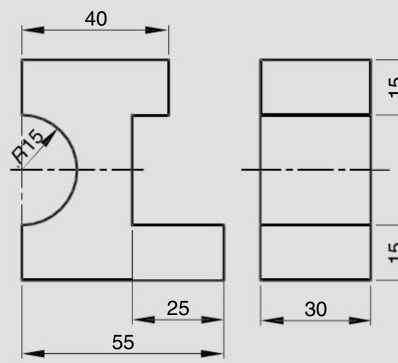


Fig. 12.37(a)

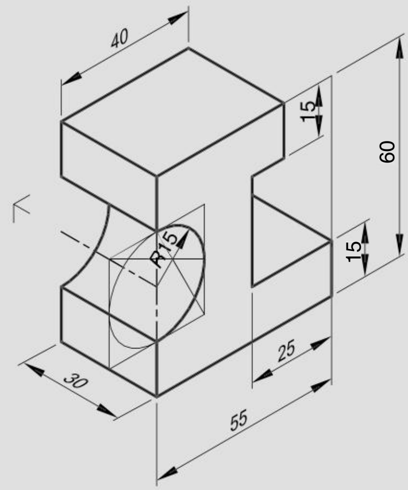


Fig. 12.37(b)

Fig. 12.37(b) shows the required isometric view. Construction lines are left intact for guidance.

Example 12.33 (Fig 12.38)

Front and right-hand side views of a casting are shown in Fig. 12.38(a). Draw its isometric view.

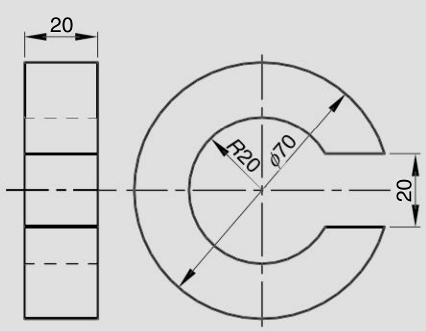


Fig. 12.38(a)

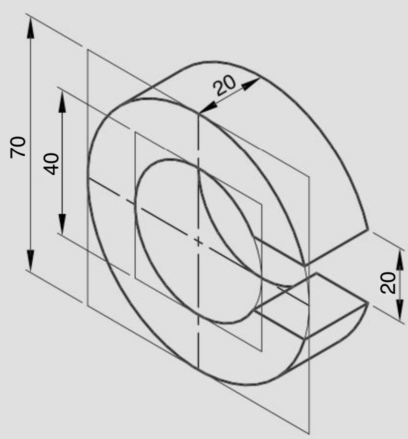


Fig. 12.38(b)

Fig. 12.38(b) shows the required isometric view. Construction lines are left intact for guidance.

Example 12.34 (Fig 12.39)

Fig. 12.39(a) shows the orthographic projections of an object. Draw its isometric view.

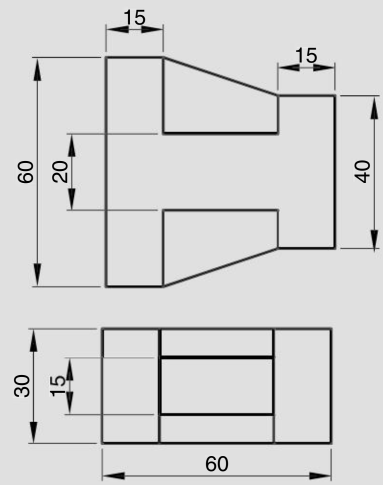


Fig. 12.39(a)

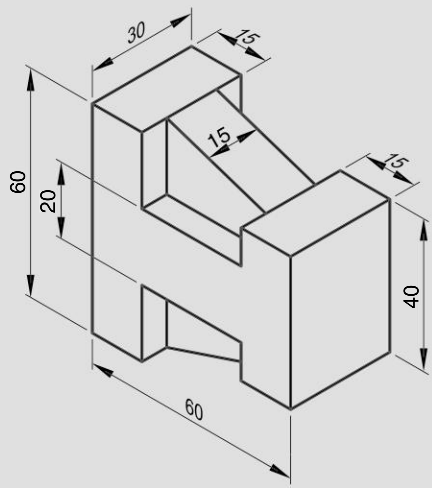


Fig. 12.39(b)

Fig. 12.39(b) shows the required isometric view.

Example 12.35 (Fig 12.40)

Two orthographic views of a block are given in Fig. 12.40(a). Draw the isometric view of the block. [RGPV Feb. 2007]

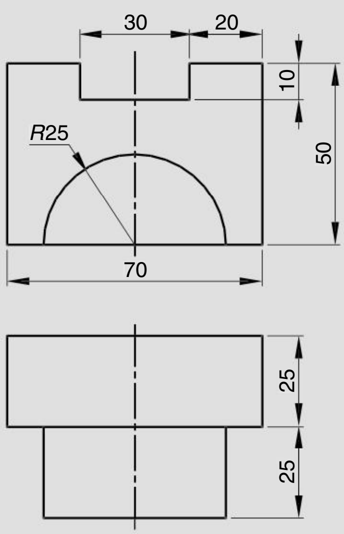


Fig. 12.40(a)

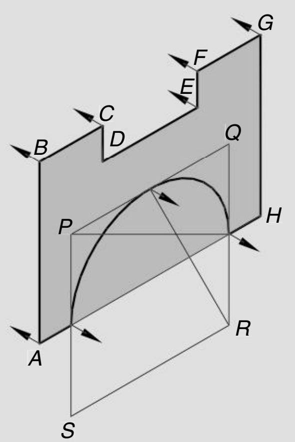


Fig. 12.40(b)

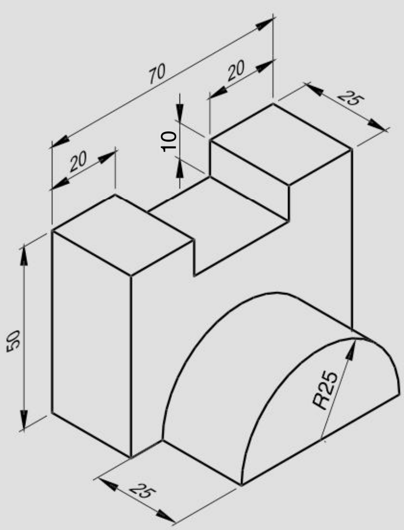


Fig. 12.40(c)

Construction: Fig. 12.40(c)

1. Draw the polygon *ABCDEFGH* on the *x-z* plane. Also, draw a rhombus *PQRS* and mark semi-ellipse, as shown in Fig. 12.40(b).
2. Extrude the points of the polygon 25 mm along the *y*-axis in a direction towards V.P. and complete the back plate.
3. Extrude the points of the semi-ellipse 25 mm along the *y*-axis in a direction away from the V.P. Join the semi-ellipses by tangent lines.
4. Darken the visible edges and dimension the figure to obtain Fig. 12.40(c).

Example 12.36 (Fig 12.41)

Draw the isometric projection of the object shown in Fig. 12.41(a). **[RGPV Feb. 2007]**

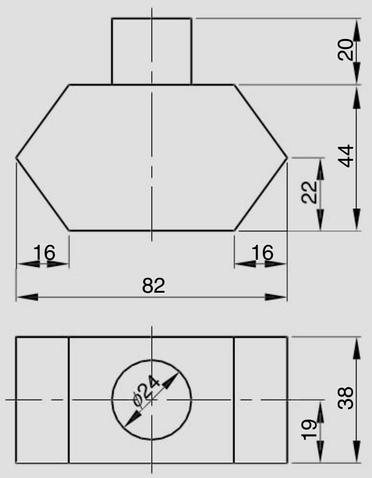


Fig. 12.41(a)

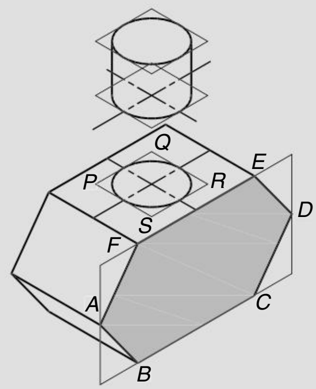


Fig. 12.41(b)

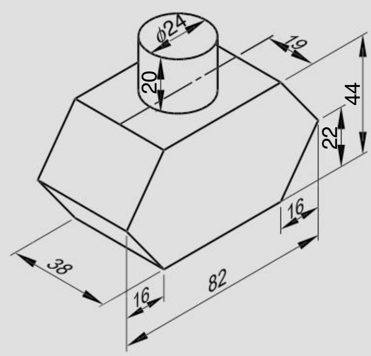


Fig. 12.41(c)

Construction: Fig. 12.41(c)

1. Draw the polygon *ABCDEF* on the *x-z* plane, as shown in Fig. 12.40(b). Extrude the points of the polygon 38×0.816 mm along the *y*-axis and complete the base plate.
2. On the top surface of the main block mark a rhombus *PQRS* and inscribe an ellipse using four-centre method.
3. Extrude the ellipse 20×0.816 mm along *z*-axis in upward direction. Join the ellipses by tangent lines.
4. Darken the visible edges and dimension the figure to obtain Fig. 12.40(c).

Example 12.37 (Fig 12.42)

Fig. 12.42(a) shows the orthographic projections of an object. Draw its isometric view.

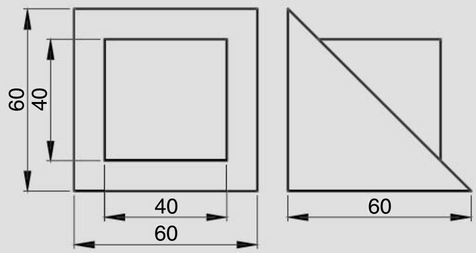


Fig. 12.42(a)

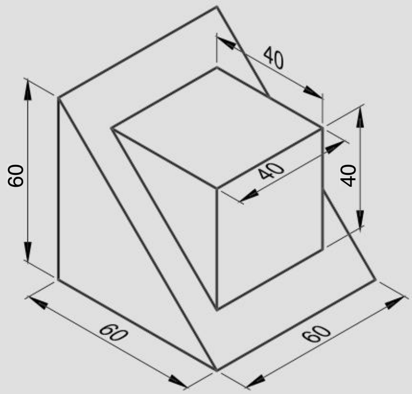


Fig. 12.42(b)

Fig. 12.42(b) shows the required isometric view.

Example 12.38 (Fig 12.43)

An object has its front, top and side views as shown in Fig. 12.43(a). Draw its isometric view.

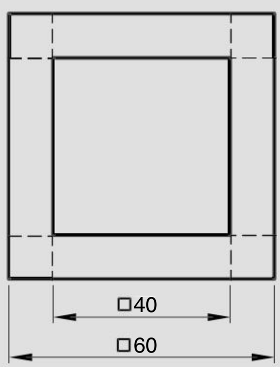


Fig. 12.43(a)

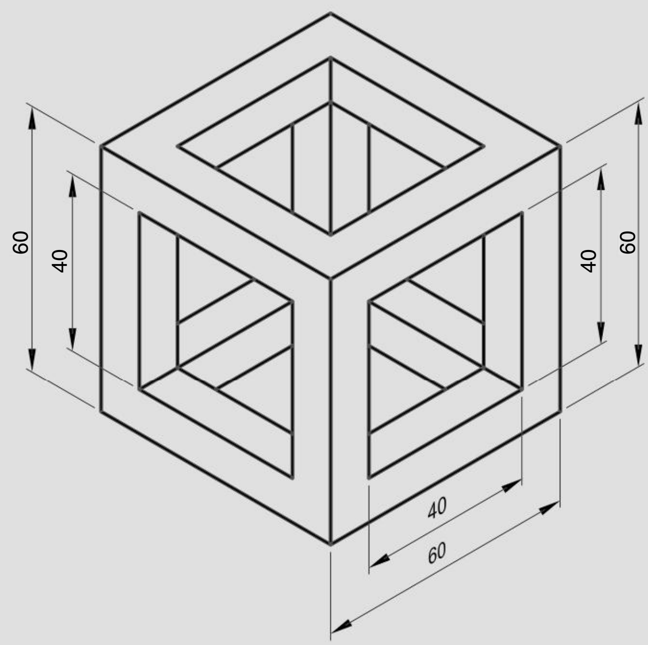


Fig. 12.43(b)

Fig. 12.43(b) shows the required isometric view.

Example 12.39 (Fig 12.44)

Draw isometric view of the casting shown in two views of Fig. 12.44(a). [RGPV Dec. 2005]

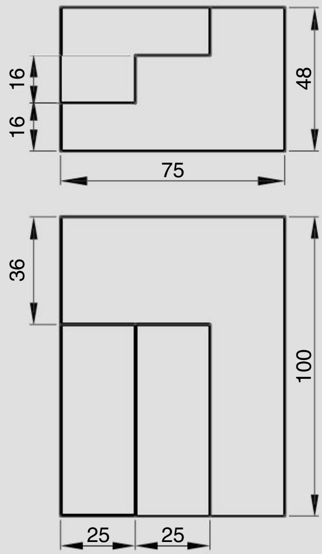


Fig. 12.44(a)

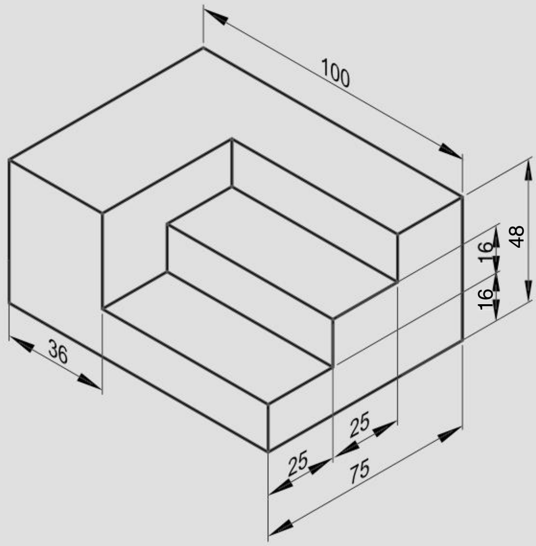


Fig. 12.44(b)

Fig. 12.44(b) shows the required isometric view

Example 12.40 (Fig 12.45)

Fig. 12.45(a) shows three orthographic views of an object. Draw its isometric view.

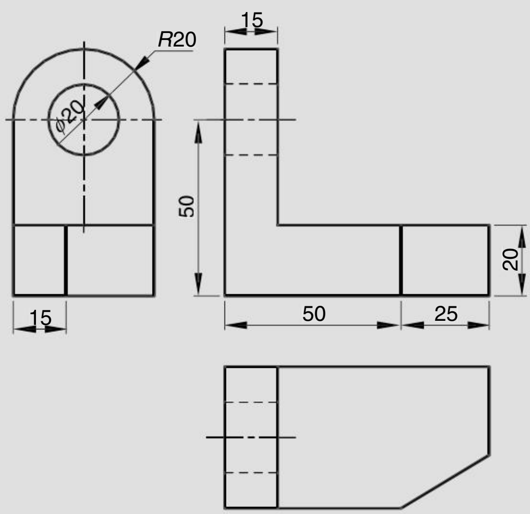


Fig. 12.45(a)

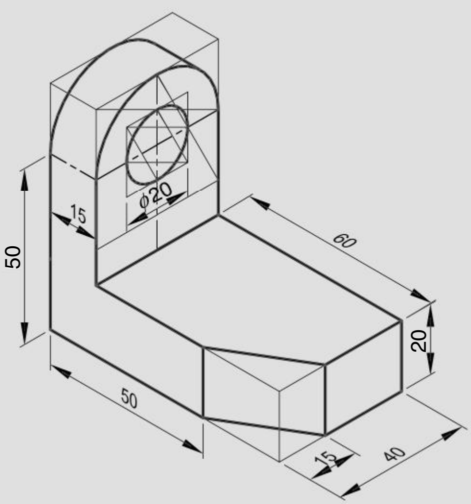


Fig. 12.45(b)

Fig. 12.45(b) shows the required isometric view. Length is taken on the left hand side to visualize its major portion. Construction lines are left intact for guidance.

Example 12.41 (Fig 12.46)

Fig. 12.46(a) shows the orthographic projections of an object. Draw its isometric view.

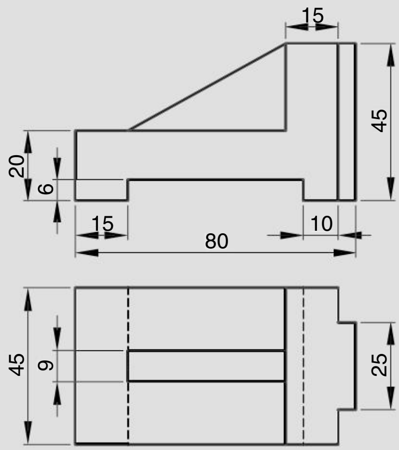


Fig. 12.46(a)

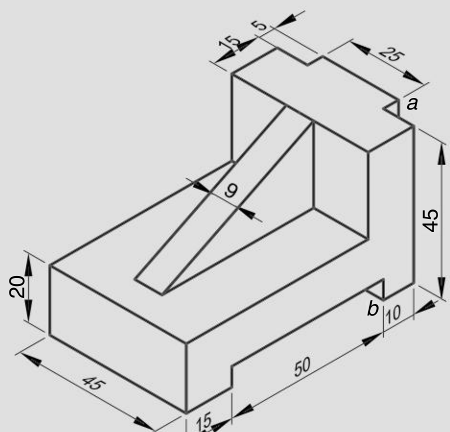


Fig. 12.46(b)

Fig. 12.46(b) shows the required isometric view. The following may be noted :

1. Length should be taken on the right-hand side as to visualize its major portion.
2. Draw all the three lines converging from both points *a* and *b* carefully.

Example 12.42 (Fig 12.47)

Fig. 12.47(a) shows the orthographic projections of an object. Draw its isometric view.

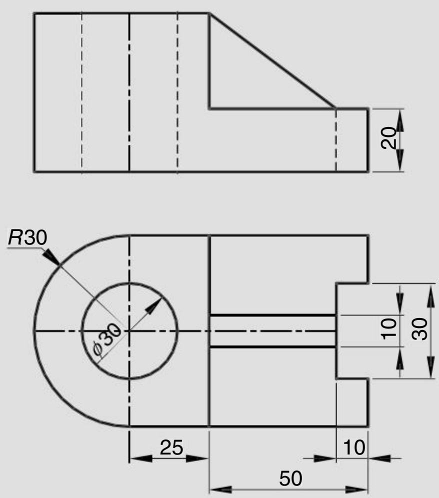


Fig. 12.47(a)

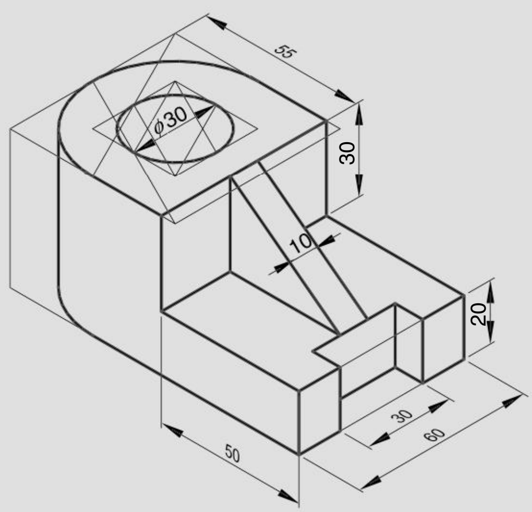


Fig. 12.47(b)

Fig. 12.47(b) shows the required isometric view. Construction lines are left intact for guidance.

Example 12.43 (Fig 12.48)

Fig. 12.48(a) shows the orthographic projections of an object. Draw its isometric view.

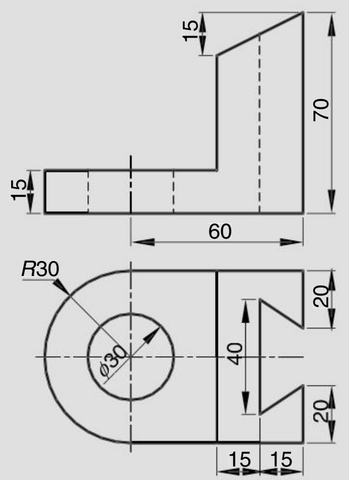


Fig. 12.48(a)

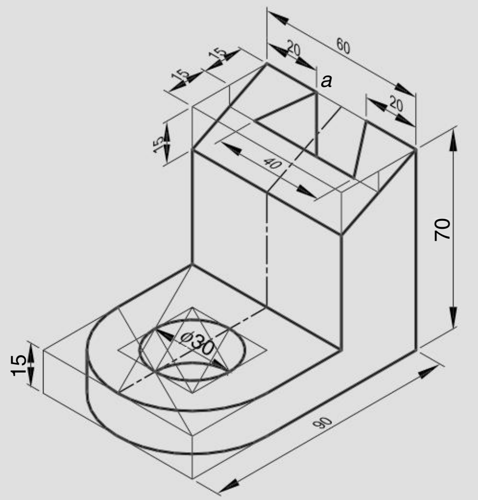


Fig. 12.48(b)

Fig. 12.48(b) shows the required isometric view. Construction lines are left intact for guidance. The following may be noted :

1. All the three lines converging from point *a* should be drawn.
2. Lines for the visible lower edges of the circular hole should be drawn.

Example 12.44 (Fig 12.49)

Fig. 12.49(a) shows the orthographic projections of an object. Draw its isometric view.

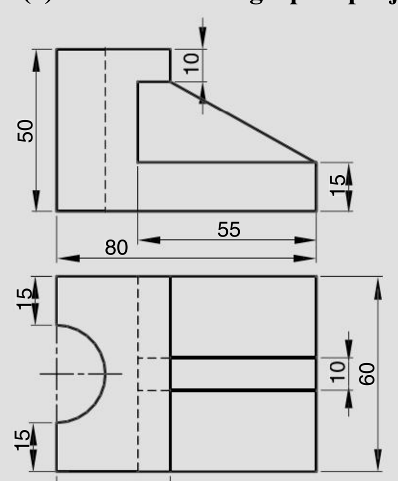


Fig. 12.49(a)

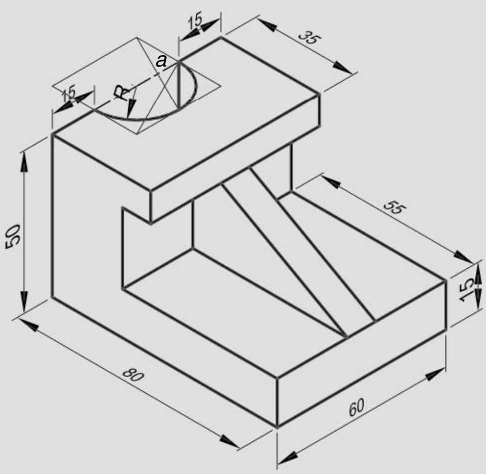


Fig. 12.49(b)

Fig. 12.49(b) shows the required isometric view. Construction lines are left intact for guidance. Two lines and a curve converging from point *a* should be drawn.

Example 12.45 (Fig 12.50)

Fig. 12.50(a) shows the orthographic projections of an object. Draw its isometric view.

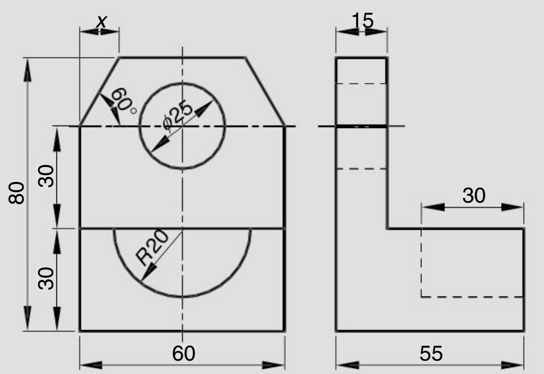


Fig. 12.50(a)

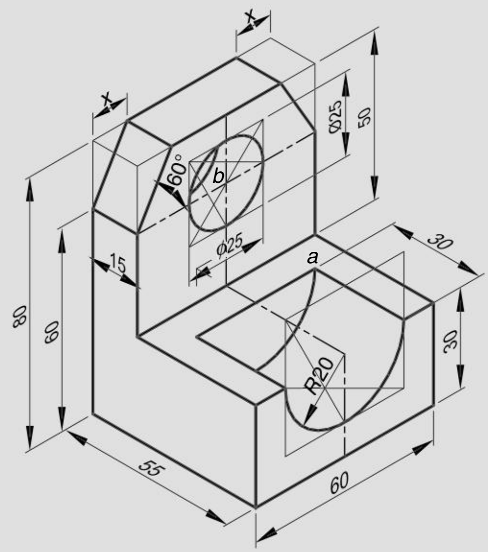


Fig. 12.50(b)

Fig. 12.50(b) shows the required isometric view. Construction lines are left intact for guidance. The following may be noted:

1. Length *x* has to be determined in order to transfer it in the isometric view.
2. Two lines and a curve converging from point *a* should be drawn.
3. Lines for the visible edge of the circular hole should be drawn at point *b*.

Example 12.46 (Fig 12.51)

The front view and top view of an object are shown in Fig. 12.51(a). Draw the isometric view of the object.

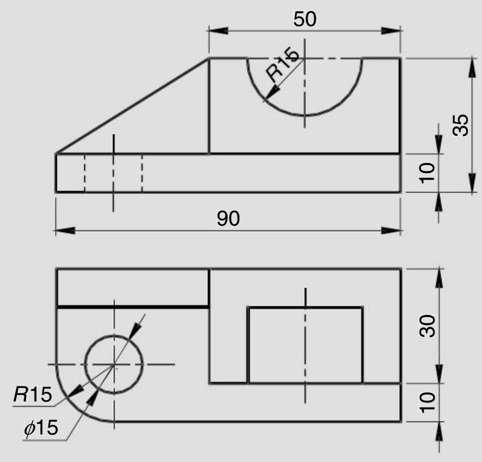


Fig. 12.51(a)

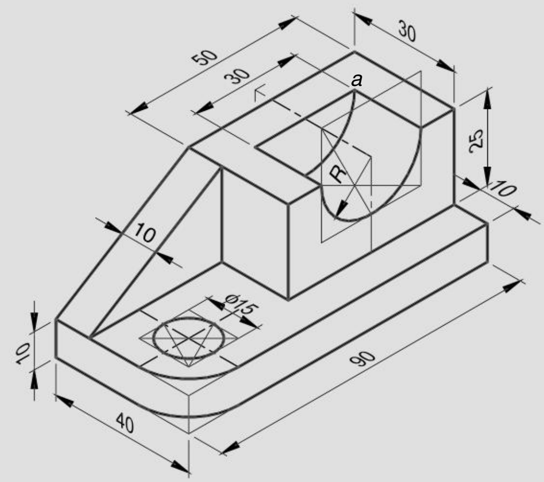


Fig. 12.51(b)

Fig. 12.51(b) shows the required isometric view. Construction lines are left intact for guidance. Two lines and a curve converging from point *a*, need be drawn.

Example 12.47 (Fig 12.52)

The front view and top view are shown in Fig. 12.52(a). Draw the isometric view of the object. [RGPV Sep. 2009]

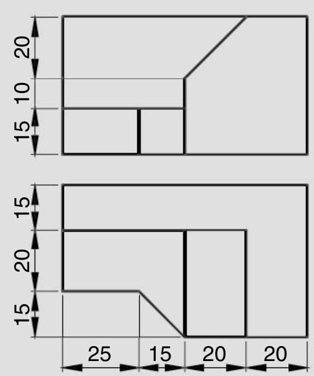


Fig. 12.52(a)

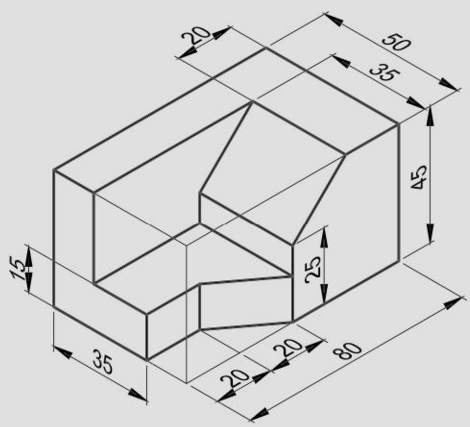


Fig. 12.52(b)

Fig. 12.52(b) shows the required isometric view.

Example 12.48 (Fig 12.53)

Fig. 12.53(a) shows the orthographic projections of an object. Draw its isometric view.

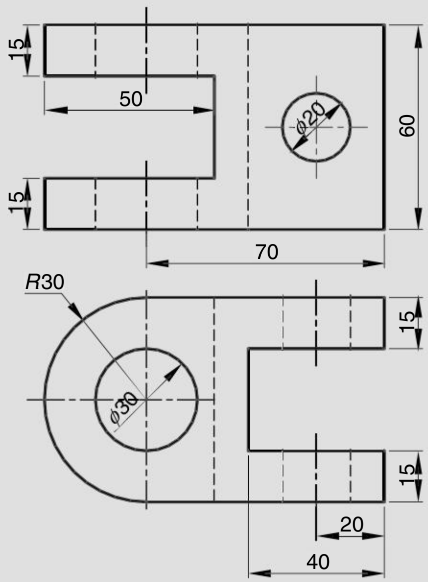


Fig. 12.53(a)

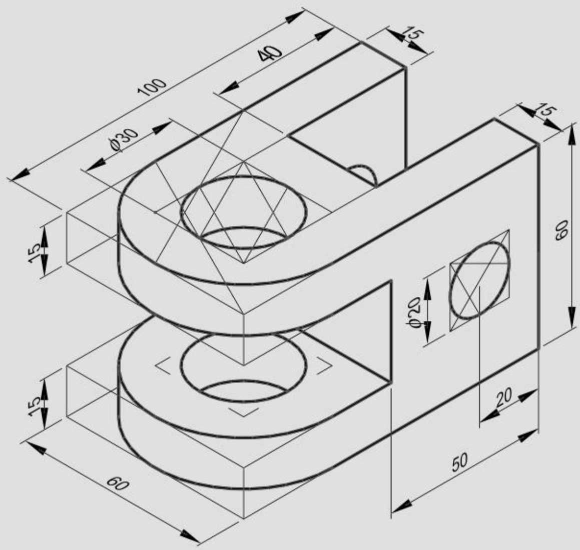


Fig. 12.53(b)

Fig. 12.53(b) shows the required isometric view. Construction lines are left intact for guidance.

Example 12.49 (Fig 12.54)

Fig. 12.54(a) shows the orthographic projections of an object. Draw its isometric view.

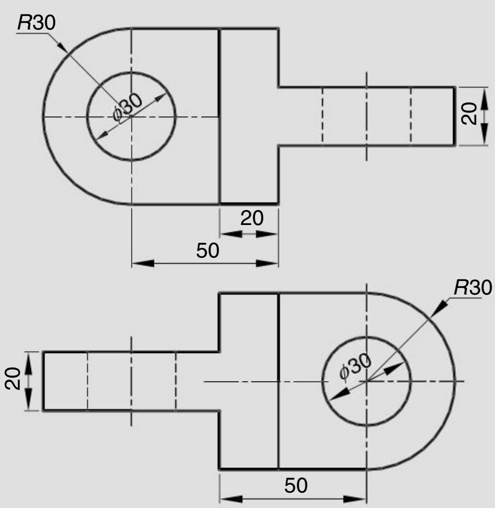


Fig. 12.54(a)

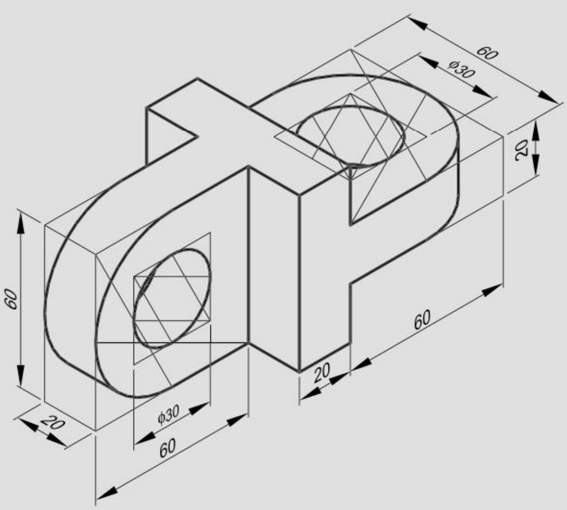


Fig. 12.54(b)

Fig. 12.54(b) shows the required isometric view. Construction lines are left intact for guidance.

Example 12.50 (Fig 12.55)

Fig. 12.55(a) shows the orthographic projections of an object. Draw its isometric view.

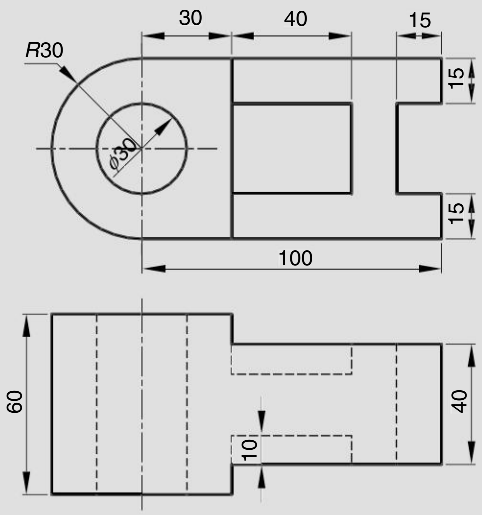


Fig. 12.55(a)

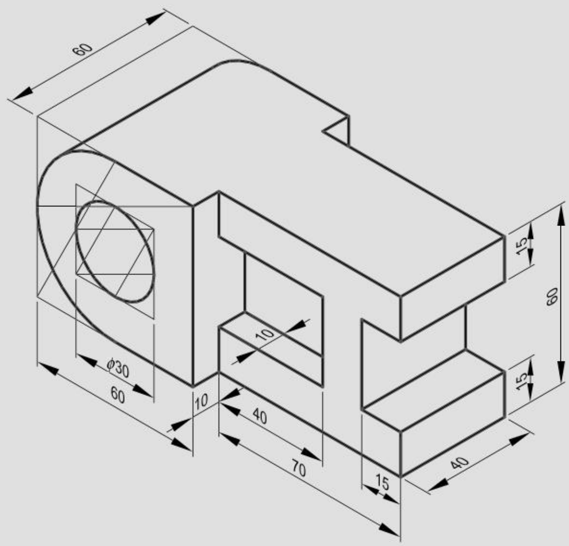


Fig. 12.55(b)

Fig. 12.55(b) shows the required isometric view. Construction lines are left intact for guidance.

EXERCISE 12

Simple Solids

1. Draw an isometric view of a pentagonal prism with 30 mm base side and 60 mm long axis resting on its base in the H.P. with a face parallel and nearer to the V.P.
2. Draw an isometric view of a hexagonal prism with 30 mm base side and 70 mm long axis, which is lying on its face in the H.P. with axis parallel to both H.P. and V.P.
3. A cylindrical block of base 60 mm diameter and height 90 mm, standing on the H.P. with its axis perpendicular to the H.P. Draw its isometric view. **[RGPV Feb. 2006]**
4. Draw isometric view of a pentagonal pyramid with the 25 mm base side and 60 mm long axis. The pyramid is resting on its base on the H.P. with an edge of the base parallel to V.P. **[RGPV June 2008]**
5. Draw isometric projection of the frustum of a pentagonal pyramid of 40 mm base side, 20 mm top side and 35 mm height resting on its base in the H.P.
6. Draw isometric projection of the pentagonal pyramid, the projections of which is given in Fig. E12.1. **[RGPV June 2007]**

7. Draw the isometric view of the pentagonal pyramid shown in Fig. E12.2. **[RGPV June 2009]**
8. Draw isometric projection of a frustum of the hexagonal pyramid shown in Fig. E12.3. **[RGPV June 2007]**
9. Draw the isometric view of the frustum of the cone shown in Fig. E12.4. **[RGPV June 2009]**

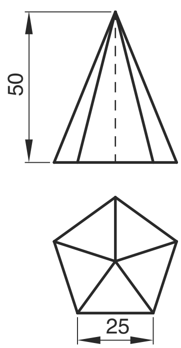


Fig. E12.1

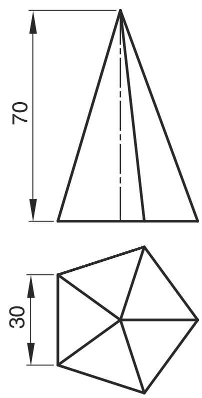


Fig. E12.2

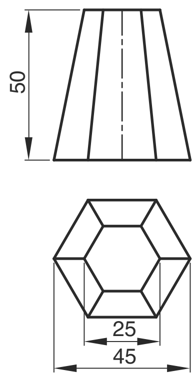


Fig. E12.3

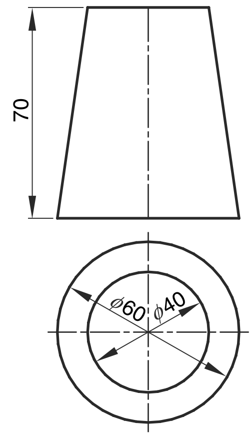


Fig. E12.4

Composite Solids

10. A composite solid is made up of a rectangular block of 80 mm × 40 mm × 30 mm high and semi-cylinders of 80 mm diameter 30 mm high at both the rectangular faces (80 mm × 30 mm) of the block. Draw its isometric projection.
11. A cone with a base diameter of 30 mm and 50 mm long axis, rests centrally over a square prism of base 50 mm side and 30 mm thick. Draw an isometric projection of the arrangement.
12. The frustum of a cone of 60 mm base diameter, 40 mm top diameter and 50 mm height is surmounted centrally over a cylindrical block of 80 mm diameter and thickness 30 mm. Draw its isometric projection.
13. A cube of 40 mm sides rests centrally on a square block of 60 mm edge and 20 mm thickness. Draw the isometric projections of the two objects with the edges of the two blocks mutually parallel to each other. **[RGPV June 2005]**
14. Draw the isometric projection of a sphere (R = 25 mm) resting centrally on the top of a square prism (base = 60 mm, height = 20 mm). **[RGPV Apr. 2009]**
15. Draw the isometric view of a sphere of 20 mm radius which rests centrally on top of a square prism of 50 mm base and 60 mm height. **[RGPV Aug. 2010]**
16. Draw the isometric projection of a spherical ball of 40 mm diameter resting centrally on the top of a pentagonal disc of 30 mm base side and 50 mm height. **[RGPV Dec. 2008]**
17. A sphere of radius 15 mm is placed on the top base of a square prism of side base 40 mm and height 50 mm. The square prism is place on the top of a cylinder of 30 mm height and 65 mm diameter. All the three solids have the common axis. Draw the isometric view of the combination of solids. **[RGPV June 2011]**

Sectioned Solids

18. A hexagonal prism of 30 mm base and 45 mm axis has an axial hole of 30 mm diameter. Draw its isometric projection.

19. A triangular pyramid having a 60 mm base side and 80 mm long axis is resting on its base in the H.P. with a side of the base parallel to the V.P. It is cut by an A.I.P. inclined at 45º with the H.P. and bisecting the axis. Draw its isometric view.
20. A cone of 50 mm base diameter and 60 mm axis rests with its base on H.P. A section plane perpendicular to V.P. and inclined at 30º to H.P. passes through the axis at a distance of 25 mm above base. Draw the isometric view of the truncated cone. **[RGPV Feb. 2011]**
21. A cube of 60 mm side has square holes of 30 mm side, cut through from all the six faces. The sides of the square holes are parallel to the edges of the cube. Draw the isometric view of the cube.

Solids Extruded in One Direction

22. Figs. E12.5-E12.16 show shows the orthographic projections of an object extruded in single direction. Draw their isometric views.

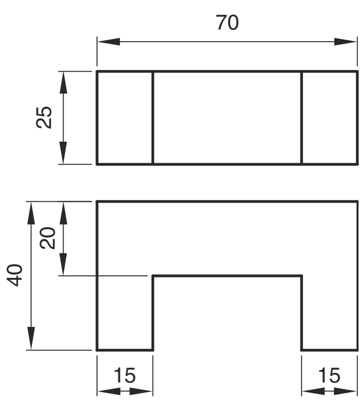


Fig. E12.5

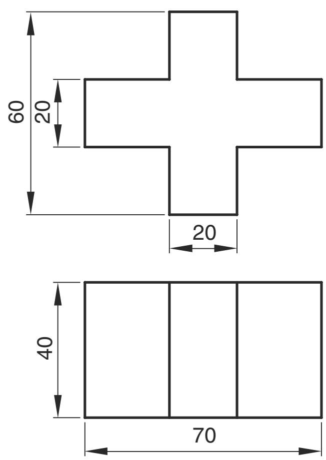


Fig. E12.6

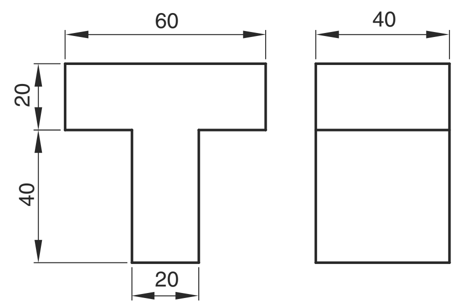


Fig. E12.7

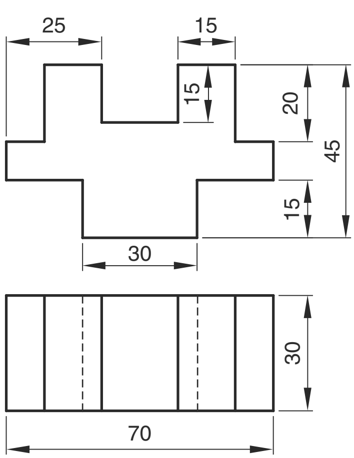


Fig. E12.8

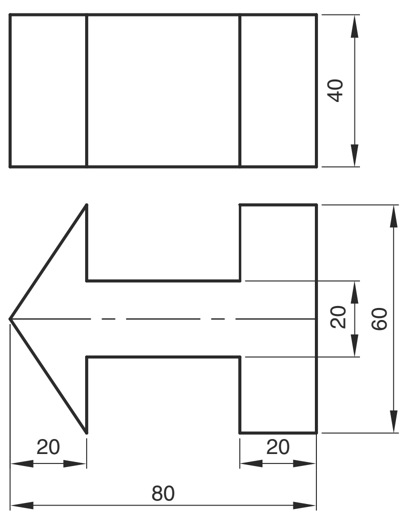


Fig. E12.9

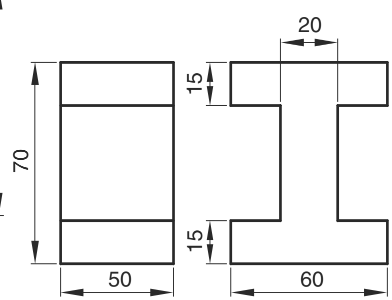


Fig. E12.10

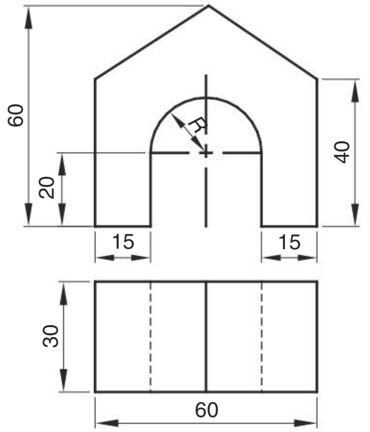


Fig. E12.11

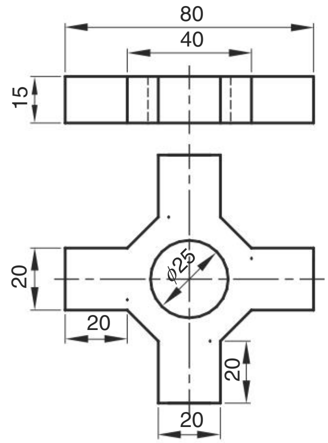


Fig. E12.12

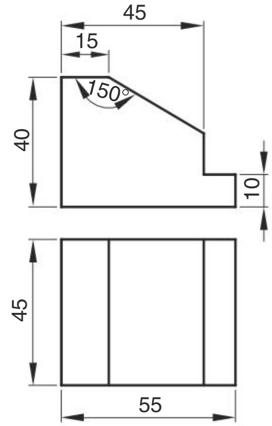


Fig. E12.13

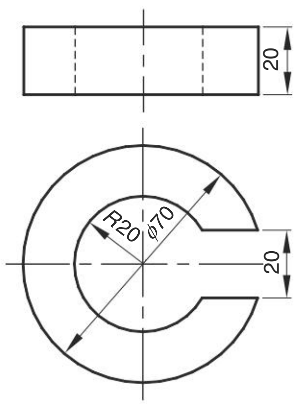


Fig. E12.14

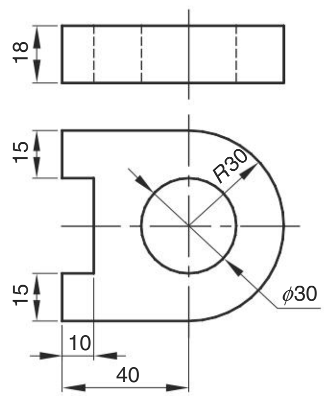


Fig. E12.15

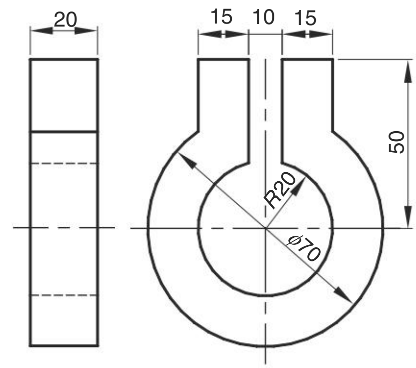


Fig. E12.16

23. Figs. E12.17-E12.19 show the orthographic projections of an object extruded in single direction and having hole, web or rib in the perpendicular direction. Draw their isometric views.

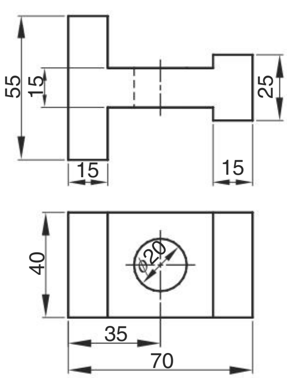


Fig. E12.17

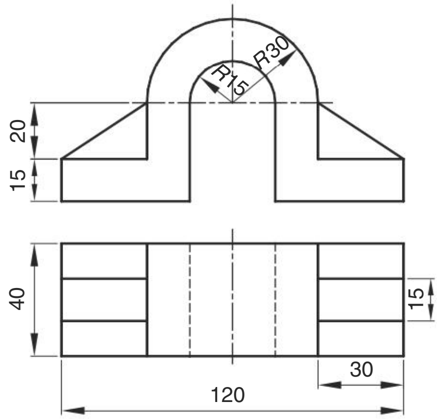


Fig. E12.18

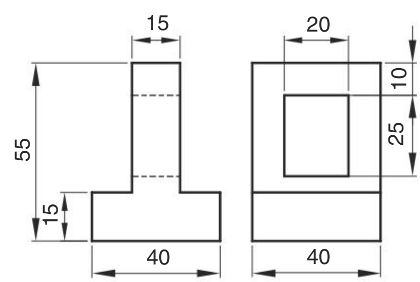


Fig. E12.19

Angle Plates

24. Figs. E12.20-E12.22 show the orthographic projections (in first angle) of angle plates with certain additional features such as ribs, slots, holes, etc. Draw their isometric views.

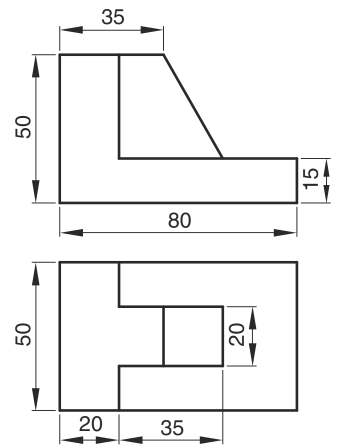


Fig. E12.20

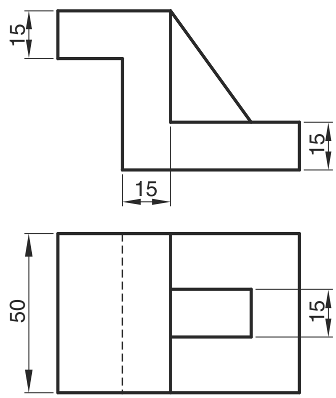


Fig. E12.21

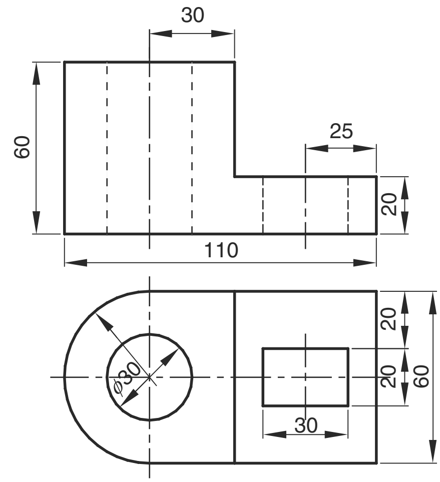


Fig. E12.22

Typical Problems

25. Figs. E12.23-E12.30 show the orthographic projections of objects in first angle projections. Draw their isometric views.

