PIXEL
EXAM GUIDE

on

ENGINEERING
THERMODYNAMICS

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McGraw Hill Education (India) Private Limited

Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Engineering Thermodynamics

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This edition can be exported from India only by the publishers,
McGraw Hill Education (India) Private Limited.

1 2 3 4 5 6 7 8 9 D103074 22 21 20 19 18

Printed and bound in India.

ISBN (13): 978-93-5260-796-9
ISBN (10): 93-5260-796-1

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Printer:

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Preface

This book focuses on first and second-year undergraduate students of Mechanical Engineering, Aerospace Engineering and Chemical Engineering who are pursuing a course in Engineering Thermodynamics. It will prove to be a great aid in preparation for various universities semesterial examination. Readers who want to refresh, deepen and systemize their knowledge on this course will find this book as an ultimate resort.

Students often find Engineering Thermodynamics a back-breaking course, because of the rigor that this course offers and importance that it enjoys in later semesters of Mechanical Engineering. The market is flooded with exhaustive and heavy volumes on Engineering Thermodynamics, but there is no individual textbook that provides holistic, simple yet concise and quality coverage on all the key topics. This further adds up to the challenge specially during '*the exam days*'.

This book has a perfect blend of focused content coverage and key pedagogical aids following *question-and-answer format*, *based on real questions that real students ask*. The pedagogical aids have been designed using ***5Es Approach***: ***E****ngage,* ***E****xplore,* ***E****xplain,* ***E****laborate and* ***E****valuate*. Hence, this book from MHE fills the void in the market.

We may consider this book to be an attempt to supplement excellent comprehensive engineering thermodynamics textbooks like Engineering Thermodynamics (PK Nag, Sixth Edition, 2018), Basic and Applied Thermodynamics (PK Nag, Second Edition, 2009)

Highlights

- Focus on basics
- **Just enough theory** with emphasis on special topics such as first law of thermodynamics, second law of thermodynamics, entropy, etc., which are frequently asked in exams
- **Solutions to previous year questions** given from universities such as AKTU, GTU, RGPV, AU, JNTU, VTU, etc.
- **Summary** at the end of each chapter to quickly review the concepts
- Well-labelled **illustrations** and graphical explanation of difficult topics like Entropy and Steam Cycles
- **Examination-oriented** pedagogy:
  - **158** step-wise Solved Examples
  - **124** Practice Problems
  - **115** Multiple Choice Questions

Organization of the Book

This book comprises of 15 chapters. **Chapter 1** starts with the description of basic concepts of the subject and its applications in the vast field of engineering. **Chapter 2** discusses the core aspects of thermodynamics, namely, work and heat. **Chapter 3** takes into account the fundamentals of first law of thermodynamics, its properties and applications. **Chapter 4** deals with the application aspects of the first law when applied to various flow processes. **Chapter 5** is devoted to the description of second law of thermodynamics and covers fundamental theorems and heat engines. **Chapter 6** covers another important property of thermodynamics—entropy—in detail. **Chapter 7** deals with the outcome of the second law of thermodynamics, i.e. exergy (available energy), its analysis and concepts of irreversibility. **Chapter 8** is based on the properties of pure substance, gases and mixtures. **Chapter 9** touches upon the different thermodynamic relations, equilibrium and stability. **Chapter 10** elaborates on the first type of power cycle, i.e. vapour power cycle while **Chapter 11** elucidates another type of power cycle, i.e. gas power cycle. **Chapter 12** describes in detail the refrigeration cycles and their components. **Chapter 13** encompasses the concept of psychrometrics where the properties of atmospheric air, psychrometric chart and its different process are discussed. **Chapter 14** deals with reactive systems and covers chemical reactions, heat and mass laws, and third law of thermodynamics. **Chapter 15** describes the gas compressors along with its varied applications.

Web Supplements

The text is supported by additional content which can be accessed from the weblink *http://www.mhhe.com/exam_prep/et*.

- *Solutions Manual (for Instructors and Students)*

Acknowledgements

The content was extensively reviewed all across the country. Special thanks to faculty members for providing valuable suggestions and encouraging comments regarding the development of the manuscript.

Publisher Note

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Introduction and Basic Concepts

1

Chapter Outline

- ☞ Applications of Thermodynamics
- ☞ Macroscopic and Microscopic Points of View
- ☞ Thermodynamic Definitions
- ☞ Thermodynamic Properties
- ☞ Equilibrium
- ☞ Processes and Cycles
- ☞ Continuum
- ☞ Units and Dimensions
- ☞ Pressure and its Measurement
- ☞ Temperature
- ☞ Zeroth Law
- ☞ Thermometer
- ☞ Temperature Scale

INTRODUCTION

Thermodynamics is a natural science that deals with the energy and its transfer from one form to another. It studies the effect of energy manipulation in different circumstances. The laws of thermodynamics are mere statements expressed based on experiments and natural phenomena observed over a long period. Hence, there are no mathematical proofs that exist for these laws. It finds its applications in varied industries such as power, transportation and simply wherever energy transfer is present. The design and manufacture of such machineries must obey the laws of thermodynamics, however small or big the machines are.

1.1 APPLICATIONS OF THERMODYNAMICS

Typical applications of thermodynamics are:

1. ***Steam power plant:*** In a steam power plant, steam is generated in a boiler and is directed towards a turbine at high pressure and high temperature. This superheated steam does work in the turbine which drives the electric generator to produce electricity. After the contact of steam

with the turbine blades, it loses the pressure and enters the condenser. Here, the remaining heat is transferred from the steam to the cooling water circulating through the tubes. Steam is condensed to water which is then pumped back to the boiler. All these processes require energy transfer between different states of water and hence, thermodynamics becomes more important (Figure 1.1).

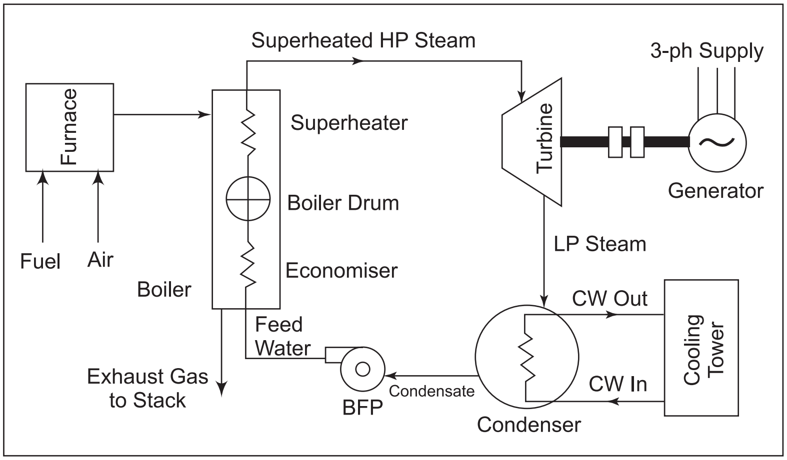


Figure 1.1 *Steam power plant*
(Abbreviations: HP, High pressure; LP, Low pressure; CW, Cooling water; BFP, Boiler feed pump)

2. ***Internal combustion engines:*** Internal combustion engines are heat engines that may be used in vehicles that operate on road, sea or in air/sea. As the name is suggestive, these engines use an internal combustion chamber to burn a fuel or a fuel mixture whose nature could be liquid, solid or gaseous (Figure 1.2). The energy transfer is important as that solely determines the efficiency and work performance of these engines. The design is limited by the laws of thermodynamics. Examples of internal combustion engines are high speed rail, ships and submarines, aircraft and spacecraft.
3. ***Refrigerators:*** A refrigerator is a device which produces cooling effect and can be used at a domestic or an industrial scale (Figure 1.3). In either case, the goal is to remove heat from objects kept inside it in order to produce a cooling effect. This device finds application in household, pharmaceutical companies, hospitals, offices and space missions. The calculations involved must be right otherwise which the drain of energy will terminate the mission and affect economically and environmentally. In a similar way, all discomfort shall be experienced due to wrong calculations in other places.

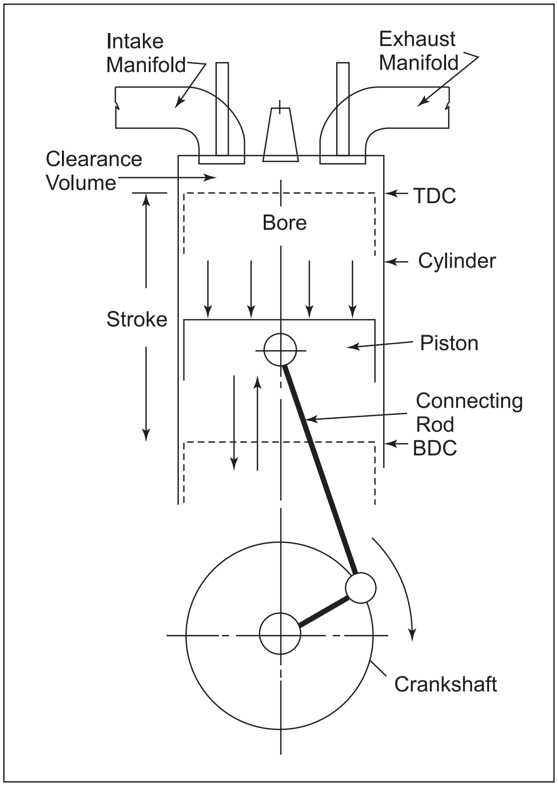


Figure 1.2 *An internal combustion engine*
(Abbreviations: TDC, Top dead center; BDC, Bottom dead center)

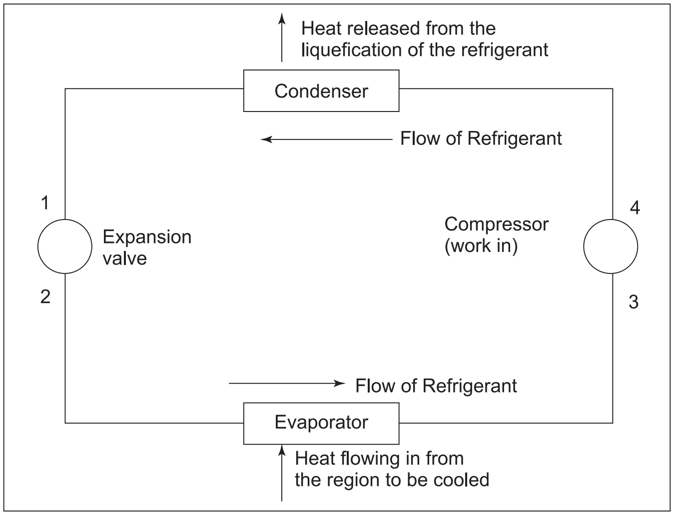


Figure 1.3 *Working principle of a refrigerator*

4. ***Air conditioners:*** This is another important application of thermodynamics. Air conditioning is very much needed in large buildings such as an office, a computer lab, a research lab or a hospital operation theater. Different technologies and methods have evolved over years such as split air conditioning, central air conditioning, etc. Again, the concepts of humidity ratio and partial pressure are essential to achieve the required goal. The functioning of an air conditioner is shown in Figure 1.4.

