Engineering Physics

JNTU Anantapur Series (2015)

About the Author

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Dr. K Thyagarajan has published numerous research papers in journals of international repute and has presented many papers in international and national conferences. His research areas of interest include Nanomaterials and Thin Films.

To his credit, he established a research lab in the department with the financial assistance of JNTU Anantapur. At present, seven students are pursuing PhD under his guidance. He organised DST INSPIRE Science Camps sponsored by DST, Govt. of India, and a National Conference on Nanomaterials and Nanotechnology (NCNN) sponsored by UGC and co-sponsored by APSCHE, Govt. of Andhra Pradesh. He conducted National Workshop on Recent trends in Nanomaterial synthesis and Characterization Techniques under TEQIP II programme.

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Pulivendula, Andhra Pradesh
India

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PREFACE

Engineering Physics provides basic knowledge in the application of physical concepts in various fields of engineering. It covers the complete syllabus on Engineering Physics for different branches of engineering under the prescribed syllabus of JNTU Anantapur, effective as per the Regulation of 2015.

The highlight of this book is the simple and clear way in which all the topics have been dealt with to suit the requirements of undergraduate students. Besides these, a large number of illustrations lend clarity to the topics. The excellent pedagogy includes solved examples, multiple-choice questions and review questions which help students to have a firm grasp on the topics.

Key Features
- Complete syllabus coverage
- Comprehensive coverage of Physical Optics, Superconductivity, Quantum Mechanics, etc. (essential as per the syllabus)
- Lucid style of presentation
- Model Question Papers
- Rich pedagogy
  - 192 Illustrations
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Chapter Organisation
The book is divided into five units spanning 10 chapters. **Chapters 1** and **2** describe the optical phenomena of interference and diffraction. The theory of lasers, laser systems and their applications in various fields are elucidated in **Chapter 3**. An introduction to fibre optic principles and communication is presented in **Chapter 4**. The basic concepts of periodic arrangement of atoms in crystals, Bragg's law, crystal-structure determinations by X-ray diffraction technique and the role of different types of defects in crystals in understanding

their properties, including non-destructive testing of materials by ultrasonic waves, are discussed in **Chapter 5**.

Chapter 6 deals with classical and quantum free-electron theory for electrical conductivity, the effect of periodic potential on electron motion, origin of band theory and classification of material based on it. The basic concepts and applications of semiconductors are presented in **Chapter 7**. The magnetic properties of materials are discussed in depth in **Chapter 8**. The properties and applications of superconductors are introduced in **Chapter 9**. **Chapter 10** includes the basic concepts of nanomaterials, their synthesis, properties and applications.

Acknowledgements

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I would also like to thank the editorial team at McGraw Hill Education (India) for their support and cooperation.

I hope that this book will be well received by students and teachers alike.

K Thyagarajan

Feedback

Criticism and suggestions from the users of this book for making further improvements are most welcome. Readers can write to me at *ktrjntu@gmail.com.*

K Thyagarajan

Publisher's Note

Constructive suggestions and criticism always go a long way in enhancing any endeavour. To improve the book, we request all readers to email us their valuable comments/views/feedback at *info.india@mheducation.com* mentioning the title and author name in the subject line. Readers can also report any piracy of the book spotted by them.

Roadmap to the Syllabus

This text is useful for Engineering Physics–JNTU Anantapur–2015

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Chapter 5: Crystallography and Ultrasonics

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Chapter 6: Quantum Mechanics and Free Electron Theory

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CHAPTER 1

PHYSICAL OPTICS– INTERFERENCE

INTRODUCTION

Wave theory of light attempts to understand the various optical phenomena exhibited by light waves. Interference constituted the first proof of the wave nature of light. Thomas Young first experimentally demonstrated interference in light waves. The superposition principle forms the conceptual basis for the explanation of interference. To produce interference, the light waves should be coherent, i.e., the light waves should have constant phase difference and same frequencies.

1.1 INTERFERENCE

When two or more light waves superimpose in the medium then according to superposition principle, the resultant displacement at any point is equal to the algebraic sum of the displacements due to individual waves. If y_1 and y_2 are the displacements due to two light waves 1 and 2, then the resultant displacement of the resultant wave is given by $y = y_1 + y_2$. The variation of resultant displacement influences amplitude variation, which causes intensity variations. This modification in the distribution of intensity in the region of superposition is known as *interference*. When the resultant amplitude is the sum of the amplitudes due to
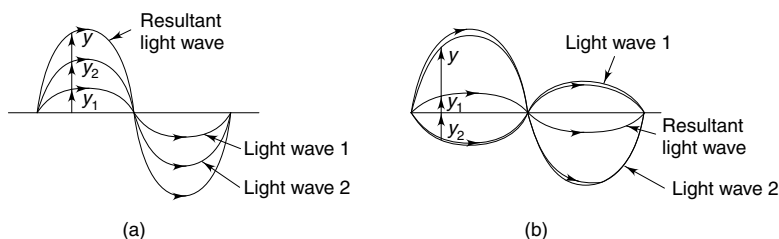
Figure 1.1　(a) Constructive interference ($y' = y_1 + y_2$);
(b) Destructive interference ($y = y_1 - y_2$)

two light waves, the interference is *constructive interference* (Fig. 1.1(a)). If the resultant amplitude is equal to the difference of two amplitudes, the interference becomes *destructive interference* (Fig. 1.1(b)). The intensity variations are studied as interference patterns or fringes or bright and dark fringes.

1.1.1 Coherence

To observe interference, the two light waves should be coherent. The two light waves are said to be coherent if they have the same frequency (wavelength) and they are in phase or maintain a constant phase relationship. When such light waves superimpose, they give, rise to an interference pattern. The light waves emitted by two independent light sources do not have a constant phase relationship for a long time and they have randomly varying phase relationship. Due to which, they do not produce stationary interference pattern.

When a light from a single source is made to split into two wavefronts by using two slits, then these split wavefronts act as if they emerged from two sources having a fixed phase relationship and by superimposition, they produce a stationary interference pattern. The process of maintaning constant phase relation is known as coherence.

Coherence is of two types.

1. **Temporal Coherence**   If it is possible to predict the phase relation at a point on the wave w.r.t. another point on the same wave then the wave has temporal coherence.

2. **Spatial Coherence**   If it is possible to predict the phase relation at a point on a wave w.r.t. another point on a second wave, then the wave is said to have spatial coherence.

1.1.2 Conditions for Interference

1. The two light sources emitting light waves should be coherent.
2. The two sources must emit continuous light waves of same wavelengths and frequency.
3. The separation between the two sources should be small.
4. The distance between the two sources and the screen should be large.
5. To view interference fringes, the background should be dark.
6. The amplitudes of the light waves should be equal or nearly equal.
7. The sources should be narrow, i.e., they must be small.
8. The sources should be monochromatic.

1.1.3 Interference in Thin Film by Reflection

When light is incident on a plane parallel thin film, some portion gets reflected from the upper surface and the remaining portion is transmitted into the film. Again, some portion of the transmitted light is reflected back into the film by the lower surface and emerges through the upper surface. These reflected light

beams superimpose with each other, producing interference and forming interference patterns.

Consider a transparent plane parallel thin film of thickness '*t*' with refractive index μ. Let a monochromatic light ray *AB* be incident at an angle of incidence of '*i*' on the upper surface of the film. *BE* and *BC* are the reflected and transmitted light rays. Let the angle of refraction is '*r*'. The ray *BC* will be reflected into the

Figure 1.2   Interference in thin film

film and emerge through the film in the form of the light ray *DF*. These two light rays superimpose and depending upon path difference between them, they produce interference patterns.

To know the path difference, let us draw the normal *DG* to *BE*. From the points *D* and *G* onwards, the light rays travel equal distances. By the time the light ray travels from *B* to *G*, the transmitted light ray has to travel from *B* to *C* and *C* to *D*.

The path difference between light rays (1) and (2) is

$$\text{Path difference} = \mu(BC + CD) \text{ in film} - BG \text{ in air} \tag{1}$$

Consider the ΔBCH, $\qquad \cos r = \dfrac{HC}{BC}$

$$BC = \frac{HC}{\cos r} = \frac{t}{\cos r}$$

Similarly, from ΔDCH, $\quad CD = \dfrac{t}{\cos r}$

$$BC = CD = \frac{t}{\cos r} \tag{2}$$

To calculate *BG*, first *BD* which is equal to (*BH* + *HD*) has to be obtained.

From ΔBHC, $\qquad \tan r = \dfrac{BH}{CH} = \dfrac{BH}{t}$

$$BH = t \tan r$$

Similarly, $\qquad\qquad HD = t \tan r$

$$BD = BH + HD = 2t \tan r \quad (\because \ BH = HD) \tag{3}$$

From ΔBGD, $\qquad \sin i = \dfrac{BG}{BD}$

$$BG = BD \sin i = 2t \tan r \sin i$$

From Snell's law, $\sin i = \mu \sin r$

$$BG = 2\mu t \tan r \sin r \tag{4}$$

Substituting the above values in Eq. (1)

$$\text{Path difference} = \frac{2\mu t}{\cos r} - 2\mu t \tan r \sin r$$

$$= \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} (1 - \sin^2 r) = \frac{2\mu t}{\cos r} \cos^2 r$$

$$= 2\mu t \cos r \tag{5}$$

At the point B, reflection occurs from the upper surface of the thin film (denser medium). Light ray (1) undergoes an additional phase change of π or an additional path difference of $\frac{\lambda}{2}$.

Total path difference $= 2\mu t \cos r + \frac{\lambda}{2}$

When the path difference is equal to integral multiples of λ then the rays (1) and (2) meet in phase and undergo constructive interference.

The condition for bright fringe is

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n - 1) \frac{\lambda}{2} \quad \text{where } n = 0, 1, 2, 3\ldots \tag{6}$$

When the path difference is equal to half integral multiples of λ then the rays (1) and (2) meet in out of phase and undergo destructive interference.

The condition for dark fringe is

$$2\mu t \cos r + \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \quad \text{where } n = 0, 1, 2, 3\ldots \tag{7}$$

Depending on the above conditions, the interference pattern consists of bright and dark fringes.

1.2   NEWTON'S RINGS

Newton's rings are one of the best examples for the interference in a nonuniform thin film. When a plano-convex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate

dark and bright rings concentric around the point of contact between the lens and glass plate are seen. These circular rings were discovered by Newton and are called Newton's rings.

1.2.1 Experimental Arrangement

The experimental arrangement is as shown in Fig. 1.3. The plano-convex lens (*L*) of large radius of curvature is placed with its convex surface on a plane glass plate (*P*). The lens makes the contact with the plate at '*O*'. The monochromatic light falls on a glass plate *G* held at an angle of 45° with the vertical. The glass plate *G* reflects normally a part of the incident light towards the air film enclosed by the lens *L* and the glass plate *P*. A part of the light is reflected by the curved surface of the lens *L*

Figure 1.3   Newton's rings (Experimental set up)

and a part is transmitted which is reflected back from the plane surface of the plate. These reflected rays interfere and give rise to an interference pattern in the form of circular rings. These rings are seen near the upper surface of the air film through the microscope.

1.2.2 Explanation of Newton's Rings

Newton's rings are formed due to interference between the light rays reflected from the top and bottom surfaces of air film between the plate and the lens. The formation of Newton's rings can be explained with the help of Fig. 1.4. A part of the incident monochromatic light *AB* is reflected at *B* (glass–air boundary) in the form of the ray (1) with any additional phase (or path)

Figure 1.4   Formation of Newton's rings

change. The other part of the light is refracted along *BC*. Then at *C* (air–glass boundary), it is again reflected in the form of the ray (2) with additional phase change of π or path change of $\frac{\lambda}{2}$.

As the rings are observed in the reflected light, the path difference between them is $2\mu t \cos r + \dfrac{\lambda}{2}$. For air film $\mu = 1$ and for normal incidence $r = 0$, path difference is $2t + \dfrac{\lambda}{2}$.

At the point of contact $t = 0$, path difference is $\dfrac{\lambda}{2}$, i.e., the reflected light at the point of contact suffers phase change of π. Then the incident and reflected lights are out of phase and interface destructively. Hence the central spot is dark.

The condition for bright ring is $2t + \dfrac{\lambda}{2} = n\lambda$.

$$2t = (2n - 1)\frac{\lambda}{2} \quad \text{where } n = 1, 2, 3\ldots$$

The condition for dark ring is

$$2t + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

$$2t = n\lambda \quad \text{where } n = 0, 1, 2, 3\ldots$$

For monochromatic light, the bright and dark rings depends on thickness of the air film. For a Newton's rings system, the locus of points having same thickness lie on a circle having its centre at the point of contact. Thus, we get bright and dark circular rings with the point of contact as the centre.

1.2.3   Theory of Newton's Rings

To find the diameters of dark and bright rings, let 'L' be a lens placed on a glass plate P. The convex surface of the lens is the part of spherical surface (Fig. 1.5) with centre at 'C'. Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the film thickness 't'.

From the property of a circle,

$$NA \times NB = NO \times ND$$

Substituting the values,

$$r \times r = t \times (2R - t)$$
$$r^2 = 2Rt - t^2$$

As t is small, t^2 will be negligible

$$r^2 = 2Rt$$
$$t = \frac{r^2}{2R}$$
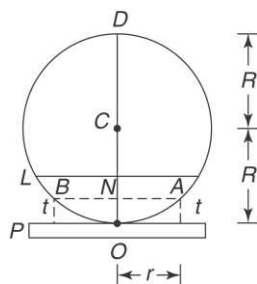
Figure 1.5   Theory of Newton's rings

For bright ring, the condition is

$$2t = (2n - 1) \frac{\lambda}{2}$$

$$2 \frac{r^2}{2R} = (2n - 1) \frac{\lambda}{2}$$

$$r^2 = \frac{(2n - 1)\lambda R}{2}$$

Replacing r by $\frac{D}{2}$, the diameter of nth bright ring will be

$$\frac{D^2}{4} = \frac{(2n - 1)}{2} \lambda R$$

$$D = \sqrt{2n - 1} \sqrt{2\lambda R}$$

$$D \propto \sqrt{2n - 1}$$

$$D \propto \sqrt{\text{odd natural numbers}}$$

Thus the diameter of the bright rings are proportional to the square root of odd natural numbers. For dark ring, the condition is

$$2t = n\lambda$$

$$2 \frac{r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D = 2\sqrt{n\lambda R}$$

$$D \propto \sqrt{n}$$

$$D \propto \sqrt{\text{natural numbers}}$$

Thus, the diameters of dark rings are proportional to the square root of natural numbers.

With increase in the order (n), the rings get closer and the fringe width decreases and is shown in Fig. 1.6.
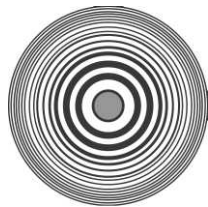
Figure 1.6 Newton's ring pattern

1.3 DETERMINATION OF WAVELENGTH OF A LIGHT SOURCE

The wavelength of a light source can be calculated by measuring the diameters of the various rings and knowing the radius of curvature of the plano convex lens.

Theory

Let R be the radius of curvature of a plano-convex lens, λ be the wavelength of light used. Let D_m and D_n are the diameters of mth and n^{th} dark rings respectively. Then

$$D_m^2 = 4m\lambda R$$

and

$$D_n^2 = 4n\lambda R$$

$$D_n^2 - D_m^2 = 4(n - m)\lambda R$$

$$\lambda = \frac{D_n^2 - D_m^2}{4(n - m)R}$$

Using the above formula, λ can be calculated.

Experiment

Experimental arrangements are made as given in Section 1.2.1. After forming the Newton's rings, the microscope is adjusted so that the centre of the crosswire is adjusted at the central dark spot of the ring pattern. By counting the number of rings, the microscope is moved to the extreme left of the pattern and the cross wire is adjusted tangentially in the middle of the nth (21st) dark ring. The reading of the microscope is noted. Now the microscope is moved to the right and its readings are noted successively at $(n - 3)^{th}$ (18th), $(n - 6)^{th}$ (15th) … rings,

Figure 1.7   Plot of D^2 w.r.t. the number of rings

etc., with a difference of three rings up to the central dark spot. Again crossing the central dark spot in the same direction, the readings corresponding to $(n - 6)^{th}$ (15th), $(n - 3)^{th}$ (18th) and nth (21st) rings are noted. The difference between the left and right reading gives the diameter of the particular ring. A graph is plotted between the number of the rings and the square of the corresponding diameter.

From the graph,

$$\frac{D_n^2 - D_m^2}{n - m} = \frac{AB}{CD}$$

The radius R of the plano convex lens can be obtained with the help of a spherometer. Substituting these values in the formula, the wavelength of the light source can be known.

1.4   DETERMINATION OF REFRACTIVE INDEX OF A LIQUID

The experiment is performed when there is an air film between the glass plate and the plano-convex lens. The diameters of mth and n^{th} dark rings are determined with the help of a travelling microscope. We have

$$D_n^2 - D_m^2 = 4(n - m)\lambda R \tag{8}$$

The plano convex lens and plane glass plate is placed into the container which consists of the liquid whose refractive index (μ) is to be determined. Now, the air film is replaced by the liquid film. Again, the diameters of the same m^{th} and n^{th} dark rings are to be obtained. Then we have

$$D_n'^2 - D_m'^2 = \frac{4(n-m)\lambda R}{\mu} \tag{9}$$

From equations (8) and (9), we get

$$\mu = \frac{D_n^2 - D_m^2}{D_n'^2 - D_m'^2} \tag{10}$$

Using the above formula, 'μ' can be calculated.

SOLVED PROBLEMS

1. A parallel beam of light of 6000Å is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction into the plate is 50°. Find the least thickness of the glass plate which will appear dark by reflection.

Given data   Wavelength of the given light $\lambda = 6000\text{Å} = 6000 \times 10^{-8}$ cm

Refractive index of the glass plate $\mu = 1.5$

Angle of refraction $r = 50°$

Solution $2\mu t \, \cos r = n\lambda$

For least thickness, $n = 1$

$$\text{Thickness } t = \frac{n\lambda}{2\mu \cos r}$$

$$t = \frac{1 \times 6000 \times 10^{-8}}{2 \times 1.5 \times \cos 50°}$$

$$t = \frac{6 \times 10^{-5}}{3 \times 0.6428} = 3.1114 \times 10^{-5} \text{ cm}$$

$$= 3.11 \times 10^{-5} \text{ cm}$$

2. In a Newton's rings experiment, the diameter of the 5th ring is 0.30 cm and the diameter of the 15th ring is 0.62 cm. Find the diameter of the 25th ring.

Given data   Diameter of the 5th ring = 0.3 cm

Diameter of the 15th ring = 0.62 cm

Solution   We want diameter of the 25th ring.

We know that $D_n^2 - D_m^2 = 4(n - m)\lambda R$

$$D_{15}^2 - D_5^2 = 4(15 - 5)\lambda R$$

$\Rightarrow$ $(0.62)^2 - (0.3)^2 = 40\ \lambda R$

$0.3844 - 0.09 = 40\ \lambda R$ (i)

Similarly, $D_{25}{}^{th}$ and $D_5{}^{th}$

$$D_{25}^2 - D_5^2 = 4(25 - 5)\lambda R$$

$$D_{25}^2 - 0.09 = 80\lambda R$$ (ii)

Dividing Eq. (ii) by Eq. (i)

$$\frac{D_{25}^2 - 0.09}{0.3844 - 0.09} = \frac{80\ \lambda R}{40\ \lambda R}$$

$\Rightarrow$ $D_{25}^2 = 2 \times 0.2944 + 0.09 = 0.6788$

$D_{25} = \sqrt{0.6788} = 0.824$ cm

Diameter of the 25th ring is 0.824 cm.

3. A convex lens on a plane glass plate is exposed to a monochromatic light. The diameter of the 10th dark ring is 0.433 cm. Find the wavelength of the light used if the radius of curvature of the lens is 70 cm.

Given data   Radius of curvature, $R = 70$ cm

Diameter of the 10th ring = 0.433 cm

Solution $\lambda = \dfrac{D_n^2}{4nR}$

$$= \frac{(0.433)^2}{4 \times 10 \times 70} = \frac{0.187}{2800} = 6.6786 \times 10^{-5} \text{ cm}$$

4. A soap film with a refractive index of 1.33 and thickness of 5000 Å is exposed to white light. What wavelengths in the visible region are reflected?

Given data   Refractive index of the soap film, $\mu = 1.33$

Thickness of the film, $t = 5000$ Å $= 5000 \times 10^{-10}$ m

Solution   Condition for maximum reflection is

$$2\mu t \cos r = (2n + 1)\frac{\lambda}{2}, \quad \text{where } n = 0, 1, 2, \text{ etc.}$$

Assume $\cos r = 1$

Then $\lambda = \dfrac{4\mu t}{(2n + 1)} = \dfrac{4 \times 1.33 \times 5000 \times 10^{-10}}{(2n + 1)}$

$$= \frac{26600 \times 10^{-10}}{(2n + 1)} \text{ m}$$

When $n = 0$, $\lambda_1 = 26600 \times 10^{-10}$ m (infrared region)

When $n = 1$, $\lambda_2 = \dfrac{26600 \times 10^{-10}}{3} = 8866.7 \times 10^{-10}$ m (I R)

$n = 2$, $\lambda_3 = \dfrac{26600 \times 10^{-10}}{5} = 5320 \times 10^{-10}$ m (visible region)

$n = 3$, $\lambda_4 = \dfrac{26600 \times 10^{-10}}{7} = 3800 \times 10^{-10}$ m

(ultraviolet region)

All wavelengths reflected, at wavelength 5320Å the visible regions are reflected.

5. What is the thickness of the thinnest film of 1.33 refractive index in which destructive interference of the yellow light (6000 Å) of a normally incident beam in air can take place by reflection?

Given data   Refractive index of the film, $\mu = 1.33$

Wavelength of yellow light $\lambda = 6000$ Å $= 6000 \times 10^{-8}$ cm

Angle of incidence $i = 0°$, $\therefore r = 0°$ (reflection)

For least thickness, $n = 1$

Solution $2\mu t \cos r = n\lambda$

$$t = \frac{n\lambda}{2\mu \cos r}$$

$$= \frac{1 \times 6000 \times 10^{-8}}{2 \times 1.33 \times \cos 0°}$$

$$= \frac{6 \times 10^{-5}}{2.66 \times 1} = 2.255 \times 10^{-5} \text{ cm}$$

$\therefore$ the thickness of the thinnest film $= 2.255 \times 10^{-5}$ cm

6. In a Newton's rings experiment, the diameter of the 8th ring was 0.35 cm and the diameter of the 18th ring was 0.65 cm. If the wavelength of the light used is 6000Å then find the radius of curvature of the plano-convex lens.

Given data   Diameter of the 18th ring $= 0.65$ cm

Diameter of the 8th ring $= 0.35$ cm

Wavelength of the light, $\lambda = 6000$ Å

Solution   Radius of curvature of the plano-convex lens is

$$R = \frac{D_n^2 - D_m^2}{4(n - m)\lambda}$$

$\Rightarrow$ $$R = \frac{(0.65)^2 - (0.35)^2}{4(18 - 5) \times 600 \times 10^{-8}}$$

$$= \frac{0.4225 - 0.1225}{24 \times 10^{-4}} = \frac{0.3}{24} \times 10^4 = 125 \text{ cm}$$

∴ radius of curvature of the plano-convex lens is 125 cm.

7. In a Newton's rings experiment, the diameter of the 12th ring changes from 1.45 cm to 1.25 cm when a liquid is introduced between the lens and the glass plate. Find the refractive index of the liquid.

Given data Diameter of the 12th ring in air medium, $D_{12} = 1.45$ cm

Diameter of the 12th ring in a liquid, $D'_{12} = 1.25$ cm

Solution $$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$(D_{12})^2 = \frac{4n\lambda R}{\mu} = \frac{4 \times 12\lambda R}{1} \quad \therefore \mu = 1 \text{ for air}$$

$$(D'_{12})^2 = \frac{4 \times 12\lambda R}{\mu}$$

From the above relation,

$$\mu = \frac{(D_{12})^2}{(D'_{12})^2} = \frac{(1.45)^2}{(1.25)^2} = \frac{2.1025}{1.5625} = 1.3456$$

∴ refractive index of the liquid is 1.3456.

8. In a Newton's ring experiment, the diameter of the 5th dark ring is 0.3 cm and the diameter of the 25th ring is 0.8 cm. If the radius of curvature of the plano convex lens is 100 cm, find the wavelengths of the light used.

Given Data Diameter of 5th dark ring $D_5 = 0.3$ cm

Diameter of 25th dark ring $D_{25} = 0.8$ cm

Radius of curvature of plano convex lens $R = 100$ cm

Solution $$\lambda = \frac{D_n^2 - D_m^2}{4(n - m)R}$$

$$= \frac{(0.8)^2 - (0.3)^2}{4(8 - 3) \times 100}$$

$$= 4.87 \times 10^{-5} \text{ cm}$$

MULTIPLE CHOICE QUESTIONS

1. Two sources are said to be coherent if their emitted waves have
   - (a) same amplitude
   - (b) same wavelength
   - (c) constant phase difference
   - (d) All of these

2. When a light wave is reflected at the glass–air interface, the change of phase of the reflected wave is equal to
   - (a) 0
   - (b) $\pi/2$
   - (c) $\pi/4$
   - (d) π

3. When a light wave is reflected at the air–glass interface, the change of phase of the reflected wave is equal to
   - (a) 0
   - (b) $\pi/2$
   - (c) $\pi/4$
   - (d) π

4. In the presence of a plane parallel film, the path difference is equal to
   - (a) $2\mu \sin i$
   - (b) $2\mu t \cos r$
   - (c) $2\mu \sin r$
   - (d) $2\mu t \cos i$

5. If a light wave is refracted from air to denser medium then the phase and path difference is
   - (a) π and λ
   - (b) π and $\lambda/2$
   - (c) $\pi/2$ and λ
   - (d) $\pi/2$ and $\lambda/2$

6. In a plane parallel film due to reflected light, what is the condition for maxima for the air film to appear bright?
   - (a) $2\mu t \cos r = n\lambda$
   - (b) $2\mu t \cos r = (2n + 1)\lambda/2$
   - (c) $2\mu t \cos r = n\lambda/2$
   - (d) $2\mu t \cos r = (2n - 1)\lambda/2$

7. In a plane parallel film due to reflected light, what is the condition for minima for the air film to appear dark?
   - (a) $2\mu t \cos r = n\lambda/2$
   - (b) $2\mu t \cos r = (2n + 1)\lambda/2$
   - (c) $2\mu t \cos r = n\lambda$
   - (d) $2\mu t \cos r = (2n - 1)\lambda/2$

8. In Newton's rings, interference is due to light rays reflected from
   - (a) upper surface of lens and upper surface of glass plate
   - (b) lower surface of lens and upper surface of glass plate
   - (c) lower surface of glass plate and upper surface of lens
   - (d) lower surface of lens and lower surface of glass plate

9. In Newton's rings experiment, what is the condition for bright fringes in case of reflected light?
   - (a) $D \propto \sqrt{2n + 1}$
   - (b) $D \propto n$
   - (c) $D \propto \sqrt{n}$
   - (d) $D \propto \sqrt{2n - 1}$

10. In Newton's rings experiment, what is the condition for dark fringes in case of reflected light?
    - (a) $D \propto \sqrt{2n+1}$
    - (b) $D \propto n$
    - (c) $D \propto \sqrt{n}$
    - (d) $D \propto \sqrt{2n-1}$

11. If the thickness of the parallel thin film increases, the path difference between the reflected lights
    - (a) decreases
    - (b) increases
    - (c) first decreases and then increases
    - (d) first increases and then decreases

12. The phenomenon of intensity modification due to superposition of light waves is known as
    - (a) interference
    - (b) diffraction
    - (c) polarisation
    - (d) coherence

13. When a light wave is reflected at the air-glass interface, the change of path of the reflected wave is equal to
    - (a) $\lambda/2$
    - (b) $\lambda/4$
    - (c) $\lambda/6$
    - (d) $\lambda/8$

14. When a liquid film is introduced in the place of air film, then the diameters of the rings
    - (a) increases
    - (b) decreases
    - (c) first increases and then decreases
    - (d) first decreases and then increases

15. When the film is observed by white light, colours are observed due to
    - (a) interference
    - (b) diffraction
    - (c) polarisation
    - (d) coherence

16. Wave nature of light is evidenced by
    - (a) photoelectric effect
    - (b) interference
    - (c) blackbody radiation
    - (d) nuclear emission

17. In the interference pattern energy is
    - (a) created at the maxima
    - (b) destroyed at the minima
    - (c) converted but is redistributed
    - (d) converted into matter

18. Two waves having their intensities in the ratio $9:1$ produce interference. In the interference pattern, the ratio of maximum to minimum intensity is equal to
    - (a) $2:1$
    - (b) $9:1$
    - (c) $3:1$
    - (d) $4:1$

19. Two beams interfere have their amplitudes ratio $2:1$. Then the intensity ratio of bright and dark fringes is
    - (a) $2:1$
    - (b) $1:2$
    - (c) $9:1$
    - (d) $4:1$

20. When a thin film of oil or soap bubble is illuminated with white light, multiple colours appear. This is due to

(a) diffraction
(b) polarisation
(c) total internal reflection
(d) interference

21. When the Newton's rings are viewed through a microscope, the Central ring is seen dark because
    (a) Phase difference of π due to phase change on reflection.
    (b) Air film thickness is nearer to zero.
    (c) Path difference of λ due to phase change on reflection.
    (d) Phase change of 2π due to phase change on reflection.

22. The convex lense in Newton's ring apparatus is replaced by an ordinary glass plate, then
    (a) non-interference occurs
    (b) circular rings are still obtained
    (c) interference takes place but the shape of fringes is irregular
    (d) straight line fringes are observed.

23. In Newton's ring experiment the diameter of 40th ring is 0.1 m air film. When an oil film is formed the diameter of the same ring becomes 0.089 m the refractive index of oil is
    (a) 1.26
    (b) 0.80
    (c) 1.10
    (d) 1.33

Answers

1. (d)	2. (a)	3. (d)	4. (b)	5. (b)	6. (d)
7. (c)	8. (b)	9. (a)	10. (c)	11. (b)	12. (a)
13. (a)	14. (b)	15. (a)	16. (b)	17. (c)	18. (d)
19. (c)	20. (d)	21. (a)	22. (c)	23. (a)	

Fill in the Blanks

1. The resultant displacement is due to the presence of both the waves of displacements y_1 and y_2 is given by $y = y_1 + y_2$. This is known as the principle of _____.

2. Two waves are said to be _____ if their waves have same ratio of bright fringe to dark fringe.

3. If $5 : 1$ is the amplitude ratio of the source, the intensity ratio of maximum to minimum is
    (a) $25 : 1$
    (b) $6 : 4$
    (c) $4 : 9$
    (d) $9 : 4$

4. Due to interference in thin films, if the film thickness is extremely small when compared to λ, the film will appear _____.

5. Newton's ring experiment, the radius of nth dark ring is given by _____ where λ is the wavelength of light and R is the radius of the ring observed.

6. On introducing the liquid in between the lense and the glass plate in Newton's ring experiment, the diameter of the rings observed _____.

7. Superposition of two coherent wavefronts originating from coherent sources result in _____.

8. In interference pattern fringes have _____ width.

Answers to Fill in the blanks

1. Superposition
2. Coherent
3. 9:4
4. dark
5. $\sqrt{nR\lambda}$
6. decreases
7. Interference
8. equal

True or False

1. Two monochromatic sources of same amplitude and same wavelength can act as coherent sources. (T/F)

2. If the path difference between the two interfering waves is integral multiple of λ, constructive interference occurs. (T/F)

3. In Newton's ring experiment reflected light the point of contact of lens with the glass plate. Where thickness of air is zero, appears dark. (T/F)

4. Radius of fourth dark ring in Newton's ring experiment is $4\sqrt{R\lambda}$. (T/F)

5. Thin films when viewed under white light, appears multicolored due to dispersion. (T/F)

Answers to True or False

1. T
2. T
3. T
4. F
5. F

REVIEW QUESTIONS

Short Answer Questions

1. What is interference?
2. What is constructive interference?
3. What is destructive interference?
4. What is coherence?
5. What is temporal coherence?
6. What is spatial temporal coherence?
7. Mention any two important conditions to get interference.
8. Mention the conditions to get bright and dark fringes in the interference due to reflected light by thin film.
9. Why the centre spot is dark in the Newton's rings formed by the reflected light?
10. Why does Newton's rings consist of concentric rings?
11. Mention the expression for diameter of bright and dark rings in Newton's rings by the reflected light.

Essay Type Questions

1. Explain the interference in the films by reflection.
2. Describe the formation of Newton's rings with necessary theory.
3. Explain how the wavelength of light sources is determined by forming Newton's rings.
4. Explain how the refractive index of liquid is determined by forming Newton's rings.

CHAPTER 2

PHYSICAL OPTICS– DIFFRACTION

INTRODUCTION

The wave nature of light is further confirmed by the optical phenomenon of diffraction. The word 'diffraction' is derived from the Latin word *diffractus* which means to break to pieces. It is common experience that waves bend around obstacles placed in their path. When light waves encounter an obstacle, they bend round the edges of the obstacle. This bending is predominant when the size of the obstacle is comparable to the wavelength of light. The bending of light waves around the edge of an obstacle is diffraction. It was first observed by Gremaldy.

2.1 DIFFRACTION

As shown in Fig. 2.1, when light falls on an obstacle then the corresponding geometrical shadow on the screen should be completely dark. In practice, the geometrical shadow consists of bright and dark fringes. These fringes are due to the superimposition of bended light waves around the corners of an obstacle. The amount of bending depends upon the size of an obstacle and wavelength of light.
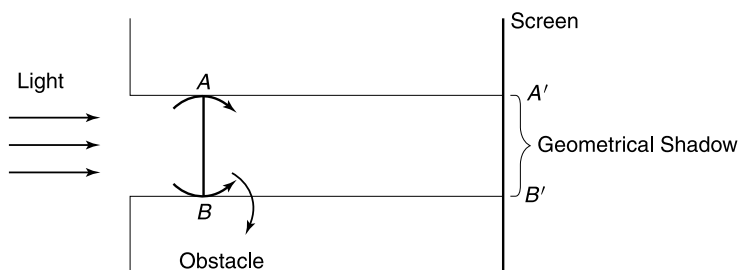
Figure 2.1   Diffraction

When light falls on an obstacle whose size is comparable with the wavelength of light then light bends around the edges or corners of an obstacle and enters

into the geometrical shadow. This bending of light is known as *diffraction*. The bright and dark fringes in the shadow forms a diffraction pattern.

The diffraction phenomena are broadly classified into two types:

Fresnel diffraction

To study diffraction, there should be a light source, obstacle and screen. In this class of diffraction, the source and screen are placed at finite distances from the obstacle. To study this diffraction, lenses are not necessary as the source and screen are at a finite distance. This diffraction can be studied in the direction of propagation of light. The incident wave fronts are either spherical or cylindrical.

Fraunhofer diffraction

In this class of diffraction, the source and screen are placed at infinite distances from the obstacle. Due to the above fact, we need lenses to study the diffraction. This diffraction can be studied in any direction. In this case, the incident wave front is plane.

2.1.1   Comparison between Fresnel's and Fraunhofer's Diffraction

Fraunhofer diffraction	*Fresnel's diffraction*
1. For diffraction to occur, the light source and screen are at infinite distance from the obstacle.	1. For diffraction to occur, the light source and screen are at finite distance from the obstacle.
2. To study diffraction, lenses are necessary.	2. No lenses are necessary to study the diffraction.
3. Study of the diffraction is easy.	3. Study of the diffraction is complicated.
4. Diffraction can be studied in any direction of propagation of light.	4. Diffraction can be studied only in the direction of propagation of light.
5. In this case, the incident wavefronts are plane.	5. In this case, the incident wavefronts are either spherical or cylindrical.

2.1.2   Difference between Interference and Diffraction

When two or more light waves superimpose then it causes intensity modifications in the resultant light wave, which causes the interference phenomenon. In diffraction, the bended light waves superimpose and cause intensity variations in the form of a diffraction pattern. The diffraction phenomenon also involves interference effect. Let us study the differences between interference and diffraction phenomena.

Interference	*Diffraction*
1. It is due to superposition of two source different wavefronts originating from two coherent sources.	1. It is due to superposition of secondary wavelets originating from the different parts of the same wavefront.

Contd..

2. Interference bands are of equal width.	2. Diffraction bands decrease in their widths as the order increases.
3. All the bright fringes are of the same intensity.	3. The bright fringes are of varying intensity.
4. All the dark fringes have zero intensity.	4. The intensity of dark fringes is not zero.

2.2 FRAUNHOFER DIFFRACTION AT SINGLE SLIT

Consider a slit AB of width 'e'. Let a plane wave front WW' of monochromatic light of wavelength λ propagating normally towards the slit is incident on it. The diffracted light through the slit is focussed by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygen–Fresnel, every point on the wave front in the plane of the slit is a source of secondary wavelets, which spread out to the right in all directions. These wavelets travelling normal to the slit, i.e., along the direction OP_0 are brought to focus at P_0 by the lens. Thus, P_0 is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focussed at a point P_1 on the screen. Depending on path difference, the point P_1 may have maximum or minimum intensities. To find intensity at P_1, let us draw the normal AC from A to the light ray at B.
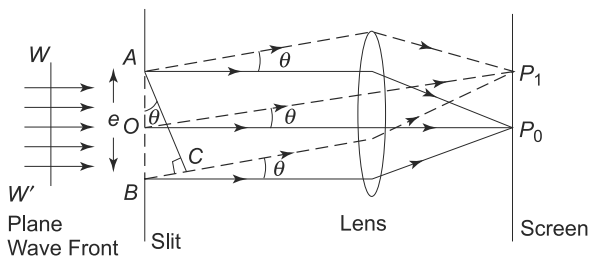
Figure 2.2 Fraunhofer diffraction—single slit

The path difference between the wavelets from A and B in the direction θ is given by

$$\text{Path difference} = BC = AB \sin \theta = e \sin \theta$$

$$\text{Corresponding phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} e \sin \theta$$

Let the width of the slit be divided into n equal parts and the amplitude of the wave from each part is 'a'. The phase difference between any two successive waves from these parts would be

$$\frac{1}{n} [\text{Total phase}] = \frac{1}{n} \left[\frac{2\pi}{\lambda} e \sin \theta \right] = d \text{ (say)}$$

Using the method of vector addition of amplitudes, the resultant amplitude R is given by

$$R = \frac{a \sin nd/2}{\sin d/2}$$

$$= \frac{a \sin (\pi e \sin \theta/\lambda)}{\sin (\pi e \sin \theta/n\lambda)}$$

$$= a \frac{\sin \alpha}{\sin \alpha/n} \quad \text{where } \alpha = \pi e \sin \theta/\lambda$$

$$= a \frac{\sin \alpha}{\alpha/n} \quad (\because \alpha/n \text{ is very small})$$

$$= n \frac{a \sin \alpha}{\alpha} \quad (\because na = A)$$

$$= A \frac{\sin \alpha}{\alpha}$$

Intensity $\quad\quad I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \quad\quad\quad (1)$

2.2.1 Principal Maximum

The resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

I will be maximum, when the value of R is maximum. For maximum value of R, the negative terms must vanish, i.e., $\alpha = 0$

$$\frac{\pi e \sin \theta}{\lambda} = 0$$

$$\sin \theta = 0$$

$$\theta = 0 \quad\quad\quad (2)$$

Then $\quad\quad R = A$

$$I_{\max} = R^2 = A^2 \quad\quad\quad (3)$$

The condition $\theta = 0$ means that the maximum intensity is formed at P_0 and is known as *principal maximum*.

2.2.2 Minimum Intensity Positions

I will be minimum, when $\sin \alpha = 0$

$$\alpha = \pm \pi, \pm 2\pi, \pm 3\pi$$

$$\alpha = \pm m\pi$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

$$e \sin \theta = \pm m\lambda \qquad (4)$$

where $m = 1, 2, 3.......$

Thus, we obtain the points of minimum intensity on either side of the principal maximum. For $m = 0$, $\sin \theta = 0$, which corresponds to principal maximum.

2.2.3 Secondary Maxima

In between these minima, we get secondary maxima. The positions can be obtained by differentiating the expression of I w.r.t. a and equating to zero. We get

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \right] = 0$$

$$A^2 \cdot \frac{2 \sin \alpha}{\alpha} \cdot \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$$

either $\sin \alpha = 0$ or $\alpha \cos \alpha - \sin \alpha = 0$

$\sin \alpha = 0$ gives positions of minima.

Hence the positions of secondary maxima are given by

$$\alpha \cos \alpha - \sin \alpha = 0$$
$$\alpha = \tan \alpha \qquad (5)$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan \alpha$ on the same graph. The points of intersection of the two curves gives the values of α which satisfy the above equation. The plots of $y = \alpha$ and $y = \tan \alpha$ are shown in Fig. 2.3.
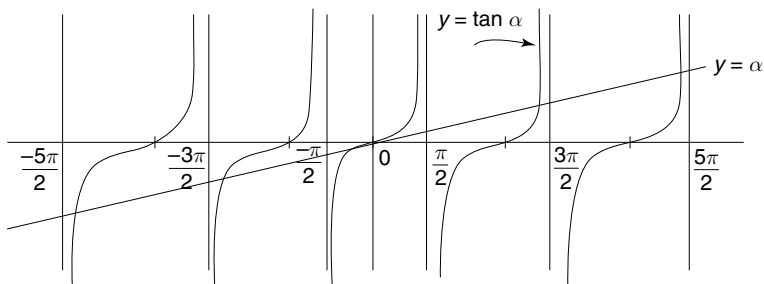
Figure 2.3 Plots of $y = \alpha$ and $y = \tan \alpha$

The points of intersections are

$$\alpha = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, ...$$

Substituting the above values in Eq. (1), we get the intensities in various maxima.

$$\alpha = 0, I_0 = A^2 \quad \text{(principal maximum)}$$

$$\alpha = \frac{3\pi}{2}, \quad I_1 = A^2 \left[\frac{\sin(3\pi/2)}{3\pi/2} \right]^2 \approx \frac{A^2}{22} \quad \text{(1}^{\text{st}}\text{ secondary maximum)}$$

$$\alpha = \frac{5\pi}{2}, \quad I_2 = A^2 \left[\frac{\sin(5\pi/2)}{5\pi/2} \right]^2 \approx \frac{A^2}{62} \quad \text{(2}^{\text{nd}}\text{ secondary maximum)}$$

and so on.

From the above expressions, it is evident that most of the incident light is concentrated in the principal maximum and the rest is redistributed into the secondary maxima.

2.2.4 Intensity Distribution

The variation of I w.r.t. α is shown in Fig. 2.4. The diffraction pattern consists of a central principal maximum for $\alpha = 0$. There are secondary maxima of decreasing intensity on either sides of it at positions $\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$. Between secondary maxima, there are minima at positions $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, ...$
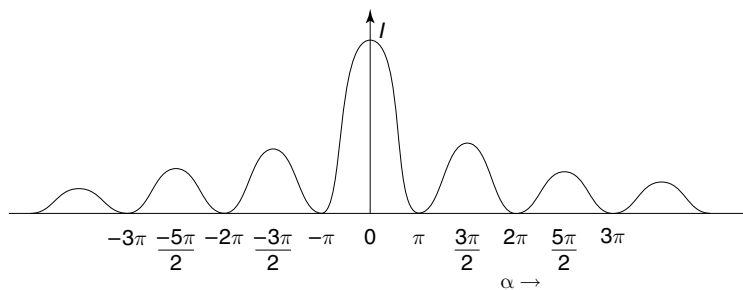
Figure 2.4   Intensity distribution

2.3 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT

Let S_1 and S_2 be double slits of equal widths e and separated by a distance d. The distance between the middle points of the two slits is $(e + d)$. A monochromatic light of wavelength λ is incident normally on the two slits. The light diffracted from these slits is focussed by a lens on the screen placed in the focal plane of the lens. The diffraction at two slits is the combination of diffraction as well as interference, i.e., the pattern on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superimposed. When a plane wavefront is incident normally on two slits, the secondary wavelets from the slits travel uniformly in all directions. The wavelets travelling in the direction of incident light come to a focus at P_0 while the wavelets travelling in a direction making an angle θ, come to focus at P_1.
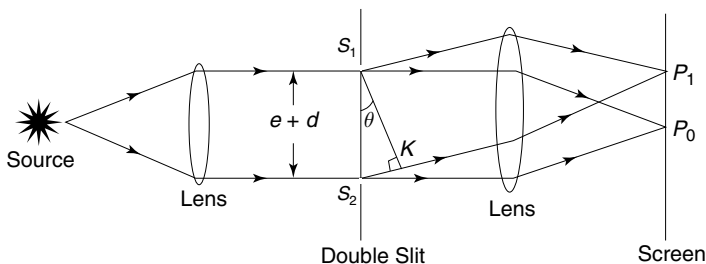
Figure 2.5 Fraunhofer diffraction—double slit

From the study of diffraction due to single slit, the resultant amplitude

$$= \frac{A \sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi e \sin \theta}{\lambda}.$$

Since we use double slit, from each slit we get a wavelet of amplitude $\frac{A \sin \alpha}{\alpha}$ in a direction θ. These two wavelets interfere and meet at a point P_1 on the screen. To calculate the path difference between the wavelets, let us draw a normal S_1K to the wavelet through S_2.

Figure 2.6 Vector addition method

Path difference $= S_2K$

$$= (e + d) \sin \theta$$

Phase difference $\qquad \delta = \frac{2\pi}{\lambda} (e + d) \sin \theta$ \hfill (6)

To find the resultant amplitude at P_1 we use vector addition method (Fig. 2.6) in which the two sides of a triangle are represented by the amplitudes through S_1 and S_2. The third side gives the resultant amplitude.

From the figure,

$$(OH)^2 = (OG)^2 + (GH)^2 + 2 (OG)(GH) \cos \delta$$

$$R^2 = \left(\frac{A \sin \alpha}{\alpha}\right)^2 + \left(\frac{A \sin \alpha}{\alpha}\right)^2 + 2 \left(\frac{A \sin \alpha}{\alpha}\right)\left(\frac{A \sin \alpha}{\alpha}\right) \cos \delta$$

$$= A^2 \frac{\sin^2\alpha}{\alpha^2} [2 + 2 \cos \delta]$$

$$= 2 \left(\frac{A \sin \alpha}{\alpha}\right)^2 (1 + \cos \delta)$$

$$= 2 \left(\frac{A \sin \alpha}{\alpha}\right)^2 (1 + 2 \cos^2\delta/2 - 1)$$

$$R^2 = 4A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \cos^2 \left[\frac{\pi (e + d) \sin \theta}{\lambda}\right]$$

Let $\qquad \beta = \frac{\pi (e + d) \sin \theta}{\lambda}$

$$R^2 = 4A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \cos^2\beta$$

The resultant intensity $I = R^2 = 4A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \cos^2\beta$ \hfill (7)

From the above expression, it is clear that the resultant intensity is the product of two factors, i.e.,

1. $A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2$ which represents the diffraction pattern due to a single slit

2. $\cos^2\beta$ which gives the interference pattern due to wavelets from double slits

The resultant intensity is due to both Diffraction and Interference effects.

2.3.1 Diffraction Effect

The diffraction term $A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2$ gives the principal maximum at the centre of the

screen with alternate minima and secondary maxima of decreasing intensity. We

get principal maximum for $\theta = 0$. We get minima for $\sin \alpha = 0$

$$\alpha = \pm m\pi, \quad \text{where } m = 1, 2, 3, ...$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

$$e \sin \theta = \pm m\lambda$$

The positions of secondary maxima occurs for

$$\alpha = \frac{\pm 3\pi}{2}, \frac{\pm 5\pi}{2}, \frac{\pm 7\pi}{2}, ...$$

2.3.2 Interference Effect

The interference term $\cos^2 \beta$ gives the equidistant bright and dark fringes.

The maxima will occur for $\cos^2 \beta = 1$

$$\beta = \pm n\pi, \quad \text{where } n = 0, 1, 2, 3, ...$$

$$\beta = 0, \pm \pi, \pm 2\pi, \pm 3\pi ...$$

$$\frac{\pi (e + d) \sin \theta}{\lambda} = \pm n\pi$$

$$(e + d) \sin \theta = \pm n\lambda$$

The minima will occur for $\cos^2 \beta = 0$

$$\beta = \pm (2n + 1)\pi/2 \quad \text{where } n = 0, 1, 2, 3, ...$$

$$(e + d) \sin \theta = \pm (2n + 1) \lambda/2$$

2.3.3 Intensity Distribution

Figure 2.7 (a) and (b) represents the intensity variations due to diffractions as well as interference effect individually. When both effects are combined then we get the resultant intensity variation (Fig. 2.7 (c)). From the figure, it is clear that the resultant minima are not equal to zero, still they have some minimum intensity due to interference effect.
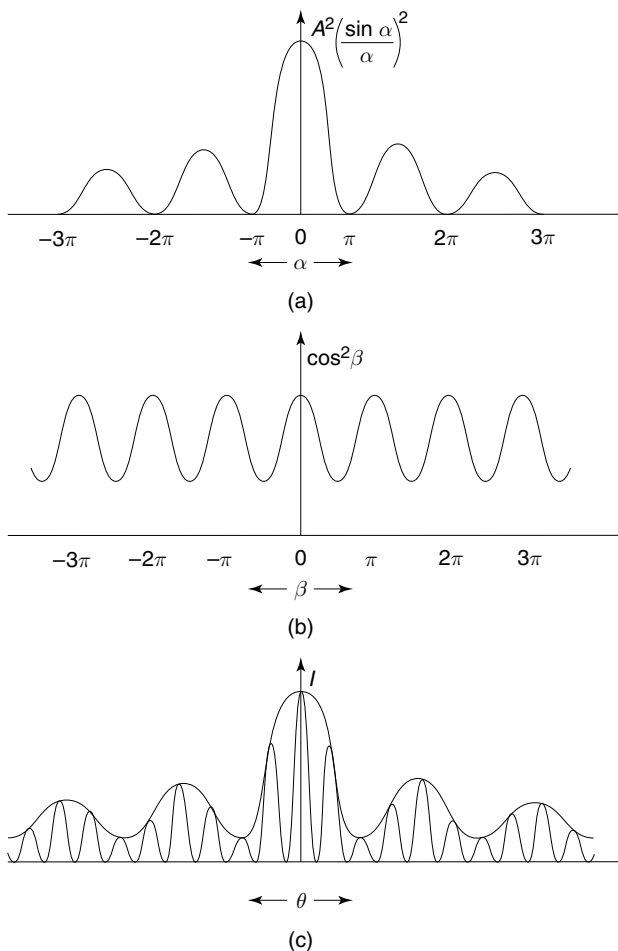
Figure 2.7 (a) Diffraction effect, (b) Interference effect, (c) Resultant intensity

2.4 DIFFRACTION GRATING

An arrangement which consists of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular intervals. Now gratings are constructed

by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit (Fig. 2.8).
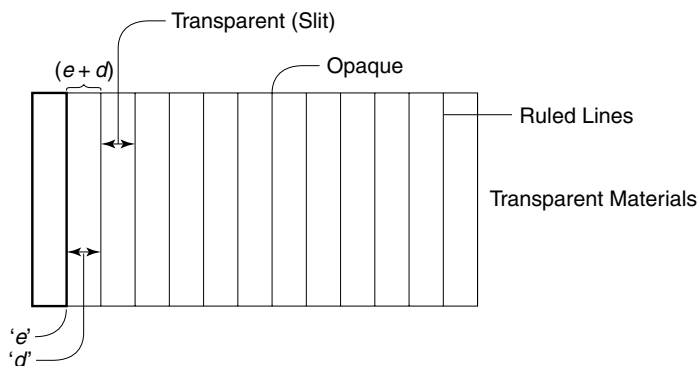
Figure 2.8   Diffraction grating

Commercial gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate. Solution of cellulose acetate is poured on the ruled surface and allowed to dry to form a thin film, detachable from the surface. These impressions of a grating are preserved by mounting the film between two glass sheets.

Let 'e' be the width of the line and 'd' be the width of the slit. Then $(e + d)$ is known as *grating element*. If 'N' is the number of lines per inch on the grating then

$$N (e + d) = 1'' = 2.54 \text{ cm}$$

$$e + d = \frac{2.54}{N} \text{ cm}$$

There will be nearly 30,000 lines per inch of a grating. Due to the above fact, the width of the slit is very narrow and is comparable to wavelengths of light. When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as *diffraction spectrum*.

2.5   FRAUNHOFER DIFFRACTION DUE TO GRATING (N PARALLEL SLITS)

Consider a grating AB consisting of N parallel slits with grating element $(e + d)$. Let a plane wavefront $\omega\omega'$ of monochromatic light of wavelength λ be incident normally on the grating. The diffracted light through the N parallel slits is focussed by means of a convex lens on a screen placed in the focal plane of the lens. The secondary wavelets travelling normal to the slit or along the direction OP_0 are brought to focus at P_0 by the lens. The point P_0 will be the central

maximum. The secondary wavelets travelling at an angle θ with the direction of the incident light are focussed at a point P_1 on the screen.
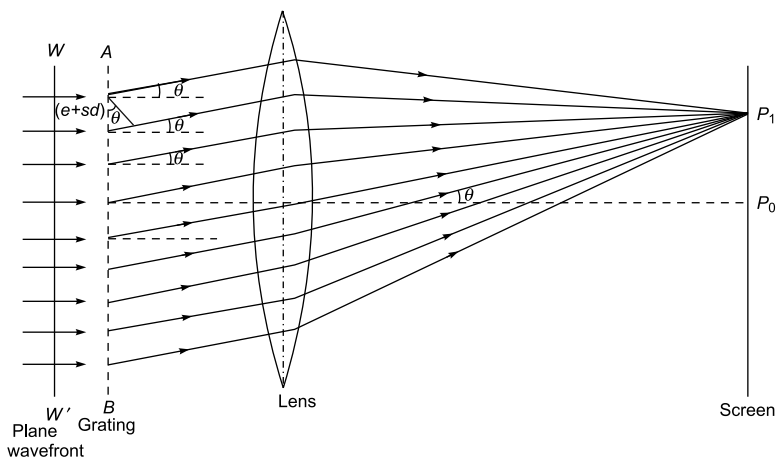
Figure 2.9   Fraunhofer diffraction-grating

Since the grating consists of N parallel slits from each slit are light wave emerges and we have N waves through the slits. From the concepts of single slit, we get N waves of amplitude $\dfrac{A \sin \alpha}{\alpha}$ where $\alpha = \dfrac{\pi e \sin \theta}{N}$ emerges in the direction of θ.

The path difference between the waves emerging through two successive slits

$$= (e + d) \sin \theta$$

The corresponding phase difference

$$= \frac{2\pi}{\lambda} (e + d) \sin \theta = 2\beta$$

From the method of vector additions of amplitudes, the resultant amplitude is given by

$$R = \frac{a \sin nd/2}{\sin d/2}$$

For the grating $a = \dfrac{A \sin \alpha}{\alpha}$, $n = N$ and $d = 2\beta$

$$R = \frac{A \sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta}$$

and

$$I = R^2 = \left(\frac{A \sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

The factor $\left(\dfrac{A \sin \alpha}{\alpha} \right)^2$ gives the diffraction effect due to single slit and $\left(\dfrac{\sin N\beta}{\sin \beta} \right)^2$ represents the combined effects of interference and diffraction due to all the slits of grating.

2.5.1 Principal Maxima

I will be maximum when $\sin \beta = 0$

$$\beta = \pm n\pi \, (n = 0, 1, 2, 3....)$$

At the same time $\sin N\beta = 0$, then $\dfrac{\sin N\beta}{\sin \beta}$ becomes indeterminate.

By using L Hospital's rule, we set

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \to \pm n\pi} \frac{\dfrac{d}{d\beta}(\sin N\beta)}{\dfrac{d}{d\beta}(\sin \beta)}$$

$$= \lim_{\beta \to \pm n\pi} \frac{N \cos N\beta}{\cos \beta} = \pm N$$

$$\lim_{\beta \to \pm n\pi} \left(\frac{\sin N\beta}{\sin \beta} \right)^2 = N^2$$

$$I = \left(\frac{A \sin \alpha}{\sin \alpha} \right)^2 N^2$$

The above intensity represents principal maximum and are obtained for $\beta = \pm n\pi$

$$\frac{\pi}{\lambda}(e + d)\sin \theta = \pm n\pi$$
$$(e + d)\sin \theta = \pm n\lambda$$

where $n = 0, 1, 2, 3$ ….

$n = 0$ corresponds to zero order principal maximum. For $n = 1, 2, 3, ….$ we get first, second, third …. order principal maxima. The $\pm$ sign shows that there are two principal maxima of the same order lying on either side of zero order maximum.

Minima   A series of minima occur, when

$$\sin N\beta = 0 \text{ but } \sin \beta \neq 0$$

For minima $\sin N\beta = 0$ or $N\beta = \pm m\pi$

$$N\frac{\pi}{\lambda}(e + d)\sin \theta = \pm m\pi.$$

$$N(e + d)\sin \theta = \pm m\lambda,$$

where *m* has all integral values except 0, *N*, 2*N*, … *nN*, because for these values $\sin \beta$ becomes zero and we get principal maxima. Thus, $m = 1, 2, 3 … (N - 1)$. Hence, there are adjacent principal maxima.

2.5.2 Secondary maxima

As there are $(N - 1)$ minima between two adjacent principal maxima, there must be $(N - 2)$ other maxima between two principal maxima. These are known as secondary maxima. To find out the position of these secondary maxima. Let us differentiate the intensity equation with respect to β and then equate it to zero. Thus

$$\frac{dI}{d\beta} = \left(\frac{A \sin \alpha}{\alpha} \right)^2 \cdot 2 \left(\frac{\sin N\beta}{\sin \beta} \right) \times \left[\frac{N \cos N\beta \sin \beta - \sin N\beta \cos \beta}{\sin^2 \beta} \right] = 0$$

or
$$N \cos N\beta \sin \beta - \sin N\beta \cos \beta = 0$$
$$N \tan \beta = \tan N\beta$$

The roots of this equation other than those for which $\beta = \pm n\pi$ (principal maxima) give the positions of secondary maxima. To find out the value of $(\sin^2 N\beta / \sin^2 \beta)$ from equation $N \tan \beta = \tan N\beta$, we make use of the triangle shown in Fig. 2.10 It gives

$$\sin N\beta = \frac{N}{\sqrt{N^2 + \cot^2 \beta}}$$
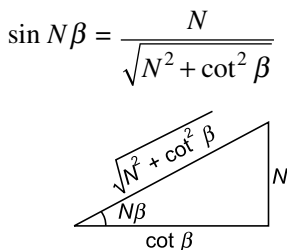
Figure 2.10

$\therefore$
$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2}{(N^2 + \cot^2 \beta) \times \sin^2 \beta}$$

$$= \frac{N^2}{N^2 + \sin^2 \beta + \cos^2 \beta}$$

$$= \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

$\therefore$
$$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}}$$

$$= \frac{1}{1 + (N^2 - 1) \sin^2 \beta}$$

As N increases, the intensity of secondary maxima relative to principal maxima decreases and becomes negligible when N becomes large.
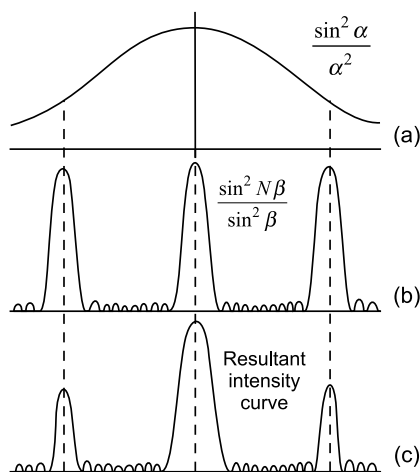
Figure 2.11

Figure 2.11 (a) and (b) show the variation of intensity due to the factors $\sin^2 \alpha / \alpha^2$ and $\sin^2 N\beta / \sin^2 \beta$ respectively. The resultant intensity is shown in Fig. 2.3(c).

SOLVED PROBLEMS

1. A plane transmission grating having 4250 lines per cm is illuminated with a sodium light normally. In the second-order spectrum, the spectral lines are deviated by 30°. What is the wavelength of the spectral line?

Given data   Number of lines in the grating $N = 4250$ lines/cm

Angle of diffraction $\theta = 30°$

Order of spectrum $n = 2$

Solution $(e + d) \sin \theta = n\lambda$ (i)

Each slit width $(e + d) = \dfrac{1}{N} = \dfrac{1}{4250} = 2.353 \times 10^{-4}$ cm

Then substituting the value of e in Eq. (i)

$2.353 \times 10^{-4} \times \sin 30° = 2\lambda$

$2.353 \times 10^{-4} \times 0.5 = 2\lambda$

$1.1765 \times 10^{-4} = 2\lambda$

$\Rightarrow$ $\lambda = 0.5882 \times 10^{-4}$ cm

$= 5.882 \times 10^{-5}$ cm

$\lambda = 5882$ Å

2. A source of light having a wavelength of 600 nm is incident on a slit with a width of 1 μm. Find the angular separation between the first-order minima and central maxima of either side.

Given data   Wavelength of the light, $\lambda = 600 \, nm = 600 \times 10^{-9}$ m

Width of the slit, $e + d = 1 \times 10^{-6}$ m

Order $n = 1$

Solution $(e + d) \sin \theta = n\lambda \Rightarrow \sin \theta = \dfrac{n\lambda}{(e + d)} \Rightarrow \theta = \sin^{-1}\left(\dfrac{n\lambda}{e + d}\right)$

$$\theta = \sin^{-1}\left(\frac{1 \times 600 \times 10^{-9}}{1 \times 10^{-6}}\right) = \sin^{-1}(0.6) = 36°52'$$

Angular separation is 36°52′.

3. A plane grating having 10520 lines per cm is illuminated with light having a wavelength of 5×10^{-5} cm at normal incidence. How many orders are visible in the grating spectra?

Given data   Number of lines in the grating $N = 10520$ lines/cm

Width of each slit, $(e + d) = \dfrac{1}{N}$

Wavelength of the light, $\lambda = 5 \times 10^{-5}$ cm

Angle $\theta = 90°$

Solution $(e + d) \sin \theta = n\lambda$

where $(e + d) = \dfrac{1}{N} = \dfrac{1}{10520} = 9.506 \times 10^{-5}$

$$n = \frac{e \sin \theta}{\lambda}$$

$$n = \frac{9.506 \times 10^{-5} \times \sin 90°}{5 \times 10^{-5}} = 1.901$$

$$n = 2$$

In this grating, two orders can be seen.

4. A grating has 6000 lines/cm. Find the angular separation between two wavelengths of 500 nm and 510 nm in the 3rd order.

Given data   Number of lines in the grating,

$$N = \frac{1}{e + d} \, 6000 \text{ lines/cm}$$

$$= 6000 \times 10^2 \text{ lines/m}$$

Order $n = 3$

Solution We know that $(e + d) \sin \theta = n\lambda$

$$\sin \theta = \frac{n\lambda}{e + d} = nN\lambda$$

For first wavelength, $\lambda_1 = 500$ nm $= 500 \times 10^{-9}$ m

$$\sin \theta_1 = 3 \times 6000 \times 10^2 \times 500 \times 10^{-9}$$

$$\sin \theta_1 = 0.9$$

$$\theta_1 = \sin^{-1}(0.9) = 64°9'$$

Similarly, $\lambda_2 = 510$ nm $= 510 \times 10^{-9}$ m

$$\sin \theta_2 = 3 \times 6000 \times 10^2 \times 510 \times 10^{-9}$$

$$= 0.918$$

$$\theta_2 = \sin^{-1}(0.918)$$

$$\theta_2 = 66°38'$$

Angular separation between two wavelengths in third order

$$= \theta_2 - \theta_1$$
$$= 66°38' - 64°9'$$
$$= 2°29'$$

5. Find the highest order that can be seen with a grating having 15000 lines/inches. The wavelength of the light used is 600 nm.

Given data Wavelength of the given light $\lambda = 600$ nm $= 600 \times 10^{-9}$ m

Number of lines in the grating $N = \dfrac{1}{e + d} = 15000$ lines/inch

$$= 15000/2.54 \text{ lines/cm}$$
$$= 5906 \text{ lines/cm}$$
$$= 5906 \times 10^2 \text{ lines/m}$$

Solution We know that

$$n_{max} = \frac{e + d}{\lambda} = \frac{1}{N\lambda}$$

$$n_{max} = \frac{1}{5906 \times 10^2 \times 600 \times 10^{-9}}$$

$$n_{max} = 2.822$$

∴ the highest order that can be seen is 2.

6. For a grating, the angle of diffraction for the second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Find the number of lines per cm of the grating.

Given data Wavelength of light $\lambda = 5 \times 10^{-5}$ cm

Diffraction angle $\theta = 30°$

Order $n = 2$

Solution $(e + d) \sin \theta = n\lambda$

$$e + d = \frac{n\lambda}{\sin \theta} = \frac{2 \times 5 \times 10^{-5}}{\sin 30°} = \frac{2 \times 5 \times 10^{-5}}{0.5}$$

$$= 10^{-3} \text{ cm}$$

$\therefore$ Number of lines per cm, $N = \dfrac{1}{e + d} = 10^3$.

7. The first diffraction minima due to a single slit diffraction is at $\theta = 30°$ for a light of wavelength 5000 Å. Find the width of the slit.

Given data   Diffraction angle $\theta = 30°$

Wavelength of light $\lambda = 5000 \text{ Å} = 5000 \times 10^{-8}$ cm

Order $m = 1$

Solution   Condition for minima due to single slit is

$$e \sin \theta = m\lambda$$

$$e = \frac{m\lambda}{\sin \theta} = \frac{1 \times 5000 \times 10^{-8}}{\sin 30°} = \frac{1 \times 5000 \times 10^{-8}}{0.5}$$

$$= 10 \times 10^{-5} \text{ cm}$$

$\therefore$ width of the slit $= 10 \times 10^{-5}$ cm

8. A monochromatic light of wavelength 6.56×10^{-7} m incidents normally on a grating of 2 cms wide. The first odder spectrum is produced at an angle of 18°14′ from the normal. Calculate the total number of lines in the grating.

Solution $\sin \theta = Nn\lambda$

$$N = \frac{\sin \theta}{n\lambda}$$

but

$$N = \frac{\text{total no. of lines}}{\text{width}} = \frac{M}{0.02}$$

Hence, $M = \dfrac{0.02 \sin \theta}{n\lambda} = \dfrac{0.02 \times \sin 18°14'}{1 \times 6.56 \times 10^{-7}} = 9540$

9. A parallel beam of sodium light of wavelength 5890 Å is incident on a thin glass plate of refractive index 1.5, such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection?

Given data   Wavelength of light $\lambda = 5890$ A°

Refractive index of glass plate $\mu = 1.5$

Angle of refraction $r = 60°$

Solution   The plate will appear dark in the reflected system if

$$2\mu t \cos r = n\lambda$$

for the smallest thickness $n = 1$

$$\text{Hence } t = \frac{\lambda}{2\mu \cos r} = \frac{5890 \times 10^{-10}}{2 \times 1.5 \times \cos 60} = 3927 \times 10^{-10}$$

$$= 3927 \text{ Å}.$$

10.   A single slit is illuminated by white light. The first minimum for red light of wavelength $\lambda = 6500$ Å falls at an angle $\theta = 30º$. What is the value of slit width?

Given data   Wavelength of red colour $\lambda = 6500$ Å $= 6500 \times 10^{-10}$ m.

angle $\theta = 30º$, slit width $2d = ?$

Solution $2d \sin \theta = \lambda$

$$2d = \frac{\lambda}{\sin \theta} = \frac{6500 \times 10^{-10}}{1/2} = 13000 \times 10^{-10} = 13000 \text{ Å}$$

11.   Light from sodium vapour lamp is normally falls on a grating of 2 cm having 10000 lines. Find the angular separation of the two lines of sodium of wavelengths 5890 and 5896 Å in the first order spectrum?

Given data   Wavelengths of sodium lamps $\lambda_1 = 5890$ Å

$$\lambda_2 = 5896 \text{ Å}$$

Width of the Grating $= 2$ cm, order $n = 1$

Solution $N = \dfrac{10000}{2} = 5000$ lines/cm $= 5000 \times 10^2$ lines/m.

We know that $\sin \theta = Nn\lambda$

For $\lambda_1 \rightarrow \sin \theta_1 = 1 \times 5 \times 10^5 \times 5890 \times 10^{-10} = 0.29450$

$$\theta_1 = 17.1275º$$

for λ_2 $\sin \theta_2 = nN\lambda_2$

$$= 1 \times 5 \times 10^5 \times 5896 \times 10^{-10} \text{ m}$$

$$= 0.29480$$

$$\theta_2 = 17.1458º$$

$$\theta_2 - \theta_1 = 0.018º$$

MULTIPLE CHOICE QUESTIONS

1. Diffraction phenomena are usually divided into _____ classes.
   - (a) one
   - (b) two
   - (c) three
   - (d) four

2. In which experiment the screen and sources are at finite distance?
    (a) Fraunhofer diffraction
    (b) Fresnel's diffraction
    (c) Both (a) and (b)
    (d) None of these

3. In Fraunhofer's diffraction, incident light waves have _____ type of wave front.
    (a) circular
    (b) spherical
    (c) cylindrical
    (d) plane

4. In which diffraction are lenses required?
    (a) Fraunhofer diffraction
    (b) Fresnel's diffraction
    (c) Young's diffraction
    (d) Newton's diffraction

5. In single-slit experiment, if the red colour is replaced by blue then _____
    (a) the diffraction pattern becomes narrower and crowded together
    (b) the diffraction bands become wider
    (c) the diffraction pattern does not change
    (d) the diffraction pattern disappears

6. Diffraction grating has a
    (a) large number of equidistant slits
    (b) only one slit
    (c) large number of random distant slits
    (d) circular slit

7. In a diffraction grating, the condition for principal maxima is
    (a) $e \sin \theta = n\lambda$
    (b) $(e + d) \sin \theta = n\lambda$
    (c) $d \sin \theta = n\lambda$
    (d) $\sin \theta = n\lambda$

8. What are the characteristics of grating spectra?
    (a) Spectral lines are almost straight and outer sharp.
    (b) The spectral lines are more and more dispersed as order increases.
    (c) It is situated symmetrically on both sides of zero order image.
    (d) All of the above

9. In Fraunhofer diffraction the wavefront undergoing diffraction has to be
    (a) cylindrical
    (b) spherical
    (c) elliptical
    (d) plane

10. The penetration of waves into the regions of the geometrical shadow is
    (a) dispersion
    (b) diffraction
    (c) interference
    (d) polarization

11. In single-slit diffraction, the first diffraction minima is observed at an angle of 30°, when a light having a wavelength of 500 nm is used. The width of the slit is
    (a) 1.25×10^{-5} cm
    (b) 10×10^{-5} cm
    (c) 5×10^{-5} cm
    (d) 2.5×10^{-15} cm

12. In a single-slit experiment if the slit width is reduced,
    (a) the fringes become narrower
    (b) the fringes becomes brighter

(c) the fringes become wider

(d) the colour of the fringes change

13. The diffraction pattern of a single slit consists of

(a) narrow bright band at the centre with alternate dark and bright bands of equal intensity on either side

(b) wider bright band at the centre with alternate dark and bright bands of equal intensity on either side

(c) wider dark band at the centre with alternate bright and dark bands on either side

(d) wider and brighter band at the centre with alternate dark and bright bands of decreasing intensity on either side

14. In diffraction due to double slit, we observe

(a) wider diffraction bands and within that narrower interference fringes

(b) wider interference fringes and narrower diffraction bands

(c) interference and diffraction fringes of equal width

(d) diffraction pattern due to both the slits independently

15. A parallel beam of monochromatic light falls normally on a plane diffraction grating having 5000 lines/cm. A second-order spectral line is diffracted through an angle of 30°. The wavelength of light is

(a) 5×10^{-6} cm   (b) 5×10^{-7} cm

(c) 5×10^{-5} cm   (d) 5×10^{-4} cm

16. When white light is incident on a diffraction grating, the light diffracted more will be

(a) blue   (b) yellow

(c) violet   (d) red

17. Maximum number of orders possible with a grating is

(a) independent of grating element

(b) inversely proportional to grating element

(c) directly proportional to grating element

(d) directly proportional to wavelength

18. When white light is incident on a grating, the light diffracted less will be

(a) blue   (b) yellow

(c) violet   (d) red

19. For a grating, if the width of the grating element is less than twice the wavelength of light, then the possible number of diffraction orders are

(a) one   (b) two

(c) three   (d) four

20. Diffraction effect is predominant when

(a) size of the obstacle is less than the wavelength of light

(b) size of the obstacle is nearly equal to the wavelength of light

(c) size of the obstacle is greater than the wavelength of light

(d) None of the above

Answers

1. (b)	2. (b)	3. (d)	4. (a)	5. (a)	6. (a)
7. (b)	8. (d)	9. (d)	10. (b)	11. (b)	12. (c)
13. (d)	14. (a)	15. (c)	16. (d)	17. (c)	18. (c)
19. (a)	20. (b)				

Fill in the Blanks

1. The bending of light around the corners of obstacles is called _____.
2. In Frensel's diffraction _____ source is used.
3. In diffraction pattern fringes have _____ width.
4. With decreasing of slit width in single slit, the width of the fringes _____.
5. In a grating, the combined width of a ruling and a slit is called _____.
6. Points on successive slits Separated by a distance equal to the grating element is called _____.
7. In Fraunhofer diffraction at a plane transmission grating, when white light source is used, the angle of diffraction for violet is _____ than that of red.

Answers to Fill in the blanks

1. diffraction
2. point
3. unequal
4. increases
5. Grating element
6. Corresponding points
7. lesser

True or False

1. For diffraction to occur, the size of the obstacle must be comparable with wavelength. (T/F)
2. Converging lense is used to focus parallel rays in Fresnel diffraction. (T/F)
3. The source and Screen are at infinite distance from the obstacle producing Fraunhofer diffraction. (T/F)
4. Diffraction is due to superposition of secondary waves originating from different parts of the same wave front. (T/F)
5. In diffraction all bright fringes have same intensity. (T/F)
6. In diffraction intensity of dark fringes is not zero. (T/F)
7. In Fraunhofer diffraction at a double slit, the angular separation between two consecutive maxima decreases if the slits are narrower and closer. (T/F)

Answers to True or False

1. T	2. F	3. T	4. T	5. F	6. T
7. T					

REVIEW QUESTIONS

Short Answer Questions

1. What is diffraction?
2. Why lenses are necessary for the study of Fraunhofer diffraction?
3. Mention any two differences between interference and diffraction.
4. Derive the relationship between the phase difference and path difference of two light waves.
5. Mention the conditions to get principal maximum and minimum intensity positions due to single slit by Fraunhofer diffraction.
6. Explain why interference effects are significant along with the diffraction effect for double slits Fraunhofer diffraction?
7. What is diffraction grating?
8. Explain why interference effects are more significant along with the diffraction effect in grating.

Essay Type Questions

1. Distinguish between Fraunhofer and Fresnel's diffraction.
2. Distinguish between interference and diffraction.
3. Describe Fraunhofer diffraction due to single slit.
4. Describe Fraunhofer diffraction due to double slits.
5. Describe Fraunhofer diffraction due to grating.

CHAPTER 3

LASERS

INTRODUCTION

Laser is an acronym for Light Amplification by Stimulated Emission of Radiation. LASER radiation is due to stimulated emission of radiation process which improves (amplifies) the intensity of radiation. In general, when an electron moves from a higher energy orbit to a lower energy orbit, it emits radiation. This emission of radiation can be explained in terms of energy levels as when the electron transits from a higher energy level to a lower energy level, it emits radiation. According to Planck's quantum theory, the emission of radiation will be in the form of photon of energy of $h\nu$. The frequency (ν) term in the energy of the photon indicates the wave characteristics of the photon. The energy of the emitted photon is equal to the energy difference between the energy levels. This loss of energy is attributed to the entire atom. As a result, it can be thought that the atom is moving from a higher energy state to a lower energy state.

When a large number of atoms transit to lower energy level, then each transition emits photons of same energy, same frequency and same wavelength $\nu = \left[\dfrac{E_2 - E_1}{h}\right]$. All the photons are in phase, and after reinforcement emit a high intensity monochromatic coherent radiation, i.e., LASER radiation.

3.1 CHARACTERISTICS OF LASER

3.1.1 Monochromaticity

In laser radiation, all the photons emitted between discrete energy levels will have same wavelength. As a result, the radiation is monochromatic in nature. If the higher energy level has closely spaced energy levels then from the transition from each level to lower energy level emits photons of different frequencies and wavelengths. Let the spread in frequency and wavelength be $\nu + \Delta\nu$ and $\lambda + \Delta\lambda$. The frequency spread $\Delta\nu$ is related to its wavelength spread $\Delta\lambda$ as

$$\Delta\lambda = -\left[\frac{c}{v^2}\right]\Delta v$$

For laser, $\Delta\lambda$ = 0.001nm. It is clear that laser radiation is highly monochromatic.

3.1.2 Directionality

In ordinary light, divergence of light takes place as it propagates through the medium. For laser radiation also, this divergence takes place as it propagates through the medium (Fig. 3.1).
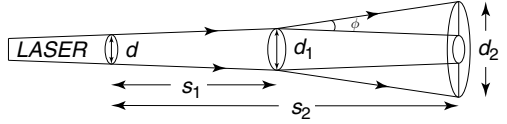
Figure 3.1   Divergence of laser beam

The laser light of wavelength λ emerges through a laser source aperture diameter d, then it propagates as a parallel beam up to d^2/λ (small value) and gets diverged. The angle of divergence of a laser beam is expressed as

$$\phi = \frac{\text{arc}}{\text{radius}} = \frac{(d_2 - d_1)}{(s_2 - s_1)}$$

where $d_1 d_2$ are the diameters of the laser spots measured at distances s_1 and s_2 from the laser aperture.

