Engineering Thermodynamics and Fluid Mechanics

Fifth Edition

WBUT–2016

About the Authors

P K Nag had been with the Indian Institute of Technology Kharagpur, for about four decades, almost since his graduation. After retirement from IIT, he was an Emeritus Fellow of AICTE, New Delhi, stationed at Jadavpur University, Kolkata, till June 2005. He was a Visiting Professor in the Technical University of Nova Scotia (now Dalhousie University), Halifax, Canada, for two years during 1985–86 and 1993–94.

Dr Nag has been a recipient of the President of India medal (1995) from the Institution of Engineers (India). He was a Fellow of the National Academy of Engineers (FNAE) and a Fellow of the Institution of Engineers (India). In his early days, he was also an NCC Officer at IIT Kharagpur. Besides these, Dr Nag was a Life Member of the Indian Society for Technical Education, Indian Society for Heat and Mass Transfer, and the Combustion Institute, USA (Indian section). He was also a member of the New York Academy of Sciences, USA.

He was interested in studying circulating fluidized bed boilers, combined cycle power generation, second law analysis of thermal systems, and waste heat recovery. He authored four books and more than 150 research papers in several national and international journals and proceedings. He had attended several international conferences in the US, Japan, Canada and Bangladesh where he presented several papers.

Sukumar Pati is currently working as Assistant Professor in the Department of Mechanical Engineering at National Institute of Technology, Silchar. Prior to joining NIT Silchar, he worked as a Post-doctoral Fellow for almost one year at Indian Institute of Technology Kharagpur after completion of his PhD from Mechanical Engineering Department of the same Institute. He received his BE in Mechanical Engineering from Jalpaiguri Government Engineering College (University of North Bengal) where he was awarded the Silver Medal for attaining the second rank. He obtained his ME in Fluid Mechanics from Bengal Engineering College (presently Indian Institute of Engineering Science and Technology, Shibpur, West Bengal).

Dr Pati joined Haldia Institute of Technology, Haldia, in the Department of Mechanical Engineering in 2001 and served till 2007 at various positions. He is author of the textbook, *A Textbook on Fluid Mechanics and Hydraulic Machines* and co-author of the textbook *Engineering Mechanics (WBUT)* both published by McGraw Hill Education (India). He has published several research papers in international journals of repute. His active areas of interest include evaporation and condensation, micro-scale fluid flow and heat transfer, natural convection and non-Newtonian fluid mechanics.

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Dr Jana has published several papers in national and international journals, and conferences. His areas of interest include CNC machining, Holonic manufacturing systems, and agent-based systems.

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Dedicated to my grandchildren
Madhurima Basu Thakur
and
Abhiroop Guha

P K Nag

Dedicated to my parents
Late Sakti Pada Pati
and
Usha Rani Pati

Sukumar Pati

Dedicated to my parents
Late Ananta Kumar Jana
and
Late Jyotsna Jana

T K Jana

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Appendices

Solved Question Papers

PREFACE

Subjects like ***Engineering Thermodynamics*** and ***Fluid Mechanics*** are envisaged as one of the fundamental subjects of many engineering disciplines. The two subjects, although have proliferated separately to a great extent, are still considered complementary to each other. Thermodynamics is the branch of Science that deals with the study of energy, predominantly in the form of *heat and work*, their mutual transformations and consequence on various properties that define the status of the system. *Fluid Mechanics*, on the other hand, deals with the behaviour of fluids under the conditions of rest and motion. The close relationship that exists between the two subjects can be understood from the fact that in a system, simple or complex, energy interactions take place between the system and surroundings by means of a working medium, which is essentially a fluid. Both the subjects find wide applications in areas such as steam power plants, gas turbine power plants, refrigeration and air conditioning, internal combustion engines and process plants, to name just a few. It, therefore, becomes immensely important and is worthwhile to study both the subjects simultaneously including their vast application potential in a unified manner for developing a strong engineering background.

Numerous textbooks on these subjects are available in the market separately for Thermodynamics and Fluid Mechanics. However, in the course curricula of several universities, like in WBUT, the two subjects are offered as one paper at the preliminary level (first year). We have been associated with the teaching fraternity for more than 10 years. During this period, we have had the onus of teaching these subjects as one regularly, conforming to the syllabus of WBUT as well. In our long-standing experience, we have understood the needs of the students and difficulties encountered by them. In dearth of a good combined book on this subject, we felt the need to write one that would be a combined and comprehensive textbook and shall form the backbone of higher study for the students. With this book we wish to groom the students with the fundamental knowledge of thermodynamics and fluid mechanics. Basic knowledge fortified with application-specific objective and associated analysis would help not only first-year students of WBUT but all students pursuing mechanical engineering, manufacturing or production engineering, chemical or process engineering, civil engineering to acquire ample benefit out of this text.

This book emphasises on fundamental concept building, necessary mathematical analysis coupled with graphical representations of the theme, applications and numerical problem-solving ability. Lucidity of the text, ample illustrations, and summaries highlighted to understand the objective of the topics and findings and conclusion are some unique chapter features. Throughout the text, SI units have been used. Solved examples find their place immediately after the relevant topic is discussed to help students proceed through topics step by step. Further, several questions, viz. objective, subjective and numerical problems, are provided at the end of each chapter for practice, better understanding and to imbibe problem-solving ability.

Salient Features

- Complete coverage of latest WBUT syllabus
- Solutions of 2015, 2014, 2013 and 2012 WBUT questions papers
- Separate chapters dedicated to elucidate the three laws of thermodynamics
- Detailed coverage of important topics such as fluids properties, fluid statics, kinematics of fluid flow and dynamics of ideal fluids
- Thoroughly updated solved examples and chapter-end exercises with answers to enhance problem-solving skills
- Excellent pedagogy:
    - *Solved Examples: 154*
    - *Multiple Choice Questions: 171*
    - *Review Questions: 146*
    - *Numerical Problems: 136*
    - *Illustrations: 158*

Chapter Organisation

The text is written taking utmost care regarding the chronology of the presentations. The book comprises 14 chapters out of which the first 10 cover thermodynamics and the rest cover fluid mechanics. Each chapter begins with the fundamental aspects of the topics and related areas.

Chapter 1 deals with the basis of thermodynamics such as applications, dimensions, and processes and cycles. **Chapter 2** is a dedicated chapter on heat and work. **Chapter 3** explains the first law of thermodynamics. Concepts like flow volume, mass and energy balance are dealt with in **Chapter 4**. The second law of thermodynamics covering topics like thermal energy, heat engines, Kelvin–Planck statement are explained in **Chapter 5**. **Chapters 6** and **7** elucidate on entropy, its properties and principles, and properties of pure substances. Properties of gas are explained in **Chapter 8**. **Chapters 9** and **10** deal with air standard cycles like Otto cycle, diesel cycle, and analysis of power cycles, and **Chapters 11** to **14** deal with the concept of fluids. This includes concept of continuity, fluid properties, capillarity, and Pascal's law of pressure, kinematics of fluid flow and dynamics of fluids.

Appendix I comprises Thermodynamic Proporties of Water (Steam Tables), **Appendix II** comprises answers to multiple-choice questions, and **Appendix III** comprises answers to numerical problems given in each chapter. Solved WBUT question papers (2012–2015) have been provided at the book end for students to practice on.

Note: Wherever required, necessary mathematics is incorporated. All the mathematical analysis is accompanied by necessary schematic arrangements and graphical plots. Representations of thermodynamic processes on various coordinate systems (P-V, T-s, and P-h) are indispensable to its study and there is no dearth of such representations in this text. Variations of several performance indexes (e.g., efficiency, work output, COP, etc.) are plotted with respect to the independent variable(s), to study the behaviour of the system(s). There can be quite a good number of situations where more than one independent variable governs the desired output and, hence, extensive study of such situations helps trade-off amongst these variables and their level in the light of desired output. Adequate attention has been paid in this direction in the present text.

Acknowledgements

We humbly remember the contributions by the top management of the institutions who have created the right ambience for the pursuit of higher studies. Constant inspiration from all the well wishers has made it possible to come out with this text for the benefit of the student community. A majority of the text is taken from our lecture notes with necessary modifications. We are indebted to our teachers, colleagues and staff in our journey towards quality education.

We are grateful to the following reviewers who took out time and gave their comments on certain chapters of the text:

Palash Mondal	*Adamas Institute of Technology, Kolkata, West Bengal*
Manoj Kumar Barai	*Future Institute of Engineering and Management, Kolkata, West Bengal*

We must express our gratitude to our family and relatives for their constant support and encouragement in the present endeavour. We also thank the editorial team at McGraw Hill Education (India) for arranging the publication of this revised edition in a short span of time.

We shall be extremely thankful to the readers for sharing their constructive suggestions and criticism with a view to enhance the utility of the book. If any reader is interested to pursue discussion regarding the contents of this book, they are most welcome to communicate with us by email at *sukumarpati@gmail.com* and *tarun.jana2000@gmail.com.*

Readers can also share their feedback with our publisher, McGraw Hill Education India. Please send your comments at *info.india@mheducation.com*. Do mention the title and authors' names as the subject and also please inform of any piracy of this book spotted by you.

P K Nag

Sukumar Pati

T K Jana

Publisher's Note

Dr P K Nag's sad demise in October 2013 has left a void seldom completely filled by another academician so accomplished. An academician, teacher and author par excellence, Dr P K Nag will be fondly remembered and dearly missed.

ROADMAP TO THE SYLLABUS

This text is useful for Subject Code: Engineering Thermodynamics and Fluid Mechanics (ME 201)

Module 1: Basic Concepts of Thermodynamics

Introduction: Microscopic and macroscopic viewpoints, Definition of thermodynamic systems: Closed, open and isolated systems, Concept of thermodynamic state; State postulate, Definition of properties: Intensive, extensive and specific properties, Thermodynamic equilibrium, Thermodynamic processes: Quasi-static, reversible and irreversible processes, Thermodynamic cycles, Zeroth law of thermodynamics, Concept of empirical temperature

Heat and Work: Definition and units of thermodynamic work, Examples of different forms of thermodynamic work, Example of electricity flow as work, Work done during expansion of a compressible simple system, Definition of heat, Unit of heat, Similarities and dissimilarities between heat and work.

Ideal Equation of State: Processes; Real Gas—Definition of ideal gas; Ideal gas equations of state, Thermodynamic processes for ideal gas, *P-V* plots, Work done, Heat transferred for isothermal, isobaric, isochoric, isentropic and polytropic processes, Equations of state of real gases: van der Waal's equation, Virial equation of state.

Properties of Pure Substances: *p-v* and *P-T* diagrams of pure substance like H_2O, Introduction to steam table with respect to steam generation process, Definition of saturation, Wet and superheated status, Definition of dryness fraction of steam, Degree of superheat of steam.

GO TO
CHAPTER 1. Thermodynamic Concepts and Zeroth Law
CHAPTER 2. Heat and Work
CHAPTER 3. First Law of Thermodynamics
CHAPTER 7. Properties of Pure Substances
CHAPTER 8. Properties of Gas

Module 2: First Law of Thermodynamics

Definition of stored energy and internal energy, First Law of Thermodynamics for cyclic processes, Non-flow energy equation, Flow energy and definition of enthalpy, Conditions for steady state steady flow: Steady state steady flow energy equation.

GO TO
CHAPTER 2. Heat and Work
CHAPTER 3. First Law of Thermodynamics
CHAPTER 4. First Law Applied to Flow Processes

Module 3: Second Law of Thermodynamics

Definition of sink, Source reservoir of heat, Definition of sink, Source reservoir of heat, Thermal efficiency of heat, Heat Engines and coefficient of performance of refrigerators, Kelvin – Planck and Clausius statements of second law of thermodynamics, Absolute or thermodynamic scale of temperature, Clausius integral, Entropy, Entropy change calculation for ideal gas processes, Carnot cycle and Carnot efficiency, PMM-2; Definition and its impossibility.

GO TO
CHAPTER 5. Second Law of Thermodynamics
CHAPTER 6. Entropy
CHAPTER 8. Properties of Gas

Module 4: Air Standard Cycles for IC Engines

Otto cycle; Plot on *P-V*, *T-S* planes; Thermal efficiency, Diesel cycle; Plot on *P-V*, *T-S* planes; Thermal efficiency.

Rankine Cycle of Steam - *h-s* chart of steam (Mollier's Chart), Simple Rankine cycle plot on *P-V*, *T-S*, *h-s* planes, Rankine cycle efficiency with and without pump work.

GO TO
CHAPTER 7. Properties of Pure Substances
CHAPTER 9. Air Standard Cycles
CHAPTER 10. Power Cycle

Module 5: Properties and Classification of Fluids

Ideal and real fluids, Newton's law of viscosity; Newtonian and Non-Newtonian fluids, Compressible and incompressible fluids.

Fluid Statics - Pressure at a point.

Measurement of Fluid Pressure - Manometers: Simple and differential, U-tube, Inclined tube.

Fluid Kinematics - Stream line, Laminar and turbulent flow, External and internal flow, Continuity equation.

Dynamics of Ideal Fluids - Bernoulli's equation, Total head; Velocity head; Pressure head, Application of Bernoulli's equation.

Measurement of Flow Rate: Basic Principles - Venturimeter, Pilot tube, Orifice meter.

GO TO
CHAPTER 11. Properties of Fluids
CHAPTER 12. Fluid Statics
CHAPTER 13. Kinematics of Fluid Flow
CHAPTER 14. Dynamics of Ideal Fluids

LIST OF IMPORTANT SYMBOLS

A	Area, m^2
BDC	Bottom dead center
COP	Coefficient of performance
COP_R	Coefficient of performance of a refrigerator
COP_HP	Coefficient of performance of a heat pump
C	Coefficient of Pitot tube
C_c	Coefficient of contraction
C_d	Coefficient of discharge
C_p	Specific heat at constant pressure, kJ/kgK
C_v	Specific heat at constant volume, kJ/kgK
C_v	Coefficient of velocity
e	Specific energy, kJ/kg
E	Total energy, kJ
E	Compressibility, N/m^2
g	Acceleration due to gravity, m^2/s
h	Specific enthalpy, kJ/kg
h_f	Specific enthalpy of saturated liquid, kJ/kg
h_{ig}	Latent heat of sublimation, kJ/kg
h_{fg}	Latent heat of vapourisation, kJ/kg
h_g	Specific enthalpy of saturated vapour, kJ/kg
H	Enthalpy, kJ
KE	Kinetic energy, kJ
m	Mass, kg
$\dot{m}$	Mass flux, kg/s
m_f	Mass of liquid, kg

m_g	Mass of vapour, kg
M	Molecular weight, kg/kmol
MEP	Mean effective pressure
n	Polytropic index
N	Number of moles
P	Pressure, kN/m^2 or kPa
P_{atm}	Atmospheric pressure, kPa
P_{cr}	Critical pressure, kPa
PE	Potential energy, kJ
Q	Heat transfer, kJ
Q_1	Heat transfer from high-temperature reservoir, kJ
Q_2	Heat transfer to low-temperature reservoir, kJ
r	Compression ratio
r_c	Cut-off ratio
R	Characteristic gas constant, kJ/kg.K
$\bar{R}$	Universal gas constant, kJ/kmol.K
Re	Reynolds number
s	Specific entropy, kJ/kg.K
s_f	Specific entropy of saturated liquid, kJ/kg.K
s_{fg}	Change in the specific entropy due to vapourisation, kJ/kg.K
s_g	Specific entropy of saturated vapour, kJ/kg.K
S	Entropy, kJ/K
t	Temperature, °C
T	Temperature, K
T_{cr}	Critical temperature, K
T_1	Temperature of source, K
T_2	Temperature of sink, K
TDC	Top dead center
u	Specific internal energy, kJ/kg

u_f	Specific internal energy of saturated liquid, kJ/kg
u_{fg}	Change in the specific internal energy due to vapourisation, kJ/kg
u_g	Specific internal energy of saturated vapour, kJ/kg
U	Internal energy, kJ
v	Specific volume, m^3/kg
v_{cr}	Critical specific volume, m^3/kg
v_f	Specific volume of saturated liquid, m^3/kg
v_{fg}	Change in the specific volume due to vapourisation, m^3/kg
v_g	Specific volume of saturated vapour, m^3/kg
V	Volume, m^3
V	Velocity, m/s
W_p	Pump work, kJ
W_T	Turbine work, kJ
W	Work done, kJ
$\dot{W}$	Rate of work done, kW
x	Quality or dryness fraction
Z	Elevation, m

Greek Symbols

ρ	Density, kg/m^3
ρ_m	Density of manometric fluid, kg/m^3
τ	Shear stress, N/m^2
γ	Specific weight
η	Thermal efficiency
μ	Dynamic viscosity, Ns/m^2
ν	Kinematic viscosity, m^2/s
σ	Surface tension, N/m

LIST OF IMPORTANT FORMULAE

- For simple compressible closed system, quasi-equilibrium work due to moving boundary between two end points 1 and 2 can be expressed as

$$W_{1-2} = \int_{V_1}^{V_2} P\,dV$$

- The expression for quasi-equilibrium work due to moving boundary for different processes are given as

  Constant pressure process: $W_{1-2} = P(V_2 - V_1)$

  Constant volume process: $W_{1-2} = 0$

  Hyperbolic process: $\mathrm{W}_{1-2} = P_1V_1 \ln\frac{V_2}{V_1} = P_1V_1 \ln\frac{P_1}{P_2}$

  Polytropic process: $W_{1-2} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{P_1V_1 - P_2V_2}{n-1}$

  Adiabatic process: $W_{1-2} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

- First law of thermodynamics for a closed system undergoing a cycle is given by $\oint \delta W = \oint \delta Q$
- First law of thermodynamics for any system undergoing any process can be expressed in differential form as

$$\delta Q - \delta W = dU$$

- The mass balance equation for a single stream entering and a single stream leaving the control volume when the flow is steady can be written as

$$\frac{A_1V_1}{v_1} = \frac{A_2V_2}{v_2}$$

  where A_1 and A_2 are the cross-sectional area of fluid stream at inlet and outlet respectively, V_1 and V_2 are the average velocity of fluid stream at inlet and outlet respectively, v_1 and v_2 are the specific volume of fluid at inlet and outlet respectively.
- The steady flow energy balance equation for a single stream entering and a single stream leaving the control volume can be written as

$$\dot{m}(h_1 + \frac{V_1^2}{2} + gz_1) + \dot{Q} = \dot{m}(h_2 + \frac{V_2^2}{2} + gz_2) + \dot{W}$$

where $\dot{m}$ is the mass flux, h_1 and h_2 are the specific enthalpies of fluid at inlet and outlet respectively, z_1 and z_2 are the elevation of inlet and outlet with respect to some arbitrary datum respectively, $\dot{Q}$ is the rate of heat transfer and $\dot{W}$ is the rate of work done.

- Thermal efficiency of a cyclic heat engine is given by

$$\eta_{\text{ther}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

where Q_1 is the heat transfer from source to heat engine, Q_2 is the heat rejection from the heat engine to sink and W is the work done by the heat engine.

- The efficiency of a Carnot heat engine is given by

$$\eta_{\text{ther, Carnot}} = 1 - \frac{T_2}{T_1}$$

where T_1 and T_1 are the temperature of source and sink respectively.

- The coefficient of performance (COP) of refrigerator and heat pump are given by

$$\text{COP}_\text{R} = \frac{Q_2}{Q_1 - Q_2}$$

$$\text{COP}_\text{HP} = \frac{Q_1}{Q_1 - Q_2}$$

where Q_2 is the heat transfer from sink to the device (refrigerator or heat pump) and Q_1 is the heat transfer from the device to the source.

- The coefficient of performance of Carnot refrigerator and heat pump are given by

$$\text{COP}_\text{R} = \frac{T_2}{T_1 - T_2}$$

$$\text{COP}_\text{HP} = \frac{T_1}{T_1 - T_2}$$

- For a cyclic process, $\oint \frac{\delta Q}{T} \le 0$

If $\oint \frac{\delta Q}{T} = 0$ then the cyclic process is possible and reversible.

If $\oint \frac{\delta Q}{T} < 0$ then the cyclic process is possible and irreversible.

If $\oint \frac{\delta Q}{T} > 0$ then the cyclic process is impossible.

- The change in the entropy of a system is given by $\mathrm{dS} = \frac{\delta \mathrm{Q_R}}{\mathrm{T}}$ for reversible process.
- Thermodynamic property relations are given by

$$Tds = du + Pdv$$

$$Tds = dh - vdP$$

which are applicable to all processes whether reversible or irreversible.

- Properties of a liquid-vapour mixture may be found by relations such as

$$v = v_f + xv_{fg}$$

$$h = h_f + xh_{fg}$$

$$s = s_f + xs_{fg}$$

$$u = u_f + xu_{fg}$$

where x is the quality or dryness fraction of liquid-vapour mixture, and the subscripts f and g denote the properties of liquid and vapour respectively.

- The equation of state of an ideal gas can be expressed as

$$PV = n\bar{R}T$$

$$P\bar{v} = \bar{R}T$$

$$PV = mRT$$

$$Pv = RT$$

where P is the pressure, V is the total volume of the gas, $\bar{v}$ is the molar volume (i.e., volume per unit mole) , n is the number of moles of the gas, m is the mass of the gas, v is specific volume of the gas, $\bar{R}$ is the universal gas constant, R is the characteristic gas constant and T is the temperature of the gas in K.

- Change in specific internal energy of a calorically perfect gas between states 1 and 2 is

$$u_2 - u_1 = C_v (T_2 - T_1)$$

- Change in specific enthalpy of a calorically perfect gas between states 1 and 2 is

$$h_2 - h_1 = C_p (T_2 - T_1)$$

- Change in specific entropy of a calorically perfect gas between states 1 and 2 is

$$s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

$$= C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= C_p \ln\frac{V_2}{V_1} + C_v \ln\frac{P_2}{P_1}$$

- The thermal efficiency of the Otto cycle can be expressed as

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}}$$

where r is the compression ratio and γ is the specific heat ratio.
- The thermal efficiency of the Diesel cycle can be expressed as

$$\eta_{Diesel} = 1 - \frac{1}{r^{\gamma-1}}\frac{1}{\gamma}\frac{r_c^{\gamma}-1}{r_c-1}$$

where r_c is the cut-off ratio, r is the compression ratio and γ is the specific heat ratio.
- The thermal efficiency of the Rankine cycle is given by

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

where W_T is the turbine work, W_P is the pump work and Q_1 is the heat input in the boiler.
- According to Newton's law of viscosity, for one-dimensional flow shear stress is given by

$$\tau = \mu\frac{du}{dy}$$

where μ is the coefficient of viscosity.
- Capillary rise of or depression is given by

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

where σ is the surface tension coefficient, θ is the area wetting contact angle, ρ is the density of fluid, and d is the diameter of tube.
- Three-dimensional continuity equations in differential form is given by

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

- Euler's equation of motion along a streamline is given by

$$\frac{dP}{\rho} + VdV + gdz = 0$$

which is valid for steady and inviscid flow.

- Bernoulli's equation along a streamline is given by

$$\frac{P}{\rho}+\frac{V^2}{2}+gz=\text{constant}$$

- Discharge by Venturimeter is given by

$$Q=\frac{C_d A_1 A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh\left(\frac{\rho_m}{\rho_w}-1\right)}$$

where A_1 and A_2 are the cross-sectional areas of the venturimeter at its inlet and throat respectively, ρ_m and ρ_w are the density of the manometric fluid and the working fluid respectively, h is the difference in height of the manometric fluid in the two limbs of the manometer, and C_d is the coefficient of discharge of the venturimeter.

- Volume flow rate by orificemeter is given by

$$Q=\frac{C_v C_C A_o A_1}{\sqrt{A_1^2-C_C^2 A_o^2}}\sqrt{2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]}$$

where C_c is the coefficient of contraction , C_v is the coefficient of velocity, and A_o is the area of orifice.

- The velocity measured by Pitot tube is given by

$$V=C\sqrt{2gh}$$

where C is the coefficient of Pitot tube and h is the difference in stagnation and static pressure head.

- The velocity measured by Pitot - static tube is given by

$$V=C\sqrt{2g\frac{\Delta P}{\rho}}$$

CHAPTER

1

Thermodynamic Concepts and the Zeroth Law

1.1 INTRODUCTION

Thermodynamics is the science which deals with energy in the form of heat and work and its transformation into each other and the effect of energy transformation on the properties of systems.

Thermodynamics is the science of three ***E*** namely, ***energy***, ***equilibrium and entropy***. But since these terms have not yet been defined, an alternative definition with which we are already familiar is: *Thermodynamics is the science that deals with the relations between heat, work and those properties of systems that bear relation to heat and work.*

The name thermodynamics stems from the Greek words *therme* (heat) and *dynamics* (force), which were used in early days to describe the effort to convert heat into power. But in the present context, the name thermodynamics is used to interpret all aspects of energy and energy transformations, including power production, refrigeration, and relationships among the properties of matter. Like all sciences, thermodynamics is also based on experimental observations. The findings from these observations have been expressed in the form of some fundamental laws.

The principles of thermodynamics are summarized in the form of four fundamental laws known as the ***zeroth, first, second and third law of thermodynamics.***

The ***zeroth law of thermodynamics*** deals with thermal equilibrium and provides a means for measuring temperature. *The* ***first law of thermodynamics*** deals with the conservation of energy and introduces the concept of internal energy. The ***second law of thermodynamics*** asserts that energy has quality as well as quantity and actual processes occur in a certain direction from high-quality energy to low-quality energy but not in the reverse direction spontaneously. It introduces the important concept of entropy. The ***third law of thermodynamics*** defines the absolute zero of entropy.

1.2 APPLICATIONS OF THERMODYNAMICS

All engineering activities involve an interaction between energy and matter. It is difficult even to imagine an area which does not relate to thermodynamics in some respect. In fact, the human body itself is an interesting application in the area of thermodynamics.

Engineering thermodynamics finds applications in diverse areas such as power-producing devices, refrigeration and air-conditioning, compressors and turbines, jet engines and rockets, the combustion of hydrocarbon fuels such as coal, oil and natural gases and cooling of electronic devices such as TVs and computers.

1.3 CLASSICAL AND STATISTICAL THERMODYNAMICS

The study of thermodynamics can be classified into two: ***classical thermodynamics*** and ***statistical thermodynamics***. In classical thermodynamics, we study the time averaged effects of the molecules. For example, the pressure of a gas in a container is the result of momentum transfer between the molecules and the walls of the container. But to determine the pressure in the container, we need not know the behaviour of the individual molecules of the gas. This macroscopic approach to the study of thermodynamics is called ***classical thermodynamics***.

An elaborate approach, considering the behaviour of a large number of individual particles is called ***statistical thermodynamics***. In short, the ***macroscopic approach*** and the ***microscopic approach*** to the study of energy are, respectively, called the classical thermodynamics and statistical thermodynamics.

1.4 DIMENSIONS AND UNITS

A dimension is a physical variable used to specify some characteristic of a system. Mass, length and time are examples of dimension, whereas a unit is a particular amount of a physical quantity. For example, time can be measured in seconds, minutes, hours, etc.

The dimension mass (M), length (L) and time (T) are considered to be the basic dimensions, from which other dimensions are derived. For example, the dimension of velocity is LT^{-1}, and of acceleration is LT^{-2}.

In mentioning a unit, it is recommended that small letters (lower case) be used when abbreviated, and complete word using only small letters when expanded. For example, the unit of time is s or second, and for length is m or metre. But when a unit is named after a person, a capital letter is used when it is abbreviated. For example, Newton (N), Pascal (Pa), Watt (W). Multiples in powers of 10 are indicated by prefixes, which are also abbreviated. The internationally accepted prefixes are given in Table 1.1.

Table 1.1 *SI Unit Prefixes*

Multiple	*Prefix*	*Abbreviation*	*Multiple*	*Prefix*	*Abbreviation*
10	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a

Dimensions are those names which are used to characterize physical qualities. Common examples of dimensions include mass (M), length (L), time (t), temperature (T), force (F), etc. The most important reasons for the importance of dimensions in engineering analysis is that any equation which relates physical quantities must be dimensionally homogeneous. By dimensionally homogeneous we mean that the dimensions of terms on one side of an equation equal to those on the other side. Equations relating physical quantities which do not fulfill the condition of dimensional homogeneity are not correct.

In order to make numerical computation with equations involving physical quantities, there is the additional requirement that units, as well as the dimensions, be homogeneous. Units are those arbitrary magnitudes and names assigned to dimensions which are adopted as standards for measurements. For example, the primary dimension of time may be measured in units of second, minute, hour, etc.

1.4.1 Base Units

The basic units of mass, length, time and temperature in the SI units are described below:

(i) **Mass:** The basic SI unit for mass is kilogram (abbreviated to kg). A standard alloy block of platinum and iridium maintained at the International Bureau of Weights and Measures at Sevres, Paris, is taken as the base unit of mass.

(ii) **Length:** The basic SI unit for length is metre (abbreviated to m). The distance between two marks on a platinum–iridium bar, kept at the International Bureau of Weights and Measures at Sevres, Paris, France, when measured at 0°C is taken as the base unit of length.

(iii) **Time:** The basic SI unit for time is the second (abbreviated to s). For many years, the accepted basic unit, second was defined as $\frac{1}{86400}$ of the mean solar day.

(iv) **Temperature:** The basic SI unit for temperature is the Kelvin (abbreviated to K). In 1967, the Thirteenth Conference Generale des Poids et Measures defined Kelvin as the fraction $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.

1.4.2 Derived Units

The secondary quantities are expressed in terms of the derived units, which in turn are formed from the base units. The relation between the derived units and the base units depends on a definition or a law. For example, velocity is defined as $v = \frac{\mathrm{d}L}{\mathrm{d}t}$; where L is length and t is time. The unit of velocity is m/s. The units of force can be derived from the equation $F = mf$ (Newton's second law) as kg-m/s^2. In honour of Newton, the unit of force has been named Newton and is abbreviated to N. The dimensions and units of some of the physical quantities commonly used in thermodynamics are given in Table 1.2.

Table 1.2 *Dimensions and Units of some physical quantities*

Quantity	*Dimensions*	*Units*	*Abbreviation*
Mass	M	kilogram	kg
Length	L	metre	m
Time	T	second	s
Temperature	θ	Kelvin	K
Velocity	LT^{-1}	m/s	—
Acceleration	LT^{-2}	m/s^2	—
Force	MLT^{-2}	kg m/s^2(N)	Newton
Pressure	$ML^{-1}T^{-2}$	kg/ms^2(N/m^2)	Pascal
Energy	ML^2T^{-2}	kg m^2/s^2(N-m)	Joule
Power	ML^2T^{-3}	kg m^2/s^3(J/s)	Watt
Density	ML^{-3}	kg/ m^3	—
Viscosity	$ML^{-1}T^{-1}$	kg /ms(Ns/m^2)	Pas
Kinematic viscosity	L^2T^{-1}	m^2/s	
Surface tension	MT^{-2}	N/m	

1.5 THERMODYNAMIC SYSTEM

A system is defined as a fixed identifiable quantity of mass or region in space upon which attention is focused for the study of energy transfer.

The mass or region outside the system is called its ***surroundings***. The real or imaginary surface that separates the system from its surroundings is called the ***system boundary*** (refer Fig 1.1). The system boundary has zero thickness and thus it can neither contain any mass nor occupy any volume in space. The system boundary can be classified as the following:

- Rigid or movable depending upon whether it allows a change in volume of the system. A **rigid boundary** is said to be restrictive with respect to the volume, whereas a movable boundary is said to be nonrestrictive with respect to the volume.
- Adiabatic or diathermal depending upon whether it allows the exchange of energy in the form of heat. An **adiabatic boundary** is said to be restrictive with respect to energy flow in the form of heat whereas a **diathermal boundary** is said to be non-restrictive with respect to energy flow in the form of heat.

The system and the surroundings as a whole is called the ***universe***.

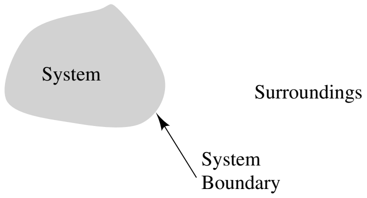


Figure 1.1 *A thermodynamic system*

- On the basis of mass and energy transfer through system boundary, a system may be considered as an open, closed or isolated system.

A **closed system** (also known as control mass system) consists of a fixed amount of mass, and no mass can cross its boundary. But, energy in the form of heat and work, can cross the boundary. Volume of a closed system does not have to be fixed. A certain quantity of fluid in a cylinder bounded by a piston constitutes a closed system as shown in Fig. 1.2. The inner surfaces of the piston and the cylinder form the boundary, and since no mass is crossing this boundary, it is a closed system. Everything outside the fluid, including the piston and the cylinder, is the surroundings.

Thus, a closed system has the following characteristics:

1) It has fixed mass. The mass does not cross the system boundary.
2) Energy, both in the form of heat and work, can cross the boundary.
3) The boundary can move.

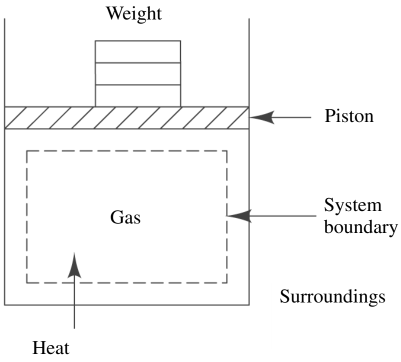


Figure 1.2 *Example of a closed system*

An open system (also known as control volume) is a properly selected region in space. The boundary, which remains fixed in space, enveloping this control volume is called the ***control surface***. Both mass and energy can cross the control surface. An example of open system is sketched in Fig. 1.3. In most engineering problems in thermodynamics, the mass of the system is not fixed, for example, an air compressor, a turbine, a car radiator, a water heater. The characteristics of an open system are the following:

(1) The system has a fixed volume called the control volume, and a fixed control surface.
(2) Both heat and work can flow across the control surface.
(3) Mass can also enter and leave the control surface.

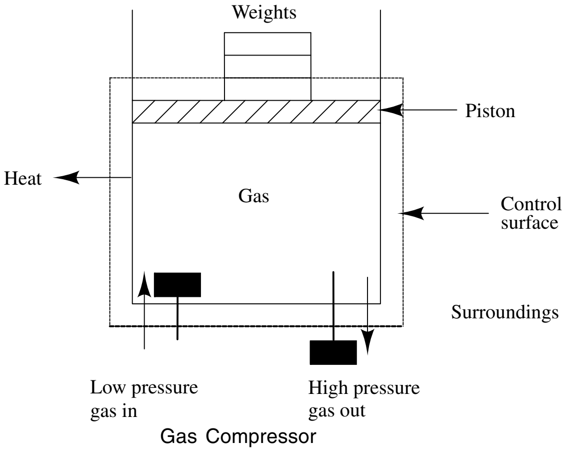


Figure 1.3 *Example of an open system*

A system which exchanges neither mass nor energy with the surroundings is called an **isolated system.** It is thus a closed system having no energy interaction with the surroundings (Fig. 1.4). By definition, the universe is an isolated system.

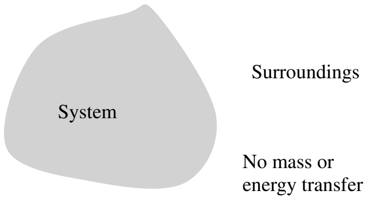


Figure 1.4 *Example of an isolated system*

On the basis of phase (physically distinct, chemically homogeneous and mechanically separable portion of a substance), a system can be divided into ***homogeneous and heterogeneous systems.*** A system consisting of a single phase is called a homogeneous system. A mixture of air and water vapour is an example of a homogeneous system. On the other hand, a system consisting of more than one phase is known as a heterogeneous system. Ice plus water is an example of a heterogeneous system.

1.6 THERMODYNAMIC PROPERTIES

Any identifiable/observable characteristic of a system by which the physical condition of the system may be described is called property of the system. Some familiar properties are pressure, temperature, volume and mass.

Properties may be divided into two categories, viz. **extensive** and **intensive.** The properties that depend on the extent of the system are known as *extensive properties*. A property is called *extensive* if its value for an overall system is the sum of its values for the parts into which the system is divided. Mass, volume, momentum are extensive properties.

Intensive properties are those that are independent of the mass of a system, such as temperature, pressure, and density. Intensive properties are not additive. Their values are independent of the size or extent of a system and may vary from place to place within the system at any instant. *Thus, intensive properties may be functions of both position and time, whereas extensive properties vary at most with time.*

Mass of the whole system is equal to the sum of the masses of the various parts. But, temperature of the whole system is not equal to the sum of the temperatures of the constituent parts.

Generally, **upper-case letters are used to denote extensive properties (with mass *m* being a major exception), and lower-case letters are used for intensive properties (with pressure *P* and temperature *T* being exceptions)**.

Extensive properties per unit mass are called ***specific properties,*** e.g., specific volume, specific energy, etc. and hence becomes intensive properties.

1.7 STATE AND EQUILIBRIUM

The thermodynamic state of a system at any instant of time is its condition as characterized by certain identifiable/observable thermodynamic properties, i.e., the set of properties completely describes the state of a system.

Thermodynamics deals with equilibrium states. The word equilibrium implies a state of balance. In an equilibrium state, there are no unbalanced potentials or driving forces within the system. A system is said to be in equilibrium when it manifests no changes with time. There are many types of equilibrium, and a system is not in thermodynamic equilibrium unless the conditions of all the relevant types of equilibrium are satisfied.

In ***thermal equilibrium***, all parts of a system are at the same temperature; also this temperature is the same as that of the surroundings. When these conditions are not satisfied, a change of state will take place until thermal equilibrium is reached.

When there is no unbalanced force or torque between a system and its surroundings and in the interior of a system, the system is said to be in a state of **mechanical equilibrium**. A system will be in **chemical equilibrium** only if its chemical composition does not change with time, i.e., no net chemical reaction occurs.

A system will be in thermodynamic equilibrium only when it satisfies the conditions for all modes of equilibrium.

When the conditions of any one of the equilibrium is not satisfied, a system is said to be in a non-equilibrium state. If the non-equilibrium is due to temperature difference between the system and surroundings then the temperature distribution is not uniform within the system. Similarly, if the nonequilibrium is due to an unbalanced pressure difference between the system and its surroundings, the pressure varies from one part of the system to the other.

1.8 STATE POSTULATE

A system is said to be comprised of a *simple compressible substance* if the electrical, magnetic, gravitational, and surface tension effects are insignificant in comparison to pressure, volume and temperature change effects characterizing the system. A relatively small number of *independent properties* suffice to fix all other properties and thus the state of the system. The number of properties required to fix the state of a system is given by the *state -postulate* which states that *the state of a simple compressible substance is completely specified by two independent, intensive properties.*

1.9 PROCESSES AND CYCLES

If the value of even one property changes, the state will change to a different one. Any change that a system undergoes from one equilibrium state to another is called a process. The series of states through which a system passes during a process is called the ***path of the process***. Figure 1.5 depicts a process between the states 1 and 2.

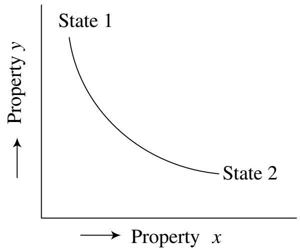


Figure 1.5 *Process*

To describe a process completely, it is necessary to specify the initial and final states of the process and the path the system followed during the change of state.

The prefix *iso* is often used to designate a process for which a particular property remains constant. For example, ***isothermal process*** is one during which the temperature remains constant, ***isobaric*** is a process during which the pressure remains constant, and ***isochoric*** is a constant volume process.

A system is said to have undergone a thermodynamic cycle, (Fig. 1.6) if it returns to its initial state at the end of the process. Thus, for a cycle, the initial and final states are identical. The change in the value of any property for a cyclic process is zero.

For example $\oint dP = 0, \oint dV = 0, \oint dT = 0$

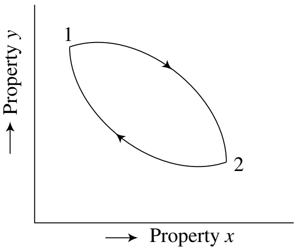


Figure 1.6 *Cycle*

1.9.1 Quasi-Static (or Quasi-equilibrium) Process

When a process proceeds in such a manner that its state deviates to an infinitesimally small extent from the corresponding equilibrium state at all times, it is called a ***quasi-equilibrium or quasi-static process.*** *Quasi* means almost.

A quasi-static process is a sufficiently slow process that allows the system to adjust itself internally in such a manner that all the properties in one part of the system do not change faster than those at any other parts.

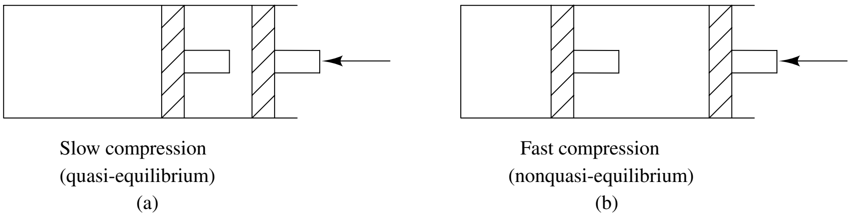


Figure 1.7 *Quasi-equilibrium and nonquasi-equilibrium compression processes*

When the piston is moved slowly (Fig. 1.7a), the molecules have a sufficient time to redistribute themselves such that they do not pile up in front of the piston. As a result, the pressure inside the cylinder will always be nearly uniform and will rise at the same rate at all locations. This is a quasi-equilibrium process. When the piston is moved fast (Fig. 1.7b), the molecules will pile up in a small region in front of the piston, thus creating a high pressure region. Therefore, the pressure inside the cylinder will not be uniform and will make the process nonquasi-equilibrium.

All actual processes are nonquasi-equilibrium because they take place with finite pressure or temperature difference between a system and surroundings. Engineers are interested in quasi-static processes for the following reasons:

(a) They are easy to analyze.

(b) Work producing devices deliver maximum work and work absorbing devices consume minimum work when they operate on quasi-equilibrium processes.

A quasi-equilibrium compression process is shown in Fig. 1.8. A nonquasi-equilibrium process is usually denoted by a dashed line between the initial and final states.

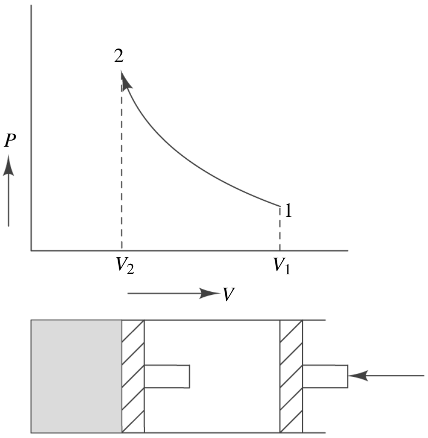


Figure 1.8 *Quasi-equilibrium compression process on a P–V diagram*

1.9.2 Reversible Process

A process is said to be ***reversible*** if at the conclusion of the process, both the system and the surroundings can be restored to their respective initial states without producing any changes in the rest of the universe. Otherwise, the process is said to be ***irreversible.***

A system can, however, always be restored to its initial state, but this does not imply that the process is reversible. Only if the surroundings are also simultaneously restored to their initial states, the process becomes reversible.

If the piston moves under such conditions so that the forces are exactly balanced, the system is said to undergo a *reversible process.*

The characteristics of irreversible processes are the following:

(a) an irreversible process can be carried out in one direction only.

(b) an irreversible process occurs at a finite rate.

1.10 ZEROTH LAW AND TEMPERATURE

Although we know that temperature is a measure of hotness or coldness, it is not easy to give a precise definition for it. Based on physiological sensations, we express the level of temperature qualitatively with words like red-hot, hot, warm, cold and freezing cold. However, from physiological sensations, we cannot assign numerical values to temperatures.

The zeroth law of thermodynamics states that ***if two bodies are in thermal equilibrium with a third body separately, they are also in thermal equilibrium with each other***. The zeroth law of thermodynamics was first formulated by R.H. Fowler in 1931. The first and second laws of thermodynamics were formulated much before 1931. It was named the zeroth law of thermodynamics since it should have preceded the first and the second laws of thermodynamics.

The temperature of a system is a property that determines whether or not a system is in thermal equilibrium with other systems.

Temperature is a property of great importance in thermodynamics, and its value can be obtained easily by indirect measurement with calibrated instruments. The temperature of a system is determined by bringing a second body, a ***thermometer***, into contact with the system and allowing thermal equilibrium to be reached. The value of the temperature is found by measuring some temperature dependent property of the thermometer. Any such property is called a ***thermometric property***. Commonly used property of materials in different thermometers are given in Table 1.3.

Table 1.3 *Thermometers and thermometric properties*

Thermometer	*Thermometric Property*
1. Mercury-in-glass gas	Length
2. Electric resistance	Resistance
3. Thermocouple	Electromotive force of two dissimilar solids
4. Constant pressure gas	Volume
5. Constant volume gas	Pressure
6. Radiation	Black body radiation

1.11 TEMPERATURE SCALES

Consider the temperature t of a system, which is a dependent property and is a function of any two independent properties of the system, say, x and y, viz.,

$$t = f(x, y)$$

Consider thermometer to be the system. The most simple and convenient way to measure the temperature is to keep one property, say y, as constant that the temperature becomes a function of x only, viz.,

$$t = f(x)$$

In this way, temperatures t_1, t_2, t_3, etc., can be measured in terms of values x_1, x_2, x_3, etc., of the thermometric property as shown in Fig. 1.9.

For example, in the case of liquid-in-glass thermometer, the two independent properties are the length (L) of the liquid and the pressure (p) of the gas above. By keeping vacuum inside, the pressure is maintained constant as zero value. The temperature becomes a function of the length L of the thermometric substance.

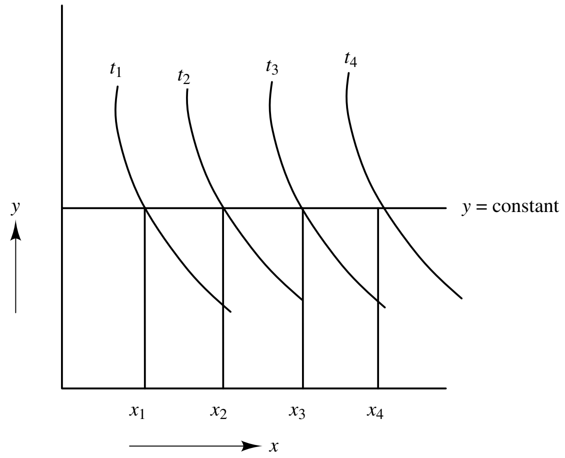


Figure 1.9 *Temperature t as a function of single thermometric property*

Therefore,

$$t = f(L)$$

To establish a temperature scale, the relationship between the temperature and the thermometric property must be expressed. For this purpose, we have the following different mathematical forms:

Direct proportionality: $t = ax$

Linear relation: $t = bx + c$

Polynomial: $t = d + ex + fx^2 + gx^3$

In each relation, a certain number of constants are to be determined.

A familiar device for temperature measurement is the liquid-in-glass thermometers as shown in Fig. 1.10. It consists of glass capillary tube connected to a bulb filled with a liquid (such as mercury or alcohol) and sealed at the other end. The space above the liquid is occupied by the vapour of the liquid or an inert gas. As temperature increases, the liquid expands in volume and rises in the capillary. The length (L) of the liquid in the capillary depends on the temperature. The liquid is the thermometric substance and the length is the thermometric property. Although, this type of thermometer is commonly used for ordinary temperature measurements, it is not suitable for applications where extreme accuracy is required.

Any body with at least one measurable property that changes as its temperature changes can be used as a thermometer. Such a property is called a thermometric property. The particular substance that exhibits change in the thermometric property is known as a thermometric substance.

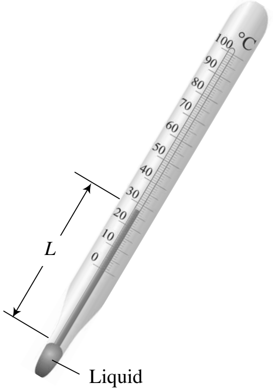


Figure 1.10 *Liquid-in-glass thermometer*

All temperature scales are based on some easily reproducible states, such as freezing and boiling point of water, also known as the ice point and steam point.

A mixture of ice and water which is in equilibrium with air saturated with vapour at 1 atm pressure is said to be the ***ice point***.

A mixture of liquid water and water vapour with no air in equilibrium at 1 atm pressure is said to be at the ***steam point***.

The thermodynamic temperature scale in the SI system is the ***Kelvin scale***, named after Lord Kelvin (1824–1907). The temperature unit on this scale is the Kelvin, which is designated by K.

The thermodynamic temperature scale in the English system is the ***Rankine scale***, named after William Rankine (1820–1872). The temperature unit on this scale is the Rankine, which is designated by R.

1.11.1 Method in Use Before 1954

Until 1954, the temperature scales were based on two fixed points, the steam point (the temperature of equilibrium between pure water and pure steam at 1 atm pressure) and the ice point (the temperature at which pure ice coexisted in equilibrium with air-saturated water at 1 atm pressure).

The temperature scales used in the SI and in the English system are the ***Celsius scale*** and the ***Fahrenheit scale***, respectively. On the Celsius scale, the ice and steam points were originally assigned the values of 0°C and 100°C, respectively. The corresponding values on the Fahrenheit scale are 32 and 212°F.

The relations for the temperature in terms of values of the thermometric property in the Celsius and Fahrenheit scale are given as

$$t, ^\circ\mathrm{C} = 100\frac{X - X_i}{X_s - X_i}$$

$$t,\ ^\circ\text{F} = 32 + 180\ \frac{X - X_i}{X_s - X_i}$$

where X_i and X_s are the values of the thermometric property at the ice point ad the steam point respectively and X is the value of the thermometric property at the temperature to be measured.

1.11.2 Method in Use After 1954

Kelvin suggested that only a single fixed point was necessary to establish a temperature scale. He pointed out that the ***triple point of water*** (the states in which the solid, liquid and the vapour phases of water coexist in equilibrium) could be used as the single fixed point. At the tenth CGPM in 1954, the value of the triple point of water was set at 0.01°C or 273.16 K in the Kelvin scale.

$$t = 273.16\ \frac{X}{X_{TP}}$$

1.12 IDEAL GAS THERMOMETER

In an ideal gas thermometer, either change of pressure of an ideal gas at constant volume, or the change in volume of an ideal gas at constant pressure as a function of temperature, serves as the thermometric substance.

Consider a fixed mass of an ideal gas that undergoes a change in temperature. If the gas is enclosed in a bulb, and the pressure readings at two different temperatures are taken in such a way that the volume of the gas is kept constant, then

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}$$

Taking the volume readings keeping pressure as constant, we have

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

In both the cases, one temperature is taken as a known reference temperature and the other as the temperature to be measured.

When volume is kept constant, the thermometer is known as a ***constant volume gas thermometer***. When pressure is kept constant, it is known as a ***constant pressure gas thermometer***.

We will discuss constant-volume gas the thermometer, used to measure temperature, in this chapter.

1.12.1 Constant-Volume Gas Thermometer

The constant-volume gas thermometer is shown in Fig. 1.11. The thermometric substance is the gas (normally hydrogen or helium), and the thermometric property is the pressure exerted by the gas. The gas is contained in a bulb, and the pressure exerted by it is measured by an open-tube mercury manometer. As temperature increases, the gas expands forcing up in the pen tube. The gas is kept at constant volume by raising or lowering the reservoir.

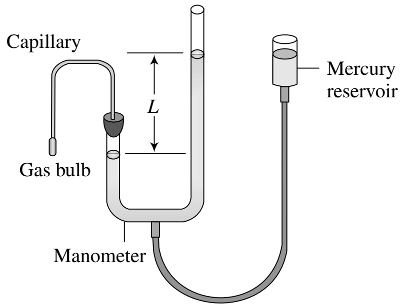


Figure 1.11 *Constant-volume gas thermometer*

For measuring a temperature, the gas bulb is first placed in a constant temperature bath at the triple point of water. Let the pressure reading taken at this temperature T_{TP} be P_{TP}. Then the bulb is placed in a location where the temperature is to be measured. Let the new pressure reading be P. Then the required temperature is given by

$$T = 273.16 \frac{P}{P_{TP}}$$

SUMMARY

- Thermodynamics is the science which deals with energy in the form of heat and work and its transformation into each other and the effect of energy transformation on the properties of systems.
- A thermodynamic system is defined as a fixed identifiable quantity of mass or region in space upon which attention is focused for the study of energy transfer. The mass or region outside the system is called its surroundings. The real or imaginary surface that separates the system from its surroundings is called the system boundary.
- The system and the surroundings as a whole is called the *universe.*
- A closed system (also known as control mass system) consists of a fixed amount of mass, and no mass can cross its boundary. But, energy in the form of heat and work can cross the boundary.
- An open system (also known as control volume) is a properly selected region in space. The boundary, which remains fixed in space, enveloping this control volume is called the control surface. Both mass and energy can cross the control surface.
- A system which exchanges neither mass nor energy with the surroundings is called an isolated system.
- Any identifiable/observable characteristic of a system by which the physical condition of the system may be described is called a property of the system. The properties that depend on the extent of the system are known as extensive properties. Intensive properties are those that are independent of the mass of a system.

- The thermodynamic state of a system at any instant of time is its condition as characterized by certain identifiable/observable thermodynamic properties, i.e., the set of properties completely describes the state of a system.
- A system is said to be in equilibrium when it involves no changes with time. In *thermal equilibrium,* all parts of a system are at the same temperature; also this temperature is the same as that of the surroundings. When there is no unbalanced force or torque between a system and its surroundings and in the interior of a system, the system is said to be in a state of *mechanical equilibrium.* A system will be in *chemical equilibrium* only if its chemical composition does not change with time, i.e., no net chemical reaction occurs. A system will be in thermodynamic equilibrium only when it satisfies the conditions for all modes of equilibrium.
- If the value of even one property changes, the state will change to a different one. Any change that a system undergoes from one equilibrium state to another is called a process. The series of states through which a system passes during a process is called the path of the process.
- A system is said to have undergone a thermodynamic cycle, if it returns to its initial state at the end of the process.
- When a process proceeds in such a manner that its state deviates infinitesimally small from corresponding equilibrium state at all times, it is called a quasi-static process. Quasi means almost.
- A process is said to be reversible if at the conclusion of the process both the system and the surroundings can be restored to their respective initial states without producing any changes in the rest of the universe. Otherwise, the process is said to be irreversible.
- The zeroth law of thermodynamics states that *if two bodies are in thermal equilibrium with a third body separately, they are also in thermal equilibrium with each other.*

REVIEW QUESTIONS

1.1 Define the terms system, boundary, surroundings and universe. Discuss various types of systems giving examples of each.

1.2 Define state, property, change of state, path, process and cycle.

1.3 Discuss briefly about the concept of thermodynamic equilibrium and its importance in engineering thermodynamics.

1.4 Distinguish between an adiabatic system and isolated system.

1.5 Distinguish between extensive property and intensive property.

1.6 What is a quasi-static process and how does it differ from a reversible process?

1.7 What is the basic difference between a process and a cycle?

1.8 Discuss whether or not the following quantities can be used as properties:

(a) $\int P\mathrm{d}V$

(b) $\int V\mathrm{d}P$

(c) $\int P\mathrm{d}V + \int V\mathrm{d}P$

(d) $\dfrac{\mathrm{d}T}{T} + P\dfrac{\mathrm{d}V}{V}$

1.9 State the zeroth law of thermodynamics.

1.10 Explain mechanical, chemical and thermal equilibrium.

1.11 Specify whether the following properties are intensive or extensive:

(a) mass (b) weight (c) volume (d) density
(e) velocity (f) energy (g) pressure (h) temperature

MULTIPLE-CHOICE QUESTIONS

1.1 Mixture of ice and water form a
(a) closed system
(b) open system
(c) isolated system
(d) heterogeneous system

1.2 A closed system is one in which
(a) mass does not cross boundaries of the system, though energy may do so
(b) mass crosses the boundary, but not the energy
(c) neither mass nor energy crosses the boundaries of a system
(d) both energy and mass cross the boundary of the system

1.3 An open system is one in which
(a) mass does not cross boundaries of the system, though energy may do so
(b) mass crosses the boundary, but not the energy
(c) neither mass nor energy crosses the boundaries of a system
(d) both energy and mass cross the boundary of the system

1.4 An isolated system is one in which
(a) mass does not cross boundaries of the system, though energy may do so
(b) mass crosses the boundary, but not the energy
(c) neither mass nor energy crosses the boundaries of a system
(d) both energy and mass cross the boundary of the system

1.5 Extensive property of a system is one whose values
(a) depends on the mass of the system, like volume
(b) does not depend on the mass of the system, like temperature , pressure, etc.
(c) is not dependent on the path followed but on the state
(d) is dependent on the path followed and not on the state

1.6 Intensive property of a system is one whose values
(a) depends on the mass of the system, like volume
(b) does not depend on the mass of the system, like temperature , pressure, etc.
(c) is not dependent on the path followed but on the state
(d) is dependent on the path followed and not on the state

1.7 A system will be in thermodynamic equilibrium only if it is in
(a) thermal equilibrium
(b) mechanical equilibrium
(c) chemical equilibrium
(d) all of the above

1.8 Which of the following is a microscopic property?
(a) pressure
(b) temperature

(c) density
(d) molecular internal energy

1.9 Centrifugal fan is an example of
(a) closed system
(b) open system
(c) isolated system.
(d) none of these

1.10 Which of the following quantities is not a property of a system?
(a) pressure
(b) temperature
(c) specific volume
(d) heat

1.11 Measurement of temperature is based on
(a) first law of thermodynamics
(b) second law of thermodynamics
(c) zeroth law of thermodynamics
(d) third law of thermodynamics

CHAPTER

2

Heat and Work

2.1 ENERGY

Energy can exist in various forms such as mechanical, chemical, thermal, kinetic, potential, electrical, magnetic etc. and the sum of all forms of energy is called the total energy of the system. Thermodynamics deals with the change of total energy, not with the absolute value of the total energy. Thermodynamics is the science of study of energy in the form of heat and work. In fact energy can cross the system boundary of a closed system in two distinct forms: *heat* and *work*. Therefore, it is important to distinguish between these two forms of energy.

2.2 ENERGY TRANSFER BY HEAT

Heat is defined as the form of energy transferred across the boundaries of a system because of temperature difference between a system and its surroundings, and in the direction from higher temperature to lower temperature

Heat is energy in transit. It is recognized only as it crosses the system boundary. Consider a hot metal body as a system. The outer surface of the body represents the system boundary. It contains energy but this energy is called heat only as it passes through the outer surface of the body (the system boundary) to reach the surroundings. Once in the surroundings, the transferred heat becomes part of the internal energy of the surroundings. Thus, in thermodynamics, heat simply means the heat transfer.

2.3 UNITS OF HEAT

The transfer of heat into a system is called ***heat addition***, and the transfer of heat out of a system is called ***heat rejection***. A process during which there is no heat transfer is called an adiabatic process. The word *adiabatic* comes from the Greek word *adiabatos,* which means not to be passed. An adiabatic boundary/wall, commonly known as heat insulator, prevents heat transfer just as a rigid wall prevents work transfer in a hydrostatic system. A diathermic wall, commonly called a heat conductor, allows heat transfer, just as a deformable wall allows work transfer. An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the energy content and thus the temperature of a system can still be changed by other means such as work.

The amount of heat transferred during a process between two states 1 and 2, is denoted by $Q_{1\text{-}2}$. The universally accepted sign convention for heat is as follows (refer Fig. 2.1):

- Heat transfer to a system is positive.
- Heat transfer from a system is negative

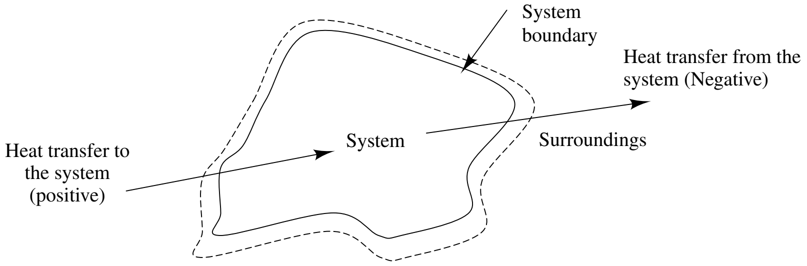


Figure 2.1 *Sign convention for heat transfer*

2.4 HEAT TRANSFER MODES

Heat transfer takes place in three different modes: conduction, convection and radiation.

Conduction is the mode of heat transfer in which a medium transporting the heat remains at rest. When a temperature gradient exists in a stationary medium, which may be solid or a fluid, the heat transfer which occurs across the medium is referred to as conduction.

Conduction is the mode of heat transfer in which energy exchange takes place from the region of high temperature to that of low temperature by the kinetic motion or direct impact of molecules, as in the case of a fluid at rest, and by the drift of electrons, as in the case of metals. In a solid which is a good electric conductor, a large number of free electrons move about in the lattice; hence materials that are good electric conductors are generally good heat conductors (i.e., copper, silver, etc.)

The empirical law of heat conduction based on experimental observations originates from Biot but is generally named after the French mathematical physicist Joseph Fourier who used it in his analytic theory of heat. This law states that the rate of heat flow by conduction in a given direction is proportional to the area normal to the direction of heat flow and to the gradient of temperature in that direction. For heat flow in the x direction, for example, the Fourier law is given as

$$Q_x = \quad k_x A \frac{\mathrm{d}T}{\mathrm{d}x}$$

or,
$$q_x = \frac{Q_x}{A} = \quad k_x \frac{\mathrm{d}T}{\mathrm{d}x}$$

where Q_x is the rate of heat flow through area A in the positive x direction and q_x is called the heat flux in the x direction. The constant of proportionality is called the ***thermal conductivity***, which is a physical property of the substance and is defined as the ability of a substance to conduct heat.

Similarly, the rate of heat transfer along y and z directions are given by

$$Q_y = \quad k_y A \frac{\mathrm{d}T}{\mathrm{d}y}$$

$$Q_z = \quad k_z A \frac{\mathrm{d}T}{\mathrm{d}z}$$

A material having $k_x = k_y = k_z = k$, is called an ***isotropic material***. For an isotropic material, the heat transfer equation $Q_x = \quad kA \frac{\mathrm{d}T}{\mathrm{d}x}$.

If temperature decreases in the positive x direction, then $\mathrm{d}T/dx$ is negative; hence q_x becomes a positive quantity because of the presence of the negative sign. Therefore, the minus sign is included to ensure that q_x is a positive quantity when the heat flow is in the positive x direction. Conversely, when the right-hand side is negative, the heat flow is in the negative x direction.

The unit of thermal conductivity k in SI units is W/m-K.

Heat convection is due to the capacity of moving matter to carry heat energy such as transporting a load from one place to another. The heat transfer that occurs between a surface and a moving fluid when they are at different temperatures is called convection.

In engineering applications, to simplify the heat transfer calculations between a heat surface at T_w and a cold fluid flowing over it at a bulk temperature T_f, a heat transfer coefficient h is defined as

$$q = h(T_w - T_f)$$

where q is the heat flux (W/m^2) from the hot wall to the cold fluid. The unit of heat transfer coefficient in SI system is W/m^2-K.

Heat radiation is due to the property of matter to emit and to absorb different kinds of radiation and to the fact that an empty space is perfectly permeable to radiation and that matter allows them to pass either more radiation or less of it.

2.5 ENERGY TRANSFER BY WORK

The concept of work is usually introduced in the study of mechanics. Mechanical work is defined as the product of a force F and the displacement caused by the force when both are measured in the same direction. The expression for differential quantity of work δW resulting from differential displacement ds is given by

$$\delta W = F\mathrm{d}s$$

Work is a scalar quantity. The total work for a finite displacement is obtained from the integration of $F\mathrm{d}s$.

Work in the thermodynamic sense is defined as the energy transformed because of a property difference other than temperature difference. Thermodynamic work is also sometimes defined as follows: "Work is transferred from the system during a given operation if the sole effect of the system behaviour can be reduced to lifting a weight".

We define work as follows: ***Work is done by a system if the sole effect on the surroundings could be the raising of a weight***. This definition of work is based on the definition first suggested by M. Planck and later developed by J.H. Keenan. This definition does not state that the mass is actually raised by the application of the force—what it states is that the sole effect, external to the system, could be the raising of a mass through a distance against gravitational force.

The basic unit of work in the SI system is the Newton-metre (N-m) called the Joule (J).We shall use the more conventional kilojoule (kJ) which is 10^3 N-m.

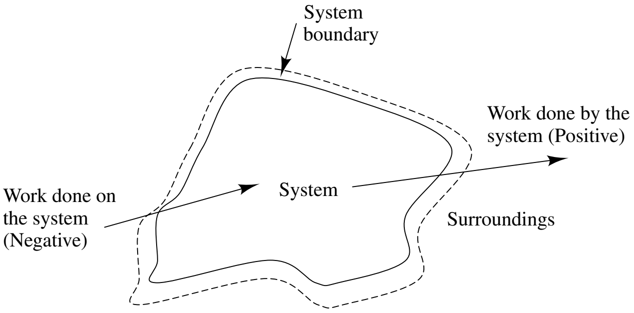


Figure 2.2 *Sign convention for work transfer*

The universally accepted sign convention for work transfer is as follows (refer Fig. 2.2):

- Work done by a system is positive.
- Work done on a system is negative.

2.6 PATH AND POINT FUNCTIONS

The magnitude of a path function depends on the path followed during a process as well as on the end states. On the other hand, the magnitude of a point function depends only on the end states. Path functions are designated by the symbol δ. Therefore, a differential amount of heat or work is represented by δQ or δW, respectively, instead of $\mathrm{d}Q$ or $\mathrm{d}W$. Properties are point functions and they have exact differentials designated by the symbol d.

For example, a small change in pressure is represented by $\mathrm{d}P$ and the total pressure change during a process 1–2 between states 1 and 2 is

$$\int_1^2 \mathrm{d}P = P_2 - P_1$$

But the total heat transfer during the same process 1–2 is

$$\int_1^2 \delta W = W_{1-2} \neq W_2 - W_1$$ and hence is an *inexact differential*.

2.7 COMPARISON OF HEAT AND WORK

Heat and work are energy transfer mechanisms between a system and its surroundings, and there are many similarities between them:

(a) Heat and work are both transient phenomena. Systems never possess heat or work, but when a system undergoes a change of state heat and work cross the system boundary.
(b) Both heat and work are boundary phenomena i.e., both are recognized at the boundaries of a system as they cross the boundaries.
(c) Both are associated with a process, not a state. Unlike properties, heat or work has no meaning at a state.
(d) Both heat and work are path functions and inexact differentials.

2.8 *P*–d*V* WORK OR DISPLACEMENT WORK OR MOVING BOUNDARY WORK

Let the gas in the cylinder be a system (Fig. 2.3) having initially the pressure P_1 and volume V_1. The inner surfaces of the piston and the cylinder form the boundary. The piston is the only boundary which moves due to gas pressure. Let the piston moves out to a new position 2, where the pressure and volume are P_2 and V_2 respectively.

At any intermediate point in the travel of the piston, let the pressure be P and the volume V. When the piston moves an infinitesimal distance dl and if A be the area of the piston, the force acting on the piston $F = PA$ and the infinitesimal amount of work done by the gas on the piston

$$\delta W = F.\mathrm{d}l = PA.\mathrm{d}l = P\mathrm{d}V$$

where, $\mathrm{d}V = A\mathrm{d}l$ = infinitesimal displacement volume.

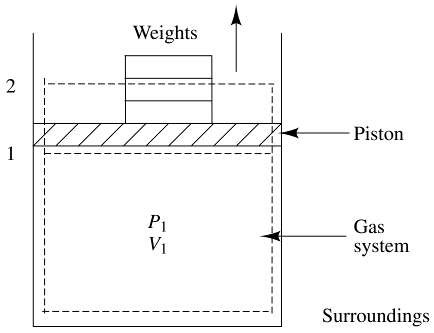


Figure 2.3 *A piston–cylinder device*

When the piston moves out from position 1 to 2 with the volume changing from V_1 to V_2, the amount of work W done by the system will be

$$W_{1-2} = \int_{V_1}^{V_2} P\mathrm{d}V \tag{2.1}$$

This integral can be evaluated only if we know the functional relationship between P and V during the process.

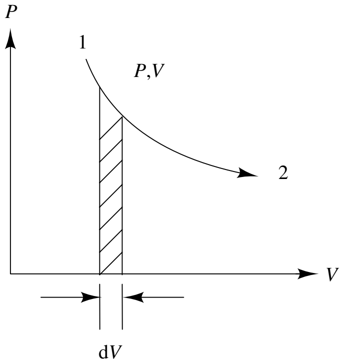


Figure 2.4 *P–V diagram for a quasi-equilibrium expansion process*

The area under the process curve on a P–V diagram is equal in magnitude to the work done during a quasi-equilibrium expansion or compression process of a closed system.

2.8.1 Limitations of the Use of $\int P\mathrm{d}V$

For $\int P\mathrm{d}V$ *to* represent work, the following conditions must be satisfied:

(i) The system is a closed one and the process taking place is a non-flow process.
(ii) The process is quasi-static. Such a process is often referred to as a reversible process since a quasi-static expansion process according to a particular P–V relationship between two states can be reversed as a quasi-static compression process according to the same P–V relationship between the same states.
(iii) The boundary of the system should move in order that work may be transferred.

2.9 *P*–d*V* WORK OR DISPLACEMENT WORK OR MOVING BOUNDARY WORK FOR DIFFERENT QUASI-STATIC PROCESSES

Most of the cycles or processes that are normally encountered in thermodynamics analysis of systems can be identified any one or a combination of the following processes.

(a) Constant pressure process
(b) Constant volume process
(c) Hyperbolic process
(d) Polytropic process
(e) Adiabatic process

2.9.1 Constant-Pressure Process

The work is given by

$$W_{1-2} = \int_{V_1}^{V_2} PdV = P\left(V_2 \quad V_1\right) \tag{2.2}$$

2.9.2 Constant-Volume Process

The work for the constant volume process is zero, since $dV = 0$. Therefore,

$$W_{1-2} = \int_{V_1}^{V_2} PdV = 0 \tag{2.3}$$

2.9.3 Hyperbolic Process

Hyperbolic process is one for which the quantity PV remains constant. Thus, from Eq. (2.1), one may write

$$\begin{aligned} W_{1-2} &= \int_{V_1}^{V_2} PdV \\ &= \int_{V_1}^{V_2} \frac{C}{V} dV \end{aligned}$$

where $PV = C = P_1V_1 = P_2V_2$.

Integrating above Eq. and substituting the constant, C, we get

$$W_{1-2} = C \ln \frac{V_2}{V_1} = P_1V_1 \ln \frac{V_2}{V_1} = P_2V_2 \ln \frac{V_2}{V_1} \tag{2.4}$$

Note: For an ideal gas the hyperbolic process becomes an isothermal process.

2.9.4 Polytropic Process

When a gas undergoes a process in which energy is transferred as heat, the process frequently occurs in such a manner that a plot of ln P versus ln V yields a straight line. For such a process, pressure and volume are related by PV^n = constant, where n is the polytropic index of expansion or compression. Such processes are called polytropic processes. Polytropic index n may be of any value from $-\infty$ to $+\infty$, depending on the particular process.

For a polytropic process between two end states 1 and 2, functional relationship between pressure and volume can be expressed as

$$P_1V_1^n = P_2V_2^n \tag{2.5}$$

Taking logarithms of both sides of Eq. (2.5), we get

$$\ln P_1 + n\ln V_1 = \ln P_2 + n\ln V_2$$

or,

$$n = \frac{\ln(P_1/P_2)}{\ln(V_2/V_1)} \tag{2.6}$$

For a process in which PV^n = constant, pressure, P can be expressed as $P = \frac{C}{V^n}$, where C is a constant. Then Eq. (2.1) becomes

$$W_{1-2} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{C}{V^n}dV$$

$$= C\frac{V_2^{-n+1} - V_1^{-n+1}}{1-n}$$

$$= \frac{CV_1^{-n+1} - CV_2^{-n+1}}{n-1}$$

Since, the constant, C, can be written as $P_1V_1^n$ or as $P_2V_2^n$, we get

$$W_{1-2} = \frac{P_1V_1^nV_1^{-n+1} - P_2V_2^nV_2^{-n+1}}{n-1}$$

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1} \tag{2.7}$$

This expression is valid for all values for n except n = 1. The case n = 1 is discussed in Section 2.9.5.

2.9.5 Adiabatic Process

The relationship between pressure and volume for an adiabatic process is given by the expression $PV^\gamma = C$ where γ is the specific heat ratio .

Following a procedure similar to the one adopted for polytropic process, one may obtain the displacement work done during an adiabatic expansion process as

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{\gamma-1} \tag{2.8}$$

Example 2.1 Obtain an expression for displacement work done by a system undergoing an isothermal expansion from initial volume V_1 to final volume V_2. The macroscopic properties of the system obey the following relationship:

$\left(P+\frac{a}{V^2}\right)(V-b) = mRT$, where a, b and R constants.

Solution Given that

$$\left(P+\frac{a}{V^2}\right)(V-b)=mRT$$

or

$$P=\frac{mRT}{V-b}-\frac{a}{V^2}$$

The displacement work is found from equation (2.1) to be

$$W_{1-2}=\int_{V_1}^{V_2} PdV$$

$$=\int_{V_1}^{V_2}\left(\frac{mRT}{V-b}-\frac{a}{V^2}\right)dV$$

$$=mRT\ln\left|\frac{V_2-b}{V_1-b}\right|+a\left[\frac{1}{V_2}-\frac{1}{V_1}\right]$$

Example 2.2 1 kg of fluid initially at 500 kPa with 0.01 m^3 volume undergoes a reversible expansion to volume 0.05 m^3 and pressure 100 kPa according to a linear law. Calculate the work done.

Solution The expansion takes place according to a linear law. Let the pressure and volume relationship is $P = aV + b$

where a and b are constants and can be evaluated from the initial and final conditions.

$$500 = 0.01a + b$$

$$100 = 0.05a + b$$

After solving the above two equations, we get

$$a = -10000,\ b = 600$$

The pressure and volume relationship can be written as

$$P = -10000V + 600$$

Work done is found to be

$$W_{1-2}=\int_{V_1}^{V_2} PdV$$

$$= \int_{0.01}^{0.05} \left(-10000V + 600\right) dV$$

$$= \left| -10000\frac{V^2}{2} + 600V \right|_{0.01}^{0.05}$$

$$= -5000\left(0.05^2 - 0.01^2\right) + 600\left(0.05 - 0.01\right)$$

$$= 12 \text{ kJ}$$

Example 2.3 Determine the total work done by a gas system following an expansion as shown in Fig. 2.5.

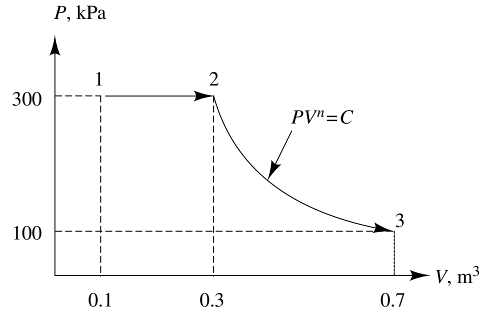


Figure 2.5

Solution Work done for the process 1–2 :

$$W_{1-2} = \int_{V_1}^{V_2} P\mathrm{d}V = P(V_2 - V_1) = 300(0.3 - 0.1) = 60 \text{ kJ}$$

For the process 2–3 :

$$P_2V_2^n = P_3V_3^n$$

$$n = \frac{\ln\frac{P_2}{P_3}}{\ln\frac{V_3}{V_2}} = \frac{\ln\frac{300}{100}}{\ln\frac{0.7}{0.3}} = 1.3$$

Work done for the process 2–3 :

$$W_{2-3} = \int_{V_1}^{V_2} P\mathrm{d}V = \frac{P_2V_2 \quad P_3V_3}{n \quad 1} = \frac{300\times 0.3 \quad 100\times 0.7}{1.3 \quad 1} = 66.67 \text{ kJ}$$

Total work done : $W_{1-3} = W_{1-2} + W_{2-3} = 60 + 66.67 = 126.67$ kJ

Example 2.4 2 kg of a gas is contained in a piston–cylinder assembly at initial conditions of 2 m^3 and 100 kPa. The gas is allowed to expand to a final volume of 5 m^3. Determine the amount of work done for the following processes:

(a) Pressure remains constant
(b) Product PV is a constant
(c) Product PV^2 is a constant

Solution (a) Work done, $W_{1-2} = P(V_2 - V_1) = 100(5 - 2) = 300$ kJ

(b) $P_1V_1 = P_2V_2$

$$\text{Final pressure, } P_2 = P_1\frac{V_1}{V_2} = 100 \times \frac{2}{5} = 40 \text{ kPa}$$

$$\text{Work done} = P_1V_1 \ln\frac{V_2}{V_1} = 100 \times 2 \ln\frac{5}{2} = 183.26 \text{ kJ}$$

(c) $P_1V_1^{\,2} = P_2V_2^{\,2}$

$$\text{Final pressure, } P_2 = P_1\left(\frac{V_1}{V_2}\right)^2 = 100 \times \left(\frac{2}{5}\right)^2 = 16 \text{ kPa}$$

$$\text{Work done, } W_{1-2} = \frac{P_1V_1 \quad P_2V_2}{n \quad 1} = \frac{100 \times 2 \quad 16 \times 5}{2 \quad 1} = 120 \text{ kJ}$$

Note: Example 2.4 illustrates that out of the three processes (P = const. PV = const. PV^2 = const.) between the same end states, constant pressure process gives the maximum output.

Example 2.5 Consider a gas contained in a piston–cylinder assembly as the system. The gas is initially at a pressure of 1000 kPa and occupies a volume of 0.1 m^3. The gas is taken to the final state where pressure is equal to 200 kPa, by the following two different processes.

(i) The volume of the gas inversely proportional to the pressure.
(ii) The process follows the path =constant, where n =1.4.

Calculate the work done by the gas in each case.

Solution

Initial pressure $P_1 = 1000$ kPa

Initial volume $V_1 = 0.1$ m^3

Final pressure $P_2 = 200$ kPa

(i) From the given condition, we get

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

or $$V_2 = \frac{P_1}{P_2}V_1 = \frac{1000}{200} \times 0.1 = 0.5 \text{ m}^3$$

The work done is computed from equation (2.4) to be

$$W_{1-2} = P_1V_1 \ln\frac{V_2}{V_1}$$

$$= 1000 \times 0.1 \times \ln\frac{0.5}{0.1} = 160.94 \text{ kJ}$$

(ii) From the given condition, we have

$$P_1V_1^n = P_2V_2^n$$

or $$1000 \times 0.1^{1.4} = 200 \times V_2^{1.4}$$

or $$V_2 = 0.3157 \text{ m}^3$$

The work done is found from equation (2.7) to be

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{1000 \times 0.1 - 200 \times 0.3157}{1.4 - 1} = 92.15 \text{ kJ}$$

Example 2.6 A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m^3 to 0.4 MPa, 0.03 m^3. Assuming that the pressure and volume are related by PV^n = constant, find the workd done by the gas system.

Solution

Initial pressure $P_1 = 80 \text{ kPa} = 80 \times 10^3 \text{ Pa}$

Initial volume $V_1 = 0.1 \text{ m}^3$

Final pressure $P_2 = 0.4 \text{ MPa} = 0.4 \times 10^6 \text{ Pa}$

Final volume $V_2 = 0.03 \text{ m}^3$

The polytropic index of compression is found from Eq. (2.6) as

$$n = \frac{\ln\frac{P_1}{P_2}}{\ln\frac{V_2}{V_1}} = \frac{\ln\frac{80}{0.4 \times 10^3}}{\ln\frac{0.03}{0.1}} = 1.337$$

The work done is found from equation (2.7) to be

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{80\times10^3 \times 0.1 - 0.4\times10^6 \times 0.03}{1.337-1}$$

$$= -11869.44 \text{ J} = -11.869 \text{ kJ}$$

Negative sign indicates that work is done on the gas.

Example 2.7 A piston–cylinder device with air at an initial temperature of 30°C undergoes an expansion process for which pressure and volume are related as given below:

P(kPa)	100	37.9	14.4
V(m^3)	0.1	0.2	0.4

Calculate the work done by the system.

Solution Let the expansion process be represented by a polytropic process

$$P_1V_1^n = P_2V_2^n$$

where n is the polytropic index of expansion.

The polytropic index of expansion is found from Eq. (2.6) as

$$n = \frac{\ln\frac{P_1}{P_2}}{\ln\frac{V_2}{V_1}}$$

$$n = \frac{\ln\frac{100}{37.9}}{\ln\frac{0.2}{0.1}} = 1.4$$

$$n = \frac{\ln\frac{37.9}{14.4}}{\ln\frac{0.4}{0.2}} = 1.4$$

The process obeys the relation $PV^{1.4}$ = constant.

The work done is found from Eq. (2.7) to be

$$W_{1-3} = \frac{P_1V_1 - P_3V_3}{n-1}$$

$$= \frac{100\times0.1 - 14.4\times0.4}{1.4-1} = 10.6 \text{ kJ}$$

Example 2.8 A piston–cylinder device contains 1 kg of fluid at 20 atmospheric pressure. The initial volume is 0.04 m^3. The fluid is allowed to expand reversibly following a process $PV^{1.45} = C$ so that the volume becomes double. The fluid is then cooled reversibly at constant pressure until the piston comes back to the original position. Keeping the position of the piston unaltered, heat is added reversibly to restore the initial pressure. Calculate the cyclic work done. Plot the process in P–V coordinates, given that 1 atmospheric pressure = 101.325 kN/m^2.

Solution The processes are shown in Fig. 2.6.

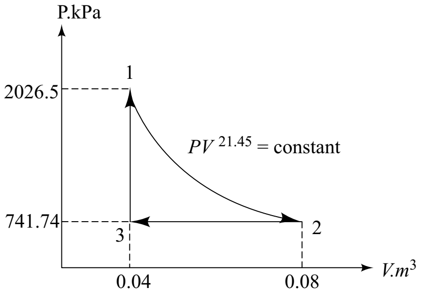


Figure 2.6

From the given data, P_1 = 20 atm = 20 × 101.325 = 2026.5 kN/m^2

$$V_1 = 0.04 \text{ m}^3;\ V_2 = 2\ V_1 = 0.08 \text{ m}^3,$$

$$P_1V_1^n = P_2V_2^n$$

$$\therefore \quad P_2 = \left(\frac{V_1}{V_2}\right)^n \times P_1 = \left(\frac{1}{2}\right)^{1.45} \times 2026.5 \text{ kPa} = 741.74 \text{ kPa}.$$

The work done during the polytropic expansion 1-2 is

$$\therefore \quad W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{2026.5 \times 0.04 - 741.74 \times 0.08}{1.45 - 1} \text{ kJ}$$

$$= 48.27 \text{ kJ}$$

The work done during the constant pressure process 2-3 is

$$W_{2-3} = P_2(V_3 - V_2) = P_2(V_1 - V_2) = 741.74\ (0.04 - 0.08) \text{ kJ}$$

$$= -\ 29.67 \text{ kJ}$$

Since the volume does not change during the process 3-1, the work done $W_{3\text{-}1}$ is zero.

The cyclic work done is $\sum W = W_{1-2} + \mathrm{W}_{2-3} + W_{3-1} = 48.27 - 29.67 + 0 = 18.6 \text{ kJ}$

Example 2.9 An ideal gas is heated at a constant volume until its temperature is doubled and then cooled at constant pressure until it is returned to the original temperature. Finally, the gas is allowed to expand at constant temperature until it is returned to the original volume. Derive an expression for net work done by the gas.

Solution The processes are shown in Fig. 2.7.

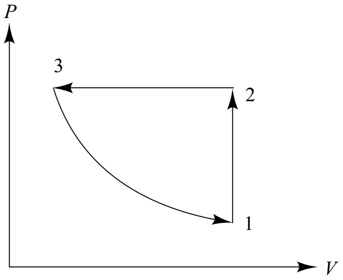


Figure 2.7

It is given that $T_2 = 2T_1$

For the constant volume process 1-2, we have

$$\frac{P_1}{P_2} = \frac{mRT_1}{mRT_2} = \frac{1}{2} \qquad [\because T_2 = 2T_1]$$

Work done for the constant volume process 1-2 is $W_{1-2} = 0$

It is further given that $T_3 = T_1$ and $P_3 = P_2$

From the characteristics equation of state for an ideal gas, we can write

$$P_1V_1 = mRT_1$$

$$P_2V_2 = mRT_2 = 2mRT_1 \qquad [\because T_2 = 2T_1]$$

$$P_3V_3 = mRT_3 = mRT_1 \qquad [\because T_3 = T_1]$$

The work done for the constant pressure process 2-3 is

$$W_{2\ 3} = P_2(V_3 - V_2) = P_3V_3 - P_2V_2$$

$$= mRT_1 - 2mRT_1 = -mRT_1$$

For the isothermal process 3-1, we have

$$P_3V_3 = P_1V_1$$

or

$$\frac{V_1}{V_3} = \frac{P_3}{P_1} = \frac{P_2}{P_1} = 2 \qquad [\because P_3 = P_2]$$

The work done for the process 3-1 is found from Eq. (2.7) to be

$$W_{3-1} = P_3 V_3 \ln \frac{V_1}{V_3}$$

$$= mRT_1 \ln 2 = 0.6931 mRT_1$$

$$\text{Total work done} \quad = -mRT_1 + 0.6931 mRT_1 = -0.3069 mRT_1$$

2.10 DIFFERENT PROCESSES IN THE PROCESS-DIAGRAM

It is important to mention here that the quasi-equilibrium processes presented in this chapter can be expressed as

$$PV^n = \text{constant}$$

Each quasi-equilibrium process is associated with a particular value for n as follows:
Constant pressure: $n = 0$
Constant temperature: $n = 1$
Adiabatic : $n = \gamma$
Constant volume : $n = \infty$
The processes are shown on P–V diagram in Fig. 2.8.

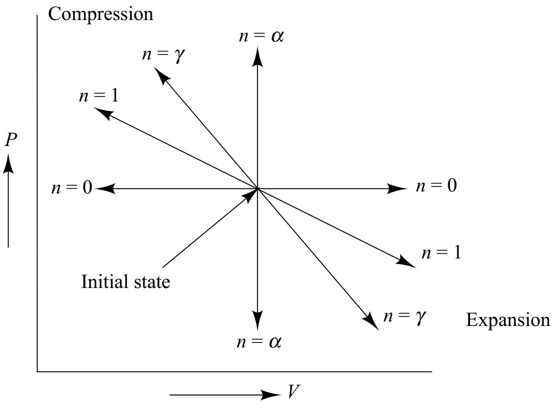


Figure 2.8 *Different processes on a P–V diagram*

Starting from the initial state the lower right quadrant shows the expansion processes and the upper left quadrant shows the compression processes.

The important observations from the process diagram are summarized below:

(i) As the value of polytropic index increases, the area included by the curve in the P-V diagram decreases, hence the work done by the process decreases.

(ii) As the value of polytropic index increases, the process curve comes closer to y-axis.

The larger the value of *n*, the more nearly does the polytropic curve approach the vertical line representing the constant volume process. This can be demonstrated by differentiating $PV^n = \text{constant}$ to give

$$V^n dP + PnV^{n-1} dV = 0$$

or, $$\frac{dP}{dV} = n\frac{P}{V}$$

Thus the slope of the curve increases in the negative direction with increase of *n*.

2.11 WORK TRANSFER: NOT A PROPERTY OF A SYSTEM

Let the system be taken from the state 1 to the state 2 by different quasi-equilibrium paths such as 1-*a*-2, 1-*b*-2 or 1-*c*-2 (refer Fig. 2.9). Since the area under the curve on the process diagrams represents the quasi-equilibrium work done and the area under the curve is different for different paths, so the work done is different for each path. The work done in a process depends not only on the initial and final states, but also the path followed by a system during a change of state, i.e., work transfer is a path function and not a property of a system.

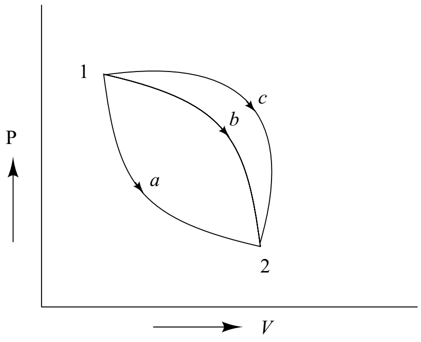


Figure 2.9

2.12 OTHER MODES OF WORK

2.12.1 Flow Work

The flow work is the work required to make the fluid flow across the control volume. The flow work is significant only in a flow process.

Consider an element of fluid mass *m* and volume *V* (Fig. 2.10). If the fluid pressure is *P* and the cross-sectional area of the fluid element is *A*, the force applied on the fluid element by the imaginary piston is *F* = *PA*.

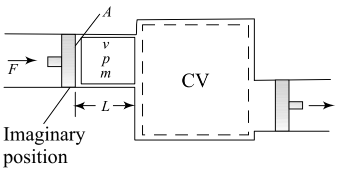


Figure 2.10 *Flow work*

To push the entire fluid element into the control volume, the force must act through a distance *L*. Thus, the flow work or the work done in pushing the fluid element across the boundary,

$$W_{\text{flow}} = F.L = PAL = PV$$

The flow work per unit mass

$$W_{\text{flow}} = Pv$$

2.12.2 Spring Work

When a force is applied on a spring, the length of the spring changes. When the length of the spring changes by a differential amount d*x* under the influence of a force *F*, the work done is

$$\delta W_{\text{spring}} = F\mathrm{d}x$$

For linear elastic springs, the displacement is proportional to the force applied. That is, $F = kx$, where *k* is spring constant.

$$\delta W_{\text{spring}} = kx\mathrm{d}x$$

Total work done necessary to stretch the spring from a length x_1 to x_2 is,

$$W_{\text{spring}} = \int_{x_1}^{x_2} Fdx = \int_{x_1}^{x_2} kxdx = \frac{1}{2}k\left(x_2^2 \quad x_1^2\right)$$

2.12.3 Electrical Work

Whenever there is a current flow through a resistor (taken as a system), work transfer takes place into the system. The potential difference is the force that drives the charge through the resistor. The current *I* is related to the charge by

$$I = \frac{dC}{dt}$$

where *C* is the charge and is the time. Thus, during the time interval *dt*, *dC*, is the charge that crosses the system boundary.

If *V* is the potential (voltage) difference, the non-equilibrium work done is

$$\begin{aligned}\delta W &= VdC \\ &= VIdt\end{aligned}$$

Total work done is

$$W = \int_1^2 VIdt$$

The electrical power (or the rate of doing work) will be

$$\dot{W} = \lim_{dt \to 0} \frac{\partial W}{\partial t} = VI$$

2.13 SOME EXCEPTIONAL CASES

2.13.1 Free Expansion Process—Situation in which $\int P\mathrm{d}V$ is Finite but Work Done is Zero

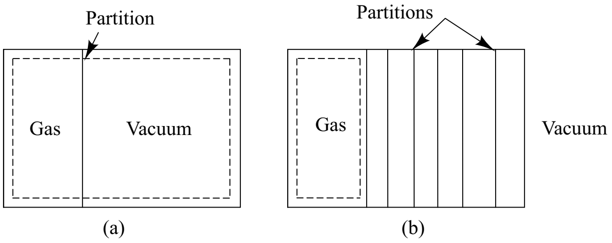
Let us consider a gas in a compartment with an initial pressure P_1 and volume V_1 that is separated from a vacuum. Let both the compartments be properly insulated so that heat transfer is zero. As the partition is removed, the gas rushes to fill the vacuum. The expansion of gas against a vacuum is known as ***free expansion***. Is any work done in the free expansion process?

Let us consider the gas and vacuum together as the system as shown in Fig. 2.11 (a) . We know that work transfer is a boundary phenomena, i.e., work transfer is identified only when work crosses the system boundary. During this free expansion, work done is zero since no work crosses the system boundary.

Let consider the gas in the compartment to be a system as shown in Fig. 2.11 (b). If the partition is removed, the volume of the system changes from V_1 to V_2. However, this is not a quasi-equilibrium process. Therefore, work cannot be calculated from the expression $\int_1^2 P\mathrm{d}V$.

If we divide the vacuum into a large number of small compartments by partitions and the partitions are removed slowly one by one then the process will be a quasi-equilibrium process. For that, work can be calculated from the expression $\int_1^2 P\mathrm{d}V$. However, $P_{\text{ext}} = 0$, i.e., external pressure is zero at the system boundary. No work is done in the process, since external pressure is zero.

Therefore, free expansion is an example of an expansion process in which $\int P\mathrm{d}V$ is finite but work transfer is zero.



(a) (b)

Figure 2.11 *Free expansion*

2.13.2 Paddle-Wheel Work—Situation in which $\int P\mathrm{d}V$ is Zero but Work Done is Finite

Consider a fixed mass of fluid in a rigid and insulated vessel as a system as shown in Fig. 2.12.. The vessel is provided with a paddle wheel. As the paddle wheel runs, work is done on the system. It increases the stored energy of the system. The temperature of the fluid increases as the work is done on the system. The system changes its state from 1 to 2. But there is no movement of the system boundary. Hence $\int P\mathrm{d}V$ is zero, although work has been done on the system. This form of work can be done in one direction only, that is, work is done on the system by the surroundings.

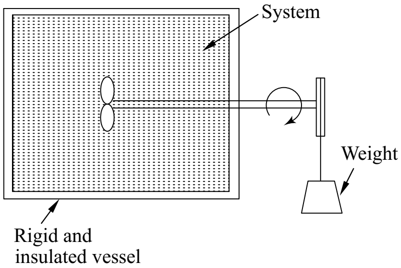


Figure 2.12 *Paddle-wheel work*

SUMMARY

- Heat is the form of energy transferred across the boundaries of a system because of temperature difference between a system and its surroundings, and in the direction of higher temperature to lower temperature.
- A process during which there is no heat transfer is called an adiabatic process.
- Work in the thermodynamic sense is defined as the energy transformed because of a property difference other than temperature difference. Thermodynamic work is also defined as follows: "Work is transferred from the system during a given operation if the sole effect of the system behaviour can be reduced to lifting a weight".
- P–dV work or displacement work, or moving boundary work, is a form of mechanical work which is associated with the expansion or compression of a fluid in a piston–cylinder device.
- When the piston moves out from position 1 to 2 with the volume changing from V_1 to V_2, the amount of displacement work done by the system can be expressed as $W_{1-2} = \int_{V_1}^{V_2} P\mathrm{d}V$.
- The expression for displacement work done for different processes are given as follows:

$$\text{Isobaric process: } W_{1-2} = P(V_2 - V_1)$$

$$\text{Isochoric process: } W_{1-2} = 0$$

$$\text{Isothermal process: } W_{1-2} = P_1V_1 \ln\frac{V_2}{V_1} = P_1V_1\ln\frac{P_1}{P_2} = P_1V_1(\ln P_1 - \ln P_2)$$

$$\text{Polytropic process: } W_{1-2} = \frac{P_2V_2 \quad P_1V_1}{1 \quad n} = \frac{P_1V_1 \quad P_2V_2}{n-1}$$

$$\text{Adiabatic process: } W_{1-2} = \frac{P_1V_1 \quad P_2V_2}{\gamma \quad 1}$$

- The flow work is the work required to make the fluid flow across the control volume.

$$W_{\text{flow}} = F.L = PAL = PV$$

- Free expansion is a process in which $\int P\mathrm{d}V$ is finite but work done is zero
- Paddle-wheel work is an example in which $\int P\mathrm{d}V$ is zero but work done is finite.

REVIEW QUESTIONS

2.1 What is a polytropic process?

2.2 Derive an expression for displacement work in a process where PV^n = constant

2.3 Prove that for a polytropic process, $W_{1-2} = \dfrac{P_2V_2 \quad P_1V_1}{1 \quad n}$.

2.4 Show that work done in isothermal process from the state 1 to the state 2 is given by

$$W_{1-2} = P_1V_1(\ln P_1 - \ln P_2)$$

2.5 What do you understand by path function and point function?

2.6 Show that work transfer is not a property of a system.

2.7 Show that heat transfer is not a property of a system.

2.8 Why does free expansion have zero work transfer?

2.9 What do you understand by flow work?

NUMERICAL PROBLEMS

2.1 Air of 0.02 m^3 at 200 kPa and 30 C is compressed to a volume of 0.002 m^3 according to the law $PV^{1/3}$ = constant. What is the final temperature and work done during compression?

2.2 1 kg of fluid initially at 6 bar with 0.01 m^3 volume undergoes the following operations.

(i) reversible expansion to volume 0.05 m^3 and pressure 2 bar according to a linear law
(ii) reversible cooling at constant pressure
(iii) reversible compression according to law PV = constant.

This brings the fluid back to initial conditions of 6 bar and 0.01 m^3.

Calculate (a) work done in each process. State whether the work is done on or by the fluid, and (b) net-work of the cycle.

2.3 Helium contained in a cylinder fitted with a piston expands reversibly according to the law PV = constant. The initial pressure, temperature and volume are 5 bar, 222 K and 0.055 m^3. After expansion, the pressure is 2 bar, calculate the work done during the process.

2.4 A gas is contained in a cylinder fitted with a piston loaded with a small number of weights. The initial pressure of the gas is 1.3 bar, and the initial volume is 0.03 m^3. The gas is now heated until the volume of the gas increases to 0.1 m^3. Calculate the work done by the gas in the following processes
(a) pressure remains constant
(b) temperature remains constant
(c) pv = constant during the process.
Show the processes on P-V diagram.

2.5 A system contains 0.15 m^3 of air at 3.8 bar and 150°C. A reversible adiabatic expansion takes places till the pressure falls to 1.03 bar. Determine the total work done.

2.6 Air at 300°C and 10 bar expands to 3 bar reversibly following the law $PV^{1.35}$ = constant. Determine the work done per kg of air if C_p = 1kJ/kg-K and C_v = 0.714 kJ/kg-K.

2.7 Three kg of air undergoes a three-process cycle as shown in Fig. 2.13. Calculate the network.

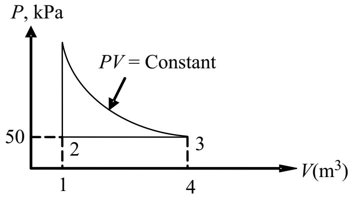


Figure 2.13

2.8 The force necessary to compress a linear spring is given by $F = 10x$ N, where x is the distance the spring is compressed, measured in m. Calculate the work required to compress the spring from 0.3 to 0.6 m.

MULTIPLE-CHOICE QUESTIONS

2.1 Work done in an adiabatic process between a given pair of end states depends on
(a) the end states only
(b) particular adiabatic process
(c) the value of index n
(d) mass of the system

2.2 Which of the following is true for reversible polytropic process
(a) temperature remains constant
(b) entropy remains constant
(c) some heat transfer takes place
(d) internal energy remains constant

2.3 Maximum work done by an expansion of a gas in a closed system is possible when process takes place at constant
(a) pressure
(b) temperature
(c) volume
(d) entropy

2.4 A diathermic wall is one in which
(a) prevents thermal interaction
(b) permits thermal interaction
(c) encourages thermal interaction
(d) discourage thermal interaction

2.5 An adiabatic wall is one in which
(a) prevents thermal interaction
(b) permits thermal interaction
(c) encourages thermal interaction
(d) discourages thermal interaction

2.6 Which of the following have the same unit?
(a) work and power
(b) work and energy
(c) power and energy
(d) all of these

2.7 In a closed system, a gas undergoes a reversible process as per the law $P = (-4V + 10)$ N/m^2 and the volume of the gas, V changes from 1 m^3 to 2 m^3. The work done will be

(a) 10 J output (b) 4 Joutput

(c) 6 J input (d) 4 J input

2.8 Work done in a free expansion process is

(a) positive (b) negative

(c) zero (d) maximum

2.9. In free expansion process

(a) $\int \partial W \neq 0, \int pdV = 0$ (b) $\int \partial W = 0, \int pdV = 0$

(c) $\int \partial W = 0, \int pdV \neq 0$ (d) $\int \partial W \neq 0, \int pdV \neq 0$

2.10. Isochoric process is one in which

(a) free expansion takes place

(b) no mechanical work is done by the system

(c) very little mechanical work is done by the system

(d) all parameters remain constant

2.11 Polytropic index n is given by

(a) $\ln P_2/P_1/\ln V_1/V_2$ (b) $\ln P_1/P_2/\ln V_1/V_2$

(c) $\ln V_1/V_2/\ln P_2/P_1$ (d) $\ln V_2/V_1/\ln P_2/P_1$

CHAPTER

3

First Law of Thermodynamics

3.1 JOULE'S EXPERIMENT

A series of experiments carried out by the English Scientist J. P. Joule between 1843 to 1848, forms the basis for the first law of thermodynamics. In Joule's paddle-wheel experiment, a known amount of water was taken in a rigid and insulated vessel as shown in Fig. 3.1. The vessel was provided with a paddle-wheel driven by a free-falling mass *m* by means of a pulley. Work was done on the system by lowering the mass *m* through a distance *h*. The work done on the system was computed in terms of the change in the potential energy of the mass *m*. The thermometer, immersed in the water, was used to determine the change in the state of the system. The temperature of the water was found to increase after work had been performed on the system. Once the system had come to rest, it was brought into contact with a water bath and the system was allowed to come to the initial state. The amount of energy transferred as heat from the system to the bath was estimated in terms of the temperature rise of the bath. The system underwent a series of processes that constitute a cycle.

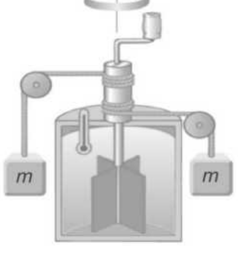


Figure 3.1 *Schematic arrangement of Joule's paddle-wheel experiment*

By repeating the experiment for different systems and for different amounts of work interactions and measuring the corresponding amounts of heat transfers in each case for bringing the system back to the initial state, Joule found that the net work input was always proportional to the net amount of heat transfer from the system measured in their conventional units. It is now found that the two were equal in magnitude when expressed in the same units. It is concluded that whenever a system undergoes a cyclic change, the algebraic sum of the work transfers is equal to the algebraic sum of the energy transfers as heat.

3.2 FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics, also known as the principle of conservation of energy, provides a basis for the study of the relationship among the various forms of energy and energy transformation. Based on the experimental observations, ***the first law of thermodynamics states that energy can neither be created nor destroyed, it can only change forms. In other words, during an interaction between a system and its surroundings, the amount of energy gained by the system is exactly equal to the amount of energy lost by the surroundings.***

For a closed system or a fixed mass, the first law may be expressed as follows:

Net energy transferred to (or from) the system as heat and work=Net increase (or decrease) in the total energy of the system

$$\begin{aligned} Q - W &= \Delta E \\ &= \Delta U + \Delta KE + \Delta PE + \ldots \end{aligned} \tag{3.1}$$

Neglecting the effects of magnetic and electric fields and surface tension, the above equation reduces to

$$Q - W = \Delta U + \Delta KE + \Delta PE \tag{3.2}$$

For a stationary closed system (the extrinsic effects on a closed system, such as the effects of motion and gravity, are neglected)

$$Q - W = \Delta U \tag{3.3}$$

Eq. (3.3) can be written in differential form as

$$\delta Q - \delta W = \mathrm{d}U \tag{3.4}$$

And per unit mass basis, Eqs (3.3) and (3.4) become

$$q - w = u \tag{3.5}$$

$$\delta q - \delta w = \mathrm{d}u \tag{3.6}$$

Internal energy is defined as the sum of all the microscopic forms of energy of a system. It is the energy associated with the molecular structure, and the molecular activity of the constituent particles of the system. It may be viewed as the sum of the kinetic and potential energies of the molecules.

3.2.1 First Law of Thermodynamics for a Closed System Undergoing a Cycle

If a system executes a cycle transferring heat and work through its boundary, the net work transfer is equivalent to the net heat transfer.

Mathematically,

$$\oint \delta W = \oint \delta Q \tag{3.7}$$

3.2.2 First Law of Thermodynamics for a Non-flow, Non-cyclic Process

The net algebraic sum of heat and work during a process is equal to the change in internal energy during the same process.