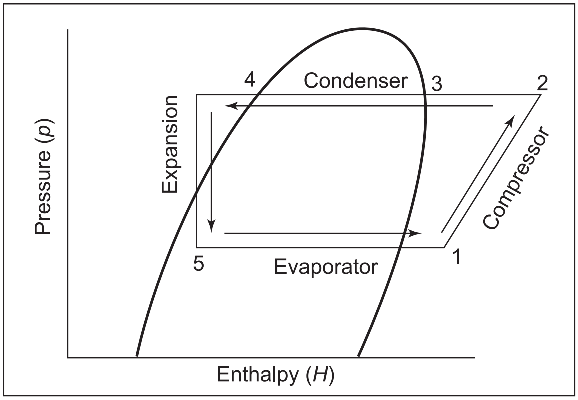


Figure 1.4 *A typical pressure-enthalpy diagram for an air conditioner*

5. ***Energy transfers:*** Devices which economically extract energy from a source and convert it to a usable form are always in demand. The more economical and simple a device is, greater is the demand. Research work is going on across the globe to create devices so that higher-grade energy can be recovered to the maximum level with minimum effort. These devices need to adhere to the laws of thermodynamics and a good knowledge of thermodynamics together with other branches of science can produce less pollution during the desired energy transfer (Figure 1.5).

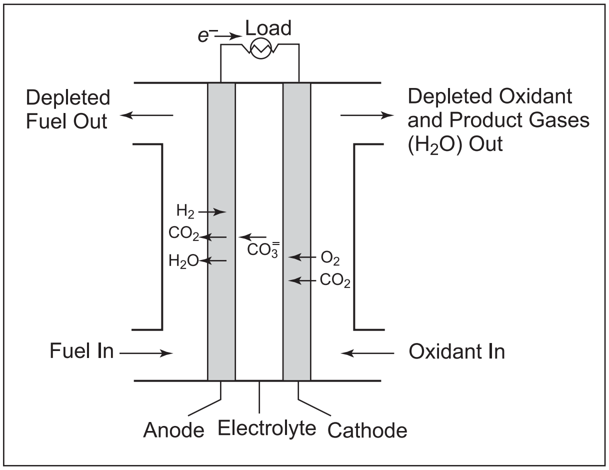


Figure 1.5 *A schematic diagram of fuel cell*

1.2 MACROSCOPIC AND MICROSCOPIC POINTS OF VIEW

University Question

1. Distinguish between microscopic and macroscopic approaches of thermodynamics. **(GTU, 2013, 2014, 2015)**

In thermodynamics, matter can be examined in two different ways—at the microscopic level (molecular level) and at the gross level. The former is termed microscopic approach and the latter is termed as macroscopic approach respectively. Both these approaches differ significantly as shown in Table 1.1.

Table 1.1 *Differences between Microscopic and Macroscopic approaches.*

Macroscopic View	Microscopic View
1. Looks at the gross picture	1. Studies at the atomic and molecular level
2. Needs only final formulae and relatively simpler mathematics	2. Needs detailed study and is statistical in approach.
3. Can be certain as the entire mass is a single entity	3. Uncertainty as each of the molecules/atoms behaves differently
4. Can be sensed by human touch	4. Cannot be seen without the help of powerful electron microscopes
5. Independent of the behaviour of molecules	
6. The behaviour can be derived from the behaviour of statistical study of molecules.	

1.3 THERMODYNAMIC DEFINITIONS

University Questions

1. Differentiate between open system, closed system and an isolated system.
2. Define a thermodynamic system. Differentiate between open system, closed system and an isolated system. **(GTU, 2016)**
3. What do you mean by an isolated system?

1. ***System:*** A region or volume under study for thermodynamic properties in relation to the rest of the universe is called a system (Figure 1.6).
2. ***Boundary:*** An imaginary or real contour separating system from the surroundings. It may be flexible or fixed. All energy interactions from/to the system take place across this boundary (Figure 1.6). Boundary does not store any energy whatsoever.
3. ***Surroundings:*** The region beyond the boundary of a system is called surrounding (Figure 1.6).
4. ***Open system:*** A system wherein both mass and energy transfer take place between system and surroundings is called an open system (Figure 1.7). For example, human body receives both energy and mass in the form of food and air.
5. ***Closed system:*** A system which allows only the transfer of energy from/to the system is known as closed system

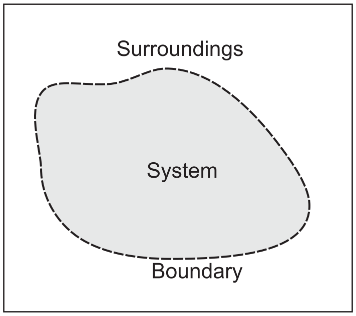


Figure 1.6 *System, boundary and surroundings*

(Figure 1.7). For example, a bottle filled with water and closed by a lid. There is no mass transfer but there could be energy transfer.

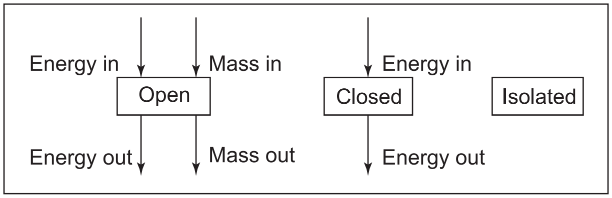


Figure 1.7 *Different types of systems—open, closed and isolated*

6. ***Isolated system:*** This is a system in which neither mass nor energy can be transferred from/to the system. For example, a thermos flask in its initial stages (Figure 1.7).
7. ***Control volume:*** When the volume of the system is fixed, it is called control volume. An example is an open system (Figure 1.8).

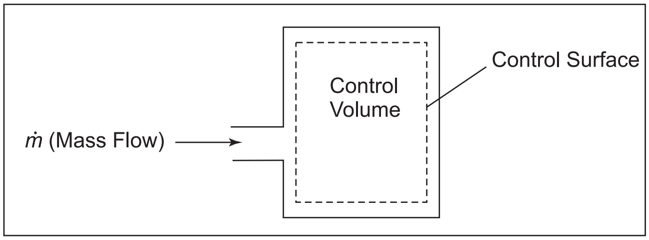


Figure 1.8 *Control volume*

Example 1.1

Classify the following into open, closed or isolated systems:

1. Evaporator
2. Thermos flask
3. Passenger train when stopped at platform
4. Refrigerant in a refrigerator
5. Pressure cooker
6. IC Engine during compression/Expansion stroke
7. Boiler
8. Throttle valve
9. TV
10. Mobile
11. Computer

○○●

Solution

- Examples of an open system are evaporator, passenger train when stopped at platform, IC engine during compression/expansion stroke, boiler, and throttle valve.
- Examples of a closed system are refrigerant in a refrigerator, pressure cooker, TV, mobile, Computer
- Example of an isolated system is a thermos flask.

Example 1.2

Show that the energy of an isolated system is constant. **(GTU, 2015)** ○●●

Solution Isolated systems can neither exchange energy nor matter with an outside system. While they may be portions of larger systems, they do not communicate with the outside in any way. The physical universe is an isolated system; a closed thermos bottle is essentially an isolated system. As there is no interaction, no other kind of energy is present except the internal energy. The internal energy remains the same as there is no change in temperature.

1.4 THERMODYNAMIC PROPERTIES

University Question

1. Define property. What is meant by intensive and extensive property? **(GTU, 2015)**

A property is a characteristic of a system and has a fixed value at a given point and at a given time. The physical condition of the system can be described by these characteristics. The definite value of a set of characteristics denotes the 'state' of a system. Any operation may result in the change of value of a characteristic. This is referred to as change of state. Common thermodynamic properties are pressure, volume, density, energy, etc. These properties can be classified into two types—intensive and extensive. Intensive thermodynamic properties are those properties which are independent of mass such as pressure, temperature, current, etc. Those properties which depend on mass are called extensive thermodynamic properties. Some examples of extensive thermodynamic properties are volume, density, etc. The extensive properties can be changed to intensive properties when divided by mass. In such a case, they are called specific properties.

A quantity of matter homogeneous in chemical composition and physical structure is called a phase. Every substance can exist in any one of the three phases, viz. solid, liquid and gas. A system consisting of a single phase is called a homogeneous system, while a system consisting of more than one phase is known as a heterogeneous system.

1.5 EQUILIBRIUM

University Question

1. What is meant by thermodynamic equilibrium? **(AKTU online, 2012–2013)**

Equilibrium is said to be achieved when there is no change in the existing state. Thus, there are two types of equilibria—static and dynamic. Depending upon the situation, the type of equilibrium can be considered. Either way, in thermodynamics, it is viewed from considering the mechanical equilibrium, chemical equilibrium and thermal equilibrium.