For laser light $\phi = 10^{-3}$ radians.

It is clear from the above value, that the divergence is low and it is highly directional.

3.1.3 Coherence

Coherence is a property of a wave being in phase with itself and also with another wave over a period of time, and space or distance. Coherence is the predictability of the amplitude and phase at any point on the wave knowing the amplitude and phase at any other point either on the same wave or on a neighbouring wave. Coherence is of two types:

Temporal coherence   If it is possible to predict the amplitude and phase at a point on the wave w.r.t. another point on the same wave, then the wave is temporal coherence. For laser radiation, as all the emitted photons (waves) are in phase, the resultant radiation has temporal coherence.

Spatial coherence   If it is possible to predict the amplitude and phase at a point on a wave w.r.t. another point on a second wave then the waves are said to be spatially coherence. The emitted photons (waves) in the laser radiation satisfies spatial coherence.

Thus, laser radiation is highly coherent.

3.1.4 Intensity or Brightness

Let there be 'n' number of coherent photons of amplitude 'a' in the emitted laser radiation. These photons reinforce with each other and the amplitude of the

resulting wave becomes *na* and hence the intensity is proportional to n^2a^2. Thus, due to coherent additions of amplitude and negligible divergence the intensity or brightness increases enormously.

3.2 SPONTANEOUS AND STIMULATED EMISSION OF RADIATION

When the incident radiation (photon) interacts with the atoms in the energy levels then three distinct processes can take place:

1. Stimulated absorption of radiation
2. Spontaneous emission of radiation
3. Stimulated emission of radiation

3.2.1 Stimulated Absorption of Radiation

Consider two energy levels of energies E_1 and E_2 such that $E_1 > E_2$ for an atom. Let N_1 and N_2 are the number of atoms per unit volume in the energy levels E_1 and E_2 shown in Fig. 3.2.
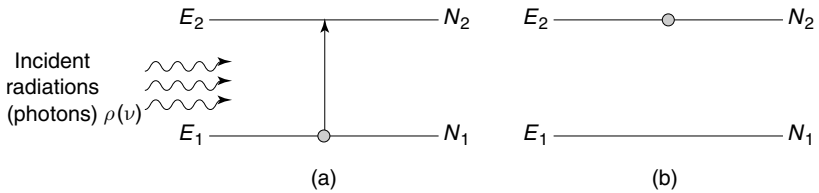
Figure 3.2   (a) Before absorption, (b) After absorption

The incident radiation consists of photons of energy equal to the energy difference between E_1 and E_2. The number of photons per unit volume of incident radiation is known as *incident radiation density* $\rho(v)$. The incident photon stimulates the atom in the lower energy level E_1 to absorb its energy. After absorbing energy, the atom transists to E_2. This process of exciting the atom to higher energy level by absorbing the stimulating incident photon is known as stimulated absorption of radiation. The number of stimulated absorptions depend upon the number of atoms per unit volume N_1 in E_1 and the number of photons per unit volume of incident radiation, i.e., incident radiation density $\rho(v)$, i.e.,

Number of stimulated absorptions $\alpha\, N_1$

$$\alpha\, \rho(v)$$
$$\alpha\, N_1\rho(v)$$
$$= N_1\, B_{12}\, \rho(v)$$

where B_{12} is known as Einstein's coefficient of stimulated absorption of radiation.

If this excited atom transits to lower energy level E_1 then it emits radiation. The emitted radiation are of two types, i.e.,

1. Spontaneous emission of radiation
2. Stimulated emission of radiation

3.2.2   Spontaneous Emission of Radiation

The excited atom in the higher energy level E_2 can stay up to 10^{-8} second. This average time up to which an excited atom can stay at E_2 is known as lifetime of the excited atom. After the lifetime of the excited atom, it transits to the lower energy level E_1 by itself emitting a photon of energy

$$E = E_2 - E_1$$
$$h\nu = E_2 - E_1$$
$$\nu = \frac{E_2 - E_1}{h}$$

This process where emission of radiation takes place by the transition of an excited atom by itself to lower energy level is known as spontaneous emission of radiation. The number of spontaneous emission of radiation depends on the number of atoms per unit volume in E_2, i.e., N_2.
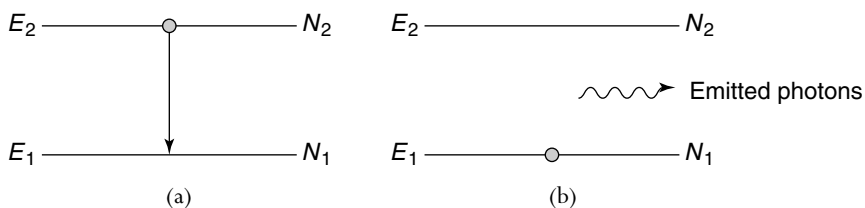
Figure 3.3   (a) Before emission, (b) After emission

Number of spontaneous emissions $\alpha\, N_2$

$$= N_2 A_{21}$$

where A_{21} is known as Einstein's coefficient of spontaneous emission of radiation.

3.2.3   Stimulated Emission of Radiation

The excited atom after being stimulated by the incident photon transits to a lower energy level E_1 in a time less than the lifetime (10^{-8} second) of an excited atom. This transition emits a photon of energy

$$E = E_2 - E_1$$
$$h\nu = E_2 - E_1$$
$$\nu = \frac{E_2 - E_1}{h}$$

This process where the excited atom after getting stimulated by the incident photon transits to lower energy level by emitting photons is known as stimulated emission of radiation.
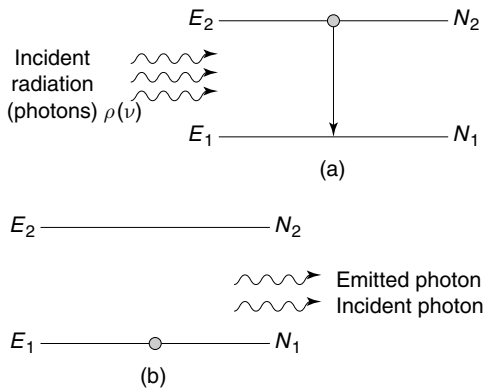
Figure 3.4 (a) Before emission, (b) After emission

The number of stimulated emissions depends on the number of atoms in the higher energy level E_1, i.e., N_2 and the incident radiation density $\rho(v)$

No. of stimulated emissions $\alpha\, N_2$

$$\alpha\, \rho(v)$$
$$\alpha\, N_2\, \rho(v)$$
$$= N_2\, B_{21}\, \rho(v)$$

where B_{21} is known as *Einstein's coefficient of stimulated emission of radiation*. In this process, in addition to emitted photons, there will be incident photons. All these photons have same energy and same frequency and are in phase. As they reinforce together, it results in high-intensity monochromatic and coherent radiation, i.e., laser radiation.

3.2.4   Distinction between Spontaneous and Stimulated Emission of Radiation

Spontaneous emission	*Stimulated emission*
1. Spontaneous emission takes place when an atom in higher energy state transits to lower energy state by itself.	1. Stimulated emission takes place when the atom in higher energy state gets stimulated by the incident photon and transits to lower energy state.
2. Spontaneous emission is independent of incident radiation density.	2. Stimulated emission depends upon the incident radiation density.
3. Spontaneous emission takes place after 10^{-8} second.	3. Stimulated emission takes place within a time of 10^{-8} second.
4. Spontaneous emission is a slow process when compared to stimulated emission.	4. Stimulated emission is a fast process when compared to spontaneous emission.
5. A_{21} is the Einstein's coefficient of spontaneous emission.	5. B_{21} is the Einstein's coefficient of stimulated emission.

3.3   EINSTEIN'S COEFFICIENTS

Consider two energy levels of energies E_1 and E_2 such that $E_2 > E_1$. Let N_1 and N_2 be the number of atoms per unit volume of E_1 and E_2. In the presence of incident radiation (photon), the atoms in E_1 get excited and we have stimulated absorption of radiation.

Figure 3.5   Absorption and emission process

The number of stimulated absorption of radiation = $N_1 B_{12} \rho(v)$.

The excited atom while transiting to lower energy level E_1 emits a radiation (photon). Thus, we get a spontaneous and stimulated emission of radiation.

The number of spontaneous emission of radiation = $N_2 A_{21}$

The number of stimulated emission of radiation = $N_2 B_{21} \rho(v)$

The upward and downward transitions are continuous and at thermal equilibrium state, both the transitions are equal

i.e., $$N_1 B_{12} \rho(v) = N_2 A_{21} + N_2 B_{21} \rho(v)$$

$$\rho(v) [N_1 B_{12} - N_2 B_{21}] = N_2 A_{21}$$

$$\rho(v) = \frac{N_2 A_{21}}{[N_1 B_{12} - N_2 B_{21}]}$$

Dividing the numerator and denominator of RHS of the above equation by N_2, we get

$$\rho(v) = \frac{A_{21}}{\left[\dfrac{N_1}{N_2} B_{12} - B_{21} \right]}$$

$$\rho(v) = \frac{A_{21}}{B_{12} \left[\dfrac{N_1}{N_2} - \dfrac{B_{21}}{B_{12}} \right]} \tag{1}$$

From Boltzmann's equation, we get

$$N_2 = N_1 e^{-(E_2 - E_1)/K_B T}$$

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/K_B T}$$

But $$E_2 - E_1 = hv$$

$$\frac{N_1}{N_2} = e^{hv/K_B T} \tag{2}$$

Substituting the above value in equation (1), we get

$$\rho(v) = \frac{A_{21}}{B_{21}\left[e^{hv/K_BT} - \dfrac{B_{21}}{B_{12}}\right]} \quad (3)$$

from Planck's radiation law, we get

$$\rho(v) = \frac{8\pi h v^3}{c^3}\left[\frac{1}{e^{(hv/K_BT)} - 1}\right] \quad (4)$$

where h is Planck's constant, v is the frequency of radiations and c, velocity of light.

Comparing the above two equations, we get

$$\frac{B_{21}}{B_{12}} = 1$$

$$B_{21} = B_{12} \quad (5)$$

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h v^3}{c^3}$$

$$A_{21} = B_{12}\left[\frac{8\pi h v^3}{c^3}\right] \quad (6)$$

$$A_{21} = B_{21}\left[\frac{8\pi h v^3}{c^3}\right] \quad (7)$$

The above equations represent the relation between Einstein's coefficients.

3.4   POPULATION INVERSION

Consider a two-level energy system of energies E_1 and E_2 (Fig. 3.6). Let N_1, N_2 be the number of atoms per unit volume of energy levels E_1 and E_2. The number of atoms per unit volume in an energy level is known as population of that energy level. That means N_1 and N_2 are the populations of E_1 and E_2.

Figure 3.6   Energy level system

The population of any energy level E, at temperature T is given by Boltzmann's distribution as

$$N = N_0\, e^{-E/K_BT} \quad (8)$$

where N_0 is the population of the lower energy level or ground state and K_B is the Boltzmann's constant. Using the above relation, the populations of E_1 and E_2 are given by

$$N_1 = N_0\, e^{-E_1/K_BT} \quad (9)$$

$$N_2 = N_0 \, e^{-E_2/K_B T} \tag{10}$$

From the above relation, we have

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/K_B T}$$

$$N_2 = N_1 \, e^{-(E_2 - E_1)/K_B T} \tag{11}$$

Since $E_2 > E_1$ so $N_2 < N_1$

To get laser emission (stimulated emission) $N_2 > N_1$, i.e., the population of the higher energy level (E_2) should be greater than the population of the lower energy level (E_1). In general, the population of a lower energy level will be greater than that of the higher energy level. The stage of making the population of the higher energy level to be greater than the population of the lower energy level is known as *population inversion*. The process of sending atoms from E_1 to E_2 to get population inversion is known as *pumping*.

Consider a three-energy level system with energies E_1, E_2 and E_3 of populations N_1, N_2, N_3.
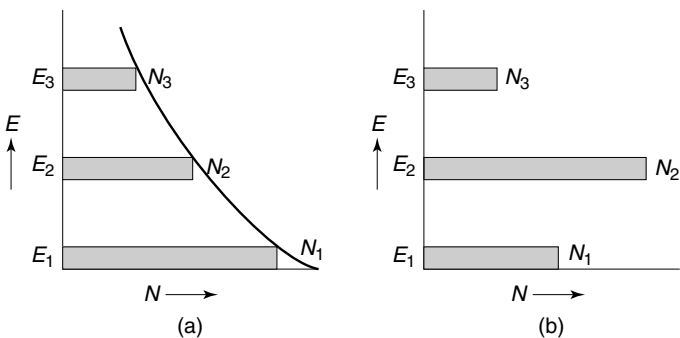
Figure 3.7   (a) Boltzmann's distribution, (b) Population inversion between E_1 and E_2

In normal conditions, $E_1 < E_2 < E_3$ and $N_1 > N_2 > N_3$ obeying Boltzmann's distribution [(Fig. 3.7 (a))]. E_1 is the lower energy state (ground state) with more lifetime of an atom. E_3 is the highest energy state with less lifetime of an atom (10^{-8} s). E_2 is the intermediate energy state with more lifetime of an atom (10^{-3} s) compared to that of E_3. This intermediate energy state with more lifetime of an atoms is known as *metastable state*. This state provides necessary population inversion for the laser emission. When a suitable energy is supplied to the system, atoms get excited to E_3 and transit to E_2. Due to more lifetime of an atom, the atoms stay for a longer time in E_2 when compared to E_3. Due to accumulation of atoms in E_2, a stage will be reached where $N_2 > N_1$. Thus, population inversion is established between E_1 and E_2.

3.5   EXCITATION MECHANISM

For laser action to take place, we need population inversion between the available energy levels of a system. To obtains population inversion, the atoms in the lower energy level should be excited to higher energy level by supply of additional energy in the form of excitation mechanism. The generally used various excitation mechanisms are as detailed below.

1. Optical pumping   In optical pumping, a light source is used to supply luminous energy to excite the atoms to higher energy levels to create population inversion for further laser emission of radiation. This type of pumping is widely used in solid state lasers or ruby laser.

2. Electric discharge   In this method, atoms are excited by collisions with fast moving accelerated electrons in an electric discharge. This mechanism is well suited for gaseous ion lasers. The bast example is the argon ion laser. The electric field (kV/m) between the cathode and anode of discharge tube causes the emitted electrons by the cathode to be accelerated towards the anode. These electrons collide with the gaseous atoms, ionise the gas and raise them to higher energy levels causing population inversion.

3. Inelastic atoms atom collisions   When the gaseous medium consists of two different atoms, then one atom is excited to the corresponding higher energy level by electric discharge. This excited atom collide with the different atoms in the medium inelastically transferring its energy to it and pumping it to the equivalent its higher energy levels. Population inversion is caused by the inelastic collisions between two various atoms of the gaseous medium. The best example of this method is He–Ne laser.

4. Chemical reaction   In this method, the chemical energy released during the chemical process will excite the atoms to higher energy level causing population inversion in the lasing system. For example, hydrogen can react with fluorine to produce hydrogen fluoride liberating heat energy. This heat energy will try to excite the atoms to higher energy level.

5. Direct conversion   When a *p-n* junction is forward biased, then the recombination of electron hole pair across the junction emits radiation. In this process, the applied electric energy promotes the emission of radiation. The suitable example for this method is semiconductor lasers.

3.6   OPTICAL RESONATOR

Optical resonator consists of a pair of mirrors in which one is fully reflective and other is partially reflective facing each other having common principal axes enclosing active medium when one of the exited atoms in the active medium
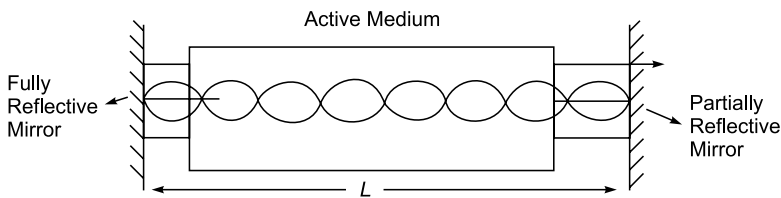
Figure 3.8 Optical resonator

emits a photon by de excitation, that photon acts as a stimulating photon for further emission of photons by stimulated emissions. To sustain stimulated emission acts and to amplify the radiation Intensely in an acumulative way, a positive feedback of emitted photons is must. This is achieved with the help of optical resonator.

The reflected photons into the medium by the mirrors enhances the stimulated emission process and in turn radiation intensity. Due to multiple reflections of photons gains its intensity. At the same time, photon losses its intensity due to some absorptions at the mirror and in active, medium. Laser beam oscillation begins when the gain in its intensity equal to the losses. As the oscillations build up to high intensity it results into laser beam.

The radiation propagation in the resonator will be in the form of standing waves, so as to enhance the wave amplitude. The condition to form standing wave is that the total phase change suffered by a wave in one complete round trip must be equal to an integral multiple of 2π. If L is the lengths of the resonator then

$$\text{phase change} = \left[\frac{2\pi}{\lambda}\right] 2L = 2n\pi, \; n = 1, 2, 3, \ldots$$

where λ is the wave lengths of radiation

$$L = \frac{n\lambda}{2}$$

But $\lambda = \frac{c}{\gamma}$ where c is the velocity of radiation and γ is the frequency of radiation.

$$L = \frac{nc}{2\gamma}$$

$$\gamma = \frac{nc}{2L}$$

The frequency of the radiation will have discrete values depending upon the value of 'n' known as mode number.

3.7   CONSTRUCTION AND COMPONENTS OF A LASER

As shown in Fig. 3.9, any laser system consists of three important components.
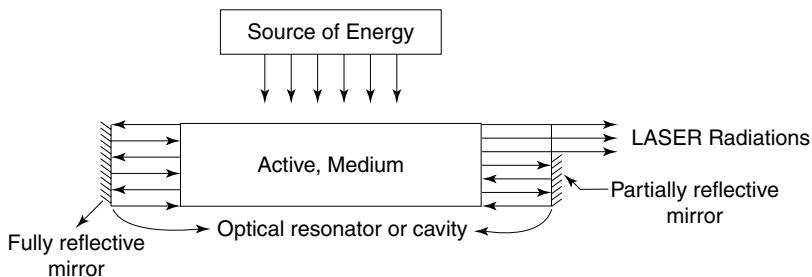
Figure 3.9   Basic components of a laser system

3.7.1   Source of Energy

To get laser emission, first we must have population inversion in the system. The source of energy supplies sufficient amount of energy to the active medium by which the atoms or molecules in it can be excited to the higher energy level. As a result we get population inversion in an active medium. That means the source of energy supply energy and pumps the atoms or molecules in the active medium to excited states.

3.7.2   Active Medium

This is the medium where stimulated emission of radiation takes place. After receiving energy from the source, the atoms or molecules get excited to higher energy levels. While transiting to a lower energy level, the emitted photons start the stimulated emission process which result in laser emission. Depending upon the type of active medium, we have solid state, liquid state, gaseous state and semiconductor lasers.

3.7.3   Optical Cavity or Resonator

The active medium is enclosed between a fully reflective mirror and a partially reflective mirror. These mirrors constitute an optical cavity or resonator. The reflecting portions of the mirrors reflect the incident radiation back into the active medium. These reflected radiation enhance the stimulated emission process within the active medium. As a result, we get high-intensity monochromatic and coherent laser light through the nonreflecting portion of the mirror.

3.8   RUBY LASER

The ruby laser was the first solid–state laser fabricated by Maiman. The experimental set-up is as shown in Fig. 3.10.
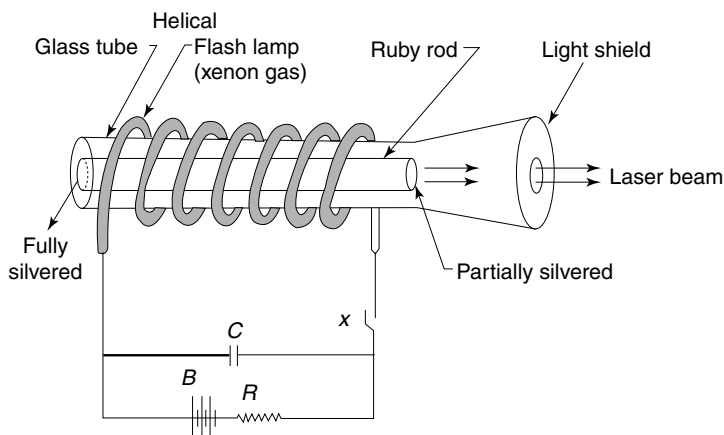
Figure 3.10 The ruby laser

3.8.1 Construction

A ruby laser consists of a cylindrical ruby rod made up of aluminium oxide (Al_2O_3) which is doped with 0.05% weight of chromium oxide (Cr_2O_3). One end of the rod is fully silvered and the other one is partially silvered so that the two ends will act as optical resonator or cavity. The ruby rod is surrounded by a glass tube which in turn is surrounded by helical flash lamp filled with xenon gas. The ends of the flash lamp are connected to the circuit consisting of a capacitor and key. In the ruby rod, Cr^{3+} ions are the active ions taking part in the lasing action. The ruby rod will act as an active medium. The flash lamp which flashes the light onto the ruby rod serves as a source of energy.

3.8.2 Working

When the discharge of the capacitor takes place by closing the key through the flash lamp, light will be flashed on the ruby rod. The Cr^{3+} ions absorb the particular wavelength of the incident light energy and are excited to higher energy states which is shown in Fig. 3.11.

Cr^{3+} ions are excited to energy bands E_2 and E_3 which are due to solid–state active medium from E_1. These ions get transmitted to the metastable state M after staying for up to 10^{-8} second in these bands and are nonradiative. Since the lifetime of atom Cr^{3+} atom is more (10^{-3} s) in the metastable state, accumulation of ions takes places in M. A stage is reached so that population inversion takes place between E_1 and the metastable state M. The transition of Cr^{3+} ion to E_1 emits a photon which stimulates the other ions to transit to E_1, resulting in the stimulated emission of radiation, i.e., laser radiation of a wavelength of 6943 Å. This wavelength corresponds to the red region of the visible spectrum, and the laser beam will be red in colour. Charging and discharging of a capacitor takes

a certain amount of time, hence the flashing of light is not a continuous process. Hence the laser emission is pulsed one.
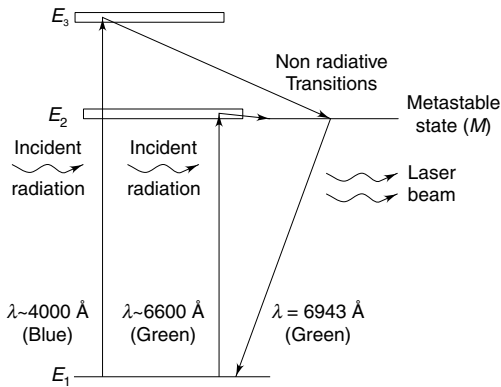
Figure 3.11   The energy levels of the chromium ion

3.8.3   Drawbacks

1. Laser requires a high pumping power because the laser transition terminates at the ground state and more than one half of the ground-state atoms must be pumped to the higher state to achieve population inversion.
2. The efficiency is very low. Only the blue and green components of the flash light are utilised and the remaining are unused.
3. The laser output is not continuous but occurs in the form of pulses of microsecond duration.

3.9   HELIUM–NEON LASER

For increased efficiency and continuous emission of laser, we prefer the helium–neon laser. This is a gaseous state laser, fabricated by Ali Javan. The experimental set-up is as shown in Fig. 3.12.
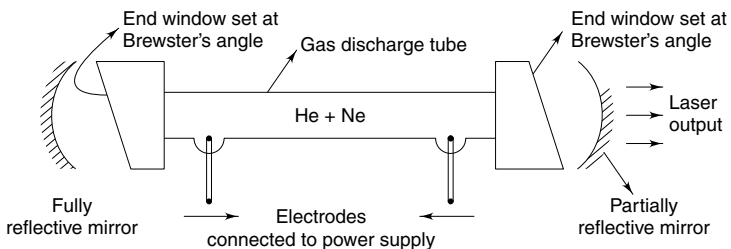
Figure 3.12   He–Ne laser

3.9.1 Construction

It consists of a large and narrow discharge tube which is filled with a mixture of helium and neon gases in the ratio of 10 : 1 with partial pressures of 1 torr and 0.1 torr. The actual lasing atoms are neon atoms and helium is used for selective pumping of the upper laser levels of neon. The gas mixture will serve as active medium. The discharge that takes place between the two electrodes exciting the helium atoms and in turn neon atoms will act as a source of energy. The discharge tube is enclosed between fully and partially reflective mirrors which serve as optical cavity or resonator. The two end windows are set at Brewster's angle, so that the reflected radiations from the mirror entering into the tube become polarised. Thus, only the component having vibrations parallel to the plane of incidence become dominent and sustains laser emission. The emerging laser is linearly polarised.

3.9.2 Working

While discharge takes places through the gas mixture, the electrons collide with the helium atoms which are more in number raising them to the energy levels F_2 and F_3 as shown in Fig. 3.13.
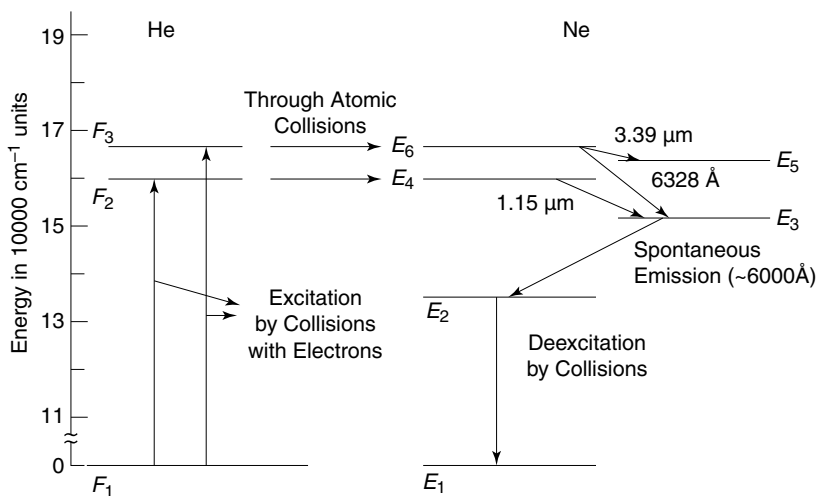
Figure 3.13 Relevant energy levels of helium and neon

Helium atoms after transferring their energies to neon atoms by inelastic collisions, get de-excited to F_1. Now neon atoms are excited to E_4 and E_6 which have nearly the same energy as the levels F_2 and F_3 of helium. This results in sizeable populations of the levels E_4 and E_6. The population in these levels happens to be more than those in the lower levels E_3 and E_5. Thus, a state of population inversion is achieved and any spontaneous emitted photon can trigger laser action in any of the three transitions shown in the figure. The neon atoms then drop down from the lower laser levels to the level E_2 through spontaneous emission and are brought

back to the ground state through collisions with the walls. The transitions from E_6 to E_5, E_4 to E_3 and E_6 to E_3 result in the emission of radiation having wavelengths 3.39 μm, 1.15 μm and 6328 Å respectively. The radiations of wavelengths 3.39 μm and 1.15 μm are invisible, whereas the radiation with 6328 Å is red in colour. The transitions from E_3 to E_2 is spontaneous. From E_2, the neon atoms get de-excited by collision with the walls of the discharge tube.

The excitations and de-excitations of helium and neon atoms is a continuous process and thus it gives continuous emission of laser radiations.

Advantages

1. Gas lasers emit more monochromatic and directional laser radiations when compared to solid–state lasers.
2. He–Ne laser emits continuous laser radiations
3. Due to setting of end windows at Brewster's angle, the output laser is linearly polarised.

3.10   APPLICATIONS OF LASER

Industry

1. With increased power output, lasers can be used as a welding tool. Dissimilar metals can be welded using lasers with minimum distortions.
2. Lasers are used to cut glass and quartz.
3. Lasers are used to drill holes in ceramics.
4. Lasers are used to drill aerosol nozzles and control orifices within the required precision.
5. Lasers are used for heat treatment in the tooling and automotive industry.
6. Lasers are used in electronic industry in trimming the components of ICS.
7. In plastic industry, polymers are obtained by irradiating monomers by lasers.

Medicine

1. Opthalmologists use laser for attaching the retina in retinal-detachment cases.
2. Lasers are used for cataract removal.
3. Lasers are used for eye lens curvature corrections.
4. Lasers are used for bloodless surgery.
5. Lasers are used in angioplasty for removal of artery block.
6. Lasers are used in cancer diagnosis and therapy.
7. Lasers are used in destroying kidney stones and gallstones.

8. Lasers are used in plastic surgery, skin injuries and to remove moles and tumours developed in skin tissue.
9. Lasers are used in the treatment of mouth diseases.
10. Lasers are used in the treatment of liver and lung diseases.
11. Laser doppler velocimetry is used to measure blood velocity in blood vessels.

Scientific Field

1. Lasers are used for isotope separation.
2. Lasers are used in recording and reconstruction of a hologram.
3. Lasers are used to create plasma.
4. Lasers are used to produce chemical reactions.
5. Lasers are used to study the internal structure of microorganisms and cells.
6. Lasers are used in air pollution, to estimate the size of dust particles.
7. Lasers are used to develop hidden fingerprints and to clean delicate pieces of art.

SOLVED PROBLEMS

1. A semiconductor diode laser has a peak emission wavelength of 1.55 μm. Find its band gap in eV.

Given data   Wavelength of semiconductor laser,
$$\lambda = 1.55 \ \mu m = 1.55 \times 10^{-6} \ m$$
Energy gap $E_g = ?$

Solution   Energy gap of semiconductor,
$$E_g = \text{energy of emitted photon, } h\upsilon$$
$$E_g = h\upsilon = h \times \frac{c}{\lambda}$$

where c = velocity of light = 3×10^8 m/s

$$E_g = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.55 \times 10^{-6}} \ J$$

$$E_g = \frac{19.89}{1.55} \times 10^{-20} \ J$$

$$E_g = 12.8322 \times 10^{-20} \ eV = 0.8 \ eV$$

2. Calculate the wavelength of emitted radiation from GaAs which has a band gap of 1.44 eV.

Given data   Energy gap of GaAs, $E_g = 1.44$ eV $= 1.44 \times 1.6 \times 10^{-19}$ J

Solution   Energy gap of semiconductor,

$$E_g = h\upsilon = \frac{h_c}{\lambda}$$

$\Rightarrow$ $$\lambda = \frac{h_c}{E_g}$$

$\Rightarrow$ $$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.44 \times 1.6 \times 10^{-19}} = 8.6328 \times 10^{-7}$$

$$= 8633 \times 10^{-10} \text{ m}$$
$$= 8633 \text{ Å}$$

3.   The ratio of population of two energy levels at 300 K is 10^{-30}. Find the wavelength of the emitted radiation.

Given data   Population ratio; $\dfrac{N_2}{N_1} = 10^{-30}$

Temperature $T = 300$ K

Boltzmann Constant $K_B = 1.38 \times 10^{-23}$ JK^{-1}

Planck's constant $h = 6.63 \times 10^{-34}$ Js

Velocity of light $c = 3 \times 10^8$ ms^{-1}

Solution

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/K_B T}$$

But $E_2 - E_1 = h\gamma = \dfrac{h_c}{\lambda}$

$$\frac{N_2}{N_1} = e^{-h_c/\lambda K_B T}$$

$$10^{-30} = e^{-\left[\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.38 \times 10^{-23} \times 300}\right]}$$

Taking log on both sides

$$-30 \log_e (10) = -\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.38 \times 10^{-23} \times 300}$$

$$-30 \times 2.302 \times 1 = -\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 1.38 \times 10^{-23} \times 300}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{30 \times 2.302 \times 1.38 \times 10^{-23} \times 300}$$

$$= 69.5 \ \mu\text{m}$$

4. Calculate the number of photons emitted by a ruby laser of output power 1 W. The lasing frequency of the ruby laser is 694.3 nm.

Given data   Power output, $P = 1$ W

Frequency of laser $\lambda = 694.3$ μm

$$= 694.3 \times 10^{-9} \text{ m}$$

Solution   Power output $P = nh\gamma$

where 'n' is the number of photons emitted.

$$n = \frac{p}{h\gamma} = \frac{p\lambda}{hc}$$

$$= \frac{1 \times 694.3 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= 3.49 \times 10^{18}$$

MULTIPLE CHOICE QUESTIONS

1. Laser is an acronym for
    (a) Light Amplification by Spontaneous Emission of Radiation
    (b) Light Amplification by Stimulated Emission of Radiation
    (c) Light Amplification by Systematic Emission of Radiation
    (d) None of the above

2. If an electron excites from lower state to higher state then that process is called
    (a) spontaneous emission   (b) stimulated emission
    (c) absorption   (d) systematic emission

3. Laser has a high degree of
    (a) monochromacity   (b) coherence
    (c) intensity   (d) All of these

4. The ratio of Einstein's coefficients $\dfrac{A_{21}}{B_{21}} =$

    (a) $\dfrac{8\pi h\upsilon^3}{c^3}$ (b) $\dfrac{8\pi h\upsilon^3}{c^2}$

    (c) $\dfrac{8\pi h\upsilon^3}{c}$ (d) $\dfrac{2\pi h\upsilon^3}{c^3}$

5. The active element in a ruby laser is
    (a) oxygen   (b) aluminium
    (c) chromium   (d) None of these

6. The pumping process used in a ruby laser is
    (a) electric discharge   (b) optical pumping
    (c) passing forward bias   (d) chemical reactions

7. A ruby laser emits light of wavelength
   - (a) 6328 Å
   - (b) 6943 Å
   - (c) 8370 Å
   - (d) 8628 Å

8. If the ruby rod contains 0.05% of chromium atoms, then it appears in _____ colour.
   - (a) yellow
   - (b) pink
   - (c) green
   - (d) red

9. In conventional light sources, the ratio of spontaneous emission rate to stimulated emission rate is nearly
   - (a) 10^3
   - (b) 10^5
   - (c) 10^{10}
   - (d) 10^{20}

10. The gas lasers give _____ coherent beam compared to semiconductor laser.
    - (a) equal
    - (b) less
    - (c) more
    - (d) none

11. In He–Ne laser, the ratio of He and Ne in gas mixture is
    - (a) 1:10
    - (b) 10:1
    - (c) 20:1
    - (d) 1:20

12. Which colour of light comes out from a He–Ne laser?
    - (a) Blue
    - (b) Green
    - (c) Red
    - (d) White

13. The pumping process used in a He–Ne gas laser is
    - (a) optical pumping
    - (b) electric discharge
    - (c) chemical reactions
    - (d) passing forward bias

14. Emission of laser radiation is observed in——type of semiconductor.
    - (a) band gap
    - (b) direct band gap
    - (c) indirect band gap
    - (d) linear band gap

15. Under population inversion, the number of atoms in the higher energy state is compared to the lower energy state is
    - (a) equal
    - (b) less
    - (c) greater
    - (d) zero

16. In excited state, the atoms will remain for a time period of
    - (a) 10^{-4} sec
    - (b) 10^{-6} sec
    - (c) 10^{-8} sec
    - (d) 10^{-10} sec

17. He–Ne laser gives—coloured laser light.
    - (a) red
    - (b) green
    - (c) blue
    - (d) violet

18. The population of the various energy levels of a system in thermal equilibrium is given by
    - (a) Einstein's relation
    - (b) Planck's law
    - (c) Boltzmann law
    - (d) Beer's law

19. Ruby laser was invented by
    - (a) Ali Javan
    - (b) T Maiman
    - (c) Einstein
    - (d) C K N Patel

20. The wavelengths of the laser emitted by He–Ne laser is
    - (a) 694.3 nm
    - (b) 632.8 nm
    - (c) 652.5 nm
    - (d) 671.6 nm

21. The He–Ne laser was invented by
    - (a) Einstein
    - (b) T. Maiman
    - (c) Ali Javan
    - (d) C K N Patel

22. Emission of photon when an electron jumps from higher energy state to lower energy state due to interaction with a photon is known as
    - (a) spontaneons emersion
    - (b) stimulated emission
    - (c) amplified emission
    - (d) induced emission

23. A He–Ne laser emits light of wavelength 632.8 nm and has an output power of 2.3 mw. Then the number of photons emitted per second is
    - (a) 73.3×10^{14}
    - (b) 43.4×10^{14}
    - (c) 25.5×10^{14}
    - (d) 65.4×10^{14}

24. In an He–Ne laser, atoms involved in laser emission are
    - (a) neon
    - (b) helium
    - (c) hydrogen
    - (d) chlorine

25. Rate of stimulated emission is proportional to
    - (a) population of lower energy state
    - (b) population of excited state
    - (c) incident radiation density
    - (d) population of excited state and incident radiation density

26. Emission of photon when an electron jumps from higher energy state to lower energy state by itself is known as
    - (a) induced emission
    - (b) stimulated emission
    - (c) spontaneons emission
    - (d) stimulated absorption

27. Any laser device consists of
    - (a) active medium
    - (b) source of energy
    - (c) continuous band gap
    - (d) discontinuous band gap

28. In ruby laser the material percentage of chromium ions in aluminium oxide is
    - (a) 0.005
    - (b) 0.5
    - (c) 0.05
    - (d) 5

29. Measurement of variation of divergence of laser beam with distance is used to determine.
    - (a) directionality
    - (b) coherence
    - (c) monochromativity
    - (d) brightness

30. Coherence of light is measured from
    (a) variation in spot size with distance
    (b) visibility of interference fringes it produces
    (c) brightness of the beam
    (d) wavelength of the beam

31. According to Boltzmann distribution law
    (a) $N_i = g_i N_o \exp\left(\dfrac{KT}{Ei}\right)$
    (b) $N_i = g_i N_o \exp\left(\dfrac{Ei}{KT}\right)$
    (c) $N_i = \dfrac{N_o}{g_i} \exp\left(\dfrac{-Ei}{KT}\right)$
    (d) $N_i = g_i N_o \exp\left(\dfrac{-Ei}{KT}\right)$

32. Choose the correct answer.
    Under population inversion condition intensity of light passing through the medium.
    (a) increases
    (b) decreases
    (c) remains the same
    (d) first decrease and then increases

33. Optical feedback in a laser is to
    (a) change the wavelength of output emission
    (b) increase the lift time of emission
    (c) effectively increase the length of the active medium
    (d) None of the above

34. Population inversion cannot be achieved by
    (a) optical pumping
    (b) chemical reaction
    (c) electric discharge
    (d) thermal process

35. He–Ne gas laser is
    (a) solid state laser
    (b) semiconductor laser
    (c) continuous laser
    (d) pulsed laser

36. In computer printer _____ laser is used.
    (a) Ruby
    (b) semiconductor
    (c) He–Ne
    (d) None of these

37. A direct conversion of electrical energy into radiation occurs in
    (a) LEDs
    (b) Ruby laser
    (c) He-Ne laser
    (d) GAS laser

38. The lasing action is possible only if there is
    (a) a black body
    (b) population inversion
    (c) a set of reflecting mirrors
    (d) oscillation of laser source

39. The source of excitation in ruby laser is
    (a) mercury vapour lamp
    (b) xenon flash lamp
    (c) sodium vapour lamp
    (d) incandescent lamp

40. The units of planks constant is
    - (a) Joule-sec^{-1}
    - (b) m-sec
    - (c) watts
    - (d) Joule-sec

41. The source of excitation in the He–Ne gas laser is
    - (a) optical pumping
    - (b) xenon flash lamp
    - (c) electric discharge
    - (d) direction conversion

42. Which of the following is coherent?
    - (a) stimulated emission
    - (b) spontaneous emission
    - (c) Both (a) and (b)
    - (d) None of these

43. Laser action is found in which of the following semiconductors?
    - (a) direct band gap
    - (b) indirect band gap
    - (c) germanium
    - (d) silicon

Answers

1. (b)	2. (c)	3. (d)	4. (a)	5. (c)	6. (b)
7. (b)	8. (b)	9. (c)	10. (c)	11. (b)	12. (c)
13. (b)	14. (b)	15. (c)	16. (c)	17. (a)	18. (c)
19. (b)	20. (b)	21. (c)	22. (b)	23. (a)	24. (a)
25. (d)	26. (c)	27. (d)	28. (c)	29. (a)	30. (b)
31. (d)	32. (a)	33. (c)	34. (d)	35. (c)	36. (b)
37. (a)	38. (b)	39. (b)	40. (d)	41. (d)	42. (c)
43. (a)					

Fill in the Blanks

1. "Laser" is an acronym for light amplification by _____ emission of radiation.
2. In 1954, Charls H. Towers and his group operated microwave device called _____.
3. In 1960, Maiman achieved first laser action in _____.
4. The law governing the distribution of atoms at various energy levels of a system is called _____.
5. When the population of the excited state is larger than the population of the lower state, the condition is called _____.
6. Ruby laser is the best example for a _____ level system.
7. He–Ne laser is a good example for a _____ level system.
8. The mechanism applied to create population inversion in semiconduction laser is _____ through junction.
9. Resonator mirrors in a laser provide optical _____ to the photons.

10. A predictable correlation of the amplitude and phase at any one point with any other point is called _____.

11. The maximum length of a wave train on which any two points can be correlated is called _____.

12. Wavelength of emission of ruby laser is _____.

13. Wavelength of emission of He-Ne laser is _____.

14. In Ruby laser, the emission is from _____.

15. In He–Ne laser, the emission is from _____.

16. Laser radiation is highly _____.

17. In He–Ne laser, He : Ne ratio is _____.

18. Ruby laser output is _____ in colour.

Answers to Fill in the blanks

1. Stimulated
2. MASER
3. ruby
4. Boltzman distribution law
5. population inversion
6. Three
7. Four
8. injection current
9. feed back
10. coherence
11. coherent length
12. 694.3 nm
13. 632.8 nm
14. chromium ions
15. Neon atoms
16. Coherent
17. 10 : 1
18. red

True or False

1. Three level laser systems work in pulsed mode only. (T/F)
2. Four level laser systems work always in continuous wave mode. (T/F)
3. Gas lasers are pumped with flash lamp. (T/F)
4. In solid state lasers optical pumping is done. (T/F)
5. He–Ne laser is the best example for creation of population inversion by chemical reaction. (T/F)
6. Typical concentration of Cr^{3+} ions in Ruby laser material is around 5% by weight. (T/F)
7. Ruby laser output consists of a large number of spikes. (T/F)
8. By modulating the current through the junction, the output of semiconductor laser can be modulated. (T/F)
9. Laser surgery is called blood less surgery since the blood is vaporised. (T/F)

Answers to True or False

1. T   2. F   3. F   4. T   5. F   6. F
7. T   8. T   9. F

REVIEW QUESTIONS

Short Answer Questions

1. Why is laser light highly monochromatic?
2. Why is laser light highly directional?
3. Why is laser light highly coherent?
4. Why is laser light of high intensity?
5. What is stimulated absorption of radiation?
6. What is spontaneous emission of radiation?
7. What is simulated emission of radiation?
8. Mention the relation between Einstein's coefficients.
9. What is population inversion?
10. What is optical pumping excitation mechanism?
11. What is chemical reaction excitation?
12. What is electric discharge excitation?
13. Mention the drawbacks of Ruby laser.
14. Mention the advantages of He–Ne laser.
15. What is an optical resonator?
16. Mention the important components of laser device.

Essay Type Questions

1. Explain the characteristics of laser lights.
2. Explain the spontaneous stimulated emission of radiation.
3. Derive the relation between various Einstein's coefficients.
4. Describe population inversion mechanics in the emission of laser radiation.
5. Explain various excitation mechanisms.
6. Describe optical resonator.
7. Explain the construction and working of Ruby.
8. Explain the construction He–Ne laser.
9. Mention the applications of laser.

CHAPTER 4

FIBRE OPTICS

INTRODUCTION

Generally, communication is transferred through carrier waves in any communication system. When the frequency of the carrier waves are high then the information carrying capacity also enhances. As the propagation of light takes place in the form of high frequency waves, these light waves can be used to carry information, i.e., as carrier waves. For the proper guiding of information carrying light waves, we need a proper guiding medium or material. That material is the optical fibre.

4.1   OPTICAL FIBRE

Optical fibre is a guiding medium or material which guides the information carrying light waves. To guide the light waves, optical fibre should be transparent. To minimise the transmission losses through the optical fibre it is made thin. Thus *"optical fibre is a thin and transparent guiding medium or material which guides the information carrying light waves"*.

John Tyndall observed that the propagation of light through the optical fibre will be in the form of multiple total internal reflections. Even in the bended optical fibre also, it is in the form of multiple total internal reflections. As the information carrying capacity of light waves are high, a single optical fibre can carry 140 M bytes of information up to 220 km in one sec.

4.2   WORKING PRINCIPLE OF OPTICAL FIBRE

Optical fibre consists of inner most layer known as core, a denser medium and next layer is known as cladding a rarer medium.

Consider an optical fibre consisting of a core and cladding of refractive indices n_1 and n_2 respectively ($n_1 > n_2$). Let a light ray move from core to cladding medium with 'i' as the angle of incidence and 'r' as the angle of refraction. The refracted ray bends away from the normal as it travels from core to cladding with increase of angle of incidence. In this we get three cases:
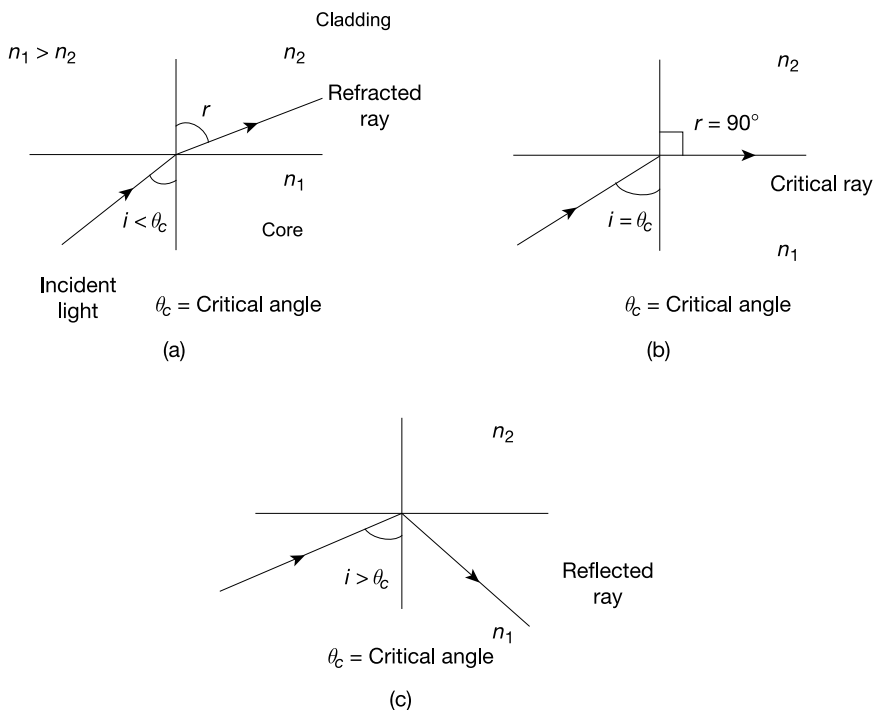
Figure 4.1   (a) Normal refraction (b) Critical refraction (c) Total internal reflection

Case 1   When $i < \theta_c$, the light ray refracts into core [Fig. 4.1(a)].

Case 2   When $i = \theta_c$ then the ray traverses along the interface of the core and cladding, θ_c is known as critical angle and the refracted ray as critical ray [Fig. 4.1(b)].

Case 3   When $i > \theta_c$, then the light ray will be reflected back into the core and undergoes total internal reflection.

For core and cladding, applying Snell's law, we get

$$n_1 \sin i = n_2 \sin r$$

When $i = \theta_c$ = critical angle then $r = 90°$

Therefore, $n_1 \sin \theta_c = n_2 \sin 90°$

$$n_1 \sin \theta_c = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1} \quad \text{where } n_1 > n_2$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

4.2.1 Conditions for the Propagation of Light Through Optical Fibre

1. When the incident light makes an angle greater than the critical angle at core cladding interface at the top of the fibre it will be reflected down into core due to total internal reflection.

2. When the reflected light when it strikes the core cladding interface at the bottom of the fibre again, it will be reflected up due to total internal reflection.

3. Thus the propagation of light will be in the form of multiple total internal reflection through the fibre.

4.3 FIBRE STRUCTURE AND CONSTRUCTION

An optical fibre is a very thin, flexible thread of transparent plastic or glass in which light is transmitted through multiple, total internal reflection. It consists of a central cylindrical layer known as *core* surrounded by a second layer called the *cladding.* These layers are known as optical fibre. Light is transmitted within the core. The cladding keeps the light within the core because the refractive index of the cladding is less than that of the core, i.e., core acts as denser medium and cladding as rarer medium. Silicon coating is provided between buffer jacket and cladding in order to improve the quality of transmission of light. The buffer jacket protects the fibre from moisture and abrasion. To provide necessary toughness and tensile strength, a layer of strength member is arranged surrounding the buffer jacket. Finally, it is covered with a black polyurethane outer jacket (Fig. 4.2).
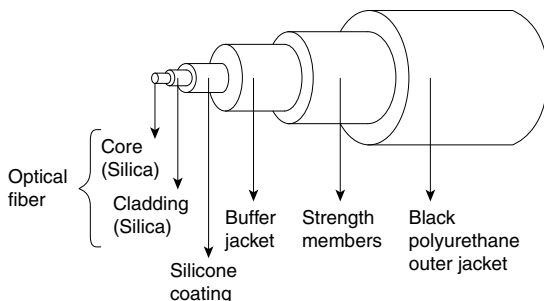
Figure 4.2 Typical construction of an optical fibre

The core as well as cladding is made of either glass or plastic. Thus, we have two types of optical fibres:

1. Glass fibre
2. Plastic fibre

4.3.1   Glass Fibre

If the optical fibres are made by fusing mixtures of metal oxides and silica glasses, then it is known as *glass fibre*. The resulting material is randomly connected by molecular network rather than well-defined ordered structures as found in crystalline materials.

The most common material used in glass fibre is silica (oxide glasses). It has a refractive index of 1.458 at 850 nm. To produce two similar materials having slightly different indices of refraction for the core and cladding, either fluorine or various oxides such as B_2O_3, GeO_2 or P_2O_5 are added to silica.

Examples of fibre compositions:

1. GeO_2–SiO_2 core; SiO_2 cladding
2. P_2O_5–SiO_2 core; SiO_2 cladding
3. SiO_2 core; P_2O_5–SiO_2 cladding

Another type of silica glasses are the low-melting silicates. Such optical fibres are made of soda-lime silicates, germanosilicates and various borosilicates.

4.3.2   Plastic Fibre

The plastic fibres are typically made of plastics, are of low cost and can be handled without special care due to their toughness and durability.

Examples of plastic fibres are as follows:

1. A polystyrene core ($n_1 = 1.60$) and a methylmethacrylate cladding ($n_2 = 1.49$).
2. A polymethylmethacrylate core ($n_1 = 1.49$) and a cladding made of its co-polymer ($n_2 = 1.40$).

4.3.3   Fibre Dimensions

Optical fibres are typically made in lengths of 1 km. Optical fibres can be joined with suitable connectors. Its outer diameter ranges from 0.1 to 0.15 mm. Core diameters range from 5 to 600 μm whereas cladding diameters vary from 125 to 750 μm. To keep the light within the core, the cladding must have a minimum thickness of wavelength of light. The protective jackets may add as much as 100 μm in diameter to the fibre's total diameter. The fibre dimensions are such that inside the fibre, light gets total internal reflection at core cladding interface. The propagation of light in the optical fibre in the form of multiple total internal reflection is as shown in Fig. 4.3.
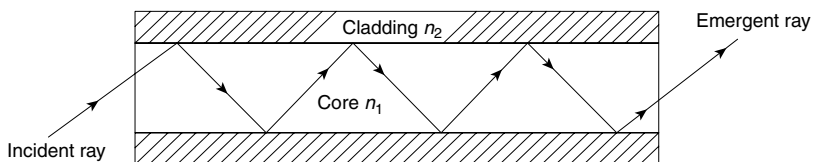
Figure 4.3   Propagation of light in optical fibre

4.3.4 Acceptance Angle and Acceptance Cone

Consider a cross-sectional view of an optical fibre having core and cladding of refractive indices n_1 and n_2 respectively such that $n_1 > n_2$. Let the fibre be in air medium (n_0). The incident light while entering into the core at A makes an incident angle of θ_i with the fibre axis. In core it travels along AB and is incident at point B on core cladding interface. Let θ_r be the angle of refraction at point 'A' and θ be the angle of incidence at 'B'. When θ is greater than the critical angle (θ_c) then total internal reflection takes place into the core and light takes the path BD. Due to multiple total internal reflection the propagation of light ray takes place through the fibre.
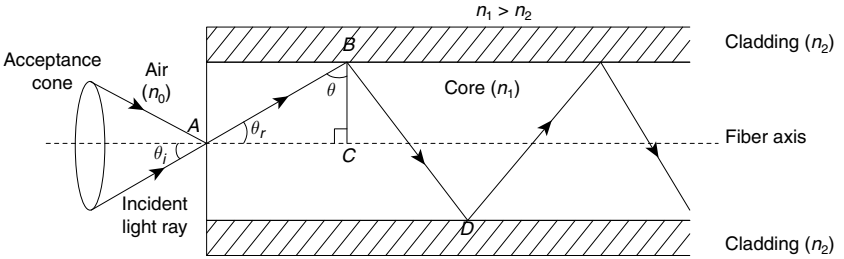
Figure 4.4   Cross-sectional view of optic fibre

Applying Snell's law at A (core-air interface)

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_1}{n_0}$$

$$n_0 \sin \theta_i = n_1 \sin \theta_r \tag{1}$$

Let a normal BC be drawn from the point B to fibre axis. Then from ΔABC, we get

$$\theta_r + \theta = 90°$$
$$\theta_r = 90° - \theta \tag{2}$$

Substituting the above value in Eq. (1),

$$n_0 \sin \theta_i = n_1 \sin (90° - \theta)$$
$$n_0 \sin \theta_i = n_1 \cos \theta \tag{3}$$

To get total internal reflection at point B (core, cladding interface) the incident angle θ should be greater than or equal to θ_c (critical angle).

Let the maximum angle of incidence at point A be θ_a for which $\theta \geq \theta_c$

From Eq. (3), we get

$$n_0 \sin \theta_a = n_1 \cos \theta_c$$

$$\sin \theta_a = \frac{n_1}{n_0} \cos \theta_c \tag{4}$$

But
$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\cos \theta_c = \sqrt{1 - \sin^2 \theta_c} = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$= \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \tag{5}$$

Substituting the value in Eq. (4),

$$\sin \theta_a = \frac{n_1}{n_0} \times \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\sin \theta_a = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \tag{6}$$

For air medium, $n_0 = 1$

∴
$$\sin \theta_a = \sqrt{n_1^2 - n_2^2} \tag{7}$$

$$\theta_a = \sin^{-1} \sqrt{n_1^2 - n_2^2} \tag{8}$$

In the above expression, θ_a is the maximum incidence angle of light at the core so that light can be guided through the fibre. This angle is known as acceptance angle.

Acceptance angle

It is the maximum angle of incidence at the core of an optical fibre so that the light can be guided through the fibre, and is given by Eq (8).

Acceptance cone

Rotating the acceptance angle about the fibre axis, we get an incident cone with semi-vertical angle as θ_a. This incident light cone at core of an optical fibre will be accepted by the fibre for guidance through it, and is known as acceptance cone.

4.3.5 Numerical Aperture

Numerical aperture represents the light gathering power of an optical fibre. Numerical aperture is proportional to acceptance angle. So, numerical aperture is equal to the sine of acceptance angle.

Numerical aperture $NA = \sin \theta_a \sqrt{n_1^2 - n_2^2}$ \hfill (9)

$$NA = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

$$NA = \sqrt{(n_1 + n_2)n_1\Delta}$$

where $\Delta = \dfrac{n_1 - n_2}{n_1} =$ Fractional change in refractive indices of core and cladding.

For all optical fibres, $n_1 \approx n_2$ so $n_1 + n_2 = 2n_1$

$$NA = \sqrt{2n_1^2\Delta} = n_1\sqrt{2\Delta} \tag{10}$$

'*NA*' can be increased by increasing Δ and thus enhancing the light gathering capacity of the fibre.

4.4   TYPES OF OPTICAL FIBRES

Based on the variation of refractive index of core of an optical fibre, fibres are classified into two types:

1. Step index optical fibre
2. Graded index optical fibre

Again based on the number of paths (modes) available for the light rays inside the core, these optical fibres are further divided into two categories:

1. Single-mode optical fibre
2. Multimode optical fibre

In single-mode optical fibre, the width or diameter of the core is smaller when compared to the width of the cladding. As a result, only a single path (mode) is available for the light ray through the fibre.

When the width of the core is greater than the cladding, then large number of paths (modes) are available for the light ray through the fibre and it is known as multimode optical fibre.

4.4.1   Step-index Optical Fibre–Refractive Index Profile

In a step-index optical fibre, the refractive index of the core remains constant throughout the core and decreases from step n_1 to n_2 at the core cladding interface. Thus, it is known as step-index optical fibre. The transmission of information will be in the form of signals or pulses. For a single-mode, step-index optical fibre, a single light ray from the signal enters into the fibre and traverses a single path and forms the output signal. In this case, two signals match with each other [Fig. 4.5].
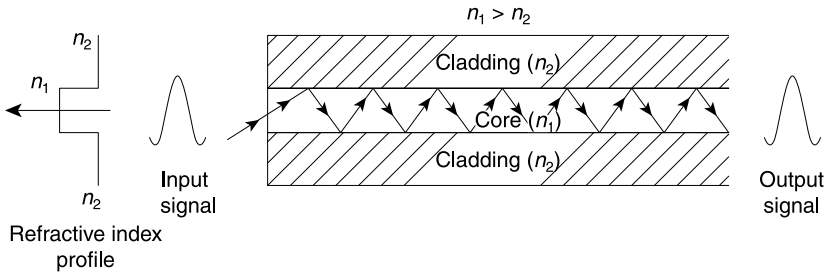
Figure 4.5   Single-mode step-index optical fibre

In a multimode step index fibre, due to large width of core, greater number of light rays from the input signals enters into the core and takes multipaths as shown in Fig. 4.5. The light ray (1) which makes greater angle with the fibre axis suffers more reflections through the fibre and takes more time to traverse the optical fibre, whereas the light ray (2) makes less angle with the axis, suffers less number of reflections and within a short time, it traverses the optical fibre. At the output end we receive ray (2) first, and later we get ray (1). Due to the path difference between the light rays when they superimpose to form the output signals, the signals are overlapped. In this, we get signal distortion known as *intermodal dispersion.* It is difficult to retrieve the information carried by the distorted output signal. In a step-index fibre, the propagation of light ray is due to multiple reflections, so it is of reflective type.
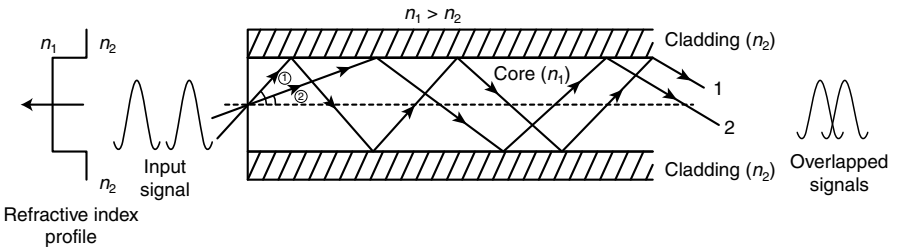
Figure 4.6   Multimode step index fibre

The number of possible propagation modes in the core is given by the *V*-number as

$$V = \frac{2\pi}{\lambda} a(NA)$$

where λ – wavelength of light

           a – radius of the core

      NA – Numerical aperture

Number of modes through step index fibre $= \dfrac{V^2}{2}$.

4.4.2 Graded Index Optical Fibre–Refractive Index Profile

In graded index optical fibre, the refractive index of the core decreases from the fibre axis to the cladding interface in a parabolic manner. When light ray enters into the core and moves towards the cladding interface, it encounters a more and more rarer medium due to decrease of refractive index. As a result, the light ray bends more away from the normal and finally bends towards the axis and moves the core-cladding interface at the bottom. Again, it bends in the upward direction. Thus, the light due to refraction takes sinusoidal paths. This fibre is of refractive type. When two light rays (1) and (2) making different angles with the axis enters into the fibre, they adjust their velocities (due to variations of refractive index) and come to focus at the same point. As a result, all the light rays will be received at the output end at the same time. There is no intermodal dispersion and the output signals match with input signal. It is easy to retrieve the information from the signals. In this fibre, we get a refocusing effect of light rays. The number of possible modes through graded index fibre $= \dfrac{V^2}{4}$ where V is the V-number of fibre.
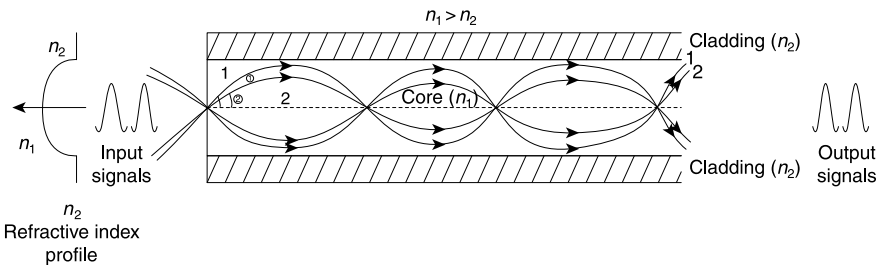
Figure 4.7 Graded index optical fibre

4.4.3 Differences between Step Index and Graded Index Optical Fibres

Step index	*Graded index*
1. Refractive index of the core remains constant and decreases a step at cladding.	1. Refractive index of the core decreases parabolically from the axis of the fibre to cladding.
2. It is of reflective type.	2. It is refractive type.
3. Signal distortion is high in multimode step index fibre.	3. Signal distortion is very low in graded index fibre.
4. NA is more for multimode step index fibre.	4. NA is less for graded index fibre.
5. No focusing of light takes place.	5. Focusing of light rays takes place.

4.5   ATTENUATION AND LOSSES IN FIBRES

When the light signal propagates in the optical fibre losses arises due to different factors and these losses are referred to attenuation in optical fibre. The various factors causing attenuation in optical fibre are:

1. Material or impurity losses
2. Scattering losses
3. Absorption losses
4. Bending losses
5. Radiation induced losses
6. Inherent defect losses
7. Inverse sequence law losses
8. Transmission losses
9. Core and cladding losses

Losses are expressed in decibels per kilometer (dB/km). The attenuation loss is given by

$$P_{out} = P_{in} \, 10^{-2L/10} \tag{11}$$

where P_{aut} = Power at a distance L from the input

P_{in} = Amount of power coupled in to the fibre

α = Fibre attenuation in dB/km and

L = Length of the optical fibre

Therefore, attenuation in the fibre is defined as the ratio of the optical power output P_{out} obtained from a fibre of length 'L' to the optical power P_{in} fed to the input of the fibre.

$\therefore$ from Eq. (11), we have

$$\alpha = \frac{10}{L} \, \log \left(\frac{P_{in}}{P_{out}} \right) \text{dB/km}$$

1. *Material or impurity losses*   The doped impurities present in the fabrication of an optical fibre in order to vary the refractive index causes losses in the light signal propagation through the fibre.

2. ***Scattering losses*** In glass fibre the glass contains many microscopic inhomogeneities and material content. Due to this, a portion of light signal passing through the glass fibre gets scattered. This scattering losses vary inversely with the fourth power of the wavelength.

3. ***Absorption losses*** Absorption loss is caused by the nature of the core material and varies inversely to the transparency of the material. For glass fibres, ion-resonance absorption, ultraviolet absorption and infrared absorption are the three separate mechanisms which contribute to total absorption losses.

4. ***Bending losses*** Whenever a fibre deviates from a straight line path, radiative losses occur. These losses are prominent for improperly installed single mode optical cable.

5. ***Radiation induced losses*** When the glass molecular matrix interacts with electrons, neutrons, X-rays and gamma rays, the structure of the glass molecules is altered and the fibre darkens. This introduces additional losses which increase with amount, type, dose and exposure time of radiation

6. ***Inherent defect losses*** The inherent defect present in core and cladding causes losses of the propagating light signal through it. The surface defect in the core causes losses in the light signal. Grease, oil and other contaminates on the surface of the fibre also causes signal losses due to variation of refractive index.

7. ***Inverse sequence law losses*** In all light systems, there is the possibility of losses caused by divergence of the beam. The illuminance per unit area is inversely proportional to the square of the distance $\left(\dfrac{1}{r^2} \right)$.

8. ***Transmission losses*** These losses are caused by light which is caught in the cladding material of optical fibres. This light is either lost to the outside, or is trapped in the cladding layer and is thus not available for propagation in the core of the fibre.

9. ***Core and cladding losses*** In a fibre core and cladding have different retractive indices, as they have different compositions. So the core and the cladding have different attenuation coefficients, causing the power losses to the fibre.

4.6 FIBRE OPTIC COMMUNICATION SYSTEM

A fundamental digital fibre optical communication system is shown in Fig. 4.8.
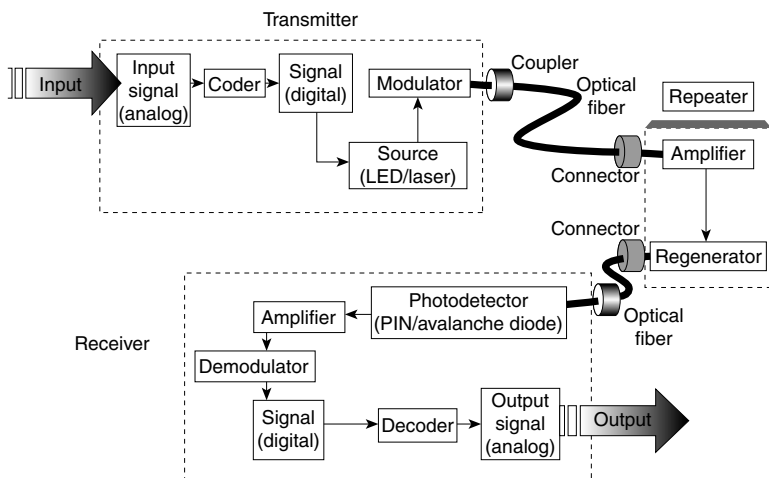
Figure 4.8   Fibre optic communication system

Fibre optic communication system consists of three important components: They are

1. Optical transmitter
2. Fibre repeater
3. Optical receiver

4.6.1   Optical Transmitter

An optical transmitter converts an analogue or digital signal into optical form. It consists of an encoder, light source and modulator. The input analogue signal is converted into a digital signal by means of an encoder. The converted digital signal is fed to the source. The source can be a light emitting diode (LED) or a semiconductor laser diode. The optical carrier wave from the source is modulated based on intensity, amplitude or frequency with the help of a modulator. This optical signal is coupled to the optical fibre by means of couplers. The couplers launch the optical signal into the fibre without any distortion and loss. The optical signal through fibre is properly connected to a repeater with the help of a connector.

4.6.2   Fibre Repeater

The optical signal while travelling through very long optical fibres through long distances can suffer transmission losses and fibre losses like dispersion. As a result, we get a weak optical signal at the output end of the fibre. To minimise the losses, we use fibre repeaters at regular intervals between the fibres. The repeater

consists of an amplifier and regenerator. The amplifier amplifies the weak optical signal, it is reconstructed to original optical signal with the help of regenerator and it is transmitted through the optical fibre. At the last stage, it is received by optical receiver.

4.6.3   Optical Receiver

The receiver unit consists of a photodetector, amplifier, demodulator and decoder. The photodetector consists of PIN photo diode or avalanche photo diode. This works on the principle of creation of an electron-hole pair at the *p-n* junction by successive collisions of the incident optical signal (photons). The released electrons output a current which is in direct relationship with the incident optical signal. This electric current (signal) is then amplified and demodulated to obtain a digital signal. This signal is then decoded and the transmitted signal is outputted.

4.7   ADVANTAGES OF OPTICAL FIBRES IN COMMUNICATION

1. *Extremely wide band width*   The rate at which information can be transmitted is directly related to signal frequency. Light has very high frequency in the range of 10^{14} to 10^{15}, Hz (wide bandwidth) so the optical signal can transmit information at a higher rate.

2. *Smaller diameter, lighter weight cables*   Optical fibres due to light weight and flexibility can be handled easily than copper cables.

3. *Lack of cross talk between parallel fibres*   In ordinary cables, signals often stray from one another resulting in cross talk. But in optical fibres this is negligible.

4. *Immunity to inductive interference*   Fibre cables are immune to interference caused by lightning and other equipments.

6. *Potential of delivering signals at low cost*

7. *Much safer than copper cables*

8. *Longer life span*

9. *High temperature resistance*

10. *Optical fibres are more reliable and easy to maintain than copper cables.*

4.8   APPLICATIONS OF OPTICAL FIBRES IN MEDICINE

Optical fibres were extensively used in medical field.

1. Fibre scope in endoscopy is one of the widely used optical technique to view the internal parts of the disease affected body. In this optical fibres

plays a major role in visualisation of internal portions of human body but also in the selective cauterisation of tissues, using laser beam.

The basic principle in this fibre endoscope is the light transmitted though the outer fibres falls on the inner portions of the body. The reflected light from the inner portion is carried by the inner fibres to the observer. The flexible fibrescope is as shown in Fig. 4.9. The light from the source is carried by the outer fibres to illuminate the portion of the body to be viewed. The reflected light from the same will be carried by the inner fibres to eye to observe the image of that portion.
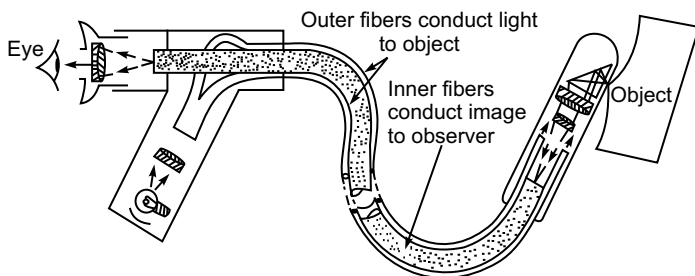
Figure 4.9   Flexible fibrescope using optical fibres

2. This technique is widely used for the diagnoses of interior of lungs, stomach and other human body parts.
3. This method is used for the examination of gastrointestinal tract for diagnoses of ulcers, cancers, etc.
4. Optical fibres are used in photodynamic therapy for cancer.
5. They are used in the treatment of lung disorders.
6. They are used in the treatment of bleeding ulcers.
7. They are used in arthroscopic surgery for damaged cartilage, ligaments and tendons in major joints such as knees and shoulders.
8. They are used in the investigation of heart, respiratory system and pancreas.

4.9   APPLICATIONS OF OPTICAL FIBRES IN SENSORS

Optical fibres were widely used in sensors for sensing and measuring of acoustic fields, magnetic fields, currents, acceleration, strain, pressure, temperature, rotation, etc. Rotation sensing can be done with the help of fibre optic gysoscope.

A fibre optic sensor consists of a light source which generate light signals. These signals passes through the optical fibre placed in the sensing fields and then passes through the light detector. The variation in the light signal is caused by the sensing field and is detected by the detector as shown in the Fig. 4.10
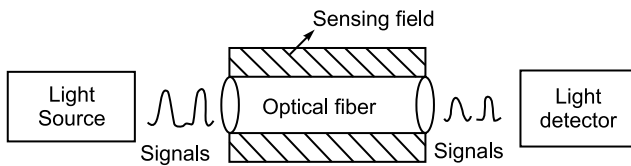
Figure 4.10 Fibre optic sensor

The optical fibre may be of single mode or multimode type.

1. Acoustic sensor   As shown in Fig. 4.11, when there is no sound signals the light gets transmitted into the movable fibre through the fixed fibre and is detected in the detector. In the pressure of sound signal the movable fibre vibrates due to which there will be misalignment of the fibres. This leads to coupling loss which inturn changes the intensity of transmitted light through the moving fibre which in turn is detected by the detector. By knowing the variation in light intensity one can sense the presence of sound signal.
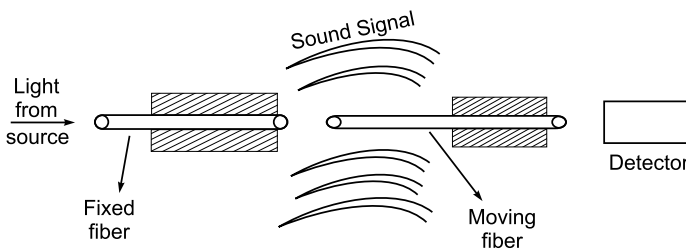
Figure 4.11

2. Displacement sensor   The light from the source passes through the upper fibre and falls on a reflective surface at a distance *d* from it. The reflected light is carried by the bottom fibre to detector to detect its intensity. When the
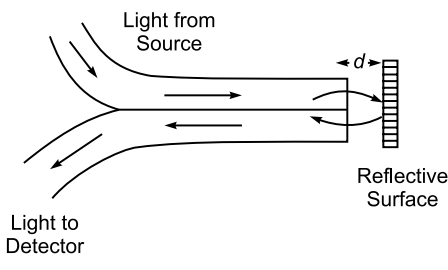
Figure 4.12   Displacement sensor
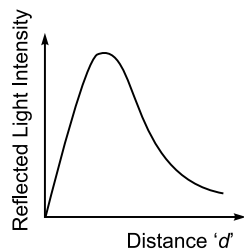
Figure 4.13   Reflected intensity verses '*d*'

displacement (*d*) of the reflective surface varies them it causes variation in the reflected light intensity as shown in Fig. 4.12. The variation is linear up to certain displacements after that is nonlinear. Generally, sensor can be used for measuring the displacement in linear region.

3. Temperature sensor   The light from the sources passes through the upper fibre and then through silicon layer having reflective coating at the end placed in the temperature field to be sensed. The reflected light after passing through the bottom fibre is detected by the detector. When light passes through the silicon layer it absorbs certain light and hence the reflected light intensity varies. The amount of light absorbed by silicon varies with temperature which in turn modifies the intensity of reflected light. Using this technique, temperature less than 0.001°C can be sensed.
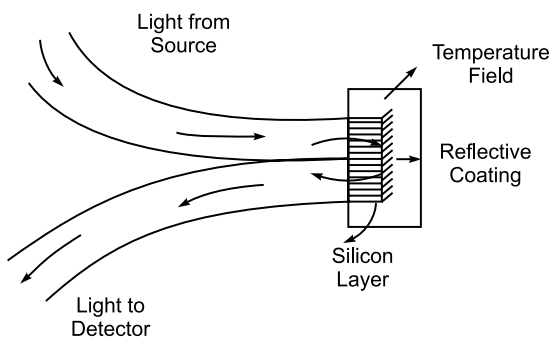
Figure 4.14   Temperature sensor

4. Pressure sensor   When the pressure field is present on the fibre, then the distortion in the fibre occurs due to which bending loss of the fibre takes place. This loss in turn modifies the received light intensity by the detector. Thus pressure can be sensed with the change in intensity of light in the detector.
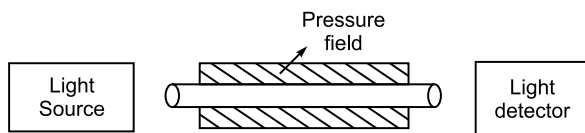
Figure 4.15

Applications of Optical Fibre in Communication

1. Optical fibres are extensively used in optical communication systems.
2. Nearly 10,000 information carries signals can be transmitted simultaneously through the optical fibre.

3. Due to higher band widths optical fibre carries more information.

4. As the optical fibre is highly immune to temperature, moisture, etc., without any environmental effects the information can be delivered.

5. Without any crosstalk the information can be safely delivered.

6. During war time they are used for secret communication.

7. They are used for guiding weapons and submarine communication systems.

SOLVED PROBLEMS

1. As optical fibre has a core material of refractive index 1.55 and cladding material of refractive index 1.50. The light is launched into it in air. Calculate its numerical aperture.

Given data   Refractive index of core, $n_1 = 1.55$

Refractive index of cladding, $n_2 = 1.50$

Solution   Numerical aperture of the optical fibre,

$$NA = \sqrt{n_1^2 - n_2^2}$$

$\Rightarrow$ $$\sqrt{1.55^2 - 1.50^2}$$

$$\sqrt{2.40 - 2.25} = \sqrt{0.15} = 0.39$$

2. Calculate the angle of acceptance of a given optical fibre, if the refractive indices of the core and cladding are 1.563 and 1.498, respectively.

Given data   Refractive index of core, $n_1 = 1.563$

Refractive index of cladding, $n_2 = 1.498$

Solution

Acceptance angle $\theta_a = \sin^{-1}(NA)$

Where NA is numerical aperature

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.563^2 - 1.498^2} = \sqrt{0.1989} = 0.446$$

Acceptance angle, $\theta_a = \sin^{-1}(0.446)$

$$= 26° 29'$$

3. Calculate the fractional index change for a given optical fibre if the refractive indices of the core and cladding are 1.563 and 1.498 respectively.

Given data   Refractive index of the core, $n_1 = 1.563$

Refractive index of the cladding, $n_2 = 1.498$

Solution   The fractional refractive indices change,

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\Delta = \frac{1.563 - 1.498}{1.563} = \frac{0.065}{1.563}$$

$$= 0.0416$$

4. The refractive indices of core and cladding materials of a step index fibre are 1.48 and 1.45, respectively. Calculate (i) numerical aperature, (ii) acceptance angle, (iii) the critical angle at the core–cladding interface, and (iv) fractional refractive indices change.