Mathematically,

$$\delta Q - \delta W = \mathrm{d}U \tag{3.8a}$$

Since for a quasi-equilibrium process, $\delta W = PdV$, Eq. (3.8a) becomes

$$\delta Q - P\mathrm{d}V = \mathrm{d}U \tag{3.8b}$$

Example 3.1 A system undergoes a cycle composed of four processes, 1–2, 2–3, 3–4 and 4–1. The rate of energy transfers are tabulated below:

Process	*Q(kW)*	*W(kW)*	*ΔU(kW)*
1–2	400	150	*A*
2–3	200	*B*	300
3–4	–200	*C*	*D*
4–1	0	75	*E*

(i) Calculate the value of *A*, *B*, *C*, *D* and *E*.

(ii) Determine the rate of work in kW.

Solution For process 1–2,

$$Q_{1-2} = \Delta U + W_{1-2}$$
$$400 = A + 150$$
$$A = 250 \text{ kW}$$

For process 2–3,

$$Q_{2-3} = \Delta U + W_{2-3}$$
$$200 = 300 + B$$
$$B = -100 \text{ kW}$$

For process 4–1,

$$Q_{4-1} = \Delta U + W_{4-1}$$
$$0 = E + 75$$
$$E = -75 \text{ kW}$$

Now,

$$\oint U = 0$$

$$A + 300 + D + E = 0$$
$$250 + 300 + D - 75 = 0$$

$$D = -\,475 \text{ kW}$$

For process 3–4, $Q_{3-4} = \Delta U + W_{3-4}$

$$-\,200 = D + C$$

$$-\,200 = -\,475 + C$$

$$C = 275 \text{ kW}$$

Since $\oint \delta Q = \oint \delta W$

Net cyclic work done $= 400 + 200 - 200 + 0 = 400$ kW

Negative sign indicates that heat is lost from the system.

Example 3.2 A system undergoes a process 1-2 in which it absorbs 200 kJ energy as heat while it does 100 kJ work. Then it follows path 2-3 in which it rejects 50 kJ energy as heat when 80 kJ work is done on it. If it is required to restore the system to state 1 through an adiabatic path, calculate the work and heat transfer along the adiabatic path. Also calculate net heat transfer.

Solution From the given data, we have

$$Q_{1-2} = 200 \text{ kJ}\,,\ W_{1-2} = 100 \text{ kJ}\,,$$

$$Q_{2-3} = -50 \text{ kJ}\,,\ W_{2-3} = -80 \text{ kJ}$$

$$Q_{3-1} = 0$$

The processes 1-2, 2-3 and 3-1 together constitute a cycle. Thus from Eq. (3.7), we have

$$\sum Q = \sum W$$

or, $Q_{1-2} + Q_{2-3} + Q_{3-1} = W_{1-2} + W_{2-3} + W_{3-1}$

or, $200 - 50 + 0 = 100 - 80 + W_{3-1}$

or, $W_{3-1} = 130$ kJ W_{31}=130 kJ

Net heat transfer is $Q_{1-2} + Q_{2-3} + Q_{3-1} = 200 - 50 + 0 = 150$ kJ

Example 3.3 One kg of fluid initially at 1000 kPa and 0.2 m^3 undergoes a quasi-equilibrium expansion to 200 kPa and 1.2 m^3 according to a linear relationship between pressure and volume. The internal energy of the fluid is given by the relation

$$U = 2PV + 45 \text{ kJ}$$

where P is in kPa and V is in m^3. Calculate the net work done, heat transfer, and the change in internal energy.

Solution The expansion takes place according to a linear law. Let the pressure and volume relationship be $P = aV + b$

where a and b are constants and can be evaluated from the initial and final conditions as follows

$$1000 = 0.2a + b$$
$$200 = 1.2a + b$$

After solving the above two equations, we get

$$a = -800,\ b = 1160$$

Thus, the pressure and volume relationship can be written as

$$P = -800V + 1160$$

The work done for the quasi-equilibrium expansion is found to be

$$W_{1-2} = \int_{V_1}^{V_2} PdV$$

$$= \int_{0.2}^{1.2} (-800V + 1160)\,dV$$

$$= \left| -800\frac{V^2}{2} + 1160V \right|_{0.2}^{1.2}$$

$$= -400\left(1.2^2 - 0.2^2\right) + 1160(1.2 - 0.2)$$

$$= 600 \text{ kJ}$$

Change in internal energy is

$$U_2 - U_1 = (2PV + 45)|_2 - (2PV + 45)|_1 = 2(P_2V_2 - P_1V_1)$$

$$= 2(200 \times 1.2 - 1000 \times 0.2)$$

$$= 80 \text{ kJ}$$

Heat transfer is then

$$Q_{1\text{-}2} = U_2 - U_1 + W_{1\text{-}2}$$

$$= 80 + 600 = 680 \text{ kJ}$$

Example 3.4 A domestic refrigerator is loaded with food and the door is closed. During a certain period the refrigerator consumes 1kWh of energy and the internal energy of the system drops by 5000 kJ. Find the net heat transfer for the system.

Solution From the given data, we have

$$\Delta U = -5000 \text{ kJ}$$

$W = -1$ kWh $= -3600$ kJ (According to sign convention of work transfer, energy consume by the refrigerator is negative)

From first law of thermodynamics, we have

$$Q = \Delta U + W$$

$$= -5000 - 3600 = -8600 \text{ kJ} = -8.6 \text{ MJ}$$

Negative sign indicates that the heat transfer takes place from the refrigerator to the surroundings.

3.3 COROLLARIES OF THE FIRST LAW OF THERMODYNAMICS

Corollary-1 There exists a property of a closed system such that a change in its value is equal to the difference between heat supplied and work done during the change of state.

Proof

Let the system be taken from state 1 to state 2 by the two different processes 1-a-2 and 1-b-2 as shown in Fig. 3.2.

Let us consider,

$$(\delta Q - \delta W)_a \neq (\delta Q - \delta W)_b \tag{3.9}$$

where δQ the net heat is supplied to the system and δW is the net work done by the system during a process. Let the system be taken from state 2 to 1 through path 2-c-1.Now the processes 1-a-2 and 2-c-1 together constitute a cycle. From the first law of thermodynamics for a cyclic process, we can write

$$\oint \delta Q = \oint \delta W$$

$$\delta Q_a + \delta Q_c = \delta W_a + \delta W_c$$

$$(\delta Q - \delta W)_a = -(\delta Q - \delta W)_c \tag{3.10}$$

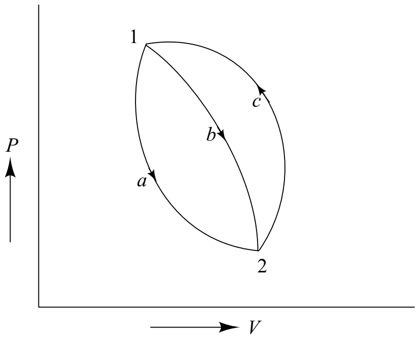


Figure 3.2

Similarly, the processes 1-a-2 and 2-c-1 together constitute a cycle for which the first law of thermodynamics becomes

$$\oint \delta Q = \oint \delta W$$

$$\delta Q_b + \delta Q_c = \delta W_b + \delta W_c$$

$$(\delta Q - \delta W)_b = -(\delta Q - \delta W)_c \tag{3.11}$$

If inequality (3.9) is true, then Eqs (3.10) and (3.11) contradict each other which implies that these quantities must be equal. Therefore $(\delta Q - \delta W)$ is independent of the path the system follows during a change of state. If the property denoted by U, the corollary can be expressed mathematically as $\delta Q - \delta W = dU$.

The property U is called the internal energy of the system. The adjective “internal” has been adopted to distinguish this form of energy from the mechanical forms of energy.

Corollary-2 The internal energy of a closed system remains unchanged if the system is isolated from the surroundings.

Proof

A system which exchanges neither mass nor energy with the surroundings is called an isolated system. It is thus a closed system having no energy interaction ($\delta Q = 0$, $\delta W = 0$) with the surroundings.

Then the first law of thermodynamics in differential form becomes

$$dU = 0$$

or,

$$U = \text{Constant}$$

Figure 3.3 shows an isolated system. All that happens during a process is a spontaneous redistribution of energy between parts of the system which continues until a state of equilibrium is reached. There is no change in the total energy within the system during the process.

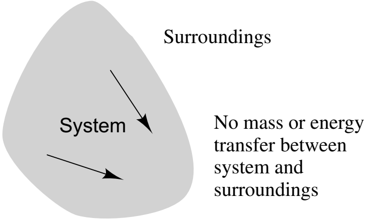


Figure 3.3 *An isolated system*

Corollary-3 A perpetual motion machine of first kind is impossible.

Proof A hypothetical device which would produce work continuously without absorbing any energy from its surroundings is called a perpetual motion machine of the first kind. A perpetual motion machine of the first kind must operate on a cycle to produce work continuously. If it does not operate on a cycle, its state would change continuously and it could not go on indefinitely. For such a device there cannot be any energy transfer in the form of heat from the surroundings, hence

$\oint \delta Q = 0$. Therefore, $\oint \delta W = 0$, that means the work delivered by it is zero. Therefore, it is impossible to construct a perpetual motion machine of the first kind. *A perpetual motion machine of the first kind is a machine which violates the first law of thermodynamics.*

It is always possible to devise a machine to deliver a limited (certain) quantity of work without requiring a source of energy in the surroundings. For example, a compressed gas in a piston-cylinder arrangement will expand and do work at the expense of the internal energy of the gas. Such a device can not produce work continuously.

3.4 HEAT TRANSFER: NOT A PROPERTY OF A SYSTEM

Let the system be taken from state 1 to state 2 by the two different processes 1-a-2 and 1-b-2. Let the system be taken from state 2 to 1 through path 2-c-1.Now the processes 1-a-2 and 2-c-1 together constitute a cycle.

Applying the first law of thermodynamics for the cyclic process

$$\oint \delta Q = \oint \delta W$$

$$\int_{1-a-2} \delta Q + \int_{2-c-1} \delta Q = \int_{1-a-2} \delta W + \int_{2-c-1} \delta W \tag{3.12}$$

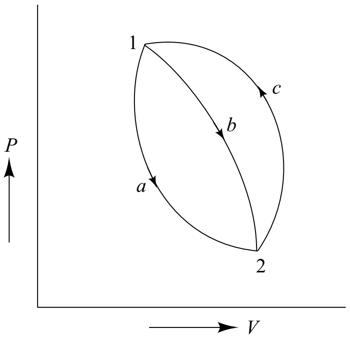


Figure 3.4

Similarly, the processes 1-b-2 and 2-c-1 together constitute a cycle for which

$$\int_{1-b-2} \delta Q + \int_{2-c-1} \delta Q = \int_{1-b-2} \delta W + \int_{2-c-1} \delta W \tag{3.13}$$

Subtracting Eq. (3.13) from Eq. (3.12), we get

$$\int_{1-a-2} \delta Q - \int_{1-b-2} \delta Q = \int_{1-a-2} \delta W - \int_{1-b-2} \delta W \tag{3.14}$$

Since work transfer is a path function,

$$\int_{1-a-2} \delta W - \int_{1-b-2} \delta W \neq 0 \tag{3.15}$$

Therefore,

$$\int_{1-a-2} \delta Q - \int_{1-b-2} \delta Q \neq 0 \tag{3.16}$$

That is the heat transfer in a process depends not only the initial and final states, but also the path followed by system during a change of state, i.e., *heat transfer is a path function and not a property of a system.*

3.5 SPECIFIC HEATS, INTERNAL ENERGY AND ENTHALPY

The specific heat or heat capacity of a substance is the amount of heat required to change temperature of the unit mass of the substance by one degree. Denoting by C, from definition, we have

$$C = \frac{1}{m}\left(\frac{\delta Q}{\delta T}\right) = \frac{\delta q}{\delta T}$$

The unit of specific heat in SI system is J/kg-K (or kJ/kg-K). In general, this energy needed to change the temperature depends on how the process is executed. In thermodynamics, we are interested in two kinds of specific heats, namely specific heat at constant volume, C_v and specific heat at constant pressure, C_p.

Physically, the constant volume specific heat, C_v is defined as the energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant. The energy required to do the same as the pressure is maintained constant is the specific heat at constant pressure, C_p. *The C_p is always greater than C_v because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system.* Both C_v and C_p can be expressed in terms of thermodynamic properties as discuss in the next Section for an ideal gas (Section 3.5.1).

According to the state -postulate we know that the state of a simple compressible substance is completely specified by two independent, intensive properties. Consider the specific internal energy to be a function of temperature and specific volume, that is

$$u = u(T, v) \tag{3.17}$$

Using the chain rule, we can write

$$du = \left.\frac{\partial u}{\partial T}\right|_v dT + \left.\frac{\partial u}{\partial v}\right|_T dv \tag{3.18}$$

The *constant-volume specific heat*, C_v is defined as

$$C_v = \left.\frac{\partial u}{\partial T}\right|_v \tag{3.19}$$

It has been demonstrated mathematically and experimentally that for an ideal gas both the internal energy and enthalpy is a function of the temperature only. That is for an ideal gas

$$\left.\frac{\partial u}{\partial v}\right|_T = 0 \tag{3.20}$$

Combining Eq. (3.18), (3.19) and (3.20), we have

$$du = C_v dT \tag{3.21}$$

Now, consider a constant-pressure quasi-equilibrium process for which the work done between two end states 1 and 2 is given by

$$W_{1-2} = P(V_2 - V_1) \tag{3.22}$$

Then, the first of thermodynamics for the process can be expressed in the form

$$\begin{aligned} Q_{1-2} &= U_2 - U_1 + W_{1-2} \\ &= U_2 - U_1 + P_2V_2 - P_1V_1 \\ &= (U + PV)_2 - (U + PV)_1 \end{aligned}$$

The quantity in the parentheses occurs frequently in thermodynamics, and is called the enthalpy, H of the system, that is

$$H = U + PV \tag{3.23}$$

It is important to note that *the enthalpy is a property of the system since it is a combination of the properties.*

The specific enthalpy, h is found by dividing Eq. (3.23) by the mass. Thus

$$h = u + Pv \tag{3.24}$$

The first of thermodynamics for a constant-pressure quasi-equilibrium process is then

$$Q_{1-2} = H_2 - H_1 \tag{3.25}$$

Similar to internal energy, considering specific enthalpy to be a function of temperature and pressure, we have

$$dh = \left.\frac{\partial h}{\partial T}\right|_P dT + \left.\frac{\partial h}{\partial P}\right|_T dP \tag{3.26}$$

The *constant pressure specific heat*, C_p is defined as

$$C_p = \left.\frac{\partial h}{\partial T}\right|_P \tag{3.27}$$

Likewise internal energy, enthalpy of an ideal gas is also a function of the temperature only. That is for an ideal gas

$$\left.\frac{\partial h}{\partial P}\right|_T = 0 \tag{3.28}$$

Combining Eqs (3.26), (3.27) and (3.28), we have

$$dh = C_p dT \tag{3.29}$$

The change in specific internal energy and enthalpy can be found by integrating the Eqs (3.21) and (3.29) respectively. For an ideal gas with constant specific heats, one can write

$$u_2 - u_1 = C_v (T_2 - T_1) \tag{3.30}$$

$$h_2 - h_1 = C_p (T_2 - T_1) \tag{3.31}$$

3.5.1 Relation between C_p and C_v for an Ideal Gas

Equation (3.24) can be written in differential form as

$$dh = du + d(Pv) \tag{3.32}$$

From the specific heat relations for an ideal gas (Eqs (3.21)and (3.29)), we have

$$dh = C_p dT \text{ and } du = C_v dT$$

From the characteristic equation of state for an ideal gas, one can write

$$Pv = RT$$

or,

$$d(Pv) = RdT$$

Thus, from Eq. (3.32), one can write

$$C_p dT = C_v dT + RdT \tag{3.33}$$

Dividing by dT, Eq. (3.33) becomes

$$C_p = C_v + R \tag{3.34}$$

It is evident from Eq. (3.34) that although C_p and C_v for ideal gases are function of temperatures only, the difference between them is always a constant (since R is constant).

The specific heat ratio, γ is defined as

$$\gamma = \frac{C_p}{C_v} \tag{3.35}$$

From Eqs (3.34) and (3.35), we have

$$C_p = \frac{\gamma R}{\gamma - 1} \tag{3.36}$$

$$C_v = \frac{R}{\gamma - 1} \tag{3.37}$$

For air we will use $C_p = 1.005$ kJ/kg-K and $C_v = 0.718$ kJ/kg-K unless otherwise stated.

At low pressures all real gases approach ideal-gas behaviour, and therefore, their specific heats depend on temperature only. The specific heats of gases with complex molecules (molecules with two or more atoms) are higher and increase with temperature. The variation of specific heats with temperature is smooth and may be approximated as linear over small temperature intervals. The ideal-gas specific heats of monatomic gases such as argon, neon, and helium remain constant over the entire range of temperature.

It can be shown by the classical kinetic theory of gases that the values of γ are $\frac{5}{3}$ (1.67) for monatomic gases and $\frac{7}{5}$ (1.4) for diatomic gases. For polyatomic gases γ is $\frac{4}{3}$ (1.67).

3.6 FIRST LAW OF THERMODYNAMICS FOR DIFFERENT PROCESSES

A thermodynamic process can be defined as the evolution of a thermodynamic system encouraged by energy transfer that causes a change of state. Paths through the space of thermodynamic variables are often specified by holding certain thermodynamic variables constant. Here, we will discuss the analysis of first law of thermodynamics for different processes that are normally encountered in thermodynamics analysis of systems.

3.6.1 Constant Volume Process (Isochoric)

An **isochoric process** is one during which the volume remains constant, implying that the work done by the system will be zero. It therefore follows that any heat energy transferred to the system externally will be absorbed as internal energy. An isochoric process is also known as an **isometric** process or an **isovolumetric** process and is represented by a vertical line in the *P*-*V* diagram.

Consider a gas confined in a rigid vessel of volume *V* as the system. Since the vessel has rigid walls, the displacement work done by the system is zero. Let the system be brought into contact with a heat source so that it can exchange energy in a quasi-equilibrium manner.

First law of thermodynamics for the constant volume process in differential form becomes

$$\delta Q = dU \tag{3.38}$$

When the system changes its state from 1 to 2, Eq. (3.38) becomes

$$Q_{1-2} = U_2 - U_1 \tag{3.39}$$

Hence, the heat interaction is equal to the change in the internal energy of the system.

For constant C_v, we get

$$Q_{1-2} = mC_v(T_2 - T_1) \tag{3.40}$$

Example 3.5 Air is contained in a 1 m^3 rigid volume at 40°C and 200 kPa. Calculate the heat transfer needed to increase the pressure to 500 kPa. The C_v for air is constant and equal to 0.718 kJ/kg-°C.

Solution The mass of air (considering an ideal gas) is found to be

$$m = \frac{PV}{RT} = \frac{(200 \text{ kPa})(1 \text{ m}^3)}{(0.287\text{kJ/kg.K})(313\text{K})} = 2.226 \text{ kg}$$

The work is zero for this constant-volume process. Consequently, the first law of thermodynamics gives

$$Q = m\Delta u = mC_v\Delta T = mC_v(T_2 - T_1)$$

The ideal-gas law, $PV = mRT$, allows us to write

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

or, $$\frac{200}{313} = \frac{500}{T_2}$$

$$\therefore T_2 = 782.5 \text{ K}$$

The heat transfer is then found from Eq. (3.40) as

$$Q = (2.226 \text{ kg})(0.718 \text{ kJ/kg-K})(782.5 - 313) \text{ K} = 750.39 \text{ kJ}$$

3.6.2 Constant Pressure Process (Isobaric)

Consider a certain quantity of gas in a cylinder bounded by a piston as a system. Let the system undergoes a quasi-equilibrium constant-pressure heating process without changes in potential and kinetic energy. The only work done associated with the process is the displacement work done, which is

$$W_{1-2} = P(V_2 - V_1) \tag{3.41}$$

The heat transfer is found by applying the first of thermodynamics for a constant-pressure quasi-equilibrium process as (see Eq. (3.25))

$$Q_{1-2} = H_2 - H_1 \tag{3.42}$$

It is important to note that from Eq. (3.42) that the heat transfer in the constant pressure quasi-equilibrium process is equal to the change in enthalpy. It includes the change in internal energy and the work for the process.

For constant C_p, we get

$$Q_{1-2} = mC_p(T_2 - T_1) \tag{3.43}$$

Example 3.6 The gas in a system receives heat which causes expansion against a constant pressure of 4 bar. An agitator in the system is driven by an electric motor using 200 W. For 6 kJ of heat supplied, the volume increase of the system in 30 s is 0.05 m^3. Estimate net change in the energy of the system.

Solution The rate of work on the system through agitator is = –200 W = –200 J/s (According to the sign convention of work transfer, work done on the system is negative).

Thus, during the 30 s of operation the work done is $= -200 \times 30 = -6000 \text{ J} = -6 \text{ kJ}$

Quasi-equilibrium expansion work is

$$W_{1-2} = P(V_2 - V_1) = 4 \times 10^5 \times 0.05 = 20 \times 10^3 \text{ N-m} = 20 \text{ kJ}$$

Net work done is therefore = 20 – 6 =14 kJ

From first law of thermodynamics, we can write

$$Q_{1-2} = \Delta U + W_{1-2}$$

or, $6 = \Delta U + 14$

or, $\Delta U = -8$ kJ

Note that the negative sign indicates that the energy of the system decreases.

3.6.3 Constant Temperature Process (Isothermal)

Consider certain quantity of gas in a cylinder bounded by a piston as a system. The system is allowed to undergo a quasi-equilibrium expansion process while in contact with a constant temperature bath. Let the system change its state from 1 to 2. Applying the first law of thermodynamics for the path 1-2, we get

$$Q_{1-2} - W_{1-2} = \Delta U \tag{3.44}$$

Suppose the gas under consideration is an ideal gas. Since, for an ideal gas the internal energy is a function of temperature only, change in internal energy for constant temperature process is zero, that is $\Delta U = 0$.

Then the Eq. (3.44) becomes

$$Q_{1-2} = W_{1-2} \tag{3.45}$$

The heat transfer as well as the work transfer for a quasi- equilibrium process then can be found to be

$$Q_{1-2} = W_{1\ 2} = P_1V_1 \ln\frac{V_2}{V_1} = mRT_1 \ln\frac{V_2}{V_1} = mRT_1 \ln\frac{P_1}{P_2} \tag{3.46}$$

Example 3.7 An ideal gas undergoes a thermodynamic cycle consisting of the following quasi-equilibrium processes

(i) Process 1-2: Constant volume $U_2 - U_1 = 30$ kJ

(ii) Process 2-3: Isothermal expansion

(iii) Process 3-1: Constant pressure P=1 bar, $W_{3-1} = -10$ kJ, $V_3 = 0.2$ m^3

The changes of KE and PE are negligible. Sketch the cycle on the *P-V* diagram. Calculate the net work for the cycle. Also calculate the heat transfer for the process 3-1.

Solution The cycle on the *P-V* diagram is shown in Fig. 3.5

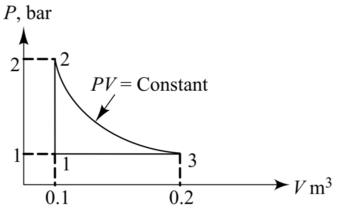


Figure 3.5

For the constant-volume process 1-2, the work done is zero that is $W_{1-2} = 0$
The first law of thermodynamics for the constant-volume process 1-2 is

$$Q_{1-2} = U_2 - U_1$$

or $$Q_{1-2} = 30 \text{ kJ}$$

For an ideal gas, isothermal expansion process 2-3 gives

$$U_2 = U_3$$

Work done for the isothermal expansion 2-3 is (Eq. (3.46))

$$W_{2\text{-}3} = P_2V_2 \ln\frac{V_3}{V_2} = P_3V_3 \ln\frac{V_3}{V_2}$$

$$= 1 \text{ bar} \times 0.2\text{m}^2 \times \ln\frac{0.2}{0.1} = 100\text{kPa} \times 0.2\text{m}^2 \times \ln\frac{0.2}{0.1} = 13.86 \text{ kJ}$$

Net work done for the cycle is $W_{net} = W_{1-2} + W_{2-3} + W_{3-1} = 0 + 13.86 - 10 = 3.86 \text{ kJ}$
The first law of thermodynamics for the constant-pressure process 3-1 is

$$Q_{3-1} = U_3 - U_1 + W_{3-1} = U_2 - U_1 + W_{3-1} \qquad [\because U_3 = U_2]$$

$$= 30 - 10 = 20 \text{ kJ}$$

For the constant-pressure process 3-1, we have

$$W_{3-1} = P(V_1 - V_3)$$

$$-10 = 100 \times (V_1 - 0.2)$$

or $$V_1 = 0.1 \text{ m}^3$$

For the isothermal process 2-3, we can write

$$P_2V_2 = P_3V_3$$

or $$P_2 = P_3\frac{V_3}{V_2} = 1 \times \frac{0.2}{0.1} = 2\text{bar}$$

3.6.4 Polytropic Process

Consider certain quantity of gas in a cylinder bounded by a piston as a system. The system is allowed to undergo a quasi-equilibrium expansion process in such a way that the functional relationship between pressure and volume during the expansion process follow the relation $PV^n = \text{constant}$. Such process in thermodynamics is called as polytropic process. For a polytropic process ($PV^n = \text{constant}$) between two end states 1 and 2, the work done is calculated as

$$W_{1-2} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{C}{V^n} dV \quad (\text{where } C = P_1V_1^{\,n} = P_2V_2^{\,n})$$

$$= C\frac{V_2^{-n+1} - V_1^{-n+1}}{1-n}$$

$$= \frac{CV_1^{-n+1} - CV_2^{-n+1}}{n-1}$$

$$= \frac{P_1V_1^nV_1^{-n+1} - P_2V_2^nV_2^{-n+1}}{n-1}$$

or,
$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1} \tag{3.47}$$

By applying the first law of thermodynamics to a quasi-equilibrium polytropic process between states 1 and 2, we get

$$Q_{1-2} = \Delta U + \int_1^2 PdV = (U_2 - U_1) + \frac{P_2V_2 - P_1V_1}{1-n}$$

$$Q_{1-2} = mC_v(T_2 - T_1) + \frac{P_2V_2 - P_1V_1}{1-n} \tag{3.48}$$

(since $(U_2 - U_1) = mC_v(T_2 - T_1)$ for an ideal gas with constant specific heats).

Using the ideal-gas equation $PV = mRT$, the Eq. (3.48) becomes

$$Q_{1-2} = mC_v(T_2 - T_1) + \frac{mR(T_2 - T_1)}{1-n}$$

$$= m\left(C_v + \frac{R}{1-n}\right)(T_2 - T_1)$$

Using the relationships $C_p - C_v = R$ and $\gamma = \frac{C_p}{C_v}$, it yields

$$Q_{1-2} = m\left(C_v + \frac{C_p - C_v}{1-n}\right)(T_2 - T_1)$$

$$Q_{1-2} = m\left(1 + \frac{\gamma - 1}{1-n}\right)C_v(T_2 - T_1) = m\left(\frac{1 - n + \gamma - 1}{1-n}\right)C_v(T_2 - T_1)$$

$$Q_{1-2} = m\left(\frac{\gamma - n}{1-n}\right)C_v(T_2 - T_1) \tag{3.49}$$

Equation (3.49) gives the heat transfer for a quasi-equilibrium polytropic process between states 1 and 2. From Eq. (3.49), specific heat, C_n, of an ideal gas during a quasi-equilibrium polytropic process can be expressed as

$$C_n = \left(\frac{\gamma - n}{1-n}\right)C_v \tag{3.50}$$

The functional relationship among temperature-volume and temperature-pressure for an ideal gas during a quasi-equilibrium polytropic process can be expressed as

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} \tag{3.51}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \tag{3.52}$$

(Equations (3.51) and (3.52) can be derived by using $P_1V_1^n = P_2V_2^n$, $P_1V_1 = mRT_1$ and $P_2V_2 = mRT_2$)

Note that Eq. (3.50) has been derived for an ideal gas with constant specific heats.

Example 3.8 A mass of 8 kg gas expands within a flexible container so that the *P*-*V* relationship is of the form $PV^{1.2}$ = constant. The initial pressure is 1000 kPa and the initial volume is 1 m^3. The final pressure is 5 kPa. If specific internal energy of the gas decreases by 40 kJ/kg, find the heat transfer in magnitude and direction.

Solution From the given data, we have

$P_1 = 1000$ kPa, $V_1 = 1$ m^3, and $P_2 = 5$ kPa

Final volume can be found as follows

$$P_1V_1^{1.2} = P_2V_2^{1.2}$$

or, $$V_2 = 82.7 \text{ m}^3$$

Work done during polytropic expansion is

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$= \frac{1000 \times 1 - 5 \times 82.7}{1.2 - 1}$$

$$= 2932.5 \text{ kJ}$$

Change in internal energy is $\Delta U = 8 \times (-40 \text{ kJ}) = -320 \text{ kJ}$ (Negative sign signifies that the energy decreases)

From first law of thermodynamics, we get

$$Q_{1-2} = \Delta U + W_{1-2}$$

$$= -320 + 2932.5 = 2612.5 \text{ kJ}$$

Since heat transfer is positive, heat is transferred to the gas.

Example 3.9 Air at 227°C and 800 kPa expands to 200 kPa in a quasi-equilibrium process following the law $PV^{1.3} = C$, where C is a constant. Determine the work done and heat transfer per kg of air. Specific heats of air are given as $C_p = 1.0$ kJ/kg-K and $C_v = 0.714$ kJ/kg-K ·

Solution Characteristic gas constant for air is found to be

$$R = C_p - C_v = 1 - 0.714 = 0.286 \text{ kJ/kg-K}$$

Ratio of specific heat is $\gamma = \dfrac{C_p}{C_v} = \dfrac{1}{0.714} = 1.4$

Initial temperature of air is $T_1 = 227°\text{C} = 500$ K

Final temperature of air is found to be (Eq. (3.52))

$$T_2 = T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 500\left(\frac{200}{800}\right)^{\frac{1.3-1}{1.3}} = 363.1 \text{ K}$$

Quasi-equilibrium work done is

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

$$= \frac{(1)(0.286)(500 - 363.1)}{1.3 - 1} = 130.51 \text{ kJ/kg}$$

Heat transfer during the process1-2 is found to be (Eq. (3.49))

$$Q_{1-2} = m\left(\frac{\gamma - n}{1 - n}\right)C_v(T_2 - T_1)$$

$$= 1\left(\frac{1.4 - 1.3}{1 - 1.3}\right)0.714(363.1 - 500) = 32.58 \text{ kJ/kg}$$

3.6.5 Adiabatic Process

Consider certain quantity of gas in a cylinder bounded by a piston as a system. The system is allowed to undergo an expansion process while the cylinder wall is insulated from all sides so that during the expansion process there is no heat transfer. For an adiabatic process, the first law of thermodynamics in differential form can be written as

$$-\delta W = dU \qquad (3.53)$$

For a quasi-equilibroum process, Eq. (3.53) becomes

$$-PdV = dU$$

or, $$PdV + dU = 0 \tag{3.54}$$

For an ideal gas ($PV = mRT$), Eq. (3.54) can be written as

$$\frac{mRT}{V} dV + mC_v dT = 0$$

or, $$\frac{C_v}{R}\frac{dT}{T} = -\frac{dV}{V} \tag{3.55}$$

Integrating Eq. (3.55) (assuming constant specific heats), we get

$$\frac{C_v}{R}\ln\frac{T_2}{T_1} = -\ln\frac{V_2}{V_1} = \ln\frac{V_1}{V_2}$$

or, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R}{C_v}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \tag{3.56}$$

Using the ideal gas law, Eq. (3.56) can be written as

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{V_2}{V_1} = \frac{P_2}{P_1}\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$$

or, $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{3.57}$$

Combining Eqs (3.56) and (3.57), we get

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \tag{3.58}$$

Example 3.10 One liter of hydrogen at 273 K is adiabatically compressed to one-half of it initial volume. Find the change in temperature of the gas, if the ratio of two specific heats for hydrogen is 1.4.

Solution The temperature-volume relationship between two states for an adiabatic process is found from Eq. (3.56) as

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (2)^{1.4-1}$$

or, $T_2 = 360.22$ K

The change in temperature is $\Delta T = T_2 - T_1 = 360.22 - 273 = 87.22$ K

Example 3.11 Three grams of air at 600 kPa and 160°C is expanded adiabatically to double of its initial volume, then compressed at constant pressure to its initial volume and then compressed it again at constant volume to its initial state. All the processes are quasi-equilibrium. Calculate the net work done on the gas. Sketch the processes on the *P-V* diagram.

Solution The processes on the *P-V* diagram are shown in Fig. 3.6.

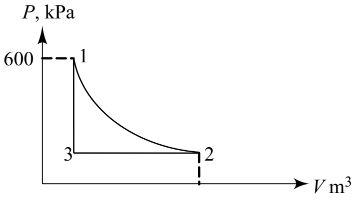


Figure 3.6

The temperature at state 2 is found from Eq. (3.56) as

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 433\times(0.5)^{1.4-1} = 328.15\text{ K}$$

The work done for the quasi-equilibrium adiabatic expansion process 1-2 is

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{\gamma-1} = \frac{mR(T_1-T_2)}{\gamma-1}$$

$$= \frac{0.003\times0.287\times(433-328.15)}{1.4-1} = 0.225\text{ kJ}$$

Work done for the quasi-equilibrium constant pressure process 2-3 is

$$W_{2-3} = P_2(V_3-V_2) = P_2(V_1-V_2) = P_2V_2\left(\frac{V_1}{V_2}-1\right) = mRT_2\left(\frac{V_1}{V_2}-1\right)$$

$$= P_2V_2\left(\frac{V_1}{V_2}-1\right) = mRT_2\left(\frac{V_1}{V_2}-1\right)$$

$$= 0.003\times0.287\times328.15\times(0.5-1) = -0.141\text{ kJ}$$

For the constant-volume process 3-1, $W_{3-1} = 0$

Thus, the net work done is $\oint dW = 0.225 - 0.141 = 0.084\text{ kJ}$

SUMMARY

- The first law of thermodynamics also known as the principle of conservation of energy, provides a basics for the study of the relationship among the various forms of energy and energy transformations. Based on the experimental observations, the first law of thermodynamics states that energy can neither be created nor be destroyed, it can only change forms. In other words, during an interaction between a system and its surroundings, the amount of energy gained by the system is exactly equal to the amount of energy lost by the surroundings.
- For a closed system or a fixed mass, the first law may be expressed as net energy transferred to (or from) the system as heat and work = net increase (or decrease) in the total energy of the system. In differential form, it can be expressed as
$$\delta Q - \delta W = dE$$
- For a stationary closed system (the extrinsic effects on a closed system, such as the effects of motion and gravity, are neglected)
$$\delta Q - \delta W = \mathrm{d}U$$
- Internal energy is defined as the sum of all the microscopic forms of energy of a system. It is the energy associated with the molecular structure, and the molecular activity of the constituent particles of the system. It may be viewed as the sum of the kinetic and potential energies of the molecules.
- The first law of thermodynamics for a closed system undergoing a cycle states that if a system executes a cycle transferring heat and work through its boundary, the net work transfer is equivalent to the neat heat transfer.
$$\oint \delta W = \oint \delta Q$$
- The first law of thermodynamics for a non-flow non-cyclic quasi-equilibrium process is as follows: The net algebraic sum of heat and work during a process (non-flow, non-cyclic, quasi-equilibrium) is equal to change in internal energy during the same process.
$$\delta Q - P\mathrm{d}V = \mathrm{d}U$$

REVIEW QUESTIONS

3.1 State the first law of thermodynamics and prove that for non-flow process, it leads to $Q = W + \Delta U$

3.2 State the first law of thermodynamics for a closed system undergoing a cycle.

3.3 Define internal energy and prove that it is a property of the system.

3.4 Define enthalpy of a system and prove that the change in enthalpy for a non-flow constant pressure process is equal to the heat exchange.

3.5 What is a perpetual motion machine of first kind (PMM1)? Is it possible to devise a PMM1? If not, explain why?

3.6 What are the limitations of the first law of thermodynamics? Illustrate with examples.

3.7 Prove that heat transfer is a path function.

3.8 Is the first law of thermodynamics applicable to irreversible processes also? Explain.

3.9 Derive an expression for the heat transfer in a non-flow reversible polytropic process.

3.10 Define the specific heats in terms of property changes.

3.11 What is the relation between temperature and pressure for an ideal gas, if it undergoes a reversible adiabatic process?

NUMERICAL PROBLEMS

3.1 A system contains air initially at 40 kPa, 0.2 m^3 and 150°C. It is expanded to10 kPa according to the law $PV^{1.35}$ = constant. The gas is then heated at constant pressure till its enthalpy increases by 60 kJ. Calculate the total work done.

3.2 Starting from the non-flow energy equation, show that the specific heat, C_n, of an ideal gas with constant specific heats during a quasi-equilibrium polytropic expansion may be expressed by

$$C_n = \left(\frac{\gamma - n}{1 - n}\right) C_v$$

3.3 One kg of air expands from 6.5 bar and 0.0135 m^{3}to a final volume of 0.1 m^3 in a quasi-equilibrium isothermal process. Find the final pressure, final temperature, work done and change in internal energy and heat interaction.

3.4 Air at 300°C and 10 bar expands to 3 bar reversibly following the law $pv^{1.35} = C$. Determine the work done per kg of air and heat transfer if C_p = 1 kJ/kg-K and C_v = 0.714 kJ/kg-K.
If 15 % of the work is dissipated in friction with heat transfer with surroundings unaltered, determine the final temperature and work done.

3.5 1.5 kg of a certain gas is contained in a frictionless piston–cylinder system. The gas expands from initial volume of 0.3 m^3 and 5-bar pressure through a pressure ratio of 5. The process of expansion follows the law $Pv^{1.2}$ = constant. The specific internal energy u is related to the pressure P and specific volume v by the following relation $u = 60\ Pv + 4500$, where P is in bar and V is in m^3/kg
For a quasi-equilibrium expansion, determine the changes of internal energy, heat transfer to or from the system and work done on or by the system.
If during expansion, 10 kJ of heat were transferred to the system, what would then be the work done?

3.6 One kg of gas expands reversibly and adiabatically, its temperature falling from 240°C to 115°C while its volume is doubled. The gas does 90 kJ of work in the process. Find the values of specific heats C_p and C_v, and the molecular weight of the gas.

3.7 A quantity of air occupying a volume of 1 m^3 at 4 bar and 150°C is allowed to expand isentropically to 1 bar. Its enthalpy is then raised by 70 kJ by heating at constant pressure. What is the total work done during the pocess?
If the process is to be replaced by a reversible polytropic expansion which will result in the same final state being reached, what index of expansion is required? Will the work done be greater or less than in the original process?

3.8 A system contains 0.15 m^3 of air at 3.8 bar and 150°C. A reversible adiabatic expansion takes place till the pressure falls to 1.03 bar. The gas is then heated at constant pressure till enthalpy increases by 60.7 kJ. Determine the total work done.
If these processes are replaced by a single reversible polytropic process giving the same work between the same initial and final states, determine the index of expansion. Given, C_p = 1kJ/kg-K and C_v = 0.714 kJ/kg-K.

3.9 One kg of air is compressed in a quasi-static way from an initial state of 1 kPa and 300 K to a final state of 400 kPa and 300 K. Compare the work required for the following processes:
(i) Isothermal compression
(ii) Heating at constant volume followed by cooling at constant pressure.
For air, C_p = 1.005 kJ/kg.K and C_v = 0.718 kJ/kg.K.

3.10 A system undergoes a cycle consisting of four processes. The energy transfers are given in the table below:

Process	*Q(kW)*	*W(kW)*	*ΔU(kW)*
1–2	–100	*A*	0
2–3	400	*B*	*C*
3–4	*D*	300	200
4–1	0	*E*	–600

Find the value of A, B, C, D and E?

3.11 One kg of air is initially at 400 kPa and 400°C. It expands in a quasi-equilibrium constant pressure to a volume of 2.5 m^3. Then the pressure of the air falls to 80 kPa in a constant volume process. Finally the system restores its initial state in a quasi-equilibrium isothermal process. Find the work output and the net heat transfer.

MULTIPLE-CHOICE QUESTIONS

3.1 The internal energy of a system is a function of only
(a) pressure
(b) temperature (absolute)
(c) volume
(d) pressure and temperature

3.2 In an isothermal process, the internal energy of an ideal gas
(a) increases
(b) decreases
(c) remains constant
(d) shows unpredictable behaviour

3.3 Work done is zero for the following processes:
(a) Constant volume (b) Free expansion (c) Constant pressure (d) Both (a) and (b)

3.4 Change of enthalpy of a system is the heat supplied at
(a) constant pressure
(b) constant temperature
(c) constant volume
(d) constant entropy

3.5 The total heat of a substance is also known as
(a) entropy (b) enthalpy (c) internal energy (d) thermal capacity

3.6 First law of thermodynamics is the law of
(a) conservation of mass
(b) conservation of energy
(c) conservation of momentum
(d) conservation of heat

3.7 A perpetual motion machine is a
(a) thermodynamic machine
(b) non-thermodynamic machine
(c) hypothetical machine
(d) hypothetical machine whose operation would violate the laws of thermodynamics

3.8 When 2 kg of ice at 0°C is mixed with 2 kg of water at 80°C, the temperature of the mixture becomes
(a) 40°C (b) 80°C (c) 0°C (d) 60°C

3.9 First law of thermodynamics furnishes the relationship between
   (a) heat, work and properties of the system
   (b) various properties of the system
   (c) various thermodynamic processes
   (d) heat and internal energy

3.10 Change in the specific internal energy of small temperature change ΔT for ideal gases is expressed by the relation
   (a) $\Delta u = C_v \Delta T$
   (b) $\Delta u = C_p \Delta T$
   (c) $\Delta u = R\Delta T$
   (d) $\Delta u = k\Delta T$

3.11 Change in the specific enthalpy of small temperature change ΔT for ideal gases is expressed by the relation
   (a) $\Delta h = C_v \Delta T$
   (b) $\Delta h = C_p \Delta T$
   (c) $\Delta h = R\Delta T$
   (d) $\Delta h = k\Delta T$

CHAPTER

4

First Law Applied to Flow Processes

4.1 CONTROL VOLUME

In flow processes we normally are concerned with the flow of fluids through devices such as an air compressor, a turbine, a nozzle, a pump, a blower, a fan, a car radiator, a water heater etc. In these cases it is difficult to focus attention on a fixed identifiable quantity of mass. It is much more convenient, for analysis, to focus attention on a volume in space through which the fluid flows.

A control volume is a properly selected region in space. The boundary, which remains fixed in space, enveloping this control volume is called the control surface.

4.2 STEADY FLOW PROCESSES AND DEVICES

A steady flow is defined for a control volume as that type of flow in which the thermodynamic properties at a given position within or at the boundaries of the control volume are invariant with time. The properties include temperature, pressure, density, internal energy as well as velocity and acceleration of the flow stream. However, in a steady flow process the state of the fluid can change as it passes through the control volume.

Many engineering devices such as turbines, pumps, compressors, boilers, condensers, etc., operate over long periods of time under the same working conditions and they are classified as steady-flow devices.

4.3 MASS BALANCE AND ENERGY BALANCE FOR A STEADY FLOW PROCESS

The following assumptions are made in the analysis.

(a) The properties of the fluid at any point within the control volume do not vary with time.

(b) The fluid properties are uniform over the inlet and outlet flow areas.
(c) The potential, kinetic, internal and flow energies are only considered in the analysis.
(d) There is no change in chemical composition of the fluid.

Let us consider a steady flow process as shown in Fig. 4.1 in which there is a single stream entering and a single stream leaving the control volume. Fluid enters at the Section 1 and leaves at the Section 2.

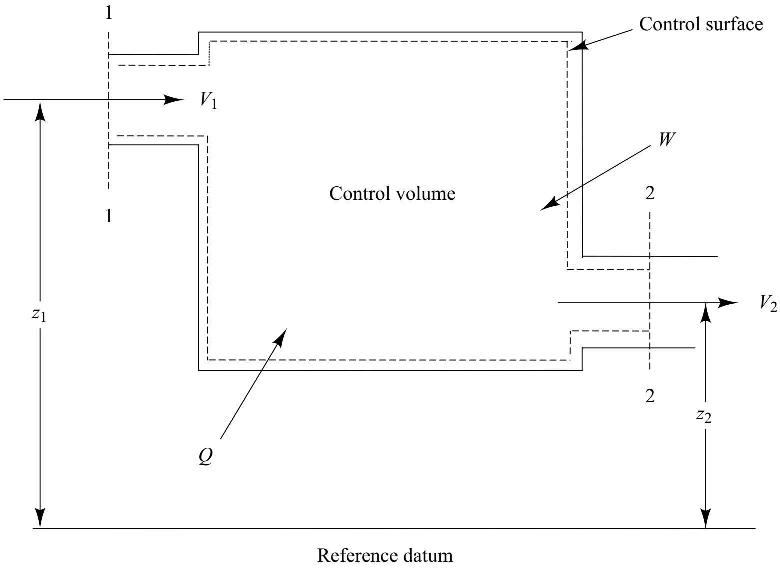


Figure 4.1 *Steady flow through a control volume*

From the law of conservation of mass, for a control volume, we can write

$$\left(\begin{array}{l}\text{Net amount of mass}\\ \text{added to the control volume}\end{array}\right) = \left(\begin{array}{l}\text{Net increased in mass}\\ \text{of the control volume}\end{array}\right) \quad (4.1)$$

For a steady flow process the right-hand side of Eq. (4.1) becomes zero, and we have

$$\left(\begin{array}{l}\text{Net amount of mass}\\ \text{added to the control volume}\end{array}\right) = 0$$

$$\left(\begin{array}{l}\text{Amount of mass}\\ \text{entering the control volume}\end{array}\right) - \left(\begin{array}{l}\text{Amount of mass}\\ \text{leaving from the control volume}\end{array}\right) = 0$$

$$\left(\begin{array}{l}\text{Amount of mass}\\ \text{entering the control volume}\end{array}\right) = \left(\begin{array}{l}\text{Amount of mass}\\ \text{leaving the control volume}\end{array}\right) \quad (4.2)$$

$$m_1 = m_2$$

where m_1 = total mass flow at inlet

m_2 = total mass flow at outlet

In engineering applications, it is more convenient to use the mass flow rate. Then the Eq. (4.2) becomes

$$\begin{pmatrix}\text{Rate of mass} \\ \text{flow into the control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass} \\ \text{flow out of the control volume}\end{pmatrix}$$

$$\dot{m}_1 = \dot{m}_2 \tag{4.3}$$

$$\frac{A_1V_1}{v_1} = \frac{A_2V_2}{v_2} \tag{4.3a}$$

where $\dot{m}_1$ = mass flow rate at inlet

$\dot{m}_2$ = mass flow rate at outlet

A_1 = cross-sectional area of fluid stream at inlet

A_2 = cross-sectional area of fluid stream at outlet

V_1 = average velocity of fluid stream at inlet

V_2 = average velocity of fluid stream at outlet

v_1 = specific volume of fluid at inlet

v_2 = specific volume of fluid at outlet

Equations (4.3) and (4.3a) are all forms of mass balance equations for a single stream of inlet and a single stream of outlet.

When more than one stream of fluid enters or leaves the control volume, the mass balance equation becomes

$$\sum_{\substack{\text{all streams}\\ \text{entering}}} \dot{m} = \sum_{\substack{\text{all streams}\\ \text{leaving}}} \dot{m} \tag{4.3b}$$

$$\sum_{\substack{\text{all streams}\\ \text{entering}}} \frac{AV}{v} = \sum_{\substack{\text{all streams}\\ \text{leaving}}} \frac{AV}{v} \tag{4.3c}$$

Equations (4.3b) and (4.3c) are all forms of mass balance equations for more than one stream of inlet and outlet.

From the first law of thermodynamics, we can conclude for a control volume.

$$\begin{pmatrix}\text{Net amount of energy} \\ \text{added to the control volume}\end{pmatrix} = \begin{pmatrix}\text{Net increased in stored energy} \\ \text{of the control volume}\end{pmatrix} \tag{4.4}$$

When a fluid enters a control volume, the stored energy of the control volume is increased by an amount equal to the stored energy of the entering fluid. Similarly, when a fluid leaves a control volume, the stored energy of the control volume is decreased by an amount equal to the stored energy of the leaving fluid. If we distinguish this transfer of stored energy of fluid crossing the control surface from heat and work, Eq. (4.4) becomes

$$\begin{pmatrix}\text{Net amount of}\\ \text{energy added to}\\ \text{the control volume}\\ \text{as heat and work}\end{pmatrix} + \begin{pmatrix}\text{Stored energy}\\ \text{of the fluid}\\ \text{entering the control}\\ \text{volume}\end{pmatrix} - \begin{pmatrix}\text{Stored energy}\\ \text{of the fluid}\\ \text{leaving the control}\\ \text{volume}\end{pmatrix} = \begin{pmatrix}\text{Net increased in}\\ \text{stored energy}\\ \text{of the control}\\ \text{volume}\end{pmatrix} \tag{4.5}$$

For a steady-flow process, the right-hand side of Eq. (4.5) becomes zero, and we have

$$\begin{pmatrix}\text{Net amount of}\\ \text{energy added to}\\ \text{the control volume}\\ \text{as heat and work}\end{pmatrix} + \begin{pmatrix}\text{Stored energy}\\ \text{of the fluid}\\ \text{entering the}\\ \text{control volume}\end{pmatrix} - \begin{pmatrix}\text{Stored energy of}\\ \text{the fluid leaving}\\ \text{the control volume}\end{pmatrix} = 0 \tag{4.6}$$

For the control volume as shown in Fig. 4.1, Eq. (4.6) becomes

$$Q - W + mP_1v_1 + E_1 - mP_2v_2 - E_2 = 0 \tag{4.7}$$

where Q = net amount of heat added to the control volume
W = net amount of work done by the control volume during the same time
m = total mass flow through the control volume
mP_1v_1 = flow work done on the control volume by the entering fluid
mP_2v_2 = flow work done by the control volume on the leaving fluid
E_1 = stored energy of the fluid entering the control volume
E_2 = stored energy of the fluid leaving the control volume

The stored energy of a simple compressible system consists of three parts—internal, kinetic and potential energy—and is given by

$$E = U + \frac{mV^2}{2} + mgz$$

where U is the internal energy, V is the velocity and z is the elevation of the system relative to some external reference point. On a unit mass basis

$$e = u + \frac{V^2}{2} + gz \quad (u \text{ is the specific internal energy})$$

$$E = m\left(u + \frac{V^2}{2} + gz\right)$$

Now Eq. (4.7) can be written as

$$Q - W + mP_1v_1 + m\left(u_1 + \frac{V_1^2}{2} + gz_1\right) - mP_2v_2 - m\left(u_2 + \frac{V_2^2}{2} + gz_2\right) = 0 \tag{4.8}$$

$$m\left(u_1 + \frac{V_1^2}{2} + gz_1\right) + mP_1v_1 + Q = m\left(u_2 + \frac{V_2^2}{2} + gz_2\right) + mP_2v_2 + W$$

$$m\left(u_1 + P_1 v_1 + \frac{V_1^2}{2} + gz_1\right) + Q = m\left(u_2 + P_2 v_2 + \frac{V_2^2}{2} + gz_2\right) + W$$

$$m\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + Q = m\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + W \tag{4.9}$$

$(h = u + Pv)$

Per unit mass passing through the control volume basis, Eq. (4.9) can be written as

$$h_1 + \frac{V_1^2}{2} + gz_1 + q = h_2 + \frac{V_2^2}{2} + gz_2 + w \tag{4.9a}$$

$$q - w = h_2 \quad h_1 + \frac{V_2^2 \quad V_1^2}{2} + g\left(z_2 \quad z_1\right) \tag{4.9b}$$

Per unit time basis, Eq. (4.9) can be written as

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + \dot{W} \tag{4.10}$$

Equations (4.9), (4.9a), (4.9b), (4.10) are all forms of steady flow energy equations for a single stream of inlet and a single stream of outlet.

All the terms in Eq. (4.9a) represent energy per unit mass of fluid and in Eq. (4.10) represent energy per unit time.

When there is more than one inlet and outlet, the steady flow energy equation becomes

$$\sum_{\substack{\text{all streams}\\ \text{entering}}} m\left(h + \frac{V^2}{2} + gz\right) + Q = \sum_{\substack{\text{all streams}\\ \text{leaving}}} m\left(h + \frac{V^2}{2} + gz\right) + W \tag{4.11}$$

$$\sum_{\substack{\text{all streams}\\ \text{leaving}}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) + \dot{Q} = \sum_{\substack{\text{all streams}\\ \text{leaving}}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) + \dot{W} \tag{4.12}$$

4.4 APPLICATIONS OF ENERGY EQUATION

The steady flow energy equation (SFEE) developed earlier is a generalized one and the same can be applied to numerous devices such as boiler, compressor, turbine heat exchanger, nozzle, diffuser - intended for specific purpose(s). In essence, all the aforesaid devices are open system and hence mass as well as energy transfer takes place across the boundary.

4.4.1 Nozzles and Diffusers

A nozzle is a device of uniformly varying cross-section (either converging or converging followed by diverging) to increase the velocity of the working fluid passing through it at the cost of pressure drop. The pressure drop is accompanied by fall in enthalpy which is converted to gain the kinetic energy of the fluid. The inner wall of the device is made extremely smooth and outside wall is thermally insulated to prevent heat loss. Nozzle finds its application in thermal power plant, jet engines, rockets, space-craft, etc.

Diffuser is a device that functions just opposite to that of a nozzle that is the working fluid passes through it gains pressure at the outlet by loosing its kinetic energy. Diffuser is used in hydraulic power pack and mounted at the exit of the return line that brings the oil back to the tank (to reduce the speed of the oil so as to avoid turbulence).

We have $\dot{Q} = 0$ because in nozzles and diffusers, the rate of heat transfer between the fluid flowing through them and the surroundings is usually insignificant, even when they are not insulated from one another. This is mainly because of the fact that the fluid has high velocity and thus passes through a nozzle or diffuser quickly without any significant heat transfer taking place. Therefore, the flow through a nozzle or diffuser can be assumed to be adiabatic.

We have $\dot{W} = 0$ because the nozzles and diffusers are properly shaped ducts to achieve flow acceleration and deceleration, without involving work like shaft work, and so on.

We have $\Delta KE \neq 0$, since the devices involve very high velocity and the fluid experiences large changes in the velocity.

We have $\Delta PE = 0$, that is, the fluid usually experiences little or no changes in its elevation as it flows through a nozzle or diffuser. Moreover, if the fluid is in gaseous state, gravitational effects can be neglected even if there is a significant variation in elevation.

Thus, the steady flow energy equation reduces to

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \tag{4.13}$$

Example 4.1 The steam flows through a nozzle with negligible heat transfer. At the inlet the enthalpy of steam is 3159.3 kJ/kg and the velocity is 30 m/s. At the exit, the enthalpy is 2855.4 kJ/kg. The mass flow rate of steam is 7 kg/s. At the exit the specific volume of steam is 0.2275 m^3/kg. Determine the velocity at exit from the nozzle and the exit area of the nozzle.

Solution

Mass flow rate $\dot{m} = 7$ kg/s

Enthalpy at inlet $h_1 = 3159.3 \text{ kJ/kg} = 3159.3 \times 10^3 \text{ J/kg}$

Velocity at inlet $V_1 = 30$ m/s

Enthalpy at exit $h_2 = 2855.4 \text{ kJ/kg} = 2855.4 \times 10^3 \text{ J/kg}$

Specific volume at exit $v_2 = 0.2275 \text{ m}^3\text{/kg}$

The steady flow energy equation for the nozzle is given by Eq. (4.13) as

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

or $$3159.3 \times 10^3 + \frac{30^2}{2} = 2855.4 \times 10^3 + \frac{V_2^2}{2}$$

or $$V_2 = 780.19 \text{ m/s}$$

Mass flow rate can be expressed as

$$\dot{m} = \frac{A_2 V_2}{v_2}$$

or $$7 = \frac{A_2 \times 780.19}{0.2275}$$

or $$A_2 = 0.002 \text{ m}^2$$

Example 4.2 Fluid flows through a horizontal nozzle with negligible heat loss from it. At the inlet, the enthalpy of the fluid passing is 3000 kJ/kg and the velocity is 60 m/s. At the exit, the enthalpy is 2762 kJ/kg. (a) Find the velocity at exit from the nozzle, (b) If the inlet area is 0.1 m^2 and the specific volume at inlet is 0.187 m^3/kg , find the mass flow rate, and (c) If the specific volume at the nozzle exit is 0.498 m^3/kg, find the exit area of the nozzle.

Solution

Velocity at inlet	$V_1 = 60$ m/s
Enthalpy at inlet	$h_1 = 3000 \text{ kJ/kg} = 3000 \times 10^3 \text{ J/kg}$
Enthalpy at exit	$h_2 = 2762 \text{ kJ/kg} = 2762 \times 10^3 \text{ J/kg}$
Nozzle inlet area	$A_1 = 0.1 \text{ m}^2$
Specific volume at inlet	$v_1 = 0.187 \text{ m}^3\text{/kg}$
Specific volume at exit	$v_2 = 0.498 \text{ m}^3\text{/kg}$

(a) The steady flow energy equation for the nozzle is given by Eq. (4.13) as

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

or $$3000 \times 10^3 + \frac{60^2}{2} = 2762 \times 10^3 + \frac{V_2^2}{2}$$

or $$V_2 = 692.53 \text{ m/s}$$

(b) Mass flow rate is found to be

$$\dot{m} = \frac{A_1 V_1}{v_1}$$

$$= \frac{0.1 \times 60}{0.187} = 32.08 \text{ kg/s}$$

(c) Mass flow rate can also be expressed in terms of exit area of the nozzle as

$$\dot{m} = \frac{A_2 V_2}{v_2}$$

or $$32.08 = \frac{A_2 \times 692.53}{0.498}$$

or $$A_2 = 0.023 \text{ m}^2$$

Example 4.3 Air expands through a nozzle from a pressure of 600 kPa to a final pressure of 100 kPa. The enthalpy decreases by 150 kJ/kg during the flow of air. If the entering velocity and loss of heat rate are neglected, compute the exit velocity.

Solution

Change in enthalpy $h_1 - h_2 = 150 \text{ kJ/kg} = 150 \times 10^3 \text{ J/kg}$

The steady flow energy equation for the nozzle is given by Eq. (4.13) as

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

or $$\frac{V_2^2}{2} = h_1 - h_2 \qquad [\because V_1 \text{ neglected}]$$

or $$\frac{V_2^2}{2} = 150 \times 10^3$$

or $$V_2 = 547.72 \text{ m/s}$$

4.4.2 Turbines, Compressors, Pumps, Fans and Blowers

In steam, gas or hydroelectric power plants, the turbine is the device used to drive the electric generators. As the fluid passes through the turbine, work is done against the turbine blades which are attached to its shaft. As a result, the shaft rotates and the turbine produces work. Compressors, pumps, fans, etc., are devices used to increase the pressure of a fluid. A compressor is capable of compressing a gas to a very high pressure. Pumps work very much like compressors except that they handle liquids instead of gases.

We have $\dot{Q} = 0$, because the rate of heat transfer in respect of these devices is generally small compared to the shaft work unless there is internal cooling (as in the case of some compressors).

We have $\dot{W} \neq 0$, since there is shaft work involved.

The change of potential energy $\Delta PE \approx 0$.

The steady-flow energy equation then becomes

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W} \tag{4.14}$$

Example 4.4 A turbine operates under steady flow conditions. The rate of steam flow through the turbine is 1.1 kg/s. At the entry the steam velocity and enthalpy are 60 m/s and 2700 kJ/kg, respectively. At the turbine exists, the steam leaves at 110 m/s with an enthalpy of 1800 kJ/kg. Determine the power output of the turbine. Neglect the changes in potential energy.

Solution

Mass flow rate of steam	$\dot{m} = 1.1$ kg/s
Enthalpy of steam at inlet	$h_1 = 2700$ kJ/kg $= 2700 \times 10^3$ J/kg
Velocity of steam at inlet	$V_1 = 60$ m/s
Enthalpy of steam at exit	$h_2 = 1800$ kJ/kg $= 1800 \times 10^3$ J/kg
Velocity of steam at exit	$V_2 = 110$ m/s

The steady flow energy equation for the turbine is given by Eq. (4.14) as

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}$$

or $$1.1 \times \left(2700 \times 10^3 + \frac{60^2}{2}\right) = 1.1 \times \left(1800 \times 10^3 + \frac{110^2}{2}\right) + \dot{W}$$

or $$\dot{W} = 985325 \text{ J/s} \quad = 985.325 \text{ kW}$$

Example 4.5 Steam flows steadily through a turbine at the rate of 0.42 kg/sec. The steam enters the turbine at the following state: pressure 1.2 MPa, temperature 188°C, enthalpy 2785 kJ/kg, velocity 33.3 m/sec and elevation 3 m. The steam leaves the turbine at the following state: pressure 20 kPa, enthalpy 2512 kJ/kg, velocity 100 m/sec and elevation 0 m. Heat is lost to the surroundings at the rate of 0.29 kJ/kg. Compute the power output of the turbine.

Solution

Mass flow rate of steam $\dot{m} = 0.42$ kg/s

Enthalpy of steam at inlet $h_1 = 2785 \text{ kJ/kg} = 2785 \times 10^3 \text{ J/kg}$

Velocity of steam at inlet $V_1 = 33.3 \text{ m/s}$

Elevation at inlet $z_1 = 3 \text{ m}$

Enthalpy of steam at exit $h_2 = 2512 \text{ kJ/kg} = 2512 \times 10^3 \text{ J/kg}$

Velocity of steam at exit $V_2 = 100 \text{ m/s}$

Elevation at exit $z_2 = 0 \text{ m}$

Rate of heat transfer $\dot{Q} = -0.29 \text{ kJ/kg} = -0.29 \times 0.42 \text{ kJ/s} = -0.29 \times 0.42 \times 10^3 \text{ J/s}$

(negative sign because heat is lost to the surroundings)

The steady flow energy equation for the turbine is given by

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + \dot{W}$$

or

$$0.42 \times \left(2785 \times 10^3 + \frac{33.3^2}{2} + 9.81 \times 3\right) - 0.29 \times 0.42 \times 10^3$$

$$= 0.42 \times \left(2512 \times 10^3 + \frac{100^2}{2} + 0\right) + \dot{W}$$

or $\dot{W} = 112683.4 \text{ W} = 112.68 \text{ kW}$

Example 4.6 A steam turbine receives a steam flow rate of 1.6 kg/s and delivers 700 kW of power. The velocity of steam at the entrance and exit are 50 m/s and 350 m/s respectively. The inlet pipe is 4 m above the exhaust pipe. The heat loss from the turbine casing is negligible. Find the change of enthalpy across the turbine.

Solution

Mass flow rate of steam $\dot{m} = 1.6 \text{ kg/s}$

Rate of work transfer $\dot{W} = 700 \text{ kW} = 700 \times 10^3 \text{ W}$

Velocity of steam at inlet $V_1 = 50 \text{ m/s}$

Velocity of steam at exit $V_2 = 350 \text{ m/s}$

Elevation difference $z_1 - z_2 = 4 \text{ m}$

The steady flow energy equation for the turbine gives

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + \dot{W}$$

or $$h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2 + \frac{\dot{W}}{\dot{m}}$$

or $$h_1 - h_2 = \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) + \frac{\dot{W}}{\dot{m}}$$

$$= \frac{350^2 - 50^2}{2} + 9.81 \times (-4) + \frac{700 \times 10^3}{1.6}$$

$$= 497460.76 \text{ J/kg} = 497.46 \text{ kJ/kg}$$

Example 4.7 A steam turbine in a power plant develops 5000 kW. The heat supplied to the steam in the boiler is 4700 kJ/kg, the heat rejected by the steam to the cooling water in the condenser is 2200 kJ/kg. The feed-pump work required to pump the condensate back into the boiler is 10 kW. Calculate the mass flow rate of the steam.

Solution

Heat supplied to the steam in the boiler	$Q_1 = 4700$ kJ/kg
Heat rejected by the steam in the condenser	$Q_2 = 2200$ kJ/kg
Power developed by the turbine	$\dot{W}_T = 5000$ kW
Power required to run the pump	$\dot{W}_P = 10$ kW

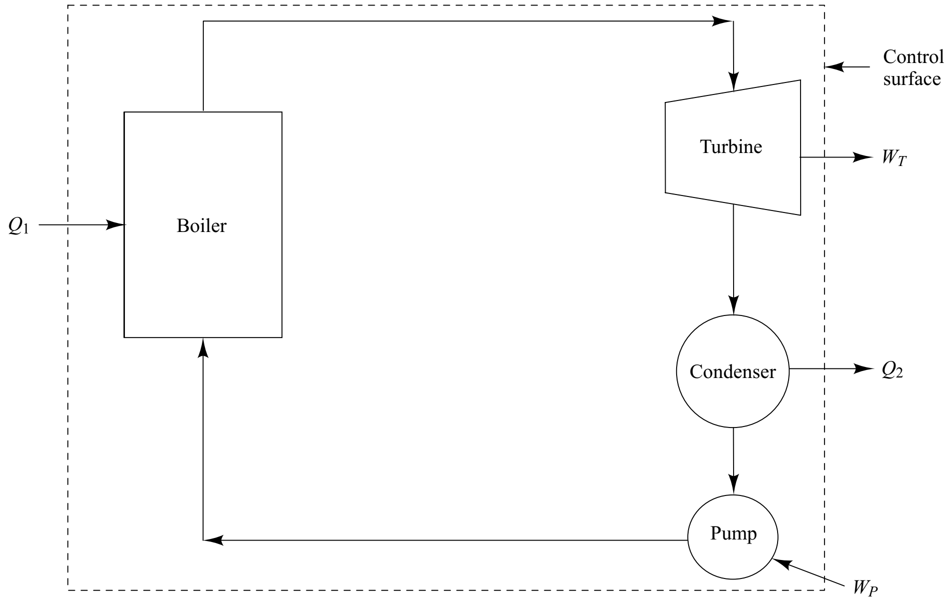


Figure 4.2

The steady flow energy equation for the control volume as shown in Fig. 4.2 is

$$\dot{m}\left(Q_1 - Q_2\right) = \left(\dot{W}_T - \dot{W}_P\right)$$

or $$\dot{m}\left(4700 - 2200\right) = \left(5000 - 10\right)$$

or $$\dot{m} = 1.996 \text{ kg/s}$$

Example 4.8 The compressor of a large gas turbine receives air from the ambient at 100 kPa, 20°C, with a negligible velocity. At the compressor discharge, air exits at 1 MPa, 400°C, with velocity of 100 m/s. The power input to the compressor is 4000 kW. Determine the mass flow rate of air through the unit.

Solution

Temperature of air at inlet $T_1 = 20°\text{C}$

Rate of work transfer $\dot{W} = -4000 \text{ kW} = -4000 \times 10^3 \text{ W}$

(negative sign, because it consumes power)

Velocity of air at inlet $V_1 \approx 0$

Velocity of air at exit $V_2 = 100 \text{m/s}$

Temperature of air at exit $T_2 = 400°\text{C}$

Specific heat of air $C_p = 1.005 \text{ kJ/kg.K} = 1005 \text{ J/kg.K}$

The steady flow energy equation for the compressor gives

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}$$

or $$h_1 = h_2 + \frac{V_2^2}{2} + \frac{\dot{W}}{\dot{m}}$$

or $$-\frac{\dot{W}}{\dot{m}} = C_p\left(T_2 - T_1\right) + \frac{V_2^2}{2}$$

or $$-\frac{(-4000 \times 10^3)}{\dot{m}} = 1005(400 - 20) + \frac{100^2}{2}$$

or $$\dot{m} = 10.34 \text{ kg/s}$$

Example 4.9 A compressor receives air at 100 kPa and 20°C. The air is compressed adiabatically to 800 kPa. The power input to the compressor is 400 kW. Determine the mass flow rate. The changes in kinetic and potential energies are negligible.

Solution

Inlet pressure $P_1 = 100 \text{ kPa}$

Inlet temperature $T_1 = 20°\text{C} = 20 + 273 = 293 \text{ K}$

Outlet pressure $P_2 = 800 \text{ kPa}$

Rate of work transfer $\dot{W} = -400 \text{ kW} = -400 \times 10^3 \text{ W}$ (Since it consumes power)

The temperature after compression is found to be

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or $$T_2 = 293 \times \left(\frac{800}{100}\right)^{\frac{1.4-1}{1.4}} = 530.75 \text{ K}$$

The steady flow energy equation for the compressor gives

$$\dot{m}h_1 = \dot{m}h_2 + \dot{W}$$

or $$\dot{m}\left(h_1 - h_2\right) = \dot{W}$$

or $$\dot{m}C_p\left(T_1 - T_2\right) = \dot{W}$$

or $$\dot{m} \times 1005\left(293 - 530.75\right) = -400 \times 10^3$$

or $$\dot{m} = 1.674 \text{ kg/s}$$

Example 4.10 A blower handles 1 kg/s of air at 293 K and consumes a power of 15 kW. The inlet and outlet velocities of air are 100 m/s and 150 m/s respectively. Find the exit air temperature, assuming adiabatic conditions. Take C_p of air as 1.005 kJ/kg-K.

Solution

Mass flow rate of air $\dot{m} = 1 \text{ kg/s}$

Temperature of air at inlet $T_1 = 293 \text{ K}$

Rate of work transfer $\dot{W} = -15 \text{ kW} = -15000 \text{ W}$ (negative sign, because it consumes power)

Velocity of air at inlet $V_1 = 100 \text{m/s}$

Velocity of air at exit $V_2 = 150 \text{m/s}$

Specific heat of air $C_p = 1.005 \text{ kJ/kg.K} = 1005 \text{ J/kg.K}$

The steady flow energy equation for the blower under the adiabatic condition ($Q = 0$) is given by Eq. (4.14) as

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}$$

or $$\dot{m}\left(h_2 - h_1\right) = \dot{m}\left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) - \dot{W}$$

or $$\dot{m}C_p(T_2 - T_1) = \dot{m}\left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) - \dot{W}$$

or $$1 \times 1005 \times (T_2 - 293) = 1 \times \left(\frac{100^2}{2} - \frac{150^2}{2}\right) - (-15000)$$

or $$T_2 = 301.7 \text{ K}$$

4.4.3 Throttling Devices

The main purpose of the throttling process is a significant pressure drop without any work interactions or changes in kinetic or potential energy. Flow through a restriction such as a valve or a porous plug fulfills the necessary condition. A throttling valve is shown in Fig. 4.3. By decreasing the cross-sectional area for flow, a greater flow resistance is introduced. For a given mass flow rate, the greater flow resistance requires a greater pressure drop across the valve.

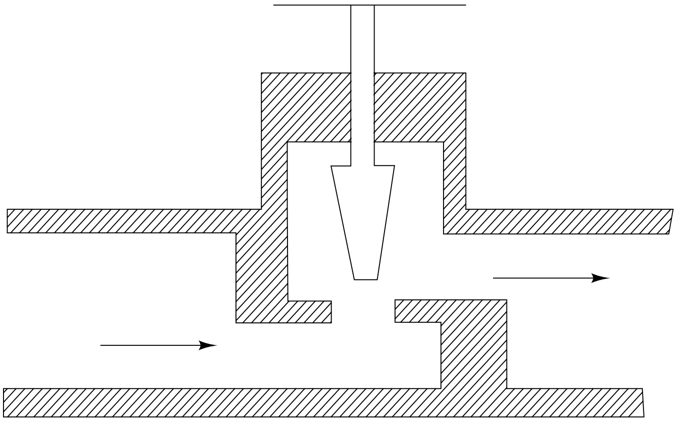


Figure 4.3 *Throttling valve*

Although the velocity may be quite high in the region of the restriction, measurements upstream and downstream from the actual valve area will indicate that the change in velocity, and hence the change in kinetic energy, across the restriction is very small. Since no rotating shaft is present, no work interaction is involved.

In most applications, either the throttling devices are properly insulated or the heat transfer is insignificant.

$$h_1 = h_2$$

Thus for a throttling process, the enthalpy change is zero.

Throttling devices are in commonly used in most domestic refrigerators.

4.4.4 Heat Exchangers

A heat exchanger is one of the important steady-flow devices in engineering applications. A heat exchanger is a device where heat is transferred between two or more fluids. The radiators of an automobile, steam condenser in the steam power plant are the example of heat exchangers. The simplest form of a heat exchanger is shown in Fig. 4.4 where heat transfer takes place between two fluids.

The main objective of the heat exchanger is to transfer heat from one fluid to another. The changes of kinetic and potential energies are usually negligible. A heat exchanger involves no work interactions. The pressure drop through the heat exchanger is usually small. The heat exchanger is usually properly insulated to prevent heat loss to the surroundings.

The steady flow energy balance equation for the heat exchanger shown in Fig. 4.4 becomes

$$\dot{m}_h \left(h_1 \quad h_2\right) = \dot{m}_c \left(h_4 \quad h_3\right)$$

$\dot{m}_h$ = mass flow rate of hot fluid

$\dot{m}_c$ = mass flow rate of cold fluid

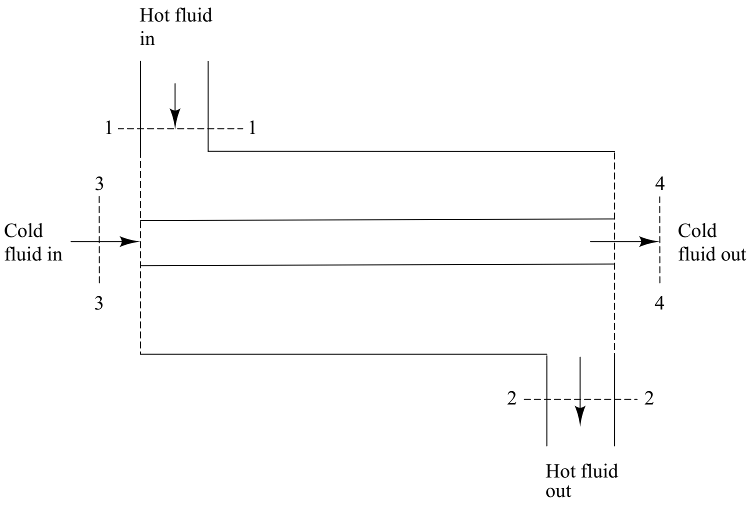


Figure 4.4 *Heat exchanger*

SUMMARY

- A control volume is a properly selected region in space. The boundary, which remains fixed in space, enveloping this control volume, is called the control surface.
- The mass balance equation for a single stream entering and a single stream leaving the control volume when the flow is steady can be written as

$$m_1 = m_2$$

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

- When more than one stream of fluid enters or leaves the control volume, the mass balance equation becomes

$$\sum_{\substack{\text{all streams}\\\text{entering}}} \dot{m} = \sum_{\substack{\text{all streams}\\\text{leaving}}} \dot{m}$$

$$\sum_{\substack{\text{all streams}\\\text{entering}}} \frac{AV}{v} = \sum_{\substack{\text{all streams}\\\text{leaving}}} \frac{AV}{v}$$

- The steady flow energy balance equation for a single stream entering and a single stream leaving the control volume can be written as

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + \dot{W}$$

  All the terms in the above energy equation represent energy per unit time.
- When there is more than one inlet and outlet, the steady flow energy equation becomes

$$\sum_{\substack{\text{all streams}\\\text{entering}}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) + \dot{Q} = \sum_{\substack{\text{all streams}\\\text{leaving}}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) + \dot{W}$$

- The steady flow energy equation for nozzles and diffusers reduces to

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

- The steady flow energy equation turbines, compressors, pumps, fans and blowers reduces to

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}$$

- For a throttling process, the enthalpy change is zero.

REVIEW QUESTIONS

4.1 What do you mean by 'control volume'?
4.2 What is steady-flow process?
4.3 Write the steady-flow energy equation for a single stream entering and a single stream leaving a control volume and explain the various terms in it.
4.4 Show that enthalpy remains constant during a throttling process.

NUMERICAL PROBLEMS

4.1 The steam flows through a nozzle with negligible heat transfer. The mass flow rate of steam is 0.15 kg/s. The initial and final pressures are 1.2 MPa and 12 kPa. The initial and final velocities are 100 and 1100 m/s. Compute the change in enthalpy.

4.2 Air expands adiabatically and quasi-statically through a nozzle. At the inlet to the nozzle, pressure is 800 kPa and temperature is 400°C. The exit pressure is 100 kPa and the area is 0.2 m^2. The inlet velocity is negligible. Determine the exit velocity and the mass flow rate.

4.3 A fluid passes through a turbine at a rate of 1.75 kg/s. The inlet and exit velocities are 40 m/s and 130 m/s, respectively. The initial and final enthalpies are 2900 and 2600 kJ/kg, respectively. The heat loss to the surroundings is 40 kW. Compute the power output from the turbine.

4.4 Calculate the power required by a compressor if air flows at a rate of 0.7 kg/s. Air enters at 100 kPa, 20°C, with a velocity of 50 m/s, and leaves at 250 kPa, 100°C, with a velocity of 110 m/s. The enthalpy of the air increases by 70 kJ/kg as it passes through the compressor. The internal energy increases by 50 kJ/kg. There is a heat transfer of 20 kJ/kg from the air to the cooling water.

4.5 A blower supplying air to an engine takes in air at 90 kPa, 20°C with a velocity of 0.2 m/s. It discharges air into the engine at 106 kPa, 30°C with a velocity of 30 m/s. Calculate the power consumption of the blower. The properties of air at 90 kPa, 20°C are: specific volume = 0.93 m^3/kg, enthalpy = 209.2 kJ/kg, and at 106 kPa, 30°C are: specific volume = 0.81 m^3/kg, enthalpy = 303.3 kJ/kg.

4.6 Steam flows through a turbine at a rate of 2.5 kg/s. The inlet and exit enthalpy of steam are 2700 kJ/kg and 1800 kJ/kg respectively. Velocity of steam at inlet and exit are 35 m/s and 250 m/s respectively. There is a heat loss to the surroundings of 40 kW. Calculate the power output from the turbine.

4.7 A gas flows steadily through a turbine at a rate of 1.2 kg/s, through a cross- sectional area of 0.02 m^2. At the inlet to the turbine the conditions are found to be pressure = 5 bar, temperature = 650°C, enthalpy = 875 kJ/kg, internal energy = 704 kJ/kg. At the exit from the turbine the conditions are found to be: pressure = 1 bar, temperature = 500°C, enthalpy = 675 kJ/kg, internal energy = 543 kJ/kg. The inlet velocity is negligible. The power output from the turbine is 200 kW. Calculate the rate of heat transfer.

4.8 A hydraulic turbine is located slightly lower than the level of the water surface downstream of the dam in which the turbine is placed. The flow rate through the turbine is 1.12 m^3/s. Water entering the turbine has a velocity of 3 m/s, and that leaving has a velocity of 1.5 m^3/s. The difference in water level between the upstream and downstream sides of the dam is 16 m. Calculate the power output of the turbine.

4.9 In an adiabatic flow through nozzle, air flows at the rate of 500 kg/hr. The initial velocity of air is 200 m/s. At inlet to the nozzle, pressure is 2 MPa and temperature is 127°C. The exit pressure is 0.4 MPa. Determine (a) the exit velocity of air and (b) inlet and exit area of nozzle.

4.10 A centrifugal pump delivers water at the rate of 2500 kg/min from initial pressure of 80 kPa (absolute) to a final pressure of 250 kPa (absolute). The suction is 1 m below and the delivery is 4 m above the center of pump. The diameters of the suction and delivery pipes are 15 cm and 10 cm respectively. Determine the power required to run the pump.

MULTIPLE-CHOICE QUESTIONS

4.1 During throttling process
  (a) heat exchange does not take place
  (b) no work is done by the expanding steam
  (c) there is no change in internal energy of steam
  (d) all of the above

4.2 During throttling process
  (a) internal energy does not change
  (b) pressure does not change
  (c) enthalpy does not change
  (d) volume does not change

4.3 The work done in a steady flow process is given by
  (a) $\int PdV$ (b) $\int -PdV$ (c) $\int VdP$ (d) $\int -VdP$

CHAPTER

5

Second Law of Thermodynamics

5.1 INTRODUCTION

The first law of thermodynamics gives a quantitative relationship of heat and work interactions between a system and the surroundings if the system undergoes a thermodynamic process or a cycle. However, it does not say whether the process or the cycle in a particular direction would occur at all or not.

All spontaneous processes in nature proceed in one direction only. Reversal of these processes is not possible without the assistance of any external agency. For example, a hot cup of tea left in a cool surrounding eventually cools down. The reverse process, i.e., hot cup of tea getting even hotter in a cool surrounding can never take place, even though doing so would not violate the first law of thermodynamics.

There exists a directional law which determines the direction in which a spontaneous process will take place. This law is called the second law of thermodynamics.

Work can be converted to heat completely, but heat cannot be completely converted to work. The first law of thermodynamics does not say anything regarding it. The first law of thermodynamics is concerned with the energy transformation from one form to another quantitywise with no regard of its quality. The second law of thermodynamics asserts that energy has quality as well as quantity.

5.2 THERMAL ENERGY RESERVOIR

A thermal energy reservoir (TER) is a hypothetical body with a relatively large thermal energy capacity that can supply or absorb finite quantities of energy as heat without changing its temperature.

A thermal energy reservoir at high temperature, which supplies energy in the form of heat is called ***source***, and on the other hand, one which absorbs energy in the form of heat is called a ***sink***.

Large bodies of water such as lakes and oceans and the atmosphere behave as thermal energy reservoirs.

5.3 HEAT ENGINES

Work can easily be converted to other forms of energy but converting other forms of energy to work is not easy. Work can be converted to heat directly and completely, but converting heat to work requires the use of special devices. These devices are called ***heat engines***. A schematic diagram of a heat engine is shown in Fig. 5.1.

Although heat engines differ considerably from one another, there are some common characteristic feature of all heat engines.

(a) They receive heat from high temperature sources.
(b) They convert part of this heat to useful work.
(c) They reject the remaining heat to a low temperature sink.
(d) They operate on a cycle.

Heat engines usually involve a substance which receives energy from the source and which rejects energy to the sink. This substance is called the ***working fluid*** or ***working medium.***

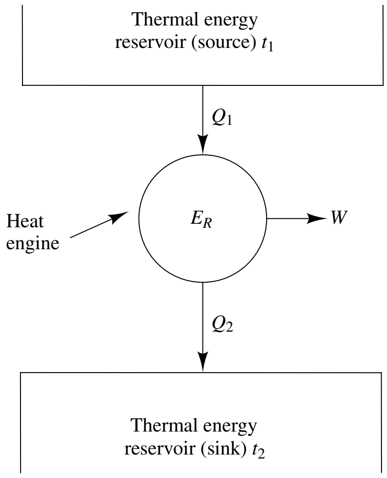


Figure 5.1 *Schematic diagram of a heat engine*

5.3.1 Thermal Efficiency of Heat Engine

An index of performance of a heat engine is expressed by the thermal efficiency η_{ther}, which is defined as the ratio of the net work done to the energy absorbed as heat.

$$\text{Thermal efficiency,} \qquad \eta_{\text{ther}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

Let Q_1 = magnitude of heat transfer between the heat engine and the source
Q_2 = magnitude of heat transfer between the heat engine and the sink
W = Work done by the heat engine

$$\eta_{\text{ther}} = \frac{W}{Q_1} \tag{5.1}$$

Again, applying the first law of thermodynamics, $W = Q_1 - Q_2$

$$\eta_{\text{ther}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \tag{5.2}$$

The thermal efficiency of a heat engine is a measure of how successfully it converts the heat received to work. It is always less than unity. Engineers are constantly trying to improve the efficiencies of heat engines.

5.4 REFRIGERATORS AND HEAT PUMPS

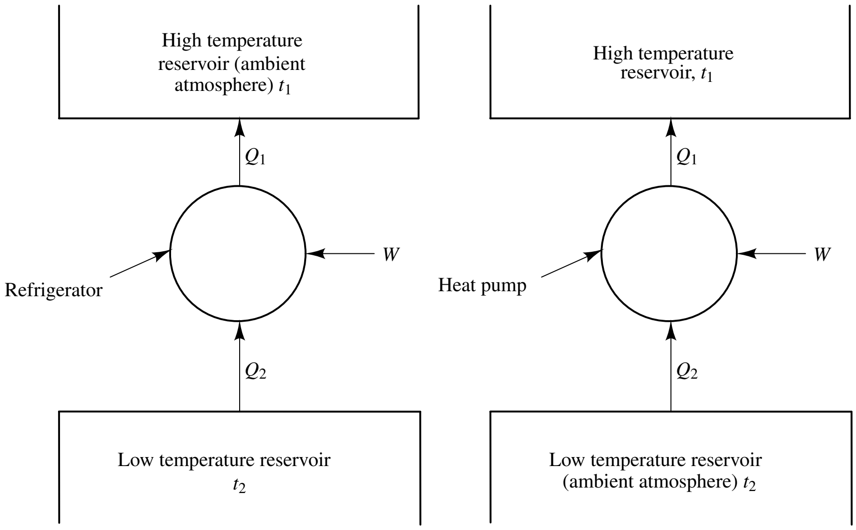
We know that heat is transferred spontaneously from a high temperature medium to a low temperature medium. The reverse process, however, cannot occur by itself. The transfer of heat from a low-temperature medium to a high temperature medium requires special devices called ***refrigerators*** and ***heat pumps***.

Refrigerators and heat pumps are simply heat engines operated in the reverse direction.

A heat pump is a device which works on a cycle, maintains temperature of a body which is more than the temperature of surroundings.

The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. A refrigerator operates between the ambient temperature and a low temperature (Fig. 5.2(a)).

The objective of a heat pump is to reject heat to a high temperature body. A heat pump operates between the ambient temperature and a high temperature (Fig. 5.2(b)).



(a) (b)

Figure 5.2 *Schematic diagram of (a) a refrigerator and (b) a heat pump*

The most popular refrigeration cycle is the vapour-compression cycle (Fig. 5.3). In this cycle, the working fluid, known as the ***refrigerant***, is compressed in the vapour phase, then condensed to a liquid, following which the pressure is dropped so that the refrigerant can evaporate at a low pressure.

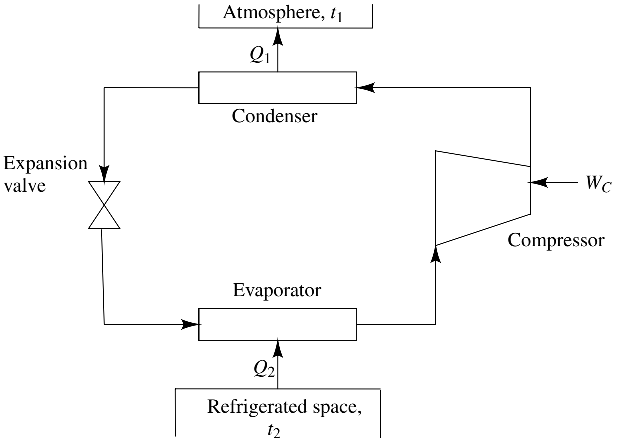


Figure 5.3 *Schematic arrangement of vapour compression refrigeration plant*

5.4.1 Coefficient of Performance of a Refrigerator and a Heat Pump

For a refrigerator, the performance parameter is not called efficiency, because that term is usually reserved for the ratio of output to input. The ratio of output to input would be misleading applied to a refrigeration system because the output is usually wasted. The concept of performance parameter of the refrigeration cycle is tantamount to efficiency of the heat engine. However, any such parameter is defined keeping the desired objective in mind.

In case of refrigeration, the prime objective is cooling and this is possible only at the expense of some work. The performance index in the refrigeration cycle is therefore, expressed (quantified) by a separate parameter called the coefficient of performance (abbreviated as COP).

Hence,
$$[\text{COP}]_R = \frac{\text{Useful refrigeration}}{\text{Net work}}$$

In a household refrigerator, the freezer compartment where heat is absorbed by the refrigerant serves as the evaporator and the coils usually behind the refrigerator where heat is dissipated to the surroundings serve as the condenser. For refrigerator, COP an be expressed as

$$\text{COP}_R = \frac{Q_2}{Q_1 - Q_2} \tag{5.3}$$

where Q_1 is the heat rejected to the high-temperature medium and Q_2 is the heat absorbed from the low temperature medium.

Similarly, the objective of a heat pump is to pump in heat to a chamber for the purpose of heating and this is also possible only when some work is expended.

Hence, $$[\text{COP}]_{HP} = \frac{\text{Heat delivered to the chamber}}{\text{Net work}}$$

$$\text{COP}_{HP} = \frac{Q_1}{Q_1 - Q_2} \tag{5.4}$$

From the expression of COP of refrigerator and heat pump, it is evident that the two are related by the following expressions.

$$\text{COP}_{HP} = \text{COP}_R + 1$$

The COP of a heat pump is greater than the COP of a refrigerator by unity.

The coefficient of performance of a heat pump is always greater than unity since COP_R is a positive quantity.

5.4.2 Heat Pump vs Electric Resistance Heater

For the purpose of heating, the use of a heat pump is more economical than an electric resistance heater.

$$\text{COP}_{HP} = \text{COP}_R + 1 \tag{5.5}$$

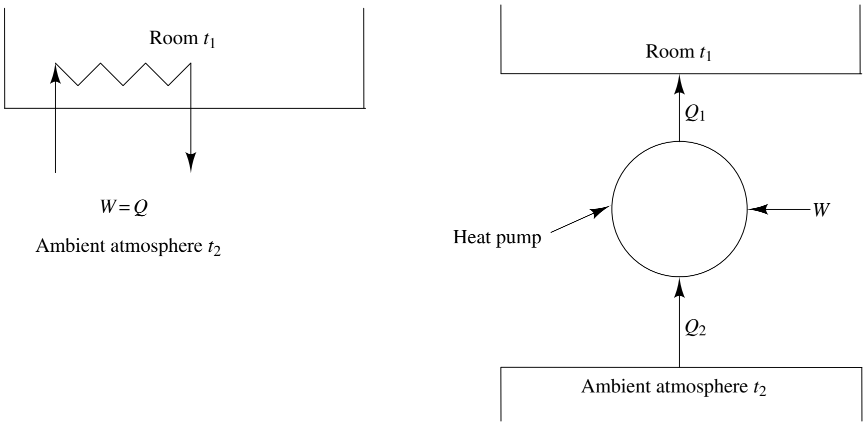
Since, COP_R is a positive quantity, from Eq. (5.5) it is clear that COP_{HP} is always greater than unity.

If W is the energy consumption of an electric resistance heater, the heat released by the heater to the space will be W only.

Now, if the same amount of energy is utilized to run a heat pump, the heat pumped to the space will be

$$Q_1 = \text{COP}_{HP} \,.\, W \tag{5.6}$$

From Eq. (5.6), Q_1 will always be greater than W.



(a) (b)

Figure 5.4 *Heating of a room by (a) an electric resistance heater and (b) a heat pump*

Let the COP_{HP} be 3.

The power consumption by an electric resistance heater

$$\dot{W} = \dot{Q}_1$$

The power consumption by a heat pump is

$$\dot{W} = \frac{\dot{Q}_1}{\text{COP}_{HP}} = \frac{\dot{Q}_1}{3}$$

Thus, the power consumption of the heat pump is lower than that of the electric resistance heater. Therefore, heat pump is more effective than electric resistance heater.

5.5 KELVIN–PLANCK STATEMENT OF THE SECOND LAW

The Kelvin-Planck statement of the second law of thermodynamics states that "*It is impossible to construct a device, which operating in a cycle, will produce no effect other than the absorption of energy as heat from a single reservoir and produces an equivalent amount of work*."

The statement implies that a heat engine which will receive heat Q_1 from a high temperature reservoir, and convert it completely into work, is impossible (Fig. 5.5). The alternative is that there must be at least another low-temperature reservoir to which heat must be rejected by the heat engine.

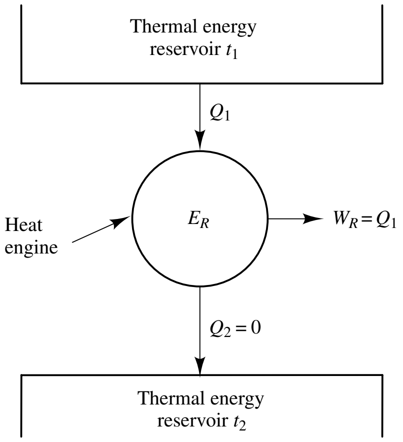


Figure 5.5 *Heat engine exchanging heat with one TER: Impossible*

5.5.1 Perpetual Motion Machine of the Second Kind

Wilhelm Ostwald introduced the concept of a perpetual motion machine of the second kind. A perpetual motion machine of the second kind is a device which would perform work solely by absorbing energy as heat from a single reservoir. Such a device does not violate the first law of thermodynamics because it would perform work at the expense of the internal energy of a body. It is to be noted that the efficiency of the perpetual motion machine of the second kind is 100%. The heat engine shown in Fig. 5.5 is an example of a perpetual motion machine of second kind. *A perpetual motion machine of the second kind* is a machine *which violates the second law of thermodynamics*.

5.6 CLAUSIUS STATEMENT OF THE SECOND LAW

The Clausius statement of the second law of thermodynamics states that "*It is impossible to construct a device, which, operating in a cycle, will produce no effect other than transfer of energy in the form of heat from a low temperature body to a high temperature body*."

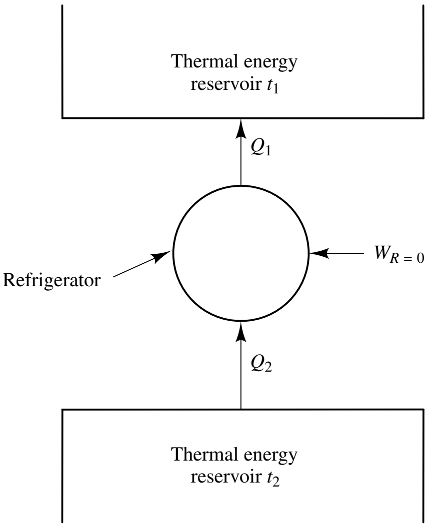


Figure 5.6 *Refrigerator without any work input: Impossible*

The statement implies that a refrigerator which will receive heat from a low-temperature reservoir and transfer it to a high-temperature reservoir is impossible (Fig. 5.6). The alternative is that there must be some work input.

5.7 EQUIVALENCE OF KELVIN–PLANCK AND CLAUSIUS STATEMENTS

At first, it appears as if the Kelvin–Planck and Clausius statements are different together. However, it can be shown that the Kelvin-Planck and Clausius statements are in fact, equivalent.

The equivalence of the Kelvin–Planck and Clausius statements is demonstrated by showing that the violation of each statement implies the violation of the other.

To prove that violation of the Kelvin–Planck statement leads to a violation of the Clausius statement, let us suppose that the Kelvin–Planck statement is wrong. That is, it is possible to construct a heat engine, which, operating on a cycle, absorbs heat (Q_1) from a source at temperature t_1 and produces an equivalent amount of work ($W = Q_1$) (Fig. 5.7).

Suppose that a heat pump also operates between the same two reservoirs and uses up all the work produced by the heat engine. The engine and pump together constitute a heat pump that transfers heat from the low-temperature reservoir to the high-temperature reservoir without producing any changes elsewhere. Therefore, the engine and the pump together constitute a heat pump (Fig. 5.7) that violates the Clausius statement.

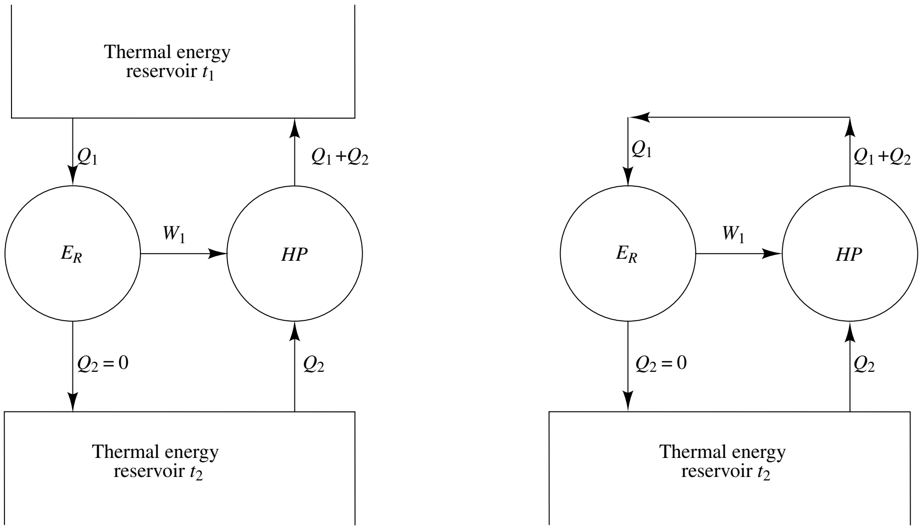


Figure 5.7 *Violation of the Kelvin–Planck statement leads to a violation of the Clausius statement*

To prove that the violation of the Clausius statement leads to a violation of the Kelvin–Planck statement, let us suppose that the Clausius statement is wrong. Consider a heat pump (Fig. 5.8) that requires no work to transfer heat from a low-temperature to a high-temperature reservoir and that, therefore, violates the Clausius statement.

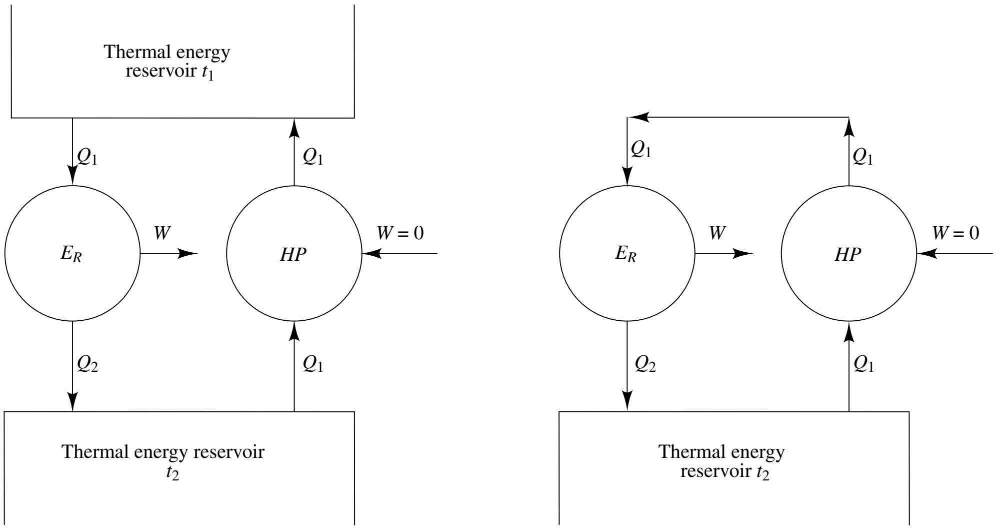


Figure 5.8 *Violation of the Clausius statement leads to a violation of the Kelvin–Planck statement*

Let us assume a heat engine operates between the same two reservoirs in such a way that the engine draws an amount of heat which is delivered by the heat pump. The engine and pump together constitute a heat engine (Fig. 5.8) that produces no effect other than the absorption of energy as heat from a single reservoir and produces an equivalent amount of work. Therefore, the engine and the pump together constitute a heat engine that violates the Kelvin–Planck statement.

5.8 REVERSIBLE AND IRREVERSIBLE PROCESSES

A process is said to be reversible if at the conclusion of the process, both the system and the surroundings can be restored to their respective initial states without producing any changes in the rest of the universe. Otherwise, the process is said to be irreversible.

A system can, however, always be restored to its initial state, but this does not imply that the process is reversible. Only if the surroundings are also simultaneously restored to their initial states, the process becomes reversible.

A process is called ***internally reversible*** if no irreversibilities occur within the system boundary during the process. The quasi-equilibrium process is an example of an internally reversible process.

A process is called ***externally reversible*** if no irreversibilities occur outside the system boundary during the process.

A process is called ***reversible*** if no irreversibilities occur within or outside the system boundary during the process. That is, a process is reversible if it is *internally* as well as *externally reversible.*

The characteristics of irreversible processes are the following:

(a) An irreversible process can be carried out in one direction only.
(b) An irreversible process occurs at a finite rate.

An irreversible process cannot be reversed without causing permanent changes in the surroundings.

During an irreversible process, the system is not in equilibrium.

Factors that make processes irreversible are

(i) heat transfer through normal a temperature difference
(ii) friction
(iii) free expansion
(iv) mixing of substances
(v) combustion

5.9 THE CARNOT CYCLE

According to the second law of thermodynamics, a perpetual motion machine of the second kind is impossible, i.e., without exchanging with two thermal energy reservoirs at two different temperatures, it is impossible to produce work in a cycle.

From the second law of thermodynamics, it has been observed that the efficiency of a heat engine cannot be equal to unity. It is, therefore, important to answer the following questions:

(a) What is the maximum thermal efficiency of a heat engine?
(b) What are the characteristics of such an engine?

These can be answered by considering the Carnot cycle. French military engineer, Nicholas Sadi Carnot (1796–1832), was the first to introduce the idea of a reversible cycle which is named after him. The theoretical heat engine that operates on the Carnot cycle is called the Carnot heat engine. The Carnot cycle has four basic processes. These are

- a reversible isothermal process in which heat is transferred from the high temperature reservoir
- a reversible adiabatic process in which the temperature of the working fluid decreases from high temperature to low temperature
- a reversible isothermal process in which heat is transferred to the low temperature reservoir
- a reversible adiabatic process in which the temperature of the working fluid increases from the low temperature to the high temperature.

A Carnot cycle on the *P–V* diagram is shown in Fig. 5.9.

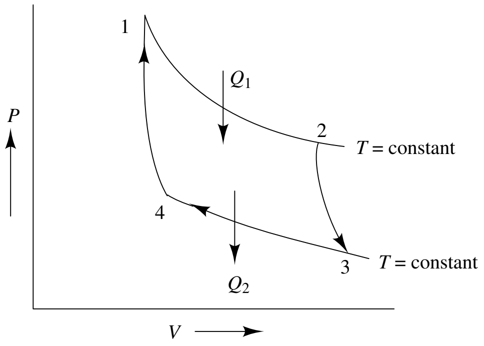


Figure 5.9 *Carnot cycle on P–V diagram*

5.9.1 The Reversed Carnot Cycle

All the processes of the Carnot heat engine cycle are reversible. If the directions of energy interactions are reversed, then the cycle becomes Carnot refrigeration cycle (or reversed Carnot cycle). A reversed Carnot cycle on the *P–V* diagram is shown in Fig. 5.10.

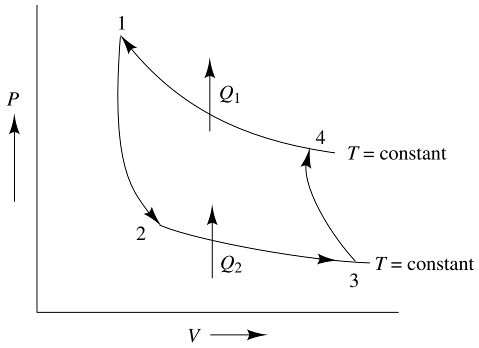


Figure 5.10 *Reversed Carnot cycle on P–V diagram*

5.10 COROLLARIES OF THE SECOND LAW OF THERMODYNAMICS

Clausius statement of the second law is known as the first corollary of the second law of thermodynamics.

