Engineering Physics - II

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Contents

Preface

Today's technological developments are the results of the joint efforts of physicists and engineers. A study of Physics is, therefore, indispensible for students of engineering in order to excel in their respective fields. This book has been written with a view to provide a comprehensive text on Engineering Physics for the first-year BE/B.Tech degree students studying in engineering colleges affiliated to the Anna University stream.

About the Book

This book covers topics like conducting materials, semiconducting materials, magnetic materials, superconducting materials, dielectric materials and modern engineering materials such as metallic glasses, shape memory alloys, nanomaterials and carbon nanotubes. Conducting material concepts have been explained in detail in **Chapter 1.** Various types of semiconducting materials along with their applications are covered in **Chapter 2**. **Chapter 3** describes the basics of various types of magnetic and superconducting materials and their applications. **Chapter 4** discusses dielectric materials and their applications. **Chapter 5** elucidates modern engineering materials such as metallic glasses, shape memory alloys, nanomaterials and carbon nanotubes along with their applications.

The text presents the fundamental principles of physics and their applications in a simple language. At the end of each unit, numerous solved problems including problems from latest question papers, review questions and exercise problems have been given. Questions that appeared in various examination papers have been marked to notify their respective year of appearance.

Salient Features

This book is designed for a first-year undergraduate course on Engineering Physics offered to the second semester students of engineering. It provides coverage of all the topics in appropriate depth, well supported with examples and exercises. A lucid writing style has been followed, which makes it easier for students to understand the concepts. Equal attention is devoted to elucidating engineering applications of the various topics in every chapter. To summarize, the salient features of this book are

- Complete syllabus coverage
- Simple and lucid writing style
- Rich pedagogy
  - ☆ Illustrations
  - ☆ Solved Examples
  - ☆ Exercise Questions
  - ☆ Practice Questions
  - ☆ Short-Answer Questions

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We would like to extend our heartfelt thanks to the Management, Principal and friends at Kongu Engineering College, Tamil Nadu, for providing a wonderful environment and the encouragement given to us in bringing out this book. We are also thankful to our publisher, Tata McGraw Hill, for the cooperation and support.

We hope that this book will be well received by the students and teachers alike. We invite criticism and suggestions from the readers for making further improvements to the text.

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Remember to write to us! We at Tata McGraw-Hill would be glad to receive your comments and suggestions, all of which can be sent to tmh.sciencemathsfeedback@gmail.com. Also, let us know if you spot any piracy of the book.

UNIT I

CONDUCTING MATERIALS

Freedom is not worth having if it does not include the freedom to make mistakes. **—Mahatma Gandhi**

1.1 INTRODUCTION

Solids are characterized by a strong bonding between atoms and they are classified as metals and non-metals. The properties of materials depend on the nature of bonding and the energy distribution of bonding electrons. In general, the bonding electrons in a solid are subjected to the potential of their parent nuclei, other nuclei and other electrons. Hence, the study of solids is complex. However, it may be simplified by assuming simple models for the potential of solids. For example, in a metal, the potential is assumed to be zero.

It is so because valence electrons in a metal are free. However, the free electrons cannot leave the metal. In this respect, conduction electrons in a metal resemble the particles in a box. Thus, the conduction electrons in a metal are modeled by 'free electron gas'. This theory is called the free electron theory of metals.

1.2 CONDUCTION IN METALS

Due to the presence of a large number of free electrons, the metals are good electrical and thermal conductors. The study of conduction in metals involves the concepts such as mobility, conductivity, current density, drift velocity, mean free path, mean free time and relaxation time of free electrons.

1.2.1 Drift Velocity of Free Electrons

It is defined as the steady-state average velocity of free electrons when subjected to an external electric field.

1.2.2 Current Density

It is the current flowing across unit area of a metal in the presence of an external electric field. It is given by

$$j = nev_d$$

where n is the carrier concentration, e is the charge of electron and v_d is the drift velocity of free electrons.

Its unit of measurement is A/m^2.

1.2.3 Electrical Conductivity

It is defined as the current density in a metal when subjected to unit electric field. That is,

$$\sigma = \frac{j}{E}$$

Its unit of measurement is $\text{ohm}^{-1} \cdot \text{m}^{-1}$.

1.2.4 Mobility

The mobility of a free electron in a metal is its drift velocity when subjected to unit electric field. That is,

$$\mu = \frac{v_d}{E}$$

Its unit of measurement is $m^2/V.s$.

1.2.5 Mean Free Path

It is the average distance travelled by free electrons between two successive collisions with lattice ions in a metal subjected to electric field.

1.2.6 Mean Free Time

It is the average time taken by the free electrons between two successive collisions with the lattice ions in a metal subjected to electric field.

1.2.7 Relaxation Time

It is the time taken by free electrons to have their drift velocity equal to zero, when the electric field is switched off.

1.3 CLASSICAL FREE ELECTRON THEORY OF METALS

Many of the properties of metals can be attributed to the free nature of the valence electrons. In the presence of an external electric field, these electrons drift resulting in the electrical conductivity. This classical approach, also called the Drude–Lorentz theory, was developed in 1900 and explains the properties of metals such as their electrical and thermal conductivities. The classical free electron theory is based on the following postulates.

1.3.1 Postulates of Classical Free Electron Theory

- Metals consist of atoms, which in turn, consist of a nucleus and electrons.
- Valence electrons are free and the metal is visualized as an array of ions permeated by a gas of free electrons.
- In the absence of an electrical field, electrons move in random directions colliding with ions or free electrons.
- In an electrical field, free electrons are accelerated and move in the opposite direction compared to that of the electric field.
- Electron velocities obey Maxwell–Boltzmann distribution function.
- Free electrons move without interaction between them and they obey the laws of classical kinetic theory of gases.
- Free electrons move in a uniform potential field since ions are fixed in the lattice.

Based on these postulates, the expressions for electrical and thermal conductivities can be derived.

1.4 ELECTRICAL CONDUCTIVITY

Electrical conductivity refers to the ability of the metal to conduct electrical energy. Now, we will derive an expression for electrical conductivity of a metal. The metal is considered to be an array of positive ions permeated by the gas of free electrons (Fig. 1.1).

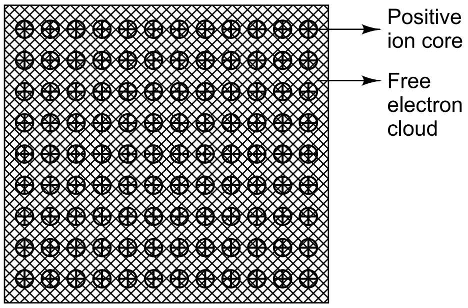


Fig. 1.1 *Arrangement of Ions in a Metal*

Let the metal be subjected to an electric field E_x. Then, the free electrons are accelerated and they move with an average velocity $< v_x >$. Then, Newton's second law gives the equation of motion for the free electrons as

$$m\frac{d < v_x >}{dt} + m\frac{< v_x >}{\tau} = eE_x \tag{1.1}$$

where m is the mass of electron, e is the charge of electron and τ is the relaxation time of free electrons in the metal,

i.e. $$\tau\frac{d < v_x >}{dt} = \frac{eE_x\tau}{m} - < v_x >$$

The above equation may be rewritten as

$$\frac{d < v_x >}{\frac{eE_x\tau}{m} - < v_x >} = \frac{dt}{\tau}$$

Integration of the above equation gives

$$\ln\left[\frac{eE_x\tau}{m} - < v_x >\right] = \frac{-t}{\tau} + \text{const.}$$

or $$\frac{eE_x\tau}{m} - < v_x > = A\exp(-t/\tau) \tag{1.2}$$

where, A is a constant.

The value of the constant A may be determined through the initial condition. At the initial moment of time, the average velocity of free electrons is zero. Thus, we have the initial condition as

At $t = 0$, $< v_x > = 0$

Application of the initial condition to Eq. (1.2) gives

$$A = \frac{eE_x\tau}{m}$$

Therefore, Eq. (1.2) is rewritten as

$$< v_x > = \frac{eE_x\tau}{m}[1 - \exp(-t/\tau)] \tag{1.3}$$

Equation (1.3) gives the steady-state velocity of free electrons called the drift velocity as

$$< v_x > \equiv v_d = \frac{eE_x\tau}{m} \tag{1.4}$$

It is the steady-state value of the average velocity of an electron in a metal subjected to an electric field.

On the other hand, current density is given as

$$j_x = ne < v_x > \tag{1.5}$$

where n is the electron concentration in the metal.

Substitution of Eq. (1.4) into Eq. (1.5) gives

$$j_x = \frac{ne^2 E_x \tau}{m} \tag{1.6}$$

Electrical conductivity is given by

$$\sigma = \frac{j_x}{E_x} \tag{1.7}$$

Substitution of Eq. (1.6) into Eq. (1.7) then gives the electrical conductivity as

$$\sigma = \frac{ne^2\tau}{m} \tag{1.8}$$

As seen, electrical conductivity of a metal is directly proportional to the concentration of free electrons and the relaxation time. Now, the kinetic theory of gases can be applied to the free electron gas and the expression for electrical conductivity, Eq. (1.8), may be rewritten to reveal the temperature dependence of electrical conductivity.

1.4.1 Application of Kinetic Theory of Gases to Free Electrons

Let $\bar{c}$ be the root mean square velocity (also called the mean thermal velocity) of free electrons. As the free electrons are assumed to obey the Maxwell–Boltzmann distribution, pressure exerted by electron gas in unit volume of metal is

$$p = \frac{1}{3}\rho \bar{c}^2 = \frac{1}{3} mn\bar{c}^2 \tag{1.9}$$

where ρ is the density of free electron gas.

Then, pressure exerted by electron gas in molar volume of metal is

$$p = \frac{1}{3} m \frac{N_A}{V_m} \bar{c}^2 \tag{1.10}$$

where N_A is the Avogadro number and V_m is the molar volume. Eq. (1.10) may be rewritten as

$$pV_m = \frac{1}{3} m\, N_A\, \bar{c}^2 \tag{1.11}$$

Assuming that electron gas is an ideal gas, we have

$$RT \equiv pV_m = \frac{1}{3} m\, N_A\, \bar{c}^2$$

where R is the universal gas constant and T is absolute temperature,

i.e. $$mc^2 = \frac{3RT}{N_A}$$

On the other hand, $k \equiv R/N_A$ is the Boltzmann constant. Hence, we have

$$mc^2 = 3kT$$

$$\frac{1}{2} mc^2 = \frac{3}{2} kT$$

Hence, we get the expression for $\bar{c}$ as

$$\bar{c} = \left(\frac{3kT}{m} \right)^{1/2} \tag{1.12}$$

On the other hand, relaxation time is given as

$$\tau = \frac{\lambda}{\bar{c}} \tag{1.13}$$

where λ is the mean free path of free electrons.

Substituting Eq. (1.12) into Eq. (1.13), we get

$$\tau = \lambda \left(\frac{m}{3kT} \right)^{1/2} \tag{1.14}$$

Now, Eq. (1.14) may be substituted into Eq. (1.8) to get the electrical conductivity as

$$\sigma = \frac{ne^2\lambda}{\sqrt{3mkT}} \tag{1.15}$$

As seen, electrical conductivity is directly proportional to mean free path and inversely proportional to temperature.

1.5 FACTORS AFFECTING THE RESISTIVITY OF METALS

Resistivity of a metal is the reciprocal of its electrical conductivity. It depends on various factors such as

- Temperature
- Impurity concentration
- Strain
- Magnetic field

1.5.1 Dependence on Temperature

The temperature dependence of resistivity in metals is mainly due to the electron-phonon scattering. The relaxation time τ is a measure of the electron-phonon scattering and it is temperature dependent.

The factor $\frac{1}{\tau}$ has two contributions in metals. That is,

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{phonon}}} + \frac{1}{\tau_{\text{imp}}} \tag{1.16}$$

where τ_{phonon} is the relaxation time of due to the scattering by phonons and τ_{imp} is the relaxation time due to the scattering by imperfections in the solid.

When Eq. (1.16) is taken into account, Eq. (1.8) gives the resistivity as

$$\rho = \frac{m}{ne^2}\left(\frac{1}{\tau_{\text{phonon}}} + \frac{1}{\tau_{\text{imp}}}\right) \tag{1.17}$$

Moreover, τ_{phonon} depends on temperature and τ_{imp} is independent of temperature. Hence, Eq. (1.17) can be rewritten as

$$\rho = \rho_o + \rho(T) \tag{1.18}$$

where

$$\rho_o = \frac{m}{ne^2\tau_{\text{imp}}} \tag{1.19}$$

$$\rho(T) = \frac{m}{ne^2\tau_{\text{phonon}}} \tag{1.20}$$

Equation (1.18) is called the **Matthiessen's rule of resistivity**. The temperature-independent term ρ_o is called the **residual resistivity**.

The temperature-dependent term $\rho(T)$ is called the **lattice resistivity**. At normal temperatures, the variation of resistivity with temperature is linear. At very low temperatures, it is non-linear.

1.5.2 Dependence on Impurity Concentration

The relaxation time τ_{imp} decreases with increasing impurity concentration, resulting in higher resistivity. It can be expressed as

$$\tau_{\text{imp}} = \frac{\lambda_{\text{imp}}}{v_d} \tag{1.21}$$

where λ_{imp} is the mean free path of electrons due to impurities and v_d is the drift velocity of electrons.

Therefore, Eq. (1.19) can be rewritten as

$$\rho_o = \frac{mv_d}{ne^2\lambda_{\text{imp}}} \tag{1.22}$$

On the other hand, the mean free path is given as

$$\lambda_{\text{imp}} = \frac{a}{x_i} \tag{1.23}$$

where x_i is the atomic fraction of the impurity atom and a is the lattice constant of the metal.

From Eqs (1.22) and (1.23), it follows that

$$\rho_o = \frac{mv_d x_i}{ne^2 a} \tag{1.24}$$

That is, the resistivity of a metal increases linearly with increasing atomic fraction of the impurity atom.

1.5.3 Dependence on Strain

When a metal is stressed, its lattice is distorted. It causes imperfections in the lattice, increasing the resistivity proportional to the strain. It can be expressed as

$$\frac{\Delta R}{R} \alpha \text{ strain} \tag{1.25}$$

where R is the resistance of the unstressed metal and ΔR is the increase in resistance of the metal under stress.

1.5.4 Dependence on Magnetic Field

The magneto-resistance effect is responsible for the dependence of resistivity of a metal on the applied magnetic field.

At smaller magnetic fields, the variation of resistance is given by

$$\frac{\Delta R}{R} \alpha H^2 \tag{1.26}$$

where H is the magnetic field strength.

At larger magnetic fields, the variation in resistivity is given by

$$\frac{\Delta R}{R} \alpha H \tag{1.27}$$

1.6 THERMAL CONDUCTIVITY

When there is a temperature gradient in a material, conduction of heat takes place. In metals, thermal conduction is due to both free electrons and phonons (lattice vibrations). However, free electron contribution to thermal conduction is much more than the phonon contribution. Consider a metallic rod in which a temperature gradient dT is maintained (Fig. 1.2). Let the reference planes A and B of the metallic rod be maintained at temperatures T and $T - dT$, respectively. Let the planes A and B be away by a distance of λ.

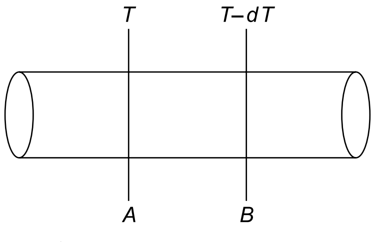


Fig. 1.2 *Metallic Rod*

Due to the temperature gradient, energetic free electrons move from the plane A to the plane B.

At the plane *A*, kinetic energy of free electron is

$$E_1 = \frac{3}{2}kT \tag{1.28}$$

At the plane *B*, kinetic energy of free electron is

$$E_2 = \frac{3}{2}k(T - dT) \tag{1.29}$$

In their motion from the plane *A* to the plane *B*, the free electrons carry an excess energy given by

$$\Delta E = E_1 - E_2 = \frac{3}{2}kdT \tag{1.30}$$

On the other hand, the number of such electrons crossing unit area in unit time is

$$N = \frac{1}{6}n\bar{c} \tag{1.31}$$

Therefore, excess energy transported from the plane *A* to the plane *B* across unit area per unit time is

$$Q_{AB} = \frac{1}{6}n\bar{c}\,\frac{3}{2}kdT \tag{1.32}$$

$$\therefore \qquad Q_{AB} = \frac{n\bar{c}\,kdT}{4} \tag{1.33}$$

Moreover, less energetic free electrons driven by the concentration gradient, move from the plane *B* to the plane *A*.

Deficiency of energy carried by free electrons from the plane *B* to the plane *A* across unit area per unit time is

$$Q_{BA} = -\frac{n\bar{c}\,kdT}{4} \tag{1.34}$$

Hence, net energy transported from *A* to *B* per unit area per unit time is

$$Q = Q_{AB} - Q_{BA}$$

From Eqs (1.33) and (1.34), we get

$$Q = \frac{n\bar{c}\,kdT}{4} - \left(-\frac{n\bar{c}\,kdT}{4}\right)$$

$$\text{i.e.} \qquad Q = \frac{n\bar{c}\,kdT}{2} \tag{1.35}$$

Multiplying Eq. (1.35) by λ/λ, we get

$$Q = \frac{n\bar{c}\,k\lambda}{2}\,\frac{dT}{\lambda} \tag{1.36}$$

Comparing Eq. (1.36) with the definition of coefficient of thermal conductivity, we get the expression for thermal conductivity as

$$K = \frac{n\bar{c}\,k\lambda}{2} \tag{1.37}$$

Substitution of Eq. (1.12) into Eq. (1.37) gives

$$K = \frac{nk\lambda}{2}\left(\frac{3kT}{m}\right)^{1/2} \tag{1.38}$$

As seen, thermal conductivity of a metal is directly proportional to the concentration of free electrons, mean free path and temperature.

1.7 WIEDEMANN–FRANZ LAW

The Wiedemann–Franz law establishes correlation between the electrical and thermal conductivities of metals. It states that the ratio of thermal conductivity of a metal to its electrical conductivity is directly proportional to temperature. Moreover, the proportionality constant is the same for all metals. That is,

$$\frac{K}{\sigma}\,\alpha\,T \tag{1.39}$$

i.e.

$$\frac{K}{\sigma} = LT \tag{1.40}$$

where L is the proportionality constant called the Lorentz number and it is independent of the nature of the metal.

This law can be theoretically verified using the previously derived expressions for electrical and thermal conductivities.

From Eq. (1.15) and (1.38), we get

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k}{e}\right)^2 T \tag{1.41}$$

Hence, the Wiedemann–Franz law is verified.

1.7.1 Lorentz Number

The Lorentz number is the ratio of thermal conductivity of a metal to the product of its electrical conductivity and its absolute temperature.

Moreover, comparison of Eqs (1.40) and (1.41) gives the Lorentz number as

$$L = \frac{3}{2}\left(\frac{k}{e}\right)^2 \tag{1.42}$$

Substitution of the values for K and σ into Eq. (1.42) gives $L = 1.12 \times 10^{-8}$ W.ohm.K^{-2}. On the other hand, for copper at 20°C, K = 386 Wm^{-1} K^{-1} and $\sigma = 5.81 \times 10^{7}$ ohm^{-1}. m^{-1}. Then, Eq. (1.40) gives a value of $L = 2.266 \times 10^{-8}$ W.ohm.K^{-2}. That is, the value of L predicted by classical free electron theory disagrees with its experimental value. To overcome this discrepancy, the quantum free electron theory assumes that only few free electrons in the vicinity of Fermi level conduct heat.

1.8 MERITS OF CLASSICAL FREE ELECTRON THEORY

- It explains the Ohm's law.
- It explains the electrical and thermal conductivities of metals.
- It derives Wiedemann–Franz law.
- It explains the optical properties of metals.

1.9 DRAWBACKS OF CLASSICAL FREE ELECTRON THEORY

- It fails to explain the photoelectric effect, Compton effect, black body radiation, etc.
- It gives a value of 4.5R for the specific heat of metals. But, the experimental value is only 3R.
- It gives a value of $\left(\frac{3}{2}\right)R$ for the electronic specific heat. But, the experimental value is only 0.01R.
- It does not explain the electrical conductivity of semiconductors and insulators.
- It gives $\frac{k}{\sigma T}$ = const. for all temperatures. But, it is so only at low temperatures.
- It does not explain the ferromagnetism.

- The long mean free paths (more than one cm) of the free electrons at low temperatures cannot be explained on the basis of the classical theory.
- This theory predicted that resistivity varies as $\sqrt{T}$, whereas actually it is found to vary linearly with temperature.
- Experimental results shows that paramagnetism of metals is independent of temperature which deviates from classical result that paramagnetic susceptibility is inversely proportional to the temperature.
- The resistivity of metals increases with increasing impurity concentration. On the other hand, in semiconductors even a very small amount of impurity causes a drastic decrease in their resistivity. The above feature cannot be explained by the classical theory.

1.10 QUANTUM FREE ELECTRON THEORY OF METALS

The drawbacks of the classical free electron theory were removed by Sommerfeld in 1928. He applied the Schroedinger's wave equation and de Broglie's concept of matter waves to obtain the expression for electron energies. Sommerfeld treated the problem quantum mechanically using the Fermi–Dirac statistics rather than the classical Maxwell–Boltzmann statistics. The important assumptions made by Sommerfeld are given below.

- The free electrons move in a constant potential inside the metal and are confined within defined boundaries.
- The eigen values of the conduction electron are quantized.
- The electrons are considered to posses wave nature.
- In the various allowed energy levels, distribution of electrons takes place according to Pauli's exclusion principle.
- Mutual attraction between electrons and lattice ions and the repulsion between individual electrons may be ignored.

1.11 FERMI–DIRAC DISTRIBUTION FUNCTION

The distribution statistics obeyed by quantum particles like electrons is called the Fermi–Dirac statistics. The Fermi–Dirac distribution function is given by

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \tag{1.43}$$

where E_F is the Fermi energy, k is the Boltzmann constant and T is the temperature. The function $f(E)$ gives the probability that the energy level E is occupied by an electron at a temperature T. The plot of $f(E)$ at different temperatures is shown in Fig. 1.3.

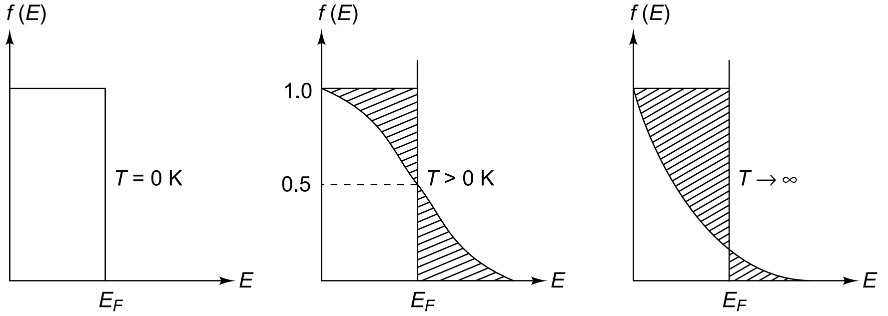


Fig. 1.3 *Fermi–Dirac Distribution Function at Zero and Non-zero Temperatures*

As seen from Eq. (1 .43) at $T = 0$ K, $f(E) = 1$ for values of E below E_F and $f(E) = 0$ for values of E above E_F. That is, at $T = 0$ K, all the energy levels below Fermi energy are occupied by electrons and all the levels above Fermi energy are unoccupied. Hence, **Fermi energy is defined as the maximum energy of filled states at $T = 0$ K**.

For $T > 0$ K, some states below E_F are unoccupied and some states above E_F are occupied. This fact is illustrated by Eq. (1.43) and Fig. 1.3. For $E = E_F$, Eq. (1.43) gives $f(E) = \frac{1}{2}$ at $T > 0$ K. Thus, **Fermi energy may also be defined as the energy of the state at which the probability of electron occupancy is 1/2 at $T > 0$ K.**

From Fig. 1.3, it may be noted that only those states close to E_F are affected by temperature while the states far away from E_F are unaffected. The energy range over which the changes in Fermi–Dirac distribution function take place is about kT.

1.11.1 Temperature Dependence of Fermi Energy

Fermi energy is temperature dependent. To find the Fermi energy at a temperature T, Fermi–Dirac distribution function is used.

Fermi Energy at $T = 0$ K

At $T = 0$ K, Eq. (1.44) gives

$$f(E) = \begin{cases} 1 & , \text{for } E < E_F \\ 0 & , \text{for } E > E_F \end{cases} \tag{1.44}$$

Carrier concentration (i.e. electron concentration) is then given as

$$n = \int_0^{E_F} f(E) N(E)\, dE \tag{1.45}$$

where $N(E) \equiv \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} dE$ is the density of states, m is the mass of electron and h is Planck's constant.

As seen from Eq. (1.45), the knowledge of Fermi energy E_F is of use in determining the carrier concentration.

Equations (1.44) and (1.45) then give

$$n = \int_0^{E_F} \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} dE$$

$$= \frac{4\pi}{h^3} \frac{(2m)^{3/2}(E_F)^{3/2}}{3/2}$$

$$n = \frac{8\pi}{3h^3}(2mE_F)^{3/2} \tag{1.46}$$

Hence, Fermi energy at $T = 0$ K is

$$E_{F0} = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3} \tag{1.47}$$

Fermi Energy at $T > 0$ K

Carrier concentration is given as

$$n = \int_0^{\infty} f(E) N(E)\, dE$$

i.e.

$$n = \frac{4\pi}{h^3}(2m)^{3/2} \int_0^{\infty} \frac{E^{1/2}}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE$$

Evaluating the above integral using the approximation $E - E_F > kT$ we get the expression for Fermi energy at a temperature T as

$$E_F = E_{F0}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F0}}\right)^2\right] \tag{1.48}$$

The factor (kT/E_{F0}) in Eq. (1.48) is very small. For a metal of $E_{F0} = 5$ eV, $(kT/E_{F0})^2 \approx 2.6 \times 10^{-5}$ at room temperature. That is, the temperature dependence of E_F is insignificant. Therefore, E_F can be treated as almost a constant.

1.11.2 Average Energy of Free Electrons at $T = 0$ K

The average energy of a free electron is obtained by finding the total energy of all free electrons per unit volume of the metal and dividing it by the free electron concentration.

Let E be the energy of an electron at $T = 0$ K and E_0 be the total energy of electrons at $T = 0$ K. Then, we have

$$E_0 = \int_0^{E_F} f(E)N(E)\,dE \times E \tag{1.49}$$

Substitution of Eq. (1.44) and the expression for the density of states into Eq. (1.49) gives

$$E_0 = \frac{4\pi}{h^3}(2m)^{3/2}\int_0^{E_F} E^{3/2}\,dE$$

i.e.
$$E_0 = \frac{8\pi}{5h^3}(2m)^{3/2}E_F^{5/2}$$

Average energy of electrons at 0 K is given as

$$\bar{E}_0 = \frac{E_0}{n} \tag{1.50}$$

where n is the concentration of electrons.

Substituting Eq. (1.46) into Eq. (1.50), we get the average energy of electron at $T = 0$ K as

$$\bar{E}_0 = \frac{3}{5}E_F \tag{1.51}$$

Thus, the knowledge of Fermi energy is found to be used in determining the average energy of electrons at $T = 0$ K.

1.11.3 Average Energy of a Free Electron at $T \neq 0$ K

The average energy of a free electron at $T \neq 0$ K is given as

$$\bar{E} = \frac{1}{n}\int_0^\infty E\, f(E) N(E)\, dE \tag{1.52}$$

$$\bar{E} = \frac{4\pi}{nh^3}(2m)^{3/2}\int_0^\infty \frac{E^{3/2}dE}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} \tag{1.53}$$

On simplifying and expressing in terms of $\bar{E}_0$, we get

$$\bar{E} = \bar{E}_0\left(1+\frac{5\pi^2}{12}\left(\frac{kT}{E_{F0}}\right)^2\right) \tag{1.54}$$

1.12 DENSITY OF STATES

The density of states is defined as the number of energy states in an energy range of E and $E + dE$ states per unit volume of a metal.

Let n_1, n_2 and n_3 be the quantum numbers specifying an energy state E. As known, n_1, n_2 and n_3 can have only positive values. To find the density of states, construct a sphere of radius n, where $n^2 = \left(n_1^2 + n_2^2 + n_3^2\right)$. Each point (n_1, n_2, n_3) in the sphere represents a quantum state. As each integer n represents an energy state, the unit volume of the sphere contains only one state.

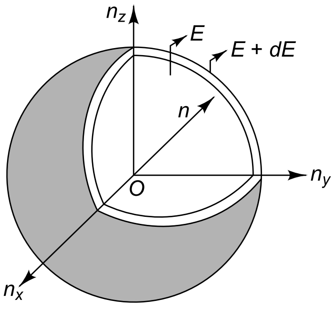


Fig. 1.4 *Sphere of Radius n*

Therefore, the number of energy states in the sphere is equal to the volume of the sphere. That is,

$$N'(E) = \frac{4}{3}\pi n^3$$

As the quantum numbers n_1, n_2 and n_3 can have positive values only in one octant of the sphere, we have

$$N'(E) = \frac{1}{8}\times\frac{4}{3}\pi n^3$$

i.e. $$N'(E) = \frac{\pi}{6} n^3$$

To find the number of energy states in a small interval dE, construct another sphere of radius $n + dn$.

Number of energy states having energies in the interval of E and $E + dE$ is

$N'(E)\, dE$ = Volume available between the two spheres

$$N'(E)\,dE = \frac{\pi}{6}(n+dn)^3 - \frac{\pi}{6}n^3$$

Neglecting the high-order terms of dn (since dn is small), we get

$$N'(E)\,dE = \frac{\pi}{6}n^3 + \left(\frac{\pi}{6}\times 3n^2\, dn\right) - \frac{\pi}{6}n^3$$

$$N'(E)\,dE = \frac{\pi}{2}n^2 dn \tag{1.55}$$

On the other hand, considering the electrons in a metal as particles in a box, we have

$$E = \frac{n^2h^2}{8ma^2}$$

$$n^2 = \frac{8ma^2E}{h^2} \tag{1.56}$$

$$n = \left(\frac{8ma^2E}{h^2}\right)^{1/2} \tag{1.57}$$

Differentiating Eq. (1.56), we get

$$2n\,dn = \frac{8ma^2}{h^2}dE$$

i.e. $$dn = \frac{1}{2n} \cdot \frac{8ma^2}{h^2} dE \tag{1.58}$$

Substitution of Eq. (1.58) into Eq. (1.55) gives

$$N'(E)\,dE = \frac{\pi}{2} \times n^2 \times \frac{1}{2n} \cdot \frac{8ma^2}{h^2} dE$$

$$N'(E)\,dE = \frac{\pi}{4} \times n \times \frac{8ma^2}{h^2} dE \tag{1.59}$$

$$N'(E)\,dE = \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2} E^{1/2} dE$$

Moreover, Pauli's exclusion principle allows two electrons with opposite spins to occupy the same energy state. Hence, the number of states available for electron occupancy is

$$N(E)\,dE = 2 \times \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2} E^{1/2} dE$$

$$N(E)\,dE = \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} a^3 dE$$

The density of states is the number of states per unit volume.

Hence, the density of states in the energy range E and $E + dE$ is

$$N(E)\,dE = \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} dE \tag{1.60}$$

The density of states increases with energy as $E^{1/2}$ from the bottom of the conduction band. The variation of $N(E)$ with E is shown in Fig. 1.5. It is seen that the density of states is almost constant near the Fermi energy.

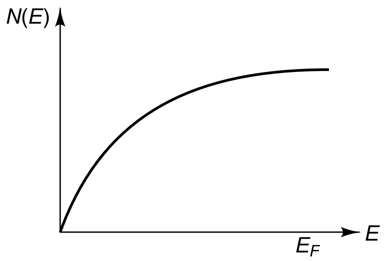


Fig. 1.5 *Variation of the Density of States with Energy*

Density of States in terms of Momentum

The density of states in the momentum range p and $p + dp$ is obtained by substituting $E = \frac{p^2}{2m}$ into Eq. (1.60). That is, we have

$$N(p)\,dp = \frac{8\pi}{h^3}\,p^2 dp \tag{1.61}$$

1.13 CARRIER CONCENTRATION IN METALS

The number of free electrons (i.e. carrier concentration) in a metal can be obtained by integrating the product of the density of states and the Fermi–Dirac distribution function in the limits of 0 to ∞. That is,

$$n = \int_0^\infty f(E)N(E)dE \tag{1.62}$$

Substituting Eqs (1.44) and (1.60) into Eq. (1 .62), we get the carrier concentration in a metal at $T = 0$ K as

$$n = \frac{8\pi}{3h^3}(2mE_{F0})^{3/2} \tag{1.63}$$

where E_{F0} is the Fermi energy at $T = 0$ K.

Similarly, we get the carrier concentration in a metal at $T > 0$ K as

$$n = \frac{4\pi}{h^3}(2m)^{3/2}\int_0^\infty \frac{E^{1/2}dE}{1 + \exp\left(\frac{E - E_F}{KT}\right)} \tag{1.64}$$

1.14 ENERGY DISTRIBUTION OF ELECTRONS

The density of states respresents the number of states that can be occupied by charge carriers. However, all the available energy states are not filled in an energy band. The Fermi–Dirac distribution function $f(E)$ determines the probability of having a given energy level E filled with the charge carriers. Therefore, the carrier concentration in a particular energy range is determined by the density of states in the given range and by the probability that the carriers occupy these energy states. Hence, the carrier concentration in an energy range dE is obtained by multiplying the density of states in that range with the probability of their occupation. Thus, the carrier concentration in the energy range dE is given by

$$n(E)\,dE = f(E)N(E)\,dE \tag{1.65}$$

$n(E)$ is known as the carrier distribution function.

The electron distribution function in a conductor at $T = 0$ K is shown in Fig. 1.6.

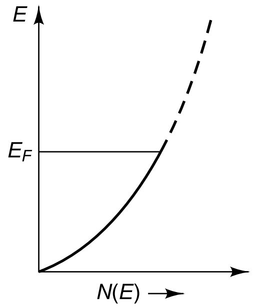


Fig. 1.6 *Electron Distribution Function in a Conductor at T = 0 K*

Similarly, electron distribution function in a conductor at $T > 0$ K is shown in Fig. 1.7.

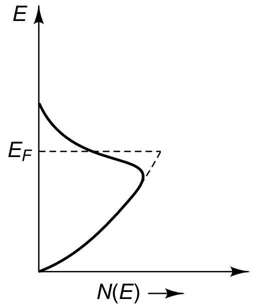


Fig. 1.7 *Electron Distribution Function in a Conductor at T > 0 K*

SOLVED EXAMPLES

Example 1.1 *A copper wire of 1 mm diameter carries a current of 5 mA. Calculate the average drift velocity of free electrons. Copper is monovalent and it crystallizes into FCC structure with a lattice constant of 3.61 Å.*

Given Data

$d = 1 \text{ mm} = 1 \times 10^{-3}$ m

$I = 5 \text{ mA} = 5 \times 10^{-3}$ A

$a = 3.61 \text{ Å} = 3.61 \times 10^{-10}$ m

$< v > = ?$

As copper has FCC structure, the number of atoms per unit cell is Unit cell volume is

$$V \equiv a^3 = (3.61 \times 10^{-10})^3 \text{ m}^3$$

Therefore, number of atoms per unit volume (i.e. concentration of atoms) is

$$n' = \frac{4}{(3.61 \times 10^{-10})^3}$$

i.e. $n' = 8.5 \times 10^{28} \text{ m}^{-3}$

As copper is monovalent, we have concentration of free electrons = concentration of atoms

Thus, we have the concentration of free electrons as

$$n = 8.5 \times 10^{28} \text{ /m}^3$$

On the other hand, current density is

$$j \equiv \frac{I}{A} = \frac{5 \times 10^{-3}}{\pi (0.5 \times 10^{-3})^2}$$

i.e. $j \equiv 6366 \text{ A/m}^2$

As known, the average drift velocity and the current density in a metal are related as

$$< v > = \frac{j}{ne} \tag{1}$$

where e is the charge of electron.

Substituting the given values for j, n and e into Eq. (1), we get

$$< v > = \frac{6366}{8.5 \times 10^{-28} \times 1.6 \times 10^{-19}}$$

i.e. $\mathbf{< v > = 4.68 \times 10^{-7} \text{ m/s}}$

Example 1.2 *Taking the required data for a typical metal, show that the relaxation time of free electrons is of the order of* 10^{-14} *s.*

Given Data

For a typical metal (say, copper), at room temperature, we have

$\sigma = 6 \times 10^7$ ohm^{-1} m^{-1}

$n = 8.5 \times 10^{28}$ m^{-3}

Classical free electron theory gives the relaxation time as

$$\tau = \frac{m\sigma}{ne^2} \tag{1}$$

where m is mass of electron and e is the change of electron. Substituting the given values of m, σ, n and e into Eq. (1), we get the relaxation time as

$$\tau = \frac{9.1 \times 10^{-31} \times 6 \times 10^{-7}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2}$$

$$\boldsymbol{\tau = 2.5 \times 10^{-14}\ \mathbf{s}}$$

Example 1.3 *Determine the mean free path of free electrons in copper. Given that the electrical conductivity of copper at the given temperature is 5 × 10^7 ohm^{-1} m^{-1}, the concentration of free electrons is 8.5 × 10^{28} m^{-3} and the mean thermal velocity (i.e. root mean square velocity) of free electrons is 1.6 × 10^6 m/s.*

Given Data

$\sigma = 5 \times 10^7$ ohm^{-1} m^{-1}

$\bar{c} = 1.6 \times 10^6$ m/sec

$n = 8.5 \times 10^{28}$ m^{-3}

$\lambda = ?$

Classical free electron theory gives the electrical conductivity of a metal as

$$\sigma = \frac{ne^2\tau}{m} \tag{1}$$

On the other hand, relaxation time τ is given by

$$\tau = \frac{\lambda}{\bar{c}} \tag{2}$$

From Eqs (1) and (2), we get

$$\lambda = \frac{m\bar{c}\sigma}{ne^2} \tag{3}$$

Substituting the given values of m, σ, n and e into Eq. (3), we get the mean free path as

$$\lambda = \frac{9.1 \times 10^{-31} \times 1.6 \times 10^{6} \times 5 \times 10^{7}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2}$$

$$\boldsymbol{\lambda = 3.35 \times 10^{-8}\ \mathrm{m}}$$

Example 1.4 *Calculate the mobility of electrons in copper assuming that each atom contributes one free electron for conduction. Given: resistivity of copper is 1.7 × 10^{-8} ohm.m, atomic weight of copper is 63.54, its density is 8.9 × 10^{3} kg/m^3 and Avogadro number is 6.025 × 10^{23}/mol.*

(A.U., B.E./B. Tech., May/June, 2005)

Given Data

$\rho = 1.7 \times 10^{-8}$ ohm.m

$m = 63.54$ g/mol $= 63.54 \times 10^{-3}$ kg/mol

$\rho' = 8.9 \times 10^{3}$ kg/m^3

$N_A = 6.025 \times 10^{23}$ mol^{-1}

$\mu = ?$

Mobility and electrical conductivity of metals are related by

$$\mu = \frac{\sigma}{ne} \tag{1}$$

where n is the concentration of electrons and e is the charge of an electron.

Let n' be the concentration of atoms. Density of a material is given as

$$\rho' = \frac{\text{No. of atoms} \times \text{Mass of 1 atom}}{\text{Volume}}$$

$$\rho' = n' \times \text{Mass of 1 atom}$$

or

$$\rho' = n' \times \frac{M}{N_A}$$

i.e. $$n' = \frac{\rho' N_A}{M} \tag{2}$$

As copper is monovalent, we have

Concentration of free electrons = Concentration of atoms

i.e. $n = n'$

Hence, Eq. (2) may be rewritten as

$$n = \frac{\rho' N_A}{M} \tag{3}$$

Substituting Eq. (3) into Eq. (1), we get

$$\mu = \frac{\sigma M}{\rho' N_A e}$$

or $$\mu = \frac{M}{\rho \rho' N_A e} \tag{4}$$

Substituting the values of M, ρ, ρ', N_A and e into Eq. (4), we get

$$\mu = \frac{63.54 \times 10^{-3}}{1.7 \times 10^{-8} \times 8.9 \times 10^{3} \times 6.025 \times 10^{23} \times 1.6 \times 10^{-19}}$$

i.e. $\boldsymbol{\mu = 4.36 \times 10^{-3}}$ **m^2 V^{-1} s^{-1}**

Example 1.5 *Calculate the drift velocity of the free electrons (with a mobility of 3.5×10^{-3} m^{2}V^{-1}s^{-1}) in copper for an electric field strength of 0.5 V/m.*

(A.U., B.E./B.Tech., May/June, 2005)

Given Data

$\mu = 4.36 \times 10^{-3}$ m^{2}V^{-1}s^{-1}

$E = 0.5$ V/m

$v_d = ?$

Mobility of free electrons in a metal is defined as

$$\mu = \frac{v_d}{E}$$

i.e. $$v_d = \mu E \tag{1}$$

Substitution of the values of μ and E into Eq. (1) gives

$$v_d = 3.5 \times 10^{-3} \times 0.5$$

i.e. $$\mathbf{v_d = 1.75 \times 10^{-3}\ m/s}$$

Example 1.6 *Copper at 300 K has an electrical conductivity of 6.4 × 10^7 mho/m. Calculate the thermal conductivity of copper. Lorentz number is L = 2.45 × 10^{-8} W.ohm.K^{-2}.*

(A.U., B.E.(ECE), May/June, 2006)

Given Data

$T = 300$ K

$\sigma = 6.4 \times 10^7$ mho/m

$L = 2.45 \times 10^{-8}$ W.ohm.K^{-2}

$K = ?$

The Wiedemann–Franz law gives

$$\frac{K}{\sigma} = LT$$

i.e. $$K = L\sigma T \qquad (1)$$

Substitution of the values of L, σ and T into Eq. (1) gives

$$K = 2.45 \times 10^{-8} \times 6.4 \times 10^7 \times 300$$

$$\mathbf{K = 470.4\ W/m/K}$$

Example 1.7 *The thermal conductivity of a metal at 300 K is 123.92 W/m/K. Find the electrical conductivity and Lorentz number when the metal posseses the relaxation time of 10^{-14} seconds at 300 K. Density of electrons is 6 × 10^{28}/m^3.*

(A.U, B.Tech. (Chemical), May/June, 2006)

Given Data

$K = 123.92$ W/m/K

$\tau = 10^{-14}$ sec

$T = 300$ K

$n = 6 \times 10^{-28}$/m^3

$\sigma = ?$; $L = ?$

The classical free electron theory of metals gives the electrical conductivity of a metal as

$$\sigma = \frac{ne^2\tau}{m} \tag{1}$$

where e is the charge of the electron and m is the mass of the electron. Substituting the values of n, e, τ and m into Eq. (1), we get

$$\sigma = \frac{6\times10^{28}\times(1.6\times10^{-19})^2\times10^{-14}}{9.11\times10^{-31}}$$

i.e. $\boldsymbol{\sigma = 1.69 \times 10^7}$ **ohm^{-1}. m^{-1}**

On the other hand, the Wiedemann–Franz law gives

$$\frac{K}{\sigma} = LT$$

i.e.
$$L = \frac{K}{\sigma T} \tag{2}$$

Substituting the values of K, σ and T into Eq. (2), we get

$$L = \frac{123.92}{1.69\times10^7\times300}$$

$\mathbf{L = 2.44 \times 10^{-8}}$ **W.ohm. K^{-2}**

Example 1.8 *Fermi energy of electrons in a solid is 5 eV. Find the temperature at which we have 1% probability for finding the electrons in an energy level 0.5 eV above the Fermi level.*

Given Data

$E_F = 5 \text{ eV} = 5 \times 1.6 \times 10^{-19}$ J

$E = E_F + 0.5 \text{ eV} = 5.5 \times 1.6 \times 10^{-19}$ J

$f(E) = 0.01$

$k = 1.38 \times 10^{-23}$

$T = ?$

Fermi–Dirac distribution function is given as

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

or
$$T = \frac{E - E_F}{k \cdot \ln\left(\frac{1}{f(E)} - 1\right)} \qquad (1)$$

Substitution of the given values for E, E_F, $f(E)$ and the Boltzmann constant k into Eq. (1) gives

$$T = \frac{0.5 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times \ln(100 - 1)}$$

i.e. $\mathbf{T = 1260\ K}$

Example 1.9 *Calculate the Fermi energy of sodium at 0 K. The density and the atomic weight of sodium are 970 kg/m³ and 23, respectively.*

Given Data

$\rho' = 970$ kg/m^3

$M = 23$ g/mol $= 23 \times 10^{-3}$ kg/mol

$T = 0$ K

$E_{F_0} = ?$

Concentration of atoms is given as

$$n' = \frac{\text{Density} \times \text{Avogadro's number}}{\text{Atomic weight}}$$

Hence, for sodium, we have

$$n' = \frac{970 \times 6.023 \times 10^{23}}{23 \times 10^{-3}}$$

i.e. $n' = 2.5 \times 10^{28}$/m^3

As sodium is a monovalent metal, its carrier concentration is

$$n = n'$$

$\therefore$ $n = 2.5 \times 10^{28}$/m^3

On the other hand, Fermi energy at $T = 0$ K is given by

$$E_{F_0} = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3} \qquad (1)$$

Substitution of the given data into Eq. (1) gives

$$E_{F_0} = \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}}\cdot\left(\frac{3\times2.5\times10^{28}}{8\times3.14}\right)^{2/3}$$

i.e. $\mathbf{E_{F_0} = 5.01 \times 10^{-19}\ J}$

Example 1.10 *A solid is maintained at a temperature of T K. Determine the probability of finding electrons at an energy level kT above the Fermi energy level.*

Given Data

$E = E_F + kT$

$f(E) = ?$

Fermi–Dirac distribution function gives

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} \qquad (1)$$

Substituting the given data into Eq. (1), we get

$$f(E) = \frac{1}{1+\exp\left(\dfrac{kT}{kT}\right)}$$

i.e. $$f(E) = \frac{1}{1+\exp(1)}$$

or $\mathbf{f(E) = 0.269}$

Example 1.11 *The Fermi energy of sodium or 0 K is 3.1 eV. Determine the number of electrons per unit volume of metal with energies in an energy interval of 0.02 eV above the Fermi energy.*

Given Data

$E_F = 3.1\ \text{eV} = 3.1 \times 1.6 \times 10^{-19}\ \text{J}$

$E = E_F$

$\Delta E = 0.02\ \text{eV} = 0.02 \times 1.6 \times 10^{-19}\ \text{J}$

$T = 0\ \text{K}$

$n = ?$

Carrier concentration in the metal is given as

$$n = \int_{E_F}^{E_F+\Delta E} f(E)\,N(E)\,dE$$

$$n = \int_{E_F}^{E_F+\Delta E} \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} dE$$

$$n = \frac{4\pi}{h^3}\left(2m\right)^{3/2}\left(E^{3/2}\right)_{E_F}^{E_F+\Delta E} \qquad (1)$$

Substituting the given data into Eq. (1), we get

$$n = \frac{4\times 3.14\times(2\times 9.1\times 10^{-31})^{3/2}}{(6.63\times 10^{-34})^3}(0.02\times 1.6\times 10^{-19})^{3/2}$$

i.e. $\mathbf{n = 0.192 \times 10^{26}/m^3}$

Example 1.12 *A solid is maintained at 300 K. Calculate the probability that the energy level 0.02 eV below Fermi level is not occupied by an electron.*

Given Data

$E_F - E = 0.02$ eV $= 0.02 \times 1.6 \times 10^{-19}$ J

$T = 300$ K

$1 - f(E) = ?$

The probability of an energy level being not occupied by an electron is

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \qquad (1)$$

where k is the Boltzmann constant.