A system is said to have achieved mechanical equilibrium when the pressure/force remains the same throughout the system. If the system does not vary in chemical composition and does not have any modification in mass, the system is said to possess chemical equilibrium. If the system experiences the same temperature throughout, then it is said to be in thermal equilibrium. Any system which enjoys all these three equilibria is said to be in thermodynamic equilibrium. Therefore, for all analyses at the macroscopic levels, the properties of a system refer to thermodynamic equilibrium conditions.

Note: Although one can consider other types of equilibria like electric, magnetic or radiative types, it is sufficient to consider in most cases only the ones mentioned above.

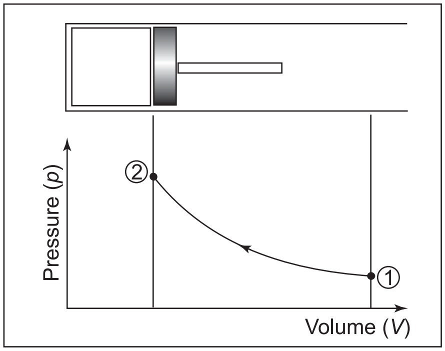
1.6 PROCESSES AND CYCLES

University Question

1. List the thermodynamic processes. **(GTU, 2016)**

At this stage, it is assumed that any gas will behave in an ideal way and, hence, obey the ideal gas law, $pV = RT$ with usual notations. Some common definitions are:

1. ***Path:*** The different states connected by a curve as the system changes its state due to external or internal disturbance.
2. ***Process:*** A path defined by a distinct mathematical rule or relation (Figure 1.9).
3. ***Reversible process:*** A reversible process is a process in which the path can be traversed in either direction without any change to the system or surroundings (Figure 1.10).



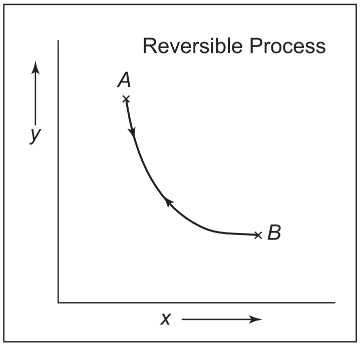


Figure 1.9 *A process*

Figure 1.10 *A reversible process*

4. ***Irreversible process:*** An irreversible process is the one in which the path can be traversed in one direction only.
5. ***Isothermal process:*** An isothermal process is the one in which temperature remains constant (pV = constant); $n = 1$.
6. ***Isobaric process:*** An isobaric process is the one in which pressure remains constant (V/T = constant); $n = 0$.
7. **Isochoric process:** An isochoric process is the one in which volume remains constant (p/T = constant).
8. ***Isenthalpic process:*** An isenthalpic process is the one in which enthalpy (S, to be defined later) remains constant.
9. ***Adiabatic process:*** An adiabatic process is the one in which no exchange of heat energy takes place ($\delta Q = 0$); pV^{γ} = constant; $n = \gamma$.
10. ***Isentropic process:*** An isentropic process is the one in which entropy (S, to be defined later) remains constant.
11. ***Polytropic process:*** A polytropic process involves a series of change of states that the system undergoes continuously without any gap between successive states. This is an ideal situation, where pV^n = constant.
12. ***Quasi-static process:*** A quasi-static process is the one which proceeds infinitesimally slowly so that there is no disturbance to the conditions of the system and/or surroundings. Therefore, by definition, it is also a reversible process (Figure 1.11).

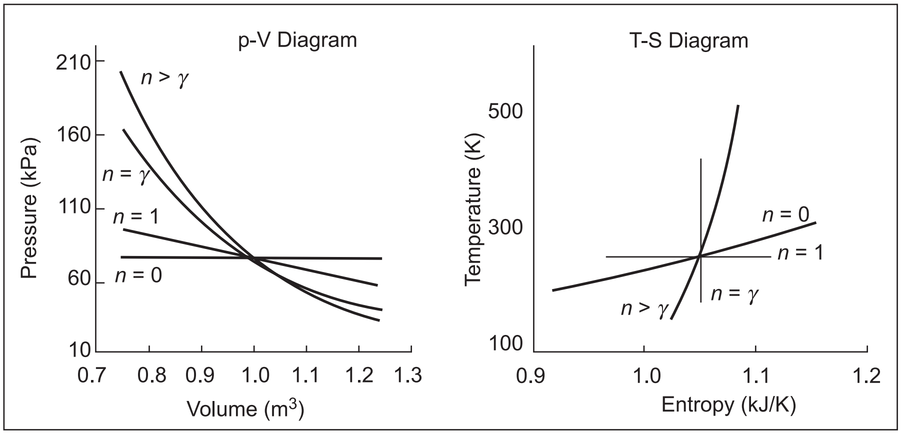


Figure 1.11 *Example of quasi-static process*

13. ***Cycle:*** A series of processes is known as cycles. A complete cycle is one in which the system returns to its original (starting) state (point). In such a case, there is no variation in the properties of the system at the beginning and conclusion of the cycle. If all the constituent processes are reversible individually then the whole cycle is said to be a reversible cycle. Otherwise, even if one of them turns out to be irreversible, the entire cycle becomes irreversible (Figure 1.12).

At any point in the process or cycle, the condition of the system can be described by any of the two independent properties only—two property rule. Therefore, the change in the conditions of the system can be plotted using any two of them as coordinates (p-T, V-T, p-V, T-S, etc.).

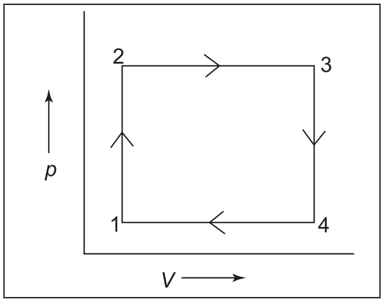


Figure 1.12 *Example of a cycle (cyclic process)*

Example 1.3

What do you understand by cycle and quasi-static process? **(PTU, 2012–2013)** ○●●

Solution A series of processes is known as cycles. A complete cycle is one in which the system returns to its original (starting) state (point). In such a case, there is no variation in the properties of the system at the beginning and conclusion of the cycle. If all the constituent processes are reversible individually then the whole cycle is said to be a reversible cycle. Otherwise, even if one of them turns out to be irreversible, the entire cycle becomes irreversible.

Quasi-state process is a process which proceeds infinitesimally slowly so that there is no disturbance to the conditions of the system and/or surroundings. Therefore, by definition, it is also a reversible process.

1.7 CONTINUUM

At the macroscopic level, the matter is assumed to be continuous. However, at the molecular level, there exists lot of space as gap between molecules. Therefore, for a true consideration, the intermolecular distances need to be compared to the dimensions of the vessel. If they are of the same order, then it is said to be discontinuous otherwise not. Thus, there exists a minimum value of volume which separates the continuous and discontinuous flow. At values, greater than the threshold value, the density is uniform and hence continuous. Truly speaking, one needs to consider Knudsen number which is beyond the scope of the present article (Figure 1.13).

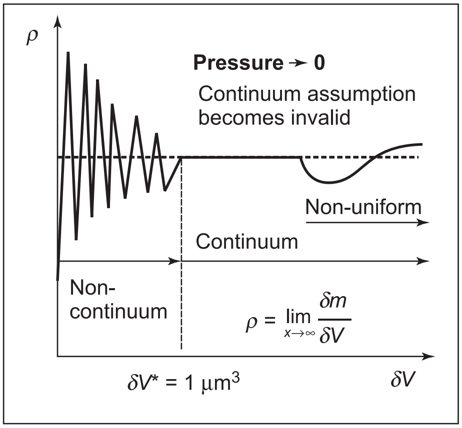


Figure 1.13 *Concept of Continuum*

Example 1.4

What is the concept of continuum? **(AKTU Online 2011–2012; RTU, 2011–2012)** ○●●

Solution Matter behaves differently at atomic/molecular level than at the macroscopic level. It is to be noted that the concept of continuum is used only in macroscopic approach of thermodynamics and not in microscopic approach.

At the microscopic level the effect of forces is to be observed on individual molecules. The study at molecular level is from a statistical point of view to observe its effect overall. There exists a discontinuity which becomes important due to intermolecular spacing and thus the system under consideration is not continuous.

In the macroscopic approach, where gross effect is of importance rather than at the molecular level, the system is continuous. The density is uniform so the order of analysis or scale of analysis becomes very important. Thus, in case the scale of analysis is large enough and the discontinuities are of the order of intermolecular spacing or mean free path then due to relative order of discontinuity being negligible it may be treated continuous.

1.8 UNITS AND DIMENSIONS

The SI (Système International d'unités) system of units is the widely-accepted system of measurement and shall be used throughout the text. The basic units in this system are given in Table 1.2.

Table 1.2 *Basic SI units*

Quantity	Unit	Symbol
Length (*l*)	Metre	m
Mass (*m*)	Kilogram	kg
Time (*t*)	Second	s
Amount of substance	Mole	Mol
Temperature (*T*)	Kelvin	K
Electric current	Ampere	A
Luminous intensity	Candela	Cd
Plane angle	Radian	Rad
Solid angle	Steradian	sr

All other units are derived from these and a few of these are listed as follows (Table 1.3):

Table 1.3 *Derived SI units*

Quantity	Unit	Symbol	In basic units
Force (*F*)	Newton	N	kg m/s^2
Energy (*E*)	Joule	J	kg m^2/s^2
Power (P)	Watt	W J/s	kg m^2/s^3
Pressure (p)	Pascal	Pa	kg/ms

1.9 PRESSURE AND ITS MEASUREMENT

Pressure is defined as the normal force exerted by a system against unit area of the bounding surface. Therefore, in the limiting case, it becomes the force per unit area. There are several units of pressure in use and some of the common ones are listed in Table 1.4.

Table 1.4 *Different units of pressure*

Unit	Pa (N/m^2)	Bar	Atmosphere	kg/cm^2	pound square inches (psi)
Pa (N/m^2)	1	10^{-5}	9.87 10^{-6}	1.02 10^{-5}	1.45 10^{-4}
Bar	10^5	1	0.987	1.0197	14.5
Atmosphere	1.01 10^5	1.013	1	1.03	14.7
kg/cm^2	9.8 10^4	0.98	0.97	1	14.3
pound square inches (psi)	6894.76	0.069	0.068	0.07	1

- More can be found online or other text books.
- Pressure can also be measured as a column head of any fluid such as mm of Hg or water or air.