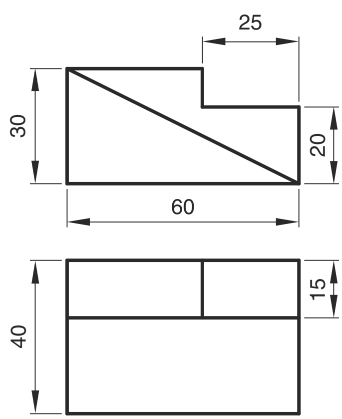


Fig. E12.23

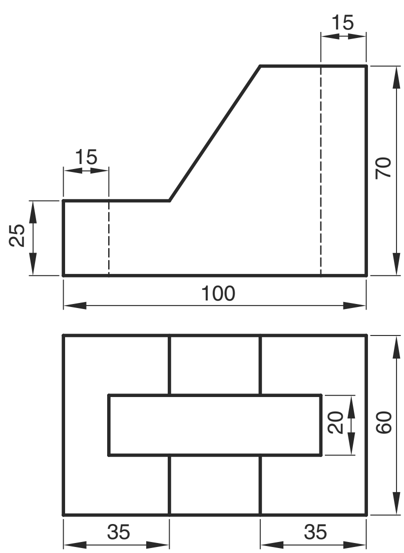


Fig. E12.24

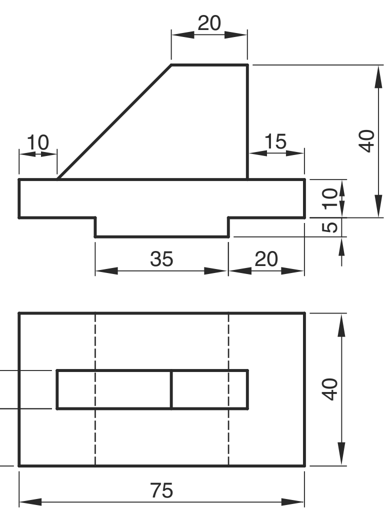


Fig. E12.25

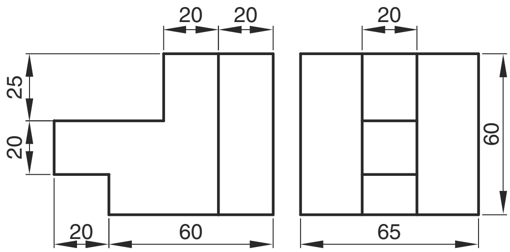


Fig. E12.26

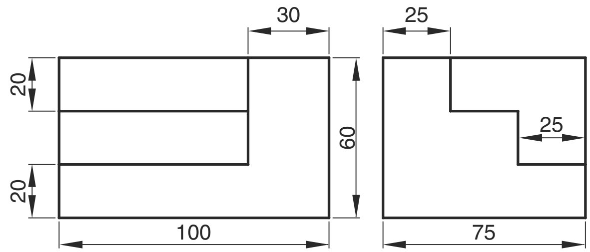


Fig. E12.27

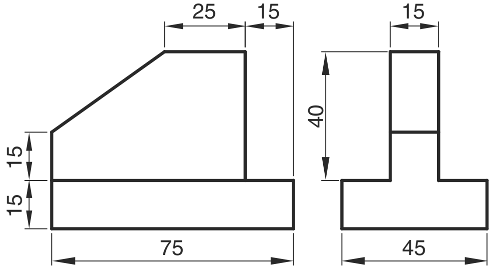


Fig. E12.28

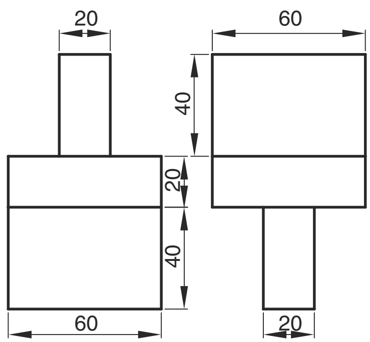


Fig. E12.29

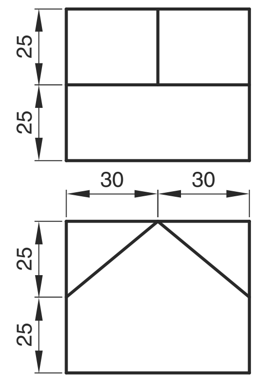


Fig. E12.30

REVIEW QUESTIONS

1. What is the R.F. of an isometric scale?
2. How would you construct an isometric scale?
3. Differentiate between isometric lines and non-isometric lines.
4. What is the relation among projectors in isometric projection?
5. State the relation between true length and isometric length?
6. Differentiate between isometric projection and isometric view.
7. Name the methods preferred for drawing ellipse in isometric projections.
8. Define isometric axes and isometric planes.
9. What are the principles of dimensioning in isometric projections?
10. What are the advantages of drawing isometric views?

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the given alternatives:

i) The projectors in isometric view are
(a) converging (b) diverging
(c) parallel to plane of projection (d) perpendicular to plane projection

ii) Pictorial views drawn on isometric scale are called
(a) isometric drawings (b) isometric projections
(c) isometric views (d) any of these

iii) The exact value of R.F. of an isometric scale is
(a) 9/11 (b) 0.815 (c) 0.8165 (d) $\sqrt{2} / \sqrt{3}$

iv) The angle that isometric lines make with each other is
(a) 45º (b) 60º (c) 90º (d) 120º

v) A square in a regular multi-view projection appears in an isometric view as
(a) box (b) square (c) parallelogram (d) rhombus

vi) In comparison to an isometric projection, the appearance of an isometric view is
(a) larger (b) smaller (c) more accurate (d) more realistic

vii) On isometric plane, a circle appears as
(a) an obloid (b) a circle (c) an ellipse (d) an involute

viii) While making isometric projections, the ellipse is preferably drawn by
(a) four-centre method (b) oblong method
(c) concentric circles method (d) parallelogram method

ix) Isometric projections cannot be drawn by
(a) box method (b) coordinate method
(c) offset method (d) zone method

x) A sphere in isometric projection appears as a circle of diameter
(a) equal to the diameter of sphere (b) 0.816 times the diameter of sphere
(c) less than 0.816 diameter of sphere (d) greater than the diameter of sphere

xi) Select the correct isometric view corresponding to the orthographic view shown in Fig. M12.21.

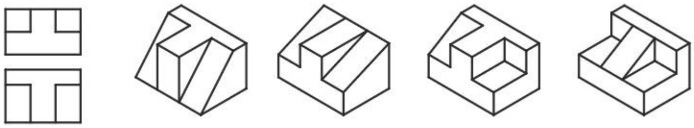


Fig. M12.21 **(a)** **(b)** **(c)** **(d)**

xii) Select the correct isometric view corresponding to the orthographic view shown in Fig. M12.22.

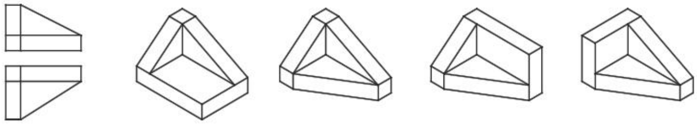


Fig. M12.22 **(a)** **(b)** **(c)** **(d)**

xiii) Select the correct isometric view corresponding to the front view shown in Fig. M12.23.

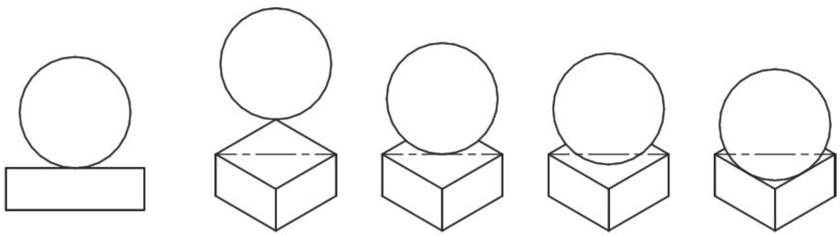


Fig. M12.23 **(a)** **(b)** **(c)** **(d)**

xiv) Select the correct isometric view corresponding to the front view shown in Fig. M12.24.

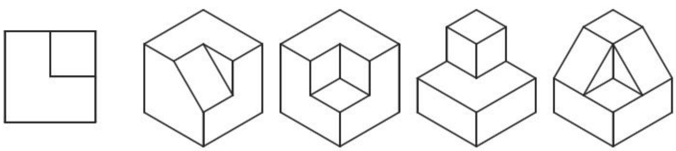
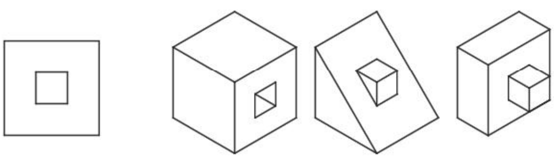


Fig. M12.24 **(a)** **(b)** **(c)** **(d)**

xv) Select the correct isometric view corresponding to the orthographic view shown in Fig. M12.25.



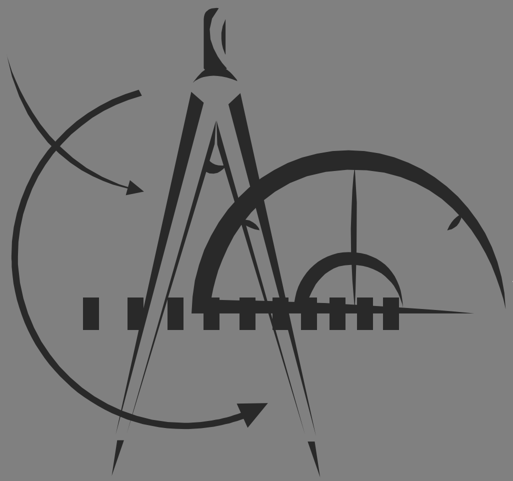
All of these

Fig. M12.25 **(a)** **(b)** **(c)** **(d)**

Answers

(i) d (ii) b (iii) d (iv) d (v) d (vi) a (vii) c (viii) a (ix) d (x) a (xi) c (xii) b (xiii) c (xiv) b (xv) d

13



Computer Aided Drafting (CAD)

- Introduction
- CAD Application
- Basic Components of a Computer
- Introduction to AutoCAD
- Setting Up Drawing Space
- Sheet Layout
- Methods of Locating a Point
- Regulating the Cursor Movement
- Drawing Lines and Curves
- Editing a Drawing

13.1 INTRODUCTION

The drafting work can be automated and accelerated through the use of Computer Aided Drafting (CAD) systems. It may be applied for a wide variety of products in the field of automotive, electronics, aerospace, naval, architecture, civil and other disciplines of engineering. CAD systems were originally used for automated drafting only, but now they also include three-dimensional modeling and computer-simulated operations of the models. Sometimes CAD is translated as "computer-assisted drafting", "computer-aided drafting", or a similar phrase. Related acronyms are CADD, which stands for "computer-aided design and drafting", CAID, for Computer-aided Industrial Design, and CAAD, for "computer-aided architectural design". All these terms are essentially synonymous, but there are some subtle differences in meaning and application.

13.2 CAD APPLICATION

CAD is used to design, develop and optimize products, which can be goods used by end consumers or intermediate goods used in other products. CAD is also extensively used in the design of tools and equipment required in the manufacturing process, and in the drafting and design of all types of buildings, ranging from small residential houses to the largest commercial or industrial complexes. CAD enables designers to layout and to develop their work on a computer screen, print and save it for future editing, thus saving a lot of time on their drawings. CAD is mainly used for detailed engineering of 3D models and/or 2D drawings of physical components, but it is also used throughout the engineering process, from conceptual design and layout of products to definition of manufacturing methods of components. Rather than building prototypes and changing components to determine the effects of tolerance ranges, engineers can use CAD systems to simulate operation to determine loads and stresses. The major benefits of such systems include lower product development costs and a greatly shortened design cycle. The CAD systems running on workstations and mainframe computers are increasingly integrated with computer-aided manufacturing systems.

13.3 SOFTWARE PROVIDERS

There are many CAD software products currently in the market. They can be classified into three types, which are, 2D drafting systems (e.g., AutoCAD, General CADD Pro), mid-range 3D solid feature modelers (e.g., Inventor, TopSolid, IronCAD, SolidWorks, SolidEdge, Alibre Design, VariCAD, ArchiCAD) and high-end 3D hybrid systems (e.g., CATIA, NX (Unigraphics), Pro/ENGINEER). However, these classifications cannot be applied too strictly as many 2D systems have 3D modules, the mid-range systems are increasing their surface functionality, and the high-end systems have developed their user interface in the direction of interactive Windows systems. More than half of the market is however covered by Autodesk Inc., Dassault Systems, PTC, and UGS Corp.

13.4 HARDWARE AND OPERATING SYSTEM TECHNOLOGIES

Today, most CAD computer workstations are Windows based PCs. However, some CAD systems also run on Unix or Linux operating systems. Generally no special hardware is required except a high end

Graphics card. However, for complex product design, machines with high speed CPUs and large capacity of RAM are recommended. The human-machine interface is generally via a computer mouse or keyboard, but can also be via a pen and digitizing graphics tablet.

13.5 BASIC COMPONENTS OF A COMPUTER

A general purpose computer has four main sections, which are the arithmetic and logic unit (ALU), the control unit, the memory, and the input and output devices. These parts are inter-connected by busses, often made of groups of wires. The basic components of a computer have been presented in Fig. 13.1.

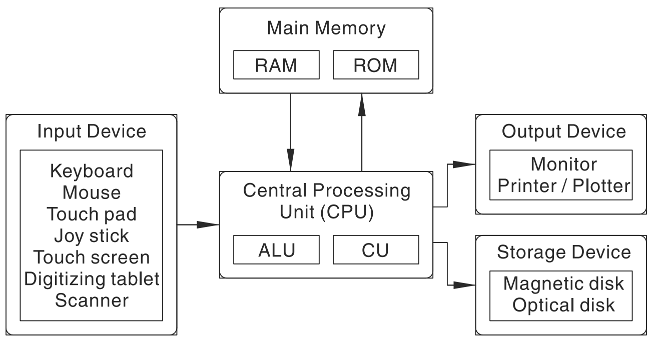


Fig. 13.1 *Basic components of a computer*

13.5.1 Central Processing Unit (CPU)

A CPU, also known as processor, interprets computer program instructions and processes data. In general, a CPU consists of:

- Arithmetic logic unit = ALU (addition, multiplication, comparison, etc.)
- Registers (extremely fast special memory for commands and operands)
- Control unit (gets commands, moves data to/from ALU and memory)

The arithmetic logic unit (ALU) is a digital circuit that calculates an arithmetic operation (addition, subtraction, etc.) and logic operations (Exclusive OR, AND, etc.) between two numbers. Processor registers are the top of the memory hierarchy, and provide the fastest way for the system to access data. The control unit fetches instructions from memory and decodes them to produce signals which control the other part of the computer. This may cause it to transfer data between memory and ALU or to activate peripherals to perform input or output.

13.5.2 Memory

It is a very fast storage device used to hold data connecting directly to the microprocessor. There are several types of memory in a computer.

1. ***Random-access Memory (RAM)*** Used to temporarily store information that the computer is currently working with.
2. ***Read-only Memory (ROM)*** A permanent type of memory storage used by the computer for important data that does not change.
3. ***Basic Input/Output System (BIOS)*** A type of ROM that is used by the computer to establish basic communication when the computer is first turned on.
4. ***Cach (pronounced as 'Cash') Memory*** The storage of frequently used data in extremely fast RAM that connects directly to the CPU.
5. ***Virtual Memory*** Space on a hard disk used to temporarily store data and swap it in and out of RAM as needed.

13.5.3 Input Device

Input refers to the process of entering data, programs, commands, and user responses into computer's memory. Following are some of the common input devices.

1. ***Keyboard*** It is the most commonly used input device which enables to enter data into a computer by pressing keys.
2. ***Mouse*** It is basically a pointing or picking device. The optical mouse has two buttons and a scroll wheel. Electronic circuits translate the mouse's movement into signals that are sent to the computer and used to direct the pointer.
3. ***Trackball*** It is a pointing device like a mouse with the ball on top. There are usually one to three buttons next to the ball, which are used just like mouse buttons. The only advantage of trackballs over a mouse is that the trackball is stationary so it does not require much space to operate it.
4. ***Touch Pad*** It is a flat surface that controls the movement of the pointer by sensing the motion of a finger on its exterior. The position of the pointer can be clicked by tapping the pad. It is generally integral part of lap-top computers.
5. ***Pointing Stick*** It is a device shaped like a pencil eraser that moves the pointer as pressure is applied.
6. ***Joystick*** A joystick uses the movement of a vertical stem to direct the pointer. It is a lever that moves in all direction and controls the movement of a pointer or some other display symbol. Joysticks are used mostly for computer games, but they are also used occasionally for CAD/CAM systems and other applications.
7. ***Pen Input*** Pen input devices can input data with hand written characters, select items by pressing the pen against the screen, and use gestures, which are special symbols, to issue commands. A light pen can be used to select processing options or to draw on the screen.
8. ***Touch Screen*** Touch screen allows users to touch areas of the screen to enter data. The key benefit of touch screens is enabling users to work interactively and more intuitively than through separate keypads or other input devices.
9. ***Digitizing Tablet*** It consists of an electronic tablet and a cursor or pen. It is used to convert shapes from a drawing or photograph to digital impulses and transmits them to a computer. Digitizing tablets are also called digitizers, graphics tablets, touch tablets, or simply tablets.
10. ***Scanner*** It captures an entire page containing image and/or text and converts it into a digital file allowing the computer to read and/or display the scanned object.
11. ***Magnetic Ink Character Recognition (MICR)*** Magnetic Ink Character Recognition is a char-

acter recognition system that uses special ink and characters. When a document that contains this ink needs to be read, it passes through a machine, which magnetizes the ink and then translates the magnetic information into characters.

12. ***Terminals*** It is a device that enables you to communicate with a computer. Generally, a terminal is a combination of keyboard and display screen.
13. ***Sound Card*** Sound cards enable the computer to output sound through speakers connected to the board, to record sound input from a microphone connected to the computer, and manipulate sound stored on a disk.
14. ***Voice Input*** It allows data and commands to be entered with spoken words. Digital cameras record photographs in the form of digital data. Video material is inputted using a video camera or a video recorder.
15. ***Electronic Whiteboard*** Electronic whiteboard is a modified whiteboard that captures text and drawing an area on a display screen that multiple users can write or draw on. Whiteboards are a principal component of tele-conferencing applications because they enable visual as well as audio communications in a file on an attached computer.

13.5.4 Output Device

Output refers to the processed result that is to be communicated by the computer. Following are some of the common output devices.

1. ***Monitor*** It is a display device that looks like a television screen. The monitor is an external component of the computer that displays the information produced by it to the user.
2. ***Printers and Plotters*** A printer allows the computer to provide the result in a printed form while a plotter is used to produce high-quality line drawings on paper.

13.5.5 Storage Device

It is a device used for storing input or output data.

1. ***Magnetic Disk*** It refers to the storage device on a magnetized medium. These devices are cheaper but are slow to access. Hard disk is currently in use. Floppies $\left(5\frac{1}{4} \text{ and } 3\frac{1}{2}\right)$ have been almost completely phased out due to their low capacity, low speed, and low durability.
2. ***Semi-conductor Storage*** It uses semi-conductor-based integrated circuits to store information. A semiconductor memory chip may contain millions of tiny transistors or capacitors. A type of non-volatile semi-conductor memory known as flash memory has steadily gained share as off-line storage for home computers.
3. ***Optical Disk*** It uses tiny pits etched on the surface of a circular disc to store information, and reads this information by illuminating the surface with a laser diode and observing the reflection. Compact disk (CD), Digital video disk (DVD) and Blue-violet disk (Blu-ray) are currently in common use.
4. ***Ultra Density Optical Disc (UDO) Storage*** It uses blue laser-based technology. This offers a revolutionary 30GB of storage capacity on a single 130 mm cartridge.

13.6 INTRODUCTION TO AUTOCAD

AutoCAD is one of the most versatile drafting software. It is widely applicable to all disciplines of engineering, architecture and construction fields. Among its many versions, AutoCAD 2011 is the latest and is highly user-friendly. The present chapter highlights some of its basic features applied to generate 2-D drawings in engineering practice. It is presumed that the readers have basic knowledge of operating computers as well as they are conversant with the Windows environment.

13.7 STARTING WITH AUTOCAD 2007

When the AutoCAD icon is clicked on, a Workspaces dialog box appears as shown in Fig. 13.2(a). This offers two options (a) 3D Modeling or (b) AutoCAD Classic. The 3D Modeling is used for three-dimensional drawing while AutoCAD Classic is suitable for two-dimensional drawing. On clicking OK, New Features Workshop dialog box appears as shown in Fig. 13.2(b). It offers option to learn some new features. To view these features, invoke option 'Yes' or else option 'Maybe later' and click OK. AutoCAD opens the screen as shown in Fig. 13.3 or Fig 13.4 depending upon the workspace selected, 3D Modeling or AutoCAD Classic, respectively. Several other items may also appear on the screen, such as the Sheet Set Manager, the Info Palette, and so on, which may be closed by clicking Close (X) button.

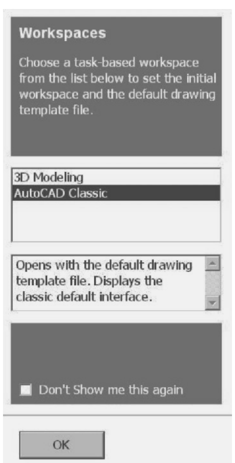


Fig. 13.2(a)

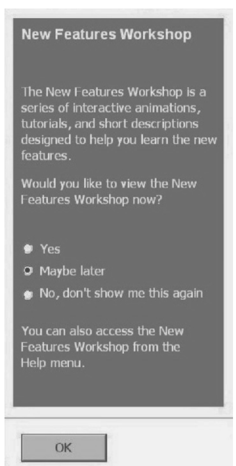


Fig. 13.2(b)

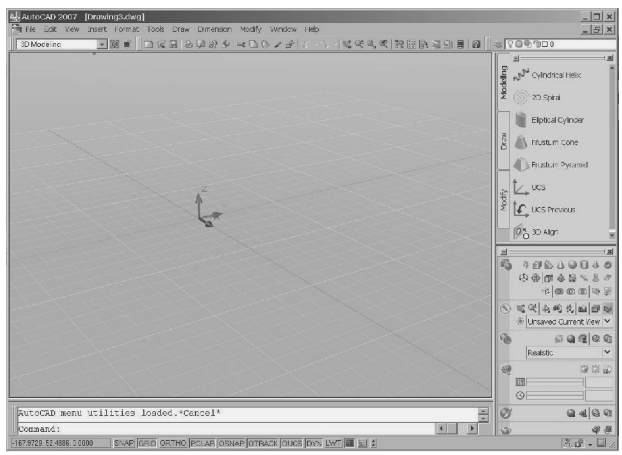


Fig. 13.3 *3D Modeling workspace*

In case 3D Modeling Workspaces is selected, and one desires to switch over to AutoCAD Classic, it can be done through the Workspaces toolbar at the upper left corner of the screen and then File ⇨ New, followed by acad.dwt in the Select Template dialog box.

13.8 AUTOCAD CLASSIC WORKSPACE

The AutoCAD Classic Workspace screen as shown in Fig. 13.4 consists of the following features.

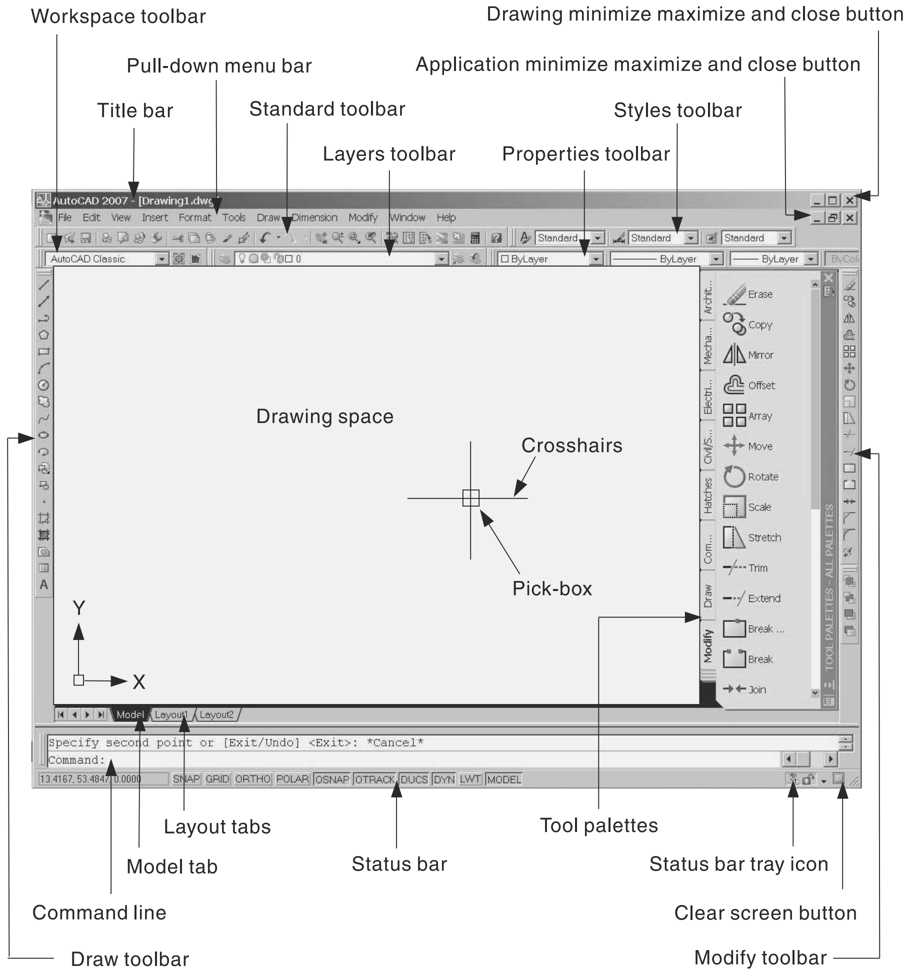


Fig. 13.4 *AutoCAD classic workspace*

1. ***Drawing Area*** The big rectangular space in the middle of the screen is called the drawing space or drawing area where drawing work is done. There is an almost infinite area to draw and what appears is just a portion of that.
2. ***WCS Icon*** The symbol consisting of two arrows in the bottom-left corner of the drawing area is called the WCS (World Coordinate System) icon. This shows the direction of positive X-axis and positive Y-axis. It can be changed to a User Co-ordinate System.
3. ***Pick-box and Crosshairs*** A small box in the drawing area is called pick-box and the super-imposed cross lines over the pick-box are called the crosshairs. The pick-box helps to pick or select objects, whereas the crosshairs show the location of the cursor. The movement of the pick-box and crosshairs are guided by the mouse. The X, Y coordinates of the current position of the crosshairs is indicated on the status bar at the bottom left corner of the screen.
4. ***Menus and Toolbars*** The following main toolbars which appear on the screen are shown in Fig. 13.4.

Title Bar It shows the name of the running program and the current filename.

Menu Bar These are the standard pull-down menus through which almost all commands are accessible.

Standard Toolbar This contains commonly used AutoCAD commands as well as standard Windows icons.

Properties Toolbar It gives a way to quickly modify an object's properties, such as layer and linetype.

Floating Toolbar A few toolbars which can be docked at any convenient positions of the screen are known as floating toolbars. Amongst these, the Draw toolbar and the Modify toolbar appear at the left and right columns of the screen. More floating toolbars may be opened or customized by choosing View ⇨ Toolbars from the menu bar.

5. ***Command Line and Dynamic Input Tooltip*** A pair of windows which appears near the bottom of the screen is called the command line. They prompt for the desired information and accept the actual command. All commands can be executed by typing them on the command line. The Dynamic Input tool-tip allows seeing the text that is typed at the cursor.
6. ***Status Bar*** The status bar appears at the bottom of the screen. At the left, it shows the *X*, *Y* coordinates of the current position of the crosshair. At the middle, it allows seeing and changing different modes of drawing such as Snap, Grid, Ortho, Polar, Osnaps, Otrack, etc. At the right side, the **Tray Icons** give the updates on items like reference files program updates and print status.
7. ***Tool Palettes*** Tool palettes can be seen towards the right side of the drawing space. It can be invoked through menu bar by Tools ⇨ Palettes ⇨ Tool Palettes. It provides an alternative way for selecting a command.

13.9 SETTING UP DRAWING SPACE

To start a new drawing, it is necessary to setup its size and units. These settings can be saved in a template for new drawings in future.

13.9.1 Units Command

This command is used to define the unit and angle formats. It can be invoked by any one of the following methods.

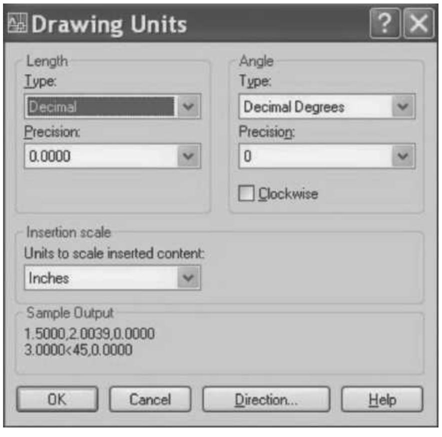


Fig. 13.5(a) *Drawing units dialog box*

Fig. 13.5(b) *Direction control dialog box*

(a) Menu bar : Format ⇨ Units
(b) Command line : Units

This will open Drawing Units dialog box as shown in Fig. 13.5(a). The parameters required to be set are given below.

1. ***Length*** It specifies the current unit format and precision for linear measurement as shown in Table 13.1. While the unit type sets the units in architectural, decimal, engineering, fractional or scientific type, the precision sets the number of decimal places or the fractional size.

Table 13.1

Unit Type	*Sample*	*Precision upto*	*Description*
Architectural	1′–4 1D2″	0′-01/256	Feet and inches in fractions
Decimal	16.50	0.00000000	Partial units in decimals
Engineering	1′–4.50″	0′-0.00000000	Feet and inches in decimals
Fractional	16 1⁄2	0 1/256	Partial units in fractions
Scientific	1.65E+01	0.00000000E+01	Base number + exponent

2. ***Angle*** It specifies the current angle format and precision for angular measurement as shown in Table 13.2. While the angle type sets the units in Decimal Degrees, Deg/Min/Sec, Grads, Radians or Surveyor type, the precision sets the number of decimal places. The default direction for positive angles is counter-clockwise. The clockwise option takes positive angles in the clockwise direction.

Table 13.2

Angle Type	*Sample*	*Precision upto*	*Measurement Description*
Decimal Degrees	16.5	0.00000000	Degrees in decimals
Deg/Min/Sec	16°30′0″	0d00′00.0000″	Degrees, minutes, and seconds
Grads	18.33333g	0.00000000g	Grads in decimals
Radians	0.287979r	0.00000000r	Radians in decimals
Surveyor	N 73d30′ E	N0d00′00.0000″E	Surveyor's units

3. ***Insertion Scale*** It is defined as the ratio of the units used in the source drawing and the units used in the target drawing. This is used when a drawing created with different units is to be inserted into the current drawing. Unitless option is used to insert the drawing or block without scaling.
4. ***Sample Output*** It displays an example of the current settings for units and angles.
5. ***Direction*** It displays the Direction Control dialog box as shown in Fig. 13.5(b). It defines the angle for zero degrees and specifies the direction in which angles are measured. The other ⇨ angle option is used to specify it by locating points or by entering a value.

13.9.2 Limits Command

This command creates an artificial and invisible boundary for the drawing. It affects the size of the grid, when displayed. However, drawing can be made outside the limits also. The default value of the lower-left corner and the upper right corners are (0,0) and (12,9) respectively. It can be invoked by any one of the following methods.

(a) Menu bar : Format ⇨ Drawing Limits
(b) Command line : Limits

Reset Model space limits:
Specify lower left corner or [ON/OFF] <0.0000,0.0000>: (Pick a point or press Enter for current)
Specify upper right corner <12.0000,9.0000>: (Pick a point or press Enter for current)

13.9.3 Scale

A scale is often indicated in the format (plotted size: actual size) known as representative fraction or scale factor. A scale is chosen during the process of laying out a drawing to ensure text, annotations and dimensions remains readable in its final plotted form. It can be setup through Plot Scale section in the Page Setup dialog box or Plot dialog box.

13.10 SHEET LAYOUT

It is a rectangle that includes boundary, frames, various marks, title block, etc. It can be created and saved in the form of a template for future drawings or selected from a large number of the standard templates provided with AutoCAD.

For a new drawing, the Select template dialog box, shown in Fig 13.6(a), can be opened through File ⇨ New in the menu bar. For example, select "ISO A3–Named plot styles.dwt" to open an A3-size sheet with layout marked as shown in Fig 13.6(b).

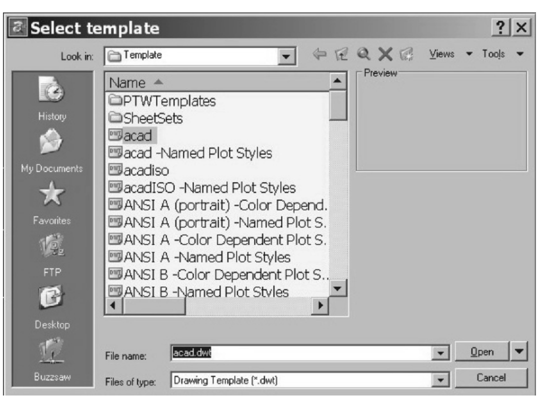


Fig. 13.6(a) *Select template dialog box*

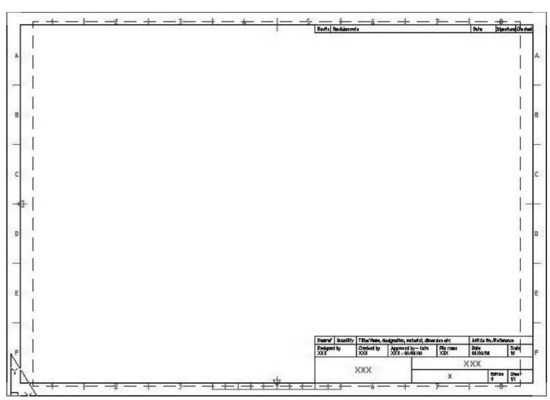


Fig. 13.6(b) *ISO A3-Named plot styles template*

13.11 MVSETUP COMMAND

This command provides a series of command lines for setting up the drawing units, limits, scale factor, etc. The prompts displayed for Model tab (model space) is given below.

Command: MVSETUP
Enable paper space? [No/Yes] <Y>: (Type N for model space)
Enter units type [Scientific/Decimal/Engineering/Architectural/Metric]: (Enter an option)
Enter the scale factor: (Type in a numeric value)
Enter the paper width: (Enter a number based on the width of the paper)
Enter the paper height: (Enter a number based on the height of the paper)

However, all these can also be done easily with the help of Page Setup dialog box.

13.12 COMMAND EXECUTION

Depending upon the user's convenience, a command can be executed in any one of the following ways.

1. Menu bar
2. Floating toolbars
3. Command line
4. Dynamic input tooltip
5. Tool palettes

It can be terminated by pressing either ESC, ENTER or the SPACEBAR.

13.13 METHODS OF LOCATING A POINT

A point in the drawing space can be located either by specifying its coordinates or by directly picking it up at the point of crosshairs on the screen by a click of mouse. Coordinates may be entered either at the command line or at the dynamic input tool-tip. The following coordinate systems may be used to locate a point in *X*-*Y* plane.

13.13.1 Absolute Coordinate System

In this system, the point is located using the Cartesian coordinate system. It is specified by entering the *X* and *Y* coordinates separated by comma with respect to the origin (0,0). The following example illustrates the use of absolute coordinate system for drawing the object shown in Fig. 13.7(a).

Command: L↵ (L refers to Line command and symbol ↵ to press enter)
Specify first point: 4,1↵ (Specify the coordinates of *A*)
Specify next point or [Undo]: 8,1↵ (Specify the coordinates of *B*)

Specify next point or [Undo]: 10,7↵ (Specify the coordinates of *C*)
Specify next point or [Close/Undo]: 2,7↵ (Specify the coordinates of *D*)
Specify next point or [Close/Undo]: C↵ (Enter *C* to join *DA* and terminate the command)

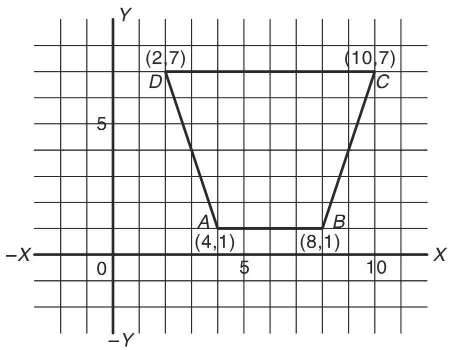


Fig. 13.7(a) *Absolute coordinates*

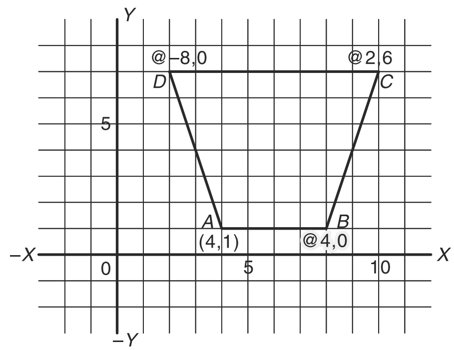


Fig. 13.7(b) *Relative rectangular coordinates*

13.13.2 Relative Rectangular Coordinate System

This system uses the Cartesian coordinates of the second and the subsequent points with respect to the previous point rather than the origin. It is designated by the symbol @ and should precede any entry. The following example illustrates the use of relative rectangular coordinate system for drawing the object shown in Fig. 13.7(b).

Command: L↵ (Line command)
Specify first point: 4,1↵ (Specify the coordinates of *A*)
Specify next point or [Undo]: @4,0↵ (Specify the coordinates of *B* relative to *A*)
Specify next point or [Undo]: @2,6↵ (Specify the coordinates of *C* relative to *B*)
Specify next point or [Close/Undo]: @–8,0↵ (Specify the coordinates of *D* relative to *C*)
Specify next point or [Close/Undo]: C↵ (Enter *C* to join *DA* and terminate the command)

13.13.3 Relative Polar Coordinate System

In this system, the point is located by defining the distance of the point from the current point, and the angle that the line between the two points makes with the positive *X* axis. The symbol < is used before giving the numerical value of the angle. The following example illustrates the use of relative polar co-ordinate system for drawing the object shown in Fig. 13.7(c).

Command: L↵ (Line command)
Specify first point: 4,1↵ (Specify the coordinates of *A*)
Specify next point or [Undo]: @4<0↵ (Specify the coordinates of *B* relative to *A*)
Specify next point or [Undo]: @6.32<72↵ (Specify the coordinates of *C* relative to *B*)
Specify next point or [Close/Undo]: @–8<0↵ (Specify the coordinates of *D* relative to *C*)
Specify next point or [Close/Undo]: C↵ (Enter *C* to join *DA* and terminate the command)

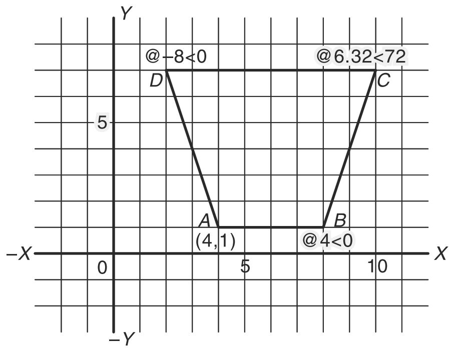


Fig. 13.7(c) *Relative polar coordinate system*

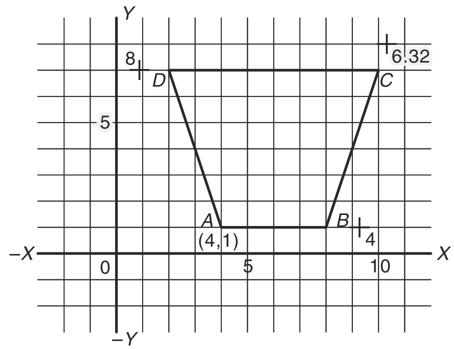


Fig. 13.7(d) *Direct distance entry system*

13.13.4 Direct Distance Entry System

In this system, the point is located by specifying the length of the line and its direction. The length is entered from the keyboard and direction is determined by position of the cursor. The following example illustrates the use of direct distance entry system for drawing the object shown in Fig. 13.7(d). Switching on the Ortho in the status bar helps to draw horizontal and vertical lines quickly.

Command: L↵ (Line command)
Specify first point: 4,1↵ (Specify the coordinates of *A*)
Specify next point or [Undo]: 4↵ (Position the cursor and then enter the distance for *B*)
Specify next point or [Undo]: 6.32↵ (Position the cursor and then enter the distance for *C*)
Specify next point or [Close/Undo]: 8↵ (Position the cursor and then enter the distance for *D*)
Specify next point or [Close/Undo]: C↵ (Enter *C* to join *DA* and terminate the command)

13.13.5 Picking Points Directly on Screen

The easiest and the quickest way to specify the coordinates of a point is to pick it directly on the screen at the point of crosshairs by a click of mouse. Users may take advantage of various settings like snap, grid, ortho, polar tracking, etc., to accurately regulate the cursor movement and work fast.

13.14 REGULATING THE CURSOR MOVEMENT

The movement of the cursor can be regulated or restricted for locating the points quickly by snap and grid settings with the help of drafting setting dialog box.

13.14.1 Snap and Grid

Snap with grid is one of the quickest and most accurate cursor regulating tools available. The Snap restricts the cursor movement to the specified incremental distance. If all the measurements are in the multiples of 5 units, the snap can be set to 5. The grid helps to visualize the snap points. However, the grid dots need not be set to the same spacing as the snap points. Setting of the snap and grid can be invoked by any one of the following methods.

(a) Menu bar : Tools ⇨ Drafting Settings
(b) Status bar : Right-click the SNAP or GRID button and choose Settings

This opens the Drafting Settings dialog box as shown in Fig. 13.8. To turn snap on, click the 'Snap On' check box. However, the most common way to turn snap on or off is by clicking SNAP button on the status bar or by pressing F9 key. Similarly, to turn grid on, click the 'Grid On' check box or click GRID button on the status bar or press F7 key.

Snap and Grid Spacing In the Drafting Setting dialog box, 'Snap *X* Spacing' specifies the snap spacing in the *X* direction and 'Snap *Y* Spacing' specifies the snap spacing in the *Y* direction. In case only the value of *X* spacing is supplied, it automatically assumes the same value of *Y* spacing by default. However, the 'Equal *X* and *Y* Spacing' check box may be turned off and a different value may be keyed in the 'Snap *Y* spacing' box for keeping unequal *X* and *Y* spacing.

Similarly, 'Grid *X* Spacing' specifies the grid spacing in the *X* direction while 'Grid *Y* Spacing' specifies that in the *Y* direction. However, it is not necessary that the grid be set to the same spacing as the snap points. *X* and *Y* spacing will be different only if 'Equal *X* and *Y* Spacing' check box is turned off and a different value is entered in the *Y* Spacing box.

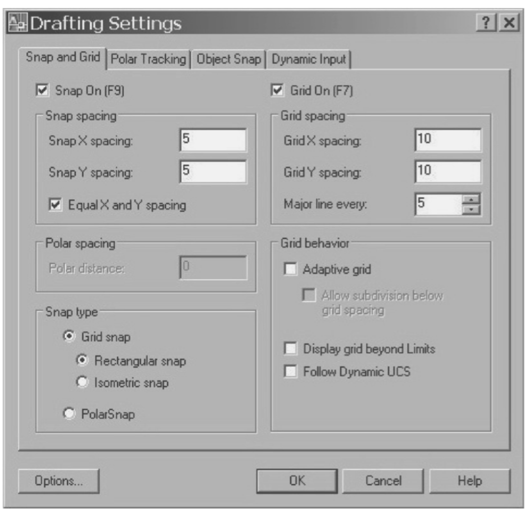


Fig. 13.8 *Snap and Grid settings through drafting setting dialog box*

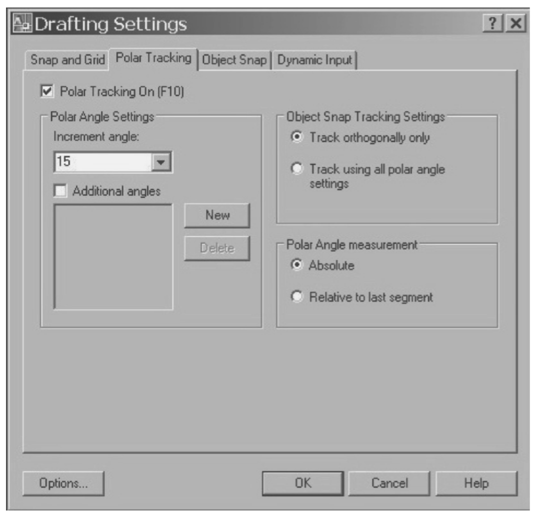


Fig. 13.9 *Polar tracking settings through drafting setting dialog box*

13.14.2 Polar Tracking

This regulates the cursor movement to specified increments along a polar angle. It also facilitates creating or modifying objects conveniently and precisely for many angles using direct distance entry system rather than other system of specifying coordinates. It can be invoked by any one of the following methods.

(a) Menu bar : Tools ⇨ Drafting Settings
(b) Status bar : Right-click the POLAR button and choose Settings

This opens Drafting Settings dialog box as shown in Fig. 13.9.

1. ***Polar Angle Settings*** This setting can be done in the ways given below.
   (a) ***Increment Angle*** To set the increment angle, type a desired value in the text box or select it from the drop-down arrow.
   (b) ***Additional Angles*** It is used to set additional angles other than those defined by increment angles. For this, tick the additional angles check box and click New. Then supply the value of the desired angle. It can accept 10 additional angles. To delete an additional angle, select it and click Delete.
2. ***Object Snap Tracking Settings*** This option is used to set object snap tracking to work with all polar angles or can be limited to orthogonal angles only.
3. ***Polar Angle Measurement*** This option is used to set polar angles to be taken either with respect to positive *X*-axis, i.e., absolute angles or relative to the recently drawn segment.
   To turn Polar tracking on, click the 'Polar Tracking On' check box. However, the most common way to turn Polar tracking on or off is by clicking POLAR button on the status bar or pressing F10 key.
   The cursor should be moved slowly through the angles to allow time for the calculation and display of the vector and tooltip as shown in Fig. 13.10. In case Dynamic Input is on, the word "Polar" is displayed to distinguish between the Dynamic Input and the polar tooltips.

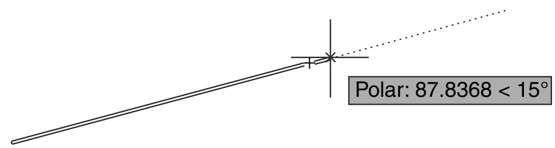


Fig. 13.10 *Polar tooltip*

13.14.3 Ortho

This mode is used to regulate the cursor movement along the horizontal and vertical axes only. This helps to create and modify objects consisting of mostly horizontal and vertical lines in a convenient way.

To turn Ortho on, type ortho in the command line. However, the most common way to turn ortho on or off is by clicking ORTHO button on the status bar or pressing F8 key.

Orthogonal mode only helps to pick up points directly on the screen and also through direct distance entry system. Any relative or absolute coordinates that is typed in the Dynamic Input tooltip, or on the command line, overrides this mode. It may be noted that Ortho mode and polar tracking cannot be on at the same time. Turning on Ortho automatically turns off the polar tracking.

13.15 DRAWING LINES AND CURVES

13.15.1 LINE Command

This command is used to create a series of contiguous line segments. It can be invoked by any of the following methods depending upon the preference of the individual user.

(a) Draw toolbar : 
(b) Command line : LINE or L
(c) Menu bar : DRAW ⇨ LINE
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.11.

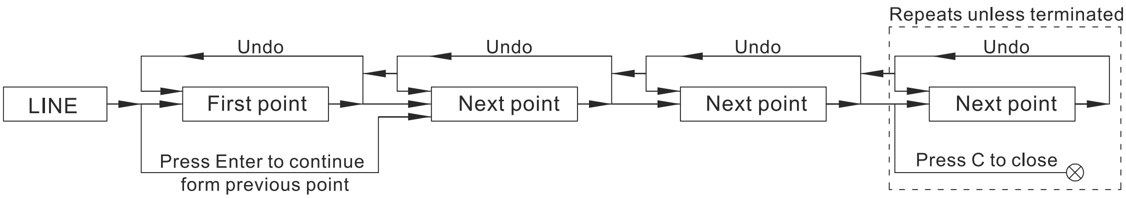


Fig. 13.11 *Ray diagram for LINE command*

Once this command is invoked, it prompts for specifying the position of a series of points one after the other through the Command window which shows the following command prompts unless it is terminated by pressing ESC, ENTER or SPACEBAR.