Corollary 2: *It is impossible to construct a heat engine operating between only two reservoirs, which will have a higher efficiency than a reversible heat engine operating between the same two reservoirs.*

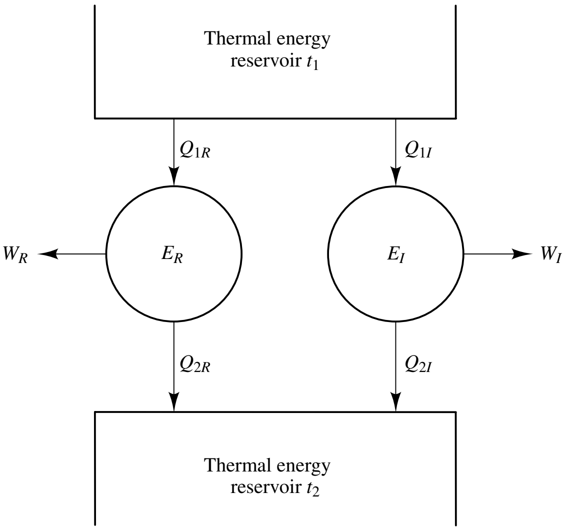


Figure 5.11 *Two heat engines operating between the same two reservoirs*

Let us assume that Corollary 2 (also known as Carnot Theorem 1) is wrong, i.e., it is possible to construct an irreversible heat engine E_I, which operates between the same thermal energy reservoirs as that of a reversible heat engine E_R such that the efficiency of the irreversible engine is greater than that of the reversible engine (Fig. 5.11). By assumption, the efficiency of the reversible engine E_R is less than the efficiency of the irreversible engine E_I.

$$\eta_I > \eta_R$$

Let the rate of working of the engines be such that $Q_{1R} = Q_{1I}$

Thermal efficiency of reversible engine $\eta_R = \dfrac{W_R}{Q_{1R}}$

Thermal efficiency of irreversible engine $\eta_I = \dfrac{W_I}{Q_{1I}}$

Since, $\eta_I > \eta_R$

$$W_I > W_R$$

Therefore, for the same amount of energy input, the reversible engine E_R does less work than the irreversible engine E_I. Since the heat engine E_R is a reversible, it can be executed in a reversed order i.e., the magnitudes of energy transfer will remain the same, but the direction will be the reversed. Since W_I is greater than W_R, some part of W_I (equal to W_R) may be fed to execute the reversed heat engine E_R.

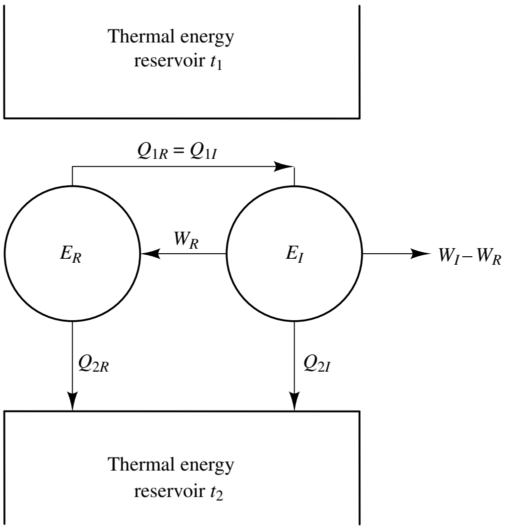


Figure 5.12 *Combined system violates the Kelvin–Planck statement of the second law*

Now, E_R and E_I together constitute a heat engine as shown in Fig. 5.12, which, operating in a cycle, delivers net work $W_I - W_R$. The combined device absorbs energy as heat $(Q_{2R} - Q_{2I})$ from the thermal energy reservoir at t_2, and does equivalent amount of work without rejecting energy to thermal energy reservoir at t_1. Thus, it becomes a perpetual motion machine of the second kind which violates the Kelvin–Planck statement of the second law of thermodynamics. Hence, the assumption that the efficiency of the irreversible engine is greater than the efficiency of the reversible engine is wrong.

Therefore, $\eta_R \geq \eta_I$

Corollary 3**:** *All reversible heat engines operating between the same two reservoirs have the same efficiency.*

Consider two reversible heat engines E_{R1} and E_{R2} are operating between the same two thermal energy reservoirs as shown in Fig. 5.13.

Let us assume that the efficiency of the reversible heat engine E_{R1} is greater than the efficiency of the reversible heat engine E_{R2}.

$$\eta_{R1} > \eta_{R2}$$

Let the rate of working of the engines be such that $Q_{11} = Q_{12}$

Thermal efficiency of reversible engine E_{R1}, $\eta_{R1} = \dfrac{W_R}{Q_{11}}$

Thermal efficiency of reversible engine E_{R2}, $\eta_{R2} = \dfrac{W_2}{Q_{12}}$

Since, $\eta_{R1} > \eta_{R2}$

$$W_1 > W_2$$

Since the heat engine E_{R2} is a reversible one, it can be executed in a reverse order, i.e., the magnitudes of energy transfer will remain the same, but the direction will be reversed. Since W_1 is greater than W_2, some part of W_1 (equal to W_2) may be fed to execute the reversed heat engine E_{R2}.

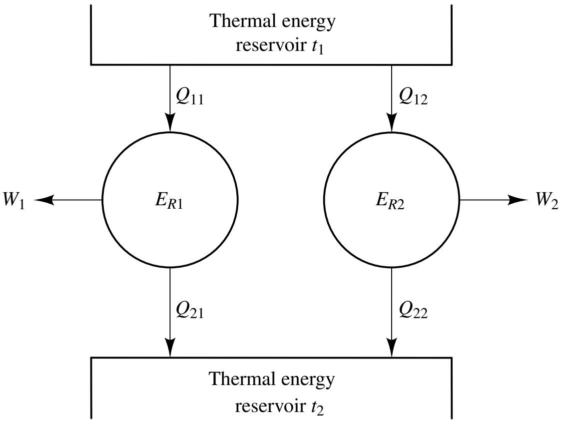


Figure 5.13

Now, E_{R1} and E_{R2} together constitute a heat engine as shown in Fig. 5.14, which operating in a cycle, delivers net work $W_1 - W_2$. The combined device absorbs energy as heat ($Q_{22} - Q_{21}$) from the thermal energy reservoir at t_2, and does equivalent amount of work without rejecting energy to the thermal energy reservoir at t_1. Thus, it becomes a perpetual motion machine of second kind which violates the Kelvin–Planck statement of the second law of thermodynamics. Hence the assumption that the efficiency of the reversible engine E_{R1} is greater than the efficiency of the reversible engine E_{R2} is wrong.

Therefore, $$\eta_{R2} \geq \eta_{R1} \tag{5.7}$$

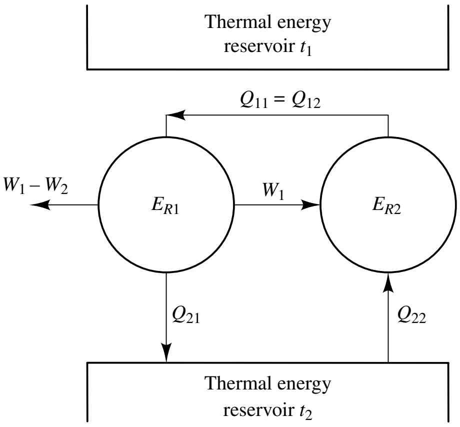


Figure 5.14 *Combined heat engines E_{R1} and E_{R2} violates the Kelvin–plank statement of the second law*

Now, let us assume that the efficiency of the reversible heat engine E_{R2} is greater than the efficiency of the reversible heat engine E_{R1}.

The reversible heat engine E_{R1} can be executed in a reversed, utilizing part of work done by E_{R2}. By following a similar argument we can arrive at the result that

$$\eta_{R1} \geq \eta_{R2} \tag{5.8}$$

From Eqs (5.7) and (5.8), it can be concluded that

$$\eta_{R1} = \eta_{R2} \tag{5.9}$$

This is also known as Carnot Theorem 2. This theorem states *that the efficiency of a reversible heat engine is independent of the nature of the working substance and depends only on the temperature of the reservoirs between which it operates.*

Corollary 4: *A scale of temperature can be defined which is independent of any thermometric substance, and which provides an absolute zero of temperature.*

Consider three reversible heat engines E_1, E_2 and E_3 operating between temperatures t_1 and t_3 as shown in Fig. 5.15. Heat engine E_3 extracts heat from high temperature at t_1 and rejects it to the low temperature reservoir at t_3. Heat engine E_1 operates between the high temperature reservoir at t_1 and an intermediate reservoir at t_2, while the heat engine E_2 operates between the intermediate reservoir at t_2 and the low-temperature reservoir at t_3. Let heat engines E_1 and E_3 receive the same amount of heat from the high temperature source at t_1.

The efficiency of the combined reversible heat engines (E_1 and E_2) must be the same as the single reversible heat engine (E_3), since as per Corollary 3, the efficiency of all reversible heat engines operating between the same source and sink is equal.

The amount of heat rejected by the combined reversible heat engines (E_1 and E_2) will be same as that of E_3 i.e., Q_3

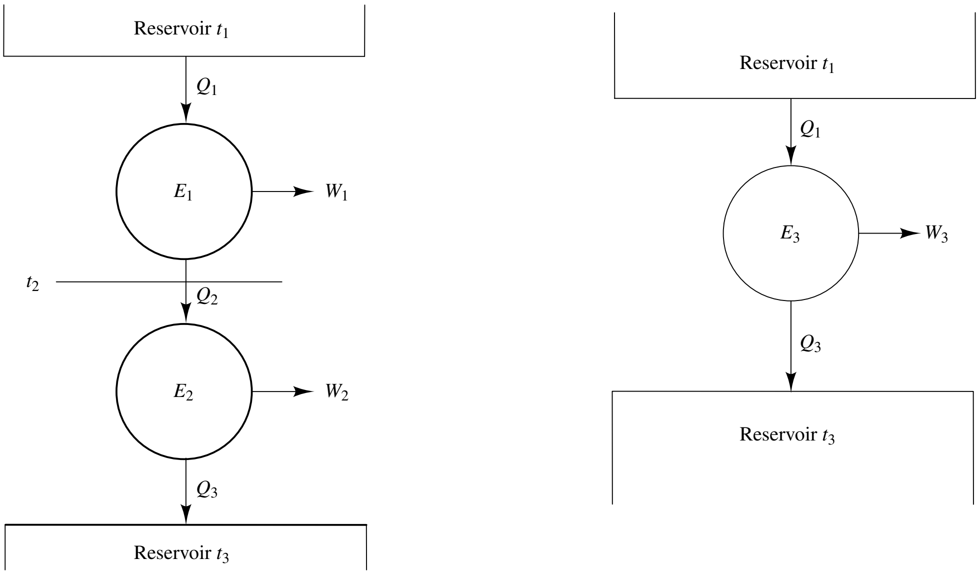


Figure 5.15 *Schematic arrangement of heat engines to develop the thermodynamic temperature scale*

As per Corollary 3, the efficiency of reversible heat engine E_1

$$\eta_1 = 1 - \frac{Q_2}{Q_1} = f\left(t_1, t_2\right)$$

In terms of a new function,

$$\frac{Q_1}{Q_2} = \varphi(t_1, t_2)$$

Similarly, for reversible heat engines E_2 and $E_{3,}$ we have

$$\frac{Q_2}{Q_3} = \varphi(t_2, t_3)$$

$$\frac{Q_1}{Q_3} = \varphi(t_1, t_3)$$

Now,

$$\frac{Q_1}{Q_2} = \frac{\frac{Q_1}{Q_3}}{\frac{Q_2}{Q_3}} = \frac{\varphi\left(t_1, t_3\right)}{\varphi\left(t_2, t_3\right)}$$

$$\frac{Q_1}{Q_2} = \varphi\left(t_1, t_2\right) = \frac{\varphi\left(t_1, t_3\right)}{\varphi\left(t_2, t_3\right)} \tag{5.10}$$

Since t_3 does not appear in the left hand side of the Eq. (5.10), t_3 must, therefore, drop out from the ratio on the right hand side of equation. The numerator can be written as $\phi(t_1)$ and the denominator as $\phi(t_2)$, where ϕ is another unknown function of one temperature.

The ratio $\frac{Q_1}{Q_2}$ depends only on t_1 and t_2 and is independent of t_3.

$$\frac{Q_1}{Q_2} = \varphi(t_1, t_2) = \frac{\phi(t_1)}{\phi(t_2)}$$

In 1848, Kelvin proposed a linear function of temperature because all the scientific and engineering data had been obtained from the mercury-in-glass thermometer, which is essentially linear over its useful range.

$$\frac{Q_1}{Q_2} = \frac{t_1 + c}{t_2 + c} = \frac{T_1}{T_2} \tag{5.11}$$

where c is a constant and T is the absolute temperature.

This temperature scale is called the Kelvin scale, and the temperatures on this scale are called absolute temperature. The triple point of water is taken as the standard reference point. At the International Conference on Weights and Measures held in 1954, the triple point of water was assigned the value of 273.16 K.

$$\frac{Q_1}{Q_{TP}} = \frac{T}{T_{TP}}$$

$$T = 273.16 \frac{Q}{Q_{TP}} \tag{5.12}$$

Consider, a series of reversible heat engines (Fig. 5.16), each operating between only two reservoirs and each producing same quantity of work:

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$$

$$Q_1 - Q_2 = (T_1 - T_2)\ \frac{Q_2}{T_2}$$

Similarly,

$$Q_2 - Q_3 = (T_2 - T_3)\ \frac{Q_3}{T_3} = \left(T_2 - T_3\right)\frac{Q_2}{T_2}$$

$$Q_3 - Q_4 = (T_3 - T_4)\ \frac{Q_2}{T_2}$$

$$T_1 - T_2 = T_2 - T_3 = T_3 - T_4 = \ldots$$

If sufficient number of heat engines are placed in series to make the total work output equal to the heat input to the first heat engine then the heat rejected from the last engine will be zero.

If the heat rejected from the last heat engine is zero then the temperature of the last reservoir will be zero. However, if the heat rejection from a cyclic heat engine is zero, then it violates the Kelvin–Planck statement of the second law of thermodynamics.

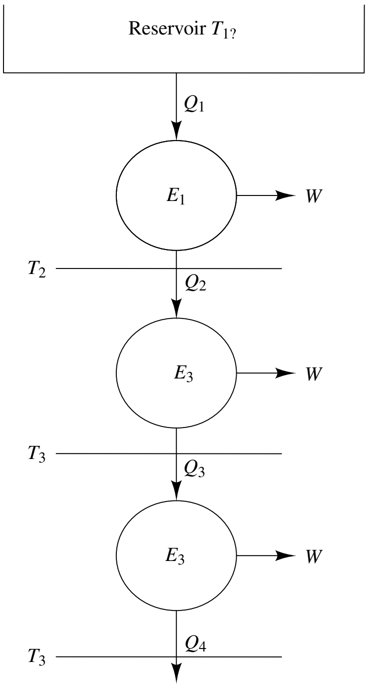


Figure 5.16 *Heat engines in series*

5.11 CARNOT HEAT ENGINE, REFRIGERATOR AND HEAT PUMP

5.11.1 Carnot Heat Engine

The heat engine that operates on the Carnot cycle is called the *Carnot heat engine*. The thermal efficiency of any heat engine is given by

$$\eta_{\text{ther}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

where Q_1 is the heat transferred to the heat engine from a high temperature reservoir at T_1 and Q_2 is the heat rejected to a low temperature reservoir at T_2.

From the thermodynamic temperature scale

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

Then the efficiency of a Carnot heat engine becomes

$$\eta_{\text{ther, Carnot}} = 1 - \frac{T_2}{T_1} \tag{5.13}$$

5.11.2 Carnot Refrigerator and Carnot Heat Pump

A refrigerator of a heat pump that operates on the reversed Carnot cycle is called a Carnot refrigerator or a Carnot heat pump. The coefficient of performance of any refrigerator or heat pump is given by

$$\text{COP}_R = \frac{Q_2}{Q_1 - Q_2} \tag{5.14}$$

$$\text{COP}_R = \frac{Q_1}{Q_1 - Q_2} \tag{5.15}$$

where Q_1 is the heat rejected to the high-temperature medium (T_1) and Q_2 is the heat absorbed from the low-temperature medium (T_2).

From the thermodynamic temperature scale

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

Then the coefficient of performance becomes

$$\text{COP}_R = \frac{T_2}{T_1 - T_2} \tag{5.16}$$

$$\text{COP}_{HP} = \frac{T_1}{T_1 - T_2} \tag{5.17}$$

Example 5.1 Which is the more effective way to increase the efficiency of a Carnot Engine?

(i) to increase T_1, keeping T_2 constant
(ii) to decrease T_2, keeping T_1 constant.

Solution: The efficiency of a Carnot engine is given by $\eta = 1 - \frac{T_2}{T_1}$

Let T_1 be increased by ΔT, keeping T_2 constant. Then, efficiency becomes

$$\eta_1 = 1 - \frac{T_2}{T_1 + \Delta T}$$

Let T_2 be decreased by same ΔT, keeping T_1 constant. Then, efficiency becomes

$$\eta_2 = 1 - \frac{T_2 - \Delta T}{T_1}$$

$$\eta_2 - \eta_1 = 1 - \frac{T_2 - \Delta T}{T_1} - 1 + \frac{T_2}{T_1 + \Delta T}$$

$$= \frac{T_1 T_2 - (T_1 + \Delta T)(T_2 - \Delta T)}{T_1 (T_1 + \Delta T)}$$

$$= \frac{(T_1 + T_2)\Delta T + (\Delta T)^2}{T_1 (T_1 + \Delta T)}$$

Since $T_1 > T_2$, $\eta_2 - \eta_1 > 0$
The more effective way to increase the efficiency of Carnot engine is to decrease the lower temperature T_2.

Example 5.2 If the thermal efficiency of a Carnot engine is $\frac{1}{6}$, calculate the coefficient of performance of (i) a Carnot heat pump, and (ii) a Carnot refrigerator.

Solution The efficiency of a Carnot engine is given by Eq. (5.13) as $\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1}$

where, T_1 is the source temperature and T_2 is the sink temperature.

Therefore, $\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1} = \frac{1}{6}$

or, $$\frac{T_1 - T_2}{T_1} = \frac{1}{6}$$

or, $$\frac{T_1}{T_1 - T_2} = 6$$

The coefficient of performance of Carnot heat pump is (Eq. (5.17))

$$\text{COP}_{HP} = \frac{T_1}{T_1 - T_2} = 6$$

From Eq. (5.5), we have

$$\text{COP}_{HP} = \text{COP}_R + 1$$

Therefore, the coefficient of performance of Carnot refrigerator is $\text{COP}_R = 5$.

Example 5.3 A Carnot heat engine operates between a source at 1000 K and a sink at 300 K. If the heat engine is supplied with heat at a rate of 800 kJ/min. Determine (a) the thermal efficiency, and (b) the power output of this heat engine.

Solution Temperature of source, $T_1 = 1000$ K
Temperature of sink, $T_2 = 300$ K

(a) Thermal efficiency of a Carnot cycle is to be (Eq. (5.13))

$$\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{300}{1000} = 0.7 \text{ or } 70\%$$

(b) From Eq. (5.1), one can write

$$\eta_{\text{th, Carnot}} = 0.7 = \frac{\dot{W}_{\text{net}}}{\dot{Q}_1} = \frac{\dot{W}_{\text{net}}}{\frac{800}{60}}$$

or, $$\dot{W}_{\text{net}} = 9.33 \text{ kJ/s} = 9.33 \text{ kW}$$

Example 5.4 A Carnot heat engine receives 650 kJ of heat from a source of unknown temperature and rejects 250 kJ of it to a sink at 24°C. Determine the thermal efficiency of the heat engine and the temperature of the source.

Solution Temperature of sink is given, $T_2 = 24°\text{C} = 297$ K
Let the temperature of source be T_1 K
Heat transferred to the Carnot engine is, $Q_1 = 650$ kJ

Heat rejected by the Carnot engine is $Q_2 = 250$ kJ

Thermal efficiency of the heat engine is found to be (Eq. (5.1))

$$\eta_{\text{th,Carnot}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{250}{650} = 0.6154$$

The efficiency of a Carnot cycle is found to be (Eq. (5.13))

$$\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1}$$

Thus, $$1 - \frac{297}{T_1} = 0.6154$$

or, $$T_1 = 772.23 \text{ K} = 499.23 \text{ C}$$

Example 5.5 A heat engine operates between a source at 550°C and a sink at 25°C. If heat is supplied to the heat engine at a steady rate of 1200 kJ/min. Determine the maximum power output of this heat engine.

Solution A heat engine delivers maximum power only when it is a Carnot heat engine.

Thermal efficiency of Carnot heat engine is

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{25+273}{550+273} = 1 - \frac{298}{823} = 0.6379$$

Rate of heat transferred to the engine is given as $\dot{Q}_1 = 1200 \text{ kJ/min} = 1200 \text{ kJ/60s} = 20 \text{ kJ/s} = 20 \text{ kW}$

Now, from the condition of maximum power delivery, we get

$$\eta_{\text{th, Carnot}} = \frac{\dot{W}_{\max}}{Q_1}$$

or, $$\dot{W}_{\max} = \dot{\eta}_{\max} \times \dot{Q}_1 = 0.6379 \times 20 = 12.76 \text{ kW}$$

Example 5.6 A Carnot heat engine is operating between a source at T_1 and a sink at T_2. If it is desired to double the thermal efficiency of this engine by changing only the source temperature keeping sink temperature constant, what should be the new source temperature be?

Solution The efficiency of a Carnot engine is given by Eq. (5.13) as $\eta_{\text{Carnot}} = 1 - \dfrac{T_2}{T_1}$

Let T_{new} be the source temperature so that the efficiency becomes $2\eta_{\text{Carnot}}$. Thus,

$$1-\frac{T_2}{T_{\text{new}}} = 2\eta_{\text{Carnot}}$$

or, $$1-\frac{T_2}{T_{\text{new}}} = 2\left(1-\frac{T_2}{T_1}\right)$$

or $$1-\frac{T_2}{T_{\text{new}}} = 2-\frac{2T_2}{T_1}$$

or, $$\frac{T_2}{T_{\text{new}}}-\frac{2T_2}{T_1} = 1$$

or, $$\frac{T_2}{T_{\text{new}}} = \frac{2T_2}{T_1}-1$$

or, $$T_{\text{new}} = \frac{T_2}{\dfrac{2T_2}{T_1}-1}$$

Example 5.7 Consider a heat engine that receives heat at the rate of 1 MW at a high temperature of 550°C and rejects energy to the ambient at 27°C while work is produced at the rate of 450 kW. Find out how much heat is discarded to the ambient and engine efficiency. Also compare both of these values with a Carnot heat engine operating between the same two reservoirs.

Solution Rate of Heat transferred to the engine is $\dot{Q}_1 = 1\text{ MW} = 1000\text{ kW}$

Rate of work output is $\dot{W} = 450\text{ kW}$

Thermal efficiency of engine is found to be

$$\eta_{\text{th, Engine}} = \frac{\dot{W}}{\dot{Q}_1} = \frac{450}{1000} = 0.45 \text{ or } 45\%$$

Rate of heat discarded by the engine is $\dot{Q}_2 = \dot{Q}_1 - \dot{W} = 1000 - 450 = 550\text{ kW}$

Thermal efficiency of Carnot heat engine is found to be

$$\eta_{\text{th, Carnot}} = 1-\frac{T_2}{T_1} = 1-\frac{27+273}{550+273} = 1-\frac{300}{823} = 0.6355 \text{ or } 63.55\%$$

Rate of work output by Carnot heat engine

$$\dot{W}_{\text{Carnot}} = \eta_{\text{Carnot}}\dot{Q}_1 = 0.6355\times 1000 = 635.5\text{ W}$$

$$\frac{\eta_{\text{engine}}}{\eta_{\text{Carnot heat engine}}} = \frac{0.45}{0.6355} = 0.708$$

Example 5.8 An inventor comes to an industrialist and claims to have developed a heat engine that receives 700 kJ of heat from a source at 500 K and produces 300 kJ of net work while rejecting the waste heat to a sink at 290 K. What would you advise to the industrialist, he should invest or not.

Solution The maximum possible efficiency of a heat engine is the same as the efficiency of a Carnot heat engine operating between the same source and sink and is given by

$$\eta_{\text{th, max}} = \eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1}$$

where, T_1 is the source temperature and T_2 is the sink temperature.

The efficiency of a Carnot heat engine is

$$\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1} = 1 - \frac{290}{500} = 0.42$$

The efficiency of the heat engine is

$$\eta_{\text{ther}} = \frac{W}{Q_1} = \frac{300}{700} = 0.4285$$

Since the efficiency of the heat engine is higher than the efficiency of Carnot heat engine operating between the same source and sink, his claim is not reasonable.

Example 5.9 An inventor claims to have developed a refrigeration system that removes heat from the closed region at –12°C and transfers it to the surroundings at 25°C while maintaining a COP of 6.5. Is this a reasonable claim? Why?

Solution The maximum COP of a refrigeration system is the same as the COP of a Carnot refrigeration system operating between the same source (T_1) and sink (T_2) and is given by (see Eq. (5.16))

$$\text{COP}_{\text{Carnot}} = \frac{T_2}{T_1 - T_2} = \frac{261}{298 - 261} = 7.05$$

Actual COP of the refrigeration system is given as 6.5.

Since the actual COP is less than that of Carnot refrigeration system, the claim is reasonable.

Example 5.10 Two Carnot heat engines work in series between the source and sink temperatures of 600 K and 300 K respectively. If both engines develop equal power, determine the intermediate temperature.

Solution The arrangement is shown in Fig. 5.17.

Let the intermediate temperature be T_2.

For the heat engine 1,

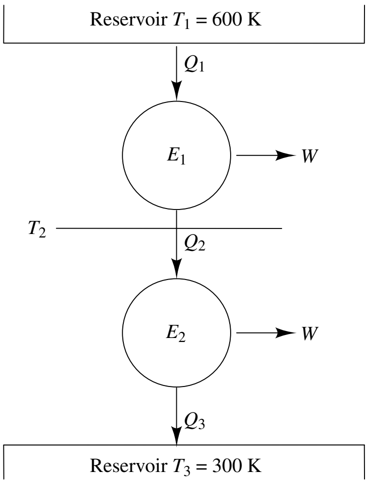


Figure 5.17

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$$

$$\frac{W}{Q_2} = \frac{T_1 - T_2}{T_2} \quad (5.18)$$

For the heat engine 2,

$$\frac{Q_3}{Q_2} = \frac{T_3}{T_2}$$

$$\frac{Q_2 - Q_3}{Q_2} = \frac{T_2 - T_3}{T_2}$$

$$\frac{W}{Q_2} = \frac{T_2 - T_3}{T_2} \quad (5.19)$$

Equating Eqs (5.18) and (5.19), we get

$$T_1 - T_2 = T_2 - T_3$$

$$T_2 = \frac{T_1 + T_3}{2} = \frac{600 + 300}{2} = 450 \text{ K}$$

The intermediate temperature is 450 K.

Example 5.11 Two Carnot heat engines work in series between the source and sink temperatures of 900 K and 400 K respectively. If both engines have equal efficiencies, determine the intermediate temperature.

Solution The arrangement is shown in Fig. 5.18.

For the heat engine 1,

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$\eta_1 = \frac{T_1 - T_2}{T_1} \tag{5.20}$$

For the heat engine 2,

$$\frac{Q_3}{Q_2} = \frac{T_3}{T_2}$$

$$\frac{Q_2 - Q_3}{Q_2} = \frac{T_2 - T_3}{T_2}$$

$$\eta_2 = \frac{T_2 - T_3}{T_2} \tag{5.21}$$

Equating Eqs (5.20) and (5.21), we get

$$T_2^2 = T_1 T_3$$

$$T_2 = \sqrt{T_1 T_3} = \sqrt{900 \times 400} = 600 \text{ K}$$

The intermediate temperature is 600 K.

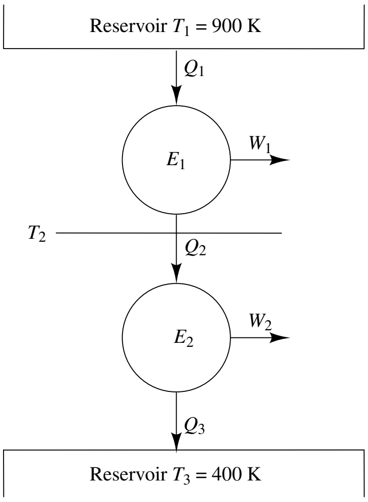


Figure 5.18

Example 5.12 A reversible power cycle is used to drive a reversible heat pump cycle. The power cycle takes in Q_1 heat units at T_1 and rejects Q_2 heat units at T_2. The heat pump abstracts Q_4 from the sink at T_4 and discharges Q_3 at T_3. Develop an expression for the ratio $\frac{Q_3}{Q_1}$ in terms of the four temperatures.

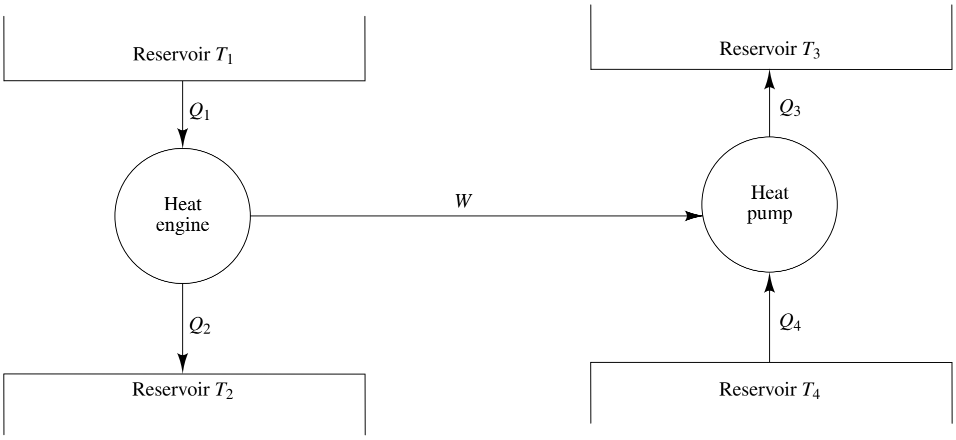


Figure 5.19

Solution The arrangement is depicted in Fig. 5.19. From reversible heat engine cycle, we get

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

or,

$$W = Q_1\left(1 - \frac{T_2}{T_1}\right)$$

From reversible heat pump cycle, we have

$$\text{COP}_{HP} = \frac{Q_3}{W} = \frac{T_3}{T_3 - T_4}$$

or,

$$W = Q_3\left(1 - \frac{T_4}{T_3}\right)$$

Since heat engine drives the heat pump, therefore

$$Q_3\left(1 - \frac{T_4}{T_3}\right) = Q_1\left(1 - \frac{T_2}{T_1}\right)$$

$$\frac{Q_3}{Q_1} = \frac{\dfrac{T_1 - T_2}{T_1}}{\dfrac{T_3 - T_4}{T_3}} = \frac{T_3(T_1 - T_2)}{T_1(T_3 - T_4)}$$

Example 5.13 A reversible heat engine which takes in heat from a reservoir at 840ºC and rejects heat to a reservoir at 60 ºC. The heat engine drives a reversible heat pump which takes in heat from a reservoir at 5 ºC and delivers heat to a reservoir at 60 ºC. The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from the 5 ºC reservoir, determine (i) the rate of heat supply from the 840 ºC source, and (ii) the rate of heat ejection to 60 ºC sink.

Solution The arrangement is schematically presented in Fig. 5.20.

The efficiency of the reversible heat engine is

$$\eta_{\text{HE}} = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{333}{1113} = 0.7008$$

The coefficient of performance of the reversible heat pump is

$$\text{COP}_{\text{HP}} = \frac{Q_3}{W_{\text{HP}}} = \frac{T_3}{T_3 - T_4} = \frac{333}{333 - 278} = 6.05$$

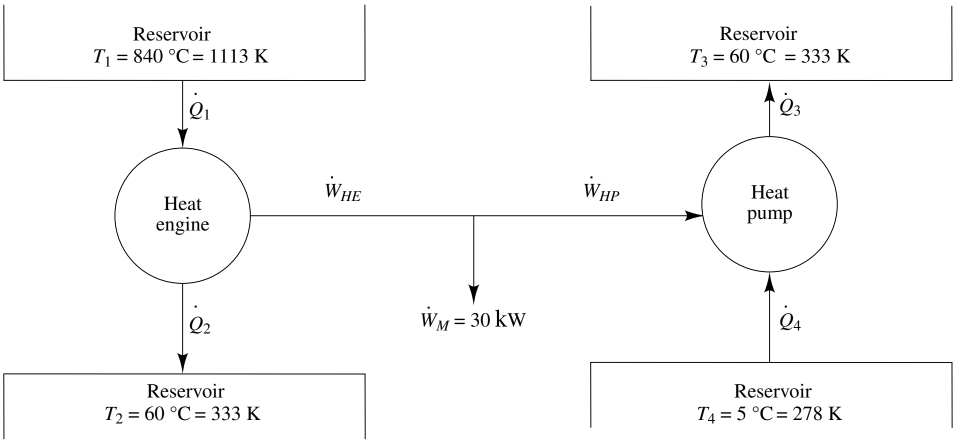


Figure 5.20

Further, COP can be expressed as

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_3}{\dot{Q}_3 - \dot{Q}_4} = 6.05$$

or $$\dot{Q}_3 = 30.37 \text{ kW} \qquad \left[\because \dot{Q}_4 = 17 \text{ kW}\right]$$

Thus, $$\dot{W}_{\text{HP}} = \dot{Q}_3 = \dot{Q}_4 = 20.37 - 17 = 3.37 \text{ kW}$$

The rate of work done by the heat engine is

$$\dot{W}_{\text{HE}} = 30 + 3.37 = 33.37 \text{ kW}$$

Further, the efficiency of the heat engine can be expressed as

$$\eta_{\text{HE}} = \frac{\dot{W}_{\text{HE}}}{Q_1} = 0.7008$$

or $$\dot{Q}_1 = \frac{\dot{W}_{\text{HE}}}{\eta_{\text{HE}}} = \frac{33.37}{0.7008} = 47.62 \text{ kW}$$

Thus $$\dot{Q}_2 = \dot{Q}_1 - \dot{W}_{\text{HE}} = 47.62 - 33.37 = 14.25 \text{ kW}$$

Rate of heat rejection to 60 ºC sink is $\dot{Q}_2 + \dot{Q}_3 = 14.25 + 20.37 = 34.62$ kW

Example 5.14 Electric solar cells can produce power with 12 % efficiency. Assume a heat engine with a low temperature heat rejection at 27°C driving an electric generator with 85 % efficiency. What should the effective high temperature in the heat engine be to have the same overall efficiency as the solar cells?

Solution Given data:

Efficiency of solar cell,

$$\eta_{SC} = 0.12$$

Efficiency of electric generator,

$$\eta_{EG} = 0.85$$

Temperature of sink of heat engine,

$$T_2 = 27°\text{C} = 300\text{K}$$

Efficiency of heat engine,

$$\eta_{HE} = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

Now, overall efficiency of heat engine and electrical generator,

$$\eta_{\text{overall}} = \eta_{HE} \times \eta_{EG}$$

Again, $$\eta_{\text{overall}} = \eta_{SC} = 0.12$$

$$0.12 = \eta_{HE} \times \eta_{EG}$$

$$\eta_{HE} = \frac{\eta_{\text{overall}}}{\eta_{EG}} = \frac{0.12}{0.85} = 0.141$$

$$1 - \frac{T_2}{T_1} = 0.141$$

$$\frac{T_2}{T_1} = 0.859$$

$$T_1 = \frac{T_2}{0.859} = \frac{300}{0.859} = 349.24\text{ K}$$

Corollary 5: *The efficiency of any reversible engine operating between more than two reservoirs must be less than that of a reversible engine operating between two reservoirs which have temperatures equal to the highest and lowest temperatures of the fluid in the original engine.*

Let a reversible heat engine operates between more than two reservoirs say 3 as shown in Fig. 5.21. Reversible heat engine E_1 operates between heat reservoirs A, B and C at temperatures T_A, T_B and T_C ($T_A > T_B > T_C$) respectively. The engine receives Q_A and Q_B quantities of heat from reservoir A and B respectively and rejects Q_C heat to reservoir C. Let the efficiency of the above mentioned engine be η_1.

Then

$$\eta_1 = \frac{W_1}{Q_A + Q_B}$$

$$= 1 - \frac{Q_C}{Q_A + Q_B} \tag{5.22}$$

From fourth corollary of the second law of thermodynamics,

$$\frac{Q_A}{T_A} + \frac{Q_B}{T_B} = \frac{Q_C}{T_C}$$

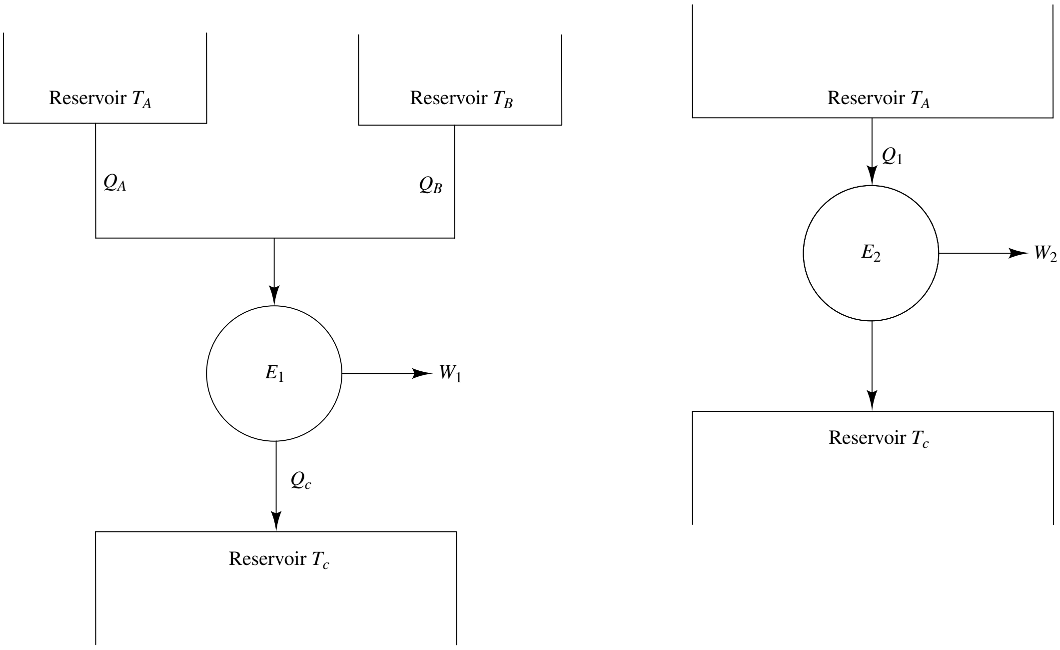


Figure 5.21

Equation (5.22) becomes

$$\eta_1 = 1 - \frac{T_C\left(\dfrac{Q_A}{T_A} + \dfrac{Q_B}{T_B}\right)}{Q_A + Q_B}$$

$$= 1 - \frac{\dfrac{T_C}{T_A}\left(Q_A + \dfrac{T_A}{T_B}Q_B\right)}{Q_A + Q_B}$$

Let a reversible heat engine E_2 operates between heat reservoirs A and C at temperatures T_A and T_C respectively. Let the efficiency of the above mentioned engine is η_2, then

$$\eta_2 = 1 - \frac{T_C}{T_A}$$

Since, $T_A > T_B$, therefore $\left(Q_A + \frac{T_A}{T_B} Q_B\right) > (Q_A + Q_B)$. Thus, from the above two equations, we get

$$\eta_1 < \eta_2.$$

Example 5.15 A reversible engine operates between heat reservoirs A, B and C. The engine receives equal quantities of heat from reservoir A and B at temperatures T_a and T_b respectively and rejects heat to reservoir C at temperature T_c. If the efficiency of the above mentioned engine is α times the efficiency of another reversible engine operating between the reservoir A and C only at temperature T_a and T_c, prove that $\alpha = \frac{1}{2}T_a/T_b\,[(T_b - T_c)/(T_a - T_c) + T_b/T_a]$.

Solution

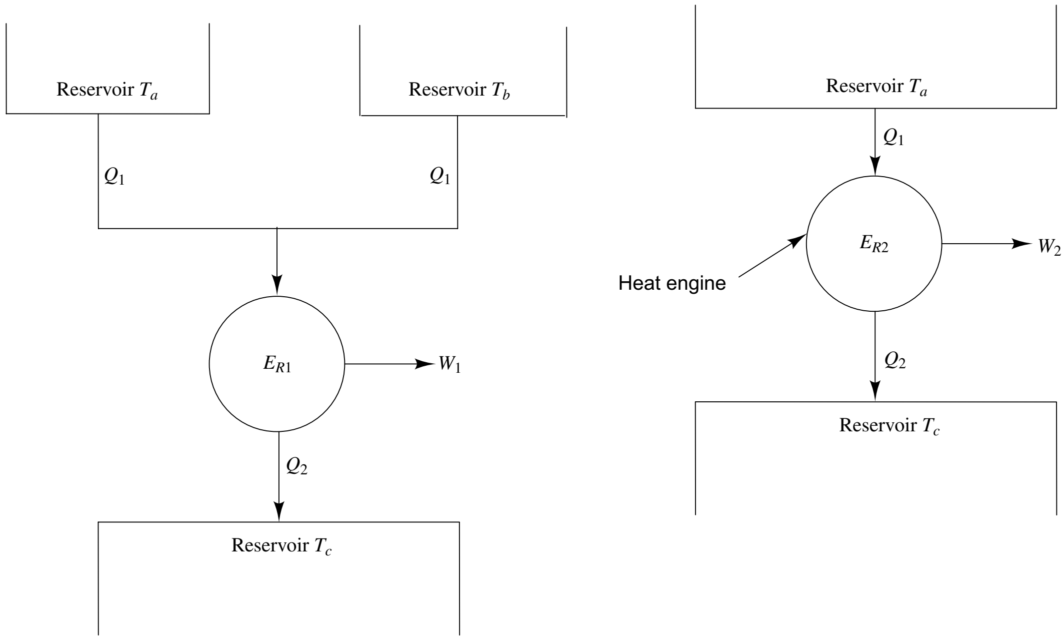


Figure 5.22

The arrangement is depicted in Fig. 5.22.

Efficiency of reversible heat engine E_{R1} operating between reservoirs A, B and C (Fig. 5.22) is given by

$$\eta_1 = \frac{W_1}{2Q_1}$$

From the thermodynamic temperature scale, we have

$$\frac{Q_1}{T_a} + \frac{Q_1}{T_b} = \frac{Q_2}{T_c}$$

Applying first law of thermodynamics to heat engine E_{R1}, we get

$$Q_2 = 2Q_1 - W_1$$

or, $$Q_1\left[\frac{1}{T_a} + \frac{1}{T_b}\right] = \frac{2Q_1 - W_1}{T_c}$$

or, $$\frac{W_1}{T_c} = Q_1\left[\frac{2}{T_c} - \frac{1}{T_a} - \frac{1}{T_b}\right]$$

or, $$\frac{W_1}{2Q_1} = \frac{T_c}{2}\left[\frac{2}{T_c} - \frac{1}{T_a} - \frac{1}{T_b}\right]$$

Efficiency of reversible heat engine E_{R2} operating between reservoirs A and C (Fig. 5.22) is given by

$$\eta_2 = 1 - \frac{T_c}{T_a}$$

Since, $\eta_1 = \alpha\eta_2$, we have

$$\frac{T_c}{2}\left[\frac{2}{T_c} - \frac{1}{T_a} - \frac{1}{T_b}\right] = \alpha\left(1 - \frac{T_c}{T_a}\right)$$

or, $$\alpha = \frac{T_a}{T_a - T_c}\frac{T_c}{2}\left[\frac{2}{T_c} - \frac{1}{T_a} - \frac{1}{T_b}\right]$$

or, $$\alpha = \frac{1}{2}\frac{1}{T_a - T_c}\frac{T_a}{T_b}\left[2T_b - \frac{T_c T_b}{T_a} - T_c\right]$$

or, $$\alpha = \frac{1}{2}\frac{1}{T_a - T_c}\frac{T_a}{T_b}\left[(T_b - T_c) + T_b\left(\frac{T_a - T_c}{T_a}\right)\right]$$

or, $$\alpha = \frac{1}{2}\frac{T_a}{T_b}\left[\frac{(T_b - T_c)}{(T_a - T_c)} + \frac{T_b}{T_a}\right]$$

Example 5.16 Two reversible heat engines are arranged in series in such a way that the heat rejected by the first engine is absorbed by the second engine. The first engine receives 400 kJ of heat from a reservoir maintained at temperature 600°C while the second engine rejects heat to a reservoir having temperature 0°C. If the work output of the first engine is twice that of the second, determine

(a) efficiency of both the engines
(b) heat rejected by the second engine
(c) intermediate temperature.

Solution The above arrangement is schematically presented in Fig. 5.23. An imaginary reservoir having temperature T_2 is incorporated which will act as sink for the first engine as well as source for the second engine.

The heat rejected by the first engine (Q_2) to the intermediate reservoir will be utilized for heat supply to the second engine.

From the given data, we have. Q_1 = 400 kJ; T_1 = 873 K, T_3 = 273 K and $W_1 = 2W_2$.

Since the heat engines are reversible, we have

$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$ and $\frac{Q_2}{T_2} = \frac{Q_3}{T_3}$ which yields

$$\frac{Q_3}{Q_1} = \frac{T_3}{T_1}$$

$$= \frac{273 \times 400}{873} \text{ kJ} = 125 \text{ kJ}$$

∴ heat rejected by the second engine is 125 kJ.

Now $$\eta_1 = 1 - \frac{T_2}{T_1} = \frac{W_1}{Q_1}$$

and $$\eta_2 = 1 - \frac{T_3}{T_2} = \frac{W_2}{Q_2}$$

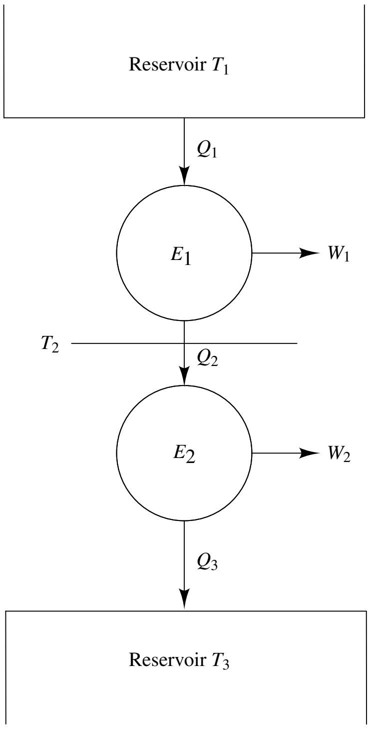


Figure 5.23

And since $W_1 = 2 \times W_2$

$$Q_1 \times \left(1 - \frac{T_2}{T_1}\right) = 2 \times Q_2 \times \left(1 - \frac{T_3}{T_2}\right)$$

$$Q_2 \times \frac{T_1}{T_2} \times \left(1 - \frac{T_2}{T_1}\right) = 2 \times Q_2 \left(1 - \frac{T_3}{T_2}\right)$$

$$\frac{T_1}{T_2} - 1 = 2 \times \left(1 - \frac{T_3}{T_2}\right)$$

$$T_2 = \frac{T_1 + 2T_3}{3} = \frac{873 + 2 \times 273}{3} = 473\,\text{K} = 200°\text{C}$$

∴ temperature of the intermediate reservoir is 200°C.

Hence, $\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{473}{873} = 0.4582$ and $\eta_2 = 1 - \frac{T_3}{T_2} = 1 - \frac{273}{473} = 0.423$

∴ Efficiency of the two engines becomes 45.82% and 42.3% respecively.

Example 5.17 A reversible heat engine operates between three reservoirs at temperatures 600 K, 300 K and 200 K. The engine receives 3200 kJ of heat from the reservoir at 600 K and gives an output of 1800 kJ. Calculate the heat interactions with other reservoirs.

Solution The arrangement is shown in Fig. 5.24.

Since the reservoirs having temperatures of 300 K and 200 K are not categorically told as whether source or sink, these can be considered anything—either a source or sink.

Let us consider the reservoir having a temperature of 300 K as source and a reservoir having a temperature of 200 K as sink.

Then from 1st law of thermodynamics; $Q_1 + Q_2 - Q_3 = W$

∴ $$3200 + Q_2 - Q_3 = 1800$$

$$Q_2 - Q_3 = -1400 \tag{5.23}$$

Further, for the complete system, according to Corollary 5, $\oint \frac{dQ}{T} = 0$

∴ $$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = 0$$

$$\frac{3200}{600} + \frac{Q_2}{300} - \frac{Q_3}{200} = 0$$

$$2Q_2 - 3Q_3 = -3200 \tag{5.24}$$

Solving Eqs (5.23) and (5.24), we have

$Q_2 = -1000$ kJ and $Q_3 = 400$ kJ.

Since Q_2 is of negative magnitude, this implies our consideration of reservoir having temperature 300 K as source, is wrong.

So both the reservoirs would be sinks.

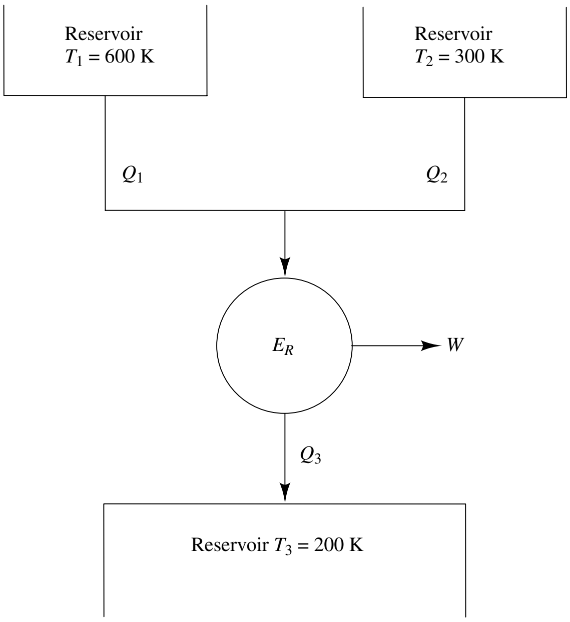


Figure 5.24

5.12 CARNOT EFFICIENCY

We have already seen that the efficiency of Carnot heat engine is independent of the working substance and is dependent on the temperature of source and sink. In this section, we will derive the expression for efficiency of Carnot engine. A Carnot cycle on the *P*-*V* diagram is shown in Fig. 5.25.

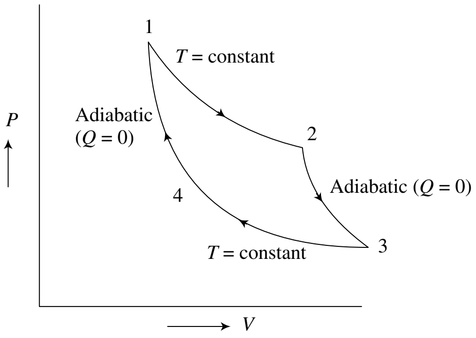


Figure 5.25 *Carnot cycle on P–V diagram*

The heat transfer for each of the four processes is found as follows:

$$Q_{1-2} = W_{1-2} = \int_{V_1}^{V_2} PdV = mRT_1 \ln \frac{V_2}{V_1}$$

$$Q_{2-3} = 0$$

$$Q_{3-4} = -W_{3-4} = -\int_{V_3}^{V_4} PdV = -mRT_2 \ln \frac{V_4}{V_3}$$ (Negative sign is given, since in thermal efficiency Q_{3-4} is a positive quantity)

$$Q_{4-1} = 0$$

The thermal efficiency is then

$$\eta_{\text{Carnot}} = 1 - \frac{Q_{3-4}}{Q_{1-2}} = 1 - \frac{mRT_2 \ln \frac{V_4}{V_3}}{mRT_1 \ln \frac{V_2}{V_1}} \tag{5.25}$$

For the reversible adiabatic processes 2-3 and 4-1, we know that (see Eq. (3.56))

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \tag{5.26}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \tag{5.27}$$

From Eqs (5.26) and (5.27), we get

$$\frac{V_2}{V_3} = \frac{V_1}{V_4}$$

or,

$$\frac{V_4}{V_3} = \frac{V_1}{V_2} \tag{5.28}$$

From Eqs (5.25) and (5.28), we get

$$\eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1} \tag{5.29}$$

Example 5.18 A Carnot engine operates on air with the cycle shown in Fig. 5.26. Determine the thermal efficiency and the power output for each cycle of operation,

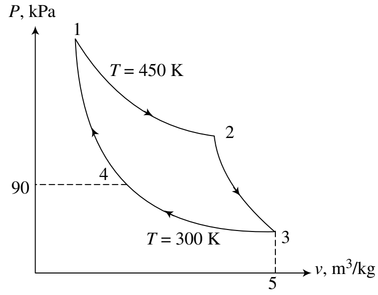


Figure 5.26

Solution The thermal efficiency of the Carnot engine is given by

$$\eta_{\text{Carnot}} = 1-\frac{T_2}{T_1} = 1-\frac{300}{450} = 0.3333 \text{ or } 33.33\%$$

Specific volume at 4 is

$$v_4 = \frac{RT_2}{P_4} = \frac{0.287\times 300}{90} = 0.957 \text{ m}^3/\text{kg}$$

Specific volume at 1 is (see Eq. (3.56))

$$v_1 = v_4\left(\frac{T_4}{T_1}\right)^{\frac{1}{\gamma-1}} = 0.957\left(\frac{300}{450}\right)^{\frac{1}{1.4-1}} = 0.347 \text{ m}^3/\text{kg}$$

Specific volume at 2 is

$$v_2 = v_3\left(\frac{T_3}{T_2}\right)^{\frac{1}{\gamma-1}} = 5\left(\frac{300}{450}\right)^{\frac{1}{1.4-1}} = 1.814 \text{ m}^3/\text{kg}$$

The heat transfer during the process 1-2 is

$$q_{1-2} = w_{1-2} = \int_{V_1}^{V_2} Pdv = RT_1 \ln\frac{v_2}{v_1}$$

$$= 0.287\times 450 \ln\frac{1.814}{0.347} = 213.61 \text{ kJ/kg}$$

The work done for each cycle is

$$w_{cycle} = \eta_{\text{Carnot}} q_{1-2} = 0.3333\times 213.61 = 71.2 \text{ kJ/kg}$$

SUMMARY

- All spontaneous processes in nature proceed in one direction only and never in the reverse direction. Reversal of these processes is not possible without the assistance of any external agency. There exists a directional law which determines the direction in which a spontaneous process will take place. This law is called the second law of thermodynamics.
- A thermal energy reservoir (TER) is a hypothetical body with a relatively large thermal energy capacity that can supply or absorb finite quantities of energy as heat without changing its temperature.
- A thermal energy reservoir at high temperature, which supplies energy in the form of heat is called source, and on the other hand one which absorbs energy in the form of heat is called a sink.

- *Heat engines* are cyclic devices used to convert heat to work. The thermal efficiency of the heat engine is defined as the ratio of the net work done to the energy absorbed as heat and given by.

$$\eta_{\text{ther}} = \frac{W}{Q_1} = \frac{Q_1 \quad Q_2}{Q_1} = 1 \quad \frac{Q_2}{Q_1}$$

  where, Q_1 is the heat transfer from source to heat engine , Q_2 is the heat rejection from the heat engine to sink and W is the work done by the heat engine.

- Refrigerators and heat pumps are simply heat engines operated in the reverse direction. The transfer of heat from a low-temperature medium to a high temperature medium requires special devices called *refrigerators*. The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. A refrigerator operates between the ambient temperature and a low temperature. A heat pump is a device which works on a cycle, absorbing heat from a low temperature reservoir and rejecting heat to a high temperature reservoir. The objective of a heat pump is to reject heat to a high temperature body. A heat pump operates between the ambient temperature and a high temperature. The coefficients of performance of the refrigerator and the heat pump are defined as

$$\text{COP}_{\text{R}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 \quad Q_2}$$

$$\text{COP}_{\text{HP}} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 \quad Q_2}$$

  The COP of a heat pump is greater than then COP of a refrigerator by unity. The coefficient of performance of a heat pump is always greater than unity.

- The Kelvin–Planck statement of the second law is as follows:

  It is impossible to construct a device, which operating in a cycle, will produce no effect other than the absorption of energy as heat from a single reservoir and produces an equivalent amount of work.

- A perpetual motion machine of the second kind is a device which would perform work solely by absorbing energy as heat from a single reservoir. Such a device does not violate the first law of thermodynamics because it would perform work at the expense of the internal energy of a body.

- The Clausius statement of the second law is as follows:

  It is impossible to construct a device, which operating in a cycle, will produce no effect other than transfer of energy in the form of heat from a low temperature body to a high temperature body.

- A process is said to be reversible if at the conclusion of the process both the system and the surroundings can be restored to their respective initial states without producing any changes in the rest of the universe. Otherwise, the process is said to be irreversible.

- A process is called *internally reversible* if no irreversibilities occur within the system boundary during the process. The quasi-equilibrium process is an example of an internally reversible process.
- A process is called *externally reversible* if no irreversibilities occur outside the system boundary during the process.
- A process is called *reversible* if no irreversibilities occur within or outside the system boundary during the process. That is, a process is reversible if it is *internally* as well as *externally reversible.*
- The Carnot cycle consists of four processes—two reversible isothermal and two reversible adiabatic processes.
- *It is impossible to construct a heat engine operating between only two reservoirs, which will have a higher efficiency than a reversible heat engine operating between the same two reservoirs.*
- *All reversible heat engines operating between the same two reservoirs have the same efficiency.*
- *The efficiency of a reversible heat engine is independent of the nature of the working substance and depends only on the temperature of the reservoirs between which it operates.*
- *A scale of temperature can be defined which is independent of any thermometric substance, and which provides an absolute zero of temperature.*
- From the thermodynamic temperature scale

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

- The efficiency of a Carnot heat engine is given by

$$\eta_{\text{ther, Carnot}} = 1 - \frac{T_2}{T_1}$$

where T_1 and T_2 are the temperatures of source and sink respectively.

- The efficiency of the Carnot engine is the maximum possible efficiency of any real engine operating between same two reservoirs.
- The coefficients of performance of a Carnot refrigerator and heat pump are given by

$$\text{COP}_R = \frac{T_2}{T_1 - T_2}$$

$$\text{COP}_{HP} = \frac{T_1}{T_1 - T_2}$$

REVIEW QUESTIONS

5.1 Define heat engine, refrigerator and heat pump.
5.2 What is the difference between a refrigerator and a heat pump?
5.3 What are the characteristics of all heat engines?

5.4 What is a thermal energy reservoir? Give some examples.

5.5 Is there any way to increase the efficiency of a Carnot heat engine other than by increasing T_1 or by decreasing T_2?

5.6 Show that $COP_{HP} = COP_R + 1$, when both the heat pump and refrigerator have the same Q_1 and Q_2.

5.7 What is a perpetual motion machine of the second kind?

5.8 State the second law of thermodynamics.

5.9 Define thermal efficiency of a heat engine.

5.10 Define the coefficient of performance of a refrigerator. Can it be greater than unity?

5.11 Define the coefficient of performance of a heat pump. Can it be greater than unity?

5.12 Show that the Kelvin–Planck statement and the Clausius statement of the second law are equivalent.

5.13 Prove that it is impossible to construct an engine operating between two reservoirs, which will have higher efficiency than a reversible engine operating between the same reservoirs.

5.14 Prove that all reversible engines operating between the same two heat reservoirs have the same efficiency.

5.15 Explain why the performance of heat engines is measured in terms of efficiency but that of refrigerators and heat pumps in terms of COP? Why does the expression for COP differ for refrigerators and heat pumps?

5.16 Why is direct heating thermodynamically wasteful?

5.17 What is a reversible process?

5.18 What are the causes of irreversibility of a process?

5.19 What is a Carnot cycle?

5.20 Show that the efficiency of a reversible heat engine is independent of the nature of the working substance.

NUMERICAL PROBLEMS

5.1 Two Carnot engines work in series between the sources and sink temperatures of 500 K and 300 K. If both engines develop equal power, determine the intermediate temperature.

5.2 Three Carnot engines *A*, *B* and *C* work in series between the temperature limits of 1000 K and 300 K. The amount of work developed is in the proportion of $W_a : W_b : W_c = 5:4:3$. Determine the intermediate temperatures.

5.3 A Carnot cycle operates between two heat sources at temperatures T_1 and T_2 ($T_1 > T_2$). However, the temperature of the working fluid is θ_1 and θ_2, where $\theta_1 < T_1$ and $\theta_2 > T_2$. If $T_1 = (\theta_1 + kQ_1)$ and $T_2 = (\theta_2 - kQ_2)$ where Q_1 and Q_2 are the amounts of heat extracted from hot source and rejected to sink respectively and K is a constant. Show that the efficiency of the engine is given by $\eta = 1 - T_2/(T_1 - 2\,KQ_1)$.

5.4 Consider an engine in outer space, which operates on the Carnot cycle. The only way in which heat can be transferred from the engine is by radiation. The rate at which heat is radiated is proportional to the fourth power of the absolute temperature and to the area of the radiating surface. Show that for a given power output and at given T_1, the area of the radiator will be minimum when $T_2/T_1 = 3/4$.

5.5 Prove that the COP of a reversible refrigerator operating between two given temperatures is the maximum.

5.6 A car engine operates with a thermal efficiency of 35 %. Assume the air-conditioner has a coefficient of performance that is one-third of the theoretical maximum and it is mechanically pulled by the engine. How much fuel energy should be spent extra to remove 1 kJ at 15°C when the ambient is at 35°C ?

5.7 We propose to heat a house in the winter with a heat pump. The house is to be maintained at 20ºC at all times. When the ambient temperature outside drops to –10ºC, the rate at which heat is lost from the house is estimated to be 25 kW. What is the minimum electrical power required to drive the heat pump?

5.8 A heat pump has a COP that is 50 % of the theoretical maximum. It maintains a house at 20ºC, which leaks energy of 0.6 kW per degree temperature difference to the ambient. For a maximum of 1.0 kW power input, find the minimum outside temperature for which the heat pump is a sufficient heat source.

5.9 A heat pump cools a house at 20ºC with a maximum of 1.2 kW power input. The house gains 0.6 kW per degree temperature difference to the ambient and the heat pump COP is 60 % of the theoretical maximum. Find the maximum outside temperature for which the heat pump provides sufficient cooling.

5.10 At certain locations geothermal energy in underground water is available and used as the energy source for a power plant. Consider a supply of saturated liquid water at 150ºC. What is the maximum possible thermal efficiency of the cyclic heat engine using this source of energy with the ambient at 20ºC ? Would it be better to locate a source of saturated vapour at 150ºC than to use the saturated liquid at 150ºC?

5.11 A refrigerator is to remove heat from the cooled space at a rate of 300 kJ/min to maintain its temperature at –8ºC. If the air surrounding the refrigerator is at 25ºC, determine the minimum power input required for this refrigerator.

5.12 A Carnot refrigerator operates in a room in which the temperature is 22ºC and consumes 2 kW of power when operating. If the food compartment of the refrigerator is to be maintained at 3ºC, determine the rate of heat removal from the food compartment.

5.13 A heat engine that receives heat at the rate of 800 kW at a high temperature of 427°C and rejects heat to the ambient at 27°C while work is produced at the rate of 350 kW. Find out how much heat is discarded to the ambient and the engine efficiency. Also compare both of these values with a Carnot heat engine operating between the same two reservoirs.

5.14 A heat engine operating between two reservoirs at 100 K and 300 K is used to drive heat pump which extracts heat from reservoir at 300 K at a rate twice that at which engine rejects heat to it. If the efficiency of the engine is 40% of the maximum possible and the coefficient of performance of the heat pump is 50% of the maximum possible, determine the temperature of the reservoir to which the heat pump rejects heat. Also calculate the rate of heat rejection from the heat pump if the rate of heat supply to the engine is 50 kW.

5.15 A house hold refrigerator is maintained at a temperature of 275 K. Every time the door is opened, warm material is placed inside, introducing an average of 300 kJ, but making only a small change in the temperature of the refrigerator. The door is opened 25 times a day, and the refrigerator operates at 20% of the ideal COP. The cost of work is Rs. 4.00 per kWhr. What is the bill for the month for this refrigerator? The atmosphere is at 300 K.

MULTIPLE-CHOICE QUESTIONS

5.1 The efficiency of a Carnot heat engine operating between absolute temperatures T_1 and T_2 is

(a) $\dfrac{T_1}{T_2}$ (b) $\dfrac{T_1 - T_2}{T_1}$ (c) $\dfrac{T_2}{T_1}$ (d) $\dfrac{T_1 - T_2}{T_2}$

5.2 The more effective way of increasing efficiency of a Carnot engine is

(a) increase higher temperature (b) decrease higher temperature

(c) increase lower temperature (d) decrease lower temperature

5.3 Kelvin–Planck's law deals with
(a) conservation of heat (b) conservation of work
(c) conversion of heat into work (d) conversion of work into heat

5.4 In a Carnot cycle, heat is transferred at
(a) constant pressure (b) constant volume
(c) constant temperature (a) any one of the above

5.5 If a system after undergoing a series of processes, returns to the initial state then
(a) processes are thermodynamically in equilibrium
(b) its entropy will change due to irreversibility
(c) sum of heat and work transfer will be zero
(d) process are executed in close system cycle

5.6 According to Clausius statement,
(a) heat flows from a cold surface to a hot surface, unaided
(b) heat flows from a hot surface to a cold surface, unaided
(c) heat can flow from a cold surface to a hot surface with the aid of external work
(d) both b and c above

5.7 Efficiency of a Carnot engine with $t_1 = 200$°C and $t_2 = 30$°C is
(a) 86 % (b) 47 % (c) 36 % (d) 15 %

5.8 A heat engine supplied heat at the rate of 30 kJ/sec and gives an output of 9 kW. The thermal efficiency of the engine will be
(a) 30% (b) 33% (c) 40% (d) 50%

5.9 If Q_1 is the heat transfer between a hot temperature source and a machine, and Q_2 is the heat transfer between cold temperature source and machine, the COP of the heat pump will be equal to
(a) $Q_1/(Q_1 - Q_2)$ (b) $Q_2/(Q_1 - Q_2)$ (c) $(Q_1 - Q_2)/Q_1$ (d) $(Q_1 - Q_2)/Q_2$

5.10 The efficiency of a Carnot engine is 0.75. If cycle direction is reversed, COP of the reversed Carnot refrigeration cycle will be
(a) 0.25 (b) 1.25 (c) 1.33 (d) 0.33

5.11 Which of the following relationships is valid for reversible processes undergone by a closed system of simple compressible substance (neglect changes in kinetic and potential energy)?
(a) $\delta Q = dU + \delta W$ (b) $TdS = dU + pdV$ (c) $TdS = dU + \delta W$ (d) $\delta Q = dU + pdV$

5.12 If a heat engine attains 100% thermal efficiency, it violates the
(a) zeroth law of thermodynamics (b) first law of thermodynamics
(c) second law of thermodynamics (d) none of the above.

5.13 A refrigerator and a heat pump operate between the same temperature limits. If the COP of the refrigerator is 4, the C.O.P. of the heat pump would be
(a) 3 (b) 4 (c) 5 (d) none of these

CHAPTER

6

Entropy

6.2 TWO REVERSIBLE ADIABATIC PATHS CANNOT INTERSECT EACH OTHER

Consider a reversible adiabatic process represented by the path *AB* as shown in Fig. 6.1. Let us assume that it is possible for the system to undergo a reversible isothermal process *BC* and then the system be restored to its original state by another reversible adiabatic process *CA*. These three reversible processes together constitute a cycle. The area enclosed by the cycle represents the reversible work done. However, there is no heat transfer during the processes *AB* and *CA*. We therefore have a system undergoing a cycle and developing net work while exchanging heat with a single reservoir. It violates the Kelvin–Planck statement of the second law of thermodynamics. Therefore, the process *AB* and *CA* cannot intersect each other at the point *A*. There can be only one reversible adiabatic path passing through one point.

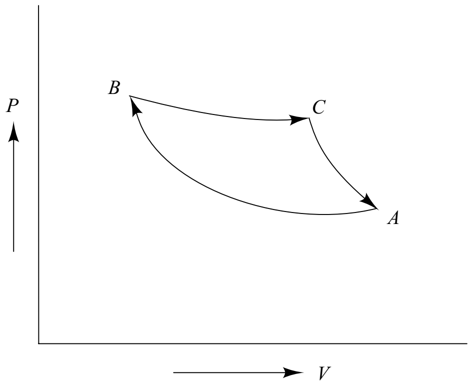


Figure 6.1

6.2 CLAUSIUS INEQUALITY

Consider a reversible heat engine operating between two reservoirs at temperatures T_1 and T_2 as shown in Fig. 6.2.

Considering the heat engine as a closed system undergoing a cyclic process, we have

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} \tag{6.1}$$

Since heat transfer from a system is negative, therefore Q_2 is negative.

From fourth Corollary, for a reversible heat engine, we have (see Eq. (5.11))

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

or

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0 \tag{6.2}$$

Combining Eq. (6.1) and (6.2), we have

$$\oint \frac{\delta Q}{T} = 0 \tag{6.3}$$

It physically implies that the cyclic integral of $\frac{\delta Q}{T}$ for a reversible cycle is equal to zero. This is known as ***Clausius' theorem.***

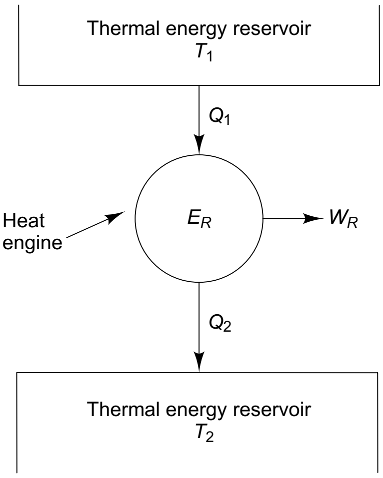


Figure 6.2

Now, consider an irreversible heat engine operating between the same two reservoirs at temperatures T_1 and T_2 as shown in Fig. 6.3.

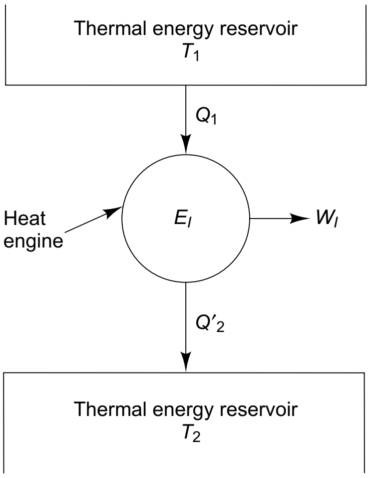


Figure 6.3

Since the efficiency of the irreversible engine is less than that of the reversible engine, we have

$$W_I < W_R$$

or $$Q_1 - Q'_2 < Q_1 - Q_2 \qquad \left[\because W_I = Q_1 - Q'_2, \text{ and } W_R = Q_1 - Q_2\right]$$

or $$Q'_2 > Q_2$$

Equation (6.3) is then modified to

$$\frac{Q_1}{T_1} - \frac{Q_2'}{T_2} < 0$$

For the irreversible heat engine cycle

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2'}{T_2} < 0 \tag{6.4}$$

Combining Eqs (6.3) and (6.4), we have generalized expression

$$\oint \frac{\delta Q}{T} \le 0 \tag{6.5}$$

Similarly, it can be proved that $\oint \frac{\delta Q}{T} \le 0$ for any cyclic device operating between more than two reservoirs.

Equation (6.5) is known as the ***Clausius inequality***. The significance of the Clausius Inequality is as follows:

From Clausius inequality we can conclude

(i) whether a cyclic process is feasible or not, and
(ii) even if feasible then whether it is reversible or irreversible.

If $\oint \frac{\delta Q}{T} = 0$, then the cyclic process is possible and reversible.

If $\oint \frac{\delta Q}{T} < 0$, then the cyclic process is possible and irreversible.

If $\oint \frac{\delta Q}{T} > 0$, then the cyclic process is not possible .

The Clausius inequality is also known as the 6th corollary of the second law of thermodynamics which states that ***whenever a system undergoes a cycle***, $\oint \frac{\delta Q}{T}$ ***is zero if the cycle is reversible and negative if irreversible i.e., in general*** $\oint \frac{\delta Q}{T} \le 0$.

6.3 ENTROPY—A PROPERTY OF A SYSTEM

Let a closed system undergo a reversible process from the state 1 to the state 2 along a path *A*, and let the cycle be completed along path *C*, which is also reversible as shown in Fig. 6.4.

According to the Clausius theorem, for a reversible cycle, we can write

$$\oint \frac{\delta Q}{T} = 0$$

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_A + \int_2^1 \left(\frac{\delta Q}{T}\right)_C = 0 \tag{6.6}$$

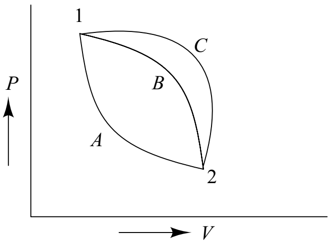


Figure 6.4

Let the system be taken from state 1 to state 2 by another reversible path B.

Let us consider $\int_1^2 \left(\frac{\delta Q}{T}\right)_A \neq \int_1^2 \left(\frac{\delta Q}{T}\right)_B$ (6.7)

Now the reversible processes 1-B-2 and 2-C-1 together constitute a cycle for which

$$\oint \frac{\delta Q}{T} = 0$$

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_B + \int_2^1 \left(\frac{\delta Q}{T}\right)_C = 0 \tag{6.8}$$

If the inequality (6.7) is true, then equations (6.6) and (6.8) contradict each other which imply that these quantities must be equal. Therefore, $\int_1^2 \left(\frac{\delta Q}{T}\right)$ is independent of the path the system follows during a change of state.

$$\therefore \qquad \int_1^2 \left(\frac{\delta Q}{T}\right)_A = \int_1^2 \left(\frac{\delta Q}{T}\right)_B$$

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{rev}} = S_2 - S_1 \tag{6.9}$$

In differential form,

$$\mathrm{d}S = \left(\frac{\delta Q}{T}\right)_{\text{rev}} \tag{6.10a}$$

or

$$\delta Q_{\text{rev}} = TdS \tag{6.10b}$$

The property S is called the entropy of the system. The unit of entropy is J/K and that of specific entropy is J/kg-K. Entropy is an extensive property of a system. Unlike energy, entropy is a non-conserved property.

This is also known as the 7th corollary of the second law of thermodynamics which states that ***there exists a property of a closed system such that a change in its value is equal to*** $\int_1^2 \frac{\delta Q}{T}$ ***for any reversible process undergone by the system between states 1 and 2***.

Example 6.1 A lump of steel of mass 15 kg at 557°C is dropped in 120 kg of oil at 25°C. The specific heats of steel and oil are 0.5 kJ/kg-K and 3.5 kJ/kg-K respectively. Calculate the entropy change of the steel, the oil and the universe.

Solution Let the final temperature be T_f. From the principle of calorimetry, we have

$$15 \times 0.5(557 - T_f) = 120 \times 3.5(T_f - 25)$$

$$T_f = 34.33°\text{C} = 307.33 \text{ K}$$

$$\text{Entropy change of steel} = \int_{T_1}^{T_f} \frac{\delta Q}{T} = mC_p \ln \frac{T_f}{T_1}$$

$$= 15 \times 0.5 \ln \frac{307.33}{830} = -7.45 \text{ kJ/K}$$

$$\text{Entropy change of oil} = \int_{T_2}^{T_f} \frac{\delta Q}{T} = mC_p \ln \frac{T_f}{T_2}$$

$$= 120 \times 3.5 \ln \frac{307.33}{298} = 12.95 \text{ kJ/K}$$

Since there is no heat transfer to the surroundings, entropy change of the surroundings is zero.

Entropy change of universe = Entropy change of steel + Entropy change of oil

$= -7.45 + 12.95 = 5.5$ kJ/K

Example 6.2 Consider 1 kg of ice at – 20 °C as a system. It is exposed to surroundings at 25 °C. The ice melts to water ultimately coming to equilibrium with the surroundings. Calculate the entropy change of the system, the surroundings and the universe. Specific heat of ice and water are 2.1 kJ/kg-k and 4.2 kJ/kg-K respectively and the latent heat of fusion of ice is 333.5 kJ/kg.

Solution **Process 1–2** Sensible heating of ice from – 20 °C to 0 °C

Process 2–3 Melting of ice from and at 0 °C

Process 3–4 Sensible heating of water from 0 °C to 25 °C

Total heat transfer from the surroundings = 1[2.1(0 + 20) + 333.5 + 4.2(25 – 0)]

= 480.5 kJ

Change of entropy of ice during process 1–2, $(\Delta S)_{1-2} = mC_p \ln \frac{T_f}{T_i} = 1 \times 2.1 \ln \frac{273}{253}$

$= 0.1598$ kJ/K

Entropy change during process 2–3, $(\Delta S)_{2-3} = \frac{1 \times 335.5}{273} = 1.2216$ kJ/K

Entropy change during process 3–4, $(\Delta S)_{3-4} = mC_p \ln \frac{T_f}{T_i} = 1 \times 4.2 \ln \frac{298}{273} = 0.368$kJ/K

Entropy change of system $(\Delta S)_{\text{system}} = (\Delta S)_{1-2} + (\Delta S)_{2-3} + (\Delta S)_{3-4}$

$= 0.1598 + 1.2216 + 0.368 = 1.7494$kJ/K

Entropy change of surroundings $(\Delta S)_{\text{surr}} = \frac{-480.5}{298} = -1.6124$kJ/K (negative sign because heat is transferred from the Surroundings)

Entropy change of universe $(\Delta S)_{\text{Uni}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{surr}}$

$= 1.7494 - 1.6124 = 0.137$kJ/K

Example 6.3 Calculate the entropy change of 1 kg of water at 27 °C, when it is converted to ice at –20 °C. Specific heat of ice and water are 2.18 kJ/kg-K and 4.18 kJ/kg-K respectively. The latent heat of fusion of ice at 0°C is 335 kJ / kg.

Solution

Mass of water, $m = 1$ kg
Initial temperature of water, $T_1 = 27$ °C = 300 K
Final temperature of ice, $T_2 = -20$ °C = 253 K
Specific heat of water, $C_{P\text{water}} = 4.18$ kJ/kg-K
Specific heat of ice, $C_{P\text{ice}} = 2.18$ kJ/kg-K
Latent heat of fusion of ice, $L_{\text{ice}} = 335$ kJ/gm

Change of entropy when water changes its temperature from 27 °C to 0 °C

$$= \int_{300}^{273} \frac{mC_p \mathrm{d}T}{T} = 1 \times 4.18 \ln \frac{273}{300} = -0.3942 \text{ kJ/K}$$

Change of entropy when water solidifies into ice from and at 0 °C

$$= -\frac{1 \times 335}{273} = -1.227 \text{kJ/K}$$ (negative sign because heat is transferred from the water during solidification)

Change of entropy when water changes its temperature from 0 °C to –20 °C

$$= \int_{273}^{253} \frac{mC_p \mathrm{d}T}{T} = 1 \times 2.18 \ln \frac{253}{273} = -0.1659 \text{ kJ/K}$$

Total entropy change = – 0.3942 – 1.227 – 0.1659 = –1.7871 kJ/K

Example 6.4 A Carnot cycle receives heat at 427 °C causing an increase in entropy equal to 10 kJ/kg-K. The engine delivers 4000 kJ/kg of work. Determine the efficiency of the cycle and temperature of the sink.

Solution

Temperature of source $T_1 = 427°\text{C} = 427 + 273 = 700$ K

Change of entropy $s_2 - s_1 = 10 \text{kJ/kgK}$

Net work done $W = 4000$ KJ/kg

Now, $Q = T_1(s_2 - s_1) = 700 \times 10 = 7000$ kJ/kg
= 700 × 10 = 7000 kJ/kg

Efficiency of the cycle is

$$\eta = \frac{W}{Q_1} = \frac{4000}{7000} = 0.5714$$

The efficiency of a Carnot cycle can be expressed in terms of source temperature and sink temperature (T_2) as (see Eq. (5.13))

$$\eta = 1 - \frac{T_2}{T_1}$$

or $$1 - \frac{T_2}{700} = 0.5714$$

or $$T_2 = 300.02 \text{ K} = 27.02°\text{C}$$

Example 6.5 A body of mass m at temperature T_i and of constant heat capacity C_p is brought in contact with a thermal reservoir at temperature T_f $(T_f > T_i)$. The pressure remains constant while the body attains equilibrium with the reservoir. Show that change in entropy of the universe is equal to $mC_p[x - \ln(1+x))]$, where $x = -\frac{T_f - T_i}{T_f}$

Solution The heat transfer from the reservoir to the body will continue until the later attains the temperature of the reservoir. Since the heat transfer takes place at constant pressure,

$$Q = mC_p\left(T_f - T_i\right)$$

From the very definition of reservoir, its temperature remains unchanged, irrespective of the amount of heat transfer. Thus, the entropy change of the reservoir is

$$(\Delta S)_{\text{res}} = -\frac{mC_p\left(T_f - T_i\right)}{T_f}$$ (Negative sign because heat is transferred from the reservoir)

Entropy change for the body is $(\Delta S)_{\text{body}} = \int_{T_i}^{T_f} \frac{mC_p dT}{T} = mC_p \ln\frac{T_f}{T_i}$

Since there is no heat transfer to the surroundings, entropy change of the surroundings is zero.

Hence entropy changes of the universe is then

$$(\Delta S)_{\text{univ}} = (\Delta S)_{\text{body}} + (\Delta S)_{\text{res}}$$

$$= mC_p\left[\ln\frac{T_f}{T_i} - \frac{\left(T_f - T_i\right)}{T_f}\right] = mC_p\left[\ln\frac{1}{1+x} + x\right] = mC_p[x - \ln(1+x)]$$

6.4 TEMPERATURE ENTROPY DIAGRAM

The infinitesimal change in entropy dS due to infinitesimal heat transfer δQ of a reversible process at temperature T (see Eq. 6.10b) is

$$\delta Q_R = T\text{d}S$$

Total amount of heat transfer in a reversible process 1–2 (Fig. 6.5) is given by

$$Q_{1-2/R} = \int_1^2 T\text{d}S \tag{6.11}$$

The integral can be interpreted graphically as the curve under a process curve on a diagram in which T is plotted along the y-axis and S along the x-axis. Such a diagram is called a temperature–entropy (T–S) diagram, i.e., the area under a curve on the T–S diagram represents the reversible heat transfer during the process.

The shape of the curve on the T–S diagram is determined by the kind of reversible process that the system undergoes.

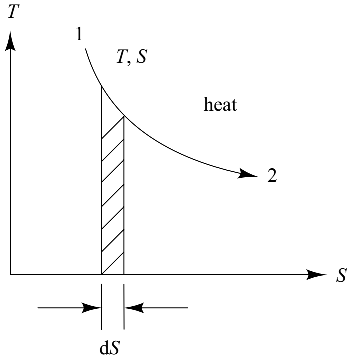


Figure 6.5

6.4.1 Reversible Isothermal Process on *T*–*S* Diagram

An isothermal process is represented by a horizontal line as shown in Fig. 6.6.

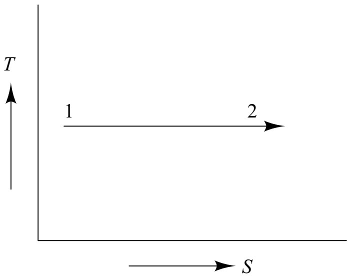


Figure 6.6

$$Q = T\int_{1}^{2} \mathrm{d}S = T(S_2 - S_1)$$

$$S_2 - S_1 = \frac{Q}{T}$$

6.4.2 Reversible Adiabatic Process on *T–S* Diagram

For reversible adiabatic process, $\delta Q_R = 0$

If T is not zero then $dS = 0$, S = constant.

A reversible adiabatic process is represented by a vertical line as shown in Fig. 6.7.

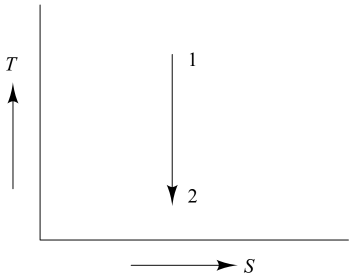


Figure 6.7

6.4.3 Carnot Cycle on *T–S* Diagram

It consists of two reversible isothermal processes and two reversible adiabatic processes.

Process 1–2 Reversible isothermal heat addition process
Process 2–3 Reversible adiabatic expansion
Process 3–4 Reversible isothermal heat rejection process
Process 4–1 Reversible adiabatic compression

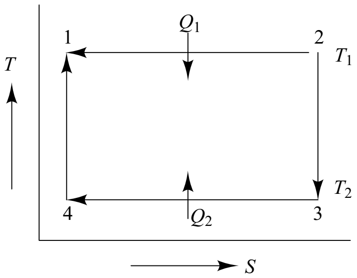


Figure 6.8 *Carnot cycle on T–S diagram*

Efficiency of Carnot cycle is then,

$$\eta_{\text{Carnot}} = 1-\frac{Q_2}{Q_1} = 1-\frac{T_2(S_3-S_4)}{T_1(S_2-S_1)} = 1-\frac{T_2}{T_1}$$

6.5 ENTROPY PRINCIPLE AND APPLICATIONS

The 8th corollary of the second law of thermodynamics states that ***the entropy of any closed system which is thermally isolated from the surroundings either increases***, ***or if the process undergone by the system is reversible remains constant***.

The universe is an example of an isolated system.

Mathematically,

$$\mathrm{d}S_{\text{universe}} = \mathrm{d}S_{\text{system}} + \mathrm{d}S_{\text{surr}} \geq 0 \tag{6.12}$$

The equality holds if the process is reversible and the inequality holds if the process is irreversible.

Since all the natural processes in the universe are irreversible, it implies from the equation (6.12) that ***the entropy of the universe is continuously increasing. The only processes that can actually occur in nature are those in which the entropy of the universe increases. The processes in which the entropy of the universe decreases, cannot take place at all***.

6.5.1 Heat Transfer Through a Finite Temperature Difference

Consider the steady-state heat transfer from the reservoir 1 at the temperature T_1 to the reservoir 2 at temperature T_2 ($T_1 > T_2$) through an intermediate body connecting the reservoirs as shown in Fig. 6.9. Let Q be the rate of heat transfer.

Change in entropy of the reservoir 1, $\Delta S_1 = -\frac{Q}{T_1}$ (Negative sign because heat is transferred from reservoir 1).

Change in entropy of the reservoir 2, $\Delta S_2 = +\frac{Q}{T_2}$.

The entropy change of the intermediate body is zero since it neither gains nor loses heat under steady-state conditions.

Net change in the entropy of the universe $\Delta S_{\text{uni}} = \Delta S_1 + \Delta S_2 = -\frac{Q}{T_1} + \frac{Q}{T_2}$

$$= Q\frac{T_1 - T_2}{T_2}$$

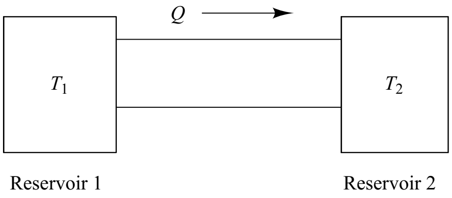


Figure 6. 9 *Heat transfer through a finite temperature difference*

Since $T_1 > T_2$, ΔS_{uni} is positive. Hence, heat transfer through a finite temperature difference is an irreversible process.

For the case when $T_1 = T_2$, ΔS_{uni} is zero and the process is reversible.

For the case when $T_1 < T_2$, ΔS_{uni} is negative and the process is impossible.

Example 6.6 A mass of "m" kg of liquid at a temperature T_1 is mixed with an equal mass of same liquid at temperature T_2 ($T_1 > T_2$), and then the system is thermally insulated. Show that the change in entropy of the universe is given by $(\Delta S)_{\text{net}} = 2mC_p \ln\left\{(T_1 + T_2)/2\sqrt{T_1T_2}\right\}$. Also prove that it is necessarily positive

Solution Let T_f be the final temperature after mixing. Thus,

$$mC_p(T_1 - T_f) = mC_p(T_f - T_2)$$

or,

$$T_f = \frac{T_1 + T_2}{2}$$

The change in entropy of liquid of initial temperature T_1 is $(\Delta S)_1 = mC_p \ln\dfrac{T_f}{T_1}$.

The change in entropy of liquid of initial temperature T_2 is $(\Delta S)_2 = mC_p \ln\dfrac{T_f}{T_2}$.

Since, the system is thermally insulated, therefore, there is no heat transfer to the surroundings and hence, the change in entropy of the surroundings is zero.

The entropy change for the universe is calculated as

$$\begin{aligned}(\Delta S)_{\text{net}} &= (\Delta S)_1 + (\Delta S)_2 + 0\\ &= mC_p \ln\frac{T_f}{T_1} + mC_p \ln\frac{T_f}{T_2}\\ &= mC_p \ln\frac{T_f^{\,2}}{T_1T_2} = 2mC_p \ln\frac{T_f}{\sqrt{T_1T_2}} = 2mC_p \ln\frac{\dfrac{T_1+T_2}{2}}{\sqrt{T_1T_2}}\end{aligned}$$

Note that $\dfrac{T_1 + T_2}{2}$ and $\sqrt{T_1T_2}$ are the arithmetic and geometric mean of T_1 and T_2 respectively.

As we know that arithmetic mean is greater than that of geometric mean, therefore $\frac{T_1+T_2}{2}$ is greater than $\sqrt{T_1T_2}$.

$\ln \frac{\frac{T_1+T_2}{2}}{\sqrt{T_1T_2}}$ is greater than zero.

Net entropy change is positive.

Example 6.7 Two bodies, each of equal mass m and heat capacity C_P, are at temperatures T_1 and T_2 respectively ($T_1 > T_2$). The first body is used as a source of heat for reversible engine and the second body as the sink. Show that the maximum work obtainable from such an arrangement is $mC_P\left(\sqrt{T_1}-\sqrt{T_2}\right)^2$.

Solution As heat is transferred from the first body and heat is rejected to the second body, the temperature of the first body will be decreasing and that of the second body will be increasing. When both the bodies attain the same temperature, the heat engine will stop operating.

Let T_f be the final temperature.

Total heat transfer from the first body, $Q_1 = mC_P\,(T_1 - T_f)$

Total heat rejected to second body, $Q_2 = mC_P\,(T_f - T_2)$

Work done by the heat engine, $W = Q_1 - Q_2$

$$W = mC_P\,(T_1 + T_2 - 2T_f)$$

For minimum value of T_f value of W will be maximum.

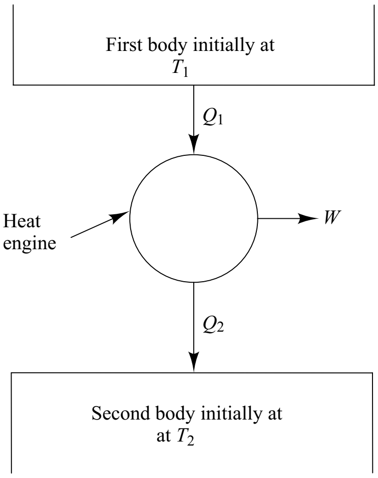


Figure 6.10

Change of entropy of the first body $\Delta S_1 = \int_{T_1}^{T_f} \frac{mC_p \mathrm{d}T}{T} = mC_p \ln \frac{T_f}{T_1}$

Change of entropy of the second body $\Delta S_2 = \int_{T_2}^{T_f} \frac{mC_p \mathrm{d}T}{T} = mC_p \ln \frac{T_f}{T_2}$

We know that the heat engine is a cyclic device, and cyclic integral of any property is zero. Since entropy is a property of a system,

entropy change for the heat engine, $\Delta S_{HE} = 0$

Total entropy change of the universe

$$(\Delta S)_{\text{uni}} = \Delta S_1 + \Delta S_2 + \Delta S_{HE}$$

$$= mC_p \ln \frac{T_f}{T_1} + mC_p \ln \frac{T_f}{T_2}$$

$$= mC_p \ln \frac{T_f^{\,2}}{T_1 T_2}$$

From the entropy principle (Eq. 6.12), we have

$$(\Delta S)_{\text{uni}} \geq 0$$

$$mC_p \ln \frac{T_f^{\,2}}{T_1 T_2} \geq 0$$

For minimum value of T_f

$$mC_p \ln \frac{T_f^{\,2}}{T_1 T_2} = 0$$

$$\ln \frac{T_f^{\,2}}{T_1 T_2} = 0 = \ln 1$$

$$T_f = \sqrt{T_1 T_2}$$

Maximum work obtainable

$$W_{\max} = mC_p (T_1 + T_2 - 2\sqrt{T_1 T_2})$$

$$= mC_p \left(\sqrt{T_1} - \sqrt{T_2}\right)^2$$

6.6 ENTROPY GENERATION IN A CLOSED SYSTEM

The entropy generation is one of the most important attributes in thermodynamics. Entropy generation is a measure of the entropy created during the irreversible processes. The entropy of any closed system can be changed by two ways:

(a) By heat exchange between system and the surroundings.
(b) Due to internal irreversibility arising from dissipative effects.

Mathematically, one can write

$$(dS)_{\text{total}} = (dS)_{\text{ext}} + (dS)_{\text{int}} \tag{6.13a}$$

or,

$$(dS)_{\text{total}} = \frac{\delta Q}{T} + (dS)_{\text{int}} \tag{6.13b}$$

The term $\frac{\delta Q}{T}$ is associated with external heat transfer whereas $(dS)_{\text{int}}$ is realized due to internal dissipative effects like friction, if the process is irreversible. The sign of the term $\frac{\delta Q}{T}$ depends on the direction on heat transfer (positive when heat is transferred to the system, whereas negative when heat is rejected from the system), whereas $(dS)_{\text{int}}$ is always positive.

It is therefore concluded that in case of heat rejection $(dS)_{\text{total}}$ may be zero implying that entropy remains constant. Hence, irreversible process may also be an isentropic process. But if the process is reversible, $(dS)_{\text{int}}$ is zero. Under such condition, $(dS)_{\text{total}}$ would be zero if $\frac{\delta Q}{T}$ is zero, i.e., the process is no other than adiabatic. In a summary, we can conclude that an isentropic process needs not to be an adiabatic process, but if the isentropic process is reversible, it must be adiabatic.

6.7 ENTROPY AND DISORDER

Entropy can be viewed as a measure of molecular disorder or randomness. The entropy of a substance is the highest in the gas phase and lowest in the solid phase. This is because the molecules of a substance are less predictable when they are in gas phase. However, in the solid phase, the position of the molecules can be predicted with a good certainty. Actually, in the solid phase the molecules of a substance continually oscillate about their equilibrium positions and these oscillations fade as the temperature is decreased. At absolute zero temperature, molecules supposedly become motionless. ***Therefore, the entropy of a pure crystalline substance at absolute zero temperature is zero.*** This is known as the ***third law of thermodynamics***. The third law of thermodynamics provides an absolute reference point for the determination of entropy.

6.8 THERMODYNAMIC PROPERTY RELATIONS

Mathematically, first law of thermodynamics for a reversible infinitesimal change can be expressed as

$$\delta Q_{\text{rev}} = dU + PdV$$

or, $$\delta Q_{\text{rev}} = dH - VdP \qquad (\because H = U + PV, \text{ or } dH = dU + PdV + VdP)$$

The heat transfer during a reversible infinitesimal change can be expressed as (see Eq. (6.10a))

$$\delta Q_{\text{rev}} = TdS$$

Comparing the above equations, we have

$$TdS = dU + PdV \tag{6.14}$$

$$TdS = dH - VdP \tag{6.15}$$

Dividing both sides of equations (6.14) and (6.15) by mass *m*, one can write

$$Tds = du + Pdv \tag{6.16}$$

$$Tds = dh - vdP \tag{6.17}$$

The Eqs (6.16) and (6.17) are the well-known *T-ds* equations. *Although the equations are derived for a reversible process only, but they are applicable to all processes whether reversible or irreversible, since the equation establishes relation amongst properties, which are independent of the path. These equations are referred to* ***thermodynamic property relations.***

If the relationship among various properties such as *P*,*V*, *U* (or *H*) and *T* is known, then Eqs (6.14) and (6.15) can be used to determine the change in entropy between any two equilibrium states.

Example 6.8 One kg of air is initially contained in a piston–cylinder arrangement at 15 kPa and 40 °C. The air undergoes the following cyclic process:

1–2 Constant volume heating until pressure is 60 kPa
2–3 Isentropic expansion to the initial pressure
3–1 Constant pressure cooling to the initial state
(i) Sketch the cycle on the *P-V* and *T-S* diagram.
(ii) Determine the thermal efficiency of the cycle.

Take $C_p = 1.005$ kJ/kg-K and $C_v = 0.718$ kJ/kg-K

Solution Initial pressure, $P_1 = 15$ kPa

Initial temperature, $T_1 = 40$ °C = 313K
$P_2 = 60$ kPa
The processes on *P–V* and *T–S* diagrams are shown in Fig. 6.11

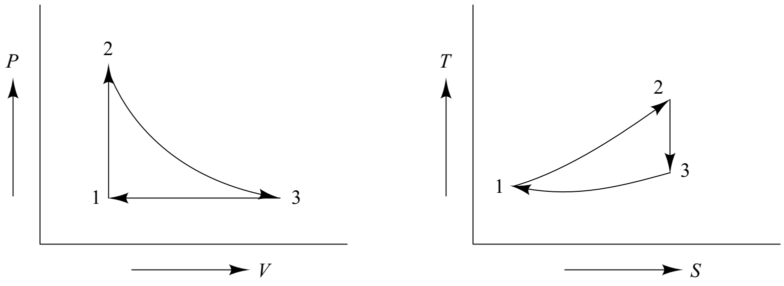


Figure 6.11

For the constant volume process, 1–2

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} = \frac{60}{15} = 4$$

or $$T_2 = 4T_1 = 4 \times 313 = 1252 \text{ K}$$

$$\frac{T_2}{T_3} = \left(\frac{P_2}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 4^{\frac{1.4-1}{1.4}}$$

$$T_3 = \frac{T_2}{4^{\frac{1.4-1}{1.4}}} = \frac{1252}{4^{\frac{1.4-1}{1.4}}} = 842.53 \text{ K}$$

Heat transfer during the constant volume process 1–2

$$Q_{1-2} = mC_v\,(T_2 - T_1)$$

$$Q_{1-2} = (1)(0.718)(1252 - 313) = 674.202 \text{ kJ}$$

Work done during constant volume process 1–2

$$W_{1-2} = 0$$

Heat transfer during isentropic expansion process 2–3

$$Q_{2-3} = 0$$

Work done during isentropic expansion process 2–3

$$W_{2-3} = \frac{P_2V_2 - P_3V_3}{\gamma - 1} = \frac{mR(T_2 - T_3)}{\gamma - 1}$$

$$= \frac{1 \times 0.287(1252 - 842.53)}{1.4 - 1} = 293.795 \text{ kJ}$$

Heat transfer during constant pressure process 3–1

$$Q_{3-1} = mC_P\,(T_1 - T_3)$$

$$= 1 \times 1.005(313 - 842.53)$$

$$= -532.178 \text{ kJ}$$

Work done during constant pressure process 3–1

$$W_{3-1} = P(V_1 - V_3) = mR(T_1 - T_3)$$

$$= 1 \times 0.287(313 - 842.53) = -151.975 \text{ kJ}$$

Thermal efficiency is found to be $\eta = \frac{W_{\text{net}}}{Q_{1-2}} = \frac{293.795 - 151.975}{674.2}$ = 0.210 or 21%

Example 6.9 Prove the following relationship for an ideal gas. $ds = C_p \frac{dv}{v} + C_v \frac{dP}{P}$ and hence prove that for an isentropic change of state $Pv^{\gamma} = \text{constant}$.

Solution From the property relationship (Eq. (6.17)), we have

$$Tds = dh - vdP$$

or, $ds = \frac{dh}{T} - \frac{vdP}{T}$ since for ideal gas $Pv = RT$

or, $$ds = C_p \frac{dT}{T} - R\frac{dP}{P} \tag{6.18}$$

$\left(\text{since for an ideal gas } dh = RdT \text{ and } \frac{v}{T} = \frac{R}{P}\right)$

Further, from the characteristic equation of state for an ideal gas, we have

$$Pv = RT$$

Differentiating, we get

$$Pdv + vdP = RdT$$

or, $$\frac{dv}{v} + \frac{dP}{P} = \frac{dT}{T} \tag{6.19}$$

From Eqs (6.18) and (6.19), we get

$$ds = C_p\left[\frac{dv}{v} + \frac{dP}{P}\right] - \frac{(C_p - C_v)dP}{P}$$ (since for an ideal gas $C_p - C_v = R$)

or, $$ds = C_p \frac{dv}{v} + C_v \frac{dP}{P}$$

Since, for an isentropic process, $ds = 0$, we get

$$C_p \frac{dv}{v} + C_v \frac{dP}{P} = 0$$

Dividing both sides of the above equation by C_v, we have

$$\gamma \frac{dv}{v} + \frac{dP}{P} = 0$$

or, $$\ln \gamma \frac{dv}{v} + \ln \frac{dP}{P} = \ln \text{ (constant)}$$

or, $$Pv^{\gamma} = \text{constant}$$

SUMMARY

- Whenever a system undergoes a cyclic process, the algebraic sum of all the heat interactions divided by the absolute temperature at which the heat interactions are taking place, for the whole cycle, is less than or equal to zero.

  Mathematically, $\oint \frac{\delta Q}{T} \leq 0$

  If $\oint \frac{\delta Q}{T} = 0$ then the cyclic process is possible and reversible.

  If $\oint \frac{\delta Q}{T} < 0$ then the cyclic process is possible and irreversible.

  If $\oint \frac{\delta Q}{T} > 0$ then the cyclic process is not possible .
- The change in the entropy of a system is given by

  $dS = \frac{\delta Q_R}{T}$ for reversible process.
- The entropy of any closed system which is thermally isolated from the surroundings either increases, or if the process undergone by the system is reversible, remains constant.
- Entropy can be viewed as a measure of molecular disorder or randomness. The entropy of a pure crystalline substance at absolute zero temperature is zero. This is known as the *third law of thermodynamics.*
- The thermodynamics property relations can be expressed as

  $$T\text{d}s = \text{d}u + P\text{d}v$$

  $$T\text{d}s = \text{d}h - v\text{d}P$$

  Although the equations are derived for a reversible process only, but they are applicable to all processes whether reversible or irreversible.

REVIEW QUESTIONS

6.1 How is entropy defined?
6.2 State the Clasius inequality and explain its significance.
6.3 Why is the Carnot cycle on the T–s plot a rectangle?
6.4 What do you understand by the entropy principle?

6.5 Why does an isochoric curve plotted on a *TS* diagram have a greater slope than an isobaric curve at the same temperature?

6.6 Establish the Clasius inequality.

6.7 Show that entropy is a property of a system.

6.8 Why is an isentropic process not necessarily an adibatic process?

6.9 Show that heat transfer through a finite temperature difference is an irreversible process.

6.10 Can the entropy of a system ever decrease? If yes, explain with an example.

6.11 Show that a reversible adiabatic process is an isentropic process.

NUMERICAL PROBLEMS

6.1 Consider 1 kg of ice at 30 °C as a system. It is exposed to surroundings at 20 °C. The ice melts to water ultimately coming to equilibrium with the surroundings. Calculate the entropy change of the system, the surroundings and the universe. Specific heat of ice and water are 2.187 kJ/kgK and 4.187 kJ/kgK respectively and the latent heat of fusion of ice is 333.5 kJ/kg.

6.2 An aluminium block of 8 kg at 350 °C is dropped in a container having 8 kg of water at 25 °C. The container is open to the atmosphere. Calculate the entropy change of the aluminium block and the universe. Specific heat of aluminium and water are 0.9 kJ/kgK and 4.2 kJ/kgK respectively.

6.3 An insulated container initially contains 6 kg of water at 30 °C. If 2 kg of ice at 0 °C is dropped into the container, determine the final equilibrium temperature and the entropy change associated with the process. Specific heat of water is 4.187 kJ/kgK respectively and the latent heat of fusion of ice is 333.5 kJ/kg.

6.4 One kg of air is compressed in a closed system from 100 kPa, 30 C to 600 kPa isothermally. Find the change in entropy. Assume that C_p and C_v are constant over this range of temperature.

6.5 One kg of air initially at 100 kPa and a volume of 1 m^3 is heated at constant pressure until the volume is doubled. It is then compressed isothermally until the volume is back to its initial value. It is then cooled at constant volume until the pressure is again 100 kPa. Sketch the processes on *P-V* and *T-S* diagrams. Compute the entropy changes for the individual processes, and also find the total entropy change.

6.6 A Carnot cycle rejects heat at 27 °C causing a decrease in entropy equal to 5 kJ/kgK. The engine receives 2500 kJ/kg of heat from a source. Determine the efficiency of the cycle and temperature of the source.