Equation (1) may be rewritten as

$$1 - f(E) = \frac{\exp\left(\frac{E - E_F}{kT}\right)}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \qquad (2)$$

Substituting the given data for $E - E_F$, T and for k into Eq. (2), we get

$$1 - f(E) = \frac{\exp\left(-\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}\right)}{1 + \exp\left(-\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}\right)}$$

i.e. **1 – f (E) = 0.316**

Example 1.13 *The Fermi energy of potassium is 2.1 eV. Determine the velocity of Fermi-level electrons.*

Given Data

$E_F = 2.1 \text{ eV} = 0.02 \times 1.6 \times 10^{-19}$ J

$V_F = ?$

Energy of electrons at Fermi level is given as

$$E_F = \frac{1}{2} m v_F^2 \tag{1}$$

where m is the mass of electron.

Equation (1) gives the velocity of Fermi-level electrons as

$$V_F = \left(\frac{2E_F}{m}\right)^{\frac{1}{2}} \tag{2}$$

Substituting the given data for E_F and for m into Eq. (2), we get

$$V_F = \left(\frac{2 \times 2.1. \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}\right)^{\frac{1}{2}}$$

i.e. $\mathbf{V_F = 8.6 \times 10^5}$ **m/s**

Example 1.14 *Calculate the concentration of free electrons per unit volume of silver at 0 K. The Fermi energy of its free electrons is 5.5 eV. (Given value of Planck's constant h = 6.63 × 10^{-34} J.s, mass of electron = 9.11 × 10^{-31} kg).*

(A.U., B.E. (ECE) May/June 2005)

Given Data

$E_{F_0} = 5.5 \text{ eV} = 5.5 \times 1.6 \times 10^{-19}$ J

$h = 6.63 \times 10^{-34}$ J.s.

$m = 9.11 \times 10^{-31}$ kg

$n = ?$

The Fermi energy of a metal at 0 K is

$$E_{F_0} = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{\frac{3}{2}} \tag{1}$$

where n is the concentration of free electrons. Equation (1) gives the concentration of free electrons as

$$n = \frac{8\pi}{3}\left(\frac{2mE_{F_o}}{h^2}\right)^{\frac{3}{2}} \tag{2}$$

Substituting the given data for E_{F_o}, m, h into Eq. (2), we get

$$n = \frac{8\pi}{3}\left(\frac{2 \times 9.11 \times 10^{-31} \times 5.5 \times 1.6 \times 10^{-19}}{(6.63 \times 10^{-34})^2}\right)^{\frac{3}{2}}$$

i.e. $\mathbf{n = 58.36 \times 10^{27}/m^3}$.

Example 1.15 *Calculate the relaxation time for aluminium at 273 K, whose conductivity is $4 \times 10^7/\Omega m$, and number of atoms/unit volume is $18.1 \times 10^{28}/m^3$.*

(A.U., B.E./B.Tech, May/June 2011)

Given Data

$n = 18.1 \times 10^{28}$ m^3

$\sigma = 4 \times 10^7/\Omega$m

$\tau = ?$

Electrical conductivity of a metal

$$\sigma = \frac{ne^2\tau}{m}$$

$$\tau = \frac{m\sigma}{ne^2}$$

$$= \frac{9.1 \times 10^{-31} \times 4 \times 10^{7}}{18.1 \times 10^{28} \times (1.6 \times 10^{-19})^2}$$

Relaxation time = 7.855×10^{-15} s

QUESTIONS AND ANSWERS

1. **What are the postulates of the classical free electron theory?** ***(A.U., B.E./B.Tech, May/June, 2005)***

   - Metals consist of atoms, which in turn, consist of a nucleus and electrons.
   - Valence electrons are free and the metal is visualized as an array of ions permeated by a gas of free electrons.
   - In the absence of an electrical field, electrons move in random directions colliding with ions or free electrons.
   - In an electrical field, free electrons are accelerated and move in the opposite direction compared to that of the electric field.
   - Electron velocities obey Maxwell–Boltzmann distribution function.
   - Free electrons move without interaction between them and they obey the laws of classical kinetic theory of gases.
   - Free electrons move in a uniform potential field since ions are fixed in the lattice.

2. **Define the term 'mobility' of free electrons in a metal.** ***(A.U., B.E./B.Tech, May/June, 2005)***

   The mobility of a free electron in a metal is its drift velocity when subjected to unit electric field. That is,

   $$\mu = \frac{v}{E}$$

   Its unit of measurement is $m^2/V.s$.

3. **Define the term 'relaxation time' of free electrons in a metal.** ***(A.U., B.E./B.Tech, May/June, 2005)***

   It is the average time taken by the free electrons between two successive collisions with the lattice ions in a metal subjected to an electric field.

4. **Define the term 'mean free path' of free electrons in a metal.**
   It is the average distance travelled by free electrons between two successive collisions with lattice ions in a metal subjected to electric field.

5. **What are the merits of classical free electron theory of metals?**
   - It explains the Ohm's law.
   - It explains the electrical and thermal conductivities of metals.
   - It derives Wiedemann–Franz law.
   - It explains the optical properties of metals.

6. **What are the drawbacks of classical free electron theory of metals?** ***(A.U., B.E/B.Tech, May/June, 2011)***
   - Does not explain the photoelectric effect, Compton effect, black body radiation, etc.
   - It gives a value of $4.5R$ for the specific heat of metals. But the experimental value is only $3R$.
   - It gives a value of $\left(\frac{3}{2}\right) R$ for the electronic specific heat. But, the experimental value is only $0.01R$.
   - It does not explain the electrical conductivity of semiconductors and insulators.
   - It gives $\frac{K}{\sigma T}$ = const. for all temperatures. But, it is so only at low temperatures.
   - It does not explain ferromagnetism.

7. **State the Wiedemann–Franz law. What is meant by Lorentz number?**
   (A.U., B.E/B.Tech, May/June, 2005; A.U., B.E(ECE) May/June, 2006)

   The Wiedemann–Franz law states that the ratio of thermal conductivity of a metal to its electrical conductivity is directly proportional to temperature. And, the proportionality constant is the same for all metals. That is,

$$\frac{K}{\sigma} \alpha T$$

i.e. $$\frac{K}{\sigma} = LT$$

where L is the proportionality constant called the Lorentz number and it is independent of the nature of the metal.

8. **State Matthiessen's rule for the resistivity of metals.**

Matthiessen's rule of resistivity gives the resistivity of metals as

$$\rho = \rho_o + \rho(T)$$

where $$\rho_o = \frac{m}{ne^2\tau_{\text{imp}}} \text{ and } \rho(T) = \frac{m}{ne^2\tau_{\text{phonon}}}$$

The temperature-independent term ρ_o is called the residual resistivity. The temperature-dependent term $\rho(T)$ is called the lattice resistivity.

9. **Define the term 'residual resistivity' of metals.**

Matthiessen's rule of resistivity gives the resistivity of metals as

$$\rho = \rho_o + \rho(T)$$

where $$\rho_o = \frac{m}{ne^2\tau_{\text{imp}}} \text{ and } \rho(T) = \frac{m}{ne^2\tau_{\text{phonon}}}$$

The temperature-independent term ρ_o is called the residual resistivity.

10. **Define the term 'lattice resistivity' of metals.**

Matthiessen's rule of resistivity gives the resistivity of metals as

$$\rho = \rho_o + \rho(T)$$

where $$\rho_o = \frac{m}{ne^2\tau_{\text{imp}}} \text{ and } \rho(T) = \frac{m}{ne^2\tau_{\text{phonon}}}$$

The temperature-dependent term $\rho(T)$ is called the lattice resistivity.

11. **What are the factors affecting the resistivity of metals?**

The resistivity of metals depends on various factors such as temperature, impurity concentration, strain and magnetic field.

12. **Write a short note on the temperature dependence of resistivity of a metal.**

The resistivity of a metal can be expressed as

$$\rho = \rho_o + \rho(T)$$

where $$\rho_o = \frac{m}{ne^2\tau_{\text{imp}}} \text{ and } \rho(T) = \frac{m}{ne^2\tau_{\text{phonon}}}$$

The temperature-independent term ρ_o is called the residual resistivity. The temperature-dependent term $\rho(T)$ is called the lattice resistivity.

13. Write a short note on the dependence of resistivity on impurity concentration in metals.

The resistivity of a metal increases linearly with increasing atomic fraction of the impurity atom.

14. How does the resistivity of a metal depend on strain?

The dependence of resistivity of a metal on the strain can be expressed as

$$\frac{\Delta R}{R} \alpha \text{ strain}$$

where R is the resistance of the unstressed metal and ΔR is the increase in resistance of the metal under stress.

15. How does the resistivity of a metal depend on magnetic field?

At smaller magnetic fields, the variation of resistance is given by

$$\frac{\Delta R}{R} \alpha H^2$$

where H is the magnetic field strength. At larger magnetic fields, the variation in resistivity is given by

$$\frac{\Delta R}{R} \alpha H$$

16. State the merit of Wiedemann–Franz law.

It relates electrical and thermal conductivities of metals.

17. State the demerit of Wiedemann–Franz law.

The value of Lorentz number predicted by the Wiedemann–Franz law does not agree with the experimental value. That is, Wiedemann–Franz law does not quantitavely describe the electrical and thermal conductivities of metals.

18. What is Fermi–Dirac distribution function?

(A.U., B.E./B.Tech, May/June, 2011)

The Fermi–Dirac distribution function gives the probability that the energy level E is occupied by an electron at a temperature T. It is given as

$$f(E) = \frac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)}$$

where E_F is the Fermi energy, k is the Boltzmann constant and T is the temperature.

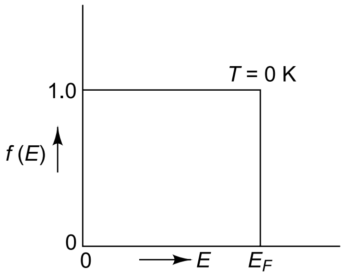
19. What is meant by Fermi energy of a solid?

Fermi energy is defined as the maximum energy of filled states at $T = 0$ K.

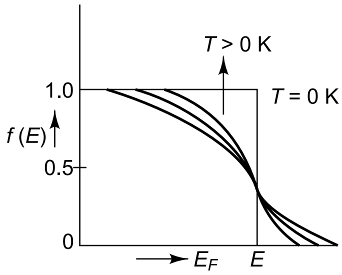
20. What is meant by Fermi energy level in a solid?

It is the uppermost energy level filled by electrons in a solid at $T = 0$ K.

21. Sketch a graph of variation of Fermi–Dirac distribution function with energy at 0 K.



22. Sketch a graph of variation of Fermi–Dirac distribution function with energy at non-zero temperature.



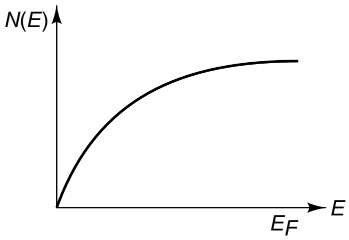
23. Define the term 'density of states'.

(A.U., B.E./B.Tech, May/June, 2011)

The density of states is defined as the number of energy states in an energy range of E and $E + dE$ states per unit volume of a metal. The density of states in the energy range E and $E + dE$ is

$$N(E)dE = \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} dE$$

24. Sketch the relation between the density of states and energy.



25. Give the expression for carrier concentration of a metal at 0 K.

The carrier concentration in a metal at $T = 0$ K is

$$n = \frac{8\pi}{3h^3}(2mE_{F0})^{3/2}$$

where E_{F0} is the Fermi energy at $T = 0$ K.

26. Give the expression for carrier concentration of a metal at non-zero temperature.

The carrier concentration in a metal at $T > 0$ K is

$$n = \frac{4\pi}{h^3}(2m)^{3/2}\int_0^\infty \frac{E^{1/2}dE}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

EXERCISE PROBLEMS

PART-A

1. Silver is monovalent and it has FCC structure with a lattice parameter of 4.07 Å. Given that the resistivity of silver is 1.6×10^{-8} ohm.m. Calculate the relaxation time of the free electrons in silver. (*Ans.* $\tau = 3.74 \times 10^{-14}$ *s*)

2. At 20°C, copper, a monovalent metal, has the density of 8960 kg/m^3 and the atomic weight of 63.54. A copper rod of diameter 0.16 cm carries a current of 10 A. Calculate the average drift velocity of free electrons in copper at 20°C.

   (*Ans.* $< v > = 3.66 \times 10^{-4}$ *m/s*)

3. At 300 K, copper, a monovalent metal, has the density of 8960 kg/m^3 and the atomic weight of 63.54. Calculate the electrical conductivity of copper at 300 K if the relaxation time of free electrons in copper is 2×10^{-14} s.

   (*Ans.* $\sigma = 3.8 \times 10^{-7}$ $ohm^{-1}.m^{-1}$)

4. The relaxation time of free electrons in silver is 4×10^{-14} s. Determine the mobility and the drift velocity of free electrons when a silver rod is subjected to a field intensity of 1 V/cm.

   $\left(Ans.\ \begin{array}{l} \mu = 7 \times 10^{-3}\, m^2/V.s \\ v_d = 0.7\ m/s \end{array}\right)$

5. Find the energy level in sodium for which the probability of occupation at 300 K is 0.75. Fermi energy of sodium is 3.13 eV.

   (*Ans.* $E = 3.10$ *eV*)

6. At what temperature, can it be expected with a 10%. Probability that electrons in silver have an energy which is 1% above the Fermi energy? The Fermi energy of silver is 5.5 eV.

   (*Ans.* $T = 290$ *K*)

7. Calculate the mobility of electrons in copper if it possesses 9×10^{28} valence electrons/m^3 and its conductivity is 6×10^7 mho/m. (*Ans.* $\mu = 4.16 \times 10^{-3}$ $m^2/V.s$)

8. The mean free collision time of copper is 3×10^{-14} seconds. Calculate the conductivity of copper at 300 K. Given that density of copper = 8960 kg/m^3, atomic weight of copper = 63.54 amu, mass of an electron = 9.1×10^{-31} kg, charge of an electron = 1.602×10^{-19} C. (*Ans.* $\sigma = 7.185 \times 10^7$ ohm^{-1} m^{-1})

9. The Fermi energy for lithium is 4.72 eV at absolute zero. Calculate the number of conduction electron per unit volume in lithium.

   (*Ans.* $N = 4.64 \times 10^{28}/m^3$)

10. There are 2.54×10^{28} free electrons per cm^3 in sodium. Calculate its Fermi energy and Fermi temperature.

(Ans. $E_F = 3.1$ eV, $T_F = 3.6 \times 10^4$ K)

QUESTIONS

PART-B

1. Derive an expression for electrical conductivity of a metal. How is it affected by temperature?
2. Derive an expression for the thermal conductivity of a metal. How is it affected by temperature?
3. Based on the classical free electron theory, derive an expression for electrical and thermal conductivity of a metal and hence arrive at the Wiedemann–Franz law.

   (A.U., B.E./B.Tech, May/June, 2011)
4. State the merits and demerits of the classical free electron theory.
5. Discuss in detail the Fermi–Dirac distribution function of electrons. Illustrate graphically the effect of temperature on the distribution.
6. Derive an expression for density of states in a metal and hence obtain the Fermi energy in terms of density of free electrons.

   (A.U., B.E./B.Tech, May/June, 2011)

SUMMARY

- Solids are broadly classified as metals and non-metals.
- The materials which allows easy passage of current through them are called conducting materials.
- The electron theory of metals is concerned with the study of valence electrons which controls the various physical properties such as electrical, thermal and magnetic properties of metals.
- The classical free electron theory is based on the motion of free electrons within the boundaries of a metal having positive ions fixed in the lattice.
- The electrical conductivity of a material based on classical free electron theory is given by

$$\sigma = \frac{ne^2\tau}{m}$$

- The thermal conductivity of a material based on classical free electron theory is given by

$$K = \frac{nK\tau}{2}\left(\frac{3kT}{m}\right)^{1/2}$$

- The Wiedemann–Franz law states that the ratio between thermal conductivity and electrical conductivity is directly proportional to the absolute temperature of a metal.
- Lorentz number is the ratio of thermal conductivity of a metal to the product of its electrical conductivity and its absolute temperature.
- Quantum free electron theory is based on dual nature (i.e. particle and wave nature) of electrons.
- Temperature, impurities, stress, deformation, etc., are some factors that affect the resistivity of a metal.
- The Fermi distribution function governs the distribution of electrons in a material among the various possible energy states. It is given by

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

- The highest energy level occupied by an electron at absolute zero is called the Fermi level and the corresponding energy as the Fermi energy. The Fermi energy is given as

$$E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3}$$

- The density of energy states is defined as the number of available energy states per unit volume in an energy interval.
- The density of energy states represents the number of states that could be occupied by charge carriers.
- The density of states in the energy range E and $E + dE$ is given by

$$N(E)\,dE = \frac{4\pi}{h^3}(2m)^{3/2}E^{1/2}dE$$

UNIT II

SEMICONDUCTING MATERIALS

The scientist is not a person who gives the right answers; he's one who asks the right question. **—Claude Lévi–Straus**

2.1 INTRODUCTION

Semiconductors form a class of materials with electrical conductivity lying in between that of conductors and that of insulators. The energy band theory explains the electrical properties of a semiconductor by the presence of a narrow energy gap of about 1 to 3 eV between the conduction and valence bands. As the forbidden energy gap is sufficiently small, thermal excitation can excite electrons in the valence band to the conduction band. When an electron is excited to the conduction band, an unfilled energy state in the valence band occurs. This unfilled state is called a hole. lt also contributes to electrical conductivity. It has the same properties as an electron, except that a positive charge is assigned to it. With the application of an electrical field, the hole moves in the direction of the field, such as a positive charge would.

2.2 TYPES OF SEMICONDUCTORS

The chemical composition, the energy–momentum profile for electrons and the chemical impurity content can serve as the criteria for classifying the semiconductors into various types.

2.2.1 Classification I

Based on the chemical composition, the semiconductors are classified as **elemental semiconductors** and **compound semiconductors**.

Elemental Semiconductors

They consist of only one chemical element. Examples are Ge, Si, Te, α–Sn, etc.

The elemental semiconductors Si and Ge are extensively used in electronic devices such as diodes, bipolar transistors, Field Effect Transistors (FET). In particular, Si is used in operational amplifiers, logic gates, LSF and VLSI chips, solar cells, etc.

Compound Semiconductors

They consist of more than one element. Examples are GaAs, InP, A1P, CdS, CdSe, etc. Due to its wider energy gap, the compound semiconductor has a range of applications. Some selected compound semiconductors are listed in Table 2.1.

Table 2.1 *Selected Compound Semiconductors*

Type	*Semiconductor*	$E_g(eV)$	*Applications*
III-IV Compounds	A1P	3	LED, Laser
	InSb	0.18	Photovoltaic cell
	AlAs	2.3	Gaussmeter
	GaAs	1.34	Hall effect devices
II-VI Compounds	CdS	2.45	LDR
	CdSe	0.24	Photoconducting cell
	CdTe	1.45	Photometer
IV-VI Compounds	PbSe	0.37	Thermoelectricity generation
	PbSe	0.27	Thermoelectric
	PbTe	0.33	cooling
Ternary Compounds	CuInS	–	Photovoltaic material
	CuInSe	–	
	CuInTe	–	
	ZnO	3.3	
Oxide Semiconductor	MgO	7.3	
	TiO	–	
	VO	–	

2.2.2 Classification II

Based on the energy–momentum profile for electrons, the semiconductors are classified as **direct bandgap semiconductors** and **indirect bandgap semiconductors.**

Direct Bandgap Semiconductors

The electrons at the conduction band bottom and at the valence band top have the same momentum. That is, when an electron in the valence band is excited to the conduction material band, its momentum does not change.

Examples: GaAs, GaSb, InP, InAs, InSb, α–Sn, Te.

Indirect Bandgap Semiconductors

The electrons at the conduction band bottom and at the valence band top have different momentum. That is, when an electron in the valence band is excited to the conduction band, its momentum changes.

Examples: Ge, Si, AlP, AlAs, AlSb, GaP.

Thus, the excitation of electrons in an indirect bandgap semiconductor is difficult. Therefore, the indirect bandgap semiconductors are not used for laser production.

2.2.3 Classification III

Based on the chemical impurity content, the semiconductors are classified as **intrinsic semiconductors** and **extrinsic semiconductors.** Further, depending upon the nature of the doped chemical impurity, the extrinsic semiconductors are classified as ***n*-type semiconductors** and ***p*-type semiconductors.**

2.3 INTRINSIC SEMICONDUCTORS

These semiconductors are chemically pure without any impurity. That is, these are non-doped semiconductors. Electron-hole pairs are generated within an intrinsic semiconductor by thermal excitation. Obviously, the intrinsic carrier concentration depends on temperature. The intrinsic semiconductor has an equal number of conduction electrons and holes, since for every conduction electron produced there must be a corresponding hole created.

That is,

$$n = p = n_i$$

where n is the electron concentration in conduction band, p is the hole concentration in valence band and n_i is the intrinsic carrier concentration.

2.3.1 Intrinsic Carrier Concentration

In an intrinsic semiconductor, both electrons and holes contribute to electrical conductivity. To find the concentration of charge carriers in an intrinsic semiconductor, the electron-concentration in the conduction band and the hole-concentration in the valence band are to be determined. For this purpose, the concepts of density of states and Fermi–Dirac distribution can be used.

2.3.2 Electron Concentration in Conduction Band

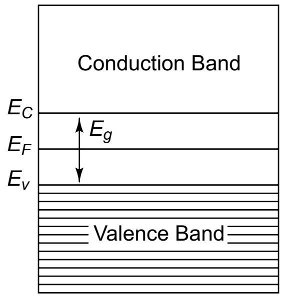


Fig. 2.1 *Energy Band Diagram of an Intrinsic Semiconductor*

The Fermi–Dirac distribution function (i.e. the probability that a quantum energy state E is occupied by an electron) for the conduction band is given as

$$f_c(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)} \tag{2.1}$$

where E_F is the Fermi energy level, k is the Boltzmann constant and T is the absolute temperature.

The density of energy states (i.e. number of energy states in the interval E and $E + dE$) in the conduction band is given as

$$N_c(E)\, dE = \gamma_n\ (E - E_c)^{1/2}\, dE \tag{2.2}$$

where E_c is the conduction band-bottom;

$$\gamma_n = \frac{4\pi}{h^3}\left(2m_n^*\right)^{3/2} = \text{const.};$$

h is the Planck's constant and m_n^* is the effective mass of electrons.

Then, the electron concentration in the energy interval E and $E + dE$ of conduction band may be regarded as the product of $f_c(E)$ and $N_c(E)\, dE$. Hence, electron concentration in conduction band is given as

$$n = \int_{E_c}^{\infty} f_c(E) N_c(E)\, dE$$

$$n = \int_{E_c}^{\infty} \frac{\gamma_n (E - E_c)^{1/2}}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE$$

i.e. $$n = \gamma_n \int_{E_c}^{\infty} \frac{(E - E_c)^{1/2}}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE$$

At room temperature, $kT = 0.025$ eV and in the conduction band $E - E_F > 1$ eV.

Therefore, we have

$$\exp\left(\frac{E - E_F}{kT}\right) >> 1$$

Then, the expression for n is reduced to

$$n = \gamma_n \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left(-\frac{E - E_F}{kT}\right) dE$$

$$n = \gamma_n \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left(-\frac{E_c - E_F}{kT}\right) \exp\left(-\frac{E - E_c}{kT}\right) dE$$

$$n = \gamma_n \exp\left(-\frac{E_c - E_F}{kT}\right) \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left(-\frac{E - E_c}{kT}\right) dE$$

Introducing a new variable $\frac{E - E_c}{kT} \equiv x$, the above equation may be rewritten as

$$n = \gamma_n \exp\left(-\frac{E_c - E_F}{kT}\right) \int_{0}^{\infty} (kT)^{3/2} x^{1/2} \exp(-x)\, dx$$

$$n = \gamma_n (kT)^{3/2} \exp\left(-\frac{E_c - E_F}{kT}\right)\int_0^\infty x^{1/2} \exp(-x)\,dx$$

On the other hand, as a standard integral, we have

$$\int_0^\infty x^{1/2} \exp(-x)\,dx = \frac{\sqrt{\pi}}{2}$$

Therefore, we get

$$n = \frac{\sqrt{\pi}}{2}\gamma_n (kT)^{3/2} \exp\left(-\frac{E_c - E_F}{kT}\right)$$

i.e.

$$n = G_c \exp\left(-\frac{E_c - E_F}{kT}\right) \tag{2.3}$$

where

$$G_c = \frac{\sqrt{\pi}}{2}\gamma_n (kT)^{3/2} \tag{2.4}$$

The factor G_c is called the effective density of states in the conduction band. Equation (2.3) gives the density or concentration of electrons in the conduction band of an intrinsic semiconductor as a function of temperature.

2.3.3 Hole Concentration in the Valence Band

By a method similar to that was employed above, we can derive the expression for the hole concentration in the valence band.

For the energy states in the valence band, Fermi–Dirac distribution function may be obtained from the fact that an electron in a semiconductor may be present in the conduction band or in the valence band. That is,

$$f_c(E) + f_v(E) = 1$$

$$f_V(E) = 1 - f_c(E) \tag{2.5}$$

Substitution of Eq. (2.1) into Eq. (2.5) gives

$$f_v(E) = \frac{\exp\left(\frac{E - E_F}{kT}\right)}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \tag{2.6}$$

At room temperature, we have

$$\exp\left(\frac{E-E_F}{kT}\right) << 1$$

Hence, the expression for $f_v(E)$ reduces to

$$f_v(E) = \exp\left(\frac{E-E_F}{kT}\right) = \exp\left(-\frac{E_F-E}{kT}\right)$$

The density of states in the valence band is given as

$$N_v\,(E)\;dE = \gamma_p\,(E_v - E)^{1/2}\;dE \tag{2.7}$$

where $\gamma_p = \frac{4\pi}{h^3}\left(2m_p^*\right)^{1/2}$ = const. and m_p^* is the effective mass of a hole.

The concentration of holes is, then, given as

$$p = \int_{-\infty}^{E_v} f_v\,(E)\,N_v(E)\,dE \tag{2.8}$$

Substitution of Eqs (2.6) and (2.7) into Eq. (2.8) gives

$$p = \int_{-\infty}^{E_v} \gamma_p\left(E_v - E\right)^{1/2} \exp\left(-\frac{E_F-E}{kT}\right) dE$$

$$p = \gamma_p \int_{-\infty}^{E_v} \left(E_v - E\right)^{1/2} \exp\left(-\frac{E_F-E}{kT}\right) dE$$

$$p = \gamma_p \int_{-\infty}^{E_v} \left(E_v - E\right)^{1/2} \exp\left(-\frac{E_F-E_v}{kT}\right) \exp\left(-\frac{E_v-E}{kT}\right) dE$$

$$p = \gamma_p \exp\left(-\frac{E_F-E_v}{kT}\right) \int_{-\infty}^{E_v} \left(E_v - E\right)^{1/2} \exp\left(-\frac{E_v-E}{kT}\right) dE$$

Introducing a new variable $(E_v - E)/kT \equiv x$, the above equation may be rewritten as

$$p = \gamma_p\;\left(kT\right)^{3/2} \exp\left(-\frac{E_F-E_v}{kT}\right) \int_O^{\infty} x^{1/2} \exp\left(-x\right) dx \tag{2.9}$$

Substituting the value of standard integral into Eq. (2.9), we get

$$p = \frac{\sqrt{\pi}}{2}\gamma_p (kT)^{3/2} \exp\left(-\frac{E_F - E_v}{kT}\right)$$

i.e.
$$p = G_v \exp\left(-\frac{E_F - E_v}{kT}\right) \tag{2.10}$$

where
$$G_v = \frac{\sqrt{\pi}}{2}\gamma_p (kT)^{3/2} \tag{2.11}$$

The factor G_v is called the effective density of states in the valence band. Equation (2.10) gives the concentration of holes in the valence band of an intrinsic semiconductor as a function of temperature.

2.3.4 Fermi Level in an Intrinsic Semiconductor

In an intrinsic semiconductor, $n = p = n_i$ (here index i implies the intrinsic semiconductor). Therefore, from Eqs (2.3) and (2.10), we get

$$G_c \exp\left(-\frac{E_c - E_F}{kT}\right) = G_v \exp\left(-\frac{E_F - E_v}{kT}\right)$$

$$\frac{G_v}{G_c} = \exp\left(\frac{E_F - E_c + E_F - E_v}{kT}\right)$$

$$\frac{G_v}{G_c} = \exp\left(\frac{2E_F - (E_c + E_v)}{kT}\right)$$

Taking logarithm on both sides and rearranging the terms, we get

$$E_F = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{G_v}{G_c}\right) \tag{2.12}$$

The ratio G_v/G_c depends upon the effective masses of hole and electron. Equation (2.12) represents the temperature dependence of Fermi energy of an intrinsic semiconductor. At T = 0 K, Eq. (2.12) reduces to

$$E_F = \frac{E_c + E_v}{2} \tag{2.13}$$

2.3.5 Variation of Fermi Level with Temperature

That is, in an intrinsic semiconductor at $T = 0$ K, the Fermi level lies at the middle of the forbidden energy gap. Figure 2.2 shows the plot of

Fermi energy of an intrinsic semiconductor versus temperature as given by Eq. (2.12).

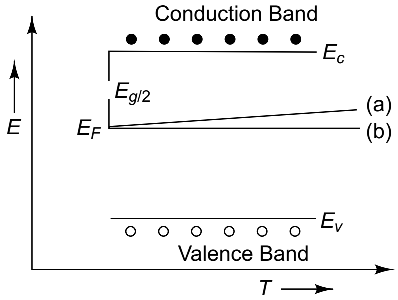


Fig. 2.2 *Plot of Fermi Energy Versus Temperature for an Intrinsic Semiconductor*

As seen, at $T = 0$ K, the Fermi level lies at the middle of the forbidden energy gap. With the increase in temperature, the Fermi level moves up towards the conduction band. At sufficiently high temperatures, the Fermi level is located in the conduction band.

2.3.6 Carrier Concentration

Considering Eq. (2.13), Eqs (2.3) and (2.10) may now be rewritten as

$$n = G_c \exp\left(-\frac{E_g}{2kT}\right) \tag{2.14}$$

$$p = G_v \exp\left(-\frac{E_g}{2kT}\right) \tag{2.15}$$

Now, it is seen that the intrinsic carrier concentration at a given temperature strongly depends upon the width of the forbidden energy gap. This is quite obvious. If E_g is large, there will be few carriers for conduction of electricity.

The multiplication of Eqs (2.14) and (2.15) leads to

$$n_i^2 \equiv np = G_c\, G_v \exp\left(-\frac{E_g}{kT}\right) \tag{2.16}$$

Substituting Eqs (2.4) and (2.11) into Eq. (2.16), we get

$$n_i^2 = BT^3 \exp\left(-\frac{E_g}{kT}\right) \tag{2.17}$$

where $B \equiv \gamma_n \gamma_p \frac{\pi}{4} k^3 = \text{constant}$

The intrinsic carrier concentration is then given as

$$n_i = AT^{3/2} \exp\left(-\frac{E_g}{2kT}\right) \tag{2.18}$$

where $A \equiv B^{1/2} = \left(\gamma_n \gamma_p \frac{\pi}{4} k^3\right)^{1/2} = \text{constant}$

This result shows that the intrinsic carrier concentration n_i is dependent on forbidden energy gap width and temperature, but not on the position of the Fermi level.

2.3.7 Intrinsic Electrical Conductivity

When an electric field is applied across an intrinsic semiconductor, in addition to the thermal motion, the free electrons and holes exhibit a steady drift constituting a current flow.

The current density contributed by electron flow is given by

$$j_n = nev_n$$

where e is the charge of electron and v_n is the drift velocity of free electrons.

The current density contributed by hole flow is given by

$$J_p = pev_p$$

where v_p is the drift velocity of holes.

Then, the conductivity due to the electrons is given by

$$\sigma_n = \frac{j_n}{E} = \frac{nev_n}{E} = ne\mu_n \tag{2.19}$$

where E is the applied electric field and $\mu_n \equiv v_n/E$ is the mobility of free electrons defined as the drift velocity of electrons when subjected to unit electric field.

Similarly, the conductivity due to the holes is given by

$$\sigma_p = \frac{j_p}{E} = \frac{pev_p}{E} = pe\mu_p \tag{2.20}$$

where $\mu_p \equiv v_p/E$ is the mobility of holes.

As both electrons and holes contribute to current flow in an intrinsic semiconductor, the total conductivity in an intrinsic semiconductor is given as

$$\sigma_i = \sigma_n + \sigma_p = en_i(\mu_n + \mu_p) \tag{2.21}$$

where σ_i is called the intrinsic conductivity.

Considering Eq. (2.18) along with the fact that the mobilities of electrons and holes are proportional to $T^{-3/2}$, we get

$$\sigma_i \propto \exp\left(-\frac{E_g}{2kT}\right)$$

i.e.

$$\sigma_i = \sigma_o \exp\left(-\frac{E_g}{2kT}\right) \tag{2.22}$$

where σ_o is a constant for the given intrinsic semiconductor which represents its electrical conductivity at $T = 0$ K.

Equation (2.22) shows that $\sigma(T)$ of an intrinsic semiconductor varies in an exponential manner. A plot ln σ versus $1/T$ for an intrinsic semiconductor is shown in Fig. 2.3.

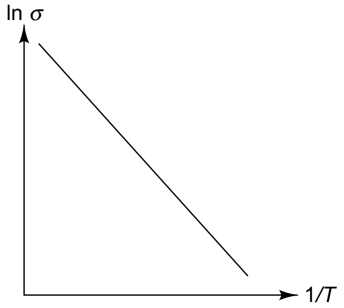


Fig. 2.3 *Plot of Electrical Conductivity versus Temperature for an Intrinsic Semiconductor*

As the temperature is increased, more and more electrons are shifted to the conduction band and an equal number of holes are generated in the valence band. It leads to an increase in carrier concentrations. As a result, electrical conductivity of the intrinsic semiconductor increases with the increase in temperature. Then, obviously, electrical resistivity ($\rho_i = 1/\sigma_i$) of an intrinsic semiconductor decreases with the increase in temperature. In other words, intrinsic semiconductors have negative temperature coefficient of resistivity. That is, slope of $\rho_i(T)$ has a negative value. This result differentiates the intrinsic semiconductors from the conductors which have the positive coefficients of resistivity.

Although in an intrinsic semiconductor $n = p$, about two-thirds of the total current is carried by the free electrons. This is because of the higher mobility of free electrons as compared to that of holes. The term 'intrinsic' simply means that it is a property of the crystal in its purest form.

Typical Intrinsic Semiconductors

The characteristic parameters of selected intrinsic semiconductors at 300 K are given in Table 2.2.

Table 2.2 *Characteristic Parameters of Selected Intrinsic Semiconductors at 300 K*

Semi conductor	E_g *eV*	n_i m^{-3}	μ_n $m^2/V.sec$	μ_p $m^2/V.sec$	σ_i $ohm^{-1}.m^{-1}$
Si	1.12	1.45×10^{16}	0.15	0.05	4.35×10^{-4}
Ge	0.66	2.40×10^{19}	0.39	0.19	2.13
GaAs	1.42	1.79×10^{12}	0.85	0.04	10^{-6}

2.3.8 Bandgap Determination

Theory As already seen, the electrical conductivity of an intrinsic semiconductor is

$$\sigma_i = \sigma_o \exp\left(-\frac{E_g}{2kT}\right)$$

where E_g is the bandgap energy, k is the Boltzmann constant, T is the absolute temperature and σ_o is a constant for the given material.

Then, its electrical resistivity of an intrinsic semiconductor is given as

$$\rho_i = \frac{1}{\sigma_o} \exp\left(\frac{E_g}{2kT}\right) \tag{2.23}$$

Let us consider a wire of intrinsic semiconductor of length ℓ. Let R_i be its resistance and A be its cross-sectional area. The resistivity of the wire is then given as

$$\rho_i = \frac{R_i A}{\ell}$$

Hence, Eq. (2.23) may be rewritten as

$$\frac{R_i A}{\ell} = \frac{1}{\sigma_o} \exp\left(\frac{E_g}{2kT}\right)$$

$$R_i = \frac{\ell}{A\sigma_o} \exp\left(\frac{E_g}{2kT}\right)$$

or $$\ln R_i = \ln\left(\frac{\ell}{A\sigma_o}\right) + \frac{E_g}{2kT} \tag{2.24}$$

That is, the plot of ln R_i versus $1/T$ is a straight line. Its slope gives the value of $E_g/2k$.

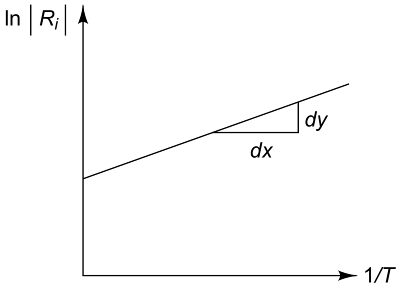


Fig. 2.4 *Plot of Resistance versus Temperature for an Intrinsic Semiconducting Wire*

The resistance of the given wire is measured at various temperatures. The results are represented as a graph of ln R_i versus $1/T$ in Fig. 2.4. The slope *dy*/*dx* of the straight line is determined. Then, the bandgap of the semiconductor is determined by

$$E_g = 2k\frac{dy}{dx} \tag{2.25}$$

2.4 EXTRINSIC SEMICONDUCTORS

By suitable doping, a semiconducting crystal can be made to have unequal values of electron and hole concentrations. The resulting semiconductor is said to be an extrinsic one. For an extrinsic semiconductor, therefore, we have

$$n \neq p$$

Most of the engineering applications of semiconductors involve semiconductors which have been doped intentionally with specific impurities, rather than intrinsic material. From X-ray diffraction data, it has been established that the impurity atoms occupy lattice positions which in the pure material are occupied by the host atoms. Of particular importance is the case of Ge or Si doped with elements from the third (B, Al, Ga, In) and the fifth (P, As, Sb) groups of the periodic table.

Addition of a fraction of a percent of such elements may increase the conductivity by several powers of ten. Also, the temperature dependence of the conductivity is strongly affected by the impurity content. Depending upon the added impurities, the resulting semiconductors become *n*-type or *p*-type semiconductors.

2.4.1 *n*-Type Semiconductors

These are doped semiconductors with greater electron concentration in the conduction band than hole concentration in the valence band.

i.e. $n > p, \qquad n > n_i$

Consider a crystal of silicon doped with a very small amount of pentavalent atoms, say, phosphor. Upon incorporation into the crystal, these impurities occupy lattice sites that are otherwise occupied by Si atoms. Having the tetravalent host atoms in its surroundings, the pentavalent atom now requires only four valence electrons to form electron-pair bonds with neighbours. Thus, the dopant has one valence electron which is not being used in chemical bonds with its neighbours. This 'extra' electron is not strongly bound as other valence electrons.

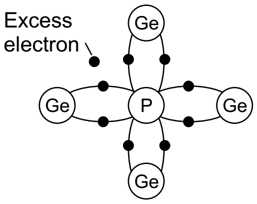


Fig. 2.5 *Extra Electron in Conduction Band*

This extra electron may be pictured as moving around a positive ion of charge $+e$, in a way similar to the motion of electron in a hydrogen atom, around a proton of charge $+e$. The extra electron moving in the field of positively charged dopant ion sees the ion embedded in a dielectric medium, viz., silicon (host material). The presence of a dielectric medium between the electron and the dopant ion reduces the field strength and hence reduces the binding energy of the electron. Thus, in silicon, the ionization energy of phosphor is found to be about 0.01 eV.

Note: Since germanium has a somewhat higher dielectric constant of $\varepsilon_r = 16$, the ionization energies of these elements in germanium are somewhat lower than in silicon with $\varepsilon_r = 12$. Moreover, the ionization energy decreases with the increase in impurity content.

2.4.2 Energy Band Structure of *n*-type Semiconductors

At $T = 0$ K, the energy band structure is similar to that of an intrinsic semiconductor in that the valence band is filled and the conduction band is empty of electrons. However, the 'extra' dectrons associated with donor atoms occupy states which lie only a fraction of a volt (≈ 0.01 eV) below the conduction band. As the temperature is increased, the lattice vibrations become more intense and by absorbing the proper amount of energy, some of the donor atoms may become ionized, i.e. they release their electrons to the conduction band (Fig. 2.6). Since the ionization energy of the donor atoms is much smaller than the energy E_g required to break one of the valence bonds, the donor atoms will donate electrons to the conduction band at much lower temperatures than the valence band will.

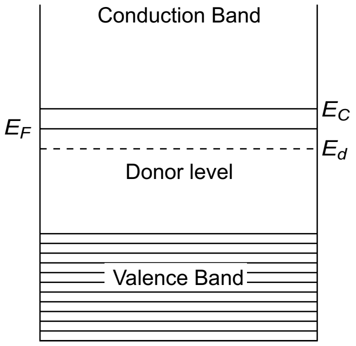


Fig. 2.6 *Conduction Band*

When $kT > \Delta E$, nearly all donor atoms are ionized, and n approaches, N_d, where n is the electron concentration in the conduction band and N_d is the donor concentration.

2.4.3 Electron Concentration in Conduction Band of *n*-Type Semiconductor

To determine the concentration of charge carriers, it is necessary to know their statistics in an impurity semiconductor. The donor states are each occupied by a single electron in contrast to the levels in the conduction band where each level is occupied by two electrons of opposite spins.

Let N_d be the density of donor energy levels (hence, N_d also represents the concentration of donor atoms).

Let $f(E_d)$ be the probability of finding the electron in the donar energy level E_d (i.e., Fermi–Dirac distribution function). Thus, we have

$$f(E_d) = \frac{1}{1 + \exp\left(\frac{E_d - E_F}{kT}\right)} \tag{2.26}$$

At low temperatures, the number of electrons in the conduction band is equal to the number of ionized donor atoms (or) to the number of empty donor energy levels.

Then, the electron concentration in the conduction band is given as

$$n \equiv N_d(1 - f(E_d)) = N_d\left[1 - \frac{1}{1 + \exp\left(\frac{E_d - E_F}{kT}\right)}\right] \tag{2.27}$$

At low temperatures, exp $[(E_d - E_F)/kT] << 1$. Hence, Eq. (2.27) reduces to

$$n \equiv N_d(1 - f(E_d)) = N_d \exp\left(\frac{E_d - E_F}{kT}\right) \tag{2.28}$$

The low temperature approximation may be understood by the fact that in an n-type semiconductor, the Fermi level is located between the conduction band bottom E_c and the donor energy level E_d.

On the other hand, we had previously derived an expression for electron concentration in the conduction band of a semiconductor, which has the form

$$n = G_c \exp\left(\frac{E_F - E_c}{kT}\right) \tag{2.29}$$

We had nowhere assumed in the derivation of Eq. (2.29) that the material is intrinsic. Therefore, the result holds in the presence of impurities as well. It should be noted that an appropriate expression for Fermi level E_F in an n-type semiconductor is to be substituted in Eq. (2.28). It is so, since the Fermi energy level has been shifted upwards in an n-type semiconductor in comparison with that of an intrinsic semiconductor.

2.4.4 Variation of Fermi Level with Temperature and Impurity Concentration

Hence, Eqs (2.28) and (2.29) give

$$N_d \exp\left(\frac{E_d - E_F}{kT}\right) = G_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

Rearranging the terms, we get

$$\exp\left(\frac{2E_F - (E_c + E_d)}{kT}\right) = \frac{N_d}{G_c}$$

Taking logarithms on both sides, we get

$$\frac{2E_F - (E_c + E_d)}{kT} = \ln\left(\frac{N_d}{G_c}\right)$$

Thus, we have

$$E_F = \frac{E_c + E_d}{2} + \frac{kT}{2}\ln\left(\frac{N_d}{G_c}\right) \tag{2.30}$$

Equation (2.30) gives the dependence of Fermi energy of an *n*-type semiconductor on temperature and on donor concentration.

At $T = 0$ K, Eq. (2.30) gives

$$E_F = \frac{E_c + E_d}{2} \tag{2.31}$$

That is, at absolute zero, the Fermi level in a *n*-type semiconductor lies exactly halfway between donor energy level and the conduction band bottom.

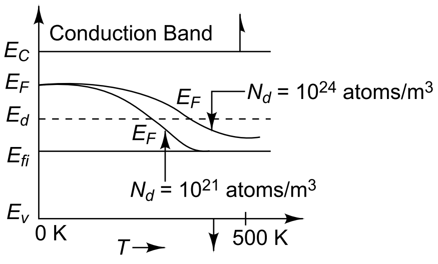


Fig. 2.7 *Fermi Energy of n-type Semiconductor vs Temperature*

Figure 2.7 shows the plot of the Fermi energy of a n-type semiconductor versus temperature. As seen, at $T = 0$ K, the Fermi level lies exactly halfway between the donor energy level and the conduction band bottom with the increase in temperature, the Fermi level drops. At sufficiently high temperatures, the Fermi level of the n-type semiconductor reaches its intrinsic level.

Using Eq. (2.30) , we get

$$\frac{E_F - E_c}{kT} = \frac{E_d - E_c}{2kT} + \frac{1}{2}\ln\left(\frac{N_d}{G_c}\right)$$

$$\exp\left(\frac{E_F - E_c}{kT}\right) = \left(\frac{N_d}{G_c}\right)^{1/2} \cdot \exp\left(\frac{E_d - E_c}{2kT}\right) \tag{2.32}$$

2.4.5 Carrier Concentration

Substituting Eq. (2.31) into Eq. (2.29), we get the electron concentration in an n-type semiconductor as

$$n = (G_c \cdot N_d)^{1/2} \exp\left(\frac{E_d - E_c}{2kT}\right) \tag{2.33}$$

Equation (2.33) shows that the electron concentration in the conduction band of a n-type semiconductor is proportional to the square root of donor concentrations. This equation is valid only at low temperatures. At high temperatures, the contribution of electrons to the conduction band from the valence band increases and at very high temperatures (≈ 500 K for Ge) far exceeds the donor contributions. Thus, the intrinsic behaviour predominates at higher temperatures.