Specify first point: (Specify the starting point)
Specify next point or [Undo]: (Specify the next point)
Specify next point or [Undo]: (Specify the next point or enter an option)
Specify next point or [Close/Undo]: (Specify the next point or enter an option)

The following options may also be used with Line command.

1. ***Undo*** Undo option is used to delete the action of the latest command sequentially.
2. ***Close*** Close option is used to draw the line between the current point and the initial point of the first line.

13.15.2 Rectangle Command

This command is used to create rectangles directly in place of LINE command. It can be invoked by any one of the following methods.

(a) Draw Toolbar : 
(b) Command line : RECTANGLE or RECTANG
(c) Menu bar : DRAW ⇨ RECTANGLE
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.12.

The following command prompt appears in the command window.

Specify first corner point or [Chamfer/Elevation/Fillet/Thickness/Width]: (Specify the position of the first corner point or enter an option)

Fig. 13.13(a) shows construction of rectangle drawn by specifying diagonally opposite corners. The advantages of the following options may be also taken for its construction.

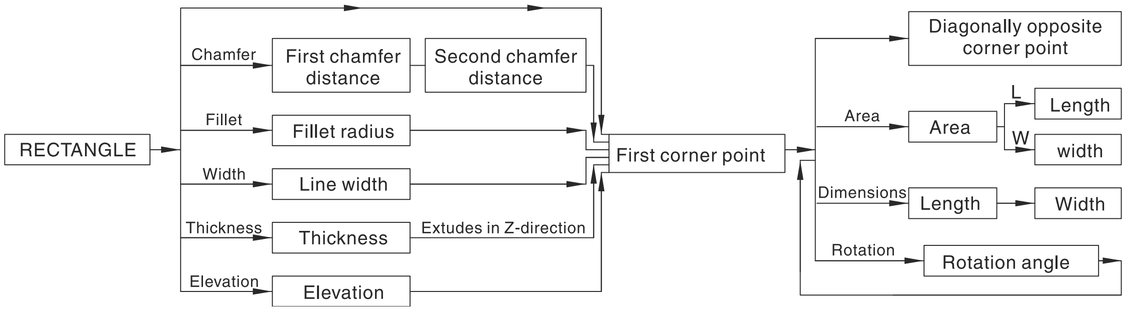


Fig. 13.12 *Ray diagram for RECTANGLE command*

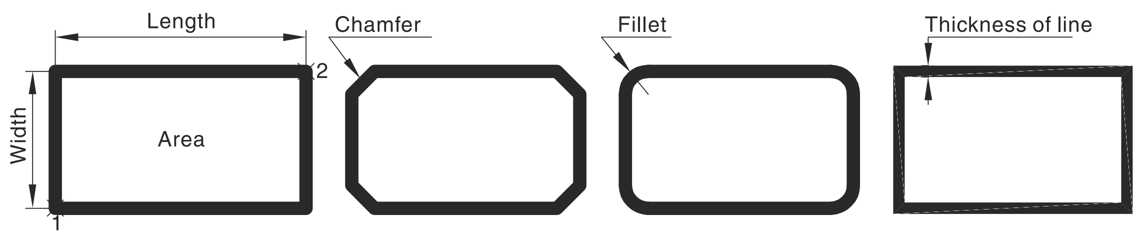


Fig. 13.13(a) **Fig. 13.13(b)** **Fig. 13.13(c)** **Fig. 13.13(d)**

1. ***Chamfer*** This option creates chamfer at all the four corners by specifying the chamfer distance. Fig. 13.13(b) shows rectangle with equal chamfers. To create unequal chamfers, two different values may be given.
   Specify first chamfer distance for rectangles <current>: (Specify a value)
   Specify second chamfer distance for rectangles <current>: (Press ENTER or same value for equal chamfer or specify a new value for unequal chamfer)
2. ***Fillet*** This option makes all the four corners rounded by specifying the fillet radius. See Fig. 13.13(c).
   Specify fillet radius for rectangles <*current*>: (Specify a value)
3. ***Width*** This option is used to specify the thickness of the line of the rectangle. See Fig. 13.13(d).
   Specify line width for rectangles <*current*>: (Specify a value)
4. ***Thickness*** This option is used to draw a rectangle that is extruded by the specified value of thickness in the Z-direction. To view such rectangle, follow VIEW ⇨ 3D VIEWS ⇨ SW ISO-METRIC from menu bar.

Specify thickness for rectangles <*current*>: (Specify a value)

5. ***Elevation*** This option is used to draw a rectangle at a specified distance from the *XY* plane along the *Z*-axis.
   Specify the elevation for rectangles <*current*>: (Specify a value)
   After specifying the first corner point, it shows the following command prompt.
   Specify other corner point or [Area/Dimensions/Rotation]: (Specify the position of the diagonally opposite corner or enter an option)
   Here one of the following options may be selected.

1. ***Area*** This option is used to create a rectangle by specifying its area followed by either length or width.
   Enter area of rectangle in current units <*current*>:
   Calculate rectangle dimensions based on [Length/Width] <Length>: (Choose length or width)
   Enter rectangle length <*current*>: (This appears if length option is selected)
   Enter rectangle width <*current*>: (This appears if width option is selected)
2. ***Dimensions*** This option is used to create a rectangle by specifying its length and width.
   Specify length for rectangles <*current*>: (Specify a value or press ENTER for current value)
   Specify width for rectangles <*current*>: (Specify a value or press ENTER for current value)
   Specify other corner point or [Dimensions]: (Move the cursor to specify one of the four possible locations)
3. ***Rotation*** This option is used to create a rectangle which is rotated at a desired angle.
   Specify rotation angle or [Pick points] <*current*>: (Specify a rotation angle)
   Specify other corner point or [Area/Dimensions/Rotation]: (Specify the position of the diagonally opposite corner or enter an option)
   The drawn rectangle is treated as a single object. The individual sides can be edited only after it is exploded by EXPLODE command. It may be noted that the value entered for fillet, width, elevation, thickness and rotation are taken as the current value for subsequent rectangle command. In case they are different from the current value, it is essential to reset them.

13.15.3 Polygon Command

This command is used to draw regular polygons. The number of sides may vary from 3 to 1,024. It can be invoked by any one of the following methods.

(a) Draw toolbar : 
(b) Command line : POLYGON or POL
(c) Menu bar : DRAW⇨POLYGON
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.14.

The following command prompt appears in the command window.

Enter number of sides <*current*>: (Specify a value between 3 and 1024)

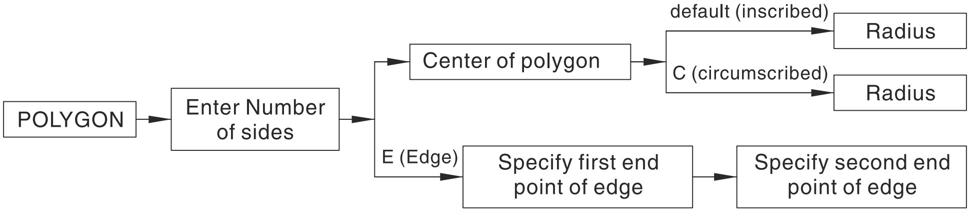


Fig. 13.14 *Ray diagram for POLYGON command*

Specify center of polygon or [Edge]: (Specify the center point or enter *E* for edge option)

1. ***Inscribed in Circle*** This defines the polygon with reference to an imaginary circle whose circumference touches all the vertices of the polygon. See Fig. 13.15(a). After specifying the centre, select inscribed option and enter the value for the radius of inscribed circle.
   Enter an option [Inscribed in circle/Circumscribed about circle] <I>: (Type I for inscribed option)
   Specify radius of circle: (Specify the radius from the centre to a vertex)
2. ***Circumscribed about Circle*** This defines the polygon with reference to an imaginary circle whose circumference touches all the mid-points of the polygon's sides. See Fig. 13.15(b). After specifying the centre select circumscribed option and enter the value for the radius of circumscribed circle.
   Enter an option [Inscribed in circle/Circumscribed about circle] <I>: (Type C for this option)
   Specify radius of circle: (Specify the radius from the centre to the mid-point of a side)
3. ***Edge*** In this option, two end points of an edge of the polygon are to be specified. See Fig. 13.15(c).

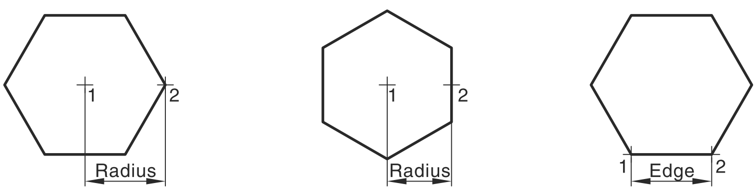


Fig. 13.15(a) *Inscribed in circle* **Fig. 13.15(b)** *Circumscribed* **Fig. 13.15(c)** *Edge*

Specify first end point of edge: (Specify first point)
Specify second end point of edge: (Specify second point)
The drawn polygon is treated as a single object. The individual sides can be edited only after the polygon is exploded by EXPLODE command.

13.15.4 PLINE command

This command is used to draw contiguous lines as a single entity called poly-lines. It can be invoked by any one of the following methods.

(a) Draw toolbar : 
(b) Command line : PLINE or PL
(c) Menu bar : DRAW ⇨ POLYLINE
(d) Tool palettes

Command window now shows the following command prompt.

Specify start point: (Specify first point)
Specify next point or [Arc/Half-width/Length/Undo/Width]: (Specify second point to create a straight line segment or enter an Option)
Specify next point or [Arc/Close/Half-width/Length/Undo/Width]: (Specify the next point or enter an Option)

The following options may be used before specifying the next point.

1. ***Arc*** This option is used to draw arcs with the help of the following sub-options.
   Specify end point of arc or [Angle/CEnter/CLose/Direction/Half-width/Line/Radius/ Second pt/ Undo/Width]: (Specify a value or enter an option)
   Most of these are similar to the ARC command options.
   (a) *Specify end Point Arc* If the end point of the arc is specified, this default option creates an arc tangent to the previous arc or line.
   (b) *Angle* Specify the included angle and its sub-options.
   (c) *Center* Specify the arc's centre and its sub-options.
   (d) *Close* This closes the poly-line by drawing an arc from the end point of the last arc to the start point of the poly-line.
   (e) *Direction* Specify the direction of the arc from the start point and its sub-options.
   (f) *Half Width* Specify the starting half-width and the ending half-width.
   (g) *Line* This returns to the main poly-line prompt to draw further line segments.
   (h) *Radius* Specify the arc's radius and its sub-options.
   (i) *Second Point* Specify the second point of the arc and its sub-options.
   (j) *Undo* To erase the last drawn entity.
   (k) *Width* Specify the starting width and the ending width.
   PLINE continues to display the arc sub-menu until the Line sub-option is opted or the command is terminated by pressing Esc, Enter or Spacebar.
2. ***Close*** It closes a poly-line by drawing a line from the end point of the last line segment to the start point of the poly-line. This appears only after specifying the second point.
3. ***Half Width*** Specify the starting half-width and the ending half-width, to create poly-lines that are tapered.
   Specify starting half-width <*current*>: (Specify a value or press ENTER)
   Specify ending half-width <*current*>: (Specify a new value or press ENTER for above value)
4. ***Length*** Specify the length of the next line segment. The option draws the line segment in the same direction as the last line segment or tangent to the last arc.
   Specify length of line: (Specify a value)
5. ***Undo*** To erase the last drawn entity.

6. ***Width*** Specify the starting width and the ending width of the poly-line.
   Specify starting width <*current*>: (Specify a value or press ENTER)
   Specify ending width <*current*>: (Specify a new value or press ENTER for above value)
   PLINE continues to prompt for more points, repeating the entire prompt each time in the same way as the LINE command. It can be terminated by pressing Esc, Enter or Spacebar. It may be noted that the value entered for ending half-width and ending width becomes the current value for subsequent PLINE command. Therefore, it is essential to reset the value incase they are different from the current value.

13.15.5 Circle Command

This command is used to draw a circle. It can be invoked by any one of the following methods.

(a) Draw toolbar : 
(b) Command line : CIRCLE or C
(c) Menu bar : DRAW ⇨ CIRCLE
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.16.

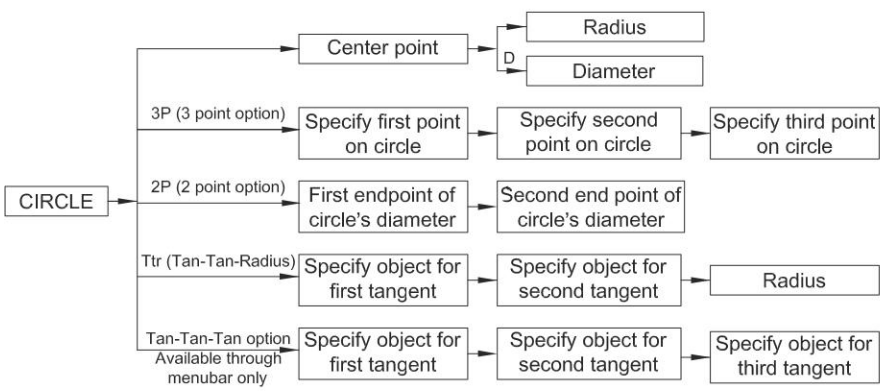


Fig. 13.16 *Ray diagram for CIRCLE command.*

Command window now shows the following command prompt.
Specify center point for circle or [3P/2P/Ttr (tan-tan-radius)]: (Specify a point or enter the option)

1. Center-radius option Draws a circle based on centre point and radius. See Fig. 13.17(a). In this method, after specifying centre point of the circle, specify its radius.
   Specify radius of circle or [Diameter] <*current*>: (Specify the radius or press ENTER to accept current radius)
2. Center-diameter option Draws a circle based on centre point and diameter. See Fig. 13.17(b). In this method, after specifying centre point of the circle, select diameter option and specify the diameter.

Specify radius of circle or [Diameter] <*current*>: (Enter d)
Specify diameter of circle <*current*>: (Specify diameter or press ENTER to accept current value)

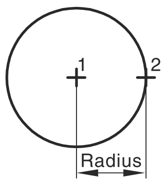


Fig. 13.17(a) *Radius*

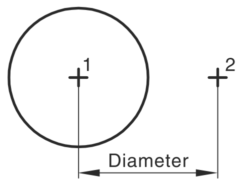


Fig. 13.17(b) *Diameter*

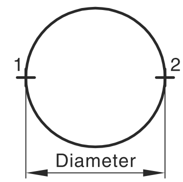


Fig. 13.17(c) *Two points*

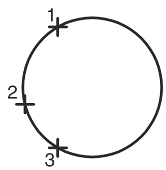


Fig. 13.17(d) *Three points*

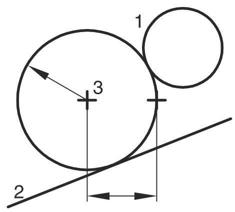


Fig. 13.17(e) *Tan-tan-radius*

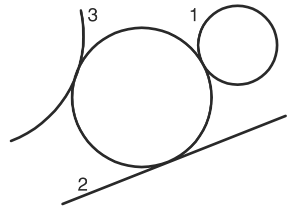


Fig. 13.17(f) *Tan-tan-tan*

3. Two-point option Draws a circle based on two end points of the diameter. See Fig. 13.17(c). In this method, value of endpoints of the circle's diameter is to be specified.
   Specify first endpoint of circle's diameter: (Specify the first end point)
   Specify second endpoint of circle's diameter: (Specify the second end point)
4. Three-point option Draws a circle based on three points on the circumference. See Fig. 13.17(d). In this method, three points are to be specified.
   Specify first point on circle: (Specify first point)
   Specify second point on circle: (Specify second point)
   Specify third point on circle: (Specify third point)
5. Tangent-tangent-radius option Draws a circle with a specified radius and tangent to two selected objects. See Fig. 13.17(e).
   Specify point on object for first tangent of circle: (Select a circle, arc, or line)
   Specify point on object for second tangent of circle: (Select another circle, arc, or line)
   Specify radius of circle <*current*>: (Specify either by picking two points or enter a value)
6. Tangent-tangent-tangent option This option is available at draw menu bar. It draws a circle which is tangent to three selected objects. See Fig. 13.17(f).
   Specify point on object for first tangent of circle: (Select a circle, arc, or line)
   Specify point on object for second tangent of circle: (Select another circle, arc, or line)
   Specify point on object for second tangent of circle: (Select last circle, arc, or line)

13.15.6 ARC Command

This command is used to draw arc which is defined as a part of a circle. The arc terminology in this context is shown in Fig. 13.18(a).

This command can be invoked by any one of the following methods.

(a) Draw Toolbar : 
(b) Command line : ARC or A
(c) Menu bar : DRAW ⇨ ARC
(d) Tool palettes

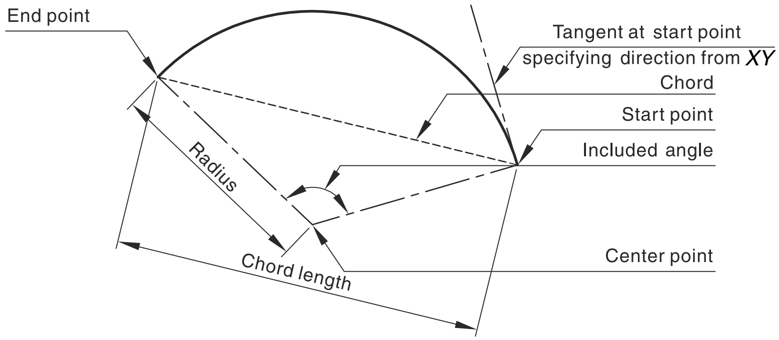


Fig. 13.18(a) *Arc terminology*

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.18(b).

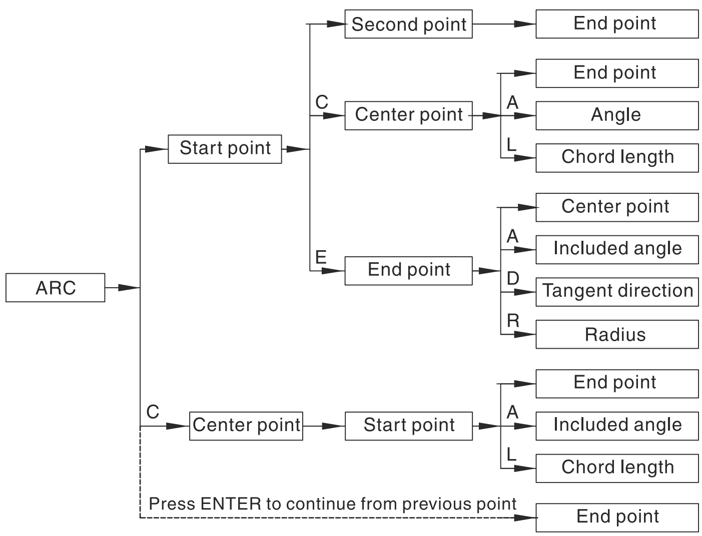


Fig. 13.18(b) *Ray diagram for ARC command*

Command window now shows the command prompt with twelve distinct options as given below.

1. The three points option See Fig. 13.19(a)

Specify start point of arc or [Center]: (Specify first point)
Specify second point of arc or [Center/End]: (Specify second point)
Specify end point of arc: (specify end point)

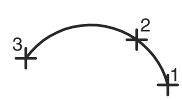


Fig. 13.19(a)

2. The start, centre, end option See Fig. 13.19(b)

   Specify start point of arc or [Center]: (Specify first point)
   Specify second point of arc or [Center/End]: (Enter C)
   Specify center point of arc: (Specify centre point)
   Specify end point of arc or [Angle/Chord length]: (Specify end point)

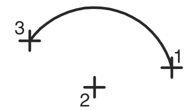


Fig. 13.19(b)

3. The start, centre, angle option See Fig. 13.19(c)

   Specify start point of arc or [Center]: (Specify first point)
   Specify second point of arc or [Center/End]: (Enter C)
   Specify center point of arc: (Specify centre point)
   Specify end point of arc or [Angle/Chord length]: (Enter A)
   Specify included angle: (Specify angle from the horizontal)

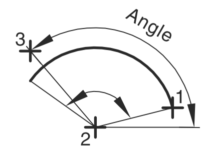


Fig. 13.19(c)

4. The start, centre, length option See Fig. 13.19(d)

   Specify start point of arc or [Center]: (Specify first point)
   Specify second point of arc or [Center/End]: (Enter C)
   Specify center point of arc: (Specify centre point)
   Specify end point of arc or [Angle/Chord length]: (Enter L)
   Specify length of chord: (Specify Chord length)

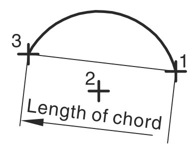


Fig. 13.19(d)

5. The start, end, centre option See Fig. 13.19(e)

   Specify start point of arc or [Center]: (Specify first point)
   Specify second point of arc or [Center/End]: (Enter E)
   Specify end point of arc: (specify end point)
   Specify center point of arc or [Angle/Direction/Radius]: (Specify centre point)

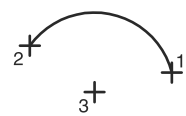


Fig. 13.19(e)

6. The start, end, angle option See Fig. 13.19(f)

   Specify start point of arc or [Center]: (Specify first point)
   Specify second point of arc or [Center/End]: (Enter E)
   Specify end point of arc: (Specify end point)
   Specify center point of arc or [Angle/Direction/Radius]: (Enter A)
   Specify included angle: (Specify angle from the horizontal)

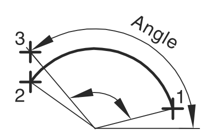


Fig. 13.19(f)

7. The start, end, direction option See Fig. 13.19(g)

   Specify start point of arc or [Center]: (Specify first point)
   Specify second point of arc or [Center/End]: (Enter E)
   Specify end point of arc: (Specify end point)
   Specify center point of arc or [Angle/Direction/Radius]: (Enter D)
   Specify tangent direction for the start point of arc: (Specify direction)

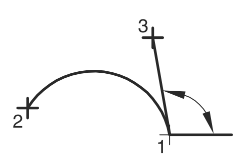


Fig. 13.19(g)

8. The start, end, radius option See Fig. 13.19(h)

   Specify start point of arc or [Center]: (Specify first point)
   Specify second point of arc or [Center/End]: (Enter E)
   Specify end point of arc: (Specify end point)
   Specify center point of arc or [Angle/Direction/Radius]: (Enter R)
   Specify radius of arc: (Specify radius)

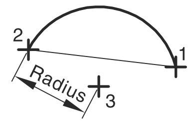


Fig. 13.19(h)

9. The centre, start, end option See Fig. 13.19(i)

   Specify start point of arc or [Center]: (Enter C)
   Specify center point of arc: (Enter centre point)
   Specify start point of arc: (Specify start point)
   Specify end point of arc or [Angle/Chord length]: (Specify end point)

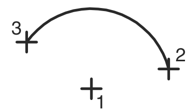


Fig. 13.19(i)

10. The centre, start, angle option See Fig. 13.19(j)

    Specify start point of arc or [Center]: (Enter C)
    Specify center point of arc: (Enter centre point)
    Specify start point of arc: (Specify start point)
    Specify end point of arc or [Angle/Chord length]: (Enter A)
    Specify included angle: (Specify angle from the horizontal)

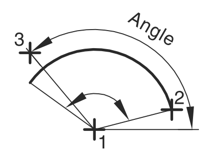


Fig. 13.19(j)

11. The centre, start, length option See Fig. 13.19(k)

    Specify start point of arc or [Center]: (Enter C)
    Specify center point of arc: (Enter centre point)
    Specify start point of arc: (Specify start point)
    Specify end point of arc or [Angle/Chord length]: (Enter L)
    Specify length of chord: (Specify Chord length)

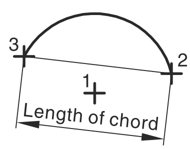


Fig. 13.19(k)

12. Continue option The arc will be tangent to the previous point. See Fig. 13.19(l)

    Specify start point of arc or [Center]: (Press ENTER)
    Specify end point of arc: (Specify end point)

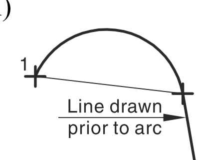


Fig. 13.19(l)

Note An arc may be closed to create a circle using JOIN command.

13.15.7 ELLIPSE Command

This command is used to draw an ellipse. It can be invoked by any one of the following methods.

(a) Draw toolbar : 
(b) Command line : ELLIPSE or EL

(c) Menu bar : DRAW ⇨ ELLIPSE
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.20.

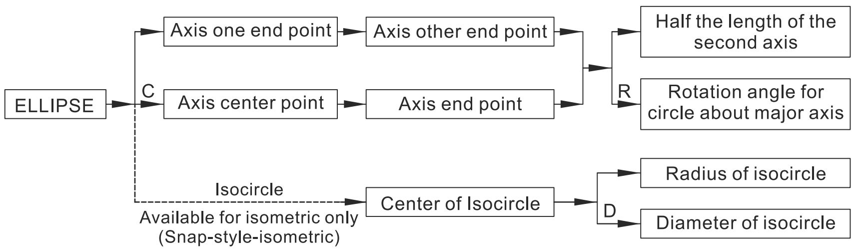


Fig. 13.20 *Ray diagram for ELLIPSE command*

Command window now shows the following command prompt.

Specify axis endpoint of ellipse or [Arc/Center]: (Specify a point or enter an option)

It offers the following ways to draw an ellipse.

1. ***Axis End Points***
   (a) In first two statements, specify the end points of the first axis. In the third statement, specify half of the length of the second axis, i.e., the distance of the end point of the second axis from the mid-point of the first axis. See Fig. 13.20(a)
   Specify axis end point of ellipse or [Arc/Center]: (Specify start point of first axis)
   Specify other end point of axis: (Specify end point of first axis)
   Specify distance to other axis or [Rotation]: (Specify end point of second axis)
   (b) In first two statements, specify the end points of the first axis. Now select rotation option and specify an angle between zero and 89.4 degrees. This draws the ellipse which is created by rotating a circle about the first axis. See Fig. 13.20(b)
   Specify axis end point of ellipse or [Arc/Center]: (Specify start point of first axis)
   Specify other end point of axis: (Specify end point of first axis)
   Specify distance to other axis or [Rotation]: (Enter R)
   Specify rotation around major axis: (Specify a point or enter a value between 0 and 89.4)
2. ***Center Option***
   (a) In this case, after specifying the center, the end point of the first axis from the center is specified. Now distance from the center to the end point of the second axis is specified next. See Fig. 13.20(c)
   Specify axis end point of ellipse or [Arc/Center]: (Enter C)
   Specify center of ellipse: (Specify center point)
   Specify other end point of axis: (Specify end point of first axis)

Specify distance to other axis or [Rotation]: (Specify end point of second axis)

(b) In this case, after specifying the center, the end point of the first axis from the center is specified. Now select the rotation option and specify an angle between zero and 89.4 degrees as earlier. See Fig. 13.20(d)

Specify axis end point of ellipse or [Arc/Center]: (Enter C)
Specify center of ellipse: (Specify center point)
Specify other end point of axis: (Specify end point of first axis)
Specify distance to other axis or [Rotation]: (Enter R)
Specify rotation around major axis: (Specify a point or enter a value between 0 and 89.4)

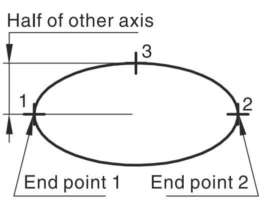


Fig. 13.20(a)

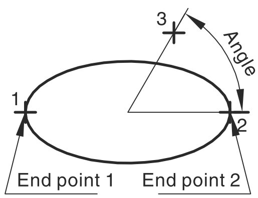


Fig. 13.20(b)

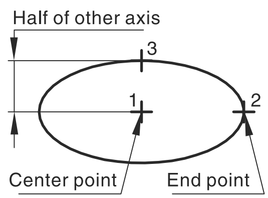


Fig. 13.20(c)

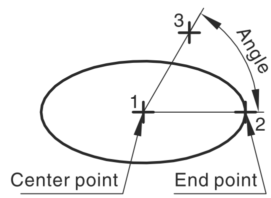


Fig. 13.20(d)

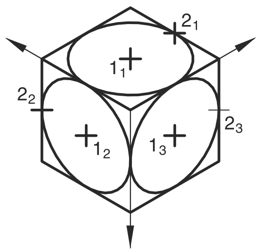


Fig. 13.20(e)

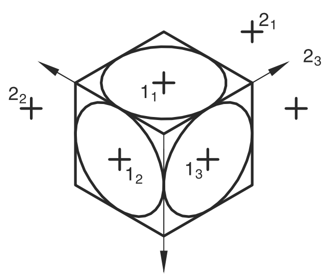


Fig. 13.20(f)

3. ***This Creates an Elliptical Arc*** The elliptical arc is beyond the scope of this text.
4. ***Isocircle*** This option is invoked by SNAP ⇨ Style ⇨ Isometric through command prompt. It is used to draw an ellipse in an isometric plane representing a circle. The following two options are available for drawing an isocircle.

(a) The option creates an isocircle by using centre and radius. Fig. 13.20(e) shows isocircles on three different isometric planes.

Specify center of isocircle: (Specify a center point)
Specify radius of isocircle or [Diameter]: (Specify a distance corresponding to radius)

(b) The other option creates an isocircle by using centre and diameter. Fig. 13.20(f) shows isocircles on three different isometric planes.

Specify center of isocircle: (Specify a center point)
Specify radius of isocircle or [Diameter]: (Enter D)
Specify diameter of isocircle: (Specify a distance corresponding to diameter)

13.15.8 SPLINE command

This command is used to draw a smooth curve called spline using NURBS (non-uniform rational B-splines) mathematics. It can be invoked by any one of the following methods.

(a) Draw toolbar : 
(b) Command line : SPLINE
(c) Menu bar : DRAW ⇨ Spline
(d) Tool palettes
Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.21.

The following command prompt appears in the command window.

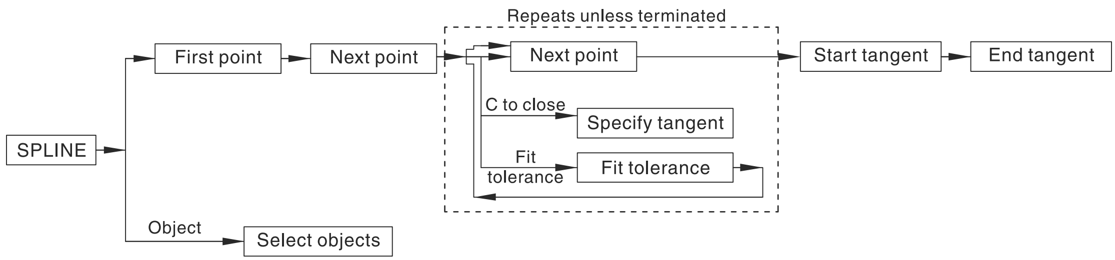


Fig. 13.21 *Ray diagram for SPLINE command*

Specify first point or [Object]: (Specify a point or select O for object option)

It can be created by specifying two or more contiguous points and tangents at both ends. Advantages of close option can be taken to close the spline by connecting the last point with the first point. Fit tolerance option specifies how closely the spline passes through the selected points. For its default value which is zero, it creates a spline that passes through each point as shown in Fig. 13.22(a). A tolerance of 5 will generate the curve passing though ±5 units from the selected points, as shown in Fig. 13.22(b).

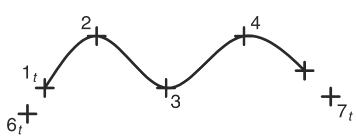


Fig. 13.22(a) *Spline with a tolerance of zero*

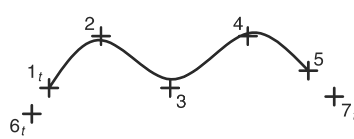


Fig. 13.22(b) *Spline with a tolerance of five*

Specify next point: (Specify a point)
Specify next point or [Close/Fit tolerance] <start tangent>: (Specify a point or enter an option)
Specify start tangent: (Specify tangent at start point)
Specify end tangent: (Specify tangent at end point)

Object Option This option changes internal definition a poly-line created with PEDIT's Spline option into a true spline.

Select objects to convert to splines...
Select objects: (Select 2D or 3D spline-fit polylines and press ENTER to finish)

13.15.9 XLINE Command

This command is used to draw lines that extend to infinity in both directions, known as construction lines, and can be used as references for creating other objects. It can be invoked by any one of the following methods.

(a) Draw Toolbar : 
(b) Command line : XLINE or XL
(c) Menu bar : DRAW ⇨ Construction Lines
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.23.

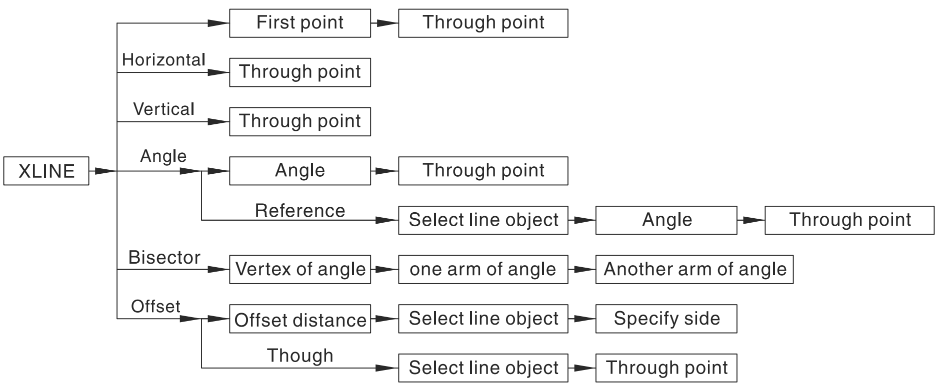


Fig. 13.23 *Ray diagram for XLINE command*

The following command prompt appears in the command window.

Specify a point or [Hor/Ver/Ang/Bisect/Offset]: (Specify a point or enter an option)

It can be created in any one of the following ways.

1. ***Two-point method*** This is default option. It draws a construction line by specifying two points.
   Specify through point: (Specify another point)
2. ***Horizontal*** It draws construction lines that pass through a given point parallel to the *X*-axis of the current UCS.
   Specify through point: (Specify a point)
3. ***Vertical*** It draws construction lines that pass through a given point parallel to the *Y*-axis of the current UCS.
   Specify through point: (Specify a point)

4. ***Angle*** It draws construction line in one of two ways. Either select a reference line and then specify the angle of the construction line from that line, or create a construction line at a specific angle to the horizontal axis by specifying an angle and then a point through which the construction line should pass.
   Enter angle of xline (0) or [Reference]: (Specify an angle or press R for reference option)
   Specify through point: (Specify a point)
5. ***Bisector*** It draws a construction line that bisects the given angle. Specify the vertex and the lines that create the angle.
   Specify angle vertex point: (Specify vertex of an angle)
   Specify angle start point: (Specify one side of the angle)
   Specify angle end point: (Specify another side of the angle)
6. ***Offset*** It draws a construction line parallel to the specified baseline. Specify the offset distance, select the baseline, and then indicate on which side of the baseline to locate the construction line.
   Specify offset distance or [Through] <*current*>: (Specify offset distance or enter an option)
   Select a line object: (Select a line)
   Specify side to offset: (Picking a point on one side the line to specify the side)

13.15.10 RAY command

This command is used to draw lines that extend to infinity in one direction, known as rays, and can also be used as referenc for creating other objects. It can be invoked by any one of the following methods.

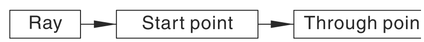


Fig. 13.24 *Ray diagram for RAY command*

(a) Command line : Ray
(b) Menu bar : DRAW ⇨ Ray

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.24.

It draws a ray by specifying two points. The command prompt appears in the command window as.

Specify start point: (Specify originating point)
Specify through point: (Specify another point for angle and direction)

13.16 EDITING A DRAWING

A drawing can be edited or modified in a number of ways with the help of the following commands

13.16.1 ERASE Command

This command is used to remove complete or a part of the drawing. To erase an object, select the object and choose Erase on the Modify toolbar or press CTRL+X or DELETE on the keyboard. Alternatively, choose Erase by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : ERASE or E
(c) Menu bar : MODIFY ⇨ ERASE
(d) Tool palettes

Fig. 13.25(a)

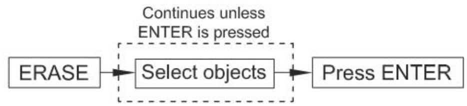


Fig. 13.25(b)

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.25(a) and (b).

The following command prompt appears in the command window.

Select objects: (Pick an object using grip or creating window box)

It prompts continuously to pick or select objects until ENTER is pressed. This erases all the picked objects.

13.16.2 MOVE Command

This command is used to move or displace a part of the drawing to a new location at a specified distance and direction. It can be invoked by any one of the following methods.

(a) Modify Toolbar : 
(b) Command line : MOVE or M
(c) Menu bar : MODIFY ⇨ MOVE
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.26.

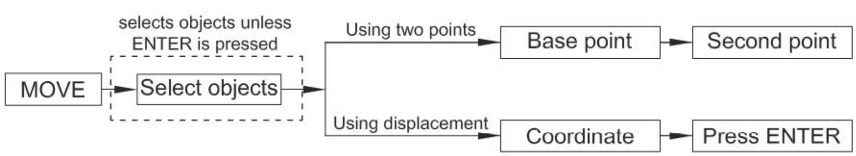


Fig. 13.26 *Ray diagram for MOVE command*

The following command prompt appears in the command window.

Select objects: (Select objects and press ENTER)

The drawn figure can be moved in any one of the following ways.

1. ***Two Points*** Continue to pick as many objects as needed to move. They all show grips (cursor) until ENTER is pressed. Now specify a base point anywhere on the drawing followed by a second point, either by picking on the screen or by specifying relative coordinates without using the symbol @.

Specify base point or [Displacement] <*Current Displacement*>: (Pick a base point)
Specify second point of displacement or <use first point as displacement>: (Specify second point)

2. ***Displacement*** Continue to pick as many objects as are needed to be moved. They all show grips (cursor) until ENTER is pressed. Now instead of specifying base point by picking, specify a co-ordinate value without using the symbol @. This value is used as a relative displacement. Press ENTER for the second point, as already all the necessary information is specified.
Specify base point or [Displacement] <*Current Displacement*>: (Specify a coordinate)
Specify second point of displacement or <use first point as displacement>: (Press ENTER)

13.16.3 COPY Command

This command is used to make multiple copies of a part or complete drawing. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : COPY or CO
(c) Menu bar : MODIFY ⇨ COPY
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.27.

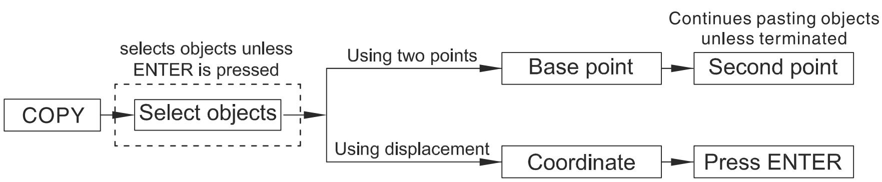


Fig. 13.27 *Ray diagram for COPY command*

The following command prompt appears in the command window.

Select objects: (Select objects and press ENTER)

The drawn figure can be copied in any one of the following ways.

1. ***Two Points*** Continue to pick as many objects as are needed to be moved. They all show grips (cursor) until ENTER is pressed. Now specify a base point anywhere on the drawing followed by successive points for multi-copies, either by picking on the screen or by specifying relative coordinates without using the symbol @.
Specify base point or [Displacement] <*Current Displacement*>: (Pick a base point)
Specify second point of displacement or <use first point as displacement>: (Specify second point)
Specify second point of displacement or <use first point as displacement>: (Press ENTER)

2. ***Displacement*** Continue to pick as many objects as are needed to be moved. They all show grips (cursor) until ENTER is pressed. Now instead of specifying base point by picking, specify a co-ordinate value without using the symbol @. This value is used as a relative displacement. Press ENTER for the second point as already all the necessary information is specified.
Specify base point or [Displacement] <*Current Displacement*>: (Specify a coordinate)
Specify second point of displacement or <use first point as displacement>: (Press ENTER)

13.16.4 ROTATE Command

This command is used to rotate the drawing or a part of it around a base point. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : ROTATE or RO
(c) Menu bar : MODIFY ⇨ ROTATE
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.28.

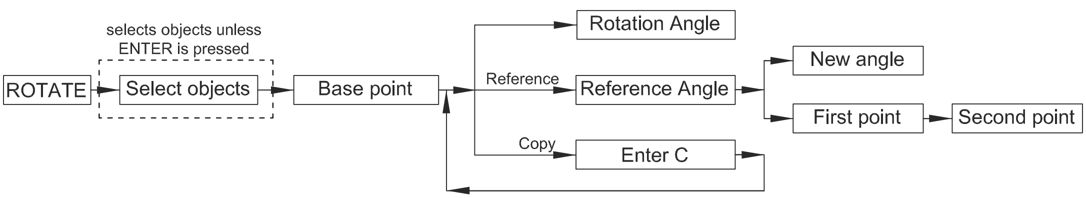


Fig. 13.28 *Ray diagram for ROTATE command*

The following command prompt appears in the command window.

Current positive angle in UCS: ANGDIR=counterclockwise ANGBASE=0
Select objects: (Select objects and press ENTER)

The first line shows the default settings. Here the settings show that the unit for angle measurement is the degree, reference is taken from *X*-axis and positive angle will turn the object counter-clockwise. These default settings can be changed with the help of 'Units' command.

At command prompts 'select objects', pick those objects that are needed to be rotated. Selecting the objects, show grips until ENTER is pressed. Now specify a fixed point about which objects are to be rotated and then specify the rotation angle, either directly specifying a value or using reference line. This erases the picked object and redraws it in a specified rotated position. Advantage of copy option may be taken in case it is desired to retain the picked objects, and proceed to specify rotation angle as earlier.

Specify base point: (Specify the base point about which the object is to be turned)
Specify rotation angle or [Copy/Reference] <*current*>: (Specify an angle or Enter an option)

13.16.5 MIRROR Command

This command is used to make a mirror copy of the selected objects about a specified line. They are mostly used for drawing symmetrical elements. One-half or one-quarter of the drawing is created and it is completed simply by mirroring the drawn elements. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : MIRROR or MI
(c) Menu bar : MODIFY ⇨ MIRROR
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.29.

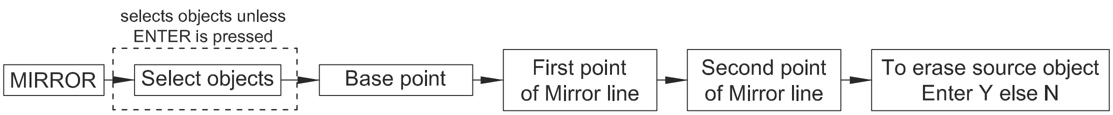


Fig. 13.29 *Ray diagram for MIRROR command*

The following command prompt appears in the command window.

Select objects: (Select objects and press ENTER)

Continue to pick as many objects as are required. They all show grips (cursor) until ENTER is pressed. Now specify two points of the mirror line which is an imaginary line across which the objects get mirrored. Finally enter Y or N, depending upon whether the picked object is to be removed or retained.

Specify first point of mirror line:
Specify second point of mirror line:
Erase source objects? [Yes/No] <*current*>: (Enter Y or N, or press ENTER)

13.16.6 OFFSET Command

This command creates concentric circles, parallel lines and parallel curves at specified distances. It can be invoked by any one of the following methods.

(a) Modify Toolbar : 
(b) Command line : OFFSET or O
(c) Menu bar : MODIFY ⇨ OFFSET
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.30.

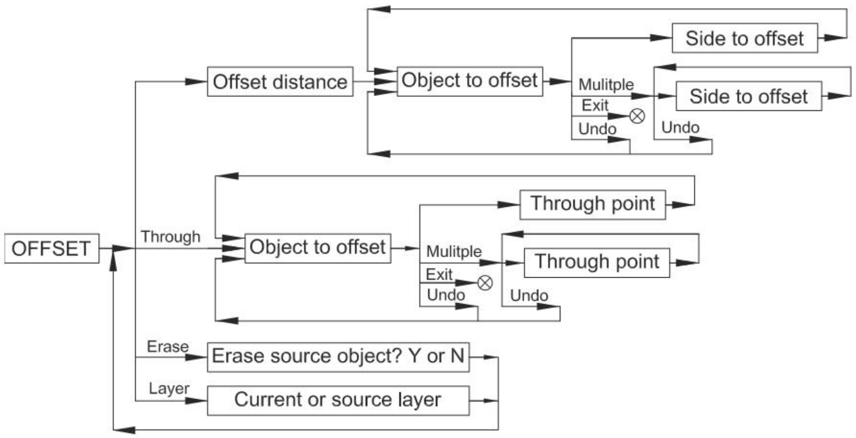


Fig. 13.30 *Ray diagram for OFFSET command*

The following command prompt appears in the command window.

Current settings: Erase source=No Layer=Source OFFSETGAPTYPE=0
Specify offset distance or [Through/Erase/Layer] <*current*>: (Specify distance or enter an option)

The first line shows the default setting. Here it shows that source object will not be erased and the new object will be on the source layer. The OFFSETGAPTYPE system variable controls how potential gaps between segments of closed poly-lines are treated when offset. The value zero fills the gaps by extending the poly-line segments, One fills the gaps with fillet arc segments and two fills the gaps with chamfered line segments.

The second line offers following ways to draw offset and change default settings.

1. ***Offset Distance*** If a value for offset distance is supplied, this option is activated. In this method, after specifying the offset distance, pick an object to offset and define the side on which offset part is to be drawn. See Fig. 13.31(a).
   Select object to offset or [Exit/Undo] <Exit>: (Select an object or an option)
   Specify point on side to offset or [Exit/Multiple/Undo] <Exit>: (Pick a point or enter an option)
2. ***Through Point*** It draws an offset object passing through a specified point. In this method, select through option and followed by selecting an object to offset. Then locate a point through which offset object should pass. See Fig. 13.31(b).
   Select object to offset or [Exit/Undo] <Exit>: (Select an object or an option)

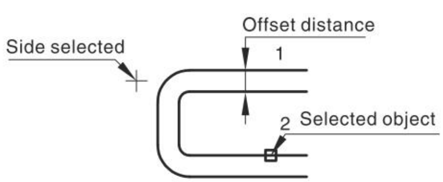


Fig. 13.31(a) *Offset distance*

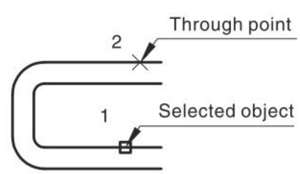


Fig. 13.31(b) *Through point*

Specify through point or [Exit/Multiple/Undo] <Exit>: (Specify point or enter an option)

In both the above cases, following options are available.

- Exit: To terminate the command.
- Multiple: To draw multiple copies of the selected object.
- Undo: To reverse the previous offset.

3. ***Erase*** This is a setting, if selected, it erases the source object after offset is complete.
   Erase source object after offsetting? [Yes/No]<*current*>: (Enter Y or N)
4. ***Layer*** This is also a setting which determines whether offset objects are created on the current layer or on the layer of the source object.
   Enter layer option for offset objects [Current/Source]<*current*>: (Give an option)

13.16.7 ARRAY Command

This command is used to create multiple copies of objects in a regular pattern either rectangular or polar (circular). It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : ARRAY or AR
(c) Menu bar : MODIFY ⇨ ARRAY
(d) Tool palettes

It displays the Array dialog box which provides two options in its top line as follows:

1. ***Rectangular Arrays*** This option is used to draw multiple copies of the selected object in a rectangular pattern. The dialog box for this is shown in Fig. 13.32(a). The following is the information to be supplied in any random sequence.
   (a) Select Objects Choose 'Select objects' button. The dialog box temporarily closes. Now start selecting objects whose array is to be drawn. Press ENTER to finish the selection. The Array dialog box is redisplayed, and the number of objects selected is shown below the Select objects button.
   (b) Rows Specify the number of rows in the array.
   (c) Columns Specify the number of columns in the array.
   By default, the maximum number of array elements that you can generate in one command is 100,000. The limit is set by the **MAXARRAY** setting in the registry.
   (d) Offset distance and direction This provides a space to specify the distance and direction of the array's offset.
   - Row offset Specify a value for the distance between rows or use the 'Pick both offsets' button or the 'Pick row offset' button.
   - Column offset Specify a value for the distance between columns or use the 'Pick both offsets' button or the 'Pick column offset' button.
   - Angle of array Specify the angle of rotation. For a value of zero, rows and columns created are orthogonal with respect to the X and Y drawing axes.