6.7 Initially, two identical bodies of constant specific heat are maintained at the same temperature. These two bodies of finite size are the used as reservoirs for a refrigerator. Heat is removed from one body and rejected to the other body. As a result, the temperature of one body continues to decrease and that of the other continually increases. The operation is at constant pressure, and it is assumed that neither body undergoes a phase change. Show that the minimum work required to decrease the temperature of the cooled body to some value T_f, which is less than T_i, the initial temperature of both bodies, is

$$W_{\min} = mc_p\left(\frac{T_i^2}{T_f} + T_f - 2T_i\right)$$

6.8 One kg of air expands isothermally from 6 MPa to 200 kPa. The initial volume and temperature of air are 500 cm^3 and . Calculate the net heat transfer and net entropy change.

6.9 Air expands from $0.2\ m^3$ to $1\ m^3$ in a cylinder while the pressure is held constant at 600 kPa. The initial temperature of air is 20 °C. Calculate the heat transfer and the entropy change. The specific heats of are $C_p = 1.005\ kJ/kg°C$ and $C_v = 0.718\ kJ/kg°C$.

6.10 A Carnot engine operates between reservoirs at 850 °C and 60 °C. It delivers 100 kW of power. Calculate the entropy change of each reservoir and the net entropy change of the reservoirs after 10 min of operation.

6.11 Two bodies of thermal capacities C_1 and C_2 at temperatures T_1 and T_2 are brought to the same temperature T by means of a reversible heat engine. Show that the final temperature can be expressed as

$$\ln T = \frac{C_1 \ln T_1 + C_2 \ln T_2}{C_1 + C_2}$$

MULTIPLE-CHOICE QUESTIONS

6.1 Entropy change depends on
(a) heat transfer (b) mass transfer
(c) change of temperature (d) thermodynamic state

6.2 For a reversible adiabatic process, change in entropy is
(a) maximum (b) minimum
(c) zero (d) negative

6.3 When a gas flows through a very long pipe of uniform cross section, the flow is approximately
(a) isentropic (b) isothermal
(c) isobaric (d) isochoric

6.4 The value of entropy at 0 K is taken as
(a) 1 (b) 0
(c) –1 (d) any value

6.5 The slope of constant pressure line on T-S diagram is given by
(a) Cp/T (b) T/Cp
(c) S/T (d) T/S

6.6 Steam flow through a nozzle is considered as
(a) constant flow (b) isothermal flow
(c) adiabatic (d) constant volume flow

6.7 An isentropic process on *T-S* diagram is represented by a
(a) horizontal line (b) vertical line
(c) inclined line (d) curved line

6.8 A process occurs spontaneously if its entropy
(a) increases (b) decreases
(c) remains same (d) becomes zero

6.9 The property of a working substance which increases or decreases as the heat is supplied or removed in a reversible manner, is known as
(a) enthalpy (b) entropy
(c) internal energy (d) external energy

6.10 For in irreversible process entropy change is
(a) greater than $\frac{\delta Q}{T}$ (b) equal to $\frac{\delta Q}{T}$
(c) less than $\frac{\delta Q}{T}$ (d) equal to zero

6.11 $Tds = dh - vdp$ equation can be applied for

(a) reversible process (b) irreversible process
(c) all of the above (d) none of the above

6.12 Choose the correct statement:

(a) entropy is an intensive property (b) density is an extensive property
(c) density is an intensive property (d) pressure is an extensive property.

6.13 Strictly speaking all engineering processes are

(a) quasi-static (b) reversible
(c) irreversible (d) thermodynamically in equilibrium

6.14 For an irreversible process, entropy change is

(a) greater than $\delta Q/T$ (b) less than $\delta Q/T$
(c) equal to $\delta Q/T$ (d) equal to zero

6.15 The value of $\Sigma(\delta Q/T)$ for reversible process is equal to

(a) +ve value (b) –value
(c) zero (d) any one of the above

6.16 The value of $\Sigma(\delta Q/T)$ for irreversible process is

(a) equal to zero (b) less than zero
(c) greater than zero (d) any one of the above

6.17 The change of entropy when heat is absorbed by the gas is

(a) positive (b) negative
(c) positive or negative (d) none of these

6.18 An isentropic process

(a) must be reversible and adiabatic (b) need not be reversible adiabatic
(c) decreases the entropy of a system (d) none of these

CHAPTER

7

Properties of Pure Substances

7.1 INTRODUCTION

A pure substance is defined as one that is homogeneous and invariable in chemical composition throughout its mass. The relative proportion of the chemical elements, constituting the substance, is also constant. Atmospheric air, steam–water mixture and combustion products of a fuel are regarded as pure substance. A phase is a physically distinct, chemically homogeneous and mechanically separable portion of a substance.

7.2 PROPERTY DIAGRAMS

The property diagrams for a pure substance are of immense importance from thermodynamic point of view. In this Chapter, we will discuss the pressure-volume ($P–V$), pressure-temperature ($P–T$), temperature-entropy ($T–S$) and enthalpy-entropy ($h–s$) diagram for water.

7.2.1 Pressure–Volume Diagram

Water is one of the commonly used pure substances that can exist in three different phases—solid, liquid and gas. Addition of heat at constant pressure causes its phase to change that result to a change of its volume. Water is a rare category of pure substance which unlike other substances manifests decrease in volume when it is converted from solid to liquid, i.e., when ice is converted to water. To study the complete behaviour of the pure substance, it is imperative to plot $P–V$ diagram for the same. This is portrayed in Fig. 7.1.

Let us consider a mass of ice at say –20ºC at atmospheric pressure. Addition of heat to ice will cause its temperature to increase up to 0ºC accompanied by increase in volume. The initial state is denoted by 1 and the final state is denoted by 2. Further heating will cause change in phase from ice to water without increase in temperature. The substance will absorb ***latent heat of fusion*** for change in phase. The final

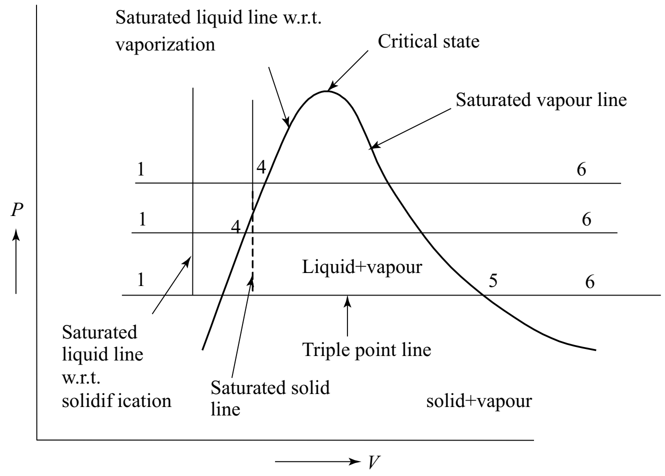


Figure 7.1 *P–V diagram of pure substance (e.g. water) that expands on freezing*

volume is represented by the point 3. It is interesting to note that this time volume decreases. Addition of further heat is responsible for rise in temperature of water from 0°C to 100°C. Consequently volume increases from 3 to 4. At the point 4, the entire liquid attains its boiling point temperature. Continuation of heating at this stage will convert water (liquid) to steam or vapour (gas) by taking ***latent heat of vapourization***. The final state is represented by the point 5 and is called *saturated vapour*. The increase in volume during this part is significantly high. Further heating will convert the saturated steam to what is called superheated steam. The temperature of superheated steam is higher than that of saturated steam. The states 2, 3, 4 and 5 are called ***saturation states*** implying that at this point change in phase occurs. Water (liquid) being at the middle of three different phases ***(solid to liquid and liquid to vapour)*** there are two saturated liquid states. In the state 3, the liquid is saturated in regard to solidification whereas at 4, the liquid is saturated with respect to vaporization. The temperature at which water is converted to complete steam is called ***saturation temperature*** corresponding to that particular pressure. The difference in temperature between the superheated steam and the corresponding saturation temperature is called ***degree of superheat***.

A liquid existing at a temperature lower than the saturation temperature corresponding to its pressure is called ***compressed liquid*** or ***subcooled liquid***. Vapour existing at a temperature higher than the saturation temperature corresponding to its pressure is called ***superheated vapour***.

If this heating is carried out at different pressures, similar states would be obtained. The Locus of all the saturated solid states is called ***saturated solid line***. Likewise, locus of all the points corresponds to point 3 and 4 are called ***saturated liquid lines***. The line joining all the points 3 is called *saturated liquids line with respect to solidification*, whereas the line joining all the points, 4 is called *saturated liquid line with respect to vaporisation*. Similarly, the line joining all the points, 5 gives rise to ***saturated vapour line***.

The zone that lies on the left side of the saturated solid line is the solid zone. Similarly, other zones are established. The zones are established by the following series.

Solid → Solid + Liquid → Liquid → Liquid + Vapour → Vapour

Triple Point It is a line on the P–V diagram where all the three phases can exist in equilibrium. Although it is a line but it is called triple point. At this line the pressure and temperature of three different phases of the substance remains same but specific volumes are different. For water, the triple point temperature and pressure are 0.01°C and 0.6117 kPa respectively. At pressure below the triple point, no substance can exist in stable equilibrium in liquid phase. Addition of heat to solid below this pressure, directly converts it to vapour by absorbing ***latent heat of sublimation***. Hence, the zone below the triple point line denotes the solid and vapour zone.

Critical Point It is interesting to note that as the pressure increases, the transition zone from liquid to vapour becomes narrow. This implies that saturated liquid line (locus of points 4) and saturated vapour line are inclined towards each other. At a specific pressure, the transition zone is converted to a point. ***The point, at which the saturated liquid state and saturated vapour state are identical, is called critical point***. At pressures above this point, the liquid, if heated, immediately flashes into vapour without manifesting a clear transition zone. Conversely, cooling of vapour converts it to water all of a sudden. The pressure, specific volume and temperature at critical point are called ***critical pressure***, ***critical specific volume*** and ***critical temperature*** respectively. The critical point data for water are given below.

$$P_{cr} = 221.2 \text{ bar} \qquad v_{cr} = 0.00317 \text{ m}^3/\text{kg} \qquad t_{cr} = 374.15°\text{C}$$

It must be remembered that phase change takes place only at constant pressure and temperature. For constant pressure heating, liquid is converted to vapour only at a particular temperature called ***saturation temperature***. Similarly, if the temperature remains constant, this phase change takes place at a definite pressure called ***saturation pressure***.

7.2.2 Pressure–Temperature Diagram

The change in volume of pure substance at constant pressure heating is analyzed by the P–V diagram. The pressure-temperature (P-T) portrays the variations in temperature at constant pressure heating. This is plotted in Fig. 7.2.

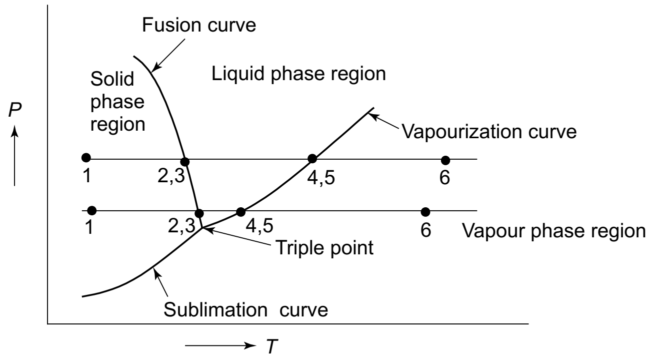


Figure 7.2 *P–T diagram of a pure substanace (e.g. water) that expands on freezing*

The variation of temperature at any particular pressure is represented by a horizontal line. Several such curves may be obtained which are parallel. However, heating at higher pressure is accompanied by increase in saturation temperature. Note that on any particular curve, points 2 and 3 and points 4 and 5 coincides. This is due to the fact that between 2 and 3 there is change in phase (from solid to liquid) and between 4 and 5, liquid is converted to vapour. Since during phase change the substance absorbs or rejects only latent heat, the temperature during phase change remains unaltered. The curve passing through the series of 2, 3 points is called the ***fusion curve*** while the curve when drawn through points 4 and 5 is called the ***vaporization curve***. The ***sublimation curve*** is the locus of vapour pressure at different temperatures. Three aforementioned curves meet at the ***triple point***.

7.2.3 Temperature–Entropy Diagram

Temperature-entropy (*T*-*S*) diagram for pure substance is of immense importance from the thermodynamic point of view. Water being one of the popular working substance for power plants, it is imperative to investigate the nature of *T–S* plot when the substance is heated gradually.

Two different situations are encountered while heating ice (at –20ºC) so as to convert it to a superheated steam at 200ºC.

(i) Temperature is increased continuously so long there is no change in phase.

(ii) Temperature remains constant during change in phase.

In the first case, change in entropy between any two finite temperature range T_f and T_i can be calculated as $\Delta S = S_f - S_i = \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mC_p dT}{T} = mC_p \ln \frac{T_f}{T_i}$

And if temperature remains constant, ΔS becomes $\frac{mL}{T}$.

The *T–S* plot is shown in Fig. 7.3. Several such curves are plotted when heating is carried out at different pressures. ***The dome–shaped region formed by the saturated liquid line and saturated vapour line is called vapour dome***.

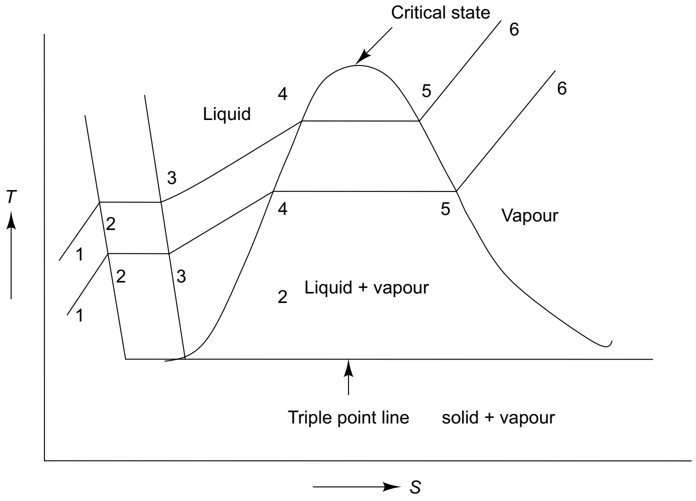


Figure 7.3 *T–S diagram of a pure substance*

7.2.4 Enthalpy–Entropy Diagram

In this context, the enthalpy vs entropy plot or *h–s* diagram is found to be valuable in the analysis of steady-flow devices like turbines, compressors, nozzles etc.The *h–s* diagram is popularly known as ***Mollier diadram.***

From the thermodynamic property relationship (see Eq. (6.17)), we have

$$Tds = dh - vdp$$

For constant pressure, the above equation becomes

$$Tds = dh$$

or, $$\left(\frac{\partial h}{\partial s}\right)_P = T \tag{7.1}$$

Equation (7.1) implies that the slope of the *constant pressure* lines is equal to the corresponding saturation temperatures. As the pressure increases, the saturation temperature also increases. This is the reason for which *constant pressure lines* (*isobar*) are divergent on the h-s diagram. The h-s diagram (Mollier diagram) for water is shown in Fig. 7.4. The saturated liquid line and saturated vapour line meets at critical point.

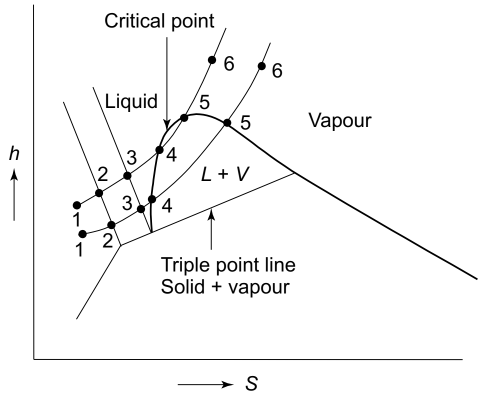


Figure 7.4 *h–s (Mollier diagram) of a pure substance (water)*

Constant temperature lines are distinctly visible in the superheat region. However, in the wet region these lines coincide with constant pressure lines. It is noteworthy that in the superheated region, enthalpy is more dependent on temperature alone. *As the degree of superheat increases, the constant temperature lines tend to become horizontal.* Constant dryness fraction lines are also added in the curve to calculate different properties of wet steam. From the curve, it is evident that *reduction in pressure at constant enthalpy results in the drying and superheating of wet vapour.*

7.3 *P-V-T* SURFACE

According to state postulate (see Section 1.8), the state of a substance is described by two independent, intensive properties. Once these are completely specified, all the other properties become dependent properties. Any equation with two independent variables in the form $f = f(x, y)$ represents a surface in space; it is possible to represent the *P-V-T* behaviour of a substance as a surface in space as shown in Fig. 7.5. Here, T and V are considered as independent variables and P as dependent. All the points in the surface represent equilibrium states. Hence the quasi-static processes must lie on the surface.

Essentially, this is a three-dimensional view where three different dimensions are P, V and T. It can be concluded therefore, that the popularly used P–V diagrams and T–V diagram are merely the orthographic projections of the original-three dimensional objects. Although such a three-dimensional view is very comprehensive, in thermodynamic analysis use of two-dimensional diagrams are more popular.

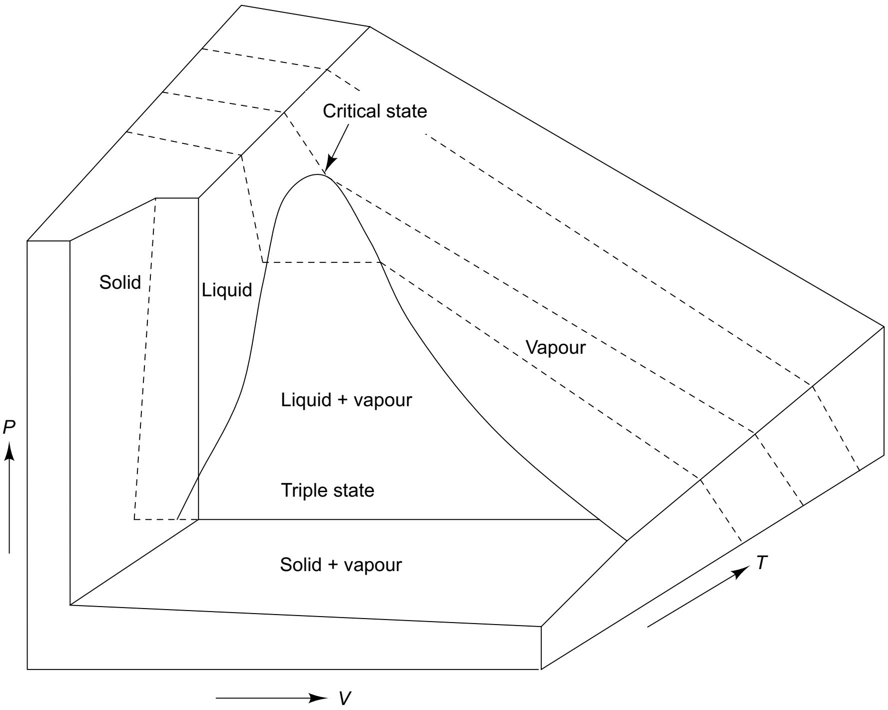


Figure 7.5 *A P-V-T surface for a substance which expands on freezing*

7.4 QUALITY OR DRYNESS FRACTION

Quality or dryness fraction of a liquid-vapour mixture is defined as the ratio of mass of saturated vapour to the total mass of mixture. That is,

$$x = \frac{\text{mass of saturated vapour}}{\text{total mass of mixture}}$$

or,

$$x = \frac{m_g}{m} = \frac{m_g}{m_f + m_g} \tag{7.2}$$

where m_f is the mass of saturated liquid, m_g is the mass of saturated vapour and m is the total mass of liquid-vapour mixture.

Let V be the total volume of the mixture, V_f be the volume of the saturated liquid and V_g be the volume of the saturated vapour.

Total volume of liquid and vapour can be expressed in terms of their specific volume as

$$V_f = m_f v_f$$

$$V_g = m_g v_g$$

where v_f and v_g are the specific volume of saturated liquid and saturated vapour respectively.

Specific volume of the mixture is then

$$\begin{aligned} v &= \frac{V}{m} = \frac{V_f + V_g}{m} \\ &= \frac{m_f v_f + m_g v_g}{m_f + m_g} \\ &= \frac{m_f v_f}{m_f + m_g} + \frac{m_g v_g}{m_f + m_g} \\ &= (1 - x)v_f + x v_g \\ &= v_f + x(v_g - v_f) \\ &= v_f + x v_{fg} \end{aligned}$$

Similarly, one can write

$$h = h_f + x h_{fg}$$

$$s = s_f + x s_{fg}$$

and

$$u = u_f + x u_{fg}$$

7.5 STEAM TABLES

From the foregoing discussion, it is possible to know the various properties of steam when it is either saturated liquid or saturated vapor provided its pressure or temperature is known. The subscripts f

and g refer to saturated liquid and saturated vapour respectively. Steam table is nothing but a complete data book that contains various properties of water in different phases for a given pressure or temperature. It is given in the Appendix I. The properties of steam are given in three different tables: saturated steam table-temperature base (Appendix I, Table A.1), saturated steam table-pressure base (Appendix I, Table A.2), and superheated steam table (Appendix I, Table A.3),. In the saturated steam tables, the properties of saturated liquid and saturated vapour are presented. It is known that the pressure and temperature both remain constant during the phase transition. That means both are not independent variables. One is independent and the other is dependent. When the temperature is chosen as independent variable and the properties of steam are tabulated, the steam table is referred to as the saturated steam table-temperature base. For saturated steam table-pressure base, pressure is the independent variable. There is a separate table for superheated steam for different amount of degree of superheat.

Example 7.1 Using the steam tables, estimate the saturation temperature and specific volume, specific enthalpy and specific entropy of saturated liquid and vapour at 3 MPa.

Solution From the saturated steam table based on pressure (Appendix I, Table A.2), it is found that saturation temperature of water corresponds to 3 MPa is $t_{\text{sat}} = 233.9$ C

Specific volume of saturated liquid is $v_f = 0.001216$ m^3/kg

Specific volume of saturated vapour is $v_g = 0.06668$ m^3/kg

Specific enthalpy of saturated liquid is $h_f = 1008.4$ kJ/kg

Specific enthalpy of saturated vapour $h_g = 2804.1$ kJ/kg

Specific entropy of saturated liquid is $s_f = 2.6462$ kJ/kg-K

Specific entropy of saturated vapour is $s_g = 6.1878$ kJ/kg-K

Example 7.2 Using the steam tables, estimate saturation pressure and the specific volume, specific enthalpy and specific entropy of saturated liquid and vapour at 200°C.

Solution From the saturated steam table based on temperature (Appendix I, Table A.1), it is found that saturation pressure of water corresponds to 200°C is $P_{\text{sat}} = 1.554$ MPa ·

Specific volume of saturated liquid is $v_f = 0.001156$ m^3/kg

Specific volume of saturated vapour is $v_g = 0.1274$ m^3/kg

Specific enthalpy of saturated liquid is $h_f = 852.4$ kJ/kg

Specific enthalpy of saturated vapour $h_g = 2793.2$ kJ/kg

Specific entropy of saturated liquid is $s_f = 2.3313$ kJ/kg-K

Specific entropy of saturated vapour is $s_g = 6.4331$ kJ/kg-K

Example 7.3 Calculate the specific volume, specific enthalpy and specific entropy of wet steam of dryness fraction or quality of 0.9 at 2 MPa pressure.

Solution From the saturated steam table (Appendix I, Table A.2), at 2 MPa

Specific volume of saturated liquid is $v_f = 0.001177\ \text{m}^3/\text{kg}$

Specific volume of saturated vapour is $v_g = 0.09963\ \text{m}^3/\text{kg}$

Specific enthalpy of saturated liquid is $h_f = 908.8$ kJ/kg

Specific enthalpy of saturated vapour $h_g = 2799.5$ kJ/kg

Specific entropy of saturated liquid is $s_f = 2.4478$ kJ/kg-K

Specific entropy of saturated vapour is $s_g = 6.3417$ kJ/kg-K

Dryness fraction of the mixture is given as $x = 0.9$
The specific volume of the wet steam is

$$\begin{aligned} v &= v_f + x(v_g - v_f) \\ &= 0.001177 + 0.9(0.09963 - 0.001177) = 0.08978\ \text{m}^3/\text{kg} \end{aligned}$$

The specific enthalpy of the wet steam is

$$\begin{aligned} h &= h_f + x(h_g - h_f) \\ &= 908.8 + 0.9(2799.5 - 908.8) = 2610.43\ \text{kJ/kg} \end{aligned}$$

The specific entropy of the wet steam is

$$\begin{aligned} s &= s_f + x(s_g - s_f) \\ &= 2.4478 + 0.9(6.3417 - 2.4478) = 5.9523\ \text{kJ/kg-K} \end{aligned}$$

Example 7.4 If the specific entropy of steam at 1 MPa is 5.82 kJ/kg-K , determine its state.

Solution From the saturated steam table based on pressure (Appendix I, Table A.2), it is found that at 1 MPa the specific entropy of saturated liquid and saturated vapour are $s_f = 2.1391$ kJ/kg-K and $s_g = 6.5873$ kJ/kg-K respectively

Since the given specific entropy lies between s_f and s_g, the state will be within the vapour dome. Thus, temperature of the steam is same as the saturation temperature corresponds to 1 MPa pressure. From Appendix I, Table A.2, we get $t_{\text{sat}} = 179.9$ C

Let x be the quality of the steam.

$$\begin{aligned} s &= s_f + x(s_g - s_f) \\ 5.82 &= 2.1391 + x(6.5873 - 2.1391) \end{aligned}$$

or, $$x = 0.8275$$

Example 7.5 Find the specific volume, specific enthalpy and specific entropy of steam at 5 bar and 400°C.

Solution From the saturated steam table (Appendix I, Table A.2), it is found that saturation temperature of water corresponds to 5 bar is $t_{\text{sat}} = 151.86\ \text{C}$

Since the temperature is greater than that of saturation temperature, the state would be in the superheated region.

From the superheated steam table (Appendix I, Table A.3),

Specific volume of steam is $v = 0.6173\ \text{m}^3/\text{kg}$

Specific enthalpy of steam is $h = 3271.9\ \text{kJ/kg}$

Specific entropy of steam is $s = 7.7938\ \text{kJ/kg-K}$

Example 7.6 A rigid vessel of volume 0.2 m^3 contains 1 kg of steam at a pressure of 0.8 MPa. Evaluate the specific volume, temperature, dryness fraction, enthalpy and entropy of steam.

Solution Specific volume of the steam $v = \dfrac{V}{m} = \dfrac{0.2}{1} = 0.2\ \text{m}^3/\text{kg}$

From the saturated steam table (Appendix I, Table A.2), it is found that at 0.8 MPa the specific volume of saturated liquid and saturated vapour are $v_f = 0.001115\ \text{m}^3/\text{kg}$ and $v_g = 0.2404\ \text{m}^3/\text{kg}$ respectively.

Since the given specific volume lies between v_f and v_g, the state will be within the vapour dome. From the saturated steam table, it is found that saturation temperature of water corresponds to 0.8 MPa is $t_{\text{sat}} = 170.4\ \text{C}$

Let x be the quality of the steam. Thus,

$$v = v_f + x(v_g - v_f)$$

$$0.2 = 0.001115 + x(0.2404 - 0.001115)$$

or, $$x = 0.8312$$

The specific enthalpy of the wet steam is

$$\begin{aligned} h &= h_f + x(h_g - h_f) \\ &= 721.1 + 0.8312(2769.1 - 721.1) = 2423.4\ \text{kJ/kg} \end{aligned}$$

The total enthalpy of the wet steam is then

$$H = mh = (1\text{kg})(2423.4\ \text{kJ/kg}) = 2423.4\ \text{kJ}$$

The specific entropy of the wet steam is

$$\begin{aligned} s &= s_f + x(s_g - s_f) \\ &= 2.0466 + 0.8312(6.6636 - 2.0466) = 5.8843 \text{ kJ/kg-K} \end{aligned}$$

The total entropy of the wet steam is

$$S = ms = (1\text{kg})(5.8843 \text{ kJ/kg-K}) = 5.8843 \text{ kJ/K}$$

Example 7.7 10 kg of wet steam at a pressure of 0.2 MPa is contained in a rigid tank of volume 6.058 m^3. The tank is heated until the steam becomes dry saturated. Determine the final pressure and the heat transfer to the tank.

Solution Specific volume of the steam $v = \frac{V}{m} = \frac{6.058}{10} = 0.6058 \text{m}^3\text{/kg}$

From the saturated steam table (Appendix I, Table A.2), it is found that at 0.2 MPa the specific volume of saturated liquid and saturated vapour are $v_f = 0.001061 \text{ m}^3\text{/kg}$ and $v_g = 0.8857 \text{ m}^3\text{/kg}$ respectively.

Since the given specific volume lies between v_f and v_g, the state will be within the vapour dome. From the saturated steam table, it is found that saturation temperature of water corresponds to 0.2 MPa is $t_{\text{sat}} = 120.2$ C

$$\begin{aligned} v &= v_f + x(v_g - v_f) \\ 0.6058 &= 0.001061 + x(0.8857 - 0.001061) \end{aligned}$$

or, $x = 0.6836$

The specific enthalpy of the wet steam is

$$\begin{aligned} h &= h_f + x(h_g - h_f) \\ &= 504.7 + 0.6836(2706.6 - 504.7) = 2009.92 \text{ kJ/kg} \end{aligned}$$

From the saturated steam table, it is found that for $v_g = 0.6058 \text{m}^3\text{/kg}$, the corresponding pressure is 0.3 MPa

At 0.3 MPa, the specific enthalpy of saturated vapour $h_g = 2725.3$ kJ/kg

Specific heat transfer to the tank is $q = h_g - h = 2725.3 - 2009.92 = 715.38$ kJ/kg

Total heat transfer is then $Q = mq = (10\text{kg})(715.38 \text{ kJ/kg}) = 7153.8 \text{ kJ}$

Example 7.8 One kg of water at 75°C is heated at a constant pressure of 0.8 MPa until it becomes superheated vapor at 200ºC. Find the change in volume, enthalpy, entropy and internal energy.

Solution From the steam table (Appendix I, Table A.2), it is found that saturation temperature of water corresponds to 0.8 MPa is $t_{\text{sat}} = 170.4$ C

The conversion of water at 75ºC to superheated steam at 200ºC can be divided into the following distinct steps

(i) Conversion of water at 75ºC to saturated water at 170.4ºC

The increase in enthalpy for this change is $\Delta H_1 = m \times C_{pw} \times (T_{\text{sat}} - T_i)$

$$= 1 \times 4.18 \times (170.4 - 75) = 398.72 \text{ kJ}$$

(ii) Conversion of saturated water to saturated steam without change in temperature.

The increase in enthalpy for this process is $\Delta H_2 = m(h_g - h_f)$

From the saturated steam table (Appendix I, Table A.2), at 0.8 MPa

$$h_f = 721.1 \text{ kJ/kg} \text{ and } h_g = 2769.1 \text{ kJ/kg}$$

Hence, $\Delta H_2 = (1\text{kg})(2769.1 - 721.1)\text{kJ/kg} = 2048 \text{ kJ}$

(iii) Conversion of saturated steam into superheated steam of 200ºC

From superheated steam table (Appendix I, Table A.3), it is found that at 0.8 MPa and 200ºC, $h = 2839.3$ kJ/kg

The increase in enthalpy associated with this change is

$$\Delta H_3 = m(h - h_g)$$

$$= (1\text{kg})(2839.3 - 2769.1) \text{ kJ/kg} = 70.2 \text{ kJ}$$

Therefore total change in enthalpy becomes

$$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$$

$$= 398.72 + 2048 + 70.2 = 2516.92 \text{ kJ}$$

From the superheated steam table it is found that $v = 0.2608$ m^3/kg and $v_f = 0.001115$ m^3/kg

Therefore change in volume becomes

$$\Delta V = m(v - v_f) = (1\text{kg})(0.2608 - 0.001115)\text{m}^3\text{/kg} = 0.2597\text{m}^3$$

From the relationship $h = u + Pv$ we have $h_1 = u_1 + Pv_1$ and $h_2 = u_2 + Pv_2$

Hence, change in internal energy is

$$U_2 - U_1 = (H_2 - H_1) - P \times \Delta V$$

$$= 2516.92 - (800 \text{ kPa})\left(0.2597 \text{ m}^3\right) = 2309.16 \text{ kJ}$$

Following the logic followed for enthalpy computations,

Change of entropy of water during conversion from 75ºC to saturated water at 170.4ºC is

$$\Delta S_1 = mC_p \ln \frac{T_{sat}}{T_i} = 1 \times 4.18 \ln \frac{170.4 + 273}{75 + 273} = 1.0127 \text{ kJ/K}$$

Change of entropy of water Conversion of saturated water to saturated steam is

$$\Delta S_2 = m \times (s_g - s_f) = 1 \times (6.6636 - 2.0466) = 4.617 \text{ kJ/K}$$

Change of entropy of water during conversion of saturated steam into superheated steam of 200ºC is

$$\Delta S_3 = m \times (s - s_g) \ = 1 \times (6.8158 - 6.6636) = 0.1522 \text{ kJ/K}$$

Hence total change in entropy becomes

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 = 1.0127 + 4.167 + 0.1522 = 5.7819 \text{ kJ/K}$$

Example 7.9 A vessel of volume 0.08 m^3 contains a mixture of saturated water and saturated steam at a temperature of 200°C. The mass of the liquid present is 10 kg. Find the pressure, the mass, the specific volume, the enthalpy, the entropy and the internal energy.

Solution From temperature based saturated steam table (Appendix I, Table A.1), at 200°C, saturation pressure is $P_{\text{sat}} = 1.554$ MPa

Specific volume of saturated liquid is $v_f = 0.001156$ m^3/kg

Specific volume of saturated vapour is $v_g = 0.1274$ m^3/kg

Specific enthalpy of saturated liquid is $h_f = 852.4$ kJ/kg

Specific enthalpy of saturated vapour is $h_g = 2793.2$ kJ/kg

Specific entropy of saturated liquid is $s_f = 2.3313$ kJ/kg-K

Specific enthalpy of saturated vapour is $s_g = 6.4331$ kJ/kg-K

Specific internal energy of saturated liquid is $u_f = 850.6$ kJ/kg

Specific internal energy of saturated vapour is $u_g = 2595.3$ kJ/kg

Total Volume of liquid is $V_f = m_f v_f$

$$= (10\text{kg})\left(0.001156\text{m}^3/\text{kg}\right) \ = 0.01156 \text{ m}^3$$

Total Volume of vapour is $V_g = V - V_f = 0.08 - 0.01156 = 0.06844 \text{ m}^3$

Mass of vapour is found to be

$$m_g = \frac{V_g}{v_g} = \frac{0.06844}{0.1274} = 0.5372 \text{ kg}$$

Total mass of liquid-vapour mixture is $m = m_f + m_g = 10 + 0.5372 = 10.5372$ kg

Quality (or dryness fraction) of the mixture is $x = \frac{m_g}{m} = \frac{0.5372}{10.5372} = 0.05$

Specific volume of the mixture is

$$\begin{aligned} v &= v_f + x(v_g - v_f) \\ &= 0.001156 + 0.05(0.1274 - 0.001156) = 0.00747 \text{ m}^3\text{/kg} \end{aligned}$$

Specific enthalpy of the mixture is

$$\begin{aligned} h &= h_f + x(h_g - h_f) \\ &= 852.4 + 0.05(2793.2 - 852.4) = 949.44 \text{ kJ/kg} \end{aligned}$$

Total enthalpy of the mixture is then

$$H = mh = (10.5372\text{kg})(949.44 \text{ kJ/kg}) = 10004.44 \text{ kJ}$$

Specific entropy of the mixture is

$$\begin{aligned} s &= s_f + x(s_g - s_f) \\ &= 2.3313 + 0.05(6.4331 - 2.3313) = 2.5364 \text{ kJ/kg-K} \end{aligned}$$

Total entropy of the mixture is then found to be

$$S = ms = (10.5372\text{kg})(2.5364 \text{ kJ/kg-K}) = 26.7266 \text{ kJ/K}$$

Specific internal energy of the mixture is

$$\begin{aligned} u &= u_f + x(u_g - u_f) \\ &= 850.6 + 0.05(2595.3 - 850.6) = 937.835 \text{ kJ/kg} \end{aligned}$$

Total internal energy of the mixture is

$$U = mu = (10.5372\text{kg})(937.835 \text{ kJ/kg}) = 9882.15 \text{ kJ}$$

SUMMARY

- A pure substance is defined as one that is homogeneous and invariable in a chemical composition throughout its mass.
- A phase is a physically distinct, chemically homogeneous and mechanically separable portion of a substance.

- The pressure and temperature under which two phases can exist in equilibrium are called *saturation pressure* and *saturation temperature*.
- A liquid existing at a temperature lower than the saturation temperature corresponding to its pressure is called *compressed liquid* or *subcooled liqui*d. Vapour existing at a temperature higher than the saturation temperature corresponding to its pressure is called *superheated vapour*.
- The condition of pressure and temperature under which all the three phases of a pure substance can exist in equilibrium is called the triple point of the substance.
- *The point, at which the saturated liquid state and saturated vapour state are identical, is called the critical point.* At pressures above this point, a liquid, if heated, immediately flashes into vapour without manifesting a clear transition zone. Conversely, cooling of vapour converts it to water all of a sudden. The pressure, specific volume and temperature at the critical point are called *critical pressure, critical specific volume and critical temperature* respectively.
- Quality or dryness fraction is defined as the ratio of mass of saturated vapour to the total mass of mixture. That is,

$$x = \frac{m_g}{m} = \frac{m_g}{m_f + m_g}$$

  where m_f is the mass of saturated liquid, m_x is the mass of saturated vapour and m is the total mass of liquid-vapour mixture.
- Properties of a liquid–vapour mixture may be found by relations such as

$$v = v_f + xv_{fg}$$

$$h = h_f + xh_{fg}$$

$$s = s_f + xs_{fg}$$

$$u = u_f + xu_{fg}$$

REVIEW QUESTIONS

7.1 What is a pure substance?
7.2 What is a phase?
7.3 What is a saturated state?
7.4 What is saturation temperature and saturation pressure?
7.5 What is a compressed liquid?
7.6 What is superheated vapour?
7.7 Define the quality of dryness fraction of a liquid–vapour mixture.
7.8 What is the critical point? State the values of critical pressure and critical temperature of water.
7.9 Sketch the P–V diagram for a pure substance and show the isotherms and constant quality lines on it.
7.10 Is it possible to convert a liquid into vapour phase without ever observing the phase transition? If so, sketch the process on a P–T diagram.

NUMERICAL PROBLEMS

7.1 Using the steam tables, estimate the specific volume, specific enthalpy and specific entropy of saturated liquid and vapour at 30 bar.

7.2 Using the steam tables, estimate the specific volume, specific enthalpy and specific entropy of saturated liquid and vapour at 250ºC.

7.3 Calculate the specific volume, specific enthalpy and specific entropy of wet steam of dryness fraction of quality 0.9 at 30 bar pressure.

7.4 If the specific entropy of steam at 30 bar is 5.22 kJ/kg-K, determine its state.

7.5 Find the saturation temperature specific volume, specific enthalpy and entropy of saturated vapour at 5 bar. Also find the latent heat of vaporisation of steam at that pressure.

7.6 A rigid vessel of volume 0.3 m^3 contains 10 kg of steam at a pressure of 5 bar. Evaluate the specific volume, temperature, dryness fraction, enthalpy and entropy of steam.

7.7 A vessel of volume 0.03 m^3 contains a mixture of saturated water and saturated steam at a pressure of 30 bar. The mass of the liquid present is 6 kg. Findthe pressure, the mass, the specific volume, the enthalpy, the entropy and the internal energy.

7.8 Suppose a closed and rigid vessel is initially filled with saturated water and saturated vapour at 100 kPa. On transferring energy as heat, the water is found to pass through the critical point. Determine the volume of saturated vapour to the volume of saturated liquid with which the vessel is initially filled.

7.9 A vessel of volume 0.03 m^3 contains a mixture of saturated water and saturated steam at a temperature of 250°C. The mass of the liquid present is 8 kg. Find the pressure, the mass, the specific volume, the enthalpy, the entropy and the internal energy.

MULTIPLE-CHOICE QUESTIONS

7.1 In a *P-V-T* surface, the zone below the triple point is known as
(a) liquid zone (b) vapour zone (c) sublimation zone (d) none of these

7.2 Triple point of a pure substance is a point at which
(a) liquid and vapour exist together (b) solid and vapour exist together
(c) solid and liquid exist together (d) solid, liquid and vapour exist together

7.3 The latent heat of vapourization at the critical point is
(a) equal to zero (b) less than zero (c) greater than zero (d) none of these

7.4 The phase change from liquid to vapour is referred to as
(a) melting (b) vapourization (c) sublimation (d) solidification

7.5 The point that connects the saturated liquid line to the saturated vapour line is called the
(a) triple point (b) superheated point (c) critical point (d) compressed liquid point

CHAPTER

8

Properties of Gas

8.1 EQUATION OF STATE OF A GAS

An equation of state is a relationship among the properties x, y and z, such that

$$f\ (x,y,z) = 0$$

In thermodynamics, the equations which relate pressure P, specific volume v and temperature T, are of particular interest. Therefore, an equation that furnishes the relationship among pressure P, specific volume v and temperature T of a substance is called an ***equation of state***.

The PvT relationship for most of the substances is quite complex, so that accurate equations of state for wide ranges of pressure and temperature have been developed for only a few substances.

8.2 IDEAL GAS

An *ideal gas* is defined as one for which the equation of state is

$$P\bar{v} = \bar{R}T \tag{8.1}$$

where $\bar{R}$ the universal gas constant and $\bar{v}$ is the molar volume (i.e., volume per unit mole).

From Boyle's, Charles, and Gay-Lussac's experimental observations, it has been found that $P\bar{v}T$ behaviour of many gases at low pressure and moderate temperatures can be approximated quite well by the ideal-gas equation. Gases deviate from ideal gas behaviour at high pressure and low temperature.

8.2.1 Ideal Gas Equation of State

A. Unit mole basis Equation (8.1) i.e. $P\bar{v} = \bar{R}T$ is the equation of state of an ideal gas based on unit moles.

The value of the universal gas constant is the same for all gases. It can be found from Avogadro's law. According to Avogadro's law at NTP, 1 kg mole of all gases occupy the same volume and the volume of 1 kg mole of all gases at NTP (273.15 K and 1.0132 bar) is 22.4146 m^3.

$$\bar{R} = \frac{P\bar{v}}{T}$$

$$= \frac{1.0132 \times 10^5 \times 22.4146}{273.15} = 8314.3 \text{ J/kg mol-K}$$

$$= 8.3143 \text{ kJ/kg mol - K}$$

B. Total number of moles basis If V is the total volume and n is the number of moles of the gas, then

$$\bar{v} = \frac{V}{n}$$

Now, the equation of state for an ideal gas can be written as

$$PV = n\bar{R}T \tag{8.2}$$

C. Total mass basis If M is the molecular mass and m is the total mass of the gas then

$$n = \frac{m}{M}$$

From the equation (8.2), we have

$$PV = \frac{m}{M}\bar{R}T$$

$$PV = m\frac{\bar{R}}{M}T$$

$$PV = mRT \tag{8.3}$$

where R is characteristic gas constant which is constant for a particular gas and different for different gases.

The characteristic gas constant for any particular gas can be obtained from

$$R = \frac{\bar{R}}{M} \tag{8.4}$$

For air,

$$R_{\text{air}} = \frac{8314.3}{28.96} = 287 \text{J/kgK} = 0.287 \text{kJ/kgK}$$

For nitrogen,

$$R_{N_2} = \frac{8314.3}{28} = 296.94 \text{ J/kgK}$$

D. Unit mass basis Dividing by the mass of the gas on both sides of the equation (8.3), we have

$$Pv = RT \tag{8.5}$$

where v is the specific volume of the gas.

8.3 SPECIFIC HEATS, INTERNAL ENERGY AND ENTHALPY OF AN IDEAL GAS

It has been demonstrated mathematically that for an ideal gas both the internal energy and enthalpy is a function of the temperature only. Thus, for an ideal gas, one can write (see Chapter 3, for more details)

$$du = C_v dT \quad \text{and} \quad dh = C_p dT$$

Further, from the definition of specific enthalpy, we have

$$h = u + Pv$$

or
$$dh = du + d(Pv)$$

For an ideal gas, the above equation becomes

$$C_p dT = C_v dT + RdT$$

or
$$C_p = C_v + R \tag{8.6}$$

The specific heat ratio, γ is defined as

$$\gamma = \frac{C_p}{C_v} \tag{8.7}$$

From equations (8.6) and (8.7), we have

$$C_p = \frac{\gamma R}{\gamma - 1} \tag{8.8}$$

$$C_v = \frac{R}{\gamma - 1} \tag{8.9}$$

Example 8.1 A certain gas has C_p = 1.005 kJ/kgK and C_v = 0.718 kJ/kgK. Find its molecular weight and the gas constant. Identify the gas.

Solution The characteristic gas constant can be found from equation (8.6) as

$$R = C_p - C_v = 1.005 - 0.718 = 0.287 \text{ kJ/kgK}$$

The molecular weight of the gas is obtained from equation (8.4) as

$$M = \frac{\bar{R}}{R} = \frac{8.3143}{0.287} = 28.96 \text{ kg/kg mol}$$

From the values of R, one can conclude that the gas is air.

Example 8.2 From experimental results, the specific heat ratio for helium is found to be 1.667. Determine the two specific heats.

Solution For helium, characteristic gas constant is

$$R_{He} = \frac{8314.3}{4} = 2.078 \text{ kJ/kgK} \qquad \left[\because M_{He} = 4\text{kg/kmol}\right]$$

The two specific heats can be found from equations (8.8) and (8.9) as

$$C_p = \frac{\gamma R}{\gamma - 1} = \frac{1.667 \times 2.078}{1.667 - 1} = 5.193 \text{ kJ/kgK}$$

$$C_v = \frac{R}{\gamma - 1} = \frac{2.078}{1.667 - 1} = 3.115 \text{ kJ/kgK}$$

Example 8.3 Calculate the mass of air contained in a room 6 × 8 × 3 m in the summer, when the temperature is 37°C and pressure is 101 kPa.

Solution

Volume of room $V = 6 \times 8 \times 3 \text{ m}^3 = 144 \text{ m}^3$

Temperature of air $T = 37°\text{C} = 37 + 273 = 310 \text{ K}$

Pressure of air $P = 101 \text{ kPa}$

Mass of air can be found from equation (8.3) as

$$m = \frac{PV}{RT}$$

$$= \frac{101 \times 144}{0.287 \times 310} = 163.47 \text{ kg}$$

Example 8.4 During an experiment, 1 kg of an unknown gas is heated from 30°C to 130°C. It is observed that 43.6 kJ of heat is required at constant pressure and 14 kJ of heat is needed at constant volume. Find the characteristic gas constant and the molecular weight of the gas.

Solution

Mass of air $m = 1 \text{ kg}$

Initial temperature $T_1 = 30°\text{C}$

Final temperature $T_2 = 130°\text{C}$

Heat needed to raise the temperature at constant pressure $Q_{p=const} = 43.6 \text{ kJ}$

Heat needed to raise the temperature at constant volume $Q_{v=const} = 14 \text{ kJ}$

From the given conditions, we have

$$Q_{p=const} = mC_p\left(T_2 - T_1\right)$$

$$Q_{v=const} = mC_v\left(T_2 - T_1\right)$$

Substituting the values, we get

$$43.6 = 1 \times C_p\left(130 - 30\right)$$

$$14 = 1 \times C_v\left(130 - 30\right)$$

From the above two equations, we find

$$C_p = 0.436 \text{ kJ/kgK} \text{ and } C_v = 0.14 \text{ kJ/kgK}$$

The characteristic gas constant can be found from equation (8.6) as

$$R = C_p - C_v = 0.436 - 0.14 = 0.296 \text{ kJ/kgK}$$

The molecular weight of the gas is obtained from equation (8.4) as

$$M = \frac{\bar{R}}{R} = \frac{8.3143}{0.296} = 28 \text{ kg/kg mol}$$

8.4 INTERNAL ENERGY, ENTHALPY AND ENTROPY CHANGE OF AN IDEAL GAS

8.4.1 Internal Energy and Enthalpy Change of an Ideal Gas

Change in specific internal energy between two states 1 and 2, for a calorically ideal gas (specific heats are constant) is

$$u_2 - u_1 = C_v(T_2 - T_1) \tag{8.10}$$

Change in specific enthalpy between two states 1 and 2, for a calorically ideal gas (specific heats are constant) is

$$h_2 - h_1 = C_p(T_2 - T_1) \tag{8.11}$$

8.4.2 Entropy Change of an Ideal Gas

For an ideal gas $Pv = RT$ and $du = C_v dT, dh = C_p dT$

From the thermodynamic property relation (equation (6.16)), we have

$$Tds = du + Pdv$$

$$ds = \frac{du}{T} + \frac{P}{T}dv$$

or $$ds = \frac{C_v dT}{T} + R\frac{dv}{v} \tag{8.12}$$

Integrating between any two states 1 and 2 for a calorically ideal gas (specific heats are constant), change in specific entropy between two states 1 and 2 can be written as

$$s_2 - s_1 = C_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \tag{8.13}$$

From the thermodynamic property relation (equation (6.17)), we have

$$Tds = dh - vdP$$

$$ds = \frac{dh}{T} - \frac{v}{T}dP$$

$$= \frac{C_P dT}{T} - R\frac{dP}{P}$$

Integrating between any two states 1 and 2 for a calorically ideal gas (specific heats are constant), change in specific entropy between two states 1 and 2 can be expressed as

$$s_2 - s_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \tag{8.14}$$

or $$s_2 - s_1 = C_p \ln\frac{T_2}{T_1} - (C_p - C_v)\ln\frac{P_2}{P_1}$$

or $$s_2 - s_1 = C_p \ln\frac{T_2 P_1}{T_1 P_2} + C_v \ln\frac{P_2}{P_1}$$

or $$s_2 - s_1 = C_p \ln\frac{v_2}{v_1} + C_v \ln\frac{P_2}{P_1} \tag{8.15}$$

Example 8.5 Air expands from 300 kPa, 227°C to 100 kPa, 127°C. Calculate the changes in specific internal energy, enthalpy ad entropy. The specific heats of air are C_p = 1.005 kJ/kgK and C_v = 0.718 kJ/kgK. Assume that C_p and C_v are constant over this range of temperature.

Solution

Initial pressure $P_1 = 300$ kPa

Initial temperature $T_1 = 227°\text{C} = 227 + 273 = 500$ K

Final pressure $P_2 = 100$ kPa

Final temperature $T_2 = 127°\text{C} = 127 + 273 = 400$ K

Change in specific entropy is found from Eq. (8.14) as

$$s_2 - s_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= 1.005\ln\frac{500}{400} - 0.287\ln\frac{100}{300} = 0.54 \text{ kJ/kgK}$$

Change in specific internal energy is obtained from Eq. (8.10) as

$$u_2 - u_1 = C_v(T_2 - T_1)$$

$$= 0.718(500 - 400) = 71.8 \text{ kJ/kg}$$

Change in specific enthalpy is found from Eq. (8.11) as

$$h_2 - h_1 = C_p(T_2 - T_1)$$

$$= 1.005(500 - 400) = 100.5 \text{ kJ/kg}$$

Example 8.6 A constant volume chamber of 0.2 m^3 capacity contains 1 kg of air initially at 10°C. Heat is transferred to the gas until the temperature becomes 80°C. Find the changes in internal energy, enthalpy and entropy. Also find the heat transferred, and the work done. The specific heats of air are $C_p = 1.005$ kJ/kgK and $C_v = 0.718$ kJ/kgK. Assume that C_p and C_v are constant over this range of temperature.

Solution

Mass of air $m = 1$ kg

Initial temperature $T_1 = 10°\text{C} = 10 + 273 = 283$ K

Final temperature $T_2 = 80°\text{C} = 80 + 273 = 353$ K

The changes in internal energy is (see Eq. (8.10))

$$U_2 - U_1 = mC_v(T_2 - T_1)$$

$$= 1 \times 0.718 \times (80 - 10) = 50.26 \text{ kJ}$$

The changes in enthalpy is (see Eq. (8.11))

$$H_2 - H_1 = mC_p(T_2 - T_1)$$

$$= 1 \times 1.005 \times (80 - 10) = 70.35 \text{ kJ}$$

Change in entropy for a constant volume process (see Eq. (8.13))

$$S_2 - S_1 = mC_v \ln\frac{T_2}{T_1}$$

$$= 1 \times 0.718 \ln\frac{353}{283} = 0.157 \text{ kJ/K}$$

The heat transferred is same as the change in enthalpy.

$$Q_{1-2} = 70.35 \text{ kJ}$$

Application of the first law to this closed system for a change of state from 1 to 2 gives

$$Q_{1-2} = W_{1-2} + U_2 - U_1$$

or

$$W_{1-2} = Q_{1-2} - (U_2 - U_1)$$

$$= 70.35 - 50.26 = 20.09 \text{ kJ}$$

Example 8.7 One kg of air is compressed in a closed system from 0.1 MPa, 20°C to 0.4 MPa isothermally. Find the changes in internal energy, enthalpy and entropy. Also find the work done, and the heat transferred. Assume that C_p and C_v are constant over this range of temperature.

Solution

Initial pressure $P_1 = 0.1$ MPa

Initial temperature $T_1 = 20°\text{C} = 20 + 273 = 293$ K

Final pressure $P_2 = 0.4$ MPa

Since internal energy and enthalpy of an ideal gas is a function of absolute temperature only, the changes in internal energy and enthalpy both are zero for an isothermal process.

Change in entropy for the isothermal process (see Eq. (8.14))

$$S_2 - S_1 = -mR \ln \frac{P_2}{P_1}$$

$$= -1 \times 0.287 \ln \frac{0.4}{0.1} = -0.398 \text{ kJ/K}$$

Work done during the isothermal process 1-2 is

$$W_{1-2} = P_1 V_1 \ln \frac{P_1}{P_2}$$

$$= mRT_1 \ln \frac{P_1}{P_2} \qquad [\because P_1V_1 = mRT_1]$$

$$= 1 \times 0.287 \times 293 RT_1 \ln \frac{0.1}{0.4} = -116.57 \text{ kJ}$$

Negative sign physically signifies that the work is done on the air.

From the first law of thermodynamics, one can write

$$Q_{1\text{-}2} = W_{1-2} + U_2 - U_1$$

$$= W_{1-2} = -116.57 \text{ kJ} \qquad [\because U_2 - U_1 = 0]$$

The heat transferred is same as the work done.

Example 8.8 One kg of air initially at 100 kPa and a volume of 1 m^3 is heated at constant pressure until the volume is tripled. It is then compressed isothermally until the volume is back to its initial value. It is then cooled at constant volume until the pressure is again 100 kPa. Sketch the processes on *P-V* and *T-S* diagrams. Compute the entropy changes for the individual processes, and also find the total entropy change. The specific heats of air are $C_p = 1.005$ kJ/kgK and $C_v = 0.718$ kJ/kgK. Assume that C_p and C_v are constant over this range of temperature.

Solution

The processes on the *P-V* and *T-S* diagrams are shown in Fig. 8.1.

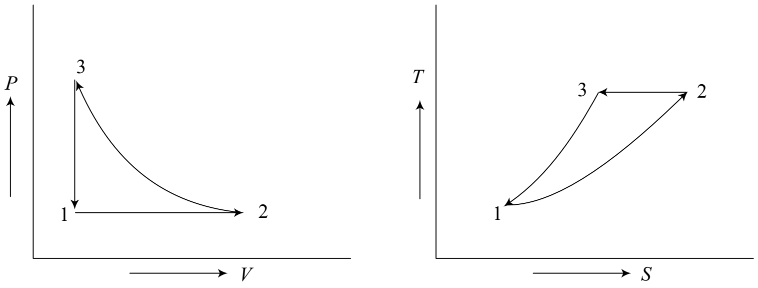


Figure 8.1

Mass of air $m = 1$ kg

Initial pressure $P_1 = 100$ kPa

Initial volume $V_1 = 1\ m^3$

From the given conditions, we have

$$V_2 = 3V_1,\ V_3 = V_1 \text{ and } T_3 = T_2$$

From Eq. (8.3), one can write

$$P_1V_1 = mRT_1$$

$$P_2V_2 = mRT_2$$

$$P_3V_3 = mRT_3$$

From the above three equations and the given conditions, we have

$$\frac{T_2}{T_1} = \frac{T_3}{T_1} = \frac{V_2}{V_1} = 3$$

Entropy change for the constant pressure process 1-2 is given by Eq. (8.14)

$$S_2 - S_1 = mC_p \ln\frac{T_2}{T_1}$$

$$= 1 \times 1.005 \ln 3 = 1.104 \text{ kJ/K}$$

Entropy change for the isothermal process 2-3 is given by Eq. (8.13)

$$S_3 - S_2 = mR \ln \frac{V_3}{V_2}$$

$$= 1 \times 0.287 \ln \frac{1}{3} = -0.315 \text{ kJ/K}$$

Entropy change for the constant volume process 3-1 is given by equation (8.13)

$$S_1 - S_3 = mC_v \ln \frac{T_1}{T_3}$$

$$= 1 \times 0.718 \ln \frac{1}{3} = -0.789 \text{ kJ/K}$$

Total entropy change $= 1.104 - 0.315 - 0.789 = 0$

Example 8.9 Show that for an ideal gas the slope of the constant volume (isochoric) line on the *T*-*s* diagram is more than that of the constant pressure (isobaric) line.

Solution The slope of constant volume line on the *T*-*s* diagram is $\left(\frac{\partial T}{\partial s}\right)_v$ and that of constant pressure line is $\left(\frac{\partial T}{\partial s}\right)_P$.

From the thermodynamic property relation (Eq. (6.16)), we have

$$Tds = du + Pdv$$

or $Tds = C_v dT + Pdv$ (Since for an ideal gas $du = C_v dT$)

For constant volume process ($dv = 0$), the above equation becomes

$$Tds = C_v dT$$

or
$$\left(\frac{\partial T}{\partial s}\right)_v = \frac{T}{C_v} \qquad (8.16)$$

From the thermodynamic property relation (Eq. (6.17)), we have

$$Tds = dh - vdP$$

or $Tds = C_p dT - vdP$ (Since for an ideal gas $dh = C_p dT$)

For constant pressure process ($dP = 0$), the above equation becomes

$$Tds = C_p dT$$

or
$$\left(\frac{\partial T}{\partial s}\right)_p = \frac{T}{C_p} \qquad (8.17)$$

Since for an ideal gas, $C_p > C_v \left(\frac{T}{C_v} > \frac{T}{C_p} \right)$, from Eq. (8.16) and (8.17), we get

$$\left(\frac{\partial T}{\partial s} \right)_v > \left(\frac{\partial T}{\partial s} \right)_P$$

Example 8.10 An ideal gas initially at temperature T_1 is heated at constant pressure to a temperature T_2. It is then expanded in a reversible process following the law PV = constant, until the temperature once again T_1. If the changes of entropy during the two processes are equal, find the value of n.

Solution Change in specific entropy for constant pressure is found from Eq. (8.14) as

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1}$$

or $$s_2 - s_1 = \frac{\gamma}{\gamma - 1} R \ln \frac{T_2}{T_1} = \ln \left(\frac{T_2}{T_1} \right)^{\frac{\gamma R}{\gamma - 1}} = \ln \left(\frac{T_1}{T_2} \right)^{-\frac{\gamma R}{\gamma - 1}}$$

Change in specific entropy for polytropic process is found from Eq. (8.13) as

$$s_3 - s_2 = C_v \ln \frac{T_3}{T_2} + R \ln \frac{v_3}{v_2}$$

or $$s_3 - s_2 = C_v \ln \frac{T_1}{T_2} + R \ln \frac{v_3}{v_2} \qquad [\because T_3 = T_1]$$

or $$s_2 - s_1 = \frac{R}{\gamma - 1} \ln \frac{T_1}{T_1} + \frac{R}{n - 1} \ln \frac{T_1}{T_2} \qquad \left[\because \frac{v_3}{v_2} = \left(\frac{T_2}{T_3} \right)^{\frac{1}{n-1}} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{n-1}} \right]$$

or $$s_2 - s_1 = \frac{n - \gamma}{(\gamma - 1)(n - 1)} R \ln \frac{T_1}{T_2} = \ln \left(\frac{T_1}{T_2} \right)^{\frac{(n-\gamma)R}{(\gamma-1)(n-1)}}$$

Since the changes in entropy during the two processes are equal, we have

$$\ln \left(\frac{T_1}{T_2} \right)^{-\frac{\gamma R}{\gamma - 1}} = \ln \left(\frac{T_1}{T_2} \right)^{\frac{(n-\gamma)R}{(\gamma-1)(n-1)}}$$

or $$-\frac{\gamma}{\gamma - 1} R = \frac{n - \gamma}{(\gamma - 1)(n - 1)} R$$

or $$-\gamma = \frac{n - \gamma}{(n - 1)}$$

or $$n = \frac{2\gamma}{\gamma + 1}$$

Example 8.11 The specific heats of a certain gas vary linearly with temperature and the variation are given by $C_v = a + kT$ and $C_p = b + kT$, where a, b and k are constants and T is in Kelvin. Show that for an isentropic expansion of this gas is given by

$$T^a e^{kT} v^{b-a} = \text{constant}$$

where v is the specific volume of the gas.

Solution The characteristic gas constant can be found from Eq. (8.6) as

$$R = C_p - C_v = b + kT - a - kT = b - a$$

Change in specific entropy is found from Eq. (8.12) as

$$ds = C_v \frac{dT}{T} + R\frac{dv}{v}$$

or
$$ds = (a + kT)\frac{dT}{T} + (b - a)\frac{dv}{v}$$

or
$$ds = a\frac{dT}{T} + kdT + (b - a)\frac{dv}{v}$$

Since for an isentropic process, change in entropy is zero, we have

$$ds = 0$$

or
$$a\frac{dT}{T} + kdT + (b - a)\frac{dv}{v} = 0$$

or
$$a \ln T + kT + (b - a)\ln v = \text{constant}$$

or
$$T^a e^{kT} v^{b-a} = \text{constant}$$

8.4 REAL GAS

As stated earlier from Boyle's, Charles's and Gay-Lussac's experimental observations, it has been found that $P\bar{v}T$ behaviour of many gases at low pressure and moderate temperatures can be approximated quite well by the ideal-gas equation. This is because of the fact that at low pressure and moderate temperature, the intermolecular force of attraction as well as the volume occupied by the individual molecules compared with the total volume of the gas is very small. But at high pressure and low temperature, the intermolecular force of attraction is noticeable and the volume occupied by the individual molecules is comparable with the total volume of the gas. Therefore, the real gases deviate from ideal gas behaviour at high pressure and low temperature.

van der Walls proposed the following equation of state for a real gas

$$\left(P + \frac{a}{V^2}\right)(V - b) = mRT \tag{8.18}$$

where a and b are constants. The above equation is known as *van der Waals equation of state*. The term $\frac{a}{V^2}$ takes into account the intermolecular forces of attraction and the constant b takes care the finite volume of the molecules. The term $\frac{a}{V^2}$ is called the *force of cohesion*, whereas the constant b is known as *co-volume*.

A *virial equation of state* presents the product PV as a series expansion. The most common expansion is

$$P = \frac{mRT}{V} + \frac{B(T)}{V} + \frac{C(T)}{V^2} + \cdots \tag{8.19}$$

Important to note that $B(T)$ represents the first-order correction to the ideal-gas law and hence attention is focused on it.

SUMMARY

- An ideal gas is defined as one for which the equation of state is

  $$P\bar{v} = \bar{R}T$$

  where P is the pressure, $\bar{v}$ is the molar volume (i.e., volume per unit mole), $\bar{R}$ the universal is gas constant, and T is the temperature of the gas in K
- The equation of state of an ideal gas can also be expressed as

  $$PV = n\bar{R}T$$
  $$PV = mRT$$
  $$Pv = RT$$

  where V is the total volume of the gas, n is the number of moles of the gas, m is the mass of the gas and v specific volume of the gas.
- Change in specific internal energy of a calorically perfect gas between states 1 and 2 is

  $$u_2 - u_1 = C_V(T_2 \quad T_1)$$
- Change in specific enthalpy of a calorically perfect gas between states 1 and 2 is

  $$h_2 - h_1 = C_P(T_2 \quad T_1)$$
- Change in specific entropy of a calorically perfect gas between states 1 and 2 is

  $$\begin{aligned} s_2 - s_1 &= C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1} \\ &= C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \\ &= C_p \ln\frac{V_2}{V_1} + C_v \ln\frac{P_2}{P_1} \end{aligned}$$

REVIEW QUESTIONS

8.1 What is an equation of state?
8.2 Define an ideal gas.
8.3 What is the characteristic gas constant?
8.4 What is the universal gas constant?

8.5 Show that specific entropy change for an ideal gas can be expressed as

$$s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

8.6 Show that the entropy change for a polytropic process between the states 1 and 2, can be expressed as

$$s_2 - s_1 = \frac{n-\gamma}{(\gamma-1)(n-1)} R \ln \frac{T_2}{T_1}$$

8.7 Why does an isochoric curve plotted on a T–S diagram have a greater slope than an isobaric curve at the same temperature?

8.8 Derive the expression of work transfer and heat transfer for an ideal gas in a reversible isothermal process.

8.9 Express the changes in internal energy and enthalpy of an ideal gas in an isentropic process in terms of the pressure ratio. Also find the expression of work transfer and heat transfer.

NUMERICAL PROBLEMES

8.1 A closed rigid tank contains 0.2 m^3 of air initially at 100 kPa and 200 °C. 10 kJ of heat is added to the air. The final pressure of air is 200 kPa. Calculate the work done.

8.2 0.03 kg of N_2 gas contained in a cylinder is compressed from 1.01 bar, 15 °C to 4.2 bar reversibly and isothermally. Sketch the process on P–v and T–S diagram. Calculate the change of entropy, heat transfer and work done.

8.3 Air initially expands from 300 kPa, 50 °C reversibly and isothermally to a pressure of 150 kPa, then reversibly and adiabatically to a pressure of 100 kPa. Sketch the processes on $P-V$ and $T-S$ diagrams. Calculate the heat transfer and changes in internal energy and entropy for individual processes.

8.4 One kg of air is contained in a piston–cylinder arrangement at 20 kPa and 50 °C. The gas undergoes the following cyclic process:
1-2: constant volume heating until pressure is 75 kPa
2-3: Isentropic expansion to the initial pressure
3-1: Constant pressure cooling to the initial state
(i) Sketch P–V and T–S diagrams for the cycle.
(ii) Calculate the change of entropy, heat transfer and work done for the individual processes and the cycle.

MULTIPLE-CHOICE QUESTIONS

8.1 The slope of constant pressure line of an ideal gas on $T-S$ diagram is given by
(a) C_p/T
(b) T/C_p
(c) S/T
(d) T/S

8.2 The slope of constant volume line of an ideal gas on a $T-S$ diagram is given by
(a) C_p/T
(b) T/C_p
(c) S/T
(d) T/C_V

8.3 The internal energy of an ideal gas is a function of only

(a) pressure
(b) temperature (absolute)
(c) volume
(d) pressure and temperature

8.4 Which one of the following is the characteristic equation of a real gas?

(a) $Pv = RT$
(b) $Pv = nRT$
(c) $\left(P + \frac{a}{v^2}\right)(v - b) = RT$
(d) $\left(P - \frac{a}{v^2}\right)(v + b) = RT$

CHAPTER

9

Air-Standard Cycles

9.1 INTRODUCTION

From the point of view of purpose of the cycles, they are classified into two: power cycles and refrigeration cycles.

The devices that produce a net power output are called ***engines*** and the cycles on which engines operate are called ***power cycles***. On the other hand, devices that produce refrigeration are called ***refrigerators*** (or air-conditioners, heat pumps). They are also operated on a cycle, which is called ***refrigeration cycle***.

On the basis of the phase of a working fluid, cycles are classified into two: ***gas cycles*** and ***vapour cycles***.

Cycles can also be classified into thermodynamic (closed) and mechanical (open) cycles. In ***thermodynamic cycles***, the working fluid is returned to the initial state at the end of the cycle and is recirculated, whereas in ***mechanical cycles***, the working fluid is renewed at end of the each cycle.

Familiar examples of gas power cycles are the Otto cycle and the Diesel cycle.

9.2 ANALYSIS OF POWER CYCLES

The study of power cycles is an exciting and important part of thermodynamics. The cycles of actual devices are difficult to analyse because of the presence of friction and non-availability of sufficient time for the establishment of equilibrium conditions during the cycle.

The complexities associated with the actual cycles can be significantly simplified with the following approximations, commonly known as ***air-standard assumptions***.

(i) The working fluid is air, which behaves like an ideal gas with constant specific heats.
(ii) The working fluid is continuously recirculated in a closed loop.
(iii) All the processes that make up the cycle are quasi-equilibrium.
(iv) The combustion process is replaced by a heat addition process from an external source.
(v) The exhaust process is replaced by a heat rejection process.

The cycle with the above-mentioned assumptions is often called an **air-standard cycle.**

9.3 ENGINE TERMINOLOGY

The basic components of a reciprocating engine are shown in Fig. 9.1. The piston reciprocates in the cylinder between two fixed positions called the ***top dead centre*** (TDC) and the ***bottom dead centre*** (BDC). The piston is said to be at the ***top dead centre*** when it has moved to the position where the cylinder volume is a minimum. When the piston has moved to the position of the maximum cylinder volume, the piston is at the ***bottom dead centre***. The ***bore*** of the cylinder is its diameter. The distance between the TDC and the BDC is the largest distance that the piston can travel in one direction. The ***stroke*** is the largest distance the piston moves in one direction. The fresh charge (air or air-fuel mixture) is drawn into the cylinder through the ***intake valve***, and the products of combustion are thrown out from the cylinder through the ***exhaust valve***. The minimum volume in the cylinder with piston at TDC is called the ***clearance volume***. The volume displaced by piston movement between TDC and BDC is called the ***displacement volume***. The ratio of the maximum volume formed in the cylinder to the minimum volume is called the ***compression ratio*** r of the engine. That is,

$$r = \frac{V_{\max}}{V_{\min}}$$

The ***mean effective pressure*** is another important term associated with reciprocating engines. It is a fictitious pressure and is defined as that pressure which when acting on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle.

$$W_{\text{net}} = \text{MEP} \times \text{piston area} \times \text{stroke}$$

$$W_{\text{net}} = \text{MEP} \times \text{displacement volume}$$

$$W_{\text{net}} = \text{MEP} \times (V_{\max} - V_{\min})$$

$$\text{MEP} = \frac{W_{\text{net}}}{V_{\max} - V_{\min}} \tag{9.1}$$

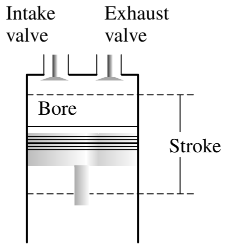


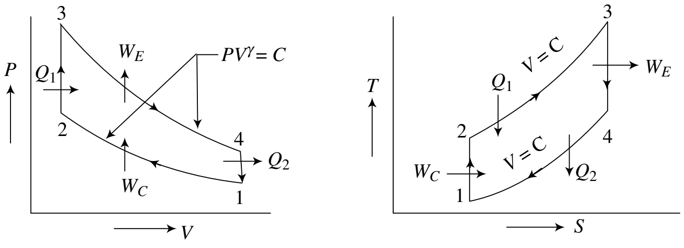
Figure 9.1 *Nomenclatures for reciprocating engines*

If the combustion of the air-fuel mixture is initiated by a spark plug, the reciprocating engine is called a spark-ignition (SI) engine, and if the combustion is by self-ignition as a result of compression of the mixture above its self-ignition temperature, the engine is called a compression-ignition (CI) engine.

9.4 OTTO CYCLE

The complexities associated with the actual spark-ignition (SI) engine can be significantly simplified with the air-standard assumptions. The resulting cycle which closely resembles the actual operating conditions is the ideal Otto cycle. It consists of two reversible adiabatic processes and two reversible constant volume (isochors) processes. Different processes of the Otto cycle on the $P-V$ and $T-S$ diagrams are shown in Fig. 9.2.

Air is compressed in the process 1–2 reversibly and adiabatically. Heat is then added to air reversibly at constant volume in the process 2–3. Work is done by air in expanding reversibly and adiabatically in process 3–4. Heat is then rejected by air reversibly at constant volume in the process 4–1, and the system comes back to its initial state. Heat transfer processes have been substituted for the combustion and blow-down processes of the engine. The intake and exhaust processes of the engine cancel each other.



(a) (b)

Figure 9.2 *Otto cycle on (a) P–V diagram and (b) T–S diagram*

Let m be the fixed mass of air undergoing the cycle of operation.

Heat supplied $Q_1 = mC_v(T_3 - T_2)$

Heat rejected $Q_2 = mC_v(T_4 - T_1)$

Thermal efficiency of the cycle is

$$\eta_{\text{Otto}} = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{mc_v(T_4 - T_1)}{mc_v(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

or
$$\eta_{\text{Otto}} = 1 - \frac{T_1}{T_2}\,\frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1} \tag{9.2}$$

Now for reversible adiabatic process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

And for reversible adiabatic process 3-4, we get

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad [\because V_4 = V_1, V_3 = V_2]$$

Therefore, from the above two equations, one can write

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

or

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Thus Eq. (9.2) become

$$\eta_{\text{Otto}} = 1-\frac{T_1}{T_2} = 1-\frac{1}{\left(\frac{V_1}{V_2}\right)^{\gamma-1}}$$

or

$$\eta_{\text{Otto}} = 1-\frac{1}{r^{\gamma-1}} \tag{9.3}$$

where $r = \frac{V_1}{V_2}$ is the compression ratio.

It is evident from Eq. (9.3) that the thermal efficiency of the Otto cycle is dependent only on the compression ratio for a given value of specific heat ratio; the higher the compression ratio, the higher the thermal efficiency. This is true for real spark-ignition engine, quantitatively. The variations of thermal efficiency of the Otto cycle with compression ratio for different values of specific heat ratio are shown in Fig. 9.3. Figure 9.3 shows that the increase in the thermal efficiency is not so pronounced beyond certain values of the compression ratios.

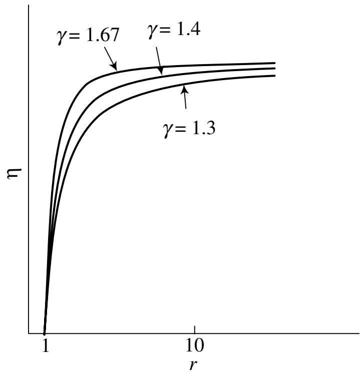


Figure 9.3 *Variation of efficiency of Otto cycle with compression ratio for different values of specific heat ratio*

Note that the compression ratio of the spark-ignition engines is restricted below a certain limit (about 8 to 10) because of the phenomenon of *knocking*. It is the premature ignition of the fuel which produces an audible noise. Knocking occurs at higher compression ratios because at higher compression ratios the temperature of the air-fuel mixture rises above the auto-ignition temperature of the fuel during the combustion process. Thermal efficiency of the spark-ignition engines can be improved by utilizing higher compression ratios (upto 12) without knocking by using fuel (gasoline) blends that have good antiknock characteristics.

Another parameter that influences the thermal efficiency of the Otto cycle is the specific heat ratio. Thermal efficiency of the Otto cycle increases with the specific heat ratio for a given compression ratio. Figure 9.3 depicts that for a fixed compression ratio, Otto cycle using a monatomic gas as the working fluid will have the highest thermal efficiency.

The actual efficiency of spark-ignition engine is much less than that of the Otto cycle.

Example 9.1 An engine equipped with a cylinder having a bore of 15 cm and a stroke of 45 cm operates on an Otto cycle. If the clearance volume is 2000 cm^3, compute the air standard efficiency.

Solution Bore D = 15 cm, stroke L = 45 cm, Clearance volume, V_c = 2000 cm^3

Displacement volume, $V_s = \frac{\pi}{4}D^2L = \frac{\pi}{4}(15)^2 \times 45 = 7952.156 \text{ cm}^3$

Compression ratio, $r = \frac{V_{\max}}{V_{\min}} = \frac{V_c + V_s}{V_c} = \frac{2000 + 7952.156}{2000} = 4.976$

Air standard efficiency, $\eta_{\text{cycle}} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{4.976^{0.4}} = 0.4737 \text{ or } 47.37\%$

Example 9.2 For an engine operating on air standard Otto cycle, the clearance volume is 10% of the swept volume. If the specific heat ratio of air is 1.4, compute the air standard cycle efficiency.

Solution

Clearance volume $V_c = 0.1\, V_s$, where V_s is swept volume.

Specific heat ratio $\gamma = 1.4$

Compression ratio of the engine is

$$r = \frac{V_{\max}}{V_{\min}} = \frac{V_c + V_s}{V_c} = 1 + \frac{V_s}{V_c} = 1 + \frac{V_s}{0.1V_s} = 11$$

The air standard cycle efficiency is then

$$\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{11^{0.4}} = 0.6168 \text{ or } 61.68\%$$

Example 9.3 A spark-ignition engine working on air standard Otto cycle, the compression ratio is 7, and compression begins at 35°C, 100 kPa. The maximum temperature of the cycle is 1100°C. Find (a) temperature and pressure at the cardinal points of the cycle, (b) the heat supplied per kg of air (c) the cycle efficiency, (d) the work done per kg of air, and (e) the mean effective pressure (MEP).

Solution The four processes that form the Otto cycle on P-V diagram is shown in Fig. 9.4.

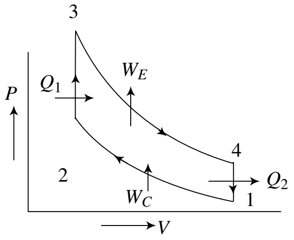


Figure 9.4

Compression ratio $r = 7$
Minimum pressure of the cycle $P_1 = 100$ kPa
Minimum temperature of the cycle $T_1 = 35\text{ °C} = 35 + 273 = 308$ K
Maximum temperature of the cycle $T_1 = 1100\text{°C} = 1100 + 273 = 1373$ K

(a) Since the process 1–2 is isentropic, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

or
$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 308 \times (7)^{0.4} = 670.82 \text{ K}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

or
$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100 \times (7)^{1.4} = 1524.53 \text{ kPa}$$

For the constant volume process 2–3, we find that

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

or
$$P_3 = P_2\frac{T_3}{T_2} = 1524.53 \times \frac{1373}{670.82} = 3120.33 \text{ kPa} = 3.12 \text{ MPa}$$

For the isentropic process 3–4, we have

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1} = 7^{0.4} = 2.178$$

or
$$T_4 = \frac{T_3}{2.178} = \frac{1373}{2.178} = 630.39 \text{ K}$$

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^{\gamma}$$

or
$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = \frac{P_3}{\left(\frac{V_4}{V_3}\right)^{\gamma}} = \frac{P_3}{\left(\frac{V_1}{V_2}\right)^{\gamma}} = \frac{P_3}{r^{\gamma}}$$

or
$$P_4 = \frac{3.12}{7^{1.4}} = 0.205 \text{ MPa}$$

(b) Heat supplied per kg of air is

$$Q_1 = C_v(T_3 - T_2) = 0.718(1373 - 670.82) = 504.17 \text{ kJ/kg}$$

(c) The cycle efficiency is

$$\eta_{\text{Otto}} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{(7)^{0.4}} = 0.5408 \text{ or } 54.08\%$$

(d) Work done per kg of air is

$$W_{\text{net}} = Q_1\eta_{\text{Otto}} = 504.17 \times 0.5408 = 272.66 \text{ kJ/kg}$$

(e) The MEP is found from Eq. (9.1) as

$$\text{MEP} = \frac{W_{net}}{V_1 - V_2}$$

We have

$$V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 308}{100} = 0.884 \text{ m}^3\text{/kg}$$

$$V_2 = \frac{V_1}{7} = \frac{0.884}{7} = 0.126 \text{ m}^3\text{/kg}$$

Thus

$$\text{MEP} = \frac{W_{net}}{V_1 - V_2} = \frac{272.66}{0.884 - 0.126} = 359.71 \text{ kPa}$$

Example 9.4 An engine working on Otto cycle is supplied with air at 1 bar, 27°C. The compression ratio is 8. Heat supplied is 1500 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency, and the mean effective pressure. For air C_p = 1.005 kJ/kgK and C_v = 0.718 kJ/kgK.

Solution The four processes that form the Otto cycle on P-V diagram is shown in Fig. 9.5.

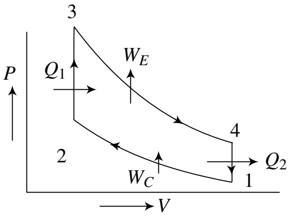


Figure 9.5

Compression ratio $r = 8$

Minimum pressure of the cycle $P_1 = 1$ bar

Minimum temperature of the cycle $T_1 = 27\ °\text{C} = 27 + 273 = 300$ K

Heat supplied to the cycle $Q_1 = 1500$ kJ/kg

For the isentropic process 1–2, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

or
$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times 8^{0.4} = 689.22 \text{ K}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

or
$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times 8^{1.4} = 18.38 \text{ bar}$$

Heat supplied per kg of air is given by

$$Q_1 = C_v(T_3 - T_2)$$

or
$$1500 = 0.718(T_3 - 689.22)$$

or
$$T_2 = 2778.36\text{K}$$

For the constant volume process 2–3, we find that

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

or
$$P_3 = P_2\frac{T_3}{T_2} = 18.38 \times \frac{2778.36}{689.22} = 74.09 \text{ bar}$$

The cycle efficiency is

$$\eta_{\text{Otto}} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{8^{1.4-1}} = 0.5647 \text{ or } 56.47\%$$

Work done per kg of air is

$$W_{\text{net}} = Q_1 \times \eta_{Otto} = 1500 \times 0.5647 = 847.05 \text{ kJ/kg}$$

The MEP is found from equation (9.1) as

$$\text{MEP} = \frac{W_{net}}{V_1 - V_2}$$

We have

$$V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{100} = 0.86 \text{ m}^3\text{/kg}$$

$$V_2 = \frac{V_1}{8} = \frac{0.86}{8} = 0.11 \text{ m}^3\text{/kg}$$

Thus

$$\text{MEP} = \frac{W_{net}}{V_1 - V_2} = \frac{847.05}{0.86 - 0.11} = 1129.4 \text{ kN/m}^2 = 11.294 \text{ bar}$$

Example 9.5 An engine working on the Otto cycle has an air standard cycle efficiency of 56% and rejects 544 kJ/kg of heat. The pressure and temperature of air at the beginning of compression are 0.1 MPa, and 30 °C respectively. Compute the compression ratio of the engine, the work done per kg of air, the pressure and temperature at the end of compression, and the maximum pressure in the cycle.

Solution The four processes that form the Otto cycle on P-V diagram is shown in Fig. 9.6.

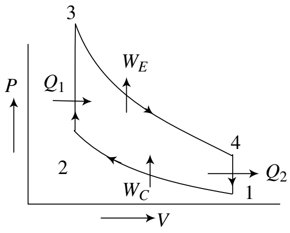


Figure 9.6

Compression ratio	$r = 8$
Minimum pressure of the cycle	$P_1 = 0.1$ MPa
Minimum temperature of the cycle	$T_1 = 30$ °C = 30 + 273= 303 K
Heat supplied to the cycle	$Q_1 = 544$ kJ/kg
Air standard cycle efficiency	$\eta_{\text{Otto}} = 0.56$

The cycle efficiency is

$$\eta_{\text{Otto}} = 1 - \frac{1}{r^{\gamma-1}} = 0.56$$

or $$r = 7.79$$

The thermal efficiency can also be expressed as

$$\eta_{\text{Otto}} = 1 - \frac{Q_2}{Q_1} = 0.56$$

or $$\frac{Q_2}{Q_1} = 1 - 0.56 = 0.44$$

or $$Q_1 = \frac{Q_2}{0.44} = \frac{544}{0.44} = 1236.36 \text{ kJ/kg}$$

Work done per kg of air is $W_{\text{net}} = Q_1 \times \eta_{\text{Otto}} = 1236.36 \times 0.56 = 692.36$ kJ/kg

For the isentropic process 1–2, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

or $$T'_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 303 \times 7.79^{0.4} = 688.74 \text{ K}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

or $$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 0.1 \times 7.79^{1.4} = 1.77 \text{ MPa}$$

Again, $$Q_1 = C_v\left(T_3 - T_2\right) = 0.718\left(T_3 - 688.74\right) = 1236.36 \text{ kJ/kg}$$

or $$T_3 = 2410.69 \text{ K}$$

For the constant volume process 2–3, we can write

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

or $$P_3 = P_2 \times \frac{T_3}{T_2} = 1.77 \times \frac{2410.69}{688.74} = 6.195 \text{ MPa}$$

Example 9.6 (a) Show that the net shaft work output from the air-standard Otto cycle per unit mass of air is given by $W_{\text{net}} = C_v\left[T_3 - T_1 r^{\gamma-1} - \frac{T_3}{r^{\gamma-1}} + T_1\right]$, where T_3 and T_1 are the maximum and minimum temperatures in the cycle and r is the compression ratio.

(b) In practical engines T_1 is fixed at ambient temperature and T_3 is fixed by the maximum temperature that can avoid the knocking phenomenon. Show that for fixed values of T_1 and T_3, the compression ratio for maximum work output per unit mass of air flowing round the cycle is given by $r = \left(\frac{T_1}{T_3}\right)^{\frac{1}{2(1-\gamma)}}$

Solution (a) The net shaft work output from the air-standard Otto cycle per unit mass of air is given by (see Fig. 9.2)

$$W_{\text{net}} = C_v\left(T_3 - T_2\right) - C_v\left(T_3 - T_2\right) \tag{9.4}$$

For the isentropic process 1 → 2, we have

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 r^{\gamma-1} \tag{9.5}$$

Similarly, for the isentropic process , we get

$$T_3 = T_4\left(\frac{V_4}{V_3}\right)^{\gamma-1} = T_4 r^{\gamma-1} \tag{9.6}$$

From Eqs. (9.4), (9.5) and (9.6), we have

$$W_{\text{net}} = C_v\left[T_3 - T_1 r^{\gamma-1} - \frac{T_3}{r^{\gamma-1}} + T_1\right]$$

(b) For fixed values of T_1 and T_3, $W_{\text{net}}t$, will be maximum when $\frac{dW_{\text{net}}}{dr} = 0$

or $$\frac{d}{dr}C_v\left[T_3 - T_1 r^{\gamma-1} - \frac{T_3}{r^{\gamma-1}} + T_1\right] = 0$$

or $$-T_1(\gamma-1)r^{\gamma-1-1} + (\gamma-1)T_3 r^{-(\gamma-1)-1} = 0$$

or $$T_3 r^{-\gamma} = T_1 r^{\gamma-2}$$

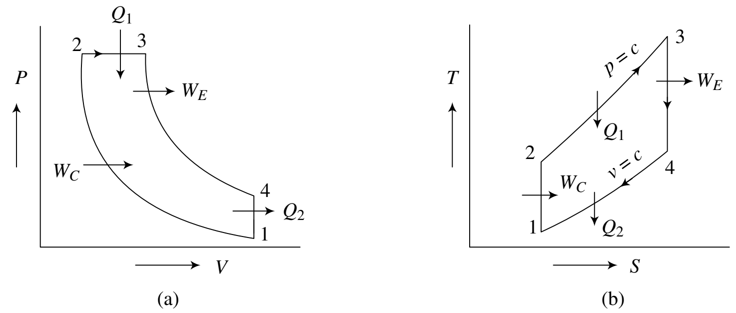
or $$r^{2(1-\gamma)} = \frac{T_1}{T_3}$$

or $$r = \left(\frac{T_1}{T_3}\right)^{\frac{1}{2(1-\gamma)}}$$

9.5 DIESEL CYCLE

The Diesel cycle was developed by Rudolf Diesel (1858–1913) in 1893. This cycle is extensively used by large and small stationary engines. Most of the present day transport trucks, buses, and cars are operate on diesel because of the low cost of the diesel oil.

Air is compressed in the process 1–2 reversibly and adiabatically. Heat is then added to air reversibly at constant pressure in the process 2–3. Work is done by air in expanding reversibly and adiabatically in the process 3–4. Heat is then rejected by air reversibly at constant volume in the process 4–1, and the system comes back to its initial state. Heat transfer processes have been substituted for the combustion and blow-down processes of the engine. The intake and exhaust processes of the engine cancel each other. Different processes of the Otto cycle on *P–V* and *T–S* diagrams are shown in Fig. 9.7.



(a) (b)

Figure 9.7 *Diesel cycle on (a) P– V diagram and (b) T– S diagram*

Let *m* be the fixed mass of air undergoing the cycle of operation.

The constant-pressure heat addition $Q_1 = mC_p(T_3 - T_2)$

The constant-volume heat rejection $Q_2 = mC_v(T_4 - T_1)$

The thermal efficiency of the Diesel cycle is expressed as

$$\eta_{\text{Diesel}} = \frac{W_{\text{net}}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_p(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)}$$

or

$$\eta_{\text{Diesel}} = 1 - \frac{1}{\gamma}\frac{T_1}{T_2}\frac{\left(\frac{T_4}{T_1} - 1\right)}{\left(\frac{T_3}{T_2} - 1\right)} \tag{9.7}$$

The expression for the thermal efficiency is often written in terms of the compression ratio $r\,(= V_1/V_2)$ and the *cut-off ratio* r_c $(= V_3/V_2)$ which is defined as the ratio of volume at cut-off to the clearance volume.

For the isentropic compression process 1–2, we have

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$$

For the constant-pressure process 2–3, one can write

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

For the isentropic expansion process 3–4, we have

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_3}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_2}\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{r_c}{r}\right)^{\gamma-1}$$

Substituting $\dfrac{T_2}{T_1}, \dfrac{T_3}{T_2}$ and $\dfrac{T_4}{T_3}$ in Eq. (9.7), we obtain

$$\eta_{\text{Diesel}} = 1 - \frac{1}{\gamma}\frac{T_1}{T_2}\frac{\left(\dfrac{T_4}{T_1} - 1\right)}{\left(\dfrac{T_3}{T_2} - 1\right)} = 1 - \frac{1}{\gamma}\frac{T_1}{T_2}\frac{\left(\dfrac{T_4}{T_3}\dfrac{T_3}{T_2}\dfrac{T_2}{T_1} - 1\right)}{\left(\dfrac{T_3}{T_2} - 1\right)}$$

$$= 1 - \frac{1}{\gamma}\frac{1}{r^{\gamma-1}}\frac{\left(\dfrac{r_c}{r}\right)^{\gamma-1} r_c r^{\gamma-1} - 1}{(r_c - 1)}$$

or

$$\eta_{\text{Diesel}} = 1 - \frac{1}{r^{\gamma-1}}\frac{1}{\gamma}\frac{r_c^{\gamma-1} - 1}{r_c - 1} \tag{9.8}$$

As cut-off ratio $r_c > 1$, $\dfrac{1}{\gamma}\dfrac{r_c^{\gamma} - 1}{r_c - 1}$ is greater than unity. Therefore, for a given compression ratio r, the efficiency of the Diesel cycle is less than that of an Otto cycle. For example, if $r = 10$ and $r_c = 2$, the Otto cycle efficiency is 60.2% and the Diesel cycle efficiency is 53.4%. In practice, however, a compression ratio of 20 or so can be achieved in a diesel engine (compression ratio of the gasoline engines is restricted below about 8 to 10); using $r = 20$ and $r_c = 2$, we would find $\eta = 64.7\%$. Thus, *because of the higher compression ratio, a diesel engine typically operates at a higher efficiency than a gasoline engine.*

As the cut-off ratio r_c increases, the Diesel cycle efficiency decreases.

Example 9.7 In a diesel engine the compression ratio is 14:1 and fuel is cut off at 7% of the stroke. Find the air-standard efficiency of the engine. Take γ for air = 1.4.

Solution The four processes that form the Otto cycle on *P–V* diagram is shown in Fig. 9.8.

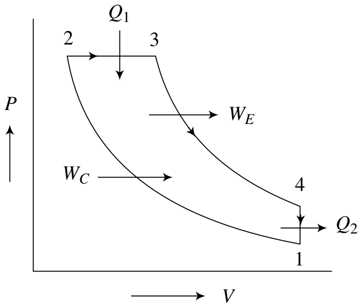


Figure 9.8

Compression ratio $r = \dfrac{V_1}{V_2} = 14$

Fuel is cut off at 7% of the stroke. It means

$$V_3 - V_2 = 0.07(V_1 - V_2)$$

or $$\frac{V_3 - V_2}{V_2} = 0.07\frac{(V_1 - V_2)}{V_2}$$

or $$\frac{V_3}{V_2} - 1 = 0.07\left(\frac{V_1}{V_2} - 1\right)$$

or $$\frac{V_3}{V_2} = 1 + 0.07(14 - 1) = 1.91$$

Thus the cut-off ratio is r_c = 1.91

Efficiency of the Diesel cycle is given by

$$\eta_{\text{Diesel}} = 1 - \frac{1}{r^{\gamma-1}}\frac{1}{\gamma}\frac{r_c^{\gamma} - 1}{r_c - 1}$$

$$= 1 - \frac{1}{14^{1.4-1}}\frac{1}{1.4}\frac{1.91^{1.4} - 1}{1.91 - 1} = 0.5973 = 59.73\%$$

Example 9.8 In an air standard Diesel cycle, the compression ratio is 15. Compression begins at 0.1 MPa, 40 °C. The heat added is 1.675 MJ/kg . Find (a) the maximum pressure and temperature of the cycle, (b) the cut-off ratio, (c) the cycle efficiency, (d) the work done per kg of air, and (e) the temperature at the end of the isentropic expansion.

Solution The four processes that form the Otto cycle on *P–V* diagram is shown in Fig. 9.9.

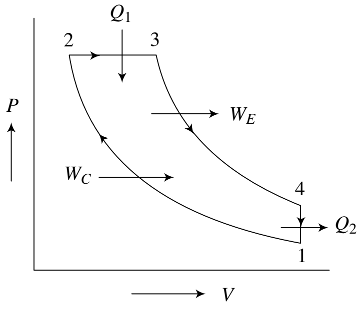


Figure 9.9

Compression ratio $r = 15$

Minimum pressure of the cycle $P_1 = 0.1$ MPa

Minimum temperature of the cycle $T_1 = 40°\text{C} = 40 + 273 = 313$ K

Heat supplied to the cycle $Q_1 = 1.675 \text{ MJ/kg} = 1.675 \times 10^3$ kJ/kg

(a) For the isentropic process 1 – 2, we have

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

or $$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 0.1 \times 15^{1.4} = 4.43 \text{ MPa}$$

This is the maximum pressure of the cycle.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

or $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 313 \times 15^{0.4} = 924.66 \text{ K}$$

Heat supplied per kg of air is given by

$$Q_1 = C_p(T_3 - T_2)$$

or $$1.675 = 10^3 = 1.005\ (T_3 - 924.66)$$

or $$T_3 = 2591.33 \text{ K}$$

This is the maximum temperature of the cycle.

(b) For the constant pressure process 2-3, we can write

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

or $$\frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2591.33}{924.66} = 2.8$$

The cut-off ratio is

(c) The cycle efficiency is $r_c = 2.8$

$$\eta_{\text{Diesel}} = 1 - \frac{1}{r^{\gamma-1}} \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1}$$

$$= 1 - \frac{1}{15^{1.4-1}} \frac{1}{1.4} \frac{2.8^{1.4} - 1}{2.8 - 1} = 0.5665 \text{ or } 56.65\%$$

(d) Work done per kg of air is found to be

$$W_{\text{net}} = Q_1 \times \eta_{Diesel} = 1.675 \times 10^3 \times 0.5665 = 948.89 \text{ kJ/kg}$$

(e) For the isentropic process 3–4, we have

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{V_2}\frac{V_2}{V_3}\right)^{\gamma-1}$$

or $$\frac{T_3}{T_4} = \left(\frac{r}{r_c}\right)^{\gamma-1} = \left(\frac{15}{2.8}\right)^{0.4}$$

or $$T_4 = \frac{2591.33}{\left(\frac{15}{2.8}\right)^{0.4}} = 1324.19 \text{ K}$$

Example 9.9 In an air standard Diesel cycle, the pressure and temperature at the intake are 100 kPa bar and 27 °C respectively. The maximum pressure in the cycle is 4 MPa and heat supplied during the cycle is 1000 kJ/kg. Determine (a) the compression ratio, (b) the temperature at the end of the compression, (c) the temperature at the end of the combustion, (d) the cut-off ratio, and (e) the air-standard efficiency. Assume $\gamma = 1.4$ and $C_p = 1.005$ kJ/kg-k for air.

Solution The four processes that form the Otto cycle on P–V diagram is shown in Fig. 9.10.

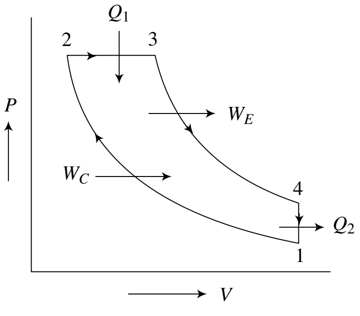


Figure 9.10

Minimum pressure of the cycle $P_1 = 100$ kPa
Minimum temperature of the cycle $T_1 = 27\ °\text{C} = 27 + 273 = 300$ K
Maximum pressure of the cycle $P_3 = P_2 = 4$ MPa $= 4000$ kPa
Heat supplied to the cycle $Q_1 = 1000$ kJ/kg

(a) For the isentropic process 1–2, we have

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

or

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} = \left(\frac{4000}{100}\right)^{\frac{1}{1.4}} = 13.94$$

Thus, the compression ratio is $r = 13.94$

(b) For the isentropic process 1–2, we get

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

or

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times 13.94^{0.4} = 860.65 \text{ K}$$

This is the temperature at the end of the compression.

(c) Heat supplied per kg of air is given by

$$Q_1 = C_p(T_3 - T_2)$$

or

$$1000 = 1.005(T_3 - 860.65)$$

or

$$T_3 = 1855.67 \text{ K}$$

This is the temperature at the end of the combustion.

(b) For the constant pressure process 2–3, we can write

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

or

$$\frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1855.67}{860.65} = 2.16$$

The cut-off ratio is $r_c = 2.16$

(c) The cycle efficiency is

$$\eta_{\text{Diesel}} = 1 - \frac{1}{r^{\gamma-1}}\frac{1}{\gamma}\frac{r_c^{\gamma}-1}{r_c-1}$$

$$= 1 - \frac{1}{13.94^{1.4-1}}\frac{1}{1.4}\frac{2.16^{1.4}-1}{2.16-1} = 0.5838 \text{ or } 58.38\%$$

Example 9.10 Show that the mean effective pressure of an air-standard Diesel cycle is

$$\text{MEP} = P_1 \frac{\gamma r^{\gamma}(r_c - 1) - r(r_c^{\gamma} - 1)}{(\gamma - 1)(r - 1)}$$

where is the compression ratio, r_c is the cut-off ratio, γ is the specific heat ratio, and P_1 is the pressure at the beginning of the compression.

Solution Let m kg be the mass of working fluid.

The heat addition for the diesel cycle is given by (see Fig. 9.7)

$$Q_1 = mC_p(T_3 - T_2)$$

Efficiency of a Diesel cycle is given by

$$\eta = \frac{W_{net}}{Q_1} = 1 - \frac{1}{r^{k-1}} \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1}$$

or

$$W_{\text{net}} = \eta Q_1 = mC_p(T_3 - T_2)\left[1 - \frac{1}{r^{k-1}} \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1}\right]$$

$$= m\frac{\gamma R}{\gamma - 1}(T_3 - T_2)\left[1 - \frac{1}{r^{\gamma-1}} \frac{1}{k} \frac{r_c^{\gamma} - 1}{r_c - 1}\right] \qquad \left[\because C_p = \frac{\gamma R}{\gamma - 1}\right]$$

$$= \frac{\gamma P_2}{\gamma - 1}(V_3 - V_2)\left[1 - \frac{1}{r^{\gamma-1}} \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1}\right]$$

Mean effective pressure is given by

$$\text{MEP} = \frac{W_{net}}{V_1 - V_2} = \frac{\gamma P_2}{(\gamma - 1)(V_1 - V_2)}(V_3 - V_2)\left[1 - \frac{1}{r^{\gamma-1}} \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1}\right]$$

$$= \frac{\gamma P_1 r^{\gamma}}{(\gamma - 1)(r - 1)}(r_c - 1)\left[1 - \frac{1}{r^{\gamma-1}} \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1}\right]$$

$$\left[\because P_2 = P_1 r^{\gamma},\ \frac{V_1}{V_2} = r \text{ and } \frac{V_3}{V_2} = r_c\right]$$

$$= P_1 \frac{\gamma r^{\gamma}(r_c - 1) - r(r_c^{\gamma} - 1)}{(\gamma - 1)(r - 1)}$$

9.6 COMPARISON OF OTTO AND DIESEL CYCLE

Otto and Diesel cycles can be compared on the basis of either the same compression ratio or the same maximum pressure and temperature.

For the same compression ratio and heat rejection, Otto cycle and Diesel cycle on the *P–V* and *T–s* diagrams are shown in Fig. 9.11. Here,

1-2-3-4 -Otto cycle
1-2-5-4 -Diesel cycle

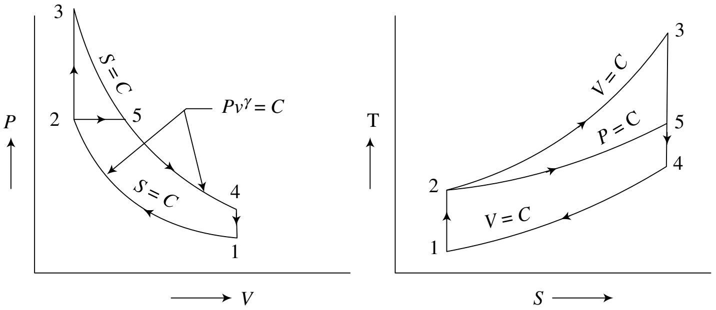


Figure 9.11 *Comparison of Otto and Diesel Cycle for the same compression ratio*

For the same heat rejection, the higher the heat addition, the higher is the cycle efficiency. In the *T–s* diagram, the area under 2-3 represents heat addition for the Otto cycle, the area under 2-5 represents heat addition for the Diesel cycle.

Since the area under 2-3 is greater than that of 2-5, the efficiency of Otto cycle is greater than that of Diesel cycle.

$$\eta_{\text{Otto}} > \eta_{\text{Diesel}}$$

For the same maximum pressure and temperature and heat rejection, Otto cycle and Diesel cycle on the *P–V* and *T–s* diagrams are shown in Fig. 9.12. Here,

1-2-3-4 -Otto cycle
1-5-3-4 -Diesel cycle

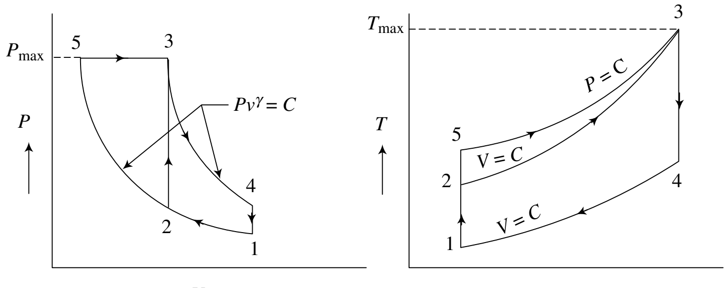


Figure 9.12 *Comparison of Otto and Diesel Cycle for the same maximum pressure and temperature*

For the same heat rejection, the higher the heat addition, the higher is the cycle efficiency. In the *T–s* diagram, the area under 2-3 represents heat addition for the Otto cycle, the area under 5-3 represents heat addition for the Diesel cycle.