2.4.6 *p*-Type Semiconductors

These are impurity semiconductors with greater hole concentration in valence band than electron concentration in conduction band,

i.e. $p > n, \quad p > n_i$

Consider a crystal of silicon doped with a very small amount of trivalent atoms, say boron. Upon incorporation into the crystal, the impurities occupy lattice sites that are otherwise occupied by silicon atoms. The three valence electrons of dopant atoms, now, complete three of the covalent bands, leaving one bond vacant which is called the hole. Initially, the hole is 'bound' to the impurity atom, at least at absolute zero.

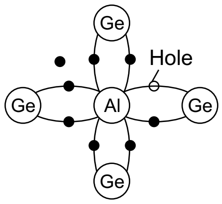


Fig. 2.8 *Electron-Hole Concentration*

Actually, the hole may be pictured as revolving around the impurity ion (negatively charged with $-e$) in a Bohr-like orbit. At higher temperatures, an electron from a normal electron-pair bond would jump into the hole. Subsequent jumps of electrons from other electron-pair bonds lead to hole conduction. The impurity atom that contributes a hole is called an acceptor atom as it accepts a bound electron from a covalent bond.

2.4.7 Energy Band Structure of *p*-Type Semiconductors

The energy required for a valence electron to be excited into the hole is approximately equal to the ionization energy of donor atoms in the same material. That is, the bound holes of the acceptor atoms may be represented in the energy level diagram (Fig. 2.9) as occupying levels slightly above the top of valence band.

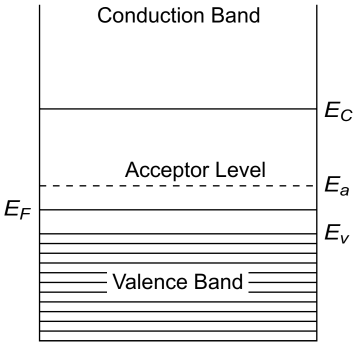


Fig. 2.9 *Energy-Level Diagram*

At $T = 0$ K, the holes remain bound to the impurities. As the temperature is raised, electrons from valence band are excited to acceptor energy levels to make the hole-concentration in the valence band higher in comparison with the intrinsic material. However, the electron-concentration in the conduction band remains the same as in the intrinsic semiconductor. At sufficiently higher temperatures with $kT > \Delta E$, nearly all the acceptor atoms have accepted electrons, and p approaches N_a, where p is the hole concentration in valence band and N_a is the concentration of acceptor atoms.

2.4.8 Hole Concentration in Valence Band of *p*-type Semiconductor

The acceptor states accept only one electron each in contrast to the valence band energy levels which accommodate two electrons with opposite spins.

Let N_a be the density of acceptor energy levels (hence, N_a also represents the concentration of acceptor atoms).

Let $f(E_a)$ be the probability of finding the electron in the acceptor energy level E_a (i.e. Fermi–Dirac distribution function).

Thus, we have

$$f(E_a) = \frac{1}{1 + \exp\left(\dfrac{E_a - E_F}{kT}\right)} \tag{2.34}$$

At low temperatures (when the transitions of electrons from valence band to conduction band are negligible), the number of holes in the valence band is equal to number of electrons in the acceptor energy level.

Thus, the hole concentration in the valence band is given as

$$p = N_a \frac{1}{1 + \exp\left(\dfrac{E_a - E_F}{kT}\right)} \tag{2.35}$$

As the Fermi level in a p-type semiconductor is located between the acceptor energy level and the valence band top at low temperature (such as room temperature), we have

$$\exp\,[(E_a - E_F)/kT] >> 1$$

Therefore, we get

$$p = N_a \exp\left(\frac{E_F - E_a}{kT}\right) \tag{2.36}$$

The previously obtained expression for hole concentration in the valence band of an intrinsic semiconductor has the form:

$$p = G_v \exp\left(\frac{E_v - E_F}{kT}\right) \tag{2.37}$$

Equation (2.37) holds even for an impurity semiconductor as nowhere in its derivation the material was assumed to be an intrinsic one. But, the appropriate expression for Fermi level in a p-type semiconductor is to be taken into account.

2.4.9 Variation of Fermi Level with Temperature and Impurity Concentration

Therefore, Eqs (2.36) and (2.37) give

$$N_a \exp\left(\frac{E_F - E_a}{kT}\right) = G_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

Rearranging the terms, we get

$$\exp\left(\frac{2E_F - (E_a + E_v)}{kT}\right) = \frac{G_v}{N_a}$$

Taking logarithms on both sides, we get

$$\frac{2E_F - (E_a + E_v)}{kT} = \ln\left(\frac{G_v}{N_a}\right)$$

Hence, we get

$$E_F = \frac{E_a + E_v}{2} + \frac{kT}{2}\ln\left(\frac{G_v}{N_a}\right) \tag{2.38}$$

At $T = 0$ K, we have

$$E_F = \frac{E_a + E_v}{2} \tag{2.39}$$

That is, at absolute zero, the Fermi level in a *p*-type semiconductor lies exactly halfway between acceptor energy level and the valence band top.

Figure (2.10) shows the plot of the Fermi energy of a *p*-type semiconductor versus temperature. As seen, at $T = 0$ K, the Fermi energy lies exactly halfway between the acceptor energy level and the valence band top. With the increase in temperature, the Fermi level moves up. At sufficiently high temperature, the Fermi level of a *p*-type semiconductor reaches its intrinsic level.

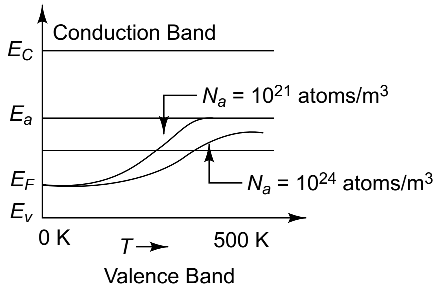


Fig. 2.10 *Fermi Energy of p-type Semiconductor versus Temperature*

Equation (2.38) express the temperature dependence of Fermi energy level in a p-type semiconductor at low temperatures. Considering Eq. (2.38), we get

$$\frac{E_v - E_F}{kT} = \frac{E_v - E_a}{2kT} - \frac{1}{2}\ln\left(\frac{G_v}{N_a}\right)$$

$$\exp\left(\frac{E_v - E_F}{kT}\right) = \left(\frac{N_a}{G_v}\right)^{1/2} \exp\left(\frac{E_v - E_a}{2kT}\right) \tag{2.40}$$

2.4.10 Carrier Concentration

Substitution of Eq. (2.40) into Eq. (2.37) gives the hole concentration in valence band of a p-type semiconductor as

$$p = (G_v \cdot Na)^{1/2} \exp\left(\frac{E_v - E_a}{2kT}\right) \tag{2.41}$$

That is, the hole concentration in valence band of a p-type semiconductor is proportional to the square root of acceptor concentrations. Equation (2.41) is valid only at low temperatures. At high temperatures, intrinsic behaviour predominates.

2.4.11 Law of Mass Action

Considering Eqs (2.3) and (2.10), we get

$$np = 4\left(\frac{kT}{2\pi h^2}\right)^3 (m_n^* \, m_p^*)^{3/2} \exp\left(-\frac{E_g}{kT}\right) \tag{2.42}$$

This result does not involve the Fermi level E_F. It is an expression of the law of mass action which implies that the product np is constant at a given temperature. In the derivation of Eq. (2.42), it is nowhere assumed that the material is intrinsic. Therefore, this results holds in the presence of impurities as well. The only assumption made is that the distance of the Fermi level from the edge of both bands is large in comparison with kT.

Suppose that the equilibrium population of electrons and holes is maintained by black-body photon radiation at temperature T. The photos generate electron-hole pairs at a rate $A(T)$, while $B(T)\ np$ is the rate of recombination reaction (i.e. electron + hole → photon). Then, we have

$$\frac{dn}{dt} = A(T) - B(T)\,np = \frac{dp}{dt}$$

In equilibrium, $dn/dt = 0$; $dp/dt = 0$.

Hence, $np = A(T)/B(T)$.

The product of the electron and hole concentrations is a constant independent of impurity concentration at a given temperature. The introduction of a small portion of a suitable impurity to increase n, say, must decrease p. This result is important. In practice, we can reduce the total carrier concentration $n+p$ in an impure crystal, sometimes enormously, by the controlled introduction of suitable impurities. Such a reduction is called compensation.

2.4.12 Physical Interpretation of Effective Mass

An electron of mass m when put into a crystal respond to applied fields as if the mass were m^*, is the effective mass. The effective mass approximation removes the quantum features of electron motion in a crystal and allows us to use Newton's classical mechanics to determine the effect of external forces on the electrons and holes within the crystal. This fact justifies the use of effective mass of charge carriers in the expression for the density of states.

2.4.13 Fermi Energy Level

For metals at temperatures below their melting point, we have

$$E_F = 3.64 \times 10^{-19}\, n^{2/3}$$

where n is the concentration of free electrons. Since the value of n varies from metal to metal, E_F will also vary among metals. For most metals, $F_F < 10$ eV. The Fermi energy level E_F also varies with temperature, but this variation is very small.

As already established, Fermi level in an intrinsic semiconductor lies at the centre of the forbidden energy gap (at absolute zero). In an extrinsic semiconductor, Fermi level lies just below the conduction band bottom of a n-type semiconductor or Fermi level lies just above the top of the valence band of a p-type semiconductor. In highly doped (with atleast 10^{18} impurity atoms/cm^3) n-type semiconductors, Fermi level is located well inside the conduction band. Similarly, in highly doped p-type semiconductors, Fermi level is located well inside the valence band.

2.4.14 Electrical Conductivity of an Extrinsic Semiconductor

The variation of electrical conductivity of an extrinsic semiconductor as a function of temperature is graphically shown in Fig. 2.11. As seen, the plot ln ρ versus $1/T$ may be divided into three regions.

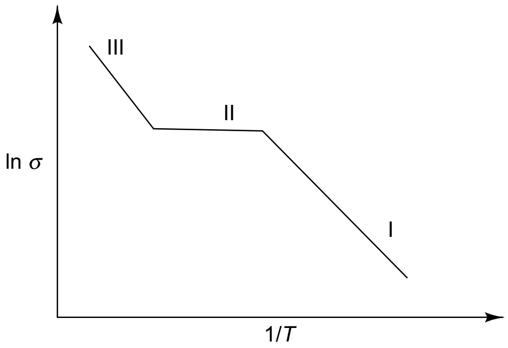


Fig. 2.11 *Electrical Conductivity of an Extrinsic Semiconductor as a Function of Temperature*

Extrinsic Region (I) The carrier concentration increases with increase in temperature. This is due to the shifting of electron from the donor level to the conduction band. Any device making use of n-type or p-type semiconductor should be operated at this temperature range.

Exhaustive Region (II) When the temperature further is increased, the carrier concentration does not change and it remains almost constant. This is so because the donor level has exhausted all its free electrons. Since the impure atoms which are responsible for donor levels are limited in number, there is no further addition of electrons from the donor level to the conduction band when the temperature is increased. This range is called exhaustive range.

Intrinsic Region (III) When the temperature is further increased, the carrier concentration again increases in the same way as exhibited in an intrinsic case. In this temperature range, the electrons from valence band start flowing into the conduction band. That is, now we get electrons not from the impurity atom but from the parent atoms. Hence, the conductivity increases with temperature.

Note: A similar behaviour is exhibited by a p-type semiconductor with the only difference that now the holes are responsible for electrical conduction.

2.5 HALL EFFECT

When a magnetic field is applied at right angles to the current flow in the specimen (of metal or semiconductor), an electric field (called the Hall field) is generated which is mutually perpendicular to the current flow and the magnetic field. This phenomenon is called the Hall effect. The Hall effect is observed in all metals, but it is very prominent in semiconductors.

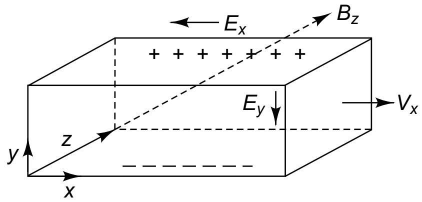


Fig. 2.12 *Rectangular n-type Semiconductor*

2.5.1 Determination of Hall Coefficient

Let the rectangular specimen of an *n*-type semiconductor be simultaneously subjected to the electric field $\vec{E}_x$ (acting along the x axis) and the magnetic field $\vec{B}_z$ (acting along z axis).

The electric field $\vec{E}_x$ exerts a force of $\vec{F} = -e\vec{E}_x$ on the electrons, making them move with a drift velocity $\vec{v}_x$. Here, e is the electronic charge. The magnetic field $\vec{H}_z$ exerts the so-called Lorentz force $\vec{F}_L = -e\,[\vec{V}_x \times \vec{B}_z]$ on electrons. The Lorentz force acting along the y axis makes the electrons to accumulate at the bottom surface and the positive charges collect at the top surface. The edge surfaces act like charged electrodes and the potential difference measured between the top and bottom edges is called the Hall potential or Hall voltage. It increases with increase in $\vec{E}_x$ and $\vec{B}_z$.

The build-up of the charges on the edge surfaces will, in turn, develop an electric field (known as the Hall field) of such polarity that counteracts the collection of charges on the surfaces. The forces on electrons due to Hall field and the Lorentz force balance each other finally. The time required to reach this equilibrium is about 10^{-14} second.

At equilibrium, we have

$$\text{Force exerted on electrons by Hall field} = \text{Force exerted on electrons by Lorentz force}$$

i.e. $$-eE_y = -ev_x B_z$$

$$E_y = v_x B_z \tag{2.43}$$

Considering the expression for mobility of charge carriers $\mu \equiv v_x/E_x$, we get

$$E_y = \mu E_x B_z \tag{2.44}$$

On the other hand, current density in the specimen is given as

$$j_x = -ne\, v_x$$

i.e. $$v_x = -j_x/(ne)$$

Thus, Eq. (2.43) may be reduced to

$$E_y = \left(-\frac{j_x}{ne}\right)B_z$$

$$R_H \equiv \frac{E_y}{j_x B_z} = -\frac{1}{ne} \tag{2.45}$$

where R_H is called the Hall coefficient. The sign of R_H is negative for *n*-type semiconductors and it is positive for *p*-type semiconductors. Thus, it follows from Eq. (2.45) that the carrier type as well as concentration can also be determined from the Hall coefficient.

2.5.2 Hall Angle

From Eq. (2.44), we get

$$\frac{E_y}{E_x} = \mu B_z \tag{2.46}$$

Hall angle ϕ is defined as

$$\phi = \frac{E_y}{E_x} \tag{2.47}$$

From Eqs (2.46) and (2.47), we have

$$\phi = \mu B_z$$

Equation (2.45) gives the Hall field as

$$E_y = R_H J_x B_z \tag{2.48}$$

From Eqs (2.44) and (2.48), we get

$$R_H j_x B_z = \mu E_x B_z$$

$$\mu = R_H \frac{j_x}{E_x} \tag{2.49}$$

As the electrical conductivity is $\sigma = j_x / E_x$, the Eq. (2.49) reduces to

$$\mu = R_H \sigma \tag{2.50}$$

Equation (2.50) gives the relation between Hall coefficient, Hall mobility and electrical conductivity of a specimen.

2.5.3 Experimental Determination of Hall Coefficient

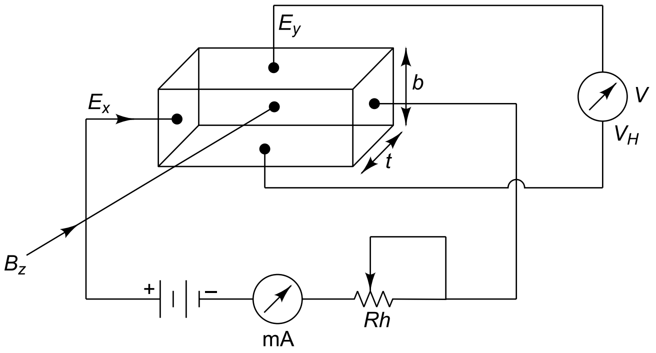


Fig. 2.13 *n-type Ge*

Let us consider a thin strip of n-type Ge being subjected to magnetic field $\vec{H}$ normal to its surface while it carries a current I along the length of the strip, but normal to $\vec{H}$. Let t be the thickness of the specimen and b be the breadth of the specimen.

In this experiment, the external electric field is applied by means of a battery and the current may be altered using a rheostat. The current I through the specimen is measured using an ammeter. The Hall voltage V_H is measured using a sensitive voltmeter and the current through the specimen is noted.

As already established, the Hall coefficient R_H is given as

$$R_H = \frac{E_y}{j_x B_z}$$

Since $j_x = I/(t.b)$ and $E_y = V_H/b$, the Hall coefficient reduces to

$$R_H = \frac{V_H \cdot t}{I B_z} m^3 c^{-1} \tag{2.51}$$

Thus, knowing thickness of the strip, the Hall coefficient may be determined using Eq. (2.51).

Table 2.3 *Experimental Determination of* R_H

S. No.	B_z *Weber/m*^{2}	I *amp*	V_H *volts*	$R_H = \frac{V_H \cdot t}{I B_z} m^3 c^{-1}$

To take number of readings, either B_z is kept constant and by varying I using rheostat, the corresponding Hall voltage is measured; or I is kept constant by varying B_z and the corresponding Hall voltage is measured. The readings are tabulated. Using the above formula, the Hall coefficient is calculated and tabulated in the last column. The values in the last column are to be constant. By taking the average value of the last column, the Hall coefficient R_H is determined.

2.5.4 Applications of Hall Effect

- The sign of the current-carrying charge is determined.
- Carrier concentration can be calculated.
- Mobility of charge carriers is determined.
- Electrical conductivity is determined.
- Hall effect can be used to determine the electronic structure of the specimen: whether the specimen is a conductor, insulator or semiconductor.
- Knowledge of R_H enables us to measure unknown magnetic fields.

- Hall elements are used in electronic multimeters and electronic wattmeters.

2.6 SEMICONDUCTOR DEVICES

2.6.1 Solar Cell

It is a semiconducting device which converts solar energy into electrical energy. It is also known as photovoltaic cell or optoelectronic device.

2.6.2 Construction

An *n*-type silicon wafer of $\frac{1}{2}$ mm thickness is taken and over it boron is diffused at 1000°C for about 20 minutes forming a *p*-type layer. The top *p* layer is of thickness of few microns. This ensures that much solar radiation is not absorbed by the top layer. The top layer is given conducting grid coatings. This enables us to collect the carriers as soon as it is produced. When a *pn* junction layer is formed, a barrier voltage develops and this is an essential criteria for power generation. The commercial silicon solar cell is shown in Fig. 2.14. Ohmic contacts are taken from the top and bottom by nickel plating. Leads are connected to the ohmic contacts and these leads are in turn connected to external loads to deliver power.

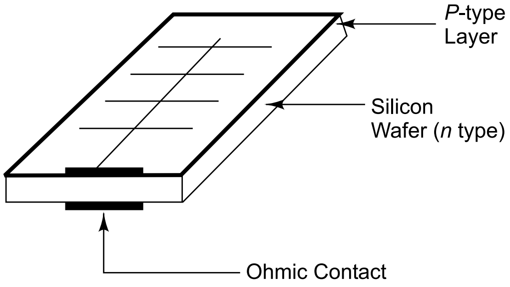


Fig. 2.14 *Solar Cell*

2.6.3 Principle of Power Generation

When solar radiation falls on the thin *p* layer of the solar cell, the radiation will reach the *pn* junction where we have the electric field produced by the barrier potential V_B.

This incident photons produce electron hole pairs. They are separated by the electric field of the barrier potential. The electrons are swept to the *n* side and holes are swept to the *p* side by the potential barrier. On the *p* side, we have the accumulation of positive charges and on the *n* side, accumulation of negative charges. These accumulated charges of opposite sign give rise to an emf which sends current through the external load (Fig. 2.15).

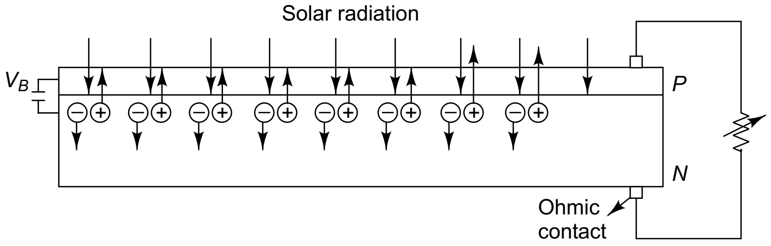


Fig. 2.15 *Power Generation in a Solar Cell*

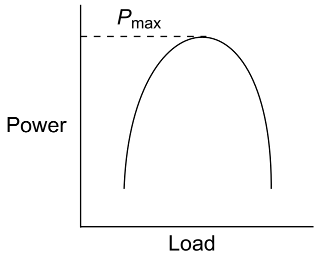


Fig. 2.16 *Load versus Power*

2.6.4 Quantum Efficiency

$$\text{Quantum efficiency} = \frac{\text{No. of electron-hole pairs produced}}{\text{No. of photons incident}}$$

Efficiency = η = Electric Power Output/Light Power Input

The graph plotted between the load and output power is shown in Fig. 2.16. The system delivers maximum power P_{max} at a specific load and it is found out.

2.6.5 Methods to Increase Efficiency

- Efficiency can be increased by giving anti-reflection coating.
- Efficiency can be increased by using Tandem cells. When radiation is incident on multilayer cells of different materials,

then each cell will absorb a particular range of wavelength suited for it. So the output will be the sum of the outputs of all the cells. Hence, for the same input power we get large output power because all the wavelengths of the incident solar radiation is utilized. So efficiency is increased.

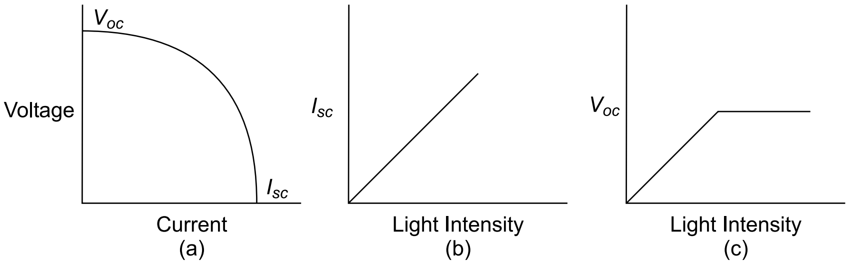


Fig. 2.17 *(a) Current versus Voltage (b) Light Intensity versus Intensity Short Circuit Current (c) Light versus Open Circuit Voltage*

- Figure 2.17(a) shows the current voltage characteristic of the solar cell. From this graph, short circuit current I_{sc}, open circuit voltage V_{oc} are measured for different light intensities. It is found from Fig. 2.17(b) that I_{sc} varies linearly with light intensity. This principle is utilized in the construction of lightmeters. Figure 2.17(c) shows that V_{oc} remains constant beyond a certain light level. This principle is utilized in charging batteries.

2.6.6 Advantages

- As solar energy is available in plenty, there is no shortage of fuel in power generation using solar cell.
- It does not cause any pollution threats unlike thermal power stations.
- It is free from any health hazards unlike nuclear reactors.
- It is noise-free unlike hydroelectric power stations.

2.6.7 Disadvantages

- Solar cells are costly and hence it not suitable for large-scale power generation.
- Efficiency is low.

2.6.8 Applications

- They are used in charging batteries.
- They are used in the operation of street lights.
- They are used in the operation of electric motors.
- They are used in calculators.
- They are used in road signals.

2.6.9 LDR

Light Dependent Resistor (LDR) is a semiconducting device and its resistance varies depending upon the intensity and wavelength of the incident light.

An LDR is made up of a piece of semiconductor material which has high sensitivity to light. When light falls on a semiconductor, its resistance decreases and hence the conductivity of the material increases. This occurs due to the rupture of covalent bonds and hence electron-hole pairs are created. It results in an increase of charge carriers in the material. As light changes the conductivity of the semiconductor material, it is known as LDR.

At room temperature, the conductivity of LDR is low and a small current flows in the circuit even in the absence of illumination. When it gets illuminated, its resistance decreases and the current in the circuit increases in proportion to the illumination.

LDR is made up of Cadmium Sulphide (CdS), Lead Sulphide (PbS) and Cadmium Selenide (CdSe). They are manufactured as a thin film by depositing a thin layer of semi-conductor on an insulating substrate with electrodes previously deposited on it.

When an electric field is applied to the semiconductor, the electrons and holes generated by the illumination of light get separated and drift towards positive and negative electrodes of the voltage source and an electric current flows in the circuit as shown in Fig. 2.18.

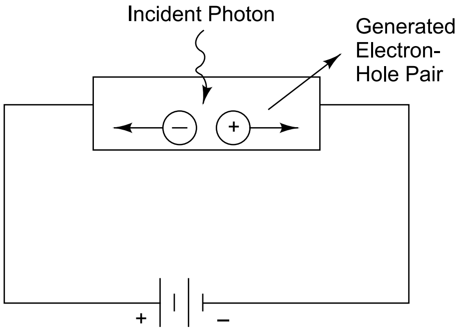


Fig. 2.18 *Electric Current in a Semiconductor*

2.6.10 Characteristics

Figure 2.19 shows the resistance of the semiconductor versus illuminated light intensity characteristics of LDR. It is seen that the resistance of LDR decreases rapidly with the increase in intensity of the incident light.

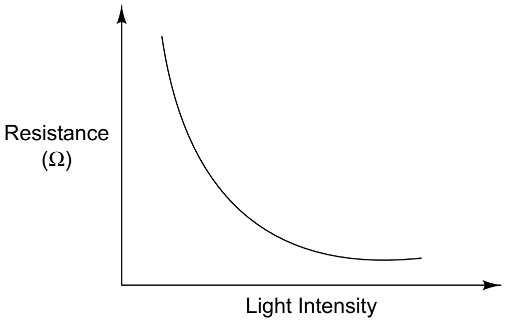


Fig. 2.19 *Resistance of a Semiconductor w.r.t. Light Intensity*

2.6.11 Applications

- They are used in street light control.
- They are used in camera exposure control.
- They are used in automatic head light control.
- They are used in optical switches.
- They are used in position sensors.

SOLVED EXAMPLES

Example 2.1 *The resistivity of silicon at 300 K is* 3.16×10^3 *ohm.m. The mobilities of electrons and holes in silicon are* $0.14\ m^2 / V.s$ *and* $0.05\ m^2/V.s$*, respectively. Calculate the intrinsic carrier concentration of silicon at 300 K.*

Given Data

$\rho_i = 3.16 \times 10^3$ ohm.m

$\mu_e = 0.14$ m^2/V.s

$\mu_p = 0.05$ m^2/V.s

$n_i = ?$

The electrical conductivity of an intrinsic semiconductor is given as

$$\sigma_i = n_i e\,(\mu_n + \mu_p)$$

i.e.
$$\frac{1}{\rho_i} = n_i e(\mu_n + \mu_p) \qquad (1)$$

where ρ_i is the electrical resistivity and e is the charge of the electron.

Equation (1) may be rewritten as

$$n_i = \frac{1}{\rho_i\, e(\mu_n + \mu_p)} \qquad (2)$$

Substituting the given data for n_i, e, μ_n, μ_p into Eq. (2), we get

$$n_i = \frac{1}{3.16\times10^{3}\times1.6\times10^{-19}\times(0.14+0.01)}$$

i.e. $\mathbf{n_i = 1.6 \times 10^{16}\ m^{-3}}$

Example 2.2 *The intrinsic carrier density at room temperature in Ge is 2.37 × 10^{19} m^3. If the electron and hole mobilities are 0.38 and 0.18 m^2/V.s, respectively, calculate the resistivity.*

(A.U., B.Tech(Chem.), May/June 2006)

Given Data

$n_i = 2.37 \times 10^{19}/\text{m}^3$

$\mu_n = 0.38\ \text{m}^2/\text{V.s}$

$\mu_p = 0.18\ \text{m}^2/\text{V.s}$

$\rho_i = ?$

Electrical conductivity of an intrinsic semiconductor is given as

$$\sigma_i \equiv \frac{1}{\rho_i} = n_i e(\mu_n + \mu_p) \qquad (1)$$

Equation (1) gives the resistivity as

$$\rho_i = \frac{1}{n_i e(\mu_n + \mu_p)} \qquad (2)$$

Substituting the given values of n_i, e, μ_n, μ_p into Eq. (2), we get

$$\rho_i = \frac{1}{2.37\times10^{19}\times1.6\times10^{-19}\times(0.38+0.18)}$$

i.e. $\boldsymbol{\rho_i}$ **= 0.47 ohm.m**

Example 2.3 *Calculate the conductivity of germanium at 200°C. Given that at 300 K,* $n_i = 23\times10^{18}\ m^{-3}$, $\mu_n = 0.364\ m^2\ V^{-1}\ s^{-1}$ *and* $\mu_p = 0.19\ m^2\ V^{-1}\ s^{-1}$.

(A.U., B.E.(Mech.), May/June 2006)

Given Data

$n_i = 23\times10^{18}/\text{m}^{-3}$

$\mu_n = 0.364\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$

$\mu_p = 0.19\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$

$\sigma_i = ?$

Electrical conductivity of an intrinsic semiconductor is given as

$$\sigma_i = n_i e\,(\mu_n + \mu_p) \qquad (1)$$

Substituting the given values of n_i, e, μ_n, μ_p into Eq. (1), we get

$$\sigma_i = 23\times10^{18}\times1.6\times10^{-19}\times(0.36+0.19)$$

i.e. $\boldsymbol{\sigma_i}$ **= 2.04 ohm^{-1}.m^{-1}**

Example 2.4 *The electron mobility and hole mobility in silicon are 0.17 m²/V. s and 0.035 m²/V. s, respectively at room temperature. If the carrier concentration is* $1.1\times10^{16}\ m^{-3}$, *calculate the resistivity of silicon at room temperature.*

(A.U., B.E. (ECE), May/June 2005)

(A.U., B.E./B.Tech, May/June 2011)

Given Data

$n_i = 1.1\times10^{16}\ \text{m}^{-3}$

$\mu_n = 0.17\ \text{m}^2/\text{V-s}$

$\mu_p = 0.035\ \text{m}^2/\text{V-s}$

$\rho_i = ?$

Electrical conductivity of an intrinsic semiconductor is given as

$$\sigma_i \equiv \frac{1}{\rho_i} = n_i e\,(\mu_n + \mu_p) \tag{1}$$

Equation (1) gives the resistivity as

$$\rho_i = \frac{1}{n_i e(\mu_n + \mu_p)} \tag{2}$$

Substituting the given values of n_i, e, μ_n, μ_p, into Eq. (2), we get

$$\rho_i = \frac{1}{1.1 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.17 + 0.035)}$$

i.e. $\boldsymbol{\rho_i = 2.77 \times 10^3}$ **ohm.m**

Example 2.5 *In an intrinsic semiconductor, the energy gap is 1.2 eV. What is the ratio between its conductivity at 600 K and that at 300 K? Given : 1 eV =* 1.602×10^{-19} *J.*

(A.U., B.E.(ECE), May/June 2005)

Given Data

$E_g = 1.2$ eV $= 1.2 \times 1.6 \times 10^{-19}$ J
$T_1 = 600$ K
$T_2 = 300$ K

$$\frac{\sigma_{T_1}}{\sigma_{T_2}} = ?$$

The electrical conductivity of an intrinsic semiconductor is given as

$$\sigma = \sigma_0 \exp\left(-\frac{E_g}{2kT}\right)$$

Hence, we have

$$\sigma_{T_1} = \sigma_0 \exp\left(-\frac{E_g}{2kT_1}\right) \tag{1}$$

$$\sigma_{T_2} = \sigma_0 \exp\left(-\frac{E_g}{2kT_2}\right) \tag{2}$$

From Eqs (l) and (2), we get

$$\frac{\sigma_{T_1}}{\sigma_{T_2}} = \exp\left(\frac{E_g}{2k}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right) \tag{3}$$

Substituting the given values of E_g, k, T_1, T_2, into Eq. (3), we get

$$\frac{\sigma_{T_1}}{\sigma_{T_2}} = \exp\left(\frac{1.2 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}}\left(\frac{1}{300} - \frac{1}{600}\right)\right)$$

i.e. $$\boldsymbol{\frac{\sigma_{T_1}}{\sigma_{T_2}} = 1.1 \times 10^5}$$

Example 2.6 *The conductivity of a semiconductor at 20°C is 250 ohm^{-1} m^{-1} and at 100°C is 110 ohm^{-1}. m^{-1}. What is its bandgap, E_g?*

(A.U., B.E.(ECE), May/June 2005)

Given Data

T_1 = 20ºC = 293 K

σ_{T_1} = 250 ohm^{-1}.m^{-1}

T_2 = 100°C = 373 K

σ_{T_1} = 1100 ohm^{-1}.m^{-1}

E_g = ?

The electrical conductivity of an intrinsic semiconductor is given as

$$\sigma = \sigma_0 \exp\left(-\frac{E_g}{2kT}\right)$$

Hence, we have

$$\sigma_{T_1} = \sigma_0 \exp\left(-\frac{E_g}{2kT_1}\right) \tag{1}$$

$$\sigma_{T_2} = \sigma_0 \exp\left(-\frac{E_g}{2kT_2}\right) \tag{2}$$

From Eqs (1) and (2), we get

$$\frac{\sigma_{T_1}}{\sigma_{T_2}} = \exp\left(\frac{E_g}{2k}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right) \tag{3}$$

i.e. $$E_g = \frac{2kT_1T_2}{(T_2 - T_1)} \times \ln\left(\frac{\sigma_{T_2}}{\sigma_{T_1}}\right) \tag{4}$$

Substituting the given values of k, T_1, T_2, σ_{T_1}, σ_{T_2}, into Eq. (4), we get

$$E_g = \frac{2 \times 1.38 \times 10^{-23} \times 293 \times 373}{80} \times \ln\left(\frac{1100}{250}\right)$$

i.e. $\mathbf{E_g = 0.56 \times 10^{-19}\ J}$

or $\mathbf{E_g = 0.35\ eV}$

Example 2.7 *Germanium at 20°C has the electrical conductivity of 2 ohm^{-1}.m^{-1}. It has a bandgap of 0.72 eV. Calculate the electrical conductivity of germanium at 40°C.*

Given Data

$\sigma_1 = 2\ \text{ohm}^{-1} \cdot \text{m}^{-1}$

$E_g = 0.72\ \text{eV} = 0.72 \times 1.6 \times 10^{-19}\ \text{J}$

$T_1 = 20°\text{C} = 293\ \text{K}$

$T_2 = 40°\text{C} = 313\ \text{K}$

$\sigma_2 = ?$

The temperature dependence of electrical conductivity of an intrinsic semiconductor is given as

$$\sigma_0 = \sigma_0 \exp\left(-\frac{E_g}{2kT}\right) \tag{1}$$

where σ_0 is a material-specific constant and k is the Boltzmann constant. Applying Eq. (1) to the given cases, we get

$$\sigma_{T_1} = \sigma_0 \exp\left(-\frac{E_g}{2kT_1}\right) \tag{2}$$

$$\sigma_{T_2} = \sigma_0 \exp\left(-\frac{E_g}{2kT_2}\right) \tag{3}$$

Equations (2) and (3) give

$$\frac{\sigma_{T_2}}{\sigma_{T_1}} = \exp\left[\frac{E_g}{2k}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

i.e. $$\sigma_{T_2} = \sigma_{T_1} \exp\left[\frac{E_g}{2k}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \tag{4}$$

Substitution of the given values for σ_1, E_g, T_1, T_2 , k into Eq. (4) gives

$$\sigma_{T_2} = 2 \times \exp\left[\frac{0.72 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}}\left(\frac{1}{293} - \frac{1}{313}\right)\right]$$

i.e. $$\boldsymbol{\sigma_{T_2} = 4.97\ \text{ohm}^{-1}.\ \text{m}^{-1}}$$

Example 2.8 *The electron mobility in a pure semiconductor is 50 m²/Volt-s at 4.2 K. What is the mobility at 300 K?*

(A.U., B.Tech(Chem.), May/June 2006)

Given Data

$\mu_{T1} = 50\ \text{m}^2\ \text{Volt-s}$

$T_1 = 4.2$ K; $T_2 = 300$ K

$\mu_{T2} = ?$

Mobility of charge carriers is given as

$$\mu \propto T^{-3/2}$$

That is, we have

$$\mu = CT^{-3/2} \tag{1}$$

where C is the proportionality constant.

In the given case, Eq. (1) gives

$$\mu_{T_1} = CT_1^{-3/2} \tag{2}$$

$$\mu_{T_2} = CT_2^{-3/2} \tag{3}$$

From Eqs (2) and (3), we get

$$\mu_{T_2} = \mu_{T_1}\left(\frac{T_1}{T_2}\right)^{3/2} \tag{4}$$

Substitution of μ_{T_1}, T_1, T_2 into Eq. (4) gives

$$\mu_{T_2} = 50 \times \left(\frac{4.2}{300}\right)^{3/2}$$

i.e. $\mathbf{\mu_{T_2} = 0.083\ m^2/\ V.\ s}$

Example 2.9 *The bandgap of diamond at 27°C is 5.6 eV. Find the probability that an electron in diamond at 27°C is thermally promoted to the conduction band.*

(A.U., B.E. (Mech) May /June 2006)

Given Data

$T_1 = 27°\text{C} = 300\ \text{K}$

$E_g = 5.6\ \text{eV} = 5.6 \times 1.6 \times 10^{-19}\ \text{J}$

$f(E_c) = ?$

The probability that an electron being thermally promoted to the conduction band is given as

$$f(E_c) = \frac{1}{1 + \exp\left(\dfrac{E_g}{2kT}\right)} \qquad (1)$$

where k is the Boltzmann constant.

Substituting the given data for E_g, T, k into Eq. (l), we get

$$f(E_c) = \frac{1}{1 + \exp\left(\dfrac{5.6 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 300}\right)}$$

i.e. $\mathbf{f\ (E_c) = 1.7 \times 10^{-47}}$

Since the probability $f_c(E_c)$ is extremely small, electrons cannot be thermally promoted to the conduction band of diamond at 27°C.

Example 2.10 *Estimate the fraction of electrons in the conduction band of germanium at 300 K. The bandgap of germanium is 0.72 eV.*

Given Data

$E_g = 0.72 \text{ eV} = 0.72 \times 1.6 \times 10^{-19}$ J

$T = 300$ K

$\frac{n}{N} = ?$

The fraction of electrons in the conduction band to that in the valence band is given as

$$\frac{n}{N} \equiv f(E_c) = \frac{1}{1 + \exp\left(\frac{E_c - E_F}{kT}\right)} \quad (1)$$

Equation (1) may be rewritten as

$$\frac{n}{N} = \frac{1}{1 + \exp\left(\frac{E_g}{2kT}\right)} \quad (2)$$

Moreover, $E_g/2 >> kT$ at room temperature. Hence, we have

$$\frac{n}{N} = \frac{1}{\exp\left(\frac{E_g}{2kT}\right)} \quad (2)$$

Substitution of the given values for E_g, T, k into Eq. (2), gives

$$\frac{n}{N} = \frac{1}{\exp\left(\frac{0.72 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 300}\right)}$$

i.e. $$\mathbf{\frac{n}{N} = 9.7 \times 10^{-7}}$$

Example 2.11 *The Hall coefficient of a specimen of doped silicon is found to be* 3.66×10^{-4} m^3 *coulomb*$^{-1}$*, and the resistivity of the specimen is* 8.93×10^{-3} *ohm . m. Find the mobility and density of charge carriers, assuming single carrier conduction.*

Given Data

$R_H = 3.66 \times 10^{-4}$ m^3 coulomb^{-1}

$\rho = 8.93 \times 10^{-3}$ ohm . m

$\mu = ?$ $\quad n = ?$

Mobility of the charge carriers is given as

$$\mu = R_H\ \sigma \equiv \frac{R_H}{\rho} \tag{1}$$

Substitution of the given data for R_H, ρ into Eq. (l) gives

μ = 0.041 m^2/V.sec.

On the other hand, Hall coefficient is given as

$$R_H = \frac{1}{ne}$$

i.e.
$$\text{n} = \frac{1}{R_H \cdot e} \tag{2}$$

Substitution of the given data for R_H, e into Eq. (l) gives

n = 1.707 × 10^{22}/m^3

Example 2.12 *A specimen of a semiconductor has a Hall coefficient of 3.66 × 10^{-4} m^3 coulomb^{-1} and has a resistivity of 8.93 × 10^{-3} ohm.m. In a Hall effect experiment, a magnetic flux density of 0.5 weber m^{-2} is used. Find the Hall angle.*

Given Data

$\rho_i = 8.93 \times 10^{-3}$ ohm.m

$R_H = 3.66 \times 10^{-4}$ m^3. coulomb^{-1}

$B_z = 0.5$ weber . m^{-2}

$\phi = ?$

Hall angle is given as

$$\phi \overset{\text{def}}{=} \frac{E_y}{E_x} \equiv \mu H_z \tag{1}$$

On the other hand, we have

$$\mu = R_H \cdot \sigma \equiv R_H \cdot \frac{1}{\rho} \qquad (2)$$

From Eqs (1) and (2), we get

$$\phi = \frac{R_H . B_z}{\rho}$$

Substitution of the given values of R_H, B_z, ρ into Eq. (3) gives

$$\phi = \frac{3.66 \times 10^{-4} \times 0.5}{8.93 \times 10^{-3}}$$

i.e. $\boldsymbol{\phi = 0.021}$

Example 2.13 *In an n-type germanium, the electron and hole concentrations are* 3.5×10^{25} m^{-3} *and* 1.5×10^{15} m^{-3}*, respectively. The electron and hole mobilities in the given semiconductor are 0.39* m^2*/V.s and 0.19* m^2*/V.s, respectively. Compare the conductivities of doped and intrinsic germanium.*

Given Data

$n_n = 3.5 \times 10^{25}$ m^{-3}

$p_n = 1.5 \times 10^{25}$ m^{-3}

$\mu_n = 0.39$ m^2/V.s

$\mu_p = 0.19$ m^2/V.s

$\frac{\sigma_n}{\sigma_i} = ?$

The electrical conductivity of an n-type semiconductor is given as

$$\sigma_n = n_n \, e \, \mu_n \qquad (1)$$

On the other hand, the law of mass action gives the concentration of the intrinsic semiconductor as

$$n_i^2 = n_n p_n$$

i.e. $$n_i = \sqrt{n_n p_n}$$

The electrical conductivity of the intrinsic semiconductor is given as

$$\sigma_i = n_i e (\mu_n + \mu_p)$$

i.e. $$\sigma_i = \sqrt{n_n p_n}(\mu_n + \mu_p)e \tag{2}$$

From Eqs (l) and (2), we get

$$\frac{\sigma_n}{\sigma_i} = \sqrt{\frac{n_n}{p_n}} \frac{\mu_n}{(\mu_n + \mu_p)} \tag{3}$$

Substituting the given values of n_n, P_n, μ_n, μ_p, into Eq. (3), we get

$$\frac{\sigma_n}{\sigma_i} = \sqrt{\frac{3.5\times10^{25}}{1.5\times10^{15}}} \frac{0.39}{(0.39\times0.19)}$$

i.e $$\boldsymbol{\frac{\sigma_n}{\sigma_i} = 1.03 \times 10^5}$$

This is, upon doping, the conductivity of the given semiconductor increases by a factor of 1.03×10^5.

Example 2.14 *The intrinsic carrier concentration of germanium at 300 K is 1.7×10^{19} m^{-3}. Density of germanium at 300 K is 5320 kg/m^3 and its atomic weight is 72.59. It is doped with a pentavalent impurity in the concentration of 1 ppm. Assume that all the impurity atoms are ionized at the given temperature. Calculate the hole concentration in the doped germanium at 300 K.*

Given Data

$n_i = 1.7 \times 10^{19}$ m^{-3}

Dopant concentration = 1 ppm

$\rho' = 5320$ kg/m^3

$M = 72.59$ g/mol $= 72.59 \times 10^{-3}$ kg/mol

$P_n = ?$

Law of mass action gives the hole concentration in the p-type semiconductor as

$$n_i^2 = n_n p_n$$

i.e. $$p_n = \frac{n_i^2}{n_n} \tag{1}$$

Obviously, the concentration of germanium atoms is

$$N = \frac{\rho' \cdot N_A}{M}$$

As the dopant concentration is 1 ppm, we get the concentration of pentavalent impurity atoms as

$$N_d = N \times 10^{-6}$$

Hence, we have

$$N_d = \frac{\rho' \cdot N_A}{M} \times 10^{-6}$$

As all the impurity atoms are ionized, we get

$$n_n = N_d$$

i.e.

$$n_n = \frac{\rho' N_A}{M} \times 10^{-6} \tag{2}$$

From Eqs (1) and (2), we get

$$p_n = \frac{n_i^2 \cdot M}{\rho' N_A \times 10^{-6}} \tag{3}$$

Substituting of the given values for n_i, M, ρ' and for N_A into Eq. (3) gives

$$p_n = \frac{(1.7 \times 10^{19})^2 \times 72.59 \times 10^{-3}}{5320 \times 6.023 \times 10^{23}}$$

i.e. $\mathbf{p_n = 6.6 \times 10^{15}\ m^{-3}}$

Example 2.15 *The Hall coefficient of a certain silicon specimen was found to be –7.35 × 10*$^{-5}$ *m*^{2}*/C from 100 to 400 K. Determine the nature of the semiconductor. If the conductivity was found to be 200 ohm*$^{-1}$*.m*$^{-1}$*. Calculate the density and mobility of the charge carrier.*

(A.U., B.Tech(Chem.), May/June 2006)

Given Data

$R_H = 7.35 \times 10^{-5}\ \mathrm{m^2/C}$

$\sigma = 200\ \mathrm{ohm^{-1} \cdot m^{-1}}$

$M = 72.59\ \mathrm{g/mol} = 72.59 \times 10^{-3}\ \mathrm{kg/mol}$

Nature of semiconductor = ?; n = ?; μ = ?

As the Hall coefficient has a negative value, the given semiconductor is an *n*-type semiconductor.