Given data   Refractive index of the core, $n_1 = 1.48$

                     Refractive index of the cladding, $n_2 = 1.45$

Solution

(i) Numerical aperture

$$(NA) = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{1.48^2 - 1.45^2}$$

$$= \sqrt{2.1904 - 2.1025}$$

$$= \sqrt{0.0879} = 0.2965$$

(ii) Acceptance angle (θ_a)

$$= \sin^{-1} NA$$

$$= \sin^{-1}(0.2965) = 17°14'$$

(iii) Critical angle $= \theta_c$

$$= \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$= \sin^{-1}\left(\frac{1.45}{1.48}\right)$$

$$= \sin^{-1}(0.9797)$$

$$= 78°26'$$

(iv) The fractional refractive indices change,

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$= \frac{1.48 - 1.45}{1.48}$$

$$= \frac{0.03}{1.48} = 0.02$$

5. The numerical aperture of an optical fibre is 0.39. If the difference in the refractive indices of the material of its core and the cladding is 0.05, calculate the refractive index of material of the core.

Given data   Numerical aperture, NA = 0.39

The difference in the refractive indices, $n_1 - n_2 = 0.05$ (i)

Solution

$$\text{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 - n_2)\,(n_1 + n_2)}$$

$$0.39 = \sqrt{0.05 \times (n_1 + n_2)}$$

$\Rightarrow$ $n_1 + n_2 = \dfrac{(0.39)^2}{0.05} = \dfrac{0.1521}{0.05} = 3.042$

$\Rightarrow$ $n_1 + n_2 = 3.042$ (ii)

Adding Eqs. (i) and (ii)

$$n_1 - n_2 = 0.05$$
$$n_1 + n_2 = 3.042$$
$$\overline{}$$
$$2n_1 = 3.092$$

$$n_1 = \frac{3.092}{2} = 1.546$$

∴ the refractive index of the core is 1.546.

6. Calculate the refractive indices of core and cladding of an optical fibre with a numerical aperture of 0.33 and their fractional difference of refractive indices being 0.02.

Given data   Numerical aperture NA = 0.33

Fractional difference of refractive index, $\Delta = 0.02$

Solution   ∴ $\Delta = \dfrac{n_1 - n_2}{n_1} = 0.02$

$$\text{NA} = n_1\sqrt{2\Delta}$$

$$n_1 = \frac{\text{NA}}{\sqrt{2\Delta}} = \frac{0.22}{\sqrt{2 \times 0.02}} = 1.1$$

From the above equation,

$$0.02 = \frac{1.1 - n_2}{1.1} \quad \text{or} \quad n_2 = 1.1 - 0.02 = 1.078$$

7. An optical fibre has a numerical aperture of 0.20 and a cladding refractive index of 1.59. Find the acceptance angle for the fibre in water which has a refractive index of 1.33.

Given data   Numerical aperture of the fibre, NA = 0.20

Refractive index of cladding, $n_2 = 1.59$

Refractive index of water, $n_0 = 1.33$

Solution

$$NA = \sqrt{n_1^2 - n_2^2}$$

$\Rightarrow$ $(0.20) = \sqrt{n_1^2 - (1.59)^2}$

$\Rightarrow$ $(0.20)^2 = n_1^2 - (1.59)^2$

$\Rightarrow$ $n_1^2 = (0.20)^2 + (1.59)^2$

$$= 0.04 + 2.53 = 2.57$$

$$n_1 = \sqrt{2.57} = 1.6031$$

$$\sin \theta_a = \sqrt{\frac{n_1^2 - n_2^2}{n_0}}$$

$$\theta_a = \sin^{-1} = \left(\sqrt{\frac{n_1^2 - n_2^2}{n_0}} \right)$$

$\Rightarrow$ $\theta_a = \sin^{-1} \left(\sqrt{\frac{(1.6031)^2 - (1.59)^2}{1.33}} \right)$

$$= \sin^{-1} \left(\sqrt{\frac{2.5699 - 2.5281}{1.33}} \right)$$

$$= \sin^{-1} \left(\sqrt{\frac{0.0418}{1.33}} \right)$$

$$= \sin^{-1} \left(\frac{0.2045}{1.33} \right)$$

$$= \sin^{-1} (0.1538)$$

$$\theta_a = 8°50'$$

8. An optical fibre has a diameter of 60 μm and a numerical aperture of 0.45. Calculate the number of modes for an operating wavelength of 10 μm.

Given data   Diameter of fibre, $d = 60\ \mu$m

$$\text{Radius of fibre, } a = \frac{d}{2} = 30 \ \mu\text{m}$$

$$= 30 \times 10^{-6} \text{ m}$$

$$\text{Numerical aperture, NA} = 0.45$$

$$\text{Wavelength } \lambda = 10 \ \mu\text{m}$$

$$= 10 \times 10^{-6} \text{ m.}$$

Solution Number of possible modes through the fibre is given by

$$V = \frac{2\pi}{\lambda} \, a(\text{NA})$$

$$= \frac{2\pi}{10 \times 10^{-6}} \times 30 \times 10^{-6} \times 0.45$$

$$= 9.21$$

Number of possible modes = 9

9. The refractive indices of the core and cladding of a fibre are 1.48 and 1.45 respectively. The diameter of the fibre is 60 μm. Calculate (a) numerical aperture (b) number of possible modes at an operating wavelength of 1.5 μm. (c) Velocity of light rays in the core and cladding of the fibre.

Given data Refractive index of core, $n_1 = 1.48$

$$\text{Refractive index of cladding, } n_2 = 1.45$$

$$\text{Diameter of the fibre, } d = 60 \ \mu\text{m}$$

$$\text{Radius of the fibre, } a = \frac{d}{2} = 30 \ \mu\text{m}$$

$$= 30 \times 10^{-6} \text{ m}$$

$$\text{Wavelength light } \lambda = 1.5 \ \mu\text{m}$$

$$= 1.5 \times 10^{-6} \text{ m}$$

Solution (a) Numerical aperture,

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{(1.48)^2 - (1.45)^2}$$

$$= 0.2964$$

(b) Number of possible modes,

$$V = \frac{2\pi}{\lambda} \, a(\text{NA})$$

$$= \frac{2\lambda}{1.5 \times 10^{-6}} \times 30 \times 10^{-6} \times 0.2964$$

$$= 37.25$$

$$= 37$$

(c) Velocity of light in the core

$$= \frac{\text{Velocity of light in air}}{\text{Refractive index of core}}$$

$$= \frac{3 \times 10^{-8}}{1.48} = 2.02 \times 10^{-8} \text{ ms}^{-1}$$

Velocity of light in the cladding

$$= \frac{\text{Velocity of light in air}}{\text{Refractive index of cladding}}$$

$$= \frac{3 \times 10^{-8}}{1.45}$$

$$= 2.06 \times 10^{-8} \text{ ms}^{-1}$$

10. An optical fibre of 1 mW is guided into optical fibre of length 100 m. If the output power at the other end is 0.3 mW, calculate the fibre attenuation.

Given data   Input power P_{in} = 1 mW
Output power P_{out} = 0.3 mW
Length of the fibre L = 100 m
= 0.1 km

Solution   Fibre attenuation

$$\alpha = \frac{10}{L} \ \log \ \left(\frac{P_{in}}{P_{out}} \right)$$

$$= \frac{10}{0.1} \ \log \ \left(\frac{1}{0.3} \right) = 52.29 \text{ dB/km}$$

11. The optical power launched into an optical fibre is 1 mW. The fibre has attenuation of 0.5 dB/km. If the power output is 0.5 mW, then calculate the fibre length.

Given data

Input power P_{in} = 1 mW
Output power P_{out} = 0.5 mW
Attenuation α = 0.5 dB/km

Solution $$\alpha = \frac{10}{L} \ \log \ \left(\frac{P_{in}}{P_{out}} \right)$$

$$0.5 = \frac{10}{L} \ \log \ \left(\frac{1}{0.5}\right)$$

$$L = \frac{10}{0.5} \ \log \ \left(\frac{1}{0.5}\right) = 13.8629 \ m$$

12. A multimode step index fibre with refractive indices of core and cladding as 1.53 and 1.50 respectively. If the wavelength of light signal is 1 µm, and the radius of core is 50 µm, then find the number of possible modes through it.

Given data   Refractive index of core n_1 = 1.53

Refractive index of cladding n_2 = 1.50

Wave length λ = 1 µm = 1×10^{-6} m

Radius of case a = 50 µm = 50×10^{-6} m.

Solution

$$V = \frac{2\pi}{\lambda} \ a(NA)$$

$$= \frac{2\pi a \sqrt{n_2^1 - n_2^2}}{\lambda}$$

$$= \frac{2 \times 3.14 \times 50 \times 10^{-6} \times \sqrt{(1.53)^2 - (1.50)^2}}{1 \times 10^{-6}}$$

$$= 94.72$$

Possible number of modes = $\dfrac{V^2}{2}$ = 4486

MULTIPLE CHOICE QUESTIONS

1. In a fibre, light travels in
   - (a) core medium
   - (b) air medium
   - (c) cladding medium
   - (d) buffer medium

2. The difference of refractive indices of core and cladding is
   - (a) less
   - (b) more
   - (c) zero
   - (d) negative

3. The refractive index of the core is
   - (a) equal to that of cladding
   - (b) Less than the cladding
   - (c) greater than the cladding
   - (d) None of these

4. Optical fibre can be satisfactorily operated if, refractive index of
   - (a) the core is lesser than that of cladding

    (b) the core is greater than that of cladding

    (c) the core is equal to that of cladding

    (d) None of the above

5. At critical angle of incidence

    (a) light ray in air is parallel to the glass surface

    (b) light ray in air is perpendicular to the glass surface

    (c) there is no such relation

    (d) None of the above

6. If the angle of incidence of a ray is equal to the critical angle at the interface of core and cladding, then the ray travels

    (a) in the cladding   (b) in the core

    (c) in the buffer   (d) along the interface

7. If the angle of incidence of a ray is less than the critical angle at the interface of core and cladding, then the ray travels

    (a) in the cladding   (b) in the core

    (c) in the buffer   (d) along the interface

8. Optical fibre guides light waves by

    (a) interference of waves

    (b) polarisation of waves

    (c) diffraction of waves

    (d) by total internal reflection of waves

9. By increasing the refractive index of core material, the number of modes of propagation in an optical fibre_____

    (a) decreases   (b) Increases

    (c) remains constant   (d) None of these

10. In optical fibres, mode means _____ available for light rays to propagate in the fibre

    (a) the change in refractive index

    (b) the number of paths

    (c) the number of fibre in optical fibre cable

    (d) None of the above

11. Numerical aperture represents _____ capacity of a optical fibre.

    (a) light gathering   (b) light dissipation

    (c) heat dissipation   (d) magnetic lines gathering

12. If n_1 and n_2 are the refractive indices of core and cladding, then numerical aperture of the fibre is

    (a) $\sqrt{n_2^2 - n_1^2}$ (b) $\sqrt{n_1^2 - n_2^2}$

    (c) $n_1^2 - n_2^2$ (d) $n_2^2 - n_1^2$

13. All the light rays which enter at a time into the multimode graded index fibre may arrive at _____
    - (a) different times at the other end of the fibre
    - (b) same time at the other end of the fibre
    - (c) Both (a) and (b)
    - (d) None of the above

14. Delay distortion of light pulses in an optical fibre is because of
    - (a) spreading of pulses with wavelength
    - (b) spreading of pulses with time
    - (c) spreading of pulses with refractive index
    - (d) None of the above

15. Optical fibres carry very large information compared to copper cables because of
    - (a) extremely wide bandwidth
    - (b) Extremely less bandwidth
    - (c) large thickness of fibre
    - (d) None of these

16. If the refractive indices of air, core and cladding are μ_0, μ_1 and μ_2 respectively then
    - (a) $\mu_0 > \mu_1 > \mu_2$
    - (b) $\mu_2 > \mu_1 > \mu_0$
    - (c) $\mu_1 > \mu_2 > \mu_0$
    - (d) $\mu_1 > \mu_0 > \mu_2$

17. Numerical aperture depends on
    - (a) critical angle
    - (b) acceptance angle
    - (c) angle of refraction
    - (d) refractive angle

18. In step-index fibre, the refractive index of the core
    - (a) changes parabolically
    - (b) changes randomly
    - (c) changes exponentially
    - (d) remains constant

19. In step-index fibre, the signal travels in a
    - (a) random manner
    - (b) skew manner
    - (c) zig-zag manner
    - (d) linear manner

20. In graded index fibre, the signal travels in a
    - (a) random manner
    - (b) skew manner
    - (c) zigzag manner
    - (d) linear manner

21. To enter an optical signal into the core, the angle of incidence at the interface of air and core should be
    - (a) less than the acceptance angle
    - (b) greater than the acceptance angle
    - (c) greater than the critical angle
    - (d) less than the critical angle

22. In graded index fibres, the refractive index of the core varies
    - (a) linearly
    - (b) exponentially
    - (c) parabolically
    - (d) None of these

23. By increasing the refractive index of the core number of modes of propagation in an optical fibre
    - (a) increases
    - (b) decreases
    - (c) zero
    - (d) remains constant

24. The acceptance angle is maximum if the critical angle is
    - (a) minimum
    - (b) maximum
    - (c) zero
    - (d) negative

25. If the angle of incidence for a light ray at the face of an optical fibre is greater than the acceptance angle then the light ray
    - (a) will propagate in the fibre
    - (b) will not propagate in the fibre
    - (c) first propagate and then stop at the middle
    - (d) first propagate and then stop at the end

26. Propagation of light through fibre core is due to
    - (a) total internal reflection
    - (c) interference
    - (c) diffraction
    - (d) refraction

27. In an optical fibre if n is the refractive index of core and n_2 the refractive index of cladding then
    - (a) $\dfrac{n_1 - n_2}{n_1} > 1$
    - (b) $\dfrac{n_1 - n_2}{n_1} < 1$
    - (c) $\dfrac{n_1 - n_2}{n_1} = 0$
    - (d) $\dfrac{n_1 - n_2}{n_1} = \infty$

28. Step index fibre can be a
    - (a) multimode fibre only
    - (b) monomode fibre only
    - (c) monomode as well as multimode fibre
    - (d) either monomode or multimode (cannot be both)

29. Intermodel dispersion is neglegible in
    - (a) monomode graded index fibre
    - (b) multimode step index fibres
    - (c) monomode step index fibres
    - (d) multimode graded index fibres

30. The process of mixing the single with the carrier is called
    - (a) attenuation
    - (b) dispression
    - (c) modulation
    - (d) demodulation

31. Total internal reflection takes place when light ray travels from
    - (a) denser to denser medium
    - (b) denser to rarer medium
    - (c) rarer to denser medium
    - (d) rarer to rarer medium

32. In step index fibres, the signals travel in a
    - (a) linear manner
    - (b) random manner
    - (c) zigzag manner
    - (d) skew manner

33. The refractive index of a cladding of a fibre with core refractive index 1.5 and numerical aperture 0.244 is
    (a) 1.656
    (b) 1.48
    (c) 1.325
    (d) 1.4

34. The angle of incidence at the core – cladding interface
    (a) varies in a straight fibre
    (b) remains constant in a bent fibre
    (c) remains constant in a graded index fibre
    (d) remains constant in a straight fibre

35. Optical fibres donot pick up electricity, because they are
    (a) magnetic materials
    (b) nonmetallic materials
    (c) transparent materials
    (d) electric material

36. Total internal reflection takes place when the angle of incidence is _____ the critical angle
    (a) equal to
    (b) less than
    (c) greater than
    (d) much less than

37. The refractive index of core and cladding are 1.50 and 1.44 respectively, and then acceptance angle is
    (a) 24°50′
    (b) 26°
    (c) 23°
    (d) 23° 45′

38. Source of light in a fibre optic system
    (a) LED
    (b) LED and Laserdiode
    (c) laserdiode
    (d) photodiode

39. The refractive index of core and cladding are 1.563 and 1.498 respectively and then NA is
    (a) 0.346
    (b) 0.246
    (c) 0.199
    (d) 0.446

40. It is not a source of light in fibre optic system
    (a) Laser
    (b) LED
    (c) photodiode
    (d) Laserdiode

41. Fractional index change $\Delta =$ _____
    (a) $\dfrac{n_1 + n_2}{n_2}$
    (b) $\dfrac{n_1 + n_2}{n_1}$
    (c) $\dfrac{n_1 - n_2}{n_1}$
    (d) $\dfrac{n_1 - n_2}{n_2}$

42. Pick out the losses present in the optical communication system
    (a) absorption losses
    (b) scattering losses
    (c) distortion losses
    (d) All of these

43. The attenuation in an optical fibre is a function of
    (a) Fibre material
    (b) Wavelength of light
    (c) Length of the fibre
    (d) All of these

44. Attenuation in optical fibres are mainly due to
    - (a) scattering losses
    - (b) absorption losses
    - (c) bending losses
    - (d) All of these

Answers

1. (a)	2. (a)	3. (c)	4. (b)	5. (a)	6. (d)
7. (a)	8. (d)	9. (b)	10. (b)	11. (a)	12. (b)
13. (b)	14. (b)	15. (a)	16. (c)	17. (b)	18. (d)
19. (c)	20. (b)	21. (a)	22. (c)	23. (a)	24. (a)
25. (b)	26. (a)	27. (b)	28. (d)	29. (d)	30. (c)
31. (b)	32. (c)	33. (b)	34. (d)	35. (b)	36. (c)
37. (a)	38. (b)	39. (d)	40. (c)	41. (c)	42. (d)
43. (d)	44. (d)				

Fill in the Blanks

1. Fibre optic communication is based on the phenomenon _____.
2. Refractive index of cladding has to be _____ than the refractive index of core.
3. For total internal reflection to occur, light must travel from _____ to _____ medium and angle of incidence must be greater than _____.
4. Light launched at the fibre end within _____ alone propagates to the other end.
5. Sine of the maximum acceptance angle is called _____ of the fibre.
6. In multimode fibre, the pulsed single recieved at the receiving is broadened due to _____.
7. In optical fibres attenuation is mainly due to two sources of losses namely _____ and _____.
8. In communication, mixing of single wave, with carrier wave is called _____.
9. At the receiving end of communication, the separation of single from the mixed carrier wave is called _____.

Answers to Fill in the blanks

1. total internal reflection
2. lesser
3. denser, rarer, critical angle
4. acceptance angle
5. Numerical aperture
6. intermodel dispersion
7. scattering losses, absorption losses
8. modulation
9. demodulation.

True or False

1. With out cladding, total internal reflection is not possible. (T/F)
2. Only if the angle of incidence at the Launching end of the fibre is greater than acceptance angle, total internal reflection is possible inside the fibre. (T/F)

3. Light collecting capacity of the fibre is expressed interms of "Numerical aperture". (T/F)

4. Numerical aperture does not depend on fibre dimensions. (T/F)

5. Thickness of the core of monomode fibre is around 50 μm. (T/F)

6. The problem of intermodel dispersion is overcome in graded index fibre. (T/F)

7. At the receiving end of the fibre, the conversion of optical singal into electrical singal is being done by light emitting diode. (T/F)

Answers to True or False

1. F   2. F   3. T   4. T   5. F   6. T   7. F

REVIEW QUESTIONS

Short Answer Questions

1. What is an optical fibre?
2. Mention the basic principle in the propagation of light signal through the optical fibre.
3. Mention the important parts of fibre structure.
4. What is acceptance angle and mention its significance?
5. What is acceptance core and mention its significance?
6. Mention the various types of optical fibres.
7. What is fibre optics endoscopy?
8. What is the role of optical fibre in sensors?

Essay Type Questions

1. Describe the construction and the working principle of optical fibre.
2. Derive expressions for numerical aperture and acceptance angle of optical fibre.
3. Explain the various types of optical fibres.
4. Explain attenuation and losses in fibres.
5. Describe optical fibre communication system.
6. Mention the applications of optical fibre in communication.
7. Mention the applications of optical fibre in sensors.
8. Mention the applications of optical fibre in medicine.

U N I T 2

Chapter 5: Crystallography and Ultrasonics

CHAPTER 5

CRYSTALLOGRAPHY AND ULTRASONICS

INTRODUCTION

A solid consists of a large number of closely packed atoms or molecules. The physical structure of a solid and its properties are related to the arrangement of atoms or molecules within the solid. The study of crystal geometry helps to understand the diverse behaviour of solids in their mechanical, metallurgical, electrical, magnetic and optical properties. For structural investigations of solids we use X-rays, neutron beams or electron beams.

Solids are classified into two categories based on the arrangement of atoms or molecules:

1. Crystalline solids
2. Amorphous solids

Crystalline Solids

In crystalline solids, atoms or molecules are arranged in a regular manner, i.e., the atomic array is periodic. Each atom is at regular intervals along arrays in all directions of the crystal. The crystalline solids have directional properties and are also called *anisotropic substances*. The structure may be made up of metallic crystals or nonmetallic crystals. Crystalline solids may be single crystalline solids or poly crystalline solids. In single crystalline solids, the periodicity of atoms or molecules is extended to the entire crystal, whereas in polycrystalline solids it is extended to small regions known as grains. Crystalline solids have

sharp melting points. The metallic crystals find wide application in engineering because of their strength, conductivity, reflection, etc.

Examples of metallic crystals are copper, silver, aluminium, tungsten, etc.

Amorphous Solids (Nonmetallic Crystals)

In amorphous solids, the atoms or molecules are arranged randomly. The amorphous solids have no regular structure (no directional property) and hence they are known as *isotropic substances*. Amorphous solids have a wide range of melting points as they have no regular shape. If an amorphous solid breaks, the broken pieces are irregular in shape. Such materials have no specific electrical property, but have only plasticity. Examples are glass, plastics and rubber.

Table 5.1   Distinction between Crystalline and Amorphous Solids

Crystalline Solid	Amorphous solid
1. In a crystalline solid, the atoms or moelcules have regular periodic arrangements.	1. In amorphous solids, the atoms or molecules are not arranged in a regular periodic manner. They have random arrangement.
2. They exhibit different magnitudes of physical properties in different directions.	2. They exhibit same magnitudes of physical properties in different directions.
3. They are an anisotropic in nature.	3. They are isotropic in nature.
4. They exhibit directional properties.	4. They do not exhibit directional properties.
5. They have sharp melting points.	5. They do not possess sharp melting points.
6. Crystal breaks along regular crystal planes and hence the crystal pieces have regular shape. Eg., Copper, silver, aluminium, etc.	6. When amorphous solid breakes, the broken pieces have irregular shape due to lack of crystal plane. Eg., Glass, plastic, rubber, etc.

5.1   FUNDAMENTAL TERMS OF CRYSTALLOGRAPHY

The structure of all crystals is described in terms of lattices with a group of atoms, each in a lattice point. The group is termed as *basis*. The basis is repeated in space to form the crystal structure. The lattice parameters give the measure of crystal structure. Let us now consider the various crystallographic terms in detail.

5.1.1   Space Lattice or Crystal Lattice

Consider a two-dimensional crystal structure, which consists of a regular periodic arrangement of atoms along the *x* and *y* directions (Fig. 5.1(a)). The distance between any two atoms remains same and each atom is surrounded by equal number of neighbouring atoms, i.e., in crystal structure, each atom will have an identical environment w.r.t. other atoms. This crystal structure can be studied in terms of an idealised geometrical concept called a space lattice, devised by
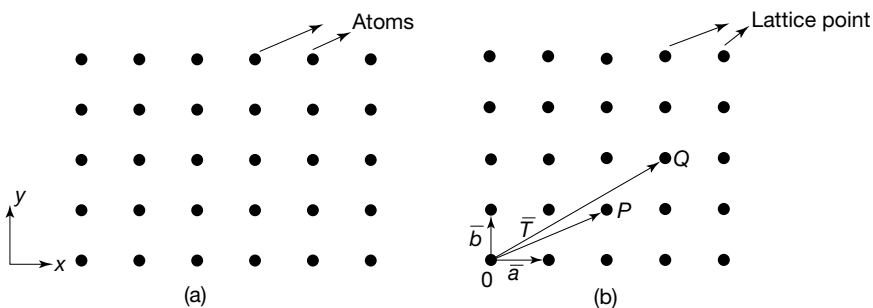
Figure 5.1 (a) Array of atoms in a two-dimensional crystal structure, (b) Array of lattice points in a space (space lattice or crystal lattice)

Bravais. According to this concept, each atom in the structure can be replaced by a point in space. These points will have a regular periodic arrangement and each point will have an identical environment w.r.t. other points. These points are known as *lattice points*. This array of lattice points in space which resembles the atoms in a crystal is known as *space lattice* (or) *crystal lattice* (Fig. 5.1(b)). A three-dimensional space lattice consists of an array of lattice points in three dimensions (*x*, *y*, and *z* directions).

Consider a lattice point at '0' as origin and join the origin to a successive lattice point along *x* and *y* directions. Let the position vectors of these lattice points be $\bar{a}$ and $\bar{b}$. When $\bar{a}$ is repeated regularly, then it gives the positions of lattice points along the *x* direction, i.e., $2\bar{a}$, $3\bar{a}$, $4\bar{a}$ … Similarly, if $\bar{b}$ is repeated periodically then it gives the positions of lattice points along the *y* direction, i.e., $2\bar{b}$, $3\bar{b}$, $4\bar{b}$, … Since, $\bar{a}$ and $\bar{b}$ when repeated regularly give the array of lattice points in space lattice, they are known as *fundamental translational vectors* or *basis vectors* or *primitive vectors*.

Now the position vector of the lattice point *P* is given by

$$\bar{T} = 2\bar{a} + \bar{b}$$

Similarly, for the lattice point *Q*,

$$\bar{T} = 3\bar{a} + 2\bar{b}$$

For a two-dimensional space lattice, the position vector of any lattice point is given by

$$\bar{T} = n_1\bar{a} + n_2\bar{b}$$

where n_1 and n_2 are the integers.

For a three-dimensional space lattice, the position vector of any lattice point is represented by

$$\bar{T} = n_1\bar{a} + n_2\bar{b} + n_3\bar{c}$$

where n_1, n_2 and n_3 are the integers and $\bar{a}\ \bar{b}\ \bar{c}$ are the translational vectors along *x*, *y* and *z* directions.

5.1.2 Basis

Consider a two-dimensional crystal structure which consists of three different atoms as shown in Fig. 5.2(a). To study the structure, first we have to construct a space lattice. With the centre atom as a lattice point, we get a space lattice. The two different atoms on either side of the centre atom can be taken as a unit assembly and when it is repeated regularly w.r.t. lattice points along *x* and *y* directions, we get the actual crystal structure. This unit assembly of atoms or molecules, identical in composition, is known as the *basis*. This basis when repeated periodically in all directions, gives the crystal structure. The crystal structure is real while the lattice is imaginary. Thus,

Crystal structure = Space lattice + Basis

Depending upon the crystal structure, the basis may be monoatomic, diatomic, etc.
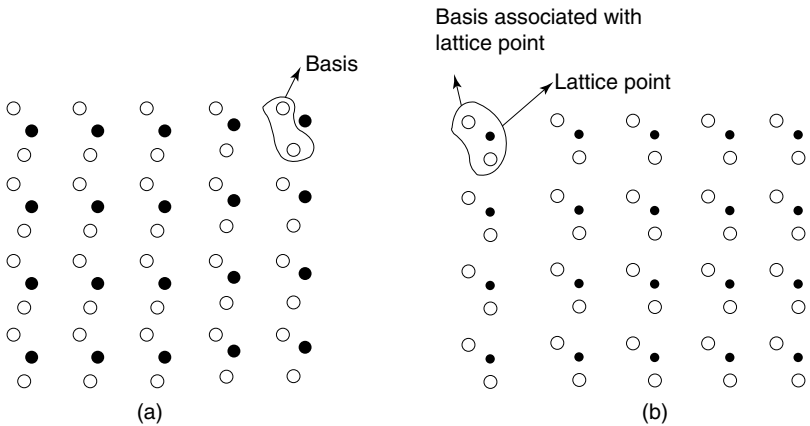
Figure 5.2 (a) Crystal structure, (b) Space lattice + Basis

5.2 UNIT CELL AND LATTICE PARAMETERS

Consider a two-dimensional crystal with periodic arrangement of atoms as shown in Fig. 5.3(a). The smallest geometrical figure which we get by joining the successive atoms along *x* and *y* directions is *ABCD* (square). When this figure is repeated regularly along *x* and *y* axis, we get the actual crystal structure. Similarly, for a three-dimensional crystal structure, the smallest geometrical figure is *ABCDEFGH* (cube), which when repeated periodically represents the crystal. Thus, the smallest possible geometrical figure which repeated regularly gives the crystal structure is known as the *unit cell*.

For a three-dimensional crystal, the unit cell may be cubic or parallel piped. Consider a cubic unit cell with crystallographic axes *x*, *y*, and *z* (Fig. 5.4). Let *OA*, *OB*, and *OC* be the intercepts made by the unit cell along the crystallographic axes. These intercepts giving the concurrent edges of the unit cell are represented
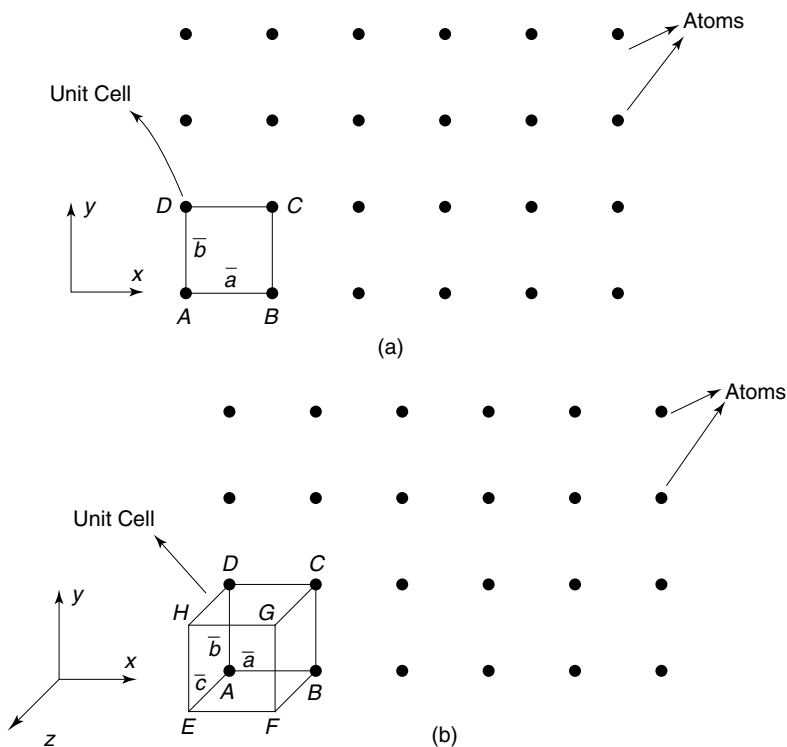
Figure 5.3   (a) Two-dimensional crystal structure (b) Three-dimensional crystal structure

by $\bar{a}$, $\bar{b}$, and $\bar{c}$ are known as *primitives*. The angles between *x*, *y*, and *z* axes are called *interaxial angles*. The angles between *x* and *y* (γ), *y* and *z* (α), *x* and *z*(β) are known as *interfacial angles*.

To present a lattice unit cell, we require the six parameters, i.e., *a*, *b*, *c* and α, β, γ. These quantities are known as *lattice parameters*. The lattice parameters *a*, *b*, *c* may or may not be equal and α, β, γ may or may not be right angles. Based on the value of the lattice parameters, we have different crystal systems.

Figure 5.4   Cubic unit cell

5.3   CRYSTAL SYSTEMS

Crystals are classified into seven systems on the basis of the shape of the unit cell or lattice parameters. These are classified in terms of lengths of unit cells and the angle of inclination between them. The seven systems are cubic, tetragonal, orthorhombic, monoclinic, triclinic, rhombohedral and hexagonal. Table 5.2 shows the crystal systems with the figures.

Table 5.2 The seven crystal systems

S.No.	Crystal system	Lattice parameters	Examples	Figure
1.	Cubic	$a = b = c$ $\alpha = \beta = \gamma = 90°$	Cu, Au, Ag, Nacl, etc.	
2.	Tetragonal	$a = b \neq c$ $\alpha = \beta = \gamma = 90°$	TiO_2, SnO_2, KH_2PO_4, etc.	
3.	Orthorhombic	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90°$	$BaSO_4$, K_2SO_4, $SnSO_4$, PbCO, etc.	
4.	Monoclinic	$a \neq b \neq c$ $\alpha = \beta = 90° \neq \gamma$	$CaSO_4$, $2H_2O$, Na_3AlF_6, etc.	
5.	Triclinic	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$	$K_2Cr_2O_7$, $CuSO_4 5H_2O$	

Contd..

Contd..
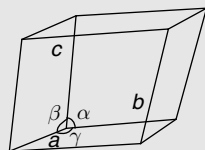
6.	Rhombohedral (Trigonal)	$a = b = c$ $\alpha = \beta = \gamma \neq 90°$	As, Bi, Sb, etc.	
7.	Hexagonal	$a = b \neq c$ $\alpha = \beta = 90°$ $\gamma = 120°$	Quartz, Zn, Cd, etc.	
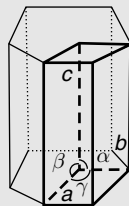
5.4   BRAVAIS LATTICES

Bravais showed that there are 14 different types of unit cells under the seven crystal systems as shown in Table 5.2. They are commonly called *Bravais lattices*. The classification of Bravais lattices is based on the following crystal lattices:

1. ***Primitive Lattice : P***

   In this lattice, the unit cell consists of eight corner atoms and all these corner atoms contribute only one effective atom for the lattice.

2. ***Body Centred Lattice : I***

   In addition to the eight corner atoms, it consists of one complete atom at the centre.

3. ***Face Centred Lattice : F***

   Along with the corner atoms, each face will have one centre atom.

4. ***Base Centred Lattice : C***

   The base and opposite face will have centre atoms along with the corner atoms.

Table 5.3 shows the 14 Bravais lattices with types and numbers along with examples.

Table 5.3   The seven crystal systems

S.No.	Name of the system	Lattice parameters	Types of Bravais lattices	No.of Bravais lattices	Examples
1.	Cubic	$a = b = c$ $\alpha = \beta = \gamma = 90°$	P I F	3	Po Na, W, $\alpha - Fe$ Ag, Au, Pb

Contd..

2.	Tetragonal	$a = b \neq c$ $\alpha = \beta = \gamma = 90°$	P I	 2	 TiO_2, SnO_2 KH_2PO_4
3.	Orthorhombic	$a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$	P I F C	4	$PbCO_3$, $BaSO_4$ KNO_3, K_2SO_4 $\alpha - S$
4.	Monoclinic	$a \neq b \neq c$ $\alpha = \beta = 90° \neq \gamma$	P C	2	$CaSO_4$, $2H_2O$ K_2MgSO_4, $6H_2O$
5.	Triclinic	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$	P	1	$K_2Cr_2O_7$
6.	Rhombohedral (Trigonal)	$a = b = c$ $\alpha = \beta = \gamma \neq 90°$	p	1	Calcite, As, Sb, Bi
7.	Hexagonal	$a = b \neq c$ $\alpha = \beta = 90°$ $\gamma = 120°$	p	1	SiO_2, Agl

Bravais lattices : Cubic

Cubic (*P*)   Cubic (*I*)   Cubic (*F*)
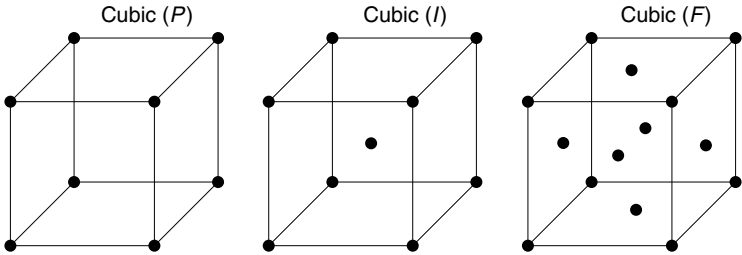
Bravais Lattices:Tetragonal

Tetragonal (*P*)   Tetragonal (*I*)
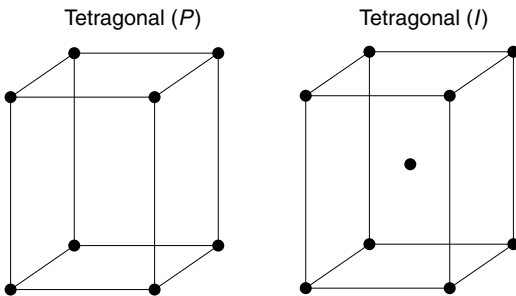
Bravais Lattices: Orthorhombic
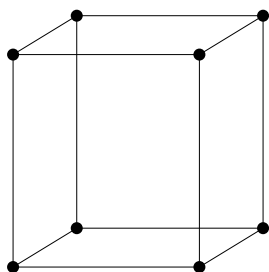
Orthorhombic (*P*)   Orthorhombic (*I*)   Orthorhombic (*F*)
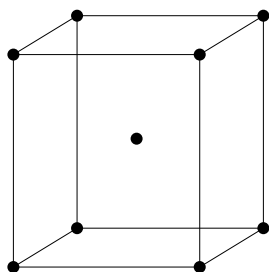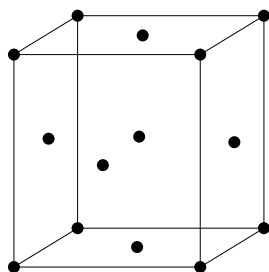
Orthorhombic (*C*)
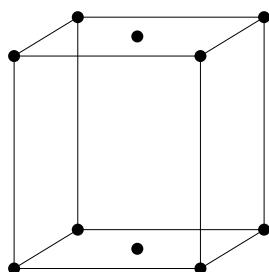
Bravais Lattices: Monoclinic
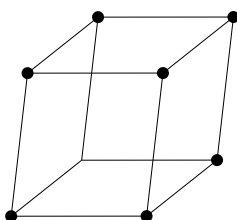
Monoclinic (*P*)   Monoclinic (*C*)
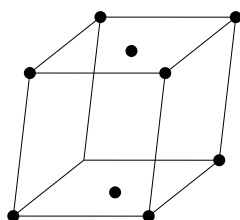
Bravais Lattices: Triclinic

Triclinic (*P*)
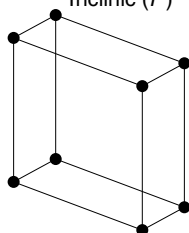
Bravais Lattices: Rhombohedral (Trigonal)

Rhombohedral (*P*)
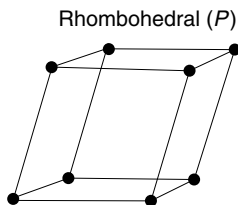
Bravais Lattices: Hexagonal

Hexagonal (*P*)
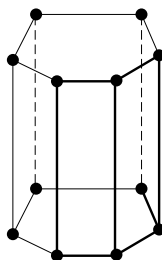
5.5   EXPRESSION FOR LATTICE CONSTANT

Consider a cubic unit cell with lattice parameters $a = b = c$ and $\alpha = \beta = \gamma = 90°$. Let ρ be the density of the cell. Since the edges of the cube are equal and constant, it is known as lattice constant, i.e., $a = b = c$ = lattice constant (a)

Figure 5.5   Cubic unit cell

Volume of cubic unit cell = a^3

Mass of cubic unit cell = $a^3 \rho$ (1)

Let M be the molecular weight of the cell and N_A be the Avagadro's number.

Mass of each atom in unit cell = $\dfrac{M}{N_A}$

If there are 'n' number of atoms in a unit cell, then

mass of the cubic unit cell = $\dfrac{nM}{N_A}$ (2)

From Eqs. (1) and (2), we have

$$a^3\rho = \frac{nM}{N_A}$$

$$a^3 = \frac{nM}{\rho N_A}$$

$$a = \left[\frac{nM}{\rho N_A}\right]^{\frac{1}{3}}$$
(3)

The above expression represents the expression for lattice constant.

5.6   CRYSTAL STRUCTURES OF MATERIALS

Consider some of the important parameters which are used to describe the crystal structure of materials.

1. Effective Number of Atoms per unit Cell

The effective number of atoms per unit cell is the total number of atoms in a unit cell by considering the contribution of corner atoms, centre atoms and face centred atoms.

2. Atomic Radius (r)

It is half the distance between any two successive atoms in a crystal lattice.

3. Coordination Number (N)

The number of equidistant neighbours that an atom has in a crystal lattice is known as the coordination number.

4. Nearest Neighbour Distance (2r)

The distance between two nearest neighbouring atoms in a crystal lattice is known as the nearest neighbour distance ($2r$).

5. Atomic Packing Factor

It is the ratio between the total volume occupied by the atoms or molecules in a unit cell to the volume of the unit cell,

i.e., Atomic packing factor $= \dfrac{\text{Total volume occupied by atoms in a unit cell}}{\text{Volume of the unit cell}}$

$= \dfrac{\text{Number of atoms present in a unit cell} \times \text{Volume of the one atom}}{\text{Volume of the unit cell}}$

6. Interstitial Space (or) Void Space

The empty space available in a crystal lattice with atoms occupying their respective positions is the interstitial or void space.

7. Density

Density of crystal lattice $= \rho = \dfrac{nM}{a^3 N_A}$

where n = number of atoms in a unit cell

M = Molecular weight

N_A = Avagadro number

a = Lattice constant

5.7   SIMPLE CUBIC CRYSTAL STRUCTURE (SCC)

A simple cubic unit cell consists of eight corner atoms as shown in Fig. 5.6. The respective crystal structure parameters are given below.

Unit cell   Cubic unit cell

Lattice parameters

Figure 5.6   Simple cubic cell

$$a = b = c = \text{lattice constant } (a)$$
$$\alpha = \beta = \gamma = 90°$$

Primitive cell   It is a primitive cell.

Effective number of atoms in unit cell   In actual crystals, each and every corner atom is shared by eight adjacent unit cells. Therefore, each and every corner atom contributes 1/8 of its part to one unit cell. Hence, the effective number of atoms present in a unit cell is $= \dfrac{1}{8} \times 8 = 1$.

Coordination number (N)   For the corner atom, there are four nearest neighbours in its own plane. There is another nearest neighbour in a plane which lies just above this atom and yet another nearest neighbour in another plane which lies just below this atom. Therefore, the total number of nearest neighbours is six and hence the coordination number is 6.

Nearest neighbour distance (2r)   Let r be the radius of spherical corner atoms. From Fig. 5.7, the nearest neighbour distance $= 2r = a$.

Atomic radius (r)   Atomic radius $r = \dfrac{a}{2}$

Atomic packing factor   For simple cubic, the total number of atoms present is one. Therefore, the atomic packing factor of the simple cubic can be written as follows.

Figure 5.7   One face of simple cubic cell

$$\text{Volume of one atom} = \frac{4}{3} \pi r^3$$

$$\text{Volume of unit cell} = a^3$$

$$\text{Numbers of atoms present in unit cell} = 1$$

$$\text{Atomic packing factor} = \frac{1 \times (4/3) \times \pi r^3}{a^3}$$

Substituting $r = a/2$ in the above relation:

$$\text{Atomic packing factor} = \frac{1 \times (4/3) \times \pi(a/2)^3}{a^3}$$

Therefore, atomic packing factor $= \dfrac{\pi}{6} = 0.52$

Thus, 52 per cent of the volume of the simple cubic unit cell is occupied by atoms.

Interstitial Space   The empty space available (or) interstitial space = 47%

Density   As $n = 1$ for simple cubic cell,

$$\rho = \frac{M}{a^3 N_A}$$

Example   Potassium crystal.

5.8   BODY CENTRED CUBIC STRUCTURE (BCC)

A body centred cubic cell consists of one body centred atom in addition to eight corner atoms (Fig. 5.8). The corresponding crystal structure parameters are as follows:
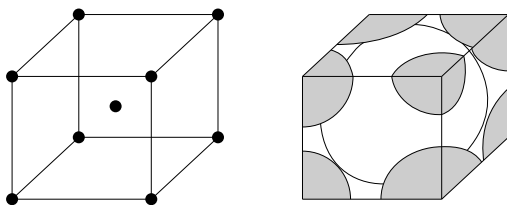
Figure 5.8   Body centred cubic cell

Unit cell   Cubic unit cell

Lattice parameters

$a = b = c =$ lattice constant (a)

$\alpha = \beta = \gamma = 90°$.

Primitive cell   As it consists of a centre atom along with corner atoms, it is not a primitive cell.

Effective number of atoms in unit cell   In a bcc unit cell, each and every corner atom is shared by eight adjacent unit cells, So, the total number of atoms contributed by the corner atoms is $(1/8)8 = 1$.

A bcc unit cell has one full atom at the centre of the unit cell. Therefore, the effective number of atoms present in a bcc unit cell is 2.

Coordination number (N)   Let us consider a body centred atom. The nearest neighbour for a body centred atom is a corner atom. A body centred atom is surrounded by eight corner atoms. Therefore, the coordination number of a bcc unit cell is 8.

Atomic radius   For a body centred cubic unit cell, the atomic radius can be calculated from Fig. 5.9.

From Fig. 5.9, $AH = 4r$ and $DH = a$
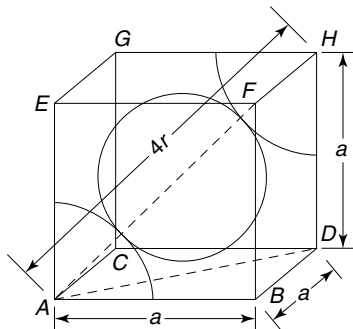
Figure 5.9   Calculation of atomic radius

From the triangle *AHD,*
$$AD^2 + DH^2 = AH^2 \tag{4}$$

To find *AD*, consider the triangle *ABD,*

From the triangle *ABD*
$$AB^2 + BD^2 = AD^2 \text{ or } a^2 + a^2 = AD^2$$

i.e., $\qquad\qquad AD^2 = 2a^2$

Therefore, $\qquad\quad AD = \sqrt{2}a \tag{5}$

Substituting *AD*, *AH* and *DH* values in Eq. (4), we get
$$AD^2 + DH^2 = AH^2$$

or $\qquad\qquad 2a^2 + a^2 = (4r)^2$

i.e., $\qquad\qquad 16r^2 = 3a^2$

$$r^2 = \frac{3}{16} a^2$$

The atomic radius $\qquad r = \sqrt{\frac{3}{4}} a \tag{6}$

Nearest neighbour distance (2r)

$$r = \frac{\sqrt{3}}{4} a$$

$$2r = \frac{2\sqrt{3}}{4} a = \frac{\sqrt{3}}{2} a$$

Atomic packing factor The atomic packing factor of a body centred cubic unit cell can be calculated as follows:

The number of atoms present in a unit cell is 2.

Volume of one atom $= \frac{4}{3} \pi r^3$

Volume of unit cell $= a^3$

Atomic packing factor $= \dfrac{2(4/3)\pi r^3}{a^3} \tag{7}$

Substituting $r = (\sqrt{3}/4)a$ in Eq. (7)

$\therefore$ atomic packing factor $= \dfrac{2(4/3)\pi \left[\left(\dfrac{\sqrt{3}}{4}\right)\right] a^3}{a^3}$

$$\frac{\sqrt{3}\,\pi}{8} = 0.68$$

Atomic packing factor = 0.68

This shows that 68 per cent of the volume of the body centered cubic unit cell is occupied by atoms. It is closely packed structure compared to simple cubic structure.

Interstitial space

Interstitial space = 32%

Density   As $n = 2$ for BCC,

$$\rho = \frac{3M}{a^3 N_A}$$

Examples: Elements like sodium, potassuim, chromium, tungsten, etc.

5.9   FACE CENTRED CUBIC STRUCTURE (FCC)

Face centred cubic structure consists of atoms at eight corners and each face has a centre atom.

The crystal parameters are as detailed below.

Unit cell   Cubic unit cell

Figure 5.10   Face centred cubic cell

Lattice parameters

$a = b = c$ = lattice constant (a)

$\alpha = \beta = \gamma = 90°$

Primitive cell   In addition to corner atoms, it consists of face centred atoms, hence it is not a primitive cell.

Effective number of atoms in unit cell   A face centred cubic unit cell consists of eight corner atoms and six face centred atoms. An fcc unit cell consists of eight corner atoms and six face centred atoms Fig. 5.11. Each and every corner atom is shared by eight adjacent unit cells. Therefore, each and every corner atom contributes 1/8 of its part to one unit cell. The total number of atoms contributed by the corner atoms is $1/8 \times 8 = 1$.

Figure 5.11   Arrangement of atoms in fcc unit cell

Each and every face centred atom is shared by two unit cells. Therefore, a face centred atom contributes half of its part to one unit cell. The total number of atoms contributed by the face centered is $\frac{1}{2} \times 6 = 3$. Therefore, the total number of atoms present in an fcc cell is 4.

Coordination Number   Let us consider a corner atom. In its own plane, that corner atom has four face centred atoms. These face centred atoms are its nearest neighbours. In a plane which lies just above this corner atom, it has four more

face centred atoms as nearest neighbours. In a plane which lies just below this corner atom, it has yet four more face centred atoms as its nearest neighbours. Therefore, for an atom in an fcc unit cell, the number of nearest neighbours is 12.

Atomic Radius The atomic radius can be calculated from Fig. 5.12 as follows.

From Fig. 5.12, consider the triangle *ABC*.

Figure 5.12 Calculation of atomic radius

$$AC^2 = AB^2 + BC^2$$
$$(4r)^2 = a^2 + a^2$$
$$16r^2 = 2a^2$$

The atomic radius $r = \dfrac{a}{2\sqrt{2}}$

Nearest Neighbour Distance (2r)

$$r = \frac{a}{2\sqrt{2}}$$
$$2r = \frac{a}{\sqrt{2}}$$

Atomic Packing Factor The atomic packing factor of the fcc unit cell can be calculated as follows. The number of atoms present in an fcc unit cell is 4.

Volume of one atom $= \dfrac{4}{3} \pi r^3$

Volume of unit cell $= a^3$

Atomic packing factor $= \dfrac{4(4/3)\pi r^3}{a^3}$ \hfill (8)

Substituting $r = a/(2\sqrt{2})$ in Eq. (8)

$\therefore$ atomic packing factor $= \dfrac{4(4/3)\pi \times (a/2\sqrt{2})^3}{a^3} = \dfrac{\pi}{3\sqrt{2}}$ \hfill (9)

$\therefore$ packing density = 0.74

That is, 74 per cent of the volume of an fcc unit cell is occupied by atoms. It is a very closely packed structure than the simple cubic and bcc.

Interstitial Space Interstitial space = 26%

Density As $n = 4$

$$\rho = \frac{4M}{a^3 N_A}$$

Examples Cu, Ag, Al, etc.

5.10 STRUCTURE OF NaCl

NaCl crystal is an ionic crystal where Na^+ and Cl^- ions were arranged alternately in a cubic pattern so that the electrostatic attraction and the repulsion between the same ions balances and the crystal will be in equilibrium (stable) state.

From Fig. 5.13, it is clear that Na^+ ions are situated at the corner as well as at the centres of the faces of the cube, i.e., Na^+ ions forms a FCC lattice. Similarly, Cl^- ions lies in a FCC lattice, but their lattice being relatively displaced half the edge of the lattice along each axis. Thus NaCl crystal is formed by interpenetration of two FCC sublattices of Na^+ and Cl^- ions exactly at the half of the edge of the lattice along each axis.

For Na^+ FCC sublattice, number of Na^+ ions = 4

For Cl^- FCC sublattice, number of Cl^- ions = 4

Hence the NaCl unit cell consists of 4 $Na^+ - Cl^-$ ion pairs. The unit cell has four NaCl molecules.

Each Na^+ ion has 6 Cl^- ions as nearest neighbours and similarly each Cl^- ion has 6 Na^+ ions. The coordination number of NaCl is 6.

The corner occupied by Cl^- ion is treated as the origin and the coordinates of remaining Cl^- ions in the unit cubic cell are expressed in terms of lattice constant

'a' as 000, $\dfrac{a}{2}\dfrac{a}{2}0$, $\dfrac{a}{2}0\dfrac{a}{2}$, $0\dfrac{a}{2}\dfrac{a}{2}$, $0\dfrac{a}{2}\dfrac{a}{2}$

Then the coordinates of Na^+ ions were

$$\dfrac{a}{2},\dfrac{a}{2}\dfrac{a}{2}, \quad 00\dfrac{a}{2}, 0\dfrac{a}{2}0, \dfrac{a}{2}00.$$
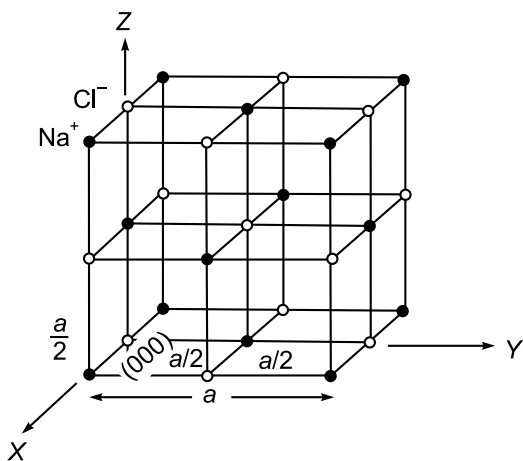
Figure 5.13 NaCl structure

5.10.1   Structure of Diamond

Generally diamond crystal crystallises with cubic symmetry and is known as diamond cubic structure. Diamond cubic structure is obtained when two FFC sublattices interpenetrates along the body diagonal by 1/4th cube edge. If one sublattice has its origin at the point (0, 0, 0) then the second sublattice will be at the point (a/4, a/4, a/4).

The basic diamond lattice and its atomic positions in the cubic cell of diamond projected on a cube face as shown in Fig. 5.14.
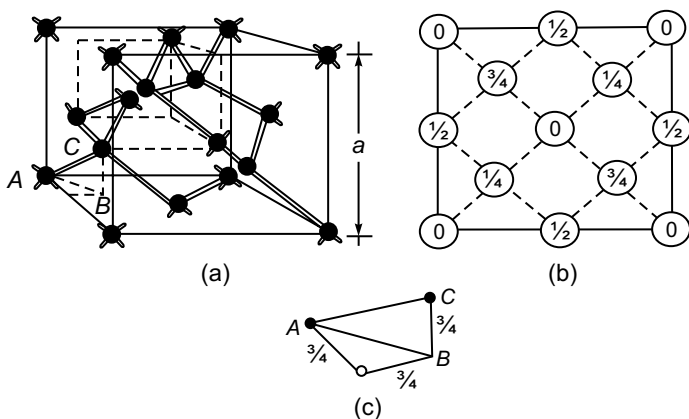
Figure 5.14

The fractions, 1/4, 1/2, 3/4 denote the height, above the base in units of cube edge. In diamond cubic structure, each atom is surrounded by 4 nearest neighbouring atoms. Hence the coordination number is 4. In addition to the eight corner atoms, there are six face centred atoms and four more atoms are located inside the unit cell. Each corner atom is shared by eight adjacent unit cells and each face centred atom is shared by two unit cells.

Total effective number of atoms per unit cell

$$= \left(\frac{1}{8} \times 8 \right) + \left(\frac{1}{2} \times 6 \right) + 4 = 8$$

By considering the projection of atom at C on to the bottom surface at B and joining B with A and to the edge of the cube.

We get Fig. 5.25(c).

From the figure, $AB^2 = \left(\dfrac{a}{4} \right)^2 + \left(\dfrac{a}{4} \right)^2 = \dfrac{a^2}{8}$

$$AC^2 = AB^2 + BC^2$$

$$= \frac{a^2}{8} + \frac{a^2}{16} = \frac{3a^2}{16}$$

But $\qquad\qquad AC = 2\,r$

The nearest neighbour distance

$$2r = \left(\frac{3a^2}{16}\right)^{1/2}$$

$$(2r)^2 = \frac{3a^2}{16}$$

Atomic radius $$r = \frac{\sqrt{3}a}{8}$$

Lattice constant $$a = \frac{8r}{\sqrt{3}}$$

Volume of the cube $= a^3$

Volume of 8 atoms $= 8 \times$ Volume of each atom

$$= 8 \times \frac{4}{3} \pi r^3$$

$$= 8 \times \frac{4}{3} \times \pi \times \left(\frac{\sqrt{3}a}{8}\right)^3$$

$$= \frac{\sqrt{3}\pi a^3}{16}$$

Atomic packing factor $= \dfrac{\text{Volume of all atoms in unit cell}}{\text{Volume of the unit cell}}$

$$= \frac{\sqrt{3}\pi a^3/16}{a^3} = \frac{\sqrt{3}\pi}{16} = 0.34$$

Thus 34% of the entire volume of diamond cubic is occupied by the atoms and the remaining volume of 66% is empty, Thus, it is loosely packed structure.

5.11   MILLER INDICES – CRYSTAL PLANES

Consider a two-dimensional lattice with a periodic array of lattice points (Fig. 5.15). The plane passing through the lattice points is known as crystal
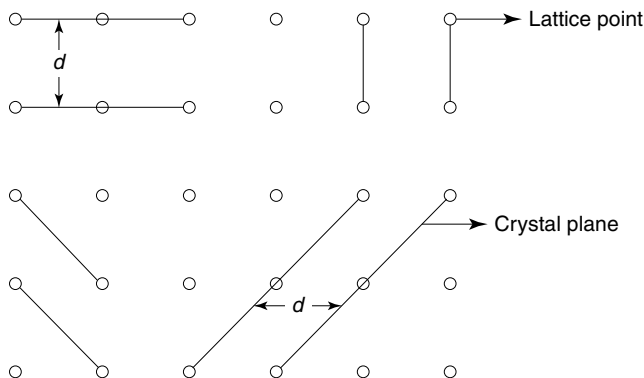
Figure 5.15   Crystal planes in two-dimensional lattice

plane. The perpendicular distance between two adjacent crystal planes is called *interplanar spacing (d)*. The crystal plane can have different orientations. Miller derived a method of representing the orientation of a crystal plane. The crystal plane is represented by three smallest integers known as Miller indices.

Miller indices are three, smallest integers which have the same ratio as the reciprocals of the intercepts of the crystal plane with the coordinate axes.

5.11.1   Procedure to Find Miller Indices

1. The intercepts made by the crystal plane along *x*, *y* and *z* axis in terms of lattice parameters *a*, *b*, *c* are noted.
2. Express the intercepts as multiples of *a*, *b*, *c*.
3. Take the reciprocals of the intercepts.
4. Convert the reciprocals into integers by multiplying each one of them with their LCM.
5. Enclose these integers in smaller parantheses. This represents the Miller indices of the crystal plane.

For example, *ABC* is the crystal plane which makes intercepts 2*a*, 3*b* and 2*c* along the *x*, *y*, and *z* axes respectively where *a*, *b*, *c* are the lattice parameters.

The intercepts as the multiples of *a*, *b*, *c* are 2, 3, 2. Reciprocals of the intercepts are 1/2, 1/3, 1/2. Their LCM is 6. Multiply the reciprocals by 6, so that they become integers 3, 2, 3.

The integers are written within the parenthesis as (323). (323) represents the Miller indices of the crystal plane *ABC*.
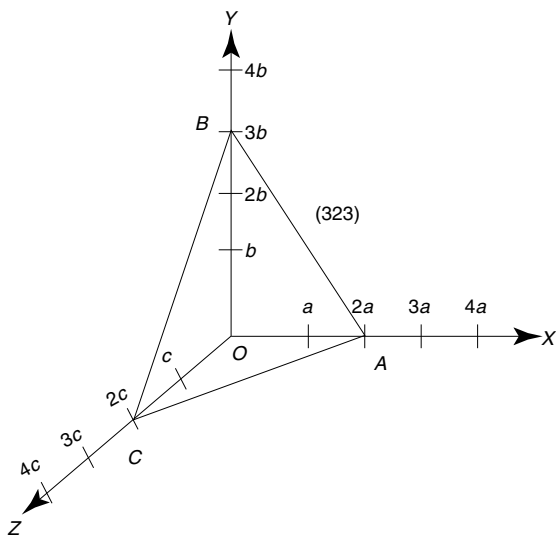
Figure 5.16   Miller indices of the crystal plane *ABC*

Note

(i) While writing Miller indices, a comma or dot between any two numbers may be avoided.

(ii) The positive X axis is represented as (100), Y axis as (010) and Z axis as (001), Similarly, the negative X axis as ($\bar{1}$00), negative Y axis as (0$\bar{1}$0) and negative Z axis as (00$\bar{1}$).

(iii) The Miller indices for a plane (101) is read as 'one zero one' and not as one hundred and one.

5.11.2 Miller Indices of the Crystal Planes in a Unit Cubic Cell

The Miller indices of various crystal planes in a unit cell are shown in Fig. 5.17.
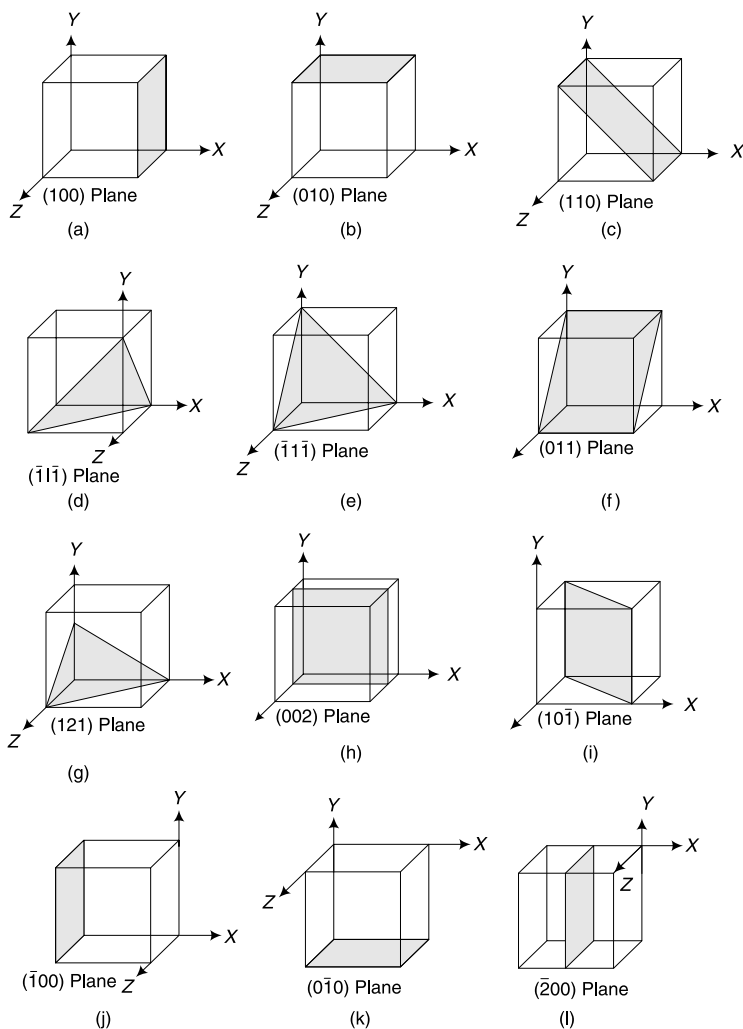
Figure 5.17 Miller indices of some crystal planes in unit cubic cell

Important Features of Miller Indices of Crystal Planes

1. Miller indices represent the orientation of crystal planes in a crystal lattice.

2. Any plane parallel to one of the coordinate axes will have infinite intercept, and the corresponding Miller index becomes zero.

3. A plane passing through the origin is defined in terms of a parallel plane having nonzero intercepts.

4. If the Miller indices of the planes have the same ratio then the planes are parallel to each other. For example, (211), (422), etc.

5. Planes with negative intercepts are represented by a bar, i.e., ($\bar{1}00$). The Miller indices ($\bar{1}00$) indicates that the plane has an intercept in the negative *x*-axis.

6. If (*hkl*) is the Miller indices of a crystal plane, then the intercepts made by the plane with the coordinate axes are *a/h, b/k, c/l* where *abc* are the lattice parameters or primitives.

7. For a cubic crystal, the interplanar spacing between the adjacent parallel plane is given by $d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$ where *a* is the lattice constant and (*hkl*) is the Miller indices of the crystal plane.

8. For a cubic crystal, $h : k : l = \dfrac{a}{p} : \dfrac{a}{q} : \dfrac{a}{r}$

   For any crystal, $h : k : l = \dfrac{a}{p} : \dfrac{b}{q} : \dfrac{c}{r}$

5.11.3   Miller Indices – Crystal Directions

In a crystal system, the line joining the origin and a lattice point represents the direction of the lattice point.

Consider a unit cubic cell with lattice points at the corners (Fig. 5.18) and *H* as the origin. For the cell, *HG*, *HE* and *HD* represents the direction of lattice points *G*, *E* and *D*. To find the Miller indices of crystal direction of a lattice point, first note down the coordinates of the lattice point, and if it is in fractions reduce to smallest integers and enclose the integers in bigger paranthesis as [*hkl*].

Figure 5.18   Unit cubic cell

For the unit cubic cell, the directions of lattice points are

HG – [100]	*HF* – [110]	*HB* – [111]
HE – [010]	*HA* – [011]	
HD – [001]	*HC* – [101]	

The line joining the origin to the crystal plane represents the directions of the crystal plane. The Miller indices of the crystal plane enclosed within the bigger paranthesis represents the Miller indices of the directions of the crystal plane.

Figure 5.19 represents the direction of crystal plane in various cells.
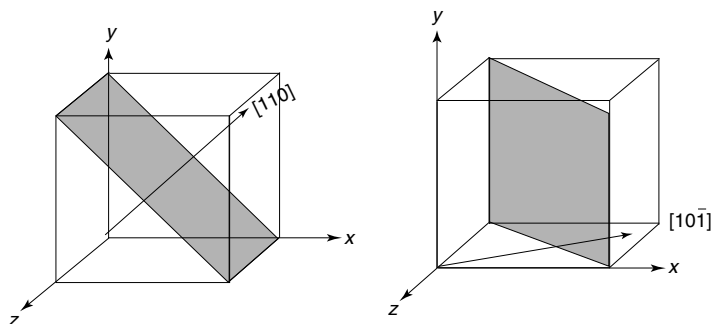
Figure 5.19 Directions of crystal planes

5.11.4 Important Features of Miller Indices of Crystal Directions

1. The line joining the origin to the lattice point represents the directions of the lattice point.
2. A normal line drawn from the origin to the plane gives the directions of the plane.
3. The directions of the plane are indicated by bigger paranthesis [*hkl*].
4. The direction [*hkl*] is perpendicular to the plane [*hkl*]
5. The angle θ between the two directions $[h_1, k_1, l_i]$ and $[h_2\ k_2\ l_2]$ is given by

$$\cos\theta = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2}\ \sqrt{h_2^2 + k_2^2 + l_2^2}}$$

5.11.5 Separation between Successive (*hkl*) Planes

Consider a plane *ABC* intercepting all three axes at distances *OA*, *OB*, and *OC* (*a/h*, *a/k*, and *a/l*, respectively) as shown in Fig. 5.20. Let the normal to the plane from the origin intercept the plane at *N*. Distance of the normal from the origin is the interplanar distance '*d*' (i.e., *ON*). Let the angles made by the normal to the *X*, *Y*, and *Z* axes be α, β, and γ, respectively. Then the interplanar distance '*d*' between the origin and the plane is derived as follows.

Let, *OA*, *OB*, and *OC* be the intercepts made on the orthogonal axes *OX*, *OY*, *OZ*; *ON*, the perpendicular distance form the origin to the plane; '*d*' the interplanar distance; and α, β, and γ be the angles between *ON* and *X*, *Y*, and *Z* axes, respectively.

Here, $OA = a/h$, $OB = b/k$, $OC = c/l$

In $\quad \Delta\, ONA$, $\cos \alpha = \dfrac{ON}{OA} = \dfrac{d}{a/h} = d\left(\dfrac{h}{a}\right)$

In $\quad \Delta\, ONB$, $\cos \beta = \dfrac{ON}{OB} = \dfrac{d}{b/k} = d\left(\dfrac{k}{b}\right)$

In $\quad \Delta\, ONC$, $\cos \gamma = \dfrac{ON}{OC} = \dfrac{d}{c/l} = d\left(\dfrac{l}{c}\right)$

We know from the law of direction cosines that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

i.e.,

$$\frac{d^2}{a^2}(h^2) + \frac{d^2}{b^2}(k^2) + \frac{d^2}{c^2}(l^2) = 1$$

Figure 5.20   Separation between successive (*hkl*) planes

For a cubic system: $a = b = c$; then, the above equation becomes

$$\frac{d^2}{a^2}(h^2 + k^2 + l^2) = 1$$

or

$$d^2 = \frac{a^2}{(h^2 + k^2 + l^2)}$$

Therefore,

$$d = \frac{a}{\sqrt{(h^2 + k^2 + l^2)}} \tag{10}$$

Consider a parallel plane intercepting the three axes at A_1, B_1, and C_1. Let it be at a distance of $2a/h$ ($= OA_1$), $2b/k$ ($= OB_1$), and $2c/l$ ($= OC_1$) from the origin O. It makes the same angles α, β, and γ with X, Y, and Z axes, respectively. Then, as discussed earlier, its planar distance from the origin O is ON_1 ($= 2d$) and is given by

$$2d = \frac{2a}{\sqrt{(h^2 + k^2 + l^2)}} \tag{11}$$

Hence, the interplanar distance between the two planes is given by (ON_1 - ON), i.e., ($2d - d$). Therefore, subtracting Eq. (11) from (10), we get

$$2d - d = \frac{2a}{\sqrt{(h^2 + k^2 + l^2)}} - \frac{a}{\sqrt{(h^2 + k^2 + l^2)}}$$

$$d = \frac{a}{\sqrt{(h^2 + k^2 + l^2)}}$$

The above expression represents the interplanar spacing between two parallel planes.

5.11.6 X-ray Diffractions by Crystal Planes

For diffraction to occur, the size of the obstacle must be comparable with wavelength of the incident light. Generally, diffraction grating is used for diffraction studies. The grating consists of a large number of closely spaced ruled lines so that the spacing between the lines may be of the order of wavelength of light.

Due to very small wavelength (0.1nm) of X-rays, grating cannot be used for X-ray diffraction studies as the spacing cannot be comparable to wavelength of X-rays. Laue suggested that a crystal which consists of a three-dimensional array of regularly spaced atoms will produce observable diffraction effects for X-rays, i.e., the crystal will act as diffraction grating for X-rays. Laue's associates, Friedrich and Knipping, succeeded in diffracting X-rays by passing X-rays through a thin crystal of zinc blend.

The diffraction pattern obtained consists of a central spot and a series of spots arranged in a definite pattern around the central spot. This pattern is known as *Laue's pattern*. The interpretation of the diffraction effect was given by Bragg, according to which these spots are due to the reflection of X-rays from parallel planes in a crystal containing a large number of atoms.

5.11.7 Bragg's Law

Consider a crystal which consists of equidistant parallel planes with the interplanar spacing 'd' as shown in Fig. 5.21. When X-rays of wavelength (λ) fall on these crystal planes, the atoms in the planes diffract (reflect) the X-rays in all directions.
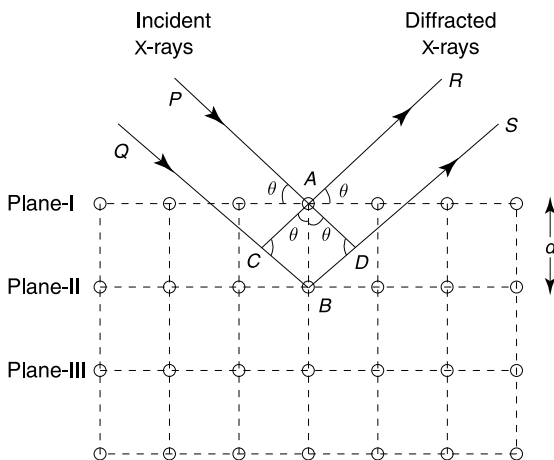
Figure 5.21 Diffraction of X-rays by crystal planes

Let the X-ray *PA* incident at an angle 'θ' with the plane *I* be reflected at atom *A* in the direction *AR* and another X-ray *QB* reflected at atom 'B' in the direction

BS. These diffracted X-rays will interfere constructively or destructively depending on the path difference between the X-rays. To calculate the path difference, two normals *AC* and *AD* are drawn from *A* to *QB* and *BS*.

$$\text{Path difference} = CB + BD$$

From ΔACB, $\qquad \sin \theta = \dfrac{CB}{AB}$

$$CB = AB \sin \theta = d \sin \theta$$

Similarly, from ΔADB, $\qquad BD = d \sin \theta$

$\therefore \qquad \text{Path difference} = d \sin \theta + d \sin \theta$

$$= 2d \sin \theta$$

To get maximum intensity, path difference should be equal to integral multiple of λ.

$$\text{Path difference} = n\lambda$$

$$2d \sin \theta = n\lambda \quad \text{where } n = 1, 2, 3$$

The above expression represents Bragg's law, according to which the diffracted X-rays interface constructively when the path difference is an integral multiple of the wavelength of X-rays. Corresponding to $n = 1, 2, 3$ we obtain, first, second, third ... order diffraction spots. Since maximum possible value of θ is 1, we get

$$n\lambda = 2d$$

$$\lambda \leq 2d$$

Thus, the wavelength λ should not exceed twice the interplanar spacing for diffraction to occur.

Importance of Bragg's law

1. From Bragg's law, $2d \sin \theta = n\lambda$, we get

$$d = \frac{n\lambda}{2 \sin \theta}$$

   Knowing wavelengths of X-rays (λ) and the glancing angle θ, 'd' can be calculated.

2. Knowing 'd', the lattice constant 'a' of the cubic crystal can be known.

3. Knowing a, ρ (density) and M (molecular weight) of the crystal, the number of atoms (or) molecules in the unit cell can be calculated.

4. For a cubic crystal,

$$d = \frac{a}{\sqrt{(h^2 + k^2 + l^2)}}$$

   Knowing d and a, the value of $h^2 + k^2 + l^2$ can be calculated.

5. Depending on the values of $h^2 + k^2 + l^2$, we can classify the crystals as BCC, FCC, simple cubic, etc.

5.11.8 Laue Method

The Laue method is one of the X-ray diffraction techniques used for crystal structure studies. The experimental arrangement is shown in Fig. 5.22.
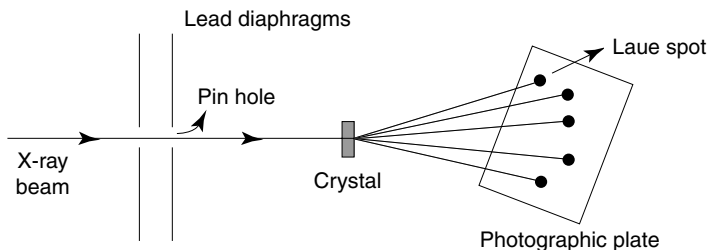
Figure 5.22 Laue method

The crystal whose structure has to be studied, is held stationary in a continuous X-ray beam. After passing through the pin holes of lead diaphragms, we obtain a fine beam of X-rays. These X-rays are allowed to fall on the crystal. The crystal planes in the crystal diffract the X-rays satisfying Bragg's law.

The diffracted X-rays are allowed to fall on a photographic plate. The diffraction pattern consists of a series of bright spots corresponding to interference maximum for a set of crystal planes satisfying the Bragg's equation $2d \sin \theta = n\lambda$ for a particular wavelength of the incident beam.

The distribution of spots depends on the symmetry of the crystal and its orientation with respect to X-ray beam. For a simple cubic crystal, the Laue photograph is shown in Fig. 5.23.
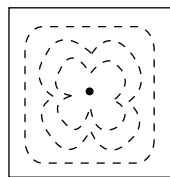
Figure 5.23 Laue photograph for simple cubic crystal

5.11.9 Merits and Demerits

This method is used for determination of crystal orientation and symmetry. It is also used to study crystalline imperfections (defects).

This method is not convenient for actual crystal structure determination, because several wavelengths of X-rays diffract in different order from the same plane, and they superimpose on a single Laue spot.

5.11.10 Powder Method

The powder method is an X-ray diffraction technique used to study the structure of microcrystals (tiny crystallites) in the form of powder. This method gives information regarding the size and orientations of the crystallites in the powder. It is also known as Debye–Scherrer method. The experimental arrangement is shown in Fig. 5.24.
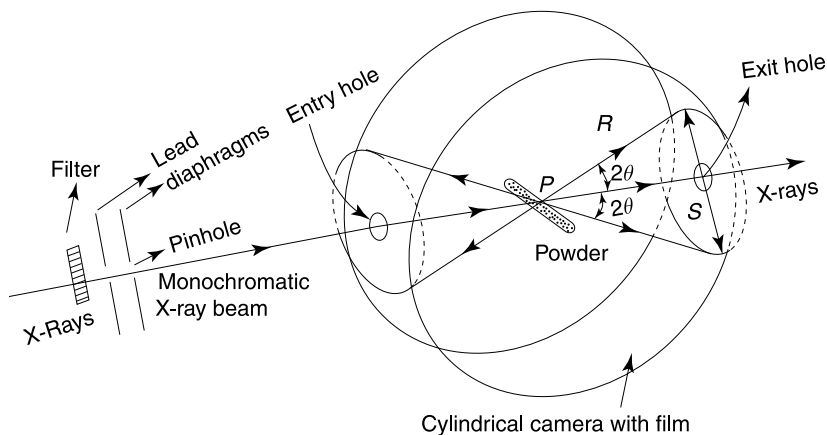
Figure 5.24   Powder method

The powder is prepared by crushing the polycrystalline material so that it consists of crystallites. These crystallites are all randomly oriented so that they make all possible angles with the incident, X-rays. Thus all orders of reflections from all possible atomic planes are re-ordered at the same time.