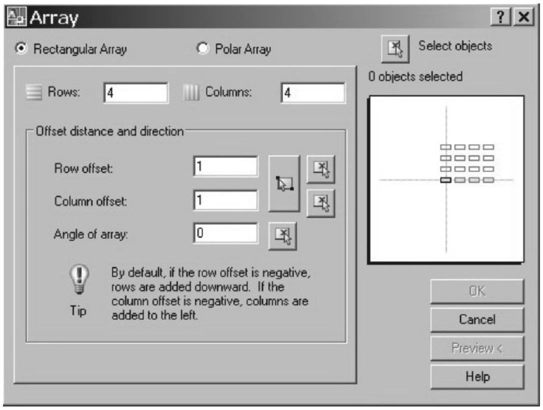


Fig. 13.32(a) *Rectangular Array dialog box*

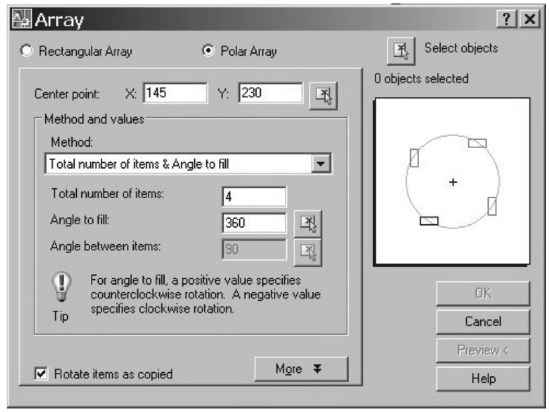


Fig. 13.32(b) *Polar Array dialog box*

2. ***Polar Arrays*** This option comes into action when polar option is selected from the first line of the array dialog box. The dialog box is shown in Fig. 13.32(b). It is used to make multi copies in the polar or circular pattern. The information to be supplied is given below.
   (a) Select objects Choose 'Select Objects' button and pick the objects whose array is to be drawn. Press ENTER to finish the selection process.
   (b) Center point Specify the centre point of the polar array. Enter coordinate values for *X* and *Y*, or use 'Pick Center Point' button for its location.
   (c) Method This setting controls the Method and Value fields available for specifying values.
   - Total number of items Specify the number of objects to be created.
   - Angle to fill Specify the included angle between the base points of the first and last elements in the array or use its button.
   - Angle between items Specify the included angle between the base points of the arrayed objects and the centre of the array or use pick button.

   (d) Rotate Items as copied: In case it is required to rotate the items when array is to be created, check this box.

13.16.8 SCALE Command

This command is used to enlarge or reduce selected objects proportionally. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : SCALE
(c) Menu bar : MODIFY ⇨ SCALE
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.33.

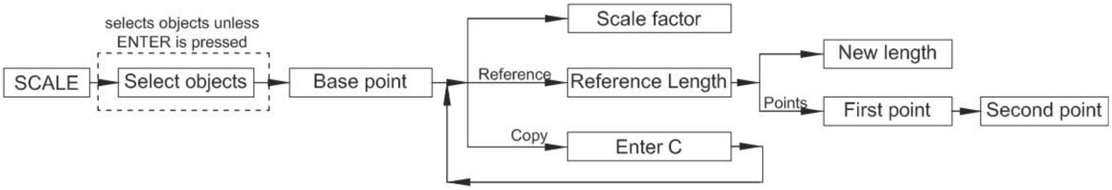


Fig. 13.33 *Ray diagram for SCALE command*

The following command prompt appears in the command window.

Select objects: (Select objects and press ENTER)
Specify base point: (Specify a point)
Specify scale factor or [Copy/Reference]: (Specify a scale or enter an option)

The first command prompts to select object(s). Continue to pick as many objects as desired and then press ENTER. Now it prompts to specify the base point and repress ENTER. This prompts to resize objects either using a scale factor or by reference option.

1. ***Scale Factor*** The common way to resize an object is to specify the scale factor. A scale factor between 0 and 1 shrinks the objects while a scale factor greater than 1 enlarges the objects. The most recent scale factor that is specified becomes the default for other scaling operations during the same session. See Fig. 13.34(a).
2. ***Reference*** Reference option scales the selected objects based on a reference length and a specified new length. See Fig. 13.34(b).
   Specify reference length <1.0000>: (Specify a distance or pick two points or press ENTER)
   Specify new length or [Points] <*current*>: (Specify a new distance or enter option)
3. ***Copy*** Advantage of copy option may be taken in case it is desired to retain the picked objects. Select this option and proceed further, to resize either by specifying the scale factor or choosing reference option.

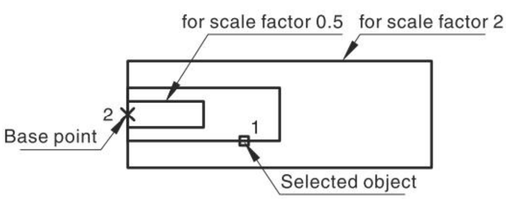


Fig. 13.34(a) *Scale factor*

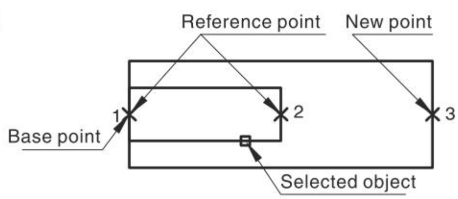


Fig. 13.34(b) *Reference*

13.16.9 TRIM Command

This command is used to trim objects at a cutting edge defined by other objects. Objects that can be trimmed include arcs, circles, elliptical arcs, lines, open 2D and 3D poly-lines, rays, splines, hatches, and xlines. To trim an object, the cutting edge is to be specified first, to define the point at which the object is to be trimmed. Valid cutting edge objects include 2D and 3D polylines, arcs, circles, ellipses, lines, layout view ports, rays, regions, splines, text, and xlines. When an object is selected, pick the object on the side that is to be trimmed (not on the side that is to be retained). An object can be used as both a cutting edge and an object to be trimmed in the same trimming process. Several cutting edges and objects may be selected at one time. The objects within blocks can also be trimmed. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : TRIM
(c) Menu bar : MODIFY ⇨ TRIM
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.35.

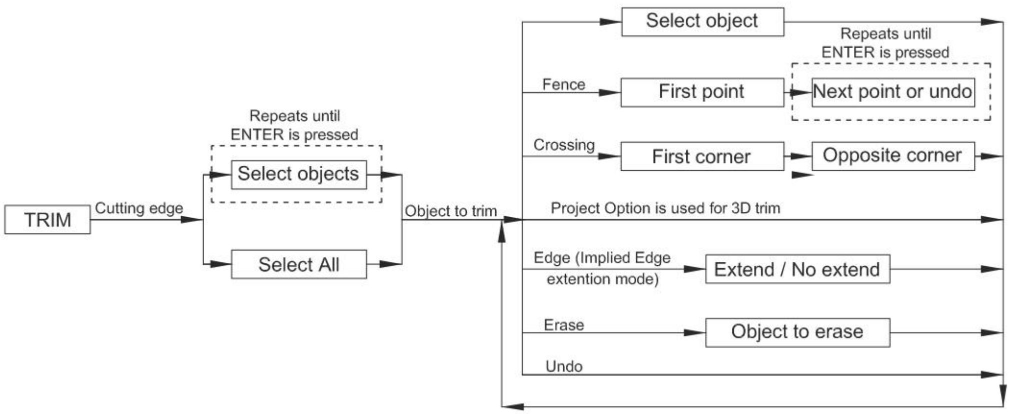


Fig. 13.35 *Ray diagram for TRIM command*

Command window prompts to select cutting edges along with default setting. The Projection setting is used only for 3D models and can trim based on either the current UCS or the current view. The Edge setting is used for implied intersections.

Current settings: Projection=UCS, Edge=None
Select cutting edges ...
Select objects: (Select objects and press ENTER)
Select object to trim or shift select to extend or [Fence/Crossing/Project/Edge/eRase/Undo]:

Continue to pick as many objects as desired as cutting edge and then press ENTER. Now it prompts to trim to an actual or an implied intersection (an intersection that would exist if objects were extended).

1. ***Actual Intersection*** To trim to an actual intersection, select the objects to trim. Fence option can be used to draw lines that criss-cross the objects to trim. Use the Crossing option to select the objects with a crossing window. Be sure to pick each object between the cutting edge and the end to trim off. Press Enter to end object selection. This action trims the object(s).
2. ***Implied Intersection*** An object can be trimmed to a cutting edge that would intersect the object if extended. This is called trimming to an implied intersection. For implied intersection, select the

Edge option by pressing E. Then select the object that is to be trimmed. Be sure to pick each object at or near the end that is to be trimmed. Press Enter to end object selection and trim the object(s).
Enter an implied edge extension mode [Extend/No extend] <No extend>: (Type E)
Select object to trim or shift select to extend or [Fence/Crossing/Project/Edge/eRase/Undo]:
Use Project option to change projection setting for 3D trim. The eRase option allows erasing an object instead of trimming it, without leaving the TRIM command.

13.16.10 EXTEND Command

This command is used to extend an object to meet another object. Objects that can be extended include circular or elliptical arcs, lines, open 2D and 3D poly-lines, and rays. Poly-lines, arcs, circles, ellipses, elliptical arcs, lines, rays, regions, splines, text, or xlines may be used as boundary edges. An object can be used as both a boundary edge and an object to be extended in the same extending process. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : EXTEND or EX
(c) Menu bar : MODIFY ⇨ EXTEND
(d) Tool palettes

Different options and sub-options available with this command are the same as those in the Trim command shown in Fig. 13.35.

Command window prompts to select boundary edges along with default setting. The Projection setting is used only for 3D models and can extend based on either the current UCS or the current view. The Edge setting is used for implied intersections.

Current settings: Projection=UCS, Edge=None
Select boundary edges ...
Select objects: (Select objects and press ENTER)
Select object to extend or shift-select to trim or [Fence/Crossing/Project/Edge/Undo]:

Continue to pick as many objects as desired as boundary edge and then press ENTER. Now it prompts to extend to an actual or an implied edge extension mode (extend an object to a boundary edge that would intersect the extended object if it were longer).

1. ***Actual Intersection*** If you want to extend to an actual intersection, select the objects that you want to extend. You can use the Fence option to draw lines that criss-cross the objects that you want to extend. Use the Crossing option to select the objects with a crossing window. Be sure to pick each object between the boundary edge and the end you want to extend. Press Enter to end object selection. This action extends the object(s).
2. ***Implied Edge Extension Mode*** You can extend an object to a boundary edge that would meet the object if extended. This is called extending to an implied edge extension mode. For Implied edge extension mode, select the Edge option by pressing E. Then select the objects that you want to extend. Be sure to pick each object at or near the end that you want to extend. Press Enter to end object selection and extend the object(s).

Enter an implied edge extension mode [Extend/No extend] <No extend>: (Type E)
Select object to extend or shift-select to trim or [Fence/Crossing/Project/Edge/Undo]:
Use Project option to change projection setting for 3D extend. Use the Undo option if the results of the trim are not what you want and then continue to select objects to trim.

13.16.11 BREAK (and Remove) Command

This command is used to break and remove the selected object between two points. A common use for BREAK is to break a wall at a door or a window in an architectural floor plan. You specify two points on the object, and the command erases whatever is between those two points. Typically, you use object snaps to specify the points. Sometimes, you can use TRIM to break an object, but if you have no convenient cutting edge, you may find BREAK more efficient. You can break lines, poly-lines, splines, xlines, rays, circles, arcs, donuts, ellipses and elliptical arcs. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : BREAK
(c) Menu bar : MODIFY ⇨ BREAK
(d) Tool palettes

The command prompt responds as:

Select object: (Select the object at first break point or select the entire object)

When breaking an object, you can either select the object at the first break point and then specify a second break point, or select the entire object and then specify two break points. Notice that you can only select one object to break. The prompt responds as.

Specify second break point or [First point]: (Specify the second point or enter F)

1. ***Specify the Second Point*** If you specify the second point, AutoCAD considers the point used towards the Select object as first break point. The command breaks the object between the two points.
2. ***First Point*** If you enter *F*, then you have to again specify the first point. It overrides the original first point with the new point that you specify. Now it prompts to specify the second break point. The command breaks the object between the two points.
   Specify first break point: (Pick the first break point)
   Specify second break point: (Pick the second point)
   To split an object in two without erasing a portion, enter the same point for both the first and second points. You can do this by entering @ to specify the second point.

13.16.12 Break (Split) Command

Sometimes you may want to split an object into two pieces at a point, without erasing any part of the object. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Tool palettes

The command prompt responds as follows.

Select object: (Select an object or specify the split point on an object)
Specify second break point or [First point]: _f
Specify first break point: (Override the split point or press ENTER)
Specify second break point: @

After selecting the object, pick where you want to split the object at the Specify second break point or [First point] prompt. The command splits the object. The two new objects look the same as before on the screen until you select one of the objects.

To split an object, you can also use break command and enter @ when command prompts to specify the second break point. Thus, the first and second break points are the same.

13.16.13 JOIN Command

This command is used to join objects to form a single, unbroken object. It is the opposite of breaking objects. You can join lines, poly-lines, arcs, elliptical arcs, splines and helixes. The objects must be along the same linear, circular, or elliptical path. The objects can overlap, have a gap between them, or touch end-to-end. The command can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : JOIN or J
(c) Menu bar : MODIFY ⇨ JOIN
(d) Tool palettes

The command prompt responds as folows.

Select source object: (Select the first object that you want to join)

AutoCAD knows which type of object you have selected for the first prompt and inserts it into the second prompt.

1. If your first object is a line, the command prompts as
   Select lines to join to source: (Select lines and press ENTER)
2. If your first object is a pline, the command prompts as
   Select objects to join to source: (Select objects and press ENTER)
3. If your first object is an arc, the command prompts as
   Select arcs to join to source or [cLose]: (Select arcs and press ENTER, or enter L)
4. If your first object is an elliptical arc, the command prompts as

Select elliptical arcs to join to source or [cLose]: (Select elliptical arcs and press ENTER, or enter L)

5. If your first object is a spline or helixe, the command prompts as
   Select splines or helixes to join to source: (Select splines or helixes and press ENTER)

You can continue to select other objects. The command joins the objects. A very nice touch is the ability to close arcs (to circles) and elliptical arcs (to ellipses). If your first object is either type of an arc, use close option to close them.

13.16.14 CHAMFER Command

This command bevels the edges of objects. It is a two-step process. First you define how you want to chamfer the corner, specifying either two distances from the corner or a distance and an angle. Then you select the two lines that you want to chamfer. The command can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : CHAMFER or CHA
(c) Menu bar : MODIFY ⇨ CHAMFER
(d) Tool palettes

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.36.

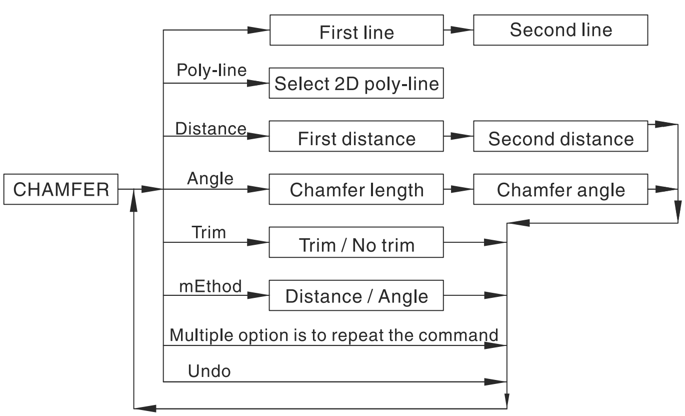


Fig. 13.36 *Ray diagram for CHAMFER command*

The following command prompt appears in the command window.

(TRIM mode) Current chamfer Dist1 = *current*, Dist2 = *current*

Select first line or [Undo/Polyline/Distance/Angle/Trim/mEthod/ Multiple]: (Select the first line or enter an option)

The first line gives information of the default setting. Here it shows that you are in trim mode and method for chamfer is distance. The second prompts to select a line or enter an option. The pick points on intersecting lines should be on the part of the lines that you want to keep, not on the part of the lines that you want to trim off. You can define two distances from a corner or one distance and an angle:

1. ***Distance Option*** It is used to define two distances from the corner.

   Specify first chamfer distance <*current*>: (Type the first chamfer distance or press ENTER)
   Specify second chamfer distance <*current*>: (Type the second distance or press ENTER)

   The default for the first chamfer distance is the distance defined previously. The default for second chamfer is always the first chamfer distance because equal chamfer distances are mostly used in practice. If you set both distances to zero, AutoCAD extends or trims the two lines so they end at the same point.

   If you are not creating a chamfer with equal distances, the order in which you select the lines is important. The command trims the first line selected by the first distance, and the second line selected based on either the second distance or the angle. At the Select second line: prompt, select the second line to chamfer the lines.

2. ***Angle Option*** It is used to define a distance from the corner and an angle.

   Specify chamfer length on the first line <*current*>: (Enter a distance or press ENTER)
   Specify chamfer angle from the first line <*current*>: (Type the angle between the first line and the chamfer line or press ENTER for default)

   By default, CHAMFER trims the original lines that it chamfers. If you want to keep the full original lines when you add the chamfer line, choose the Trim option and choose No Trim. Use the Multiple option to continue the prompts and chamfer several corners in one command. Use Polyline option to chamfers an entire 2D poly-line. The Undo option lets you undo your last chamfer and try again.

13.16.15 FILLET Command

This command is used to connect two objects with an arc that is tangent to the objects and has a specified radius. An inside corner is called a fillet and an outside corner is called a round, and you can create both by using the FILLET command. As with Chamfer, you can fillet lines, xlines, rays and poly-lines. You can also fillet circles, arcs, ellipses, elliptical arcs and splines. It can be invoked by any one of the following methods.

(a) Modify toolbar : 
(b) Command line : FILLET or F
(c) Menu bar : MODIFY ⇨ FILLET
(d) Tool palettes

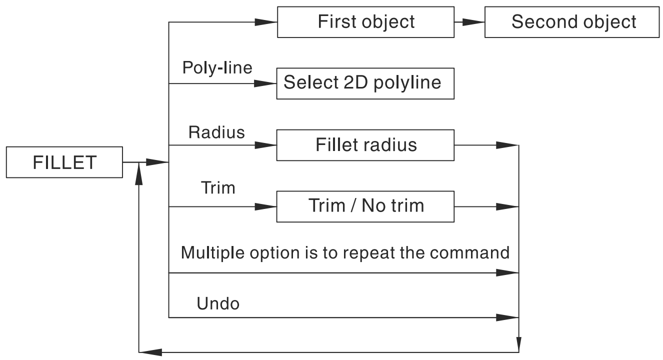


Fig. 13.37 *Ray diagram for FILLET command*

Different options and sub-options available with this command are summarized in the ray diagram shown in Fig. 13.37.

The following command prompt appears in the command window.

Current settings: Mode = TRIM, Radius = *current*

Select first object or [Undo/Polyline/Radius/Trim/Multiple]: (Use an object selection method or enter an option)

First line in the command prompts specifies the current settings. Here it shows that you are in trim mode. In the second line, the command prompts to select object or enter an option. You select the two objects or a poly-line that you want to fillet or round. Various options available with the command are given below.

1. ***Poly-line*** It is used to fillet an entire poly-line or remove fillets from an entire poly-line. If the Trim option is on, the filleted objects and the fillet arc join to form a single new poly-line.
2. ***Radius*** It is used to change the fillet radius to a new value.
   Specify fillet radius <*current*>: (Specify a value or press ENTER to accept current value)
3. ***Trim*** It is used to control whether the selected objects are trimmed or extended to the endpoints of the resulting arc or left unchanged.
   Enter Trim mode option [Trim/No trim] <*current*>: (Enter an option or press ENTER)
4. ***Multiple*** It is used to fillet more than one set of objects without leaving the command. FILLET displays the main prompt and the Select Second Object prompt repeatedly until you press ENTER to end the command.
5. ***Undo*** It reverses the previous action in the command.
   The value you enter for fillet radius and trim mode becomes the current value for subsequent FILLET commands. Changing this value does not affect existing fillet arcs.

13.17 MISCELLANEOUS EXAMPLES

Example 13.1

State a series of command steps required to reproduce Fig. 13.38 with the help of Line command, using absolute coordinate system.

Command: LINE
Specify first point: 0,0↵ (Assumption: Lower left corner of the object is at 0,0)
Specify next point or [Undo]: 90,0↵
Specify next point or [Undo]: 90,60↵
Specify next point or [Close/Undo]: 0,60↵
Specify next point or [Close/Undo]: c↵

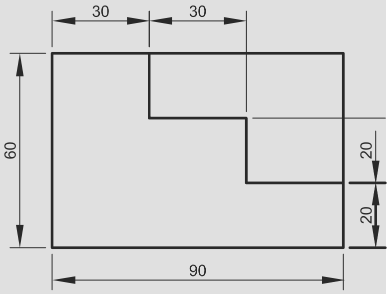


Fig. 13.38

Command: ↵ (Repeats the previous command, i.e., LINE)
Specify first point: 30,60↵
Specify next point or [Undo]: 30,40↵
Specify next point or [Undo]: 60,40↵
Specify next point or [Close/Undo]: 60,20↵
Specify next point or [Close/Undo]: 90,20↵
Specify next point or [Close/Undo]: ↵ (Press Esc, Enter or Spacebar to Exit)

Example 13.2

State a series of command steps required to reproduce Fig. 13.39 with the help of Line command, using relative rectangular coordinate system.

Command: LINE
Specify first point: 0,0↵ (Assumption: Lower left corner of the object is at 0,0)
Specify next point or [Undo]: @0,10↵
Specify next point or [Undo]: @15,0↵
Specify next point or [Close/Undo]: @0,30↵
Specify next point or [Close/Undo]: @15,0↵
Specify next point or [Close/Undo]: @15,–15↵
Specify next point or [Close/Undo]: @0,–5↵
Specify next point or [Close/Undo]: @10,0↵
Specify next point or [Close/Undo]: @0,5↵
Specify next point or [Close/Undo]: @15,15↵
Specify next point or [Close/Undo]: @15,0↵
Specify next point or [Close/Undo]: @0,–30↵
Specify next point or [Close/Undo]: @15,0↵
Specify next point or [Close/Undo]: @0,–10↵
Specify next point or [Close/Undo]: c↵

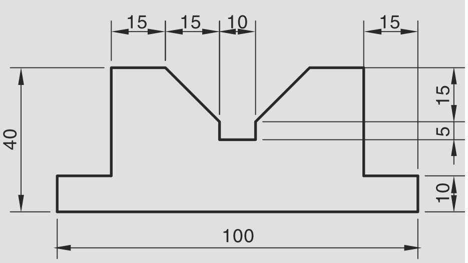


Fig. 13.39

Example 13.3

State a series of command steps required to reproduce Fig. 13.40 with the help of Line command, using relative rectangular polar coordinate system.

Command: LINE
Specify first point: 0,0↵ (Assumption: Lower left corner of the object is at 0,0)
Specify next point or [Undo]: @10<90↵
Specify next point or [Undo]: @15<0↵
Specify next point or [Close/Undo]: @20<60↵
Specify next point or [Close/Undo]: @10<0↵
Specify next point or [Close/Undo]: @10<270↵ (Type either @10<-90, @10<270 or –10<90)
Specify next point or [Close/Undo]: @20<0↵
Specify next point or [Close/Undo]: @10<90↵
Specify next point or [Close/Undo]: @10<0↵
Specify next point or [Close/Undo]: @20<300↵ (Type either @20<300 or @20<–60)
Specify next point or [Close/Undo]: @15<0↵
Specify next point or [Close/Undo]: @10<270↵
Specify next point or [Close/Undo]: @20<180↵ (Type either @20<180 or @–20<0)
Specify next point or [Close/Undo]: @5<270↵
Specify next point or [Close/Undo]: @50<180↵
Specify next point or [Close/Undo]: @5<90↵
Specify next point or [Close/Undo]: c↵

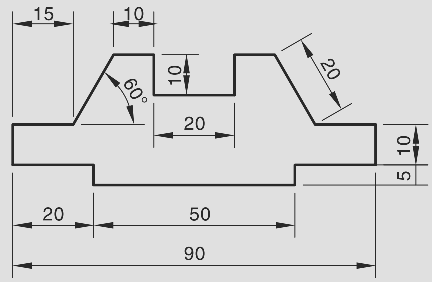


Fig. 13.40

Example 13.4

State a series of command steps required to reproduce Fig. 13.41, one-half with the help of Line command and another half using Mirror command.

Command: LINE
Specify first point: 0,0↵ (Assumption: Mid-point A of the base is at 0,0)
Specify next point or [Undo]: @25<180↵
Specify next point or [Undo]: @10<45↵
Specify next point or [Close/Undo]: @20<180↵
Specify next point or [Close/Undo]: @30<60↵
Specify next point or [Close/Undo]: @20<0↵
Specify next point or [Close/Undo]: @10<270↵
Specify next point or [Close/Undo]: @10<180↵
Specify next point or [Close/Undo]: @5<270↵
Specify next point or [Close/Undo]: @12.93<0↵
Specify next point or [Close/Undo]: ↵ (Press Esc, Enter or Space bar to Exit)

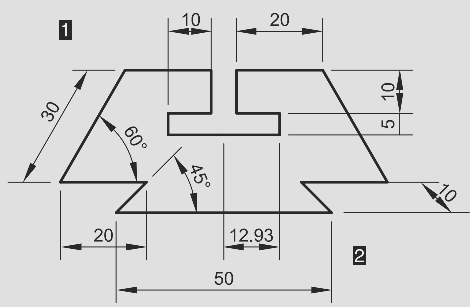


Fig. 13.41

Command: MIRROR
Select objects: Window↵ (Either select one-by-one all the nine lines or define by window)
Specify first corner: (Pick point 1 as windows first corner)
Specify opposite corner: (Pick point 2 as windows opposite corner)
13 found Select objects: ↵
Specify first point of mirror line: 0,0↵
Specify second point of mirror line: 0,15↵ (Specify any point on *Y*-axis)
Erase source objects? [Yes/No] <N>: ↵ (Considers No as default option)

Example 13.5

State a series of AutoCAD command steps required to draw Fig 13.42 with the help of Rectangle, Circle and polygon commands.

Command: RECTANGLE
Specify first corner point or [Chamfer/Elevation/Fillet/Thickness/Width]: C↵
Specify first chamfer distance for rectangles <0.0000>: 10↵
Specify second chamfer distance for rectangles <10.0000>: ↵
Specify first corner point or [Chamfer/Elevation/Fillet/Thickness/Width]: 0,0↵
Specify other corner point or [Area/Dimensions/Rotation]: 90,50↵

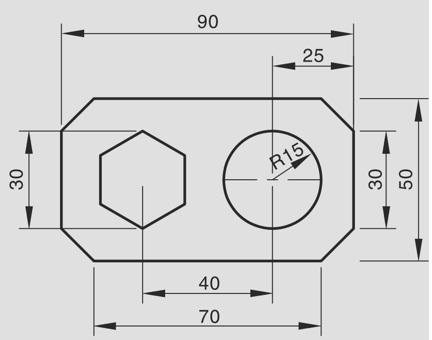


Fig. 13.42

Command: CIRCLE↵
Specify center point for circle or [3P/2P/Ttr (tan-tan-radius)]: 65,25↵
Specify radius of circle or [Diameter]: 15↵

Command: POLYGON↵
Enter number of sides <4>: 6↵
Specify center of polygon or [Edge]: 25,25↵
Enter an option [Inscribed in circle/Circumscribed about circle] <I>: ↵
Specify radius of circle: 25,10↵

Example 13.6

Two pentagons are drawn, one inscribed in a circle with a 30-mm radius and another 10 mm inside it, using OFFSET command. Then corners were joined to obtain final figure, as shown in Fig 13.43. State a series of AutoCAD command to reproduce it.

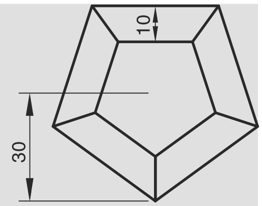


Fig. 13.43

Command: POLYGON↵

Enter number of sides <4>: 5↵
Specify center of polygon or [Edge]: 0,0↵
Enter an option [Inscribed in circle/Circumscribed about circle] <I>: ↵
Specify radius of circle: 0,–30↵

Command: OFFSET↵
Current settings: Erase source=No Layer=Source OFFSETGAPTYPE=0
Specify offset distance or [Through/Erase/Layer] <Through>: 10↵
Select object to offset or [Exit/Undo] <Exit>: 0,–30↵
Specify point on side to offset or [Exit/Multiple/Undo] <Exit>: 0,0↵
Select object to offset or [Exit/Undo] <Exit>: ↵

Command: LINE↵
Specify first point: (Pick a corner of the outer circle) ↵
Specify next point or [Undo]: (Pick the corresponding corner of the inner circle) ↵
Specify next point or [Undo]: (Press Esc, Enter or Spacebar to exit the command) ↵
Similarly, join other five corners using LINE command or use polar array as below.

Command: ARRAY↵
(Check polar array box and specify total number of items: 5 and Angle to fill: 360)
Specify center point of array: 0,0↵
Select objects: (Pick the line joining the corners of the pentagon)
Select objects: ↵ (Press ENTER and Select OK in the Array dialog box)

Example 13.7

State a series of AutoCAD command steps to draw an ellipse with 100-mm and 60-mm long axis and also to draw a curve parallel to this ellipse 20 mm away from it.

Command: ELLIPSE↵
Specify axis end point of ellipse or [Arc/Center]: 0,0↵ (Specify point 1)
Specify other end point of axis: @100<0↵ (Specify point 2)
Specify distance to other axis or [Rotation]: 30↵ (Enter 30 as its value or specify point 3)

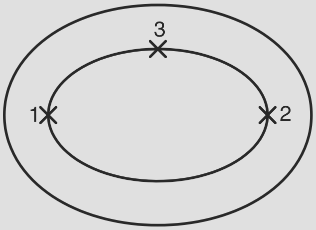


Fig. 13.44

Command: OFFSET↵
Current settings: Erase source=No Layer=Source OFFSETGAPTYPE=0
Specify offset distance or [Through/Erase/Layer] <*current*>: 20↵ (Specify 20 as its value)
Select object to offset or [Exit/Undo] <Exit>: (Select the ellipse) ↵
Specify point on side to offset or [Exit/Multiple/Undo] <Exit>: (Select point outside ellipse)

Example 13.8

State a series of AutoCAD command steps, including Array command, to reproduce Fig 13.45.

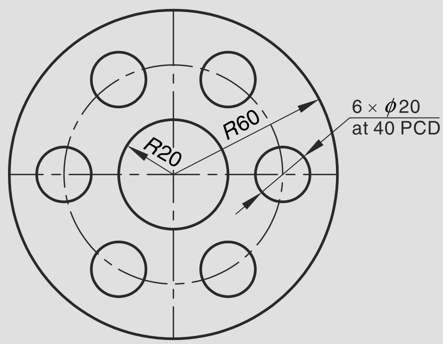


Fig. 13.45

Command: CIRCLE↵
Specify center point for circle or [3P/2P/Ttr (tan-tan-radius)]: 0,0↵
Specify radius of circle or [Diameter]: 60↵
Command: ↵ (Press Enter to repeat the previous command i.e. CIRCLE)
Specify center point for circle or [3P/2P/Ttr (tan-tan-radius)]: 0,0↵
Specify radius of circle or [Diameter] <60.0000>: 20↵

Command: ↵
Specify center point for circle or [3P/2P/Ttr (tan-tan-radius)]: 40,0↵
Specify radius of circle or [Diameter] <20.0000>: 10↵

Command: ARRAY
(Check polar array box and specify Total number of items: 6 and Angle to fill: 360)
Specify center point of array: 0,0↵
Select objects: (Pick circle of 10 diameter) ↵
Select objects: ↵ (Press ENTER and Select OK in the Array dialog box)

Example 13.9

State a series of AutoCAD command steps, including Trim command, to reproduce Fig 13.47(a).

Command: POLYGON ↵
Enter number of sides <4>: 6 ↵
Specify center of polygon or [Edge]: 0,0 ↵
Enter an option [Inscribed in circle/Circumscribed about circle] <I>: ↵
Specify radius of circle: 50 ↵

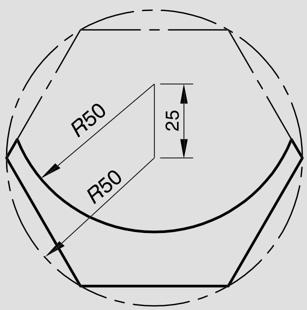


Fig. 13.46(a)

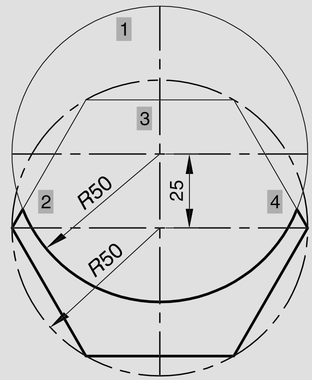


Fig. 13.46(b) *Showing construction lines*

Command: CIRCLE↵
Specify center point for circle or [3P/2P/Ttr (tan-tan-radius)]: 0,25↵
Specify radius of circle or [Diameter]: 50↵

Command: TRIM↵
Current settings: Projection=UCS, Edge=None
Select cutting edges ...
Select objects or <select all>: ALL↵
2 found
Select objects: ↵
Select object to trim or shift-select to extend or [Fence/Crossing/Project/Edge/eRase/Undo]: (Pick point 1 on the circle)
Select object to trim or shift-select to extend or [Fence/Crossing/Project/Edge/eRase/Undo]: (Pick point 2 on the hexagon)
Select object to trim or shift-select to extend or [Fence/Crossing/Project/Edge/eRase/Undo]: (Pick point 3 on the hexagon)
Select object to trim or shift-select to extend or [Fence/Crossing/Project/Edge/eRase/Undo]: (Pick point 4 on the hexagon)
Select object to trim or shift-select to extend or [Fence/Crossing/Project/Edge/eRase/Undo]:

EXERCISE 13

1. State a series of AutoCAD command steps to reproduce Figs. E13.1 and E13.2 using LINE command.

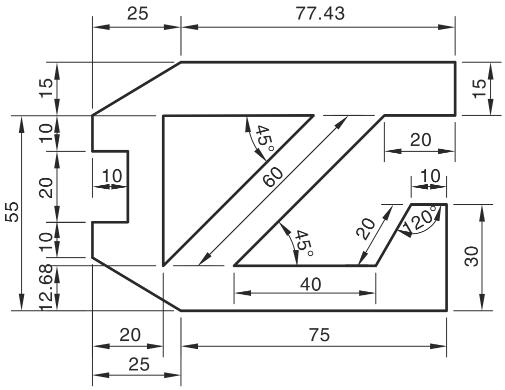


Fig. E13.1

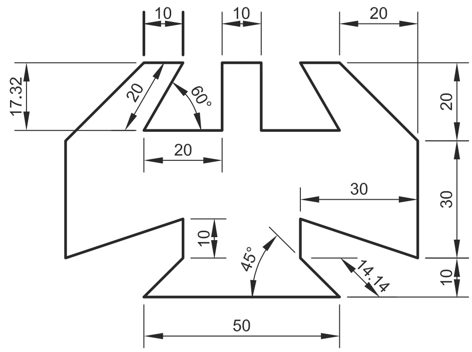


Fig. E13.2

2. State a series of AutoCAD command steps to reproduce Fig. E13.3 and E13.4

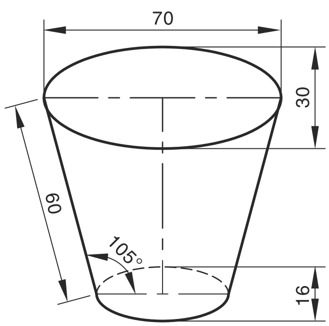


Fig. E13.3

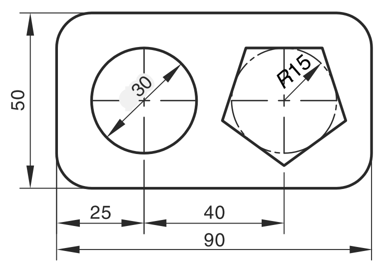


Fig. E13.4

3. State a series of AutoCAD command steps to reproduce Figs. E13.5 and Fig. E13.6, showing use of ARRAY command for making multiple copies.

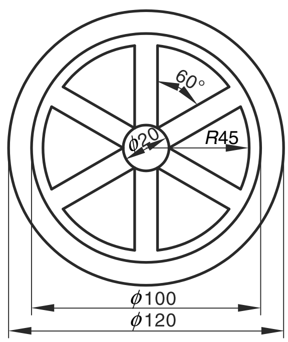


Fig. E13.5

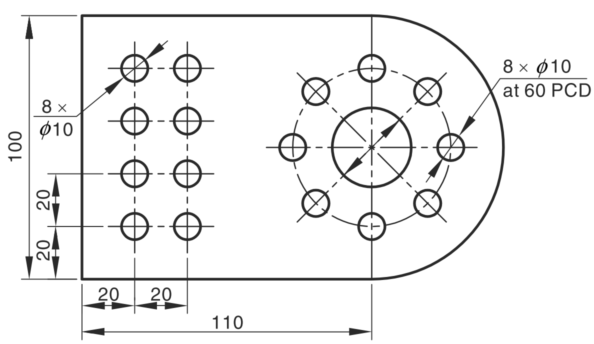


Fig. E13.6

4. Reproduce Figs. E13.7 to E13.10 on AutoCAD spread-sheet.

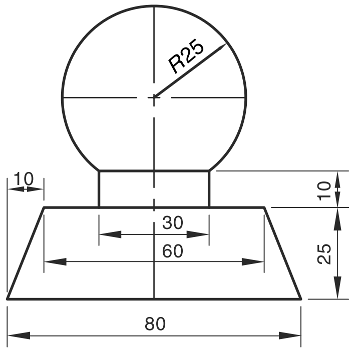


Fig. E13.7

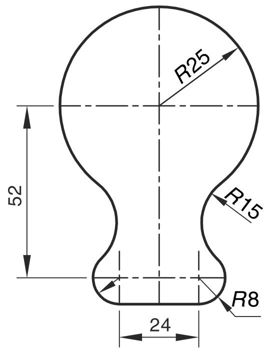


Fig. E13.8

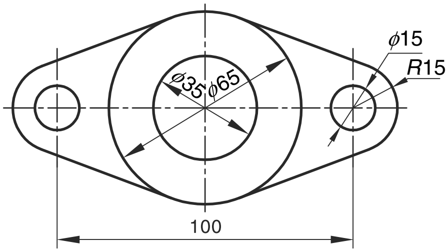


Fig. E13.9

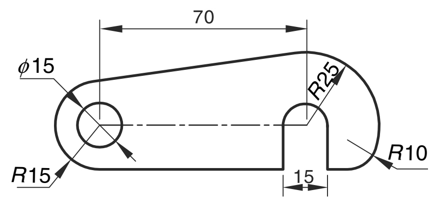


Fig. E13.10

5. Draw three orthographic views of the objects shown in Figs. E13.11 and E13.12.

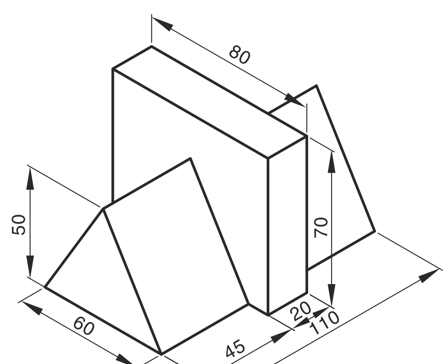


Fig. E13.11

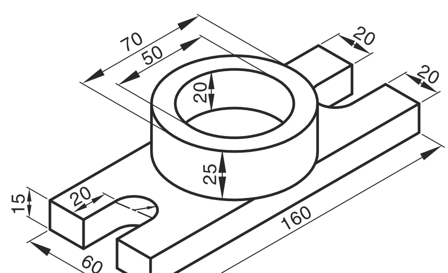


Fig. E13.12

REVIEW QUESTIONS

1. What is CAD? Name two CAD softwares. Give two limitations of manual drawing and three advantages of computer aided drawing and drafting.
[RGPV June 2008, April 2009, June 2009, Feb. 2010, April 2010, Aug. 2010, Dec. 2010, Feb. 2011]
2. Name any five input devices used in computers and describe them in brief.
3. Give a brief description of output and storage devices of a computer.
4. Give a brief description of the facilities available in the AutoCAD status bar.
5. Explain the role of Units and Limits commands in setting up the AutoCAD drawing space.
6. Describe the types of length and angle units that can be set in AutoCAD. Give suitable examples of each.
7. Explain the different settings available in the Units dialogue box of AutoCAD.
8. Name any four common methods by which a command can be executed.
9. Name various methods of locating a point in CAD and explain any two of them.
[RGPV Dec. 2008, April 2010]
10. Describe the following commands to regulate the cursor movement for locating a point quickly. (a) Snap and Grid (b) Polar tracking (c) Ortho
11. Name any five commands with their corresponding icons available under (a) draw toolbar (b) modify toolbar.
12. With the help of a ray diagram explain different options that are available in the following commands. (a) Rectangle (b) Arc (c) Xline (d) Ellipse
13. With the help of a ray diagram and suitable examples explain different options that are available in the following commands. (a) Line (b) Polygon (c) Circle (d) Spline
14. Name and explain five edit commands used in CAD.
[RGPV Feb. 2008, June 2008, April 2009, June 2009, Feb. 2010, April 2010, Aug. 2010, Dec. 2010]
15. Explain the following commands in brief. (a) Move (b) Array (c) Chamfer (d) Hatch
[RGPV Feb. 2011]
16. Explain different options that are available in the following commands with the help of ray diagram. (a) Move (b) Copy (c) Rotate (d) Mirror
17. Explain different options that are available in the following commands with the help of ray diagram. (a) Offset (b) Trim (c) Chamfer (d) Fillet
18. Which command is commonly used to create multiple copies in a rectangular or a polar pattern? Explain different settings available in the dialogue boxes for creating them.
19. Explain any two methods of drawing a circle in AutoCAD.
[RGPV Sep. 2009, April 2010]
20. Explain any four methods of drawing an arc in AutoCAD.
[RGPV Dec. 2008, June 2009]
21. State a series of AutoCAD command steps to draw a rectangle of 60 cm × 40 cm with the help of Line commands.
[RGPV Dec. 2008, Aug. 2010]

22. Write the steps for drawing a pentagon of 50 mm side.

[RGPV Feb. 2008]

23. State a series of AutoCAD command steps to draw the top view of frustum of a hexagonal pyramid assuming suitable dimensions. You may use Polygon, Offset, Line and Array commands.
24. State a series of AutoCAD command steps to draw an isometric view of frustum of a square pyramid with the help of Line command or otherwise. Assume suitable dimensions.
25. State a series of AutoCAD command steps that will be required to draw an ellipse of given major and minor axes. How would you draw another ellipse parallel to it at a specified distance?
26. State a series of AutoCAD command steps to draw a pentagon inscribed in a circle of 70 mm diameter and also to draw five circles of equal diameter, each touching one side of the pentagon and two adjacent circles.

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer out of the alternatives given below:

(i) What should you pay attention to when learning AutoCAD?
(a) The Command Line (b) The Status Bar
(c) The Title Bar (d) Floating toolbars

(ii) What does WCS stand for?
(a) Western CAD System (b) Worldwide Coordinate Sectors
(c) World Coordinate System (d) Wrong CAD Setting

(iii) Status bar does not contain
(a) snap (b) grid (c) erase (d) polar

(iv) Coordinates for the current position of crosshair of the cursor is seen in
(a) menu bar (b) standard toolbar
(c) properties toolbar (d) status bar

(v) Units command of AutoCAD is **not** used to set
(a) units for linear measurement (b) units for angular measurement
(c) limits of drawing (d) direction in which angle is to be measured

(vi) Polar coordinates are used mostly for drawing
(a) circles (b) arcs (c) vertical lines (d) angled lines

(vii) The number of points needed to draw a line using Absolute Coordinates is
(a) none (b) one (c) two (d) four

(viii) If a line is drawn between points 1,5 and –3,5, its absolute length is
(a) three units (b) four units
(c) five units (d) insufficient data

(ix) How long will a line from 1,5 to @5<10 be?
(a) One unit (b) Four units
(c) Five units (d) Ten units

(x) To move an object six units to the right, what would be the second point of displacement?
(a) @6,0 (b) @6<0 (c) Both 1 and 2 (d) Neither 1 nor 2

(xi) Which one is **not** a valid option of Units command?
(a) Architectural (b) Decimal (c) Metre (d) Metric

(xii) Snap command is used to regulate the cursor movement to the specified increments along
(a) vertical axis (b) horizontal axis
(c) cartesian coordinates (d) polar angles

(xiii) What can be contained in a template drawing?
(a) Sheet layout (b) Dimension styles
(c) Text styles (d) All of these

(xiv) How many points do you need to define for the Rectangle command?
(a) One (b) Two (c) Three (d) Four

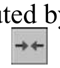
(xv) Which one of the following in **not** a valid option for drawing a circle?
(a) 3 Points (b) Tan-Tan Centre
(c) Tan-Tan-Radius (d) Tan-Tan-Tan

(xvi) 2-points option is used to draw circles by specifying the
(a) two end points of a diameter (b) two end points of a radius
(c) radius and tangent to two objects (d) center and two end points of a chord

(xvii) Xline command is used to draw line that
(a) extends up to a specified point (b) extends up to another line
(c) extends infinity in one direction only (d) extends infinity in both directions

(xviii) Offset command can be used for drawing
(a) infinite long lines (b) parallel lines
(c) intersecting lines (d) perpendicular bisectors

(xix) Objects are rotated around the
(a) base point (b) bottom right of the object
(c) centre of the object (d) origin

(xx) Join command is executed by clicking the icon
(a)  (b)  (c)  (d) 

(xxi) Scaling objects makes them
(a) bigger (b) smaller
(c) it only stretches them (d) both bigger and smaller

(xxii) When using the TRIM command, which do you select first?
(a) The cutting edges (b) The object to be trimmed
(c) Everything (d) Nothing

(xxiii) The term used by most CAD systems for "rounding corners" is
(a) chamfer (b) curve (c) fillet (d) smooth

(xxiv) The fillet command creates
(a) sharp corners (b) round corners
(c) angled corners (d) smooth corners

(xxv) Which of the following is **incorrect** statement?
(a) Chamfer command is used to bevel the edges.
(b) Fillet command is used to round the corners.
(c) Array command is used to draw multiple copies.
(d) Scale command is used to draw plain scales.

Answers

(i)	(a)	(ii)	(c)	(iii)	(c)	(iv)	(d)	(v)	(c)	(vi)	(d)
(vii)	(c)	(viii)	(b)	(ix)	(c)	(x)	(c)	(xi)	(c)	(xii)	(c)
(xiii)	(b)	(xiv)	(b)	(xv)	(b)	(xvi)	(a)	(xvii)	(d)	(xviii)	(b)
(xix)	(a)	(xx)	(b)	(xxi)	(d)	(xxii)	(a)	(xxiii)	(c)	(xxiv)	(b)
(xxv)	(d)										

B. E. (First Semester) EXAMINATION, Feb., 2010
(Common for all Branches)

BE-105 ENGINEERING GRAPHICS

Time: 3 Hrs **Max. marks: 80** **Min. Marks: 28**

Note: Attempt any five questions. All questions carry equal marks. Answer the questions in drawing sheet only.