Hall coefficient of that *n*-type semiconductor is given as

$$R_H = -\frac{1}{ne} \tag{1}$$

Equation (1) may be rewritten as

$$n = -\frac{1}{R_H \cdot e} \tag{2}$$

Substituting the given values of R_H, e into Eq. (2), we get

$$n = -\frac{1}{(-7.35 \times 10^{-5}) \times 1.6 \times 10^{-19}}$$

i.e. $n = 8.5 \times 10^{22}\ \text{m}^{-3}$

Mobility of the *n*-type semiconductor is given as

$$\mu = |R_H| \cdot \sigma \tag{3}$$

Substitution of the given values of R_H, σ into Eq. (3), gives

$$\mu = 7.35 \times 10^{-5} \times 200$$

i.e. $\boldsymbol{\mu = 0.0147\ \text{m}^2/\text{V.sec}}$

Example 2.16 *The sample of silicon is doped with 10^{16} phosphor atoms/cm^3. Find the Hall voltage in a sample with 500 mm thickness, 2.5×10^{-3} cm^{-2} area of cross-section, 1 A current and magnetic field Bz as 10 Wb/cm^2.*

(A.U., B.E.(ECE), May/June 2005)

Given Data

$N_d \equiv n = 10^{16}/\text{cm}^{+3} = 10^{22}/\text{m}^3$

$t = 500\ \mu\text{m} = 500 \times 10^{-6}\ \text{m}$

$A = 2.5 \times 10^{-3}\ \text{cm}^{-2} = 2.5 \times 10^{-2}\ \text{m}^{-2}$

$B_z = 10\ \text{Wb/cm}^2 = 10 \times 10^4\ \text{Wb/m}^2$

$I = IA$

$V_H = ?$

In a Hall effect arrangement, the Hall voltage is given as

$$V_H = \frac{R_H \cdot I.B_z}{t}$$

or $$V_H = \frac{IB_z}{\text{net}} \qquad (1)$$

Subsituting the given values of I, B_z, n, e, t into Eq (l), we get

$$V_H = \frac{1 \times 10 \times 10^4}{10^{22} \times 1.6 \times 10^{-19} \times 500 \times 10^{-6}}$$

i.e. $\mathbf{V_H = 1.25 \times 10^5}$ **volts**

Example 2.17 *The intrinsic carrier density of an unknown material is 1.5 × $10^{16}/m^3$. If the electron and hole mobilities are 0.13 and 0.05 $m^2/V/s$. Calculate its electrical conductivity.*

(A.U., B.E./B.Tech, May/June 2011)

Given Data

$n_i = 1.5 \times 10^{16}/\text{m}^3$

$\mu_n = 0.13\ \text{m}^2/\text{V/s}$

$\mu_1 = 0.05\ \text{m}^2/\text{V/s}$

$\sigma_i = ?$

$$\sigma_i = n_i e\,(\mu_n + \mu_p)$$
$$= 1.5 \times 10^{16} \times 1.6 \times 10^{-19} \times (0.13 + 0.05)$$
$$\sigma_i = 4.32 \times 10^{-4}\ \text{ohm}^{-1}\ \text{m}^{-1}$$

Example 2.18 *An n-type germanium sample has a donor density of $10^{21}/m^3$. It is arranged in a Hall experiment having B = 0.5 W/m^2 and T = 500 A/m^2. Find the Hall voltage if the sample is 3 mm thick.*

(A.U., B.E./B.Tech, May/June 2011)

Given Data

$N_d = n = 10^{21}/\text{m}^3$

$t = 3 \times 10^{-3}\ \text{m}$

$B_z = 0.5\ \text{W/m}^2$

$I_x = 500\ \text{A/m}^2$

$V_y = ?$

$$V_y = \frac{I_x B_z}{net}$$

$$= \frac{500 \times 0.5}{10^{21} \times 1.6 \times 10^{-19} \times 3 \times 10^{-3}}$$

$$V_y = 52.08 \text{ V}$$

QUESTIONS AND ANSWERS

1. What are semiconductors?

Semiconductors are the materials with electrical conductivity lying in between that of conductors and insulators. The energy band theory explains the electrical properties of a semiconductor by the presence of a narrow energy gap of about 1 to 3 eV between the conduction and valence bands.

2. What are elemental semiconductors? Give examples.

They consist of only one chemical element.

Examples : Ge, Si, Te, α–Sn etc.

3. What are compound semiconductors? Give examples.

(A.U., B.E./B.Tech, May/June 2011)

They consist of more than one element.

Examples: GaAs, InP, A1P, CdS, CdSe etc.

4. What are direct bandgap semiconductors? Give examples.

The electrons at the conduction band bottom and at the valence band top have the same momentum. That is, when an electron in the valence band is excited to the conduction material band, its momentum does not change.

Examples: GaAs, GaSb, InP, InAs, InSb, α–Sn, Te.

5. What are indirect bandgap semiconductors? Give examples.

The electrons at the conduction band bottom and at the valence band top have different momentum. That is, when an electron in the valence band is excited to the conduction band, its momentum changes.

Examples: Ge, Si, A1P, AlAs, AlSb, GaP.

6. **What are intrinsic semiconductors?**

   These semiconductors are chemically pure without any impurity in which $n = p = n_i$

   where n is the electron concentration in conduction band, p is the hole concentration in valence band and n_i is the intrinsic carrier concentration.

7. **Give the expression for the carrier concentration of an intrinsic semiconductors at low temperatures.**

   The intrinsic carrier concentration is then given as

   $$n_i = AT^{3/2}\exp\left(-\frac{E_g}{2kt}\right)$$

   where $A \equiv B^{1/2} = \left(\gamma_n\gamma_p \frac{\pi}{4}k^3\right)^{1/2} = \text{const.}$; E_g is the energy bandgap, k is the Boltzmann constant and T is the temperature.

8. **Give the expression for the electrical conductivity of an intrinsic semiconductor.**

   The electrical conductivity of an intrinsic semiconductor is

   $$\sigma_i = \sigma_o \exp\left(-\frac{E_g}{2kT}\right)$$

   where σ_o is a constant for the given intrinsic semiconductor which represents its electrical conductivity at $T = 0$ K; E_g is the energy bandgap, k is the Boltzmann constant and T is the temperature.

9. **Give the expression for the Fermi energy of an intrinsic semiconductor at 0 K.**

   Fermi energy of an intrinsic semiconductor at 0 K given as

   $$E_F = \frac{E_c + E_v}{2}$$

   where E_c is the conduction band bottom and E_v is the valence band top.

10. **Give the expression for the temperature dependence of Fermi energy of an intrinsic semiconductor at a non-zero temperature.**

    Fermi energy of an intrinsic semiconductor at a non-zero temperature is given as

$$E_F = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{G_v}{G_c}\right)$$

where E_c is the conduction band bottom and E_v is the valence band top; k is the Boltzmann constant, T is the temperature; G_v and G_c are the effective densities of states in the valence and conduction bands, respectively.

11. What are extrinsic semiconductors?

By suitable doping, a semiconducting crystal can be made to have unequal values of electron and hole concentrations. The resulting semiconductor is said to be an extrinsic one. For an extrinsic semiconductor, therefore, we have $n \neq p$.

12. What are *n*-type semiconductors?

These are doped semiconductors with greater electron concentration in conduction band than hole concentration in valence band.

i.e. $n > p, \quad n > n_i$

where n_i is the intrinsic carrier concentration.

13. What are *p*-type semiconductors?

These are impurity semiconductors with greater hole concentration in valence band than electron concentration in conduction band

i.e. $p > n, \quad p > n_i$

where n_i is the intrinsic carrier concentration.

14. Give the expression for the carrier concentration in an *n*-type semiconductor at low temperatures.

(A.U., B.E./B.Tech, May/June 2011)

The electron concentration in an *n*-type semiconductor is

$$n = (G_c \cdot N_d)^{1/2} \exp\left(\frac{E_d - E_c}{2kT}\right)$$

where G_c is the effective density of states in the conduction band, N_d is the donor concentration, E_d is the donor energy level, E_c is the conduction band bottom, k is the Boltzmann constant and T is the temperature.

15. Give the expression for the Fermi energy of an *n*-type semiconductor at 0 K. ***(A.U., B.E./B.Tech, May/June 2011)***

Fermi energy of an *n*-type semiconductor at 0 K is given as

$$E_F = \frac{E_c + E_d}{2}$$

where E_d is the donor energy level and E_c is the conduction band bottom.

16. Give the expression for the temperature dependence of Fermi energy of an *n*-type semiconductor at a non-zero temperature.

Fermi energy of an *n*-type semiconductor at a non-zero temperature is given as

$$E_F = \frac{E_c + E_d}{2} + \frac{kT}{2}\ln\left(\frac{N_d}{G_c}\right)$$

where G_c is the effective density of states in the conduction band, N_d is the donor concentration, E_d is the donor energy level, E_c is the conduction band bottom, k is the Boltzmann constant and T is the temperature.

17. Give the expression for the carrier concentration in a *p*-type semiconductor at low temperatures.

The hole concentration in valence band of a *p*-type semiconductor is

$$p = (G_v \, . \, N_a)^{1/2} \exp\left(\frac{E_v - E_a}{2kT}\right)$$

where G_v is the effective density of states in the valence band, N_a is the acceptor concentration, E_a is the acceptor energy level, E_v is the valence band top, k is the Boltzmann constant and T is the temperature.

18. Give the expression for the Fermi energy of a *p*-type semiconductor at 0 K.

Fermi energy of a *p*-type semiconductor at 0 K is given as

$$E_F = \frac{E_a + E_v}{2}$$

where E_a is the acceptor energy level and E_v is the valence band top.

19. Give the expression for the temperature dependence of Fermi energy of a *p*-type semiconductor at a non-zero temperature.

Fermi energy of a *p*-type semiconductor at a non-zero temperature is given as

$$E_F = \frac{E_a + E_v}{2} + \frac{kT}{2}\ln\left(\frac{G_v}{N_a}\right)$$

where G_v is the effective density of states in the valence band, N_a is the acceptor concentration, E_a is the acceptor energy level, E_v is the valence band top, k is the Boltzmann constant and T is the temperature.

20. Define Hall effect.

When a magnetic field is applied at right angles to the current flow in the specimen (of metal or semiconductor), an electric field (called the Hall field) is generated which is mutually perpendicular to the current flow and the magnetic field. This phenomenon is called the Hall effect.

21. Define Hall voltage.

When a magnetic field is applied at right angles to the current flow in the specimen (of metal or semiconductor), an electric potential difference is established in a direction mutually perpendicular to the current flow and the magnetic field. This electric potential difference is called the Hall voltage.

22. Define Hall coefficient.

The Hall coefficient is defined as

$$R_H \equiv \frac{E_y}{j_x B_z} = -\frac{1}{ne}$$

where E_y is the Hall electric field, j_x is the current density, B_z is the applied magnetic field, n is the electron concentration and e is the charge of the electron.

23. Define Hall angle.

The Hall angle is defined as

$$\phi = \frac{E_y}{E_x}$$

where E_y is the Hall electric field and E_x is the applied electric field.

24. Mention a few applications of Hall effect.

- Sign of the current-carrying charge is determined.
- Carrier concentration can be calculated.
- Mobility of charge carriers is determined.
- Electrical conductivity is determined.
- Hall effect can be used to determine the electronic structure of the specimen: whether the specimen is conductor, insulator or semiconductor.
- Knowledge of R_H enables us to measure unknown magnetic fields.
- Hall elements are used in electronic multimeters and electronic wattmeters.

25. Define solar cell.

A solar cell is a semiconductor device that converts the energy of sunlight directly into electricity by photovoltaic effect.

26. Define LDR.

A light dependent resistor (LDR) is a semiconductor device whose resistance decreases with increase in incident light intensity.

EXERCISE PROBLEMS

PART-A

1. Calculate the number of silicon atoms per cubic metre. The density of silicon is 2.33 Mg/m^3 (2.33 g/cm^3), and its atomic mass is 28.08 g/mol. (*Ans.* 5.00×10^{28} *atoms/m*^{3})

2. Calculate the electrical resistivity of intrinsic silicon at 300 K. For Si at 300 K, $n_i = 1.5 \times 10^{16}$ carriers/m^3. $\mu_n = 0.135$ m^2/v.s., and $\mu_p = 0.048$ m^2/v.s. (*Ans.* $2.28 \times 10^3\ \Omega\ m$)

3. The electrical resistivity of pure silicon is 2.3×10^3 Ω.m at room temperature, 27°C (300 K). Calculate its electrical conductivity at 200°C (473 K). Assume that the E_g of silicon is 1.1 eV; $k = -8.62 \times 10^{-5}$ eV/K. (*Ans.* $1.04\ (\Omega\ m)^{-1}$)

4. A silicon water is doped with 10^{21} phosphorus atoms/m^3. Calculate (a) majority-carrier concentration, (b) the minority carrier concentration, and (c) the electrical resistivity of the doped silicon at room temperature (300 K). Assume complete ionization of the dopant atoms; n_i (Si) = 1.5×10^{16} m^{-3}. $\mu_n = 0.135$ $m^2/(V.s)$, and $\mu_p = 0.048$ $m^2/(V.s)$. (*Ans.* *0.463 Ω m*)

5. A semiconducting crystal that is 10 mm long, 4 mm wide and 0.5 mm thick has a magnetic flux density of 1 Wb/m^2 applied from front to back perpendicular to largest faces. When a current of 30 mA flows lengthwise through the specimen, the voltage measured across its width is found to be 47 μV. What is the Hall coefficient of this semiconductor? (*Ans.* *7.8×10^7 $C^{-1}m^3$*)

6. The Hall coefficient of a certain silicon specimen was found to be -6.33×10^{-5} m^3 C^{-1} from 90 to 350 K. Determine the nature of the semiconductor. If the conductivity was found to be 150 m^{-1} Ω^{-1}, calculate the density and mobility of the charge carrier.

(*Ans.* *$n_e = 9.8736 \times 10^{22}/m^3$, μ_e $9.49 \times l0^{-3}$ m^2/Vs*)

7. An *n*-type semiconductor has Hall coefficient of 2.16×10^{-4} m^3 C^{-1}. The conductivity is 100 ohm^{-1} m^{-1}. Calculate its charge carrier density and electron mobility at room temperature.

(*Ans.* *$n_e = 3.40712 \times 10^{22}/m^3$,*

$\mu_e = 0.01834$ m^2/Vs)

8. The Hall coefficient of a certain silicon specimen was found to be -7.35×10^{-5} m^{-3} C^{-1} from 100 to 400 K. Determine the nature of the semiconductor. If the conductivity was found to be 200 $m^{-1}\Omega^{-1}$, calculate the density and mobility of the charge carrier.

(*Ans.* *$n_e = 8.503 \times 10^{22}/m^3$,*

$\mu_e = 0.0147$ m^2/VS)

9. The mobilities of electrons and holes in a sample of intrinsic Ge at 300 K are 0.36 $m^2V^{-1}s^{-1}$ and 0.17 $m^{-3}V^{-1}s^{-1}$ respectively. Find the intrinsic concentration of charge carriers if the resistivity of the specimen is 2.12 Ωm. (*Ans.* *$n_i = 5.5 \times 10^{18}/m^3$*)

10. For germanium at 300 K, $n_i = 2.4 \times 10^{19}/m^3$, μ_e and μ_h are 0.39 m^2 $V^{-1}s^{-1}$ and 0.19 $m^2V^{-1}s^{-1}$ respectively. Calculate the conductivity. (*Ans.* *$\sigma = 2.2$ $\Omega^{-1}m^{-1}$*)

QUESTIONS

PART-B

1. Derive an expression for carrier concentration of an intrinsic semiconductor. *(A.U., B.E./B.Tech, May/June 2011)*
2. Prove that at temperature 0 K, the Fermi level lies exactly halfway between the valence band and conduction band in an intrinsic semiconductor.
3. Derive an expression for electrical conductivity of an intrinsic semiconductor.
4. Discuss in detail the bandgap determination of an intrinsic semiconductor.
5. Derive an expression for carrier concentration of electrons in an *n*-type semiconductor. *(A.U., B.E./B.Tech, May/June 2011)*
6. Explain the variation of Fermi level with temperature and impurity atom in an *n*-type semiconductor. *(A.U., B.E./B.Tech, May/June 2011)*
7. Derive an expression for carrier concentration of holes in a *p*-type semiconductor. *(A.U., B.E./B.Tech, May/June 2011)*
8. Explain the variation of Fermi level with temperature and impurity atom in a *p*-type semiconductor. *(A.U., B.E./B.Tech, May/June 2011)*
9. Give the theory of Hall effect in the case of extrinsic semiconductors. Using Hall effect how will you determine the mobility of charge carriers? *(A.U., B.E./B.Tech, May/June 2011)*
10. Describe the experiment to find the Hall coefficient in an *n*-type semiconductor using Hall effect.
11. Explain solar cells in detail.
12. Explain LDR in detail.

SUMMARY

- Semiconductors are materials having electrical conductivities intermediate between metals and insulators.
- The resistivity of semiconductors decreases with increase in temperature.
- Intrinsic semiconductors and extrinsic conductors are the two types of semiconductors.
- Semiconductors in an extremely pure form are called intrinsic semiconductors.
- At 0 K, intrinsic semiconductor behave as perfect insulator.
- Fermi energy of an intrinsic semiconductor at 0 K is given by

  $$E_F = \frac{F_c + E_v}{2}.$$

- Semiconductors in an impure form are called extrinsic semiconductors.
- *n*-type and *p*-type semiconductors are the two types of extrinsic semiconductors.
- *n*-type semiconductors are formed by doping a pure semiconductor with pentavalent impurities.
- *p*-type semiconductors are formed by doping a pure semiconductor with trivalent impurities.
- Fermi energy of an *n*-type semiconductor at 0 K is given by

  $$E_F = \frac{E_c + E_d}{2}.$$

- Fermi energy of a *p*-type semiconductor at 0 K is given by

  $$E_F = \frac{E_a + E_v}{2}.$$

- Hall effect measurements help us in the determination of a semiconductor type, and in determining carrier concentration and carrier mobility.

- Solar cell is a semiconductor device which converts solar energy into electrical energy.
- Light Dependent Resistor (LDR) is a semiconducting device and its resistance varies depending upon the intensity and wavelength of the incident light.

UNIT III

MAGNETIC AND SUPERCONDUCTING MATERIALS

Look deep into nature and then you will understand everything better.
—Albert Einstein

3.1 INTRODUCTION—ORIGIN OF MAGNETIC MOMENT

Magnetic materials are those in which a state of magnetization can be induced. Such materials, when magnetized, create a magnetic field in the surrounding space. Magnetism arises from the magnetic moment or magnetic dipole of the magnetic materials. When the electron revolves around the positive nucleus, orbital magnetic moment arises. Similarly, when the electron spins, spin magnetic moment arises. The value of spin magnetic moment is very large when compared to orbital magnetic moment. Michael Faraday, in 1845, found out that magnetic materials can be broadly classified into three groups, namely diamagnetic, paramagnetic and ferromagnetic materials. Magnetic materials play an important role in modern technology. They are widely used in computers, electrical machines, audios, etc. These materials play a prominent role in information storage devices.

3.2 MAGNETIC INDUCTION (B)

Magnetic induction in a material is defined as the number of lines of magnetic force passing perpendicularly through unit area. Its unit is weber/m^2 or tesla.

3.3 MAGNETIC DIPOLES OR MAGNETIC MOMENT

Let us consider an electric current of i amperes flowing through one turn of circular wire as shown in Fig. 3.1. Let its area be a square metre. Then its magnetic dipole moment is $\mu = ia$ amp$\cdot$m^2. It is a vector quantity and its direction is normal to the plane of the loop when the current flow is clockwise. The magnetic dipole moment of current causes magnetic field around the wire. As the result of internal atomic current, the material possesses a magnetic dipole moment.

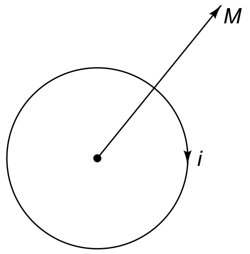


Fig. 3.1 *Current Through one Turn of a Circular Wire*

In an atom, every two electrons of opposite spins are paired and the resultant magnetic moment is zero. But there are unpaired electrons in a 3d orbital like iron, cobalt, nickel, etc. This unpaired electron's spin magnetic moment interacts with the neighbourhood atom's unpaired electron's spin magnetic moment to align in a parallel fashion resulting in large spin magnetic moment. These unpaired electrons spin results in magnetic behaviour of materials.

3.4 BOHR MAGNETON THEORY

Let us consider a circular orbit of radius r in which an electron revolves with an angular velocity ω. This constitutes a current of magnitude $\frac{e\omega}{2\pi}$ where $\frac{\omega}{2\pi}$ is the frequency and e is charge of the electron.

The magnetic moment associated with this electron,

$\mu = ia = \frac{e\omega\pi r^2}{2\pi} = \frac{1}{2}e\omega\ r^2$ where a = area of cross-section of the circular orbit. The angular momentum of the electron due to rotation of the electron of mass $m = m\ \omega\ r^2 = L$. Hence $\mu = \frac{eL}{2m}$. According to Bohr's theory, $L = n\,\hbar$ where n is an integral number. Therefore, μ, the magnetic moment of the electron, is a multiple of $\frac{e\hbar}{2m}$,

i.e. $\mu = \frac{ne\hbar}{2m}$ where $\frac{e\hbar}{2m}$ is called Bohr magneton. Here, $\hbar = h/2\pi$ where h is the Planck's constant.

Thus, a Bohr magneton is used as a practical unit for electron's magnetic moment. One Bohr magneton $\frac{e\hbar}{2m} = 9.27 \times 10^{-24}$ Am^2.

3.5 INTENSITY OF MAGNETIZATION (*I*)

Intensity of magnetization of a material is defined as the magnetic moment per unit volume. The intensity of magnetization of a magnetic specimen is the measure of its magnetization. Its unit is amp/metre.

3.6 MAGNETIC FIELD INTENSITY (*H*)

It is defined as the force experienced by a unit north pole when placed at the given point in a magnetic field. Its unit is amp/metre.

3.7 MAGNETIC SUSCEPTIBILITY (χ)

Magnetic susceptibility of a material is defined as the ratio of the intensity of magnetization produced in the sample to the magnetic field which produces the magnetization. The sign and magnitude of the magnetic susceptibility are used to determine the nature of the magnetic materials. It has no unit.

$$\chi = \frac{I}{H}.$$

3.8 MAGNETIC PERMEABILITY (μ)

Magnetic permeability of a material is defined as the ratio of the magnetic induction in the sample to the applied magnetic field intensity. The readiness of a material to accept magnetism is permeability,

i.e. $$\mu = \frac{B}{H}$$

This also can be written in another way as

$$B = \mu_0 (I + H)$$

where μ_0 is the permeability of free space.

The relative permeability $\mu_r = \frac{\mu}{\mu_0} = \frac{B/H}{B/(H+I)} = 1 + \frac{1}{H} = 1 + \chi$

where χ is the susceptibility of the medium. The relative permeability has no unit. The magnetic materials must have high permeability so that large fluxes may be produced.

3.9 CLASSIFICATION OF MAGNETIC MATERIALS

Magnetic materials are broadly classified into two categories.

- Magnetic materials not having any permanent magnetic moment (diamagnetic materials)
- Magnetic materials having any permanent magnetic moment (paramagnetic materials, ferromagnetic materials, antiferromagnetic materials, ferrimagnetic materials, ferrites).

3.10 DIAMAGNETIC MATERIALS

Diamagnetic materials are those which when placed in a magnetic field, become weakly magnetized in the direction opposite to that of the applied field (Fig. 3.2).

Hydrogen, air, water, gold, silver, germanium, silicon are some examples for diamagnetic materials.

A material consists of a large number of electrons and the orbits of the electrons are randomly oriented in space. Due to this, the magnetic moments of all such electrons are also randomly oriented. As a result, the magnetic moments of all the electrons get cancelled. The net magnetization in the material is zero. When an external field is applied to the material, it causes rotation of the individual dipoles. This results in an induced dipole moments and it opposes the applied force. This magnetism which is in the opposite direction to that of the external field is known as diamagnetism.

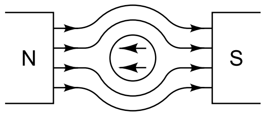


Fig. 3.2 *Diamagnetic Material*

3.10.1 Characteristics of Diamagnetic Materials

- Diamagnetic materials repel the magnetic lines of force.
- They have no permanent dipole moment.
- Diamagnetic materials exhibit negative magnetic susceptibility.
- As diamagnetic susceptibility is negative, the relative permeability μ_r is less than unity ($\mu_r < 1$).
- The magnetic susceptibility is independent of temperature and applied magnetic field strength.
- The magnetization M is a linear function of the magnetic field H, when the field is not too strong.

3.11 PARAMAGNETIC MATERIALS

Paramagnetic materials are those which when placed in a magnetic field, become weakly magnetized in the same direction as that of the applied field.

Fig. 3.3 *Paramagnetic Material*

Chromium, platinum, crown glass, copper chloride, oxygen are some examples for paramagnetic materials.

The magnetic moment created due to the orbital motion of electrons disappear due to the effect of electric field of the neighbouring charges. The magnetic moments due to spin motion of electrons remain unaffected by this field. In the absence of the external field, these magnetic moments are randomly distributed. When the external field is applied, the magnetic moments orient in the direction of the field. If there is no opposing force, all the magnetic moments line up and material acquire large magnetization and at the same thermal agitation of the atoms opposes such magnetic moments and keeps them in random. This leads to partial alignment of the magnetic moments in the direction of the external field and the material is weakly magnetized.

3.11.1 Characteristics of Paramagnetic Materials

- Paramagnetic materials attract the magnetic lines of force.
- Paramagnetic materials possess permanent dipole moment.
- They exhibit positive magnetic susceptibility.
- The relative permeability μ_r is slightly more than unity ($\mu_r > 1$).
- The magnetization M is a linear function of the magnetic field H, when the field is not too strong.
- The paramagnetic susceptibility is strongly dependent on temperature.

3.12 FERROMAGNETIC MATERIALS

Ferromagnetic materials are those which when placed in a magnetic field, become strongly magnetized in the direction of the applied field. The direction of magnetization is the same as that of the external field.

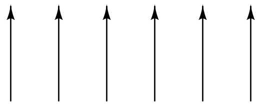


Fig. 3.4 *Ferromagnetic Material*

Iron, nickel, cobalt are some examples for ferromagnetic materials.

Ferromagnetism arises due to permanent magnetic moments in the atoms or molecules of the material. When an external field is applied, the magnetic moments orients in the same direction as that of the external field.

3.12.1 Characteristics of Ferromagnetic Materials

- Ferromagnetic materials conduct magnetic flux as much as metals conduct electric current.
- Ferromagnetic materials have permanent dipole moment.
- They exhibit magnetization even in the absence of a magnetic field which is known as spontaneous magnetization.
- They exhibit hysteresis curve.
- Ferromagnetic materials exhibit positive magnetic susceptibility of the order of 10^6.
- Magnetization of the ferromagnetic material does not vary linearly with the applied field H.
- The permeability of the ferromagnetic material does not have a constant value because of the non-linear relationship between B and H.
- Ferromagnetic materials behave like ordinary paramagnetic substances in liquid and gaseous states.
- The susceptibility of the ferromagnetic material depends on temperature in a complex manner. When the temperature of the material is greater than the Curie temperature, the ferromagnetic material is converted into paramagnetic material.
- During heating, ferromagnetic materials lose the magnetization slowly.

3.13 ANTIFERROMAGNETIC MATERIALS

Antiferromagnetic materials are crystalline materials which possess a small positive susceptibility of the order of 10^{-3} to 10^{-5}. The magnetic interactions between any two dipoles align themselves antiparallel to each other and all the dipoles are equal in magnitude and hence the resultant magnetization is zero. When an external field is applied, a kind of ferrimagnetic behaviour may be displayed in the antiferromagnetic phase, with the absolute value of one of the sublattice magnetizations differing from that of the other sublattice, resulting in a non-zero net magnetization. Antiferromagnetic materials occur less frequently in nature than ferromagnetic ones.

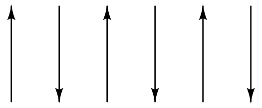


Fig. 3.5 *Antiferromagnetic Material*

MnO, MnS, Cr_2O_3 are some of the ionic compounds which exhibit antiferromagnetism.

3.13.1 Characteristics of Antiferromagnetic Materials

- The susceptibility is very small and positive. It is given by

$$\chi = \frac{C}{T + \theta}$$

  where, C is the Curie constant and θ is the paramagnetic Curie temperature.
- Due to exchange interaction, the adjacent dipoles are opposite aligned.
- With the increase in temperature, the susceptibility increases and reaches a maximum at a certain temperature called Neel temperature (T_N). When the temperature further increases, the material goes to paramagnetic state.
- The variation of susceptibility of an antiferromagnetic crystal as a function of temperature is as shown in Fig. 3.8(c).
- The $1/\chi$ verses T curves for antiferromagnetic material is shown in Fig. 3.6.

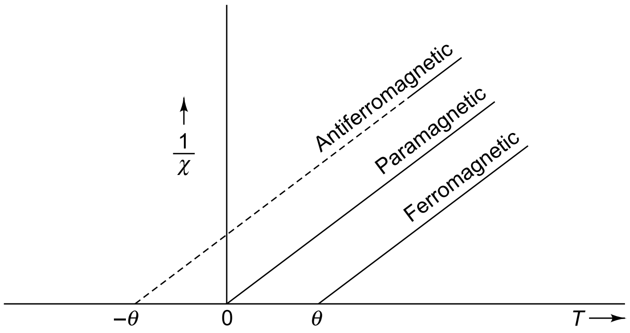


Fig. 3.6 *Plot for Magnetic Materials*

3.14 FERRIMAGNETIC MATERIALS

Ferrimagnetic materials are also known as uncompensated antiferromagnetism. They are very much identical to ferromagnetic materials in their macroscopic behaviour. The saturation magnetizations are not as high as ferromagnetic materials.

The magnetic interactions between any two dipoles align themselves antiparallel to each other and the magnitudes of any two adjacent dipoles are not equal. When a small value of magnetic field is applied, it produces a large value of magnetization. Ferrimagnetic materials have high resistivity and have anisotropic properties. The anisotropy is actually induced by an external applied field. When this applied field aligns with the magnetic dipoles, it causes a net magnetic dipole moment and causes the magnetic dipoles to precess at a frequency controlled by the applied field, called Larmor or precession frequency.

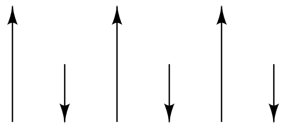


Fig. 3.7 *Ferrimagnetic Material*

Ferrous ferrite, zinc ferrite, nickel ferrite are some examples for ferromagnetic materials.

3.14.1 Characteristics of Ferrimagnetic Materials

- The susceptibility is very large and positive quantity.
- A ferromagnetic material is composed of two or more sets of different transition metals.

3.15 VARIATION OF SUSCEPTIBILITY WITH TEMPERATURE IN DIFFERENT MAGNETIC MATERIALS

Figure 3.8 illustrates the variation of susceptibility with temperature in different magnetic materials. The corresponding orientation of magnetic moments are also shown in Fig. 3.8.

The various types of magnetic materials along with their different types of interatomic interaction are listed in Table 3.1. Table 3.2 presents a brief distinction between different magnetic materials.

Table 3.1 *Types of Magnetic Materials*

Type of the material	*Permanent dipoles*	*Dipole Arrangement*
Diamagnetic	Absent	—
Paramagnetic	Present	Random orientation
Ferromagnetic	Present	Parallel orientation
Antiferromagnetic	Present	Antiparallel orientation of equal moments
Ferrimagnetic	Present	Antiparallel orientation of unequal moments

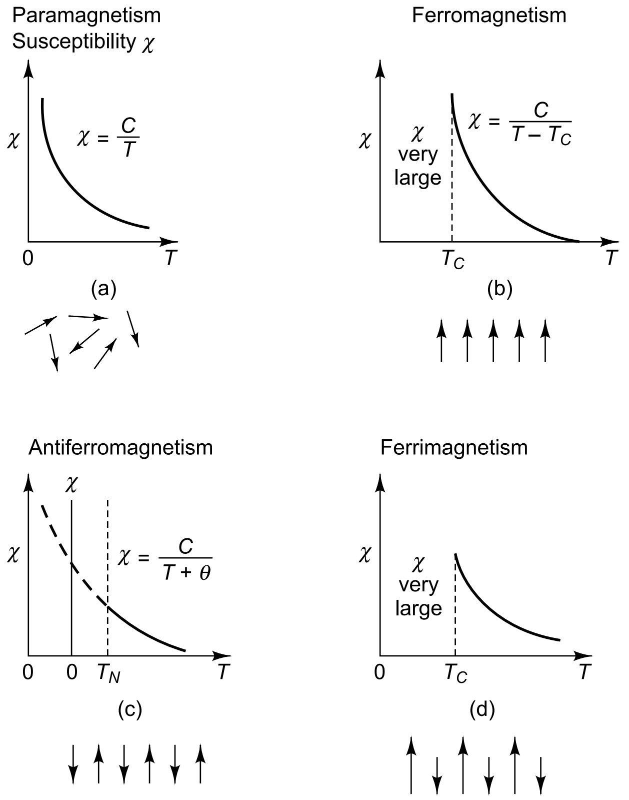


Fig. 3.8 *Magnetic Susceptibility versus T for (a) Paramagnetic, (b) Ferromagnetic, (c) Antiferromagnetic, and (d) Ferrimagnetic Materials with Magnetic Moment Alignments Indicated for Each Case*

Table 3.2 *Distinction between different Magnetic Materials*

Type	*Magnitude of susceptibility*	*Temperature dependence*	*Remarks*	*Examples*
Diagmagnetic	Small, negative	Independent	There are no permanent dipoles, consequently magnetic effects are very small	**Organic materials, light elements**
	Intermediate negative	Varies with field and temperature below 20 K		**Alkali earths, bismuth**
	Large, negative	Exists only below critical temperature T_C		**Superconducting materials**
Paramagnetic	Small, positive	Independent	They possess permanent dipoles which in the absense of field, are randomly oriented so that net magnetization in any given direction is zero	**Alkali metals, transition metals**
	Large, positive	$\chi = \dfrac{C}{T - \theta}$		**Rare earths**
Ferromagnetic	Very large, positive	$T > T_C; \chi = \dfrac{C}{T - \theta}$	Due to the large internal field, the permanent dipoles are strongly alinged in the same direction and consequently a large spontaneous magnetization results even in the absense of an applied field	**Some transition and rare earth metals**
Antiferro-magnetic	Small, positive	When $T > T_N$ $\chi = \dfrac{C}{T + \theta}$	The magnetic icons on lattice site A will be alinged antiparallel to those on lattice site B because the molecular field is negative. Also $M_A = M_B$	**Salts of transition elements**
Ferrimagnetic	Very large, positive	When $T > T_N$ $\chi = \dfrac{C}{T \pm \theta}$	It is a special case of antiferromagnetics in which opposed moments are of different magnitudes and a large magnetization thereby results	**Ferrites**

3.16 DOMAIN THEORY OF FERROMAGNETISM

The concept of domains was proposed by Weiss in order to explain the properties of ferromagnetic materials and their hysteresis effects.

3.16.1 Magnetic Domains

The group of atoms organized into tiny bounded regions in the ferromagnetic materials are called **magnetic domains**.

In each individual domain the magnetic moments of the atoms are aligned in the same direction and hence the domain is a region of the ferromagnetic material in which all the magnetic moments are aligned to produce a net magnetic moment in one direction only and hence behaves like a magnet with its own magnetic moment and axis. In a demagnetized state, the domains are randomly oriented as shown in Fig. 3.9(a) so that the magnetization of the material as a whole is zero.

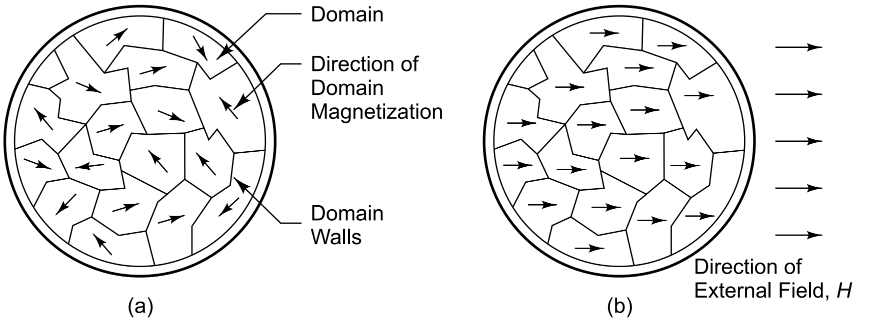


Fig. 3.9 *(a) Schematic Illustration of Magnetic Domains in a Demagnetized Ferromagnetic Material. In each Domain, the Magnetic Dipoles are Aligned but the Domains are aligned at Random so that the Net Magnetization is Zero*
(b) Domain Configuration in a Magnetized body. The Magnetic moments of Domains are Aligned Resulting in Strong Net Magnetization

The boundaries separating the domains are called *domain walls*, also known as **Bloch walls**. The domain walls are similar to the grain boundaries in a polycrystalline material and are thicker than the grain boundaries. Similar to grain growth, the domain size can also grow due to the movement of domain walls.

When a magnetic field is applied externally to a ferromagnetic material, then the domains align themselves with the field as shown in Fig. 3.9(b). This results in a large net magnetization of the material.

3.16.2 Process of Domain Magnetization

As already stated earlier, in an unmagnetized specimen, the domains are randomly oriented and the net magnetization is zero. When the external field is applied, domains align with the direction of field resulting in large net magnetization of a material. There are two possible ways to align the domains by applying an external magnetic field.

- By the motion of domain walls
- By rotation of domains

3.16.3 Motion of Domain Walls

Figure 3.10(a) shows an unmagnetized specimen where the domains are randomly aligned. When a small amount of magnetic field is applied, the domains with magnetization direction parallel or nearly parallel to the field, grow at the expense of others, as shown in Fig. 3.10(b). This domain growth occurs due to the movement of domain walls away from the minimum energy state.

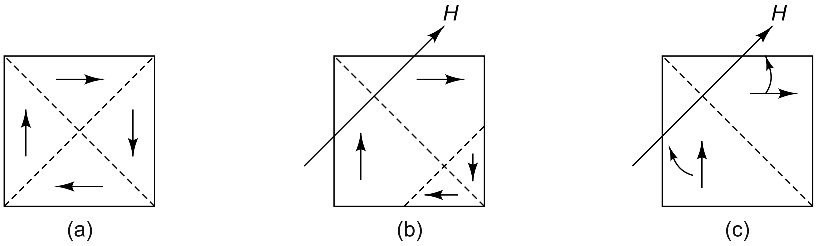


Fig. 3.10 *(a) Random Domain Alignment (b) Domain Wall Movement (c) Domain Rotation*

3.16.4 Rotation of Domains

When the magnetic field is increased further to a large value (i.e., near saturation), further domain growth becomes impossible and hence most favourably oriented and fully grown domains tend to rotate so as to be in complete alignment with the field direction, as shown in Fig. 3.10(c).

3.16.5 Types of Energy Involved in the Process of Domain Growth

There are four types of energy involved in the process of domain growth.

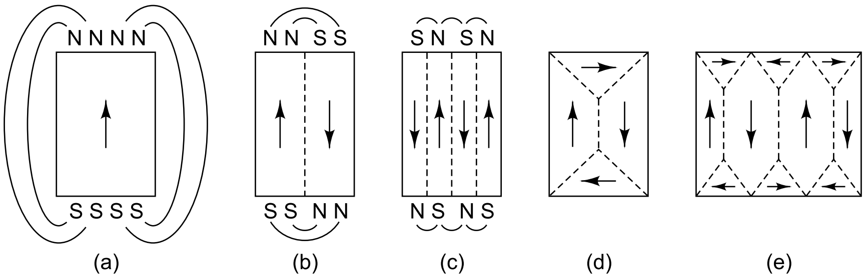


Fig. 3.11 *The Origin of Domains*

1. Exchange Energy It is defined as the energy associated with the quantum mechanical coupling that aligns the individual atomic dipoles within a single domain which arises as the result of the interaction of electron spins and depends upon the interatomic distance.

Figure 3.11(a) represents a cross-section through ferromagnetic crystal having a single domain structure established by exchange energy and is saturated.

2. Crystal Anisotropy Energy It is defined as the energy of magnetization which is the function of crystal orientation. Figure 3.12, shows the magnetization curves for iron with the applied field directed along different crystallographic directions.

It is clear from the figure, much greater fields are needed to produce magnetic saturation in [1 1 1] as compared to the field required in [1 0 0] direction. The difference in magnetic energy to produce saturation in an easy [1 0 0] direction and hard [1 1 1] direction is called crystal anisotropic energy.

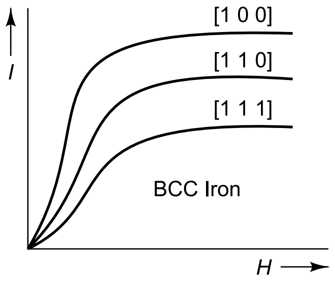


Fig. 3.12 *Hard and Easy Direction for Magnetization*

Domain Wall Energy (Bloch Wall Energy) The boundary between two domains is known as 'Bloch wall'. Bloch wall energy or domain wall energy is the sum of exchange and crystalline anisotropy energies in the domain wall region.

3. Magnetostatic Energy Magnetostatic energy, also known as magnetic potential energy, is the energy present in any ferromagnetic material when that material produces an external field. This is due to the existence of resultant dipole moment in that material even in the absence of external magnetic field.

The magnetic energy can be reduced by dividing the specimen into two domains as shown in Fig. 3.11(b). Further subdivision, as shown in Fig. 3.11(c), to *N* domains, reduces the magnetic energy to 1/*N* of the magnetic energy of the configuration of Fig. 3.11(a).

A domain structure shown in Fig. 3.11(d) and (e) have zero magnetic energies due to the introduction of triangular domains at the top and bottom of the crystal. These triangular domains are called closure domains.

4. Magnetostrictive Energy When a material is magnetized, it is found that it undergoes a change in dimensions. This phenomenon is known as magnetostriction. This deformation is different along different crystal directions. Therefore, if the domains are magnetized in different directions, they will either expand or shrink. This means that work has been done against the elastic restoring forces. The work done by the magnetic field against these elastic restoring forces is called the magnetostrictive energy also known as magneto-elastic energy.

3.17 REVERSIBLE AND IRREVERSIBLE DOMAINS

When a small external field is applied on a ferromagnetic specimen, the domain wall is displaced slightly away from the minimum energy. But it returns to the original position when the field is removed. This gives a reversible domain wall movement and the domain is known as reversible domain.

In larger external fields, the domain wall may be shifted to a more distant position where the energy curve has passed through a maximum and then diminished. On removing the field, the domain wall cannot cross the maximum energy and hence it is unable to return to its initial position. This gives an irreversible domain wall movement and the domain is known as irreversible domain.

3.18 HYSTERESIS

The lag in the changes of magnetization behind variations of the magnetic field is defined as **hysteresis**. It is also defined as a phenomenon of variation of flux density (*B*) with the change in magnetic field strength (*H*) in a ferromagnetic material.

Let us consider an unmagnetized specimen of ferromagnetic material subjected to an external magnetic field. We know that when the magnetic field strength is gradually increased from zero, the flux density increases rapidly as shown by the curve in Fig. 3.13. It is also observed that if the electric field strength is further increased, the rate of increase in flux density is reduced, until a saturation is reached as shown by the curve *AB* in the figure.

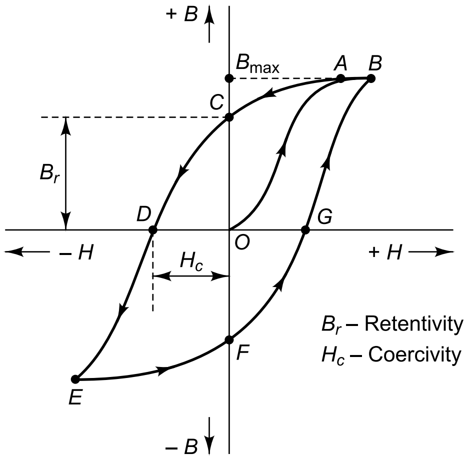


Fig. 3.13 *Hysteresis Loop for a Ferromagnetic Specimen*

When the magnetic field strength is reversed to zero, then the flux density decreases along the curve *BAC* and it does not become zero. The value of flux density (represented by *OC*), which is still left in the material is known as residual magnetism or remanent flux density or retentivity. It is designated by the symbol B_r. Thus the magnetization retained by the specimen when the magnetizing field is reduced from saturation value to zero is defined as retentivity of a specimen.

To make the flux density to zero, a certain amount of magnetic field strength (represented by *OD*) is applied in the reverse direction. The negative value of the magnetic field causes demagnetization and it is called the coercive force (or) coercivity. It is designated by the symbol H_c. It is observed that if the field strength is further increased in the negative direction, the flux density increases rapidly along the curve *DE*, until a negative saturation is reached at *E*. By proceeding in this way, the curve *EFGB* is obtained and the cycle is completed. The enclosed area formed by the curve *BACDEFGB* is called the hysteresis loop.

It may be understood from the figure that the flux density always lags behind the magnetic field strength. This phenomenon is known as magnetic hysteresis. The work done on the ferromagnetic specimen gives an energy loss known as hysteresis loss. The magnitude of the hysteresis loss is equal to the area enclosed by the hysteresis loop.

3.19 SOFT AND HARD MAGNETIC MATERIALS

The magnetic materials are classified into the following two types upon the ease with which they can be magnetized and demagnetized.

- Soft magnetic materials
- Hard magnetic materials

3.19.1 Soft Magnetic Materials

Materials which can be easily magnetized and demagnetized are called soft magnetic materials.

Iron and silicon alloys, nickel–iron alloys, iron–cobalt alloys are some soft magnetic materials.

They retain a small amount of magnetic energy, even after the magnetic field is removed. These materials are also known as **temporary magnets**.

These materials must possess easily moving domain walls. They are prepared by heating the pure material to a temperature at which sufficient movement of the atoms is possible for them to settle into an ordered lattice, followed by slow cooling.

Properties of Soft Magnetic Materials

- The nature of the hysteresis loop for a soft magnetic. material is steep and the hysteresis loop for soft magnetic materials is shown below in Fig. 3.14.

- These materials have small hysteresis loss due to small hysteresis loop area.
- These materials have a large value of susceptibility and permeability.
- The coercivity and retentivity are small.
- Due to higher resistivity, the eddy current loss is small.
- The magnetostatic energy is very small for these materials.
- These materials are free from irregularities like strain or impurities.

Applications of Soft Magnetic Materials

They are widely used in transformer cores, electric machinery cores, memory cores in computers, inductors, relays, alternators, potentiometers and small motors.

3.19.2 Hard Magnetic Materials

Materials which cannot be easily magnetized and demagnetized are called hard magnetic materials and they retain their magnetism.

High carbon steel, tungsten steel, chromium steel, cobalt steel, Al-Ni-Co alloys, barium ferrite are some examples for hard magnetic materials.

They retain considerable amount of magnetic energy, even after the magnetic field is removed. These materials are also known as permanent magnets.

The movement of the domain walls must be prevented for the permanent magnets in order to keep their magnetism for a longer time. They are prepared by heating magnetic materials to the required temperature and then suddenly cooling them by dipping in a cold liquid. The impurities are purposely introduced to these materials in order to make them hard.