Since the area under 5-3 is greater than that of 2-3, the efficiency of Diesel cycle is greater than that of Otto cycle.

$$\eta_{\text{Diesel}} > \eta_{\text{Otto}}$$

SUMMARY

- The *air-standard assumptions* reduce the complexities associated with the actual cycles.
- In reciprocating engine, the piston reciprocates in the cylinder between two fixed positions called the *top dead centre* (TDC) and the *bottom dead centre* (BDC). The piston is said to be at the *top dead centre* when it has moved to the position where the cylinder volume is a minimum. When the piston has moved to the position of maximum cylinder volume, the piston is at the *bottom dead centre*. The *bore* of the cylinder is its diameter. The distance between the TDC and the BDC is the largest distance that the piston can travel in one direction. The *stroke* is the largest distance the piston moves in one direction.
- The *compression ratio r* of the reciprocating engine is the ratio of the maximum volume formed in the cylinder to the minimum volume.

  $$r = \frac{V_{\text{max}}}{V_{\text{min}}}$$

- The *mean effective pressure* is a fictitious pressure and is defined as that pressure which when acting on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle.

  $$\text{MEP} = \frac{W_{\text{net}}}{V_{\text{max}} - V_{\text{min}}}$$

- The Otto cycle is the ideal cycle for spark-ignition (SI) engine. It consists of four processes: isentropic compression, quasi-equilibrium constant-volume heat addition, isentropic expansion and quasi-equilibrium constant-volume heat rejection.
- The thermal efficiency of the Otto cycle can be expressed as

  $$\eta_{\text{Otto}} = 1 - \frac{1}{r^{\gamma - 1}}$$

  where r is the compression ratio and γ is the specific heat ratio. The thermal efficiency of an Otto cycle is dependent only on the compression ratio for a given value of specific heat ratio; the higher the compression ratio, the higher the thermal efficiency.
- The Diesel cycle is the ideal cycle for compression-ignition (CI) engine. The difference between the diesel and the Otto cycle is that, in the Diesel cycle, the heat is added during a constant-pressure process.
- The thermal efficiency of the Diesel cycle can be expressed as

$$\eta_{\text{Diesel}} = 1 - \frac{1}{r^{\gamma-1}} \frac{1}{\gamma} \frac{r_c^{\gamma} - 1}{r_c - 1}$$

where r_c is the cutoff ratio and is defined as the ratio of cylinder volume to the volume of the cylinder when the fuel injection is cut-off. The diesel cycle efficiency decreases with increase in the cut-off ratio.

- The efficiency of the Diesel cycle is less than that of the Otto cycle for the same compression ratio and heat rejection. On contrary, for the same maximum pressure and temperature, and heat rejection, the efficiency of the Diesel cycle is greater than that of the Otto cycle.

REVIEW QUESTIONS

9.1 Define the compression ratio.

9.2 What is an air-standard cycle?

9.3 State the four processes of the Otto cycle.

9.4 State the four processes of the Diesel cycle.

9.5 Sketch an air-standard Otto cycle on $P-V$ and $T-S$ diagrams.

9.6 Sketch an air-standard Diesel cycle on $P-V$ and $T-S$ diagrams.

9.7 How does the Diesel cycle differ from the Otto cycle?

9.8 Define the cut-off ratio for an air-standard Diesel cycle.

9.9 Derive an expression for the efficiency of an air-standard Otto cycle in terms of its compression ratio.

9.10 Derive an expression for the efficiency of an air-standard Diesel cycle.

9.11 For the same compression ratio and heat rejection, explain with the help of $P–V$ and $T–s$ diagrams why the efficiency of Otto cycle is greater than that of Diesel cycle.

9.12 For the same maximum pressure and temperature of the cycle and the same heat rejection, which cycle is more efficient: Otto or Diesel? Explain with the help of $P–v$ and $T–s$ diagrams.

NUMERICAL PROBLEMS

9.1 The bore and stroke of a four-stroke spark ignition engine are 250 mm and 300 mm respectively. The clearance volume is 0.002 m^3. If the specific heat ratio k = 1.4, compute the air standard efficiency.

9.2 A four-cylinder petrol engine working on an air-standard Otto cycle has a swept volume of 2000 cm^3, and the clearance volume in each cylinder is 60 cm^3. Determine the cycle efficiency.

9.3 For an engine operating on air-standard Otto cycle, the clearance volume is 15% of the swept volume. If the specific heat ratio of air is 1.4, compute the air-standard cycle efficiency.

9.4 In an air-standard Otto cycle the compression ratio is 8.5, and compression begins at 27 °C, 0.1 MPa. The heat addition to the cycle is 1500 kJ/kg. Find (a) temperature and pressure at the cardinal points of the cycle, (b) the cycle efficiency, and (c) the work done per kg of air.

9.5 In an air-standard Otto cycle the compression ratio is 7.5, and the temperature, the pressure and volume at the beginning of he compression stroke are 27 °C, 0.1 MPa and 0.25 m^3 respectively. The maximum temperature of the cycle is 2727 °C. Find (a) the work done and (b) the cycle efficiency.

9.6 The four temperatures in an Otto cycle (Fig. 9.2) using air are $T_1 = 300$ K, $T_2 = 754$ K, $T_3 = 1600$ K, $T_4 =$ 637 K. A pressure of exists at state 1. Find (a) pressure at state 2 (b) the heat added to the cycle per kg of air, and (c) the compression ratio of the cycle. Assume constant specific heats.

9.7 In an air-standard Diesel cycle, the compression ratio is 18. Compression begins at 0.1 MPa, 27 °C. The cut-off ratio is 2. Find (a) the maximum temperature of the cycle, (b) the work done per kg of air, and (c) the cycle efficiency.

9.8 In an air standard Diesel cycle, the compression ratio is 15. Compression begins at 0.1 MPa, 27 °C. The maximum temperature of the cycle is 1727 °C. Find (a) the work done per kg of air, and (b) the cycle efficiency.

9.9 In an air-standard Diesel cycle, the compression begins at 0.1 MPa, 300 K. The maximum temperature of the cycle is 1800 K. The heat addition to the cycle is 700 kJ/kg. Find the cycle efficiency.

9.10 In an air-standard Diesel cycle, the compression ratio is 15. The compression begins at 0.1 MPa, 300 K. The maximum temperature of the cycle is 1800 K. The heat addition to the cycle is 1200 kJ/kg. Find (a) the maximum pressure and temperature of the cycle, (b) the cut-off ratio, (c) the cycle efficiency, and (d) the work done per kg of air.

MULTIPLE-CHOICE QUESTIONS

9.1 A petrol engine theoretically operates on
(a) constant pressure cycle (b) constant volume cycle
(c) constant temperature cycle (d) constant entropy cycle

9.2 A Diesel engine theoretically operates on
(a) constant pressure cycle (b) constant volume cycle
(c) constant temperature cycle (d) constant entropy cycle

9.3 The cycle with constant volume heat addition and heat rejection is also called
(a) Carnot cycle (b) Joule cycle
(c) Rankine cycle (d) Otto cycle

9.4 The cycle with constant pressure heat addition and constant volume heat rejection is also called
(a) Carnot cycle (b) Diesel cycle
(c) Rankine cycle (d) Otto cycle

9.5 Air-standard efficiency of an Otto cycle is dependent upon
(a) ratio of specific heats (b) cut off ratio
(c) compression ratio (d) both (a) and (c)

9.6 For the same compression ratio and heat rejection, the efficiency of Otto cycle is
(a) greater than Diesel cycle (b) less than Diesel cycle
(c) equal to Diesel cycle (d) none of the above

9.7 For the same maximum pressure and temperature of the cycle and for the same heat rejection, the efficiency of Otto cycle is
(a) greater than Diesel cycle (b) less than Diesel cycle
(c) equal to Diesel cycle (d) none of the above

9.8 Air-standard efficiency of a diesel cycle is dependent upon
(a) ratio of specific heats (b) cut-off ratio
(c) compression ratio (d) all of the above

9.9 A cycle consisting of two isentropic and two constant volume processes is known as
(a) Otto cycle (b) Diesel cycle
(c) Joule cycle (d) Rankine cycle

9.10 A cycle consisting of two isentropic, one constant volume and one constant pressure processes is known as

(a) Otto cycle (b) Diesel cycle
(c) Joule cycle (d) Rankine cycle

9.11 The efficiency of diesel cycle with decrease in cut off
(a) increases (b) decreases
(c) remains unaffected (d) first increases and then decreases

CHAPTER

10

Power Cycles

10.1 INTRODUCTION

Various thermodynamic cycles are developed by scientists and researchers to achieve a predetermined objective. From this point of view, they are classified into two different categories: power cycles and refrigeration cycles.

On the basis of the phase of a working fluid, cycles are further classified into two: gas cycles and vapour cycles.

Cycles can also be classified into thermodynamic (closed) and mechanical (open) cycles. In thermodynamic cycles, the working fluid is returned to the initial state at the end of the cycle and is re-circulated, whereas in mechanical cycles, the working fluid is renewed at the end of each cycle.

Since the working substance in a vapour power cycle is steam/water, it is often referred as the ***steam power cycle***. Steam has many desirable characteristics such as high enthalpy of evaporation, low cost, easy availability.

The four basic components of a steam power plant are boiler, turbine, condenser and pump—all are put in a logical sequence as shown below in Fig. 10.1.

Water at an ambient temperature is supplied to the boiler by means of a pump. Heat is added (Q_1) to the water in the boiler to convert the working substance to steam. Steam so produced, having high pressure and temperature, is allowed to expand in the turbine. Expansion of the steam in the turbine causes the turbine blades to be rotated thus giving external work as output (W_T). This mechanical work is converted to the electrical energy by coupling a generator with the rotor of the turbine. The pressure and temperature of the steam after expansion by turbine comes down and the condition of steam becomes wet. However, to facilitate complete conversion of steam into water, further cooling is required at constant pressure and temperature (by rejecting latent heat). This is accomplished in the condenser. Condenser is nothing but a heat exchanger that uses two fluids having different temperature for mutual interaction of heat. The steam rejects latent heat (Q_2) for complete conversion to water. This water at low pressure and temperature is fed back to the boiler for further use by means of a feed pump, requiring some work (W_P) to be expended. This is how the cycle is continued.

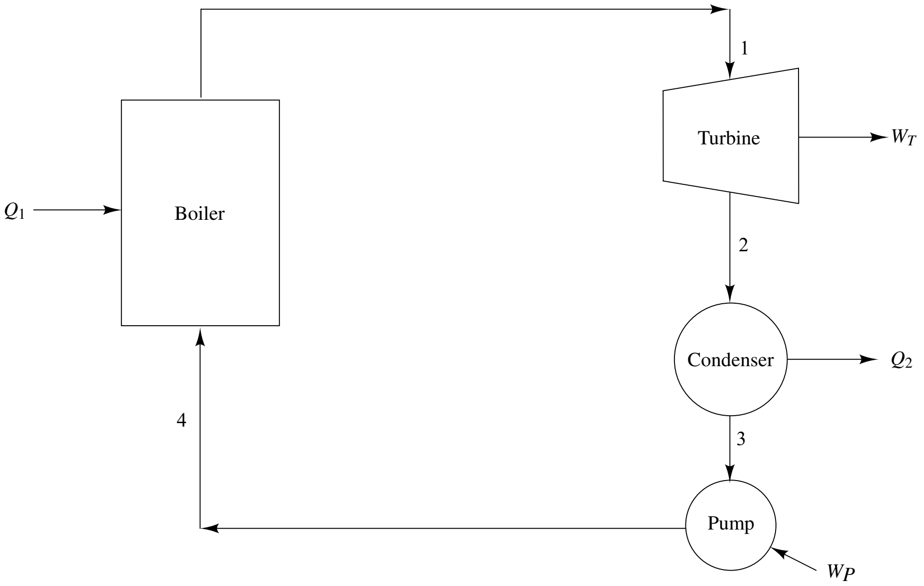


Figure 10.1 *Schematic arrangement of simple steam power plant*

10.2 CARNOT VAPOUR POWER CYCLE

A Carnot vapour power cycle comprises of two reversible isothermal and two reversible adiabatic processes as shown in Fig. 10.2. Dry saturated steam enters the turbine and expands reversibly and adiabatically to condenser pressure. The steam is then condensed at constant pressure and temperature. The wet steam leaving the condenser is then compressed reversibly and adiabatically into the boiler pressure. The saturated liquid is then evaporated in the boiler to complete the cycle.

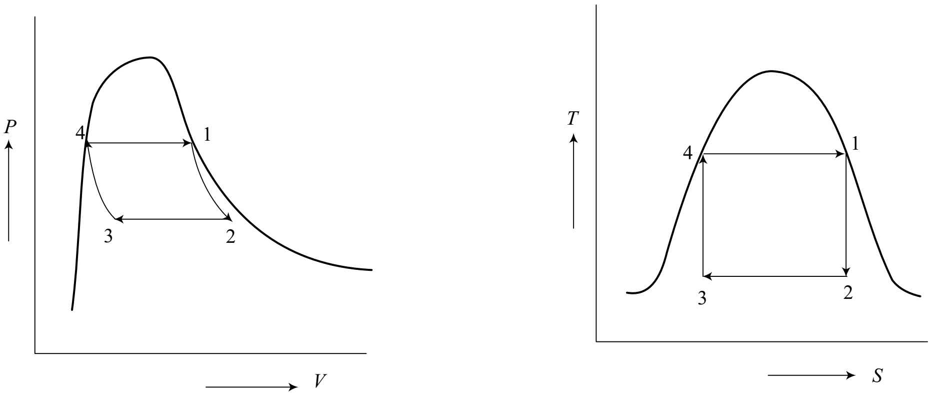


Figure 10.2 *Carnot Vapour Cycle on P–v and T–s diagrams*

A Carnot cycle is not practicable for a steam power plant because of the following drawbacks:

(i) The isentropic compression process involves the compression of wet steam to a saturated liquid. It is not practical to design a pump that can handle wet steam.

(ii) The rate of delivery of work is less, because of very large pump work.

(iii) The turbine that takes saturated vapour at the inlet produces wet steam with low quality. Thus the turbine has to handle steam with high moisture content. The impingement of liquid droplets on the turbine blades causes the erosion of the blades.

10.3 RANKINE CYCLE

The Rankine cycle is an ideal cycle for vapour cycles. The cycle is shown in Fig. 10.3 on *P*–*v*, *T*–*s*, and *h*–*s* diagrams. The Rankine cycle comprises of the following processes:

4–1: Reversible constant pressure heat addition in a boiler
1–2: Reversible adiabatic expansion in a turbine
2–3: Reversible constant pressure heat rejection in a condenser
3–4: Reversible adiabatic compression in a pump

Dry saturated steam enters the turbine and expands reversibly and adiabatically to condenser pressure. The steam is then condensed at constant pressure and temperature to a saturated liquid. The saturated liquid leaving the condenser is then pumped reversibly and adiabatically into the boiler pressure. The compressed liquid is first heated to the saturation temperature at boiler pressure and then evaporated to the state 1 to complete the cycle.

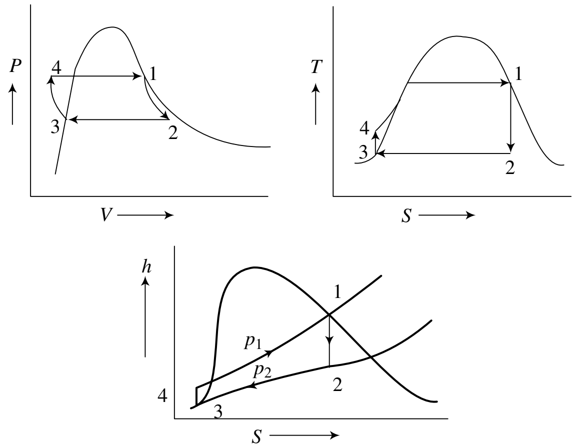


Figure 10.3 *Rankine Cycle on P–v, T–s and h–s diagram*

10.4 RANKINE CYCLE EFFICIENCY

In the thermodynamic analysis of power cycles, our main objective is to estimate the thermal efficiency. The thermal efficiency measures how successfully the energy input to the working fluid passing through the boiler is converted to net work output.

Considering unit mass of working fluid and neglecting the changes in kinetic and potential energy, the first law of thermodynamics applied to each of the steady-flow devices (Fig. 10.1) gives

Boiler:

$$Q_1 + h_4 = h_1$$

or $$Q_1 = h_1 - h_4 \tag{10.1}$$

Turbine:

$$h_1 = h_2 + W_T$$

or $$W_T = h_1 - h_2 \tag{10.2}$$

Condenser:

$$h_2 = Q_2 + h_1$$

or $$Q_2 = h_2 - h_3 \tag{10.3}$$

Pump:

$$h_3 + W_P = h_4$$

or $$W_P = h_4 - h_3 \tag{10.4}$$

The efficiency of the Rankine cycle is given by

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} \tag{10.5}$$

The way to determine the enthalpy change across the pump is to use the thermodynamic property relation $Tds = dh - vdP$ for an isentropic process. It becomes

$$dh = vdP$$

or $$\Delta h = \int vdP$$

The pump handles liquid which is incompressible (specific volume is independent of pressure). Therefore, one can write

$$\Delta h = v\Delta P$$

or $$h_4 - h_3 = v_3(P_1 - P_2) \tag{10.6}$$

Note that the work required in the pumping process (Pump work) is quite small compared to the turbine work and is sometimes neglected. Then, we have $h_4 \cong h_3$.

The efficiency of the Rankine cycle then becomes

$$\eta = \frac{h_1 - h_2}{h_1 - h_4} \tag{10.7}$$

Example 10.1 A steam power plant is designed to operate on Rankine cycle. Steam enters the turbine as saturated vapour at 30 bar and leaves as saturated liquid in the condenser at 10 kPa. The mass flow rate of steam is 1 kg/s. Determine the net power output of the cycle and the thermal efficiency of the Rankine cycle.

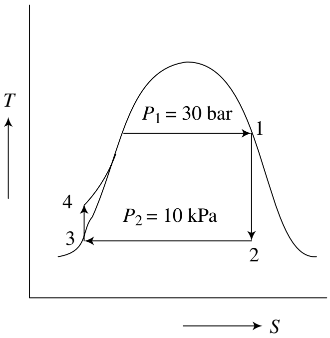


Figure 10.4

Solution

The *T-s* diagram of the Rankine cycle is shown in the Fig 10.4. From the saturated steam table based on pressure (Appendix 1.2), it is found that at 30 bar

$$h_1 = 2804.1 \text{ kJ/kg}, s_1 = 6.1878 \text{ kJ/kgK},$$

Similarly from Appendix 1.2, we get at 10 kPa

$$s_f = 0.6491 \text{ kJ/kgK}, s_g = 8.1510 \text{ kJ/kgK}, h_3 = h_f = 191.8 \text{ kJ/kg},$$

$$h_{fg} = 2392.8 \text{ kJ/kg}, v_3 = v_f = 0.00101 \text{ m}^3\text{/kg}$$

To locate state 2, we recognize that $s_1 = s_2 = 6.1878$ kJ/kgK. Hence,

$$s_1 = s_{f_{P=10kPa}} + x_2 \left(s_g - s_f\right)\Big|_{P=10kPa}$$

$$6.1878 = 0.6491 + x_2 \left(8.1510 - 0.6491\right)$$

$$\therefore \quad x_2 = 0.7383$$

This allows us to find the specific enthalpy at the exit from the turbine, h_2 to be

$$h_2 = h_f + x_2 h_{fg} = 191.8 + 0.7383 \times 2392.8 = 1958.4 \text{ kJ/kg}$$

The specific work output from the turbine is

$$W_T = h_1 - h_2 = 2804.1 - 1958.4 = 845.7 \text{ kJ/kg}$$

The pump work requirement for this ideal cycle is (refer to Fig. 10.1)

$$W_P = v_3 \left(P_1 - P_2\right) = 0.00101\left(30 \times 100 - 10\right) = 3.02 \text{ kJ/kg}$$

Net work output of the cycle is $W_{net} = W_T - W_P = 845.7 - 3.02 = 842.68$ kJ/kg

Net power output of the cycle is $= W_{net} \times$ mass flow rate of steam

$$= 842.68 \times 1 = 842.68 \text{ kW}$$

The specific enthalpy at the pump outlet, state 4, is the inlet specific enthalpy h_3 plus W_P. Thus,

$$h_4 = h_3 + W_P = 191.8 + 3.02 = 194.82 \text{ kJ/kg}$$

To calculate the thermal efficiency, we must know the boiler heat input. It is

$$Q_1 = h_1 - h_4 = 2804.1 - 194.82 = 2609.28 \text{ kJ/kg}$$

The thermal efficiency of the Rankine cycle is then calculated to be

$$\eta = \frac{W_{net}}{Q_1} = \frac{842.68}{2609.28} = 0.3230 \text{ or } 32.3\%$$

Example 10.2 A thermal power plant is to be operated on an ideal Rankine cycle. Steam enters the turbine at 30 bar and 400°C and leaves as saturated liquid in the condenser at 10 kPa. The mass flow rate of steam is 1.5 kg/s. Determine the net power output of the cycle and the thermal efficiency of the Rankine cycle.

Solution

The *T*-*s* diagram of the Rankine cycle is shown in the Fig 10.5.

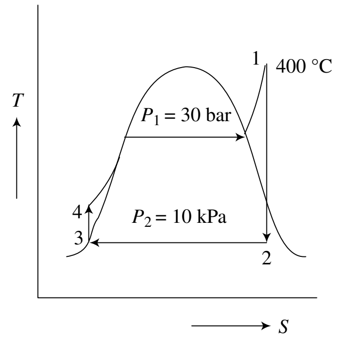


Figure 10.5

From the superheated steam table (Appendix 1.3), it is found that at 30 bar and 400°C

$$h_1 = 3230.9 \text{ kJ/kg}, s_1 = 6.9212 \text{ kJ/kgK},$$

From the saturated steam table based on pressure (Appendix 1.2), it is found that at 10 kPa

$$s_f = 0.6491 \text{ kJ/kgK}, s_g = 8.1510 \text{ kJ/kgK}, h_3 = h_f = 191.8 \text{ kJ/kg}$$

$$h_{fg} = 2392.8 \text{ kJ/kg}, v_3 = v_f = 0.00101 \text{ m}^3\text{/kg}$$

To locate state 2, we recognize that $s_1 = s_2 = 6.9212$ kJ/kgK. Hence,

$$s_1 = s_{f_{P=10kPa}} + x_2 \left(s_g - s_f\right)\Big|_{P=10kPa}$$

$$6.9212 = 0.6491 + x_2 \left(8.1510 - 0.6491\right)$$

$$\therefore \quad x_2 = 0.8361$$

This allows us to find the specific enthalpy at the exit from the turbine, h_2 to be

$$h_2 = h_f + x_2 h_{fg} = 191.8 + 0.8361 \times 2392.8 = 2192.42 \text{ kJ/kg}$$

The specific work output from the turbine is

$$W_T = h_1 - h_2 = 3230.9 - 2192.42 = 1038.48 \text{ kJ/kg}$$

The pump work requirement for this ideal cycle is (refer to Fig. 10.1)

$$W_P = v_3 \left(P_1 - P_2\right) = 0.00101\left(30 \times 100 - 10\right) = 3.02 \text{ kJ/kg}$$

$$\begin{aligned}\text{Net power output of the cycle is } &= W_{net} \times \text{mass flow rate of steam}\\ &= \left(W_T - W_P\right) \times \text{mass flow rate of steam}\\ &= \left(1038.48 - 3.02\right) \times 1.5 = 1553.19 \text{ kW}\end{aligned}$$

The specific enthalpy at the pump outlet, state 4, is the inlet specific enthalpy h_3 plus W_p. Thus,

$$h_4 = h_3 + W_P = 191.8 + 3.02 = 194.82 \text{ kJ/kg}$$

To calculate the thermal efficiency, we must know the boiler heat input. It is

$$Q_1 = h_1 - h_4 = 3230.9 - 194.82 = 3036.08 \text{ kJ/kg}$$

The thermal efficiency of the Rankine cycle is then calculated to be

$$\eta = \frac{W_{net}}{Q_1} = \frac{1038.48 - 3.02}{3036.08} = 0.3411 \text{ or } 34.11\%$$

Example 10.3 An ideal Rankine cycle operating between temperature of 500°C and 50°C. Calculate the cycle efficiency and the quality of steam at the turbine outlet if the pump outlet pressure is 2 MPa.

Solution The *T*-*s* diagram of the Rankine cycle is shown in the Fig 10.6.

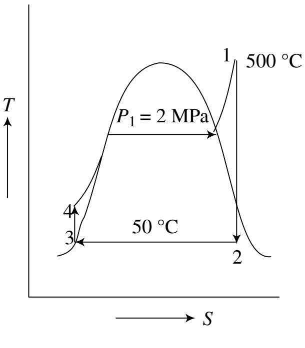


Figure 10.6

From the superheated steam table (Appendix 1.3), it is found that at 2 MPa and 500°C

$$h_1 = 3467.6 \text{ kJ/kg}, s_1 = 7.4317 \text{ kJ/kgK}$$

From the saturated steam table based on temperature (Appendix 1.1), it is found that at 50°C

$$s_f = 0.7036 \text{ kJ/kgK}, s_g = 8.0771 \text{ kJ/kgK}, h_3 = h_f = 209.3 \text{ kJ/kg}$$

$$h_{fg} = 2382.8 \text{ kJ/kg}, v_3 = v_f = 0.001012 \text{ m}^3\text{/kg}, P_2 = 0.01235 \text{ MPa}$$

To locate state 2, we recognize that Hence,

$$s_1 = s_{f_{P=10\text{ kPa}}} + x_2 \left(s_g - s_f\right)\Big|_{P=10\text{ kPa}}$$

$$7.4317 = 0.7036 + x_2 \left(8.0771 - 0.7036\right)$$

$$\therefore \quad x_2 = 0.9125$$

This allows us to find the specific enthalpy at the exit from the turbine, h_2 to be

$$h_2 = h_f + x_2 h_{fg} = 209.3 + 0.9125 \times 2382.8 = 2383.8 \text{ kJ/kg}$$

The specific work output from the turbine is

$$W_T = h_1 - h_2 = 3467.6 - 2383.6 = 1084 \text{ kJ/kg}$$

The pump work requirement for this ideal cycle is (refer to Fig. 10.1)

$$W_P = v_3 \left(P_1 - P_2\right) = 0.001012 \times \left(2 - 0.01235\right) \times 10^3 = 2.01 \text{ kJ/kg}$$

The specific enthalpy at the pump outlet, state 4, is the inlet specific enthalpy h_3 plus W_P. Thus,

$$h_4 = h_3 + W_P = 209.3 + 2.01 = 211.31 \text{ kJ/kg}$$

The thermal efficiency of the Rankine cycle is then calculated from equation (10.5) as

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{h_1 - h_4} = \frac{1084 - 2.01}{3467.6 - 211.31} = 0.3323 \text{ or } 33.23\%$$

Example 10.4 A steam power plant is designed to operate on Rankine cycle. Steam enters the turbine at 4 MPa and 500°C and leaves as saturated liquid in the condenser at 10 kPa. Calculate (a) the thermal efficiency with pump work included, (b) the thermal efficiency neglecting pump work, and (c) the percentage error in efficiency neglecting pump work.

Solution The *T-s* diagram of the Rankine cycle is shown in the Fig 10.7.

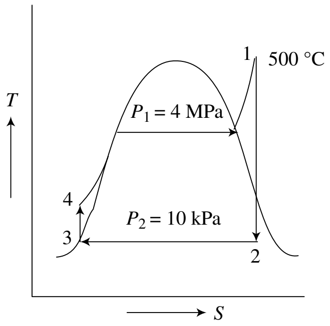


Figure 10.7

From the superheated steam table (Appendix 1.3), it is found that at 4 MPa and 500°C

$$h_1 = 3445.3 \text{ kJ/kg}, s_1 = 7.0901 \text{ kJ/kgK}$$

From the saturated steam table based on pressure (Appendix 1.2), it is found that at 10 kPa

$$s_f = 0.6491 \text{ kJ/kgK}, s_g = 8.1510 \text{ kJ/kgK}, h_3 = h_f = 191.8 \text{ kJ/kg}$$

$$h_{fg} = 2392.8 \text{ kJ/kg}, v_3 = v_f = 0.00101 \text{ m}^3\text{/kg}$$

To locate state 2, we recognize that $s_1 = s_2 = 7.4317$ kJ/kgK . Hence,

$$s_1 = s_{f_{P=10kPa}} + x_2\left(s_g - s_f\right)\Big|_{P=10kPa}$$

$$7.0901 = 0.6491 + x_2\left(8.1510 - 0.6491\right)$$

$\therefore$ $$x_2 = 0.8586$$

This allows us to find the specific enthalpy at the exit from the turbine, h_2 to be

$$h_2 = h_f + x_2 h_{fg} = 191.8 + 0.8586 \times 2392.8 = 2246.26 \text{ kJ/kg}$$

The specific work output from the turbine is

$$W_T = h_1 - h_2 = 3545.3 - 2246.26 = 1199.04 \text{ kJ/kg}$$

(a) The pump work requirement for this ideal cycle is (refer to Fig. 10.1)

$$W_P = v_3\left(P_1 - P_2\right) = 0.00101 \times \left(4 \times 10^3 - 10\right) = 4.03 \text{ kJ/kg}$$

The specific enthalpy at the pump outlet, state 4, is the inlet specific enthalpy h_3 plus W_P. Thus,

$$h_4 = h_3 + W_P = 191.8 + 4.03 = 195.83 \text{ kJ/kg}$$

The thermal efficiency of the Rankine cycle is then calculated from equation (10.5) as

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_{net}}{h_1 - h_4} = \frac{1199.04}{3445.3 - 195.83} = 0.369 \text{ or } 36.9\%$$

(b) Neglecting the pump work, we have $h_4 \approx h_3 = 191.8$ kJ/kg

Then the thermal efficiency of the Rankine cycle becomes

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_{net}}{h_1 - h_4} = \frac{1199.04}{3445.3 - 191.8} = 0.3685 \text{ or } 36.85\%$$

(c) The percentage error in efficiency neglecting pump work is

$$= \frac{0.369 - 0.3685}{0.3685} = 0.001357 \text{ or } 0.1357\%$$

Example 10.5 A thermal power plant is to be operated on an ideal Rankine cycle. Steam enters into the turbine at 2 MPa, 360°C and leaves as saturated liquid in the condenser at 8 kPa. The pump feeds the water back into the boiler. Assume ideal processes, find, per kg of steam, the net work and the cycle efficiency.

Solution The *T*-*s* diagram of the Rankine cycle is shown in the Fig 10.8.

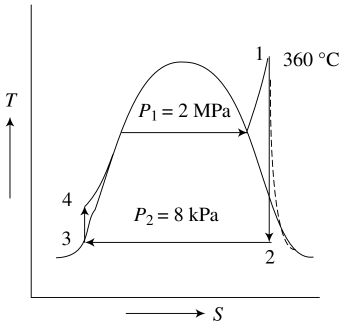


Figure 10.8

At 2 MPa and 360 °C steam is in superheated state. However, at that condition, properties are not given in the steam table (Appendix 1.3). Properties are then obtained by linear interpolation of the properties at 2 MPa , 350°C and 2 MPa, 400°C.

From the superheated steam table (Appendix 1.3), it is found that

at 2 MPa and 350 °C $h = 3137.0$ kJ/kg, $s = 6.9563$ kJ/kgK

at 2 MPa and 400 °C $h = 3247.6$ kJ/kg, $s = 7.1271$ kJ/kgK ,

Using linear interpolation, specific enthalpy and entropy at 2 MPa and 360°C are found to be

$$\frac{h_1 - 3137.0}{3247.6 - 3137.0} = \frac{s_1 - 6.9563}{7.1271 - 6.9563} = \frac{360 - 350}{400 - 350}$$

$$h_1 = 3159.12 \text{ kJ/kg}, s_1 = 6.9905 \text{ kJ/kgK}$$

From the saturated steam table based on pressure (Appendix 1.2), it is found that at 8 kPa

$$s_f = 0.5924 \text{ kJ/kgK}, s_g = 8.2295 \text{ kJ/kgK}, h_3 = h_f = 173.9 \text{ kJ/kg}$$

$$h_{fg} = 2403.1 \text{ kJ/kg}, v_3 = v_f = 0.001008 \text{ m}^3\text{/kg}$$

To locate state 2, we recognize that $s_1 = s_2 = 7.4317$ kJ/kgK . Hence,

$$s_1 = s_{f_{P=10kPa}} + x_2 \left(s_g - s_f\right)\Big|_{P=10kPa}$$

$$6.9905 = 0.5924 + x_2 (8.2295 - 0.5924)$$

$$\therefore \quad x_2 = 0.8378$$

This allows us to find the specific enthalpy at the exit from the turbine, h_2 to be

$$h_2 = h_f + x_2 h_{fg} = 173.9 + 0.8378 \times 2403.1 = 2187.22 \text{ kJ/kg}$$

The specific work output from the turbine is

$$W_T = h_1 - h_2 = 3159.12 - 2187.22 = 971.9 \text{ kJ/kg}$$

The pump work requirement for this ideal cycle is (refer to Fig. 10.1)

$$W_P = v_3 (P_1 - P_2) = 0.001008 \times \left(2 \times 10^3 - 8\right) = 2.008 \text{ kJ/kg}$$

Net work output of the cycle is $W_{net} = W_T - W_P = 971.9 - 2.008 = 969.892$ kJ/kg

The specific enthalpy at the pump outlet, state 4, is the inlet specific enthalpy h_3 plus W_p. Thus,

$$h_4 = h_3 + W_P = 173.9 + 2.008 = 175.908 \text{ kJ/kg}$$

To calculate the thermal efficiency, we must know the boiler heat input. It is

$$Q_1 = h_1 - h_4 = 3159.12 - 175.908 = 2983.212 \text{ kJ/kg}$$

The thermal efficiency of the Rankine cycle is then calculated to be

$$\eta = \frac{W_{net}}{Q_1} = \frac{969.892}{2983.212} = 0.3251 \text{ or } 32.51\%$$

Example 10.6 A steam power plant is designed to operate on Rankine cycle. Steam enters into the turbine at 2 MPa, 400°C and leaves as saturated liquid in the condenser at 10 kPa. The mass flow rate of steam is 1 kg/s. Find out the power developed by the turbine and the efficiency of the cycle. Assume the efficiencies of the turbine and the pump as 0.85 and 0.8 respectively.

Solution The *T*-*s* diagram of the Rankine cycle is shown in the Fig 10.9.

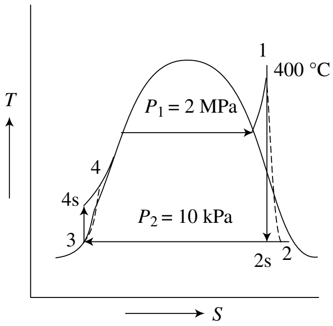


Figure 10.9

From the superheated steam table (Appendix 1.3), it is found that at 2 MPa, 400°C

$$h_1 = 3247.6 \text{ kJ/kg}, s_1 = 7.1271 \text{ kJ/kgK},$$

From the saturated steam table based on pressure (Appendix 1.2), it is found that at 10 kPa

$$s_f = 0.6491 \text{ kJ/kgK}, s_g = 8.1510 \text{ kJ/kgK}, h_3 = h_f = 191.8 \text{ kJ/kg}$$

$$h_{fg} = 2392.8 \text{ kJ/kg}, v_3 = v_f = 0.00101 \text{ m}^3\text{/kg}$$

To locate state 2, we recognize that $s_1 = s_2 = 7.1271$ kJ/kgK. Hence,

$$s_1 = s_{f_{P=10kPa}} + x_2 \left(s_g - s_f\right)\Big|_{P=10kPa}$$

$$7.1271 = 0.6491 + x_{2s}\left(8.1510 - 0.6491\right)$$

$$\therefore \qquad x_{2s} = 0.8645$$

This allows us to find the specific enthalpy at the exit from the turbine, h_2 to be

$$h_{2s} = h_f + x_{2s} h_{fg} = 191.8 + 0.8635 \times 2392.8 = 2257.98 \text{ kJ/kg}$$

The specific work output from the turbine is

$$W_T = h_1 - h_2 = \eta_T\left(h_1 - h_{2s}\right) = 0.85\left(3247.6 - 2257.98\right) = 841.18 \text{ kJ/kg}$$

The pump work is

$$W_P = \frac{v_{f_{P=10kPa}}\left(P_1 - P_2\right)}{\eta_P} = \frac{0.00101 \times \left(2 \times 10^3 - 10\right)}{0.8} = 2.51 \text{ kJ/kg}$$

This allows us to find h_4 to be

$$h_4 = h_3 + W_P = 191.8 + 2.51 = 194.31 \text{ kJ/kg}$$

The heat input is found using $Q_1 = h_1 - h_4 = 3247.6 - 194.31 = 3053.29$ kJ/kg

Net work output of the cycle is $W_{net} = W_T - W_P = 841.18 - 2.51 = 838.67$ kJ/kg

Net power output of the cycle is $= W_{net} \times \text{mass flow rate of steam}$

$$= 838.67 \times 1 = 838.67 \text{ kW}$$

The thermal efficiency of the Rankine cycle is then calculated to be

$$\eta = \frac{W_{net}}{Q_1} = \frac{838.67}{3053.29} = 0.2747 \text{ or } 27.47\%$$

SUMMARY

- A cycle which continuously converts heat into work is called a power cycle.
- A cycle which produces refrigeration effect is called a refrigeration cycle.
- The Carnot vapour cycle serves as an ideal cycle for vapour power cycle, but not practicable for a steam power plant.
- The Rankine cycle is an ideal cycle for vapour power cycle.
- The thermal efficiency of the Rankine cycle is given by

$$\eta = \frac{W_{\text{net}}}{Q_1} = \frac{W_T \quad W_P}{Q_1}$$

REVIEW QUESTIONS

10.1 What are the four basic components of a steam power plant working on Rankine cycle? Show by a block diagram.
10.2 Draw the nature of P–V and T–S plots of a Rankine cycle (with saturated steam at turbine inlet).
10.3 Draw the nature of P–V and T–S plots of a Rankine cycle (with superheated steam at turbine inlet).
10.4 Why is a Carnot cycle not practicable for a steam power plant?

NUMERICAL PROBLEMS

10.1 Steam is the working fluid in a Rankine cycle. Steam enters the turbine as saturated vapour at 40 bar and leaves as saturated liquid in the condenser at 10 kPa. Determine the thermal efficiency of the Rankine cycle.
10.2 A steam power plant is operated on an ideal Rankine cycle. Steam enters the turbine as saturated vapour at 150 bar and leaves as saturated liquid in the condenser at 20 kPa. Determine the thermal efficiency of the Rankine cycle.
10.3 A steam power plant is operated on an ideal Rankine cycle. Steam enters the turbine at 20 bar, 400 ºC and leaves as saturated liquid in the condenser at 8 kPa. Determine the thermal efficiency of the Rankine cycle.
10.4 A steam power plant operates between the pressures of 10 kPa and 2 MPa with a maximum temperature of 400°C. What is the maximum efficiency possible from the power cycle?
10.5 A steam power plant operates on a Rankine cycle with a condenser outlet temperature of 80°C and boiler outlet temperature of 500°C. The pump outlet pressure is 2 MPa. Determine the maximum possible thermal efficiency of the cycle.

MULTIPLE-CHOICE QUESTIONS

10.1 Thermal power plant works on
(a) Carnot cycle (b) Otto cycle (c) Rankine cycle (d) Joule cycle

10.2 Carnot cycle is
(a) a reversible cycle (b) an irreversible cycle
(c) a semi-reversible cycle (d) an adiabatic irreversible cycle

10.3 The working substance for a Carnot cycle is
(a) atmospheric air (b) air-fuel mixture (c) ideal gas (d) steam

CHAPTER

11

Properties of Fluids

11.1 INTRODUCTION

Mechanics is a subject dealing with the conditions under which a body can remain at rest or in motion. Mechanics can be classified into two: ***solid mechanics*** and ***fluid mechanics*.** Fluid mechanics is that branch of science which deals with the behaviour of fluids at rest as well as in motion and the subsequent effects of fluid upon its boundaries which may be either solid surfaces or interfaces with other fluids. Both liquids and gases are classified as fluids. The number of fluids in engineering applications are enormous: breathing, blood flow, swimming, pumps, fans, blowers, turbines, ships, rivers, airplanes, missiles, rockets, engines, jets, etc. Almost everything on this planet either is a fluid or moves within or near a fluid.

The field of fluid mechanics has been divided into three branches—fluid statics, fluid kinematics and fluid dynamics. ***Fluid statics*** is concerned with the behavior of a fluid at rest. ***Fluid kinematics*** deals with the motion of fluids without reference to forces that cause the motion. ***Fluid dynamics*** involves the study of a fluid motion as a consequence of forces that causes the motion.

11.2 DEFINITION OF FLUID

From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. A solid can resist a shear stress by a static deformation, a fluid cannot. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as shear stress is applied. A fluid is a substance that deforms continuously when subjected to a shear stress, however small the shear stress may be. A fluid may be either a liquid or a gas.

11.2.1 Difference between a Solid and a Fluid

- The fundamental difference between a fluid and a solid lies in the response to a shear stress of the respective materials. For a solid, the strain is a function of the applied stress, provided that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.

- The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow for as long as the force is applied and will not recover its original form when the force is removed. Consider a rectangular solid element *ABCD* as shown in Fig. 11.1 (a). Under the action of a shear force *F* the element assumes the shape $ABC'D'$. If the solid is perfectly elastic, it goes back to its shape *ABCD* when the force is withdrawn. In contrast, the element of the fluid *ABCD* (refer Fig. 11.1 (b)) confined between parallel plates deforms to shapes such as $ABC'D'$ and $ABC''D''$ as long as he force *F* is maintained on the upper plate.
- Within elastic limit, a solid has perfect memory because solid always relaxes back to its preferred shape, whereas a fluid has zero memory.
- The molecules of a solid are closer together than those of a fluid. The attractive forces between the molecules of a solid are so large that a solid tends to retain its shape. However, fluids cannot retain their shape, because the attractive forces between the molecules are smaller.

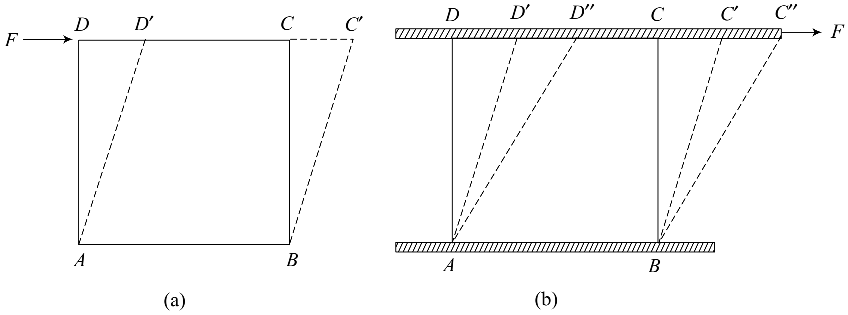


Figure 11.1 *Deformation of solid and fluid elements: (a) solid and (b) fluid*

11.2.2 Difference between Liquids and Gases

The difference between the two classes of fluids, liquids and gases, is all about the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above. Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand until it encounters confining walls. A gas has no definite volume, and when left to itself without confinement, a gas forms an atmosphere which is essentially hydrostatic.

11.3 THE CONCEPT OF CONTINUUM

The fluid flow analysis can be attempted from two different view points. One approach, popularly known as microscopic, stems from molecular point of view. It relies on the consideration that fluid essential comprises of molecules the motion of which is characterized by the laws of dynamics. The behaviour of the fluid then can be described by summing up the properties of the molecules following statistical approach. However, it has got certain limitations in regard to its applications in gases of higher density and in case of liquids. The other approach is the macroscopic one where the gross behaviour is considered rather than an individual molecule. The macroscopic approach treats the fluid as continuous and the variations of the property values of the individual molecules are not reflected. This approach gives the concept of **continuum** where fluids can be treated as a continuous medium disregarding the discontinuity in the microscopic entities. In the continuum approach at each point of the continuous fluid there is a unique value of the field variables such as pressure, velocity, density. This continuous matter follows the conservation laws of mass, momentum and energy. These laws can be derived using a set of differential equations. In most of the engineering applications, the concept of continuum yields very good results and hence accepted well.

11.4 FLUID PROPERTIES

Certain characteristics of a continuous fluid are independent of the motion of the fluid. These characteristics are called basic properties of the fluid. Here, elaborate discussions on a few such basic properties are included.

Fluid property is defined to be a characteristic of the material structure of the fluid. ***Flow property,*** is something whose value is determined in part by how the fluid is moving. The colour of a fluid is purely a fluid property, while the velocity of a fluid is purely a flow property. The density, pressure, temperature, viscosity, etc, are actually flow properties whose precise values depend on the nature of the fluid and type of flow.

11.4.1 Density

The density of a fluid is mass per unit volume. If a fluid element enclosing a point has a volume ΔV and mass Δm then density (ρ) at that point is written as

$$\rho = \lim_{\Delta V \to V^*} \frac{\Delta m}{\Delta V} = \frac{\mathrm{d}m}{\mathrm{d}V}$$

where ΔV^* is the smallest elemental volume over which the continuum hypothesis is valid.

The dimension of density is ML^{-3} and the unit of density in SI system is kg/m^3.

The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

A thermodynamic property of a fluid density has a certain value defined by the pressure and temperature of the fluid. The relationship is expressed by the characteristic equation of state. Since pressure and temperature are generally functions of position and time of flow, density must also be a function of position and time. Thus, density is flow property. The accepted value of density of air at sea level at 1.0133 bar and 288.15 K is 1.225kg/m^3.

The density of most gases is proportional to pressure and inversely proportional to temperature.

The variation of density of liquids with pressure is usually negligible. For example, at 20ºC, the density of water changes from 998 kg/m^3 at 1 atm to 1003 kg/m^3 at 100 atm (0.5% change). The density of liquids depends more strongly on temperature than it does on pressure. For example, at 1 atm, the density of water changes from 998 kg/m^3 to 975 kg/m^3 at 75ºC (only 2.3 % change).

11.4.2 Specific Weight

The specific weight is the weight of fluid per unit volume.

$$\text{Specific weight } (\gamma) = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{\text{Mass of fluid}}{\text{Volume of fluid}} \times \text{Acceleration due to gravity}$$

$$\gamma = \rho g$$

Specific weight is $ML^{-2}T^{-2}$ and the unit of specific weight in SI system is N/m^3.

11.4.3 Specific Volume

The specific volume of a fluid is the volume occupied by unit mass of fluid.

The dimension of specific volume is L^3T^{-1} and the unit of specific volume in SI system is m^3/kg.

Specific volume is the reciprocal of density i.e. $v = \frac{1}{\rho}$

11.4.4 Specific Gravity (or Relative Density)

Specific gravity or relative density is defined as the ratio of the density of some standard reference fluid at a specified temperature and pressure.

The standard fluid is water for liquid and is air for gases. For liquids, the specified temperature and pressure are 4°C and 101 kN/m^2, for which the density of water is 1000 kg/m^3.

$$S_{\text{liquid}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}}$$

$$S_{\text{gas}} = \frac{\rho_{\text{gas}}}{\rho_{\text{air}}}$$

Specific gravity is a dimensionless parameter. Engineers find these dimensionless ratios easier to remember than the actual numerical values of density of a variety of fluids.

11.5 COMPRESSIBLE FLUID AND INCOMPRESSIBLE FLUID

Fluids can be classified into incompressible and compressible on the basis of density change with change in pressure. A fluid is usually called incompressible if its density does not change significantly with change in pressure. A fluid is compressible when it has a change in density because of change in pressure. There is no fluid in reality which is incompressible. Liquids are considered incompressible fluids, since the change in density for liquids with pressure is so small as to be negligible.

11.6 IDEAL FLUID AND REAL FLUID

An ***ideal fluid*** is a fully hypothetical fluid which is assumed to have no viscosity and no compressibility. The concept of an ideal fluid has been utilized in the analytical treatment of fluid-flow problems. The mathematical analysis of the flow problem can be considerably simplified by assuming the fluid to be non-viscous and incompressible. Such a fluid does not exist in reality.

In a ***real fluid***, shear stresses occur whenever the fluid is in motion. In other words, fluid friction exists when a real fluid is in motion. Shear stresses in a real fluid in motion are possible due to a property called the ***viscosity*** of the fluid.

11.7 VISCOSITY

The property which characterizes the resistance that a fluid offers to applied shear forces is termed viscosity. The resistance depends on the rate of deformation.

Let us consider a fluid contained between two large parallel plates, separated by a distance L, as shown in Fig. 11.2. The lower plate is assumed to be stationary, while the upper one is moving parallel to it with a velocity V under the influence of the applied shearing force F.

The fluid particles sticking to the moving plate move with the same velocity V, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A}$$

where A is the area of contact between the plate and the fluid.

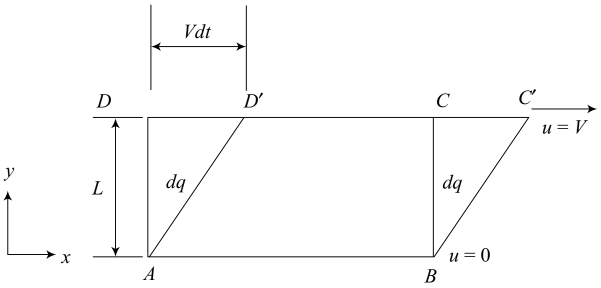


Figure 11.2

The fluid in contact with the lower plate assumes the velocity same as that of the plate, which is zero. If the gap separating the two plates is small or the velocity V is high, the velocity distribution will be a straight line. The equation of velocity can be written as

$$u(y) = \frac{y}{L}V$$

where y is the vertical distance measured from the lower plate.

The velocity gradient will be

$$\frac{\mathrm{d}u}{\mathrm{d}y} = \frac{V}{L}$$

During a differential time interval dt, the element of the fluid deforms through a differential angle dθ while the upper plate moves a differential distance dx.

$$\mathrm{d}x = V\mathrm{d}t$$

The angular deformation can be expressed as

$$\mathrm{d}\theta \approx \tan\theta = \frac{\mathrm{d}x}{L} = \frac{V\mathrm{d}t}{L} = \frac{\mathrm{d}u}{\mathrm{d}y}.\mathrm{d}t$$

The rate of angular deformation (or the shear strain) of a fluid element is equivalent to the velocity gradient $\frac{\mathrm{d}u}{\mathrm{d}y}$.

For a well-ordered flow whereby fluid particles move in straight, parallel lines (parallel flow), Newton's law of viscosity states that the for certain fluids, called Newtonian fluids, the shear stress (τ) on an interface tangent to the direction of flow is proportional to the distance rate of charge of velocity $\left(\frac{du}{dy}\right)$, wherein the differentiation is taken in a direction normal to the interface.

Mathematically, Newton's law of viscosity can be expressed as

$$\tau \propto \frac{\mathrm{d}u}{\mathrm{d}y}$$

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y} \tag{11.1}$$

where, the constant of proportionality μ is known as the ***viscosity coefficient*** or simply the viscosity which is the property of the fluid and depends on its state.

Common fluids, such as water, air, mercury, obey Newton's law of viscosity and are known as ***Newtonian fluids.*** Other fluids that do not obey Newton's law of viscosity are known as ***non-Newtonian fluids.***

A Newtonian fluid is one in which the shear stress is linearly proportional to the rate of shear deformation. The constant of proportionality is the viscosity, μ. Air would be considered a low-viscosity Newtonian fluid, while water would be a medium-viscosity Newtonian fluid. Motor oil and maple syrup are high-viscosity Newtonian fluids. Fluids that do not follow the Newtonian behaviour law include toothpaste, blood and paints. *Note that the equation 11.1 is applicable only for one–dimensional flow field and Newtonian fluid.*

11.7.1 Dimensional Formula and Units of Viscosity

The dimension of viscosity can be determined from Newton's law of viscosity (Eq. 11.1) as

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{FL^{-2}}{\frac{LT^{-1}}{L}} = FTL^{-2}$$

The dimensions of viscosity may also be expressed as $ML^{-1}T^{-1}$. The unit of viscosity if SI system is N-s/m^2, or kg/m-s.

The unit of dynamic viscosity in CGS system is Poise (P), named after Jean Louis Marie Poiseuille.

$$1 \text{ Poise} = 1 \ \frac{\text{g}}{\text{cm-s}} = \frac{10^{-3}\text{kg}}{10^{-2}\text{m-s}} = 0.1 \text{ kg/m-s}$$

$$1 \text{ centiPoise(cP)} = \frac{1}{100} \text{ poise} = 10^{-3} \text{ kg/m-s}$$

11.7.2 Variation of Viscosity with Temperature

The viscosity of a liquid decreases with temperature, but the viscosity of a gas increases with temperature (Fig. 11.3). The causes of viscosity are two—the intermolecular cohesive forces and the molecular momentum transfer. A liquid, with molecules much more closely spaced than a gas, has cohesive forces much larger than a gas. Although molecular momentum transfer exists, intermolecular cohesive forces predominate in the case of a liquid. Now, since intermolecular cohesive forces decrease with temperature, viscosity of liquids decreases with temperature. On the other hand, a gas has very small cohesive forces and molecular momentum transfer predominates. Since molecular momentum transfer increases with temperature, viscosity of gases also increases with temperature. According to kinetic theory of gases, viscosity of gases should be proportional to the square root of the absolute temperature. In practice, it increases more rapidly.

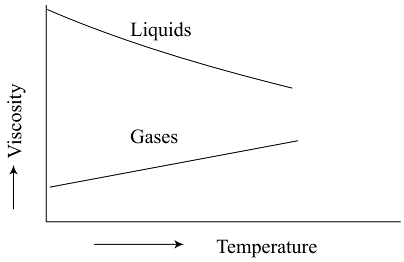


Figure 11.3 *Variation of viscosity of liquids and gases with temperature*

11.7.3 Newtonian and Non-Newtonian Fluids

Fluids for which the shear stress is proportional to the rate shear deformation are called Newtonian fluids after Sir Isacc Newton, who expressed it first in 1687. Newtonian fluids can be represented by a straight line. The slope of this line is determined by the viscosity.

The study of the response of materials to stress is called ***rheology***.

There are certain fluids where the linear relationship between the shear stress and the deformation rate (velocity gradient in parallel flow) is not valid. Because of the deviation from Newton's law of viscosity they are commonly termed non-Newtonian fluids.

Many common fluids exhibit non-Newtonian behaviour. Two familiar examples are toothpaste and paint. Toothpaste behaves as a fluid when squeezed from the tube. However, it does not run out by itself when the cap is removed. There is a yield stress below which toothpaste behaves as a solid. Paint is very thick when in the can, but becomes thin when sheared by brushing.

Non-Newtonian fluids are classified as having time-independent or time-dependent behaviour. Examples of time-independent behaviour are shown in the rheological diagram (Fig. 11.4 (a)).

The abscissa in the rheologigal diagram represents the behaviour of ideal fluids since for the ideal fluids the resistance to shearing deformation rate is always zero, and hence they exhibit zero shear stress under any condition of flow. The ordinate represents the behaviour of an ideal solid for any condition, since there is no deformation of an ideal solid for any load.

The Newtonian fluids behave according to Newton's law of viscosity that shear stress is linearly proportional to the velocity gradient for parallel flow. Thus, for Newtonian fluids, the plot of shear stress against velocity gradient is a straight line passing through the origin.

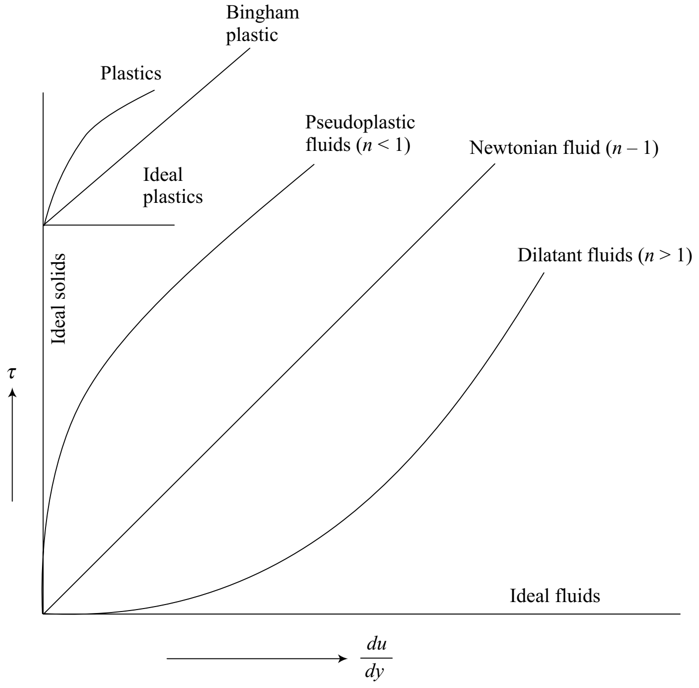


Figure 11.4 (a) *Rheological diagram*

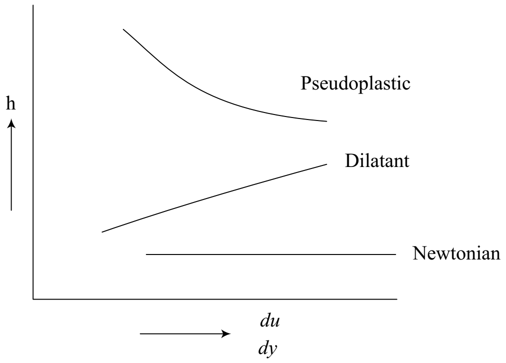


Figure 11.4 (b) *Rheological diagram*

Time-independent non-Newtonian fluids The mechanistic behaviour of a variety of non-Newtonian fluids can be described by the Power Law model.

According to Power Law model,

$$\tau = m\left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|^n = m\left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|^{n-1}\frac{\mathrm{d}u}{\mathrm{d}y} \tag{11.2}$$

where m is known as the flow consistency index and n is the flow behaviour index.

Again according to Newton's law of viscosity,

$$\tau = \mu\frac{\mathrm{d}u}{\mathrm{d}y}$$

Hence, viscosity for the Power Law fluid obeying the above model can be described as

$$\eta = m\left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|^{n-1}$$

It is readily observed that the viscosities of non-Newtonian fluids are functions of deformation rates and are often termed ***apparent*** or ***effective viscosity*** (η). Most non-Newtonian fluids have apparent viscosities that are relatively high compared with the viscosity of water.

When $n = 1$, η equals to $m(= \mu)$, the model identically satisfies Newtonian model as a special case.

Fluids in which the apparent viscosity decreases with increasing deformation rate ($n < 1$) are called pseudoplastic (or shear thinning) fluids. Most non-Newtonian fluids fall into this group. Examples of pseudoplastic fluids are blood, milk, gelatine, paper pulp, polymer solutions, colloidal suspensions, etc.

If the apparent viscosity increases with increasing deformation rate ($n > 1$) the fluid is termed dilatant (or shear thickening). Sugars in water, suspensions of starch and of sand, and butter are examples of ***dilatant fluids***.

For plastic the shear stress must reach a certain minimum value before flow commences. Thereafter, shear stress increases with the rate of shear according to the relationship

$$\tau = A + B\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^n$$

where A, B and n are constants.

A fluid that behaves as a solid until a minimum yield stress is exceeded and subsequently exhibits a linear relation between shear stress and rate of shear deformation is refered as a ***Bingham plastic*** after E.C. Bingham who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century. Toothpaste, clay suspensions, sewage sludge and drilling muds are examples of substances exhibiting this behaviour. The mathematical relationship between shear stress and rate of shear deformation is $A + B\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^n$ where $n = 1$.

Time-dependent non-Newtonian fluids The study of non-Newtonian fluids is further complicated by the fact that apparent viscosity may be time dependent.

Some fluids require a gradual increasing shear stress to maintain a constant strain rate and are called ***rheopectic*.** Otherwise, rheopectic fluids are those for which apparent viscosity increases with the time for which shearing forces are applied. Gypsum in water, whipping cream, bentonite clay solutions are examples of rheopectic fluids.

Thixotropic substances are those for which the apparent viscosity decreases with time (refer Fig. 11.5) for which the shearing forces are applied. Many paints, printer's ink, lipstick, enamels, crude oils are thixotropic.

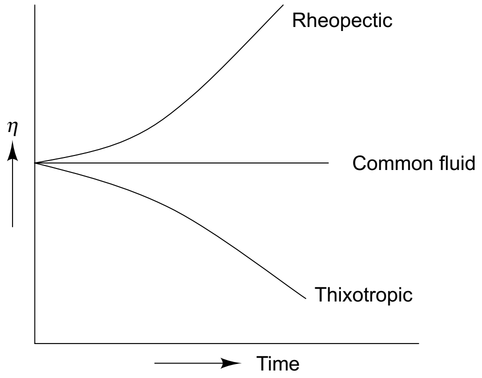


Figure 11.5 *Time-dependent non-Newtonian fluids*

11.8 KINEMATIC VISCOSITY

In fluid mechanics, the ratio of dynamic viscosity to density appears frequently. The ratio is given the name kinematic viscosity and is designated by Greek letter (nu)v. It is expressed as $\upsilon = \frac{\mu}{\rho}$.

The kinematic viscosity is considered as a kinematic quantity, since its unit does not contain any unit of mass. Physically, the kinematic viscosity represents the ratio of the ability to diffuse a disturbance in momentum relative to the ability of sustaining the original momentum.

The dimensional formula of kinematic viscosity is L^2T^{-1}. Kinematic viscosity has an SI unit of m^2-s^{-1}, and a CGS unit of cm^2-s^{-1}. The CGS unit is also known as Stokes, in honor of the famous mathematician Stokes.

$$1\ \textit{stoke} = 1\ \frac{\text{cm}^2}{\text{s}} = (10^{-2})^2\frac{\text{m}^2}{\text{s}} = 10^{-4}\ \text{m}^2/\text{s}$$

$$1\ \textit{centisoke} = \frac{1}{100}\ \textit{stoke} = 10^{-6}\ \text{m}^2/\text{s}$$

Viscosity is practically independent of pressure and depends upon temperature only. The kinematic viscosity of liquids and of gases at a given pressure is substantially a function of temperature.

Example 11.1 The specific gravity and the dynamic viscosity of a fluid are 13.6 and 0.002 N.s/m^2 respectively. Calculate its (i) density, and (ii) kinematic viscosity.

Solution

$$S_{\text{liquid}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}}$$

$$\rho\text{liquid} = \rho_{\text{water}} \times S_{\text{liquid}}$$

$$\rho_{\text{liquid}} = 1000 \times 13.6 = 13600\ \text{kg/m}^3$$

$$\text{Kinematic viscosity, } v = \frac{\mu}{\rho} = \frac{0.002}{13600} = 1.47 \times 10^{-7}\ \text{m}^2/\text{s}$$

Example 11.2 The velocity profile of a fluid over a plate is a parabola having a vertex of 10 cm from the boundary. The velocity at the vertex is 1.2 m/s. Calculate the velocity gradients for y = 0, 5 and 10 cm. Also calculate the shear stress at these points if the fluid viscosity is 0.004 Ns/m^2.

Solution Let the equation of velocity profile (parabolic) be

$$u = Ay^2 + By + C$$

where A, B and C are constants and their values are determined from boundary conditions as given by

(i) $u = 0$ at $y = 0$

(ii) $u = 1.2$ m/s at $y = 10$ cm

(iii) $\frac{\mathrm{d}u}{\mathrm{d}y} = 0$ at $y = 10$ cm

From the first boundary condition

$$0 = A \times 0 + B \times 0 + C$$

$$C = 0$$

From the second boundary condition

$$1.2 = A \times (0.1)^2 + B \times 0.1 + 0$$

$$1.2 = 0.01\,A + 0.1\,B \qquad (11.3)$$

From the third boundary condition

$$\frac{\mathrm{d}u}{\mathrm{d}y} = 2Ay + B$$

$$0 = 2A \times 0.1 + B \qquad (11.4)$$

After solving equations (11.3) and (11.4),

$$A = -\,120 \text{ and } B = 24$$

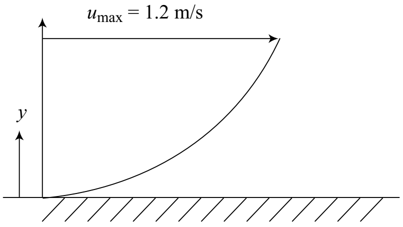


Figure 11.6

The velocity profile becomes

$$u = -\,120y^2 + 24y$$

$$\frac{\mathrm{d}u}{\mathrm{d}y} = -\,240\,y + 24$$

Velocity gradient, $\frac{\mathrm{d}u}{\mathrm{d}y} = -\,240\,y + 24$

At $y = 0$, velocity gradient $\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0} = -\,240 \times 0 + 24 = 24/\text{s}$.

At $y = 0.05$ m, velocity gradient $\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0.05\text{ m}} = -\,240 \times 0.05 + 24 = 12\ /\text{s}$.

At $y = 0.1$ m, velocity gradient $\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0.1} = -\,240 \times 0.1 + 24 = 0\ /\text{s}$.

Shear stress is given by $\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y}$

Shear stress at $y = 0$ m, $\tau = \mu \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0} = 0.004 \times 24 = 0.096\ \text{N/m}^2$

Shear stress at $y = 0.05$ m, $\tau = \mu \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0.05} = 0.004 \times 12 = 0.048\ \text{N/m}^2$

Shear stress at $y = 0.1$ m, $\tau = \mu \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=0.1} = 0.004 \times 0 = 0$

Example 11.3 The space between two parallel plates kept 5 mm apart is filled with a fluid with a dynamic viscosity of 1 Ns/m^2. The upper plate is moving with a velocity of 2 m/s. What is the shear stress on the lower plate, which is stationary?

Solution Since the gap between the plates is very small, we can assume that the velocity distribution is a linear one.

Velocity gradient, $$\frac{\mathrm{d}u}{\mathrm{d}y} = \frac{V}{h} = \frac{2}{5\times10^{-3}} = 400 \text{ per second}$$

Shear stress on the bottom plate, $$\tau = \mu\frac{\mathrm{d}u}{\mathrm{d}y} = 1\times400 = 400 \text{ N/m}^2$$

Example 11.4 A rectangular solid block of 1 m by 1 m that weighs 30 N slides down a 30° inclined plane as shown in Fig. 11.7. The plane is lubricated by a 5 mm thick film of oil of viscosity of 0.04 Ns/m^2. Calculate the terminal velocity of the block.

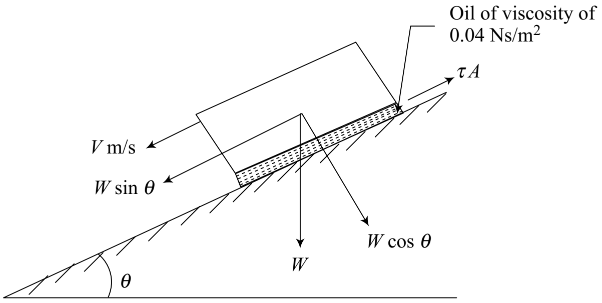


Figure 11.7

Solution

Thickness of the film $h = 5 \text{ mm} = 5\times10^{-3}\text{ m}$

Area of the block $A = 1\times1 = 1\text{m}^2$

Weight of the block $W = 30\text{ N}$

Viscosity of oil $\mu = 0.04 \text{ Ns/m}^2$

Component of weight along the slope is $W \sin \theta$.
Velocity gradient is found to be

$$\frac{du}{dy} = \frac{V-0}{h} = \frac{V}{h}$$

where h is the thickness of the oil film.
Viscous resistance F is given by

$$F = \text{shear stress} \times \text{area} = \tau A$$

or $$F = \mu \frac{du}{dy} A = \mu \frac{V}{h} A \qquad \left(\text{From Eq. (11.1), } \tau = \mu \frac{du}{dy} \right)$$

At the terminal condition, equilibrium occurs. Hence, the viscous resistance to the motion should be equal to the component of the weight of the solid block along the slope. Thus,

$$\mu \frac{V}{h} A = W \sin\theta$$

or $$0.04 \times \frac{V}{5 \times 10^{-3}} \times 1 = 30 \sin 30°$$

or $$V = 1.875 \text{ m/s}$$

Example 11.5 A liquid is filled in the annular space between two concentric cylinders 30 cm long. The inner cylinder of radius 10 cm rotates inside the outer cylinder which is stationary and has an internal radius of 10.05 cm. Determine the viscosity of the liquid if a torque of 10 N-m is required to maintain an angular velocity of 60 rpm.

Solution

Radius of the inner cylinder $R = 10 \text{ cm} = 0.1\text{m}$

Radius of the outer cylinder $R_o = 10.05 \text{ cm} = 0.1005 \text{ m}$

Length of the cylinder $L = 30 \text{ cm} = 0.3\text{m}$

Speed of inner cylinder $N = 60 \text{ rpm}$

Tangential velocity of the inner cylinder is

$$u = \frac{2\pi N}{60} \times R = \frac{2\pi \times 60}{60} \times 0.1 = 0.628 \text{ m/s}$$

Radial clearance between the cylinders is

$$h = \text{difference between radius of outer and inner cylinder}$$
$$= R_o - R = 0.1005 - 0.10 = 0.0005 \text{ m}$$

Surface area of the cylinder is

$$A = \text{circumference} \times \text{length of the pipe} = 2\pi RL$$
$$= 2\pi \times 0.1 \times 0.3 = 0.1885 \text{ m}^2$$

For the small space between the cylinders, the velocity profile may be assumed to be linear. Then, the velocity gradient is found to be

$$\frac{du}{dr} = \frac{u - 0}{h} = \frac{0.628}{0.0005} = 1256 \text{ per s}$$

Let μ be the viscosity of the liquid.

The shear stress at the wall is obtained from Eq. (11.1) as

$$\tau = \mu \frac{du}{dr} = 1256\mu$$

Shear force is given by

$$F = \text{shear stress} \times \text{area} = \tau A$$

$$= 1256\mu \times 0.1885 = 236.75\mu$$

Then, the resisting torque by the fluid is

$$T = \text{Force} \times \text{radius of inner cylinder} = F \times r$$

$$= 236.75\mu \times 0.1 = 23.675\mu$$

Applied torque should be same as the resisting torque by the fluid. Therefore, one can write

$$23.675\mu = 10$$

or $\mu = 0.42\text{N-s/m}^2$

Example 11.6 A 150 mm diameter shaft rotates at 1500 rpm in a 200 mm long journal bearing with 150.5 mm internal diameter. The uniform annular space between the shaft and the bearing is filled with oil of dynamic viscosity 0.8 poise. Calculate the power dissipated as heat.

Solution The arrangement is shown in Fig. 11.8.

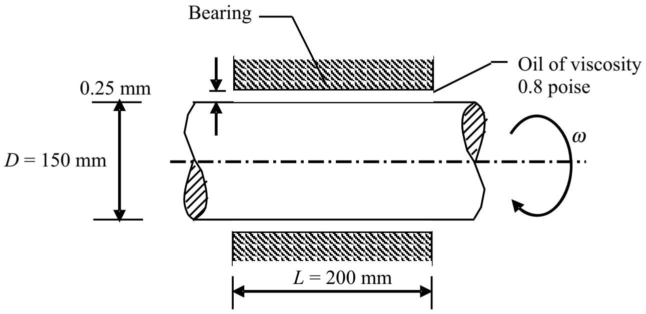


Figure 11.8

Diameter of the inner shaft $D = 150 \text{ mm} = 0.15 \text{ m}$

Radius of the shaft $R = \dfrac{0.150}{2} = 0.075 \text{ m}$

Radius of the bearing $R_o = \dfrac{150.5}{2} \text{mm} = \dfrac{0.1505}{2} \text{m} = 0.07525 \text{ m}$

Length of the bearing $L = 200 \text{ mm} = 0.2 \text{ m}$

Speed of shaft $N = 1500 \text{ rpm}$

Viscosity of oil $\mu = 0.8 \text{ poise} = 0.8 \times 10^{-1} \text{ N} - \text{s/m}^2$

Thickness of the oil film is

$$dy = \text{difference between radius of bearing and shaft}$$

$$= R_o - R = 0.07525 - 0.075 = 0.00025 \text{ m}$$

Surface area of the bearing is

$$A = \text{circumference of the shaft} \times \text{length of the bearing}$$

$$= \pi DL = \pi \times 0.15 \times 0.2 = 0.094 \text{ m}^2$$

Circumferential velocity of the shaft is

$$u = \text{angular velocity of the shaft} \times r\text{adius of the shaft}$$

$$= \frac{2\pi N}{60} \times r = \frac{2\pi \times 1500}{60} \times \frac{0.150}{2} = 11.78 \text{ m/s}$$

Velocity gradient is found to be

$$\frac{du}{dy} = \frac{11.78 - 0}{0.00025} = 47120 \text{ per s}$$

Shear stress τ is found from Newton's law of viscosity (Eq. (11.1)) as

$$\tau = \mu \frac{du}{dy} = 0.8 \times 10^{-1} \times 47120 = 3769.6 \text{ N/m}^2$$

Shear force on the shaft is then

$$F = \text{shear stress} \times \text{area} = \tau A$$

$$= 3769.6 \times 0.094 = 354.34 \text{ N}$$

Torque T on the shaft is then

$$T = \text{Force} \times \text{radius of inner cylinder} = F \times R$$

$$= 354.34 \times 0.075 = 26.58 \text{ N-m}$$

Power dissipated as heat P is found to be

$$P = \text{Torque}\times\text{angular velocity} = T \times \omega = T \times \frac{2\pi N}{60}$$

$$= 26.58 \times \frac{2 \times \pi \times 1500}{60}$$

$$= 4175.18 \text{ W} \cong 4.175 \text{ kW}$$

Example 11.7 A hydraulic ram 200 mm diameter and 1.2 m long moves within a concentric cylinder 200.4 mm diameter. The annular clearance is filled with oil of relative density 0.85 and kinematic viscosity 400 mm^2/s. What is the viscous force resisting the motion when the ram moves at a speed of 120 mm/s?

Solution

Kinematic viscosity of oil $\nu = 400 \text{ mm}^2/\text{s} = 400 \times 10^{-6} \text{ m}^2/\text{s}$

Relative density of oil $S_{oil} = 0.85$

Density of water $\rho_{water} = 1000 \text{ kg/m}^3$

Speed of ram $V = 120\text{mm/s} = 0.120\text{m/s}$

Length of ram $L = 1.2 \text{ m}$

Diameter of ram $D = 200 \text{ mm } = 0.2 \text{ m}$

Radius of ram $R = 0.1 \text{ m}$

Diameter of cylinder $D_o = 200.4 \text{ mm } = 0.2004 \text{ m}$

Radius of cylinder $R_o = \dfrac{0.2004}{2} \text{ m} = 0.1002 \text{ m}$

Radial clearance is given by $dy = R_o - R = 0.1002 - 0.1 = 0.0002 \text{ m}$

Velocity gradient is $\dfrac{du}{dy} = \dfrac{V-0}{dy} = \dfrac{0.120-0}{0.0002} = 600 \text{ per s}$

Density of the oil ρ is found as

$$\rho = \text{Specific gravity of the oil } \times \text{density of water}$$

$$= S_{oil}\rho_{water}$$

$$= 0.85 \times 1000 = 850 \text{ kg/m}^3$$

Dynamic viscosity of oil μ is found to be

$$\mu = \rho\nu$$

$$= 850 \times \left(400 \times 10^{-6}\right) = 0.34 \text{ N} - \text{s/m}^2$$

Shear stress is found from Newton's law of viscosity (Eq. (11.1)) as

$$\tau = \mu \frac{du}{dy} = 0.34 \times 600 = 204 \text{ N/m}^2$$

Shear area is

$$A = \text{circumference of the ram} \times \text{length of the ram}$$

$$= \pi DL = \pi \times 0.2 \times 1.2 = 0.754 \text{ m}^2$$

Viscous force F resisting the motion is

$$F = \text{shear stress} \times \text{area} = \tau A = 204 \times 0.754 = 153.82 \text{ N}$$

Example 11.8 The space between two large flat and parallel walls 25 mm apart is filled with a liquid of absolute viscosity 0.7 N-s/m^2. Within this space a thin flat plate is towed at a velocity of 0.15 m/s at a distance of 6 mm from one wall, the plate and its movement being parallel to the walls. Assuming linear variations of velocity between the plate and the walls, determine the force exerted by the liquid on the plate.

Solution

Area of the plate $A = 25\ \text{cm} \times 25\ \text{cm} = 625\ \text{cm}^2 = 0.0625\ \text{m}^2$

Viscosity of liquid $\mu = 0.7\ \text{N} - \text{s/m}^2$

Velocity of plate $V = 0.15\ \text{m/s}$

Let F_1 and F_2 be the shear forces on the upper surface and lower surface of the thin plate respectively. Let us also consider that the distance of the thin plate from the top wall is 6 mm as shown in Fig. 11.9.

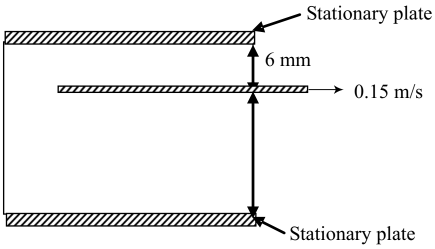


Figure 11.9

From Newton's law of viscosity (Eq. (11.1)), shear stress on the upper surface of the plate τ_1 is given by

$$\tau_1 = \mu \frac{du}{dy}$$

where dy = distance between top wall and thin plate = 6mm = 6×10^{-3} m

or $$\tau_1 = 0.7 \times \frac{0.15}{6 \times 10^{-3}} = 17.5 \text{N/m}^2$$

Shear force on the upper surface of the plate is

$$F_1 = \text{shear stress} \times \text{area} = \tau_1 A$$

$$= 17.5 \times 0.0625 = 1.094 \text{N}$$

From Newton's law of viscosity (Eq. (11.1)), shear stress on the lower surface of the plate τ_2 is given by

$$\tau_2 = \mu \frac{du}{dy}$$

where dy = distance between bottom wall and thin plate $= 25 - 6 = 19\text{mm} = 19 \times 10^{-3}\,\text{m}$

or $$\tau_2 = 0.7 \times \frac{0.15}{19 \times 10^{-3}} = 5.526\text{N/m}^2$$

Shear force on the bottom surface of the plate is

$$F_2 = \text{ shear stress} \times \text{area} = \tau_2 A$$
$$= 5.526 \times 0.0625 = 0.345\text{N}$$

Force exerted by the liquid on the plate is the sum of the forces on either side of the plate. Therefore, total force exerted by the liquid is

$$F = F_1 + F_2 = 1.094 + 0.345 = 1.439 \text{ N}$$

Example 11.9 A circular disc of radius R is kept at a small height h above a fixed bed by means of a layer of oil of dynamic viscosity, as shown in Fig. 11.10. If the disc is rotated at an angular velocity, ω, obtain an expression for the viscous torque on the disc. Assume linear variation of velocity within the oil film.

Solution Angular velocity of the disc = ω

Consider an element of disc at a radial distance r with width dr as shown in Fig.11.10. For linear variation of velocity with depth, the velocity gradient is given by

$$\frac{du}{dy} = \frac{u}{h} = \frac{\omega r}{h}$$

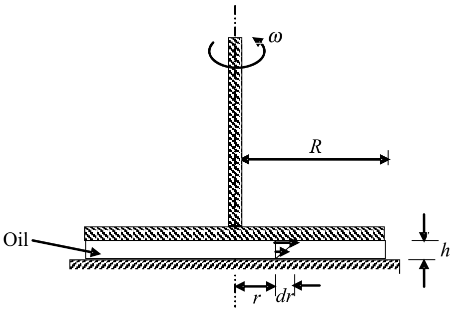


Figure 11.10

Elemental shear stress is then

$$\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{h}$$

Elemental shear force is given by

$$dF = \tau dA = \mu \frac{\omega r}{h} 2\pi r dr$$

Viscous torque acting on the element is

$$dT = dFr = \mu \frac{\omega r}{h} 2\pi r dr r = \mu \frac{\omega}{h} 2\pi r^3 dr$$

Total viscous torque on the disc is then

$$T = \int dT = \int_0^R \mu \frac{\omega}{h} 2\pi r^3 dr = \frac{2\pi\mu\omega}{h}\frac{R^4}{4} = \frac{\pi\mu\omega}{h}\frac{R^4}{2}$$

Example 11.10 A solid cone of radius *R*, and vertex angle 2θ is to rotate at an angular velocity, ω, as shown in Fig. 11.11. An oil of viscosity μ and thickness *h* fills the gap between the cone and the housing. Determine the torque *T* required to rotate the solid cone.

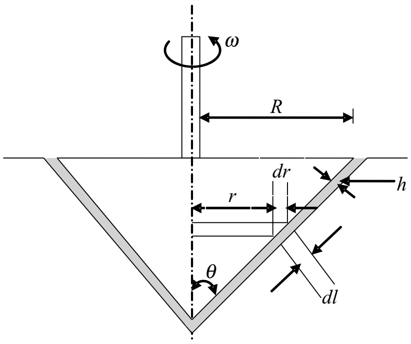


Figure 11.11

Solution

Consider an element at a radial distance *r* with width d*r* as shown in Fig. 11.11. Assuming linear variation of velocity, the velocity gradient is given by

$$\frac{du}{dy} = \frac{u}{h} = \frac{\omega r}{h}$$

From the geometry, we have

$$dl = \frac{dr}{\sin\theta}$$

Elemental area is then

$$dA = 2\pi r dl = 2\pi r \frac{dr}{\sin\theta}$$

Shear stress on the inclined wall is then

$$\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{h}$$

Elemental shear force is given by

$$dF = \tau dA = \mu \frac{\omega r}{h} 2\pi r \frac{dr}{\sin\theta}$$

Viscous torque acting on the element is

$$dT = dFr = \mu \frac{\omega r}{h} 2\pi r \frac{dr}{\sin\theta} r = \mu \frac{2\pi\omega}{h} \frac{1}{\sin\theta} r^3 dr$$

Viscous torque required to rotate the solid cone is found to be

$$T = \int dT = \int_0^R \mu \frac{2}{h} \frac{1}{\sin\theta} r^3 dr = \frac{\pi\omega\mu}{2h\sin\theta} R^4$$

11.9 NO-SLIP CONDITION

Consider the flow of fluid over a solid wall that is impervious (i.e., impermeable to the fluid). All experimental observations indicate that a fluid in motion comes to a complete stop at the wall and assumes zero velocity relative to the wall. That is, on a solid wall, the fluid can be assumed to stick to the wall and there is no slip along the solid wall the way a different solid might. This is known as the ***no-slip condition***. The physical reason for the no-slip condition is that the fluid molecules hitting the solid wall collide so frequently with the solid wall molecules that they have no average motion that is different from the wall molecules.

11.10 COMPRESSIBILITY

Compressibility of any substance is the measure of its change in volume under the action of external forces. The normal compressive stress of any fluid element at rest is known as hydrostatic pressure P and arises as a result of innumerable molecular collisions in the entire field. The degree of compressibility of a substance is characterized by the bulk modulus of elasticity E defined as

$$E = \lim_{\Delta V \to \Delta V^*} \frac{-\Delta P}{\frac{\Delta V}{V}}$$

where ΔV^* is the smallest elemental volume over which the continuum hypothesis is valid and ΔV and ΔP are the changes in the volume and pressure respectively, and V is the initial volume. The negative sign indicates that an increase in pressure is associated with a decrease in volume.

Values of E for liquids are very high compared with those of gases. Therefore, liquids are usually termed as incompressible. Density of water increases only 1 per cent if the pressure is increased by a factor of 220.

Since $\frac{\Delta V}{V}$ is a dimensionless ratio, the dimensions and units for E are the same as those for pressure i.e. FL^{-2} and N/m^2.

11.11 COHESION, ADHESION AND SURFACE TENSION

11.11.1 Cohesion

A definite amount of fluid mass is envisaged as an aggregation of several molecules of the fluid in close association with each other. Any particular molecule is attracted in all the directions by an equal amount of force exerted by the surrounding molecules. ***Cohesion is the property of the fluid by virtue of which liquid molecules are connected with each other so as to form a continuous mass***.

11.11.2 Adhesion

Adhesion is the property of the fluid by virtue of which liquid adheres another body that comes in its contact. For instance, if a rod is immersed in water and subsequently taken out, it is found that the rod becomes wet since the water molecules adhere to the rod. The same phenomenon occurs when a liquid is contained in a vessel and it is emptied.

It is noteworthy that some fluids may not exhibit adhesion. Example: mercury.

11.11.3 Surface Tension

A molecule (molecule A in Fig. 11.12) within the liquid and below the free surface is attracted equally in all directions by the other molecule surrounding it by virtue of cohesion. This ensures that such molecules remain in equilibrium. However, the same situation does not exist for the molecules in the free surface (molecules *B* and *C* in Fig. 11.12). This is due to the fact that such molecules although are attracted towards the inside mass by its neighboring molecules (lower molecules), but no liquid molecules is available on the upper side to counteract the pull force generated by the former molecules. This causes imbalances and thus molecules lying on the surface, experience a net downward pull force towards the interior of the liquid. An amount of energy/work is therefore required to bring the molecule to the free liquid surface which acts like an elastic membrane. This is called surface tension. It represents the surface energy per unit area, and is denoted by Greek letter σ (sigma). It has dimensions of FL^{-1} or MT^2 and units of N/m or kg/s^2 in SI units. Surface tension decreases slightly with increasing temperature.

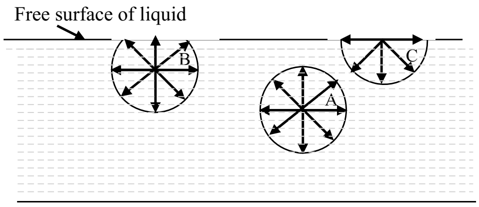


Figure 11.12

Surface tension on liquid droplet

Let the liquid droplet is cut into two halves. Half of the droplet is shown in Fig. 11.13 (a).

Tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 11.13 (a) is

$$= \text{Circumference} \times \text{Surface tension}$$

$$= \pi d \sigma$$

Pressure force acting on the half of the droplet (Fig. 11.13 (a)) is

$$= \text{Area} \times \text{pressure}$$

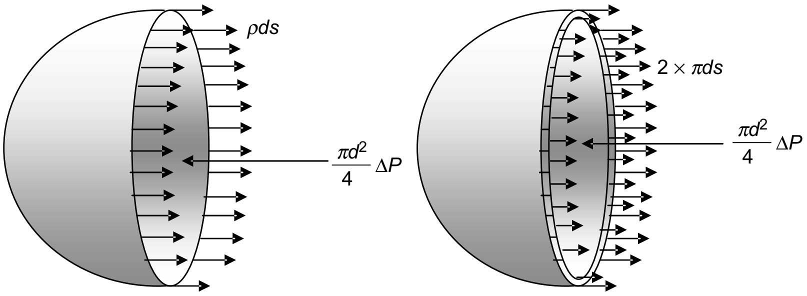
$$= \frac{\pi d^2}{4} \Delta P$$

The pressure force in the droplet is balanced by the surface tension force around the circumference. Hence, equating the above two forces, we have

$$\frac{\pi d^2}{4} \Delta P = \pi d \sigma$$

or,

$$\Delta P = \frac{4\sigma}{d} \tag{11.5}$$



(a) (b)

Figure 11.13 *Free body diagram of (a) half a droplet and (b) half a bubble*

Surface tension on a soap (or hollow) bubble

In a soap bubble, there are two interfaces as shown in Fig. 11.14 (b). Therefore, the tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 11.14 (b) is

$$= \text{Circumference} \times \text{Surface tension}$$

$$= \pi d \sigma$$

Pressure force acting on the half of the droplet (Fig. 11.14 (b)) is

$$= \text{Area} \times \text{pressure}$$

$$= \frac{\pi d^2}{4} \Delta P$$

The pressure force is balanced by the surface tension force on the two circumferences. Hence

$$\frac{\pi d^2}{4} \Delta P = 2 \times \pi d \sigma$$

or,
$$\Delta P = \frac{8\sigma}{d} \tag{11.6}$$

Surface tension on a liquid jet

Consider a liquid jet of diameter *d* and length *L*. Semi circular jet is shown in Fig. 11.15.

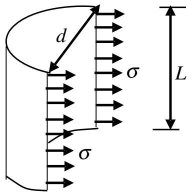


Figure 11.14 *Forces on liquid jet*

Force due to surface tension $= \sigma \times 2L = 2\sigma L$

Force due to pressure $= dL\Delta P$

Equating the above two forces, one can write

$$2\sigma L = dL\Delta P$$

or,
$$\Delta P = \frac{2\sigma}{d} \tag{11.7}$$

Example 11.11 If the pressure difference between the inside and outside of an air bubble of diameter 0.01 mm is 29.2 kPa, what will be the surface tension at the air-water interface?

Solution Pressure difference between the inside and outside of an air bubble

$$\Delta P = 29.2 \text{ kPa} = 29.2 \times 10^{3} \text{ N/m}^{2}$$

Diameter of the bubble $d = 0.01\text{mm} = 0.01 \times 10^{-3}$ m

The pressure difference between the inside and outside of an air bubble can be found from Eq. (11.5) as

$$\Delta P = \frac{4\sigma}{d}$$

$$\sigma = \frac{\Delta P d}{4} = \frac{29.2 \times 10^{3} \times 0.01 \times 10^{-3}}{4} = 0.073 \text{ N/m}$$

Example 11.12 Find the internal pressure in a soap bubble of 4 cm diameter, when the surface tension at the soap-air interface is 0.08 N/m.

Solution

Diameter of the soap bubble $d = 4\text{cm} = 4 \times 10^{-2}$ m

Surface tension at the soap-air interface $\sigma = 0.08$ N/m

Since for a soap bubble, there are two interfaces, the pressure difference is (Eq. (11.6))

$$\Delta p = \frac{8\sigma}{d}$$

$$= \frac{8 \times 0.08}{4 \times 10^{-2}} = 16 \text{ N/m}^{2} \text{ above atmospheric pressure}$$

Example 11.13 A circular jet of water 0.5 mm in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet. The surface tension at the water-air interface is 0.073 N/m.

Solution

Radius of the glass tube $r = 1 \text{ mm} = 0.001$ m

Diameter of the circular jet $d = 0.5 \text{ mm} = 0.5 \times 10^{-3}$ m

Surface tension at the water-air interface $\sigma = 0.073$ N/m

The pressure difference between the inside and outside of a circular jet can be found from Eq. (11.7) as

$$\Delta P = \frac{2\sigma}{d}$$

Substituting the values of σ and r, we have

$$\Delta P = \frac{2 \times 0.073}{0.5 \times 10^{-3}} = 292 \text{ N/m}^{2}$$

11.12 CAPILLARITY

When a glass tube of small diameter is dipped into a vessel or container that contains water, it is found that water rises in the tube to a level higher than that of the level of water in the container. Conversely, when mercury is used instead of water, the liquid level falls as compared to the level of the container. This phenomenon of rise or fall of a liquid surface in a small diameter tube relative to the adjacent general level of liquid when the tube is held vertically in liquid is called ***capillary*** or ***meniscus effect*** or ***capillarity***. Such narrow tubes are called ***capillaries***.

Rise of the liquid surface is called ***capillary rise (Fig. 11.14)*** and fall of the liquid surface is called ***capillary depression (Fig. 11.14)***.

Capillarity is due to both force of cohesion and adhesion. When the force of adhesion predominates, the liquid will wet a solid surface with which it is in contact and rise at the point of contact. If force of cohesion predominates the liquid surface will be depressed at the point of contact. The curved free surface of a liquid in a capillary tube is called the ***meniscus***. The strength of the capillary effect is quantified by the area wetting the contact angle, θ, which is defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact. A liquid is said to wet the surface when $\theta < 90°$ and not to wet the surface when $\theta > 90°$.

Weight of column of liquid h is found to be

$$= \text{mass of liquid} \times \text{acceleration due to gravity}$$

$$= \text{density of liquid } \times \text{volume of liquid having height } h \ \times g$$

$$= \rho \times \frac{\pi}{4} d^2 h \times g = \frac{\pi}{4} d^2 \rho g h$$

where d is the diameter of the capillary tube, ρ is the density of liquid and g is the acceleration due to gravity.

Vertical component of the surface tension force is

$$= (\text{Surface tension} \times \text{Circumference}) \cos\theta$$

$$= \sigma \pi d \cos\theta$$

where σ is the surface tension coefficient and θ is the area wetting contact angle.

For equilibrium, weight of column of liquid h should be equal to vertical component of the surface tension force. Thus, equating the above two forces, we get

$$\frac{\pi}{4} d^2 \rho g h = \sigma \pi d \cos\theta$$

or

$$h = \frac{4\sigma \cos\theta}{\rho g d} \tag{11.8}$$

Capillary rise is inversely proportional to the diameter of the tube. Therefore, thinner the tube is, the greater the rise or fall of the liquid in the tube. The capillary effect is usually negligible in tubes whose diameter is greater than 1 cm.

The density of the fluid should be high to avoid the capillary rise. In a tube of 5 mm diameter, the capillary rise of water will be approximately 4.5 mm, while for mercury the capillary depression will be 1.4 mm.

For pure water in contact with air in a clean glass tube, the capillary rise takes place $\theta = 0$. The value of θ may be different from zero in practice where cleanliness of a high order is seldom found. Mercury causes capillary depression with an angle of contact of about 130º in a clean glass in contact with air.

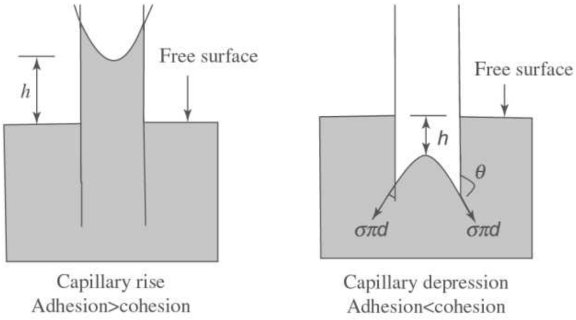


Figure 11.15 *Capillary effects*

Example 11.14 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in water. Take the surface tension of water in contact with air as 0.073 N/m and the area wetting contact angle as 0º. Density of water is 1000 kg/m^3.

Solution

Radius of the glass tube $r = 1 \text{ mm} = 0.001 \text{ m}$
Diameter of the glass tube $d = 2 \text{ mm} = 0.002 \text{ m}$
Surface tension of water in contact with air $\sigma = 0.073 \text{ N/m}$
Density of water $\rho = 1000 \text{ kg/m}^3$
Area wetting contact angle $\theta = 0°$
From Eq. (11.8), capillary rise is found to be

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$$= \frac{4 \times 0.073 \times \cos 0°}{1000 \times 9.81 \times 0.002} = 0.0149 \text{ m} = 1.49 \text{ cm}$$

Example 11.15 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in mercury. Take the surface tension of mercury in contact with air as 0.44 N/m and the area wetting contact angle as 130°. Density of mercury is 13600 kg/m^3.

Solution

Radius of the glass tube $r = 1$ mm $= 0.001$ m
Diameter of the glass tube $d = 2$ mm $= 0.002$ m
Surface tension of mercury in contact with air $\sigma = 0.44$ N/m
Density of mercury $\rho = 13600$ kg/m^3
Area wetting contact angle $\theta = 130°$
From Eq. (11.8), capillary rise is found to be

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

$$= \frac{4 \times 0.44 \times \cos 130°}{13600 \times 9.81 \times 0.002} = -0.0042 \text{ m} = -4.24 \text{ mm}$$

The minus sign indicates that there is a capillary depression.

SUMMARY

- A fluid is a substance that deforms continuously when subjected to a shear stress, however small the shear stress may be. A fluid may be either a liquid or a gas.
- A solid can resist a shear stress by a static deformation, a fluid cannot. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as shear stress is applied.
- The density of a fluid is mass per unit volume.
- The specific weight is the weight of fluid per unit volume.
- The specific volume of a fluid is the volume occupied by unit mass of fluid.
- Specific gravity or relative density is defined as the ratio of the density of some standard reference fluid at a specified temperature and pressure.
- An *ideal* fluid is a fully hypothetical fluid which is assumed to have no viscosity and no compressibility.
- In a *real* fluid, shear stresses occur whenever the fluid is in motion.
- The property which characterizes the resistance that a fluid offers to applied shear forces is termed *viscosity*.
- According to Newton's law of viscosity, the shear stress is proportional to the rate of shear strain.

  Mathematically, for one-dimensional flow, Newton's law of viscosity can be expressed as $\tau = \mu \frac{du}{dy}$

  where, the constant of proportionality μ is known as the viscosity coefficient or simply the viscosity which is the property of the fluid and depends on its state. Common fluids, such as water, air, mercury, obey Newton's law of viscosity and are known as Newtonian fluids. Other fluids that do not obey Newton's law of viscosity are known as non-Newtonian fluids.
- The viscosity of a liquid decreases with temperature, but the viscosity of a gas increases with temperature.
- The study of the response of materials to stress is called *rheology*.
- The ratio of dynamic viscosity to density is known as kinematic viscosity.

- Compressibility of any substance is the measure of its change in volume under the action of external forces. The degree of compressibility of a substance is characterized by the bulk modulus of elasticity E defined as

$$E = \lim_{\Delta V \to \Delta V^*} \frac{-\Delta P}{\frac{\Delta V}{V}}$$

- Cohesion is the property of the fluid by virtue of which liquid molecules are connected with each other so as to form a continuous mass.
- Adhesion is the property of the fluid by virtue of which a liquid adheres to another body that comes in its contact.
- When a glass tube of small diameter is dipped into a vessel or container that contains water, it is found that water rises in the tube to a level higher than that of the level of water in the container. Conversely, when mercury is used instead of water, the liquid level falls as compared to the level of the container. This phenomenon of rise or fall of a liquid surface in a small diameter tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid is called *capillary* or *meniscus effect* or *capillarity*. Such narrow tubes are called *capillaries*.Rise of the liquid surface is called *capillary rise* and fall of the liquid surface is called *capillary depression.*
- *Capillary rise* or *depression* is given by

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

where σ is the surface tension coefficient, θ is the area wetting contact angle and d is the diameter of tube.

REVIEW QUESTIONS

11.1 What is a fluid? How does it differ from a solid?

11.2 Differentiate between liquids and gases.

11.3 Define density, specific weight, specific volume and specific gravity.

11.4 State and explain Newton's law of viscosity.

11.5 What is the effect of temperature on viscosity of water and that of air?

11.6 Distinguish between ideal fluids and real fluids.

11.7 Why does the viscosity of a liquid decreases with increase in temperature while that of a gas increases with increase in temperature.

11.8 What is the difference between dynamic viscosity and kinematic viscosity?

11.9 Define Newtonian and non-Newtonian fluids.

11.10 Explain the no-slip condition of a viscous fluid.

11.11 Define compressibility. How is it related to bulk modulus of elasticity?

11.12 Discuss the shear characteristics of different fluids. Give at least one example of each type of fluid. What is the Ostwald-de Waele equation?

11.13 What do you mean by surface tension? What are the factors that affect surface tension?

11.14 What is bulk modulus of elasticity?

NUMERICAL PROBLEMS

11.1 The specific gravity and the kinematic viscosity of a fluid are 0.8 and 2×10^{-6} m^2/s respectively. Calculate its (i) density and (ii) dynamic viscosity.

11.2 A hot plate of area 0.125 m^2 is pulled at 0.25 m/s with respect to another parallel plate 1 mm distant from it the space between the plates containing water of viscosity 0.001 Ns/m^2. Find the force necessary to maintain this velocity and also the power required.

11.3 A 10 cm diameter shaft rotates at 1000 rpm in a 10 cm long journal bearing of 10.05 cm internal diameter. The annular space in the bearing is filled with oil having dynamic viscosity of 0.1 N.s/m^2. Estimate the power dissipated as heat.

11.4 Two large plane surfaces are 15 mm apart and the gap contains a liquid of viscosity 0.8 N-/m^2. Within the gap a thin plate of cross-sectional area 0.5 m^2 is to be pulled at a velocity of 0.5 m/s at a distance of 5 mm from one surface. Determine the force required for pulling the plate.

11.5 A piston of 20 cm diameter and 40 cm length works in a cylinder of 20.50 cm diameter. The annular space of the piston is filled with an oil of viscosity 0.5 N-s/m^2. If an axial load of 25 N is applied to the piston, calculate the speed of movement of the piston.

11.6 A vertical shaft has a hemispherical bottom of radius R which rotates inside a bearing of identical shape at its end. An oil film of thickness h and viscosity μ is maintained in the bearing. Estimate the viscous torque in the shaft when it rotates with an angular velocity ω.

11.7 A thin plate of very large area is placed in a gap of height h with oils of viscosities μ_1 and μ_2 on the two sides of the plate. The plate is pulled at a constant velocity V. Calculate the position of the plate so that

(i) the shear force on the two sides of the plate is equal

(ii) the force required to drag the plate is minimum

11.8 During the flow of a non-Newtonian fluid it is observed that the velocity distribution within the fluid film can be expressed by

$$\frac{u}{u_{\max}} = 2\left(\frac{y}{h}\right) - \frac{1}{2}\left(\frac{y}{h}\right)^3$$

where h is the film thickness and $u_{\max}$ the maximum velocity, y is measured from the solid surface. The viscosity of the fluid is 0.5 N-s/m^2 and $n = 1.3$. Calculate the shear stress at the solid surface when $u_{\max} = 0.4$ m/s and $h = 10$ mm. What should be the viscosity of a Newtonian fluid to induce the same shear stress value for similar velocity profile and the same maximum velocity?

11.9 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in mercury. Take the surface tension of mercury in contact with air as 0.44 N/m and the area wetting the contact angle as 130°.

11.10 A space of 2.5 cm wide between two large plane surface is filled with a liquid of absolute viscosity of 0.785 N-s/m^2. What force is required to drag a very thinplate 0.75 m^2 in area with a speed of 0.5 m/s if the plate remains equidistant fromthe two surfaces?

11.11 The space between two square flat parallel plates is filled with oil. Each side of the plate is 30 cm long the thickness of oil film is 14 mm. The upper plate whichmoves at 2.5 m/s requires a force of 120 N to maintain the speed. Determine (a) dynamic viscosity of oil, and (b) kinematic viscosity of oil, if the sp. gravity of oil is 0.8.

11.12 A thin plate is placed between two flat surfaces *h* apart such that the viscosity of liquids on the top and bottom of the plate are μ_1 and μ_2 respectively. Determinethe position of the plate such that the viscous resistance to uniform motion of theplate is minimum.

11.13 Calculate the capillary depression in a glass tube of 1 mm radius when immersed vertically in mercury. Take the surface tension of mercury in contact with air as 0.44 N/m and the area wetting contact angle as 130°.

11.14 Find the pressure difference between inside and outside of an air bubble of diameter 0.02 mm if the surface tension at air-water interface is 0.073 N/m.