Unit-I

1. (a) Draw a scale of 1:50 showing metres and decimetres and to measure up to 6 metres. 8

Similar to Example 2.1, Page 2.4

(b) The major and the minor axes of an ellipse are 125 mm and 100 mm long respectively. Draw the curve by any one of the standard method and locate its focii. 8

Similar to Example 3.27, Page 3.35

Or

2. (a) Construct a cycloid having a rolling circle diameter of 50 mm. Draw a normal and tangent to the curve at a point 35 mm above the base line. 8

Refer Example 4.1, Page 4.2

(b) A line AB of 100 mm length is inclined at an angle 30º to the H.P. and parallel to V.P. The point A is 15 mm above H.P. and 20 mm in front of V.P. Draw the front view and top view of the line. 8

Refer Example 7.10, Page 7.12

Unit-II

3. A line AB of 80 mm long has its end A, 15 mm from both H.P. and V.P. The other end B is 40 mm above H.P. and 50 mm in front of V.P. Draw the projections of the line and determine inclinations of the line with H.P. and V.P. 16

Refer Example 7.24, Page 7.31

Or

4. (a) Draw the projections of a circle of 60 mm diameter resting on V.P. on a point of the circumference. The plane is inclined at 45º to V.P. and perpendicular to the H.P. the centre of the plane is 40 mm above H.P. 8

Similar to Example 8.8, Page 8.9

(b) A cube of 40 mm side is resting with a face on H.P. such that the vertical faces are equally inclined to V.P. Draw its projections. 8

Similar to Example 9.2(c), Page 9.8

Unit-III

5. A hexagonal prism of base 25 mm and axis 45 mm long is positioned with one of its base edges on H.P. such that the axis is inclined at 30º to H.P. and 45º to the V.P. Draw the projections of the prism by change of position method. 16

Refer Example 9.40, Page 9.47

Or

6. (a) A hexagonal pyramid of side of base 30 mm and axis 60 mm long is resting on its base on H.P. with an edge of the base perpendicular to the V.P. It is cut by a section plane inclined at 30º to the H.P. and passing through the axis at 20 mm from the base. Draw the sectional view, top view and true shape of the section. 8

Similar to Example 10.15, Page 10.19

(b) A square prism of side of base 40 mm and axis 80 mm long is resting on its base on H.P. such that a rectangular face of it is parallel to V.P. Draw the development of the prism. 8

Refer Example 11.1, Page 11.3

Unit-IV

7. (a) A cone of diameter of base 60 mm and height 65 mm rests with its base on H.P. A cutting plane perpendicular to V.P. and inclined at 30º to H.P. cuts the cone such that it passes through a point on the axis at a distance 30 mm above the base. Draw the isometric projections of the cone. 8

Refer Exercise 12, Problem 20, Page 12.43

Or

8. (a) Write two advantages of CAD. 2

Refer Page 13.2

(b) Name two editing commands used in CAD. 2

Refer Page 13.30

(c) Name two CAD softwares. 2

Refer Page 13.2

(d) State whether the following statements are true or false: 10

(i) The R.F. of an enlarging scale is more than 1. True

(ii) The scale of chord is used to measure distances in mm, cm, sm. False

(iii) Conic sections like ellipse are generated by cutting a right circular cone in different positions relative to the axis of the cone by a cutting plane. True

(iv) The projecting lines meet the plane of projection at an angle of 80º to it. False

(v) When a point is above H.P., its front view lies below XY. False

(vi) When a straight line is parallel to H.P. its front view will be equal to its true length. False

(vii) Using change of position method, projections of a solid inclined to reference planes could be obtained. True

(viii) The section lines in a sectional view are drawn at an angle of 85º to the outlines. False

(x) Isometric projections is a pictorial projection of an object in which all the three dimensions of an object is revealed. True

(x) Computer aided drafting help in making speedier and accurate projection of an object. True

B. E. (First Semester) EXAMINATION, Apr., 2010

(Common for all Branches)

BE-105 ENGINEERING GRAPHICS

Time: 3 Hrs **Max. marks: 80** **Min. Marks: 28**

Note: Attempt any five questions selecting one question from each unit. Assume suitable misprint/missing data. Draw in first angle projection unless stated otherwise. Answer the questions in drawing sheet only.

Unit-I

1. (a) In a map of Bhopal, a distance of 36 km between two localities is shown by a line of 45 cm long. Calculate its R.F. and construct a plain scale to read kilometres and hectometres. Show a distance of 9.3 km on it. 8

Refer Example 2.5, Page 2.7

(b) A circle of 50 mm diameter rolls along a straight line without slipping. Draw the curve traced out by a point P on the circumference for one complete revolution of the circle. Name the curve also. 8

Refer Example 4.1, Page 4.2

Or

2. (a) Using the scale of chords, construct angles of 45º and 60º. 8

Similar to Example 2.28, Page 2.26

(b) The major axis of an ellipse is 150 mm long and the minor axis is 100 mm long. Draw the ellipse and then a tangent to the ellipse at a point on it 25 mm above the major axis. 8

Similar to Example 3.26, Page 3.35

Unit-II

3. (a) Draw the projection of the following points on the same ground lines, keeping the projectors 15 mm apart: 8

(i) A in the H.P. and 20 mm behind V.P.
(ii) B 25 mm below the H.P. and 25 mm behind V.P.
(iii) C 15 mm above the H.P. and 20 mm in front of V.P.
(iv) D 40 mm below H.P. and 25 mm in front of V.P. 8

Refer Example 6.10, Page 6.12

(b) The front view of a line, inclined at 30º to the V.P. is 65 mm long. Draw the projections and true length of the line when it is parallel to and 40 mm above the H.P. and its one end being 30 mm in front of the V.P. 8

Refer Example 7.13, Page 7.15

Or

4. The projectors of the ends of a line AB are 6 cm apart. The end A is 2 cm above H.P. and 3 cm in front of the V.P. The end B is 1 cm below H.P. and 4 cm behind the V.P. Determine the true length and traces of AB and its inclinations with the two planes. 16

Refer Example 7.56, Page 7.67

Unit-III

5. (a) A circle of 40 mm diameter is resting on H.P. on a point with its surface inclined at 30º to H.P. Draw the projections of the circle when the top view of the diameter through the resting point makes an angle of 45º with XY. 8

Similar to Example 8.24(a), Page 8.31

(b) A pentagonal pyramid of edge of base 25 mm and height 60 mm is resting on the corner of its base on H.P. and the slant edge containing that corner is inclined at 45º with H.P. Draw the projections of the solid, when its axis makes an angle of 30º with V.P. 8

Refer Example 9.41, Page 9.48

Or

6. (a) Draw the projections of a regular hexagon of 25 mm side having one of its sides in the H.P. and inclined at 60º to the V.P. and its surface making an angle of 45º with the H.P. 8

Refer Example 8.14, Page 8.19

(b) Draw the projections of a cube 20 mm long edges resting on the H.P. on one of its corners with a solid diagonal perpendicular to the V.P. 8

Refer Example 9.48, Page 9.55

Unit-IV

7. (a) A right circular cone of base diameter 45 mm and axis 55 mm long is lying on one of its generators on H.P. It is cut by a horizontal section plane passing through the midpoint of the axis. Draw the projections of the cone and its true section. 8

Refer Example 10.5, Page 10.9

(b) A cylinder has been truncated by a circular surface as shown in the following Fig. 1. Draw the development of surface of the cylinder. 8

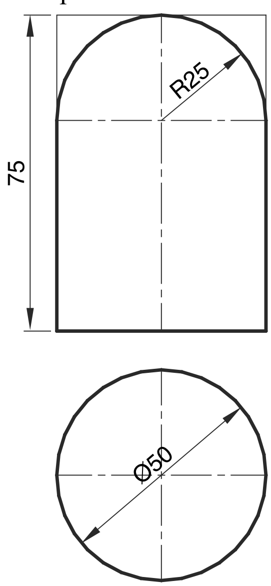


Fig. 1

Similar to Example 11.9, Page 11.11

Or

8. (a) Draw the isometric view of the block shown in Fig. 2. 8

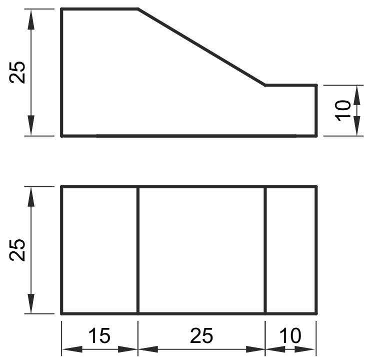


Fig. 2

Refer Example 12.31, Page 12.30

(b) A hexagonal pyramid base 30 mm side and axis 65 mm long is resting on its base on the H.P. with two edges parallel to the V.P. It is cut by a section plane perpendicular to the V.P. and inclined at 45° to the H.P. and intersecting the axis at a point 25 mm above the base. Draw front view and sectional top view of the section. 8

Refer Example 10.13, Page 10.17

Unit-V

9. (a) What is CAD. State the advantages of CAD. 6

Refer Page 13.2

(b) Explain any two methods of drawing a circle in AutoCAD. 5

Refer Page 13.21

(c) Name five edit commands used in CAD. 5

Refer Page 13.30

Or

10. (a) Name various methods of locating a point in CAD and explain any to of them. 6

Refer Page 13.11

(b) Fill in the blanks: 5

(i) Computer is an _____ device with _____ brain.

[eletronic, no]

(ii) ROM stands for _____.

[Read Only Memory]

(iii) _____ are those commands which can tell the basic functions of AutoCAD.

[Utility commands]

(iv) _____is a bright dot on the screen.

[Cursor]

(v) Commercial Computer Aided Drafting was introduced in 1964 by _____.

[International Business Machines Corporation (IBM)]

(c) State whether the statement is true or false: 5

(i) Machine language has only two words "True or false". True

(ii) Once the computer is switched off, ROM forgets everything. False

(iii) The utility commands control the basic functions of AutoCAD. True

(iv) ERASE and UNDO are Edit Commands. True

(v) Hatch command is used in drawing sectional view of an object. True

B. E. (First Semester) EXAMINATION, Aug., 2010
(Common for all Branches)

BE-105 ENGINEERING GRAPHICS

Time: 3 Hrs **Max. marks: 80** **Min. Marks: 28**

Note: (i) *Attempt any five questions selecting one question from each unit.*
(ii) *Draw in first angle projection unless specified otherwise.*
(iii) *Assume suitable missing and misprint data if any.*

Unit-I

1. (a) Construct a scale to be used with a map; the scale of which is 1 cm = 500 m. The maximum length to be read is 5 km. Mark on the scale a distance of 3.85 km. 8

Refer Example 2.14, Page 2.13

(b) Construct a hyperbola with the distance between the focus and the directrix as 50 mm and eccentricity as 3/2. Also draw normal and tangent to the curve at a point 30 mm from the axis. 8

Refer Example 3.18, Page 3.25

Or

2. (a) A rectangular plot of land of area 16 sq. m is represented on a map by a similar rectangle of 1 sq. cm. Calculate the R.F. of the scale of the map. Construct a plain scale to read metres and long enough to measure upto 60 m. Indicate a distance of 45 m on the scale. 8

Similar to Example 2.6, Page 2.7

(b) A coir is unwound from a drum of 30 mm diameter. Draw the locus of the free end of the coir for unwinding through an angle of 360º. Draw also a normal and tangent at any point on the curve. 8

Similar to Example 4.4, Page 4.7

Unit-II

3. (a) State the quadrants in which the following points are located: 4
(i) A - front view and top view are above XY.
(ii) B-front view below XY and top view above XY.
(iii) C-front view and top view below XY.
(iv) front view above xy and top view below XY.

Similar to Example 6.12, Page 6.13

(b) A line PQ of 80 mm long has its end P, 15 mm from both H.P. and V.P. The other end Q is 40 mm above H.P. and 50 mm in front of V.P. Draw the projections of the line and determine the inclinations of the line with H.P. and V.P. 12

Refer Example 7.24, Page 7.31

Or

4. The front view of a line AB is 50 mm long and it makes an angle of 35º with XY. The point A lies 10 mm above H.P. and 25 mm behind V.P. The difference between the distances of A and B from V.P. is 25 mm. The line AB is in second quadrant. Draw the projections of the line and determine its true length and inclinations with the H.P. and V.P. 16

Refer Example 7.59, Page 7.70

Unit-III

5. (a) A semicircular plate of 80 mm diameter has its straight edge on V.P. and inclined at 30º to H.P., while the surface of the plate is inclined at 45º to V.P. Draw the projection of the plate. 8

Refer Example 8.16, Page 8.21

(b) A hexagonal prism, side of base 20 mm and axis 48 mm long, rests with its base on H.P. such that an edge of the base is parallel to V.P. Draw the projections of the prism on an auxiliary plane which makes an angle of 60º with the H.P. 8

Or

6. (a) A pentagon of 30 mm side has one corner on H.P. Its plane is inclined at 60º to V.P. and perpendicular to H.P. Draw the projection of the pentagon. 8

Refer Example 8.11, Page 8.15

(b) A hexagonal pyramid of side of base 25 mm and axis 60 mm long, is resting on an edge of the base on H.P. Draw the projections of the solid when the axis makes an angle of 45º with V.P. and the base of the solid is nearer to V.P. 8

Refer Example 9.20, Page 9.25

Unit-IV

7. (a) A cylinder of diameter of base 40 mm and axis 55 mm long is resting on its base on H.P. It is cut by a section plane perpendicular to V.P. and inclined at 45º to H.P. The section plane passes through the top end of an extreme of the cylinder. Draw the development of the lateral surface of the cut cylinder. 8

Refer Example 11.7, Page 11.9

(b) A vertical cylinder of 60 mm diameter is penetrated by a horizontal square prism of 35 mm side. The axes of the two solids intersect each other. A rectangular face of the prism is inclined at 60º to the V.P. Draw the lines of intersection. 8

Or

8. (a) A pentagonal pyramid side of base 35 mm and axis 60 mm long rests with its base on the H.P. such that one of the edges of the base is perpendicular to the V.P. A section plane perpendicular to H.P. and parallel to V.P. cuts the pyramid at a distance of 20 mm from the corner of the base nearer to the observer. Draw its top and sectional front view. 8

Refer Example 10.9, Page 10.13

(b) A cone base 50 mm diameter and axis 60 mm long rests with its base on H.P. A section plane perpendicular to V.P. and inclined at 45º to H.P. bisects the axis of the cone. Draw the development of the lateral surface of the remaining portion of the cone. 8

Refer Example 11.12, Page 11.14

Unit-V

9. (a) State three advantages of computer aided drafting. 3

Refer Page 13.2

(b) State any four significant EDIT commands and its function. 8

Refer Page 13.30

(c) Write the prompt sequence for drawing a rectangle of 60 mm × 40 mm with the help of line command. 5

Refer Page 13.16

Or

10. Draw the isometric view of a sphere of radius 20 mm which rests centrally on top of a square prism of base 50 mm and height 60 mm. 16

Similar to Example 12.13, Page 12.16

B. E. (First Semester) EXAMINATION, Dec., 2010
(Non-Grading System) (Common for all Branches)

BE-105 ENGINEERING GRAPHICS

Time: 3 Hrs **Max. marks: 100** **Min. Marks: 35**

Note: *Attempt any five questions selecting one question from each Unit. All questions carry equal marks. Assume suitably a missing/misprint data, if any.*

Unit-I

1. (a) Construct a scale to be used with a map, the scale of which is 1 cm = 4 m. The scale should read in metres up to 60 m. Show on it a distance of 46 m. 10

Similar to Example 2.2, Page 2.5

(b) Draw an ellipse with major axis 120 mm and minor axis 80 mm. Draw a normal and tangent on any point on ellipse. 10

Similar to Example 3.26, Page 3.35

Or

2. (a) Construct a diagonal scale of 1/48 showing metres, decimetres and centimetres and to measure up to 6 m. Mark a length of 3·76 m on it. 10

Similar to Example 2.10, Page 2.10

(b) Draw the involute of an equilateral triangle of side 25 mm. 10

Similar to Example 4.5, Page 4.8

Unit-II

3. The top view of line AB, 70 mm long measures 55 mm and front view measures 45 mm. Its end A is 10 mm from H.P. and 15 mm from V.P. Draw the projections of the line and determine its inclination with the H.P. and V.P. Also draw its traces if the line is in first quadrant. 20

Similar to Example 7.26, Page 7.33

Or

4. Two bulbs on two poles are respectively 2 m and 3 m above the ground and 1 m and 2 m from the wall, 0·3 mm thick but on opposite side of it The distance between the bulbs measured along the ground and parallel to the wall is 3 m. Determine the real distance between the bulbs. 20

Similar to Example 7.61, Page 7.72

Unit-III

5. (a) A hexagonal plate of 30 mm side is resting on one of its comer on H.P. The plate is perpendicular to V.P. and inclined to H.P. at 45°. Draw its projections. 10

Refer Example 8.5, Page 8.6

(b) A pentagonal pyramid of base edge 30 mm and height 50 mm is resting on one of its corners in H.P. Draw the projection when the axis is inclined to 45° to H.P. 10

Refer Example 9.10, Page 9.16

Or

6. (a) A square plate of 40 mm side is perpendicular to H.P. and inclined to V.P. at 40º. One of its edge is on V.P. Draw the projections when one of the corners is 12 mm from the H.P. 10

Similar to Example 8.7, Page 8.9

(b) A right circular cone of dia. 70 mm and axis height 80 mm is resting on one of its generators in H.P. The top view of the axis is inclined at 45° to V.P. Draw the projections of the cone. 10

Similar to Example 9.34, Page 9.41

Unit-IV

7. (a) A cylinder of 45 mm dia. and 70 mm long is resting on one of its bases on H.P. It is cut by a section plane inclined at 60° with H.P. and passing through a point on the axis at 15 mm from one end. Draw the two views of the solid. Also obtain the true shape of the section. 10

Refer Example 10.12, Page 10.16

(b) A pentagonal pyramid of side of base 30 mm and axis 60 mm long is resting on its base on H.P. with an edge of the base parallel to V.P. Draw the development of the lateral surface of the pyramid. 10

Similar to Example 11.18, Page 11.17

Or

8. (a) A square prism with edge of base 45 mm and axis 90 mm long has its axis parallel to both H.P. and V.P. The lateral surfaces are equally inclined to H.P. It is cut by a vertical section plane inclined at 60° to the V.P. and passing through the axis at 65 mm from one end. Draw the projection of the solid. 10

Similar to Example 10.28, Page 10.32

(b) Draw the development of the frustum of a hexagonal pyramid of side of base 35 mm at the bottom and 15 mm at the top, the height of the frustum being 50 mm. 10

Similar to Example 11.23, Page 11.26

Unit-V

9. (a) Draw the isometric view of a hexagonal prism with side of base 25 mm and axis 60 mm long. The prism is resting on its base on H.P. with an edge of the base parallel to V.P. 15

Refer Example 12.2, Page 12.8

(b) Name and explain *five* edit commands used in AutoCAD. 5

Refer Page 13.30

Or

10. (a) Write *three* advantages and limitations of the CAD. 6

Refer Page 13.2

(b) Fill in the blanks with suitable words: 8

(i) RAM stands for _____. Random Access Memory

(ii) ALU stands for _____. Arithmetic Logic Unit

(iii) _____are those commands which can tell basic functions of the AutoCAD. Utility commands

(iv) _____is a bright spot on the screen. Cursor

(c) State whether the following statements are true *or* false: 6

(i) Machine language has only two words 'True' or 'False'. True

(ii) Command prompt means AutoCAD is' ready for you to type a command name. True

(iii) Section command allows you to draw lines at right angles only. False

(iv) Save command is used to save the drawing changes in a drawing. True

(v) Hatch command can be used to draw sectional view. True

(vi) Zoom command is used to enlarge part of a drawing to view its detail. True

B. E. (First Semester) EXAMINATION, Feb., 2011
(Grading System) (Common for all Branches)

BE-105 ENGINEERING GRAPHICS

Time: 3 Hrs **Max. marks: 70** **Min. Marks: 22 (D Grade)**

Note: Attempt five questions in all selecting one question from each Unit. All questions carry equal marks.

Unit-I

1. (a) Construct a scale to be used with a map,. the scale of which is 1 cm = 40 m. The scale should read in metres and maximum up to 500 m. Mark a distance of 456 m on it. 7

Similar to Example 2.14, Page 2.13

(b) A wheel 50 mm dia. rolls on a straight road surface without slip. Trace the path of point of contact for one complete revolution of the wheel. 7

Refer Example 4.1, Page 4.2

Or

2. (a) A rectangular plot of land of area 16 sq. m is represented on a map by a similar rectangle of 1 square centimetre. Calculate the R. F. of scale and construct plain scale to read metres and long enough to measure up to 60 m. 7

Similar to Example 2.6 Page 2.7

(b) A ball thrown from the ground level reaches a maximum height of 5 m and travels a horizontal distance of 11 m from the point of projection. Trace the path of the ball. 7

Similar to Example 3.12, Page 3.20

Unit-II

3. The distance between the projector of two ends of a straight line is 40 mm. One end is 15 mm above H.P. and 10 mm in front of the V.P. The other end is 40 mm above H.P. and 40 mm in front of V.P. Find the true length and true inclination of the line. 14

Similar to Example 7.17, Page 7.21

Or

4. A line AB is inclined at 40° to H.P. It's one end A is 25 mm above H.P. and 30 mm in front of V.P. The top view of the line is 70 mm and is inclined at 30° to XY. Draw the projections of the line and determine its true length and inclination with V.P. 14

Similar to Example 7.33, Page 7.40

Unit-III

5. (a) Draw the projections of a circle 60 mm diameter resting on V.P. on a point on the circumference. The plane is inclined at 45° to V.P. and perpendicular to H.P. The centre of the plane is 30 mm above H.P. 7

Similar to Example 8.8, Page 8.9

(b) A square pyramid of side of base 30 mm and axis 50 mm long is freely suspended from a corner of its base. Draw its projections. 7

Refer Example 9.49 (up to second stage), Page 9.56

Or

6. (a) A regular hexagonal plane of 45 mm side has a corner on H.P. and it's surface is inclined at 45° to H.P. Draw the projections when the diagonal through the corner which is. on H.P. makes 30º with the V.P. 7

Refer Example 8.21, Page 8.27

(b) Draw the projections of a cone of base 50 mm diameter and altitude 60 mm lying on one of its generators on H.P. when the top view of the axis makes an angle of 30º with XY. 7

Similar to Example 9.34, Page 9.41

Unit-IV

7. (a) A hexagonal pyramid, side of base 30 mm and axis 60 mm long rests with its base on H.P. and one of the edges of its base is parallel to V.P. It is cut by a horizontal section plane at a distance of 30 mm above the base. Draw the front and sectional top views. 7

Refer Example 10.4, Page 10.8

(b) A cylinder of 45 mm base dia. and 55 mm long axis rests with its base on H.P. It is cut by a plane perpendicular to V.P. inclined at 60º to H.P. and passing through a point on the axis 12 mm from its top. Draw the top view and development of lateral surface of the truncated cylinder. 7

Similar to Example 11.7, Page 11.9

Or

8. (a) A cylinder 50 mm dia. and 70 mm long is resting on H.P. with its axis inclined at 30° to H.P. and parallel to V.P. A section plane inclined at · 45° to V.P. passes through the axis at 25 mm from one end of it. Draw the project of the cut solid. 7

Refer Example 10.30, Page 10.34

(b) A pentagonal pyramid side of base 30 mm and height 52 mm stands with its base on H.P. and an edge of the base in parallel to V.P. It is cut by a plane perpendicular to V.P and inclined at 40° to H.P. and passing through a point 30 mm above the base. Draw the development of the lateral surface of the truncated pyramid. 7

Similar to Example 11.21, Page 11.24

Unit-V

9. A cone of base diameter 50 mm and axis 60 mm rests with its base on H.P. A section plane perpendicular to V.P. and inclined at 30° to H.P. passes through the axis at a distance of 25 mm above base. Draw the isometric projections of the truncated cone. 14

Refer Exercise 12, Problem 20, Page 12.43

Or

10. (a) What are the advantages and disadvantages of CAD? 6

Refer Page 13.2

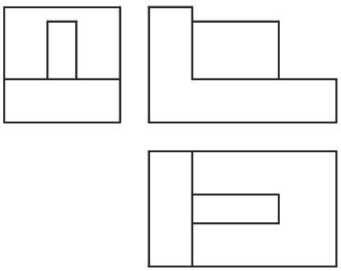
(b) Explain the following commands in brief :

(i) Move (ii) Array (iii) Chamfer (iv) Hatch 8

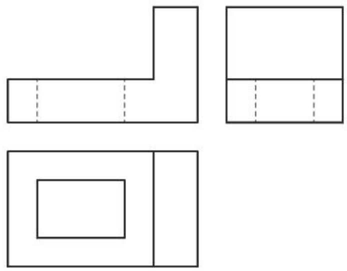
Refer Pages 13.31, 13.36, 13.43

Appendix: Solutions to Unsolved Problems

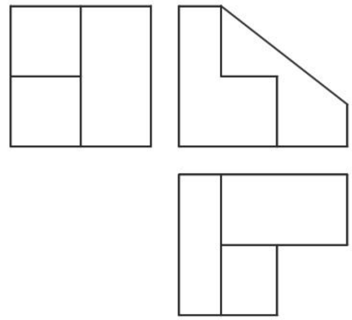
EXERCISE 5



Sol E5.1



Sol E5.2



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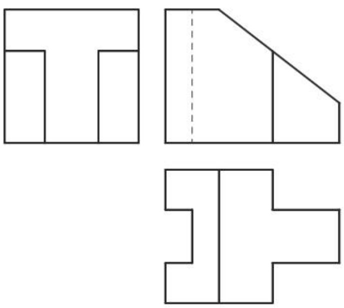

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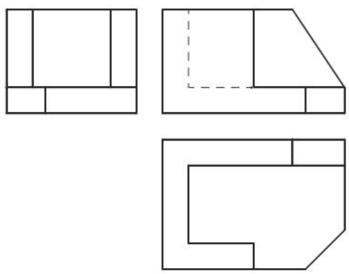

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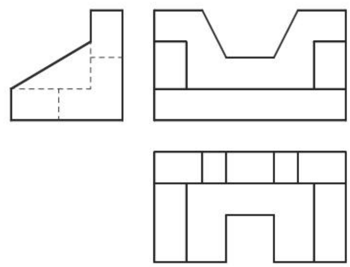

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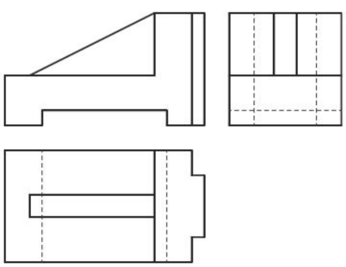
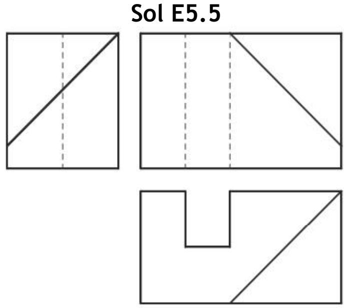
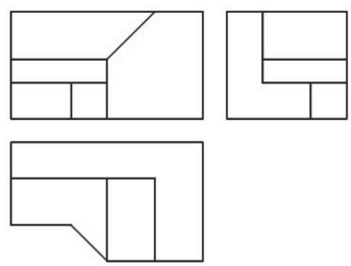


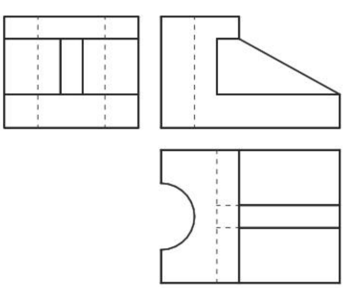
Sol E5.7



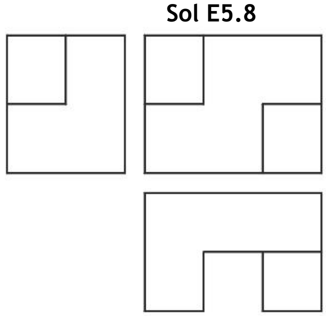
Sol E5.8



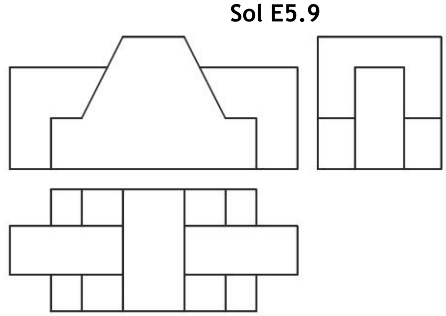
Sol E5.9



Sol E5.10



Sol E5.11



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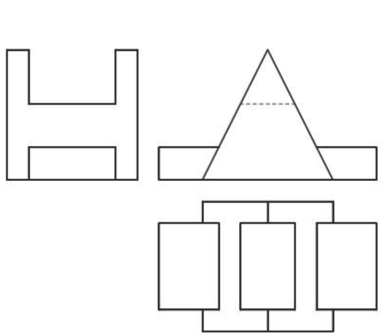
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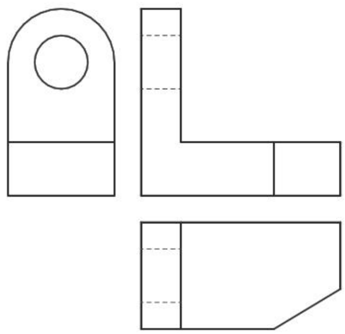
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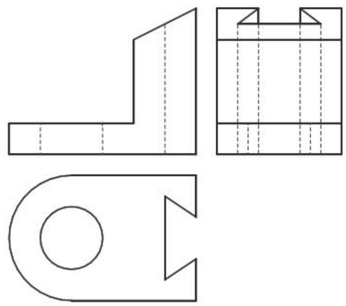
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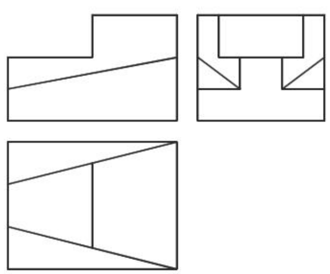
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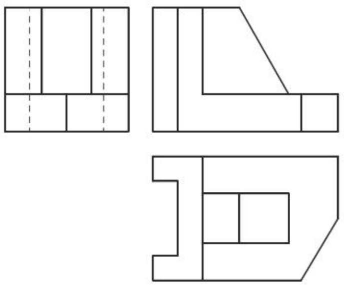
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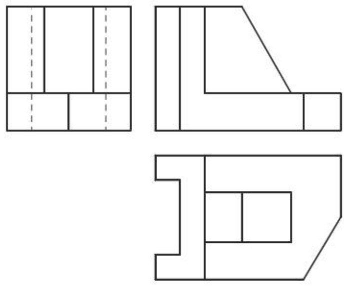
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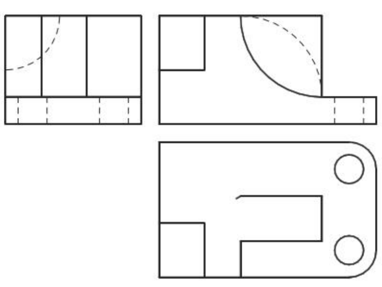
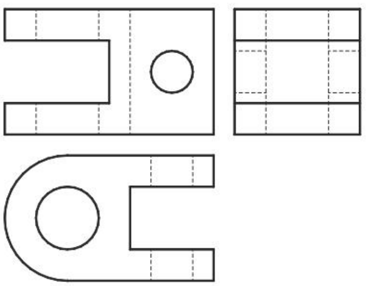
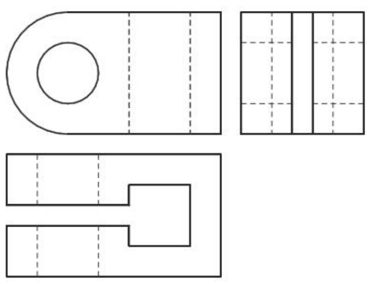
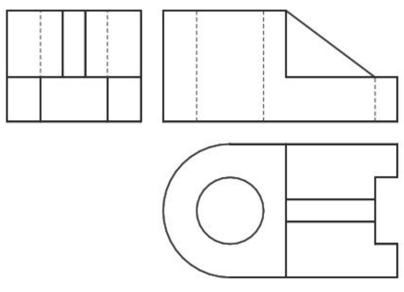

Sol E5.19


Sol E5.20


Sol E5.21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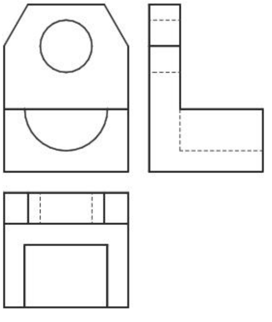
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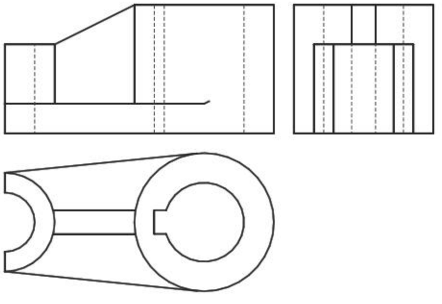
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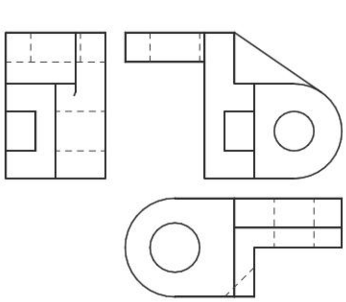

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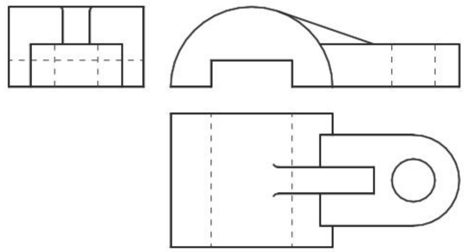

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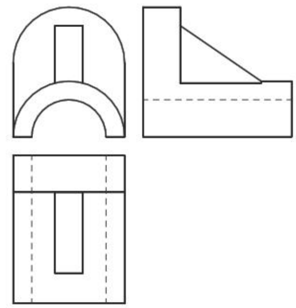

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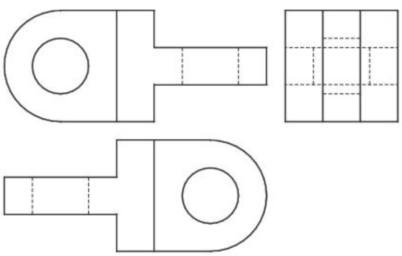
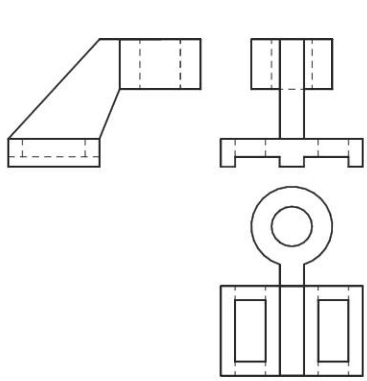


Sol E5.28



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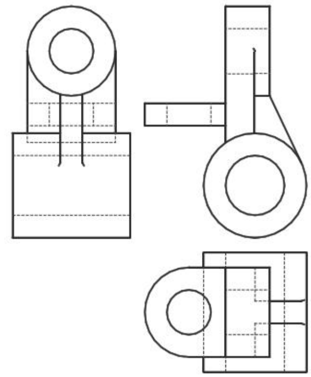
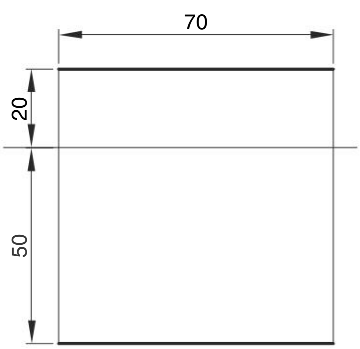
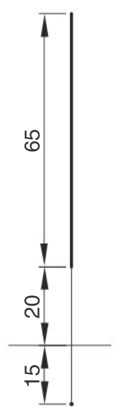


Sol E5.30

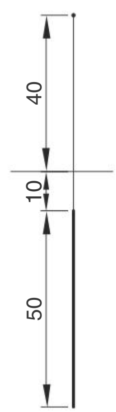
EXERCISE 7A



Sol 1



Sol 2



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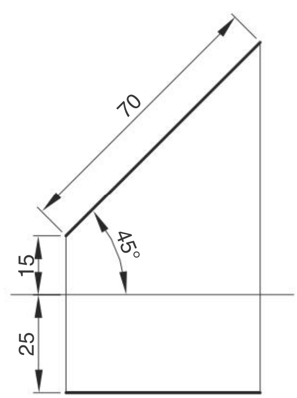

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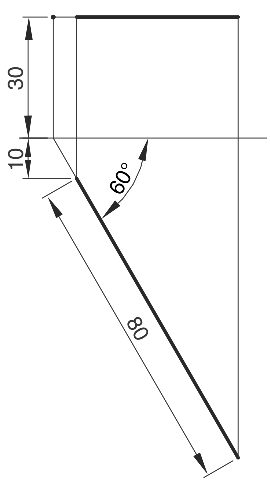

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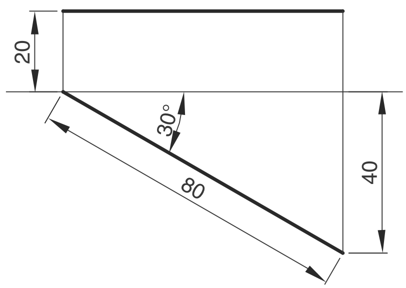
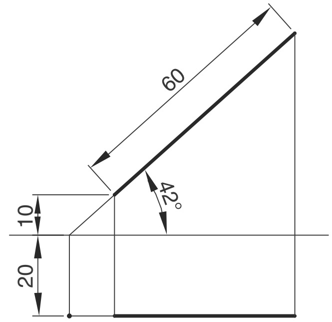
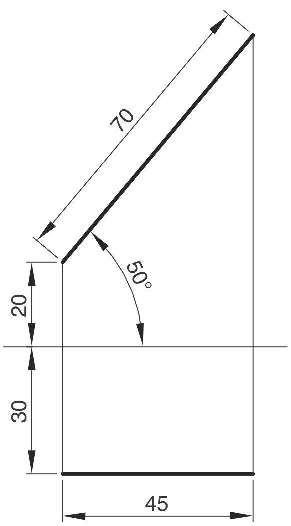


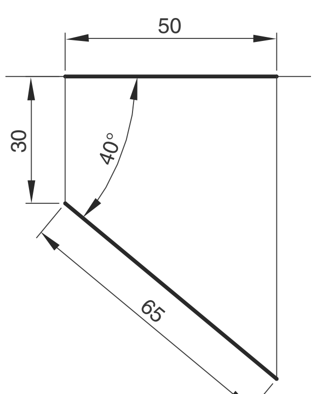
Sol 6



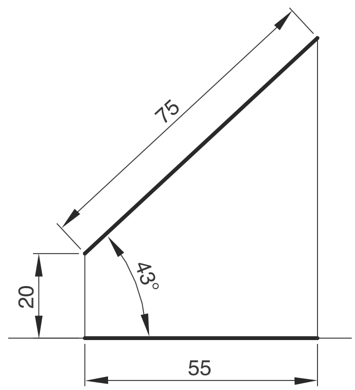
Sol 7



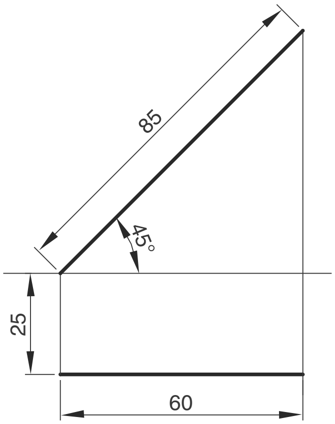
Sol 8



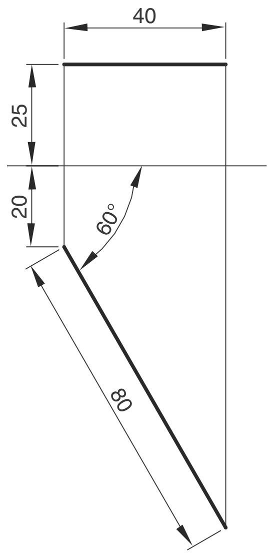
Sol 9



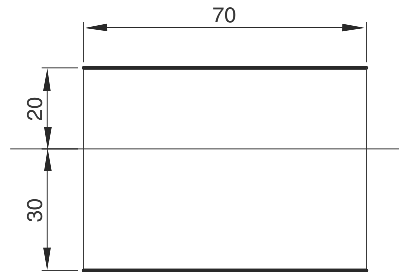
Sol 10



Sol 11



Sol 12



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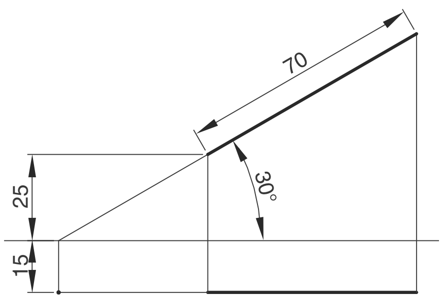

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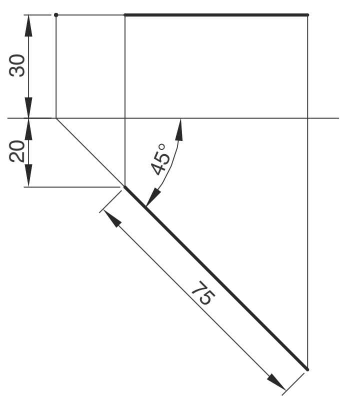

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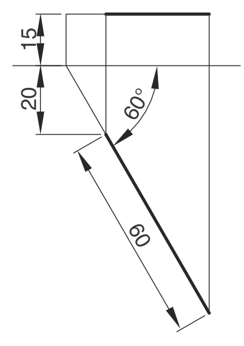
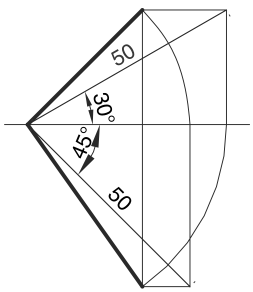


Sol 16

EXERCISE 7B



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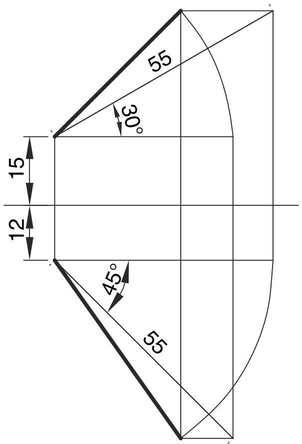

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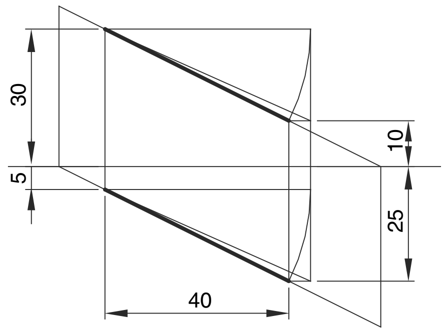

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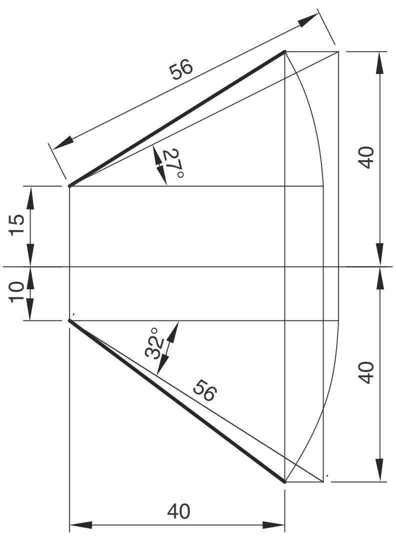

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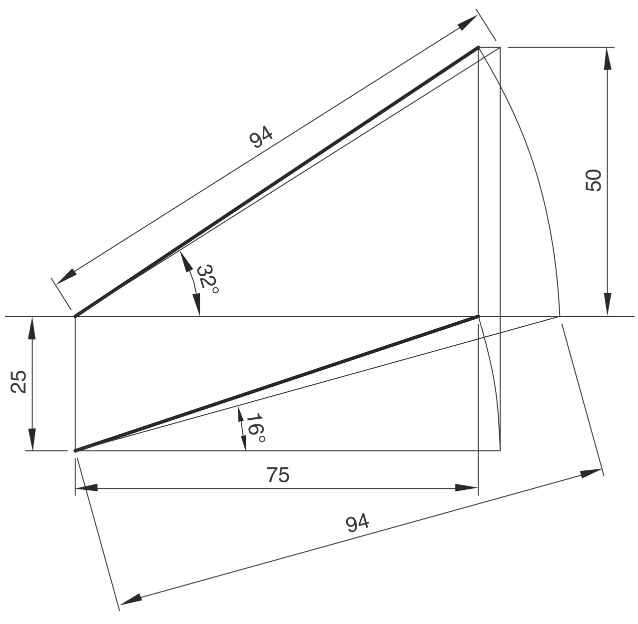

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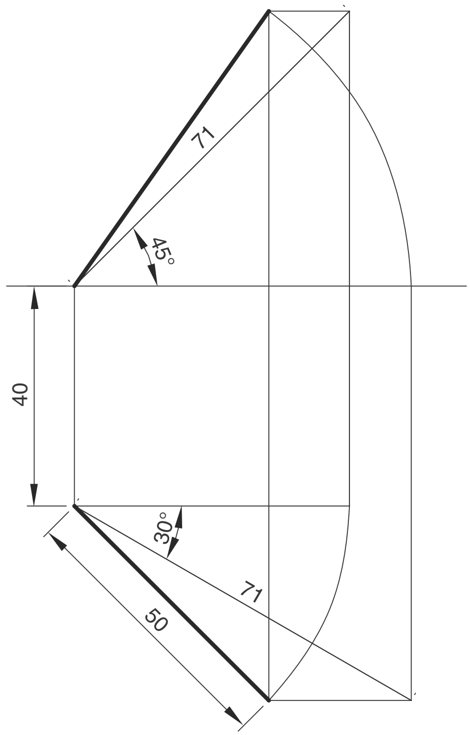

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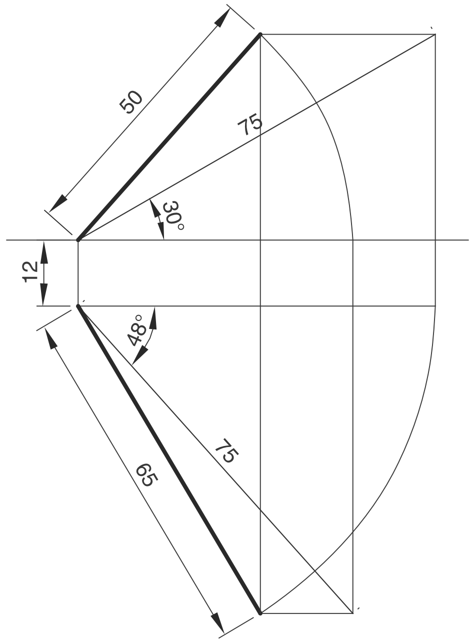

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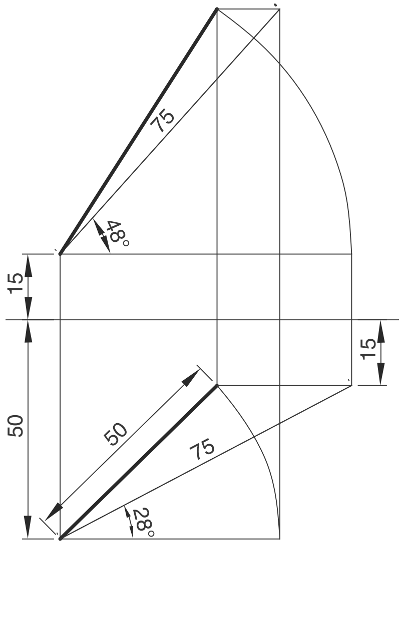

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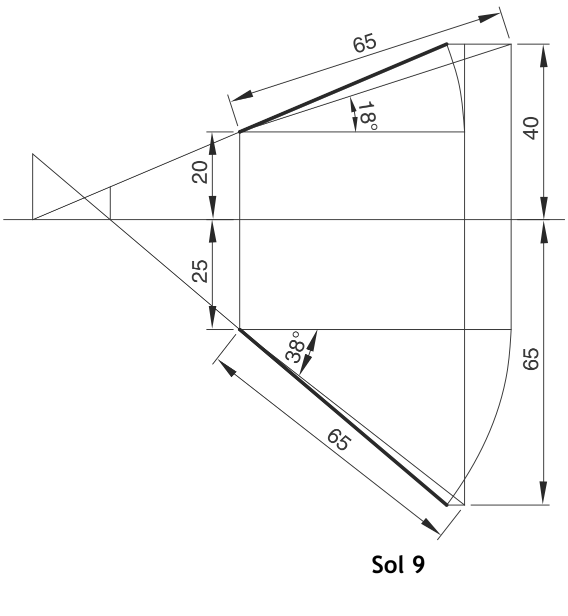

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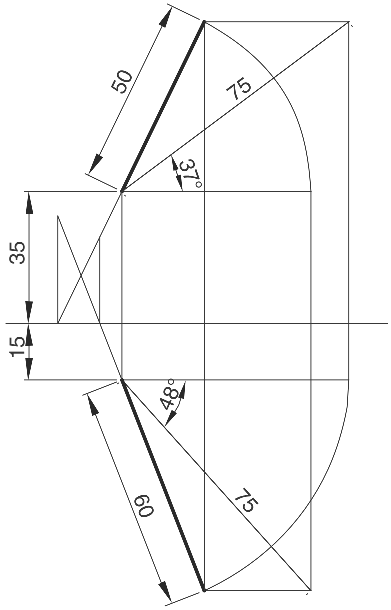

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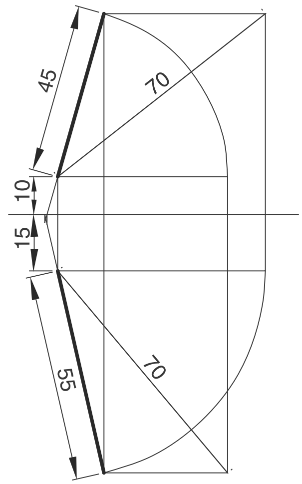