Properties of Hard Magnetic Materials

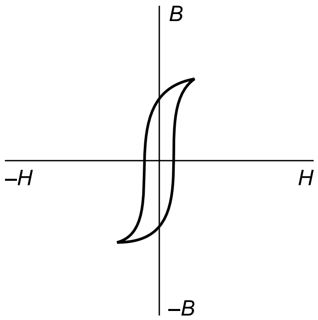
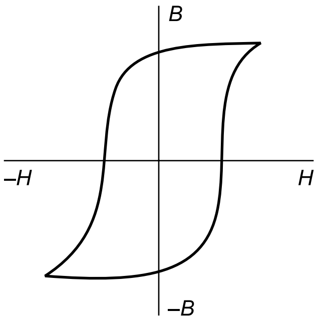
- The hysteresis curve is very broad and has a large area as shown in Fig. 3.15.
- Due to large hysteresis loop, there is large hysteresis loss.
- These materials have low value of susceptibility and permeability.

- The coercivity and retentivity are large.
- The eddy current loss is very high.
- The magnetostatic energy is very large.
- These materials have large amount of impurities and lattice defects.

Applications of Hard Magnetic Materials

They are used in meters, motors, electron tubes, transducers and latching relays.

Table 3.3 *Comparison between Soft and Hard Magnetic Materials*

S.No.	*Soft Magnetic Material*	*Hard Magnetic Material*
1.	The nature of the hysteresis loop is very steep as shown below.  **Fig. 3.14**	The nature of the hysteresis loop is very large is as shown below.  **Fig. 3.15**
2.	These materials can be very easily magnetized and demagnetized.	These materials are very difficult to magnetize and demagnetize.
3.	They are prepared by annealing process.	They are prepared by quenching process.
4.	Due to small hysteresis loop area, they have small hysteresis loss.	Due to large hysteresis loop area, they have large hysteresis loss.
5.	They have large value of permeability and susceptibility.	They have low value of permeability and susceptibility.
6.	The coercivity and retentivity are small.	The coercivity and retentivity are large.
7.	Due to higher resistivity, they possess small eddy current loss.	Due to smaller resistivity, they possess more eddy current loss.
8.	They are free from irregularities.	They have large amount of impurities and lattice defects.
9.	Its magnetostatic energy is very small.	Its magnetostatic energy is very large.
10.	They are used to produce temporary magnets.	They are used to produce permanent magnets.

3.20 FERRITES

Ferrites are modified structures of iron with no carbon and it is one of the soft magnetic materials and also called ferrimagnetic materials. They are complex compounds of various metals and oxygen which exhibit the phenomenon of ferromagnetism. The general chemical formula of a ferrite molecule is Mo Fe_2O_3, where M is a divalent metal (i.e., metal having two valence electrons). The commonly used divalent metals are iron, copper, nickel, cobalt, zinc, etc.

In ferrites, the ions are so arranged that although the magnetic moments point in opposite directions, those in one direction are larger than those in the other. This is a form of antiferromagnetism, but it produces typical ferromagnetic properties.

Nickel ferrite ($NiFe_2O_4$), manganese ferrite ($MnFe_2O_4$), zinc ferrite ($Zn\,Fe_2O_4$), barium ferrite ($Ba_{0.6}\,Fe_2\,O_3$), Mn–Zn ferrites, Ni–Zn ferrites, magnesium–manganese ferrites, manganese–copper ferrite, lithium–nickel ferrite are some examples for ferrites.

3.20.1 Structure of Ferrites

Ferrites crystalize in the form of a cubic structure. Each and every corner atom of a ferrite unit cell consists of a ferrite molecule. So in a ferrite unit cell, there are eight ferrite molecules. Therefore, in a ferrous ferrite unit cell, there are eight ferrous ions, 16 ferric ions and 32 oxygen ions. If only the oxygen ions in a ferrite crystal are considered, it is found that they constitute a close-packed face centered cubic (FCC) structure. In a ferrite unit cell, there are 16 octahedral sites (surrounded by six O^{2-ions}) and eight tetrahedral sites (surrounded by four O^{2-ions}). Metal ions occupy these octahedral and tetrahedral sites. Thus, in ferrites, the number of octahedral sites is twice the number of tetrahedrasites.

3.20.2 Types of structures

Two types of structures in ferrite are regular spinel and inverse spinel.

1. Regular Spinel In the regular spinel, each trivalent metal ion occupies an octahedral site (B) and each divalent ion occupies a tetrahedral site (A). The structural arrangement is shown in Fig. 3.16.

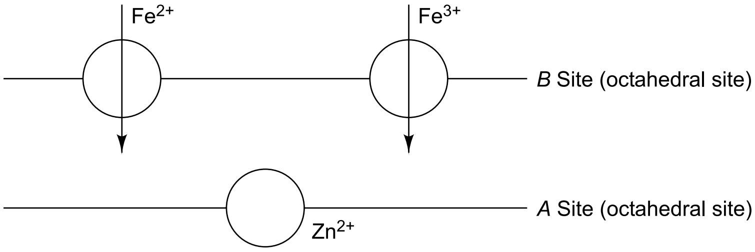


Fig. 3.16 *Schematic Representation of* $Zn^{2+}\ Fe_2^{3+}O_4^{2-}$

2. Inverse Spinel In the inverse spinel, ferrous ion occupy half of the octahedral site (*B* site), whereas the remaining half are occupied by ferric ions. The remaining ferric ions occupy the tetrahedral site (*A* site). The structural arrangement is shown in Fig. 3.17.

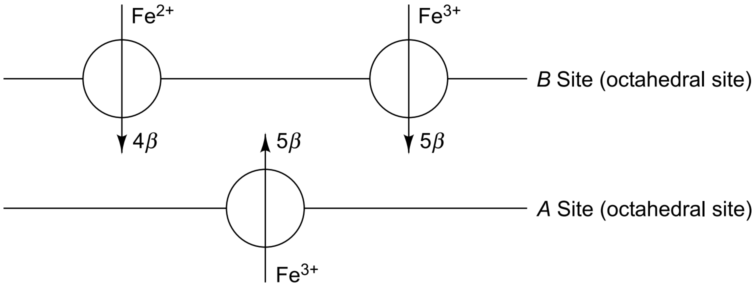


Fig. 3.17 *Diagrammatic Representation of* Fe^{2+} *and* Fe^{3+} *Ions in Ferrous Ferrite*

Example

$$M^{2+}\ Fe_2^{3+}\ O_4^{2-}$$

where M^{2+} is a magnetic divalent metal ion such as Fe^{2+}

3.20.3 Preparation

Ferrites are prepared by powder metallurgical process by preparing the powder, mixing compacting and sintering followed by age hardening in magnetic fields.

3.20.4 Powder Preparation

The following are the four common methods for producing the metal powder.

1. *Mechanical Pulverisation* Heavy crushing machines, crushing rolls and ball mills are employed to crush the metal of desired possible fineness.

2. *Atomization* Molten metal is forced through a nozzle into a stream of water, air or inert gas which on cooling, solidifies into tiny particles of various sizes.
3. *Electrolytic Method* On the surface of the electrode, metallic particle are deposited by electrolysis. The deposit is scraped off and pulverized after drying to produce powder of required fineness.
4. *Chemical Reduction* A metal oxide is reduced in an appropriate atmosphere such as hydrogen, which combines with the oxygen to leave the metal free. By mechanical means the basic powder is subsequently finished to a finer and more uniform size.

3.20.5 Mixing

By double-cone apex mixers powders in right proportions are thoroughly mixed. These mixing may be dry or wet. Graphite, stearic acid and lithium stearate lubricants are added during mixing in order to reduce friction.

3.20.6 Compacting

The following are the two common methods for compacting (final product of required size and shape) the metal powder.

1. *Die-Pressing* This process includes a feed hopper to press the powder, shaping die to form the product and a ram to apply the correct pressure in the right direction.
2. *Roll-Pressing* A regulated stream of powder is guided by two rolls of appropriate size, into which a regulated stream of powder is guided and they provide necessary compacting pressure in a continuous sequence.

3.20.7 Sintering

The compacted powder is heated to an elevated temperature below its melting point with controlled atmospheric conditions to avoid oxidation and other undesirable reactions which results bonding the individual powder particles.

3.20.8 Properties of Ferrites

- They are non-magnetic.
- These materials have high permeability.

- They have high resistivity of value greater than 10^5 ohm-cm.
- They have extremely low dielectric loss.
- They have low eddy current loss, but large hysteresis loss.
- They are brittle and difficult to machine.
- They are essentially ceramics, i.e. refractory materials.
- They have low tensile strength.

3.20.9 Applications of Ferrites

- Ferrites are used in radio receivers to increase the sensitivity and selectivity of the receiver.
- Ferrites are used as cores in audio and TV transformers.
- Ferrites are used in digital computers and data processing circuits.
- Ferrites are used to produce low-frequency ultrasonic waves by magnetostriction principle.
- Ferrites are widely used in non-reciprocal microwave devices such as gyrator, isolator and circulator.
- Ferrites are also used in power limiting and harmonic generation devices.
- Ferrites can also be used in the design of ferromagnetic amplifiers of microwave signals.
- Ferrite core can be used as a bistable element.
- The rectangular shape ferrite cores can be used as magnetic shift register.
- Hard ferrites are used to make permanent magnets.

3.21 MAGNETO-OPTICAL RECORDING AND READ OUT

The active medium consist of a thin layer of amorphous terbium iron cobalt (TbFeCo) magnetic film coated on the substrate. Using magnetic storage media and by using optical techniques, storage of data is done. Thermally assisted magnetic (thermo-magnetic) recording results in atomic magnet reorientation in the magnetic film magnetic polarization exists perpendicular to the film plane in the upward direction.

3.21.1 Recording Process

In the recording process, the magnetic field is applied in the downward direction, i.e. opposite to the polarization direction. The field strength is insufficient to cause reversal of polarization at room temperature. The focusing of laser pulse causes strong localized heating. At high temperature reversed magnetic polarization is created and preserved on cooling. Thus the data is recorded as regions of opposite magnetization.

3.21.2 Read-out Process

When plane polarized light is focused on the written spot, depending on the orientation of the magnetic domain either up or down, the plane of polarization of the reflected light rotates either clockwise or anti-clockwise by magneto-optic Kerr effect. The data can be retrieved back by analyzing the reflected light data.

3.21.3 Erasure Process

When the laser light is focused, magnetic field is applied along the signal polarization direction. As we know, laser pulse cause localized heating and retrieval of magnetic polarization is achieved on cooling.

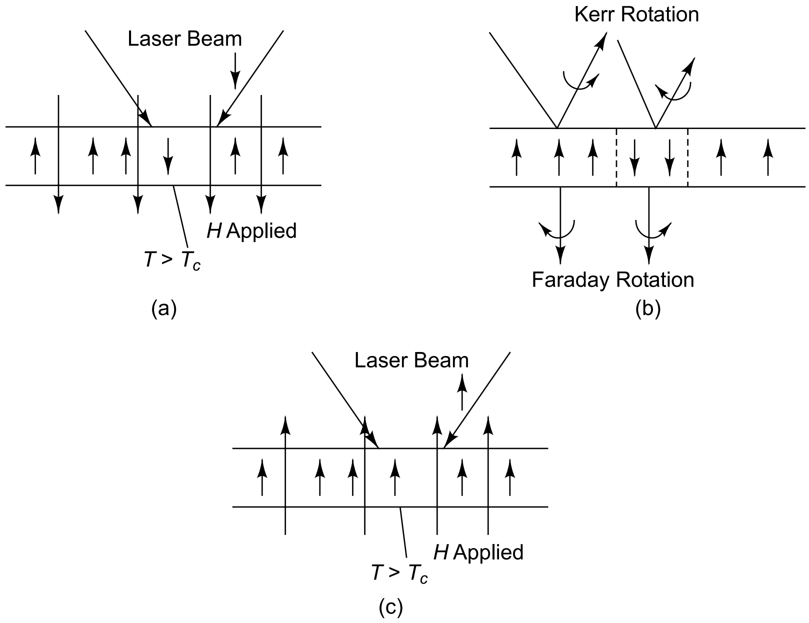


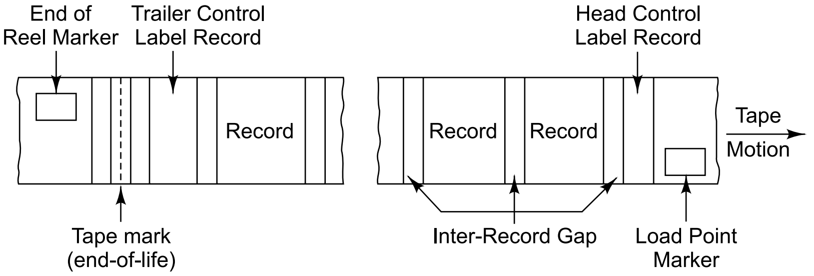
Fig. 3.18 *(a) Record (b) Read-out (c) Erase Process*

3.22 MAGNETIC STORAGE

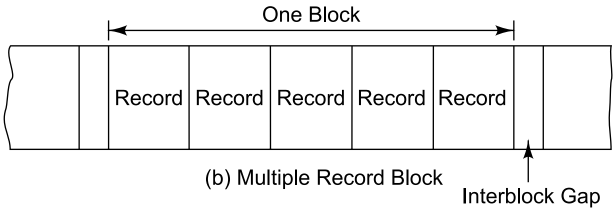
Magnetic materials are widely used for storage of information. In 1888, Oberlin Smith first suggest the magnetic storage.

3.22.1 Magnetic Tape

The cross-section of the magnetic tape is shown in Fig. 3.19. It consists of a thin layer of plastic (called mylar) over which a thin layer of magnetic material such as ferrous or ferric oxide is coated. The standard tape is 0.5 inch wide and contains 9 tracks. The newer tape, which contains chromium chloride, has 18 tracks for some 0.5 inch width. The data density of a 9 track tape is 250 characters per inch, while for 18 track tape is 3800 characters per inch. These tapes are available in the market in the form of large reel or a small catridge or cassette.



(a) Marker and Labels



(b) Multiple Record Block

Fig. 3.19 *Magnetic Tape*

Working

The schematic arrangement of a recording (read out) head and a magnetic tape is shown in Fig. 3.20.

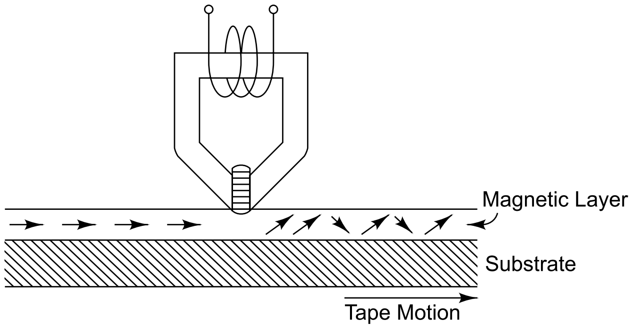


Fig. 3.20 *Recording Head and a Magnetic Tape*

The magnetic storage medium consists of magnetic oxide particles dispersed in a binder which strongly bonds to a plastic film. The oxide particles are acicular (needle-like) and about 0.1 × 0.5 µm size. Each particle is a single domain magnetized to saturation along its major axis. The most commonly used magnetic particles are γ-Fe_2O_3 crystallites. The write head (i.e. recording head) and read head consist of a laminated electromagnet made of permalloy or soft ferrite which has an air gap of about 0.3 m wide.

A wire coil is wound around the magnetic material core. **Data are written by the electrical signal within the coil which generates a magnetic field across the gap**. As the tape is passed along this electromagnet, the field magnetizes a very small area of the tape in a certain pattern proportional to the current in the coil. The magnetization is retained upon the removal of the field. Thus, the signal is stored.

The data stored on the magnetic tape is read out by the read head. In the **read mode**, the moving tape induces an alternating emf in the coil of the head. This emf is amplified and fed to a suitable output device.

In some of the above applications, ferromagnetic chromium dioxide (CrO_2) is used instead of γ-Fe_2O_3.

3.22.2 Floppy Disks (Diskettes)

Floppy disks or diskettes are popularly used as a secondary and backup memory devices. Sizes and storage capacity of the floppy diskettes vary. Conventional sizes are 3½ and 5¼ inches in diameter. Storage capacities range from 360 kilobytes to about 1.44 megabytes.

Floppies are made of very thin and flexible plastic (mylar) material and are coated with magnetic material such as iron oxide.

Description

The schematic diagram of a floppy disk is shown in Fig. 3.21. To read or write, the floppy disk is inserted into the floppy drive and hence the mechanism starts to rotate the disk at 300 rpm. The read/write window on the disk allows the head to touch the disk surface for read/write operation.

The disk also contains a write protect notch to protect the stored information. A black or silver metallic tab known as write protect tab is fixed on the notch to protect the disk from any more writing or erasing the information already stored on the disk.

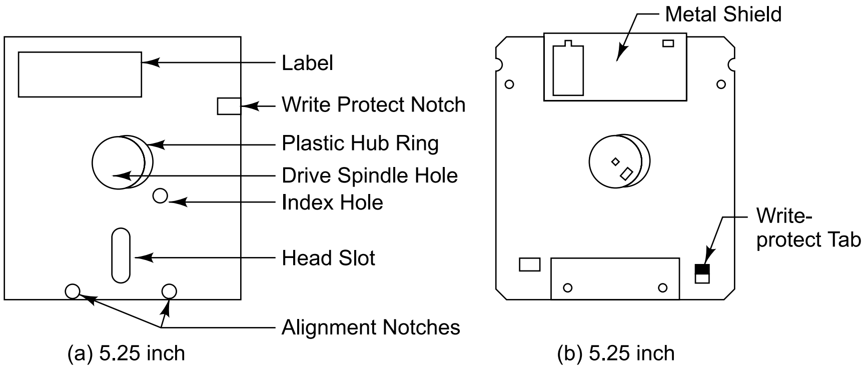


Fig. 3.21 *Floppy Disk*

Tracks and Sectors of a Magnetic Floppy

The data in the floppy disk are organized into a number of concentric circles or tracks, each of which has a designated location number. There are typically 200 to 800 tracks on the disk surface. The information is recorded on tracks. A typical data recording pattern for a floppy disk is shown in Fig. 3.22. Each track is divided into a number of sectors. The tracks and sectors are logical areas on disk and not physical areas.

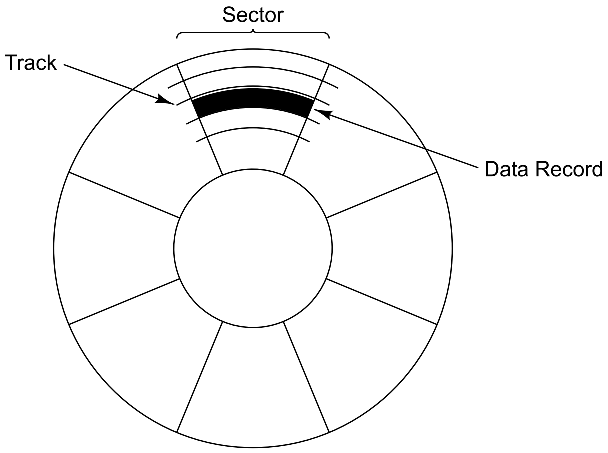


Fig. 3.22 *The data-recording Pattern for an Individual Disk Surface, including Relationships of Sectors, Tracks and Records*

Working Principle

Read-write head is a small electromagnet with minute gaps between the poles. In writing operation, the disk moves over the gaps while the electric pulses from the central processing unit (CPU), flow through the write coil of the head. It causes the iron oxide coating of the disk to be magnetised in the proper pattern.

When the disk is being read, the magnetized patterns induce pulses of current in the read coil that feed the data into the CPU. Data can be stored on both sides of the floppy disk.

Types of Diskettes

Floppy disks are typically 3½, 5¼ or 8 inches in diameter. They are available in either single or double density versions. Thus, the four types of diskettes are

- Single sided-single density
- Single sided-double density
- Double sided-single density
- Double sided-double density

Frequency modulation (FM) is used to write/read data in single density format. But a modified frequency modulation (MFM) is used in double density format.

Advantages of Floppy Disks

- A floppy disk is a non-volatile storage medium.
- They are inexpensive.
- They have low capacity online storage.
- Diskettes are not flexible and they can be easily handled.

3.23 MAGNETIC DISK DRIVES

A hard disk drive is a non-volatile storage device for digital data. It consists of a flat, circular plate (called platter) whose surface is coated with magnetic iron oxide particles and is known as magnetic disk. They possess large storage capabilities with moderate operating speed. It contains a read-write head that hovers over the surface to read data. A hard disk consists of one or more platters and their associated read-write heads. These platters rotate at very high speed (3000 rpm). The data is stored on their surface by magnetic heads mounted on access arms. Data is recorded in the form of bands. Each band of data on a particular disc is called a *track*. These tracks are divided into *sectors*. Tracks and sectors are created when the hard disk is first formatted before it is used. The tracks are arranged in concentric rings. There will be several data tracks on one side of a disk in which the bits are recorded. A typical outer track contains more bits than the inner tracks since the circumference of an outer track is greater. The disk rotates like a record and the head moves to the required track is direct-access storage. The drive is enclosed by a sealed unit to avoid dust causing crushing of heads. The storage capacity in increased by having several disks on a common drive unit called as disk pack as shown in Fig. 3.23.

Access time is the total time taken to begin reading or writing on a selected track. Seek time is the time take to position on the selected track which is of the order of milliseconds. Rotational delay is the time required for the desired data to reach the magnetic head after the positioning of the head which will be of the order of milliseconds. The sum of the seek time and rotational delay is the total access time for a disk. The number of bits transferred per second while reading or writing begins is called transfer rate of disk.

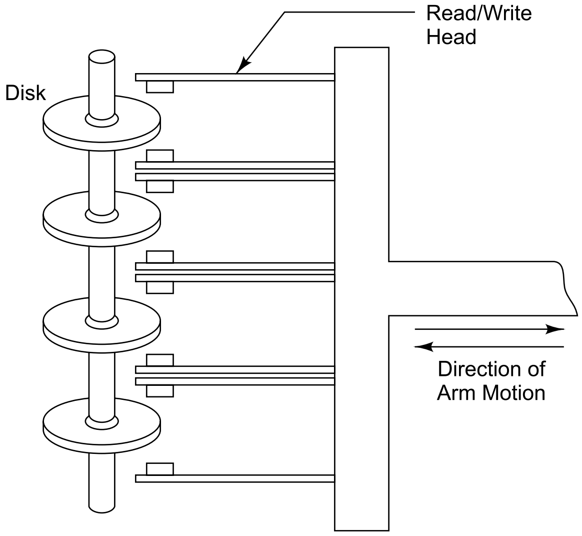


Fig. 3.23 *Magnetic Disk Drive*

3.24 MAGNETIC DEVICES

3.24.1 Transformer

Transformers are used for safe and effective transfer of energy. The core plays an important role in a transformer. Two coils of wire are wound on a magnetic material, magnetic core, known as primary and secondary winding. **Primary winding** receives energy from the ac source. **Secondary winding** receives energy from primary winding and delivers it to the load. The whole arrangement is protected by an enclosure to protect it from dirt, moisture and mechanical damage.

3.24.2 Transformer Core

A transformer core consists of laminated sheets of steel insulated with a non-conducting material. There are about 50 such laminations to form an inch-thick core. This arrangement dissipates heat readily and to reduce losses and hence provides efficient transfer of power.

The purpose of the core is to provide an effective path for more magnetic lines of flux without any loss in magnetic and electrical energy. The used magnetic material must have high permeability used to confine and guide more magnetic flux lines. The core can increase the magnetic field by a factor of several thousand when compared to the condition of without core.

3.24.3 Characteristics of a Core

Voltage, current and frequency are the important factors that decide the composition, size and shape of a transformer core. Air, soft iron and steel are the widely used core materials. When the voltage source has a high frequency of above 20 kHz, air-core transformers are used, and when the frequency is below 20 kHz, iron-core transformers are used. Soft iron is better when compared with steel for these purposes. It has a high initial permeability of 250 and it can be increased by adding 4 per cent silicon, and this alloy is known as transformer steel which is a very good material for cores of transformers. Adding more amount of silicon causes the material to become more brittle. Radio metal has an initial permeability value of 2000 to 15000. Mumetal has an initial permeability value of 10000 to 100000.

3.24.4 Types of a Transformer Core

There are two shapes of cores used in laminated steel-core transformers.

3.24.5 Hollow Core Transformers

The core is shaped with a hollow square through the center and hence it is named as hollow core transformer. It is shown in Fig. 3.24.

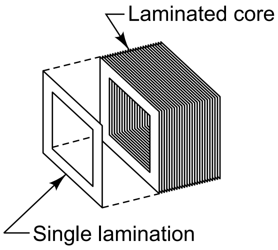


Fig. 3.24 *Hollow-core Construction*

3.24.6 Shell Core Transformers

The core consists of the *E* and *I* shaped sections butted together to form laminations. Each lamination is insulated from the other one and then pressed together to form a core. It is shown in Fig. 3.25.

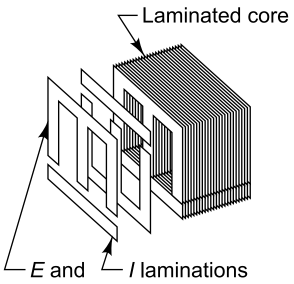


Fig. 3.25 *Shell-type Core Construction*

3.24.7 Requirement of a Transformer Core

The chief requirement of the transformer core materials are

- High permeability to allow many magnetic flux lines through it
- Low hysteresis loss which results in small heating effect and the winding is protected by any breakdown of insulation
- High specific resistance to reduce eddy current loss

3.25 SUPERCONDUCTING MATERIALS—INTRODUCTION

The property of certain ultracold substances to conduct electricity without any resistance is termed as **superconductivity.**

The temperature at which the transition from the normal state to the superconducting state occurs is called the **critical temperature** or the **transition temperature**.

The phenomenon of superconductivity was discovered in 1911 by Kamerlingh Onnes when he observed that the electrical resistivity of mercury disappeared completely at temperatures below 4.2 K. It had defined a satisfactory atomic interpretation for many years. Discovery of superfluidity of liquid helium by P.L. Kapitsa of Russia in 1938 and the theory of superfluidity subsequently developed by L.D. Landau of Russia in 1941 provided a significant amount of insight into the nature of superconductivity. It became clear that superconductivity is some sort of superfluidity of the electron gas. But, this interpretation could not explain how the electron ensembles with half-spin can contribute to superfluidity. The other attempts to find an atomic interpretation made by Frohlich, Bardeen, Cooper, Schrieffer and Bogolubov since 1950 appeared to be promising.

Since the discovery of the superconducting property of mercury, many elements, compounds and alloys have been found to exhibit the same behaviour. The superconducting transition is found to be very sharp for a pure metal and it is broad if the metal is impure.

3.25.1 Properties of Superconductors

Superconductors exhibit peculiar electrical, magnetic, thermal and mechanical properties. Now, we will consider some of the characteristic properties of superconductors. These properties show that the superconductors belong to a new class of materials.

1. Zero Resistivity The electrical resistivity of the superconductor suddenly vanishes, when cooled below its critical temperature. The critical temperature is defined as the temperature at which the electrical resistivity of the superconductor suddenly vanishes. That is, $\rho = 0$ at $T < T_c$. The zero resistivity of a superconductor is schematically shown in Fig. 3.26.

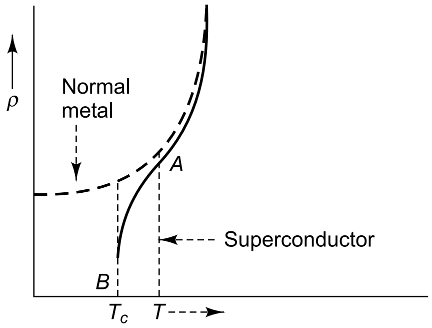


Fig. 3.26 *Zero Resistivity of a Superconductor*

Hence, a superconductor may be considered an ideal conductor with infinite electrical conductivity below its critical temperature. Table 3.4 shows the critcal temperatures of selected superconductors.

Table 3.4 *Critical Temperatures of Some Superconductors*

Superconductor	T_c *(K)*
In	3.40
Sn	3.72
Hg	4.20
Pb	7.19
Nb	9.50
La_3In	10.40
Nb_3Ge	23.20

2. Meissner Effect (or) Perfect Diamagnetism When subjected to magnetic field $H_{ext} < H_c$, the superconductor excludes the magnetic flux lines from its interior. This phenomenon is called the Meissner effect. The magnetic field H_c is a constant for the given superconductor and it is called the critical magnetic field. It is defined as the minimum magnetic field strength at and above which the magnetic flux lines penetrate into the superconductor. Figure 3.27 shows magnetic behaviour of superconductors and normal conductors.

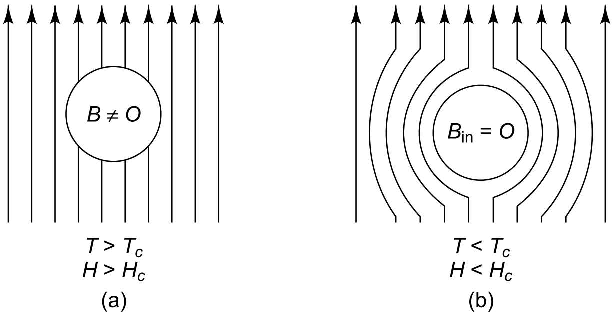


Fig. 3.27 *(a) Normal Conductor (b) Superconductor*

Inside the superconductor, we have

$$\vec{B} \equiv 0$$

i.e. $$\vec{B} \equiv \mu_o (\vec{H} + \vec{M}) = 0 \qquad (3.1)$$

where μ_o is the permeability of free space, M is the intensity of magnetization and H is the magnetic field strength.

Equation (3.1) may be rewritten as

$$\vec{B} \equiv \mu_o \left(1 + \frac{\vec{M}}{\vec{H}}\right) \vec{H} = 0$$

i.e. $$\mu_o(1+\chi)\vec{H} = 0 \tag{3.2}$$

where $\chi = \overline{M}/\overline{H}$ is the susceptibility of the material.

For a superconductor, Eq. (3.2) gives

$$\chi = -1$$

Hence, a superconductor can be considered as an ideal diamagnet when the applied magnetic field is less than its critical magnetic field.

3. Effect of Strong Magnetic Fields When subjected to a strong magnetic field $H_{\text{ext}} > H_c$, the magnetic flux lines penetrate into the superconductor. That is, the superconductor loses its perfect diamagnetism. The critical magnetic field is given as

$$H_c = H_o\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

where H_o is the critical magnetic field of the given superconductor at 0 K.

Table 3.5 shows the critical magnetic fields of selected superconductors at 0 K.

Table 3.5 *Critical Magnetic Fields of Some Superconductors*

Superconductor	*H_o (10^3 amp/m)*
In	2.33
Sn	2.46
Hg	3.28
Pb	6.39
Nb	15.76
Nb_3Sn	1.81
$BaBi_3$	5.90

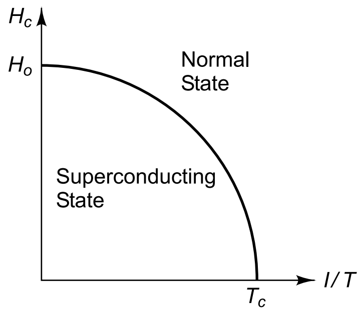


Fig. 3.28 *Temperature Dependence of the Critical Magnetic Field*

4. Effect of Heavy Currents A superconducting material loses its superconductivity when a heavy current $I \geq I_c$ flows through it. The critical current is defined as the minimum current to flow through a superconductor so that its superconductivity is destroyed. The critical current of a superconducting wire of radius r is given as

$$I_c = 2\pi r H_c$$

The above equation express the so-called **Silsbee's rule.**

5. Pressure Effect With the increase in pressure, the critical temperature of the superconductor increases. Table 3.6 shows the superconducting behaviour of silicon at an elevated pressure.

Table 3.6 *Superconducting Behaviour of Silicon at High Pressure*

Material	*P* *(MPa)*	T_c *(K)*
Si	0.1013	0
Si	16500	8.3

6. Isotope Effect The critical temperature of an isotope depends on the isotopic mass. The critical temparature of an isotope is related to the isotopic mass as

$$M^{\alpha} T_c = \text{constant}$$

where M is the isotopic mass and α is a characteristic constant for the given element. The values of α for some selected superconductors are shown in Table 3.7.

Table 3.7 α *Values of Some Superconductors*

Superconductor	α
Hg	0.5
Zn	0.3
Cd	0.5
Pb	0.48
Os	0.2

7. Specific Heat Below the superconducting transition temperature, the specific heat of a superconductor falls more rapidly than that of a conductor (Fig. 3.29). Since specific heat is a measure of the change in entropy with temperature, it may be concluded that the superconducting electron system is more ordered than the normal electron system.

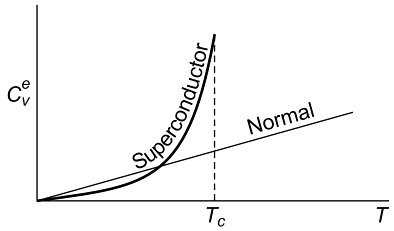
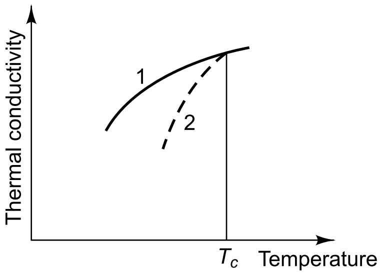


Fig. 3.29 *Specific Heats vs Time*

8. Thermal Conductivity In case of superconductors, a considerable decrease in the thermal conductivity is observed when the material makes the transition from the normal state to the superconducting state (Fig. 3.30). Such a behaviour of thermal conductivity shows that the superconducting electrons do not take part in the transportation of heat.



1-Normal state; 2-Superconducting state

Fig. 3.30 *Thermal Conductivity of Tin*

9. Effect of Impurities Addition of chemical impurities modifies the superconducting properties of a material, particularly, its magnetic properties.

10. Presence of Energy Gap The formation of Cooper pair of electrons in a superconductor results in the minimization of the total energy of the system. It leads to a small energy gap Δ near the Fermi energy E_F. The energy of the Cooper pair at E_F is taken as $E_F - \Delta$. The energy gap depends on temperature. At 0 K, it has the maximum value and it decreases with increasing temperature. At the critical temperature, the energy gap vanishes.

11. Microwave Transmission Microwave photons have energies less than the energy gap in superconductors. Hence, the superconductors do not absorb microwave radiation.

12. Magnetic Flux Quantization The magnetic flux passing through a superconducting ring is quantized. That is, the magnetic flux can have only discrete values. It is given as

$$\phi = n\left(\frac{h}{2e}\right)$$

where n is an integer, h is the Planck's constant and e is the charge of electron.

3.25.2 Types of Superconductors

The superconducting materials are classified as **Type I superconductors** (or soft superconductors) and **Type II superconductors** (or hard superconductors) according their behaviour in an external magnetic field.

Type I Superconductors

The materials for which the magnetization varies in a manner shown in Fig. 3.31 are called Type I superconductors.

In the superconducting state ($T < T_c$), the magnetic flux vanishes abruptly if the applied magnetic field becomes equal to the critical field. That is, perfect diamagnetism (or complete Meissner effect) is observed for Type I superconductors. The transition between the normal and the superconducting states is reversible. The material acquires its superconducting property when the external magnetic field is reduced below the critical magnetic field.

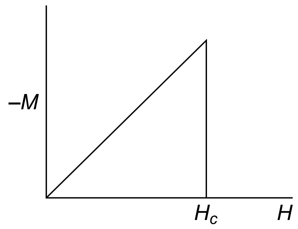


Fig. 3.31 *Magnetization Plot for Type I Superconductor*

The criterion for the Type I behaviour of the materials is the small value of the Landau–Ginsburg parameter. That is, for a Type I superconductor, we have

$$\frac{\lambda_L}{\varepsilon} < 0.71 \tag{3.3}$$

where λ_L is the London penetration depth and ε is the coherence length of the material.

In general, pure metals such as mercury, zinc, lead, etc., behave as Type I superconductors since they satisfy the criterion given by Eq. (3.3).

Type II Superconductors

The materials for which the magnetization varies with the applied field in a manner shown in Fig. 3.32 are called Type II superconductors.

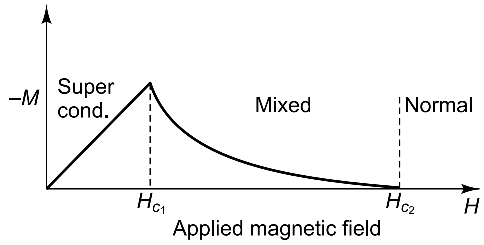


Fig. 3.32 *Magnetization Plot for Type II Superconductors*

In these materials, the magnetic flux does not penetrate $\left(\text{i.e. } \vec{B} = 0\right)$ up to a critical field H_{c1}. For fields lying between H_{c1} and H_{c2} the magnetic flux partially penetrates the material although it is still in the superconducting state. This occurs because for $H_{c1} < H < H_{c2}$ the material is divided into two types of regions, one superconducting and the other normal. The magnetic flux penetrates the material through the normal regions. The material still has zero resistance as the superconducting state electrons carry the electric current. The state within the fields H_{c1} and H_{c2} is called the mixed state. The maximum value observed for the upper critical field H_{c2} of about 800000 Gauss.

The criterion for a superconductor to be of Type II is

$$\frac{\lambda_L}{\varepsilon} > 0.71 \tag{3.4}$$

A material can change from Type I to Type II on the substitution of some impurities. For example, lead is a Type I superconductor with $H_c \cong 600$ G at 4 K. When 2% (by weight) of indium is added to it, the material becomes a Type II superconductor with $H_{c1} \cong 400$ G and $H_{c2} \cong 1000$ G. On adding 20% (by weight) of indium to lead, $H_{c1} \cong 70$ G and $H_{c2} \cong 3600$ G. It is found that the area within the M-H curve remains constant as the material goes from Type I to Type II on substitution of impurities.

3.25.3 BCS Theory of Superconductivity

The American physicists, John Bardeen (one of the inventors of transistor), Leon N. Cooper and John Robert Schrieffer developed in

1957 the quantum theory of superconductivity. Starting from the two experimental results, namely the isotope effect and the variation of electronic specific heat with temperature, they based their theory on interaction of two electrons through the intermediary phonons. When an electron approaches an ion in the lattice, there is a coulomb attraction between the electron and the lattice ion. This produces a distortion in the lattice. The distortion causes an increase in the density of ions in the region of distortion. The higher density of ions in the distorted region attracts, in its turn, another electron. Thus, a free electron exerts a small attractive force on another electron through phonons which are quanta of lattice vibrations. A pair of free electrons thus coupled through a phonon is a called Cooper pair. At normal temperatures, the attractive force is too small and pairing of electrons dose not takes place. Each Cooper pair consists of two electrons of opposite momenta. At room temperatures, such pairing is energetically advantageous.

In a typical superconductor, the volume of the given pair encompasses as many as 10^6 other pairs. These dense clouds of Cooper pairs form a collective state where strong correlations arise among the motions of all pairs because of which they drift cooperatively through the material. Thus, the superconducting state is an ordered state of the conduction electrons. The motion of all Cooper pairs is the same. Either they are at rest, or if the superconductor carries a current, they drift with identical velocity. The small velocity of Cooper pairs is quite high, even large currents require only a small velocity. The small velocity of Cooper pairs combined with their precise ordering minimizes collision process. The extremely rare collision of Cooper pairs with the lattice leads to vanishing resistivity. The BCS theory gives two important results, namely existence of energy gap and flux quantization.

3.25.4 Applications of Superconductors

Superconductors have enormous actual and potential applications. In this chapter, we discuss the important applications of superconductors.

3.25.5 Cryotron

Superconductors find applications in switching elements called cryotrons. The disappearance of superconductivity for fields above the critical field is the principle on which a cryotron operates. Let us consider a wire made of a superconducting material *A*, inside a coil of another superconducting material *B* (Fig. 3.33). Let the critical field of the material *A* be less than that of the material *B*. Also, let the temperature of the system be below the transition temperature of the two materials so that both are in the superconducting state.

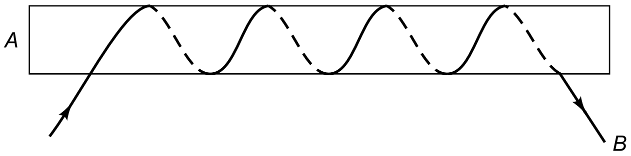


Fig. 3.33 *Wire Made of Two Superconducting Materials*

The current in the central wire *A* can then be controlled by a current in the coil because the magnetic field produced by the latter may exceed the critical field of the core material at the operating temperature. The control current required to make the core "normal" depends on the dc current flowing though the core because this current also produces a magnetic field. A metal such as tantalum is useful as a core material if the operating temperature is that of a liquid helium bath (4.2 K). The reason is that for tantalum, $T_c = 4.38$ K so that the critical field required to produce the normal state is relatively small. The coil material must be chosen so that it remains superconducting even if the control current flows; niobium or lead are, thus, suitable coil materials.

The single cryotron represented in Fig. 3.33 may be used an element in a more complicated device such as flip-flop. Since the elements are superconductors, the power consumption is very low and it has been estimated that a large-scale digital computer could be built requiring no more than 0.5 watt (not counting the helium cryostat and terminal equipment). Also, since the elements can be made small, the essential components of such a computer would occupy only one cubic foot.

3.25.6 Superconducting Levitation

Superconducting magnets are used to levitate a train above the track. This reduces the problem of friction. A levitated train can reach speeds upto 550 km/h. Superconducting magnets are longitudinally arranged on both sides of the bottom surface of the train. Superconducting magnets are arranged parallel to the track and these magnets lie just above the track. The track is made of concrete and coated with aluminium sheets.

An air-cored Linear Synchronous Motor (LSM) with superconducting coils act as the propulsion system. The middle part of the bottom surface of the train is fitted with coils which are parallel to the track. The interaction between the field coil of LSM and the travelling waves generates a propulsion force which makes the train to move.

When the train moves, the flux of the superconducting magnet induces current in the aluminium sheets. This induced current flows in the opposite direction to that passing through the superconducting coil of the magnet. Hence, a repulsive force is developed between these two current-carrying conductors which lifts the trains above the track. When the train moves at very low speeds (≈50 km/h) or when it is rest, the lifting force is not sufficient or zero. In this mode of operation, the train should be supported by wheels.

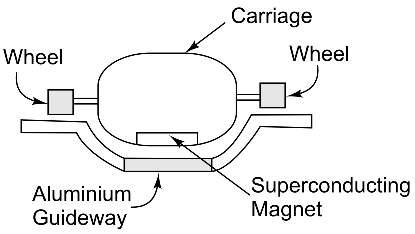


Fig. 3.34 *Guideway of Maglev Train (Cross-sectional View)*

Characteristics

- Absence of friction makes it possible for the train to move at great speeds with low consumption of power.
- Wide air gaps (exceeding 10 cm) are available for levitation.
- As the levitation and guidance are stable, no gap control is required.

3.25.7 Superconducting Quantum Interference Device (SQUID)

Josephson effect reveals us that tunneling of superconducting electron pair (Cooper pair) occurs through a very thing insulating barrier of 10–20 Å between two superconductors. The arrangement for observing Josephson effect is shown in Fig. 3.35. S_1 and S_2 are the two superconductors separated by a thin insulator. The tunneling of superconductor electron pairs through the Josephson junction leads to the flow of current without a voltage drop and this is known as dc Josephson effect. When a dc voltage is impressed across the junction through which supercurrent is flowing, an ac current is observed. Hence when a dc voltage exists across a Josephson junction an ac current is produced and is known as ac Josephson effect. By analyzing the current flow between two Josephson junctions in parallel, one obtains quantum interference effects and this is used to detect very low value of magnetic fields. This device is called SQUID (Superconducting Quantum Interference Device) and it is one of the applications of Josephson effect. It is a very sensitive device to detect very low value of magnetic fields. It is also used to detect small voltages as 10^{-15} V and small current as 10^{-12} A. They are used to detect neural electrical activity in the brain.

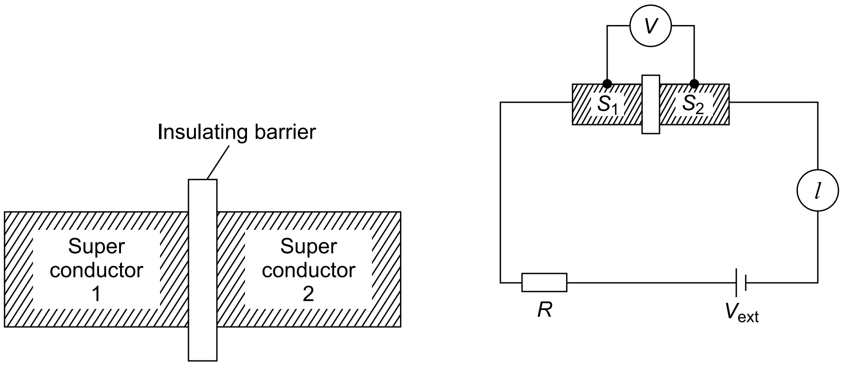


Fig. 3.35 *Josephson Effect*

3.25.8 High Temperature Superconductors

High T_c values are desirable in superconductors in order to reduce the amount of refrigeration needed to keep it in the superconducting state. According to BCS theory of superconductivity, the two factors that mainly decide the superconducting transition temperature are the electron-phonon coupling constant (V) and the density of electron states at the Fermi level $D(E_F)$. Both V and $D(E_F)$ must be high for a high value of T_c.

Some of the ternary compounds having the typical formula A_x Mo_6 Ch_8 where A is a metal atom and Ch is a chalcogen atom (i.e. S, Se or Te) satisfy the condition for high T_c. Experimentally they have been found to have moderately high T_c in the range of 14–20 K. These materials are also found to have high H_{c2} values. For example the superconductor $PbMo_6S_8$ has a T_c of 15 K and H_{c2} of 50 Tesla at 4.2 K.

The first high T_c ceramic superconductor was discovered in 1986, in a compound of barium, lanthanum, copper and oxygen. Since then a large number metal oxide compounds (ceramics), with a range of highT_c values have been reported. The oxide $YBa_2Cu_3O_7$, with a T_c of 90 K is the most extensively studied high T_c superconductor. Some of the high T_c superconductors and their T_c values are tabulated in Table 3.8. These are known as phase superconductors.

Table 3.8

High T_c Superconductors	*T_c (K)*
$La,_{1.85}$ $Ba_{0.15}$ CuO_4	36
YBa_2 Cu_3 $0_{6.9}$	90
Tl_2 Ba_2 Ca_2 Cu_3 O_{10}	125
$HgBa_2$ $CaCu_2$ 0_{6+}	133

As mentioned earlier, superconductors for use in practical applications need to have not only high T_c but also high H_c, and J_c, and they must also have favourable mechanical properties. They must be ductile so that they may be drawn in the form of wires. Many of the high T_c superconductors are brittle and so it is difficult to get them in the form of wires.