MULTIPLE-CHOICE QUESTIONS

11.1 An ideal fluid is defined as a fluid which
(a) is incompressible
(b) is compressible
(c) is incompressible and non-viscous
(d) has negligible surface tension

11.2 Newton's law of viscosity states that
(a) shear stress is directly proportional to the velocity
(b) shear stress is directly proportional to the velocity gradient
(c) shear stress is directly proportional to shear strain
(d) shear stress is directly proportional to the viscosity

11.3 A Newtonian fluid is defined as a fluid which
(a) is incompressible and non-viscous
(b) obeys Newton's law of viscosity
(c) is highly viscous
(d) is compressible and non-viscous

11.4 Kinematic viscosity is defined as equal to
(a) dynamic viscosity × density
(b) dynamic viscosity / density
(c) dynamic viscosity × pressure
(d) pressure × density

11.5 Dynamic viscosity has the dimensions
(a) MLT^{-2}
(b) $ML^{-1}T^{-1}$
(c) $ML^{-1}T^{-2}$
(d) $M^{-1}L^{-1}T^{-1}$

11.6 Poise is the unit of
(a) density
(b) kinematic viscosity
(c) viscosity
(d) velocity gradient

11.7 The increase in temperature
(a) increases the viscosity of a liquid
(b) decreases the viscosity of a liquid
(c) increases the viscosity of a gas
(d) decreases the viscosity of a gas
(e) both (b) and (d)

11.8 Stoke is the unit of
(a) surface tension
(b) kinematic viscosity
(c) viscosity
(d) none of the above

11.9 Surface tension has the unit of
(a) force per unit area
(b) force per unit length
(c) force per unit volume
(d) none of the above

11.10 Fluid is a substance that
(a) cannot be subjected to shear stress
(b) always expands until it fills any container
(c) has the same shear stress at a point regardless of its motion
(d) cannot remain at rest under action of any shear stress

11.11 Practical fluids
(a) are viscous (b) possess surface tension
(c) are compressible (d) possess all the above properties

11.12 Property of a fluid by which its own molecules are attracted is called
(a) adhesion (b) cohesion (c) surface tension (d) viscosity

11.13 Property of a fluid by which molecules of different kinds of fluids are attracted is called
(a) adhesion (b) cohesion (c) surface tension (d) viscosity

11.14 The conditions of no-slip at rigid boundaries is applicable to
(a) flow of Newtonian fluids only (b) flow of ideal fluids only
(c) flow of all real fluids (d) flow of non-Newtonian fluids only

11.15 Typical example of a non-Newtonian fluid of pseudoplastic variety is
(a) water (b) air (c) blood (d) printing ink

11.16 The relationship between the shear stress τ and the rate of shear strain $\frac{\mathrm{d}u}{\mathrm{d}y}$ is expressed as $\tau = m\left[\frac{\mathrm{d}u}{\mathrm{d}y}\right]^n$. The fluid with the exponent $n < 1$ is known as
(a) Pseudoplastic fluid (b) Bingham fluid (c) Dilatant fluid (d) Newtonian fluid

11.17 Shear stress for a general fluid motion is represented by $\tau = \mu\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^n + A$, where n and A are constants. A Newtonian fluid is given by
(a) $n > 1$ and $A = 0$ (b) $n = 1$ and $A = 0$ (c) $n > 1$ and $A \neq 0$ (d) $n < 1$ and $A = 0$

11.18 A fluid which obeys the relation $\mu = \frac{\tau}{\frac{\mathrm{d}u}{\mathrm{d}y}}$
(a) real fluid (b) perfect fluid (c) Newtonian fluid (d) dilatant fluid

11.19 Newton's law of viscosity depends upon the
(a) viscosity and shear stress (b)stress and strain in a fluid
(c) shear stress and rate of strain (d) shear stress, pressure and velocity

11.20 The coefficient of viscosity is a property of the
(a) fluid (b) boundary condition
(c) flow velocity (d) body over which flow occurs

11.21 Paper pulp can be regarded as
(a) Dilatant (b) Newtonian fluids (c) Bingham plastic (d) Pseudoplastic fluids

11.22 Which of the following is the bulk modulus of elasticity K of fluid?
(a) $\rho\frac{\mathrm{d}P}{\mathrm{d}\rho}$ (b) $\frac{\mathrm{d}P}{\rho\mathrm{d}\rho}$ (c) $\frac{\mathrm{d}\rho}{\rho\mathrm{d}P}$ (d) $\frac{\rho\mathrm{d}\rho}{\mathrm{d}P}$

11.23 The dimension of surface tension is
(a) $FL^{-1}T^{-1}$ (b) FL^{-1} (c) FL^2T^{-1} (d) FLT^{-2}

11.24 The dimension of viscosity is
(a) L^2T^{-1} (b) $ML^{-1}T^{-1}$ (c) MT^{-2} (d) L^2T^{-1}

11.25 The unit of surface tension is

(a) J/m (b) J/m^2 (c) W/m (d) N/m^2

11.26 If p is the gauge pressure within a spherical droplet, the gauge pressure within a bubble of the same fluid and of same size will be

(a) $P/4$ (b) $P/2$ (c) $2P$ (d) P

11.27 Spherical shape of droplets of mercury is due to

(a) high density (b) high surface tension
(c) high adhesion (d) low vapour pressure

11.28 Which fluid does not experience shear sress during flow?

(a) Pseudoplastic (b) Dilatant (c) Inviscid (d) Newtonian

11.29 The bulk modulus of elasticity

(a) is independent of temperature
(b) increases with the pressure
(c) has the dimensions of $\frac{1}{\rho}$
(d) is larger when the fluid is more compressible
(e) is independent of pressure and viscosity

11.30 The bulk modulus of elasticity for a gas at a constant temperature T is given by

(a) $\frac{P}{\rho}$ (b) RT (c) $P\rho$ (d) ρRT
(e) none of these

11.31 The property of a fluid by virtue of which it offers resistance to shear strain is called

(a) surface tension (b) viscosity (c) adhesion (d) cohesion

CHAPTER

12

Fluid Statics

12.1 INTRODUCTION

Fluid statics deals with problems associated with fluids at rest. Fluid statics is generally referred to as ***hydrostatics*** when the fluid is a liquid and as ***aerostatics*** when the fluid is a gas. In fluid statics, there is no relative motion between adjacent fluid layers. That means, there is no shear stress acting on the fluid. The only stress acting on the fluid element is the normal stress, which is manifested in the form of pressure.

12.2 PRESSURE

A fluid will exert a force normal to a solid boundary or any plane drawn through the fluid. Consider a small area δA in a stationary fluid. Let δF be the force acting on the area δA in the normal direction. Mathematically, the pressure at a point in a stationary fluid is

$$P = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$$

If the force is uniformly distributed over the area A, then the pressure at any point is given by

$$P = \frac{F}{A}$$

12.3 PASCAL'S LAW FOR PRESSURE AT A POINT

Pascal's law stats that pressure (or intensity of pressure) at a point in a static fluid is equal in magnitude in all directions.

To demonstrate it, let a small wedge shaped fluid element in static condition be considered. Let us assume that P_1, P_2, and P_3 are the pressures acting on the three surfaces as shown in Fig. 12.1. Since static condition, the tangential force exerted by the surrounding fluid elements is zero, the forces acting on the fluid element are pressure forces on the surfaces and the gravity forces. Let us also assume that the gravity forces are acting along negative y direction.

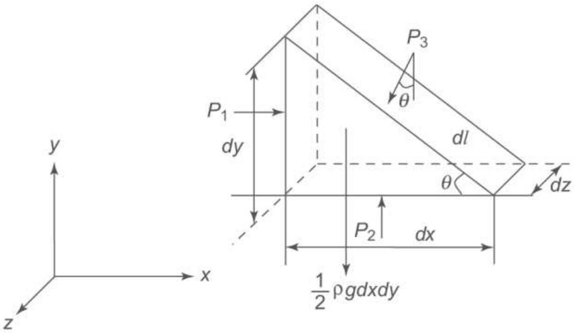


Figure 12.1 *Static equilibrium of a fluid element*

From Newton's second law, a force balance in the x and y direction gives:

$$\sum F_x = 0$$

$$P_1 dydz - P_3 dzdl\sin\theta = 0 \tag{12.1}$$

$$\sum F_y = 0$$

$$P_2 dxdz - P_3 dldz\cos\theta - \frac{1}{2}\rho gdl(dy\cos\theta)dz = 0 \tag{12.2}$$

where ρ is the density and $\frac{1}{2}\rho gdl(dy\cos\theta)dz$ is the weight of the fluid element. From the figure (right angle triangle) $dx = dl\cos\theta$ and $dy = dl\sin\theta$.

Substituting these on Eq. (12.1) and (12.2)

$$P_1 - P_3 = 0$$

or,

$$P_1 = P_3 \tag{12.3}$$

$$P_2 - P_3 - \frac{1}{2}\rho gdl\sin\theta = 0 \tag{12.4}$$

As $dl \rightarrow 0$ (the fluid element shrinks to a point), we have

$$P_2 = P_3 \tag{12.4a}$$

Therefore, from Eq. (12.3) and (12.4a) $P_1 = P_2 = P_3$ i.e., the pressure at a point in a fluid has the same magnitude in all possible directions.

12.4 BASIC EQUATION OF FLUID STATICS

Consider a differential fluid element at rest in rectangular Cartesian Co-ordinates with z axis vertically upward, as shown in Fig. 12.2. The forces acting on the fluid element at rest, are of surface forces and body forces. The only body force acting is the gravity force.

The body force due to gravity acting on the fluid element is $\rho g \Delta x \Delta y \Delta z$ which is acting vertically downward. Therefore, the body force acting along the z direction is $-\rho g \Delta x \Delta y \Delta z$.

Since the fluid element is at rest, the shear stress acting on the element will be zero. The only surface force is the pressure force.

Let the pressure at the center of the element be P. the surface forces acting on the different faces are shown in Fig. 12.2.

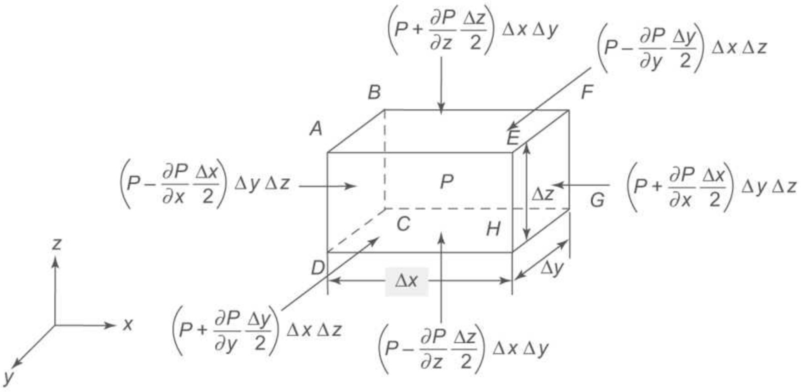


Figure 12.2 *Variation of fluid pressure*

The pressure at the bottom face (normal to z axis) of the element is $P-\frac{\partial P}{\partial z}\frac{\Delta z}{2}$

The pressure at the top face (normal to z axis) of the element is $P+\frac{\partial P}{\partial z}\frac{\Delta z}{2}$

The forces acting on faces normal to y direction are $\left(P-\frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta z$ and $\left(P+\frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta y$ respectively.

Similarly the surface forces acting on the left and right faces (normal to x direction) are $\left(P-\frac{\partial P}{\partial x}\frac{\Delta x}{2}\right)\Delta y\Delta z$ and $\left(P+\frac{\partial P}{\partial x}\frac{\Delta x}{2}\right)\Delta y\Delta z$ respectively

The forces acting on faces normal to y direction are $\left(P-\frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta z$ and $\left(P+\frac{\partial P}{\partial y}\frac{\Delta y}{2}\right)\Delta x\Delta z$ respectively.

Net forces acting on the element in the x direction is $\delta F_x = -\frac{\partial P}{\partial x}\Delta x\Delta\, y\Delta z.$

Similarly, net forces acting on the fluid element in the y and z direction are $\delta F_y = -\frac{\partial P}{\partial y}\Delta x\Delta y\Delta z$, and $\delta F_z = -\frac{\partial P}{\partial z}\Delta x\Delta y\Delta z - \rho g\Delta x\Delta y\Delta z$ respectively.

From Newton's second law, a force balance in the x, y and z directions gives:

$$\sum F_x = ma_x = 0$$

or,

$$\frac{\partial P}{\partial x} = 0 \tag{12.5}$$

$$\sum F_y = ma_y = 0$$

or,

$$\frac{\partial P}{\partial y} = 0 \tag{12.6}$$

$$\sum F_z = ma_z = 0$$

or,

$$\frac{\partial P}{\partial z} = -\rho g \tag{12.7}$$

It implies that any two points at the same elevation in the same continuous mass of fluid at rest have the same pressure. From Eq. (12.5) to (12.7), it can be concluded that the pressure P is a function of z only. Therefore, Eq. (12.7) can be written as

$$\frac{dp}{dz} = -\rho g \tag{12.8}$$

Equation (12.8) relates the change of pressure to change of elevation and is applicable to both compressible and incompressible fluids.

12.4.1 Pressure Variation in an Incompressible Fluid

For most problems involving liquids, it is usual to assume that the density ρ is constant, and the same assumption can also be made for a gas if pressure differences are very small. Equation (12.8) can be written as

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g$$

For an incompressible fluid, the density ρ does not change with change in pressure P.

If the pressure at the reference level, z_0, is designated as P_0 then the pressure P at the location z is found by integrating the Eq. (12.8),

$$\int_{p_0}^{p} \mathrm{d}P = \int_{z_0}^{z} -\rho g \mathrm{d}z$$

$$P - P_0 = -\rho g(z - z_0) = \rho g(z_0 - z)$$

For liquids, it is convenient to take the origin of the coordinate system at the reference level and to measure distances as positive downward from the reference level.

$$z_0 - z = h$$

$$P - P_0 = \rho g(z_0 - z) = \rho g h \tag{12.9}$$

Equation (12.9) indicates that the pressure difference between two points in a static fluid can be determined by measuring the elevation difference between the two points.

Example 12.1 Calculate the pressure at an altitude of 5 km above sea level if the atmospheric pressure and density at sea level are 101.3 kPa and 1.22 kg/m^3 respectively. Assume air as an incompressible fluid and neglect the variation of g with altitude.

Solution

Atmospheric pressure at the sea level $P_0 = 101.3 \text{ kPa} = 101.3 \times 10^3 \text{ N/m}^2$,

Density of air at the sea level $\rho_0 = 1.22 \text{ kg/m}^3$

Altitude from the sea level $z - z_0 = 5 \text{ km} = 5000 \text{ m}$

Pressure at any point can be found from Eq. (12.9) as

$$P = P_0 - \rho g(z - z_0)$$

Substituting the values in the above equation, we get

$$P = 101.3 \times 10^3 - 1.22 \times 9.81 \times 5000$$

$$= 41459 \text{ N/m}^2 = 41.459 \text{ kN/m}^2$$

Note: *Example 12.1 illustrates that pressure decreases with an altitude from the sea level.*

12.4.2 Pressure Variation in a Compressible Fluid

So far we have considered the variation of pressure for incompressible fluid for which density variation with change in pressure is not significant and is neglected. Now, we will consider the case of pressure variation in a compressible fluid for which density varies with pressure.

For isothermal fluid

For isothermal fluid, one can write

$$\frac{P}{\rho} = \frac{P_0}{\rho_0}$$

where P_0 and ρ_0 are the pressure and density at some arbitrary reference level z_0.

or, $$\rho = \frac{\rho_0}{P_0}P$$

From Eq. (12.8), we have

$$\frac{dP}{dz} = -\rho g$$

Substituting the value of ρ in the above equation, we get

$$\frac{dP}{dz} = -\frac{\rho_0}{P_0}Pg$$

or, $$\frac{dP}{P} = -\frac{\rho_0}{p_0}gdz$$

Integrating, we obtain

$$\int_{P_0}^{P}\frac{dP}{P} = \int_{z_0}^{z}-\frac{\rho_0}{P_0}gdz \qquad [P = P_0 \text{ at } z = z_0]$$

or, $$\ln\frac{P}{p_0} = \left[-\frac{\rho_0}{P_0}g(z - z_0)\right]$$

or, $$P = P_0 \exp\left[-\frac{\rho_0}{P_0}g(z - z_0)\right] \tag{12.10}$$

Equation (12.10) shows that the pressure decreases exponentially with the elevation (altitude) for an isothermal fluid.

Example 12.2 Calculate the pressure and density of air at an altitude of 10 km above sea level if the atmospheric pressure and density at sea level are 101.3 kPa and 1.22 kg/m^3 respectively. Assume isothermal process and neglect the variation of g with altitude.

Solution

Atmospheric pressure at the sea level $P_0 = 101.3$ kPa

Density of air at the sea level $\rho_0 = 1.22$ kg/m^3

Altitude from the sea level $z - z_0 = 10$ km $= 10000$ m

Pressure at any point can be found from Eq. (12.10) as

$$P = P_0 \exp\left[-\frac{\rho_0}{P_0} g(z - z_0)\right]$$

$$= 101.3 \times 10^3 \exp\left[-\frac{1.22}{101.3 \times 10^3} \times 9.81 \times 10000\right]$$

$$= 31.08 \times 10^3 \text{ Pa} = 31.08 \text{ kPa}$$

For isothermal process, we have

$$\frac{P}{\rho} = \frac{P_0}{\rho_0}$$

Substituting the values of p_0, ρ_0 and p, one can get

$$\frac{31.08}{\rho} = \frac{101.3}{1.22}$$

or $$\rho = \frac{31.08 \times 1.22}{101.3} = 0.3743 \text{ kg/m}^3$$

For non-isothermal fluid

From the characteristic equation of ideal gas, one can write

$$P = \rho RT$$

where R is the characteristic gas constant.

Substituting the value of ρ in Eq. (12.8), we obtain

$$\frac{dP}{dz} = -\rho g = -\frac{P}{RT} g \tag{12.11}$$

It has observed that up to a certain altitude temperature varies (decreases) linearly with elevation. The temperature variation can be expressed as

$$T = T_0 + \beta(z - z_0)$$

where β is the temperature lapse rate (negative), and T_0 is the temperature at $z = z_0$.

Substituting the value of T in Eq. (12.11), we have

$$\frac{dP}{dz} = -\frac{P}{R[T_0 + \beta(z - z_0)]} g$$

or, $$\frac{dP}{P} = -\frac{g}{R[T_0 + \beta(z - z_0)]} dz \tag{12.12}$$

Integration of Eq. (12.12) yields

$$\ln\frac{P}{P_0} = -\frac{g}{R\beta} \ln\frac{T_0 + \beta(z - z_0)}{T_0}$$

or,
$$\frac{P}{P_0} = \left(\frac{T_0 + \beta(z - z_0)}{T_0}\right)^{-\frac{g}{R\beta}} \tag{12.13}$$

or,
$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{-\frac{g}{R\beta}} \tag{12.14}$$

Equation (12.13) or (12.14) shows that the pressure decreases with elevation and depends on g, R and β.

Example 12.3 Calculate the atmospheric pressure at the end of troposphere, which extends up to a height of 9 km from sea level. Consider a temperature variation in the troposphere as $T = 288–0.006z$, where z is the elevation in metres and T is the temperature in Kelvin. The atmospheric pressure and temperature at sea level are 101.3 kN/m^2 and 288 K respectively. For air, R = 287 J/kg-K.

Solution

Altitude from the sea level	$z = 9\text{ km} = 9000\text{ m}$
Atmospheric pressure at the sea level	$p_0 = 101.3\text{ kPa}$
Atmospheric temperature at the sea level	$T_0 = 288\text{ K}$
Characteristic gas constant of air	$R = 287\text{ J/kg-K}$
Temperature variation	$T = 288 - 0.006z$
Temperature lapse rate	$\beta = -0.006$

Temperature at an altitude of 9 km from sea level is

$$T = 288 - 0.006 \times 9000 = 234\text{ K}$$

Pressure at any point can be found from Eq. (12.14) as

$$P = P_0\left(\frac{T}{T_0}\right)^{-\frac{g}{R\beta}}$$

$$P = 101.3\left(\frac{234}{288}\right)^{-\frac{9.81}{287\times(-0.006)}}$$

$$= 30.998\text{ kN/m}^2$$

The atmospheric pressure at the end of troposphere is 30.998 kN/m^2.

12.5 UNITS AND SCALE OF PRESSURE MEASUREMENT

Pressure may be expressed with reference to any arbitrary datum. The usual data are absolute zero and local atmospheric pressure. When a pressure is expressed as a difference between its value and a complete vacuum, it is called an *absolute pressure*. When it is expressed as a difference between its value and the local atmospheric pressure, it is called a *gauge pressure*.

Mathematically, one can write (refer Fig. 12.3)

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}} \tag{12.15}$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}} \tag{12.16}$$

At sea level and 15°C, the international standard atmosphere has been chosen as 1.01325 bar (= 10.34 m of water = 760 mm of mercury). In the SI system, the unit of pressure is N/m^2, also known as ***Pascal***. To express large magnitudes, bar is also used as a unit of pressure.

$$1 \text{ bar} = 10^5 \text{ Pascal}$$

The unit mm of Hg is also called *torr* in honour of Torricelli. Therefore,

$$1 \text{ atm} = 760 \text{ torr}$$

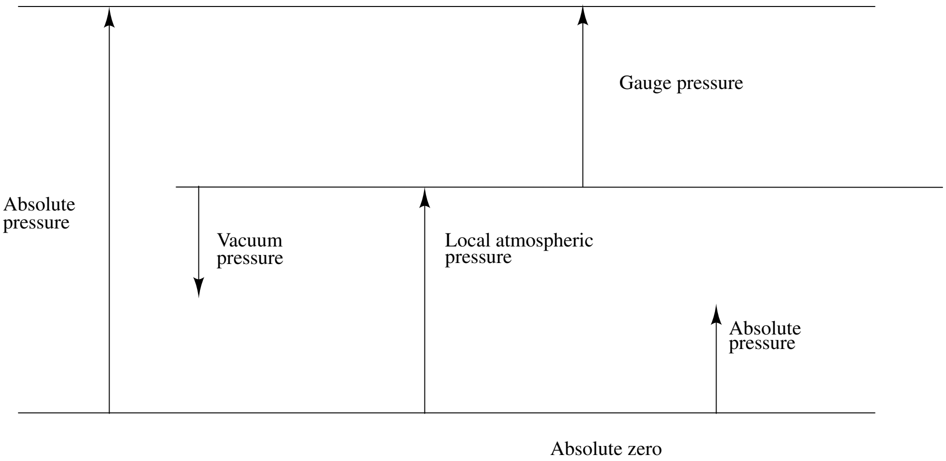


Figure 12.3 *Illustration of absolute, gauge and vacuum pressure readings*

Example 12.4 Calculate the pressure in N/m^2 corresponding to

(i) 6 m of water column

(ii) 10 cm of mercury column

(iii) 4 cm of column of a fluid of specific gravity 0.7

Solution

(i) h = 6 m of water column

$$P = \rho g h$$
$$= 1000 \times 9.81 \times 6 = 58860 \text{ N/m}^2$$

(ii) h = 10 cm of mercury column = 0.1 m of mercury column

$$P = \rho g h$$
$$= 13600 \times 9.81 \times 0.1 = 13341.6 \text{ N/m}^2$$

(iii) h = 4 cm of column of a fluid of specific gravity 0.7 = 0.4 m column of fluid
Density of fluid = 0.7 × 1000 kg/m^2

$$P = \rho g h$$
$$= 0.7 \times 1000 \times 9.81 \times 0.4 = 274.68 \text{ N/m}^2$$

12.6 MEASUREMENT OF PRESSURE

The relationship between pressure and the head is utilized for pressure measurement in the manometer. ***Manometers*** are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere. Manometers are extensively used in flow measurements.

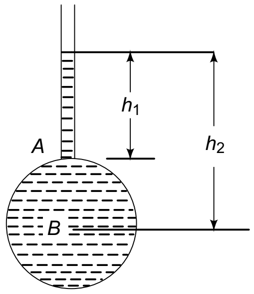


Figure 12.4 *Piezometer*

12.6.1 Piezometer

The most elementary manometer is called a piezometer. A piezometer is essentially a glass or plastic tube mounted vertically so that it is connected to the space within the container (refer Fig. 12.4). Liquid rises in the tube until equilibrium is reached. The pressure is given by the vertical distance from the meniscus (liquid surface) to the point where the pressure is to be measured. It is expressed in units of the length of liquid in the glass tube.

If the liquid is moving in the pipe or vessel, the bottom of the tube must be flush with the inside of the vessel, otherwise the reading will be affected by the velocity of the fluid. This instrument can only be used with liquids, and the height of the tube which can conveniently be employed limits the maximum pressure that can be measured.

If the top of the tube is open to atmosphere, the pressure measured is ***gauge pressure***.

Pressure at A = Pressure due to column of liquid of height h_1

$$P_A = \rho g h_1$$

Similarly,

Pressure at B, $$P_B = \rho g h_2$$

There are certain drawbacks of a piezometer:

1. It measures only the positive gauge pressure (*i.e.*, the pressure in the liquid is above atmospheric pressure). The piezometer would not work for negative gauge pressures, because air would flow into the container through the tube.
2. The use of a piezometer is also impractical for measuring large pressures, since for that the vertical tube would need to be very long.
3. If the working fluid is a gas, the usage of a piezometer is not possible, because gases do not have a free surface.

The above mentioned drawbacks of the piezometer can be overcome by modifying the tube and this modified tube is known as a manometer.

12.6.2 U-tube Manometer

For the measurement of small, negative or larger gauge pressures, some modifications in the tube of a piezometer are incorporated and this modified tube is known as a manometer. The U-tube manometer is very commonly used. The lower part of the U-tube contains a liquid which is immiscible with the working fluid. This fluid is called the ***manometric fluid***.

Desirable properties of manometric fluid

(a) The fluid should have a low vapour pressure.
(b) It should have a defined meniscus at the interface for good readability.
(c) It should have low surface tension to avoid capillary rise.
(d) The fluid should be immiscible with the working fluid.

The choice of a measuring fluid is guided by the range of pressure to be measured; higher the range, heavier the fluid.

Mercury is widely used as a manometric fluid because it has properties like

(a) low vapour pressure (≈ 0.17 N/m^2 at 20 °C) and thus for all intents and purposes it can be neglected in comparison with atmospheric pressure
(b) high density

Now, consider the two cases one by one.

Case 1 Measurement of large gauge pressure

One end of the U-tube is connected to the pipe or the container whose pressure is to be measured. The other end of the tube is open to atmosphere. The level of the manometric fluid on the left limb will fall and on the right limb, it will rise (refer Fig. 12.5).

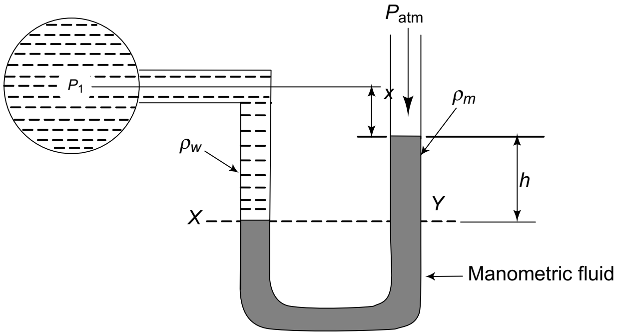


Figure 12.5 *U-tube manometer to measure gauge pressure*

Applying the fundamental law of fluid statics, the pressures at two points, X and Y, at the same elevation in the same continuous mass of fluid at rest must be equal.

For the left hand side,

$$P_X = P_1 + \rho_w g(x + h)$$

For the right hand side,

$$P_Y = P_{atm} + \rho_m gh$$

Since

$$p_X = p_Y$$

$$P_1 + \rho_w g(x + h) = P_{atm} + \rho_m gh$$

$$P_1 - P_{atm} = (\rho_m - \rho_w)gh - \rho_m gx \tag{12.17}$$

where P_1 is the absolute pressure of the fluid in the pipe or the container, and P_{atm} is the local atmospheric pressure, ρ_m is the density of the manometric fluid and ρ_w is the working fluid.

Case 2 Measurement of negative gauge pressure

The level of the manometric fluid on the left limb will rise and on the right limb will fall (Fig. 12.6). After attaining the equilibrium, if we apply the fundamental law of fluid statics, the pressures at two points, *X* and *Y*, at the same elevation in the same continuous mass of fluid at rest must be equal.

Equating the pressure at the level *XY* (pressure at the same level in a continuous body of static fluid is equal),

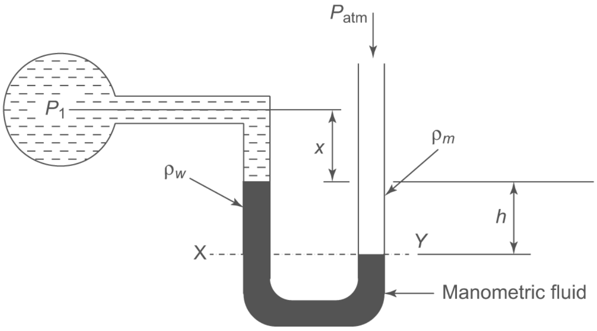


Figure 12.6 *U-tube manometer to measure vacuum pressure*

For the left hand side,

$$P_x = P_1 + \rho_w gx + \rho_m gh$$

For the right hand side,

$$P_Y = P_{\text{atm}}$$

Since $$P_X = P_Y$$

$$P_1 + \rho_w gx + \rho_m gh = P_{\text{atm}}$$

$$P_{\text{atm}} - P_1 = \rho_w gx + \rho_m gh \qquad (12.18)$$

12.6.3 U-tube Differential Manometer

To measure the pressure difference between two points in the flow field, the manometer which is frequently used, is called the differential U-tube manometer.

A differential U-tube manometer is very handy to measure the pressure difference directly and is basically similar to the U-tube manometer discussed above. What the open end was before is now connected to a different pressure P_2 in the flow field as shown in Fig. 12.7.

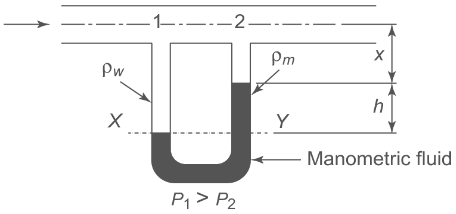


Figure 12.7 *U-tube differential manometer*

$$P_X = P_1 + \rho_w g(x + h)$$

$$P_Y = P_2 + \rho_w gx + \rho_m gh$$

Equating the pressure at the level *XY* (pressure at the same level in a continuous body of static fluid is equal),

$$P_X = P_Y$$

$$P_1 + \rho_w g(x + h) = P_2 + \rho_w gx + \rho_m gh$$

$$P_1 - P_2 = (\rho_m - \rho_w)gh \tag{12.19}$$

In forming the connection from a manometer to a pipe or vessel in which a fluid is flowing, care must be taken to ensure that the connection is perpendicular to the wall and flush internally. Any burr or protrusion on the inside of the wall will disturb the flow and cause a local change in pressure so that the manometer reading will not be correct.

Example 12.5 Determine the pressure difference between points *A* and *B*, as shown in Fig. 12.8.

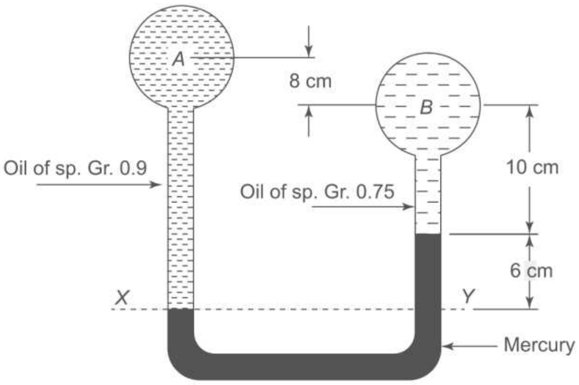


Figure 12.8

Solution Equating the pressures of both the limb along the horizontal plane *XY*

$$P_A + (0.06 + 0.1 + 0.08) \times 0.9 \times 1000 \times 9.81 = P_B + 0.06 \times 13600 \times 9.81 \times 0.1 \times 0.75 \times 1000 \times 9.81$$

$$P_A - P_B = 4502.79 \text{ Pa} = 4.503 \text{ kPa}$$

Example 12.6 A differential U-tube mercury manometer is used to measure the pressure difference between points 1 and 2 in a pipeline conveying water. The point 1 is 0.5 m lower than the point 2. The difference in the level of manometric fluid on the two limbs is 0.8 m. Calculate the pressure difference between points 1 and 2.

Solution The problem is shown in Fig. 12.9.

Here,

$$z_2 - z_1 = 0.5 \text{ m}$$

$$h = 0.8 \text{ m}$$

$$P_X = P_1 + \rho_w g(z_1 - z)$$

$$P_Y = P_2 + \rho_w g(z_2 - z - h)x + \rho_m gh$$

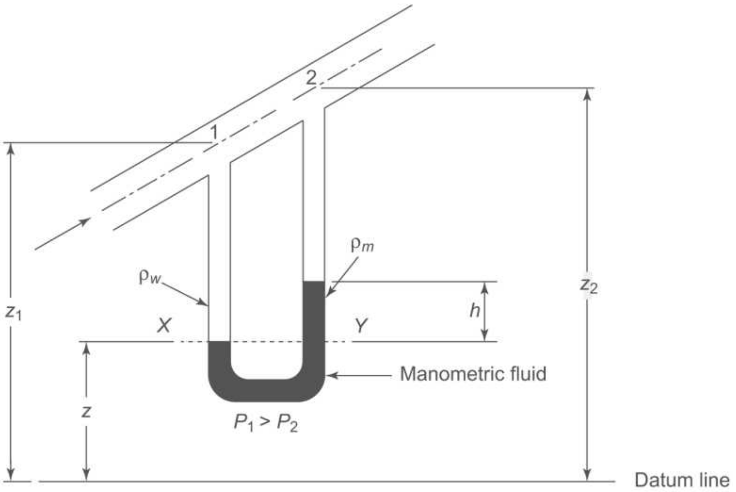


Figure 12.9

Equating the pressure at the level XY (pressure at the same level in a continuous body of static fluid is equal),

$$P_X = P_Y$$

$$P_1 + \rho_w g(z_1 - z) = P_2 + \rho_w g(z_2 - z - h)x + \rho_m gh$$

$$P_1 - P_2 = (\rho_m - \rho_w)gh + \rho_w g(z_2 - z_1)$$

$$P_1 - P_2 = (13600 - 1000)9.81 \times 0.8 + 1000 \times 9.81 \times 0.5$$

$$= 103789.8 \text{ N/m}^2 = 103.79 \text{ kN/m}^2$$

12.6.4 U-Tube Manometer with One Leg Enlarged

The disadvantage of the simple U-tube manometer is that movement of the liquid in both limbs must be read. By making the diameter of one leg very large as compared with the other, it is possible to make the movement in the large leg very small, so that it is only necessary to read the movement of the liquid in the narrow leg. Let *M–N* be the level of the liquid surface, when the pressure difference is zero and the working fluid is a gas.

Then when pressure is applied ($P_1 > P_2$), the level in the right-hand limb will rise a distance h vertically as shown in Fig. 12.10.

Volume of liquid transferred from left-hand leg to right-hand leg $V = \frac{\pi}{4}\mathrm{d}^2 h$.

∴ Fall in level of the left-hand leg

$$= \frac{\text{Volume transferred}(V)}{\text{Area of left hand leg}(A)} = \frac{\frac{\pi}{4}\mathrm{d}^2 h}{\frac{\pi}{4}D^2} = \left(\frac{\mathrm{d}}{D}\right)^2 h.$$

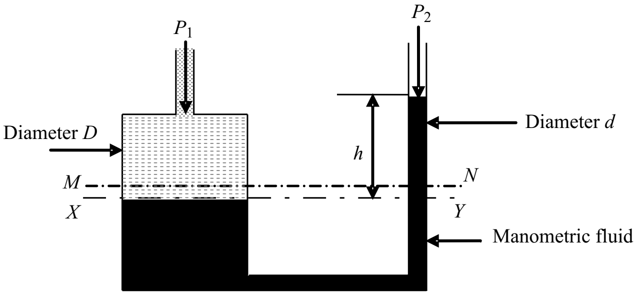


Figure 12.10 *U-tube with one leg enlarged*

The pressure difference, $P_1 - P_2$, is represented by the height of the manometric liquid corresponding to the new difference of level.

$$\therefore \qquad P_1 - P_2 = \rho_m g\left[h + \left(\frac{\mathrm{d}}{D}\right)^2 h\right] = \rho_m gh\left[1 + \left(\frac{\mathrm{d}}{D}\right)^2\right]$$

If D>>d then neglecting $(\mathrm{d}/D)^2$, the above equation yields to

$$P_1 - P_2 = \rho_m gh \tag{12.20}$$

12.6.5 Inclined Tube Manometer

If the pressure difference to be measured is small, the leg of the U tube may be inclined. The movement of the meniscus along the inclined leg, read off on the scale, is considerably greater than the change in level h as shown in Fig. 12.11.

∴ pressure difference,

$$P_1 - P_2 = \rho gh = pgs \sin\theta \tag{12.21}$$

The manometer can be made as sensitive as may be required by adjusting the angle of inclination of the leg and selecting a liquid with an appropriate value of density ρ to give a scale reading s of the desired size for a given pressure difference.

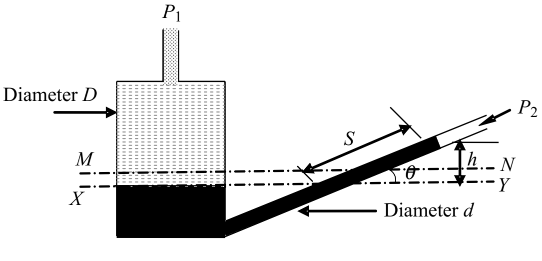


Figure 12.11 *Inclined manometer*

12.6.6 Inverted U-tube Manometer

The inverted U-tube manometer is used for measuring pressure differences in liquids. The top of the U-tube is filled with a fluid which has a density less than that of the working fluid as depicted in Fig. 12.12. For an inverted U-tube manometer, the manometric fluid is usually air. Air can be admitted or expelled through the tap on the top, in order to adjust the level of the liquid in the manometer.

If the density of manometric fluid in the top of the tube is very close to that of the working fluid then the result will be very sensitive giving a large value of *h* for a small pressure difference.

$$P_X = P_1 - \rho_w g(x + h)$$

$$P_Y = P_2 - \rho_w gx - \rho_m gh$$

Equating the pressure at the level *XY* (pressure at the same level in a continuous body of static fluid is equal),

$$P_X = P_Y$$

$$P_1 - \rho_w g(x + h) = P_2 - \rho_w gx - \rho_m gh$$

$$P_1 - P_2 = (\rho_w - \rho_m)gh \tag{12.22}$$

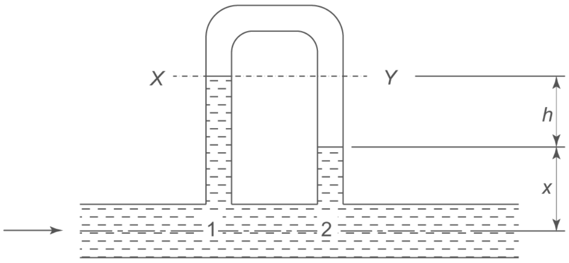


Figure 12.12 *Inverted U-tube manometer*

A major disadvantage of the manometer is its slow response, which makes it unsuitable for measuring fluctuating pressures. Even under comparatively static conditions, slight fluctuations of pressure can make the liquid in the manometer oscillate, so that it is difficult to get a precise reading of the levels of the liquid in the gauge. These oscillations can be reduced by putting restrictions in the manometer connections. It is also essential that the pipes connecting the manometer to the pipe or vessel containing the liquid under pressure should be filled with this liquid and that there should be no air bubbles in the liquid.

Example 12.7 Determine the pressure difference between points *A* and *B* for the inverted U-tube manometer as shown in Fig. 12.13.

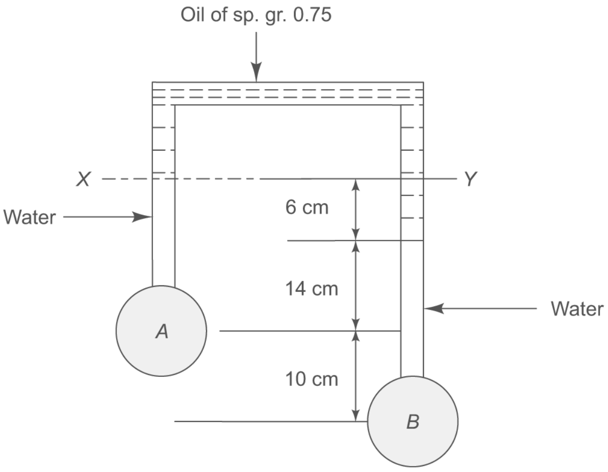


Figure 12.13

Solution Equating the pressures of both the limbs along the horizontal plane *XY*

$P_A - 1000 \times 9.81(0.14 + 0.06) = P_B - 1000 \times 9.81 \times (0.1 + 0.14) - 0.06 \times 0.8 \times 1000 \times 9.81$

$P_A - P_B = -863.28 \text{ Pa} = -0.863 \text{ kPa}$

Pressure at *B* is higher than at *A* by 0.863 kPa.

Example 12.8 Determine the pressure difference between the pipes *A* and *B* for the inverted U-tube manometer as shown in Fig. 12.14.

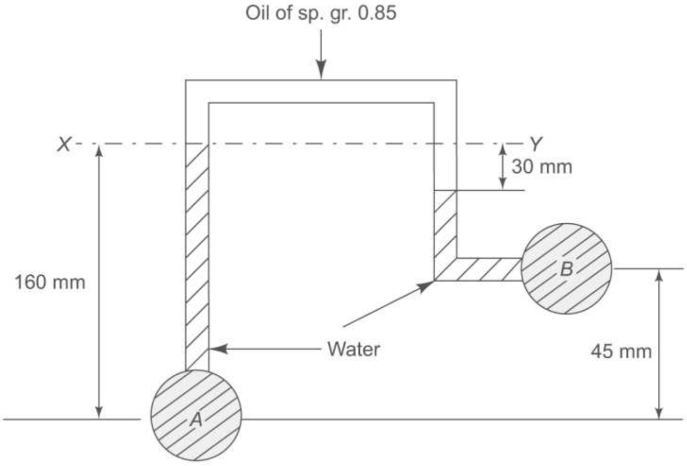


Figure 12.14

Solution Equating the pressures of both the limb along the horizontal plane *XY*

$P_A - 1000 \times 9.81 \times 0.16 = P_B - 1000 \times 9.81 \times (0.16 - 0.03 - 0.045) - 0.85 \times 1000 \times 0.03 \times 9.81$

$P_A - P_B = 485.595$ Pa

Pressure at A is higher than at B by 485.595 Pa.

Example 12.9 Two pipes A and B are in the same elevation. Water is contained in A and rises to a level of 2 m above it. Pipe B contains an oil of sp. gr. 1.7. The inverted U-tube is filled with compressed air at 350 kN/m^2 and 20 °C. Determine

(i) the pressure difference between A and B and

(ii) the absolute pressure in B

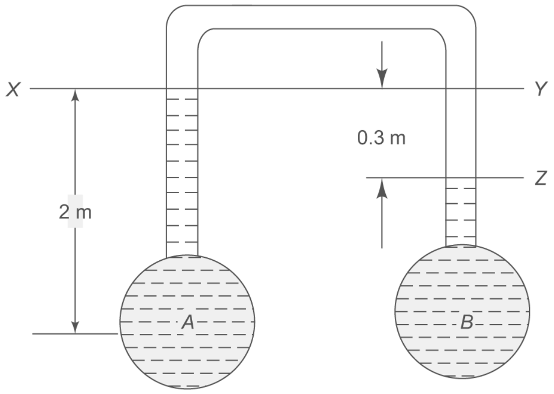


Figure 12.15

Solution Pressure at Y is the same as that at Z.

Equating the pressures of both the limbs along the horizontal plane XY,

$$P_A - 1000 \times 9.81 \times 2 = P_B - 1.7 \times 1000 \times 9.81 \times (2 - 0.3)$$

$$P_A - P_B = 873.09 \text{ N/m}^2$$

$$P_A - P_B = 0.87 \text{ kN/m}^2$$

Pressure at B,

$$P_B = 350 \text{ kN/m}^2 + \text{presure of } (2 - 0.3) \text{ m of oil column}$$

$$P_B = 350 \times 10^3 + 1000 \times 1.7 \times 9.81 \times (2 - 0.3) \text{ N/m}^2$$

$$P_B = 378350.9 \text{ N/m}^2$$

$$P_B = 378.35 \text{ kN/m}^2$$

12.6.7 Micro Manometers

A micro manometer is used for measuring small differences of pressures in the order of 0.001 mm of Hg. It utilizes two liquids which are immiscible with each other. Let ρ_m and ρ_g be the density of the two manometric fluids (let $\rho_m > \rho_g$) the denser liquid will fill the bottom of the U-tube.

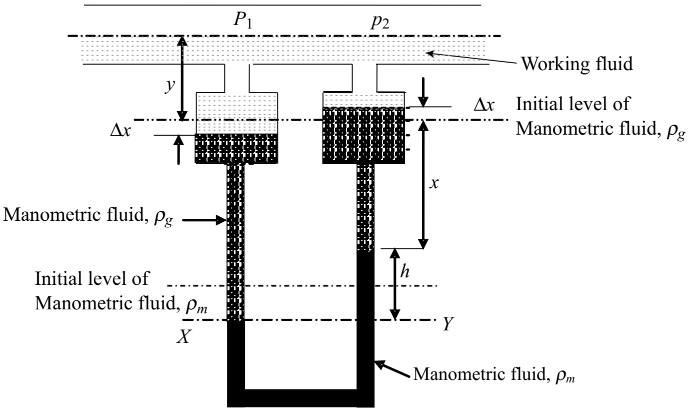


Figure 12.16 *Micro manometer*

Let A = area of each enlarged end

a = area of the tube

Equating the pressure at the level XY (pressure at the same level in a continuous body of static fluid is equal),

$$P_X = P_Y$$

$$P_1 + \rho_w g(y + \Delta x) + \rho_g g(x + h - \Delta x) = P_2 + \rho_w g(y - \Delta x) + \rho_g g(x + \Delta x) + \rho_m gh \qquad (12.23)$$

The volume of the manometric liquid of density ρ_g displaced in the enlarged section equals to the displacement in the U-tube.

$$A\Delta x = a\frac{h}{2}$$

$$\Delta x = \frac{a}{A}\frac{h}{2}$$

Substituting the value of Δx in the Eq. (12.23), we have

$$P_1 + \rho_w g\left(y + \frac{a}{A}\frac{h}{2}\right) + \rho_g g\left(x + h - \frac{a}{A}\frac{h}{2}\right) = P_2 + \rho_w g\left(y - \frac{a}{A}\frac{h}{2}\right) + \rho_g g\left(x + \frac{a}{A}\frac{h}{2}\right) + \rho_m gh$$

$$P_1 - P_2 = -\rho_w g\frac{a}{A}h - \rho_g g\left(h - \frac{a}{A}h\right) + \rho_m gh \qquad (12.24)$$

If D>>d then neglecting (a/A), above equation yields to

$$P_1 - P_2 = \rho_m gh - \rho_g gh = (\rho_m - \rho_g)gh \qquad (12.25)$$

If the densities of the two manometric fluids are close to each other then for a small pressure difference we can achieve a reasonable value of h.

12.6.8 Barometer

The barometer is a special manometer used for measuring atmospheric air pressure. Mercury is used as the manometric fluid. The tube is evacuated of all gas so that no atmospheric pressure acts on the top of the mercury column. Because atmospheric pressure acts on the bottom of the mercury, the height to which the mercury column is lifted represents atmospheric pressure.

The pressure at A is the local atmospheric pressure and is expressed by the following equation:

$$P_{\text{atm}} - P_v = \rho g h \tag{12.26}$$

where P_v is the vapour pressure of mercury. Since mercury has a low vapour pressure ($\approx$ 0.17 N/m^2 at 20 °C) it can be neglected in comparison with atmospheric pressure for all intents and purposes.

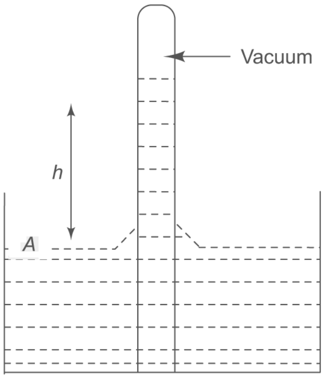


Figure 12.17 *Barometer*

SUMMARY

- Pascal's law states that pressure (or intensity of pressure) at a point in a static fluid is equal in magnitude in all directions.
- Two points at the same elevation in the same continuous mass of fluid at rest have the same pressure.

$$\frac{dp}{dz} = -\rho g$$

- When a pressure is expressed as a difference between its value and a complete vacuum, it is called an absolute pressure. When it is expressed as a difference between its value and the local atmospheric pressure, it is called a gauge pressure.

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}}$$

- Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere.
- A simple U-tube manometer is used to measure pressure at a point.
- A differential manometer is used to measure difference of pressure between two points.
- Micro manometers are used for measuring small differences of pressures in the order of 0.001 mm of Hg.
- The barometer is a special manometer used for measuring atmospheric air pressure.

REVIEW QUESTIONS

12.1 State and prove Pascal's law of hydrostatics.

12.2 What is a manometer? How are manometers classified?

12.3 Differentiate between the following:

  (i) Absolute pressure and gauge pressure

  (ii) Piezometer and simple manometer

  (iii) Simple manometer and differential manometer

  (iv) U-tube differential manometer and inverted U-tube differential manometer

12.4 Explain how the static pressure is isotropic at any horizontal cross section in a fluid.

12.5 State the hydrostatic law and derive the expression for the same.

12.6 Prove that pressure varies exponentially with elevation for isothermal condition.

NUMERICAL PROBLEMS

12.1 Calculate the pressure and density of air at an altitude of 6 km from the sea level. The pressure, temperature and density of the air at sea level are 101.3 kPa, 288 K and 1.22 kg/m^3 respectively. The temperature lapse rate is 0.00065 K/m.

12.2 Calculate the pressure at an altitude of 2 km above sea level if the atmospheric pressure and density at sea level are 101.3 kPa and 1.22 kg/m^3 respectively. Assume air as an incompressible fluid and neglect the variation of g with altitude.

12.3 Determine the pressure difference between points A and B, as shown in Fig. 12.18.

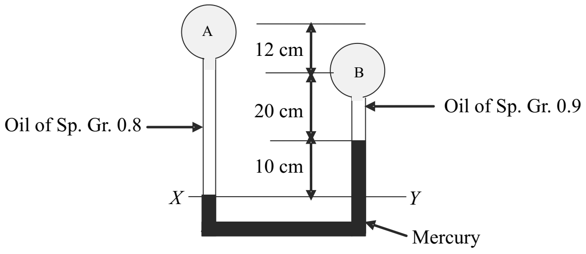


Figure 12.18

12.4 A differential U-tube mercury manometer is used to measure the pressure difference between points 1 and 2 in a pipeline conveying water. The point 1 is 0.5 m lower than the point 2. The difference in the level of manometric fluid on two limbs is 0.8 m. Calculate the pressure difference between points 1 and 2.

12.5 Water is flowing through two different pipes A and B to which an inverted differential manometer having an oil of sp.gr. 0.9 is connected. The pressure in the pipe A is 2.5 m of water. Find the pressure in the pipe B for the manometer readings as shown in Fig. 12.19.

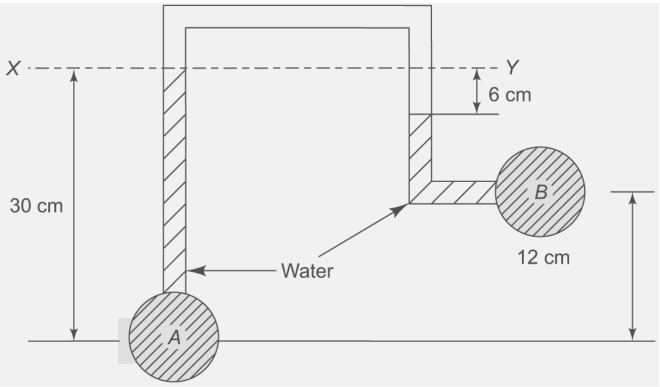


Figure 12.19

12.6 While one end of a U-tube mercury manometer is connected to a horizontal pipe in which water is flowing, its other end is open to the atmosphere. If the difference of mercury levels in the two limbs of this U-tube manometer is found to be 25 cm and the vertical height of water above mercury remains 10 cm below the pipe axis, find the pressure in the pipe.

12.7 A multi-tube manometer is used to determine the pressure difference between points *A* and *B* as shown in Fig.12.20. For the given values of heights, determine the pressure difference between points *A* and *B*. Specific gravities of benzene, kerosene and mercury are 0.88, 0.82 and 13.6 respectively.

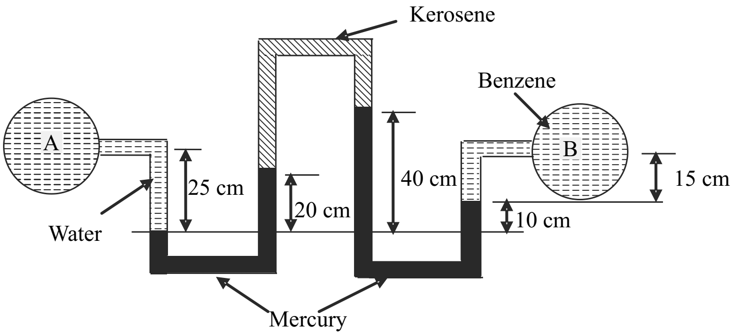


Figure 12.20

12.8 A differential U-tube mercury manometer is used to measure the pressure difference between two sections of a vertical pipe through which water flows upwards and shows a deflection of 10 cm. The distance between the two sections is 30 cm. Determine the difference of pressure between the two sections. Assume density of mercury as 13600 kg/m^3 and density of water as 1000 kg/m^3.

12.9 The pressure and density of air at an altitude of 2000 m above sea level are 79.98 kPa and 0.963 kg/m^3 respectively. Find the atmospheric pressure and density of air at sea level. Assume an isothermal process and neglect the variation of g with altitude.

MULTIPLE-CHOICE QUESTIONS

12.1 The intensity of pressure at a point in a fluid is the same in the directions, only when
   (a) the fluid is frictionless and incompressible
   (b) the fluid is frictionless
   (c) there is no motion of one fluid layer relative to an adjacent layer
   (d) the fluid has zero viscosity and is at rest

12.2 In a static fluid, with y as the vertical direction, the pressure variation is given by
   (a) $\frac{dp}{dy} = \rho$ (b) $\frac{dp}{dy} = -\rho$
   (c) $\frac{dp}{dy} = \gamma$ (d) $\frac{dp}{dy} = -\gamma$

12.3 In an isothermal atmosphere, the pressure
   (a) remains constant
   (b) decreases linearly with elevation
   (c) decreases exponentially with elevation
   (d) varies in the same way as the density

12.4 The piezometric head in a static liquid
   (a) remains constant at all points in the liquid
   (b) increases linearly with depth below a free surface
   (c) decreases linearly with depth below a free surface
   (d) remains constant only on a horizontal plane

12.5 Local atmospheric pressure is measured by
   (a) thermometer (b) manometer
   (c) barometer (d) hydrometer

12.6 In a barometer, mercury is preferred over water because
   (a) it has higher vapour pressure and bulk modulus of elasticity
   (b) it has higher thermal conductivity
   (c) it has higher density and lower vapour pressure
   (d) its surface is easier to read

12.7 A differential manometer is used to measure
   (a) atmospheric pressure (b) very low pressure
   (c) difference of pressure between two points (d) velocity in pipes

12.8 Gauge pressure is equal to
   (a) absolute pressure – atmospheric pressure (b) atmospheric pressure – absolute pressure
   (c) atmospheric pressure + absolute pressure (d) atmospheric pressure + vacuum

12.9 Mercury is used in barometers on account of its
   (a) negligible capillary effect (b) high density
   (c) very low vapour pressure (d) low compressibility

CHAPTER

13

Kinematics of Fluid Flow

13.1 INTRODUCTION

In Chapter 12, fluid at rest was discussed. In this chapter, kinematics of fluid flow will be discussed. *Kinematics of fluid flow is the study of motion of fluids without considering the forces and moments that cause the motion*. We begin this chapter with the different methods of analysis of fluid flow. Different types of flows and various ways to visualise flow fields are next illustrated with examples. Finally, generalised form of continuity equation in differential form is derived.

13.2 METHODS OF DESCRIPTION/ANALYSIS

There are two different points of view in analysing problem in mechanics. The first method, which follows an individual particle moving through the flow, is called the *Lagrangian approach* after the Italian mathematician Joseph Louis Lagrange (1736–1813). In this description, the fluid motion is described by tracing the kinematic behaviour of each and every individual particle constituting the flow. The position of a particle at any instant of time becomes a function of its identity and time is given by

$$S = S\left(\vec{S}_0, t\right) \tag{13.1}$$

The second approach, appropriate to fluid mechanics, is concerned with the field of flow and is called the *Eulerian approach* named after Swiss mathematician Leonhard Euler (1707–1783). In the Eulerian method of description, the properties of a flow field are described as functions of space coordinates and time. For example, the pressure field is a *scalar field* variable. For three-dimensional unsteady fluid flow in Cartesian coordinate system, one can write

$$P = P\,(x, y, z, t) \tag{13.2}$$

The velocity field is a *vector field* variable and is represented by

$$\vec{V} = \vec{V}\left(x, y, z, t\right) \tag{13.3}$$

The velocity components can be expressed as

$$u = u(x, y, z, t) \tag{13.3a}$$

$$v = v(x, y, z, t) \tag{13.3b}$$

$$w = w(x, y, z, t) \tag{13.3c}$$

where u, v and w are the velocity components along x, y and z directions respectively.

The Lagrangian approach is more appropriate to solid mechanics. In the study of fluids, the Eulerian approach is preferred because it is difficult to follow a fluid particle.

13.3 CLASSIFICATIONS OF FLOWS

According to the type of variations of properties, different categories of fluid flows are as follows:

(a) Steady and unsteady flows
(b) Uniform and non-uniform flows
(c) Laminar and turbulent flows
(d) Incompressible and compressible flows
(e) One, two and three-dimensional flows
(f) Internal and external flows
(g) Inviscid and viscous flows
(h) Irrotational and rotational flows

13.3.1 Steady and Unsteady Flows

Fluid flow can be classified into steady and unsteady on the basis of variations of fluid properties and flow characteristics with time.

A flow is said to be *steady* during which the fluid properties (such as density, pressure, temperature) and flow quantities (such as velocity, acceleration, etc.) at any point does not change with time. The term steady implies no change at a point with time. Mathematically, for steady flow, one can write

$$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in the flow field.

A flow is said to be *unsteady* during which the fluid properties and flow characteristics at any point changes with time. Mathematically, for unsteady flow, one can write

$$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

Liquid flow through a pipe at a constant rate is steady flow, whereas liquid flow through a pipe at a variable (increasing or decreasing) rate is unsteady flow.

13.3.2 Uniform and Non-uniform Flows

Fluid flow can be classified into uniform and non-uniform on the basis of variations of fluid properties and flow characteristics with space at a given instant of time.

A *uniform flow* is defined as that type of flow in which the fluid properties and flow characteristics at any instant of time do not change with space. The term uniform implies no change with location over a specified region. Mathematically, for uniform flow, we have

$$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} = 0$$

where t_0 is any fixed instant of time during the flow.

A *non-uniform flow* is defined as that type of flow in which the fluid properties and flow characteristics at any instant of time do not change with space. Mathematically, for non-uniform flow, one can write

$$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} \neq 0$$

The liquid flow through a uniform cross-sectional area is uniform flow. Uniform flow at a section is shown in Fig. 13.1. The liquid flow through an expanding tube is non-uniform flow. Figure 13.2 shows flow through a circular pipe, which is non-uniform in nature.

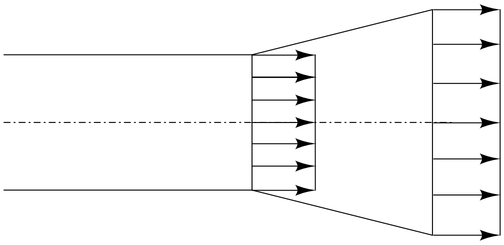


Figure 13.1 *Uniform flow at a section*

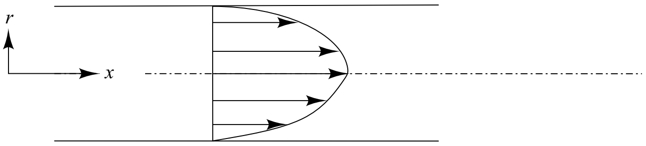


Figure 13.2 *Non-uniform flow at a section*

Note that any combinations of the above-mentioned four types of flows are possible, viz., (a) steady-uniform flow, (b) unsteady-uniform flow, (c) steady non-uniform flow, and (d) unsteady-non-uniform flow. Some of the common examples of these combinations of flows are listed in Table 13.1.

Table 13.1

Type of flow	Example
Steady-uniform flow	Flow of liquid through a long pipe of constant diameter at a constant rate
Unsteady-uniform flow	Flow of liquid through a long pipe of constant diameter at either increasing or decreasing rate
Steady non-uniform flow	Flow of liquid through a tapering pipe at a constant rate
Unsteady non-uniform flow	Flow of liquid through a tapering pipe at either increasing or decreasing rate

13.3.3 Laminar and Turbulent Flows

On the basis of flow structure, flow regimes are classified as laminar and turbulent.

A *laminar flow* is one in which the fluid particles move along smooth, regular paths which can be predicted well in advance. The fluid particles thus move in layers, gliding smoothly over adjacent layers. There is no transformation of fluid particles from one layer to another.

On the other hand, a flow is said to be *turbulent*, when the fluid particles move in very irregular paths. The velocities in turbulent flow vary from point to point in magnitude and direction as well as from instant to instant. All the fluid particles are disturbed and they mix with each other. Thus there is a continuous transfer of momentum to adjacent layers.

A familiar example is the flow of water from water tap. Whenever water is allowed to flow at a low velocity by opening the tap a little, the water flows out smoothly and the flow is laminar. However, as the tap is gradually opened to let the water velocity increase, the flow becomes turbulent.

The behaviour of flow is governed by the magnitude of a non-dimensional *Reynolds number*, which is defined as the ratio of inertia force to viscous force and is given by

$$\text{Re} = \frac{\rho VL}{\mu} \tag{13.4}$$

where ρ is the fluid density, V is the characteristic velocity of flow, L is the characteristic length, and μ is the fluid viscosity. For flow through circular pipe, Reynolds number is expressed as

$$\text{Re} = \frac{\rho VD}{\mu} \tag{13.5}$$

where V is the average flow velocity, and D is diameter of the pipe. The Reynolds number at which the flow becomes turbulent is called the *critical Reynolds number*. The value of the critical Reynolds number is different for different geometries and flow conditions. For internal flow in a circular pipe, critical Reynolds number is 2000. The flow in a circular pipe is laminar for $\text{Re} \le 2000$, turbulent for $\text{Re} \ge 4000$, and transitional in between. That is

$\text{Re} \le 2000$	laminar flow
$2000 \le \text{Re} \le 4000$	transitional flow
$\text{Re} \ge 4000$	turbulent flow

Example 13.1 Water is flowing through a circular pipe of diameter 20 mm at a uniform velocity of 3 m/s. The kinematic viscosity of water is 1×10^{-6} m^2/s. Determine whether the flow field is laminar or turbulent.

Solution

Diameter of pipe $D = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} = 0.02 \text{ m}$

Velocity of flow $V = 3 \text{ m/s}$

Kinematic viscosity of water $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$

The Reynolds number is found to be (see Eq. (13.5))

$$\text{Re} = \frac{\rho VD}{\mu}$$

or

$$\text{Re} = \frac{VD}{\nu} \qquad \left[\because \nu = \frac{\mu}{\rho}\right]$$

$$= \frac{3 \times 0.02}{1 \times 10^{-6}} = 60000$$

Since the Reynolds number is more than 4000, the flow is turbulent.

13.3.4 Compressible and Incompressible Flows

Flows in which variations in density are negligible are termed as *incompressible flows*. When the density changes significantly within a flow, then the flow is called *compressible*. Liquid flows are considered as incompressible flows. Transmission of gases in pipelines at high pressure, flow in high-speed aircraft and missiles, fans and compressors are compressible flows.

Note: *There is a subtle difference between incompressible fluid and incompressible flow. A fluid is called incompressible if its density does not change significantly with change in pressure. The change in density may not be always due to change in pressure, it may be due to change in temperature. That means there may be compressible flow of an incompressible fluid.*

13.3.5 One-, Two- and Three-Dimensional Flows

All general flows such as flow around a moving car have velocity components in x, y and z directions. They are called three-dimensional flows. Thus, a *three-dimensional flow* is the one in which the velocity vector depends on three space variables and time. The velocity components can be expressed as

$$u = u(x, y, z, t) \tag{13.6a}$$

$$v = v(x, y, z, t) \tag{13.6b}$$

$$w = w(x, y, z, t) \tag{13.6c}$$

A *two-dimensional* flow is one in which the velocity vector depends on two space variables and time. Steady flow between two parallel plates close to the inlet (entrance region) as shown in Fig. 13.3 is two-dimensional. The velocity components can be expressed as

$$u = u(x, y, t) \tag{13.7a}$$

$$v = v(x, y, t) \tag{13.7b}$$

A *one-dimensional flow* is one in which velocity vector depends on only one space variable and time. The velocity can be expressed as

$$u = u(x,t) \tag{13.8}$$

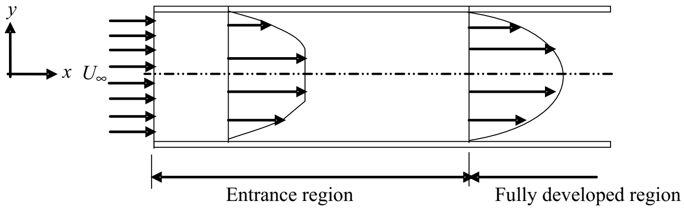


Figure 13.3 *Parallel flow between two infinite parallel plates*

Few real flows are strictly one-dimensional. Steady flow between two parallel plates in the fully developed region [$u = u(y)$] as shown in Fig.13.3 is one-dimensional.

Example 13.2 The velocity components for a flow field are given as

$$u = axy,\ v = -byzt,\ w = 0.$$

Determine

(a) whether the flow field is one-, two-, or three-dimensional.
(b) whether the flow is steady or unsteady.

Solution

(a) Since the velocity field is a function of x, y and z only, the flow field is three-dimensional.
(b) Since time t appears explicitly in the velocity, the flow is unsteady.

13.3.6 Internal and External Flows

On the basis of flow in a confined channel or over a surface, a fluid flow can be classified as internal or external flow.

The flow of fluid in a pipe or duct is *internal flow* if the fluid is completely bounded by the solid surfaces. The flow of an unbounded fluid over a surface such as flat plate is *external flow*. Water flow in a pipe is an internal flow, whereas flow over a flat plate is an external flow.

13.3.7 Inviscid and Viscous Flows

An *inviscid flow* is one in which the effect of viscosity is negligible. In inviscid flow, the fluid viscosity is assumed to be zero. Inviscid flow does not exist in reality; however, in many situations the flow can be simplified by neglecting the viscous forces. .

All fluids possess viscosity, and accordingly, *viscous flows* are important in the study of fluid mechanics.

13.3.8 Irrotational and Rotational Flows

A flow is said to be *irrotational* if the fluid particles while flowing do not rotate about their mass centres. On the other hand, a flow is said to be rotational when the fluid particles while flowing also rotate about their mass centres.

When fluid flows over a flat plate as shown in Fig. 13.4, the flow field can be separated into two regions, namely the boundary layer region (where the viscous effects are significant) and the outer region (where the viscous effects are not significant). In the boundary layer region the flow is rotational, whereas in the outer region, the flow is irrotational.

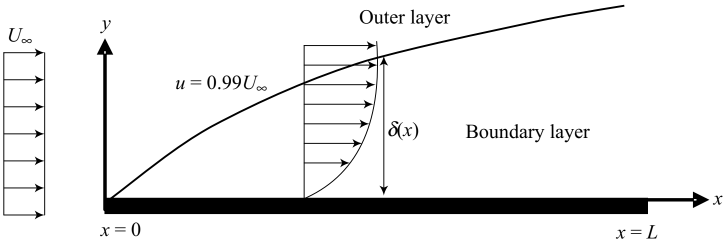


Figure 13.4 *Flow over a flat plate*

13.4 STREAMLINES, PATHINES AND STREAKLINES

The pictorial representation of fluid flow is very helpful to describe the flow characteristics, whether this be done by experimental flow visualisation or by numerical solution. Streamlines, streaklines and pathlines are widely used to describe the flow behaviour. Here, the important characteristics of the above-mentioned three lines are discussed.

13.4.1 Streamline and Streamtube

Streamline at any instant can be defined as an imaginary line in the flow field so that the tangent to the line at any point represents the direction of the instantaneous velocity of that point. For unsteady flows the streamline pattern changes with time. From the definition of streamline, it can be written

$$\vec{V} \times d\vec{S} = 0 \tag{13.9}$$

where $d\vec{S}$ is the length of an infinitesimal line segment along a streamline at a point where $\vec{V}$ is the instantaneous velocity vector.

Consider an infinitesimal arc length $d\vec{S} = dx\hat{i} + dy\hat{j}$ along a streamline in the xy-plane as shown in Fig. 13.5. From the definition of streamline, $d\vec{S}$ must be parallel to the local velocity vector $\vec{V} = u\hat{i} + v\hat{j}$. Thus, one can write

$$\vec{V} \times d\vec{S} = 0$$

or $$v\,dx - u\,dy = 0$$

or $$\frac{dx}{u} = \frac{dy}{v} \tag{13.10}$$

Equation (13.10) represents the equation of a streamline in x-y plane.

The general differential equation for streamlines in three-dimensional flow field can be obtained as

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \tag{13.11}$$

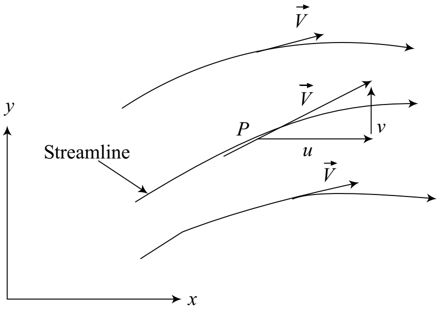


Figure 13.5 *Streamline in a two-dimensional flow field*

A bundle of neighbouring streamlines (Fig. 13.6) may be imagined to form a passage through which the fluid flows. This passage is known as a ***stream tube***. Since the stream tube is bounded on all sides by streamlines and by definition, velocity does not exist across a streamline, no fluid may enter or leave a stream tube except through its ends.

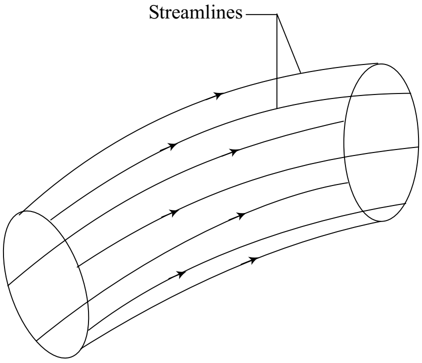


Figure 13.6 *Stream tube*

Example 13.3 For the following flow fields find the equation of streamline:

(i) $\vec{V} = x\hat{i} - y\hat{j}$ passing through the point (1, 1)

(ii) $\vec{V} = 3y\hat{i} - 2x\hat{j}$ passing through the point (2, –1)

Solution The equation of a streamline in two-dimensional flow is (see Eq. (13.10))

$$\frac{dx}{u} = \frac{dy}{v}$$

(i) x-component of velocity, $u = x$

y-component of velocity, $v = -y$

Substituting the x and y component of velocities in the equation of streamline, we have

$$\frac{dx}{x} = -\frac{dy}{y}$$

Integrating the above equation, we get

$$\ln x = -\ln y + \ln C$$

where C is integration constant

or $$\ln xy = \ln C$$

or $$xy = C$$

Streamline is passing through point (1,1)

Thus, $$C = 1 \times 1 = 1$$

The equation of streamline is $xy = 1$

(ii) x-component of velocity, $u = 3y$

y-component of velocity, $v = -2x$

Substituting the x and y component of velocities in the equation of streamline, we get

$$\frac{dx}{3y} = \frac{dy}{-2x}$$

$$2xdx = -3ydy$$

Integrating the above equation, we obtain

$$2\frac{x^2}{2} = -3\frac{y^2}{2} + \frac{C}{2}$$

where C is integration constant

$$2x^2 + 3y^2 = C$$

Streamline is passing through point (2, –1)

Thus, $$C = 2 \times 2^2 + 3 \times (-1)^2 = 11$$

The equation of streamline is $2x^2 + 3y^2 = 11$

Example 13.4 A two-dimensional flow field has velocities along the x and y directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. Find the equation of streamline.

Solution The equation of a streamline in two-dimensional flow is (see Eq. (13.10))

$$\frac{dx}{u} = \frac{dy}{v}$$

Substituting $u = x^2 t$ and $v = -2xyt$ in the equation of streamline, we have

$$\frac{dx}{x^2 t} = -\frac{dy}{2xyt}$$

or $$2\frac{dx}{x} = -\frac{dy}{y}$$

Integrating the above equation, we get

$$2\ln x = -\ln y + \ln C$$

where C is integration constant

$$\ln x^2 y = \ln C$$

or $$x^2 y = \text{constant}$$

The equation of streamline is $x^2 y$ = constant

Example 13.5 In a flow the velocity vector is given by $V = 2x\hat{i} - 3y\hat{j} + 5z\hat{k}$. Determine the equation of the streamline passing through point (1, 4, 5).

Solution The equation of a streamline in three-dimensional flow is (see Eq. (13.11))

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

x-component of velocity, $u = 2x$
y-component of velocity, $v = -3y$
z-component of velocity, $w = 5z$
Streamline in the xy-plane is given by (see Eq. (13.10))

$$\frac{dx}{u} = \frac{dy}{v}$$

or $$\frac{dx}{2x} = \frac{dy}{-3y}$$

or $$\frac{dy}{3y} = -\frac{dx}{2x}$$

or $$\frac{1}{3}\ln y = -\frac{1}{2}\ln x + \ln\ C_1$$

or $$y = C_1 x^{-\frac{3}{2}}$$

Streamline is passing through point (1, 4, 5)

Thus, $C_1 = 4$

The equation of streamline passing through point (1, 4, 5) is

$$y = 4x^{-\frac{3}{2}}$$

Streamline in the xz-plane is given by (see Eq. (13.11))

$$\frac{dx}{u} = \frac{dz}{w}$$

or $$\frac{dx}{2x} = \frac{dz}{5z}$$

or $$\frac{1}{5}\ln z = \frac{1}{2}\ln x + \ln C_2$$

or $$z = C_2 x^{\frac{5}{2}}$$

Streamline is passing through point (1, 4, 5).

Thus, $C_2 = 5$

The equation of streamline passing through point (1, 4, 5) is

$$z = 5x^{\frac{5}{2}}$$

13.4.2 Pathline

In experimental fluid mechanics, the concept of pathline is important. A *pathline* is the actual trajectory through space of a selected fluid article during a time of interval. Pathine and streamlines are identical in a steady flow, but not in an unsteady flow. A pathline is a Lagrangian concept because it is defined by the motion of fluid particles as shown in Fig.13.7.

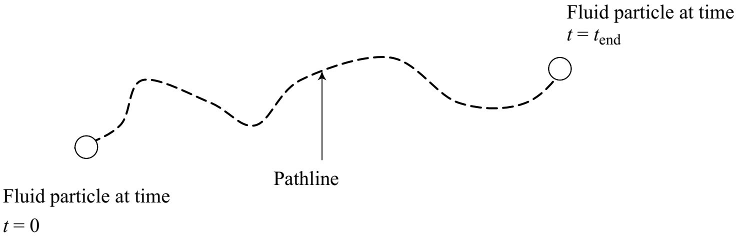


Figure 13.7 *Pathline*

Example 13.6 A two-dimensional flow field is described in the Lagrangian system as

$$x = x_0 e^{ct}, \text{ and } y = y_0 e^{-ct}$$

Find the equation of path line of the particle.

Solution The equation of pathline can be found from the equations of motion describing the flow by eliminating t.

From the given flow field, we have

$$x = x_0 e^{ct}$$

or
$$e^{ct} = \frac{x}{x_0} \tag{13.12}$$

$$y = y_0 e^{-ct}$$

or
$$e^{ct} = \frac{y_0}{y} \tag{13.13}$$

Eliminating t from Eqs. (13.12) and (13.13), we have

$$\frac{x}{x_0} = \frac{y_0}{y}$$

or
$$xy = x_0 y_0$$

This is the required equation of pathline.

13.4.3 Streakline

A *streakline* at any instant of time is the temporary locations of all particles that have passed through a fixed point in the flow field. Note that *smoke emitting from a lighted cigarette represents streakline*. Suppose x, y, z are the fluid particles which have passed through a reference point say A as shown in Fig. 13.8. Further, at an instant of time t, the fluid particle x, y, z are at B, C, D. Then the line joining A, B, C and D is the streakline, at time t as shown in Fig. 13.8.

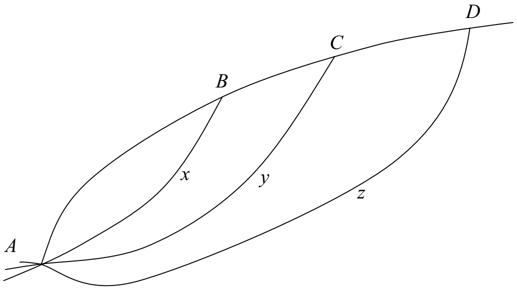


Figure 13.8 *Streakline*

Note that streamlines, pathlines and streaklines are identical in steady flow. Streamline is instantaneous line while the streakline and pathline are generated by the passage of time.

13.5 CONTINUITY EQUATION

13.5.1 Continuity Equation for Steady One-Dimensional Flow in a Conduit

Consider fluid flows steadily through a portion of the stream tube having a cross-sectional area small enough for the velocity to be considered as constant over any cross-section, for the setions 1 and 2 as shown in the Fig. 13.9. Suppose that at section 1, the area of the stream tube is A_1, the uniform velocity of the fluid is V_1 and the density is ρ_1, while at section 2 the corresponding values are A_2, V_2 and ρ_2 respectively.

According to the principle of conservation of mass for a control volume, we have

rate of mass flow entering into the control volume = *rate of mass leaving from the control volume* + *rate of increase of mass of fluid in the control volume* (13.14)

For steady flow, the mass of fluid in the control volume remains constant and Eq. (13.14) simplifies to

rate of mass flow entering into the control volume = *rate of mass leaving from the control volume* (13.15)

Since there cannot be any flow across the walls of a stream tube, for flow through a streamtube as shown in Fig. 13.9, Eq. (13.15) becomes

rate of mass entering at section 1 = *rate of mass flow leaving at section* 2 (13.16)

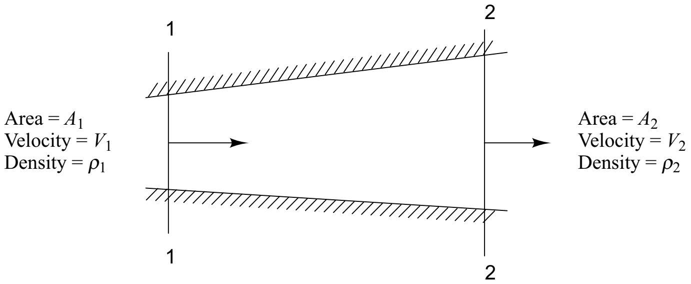


Figure 13.9 *Steady flow through a streamtube*

Rate of mass flow entering at section 1 = $\rho_1 A_1 V_1$
Rate of mass flow leaving at section 2 = $\rho_2 A_2 V_2$
For steady flow, Eq. (13.16) simplifies to

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \tag{13.17}$$

Equation (13.17) is the *continuity equation* for the steady flow of a compressible fluid applied to two sections along a stream tube.

For the flow of a real fluid, velocity is not uniform at any section varies from wall to wall. The *average velocity* over a cross section is given by

$$\bar{V} = \frac{\int_A V dA}{A} \tag{13.18}$$

The average velocity is physically an equivalent uniform velocity that could have given rise to the same volume flow rate as the actual one.

Using the average velocity, continuity equation for steady flow can be written as

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2 \tag{13.19}$$

For constant density, steady flow continuity equation becomes

$$A_1 \bar{V}_1 = A_2 \bar{V}_2 \tag{13.20}$$

The volume of the fluid flowing through a cross section per unit time is called the volume flow rate Q and is given by

$$Q = A\bar{V} \tag{13.21}$$

where A is the cross sectional area normal to the flow direction and $\bar{V}$ is the average velocity across the section.

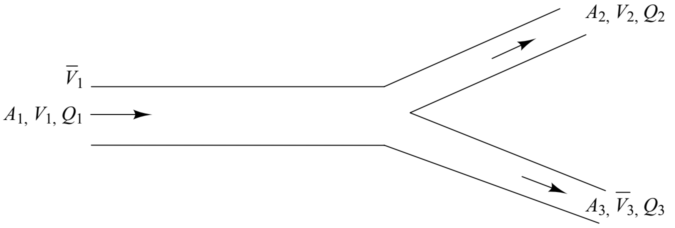


Figure 13.10 Flow through a branched pipe

The continuity equation can also be applied to determine the relation between the inflow and outflow of a junction. For steady flow, continuity equation for the branched pipe shown in Fig. 13.10 can be written as

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3 \tag{13.22}$$

For a constant density fluid, it becomes

$$Q_1 = Q_2 + Q_3$$

or

$$A_1 \bar{V}_1 = A_2 \bar{V}_2 + A_3 \bar{V}_3 \tag{13.23}$$

Note: *The expression* $A_1 \bar{V}_1 = A_2 \bar{V}_2$ *between any two sections holds valid only for constant density flow.*

Example 13.7 For the pipe shown in Fig. 13.11, the diameters of the pipe at sections 1-1 and 2-2 are 10 and 20 cm respectively. If the volume flow rate through the pipe is 0.005 m^3/s, find the average velocity at the two sections.

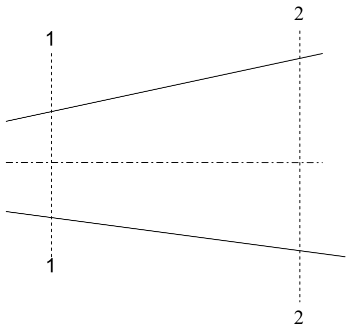


Figure 13.11

Solution

Diameter of pipe at section 1-1 $D_1 = 10\text{ cm} = 0.1\text{ m}$

Diameter of pipe at section 2-2 $D_2 = 20\text{ cm} = 0.2\text{ m}$

Cross-sectional area at section 1 is $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.00785\text{ m}^2$

Cross-sectional area at section 2 is $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\text{ m}^2$

Volume flow rate $Q = 0.005\text{ m}^3/\text{s}$

From continuity equation, we have

$$Q = A_1\bar{V}_1 = A_2\bar{V}_2$$

Thus, the average velocity at section 1-1 is $\bar{V}_1 = \frac{Q}{A_1} = \frac{0.005\text{ m}^3/\text{s}}{0.00785\text{ m}^2} = 0.637\text{ m/s}$

Average velocity at section 2-2 is $\bar{V}_2 = \frac{Q}{A_2} = \frac{0.005\text{ m}^3/\text{s}}{0.0314\text{ m}^2} = 0.16\text{ m/s}$

Example 13.8 Air having a mass density of 1.23 kg/m^3 flows in a pipe with a diameter of 20 cm at a mass flow rate of 2 kg/s. What are the mean (or average) velocity of flow in this pipe and the volume flow rate?

Solution

Density of air $\rho = 1.23\text{ kg/m}^3$

Mass flow rate $\dot{m} = 2\text{ kg/s}$

Diameter of pipe $D = 20\text{ cm} = 0.2\text{ m}$

Cross-sectional area of pipe $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\text{ m}^2$

Volume flow rate is $$Q = \frac{\dot{m}}{\rho} = \frac{2}{1.23} = 1.63\ \text{m}^3/\text{s}$$

Average velocity of air is found to be

$$\bar{V} = \frac{Q}{A} = \frac{1.63}{0.0314} = 51.88\ \text{m/s}$$

Example 13.9 A diffuser consists of two circular parallel plates 20 cm in diameter and 5 mm apart and connected to a 30 mm diameter pipe as shown in Fig. 13.12. If the streamlines are assumed to be radial in the diffuser, what mean velocity in the pipe will correspond to an exit velocity of 0.5 m/s.

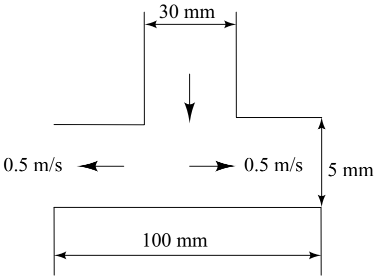


Figure 13.12

Solution

Diameter of the plates $d = 20$ cm $= 0.2$ m

Diameter of the pipe $D = 30$ mm $= 0.03$ m

Distance of separation of plates $t = 5$ mm $= 0.005$ m

Exit velocity from plates $U = 0.5$ m/s

From continuity equation, we have

$$\pi d t \times U = \frac{\pi}{4} D^2 \times \bar{V}$$

Substituting the respective values, we get

$$\pi \times 0.2 \times 0.005 \times 0.5 = \frac{\pi}{4} \times (0.03)^2 \times \bar{V}$$

or $$\bar{V} = 2.222\ \text{m/s}$$

Example 13.10 A pipe 40 cm in diameter branches into two pipes of diameters 25 cm and 20 cm respectively as shown in Fig. 13.13. The average velocity in 40 cm diameter pipe is 4 m/s. Find

(a) the discharge through 40 cm diameter pipe, and

(b) the average velocity in 25 cm diameter pipe if the average velocity in 20 cm pipe is 2 m/s.

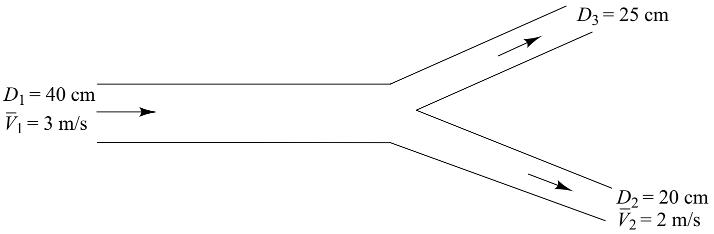


Figure 13.13

Solution

(a) Diameter of main pipe, $D_1 = 40 \text{ cm} = 0.4 \text{ m}$

Average velocity of flow in main pipe, $\bar{V}_1 = 3$ m/s
Diameter of branched pipe 2, $D_2 = 20 \text{ cm} = 0.2 \text{ m}$

Average velocity of flow in branched pipe 2, $\bar{V}_2 = 2$ m/s
Diameter of branched pipe 3, $D_3 = 25 \text{ cm} = 0.25 \text{ m}$
The discharge through 40 cm diameter pipe is given by

$$Q = \text{Area of main pipe} \times \text{average velocity of flow in main pipe}$$

$$= A_1\bar{V}_1 = \frac{\pi}{4}D_1^2\bar{V}_1 = \frac{\pi}{4} \times 0.4^2 \times 3 = 0.377 \text{ m}^3/\text{s}$$

(b) For incompressible steady flow continuity equation becomes

Area of main pipe × average velocity in main pipe = Area of pipe 2 × average velocity in pipe 3 + Area of pipe 2 × average velocity in pipe 3

or $$A_1\bar{V}_1 = A_2\bar{V}_2 + A_3\bar{V}_3$$

or $$\frac{\pi}{4}D_1^2\bar{V}_1 = \frac{\pi}{4}D_2^2\bar{V}_2 + \frac{\pi}{4}D_3^2\bar{V}_3$$

or $$\frac{\pi}{4} \times 0.4^2 \times 3 = \frac{\pi}{4} \times 0.2^2 \times 2 + \frac{\pi}{4} \times 0.25^2\bar{V}_3$$

or $$\bar{V}_3 = 6.4 \text{ m/s}$$

Example 13.11 A jet of water issuing from a 20 mm diameter nozzle is directed vertically upwards. The diameter of the water jet at a point 3 m above nozzle is 40 mm. Find the velocity of jet at point 3 m above nozzle. Assume that jet remains steady and there is no loss of energy.

Solution Let the nozzle exit and the point 3 m above nozzle be designated by point 1, and 2 respectively, as shown in Fig. 13.14.

Diameter at nozzle exit is $D_1 = 20$ mm $= 0.02$ m
Diameter of the water jet at 2 is $D_2 = 40$ mm $= 0.04$ m
Let the average velocity of jet at point 1 and 2 be V_1 and V_2, respectively.
Considering the motion of the jet from the exit of the nozzle to point 2, we have

$$V_2^2 = V_1^2 - 2gh$$

where h is the distance between point 1 and 2 (here $h = 3$ m)
Putting the value h, we get

$$V_2^2 = V_1^2 - 2 \times 9.81 \times 3$$

or $$V_2^2 = V_1^2 - 58.86 \qquad (13.24)$$

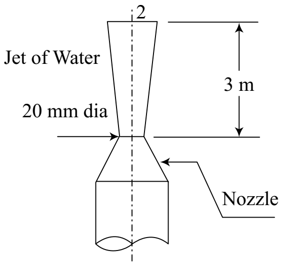


Figure 13.14

Applying the continuity equation between the exit of the nozzle 1 and the point 2, we have

$$A_1 V_1 = A_2 V_2$$

where A_1 and A_2 are the cross-sectional area at sections 1 and 2 respectively.

or $$\frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} D_2^2 V_2$$

or $$V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2 = \left(\frac{0.04}{0.02}\right)^2 V_2 = 4V_2 \qquad (13.25)$$

From Eqs. (13.24) and (13.25), we have

$$V_2^2 = (4V_2)^2 - 58.86$$

or, $$V_2 = 1.98 \text{ m/s}$$

13.5.2 Continuity Equation-Differential Form

A rectangular parallelepiped with sides dx, dy and dz in the x, y and z directions, respectively, is considered as the control volume in three-dimensional Cartesian coordinate system as shown in Fig. 13.15.

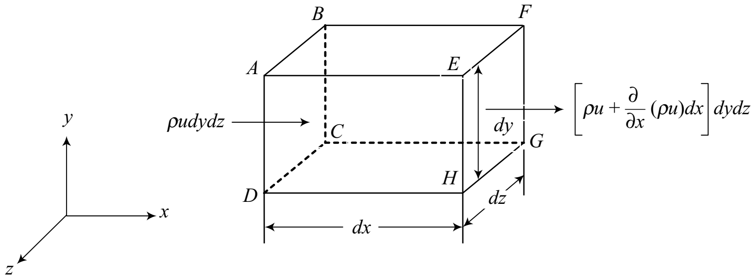


Figure 13.15 *Differential control volume in rectangular cartesian coordinate system*

Let the fluid enter through the surface *ABCD* (normal to the x-axis) with a velocity u and a density ρ.

Rate of mass inflow through the surface *ABCD* (normal to x axis) = $\rho udydz$

Rate of mass outflow through the surface *EFGH* (normal to x-axis) = $\left[\rho u+\frac{\partial}{\partial x}(\rho u)dx\right]dydz$

Net rate of mass outflow in x-direction $=\left[\rho u+\frac{\partial}{\partial x}(\rho u)dx\right]dydz-\rho udydz=\frac{\partial}{\partial x}(\rho u)dxdydz$

Similarly,

Net rate of mass outflow in y-direction $=\frac{\partial}{\partial y}(\rho v)dx\,dy\,dz$

Net rate of mass outflow in z-direction $=\frac{\partial}{\partial z}(\rho w)dxdydz$

Therefore, total net rate of mass outflow in x-, y- and z-direction

$$=\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dxdydz \tag{13.26}$$

The effect of mass loss in Eq. (13.26) is to cause the time rate of decrease of mass encompasses by the volume.

Since $\frac{\partial \rho}{\partial t}$ is the rate of change of mass density, the rate of change of mass in control volume $=-\frac{\partial \rho}{\partial t}dxdydz$.

Therefore, according to the principle of conservation of mass,

Total net rate of mass outflow in x-, y- and z-direction = Rate of change of mass in control volume

$$\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dxdydz=-\frac{\partial \rho}{\partial t}dxdydz$$

or
$$\left[\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dxdydz=0 \tag{13.27}$$

Since the volume of a control volume cannot be zero, Eq. (13.27) becomes

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \tag{13.28}$$

Equation (13.28) is differential form of *continuity equation* in three-dimensional flow field. Equation (13.28) can be written in a vector form as

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\vec{V})=0 \tag{13.29}$$

where $\vec{V}$ represents the velocity vector and $\nabla\cdot(\rho\vec{V})$ represents divergence of $(\rho\vec{V})$.

For steady flow, $\frac{\partial \rho}{\partial t}=0$. Then, Eq. (13.28) simplifies to

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \tag{13.30}$$

For two-dimensional flow, Eq. (13.30) simplifies to

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \tag{13.31}$$

Equation (13.30) can be written in a vector form as

$$\nabla\cdot(\rho\vec{V})=0 \tag{13.32}$$

From Eq. (13.28), we get

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}+w\frac{\partial \rho}{\partial z}+\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=0$$

$$\frac{D\rho}{Dt}+\rho\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]=0$$

$$\frac{1}{\rho}\frac{D\rho}{Dt}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

For incompressible flow, the rate of volumetric dilation per unit volume $\left(\frac{1}{\rho}\frac{D\rho}{Dt}\right)$ of a fluid element in motion is zero. Then the above equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (13.33)$$

For two-dimensional flow, Eq. (13.33) simplifies to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (13.34)$$

Equation (13.33) can be written in a vector form as

$$\nabla \cdot \vec{V}=0 \qquad (13.35)$$

Equation (13.33) or (13.35) holds for incompressible (both steady as well as unsteady) flow.

Note: *Any velocity field representing the motion of a fluid should satisfy the continuity equation.*

Note:

The equation $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ *(equivalently* $\nabla\cdot\bar{V}=0$ *) is applicable for incompressible flow.*

The equation $\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$ *is applicable for steady flow.*

The equation $\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$ *holds for steady and two-dimensional flow.*

The equation $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ *holds for two-dimensional and incompressible (both steady as well as unsteady) flow.*

Example 13.12 The velocity field in a two-dimensional flow field is given by $\vec{V}=(x^2y+y^2)\hat{i}-xy^2\hat{j}$.

Check whether the velocity field describes the motion of an incompressible flow or not?

Solution For a two-dimensional, incompressible flow, the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Given that $u=x^2y+y^2$, and $v=-xy^2$

Hence, $\frac{\partial u}{\partial x}=2xy$, and

$$\frac{\partial v}{\partial y} = -2xy$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2xy - 2xy = 0$$

This shows that the above velocity field satisfies the continuity equation.

Example 13.13 Check whether the following sets of velocity components satisfy the continuity equation of steady, incompressible flow:

(a) $u = x^2 - y^2$, $v = x - 2xy$

(b) $u = -\ln xy$, $v = \frac{y}{x}$

(c) $u = 2x^2 - xy + z^2$, $v = x^2 - 4xy + y^2$ and $w = 2xy - yz + y^2$

Solution For a two-dimensional, incompressible flow the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(a) Given that $u = x^2 - y^2$ and $v = x - 2xy$

Hence, $\frac{\partial u}{\partial x} = 2x$ and $\frac{\partial v}{\partial y} = -2x$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we find

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2x - 2x = 0$$

Hence the continuity equation is satisfied.

(b) Here, $u = -\ln xy$ and $v = \frac{y}{x}$

Hence, $\frac{\partial u}{\partial x} = -\frac{1}{xy} y = -\frac{1}{x}$ and $\frac{\partial v}{\partial y} = \frac{1}{x}$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we find

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{x} + \frac{1}{x} = 0$$

Hence the continuity equation is satisfied.

(c) Here, $u = 2x^2 - xy + z^2$, $v = x^2 - 4xy + y^2$ and $w = 2xy - yz + y^2$

Hence, $\frac{\partial u}{\partial x} = 4x - y$, $\frac{\partial v}{\partial y} = -4x + 2y$, and $\frac{\partial w}{\partial z} = -y$

For a three-dimensional, incompressible flow the continuity equation can be written in a differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation, we find

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4x - y - 4x + 2y - y = 0$$

Hence the continuity equation is not satisfied.

Example 13.14 The velocity components for a two-dimensional, incompressible flow are given as

$$u = -\frac{x}{x^2 + y^2}, v = -\frac{y}{x^2 + y^2}$$

Show that the velocity field satisfies the continuity equation.

Solution For a two-dimensional, incompressible flow the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Given that $u = -\frac{x}{x^2 + y^2}$, and $v = -\frac{y}{x^2 + y^2}$

Hence, $\frac{\partial u}{\partial x} = -\frac{1}{x^2 + y^2} + \frac{2x^2}{\left(x^2 + y^2\right)^2}$, and $\frac{\partial v}{\partial y} = -\frac{1}{x^2 + y^2} + \frac{2y^2}{\left(x^2 + y^2\right)^2}$

Substituting $\frac{\partial u}{\partial x}$, and $\frac{\partial v}{\partial y}$ in the continuity equation, we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

This shows that the above velocity field satisfies the continuity equation.

Example 13.15 For a two-dimensional incompressible flow, the x-component of velocity is $u = 2xy$. What is the y component that will satisfy continuity equation?

Solution Given that $u = 2xy$

Hence, $$\frac{\partial u}{\partial x} = 2y$$

For a two-dimensional, incompressible flow the continuity equation can be written in a differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

or $$2y + \frac{\partial v}{\partial y} = 0$$

or $$\frac{\partial v}{\partial y} = -2y$$

or $$v = -y^2 + f(x)$$

where $f(x)$ is a constant of integration.

Example 13.16 In a three-dimensional incompressible fluid flow, the flow field is given by expression $V = (x + y + z)\hat{i} - (xy + yz + zx)\hat{j} + (w)\hat{k}$. Find the w component of velocity so that the case is possible for an incompressible fluid flow.

Solution For an incompressible flow the continuity equation can be written in differential form as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Given that $u = x + y + z$, and $v = -xy - yz - zx$

Hence, $$\frac{\partial u}{\partial x} = 1, \text{ and}$$

$$\frac{\partial v}{\partial y} = -x - z$$

Substituting $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in the continuity equation, we get

$$1 - x - z + \frac{\partial w}{\partial z} = 0$$

or $$\frac{\partial w}{\partial z} = x + z - 1$$

Integrating with respect to z, we have

$$w = xz + \frac{z^2}{2} - z + f(x, y)$$

SUMMARY

- In Lagrangian description, the fluid motion is described by tracing the kinematic behaviour of each and every individual particle constituting the flow. The position of a particle at any instant of time becomes a function of its identity and time and is given by

  $$S = S(\vec{S}_0, t)$$

  The Lagrangian approach is more appropriate to solid mechanics.
- In *Eulerian description*, the properties of a flow field are described as functions of space coordinates and time. For example, the pressure field is a scalar field variable. For three-dimensional unsteady fluid flow in Cartesian coordinates, the velocity field is given by

  $$\vec{V} = \vec{V}(x, y, z, t)$$

  In fluid mechanics, the Eulerian approach is preferred because it is difficult to follow a fluid particle.
- A *steady flow* is that type of flow in which the fluid properties (such as density, pressure, temperature) and flow characteristics (such as velocity, acceleration, etc.) at a point do not change with time. If they change with time, the flow is called *unsteady flow*.

  $$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} = 0, \text{ for steady flow}$$

  $$\left(\frac{\partial \vec{V}}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \text{ for unsteady flow}$$
- A *uniform flow* is that type of flow in which the fluid properties (such as density, pressure, temperature) and flow characteristics (such as velocity, acceleration, etc.) at any instant of time do not change with space. If they change with space, the flow is called *non-uniform flow*.

  $$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} = 0, \text{ for uniform flow}$$

$$\left(\frac{\partial \vec{V}}{\partial s}\right)_{t=t_0} \neq 0, \text{ for non-uniform flow}$$

- A *laminar flow* is one in which the fluid particles move along smooth, regular paths which can be predicted well in advance. The fluid particles thus move in layers, gliding smoothly over adjacent layers. In turbulent flow, the fluid particles move in very irregular paths. In the *turbulent regime*, the flow structure is characterised by random three-dimensional motions of fluid particles in addition to the mean motion.
  The flow in a circular pipe is laminar for Re ≤ 2000, turbulent for Re ≥ 4000, and transitional in between. That is

$Re \le 2000$	laminar flow
$2000 \le Re \le 4000$	transitional flow
$Re \ge 4000$	turbulent flow

- Flows in which variations in density are negligible are termed as *incompressible flows*. When the density changes significantly within a flow, then the flow is called compressible.
- A *one-dimensional flow* is one in which velocity vector depends on only one space variable and time. A *two-dimensional flow* is one in which the velocity vector depends on two space variables and time, that is, $\vec{V} = \vec{V}(x, y, t)$. A *three-dimensional flow* is the most general flow in which the velocity vector depends on three space variables and time, that is, $\vec{V} = \vec{V}(x, y, z, t)$
- The flow of fluid in a pipe or duct is *internal flow* if the fluid is completely bounded by the solid surfaces. The flow of an unbounded fluid over a surface such as flat plate is *external flow*.
- An *inviscid flow* is one in which the effect of viscosity is negligible. In inviscid flow, the fluid viscosity is assumed to be zero. All fluids possess viscosity, and accordingly, all real flows are *viscous*.
- A flow is said to be *irrotational* if the fluid particles while flowing do not rotate about their mass centres. On the other hand, a flow is said to be *rotational* when the fluid particles while flowing also rotate about their mass centres.
- *Streamline* at any instant can be defined as an imaginary line in the flow field so that the tangent to the line at any point represents the direction of the instantaneous velocity of that point. For unsteady flows the streamline pattern changes with time. From the definition of streamline, it can be written

  $$\vec{V} \times d\vec{S} = 0$$

  A bundle of neighbouring streamlines may be imagined to form a passage through which the fluid flows. This passage is known as a *stream tube*.
- A *pathline* is the actual trajectory through space of a selected fluid article during a time of interval.
- A *streakline* at any instant of time is the temporary locations of all particles that have passed through a fixed point in the flow field.
- In a steady flow, the streamlines, path lines and streak lines are identical.

- Three-dimensional continuity equation in differential form is given by

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)+\frac{\partial \rho}{\partial t}=0$$

For steady flow, the continuity equation is given by

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

For incompressible (*both steady as well as unsteady*) flow, the continuity equation is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

For two-dimensional incompressible (*both steady as well as unsteady*) flow continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

REVIEW QUESTIONS

13.1 Explain Lagrangian and Eulerian methods of describing fluid flow.
13.2 Distinguish between:
  (a) Steady flow and unsteady flow
  (b) Uniform flow and non-uniform flow.
  (c) Incompressible and compressible flow.
13.3 Give examples of laminar flow, turbulent flow, steady flow, unsteady flow, uniform flow and non-uniform flow.
13.4 What do you mean by one-, two- and three-dimensional flows?
13.5 Define Reynolds number. State its significance regarding the determination of type of flow-laminar and turbulent.
13.6 Explain the terms:
  (a) Streamline
  (b) Streakline
  (c) Pathline
13.7 Define streamline. What do streamlines indicate?
13.8 Distinguish between a pathline and a streak line.
13.9 What does the smoke emitting from a lighted cigarette represent, streamline or pathline or streakline? Why?
13.10 Derive an expression for continuity for three dimensional flow and reduce it for steady, incompressible two dimensional flow.

NUMERICAL PROBLEMS

13.1 A 30 cm diameter pipe 50 km long transport oil from a tanker to the shore at 0.02 m^3/s. Find the Reynolds number and comment on the type the flow. The dynamic viscosity and density of oil are 0.1 Ns/m^2 and 850 kg/m^3 respectively.

13.2 A fluid flow is represented by the velocity field $\vec{V} = a x \hat{i} + a y \hat{j}$, where is a constant. Find the equation of streamline passing through a point (1, 2).

13.3 For the following flows find the equation of streamline:

(i) $\vec{V} = 2y\hat{i} - x^2\hat{j}$ passing through the point (1, 2)

(ii) $\vec{V} = 4x\hat{i} + 3y\hat{j}$ passing through the point (1, 4).

13.4 Obtain the equation of the streamlines for the velocity field given as $\vec{V} = 2x^3\hat{i} - 6x^2 y\hat{j}$

13.5 In a flow the velocity vector is given by $\vec{V} = 4x\hat{i} - 3y\hat{j} - 5z\hat{k}$. Determine the equation of the streamline passing through a point (1, 1, 1).

13.6 A three-dimensional velocity field is given by $u = -x$, $v = 2y$, $w = 5 - z$. Find the equation of streamline through (1, 2, 1).

13.7 A two-dimensional flow is described in the Lagrangian system as

$$x = x_0 e^{-kt} + y_0(1 - e^{-2kt}) \text{ and } y = y_0 e^{kt}$$

Find the equation of path line of the particle.

13.8 A 40 cm diameter pipe is in series with a 30 cm diameter pipe. The volume flow rate of water in the system is 4 m^3/s. What is the average velocity of flow in each pipe?

13.9 A pipe 30 cm diameter is carrying oil of density 900 kg/m^3 with an average velocity of 2 m/s. Calculate the discharge. If the pipe bifurcates into two pipes of 15 cm each, find the average velocity of oil in the 15 cm diameter pipe.