Most instruments that are available indicate pressure relative to the atmospheric pressure, whereas the pressure of a system is its pressure above zero, or relative to a perfect vacuum. The pressure relative to the atmosphere is called gauge pressure. The pressure relative to a perfect vacuum is called absolute pressure (Figure 1.14). The relation between the two can be given as:

Absolute pressure = Gauge pressure + Atmospheric pressure

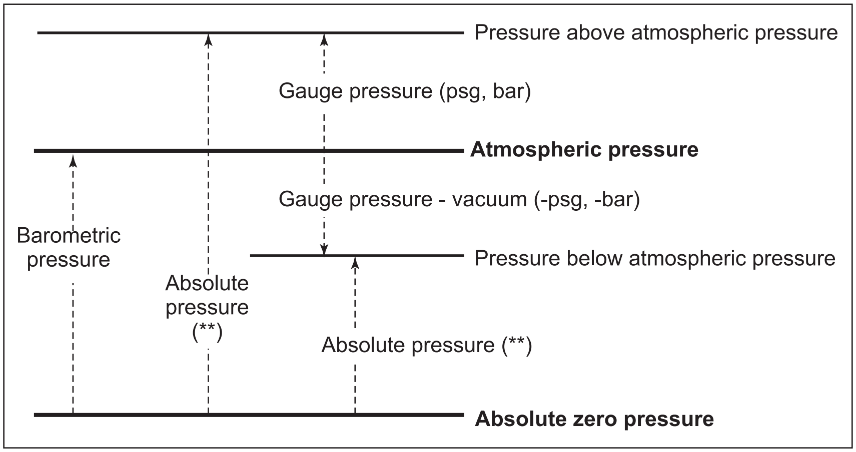
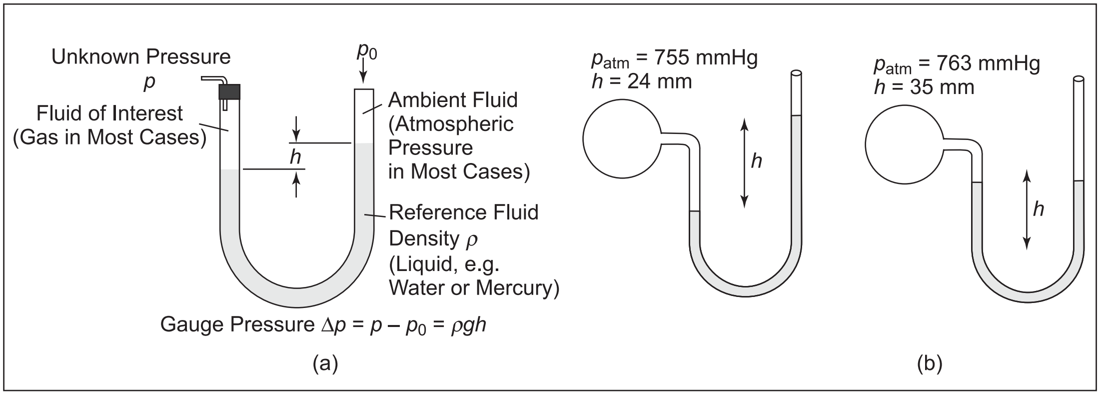
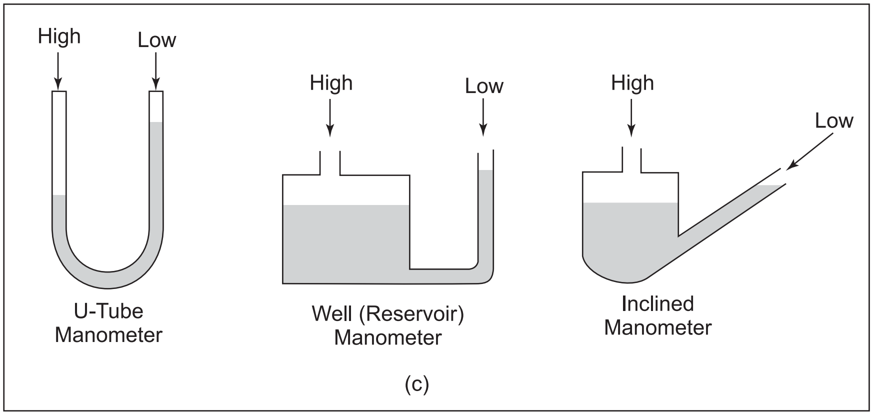


Figure 1.14 *Pressure terminology*

Bourdon gauge is a common pressure measuring device that measures the difference between the system pressure inside the tube and atmospheric pressure. It relies on the deformation of a bent hollow tube of suitable material which when subjected to the pressure to be measured on the inside (and atmospheric pressure on the outside), tends to unbend. This moves a pointer through a suitable gear and lever mechanism against a calibrated scale. Then there are open and closed U-tubes indicating gauge pressure, and a closed U-tube indicating absolute pressure. If p is atmospheric pressure, this is a barometer. These are called U-tube manometers. All these devices measure pressure by using the fact that the pressures at the same heights must be equal. The exact design of manometer is beyond the scope of the present article [Figures 1.15 (a to c)].





Figures 1.15 (a to c) *Different types of manometers*

1.10 TEMPERATURE

It is commonly observed that two vessels filled with liquid will have the same level. Similarly, thermal equilibrium refers to the common state of the two objects or systems. When two objects at different temperatures are brought in contact with each other, eventually, they attain a common temperature. At this stage, they are said to be in thermal equilibrium.

Example 1.5

If an object is at 20°C then what is its temperature in Fahrenheit scale? ○○●

Solution Use the formula C = 5/9 (F – 32); Temperature = 68°F

Example 1.6

If an object is at 20°F then what is its temperature in Celsius scale? ○○●

Solution Use the formula C = 5/9 (F – 32); Temperature = –6.67°C

Example 1.7

In the above problems, determine the absolute temperatures for the solution obtained. ○●●

Solution Fahrenheit to Rankine Temperature, $R = 459.67 + F = 459.67 + 68 = 527.67$;

Celsius to Rankine Temperature, $K = 273.15 + C = 273.15 + (-6.67) = 266.48$

Note: Unless an exact value is required, usually the additions are done with 460 and 273 respectively.

Example 1.8

1 kg of water at 10°C is added to 10 kg of water at 50°C. What is the temperature of the water when it reaches thermal equilibrium? ○●●

Solution Heat gained by cold water should be the same as heat lost by the hot water. Let T be the equilibrium temperature obtained. This means, $q_{\text{lost}} = q_{\text{gain}}$

However: $q = \text{Mass} \cdot (\Delta T) \cdot s$

Substituting values into the above, we have

$1 \times s \times (T - 10) = 10 \times s \times (50 - T)$, where s is the specific heat of water.

Solving, $T = 46.36°\text{C}$

Example 1.9

In a constant-volume gas thermometer how can you measure temperature by measuring the pressure of the gas? ○●●

Solution The pressure of a gas increases when it is heated, provided the volume is constant and decreases when it is cooled. This effect is used to measure temperature by measuring the pressure of the gas when it is placed in thermal contact with the substance whose temperature is being measured. If the pressure of a gas in a constant-volume thermometer is plotted versus temperature a straight line is obtained. If the line is extended down to zero pressure, the temperature at this point shall be –273.15°C.

1.11 ZEROTH LAW

If a body *A* is in thermal equilibrium with a body *B* and also separately with body *C*, then *B* and *C* will be in thermal equilibrium with each other (Figure 1.16). This is known as the zeroth law of thermodynamics. This is an important law of thermodynamics.

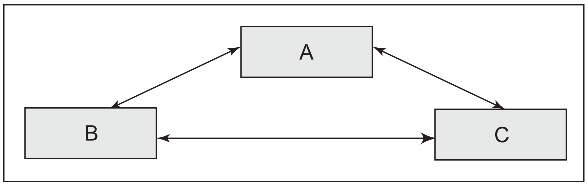


Figure 1.16 *Pictorial representation of zeroth law of thermodynamics*

This is called as zeroth law because the first law was discovered before this law. There is no mathematical proof for this law. It is derived purely from observation of experiments and events happening naturally. This natural law forms the basis of temperature measurement.

Example 1.10

State the zeroth law of thermodynamics. ○○●

Solution If two systems are each in thermal equilibrium with a third system separately, then they are also in thermal equilibrium with each other.

Example 1.11

Why is the zeroth law of thermodynamics named so? ○○●

Solution The first and second laws of thermodynamics were discovered earlier than this law, hence, it is named as zeroth law of thermodynamics.

Example 1.12

How do you prove the zeroth law? ○○●

Solution There is no proof for the zeroth law of thermodynamics. This law has been framed purely on observation and experience.

Example 1.13

Relate the zeroth law to daily life experience. ○○●

Solution Boiled milk in a cup cools down to room temperature after some time. Food taken out of refrigerator attains room temperature after some time.

Example 1.14

What is the concept that the zeroth law leads to? ○○●

Solution It leads us to define the property temperature and a scale to measure the same.

1.12 THERMOMETER

As the temperature changes, certain characteristics of the body will also change. Such a property is termed thermometric property. The reference body itself is called thermometer (Figure 1.17).

The common thermometers are mercury-in-glass, resistance, thermocouple, constant volume gas thermometer and constant pressure gas thermometer. The easy reference points that can be fixed to quantify the temperature of a system are the ice point and steam point of water. However, since 1954, only one fixed point, the triple point of water (0.01°C) is used for convenience and it is the standard fixed point of thermometry.

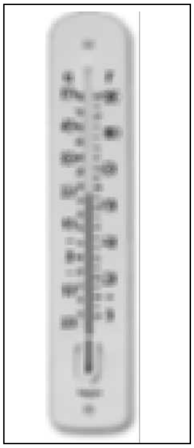


Figure 1.17 *A typical thermometer*

Example 1.15

Define control volume. **(VTU, 2016)** ○○●

Solution It is a fixed region with definite or flexible boundary across which the energy interaction takes place. An example of definite boundary is the piston cylinder arrangement while the flexible boundary can be thought of as in case of a shrinking or expanding balloon.