3.25.9 Crystal Structure of High T_c Ceramic Superconductors

Most of the oxide superconductors crystallize in a modified perovskite structure. Oxides of the type ABO_3 ($CaTiO_3$, $BaTiO_3$) crystallize in the perovskite structure. The unit cell of a compound belonging to the perovskite structure is shown in Fig. 3.36. The unit cell is a cube in which the large metal atom *A* lies at the body centre, the smaller metal atoms *B* occupy the corners of the cube and the oxygen atoms are at the

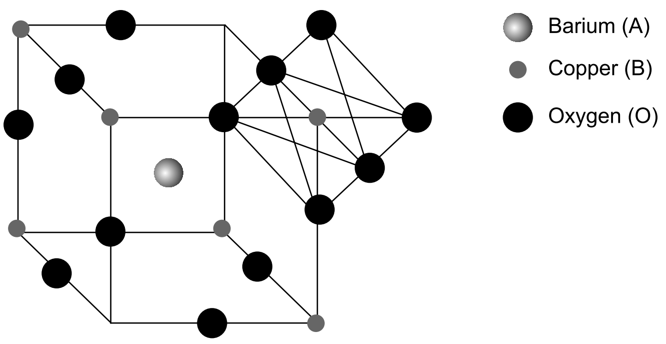


Fig. 3.36 *Cubic Perovskite Structure. Unit cell of ABO_3 type crystals*

centre of all the cube edges. There is one molecule per unit cell. The smaller metal atom B is surrounded by six oxygen atoms in an octahedral coordination shown by thick lines in Fig. 3.36. The lattice can be viewed as made up of a large number of octahedral units, with each unit joined at octahedral being occupied by the bigger metal atom A.

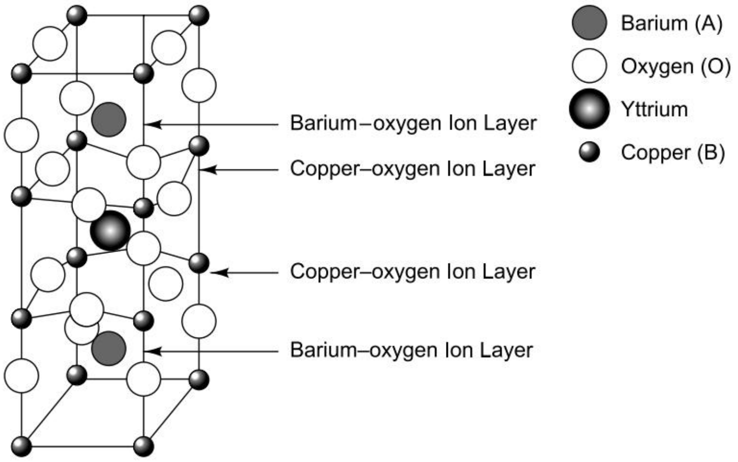


Fig. 3.37 *Structure of* $YBa_2\,Cu_3\,O_7$

The structure of $YBa_2\,Cu_3\,O_7$ is a modified perovskite structure as shown in Fig. 3.37. In this structure, the unit cell is bigger, with three perovskite unit cell cubes arranged along the direction, with Ba and Y atoms alternately occupying the cube centres. The copper atoms are at the cube corners, and some of the oxygen sites are unoccupied. The structure may be called quasi-tetragonal perovskite structure. This structure can be viewed as a layered structure, with alternating copper–oxygen ion layers, and Ba (or Y)-oxygen ion layers. The charge carriers in the conducting layers form Cooper pairs which move freely without getting scattered. Due to the layered structure the oxide superconductors are strongly anisotropic in their electrical properties.

SOLVED EXAMPLES

Example 3.1 *A magnetic material has a magnetization of 2000 A/M and produces a flux density of 0.00314* Wb/m^2*. Calculate magnetizing force and relative permeability of the material.* ($\mu_o = 12.57 \times 10^{-7}$ *H/m)*

Given Data

$I = 2000$ A/m

$B = 0.00314$ Wb/m^2

$\mu_o = 12.57 \times 10^{-7}$ H/m

Magnetizing force

$$H = \frac{B}{\mu_o} - 1$$

$$= \frac{0.00314}{12.57 \times 10^{-7}} - 2000$$

$H = 498$ A/m

Relative permeability

$$\mu_r = \frac{\mu}{\mu_o} = \frac{B}{B_o} = \frac{B}{\mu_o H}$$

$$\mu_r = \frac{0.00314}{12.57 \times 10^{-7} \times 498}$$

$\mu_r = 5.016$

Example 3.2 *Diamagnetic* Al_2O_3 *is subjected to an external magnetic field of* 10^6 *A/m. Calculate magnetization and magnetic flux density in* Al_2O_3*. Take susceptibility of* $Al_2O_3 = -5 \times 10^{-5}$ *and* $\mu_o = 12.57 \times 10^{-7}$ *H/m.*

Given Data

$H = 10^5$ A/m

$\chi = -5 \times 10^{-5}$

$\mu_o = 12.57 \times 10^{-7}$ H/m

Magnetization

$M = \chi H$

$= -5 \times 10^{-5} \times 10^6$

$= -50$ A/m

Magnetic flux density

$B = \mu_0 (H + M)$

$= 12.57 \times 10^{-7} (10^5 + (-50))$

$= 0.125$ Wb/m^2

Example 3.3 *Lead has two isotopes of atomic masses 206 and 210 which have the critical temperatures of 7.193 K and 7.126 K, respectively. Determine the value of the constant* α *for lead.*

Given Data

$M_1 = 206$

$M_2 = 210$

$T_{C1} = 7.193$ K

$T_{C2} = 7.126$ K

$\alpha = ?$

Isotope effect of superconductors gives

$$M^{\alpha} T_c = \text{const} = C \tag{1}$$

Therefore, for one isotope, we have

$$M_1^{\alpha} T_{c1} = C \tag{2}$$

For the other isotope, we have

$$M_2^{\alpha} T_{c2} = C \tag{3}$$

From Eqs (2) and (3), we get

$$\left(\frac{M_1}{M_2}\right)^{\alpha} \frac{T_{c_1}}{T_{c_2}} = 1 \tag{4}$$

i.e.

$$\alpha = \frac{\ln\left(\dfrac{T_{c2}}{T_{c1}}\right)}{\ln\left(\dfrac{M_1}{M_2}\right)}$$

Substituting the given values of M_1, M_2, T_{c1}, T_{c2} into Eq. (4), we get

$$\alpha = \frac{\ln\left(\dfrac{7.126}{7.193}\right)}{\ln\left(\dfrac{206}{210}\right)}$$

i.e. $\boldsymbol{\alpha = 0.487}$

Example 3.4 *Indium has the critical temperature of 3.4 K and the critical magnetic field at 0 K of 0.0293 tesla. Determine the critical current at 2 K for an indium wire of 6 mm radius.*

Given Data

$T_c = 3.4$ K

$B_0 = \mu_o \mathrm{H} = 0.0293$ tesla

$H_0 = 2.333 \times 10^4$ A/m

$r = 6$ mm $= 6 \times 10^{-3}$ m

$T = 2$ K

$I_c = ?$

The critical current flowing through a superconducting wire is given as

$$I_c = 2\pi r H_c \tag{1}$$

The critical magnetic field is given as

$$H_c = H_0\left(1 - \frac{T^2}{T_c^2}\right) \tag{2}$$

From Eqs (1) and (2), we get

$$I_c = 2\pi r H_o\left(1 - \frac{T^2}{T_c^2}\right) \tag{3}$$

Substitution of the given values for r, H_o, T, T_c into Eq. (3) gives

$$I_c = 2 \times 3.14 \times 6 \times 10^{-3} \times 2.333 \times 10^4 \times \left(1 - \frac{2^2}{(3.4)^2}\right)$$

i.e. $\mathbf{I_c = 574.9\ A}$

Example 3.5 *The superconducting transition temperature of lead is 7.26 K. The initial field at 0 K is 64 × 10^{3}amp/m. Calculate the critical field at 5 K.* ***(A.U., B.E. (EEE), May/June 2006)***

(A.U., B.E./B.Tech, May/June 2011)

Given Data

$T_c = 7.26$ K

$H_0 = 64 \times 10^3$ amp/m

$T = 5$ K

$H_c(T) = ?$

The critical magnetic field of a superconductor is given as

$$H_c(T) = H_0\left(1 - \frac{T^2}{T_c^2}\right) \qquad (1)$$

Substitution of the given values of H_0, T_c, T into Eq. (1) gives

$$H_c(5\text{ K}) = 64 \times 10^3 \times \left(1 - \frac{5^2}{(7.26)^2}\right)$$

i.e. $\mathbf{H_c(5\ K) = 33.6 \times 10^3\ amp/m}$

QUESTIONS AND ANSWERS

1. **Define magnetic field intensity. Name the unit to measure it.**

   It is defined as the force experienced by a unit north pole when placed at the given point in a magnetic field. Its unit is amp/metre.

2. **Define magnetic induction. Name the unit to measure it.**
   (A.U., B.E./B.Tech, May/June 2011)

   Magnetic induction in a material is defined as the number of lines of magnetic force passing perpendicularly through unit area. Its unit is weber/m^2 or tesla.

3. **Define intensity of magnetization. Name the unit to measure it.**

   Intensity of magnetization of a material is defined as the magnetic moment per unit volume. The intensity of magnetization of a magnetic specimen is the measure of its magnetization. Its unit is amp/metre.

4. **Define magnetic susceptibility.**

   Magnetic susceptibility of a material is defined as the ratio of the intensity of magnetization produced in the sample to the magnetic field which produces the magnetization. The sign and magnitude of the magnetic susceptibility are used to determine the nature of the magnetic materials. It has no unit.

5. **Define permeability.**

   Magnetic permeability of a material is defined as the ratio of the magnetic induction in the sample to the applied magnetic field intensity. The readiness of a material to accept magnetism is permeability.

6. **What are diamagnetic materials?**

   Diamagnetic materials are those which when placed in a magnetic field, become weakly magnetized in the direction opposite to that of the applied field.

7. **Mention any four properties of diamagnetic materials.**

   1. Diamagnetic materials repel the magnetic lines of force.
   2. They have no permanent dipole moment.
   3. Diamagnetic materials exhibit negative magnetic susceptibility.
   4. As diamagnetic susceptibility is negative, the relative permeability μ_r is less than unity. ($\mu_r < 1$)

8. **What are paramagnetic materials?**

   Paramagnetic materials are those which when placed in a magnetic field, become weakly magnetized in the same direction as that of the applied field.

9. **Mention any four properties of paramagnetic materials.**

   1. Paramagnetic materials attract the magnetic lines of force.
   2. Paramagnetic materials possess permanent dipole moment.
   3. They exhibit positive magnetic susceptibility.
   4. The relative permeability μ_r is slightly more than unity. ($\mu_r > 1$)

10. **What are ferromagnetic materials?**

    Ferromagnetic materials are those which when placed in a magnetic field, become strongly magnetized in the direction of the applied field. The direction of magnetization is the same as that of the external field.

11. **Mention any four properties of ferromagnetic materials.**

    1. Ferromagnetic materials conduct magnetic flux as much as metals conduct electric current.
    2. Ferromagnetic materials have permanent dipole moment.
    3. They exhibit magnetization even in the absence of magnetic field known as spontaneous magnetization.
    4. They exhibit hysteresis curve.

12. **What are ferrimagnetic materials?**

    Ferrimagnetic materials are also known as uncompensated antiferromagnetism. They are very much identical to ferromagnetic materials in their macroscopic behaviour. The saturation magnetization are not as high as ferromagnetic material.

13. What is Curie temperature?

The critical temperature at which a ferromagnetic material becomes a paramagnetic material is known as Curie temperature.

14. What are antiferromagnetic materials?

Antiferromagnetic materials are crystalline materials which posses a small positive susceptibility of the order of 10^{-3} to 10^{-5}. The magnetic interactions between any two dipoles align themselves antiparallel to each other and all the dipoles are equal in magnitude and hence the resultant magnetization is zero.

15. What is hysteresis? ***(A.U., B.E./B.Tech, May/June 2011)***

When a magnetic material is taken through a cycle of magnetization, the intensity of magnetization (I) and the magnetic flux density (B) lag behind the applied magnetic field (H). This lagging of I and B behind H is known as hysteresis.

16. What is hysteresis loop?

The closed curve obtained when a magnetic material is taken through a cycle of magnetization is known as hysteresis loop.

17. What is hysteresis loss? ***(A.U., B.E./B.Tech, May/June 2011)***

The energy loss due to the work done on the magnetic material during the hysteresis process is known as hysteresis loss. The magnitude of the hysteresis loss is given by the area enclosed by the hysteresis loop.

18. Define retentivity of a ferromagnetic material.

Retentivity is defined as the magnetization retained by the specimen when the magnetizing field is reduced from saturation value to zero.

19. Define coercivity of a ferromagnetic material.

The negative value of the magnetic field required to demagnetize a ferromagnetic specimen is defined as coercivity.

20. What are soft magnetic materials? Give examples.

Materials which are easy to magnetize and demagnetize are called soft magnetic materials.

Examples

Iron-sillicon alloys, Iron-cobalt alloys, etc.

21. What are hard magnetic materials? Give examples.

Materials which are difficult to magnetize and demagnetize are called hard magnetic materials.

Examples

Carbon steel, tungsten steel, Al-Ni-Co alloys, etc.

22. What are ferrites? Give examples.

Ferrimagnetic materials are called ferrites. Ferrites have an un-compensated anti-ferro magnetism.

Examples

Ferrous ferrite, zinc ferrite, nickel ferrite, etc.

23. What are magnetic domains?

The group of atoms organized into tiny bounded regions in a ferromagnetic material are called magnetic domains. A magnetic domain will behave like a magnet having its own magnetic moment.

24. Mention any two applications of ferrites.

1. Ferrites are used as cores in audio and TV transformers.
2. Ferrites are used in digital computers and data processing circuits.

25. What are the requirements of a transformer core material?

Transformer core material should have high resistivity to reduce eddy current losses and be magnetically soft to reduce hysteresis losses.

26. Mention any four characteristics of a hard magnetic material.

1. It has a large area hysteresis loop.
2. It has high coercivity and high retentivity.
3. It has lesser permeability.
4. It is used for making permanent magnets.

27. Mention any four characteristics of a soft magnetic material.

1. It has smaller area hysteresis loop.
2. It has lesser coercivity and retentivity.

3. It has large permeability.
4. It is used for making electromagnets.

28. Determine the power loss due to hysteresis in a transformer core of 0.01 m^3 volume and 50 Hz frequency. Take area of hysteresis loop as 600 Jm^{-3}.

Given Data

Volume of the material $V = 0.01$ m^3
Frequency $f = 50$ Hz
Area of the loop $A = 600$ Jm^{-3}

We know the power loss due to hysteresis,

W_h = (Area of the loop) × (volume of the material) × (frequency of the reversal of magnetization)

$= 600 \times 0.01 \times 50$

$= 300$ watts

29. What is the role of a core in a transformer?

A transformer core provides a path for the magnetic lines of flux.

30. Define critical magnetic field in superconductors.

It is minimum magnetic field strength at and above which the magnetic flux lines penetrate into the superconductor.

31. What are superconductors?

The substances exhibiting zero resistivity below the critical temperature and the Meissner effect below the critical magnetic field are called superconductors.

32. Define the critical temperature of superconductors.

The critical temperature is defined as the temperature at which the electrical resistivity of the superconductor suddenly vanishes.

33. Define the critical magnetic field of superconductors.

The critical magnetic field is defined as the minimum magnetic field strength at and above which the magnetic flux lines penetrate into the superconductor.

34. Give the expression for the critical magnetic field of superconductors.

The critical magnetic field is given as

$$H_c = H_o\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

where H_o is the critical magnetic field of the given superconductor at 0 K.

35. Define the critical current flowing through a superconduting wire.

The critical current is defined as the minimum current to flow through a superconductor so that its superconductivity is destroyed.

36. Give the expression for the critical current flowing through a superconducting wire.

The critical current of a superconducting wire of radius r is given as

$$I_c = 2\pi r H_c$$

37. What is Meissner effect in superconductors?

(A.U., B.E./B.Tech, May/June 2011)

When subjected to magnetic field $H_{\text{ext}} < H_c$, the superconductor excludes the magnetic flux lines from its interior. This phenomenon is called the Meissner effect.

38. What is pressure effect in superconductors?

With the increase in pressure, the critical temperature of the superconductor increases.

39. What is isotope effect in superconductors?

The critical temperature of an isotope depends on the isotopic mass. The critical temperature of an isotope is related to the isotopic mass as

$$M^{\alpha} T_c = \text{const.}$$

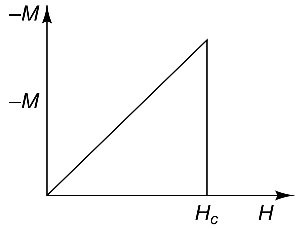
where M is the isotopic mass and α is a characteristic constant for the given element.

40. What is impurities effect in superconductors?

Addition of chemical impurities modifies the superconducting properties of a material, particularly, its magnetic properties.

41. What are Type I superconductors?

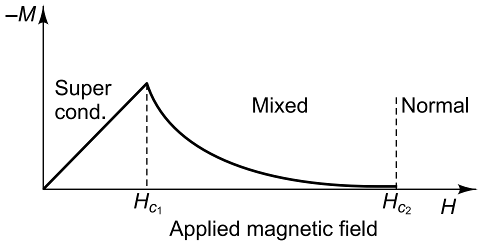
The materials for which the magnetization varies in a manner shown below are called Type I superconductors.



In the superconducting state ($T < T_c$), the magnetic flux vanishes abruptly if the applied magnetic field becomes equal to the critical field.

42. What are Type II superconductors?

The materials for which the magnetization varies with the applied field in a manner shown below are called Type II superconductors.



43. What are high temperature superconductors?

They are the superconductors with higher values of critical temperatures. ***(A.U., B.E./B.Tech, May/June 2011)***

EXERCISE PROBLEMS

PART-A

1. The mercury isotope of mass number 202 has the critical temperature 4.153 K. The constant α for mercury is 0.5. Determine the critical temperature of another isotope of mercury with a mass number 200. (*Ans.* $T_{c2} = 4.174$ *K*)

2. Determine the temperature to which tantalum is to be cooled so that it becomes a superconductor in a magnetic field of 15×10^3 A/m. The critical temperature of tantalum is 4.483 K and its critical magnetic field at 0 K is 6.608×10^4 A/m. (*Ans.* $T = 3.941$ *K*)

3. The slope of the magnetization curve of a Type I superconductor at $T_c = 7$ K is -1.991×10^4 A/m/K. Determine the critical magnetic field of the superconductor at 6 K. (*Ans.* $H_c = 1.849 \times 10^4$ *A/m*)
4. Calculate the relative permeability of a magnetic material if the material has a magnetization of 3000 A/m and flux density of 0.044 Wb/m^2. (*Ans.* $\mu_r = 17.258$)
5. Prove that a superconductor is a perfect diamagnet. (*Ans.* $x = -1$)

QUESTIONS

PART-B

1. Explain the classification of magnetic materials with their properties in detail.
2. Distinguish between dia, para and ferromagnetic materials.
3. Explain domain theory of ferromagnetism and the types of energy involved in the process of domain growth in detail. *(A.U., B.E./B.Tech, May/June 2011)*
4. Discuss ferromagnetic hysteresis in detail.
5. Distinguish between soft and hard magnetic materials.
6. What are ferrites? Discuss the structure, preparation, properties and application of ferrites in detail. *(A.U., B.E./B.Tech, May/June 2011)*
7. Explain the magneto-optical recording, reading and erasure process.
8. Discuss in detail the principle and working of magnetic tape of magnetic storage.
9. Discuss the principle and working of floppy disk of magnetic storage in detail.
10. Discuss the principle and working of magnetic disk drives of magnetic storage in detail.
11. Explain the properties of superconductors. *(A.U., B.E./B.Tech, May/June 2011)*

12. Discuss type-I and type-II superconductors in detail.
(A.U., B.E./B.Tech, May/June 2011)

13. Explain BCS theory of superconductivity.
(A.U., B.E./B.Tech, May/June 2011)

14. Comment on high temperature superconductor.
(A.U., B.E./B.Tech, May/June 2011)

15. Discuss the application of superconductors in SQUID, cryotron and magnetic levitation in detail.
(A.U., B.E./B.Tech, May/June 2011)

SUMMARY

- The materials which get magnetized in a magnetic field are known as magnetic materials.
- A bar magnet in which the two opposite poles are separated by a distance ‘l’ is known as a magnetic dipole.
- Intensity of magnetization (I) is defined as the magnetic moment per unit volume.
- Magnetic susceptibility χ_m is the ease with which a material becomes magnetized.

$$\chi_m = \frac{1}{H}$$

- Diamagnetic materials exhibit weak magnetism.
- Paramagnetic materials possess weak magnetism.
- Ferromagnetic materials possess very high susceptibility and high relative permeabilities.
- Ferromagnetic materials exhibit hysteresis when they are subjected to cyclic magnetization and demagnetization.
- Orbitting electrons and spinning electrons in an atom are the ultimate source of magnetic properties of materials.
- Atoms (or molecules) in diamagnetic materials do not possess permanent magnetic moment.

- The spontaneous magnetization is a consequence of exchange forces operating between the spin moments.
- When the ferromagnetic domains in a specimen are at random orientations, the sample is in a demagnetized state. When a magnetic field is applied, the domains are aligned and the material becomes magnetized and remains magnetized to a certain extent when the field is switched off.
- In antiferromagnetic materials, the adjacent electron spins are aligned in opposite directions so that the net magnetization is zero.
- In ferrimagnetic materials, unequal spin moments are aligned in opposite directions, so that there is a net magnetic moment.
- The antiferromagnetic and ferrimagnetic materials become paramagnetic when they are heated above Neel temperature.
- Soft magnetic materials are characterized by large hysteresis loop, low permeability and high coercive force.
- Hard magnetic materials are characterized by large hysteresis loop, low permeability and high coercive force.
- The phenomenon in which there is no resistance to current flow at very low temperatures is known as superconductivity.
- The basis of superconductivity is the formation of cooper pairs at very low temperatures.
- Bound pair of electrons are called Cooper pairs.
- Type I (or soft) and Type II (or hard) superconductor are the two types of superconductors.
- Superconductors having high value of critical temperatures are known as high temperature superconductors.

UNIT IV

DIELECTRIC MATERIALS

Education is what remains after one has forgotten what one has learned in school. **—Albert Einstein**

4.1 INTRODUCTION

Dielectric materials belong to an important class of engineering materials. Dielectric materials are insulating materials and are used to store electrical energy. Dielectric materials differ from the insulating materials in the way that insulating materials resist the flow of an electric current while the function of a dielectric material is to store electrical energy. The essential characteristic of a material to be a good dielectric is that it must be an insulator. Hence, all the dielectric materials are insulators but all the insulators need not to be dielectric. Dielectric materials are also known as dielectrics. In general the resistivities of dielectrics are in the range of 10^{10} to 10^{20} ohm.m.

A dielectric material is characterized by the presence of large forbidden energy gap between the valence and conduction bands. Moreover, the valance band is completely filled and the conduction band is unfilled with electrons. Hence under normal conditions excitation of electrons from the valence band to the conduction band is not possible and no electrical conduction takes place. Generally dielectric materials are non-metallic materials of high specific resistance and has negative temperature coefficient of resistance.

4.2 CHARACTERISTIC PARAMETERS OF DIELECTRICS

Dielectric materials are characterized by the parameters such as

- Dielectric constant
- Electric dipole moment
- Polarization
- Polarizability

4.2.1 Dielectric Constant

The dielectric constant of a material is defined as the ratio of the permittivity of the medium (ε) to the permittivity of free space (ε_0) That is,

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \tag{4.1}$$

where ε_r is the dielectric constant and is a dimensionless quantity. The measure of dielectric constant or relative permittivity gives the properties of a dielectric material. Permittivity is a measure of how easily a dielectic polarizes in response to an applied electric field. The dielectric constant of air is 1. Table 4.1 shows the values of dielectric constants of selected dielectrics.

Table 4.1 *Dielectric Constants of some Materials*

Dielectrics	ε_r
Teflon	2.1
Polyethylene	2.3
PVC	3.5
Sodalime glass	7
Mica	> 7
$BaTiO_3$	> 3000

4.2.2 Electric Dipole Moment

Let us consider two charges of equal magnitudes but opposite polarities separated by a distance *r* as shown in Fig. 4.1.

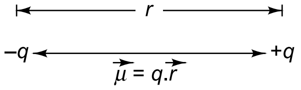


Fig. 4.1 *Electric Dipole Moment*

The arrangement of two equal, opposite charges of + *q* and – *q* separated by a distance *r* is known as **electric dipole**. The product of magnitude of the charge and the distance of separation is known as electric dipole moment. Its unit is coulomb-metre. That is,

$$\vec{\mu} = q \times \vec{r} \tag{4.2}$$

Since the dipole moment is a vector, it points from negative to the positive charges as shown in Fig. 4.1.

The total dipole moment of a system constituting point charges q_1, q_2, … q_n and the distances of separation r_1, r_2 … r_n is given by

$$\mu_{\text{total}} = \sum_{i=1}^{n} q_i r_i \tag{4.3}$$

4.2.3 Polarization

When an electric field is applied to a solid material consisting of positive and negative charges, the positive charges are displaced in the direction of the field while negative charges are displaced opposite to the direction of the field. The displacement of these two charges create a local dipole in the solid. This type of displacement of positive and negative charges by the application of the electric field leads to polarization.

P = Nμ (4.4), where P is the polarization in coulomb m^{-2}, μ is the dipole moment per atom and N is the number of atoms per unit volume.

4.2.4 Electric Susceptibility

The polarization vector $\vec{P}$ is proportional to the total electric field intensity $\vec{E}$ and is in the direction of $\vec{E}$. Therefore, the polarization vector can be written as

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \tag{4.5}$$

where the constant χ_e is referred to as the electric susceptibility and is a characteristic of a dielectric.

Equation (4.5) may be rewritten, in the scalar form, as

$$\chi_e = \frac{P}{\varepsilon_0 E} \tag{4.6}$$

On the other hand, we have

$$\frac{P}{E} = \varepsilon = \varepsilon_0 \equiv \varepsilon_0 (\varepsilon_r - 1) \tag{4.7}$$

From Eqs (4.6) and (4.7), we get

$$\chi_e = \varepsilon_r - 1 \tag{4.8}$$

4.2.5 Polarizability

When a dielectric material is placed in an electric field, dipoles are established in the material. The polarization P of an elementary particle is directly proportional to the electric field strength E.

i.e. $P\, \mu \propto E$

or $\mu = \alpha E$ (4.9)

where α is a proportionality constant known as polarizability. The unit of α is Farad·m^2.

If the solid material contains N number of particles per unit volume, then the polarization can be written as

$$P = N \alpha E \tag{4.10}$$

where $\alpha = \alpha_e + \alpha_i + \alpha_o$. Here, α_e, α_i, and α_o are the electronic, ionic and orientational polarizabilities, respectively.

4.3 DIFFERENT TYPES OF POLARIZATIONS

When an electric field is applied to a dielectric material, the electric charges of the constituting atoms are displaced from their equilibrium position and this results in the separation of charges. Polarization occurs due to several microscopic mechanisms such as

- Electronic polarization
- Ionic polarization
- Orientational polarization
- Space-charge polarization

4.3.1 Electronic or Induced Polarization (α_e)

On the application of an electric field, the displacement of the positively charged nucleus and the negatively charged electrons of an atom in the opposite directions results in electronic polarization. It occurs in all dielectrics for any state of aggregation. The phenomenon is illustrated in Fig. 4.2. The electronic polarization takes place in a very short span of time, of the order of 10^{-14} to 10^{-15} s and it is independent of temperature.

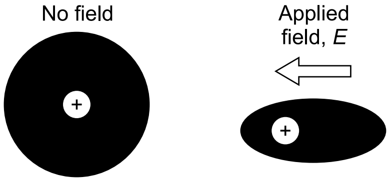
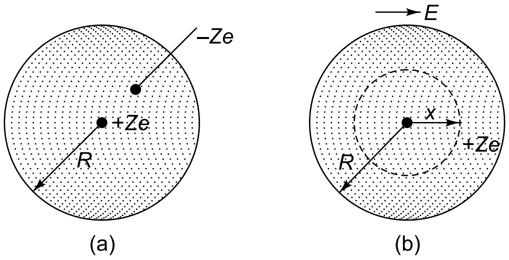


Fig. 4.2 *Electronic Polarization*

Calculation of Electronic Polarizability

Let us consider an atom of a dielectric material subjected to an electric field *E*. The nucleus of that atom moves towards the field direction and the electron cloud of the atom moves in opposite direction. When this happens, an attractive coulomb force is created and it will try to maintain the original equilibrium positions and at the same time, the Lorentz force will try to separate the nucleus and electron cloud of that atom from their equilibrium positions. When there is no external field, the centre of the electron cloud and nucleus of the atom are same and one as shown in Fig. 4.3(a). When the field is applied, the electron cloud centre gets a displacement *x* with respect to the centre of the nucleus. It is assumed that on the application of the electric field, the spherical shape of the electron cloud is not altered and the electron cloud is uniformly distributed over a sphere of radius *R* as shown in Fig. 4.3(b).



(a) (b)

Fig. 4.3 *Shape of Electron Clouds*

The charge density of the charged sphere is ($x << R$)

$$q' = \frac{-Ze}{\frac{4}{3}\pi R^3} \tag{4.11}$$

where *Ze* is the total negative charge.

The total number of negative charges in the sphere of radius *x* is

$$N' = \frac{-Ze\frac{4}{3}\pi x^3}{\frac{4}{3}\pi R^3} = -\frac{Zex^3}{R^3} \tag{4.12}$$

The coulomb force between the nucleus with positive charge *Ze* and the electron cloud which is displaced by a distance *x* from the centre of the nucleus is given by

$$F = \frac{\frac{-Zex^3}{R^3}Ze}{4\pi\varepsilon_0 x^2} = \frac{-Z^2e^2x}{4\pi\varepsilon_0 R^3} \tag{4.13}$$

The Lorentz force between nucleus and the electron cloud is given by

$$F = ZeE \tag{4.14}$$

Under equilibrium conditions, the coulomb attractive force and the Lorentz repulsive force are equal and opposite.

Hence,

$$ZeE = \frac{-Z^2e^2x}{4\pi\varepsilon_0 R^3}$$

i.e.

$$E = \frac{Zex}{4\pi\varepsilon_0 R^3} \tag{4.15}$$

The induced dipole moment is given by

$$\mu = Zex \tag{4.16}$$

Moreover, dipole moment is given as

$$\mu = \alpha_e E \tag{4.17}$$

From Eqs (4.16) and (4.17), we get

$$E = \frac{Zex}{\alpha_e} \tag{4.18}$$

Comparing Eqs (4.15) and (4.18), we get

$$\alpha_e = 4\pi\varepsilon_0 R^3 \tag{4.19}$$

4.3.2 Atomic or Ionic Polarization (α_i)

Ionic polarization occurs in ionic crystals. On the application of an electric field, the displacement of the positively and negatively charged ions in the ionic bonded solid in the opposite directions results in ionic polarization. The phenomenon is illustrated in Fig. 4.4.

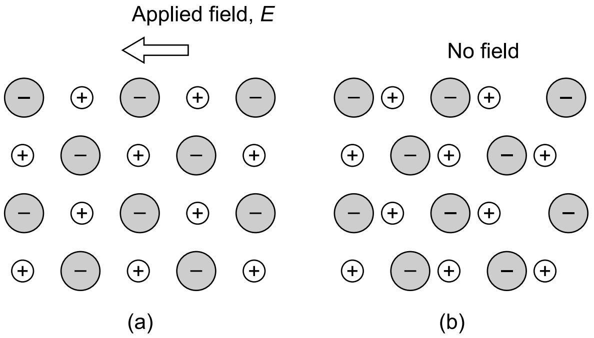


Fig. 4.4 *Ionic Polarization*

Let us assume an ionic crystal in which each of its unit cell contains one cation and one anion. When the electric field is applied in the negative direction, as shown in Fig. 4.4(a), then the positive ions move to the left and negative ions move to the right. The ionic polarization occurs when the frequency of the applied electric field is below the optical range of frequencies and it also independent of temperature.

Calculation of Ionic Polarizability

Let us consider an electric field applied in the positive x direction. Then the positive ions moves to the right and the negative ions move to the left.

The resultant dipole moment per unit cell due to ionic displacement is

$$\mu = e\,(x_1 + x_2) \tag{4.20}$$

where x_1 is the shift of positive ion and x_2 is the shift of negative ion with respect to their equilibrium position.

Let F be the force due to the application of electric field. Let $\beta_1 x_1$ be the restoring force on the positive ion and $\beta_2 x_2$ be the restoring force on the negative ion. β_1 and β_2 are restoring force constants.

Under equilibrium,

$$F = \beta_1 x_1 = \beta_2 x_2 \tag{4.21}$$

i.e.

$$x_1 = \frac{F}{\beta_1} = \frac{eE}{M\omega_0^{\;2}} \tag{4.22}$$

where M is the mass of the positive ion.

Also,

$$F = eE \text{ and } \beta_1 = M\omega_0^2$$

For negative ion we have,

$$x_2 = \frac{eE}{m\omega_0^2} \tag{4.23}$$

where m is the mass of the negative ion.

Now,

$$(x_1 + x_2) = \frac{eE}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) \tag{4.24}$$

From Eqs (4.20) and (4.24)

$$\mu = \frac{e^2 E}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) \tag{4.25}$$

Hence,

$$\alpha_i \equiv \frac{\mu}{E} = \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) \tag{4.26}$$

Thus, the ionic polarizability α_i is inversely proportional to the square of the natural frequency of the ionic molecule and to its reduced mass where reduced mass is mathematically equal to

$$M_r = \left(\frac{1}{M} + \frac{1}{m}\right)^{-1} \tag{4.27}$$

4.3.3 Orientation Polarization (α_0)

The existence of a permanent dipole moment in the dielectric medium results in orientation polarization. On the application of electric field, the permanent dipoles will tend to align along its own direction. This phenomenon is illustrated in Fig. 4.5.

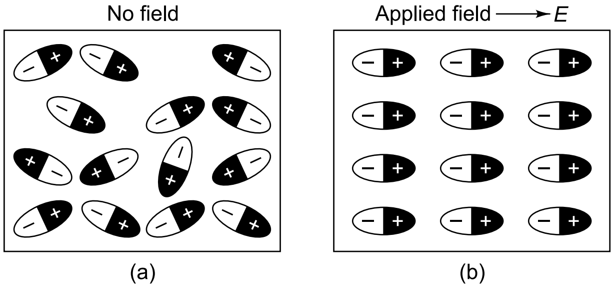


Fig. 4.5 *Orientation Polarization*

The orientation polarization occurs when the frequency of applied field is in the range of audio frequencies. The orientation polarization decreases with the increase in temperature.

Calculation of Orientation Polarizability

Orientation polarization is due to the presence of polar molecules in the dielectric medium. The dielectrics can be further subdivided into two as polar dielectrics and non-polar dielectrics. The arrangement or geometry of the atoms in some molecules is such that one end of the molecule has a positive electrical charge and the other side has a negative charge. If this is the case, the molecule is called a polar molecule. Otherwise, it is called a non-polar molecule. Polar molecules have permanent dipoles even in the absence of an electric field. When an electric field is applied on the dielectric medium with polar molecules, the electric field tries to align these dipoles along its direction. Due to this, there results a dipole moment in that material and this process is called orientational polarization. This concept is similar to the alignment of permanent magnetic moments of atoms in a magnetic field giving rise to paramagnetism. Following the procedure adopted in the Langevin theory of paramagnetism, we get orientational polarization as

$$P_0 = N \mu^2 E/3 kT \tag{4.28}$$

Moreover, we have

$$P_0 = N \alpha_0 E \tag{4.29}$$

From Eqs (4.28) and (4.29), we get

$$\alpha_0 = \frac{P_0}{NE} = \mu^2/3 kT \tag{4.30}$$

Thus, the orientational polarizability α_0 is inversely propotional to the absolute temperature of the material.

4.3.4 Space Charge Polarization or Interfacial Polarization (α_s)

The accumulation of charges at the electrodes or interfaces in a multiphase material results in space charge polarization. This phenomenon is illustrated in Fig. 4.6.

The space charge polarization occurs when the frequency of applied field is in the range of power frequencies. It increases with the increase in temperature.

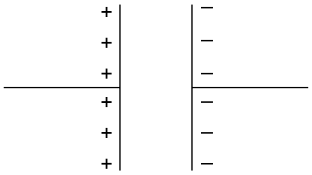


Fig. 4.6 *Space Charge Polarization*

4.3.5 Total Polarization of a Dielectric Material

The total polarization of a dielectric material is the sum or contribution of the electronic, ionic, orientational and space charge polarizations. Since the value of the space charge polarizability is very small when compared to other polarizabilities, we are neglecting it in the calculation of total polarizability.

The total polarizability in a dielectric material is given as

$$\alpha = \alpha_e + \alpha_i + \alpha_o \tag{4.31}$$

From Eqs (4.19), (4.26), (4.30), we get

$$\alpha = 4\pi\,\varepsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT} \tag{4.32}$$

The total polarization P can be written as

$$P = N\alpha E = NE\left[4\pi\varepsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT}\right] \tag{4.33}$$

This equation is known as **Langevin–Debye Equation**.

4.4 FREQUENCY DEPENDENCE OF POLARIZATION

The polarization and dielectric constant depends on the frequency of the applied field. This phenomenon is called dielectric dispersion. On the application of an electric field, a polarization process occurs as a function of time. The polarization $P(t)$ as a function of time t is given by

$$P(t) = P[1 - e^{(-t/t_r)}] \tag{4.34}$$

where, P is the maximum polarization attained in time t, and t_r is the relaxation time for the particular polarization process.

The relaxation time t_r is defined as the time taken for a polarization process to reach 0.63 of the maximum value. This relaxation time differs for different polarization processes.

Electronic polarization is extremely rapid and it takes place at the instant the voltage is applied. It occurs at all frequencies of the applied voltage. Ionic polarization takes place slower than electronic polarization. Ionic polarization does not occur at optical frequencies. When the frequency of the applied voltage is less than 10^{13} Hz, the ions possess enough time to respond during each cycle. Orientation polarization is further slower than ionic polarization. Orientation polarization occurs when the frequency of the applied field is in the range of audio frequencies. Space charge polarization is the slowest process as it involves the diffusion of ions over several interatomic distances. The relaxation time for this process is related to the frequency of successful jumps of ions under the influence of the applied field, a typical value being 100 Hz.

Correspondingly, space charge polarization occurs at machine frequencies of about 50–60 Hz. The plot of polarization and loss of power is shown in Fig. 4.7.

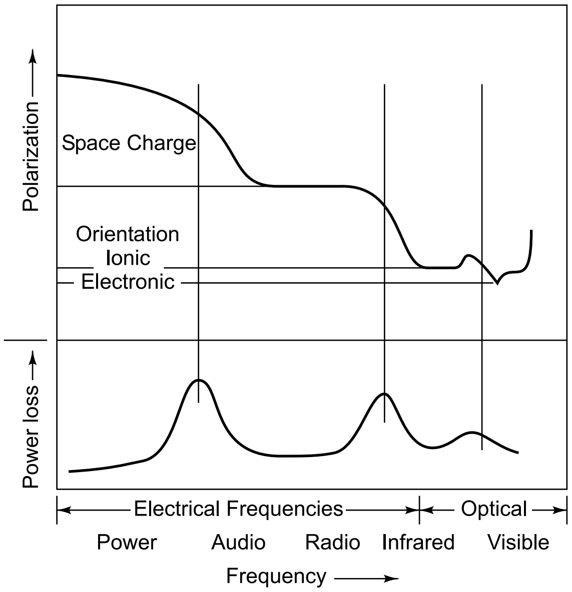


Fig. 4.7 *Frequency Dependence of Polarization Processes and Peak Power Losses Corresponding to each Process*

Figure 4.7 reveals that when there is an increase in frequency of the applied voltage the dielectric constant decreases. The value of total polarization is very high at low frequencies and at high frequencies the value of the total polarization is very high.

4.5 TEMPERATURE DEPENDENCE OF POLARIZATION

The effect of temperature on the dielectric constant of a material is discussed below.

- The electronic polarization is independent of the temperature.
- The ionic polarization is also independent of the temperature.
- The orientation polarization decreases with the increase in temperature. The reason is that the thermal energy tends to randomize the alignment of dipole moment. Hence with the increase in temperature, the dielectric constant decreases.
- The space charge polarization increases with the increase in temperature. The reason is that the thermal energy also helps in overcoming the activation barrier for the orientation of polar molecules in the direction of the field. Hence with the increase in temperature, the dielectric constant increases.

4.6 INTERNAL FIELD

When an electric field is applied, the atoms of a dielectric material are polarized. As a result, the atom at a given point experiences both the applied electric field and the electric field of neighbouring polarized atoms. Hence, the atom is considered to experience the so-called internal field or local field. The internal field is defined as the electric field acting on the atom at a given point of the dielectric material.

Let us consider a dielectric material placed between the plates of a capacitor. Let $\vec{E}$ be the electric field applied across the plates. Let *A* be a reference point of the dielectric material. For determining the internal field at *A*, let us construct a spherial cavity with its centre at *A*. As seen (Fig. 4.8), the internal field acting at *A* is given as

$$\vec{E}_1 = \vec{E}_1 - \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \tag{4.35}$$

where $\vec{E}_1$ is the electric field acting between the plates of the capacitor, $\vec{E}_2$ is the electric field inside the dielectric material, $\vec{E}_3$ is the electric field due to the charges on the spherical cavity surface and $\vec{E}_4$ is the electric field due to other dipoles inside the cavity.

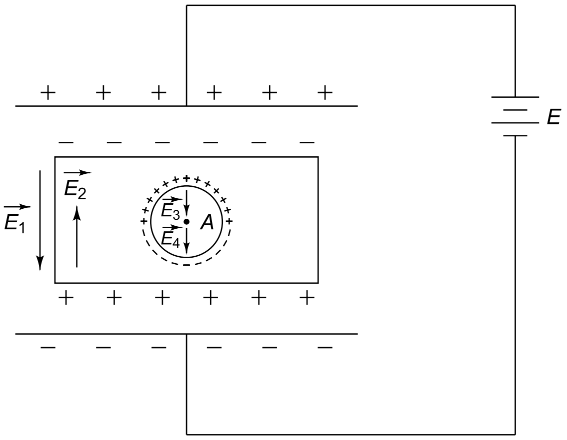


Fig. 4.8 *Internal Field Inside a Spherical Cavity*

Determination of $\vec{E}_1$

The field theory gives the expression for $\vec{E}_1$ as

$$\vec{E}_1 = \frac{\vec{D}}{\varepsilon_0} \tag{4.36}$$

where $\vec{D} \equiv \vec{P} + \varepsilon_0 \vec{E}$ is the electric displacement vector and $\vec{P}$ is the polarization.

Hence, Eq. (4.36) may be rewritten as

$$\vec{E}_1 = \vec{E} + \frac{\vec{P}}{\varepsilon_0} \tag{4.37}$$

Determination of $\vec{E}_2$

The electric field due to the polarization charges on the dielectric surface is given as

$$\vec{E}_2 = \frac{\vec{P}}{\varepsilon_0} \tag{4.38}$$

Determination of $\vec{E}_3$

Let r be the radius of the cavity. Consider an elementary ring of radius $r \sin \theta$ and of width $rd\theta$ (Fig. 4.9).

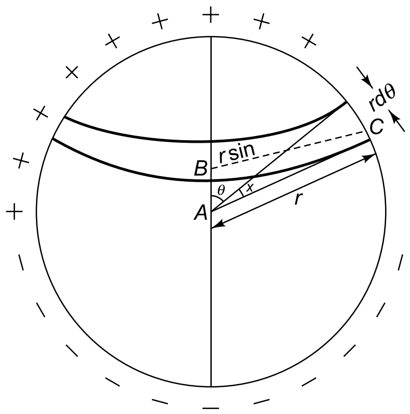


Fig. 4.9 *Internal Field Inside a Ring*

The surface area of the ring is

$$dS = 2\pi \times r \sin\theta \times r d\theta$$

i.e. $$dS = 2\pi r^2 \sin\theta d\theta$$

Field theory gives the surface charge density on the spherical surface as

$$q' = P \cos \theta$$

Hence, the charges on the ring is

$$dq = q' \times 2\pi r^2 \sin \theta d\theta$$

$$dq = P \cos \theta \times 2\pi r^2 \sin \theta d\theta$$

i.e. $$dq = 2\pi r^2 P \sin \theta \cos \theta d\theta \tag{4.39}$$

Electric field at *A* due of the charges on the ring is given as

$$dE = \frac{dq \cos\theta}{4\pi\varepsilon_0 r^2} \tag{4.40}$$

Substitution of Eq. (4.39) into Eq. (4.40) gives

$$dE = \frac{2\pi r^2 P \sin\theta \cos\theta \times \cos\theta}{4\pi\varepsilon_0 r^2}$$

i.e. $$dE = \frac{P}{2\varepsilon_0} \cos^2\theta \sin\theta d\theta$$

The electric field at *A* due to charges on the cavity surface is then given as

$$\vec{E}_3 = \int_0^{\pi} \frac{P}{2\varepsilon_0} \cos^2\theta \sin\theta d\theta$$

Integration of the above equation gives

$$E_3 = \frac{P}{2\varepsilon_0} \times \frac{2}{3}$$

i.e.
$$\vec{E}_3 = \frac{P}{3\varepsilon_0} \tag{4.41}$$

Determination of $\vec{E}_4$

Assuming the cavity to have spherical and cubic symmetries, we get

$$\vec{E}_4 = 0 \tag{4.42}$$

Now, substituting the Eqs (4.37), (4.38), (4.41) and (4.42) into Eq. (4.35), we get

$$\vec{E}_i = \vec{E} + \frac{\vec{P}}{\varepsilon_0} - \frac{\vec{P}}{\varepsilon_0} + \frac{\vec{P}}{3\varepsilon_0} + 0$$

i.e.
$$\vec{E}_i = \vec{E} + \frac{\overleftarrow{P}}{3\varepsilon_0} \tag{4.43}$$

The internal field $\vec{E}_i$ is also called the **Lorentz field**.

4.7 CLAUSIUS–MOSOTTI EQUATION

The Clausius–Mosotti equation relates the polarizability of a dielectric material to its dielectric constant. Consider an elemental dielectric in which ionic, orientational and space-charge polarizations are absent. That is,

$$\alpha_i = 0,\ \alpha_o = 0,\ \alpha_s = 0$$

Let E be the electric field applied to the dielectric material. Then, the polarization is given as

$$P = N\alpha_e E_i \tag{4.44}$$

where N is the concentration of atoms in the dielectric material, α_e is the electronic polarizability and E_i is the internal field.