The experimental arrangement consists of a cylindrical camera, called the Debye–Scherrer camera consisting of a film in the inner portion. The finely powdered sample is filled in a thin capillary tube made of nondiffracting material. The incident X-rays after passing through the filter and pin holes of lead diaphrgrams, produce a fine beam of monochromatic X-rays. The fine beam after passing through the entry hole falls on the capillary tube P containing the powdered crystals. Since the powder consists of randomly oriented crystallites, all possible θ and d values are available for diffraction of incident X-rays. The diffraction takes place for these values of d and θ which satisfy the Bragg's relation $2d \sin \theta = n\lambda$, where λ is constant for monochromatic X-rays. For a particular value of the angle of incidence 'θ', various orientations of a particular set of planes are possible. The diffracted rays corresponding to fixed values of θ and d lie on the surface of a cone with its apex at the tube P and the semivertical angle 2θ. Different cones are observed for different sets of d and θ for a particular order of n and also for different combinations of θ and n for a particular value of d.

The transmitted X-rays come out through the exit hole. The diffracted X- ray cones make impressions on the film in the form of arcs on either side of the exit and entry holes with their centres coinciding with the hole. The diffraction pattern is shown in Fig. 5.25.

Entry hole   Arcs on the film   Exit hole
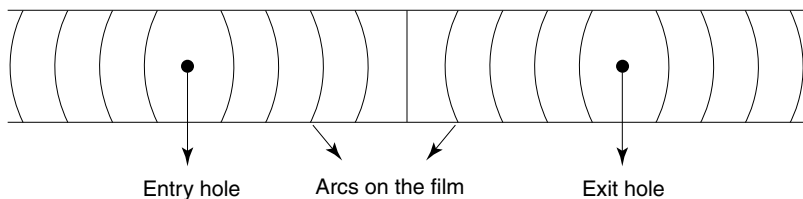
Figure 5.25   Diffraction pattern

The angle θ corresponding to a particular pair of arcs is related to the distance 's' between the arcs as

$$4\theta \text{ (radiaus)} = \frac{s}{R} \quad \left[\because \text{angle} = \frac{\text{arc}}{\text{radius}} \right]$$

where R is the radius of the camera.

$$4\theta \text{ (degrees)} = \frac{s}{R} \left(\frac{180}{\pi} \right)$$
$$= \frac{57.296 \, s}{R}$$

From the above expression, θ can be calculated. Then the interplanar spacing for first-order diffraction is

$$d = \frac{\lambda}{2 \sin \theta}$$

Knowing all the parameters, the crystal structure can be studied.

5.11.11   Merits
1. Using filter, we get monochromatic X-rays (λ remains constant).
2. All crystallites are exposed to X-rays and diffractions take place with all available planes.
3. Knowing all parameters, crystal structure can be studied completely.

5.11.12   Defects in Solids (Crystals)
Crystals will have a regular periodic arrangement of atoms. Any deviation from this periodicity in known as imperfections or defects in crystals, i.e., when there is an irregularity or departure from the perfect periodicity of atomic arrays in a crystal then it is known as crystal defects or imperfection. These defects in crystals affects their structure sensitive properties such as mechanical strength, ductility, crystal growth, dielectric strength, magnetic hysteresis, conduction in semiconductors, etc. The imperfections in crystals may be confined to a point, line, surface and volume of a crystal. Based on the type of imperfections they are classified as follows.

1. Point defects
      (a) Vacancies (Schottky defect)
      (b) Interstitialies (Frenkel defect)
      (c) Compositional defects

1. Substitutional impurity defect
2. Interstitial impurity defect
    (d) Electronic defects
3. Line defects:
    (a) Edge dislocation
    (b) Screw dislocation
4. Surface defects:
    (a) Grain boundaries
    (b) Twin boundaries
    (c) Tilt boundaries
    (d) Stacking fault
5. Volume defects.

Point defects   If a defect is localised near a point or atom in the crystal structure then the defect is known as point defect. This is due to imperfect packing of atoms during crystallisation. When a regular atom is absent at its periodic position (or) present at irregular position creates point defects. This defect may also be due to the presence of impurity atoms in the crystal. Point defect occur in metallic and ionic crystals. These defects are classified into various types.

(a) Vacancies

A vacancy is the simplest point defect in a crystal which refers to a missing atom at its site. As shown in Fig. 5.26, in a crystal lattice, when an atom is missing from its position in a regular periodic arrangements of atoms, it creates a vacancy in its position and the periodicity of atoms is lost. This defect is known as vacancy. Such vacancies may arise either from imperfect packing during the formation of crystals or from thermal vibrations of the atoms at high temperatures. When the thermal energy due to vibration is increased the atoms will jump out of their positions of lowest energy and move over to the surface. For most crystals, this thermal energy is of the order of

Figure 5.26   Vacancy defect

1 eV per vacancy. The number of vacancies per unit volume of the crystal depends upon temperature. Depending upon the number of atoms missing there may be single vacancy (if one atom is missing), divacancies (if two atoms are missing), trivacancies (if three atoms are missing) and so on.

(b) Interstitials

When an extra atom is inserted into the crystal, it occupies interstitial positions i.e., void or space between the regularly positioned parent atoms without dislodging the parent atom as shown in Fig. 5.27. The defect caused is known as
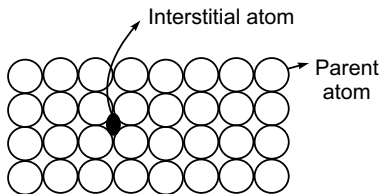
Figure 5.27   Interstitial defect

interstitial defect and the extra atom becomes interstitial atom. This happens when the interstitial atom is smaller than the parent atoms and crystal has low package factor. Depending on the number of extra atoms, interstitials may be single, di, tri and so on.

Schottky defect Schottky defects is a special case of vacancy defect. In ionic crystal, let us consider the periodic distribution of +ve and –ve ions carrying equal changes.

There are two types of possible vacancies in ionic crystal:

1. Cation Vacancies (+ve ion Vacancies)
2. Anion Vacancies (–ve ion Vacancies).

A +ve ion vacancy is formed when a +ve ion from the interior of the crystal moves out of the crystal to

Figure 5.28 Schottky defect

its surface as shown in Fig. 5.28 when a +ve ion vacancy is formed inside the crystal it results in excess negative change inside the crystal. To main change neutrality of the crystal a –ve ion moves to the crystal surface as shown in Fig. 5.28. Thus a +ve and –ve, ion vacancy pair is created in the crystal. Such a pair of vacant sites is known as Schottky defect. In NaCl crystal, each vacancy of Na ion tends to result in the formations of one chloride ion vacancy. In $MgCl_2$, Mg ion is doubly changed, so each vacancy at Mg site is compensated by two anion vacancies in chloride sites.

Frenkel defect Frenkel defect is a special case of interstitial defect.
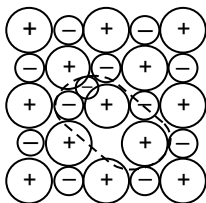
Figure 5.29 Frenkel defect

Consider a periodic distribution of +ve and –ve ions in an ionic crystal when a –ve ion leaves its site and settles in the interstitial position then it creates a vacancy in its position. Thus a vacancy and interstitial defects are created. This pair of defect is known as Frenkel defect. This defect is exhibited by ionic crystals whose anions are larger in size than cations. for example, silver halides, AgBr, AgCl etc.

(c) Compositional defects

When an impurity atom is dopped into the crystal, then it will try to settle in the position of parent atom if the vacancy is present. Otherwise, if the size of the

impurity atom is less than the parent atom then it will try to settle in the interstitial spaces between the parent atoms. This defects are known as compositional defects. They are of two types:

(i) Substitutional impurity defect   If the impurity atoms occupies the one of the positions of parents atoms in a crystal, then it becomes a substitutional defect.
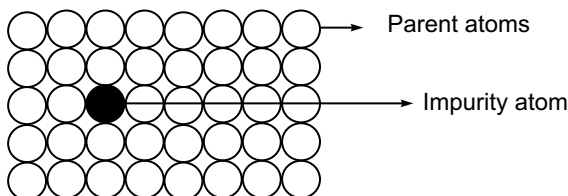
Figure 5.30   Substitutional defect

(ii) Interstitial impurity defect   When the impurity atom tries to settle in the interstitial space between the parent atoms of the crystal then it gives rise to interstitial impurity defect.
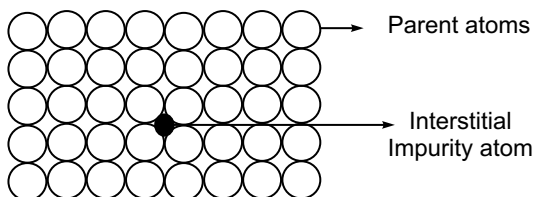
Figure 5.31   Interstitial impurity defect

Electronic defects   Due to the defect present in the crystal the change distribution is not uniform i.e., it varies in the crystal. Such non-uniform distribution of charge in a crystal due to the presence of various defects is known as electronic defect. When the +ve ions occupy the interstitial spaces in a crystal then the amounts of +ve change will be more when compared to –ve change of the crystal and vice versa.

Line defects   When the periodicity of atomic arrangement is interrupted along a line or along a certain directions in the crystal, then the defect is known as line defects or dislocations. Dislocation refers to a linear disturbance of the atomic arrangement in a crystal. This defect occurs in the process of formation of the crystal and also during slipping of the atomic planes over the others. The dislocation influences the structure sensitive properties of crystals like ductility in metals, ceramics and crystalline polymers.

The dislocations are of two types:

    1. Edge dislocation
    2. Screw dislocation

Edge dislocation

(a) Perfect crystal

(b) Incomplete plane in a crystal (edge dislocation)

(c) Perfect crystal face

(d) Dislocated crystal face
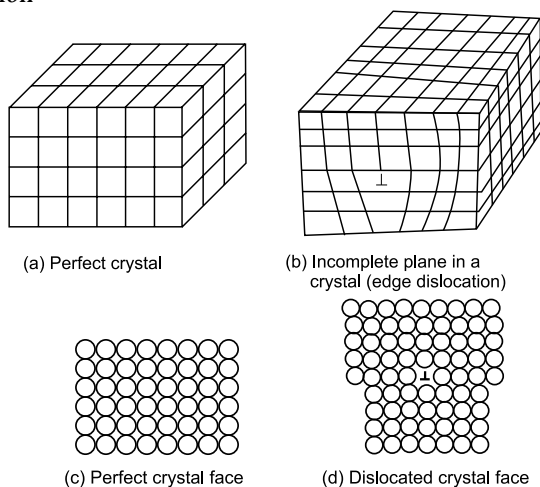
Figure 5.32   Dislocated Crystal face

Figure 5.32 (a) and (c) shows the perfect crystal and its face having a regular periodic atomic arrangement. The perfect crystal is made up of vertical planes parallel to one another and to the side faces. If one of these vertical planes does not extend from top to the bottom of the crystal but ends partway within the crystal, as shown in Fig. 5.32(b) a discontinuity or a dislocation is present along the edge of that vertical plane and Fig. 5.32(d) shows the dislocated crystal face. In perfect crystal, the atoms are in equilibrium positions and all the bond lengths are of the equilibrium value. In the imperfect crystal, above the edge of the incomplete plane, the atoms are squeezed and are in a state of compression. The bond length have been compressed to smaller than the equilibrium value. Below the edge the atoms are pulled apart and are in a state of tension. The bond lengths have been stretched to above the normal values. This distorted configuration
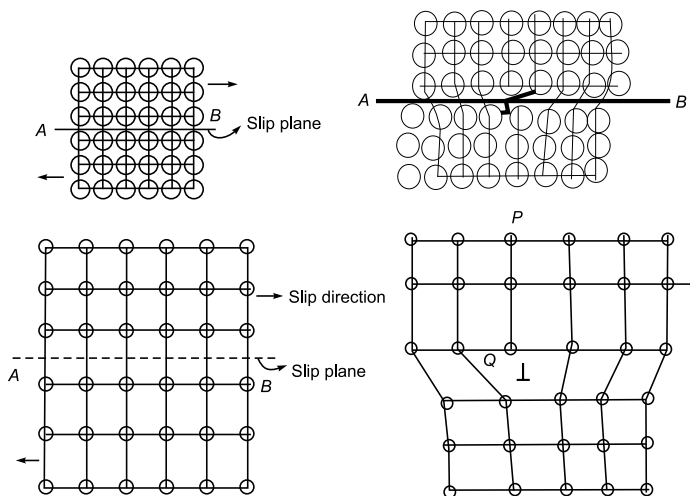
Figure 5.33   Slip causing edge dislocation

extends all along the edge into the crystal. Thus mass distortion is centered around the edge of the in complete plane, it represents a line imperfection and is called an edge dislocation and is represented by ⊥. This dislocation is due to the imperfects matching of vertical planes of upper and lower surfaces of a crystal while preparing crystal. Edge dislocation may also due to the slipping of atomic planes over the other.

As shown in the Fig. 5.34, when the two atomic planes of regular crystal are made to slip along *AB*, due to mismatching of upper and lower planes. There is a discontinuity at the edge of the vertical plane *PQ* causing the edge dislocation ⊥. A state of and to have bonding between the two portions the atoms in the upper portions are in a state of compression and the atoms in the lower portion are in a state of extension.
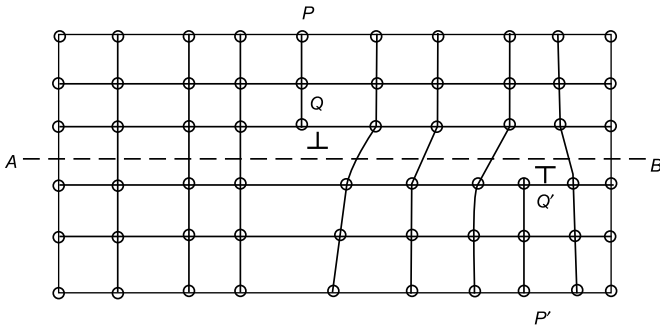
Figure 5.34   +ve and –ve edge distlocation

As shown in Fig. 5.34 if the edge dislocation is above the slip plane then it is known as +ve edge dislocation (*PQ*) and is denoted by the symbol ⊥. If it is below the slip plane then it becomes –ve edge dislocation (*P′ Q′*) and is denoted by the symbol T.

Screw dislocation   Consider a perfect crystal having regular periodic atomic arrangements when one face is cut midway along *AB*, and applying a shear stress parallel to the cutting plane such that the right portion (2) moves down with
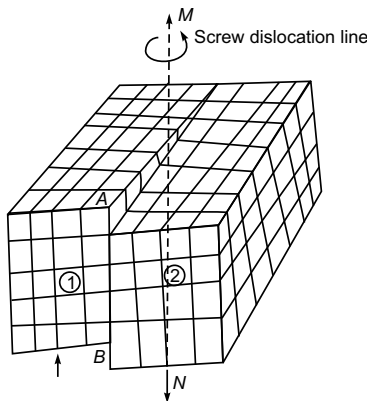
Figure 5.35   Screw dislocation

respect to left portion (1) of the crystal then along the cut boundary the atoms get displaced and the two regions will be in the mismatched condition, i.e., Dislocation of atoms take place around the edge line *AB* known as dislocation line. From the top surface, the dislocation of atoms will be in the form of screw or spiral around *AB*. Hence the dislocations is called screw dislocation.

Burger's vector　　The strength of any dislocation is described in terms of a parameter known as Burger's vector (or) Slip vector Burger's vector is an important property of a dislocation because if the Burger's vector and the orientation of a dislocation line are known the dislocation is completely described. Burger's vector indicates how much above the slip plane and in what direction the lattice appears to have been shifted w.r.t. the lattice below the slip plane.

A convenient way of defining the Burger's vector of a dislocation is by means of a Burger's circuit. The Burger's circuit can be obtained by considering atomic arrangement around the edge dislocation or screw dislocation. Starting from a lattice point, trace path from atom to atom, equal distance in each direction. If the region enclosed by the path does not continue a dislocation, the Burger's circuit will close. If the path encloses a dislocation, the Burger's circuit will not close. The closure failure of the Burger's circuit is the Burger's vector '*b*'.

Burger's vector of edge dislocation　　Burger's circuit is the path traced by counting equal number of atoms in horizontal and vertical directions. In a perfect crystal, the circuit be started from the lattice point *P* and covers 4 atoms and reach *Q*. From *Q* by covering 5 atoms reaches *R*. By covering 4 and 5 atoms respectively it covers *S* and *P*. The circuit is closed at *P*. Thus the Burger's circuit is closed in a perfect crystal. In the crystal containing dislocation, the circuit is started from *P* and reaches *P'* by counting equal number number atoms in vertical and horizontal directions around the edge dislocation. To close the circuit, we need an extra step '*b*'. This step '*b*' is known as Burgers vector. The Burger vector is perpendicular to the edge dislocation.

(a) Burger's circuit in perfect crystal

(b) Burger's circuit in edge dislocation
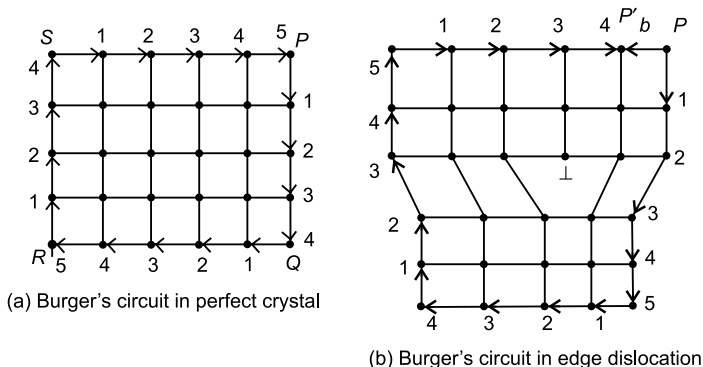
Figure 5.36

Burger's vector of screw dislocation   By tracing equal number of atoms along the screw dislocation of a crystal from *A* to *F*, we need one step from *F* to *A* to close the circuit. This additional step represents Burger's vector '*b*' of screw dislocation. The Burger's vector is parallel to the screw dislocation.
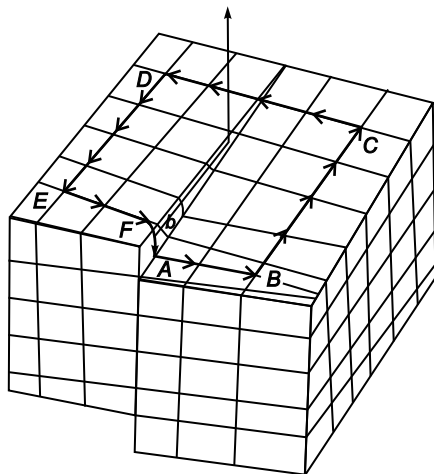
Figure 5.37   Burger's vector in screw dislocation

5.12   ULTRASONICS

We know that the audible range of sound waves is 20 Hz to 20 MHz. The sound waves having frequencies above the audible range are known as *ultrasonic waves*. Since frequency is high the wavelengths of ultrasonic waves are very low. This property of ultrasonic waves made them more versatile in the nondestructive testing of materials, i.e., without any destruction we can test the material.

5.12.1   Properties of Ultrasonic Waves

1. Ultrasonic wave are highly energetic, high frequency in audible sound waves.
2. They show negligible diffraction due to their smaller wavelength. Hence, they can be transmitted over long distances without any loss of energy.
3. Intense ultrasonic radiation has a disruptive effect in liquids by causing bubbles to be formed.
4. Ultrasonic wave requires a material medium for its propagation.
5. Their velocity of propagation is directly proportional to its frequency.
6. During the propagation of ultrasonic wave they are reflected, scattered, retracted and diffracted.

7. Ultrasonic waves set up standing waves in liquid baths and in turn produces an acoustical grating.

5.12.2 Production of Ultrasonics by Piezoelectric Method

Piezoelectric method is based on Piezoelectric effect. As shown in Fig. 5.38 when crystals like quartz, rochelle salt, tourmaline, etc., undergo mechanical deformation (stretching or compression) along the mechanical axis then an electric potential difference is produced along electric axis perpendicular to mechanical axis.
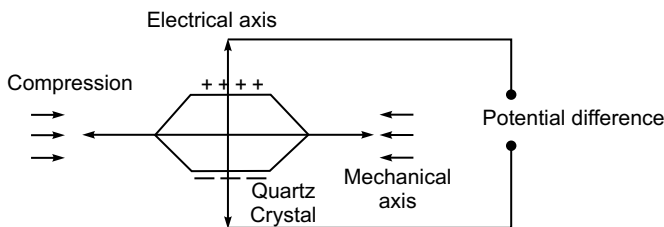
Figure 5.38   Piezoelectric effect

The converse of this effect is also possible. When an alternating potential difference is applied along the electrical axis, the crystal will be set into elastic vibrations along the mechanical axis. If the frequency of the applied field coincides with the natural frequency of the crystal, it vibrates with large amplitude producing ultrasonic waves. This phenomenon is utilised for the productions of ultrasonic waves.

The experimental, setup for the production of ultrasonic waves using Piezoelectric generator is shown in Fig. 5.39.
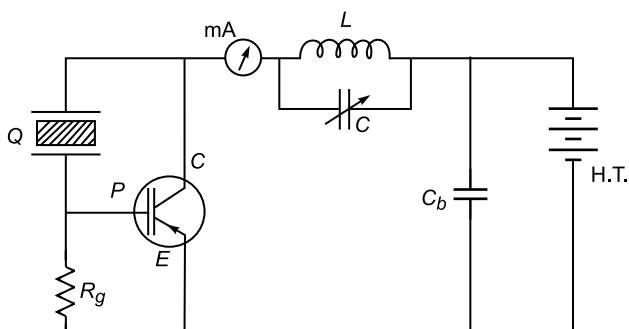
Figure 5.39   Piezoelectric generator

Construction   The quartz crystal between two metal plates is connected to collector and base of a transistor. Collector is also connected to LC circuit and high tension source shunted through a by pass capacitor C_b. C_b is used to stop

high frequency currents from passing through battery. The capacity of variable capacitor is adjusted so that the frequency of the oscillating circuits is equal to the natural frequency of the crystal. R_g provides necessary biasing for base and emitter circuit.

Working   When the circuit starts functioning slowly an alternating potential difference is built across the quartz crystal which sets the crystal into vibrations. By varying the capacity of capacitor C, at a particular stage the frequency of the alternating potential across the crystal coincides with the natural frequency of quartz crystal setting it to produce ultrasonic waves. This stage is indicated by milliameter by showing maximum current.

The natural frequency of quartz crystal of thickness t is given by

$$f = \frac{n}{2t}\sqrt{y/\rho} \ \text{ where } n = 1, 2, 3\ldots$$

y is the Young's modulus and ρ is the density of the crystal.

5.12.3   Detections of Ultrasonics

The presence of ultrasonic waves can be detected by the following methods:

1. Piezoelectric detector   By using the piezoelectric effect, one can defect the presence of ultrasonic waves. As shown in Fig. 5.40, when the faces of a quartz crystal along the mechanical axis is subjected to ultrasonic, then it undergoes compression and expansion. The opposite faces along the electrical axis, will have induced charges which establishes a potential difference across the faces. This potential difference indicates the presence of ultrasonic waves.
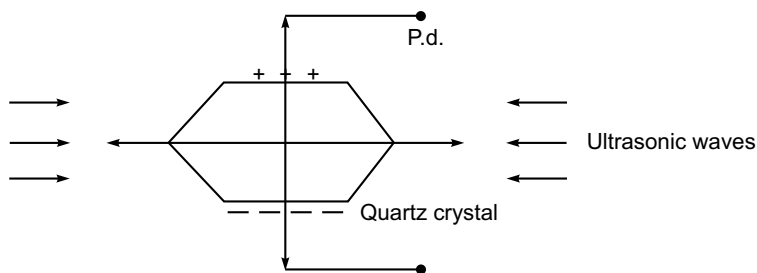
Figure 5.40

2. Kundt's tube method   As shown in Fig. 5.41, the transmitted and reflected ultrasonic wave in a Kundt's tube forms a stationary wave pattern with modes and antinodes. The Lycopodium powder present in the bottom portion of the tube will be collected as a leaps at nodes and dispensed at antinodes. By observing the change in the positions of powder, we can detect ultrasonic waves in the tube.
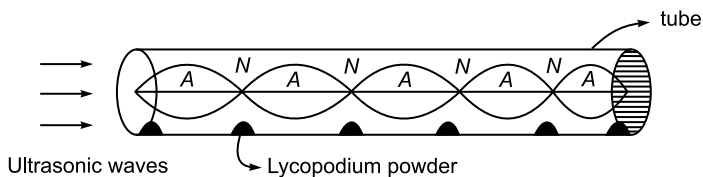
Figure 5.41

3. Sensitive flame method   When a narrow sensitive flame is moved in a medium where ultrasonic waves are present. The flame remains stationary at antinode and flickers at nodes.
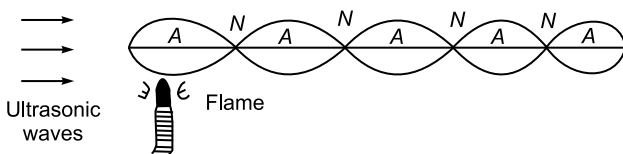
Figure 5.42

4. Thermal detector method   When a platinum wire is moved in the medium consists of standing waves of ultrasonics due to variations of temperature at nodes and antinodes, the resistance of wire also changes. By noticing the changing resistance of wire one can detect the presence of ultrasonic waves.
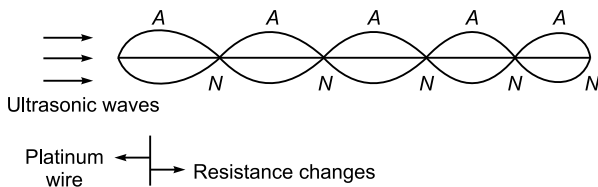
Figure 5.43

5.12.4   Application in Nondestructive Testing

Ultrasonic waves were extensively used for nondestructive testing of the material, i.e., detecting the defects (flaws) inside the material without disturbing material properties. Nondestructive testing systems consists of transducers for generations and transmission of ultrasonic waves into the material and also to receive the reflected waves from the flaws or defects. To identify the defects, cathode ray oscilloscope is used.
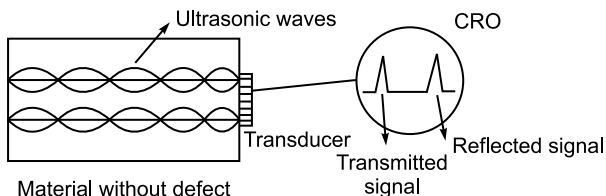
Figure 5.44

When the transducer generates and transmits the ultrasonic waves into the testing material it will be reflected by the other end of the material and is received by the transducer. Corresponding to transmitted and reflected waves we can observe two well-resolved signals on the screen of CRO.
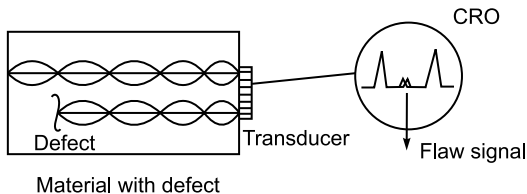
Figure 5.45

When the material is having a defect, then in the CRO screen in addition to regular transmitted and reflected signal we get a flaw signal. This signal indicates the presence of defect inside the material. By adjusting the time base scale to the transmitted signal, the time elapsed from this signal to the flaw signal can be known. By knowing the velocity and time taken by the ultrasonic waves, the flaw location can be identified.

SOLVED PROBLEMS

1. Chromium has bcc structure. Its atomic radius is 0.1249 nm. Calculate the free volume/unit cell.

Given data   Atomic radius of chromium, $r = 0.1249$ nm

Solution   Volume of the unit cell, $V = a^3$

where 'a' is the lattice constant

$$a = \frac{4r}{\sqrt{3}} \quad \Rightarrow \quad \frac{4}{\sqrt{3}} \times 0.1249 \text{ nm} = 0.28845 \text{ nm}$$

$\therefore$ volume of the unit cell $V = a^3 = (0.28845)^3 \text{ nm}^3 = 0.024 \text{ nm}^3$

Number of atoms in a BCC unit cell = 2

Hence, volume of atoms in the unit cell $= \frac{4}{3} \pi r^3 \times 2 = 0.01633 \text{ nm}^3$

Free volume/unit cell $= V - v = 0.00767 \text{ nm}^3$

2. Iron crystallises in bcc structure. Calculate the lattice constant, given that the atomic weight and density of iron are 55.85 and 7860 kg/m³, respectively.

Given data   Atomic weight of iron, $M = 55.85$

     Density of iron, $r = 7860 \text{ kg/m}^3$

Solution Number of atoms in bcc unit cell, $n = 2$

$\therefore$ lattice constant $a = \left[\dfrac{nM}{\rho N_A} \right]^{\frac{1}{3}}$

where N_A is the Avogadro number $= 6.02 \times 10^{26}$ kg/mole.

$$a = \left[\frac{2 \times 55.85}{7860 \times 6.02 \times 10^{26}} \right]^{1/3} = 2.87 \times 10^{-10} \text{ m}$$

$$= 2.87 \text{Å}$$

3. A beam of X-rays of wavelength 0.071 nm is diffracted by a (110) plane of rock salt with a lattice constant of 0.28 nm. Find the glancing angle for the second-order diffraction.

Given data Wavelength of X-rays, $\lambda = 0.071$ nm

Lattice constant, $a = 0.28$ nm

Plane $(hkl) = (110)$

Diffraction order $n = 2$

Solution According to Bragg's law,

$$2d \sin \theta = n\lambda$$

$$\sin \theta = \frac{n\lambda}{2d}$$

$$\theta = \sin^{-1} \left(\frac{n\lambda}{2d} \right)$$

But

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d = \frac{0.28 \times 10^{-9} \text{ m}}{\sqrt{1^2 + 1^2 + 0^2}} = \frac{0.28 \times 10^{-9} \text{ m}}{\sqrt{2}} \quad \text{(i)}$$

Substitute all the values, we get

$$\theta = \sin^{-1} \left(\frac{2 \times 0.071 \times 10^{-9}}{2 \times 0.28 \times 10^{-9}} \times \sqrt{2} \right)$$

$$\theta = \sin^{-1} \left(\frac{0.081}{0.28} \times \sqrt{2} \right) = \sin^{-1} (0.3586)$$

$$= 21°01'$$

4. A beam of X-rays is incident on an NaCl crystal with lattice plane spacing as 0.282 nm. Calculate the wavelength of X-rays, if the first-order Bragg reflection takes place at a glancing angle of 8°35′. Also, calculate the maximum order of diffraction possible.

Given data   Lattice plane spacing $d = 0.282$ nm

   Order of diffraction $n = 1$

   Glancing angle $\theta = 8°35'$

Solution   According to Bragg's equation,

$$2d \sin \theta = n\lambda$$

$$\lambda = \frac{2d \sin \theta}{n}$$

$$\Rightarrow 2 \times 0.282 \times 10^{-9} \times \frac{\sin (8°35')}{1}$$

$$= 0.0842 \text{ nm}$$

For $n_{\max}$, $\theta = 90°$ then

$$2d \sin 90° = n_{\max}\lambda$$

$\Rightarrow$
$$n_{\max} = \frac{2d}{\lambda} = \frac{2 \times 0.282 \text{ nm}}{0.0842 \text{ nm}} = 6.7$$

$$\approx 7$$

5. The Bragg's angle in the first order for (220) reflection from nickel (FCC) is 38.2º. When X-rays of wavelength 1.54Å are employed in a diffraction experiment, determine the lattice parameter of nickel.

Given data   Order of diffraction $n = 1$

       Plane of reflection $(hkl) = (220)$

       Diffraction angle, $\theta = 38.2°$

       Wavelength of X-rays, $\lambda = 1.54$ Å

Solution   According to Bragg's law,

$$2d \sin \theta = n\lambda$$

$$d = \frac{n\lambda}{\sin \theta} = \frac{1 \times 1.54}{2 \times \sin (38.2)} \text{ Å}$$

$$= \frac{1.54}{2 \times 0.6184} \text{ Å} = 1.2451 \text{ Å}$$

Then
$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$\Rightarrow$
$$a = d \times \sqrt{h^2 + k^2 + l^2} = 1.245 \text{ Å} \times \sqrt{2^2 + 2^2 + 0^2}$$
$$= 3.522 \text{ Å}$$

6. Calculate the glancing angle at (110) plane of a cubic crystal having axial length of 0.26 nm corresponding to the second-order diffraction maximum for the X-rays of wavelength 0.065 nm.

Given data   Axial length of cubic system, $a = 0.26$ nm

         Wavelength of X-rays, $\lambda = 0.065$ nm

Order of diffraction, $n = 2$

Diffraction plane $(hkl) = (110)$

Solution

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{0.26}{\sqrt{1^2 + 1^2 + 0^2}} \text{ nm}$$

$$= \frac{0.26}{\sqrt{2}} = 0.184 \text{ nm}$$

From Bragg's law,

$$2d \sin \theta = n\lambda$$

$$\sin \theta = \frac{n\lambda}{2d}$$

$$\theta = \sin^{-1}\left(\frac{n\lambda}{2d}\right) = \sin^{-1}\left(\frac{2 \times 0.065}{2 \times 0.184}\right)$$

$$= 20°40'$$

7. The Bragg's angle for reflection from the (111) plane in an fcc crystal is 19.2º for an X-ray wavelength of 1.54 Å. Complete the cube edge of the unit cell.

Given data Bragg's angle, $\theta = 19.2°$

Wavelength of X-rays $\lambda = 1.54$ Å

Order of diffraction, $n = 1$.

Diffraction plane $(h, k, l) = (111)$

Solution

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \Rightarrow \quad a = d \times \sqrt{h^2 + k^2 + l^2}$$

$$2d \sin \theta = n\lambda \quad \Rightarrow \quad d = \frac{n\lambda}{2 \times \sin \theta}$$

$$= \frac{1 \times 1.54}{2 \times \sin 19.2°} \text{ Å} = \frac{1.54}{2 \times 0.3289} \text{ Å}$$

$$= 2.3411 \text{ Å}$$

$$\Rightarrow \qquad a = 2.3411 \text{ Å} \times \sqrt{1^2 + 1^2 + 1^2}$$

$$= 2.3411 \text{ Å} \times \sqrt{3} = 4.05 \text{ Å}$$

8. Calculate the ratio $d_{100}:d_{110}:d_{111}$ for a simple cubic structure. (or) Show that in a simple cubic lattice, the separation between the successive lattice points (100), (110) and (111) are in the ratio of $1:\dfrac{1}{\sqrt{2}}:\dfrac{1}{\sqrt{3}}$.

Given data Crystal planes $(hkl) = (100), (110), (111)$

Solution

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d_{100} = \frac{a}{\sqrt{1^2 + 0^2 + 0^2}} = a$$

$$d_{110} = \frac{a}{\sqrt{1^2 + 1^2 + 0^2}} = \frac{a}{\sqrt{2}}$$

$$d_{111} = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{a}{\sqrt{3}}$$

$$d_{100} : d_{110} : d_{111} = a = \frac{a}{\sqrt{2}} = \frac{a}{\sqrt{3}} = 1 = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{3}}$$

9. Monochromatic X-rays of wavelength 1.5 Å is incident on a crystal face having an interplanar spacing of 1.6 Å. Find the highest order for which Bragg's reflection maximum can be seen.

Given Data Wavelengths of X-rays $\lambda = 1.5$ Å

  Interplanar spacing $d = 1.6$ Å

Solution From Bragg's law, $2d \sin \theta = n\lambda$

  For n_{max} $\sin \theta = \sin 90° = 1$

∴ $n_{max} = \frac{2d}{\lambda} = \frac{2 \times 1.6}{1.5} = 2.133$

  Maximum possible diffraction order = 2

10. What is the angle at which the third-order reflection of X-rays of 0.79 Å wavelength can occur in a calcite crystal of 3.04×10^{-10} spacing?

Given data Wavelength of X-ray $\lambda = 0.79$ Å

                                        $= 0.79 \times 10^{-10}$ m

                Interplanar spacing, $d = 3.04 \times 10^{-10}$ m

                        Order $n = 3$

Solution From Bragg's law, $2d \sin \theta = n\lambda$

$$\theta = \sin^{-1} \left(\frac{n\lambda}{2d} \right)$$

$$\theta = \sin^{-1} \left(\frac{3 \times 0.79 \times 10^{-10}}{2 \times 3.04 \times 10^{-10}} \right)$$

$$\theta = \sin^{-1} (0.3898)$$

$$\theta = 22.94°$$

11. Calculate the number of atoms per unit cell of a metal with a lattice parameter 2.9 Å. Molecular weight 55.85, density = 7870 kgm⁻³ and Avagadro number $6.02 \times 10^{+23}$ mol⁻¹.

Given data   Lattice parameter $a = 2.9$ Å
Molecular weight = 55.85 kg/m³
Density $r = 7870$ kgm⁻³
Avagadro number $N_A = 6.02 \times 10^{26}$ kg/mol.
Number of atoms per $m^3 (n) = ?$

Solution
$$a^3 = \frac{nM}{\rho N_A}$$

$$n = \frac{a^3 \rho N_A}{M} = \frac{(2.9 \times 10^{-10})^3 \times 7870 \times 6.02 \times 10^{26}}{55.85}$$

$$n \simeq 2$$

12. x-ray of wavelength 1.5418 Å are diffracted by (111) planes in a crystal at an angle 30º in the first order. Calculate the interatomic raping?

Given data   x-ray wavelength $\lambda = 1.5418$ Å $= 1.5418 \times 10^{-10}$ m.
$$\theta = 30º$$
$$n = 1$$

Solution   Bragg's law $2d \sin \theta = n\lambda$

$\therefore$
$$d = \frac{n\lambda}{2 \sin \theta} \Rightarrow d = \frac{a}{(h^2 + k^2 + l^2)^{1/2}}$$

$$d = \frac{1.5418 \times 10^{-10}}{2 \sin (30º)} = 1.5418 \times 10^{-10}$$

$$a = d(h^2 + k^2 t + l^2)^{1/2} = 1.5418 \times 10^{-10} (1^2 + 1^2 + 1^2)^{1/2}$$
$$= 2.670 \text{ Å}$$

13. Lithium crystallises in BCC structure. Calculate the lattice constant given that the atomic weight and density for lithium 6.94 and 530 kg/m³ respectively.

Given data   Atomic weight, $M = 6.94$ amu.
Density $d = 530$ kg/m³
No. of atoms per unit cell, $n = 2$
Avagadro number $= 6.02 \times 10^{26}$ kg/mol.
Lattice constant $a = ?$

Formula $a = \left(\dfrac{nM}{\rho N_A}\right)^{1/3} = \left(\dfrac{2 \times 6.94}{530 \times 602 \times 10^{26}}\right)^{1/3}$

$a = (4.35 \times 10^{-29})^{1/3} = 3.516 \times 10^{-10}$

$a = 3.52$ Å

14. Ge crystallises in diamond (form) structure with 8 atoms per unit cell. If the lattice constant is 5.62 Å, calculate its density?

Given data No. of atoms per unit cell, $n = 8$

Lattice constant, $a = 5.6$ Å $= 5.6 \times 10^{-10}$ m.

Atomic weight of Ge, $M = 710.59$ amu.

$$N = 6.02 \times 10^{26} \text{ k/mol.}$$

$$a = \left(\frac{nM}{\rho N_A}\right)^{1/3} \Rightarrow a^3 = \frac{nM}{\rho N_A} \Rightarrow \rho = \frac{nM}{a^3 N_A}$$

$$\rho = \frac{8 \times 710.59}{(5.6 \times 10^{-10})^3 (6.02 \times 10^{26})} = \frac{5684.72}{1057.208 \times 10^{-4}}$$

Density $\rho = 5.38 \times 10^4$ kg/m^3.

15. A quartz crystal has a thickness of 4×10^{-3} m and density 3×10^3 kg m^{-3}. Calculate its fundamental frequency. Given the Young's modulus of the crystal is 8.2×10^{10} Nm^{-2}.

Given data

$$\text{Thickness } t = 4 \times 10^{-3} \text{ m}$$

$$\text{Density } \rho = 3 \times 10^3 \text{ kg m}^{-3}$$

$$\text{Young's modulus } y = 8.2 \times 10^{10} \text{ Nm}^{-2}$$

$$\text{Frequency } f = \frac{n}{2t}\sqrt{y/\rho}$$

For fundamental frequency $n = 1$

$$\therefore \qquad f = \frac{1}{2t}\sqrt{y/\rho}$$

$$= \frac{1}{2 \times 4 \times 10^{-3}}\sqrt{\frac{8.2 \times 10^{10}}{3 \times 10^3}}$$

$$= 0.653 \text{ MHz}$$

MULTIPLE CHOICE QUESTIONS

1. X-rays are used to study the crystal structure by means of diffraction because
   (a) the X-rays are electromagnetic waves
   (b) the wavelength of X-rays is of the same order of interatomic spacing
   (c) the wavelength of X-rays is larger than the interplanar distance
   (d) the X-rays can be scattered by ions in the lattice

2. The smallest part of the crystal is
   - (a) primitive
   - (b) crystal
   - (c) basis
   - (d) unit cell

3. Lattice + Basis =
   - (a) unit cell
   - (b) crystal
   - (c) amorphous solid
   - (d) None of these

4. The possible number of Bravais lattices is
   - (a) 5
   - (b) 14
   - (c) 7
   - (d) 21

5. The number of atoms per unit cell in a simple cubic lattice is
   - (a) 1
   - (b) 2
   - (c) 4
   - (d) 8

6. The number of atoms per unit cell in a bcc lattice is
   - (a) 1
   - (b) 2
   - (c) 4
   - (d) 8

7. The number of atoms per unit cell in an fcc lattice is
   - (a) 1
   - (b) 2
   - (c) 4
   - (d) 8

8. Which of the following has the greatest packing fraction?
   - (a) Simple cubic
   - (b) Body centred cubic
   - (c) Face centred cubic
   - (d) All have equal packing fraction

9. The three axes of a crystal are mutually perpendicular but all lattice parameters are unequal. The crystal system is
   - (a) cubic
   - (b) tetragonal
   - (c) orthorhombic
   - (d) hexagonal

10. The coordination number of a simple cubic is
    - (a) 6
    - (b) 8
    - (c) 12
    - (d) 3

11. The coordination number of bcc is
    - (a) 6
    - (b) 8
    - (c) 12
    - (d) 13

12. The coordination number of fcc is
    - (a) 6
    - (b) 8
    - (c) 12
    - (d) 3

13. A plane cuts the x-axis at $2a$, the y-axis at $3b$ and z-axis at $4c$. Then Miller indices are
    - (a) (234)
    - (b) (643)
    - (c) (634)
    - (d) (346)

14. The Miller indices of the plane parallel to z-axis are
    - (a) (001)
    - (b) (101)
    - (c) (110)
    - (d) (100)

15. If Miller indices are (101), then it represents
    (a) plane parallel to x-axis
    (b) plane parallel to y-axis
    (c) plane parallel to z-axis
    (d) plane parallel to x-axis and z-axis

16. Interplanar distance for (111) plane is

    (a) $a\sqrt{3}$ (b) $\dfrac{a}{\sqrt{3}}$

    (c) $3a$ (d) $\dfrac{a}{3}$

17. In a simple cubic lattice $d_{111}: d_{110}: d_{100}$
    (a) $\sqrt{2} : \sqrt{3} : \sqrt{6}$ (b) $\sqrt{6} : \sqrt{3} : \sqrt{2}$
    (c) $\sqrt{2} : \sqrt{6} : \sqrt{3}$ (d) $\sqrt{3} : \sqrt{6} : \sqrt{2}$

18. Which radiation produces diffraction as they pass through the crystals?
    (a) X-rays   (b) IR-rays
    (c) γ-rays   (d) ultraviolet rays

19. Bragg's reflection can occur only for wavelengths
    (a) $\lambda \le 2d$ (b) $\lambda \le d$
    (c) $\lambda \ge 2d$ (d) for all wavelengths

20. The maximum wavelength of x-rays which can be diffracted by a crystal of spacing $d = 2.5$Å is
    (a) 2.5 Å   (b) 1.25 Å
    (c) 5 Å   (d) 10 Å

21. Laue's method is used for the determination of
    (a) crystal structure   (b) electric charge distribution
    (c) unit cell   (d) values of primitives

22. In the powder method, incident X-rays are reflected back when the incident angle $\theta =$

    (a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$

    (c) $\dfrac{3\pi}{4}$ (d) π

23. Single crystal is used in
    (a) Bragg's method   (b) powder method
    (c) Laue's method   (d) All of these

24. The relation between atomic radius 'r' and lattice constant 'a' in the case of simple cubic structure is

    (a) $a = 2r$ (b) $a = \dfrac{r}{2}$

    (c) $\dfrac{4r}{\sqrt{3}}$ (d) $a = 2\sqrt{2}r$

25. In a crystal if the primitives $a = b \neq c$ and interfacial angles $\alpha = \beta = \gamma = 90°$ then it belongs to the _____ system.
    - (a) cubic
    - (b) tetragonal
    - (c) orthorhombic
    - (d) triclinic

26. In a crystal if the primitives $a = b \neq c$ and interfacial angles $\alpha = \beta = 90°$, $\gamma = 120°$ then it belongs to the _____ system.
    - (a) cubic
    - (b) tetragonal
    - (c) monoclinic
    - (d) hexagonal

27. In a crystal if the primitives $a \neq b \neq c$ and interfacial angles $\alpha = \beta = \gamma = 90°$ then it belongs to the _____ system.
    - (a) cubic
    - (b) tetragonal
    - (c) orthorhombic
    - (d) triclinic

28. If a is the lattice constant for a bcc crystal structure, the nearest neighbour distance is given by
    - (a) $\dfrac{a}{\sqrt{2}}$
    - (b) $\dfrac{a\sqrt{3}}{2}$
    - (c) $\dfrac{a\sqrt{3}}{4}$
    - (d) a

29. If a is the lattice constant for an fcc crystal structure, the nearest neighbour distance is given by
    - (a) $\dfrac{a}{\sqrt{2}}$
    - (b) $\dfrac{a\sqrt{3}}{2}$
    - (c) $\dfrac{a\sqrt{3}}{4}$
    - (d) a

30. In a cubic crystal a plane makes intercepts, 1, −3, 1 on the x, y and z axes respectively. The Miller indices of the plane are
    - (a) $(\bar{3}\,1\,\bar{3})$
    - (b) $(3\bar{1}3)$
    - (c) $(1\bar{3}1)$
    - (d) $(\bar{1}3\bar{1})$

31. For a cubic system, if 'a' is the lattice constant then the interplanar separation for (111) planes is
    - (a) $\dfrac{a}{\sqrt{3}}$
    - (b) a
    - (c) $\sqrt{3}a$
    - (d) $3a$

32. For a cubic system, if 1Å is the lattice constant then the interplanar separation for (212) plane is
    - (a) 5 Å
    - (b) 0.33 Å
    - (c) 9 Å
    - (d) 0.11 Å

33. Using powder diffraction _____ of a crystal can be determined.
    - (a) the interatomic spacing
    - (b) the interplanar spacing
    - (c) Both (a) and (b)
    - (d) None of these

34. Laue's method is useful to decide the _____ and orientation of the internal arrangement of atoms/molecules in the crystal.
    - (a) cell parameter
    - (b) crystal symmetry
    - (c) Both (a) and (b)
    - (d) None of these

35. Miller indices of the crystal plane parallel to *y* and *z* axes are
    - (a) (100)
    - (b) (010)
    - (c) (001)
    - (d) (110)

36. If the lattice parameter of a cubic crystal is 1 nm and the distance between two parallel planes is 0.58 nm, the Miller indices of the planes are
    - (a) (110)
    - (b) (101)
    - (c) (011)
    - (d) (111)

37. Lattice constant of cospper is 0.39 nm. The distance between (221) planes is
    - (a) 0.08 nm
    - (b) 0.13 nm
    - (c) 0.43 nm
    - (d) 1.95 nm

38. The crystal direction that connects the origin and $\left(1, \frac{1}{2}, \frac{1}{3}\right)$ points is
    - (a) (122)
    - (b) (113)
    - (c) (223)
    - (d) (123)

39. [211] is the crystal direction that connects the origin (0, 0, 0) and
    - (a) (1, 2, 2)
    - (b) $\left(\frac{1}{2}, 1, 1\right)$
    - (c) (1, 1, 2)
    - (d) $\left(1, \frac{1}{2}, \frac{1}{2}\right)$

40. A cubic system has lattice parameters of
    - (a) $a = b = c$
      $\alpha = \beta = \gamma = 90°$
    - (b) $a = b \neq c$
      $\alpha = \beta = \gamma = 90°$
    - (c) $a \neq b \neq c$
      $\alpha = \beta = \gamma = 90°$
    - (d) $a \neq b \neq c$
      $\alpha \neq \beta \neq \gamma \neq 90°$

41. The materials having different properties along different directions are called
    - (a) anisotropic
    - (b) isotropic
    - (c) amorphous
    - (d) None of these

42. There are ____ basic crystal systems.
    - (a) six
    - (b) five
    - (c) seven
    - (d) four

43. The number of lattice points in a primitive cell are
    - (a) 2
    - (b) 3
    - (c) 1
    - (d) 0

44. Atomic packing facfor of simple cubic structure is
    - (a) 0.52
    - (b) 1.00
    - (c) 0.74
    - (d) 0.68

45. The atomic packing factor of BCC structure is
    - (a) 0.74
    - (b) 0.68
    - (c) 1.00
    - (d) 0.52

46. The atomic packing factor of FCC structure is
    - (a) 0.52
    - (b) 1.00
    - (c) 0.68
    - (d) 0.74

47. The relation between atomic radius 'r' and lattice constant 'a' in the case of BCC structure is
    - (a) $a = 2\sqrt{2}\, r$
    - (b) $a = \dfrac{\sqrt{3}}{4} r$
    - (c) $a^2 = \dfrac{4}{\sqrt{3}} r$
    - (d) $a = 2r$

48. The relation between atomic radius r and lattice constant a in the case of FCC structure is
    - (a) $a = 2\sqrt{2}\, r$
    - (b) $a = 2r$
    - (c) $a = \dfrac{4}{\sqrt{3}} r$
    - (d) $a = \dfrac{r}{2\sqrt{2}}$

49. The nearest neighbour distance in the case of SCC structure is
    - (a) $\dfrac{a}{\sqrt{2}}$
    - (b) a
    - (c) $\dfrac{a\sqrt{3}}{4}$
    - (d) $\dfrac{a\sqrt{3}}{2}$

50. In a crystal if the primitives $a = b = c$ and interfacial angles $\alpha = \beta = \gamma \neq 90º$ then it belongs to the system.
    - (a) trigonal
    - (b) hexagonal
    - (c) cubic
    - (d) tetragonal

51. In a crystal if the primitives $a \neq b \neq c$ and interfacial angles $\alpha = \beta = 90 \neq \gamma$, then it belongs to the system.
    - (a) hexagonal
    - (b) tetragonal
    - (c) trigonal
    - (d) monoclinic

52. The direction that connects the origin and (1/3, 1/3, 2/3) point is
    - (a) [322]
    - (b) [212]
    - (c) [112]
    - (d) [211]

53. The planes (112) and (224) are
    - (a) perpendicular to each other
    - (b) intersecting at an angle other than 90º
    - (c) parallel to each other
    - (d) None of these

54. If (123) are the miller indices of the plane, the intercepts made by the plane on the three crystallographic axes are
    - (a) $(a, 2b, 3c)$
    - (b) $(6a, 3b, 2c)$
    - (c) $(3a, 2b, c)$
    - (d) None of these

55. The miller indices of the plane parallel to the *x*-axis and *y*-axis are
    - (a) (110)
    - (b) (011)
    - (c) (100)
    - (d) (001)

56. In a simple cubic lattice $1/d_{100} : 1/d_{110} : 1/d_{111}$ is
    - (a) $\sqrt{2} : \sqrt{1} : \sqrt{3}$
    - (b) $\sqrt{3} : \sqrt{1} : \sqrt{2}$
    - (c) $\sqrt{1} : \sqrt{2} : \sqrt{3}$
    - (d) $\sqrt{1} : \sqrt{3} : \sqrt{2}$

57. In a body centred cubic lattice $1/d_{100} : 1/d_{110} : 1/d_{111}$ is
    - (a) $1 : \dfrac{1}{\sqrt{2}} : \sqrt{3}$
    - (b) $1 : \sqrt{2} : \dfrac{\sqrt{3}}{2}$
    - (c) $\sqrt{6} : \sqrt{3} : \sqrt{2}$
    - (d) $\sqrt{3} : \sqrt{2} : \sqrt{1}$

58. In a face centred cubic lattice $1/d_{100} : 1/d_{110} : 1/d_{111}$ is
    - (a) $\sqrt{1} : 1/\sqrt{2} : \sqrt{3}$
    - (b) $1 : \sqrt{2} : \sqrt{3}/2$
    - (c) $1 : \sqrt{2} : \sqrt{3}$
    - (d) $1 : 2 : 3$

59. The miller indices (*hkl*) represents
    - (a) the direction
    - (b) a plane
    - (c) system of planes
    - (d) a set of parallel plane

60. For a cubic system with lattice constant *a*, if (hkl) is the miller indices, then the interplanar spacing between adjacent planes is given by
    - (a) $d = \dfrac{a}{h^2 + k^2 + l^2}$
    - (b) $d = \sqrt{\dfrac{a}{h^2 + k^2 + l^2}}$
    - (c) $d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$
    - (d) $d = \dfrac{a^2}{h^2 + k^2 + l^2}$

61. For a cubic system, if 1 Å is the lattice constant, then the interplanar separation for (212) planes is
    - (a) 5 Å
    - (b) 0.11 Å
    - (c) 0.33 Å
    - (d) 9 Å

62. A plane cuts the three axes at 4*a*, 4*b* and 4*c* where *a*, *b* and *c* are lattice constants along *x*, *y* & *z* directions. The miller indices of this plane is
    - (a) (111)
    - (b) (100)
    - (c) (000)
    - (d) (101)

63. For a cubic unit cell, the lattice constant is given by
    - (a) $a = \left(\dfrac{\eta M}{\rho N_A} \right)$
    - (b) $a = \left(\dfrac{\rho n}{NM} \right)$
    - (c) $a = \left(\dfrac{\eta M}{\rho N_A} \right)^3$
    - (d) $a = \left(\dfrac{\eta M}{\rho N_A} \right)^{1/3}$

64. In powder method, if *R* is the radius of the of the Debye–Scherrer Camera and *S* is the distance between two diffraction lines corresponding to a particular plane then the full opening angle of the diffracted cone 4θ is related to *S* and *R* by
    - (a) $4\theta = SR$ radian
    - (b) $4\theta = \dfrac{S}{R}$ radian
    - (c) $4\theta = \dfrac{R}{S}$ radian
    - (d) $4\theta = \left(\dfrac{S}{R} \right)^2$ radian.

65 The coordinations number of NaCl structure is

    (a) 4   (b) 5

    (c) 6   (d) 7

66. The effective number of atoms per unit cell of diamond structure is

    (a) 4   (b) 6

    (c) 8   (d) 10

67. Atomic packing factor of diamond structure is

    (a) 0.34   (b) 0.48

    (c) 0.54   (d) 0.74

68. Vacancy defects corresponds to

    (a) point defects   (b) line defects

    (c) surface defects   (d) volume defects

69. Shottky defects is a special case of

    (a) Interstitial defects   (b) Vacancy defects

    (c) Electronic defects   (d) Volume defects

70. Which of the following frequencies lies in the range of ultrasonic waves

    (a) 10 kHz   (b) 8 kHz

    (c) 6 kHz   (d) 1 MHz

71. The principle used for the production of ultrasonic waves is

    (a) Hall effects   (b) Comption effects

    (c) piezoelectric effects   (d) Photoelectric effects

72. Ultrasonic waves can be detected by

    (a) Kundt's tube method   (b) Sensitive flame method

    (c) thermal detector method   (d) All of these

73. A quartz crystal of thickness *t*, with Young's modulus *y* and density ρ has the fundamental frequency of vibration given by

    (a) $f = \dfrac{1}{2t}\sqrt{\dfrac{y}{\rho}}$ (b) $f = 2t\sqrt{\dfrac{y}{\rho}}$

    (c) $f = \dfrac{1}{2t}\sqrt{\dfrac{\rho}{y}}$ (d) $f = \dfrac{1}{t}\sqrt{\dfrac{\rho}{y}}$

74. The velocity of ultrasonic waves in air is

    (a) 120 ms^{-1} (b) 420 ms^{-1}

    (c) 330 ms^{-1} (d) 480 ms^{-1}

75. What will be the wave length of ultrasonic waves in air, if the frequency is 20 kHz?

    (a) $2.65 \times 10^{-2} \text{ m}$ (b) $1.65 \times 10^{-2} \text{ m}$

    (c) $0.65 \times 10^{-2} \text{ m}$ (d) $3.65 \times 10^{-2} \text{ m}$

Answers

1. (b)	2. (d)	3. (b)	4. (b)	5. (a)	6. (b)
7. (c)	8. (c)	9. (c)	10. (a)	11. (b)	12. (c)
13. (b)	14. (c)	15. (b)	16. (b)	17. (a)	18. (a)
19. (a)	20. (c)	21. (b)	22. (b)	23. (c)	24. (a)
25. (b)	26. (d)	27. (c)	28. (b)	29. (a)	30. (b)
31. (a)	32. (b)	33. (b)	34. (b)	35. (a)	36. (d)
37. (b)	38. (d)	39. (b)	40. (a)	41. (a)	42. (c)
43. (c)	44. (a)	45. (b)	46. (d)	47. (c)	48. (a)
49. (b)	50. (b)	52. (c)	53. (c)	54. (b)	55. (d)
56. (c)	57. (a)	58. (b)	59. (d)	60. (c)	61. (c)
62. (a)	63. (d)	64. (b)	65. (c)	66. (c)	67. (a)
68. (a)	69. (b)	70. (d)	71. (c)	72. (d)	73. (a)
74. (c)	75. (b)				

Fill in the Blanks

1. An infinite array of periodically arranged points in three dimension is called _____.
2. For crystals the physical properties vary with direction and therefore are called _____ substances.
3. Amorphous substances have no directional properties and therefore are called _____ substances.
4. Lattice + basis _____.
5. The unit cell when repeated in space indefinitely, generates _____.
6. Cubic crystal system has _____ Bravais lattices.
7. The unit cell having only one lattice point is called _____.
8. The total number of space lattices possible are _____.
9. The total number of crystal systems are _____.
10. Atomic packing factor is the ratio of volume occupied by the atoms in unit cell to the _____.
11. Body centred cubic structure has coordination number _____.
12. Atomic packing factor of simple cubic crystal system is _____.
13. Separation between adjacent lattice planes in a cubic crystal is given by _____.
14. In laue X-ray diffraction method, the specimen used is _____.
15. All equally spaced parallel planes have the same _____.
16. If the defect is confined to a point or atom in the crystal then the defect is known as _____.
17. Frenkel defects is a special case of _____.
18. If the edge dislocation is above the slip plane then it becomes _____.
19. If the edge dislocation is below the slip plane then it becomes _____.

20. The strengths of dislocation is described by a parameter known as _____.
21. The audible range of sound waves is _____.
22. Piezoelectric generators are used for the production of _____.
23. Ultrasonic waves have high energy due to their _____ frequency.
24. The speed of propagates of ultrasonic waves _____ with the increase of frequency.

Answers to Fill in the blanks

1. Space Lattice	2. Anisotropic
3. Isotropic	4. Crystal structure
5. Space Lattice	6. 3
7. Perimitive cell	8. 14
9. 7	10. Total volume in the unit cell
11. 8	12. 0.52
13. $d = \dfrac{a}{\sqrt{h^2 + k^2 + l^2}}$	14. Single Crystal
15. miller indices.	16. Point defects
17. Interstitial defects	18. Positive edge dislocation
19. Negative edge dislocations	20. Burger vector
21. 20 Hz – 20 kHz	22. Ultrasonic waves
23. high	24. Increases

True or False

1. The unit cell having two lattice point is called primitive cell. (T/F)
2. Orthorhombic crystal system has four bravais lattices. (T/F)
3. Amorphous substances have wide range of melting point. (T/F)
4. Copper, silver and aluminum are examples of nonmetallic crystals. (T/F)
5. BCC crystal structure has coordination number 12. (T/F)
6. In anisotropic crystals, the physical properties do not vary with direction. (T/F)
7. The number of atoms belonging to the unit cell of FCC structure is 4. (T/F)
8. In Bragg's law $2d \sin \theta = n\lambda$, d is the distance between two atoms. (T/F)
9. The wavelength λ should not exceed twice the inter planar spacing for diffraction to occur. (T/F)
10. The number of atoms belonging to the unit cell of hexagonal close packed structure is 8. (T/F)
11. In Laue method of X-ray diffraction, mono chromatic X-ray is used. (T/F)
12. Powder X-ray diffraction method is used for determinations of crystals structure. (T/F)

Answers to True or False

1. F	2. T	3. T	4. F	5. F	6. F
7. T	8. F	9. T	10. F	11. F	12. T

REVIEW QUESTIONS

Short Answer Questions

1. What are space lattices?
2. What is unit cell?
3. What are lattice parameters?
4. Mention the lattice parameter of cubic crystal alongwith its diagram.
5. Mention the lattice parameters of tetragonal crystal alongwith its diagram.
6. Mention the lattice parameters of orthorhombic crystal alongwith its diagram.
7. Mention the lattice parameters of monoclinic crystal alongwith its diagram.
8. Mention the lattice parameters of triclinic crystal alongwith its diagram.
9. Mention the lattice parameters of rhombohedral crystal alongwith its diagram.
10. Mention the lattice parameters of hexagonal crystal alongwith its diagram.
11. What is primitive lattice?
12. What is body centred lattice?
13. What is face centred lattice?
14. What is base centred lattice?
15. Mention the expression for lattice constant of cubic crystal.
16. Define coordination number.
17. Define nearest neighbour distance.
18. Define atomic packing factor.
19. How do you find the effective number of atoms per unit cell for simple cubic structure?
20. How do you find the effective number of atoms per unit cell for BCC structure?
21. How do you find the effective number of atoms per unit cell for FCC structure?
22. How do you find the coordination number for simple cubic structure?
23. How do you find the coordination number for BCC structure?
24. How do you find the coordination number for FCC structure?
25. Find nearest neighbour distance for simple cubic structure.
26. Find nearest neighbour for BCC structure.
27. Find nearest neighbour for FCC structure.

28. Find the atomic packing factor for simple cubic.
29. Find the atomic packing factor for BCC.
30. Find the atomic packing factor for FCC.
31. How do NaCl crystal structure is formed?
32. Find the coordination number for NaCl structure.
33. How do diamond crystal structure is formed?
34. Find the coordination number of diamond structure.
35. Find the effective number of atoms per unit cell of diamond structure.
36. Find the nearest neighbour distance for diamond structure.
37. Find the atomic packing factor for diamond structure.
38. Define Miller indices.
39. Mention the procedure to find miller indices.
40. How do you find the direction of lattice point?
41. How do you find the direction of crystal plane?
42. Mention the expression for interplannar spacing for a cubic crystal.
43. State Bragg's law and mention its expression.
44. What are the disadvantages of Laue method?
45. What are the advantages of powder method?
46. What is point defect?
47. What is vacancy defect?
48. What is interstitial defect?
49. What is Schottky defect?
50. What is Frenkel defect?
51. What is compositional defect impurity?
52. What is substitutional defect?
53. What is interstitial impurity defect?
54. What is line defect?
55. What is edge dislocation?
56. What is screw dislocation?
57. What is Burger vector?
58. What are the main use of ultrasonic waves?
59. What is piezoelectric effect?
60. What are the basic principle involved in the non destructive testing by using ultrasonic waves?
61. Mention any one method of detection of ultrasonics?

Essay Type Questions

1. Describe seven crystal systems with diagrams.
2. Describe the Bravis lattices.

3. Explain the crystal structure of simple cubic.
4. Explain the crystal structure of BCC.
5. Explain the crystal structure of FCC.
6. Describe NaCl crystal structure.
7. Describe diamond crystal structure.
8. Describe the procedure to find the Miller indices and mention its significance.
9. How do you find the crystal directions of a lattice point and crystal plane.
10. State and explain Bragg's law of X-ray diffraction.
11. Describe Laue's method for determination of crystal structure.
12. Describe powder method for determination of crystal structure.
13. Derive the expression for inter planner spacing in cubic crystal.
14. Explain various point defect.
15. Explain screw and edge dislocations.
16. How do you find Burger vectors for screw and edge dislocation?
17. Mention the properties and detection of ultrasonic waves.
18. Describe the production of ultrasonic waves by piezoelectric method.
19. Explain the role of ultrasonic in nondestructive testing.

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Chapter 6:   Quantum Mechanics and
Free Electron Theory

CHAPTER 6

QUANTUM MECHANICS AND FREE ELECTRON THEORY

INTRODUCTION

The phenomena like interference, diffraction, polarisation of light, etc., are explained on the basis of wave nature of light. On the other hand, photoelectric effect, Compton effect, etc., are explained on the basis of particle nature of light. Thus, radiation or light wave has dual nature, both wave and particle. The concept of dual nature of radiation can be understood by knowing the relation between the particle as well as wave characteristics.

6.1   MATTER WAVES

If a certain amount of quantity occupies a certain space then it becomes a particle. The particle has mass. As it is occupying a certain space, its position can be identified. When its position changes then it has velocity. Due to its mass and velocity, the particle has momentum and energy. From all these items it is clear that the characteristic parameters of a particle are (1) mass, (2) velocity, (3) momentum, and (4) energy.

A wave is nothing but spreading of disturbance in a medium. Thus the characteristics of waves are (1) Amplitude, (2) time period, (3) frequency, (4) wavelength, (5) phase, and (6) intensity.

To explain the phenomena which involves both particle and wave nature, we use Planck's quantum theory. According to the quantum theory, the emission of radiation will be in the form of photons. A photon has a velocity of light and mass in motion. Thus, it has momentum and energy. That means a photon behaves as a particle. The energy of a photon is given by $E = h\nu$, where ν is the frequency of radiation. Along with frequency, the other parameters attribute a wave nature to photon, i.e., a photon has dual nature of behaving—particle, as well as wave. This concept of dual nature of radiation provoked de Broglie to suggest the idea of matter waves, i.e., particles like electron, proton, neutron, atom or molecule have an associated wave with them.

6.2   de BROGLIE HYPOTHESIS

de Broglie extended the idea of dual nature of radiation to matter assuming matter possesses wave as well as particle nature.

The associated wave is known as *de Broglie matter wave* and its wavelength as *de Broglie wavelength*. This is the de Broglie hypothesis.

Figure 6.1   Motion of particle (photon)

Consider a photon of mass 'm', moving with velocity of light 'c'. Then the energy of the photon is given by

$$E = h\nu \tag{1}$$

where h is Planck's constant, and ν is frequency of photon.

From Eq. (1), it is clear that the photon is associated with a wave. From Einstein's mass energy relation,

$$E = mc^2 \tag{2}$$

From Eqs. (1) and (2), we have

$$h\nu = mc^2 \tag{3}$$

If λ is the wavelength of the photon

$$\nu = \frac{c}{\lambda}$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} = \frac{h}{p} \tag{4}$$

where p is the momentum of the photon.

From the above concept, it is clear that when a particle of mass 'm' moves with a velocity 'v' then it is associated with a wave of wavelength λ known as de Broglie wavelength, given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \tag{5}$$

The above equation is known as the *de Broglie wave equation* for wavelength of matter waves.

6.2.1   de Broglie Wavelength in Terms of KE

The KE of a moving particle

$$E = (1/2)mv^2$$

$$= \frac{1}{2m}\, m^2v^2 = \frac{p^2}{2m}$$

or $\qquad\qquad p = \sqrt{2mE}$ $\qquad\qquad$ (6)

since $p = mv$.

Substituting the value of p in Eq. (5), we get

$$\lambda = \frac{h}{\sqrt{2mE}} \qquad\qquad (7)$$

Equation (7) gives the de Broglie wavelength in terms of kinetic energy.

6.2.2   de Broglie Wavelength of Electrons

Let us consider that an electron of mass m and charge e is accelerated through a potential difference of V volts. The energy acquired by the electron is eV and is related to $1/2\, mv^2$, i.e.,

$$\frac{1}{2}\, mv^2 = eV \quad \text{or} \quad mv^2 = 2eV$$

$$m^2v^2 = 2\, meV$$

or $\qquad\qquad mv = \sqrt{2meV}$ $\qquad\qquad$ (8)

Substituting the value of mv in Eq. (5), we get

$$\text{de Broglie wavelength of electron} = \frac{h}{\sqrt{2meV}} \qquad (9)$$

Substituting the values of h, m, and e in Eq. (9), we get

$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}} = \frac{12.26 \times 10^{-10}}{\sqrt{V}}\ \text{m}$$

$$= \frac{12.26}{\sqrt{V}}\ \text{Å} \qquad\qquad (10)$$

Equation (10) gives the de Broglie wavelength of electrons.

6.2.3   Wave Number

The wave number of an electron is given by,

$$\frac{1}{\lambda} = \frac{\sqrt{2meV}}{h}\ \text{m}^{-1} \qquad\qquad (11)$$

6.2.4   Wave Velocity

The wave velocity *u* of the matter waves can be obtained from the photon energy as,

$$E = h\nu$$

or

$$\nu = \frac{E}{h} = \frac{(1/2)m v^2}{h} = \frac{p^2}{2m} \times \frac{1}{h} \qquad (12)$$

Multiplying and dividing the numerator and the denominator by *h*, we get

$$\nu = \frac{h}{2m} \times \frac{p^2}{h^2} = \frac{h}{2m} \times \frac{1}{\lambda^2} \qquad (13)$$

Therefore, the wave velocity,

$$u = \text{frequency} \times \text{wavelength}$$

$$u = \frac{h}{2m} \times \frac{1}{\lambda^2} \times \lambda$$

$$u = \frac{h}{2m\lambda} \qquad (14)$$

The wave velocity of the electron,

$$u = \frac{h}{2m\lambda}$$

6.2.5   Properties of de Broglie Waves

The de Broglie wavelength $\lambda = \dfrac{h}{mv}$

1. The lighter the particle, the greater the wavelength associated with it.
2. Lesser the velocity of the particle, longer the wavelength associated with it.
3. When $V = 0$, $\lambda = \infty$ and if $V = \infty$, $\lambda = 0$. This shows that matter waves are generated by the motion of particles.
4. Matter waves are produced when the particles in motion are charged or uncharged.
5. Matter waves are not electromagnetic waves, they are pilot waves guiding the particle.
6. Matter waves travel faster than velocity of light, i.e., $u = \dfrac{c^2}{V}$ as particle velocity (*V*) cannot exceed the velocity of light (*c*).
7. While position of a particle is confined to a particular location at any time, the matter wave associated with it has some spread as it is a wave. Thus, the wave nature of matter introduces an uncertainity in the location of the position of the particle.

6.3   HEISENBERG'S UNCERTAINTY PRINCIPLE

Consider a particle of mass 'm' moving with velocity 'v' along x direction. According to classical theory, this moving particle will have specific position and momentum at any time t i.e., its position is given by $x = vt$ and its momentum $p = mv$. Relating the position and momentum, we get

$$x = \frac{p}{m}t$$

Figure 6.2   Moving particle

From this it is clear that any time, t, particle position and momentum can be measured accurately.

According to de Broglie hypothesis, this moving particle is associated with a matter wave. As this wave has some spread in the moving region, and even the particle is some where with in the wave spread region, it is difficult to locate its exact portion. Therefore, there is an uncertainty Δx in its position. As a result, the momentum of the particle, cannot be determined precisely.

It means that the position and momentum of a matter wave associated particle cannot be simultaneously determined with accuracy. Any attempt to determine these parameters will lead to uncertainities in each of the parameter. This is known as Heisenberg's uncertainity principle for position and momentum and is stated as, "It is not possible to find simultaneously with the exact accuracy of both the position and momentum of a moving particle associated with matter wave."

Figure 6.3   Moving particle associated with matter wave

If Δx and Δp are the uncertainities in the position and momentum of a moving particle then according to Heisenberg's uncertainity principle, we get

$$\Delta x \Delta p \geqslant \frac{h}{4\pi}$$

or
$$\Delta x \, \Delta p \geqslant \frac{\hbar}{2} \quad \left[\text{where } \hbar = \frac{h}{2\pi} \right]$$

From this it is clear that, if one parameter is measured accurately then the other associated parameter can not be measured accurately and vice versa. This gives only the probability of finding the particle in a spread region in stead of certainity.

The uncertainity relation for the simultaneous measurement of energy and time is given by

$$\Delta E \, \Delta t \geqslant \frac{\hbar}{2}$$

where ΔE and Δt are the uncertainity in energy and time of a particle.

Applications

1. Using uncertainity principle, non-existence of electrons in the nucleus can be proved.

2. Existence of protons, neutrons and α particles in the nucleons is conformed by the uncertainity principle.

3. Binding energy of an electron in an atom can be calculated from this uncertainity principle.

4. Emission of radiation of light from an excited atom is also conformed by this principle.

6.4 SCHRÖDINGER'S ONE-DIMENSIONAL TIME-INDEPENDENT WAVE EQUATION

According to de Broglie hypothesis, the particle in motion is always associated with a wave. To describe the motion of a particle in terms of its associated wave, Schrödinger derived a wave equation which is termed as *Schrödinger's wave equation.*

Consider a particle of mass 'm' moving with velocity 'v' along the x direction. It is associated with a wave. The displacement of the wave is given by the wave function ψ.

Since the wave function depends upon the x coordinate of the moving particle and time 't', it is given by the complex form

$$\psi(x, t) = Ae^{i(kx - \omega t)} \tag{15}$$

where A is the amplitude
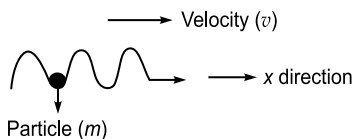
Figure 6.4   One dimensional particle (wave) motion

Differentiating Eq. (15) w.r.t, x we get

$$\frac{d\psi}{dx} = ik\, Ae^{i(kx - \omega t)}$$

Again differentiating, we have

$$\frac{d^2\psi}{dx^2} = i^2 k^2 Ae^{i(kx - \omega t)}$$

$$= i^2 k^2\, \psi$$

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

Since $k = \dfrac{2\pi}{\lambda}$, we get

$$\frac{d^2\psi}{dx^2} = \frac{-4\pi^2}{\lambda^2}\psi \tag{16}$$

de Broglie wavelength associated with the particle is

$$\lambda = \frac{h}{mv}$$

$$\frac{1}{\lambda^2} = \frac{m^2v^2}{h^2} = \frac{2m\,(1/2mv^2)}{h^2} \tag{17}$$

Let E be the total energy of the particle and V be the potential energy of the particle. Then

$$KE = \frac{1}{2}mv^2 = E - V \tag{18}$$

Substituting the above value in Eq. (17), we get

$$\frac{1}{\lambda^2} = \frac{2m}{h^2}(E - V) \tag{19}$$

Making this substitution in (15), we get

$$\frac{d^2\psi}{dx^2} = -\frac{8\pi^2 m}{h^2}(E - V)\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \tag{20}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad \left[\because \hbar = \frac{h}{2\pi}\right] \tag{21}$$

The above equation represents Schrödinger's one-dimensional time-independent wave equation.

For three-dimensional motion of the particle the above equation becomes

$$\nabla^2\psi + \frac{2m}{h^2}(E - V)\psi = 0 \tag{22}$$

where $\nabla^2 = \dfrac{d^2}{dx^2} + \dfrac{d^2}{dy^2} + \dfrac{d^2}{dz^2}$

6.4.1 Schrödinger's One-dimensional Time Dependent Wave Equation

For deriving time, dependent Schrödinger's one dimensional wave equation let us eliminate the total energy E from time, independent wave equation. The wave function is given by

$$\psi(x, t) = A e^{i(kx - \omega t)} \tag{23}$$

Differentiating the above equation w.r.t 't', we get

$$\frac{d\psi}{dt} = -i\omega A e^{i(kx - \omega t)}$$

$$= -i(2\pi\nu) A e^{i(kx - \omega t)}$$

$$= -2\pi i\nu \, \psi \qquad (\because w = 2\pi\upsilon)$$

Since

$$E = h\nu, \nu = \frac{E}{h}$$

$$\frac{d\psi}{dt} = -\frac{2\pi i E}{h}\psi$$

Since

$$\hbar = \frac{h}{2\pi}, h = 2\pi\hbar$$

$$\frac{d\psi}{dt} = -\frac{2\pi i E}{2\pi\hbar}\psi$$

$$\frac{d\psi}{dt} = \frac{-iE}{\hbar}\psi = \frac{E\psi}{i\hbar}$$

$$E\psi = i\hbar\frac{d\psi}{dt} \tag{24}$$

Substituting the value of $E\psi$ in the equation

$$\frac{d\psi}{dx^2} + \frac{2m}{\hbar}(E - V)\psi = 0$$

we get

$$\frac{d\psi}{dx^2} + \frac{2m}{\hbar^2}\left[i\hbar\frac{d\psi}{dt} - V\psi\right] = 0$$

$$\frac{d^2\psi}{dx^2} = \frac{-2m}{\hbar^2}\left[i\hbar\frac{d\psi}{dt} - V\psi\right]$$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = i\hbar\frac{d\psi}{dt} \tag{25}$$

The above equation represents time, dependent Schrödinger's one-dimensional wave equation. For three-dimensional motion, the above equation becomes

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{d\psi}{dt}$$

where
$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \qquad (26)$$

6.4.2 Physical Significance of Wave Function ψ

Consider a particle having three-dimensional motion along x, y, z coordinates.