1.13 TEMPERATURE SCALE

University Question

1. Explain different types of temperature scales. **(AKTU, 2011–2012)**

The scale for a thermometer is usually assumed to be a linear variation of temperature with the reading of some arbitrarily selected standard thermometer. This is only for ease and convenience. Any other relation such as a quadratic or cubic function may also be used. However, if there is not much of a variation then linear relation should do.

Of all the available types of thermometers, the gas thermometer, used in laboratories to determine the reference points at which other thermometers are calibrated, is found to be the most accurate. An absolute temperature scale may be constructed by considering the linear variation of temperature with the volume of the gas in a constant pressure gas thermometer. As there cannot be any negative pressure, the scale is based upon the reference gas at zero pressure. This leads us to zero absolute temperature at –273.15°C.

The metric system uses degree centigrade, °C, while the new SI system uses Celsius, C. The absolute unit is K with the relation $K = C + 273.15$. The imperial system uses Fahrenheit, F, with its absolute unit as Rankine, R, with the relation $R = F + 459.67$. Neither K nor R can be negative (Table 1.5).

Table 1.5 *Standard values as per different scales*

Comment	Kelvin (K)	Celsius (°C)	Fahrenheit (°F)	Rankine (°Ra or °R)
Absolute zero	0	–273.15	–459.67	0
Celsius/Fahrenheit's 'cross-over' temperature	233.15	–40	–40	419.67
Fahrenheit's ice/salt mixture	255.37	–17.78	0	459.67
Water freezes (at standard pressure)	273.15	0	32	491.67
Average surface temperature on Earth	287	14	57	517
Average human body temperature	310.0 ± 0.7	36.8 ± 0.7	98.2 ± 1.3	557.9 ± 1.3
Water boils (at standard pressure)	373.15	100	212	672

POINTS TO REMEMBER

- ☞ Thermodynamics is a branch of science that deals with the study of energy and its manipulations.
- ☞ Thermodynamics has various applications in real life both domestic and industrial.
- ☞ A system is a focused region for thermodynamic studies and is of three types—open, closed and isolated.
- ☞ A system is classified is based on energy flow into it or out of it.

- Boundary separates system from the surroundings.
- Control volume is an example of open system.
- A system is characterised by properties.
- A state is the condition of a system described by a set of properties.
- A property is either intensive (independent of mass) or extensive (dependent on mass).
- All specific properties are intensive.
- Mechanical equilibrium refers to pressure or force experienced.
- Thermal equilibrium refers to equality of temperature throughout.
- Chemical equilibrium refers to the homogeneity of the species.
- Thermodynamic equilibrium exists only when mechanical, thermal and chemical equilibria are existing.
- A process can be reversible or irreversible.
- A process can be well defined such as isothermal.
- A cycle is a combination of processes such that the start state and end state are the same.
- A cycle becomes irreversible even if one of the processes turns out to be irreversible and the rest all are reversible.
- A process can be indicated on an indicator diagram to denote the system conditions using only two properties.
- Continuum refers to the continuity in fluid flow.
- Units are important and prove to be crucial while solving numerical examples.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Pressure measurement is necessary as it affects the mechanical equilibrium.
- Different kinds of manometers are available for pressure measurement.
- Temperature is a property and NOT the same as heat.
- There are different devices to measure temperature—thermometer—most common among them.
- Triple point of water is used as reference to calibrate the thermometer
- It is better to convert temperature to its absolute value in numerical examples unless stated otherwise.
- Zeroth law was discovered after the first law of thermodynamics.
- Zeroth law leads to the concept of temperature and its measurement.
- Different scales of temperature measurement are available and it is always good to know the common conversions.

PRACTICE PROBLEMS

1. Define extensive property. ●●●
2. Differentiate between the intensive properties and extensive properties. ○●●
3. Which of the following is the extensive property of a thermodynamic system? ●●●
   (a) Pressure (b) Volume (c) Temperature (d) Density
4. What is meant by the term equilibrium? ○○●
5. What are the different types of equilibrium? Explain briefly. ○●●

6. What is meant by thermodynamic equilibrium? ○●●

7. What is the value at which the absolute temperatures in the two systems are equal? ○○●
(**Ans:** –3/4)

8. Temperature of an object varies linearly as $T = aX + b$ where X is some property and a and b are constants. The thermometer measures 10° when X is 10 and 35° when X is halved. Determine the temperature when $X = 20$. (**Ans:** $T = -40°$) ○●●

9. In a furnace, the temperature varies as $T = a + bX + cX^2$ where X is some property and a, b and c are constants. The thermometer measures 320° when X is 10 and 85° when X is halved. Determine the temperature when X = 20. (**Ans:** Solving, $a = 2$, $b = 3$ and $T = 1280$) ●●●

10. Consider a scale where the temperature varies as $T = a - b \ln(X)$. The scale reads –6.212 when $X = 10$ and –3.436 when X is halved. Calculate T when X is doubled. ○●●
(**Ans:** $a = 3$, $b = -4$ and $T = -8.984$)

11. Name a few measurements or quantities that can be conveniently used as thermometric properties to quantify temperature. ○●●

12. A scientist finds a thermometer frozen and faded in Antarctica. The marking is at –40. What type of scale could that be? ○●●

MULTIPLE CHOICE QUESTIONS

1. What is the name of the following statement: "When two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other"? ○○●
(a) First Law of Thermodynamics (b) Second Law of Thermodynamics
(c) Mechanical equivalent of heat (d) Zeroth law of thermodynamics
(e) Thermal expansion of solids

2. An aluminium plate has a circular hole. If the temperature of the plate increases, what happens to the size of the hole? ○●●
(a) Increases (b) Decreases
(c) Stays the same (d) Increases the top half of the hole
(e) More information is required

3. Which of the following temperature scales does not have negative numbers? ○●●
(a) Celsius (b) Kelvin (c) Reaumur (d) Fahrenheit
(e) Galileo

4. Which of two temperature changes are equivalent? ○●●
(a) 1 K = 1 F (b) 1 F = 1 C (c) 1 R = 1 F (d) 1 K = 1 C
(e) 1 R = 1 K

5. The average molecular kinetic energy of a gas depends on: ○●●
(a) Pressure (b) Volume (c) Temperature (d) Number of moles
(e) None of the above

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (d) 2. (a) 3. (b) 4. (c) and (d) 5. (c)

Work and Heat Transfer

2

CHAPTER OUTLINE

- Work
- Different Types of Work
- Heat
- Comparison between Work and Heat
- Specific Heat and Latent Heat
- Energy and its Various Forms

INTRODUCTION

Both heat and work represent energy in transition. However, they are not the same. There is a clear distinction between the two. While work is energy transfer that is triggered by a force applied from a distance, heat is the energy transfer caused by temperature difference. Both heat and work are path functions and hence not a property. An interesting fact is that heat can be converted into work and work can be also be converted into heat. As we proceed further we will understand the definition, the unique properties and types of each of these two forms of energy.

2.1 WORK

Work is a form of energy. Work in thermodynamics is of two types—flow work and non-flow work. Flow work is applied for open systems while the non-flow work is applied for closed systems. It is important to remember that work is also a type of energy. Thus, it can be defined as the transformation of energy by any process except from heat in the field of thermal engineering. It is calculated by the scalar product of force (F) and displacement (s). Displacement will be in the direction of the force applied as has been learnt through vectors.

University Questions

1. State work. **(AU, 2013–2014)**
2. Distinguish between flow work and non-flow work. **(AU, 2012–2013)**
3. What do you understand by flow work? Is it different from displacement work? **(AU, 2011–2012)**

Consider a system at equilibrium. To change the state of the system, work will be done on the system or work will be done by the system and heat will be supplied to the system or heat will be rejected from the system respectively. Let us assume that initially the system is at state 1 and by the end of the process, it will be at state 2. The changes can be easily plotted on a graph and this representation is referred to as an indicator diagram. If we consider a pV diagram, then $p = f(V)$. Then, work done over the system will be determined by calculating the area under the curve. From calculus, we can state that the work done will be given by:

$$W_{12} = \int_1^2 p d\mathrm{V}$$

This is the displacement work and hence, an expression for the non-flow work. One should remember that before integration, p should be replaced by $f(V)$ as applicable.

On the other hand, flow work is basically defined as the work which will be needed to maintain a continuous flow through a control volume. From the same pV diagram, the flow work shall be given by:

$$W = -\int_1^2 V dp$$

It can be verified that for an isothermal process, the two expressions are equal.

Example 2.1

During testing, compressed air from a cylinder is used to inflate an inelastic flexible fire hose, originally folded completely flat, to a volume 0.48 m^3. If the barometer reads 749 mm Hg, what is the amount of work done upon the atmosphere by the fire hose? ○●●

Solution The displacement work,

$$W = W_{cyl} + W_{hose} = 0 + \int p dV_{hose} = \frac{749}{760} \times 101.325 \times 0.48 = 47.93\,\text{kJ}$$

Example 2.2

Two cars are parked in a closed garage. The first car is losing fuel due to leakage at the rate of 0.005 m^3. The second car is also losing steadily at half the rate. What is the work done? ○○●

Solution The work done is zero since the cars are inside the garage (system).

Example 2.3

During a long journey in a train, two young monks are in a closed compartment, specially designed for meditation. One is doing normal breathing at the rate of 6 litres per minute, while the other is attempting heavy breathing at the rate of 12 litres per minute. What is the work done if the monks meditate for 1 hour? ○○●

Solution The work done is zero as the monks are inside the train compartment (system).