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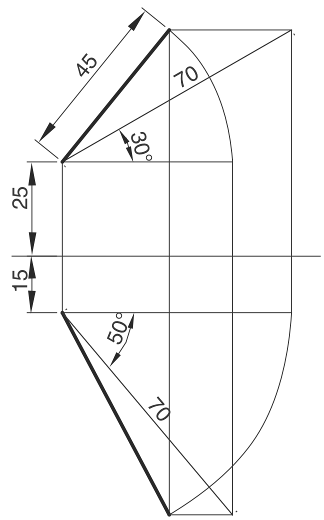

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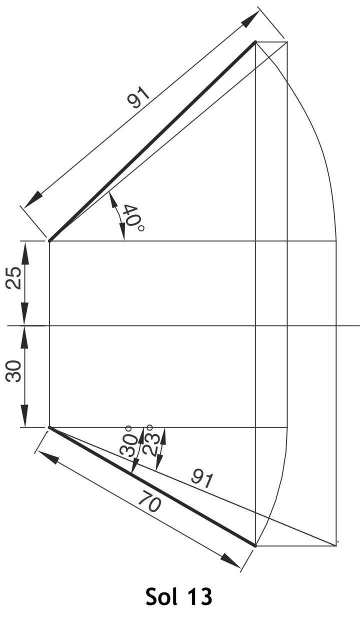

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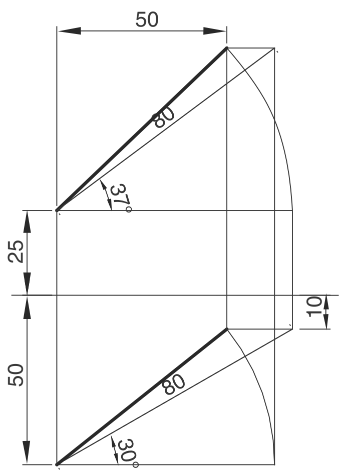

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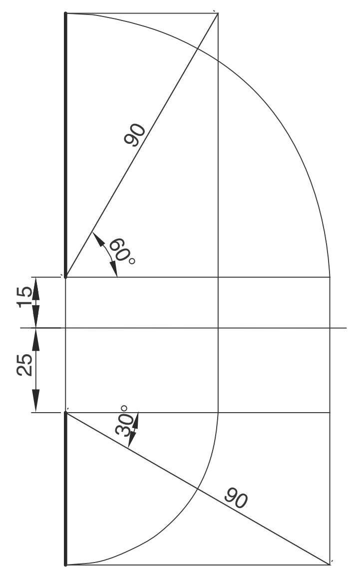

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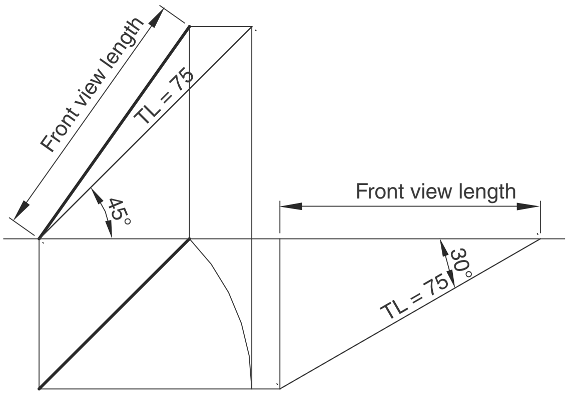

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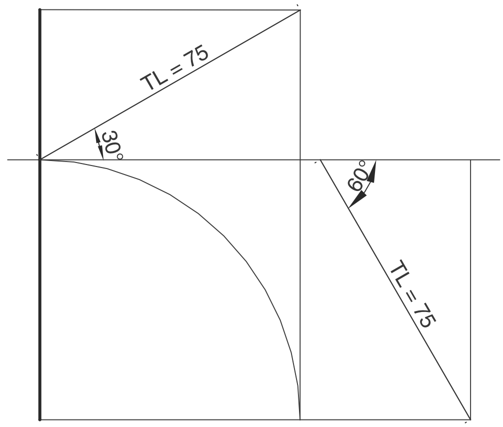

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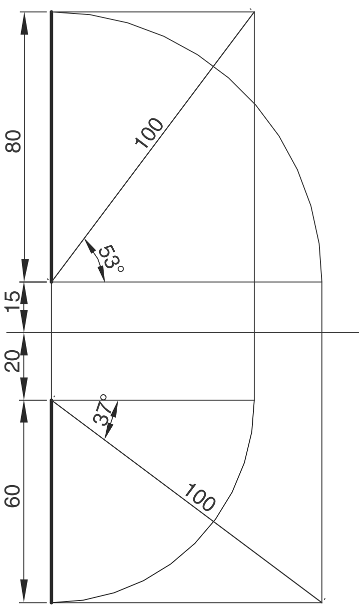

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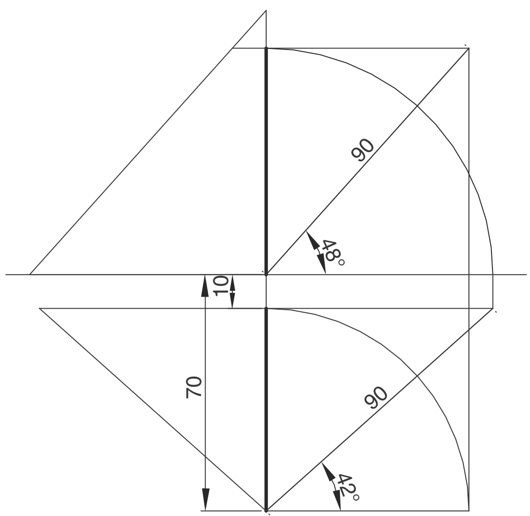

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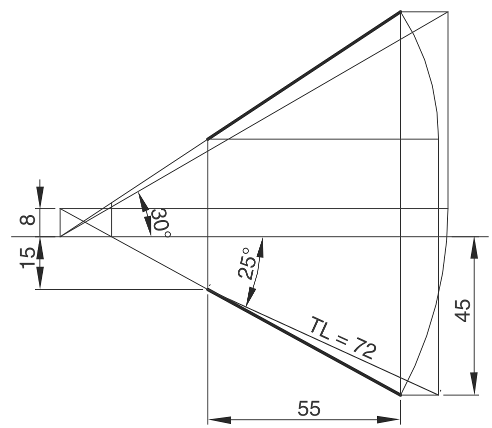

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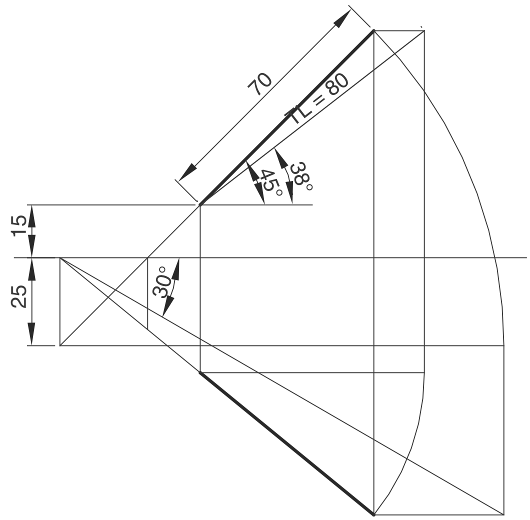

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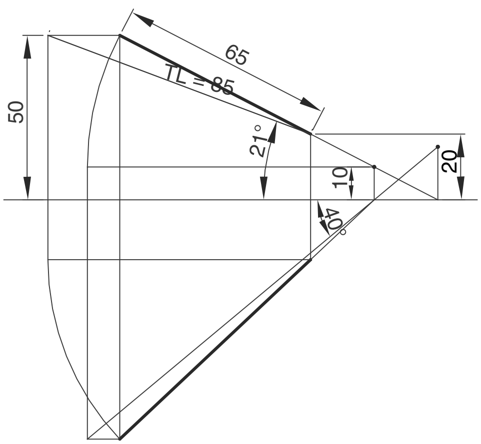

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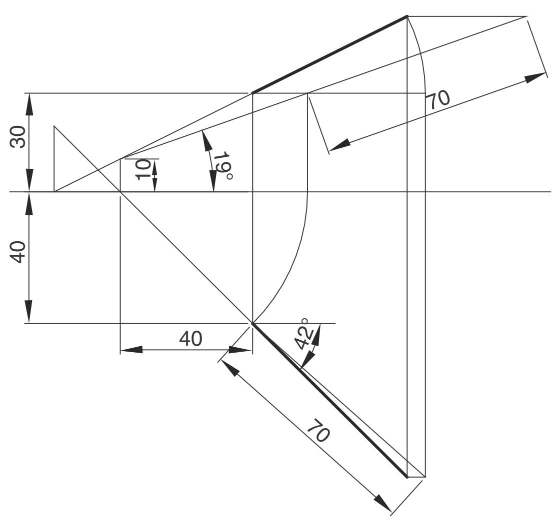

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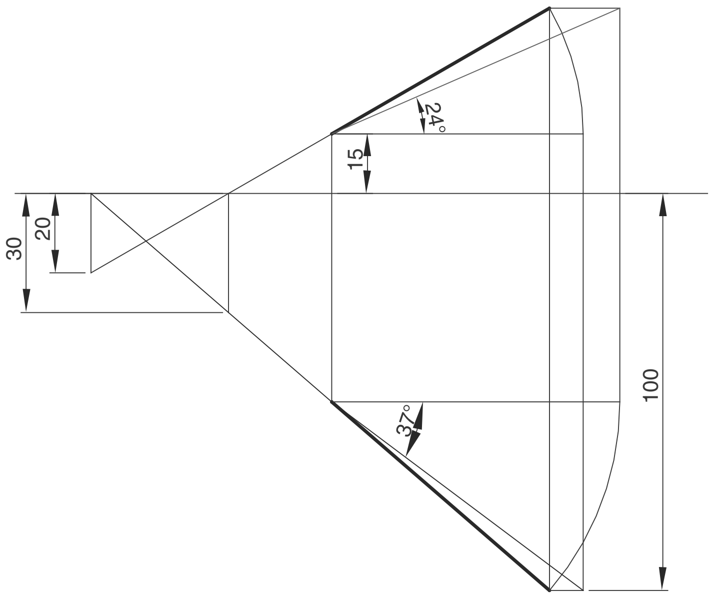

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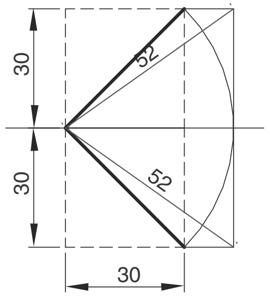

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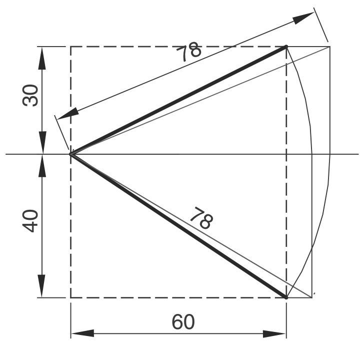

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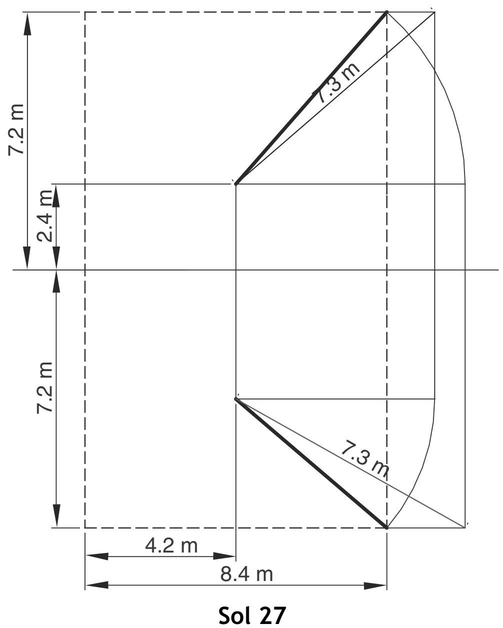

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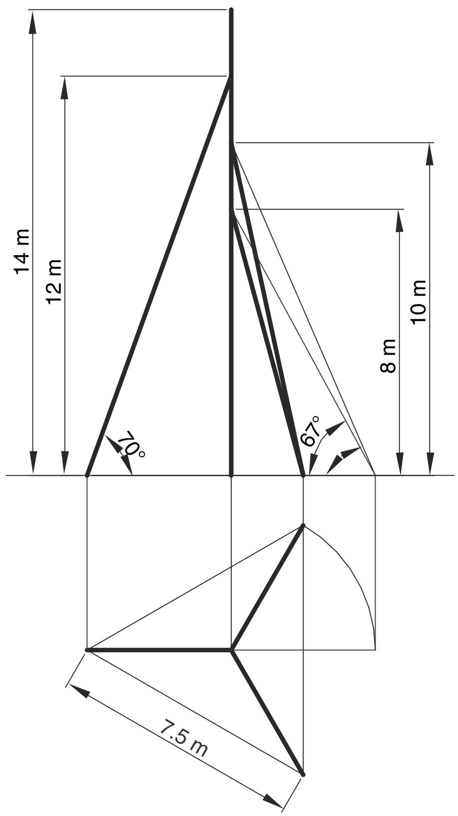
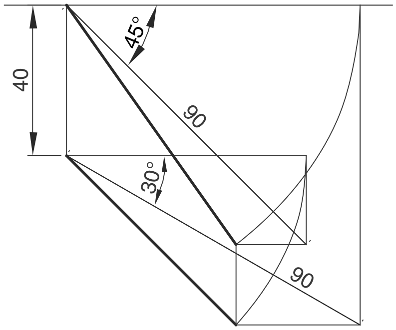


Sol 28

EXERCISE 7C



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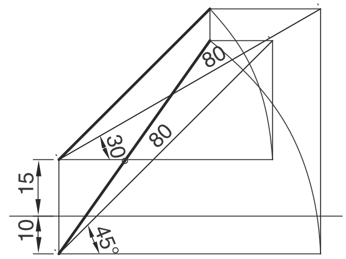
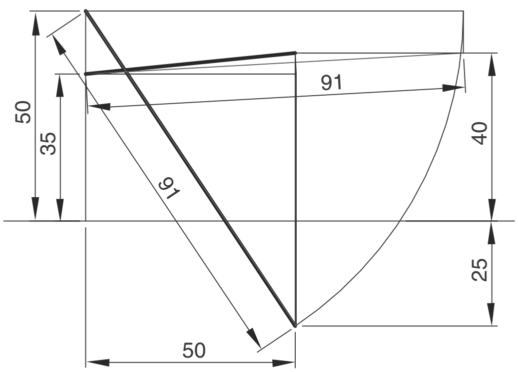
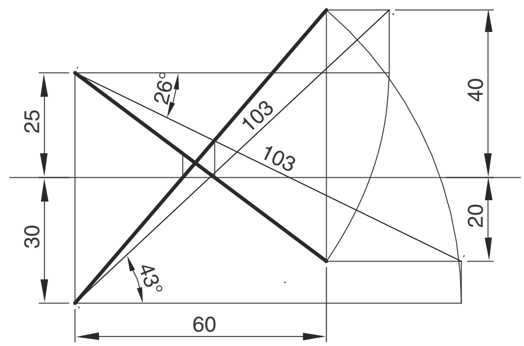


Sol 2



Sol 3



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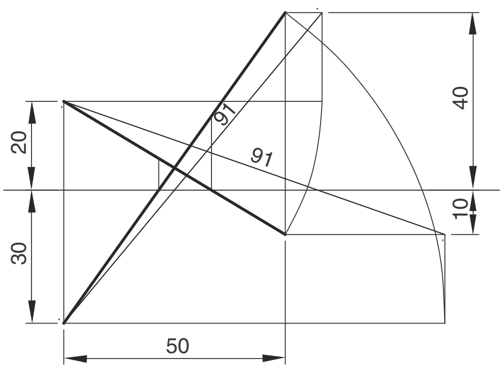

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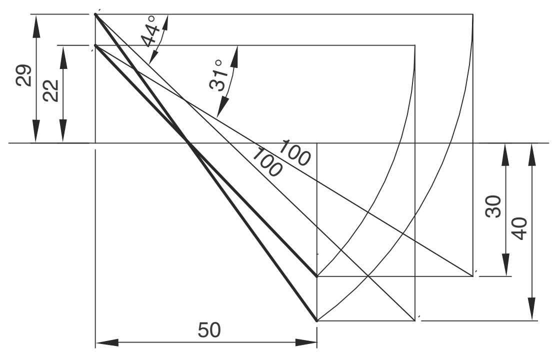

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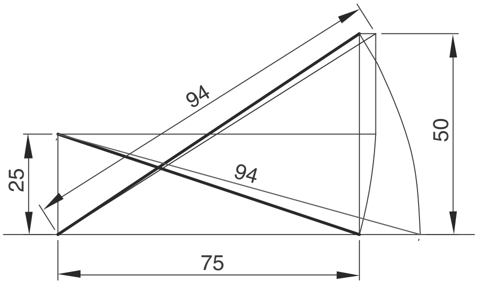

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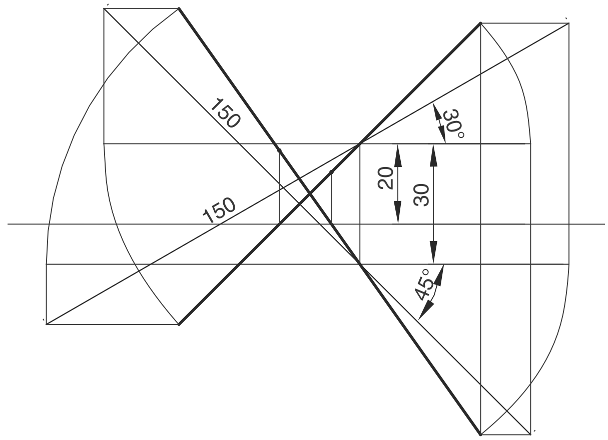

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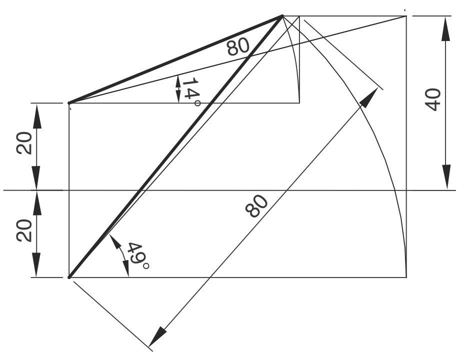

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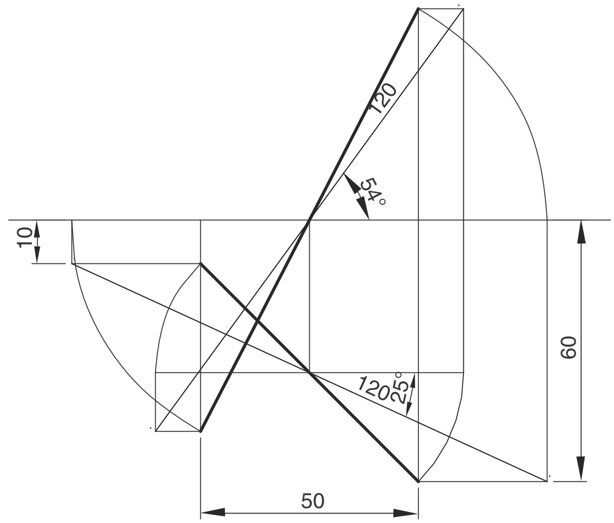

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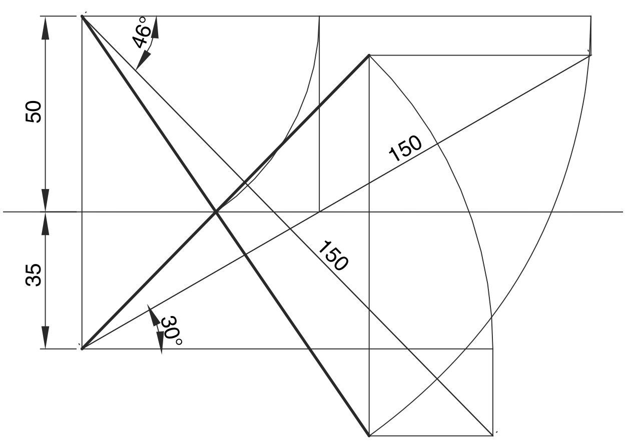

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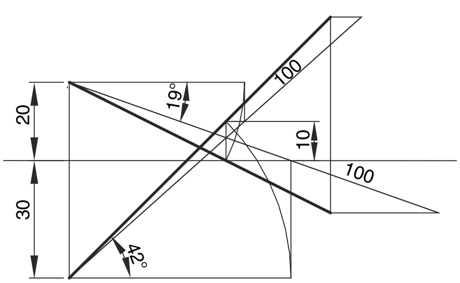

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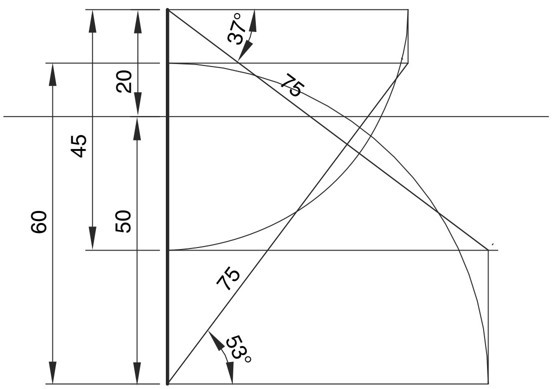

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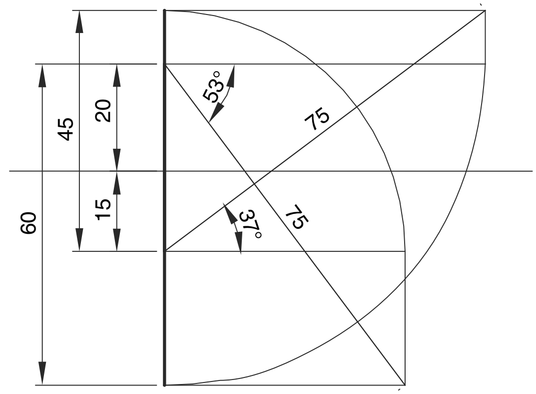

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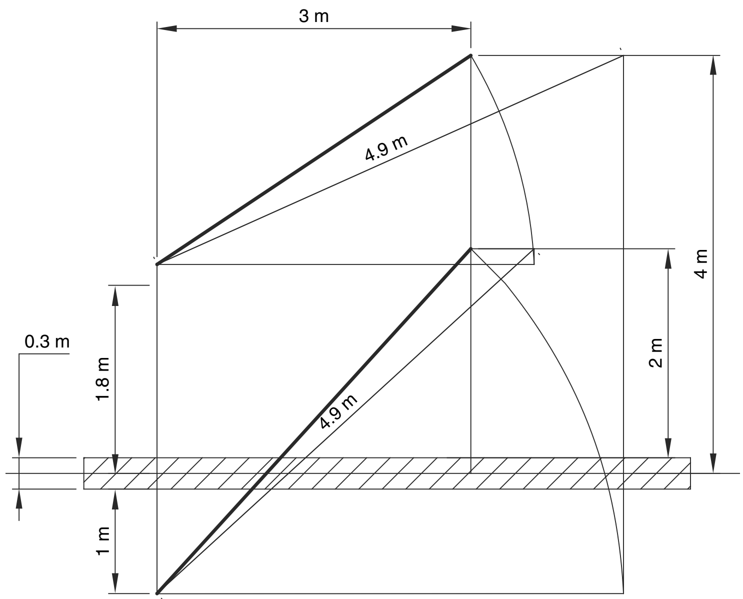
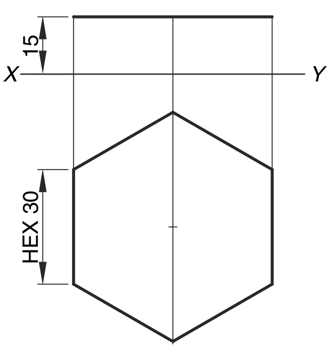


Sol 15

EXERCISE 8A



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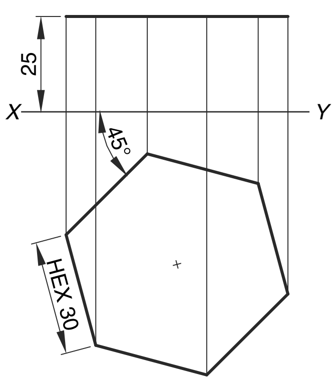

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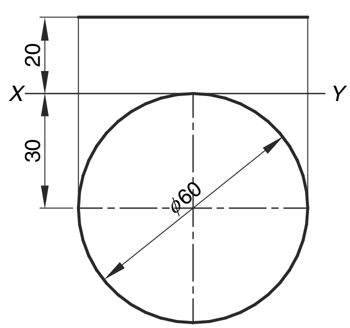

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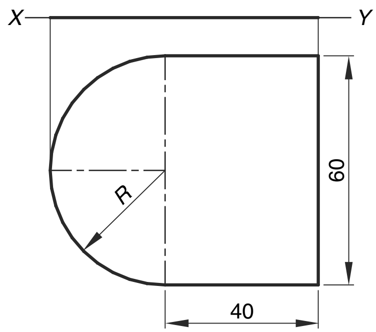
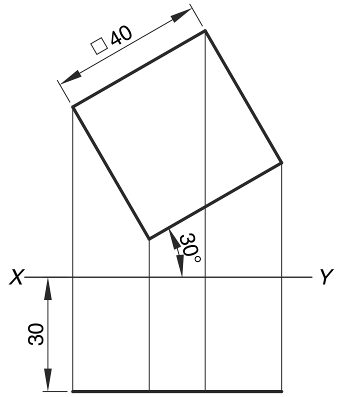
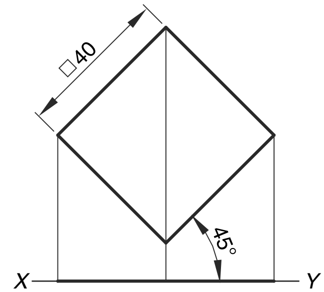


Sol 4



Sol 5



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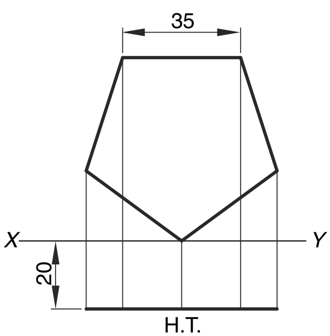

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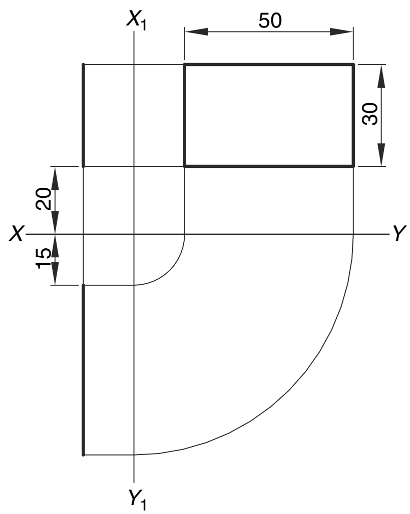

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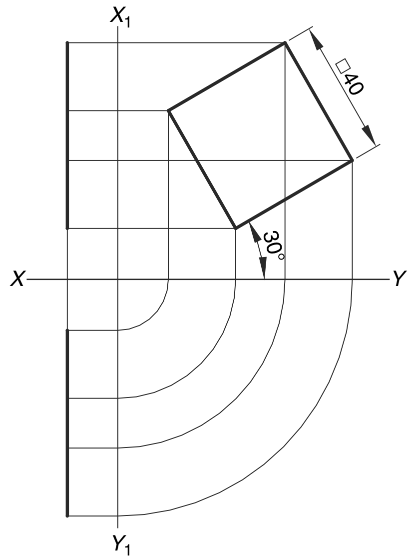

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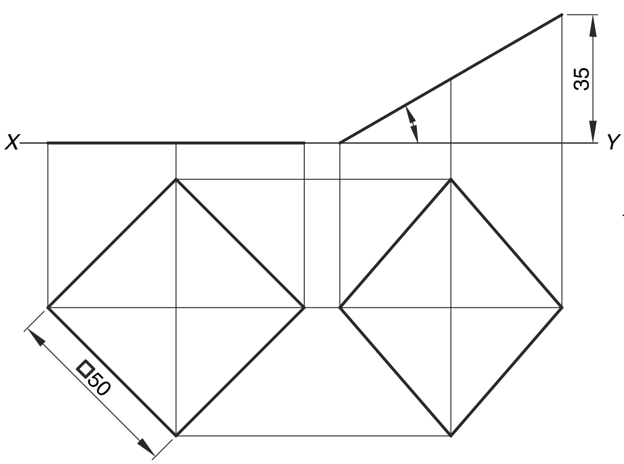

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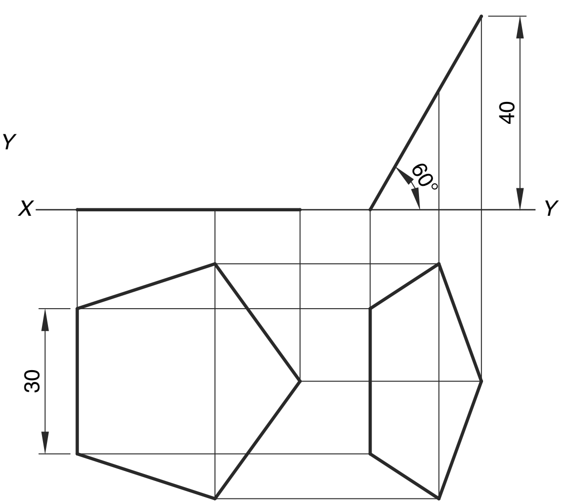

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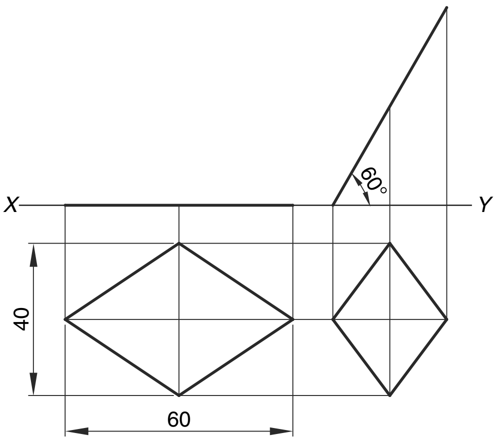

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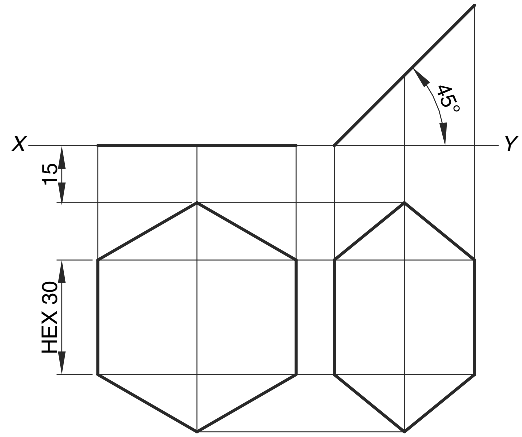

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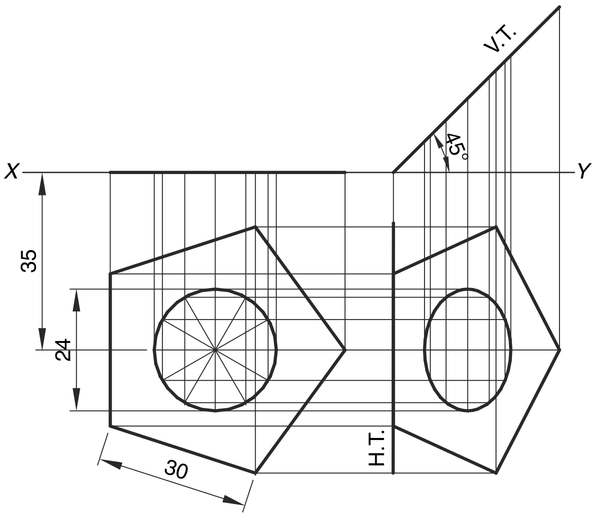

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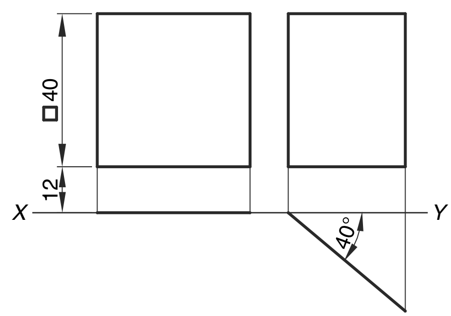

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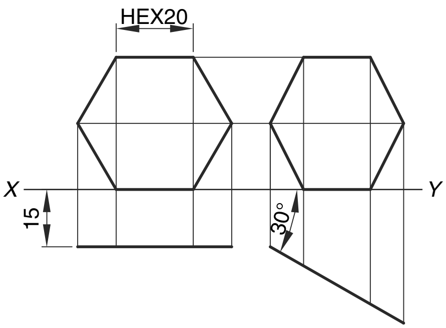

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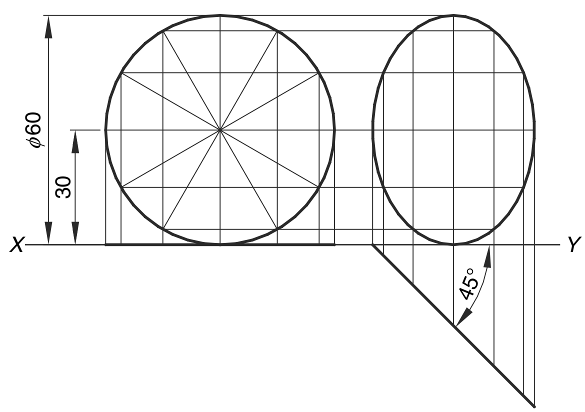

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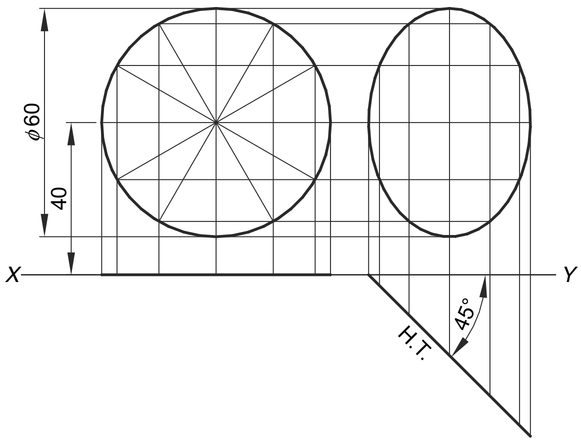

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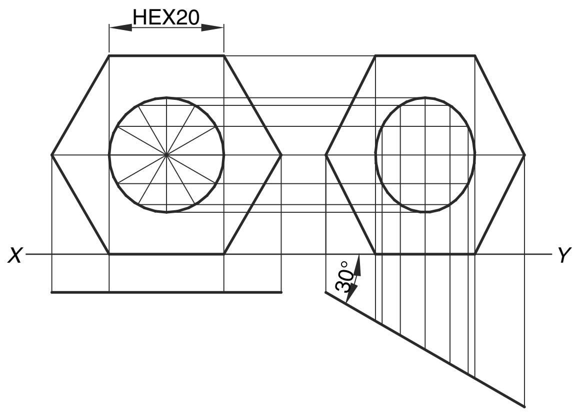

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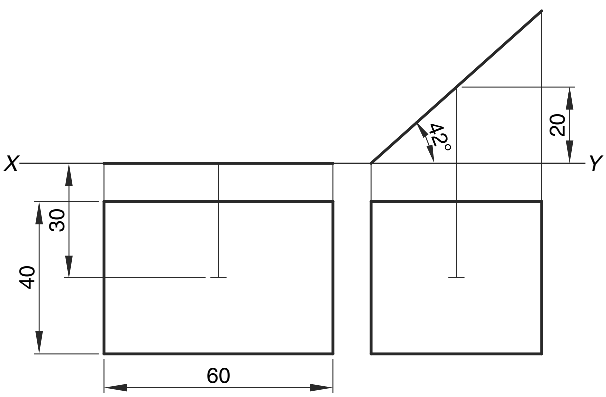

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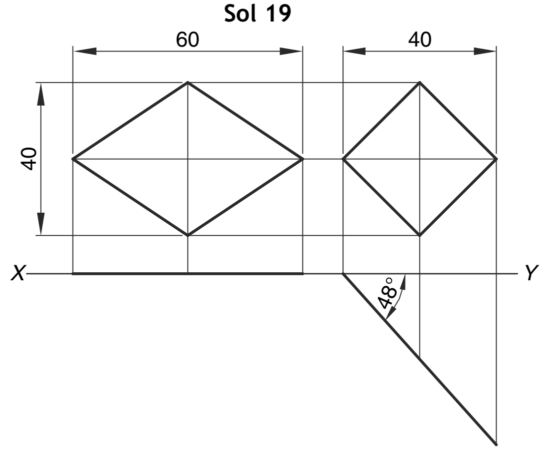

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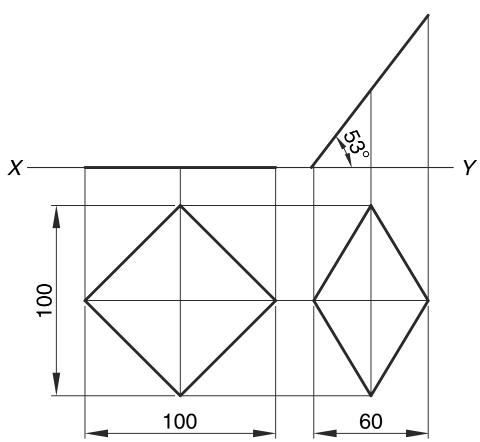

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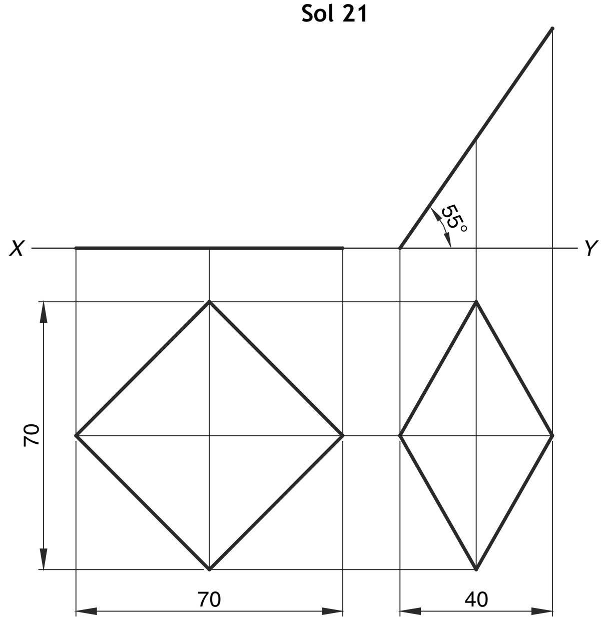

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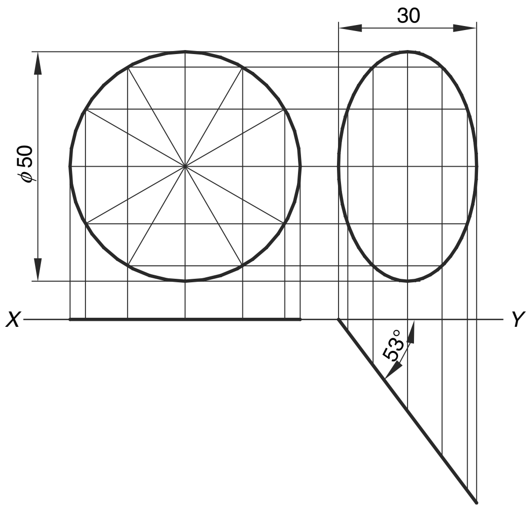

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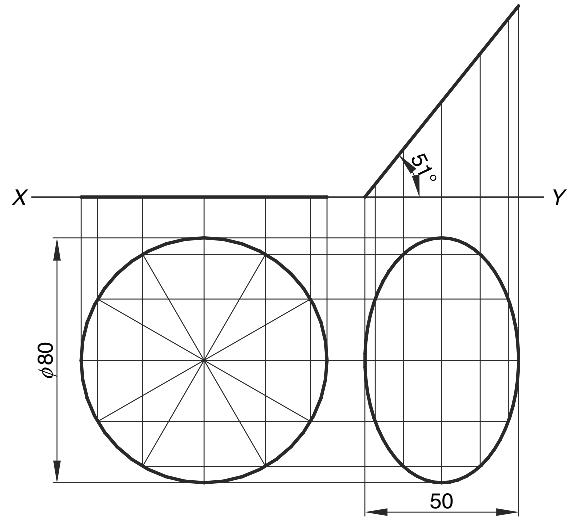

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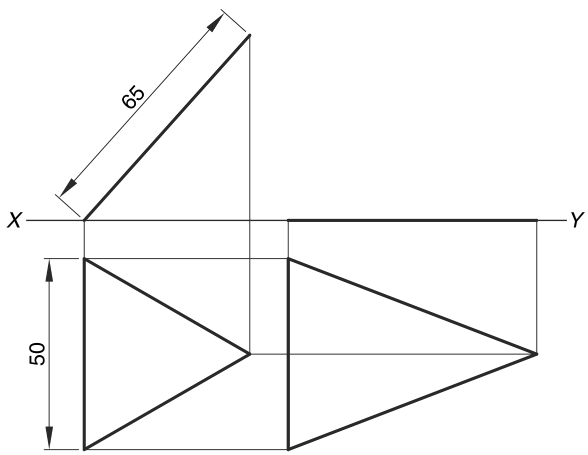

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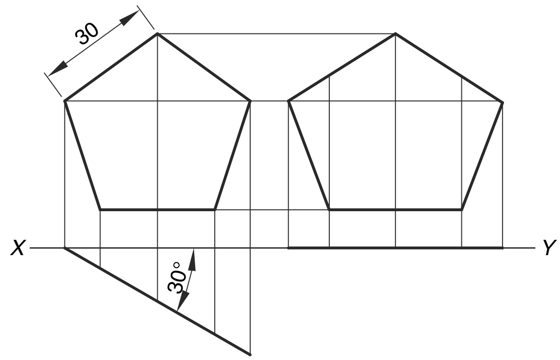
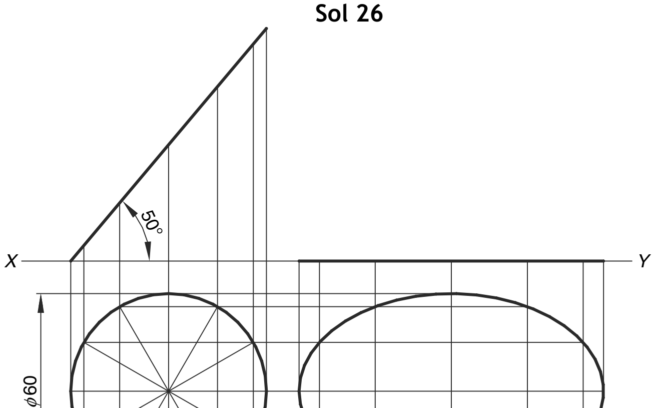
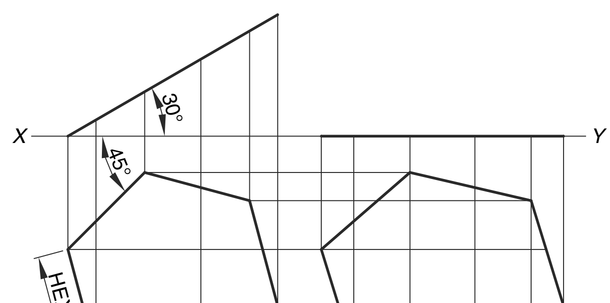


Sol 27

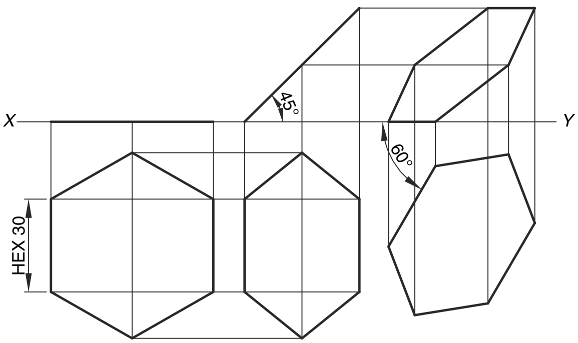


Sol 28



Sol 29

EXERCISE 8B



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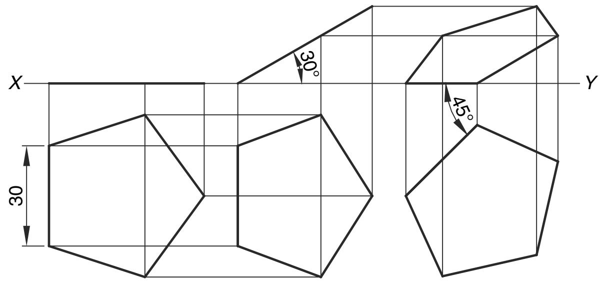

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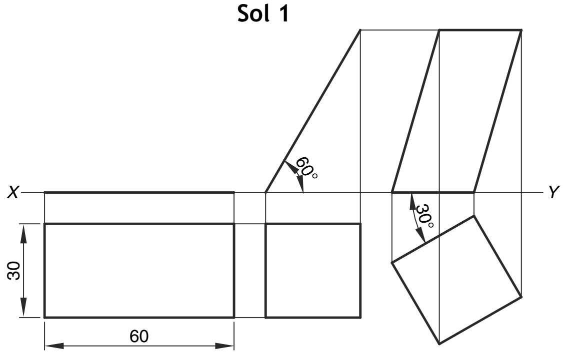

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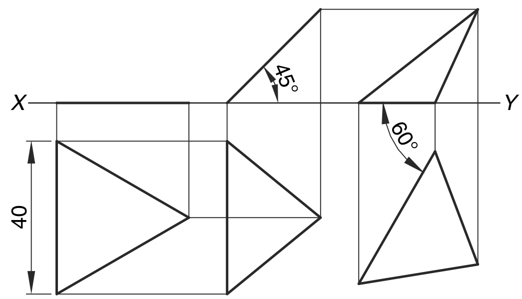

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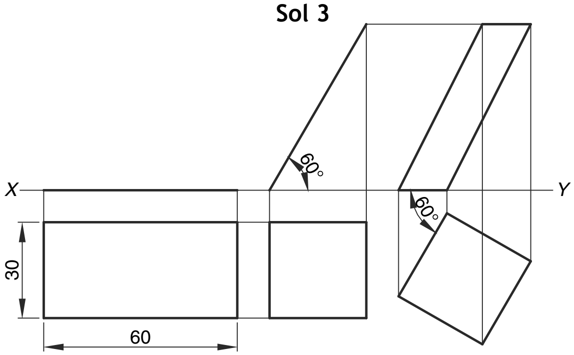

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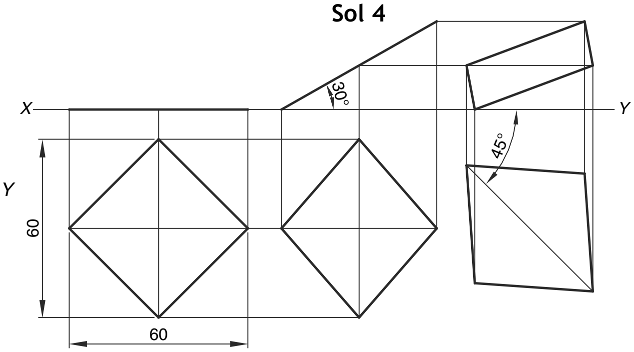

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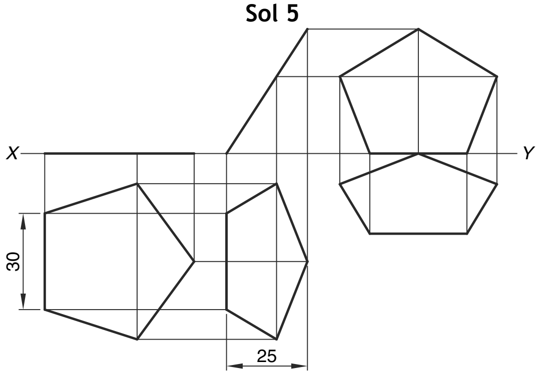

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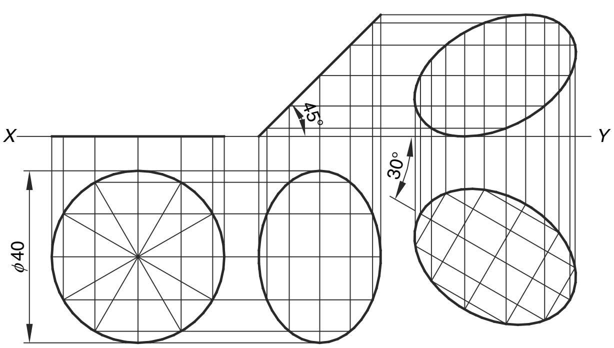