13.10 Fluid flows through a pipeline which contracts from 45 cm diameter at *A* to 30 cm diameter at *B* and then branches into two pipes *C* and *D* (Fig. 13.16). The diameter of the pipe *C* is 15 cm and diameter of the pipe *D* is 20 cm. If the velocity at *A* be 1.8 m/s and that at *D* be 3.6 m/s. Determine

(a) Velocity at *B*, and

(b) Discharge at *C* and *D*

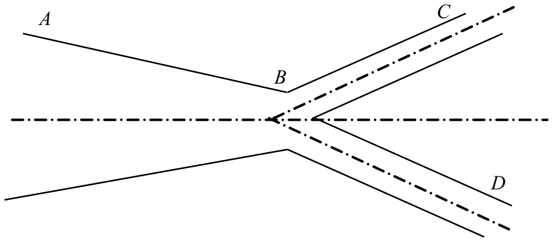


Figure 13.16

13.11 In a two-dimensional incompressible low, the velocity component in the *x*-direction is given by $u = \dfrac{2x}{x^2 + y^2}$. Evaluate the velocity field, if $v = 0$ at $y = 0$.

13.12 If $u = x^2 + y^2z^2$, $v = -xy - yz - zx$, determine the velocity component w, which will satisfy continuity for incompressible flow.

13.13 Check whether the following sets of velocity components satisfy the continuity equation of incompressible flow

(i) $u = x + y, v = x - y$

(ii) $u = 3xy, v = x^3 - xy^3$

(iii) $u = 2x^2 - xy + z^2, v = x^2 - 4xy + y^2, w = 2xy - yz + y^2$

MULTIPLE-CHOICE QUESTIONS

Choose the most appropriate answer.

13.1 The necessary condition for the flow to be steady is that

(a) the velocity does not change from place to place at any instant
(b) the velocity is constant at a point with respect to time
(c) the velocity changes at a point with respect to time
(d) the velocity changes with location at any instant

13.2 Uniform flow occurs when

(a) the velocity does not change from place to place at any instant
(b) the velocity is constant at a point with respect to time
(c) the velocity changes at a point with respect to time
(d) the velocity changes with location at any instant

13.3 During the opening of a valve in a pipeline, the flow is

(a) steady (b) unsteady (c) uniform (d) laminar

13.4 One-dimensional flow is

(a) uniform flow
(b) steady flow
(c) restricted to flow in a straight line
(d) one which neglects changes in fluid properties in a transverse direction

13.5 For pipes, turbulent flow occurs when Reynolds number is

(a) less than 2000 (b) between 2000 and 4000
(c) more than 4000 (d) less than 4000

13.6 For an ideal fluid flow the Reynolds number is

(a) 2100 (b) 100 (c) zero (d) infinity

13.7 In laminar flow

(a) Newton's law of viscosity applies
(b) fluid particles move in irregular and haphazard paths
(c) the viscosity is unimportant
(d) All of the above

13.8 Laminar flow generally occurs for cases involving

(a) very slow motions (b) highly viscous fluids
(c) very narrow passages or capillary tubes (d) All of the above

13.9 Velocity vector of a flow field is given as $\vec{V} = 2xy\vec{i} - x^2z\vec{j}$. The velocity vector at (1, 2, 1) is

(a) $4\vec{i} - \vec{j}$ (b) $4\vec{i} - \vec{k}$ (c) $\vec{i} - 4\vec{j}$ (d) $\vec{i} - 4\vec{k}$

13.10 A streamline is a line

(a) which is along the path of a particle
(b) across which there is no flow
(c) on which tangent drawn at any point gives the direction of velocity
(d) Both (b) and (c)

13.11 The velocity field is given by $\vec{V} = 3xy\vec{i} + \frac{3}{2}\left(x^2 - y^2\right)\vec{j}$. What is the relevant equation of a streamline?

(a) $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$
(b) $\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$
(c) $\frac{dy}{dx} = \frac{x^2 - y^2}{3xy}$
(d) $\frac{dy}{dx} = \frac{xy}{x^2 - y^2}$

13.12 The streamline shapes of the following 2-D velocity field: $u = -y$, $v = x$ will be

(a) circle
(b) parabola
(c) ellipse
(d) rectangular hyperbola

13.13 A fluid flow is represented by the velocity field $\vec{V} = ax\hat{i} + ay\hat{j}$, where a is a constant. The equation of streamline passing through a point (1, 2) is

(a) $x - 2y = 0$
(b) $2x - y = 0$
(c) $2x + y = 0$
(d) $x + 2y = 0$

13.14 A two-dimensional flow field has velocities along the x and y directions given by $u = x^2t$ and $v = -2xyt$ respectively, where t is time. The equation of streamlines is

(a) xy = constant
(b) xy^2 = constant
(c) x^2y = constant
(d) Not possible to determine

13.15 The equation of a streamline passing through the origin in a flow field $u = \cos\theta$; $v = \sin\theta$ for a constant is

(a) $y = x^3$
(b) $y = x\cos^2\theta$
(c) $y = x\tan\theta$
(d) $y = \sin\theta$

13.16 A path line describes

(a) the velocity direction at all points on the line
(b) the path followed by particles in a flow
(c) the path over a period of times of a single particle that has passed out at a point
(d) the instantaneous position of all particles that have passed a point

13.17 Streamline, pathline and streakline are identical when
- (a) the flow is uniform
- (b) the flow is steady
- (c) the flow velocities do not change steadily with time
- (d) the flow is neither steady nor uniform.

13.18 The continuity equation is the result of application of the following law to the flow field
- (a) Conservation of momentum
- (b) Conservation of energy
- (c) Conservation of force
- (d) Conservation of mass

13.19 The continuity equation (at two sections 1 and 2) for an incompressible fluid is given as
- (a) $\rho_1 A_1 V_1^2 = \rho_2 A_2 V_2^2$
- (b) $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
- (c) $A_1 V_1 = A_2 V_2$
- (d) $\rho_1^2 A_1 V_1 = \rho_2^2 A_2 V_2$

13.20 An ideal fluid flow must satisfy
- (a) continuity equation
- (b) Newton's law of viscosity
- (c) Pascal's law
- (d) None of the above

13.21 For two-dimensional incompressible flow, if the x component of velocity is $u = Ae^x$, then what is the y-component of velocity?
- (a) $-Ae^{-x}$
- (b) $-Ae^{-x}y$
- (c) Ae^y
- (d) $-Ae^x y$

13.22 For the continuity equation given by $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ ($\nabla . \vec{V}=0$, where $\vec{V}$ is the velocity vector) to be valid, which one of the following is a necessary condition?
- (a) steady flow
- (b) incompressible flow
- (c) inviscid flow
- (d) irrotational flow

13.23 The general form of expression for the continuity equation in a Cartesian coordinate system for incompressible or compressible flow is given by
- (a) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$
- (b) $\frac{\partial (pu)}{\partial x}+\frac{\partial (pv)}{\partial y}+\frac{\partial (pw)}{\partial z}=0$
- (c) $\frac{\partial p}{\partial t}+\frac{\partial (pu)}{\partial x}+\frac{\partial (pv)}{\partial y}+\frac{\partial (pw)}{az}=0$
- (d) $\frac{\partial p}{\partial t}+\frac{\partial (pu)}{\partial x}+\frac{\partial (pv)}{\partial y}+\frac{\partial (pw)}{az}=1$

CHAPTER

14

Dynamics of Ideal Fluids

14.1 INTRODUCTION

In Chapter 13, we dealt with the kinematics of fluid flow that includes the motion of the fluid without considering the force that causes the flow. In this chapter, the force analysis along with the motion attributes and their relationships are established. The forces that are present in fluid motion are the body forces and the surface forces. Differential equation is developed neglecting the viscous effects. It is a simplified form which is far reaching in reality. Nevertheless, it provides a lot of insight to understand the topic in its basic form. Without consideration of the viscous forces, the surface forces acting on the fluid element are the normal forces in the form of pressure.

14.2 EULER'S EQUATION OF MOTION ALONG A STREAMLINE

Consider the flow of an inviscid fluid along a streamline as shown in Fig. 14.1. The equations of motion are to be written in terms of the coordinates, distance along a streamline and the coordinate normal to the streamline. Applying Newton' second law in the streamwise direction (s-direction) to the fluid element of area , we have

$$\sum F_s = ma_s \tag{14.1}$$

where a_s is the acceleration of the fluid particle along the streamline.

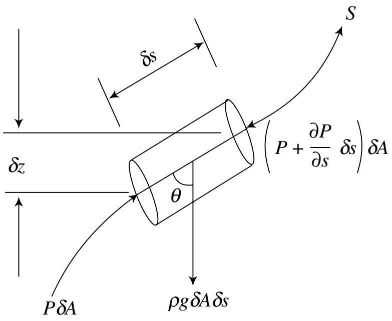


Figure 14.1

In the absence of viscous forces, forces acting in the s-direction are the pressure forces and the component of gravity force in the *s*-direction. Let the pressure at the lower face of the fluid element be *P*. The pressure acting on the other face is $P+\frac{\partial P}{\partial s}\delta s$. Therefore, the pressure force acting on the lower and upper faces are $P\delta A$ and $\left(P+\frac{\partial P}{\partial s}\delta s\right)\delta A$ respectively. Considering gravity as the only body force, the body force acting on the element is $\rho g\delta s\delta A$. Thus the component of body force along the streamline is $-\rho g\sin\theta\delta s\delta A$ (where θ is the angle between the tangent to the streamline and the direction of the gravity force). Hence, Eq. (14.1) becomes

$$P\delta A-\left(P+\frac{\partial P}{\partial s}\delta s\right)\delta A-\rho g\sin\theta\delta s\delta A=\rho a_s\delta s\delta A \tag{14.2}$$

Simplifying Eq. (14.2), we have

$$-\frac{\partial P}{\partial s}-\rho g\sin\theta=\rho a_s$$

or

$$-\frac{1}{\rho}\frac{\partial P}{\partial s}-g\frac{\partial z}{\partial s}=a_s \tag{14.3}$$

Along any streamline since the velocity *V* is a function of space and time i.e., $V=V(s,t)$, one can write

$$dV=\frac{\partial V}{\partial t}dt+\frac{\partial V}{\partial s}ds$$

or

$$\frac{dV}{dt}=\frac{\partial V}{\partial t}+\frac{\partial V}{\partial s}\frac{ds}{dt}$$

or
$$a_s = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \qquad \left[\because a_s = \frac{dV}{dt}\right]$$

Substituting the value of a_s in Eq. (14.3), we get

$$-\frac{1}{\rho}\frac{\partial P}{\partial s} - g\frac{\partial z}{\partial s} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \tag{14.4}$$

Equation (14.4) is known as the *Euler's equation of motion* along a streamline.

For steady flow, $\frac{\partial V}{\partial t} = 0$, then Eq. (14.4) becomes

$$-\frac{1}{\rho}\frac{\partial P}{\partial s} - g\frac{\partial z}{\partial s} = V\frac{\partial V}{\partial s}$$

or
$$\frac{1}{\rho}\frac{\partial P}{\partial s} + V\frac{\partial V}{\partial s} + g\frac{\partial z}{\partial s} = 0$$

Since, *s* is the only independent variable; the total differential may replace the partial

$$\frac{dP}{\rho} + VdV + gdz = 0 \tag{14.5}$$

Equation (14.5) is another form of Euler's equation of motion along a streamline, which is valid for steady flow.

Note: *Eq. (14.5) is valid for inviscid, steady flow along a streamline.*

14.3 BERNOULLI'S EQUATION

Euler's equation of motion along a streamline for steady flow can be written as (Eq. 14.5)

$$\frac{dP}{\rho} + VdV + gdz = 0$$

Integrating the above equation, we have

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = C \tag{14.6}$$

where *C* is an integration constant.

In case of an incompressible fluid (density does not change with change in pressure), Eq. (14.6) can be written as

$$\frac{P}{\rho}+\frac{V^2}{2}+gz=C \tag{14.7}$$

Equation (14.7) is known as *Bernoulli's equation*. The constant C is known as Bernoulli's constant which is constant for a streamline and varies from one streamline to another. Each term of Eq. (14.7) can be interpreted as a form of energy per unit mass. Here, $\frac{P}{\rho}$ represents the flow energy (or flow work) per unit mass, $\frac{V^2}{2}$ represents the kinetic energy per unit mass and gz represents the potential energy per unit mass.

Dividing by g, Eq. (14.7) becomes

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=C \tag{14.8}$$

Equation (14.8) is another form of Bernoulli's equation. Each term of Eq. (14.8) can be interpreted as a form of energy per unit weight (also known as head in fluid mechanics). Here, $\frac{P}{\rho g}$ is the *pressure head* (flow energy per unit weight), which represents the height of a fluid column that produces the static pressure P. The term $\frac{V^2}{2g}$ is the *velocity head* (kinetic energy per unit weight) and z and is the *potential head* (potential energy per unit weight). Therefore, the Bernoulli's equation can be viewed as an expression of mechanical energy balance and can be stated as follows:

During steady, inviscid flow of an incompressible fluid along a streamline, total mechanical energy at any point is constant. The total mechanical energy consists of flow energy, kinetic energy and potential energy.

Applying Bernoulli's equation between two points 1 and 2 along the same streamline, we have

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \tag{14.9}$$

Note: *The assumptions made in the derivation of Bernoulli's equation are*

(i) The flow is inviscid.
(ii) The flow is along a streamline.
(iii) The flow is steady.
(iv) The fluid is incompressible (density does not change with change in pressure).

Note: *Bernoulli's equation deals with the law of conservation of mechanical energy.*

Note: The equation $\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$ *holds true between any two points in the flow field provided that the flow is irrotational, inviscid, steady and fluid is incompressible.*

14.4 STATIC, DYNAMIC, STAGNATION AND TOTAL PRESSURES

Bernoulli's equation can be written as

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=\text{Constant}$$

or

$$P+\rho\frac{V^2}{2}+\rho gz=\text{Constant} \tag{14.10}$$

The physical meaning of different terms appearing in the left hand side of Eq. (14.10) is as follows:

14.4.1 Static Pressure

The pressure P as appear in Eq. (14.10) is often referred to as the *static pressure*. It is the pressure caused by molecular collisions and can be felt at any point by an observer moving with the flow. To such an observer, the fluid appears to be static or stationary, so this pressure is often called the static pressure.

14.4.2 Dynamic Pressure

The term $\rho\frac{V^2}{2}$ in Eq. (14.10) is called the *dynamic pressure*. It represents the pressure increase that would occur if all the kinetic energy of a fluid particle in a frictionless flow were converted into a corresponding increase in pressure energy.

14.4.3 Hydrostatic Pressure

The term ρgz in Eq. (14.10) is called the *hydrostatic pressure*. It represents the change in the static pressure that would occur if the fluid moved along the streamline to an elevation of zero.

14.4.4 Stagnation Pressure

It is the pressure that could result at a point in the flow, if the flow were brought to rest in an isentropic process. Applying Bernoulli's equation between two points, one just upstream of the stagnation point and the other, the stagnation point itself, we have

$$\frac{P}{\rho g}+\frac{V^2}{2g}+z=\frac{P_s}{\rho g}+\cancel{\frac{V_s^2}{2g}}^{\,0}+z \quad (p_s \text{ is the stagnation pressure})$$

where V_s is the velocity at stagnation point which is zero from its definition.

or $$P_s = P+\frac{1}{2}\rho V^2 \qquad (14.11)$$

Therefore, the sum of the static pressure and dynamic pressure is called the *stagnation pressure*.

14.4.5 Total Pressure

The sum of the static, dynamic and hydrostatic pressures $\left(P+\rho\frac{V^2}{2}+\rho g z\right)$ is referred to as the *total pressure*. Therefore, *Bernoulli's equation states that the total pressure along a streamline is constant*. It is important to mention here that the sum of the pressure head $\left(\frac{P}{\rho g}\right)$ and the potential head (z) is called as *piezometric pressure head*.

> ***Note:*** *Fluid flows from higher piezometric pressure to lower piezometric pressure (not necessarily from higher static pressure to lower static pressure).*

Example 14.1 Water is flowing in a pipe of 200 mm diameter with an average velocity of 5 m/s. At a particular section 1, the pressure is measured to be 250 kN/m^2. If the section 1 is 7 m above the datum, determine total head of water.

Solution

Average velocity of flow $V = 5$ m/s

Pressure at section 1 $P = 250 \text{ kN/m}^2 = 250\times 10^3 \text{ N/m}^2$

Pressure head at section 1 is $\frac{P}{\rho g} = \frac{250\times 10^3}{1000\times 9.81} = 25.484$ m of water

Velocity head at section 1 is $\frac{V^2}{2g} = \frac{5^2}{2\times 9.81} = 1.274$ m of water

Datum head at section 1 is $z = 7$ m

Total head of water at section 1 is $\frac{P}{\rho g}+\frac{V^2}{2g}+z = 25.484+1.274+7 = 33.758$ m

Example 14.2 Water is flowing through a pipe having diameters 30 cm and 20 cm at section 1 and 2 respectively. The average velocity of water at section 1 is 4 m/s. Find the velocity head at the section 1 and 2 and also rate of discharge.

Solution

Diameter of pipe at section 1 $D_1 = 30 \text{ cm} = 0.3 \text{ m}$

Diameter of pipe at section 2 $D_2 = 20 \text{ cm} = 0.2 \text{ m}$

Cross-sectional area at section 1 is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.0707 \text{ m}^2$

Cross-sectional area at section 2 is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$

Average velocity of water at section 1 $V_1 = 4 \text{ m/s}$

Velocity head at section 1 is $\frac{V_1^2}{2g} = \frac{4^2}{2 \times 9.81} = 0.815$ m of water

The rate of discharge is found to be

$$Q = A_1 V_1 = 0.0707 \times 4 = 0.2828 \text{ m}^3/\text{s}$$

Let the average velocity at section 2 be V_2

From continuity equation, we have

$$Q = A_1 V_1 = A_2 V_2$$

or $$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0707 \times 4}{0.0314} = 9 \text{ m/s}$$

Velocity head at section 2 is $\frac{V_2^2}{2g} = \frac{9^2}{2 \times 9.81} = 4.128$ m of water

Example 14.3 A vertical tapering pipe is 2 m long. The diameter of the pipe is 20 cm at the top end and 10 cm at the bottom end. If 30 litres/sec of water flows through the pipe, find the difference in pressure between the two ends of the pipe. Neglect losses.

Solution

The pipeline is schematically shown in Fig. 14.2. Let 1 and 2 respectively designate the bottom and top end of the pipeline.

Diameter of pipe at section 1 $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

Diameter of pipe at section 2 $D_2 = 20 \text{ cm} = 0.2 \text{ m}$

Cross-sectional area at section 1 is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$

Cross-sectional area at section 2 is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$

Discharge $Q = 30 \text{ litres/s} = 30 \times 10^{-3} \text{m}^3/\text{s} = 0.03 \text{ m}^3/\text{s}$

Difference in datum head between sections 1 and 2 $z_2 - z_1 = 2 \text{ m}$

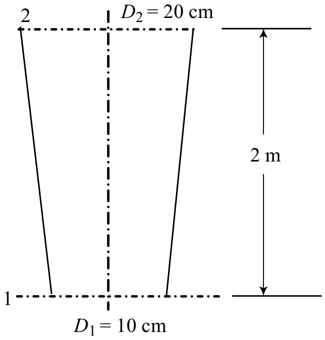


Figure 14.2

Average velocity at section 1 is $$V_1 = \frac{Q}{A_1} = \frac{0.03 \text{ m}^3/\text{s}}{0.00785 \text{ m}^2} = 3.82 \text{ m/s}$$

Average velocity at section 2 is $$V_2 = \frac{Q}{A_2} = \frac{0.03 \text{ m}^3/\text{s}}{0.0314 \text{ m}^2} = 0.955 \text{ m/s}$$

Applying Bernoulli's equation between sections 1 and 2 along a streamline, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + z_2 - z_1$$

or $$\frac{P_1 - P_2}{\rho g} = \frac{0.955^2 - 3.82^2}{2 \times 9.81} + 2 = -0.697 + 2 = 1.303 \text{ m of water}$$

or $$P_1 - P_2 = 1.303\rho g = 1.303 \times 1000 \times 9.81 \text{N/m}^2$$

or $$P_1 - P_2 = 12.78 \times 10^3 \text{ N/m}^2 = 12.78 \text{ kN/m}^2$$

Example 14.4 An oil of density 900 kg/m^3 is flowing through a vertical pipe having diameters 30 cm and 20 cm at section 1 and 2 respectively. The rate of flow through pipe is 50 litres/s. The section 1 is 9 m above datum and section 2 is 5 m above datum. If the pressure at section 1 is 300 kN/m^2, find the intensity of pressure at section 2. Neglect friction.

Solution

The pipeline is schematically shown in Fig. 14.3.

Density of oil $\rho = 900\ \text{kg/m}^3$

Diameter of pipe at section 1 $D_1 = 30\ \text{cm} = 0.3\ \text{m}$

Diameter of pipe at section 2 $D_2 = 20\ \text{cm} = 0.2\ \text{m}$

Cross-sectional area at section 1 is $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.0707\ \text{m}^2$

Cross-sectional area at section 2 is $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\ \text{m}^2$

Discharge $Q = 50\ \text{litres/s} = 50\times10^{-3}\text{m}^3/\text{s} = 0.05\ \text{m}^3/\text{s}$

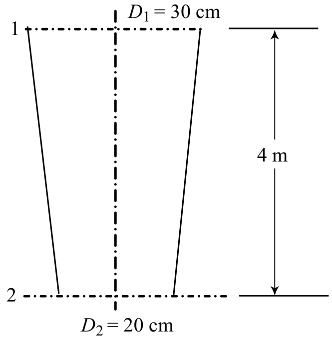


Figure 14.3

Difference in datum head between sections 1 and 2 $z_1 - z_2 = 9 - 5 = 4\ \text{m}$

Pressure at 1 $P1 = 300\ \text{kN/m}^2 = 300\times10^3\ \text{N/m}^2$

Average velocity at section 1 is $V_1 = \frac{Q}{A_1} = \frac{0.05}{0.0707} = 0.707\ \text{m/s}$

Average velocity at section 2 is $V_2 = \frac{Q}{A_2} = \frac{0.05}{0.0314} = 1.592\ \text{m/s}$

Applying Bernoulli's equation between sections 1 and 2 along a streamline, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{P_1}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + z_1 - z_2 = \frac{P_2}{\rho g}$$

or $$\frac{300\times10^3}{1000\times9.81} + \frac{0.707^2 - 1.592^2}{2\times9.81} + 4 = \frac{P_2}{\rho g}$$

or $$33.979 - 0.1037 + 4 = \frac{P_2}{\rho g}$$

or $$\frac{P_2}{900 \times 9.81} = 37.8753$$

or $$P_2 = 900 \times 9.81 \times 37.8753 = 336232 \text{N/m}^2 = 336.23 \text{ kN/m}^2$$

Example 14.5 Water flows through a tapering pipe as shown in Fig. 14.4. The diameter at sections 1 and 2 are 10 cm and 20 cm respectively, and the heights above a horizontal datum are 3 and 5 m respectively. The pressure at 1 is 30 kN/m^2. Water flow rate is 0.03 m^3/s. Estimate the pressure at section 2.

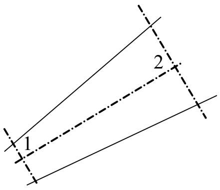


Figure 14.4

Solution

Diameter of pipe at section 1 D_1 = 10 cm = 0.1 m
Diameter of pipe at section 2 D_2 = 20 cm = 0.2 m
Pressure at section 1 P_1 = 10 kN/m^2 = 30 × 10^3 N/m^2
Height of section 1 above datum z_1 = 3 m
Height of section 2 above datum z_2 = 5 m
Volume flow rate of water Q = 0.03 m^3/s

Cross-sectional area at section 1 is $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$$

Cross-sectional area at section 2 is $$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$$

From continuity equation, we have

$$Q = A_1 V_1 = A_2 V_2$$

Thus, the average velocity at section 1 is $$V_1 = \frac{Q}{A_1} = \frac{0.03 \text{ m}^3/\text{s}}{0.00785 \text{ m}^2} = 3.82 \text{ m/s}$$

Average velocity at section 2 is $$V_2 = \frac{Q}{A_2} = \frac{0.03 \text{ m}^3/\text{s}}{0.0314 \text{ m}^2} = 0.955 \text{ m/s}$$

Applying Bernoulli's equation between sections 1 and 2 along a streamline, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{30 \times 10^3}{1000 \times 9.81} + \frac{3.82^2}{2 \times 9.81} + 3 = \frac{P_2}{1000 \times 9.81} + \frac{0.955^2}{2 \times 9.81} + 5$$

or $$3.058 + 0.744 + 3 = \frac{P_2}{1000 \times 9.81} + 0.046 + 5$$

or $$\frac{P_2}{1000 \times 9.81} = 1.756$$

or $$P_2 = 1.756 \times 1000 \times 9.81 = 17226.36 \text{ N/m}^2 = 17.226 \text{ kN/m}^2$$

Example 14.6 A 500 m long pipe has a slope of 1 in 100 and tapers from 1 m diameter at higher end to 0.5 m diameter at the lower end. It carries an oil of specific gravity 0.85 at a rate of 100 litres/s. If the pressure at high end is 250 kN/m^2, find the pressure at the lower end. Neglect losses due to friction.

Solution The pipe is schematically shown in Fig. 14.5. Let 1 and 2 respectively designate the lower and higher end of the pipe.

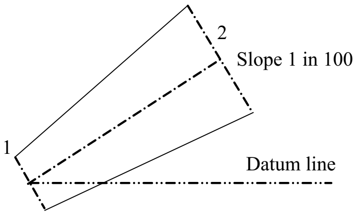


Figure 14.5

Specific gravity of oil $S = 0.85$

Density of oil $\rho = 0.85 \times 1000 = 850 \text{ kg/m}^3$

Diameter of pipe at lower end $D_1 = 0.5$ m

Diameter of pipe at higher end $D_2 = 1$ m

Pressure at higher end $P_2 = 250 \text{ kN/m}^2 = 250 \times 10^3 \text{ N/m}^2$

Volume flow rate of oil $Q = 100 \text{ lit/s} = \frac{100}{1000} \text{m}^3\text{/s} = 0.1 \text{ m}^3\text{/s}$

Cross-sectional area at lower end is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$

Cross-sectional area at higher end is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ m}^2$

Length of pipe $L = 500$ m

Slope of pipe = 1 in 100

Let the datum line passes through the centre of the lower end. Then

Height of lower end above datum $z_1 = 0$

Height of higher end above datum $z_2 = \frac{1}{100} \times 500 = 5$ m

From continuity equation, we have

$$Q = A_1 V_1 = A_2 V_2$$

Thus, the average velocity at lower end is $V_1 = \dfrac{Q}{A_1} = \dfrac{0.1}{0.1963} = 0.509$ m/s

Average velocity at higher end is $V_2 = V_2 = \dfrac{Q}{A_2} = \dfrac{0.1}{0.7854} = 0.127$ m/s

Applying Bernoulli's equation between sections 1 and 2 along a streamline, we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{P_1}{\rho g} + \frac{0.509^2}{2 \times 9.81} + 0 = \frac{250 \times 10^3}{1000 \times 9.81} + \frac{0.127^2}{2 \times 9.81} + 5$$

or $$\frac{P_1}{\rho g} + 0.0132 + 0 = 29.98 + 0.00082 + 5$$

or $$\frac{P_1}{850 \times 9.81} = 34.9676 \text{ m of oil}$$

or $$P_1 = 34.9676 \times 850 \times 9.81 = 291577 \text{ N/m}^2 = 291.577 \text{ kN/m}^2$$

Example 14.7 Water flows through a conical tube fixed vertically with its smaller end upwards. The average velocities at the smaller and larger end are 4.5 m/s and 1.5 m/s respectively. Length of the conical tube is 1.5 m. The pressure at the upper end is equivalent to a head of 10 m of water. Neglecting losses, determine the pressure at the lower end of the tube. Also find the piezometric head at both the ends.

Solution The conical tube is schematically shown in Fig. 14.6. Let 1 and 2 respectively designate the smaller and larger end of the pipe.

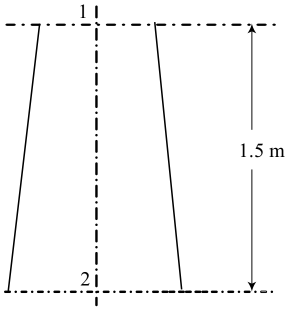


Figure 14.6

Length of the pipe $L = z_1 - z_2 = 1.5$ m

Pressure head at smaller end $\dfrac{P_1}{\rho g} = 10$ m of water

Average velocity at smaller end $V_1 = 4.5$ m/s

Average velocity at larger end $V_2 = 1.5$ m/s

Applying Bernoulli's equation between sections 1 and 2 along a streamline, we have

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

or $$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1-z_2=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}$$

or $$10+\frac{4.5^2}{2\times 9.81}+1.5=\frac{P_2}{\rho g}+\frac{1.5^2}{2\times 9.81}$$

or $$\frac{P_2}{1000\times 9.81}=12.42 \text{ m of water}$$

or $$P_2=12.42\times 1000\times 9.81=121.84\times 10^3 \text{ N/m}^2=121.84 \text{ kN/m}^2$$

Let us consider that the larger end (section 2) represents the reference datum.

Datum head at 1 $z_1 = 1.5$ m

Datum head at 2 $z_2 = 0$ m

Piezometric head at 1 is then $\dfrac{P_1}{\rho g}+z_1 = 10+1.5=11.5$ m of water

Piezometric head at 2 is then $\dfrac{P_2}{\rho g}+z_2 = 12.42+0=12.42$ m of water

14.5 ENERGY EQUATION FOR REAL FLUID

For flow of real fluids, there is a viscous force that resists the flow. Because of the resistive force there is a loss of energy during the flow of fluids. This energy loss is to be taken into consideration during the derivation of energy equation. Applying energy equation between 1 and 2 with consideration of loss due to fluid friction, one can write

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f \tag{14.12}$$

where h_f is the loss of head between 1 and 2. Detailed analysis on loss of energy due to fluid friction is not included in this text. Equation (14.12) looks similar to Bernoulli's equation (Eq. (14.8)) with an additional term h_f on the right-hand side which represents the loss of head between points 1 and 2.

Example 14.8 Water flows vertically upwards through a pipe of 1 m diameter and 10 m length. The pressure at the upper end of the pipe is 5 m of water and the head loss due to friction is 1 m of water column. When water flows at an average velocity of 5 m/s, find the pressure at the lower end of the pipe.

Solution The flow arrangement is schematically shown in Fig. 14.7.

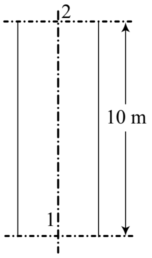


Figure 14.7

Applying energy equation between sections 1 and 2, one can write

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f$$

or $$\frac{P_1}{\rho g}+\frac{V^2}{2g}+0=5+\frac{V^2}{2g}+10+1 \qquad [\because V_1=V_2=V]$$

or $$\frac{P_1}{\rho g}=16 \text{ m of water column}$$

or $$P_1=1000\times 9.81\times 16=156.96\times 10^3 \text{ N/m}^2=156.96 \text{ kN/m}^2$$

Example 14.9 Water is flowing through a pipeline at a rate of 0.04 m^3/s as shown in Fig. 14.8. The pipeline is 10 cm in diameter and it is at an elevation of 80 m at section *A* (Fig. 14.8). At section *B* it is at an elevation of 82 m and has diameter of 20 cm. The pressure of water at *A* is 40 kN/m^2 and the energy loss in pipe between section *A* and *B* is 0.3 m of water. Calculate pressure at *B* if flow is from *A* to *B*.

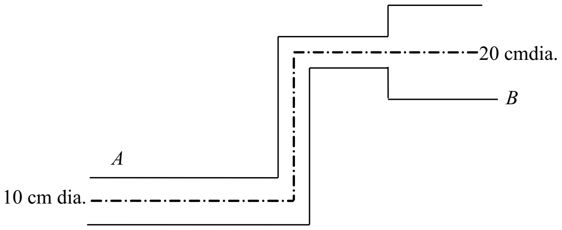


Figure 14.8

Solution

Diameter of pipe at section A $\quad D_A = 10\text{ cm} = 0.1\text{ m}$

Diameter of pipe at section B $\quad D_B = 20\text{ cm} = 0.2\text{ m}$

Pressure at section A $\quad P_A = 40\text{ kN/m}^2 = 40 \times 1000\text{ N/m}^2$

Height of section A above reference datum $\quad z_A = 80\text{ m}$

Height of section B above reference datum $\quad z_B = 82\text{ m}$

Volume flow rate of water $\quad Q = 0.04\text{ m3/s}$

Loss of energy between A and B $\quad h_{A-B} = 0.3\text{ m of water}$

Cross-sectional area at section A is $\quad A_A = \dfrac{\pi}{4} D_A{}^2 = \dfrac{\pi}{4}(0.1)^2 = 0.00785\text{ m}^2$

Cross-sectional area at section B is $\quad A_B = \dfrac{\pi}{4} D_B{}^2 = \dfrac{\pi}{4}(0.2)^2 = 0.0314\text{ m}^2$

Average velocity at section A is $\quad V_A = \dfrac{Q}{A_A} = \dfrac{0.04}{0.00785} = 5.096\text{ m/s}$

Average velocity at section B is $\quad V_B = V_B = \dfrac{Q}{A_B} = \dfrac{0.04}{0.0314} = 1.274\text{ m/s}$

Applying energy equation between sections A and B, we have

$$\frac{P_A}{\rho g} + \frac{V_A{}^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B{}^2}{2g} + z_B \ + h_{A-B}$$

or
$$\frac{40 \times 1000}{1000 \times 9.81} + \frac{5.096^2}{2 \times 9.81} + 80 = \frac{P_B}{\rho g} + \frac{1.274^2}{2 \times 9.81} + 82 + 0.3$$

or
$$\frac{P_B}{1000 \times 9.81} = 3.018$$

or
$$P_B = 1000 \times 9.81 \times 3.018\text{ N/m}^2 = 29.61 \times 10^3\text{ N/m}^2 = 29.61\text{ kN/m}^2$$

Example 14.10 Water is flowing steadily in a 30 cm diameter pipe at an average velocity of 4 m/s. At points A and B measurements of pressure and elevation are 200 kN/m^2 and 150 kN/m^2 and 12 m and 17 m respectively. Find the loss between the two points.

Solution

Diameter of pipe $\quad D = 30\text{ cm} = 0.3\text{ m}$

Average velocity $\quad V = 4\text{ m/s}$

Pressure at point A $\quad P_A = 200\text{ kN/m}^2 = 200 \times 1000\text{ N/m}^2$

Pressure at point B $\quad P_B = 150\text{ kN/m}^2 = 150 \times 1000\text{ N/m}^2$

Elevation of point A above reference datum $\quad z_A = 12\text{ m}$

Elevation of point B above reference datum $\quad z_B = 17\text{ m}$

Total mechanical energy at point *A* is

$$= \frac{P_A}{\rho g} + \frac{V_A^{\,2}}{2g} + z_A$$

$$= \frac{200 \times 10^3}{1000 \times 9.81} + \frac{4^2}{2 \times 9.81} + 12 = 20.387 + 0.815 + 12 = 33.202 \text{ m}$$

Total mechanical energy at point *B* is

$$= \frac{P_B}{\rho g} + \frac{V_B^{\,2}}{2g} + z_{\text{B}}$$

$$= \frac{150 \times 10^3}{1000 \times 9.81} + \frac{4^2}{2 \times 9.81} + 17 = 15.29 + 0.815 + 17 = 33.105 \text{ m}$$

Loss of head between points *A* and *B* is then

$$= 33.202 - 33.105 = 0.097 \text{ m of water column}$$

Example 14.11 At a certain location *A* of a pipeline carrying an oil of density 850 kg/m^3, the diameter is 80 cm, the pressure is 180 kN/m^2 and the average velocity is 5 m/s. At another section *B* which is 3 m higher than *A*, the diameter is 50 cm and the pressure is 100 kN/m^2. What is the direction of flow?

Solution

Density of oil	$\rho = 850$ kg/m^3
Diameter of pipe at location *A*	$D_A = 80$ cm = 0.8 m
Diameter of pipe at location *B*	$D_B = 50$ cm = 0.5 m
Average velocity at location *A*	$V_A = 5$ m/s
Pressure at location *A*	$P_A = 180$ kN/m^2 = 180 × 1000 N/m^2
Pressure at location *B*	$P_B = 100$ kN/m^2 = 100 × 1000 N/m^2

Let the datum line passes through the location *A*.

Height of location *A* above reference datum	$z_A = 0$
Height of location *B* above reference datum	$z_B = 3$ m
Cross-sectional area at location *A* is	$A_A = \frac{\pi}{4} D_A^{\,2} = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ m}^2$
Cross-sectional area at location *B* is	$A_B = \frac{\pi}{4} D_B^{\,2} = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$

From continuity equation, we have

$$Q = A_A V_A = A_B V_B$$

Thus, the average velocity at location *B* is

$$V_B = \frac{A_A V_A}{A_B} = \frac{0.5026 \times 5}{0.1963} = 12.8 \text{ m/s}$$

Total mechanical energy at location *A* is

$$= \frac{P_A}{\rho g} + \frac{{V_A}^2}{2g} + z_A$$

$$= \frac{180 \times 10^3}{1000 \times 9.81} + \frac{5^2}{2 \times 9.81} + 0 = 18.349 + 1.274 + 0 = 19.623 \text{ m}$$

Total mechanical energy at location *B* is

$$= \frac{P_B}{\rho g} + \frac{{V_B}^2}{2g} + z_{\text{B}}$$

$$= \frac{100 \times 10^3}{1000 \times 9.81} + \frac{12.8^2}{2 \times 9.81} + 3 = 10.194 + 8.35 + 3 = 21.544 \text{ m}$$

Since the total mechanical energy at location *B* is higher than that at location *A*, the flow takes place from location *B* to location *A*.

Note: *Example 6.11 illustrates that fluid flows from higher mechanical energy to lower mechanical energy (not necessarily from higher pressure to lower pressure).*

Example 14.12 A pipeline carrying water changes in diameter from 20 cm at section 1 to 40 cm diameter at section 2 which is 6 m at higher level. If the pressure at section 1 and 2 are 120 kN/m^2 and 80 kN/m^2 respectively and the discharge is 200 litres/s, determine the loss of head and the direction of flow.

Solution

Diameter of pipe at section 1 D_1 = 20 cm = 0.2 m
Diameter of pipe at section 2 D_2 = 40 cm = 0.4 m
Pressure at section 1 P_1 = 120 kN/m^2 = 130 × 1000 N/m^2
Pressure at section 2 P_2 = 80 kN/m^2 = 80 × 1000 N/m^2
Let us consider the reference datum for elevation is at section 1.
Height of section 1 above reference datum z_1 = 0 m
Height of section 2 above reference datum z_2 = 6 m
Volume flow rate of water Q = 200 litres/s = 0.2 m^3/s

Average velocity at section1 is $V_1 = \dfrac{Q}{A_1} = \dfrac{0.2}{\frac{\pi}{4}(0.2)^2} = 6.366$ m/s

Average velocity at section 2 is $V_2 = \dfrac{Q}{A_2} = \dfrac{0.2}{\frac{\pi}{4}(0.4)^2} = 1.59$ m/s

Total mechanical energy at section 1 is

$$= \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

$$= \frac{120 \times 10^3}{1000 \times 9.81} + \frac{6.366^2}{2 \times 9.81} + 0 = 14.298 \text{ m}$$

Total mechanical energy at section 2 is

$$= \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$= \frac{80 \times 10^3}{1000 \times 9.81} + \frac{1.59^2}{2 \times 9.81} + 6 = 14.284 \text{ m}$$

Loss of head between sections 1 and 2 is then

$$= 14.298 - 14.284 = 0.014 \text{ m of water column}$$

Since the total mechanical energy at section 1 is higher than that at section 2, the flow takes place from section 1 to section 2.

Example 14.13 Water is flowing vertically upwards through a pipeline having diameter 1 m and 0.5 m at the base and top respectively. The pressure at the lower end is 450 mm of mercury, while the pressure at the upper end is 20 kN/m^2. If the loss of head is 20% of difference in velocity head, calculate the discharge. The difference in the elevation is 4 m. The density of mercury is 13600 kg/m^3.

Solution

Let 1 and 2 respectively designate the lower end and upper end of the pipe.

Diameter of pipe at lower end $D_1 = 1$ m

Diameter of pipe at upper end $D_2 = 0.5$ m

Cross-sectional area at lower end is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ m}^2$

Cross-sectional area at upper end is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$

Pressure at the lower end $P_1 = 450 \text{ mm of Hg} = 0.45 \times 13600 \times 9.81 = 60037.2 \text{N/m}^2$

Pressure at the upper end $P_2 = 20 \text{kN/m}^2 = 20 \times 1000 \text{N/m}^2$

Difference in datum head between the upper end and lower end $z_2 - z_1 = 4$ m

Loss of energy head $h_{1-2} = 0.2 \frac{V_2^2 - V_1^2}{2g}$

Applying continuity equation between the lower end and upper end, we have

$$A_1V_1 = A_2V_2$$

or $$V_2 = \frac{A_1}{A_2}V_1 = \frac{0.7854}{0.1963}V_1 = 4V_1$$

Applying energy equation between the lower end and upper end, one can write

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{1-2}$$

or $$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1) + 0.2\frac{V_2^2 - V_1^2}{2g}$$

or $$1.2\frac{V_2^2 - V_1^2}{2g} = \frac{P_1 - P_2}{\rho g} - (z_2 - z_1)$$

or $$1.2\frac{(4V_1)^2 - V_1^2}{2g} = \frac{P_1 - P_2}{\rho g} - (z_2 - z_1) \qquad [\because V_2 = 4V_1]$$

or $$1.2\frac{15V_1^2}{2 \times 9.81} = \frac{60037.2 - 20000}{1000 \times 9.81} - 4 = 4.08 - 4 = 0.08$$

or $$V_1^2 = \frac{0.08 \times 2 \times 9.81}{1.2 \times 15} = 0.0872$$

or $$V_1 = \sqrt{0.0872} = 0.295 \text{ m/s}$$

The discharge is then found to be

$$Q = A_1V_1 = 0.7854 \times 0.295 = 0.2317 \text{ m}^3/\text{s}$$

14.6 APPLICATION OF BERNOULLI'S EQUATION FOR MEASUREMENT OF FLOW RATE THROUGH PIPES

Flow rate through a pipe is usually measured by providing a coaxial area contraction within the pipe and by recording the pressure drop across the contraction. Determination of the flow rate from the measurement of the concerned pressure drop depends on the straight forward application of Bernoulli's equation. Three different flow meters primarily operate on this principle. These are: (i) Venturimeter, (ii) Orificemeter and (iii) Flow nozzle. Here, we discuss the working principle of Venturimeter and Orificemeter. Further, the working principle of Pitot tube, commonly used to measure the velocity at a point in the flow field is discussed.

14.6.1 Venturimeter

The venturimeter, invented by the American Engineer Clemans Herschel (1842–1932) and named by him after Italian Giovanni Venturi (1746–1822). The venturimeter is one of the popular devices for measuring rate of flow in a pipe. It consists of a short converging conical tube leading to a cylindrical portion, called the throat, of smaller diameter than that of the pipeline, which is followed by a diverging section in which the diameter increases again to that of the main pipeline (Fig.14.9). The inlet and outlet diameters are the same as the diameter of the pipe in which it is to be installed. The velocity of flow increases in course of flow from pipe to throat (converging cone) and the pressure correspondingly decreases. The velocity reaches the maximum value at the throat and the pressure a minimum. The throat is followed by a diffuser which restores the pressure as nearly as possible to the original value. *The expansion angle of the diffuser is very small (usually 5° to 7°) to reduce the possibility of flow separation*. The size of a venturimeter is specified by the pipe and throat diameter, e.g., a 300 by 150 mm venturimeter fits a 300 mm diameter pipe and has a 150 mm diameter throat. For accurate results the venturimeter should be preceded by at least 10 diameters of straight pipe. The pressure difference from which the volume flow rate can be determined is measured between the entry section 1 and the throat section 2, by means of a differential U-tube manometer (Fig. 14.9).

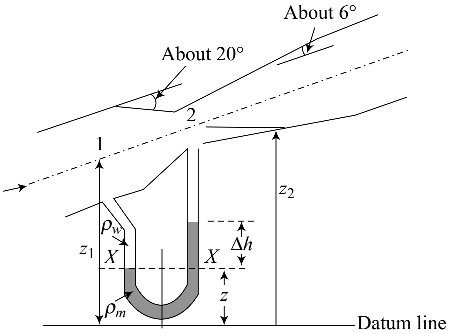


Figure 14.9 *Venturimeter*

Assuming that there is no loss of energy and applying Bernoulli's equation across sections 1 and 2, along a streamline, we get

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

$$V_2^2-V_1^2=2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right] \tag{14.13}$$

Applying continuity equation between sections 1 and 2, one can write

$$A_1V_1=A_2V_2$$

or $$V_2=\frac{A_1}{A_2}V_1$$

Substituting the expression of V_2 in Eq. (14.13), we have

$$V_1^2\left[\left(\frac{A_1}{A_2}\right)^2-1\right]=2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]$$

$$V_1 = \frac{A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

Volume flow rate then can be found to be

$$Q_{th} = A_1 V_1 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]} \quad (14.14)$$

or $$Q_{th} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gH} \quad (14.15)$$

where $$H = \left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right] \quad (14.16)$$

The value of H in Eq. (14.16) can be found from the reading of the U-tube differential manometer (Fig. 14.9). Assuming that the connections to the gauge are filled with the fluid flowing in the pipeline, which has a density ρ_w and that the density of manometric fluid is ρ_m. Then, since pressures at level *XX* must be same in both limbs

$$p_X = P_1 + \rho_w g(z_1 - z) = P_2 + \rho_w g(z_2 - z - \Delta h) + \rho_m \Delta h g$$

or $$H = \left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right] = \left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h \quad (14.17)$$

From Eqs. (14.16) and (14.17), one can write

$$Q_{th} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h} \quad (14.18)$$

In practice, some loss of energy will occur between section 1 and 2. The value of Q_{th} given by Eq. (14.18) is a theoretical value which will be slightly greater than the actual value. A coefficient of discharge C_d is, therefore, introduced, which is defined as the ratio of actual discharge (Q) to that of theoretical discharge (Q_{th}) and is given by

$$C_d = \frac{Q}{Q_{th}}$$

The usual value of C_d for venturimeter varies from 0.95 to 0.99.

Actual discharge is then given by

$$Q = C_d \times Q_{th}$$

or $$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h} \quad (14.19)$$

Note: *Manometer connected across two sections of a venturimeter actually measures the piezometric pressure differences, not the static pressure difference.*

Note: *The angle of the converging cone is steeper than the diffuser angle to minimize the loss due to flow separation.*

Example 14.14 A venturimeter having throat diameter of 150 mm is set in a vertical pipe of 300 mm diameter to measure the discharge of an oil of specific gravity 0.85 which is flowing through the pipe in upward direction. The difference in elevations of the throat section and entrance section of the venturimeter is 4 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. If the coefficient of discharge of the venturimeter is 0.95, calculate the discharge of oil flowing through the pipe.

Solution

Diameter at inlet $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

Diameter at throat $D_2 = 150 \text{ mm} = 0.15 \text{ m}$

Differential manometer reading $\Delta h = 25 \text{ cm} = 0.25 \text{ m}$

Coefficient of discharge $C_d = 0.95$

Cross-sectional area of pipe is $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}\times(0.3)^2 = 0.07068 \text{ m}^2$

Cross-sectional area of venturimeter at throat is $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}\times(0.15)^2 = 0.01767 \text{ m}^2$

The discharge is given by Eq. (14.19) as

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$= \frac{0.95 \times 0.07068 \times 0.01767 \times \sqrt{2 \times 9.81 \times \left(\frac{13.6}{0.85} - 1\right) \times 0.25}}{\sqrt{(0.07068)^2 - (0.01767)^2}}$$

$$= 0.1487 \text{ m}^3/\text{s}$$

Example 14.15 A vertical venturimeter has an area ratio of 5. It has a throat diameter of 1 cm. When oil of specific gravity 0.85 flows through it the mercury in the differential gauge indicates a difference in height of 20 cm. Find the discharge through the venturimeter. Take coefficient of discharge of the venturimeter as 0.98.

Solution

Diameter at throat $D_2 = 1\text{ cm} = 0.01\text{ m}$
Differential manometer reading $\Delta = 20\text{ cm} = 0.2\text{ m}$
Coefficient of discharge $C_d = 0.98$

Cross-sectional area of venturimeter at throat is $A_2 = \frac{\pi}{4} \times (0.01)^2 = 7.854 \times 10^{-5}\text{ m}^2$

The discharge through the venturimeter is given by Eq. (14.19) as

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$= \frac{C_d A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

$$= \frac{0.98 \times 7.854 \times 10^{-5} \times \sqrt{2 \times 9.81 \times \left(\frac{13.6}{0.85} - 1\right) \times 0.2}}{\sqrt{1 - \left(\frac{1}{5}\right)^2}}$$

$$= 0.00059\text{ m}^3/\text{s}$$

Example 14.16 Water flows through a 300 mm × 150 mm venturimeter at the rate of 0.065 m^3/s and the differential gauge is deflected 1.2 m. Specific gravity of the manometric liquid is 1.6. Determine the coefficient of discharge of the venturimeter.

Solution

Diameter at inlet $D_1 = 300\text{ mm} = 0.3\text{ m}$
Diameter at throat $D_2 = 150\text{ mm} = 0.15\text{ m}$
Differential manometer reading $\Delta h = 1.2\text{ m}$
Specific gravity of manometric liquid $S_m = 1.6$
Volume flow rate $Q = 0.065\text{ m}^3/\text{s}$

Cross-sectional area of pipe is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07068\text{ m}^2$

Cross-sectional area of venturimeter at throat is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.15)^2 = 0.01767 \text{ m}^2$

The discharge through the venturimeter is given by Eq. (14.19) as

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

or $$0.065 = \frac{C_d \times 0.07068 \times 0.01767 \times \sqrt{2 \times 9.81 \times \left(\frac{1.6}{1} - 1\right) \times 1.2}}{\sqrt{(0.07068)^2 - (0.01767)^2}}$$

or $$C_d = 0.95$$

Example 14.17 An oil of relative density 0.8 flows through a vertical pipe of diameter 24 cm. The flow is measured by a 24 cm × 12 venturimeter. The throat is 30 cm above the inlet section. A differential mercury U-tube manometer is connected to the inlet and throat. The manometer shows a deflection of 12 cm. Calculate the flow rate through the pipe. Take coefficient of discharge of the venturimeter as 0.98.

Solution

Diameter at inlet	D_1 = 24 cm = 0.24 m
Diameter at throat	D_2 = 12 cm = 0.12 m
Differential manometer reading	Δh = 12 cm = 0.12 m
Coefficient of discharge	C_d = 0.98

Cross-sectional area of pipe is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.24)^2 = 0.0452 \text{ m}^2$

Cross-sectional area of venturimeter at throat is $A_1 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times (0.12)^2 = 0.0113 \text{ m}^2$

The discharge through the venturimeter is given by Eq. (14.19)

$$Q = \frac{C_d A_1 A_2 \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$= \frac{0.98 \times 0.0452 \times 0.0113 \times \sqrt{2 \times 9.81 \times \left(\frac{13.6}{0.8} - 1\right) \times 0.12}}{\sqrt{(0.0452)^2 - (0.0113)^2}}$$

$$= 0.07 \text{ m}^3/\text{s}$$

Example 14.18 A venturimeter with inlet and throat diameters are 150 mm and 75 mm respectively, is mounted in a vertical pipe carrying water, the flow being upwards. The throat section is 250 mm above the inlet of the venturimeter. The discharge of the venturimeter is 40 litres/s and the coefficient of discharge is 0.96. Calculate (a) the static pressure difference between inlet and throat, and (b) the difference in levels of mercury in a vertical U-tube manometer connected between these points.

Solution

The schematic of the venturimeter is shown in Fig. 14.10.

Diameter at inlet	$D_1 = 150 \text{ mm} = 0.15 \text{ m}$
Diameter at throat	$D_2 = 75 \text{ mm} = 0.075 \text{ m}$
Volume flow rate	$Q = 40 \text{ litres/s} = 40 \times 10^{-3} \text{ m}^3\text{/s}$
Coefficient of discharge	$C_d = 0.96$
Cross-sectional area of pipe is	$A_1 = \dfrac{\pi}{4} D_1^2 = \dfrac{\pi}{4} \times (0.15)^2 = 0.01767 \text{ m}^2$
Cross-sectional area of venturimeter at throat is	$A_2 = \dfrac{\pi}{4} D_2^2 = \dfrac{\pi}{4} \times (0.075)^2 = 0.0044 \text{ m}^2$

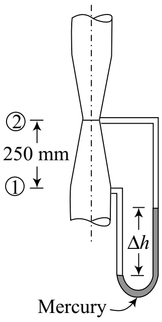


Figure 14.10

The discharge through the venturimeter is given by Eq. (14.19)

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

Substituting the values, we have

$$40 \times 10^{-3} = \frac{0.96 \times 0.01767 \times 0.0044 \times \sqrt{2 \times 9.81\left[\dfrac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}}{\sqrt{(0.01767)^2 - (0.0044)^2}}$$

or $$\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]=4.28$$

or $$\left[\frac{(P_1-P_2)}{\rho g}-0.25\right]=4.28$$

or $$\frac{(P_1-P_2)}{1000\times 9.81}=4.28+0.25=4.53\text{ m}$$

or $$P_1-P_2=4.53\times 1000\times 9.81=44439\text{N/m}^2=44.439\text{ kN/m}^2$$

Static pressure difference between inlet and throat section is 44.439 kN/m^2.

(b) From Eq. (14.17), we have

$$\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right]=\Delta h\left(\frac{\rho_m}{\rho_w}-1\right)$$

or $$4.28=\Delta h\left(\frac{13600}{1000}-1\right)$$

or $$\Delta h=0.3397\text{ m}=33.97\text{ cm}$$

The difference in levels of mercury in a vertical U-tube manometer connected between these points is 33.97 cm.

14.6.2 Orifice Meter

The venturimeter is relatively complex to construct and hence expensive. Especially, for small pipelines, its cost seems prohibitive, so simpler devices have been invented, such as orificemeter. The orifice meter is a simple device for the measurement of flow. The orificemeter consists of a thin circular plate with sharp edge circular hole drilled in it. The orifice plate produces a constriction of the flow as shown in Fig. 14.11.

The streamlines continue to converge short distance downstream of the plane of the orifice, where the vena contract is formed, and then expand. Hence the minimum flow area is actually smaller than the area of the orifice. Applying energy equation between section 1 and 2(vena contracta), we have

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

or $$V_2^2-V_1^2=2g\left[\frac{(P_1-P_2)}{\rho g}+(z_1-z_2)\right] \qquad (14.20)$$

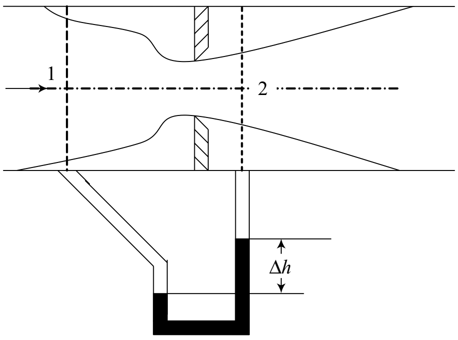


Figure 14.11 *Orificemeter*

From continuity equation, we have

$$A_1 V_1 = A_2 V_2$$

or
$$V_1 = \frac{A_2}{A_1} V_2$$

Substituting the value of in Eq. (14.20), we have

$$V_2^2 \left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] = 2g \left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2) \right]$$

or
$$V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g \left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2) \right]}$$

Area of vena contracta (A_2) is less than that of orifice (A_o) and coefficient of contraction (C_c) is defined as

$$C_c = \frac{A_2}{A_o}$$

or
$$V_2 = \frac{A_1}{\sqrt{A_1^2 - C_C^2 A_o^2}} \sqrt{2g \left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2) \right]}$$

Volume flow rate can be expressed as

$$Q = A_2 V_2 = C_c A_o V_2$$

or
$$Q = \frac{C_C A_o A_1}{\sqrt{A_1^2 - C_C^2 A_o^2}} \sqrt{2g \left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2) \right]} \qquad (14.21)$$

Equation (14.21) is also expressed in a simplified form as

$$Q = \frac{C_d A_o A_1}{\sqrt{A_1^2 - A_o^2}} \sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]} \tag{14.22}$$

where $$C_d = \frac{C_C\sqrt{A_1^2 - A_o^2}}{\sqrt{A_1^2 - C_C^2 A_o^2}}$$

The usual value of C_d for orificemeter varies from 0.60 to 0.70.

$$\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2) = \left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h$$

From Eq. (14.17), the piezometric pressure difference can also be expressed in terms of the manometer reading as

$$\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2) = \left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h$$

Then, Eq. (14.22) can be written as

$$Q = \frac{C_d A_o A_1}{\sqrt{A_1^2 - A_o^2}} \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h} \tag{14.23}$$

Example 14.19 An orificemeter with orifice diameter 12 cm is inserted in a pipe of 24 cm diameter through which an oil of density 850 kg/m^3 is flowing. A differential mercury U-tube manometer is connected to the two sides of the orificemeter to measure the pressure difference. The manometer shows a deflection of 20 cm. If the coefficient of discharge for the orificemeter is 0.6, find the discharge through the pipe.

Solution

Diameter of orifice	D_o = 12 cm = 0.12 m
Diameter of pipe	D_1 = 24 cm = 0.24 m
Coefficient of discharge	C_d = 0.6
Density of oil	ρ_w = 850 kg/m^3
Differential manometer reading	Δh = 20 cm = 0.20 m
Cross-sectional area of orifice is	$A_o = \frac{\pi}{4}D_o^2 = \frac{\pi}{4}\times(0.12)^2 = 0.0113 \text{ m}^2$
Cross-sectional area of pipe is	$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}\times(0.24)^2 = 0.0452 \text{ m}^2$

The discharge through the orificemeter is given by Eq. (14.23) as

$$Q = \frac{C_d A_o A_1}{\sqrt{A_1^2 - A_o^2}} \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}$$

$$= \frac{0.6 \times 0.0113 \times 0.0452}{\sqrt{0.0452^2 - 0.0113^2}} \sqrt{2 \times 9.81 \times \left(\frac{13600}{850} - 1\right) \times 0.2}$$

$$= \frac{0.0003}{0.0438} \times 7.672 = 0.0525 \text{ m}^3/\text{s}$$

Example 14.20 A horizontal orificemeter with orifice diameter 20 cm is inserted in a pipe of 30 cm diameter through which water is flowing. Coefficient of discharge for the orificemeter is 0.62. If the pressure gauges fitted upstream and downstream of the orificemeter show pressure and respectively, find the discharge through the pipe.

Solution

Diameter of orifice	D_o = 20 cm = 0.2 m
Diameter of pipe	D_1 = 30 cm = 0.3 m
Coefficient of discharge	C_d = 0.62
Density of water	ρ_w = 1000 kg/m^3
Pressure at upstream of the orificemeter	P_1 = 290 kN/m^2 = 290 × 10^3 N/m^2
Pressure at downstream of the orificemeter	P_2 = 195 kN/m^2 = 195 × 10^3 N/m^2
Cross-sectional area of orifice is	$A_o = \frac{\pi}{4} D_o^2 = \frac{\pi}{4} \times (0.20)^2 = 0.0314 \text{ m}^2$
Cross-sectional area of pipe is	$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0707 \text{ m}^2$

The discharge through the orificemeter is given by Eq. (14.22) as

$$Q = \frac{C_d A_o A_1}{\sqrt{A_1^2 - A_o^2}} \sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

$$= \frac{0.62 \times 0.0314 \times 0.0707}{\sqrt{0.0707^2 - 0.0314^2}} \sqrt{2 \times 9.81\left[\frac{290 \times 10^3 - 195 \times 10^3}{1000 \times 9.81} + 0\right]}$$

$$= \frac{0.001376}{0.0633} \times 13.784 = 0.2996 \text{ m}^3/\text{s}$$

14.6.3 Pitot Tube

Pitot tube is commonly used to measure the velocity at a point in the flow field. The working principle of Pitot tube is based on the Bernoulli's equation. The simplest Pitot tube consists of a right angled transparent tube with one vertical leg projecting out of the flow and another leg pointing directly upstream in the flow as shown in Fig 14.12. At location 1, the flow is practically undisturbed by the presence of the tube. At location 2, the flow has been completely stopped by the tube which has been inserted i.e., velocity at 2 is zero.

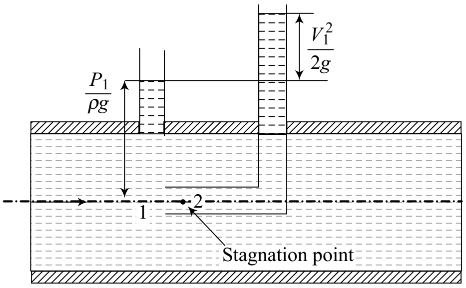


Figure 14.12 *Pitot tube*

Applying Bernoulli's equation between points 1 and 2 along a streamline, we have

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}=\frac{P_{stag}}{\rho g}$$

or
$$\frac{V_1^2}{2g}=\frac{P_{stag}}{\rho g}-\frac{P_1}{\rho g}$$

or
$$\frac{V_1^2}{2g}=h_{stag}-h_{stat}$$

or
$$V_1=\sqrt{2g\left(h_{stag}-h_{stat}\right)} \tag{14.24}$$

With consideration of loss, the velocity is given by

$$V_1=C\sqrt{2g\left(h_{stag}-h_{stat}\right)} \tag{14.25}$$

where C is the coefficient of the tube.

Example 14.21 A Pitot tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 8 m and static pressure head is 6 m. Calculate the velocity of flow, assuming the coefficient of the tube equal to 0.98.

Solution

Stagnation pressure head h_{stag} = 8 m

Static pressure head h_{stat} = 6 m

Coefficient of tube C = 0.98

Velocity of flow is given by Eq. (14.25) as

$$V = C\sqrt{2g(h_{stag} - h_{stat})}$$

$$= 6.14 \text{ m/s}$$

Pitot Static Tube The main limitation associated with the Pitot tube as described above is the necessity of two piezometers. Connecting the pizometers to a manometer would simplify things. Pitot static tube consists of a slender double-tube aligned with the flow and connected to a differential manometer as shown in Fig.14.13. The inner tube is fully open to flow at the nose, and thus it measures the stagnation pressure at that location. The outer tube is sealed at the nose, but it has holes on the side of the outer wall and thus it measures the static pressure.

Using the theory of manometer, we have

$$P_X = P_2 + \rho_w gx$$

$$P_Y = P_1 + \rho_w g(x - \Delta h) + \rho_m g\Delta h$$

Equating the pressures of both the limb along the horizontal plane *XY*, we obtain

$$P_X = P_Y$$

$$P_2 + \rho_w gx = P_1 + \rho_w g(x - \Delta h) + \rho_m g\Delta h$$

From Eq. (14.11), we know that

$$P_2 = P_1 + \frac{1}{2}\rho_w V_1^2$$

Substituting the value of P_2 to the above equation, we have

$$P_1 + \frac{1}{2}\rho_w V_1^2 + \rho_w gx = P_1 + \rho_w g(x - \Delta h) + \rho_m g\Delta h$$

or $$\frac{1}{2}\rho_w V_1^2 = (\rho_m - \rho_w)g\Delta h$$

or $$V_1^2 = 2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h$$

or $$V_1 = \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h} \qquad (14.26)$$

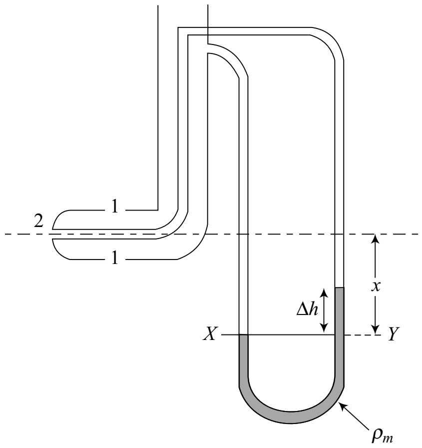


Figure 14.13 *Pitot static tube*

The Pitot static tube is a simple, inexpensive and highly reliable device since it has no moving parts. It also causes very small pressure drop and usually does not disturb the flow appreciably. However, it is important that it be properly aligned with the flow to avoid significant errors that may be caused by misalignment.

Example 14.22 An oil of specific gravity 0.85 flows through a horizontal pipe of diameter 20 cm. A Pitot static tube is inserted at the center of a pipe and its leads are filled with the same oil and attached to a U-tube containing water. The reading on the manometer is 15 cm. Determine the quantity of oil flowing through the pipe. The coefficient of Pitot static tube is unity.

Solution

Diameter of pipe $D = 20 \text{ cm} = 0.2 \text{ m}$

Specific gravity of oil $Sw = 0.85$

Density of oil $\rho_w = 0.85 \times 1000 = 850 \text{ kg/m}^3$

Density of manometric fluid (water) $\rho_m = 1000 \text{ kg/m}^3$

Manometer reading $\Delta h = 15 \text{ cm} = 0.15 \text{ m}$

Coefficient of tube $C = 1$

Cross-sectional area of pipe is $A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$

Since the coefficient of the tube is unity, the velocity of flow is given by Eq. (14.26) as

$$V = \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}$$

$$= \sqrt{2 \times 9.81\left(\frac{1000}{850} - 1\right) \times 0.15}$$

$$= 0.72 \text{ m/s}$$

The quantity of oil flowing through the pipe is

$$Q = AV = 0.0314 \times 0.72 = 0.0226 \text{ m}^3/\text{s}$$

SUMMARY

- Euler's equation of motion along a streamline is given by

  $$\frac{dP}{\rho} + VdV + gdz = 0$$

  which is valid for steady and inviscid flow.
- Bernoulli's equation along a streamline is given by

  $$\frac{P}{\rho g} + \frac{V^2}{2g} + z = C$$

  Here, $\frac{P}{\rho g}$ is the pressure head (flow energy per unit weight), $\frac{V^2}{2g}$ is the velocity head (kinetic energy per unit weight) and z is the potential head (potential energy per unit weight).
- Bernoulli equation can be stated as follows:
  During steady, inviscid flow of an incompressible fluid along a streamline, total mechanical energy at any point is constant. The total mechanical energy consists of flow energy, kinetic energy and potential energy.
- Bernoulli's equation deals with the law of conservation of mechanical energy. The equation holds true between any two points in the flow field provided that the flow field is irrotational and the flow is inviscid, steady and incompressible.
- The assumptions made in the derivation of Bernoulli's equation are
  - i) The flow is inviscid
  - ii) The flow is along a streamline
  - iii) The flow is steady
  - iv) The fluid is incompressible (density does not change with change in pressure)
- The discharge through a venturimeter is given by

  $$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}$$

  where A_1 and A_2 are the cross-sectional areas of the venturimeter at its inlet and throat respectively, ρ_m and ρ_w are the density of the manometric fluid and the working fluid respectively, Δh is the difference in height of the manometric fluid, and C_d is the co-efficient of discharge of the venturimeter.
- The discharge through an orificemeter is given by

  $$Q = \frac{C_d A_o A_1}{\sqrt{A_1^2 - A_o^2}} \sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}$$

  where Ao and A_1 are the cross-sectional areas of the orifice and pipe respectively, and are the density of the manometric fluid and the working fluid respectively, is the difference in height of the manometric fluid, and is the coefficient of discharge of the orificemeter.

- The velocity at a point in the flow filed measured by a Pitot tube given by

$$V = C\sqrt{2g\left(h_{stag} - h_{stat}\right)}$$

where h_{steg} is the stagnation pressure head and h_{stat} is the static pressure head and is the coefficient of the tube.

REVIEW QUESTIONS

14.1 Name the different forces present in a fluid flow. For the Euler's equation of motion, which forces are taken into consideration?

14.2 Derive Euler's equation of motion along a streamline.

14.3 Bernoulli's theorem is based on which principle? Give its statement. Name three devices where Bernoulli's equation is applied.

14.4 State the Bernoulli equation. List out the assumptions and limitations of Bernoulli's equation. How is it modified while applying in practice?

14.5 Define potential head, velocity head and datum head.

14.6 Draw a neat sketch of venturimeter. State why the length of divergent cone is made longer?

14.7 Starting with the Continuity and Bernoulli's equations, derive the following expression that can be used to measure flow rate with a venturimeter:

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left[\frac{(P_1 - P_2)}{\rho g} + (z_1 - z_2)\right]}$$

Also show that when the pressure difference is measured using a manometer the following expression can be used:

$$Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\left(\frac{\rho_m}{\rho_w} - 1\right)\Delta h}$$

14.8 Derive an expression for the rate of flow through an inclined venturimeter and show that, if a U-tube gauge is used to measure the pressure difference, the gauge reading will be same for a given discharge irrespective of the inclination of the meter.

14.9 Why is the angle of the converging cone in a venturimeter steeper than the diffuser angle?

14.10 Is it possible that the flow in a converging section of a vertical venturimeter takes place in a direction from lower pressure to higher pressure, if (i) the flow is in the direction of gravity, and (ii) the flow is opposite to the direction of gravity? Give reasons. Neglect viscous effects.

14.11 Coefficient of discharge of venturimeter is always greater than orificemeter. Why?

14.12 Derive an expression for the volumetric flow rate of a fluid flowing through an orificemeter. Write down the advantages and disadvantages of using orificemeter over a venturimeter.

14.13 How does a venturimeter differ from an orificemeter?

14.14 Explain in brief the working principle of a Pitot-tube with the help of a neat sketch.

14.15 Differentiate between Pitot tube and Pitot static tube

NUMERICAL PROBLEMS

14.1 An oil of relative density 0.9 is flowing in a pipe of 10 cm diameter with an average velocity of 3 m/s. At a particular section 1 the pressure is measured to be 300 kN/m^2. If the section 1 is 6 m above the datum, determine total head of oil.

14.2 A vertical pipeline 10 cm diameter at the top tapers uniformly to 20 cm at bottom. The length of the pipeline is 2 m. If the discharge through the pipeline is 30 litres/s, find the difference in pressure. Neglect friction.

14.3 An oil of specific gravity 0.85 is flowing through a vertical pipe having diameter 30 cm and 15 cm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 200 kN/m^2 and at the upper end is 98 kN/m^2. If the rate of flow through pipe is 50 lit/s, find the difference in datum head. Neglect friction.

14.4 A 300 m long pipe has a slope of 1 in 100 and tapers from 1.2 m diameter at higher end to 0.6 m diameter at the lower end. It carries water at a rate of 100 litres/s. Find the average velocities at the higher and lower end. If the pressure at high end is 150 kN/m^2, find the pressure at the low end. Neglect the losses due to friction.

14.5 At a certain location X of a pipe line carrying water, the diameter is 70 cm, the pressure is 200 kN/m^2 and the average velocity is 6 m/s. At another section Y which is 3 m higher than X, the diameter is 40 cm and the pressure is 120 kN/m^2. What is the direction of the flow?

14.6 An oil with density 900 kg/m^2 and viscosity 0.18 Ns/m^2 flows through a pipeline. The diameter of the pipe changes from 20 cm at a position A to 50 cm at a position B which is 4 m above A. If the pressure at A and B are 300 kPa and 200 kPa respectively and the discharge is 0.3 m^3/s, determine the direction of flow and the loss of head.

14.7 In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at section 1 which was at elevation 10 m. At another section 2 at elevation 12 m, the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water is 998 kg/m^3.

14.8 A venturimeter of throat diameter 6 cm is fitted into a 12 cm diameter pipeline carrying water. Calculate the discharge in the pipeline when the reading on a U-tube mercury manometer connected to the upstream and throat sections shows a reading of 25 cm. Take coefficient of discharge of the venturimeter as 0.96.

14.9. A venturimeter is used for the measurement of discharge of water in a horizontal pipeline. The pipe diameter is 250 mm and the throat diameter is 125 mm. The difference in pressure between the inlet and the throat is 3 m of head of water. Calculate the discharge in the pipe. Take coefficient of discharge of the venturimeter as 0.98.

14.10 An oil of relative density 0.8 flows through a vertical pipe of diameter 24 cm. The flow is measured by a 240 mm × 120 mm venturimeter. The throat is 200 mm above the inlet section. A differential mercury U-tube manometer is connected to the inlet and throat. The manometer shows a deflection of 130 mm. Calculate the flow rate through the pipe. Take coefficient of discharge of the venturimeter as 0.98.

14.11 A vertical venturimeter measures the flow of oil of specific gravity 0.82 and has an entrance diameter of 125 mm and a throat diameter of 50 mm. There are pressure gauges at the entrance and at the throat, which is 50 mm above the entrance. If the coefficient of the meter is 0.97, find the flow in m^3/sec when the pressure difference is 3 kPa.

14.12 A horizontal venturimeter of 24 cm × 12 cm is used to measure the discharge of an oil of density 850 kg/m^3. A differential mercury manometer is connected to the inlet and throat for the purpose. If the discharge is 100 litres/s, find the difference of mercury level in between two limbs of manometer. The coefficient of discharge of the venturimeter is 0.98.

14.13 A venturimeter of throat diameter 50 mm is fitted into a 125 mm diameter water pipeline. The coefficient of discharge is 0.96. Calculate the flow and velocity in the pipeline when the reading on a mercury-water differential U-tube manometer connected to the upstream and throat sections shows a reading of 200 mm. The piezometer head difference H is related to the gauge reading Δh by

$$H = \Delta h\left(\frac{\rho_m}{\rho_w} - 1\right)$$

where ρ_m and ρ_w are the densities of mercury and water respectively.

14.14 An orificemeter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure gauges fitted upstream and downstream of the orificemeter gives readings of N/cm^2 and 19.62 N/cm^2 respectively. Coefficient of discharge for the meter is given as 0.64. Find the discharge of the water through the pipe.

14.15 A pitot-static tube is used to measure air velocity. If a manometer connected to the instrument indicates a difference in pressure head between the tappings of 4 mm of water. Calculate the air velocity assuming the coefficient of the pitot tube to be unity. Density of air = 1.2 kg/m^3.

14.16 In a pitot - static tube stagnation pressure and static pressure are 6 kPa ad 1 kPa respectively. Calculate the velocity of flow of air assuming the coefficient of the tube equal to 0.98 and the water density as 1000 kg/m^3.

14.17 A pitot-static tube at the centre of a 10 cm diameter pipe is aligned in the direction of flow. When air flows through the pipe, the differential manometer across the pitot tube reads 6 mm of water gauge. It is known that for the air flow under consideration, the centre line velocity is 18 % higher than the average. Calculate the flow rate of air considering the coefficient of the pitot tube as unity and air density 1.2 kg/m^3.

MULTIPLE-CHOICE QUESTIONS

14.1 Bernoulli's theorem deals with the law of conservation of
(a) mass (b) momentum (c) energy (d) concentration

14.2 All the terms of energy in Bernoulli's equation $\left(\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}\right)$ have dimension of
(a) mass (b) energy (c) length (d) work

14.3 Bernoulli's equation relates
(a) various forms of mechanical energy
(b) various forces involved in fluid flow
(c) torque to change in angular momentum
(d) various forces with change in momentum

14.4 Each term of Bernoulli's equation stated in the form $\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$, has units of
(a) N (b) Nm/s (c) Nm/kg (d) Nm/N

14.5 In the most general form of Bernoulli's equation $\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$, each term represents
(a) energy per unit mass (b) energy per unit weight
(c) energy per unit volume (d) none of these

14.6 Euler's equation is written as

(a) $\dfrac{dP}{\rho}+V^2dV+gdz=0$ (b) $\dfrac{dP}{\rho}+VdV+gdz=0$

(c) $\dfrac{dP}{\rho}+V^2dV+gdz=0$ (d) $\dfrac{dP}{\rho^2}+V^2dV+gdz=0$

14.7 A stagnation point is a point in fluid flow where

(a) total energy is zero (b) pressure is zero

(c) velocity of flow is zero (d) total energy is maximum.

14.8 Bernoulli equation can be derived from

(a) momentum balance only (b) energy balance only

(c) either momentum balance or energy balance (d) conservation of mass only

14.9 Pitot tube is used for the measurement of

(a) flow (b) velocity at a point

(c) discharge (d) pressure

14.10 The range for co-efficient of discharge for a venturimeter is

(a) 0.6 to 0.7 (b) 0.7 o 0.8 (c) 0.8 to 0.9 (d) 0.95 to 0.99

14.11 The range for co-efficient of discharge for a venturimeter is

(a) 0.6 to 0.7 (b) 0.7 o 0.8 (c) 0.8 to 0.9 (d) 0.95 to 0.99

14.12 A venturimeter is a device used to measure

(a) pressure in a fluid (b) velocity at a point

(c) flow rate (d) temperature of the fluid

14.13 A pitot tube measures in a pipe

(a) the average velocity of the fluid (b) the local velocity of flow

(c) the maximum velocity of flow only (d) the flow rate in the pipe

14.14 Which of the following instrument is used to measure flow by application of Bernoulli's theorem

(a) Venturimeter (b) Orificemeter (c) Pitot tube (d) all of the above

14.15 When is Bernoulli's equation applicable between any two points in a flow field?

(a) Steady, inviscid and irrotational flow of a compressible fluid

(b) Unsteady, inviscid and irrotational flow of a compressible fluid

(c) Steady, inviscid and rotational flow of an incompressible fluid

(d) Steady, inviscid and irrotational flow of an incompressible fluid

14.16 It is recommended that the diffuser angle should be kept less than 6° because

(a) pressure decreases in flow direction and flow separation may occur

(b) pressure decreases in flow direction and flow may become turbulent

(c) pressure increases in flow direction and flow separation may occur

(d) pressure increases in flow direction and flow may become turbulent

14.17 The difference of pressure head, H measured by a mercury-oil differential manometer is expressed as

(a) $H=\left(\dfrac{\rho_m}{\rho_w}-1\right)\Delta h$ (b) $H=\left(1-\dfrac{\rho_m}{\rho_w}\right)\Delta h$

(c) $H=(\rho_m-\rho_w)\Delta h$ (d) $H=(\rho_w-\rho_m)\Delta h$

where Δh = manometer reading; ρ_m and ρ_w are the densities of mercury and oil respectively.

14.18 A pitot-static tube ($C = 1$) is used to measure air flow. With water in the differential manometer and a gauge difference of 75 mm, what is the value of air speed if the density of air is 1.16 kg/m^3?

(a) 1.21 m/s (b) 16.2 m/s (c) 5.6 m/s (d) 71.2 m/s

APPENDIX I

Thermodynamic Properties of Water (Steam Tables)

TABLE A.1 PROPERTIES OF SATURATED WATER-TEMPERATURE BASE

		Volume, m^3/kg		Energy, kJ/kg		Enthalpy, kJ/kg			Entropy, kJ/kg-K		
T,°C	P, MPa	v_f	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0.010	0.00061	0.001000	206.1	0.0	2375.3	0.0	2501.3	2501.3	0.0000	9.1571	9.1571
2	0.00071	0.001000	179.9	8.4	2378.1	8.4	2496.6	2505.0	0.0305	9.0738	9.1043
5	0.00087	0.001000	147.1	21.0	2382.2	21.0	2489.5	2510.5	0.0761	8.9505	9.0266
10	0.00123	0.001000	106.4	42.0	2389.2	42.0	2477.7	2519.7	0.1510	8.7506	8.9016
15	0.00171	0.001001	77.93	63.0	2396	63.0	2465.9	2528.9	0.2244	8.5578	8.7822
20	0.00234	0.001002	57.79	83.9	2402.9	83.9	2454.2	2538.1	0.2965	8.3715	8.668
25	0.00317	0.001003	43.36	104.9	2409.8	104.9	2442.3	2547.2	0.3672	8.1916	8.5588
30	0.00425	0.001004	32.9	125.8	2416.6	125.8	2430.4	2556.2	0.4367	8.0174	8.4541
35	0.00563	0.001006	25.22	146.7	2423.4	146.7	2418.6	2565.3	0.5051	7.8488	8.3539
40	0.00738	0.001008	19.52	167.5	2430.1	167.5	2406.8	2574.3	0.5723	7.6855	8.2578
45	0.00959	0.001010	15.26	188.4	2436.8	188.4	2394.8	2583.2	0.6385	7.5271	8.1656
50	0.01235	0.001012	12.03	209.3	2443.5	209.3	2382.8	2592.1	0.7036	7.3735	8.0771
55	0.01576	0.001015	9.569	230.2	2450.1	230.2	2370.7	2600.9	0.7678	7.2243	7.9921
60	0.01994	0.001017	7.671	251.1	2456.6	251.1	2358.5	2609.6	0.8310	7.0794	7.9104
65	0.02503	0.001020	6.197	272.0	2463.1	272.0	2346.2	2618.2	0.8934	6.9384	7.8318
70	0.03119	0.001023	5.042	292.9	2469.5	292.9	2333.8	2626.8	0.9549	6.8012	7.7561
75	0.03858	0.001026	4.131	313.9	2475.9	313.9	2321.4	2635.3	1.0155	6.6678	7.6833

TABLE A.1 (Continued)

		Volume, m^3/kg		Energy, kJ/kg		Enthalpy, kJ/kg			Entropy, kJ/kg-K		
T,°C	P, MPa	v_f	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
80	0.04739	0.001029	3.407	334.8	2482.2	334.9	2308.8	2643.7	1.0754	6.5376	7.6130
85	0.05783	0.001032	2.828	355.8	2488.4	355.9	2296.0	2651.9	1.1344	6.4109	7.5453
90	0.07013	0.001036	2.361	376.8	2494.5	376.9	2283.2	2660.1	1.1927	6.2872	7.4799
95	0.08455	0.00104	1.982	397.9	2500.6	397.9	2270.2	2668.1	1.2503	6.1664	7.4167
100	0.1013	0.001044	1.673	418.9	2506.5	419.0	2257.0	2676.0	1.3071	6.0486	7.3557
110	0.1433	0.001052	1.210	461.1	2518.1	461.3	2230.2	2691.5	1.4188	5.8207	7.2395
120	0.1985	0.00106	0.8919	503.5	2529.2	503.7	2202.6	2706.3	1.5280	5.6024	7.1304
130	0.2701	0.00107	0.6685	546	2539.9	546.3	2174.2	2720.5	1.6348	5.3929	7.0277
140	0.3613	0.00108	0.5089	588.7	2550.0	589.1	2144.8	2733.9	1.7395	5.1912	6.9307
150	0.4758	0.00109	0.3928	631.7	2559.5	632.2	2114.2	2746.4	1.8422	4.9965	6.8387
160	0.6178	0.001102	0.3071	674.9	2568.4	675.5	2082.6	2758.1	1.9431	4.8079	6.7510
170	0.7916	0.001114	0.2428	718.3	2576.5	719.2	2049.5	2768.7	2.0423	4.6249	6.6672
180	1.002	0.001127	0.1941	762.1	2583.7	763.2	2015.0	2778.5	2.1400	4.4466	6.5866
190	1.254	0.001141	0.1565	806.2	2590.0	807.5	1978.8	2786.4	2.2363	4.2724	6.5087
200	1.554	0.001156	0.1274	850.6	2595.3	852.4	1940.8	2793.2	2.3313	4.1018	6.4331
210	1.906	0.001173	0.1044	895.5	2599.4	897.7	1900.8	2798.5	2.4253	3.9340	6.3593
220	2.318	0.00119	0.0862	940.9	2602.4	943.6	1858.5	2802.1	2.5183	3.7686	6.2869
230	2.795	0.001209	0.0716	986.7	2603.9	990.1	1813.9	2804.0	2.6105	3.6050	6.2155
240	3.344	0.001229	0.0598	1033.2	2604.0	1037.3	1766.5	2803.8	2.7021	3.4425	6.1446
250	3.973	0.001251	0.0501	1080.4	2602.4	1085.3	1716.2	2801.5	2.7933	3.2805	6.0738
260	4.688	0.001276	0.0422	1128.4	2599.0	1134.4	1662.5	2796.9	2.8844	3.1184	6.0028
270	5.498	0.001302	0.0357	1177.3	2593.7	1184.5	1605.2	2789.7	2.9757	2.9553	5.9310
280	6.411	0.001332	0.0302	1227.4	2586.1	1236.0	1543.6	2779.6	3.0674	2.7905	5.8579
290	7.436	0.001366	0.0256	1278.9	2576.0	1289.0	1477.2	2766.2	3.1600	2.6230	5.7830
300	8.58	0.001404	0.0217	1332	2563.0	1344.0	1405.0	2749.0	3.2540	2.4513	5.7053
310	9.856	0.001447	0.0184	1387	2546.4	1401.3	1326.0	2727.3	3.3500	2.2739	5.6239
320	11.27	0.001499	0.0155	1444.6	2525.5	1461.4	1238.7	2700.1	3.4487	2.0883	5.5370
330	12.84	0.001561	0.013	1505.2	2499.0	1525.3	1140.6	2665.9	3.5514	1.8911	5.4425
340	14.59	0.001638	0.0108	1570.3	2464.6	1594.2	1027.9	2622.1	3.6601	1.6765	5.3366
350	16.51	0.00174	0.0088	1641.8	2418.5	1670.6	893.4	2564.0	3.7784	1.4338	5.2122
360	18.65	0.001892	0.0069	1725.2	2351.6	1760.5	720.7	2481.2	3.9154	1.1382	5.0536
370	21.03	0.002213	0.0049	1844	2229.0	1890.5	442.2	2332.7	4.1114	0.6876	4.7990
374	22.088	0.003155	0.0032	2029.6	2029.6	2099.3	0.0	2099.3	4.4305	0.0000	4.4305

SOURCES: Keenan, Keyes, Hill, and Moore, *Steam Tables*, Wiley, New York, 1969; G.J. Van Wylen and R. E. Sonntag, *Fundamentals of Classical Thermodynamics*, Wiley, New York, 1973.

TABLE A.2 PROPERTIES OF SATURATED WATER-PRESSURE BASE

		Volume, m^3/kg		Energy, kJ/kg		Enthalpy, kJ/kg			Entropy, kJ/kg-K		
P, MPa	T,°C	v_f	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0.000611	0.01	0.001000	206.1	0.0	2375.3	0.0	2501.3	2501.3	0.0000	9.1571	9.1571
0.0008	3.8	0.001000	159.7	15.8	2380.5	15.8	2492.5	2508.3	0.0575	9.0007	9.0582
0.001	7.0	0.001000	129.2	29.3	2385.0	29.3	2484.9	2514.2	0.1059	8.8706	8.9765
0.0012	9.7	0.001000	108.7	40.6	2388.7	40.6	2478.5	2519.1	0.1460	8.7639	8.9099
0.0014	12.0	0.001001	93.92	50.3	2391.9	50.3	2473.1	2523.4	0.1802	8.6736	8.8538
0.0016	14.0	0.001001	82.76	58.9	2394.7	58.9	2468.2	2527.1	0.2101	8.5952	8.8053
0.0018	15.8	0.001001	74.03	66.5	2397.2	66.5	2464.0	2530.5	0.2367	8.5259	8.7626
0.002	17.5	0.001001	67.00	73.5	2399.5	73.5	2460.0	2533.5	0.2606	8.4639	8.7245
0.003	24.1	0.001003	45.67	101.0	2408.5	101.0	2444.5	2545.5	0.3544	8.2240	8.5784
0.004	29.0	0.001004	34.8	121.4	2415.2	121.4	2433.0	2554.4	0.4225	8.0529	8.4754
0.006	36.2	0.001006	23.74	151.5	2424.9	151.5	2415.9	2567.4	0.5208	7.8104	8.3312
0.008	41.5	0.001008	18.1	173.9	2432.1	173.9	2403.1	2577.0	0.5924	7.6371	8.2295
0.01	45.8	0.001010	14.67	191.8	2437.9	191.8	2392.8	2584.6	0.6491	7.5019	8.1510
0.012	49.4	0.001012	12.36	206.9	2442.7	206.9	2384.1	2591.0	0.6961	7.3910	8.0871
0.014	52.6	0.001013	10.69	220.0	2446.9	220.0	2376.6	2596.6	0.7365	7.2968	8.0333
0.016	55.3	0.001015	9.433	231.5	2450.5	231.5	2369.9	2601.4	0.7719	7.2149	7.9868
0.018	57.8	0.001016	8.445	241.9	2453.8	241.9	2363.9	2605.8	0.8034	7.1425	7.9459
0.02	60.1	0.001017	7.649	251.4	2456.7	251.4	2358.3	2609.7	0.8319	7.0774	7.9093
0.03	69.1	0.001022	5.229	289.2	2468.4	289.2	2336.1	2625.3	0.9439	6.8256	7.7695
0.04	75.9	0.001026	3.993	317.5	2477.0	317.6	2319.1	2636.7	1.0260	6.6449	7.6709
0.06	85.9	0.001033	2.732	359.8	2489.6	359.8	2293.7	2653.5	1.1455	6.3873	7.5328
0.08	93.5	0.001039	2.087	391.6	2498.8	391.6	2274.1	2665.7	1.2331	6.2023	7.4354
0.1	99.6	0.001043	1.694	417.3	2506.1	417.4	2258.1	2675.5	1.3029	6.0573	7.3602
0.12	104.8	0.001047	1.428	439.2	2512.1	439.3	2244.2	2683.5	1.3911	5.9378	7.2980
0.14	109.3	0.001051	1.237	458.2	2517.3	458.4	2232.0	2690.4	1.4112	5.8360	7.2472
0.16	113.3	0.001054	1.091	475.2	2521.8	475.3	2221.2	2696.5	1.4553	5.7472	7.2025
0.18	116.9	0.001058	0.9775	490.5	2525.9	490.7	2211.1	2701.8	1.4948	5.6683	7.1631
0.2	120.2	0.001061	0.8857	504.5	2529.5	504.7	2201.9	2706.6	1.5305	5.5975	7.1280
0.3	133.5	0.001073	0.6058	561.1	2543.6	561.5	2163.8	2725.3	1.6722	5.3205	6.9927
0.4	143.6	0.001084	0.4625	604.3	2553.6	604.7	2133.8	2738.5	1.7770	5.1197	6.8967
0.6	158.9	0.001101	0.3157	669.9	2567.4	670.6	2086.2	2756.8	1.9316	4.8293	6.7609
0.8	170.4	0.001115	0.2404	720.2	2576.8	721.1	2048.0	2769.1	2.0466	4.6170	6.6636
1	179.9	0.001127	0.1944	761.7	2583.6	762.8	2015.3	2778.1	2.1391	4.4482	6.5873
1.2	188.0	0.001139	0.1633	797.3	2588.8	798.6	1986.2	2784.8	2.2170	4.3072	6.5242
1.4	195.1	0.001149	0.1408	828.7	2592.8	830.3	1959.7	2790.0	2.4847	4.1854	6.4701

TABLE A.2 (Continued)

		Volume, m^3/kg		Energy, kJ/kg		Enthalpy, kJ/kg			Entropy, kJ/kg-K		
P, MPa	T,°C	v_f	v_g	u_f	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
1.6	201.4	0.001159	0.1238	856.9	2596.0	858.8	1935.2	2794.0	2.3446	4.0780	6.4226
1.8	207.2	0.001168	0.1104	882.7	2598.4	884.8	1912.3	2797.1	2.3986	3.9816	6.3802
2	212.4	0.001177	0.0996	906.4	2600.3	908.8	1890.7	2799.5	2.4478	3.8939	6.3417
3	233.9	0.001216	0.0667	1004.8	2604.1	1008.4	1795.7	2804.1	2.6462	3.5416	6.1878
4	250.4	0.001252	0.0498	1082.3	2602.3	1087.3	1714.1	2801.4	2.7970	3.2739	6.0709
6	275.6	0.001319	0.0324	1205.4	2589.7	1213.3	1571.0	2784.3	3.0273	2.8627	5.8900
8	295.1	0.001384	0.0235	1305.6	2569.8	1316.6	1441.4	2758.0	3.2075	2.5365	5.7440
10	311.1	0.001452	0.018	1393.0	2544.4	1407.6	1317.1	2724.7	3.3603	2.2546	5.6149
12	324.8	0.001527	0.0143	1472.9	2513.7	1491.3	1193.6	2684.9	3.4970	1.9963	5.4933
14	336.8	0.001611	0.0115	1548.6	2476.8	1571.1	1066.5	2637.6	3.6240	1.7486	5.3726
16	347.4	0.001711	0.0093	1622.7	2431.8	1650.0	930.7	2580.7	3.7468	1.4996	5.2464
18	357.1	0.00184	0.0075	1698.9	2374.4	1732.0	777.2	2509.2	3.8722	1.2332	5.1054
20	365.8	0.002036	0.0058	1785.6	2293.2	1826.3	583.7	2410.0	4.0146	0.9135	4.9281
22.088	374.136	0.003155	0.0032	2029.6	2029.6	2099.3	0.0	2099.3	4.4305	0.0000	4.4305

SOURCES: Keenan, Keyes, Hill, And Moore, *Steam Tables*, Wiley, New York, 1969; G.J. Van Wylen and R. E. Sonntag, *Fundamentals of Classical Thermodynamics*, Wiley, New York, 1973.