The value of the wave function (ψ) associated with a moving particle, at a point x, y, z in space is related to finding of the particle there.

Since wave function is a complex quantity, it may be expressed in the form

$$\psi(x, y, z) = (a + ib)$$

where a and b are real functions of the variables (x, y, z, t) and $i = \sqrt{-1}$.

Figure 6.5 Particle motion (three dimension)

The complex conjugate of ψ is given by

$$\psi^*(x, y, z) = (a - ib)$$

Multiplying the above two equations, we have

$$\psi\psi^* = (a + ib)(a - ib) = a^2 + b^2$$
$$P = \psi\psi^* = |\psi(x, y, z)|^2 = a^2 + b^2 \qquad (27)$$

Thus the product of ψ and ψ^* is real and positive if $\psi \neq 0$ and is known as probability density of the particle associated with the de Broglie wave.

When the motion of a particle is confined to a volume element dv then the quantity

$$Pdv = \psi(x, y, z)\,\psi^*(x, y, z, t)dv = |\psi(x, y, z)|^2\, dv \qquad (28)$$

is the probability of finding the particle in dv, surrounding the point at (x, y, z).

For the motion of a particle in one dimension, the quantity

$$Pdx = \psi(x)\psi^*(x)dx$$
$$= |\psi(x)|^2\, dx \qquad (29)$$

is the probability of finding the particle over a small distance dx.

6.5   PARTICLE IN A ONE-DIMENSIONAL POTENTIAL BOX

Consider a particle of mass *m* moving back and forth between the walls of a one-dimensional box separated by a distance '*a*' as shown in Fig. 6.6. Let the particle travel only along the *x*-axis and is confined between $x = 0$ and $x = a$ by two infinitely hard walls so that the particle has no chance of penetrating them.
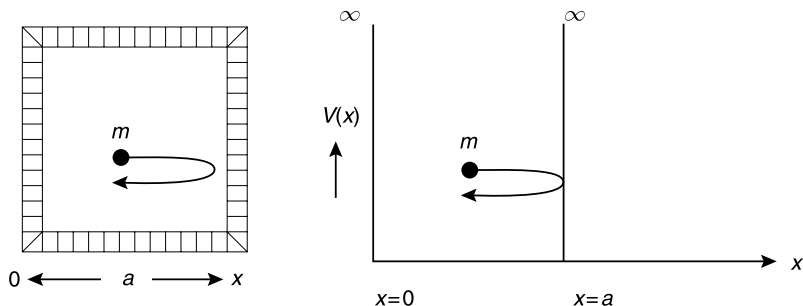
Figure 6.6   Particle in one-dimensional potential box

Thus, it does not lose energy when it collides with walls and its total energy remains constant. This box can be represented by a potential box of width *a* with potential walls of infinite height at $x = 0$ and $x = a$ so that the potential energy, *V* of the particle is infinitely high on both sides of the box and inside it, *V* is uniform. Let us assume that $V = 0$ inside the box,

$$V(x) = 0, \qquad \text{for } 0 < x < a,$$
$$V(x) = \infty, \qquad \text{for } x \leq 0 \text{ and } x \geq a.$$

Then the wave function $\psi(x)$ of the particle in the region $0 < x < a$ where $V = 0$ is described by the Schrödinger's equation

$$\frac{d^2\psi}{dx^2} + \left(\frac{2m}{\hbar^2}\right) E\psi = 0$$

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \tag{30}$$

where $\qquad\qquad K = \sqrt{\dfrac{2mE}{\hbar^2}}$

The solution for the above equation is given by

$$\psi(x) = A \sin Kx + B \cos Kx \tag{31}$$

where *A* and *B* are arbitrary constants.

Applying the first boundary condition, i.e., at $x = 0$, $\psi(x) = 0$, we get

$\therefore \qquad\qquad\qquad B = 0 \tag{32}$

Applying the second boundary condition, we have at $x = a$, $\psi(x) = \psi(a) = 0$

$\therefore \qquad\qquad \psi(a) = A \sin Ka = 0 \qquad\qquad (33)$

$A \neq 0$ since there will not be any solution. Hence, we have

$$\sin Ka = 0$$

$$Ka = n\pi$$

$$K = \frac{n\pi}{a} \qquad\qquad (34)$$

where $n = 1, 2, 3\ldots$

We cannot take $n = 0$ because for $n = 0$, $K = 0$, $E = 0$ and hence $\psi(x) = 0$ everywhere in the box. Thus a particle with zero energy cannot be present in the box, i.e., a particle in the box cannot have zero energy.

The wave functions for the motion of the particle in the region $0 < x < a$ are given by

$$\psi_n(x) = A \sin Kx$$

$$\psi_n = A \sin \frac{n\pi x}{a} \qquad\qquad (35)$$

6.5.1 Eigenvalues of Energy

We know that $K^2 = \dfrac{2mE}{h^2}$

$$E = \frac{K^2 \hbar^2}{2m}$$

Substituting the value of K, we get

$$E = \left[\frac{n^2 \pi^2}{a^2} \right] \frac{\hbar^2}{2m}$$

$$= \frac{n^2 \pi^2}{a^2} \left[\frac{h^2}{8\pi^2 m} \right] \quad \left[\because \hbar = \frac{h}{2\pi} \right] \qquad\qquad (36)$$

$$E_n = \frac{n^2 h^2}{8ma^2}$$

From the above equation, it is clear that

1. The lowest energy of the particle is obtained by putting $n = 1$

   i.e., $\qquad\qquad E_1 = \dfrac{h^2}{8ma^2} \quad$ and $\quad E_n = n^2 E_1$

   This is known as *zero point energy* of the system.

2. For $n = 1, 2, 3$, we get discrete energy values of the particle in the box.

$$E_1 = \frac{h^2}{8ma^2}$$

$$E_2 = 2^2 \frac{h^2}{8ma^2} = 4 E_1$$

$$E_3 = 3^2 \frac{h^2}{8ma^2} = 9 E_1$$

3. The spacing between the nth energy level and the next higher level increases as

$$(n + 1)^2 E_1 - n^2 E_1 = (2n + 1)E_1.$$

4. These values of E_n are known as *eigen values* of energy and the corresponding wave functions ψ_n as *eigen functions* of the particle. The integer 'n' is the quantum number of energy level E_n. Figure 6.7 shows the energy-level diagram for the particle.

Figure 6.7   Energy-level diagram of particle in a one-dimensional box

6.6   EIGENWAVE FUNCTIONS

The eigenwave function for the motion of the particle are

$$\psi_n = A \sin \frac{n\pi x}{a} \qquad \text{in the region } 0 < x < a$$

$$\psi_n = 0 \qquad \text{in the region } x \leq 0 \text{ and } x \geq a$$

The total probability that the particle is somewhere in the box must be unity.

$$\int_0^a P_x \, dx = \int_0^a |\psi_n|^2 \, dx = 1$$

$$\int_0^a A^2 \sin^2 \frac{n\pi x}{a} \, dx = 1$$

$$A^2 \int_0^a \frac{1}{2} \left[1 - \cos \frac{2\pi n x}{a} \right] dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{a}{2\pi n}\sin\frac{2\pi nx}{a}\right]_0^a = 1$$

The second term becomes zero at both the limits

$$\frac{A^2 a}{2} = 1$$

$$A = \sqrt{\frac{2}{a}}$$

The normalised wave function is

$$\psi_n = \left[\sqrt{\frac{2}{a}}\right]\sin\frac{n\pi x}{a} \qquad (37)$$

The wave functions are shown in Fig. 6.8.

The wave function ψ_1 has two nodes at $x = 0$ and $x = a$.

ψ_2 has three nodes at $x = 0$, $x = a/2$ and $x = a$.

ψ_3 has four nodes at $x = 0$, $x = a/3$, $x = 2a/3$, $x = a$.

Thus ψ_n has $(n + 1)$ nodes.

Figure 6.8 Variation of wave function

6.6.1 Probability of the Location of the Particle

The probability of finding a particle over a small distance dx at x is given by

$$P(x)dx = |\psi_n|^2\,dx$$

$$P(x)dx = \frac{2}{a}\sin^2\frac{n\pi x}{a}\,dx$$

Thus, the probability density for one-dimensional motion is

$$P(x) = \frac{2}{a}\sin^2\frac{n\pi x}{a} \qquad (38)$$

The probability density is maximum when

$$\frac{n\pi x}{a} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$x = \frac{a}{2n}, \frac{3a}{2n}, \frac{5a}{2n}$$

As shown in Fig. 6.9,

- for $n = 1$, the most probable position of the particle is at $x = a/2$
- for $n = 2$, the most probable position of the particle is at $x = a/4$ and $x = 3a/4$

Figure 6.9 Variation of probability density function

- for $n = 3$, the most probable position of the particle is at $x = a/6$, $x = 3a/6$, $x = 5a/6$

Thus, the wave functions and the probability density functions for the particle in a one-dimensional box are given by

$$\psi_1(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} \text{ and } P_1(x) = |\psi_1(x)|^2 = \frac{2}{a} \sin^2 \frac{\pi x}{a}$$

$$\psi_2(x) = \sqrt{\frac{2}{a}} \sin \frac{2\pi x}{a} \text{ and } P_2(x) = |\psi_2(x)|^2 = \frac{2}{a} \sin^2 \frac{2\pi x}{a}$$

$$\psi_3(x) = \sqrt{\frac{2}{a}} \sin \frac{3\pi x}{a} \text{ and } P_3(x) = |\psi_3(x)|^2 = \frac{2}{a} \sin^2 \frac{3\pi x}{a}$$

Energy levels   We know that for a particle in a potential box, the energy of the particle is given by

$$E_n = \frac{n^2 h^2}{8ma^2}$$

where $n = 1, 2, 3, \ldots$

In the above relation, h is Planck's constant, m is mass of the particle, a is width of the potential box. For discrete value of n, we have discrete energy values. These energies are quantised. These energies can be represented by discrete energy levels within the box.

For higher energy values, the separation between the energy levels increases. The probability of occupation of these energy levels is given by the Fermi–Dirac distribution.
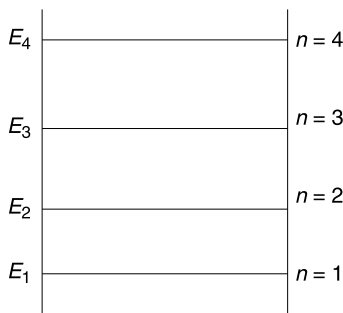
Figure 6.10   Energy levels

6.7   FREE ELECTRON THEORY

In general, the physical properties of solid are governed by the valence electrons of atoms in a solid. The mobility of these valence electrons in a solid decides the electrical, thermal and magnetic properties of solid. To study these properties electron theories were proposed. These electron theories were used to explain the structure and properties of solids.

According to free electron theory the free electrons available in the metal are responsible for its electrical conductivity based on classical mechanics. In order to overcome the draw of classical free electron theory, Quantum free electron theory was developed based on quantum laws. Later band theory was developed to explain the various properties of metals.

Classical free electron theory – Electrical conductivity
Classical free electron theory was proposed by Drude and Lorentz. According to this theory a metal is supposed to consists of +ve ions fixed in the lattice whose

free electrons move freely and randomly in the metal like gas molecules in a vessel. When an electric field is applied the free electrons experiences an electric force and gets accelerated. These accelerated electrons collide with the +ve ions loosing some of their energy and acquires a constant velocity along the metal. This constant velocity of electrons is known as drift velocity v_d. The average distance travelled by the face electron between two successive collisions is called mean free path λ. The time, taken to travel means free paths λ is known as mean free time or relation time denoted by τ.
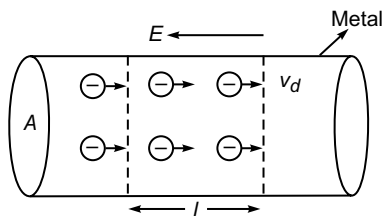
Figure 6.11   Electrical Conductivity

As shown in the Fig. 6.11, let an electric field E is maintained between two ends of a metal of area of cross section A.

Force acting on the electron in electric field $= eE$

where e is the change of an electron

Then the acceleration of electron 'a' $= \dfrac{\text{Fore}}{\text{mass}} = \dfrac{eE}{m}$

If the electron collide with a +ve ion then its drifting tendency decreases and random motion increases. At its next collision its velocity changes. On an average its velocity = acceleration × time

$$= a\,\tau$$

where τ is the time between two collisions. This average velocity is the drift velocity of electron

$\therefore$ $$v_d = a\,\tau$$

$$= \left[\frac{eE}{m}\right]\tau \tag{39}$$

Free electron theory is based on kinetic theory of gases according to which in the absence of field during collisions the kinetic velocities of electrons changes both in magnitude and direction and is assumed to be equal to root mean square velocity $\bar{c}$ of electron and the electron travel with a velocity of $\bar{c}$ before collision. If λ is the mean free path traversed by electron with a velocity $\bar{c}$ before collision. Then

$$\tau = \frac{\lambda}{\bar{c}} \tag{40}$$

$\therefore$ $$v_d = \left[\frac{eE}{m}\right]\left[\frac{\lambda}{\bar{c}}\right] \tag{41}$$

Let there are n electrons per unit volume with each of change e and mass 'm'. In a time dt, if all the electrons travels a distance l then $l_1 = v_d \, dt$

Number of electrons in the volume element

$$= \text{Volume} \times \text{Number of electrons per unit volume}$$

$$= Al \times n$$

$$= An \, v_d \, dt$$

Total change crossing in time dt = Number of electrons in the volume element $\times$ Change on the electron

$$= An \, ev_d \, dt$$

Current $i = \dfrac{\text{Total charge crossing in time } dt}{\text{time}, dt}$

$$= \frac{An e \, v_d \, \cancel{dt}}{\cancel{dt}} = Anev_d$$

Current density $J = \dfrac{i}{A} = \dfrac{\cancel{A} nev_d}{\cancel{A}} = nev_d$ (42)

$$J = ne \left[\frac{eE}{m} \right] \tau \quad \text{Substituting the value of } v_d \text{ from Eq. (50)}$$

we get

$$J = \frac{ne^2 E\tau}{m} \tag{43}$$

Conductivity $\sigma = \dfrac{J}{E} = \dfrac{ne^2 \cancel{E} \tau}{m \cancel{E}} = \dfrac{ne^2 \tau}{m}$ (44)

Resistivity $\rho = \dfrac{1}{\sigma} = \dfrac{m}{ne^2 \tau} = \dfrac{m\bar{c}}{ne^2 \lambda}$ $\left[\because \tau = \dfrac{\lambda}{c} \right]$

Mobility of electron in a metal is defined as the steady state drift velocity per unit electric field

$$\mu = \frac{v_d}{E} = \frac{e \cancel{E} \tau}{m \cancel{E}} = \frac{e\tau}{m} \tag{45}$$

$$\sigma = \frac{ne^2 \tau}{m} = ne\mu \tag{46}$$

According to kinetic theory of gases, $\overline{C}$ is given by

$$\bar{c} = \sqrt{\frac{3K_B T}{m}}$$

$\therefore$
$$\rho = \frac{m\bar{c}}{ne^2\lambda}$$

$$= \frac{m\bar{c}}{ne^2\lambda}\sqrt{\frac{3K_BT}{m}}$$

$$\rho = \frac{\sqrt{3mK_BT}}{ne^2\lambda} \tag{47}$$

$$\sigma = \frac{1}{\rho} = \frac{ne^2\lambda}{\sqrt{3mK_BT}} \tag{48}$$

The above equation represents electrical conductivity in metals.

Merits

1. It explains the electrical resistivity in a metal.
2. It explains the electrical conductivity in a metal.
3. A relationship between resistively and conductivity with temperature is resistivity established.
4. A relationship between resistivity and conductivity with mean free path of electrons is established.
5. It explains thermal conductivity of metals.
6. It provides verification of Ohm's law.

Drawbacks

1. This theory fails to explain the specific heat of metals.
2. This theory cannot explain the magnetic suspectability of metals.
3. This theory is unable to predict the accurate dependence of resistance and conductivity of metals w.r.t temperature, i.e., according to the theory, $\rho \propto T^{1/2}$ but experimentally it is observed to be $\rho \propto T$. Similarly, $\sigma \propto T^{-1/2}$ but experimentally it is $\sigma \propto T^{-1}$.
4. This theory cannot explain the electrical conductivity of nonmetals, i.e., semiconductors or insulators.
5. This theory fails to explain photoelectric effect compton effect and thermionic emission of electrons from metals.
6. Theoretical and experimental values of mean free path 'λ' does not coincide.
7. This theory is unable to explain thermal conductivity at low temperature.

Relaxation time, Mean free path and drift velocity

Relaxation time

Let at a given time $t = 0$ the average velocity of the electron in x direction is $<V_o>$. When the electric field is switched off the average velocity $<V_o>$ tends to zero,

because of collisions between electrons with +ve ions in a metal. The decreasing tendency of $<V_o>$ is according to the expression

$$<V> = <V_o>\, e^{-t/\tau} \qquad (49)$$

where τ is known as relaxation time since it gives a measure of the time, that the system takes to relax when the field is removed.

when $t = \tau$, we get $<V> = \dfrac{<V_o>}{e}$ (50)

Thus the relaxation time is defined as the time taken for the average velocity decays to $\dfrac{1}{e}$ of its initial value. The variation of $<V>$ is as shown in the Fig. 6.12.
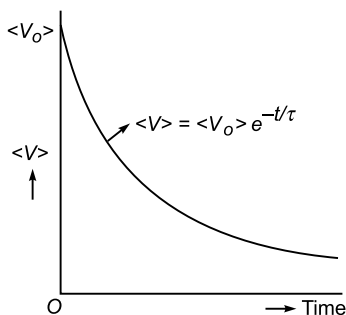
Figure 6.12   Drift velocity

Drift Velocity

When a metal is subjected to an electric field E along $-x$ direction then all the electrons experiences a force along $+x$ direction given by

Force $= -eE$

Acceleration of electrons $= \dfrac{\text{Force}}{\text{Mass}} = \dfrac{eE}{m}$

All the electrons acquire average velocity $<V>$ due to E.
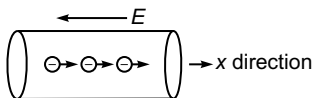
Figure 6.13   Drift Velocity

Then the change in average velocity of electrons due to interaction with field is

$$\left(\frac{d <V>}{dt}\right)_{\text{field}} = \frac{-eE}{m} \qquad (51)$$

The electrons due to collisions with the +ve ions acquires an average velocity given by $<V>$. When the field is switched off the variation of $<V>$ is given by

$$<V> = <V_o>\, e^{-t/\tau} \qquad (52)$$

where τ is the initial relaxation time of electrons and $<V_o>$ is the initial average velocity of electrons.

From the above equation it is clear that the average velocity decreases due to collisions with +ve ions, then the change in average velocity of electrons due to collisions is given by

$$\left[\frac{d}{dt} <V>\right]_{Collesion} = \frac{-<V_o>e^{-t/\tau}}{\tau}$$

$$= \frac{-<V>}{\tau} \quad (53)$$

Total rate of change of average velocity is given by

$$\left(\frac{d<V>}{dt}\right)_{Field} + \left(\frac{d <V>}{dt}\right)_{Collision} = \frac{d}{dt} < \text{constant average velocity acquired due to}$$

field and collision $V>$

$$\left(\frac{d<V>}{dt}\right)_{Field} + \left(\frac{d <V>}{dt}\right)_{Collision} = \frac{d}{dt} <V_o> = 0$$

Substituting the corresponding values

$$\frac{-eE}{m} - \frac{<V>}{\tau} = 0$$

$$<V> = -\left(\frac{e\tau}{m}\right)E$$

This steady average velocity of the electrons in the field E is known as drift velocity of electrons v_d.

$$\therefore \qquad\qquad v_d = -\left(\frac{e\tau}{m}\right)E \qquad (54)$$

Mean free path   The distance travelled by electrons before collision is known as mean free path and is given by

$$\lambda = \bar{c} \ \tau$$

where $\bar{c}$ is the root mean square velocity of electrons and τ is the relaxation time or mean free time.

Quantum free electron theory – Electrical conductivity

To overcome the drawbacks of classical free electron theory, Sommerfeld proposed quantum free electron theory. He treated electron as a quantum particle. The electron follows the principles of quantum theory. Under this, the velocity of free electrons are plotted in the velocity space with dots in Fig. 6.14 from the

origin of sphere. The maximum velocity of electron at highest occupied level, i.e., fermi level is known as the fermi velocity v_F and is represented by the radius of Fermi sphere.

In the absence of electric field, these velocities cancel each other and net velocity in all directions in zero.
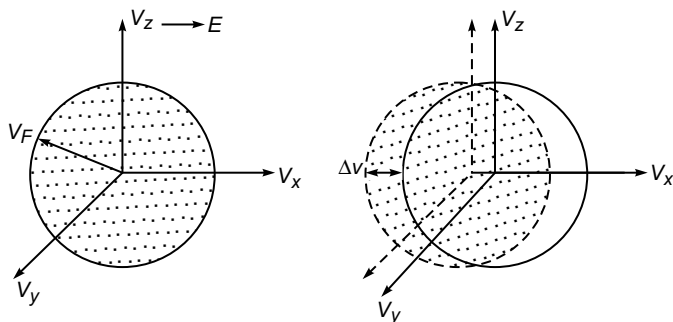
Figure 6.14   Displacement of Fermi sphere under applied electric field

If the electric field E is applied along x direction, then the electrons in the sphere experiences a force along $-x$. Direction only the electrons present near the surface (Fermi) can occupy the higher energy levels as a result the sphere slightly displaces as shown in Fig. 6.14

The relation between momentum p and wave vector $\left(k = \dfrac{2\pi}{\lambda} \right)$ is given by

$$P = \hbar k$$
$$mv = \hbar k$$
$$v = \frac{\hbar k}{m} \tag{55}$$

Differentiating the above equation w.r.t 't' we get

$$a = \frac{dv}{dt} = \frac{\hbar}{m} \frac{dk}{dt} \tag{56}$$

Force on the electron in the applied field E is given by

$$F = eE$$
$$ma = eE$$
$$\cancel{m} \times \frac{\hbar}{\cancel{m}} \frac{dk}{dt} = eE$$
$$\hbar \frac{dk}{dt} = eE$$
$$dk = \frac{eE}{\hbar} dt \tag{57}$$

Integrating the above equation, we get

$$k_{(t)} - k_{(o)} = \frac{eEt}{\hbar} \tag{58}$$

Let τ_F and λ_F are the mean collision time and mean free path of electron at Fermi surface then

$$\tau_F = \frac{\lambda_F}{V_F} \tag{59}$$

on Fermi surface, $t = \tau_F$ and $k_{(t)} - k_{(o)} = \Delta k$

Then
$$\Delta k = \frac{eE\tau_F}{\hbar} = \frac{eE}{\hbar}\left[\frac{\lambda_F}{V_F}\right] \tag{60}$$

The current density $\qquad J = ne\,\Delta v \tag{61}$

where Δv is the change in velocity

From Eq. (55), $\qquad \Delta v = \frac{\hbar\Delta k}{m} \tag{62}$

$$J = ne\,\frac{\hbar\Delta k}{m^*} \tag{63}$$

where m^* is the effective mass of electron. Substituting the value of Δk from (Eq. 62), we get

$$J = \frac{ne\hbar}{m^x}\left[\frac{eE\tau_F}{\hbar}\right]$$

$$J = \frac{ne^2 E}{m^*}[\tau_F] \tag{64}$$

But $\qquad J = \sigma E \tag{65}$

Comparing the above equations, we get

$$\sigma = \frac{ne^2\tau_F}{m^*} \tag{66}$$

Equation (66) represents electrical conductivity.

Merits of quantum free electron theory

1. It successfully explains the electrical and thermal conductivity of metals.
2. Phenomena of theormionic emission can be explained by this theory.
3. Temperature dependence of conductivity of metals can be explained by this theory.

4. It explain the specific heat of metals.

5. It explain the magnetic susceptibility of metals.

Demerits of quantum free electron theory

1. It is unable to explain the metallic properties exhibited by only certain crystals.

2. It is unable to explain why the atomic arrays in metallic crystals should prefer certain structures only.

6.8   FERMI–DIRAC DISTRIBUTION

We know that in a metal, in the absence of an electric field, free electrons move at random, similar to the behaviour of the electron gas. Since electrons are indistinguishable particles, they are known as *fermions* or *Fermi particles*. Hence, such an electron gas obeys Fermi–Dirac statistics or distribution. The Fermi–Dirac distribution describes the behaviour of free electron gas, taking into account the quantum theory and Pauli's exclusion principle.

As shown in Fig. 6.15, according to quantum theory, the electrons will have discrete energy states or levels and the occupation or distribution of electrons among these energy levels will be governed by Pauli's exclusion principle,

Figure 6.15   Occupation of energy levels at 0 K

i.e., not more than two electrons can occupy the same energy level. At absolute zero temperature, two electrons occupy the ground state and two into each state of next higher energy levels. The highest energy level occupied by electrons at absolute zero is known as Fermi energy level which divides or separates the occupied states from the unoccupied states. The energy of the Fermi level is E_F. The probability of the occupation of an energy level E by an electron at temperature T is given by

$$F(E) = \frac{1}{1 + e^{E - E_F/K_B T}}$$

where $F(E)$ is the Fermi–Dirac distribution function and K_B is the Boltzmann's constant.

6.8.1   Effect of Temperature on Fermi–Dirac Distribution Function

Figure 6.16 shows plots of the distribution function against energy at different temperatures. The special features of the plots are discussed below.

Figure 6.16 Variation of Fermi distribution function with energy

Case I – At temperature T = 0 K

(a) For $E < E_F$, $\dfrac{E - E_F}{K_B T} \to -\infty$ and $F(E) = 1$

(b) For $E > E_F$, $\dfrac{E - E_F}{K_B T} \to +\infty$ and $F(E) = \dfrac{1}{\infty} = 0$

Thus, at absolute zero temperature $F(E)$ is a step function. All the states with energies up to E_F are filled with electrons and the states with energies greater than E_F are empty. Thus the Fermi level may be defined as the topmost filled level at absolute temperature.

Case II – At temperature T > 0 K

(a) For $E < E_F$, $\dfrac{E - E_F}{K_B T}$ is negative and

$F(E) \cong 1$ for $E \ll E_F$

(b) For $E > E_F$, $\dfrac{E - E_F}{K_B T}$ is positive and

$F(E) \cong 0$ for $E \gg E_F$

(c) For $E = E_F$, $\dfrac{E - E_F}{K_B T} = 0$ and $F(E) = 1/2$.

Thus, at finite temperature the probability of occupancy of the Fermi level is ½. The Fermi function when multiplied with the density of states gives the number of states per unit volume actually occupied by the electrons.

6.9   SOURCES OF ELECTRICAL RESISTANCE

According to quantum free electron theory, the free electrons while moving at random within the metal gets scattered by +ve ions present in the metal.

The scattering of conduction electrons are due to

1. Lattice vibrations (phonon scattering)
2. By the impurities
3. By structural imperfections

Under normal condition, without applied electric field, the +ve ions will be in vibrations about their mean positions due to thermal energy. These vibrations are known as *lattice vibrations*. When the moving electrons encounter the vibrating +ve ions, they get scattered. This type of scattering is known as *phonon scattering*. Due to impurities present in the metal and due to defects, the electrons also gets scattered. The above scatterers acts as sources of electrical resistance for metals and are shown in Fig. 6.17.
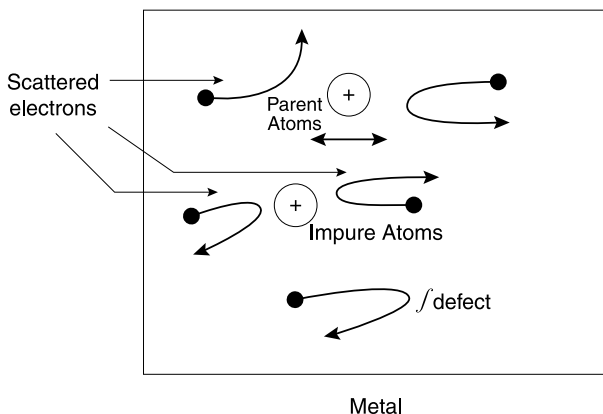
Figure 6.17   Scattering phenomena of electrons in a metal

At room temperature the resistivity of metals is largely independent of impurities or lattice defects. At very low temperatures, however, the lattice vibrations are not limiting; in fact, one would expect them to die out at 0 K and the conductivity to become infinite, in a pure, perfect crystal. As shown in Fig. 6.18(a), the resistivity in such a crystal should go as T^5 near 0 K as this is the temperature dependence of the scattering due to lattice vibrations, and the electron concentration is independent of T. No real crystal is completely pure or perfect, however, and imperfections of any kind of chemical impurities, lattice defects, even the crystal surface act as scattering centres for the electrons even in the absence of lattice vibrations. Thus, the resistivity does not fall to zero at 0 K, and the resistivity curve is displaced upward by an amount which reflects the extent to which these imperfections are present and effective as scatterers. At high temperatures, this displacement is insignificant compared to the much larger effect of the lattice vibrations. For this reason, the resistance ratio, the ratio of the resistivities at room temperature to liquid helium temperature is used as a simple measure of the purity and perfection of a metal crystal. Actually the sensitivity of the resistivity to different imperfections varies widely, as seen in Fig. 6.18(b), where the effect of certain impurities in copper is shown. It can be

seen that as little as one part per million of iron increases the resistivity at 4.2 K by more than an order of magnitude. The resistivity of metals at relatively high temperatures varies slowly with temperature, increasing with the first power of T.
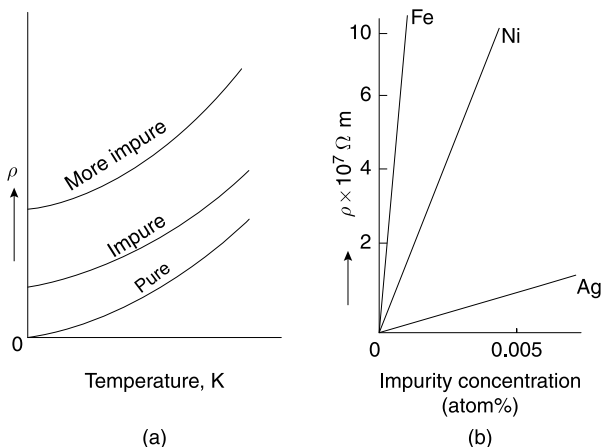
Figure 6.18   Resistivity of metal (a) its temperatures dependence at very low temperature (b) its dependence upon small concentrations of selected impurities (in Cu), at low temperatures

The general characteristics of electrical resistance in metals are summarised as follows:

1. The electric current density in the steady state is proportional to the electric field strength (Ohm's law).
2. For pure specimens, the electrical resistivity varies with temperature as

    $\rho \propto T$ at high temperature

    $\rho \propto T^5$ at low temperature

3. For metals containing small amounts of impurities, the electrical resistivity (ρ) may be written as

    $$\rho = \rho_0 + \rho_p(T)$$

    where ρ_0 is a constant that increases with increasing impurity content. $\rho_p(T)$ is the temperature dependent part of the resistivity. This is known as *Matthiessen's rule*.

4. For most metals, the electrical resistivity decreases with increasing pressure.

6.10   KRONIG–PENNEY MODEL

According to free electron theory, the potential inside a crystal is uniform. According to Kronig–Penney model, the electrons have variable potential energy due to the presence of immobile lattice ions (atoms) in crystals and hence the potential inside a crystal is not uniform. The potential energy variations in a one-dimensional lattice (crystal) is as shown in Fig. 6.19.
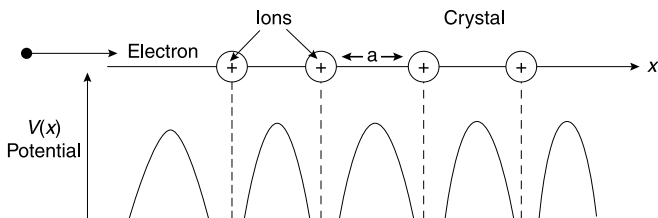
Figure 6.19   Periodic potential variations

When an electron approaches the +ve ion in the crystal then its potential energy is minimum at the middle of any two ions (atoms) and is maximum as the position of ion is approached as shown in Fig. 6.19. Thus between any two ions we get the same variations of potential energy curves and they are periodic in nature. The periodic potential is given by $V(x) = V(x + a)$ where 'a' is the lattice constant.

To study the behaviour of electron in such varying potential fields, Kronig and Penney modified the above curves as rectangular square potential wells, which is as shown in Fig. 6.20.
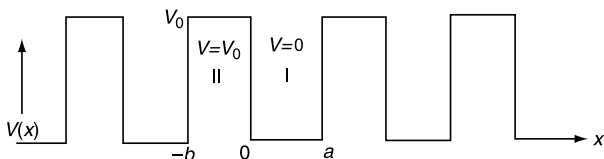
Figure 6.20   Kronig–Penney model potential variation

Let the width of the potential well and potential barrier be 'a' and 'b' respectively. Let the PE of an electron be $V = 0$ in the well and in the barrier be $V = V_0$. The periodicity of the potential is $(a + b)$.

The energies and wave functions of electrons associated with this model can be calculated by solving time-independent one-dimensional Schrödinger's wave equations for the two regions I and II.

The Schrödinger's equations are

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{\hbar^2}\right] E\psi = 0 \quad \text{for } 0 < x < a \tag{67}$$

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{\hbar^2}\right] (E - V_0)\psi = 0 \quad \text{for } -b < x < 0 \tag{68}$$

Let $\alpha^2 = \dfrac{2mE}{\hbar^2}$ and $\beta^2 = \dfrac{2m}{\hbar^2}(V_0 - E)$ $\tag{69}$

Hence, Eqs. (67) and (68) becomes

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad \text{for } 0 < x < a \tag{70}$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad \text{for} -b < x < 0 \tag{71}$$

The solution that will be appropriate for both the regions can be written in Bloch's form as

$$\psi(x) = e^{iKx}u_K(x) \quad \text{where } u_K(x) = u_K(x+a) \tag{72}$$

and $K = \dfrac{2\pi}{\lambda} =$ wave vector or propagation vector along the x direction.

Differentiating the above equation twice w.r.t x and substituting in Eq. (70) and (71) and after solving, we get

$$\left[\frac{mV_0ab}{\hbar^2}\right]\frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos Ka \tag{73}$$

Let $\quad P = \dfrac{mV_0ab}{\hbar^2}$, then $P\dfrac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos Ka \tag{74}$

where P is known as *potential barrier strength*.

In the above equations we have two variables, α and x.

The left-hand side of Eq. (72) is plotted as a function of αa for $P = 3\pi/2$ from which permissible values of α can be known. The right-hand side can take only values between -1 to $+1$. The above equation is satisfied only for values of αa for which the left-hand side lies between ± 1. The plots are as shown in Fig. 6.21.
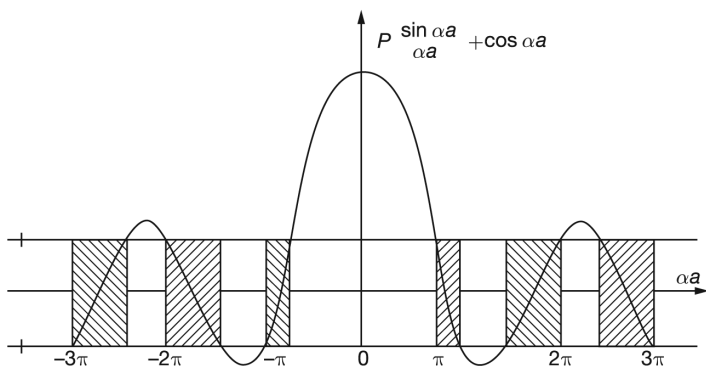
Figure 6.21 A plot of αa versus $\left[P\dfrac{\sin \alpha a}{\alpha a} + \cos \alpha a\right]$

From Fig. 6.21, the following conclusions can be drawn:

1. The allowed range of αa satisfying Eq. (55) are shown by the shaded portions which are allowed energy bands for electrons separated by forbidden regions.

2. As the value of αa increases, the width of allowed energy bands also increases as width of forbidden band decreases due to the decrease of the first term in Eq. (74).

3. If *P* is large, the width of allowed bands decreases and the width of forbidden band increases.

For $P \to \infty$, Eq. (71) has only solution of

$$\sin \alpha a = 0$$

$$\alpha a = \pm n\pi$$

$$\alpha = \pm \frac{n\pi}{a}$$

$$\alpha^2 = \frac{n^2\pi^2}{a^2} = \frac{2mE}{h^2} \Rightarrow E = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$E = \left(\frac{n^2}{2m}\right)\left(\frac{\pi^2 h^2}{4\pi^2}\right)$$

$$E = \frac{n^2 h^2}{8ma^2}$$

E is independent of *K*. Energy levels are discrete indicating the electron behaviour as a particle in a potential box as shown in Fig. 6.21.

For $P \to 0$,

$$\cos \alpha a = \cos Ka$$

$$\alpha = K$$

$$\alpha^2 = K^2 = \frac{2mE}{\hbar^2}$$

$$E = \left[\frac{\hbar^2}{2m}\right] K^2$$

The above energy represents the energy of a free particle.

When $P \to 0$, then the left-hand side of Eq. (74) will not cross ± 1 line as shown in Fig. 6.22. Hence, all the energies are allowed to the electrons. Thus, by varying *P* from zero to infinity, the energies of electrons will vary from continuous to bound, i.e., free electrons to bound electrons.
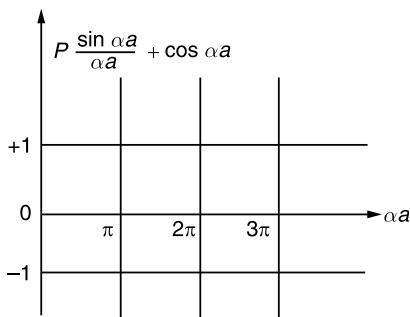
Figure 6.22   Discrete energy levels of electron
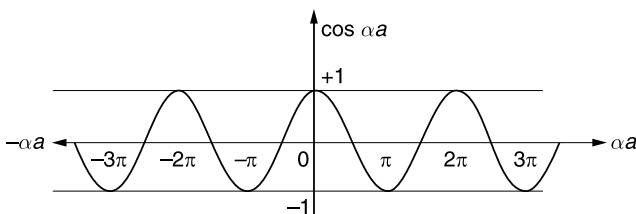
Figure 6.23 Energy curve for free electron

6.10.1 E – K Diagram

The free electrons moving in periodic potentials of lattice can have energy values only in the allowed regions or zones. It is possible to plot the total energies of free electrons versus their wave number or propagation vector K as shown in Fig. 6.24.
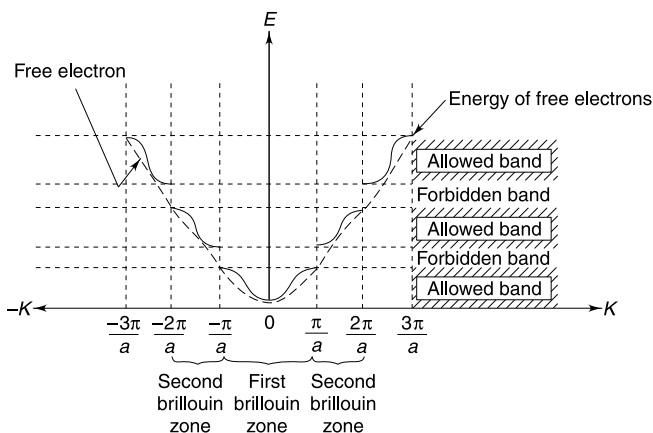
Figure 6.24 $E – K$ diagram

We observe that the curve is not continuous; it has discontinuities at

$$K = \pm \frac{n\pi}{a},$$

where $n = 1, 2, 3, \ldots$. The dotted parabolic curve shows $E – K$ relation for completely free electrons. From the graph, we see that the electrons have allowed energy values in the region or zone extending from $K = \frac{-\pi}{a}$ to $\frac{+\pi}{a}$. This is called the *first Brillouin zone*. After a break in the energy values, called the forbidden region or band, the electrons have another allowed zone of energy values in the region extended from $K = \frac{-\pi}{a}$ to $\frac{-2\pi}{a}$ and from $\frac{\pi}{a}$ to $\frac{2\pi}{a}$. This zone is called the *second Brillouin zone*. Similarly, the higher order of Brillouin zones can be defined.

The discontinuities occur at the boundaries of the Brillouin zones. Each portion of the curve gives a number of allowed energies called allowed band of

energies. The curves are horizontal at the top and bottom and they are parabolic near the top and bottom with curvatures in opposite directions. As 'P' decreases, the discontinuous $E - K$ graph will reduce to a continuous parabolic graph as shown by the dotted lines and the forbidden bands disappear. Then, the energy values are practically continuous.

Origin of energy bands in solids   Energy band theory of the crystal is useful in determining the electrical properties of crystalline solid since it shows how electrons can move in the crystal. When the atoms are at infinite distance there is no interaction between the electron orbits of neighbouring atoms, atoms have independent energy levels permitted by quantum rules. Each energy levels accommodates electrons as per Pauli's exclusion principle when the atoms are brought closer to each other, interaction between electron orbits takes place due to which single energy level of isolated atom broadens out into a large number of closely spaced levels of the atoms. Such closely packed energy levels is known as energy band. Figure 6.25 shows the energy band formation for diamond which is the crystalline form of carbon.
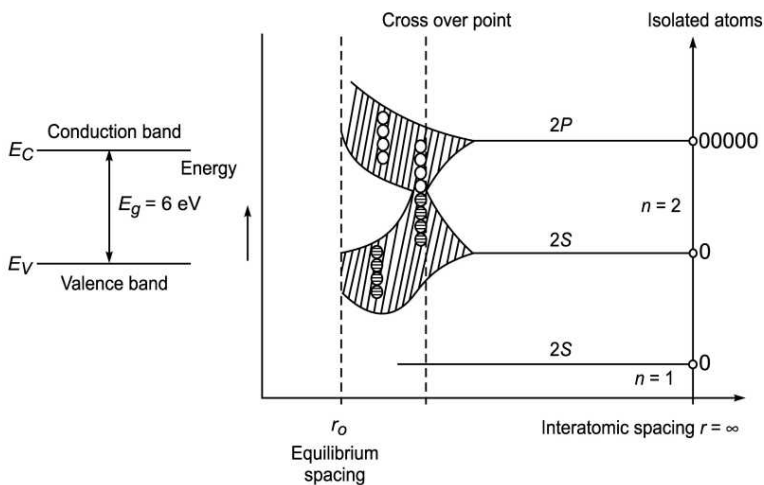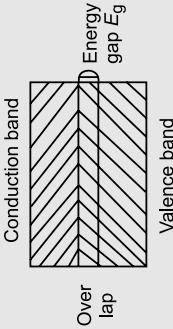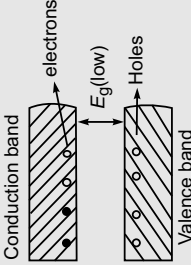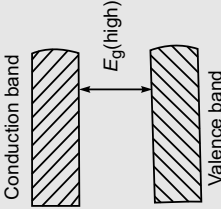
Figure 6.25

In carbon there are two electrons in the $n = 1$ shell and four in the $n = 2$ shell. There are eight energy states in the complete $n = 2$ shell. In free carbon atom or at $r = \infty$, six electron states have been filled and four states at $n = 2$ shell are empty. As the interatomic distance is decreased the discrete levels broaden out into bands and the bands corresponding to the subshells of $n = 2$ overlap. As the interatomic distance is further decreased a split occurs between the upper four states and the lower four states and at the equilibrium spacing the two bands are widely separated. The upper and lower bands are called conduction band and valence band respectively. The gap between two bands is known as forbidden energy gap.

Table 6.1 Classification of solids into conductor, semiconductor and insulator

Conductor	Semiconductor	Insulator
Figure 6.26 Energy band structure	Figure 6.27 Energy band structure	Figure 6.28 Energy band structure
1. For conductor the energy band structure will be overlapped valence and conduction band.	1. For semiconductor the energy band structure consists of separated valence and conductor bands with band gap.	1. For insulator the energy band structure consists of well-separated valence and conduction bands with band gap.
2. Due to overlapping of bands there is no band gap, i.e., $E_g = 0$.	2. The band gap (E_q) is low.	2. The band gap (E_q) is high.
3. The electrons from the valence band freely enter into the conduction band due to overlapping of bands.	3. In the pressure of external electric field, the electrons enter into the conduction band by acquiring sufficient energy to overcome the band gap.	3. Even in the presence of external electric field the electrons does not enter into the conduction band since they are tightly bound to their nucleus of respective atoms.
4. Due to the absence of band gap, free electrons are available in the conduction band at room temperature.	4. Due to low band gap, only few electrons are available in the conductions band at room temperature.	4. Due to high band gap, no electrons are available in the conduction band at room temperature.
5. The electrical conductivity is mainly due to electrons only.	5. The electrical conductivity may be either electrons or holes depending upon the type of semiconductors.	5. No change carriers are available for the electrical conductivity in insulator.
6. The large free electrons of conductor contributes to high electrical conductivity.	6. The large number of electrons or holes contribute to the electrical conductivity depending upon the type of semiconductors.	6. Electrical conductor is almost zero for insulator.

SOLVED PROBLEMS

1. Calculate the wavelength associated with an electron with energy 2000 eV.

Given data   Energy, $E = 2000$ eV $= 2000 \times 1.6 \times 10^{-19}$ J

Solution   Wavelength, $\lambda = \dfrac{h}{\sqrt{2mE}}$

where m = mass of electron

where h = Planck's constant

Kinetic energy $(E) = \dfrac{1}{2} MV^2 \implies \dfrac{M^2V^2}{2M} = \dfrac{p^2}{2M}$ $(\because p = MV)$

$\implies \qquad\qquad p = \sqrt{2ME}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2000 \times 1.6 \times 10^{-19}}} \text{ m}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{58240 \times 10^{-50}}} \text{ m} = \frac{6.63 \times 10^{-34}}{241.33 \times 10^{-25}} \text{ m}$$

$$= 0.0275 \times 10^{-9} \text{ m} \quad \text{or} \quad 0.0275 \text{ nm}$$

2. An electron is bound in a one-dimensional infinite well having a width of 1×10^{-10} m. Find the energy values in the ground state and the first two excited states.

Given data   Width of the potential well, $a = 1 \times 10^{-10}$ m

For ground state, $n = 1$

Solution $E_n = \dfrac{n^2h^2}{8ma^2}$

For ground state $n = 1$

$$E_1 = \frac{n^2h^2}{8ma^2} = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 10^{-10} \times 10^{-10}} \text{ J}$$

$$= \frac{(6.63)^2 \times 10^{-68}}{8 \times 9.1 \times 10^{-51}} \text{ J}$$

$$= \frac{43.9569}{72.8} \times 10^{-17} \text{ J} = 0.6038 \times 10^{-17} \text{ J}$$

or $\dfrac{0.6038 \times 10^{-17}}{1.6 \times 10^{-19}}$ eV $= 37.737$ eV

Similarly, $E_2 = 4E_1 = 4 \times 0.6038 \times 10^{-17}$ J $= 2.415 \times 10^{-17}$ J $= 150.95$ eV.

$\qquad\qquad E_3 = 9E_1 = 9 \times 0.6038 \times 10^{-17}$ J $= 5.434 \times 10^{-17}$ J $= 339.639$ eV.

3. An electron is bound in a one-dimensional box having size of 4×10^{-10} m. What will be its minimum energy?

Given data Size of the potential box, $a = 4 \times 10^{-10}$ m

Solution
$$E_n = \frac{n^2 h^2}{8ma^2}$$

$$E_{\min} = E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 4 \times 10^{-10} \times 4 \times 10^{-10}} \text{ J}$$

$$= \frac{43.9569 \times 10^{-68}}{1164.8 \times 10^{-51}} = 0.037710^{-17} \text{ J}$$

or $\Rightarrow$ $\quad \dfrac{0.0377 \times 10^{-17}}{1.6 \times 10^{-19}}$ eV = 2.3586 eV

4. An electron is confined in a one-dimensional potential well having width of 3×10^{-10} m. Find the kinetic energy of electron when it is in the ground state.

Given data Width of the one-dimensional potential well, $a = 3 \times 10^{-10}$m. Electron is present in the ground state, $n = 1$.

Solution

$$E_n = \frac{n^2 h^2}{8ma^2}$$

For ground state ($n = 1$)

$$E_1 = \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (3 \times 10^{-10})^2} \text{ J}$$

$$= \frac{(6.63)^2 \times 10^{-68}}{(8 \times 9.1 \times 9) \times 10^{-51}} \text{ J}$$

$$= \frac{43.9569 \times 10^{-68}}{655.2 \times 10^{-51}} \text{ J} = 0.067 \times 10^{-17} \text{ J}$$

or $\quad E_1 = \dfrac{0.067 \times 10^{-17}}{1.6 \times 10^{-19}}$ eV = 4.2 eV

5. An electron is confined to a one-dimensional potential box of 2Å length. Calculate the energies corresponding to the second and fourth quantum states (in eV).

Given data Length of the one-dimensional potential box,
$$a = 2 \text{ Å} = 2 \times 10^{-10} \text{ m}$$

Solution   Energy of electron in n^{th} level is

$$E_n = \frac{n^2 h^2}{8ma^2} = n^2 E_1$$

where

$$E_1 = \frac{h^2}{8ma^2} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 2 \times 10^{-10} \times 2 \times 10^{-10}} \text{ J}$$

$$= \frac{43.9569 \times 10^{-68}}{291.2 \times 10^{-51}} = 0.15095 \times 10^{-17} \text{ J}$$

or $\dfrac{0.15095 \times 10^{-17}}{1.6 \times 10^{-19}}$ eV= 9.43 eV

Energy corresponding to second and fourth quantum states is

$$E_2 = 2^2 E_1 = 4 \times 9.43\,\text{eV} = 37.72\,\text{eV}$$

and

$$E_4 = 4^2 E_1 = 16 \times 9.43\,\text{eV} = 150.88\,\text{eV}$$

6.   Find the temperature at which there is 1% probability that a state with energy 0.5 eV is above Fermi energy.

Given data   Probability, $F(E) = 1\% = \dfrac{1}{100} = 0.01$

$$E = E_F + 0.5 \text{ eV} \quad \Rightarrow \quad E - E_F = 0.5 \text{ eV}$$

Solution

Probability $F(E) = \dfrac{1}{1 + \exp (E - E_F)/K_B T}$

$\Rightarrow$

$$0.01 = \frac{1}{1 + \exp \left(\dfrac{0.5}{K_B T} \right)}$$

$$0.01 \exp \left(\frac{0.5}{K_B T} \right) = (1 - 0.01)$$

$$\exp \left(\frac{0.5}{K_B T} \right) = \frac{0.99}{0.01} = 99$$

$$\frac{0.5}{K_B T} = 2.303 \times \log_{10} 99$$

$$K_B T = \frac{0.5}{2.303 \times \log_{10} 99}$$

$$= \frac{0.5}{2.303 \times 1.9956}$$

$$= \frac{0.5}{4.5959} = 0.1087 = 0.109 \text{ eV}$$

$$T = \frac{0.109 \times 1.6 \times 10^{-9}}{1.38 \times 10^{-23}}$$

$$T = 1263 \text{ K}$$

7. The position of an electron in an atom is located within a distance of 0.1 Å using a microscope. What is the uncertainty in the momentum of the electron located in this way?

Given data   The uncertainty in the position of an electron,

$$\Delta x = 0.1 \times 10^{-10} \text{ m}$$

Solution   According to the uncertainty principle

$$\Delta p \cdot \Delta x = \frac{h}{2\pi}$$

or

$$\Delta p = \frac{h}{2\pi \times \Delta x}$$

$$= \frac{6.626 \times 10^{-34}}{2\pi \times 1 \times 10^{-11}}$$

$$= 1.054 \times 10^{-23} \text{ kg ms}^{-1}$$

The uncertainty in the momentum of the electron located

$$= 1.054 \times 10^{-23} \text{ kg ms}^{-1}$$

8. Calculate the wavelength associated with an electron raised to a potential of 1600 V.

Given data   Potential applied to electron $V = 1600$ V

Solution   de Broglie wavelength

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}$$

$$= \frac{12.26}{\sqrt{1600}}$$

$$= \frac{12.26}{40}$$

$$= 0.3065 \text{ Å}$$

9. Calculate the velocity and kinetic energy of an electron of wavelength 1.66×10^{-10} m.

Given data   Wavelength of electron $\lambda = 1.66 \times 10^{-10}$ m

Mass of electron $m = 9.1 \times 10^{-31}$ kg

Planck's constant $\lambda = 6.63 \times 10^{-34}$ Js

Solution Wavelength $\lambda = \dfrac{h}{mv}$

$$\text{Velocity } v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.66 \times 10^{-10}}$$

$$= 438.9 \times 10^4 \text{ ms}^{-1}$$

Kinetic energy $E = \dfrac{1}{2} mv^2$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (438.9 \times 10^4)^2$$

$$= 8.754 \times 10^{-18} \text{ J}$$

$$= \frac{8.754 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$= 54.71 \text{ eV}$$

10. An electron is moving under a potential field of 15 kV. Calculate the wavelength of the electron wave.

Given data Potential field

$$V = 15 \text{ kV} = 15000 \text{ V}$$

Solution de Broglie wavelength

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}$$

$$= \frac{12.26}{\sqrt{15000}} \text{ Å}$$

$$= 0.1 \text{ Å}$$

Wavelength of electron wave = 0.1Å

11. A quantum particle confined to one-dimensional box of width 'a' is known to be in its first excited state. Determine the probability of the particle in the central half.

Given data Width of one-dimensional box = a

For first excited state, $n = 2$.

Solution Probability of location of quantum particle is

$$P(x) = \frac{2}{a} \sin^2 \frac{n\pi x}{a}$$

Probability in the central half of the width of the box is

$$P\left(\frac{a}{2}\right) = \frac{2}{a} \sin^2 \frac{n\pi a/2}{a}$$

$$= \frac{2}{a} \sin^2 \frac{2\pi a}{2a}$$

$$= \frac{2}{a} \sin 2\pi = 0$$

The probability of the quantum particle at the centre of the box is zero.

12. Calculate the de Broglie wavelength of a neutron whose kinetic energy is two times the rest mass of the electron.

Given, $m_n = 1.676 \times 10^{-27}$ kg, $m_e = 9.1 \times 10^{-31}$ kg, and $h = 6.63 \times 10^{-34}$ J sec.

Given data   Mass of neutron $m_n = 1.676 \times 10^{-27}$ kg

Mass of electron $m_e = 9.1 \times 10^{-31}$ kg

Planck's constant $\lambda = 6.63 \times 10^{-34}$ Js

Solution   Kinetic energy of neutron $= \frac{1}{2} m_n v^2 = \frac{p_n^2}{2m_n} = 2m_e$

$$P_n = \sqrt{4m_n m_e}$$

de Broglie wavelength of neurtron,

$$\lambda_n = \frac{h}{p_n}$$

$$= \frac{h}{\sqrt{4m_n m_e}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{4 \times 9.1 \times 10^{-31} \times 1.676 \times 10^{-27}}}$$

$$= 0.8488 \times 10^{-5} \text{ m}$$

$$= 8488 \text{ nm}$$

13. Calculate the de Broglie wavelength associated with a proton moving with a velocity of $\frac{1}{10}$th of velocity of light. (Mass of proton = 1.67 $\times 10^{-27}$ kg).

Given data   Velocity of proton $v = \frac{1}{10} \times c = 3 \times 10^7$ m/sc.

Mass of proton $m = 1.67 \times 10^{-27}$ kg

Hence, de Broglie wavelength $\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}}{1.67 \times 10^{-27} \times 3 \times 10^7}$

$$\lambda = 1.32 \times 10^{-14} \text{ m}$$

14. A particle is moving in a one-dimensional box of infinite height of with 10 Å. calculate the probability of finding the particle with in an interval of 1 Å at the centre of the box when it is in its state of lowest energy?

Given data   For lowest state $n = 1$

The wave function of the particle in the lowest state is

$$\psi_1 = \sqrt{\frac{2}{a}}\, \sin\left(\frac{\pi x}{a}\right)$$

The probability of finding the particle in unit interval at the centre of the box $(x = a/2)$ is given by

$$|\psi_1|^2 = \left[\sqrt{\frac{2}{a}}\, \sin\left(\frac{\pi a}{2a}\right)\right]^2$$

$$= \frac{2}{a} \sin^2 \frac{\pi}{2} = \frac{2}{a} \sin^2 \frac{\pi}{2} = \frac{2}{a}$$

The probability of finding the particle within an interval of Δx at the centre of the box.

$$= |\psi_1|^2\, \Delta x = \left(\frac{2}{a}\right) \Delta x$$

$$= \frac{2 \times 10^{-10}}{10 \times 10^{-10}} = 0.2$$

The probability of finding particle $= 0.2$

15. At what temperature we can expect a 10% probability that electrons in silver have energy which is 1% above the Fermi energy? The Fermi energy of silver is 5.5 eV.

Given data $F(E) = 10\% = 0.1$

$$E_F = 5.5 \text{ eV}$$

$$E = E_F + \frac{1}{100} E_F$$

$$E = (5.5 + 0.055) = 5.555 \text{ eV}.$$

$$E - E_F = 0.055 \text{ eV} = 0.055 \times 1.6 \times 10^{-26} \text{ J}$$

We know that probability function is given by

$$F(E) = \frac{1}{1 + e^{(E - E_F/K_B T)}}$$

$$0.1 = \frac{1}{1 + e^{\left(\frac{0.055 \times 1.5 \times 10^{-19}}{1.38 \times 10^{-23} \times T}\right)}}$$

$$0.1 = \frac{1}{1 + e^{\left(\frac{637.7}{T}\right)}}$$

$$1 + e^{\left(\frac{637.7}{T}\right)} = 10$$

$$e^{\left(\frac{637.7}{T}\right)} = 9 \Rightarrow \frac{637.7}{T} = \ln 9$$

$$T = \frac{637.7}{\ln 9} = \frac{637.7}{2.197} = 290.2 \text{ K}$$

16. Evaluate fermi function for an energy K_BT above Fermions.

Given data $\quad E - E_F = K_BT$

Fermi function $\quad F-(E) = \dfrac{1}{1 + e^{\left(\frac{E - E_F}{K_BT}\right)}}$

$$= \frac{1}{1 + \exp(1)} = \frac{1}{1 + 2.78}$$

$$= \frac{1}{3.78} = 0.27$$

The probability of occupation of electron at the energy is about 0.269 since the probability is so small, the presence of electron at that energy level is not certain.

17. Calculate the mean free path of electron in copper of density 8.5×10^{28} m^{-3} and resistivity 1.69×10^{-8} Ω-m. Given $m = 9.11 \times 10^{-39}$ kg, $T = 300$ K, $e = 1.6 \times 10^{-19}$ J, $K_B = 1.38 \times 10^{-23}$ JK^{-1}.

Given data $\qquad$ Density $n = 8.5 \times 10^{28}$ m^{-3}

$$\text{Resistivity } \rho = 1.69 \times 10^{-19} \text{ Ω-m}$$

$$\text{Mass of electron } m = 9.11 \times 10^{-39} \text{ kg}$$

$$\text{Charge of electron } e = 1.6 \times 10^{-19} \text{ J}$$

$$\text{Temperature } T = 300 \text{ K}$$

$$K_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Solution $\qquad$ We know, $\rho = \dfrac{\sqrt{3mk_BT}}{ne^2\lambda}$

$$\lambda = \frac{\sqrt{3mk_B\pi}}{ne^2\rho}$$

$$= \frac{\sqrt{3 \times 9.11 \times 10^{-39} \times 1.38 \times 10^{-23} \times 300}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.69 \times 10^{-19}}$$

$$= 2.88 \text{ m}$$

18. Find the relaxation time of conduction electrons in a metal of resistivity 1.54×10^{-8} Ω-m, if the metal has 5.8×10^{28} conduction electrons per m^3. Given $m = 9.1 \times 10^{-34}$ kg, $e = 1.6 \times 10^{-19}$ C.

Given data

$$\text{Mass of electron } m = 9.1 \times 10^{-34} \text{ kg}$$

$$\text{Charge of electron } e = 1.6 \times 10^{-19} \text{ C}$$

$$\text{Number of electron/m}^3, n = 5.8 \times 10^{28} \text{ atm/m}^3$$

$$\text{Density } \rho = 1.54 \times 10^{-8} \ \Omega\text{m}$$

Solution   We know, $\rho = \dfrac{m}{ne^2 \tau}$

$$\tau = \frac{m}{ne^2 \rho}$$

$$= \frac{9.1 \times 10^{-14}}{5.8 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.54 \times 10^{-8}}$$

$$= 3.97 \times 10^{-14} \text{ sec.}$$

MULTIPLE CHOICE QUESTIONS

1. Dual nature [particle and wave] of matter was proposed by
   - (a) de Broglie
   - (b) Planck
   - (c) Einstein
   - (d) Newton

2. The wave function ψ associated with a moving particle
   - (a) is not an observable quantity
   - (b) does not have direct physical meaning
   - (c) is a complex quantity
   - (d) All of the above

3. The most probable position of a particle in a one-dimensional potential well of width 'a' in the first quantum state is
   - (a) $a/4$
   - (b) $a/3$
   - (c) $a/2$
   - (d) $2a/3$

4. The cause for electrical resistance of a metal is
   - (a) impurities and crystal defects
   - (b) thermal vibrations
   - (c) electron scattering and non-periodicity of lattice potentials
   - (d) All of the above

5. In $E - K$ diagram
   - (a) each portion of the curve represents allowed band of energies
   - (b) the curves are horizontal at the top and bottom
   - (c) the curves are parabolic near the top and bottom with curvatures in opposite directions
   - (d) All of the above

6. The velocity of a free electron in a metal is maximum when
   - (a) it is present in the bottom energy levels of an allowed band
   - (b) it is present in the top energy levels of an allowed band
   - (c) it is present in an energy level corresponding to a point of inflection in an allowed band
   - (d) None of these

7. As the temperature of a metal is raised from absolute zero temperature, the Fermi–Dirac distribution curve begins to depart from _____ function.
   - (a) wave-like
   - (b) tail-like
   - (c) step-like
   - (d) None of these

8. The applied electric field on a metal _____ the velocity of electrons present near the Fermi level.
   - (a) decreases
   - (b) enhances
   - (c) Both (a) and (b)
   - (d) None of these

9. In the Kronig–Penney model, the width of allowed bands _____ and the width of forbidden bands _____ with increase of energy (α).
   - (a) increases, decreases
   - (b) increases, increases
   - (c) decreases, decreases
   - (d) decreases, increases

10. The discontinuities in the energies of free electrons of a metal occur at the _____ of the Brillouin zones.
    - (a) middle
    - (b) boundaries
    - (c) Both (a) and (b)
    - (d) None of these

11. An electron, neutron and a proton have the same wavelength. Which particle has greater velocity?
    - (a) Neutron
    - (b) Proton
    - (c) Electron
    - (d) All the particles have the same velocity

12. The wavelength of matter waves λ associated with an electron moving under partial V is proportional to
    - (a) $\sqrt{V}$
    - (b) V
    - (c) $\dfrac{1}{\sqrt{V}}$
    - (d) $\dfrac{1}{V}$

13. According to Fermi–Dirac statistics, the probability of an electron occupying an energy level E is given by
    - (a) $F(E) = \dfrac{1}{1 - \exp\left(\dfrac{E - E_F}{K_B T}\right)}$
    - (b) $F(E) = 1 + \exp\left(\dfrac{E - E_F}{K_B T}\right)$
    - (c) $F(E) = \dfrac{1}{1 + \exp\left(\dfrac{E - E_F}{K_B T}\right)}$
    - (d) $F(E) = 1 - \exp\left(\dfrac{E - E_F}{K_B T}\right)$

14. Fermi level is that state at which the probability of electron occupation is _____ at any temperature above 0 K.

    (a) 1   (b) 0

    (c) 1/2   (d) none of these

15. At any temperature other than 0 K, the probability of finding an electron at Fermi level is

    (a) 1   (b) 0

    (c) 0.5   (d) any positive value

16. The de Broglie wavelength of a particle is

    (a) proportional to its mass

    (b) proportional to its energy

    (c) proportional to its momentum

    (d) inversely proportional to its momentum

17. The wavelength of de Broglie waves associated with a moving particle is independent of its

    (a) mass   (b) charge

    (c) velocity   (d) momentum

18. The wavelength of an electron moving with a velocity of 500 ms^{-1} is

    (a) 1.45 nm   (b) 0.50 nm

    (c) 2.90 nm   (d) 3.00 nm

19. If a particle of mass 'm' moves with a velocity 'v' and momentum 'p' then the wavelength associated with this particle is

    (a) $\lambda = hp$ (b) $\lambda = h/p^2$

    (c) $\lambda = hp^2$ (d) $\lambda = h/p$

20. The uncertainty principle is applicable to

    (a) only small particles   (b) microscopic particles

    (c) all material particles   (d) only tiny particles

21. If an electron of mass 'm' and charge 'e' is accelerated by a potential of 100 V from rest to velocity 'v', then the wavelength 'λ' equal to

    (a) 122.6 Å   (b) 12.26 Å

    (c) 1.226 Å   (d) 0.1226 Å

22. When an electron is accelerated by a potential 'V' volts, then the de Broglie wavelength is

    (a) $\dfrac{1.226 \text{ Å}}{\sqrt{v}}$ (b) $\dfrac{12.26 \text{ Å}}{\sqrt{v}}$

    (c) $\dfrac{122.6 \text{ Å}}{\sqrt{v}}$ (d) $\dfrac{1226 \text{ Å}}{\sqrt{v}}$

23. Probability density of a wave function ψ is

    (a) ψ (b) $|\psi^2|$

    (c) $\int \psi \psi \, dt \, d\psi$ (d) none of these

24. In a one-dimensional potential box, particle energy $E_n =$

   (a) $\dfrac{nh}{8ma^2}$ (b) $\dfrac{nh}{8ma}$

   (c) $\dfrac{n^2h^2}{ma^2}$ (d) $\dfrac{n^2h^2}{8ma^2}$

25. The value of Fermi function in Fermi level is

   (a) 0.5   (b) 0

   (c) 1   (d) 0.75

26. The various energy bands in a solid

   (a) overlap   (b) do not overlap

   (c) may or may not overlap   (d) nothing can be said

27. If the valence band is partially filled then the solid is a/an

   (a) conductor   (b) semiconductor

   (c) insulator   (d) None of these

28. At absolute zero, semiconductors have the band structure similar to

   (a) conductors   (b) insulators

   (c) Neither (a) nor (b)   (d) Both (a) and (b)

29. An electron in a solid moves in a region of

   (a) constant potential   (b) negative potential

   (c) periodically varying potential   (d) None of these

30. In an allowed band, the velocity of an electron is zero at

   (a) bottom   (b) top

   (c) bottom and top   (d) None of these

31. The wavelength of de Broglie wave associated with an electron when accelerated in a potentaial field 'V' is

   (a) $\dfrac{h}{meV}$ (b) $\dfrac{h}{2meV}$

   (c) $\dfrac{h}{\sqrt{2meV}}$ (d) $\dfrac{h}{\sqrt{meV}}$

32. By solving the one dimensional Schrödinger's time-independent wave equation for a particle in the well gives

   (a) discrete values of energy levels

   (b) eigen functions of the particle

   (c) probability of location of the particle in the well

   (d) All of the above

33. When an electron is accelerated through a potential field of 100 V, then it is associated with a wave of wavelength equal to

   (a) 0.1226 nm   (b) 1.226 nm

   (c) 12.26 nm   (d) 122.6 nm

34. The normalised wave function of a particle in a one-dimensional potential well of width '*a*' is

    (a) $\dfrac{2}{a} \sin \dfrac{n\pi x}{a}$ (b) $\dfrac{a}{2} \sin \dfrac{n\pi x}{a}$

    (c) $\sqrt{\dfrac{a}{2}} \sin \dfrac{n\pi x}{a}$ (d) $\sqrt{\dfrac{2}{a}} \sin \dfrac{n\pi x}{a}$

35. First Brillouin zone corresponds to *K* value extending from

    (a) 0 to $\dfrac{\pi}{a}$ (b) $\dfrac{\pi}{a}$ to $\dfrac{2\pi}{a}$

    (c) $\dfrac{-2\pi}{a}$ to $\dfrac{2\pi}{a}$ (d) $\dfrac{-\pi}{a}$ to $\dfrac{\pi}{a}$

36. Potential barrier strength depends on
    (a) mass of the particle (b) potential of the barrier
    (c) width of the barrier (d) All of the above

37. de Broglie's wave length associated with a particle of mass *m* moving with velocity '*V*' is given by

    (a) $\lambda = \dfrac{mv}{h}$ (b) $\lambda = hmv$

    (c) $\lambda = \dfrac{v}{m}$ (d) $\lambda = \dfrac{h}{mv}$

38. We do not observe matter waves in heavy particles because
    (a) The wavelength associated is very large.
    (b) They travel with lesser velocity.
    (c) The wavelength associated is very small.
    (d) None of these

39. If *E* is the kinetic energy of the material particle of mass *m*, then the de Broglie wavelength is given by

    (a) $\dfrac{h}{2mE}$ (b) $h\sqrt{2mE}$

    (c) $\dfrac{h}{\sqrt{2mE}}$ (d) $\dfrac{\sqrt{2mE}}{h}$

40. Velocity of matter waves is always
    (a) greater than velocity of light.
    (b) lesser than velocity of light.
    (c) equal to velocity of light.
    (d) None of these

41. When an electron is accelerated, if de Broglie wave length is 1 Å then the applied voltage is nearly equal to
    (a) 12 volts (b) 150 volts
    (c) 15 volts (d) 500 volts

42. If an electron moves under a potential field of 1.6 kV the wavelength of the electron wave is

(a) 0.76 Å

(b) 0.31 Å

(c) 19.6 Å

(d) 12.26 Å

43. The drift velocity v_d of electrons in a metal is related to the electric field E and collision time τ as

(a) $\dfrac{e\tau}{mE}$

(b) $\dfrac{em\tau}{m}$

(c) $\dfrac{E\tau}{me}$

(d) $\dfrac{m\tau}{eE}$

44. Classical free electron theory failed to explain

(a) Specific heat of metals

(b) Magnetic susceptability of metals

(c) Thermionic emission

(d) All the above

45. According to quantum free electron theory the expression for electrical conductivity is

(a) $\dfrac{ne^2\tau}{m}$

(b) $\dfrac{ne^2\tau_F}{m}$

(c) $\dfrac{ne^2\tau_F}{m^*}$

(d) $\dfrac{e^2\tau_F}{nm}$

Answers

1. (a)	2. (d)	3. (c)	4. (d)	5. (d)	6. (c)
7. (c)	8. (c)	9. (a)	10. (b)	11. (c)	12. (c)
13. (c)	14. (c)	15. (c)	16. (a)	17. (b)	18. (a)
19. (d)	20. (c)	21. (c)	22. (b)	23. (b)	24. (d)
25. (a)	26. (c)	27. (a)	28. (b)	29. (c)	30. (c)
31. (c)	32. (d)	33. (a)	34. (d)	35. (d)	36. (d)
37. (d)	38. (c)	39. (c)	40. (a)	41. (b)	42. (b)
43. (b)	44. (d)	45. (c)			

Fill in the Blanks

1. The concept of matter waves was suggested by _____.

2. Quantum theory of radiation was proposed by _____.

3. Uncertainty principle was proposed by _____.

4. According to Heisenberg's uncertainity principle. 'It is impossible to know both the exact position and exact _____ of an object at the same time.

5. Several combinations of the three quantum numbers may given different wave functions, but of the same energy value. Such states and energy levels are said to be _____.

6. According to classical free electron theory the electron velocities in a metal obey _____ distribution of velocities.

7. The average distance travelled by an electron between two successive collisions in the presence of applied field is known of _____.

8. Energy of the state at which the probability of electron occupation is 1/2 at any temperature above 0 K is known as _____.

9. Insulators have relatively wide forbidden band gaps of _____.

10. _____ have relatively narrow forbidden band gaps of less than of equal 1 eV.

Answers to Fill in the blanks

1. de Broglie	2. Max planck	3. Heisenberg
4. momentum	5. degenerate	6. Maxwell–Boltzman
7. Mean freepath	8. Fermi energy	9. greater than 3eV
10. Semiconductors		

True or False

1. According to Planck, the energy is emitted in the form of packets of quanta called phonons. (T/F)

2. According to the de Broglie matter waves concept, with increases of velocity of a moving particle, the de Broglie wave length also increases. (T/F)

3. For a particle at rest, de Broglie wavelength is infinity. (T/F)

4. de Broglie waves are electromagnetic waves. (T/F)

5. Matter waves can travel faster than velocity of light. (T/F)

6. No single phenomena exhibits both particle nature and wave nature simultaneously. (T/F)

7. The wave function ψ has direct physical meaning. (T/F)

8. According to Pauli's exclusion principle no two electrons having the same four quantum numbers can occupy any one state. (T/F)

9. The wave function ψ can be considered as probability amplitude since it is used to find the location of the particle. (T/F)

10. The success of classical free electron theory is that it could explain the phenomena such as photo electric effect, Compton effect, etc. (T/F)

11. Classical free electron theory could explain optical properties of metals. (T/F)

12. According to quantum theory of free electrons, electrons have continuous energy. (T/F)

13. Electron gas inside a metal behaves like a system obeying Fermi–Dirac statistics. (T/F)

14. Fermi level is that state at which the probability of electron occupation is 1 at any temperature above 0 K. (T/F)

15. Kronig and Penny model proposes a uniform constant potential inside a metallic crystal. (T/F)

16. According to Kronig–Penny model, electrons move inside an energy spectrum which consists of allowed and for bidden bands. (T/F)

Answers to True or False

1. F	2. F	3. T	4. F	5. T	6. T
7. F	8. T	9. T	10. F	11. T	12. F
13. T	14. F	15. F	16. T		

REVIEW QUESTIONS

Short Answer Questions

1. What are matter waves?
2. What is de Broglie hypothesis of matter waves?
3. Mention any two properties of matter waves.
4. What is Heisenberg's uncertainty principle?
5. Mention any two applications of Heisenberg's uncertainty principle.
6. What is the significance of wave function?
7. How are eigen energy values of a particle in one-dimensional potential box quantised?
8. What are eigen functions?
9. What are the drawbacks of classical free electron theory?
10. What are the advantages of quantum free electron theory?
11. Mention the expression for Fermi–Dirac distribution function.
12. What is drift velocity?
13. What is relaxation time?
14. What is mean free path?
15. What are the allowed and forbidden energy bands?
16. What are Brilloin zones?
17. On what factors the energy band structure of solid depends?
18. What is the significance of wave function?

Essay Type Questions

1. State and explain de Broglie hypothesis of matter waves. Mention its properties.
2. State and explain Heisenberg's uncertainty principle along with its applications.
3. Derive Schröndger's time, independent and time dependent wave equations.
4. Describe the behaviour of a particle in a one-dimensional infinite potential well in terms of its eigen values and functions.

5. Describe the electrical conductivity in metals using classical free electron theory.

6. Describe the various sources of electrical resistance in metals.

7. Describe the electrical conductivity of metals using quantum free electron theory.

8. Explain Fermi–Dirac distribution function along with its temperature dependence.

9. Describe Kronig–Penny model to understand the behaviour of electrons in a varying periodic potential fields of a crystal.

10. Explain the origin of energy bands in solids.

11. Classify the solids into conductors semiconductors and insulators based on energy band structure.

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CHAPTER 7

SEMICONDUCTORS

INTRODUCTION

The conductivity of semiconductors varies between conductors and insulators. The interesting feature about semiconductors is that they are bipolar and current is transported by two charge carriers, i.e., electron and hole. The number of carriers can be enhanced by doping the semiconductor with suitable impurities converting into extrinsic semiconductor. The remarkable feature of extrinsic semiconductor is that current is transported in them through two different processes, namely, drift and diffusion. These extrinsic semiconductors are widely used in solid-state electronic devices. For analysis of these electronic devices, it is important to study the fundamental electronic transport properties in semiconductors.

7.1   INTRINSIC SEMICONDUCTOR

Pure semiconductors are known as intrinsic semiconductors. Frequently available semiconductors are germanium (Ge) and silicon (Si) belonging to IV group in the periodic table. Each semiconductor has four valence electrons in their outermost orbit. To set stability, each of these atoms makes four covalent bands with the surrounding four neighbouring atoms in the semiconductor crystal. The two-dimensional representation of silicon intrinsic semiconductor at $0\,$K along with the energy band structure is as shown in Fig. 7.1.