Substituting the expression for internal field into Eq. (4.44), we get

$$P = N\alpha_e\left(E + \frac{P}{3\varepsilon_0}\right)$$

Rearranging the terms, we get

$$\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)P = N\alpha_e E$$

i.e.
$$P = \frac{N\alpha_e E}{\left(1-\frac{N\alpha_e}{3\varepsilon_0}\right)} \tag{4.45}$$

On the other hand, polarization is given as

$$P = \varepsilon_0(\varepsilon_r - 1)E \tag{4.46}$$

From Eqs (4.45) and (4.46), we get

$$\varepsilon_0(\varepsilon_r - 1)E = \frac{N\alpha_e E}{1-\frac{N\alpha_e E}{3\varepsilon_0}}$$

i.e.
$$\varepsilon_0(\varepsilon_r - 1) = \frac{N\alpha_e}{1-\frac{N\alpha_e E}{3\varepsilon_0}}$$

Rearranging the terms, we get

$$1 - \frac{N\alpha_e E}{3\varepsilon_0} = \frac{N\alpha_e}{\varepsilon_0(\varepsilon_r - 1)}$$

i.e.
$$\frac{N\alpha_e}{\varepsilon_0(\varepsilon_r - 1)} + \frac{N\alpha_e E}{3\varepsilon_0} = 1$$

$$\frac{N\alpha_e}{3\varepsilon_0}\left(1+\frac{3}{\varepsilon_r - 1}\right) = 1$$

i.e.
$$\frac{N\alpha_e}{3\varepsilon_0}\left(\frac{\varepsilon_r + 2}{\varepsilon_r - 1}\right) = 1$$

i.e.
$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r + 2}{\varepsilon_r - 1} \tag{4.47}$$

Equation (4.47) is called the **Clausius–Mosotti equation.**

For a compound dielectric material, the Clausius–Mosotti equation is rewritten as

$$\frac{1}{3\varepsilon_0}\sum_j N_j\, \alpha_{ej} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \tag{4.48}$$

where *j* refers to various constituent elements of the given dielectric material.

4.8 DIELECTRIC LOSS

When an ac voltage is applied to a dielectric, the electrical energy is absorbed by the material and is dissipated in the form of heat. Due to its high resistance, the power loss is small in a dielectric material subjected to a dc field. The dissipation of energy is called dielectric loss. However, the power loss is significant in an ac field. The dielectric loss is an engineering problem assuming a dominant role in high voltage applications.

When an ac voltage is applied to a perfect insulator, there is no consumption of energy and the charging current leads the applied voltage by 90° as shown in Fig. 4.10(a). It occurs only in vacuum and purified gases and in the case of commercial dielectrics, the leakage current does not lead the applied voltage by exactly 90° as shown in Fig. 4.10(b). This complementary angle $\delta = 90° - \theta$ is called the **dielectric loss angle**. In a majority of cases of dielectrics, the angle δ is negligibly small. Temperature, humidity, voltage, frequency of the voltage increases dielectric loss. Lossy dielectric is a dielectric material having very large value of dielectric loss.

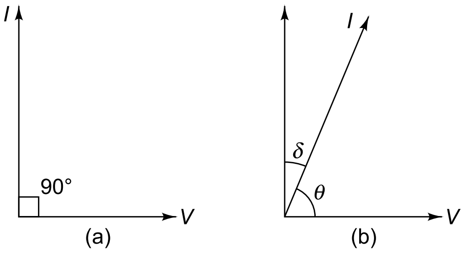


Fig. 4.10 *Voltage vs Current*

When a dielectric of capacitance *C* is subjected to an ac voltage of frequency *v*, the dielectric loss is given as

$$P' = VI \cos \theta \tag{4.49}$$

where $I = \dfrac{V}{X_c}$ and $X_c = \dfrac{1}{(2\pi v C)}$ is the capacitive reactance of the dielectric to ac voltage.

Then, Eq. (4.49) is rewritten as

$$P' = V^2 2\pi\nu C \cos\theta \tag{4.50}$$

Equation (4.50) may be rewritten as

$$P' = V^2 2\pi\nu C \sin\delta \tag{4.51}$$

As δ is very small in dielectric materials, we get $\sin\delta \approx \tan\delta$. Hence, Eq. (4.51) is rewritten as

$$P' = V^2 2\pi\nu C \tan\delta \tag{4.52}$$

The parameter $\tan\delta$ is called the **dielectric loss factor**. The dielectric loss increases with the increase in the applied voltage, the frequency, temperature and humidity.

The dielectric loss can be attributed to the fact that the dielectric constant becomes a complex quantity in these circumstances. That is,

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \tag{4.53}$$

The dielectric loss factor is then given as

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'} \tag{4.54}$$

Table 4.2 shows the values of loss tangent of selected dielectric materials.

Table 4.2 *Values of Loss Tangent of some Dielectric Materials*

Dielectrics	*tan δ*
Teflon	7×10^{-5}
Polyethylene	1×10^{-4}
PVC	5×10^{-2}
Sodalime glass	9×10^{-3}

4.9 DETERMINATION OF DIELECTRIC CONSTANT

A potential difference V is applied to the plates of a capacitor. Then, charges develop on the plates of the capacitor. The capacitance of the capacitor is given as

$$C_1 = \frac{\varepsilon_0 A}{d} \tag{4.55}$$

where A is the surface area of the plate and d is the distance of separation between the plates.

Now, the given dielectric is introduced between the dielectric plates. Due to polarization of the material, charges on the plates increase. Therefore, the capacitance of the capacitor changes and it is given as

$$C_2 = \frac{\varepsilon A}{d} \tag{4.56}$$

where ε is the permittivity of the dielectric material.

From Eqs (4.55) and (4.56), we get

$$\frac{\varepsilon}{\varepsilon_0} = \frac{C_2}{C_1}$$

Hence, the dielectric constant of the material is

$$\varepsilon_r = \frac{C_2}{C_1} \tag{4.57}$$

Thus, the dielectric constant may be determined by measuring the capacitance of the capacitor, both in the absence and presence of the given dielectric material in between the plates of the capacitor.

4.10 DIELECTRIC BREAKDOWN

When the strength of the applied electric field to a dielectric exceeds a critical value, the dielectric loses its insulating properties and hence a large current flows through it. This phenomenon is known as dielectric breakdown. The strength of the applied electric field strength at which dielectric breakdown occurs is known as **dielectric strength**. In practical applications, dielectric breakdown is of great concern to the engineer.

4.10.1 Types of Dielectric Breakdown

There are many causes for dielectric breakdown. The different types of dielectric breakdown are

- Intrinsic breakdown
- Thermal breakdown
- Electrochemical breakdown
- Defect breakdown
- Discharge breakdown

1. Intrinsic Breakdown or Avalanche Breakdown

In the case of a perfect dielectric, there are no free electrons and the conductivity is almost zero. But in general, all crystals contain some imperfections like vacancies or interstitials, non-stoichiometry, etc. These impurities act as traps for the conduction of electrons. These electrons gain high velocity and energy due to the high value of applied electric field. It is noticed that these electrons collide with the valence electrons and transfer their own energy to the valence electrons. Now the valence electrons, after acquiring this energy, jump from the valence band to the conduction band. This process proceeds as a chain reaction. Due to this, a large amount of current flows through the dielectric and breakdown occurs. This type of breakdown is called intrinsic breakdown or avalanche breakdown.

Characteristics

- It also occurs at low temperatures.
- It requires relatively large electric field.
- It does not depend on the shape of the material and electrode configuration.
- This kind of breakdown mostly occurs in thin samples.

2. Thermal Breakdown

When an electric field is applied to a dielectric material, some amount of heat is produced. The heat produced in the material should be dissipated from it. In some cases, the amount of heat generated is very high compared to the heat dissipated and this excess of heat may cause breakdown of the dielectric material. This type of breakdown is known as thermal breakdown.

Characteristics

- It occurs at high temperatures.
- The breakdown field strength depends on the size and shape of the dielectric material and on the geometry and the thermal properties of the electrodes and the ambient medium.
- The breakdown time is in milliseconds or much longer.
- In alternating fields, the breakdown strength is lower than the dc breakdown strength as the power loss increases with frequency.

3. Electrochemical Breakdown

The chemical and electrochemical breakdown has inter-relationship with thermal breakdown. When the temperature of a dielectric material increases, it results in increase in the mobility of the ions and hence electrochemical reaction occurs. When there is an increase in ionic mobility, leakage current also increases thereby decreasing the insulation resistance and this results in dielectric breakdown. Thus due to chemical reactions, the electrical and mechanical properties of insulation materials are changed and finally these causes breakdown. This type of breakdown is known as electrochemical breakdown.

Characteristics

- Electrochemical breakdown is determined by the leakage current, density of ions, temperature and permanent dipoles in the material.
- To avoid the electrochemical breakdown, impurities should not be mixed with pure dielectric materials.
- Electrochemical breakdowns are accelerated by temperature. To avoid electrochemical breakdown, the dielectric material should not be operated at high temperatures.

4. Discharge Breakdown

Some dielectric materials may have occluded gas bubbles. If these dielectric materials are subjected to high voltages, the gaseous substances are easily ionized and hence, they result in a large ionization current. Thus, large ionization current may cause electric breakdown. This type of breakdown is known as discharge breakdown.

Characteristics

This type of breakdown can occur at low voltage where there is large number of occluded gas bubbles.

5. Defect Breakdown

Some dielectric materials may have surface defects like cracks, pores, etc. Moisture and other impurities can occupy these places leading to breakdown. This is known as defect breakdown.

4.11 USES OF DIELECTRIC MATERIALS

The important applications of dielectric materials are as insulating materials and as capacitors. The insulating materials are classified into three categories.

- Solid insulating materials
- Liquid insulating materials
- Gaseous insulating materials

Low dielectric constant, low dielectric loss, high resistance and high dielectric strength are the required parameters of a dielectric to act as insulating materials. High dielectric constant, high specific resistance, high dielectric strength and low dielectric loss are the required parameters of a dielectric to act as capacitors.

4.11.1 Solid Insulating Materials—Capacitors

A capacitor is an electronic component that stores energy in the form of electric field. It is made up of two conducting plates separated by a dielectric.

1. Paper Capacitors

These capacitors use linen paper as the dielectric medium. The paper is kept between aluminium foils which act as the metal plates. The whole assembly is rolled into a cylindrical element.

2. Glass Capacitors

Very thin plates of glass are used as dielectrics in these capacitors.

3. Plastic Capacitors

These capacitors use a thin, uniform and non-porous film of plastic as a dielectric medium. Polyester, polystyrene, polypropylene are some of the materials commonly used.

4. Mica Capacitors

These capacitors use mica as the dielectric medium. Muscovite mica is a naturally occurring material used as the dielectric medium.

5. Ceramic Capacitors

These capacitors use ceramic as the dielectric medium. Low permittivity capacitors use steatite as the dielectric material and high permittivity capacitors use barium titanate as the dielectric material.

6. Electrolytic Capacitors

In electrolytic capacitors, a metallic anode has an oxide film grown over it and this oxide layer acts as a dielectric. The anode is surrounded by an electrolytic solution of ammonium borate or sodium phosphate which acts as the cathode.

4.11.2 Liquid Insulating Materials—Transformers

Mineral oils and synthetic oils are the commonly used liquid insulating materials used in transformers for the purpose of insulation and cooling.

In high voltage transformers, heat produced due to very high voltages is present in it. Arrangement has to be made to carry away the heat produced. So it is commonly immersed in liquid dielectrics.

1. Mineral Insulating Oil

Mineral oil has the desirable properties of high dielectric strength and is highly viscous. It is responsible for the dissipation of the heat generated in the transformer. Mineral oil must be free from moisture and sludge formation.

2. Synthetic Insulating Oil

Sovol and sovotol are the commonly used synthetic insulating oils, having the desirable properties of resistant to oxidation and fire hazards making them superior over mineral insulating oils.

3. Miscellaneous Insulating Oils

Petroleum oils, silicone oils, and vegetable oils are the few miscellaneous insulating oils. They have the desirable property of high thermal stability.

4.12 FERROELECTRIC MATERIALS

Ferroelectric materials constitute a very important group of dielectrics. Materials which exhibit electric polarization even in the absence of the applied electric field are known as ferroelectric materials. These materials exhibit hysteresis effects similar to ferromagnetic materials, and are known as ferroelectric materials. They have permanent dipole moment in each atom or molecule. The dielectric constants of these materials are higher than that of ordinary dielectrics. In the creation of very high value of induced dipole moment in a weak field and even in the absence of the applied electric field, the existing electric polarization is known as **ferroelectricity**. Rochelle salt, Barium titanate, lithium niobate are some examples for ferromagnetic materials.

4.12.1 Properties

- By very weak electric field, they can be easily polarized.
- They exhibit ferroelectric hysteresis (Fig. 4.11).

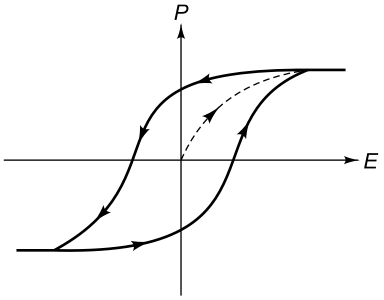


Fig. 4.11 *Dielectric Hysteresis*

- They possess spontaneous magnetization even when the applied field is zero.
- At ferroelectric Curie temperature (T_c), they exhibit spontaneous polarization due to permanent electric dipoles and internal electric field.
- When $T > T_c$, they are converted into paraelectric materials.
- They exhibit domain structure similar to ferromagnetic materials.
- Ferroelectric materials exhibit piezoelectricity. Piezoeletricity is defined as the creation of electric polarization by mechanical stress.
- Ferroelectric materials exhibit pyroelectricity. Pyroelectricity is defined as the creation of electric polarization by thermal stress.
- They possess very high values of permittivity in the range of 1000–10000.

4.12.2 Applications

- Ferroelectric crystals exhibit the pyroelectric effect, i.e. a change in the temperature of the crystal produces a change in its polarization. Using this effect, one can convert heat energy into useful electrical energy.

- The high dielectric constant of ferroelectric crystals is also useful for storing energy in small sized capacitors in electrical circuits.
- In optical communication, the ferroelectric crystals are used for optical modulation.
- These are used in electro-acoustic transducers such as microphones.
- Ferroelectric crystals exhibit piezoelectric property. Using this, we can find enormous applications of ferroelectric materials.

SOLVED EXAMPLES

Example 4.1 *Calculate the electronic polarizability of neon. The radius of the neon atom is 0.158 nm.*

(A.U., B.Tech (Chem), May/June 2005)

Given Data

$r = 0.158 \text{ nm} = 0.158 \times 10^{-9} \text{ m}$

$\alpha_e = ?$

Electronic polarizability of a dielectric is given as

$$\alpha_e = 4\pi\varepsilon_0 R^3 \tag{1}$$

where ε_0 is the permittivity of free space.

Substitution of the given values of ε_0, R into Eq. (1) gives

$$\alpha_e = 4 \times 3.14 \times 8.845 \times 10^{-12} \times (0.158 \times 10^{-9})^3$$

i.e. $\boldsymbol{\alpha_e = 0.439 \times 10^{-39}}$ **F.m²**

Example 4.2 *The dielectric constant of helium gas at NTP is 1,0000684. Calculate the electronic polarizability of the atoms if the gas contains $2 \times 7 \times 10^{25}$ atoms/m^3.*

(A.U., B.E. (ECE), May/June 2005)

Given Data

$\varepsilon_r = 1.0000684$

$N = 2.7 \times 10^{25}$ atoms/m^3

$\alpha_e = ?$

Polarization of a dielectric material is given as

$$P = N\alpha_e E \tag{1}$$

Alternatively, polarization is also given as

$$P = \varepsilon_0 (\varepsilon_r - 1)E \quad (2)$$

From Eqs (1) and (2), we get

$$N\alpha_e E = \varepsilon_0 (\varepsilon_r - 1)E$$

i.e. $$\alpha_e = \frac{\varepsilon_0 (\varepsilon_r - 1)}{N} \quad (3)$$

Substitution of the given values of ε_r, ε_0, N into Eq. (3) gives

$$\alpha_e = \frac{8.854 \times 10^{-12} \times (1.0000684 - 1)}{2.7 \times 10^{-25}}$$

i.e. $\boldsymbol{\alpha_e = 2.24 \times 10^{-41}}$ **F.m^2**

Example 4.3 *Argon gas of 1 × 8 × 10^{-40} F × m^2 polarizability is subjected to a field of 10^5 V/m. Determine the shift of the electric cloud with respect to the nucleus in the argon atom.*

Given Data

$\alpha = 1.8 \times 10^{-40}$ F.m^2

$E = 10^5$ V/m

$d = ?$

Dipole moment of a dielectric material is given as

$$\mu = \alpha E \quad (1)$$

On the other hand, dipole moment is defined as

$$\mu = Zed \quad (2)$$

where Z is the atomic number and e is the charge of the electron. From Eqs (1) and (2), we get

$$d = \frac{\alpha E}{Ze} \quad (3)$$

Substitution of the given values of α, E, Z, e into Eq. (3) gives

$$d = \frac{1.8 \times 10^{-40} \times 10^5}{18 \times 1.6 \times 10^{-19}}$$

i.e. $\mathbf{d = 6.25 \times 10^{-18}}$ **m**

Example 4.4 *Ammonium chloride has a dielectric constant of 1.0083 at 0°C and 1.0049 at 100 °C. The concentration of molecules at 0°C is* $2 \times 7 \times 10^{25}/m^3$. *Calculate the permanent dipole moment of ammonium chloride.*

Given Data

$(\varepsilon_r)_{T_1} = 1.0083$
$T_1 = 0°\text{C} = 273$ K
$(\varepsilon_r)_{T_2 =}\ 1.0049$
$T_2 = 100°\text{C} = 373$ K
$N = 2.7 \times 10^{25}/\text{m}^3$

$\mu = ?$

Total polarization in a dielectric material is given as

$$P = N\left(\alpha_e + \alpha_i + \frac{\mu^2}{3kT}\right)E \tag{1}$$

Moreover, the total polarization also given as

$$P = \varepsilon_0(\varepsilon_r - 1)E \tag{2}$$

From Eqs (1) and (2), we get

$$\varepsilon_r - 1 = \frac{N}{\varepsilon_0}\left(\alpha_e + \alpha_i + \frac{\mu^2}{3kT}\right) \tag{3}$$

Hence, we have

$$(\varepsilon_r)_{T_1} - 1 = \frac{N}{\varepsilon_0}\left(\alpha_e + \alpha_i + \frac{\mu^2}{3kT_1}\right) \tag{4}$$

$$(\varepsilon_r)_{T_2} - 1 = \frac{N}{\varepsilon_0}\left(\alpha_e + \alpha_i + \frac{\mu^2}{3kT_2}\right) \tag{5}$$

Subtracting Eq. (5) from Eq. (4), we get

$$(\varepsilon_r)_{T_1} - (\varepsilon_r)_{T_2} - 1 = \frac{N\mu^2}{3k\varepsilon_0}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

i.e.
$$\mu = \left(\frac{3k\varepsilon_0\left[(\varepsilon_r)_{T_1} - (\varepsilon_r)_{T_2}\right]}{N\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \right)^{\frac{1}{2}} \tag{6}$$

Substitution of the given values of

$(\varepsilon_r)_{T_1}$, $(\varepsilon_r)_{T_2}$, ε_0, T_1, T_2, N, k into Eq. (6) gives

$\mu = 6.85 \times 10^{-35}$ coulomb.m

Example 4.5 *Calculate the relative dielectric constant of a barium titanate crystal, which, when inserted in a parallel plate condenser of 10 m × 10 m area and a 2 mm difference of separation gives a capacitance of 10^{-9} F.*

(A.U., B.E. (Mech), May/June 2006;
A.U., B.E. (ECE) May/June 2005)

Given Data

$A = 10 \text{ m} \times 10 \text{ m} = 100 \text{ m}^2$

$d = 2 \text{ mm} = 2 \text{ m} \times 10^{-3} \text{ m}$

$C = 10^{-9}$ F

$\varepsilon_r = ?$

The capacitance of a capacitor is given as

$$C = \frac{\varepsilon A}{d} \tag{1}$$

where ε is the permittivity of the dielectric material.

Equation (l) may be rewritten as

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} \tag{2}$$

where $\varepsilon = \frac{\varepsilon}{\varepsilon_0}$ and ε_0 is the permittivity of free space.

Equation (2) gives the dielectric constant as

$$\varepsilon_r = \frac{Cd}{\varepsilon_0 A} \tag{3}$$

Substitution of the given values of C, d, ε_0 into Eq. (3) gives

$$\varepsilon_r = \frac{10^{-19} \times 2 \times 10^{-3}}{8.854 \times 10^{-12} \times 100}$$

i.e. $\boldsymbol{\varepsilon_r = 0.0023}$

Example 4.6 *Calculate the electronic polarizability produced in a dielectric medium of dielectric constant 6 when it is subjected to an electric field of 100 V/m.* ***(A.U., B.E. (ECE), May/June 2005)***

Given Data

$\varepsilon_r = 6$

$E = 100$ V/m

$\alpha = ?$

Polarization in a dielectric material is given as

$$P = \varepsilon_0 (\varepsilon_r - 1) E \qquad (1)$$

where ε_0 is the permittivity of free space.

Substitution of the given values of ε_0, ε_r, E into Eq. (1) gives

$$P = 8.854 \times 10^{-12} \times (6 - 1) \times 100$$

i.e. $\mathbf{P = 44.27 \times 10^{-10}}$ **coulomb/m^2**

Example 4.7 *Determine the dipole moment and displacement of the centeriods of positive and negative charges for a neon atom in an electric field of 5 × 10^{-4} V/m. The atomic polarizability of neon is 4.3 × 10^{-4} Fm^2.* ***(A.U., B.E./B.Tech, May/June 2011)***

Given Data

$\alpha_i = 4.3 \times 10^{-4}$ Fm^2

$E = 5 \times 10^{-4}$ V/m

$\mu = ?$

$\Delta x = ?$

Dipole moment

$$\begin{aligned} \mu &= \alpha_i \cdot E \\ &= 4.3 \times 10^{-4} \times 5 \times 10^{-4} \\ &= 2.15 \times 10^{-7} \text{ m} \end{aligned}$$

Let $\Delta\mu$ be the change in dipole moment for a change in distance Δx.

$$\Delta\mu = q \cdot \Delta x$$

$$\Delta\mu = \alpha_i E$$

where q is the charge of in electron.

$$\Delta x = \frac{\alpha_i E}{q}$$

$$= \frac{4.3 \times 10^{-4} \times 5 \times 10^{-4}}{1.6 \times 10^{-17}}$$

$$\Delta x = 1.343 \times 10^{12} \text{ m}$$

QUESTIONS AND ANSWERS

1. **What are dielectric materials?**

   Dielectrics are materials of high electrical resistivities of above 10^{10} to 10^{20} ohm.m which exhibit polarization when subjected to an electric field.

2. **What is permittivity of a material? Give its unit of measurement.**

   Permittivity is defined as the capacitance of the material per unit length. Its unit of measurement is F.m^{-1}.

3. **What is polarization of charges?**

   Polarization of charges refers to the separation of charges from their equilibrium positions when the dielectric material is subjected to an electric field.

4. **What is electric dipole?**

   The arrangement of two equal, opposite charges $+q$ and $-q$ separated by a distance r is known as electric dipole.

5. **Define the dipole moment and give its unit of measurement.**

   The product of magnitude of the charge and the distance of separation is known as electric dipole moment. Its unit is coulomb-metre. That is,

   $$\vec{\mu} = q \times \vec{r}$$

Since the dipole moment is a vector, it points from negative to the positive charges.

6. **Define the electric susceptibility of a dielectric.**

   The electric susceptibility of a dielectric is defined as

   $$\chi_e = \frac{P}{\varepsilon_0 E}$$

   where P is the polarization, E is the electric field and ε_0 is the permittivity of free space.

7. **Give the correlation of electric susceptibility and dielectric constant.**

   The correlation of electric susceptibility and dielectric constant is given as

   $$\chi_e = \varepsilon_r - 1$$

8. **Mention the types of polarization in dielectric materials.**

   Electronic polarization; Ionic polarization; Orientational polarization; Space-charge polarization.

9. **Define electronic polarization.**

   Electronic polarization is defined as the displacement of the positively charged nucleus and the negative electrons of an atom in opposite directions on the application of the electric field.

10. **Give the expression for electronic polarizability in terms of atomic radius of the dielectric materials.**

    The expression for electronic polarizability in terms of atomic radius of the dielectric materials is given as $\alpha_e = 4\pi\varepsilon_0 R^3$.

11. **Define ionic polarization in a dielectric.**

    The ionic polarization is the displacement of cations and anions in opposite directions in an ionic solid subjected to electric field.

12. **Give the expression for ionic polarizability.**

    Ionic polarizability is given as

    $$\alpha_i \equiv \frac{\mu}{E} = \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right)$$

    where ω_0 is the natural frequency of the ionic molecule, M and m are the masses of positive and negative ions, respectively.

13. What are polar dielectrics?

(A.U., B.E./B.Tech, May/June 2011)

These are the dielectrics having permanent dipole moments even in the absence of an electric field.

14. What are non-polar dielectrics?

These are the dielectrics with no permanent dipole moments in the absence of an electric field.

15. Define orientational polarization in a dielectric.

(A.U., B.E./B.Tech, May/June 2011)

When subjected to an electric field, the permanent dipoles of polar dielectrics align their dipoles in the field direction. It is called orientational polarization.

16. Give the expression for orientational polarizability.

The orientational polarizability is given as

$$\alpha_0 = \frac{P_0}{NE} = \mu^2 / 3kT$$

where μ is the dipole moment, k is the Boltzmann constant and T is the temperature.

17. Define the space-charge polarization in a dielectric.

Space-charge polarization is the accumulation of charges at the interfaces in multiphase dielectrics.

18. Give the Langevin–Debye equation for the total polarization in a dielectric.

The Langevin–Debye equation for the total polarization in a dielectric is given as

$$P = N\alpha E = NE\left[4\pi\varepsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT}\right]$$

where N is the concentration, E is the applied electric field, ε_0 is the permittivity of free space, R is the atomic radius, ω_0 is the natural frequency of the ionic molecule, M and m are the masses of positive and negative ions, respectively; e is the charge of the electron, μ is the dipole moment, k is the Boltzmann constant and T is the temperature.

19. Define dielectric breakdown.

(A.U., B.E./B.Tech, May/June 2011)

When a dielectric loses its insulation resistance and permits large currents to pass through it, it is said to have a breakdown.

20. Define dielectric strength.

(A.U., B.E./B.Tech, May/June 2011)

The dielectric strength is defined as the breakdown voltage per unit thickness of the material.

21. Mention the various types of dielectric breakdown mechanisms.

Intrinsic breakdown; Thermal breakdown; Electrochemical breakdown; Discharge breakdown; Defect breakdown.

22. What is intrinsic breakdown?

When a dielectric is subjected to strong electric fields, electrons are liberated from the valence band. Thus, electrical conduction takes place. This is called intrinsic breakdown.

23. What is thermal breakdown?

When a dielectric is subjected to heavy heating, local melting of the material takes place. It results in the flow of current through the dielectric. It is called thermal breakdown.

24. What is electrochemical breakdown?

When temperature rises, mobility of ions increases and hence electrochemical reactions take place. When ionic mobility increases, leakage current also increases. This is called electrochemical dielectric breakdown.

25. What is discharge breakdown?

Gas discharge breakdown occurs when the insulator contains occluded gas bubbles. Since gases require smaller ionization potential than solids, the gases ionize first and the gaseous ions bombard the solid dielectric causing electrical deterioration. This is called discharge breakdown.

26. What is defect breakdown?

Moisture and other impurities collect at the cracks and pores on the surface of the materials. It leads to breakdown of the dielectric material. It is called defect breakdown.

27. Define internal field of a dielectric.

The internal field is defined as the electric field acting on the atom at a given point of the dielectric material.

28. Give the expression for the internal field of a dielectric.

The internal field of a dielectric is given as $\vec{E_i} = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$ where P is the polarization, E is the electric field and ε_0 is the permittivity of free space.

29. Give the Clausius–Mosotti equation for an elemental dielectric.

The Clausius–Mosotti equation for an elemental dielectric is given as

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r + 2}{\varepsilon_r - 1}$$

where N is the concentration of atoms, α_e is the electronic polarizability, ε_0 is the permittivity of free space and ε_r is the dielectric constant.

30. Give the Clausius–Mosotti equation for a compound dielectric.

For a compound dielectric material, the Clausius–Mosotti equation is rewritten as

$$\frac{1}{3\varepsilon_0}\sum_j N_j\alpha_{ej} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}$$

where N is the concentration of atoms, α_e is the electronic polarizability, ε_0 is the permittivity of free space and ε_r is the dielectric constant. Here, j refers to various constituent elements of the given dielectric material.

31. Define the dielectric loss.

(A.U., B.E./B.Tech, May/June 2011)

When a dielectric is subjected to an ac voltage, some amount of electrical energy is absorbed by the material and is dissipated as heat. This is called dielectric loss.

32. Give the expression for dielectric loss.

The dielectric loss is given as $P = V^2\, 2\pi\nu C \tan\delta$

where P is the applied ac voltage of frequency ν, C is the capacitance of the dielectric and $\tan\delta$ is the dielectric loss factor.

33. Define the dielectric loss factor.

The dielectric loss factor is then given as $\tan \delta = \frac{\varepsilon_r''}{\varepsilon_r'}$ where ε_r' and ε_r'' are the real and imaginary parts of the dielectric constant of dielectric subjected to an alternating electric field.

EXERCISE PROBLEMS

PART-A

1. Calculate the polarization when the plates of a parallel plate capacitor are immersed in benzene at 300 K. An electric field of 300 V/cm is applied across the plates.

(*Ans.* $P = 3.398 \times 10^{7}$ *Coulomb.m*$^{-2}$)

2. NaCl has a static dielectric constant of 5.6 and a refractive index of 1.5. Calculate the difference between the static dielectric constant and the dielectric constant at the optical frequency.

(*Ans.* $\Delta\varepsilon_r = 3.35$)

3. A dielectric solid contains 5×10^{28} atoms per m^3. Each atom has a polarizability of 2×10^{-40} F.m^2. Assuming that internal field is given by the Lorentz relation, calculate the ratio of the internal field to the applied field. (*Ans.* $E_i/E = 1.6$)

4. A parallel plate capacitor is filled with a material which has a real part of dielectric constant 2.56 and a loss tangent of 0.7×10^{-4} at a frequency of 1 MHz. The area of the plate is 8 cm^2 and the separation between the plates is 0.08 mm. Calculate the capacitance.

(*Ans.* $C = 2.26 \times 10^{-10}$ *F*)

5. Calculate the change in distance between the centeres of positive and negative ions in a molecule of $LiCl_4$ when it is placed in a field of 5×10^{4} Vm^{-1}. Given $\alpha_i = 6.6 \times 10^{-40}$ Fm^2.

(*Ans.* $\Delta x = 2.06 \times 10^{-6}$ *m*)

QUESTIONS

PART-B

1. Explain the different types of polarization in dielectrics. *(A.U., B.E./B.Tech, May/June 2011)*
2. Derive an expression for electronic polarizability. *(A.U., B.E./B.Tech, May/June 2011)*
3. Derive an expression for ionic polarizability. *(A.U., B.E./B.Tech, May/June 2011)*
4. Discuss in detail the orientation and space charge polarizability.
5. Discuss in detail the frequency and temperature dependence of polarization. *(A.U., B.E./B.Tech, May/June 2011)*
6. Summarize the various factors contributing to breakdown in dielectrics. *(A.U., B.E./B.Tech, May/June 2011)*
7. Discuss in detail the concept of dielectric loss.
8. Derive an expression for internal field in a dielectric and hence deduce the Clausius–Mosotti equation. *(A.U., B.E./B.Tech, May/June 2011)*
9. Explain the uses of dielectric materials in capacitors and transformers.
10. Discuss ferroelectricity along with its properties and applications.

SUMMARY

- Dielectric materials are insulators and possess high electrical resistivity.
- Dielectric materials are used to store electrical energy.
- Dielectric constant is a measure of polarization of the dielectric subjected to an electric field.
- Polarization of charges refer to the seperation of charges from their equilibrium position when the dielectric material is subjected to electric field.

- The polarization phenomenon accounts for the ability of the dielectrics to increase the charge-storing capability of capacitors.
- Dielectric susceptibility is defined as the case with which a dielectric is polarized in the presence of an electric field.
- Electronic polarization, ionic polarization, orientation polarization and space-charge polarization are the types of polarization in dielectric materials.
- Polar dielectrics have permanent dipole moment even in the absence of an electric field.
- When a dielectric is subjected to an ac voltage, some amount of electrical energy is absorbed by the material and is dissipated as heat. This is called dielectric loss.
- When a dielectric loses its insulation resistance and permits large currents to pass through it, it is said to have dielectric breakdown.
- Materials which exhibit electric polarization even in the absence of the applied electric field are known as ferroelectric materials.
- The Clausius–Mosotti equation relates the polarizability of a dielectric material to its dielectric constant.
- The Clausius–Mosotti equation is given by

$$\frac{N\alpha_e}{3\varepsilon_o} = \frac{\varepsilon_r + 2}{\varepsilon_r - 1}$$

- Intrinsic breakdown, thermal breakdown, electrochemical breakdown, discharge breakdown and defect breakdown are the various types of dielectric breakdown mechanisms.
- The internal field is defined as the electric field acting on the atom at a given point of the dielectric material.

UNIT V

MODERN ENGINEERING MATERIALS

If you want to succeed in the world, you don't have to be much cleverer than other people. You just have to be one earlier. **— Leo Szilard**

5.1 INTRODUCTION

Due to the advancement in the field of materials science, today there are so many new materials. They possess improved and new electrical, optical and magnetic properties in addition to high mechanical strength, toughness, ductility and lightness. Metallic glasses, Shape Memory Alloys (SMA), nanomaterials, carbon nanotubes are few examples for new materials. Metallic glasses possess the properties of both metals and glasses. Shape memory alloys, after being deformed, can remember and recover the original shape when heated. Nanophase materials are the nano structured materials having a characteristic length less than 100 nm. They exhibit greatly altered physical, chemical and mechanical properties. A carbon nanotube is a cylindrical rolled-up sheet of graphene which has many remarkable properties.

5.2 METALLIC GLASSES

Metallic glasses belong to a new class of materials which exhibit the properties of both metals and glasses. In general, metals have crystalline structure. That is, they have a long-range atomic order. In contrast, glasses have amorphous structure with no long-range atomic order. Metallic glasses are amorphous alloys with an atomic arrangement similar to that of a liquid (i.e. with no translational symmetry) and with ductility, malleability, opaqueness and strength similar to that a metal.

$Fe_{75}\,P_{15}\,C_{10}$, $Fe_{24}\,Zr_{76}$, $Ni_{60}\,Nb_{40}$, $Fe_{80}\,B_{20}$ are some examples for metallic glasses.

5.2.1 Preparation

Metallic glasses are produced by rapidly quenching the metal alloys at the rate of 10^6 K/s. The glass transition temperature is the temperature at which the liquid like atomic configuration is frozen into solid in the process of rapid cooling. There are different techniques to produce ribbons of metallic glasses and among them melt spinning is the most commonly used technique for commercial production in a large scale. A melt spinner consists of a copper disc which rotates at a high speed as shown in Fig. 5.1. The alloy in the refractory tube is melted by induction heating under an inert helium atmosphere. Now the molten alloy is ejected through a nozzle from the bottom of the refractory tube on to the spinning disc by increasing the gas pressure. The molten alloy falling on the moving substrate is solidified and comes out of the roller. Thus a continuous ribbon is obtained and wound on a spool. By increasing the ejection pressure and making the disc speed constant, the width and thickness of the ribbon are increased. By increasing the disc speed and making the ejection pressure constant, increases the width of the ribbon and decreases the thickness of the ribbon. The ribbon produced by this process has no irregularities on the upper surface but the lower one which is in contact with the substrate has a large number of small irregularities due to the trapped gas bubbles.

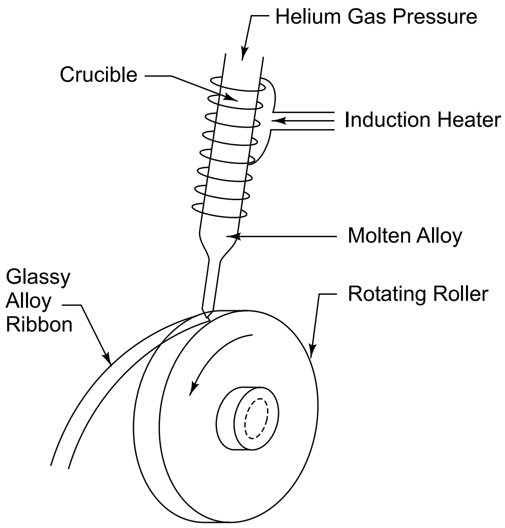


Fig. 5.1 *Melt Spinning Unit*

5.2.2 Properties

- They possess non-crystalline structure.
- They exhibit ferromagnetic behaviour.
- They possess high strength.
- They possess high tensile strength.
- They possess high hardness.
- They possess high workability.
- They have high electrical resistance.
- They exhibit the property of zero temperature coefficient of resistivity.
- Some metallic glasses are superconducting.
- They are insensitive to radiation.
- They have high corrosion resistance.

5.2.3 Applications of Metallic Glasses

Due to their unusual physical and chemical properties, metallic glasses play an important role as engineering materials. The applications of metallic glasses are listed below.

- Metallic glasses find applications in electronic circuits because of their insensitivity to temperature variations.
- They are widely used as resistance elements in electric circuits due to their high electrical resistivity.
- The metallic glass Pd_{80} Si_{20} in which Pd was replaced by Cr up to 7%, is used as a thermometer for measuring very low temperatures. Such a thermometer is called cryothermometer.
- As metallic glasses can sustain high dose of radiation without a change in electric properties, they are very useful in fission or fusion reactor environments.
- As metallic glasses possess high hardness of about 8 GPa and good corrosion resistance, they have found application as materials for magnetic tape-recording heads.
- The high magnetic permeability makes metallic glass ribbons and wires ideal for generating harmonics when they are placed in suitable ac and dc fields and useful as antitheft tags for protecting store merchandise.

- The use of metallic glasses in motors can reduce core loss by as much as 90% as compared with conventional crystalline magnets.
- Superconducting metallic glasses such as Zr_{75} Rh_{25} can be used in producing high magnetic fields and for magnetic levitation effect.
- Since metallic glasses maybe regarded as liquids whose structure has been frozen, they constitute ideal materials for low temperature transport and critical behaviour studies and they are most suited for the study of electrons in non-crystalline metals.
- Possible applications of metallic glasses include sensitive and quick response magnetic sensors or transducers, security systems and power transformer cores.

5.3 SHAPE MEMORY ALLOYS

Shape Memory Alloys (SMA) are alloys which after being deformed can remember and recover their original shape when heated. The heating in effect reminds the alloy that it prefers a different crystal structure and thus an associated shape at higher temperature. This unusual behaviour is called shape memory effect. Moreover, certain alloys exhibit the shape memory effect only upon heating and are known as one-way shape memory alloys. Some alloys also undergo a change in shape upon recooling and are known as a two-way shape memory alloys. Shape memory alloys exhibit two unique properties such as shape memory effect and pseudo-elasticity.

5.3.1 Crystal Structure of Shape Memory Alloys

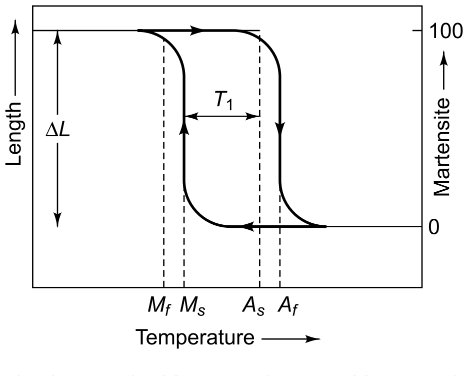
Shape memory alloys have two distinct crystal structures or phases. Temperature and internal stresses determines the phase of the SMA. The crystal structure or phase of a SMA at lower temperatures is called **martensite**, and **austenite** refers to the crystal structure or phase of a SMA at higher temperatures. When an SMA is in martensite form at lower temperatures, the metal can be easily deformed into any shape. When the alloy is heated, it goes through transformation from martensite to austenite. In the austenite phase, the memory metal 'remembers' the shape it had before it was deformed. Thus, when a metallic object of a given shape is cooled from a given temperature T_1 to a lower temperature T_2, it deforms and changes its shape. Upon reheating from T_2 to T_1, the shape change accomplished at T_2 is recovered so that the object returns to its original configuration. This thermo-elastic property of shape memory alloys is inherently connected with that type of solid-state phase transformation known as **martensitic transformation**. This is a type of microstructural change wherein the initial crystal structure transforms to a new crystal structure by a mechanism that does not involve diffusional mixing of atoms but rather is dominated by coordinated shear displacements of sections of crystals.

5.3.2 Pseudo-elasticity

Pseudo-elasticity occurs in an SMA when it consists of only austenite structure (i.e. $T > A_f$). With no change in temperature, the load on the SMA is increased until the austenite structure is transformed into martensite. The stress due to load is absorbed by the softer martensite. When the load is decreased, the martensite begins to transform to austenite. The SMA regains its original shape.

5.3.3 Hysteresis

The temperature range for the martensite to austenite transformation (upon heating) is higher than that for the reverse austenite to martensite transformation (upon cooling). The difference between the transition temperatures upon heating and cooling is called hysteresis. The hysteresis curve for an SMA is shown in Fig. 5.2.



T_1 transformation hysteresis; M_s martensite start; M_f martensite finish; A_s austenite start; A_f austenite finish

Fig. 5.2 *Typical Transformation versus Temperature Curve for a Specimen under Constant Load (Stress) as it is Cooled and Heated*

Most of the transformation occurs over a relatively narrow temperature range, although the beginning and end of the transformation during heating or cooling actually extends over a much larger temperature range. The transformation also exhibits hysteresis in that the transformations on heating and on cooling do not overlap.

5.3.4 Training Shape Memory Alloys

Heating is the only way that most memory metals retain their original shape. Since heat is the property that determines the shape of the metal, heat is the first property used for manipulation for formation. In the above example the wire is said to be **'one-way' trained,** that is, it can recover its shape in only one way, heating. This property of SMA's was the first to be exploited. The first commercial use of SMA was in forming coupling in the hydraulic lines of aircraft.

If any SMA is repeatedly subjected to the same deformation, i.e. heated and then cooled, it will begin to acquire **two-way training.** For example, a wire which is stretched by 3% in the martensite phase is heated to austenite finish (A_f) temperature to recover its shape. If this process is repeated many times then the wire will without any load stretch out 3% of its length when in martensite and will contract back in austenite. Hence an SMA can be used as a thermal actuator.

5.3.5 Thermomechanical Behaviour

The mechanical properties of shape memory alloys vary greatly over the temperature range spanning their transformation. This is seen in Fig. 5.3, where simple stress-strain curves are shown for a nickel titanium alloy that was tested in tension below, in the middle of, and above its transformation temperature range. The martensite is easily deformed to several per cent strain at quite a low stress, whereas the austenite has much higher yield and flow stresses. The dashed line on the martensite curve indicates that upon heating after removing the stress, the sample remembered its unstrained shape and the material transforms to austenite. No such shape recovery is found in the austenite phase upon straining and heating, because no phase change occurs.

An interesting feature of the stress-strain behaviour is seen in Fig. 5.3(c), where the material is tested slightly above its transformation temperature. At this temperature, martensite can be stress-induced. It then immediately strains and exhibits the increasing strain at constant stress behaviour, as seen in *AB*. Upon unloading, though, the material reverts to austenite at a lower stress, as seen in the line *CD*, and shape recovery occurs, not upon the application of heat but upon a reduction of stress. This effect, which causes the material to be extremely elastic, is known as pseudoelasticity or super-elasticity. Pseudoelasticity is non-linear. The Young's modulus is therefore difficult to define in the temperature range as it exhibits both temperature and strain dependence.

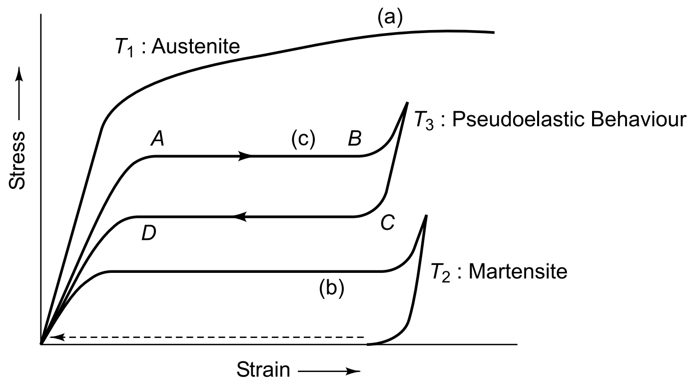


Fig. 5.3 *Typical Stress-strain Curves at Different Temperatures Relative to the Transformation, Showing (a) Austenite (b) Martensite, and (c) Pseudoelastic Behaviour*

Memory alloys demonstrate great rates of super-elastisity. For example, eyeglass frames are in a martensite phase. Bending the arms in half (at room temperature) introduces a phase change at the bend to austenite. Austenite is not stable at room temperature and because the system always seek lower energy states, the austenite will change back to the martensite phase and to do this, the arm must bend back.

5.3.6 Alloys having Shape Memory Effect

The most common memory metal is Ni-Ti consisting of equal part of nickel and titanium. Table 5.1 below displays alloys having shape memory effects.

Table 5.1 *Alloys having Shape Memory Effect*

Alloys	*Transformation Temperature (°C)*
Ag–Cd	–190 to 150
Au–Cd	30 to 100
Cu–Al–Ni	–140 to 100
Cu–Sn	–120 to 30
Ni–Al	–180 to 100
Ni–Ti	–50 to 110
Mn–Cu	–250 to 180

5.3.7 Characteristics and Properties of Ni–Ti Alloy

Let us consider a Ni–Ti wire constant tensile load as shown in Fig. 5.4. On the cooling process of Ni–Ti wire from austenite phase (A), at a particular temperature it is converted to martensite phase (B) due to elongation and also simultaneously there is a change in shape. When it is heated from martensite state (B) at a particular temperature it regains its original shape. Meanwhile its phase changes into austenite (A). Hence nickel–titanium alloy can switch from a temporary shape to a parent shape above a certain transition temperature. Below that temperature, it can be bent into various configurations. The change in shape and recovering original shape of the material are due to twinning mechanisms. The value of the transformation temperature depends on the constituents of the alloy and it varies from–200 to 110°C. The phase transformation in Ni–Ti alloy is a thermoelastic process.