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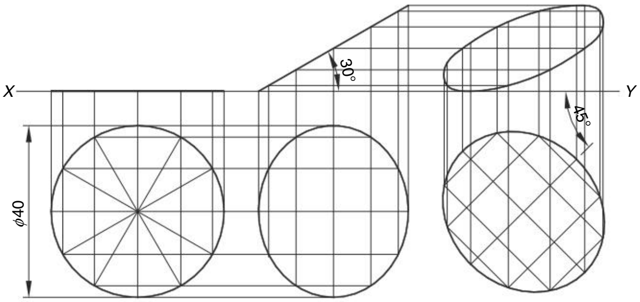

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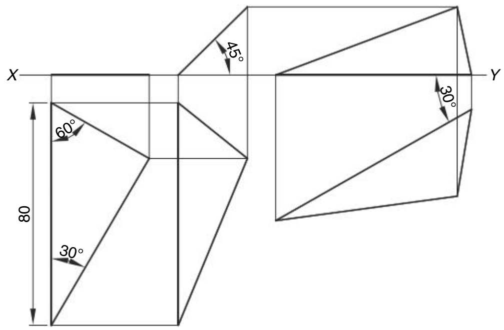

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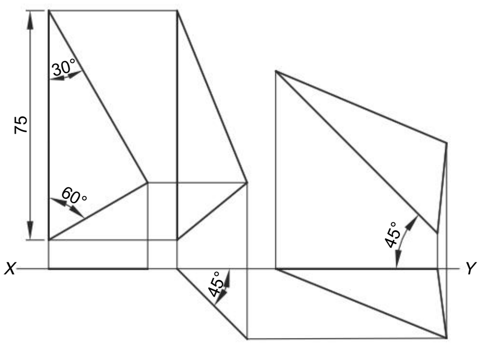

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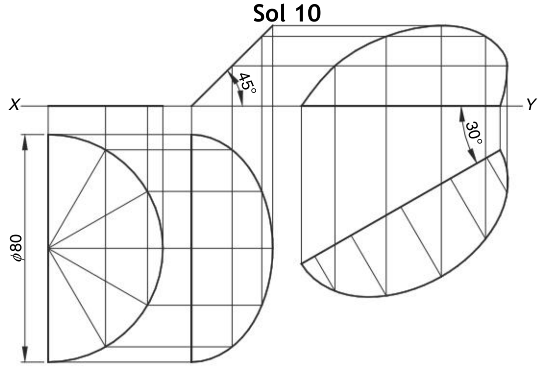

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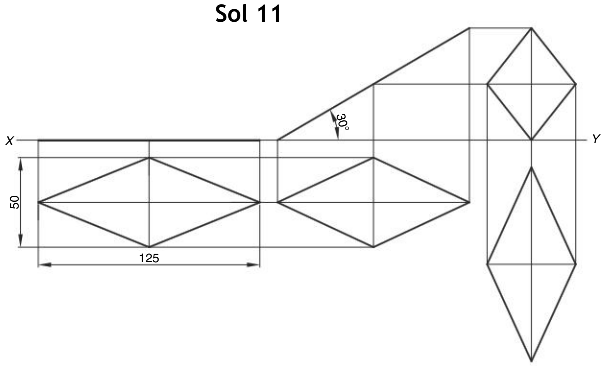

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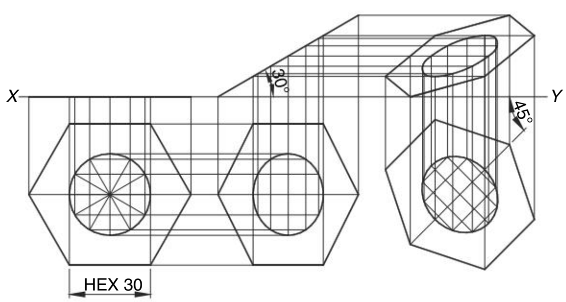

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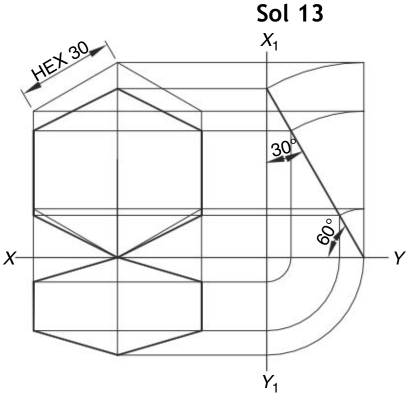

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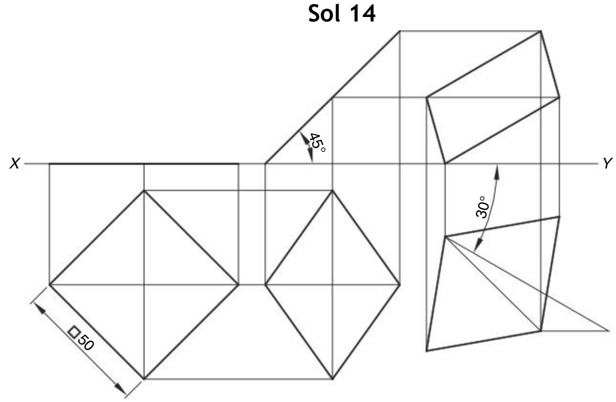

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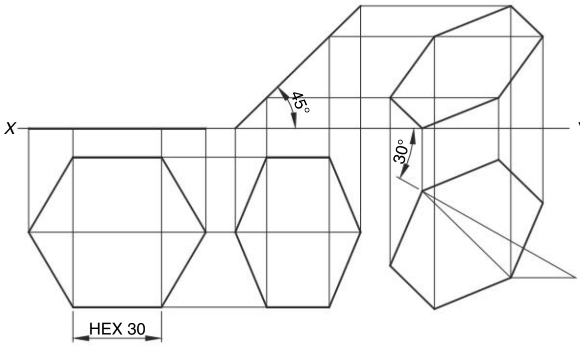

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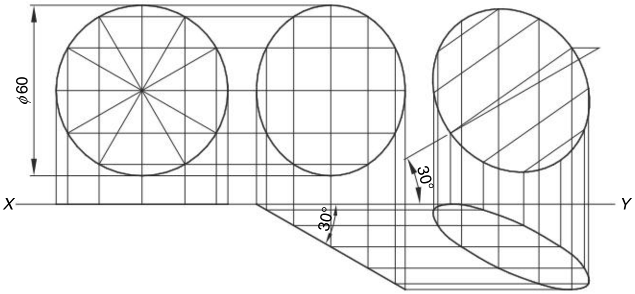

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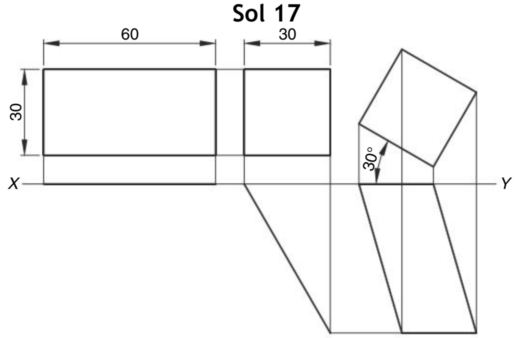

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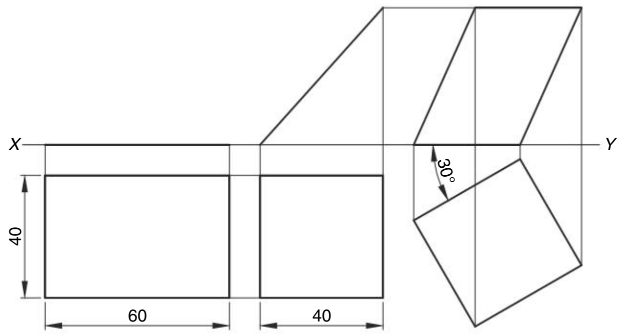

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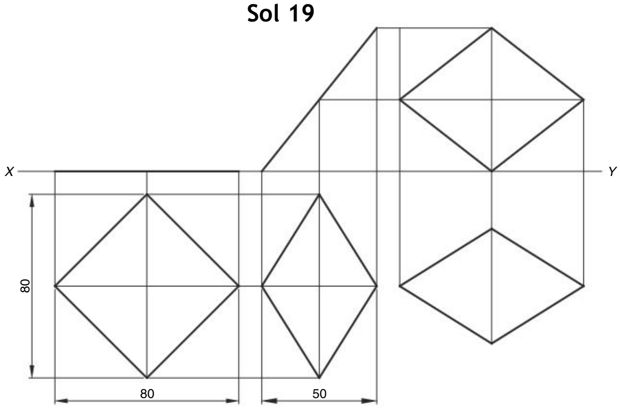
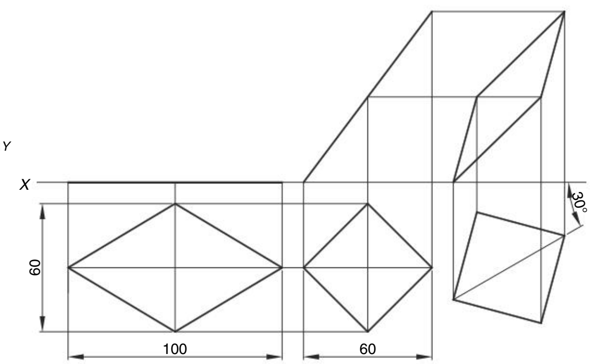


Sol 21



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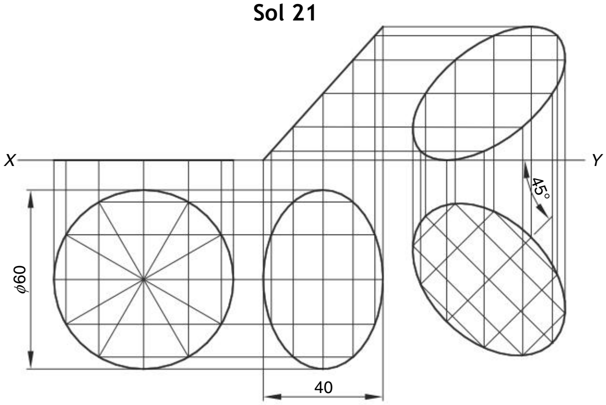

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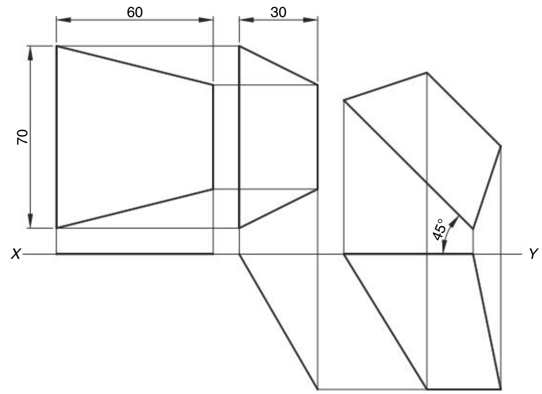

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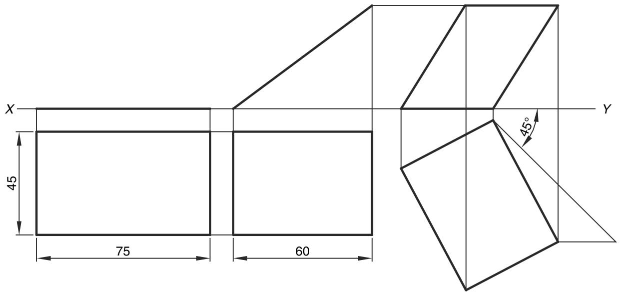

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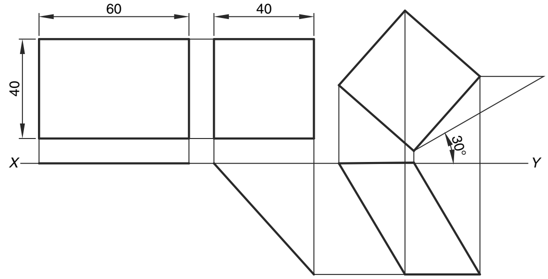
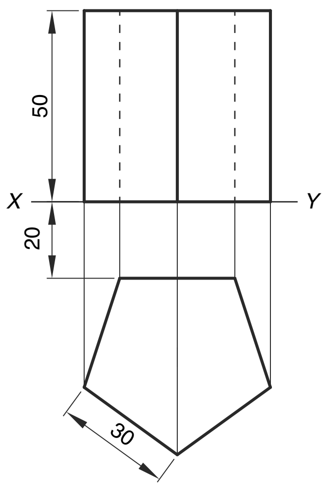


Sol 26

EXERCISE 9A



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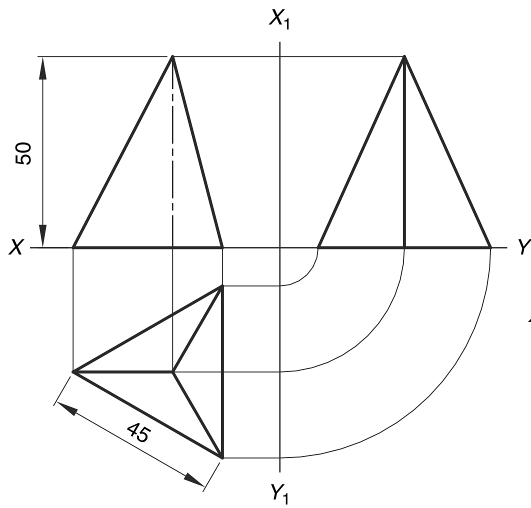

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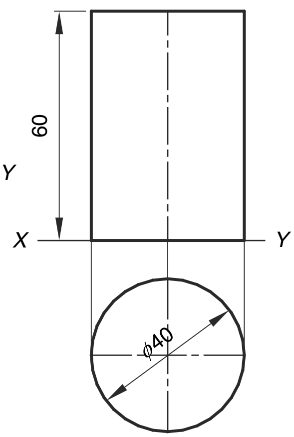

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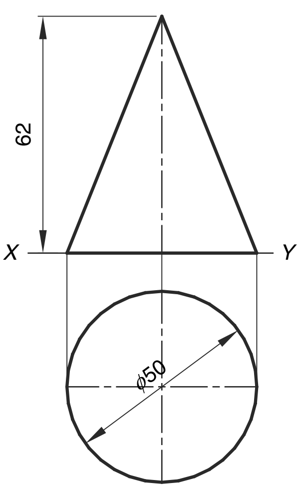

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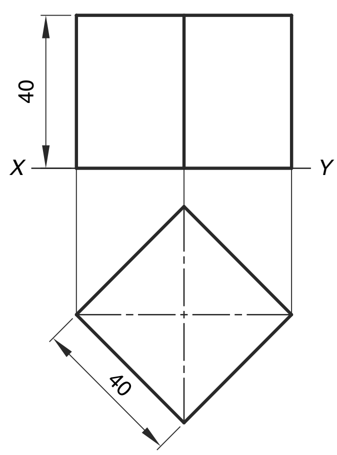
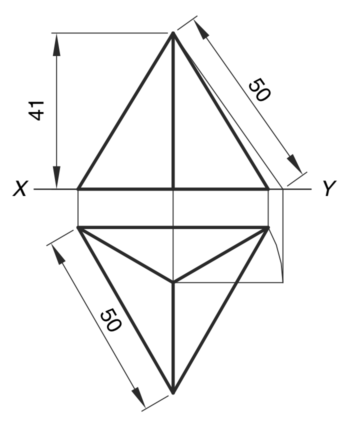


Sol 5



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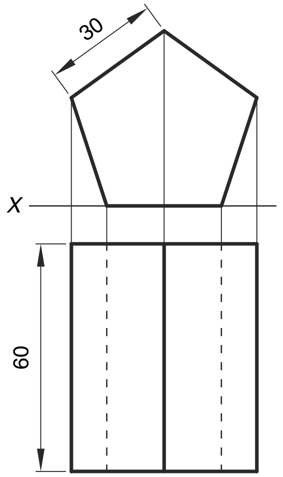

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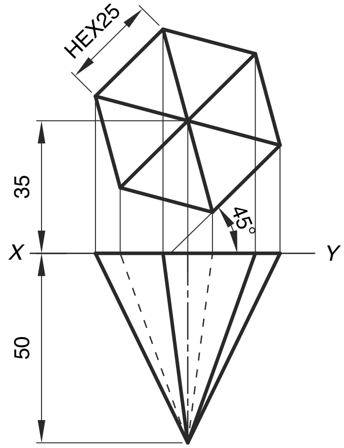

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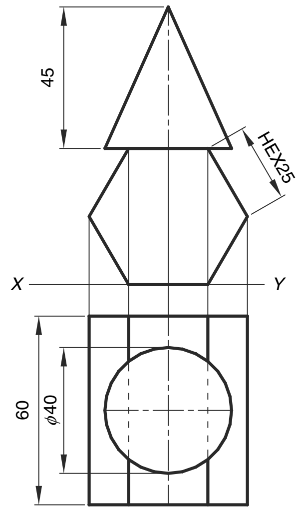
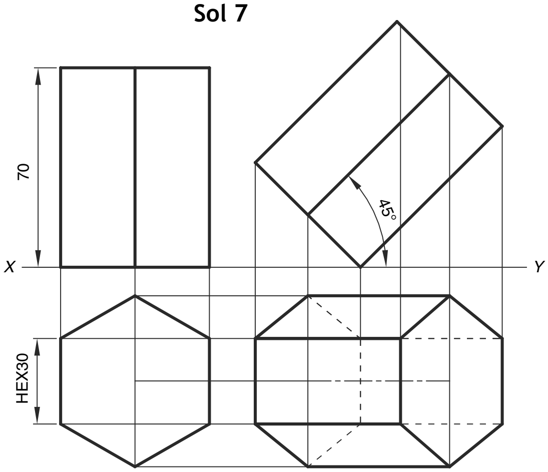


Sol 9



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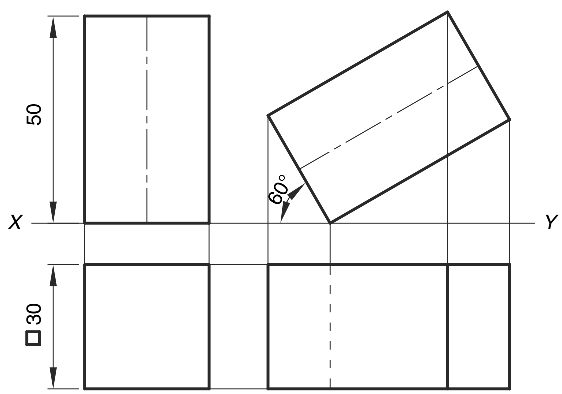

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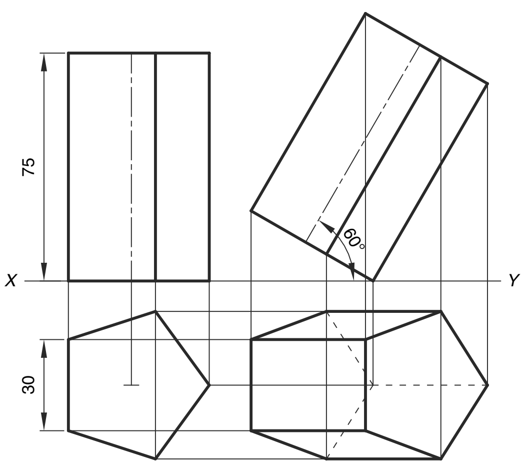

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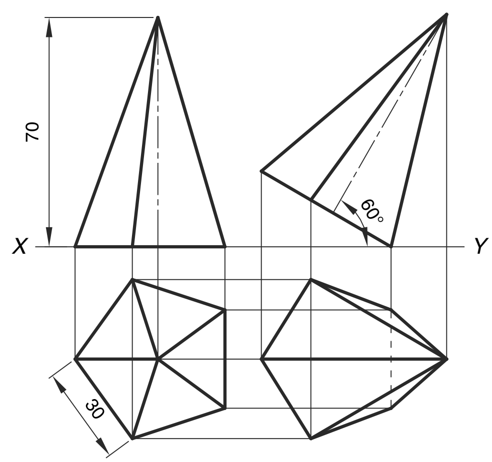

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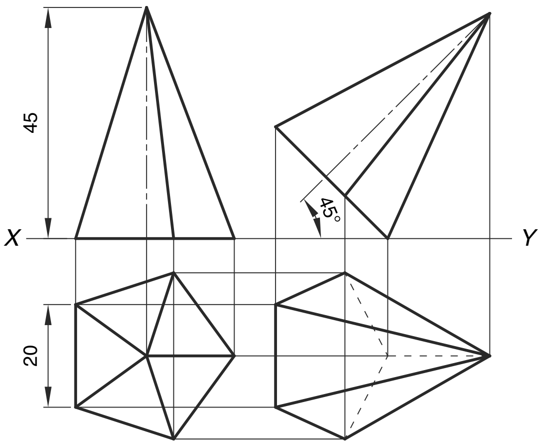

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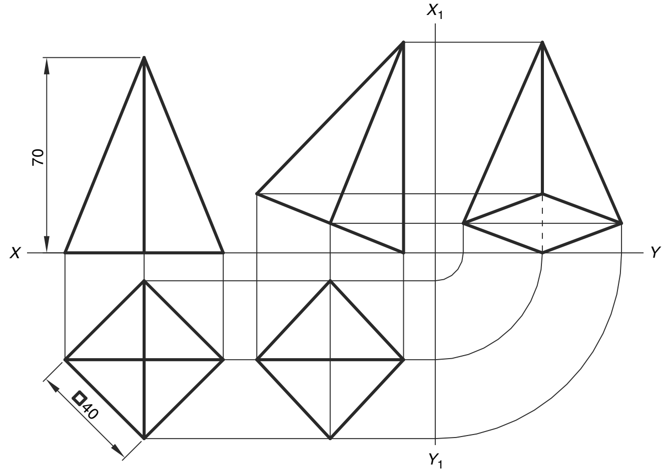

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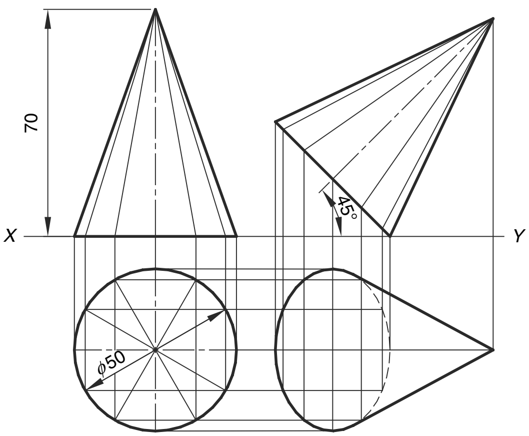

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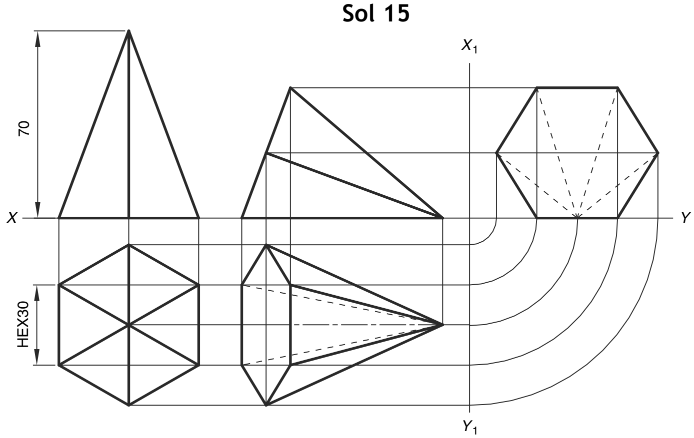

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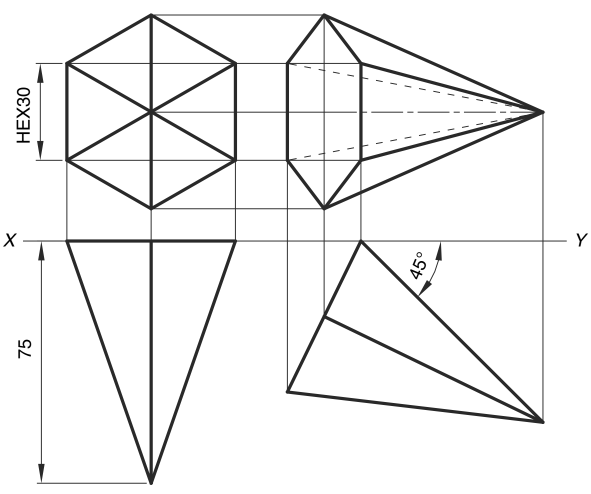

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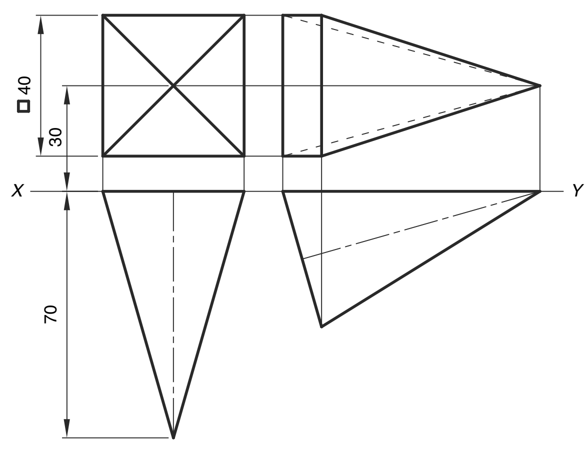

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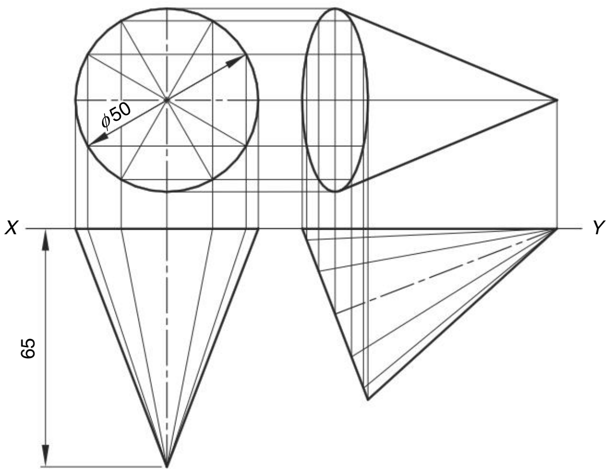

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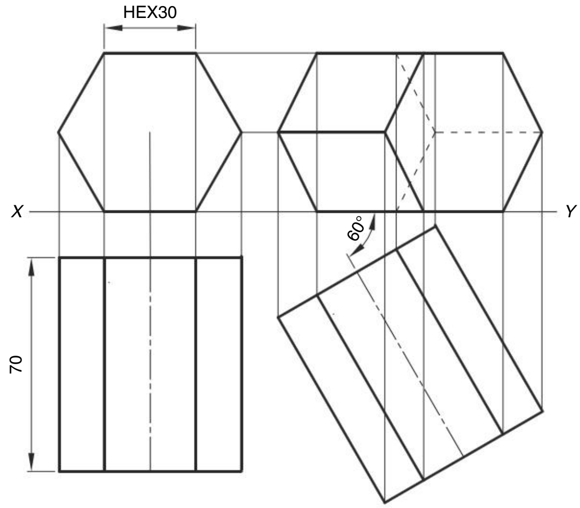

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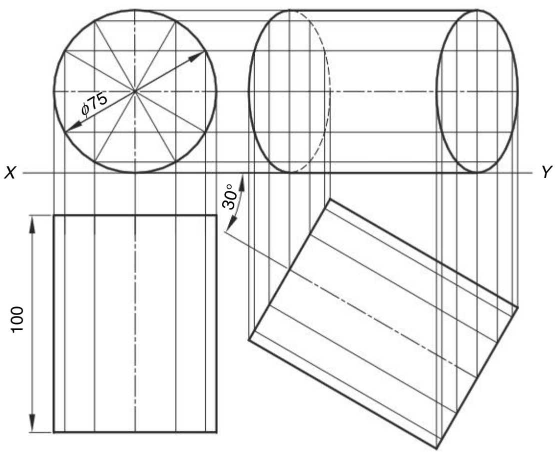

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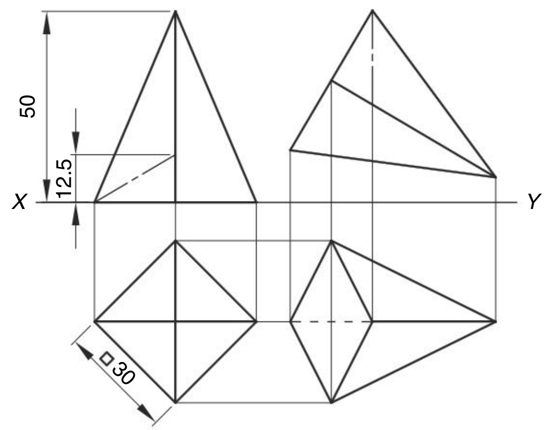

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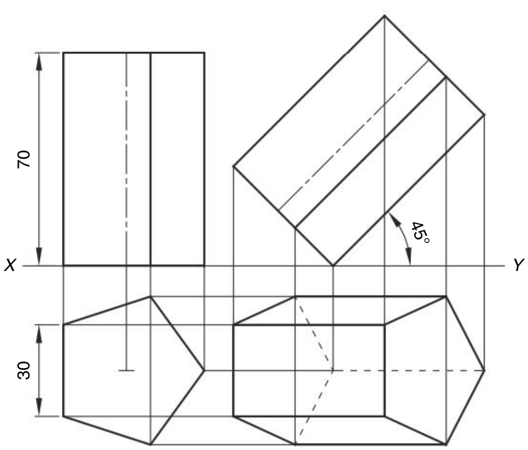

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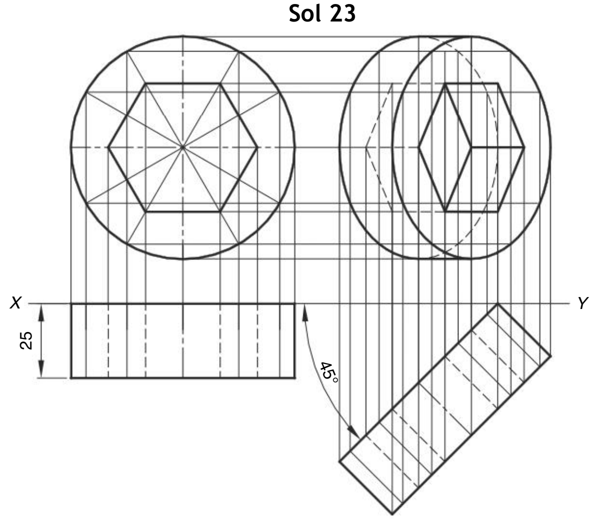
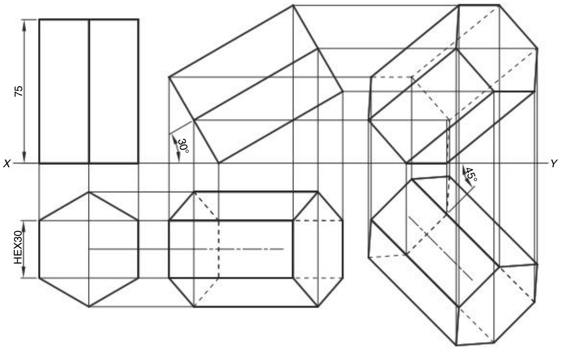
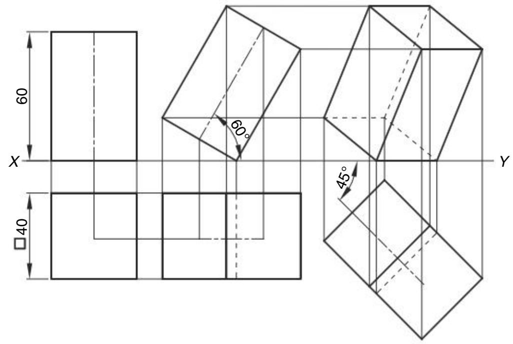


Sol 25

EXERCISE 9B



Sol 1



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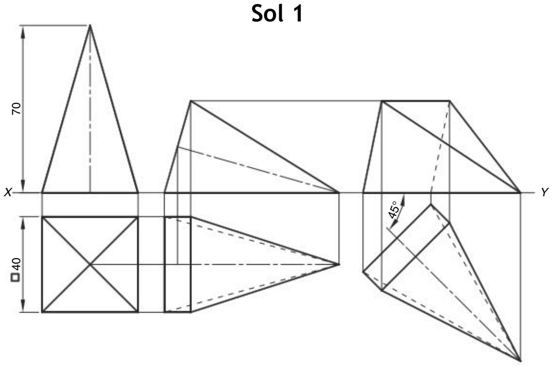
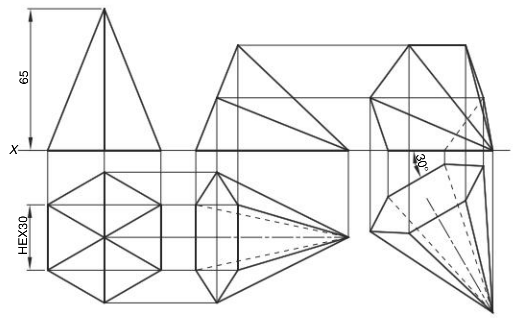
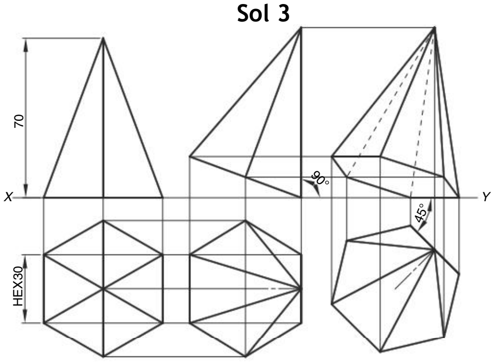


Sol 3



Sol 4



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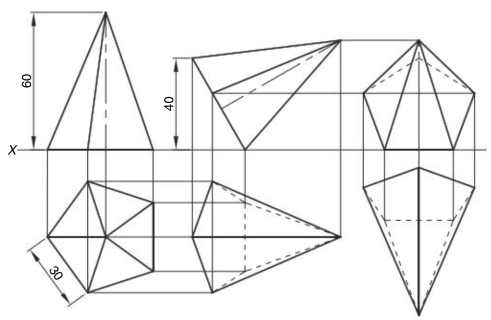
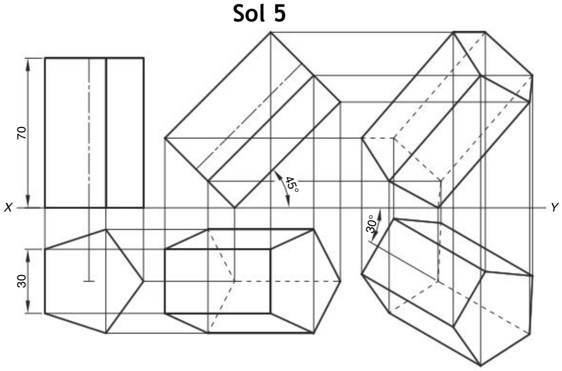


Sol 6



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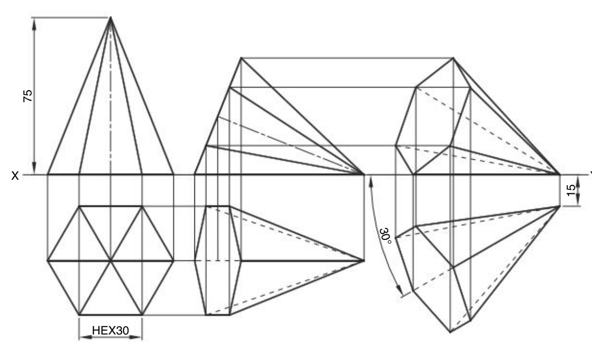

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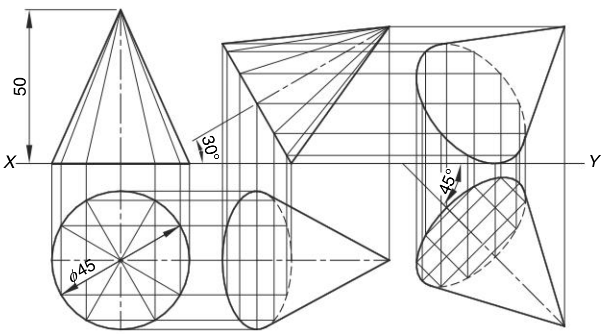

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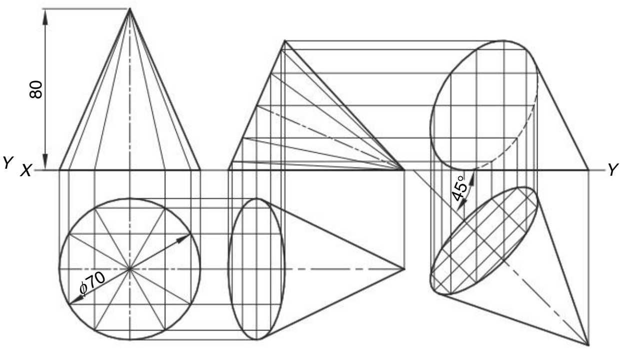

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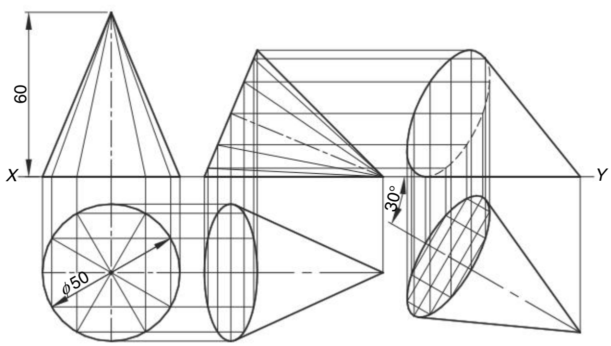

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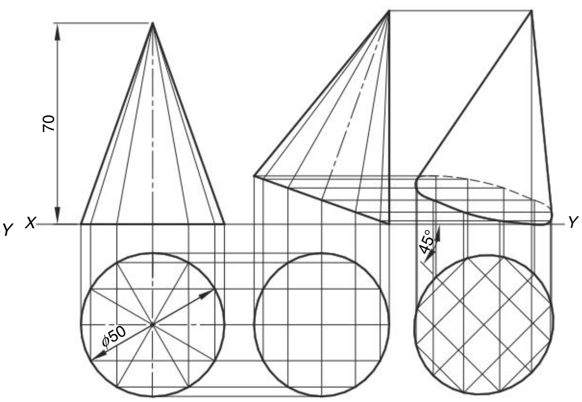

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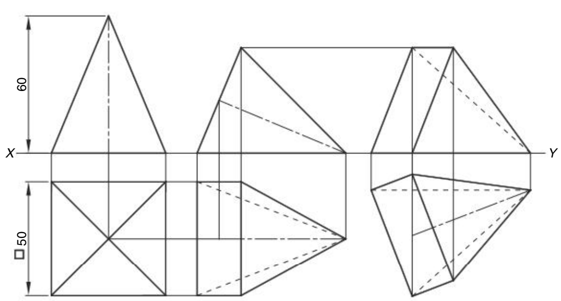

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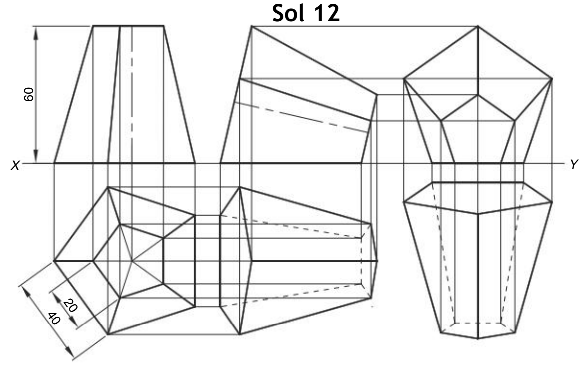

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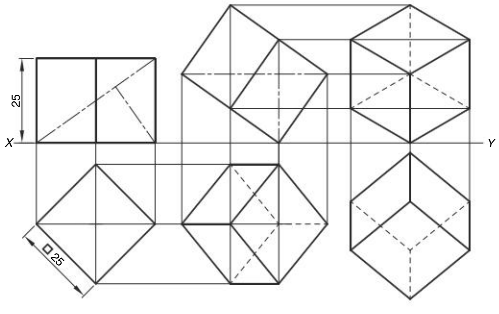
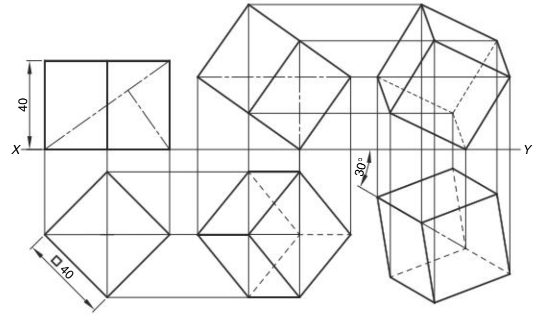


Sol 15



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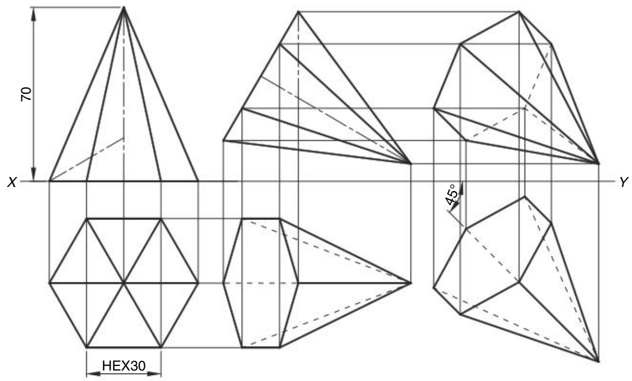

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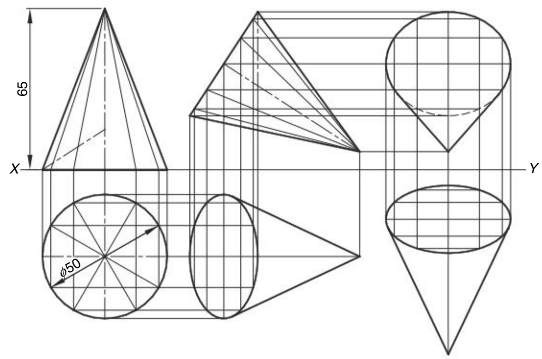

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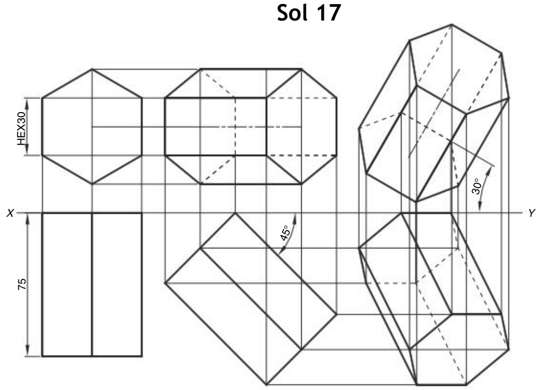

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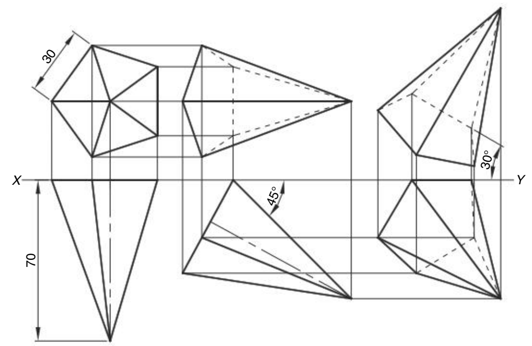
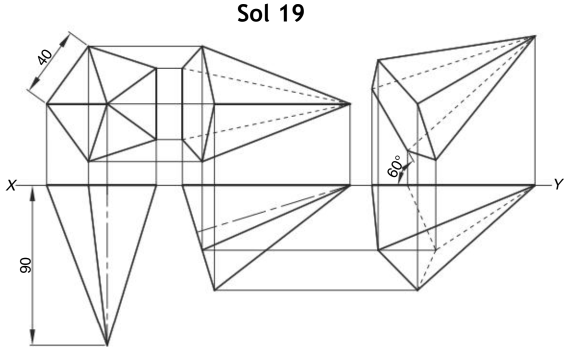
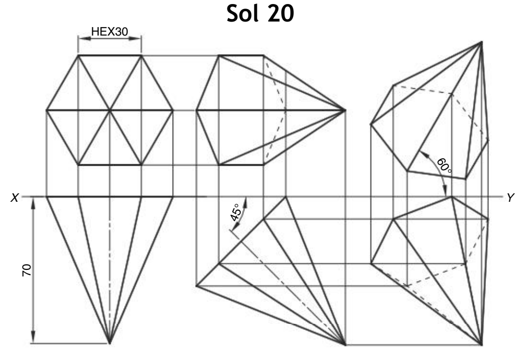


Sol 20



Sol 21



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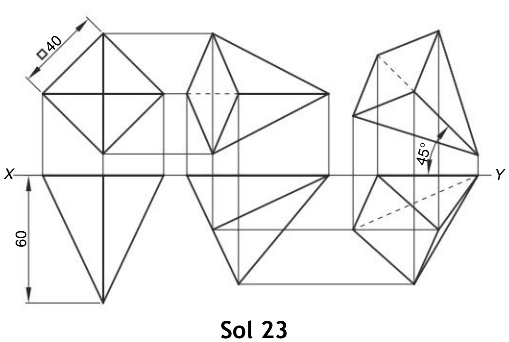

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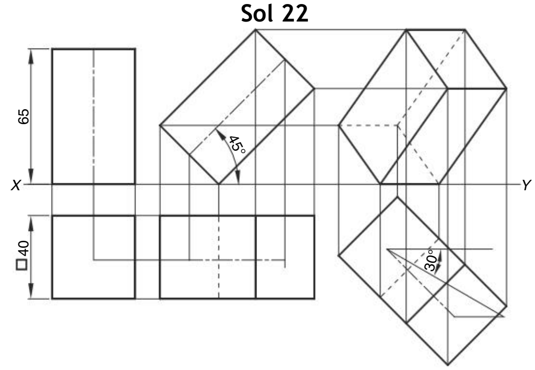

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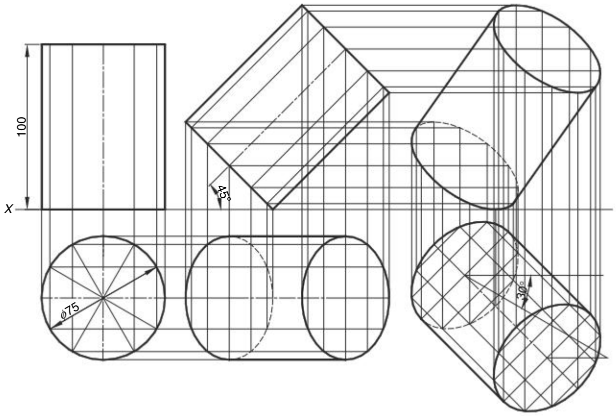

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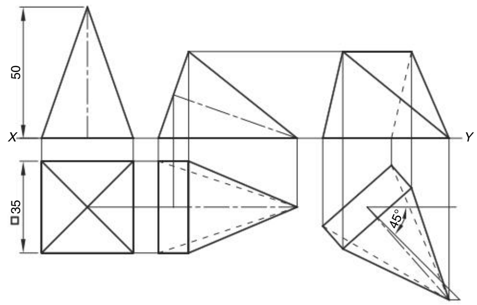

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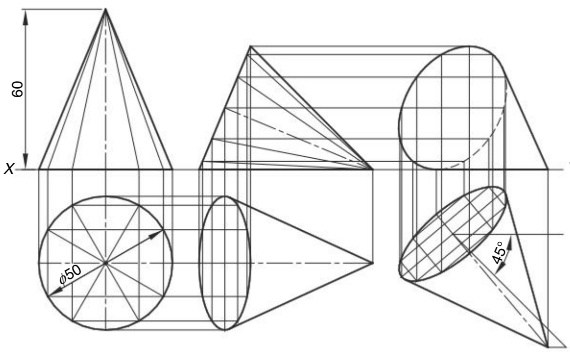
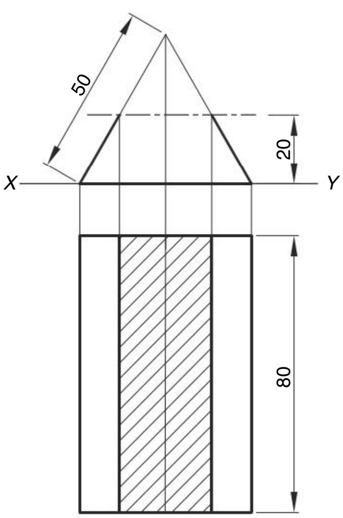