At $0\,$K, all valence electrons are strongly bound to their atoms and are actively participating in the covalent bond formation. As a result, no free electrons are available for conduction and it acts as an insulator. The conductivity phenomenon can be well understood with the help of the energy band structure. The energy band structure consists of conduction and valence bands separated by a forbidden energy gap. In Fig. 7.1, E_c corresponds to energy of the energy level of lower portion of the conduction band, E_v represents the energy level of the upper portion of the valence band. E_g is the energy gap between valence and conduction band. For Si, $E_g = 1.2$ eV and for Ge, $E_g = 0.07$ eV. E_F is the Fermi energy level. At $0\,$K, as all the valence electrons are bound to their atoms, these electrons occupy all energy levels of the valence band while the conduction band is empty.
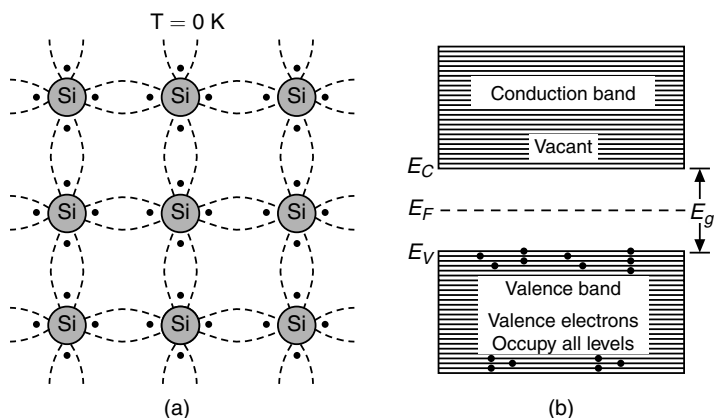
Figure 7.1 Intrinsic silicon crystal at 0 K (a) Two-dimensional representation of silicon crystal, (b) Energy band representation

At room temperature ($T > 0$ K), the valence electron acquires sufficient amount of thermal energy. As a result, breakage of covalent bonds takes place releasing free electrons. These free electrons create a vacancy in its initial position in the crystal. This vacancy is known as hole and is assumed to carry a positive charge equivalent to the charge of the electron. These free electrons, due to acquiring of sufficient thermal energy, cross the energy gap and enter into the conduction band from the valence band and occupy the energy levels in the conduction band. The electrons leaving the valence band create holes in its place. Thus, the valence band has holes while the conduction band has electrons. The corresponding crystal structure along with the energy band structure are as shown in Fig. 7.2.
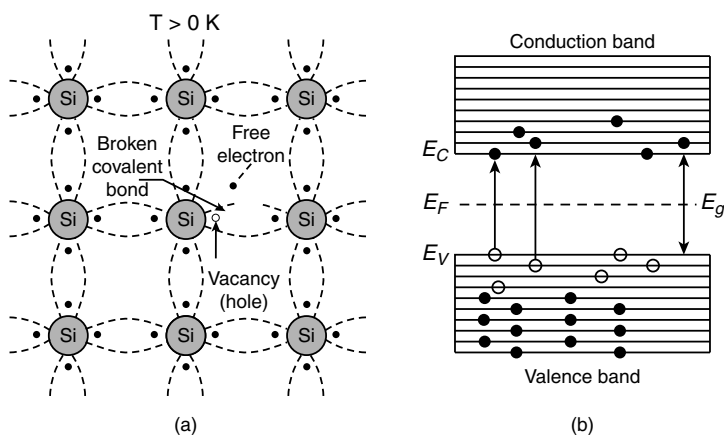
Figure 7.2 Silicon crystal at temperature above 0 K (a) Due to thermal energy breaking of covalent bonds takes place, (b) Energy band representation

7.2 INTRINSIC CARRIER CONCENTRATION AND FERMI LEVEL

In an intrinsic semiconductor, each broken bond leads to generation of two carriers, namely, an electron and a hole. At any temperature T, the number of electrons generated will be therefore equal to the number of holes generated.

Let 'n' be the number of electrons per unit volume or electron-carrier concentration in the conduction band, and 'p' be the number of holes per unit volume or hole-carrier concentration in the valence band.

For an intrinsic semiconductor

$$n = p = n_i \tag{1}$$

where n_i is known as *intrinsic carrier concentration*.

Electron concentration in the conduction band is

$$n = N_c \, e^{-(E_c - E_F)/K_B T} \tag{2}$$

Hole concentration in the valence band is given by

$$p = N_v \, e^{-(E_F - E_v)/K_B T} \tag{3}$$

where N_c and N_v are known as *pseudo-constants* depending on temperature.

K_B is the Boltzmann constant,

T is the absolute temperature of an intrinsic semiconductor

$$
\begin{aligned}
n_i^2 &= np \\
&= \left[N_c \, e^{-(E_c - E_F)/K_B T} \right] \left[N_V \, e^{-(E_F - E_V)/K_B T} \right] \\
&= (N_C \, N_V) \, e^{-(E_C - E_V)/K_B T}
\end{aligned}
$$

But

$$
\begin{aligned}
E_C - E_V &= E_g \\
n_i^2 &= N_C \, N_V \, e^{-E_g/K_B T}
\end{aligned}
$$

$$n_i = (N_C \, N_V)^{\frac{1}{2}} \, e^{-E_g/2K_B T} \tag{4}$$

From the above relation, it is clear that

1. The intrinsic carrier concentration is independent of the Fermi level position.
2. The intrinsic carrier concentration is a function of the band gap E_g.
3. The intrinsic carrier concentration depends on the temperature T.

7.2.1 Fermi Level

The Fermi energy level indicates the probability of occupation of energy levels in conduction and valence bands. For an intrinsic semiconductor, hole and electron concentrations are equal, and it indicates that the probability of occupation of energy levels in conduction and valence bands are equal. Thus, the Fermi level lies in the middle of the energy gap E_g.

For an intrinsic semiconductor, $n = p$

$$N_c\, e^{-(E_C - E_F)/K_B T} = N_v\, e^{-(E_F - E_V)/K_B T}$$

$$\frac{e^{-(E_C - E_F)/K_B T}}{e^{-(E_F - E_V)/K_B T}} = \frac{N_v}{N_c}$$

$$e^{[2E_F - (E_C + E_V)]/K_B T} = \frac{N_V}{N_C}$$

Taking log on both sides, we have

$$\frac{2E_F - (E_C + E_V)}{K_B T} = \log\left(\frac{N_V}{N_C}\right)$$

$$E_F = \frac{E_C + E_V}{2} + \frac{1}{2}\,K_B T \log\left(\frac{N_V}{N_C}\right)$$

when $N_V = N_C$, then $\quad E_F = \dfrac{E_C + E_V}{2}$

Thus, the Fermi energy level in an intrinsic semiconductor lies in the middle or centre of the energy gap. In practice, the Fermi energy level is independent of temperature.

7.3 INTRINSIC CONDUCTIVITY

Consider an intrinsic semiconductor to which a potential difference V is applied. It establishes an electric field E and the change carriers are forced to drift in the respective directions to constitute an electric current I. The drift velocity acquired by the change carrier is given by

$$V_d = \mu E \tag{5}$$

where, μ is the mobility of change carriers.

Figure 7.3 Conductivity in an intrinsic semiconductor

Let n be the concentration of electrons in the semiconductor. Then the current density due to an electron is given by

$$J_n = ne\, V_d$$
$$= ne\, \mu_n\, E \tag{6}$$

where μ_n is the mobility of the electron.

Similarly, current density due to holes is given by

$$J_p = pe\, \mu_p\, E \tag{7}$$

where p is the hole concentration and μ_p is the mobility of the hole.

Total current density $J = J_n + J_p$

$$J = ne\,\mu_n\,E + pe\,\mu_p\,E$$
$$= (n\mu_n + p\mu_p)\,eE \tag{8}$$

But total current density

$$J = \sigma E \tag{9}$$

where σ is the total conductivity.

From Eqs. (8) and (9), we get

$$\sigma = (n\mu_n + p\mu_p)\,e \tag{10}$$

For an intrinsic semiconductor, $n = p = n_i$:

$$\sigma = n_i e(\mu_n + \mu_p) \tag{11}$$

But 　　　　　　$n_i = (N_c N_v)^{\frac{1}{2}}\,e^{-E_g/2K_BT} \tag{12}$

$\therefore$ 　　　　$\sigma = (N_C N_V)^{\frac{1}{2}}\,e(\mu_n + \mu_p)\,e^{-E_g/2K_BT}$

　　　　　　　　$\sigma = Ae^{-E_g/2K_BT}$

where 　　　　$A = (N_c N_v)^{\frac{1}{2}}\,e\,(\mu_n + \mu_p) = \text{constant} \tag{13}$

7.4　DETERMINATION OF ENERGY BAND GAP (E_g)

The energy gap between valence and conduction bands is the energy band gap E_g.

From the equation $\sigma = Ae^{-E_g/2K_BT}$, we get

$$\rho = \frac{1}{\sigma}$$
$$= \frac{1}{A}\,e^{E_g/2K_BT}$$
$$= B\,e^{E_g/2K_BT}$$

where 　$B = \dfrac{1}{A} = \text{constant}$.

Figure 7.4 　Variation of log ρ with $\dfrac{1}{T}$

Taking log on both sides, we get

$$\log \rho = \log B + \frac{E_g}{2K_BT}$$

Log of the resistivity when plotted with $1/T$, produces a straight line as shown in Fig. 7.4. The slope of the line gives the energy band gap of the semiconductor.

From Fig. 7.4, 　　　$\dfrac{E_g}{2K_B} = \dfrac{dy}{dx}$
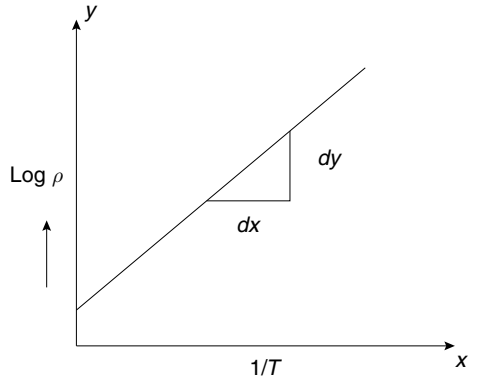
$$E_g = 2K_B \left(\frac{dy}{dx} \right)$$

7.5   EXTRINSIC SEMICONDUCTOR

When impurities are added to an intrinsic semiconductor then it becomes an extrinsic semiconductor. An extrinsic semiconductor shows remarkable variations in conductivity due to the presence of impurities.

Depending upon the type of impurity added to the intrinsic semiconductors, extrinsic semiconductors are of two types:

1. *n*-type extrinsic semiconductor
2. *p*-type extrinsic semiconductor

7.5.1   *n*-Type Extrinsic Semiconductor

When a small amount of pentavalent impurity such as phosphorous (or arsenic or antimony) is added to the intrinsic semiconductor (Si), then the impurity atoms occupy one of the positions of the Si atom. Four of the valence electrons of phosphorous form covalent bonds with the adjacent silicon atoms. The fifth electron is left free. At 0 K, this electron is bound to phosphorous with 0.045 eV. The corresponding energy band diagram will have valence and conduction bands separated by E_g. Near to the conduction band, we have donor energy level (E_d) corresponding to phosphorous which is going to donate an electron at higher temperature. It acts as a donor. The E_d level is finally occupied and is shown in Fig. 7.5.
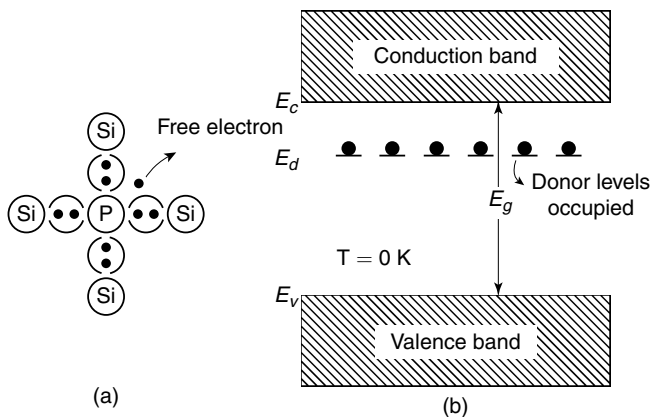
Figure 7.5   (a) Representation of *n*-type silicon (b) Energy band diagram at 0 K

When the temperature is increased (T > 0 K) then the bound electron becomes a free electron and enters into the conduction band. As a result, donor levels get ionized. At still higher temperature, breakage of covalent bond releases an electron hole pair, i.e., electrons move from valence band to conduction band

leaving holes in the valence band. As a result, concentration of electrons increases in the conduction band rather than holes. Hence, electrons become majority charge carriers and holes becomes minority charge carriers and are as shown in Fig. 7.6, i.e., concentration of electrons 'n' is greater than the concentration of holes 'p'.
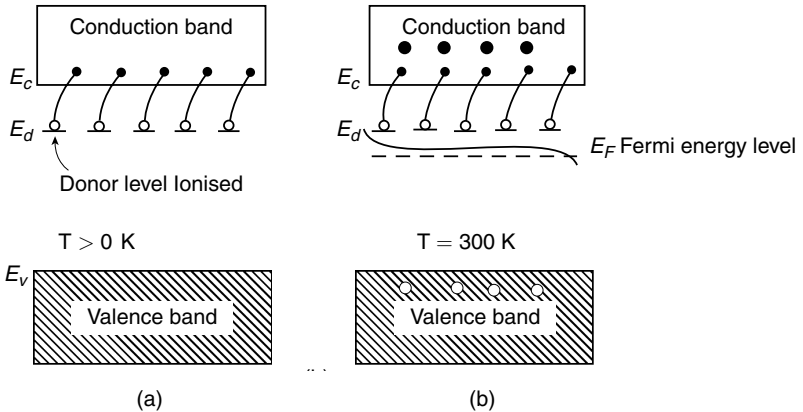
Figure 7.6 (a) Energy band diagram of n-type semiconductor at T > 0 (b) At $T = 300$ K

7.5.2   *p*-Type Extrinsic Semiconductor

When a small amount of trivalent impurity such as boron (or indium or aluminium) is added to a pure Si crystal, the boron settles one of Si atom's position. This boron will try to form three covalent bonds with the neighbouring Si atoms and one bond is left over with the deficiency of an electron, i.e., a hole is created at this position. Since boron is in a position to accept an electron, it acts as an acceptor. Correspondingly, we have an acceptor level (E_a) near the valence band and is shown in Fig. 7.7.
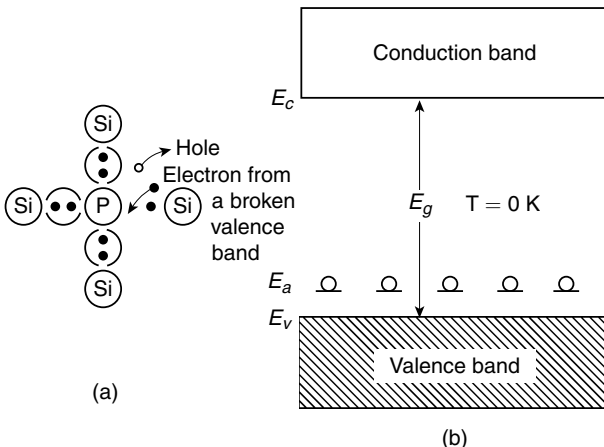
Figure 7.7 Representation of (a) *p*-type silicon (b) Energy band diagram at 0 K

When temperature is raised (T > 0 K) the electrons from the valence band try to occupy the acceptor and it gets ionised creating holes in the valence band. If the temperature is further increased then due to breakage of covalent bonds, electrons enter the conductor band leaving more holes in the valence band. As a result, holes becomes majority carriers and electrons becomes minority carriers, i.e., $p > n$ and are shown in Fig. 7.8.
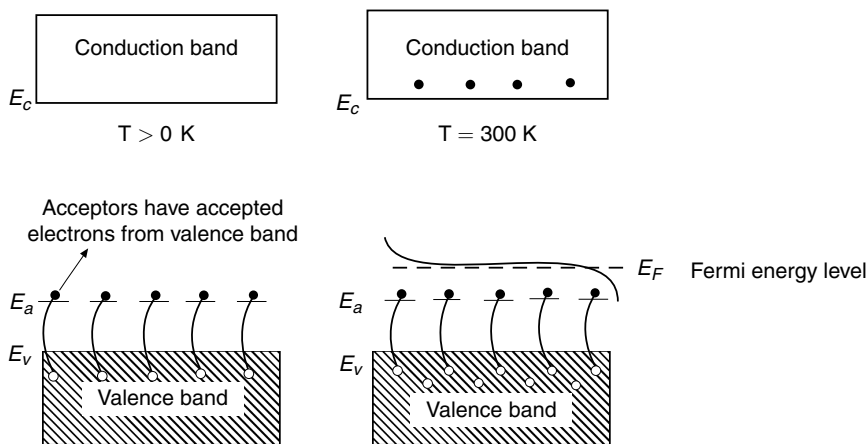
Figure 7.8   Energy band diagram of *p*-type semiconductor at T > 0 and 300 K

7.6   FERMI ENERGY LEVEL (E_F)

For an *n*-type semiconductor, the Fermi energy level is given by

$$E_F = E_C - K_B T \log \frac{N_C}{N_D}$$

where N_D is the concentration of donor atoms.

From the above relation, it is clear that Fermi level lies below the conductor band and is shown in Fig. 7.8.

For a *p*-type semiconductor, the Fermi energy level is given by

$$E_F = E_V + K_B T \log \frac{N_V}{N_A}$$

where N_A is the concentration of acceptor atoms. From the above relation, it is clear that Fermi level lies above the valence band and is as shown in Fig. 7.8.

7.6.1   Effects of Temperature on E_F

In an *N*-type semiconductor, as temperature *T* increases, more number of electron-hole pairs are formed. At a very high temperature *T*, the concentration of thermally generated electrons in the conduction band will be far greater than the concentration of donor electrons. In such a case, as concentration of electrons

and holes become equal, the semiconductor becomes essentially intrinsic and E_F returns to the middle of the forbidden energy gap. Hence, it is concluded that as the temperature of the *p*-type and *n*-type semiconductor increases, E_F progressively moves towards the middle of the forbidden energy gap.

7.6.2 Law of Mass Action

The electron and hole concentrations in an intrinsic semiconductor are given by the following expressions:

$$n = N_C e^{-(E_C - E_F)/K_B T} \tag{14}$$

and
$$p = N_V e^{-(E_F - E_V)/K_B T} \tag{15}$$

As $n = p = n_i$, we get

$$n_i = (N_C N_V)^{\frac{1}{2}} e^{-E_g/2K_B T} \tag{16}$$

Relation (16) shows that for any arbitrary value of E_F, the product of n and p is a constant. This conclusion is known as the law of mass action. The electron and hole concentrations in an extrinsic semiconductor are given by expressions similar to Eqs. (14) and (15). Thus, for an *n*-type semiconductor,

$$n_n = N_C e^{-(E_C - E_F)/K_B T} \tag{17}$$

$$p_n = N_V e^{-(E_F - E_V)/K_B T} \tag{18}$$

$$\therefore \qquad n_n p_n = (N_C N_V) e^{-E_g/K_B T} \tag{19}$$

$$\therefore \qquad n_n p_n = n_i^2$$

The above relation represents law of mass action. For a *p*-type semiconductor, the above relation becomes

$$p_p n_p = n_i^2 \tag{20}$$

The Eqs. (19) and (20) imply that the product of majority and minority carrier concentrations in an extrinsic semiconductor at a particular temperature is equal to the square of intrinsic carrier concentration at that temperature.

The law of mass action is a very important relation because it in conjunction with charge neutrality condition enables us to calculate minority carrier concentration. *The law suggests that the addition of impurities to an intrinsic semiconductor increases the concentration of one type of carrier, which consequently becomes majority carrier and simultaneously decreases the concentration of the other carrier, which as a result becomes the minority carrier*. The minority carriers decrease in number below the intrinsic value because the majority carriers increase the rate of recombinations. The law of mass action states that the product of majority and minority carriers remains constant in an extrinsic semiconductor and it is independent of the amount of donor and acceptor impurity concentrations. It may be noted that when the doping is heavy, the minority concentration will be low and if doping is lighter, the minority concentration will be larger.

7.6.3 Charge Densities in *n*-and *p*-type Semiconductors

A semiconductor, whether intrinsic or extrinsic, is an electrically neutral body in its equilibrium condition. In an *n*-type semiconductor, the total number of electrons in the conduction band must be equal to the sum of electrons originated from the donor atoms and electrons excited from the valence band. Electrons coming from donor levels leave behind positive donor ions while electrons excited from valence band leave behind holes. These changes have not created any additional charges, so that equality between positive and negative charges remains undisturbed. The ***charge neutrality condition*** applied to the *n*-type semiconductor implies that the total negative charge of mobile electrons is equal to the total positive charge created in the crystal. Thus,

$$n_D = N_D + p_n \qquad (21)$$

where N_D is the donor concentration.

The charge neutrality for a *p*-type semiconductor requires that

$$p_p = N_A + n_p \qquad (22)$$

where N_A is the acceptor concentration

Since in an *n*-type semiconductor,

$$n_n \gg p_n; \quad n_n \approx N_D$$

where n_n, p_n are the electron and hole concentration in an *n*-type semiconductor

Therefore, from the law of mass action, we get

$$p_n = \frac{n_i^2}{N_D} \quad \text{which is} \ll n_n \text{ or } N_D \qquad (23)$$

For a *p*-type semiconductor,

$$p_p \gg n_p; \quad \therefore p_p \approx N_A$$

which gives $n_p = \dfrac{n_i^2}{N_A}$ which is $\ll p_p$ or N_A

where N_A, p_p and n_p are the concentrations of acceptor impurities, holes and electrons in a *p*-type semiconductor.

7.7 DRIFT AND DIFFUSION

7.7.1 Drift

Under the influence of an external electric field, the charge carriers are forced to move in a particular direction constituting electric current (conductivity). This phenomena is known as the *drift*. Let there be '*n*' electrons in a semiconductor. Under the electric field '*E*', they are drifted with a drift velocity V_d.
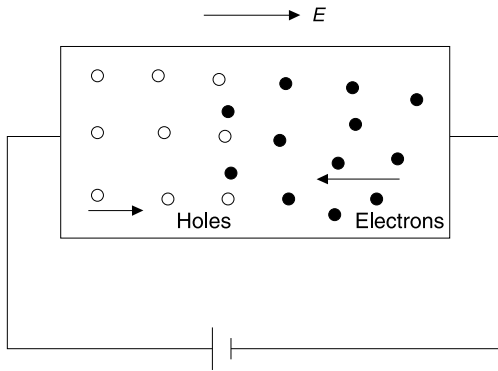
Figure 7.9   Drifting charge carrier in a semiconductor

Then the current density

$$J = neV_d$$

where e is charge of an electron.

Then conductivity $\sigma = \dfrac{J}{E} = \dfrac{neV_d}{E}$

The drift velocity is also given by $V_d = \mu_n E$ where μ_n is the mobility of electrons. Substituting the above values,

$$J = ne\,\mu_n\,E$$

$\therefore$ $\sigma = \dfrac{J}{E} = ne\,\mu_n$

and $\rho = \dfrac{1}{\sigma} = \dfrac{1}{ne\,\mu_n}$

In the case of a semiconductor, the drift current density due to electrons is given by $J_n(\text{drift}) = n\mu_n eE$ and the drift current density due to holes is

$$J_p(\text{drift}) = p\,\mu_p eE$$

Then the total drift current density

$$J(\text{drift}) = J_n(\text{drift}) + J_p(\text{drift})$$

$$= n\mu_n eE + p\mu_p eE$$

$$J(\text{drift}) = eE\,(n\mu_n + p\mu_p)$$

$$\sigma(\text{drift}) = \dfrac{J(\text{drift})}{E} = en\mu_n + epu_p$$

For an intrinsic semiconductor, $n = p = n_i$ then $\sigma_i(\text{drift}) = n_i e\,(\mu_n + \mu_p)$

7.7.2 Diffusion

Due to non-uniform carrier concentration in a semiconductor, the charge carriers moves from a region of higher concentration to a region of lower concentration. This process is known as *diffusion of charge carriers*.

Figure 7.10 Diffusion of charge carriers in a semiconductor

Let Δn be the excess electron concentration. Then according to Fick's law, the rate of diffusion of electrons

$$\propto \frac{-\partial (\Delta n)}{\partial x}$$

$$= -D_n \frac{\partial (\Delta n)}{\partial x}$$

where D_n is the diffusion coefficient of electrons, the diffusion current density due to electrons is given by J_n (diffusion)

$$= -e \left[-D_n \frac{\partial}{\partial x} (\Delta n) \right]$$

$$= eD_n \frac{\partial}{\partial x} (\Delta n)$$

The diffusion current density due to holes

$$J_p \text{ (diffusion)} = +e \left[-D_p \frac{\partial}{\partial x} (\Delta p) \right]$$

$$= -eD_p \frac{\partial}{\partial x} (\Delta p)$$

The total current density due to electrons is the sum of the current densities due to drift and diffusion of electrons

$$J_n = J_n \text{ (drift)} + J_n \text{ (diffusion)}$$

$$= ne\mu_n E + eD_n \frac{\partial}{\partial x} (\Delta n)$$

Similarly, $\qquad J_p = pe\mu_p E - eD_p \frac{\partial}{\partial x} (\Delta p)$

The variation of excess electron density Δn w.r.t. x is shown in Fig. 7.11.

Figure 7.11   Carrier diffusion

7.8   EINSTEIN'S RELATION

The relation between mobility μ and diffusion coefficient D of charge carriers in a semiconductor is known as Einstein's relation. At equilibrium with no applied electric field, if the charge disribution is uniform, there is no net current flow. Any disturbance in equilibrium state leads to diffusion of charge carriers resulting in a diffusion current

Figure 7.12   Drifting and diffusion of charge carriers in a semiconductor

which creates an internal electric field. This field causes the drifting of charge carriers resulting in a drift current. At equilibrium condition, the drift current and diffusion current balance each other.

Let Δn be the excess electron concentration of a semiconductor. Then at equilibrium the drift and diffusion current densities due to excess electrons are equal

i.e.,
$$\Delta n \, eE\mu_n = eD_n \frac{\partial(\Delta n)}{\partial x} \tag{24}$$

where E is internal electric field.

The force on excess electrons restoring equilibrium is equal to the product of excess charge and electric field, i.e., $F = (\Delta n) \, eE$. From Eq. (24),

$$\left[\frac{eD_n}{\mu_n}\right] \frac{\partial(\Delta n)}{\partial x} \tag{25}$$

From kinetic theory of gases, the force on gas molecules (charge carriers) is given by

$$F = K_B T \frac{\partial(\Delta n)}{\partial x} \tag{26}$$

Comparing Eqs. (25) and (26), we get

$$K_B T = \frac{e D_n}{\mu_n}$$

$$D_n = \frac{\mu_n K_B T}{e} \tag{27}$$

$$\frac{D_n}{\mu_n} = \frac{K_B T}{e} \tag{28}$$

Similarly for holes, we get

$$\frac{D_p}{\mu_p} = \frac{K_B T}{e} \tag{29}$$

$\therefore$ $$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p}$$

$$\frac{D_n}{D_p} = \frac{\mu_n}{\mu_p} \tag{30}$$

The above relation is known as Einstein's relation.

7.9 HALL EFFECT

When a current carrying semiconductor is subjected to a transverse magnetic field, then a potential difference or electric field is developed across the semiconductor or conductor in a direction perpendicular to both current and the applied magnetic field. This phenomenon is known as Hall effect. The established potential difference is known as Hall voltage and the electric field becomes Hall electric field.
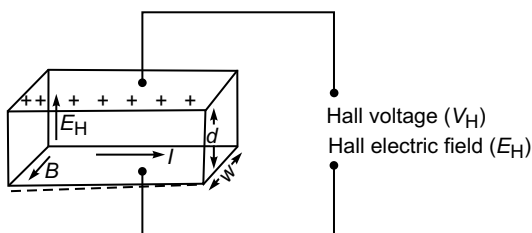
Figure 7.13   Hall effect

As shown in Fig. 7.13, consider a semiconductor slab of thickness 'd' and width 'w' in which a current I is flowing along X direction. It is subjected to a magnetic field of strength 'B' along Y direction. The charge carriers inside the semiconductor experiences a force due to the applied magnetic field. As a result, electrons will be accumulated at the bottom surface of the semiconductor due to the magnetic force on them along $-Y$ direction. Thus, negative charge builds up at the

bottom surface and the corresponding positive charge on upper surface of the semiconductor. Thus a potential difference and electric field is established across the surfaces. This developed electric field along z direction exerts a force on the electrons to prevent further accumulation at the bottom surface. At steady state, both the forces balances each other and the electrons can move freely to constitute an electric current.

Let E_H be the Hall electric field on the semiconductor. The force on the electron due to E_H.

$$= eE_H \tag{31}$$

where 'e' is the charge of electron.
The force on electron due to magnetic field B

$$= Bev \tag{32}$$

where v is the electron velocity.
At steady state, $\qquad eE_H = Bev$

$$E_H = Bv \tag{33}$$

If v_H is the Hall voltage, then $E_H = \dfrac{V_H}{d}$ $\tag{34}$

From the above equations, we get

$$\frac{V_H}{d} = Bv$$
$$V_H = Bvd \tag{35}$$

If 'n' is the concentration of electrons in the semiconductor, then
Current density $\qquad J = nev$ $\tag{36}$

$$V = \frac{J}{ne} \tag{37}$$

$\therefore \qquad\qquad V_H = \dfrac{BJd}{ne}$ $\tag{38}$

But $\qquad\qquad J = \dfrac{I}{A}$

where A is the area of cross section of a semiconductor

$$= \frac{I}{dw} \quad [\because A = dw] \tag{39}$$

$\therefore \qquad\qquad V_H = \dfrac{Bd}{ne} \times \dfrac{I}{dw} = \dfrac{BI}{new}$ $\tag{40}$

The Hall coefficient R_H is given by

$$R_H = \frac{1}{ne} \tag{41}$$

$\therefore \qquad\qquad v_H = \dfrac{BIR_H}{w}$ $\tag{42}$

$$R_H = \frac{VHw}{BI} \tag{43}$$

The conductivity in a semiconductor due to electrons is given by

$$\sigma = ne\mu \tag{44}$$

$$\mu = \frac{\sigma}{ne} = R_H\sigma \tag{45}$$

The above expressions represents Hall voltage, Hall coefficients and conductivity in terms of Hall coefficient.

Applications

1. Knowing R_H and σ, the mobility 'μ' of charge carriers can be determined.
2. By determining the sign of R_H, we can know the type of charge carrier as well as semiconductor. If the sign is negative then the charge carriers are electron and it is a *n*-type semiconductor and vice versa.
3. Carrier concentration can be determined by knowing R_H.
4. We can measure the conductivity of the given specimen knowing μ and R_H.
5. We can measure the magnetic field strength B by measuring the relative quantities.

7.10   DIRECT AND INDIRECT BAND-GAP SEMICONDUCTORS

7.10.1   Direct Gap Semiconductors

In a semiconductor, a direct band gap means that the minimum of the conduction band lies directly above the maximum of the valence band in the E–K space. In a direct band-gap semiconductor, electrons at the conduction-band minimum can combine directly with holes at the valence-band maximum, while conserving momemtum. (Fig. 7.14).The energy of the recombination across the band gap will be emitted in the form of a photon of light. This is *radiative recombination* which is also called *spontaneous emission*.

Figure 7.14   Direct band gap

The example of a direct band-gap semiconductor is gallium arsenide which is commonly used in laser diodes.

7.10.2   Indirect Gap Semiconductors

Not all semiconductor materials have the minimum of the conduction band above the top of the valence band in the E–K band structure. These materials are known as indirect semiconductors. The examples are Si, Ge, etc.

Indirect band gap is a band gap in which the minimum energy in the conduction band is shifted by a k-vector relative to the valence band. The k-vector difference represents a difference in momentums (Fig. 7.15).
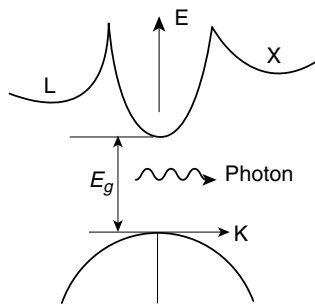
(b) Indirect band gap

Figure 7.15　Indirect band gap

Semiconductors that have an indirect band gap are inefficient at emitting light. This is because any electrons present in the conduction band quickly settle into the energy minimum of that band. Electrons in this minimum require some source of momentum allowing them to overcome the offset and fall into the valence band. Photons have very little momentum compared to this energy offset. Since the electrons cannot rejoin the valence band by radiative recombination, conduction band electrons last quite some time before recombining through less efficient means.

The indirect (nonradiative) recombination takes place at point defects or at grain boundaries in Si. If the excited electrons are prevented from reaching these recombination places, they will fall back into the valence band by radiative recombination.

The absorption of light at an indirect gap is much weaker than at a direct one. As in the emission process, both the laws of conservation of energy and of momentum must be observed. The only way to promote an electron from the top of the valence band to the bottom of the conduction band is to simultaneously emit (or absorb) a phonon that compensates for the missing momemtum vector. However, such a combined transition has a much lower probability.

The absorption of an indirect band gap material usually depend more on temperature than that of a direct material, because at low temperatures phonons are not available for a combined process.

From Fig. 7.15, we have

$$E_g = h\gamma + E_{phonon}$$

where E_{phonon} is the energy of a phonon and $h\gamma$ is the energy of the emitted photon.

Let us consider the difference between direct and indirect bandgap semiconductors

Direct band-gap semiconductor	*Indirect band-gap semiconductor*
1. As shown in the band diagram, the minimum energy of conduction band	1. As shown in the band diagram, the minimum energy of the conduction band and maxi-

Contd.

and maximum energy of valence band are have the same value of wave vector.	mum energy of the valence band having different values of wave vector.
2. An electron from the conduction band can recombine with a hole in the valence band directly emitting a light photon of energy $h\gamma$.	2. An electron from the conduction band can recombine with a hole in the valence band indirectly through traps. Here, there is emission of photon along with phonon. The emission of phonon leading to the rise of temperature of the material.
3. Lifetime (i.e., recombination time) of charge carriers is very less.	3. Lifetime of charge carriers is more.
4. Due to emission of light photon during recombination of charge carriers, these are used to fabricate LEDs and laser diodes.	4. Due to longer lifetime of charge carriers, these are used to amplify the signals as in the case of diodes and transistors.
5. These are mostly from the compound semiconductors.	5. These are mostly from the elemental semiconductors.
6. *Examples:* In P, Ga As	6. *Examples:* Si, Ge

7.11 WORKING PRINCIPLE OF P-N JUNCTION DIODE

When *p*-type and *n*-type semiconductors are formed by doping trivalent and pentavalent impurity atoms on either end regions of a semiconductor, such a way that the crystal structure remains continuous at the boundary of two doped regions, then *p–n* junction is formed. Such *p–n* junctions were widely used for various electronic applications.

Formation of p-n junction
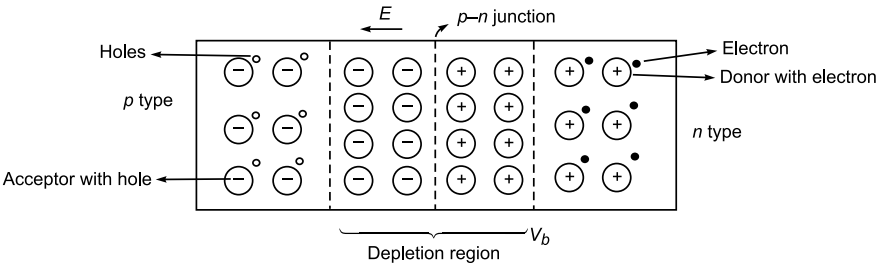
Figure 7.16   *p–n* junction

By doping appropriate dopants, the two end regions of a semiconductor were converted into *p* and *n* type. The *p* type region consists of acceptor atoms with holes and *n*-type region have donor atoms with electrons along the common boundary across these two types we get the formation of *p–n* junction. Across the junction diffusion of charge carriers takes place, i.e., electrons from *n* region try to cross the junction and diffuse over *p* region to recombine with a hole and vice versa. Due to this diffusion process, accumulation of donor and acceptor atom

takes place across the junction. This process of accumulation further stops the diffusion of carriers and recombination process, i.e., the net negative and positive charges across the junction develops a potential difference 'V_b' or electric field 'E', which opposes the further diffusion and is known as potential barrier. The region across the junction where change carriers (electrons and holes) were depleted is known as depletion region. The width of this depletion region depends upon the dopants concentration and v_b.

Forward biased p–n junction   In forward biased *p–n* junction, *p* type is connected to positive terminal of battery and *n* type is connected to negative terminal of battery. This forward biased applied potential difference decreases the potential barrier which in turn reduces the width of the depletion region. As a result more number of charge carriers cross the junction and are accelerated by the forward biased voltage, which in turn causes the flow of current and we get the conductivity through the *p–n* junction and is as shown in the corresponding band structure.
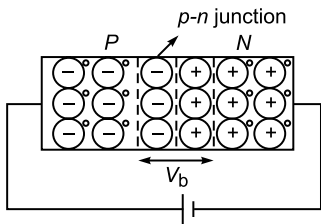
Figure 7.17   Forward biased *p-n* junction
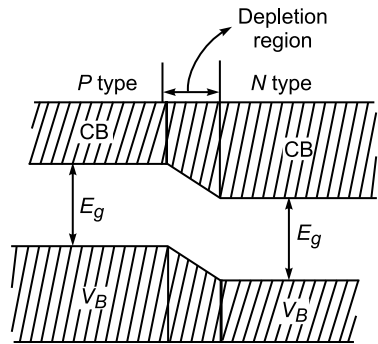
Figure 7.18   Reduced potential barrier

Reverse biased p-n junction   In reverse biased *p–n* junction, *p* type is connected to negative terminal and *n* type is connected to positive terminal of battery. This reverse biased applied voltage increases the potential barrier with increase in the width of depletion region as shown in the band diagram. As a result, the
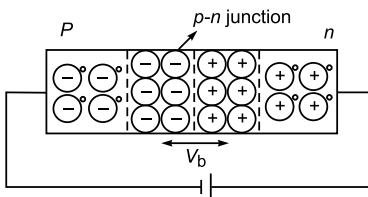
Figure 7.19   Reverse biased *p-n* junction
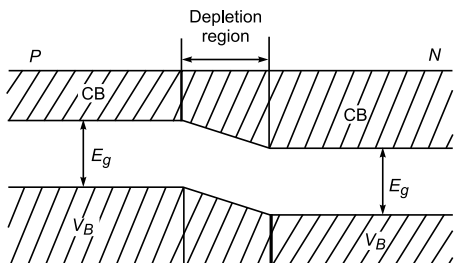
Figure 7.20   Enhanced potential barrier

change carriers are forced to move away from the junction which in turn causes no current flow through the junction and conductivity vanishes.

7.11.1   *p-n* Junction Diode

When the *pn* junction is forward biased conductivity takes place through the junction and in reverse biased no conductivity exists. Thus it can act as a switch, hence it is known as *p-n* junction diode. This diode is widely used in electron applications and its symbol is as shown in Fig. 7.21.
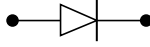
Figure 7.21   Diode symbol

7.12   LIGHT EMITTING DIODE (LED)

Light Emitting Diode (LED) is a *pn* junction device which emits light when forward biased, by a phenomenon called *electroluminescence*. In all semiconductor *pn* junctions, some of the energy will be radiated as heat and some in the form of photons. In silicon and germanium, greater percentage of energy is given out in the form of heat and the emitted light is insignificant. In other materials such as gallium phosphide (GaP) or gallium arsenide phosphide (GaAsP), the number of photons of light energy emitted is sufficient to create a visible light source. Here, the charge carrier recombination takes place when electrons from the *n*-side cross the junction and recombine with the holes on the *p*-side.
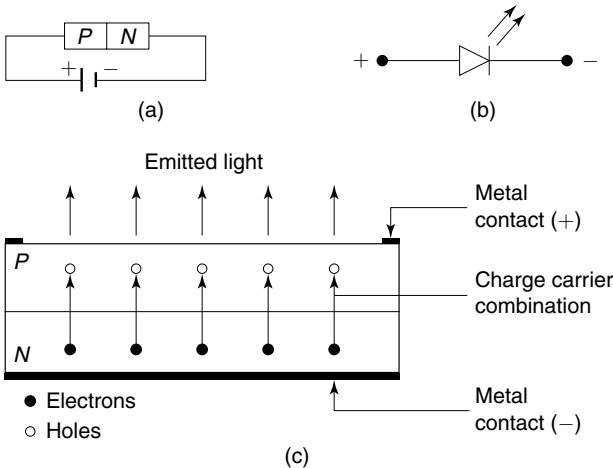
Figure 7.22   (a) LED under forward bias (b) Symbol (c) Recombinations and emission of light

LED under forward bias and its symbol are shown in Fig. 7.22 (a), (b), respectively. When an LED is forward biased, the electrons and holes move towards the junction and recombination takes place. As a result of recombination, the electrons lying in the conduction bands of an *n*-region fall into the holes

lying in the valence band of a *p*-region. The difference of energy between the conduction band and the valence band is radiated in the form of light energy. Each recombination causes radiation of light energy. Light is generated by recombination of electrons and holes whereby their excess energy is transferred to an emitted photon. The brightness of the emitted light is directly proportional to the forward bias current. The wave length of emitted photon is given by $\lambda = hc/E_g$, where E_g is the band gap energy.

Figure 7.22(c) shows the basic structure of an LED showing recombinations and emission of light. Here, an *n*-type layer is grown on a substrate and a *p*-type is deposited on it by diffusion. Since carrier recombination takes place in the *p*-layer, it is kept uppermost. The metal anode connections are made at the outer edges of the *p*-layer so as to allow more central surface area for the light to escape. LEDs are manufactured with domed lenses in order to reduce the reabsorption problem. A metal (gold) film is applied to the bottom of the substrate for reflecting as much light as possible to the surface of the device and also to provide cathode connection. LEDs are always encased to protect their delicate wires.

The efficiency of generation of light increases with the injected current and with a decrease in temperature. The light is concentrated near the junction as the carriers are available within a diffusion length of the junction.

LEDs radiate different colours such as red, green, yellow, orange and white. Some of the LEDs emit infrared (invisible) light also. The wavelength of emitted light depends on the energy gap of the material. Hence, the colour of the emitted light depends on the type of material used and is given as follows:

Gallium arsenide (GaAs) – infrared radiation (invisible)

Gallium phosphide (GaP) – red or green

Gallium arsenide phosphide (GaAsP) – red or yellow

In order to protect LEDs, resistance of 1 kΩ or 1.5 kΩ must be connected in series with the LED. LEDs emit no light when reverse biased. LEDs operate at voltage levels from 1.5 to 3.3 V, with the current of some tens of milliamperes. The power requirement is typically from 10 to 150 mW. LEDs can be switched ON and OFF at a very fast speed of 1 ms.

They are used in burglar alarm systems, picture phones, multimeters, calculators, digital meters, microprocessors, digital computers, electronic telephone exchange, intercoms, electronic panels, digital watches, solid-state video displays and optical communication systems. Also, there are two-lead LED lamps which contain two LEDs, so that a reversal in biasing changes the colour from green to red, or vice versa.

7.12.1 Applications

LEDs are used

1. for instrument display
2. in calculators

3. digital clocks
4. for indicating power ON/OFF
5. for optical switching applications
6. for solid-state video display
7. in optical communication for energy coupling circuits
8. to detect light
9. in opto-isolation circuits

7.13   PHOTODIODE

A silicon photodiode is a light-sensitive device, also called a *photodetector*, which converts light signals into electrical signals. The construction and symbol of a photodiode are shown in Fig. 7.23. The diode is made of a semiconductor *pn* junction kept in a sealed plastic or glass casing. The cover is so designed that the light rays are allowed to fall on one surface across the junction. The remaining sides of the casing are painted to restrict the penetration of light rays. A lens permits light to fall on the junction. When light falls on the reverse-biased *pn* photodiode junction, hole-electron pairs are created. The movement of these hole-electron pairs in a properly connected circuit results in current flow. The magnitude of the photocurrent depends on the number of charge carriers generated and hence, on the illumination on the diode element. This current is also affected by the frequency of the light falling on the junction of the photodiode. The magnitude of the current under large reverse bias is given by

$$I = I_s + I_0 \left(1 - e^{V/\eta V_T}\right)$$

where I_o = reverse saturation current

I_s = short-circuit current which is proportional to the light intensity

V = voltage across the diode

V_T = volt equivalent of temperature

η = parameter, 1 for Ge and 2 for Si.
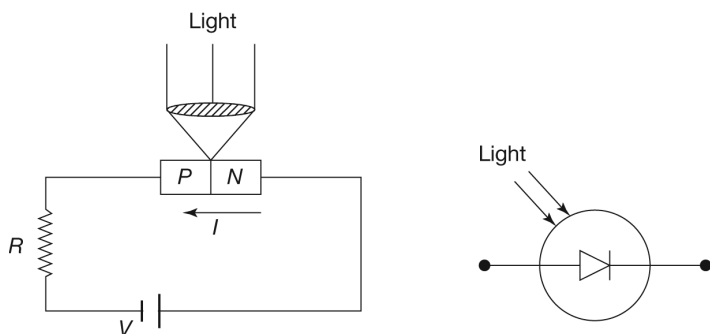
Figure 7.23   Photodiode (a) Construction (b) Symbol

The characteristics of a photodiode are shown in Fig. 7.24. The reverse current increases in direct proportion to the level of illumination. Even when no light is applied, there is a minimum reverse leakage current called *dark current*, flowing through the device. Germanium has a higher dark current than silicon, but it also has a higher level of reverse current.

Figure 6.24 Characteristics of photodiode

Applications

Photodiodes are used as light detectors, demodulators and encoders. They are also used in optical communication systems, high-speed counting and switching circuits. Further, they are used in computer card punching and tapes, light operated switches, sound track films and electronic control circuits.

7.14   LASER DIODE

Laser diode is a specially fabricated *p-n* junction device that emits coherent radiations when it is forward biased.

7.14.1   Principle

When a *p-n* junction is formed across a *p*-and-*n* type semiconductor, then it results in the formation of a depletion region across the junction. When the junction is forward biased, the width of the depletion region decreases allowing more number of electrons from *n* type to cross the junction and recombine with hole in *p* type. Thus, recombination of electron hole pairs across the junction emits the radiation [Fig. 7.25(a)].
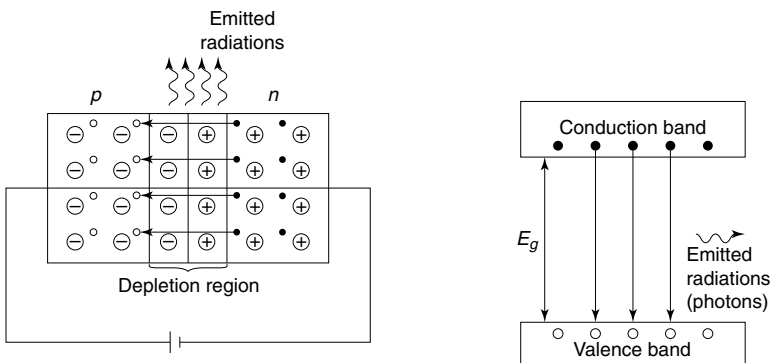
Figure 7.25   (a) Forward biased *p-n* junction, (b) Energy band diagram

From the energy band diagram, it is clear that when electrons from the conductor band recombines with the hole in the valence band then the energy of the emitted radiation (photons) is given by

$$E = h\nu = E_g$$

$$\nu = \frac{E_g}{h}$$

$$\frac{c}{\lambda} = \frac{E_g}{h}$$

$$\lambda = \frac{hc}{E_g}$$

where h is Planck's constant, c is the velocity of light and E_g is the band gap energy.

The above relations shows that the frequency and wavelength of emitted radiations depend on E_g. When E_g remains constant, all the emitted photons reinforce with each other. As a result, we get laser radiation.

In an indirect band gap, semiconductors like Ge and Si, the recombinations of electron holes will result in incoherent radiation and will not be in the optical region. In direct band gap semiconductors like GaAs, the recombination process results in coherent radiation and lies in the optical region. Due to the above fact, a GaAs semiconductor is used for semiconductor laser device.

7.14.2   Construction

A typical laser diode is as shown in Fig. 7.26. A rectangular block of GaAs semiconductor is converted into p and n type by proper doping of impurities into the block. The upper region acts as p type and the lower portion as n type. Between these two regions, we have a $p–n$ junction. To achieve poplulation inversion p and n regions are heavly doped with the impurities. The $p–n$ junctions serve as active medium. The two faces of the block, one fully polished and the other partially polished, act as an optical resonator or cavity.
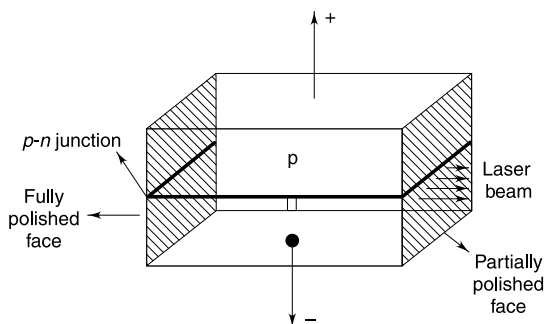
Figure 7.26   Ga–As laser

7.14.3   Working

When the p type is connected to the +ve terminal of a battery and the n type is connected to the −ve terminal then the p-n junction will be in forward–biased

condition. The recombination of electron–hole pairs takes place across the junction. Thus, laser radiation will be emitted through the *p–n* junction.

For GaAs semiconductor $E_g = 1.4$ eV

$$\lambda = 8874 \text{ Å}$$

We get light radiation in the infrared region.

Applications

1. Laser diodes are used in telecommunications as easily modulated and coupled light sources for fiber optic communication.
2. They are used in measuring instruments.
3. They are used in printing industry.
4. High power laser diodes are used in industrial applications.
5. They are used in CD players, CD-ROMs and DVD technology.

SOLVED PROBLEMS

1. The following data are given for intrinsic Ge at 300 K. $n_i = 2.4 \times 10^{19}$ m^{-3}; $\mu_e = 0.39$ m^2 $v^{-1}s^{-1}$; $\mu_p = 0.19$ m^2 $v^{-1}s^{-1}$, calculate the resistivity of the sample.

Given data   Charge of the electron, $e = 1.6 \times 10^{-19}$ c

Particle density, $n_i = 2.4 \times 10^{-19}$ m^{-3};

Electron mobility, $\mu_e = 0.39$ m^2 $v^{-1}s^{-1}$;

Hole mobility, $\mu_p = 0.19$ m^2 $v^{-1}s^{-1}$

Temperature, $T = 300$ K.

Solution $\sigma = n_i e (\mu_e + \mu_p)$

But, $\rho = \dfrac{1}{\sigma}$

$$\rho = \frac{1}{n_i^e (\mu_e + \mu_p)}$$

$$r = \frac{1}{2.4 \times 10^{-19} \times 1.6 \times 10^{-19} (0.39 + 0.19)}$$

$$\rho = 0.448 \text{ } \Omega\text{-m}$$

2. The electron and hole mobilities of Si sample are 0.135 and 0.048 m^2/Vs respectively. Determine the conductivity of intrinsic Si at 300 K. The sample is then doped with 10^{23} phosphorus atom/m^3. Determine the equilibrium hole concentration and conductivity. Given $n_i = 1.5 \times 10^{16}$ m^{-3}.

Given data   Electron mobility, $\mu_e = 0.135$ m^2/Vs

Hole mobilities, $\mu_p = 0.048$ m^2/Vs

Instrinsic carrier concentration $(n_i) = 1.5 \times 10^{16}$/m^3

Doping concentration, $N_D = 10^{23}$ phosphorous atoms/m^3

Hole concentration, $p = ?$

Conductivity $(\sigma) = ?$

Solution   Conductivity $(\sigma) = n_i e \, (\mu_e + \mu_p)$

$$= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \, [0.135 + 0.048]$$
$$= 1.5 \times 1.6 \times 0.183 \times 10^{-3} = 0.439 \times 10^{-3}$$
$$= 0.439 \times 10^{-3}/\Omega\text{-m}.$$

Hole concentration $p = \dfrac{n_i^2}{N_D} = \dfrac{(1.5 \times 10^{16})^2}{10^{23}} = 2.25 \times 10^9$/m^3

$$\sigma = N_D \, e\mu_e = 10^{23} \times 1.6 \times 10^{-19} \times 0.135$$
$$= 2.16 \times 10^3/\Omega\text{-m}.$$

3.   The R_H of a specimen is 3.66×10^{-4} m^{3}c^{-1}. Its resistivity is 8.93×10^{-3} Ω-m. Find μ and n.

Given data   Hall coefficient, $R_H = 3.66 \times 10^{-4}$ m^{3}c^{-1}

Resistivity, $\rho = 8.93 \times 10^{-3}$ m.

Mobility, $\mu = ?$

Density of atoms, $n = ?$

Solution   (i) $\mu = \sigma R_H \implies \mu = \dfrac{R_H}{\rho} \left(\text{since } \sigma = \dfrac{1}{\rho} \right)$ and

(ii) $n = \dfrac{1}{R_H e}$

1. $\mu = \dfrac{1}{8.93 \times 10^{-3} \; \Omega\text{-m}} \times 3.66 \times 10^{-4}$ m^3 c^{-1}

$$\mu = 0.4116 \text{ m}^2/\text{Vs}$$

2. $n = \dfrac{1}{R_H e}$

$$n = \dfrac{1}{3.66 \times 10^{-4} \text{ m}^3 \; c^{-1} \times 1.6 \times 10^{-19} \text{ c}}$$
$$n = 1.7 \times 10^{22}/\text{m}^3$$

4.   Find the diffusion coefficient of electron in silicon at 300 K if μ_e is 0.19 m^2/Vs.

Given data   Temperature, $T = 300$ K.

Mobility, $\mu_e = 0.19$ m^2/Vs.

Charge of the electron, $e = 1.6 \times 10^{-19}$ c

Boltzman constant, $K_B = 1.38 \times 10^{-23}$ J/K

Diffusion coefficient, D_n = ?

Solution
$$D_n = \frac{\mu_e K_B T}{e}$$

$$D_n = \frac{0.19 \text{ m}^2}{\text{Vs} \times 1.38 \times 10^{-23} \text{ J/K} \times 300 \text{ K}/1.6 \times 10^{-19} \text{ C}}$$

$$D_n = 4.9 \times 10^{-3} \text{ m}^2/\text{sec}$$

5. The resistivity of an intrinsic semiconductor is 4.5 Ω-m at 20°C and 2.0 Ω-m at 32°C. What is the energy band gap?

Given data Resistivity, $\rho_1 = 4.5$ Ω-m

Resistivity $\rho_2 = 2.0$ Ω-m

Temperature $T_1 = 20° C = 293$ K

Temperature $T_2 = 32° C = 305$ K

Boltzman constant $K_B = 13.8 \times 10^{23}$ J/K

Solution
$$\rho = A \exp\left[\frac{E_g}{2K_B T}\right]$$

$$\frac{\rho_1}{\rho_2} = \frac{A \exp\left[\dfrac{E_g}{2K_B T_1}\right]}{A \exp\left[\dfrac{E_g}{2K_B T_2}\right]} = \exp\left[\frac{E_g}{2K_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

Taking logarithm on both sides, we get

$$\ln\left(\frac{\rho_1}{\rho_2}\right) = \frac{E_g}{2K_B}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$E_g = \frac{2K_B}{\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)} \ln\left(\frac{\rho_1}{\rho_2}\right)$$

$$= \frac{2 \times 1.38 \times 10^{-23}}{\dfrac{1}{293} - \dfrac{1}{305}} \times \ln\left(\frac{4.5}{2.0}\right) \text{ J}$$

$$= 1.6669 \times 10^{-19} \text{ J} = 1.04 \text{ eV}$$

6. The following data are given for an intrinsic Ge at 300 K. Calculate the conductivity of the sample. (Given: $n_i = 2.4 \times 10^{19}$ m^{-3}, $\mu_e = 0.39$ m^2 V^{-1} s^{-1}, $\mu_h = 0.19$ m^2 V^{-1} s^{-1}.)

Given data   Intrinsic concentration $(n_i) = 2.4 \times 10^{19}$ m^{-3}

Electron mobility $(\mu_e) = 0.39$ m^2 V^{-1} s^{-1}

Hole mobility $(\mu_h) = 0.19$ m^2 V^{-1} s^{-1}

Solution   The intrinsic conductivity is

$$\sigma_i = n_i e \ (\mu_e + \mu_p)$$

$$= 2.4 \times 10^{19} \times 1.6 \times 10^{19} \ (0.39 + 0.19)$$

$$= 2.22 \ \Omega^{-1} \ m^{-1}$$

The conductivity of Ge at 300 K is 2.22 W^{-1} m^{-1}.

7. A current of 50 A is established in a slab of copper that is 0.5 cm thick and 2 cm wide. The slab is placed in a magnetic field B of 1.5 T. The magnetic field is perpendicular to the plane of the slab and to the current. The free electron concentration in Cu is 8.4×10^{28} m^{-3}. What will be the magnitude of the Hall-voltage across the width of the slab?

Given data   Current $(i) = 50$ A

Magnetic field $(B) = 1.5$ T

Thickness of the slab = 0.5 cm

Width of the slab $(d) = 2$ cm

Concentration of electrons $(N) = 8.4 \times 10^{28}$ m^{-3}

Solution   The Hall voltage is given by

$$V_H = BI/new$$

$$= \frac{1.5 \times 50}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-2}}$$

$$= 2.79 \times 10^{-7}$$

The Hall voltage is 2.79×10^{-7} V.

8. Mobilities of electrons and holes in an intrinsic germanium at 300 K are 0.36m^2/v-s and 0.17m^2/v-s respectively. If the resistivity is 2.12 Ω-m, calculate the intrinsic concentration.

Given data   Mobility of electron $\mu_e = 0.36$ m^2/v-s

Mobility of holes $\mu_p = 0.17$ m^2/v-s

Resistivity $\sigma = 2.12$ Ω-m

Solution   Conductivity

$$\sigma = \frac{1}{p} = n_i \, e \ (\mu_n + \mu_p)$$

$$\frac{1}{2.12} = n \times 1.6 \times 10^{-19} \ [0.36 + 0.17]$$

$$n_i = 556.25 \times 10^{16}/m^3$$

9. Find the conductivity of p-type Ge which is doped with acceptor atoms of concentration 2×10^{23}/m^3 and all acceptor atoms are active. Given $\mu_p = 0.19$ m^2/v-s and $e = 1.6 \times 10^{-19}$ C.

Given data Concentration of acceptor atoms $N_A = 2 \times 10^{23}$ m^3

Mobility of hole $\mu_p = 0.19$ m^2/v-s

Charge of electron $e = 1.6 \times 10^{-19}$ C

Solution For a p-type Ge,

$$Pp = N_A$$

Conductivity $\sigma_p = n_p \mu_p e$

$$= 2 \times 10^{23} \times 0.19 \times 1.6 \times 10^{-19}$$

$$= 6100 \ \Omega\text{-m}$$

10. In an intrinsic semiconductor the energy gap is 1.2 eV. What is the ratio between conductivity at 600 K and at 300 K?

Given data Energy gap $E_g = 1.2$ eV $= 1.2 \times 1.6 \times 10^{-19}$ J

Temperature $T_1 = 600$ K

Temperature $T_2 = 300$ K, $K_B = 1.38 \times 10^{-23}$ JK^{-1}

Solution Conductivity

$$\sigma = Ae^{-E_g/2K_BT}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{e^{-E_g/2K_BT_1}}{e^{-E_g/2K_BT_2}}$$

$$= e^{\frac{-E_g}{2K_B}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]}$$

$$= e^{\frac{-1.2 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}}\left[\frac{1}{600} - \frac{1}{300}\right]}$$

$$= e^{-0.6956 \times 10^4 \times -0.3316}$$

$$= e^{0.2306 \times 10^4}$$

$$= 1.25 \times 10^4$$

MULTIPLE CHOICE QUESTIONS

1. At 0 K temperature, a pure semiconductor is
   - (a) a conductor
   - (b) a resistor
   - (c) a power source
   - (d) an insulator
2. The number of valence electrons in Si atom is
   - (a) 1
   - (b) 2
   - (c) 3
   - (d) 4

3. Electric conduction in a semiconductor occurs due to the motion of
   - (a) free electrons only
   - (b) holes only
   - (c) both free electrons and holes
   - (d) neither electrons nor holes

4. When proper impurity is doped into an intrinsic semiconductor, the resistance of the semiconductor
   - (a) increases
   - (b) decreases
   - (c) remains same
   - (d) becomes zero

5. An *n*-type Ge is obtained by doping pure Ge with
   - (a) pentavalent impurity
   - (b) tetravalent impurity
   - (c) trivalent impurity
   - (d) any impurity

6. To obtain *p*-type Si out of the following impurities, the one selected for doping is
   - (a) arsenic
   - (b) antimony
   - (c) indium
   - (d) phosphorus

7. As compared to an intrinsic semiconductor, the resistivity of an extrinsic (doped) semiconductor is
   - (a) higher
   - (b) lower
   - (c) same
   - (d) resistivity can be compared

8. The majority carriers of current in a *p*-type semiconductor are
   - (a) electrons
   - (b) holes
   - (c) positive ions
   - (d) negative ions

9. Holes are charge carriers in
   - (a) intrinsic semiconductors
   - (b) ionic solids
   - (c) *n*-type semiconductors
   - (d) metals

10. In intrinsic semiconductors, the Fermi level lies
    - (a) near the conduction band
    - (b) near the valence band
    - (c) at midway of energy gap
    - (d) None of these

11. In case of intrinsic semiconductors
    - (a) $\dfrac{E_F > E_C + E_v}{2}$
    - (b) $\dfrac{E_F < E_C + E_v}{2}$
    - (c) $\dfrac{E_F = E_C + E_v}{2}$
    - (d) None of these

12. The Fermi level in an *n*-type semiconductor lies
    - (a) near the valence band
    - (b) near the conduction band
    - (c) exactly at middle of the energy gap
    - (d) None of these

13. As temperature of a semiconductor increases, the probability to find an electron in the conduction band
    - (a) increases
    - (b) decreases
    - (c) remains constant
    - (d) None of these

14. The energy gap of a semiconductor is
    - (a) constant

(b) varies with temperature

(c) varies with voltage

(d) varies with doping concentration

15. If the charge carriers are electrons, the Hall coefficient is

    (a) positive   (b) negative

    (c) zero   (d) None of these

16. The drift velocity acquired by a carrier per unit electric field strength is called

    (a) conductivity   (b) resistivity

    (c) mobility   (d) None of these

17. Minimum of the conduction band lies directly above the maximum of the valance band in momentum space in

    (a) direct gap semiconductor   (b) indirect gap semiconductor

    (c) semi-metal   (d) doped semiconductor

18. The Hall coefficient, $R_H =$

    (a) $\dfrac{1}{ne}$ (b) $\dfrac{n}{e}$

    (c) $\dfrac{e}{n}$ (d) en

19. With increase of temperature, the Fermi energy level moves_____ in *p*-type semiconductor.

    (a) downwards   (b) upwards

    (c) left   (d) right

20. The diffusion current is proportional to _____ of charge carriers.

    (a) concentration gradient   (b) drift velocity

    (c) mobility   (d) None of these

21. The carrier concentration in an intrinsic semiconductor—with increase of temperature

    (a) increases

    (b) decreases

    (c) first increases and then decreases

    (d) first decreases and then increases

22. The ratio of diffusion coefficient to mobility of charge carriers is proportional to

    (a) T (b) T^2

    (c) $\dfrac{1}{T}$ (d) $\dfrac{1}{T^3}$

23. If the Hall coefficient is negative, then the semiconductor is

    (a) *n*-type   (b) *p*-type

    (c) intrinsic   (d) extrinsic

24. With increase of temperature, the Fermi level moves—in an *n*-type semiconductor

    (a) downwards   (b) upwards

    (c) left   (d) right

25. The product np varies by changing
    - (a) pressure
    - (b) temperature
    - (c) doping trivalent impurities
    - (d) doping pentavalent impurities

26. The excess concentration of carrier injected into a semiconductor—with time.
    - (a) increass linearly
    - (b) decreases exponentially
    - (c) increases exponentially
    - (d) decreases linearly

27. In LED, if E_g is the bandgap, then the wavelength of emitted light is given by
    - (a) $\lambda = \dfrac{E_g}{h\gamma}$
    - (b) $\lambda = \dfrac{h\gamma}{E_g}$
    - (c) $\lambda = \dfrac{E_g}{hc}$
    - (d) $\lambda = \dfrac{hc}{E_g}$

28. Minimum energy gap required to obtain visible radiation from LED is
    - (a) 1 eV
    - (b) 2eV
    - (c) 3 eV
    - (d) 4 eV

29. Einstein's relation between the diffusion constant D and mobility μ of a carrier is given by
    - (a) $\dfrac{2K_BT}{e}$
    - (b) $\dfrac{e}{2K_BT}$
    - (c) $\dfrac{K_BT}{e}$
    - (d) $\dfrac{e}{K_BT}$

30. In intrinsic semiconductor the carrier concentration varies as
    - (a) $T^{3/2}$
    - (b) T^2
    - (c) T^{-2}
    - (d) T

31. At 0 K pure silicon is
    - (a) extrinsic semiconductor
    - (b) insulator
    - (c) a superconductor
    - (d) an intrinsic semiconductor

32. The majority charge carriers in p-type Ge are
    - (a) conduction electrons
    - (b) holes
    - (c) free electrons
    - (d) ions

33. In an intrinsic semiconductor
    - (a) $\sigma = n\mu_e e$
    - (b) $\sigma = p\mu_n e$
    - (c) $\sigma = e(n\mu e + p\mu_n)$
    - (d) $n\mu_e e > p\mu_n e$

34. Mobilities of electrons and holes are proportional to
    - (a) $T^{-1/2}$
    - (b) $T^{3/2}$
    - (c) $T^{-3/2}$
    - (d) $T^{1/2}$

35. For an n-type semiconductor the Hall coefficient R_H is related to carrier concentration n by
    - (a) $R_H = -ne$
    - (b) $R_H = \dfrac{-n}{e}$
    - (c) $R_H = \dfrac{-e}{m}$
    - (d) $R_H = \dfrac{-1}{ne}$

36. Electrical conductivity of a semiconductor at absolute zero of temperature is
    - (a) very small
    - (b) very large
    - (c) finite
    - (d) zero
37. In intrinsic semiconductor the electron concentration is equal to
    - (a) ion concentration
    - (b) Hole concentration.
    - (c) proton concentration
    - (d) neutron concentration
38. For *p*-type semiconductor-type of atoms are added to Si
    - (a) trivalent
    - (b) pentavalent
    - (c) tetravalent
    - (c) hectavalent
39. If the effective mass of a hole is greater than the effective mass of an electron, then the position of fermi level in intrinsic semiconductor is
    - (a) just below conduction band
    - (b) just above valence band
    - (c) just above the center of energy gap
    - (d) at the centre of energy gap
40. If the Hall coefficient is negative then the semiconductor is
    - (a) *p*-type
    - (b) intrinsic
    - (c) *n*-type
    - (d) extrinsic
41. Response time of LEDs in the order of
    - (a) 10^{-3}s
    - (b) 10^{-6}s
    - (c) 10^{-9}s
    - (d) 10^{-12}s
42. Life time of LEDs is about
    - (a) 10^2 hours
    - (b) 10^3 hours
    - (c) 10^4 hours
    - (d) 10^5 hours

Answers

1. (d)	2. (d)	3. (c)	4. (b)	5. (a)	6. (c)
7. (b)	8. (b)	9. (a)	10. (c)	11. (c)	12. (b)
13. (a)	14. (a)	15. (b)	16. (c)	17. (a)	18. (a)
19. (b)	20. (a)	21. (a)	22. (a)	23. (a)	24. (a)
25. (b)	26. (b)	27. (d)	28. (b)	29. (c)	30. (a)
31. (b)	32. (b)	33. (c)	34. (c)	35. (d)	36. (d)
37. (b)	38. (a)	39. (c)	40. (c)	41. (c)	42. (d)

Fill in the Blanks

1. Germanium and silicon posses _____ cubic crystalline structure.
2. At, 0 K the semiconductor acts as an _____.
3. As temperature increases, the fermi level in an intrinsic semiconductor shifts _____ from the middle of the forbidden band.
4. In metals with increase of temperature, resistivity _____ whereas in semiconductor it _____.

5. Phosphorous, arsenic and antimony are _____ elements.

6. Aluminium, gallium and indium are _____ elements.

7. When phosphorous is doped in silicon, the resulting semiconductor is _____.

8. When indium is doped in silicon, the resulting semiconductor is _____

9. In few semiconductors the maximum of the valence band occurs at the same value of K (wave vector) as the minimum of the conduction band. This type of semiconductor is called _____ band gap semiconductor.

10. Gallium arsenide is a _____ band gap semiconductor.

11. In few semiconductor the maximum of the valence band does not occur at the same value of K (wave vector) as minimum of the conduction band. This type of semiconductor is called _____ band gap semiconductor.

12. Silicon is a _____ band gap semiconductor.

13. Pure form of semiconductors called intrinsic semiconductors and doped form of semiconductors called extrinsic semiconductors are known as _____ semiconductors.

14. The semiconductors that are formed by combination of equal atomic fractions of fifth and third column, or sixth and second column elements are known as _____ semiconductors.

15. The relation between diffusion coefficient and mobility of a charge carrier is termed _____.

16. Recombination of an electron and hole in direct band gap semiconductor results in _____.

17. During recombination process one hole and one electron disappear and the time taken for this process is called the _____.

18. Mean life time is the time taken for the injected concentration to fall to _____ of its initial value.

19. The distance into the semiconductor at which the injected hole concentration falls to $\frac{1}{e}$ of its initial value is known as _____.

20. When a current carrying conductor is placed in a transverse magnetic field, an electric field is produced in a direction normal to the current and the magnetic field. This phenomenon is called _____.

21. The effect which help us in identifying the type of semi-conductor is _____.

22. When PN junction is formed is a small region on either side of the junction an electric field appears. This region is called _____.

23. A potential differential appears across the depletion region and this potential is called _____.

24. Under forward bias condition PN junction diode offers _____ resistance.

25. Under reverse bias condition PN junction diode offers _____ resistances.

26. Under forward bias, the width of the depletion region _____.

27. Under reverse bias, the width of the depletion region _____.

28. A device which converts alternating current (ac) to direct or unidirectional current (dc) is called as _____.

29. If a PN junction operated under forward bias gives rise to injection luminescence then it is called _____.

30. LEDs have a lifetime around _____ hours.

31. LEDs have fast response time of the order of _____.

32. The function of the photodiode junction is the opposite of _____.

Answers to Fill in the blanks

1. diamond	2. Insulator	3. upwards
4. increase, decreases	5. pentavalent	6. trivalent
7. *n*-type	8. *p*-type	9. direct
10. direct	11. indirect	12. indirect
13. elemental	14. compound	15. Einstein equation
16. emission of photon	17. minority carrier lift time	
18. $\left(\dfrac{1}{e}\right)$	19. diffusion length	20. Hall effect
21. Hall effect	22. depletion region	23. contact potential
24. Barrier potential	24. low	25. high
26. decreases	27. increases	28. rectifier
29. light emitting diode	30. 10^5	31. nano second
32. LED junction		

True or False

1. Pure germanium and silicon are called intrinsic semiconductors. (T/F)

2. The energy required to break the covalent bond in germanium is about 1.1 eV. (T/F)

3. In an intrinsic semiconductor the fermi energy level lies midway in the forbidden gas at 0 K. (T/F)

4. In intrinsic semiconductors always we have equal number of electrons and holes. (T/F)

5. In intrinsic semiconductors, since effective mass of hole is greater than the effective mass of electrons, the fermi level rises slightly from the middle with increase in temperature. (T/F)

6. Mobility is the velocity acquired by the electron or hole due to the electric field. (T/F)

7. In *n*-type semiconductors electrons are majority carriers and holes are minority carriers. (T/F)

8. At very high temperature, the extrinsic semiconductors behave in extrinsic manner only. (T/F)

9. According to Mass-action law, in any semiconductor under thermal equilibrium condition, the product of the number of electrons and number of holes is a constant. (T/F)

10. The efficiency of transition of charge carriers across the band gap is more in direct band gap than indirect band gap semiconductor. (T/F)

11. In Hall experiment, if Hall coefficient 'R_H' is measured then the carrier concentration 'n' is given by '$n = eR_H$'. (T/F)

12. In σ_n is the conductivity and R_H is the Hall coefficient, then the mobility of charge carriers is given by $\mu_n = -\sigma_n R_H$. (T/F)

13. The thickness of depletion region is in the order of 10^{-6} cm. (T/F)

14. In a PN junction diode, forward current is in μA. (T/F)

15. In a PN junction diode, reverse current is in mA. (T/F)

16. For Ge diode reverse resistance is arount 1 MΩ. (T/F)

17. LED emits light during forward bias only. (T/F)

18. For LED material, to get visible radiation, the band gap has to be around 2 eV. (T/F)

Answers to True or False

1. T	2. F	3. T	4. T	5. T	6. F
7. T	8. F	9. T	10. T	11. F	12. T
13. T	14. F	15. F	16. T	17. T	18. T

REVIEW QUESTIONS

Short Answer Questions

1. What is intrinsic semiconductor?
2. Mention the expression for intrinsic carrier concentration of intrinsic semiconductor.
3. What is Fermi level? Locate its position for intrinsic semiconductor.
4. Mention the expression for conductivity of intrinsic semiconductor.
5. What is extrinsic semiconductor?
6. Mention the effect of temperature on Fermi energy level.
7. Mention the expressions for Fermi energy levels for *n*- and *p*-type semiconductors.
8. When does drifting of charge carrier takes place in a semiconductor?
9. What is the contribution of drifting of charge carriers towards the conductivity of semiconductor?
10. When does diffusion of change carrier takes place in a semiconductor?

11. What is the contribution of diffusion of charge carriers towards the conductivity of semiconductor?
12. Mention Einstein's relation for a semiconductor.
13. What is Hall effect?
14. What are the applications of Hall effects?
15. What is direct band gap semiconductor?
16. What is indirect band gap semiconductor?
17. What is LED?
18. What is laser diode?
19. What is photodiode?

Essay Type Questions

1. Derive the expressions for intrinsic carrier concentration and Fermi level for intrinsic semiconductor.
2. Explain the conductivity of intrinsic semiconductor with relevant expressions.
3. Describe possible extrinsic semiconductors with its Fermi energy levels.
4. Describe drift and diffusion process in a semiconductor with the help of relevant expressions.
5. Derive Einstein's relation for a semiconductor.
6. Describe Hall effect in a semiconductor along with its applications.
7. Describe direct and indirect band gap semiconductors.
8. Explain the construction and working principle of LED.
9. Explain the construction and working principle of photodiode.
10. Explain the construction and working principle of laser diode.

CHAPTER 8

MAGNETIC MATERIALS

INTRODUCTION

Magnetic materials play an important role in industrial and scientific research fields. The magnetism of materials is mainly due to the interactions of uncompensated magnetic moments of the constituent atoms or molecules. Based on the response of materials in an external field, they are divided into three categories which are again classified into five important groups depending on the alignment of magnetic moments within the materials. Thus, they are known as diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic and ferrimagnetic materials. Ferromagnetic materials are further classified into soft and hard materials. These materials are widely used in information storage devices.

8.1   BASIC DEFINITIONS

1. *Magnetism*   The attracting property exhibited by the magnet is known as magnetism.

2. *Magnetic dipole*   A magnet consists of north and south poles. Any two opposite poles separated by a finite distance constitute a magnetic dipole.

3. *Magnetic dipole moment or magnetic moment*   If 'm' is the magnetic pole strength and '$2l$' is the length of the magnet then its magnetic dipole moment or magnetic moment is given by the products $m \times 2l$. It is denoted by μ_m.

   When an electric current of I ampere flows through a circular wire of one turn having an area of cross section A m^2, it is said to have a magnetic moment of

   $$\mu_m = IA$$

It is expressed in Am^2. It is a vector quantity pointing from South to North pole.

4. *Magnetic field*   The space surrounding the magnet up to which its attracting influence is felt is known as magnetic field.

5. *Magnetic field strength (H)*   The magnetic field strength H is the force experienced by a unit north pole placed at a given point in a magnetic field. It is expressed in $A\ m^{-1}$.

6. *Magnetisation or intensity of magnetisation (M)*   The magnetisation M is defined as the magnetic moment per unit volume. It is expressed in $A\ m^{-1}$.

7. *Magnetic susceptibility (χ)*   The magnetic susceptibility χ is defined as the ratio of the magnetisation produced in a sample to the magnetic field strength.

$$\chi = \frac{M}{H}$$

χ has no units.

8. *Magnetic lines of forces*   The magnetic field is characterised by magnetic lines of forces.

9. *Magnetic induction field strength (or) Magnetic flux density (B)*   The magnetic flux density B is defined as the number of magnetic lines of force passing through a unit area of cross section of magnetic material.

   If ϕ is the number of magnetic lines of forces that passes through 'A' area of cross section of a material, then

$$B = \phi/A$$

   It is expressed in $Wb\ m^{-2}$ or Tesla.

10. *Magnetic permeability (μ)*   The magnetic permeability μ is a measure of the amount of magnetic lines of forces penetrating through a material. It is defined as the ratio of the magnetic flux density (or) magnetic induction field strength B in the material to the applied magnetic field intensity H. That is,

$$\mu = \frac{B}{H}$$
$$B = \mu H$$

11. *Magnetic permeability of free space (μ₀)*   The magnetic permeability of free space μ_0 is a measure of the amount of magnetic lines of forces passing through the air medium. It is defined as the ratio of magnetic induction field strength B_0 of a free space or air to the applied magnetic field strength H.

$$\mu_0 = \frac{B_0}{H}$$
$$B_0 = \mu_0 H \quad \text{where } \mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$$

12. ***Relative magnetic permeability (μ_r)*** It is defined as the ratio of permeability of a material to the permeability of free space.

$$\mu_r = \frac{\mu}{\mu_0}$$

$$\mu = \mu_0 \mu_r$$

Relation between B, H and M

$$B = \mu H$$
$$= \mu_0 \, \mu_r \, H$$
$$B = \mu_0 \, \mu_r \, H + \mu_0 H - \mu_0 H$$

But $M = H(\mu_r - 1)$
$$B = m_0 H + \mu_0 M$$
$$B = \mu_0 \, (H + M)$$

Relation between χ and μ_r

$$B = \mu_0 \, (H + M)$$

$$\mu_0 = \frac{B}{H + M}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B/H}{B/H + M} = \frac{H + M}{H} = 1 + \frac{M}{H}$$

$$\mu_r = 1 + \chi \quad \left(\because \chi = \frac{M}{H} \right)$$

8.2   ORIGIN OF MAGNETIC MOMENT–BOHR MAGNETON

Materials are made up of atoms. These atoms consist of a nucleus surrounding which electrons are revolving in their orbits. These orbital motions of electron establish a magnetic field and hence we get orbital magnetic moment of the electrons. While revolving around the nucleus, the electrons spin about their own axes which results in a magnetic field and we get spin magnetic moment of the electrons. The nucleons in the nucleus also spin, resulting in magnetic field and we get spin magnetic moment of nucleus.