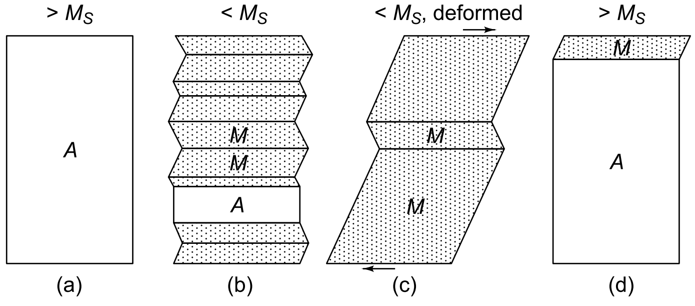


Fig. 5.4 *Explanation for Shape Memory Effect*

- Figure 5.4(a) shows the normal austentite state of the shape memory alloy and its temperature is above the martensite phase temperature M_s.

- Figure 5.4(b) shows the martensite phase at which the temperature is below the martensite phase temperature (cooling). Majority portions of the specimen are converted into martensite along with slipping and twinning. We also notice a thermoelastic phase transformation.
- Figure 5.4(c) shows the complete phase transformation into martensite along with the applied load and there is a plastic deformation or shape change.
- Figure 5.4(d) shows the phase transformation into austenite when we heat the specimen above the martensite phase temperature M_s. We notice a small martensite phase in the specimen due to its mechanical hysteresis.

5.3.8 Physical Properties of Nitinol (Ni–Ti alloy)

- Density: 6450 kg/m^3.
- Melting temperature: 1240–1310°C
- Resistivity: 82 × 10^{8}ohm-m (austenite) 72 × 10^8 ohm-m (martensite)
- Thermal conductivity: 10 W/m/K
- Heat capacity: 0.077 cal/gm-°C

5.3.9 Mechanical Properties of Nitinol (Ni–Ti alloy)

- Ultimate tensile strength: 745-960 MPa
- Elongation to fracture: 15.5%
- Yield strength: 560 MPa (austenite) 100 MPa (martensite)
- Poissons ratio: 0.3
- Elastic modulus: 75 GPa (austenite) 28 GPa (martensite).

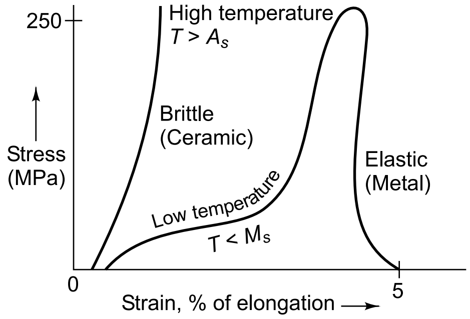


Fig. 5.5 *Stress-strain Curves of Ni-Ti Alloy at Different Temperatures*

Figure 5.5 shows the stress-strain curves of Ni–Ti alloy at different temperatures. In martensite phase, it behaves as metal with high elastic modulus and high ductility and in the austenite phase, it behaves as a brittle or ceramic material with poor elongation and low value of elastic modulus. This behaviour is due to intermetallic nature of Ni–Ti alloy.

5.3.10 Applications and Advantages of Shape Memory Alloys

Shape memory alloys have numerous applications. The following is a list of some of the applications of shape memory alloys.

Fire Security and Protection Systems Lines that carry highly flammable and toxic fluids and gases must have a great amount of control to prevent catastrophic events. Systems can be programmed with memory metals to immediately shut down in the presence of increased heat. This can greatly decrease devastating problems in industries that involve petrochemicals, semiconductors, pharmaceuticals, and large oil and gas boilers.

Helicopter Blades Performance for helicopter blades depends on vibrations. As an enhancement of memory metals in microprocessing control tabs for the trailing ends of the blades, pilots can fly with increased precision.

Eyeglass Frames Eyeglass frames made of SMA can be bent back and forth, without any deformation. These frames demonstrate super-elasticity. Eyeglass frames made of nickel–titanium alloy absorb large deformations without damage.

Force Actuators In some applications, the shape memory component is designed to exert force over a considerable range of motion, often for many cycles. One such application is the circuit-board edge connector. In this electrical connector system, the shape memory alloy component is used to open a spring when the connector is heated. This allows force-free in or withdrawal of a circuit board in the connector. Upon cooling, the nickel-titanium actuator becomes weaker, and the spring easily deforms the actuator while it closes tightly on the circuit board and forms connections.

An example based on the same principle is a fire safety valve which incorporates a copper–zinc–aluminium actuator designed to shut off toxic or flammable gas flow when fire occurs.

Medical Applications In medical applications, SMAs are required for reliable biological and chemical characteristics. These materials must not be vulnerable to degradation, decomposition, dissolution, or corrosion in the organism, and must be biocompatible. Nickel–titanium shape memory alloys have been employed in artificial joints such as artificial hip joints. These alloys have also been used for bone plates, marrow pins for healing bone fractures, connecting broken bones, teeth braces, artificial muscles, orthopaedic devices and blood clot filters.

5.3.11 Disadvantages of SMA

- Manufacturing costs of SMA is very high compared to other materials like steel and aluminium.
- SMA's possess poor fatigue properties (i.e. twisting, bending, compressing).

5.4 NANOMATERIALS

Nanomaterials are defined as the materials whose atoms are arranged in nanosize clusters and they become the building blocks of the material. At the nanoscale, the properties of materials are very different from those at a larger scale. As the grain structure decreases, there is a drastic increase in the volume fraction of grain boundaries. This is the main reason which changes the physical and chemical properties of the materials at the nanoscale. Due to these reasons, in recent years nanomaterials have drawn the attention of scientists and engineers. Nanotechnology is the design, characterization, production application of devices and materials on the nanoscale. When only one dimension of the material is reduced to the nano range and the remaining two dimensions remain large, then the structure is known as **quantum well**. When two dimensions of the material is reduced to the nano range and the remaining one dimension remains large, then the structure is known as **quantum wire**. When all the three dimensions of a material is reduced to the nano range, then the structure is known as **quantum dot**.

5.4.1 Synthesis

There are different methods used to prepare nanomaterials and are broadly classified under two groups, namely bottom-up approach and top-down approach. The top-down approach starts with large scale object and reducing its dimensions to nanoscale. In the bottom-up approach, atoms, molecules or nanoparticles itself are used as building blocks for the production of complex nanostructures.

5.4.2 Top-down Processes

The following are the methods used to produce nanostructured materials under top-down approach.

- Ball-milling method
- Lithographic method
- Plasma-arcing method

5.4.3 Bottom-up Processes

The following are the methods used to produce nanostructured materials under bottom-up approach.

- Vapour phase deposition methods
- Colloidal and sol–gel methods
- Electrodeposition method

5.4.4 Ball-Milling Method

One method of fabrication of nanomaterials of major industrial importance is high energy ball-milling method. This method consists of a stainless steel drum with hard steel or tungsten carbide balls placed inside the drum as shown in Fig. 5.6.

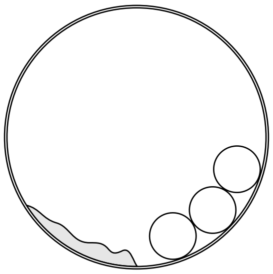


Fig. 5.6 *Rolling Ball Mill*

Argon in the stainless steel drum avoids unwanted reactions. The milling is carried out at room temperature. Now the material which has to be converted to nanomaterial is placed inside the drum in the power form. When the drum rotates, the powder materials are crushed mechanically by the hard balls as shown in Fig. 5.7. The severe plastic deformation that occurs during machining, cold rolling, drawing, cyclic deformation or sliding wear causes the reduction in grain size in the powdered particles resulting in the fabrication of nanomaterials. This method is also known as **mechanical attrition** or **mechanical alloying**. Different components can be mechanically alloyed together to produce nanostructured alloys. High energy ball milling can create structural changes of the material at room temperature.

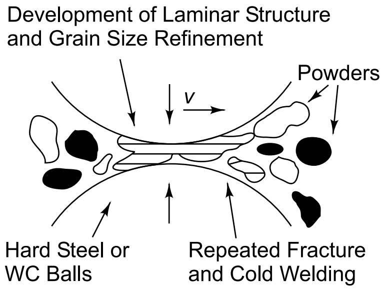


Fig. 5.7 *Schematic Representation of the Mechanical Alloying Process*

5.4.5 Lithographic Method

Lithography is a technique that creates patterns on the surface. This is one of the top-down processes. This method has tremendous requirements in electronic industry. Electron beam lithography is the method in which electron beam is used in lithography technique. The lithographic process is used to generate a quantum dot.

- The surface of the sample substrate is coated with a photoresist material.
- Now the sample is irradiated by an electron beam in the region where the nanostructure will be occupied.
- A developer removes the irradiated portion of the resist.
- An etching mask is added to these areas.
- The remaining portions of the photoresist are removed.
- The remaining areas not covered by the etching mask are chemically etched away to produce the quantum structure.
- The etching mask is removed resulting in quantum dot.

Thus the desired nanostructures are produced using electron beam lithographic technique. All the above processes are illustrated in Fig. 5.8. We can also generate more complex quantum structures such as multiple quantum well structures.

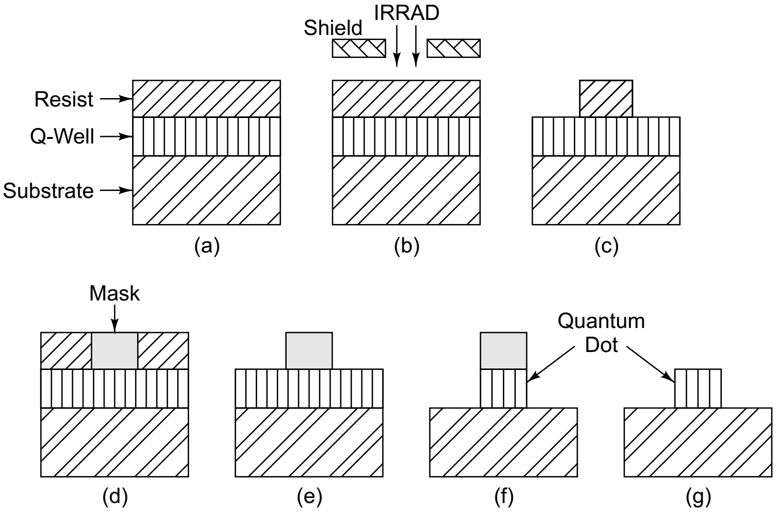


Fig. 5.8 *Steps Involved in Lithographic Method*

5.4.6 Plasma-assisted Deposition Process

The use of plasma, i.e. ionized gas during vapour deposition allows access to substantially different chemical and physical process and also higher purity final materials. The dc sputtering, RF sputtering, dc magnetron sputtering, RF magnetron sputtering and reactive sputtering are a few among them.

High energy ions striking atoms or molecules from a target and depositing them on a substrate is the basic concept of sputtering process. A sputtering unit consist of a target and a substrate facing each other as shown in Fig. 5.9. The material to be deposited acts as cathode and the substrate on which the material is to be deposited acts as cathode. Now the chamber is evacuated and the inert gas argon is introduced. When dc voltage of few kilovolts is applied between the electrodes, a discharge is initiated. Now the free electrons will be accelerated and gain sufficient energy and ionize argon atoms. These argon ions (Ar^+) strike the cathode and physically eject target atoms and they pass through the discharge and it is deposited on the substrate. Magneton sputtering introduces a magnetic field near the target in the sputtering process, increases the deposit rate and also reduces substrate heating. Reactive sputtering introduces reactive gases in the chamber to form compound films.

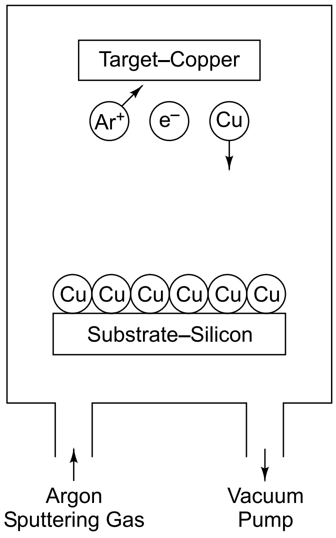


Fig. 5.9 *Plasma Assisted Deposition Process*

5.4.7 Vapour-phase Deposition Method

Vapour-phase deposition method is a technique in which materials in a vapour state are condensed through condensation, chemical reaction or conversion to form a solid nanomaterial. There are two classes of vapour phase deposition as below.

- Physical Vapour Deposition (PVD)
- Chemical Vapour Deposition (CVD)

PVD technique involves transfer of atoms from a solid target material and depositing them on the substrate and mostly involves no chemical reaction. CVD technique involves the flow of gas with diffused reactants over a hot substrate surface and invokes chemical reaction.

5.4.8 Physical Vapour Deposition

This system consists of an evaporation source and a substrate facing opposite to each other at an appropriate distance as shown in Fig. 5.10. They are located in a vacuum chamber and it prevents contamination in the system. The chamber is first evacuated. The desired material is placed in the evaporation source and is evaporated. The vapour moves from the hot source in to cool inert gas and the evaporated atoms collide with the inert gas atoms in the chamber and lose their kinetic energy and become supersaturated. Then they nucleate homogeneously with the particle size in the range of l–100 nm. Particles can be collected on a cold finger cooled by liquid nitrogen, scraped off and compacted to produce a dense material. By altering the type of carrier gas used, pressure of the gas used, the evaporation temperature and source to substrate distance of the exact size of the required nanomaterial is produced.

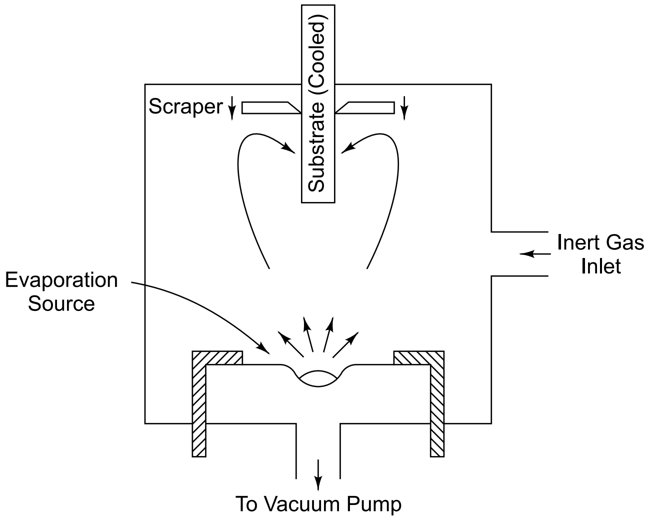
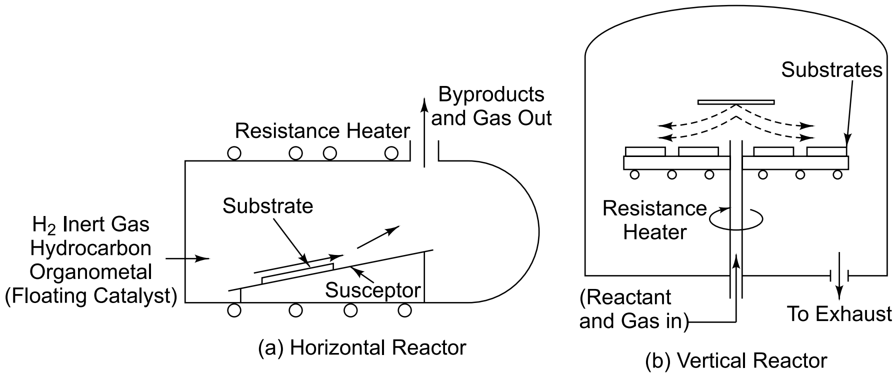


Fig. 5.10 *Physical Vapour Deposition System*

5.4.9 Chemical Vapour Deposition

This process involves the reaction or thermal decomposition of gas phase species at elevated temperatures of about 500–1000°C. Catalysts are involved in the process to enhance the speed of certain chemical reactions. This system consists of a hot substrate and diffused reactants over it as shown in Fig. 5.11. The gas carrying the reactants is known as carrier gas. When the gas flows over the hot substrate the heat energy creates chemical reactions of the reactants that result in the formation of particle size in the range of l–100 nm. The growth of nanostructures can occur either in the heating zone, before or after the heating zone. The growth time is roughly 30 minutes. The byproducts of the chemical reaction are then removed from the reactor. Inert gas is used to cool the reactor. There are two types of CVD reactors which perform the above process. They are known as horizontal reactor and vertical reactor as shown in

Fig. 5.11(a) and (b). In the reactors the substrate surface is exposed to the gas with diffused reactants. Resistance heaters surround the chamber or are kept directly under the susceptor that holds the substrate. Carbon, quartz, silicon are the commonly used substrates.



(a) Horizontal Reactor (b) Vertical Reactor

Fig. 5.11 *CVD System*

5.4.10 Colloidal Method

The colloidal particles of metal oxides can be obtained by the hydrolysis of the corresponding salt. The titanium oxide (TiO_2) nanoparticles are readily formed in the hydrolysis of titanium tetrachloride. TiO_2 hydrolysis in the presence of a base and forms a colloidal solution of TiO_2. The surface OH^- groups on TiO_2 clusters act as an electrostatic colloid stabilizer. The stabilization of colloidal solutions of nanoparticles is achieved by cutting their surfaces with polyphosphates or thoiols. The separation of nanoparticles is achieved due to different size/charge ratios of nanoparticles having different sizes.

$$TiCl_4 + 2H_2O \longrightarrow TiO_2 + 4HCl$$

5.4.11 Sol–Gel Method

Phase transformation of a sol obtained from metallic alkoxides or organometallic precursors is sol–gel method. This **sol** is a solution containing particles in suspension polymerized at low temperature to form a wet gel. The first step in this method starts with a solution of metal compounds such as metal alkoxides and acetylacetonates as sources of oxides, water as hydrolysis agent, alcohol as solvent and acid or base catalyst.

A sol can be formed through the hydrolysis and condensation of dissolved molecular precursors. This results in nanometer-sized particles to form a colloidal suspension or sol. These colloidal nanoparticles are then linked with one another in a 3D, liquid-filled solid network called **gel**. The transformation to a gel network is achieved by changing pH of the solution, solvent, the temperature and concentration of reactants.

The liquid phase from the gel is removed by supercritical drying process and produces a low density 3D porous material called **aerogel**. Aerogel densities are typically between 1–20% that of the bulk material. When the gel is dried slowly in a fluid evaporation process, the gel original network collapses, which creates a high density material known as **xerogel**. Xerogel densities are typically between 30–80% of the bulk.

This technique allows us to change the composition and structure of materials on the nanometer scale.

$$\text{Tetraethylothosilicate } (Si(OC_2H_5)_4) + \text{ethanol } (C_2H_5OH) \Big\downarrow$$
$$SiO_2 + \text{other products}$$

5.4.12 Electrodeposition Method

The process that creates solid materials directly from electrochemical reaction in liquid composites with substrate materials is known as **electrodeposition**. The arrangement for the template based growth of nanowires using electrochemical deposition is shown in Fig. 5.12. The templates are attached onto the cathode which is subsequently brought into contact with the deposition solution. The anode is placed parallel to cathode. On the application of electric field, cations diffuse resulting in the growth of nanowires. Ni, CO, Cu nanowires can be produced with nominal pore diameters between 10–200 nm.

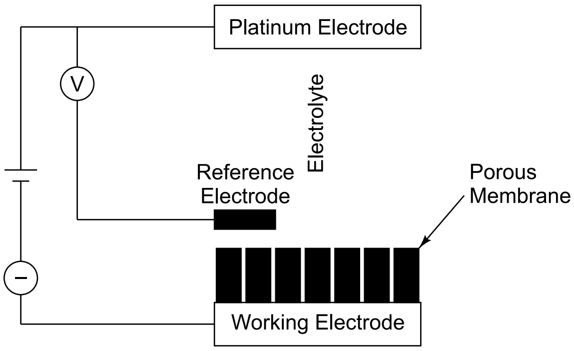


Fig. 5.12 *Schematic Diagram of Electrodeposition*

5.4.13 Properties of Nanomaterials

In the following topics properties of nanomaterials are discussed.

5.4.14 Variation of Physical Properties with Size

Let us now discuss what happens to the geometrical arrangement of atoms and their stability changes with size. From the bulk, the primary effect of reducing particle size is to create more surface sites. This results in the change of the surface pressure and causes a change in the interparticle spacing. This effect is shown in Fig. 5.13 for the case of Cu_n particles.

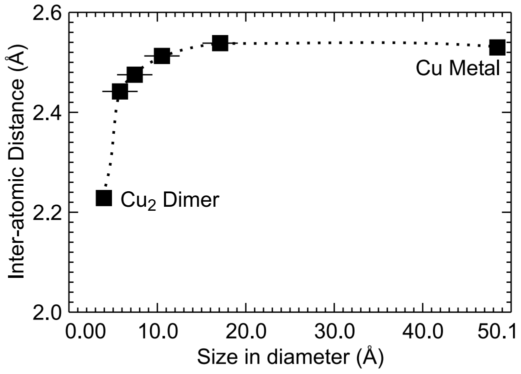


Fig. 5.13 *Inter-atomic Distance in Cu_n as a Function of Size*

It is interesting to note that the interparticle spacing decreases with size. This is due to competition between the long-range electronic forces and the short range core-core repulsion. Similarly, Fig. 5.14 shows the melting point Au_n particles as a function of size. The melting point decreases with size and rate of decrease increases substantially at very small sizes.

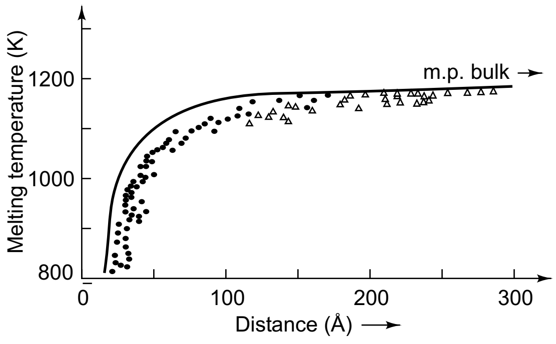


Fig. 5.14 *Melting Point of Small Au_n Particles as a Function of Size*

Variation in electronic properties with size occurs only when there is a variation in inter-particle spacing and geometry. As the size is reduced from the bulk, the electronic bands in metals become narrower and the delocalized electronic states are transformed to more localized molecular bonds.

The ionization potentials are higher at smaller sizes than at the bulk work function. The large surface-to-volume ratio and the variations in geometry and electronic structure have a strong effect on catalytic properties. Figure 5.15 shows the ionization potential and reactivity of Fe_n clusters as a function of size.

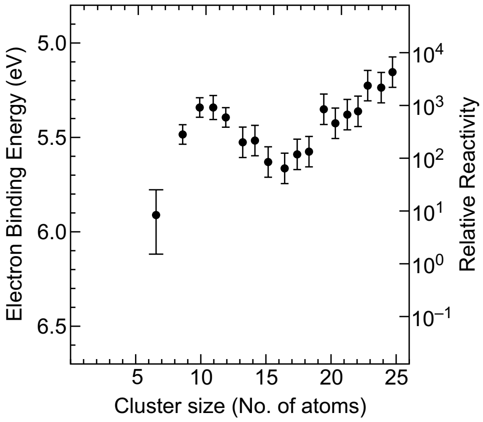


Fig. 5.15 *Ionization Potential and Reactivity of Fe_n Clusters as a Function of Size*

5.4.15 Magnetism in Nanoparticles

Nanoparticles of magnetic and even non-magnetic solids exhibit a totally new class of magnetic properties, which are marked by quantum effects that are far from being completely understood. A bulk ferromagnet is mainly characterized by domains, each containing several thousand atomic spins. Generally, the spins are aligned within the domain, but different domains point in different directions. Table 5.2 indicates the magnetic behaviour of very small particles of various metals.

Ferromagnetic and antiferromagnetic multilayers have been found to exhibit *giant magneto-resistance* (GMR). Small particles differ from the bulk in that a large fraction of the atoms reside at the surface. These atoms will have lower coordination number than the interior atoms. Figure 5.16 shows the calculated dependence of the magnetic moment on the nearest coordination number. It is clear that as the coordination number decreases, the moment increases towards the atomic value. In other words, small particles are more magnetic than the bulk material.

Table 5.2 *Magnetism in Bulk and in Nanoparticles*

Metal	*Bulk*	*Cluster*
Na, K	Paramagnetic	Ferromagnetic
Fe, Co, Ni	Ferromagnetic	Superparamagnetic
Gd,Tb	Ferromagnetic	Rotors/Superparamagnetic
Cr	Antiferromagnetic	Frustrated paramagnetic
Rh	Paramagnetic	Ferromagnetic

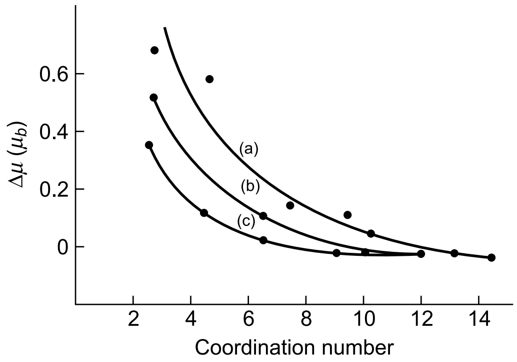


Fig. 5.16 *Deviation from the Bulk Magnetic Moment in (a) Fe, (b) Co, and (c) Ni as a Function of Coordination Number*

5.4.16 Mechanical Behaviour

The mechanical behaviour of nanostructured materials is as follows:

- 30–50% lower elastic moduli than for conventional grain size materials
- Super plastic behaviour at low homologous temperature
- 2–7 times higher hardness and strength-hardness values for nanocrystalline pure metals (~10 nm grain size) than those of large grained (> l mm) metal

(d) Decreasing hardness with decreasing grain size

Hardness and strength of conventional grain materials are a function of grain size. For ductile polycrytalline materials, the grain size dependence of flow stress is given by the empirical Hall–Petch equation

$$\sigma_o = \sigma_i + Kd^{-1/2}$$

where σ_o is the yield stress, σ_i is the friction stress which opposes the dislocation motion, K is a constant and d is the grain diameter.

Similarly, for hardness, we have

$$H_o = H_i + Kd^{-1/2}$$

The experimental results of hardness measurements show different behaviour, namely positive slope, zero and negative slope depending on the grain size, when it is less than 20 nm. Thus, the hardness, strength and deformation behaviour of nanocrystalline materials is unique.

5.4.17 Other Properties of Nanomaterials

- They have very high magneto resistance.
- They have lower melting point, high solid-state phase transition pressure, lower Debye temperature and high self-diffusion coefficient.
- They have high catalytic activity and lower ferroelectric phase transition temperature.

5.4.18 Applications

In nanophase materials, there are two types of applications such as

- Structural applications
- Functional applications

The structural applications are mainly based on the mechanical properties of the nanostructured or nanophase materials. These are used to produce plastic ceramics and extremely hard metals.

The functional applications are mainly based on the transformation of external signals such as the filtering of the incident light and the change of electrical resistance in different gas concentrations.

5.4.19 Other Applications

- They are used to produce very tiny permanent magnets with high-energy products. Hence, they are used in high-density magnetic recording.

2. Quantum wells, quantum dots and quantum wires having quantum confinements are mainly produced from semiconductor nanomaterials which act as computer storage (memory) materials with high density.
3. These materials have a large volume function of grain boundaries between surface area and volume. This property is used to improve the mechanical behaviour of the materials.
4. Nanomaterials, as catalysts, are used to increase the amount of oil which can be recovered from tarsands and used to purify water.
5. A photodynamic compound based on fullerenes for the treatment of cancer has already been put into pre-clinical trials.
6. Some nanocrystals combine with aluminum and silica which are used to produce commercial-grade coatings. These coatings provide resistance to heat and rust.
7. Zirconia is used as an oxygen sensor and for removal of poisonous gases from automobile exhaust.
8. They are better at insulation and have low cost flat-panel displays.
9. They are used in high-enegy density batteries, high-power magnets and high-senstivity sensors.
10. They are used in automobiles for greater fuel efficiency.

5.5 CARBON NANOTUBES (CNT)

Until the 1980's, carbon was known to exist as diamond and graphite only in two physical forms. Diamond and graphite have different physical structures and properties. Harry Kroto and Richard Smalley in the year 1985 found out carbon 60 (C_{60}) and it was named as buckminsterfullerene. It has a spherical shape and having 32 faces. After this, other related molecules (C36, C70, C76 and C84) made of only carbon atoms were found out. This new carbon molecules were named as fullerenes. S. Lijima, in the year 1991, discovered that carbon atoms can form long cylindrical tubes and it was named carbon nanotubes (CNT). A promising group of nanostructures with large potential application are carbon nanotubes. The hexagonal lattice of carbon is graphite. Graphene is a single sheet of graphite. CNT is a sheet of graphene that is rolled up into a cylindrical shape. The length (< 100 nm) of a carbon nanotube is much greater than (hundreds of times) its diameter.

5.5.1 Structure

Depending upon the way of rolling up of graphene sheet into tubes, there are different structures for carbon nanotubes. The various structures have different properties. Figure 5.17(a) represents an unrolled lattice. The points *O* and *A* are crystallographically equivalent on the graphene sheet where *x*-axis is parallel to one side of the honeycomb lattice. The points *O* and *A* can be connected by a vector *OA* called **chiral vector** and is given by

$$\vec{C}_h = n\vec{a}_1 + m\vec{a}_2$$

a_1 and a_2 represent the unit vectors for the honeycomb lattice of the graphene sheet and n, m are integers which indicate the position of the carbon atoms on the lattice.

The **chiral angle** is given by

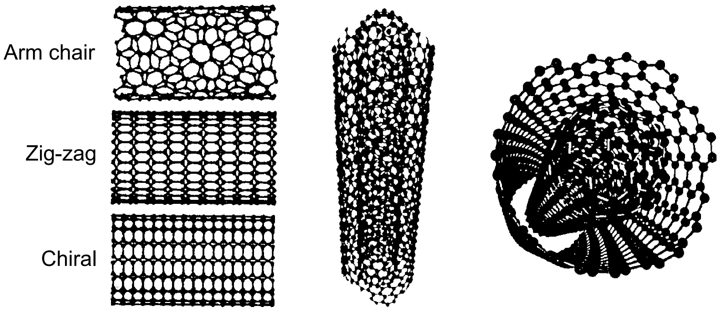
$$\theta = \tan^{-1}\left[\frac{\sqrt{3}n}{(2m+n)}\right]$$

The diameter of the nanotube is given by,

$$D = \frac{C_h}{\pi} = \frac{\sqrt{3a_{c-c}}(m^2 + 2mn + n^2)^{1/2}}{\pi}$$

where C_h is the length of the chiral vector.

a_{c-c} is the carbon-carbon bond length.



(a) Single-walled Nanotube (b) Multiwalled Nanotube (c) MWNT Front View

Fig. 5.17 *Different Types of Nanotubes*

Based on the value of the chiral vector and chiral angle, the nanotubes are classified as chiral, zigzag and armchair.

5.5.2 Types of Carbon Nanotubes

There are two types of carbon nanotubes as follows:

- Single-Walled nanotubes (SWNT)
- Multi-Walled nanotubes (MWNT)

An SWNT consists of a graphene sheet rolled on and they can be considered as nearly one-dimensional structures as shown in Fig. 5.17(b). A MWNT consists of several coaxial tubes of graphene sheet as shown in Fig. 5.17(b).

5.5.3 Fabrication of Carbon Nanotubes

Carbon nanotubes are produced by three common methods as explained below.

1. Arc-evaporation Method or Plasma Arcing

The carbon arc discharge method is the most common and simple method. This method is also known as plasma arcing. In an enclosure, two carbon electrodes, separated by approximately 1 mm, are kept at a potential difference of about 20 V serving as anode and the cathode. The enclosure is filled with inert gas at low pressure. This experimental arrangement is shown in Fig. 5.18. A direct current of 50 to 100 A, driven by the external potential difference, creates a high temperature discharge between the two electrodes. The discharge vaporizes the surface of the positive electrode, and forms a small rod-shaped nanotube deposit on the negative electrode. As the tubes form the length, the positive electrode decreases and a carbon deposit forms on the negative electrode. When a small amount of cobalt, nickel or iron is added to the positive electrode, SWNTs are produced. If no catalyst is used the tubes are of MWNT type.

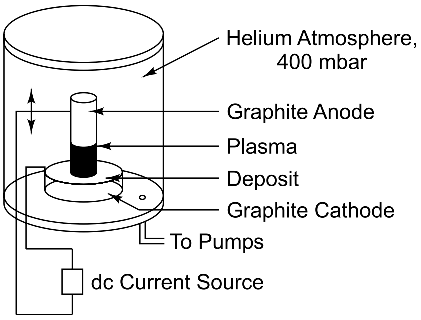


Fig. 5.18 *Electric Arc Discharge Apparatus*

2. Laser Ablation Method

This technique involves the use of laser beam. It consists of an oven. A graphite target is placed inside the oven. The oven is filled with helium or neon gas to maintain 500 torr of pressure. A small amount of cobalt and nickel are added to the graphite target which acts as catalyst. The temperature of the oven is around 1200°C. The oven consists of a copper collector which is cooled by water and is maintained at lower temperature. When an intense pulsed laser beam falls on the target, it results in the evaporating carbon atom from the graphene target. The argon guides the carbon atoms from high temperature region to the cool copper collector. In the collector, they condense into nanotubes. Nanotubes of 10–20 nm diameter can be produced by this method. By varying the growth temperature, the catalyst composition, and other process parameters, the average nanotube diameter and size distribution can be varied. This experimental arrangement is shown in Fig. 5.19.

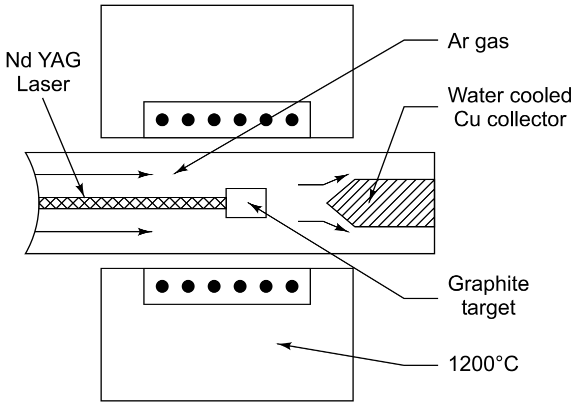


Fig. 5.19 *Laser Ablation Equipment*

3. Chemical Vapour Deposition (CVD)

In this method, multiwalled carbon nanotubes are formed. Fe Ni, Co or an alloy of these three catalytic metals is initially deposited on a substrate. The substrate with deposited film is etched using a diluted HF solution with distilled water. This etched substrate is kept in a CVD furnace. Now an additional etching of the catalystic metal film is done using NH_3 gas at a temperature of 750 to 1100°C. Now nanosize metal particles are formed resulting in the formation of carbon nanotubes due to catalyst particle nucleation. Depending on the thickness of the catalystic film, the carbon nanotube diameter is determined.

5.5.4 Properties of Carbon Nanotubes

- The tensile strength of a carbon nanotube is 20 times greater than steel.
- The thermal conductivity of a carbon nanotube is 10 times higher than that of silver.
- Even though they possess high tensile strength, they are elastic.
- Certain carbon nanotubes have conductivity higher than that of copper.
- The Young's modulus value of the carbon nanotube is as high as 1.8 TPa.
- They possess melting point three times higher than that of copper.
- Smaller carbon nanotube diameter results in increased chemical reactivity.
- Carbon nanotubes can be either metallic or semiconductor which is decided by their structure.
- Even though carbon nanotube has smaller diameter, they are very strong structures.
- Carbon nanotubes are promising in the field of optical application especially in optical communication.

5.5.5 Applications of Carbon Nanotubes

- They are used in energy storage because of their small dimensions, smooth surface topology and surface specificity.
- Carbon nanotubes are used to store hydrogen in the inner cores due to capillary effect.
- They are used in the creation of artificial muscle tissues.
- Nanotubes can replace ITO in some solar cells to act as a transparent conductive film in solar cells to allow light to pass to the active layers and generate photocurrent.
- They are used as light bulb filament as an alternative to tungsten filaments in incandescent lamps.
- Certain nanotubes exhibit superconducting effect.
- Carbon nanotubes can act as antennas for radios and other electromagnetic devices.
- Multiwalled nanotubes with magnetite can generate strong magnetic fields.

- Carbon nanotubes can be used as extremely fine electron guns, which could be used as miniature cathode ray tubes in thin high-brightness, low-energy, low-weight displays.
- Nanotube films show promise for use in displays for computers, cell phones, personal digital assistants, and automated teller machines.

QUESTIONS AND ANSWERS

1. **What are metallic glasses?**

   Metallic glasses belong to a new class of materials which exhibit the properties of both metals and glasses.

2. **Mention any four properties of metallic glasses.**

   - They possess non-crystalline structure.
   - They exhibit ferromagnetic behaviour.
   - They possess high strength.
   - They possess high tensile strength.

3. **Mention any four applications of metallic glasses.**

   - Metallic glasses find applications in electronic circuits because of their insensitivity to temperature variations.
   - They are widely used as resistance elements in electric circuits due to their high electrical resistivity.
   - When the metallic glass Pd_{80} Si_{20} in which Pd was replaced by Cr up to 7%, is used as a thermometer for measuring very low temperatures. Such a thermometer is called cryothermometer.
   - As metallic glasses can sustain high dose of radiation without a change in its electric properties, they are very useful in fission or fusion reactor environment.

4. **What are shape memory alloys?**

   Shape memory alloys (SMA) are special alloys, which after being deformed, can remember and recover their original shape when heated.

5. **What are one-way shape memory alloys?**

   Certain alloys exhibit the shape memory effect only upon heating and they are called the one-way shape memory alloys.

6. **What are two-way shape memory alloys?**

   (A.U., B.E./B.Tech, May/June 2011)

   Certain alloys exhibit the shape memory effect upon both heating and cooling. They are called the two-way shape memory alloys.

7. **What is martensite structure of shape memory alloys?**

   The crystal structure or phase of a SMA at lower temperatures is called martensite.

8. **What is austenite structure of shape memory alloys?**

   Austenite refers to the crystal structure or phase of a SMA at higher temperatures.

9. **What are nanomaterials?**

   (A.U., B.E./B.Tech, May/June 2011)

   Nanophase materials are the nanostructured materials having a characteristic length less than 100 nm.

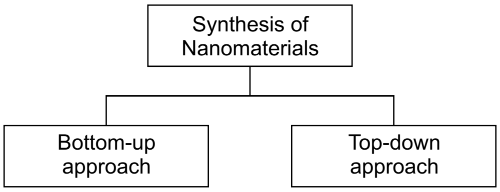
10. **What is nanotechnology?**

    It is the design, characterization, production and applications of devices and materials on the nanoscale.

11. **How are nanostructure fabrications classified?**

    The methods used for the synthesise of nanomaterials are brodly classified into two, namely

    1. Bottom up approach 2. Top-down approach.



12. **What is bottom-up approach? Mention few examples.**

    Molecules and even nanoparticles can be used as the building blocks for producing complex nanostructures. This is known as bottom-up approach.

    Examples

    - Vapour phase deposition
    - Molecular beam epitaxy

13. What is top-down process of producing nanostructure?

The removal or division of bulk material, or the miniaturization of bulk fabrication processes to produce the desired nanostructure is known as top-down approach.

14. How is vapour phase deposition broadly classified?

- Physical Vapour Deposition (PVD)
- Chemical Vapour Deposition (CVD)

15. What is chemical vapour deposition?

Chemical vapour deposition (CVD) involves the reaction or thermal decomposition of gas phase species at elevated temperatures (typically 500–1000°C) and subsequent deposition onto a substrate.

16. What is liquid phase method of producing a nanoparticle?

Liquid phase chemical synthesis adopts the reaction of a solution of precursor chemicals in an aqueous or non-aqueous solvent. These precursors react and self-assemble to form a solution supersaturated with the product.

17. What parameters determine the size of the nanoparticle in liquid phase method?

The size of the precipitated nanoparticle can be controlled by parameters such as temperature, pH, reactant concentration and time.

18. What is sol–gel method?

In this method a set of chemical reactions which irreversibly convert a homogeneous solution of molecular reactant precursors (sol) into an infinite molecular weight three-dimensional polymer (gel) filling the same volume as the solution.

19. What are the advantages of sol–gel method?

- The main advantage of sol–gel processing is the high purity and uniform nanostructure achievable at low temperatures.
- The method is often used to produce metal-oxide nanomaterials.

20. Write a note on chemical synthesis method of synthesizing nanomaterials.

This method can be used to produce both metals and ceramics by using a variety of chemical approaches such as sol–gel or thermal decomposition.

21. Write a note on mechanical deformation method of synthesizing nanomaterials.

Mechanical deformation means milling or shock deformation. It is a common method which is used to produce nanostructured powders, which in turn produce nanostructured materials. Through mechanical milling, it is possible to produce nanostructured powders of systems which are otherwise immiscible.

22. Write a note on thermal crystallization method of synthesizing nanomaterials.

This method is used to synthesize three-dimensional nanostructured materials. By controlling the nucleation and growth during annealing of an amorphous material, one can produce bulk material with an average grain size of less than 20 nm without the need for consolidation and sintering steps.

23. Write a note on thermal properties of nanomaterials.

The melting point decreases with size and rate of decrease increases substantially at very small sizes.

24. Write a note on electron affinities and chemical properties of nanomaterials.

As the size is reduced from the bulk, the electronic bands in metals become narrower and the delocalized electronic states are transformed to more localized molecular bonds.

The ionization potentials are higher at smaller sizes that at the bulk work function. The large surface-to-volume ratio and the variations in geometry and electronic structure have a strong effect on catalytic properties.

25. Write a note on magnetic properties of nanomaterials.

Metal	*Bulk Cluster*
Na, K	Paramagnetic Ferromagnetic
Fe, Co, Ni	Ferromagnetic Superparamagnetic
Gd, Tb	Ferromagnetic Rotors/Superparamagnetic
Cr	Antiferromagnetic Frustrated paramagnetic
Rh	Paramagnetic Ferromagnetic

26. Write a note on mechanical properties of nanomaterials.
(A.U., B.E./B.Tech, May/June 2011)

- 30–50% lower elastic moduli than for conventional grain size materials
- Super plastic behaviour at low homologous temperature
- 2–7 times higher hardness and strength-hardness values for nanocrystalline pure metals (>10 nm grain size) than those of large grained (>1 mm) metal
- Decreasing hardness with decreasing grain size.

27. What are carbon nanotubes?
(A.U., B.E./B.Tech, May/June 2011)

It is a sheet of graphene that is rolled into a cylindrical shape and the length of a carbon nanotube is much greater than its diameter.

28. Mention the types of carbon nanotubes based on structure.
(A.U., B.E./B.Tech, May/June 2011)

- Arm chair
- Zig zag
- Chiral

29. Mention the types of carbon nanotubes.

- Single-walled nanotubes
- Multi-walled nanotubes

30. Mention any four properties of carbon nanotubes.

- The tensile strength of the carbon nanotube is 20 times greater than steel.
- The thermal conductivity of a carbon nanotube is 10 times higher than that of silver.
- Even though they possess high tensile strength, they are elastic.
- Certain carbon nanotubes have conductivity higher than that of copper.

31. Mention any four applications of carbon nanotubes.

- They are used in energy storage.
- They are used in the creation of artificial muscle tissues.
- They can act as antennas for radios and other electromagnetic devices.
- Nanotube films show promise for use in display for computers.

QUESTIONS

1. Discuss the preparation, properties and applications of metallic glasses in detail. *(A.U., B.E./B.Tech, May/June 2011)*
2. Discuss shape memory alloys along with its advantages and disadvantages in detail. *(A.U., B.E./B.Tech, May/June 2011)*
3. Explain the characteristics and properties of Ni-Ti alloy in detail.
4. Discuss plasma assisted deposition process for the preparation of nanoparticles in detail.
5. Discuss vapour phase deposition methods for the preparation of nanoparticles in detail. *(A.U., B.E./B.Tech, May/June 2011)*
6. Discuss colloidal and sol-gel methods for the preparation of nanoparticles in detail.
7. Discuss ball milling method and electrodeposition method for the preparation of nanoparticles in detail.
8. Explain the properties and applications of nanoparticles.
9. Explain arc method, pulse laser deposition and chemical vapour deposition methods for the preparation of carbon nanotubes. *(A.U., B.E./B.Tech, May/June 2011)*
10. Discuss the structure, properties and applications of carbon nanotubes in detail. *(A.U., B.E./B.Tech, May/June 2011)*

SUMMARY

- Modern engineering materials possess improved and new electrical, optical and magnetic properties in addition to high mechanical strength, toughness, ductality and lightness.
- Metallic glasses, shpae memory alloys, nanomaterials and carbon nanotubes are some of the examples of modern engineering materials.
- Metallic glasses that belongs to a new class of materials exhibit the properties of both metals and glasses.
- Fe_{75}, P_{15}, C_{10}, Fe_{24}, Zr_{76}, Ni_{60}, Nb_{40} are few examples of metallic glasses.

- Shape memory alloys (SMA) are special alloys, which after being deformed, can remember and receiver their original shape when heated.
- One-way and two-way shape memory alloys are the two types of shape memory alloys.
- Alloys that exhibit the shape memory effect only upon heating are called as one-way shape memory alloys.
- Alloys that exhibit shape memory effect upon both heating and coding one called as two-way shape memory alloys.
- Austenite refers to the crystal structure or phase of a SMA at higher temperatures.
- Martensite refers to the crystal structures or phase of a SMA at lower temperatures.
- Ni-Ti alloy is an example for shape memory alloys.
- Materials whose atoms are arranged in nanosize dusters are known as nanomaterials.
- At the nanoscale, the properties of materials are very different from those at a larger scale.
- Nanotechnology is the design, characterization, production and application of devices and materials on nanoscale.
- When only one dimension of the material is reduced to the nanorange and two remaining dimensions remain large, then the structure is known as quantum well.
- When two dimensions of the material is reduced to nanorange and the remaining one dimension remains large, then the structure is known as quantum wire.
- When all the three dimensions of a material is reduced to the nanorange, then the structure is known as quantum dot.
- Synthesis of nanomaterials are broadly classified under two groups namely bottom-up approach and top-down approach.
- The top-down approach starts with large scale object and reducing its dimensions to nanoscale.
- The bottom-up approach, atoms, molecules or nanoparticles itself one used as the building blocks for the production of complex nanostructures.

- Ball-milling method, lithographic method. Plasma arcing method are tew methods under top-down approach.
- Vapour phase deposition methods, colloidal and sol-gel methods, electrodeposition method are few methods under bottom-up process.
- Physical vapour deposition (PVD) and chemical vapour deposition (CVD) are the two classes of vapour phase deposition method.
- Carbon nanotube (CNT) is a single sheet of graphene that is rolled up into a cylindrical shape.
- The length of a carbon nanotube is much greater than its diameter.
- Arm chair, zig zag, chiral are the types of carbon nanotubes (MWNT) are the two types of carbon nanotubes.
- Single-walled nanotubes (SWNT) and Multi-walled nanotubes (MWNT) are the two types of carbon nanotubes.
- Arc, evaporation method, laser ablation method, chemical vapour deposition method are the three common methods for the production of carbon nanotubes.

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