Example 2.4

Obtain an expression for the work done during a polytropic process. ○●●

Solution For a polytropic process, the relation is $pV^n = c$ where c is a constant. For a system change from state 1 to state 2, the work done is given by $W_{12} = \int_1^2 pdV$.

$$\therefore \qquad W_{12} = \int_1^2 \frac{c}{V^n} dv = c \left.\frac{V^{-n+1}}{-n+1}\right]_1^2 = \frac{c}{-n+1}[V_2^{-n+1} - V_1^{-n+1}]$$

$$= \frac{1}{-n+1}[p_2V_2^{-n+1} - p_1V_1^{-n+1}] = \frac{p_1V_1 - p_2V_2}{n-1}$$

Note: The formula for adiabatic process is given by replacing n by γ. However, the formula does not hold good for isothermal process when n becomes 1. The formula needs to be derived by integrating p as c/V.

Example 2.5

The properties of a closed system change as per the relation between pressure and volume as $pV = 5$ where p is in bar V is in m^3. Calculate the work done when the pressure increases from 2 bar to 8 bar. ○○●

Solution $pV = 5$; $p_1 = 2\,\text{bar}$; $p_2 = 8\,\text{bar}$

$$\therefore \qquad V_1 = \frac{5}{2} \times 101.325\,\text{m}^3 = 253.31\,\text{m}^3 \text{ and } V_2 = \frac{5}{8} \times 101.325\,\text{m}^3 = 63.328\,\text{m}^3$$

$$W_{12} = \int_1^2 pdV = \int_{V_1}^{V_2} \frac{5}{V} dV = 5 \times 101.325 \log \frac{V_2}{V_1} = -702.2\,\text{kJ/kg}$$

Example 2.6

A cylinder contains 2.4 kg of a certain fluid at an initial pressure of 16 bar. The fluid can expand reversibly behind a piston according to a law $pV^{2.6}$ = constant until the volume is 0.12 m^3. The fluid is then cooled reversibly at constant pressure until the piston regains its original position; heat is then supplied reversibly with the piston firmly locked in position until the pressure rises to the original value of 16 bar. Calculate the net work done by the fluid, for an initial volume of 0.04 m^3. ○●●

Solution Given $m = 2.4$ kg

$p_1 = 18$ bar $= 18 \times 101.325$ kPa

$V_1 = 0.04\,\text{m}^3$; $V_2 = 0.12\,\text{m}^3$

law: $pV^{2.6} = c$

During the process 1–2:

$$p_1V_1^{2.6} = p_2V_2^{2.6} \quad \rightarrow p_2 = 1.035 \times 101.325 \text{ kPa}$$

Work done, $W_{12} = \int_1^2 pdV = \int_{0.04}^{0.12} \frac{c}{V^2} dV = 6.755 \text{ kJ / kg}$

During the process 2–3:

Work done, $W_{23} = \int_2^3 pdV = 0.083 \times 101.325 = 8.41 \text{ kJ / kg}$

During the process 3–1:

Work done, $W_{31} = 0$

Total work equals $\sum W = m(W_{12} + W_{23} + W_{31}) = 2.4(6.755 - 8.41) = -3.972$ kJ.

Note: W_{12} is negative because of compression work, work is being done on the system.

2.2 DIFFERENT TYPES OF WORK

It is not necessary that only *pdV* type of work must be present in a thermodynamic system. There are other types of work as well.

University Questions

1. Why does free expansion has zero work transfer? **(AU, 2015–2016)**
2. Explain shaft work and electric work. **(AKTU, 2013–2014)**
3. What are the types of work?
4. Define the polarisation work and magnetic work.

- ***Paddle work:*** Paddle work is the work done by stirring which involves friction, and therefore, results in production of heat. Thus, there is heat transfer because of stirring work.
- ***Shaft work:*** Energy can also be transferred through a rotating shaft—a common practice. The force applied depends on the torque T applied to the shaft. For a specified constant torque, the work done during *n* revolutions is calculated as $2\pi nT$ where *n* is the number of revolutions and *T* is the torque provided.
- ***Flow work:*** This is experienced across a control volume such as in a heat exchanger, turbines, compressors and fans. The movement, if caused slowly, can be thought of as discrete and then the flow work can be computed at each section considering these as point functions.
- ***Spring work:*** A spring is manufactured so that it can contract by application of a force and expand when the force is released. For linear elastic springs, work can be calculated as $\frac{1}{2}k(x_1^2 - x_2^2)$, where *k* is the spring constant and *x* is the displacement from initial point to final point as measured from the undisturbed spring.
- ***Surface tension work:*** This is the kind of work done during the stretching of a liquid film. In such a case, the force produced is used to overcome the microscopic forces between molecules at the liquid-air interface. It is calculated as $\int_1^2 \sigma_s \, dA$.

- ***Polarisation:*** This takes place due to electrical displacement. In the presence of an electric field, a material may behave differently since charges may get realigned. The work done depends on the electric field strength, electrical displacement, polarisation due to electrical displacement and the permittivity of free space. It is given as the dot product of applied field strength and the electrical displacement.
- ***Magnetic work:*** The magnetic work is similar in nature to the polarisation work. However, in magnetism, the linear material that relates the induced magnetisation to the applied field can be either positive or negative. The material behaves in a different manner when it is subjected to a magnetic field. The alignment of the magnetic lines becomes modified due to the presence or removal of the magnetic field. The work done depends on the strengths of the magnetic field applied and induced, the type of material used and the permeability constant. It is given as the dot product of applied field strength and the induced field density.
- ***Electrical work:*** In an electrical work, the force is defined by the surroundings' voltage (the electrical potential) and the generalised displacement is the change observed in the spatial distribution of electrical charge. Work is the product of the two.
- ***Gravitational work:*** In gravitational work, the force is defined by the surroundings' gravitational field and the generalised displacement is change of the spatial distribution of the matter within the system. Work is the product of the two.
- ***Free expansion:*** Consider a vessel split into two portions. Let one contain a gas and the other be completely evacuated. The portions are separated by a thin diaphragm. Let the vessel be insulated from outside so that there is no heat interaction between the system (vessel) and the surroundings (outside). If the diaphragm is removed very slowly then the gas occupies the entire volume. Now, applying the first law to this process yields δQ as zero due to insulation and δW as zero as there is no change in the volume. Consequently, dU is zero which means there is no change in the internal energy. This is also since there is no change in the temperature. However, if the gas considered is a real gas, then the temperature will decrease after the free expansion.

Example 2.7

A three-process cycle operating with nitrogen as working substance has constant temperature compression at 34°C with initial pressure 100 kPa. Then the gas undergoes constant volume heating and then a polytropic expansion with 1.35 as index of expansion. The isothermal work requires 67 kJ/kg of work. Determine the state coordinates and net work transfer. **(AU, 2013)** ●●●

Solution For state 1:

Characteristic gas constant R of Nitrogen, $R_N = \frac{R}{28} = \frac{8.314}{28} = 0.297\,\frac{\text{kJ}}{\text{kg K}}$

$$V_1 = \frac{R_N T_1}{p_1} = \frac{0.297 \times (273 + 34)}{100} = 0.912\,\text{m}^3$$

For state 2:

$$W_{12} = p_1 V_1 \log \frac{V_2}{V_1}$$

$$\therefore \qquad -67 = 100 \times 0.912 \log \frac{V_2}{0.912} \rightarrow V_2 = 0.4375\,\text{m}^3$$

$$\frac{p_2 V_2}{T_2} = \frac{p_1 V_1}{T_1} \rightarrow p_2 = \frac{p_1 V_1}{V_2} = 208.46\,\text{kPa}; \; T_2 = T_1$$

For state 3:

$$V_3 = V_2 = 0.4375\ \text{m}^3$$

$$p_3 V_3^{1.35} = p_1 V_1^{1.35} \rightarrow p_3 = 269.65\,\text{kPa}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_3 V_3}{T_3} \rightarrow T_3 = \frac{p_3 V_3}{p_1 V_1} T_1 = 397\,\text{K}$$

Example 2.8

A piston and cylinder machine containing a fluid system has a stirring device in the cylinder. The piston is frictionless and it is held against the fluid due to the atmospheric pressure of 101.325 kPa. The stirring device is turned 10,000 revolutions with an average torque against the fluid 1.275 Nm. Meanwhile the piston of 0.6 m diameter moves out 0.8 m. Find the net work transfer for the system. **(AKTU, 2013–2014)** ○○●

Solution

$$W_{net} = W_{disp} - W_{stir} = \int p dV - 2\pi NT = p\frac{\pi}{4}d^2 l - 2\pi NT = -57.2\ \text{kJ}$$

2.3 HEAT

University Question

1. State heat. **(AU, 2013–2014)**

Heat is a form of energy. It is *not* the same as temperature. Like work, heat is also a path function and not a property.

Represented by Q, heat can flow across the boundary of the system in either direction. During any process, the change in heat energy for the system is given by Q_{12}.

2.4 COMPARISON BETWEEN WORK AND HEAT

University Question

1. Enlist the similarities between work and heat. **(AKTU, 2014)**

1. Heat and work are both transient phenomena. A system never possesses heat or work but an exchange of either heat or work or both takes place whenever a system undergoes change.

2. Both heat and work are boundary phenomena. Both are observed only at the boundary of the system and both represent the energy crossing the boundary.
3. Both heat and work are path functions and inexact differential equations.
4. Heat added to the system is positive and removed from the system is negative. However, work done by the system is positive while work done on the system is negative.
5. Only a fraction of heat can be converted to work while work can be fully converted to heat.
6. All of heat cannot be converted to work and hence heat is termed poor grade energy whereas all work can be converted to heat and hence work is termed as high grade energy.
7. While displacement is necessary for work, it may not produce heat.
8. A difference in temperature can produce heat energy flow while the work may be unaffected.

2.5 SPECIFIC HEAT AND LATENT HEAT

University Questions

1. Define heat transfer **(JNTU, 2014)**
2. Define specific heat.
3. Define latent heat.

2.5.1 Specific Heat

The specific heat of a substance is usually defined as the heat required per unit mass to raise temperature by one degree. Using, $\delta Q = mcdT$, where m is mass, c is specific heat, and dT is temperature rise, one can calculate the specific heat. For a gas, however, there are two specific heats defined, one at constant pressure (c_p) and other at constant volume (c_v). Therefore, the heat transfer rates are given as $mc_p\,(T_2 - T_1)$ for a constant pressure process and $mc_v\,(T_2 - T_1)$ for a constant volume process. They are related as:

$$c_p - c_v = R, \text{ where } R \text{ is the gas constant}$$

Their ratio is denoted by γ which is a very useful quantity in all the analyses. For a perfect gas, the value of γ is greater than unity.