8.2.1   Orbital Magnetic Moment of Electron

Consider an electron of mass 'm', charge 'e' revolving around the nucleus in a circular orbit of radius 'r' in anticlockwise direction. Let 'v' be the linear velocity of electron and 'ω' the angular velocity (angular frequency) of a revolving electron. Then the linear frequency of the revolving electron $= \dfrac{\omega}{2\pi}$

Time period of revolutions $= \dfrac{2\pi}{\omega}$

The revolving electron in circular orbit establishes a current given by

$$I = \frac{\text{charge}}{\text{time period}} = e\,\frac{\omega}{2\pi}$$

The current 'I' establishes a magnetic field around the circular orbit, so that the upper surface acts as south pole and the lower surface acts as North pole. The area of the orbit is $A = \pi r^2$.

Then the corresponding magnetic moment is given by

$$\mu_{\text{orbital}} = IA$$
$$= e\,\frac{\omega}{2\pi}\,\pi r^2$$
$$\mu_{\text{orbital}} = \frac{e\,\omega r^2}{2} \tag{1}$$
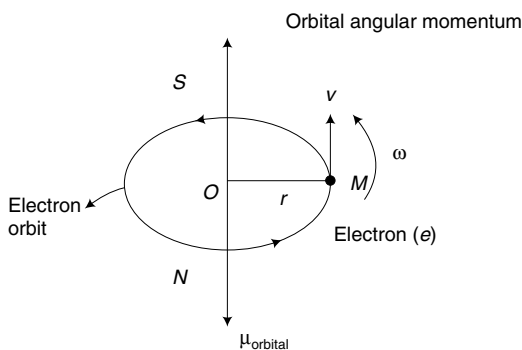
Figure 8.1   Revolving electron in an atom

This orbital magnetic moment is pointing from S-pole to N-pole, i.e., in the downward direction.

The angular momentum of the revolving electron is given by

Angular momentum = Linear momentum × Radius

$$= mvr$$
$$= m\omega r^2 \quad (\because v = \omega r) \tag{2}$$
$$\therefore \qquad \mu_{\text{orbital}} = \frac{e\,\omega r^2}{2}$$
$$= \left[\frac{e}{2m}\right][m\,\omega r^2]$$
$$= -\left[\frac{e}{2m}\right] \quad \text{(orbital angular momentum)} \tag{3}$$

The –ve sign indicates that the orbital angular momentum and magnetic moment are in opposite directions. Then the orbital gyromagnetic ratio =

$$\gamma = \frac{\text{Magnetic moment}}{\text{Orbital angular momentum}}$$

$$\boxed{\gamma = \frac{e}{2m}} \tag{4}$$

According to modern atomic theory, the angular momentum of an electron in the orbit is determined by the orbital quantum number 'l'.

Angular momentum of electrons associated with orbital quantum number 'l' is given by $l\left[\dfrac{h}{2\pi}\right]$

$$\therefore \qquad\qquad \mu_{\text{orbital}} = -\frac{e}{2m} \quad \text{(orbital angular momentum)}$$

$$= -\frac{e}{2m}\left[\frac{lh}{2\pi}\right]$$

$$\mu_{\text{orbital}} = -\left[\frac{ehl}{4\pi m}\right]$$

$$\mu_{\text{orbital}} = \mu_B l \quad \text{where } l = 1, 2, 3, \ldots$$
$$\mu_{\text{orbital}} = -\mu_B, -2\mu_B, -3\mu_B, -4\mu_B, \ldots \tag{5}$$

where $\mu_B = \dfrac{eh}{4\pi m}$ is a fundamental unit of magnetic moment known as Bohr magneton, and its value is $9.27 \times 10^{-24}\ \text{Am}^2$.

8.2.2 Spin Magnetic Moment of Electron $_{\text{spin}}$

The spinning electron about it's own axis establishes the magnetic field and we get a magnetic moment known as spin magnetic moment which is given by $\mu_{\text{spin}} = \gamma\left(\dfrac{e}{2m}\right) S$ where 'γ' is the spin gyromagnetic ratio and 'S' is the spin angular momentum. For an electron,

$$S = \frac{h}{4\pi}$$

$$\mu_{\text{spin}} = \gamma\left(\frac{e}{2m}\right)\left(\frac{h}{4\pi}\right) = -9.4 \times 10^{-24}\ \text{Am}^2$$

The value of μ_{spin} is nearly equal to the orbital magnetic moment.
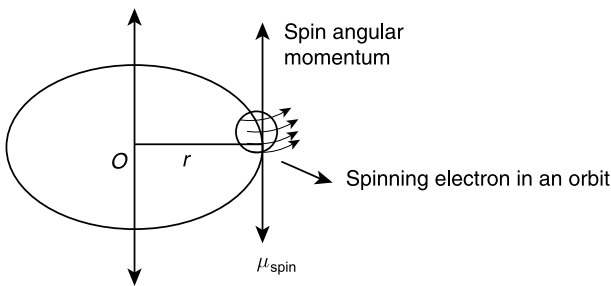
Figure 8.2 Spinning electron in an atom

8.2.3 Magnetic Moment due to Nuclear Spin

In addition to electronic contribution, nuclear spin also contributes to magnetic moment of atoms.

The nuclear spin magnetic moment is given by

$$\mu_{\text{nuclear spin}} = \frac{eh}{4\pi M_p}$$

where M_p represents the mass of the proton. Substituting all the values, we get

$$\mu_{\text{nuclear spin}} = 5.05 \times 10^{-27} \text{ Am}^2$$

8.3 CLASSIFICATION OF MAGNETIC MATERIALS

The magnetic materials have been classified into five types based on the influence of external field on them. The classification is given below:

S.No.	Diamagnetic material	Paramagnetic material	Ferromagnetic material	Antiferromagnetic material	Ferrimagnetic material
1.	Net magnetic moment exists in the presence of magnetic field in opposite direction.	Net magnetic moment exists in the direction of applied magnetic field.	Large net magnetic moment exists in the direction of applied magnetic field.	Net magnetic moment exists in the direction of applied magnetic field.	Net large magnetic moment exists in the direction of applied magnetic field.
2.	There is an induced magnetism opposite to applied field.	There is an induced magnetism in the direction of the field.	There is strong induced magnetism in the direction of the field.	There is an induced magnetism in the direction of the field.	There is a large induced magnetism in the direction of the field.
3.	It repels the magnetic lines of forces due to the external magnetic field.	It allows the magnetic lines of forces to pass through it.	It allows a large number of magnetic lines of forces to pass through it.	It allows magnetic lines of forces to pass through it.	It allows large number of lines of forces to pass through it.

Contd..

Contd..

4.	Relative permeability $\mu_r < 1$	$\mu_r > 1$	$\mu_r \gg 1$	$\mu_r > 1$	$\mu_r \gg 1$
5.	Since the induced magnetic moment is opposite to the direction of the applied field, the intensity of magnetisation 'M' is negative.	Since the induced magnetic moment is in the direction of applied field, intensity of magnetisation 'M' is positive and moderate.	Intensity of magnetisation 'M' is +ve and high, since large magnetic moment is induced in the direction of the field.	Intensity of magnetisation 'M' is positive and moderate.	Intensity of magnetisation 'M' is positive and high.
6.	Susceptibility 'χ' is negative.	'χ' is low and positive.	'χ' is high and positive.	'χ' is low and positive.	'χ' is high and positive.
7.	Susceptibility χ is independent of temperature.	$\chi = \dfrac{C}{T}$ which is Curie law where C = Curie const.	$\chi = \dfrac{C}{T \pm T_C}$ which is Curie–Weiss law where T_C = Curie temperature.	$\chi = \dfrac{C}{T + T_C}$	$\chi = \dfrac{C}{T \pm T_C}$
8.	Examples: organic materials.	Examples: Alkali metals and transition metals.	Examples: Transition and rare earth metals.	Examples: Salts of transition elements.	Examples: Ferrites.

8.3.1 Diamagnetic Material

As shown in the Fig. 8.3(a), the atoms in the diamagnetic material will have random orientations of magnetic moments in the absence of a magnetic field. Once the magnetic field is applied, all the magnetic moments of its atoms will align in the opposite direction of the applied field. As a result, it exhibits net
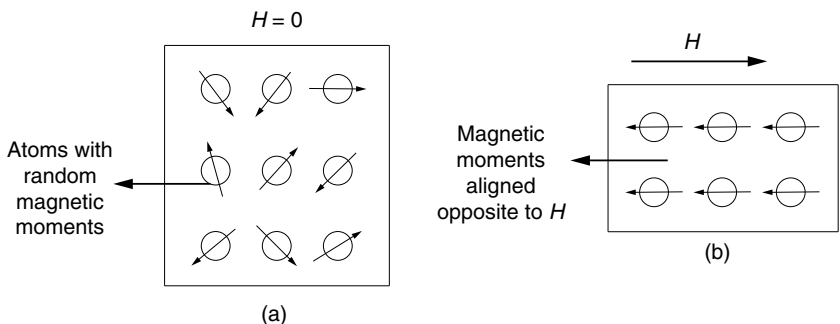

Figure 8.3 (a) Diamagnetic material in the absence of magnetic field, (b) Diamagnetic material in the presence of magnetic field

magnetic moment in a direction opposite to the applied field. It repels (does not allow) the magnetic lines of forces due to the applied field as magnetism is induced opposite to the field. The relative permeability is $\mu_r < 1$. The intensity of

magnetisation is negative and the susceptability $\chi = \dfrac{M}{H}$ is also negative.

Explanation

Diamagnetism is the result of Lenz's law in electromagnetic induction. When a coil of wire is subjected to varying magnetic flux, and electric field is set up in the wire, giving rise to induced current. The magnetic field produced by this induced current opposes the applied magnetic field. Consider two atoms which have clockwise and anticlockwise revolving electrons. If ω_0 is the angular frequency of revolving electrons then

Figure 8.4   Revolving electrons in a diamagnetic material

$$\mu_0 = IA$$
$$= -\left[\frac{e\,\omega_0}{2\pi}\right]\pi r^2$$
$$= -\frac{1}{2}\,(er^2\,\omega_0) \tag{6}$$

In the presence of magnetic induction field, B, the angular frequency is given by

$$\omega = \omega_0 \pm \frac{e}{2m}\,B \tag{7}$$

The change in orbital magnetic moment,

$$\mu_{\text{field}} = -\frac{1}{2}\,er^2\,w$$
$$= -\frac{1}{2}\,er^2\left[\omega_0 \pm \frac{e}{2m}\,B\right]$$
$$= -\frac{1}{2}\,er^2\,\omega_0 \mp \frac{e^2}{4m}\,r^2B$$
$$= \mu_0 \mp \mu_{\text{ind}} \tag{8}$$

where $\mu_{\text{ind}} = \dfrac{e^2}{4m}\,r^2B$ $\tag{9}$

From the above relation, it is clear that the angular frequency and orbital magnetic moment of anticlockwise revolving electron increases which opposes 'B'. The net magnetic moment exists opposite to B.

For a spherical symmetric atom,

The average radius of the orbit $r^2 = \dfrac{2}{3}\,r_0^2$ $\tag{10}$

where r_0 is the mean distance of electrons with respect to nucleus.

If N is the number of atoms per unit volume and each atom has Z electrons, then induced magnetic moment is given by

$$\mu_{ind} = -\frac{Ne^2}{4m} BZ \left(\frac{2}{3}\right) r_0^2$$

$$= -NZ \left[\frac{e^2}{6m} r_0^2\right] B$$

$$= -NZ\mu_0 \left[\frac{e^2}{6m} r_0^2\right] H \quad [\because B = \mu_0 H]$$

$$\chi = \frac{\mu_{ind}}{H} = -\frac{\mu_0 ZNe^2}{6m} r_0^2 \tag{11}$$

From the above equation, it is clear that χ is −ve and is independent of temperature. The above results are pertaining to diamagnetism of diamagnetic material.

8.3.2 Paramagnetic Material

In the absence of a magnetic field, all atoms in the paramagnetic material will have random magnetic moments and do not exhibit net magnetic moment [Fig. 8.5(a)].

Figure 8.5 (a) Paramagnetic material in the absence of a magnetic field, (b) Paramagnetic material in the presence of a magnetic field

When placed in a magnetic field, all the magnetic moments due to the atoms align into the direction of applied field and exhibit net magnetic moment along the field. As a result, it allows the magnetic lines to pass through it and $\mu_r > 1$. Intensity of magnetisation M is positive and susceptability χ is also positive.

Explanation

Let there be N number of magnetic dipoles per unit volume of the material. In the absence of H, the magnetic moments have random orientation. Hence $M = 0$ with H, let N_p diploes be aligned parallel to the field and N_a be aligned antiparallel to the field at temperature T.
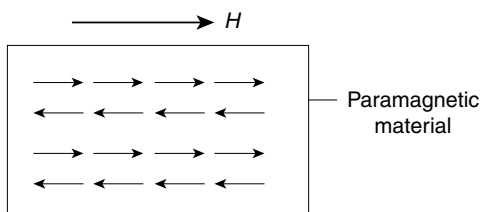
Figure 8.6 Magnetic moments

Net magnetisation $M = N_p \mu_B - N_a \mu_B$

$$= (N_p - N_a) \mu_B \tag{12}$$

where μ_B is the Bohr magneton

But $\qquad N_p - N_a = N \tan h (\alpha) \tag{13}$

where $\qquad \alpha = \mu_0 \mu_B H / K_B T \tag{14}$

$\therefore \qquad M = N \mu_B \tan h (\alpha) \tag{15}$

For strong fields and low temperature, $\alpha \gg 1$, $\tan h (\alpha) \to 1$ and $M = N \mu_B \tag{16}$

i.e., all dipoles are aligned parallel to H.

For weak fields and normal temperature, $\alpha \ll 1$, $\tan h (\alpha) \to \alpha$ then

$$M = N \mu_B \alpha$$
$$= N \mu_0 \mu_B^2 H / K_B T \tag{17}$$

$$\chi = \frac{M}{H} = \frac{N \mu_0 \mu_B^2}{K_B T} = \frac{C}{T} \tag{18}$$

where $C = \dfrac{N \mu_0 \mu_B^2}{K_B} =$ Curie constant

The above relation is the *Curie law.*

The variation of χ with T is as shown in Fig. 8.7.
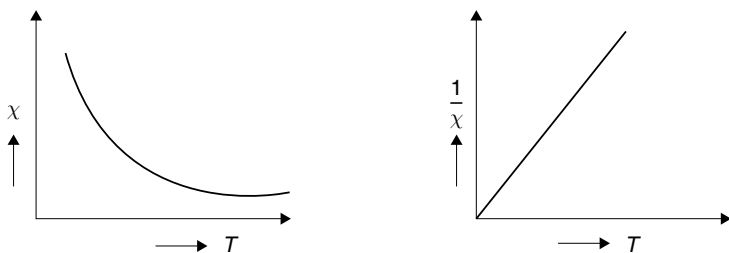
Figure 8.7 Variation of χ with T

The above facts explains the paramagnetism exhibited by paramagnetic materials.

8.3.3   Ferromagnetic Material

In ferromagnetic materials, the magnetic moments are predominently due to spin magnetic moments of electrons, which will be aligned parallel to each other to a maximum extent [Fig. 8.8(a)]. When a magnetic field (H) is applied immediately, magnetic moments align in its field direction exhibiting large net magnetic moments along H. It allows large number of magnetic lines to pass through it and $\mu_r \gg 1$. The intensity of magnetisation M and susceptibility χ are large and positive.

The magnetic susceptibility varies with temperature as per Curie–Weiss law, i.e.,

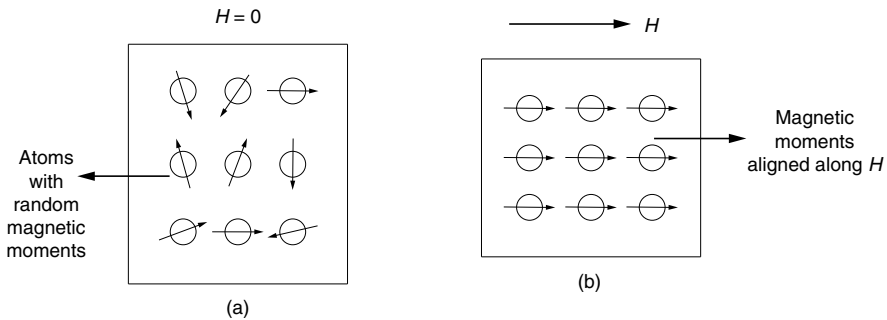$$\chi = \frac{C}{T \pm T_C}$$

where T_C is the *Curie temperature*.
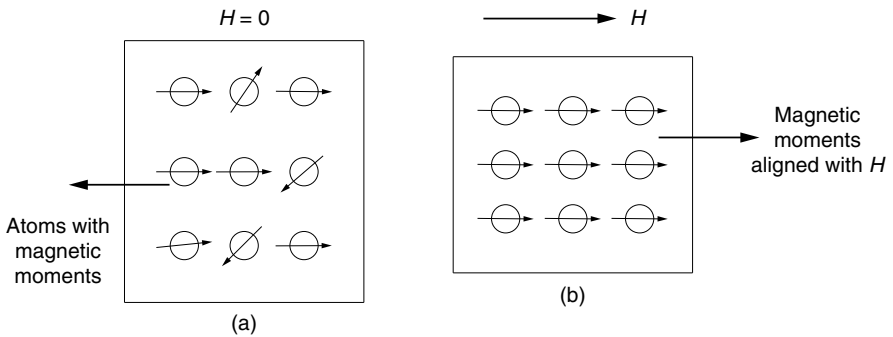
Figure 8.8   (a) Paramagnetic material in the absence of a magnetic field, (b) Paramagnetic material in the presence of a magnetic field

<u>**Case 1**</u> $$\chi = \frac{C}{T + T_C}$$

where $T < T_C$, χ is high and positive which corresponds to a ferromagnetic material.

where $T > T_C$, χ is low and positive which corresponds to a paramagnetic material.

<u>**Case 2**</u> $$\chi = \frac{C}{T - T_C}$$

where $T < T_C$, χ is negative which corresponds to a diamagnetic material.

where $T > T_C$, χ is positive which corresponds to either a paramagnetic or ferromagnetic material.

The variation of χ with temperature is shown in Fig. 8.9.

The important features of ferromagnetism are as given below:

1. A ferromagnetic material exhibits spontaneous magnetisation, i.e., even in the absence of a magnetic field, it exhibits magnetisation.

2. A ferromagnetic material exhibits hysteresis.

3. The above features can be explained based on Weiss molecular field theory (domain theory). According to this, the

Figure 8.9   Variation of χ with T

material surface consists of small regions known as *domains*. Within the domain, due to interaction between the moments, all the magnetic moments will be aligned parallel. In the weak magnetic field, the growth of favourable domain and shrinkage of unfavourable domains takes place and in the high field, the magnetic moments align parallel to the field.

Figure 8.10   Alignment of antiferromagnetic moments
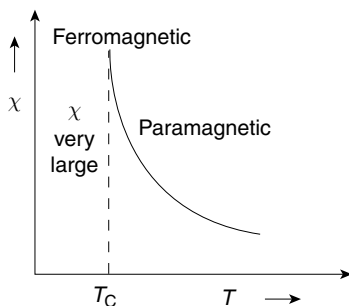
8.3.4   Antiferromagnetic Material

The materials with antiparallel magnetic moments (spin magnetic moments) are known as antiferromagnetic materials and their magnetic moments are shown in Fig. 8.10.

Examples   Oxides of Ni, Cr, Ferrous, etc.

The variation of susceptibility w.r.t. temperature is given by Curie–Wiess law, i.e., $\chi = \dfrac{C}{T + T_C}$ where C is the Curie constant and 'T_C' is Curie temperature.

The variation of χ w.r.t. T is as shown in Fig. 8.11.

'χ' increases gradually with temperature and attains a maximum value at Neel temperature and then decreases with increase in temperature.

Figure 8.11   Variation of χ with T.

Explanation

Consider a body-centred unit cubic cell of antiferromagnetic material. It consists of two

Figure 8.12   Antiferromagnetic cubic material

different atoms *A* and *B* with antiparallel magnetic moments. When it's temperature is increased, the magnetic moments of *A* and *B* get disturbed. If the disturbance is in clockwise and anti-clockwise directions, the magnetic moments will be aligned in the same direction. Hence 'χ' increases and attains a maximum value, a temperature known as *Neel temperature*. Further increase of temperature disturbs the alignment of magnetic moments in the same direction. Thus 'χ' decreases when the temperature is greater than the Neel temperature.

8.3.5 Ferrimagnetic Material

The substances which consist of antiparallel magnetic moments of different magnitudes are known as ferrimagnetic substances and the corresponding magnetic moments are shown in Fig. 8.13.

Figure 8.13 Alignment of ferromagnetic moments

Example Ferrites - Complex oxides - XFe_2O_4 where X = Mn, Mg, Cu, Ni, Co, Zn.

The variation of χ with the temperature is given by Curie–Wiess law, i.e.,

$$\chi = \frac{C}{T \pm T_C}$$

where 'C' is the Curie constant and T_C is the Curie temperature. The variations are as shown in Fig. 8.14.
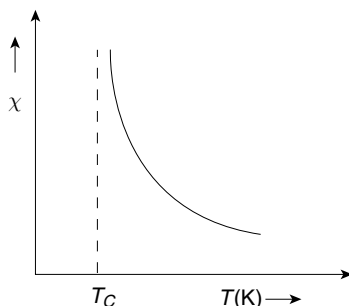
Figure 8.14 Variation of χ with T

8.4 HYSTERESIS

When a specimen of ferromagnetic material is placed in a magnetic field, the specimen is magnetised by induction. As the magnetic intensity *H* is varied, the flux density *B* in the (ferromagnetic) material does not vary linearly with *H*. The variation in *B* with variation in *H* is shown in Fig. 8.15. The point *O* represents an intitially unmagnetised specimen and a zero magnetic intensity. As *H* is increased, *B* also increases, but not uniformly, and a point such as '*a*' is reached.

Figure 8.15 Hysteresis

If *H* is now decreased, *B* also decreases but following a path *ab*. Thus, *B lags behind H*. When *H* becomes zero, *B* still has a value equal to *Ob*. This magnetic flux density remaining in the specimen in the absence of any external field is called the 'residual magnetism'. The power of retaining this magnetism is called the 'retentivity' or 'remanence' of the specimen. Thus, the *retentivity B_r of a specimen is a measure of the magnetic flux remaining in the specimen when the magnetising force (magnetic intensity) is removed*.

If the magnetic intensity *H* is now increased in the reverse direction, the value of *B* further decreases, still lagging behind *H*, and becomes zero when *H* has a value equal to *OC*. This value of the magnetic intensity is called the coercive force or coercivity of the specimen. *Thus, coercivity H_c is a measure of the magnetic intensity required to destroy the residual magnetism of the specimen*.

As *H* is increased beyond *OC*, the specimen is increasingly magnetised in the opposite direction, and a point such as *d* is reached.

By taking *H* back from its negative maximum value, through zero, to its original positive maximum value, a similar curve *defa* is obtained. At points *b* and *e* where the specimen is magnetised in the absence of any external magnetic intensity, it is said to be a *'permanent magnet'*.

It is thus found that the flux density *B* always lags behind the magnetic intensity *H*. The lagging of *B* behind *H* is called 'hysteresis'. The closed curve *abcdefa* which represents a *cycle of magnetisation* of the specimen is known as the 'hysteresis curve (or loop)' of the specimen.

Based on the area of hysteresis loop, ferromagnetic materials are classified into hard and soft magnetic materials.

8.5   SOFT AND HARD MAGNETIC MATERIALS

Depending upon the size of the hysteresis loop, ferromagnetic materials are classified into soft and hard magnetic materials.

Soft magnetic material   The ferromagnetic materials which are easily magnetised and demagnetised are known as soft magnetic materials.

Properties
1. Soft magnetic materials shows a very steep hysteresis loop as shown in Fig. 8.16.
2. The area of the loop is very small and hence the hysteresis loss is also small.
3. Soft magnetic materials can be magnetised and demagnetised easily and hence they show high values of susceptibility and permeability.
4. Due to the steep nature of the loop it is clear that it has low value of coercivity.
5. Its exhibits low value of retentivity.
6. They exhibit high resistivity.
7. They require low magnetic field for magnetisation.
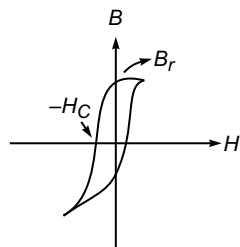
Figure 8.16   Soft magnetic material Hysteresis loop

Examples
1. Iron – Silicon alloys and demagnetisation

2. Nickel – iron alloys
3. Iron – Cobalt alloys

Applications

1. They are used in electrical equipments and magnetic cores of transformers.
2. They are used in motors, relays and sensors.
3. They are used as microwave isolators.

Hard magnetic material   The ferromagnetic materials which are hard to magnetise and demagnetise are known as hard magnetic materials.

Properties

1. They are very hard to magnetise and demagnetise.
2. They require high magnetic field for magnetisation and demagnetisation.
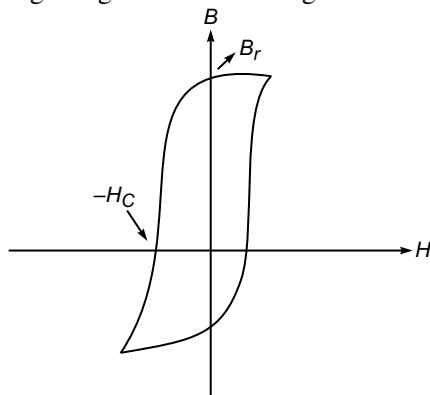
Figure 8.17   Hard magnetic material hysteresis loop

3. The corresponding hysteresis loop is very broad.
4. Hysteresis loss is large due to large area of the loop.
5. They exhibit low values of susceptibility and permeability.
6. Due to broader loop they have high coercitivity and retentivity.

Examples

Carbon steels, tungsten steels, chromium steel, alnico, etc.

Applications

1. They are used to make permanents magnets.
2. They are used in dc meters and measuring devices.
3. They are used in speedometers and sensors in automobiles, motors, etc.

8.5.1 Comparison between Soft and Hard Magnetic Materials

Soft magnetic material	*Hard magnetic material*
1. Soft magnetic materials can be easily magnetised and demagnetised.	1. Magnetisation and demagnetisation of hard magnetic materials are very difficult.
2. Small magnetic field magnetises the soft magnetic material since domain wall moves easily and reversibly.	2. Large magnetic field is required to magnetise the hard magnetic material since domain wall movement is very hard.
3. It consists of a very steep hysteresis loop.	3. It consists of a broad hysteresis loop.
4. Since the hysteresis area is very small, the corresponding loss is also small.	4. Since hysteresis area is large, the corresponding loss is also large.
5. They have large values of susceptibility and permeability.	5. They have low values of susceptibility and permeability.
6. Their coercivity and retentivity values are small.	6. Their coercivity and retentivity values are large.
7. Soft magnetic materials are used in the preparation of magnetic core materials used in transformers, electric motors, magnetic amplifiers, etc.	7. Hard magnetic materials are used in the preparation of permanent magnets which are used in loudspeakers, microphones, magnetic detectors, etc.

SOLVED PROBLEMS

1. The magnetic field in the interior of a certain solenoid has the value of 6.5×10^{-4} T when the solenoid is empty. When it is filled with iron, the field becomes 1.4 T. Find the relative permeability of iron.

Given data   Magnetic field without iron material, $B_0 = 6.5 \times 10^{-4}$ Tesla

Magnetic field with iron material, $B = 1.4$ Tesla

Solution   Relative permeability of iron is,

$$\mu_r = \frac{B}{B_0}$$

$\Rightarrow$ $\mu_r = \dfrac{1.4}{6.5 \times 10^{-4}} = 2153.85$

2. Find the relative permeability of a ferromagnetic material if a field of strength 220 amp/meter produces a magnetisation 3300 A/m in it.

Given data   Magnetisation, $M = 3300$ A/m

Field strength of given magnetic material, $H = 220$ A/m

Solution   The relative permeability,

$$\mu_r = \frac{M}{H} + 1$$

$\Rightarrow$ $\frac{3300}{220} + 1 = 15 + 1 = 16$

3. A magnetic material has a magnetisation of 3300 A/m and flux density of 0.0044 Wb/m^2. Calculate the magnetising force and the relative permeability of the material.

Given data   Magnetisation, $M = 3300$ A/m

Flux density, $B = 0.0044$ Wb/m^2

Solution $B = \mu_0 \, [H + M]$

$\Rightarrow$ $H = \frac{B}{\mu_0} - M$ where $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$H = \frac{0.0044}{4 \times 3.14 \times 10^{-7}} - 3300$$

$$H = 203 \text{ A/m}$$

$$\mu_r = \frac{M}{H} + 1 \quad \Rightarrow \quad \frac{3300}{200} + 1 = 16.5 + 1 = 17.5$$

4. A circular loop of copper having a diameter of 10 cm carries a current of 500 mA. Calculate the magnetic moment associated with the loop.

Given data   Diameter of the circular loop, $d = 10$ cm

$$d = 2r = 10 \text{ cm} = 0.1\text{m}$$

Radius of the circular loop, $r = 5$ cm $= 0.05$ m

Current in the circular loop, $I = 500$ mA $= 0.5$ A

Solution   Magnetic moment

$$\mu = AI = \pi r^2 I$$

$\Rightarrow$ $\mu = 3.14 \times (0.05)^2 \times 0.5$

$$= 3.93 \times 10^{-3} \text{ Am}^2$$

5. An electron in a hydrogen atom circulates with a radius of 0.052 nm. Calculate the change in its magnetic moment if a magnetic induction $(B) = 3$ Wb/m^2 acts at right angles to the plane of orbit.

Given data   Radius of hydrogen atom, $r = 0.052$ nm

$$= 0.52 \times 10^{-10} \text{ m}$$

Magnetic induction field, $B = 3$ Wb/m^2

Change in magnetic moment, $\Delta\mu$ = ?

Solution
$$\Delta\mu = \frac{e^2\, r^2\, B}{4m}$$

$$= \frac{[1.6 \times 10^{-19}]^2 \times [0.52 \times 10^{-10}]^2 \times 3}{4 \times 9.1 \times 10^{-31}}$$

$$= \frac{2.56 \times 0.2704 \times 3 \times 10^{-58}}{4 \times 9.1 \times 10^{-31}}$$

$$\Delta\mu = 5.705 \times 10^{-29} \text{ Am}^2$$

6. Calculate the change in magnetic moment of a circulating electron in an applied field of 2 Tesla acting perpendicular to the plane of the orbit. Given $r = 5.29 \times 10^{-11}$m for the radius of the orbit.

Given data   Magnetic induction field, B = 2 Tesla

Radius of the orbit, $r = 5.29 \times 10^{-11}$ m

Solution
$$\Delta\mu = \frac{e^2\, r^2\, B}{4m}$$

$$= \frac{[1.6 \times 10^{19})^2 \times (5.29 \times 10^{-11})^2 \times 2}{4 \times 9.1 \times 10^{-31}}$$

$$= 3.9362 \times 10^{-29} \text{ Am}^2$$

7. A paramagnetic material has 10^{28} atoms per m^3. Its susceptibility at 350 K is 2.8×10^{-4}. Calculate the susceptibility at 300 K.

Given data   Number of atoms, $N = 10^{28}$ atoms/m^3

Susceptibility at 350 K, $\chi_1 = 2.8 \times 10^{-4}$

Temperature, T_1 = 350 K

Temperature, T_2 = 300 K

Susceptibility at 300 K, χ_2 = ?

Solution $\therefore$ $\chi = \dfrac{C}{T} \implies C = \chi T$

$\therefore$ $\chi_1\, T_1 = \chi_2\, T_2$

$\implies$ $\chi_2 = \dfrac{\chi_1\, T_1}{T_2}$

$$= \frac{2.8 \times 10^{-4} \times 350}{300} = 3.267 \times 10^{-4}$$

8. When a magnetic material is subjected to a magnetic field of intensity 250 Am^{-1}, its relative permeability is 15. Calculate its magnetisation and magnetic flux density.

Given data   Intensity of magnetic field $H = 250 \text{ Am}^{-1}$

Relative permeability $\mu_r = 15$

Solution   Magnetisation, $M = H[\mu_r - 1]$

$$= 250 \, [15 - 1]$$
$$= 3500 \text{ Am}^{-1}$$

Magnetic flux density $B = \mu_0 \, [H + M]$

$$= 4\pi \times 10^{-7} \, [250 + 3500]$$
$$= 4.71 \times 10^{-3} \text{ wbm}^{-2}$$

9. Calculate magnetic moment per unit volume and flux density of a magnetic material placed in a magnetic field of intensity 1000 Am^{-1}. The magnetic susceptibility is -0.42×10^{-3}.

Given data   Intensity of magnetic field $H = 1000 \text{ Am}^{-1}$

Magnetic susceptibility $\chi = -0.42 \times 10^{-3}$

Solution   Magnetic moment per unit volume (or) magnetisation

$$M = \chi H$$
$$= -0.42 \times 10^{-3} \times 1000$$
$$= -0.42 \text{ Am}^{-1}$$

Flux density $D = \mu_0 \, (H + M)$

$$= 4\pi \times 10^{-7} \times [1000 - 0.42]$$
$$= 1.257 \times 10^{-3} \text{ wbm}^{-2}$$

MULTIPLE CHOICE QUESTIONS

1. Examples for diamagnetic materials are
   - (a) bismuth and lead
   - (b) zinc and noble gases
   - (c) gold and copper
   - (d) All of these
2. Examples for paramagnetic materials are
   - (a) manganese chloride
   - (b) salts of iron and nickel
   - (c) aluminium and platinum
   - (d) All of these
3. The magnetic dipole moments of neighbouring atoms are antiparallel and unequal for_____ magnetic material
   - (a) dia
   - (b) para
   - (c) ferri
   - (d) anti-ferro
4. Copper is
   - (a) dia
   - (b) para
   - (c) ferro
   - (d) anti-ferro

5. When a material is used in a magnetic field B, a magnetic moment proportional to B but opposite in direction is induced. The metal is
   - (a) diamagetic
   - (b) paramagnetic
   - (c) ferromagnetic
   - (d) ferrimagnetic

6. Metals getting magnetised by orientation of atomic magnetic moments in an external magnetic field are called
   - (a) diamagnetic
   - (b) paramagnetic
   - (c) ferromagnetic
   - (d) antimagnetic

7. The major contribution of magnetism in substances is due to
   - (a) spin motion of electrons
   - (b) orbital motion of electrons
   - (c) hidden magnets
   - (d) equally due to orbital and spin motion of electrons

8. The magnetic dipole moment is the product of current in the loop and
   - (a) flux enclosed by the current loop
   - (b) square of area enclosed by the current loop
   - (c) area enclosed by the current loop
   - (d) None of these

9. Magnetic susceptibility is
   - (a) torque per unit area
   - (b) dipole moment per unit volume
   - (c) magnetisation per unit magnetic field intensity
   - (d) None of these

10. Bohr magneton is defined as
   - (a) magnetic moment of a nucleus
   - (b) magnetic moment of an electron spin
   - (c) magnetic moment of an electron orbital motion
   - (d) None of these

11. One Bohr magneton μ_B is equal to
   - (a) $\dfrac{4\pi m}{eh}$
   - (b) $4\pi mhe$
   - (c) $\dfrac{me}{4\pi h}$
   - (d) $\dfrac{he}{4\pi m}$

12. Magnetic dipole moment per unit volume of material is called
   - (a) permeability
   - (b) polarisation
   - (c) magnetisation
   - (d) magnetic induction

13. The SI unit of magnetic moment is
   - (a) Wb/m^2
   - (b) Wb
   - (c) A/m^2
   - (d) $A\ m^2$

14. Relative permeability is related to magnetic susceptibility by
    - (a) $\mu_r = 1 - \chi$
    - (b) $\mu_r = 1 + \chi$
    - (c) $\mu_r = \chi - 1$
    - (d) $\mu_r = 1/\chi$

15. Magnetic induction '*B*' is related to magnetic field intensity '*H*' by the relation
    - (a) $B = \mu_0 (H + M)$
    - (b) $B = \mu_0 (H - M)$
    - (c) $B = \mu_0 \dfrac{H}{M}$
    - (d) $B = \mu_0 \dfrac{M}{H}$

16. The permeability of free space is
    - (a) $4\pi \times 10^{-7}$ H/m
    - (b) $4\pi \times 10^{-8}$ H/m
    - (c) $2\pi \times 10^{-7}$ H/m
    - (d) $2\pi \times 10^{-8}$ H/m

17. A field of strength 100 amp/metre produces a magnetisation of 2000 A/m in a ferromagnetic material. The magnetic susceptibility of the material is
    - (a) 2×10^5
    - (b) 0.05
    - (c) 2
    - (d) 20

18. A field of strength 100 A/m produces a magnetisation 2000 A/m in a ferromagnetic material. The relative permeability of the material is
    - (a) 19
    - (b) 3
    - (c) 21
    - (d) 1.05

19. The most characteristic feature of an antiferromagnetic material is
    - (a) hysteresis behaviour
    - (b) appearance of magnetic domains
    - (c) a sharp minimum in the susceptibility versus temperature curve
    - (d) a sharp maximum in the susceptibility versus temperature curve

20. The magnetisation retained by the specimen when the magnetising field is reduced from saturation value to zero is known as
    - (a) coercivity
    - (b) hysteresis
    - (c) retentivity
    - (d) spontaneous magnetisation

21. Magnetic materials which can be readily magnetised in either direction are called
    - (a) hard magnetic materials
    - (b) soft magnetic materials
    - (c) low hysterisis loss materials
    - (d) high hysterisis loss materials

22. Soft magnetic materials possess
    - (a) low coercivity and hysteresis losses
    - (b) low remanent magnetisation
    - (c) high remanent permeability and susceptibility
    - (d) All of the above

23. Hard magnetic materials possess
    - (a) low permeability
    - (b) high hysterisis loss
    - (c) high remanent magnetisation and coercivity
    - (d) All of the above

24. Hard magnetic materials are used in
    (a) magnetic separators and detectors
    (b) speakers used in audio systems and microphones
    (c) in toys and measuring meters
    (d) all the above

25. For which of the following substances is the magnetic susceptibility independent of temperature?
    (a) Diamagnetic only
    (b) Paramagnetic only
    (c) Ferromagnetic only
    (d) Dia and paramagnetic

26. Susceptibility is positive and small for a
    (a) paramagnetic substance
    (b) ferromagnetic substance
    (c) diamagnetic substance
    (d) ferrimagnetic substance

27. Above Curie temperature, a ferromagnetic substance becomes
    (a) paramagnetic
    (b) diamagnetic
    (c) antiferromagnetic
    (d) strongly ferromagnetic

28. Of dia, para, and ferromagnetism, the inherent property of all substances is
    (a) diamagnetism
    (b) paramagnetism
    (c) ferromagnetism
    (d) All of these

29. The susceptibility of a diamagnetic substance
    (a) increases with rise in temperature
    (b) decreases with rise in temperature
    (c) first decreases and then increases with temperature
    (d) does not vary with temperature

30. The susceptibility of a paramagnetic substance
    (a) increases with rise in temperature
    (b) decreases with rise in temperature
    (c) first decreases and then increases with temperature
    (d) does not vary with temperature

31. The area enclosed by hysterisis loop is a measure of
    (a) retentivity
    (b) susceptibility
    (c) permeability
    (d) energy loss per cycle

32. Equal number of opposite spins with same magnitude are present in
    (a) paramagnetic materials
    (b) ferromagnetic materials
    (c) ferrimagnetic materials
    (d) antiferromagnetic materials

33. Equal number of opposite spins with different magnitude are present in
    (a) paramagnetic materials
    (b) ferromagnetic materials
    (c) ferrimagnetic materials
    (d) antiferromagnetic materials

34. The temperature below which certain metals are antiferromagnetic and above which they are paramagnetic is called
    - (a) Curie temperature
    - (b) Weiss temperature
    - (c) Neel temperature
    - (d) transition temperature

35. Hysteresis means _____ of an effect behind the cause of effect
    - (a) lagging
    - (b) advancing
    - (c) Both (a) and (b)
    - (d) None of these

36. For an antiferromagnetic substance, the magnetic susceptibility is maximum at _____ temperature.
    - (a) Curie
    - (b) Fermi
    - (c) Debye
    - (d) Neel

37. The hysteresis loss is less for _____ magnetic materials.
    - (a) dia
    - (b) para
    - (c) soft
    - (d) hard

38. The ratio of magnetic moment to the angular momentum of an revolving electron is called _____ ratio.
    - (a) gyromagnetic ratio
    - (b) magnetic susceptibility
    - (c) permeability
    - (d) none of the above

39. Materials do not having permanent magnetic dipoles are
    - (a) diamagnetic
    - (b) paramagnetic
    - (c) ferromagnetic
    - (d) ferrimagnetic

40. Diamagnetic susceptibility is
    - (a) large, negative
    - (b) small, negative
    - (c) small, positive
    - (d) large, positive

41. The SI unit of magnetic field intensity is
    - (a) Hm^{-1}
    - (b) $wb\, m^{-2}$
    - (c) Am^{-1}
    - (d) no units

42. One Bohr magneton equals
    - (a) $9.27 \times 10^{-16}\, A\text{-}m^2$
    - (b) $9.27 \times 10^{-19}\, A\text{-}m^2$
    - (c) $9.1 \times 10^{-28}\, A\text{-}m^2$
    - (d) $9.27 \times 10^{-24}\, A\text{-}m^2$

43. In base SI units, tesla is expressed as
    - (a) $NA^{-1}\, m^{-1}$
    - (b) $Kgs^{-2}\, A^{-1}$
    - (c) $NA^{-1}\, m^{-3}$
    - (d) $NA^{-1}\, m^{-3}$

44. In a paramagnetic materials of susceptibility 10^{-3}, the induction B in wbm^{-2} at an applied field 10 KAm^{-1} is ($\mu_0 = 1.257 \times 10^{-6}\, Hm^{-1}$)
    - (a) 0.01257
    - (b) 1.257
    - (b) 0.1257
    - (d) $4\pi \times 10^{-7}$

45. The transition from the ferromagnetic to the paramagnetic state is named after
    - (a) Curie–Weiss
    - (b) Curve
    - (c) Weel
    - (d) Debye

46. Orbital magnetic depole moment μ_0 of an reveloving electrons is given by

    (a) $\mu_0 = \left(\dfrac{er^2 w_0}{2}\right)$

    (b) $-\dfrac{e^2 r}{2w_0}$

    (c) $-\left(\dfrac{er^2 w_0}{2}\right)$

    (d) $\dfrac{e^2 r}{2w_0}$

47. Upon application of external magnetic field, the orbital magnetic dipole moment changes. The induced dipole moment find is given by

    (a) $\dfrac{e^2 B}{4\,mr^2}$

    (b) $\dfrac{e^2 B}{4\pi r^2}$

    (c) $\dfrac{e^2 r}{4\pi B}$

    (d) $\dfrac{e^2 r^2 B}{4\,m}$

48. Paramagnetic susceptibility varies as

    (a) T^2

    (b) $\dfrac{1}{T}$

    (c) T

    (d) $\dfrac{1}{T^2}$

49. Diamagnetic materials possess
    (a) permanent magnetic dipoles.
    (b) no permanent magnetic dipoles.
    (c) induced dipoles along field direction.
    (d) no induced dipoles even when external field is applied.

50. One nuclear magneton equals
    (a) 9.27×10^{-24} A-m^2
    (b) 5.05×10^{-29} A-m^2
    (c) 5.05×10^{29} A-m^2
    (d) 9.27×10^{24} A-m^2

51. The susceptibility of a paramagnetic material is 3×10^{-4} at 300 K. Its susceptibility at 400 K is
    (a) 2.25×10^{-4}
    (b) 0.44×10^{-4}
    (c) 4×10^{-4}
    (d) 3×10^{-4}

Answers

1. (d)	2. (d)	3. (c)	4. (a)	5. (a)	6. (b)
7. (a)	8. (c)	9. (c)	10. (c)	11. (d)	12. (c)
13. (d)	14. (b)	15. (a)	16. (a)	17. (d)	18. (c)
19. (d)	20. (c)	21. (b)	22. (d)	23. (d)	24. (d)
25. (a)	26. (a)	27. (a)	28. (a)	29. (d)	30. (b)
31. (d)	32. (d)	33. (c)	34. (c)	35. (a)	36. (d)
37. (c)	38. (a)	39. (a)	40. (b)	41. (c)	42. (d)
43. (b)	44. (a)	45. (b)	46. (c)	47. (d)	48. (b)
49. (b)	50. (b)	51. (c)			

Fill in the Blanks

1. In any material the number of lines of magnetic force passing through unit area perpendicularly is called _____ or _____.

2. The SI unit of magnetic induction is _____.

3. At any point in the magnetic field, the force experienced by an unit north pole placed at that point is called the _____.

4. The SI unit of magnetic field intensity is _____.

5. The ratio of the magnetic induction in the sample to the applied magnetic field intensity is called _____.

6. The _____ of a sample of a material is the magnetic moment per unit volume.

7. The SI unit of intensity of magnetisation is _____.

8. The _____ of the material is the ratio of the intensity of magnetisation produced in the sample to the magnetic field intensity which produced the magnetisation.

9. Material which lack permanent dipoles are called _____.

10. If the atoms of the material carry permanent dipoles but do not interact among themselves, the material is _____.

11. If the atoms of the material carry permanent dipoles and the interaction among them is strong such that all the dipoles line up in parallel, the material is _____.

12. If the atoms of the material carry permanent dipoles and if they line up in antiparallel direction equally, the material is _____.

13. If the atoms of the materials carry permanent dipoles and if they line up in antiparallel direction unequally, the material is _____.

14. When a magnetic material is placed inside a magnetic field, if they magnetic lines of forces are repelled then the material is _____.

15. In a diamagnetic materials, magnetic susceptibility is _____ of applied magnetic field strength.

16. Ferromagnetic materials consists of number of small regions which are spontaneassly magnetised called _____.

17. If 'n' is the principal quantum number, then the maximum number of electrons in each orbit is given by _____.

18. For each value of the principle quantum number 'n' the orbital quantum number 'l' can take _____ values _____ to _____.

19. The magnetic moments of atomic systems are measured in the unit of _____.

20. The lag of magnetisation behind the magnetising field in ferromagnetic materials is known as _____.

21. Area enclosed by the hysteresis loop represents _____.

Answers to Fill in the blanks

1. magnetic induction or magenetic flux density
2. tesla
3. magnetic field intensity
4. Am^{-1}
5. magnetic permeability
6. intensity of magnetisation

7. Am^{-1}
8. magnetic susceptibility
9. diamagnetic
10. paramagnetic
11. ferromagnetic
12. antiferromagnetic
13. ferrimagnetic
14. diamagnetic
15. independent
16. domains
17. $2n^2$
18. n, 0, $(n-1)$
19. Bohr magneton
20. hysteresis
21. hysteresis loss

True or False

1. The permeability of free space (μ_0) has a value of $4\pi \times 10^{-7}$ Hm^{-1}. (T/F)
2. The SI unit of relative permeability (μ_r) is Hm^{-1}. (T/F)
3. Antiferromagnetic materials exhibit magnetisation. (T/F)
4. For diamagnetic materials the susceptibility is negative. (T/F)
5. For paramagnetic materials the susceptibility is large and positive. (T/F)
6. For ferromagnetic materials the susceptibility is very small and negative. (T/F)
7. For antiferromagnetic materials the susceptibility is large and negative. (T/F)
8. For ferrimagnetic materials the susceptibility is very large and positive. (T/F)
9. For antiferromagnetic materials spin alignment is equal and antiparallel. (T/F)
10. For ferrimagnetic materials spin alignment is antiparallel of equal magnitude. (T/F)
11. The value of Bohr magneton is 9.27×10^{-24} Am^2. (T/F)
12. Fe, Co, Ni are ferromagnetic materials. (T/F)
13. Hard magnetic materials have small value of permeability and susceptibility. (T/F)
14. Ferrites are examples of hard magnetic materials. (T/F)
15. Soft magnetic materials is used for making electromagnet. (T/F)
16. Hard magnetic material is used for making electromagnet. (T/F)
17. In hard magnetic materials, the coercivity and retentivity are large. (T/F)
18. Above the ferromagnetic curie temperature, the ferromagnetic materials exhibit hysteresis. (T/F)
19. Hysteresis loss is high for soft magnetic materials. (T/F)

Answers to True or False

1. T 2. F 3. F 4. T 5. T 6. F
7. F 8. T 9. T 10. F 11. T 12. T
13. T 14. F 15. T 16. F 17. T 18. F
19. F

REVIEW QUESTIONS

Short Answer Questions

1. What is magnetic susceptibility?
2. What is magnetic permeability?
3. Derive the relation between B, H and M.
4. Derive the relation between μ_r and χ?
5. What is Bohr magneton?
6. What is diamagnetic material?
7. What is paramagnetic material?
8. What is ferromagnetic material?
9. What is an antiferromagnetic material?
10. What is ferrimagnetic material?
11. What is hysteresis?
12. What are soft magnetic materials?
13. What are hard magnetic materials?
14. Mention the applications of soft magnetic materials.
15. Mention the applications of hard magnetic materials.

Essay Type Questions

1. Describe the origin of magnetic moment in an atom.
2. Describe the classification of magnetic materials.
3. Explain the hysteresis of ferromagnetic materials.
4. Explain soft and hard magnetic materials.

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CHAPTER 9

SUPERCONDUCTIVITY

INTRODUCTION

Resistance or *resistivity* is the inherent property exhibited by materials. This is mainly due to the scattering of electrons while interacting with the positive ions (atoms) present in the materials. When temperature of material is decreased to a low value, then due to lower energy, scattering of electrons decreases and as a result resistance or resistivity decreases. Then the conductivity increases.

The phenomenon of attaining zero resistivity or infinite conductivity at low temperature is known as superconductivity. The material becomes a superconductor and it will be in *superconducting* state.

Superconductivity was first observed by Kammerlingh Onnes in the case of mercury. When temperature of mercury is decreased then the resistance also decreases and it is nearly zero at 4.2 K temperature as shown in Fig. 9.1.

The temperature at which the material undergoes a transition from normal state to superconducting state is known as *critical temperature* or *transition temperature* (T_c). Different materials will have different T_c values.

Example	Aluminium	$T_c = 1.19\,\text{K}$
	Lead	$T_c = 7.2\,\text{K}$
	Tungsten	$T_c = 0.01\,\text{K}$
	Tin	$T_c = 0.39\,\text{K}$
	Cadmium	$T_c = 0.55\,\text{K}$

9.1 PROPERTIES OF SUPERCONDUCTORS

1. Superconductivity is a low-temperature phenomenon.
2. The transition from normal state to superconducting state occurs below the critical temperature.
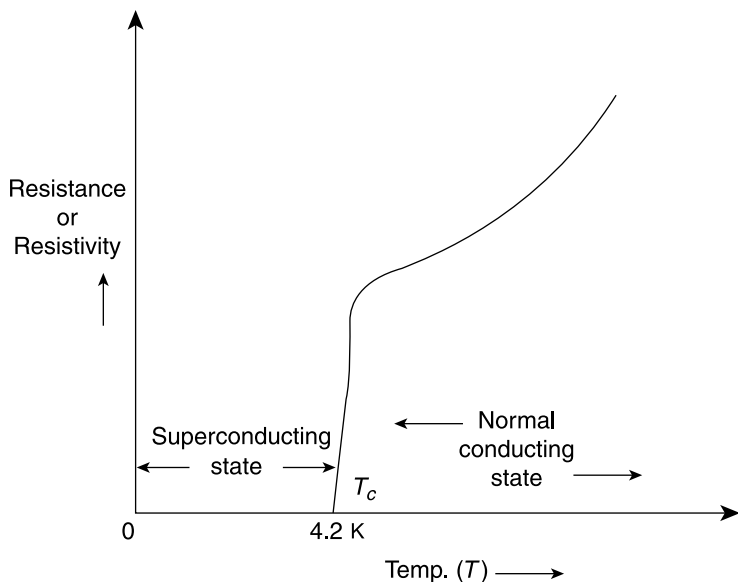
Figure 9.1   Resistivity of mercury

3. Different materials will have different critical temperatures.
4. The current once set up in a superconductor persists for a long time due to zero resistivity.
5. Superconductors do not allow magnetic field (magnetic lines) through them and behave as a diamagnetic. This property of expulsion of magnetic field is known as Meissner effect.
6. The magnetic field at which a superconductor loses it's superconductivity and becomes a normal conductor is known as *critical magnetic field H_c*.
7. The induced current in a superconductor induces a magnetic field in it. If the magnetic field is equal to the critical magnetic field then it converts into a normal conductor. The current in it is known as critical current (I_c). If 'r' is the radius of the superconductor then

$$I_c = 2\pi r H_c$$

The current density at which it occurs is known as critical current density and is given by $J_c = I_c/A$, where A is the area of cross section of the superconductor.
8. Superconductivity occurs in metallic elements in which the number of valence electrons lies between 2 and 8.
9. Materials having high normal resistivities exhibit superconductivity.
10. Superconducting materials are not good conductors at room temperature.

9.1.1 Critical Magnetic Field (H_c)

When a magnetic field is applied to a superconductor then for a particular value of applied field it loses super conductivity and becomes a normal conductor. The magnetic field for which a super conductor becomes a normal conductor is known as critical magnetic field (H_c) and is given by

$$H_c = H_0 \left| 1 - \left(\frac{T}{T_c} \right)^2 \right|$$

where H_0 is the field required to destroy the superconducting property at 0 K, T_c is the critical temperature of the super conductor and T is the temperature of the superconductor.

When $T = T_c$, then $H_c = 0$.

When $T = 0$ K then $H_c = H_0$.

The variations of H_c w.r.t. T is as shown in Fig. 9.2.
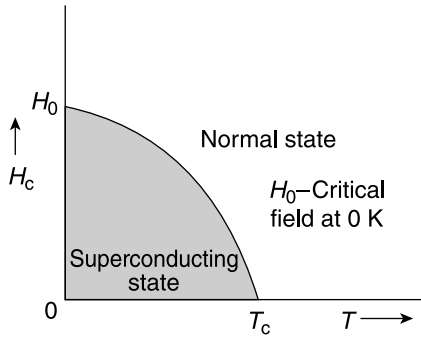
Figure 9.2 Variations of H_c with T

9.2 MEISSNER EFFECT

Consider a normal conductor at room temperature (Fig. 9.3a). When a magnetic field H is applied to it then it allows the magnetic lines to pass through it. Thus we have a magnetic induction field 'B' in a conductor (Fig. 9.3b). When the entire system is cooled so that $T < T_c$ then the normal conductor becomes a superconductor and it will not allow the magnetic lines to pass through it. It expels the magnetic lines. This effect, observed by Meissner, is known as Meissner effect. Thus, the superconductor does not allow the magnetic lines through it or expels the magnetic lines.

For a normal conductor, magnetic inductions field 'B' is given by

$$B = \mu_0(H + M)$$

where μ_0 is the permeability of free space or air,

M is magnetisation of the normal conductor

For a superconductor, $B = 0$

$$\mu_0(H + M) = 0$$

$$\boxed{H = -M}$$

i.e., applied magnetic field induces magnetisation in opposite direction.

Magnetic susceptability $\chi = \dfrac{M}{H} = -1$

$$\boxed{\chi = -1}$$

The above results clearly indicate that a superconductor behaves as a perfect diamagnetic in the presence of a magnetic field. Meissner effect proved the above fact.

9.2.1   London Penetration Depth

According to London theory, the magnetic field at one of the face of a superconductor does not vanishes, but decreases exponentially to zero after penetrating into small region inside the superconductor.

Consider a superconductor for which a magnetic field 'H_0' is applied to one of its faces as shown in Fig. 9.4. To obey Meissner effect, it will not allow magnetic lines to pass through it, but in practice a small portion of H_0 penetrates to a small distance into the superconductor.

The penetrating field at a distance x from the face is given by

$$H = H_0 e^{-x/\lambda}$$

where λ is known as London *penetration depth*.

The variation of H w.r.t. x is as shown in Fig. 9.5. When $x = \lambda$, then $H = \dfrac{H_0}{e}$. The penetration depth (λ) is the distance inside the superconductor at which the penetrating magnetic field is equal to $\dfrac{1}{e}$ times the applied magnetic field H_0. λ ranges from 10 to 100 nm. This depth is significant for a thin film superconductor rather than a bulk superconductor.

The variation of λ with temperature 'T' is given by

$$\lambda = \frac{\lambda_0}{\left[1 - \left(\dfrac{T}{T_c}\right)^4\right]^{\frac{1}{2}}}$$
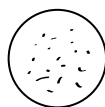
(a) Normal conductor
$(T > T_c)$
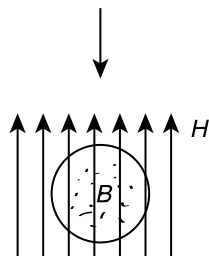
(b) Normal conductor in a magnetic field H
$(T > T_c)$

Cooled
(Temp. decreased)
$T > T_c$
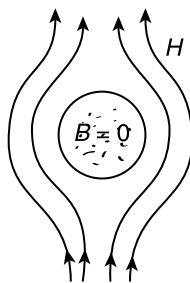
(c) Superconductor in a magnetic field H
$(T > T_c)$

Figure 9.3   Meissner effect
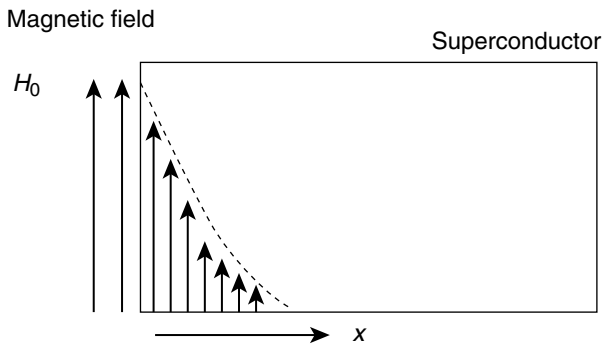
Figure 9.4 Penetration of field through one face of supercondcutor
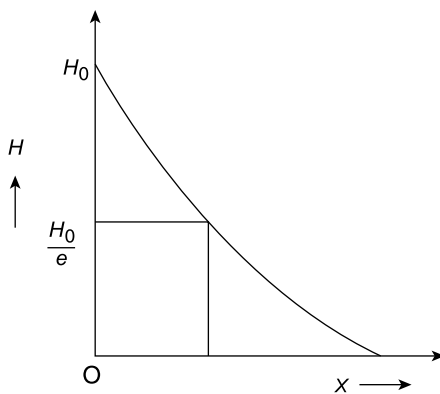
Figure 9.5 Variation of H.w.r.t.x

This variation for temperature is shown in Fig. 9.6. Near T_c, λ value is maximum, which indicates the magnetic lines penetrating all the surface converting into a normal conductor.

9.3 TYPES OF SUPERCONDUCTORS

In the presence of critical magnetic field, a superconductor converts into a normal conductor. Based on the conversion process, superconductors are classified into two types:

1. Type I superconductor
2. Type II superconductor

9.3.1 Type I Superconductor

Consider a cylindrical superconductor to which a magnetic field H is applied along the axis of it. It will not allow the magnetic lines and obeys Meissner effect (Fig. 9.7(a)), i.e., $H = -M$. Inside the superconductor, the magnetisation acting
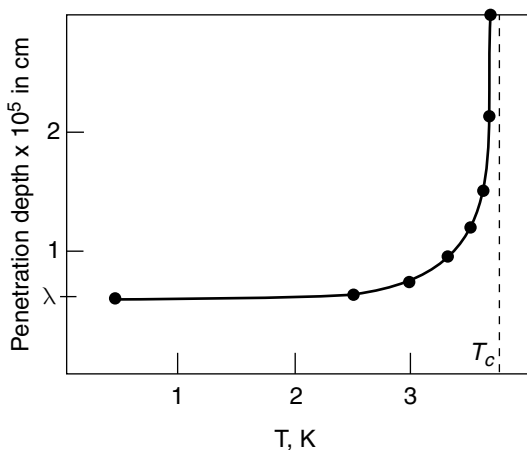
Figure 9.6   Temperature variation of penetration depth in tin

opposite to H opposes the field. When the magnetic field is equal to the critical field H_c then immediately $-M$ becomes zero and magnetic lines immediately penetrates the entire superconductor and converts it into normal conductor (Fig. 9.7(b)). The variation of $-M$ w.r.t. H is as shown in Fig 9.7(c). In this case, conversion is very fast and is known as Type I superconductor. It it also known as soft superconductor.

Examples   Sn, Hg, Nb, V, etc.

9.3.2   Type II Superconductor

Consider a spherical superconductor in the magnetic field H. As per Meissner effect, it expels the magnetic line and $H = -M$, i.e., the magnetisation inside a superconductor opposses the passage of lines through it (Fig. 9.8(a)). The curved magnetic lines are concentrated at the two ends marked as $\otimes$ in Fig. 9.8(a) such that intensity H is predominent at those ends. Up to lower critical field (H_{c1}) it behaves as a superconductor. When the magnetic field exceeds H_{c1}, then the penetration starts at those ends marked as $\otimes$. This is because the intensity at those ends reaches H_{c1} value at first. As a result, $-M$ value decreases and the penetrated portion loses superconductivity and becomes a normal conductor. This conversion is usually completed at H_{c2} (upper critical field) where the penetration covers the entire surface. Thus between H_{c1} and H_{c2} it has a mixed state of superconducting as well as normal conducting state [Fig. 9.8 (b)]. After H_{c2} it is converted into normal conductor [Fig. 9.8 (c)]. The variation of $-M$ with H is as shown in Fig. 9.8(d). In this case the conversion is slow and is known as Type II superconductors. They are also known as hard superconductors.

Examples Nb_3, Sn, Nb_3Ge, etc.

Magnetic field
H

(a) Superconductor
$H < H_c$

(b) Normal conductor
$H > H_c$

(c) Variation of $-M$ with H
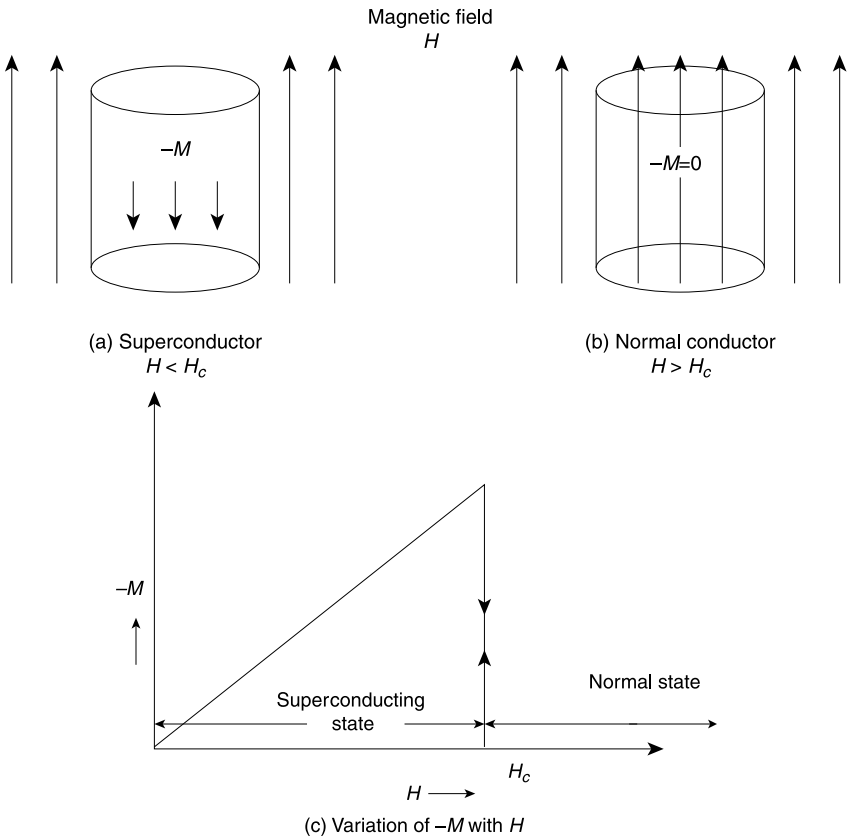
Figure 9.7 Type I superconductor

Magnetic field
H

(a) Superconductor
$H < H_{c_1}$ (Lower critical field)

(b) Superconductor
+ Normal conductor
$H \leq H_{c_2}$ (Upper critical field)

(c) Normal conductor
$H > H_{c_2}$
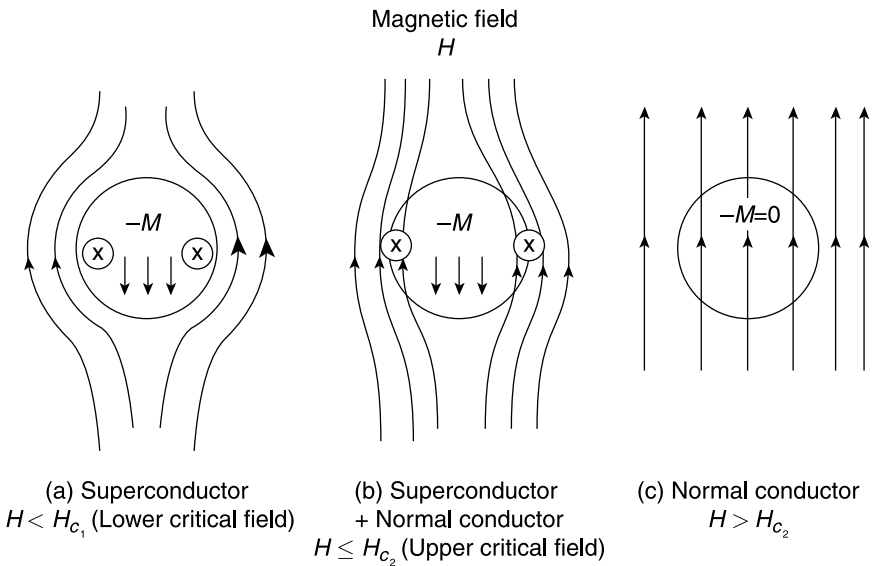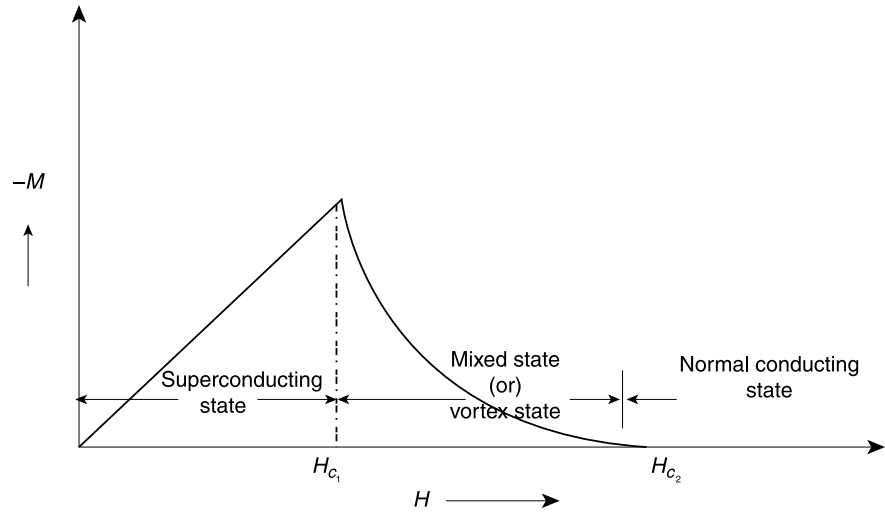
Figure 9.8 Type II superconductor

Table 9.1 Differences betweeen Type I and Type II superconductors

Type I	Type II
1. In this superconductor when the applied magnetic field is equal to critical mangetic field, then immediately it converts into normal conductor.	1. In this superconductor, when applied field reaches lower conversion starts and it is completed slowly at upper critical field.
2. It has a single critical field value (H_c).	2. It has two critical field values i.e., H_{c_1} and H_{c_2}.
3. There is no mixed state.	3. There is a mixed state.
4. They are soft superconductors.	4. They are hard superconductors.
5. Materials with pure form are Type I superconducots.	5. Materials with impurities or alloys are of Type II superconductors

9.4 FLUX QUANTISATION

Consider a hollow normal conducting ring in a magnetic field. It allows the magnetic flux due to the field. So, we observe magnetic flux outside and inner hollow space and also on the ring [Fig. 9.9(a)]. When the temperature of the ring is lowered to its critical temperature (T_c) it becomes a superconductor and it obeys Meissner effect. As a result, persistent current comes into existence so that $H = -M$ and the superconducting ring will not allow the flux to pass through it.

In this case, we observe flux outside and inner hollow space of the ring only [Fig. 9.9(b)]. When the applied magnetic field is removed, the opposing persistent

current tries to adjust itself such that some magnetic flux is trapped inside its hollow space [Fig. 9.9(c)]. The trapped flux is given by

$$\phi = \frac{nh}{2e} \qquad \text{where } n = 1, 2, 3...$$

'*h*' is Planck's constant and '*e*' is change of electron

i.e.,
$$\phi = \frac{h}{2e}, \frac{2h}{2e}, \frac{3h}{2e},...$$

ϕ is quantised in terms of $\dfrac{h}{2e}$. This $\dfrac{h}{2e}$ is treated as fundamental trapped flux and

is known as *fluxon* or *fluxoid*. This phenomenon is known as flux quantisation of superconductor.

(a) Hollow normal conducting ring in a magnetic field (*H*) (*T>T$_c$*)

(b) Hollow superconducting ring in a magnetic field (*H*) (*T<T$_c$*)

(c) Hollow superconducting ring in a removed magnetic field (*H*) (*T<T$_c$*)
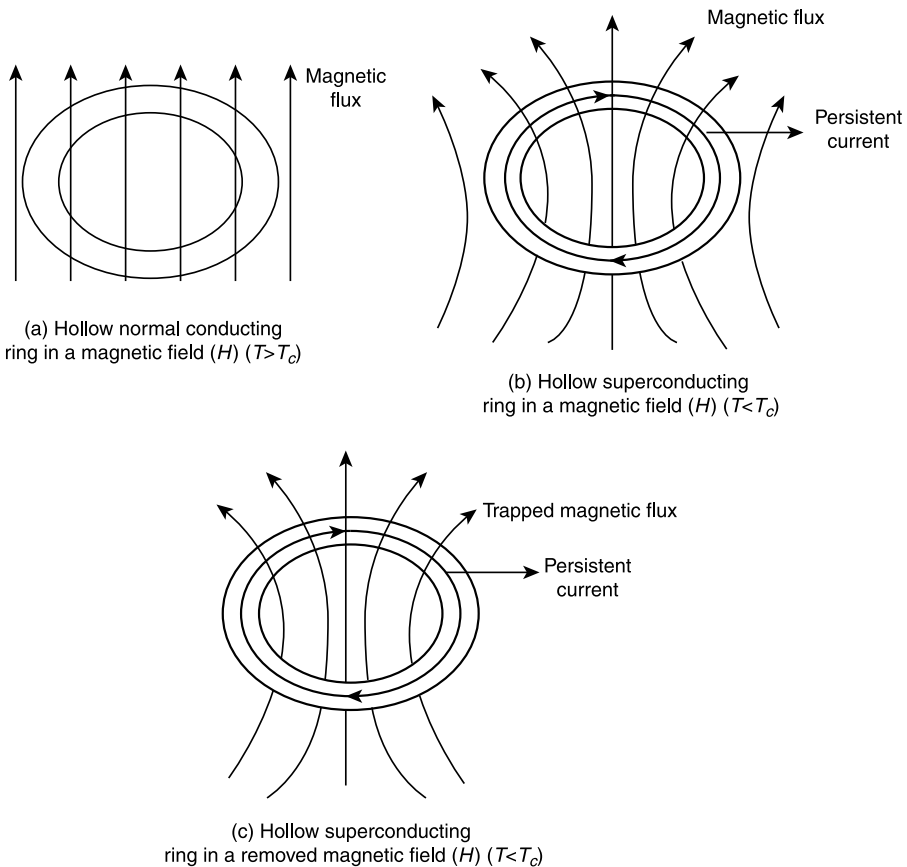
Figure 9.9   Flux quantisation in a superconductor

If ϕ_{ext} is the flux due to applied magnetic field and ϕ_{sc} is the flux due to persistent current in a superconductor, then $\phi = \phi_{ext} + \phi_{sc}$. Since ϕ is quantised ϕ_{sc} adjust its value corresponding to ϕ_{ext} so that trapped magnetic flux is always quantised.

9.5 JOSEPHSON EFFECT

Consider two superconductors which are joined together with the help of a thin insulating layer as shown in Fig. 9.10. These superconductors consist of paired electrons known as Cooper pairs in the superconducting state. These Cooper pairs will try to penetrate or tunnel through the thin insulator and constitute a small super current. The insulator which forms the junctions between superconductors is known as Josephson junction and this effect is known as Josephson effect.
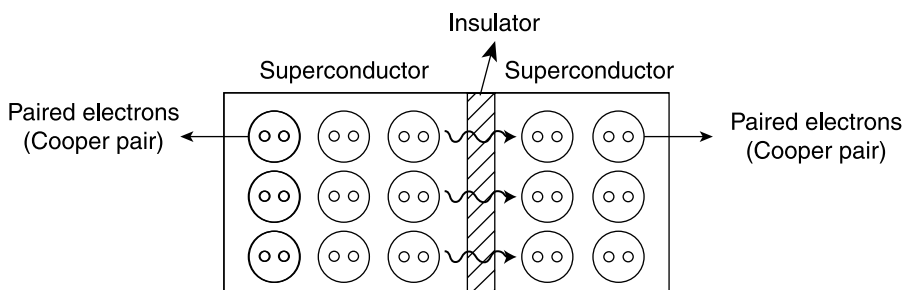
Figure 9.10   dc Josephson effect

This effect is of two types:
1. dc Josephson effect
2. ac Josephson effect

9.5.1 dc Josephson Effect

Without any applied voltage across the junction due to tunneling of Cooper pairs, a small direct super current (dc) flows across the junction. This effect is known as dc Josephson effect.

Let the propagation of Cooper pair be in the form of waves. The phase difference between the two parts of the waves on either side of the junctions in terms of wave functions is $\phi_0 = \phi_2 - \phi_1$.

The tunneling current is given by

$$I = I_0 \sin \phi_0$$

where I_0 is the maximum current that flows through the junction without any voltage across the junction. The above expression represents a direct current (dc) that flows across the junction.

9.5.2 ac Josephson Effect

When a static potential V_0 is applied across the junction then the Cooper pairs start oscillating through the insulated layer. As a result, an alternating current (ac) flows through the junction. This effect is known as ac Josephson effect.

Due to V_0, an additional phase difference of $\Delta\phi = \dfrac{Et}{\hbar}$ is introduced for the Cooper pairs, where E is the total energy of the Cooper pairs at any time 't'.

$$E = (2e)V_0$$

$$\Delta\phi = \frac{2eV_0t}{\hbar}$$

The tunneling current can be written as

$$I = I_0 \sin(\phi_0 + \Delta\phi)$$

$$= I_0 \sin\left(\phi_0 + \frac{2eV_0t}{\hbar}\right)$$

$$I = I_0 \sin(\phi_0 + \omega t)$$

where $\quad \omega = \dfrac{2eV_0}{\hbar}$ = angular frequency

Insulator
Superconductor   Superconductor

V_0

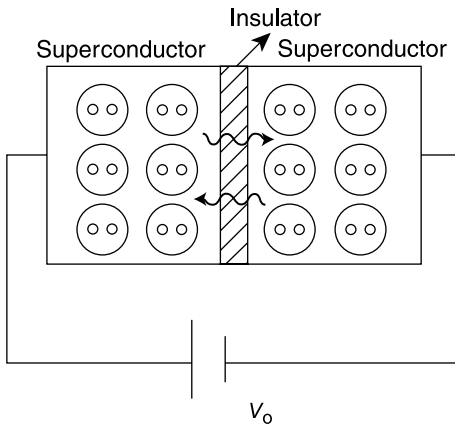**Figure 9.11** ac Josephson effect

This represents an ac with angular frequency ω. Current–voltage characteristics of a Josephson juntion is as shown in Fig. 9.12.

1. When $V_0 = 0$ there is a constant flow of dc current I_c through the junction. This current is called superconducting current and the effect is dc Josephson effect.
2. When $V_0 < V_c$, a constant dc current I_c flows.
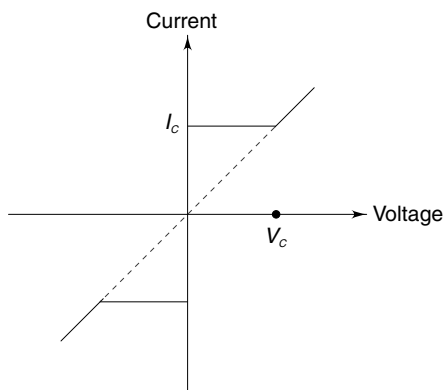
Figure 9.12 Current-voltage characteristics of Josephson junction

3. When $V_0 > V_c$, the junction has a finite resistance, and the current oscillates with frequency

$\omega = \dfrac{2eV_0}{\hbar}$. This effect is the ac Josephson effect.