TABLE A.3 SUPERHEATED STEAM TABLE (T in °C, v in m^3/kg, u in kJ/kg, in kJ/kg, s in kJ/kg-K

T	v	u	h	s	v	u	h	s	v	u	h	s
	$P = 0.010$ MPa (45.81°C)				$P = 0.050$ MPa (81.33°C)				$P = 0.1$ MPa (99.63°C)			
Sat.	14.674	2437.9	2584.7	8.1502	3.240	2483.9	2646	7.5939	1.6940	2506	2675.5	7.359
50	14.869	2443.9	2592.6	8.1749								
100	17.196	2515.5	2688	8.4479	3.418	2511.6	2683	7.6947	1.6958	2507	2676.2	7.361
150	19.512	2587.9	2783.0	8.6882	3.889	2585.6	2780.1	7.9401	1.9364	2583	2776.4	7.613
200	21.825	2661.3	2879.5	8.9038	4.356	2659.9	2877.7	8.1580	2.172	2658	2875.3	7.834
250	24.136	2736.0	2977.3	9.1002	4.820	2735.0	2976.0	8.3556	2.406	2734	2974.3	8.033
300	26.445	2812.1	3076.5	9.2813	5.284	2811.3	3075.5	8.5373	2.639	2810	3074.3	8.216
400	31.063	2968.9	3279.6	9.6077	6.209	2968.5	3278.9	8.8642	3.103	2968	3278.2	8.544
500	35.679	3132.3	3489.1	9.8978	7.134	3132.0	3488.7	9.1546	3.565	3132	3488.1	8.834
600	40.295	3302.5	3705.4	10.161	8.057	3302.2	3705.1	9.4178	4.028	3305	3704.7	9.098
700	44.911	3479.6	3928.7	10.403	8.981	3479.4	3928.5	9.6599	4.490	3479	3928.2	9.34
800	49.526	3663.8	4159.0	10.628	9.904	3663.6	4158.9	9.8852	4.952	3664	4158.6	9.565
900	54.141	3855.0	4396.4	10.84	10.828	3854.9	4396.3	10.0967	5.414	3855	4396.1	9.777
1000	58.757	4053.0	4640.6	11.039	11.751	4052.9	4640.5	10.2964	5.875	4053	4640.3	9.976
1100	63.372	4257.5	4891.2	11.229	12.674	4257.4	4891.1	10.4859	6.337	4257	4891.0	10.17

TABLE A.3 (Continued)

T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
1200	67.987	4468	5147.8	11.4091	13.597	4467.8	5148	10.666	6.799	4468	5147.6	10.3463
1300	72.602	4684	5409.7	11.5811	14.521	4683.6	5410	10.838	7.260	4684	5409.5	10.5183
	$P = 0.20$ MPa (120.23°C)				$P = 0.30$ MPa (133.55°C)				$P = 0.40$ MPa (143.63°C)			
Sat.	0.8857	2529.5	2706.7	7.1272	0.6058	2543.6	2725.3	6.9919	0.4625	2553.6	2738.6	6.8959
150	0.9596	2576.9	2768.8	7.2795	0.6339	2570.8	2761.0	7.0778	0.4708	2564.5	2752.8	6.9299
200	1.0803	2654.4	2870.5	7.5066	0.7163	2650.7	2865.6	7.3115	0.5342	2646.2	2860.5	7.1706
250	1.1988	2731.2	2971.0	7.7086	0.7964	2728.7	2967.6	7.5166	0.5951	2726.1	2964.2	7.3789
300	1.3162	2808.6	3071.8	7.8926	0.8753	2806.7	3069.3	7.7022	0.6548	2804.8	3066.8	7.5662
400	1.5493	2966.7	3276.6	8.2218	1.0315	2965.6	3275.6	8.0330	0.7726	2964.4	3273.4	7.8985
500	1.7814	3130.8	3487.1	8.5133	1.1867	3130.0	3486.0	8.3251	0.8893	3129.2	3484.9	8.1913
600	2.013	3301.4	3704.0	8.7770	1.3414	3300.8	3703.2	8.5892	1.0055	3300.2	3702.4	8.4558
700	2.244	3478.8	3927.6	9.0194	1.4957	3478.4	3927.1	8.8319	1.1215	3477.9	3926.5	8.6987
800	2.475	3663.1	4158.2	9.2449	1.6499	3662.9	4157.8	9.0576	1.2372	3662.4	4157.3	8.9244
900	2.706	3854.5	4395.8	9.4566	1.8041	3854.2	4395.4	9.2692	1.3529	3853.9	4395.1	9.1362
1000	2.937	4052.5	4640.0	9.6563	1.9581	4052.3	4639.7	9.4690	1.4685	4052.0	4639.4	9.3360
1100	3.168	4257.0	4890.7	9.8458	2.1121	4256.5	4890.4	9.6585	1.5840	4256.5	4890.2	9.5256
1200	3.399	4467.5	5147.3	10.0262	2.2661	4467.2	5147.1	9.8389	1.6996	4467.0	5146.8	9.7060
1300	3.630	4683.2	5409.3	10.1982	2.4201	4683.0	5409.0	10.0110	1.8151	4682.8	5408.8	9.8780
	$P = 0.50$ MPa (151.86°C)				$P = 0.60$ MPa (158.85°C)				$P = 0.80$ MPa (170.43°C)			
Sat.	0.3749	2561.2	2748.7	6.8213	0.3157	2567.4	2756.8	6.7600	0.2404	2576.8	2769.1	6.6628
200	0.4249	2642.9	2855.4	7.0592	0.3520	2638.9	2850.1	6.9665	0.2608	2630.6	2839.3	6.8158
250	0.4744	2723.5	2960.7	7.2709	0.3938	2720.9	2957.2	7.1816	0.2931	2715.5	2950.0	7.0384
300	0.5226	2802.9	3064.2	7.4599	0.4344	2801.0	3061.6	7.3724	0.3241	2797.2	3056.5	7.2328
350	0.5701	2882.6	3167.7	7.6329	0.4742	2881.2	3165.7	7.5464	0.3544	2878.2	3161.7	7.4089
400	0.6173	2963.2	3271.9	7.7938	0.5137	2962.1	3270.3	7.7079	0.3843	2959.7	3267.1	7.5716
500	0.7109	3128.4	3483.9	8.0873	0.5920	3127.6	3482.8	8.0021	0.4433	3126.0	3480.6	7.8673
600	0.8041	3299.6	3701.7	8.3522	0.6697	3299.1	3700.9	8.2674	0.5018	3297.9	3699.4	8.1333
700	0.8969	3477.5	3925.9	8.5952	0.7472	3477.0	3925.3	8.5107	0.5601	3476.2	3924.2	8.3770
800	0.9896	3662.1	4156.9	8.8211	0.8245	3661.8	4156.5	8.7367	0.6181	3661.1	4155.6	8.6033
900	1.0822	3853.6	4394.7	9.0329	0.9017	3853.4	4394.4	8.9486	0.6761	3852.8	4393.7	8.8153
1000	1.1747	4051.8	4639.1	9.2328	0.9788	4051.5	4638.8	9.1485	0.7340	4051.0	4638.2	9.0153
1100	1.2672	4256.3	4889.9	9.4224	1.0559	4256.1	4889.6	9.3381	0.7919	4255.6	4889.1	9.2050
1200	1.3596	4466.8	5146.6	9.6029	1.1330	4466.5	5146.3	9.5185	0.8497	4466.1	5145.9	9.3855
1300	1.4521	4682.5	5408.6	9.7749	1.2101	4682.3	5408.3	9.6906	0.9076	4681.8	5407.9	9.5575

TABLE A.3 (Continued)

T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
	$P = 1.00$ MPa (179.91°C)				$P = 1.20$ MPa (187.99°C)				$P = 1.40$ MPa (195.07°C)			
Sat.	0.19444	2583.6	2778.1	6.5865	0.16333	2588.8	2784.8	6.523	0.14084	2592.8	2790.0	6.4693
200	0.206	2621.9	2827.9	6.6940	0.16930	2612.8	2815.9	6.59	0.14302	2603.1	2803.3	6.4975
250	0.2327	2709.9	2942.6	6.9247	0.19234	2704.2	2935.0	6.829	0.16350	2698.3	2927.2	6.7467
300	0.2579	2793.2	3051.2	7.1299	0.2138	2789.2	3045.8	7.032	0.18228	2785.2	3040.4	6.9534
350	0.2825	2875.2	3157.2	7.3011	0.2345	2872.2	3153.6	7.212	0.2003	2869.2	3149.5	7.1360
400	0.3066	2957.3	3263.9	7.4651	0.2548	2954.9	3260.7	7.377	0.2178	2952.5	3257.5	7.3026
500	0.3541	3124.4	3478.5	7.7622	0.2946	3122.8	3476.3	7.676	0.2521	3321.1	3474.1	7.6027
600	0.4011	3296.8	3697.9	8.0290	0.3339	3295.6	3696.3	7.944	0.2860	3294.4	3694.8	7.8710
700	0.4478	3475.3	3923.1	8.2731	0.3729	3474.4	3922.0	8.188	0.3195	3473.6	3920.8	8.1160
800	0.4943	3660.4	4154.7	8.4996	0.4118	3659.7	4153.8	8.415	0.3528	3659.0	4153.0	8.8431
900	0.5407	3852.2	4392.9	8.7118	0.4505	3851.6	4392.2	8.627	0.3861	3851.1	4391.5	8.5556
1000	0.5871	4050.5	4637.6	8.9119	0.4892	4050.0	4637.0	8.827	0.4192	4049.5	4636.4	8.7559
1100	0.6335	4255.1	4888.6	9.1017	0.5278	4254.6	4888.0	9.017	0.4524	4254.1	4887.5	8.9457
1200	0.6798	4465.6	5145.4	9.2822	0.5665	4465.1	5144.9	9.198	0.4855	4464.7	5144.4	9.1262
1300	0.7261	4681.3	5407.4	9.4543	0.6051	4680.9	5407.0	9.37	0.5186	4680	5406.5	9.2984
	$P = 1.60$ MPa (201.41°C)				$P = 1.80$ MPa (207.15°C)				$P = 2.00$ MPa (212.42°C)			
Sat.	0.1238	2596.0	2794.0	6.4218	0.11042	2598.4	2797.1	6.3794	0.09963	2600.3	2799.5	6.3409
225	0.13287	2644.7	2857.3	6.5518	0.13673	2636.6	2846.7	6.4808	0.10377	2868.3	2835.8	6.4147
250	0.14184	2692.3	2919.2	6.6732	0.12497	2686.0	2911.0	6.6066	0.11144	2679.6	2902.5	6.5453
300	0.15862	2781.1	3034.8	6.8844	0.14021	2776.9	3029.2	6.8226	0.12547	2772.6	3023.5	6.7664
350	0.17456	2866.1	3145.4	7.0694	0.15457	2863.0	3141.2	7.0100	0.13857	2859.8	3137.0	6.9563
400	0.19005	2950.1	3254.4	7.2374	0.16847	2947.7	3250.9	7.1794	0.15120	2945.2	3247.6	7.1271
500	0.2203	3119.5	3472.0	7.5390	0.1955	3117.9	3469.8	7.4825	0.17568	3116.2	3467.6	7.4317
600	0.2500	3293.3	3693.2	7.8080	0.2220	3292.1	3691.7	7.7523	0.19960	3290.9	3690.1	7.7024
700	0.2794	3472.7	3919.7	8.0535	0.2482	3471.8	3918.5	7.9983	0.2232	3470.9	3917.4	7.9487
800	0.3086	3658.3	4152.1	8.2808	0.2742	3657.6	4151.2	8.2258	0.2467	3657.0	4150.3	8.1765
900	0.3377	3850.5	4390.8	8.4935	0.3001	3849.9	4390.1	8.4386	0.2700	3849.3	4389.4	8.3895
1000	0.3668	4049.0	4635.8	8.6938	0.3260	4048.5	4635.2	8.6391	0.2933	4048.0	4634.6	8.5901
1100	0.3958	4253.7	4887.0	8.8837	0.3518	4253.2	4886.4	8.8290	0.3166	4252.7	4885.9	8.7800
1200	0.4248	4464.2	5143.9	9.0643	0.3777	4463.7	5143.4	9.0096	0.3398	4463.3	5142.9	8.9607
1300	0.4538	4679.9	5406.0	9.2364	0.4034	4679.5	5405.6	9.1818	0.3631	4679.0	5405.1	9.1329
	$P = 2.50$ MPa (233.99°C)				$P = 3.00$ MPa (233.90°C)				$P = 3.50$ MPa (242.60°C)			
Sat.	0.07998	2603.1	2803.1	6.2575	0.06668	2604.1	2804.2	6.187	0.05707	2604	2803.4	6.125
225	0.08027	2605.6	2806.3	6.2639								
250	0.08700	2662.6	2880.1	6.4085	0.07058	2644.0	2855.8	6.287	0.05872	2624	2829.2	6.175

TABLE A.3 (Continued)

T	v	u	h	s	v	u	h	s	v	u	h	s
300	0.09890	2761.6	3008.8	6.6438	0.08114	2750.1	2993.5	6.5390	0.06842	2738.0	2977.5	6.4461
350	0.10976	2851.9	3126.3	6.8403	0.09053	2843.7	3115.3	6.7428	0.07678	2835.3	3104.0	6.6579
400	0.12010	2939.1	3239.3	7.0148	0.09936	2932.8	3230.9	6.9212	0.08453	2926.4	3222.3	6.8405
450	0.13014	3025.5	3350.8	7.1746	0.10787	3020.4	3344.0	7.0834	0.09196	3015.3	3337.2	7.0052
500	0.13998	3112.1	3462.1	7.3234	0.11619	3108.0	3456.5	7.2338	0.09918	3103.0	3450.9	7.1572
600	0.15930	3288.0	3686.3	7.5960	0.13243	3285.0	3682.3	7.5085	0.11324	3282.1	3678.4	7.4339
700	0.17832	3468.7	3914.5	7.8435	0.14838	3466.5	3911.7	7.7571	0.12699	3464.3	3908.8	7.6837
800	0.19716	3655.3	4148.2	8.0720	0.16414	3653.5	4145.9	7.9862	0.14056	3651.8	4143.7	7.9134
900	0.21590	3847.9	4387.6	8.2853	0.17980	3846.5	4385.9	8.1999	0.15402	3845.0	4384.1	8.1276
1000	0.2346	4046.7	4633.1	8.4861	0.19541	4045.4	4631.6	8.4009	0.16743	4044.1	4630.1	8.3288
1100	0.2532	4251.5	4884.6	8.6762	0.21098	4250.3	4883.3	8.5912	0.18080	4249.2	4881.9	8.5192
1200	0.2718	4462.1	5141.7	8.8569	0.22652	4460.9	5140.5	8.7720	0.19415	4459.8	5139.3	8.7000
1300	0.2905	4677.8	5404.0	9.0291	0.24206	4676.6	5402.8	8.9442	0.20749	4675.5	5401.7	8.8723
	$P = 4.0$ MPa (250.40°C)				$P = 4.50$ MPa (257.49°C)				$P = 5.0$ MPa (263.99°C)			
Sat.	0.04978	2602.3	2801.4	6.0701	0.04406	2600.1	2798.3	6.0198	0.03944	2597.1	2794.3	5.5934
275	0.05457	2667.9	2886.2	6.2285	0.04730	2650.3	2863.2	6.1401	0.04141	2631.3	2838.3	6.0544
300	0.05884	2725.3	2960.7	6.3615	0.05135	2712.0	2943.1	6.2828	0.04532	2698.0	2924.5	6.2084
350	0.06645	2826.7	3092.5	6.5821	0.05840	2817.8	3080.6	6.5131	0.05194	2808.7	3068.4	6.4493
400	0.07341	2919.9	3213.6	6.7690	0.06475	2913.3	3204.7	6.7047	0.05781	2906.6	3195.7	6.6459
450	0.08002	3010.2	3330.3	6.9363	0.07074	3005.0	3323.3	6.8746	0.06330	2999.7	3316.2	6.8186
500	0.08643	3099.5	3445.3	7.0901	0.07651	3095.3	3439.6	7.0301	0.06857	3091.0	3433.8	6.9759
600	0.09885	3279.1	3674.4	7.3688	0.08765	3276.0	3670.5	7.3110	0.07869	3273.0	3666.5	7.2589
700	0.11095	3462.1	3905.9	7.8502	0.09847	3459.9	3903.0	7.5631	0.08849	3457.6	3900.1	7.5122
800	0.12287	3650.0	4141.5	8.0647	0.10911	3648.3	4139.3	7.7942	0.09811	3646.6	4137.1	7.4440
900	0.13469	3843.6	4382.3	8.0647	0.11965	3842.2	4380.6	8.0091	0.10762	3840.7	4378.8	7.9593
1000	0.14645	4042.9	4628.7	8.2662	0.13013	4041.6	4627.2	8.2108	0.11707	4040.7	4625.7	8.1612
1100	0.15817	4248.0	4880.6	8.4567	0.14056	4246.8	4879.3	8.4015	0.12648	4245.6	4878.0	8.3520
1200	0.16987	4458.6	5138.1	8.6376	0.15098	4457.5	5136.9	8.5825	0.13587	4456.3	5135.7	8.5331
1300	0.18156	4674.3	5400.5	8.8100	0.16139	4673.1	5399.4	8.7549	0.14526	4672.0	5398.2	8.7055
	$P = 6.0$ MPa (275.64°C)				$P = 7.0$ MPa (285.88°C)				$P = 8.0$ MPa (295.06°C)			
Sat.	0.03244	2589.7	2784.3	5.8899	0.02737	2580.5	2772.1	5.8133	0.02352	2569.8	2758.0	5.7432
300	0.03616	2667.2	2884.2	6.0674	0.02947	2632.2	2838.4	3.9305	0.02426	2590.9	2785.0	5.7906
350	0.04223	2789.6	3043.0	6.3335	0.03524	2769.4	3016.0	6.2283	0.02995	2747.7	2987.3	6.1301
400	0.04739	2892.9	3177.2	6.5408	0.03993	2878.6	3158.1	6.4478	0.03432	2863.8	3138.3	6.3634
450	0.05214	2988.9	3301.8	6.7193	0.04416	2978.0	3287.1	6.6327	0.03817	2966.7	3272.0	6.5551

TABLE A.3 (Continued)

T	v	u	h	s	v	u	h	s	v	u	h	s
500	0.05665	3082.2	3422.2	6.8803	0.04814	3073.4	3410.3	6.7975	0.04175	3064.3	3398.3	6.7240
550	0.06101	3174.6	3540.6	7.0288	0.05195	3167.2	3530.9	6.9486	0.04516	3159.8	3521.0	6.8778
600	0.06525	3266.9	3658.4	7.1677	0.05565	3260.7	3650.3	7.0894	0.04845	3254.4	3521.0	7.0206
700	0.07352	3453.1	3894.2	7.4234	0.06283	3448.5	3888.3	7.3476	0.05481	3443.9	3642.0	7.2812
800	0.08160	3643.1	4132.7	7.6566	0.06981	3639.5	4128.2	7.5822	0.06097	3636.0	3882.4	7.5173
900	0.08958	3837.8	4375.3	7.8727	0.07669	3835.0	4371.8	7.7991	0.06702	3832.1	4368.3	7.7351
1000	0.09749	4037.8	4622.7	8.0751	0.08350	4035.3	4619.8	8.0020	0.07301	4032.8	4616.9	7.9384
1100	0.10536	4243.3	4875.4	8.2661	0.09027	4240.9	4872.8	8.1933	0.07896	4238.6	4870.3	8.1300
1200	0.11321	4454.0	5133.3	8.4474	0.09703	4451.7	5130.9	8.3747	0.08489	4449.5	5128.5	8.3115
1300	0.12106	4669.6	5396.0	8.6199	0.10377	4667.3	5393.7	8.5473	0.09080	4665.0	5391.5	8.4842
	$P = 9.0$ MPa (303.40°C)				$P = 10.0$ MPa (311.06°C)				$P = 12.5$ MPa (327.89°C)			
Sat.	0.02048	2557.8	2742.1	5.6772	0.01803	2544.4	2724.7	5.6141	0.013495	2505	2673.8	5.4624
325	0.02327	2646.6	2856.0	5.8712	0.01986	2610.4	2809.1	5.7568				
350	0.02580	2724.4	2956.6	6.0361	0.02242	2699.4	2923.4	5.9443	0.016128	2624.6	2826.2	5.7118
400	0.02993	2848.4	3117.8	6.2854	0.02641	2832.4	3096.5	6.6120	0.02000	2789.3	3039.3	6.0417
450	0.03350	2955.2	3256.6	6.4844	0.02975	2943.4	3240.9	6.4190	0.02299	2912.5	3199.8	6.2719
500	0.03677	3055.2	3336.1	6.6576	0.03279	3045.8	3373.7	6.5966	0.02560	3021.7	3341.8	6.4618
550	0.03987	3152.2	3511.0	6.8142	0.03564	3144.6	3500.9	6.7561	0.02801	3125.0	3475.2	6.6290
600	0.04285	3248.1	3633.7	6.9589	0.03837	3241.7	3625.3	6.9029	0.03029	3225.4	3604.0	6.7810
650	0.04574	3343.6	3755.3	7.0943	0.04101	3338.2	3748.2	7.0398	0.03248	3324.4	3730.4	6.9218
700	0.04857	3439.3	3876.5	7.2221	0.04358	3434.7	3870.5	7.1687	0.03460	3422.9	3855.3	7.0536
800	0.05409	3632.5	4119.3	7.4596	0.04859	3628.9	4114.8	7.4077	0.03859	3620.0	4103.6	7.2965
900	0.05950	3829.2	4364.3	7.6783	0.05349	3826.3	4361.2	7.6272	0.04267	3819.1	4352.5	7.5182
1000	0.06485	4030.3	4614.0	7.8821	0.05832	4027.8	4611.0	7.8315	0.04658	4021.6	4603.8	7.7237
1100	0.07016	4236.3	4867.7	8.0740	0.06312	4234.0	4865.1	8.0237	0.05045	4228.2	4858.8	7.9165
1200	0.07544	4447.2	5126.2	8.2556	0.06789	4444.9	5123.8	8.2055	0.05430	4439.3	5118.0	8.0987
1300	0.08072	4662.7	5389.2	8.4284	0.07265	4460.5	5387.0	8.3783	0.05813	4654.8	5381.4	8.2717
	$P = 15.0$ MPa (342.24°C)				$P = 17.5$ MPa (354.75°C)				$P = 20.0$ MPa (365.81°C)			
Sat.	0.010337	2455.5	2610.5	5.3098	0.007920	2390.2	2528.8	5.1419	0.005834	2293.0	2409.7	4.9269
350	0.011470	2520.4	2692.4	5.4421								
400	0.015649	2740.7	2975.5	5.8811	0.01245	2685.0	2902.9	5.7213	0.009942	2619.3	2818.1	5.5540
450	0.018445	2879.5	3156.2	6.1404	0.01517	2844.2	3109.7	6.0184	0.012695	2806.2	3060.1	5.9017
500	0.02080	2996.6	3308.6	6.3443	0.01736	2970.3	3274.1	6.2383	0.014768	2942.9	3238.2	6.1401
550	0.02293	3104.7	3448.6	6.5199	0.01929	3083.9	3421.4	6.4230	0.016555	3062.4	3393.5	6.3348
600	0.02491	3208.6	3582.3	6.6776	0.02106	3191.5	3560.2	6.5866	0.018178	3174.0	3537.6	6.5048

TABLE A.3 (Continued)

T	v	u	h	s	v	u	h	s	v	u	h	s
650	0.02680	3310.3	3712.3	6.822	0.02274	3296.0	3693.9	6.7357	0.019693	3281.4	3675.3	6.6582
700	0.02861	3410.9	3840.1	6.957	0.02434	3398.7	3824.6	6.8736	0.02113	3386.4	3809.0	6.7993
800	0.03210	3610.9	4092.4	7.204	0.02738	3601.8	4081.1	7.1244	0.02385	3592.7	4069.7	7.0544
900	0.03546	3811.9	4343.8	7.428	0.03031	3804.7	4335.1	7.3507	0.02645	3797.5	4326.4	7.2830
1000	0.03875	4015.4	4596.6	7.635	0.03316	4009.3	4589.5	7.5589	0.02897	4003.1	4582.5	7.4925
1100	0.04200	4222.6	4852.6	7.828	0.03579	4216.9	4846.4	7.7531	0.03145	4211.3	4840.2	7.6874
1200	0.04523	4433.8	5112.3	8.011	0.03876	4428.3	5106.6	7.9360	0.03391	4422.8	5101.0	7.8707
1300	0.04845	4649.1	5376.0	8.184	0.04154	4643.5	5370.5	8.1093	0.03636	4638.0	5365.1	8.0442
	$P = 25.0$ MPa				$P = 30.0$ MPa				$P = 40.0$ MPa			
375	0.001973	1798.7	1848.0	4.0320	0.0017892	1737.8	1791.5	3.9305	0.001641	1677.1	1742.8	3.8290
400	0.006004	2430.1	2580.2	5.1418	0.002790	2067.4	2151.1	4.4728	0.001908	1854.6	1930.9	4.1135
425	0.007881	2609.3	2806.3	5.4723	0.005303	2455.1	2614.2	5.1504	0.002532	2096.9	2198.1	4.5029
450	0.009162	2720.7	2949.7	5.6744	0.006735	2619.3	2821.4	5.4424	0.003693	2365.1	2512.8	4.9459
500	0.011123	2884.3	3162.4	5.9592	0.008678	2820.7	3081.1	5.7905	0.005622	2678.4	2903.3	5.4700
550	0.012724	3017.5	3335.6	6.1765	0.010168	2970.3	3275.4	6.0342	0.006984	2869.7	3149.1	5.7785
600	0.014137	3137.9	3491.4	6.3602	0.011446	3100.5	3443.9	6.2331	0.008094	3022.6	3346.4	6.0114
650	0.015433	3251.6	3637.4	6.5229	0.012596	3221.0	3598.9	6.4058	0.009063	3158.0	3520.6	6.2054
700	0.016646	3361.3	3777.5	6.6707	0.013661	3335.8	3745.6	6.5606	0.009941	3283.6	3681.2	6.3750
800	0.018912	3574.3	4047.1	6.9345	0.015623	3555.5	4024.2	6.8332	0.011523	3517.8	3978.7	6.6662
900	0.021045	3783.0	4309.1	7.1680	0.017448	3768.5	4291.9	7.0718	0.012962	3739.4	4257.9	6.9150
1000	0.02310	3990.9	4568.5	7.3802	0.019196	3978.8	4554.7	7.2867	0.014324	3954.6	4527.6	7.1356
1100	0.02512	4200.2	4828.2	7.5765	0.020903	4189.2	4816.3	7.4845	0.015642	4167.4	4793.1	7.3364
1200	0.02711	4412.0	5089.9	7.7605	0.022589	4401.3	5079.0	7.6692	0.01694	4380.1	5057.7	7.5224
1300	0.02910	4626.9	5354.4	7.9342	0.024266	4616.0	5344.0	7.8432	0.018229	4594.3	5323.5	7.6969

SOURCES: Keenan, Keyes, Hill, and Moore, *Steam Tables*, Wiley, New York, 1969; G.J. Van Wylen and R. E. Sonntag, *Fundamentals of Classical Thermodynamics*, Wiley, New York, 1973.

TABLE A.4 COMPRESSED LIQUID

	$P = 5$ MPa (263.99°C)				$P = 10$ MPa (311.06°C)				$P = 15$ MPa (342.42°C)			
T	v	u	h	s	v	u	h	s	v	u	h	s
0	0.0009977	0.04	5.04	0.0001	0.0009952	0.09	10.04	0.0002	0.0009928	0.15	15.05	0.0004
20	0.0009995	83.65	88.65	0.2956	0.0009972	83.36	93.33	0.2945	0.0009950	83.06	97.99	0.2934
40	0.0010056	167	172	0.5705	0.0010034	166.35	176.38	0.5686	0.0010013	165.8	180.78	0.5666
60	0.0010149	250.2	255.3	0.8285	0.0010270	249.36	259.49	0.8258	0.0010105	248.5	263.67	0.8232
80	0.0010268	333.7	338.9	1.0720	0.0010245	332.59	342.83	1.0688	0.0010222	331.5	346.81	1.0656
100	0.0010410	417.5	422.7	1.3030	0.0010385	416.12	426.50	1.2992	0.0010361	414.7	430.28	1.2955
120	0.0010576	501.8	507.1	1.5233	0.0010549	500.08	510.64	1.5189	0.0010522	498.4	514.19	1.5145
140	0.0010768	586.8	592.2	1.7343	0.0010737	584.68	595.42	1.7292	0.0010707	582.7	598.72	1.7242
160	0.0010988	672.6	672.1	1.9375	0.0010953	670.13	681.08	1.9317	0.0010918	667.7	684.09	1.9260
180	0.0011240	759.6	765.3	2.1341	0.0011199	756.65	767.84	2.1275	0.0011159	753.8	770.50	2.1210
200	0.0011530	848.1	853.9	2.3255	0.0011480	844.5	856.0	2.3178	0.0011433	841.0	858.2	2.3104
220	0.0011866	938.4	944.4	2.5128	0.0011805	934.1	945.9	2.5039	0.0011748	929.9	947.5	2.4953
240	0.0012264	1031	1038	2.6979	0.0012187	1026.0	1038.1	2.6872	0.0012114	1020.8	1039.0	2.6771
260	0.0012749	1128	1134	2.8830	0.0012645	1133.7	1133.7	2.8699	0.0012550	1114.6	1133.4	2.8576

	$P = 20$ MPa (365.81°C)				$P = 30$ MPa				$P = 50$ MPa			
T	v	u	h	s	v	u	h	s	v	u	h	s
0	0.0009904	0.19	20.01	0.0004	0.0009856	0.25	29.82	0.0001	0.0009766	0.20	49.03	0.0014
20	0.0009928	82.77	102.62	0.2923	0.0009886	82.17	111.8	0.2899	0.0009804	81.00	130.02	0.2848
40	0.0009992	165.17	185.16	0.5646	0.0009951	164.04	193.9	0.5607	0.0009872	161.86	211.21	0.5527
60	0.0010084	247.68	267.85	0.8206	0.0010042	246.06	276.2	0.8154	0.0009962	242.98	292.79	0.8052
80	0.0010199	330.40	350.80	1.0624	0.0010156	328.3	358.8	1.0561	0.0010073	324.34	374.70	1.0440
100	0.0010337	413.39	434.06	1.2917	0.0010290	410.78	441.7	1.2844	0.0010201	405.88	456.89	1.2703
120	0.0010496	496.76	517.76	1.5102	0.0010445	493.59	524.9	1.5018	0.0010348	487.65	539.39	1.4857
140	0.0010678	580.69	602.04	1.7193	0.0010621	576.88	608.8	1.7098	0.0010515	569.77	622.35	1.6915
160	0.0010885	665.35	687.12	1.9204	0.0010821	660.82	693.3	1.9096	0.0010703	652.41	705.92	1.8891
180	0.0011120	750.95	773.20	2.1147	0.0011047	745.59	778.7	2.1024	0.0010912	735.69	790.25	2.0794
200	0.0011388	837.7	860.5	2.3031	0.0011302	831.4	865.3	2.2893	0.0011146	819.7	875.5	2.2634
240	0.0012046	1016.0	1040.0	2.6674	0.0011920	1006.9	1042.6	2.6490	0.0011702	990.7	1049.2	2.6158
280	0.0012965	1204.7	1230.6	3.0248	0.0012755	1190.7	1228.0	2.9986	0.0012415	1167.2	1229.3	2.9537
320	0.0014437	1415.7	1444.6	3.3979	0.0013997	1390.7	1432.7	3.3539	0.0013388	1353.3	1420.2	3.2868
360	0.0018226	1702.8	1739.3	3.8772	0.0016265	1626.6	1675.4	3.7494	0.0014838	1556.0	1630.2	3.6291

SOURCES: Keenan, Keyes, Hill, and Moore, *Steam Tables*, Wiley, New york, 1969; G.J. Van Wylen and R. E. Sonntag, *Fundamentals of Classical Thermodynamics*, Wiley, New York, 1973.

APPENDIX II

ANSWERS TO MULTIPLE-CHOICE QUESTIONS

Chapter-1

1.1 (d) 1.2 (a) 1.3 (d) 1.4 (c)
1.5 (b) 1.6 (a) 1.7 (d) 1.8 (d)
1.9 (b) 1.10 (d) 1.11 (c)

Chapter-2

2.1 (a) 2.2 (c) 2.3 (a) 2.4 (b)
2.5 (a) 2.6 (b) 2.7 (b) 2.8 (c)
2.9 (c) 2.10 (b) 2.11 (a)

Chapter-3

3.1 (d) 3.2 (c) 3.3 (d) 3.4 (a)
3.5. (b) 3.6 (b) 3.7 (d) 3.8 (c)
3.9 (a) 3.10 (a) 3.11 (b)

Chapter-4

4.1 (d) 4.2 (c) 4.3 (d)

Chapter-5

5.1 (b) 5.2 (d) 5.3 (c) 5.4 (c)
5.5 (c) 5.6 (d) 5.7 (c) 5.8 (a)
5.9 (a) 5.10 (d) 5.11 (b) 5.12 (c)
5.13 (c)

Chapter-6

6.1 (a) 6.2 (c) 6.3 (b) 6.4 (b)
6.5 (b) 6.6 (c) 6.7 (b) 6.8 (a)
6.9 (b) 6.10 (a) 6.11 (c) 6.12 (c)
6.13 (c) 6.14 (a) 6.15 (c) 6.16 (b)
6.17 (a) 6.18. (a)

Chapter-7

7.1 (c) 7.2 (d) 7.3 (a) 7.4 (b)
7.5 (c)

Chapter-8

8.1	(b)	8.2	(d)	8.3	(b)	8.4	(c)

Chapter-9

9.1	(b)	9.2	(a)	9.3	(d)	9.4	(b)
9.5	(d)	9.6	(a)	9.7	(b)	9.8	(d)
9.9	(a)	9.10	(b)	9.11	(a)		

Chapter-10

10.1	(c)	10.2	(a)	10.3	(c)

Chapter-11

11.1	(c)	11.2	(b)	11.3	(b)	11.4	(b)
11.5	(d)	11.6	(c)	11.7	(e)	11.8	(b)
11.9	(b)	11.10	(d)	11.11	(d)	11.12	(b)
11.13	(a)	11.14	(c)	11.15	(c)	11.16	(a)
11.17	(b)	11.18	(c)	11.19	(c)	11.20	(a)
11.21	(d)	11.22	(a)	11.23	(b)	11.24	(b)
11.25	(b)	11.26	(c)	11.27	(b)	11.28	(c)
11.29	(b)	11.30	(d)	11.31	(b)		

Chapter-12

12.1	(c)	12.2	(d)	12.3	(c)	12.4	(d)
12.5	(c)	12.6	(c)	12.7	(c)	12.8	(a)
12.9	(c)						

Chapter-13

13.1	(b)	13.2	(a)	13.3	(b)	13.4	(d)
13.5	(c)	13.6	(d)	13.7	(a)	13.8	(d)
13.9	(a)	13.10	(d)	13.11	(b)	13.12	(d)
13.13	(b)	13.14	(c)	13.15	(c)	13.16	(b)
13.17	(b)	13.18	(d)	13.19	(b)	13.20	(a)
13.21	(d)	13.22	(b)	13.23	(c)		

Chapter-14

14.1	(c)	14.2	(c)	14.3	(a)	14.4	(d)
14.5	(b)	14.6	(b)	14.7	(c)	14.8	(c)
14.9	(b)	14.10	(d)	14.11	(a)	14.12	(c)
14.13	(b)	14.14	(d)	14.15	(d)	14.16	(c)
14.17	(a)	14.18	(c)				

APPENDIX III

ANSWERS TO NUMERICAL PROBLEMS

Chapter-2

2.1 331.5°C, –13.27 kJ
2.2 (a) (i) 27.88 kJ (ii) –4 kJ (iii) –2.863 kJ (b) 21.017 kJ
2.3 25.2 kJ
2.4 (a) 9.1 kJ (b) 4.7 kJ (c) 4.7 kJ
2.5 44.5 kJ
2.6 125.85 kJ/kg
2.7 127.26 kJ
2.8 2.7 kJ

Chapter-3

3.1 24.05 kJ
3.3 0.878 bar, 30.57 K, 17.57 kJ, 0, 17.57 kJ
3.4 126.03 kJ/kg, 16.42 kJ/kg, 209.74°C, 107.13 kJ/kg
3.5 – 2118 kJ, – 176.5 kJ, –1941.5 kJ
3.6 $C_P = 1.008$ kJ/kgK, $C_v = 0.72$ kJ/kgK, 8.0263
3.7 377.5 kJ, 1.3, greater
3.9 (i) – 515.86kJ (ii) – 34353.9 kJ
3.10 $A = -100$ kW, $B = 0$, $C = 400$ kW, $D = 500$ kW, $E = 600$ kW
3.11 478 kJ

Chapter-4

4.1 600 kJ/kg
4.2 778.44 m/s
4.3 471.6 kW
4.4 66.36 kW
4.5 – 18.82 kW
4.6 2133.4 kW
4.7 –40 kW
4.8 179.58 kW
4.10 10 kW

Chapter-5

5.1 400 K
5.2 708 K, 475 K
5.6 0.6 kJ
5.7 2.85 kW
5.8 4.37°C
5.9 38.75°C
5.10 31%, No
5.11 – 0.62 kW
5.12 29 kW
5.13 450 kW, 43.75%, 1.31, 0.7657
5.14 326.6 K, 86 kW
5.15 Rs 138.89

Chapter-6

6.1 $(\Delta s)_{surr} = -1.648$ kJ/K, $(\Delta S)_{sys} = 1.77$ kJ/K, $(\Delta S)_{univ} = 0.1226$ kJ/K
6.2 $(\Delta S)_{Al} = -4.04$ kJ/K, $(\Delta S)_{water} = 5.914$ kJ/K, $(\Delta S)_{univ} = 1.874$ kJ/K
6.3 0.14 kJ/K
6.4 0.514 kJ/K

6.5 0.697 kJ/K, 0.199 kJ/K, – 0.498 kJ/K, 0

6.6 40%, 500 K

6.8 10.2 kJ, 0.0078 kJ/K

6.9 1321.5 kJ, 2.31 kJ/K

6.10 –75.94 kJ/K , 75.94 kJ/K , 0

Chapter-7

7.1 0.0012165 m^3/kg, 0.06668 m^3/kg, 1008.4 kJ/kg, 2804.2 kJ/kg, 2.6457 kJ/kgK, 6.1869 kJ/kg-K

7.2 0.0014036 m^3/kg, 0.02167 m^3/kg, 1344 kJ/kg, 2749 kJ/kg, 3.2534 kJ/kg-K,5.7045 kJ/kg-K

7.3 0.05359 m^3/kg, 2445.04 kJ/kg, 5.4787 kJ/kg-K

7.4 $x = 0.727$

7.5 151.9°C, 0.3749 m^3/kg, 2748.7 kJ/kg, 6.8212 kJ/kg-K , 2108.5 kJ/kg

7.6 0.03 m^3/kg, 151.9°C, 0.077, 802.58 kJ/kg, 2.2427 kJ/kg-K

7.7 233.9°C, 6.3404 kg, 0.00475 m^3/kg, 1105.37 kJ/kg, 2.837 kJ/kg-K, 1091.16 kJ/kg

7.8 2.04

7.9 3.973 Mpa, 8.3988 kg, 0.00355 m^3/kg, 1166.06 kJ/kg, 2.947 kJ/kg-K, 1151.93 kJ/kg

Chapter-8

8.1 – 40.04 kJ

8.2 – 0.0127 kJ/K, – 3.66 kJ, – 3.66 kJ

8.3 64.25 kJ/kg, – 0.1989 kJ/kg-K, 0, 25.5 kJ/K, 0

8.4 0.95 kJ/K, 637.76 kJ, 0, 0, 0, 273.33 kJ, – 0.95 kJ/K, – 509.83 kJ, – 145.6 kJ

Chapter-9

9.1 57.21%

9.2 59%

9.3 55.73%

9.4 2 MPa, 706 K, 7.92 MPa, 2795 K, 0.395 MPa, 1187 K, 57.5%, 862.5 kJ/kg

9.5 2683 kJ, 55.34%

9.6 (a) 2520 kPa, (b) 607 kJ/kg, (c) 10

9.7 1906 K, 605.5 kJ, 63.2%

9.8 671.65 kJ/kg, 60%

9.9 57.65%

9.10 (a) 4.43 MPa, 2080.28 K, (b) 2.35, (c) 58.67% , (d) 704.04 kJ/k

Chapter-10

10.1 33.5 %

10.2 36.33%

10.3 25%

10.4 32.33%

10.5 31.6%

Chapter-11

11.1 800 kg/m^3, 1.6×10^{-3} N-s/m^2

11.2 0.03125 N, 0.0078 W

11.3 109.75 W

11.4 60 N

11.5 0.497 m/s

11.6 $T = \dfrac{4\pi\pi\omega\mu^4}{3h}$

11.7 $\dfrac{\mu_1}{\mu_1 + \mu_2} h, \dfrac{h}{1 + \sqrt{\dfrac{\mu_2}{\mu_1}}}$

11.8 148.93 N/m^2, 1.862 N-s/m^2

11.9 8.5 mm

11.10 47.1 N

11.11 7.467 N-s/m^2, 9.33×10^{-3} m^2/s

11.12 $\dfrac{h\sqrt{\mu_1/\mu_2}}{1+\sqrt{\mu_1/\mu_2}}$

11.13 4.24 mm

11.14 14.6 kPa

Chapter-12

12.1 49.46 kPa, 0.607 kg/m^3

12.2 77.36 kPa

12.3 $P_A - P_B = 549.36$ kPa

12.4 $P_1 - P_2 = 103.79$ kPa

12.5 $P_A - P_B = 1.24$ kPa

12.6 34.33 kPa(vac)

12.7 49.457 kPa

12.8 15.3 kN/m^2

12.9 101.3 kPa, 1.22 kg/m^3

Chapter-13

13.1 721, Laminar

13.2 $2x - y = 0)$

13.3 (a) $\dfrac{x^3}{3} + y = \dfrac{7}{3}$, (b) $y^{1/3} = 4^{1/3}\ x^{1/4}$

13.4 $x^3 y$ = constant)

13.5 $x^{1/4}y^{1/3} = 1,\ x^{1/4}z^{1/5} = 1$

13.6 $x^2y = 2, 4x + z = 5$

13.7 $(x - y_0)y^2 - x_0y_0y + y_0^3 = 0$

13.8 3.18 m/s, 5.66 m/s

13.9 0.1414 m^3/s, 3.994 m/s

13.10 (a) 4.05 m/s (b) 0.173 m^3/s, 0.113 m^3/s

13.11 $\dfrac{2y}{x^2 + y^2}$

13.12 $\dfrac{z^2}{2} - xz + f(x, y) + C$

13.13 (a) Yes, (b) No, (iii) Yes

Chapter-14

14.1 40.438 m

14.2 26.458 kN/m^2

14.3 11.85 m

14.4 179.366 kN/m^2

14.5 From *Y* to *X*

14.6 From *A* to *B*, 11.86 m of oil

14.7 From Section 1 to 2, 1.065 m

14.8 0.0218 m^3/s

14.9 0.0955 m^3/s

14.10 0.0782 m^3/s

14.11 1.039 m^3/s

14.12 26 cm

14.13 0.0134 m^3/s, 1.09 m/s

14.14 0.164 m^3/s

14.15 8.08 m/s

14.16 3.1 m/s

14.17 0.066 m^3/s

SOLVED QUESTION PAPER—2012

ENGINEERING THERMODYNAMICS AND FLUID MECHANICS

Group-A
(Multiple-Choice Questions)

1. Choose the correct alternatives for any ten of the following: $10 \times 1 = 10$

(i) During throttling process
   - (a) internal energy remains constant
   - (b) entropy remains constant
   - (c) enthalpy remains constant
   - (d) pressure remains constant.

Answer: (c) enthalpy remains constant

(ii) Which of the following is an intensive thermodynamic property?
   - (a) volume
   - (b) energy
   - (c) mass
   - (d) temperature

Answer: (d) Temperature

(iii) Newton's law of viscosity relates to
   - (a) pressure, velocity and viscosity
   - (b) shear stress and rate of angular deformation in a fluid
   - (c) shear stress, temperature, viscosity and velocity
   - (d) pressure, viscosity and rate of angular deformation

Answer: (b) shear stress and rate of angular deformation in a fluid

(iv) Stoke is the unit of
   - (a) surface tension
   - (b) viscosity
   - (c) kinematic viscosity
   - (d) None of these

Answer: (c) kinematic viscosity

(v) The first law of thermodynamics furnishes the relationship between
   - (a) heat, work and properties of the system
   - (b) heat and internal energy
   - (c) various thermodynamic properties of the system
   - (d) heat and properties of the system

Answer: (a) heat, work and properties of the system

(vi) Entropy change depends on
   - (a) change of temperature
   - (b) mass transfer
   - (c) thermodynamic state
   - (d) heat transfer

Answer: (d) heat transfer

(vii) The increase in temperature
- (a) increases the viscosity of the liquid
- (b) decreases the viscosity of the liquid
- (c) increases the viscosity of the gas
- (d) Both (b) and (c)

Answer: (d) Both (b) and (c)

(viii) A streamline is a line
- (a) which is along the path of a particle
- (b) which is always parallel to the main direction of flow
- (c) across which there is no flow
- (d) on which tangent drawn on any point gives the direction of velocity

Answer: (c) across which there is no flow
(d) on which tangent drawn on any point gives the direction of velocity

(ix) For the same compression ratio and heat rejection, the efficiency of Otto cycle is
- (a) greater than diesel cycle
- (b) less than diesel cycle
- (c) equal to diesel cycle
- (d) None of these

Answer: (a) greater than diesel cycle

(x) A refrigerator and a heat pump operate between the same temperature limits. If the COP of the refrigerator is 4, the COP of the heat pump would be
- (a) 3
- (b) 4
- (c) 5
- (d) None of these

Answer: (c) 5

(xi) Work done in a free expansion process is
- (a) positive
- (b) negative
- (c) zero
- (d) maximum

Answer: (c) zero

(xii) A stagnation point is a point in a fluid flow where
- (a) pressure is zero
- (b) velocity of flow is zero
- (c) total energy is zero
- (d) total energy is maximum

Answer: (b) velocity of flow is zero

(xiii) An inventor claims that his heat engine has the following specifications. Power developed 50 kW, Fuel burned per hour 3 kg, heating value of fuel 75000 kJ perkg. Temperature limits. His engine is
- (a) reality
- (b) impossible
- (c) costly
- (d) None of these

Answer: (b) impossible

(xiv) A flow of viscous fluid with $\mu = 1.0$ Ns/m^2 has a velocity distribution given by $u = 0.90y - y^2$. The shear stress at $y = 0.45$ m is

(a) 0.90 Ns/m^2 (b) zero

(c) infinity (d) None of these

Answer: (b) zero

(xv) For irrotational flow

(a) V = constant (b) $\nabla \times V = f(t)$

(c) $\nabla \times V = 0$ (d) $\nabla \times V = V(x, y, t)$

Answer: (c) $\nabla \times V = 0$

(xvi) Reynolds number is expressed as

(a) $\rho VD/\mu$ (b) $V^2 D/\rho$

(c) $V\rho^2 S/v$ (d) $V^2 D^2/v$

Answer: (a) $\rho VD/\mu$

Group-B
(Short-Answer Questions)
(Answer any three questions) $5 \times 3 = 15$

2. (a) Draw the rheological diagram for Newtonian and non-Newtonian fluids. 2

Answer:

Refer to Section 11.8.3.

(b) Show the pressure decreases exponentially with elevation for an isothermal compressible fluid at rest. 3

Answer:

Refer to Section 12.4.2.

3. (a) What is the basic difference between a process and a cycle? 2

Answer:

Refer to Section 1.9.

(b) Show that the work done in isothermal process from the state 1 to state 2 is given by

$$W_{1-2} = p_1 v_1 (\log_e p_1 - \text{loge}\, p_2) \qquad 3$$

Answer:

Refer to Section 2.9.3.

4. Show that the two-dimensional flow described by the equation $\psi = x + 2x^2 - 2y^2$ is irrotational. 5

Answer:

Stream function $\psi = x + 2x^2 - 2y^2$

From the definition of stream function ψ, we get

$$u = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \psi}{\partial x}$$

Thus, the velocity components become

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x + 2x^2 - 2y^2) = -4y$$

$$v = \frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(x + 2x^2 - 2y^2) = -1 - 4x$$

Hence, $\frac{\partial v}{\partial x} = -4$

Hence, $\frac{\partial u}{\partial y} = -4$

The rotation is given by

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}[-4 - (-4)] = 0$$

Since the rotation is zero, the flow is irrotational.

5. (a) State Newton's law of viscosity. 2

Answer:

Refer to Section 11.8.

(b) What are the causes of viscosity? 2

Answer:

There are two causes of viscosity are of two: the intermolecular forces of attraction (cohesive forces) and the molecular momentum transfer.

Any layer of moving fluid always tries to drag the molecules of the adjacent layer because of the strong intermolecular forces of attraction between the molecules of the adjacent layers. It produces the effect of viscosity as already discussed.

Consider molecules in two adjacent moving layers (layer 1-1 and 2-2) of fluid in unidirectional flow between two parallel plates as shown in Fig. 1 below. Assume that layer 1-1 is moving faster than that of layer 2-2. Some molecules from layer 1-1, in course of their continuous thermal agitation, migrate into layer 2-2 and collide with the molecules already prevailing in the layer 2-2. By collision the momentum of the migrated molecules is shared with the molecules of layer 2-2 and as a result, layer 2-2 is speeded up. Similarly, molecules from layer 2-2 migrate into 1-1 and tend to retard the layer 1-1. Every such migration causes forces of acceleration or deceleration so as to eliminate the difference in velocity between the layers and produces the effect of viscosity.

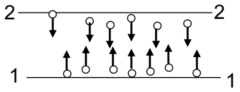


Fig. 1 *Movement of fluid molecules between two adjacent moving layers*

(c) What is no-slip condition? 1

Answer:

Refer to Section 11.10.

6. A 150 mm diameter shaft rotates at 1500 rpm in a 200 mm long journal bearing with an internal diameter 150.5 mm. The uniform annular space between the shaft and the bearing is filled with oil of dynamic viscosity 0.8 poise. Calculate the power required to rotate the shaft. 5

Answer:

The problem is worked out in Example 11.6.

GROUP-C

(Long-answer Questions)

(Answer any three questions) $3\times 15=45$

7. (a) What is PMM2? Why is it impossible? What is its difference from PMM1? 5

Answer:

Refer to Sections 5.5.1 and 3.3.

(b) Show that the COP of a heat pump is greater than the COP of a refrigerator by unity. 4

Answer:

Refer to Section 5.4.1.

(c) A Carnot heat engine draws heat from a reservoir at temperature T_A and rejects heat to another reservoir at temperature T_B. The Carnot forward cycle again drives a Carnot reversed cycle engine or Carnot refrigerator, which absorbs heat from reservoir at temperature T_C and rejects heat to reservoir at temperature T_A. Derive an expression for the ratio of heat absorbed from reservoir at temperature T_B, such that heat supplied to engine Q_A is equal to heat absorbed by refrigerator Q_C. Determine efficiency and COP (Coefficient Of Performance) of Carnot refrigerator. 6

Answer:

The arrangement is shown in Fig. 2.

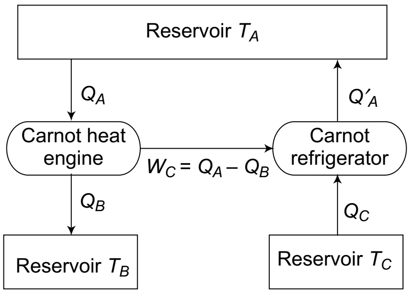


Fig. 2

Efficiency of Carnot heat engine is given by

$$\eta = \frac{W_c}{Q_A} = \frac{Q_A - Q_B}{Q_A} = \frac{T_A - T_B}{T_A}$$

or
$$W_c = Q_A \frac{T_A - T_B}{T_A}$$

Coefficient of performance of Carnot refrigerator is given by

$$COP_{Carnot} = \frac{Q_C}{W_c} = \frac{Q_C}{Q'_A - Q_C} = \frac{T_C}{T_A - T_C}$$

or
$$W_c = Q_C \frac{T_A - T_C}{T_C}$$

Comparing the above two equations, we have

$$Q_A \frac{T_A - T_B}{T_A} = Q_C \frac{T_A - T_C}{T_C}$$

or
$$\frac{T_A - T_C}{T_C} \times \frac{T_A}{T_A - T_B} = \frac{Q_A}{Q_C}$$

or
$$\frac{T_A - T_C}{T_C} \times \frac{T_A}{T_A - T_B} = 1 \qquad \left(\text{Given that } \frac{Q_A}{Q_C} = 1\right)$$

Note that the problem statement is not clear since there is no heat absorbed from reservoir at temperature T_B.

8. (a) A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of specific gravity 0.9 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference in mercury level in the two limbs is 50 cm and the height of fluid in the left limb from the centre of pipe is 10 cm below. Assume any other data required. 5

Answer:

The U-tube mercury manometer is shown in Fig. 3.

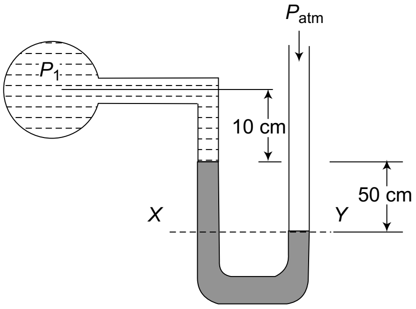


Fig. 3

Given data:

Vertical height between the pipe and the level of meniscus in the left limb $x = 10 \text{ cm} = 0.1 \text{ m}$

Difference in height of meniscus between two limbs $\Delta h = 50 \text{ cm} = 0.5 \text{ m}$

Let p_1 be the pressure in the pipe.

Now, for the left limb, pressure along the plane XY is

p_X = pressure in the pipe + pressure corresponding to 10 cm of fluid of specific gravity 0.9
+ pressure corresponding to 50 cm of mercury

$$= p_1 + \rho_w g x + \rho_m g \Delta h$$
$$= p_1 + 1000 \times 0.9 \times 9.81 \times 0.1 + 13600 \times 9.81 \times 0.5$$

For the right limb, pressure along the plane *XY* is

$$p_Y = p_{atm}$$

Equating the pressures of both the limbs along the horizontal plane *XY*, we obtain

$$p_1 + 1000 \times 0.9 \times 9.81 \times 0.1 + 13600 \times 9.81 \times 0.5 = p_{atm}$$

or
$$p_1 = -67590.9 \text{ Pa}(\text{gauge}) = 67.5909 \text{ kPa}(\text{vacuum})$$

(b) The velocity vector in a fluid flow is given by

$$V = 2x^3 i - 5x^2 yj + 2tk$$

Find the velocity and acceleration of a fluid particle at (2, 1, 3) at time $t = 1$. 10

Answer:

Velocity vector is given as

$$V = 2x^3 i - 5x^2 yj + 2tk$$

Therefore, velocity at (2,1,3) at time $t = 1$ is

$$\vec{V}\Big|_{(2,1,3)} = \left[2 \times 2^3\right]i - \left[5 \times 2^2 \times 1\right]j + \left[2 \times 1\right]k = 16i - 20j + 2k$$

The magnitude of velocity is

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{16^2 + (-20)^2 + 2^2} = 25.69 \text{ units}$$

The acceleration components a_x, a_y and a_z are given by

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Given that
$$u = 2x^3$$

Hence,
$$\frac{\partial u}{\partial t} = 0,\ \frac{\partial u}{\partial x} = 6x^2,\ \frac{\partial u}{\partial y} = 0,\ \frac{\partial u}{\partial z} = 0$$

$$v = -5x^2 y$$

$$\frac{\partial v}{\partial t} = 0,\ \frac{\partial v}{\partial x} = -10xy,\ \frac{\partial v}{\partial y} = -5x^2,\ \frac{\partial v}{\partial z} = 0$$

$$w = 2t$$

$$\frac{\partial w}{\partial t} = 2,\ \frac{\partial w}{\partial x} = 0,\ \frac{\partial w}{\partial y} = 0,\ \frac{\partial w}{\partial z} = 0$$

Substituting these values in acceleration components, we have

$$a_x = 0 + 2x^3 \times 6x^2 - 5x^2y \times 0 + 2t \times 0 = 12x^5$$

$$a_y = 0 + 2x^3 \times (-10xy) - 5x^2y \times \left(-5x^2\right) + 2t \times 0 = 5x^4y$$

$$a_z = 2 + 2x^3 \times 0 - 5x^2y \times 0 + 2t \times 0 = 2$$

Acceleration is then given by

$$\vec{a} = a_x i + a_y j + a_z k = 12x^5 i + 5x^4 yj + 2k$$

Acceleration components at (2 , 1, 3) at time $t = 1$ is

$$a_x = 12x^5 = 12 \times 2^5 = 384 \text{ units}$$

$$a_y = 5x^4y = 5 \times 2^4 \times 1 = 80 \text{ units}$$

$$a_z = 2 \text{ units}$$

Thus, the acceleration is

$$\vec{a} = 384i + 80j + 2k$$

The magnitude of acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{384^2 + 80^2 + 2^2} = 392.25 \text{ units}$$

9. (a) Derive the expression for continuity equation for a three-dimensional steady incompressible flow. 5

Answer:

Refer to Section 13.5.2.

(b) Describe the steady flow and unsteady flow. 2

Answer:

Refer to Section 13.3.1.

(c) A jet of water from a 25 mm dia nozzle is directed vertically upwards, assuming that jet remains steady and neglecting any loss of energy. What will be the dia at a point 4.5 m above the nozzle, if the velocity with which jet leaves the nozzle is 12 m/s. 8

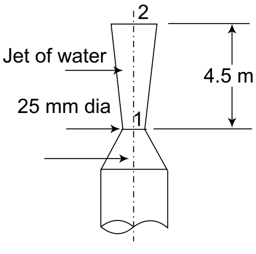


Fig. 4

Answer:

This problem is identical to worked out Example 13.11.

10. (a) An engine working on Otto cycle has an air standard cycle efficiency of 56% and rejects 544 kJ/kg of air. The pressure and temperature of air at the beginning of compression are 0.1 MPa and 60° respectively. Calculate

- (i) the compression ratio of the cycle
- (ii) work done/kg of air
- (iii) the pressure and temperature at the end of compression
- (iv) maximum pressure of the cycle 8

Answer:

The problem is worked out in Example 9.4.

(b) Find the pressure at an elevation of 3000 m above the sea level by assuming

- (i) An isothermal condition of air
- (ii) An isentropic condition of air

Pressure and temperature at sea level are 101.32 kN/m² and 293.15 K. Consider air to be an ideal gas with $R = 287$ J/kgK and $\gamma = 1.4$. 7

Solution

Given data:

Atmospheric pressure at the sea level $p_0 = 101.32 \text{ kN/m}^2 = 101.32 \times 10^3 \text{ kN/m}^2$

Temperature of air at the sea level $T_0 = 293.15$ K

Density of air at the sea level $\rho_0 = \dfrac{p_0}{RT_0} = \dfrac{101.32 \times 10^3}{287 \times 293.15} = 1.204 \text{ kg/m}^3$

Altitude from the sea level $y - y_0 = 3000$ m

(i) Pressure at an elevation of 3000 m above the sea level is found to be

$$p = p_0 \exp\left[-\frac{\rho_0}{p_0} g(y - y_0)\right]$$

$$= 101.32 \times 10^3 \exp\left[-\frac{1.204}{101.32 \times 10^3} \times 9.81 \times 3000\right]$$

$$= 71.4189 \times 10^3 \text{ N/m}^2 = 71.4189 \text{ kN/m}^2$$

(ii) Pressure at an elevation of 3000 m above the sea level is found to be

$$p = p_0 \left[1 - \frac{k-1}{k} \frac{\rho_0}{p_0} g(y - y_0)\right]^{\frac{k}{k-1}}$$

$$= 101.32 \times 10^3 \left[1 - \frac{1.4 - 1}{1.4} \times \frac{1.204}{101.32 \times 10^3} \times 9.81 \times 3000\right]^{\frac{1.4}{1.4-1}}$$

$$= 70.094 \times 10^3 \text{ N/m}^2 = 70.094 \text{ kN/m}^2$$

11. (a) Determine the quantity of heat required to produce 1 kg of steam at a pressure of 6 bar at a temperature of 25°C under the following conditions:
   (i) When the steam is wet having a dryness fractions 0.9.
   (ii) When the steam is dry saturated.
   (iii) When it is superheated at a constant pressure at 25°.

   Assume the mean specific heat of superheated steam to be 2.3 kJ/kg. 7

Solution

From the saturated steam table based on pressure (see Appendix 1.2), it is found that saturation temperature of water corresponds to 6 bar is $t_{sat} = 158.9$ C

Specific enthalpy of saturated liquid is $h_f = 670.6$ kJ/kg

Specific enthalpy of saturated vapour $h_g = 2756.8$ kJ/kg

Specific heat of water $C_{pw} = 4.18$ kJ/kg

(i) The process of steam generation can be divided into the following distinct steps:

(A) Conversion of water at 25ºC to saturated water at 158.9ºC

The heat required for this change is $\Delta H_1 = m \times C_{pw} \times (T_{sat} - T_i)$

$$= 1 \times 4.18 \times (158.9 - 25) = 559.7 \text{ kJ}$$

(B) Conversion of saturated water to wet steam having a dryness fraction 0.9 without change in temperature.

The heat required for this process is $\Delta H_2 = m \times x \times (h_g - h_f)$

$$\Delta H_2 = 1 \times 0.9 \times (2756.8 - 670.6) = 1877.58 \text{ kJ}$$

Therefore, total heat required to produce 1 kg of steam is

$$\Delta H = \Delta H_1 + \Delta H_2$$
$$= 559.7 + 1877.58 = 2437.28 \text{ kJ}$$

(ii) The process of steam generation can be divided into the following distinct steps:

(A) Conversion of water at 25ºC to saturated water at 158.9ºC

The heat required for this change is $\Delta H_1 = m \times C_{pw} \times (T_{sat} - T_i)$

$$= 1 \times 4.18 \times (158.9 - 25) = 559.7 \text{ kJ}$$

(B) Conversion of saturated water to saturated steam without change in temperature.

The heat required for this process is $\Delta H_2 = m \times (h_g - hf)$

$$\Delta H_2 = 1 \times (2756.8 - 670.6) = 2086.2 \text{ kJ}$$

Therefore, total heat required to produce 1 kg of steam is

$$\Delta H = \Delta H_1 + \Delta H_2$$
$$= 559.7 + 2086.2 = 2645.9 \text{ kJ}$$

(iii) The process of steam generation can be divided into the following distinct steps:

(A) Conversion of water at 25ºC to saturated water at 158.9ºC

The heat required for this change is $\Delta H_1 = m \times C_{pw} \times (T_{sat} - T_i)$

$$= 1 \times 4.18 \times (158.9 - 25) = 559.7 \text{ kJ}$$

(B) Conversion of saturated water to saturated steam without change in temperature.

The heat required for this process is $\Delta H_2 = m \times (h_g - h_f)$

$$\Delta H_2 = 1 \times (2756.8 - 670.6) = 2086.2 \text{ kJ}$$

(C) Conversion of saturated steam into superheated steam of 250ºC

The heat required for this change is $\Delta H_3 = m \times C_{pv} \times (T_{sat} - T_i)$

$$= 1 \times 2.3 \times (250 - 158.9) = 209.53 \text{ kJ}$$

Therefore, total heat required to produce 1 kg of steam is

$$\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$$
$$= 559.7 + 2086.2 + 209.53 = 2855.43 \text{ kJ}$$

(b) Two bodies, each of equal mass m and heat capacity C_P are of temperature T_1 and T_2 respectively ($T_1 > T_2$). The first body is used as source of heat for reversible engine and the second body as the sink. Show that the maximum work obtainable from such an arrangement is $mC_P\left(\sqrt{T_1} - \sqrt{T_2}\right)^2$. 8

Answer:

The problem is worked out in Example 6.7.

SOLVED QUESTION PAPER—2013

ENGINEERING THERMODYNAMICS AND FLUID MECHANICS

Group-A
(Multiple-Choice Questions)

1. Choose the correct alternatives for any ten of the following: $10 \times 1 = 10$

(i) If heat engine attains 100% thermal efficiency, it violets
- (a) zeroth law of thermodynamics
- (b) first law of thermodynamics
- (c) second law of thermodynamics
- (d) None of these

Answer: (c) second law of thermodynamics

(ii) The more effective way of increasing efficiency of a Carnot engine is
- (a) increase higher temperature
- (b) decrease higher temperature
- (c) increase lower temperature
- (d) decrease lower temperature

Answer: (d) decrease lower temperature

(iii) Air standard efficiency of Otto Cycle depends on
- (a) the ratio of specific heats
- (b) the cut-off ratio
- (c) the compression ratio
- (d) Both (a) and (c)

Answer: (d) Both (a) and (c)

(iv) The flow field represented by the velocity vector $V = axi + by^2 j + czt^2 k$, where a, b and c are constants is
- (a) three-dimensional and unsteady
- (b) two-dimensional and steady
- (c) three-dimensional and steady
- (d) two-dimensional and unsteady

Answer: (c) three-dimensional and unsteady

(v) Pitot tube is used to measure
- (a) dynamic viscosity
- (b) kinematic viscosity
- (c) mass density
- (d) velocity of flow

Answer: (d) velocity of flow

(vi) PMM-1 is impossible according to
- (a) second law of thermodynamics
- (b) third law of thermodynamics
- (c) first law of thermodynamics
- (d) zeroth law of thermodynamics

Answer: (c) first law of thermodynamics

(vii) During throttling, which of the following properties does not change?

(a) Internal energy (b) Entropy

(c) Pressure (d) Enthalpy

Answer: (d) Enthalpy

(viii) Which fluid does not experience shear stress during flow?

(a) Pseudo-plastic (b) Dilatant

(c) Inviscid (d) Newtonian

Answer: (c) Inviscid

(ix) The differential equation of pressure variation in a static fluid may be written as (y measured vertically upward and γ is specific weight)

(a) $dP = -\gamma dy$ (b) $\gamma dP = -dy$

(c) $\gamma dy = -\rho dP$ (d) $\gamma dP = -\rho dy$

Answer: (a) $dP = -\gamma dy$

(x) The standard atmospheric pressure is 101.32 kPa. The local atmospheric pressure at a location was 91.52 kPa. If a pressure is recorded as 22.48 kPa (gauge), it is equivalent to

(a) 123.80 kPa (abs) (b) 88.84 kPa (abs)

(c) 114.00 kPa (abs) (d) 69.04 kPa (abs)

Answer: (c) 114.00 kPa (abs)

(xi) For an ideal gas, for which of the following processes can the temperature of a system decrease even if heat is added to it?

(a) Isobaric (b) Isothermal

(c) Isentropic (d) Polytropic

Answer: Temperature of a system can never decrease when heat is added to it.

(xii) The area under a curve, representing a non-cyclic process on a temperature entropy (T-S) plane represents

(a) heat transfer for a reversible process

(b) work transfer for a reversible process

(c) heat transfer for any process

(d) work transfer for any process

Answer: (a) heat transfer for a reversible process

(xiii) Oil spreads on the surface of water because

(a) oil is less dense than water (b) oil is immiscible in water

(c) oil has less surface tension than water (d) oil has low vapour pressure

Answer: (c) oil has less surface tension than water

(xiv) Spot the odd one out in the following options:

(a) Thermal conductivity (b) Kinetic energy

(c) Work (d) Pressure

Answer: (c) Work

(xv) An engine is supplied with 1120 kJ/s of heat and the source and sink are maintained at constant fixed temperatures of 560 K and 280 K respectively. If heat rejection is 840 kJ/s, indicate the given cycle is

(a) reversible (b) irreversible

(c) impossible (d) unpredictable, insufficient data

Answer: (b) irreversible

Group-B
(Short-Answer Questions)
(Answer any three questions) $5 \times 3 = 15$

2. A 0.025 m^3 vessel contains 0.3 kg of steam at 2 MPa. Determine the quality, enthalpy and entropy of steam. Given $t_s = 212.2$ °C, $v_f = 0.001177$ m^3/kg, $v_g = 0.0995$ m^3/kg, $h_f = 908.5$ kJ/kg, $h_{fg} = 1888.7$ kJ/kg, $s_f = 2.447$ kJ/kg-K, $s_{fg} = 3.590$ kJ/kg-K

Answer:

Specific volume of steam is $v = \frac{V}{m} = \frac{0.025}{0.3} = 0.083 \text{ m}^3/\text{kg}$

Specific volume can be expressed in terms of quality of steam, x, as

$$v = v_f + x\left(v_g - v_f\right)$$

or $$0.083 = 0.001177 + x\left(0.0995 - 0.001177\right)$$

or $$x = 0.8322$$

The enthalpy of steam is found to be

$$H = m(h_f + xh_{fg}) = 0.3 \times \left(908.5 + 0.8322 \times 1888.7\right) = 744.08 \text{ kJ}$$

The entropy of steam is found to be

$$S = m(s_f + xs_{fg}) = 0.3 \times \left(2.447 + 0.8322 \times 3.590\right) = 1.6304 \text{ kJ/K}$$

3. 0.2 m^3 of an ideal gas at a pressure of 2 MPa and 600 K is expanded isothermally to 5 times the initial volume. It is then cooled to 300 K at constant volume and then compressed back polytropically to its initial state. Determine the net work done and heat transfer during the cycle

Answer: The cycle on the P-V diagram is shown in Fig. 1.

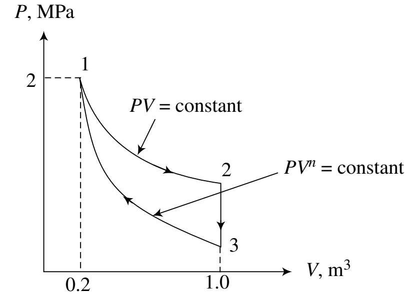


Fig. 1

Pressure after isothermal expansion is $P_2 = \frac{V_1}{V_2} P_1 = \frac{0.2}{1.0} \times 2 = 0.4 \text{ MPa}$

Work done for the isothermal expansion 1-2 is

$$W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1} = 2 \times 10^3 \times 0.2 \times \ln \frac{1}{0.2} = 643.78 \text{ kJ}$$

For the constant-volume process 2-3, the work done is zero that is $W_{2-3} = 0$

Pressure at point 3 is found to be $P_3 = \frac{T_3}{T_2} P_2 = \frac{300}{600} \times 0.4 = 0.2 \text{ MPa}$

For the polytropic compression process 3-1, we have $P_1 V_1^n = P_2 V_2^n$

The polytropic index of compression is found to be

$$n = \frac{\ln \frac{P_1}{P_3}}{\ln \frac{V_3}{V_1}} = \frac{\ln \frac{2}{0.2}}{\ln \frac{1}{0.2}} = 1.43$$

The work done for the polytropic compression 3-1 is found to be

$$W_{3-1} = \frac{P_3 V_3 - P_1 V_1}{n-1} = \frac{0.2 \times 10^3 \times 1 - 2 \times 10^3 \times 0.2}{1.43 - 1} = -465.12 \text{ kJ}$$

Net work done for the cycle is

$$W_{net} = W_{1-2} + W_{2-3} + W_{3-1} = 643.78 + 0 - 465.12 = 178.66 \text{ kJ}$$

From the first law of thermodynamics for a cycle, net heat transfer to the gas during the cycle $Q_{net} = 178.66$ kJ

4. At the inlet to a certain nozzle the specific enthalpy of fluid passing is 2800 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it. (i) Find the velocity at exit of the nozzle, (ii) If the inlet area is 900 cm^2 and specific volume at inlet is 0.187 m^3/kg, find the mass flow rate, (iii) If the specific volume at the nozzle exit is 0.498 m^3/kg, find the exit area of the nozzle.

Answer:

The problem is worked out in Example 4.2.

5. (a) Derive an expression for displacement work in a process where $PV^n = C$. 2

Answer:

Refer to Section 2.9.4.

(b) A paddle wheel used for mixing and stirring of fluids turns 600 r.p.m. when 2.5 Nm torque is applied to it. What is power transmitted to the liquid by the wheel?

Answer:

The power transmitted to the liquid by the wheel is given by

$$P = T\omega = T \times \frac{2\pi N}{60}$$

$$= 2.5 \times \frac{2\pi \times 600}{60} = 157.08 \text{ W}$$

6. (a) Establish the equivalence of Kelvin-Planck and Clausius statements.

Answer:

Refer to Section 5.7.

(b) A heat engine produces work equivalent to 80 kW with an efficiency of 40%. Determine the heat drawn from the source and rejected to the sink.

Answer:

Thermal efficiency of the heat engine is given by

$$\eta_{\text{th}} = \frac{\dot{W}_{net}}{\dot{Q}_1} = 0.4$$

where $\dot{W}_{net}$ is the rate of work done and $\dot{Q}_1$ is the rate of heat drawn from the source.

or $$\dot{Q}_1 = \frac{\dot{W}_{net}}{0.4} = \frac{80}{0.4} = 200 \text{ kW}$$

The rate of heat rejected to the sink is

$$\dot{Q}_2 = \dot{Q}_1 - \dot{W}_{net} = 200 - 80 = 120 \text{ kW}$$

Group-C
(Short-Answer Questions)

(Answer any three questions) $3 \times 15 = 45$

7. (a) Write Bernoulli's equation, stating the assumptions. 5

Answer:

Refer to Section 14.3.

(b) A two-dimensional flow is described in the Lagrangian coordinate system as

$$x = x_0 e^{-kt} + y_0\left(1 - e^{-kt}\right)$$

$$y = y_0 e^{kt}$$

Find the equation of path line of the particle and the velocity components in Eulerian system. 4

Answer:

This is given in Numerical Problem 13.7.

(c) A venturimeter has inlet and throat diameters of 300 mm and 150 mm. Water flows through it at the rate of 0.065 m^3/s and the differential gauge is deflected 1.2 m. The specific gravity of the manonmetric liquid is 1.6. Determine the coefficient of discharge of the venturimeter. 6

Answer:

The problem is worked out in Example 14.16.

8. (a) State and prove Pascal's law of pressure at a point of a fluid body. 5

Answer:

Refer to Section 12.3.

(b) The velocity vector for a two-dimensional, incompressible flow field is given by

$V = -\left(\frac{x}{x^2 + y^2}\right)i + \left(\frac{y}{x^2 + y^2}\right)j$. State whether the flow fleld is continuous or discontinuous.

5

Answer:

The problem is worked out in Example 13.14.

(c) A diffuser consists of two circular parallel plates 20 cm in diameter and 0.5 cm apart and connected to a 3 cm diameter pipe. If the streamlines are assumed to be radial in the diffuser, what mean velocity in the pipe will correspond to an exit velocity of 0.5 m/s? 5

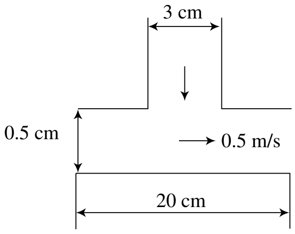


Fig. 2

Answer:

The problem is worked out in Example 13.9.

9. (a) What is pure substance? 5

Answer:

Refer to Section 7.1.

(b) What is the critical point? State the values of critical pressure and critical temperature of water? 4

Answer:

Refer to Section 7.2.1.

(c) Why is the Carnot cycle not practicable for a steam power plant?

Answer:

Refer to Section 10.2.

(d) At the inlet to a certain nozzle the specific enthalpy of fluid passing is 3000 kJ/kg and the velocity is 60 m/s. At the discharge end, the enthalpy is 2762 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.

(i) Find the velocity at exit of the nozzle,

(ii) If the inlet area is 0.1 m^2 and specific volume at inlet is 0.187 m^3/kg, find the mass flow rate,

(iii) If the specific volume at the nozzle exit is 0.498 m^3/kg, find the exit area of the nozzle.

Answer:

The problem is worked out in Example 4.2.

10. (a) In a steam turbine, steam at 20 bar, 360 °C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds the water back into the boiler. Assume ideal processes, find, per kg of steam, the net work and the cycle efficiency.

Answer:

The problem is similar to worked out Example 10.5.

(b) An air standard Otto cycle has a compression ratio of 8, temperature and pressure at the beginning of compression are 20°C and 1 bar respectively. The constant volume heat addition is 1800 kJ/kg. Calculate the maximum pressure and temperature of the cycle, temperature at the end of compression process. What are the efficiency and mean effective pressure (m.e.p.) of the cycle.

$C_v = 0.718$ kJ/kgK and $\gamma = 1.4$.

Answer:

The problem is worked out in Example 9.4.

11. (a) Two reversible heat engines are arranged in a series in such a way that the heat rejected by the first engine is absorbed by the second engine. The first engine receives 400 kJ of heat from a reservoir maintained at temperature 600°C, while the second engine rejects heat to a reservoir having temperature 0°C. If the work output of the first engine is twice than that of the second, determine

(i) the efficiency of both the engines.

(ii) the heat rejected by the second engine.

(iii) the intermediate temperature. 7

Answer:

The problem is worked out in Example 5.16.

(b) Air at temperature of 15°C passes through a heat exchanger with a velocity of 30 m/s where its temperature is raised to 800°C. It then enters a turbine with same velocity of 30 m/s and expands until the temperature falls to 650°C. On leaving the turbine, air is taken at a velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C. If the air flow rate is 2 kg/s, find out

(i) the rate of heat transfer to the air in the heat exchanger.

(ii) the power output from turbine, assuming no heat loss.

(iii) the velocity at exit from nozzle, assuming no heat loss.

Take, $C_p = 1.005$ kJ/kgK and $h = C_p t$ (h = enthalpy, t = temperature) 8

Answer:

(i) The rate of heat transfer to the air in the heat exchanger is found to be

$$\dot{Q} = \dot{m}C_p\Delta T = 2\times 1.005\times(800-15) = 1577.85 \text{ kW}$$

(ii) The steady flow energy equation for the turbine is given by

$$\dot{m}\left(h_1+\frac{V_1^2}{2}\right) = \dot{m}\left(h_2+\frac{V_2^2}{2}\right)+\dot{W}$$

or $$\dot{m}\left(C_pT_1+\frac{V_1^2}{2}\right) = \dot{m}\left(C_pT_2+\frac{V_2^2}{2}\right)+\dot{W}$$

or $$2\times\left(1005\times 800+\frac{30^2}{2}\right) = 2\times\left(1005\times 650+\frac{60^2}{2}\right)+\dot{W}$$

or $$\dot{W} = 298800 \ W = 298.8 \text{ kW}$$

(iii) The steady flow energy equation for the nozzle is given by

$$h_1+\frac{V_1^2}{2} = h_2+\frac{V_2^2}{2}$$

or $$C_pT_1+\frac{V_1^2}{2} = C_pT_2+\frac{V_2^2}{2}$$

or $$1005\times 650+\frac{60^2}{2} = 1005\times 500+\frac{V_2^2}{2}$$

or $$V_2 = 552.36 \text{ m/s}$$

SOLVED QUESTION PAPER—2014

ENGINEERING THERMODYNAMICS AND FLUID MECHANICS

Group-A
(Multiple-Choice Questions)

1. Choose the correct alternatives for any ten of the following: $10 \times 1 = 10$

(i) For an irreversible process, the entropy change is

(a) greater than $\frac{\delta Q}{T}$ (b) equal to $\frac{\delta Q}{T}$

(c) less than $\frac{\delta Q}{T}$ (d) equal to zero

Answer: (a) greater than $\frac{\delta Q}{T}$

(ii) Which of the following is an intensive thermodynamic property?

(a) Volume (b) Mass

(c) Temperature (d) Energy

Answer: (c) Temperature

(iii) Work done in a free expansion is

(a) positive (b) negative

(c) maximum (d) zero

Answer: (d) zero

(iv) The latent heat of vaporization at the critical point is

(a) equal to zero (b) greater than zero

(c) less than zero (d) none of these.

Answer: (a) equal to zero

(v) Newton's law of viscosity relates to

(a) pressure, velocity and viscosity

(b) shear stress and rate of angular deformation in a fluid

(c) shear stress, temperature, velocity and viscosity

(d) pressure, viscosity and rate of angular deformation in a fluid

Answer: (b) shear stress and rate of angular deformation in a fluid

(vi) Euler's equation is written as

(a) $\frac{dp}{\rho} + V^2 dV + gdz = 0$ (b) $\frac{dp}{\rho} + VdV + gdz = 0$

(c) $\dfrac{dp}{\rho} + V^2 dV + gdz = 0$ (d) $\dfrac{dp}{\rho^2} + V^2 dV + gdz = 0$

Answer: (b) $\dfrac{dp}{\rho} + VdV + gdz = 0$

(vii) Kinematic viscosity is defined as

(a) dynamic viscosity × density (b) dynamic viscosity/density

(c) dynamic viscosity × pressure (d) pressure × density

Answer: (b) dynamic viscosity/density

(viii) Dynamic viscosity has the dimension

(a) MLT^{-2} (b) $ML^{-1}T^{-1}$

(c) $ML^{-1}T^{-2}$ (d) $M^{-1}L^{-1}T^{-1}$

Answer: (b) $ML^{-1}T^{-1}$

(ix) The change of entropy when heat is absorbed by a gas is

(a) positive (b) negative

(c) positive or negative (d) none of these

Answer: (a) positive

(x) A stagnation point is a point in fluid flow where

(a) pressure is zero (b) velocity of flow is zero

(c) total energy is zero (d) total energy is maximum

Answer: (b) velocity of flow is zero

Group-B
(Short-Answer Questions)
(Answer any three questions) $5 \times 3 = 15$

2. A solid cube weighing 5 N and having a 45 cm edge is allowed to slide down an inclined plane surface making an angle of 30° with the horizontal. There is a uniform oil fill of 0.008 cm thickness. If the cube is having a velocity of 13 cm/s, determine the viscosity of the oil. Also find out the kinematic viscosity in stokes if the oil has a density of 850 kg/m^3.

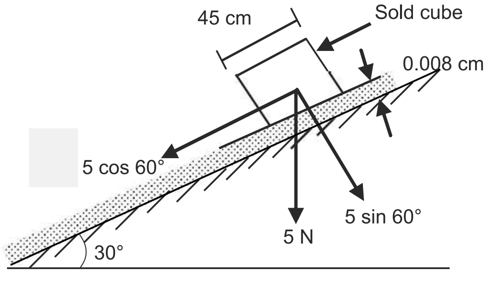


Fig. 1

Answer:

Thickness of the film $h = 0.0085 \text{ cm} = 8\times 10^{-5} \text{m}$,
Weight of the cube $W = 5$ N
Area of the cube $A = 45 \text{ cm} \times 45 \text{ cm} = 0.2025 \text{ m}^2$,
Velocity of the cube $V = 0.13$ m/s
Component of weight along the slope is $W \cos 60°$.
Considering linear velocity profile within the oil, the velocity gradient is found to be

$$\frac{du}{dy} = \frac{V-0}{h} = \frac{V}{h}$$

Viscous resistance F is given by

$$F = \text{shear stress} \times \text{area} = \tau A$$

or $$F = \mu \frac{du}{dy} A = \mu \frac{V}{h} A$$

The viscous resistance to the motion should be equal to the component of the weight of the solid cube along the slope. Thus,

$$\mu \frac{V}{h} A = W \cos 60°$$

or $$\mu \frac{0.13}{8\times 10^{-5}} \times 0.2025 = 5 \cos 60°$$

or, $$\mu = 7.6\times 10^{-3} \text{ Ns/m}^2$$

Kinematic viscosity is found to be

$$\nu = \frac{\mu}{\rho} = \frac{7.6\times 10^{-3}}{850} = 8.94\times 10^{-6} \text{ m}^2\text{/s} = 0.0894° \text{ Stokes}$$

3. (a) What is Euler's equation of motion? How will you obtain Bernoulli's equation from that?

Answer: Refer Sections 14.2 and 14.3.

(b) An incompressible fluid is flowing through a pipe of 10 cm diameter under a gauge pressure of 40 N/cm^2 and with a mean velocity of 5 m/s. Find the total head of water at a cross section which is 8 m above the line.

Answer:

Average velocity of flow $V = 5$ m/s,

Gauge pressure $p = 40 \text{ N/cm}^2 = 40\times 10^4 \text{ N/m}^2$

Height above datum $z = 8$ m

Pressure head above atmospheric pressure is $\frac{p}{\rho g} = \frac{40\times 10^3}{1000\times 9.81} = 40.775$ m of water

Velocity head is $\frac{V^2}{2g} = \frac{5^3}{2\times 9.81} = 1.274$ m of water

Datum head is $z = 8$ m

Total head of water is $\frac{p}{\rho g}+\frac{V^2}{2g}+z = 40.775+1.274+8 = 50.049$ m of water above atmospheric pressure.

4. (a) What do you mean by vacuum pressure?

Answer:

When the pressure is below the atmospheric pressure and is expressed with reference to local atmospheric pressure, it is called *vacuum pressure*. Mathematically, one can write $p_{\text{vac}} = p_{\text{atm}} - p_{\text{abs}}$, where p_{abs} is the absolute pressure.

(b) In a condenser, the vacuum pressure is found to be 145 mm of Hg and the barometer reads 735 mm of Hg. Find the absolute pressure in kPa.

Answer:

Height of mercury column $h = 15 \text{ cm} = 0.15 \text{ m}$

Pressure corresponding to 145 cm of Hg column can be found as

$$p_{\text{vac}} = \rho g h$$

$$= 13600 \times 9.81 \times 0.145 = 19345.32 \text{ N/m}^2$$

Atmospheric pressure is found to be

$$p_{\text{atm}} = 13600 \times 9.81 \times 0.735 = 98060.76 \text{ N/m}^2$$

Thus, the absolute pressure is found to be

$$p_{\text{abs}} = p_{\text{atm}} - p_{\text{vac}}$$

$$= 98060.76 - 19345.32 = 78715.44 \text{ N/m}^2 = 78.715 \text{ kPa}$$

5. The fluid pressure is given by $V = x^2 y\hat{i} + y^2 z\hat{j} - (2xyz + yz^2)\hat{k}$. Show that this is a case of possible steady incompressible flow. Calculate the velocity and acceleration at (2, 1, 3).

Answer:

Since the time *t* does not appear in the velocity, the flow is steady.

For incompressible flow, the following continuity equation must be satisfied

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0$$

Now, velocity is given by $\vec{V} = x^2 y\hat{i} + y^2 z\hat{j} - (2xyz + yz^2)\hat{k}$

Hence, $\frac{\partial u}{\partial x} = 2xy, \frac{\partial v}{\partial y} = 2yz, \frac{\partial w}{\partial z} = -(2xy+2yz)$

Thus, $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 2xy+2yz-2xy-2yz = 0$

The given velocity field satisfies the continuity equation. Therefore, this is a case of possible incompressible flow.

Velocity at (2, 1, 3)

$$\left.\vec{V}\right]_{(2,1,3)} = \left[(2^2)(1)\right]\hat{i} + \left[(1^2)(3)\right]\hat{j} - \left[(2)(2)(1)(3) + (1)(3)^2\right]\hat{k} = 4\hat{i} + 3\hat{j} - 21\hat{k}$$

Acceleration is given by

$$\vec{a} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

Acceleration at (2, 1, 3)

$$\vec{a}]_{(2,1,3)} = \left[x^2 y\{2xy\hat{i} - 2yz\hat{k}\} + y^2 z\{x^2\hat{i} + 2yz\hat{j} - (2xz + z^2)\hat{k}\} - (2xyz + yz^2)\{y^2\hat{j} - (2xy + 2yz)\hat{k}\}\right]_{(2,1,3)}$$

$$= \left[2^2.1\{2.2.1\hat{i} - 2.1.3\hat{k}\} + 1.^2 3\{2^2\hat{i} + 2.1.3\hat{j} - (2.2.3 + 3^2)\hat{k}\} - (2.2.1.3 + 1.3^2)\{1^2\hat{j} - (2.2.1 + 2.1.3)\}\hat{k}\right]$$

$$= \left[16\hat{i} - 24\hat{k} + 12\hat{i} + 18\hat{j} - 63\hat{k} - 21\hat{j} + 210\hat{k}\right] = 28\hat{i} - 3\hat{j} + 123\hat{k}$$

6. Draw a block diagram of vapour compression refrigeration cycle and also show the corresponding *p–v* and *T–s* plots.

Answer:

Presently not in the syllabus

Group-C
(Short-Answer Questions)

(Answer any three questions) $3 \times 15 = 45$

7. (a) Derive Euler's equation. How you can obtain Bernoulli's equation from it?

Answer:

Refer Sections 14.2 and 14.3.

(b) Water is flowing through a taper pipe of 100 m length having a diameter of 600 mm at the upper end and 300 mm at the lower end at the rate of 50 L/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher end is 19.62 N/m^2.

Answer:

The pipe is schematically shown in Fig. 2. Let 1 and 2 respectively designate the lower and higher ends of the pipe.

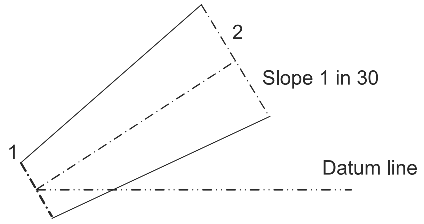


Fig. 2

Diameter of pipe at lower end $D_1 = 300$ mm $= 0.3$ m

Diameter of pipe at higher end $D_2 = 600$ mm $= 0.6$ m

Pressure at higher end $p_2 = 19.62$ N/m^2

Volume flow rate of water $Q = 50 \text{ lit/s} = \frac{50}{1000} \text{m}^3/\text{s} = 0.05 \text{ m}^3/\text{s}$

Cross-sectional area at lower end is $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.0707 \text{ m}^2$

Cross-sectional area at higher end is $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.6)^2 = 0.2827 \text{ m}^2$

Length of pipe $L = 100$ m

Slope of pipe = 1 in 30

Let the datum line pass through the centre of the lower end. Then

Height of lower end above datum $z_1 = 0$

Height of higher end above datum $z_2 = \frac{1}{30} \times 100 = 3.33$ m

From the continuity equation, we have

$$Q = A_1 V_1 = A_2 V_2$$

Thus, the average velocity at the lower end is $V_1 = \frac{Q}{A_1} = \frac{0.05}{0.0707} = 0.707$ m/s

Average velocity at the higher end is $V_2 = \frac{Q}{A_2} = \frac{0.05}{0.2827} = 0.177$ m/s

Applying Bernoulli's equation between Sections 1 and 2 along a streamline, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{p_1}{\rho g} + \frac{0.707^2}{2 \times 9.81} + 0 = \frac{19.62}{1000 \times 9.81} + \frac{0.177^2}{2 \times 9.81} + 3.33$$

or $$\frac{p_1}{\rho g} = 3.308$$

or $$p_1 = 3.308 \times 1000 \times 9.81 = 32451.48 \text{ N/m}^2$$

(c) The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid having a sp. gr. of 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Sketch the arrangement and find the pressure of fluid in the pipe. The difference of mercury level in the two limbs is 20 cm.

Answer:

The arrangement is shown in Fig. 3.

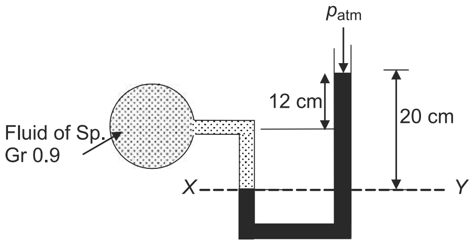


Fig. 3

Density of manometric fluid $\rho_m = 13600\ \text{kg/m}^3$

Vertical height between the pipe and the level of meniscus in the left limb $x = (20-12)\ \text{cm} = 0.08\ \text{m}$

Difference in height of meniscus between two limbs $\Delta h = 20\ \text{cm} = 0.20\ \text{m}$

Let p_1 be the pressure of fluid in the pipe.

For the left limb, pressure along the plane *XY* is

$$p_X = \text{pressure of fluid in the pipe} + \text{pressure corresponding to 8 cm of fluid}$$
$$= p_1 + \rho_w g x$$
$$= p_1 + 900 \times 9.81 \times 0.08$$

For the right limb, pressure along the plane *XY* is

$$p_Y = \text{atmospheric pressure} + \text{pressure corresponding to 20 cm of manometric fluid}$$
$$= p_{\text{atm}} + \rho_m g \Delta h$$
$$= p_{\text{atm}} + 13600 \times 9.81 \times 0.20$$

Equating the pressures of both the limbs along the horizontal plane *XY*, we have

$$p_1 + 900 \times 9.81 \times 0.08 = p_{\text{atm}} + 13600 \times 9.81 \times 0.20$$

or,
$$p_1 = p_{\text{atm}} + 25976.88\ \text{N/m}^2$$
$$= 25976.88\ \text{N/m}^2\ \text{(gauge)}$$

8. (a) Explain intensive and extensive properties.

Answer:

Refer to Section 1.6.

(b) Prove that for a polytropic process, $W = (P_2V_2 - P_1V_1)/1-n$

Answer:

Refer Section 2.9.4.

(c) Air at 14 bar having 0.085 m^3 and 627°C is supplied with heat at constant pressure till its volume becomes double. The air is then expanded isentropically till its pressure drops to 1 bar. Calculate the heat supplied and work done and change in internal energy during constant pressure heating. Also find the total work done per kg of air.

Answer:

The processes are shown in Fig. 4.

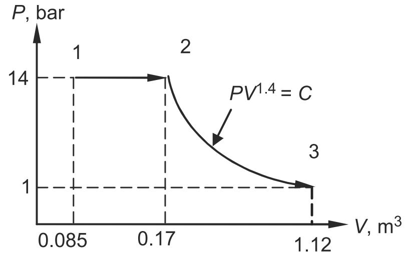


Fig. 4

From the given data, $P_1 = P_2 = 14\text{bar} = 14\times10^5\text{Pa}$, $P_3 = 1\text{ bar} = 1\times10^5\text{ Pa}$, $V_1 = 0.085\text{ m}^3$, $V_2 = 0.17\text{ m}^3$

Mass of air is $m = \dfrac{P_1V_1}{RT_1} = \dfrac{14\times10^5\times0.085}{287\times900} = 0.46\text{ kg}$

For the process 1-2, one can write

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

or $$T_2 = \frac{V_2}{V_1}T_1 = 2T_1 = 1800\text{ K}$$

For the process 2-3, one can write

$$P_2V_2^{1.4} = P_3V_3^{1.4}$$

or, $$V_3 = \left(\frac{P_2}{P_3}\right)^{\frac{1}{1.4}} V_2 = \left(\frac{14}{1}\right)^{\frac{1}{1.4}} (0.17\text{ m}^3) = 1.12\text{ m}^3$$

For the constant pressure process 1-2, the work done is

$$W_{1-2} = P_1(V_2 - V_1) = 14\times10^5(0.17-0.085) = 119000\text{ J} = 119\text{ kJ}$$

For the constant pressure process 1-2, change in internal energy is

$$U_2 - U_1 = mc_p(T_2 - T_1) = 0.46\times1005(1800-900) = 416070\text{ J} = 416.07\text{ kJ}$$

For the constant pressure process 1-2, heat transfer is found to be

$$Q_{1-2} = W_{1-2} + U_2 - U_1 = 119 + 416.07 = 535.07\text{ kJ}$$

For the isentropic process 2-3 ($PV^{1.4} = C$), the work done is

$$W_{2-3} = \frac{P_2V_2 - P_3V_3}{\gamma-1} = \frac{14\times10^5\times0.17 - 1\times10^5\times1.12}{1.4-1} = 31500\text{ J} = 315\text{ kJ}$$

Therefore, the total work done is $W_{1-2} + W_{2-3} = 119 + 315 = 434\text{ kJ}$

The total work done per kg of air $= \dfrac{434}{0.46} = 943.48\text{ kJ/kg}$

9. (a) Define Kelvin-Planck and Clausius' statements.

Answer:

Refer Sections 5.5 and 5.6.

(b) Establish the equivalence Kelvin-Planck and Clausius' statements.

Answer:

Refer Section 5.7.

(c) Two Carnot engines work in series between the source and sink temperatures of 500 K and 300 K. If both engines develop equal power, determine the intermediate temperature.

Answer:

The problem is similar to Example 5.10.

10. (a) Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.9 is connected. The pressure head in the pipe A is 2.5 m of water. Find the pressure in the pipe B for the manometer readings as shown in Fig. 5.

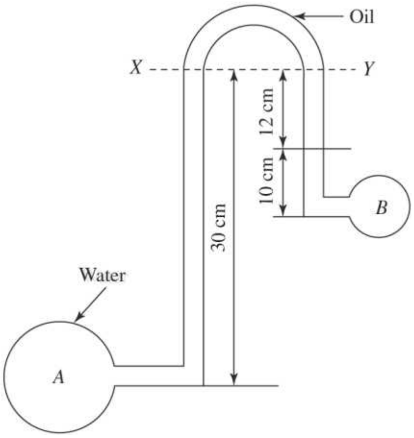


Fig. 5

Answer:

Equating the pressures of both the limb along the horizontal plane *XY*

$$P_A - (1000)(9.81)(0.3) = P_B - (1000)(9.81)(1.0) - (0.9)(1000)(9.81)(0.12)$$

$$(1000)(9.81)(2.5) - (1000)(9.81)(0.3) = P_B - (1000)(9.81)(1.0) - (0.9)(1000)(9.81)(0.12)$$

$$P_B = 32451.48 \text{ Pa} \cong 32.451 \text{ kPa}$$

Pressure at *B* is 32.451 kPa.

(b) The velocity vector for a 2D incompressible flow is given by $\vec{V} = \frac{x}{x^2+y^2}\hat{i} + \frac{y}{x^2+y^2}\hat{j}$. State whether the flow is steady or unsteady.

Answer:

Since time *t* does not appear in the velocity, the flow is steady.

(c) A vertical venturimeter shown in Fig. 6 has an area of 5 cm^2. It has a throat diameter of 1 cm. When oil of sp. gr. 0.8 flows through it, the mercury in the differential gauge indicates a difference in height of 12 cm. Find the discharge through the venturimeter.

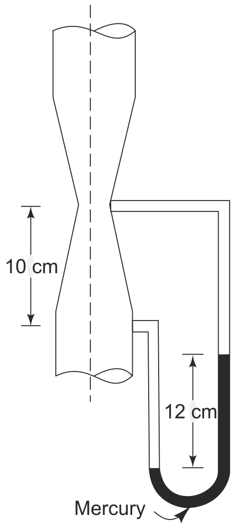


Fig. 6

Answer:

The problem is similar to Example 14.14.

11. (a) Derive the expression for efficiency of an Otto cycle and show the process on *T-s* planes.

Answer:

Refer Section 9.4.

(b) For the same compression ratio, explain why the efficiency of an Otto cycle is greater than that of a diesel cycle.

Answer:

Refer Section 9.6.

(c) An engine working on the Otto cycle is supplied with at 0.1 MPa, 35°C. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency and the mean effective pressure.

Answer:

The problem is similar to Example 9.4.

SOLVED QUESTION PAPER–2015

ENGINEERING THERMODYNAMICS AND FLUID MECHANICS

Group-A

(Multiple-Choice Questions)

1. Choose the correct alternatives for any ten of the following: $10 \times 1 = 10$

(i) For pipes, turbulent flow occurs when Reynolds number is:

(a) Less than 2000 (b) Between 2000 and 4000

(c) More than 4000 (d) Less than 4000

Answer: (c) More than 4000

(ii) Thermal power plants work on:

(a) Carnot cycle (b) Joule cycle

(c) Rankine cycle (d) Otto cycle

Answer: (c) Rankine cycle

(iii) The increase in temperature:

(a) Increases the viscosity of a liquid

(b) Decreases the viscosity of a liquid

(c) Increases the viscosity of a gas

(d) Both (b) and (c)

Answer: (d) both (b) and (c)

(iv) Internal energy for gas in general can be written as:

(a) $du = \left(\frac{du}{dt}\right)_v dt + \left(\frac{du}{dv}\right)_t dv$ (b) $du = \left(\frac{du}{dt}\right)_P dt + \left(\frac{du}{dv}\right)_t dv$

(c) $du = \left(\frac{du}{dt}\right)_v dt + \left(\frac{du}{dv}\right)_P dv$ (d) $du = \left(\frac{du}{dt}\right)_P dt + \left(\frac{du}{dv}\right)_V dv$

Answer: (a) $du = \left(\frac{du}{dt}\right)_v dt + \left(\frac{du}{dv}\right)_t dv$

(v) Velocity potential exists for:
(a) All 3-D flows (b) All irrotational flows
(c) All Newtonian fluid flow (d) Steady irrotational only

Answer: (b) All irrotational flows

(vi) Stoke is the unit of:
(a) Dynamic viscosity (b) Kinematic viscosity
(c) Both (a) and (b) (d) None of these

Answer: (b) Kinematic viscosity

(vii) The household refrigerator operates in:
(a) Absorption refrigeration cycle
(b) Cascade refrigeration
(c) Reversed Carnot cycle
(d) Ordinary vapour compression refrigerator cycle

Answer: (d) Ordinary vapour compression refrigerator cycle

(viii) The equation of a streamline passing through the origin in a flow field $u = \cos\theta$ is:
(a) $y = x^3$ (b) $y = x\cos^2\theta$
(c) $y = x\tan\theta$ (d) $y = \sin\theta$

Answer: (c) $y = x\tan\theta$

(ix) A flow field is said to be steady when:
(a) Conditions change steadily with time
(b) Conditions do not change with time at any point
(c) Conditions do not change steadily with time at any point
(d) The velocity does not change at all with time at any point

Answer: (b) Conditions do not change with time at any point

(x) Which fluid does not experience shear stress during flow?
(a) Pseudo plastic (b) Dilatant (c) Inviscid (d) Newtonian

Answer: (c) Inviscid

Group-B

(Short-Answer Questions)

(Answer any three questions) $3 \times 5 = 15$

2. Consider a gas contained in a piston-cylinder assembly as the system. The gas is initially at a pressure of 5000 kPa and occupies a volume of 0.2 m^3. The gas is taken to the final state where pressure is equal to 100 kPa, by the following two different processes.

(i) The volume of the gas inversely proportional to the pressure
(ii) The process follows the path, pv^n = constant, where $n = 1.4$.

Calculate the work done by the gas in each case.

Answer: The problem is worked out in Example 2.5.

3. (a) Prove that pressure (or intensity of pressure) at a point in a static fluid is equal in magnitude in all direction.

Answer: Refer to Section 12.3.

(b) The fluid flow velocity distribution over a flat plate is $u = 0.5y - y^2$ in which u is velocity in m/s at a distance y above the plate. Determine the shear stress at $y = 0.1$ m if the viscosity of the fluid is 5 Poise.

Answer: The problem is identical to the one worked out in Example 11.2.

4. (a) State Newton's laws of viscosity.

Answer: Refer to Section 11.7.

(b) What is the effect of temperature on viscosity of liquid and that of gas? Explain your answer.

Answer: Refer to Section 11.7.2.

5. The diameters of a pipe at the sections 1 and 2 are 15 cm and 20 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 8 m/s. Determine also the velocity at section 2.

Answer: The problem is identical to the one worked out in Example 13.7.

6. Derive the relation between the performance parameter of a refrigerator and that of a heat pump. What is the advantage of using a heat pump over that of an electric resistance heater for heating purpose?

Answer: Refer to Sections 5.4.1 and 5.4.2.

Group-C

(Long-Answer Questions)

(Answer any three questions) $3 \times 15 = 45$

7. (a) What is bulk modulus of elasticity?

Answer: Refer to Section 11.10.

(b) The space between two large flat and parallel walls, 25 mm apart is filled with a liquid of absolute viscosity 0.7 N-s/m^2. Within this space a thin flat plate 250 mm by 250 mm, is towed at a velocity of 150 mm/s at a distance of 6 mm from one wall, the plate and its movement being parallel to the walls. Assuming linear variations of velocity between the plate and the walls, determine the force exerted by the liquid on the plate.

Answer:

Given data:

Area of the plate: $A = 25\text{ cm} \times 25\text{ cm} = 625\text{ cm}^2 = 0.0625\text{ m}^2$

Viscosity of liquid: $\mu = 0.7$ N–s/m^2

Velocity of plate: $V = 0.15$ m/s

Let F_1 and F_2 be the shear forces on the upper surface and lower surface of the thin plate respectively. Let us also consider that the distance of the thin plate from the top wall is 6 mm as shown in Fig. 1.

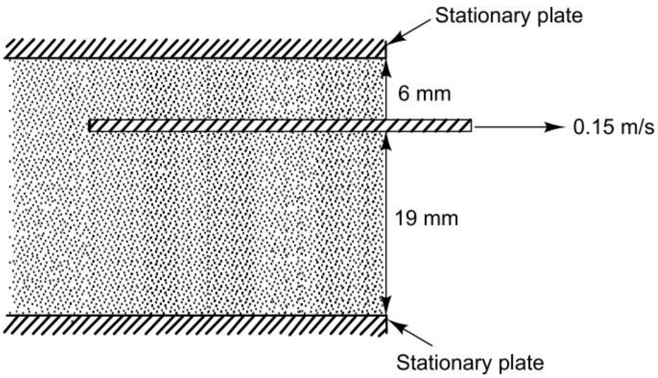


Fig. 1

From Newton's law of viscosity, shear stress on the upper surface of the plate τ_1 is given by:

$$\tau_1 = \mu \frac{du}{dy}$$

where, dy = distance between top wall and thin plate = 6 mm = 6×10^{-3} m

or $$\tau_1 = 0.7 \times \frac{0.15}{6 \times 10^{-3}} = 17.5 \text{ N/m}^2$$

Shear force on the upper surface of the plate is:

$$F_1 = \text{shear stress} \times \text{area} = \tau_1 A$$
$$= 17.5 \times 0.0625 = 1.094 \text{ N}$$

From Newton's law of viscosity, shear stress on the lower surface of the plate τ_2 is given by:

$$\tau_2 = \mu \frac{du}{dy}$$

where, dy = distance between bottom wall and thin plate = 25 – 6 = 19 mm = 19×10^{-3} m

or $$\tau_2 = 0.7 \times \frac{0.15}{19 \times 10^{-3}} = 5.526 \text{ N/m}^2$$

Shear force on the bottom surface of the plate is:

$$F_2 = \text{shear stress} \times \text{area} = \tau_2 A$$
$$= 5.526 \times 0.0625 = 0.345 \text{ N}$$

Force exerted by the liquid on the plate is the sum of the forces on either side of the plate. Therefore, total force exerted by the liquid is:

$$F = F_1 + F_2 = 1.094 + 0.345 = 1.439 \text{ N}$$

(c) What is pure substance?

Answer: Refer to Section 7.1.

8. (a) Derive an expression for continuity equation for a three dimensional steady incompressible flow.

Answer: Refer to Section 13.5.2.

(b) The fluid flow is given by $\vec{v} = x^2 y\vec{i} + y^2 z\vec{j} - (2xyz + yz^2)\vec{k}$. Show that this is the case of possible steady and incompressible flow. Calculate the velocity and acceleration at (2, 1, 3).

Answer: The flow is steady, since the velocity vector is independent of time.

The continuity equation in differential form for an incompressible flow is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$u = x^2y,\ v = y^2z,\ w = -(2xyz + yz^2)$$

Therefore, $$\frac{\partial u}{\partial x} = 2xy,\ \frac{\partial v}{\partial y} = 2yz,\ \frac{\partial w}{\partial z} = -\left(2xy + 2yz\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Therefore, the flow is incompressible.

Velocity vector is given as:

$$\vec{V} = x^2 y\hat{i} + y^2 z\hat{j} - (2xyz + yz^2)\hat{k}$$

Therefore, velocity at (2, 1, 3) is:

$$\vec{V}\Big|_{(2,1,3)} = \left[2^2 \times 1\right]\hat{i} + \left[1^2 \times 3\right]\hat{j} - \left[2 \times 2 \times 1 \times 3 + 1 \times 3^2\right]\hat{k} = 4\hat{i} + 3\hat{j} - 21\hat{k}$$

The acceleration components a_x, a_y and a_z are given by:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Given that: $$u = x^2y$$

Hence, $$\frac{\partial u}{\partial t} = 0,\ \frac{\partial u}{\partial x} = 2xy,\ \frac{\partial u}{\partial y} = x^2,\ \frac{\partial u}{\partial z} = 0$$

$$v = y^2z$$

$$\frac{\partial v}{\partial t} = 0,\ \frac{\partial v}{\partial x} = 0,\ \frac{\partial v}{\partial y} = 2yz,\ \frac{\partial v}{\partial z} = y^2$$

$$w = -(2xyz + yz^2)$$

$$\frac{\partial w}{\partial t} = 0,\ \frac{\partial w}{\partial x} = -2yz,\ \frac{\partial w}{\partial y} = -\left(2xz + z^2\right),\ \frac{\partial w}{\partial z} = -\left(2xy + 2yz\right)$$

Substituting these values in acceleration components, we have:

$$\begin{aligned}
a_x &= 0 + x^2y \times 2xy + y^2z \times x^2 - (2xyz + yz^2)(0)\\
&= 2x^3y^2 + x^2y^2z\\
a_y &= 0 + x^2y \times 0 + y^2z \times 2yz - (2xyz + yz^2) \times y^2\\
&= 2y^3z^2 - 2xy^3z - y^3z^2 = y^3z^2 - 2xy^3z\\
a_z &= 0 + x^2y\left(-2yz\right) + y^2z\left(-2xz - z^2\right) - (2xyz + yz^2)\left(-2xy - 2yz\right)\\
&= -2x^2y^2z - 2xy^2z^2 - y^2z^3 + 4x^2y^2z + 4xy^2z^2 + 2xy^2z^2 + 2y^2z^3\\
&= 2x^2y^2z + 4xy^2z^2 + y^2z^3
\end{aligned}$$

Acceleration is then given by:

$$\begin{aligned}
\vec{a} &= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\\
&= \left(2x^3y^2 + x^2y^2z\right)\hat{i} + \left(y^3z^2 - 2xy^3z\right)\hat{j} + \left(2x^2y^2z + 4xy^2z^2 + y^2z^3\right)\hat{k}
\end{aligned}$$

Acceleration at (2, 1, 3) is:

$$\begin{aligned}
\vec{a}\Big|_{(2,1,3)} &= \left[2 \times 2^3 \times 1^2 + 2^2 \times 1^2 \times 3\right]\hat{i} + \left[1^3 \times 3^2 - 2 \times 2 \times 1^3 \times 3\right]\hat{j}\\
&\quad + \left[2 \times 2^2 \times 1^2 \times 3 + 4 \times 2 \times 1^2 \times 3^2 + 1^2 \times 3^3\right]\hat{k}\\
&= 28\hat{i} - 3\hat{j} + 123\hat{k}
\end{aligned}$$

(c) The velocity vector for a 2D incompressible flow field is given by $\vec{v} = \dfrac{x}{x^2 + y^2}\vec{i} + \dfrac{y}{x^2 + y^2}\vec{j}$. State whether the flow is continuous or discontinuous.

Answer: The problem is worked out in Example 13.14.

9. (a) Derive an expression for the discharge through an orificemeter.

Answer: Refer to Section 14.6.2.

(b) A horizontal orificemeter with orifice diameter 20 cm is inserted in a pipe of 30 cm diameter through which water is flowing. Coefficient of discharge for the orificemeter is 0.62. If the pressure gauges fitted upstream and downstream of the orificemeter show pressure 290 kN/m^2 and 195 kN/m^2 respectively, find the discharge through the pipe.

Answer: The problem is worked out in Example 14.20.

10. (a) Draw the nature of $p - v$ and $T - s$ plots of a Rankine cycle.

Answer: Refer to Section 10.3.

(b) Why is the Carnot cycle not practicable for a steam power plant?

Answer: Refer to Section 10.2.

(c) A thermal power plant is to be operated on an ideal Rankine cycle. Steam enters into the turbine at 2 MPa, 400°C and leaves as saturated liquid in the condenser at 10 kPa (0.01 MPa). The mass flow rate of steam is 1 kg/s. Find out the power developed by the turbine and the efficiency of the cycle. Assume the efficiencies of the turbine and the pump as 0.85 and 0.8 respectively.

Properties of Saturated Water-Pressure Base

		Volume (m^3/kg)		Enthalpy (kJ/kg)			Entropy (kJ/kg-K)		
P (MPa)	Sat temp (°C)	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
2	212.4	0.001177	0.0996	908.8	1890.7	2799.5	2.4478	3.8939	6.3417
0.01	45.8	0.001010	14.67	191.8	2392.8	2584.6	0.6491	7.5019	8.1510

Properties of Superheated Steam Table Corresponding to 2 Mpa and 400°C

Volume (m^3/kg)	Enthalpy (kJ/kg)	Entropy (kJ/kg-K)
0.15120	3247.6	7.1271

Answer: The problem is worked out in Example 10.6.

11. (a) State the first law of thermodynamics for a closed system undergoing a cycle and a process.

Answer: Refer to Sections 3.2.1 and 3.2.2.

(b) State the Clausius inequality.

Answer: Refer to Section 6.2.

(c) Write the steady flow energy equation for single stream entering and single stream leaving a control volume and explain the various terms.

Answer: Refer to Section 4.3.

(d) A gaseous system undergoes three quasi-static processes in sequence. The gas initially at 5 bar and 0.01 m^3 is expended at constant pressure. It is then further expended according to the relation $PV^2 = C$ to 2 bar and 0.025 m^3. The gas is then returned to its initial state during which process $PV = C$. Determine the work done.

Answer: Different processes are shown in Fig. 2.

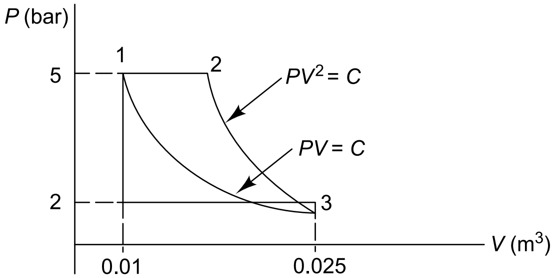


Fig. 2

For the process 2-3, we have:

$$P_2V_2^2 = P_3V_3^2$$

or

$$V_2 = \left(\frac{P_3}{P_2}\right)^{1/2} V_3 = \left(\frac{2}{5}\right)^{1/2} 0.025 = 0.0158 \text{ m}^3$$

For the constant-pressure process 1-2, the work is:

$$W_{1-2} = \int PdV = P(V_2 - V_1) = 500\big(0.0158 - 0.01\big) = 2.9 \text{ kJ}$$

For the process 2-3, work done is:

$$W_{2-3} = \frac{P_2V_2 - P_3V_3}{n-1} = \frac{500 \times 0.0158 - 200 \times 0.025}{2-1} = 2.9 \text{ kJ}$$

The work needed for the process 3-1 is:

$$W_{3-1} = P_1V_1 \ln\frac{V_1}{V_3} = 500 \times 0.01 \ln\left(\frac{0.01}{0.025}\right) = -4.58 \text{ kJ}$$

Finally, the net work done is:

$$W_{\text{net}} = W_{1-2} + W_{2-3} + W_{3-1} = 2.9 + 2.9 - 4.58 = 1.22 \text{ kJ}$$

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