2.5.2 Latent Heat

It is the heat required to convert any substance from one state to another without change of temperature. For example, h_{fg}, represents the latent heat while transforming from fluid to gaseous state.

Note: Specific heats can vary depending upon the form of existence—solid, alpha, etc. The detailed analysis is out of scope for the present article.

Example 2.9

During a hailstorm, a stone, at a height of 2.50 km has mass 3 g. What is its potential energy? If all this potential energy is converted to latent heat during the fall, calculate the mass of the hailstone on reaching the ground. Take the specific latent heat of fusion of ice to be 3.36×10^5 J/kg. ○●●

Solution $PE = mgh = 0.003 \times 9.81 \times 2500 = 73.58\text{J}$

$m \cdot L = 73.58$, where m is the mass of hail stone melted and L is the latent heat of fusion.

Therefore, melted mass = 21.9×10^{-2} g
Remaining mass that reaches the ground = $3 - 21.9 \times 10^{-2} = 2.781$ g

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2.6 ENERGY AND ITS VARIOUS FORMS

University Question

1. Justify energy. **(GTU, 2016)**

Energy is defined as the capacity to do work. Hence, it is a scalar quantity. It appears in many forms such as thermal, mechanical, chemical, electrical, magnetic and gravitational. However, traditional thermodynamics speaks of only work, heat, internal, potential and kinetic forms only. Thus, it becomes necessary to identify if there are other forms of energy in each scenario and proceed cautiously for analysis. Any two quantities of energy can be added or subtracted provided they have the same unit.

Example 2.10

Calculate the energy released when (a) 20g water at 100°C and (b) 10g of steam at 100°C are each spilt on the hand. Take the specific heat capacity of water to be 4,200 J/kg K and the specific latent heat of vaporisation of water to be 2.2 MJ/kg. Assume that the temperature of the skin is 30°C. ○●●

Solution

(a) $E = ms\delta T = 0.02 \times 4200 \times (100 - 30) = 5.88$ kJ
(b) $E = mL = 0.01 \times 2.2 \times 10^3 = 22$ kJ; Total energy released = 27.88 kJ

POINTS TO REMEMBER

- Work is a form of energy that flows across the boundary of the system.
- Work has got the same units as energy, i.e., kilo Joule (kJ).
- Work is a path function.
- Work can be of different types.
- Work done is equal to the area under the curve in the indicator diagram.
- Work done by a system is positive while the work done on the system is negative.
- For calculation purposes, the particular work type needs to be used as demanded by the scenario.
- During free expansion, the work done is zero.
- Heat is a form of energy, different from temperature. Heat has got the same units as energy (kJ). Heat is a path function.
- Heat added to a system is positive and becomes negative if removed from the system.
- Both work and heat are different forms of energy.
- Depending upon the context, the energy can be converted from one form to another.
- Specific heats for gases have two different values. One is at constant pressure and the other is at constant volume.
- For gases, c_p is always higher.

☞ The difference of the two specific heats is equal to the gas constant.
☞ Energy manifests in different types.
☞ Proper identification is necessary for analysis in any context.

PRACTICE PROBLEMS

1. At 27°C, two moles of neon gas occupy a volume 1 m^3. The gas is adiabatically expanded to a volume 2.4 m^3. Calculate the final pressure and temperature if the initial pressure is 1 atm. Also, calculate the work done during the process if the ratio of specific heats is 1.667. ○○●
(**Ans:** 67.17 kJ)

2. A certain fluid at 10 bar is contained in a cylinder behind a piston, the initial volume being 0.05 m^3. Calculate the work done by the fluid when it expands reversibly to a final volume of 0.2 m^3, a) at constant pressure, (b) according to linear law to a final pressure of 2 bar, (c) according to a law pV = constant, (d) according to law pV^γ = constant. As an engineering student, which process do you suggest? ○●●
(**Ans:** (a) 152 kJ, (b) 111.46 kJ, (c) 70.23 kJ, (d) 252.05 kJ; (c) is the suggested process as it involves least work)

3. An ideal monatomic gas is taken through a cycle which consists of a constant volume process between states 1 and 2, a linear variation between states 2 and 3 and a constant pressure process between states 3 and 1. Determine the work done given a) the pressure varies between 102 kPa and 210 kPa and b) the volume varies between 0.12 m^3 and 0.44 m^3. (**Ans:** 17.28 kJ) ○○●

4. The piston of an oil engine, of area 0.0054 m^2, moves downwards 70 mm, drawing in 0.00028 m^3 of fresh air from the atmosphere. The pressure in the cylinder is uniform during the process at 82 kPa, while the atmospheric pressure is 101.325 kPa, the difference being due to the flow resistance in the induction pipe and the inlet valve. Estimate the displacement work done by the air finally in the cylinder. (**Ans:** 31 J) ○○●

5. An engine cylinder has a piston of area 0.14 m^2 and contains gas at a pressure of 1.8 MPa. The gas expands according to a process which is represented by a straight line on a pressure-volume diagram. The final pressure is 0.16 MPa. Calculate the work done by the gas on the piston if the stroke is 0.35 m. (**Ans:** 48 kJ) ○●●

6. A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m^3 to 0.4 MPa, 0.03 m^3. If the pressure and volume are related by pV^n = constant, find the work done by the gas system. (**Ans:** 11.77 kJ) ○●●

7. A system of volume V contains a mass m of gas at pressure p and temperature T. The macroscopic properties of the system obey the following relationship: ●●●

$$\left(p + \frac{a}{V^2}\right)(V - b) = mRT$$

where a, b, and R are constants. Obtain an expression for the displacement work done by the system during a constant-temperature expansion from volume V_1 to volume V_2. Calculate the work done by a system which contains 10 kg of this gas expanding from 1 m^3 to

10 m^3 at a temperature of 293 K. Use the values $a = 15.7 \times 10\ Nm^4$, $b = 1.07 \times 10^{-2}\ m^3$ and $R = 0.278$ kJ/kg-K. (**Ans:** 1742 kJ)

8. 680 kg of fish at 5°C are to be frozen and stored at –12°C. The specific heat of fish above freezing point is 3.182, and below freezing point is 1.717 kJ/kg K. The freezing point is –2°C, and the latent heat of fusion is 234.5 kJ/kg. How much heat must be removed to cool the fish, and what per cent of this is latent heat? (**Ans:** 186.2816 MJ, 85.6%) ○●●

MULTIPLE CHOICE QUESTIONS

1. In a reversible polytropic process: ○●●
(a) Heat transfer occurs
(b) The pressure remains constant
(c) The volume remains constant
(d) The temperature remains constant

2. Work done during free expansion is ○○●
(a) Zero
(b) Minimum
(c) Maximum
(d) Positive

3. Heat is: ○●●
(a) Work
(b) A process
(c) Energy
(d) Temperature

4. Heat transfer depends on: ○○●
(a) Pressure
(b) Temperature difference
(c) State
(d) System

5. Work and heat are: ○●●
(a) Not mutually convertible
(b) Mutually convertible
(c) Dependent
(d) Different systems

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (a) 2. (a) 3. (c) 4. (b) 5. (b)

First Law of Thermodynamics

3

CHAPTER OUTLINE

- ☞ First Law Statement
- ☞ Application to a Process
- ☞ Energy—A Property of the System
- ☞ Perpetual Motion Machine of the First Kind (PMM 1)
- ☞ Specific Heat
- ☞ Limitations of First Law

INTRODUCTION

First law of thermodynamics deals with energy balance. It is called the First Law as it was discovered first although it happens to be the second in the series. This law can be applied to a single process or a cycle. It states that the energy transformations need to be balanced before and after the process. Mathematically, it describes the relationship among heat energy, work energy and internal energy and is expressed as:

$$\delta Q = \delta W + dE$$

where, Q represents heat energy, W represents work done and E is the total internal energy.

3.1 || FIRST LAW STATEMENT

First law of thermodynamics states that during a thermodynamic cycle undergone by a system, the net heat transfer will be equal to net work done. Mathematically,

$$\int \delta Q = \int \delta W$$

Or

Energy can neither be created nor destroyed but the sum of energy remains the same.

University Question

1. State the first law of thermodynamics. **(GTU, 2014)**

Or

No machine can produce energy without corresponding expenditure of energy, i.e., it is impossible to construct a perpetual motion machine of first kind.

3.2 APPLICATION TO A PROCESS

When a system undergoes a process, the difference between heat and work is not zero. This quantity is called total internal energy and is represented by E. Therefore,

$$E_2 - E_1 = \delta Q - \delta W$$

Probe

1. How do you apply the first law to a process?

This total internal energy consists of electric, magnetic, chemical, potential, kinetic and any other type of energies.

If the electric, magnetic and chemical energies are absent and changes in potential and kinetic energy for a closed system are neglected, the above equation can be written as:

$\delta Q - \delta W = \Delta U = U_2 - U_1$, where, U represents internal energy

Generally, when heat is added to a system its temperature rises and external work is performed due to increase in volume of the system. The rise in temperature is an indication of increase of internal energy.

Example 3.1

In a motor cylinder the compressed air has an internal energy of 400 kJ/kg at the beginning of the expansion and an internal energy of 210 kJ/kg after expansion. If the work done by the air during the expansion is 140 kJ/kg, calculate the heat flow to and from the cylinder. ○●●

Solution Given,

$U_1 = 400$ kJ/kg, Internal energy after expansion

$U_2 = 220$ kJ/kg

Work done, $W = 140$ kJ/kg

Applying the first law,

$$Q = (U_2 - U_1) + W$$

$$\therefore \quad Q = (220 - 450) + 120 = -230 + 120 = -110 \text{ kJ/kg}$$

3.3 ENERGY—A PROPERTY OF THE SYSTEM

Consider a system which changes its state from 1 to 2 by following the path A, and returns from 2 to 1 by following the path B or path C as shown in Figure 3.1. The system undergoes a cycle, either A–B or A–C. It is important to note that the start state and end state are the same for both cycles.