Applications

1. Josephson effect is used to generate microwaves with frequency

$$\omega = \frac{2eV_0}{\hbar}$$

2. The ac Josephson effect is used to define standard volt.
3. The ac Josephson effect is used to measure very low temperature based on the variation of frequency of the emitted radiation with temperature.
4. Josephson junction is used for switching of signals from one circuit to another.

9.6 BARDEEN, COOPER AND SCHRIEFFER (BCS) THEORY

Bardeen, Cooper and Schrieffer proposed a microscopic theory known as BCS theory. It explains the superconducting state of a superconductor. This theory involves the electron interactions through phonon as mediators.

In normal conductor, the electrons will be moving at random. When they approach vibrating atoms (lattice vibrations-phonon) the repulsive force predominates than the attractive force. As a result, they get scattered and resistance comes into existence [Fig. 9.13(a)]. When it is converted into superconductor by decreasing its temperature below the critical temperature, due to decrease in energy the scattering of electrons by lattice vibrations also decreases and to maintain stable state the electrons get paired up and are known as Cooper pairs. The pairing of electrons can be well understood by considering the electron–phonon interaction.

Figure 9.13 (a) Scattering of electrons in normal conductor, (b) Formation of superconducting paired electrons in a superconductor
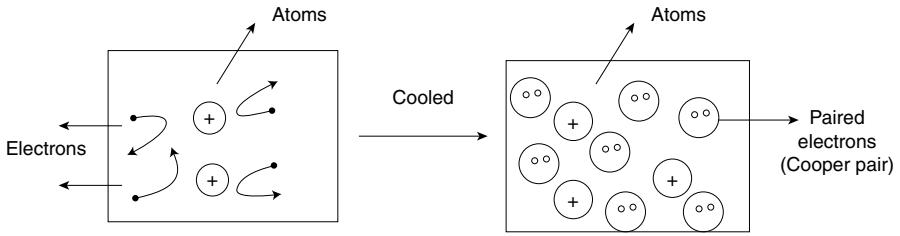
This means that electrons travelling in a solid interact with lattice vibrations by virtue of electrostatic forces between them. The oscillator distortion of the lattice is quantised in terms of phonons. This interaction is called electron–phonon interaction, which leads to scattering of electrons and hence, causes a change in the electrical resistivity. The resistivity is sensitive with temperature, particularly in the low temperature region, since the number of phonons increases with temperature.

It is assumed from the BCS theory that the electron phonon electron interaction causes the Cooper pairs.

When an electron with a wave vector K_1 encounters a lattice ion (positive ion cores), it suffers a Coulomb interaction. As a result of this interaction, the momentum of the electron decreases by an amount proportional to $K_1 - q$ and the lattice is set into motion gaining a momentum from the electron proportional to q. The lattice vibration is quantised and characterised by energy packets called

phonons (virtual). Another electron with a wave vector K_2 (Fig. 9.14) may enter this volume and this electron interacts with the distorted lattice and gains momentum, the gain depending on the phase of the vibration at that instant. The electron energy is lowered.
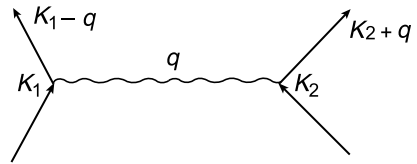
Figure 9.14 Electron-phonon interaction

The lowering of energy indicates that the force between the electrons is attractive and the interaction is through phonons. Therefore it can be thought that the electron with wave vector K_1 emits a virtual phonon q which is absorbed by the second electron with wave vector K_2 and the two electrons are scattered with wave vectors $K_1 - q$ and $K_2 + q$. The process is a virtual one and the energy is not conserved. The process is very rapid with the emission and absorption of the phonon. The short lived phonon is called virtual phonon. The two electrons should have equal and opposite spins and their energies are the same. Then the interaction between them is attractive. As a result of such attractive interaction, there exists pairs of electrons called Cooper pairs. A Cooper pair is denoted by $(K\uparrow, K\downarrow)$.

Therefore, the electrons in a superconductor are in the form of Cooper pairs. A Cooper pair is a system of two electrons having equal and opposite momenta and spin $(K\uparrow, K\downarrow)$. The energy of the pair is lowered when compared to the free separate electrons.

At lower temperature (T_C) the cooper pairs interaction with the positive ion core almost vanishes and hence the resistivity becomes zero. Thus the conductor becomes a superconductor.

9.6.1   High Temperature Superconductors

For most of the superconductors, superconductivity occurs only at low critical temperatures (T_C). For wide application of superconductors, scientists were trying to attain superconductivity at a much higher temperature. For attaining low temperature we should use liquid helium which is costly process. In attaining the superconductivity at high temperature or, to discover high temperature superconductors, scientists made the following progressive steps:

1. Superconductivity was discovered on a thin film of niobium and germanium at 23.2 K
2. Compound of the form $Ba - PbBi-O_3$ was found to be superconductor at 38 K
3. Oxide compound of the form $Y_1Ba_2Cu_3O_7$ (123 superconductor) was found to be superconductor at 92 K.
4. Oxide compound of the form Bi—Sr—Ca—Cu—O was found to be superconductor at 115 K.
5. The form Ti—Ba—Ca—Cu—O was found to be superconductor at 125 K.

In this high temperature superconductors phenomenon, liquid nitrogen (77 K) is used which is safer than liquid helium (4 K) or liquid hydrogen (23 K). It has been understood that oxygen atoms plays a major role in high temperature superconductors. Most of them have layered structure of copper and oxygen atoms.

Properties

1. They are highly anisotropic.
2. They have the presence of CuO_3 layers.
3. They have inherent metallic properties.

9.6.2   Applications of Superconductors

1. *Electric Generators* Superconducting generators are smaller in size, with less weight and consume very low energy. The low-loss superconducting coil is rotated in a strong magnetic field. This is the basis of new generation of energy-saving power systems.

2. *Low loss transmission lines and transformers* When superconducting wires are used as electric cables then the transmission losses are

minimised. If superconductors are used for winding of a transformer, the power losses will be very small.

3. *Magnetic levitation* Diamagnetic property of a superconductor is the basis of magnetic levitation. This effect can be used for high-speed transportation.

4. *Generation of high magnetic fields* Superconducting materials are used for producing high magnetic fields with low power consumption.

5. *Fast electrical switching* The application of magnetic field greater than H_c, changes the superconducting state to normal state and removal of the field reverses the process. This principle is used in switching element cryotron.

6. *Logic and storage functions in computers* The C-V characteristics of Josephson effect is used for memory elements in computers. Thus, superconductors are used to perform logic and storage functions in computers.

7. *Superconducting Quantum Interference Devices (SQUIDS)* Two Josephson junctions mounted on a superconductor ring form SQUID. Since the current through SQUID is very sensitive to magnetic field, it can be used as sensitive magnetometer. These are used to study tiny magnetic signals from the brain and heart.

8. *Superconducting Magnets* Superconducting magnets consists of coils of wires made up of superconductors. Current once introduced into the coil, remains for a very long time causing the stability of the magnetic field for a long time. These coils can be used in electric machines, transformers and MRI instruments.

9. *Magnetic bearings* Mutual repulsion between two superconducting materials due to opposite magnetic fields is used in the construction of magnetic bearings without any friction.

10. *Superconducting sensitive magnetometer* The quantisation of magnetic flux in SQUIDs is the basis for construction of superconducting sensitive magnetometer. It can measure magnetic field strengths of the order of 10^{-3}.

11. *Superconducting susceptometer* A superconducting susceptometer consists of superconducting magnet and SQUIDs. It is used to detect the variation of iron content in the human body.

12. *Magnetoencephalography* Doctors can locate the damaged portions of the brain by using SQUID magnetometers around the patient's head and the received magnetic signals are fed to a computer for analysis. This technique is known as magnetoencephalography.

SOLVED PROBLEMS

1. The critical field for niobium is 1×10^5 amp/m at 8 K and 2×10^5 amp/m at absolute zero. Find the transition temperature of the element.

Given data   Critical magnetic field at 8 K,

$$H_c = 1 \times 10^5 \text{ amp/m}$$

Temperature $T = 8$ K

Critical magnetic field at 0 K, $H_0 = 2 \times 10^5$ amp/m

Solution

$$H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right] \quad \Rightarrow \quad \frac{H_c}{H_0} = 1 - \left(\frac{T}{T_c}\right)^2$$

$$\Rightarrow \qquad \left(\frac{T}{T_c}\right)^2 = 1 - \frac{H_c}{H_0} \quad \text{(or)} \quad \frac{T^2}{T_c^2} = 1 - \frac{H_c}{H_0}$$

$$\therefore \qquad T_c^2 = \frac{T^2}{1 - \dfrac{H_c}{H_0}}$$

$$\Rightarrow \qquad T_c = \sqrt{\frac{T^2}{1 - \dfrac{H_c}{H_0}}}$$

$$T_c = \sqrt{\frac{8^2}{1 - \dfrac{1 \times 10^5}{2 \times 10^5}}} = \sqrt{\frac{64}{0.5}} = \sqrt{128} = 11.3 \text{ K}$$

2. A superconducting material has a critical temperature of 3.7 K, and a magnetic field of 0.0306 tesla at 0 K. Find the critical field at 2 K.

Given data   Magnetic field at 0 K, $H_0 = 0.0306$ T

Critical temperature, $T_c = 3.7$ K

Temperature, $T = 2$ K

Solution   Critical field,

$$H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$= 0.0306\left[1 - \left(\frac{2}{3.7}\right)^2\right]$$

$$= 0.0306 \times [1 - 0.2921]$$
$$= 0.0306 \times 0.7078$$
$$= 0.02166 \text{ Tesla}$$

3. If a Josephson junction has a voltage of 8.50 μV across its terminals, calculate the frequency of the alternating current. [Planck's constant = 6.626×10^{-34} J-sec]

Given data   Voltage across the Josephson junction,
$$V_0 = 8.50 \ \mu V = 8.5 \times 10^{-6} \text{ V}$$

Solution   Frequency of alternating current,

$$\omega = \frac{2eV_0}{h}$$

$$\upsilon = \frac{2eV}{h}$$

$$\upsilon = \frac{2 \times 1.6 \times 10^{-19} \times 8.5 \times 10^{-6}}{6.626 \times 10^{-34}} \times 27$$

$$= 1.17 \times 10^9 \text{ Hz.}$$

4. Calculate the critical current for a lead wire of 0.5 mm radius at 4.2 K. Given for lead T_c = 7.18 K and $H_o = 6.5 \times 10^4$ Am^{-1}.

Given data   Temperature T = 4.2 K

Radius of lead wire r = 0.5 mm = 0.5×10^{-3} m

Critical temperature T_c = 7.18 K

Magnetic field at *OH*, $H_o = 6.5 \times 10^4$ Am^{-1}

Solution   Critical current

$$I_c = 2\pi r \, H_c$$

$$H_c = H_o \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$= 6.5 \times 10^4 \left[1 - \left(\frac{4.2}{7.18} \right)^2 \right]$$

$$= 4.276 \times 10^4 \, \text{Am}^{-1}$$

$$I_c = 2 \times \pi \times 0.5 \times 10^{-3} \times 4.276 \times 10^4$$

$$= 134.51 \text{ A}$$

5. A lead superconductor with T_c = 7.2 K has a critical magnetic field of 6.5×10^3 Am^{-1} at absolute zero. What would be the value of critical field at 5 K?

Given data   Temperature $T = 5$ K

Magnetic field at OK, $H_o = 6.5 \times 10^3 \, \text{Am}^{-1}$

Critical temperature $T_c = 7.2$ K

Solution   Critical field

$$H_c = H_o \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$= 6.5 \times 10^3 \left[1 - \left(\frac{5}{7.2} \right)^2 \right]$$

$$= 6.5 \times 10^3 \, [1 - 0.482]$$

$$= 3.365 \times 10^3 \, \text{Am}^{-1}$$

MULTIPLE CHOICE QUESTIONS

1. The conductivity of a superconductor is
   - (a) zero
   - (b) finite
   - (c) infinite
   - (d) None of these

2. The superconducting transition temperature of mercury is
   - (a) 4.2°C
   - (b) 4.2 K
   - (c) 4.2°F
   - (d) None of these

3. At a low temperature, if resistivity of a metal vanishes then it is a
   - (a) conductor
   - (b) semiconductor
   - (c) insulator
   - (d) dielectric

4. The critical magnetic field (H_c) at temperature (T) K is
   - (a) $H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$
   - (b) $H_0 \left[1 - \left(\frac{T_c}{T} \right)^2 \right]$
   - (c) $H_0 \left[1 - \left(\frac{T}{T_c} \right) \right]$
   - (d) $H_0 \left[\left(\frac{T}{T_c} \right)^2 - 1 \right]$

5. A quantum of magnetic flux in a superconductor is equal to
   - (a) $\frac{h}{c}$
   - (b) $\frac{h}{2e}$
   - (c) $\frac{h}{4e}$
   - (d) $\frac{h}{5e}$

6. The relation between critical current and critical magnetic field is
   - (a) $H_c = \frac{I_c}{2\pi r}$
   - (b) $H_c = \frac{I_c}{\pi r^2}$
   - (c) $I_c = \frac{H_c}{2\pi r}$
   - (d) $I_c = \frac{H_c}{\pi r^2}$

7. Type – I superconductors are
   - (a) soft superconductors
   - (b) hard superconductors
   - (c) ductile superconductors
   - (d) flexible superconductors

8. Type – II superconductors are
   - (a) soft superconductors
   - (b) hard superconductors
   - (c) ductile superconductors
   - (d) flexible superconductors

9. Superconductivity mainly occurs due to
   - (a) electron–electron interaction
   - (b) electron–proton interaction
   - (c) proton–proton interaction
   - (d) None of these

10. The tunneling of Cooper pairs between two superconductors separated by an insulator even in the absence of applied voltage between the superconductor is known as
    (a) Josephson effect
    (b) ac Josephson effect
    (c) dc Josephson effect
    (d) None of these

11. If dc voltage exists across Josephson junction then _____ current passes across the junction.
    (a) dc
    (b) ac
    (c) pulsating
    (d) None of these

12. BCS theory introduced the concept of
    (a) phonon pairs
    (b) Cooper pairs
    (c) electron–hole pairs
    (d) hole pairs

13. In a superconductor
    (a) $\chi = -1$
    (b) $B = 0$
    (c) $H = -M$
    (d) All of these

14. Resistivity of a pure superconductor
    (a) gradually decreases
    (b) exponentially decreases
    (c) abruptly decreases
    (d) linearly decreases

15. The following element will not show superconductivity
    (a) Copper
    (b) Gold
    (c) Silver
    (d) All of these

16. Below transition temperature, the penetration depth is
    (a) almost constant
    (b) decreases exponentially
    (c) increases exponentially
    (d) None of these

17. Cooper pairs are broken at _____ temperature.
    (a) critical temperature
    (b) below critical temperature
    (c) above critical temperature
    (d) 0 K

18. The maximum current that can be passed through a superconductor is called
    (a) super current
    (b) critical current
    (c) optimum current
    (d) None of these

19. A material changes from normal to superconducting state below _____ temperature.
    (a) Weiss
    (b) Curie
    (c) critical
    (d) None of these

20. For a superconductor, the critical magnetic field _____ with decrease of temperature.
    (a) decreases
    (b) increases
    (c) will not change
    (d) None of these

21. Superconducting bearings operate
    (a) with contact
    (b) without contact
    (c) with lubricant
    (b) without lubricant

22. A superconductor is a perfect—material
    (a) diamagnetic
    (b) dielectric
    (c) insulator
    (d) semiconductor

23. The intersity of an applied magnetic field deceases—with depth from the surface of a superconductor

(a) exponentially   (b) linearly

(c) inversely   (d) logarithmically

24. —Josephson effect is used to generator and detect electromagnetic waves of frequencies ranging from radio wave to infrared wave

    (a) dc   (b) ac

    (c) Constant current   (d) Constant voltage

25. The magnetisations of a superconductor is

    (a) 0   (b) H (c) 1   (d) $-H$

26. Below transition temperature, a superconducting material exhibits.

    (a) only zero resistance   (b) zero resistance and diamagnetism

    (c) only diamagnetic property   (d) zero resistance and ferromagnetism.

27. Examples of type-I superconductos are

    (a) Al, Nb and Ta   (b) Ta, V and Nb

    (c) Al, zn and Hg   (d) None of these

28. In a superconducting state

    (a) entropy alone changes

    (b) electronic specific heat alone changes

    (c) both entropy and electronic specific heat changes

    (d) None of these

29. In superconducting state the energy gap

    (a) is large compared to semiconductors and insulators

    (b) is zero

    (c) is very small as compared to semiconductors and insulators

    (d) does not change

30. The peneteration depth is found to depend on temperature as given by the relation.

    (a) $\lambda(T) = \lambda(0)\left[1 - \left(\dfrac{T}{T_C}\right)^4\right]^{1/2}$ (b) $\lambda(T) = \lambda(0)\left[1 - \left(\dfrac{T}{T_C}\right)^4\right]^{-1/2}$

    (c) $\lambda(T) = \lambda(0)\left[1 - \left(\dfrac{T_C}{T}\right)^4\right]^{-1/2}$ (d) $\lambda(T) = \lambda(0)\left[1 - \left(\dfrac{T_C}{T}\right)^4\right]^{1/2}$

31. Switching times with a Josephson juction are in order of magnitude.

    (a) 10^{-3} ns   (b) 10^{-15s} (c) 1 ms   (d) 10^{-2} ns

32. Magnetic susceptibility is given by (for superconductors).

    (a) $\chi = +1$ (b) $\chi = -1$ (c) $\chi < 1$ (d) $\chi > 1$

33. The correlation of wave functions of superelectrons on both side of a thin insulating layer sandwitched between two superconductors is known as

    (a) Meissner effect   (b) London effect

    (c) Isotope effect   (d) Josephson effect

34. The distance from the surface of a superconductor to a point in the super-conductor at which the magnetic flux density falls to a value ($1/e$) at the surface is called

    (a) Josephson penetration depth   (b) penetration depth

    (c) Maxwell penetration depth   (d) London penetration depth

35. The critical field strength of a supercondutor
    (a) is inversely proportional to temperature
    (b) is proportional to temperature
    (c) varies with temperature
    (d) is independent of temperature

36. Theory which explains superconductivity is
    (a) Lattice theory   (b) Lorentz theory
    (c) BCS theory   (d) Sommerfield theory

37. A supercondutor exhibits _____ resistance.
    (a) small   (b) large   (c) zero   (d) infinite

Answers

1. (c)	2. (b)	3. (c)	4. (a)	5. (b)	6. (a)
7. (a)	8. (b)	9. (a)	10. (a)	11. (b)	12. (b)
13. (d)	14. (c)	15. (d)	16. (a)	17. (a)	18. (b)
19. (c)	20. (b)	21. (b)	22. (a)	23. (a)	24. (b)
25. (d)	26. (b)	27. (c)	28. (c)	29. (c)	30. (b)
31. (d)	321. (b)	33. (d)	34. (b)	35. (c)	36. (c)
37. (c)					

Fill in the Blanks

1. Certain metals and alloys exhibit almost zero resistivity when they are cooled to sufficiently low temperatures. This phenomenon is called _____.

2. When the temperature of mercury is cooled down to below _____ the resistivity drops to zero.

3. The temperature at which the transition from normal state to superconducting state takes place on cooling in the absence of magnetic field is called _____ or _____.

4. The critical temperature is _____ for different substances.

5. When a weak magnetic field is applied to a superconducting specimen at a temperature below critical temperature, the magnetic flux lines are expelled from the specimen. This effect is called _____.

6. Type I superconductors are called _____.

7. Type II superconductors are called _____.

8. Measure of disorder of a system is given by _____.

9. The magnetic flux enclosed by a ring is quantised. This concept is known as _____.

10. _____ is a double junction quantum inter fermeter.

11. Superconductors exhibiting complete Meissner effect is called _____.

12. Zr and Nb are the examples of _____.

Answers to Fill in the blanks

1. Superconductivity
2. 4 K
3. Critical temperature transition temperature
4. different
5. Meissner effect
6. Soft superconductors
7. hard superconductors
8. entropy
9. flux quantisation
10. SQUID
11. type I (or) soft superconductors
12. type II (or) hard superconductors.

True or False

1. For a chemically pure and structurally perfect specimen the superconducting transition is very sharp. (T/F)
2. Ferromagnetic and antiferromagnetic materials are superconductors. (T/F)
3. Materials having high normal resistivities exhibit superconductivity. (T/F)
4. Transition metals having odd number of valence electrons are favourable to exhibit superconductivity. (T/F)
5. Superconductivity does not depend on temperature. (T/F)
6. Superconductivity depends on external magnetic field. (T/F)
7. In superconducting state specimen acts as an ideal ferromagnetic. (T/F)
8. Type II superconductors can carry high current densities. (T/F)
9. Superconducting state is more ordered than the normal state. (T/F)
10. An energy gap is created at fermilevel in the superconducting state. (T/F)
11. The penetration depth is found to be independent of temperature. (T/F)
12. The magnetic flux enclosed by a superconducting ring is quantised. (T/F)

Answers to True or False

1. T	2. F	3. T	4. T	5. F	6. T
7. F	8. T	9. T	10. T	11. F	12. T

REVIEW QUESTIONS

Short Answer Questions

1. What is superconductivity?
2. What is critical temperature?
3. What is critical magnetic field?
4. What is Meissner effect?
5. What is London penetration depth?
6. What is type I superconductor?
7. What is type II superconductor?
8. What is flux quantisation?

9. What is Josephson effect?
10. What is dc Josephson effect?
11. What is ac Josephson effect?
12. What is BCS theory?
13. What are high temperature superconductors?

Essay Type Questions

1. Mention the properties of superconductors.
2. Explain Meissner effect in superconductor.
3. Explain London penetration depth in superconductor.
4. Describe various types of superconductor.
5. Distinguish between type I and type II superconductor.
6. Describe flux quantisation in superconductor.
7. Describe dc and ac Josephson effects in superconductor.
8. Describe BCS theory of superconductivity.
9. Explain high temperature superconductors.
10. Mention the applications of superconductors.

CHAPTER 10

PHYSICS OF NANOMATERIALS

INTRODUCTION

In recent years, nanoscience and technology is emerging as one of the most important and exciting areas of interest in all fields of science and technology. Nano means 10^{-9}. A nanometer [(nm) is billionth of a metre (1 nm = 10^{-9} m)]. Size of the atoms can vary from 0.1 to 0.5 nm depending on the type of the element. A red blood cell is 7000 nm in diameter and a water molecule is almost 0.3 nm across. The size of a virus is 100 nm. When the material size of an object is reduced to nanoscale, then it exhibits different properties than the same material in bulk form.

Nanoscience deals with the study of properties of materials at nanoscales where properties differ significantly than those at larger scale. The applications of nanoscience emerged as nanotechnology. Nanotechnology deals with the design, characterisation, production and applications of nanostructures, nanodevices and nanosystems.

10.1   NANOMATERIALS

All materials are composed of grains, which in turn comprise many atoms. The visibility of these grains depends on their size. Conventional materials have grains varying in size from hundreds of microns to millimeters. The materials possessing grains of size ranging from 1 to 100 nm, is known as Nanomaterials. Nanomaterials can be produced with different dimensionalities.

10.1.1   One-Dimensional Nanomaterial

In these nanomaterials, grains will be layered in the form of multilayers, such as thin films or surface coatings.

10.1.2   Two-Dimensional Nanomaterial

This consists of ultrafine grains layed over layers or buried, layers, which include nanowires and nanotubes.

10.1.3   Three-Dimensional Nanomaterial

This consists of nanometer sized grains. Some of the examples are precipitates, colloids and quantum dots.

The nanomaterials reduced to the nanoscale (1–100 nm) exhibit different properties compared to what they exhibit on a macroscale (bulk form). For example, opaque substances become transparent (copper), inert materials become catalysts (platinum), stable materials turn combustible (aluminium), solids change into liquids at room temperature (gold), insulators become conductors (silicon).

10.2   SIGNIFICANCE OF THE NANOSCALE

It is observed that the properties are different on the nanoscale from those at the larger scale. The quantum, mechanical, and thermodynamical properties become important at nanolevel, which are not seen at macroscopic level. After studying an individual molecule's properties, we can put them together in very well-defined ways to produce new materials with new and amazing properties. The science dealing with the materials of the nanoworld is an extension of the existing science into the nanoscale or a recasting of existing sciences using a newer, more modern terms. Nanoscience is based on the fact that the properties of materials change with the function of physical dimensions of the materials. The dimensions at which the changes in the properties of materials are observed depend on the specific material and the property in question, as well as on the three dimensions which are restricted in the real space. Actually, change in the properties of materials in these confined spaces are due to the changes in the electronic structure of the materials. The bulk properties of any material are merely the average of all the quantum forces affecting all the atoms. But on reducing the size of the material, we eventually reach a point where the averaging no longer holds. It is observed that the effect does not occur when we go from macro to micro dimensions. It becomes dominant only when the nano size range is reached. The properties of the materials are different at nanolevel due to two main reasons: increased surface area and quantum confinement effect.

10.2.1   Surface Area

When a bulk material is reduced to the particles of smaller size, then the surface to volume ratio becomes very high. Thus, nanomaterials have a relatively large surface area when compared with the same volume or same mass of the material produced in a large form.

Example 1   For a spherical material of radius *r*, the surface area is given by

$$\text{Surface area} = 4\pi r^2$$

and its

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface-to-volume ratio} = \frac{4\pi r^2}{4/3\pi r^3} = \frac{3}{r}$$

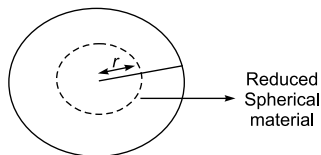**Figure 10.1** Bulk spherical material

For nanospherical material, due to decrease in size (r) the above ratio increases predominantly.

Example 2 For a one cubic volume shown in Fig. 10.2, the surface area is 6 m^2. When it is divided into eight pieces, its surface area becomes 12 m^2. Thus, when the given volume is divided into smaller pieces, the surface area increases. (For 27 pieces, surface area becomes 18 m^2).
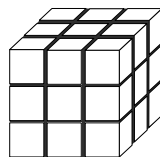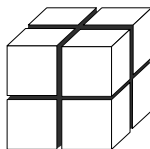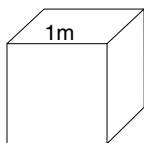
Area = 6 x 1 m^2 = 6 m^2 Area = 6 x (1/2 m)^2 x 8 = 12 m^2 Area = 6 x (1/3 m)^2 x 27 = 18 m^2

Figure 10.2 Increase in surface area for a given volume

Thus, we find that when the given volume is divided into smaller parts, surface area increases. Hence, nanoparticles have a greater surface area per given volume compared with larger particles. Since growth and catalytic chemical reactions occur at surfaces, this signifies that nanoparticles will be much more reactive than the same mass of material made up of larger particles For example, a nanomaterial of size 10 nm has 20% of its atoms on its surface and at 3 nm has 50% its atoms. Hence, at nanoscale, physical processes such as friction and sticking become more pronounced than they are in a normal system. These specific factors related to nanoparticles will facilitate their use both inside and outside the system.

10.2.2 Quantum Confinement

We known that isolated atoms have discrete energy levels. Generally, solids have split energy levels known as energy binds. But nanomaterials have the intermediate stage. As far as the physics at nanolevel is concerned, it is the quantum effect that deals with the properties. At reduced dimensions, they are said to be either a quantum well, a quantum dot, or a quantum wire. The physics at these dimensions is entirely different. Actually, when the size of the grains is reduced to nanolevel, then overlapping of wavefunction and quantum confinements occurs. If d is the diameter of the grain size, then the energy goes up by factor $1/d^2$.

From uncertainty principle, we know that

$$\Delta x \cdot \Delta p \simeq \frac{h}{4\pi}$$

where Δp is the uncertainty in momentum and h is Planck's constant. In case of nanoparticles,

$$\Delta x = d$$

$$\Delta p = \frac{h}{4\pi d}$$

If $p_{max} = \Delta p$, then energy E is given by

$$E = \frac{p^2}{2m} = \frac{p_{max}^2}{2m}$$

Now, energy in terms of the size of the grain particle can be given as

$$E = \frac{h^2}{32\pi m d^2} \tag{10.1}$$

In Eq. (10.1) at right side, except d, rest are constant. Thus, energy at nanoscale is given as

$$E \propto \frac{1}{d^2} \tag{10.2}$$

Thus, energy increases by a factor $1/d^2$.

This leads to excitons, and they are primarily responsible for enhanced optical and electrical properties at nanoscale. In case of metals, when their sizes are reduced, their properties are determined by plasmons.

Hence, we can conclude that when the dimensions of a potential well or a box concerned with a particle are reduced to the order of de Broglie wavelength of electron (within few tens of nanometres), then energy levels of electrons change. This effect is called *quantum confinement*. This can affect the properties of nanomaterials at nanoscales.

10.3   PROPERTIES OF NANOMATERIALS

10.3.1   Physical Properties

At nanoscale, surface area to volume ratio increases. This changes the surface pressure and results in a change in the interatomic spacing. As shown in Fig. 10.3(a), the interatomic spacing decreases with particle size for copper metal.

The change in interatomic spacing and large surface to volume ratio have a combined effect on material properties. Variation in the surface free energy changes the chemical potential. As a result, the melting point shows variation for a change in particle size for gold particles (Fig. 10.3(b)).
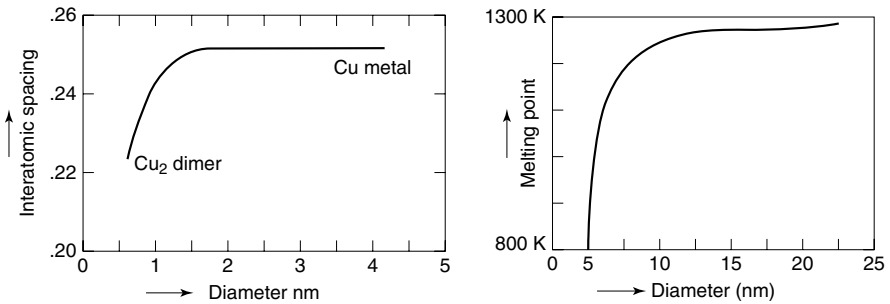
Figure 10.3 (a) Nearest neighbour distance in copper metal as a function of cluster size, (b) Melting point of gold particles as a function size

10.3.2 Mechanical Properties

The application of the nanomaterials may be on both low and high temperatures. hence, the mechanical properties of nanomaterials were studies at low and high temperatures.

Low-temperature properties

We know that grain refinement leads to an improvement in the properties of the metals and alloys. For example, a reduction in grain size lowers the transition temperature in steel from ductile to brittle. A major interest in nanomaterials is the change in mechanical properties with the reduction in grain size. The average grain size and yield strength (σ_y) are simply related by the Hall petch relation given below.

$$\sigma = \sigma_0 + \frac{k}{\sqrt{d}}$$

where σ_0 is the friction stress, k the constant and d, the average grain size.

Similarly, one can write the relation between hardness and grain size as

$$H = H + \frac{k}{\sqrt{d}}$$

where H_i, is the Vickers hardness and k a constant. From the above relation, it is clear that when the grain size changes, it in turn changes the yield strengths and hardness.

As shown in Fig. 10.4, both the Young's modulus and the shear modulus shows a decreasing variation with the decrease in particle size.
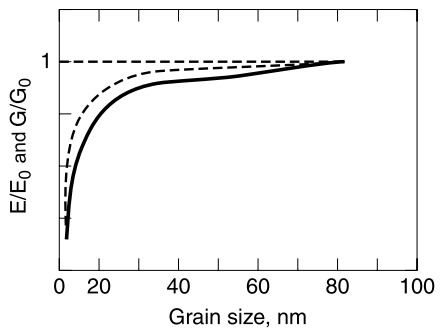

Figure 10.4 Young's modulus and shear modulus of iron as a function of particle size

High-temperature properties

The nanomaterials are also termed as super plastic materials, since they exhibit all the common microstructural features of super plastic materials, such as extensive tensile deformation without cracking or fracture. Some of the characteristics of super plastic materials are smaller grain size (less than 5 mm), equiaxed grains, high-energy grain boundaries and the presence of second phase. In case of nanophase materials, the occurrence of super plastic temperature is decreased due to the decrease in grain size, which results in an increase in the strain rate.

10.3.3   Magnetic Properties

Nanomaterials also have size-dependent magnetic behaviour. In small ferromagnetic particles, the magnetic properties are different from that of the bulk material. At nanoscale, the magnetic material has a single magnetic domain. As a result, the coercive field and the remanent magnetisation show a strong dependence on particle size (Fig. 10.5), and (Fig. 10.6).
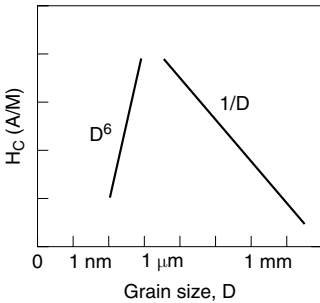
Figure 10.5   Coercive field as a of grain size function
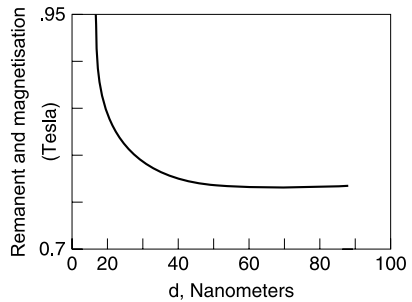
Figure 10.6   Remanent magnetisation as a of grain size function

10.3.4   Optical Properties

If semiconductor particles are made very small, quantum effects come into play which limits the energies at which electrons and holes can exist in the particles. As energy is related to wavelength (or colour) this means that the optical properties of the particle can be tuned depending on its size.

Thus, the particle can be made to emit or absorb specific wavelengths (colours) of light by controlling their size. The optical absorption spectrum of gold particles as a function of particle size is shown in Fig. 10.7.

10.3.5   Thermal Properties

In general, increasing the number of grain boundaries will enhance phonon scattering at the

Figure 10.7   Absorbance of gold particles

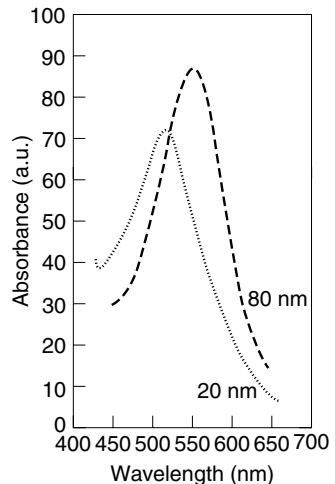disordered boundaries, resulting in lower thermal conductivity. Thus, nanocrystalline materials would be expected to have lower thermal conductivity compared to conventional materials. However, as the grain sizes assume nanodimensions, their size becomes comparable to the mean free paths of phonons that transport thermal energy. Thus, nanomaterials can show widely different properties compared to coarse-grained materials, due to the photon confinement and quantisation effects of photon transport. It has been observed that in addition to the grain size, the shape also has an influence on the thermal properties of nanomaterials. For example, one-dimensional nanowires may offer ultralow thermal conductivities, quite different from that of carbon nanotubes. In nanowires, quantum confinement of phonons in one dimensions can result in additional polarisation modes compared to that observed in bulk solids. The strong phonon-phonon interactions and enhanced scattering at grain boundaries result in a significant reduction in thermal conductivity of nanostructures. Silicon nanowires are known to exhibit thermal conductivity at least about two orders of magnitude smaller than of bulk silicon. In contrast, the tubular structures of carbon nanotubes result in an extremely high (~ 6600 W/mK) thermal conductivity along the axial direction. However, high anisotropy in their heat transport property is observed, making the thermal transport direction-dependent.

In multilayered coatings, many collective modes of phonon transport may appear besides the phonon modes in each single layer; when the phonon coherence length becomes comparable to the thickness of each layer, the transport properties are significantly influenced. When the mean free path of phonons spans multiple interfaces, the phonon dispersion relation is modified, resulting in enhanced scattering due to decrease in phonon group velocity. Further, if the multilayer is designed to have a superlattice structure, and alternate films have a large mismatch in the phonon dispersion relations, it is possible that phonons in a certain frequency range may not propagate to the neighbouring layers unless there are mode conversions at the interface. Also, the presence of interface dislocations and defects can contribute to enhanced boundary scattering. All these factors can contribute to the lower thermal conductivity of multilayered nanostructured films.

10.4   SYNTHESIS OF NANOMATERIALS

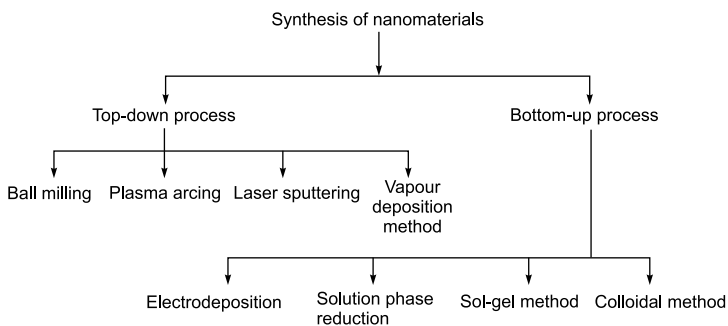As shown in Fig. 10.8, there are two main processes used for the synthesis of nanomaterials.

Figure 10.8   General processes for synthesis of nanomaterials

Top-Down Process Under this synthesis process of fabrication, bulk materials are broken into nano-sized particles. In this approach, there is no control over the size and the morphology of particles. There are many methods used in top-down approach to get nano-sized particles from bulk materials. Some methods of top-down process are as follows:

   (i) Ball milling
  (ii) Plasma arching
 (iii) Laser sputtering
 (iv) Vapour deposition

Bottom-up Process Bottom-up approach refers to the building up of a material from the bottom, i.e., atom by atom, molecule by molecule, or cluster by cluster. Colloidal dispersion is a good example of bottom-up approach in the synthesis of nanoparticles. There are different methods used for the synthesis of nanomaterials under bottom-up process. Some methods of this process are as follows:

   (i) Sol-gel
  (ii) Colloidal
 (iii) Electrodeposition
 (iv) Solution phase reductions

10.4.1 Ball Milling

Ball milling is one of the top-down method approach for the fabrication of nanoparticles. Milling technique is used in this process of fabrication. First it is necessary to explore the different types of attrition mills for various types of materials, with a variety of sizes of mills in order to up-scale the actual production from the laboratory level to the commercial production of nanoparticles. High-energy mills used for the fabrication of nanoparticles include attrition ball mill, planetary ball mill, vibrating ball mill, low-energy tumbling mill, and high-energy ball mill.
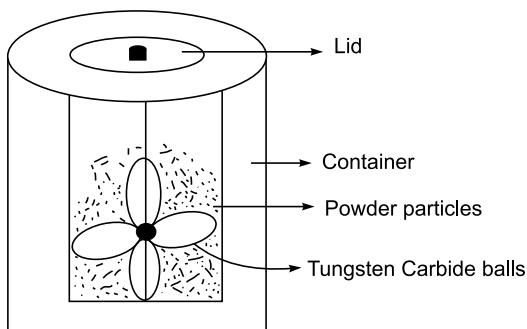

Figure 10.9   Ball milling method

Principle　　The basic principle of the ball milling method is small hard balls are allowed to rotate inside a container and then it is made to fall on a solid base with high force to crush the solid into nanoparticles.

Construction and working　　Hardened steel or tungsten carbide balls are put in a container along with the powder of particles of a desired material. The container is closed with tight lids as shown in the Fig. 10.9.

When the balls are rotated around the central axis, the material is forced to press against the balls. The milling bails impart energy on collision and produce smaller grain size of nanoparticles. Ball milling is also known as mechanical alloying or crushing.

The main advantage of this method is that few milligrams to several kilograms of nanoparticles can be synthesised in a short period of time. This technique can be operated at large scale.

Applications　　Some important applications of ball milling technique are as follows:

1. This method is useful in the preparation of elemental and metal oxide nanocrystals such as Co, Cr. AlFe, AgFe, and Fe.
2. A variety of intermetallic compounds of Ni and Al can be formed.
3. This method is useful in producing new types of building materials, fireproof materials, glass ceramics, etc.

10.4.2　Chemical Vapour Deposition

Chemical vapour deposition (CVD) is a chemical process used to produce high-purity, high-performance solid materials. The process is often used in the semiconductor industry to produce thin films. It is a technique for synthesising materials in which chemical components in vapour phase react to form a solid film at some surface (substrate). The occurrance of chemical reaction is central to this means of thin film growth, as is the requirement that the materials must start out in the vapour phase. Ability to control the components and physical conditions of the gas phase, the solid surface and the envelope that surrounds them determine the capability to control the properties of the thin film deposited by the CVD process. Micro and nano-fabrication processes widely use CVD to deposit materials in various forms, including monocrystalline, polycrystalline, amorphous, and epitaxial. These materials include silicon, carbon fibre, carbon nano-fibres/nano filaments/nano-tubes/nano-rods, SiO_2, silicon-germanium, tungsten, silicon carbide nanostructures, silicon nitride nanomaterials, silicon ogynitride, titanium nitride, and various high-k dielectrics. The CVD process is also used to produce synthetic diamonds and nano-diamonds. A schematic diagram of a CVD chamber is shown in Fig. 10.10.
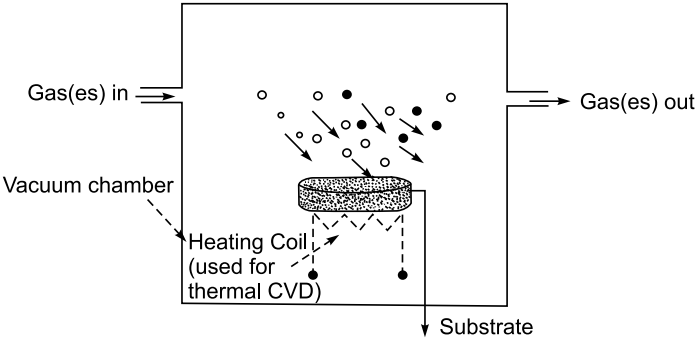
Figure 10.10 (a) Diagram of a CVD system.

CVD is a sequential process which starts from the initial vapour phase, progressess through a series of quasi-steady subprocesses, and culminates in the formation of a solid thin film of its final micro- or nano-structure. The sequence is illustrated by a flow chart in Fig. 10.10(b) and schematically represented in Fig.10 10(c). In general, the (CVD process can be divided into two broad steps one is transport of gas phase materials to the reaction tone and the reaction of materials and the other is the deposition of the film on the substrate. The transport process involves the gas supply, convection of gas-phase materials due to pressure gradient and/or buoyancy of hot gases and diffusion of gaseous reactants to the substrate surface. The deposition process involves the adsorption of the reactant species on to the substrate surface sites by surface migration followed by surface chemical reaction between the reactant species, usually catalysed by the surface, then the deception of the reaction by-products takes
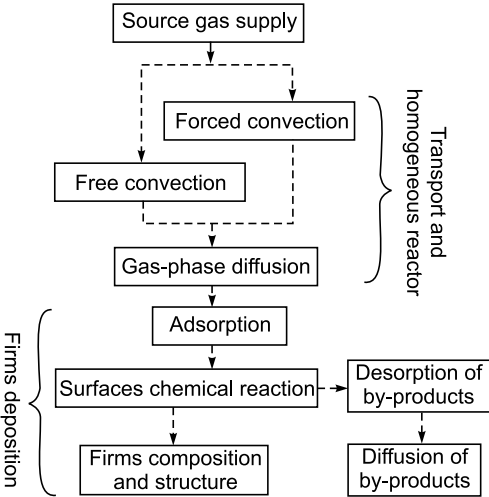

Figure 10.10 (b) The various steps involved in a CVD Process.

place followed by diffusion of the by-products away from the surface and finally incorporation of the condensed solid products into the macro/nano-structure of the growing film as shown in Fig. 10.10(c).
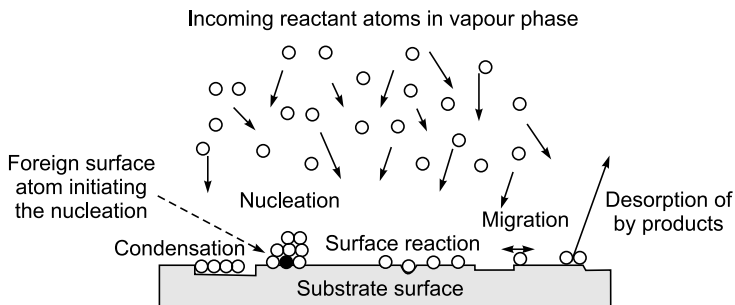

Figure 10.10 (c) Diagram of various processes involved in CVD.

Convection in a CVD chamber refers to the flow of the gaseous fluid as it moves through the reactor after being injected from the gas supply. Two types of convection generally take place inside the chamber, one is the forced convection due to pressure gradient across the chamber and the other is the free convection due to buoyancy of the hot gases. Another important process involved is the viscous friction experienced by the gas molecules, when they come very close to the substrate surface. Due to this the flow velocity shows down and the remaining transport of reactant to the surface occurs only by diffusion through the relatively stationary boundary layer of fluid [of Fig. 10.10(c)). The gas-transport flow pattern is an important aspect to determine how far from the deposition surface the transition from convection to diffusion is occurring and which of these two transport processes is limiting the reactant arrival rate at the substrate surface. Once the transition from convection to the diffusion takes place, the sourer gas molecules are adsorbed to the surface followed by the surface reaction to produce the required material. The impodant factors that influence the homogeneous reaction are the gas residence time near the surface and gas heating. both of which are functions of the gas flow rate and flow pattern. Depending on these factors, the reaction often begins in the gas phase rather than occuring entirely at the substrate surface. Generally, the gas phase reactions are homogeneous and produce powdery material and surface reactions are heterogeneous and produce thin films. For nanostructured film deposition, surface reaction is important whereas nanoparticle formation depends more on gas phase reaction. Therefore, it is very important in CVD growth to understand and control both gas phase and surface reactions.

10.4.3 Sol–Gel

Sol-gel is a wet-chemical-based self-assembly process for nanomaterial formation. The sol-gel process as the name implies, involves the evolution of networks through the formation of a colloidal suspension (sol) and gelation of the sol to form a network in a continuous liquid phase (gel). The precursors used for synthesising the colloids

generally consist of metallic ions and ligands, which arc elements surrounded by various reactive species. In some cases, few alkoxides are immiscible in water. in that situation, some homogenising agents such as alcohols are used as mutual solvents to facilitate miscibility of tbese materials. In general. solgel formation occurs in four steps:

(a) Hydrolysis and alcoholysis

(b) Water and alcohol condensation and polymerisation of monomers to form particles

(e) Growth of particles

(d) Agglomeration of particles followed by the formation of networks throughout the liquid medium resulting in thick gel

In general, all the above mentioned steps are dependent on several initial conditions such as pH value of the sol temperature of the reaction, reagent concenttations. time of reaction, nature and concentration of catalyst (if needed), $[H_2O/M^+]$ molar ratio (M^+; cation, aging temperature and time of the gel formation, etc. By controlling these parameters, it is possible to vary the structural, electrical and optical properties of the sol-gel derived inorganic network over a wide range.

Once the gel is formed, there are several ways to convert this gel (inorganic network) to the desired solid form. Depcndine on the deposition and drying processes or conditions, this gel can be converted into various forms such as aerogel, xerogel, gelled spheres, nano-powders, thin film coatings, nanostructurcd layers, etc. Figure 10.11 describes the formation of various inorganic structures by sol-gel method. The first three steps in Fig. 10.11 describe the sol preparation and gel formation via four stages described carlier. Once the gel is formed, it can be converted into acrogel by rapidly drying the gel. Also by slow drying process, the gel can be converted into xerogel which, in turn, can be converted into dense ceramic material under calcination in a similar way, the sol can be converted into small gelled spheres by using various surfactants which modify the surface tension of the dispersed particles in the sol and prevents them from agglomeration. The gelled spheres can then be converted into powders by calcination. For nanomaterial fabrication, proper use of surfactant is very important because it controls the level of agglomeration of the suspended particles in the sol. Therefore, using the right kind of surfactant and proper concentration of the surfactant will control the size of nano-particles in the powdered material produced, Another important application of sol-gel is the formation of nano-wires/rods via porous matrix, such as alumina template porous silicon, etc. The sol is first deposited on the porous matrix, and then by sonication and (or) electrodeposition process the particles suspended in the sol are guided inside the nano-pores of the matrix and then converted into nanorods by drying it. Thereafter, the porous matrix is removed by some selective etching process to get frce-standing nano-rods.

Another very important use of sol-gel method is to deposit thin film on various substrates via dip-coating/spin-coating/spraying process.
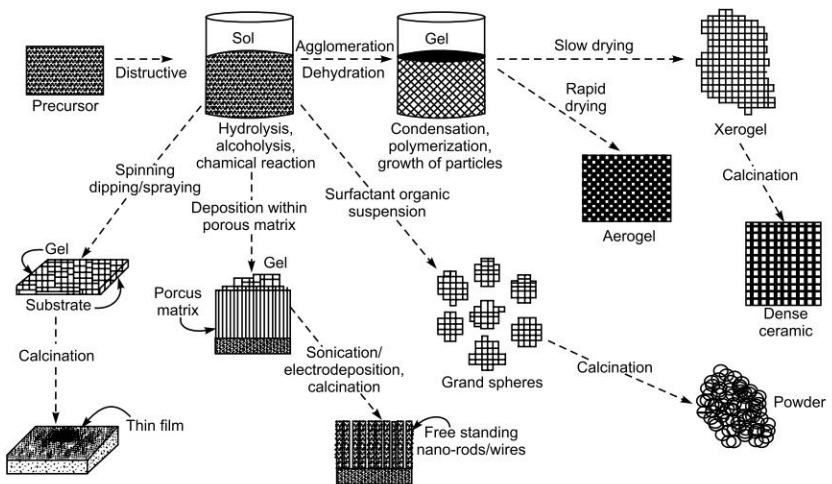
Figure 10.11   Sol-Gel process for deposition of various micro/nano-structures.

10.4.4   Thermal Evaporation

Thermal evaporation often uses a filament of high current (10-100 A, depending on the filament and material to be deposited) passing through it, producing sufficient amount of thermal energy, which heats up the material to be deposited. When the substance is heated adequately, it begins to evaporate and travel through the chamber and deposit on the substrate to be coated. The higher the vacuum, the more efficiently material will be deposited to the substrate. In a higher vacuum, there are less number of molecules in the chamber, which will increase the 'mean free path' and a longer mean free path will allow the evaporated molecules to travel further before striking an unwanted molecule in the chamber. The rate of deposition can be controlled by the current being passed through the filament. A schematic diagram of a thermal evaporation system is shown in Fig. 10.12(a).
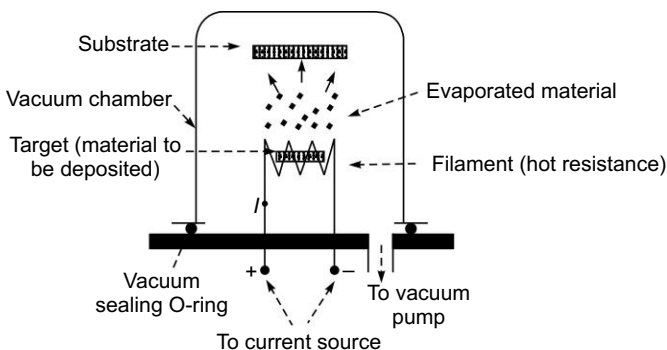

Figure 10.12(a)   Thermal evaporation system

There are different types of evaporation sources. The simplest are the metal boat, made up from refractory, metals like W, Mo, Ta, etc., and heat-shielded

crucible made up of alumina or ceramic materials. Some of the basic diagrams of these types of evaporation sources arc shown in Fig. 10.12(b). The metal boat is heated up by passing high current (–10–100 A) through it. The amount of heat generated is I^2R (Joule heating), where R is the parallel resistance of source/evaporant combination at the evaporation temperature, T. In general, T is not uniform because of heat conduction down the current contacts. Therefore, if evaporation rate control is important (and this is significant for nonmaterial synthesis), continuous flux monitoring is necessary, preferably, with feedback control of the current source. To prevent the evaporant to spread away from the hot zone of the boat, a narrower zone is always provided at each end of this zone as shown in Fig. 10.12(b)-i. The resulting higher R increases heating enough in those regions so that evaporation rate exceeds spreading rate.
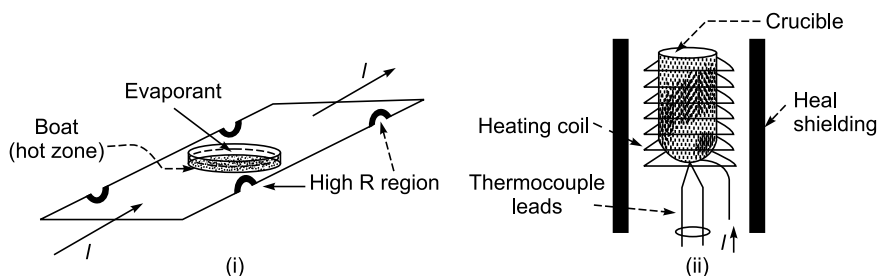

Figure 10.12(b)   Evaporation sources (i) metal boat and (ii) heat-shielded crucible.

Alloying of the evaporant with the how boat metal may result in the embrittlement or melting. If there are no metals suitable for contact with the desired evaporant, ceramic-coated boats or ceramic crucibles can be used, construction of which is shown schematically in Fig. 10.12(b)-ii. In this case, the ceramic crucible is placed inside a heating coil and the crucible along with the evaporant is heated up with the current passing through the coil.

Transport of the evaporated material from the source to the substrate is another important factor in the vacuum evaporation process, which determines the uniformity of the film deposited on the substrate. In high vacuum, the evaporant molecule, during its travel from the source to the substrate, is assumed to have very less probability to collide with background molecules along its way and consequently, the arrival-rate uniformity of the evaporant molecules at the substrate is determined solely by the geometrical factors.

10.4.5   Plasma Arching

An ionised system is considered as a plasma. When a gas is made conducting by a high potential difference via electrodes inside the gas system, so that the gas yields up its electrons and thus ionises, plasma is achieved. In general, plasma consists of electrons, ions, as well as neutral particles. Due to high conductivity the plasma, an arc is generated (due to current conduction through plasma)

between the two closely spaced electrodes in vacuum or in low-pressure inert gas atmosphere, which produces heat inside the system. This heat produced can be used to vaporise and ionise the electrodes and even some other materials present inside the discharge chamber. Plasma arc method has been used extensively in the deposition of carbon nanotube. A typical plasma arc discharge system is shown in Fig. 10.13 which consists of two electrodes inside a discharge chamber in vacuum or at low-pressure inert atmosphere. An electric arc passes from one electrode (anode) to the other (cathode due to which the anode vaporises and gets deposited to the other electrode as well as on the inside of the chamber wall. For carbon nanotube deposition, carbon electrodes are used, through which, atomic carbon cations are produced due to arcing. These carbon cations move to the cathode to pick up electrons and get deposited to form nanotubes.
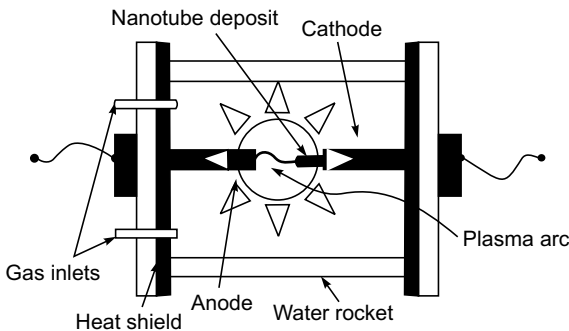

Figure 10.13 Plasma arcing process. Material is evaporated from the anode and gels deposited on the cathode.

The electrodes can be made of other materials but they must be able to conduct electricity. Boron and silicon nitride nanotubes are formed by this method using elemental boron and silicon electrodes, respectively, in an activated nitrogen atmosphere. An interesting variation is to make the electrodes from a mixture of conducting and nonconducting materials. During heating, the nonconducting material is vaporised and ionised so that it also becomes part of the plasma arc and is transported and deposited on the cathode. Another interesting variation on plasma arcing is flame ionisation. In this process, a material is sprayed into a flame and ions are produced, which can be collected and deposited in nanocrystalline form.

10.5   PROPERTIES OF CARBON NANOTUBES

The unique nature of the carbon bond in organic molecules of living beings leads to interesting nanostructures, particularly carbon nanotubes (CNT). One of the more interesting—nanostructures with huge potential applications is the carbon nanotube. The main interest in CNT is due to the existence of exotic properties like very low specific resistivities in metallic carbon nanotubes and high hole

mobilities for semiconducting nanotubes. The carbon nanotubes are formed by rolling the graphite (or graphene) sheet into tubes with the bonds at the end of the sheet. These bonds are used to close the tube. Generally, the carbon nanotubes are formed in the range of 2 to 10 nm in diameter and a length of 100 μm.

Carbon nanotubes are classified into single wall carbon nanotube (SWCNT) and Multiwall Carbon nanotube (MWCNT) depending upon the walls contained by them.

1. Carbon nanotubes are the strongest and stiffest material in terms of tensile strengths and elastic modulus respectively.
2. Multiwall carbon nanotube exhibit telescoping property whereby an inner nanotube core slide without friction with in its outer nanotube shell creating an atomically perfect linear or molecular bearing.
3. Because of the symmetry and unique electronic structure of graphene, CNT will act as metallic as well as a semiconductor.
4. Metallic CNT exhibits low specific resistivity and hence high electric current density.
5. Semiconducting CNT exhibits high carrier mobilities which inturn exhibit high current density.
6. All CNTs are good thermal conductors.
7. CNTs exhibits optical activity and hence they are used in optical devices.
8. They exhibits enhanced chemical reactivity.
9. CNT exhibit magneto resistance phenomena.

10.5.1   High Strength Applications

The CNT are very strong, that is, they are about 10 times stronger than steel. Let us consider that one end of a thin wire is nailed to the roof of a room while a weight W is attached to the other end. The stress acting on the wire is given by

$$S = W/A$$

where A is the cross-sectional area of the wire.

The strain of the wire is given by the amount of stretch ΔL of the wire of length L,

$$S = \Delta L/L$$

where L is the length of the wire before attaching the weight.

We known that stress is proportional to strain

$$S = Ee$$

where E is the proportionality constant and is equal to $LW/A\ \Delta L$, known as Young's modulus.

The Young's modulus of the material is used to characterise the elastic flexibility. For example, the larger the value of Young's modulus the lesser is the

flexibility. The Young's modulus of the carbon nanotube is in the range from 1.28 to 1.8 Tpa (1 Tpa 10^7 atm), while for steel, it is 0.21 Tpa. It means that the CNT is 10 times stronger than that of steel. This property of SWCNT and MWCNT makes the nanotubes a different material from other conventional materials which finds wide variety of high strength applications.

10.6   PROPERTIES OF GRAPHENE

Graphene is a one-atom-thick planar sheet of sp^2-bonded carbon atoms that are densely packed in a honeycomb crystal lattice. The name comes from GRAPHITE + -ENE; graphite itself consists of many graphene sheets stacked together. The carbon-carbon bond length in graphene is approximately 1.42 Å. Graphene is the basic structural element of all other graphitic materials including graphite, carbon nanotubes and fullerenes. It can also be considered as an infinitely large aromatic molecule, the limiting case of the family of flat polycyclic aromatic hydrocarbons called graphenes.
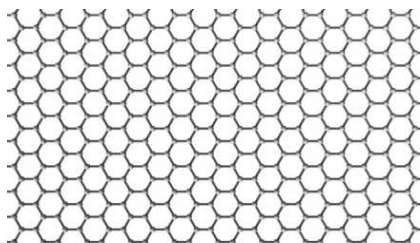
Figure 10.14   Sheet of sp^2-bonded carbon atoms of Graphene.

1. The expected structure of graphene is hexagonal lattice.
2. Intrinsic graphene is a semi-metal or zero-gap semiconductor.
3. It exhibits high electron mobility at room temperature.
4. It is an ideal material for spintronics due to small spin orbit interaction and near absence of nuclear magnetic moments in carbon.
5. It exhibits anomalous quantum hall effect.
6. It exhibit high opacity.
7. It exhibit high thermal conductivity at room temperature.
8. It exhibit high Young's modulus hence it is very strong and rigid.
9. It exhibit high carrier mobility hence it is used as the channel in a FET.

10.6.1   Graphene–Based FET
FET consists of three parts i.e., Source, Drain and Gate. It is voltage controlled device, having its wide applications in electronic devices. FETs are used in

non-inverting Amplifier, Timmer network, Fibre optic systems and relay drives. In *n* channel FET, source and drain are of *n* type and gate is of *p* type. In *p* channel FET, source and drain are of *p* type and gate is of *n* type. The channel connecting source and drain is made up of graphene then it is known as graphene-based FET. Since graphene has high carrier mobility for charge carriers (electrons or holes) it will act as effective channel for source and drain. Figure 10.15 shows the top gated FET with graphene channel.
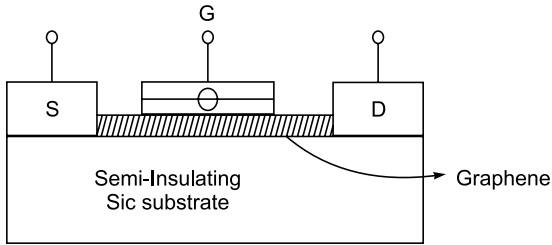
Figure 10.15   Top gate graphene-based FET

It consists of source, drain and gate. The graphene will provide a channel between source and drain. The carrier density and the type of carrier in the channel are governed by the potential difference between the channel and gate. The output characteristics of *n* channel graphene-based FET for a constant gate voltage is as shown in Fig. 10.16.
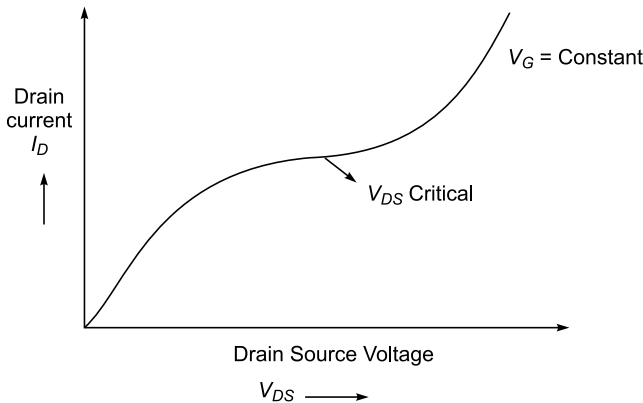
Figure 10.16   Output characteristics

From the graph it is clear that for small values of V_{DS}, it operates in the linear region and it is of *n* type. As V_{DS} is increased, the drain current starts to saturate until V_{DS} critical point is reached. Once it exceeds, then it enters into second linear region of *p* type

Applications

1. They are used in amplifiers.
2. They are used in phase shift detectors.
3. High on/off ratio has wide applications in logic gates.
4. Outstanding transconductance value find variety of applications in logic gates.

10.7 APPLICATIONS OF NANOMATERIALS

Since nanomaterials possess unique and beneficial chemical, physical and mechanical properties, they can be used for a wide variety of applications.

Material Technology

- Nanocrystalline *aerogel* are light in weight and since they are porous, air is trapped at the interstices. Aerogels are used for insulation in offices, homes, etc.
- Cutting tools made of nanocrystalline materials are much harder, much more wear-resistant, and last longer.
- Sensors made from nanocrystalline materials are sensitive to changes in their environment. Thus, they are used for smoke detectors, ice detectors on aircraft wings, etc.
- Nanocrystalline materials are used for high energy-density batteries.
- Nanoengineered membranes could potentially lead to more energy-efficient-water-purification processes.
- Nanosized titanium dioxide and zinc oxide are used in sunscreens to absorb and reflect ultraviolet rays.
- Nanoparticles in paints change colour in response to change in temperature or chemical environment, and reduce infrared absorption and heat loss.
- Nanoparticles react with pollutants in soil and ground water and convert them into harmless compounds.
- Nanocoating of highly activated titanium dioxide acts as water repellent (hydrophobic) and antibacterial. Coatings based on nano particulate oxides destroy chemical agents.
- In textile industry, breathable, waterproof and stain-resistant fabrics have been manufactured due to the control of porosity at the nanoscale.
- Nanocrystalline ceramics are used in automotive industry as high-strength springs, ball bearings and valve lifters.
- The hardness of metals can be predominantly enhanced by using nanoparticles.

Information Technology

- Nanoscale-fabricated magnetic materials are used in data storage.
- Nano computer chips reduce the size of the microprocessor with nanocrystalline starting materials, ultra-high purity materials with better thermal conductivity and durable interconnections.
- Nanocrystalline light-emitting phosphors are used for flat panel displays.
- Nanoparticles are used for information storage.
- Nanophotonic crystals are used in chemical optical computers.
- Nano thickness-controlled coating are used in optoelectronic devices.

Biomedicals

- Nanocrystalline silicon carbide is used for artificial heart valves due to its low weight, high strength and inertness.
- Biosensitive nanomaterials are used for tagging of DNA and DNA chips.
- In the medical field, nanomaterials are used for disease diagnosis, drug delivery and molecular imaging.
- Nanostructured ceramics readily interact with bone cells and hence are used as implant material.

Energy Storage

- Addition of nanoparticles to diesel fuel improves fuel economy by reducing the degradation of fuel consumption.
- Nanoparticles are used in hydrogen-storage devices.
- Nanoparticles are used in magnetic refrigeration.
- Metal nanoparticles are useful in fabrication of ionic batteries.

MULTIPLE CHOICE QUESTIONS

1. 1 nm =
   - (a) 10^{-9} m
   - (b) 10^{-8} m
   - (c) 10^{-7} m
   - (d) 10^{-6} m

2. Who first visualised the concept of nanotechnology?
   - (a) Eric Drexler
   - (b) Richard Feynman
   - (c) Norio Taniguchi
   - (d) Buckminister Fuller

3. For nanomaterials, the surface area to volume ratio is
   - (a) large
   - (b) very large
   - (c) small
   - (d) very small

4. By reducing the size of a nanomaterial, the change in the interatomic spacing is
   - (a) increased
   - (b) decreased
   - (c) first increased and then decreased
   - (d) kept constant

5. Which of the following properties change, if a bulk material is converted to a nanomaterial?
    (a) Elastic modulus
    (b) Hardness
    (c) Resistance
    (d) All of these

6. Nanomaterials are catalysts because of their enhanced
    (a) chemical activity
    (b) thermal activity
    (c) mechanical activity
    (d) optical activity

7. Carbon nanotubes can have _____ types of structures.
    (a) one
    (b) two
    (c) three
    (d) four

8. Aerogel is a
    (a) foam-type nanocrystal
    (b) nanometallic material
    (c) nonmetallic nanomaterial
    (d) All of these

9. Quantum dot is an example of
    (a) one-dimensional nanomaterial
    (b) two-dimensional nanomaterial
    (c) three-dimensional nanomaterial
    (d) All of these

10. Nanowires are examples of
    (a) one-dimensional nanomaterial
    (b) two-dimensional nanomaterial
    (c) three-dimensional nanomaterial
    (d) All of these

11. In quantum confinement effect, the energy levels of _____ changes.
    (a) electrons
    (b) atoms
    (c) molecules
    (d) nanoparticles

12. The technique used for fabrication of nanomaterials is
    (a) ball milling
    (b) sol–gel method
    (c) chemical vapour deposition
    (d) All of these

13. The carbon nanotube shows _____ effect at low temperatures.
    (a) magneto resistive
    (b) magnetic
    (c) resistive
    (d) coupling

14. The size range of nanomaterial is
    (a) 1 to 100 Å
    (b) 1 to 100 nm
    (c) 1 to 100 μm
    (d) 1 to 100 mm

15. By reducing the size of metal particles from bulk to nano, the energy bands become
    (a) narrower
    (b) broader
    (c) remain same
    (d) First become broader and then get narrower

16. Clothes made up of nanofibers are
    (a) water repellent
    (b) wrinkle free
    (c) stress resistant
    (d) All of these

17. In nanomaterials with decrease of size, the melting point
    (a) Increases
    (b) decreases
    (c) remains constant
    (d) first decreases and then increases

18. The advantage of sol-gel method in the fabrication of nanoparticles is
    (a) it is a low-temperature process
    (b) the product can be obtained in any form
    (c) it is polished to optical quality
    (d) All of the above

19. Nanosized particles are chemically very active because
    (a) of their small size
    (b) due to the lower number of atoms
    (c) number of surface atoms are more
    (d) number of surface atoms are less

20. In the fabrication of nanoparticles, bulk material is crushed into nanoparticles in _____ method
    (a) chemical vapour depositions   (b) ball milling
    (c) plasma arching   (d) Sol-gel method

21. For a sphere nano particle of radius 'r', surface area to volume ratio is given by
    (a) $\dfrac{2}{r}$ (b) $\dfrac{3}{r}$
    (c) $\dfrac{4}{r}$ (d) $\dfrac{5}{r}$

23. For a cubic nanoparticle of side 'a', surface area to volume ratio is given by
    (a) $\dfrac{3}{a}$ (b) $\dfrac{4}{a}$
    (c) $\dfrac{5}{a}$ (d) $\dfrac{6}{a}$

23. Diameter of one carbon atom is [ ]
    (a) 0.05 nm   (b) 0.5 nm
    (c) 0.15 nm   (d) 5 nm

24. In nanomaterials with decrease of size, the melting point _____ .
    (a) increase   (b) decreases
    (c) remains constant   (d) None of these

25. Gold nanospheres of 100 nm appear
    (a) blue in colour   (b) red in colour
    (c) vidol in colour   (d) orange in colour

26. At 27° the resistivity of the single walled nanotubes rope is of the order of _____ .
    (a) 10^{-6} ohm-cm   (b) 10^{-4} ohm-cm
    (c) 10^{-2} ohm-cm   (d) 10^{-3} ohm-cm

Answers

1. (a)	2. (b)	3. (b)	4. (b)	5. (d)	6. (a)
7. (b)	8. (a)	9. (c)	10. (b)	11. (a)	12. (d)
13. (a)	14. (b)	15. (a)	16. (d)	17. (b)	18. (d)
19. (c)	20. (b)	21. (b)	22. (d)	23. (c)	24. (b)
25. (d)	26. (b)				

Fill in the Blanks

1. For a sphere of radius '*r*' surface area to volume ratio is _____.
2. For a cube of side dimension '*a*' surface area to volume ratio is _____.
3. When the dimension of the nano particle is of the order of de Broglie wavelength, or mean free path of electrons,energy levels of electrons change. This effect is called _____.
4. With decrease of size of nanoparticle, the interatomic spacing _____.
5. With decrease of size of nanoparticle, the melting point _____.
6. Due to quantum confinement, in nanoparticles electronic bands become _____.
7. Gold nano spheres of 50 nm size in collidal form appear _____ in colour.
8. Nano particles are _____ magnectic than the bulk material.
9. Nano particles have _____ mechanical strength than their bulk materials.
10. The method of producing very small structures from large pieces of materials is called _____ techniques.
11. The method of synthesising nanomaterials atom by atom or molecule by molecule attachment is known as _____ technique.
12. Ball milling is an example for _____ technique.
13. Extended tubes of rolled graphite sheets are called _____.

Answers to Fill in the Blanks

1. $\frac{3}{r}$
2. $\frac{b}{a}$
3. quantum confinement
4. decreases
5. decreases
6. narrower
7. green
8. more
9. higher
10. top down
11. bottom up
12. top down
13. carbon nanotubes

True or False

1. Diameter of one carbon atom is 0.15 nm. (T/F)
2. This film coating is an example for nanoscale in two dimensional. (T/F)
3. Quantum dot is an example for nanoscale in three dimensional. (T/F)
4. Much smaller particles have lesser value of surface area-to-volume ratio. (T/F)
5. Gold nano spheres of 100 nm size in colloidal form appear pink colour. (T/F)

6. With decrease of particle size the fraction of atoms residing on the surface increases.   (T/F)
7. Plasma arching is used to produce carbon nanotubes.   (T/F)
8. Sol-gel method can be used to produce carbon nanotubes.   (T/F)
9. Carbon nano tubes exhibit better electrical conductivity.   (T/F)

Answers to True or False

1. T	2. F	3. T	4. F	5. F	6. T
7. T	8. F	9. T			

REVIEW QUESTIONS

Short Answer Questions

1. Define nanomaterial.
2. What is the significance of nanoscale?
3. How surface area-to-volume ratio changes for nanoparticles?
4. What is quantum confinement?
5. How do optical properties vary for nano materials?
6. How do thermal properties vary for nano materials?
7. How do mechanical properties vary for nano materials?
8. How do magnetic properties vary for nano materials?
9. How do physical properties vary for nano materials?
10. Mention the steps involved in sol-gel process.
11. What is carbon nanotube?
12. What is graphene?
13. Why is graphene is used in FET?

Essay Type Questions

1. Explain the basic factors involved for the exhibitions of different properties of nanomaterials.
2. Explain the variation of physical properties of nanomaterials.
3. Explain optical and thermal properties of nanomaterials.
4. Explain mechanical and magnetic properties of nanomaterials.
5. Describe the synthesis of nanomaterials by ball mill.
6. Describe the synthesis of nanomaterials by chemical vapour deposition.
7. Describe the synthesis of nanomaterials by sol-gel.
8. Describe the synthesis of nanomaterials by plasma arcing.
9. Describe the synthesis of nanomaterials by thermal evaporation.

10. Mention the properties of CNT.
11. Mention the properties of graphene.
12. Describe graphene-based FET.
13. Mention applications of nanomaterials.

MODEL QUESTION PAPER-I

B.Tech. I year (R13) Regular Examinations
June 2014
Engineering Physics
(Common to All Branches)

Time: 3 hrs.   *Max. Marks: 70*

Section-A

Answer all questions. Each question carries equal marks. $10 \times 2 = 20$

1. What is diffraction grating?
2. What is the significance of acceptance angle of optical fibre for guiding the light signal?
3. What is point defect?
4. What is the basic principle involved in ultrasonic nondestructive testing of materials?
5. What is de Broglie hypothesis of matter waves?
6. What is drift velocity?
7. What is the contribution of diffusion of charge carriers towards conductivity of semiconductor?
8. What is hysteresis?
9. What is superconductivity?
10. What is carbon nanotube?

Section-B

Answer all questions. $5 \times 10 = 50$

11. (a) Describe the formation of Newton's rings with necessary theory.
    (b) In Newton's ring experiment the diameter of 10th ring changes from 1.4 to 1.27 cm, when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

(or)

12. (a) What is the role of population inversion in laser emission?
    (b) Describe the construction and working of He-Ne laser.

13. (a) State and explain Bragg's law.
    (b) Describe various point defects.

(or)

14. (a) What is piezoelectric effect?
    (b) Describe the production of ultrasonic waves by piezoelectric method.

15. (a) What is Heisenberg's uncertainity principle.
    (b) Derive schrodinger's time independent wave equation.

(or)

16. (a) Explain the sources of electrical resistance.
    (b) Mention the merits and demerits of classical free electron theory.

17. (a) Describe the hall effect in a semiconductor.
    (b) What is *p-n* junction?

(or)

18. (a) What is Bohr magneton?
    (b) Explain soft and hard magnetic materials.

19. (a) Explain flux quantisation in super conductor.
    (b) What is Meissner effect for super conductor?

(or)

20. (a) Describe the synthesis of nanomaterials by ball mill method.
    (b) Mention the properties of graphene.

MODEL QUESTION PAPER-II

B.Tech. I year (R13) Regular Examinations
June 2014
Engineering Physics
(Common to All Branches)

Time: 3 hrs.   *Max. Marks: 70*

Section-A

Answer all questions. Each question carries equal marks. $10 \times 2 = 20$

1. Why the Newton's rings consists of concentric rings?
2. What is optical resonator?
3. How do diamond crystal structure is formed?
4. What is piezoelectric effect?
5. What is Heisenberg's uncertainity principle?
6. What are the advantages of Quantum free electron theory?
7. What is hall effect?
8. Derive the relation between B, H and M.
9. What is meissner effect in super conductor?
10. What is graphene?

Section-B

Answer all questions. $5 \times 10 = 50$

11. (a) What is the role of optical resonator in enhancing the laser emission?
    (b) Derive the relation between the various Einstein's coefficients of absorption and emissions of radiation.

(or)

12. (a) Derive the expressions for numerical aperture and acceptance angle of an optical fibre.
    (b) Calculate numerical aperture and acceptance angle for an optical fibre having core of refractive index 1-55 and cladding of refractive index 1-50 respectively.

13. (a) Describe NaCl structure.
    (b) Derive the expression for interplanar spacing in cubic crystal.

(or)

14. (a) Mention the properties of ultrasonic waves.
    (b) Describe the application of ultrasonic in nondestructive testing of material.

15. (a) What is the significance of wave function?
    (b) Describe the behaviour of particle in a one-dimensional infinite potential well in terms of eigenvalues and functions.

(or)

16. (a) Derive an equation for electrical conductivity using quantum free electron theory.
    (b) Explain the origin of bands in solids.

17. (a) Derive Einstein's equation of change carriers in a semiconductor.
    (b) What is the working principle of LED?

(or)

18. (a) Describe the origin of magnetic moments in an atom.
    (b) What is hysteresis?

19. (a) What are high T_C super conductors?
    (b) Describe Type I and Type II super conductors.

(or)

20. (a) What is quantum confinement effect of nanomaterials?
    (b) Mention the properties of carbon nanotubes.