Sol 27

EXERCISE 10



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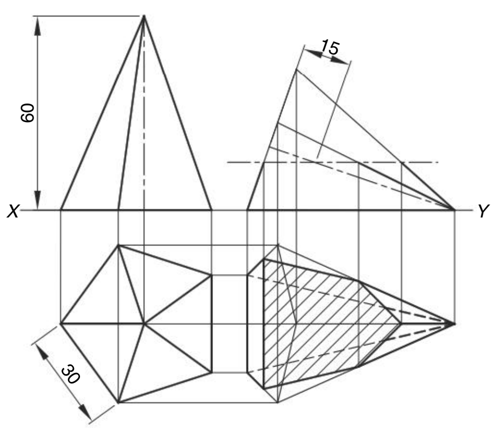

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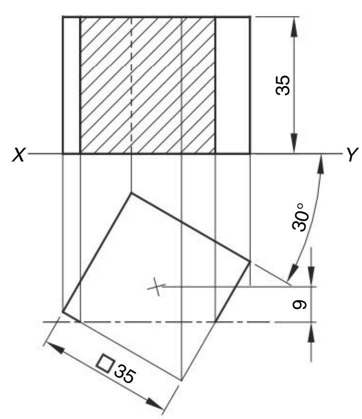

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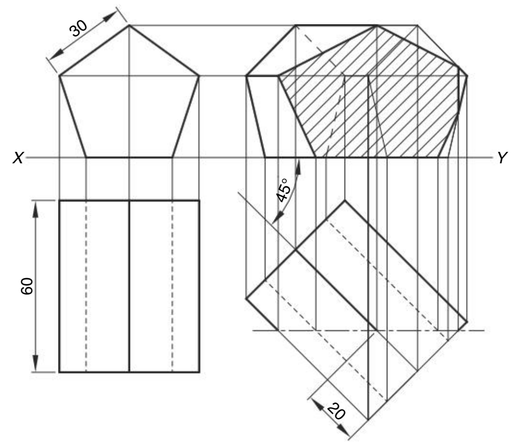

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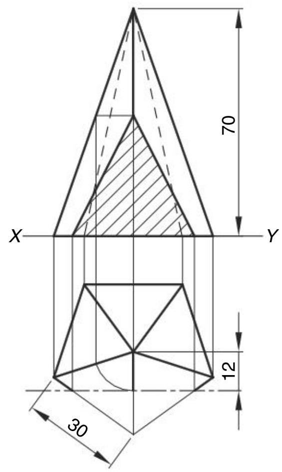

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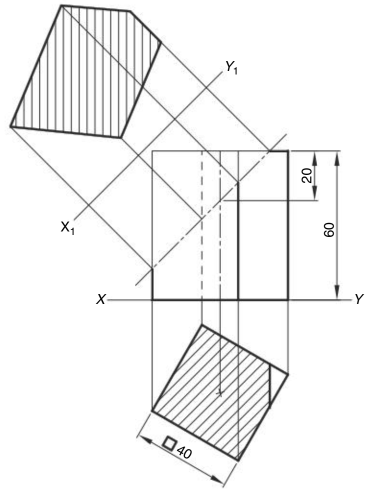

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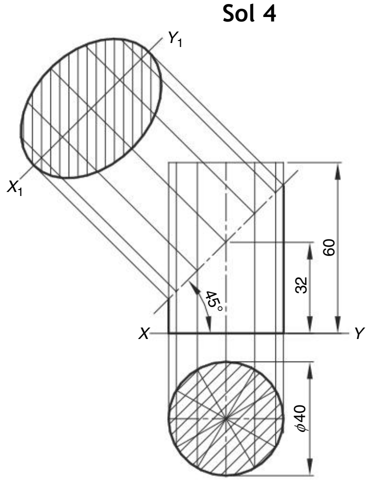

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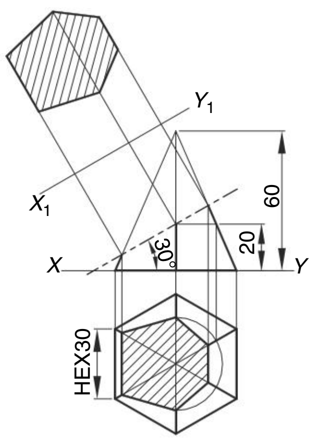

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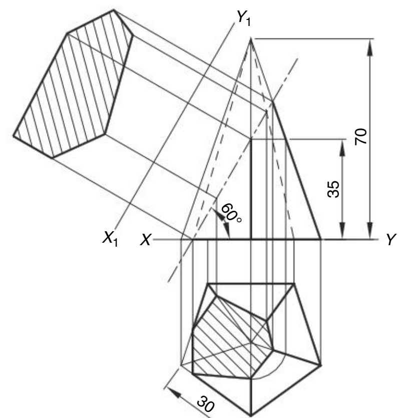

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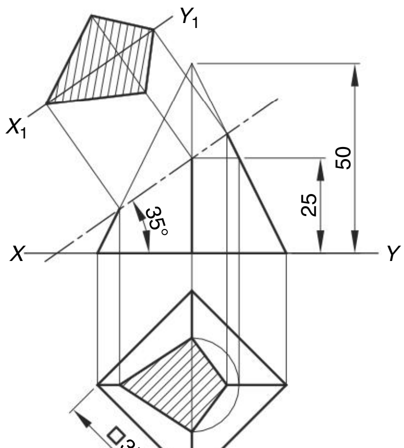

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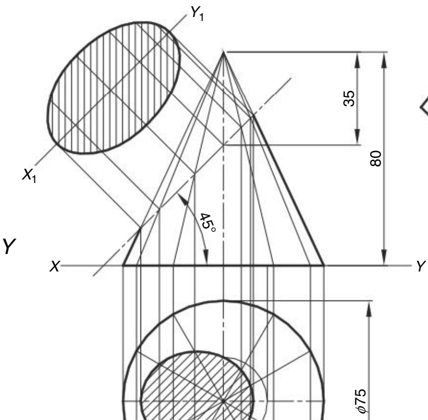

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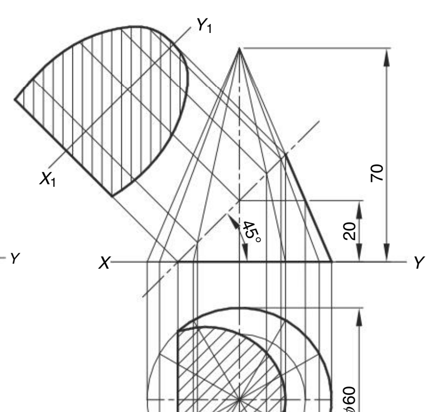

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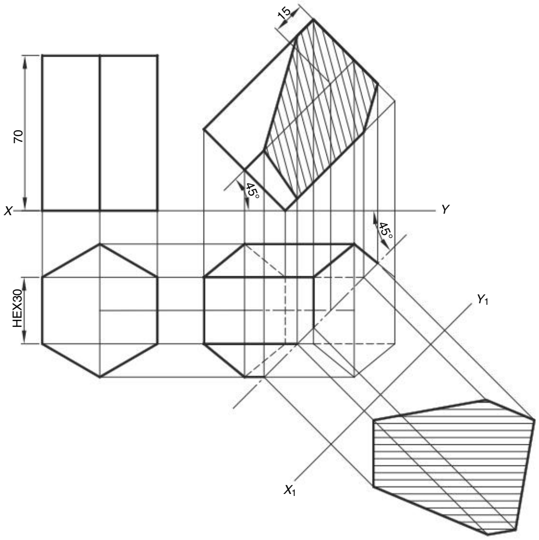

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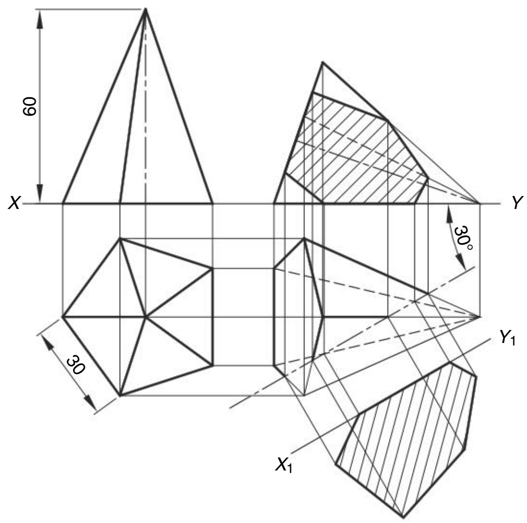

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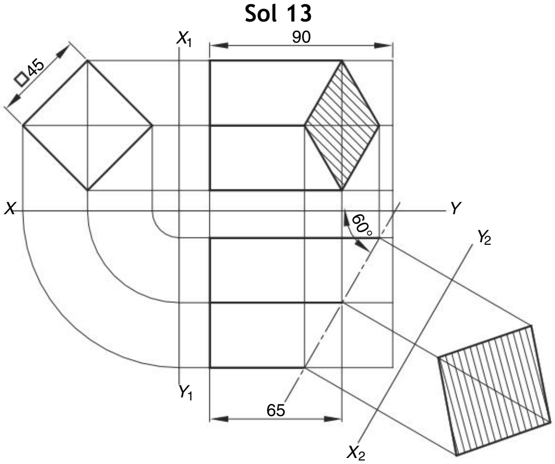

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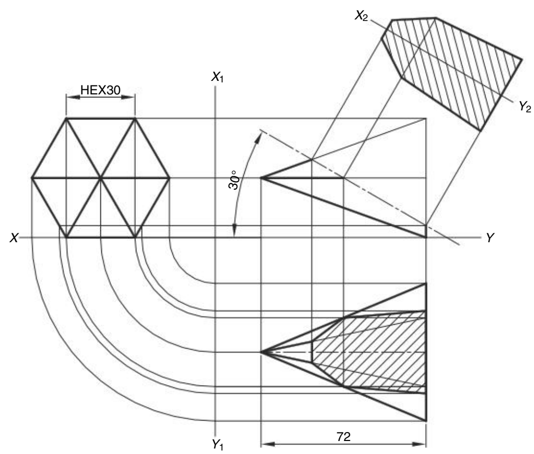

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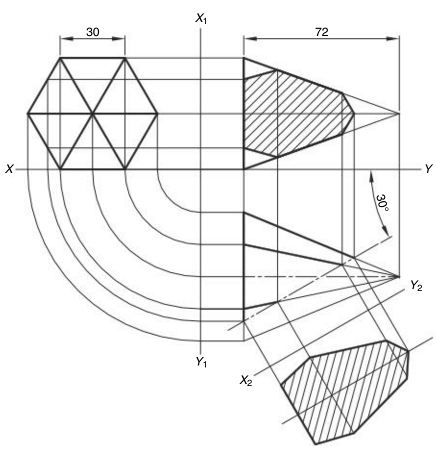

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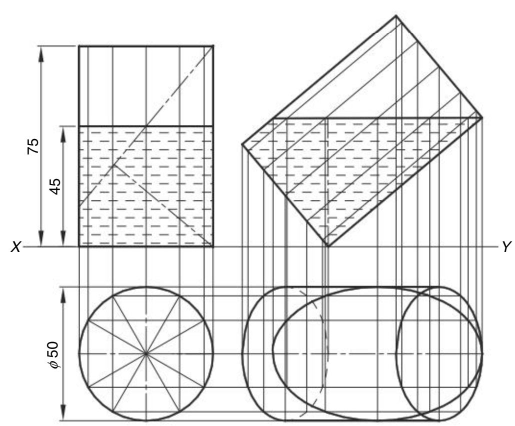

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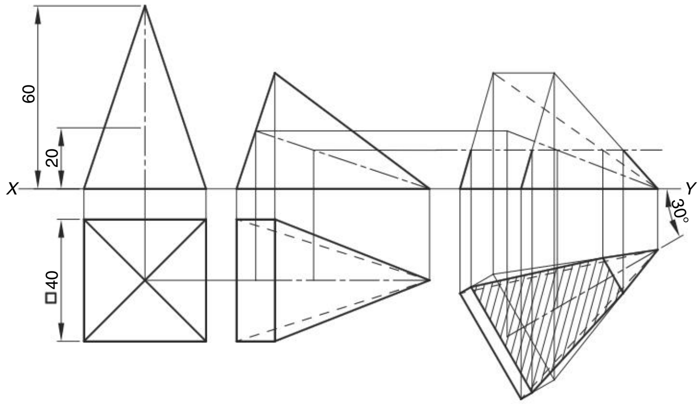
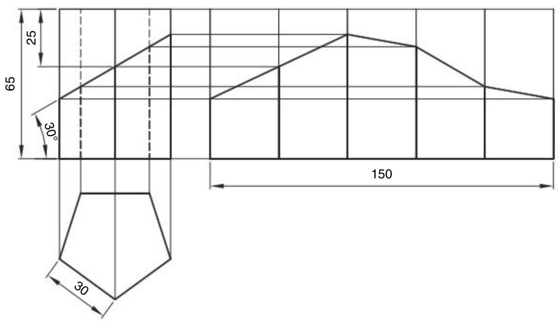


Sol 19

EXERCISE 11



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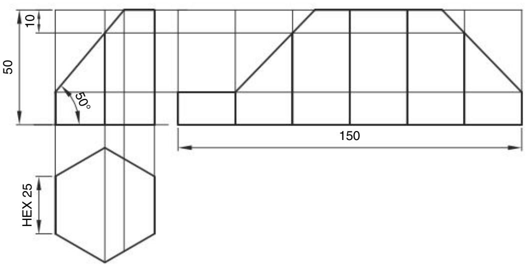

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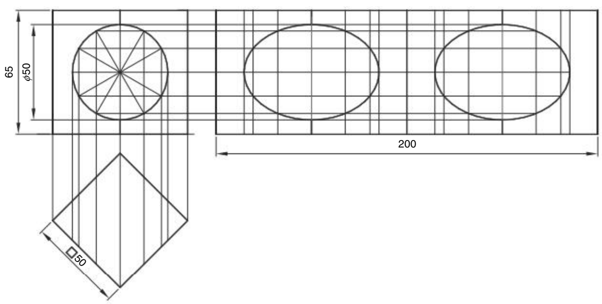

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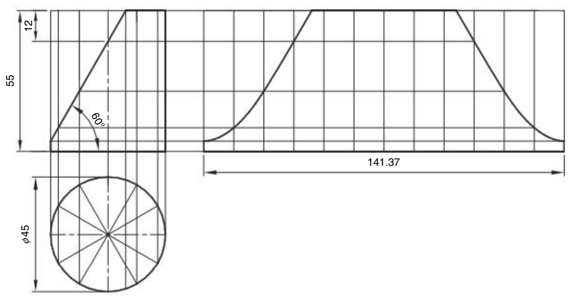

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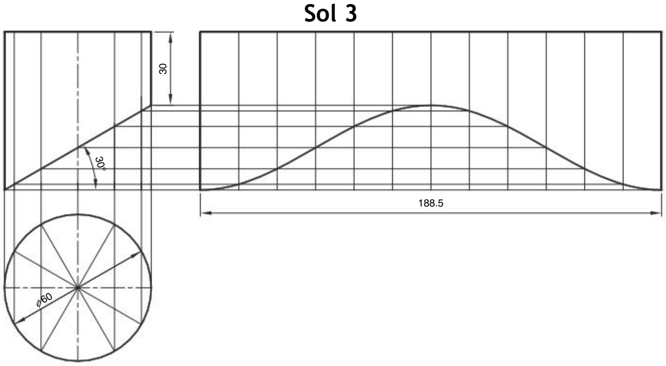

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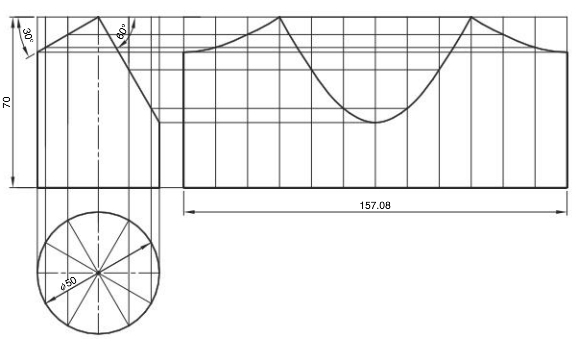

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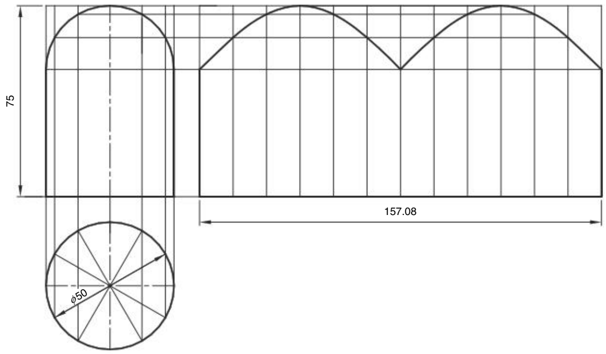

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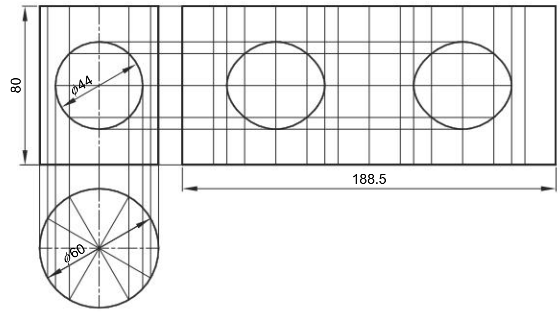

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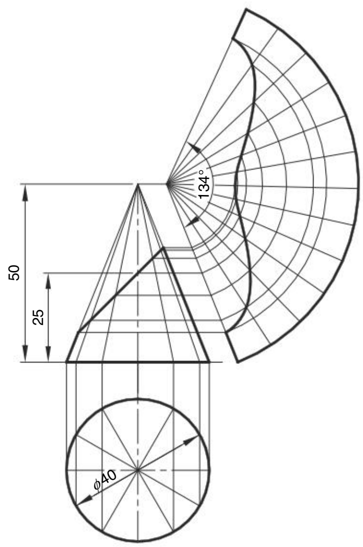

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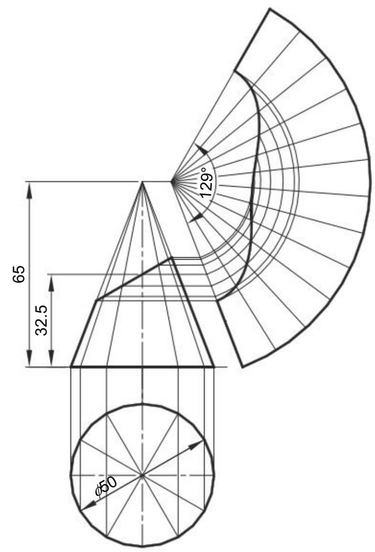

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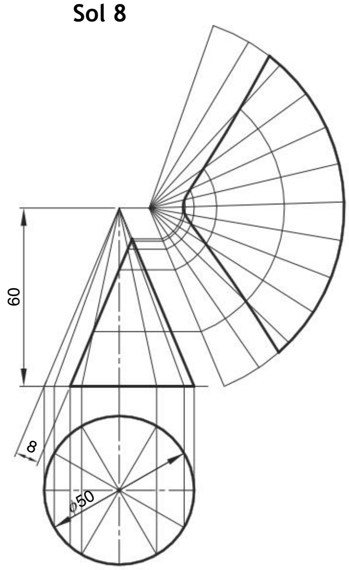

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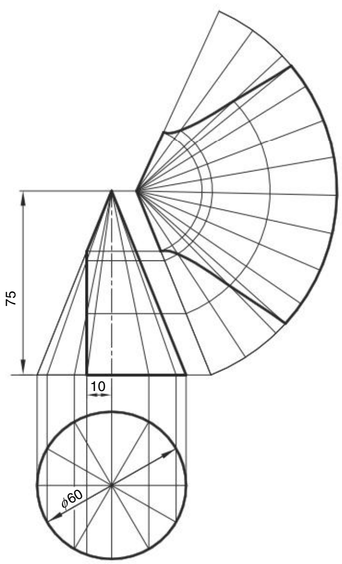

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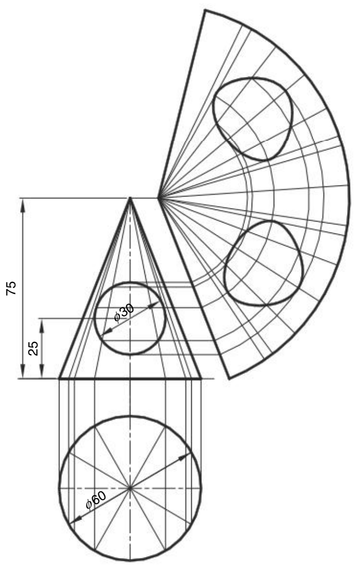

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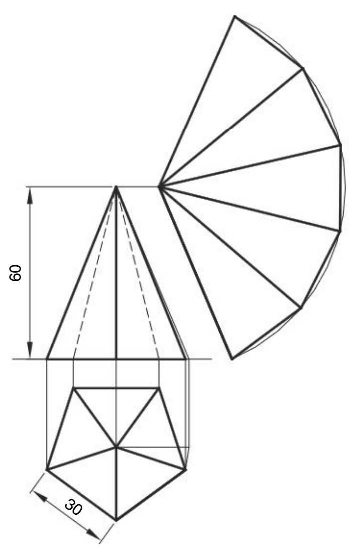

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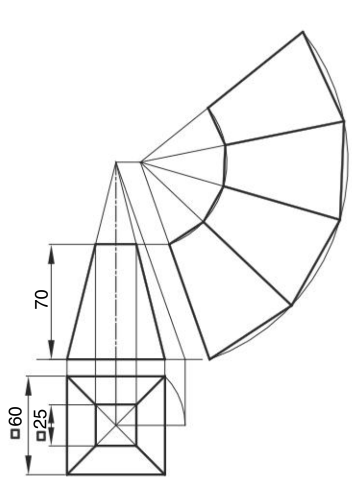

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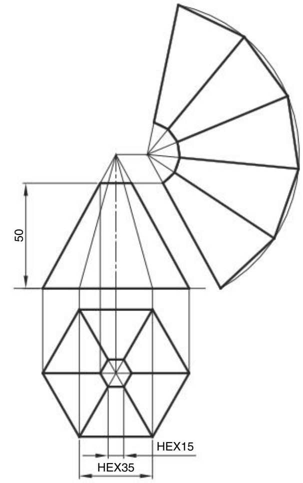

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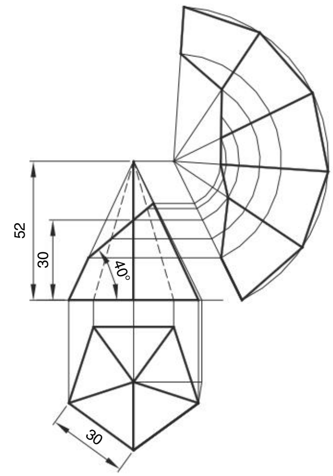

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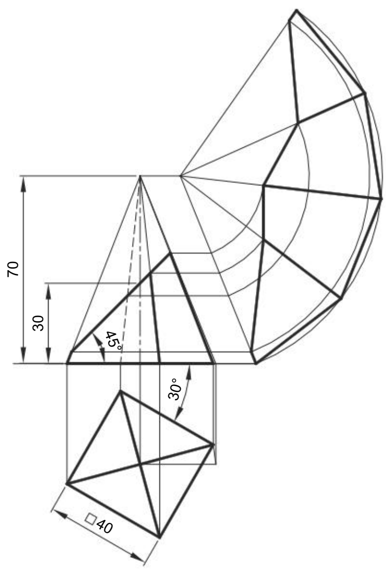

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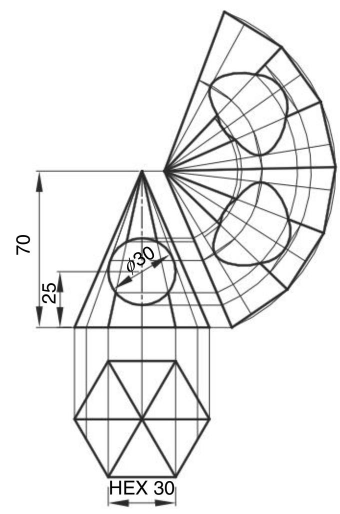

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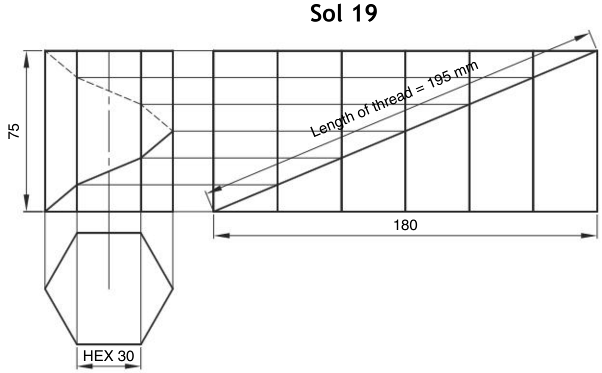

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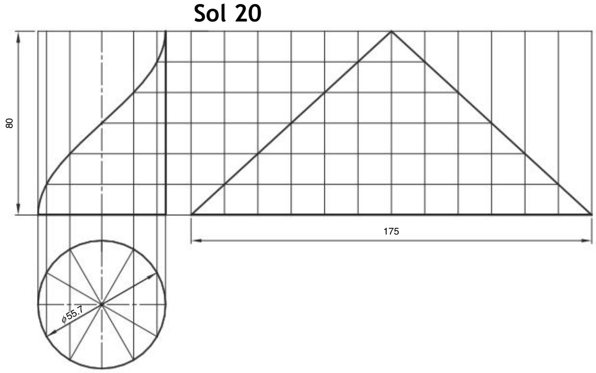

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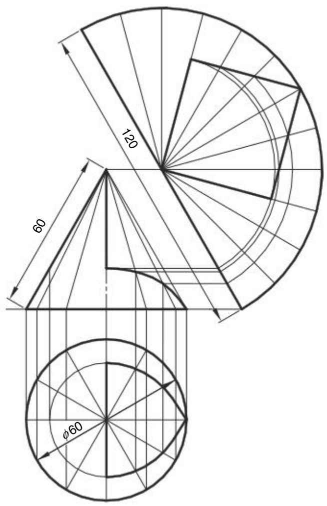

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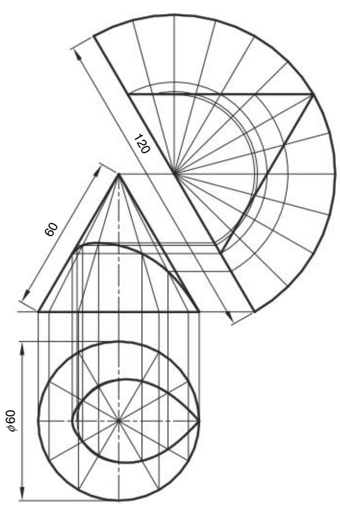

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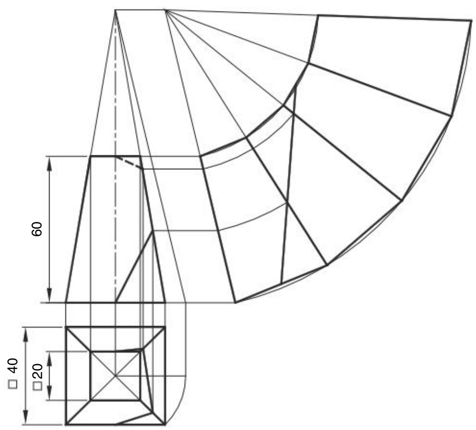

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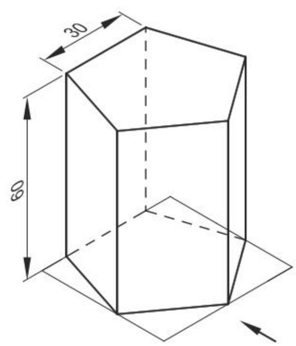
EXERCISE 12



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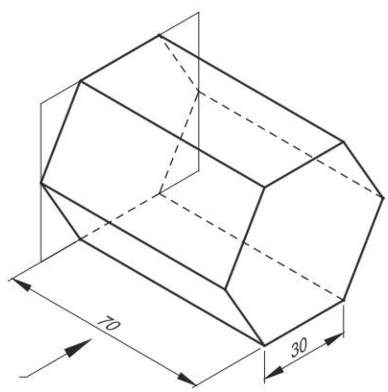

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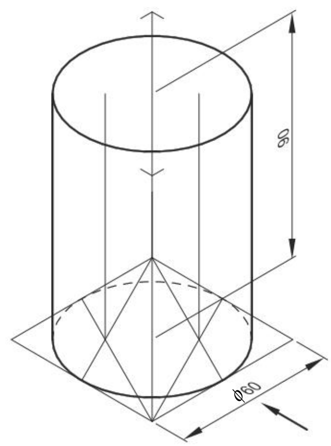

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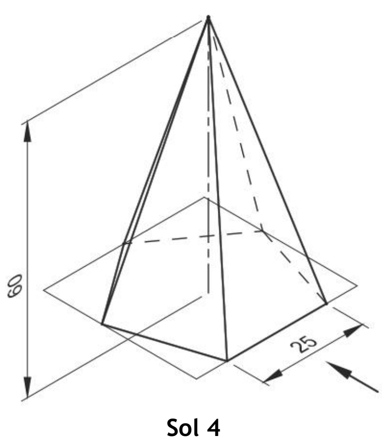

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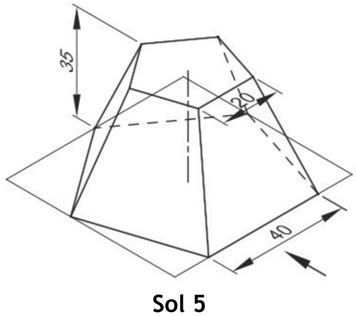

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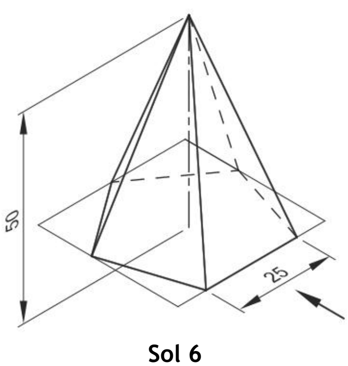

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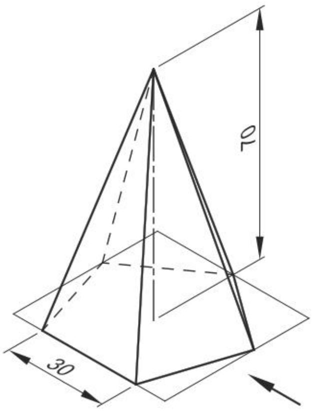

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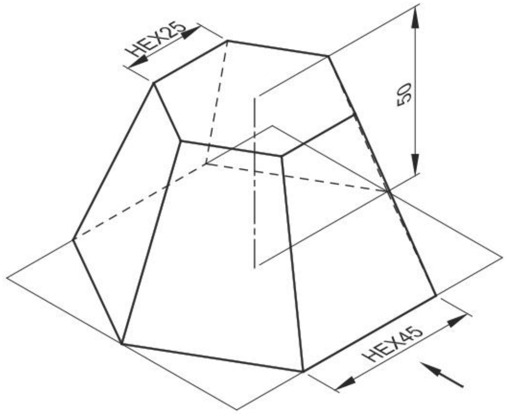

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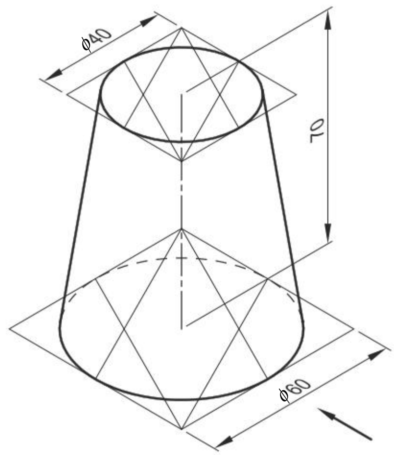

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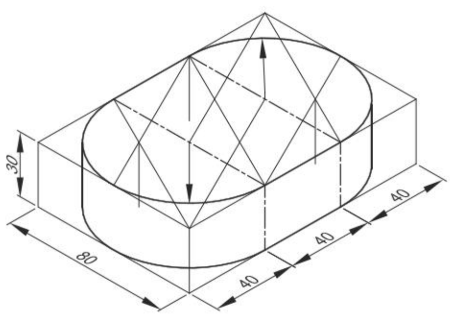

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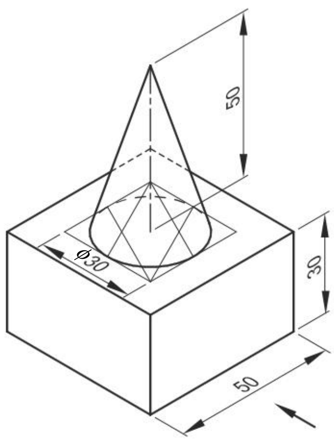

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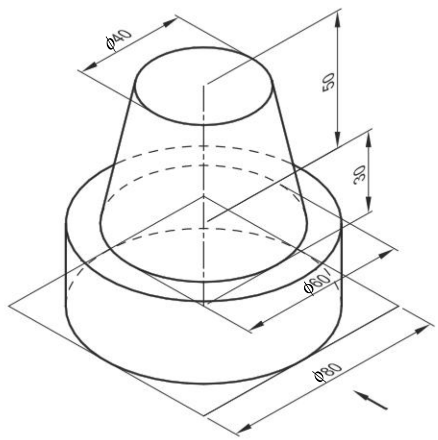

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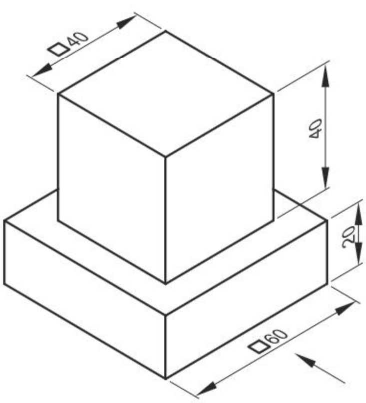

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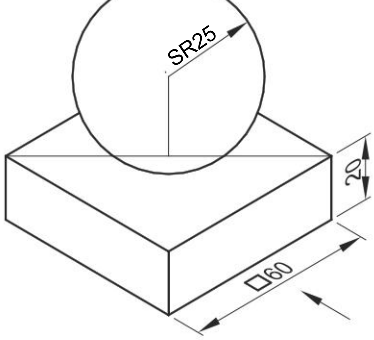

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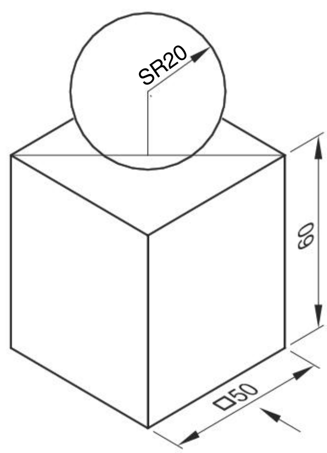

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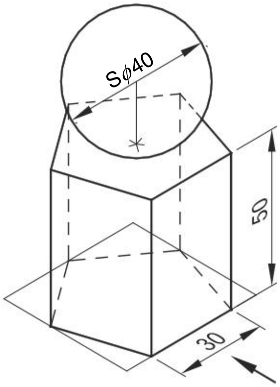

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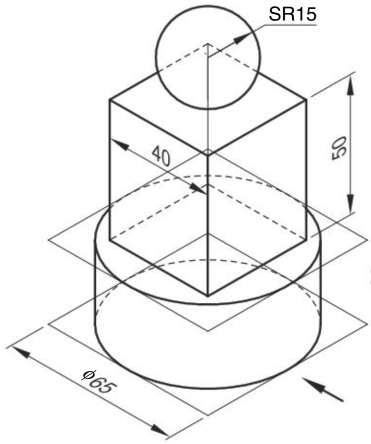

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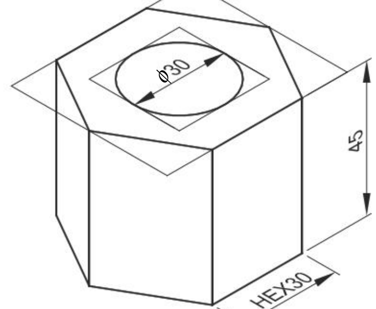

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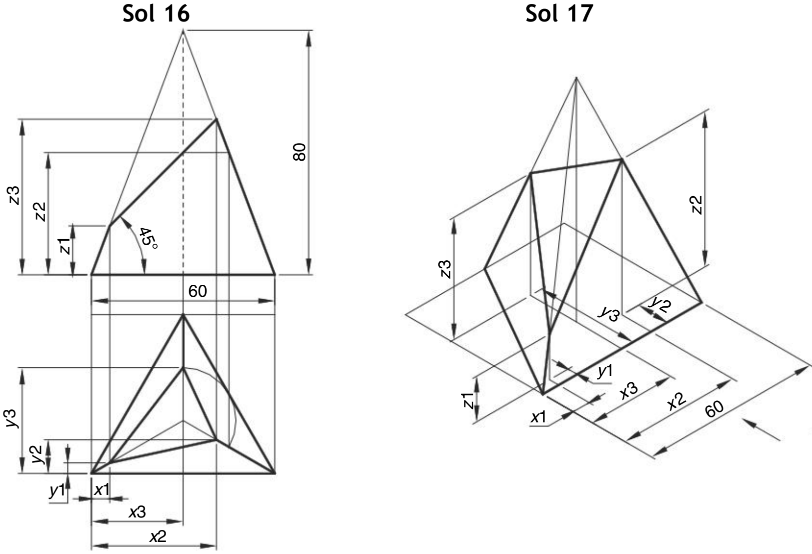

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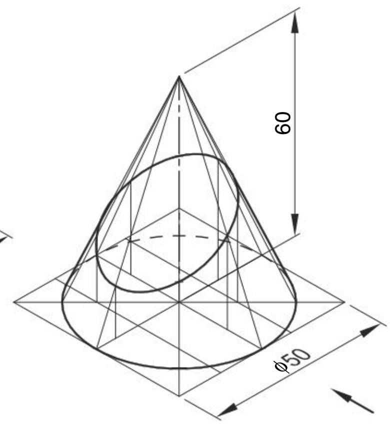

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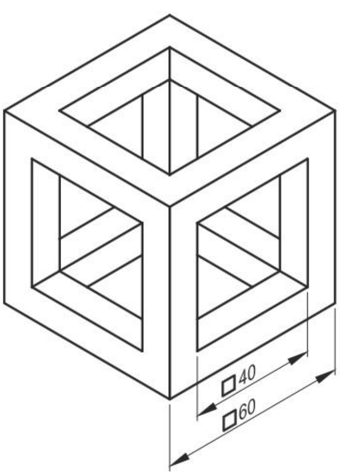

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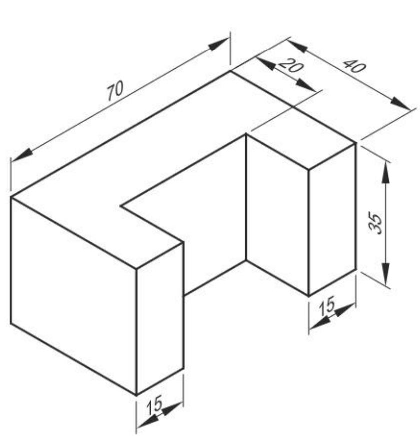
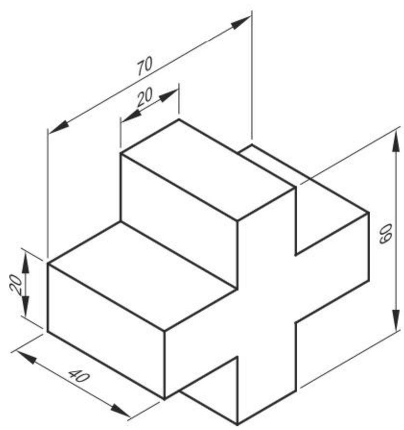
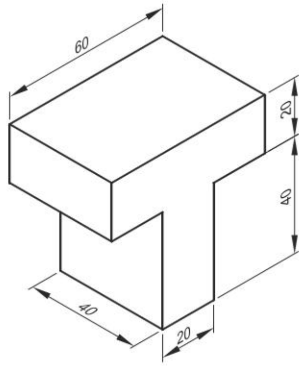


Sol 22 E12.5



Sol 22 E12.6



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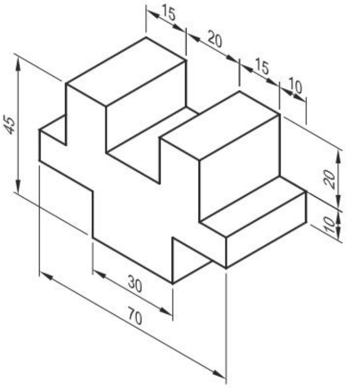
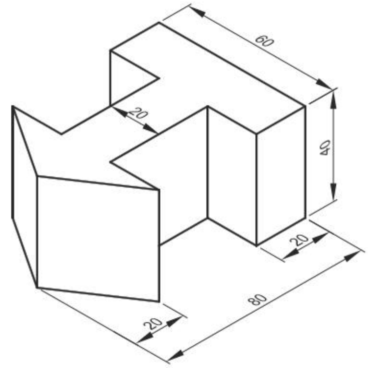


Sol 22 E12.8



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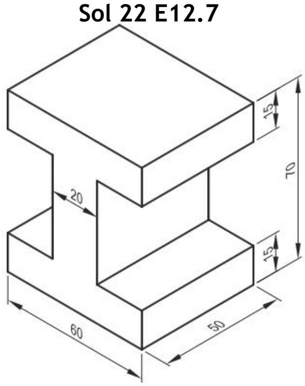

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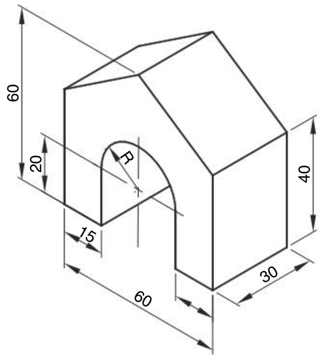

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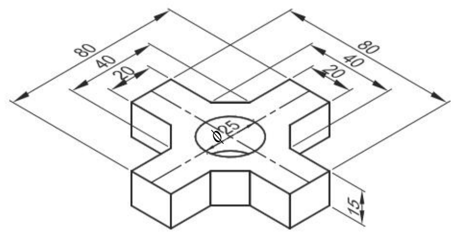

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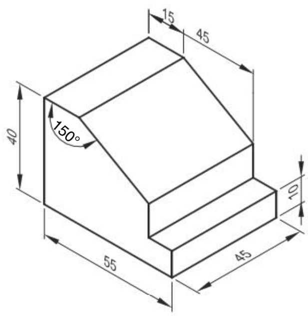

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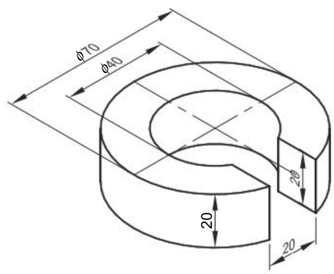

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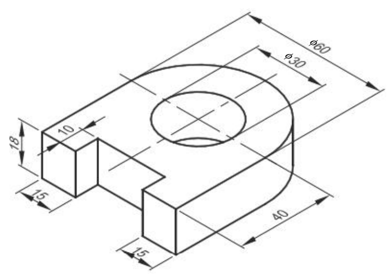

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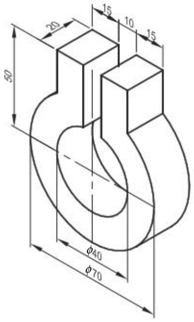

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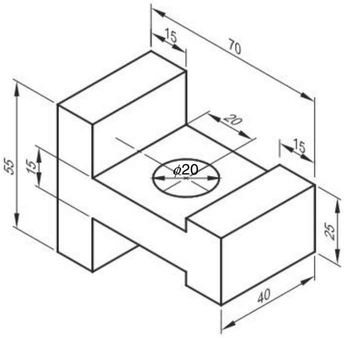

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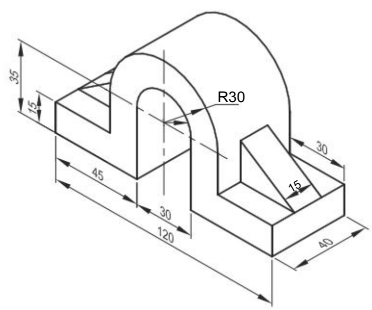

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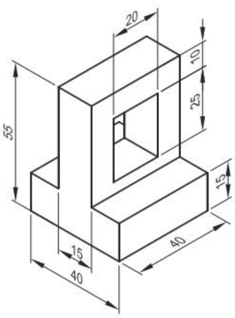

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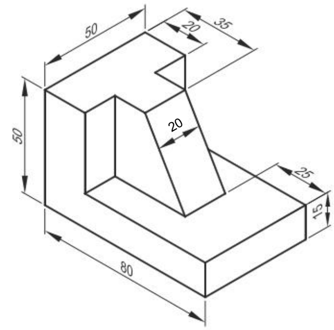

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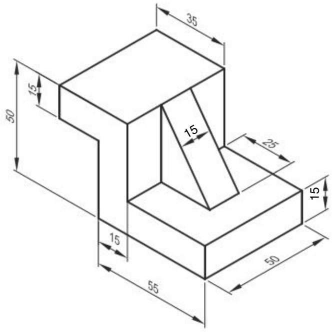

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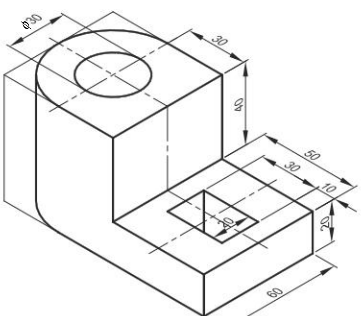

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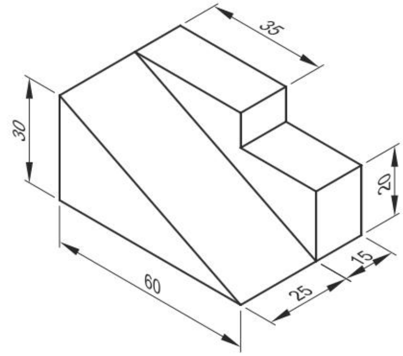

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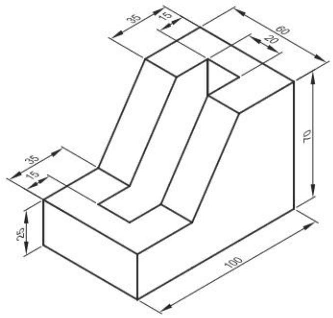

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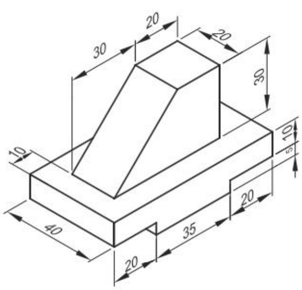

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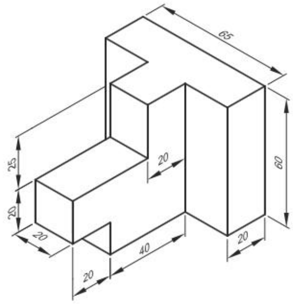

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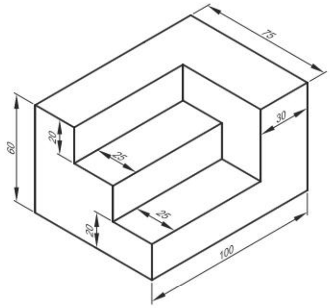

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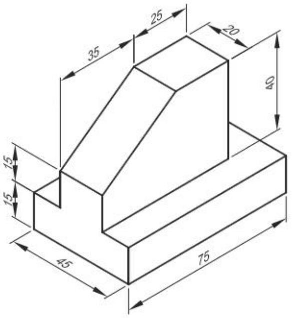

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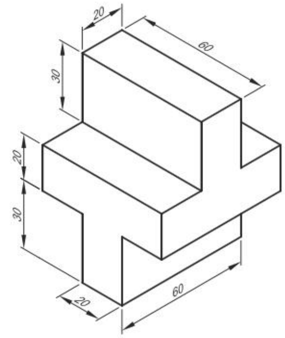

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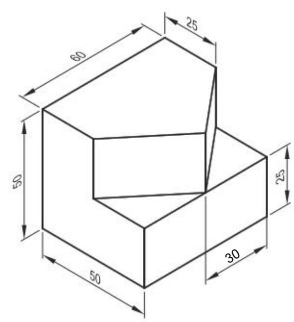

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