University Question

1. Show that the energy of a system is a property. **(GTU, 2016)**

For path A, the first law is:

$$Q_{\mathrm{A}} = \Delta E_A + W_A$$

Similarly, for the paths *B* and *C*, the first law can be written as:

$$Q_B = \Delta E_B + W_B$$

$$Q_C = \Delta E_C + W_C$$

Addition gives,

$$Q_A + Q_B = \Delta E_A + W_A + \Delta E_B + W_B$$

$\rightarrow \Delta E_A = -\Delta E_B$ since *A* and *B* constitute a cycle.

Similarly, $\Delta E_A = -\Delta E_C$

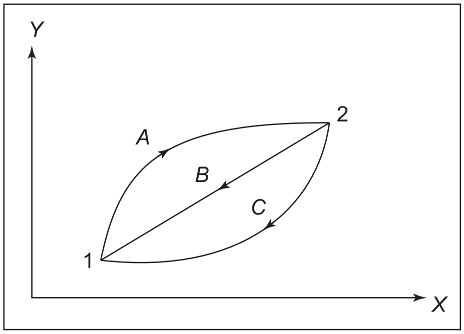


Figure 3.1 *Demonstration of internal energy as a property*

The change in energy between two states of a system is the same, irrespective of the path the system may follow in undergoing that change of state. Therefore, energy has a definite value for every state of the system. Hence, it is a point function and, therefore, a property of the system.

Example 3.2

1.2 kg of hydrogen gas at 120 kPa and 40°C is contained in a cylinder. The piston compresses hydrogen until the pressure reaches 1.2 MPa and temperature 180°C. If the work done during the process is 40 kJ, determine the heat transferred from the hydrogen to the surroundings. c_v for hydrogen = 10.16 kJ/kg K.

○●●

Solution Application of the first law gives,

$$Q = \Delta U + W = (U_2 - U_1) + W = mc_v\,(T_2 - T_1) + W$$

$$= 1.2 \times 10.16\,(180 - 40) - 40 = 1{,}667 \text{ kJ}$$

Example 3.3

In a cyclic process, heat transfers are + 15.8 kJ, – 21.2 kJ, – 13.56 kJ and + 37.5 kJ. What is the net work for this cyclic process?

○○●

Solution Net heat transfer = $\sum Q$ =15.8 – 21.2 – 13.56 + 37.5 = 18.54 kJ = Net work done

Example 3.4

A slow chemical reaction takes place in a fluid at the constant pressure of 0.1 MPa. The fluid is surrounded by a perfect heat insulator during the reaction which begins at state 1 and ends at state 2. The insulation is then removed and 105 kJ of heat flow to the surroundings as the fluid goes to state 3. The following data are observed for the fluid at states 1, 2 and 3.

State	1	2	3
***V*, m^3**	0.003	0.3	0.06
***T*, °C**	20	370	20

For the fluid system, calculate E_2 and E_3, if $E_1 = 0$.

●●●

Solution From the first law, $\delta Q = dE + \delta W = dE + \int pdV$

For the process 1–2,

$$Q_{12} = (E_2 - E_1) + \int_1^2 pdV$$

$$0\ (E_2 - 0) + 0.1 \times 10^3\ (0.3 - 0.003) \rightarrow E_2 = -29.7 \text{ kJ}$$

For the process 2–3,

$$Q_{23} = (E_3 - E_2) + \int_2^3 pdV$$

$$-105 = (E_3 + 29.7) + 0.1 \times 10^3\ (0.06 - 0.3) \rightarrow E_3 = -110.7 \text{ kJ}$$

Example 3.5

A tank containing air is stirred by a paddle wheel. The work input to the paddle wheel is 9,000 kJ and the heat transferred to the surroundings from the tank is 3,000 kJ. Determine: (a) Work done, (b) Change in internal energy of the system. ○●●

Solution From the first law, $Q_{12} = W_{12} + U_2 - U_1$

$$9{,}000 - 3{,}000 = 0 + U_2 - U_1$$

Work done is zero and all the paddle work is input as heat to the system.

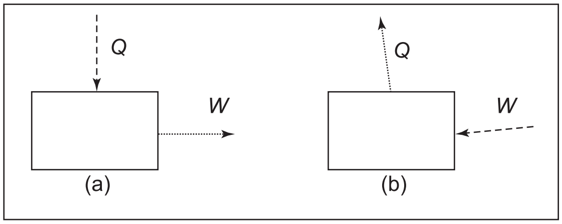
The change in internal energy is 6,000 kJ.

3.4 PERPETUAL MOTION MACHINE OF THE FIRST KIND (PMM 1)

University Question

1. Discuss the perpetual motion machine of the first kind. **(VTU, 2013)**

The first law of thermodynamics states that energy is neither created nor destroyed, but only gets transformed from one form to another. In exclusion to this principle, there can be no machine which would continuously supply mechanical work without some form of energy disappearing simultaneously. Also, there can be no machine which would continuously consume work without some other form of energy appearing simultaneously. It can only exist as a fictitious one. Such a fictitious machine is called a perpetual motion machine of the first kind, or in brief, PMM 1. A PMM 1 is thus impossible (Figures 3.2 a and b).



Figures 3.2 (a and b) *Demonstration of PMM 1*

3.5 SPECIFIC HEAT

University Questions

1. Define the specific heat at constant volume.
2. Define Enthalpy. **(GTU, 2014)**
3. Define specific heat at constant pressure.
4. Derive a relation between the two specific heats.

There are two kinds of specific heats defined, one at constant pressure, c_p and the other at constant volume, c_V.

3.5.1 Specific Heat at Constant Volume

From the first law, for a constant volume process, an increase of internal energy equals the heat energy supplied to the system as work done is zero. Therefore,

$$dU = m c_V dT \rightarrow c_V = \left(\frac{\partial U}{\partial T} \right)_V$$

This relation is derived by considering the variation of specific internal energy with respect to temperature for a fixed volume.

3.5.2 Specific Heat at Constant Pressure

Enthalpy

Enthalpy (*H*) is defined as a state function that depends only on the equilibrium state as experienced by the system at that instant and identified by the system's internal energy, pressure, and volume. It is an extensive quantity. Mathematically,

$$H = U + pV$$

where, U is the internal energy, p is the pressure and V the volume of the system.

Just as U cannot be measured absolutely, H also cannot be measured in an absolute manner since H depends on U.

From the first law, for a constant pressure process, an increase of enthalpy equals the heat energy supplied to the system. Therefore,

$$dH = m c_p dT \rightarrow c_p = \left(\frac{\partial H}{\partial T} \right)_p$$

This relation is derived by considering the variation of specific enthalpy with respect to temperature at a fixed pressure.

Relationship between c_p and c_V

From the definition of enthalpy, $h = u + pV = u + RT$ using the ideal gas law.

Differentiating,

$$\frac{dh}{dT} = \frac{du}{dT} + R \rightarrow c_p = c_v + R$$

Example 3.6

Derive the relation between the specific heats. ○●●

Solution For a constant volume process, the first law reduces to $\delta Q = dU$

$$dU = mc_V dt$$

Integrating, $U_2 - U_1 = mc_V (T_2 - T_1)$. This gives the change in internal energy for a perfect gas between two states for any process, reversible or irreversible.

For a system undergoing constant pressure process changing from T_1 to T_2, the first law gives,

$$\delta Q = dU + \delta W$$

$$mc_p dT = mc_V dt + pdV = mc_V dT + mRdT \rightarrow c_p = c_V + R$$

3.6 LIMITATIONS OF FIRST LAW

University Question

1. State the limitations of first law of thermodynamics. **(GTU, 2015)**

1. Direction of the flow of energy is not specified. The first law does not indicate whether heat can flow from a colder body to a hotter body by itself. It needs some external work.
2. The feasibility of the process is not stated.
3. Practically it is not possible to convert all the available heat energy into an equivalent amount of work.
4. There is no mention of the quality of energy used or available.

POINTS TO REMEMBER

- First law can be stated in different ways. It is basically concerned with energy transformation.
- As applied to a process, the change in internal energy is the difference of heat energy and work energy.
- Internal energy is a property and hence, an exact differential.
- Internal energy is a point function and does not depend on path.
- At the end of a cycle, the change in internal energy is zero.
- Perpetual motion of first kind is not possible as it goes against the natural law.
- Specific heat at constant volume depends only on the variation of internal energy with temperature.
- Specific heat at constant pressure depends on the variation of enthalpy with temperature.

PRACTICE PROBLEMS

1. A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is –340 kJ. The system completes 220 cycles per min. Complete the following table showing the method for each item, and compute the net rate of work output in kW. ○●●

Process	Q (kJ/min)	W (kJ/min)	ΔE (kJ/min)
1-2	0	4,340	--
2-3	42,000	0	--
3-4	–4,200	--	–73,200
4-1	--	--	--

(**Ans:** –2,469 kW)

2. During one cycle the working fluid in an engine engages in two work interactions: 15 kJ to the fluid and 44 kJ from the fluid, and three heat interactions, two of which are known: 75 kJ to the fluid and 40 kJ from the fluid. Evaluate the magnitude and direction of the third heat transfer. ○○●
(**Ans:** –6 kJ)

3. A system composed of 2 kg of air expands in a frictionless piston and cylinder machine from an initial state of 1 MPa, 100°C to a final temperature of 30°C. Find the net work for the process if the process is adiabatic. (**Ans:** –100.52 kJ) ●●●

4. A stationary system consisting of 2 kg of air expands in an adiabatic process according to $pV^{1.2}$ = constant. The initial conditions are 1 MPa and 200°C, and the final pressure is 0.1 MPa. Find W and ΔE for the process. Comment on the work transfer $\int pdV$. ●●●
(**Ans:** 72.1 kJ is different from 216.48 kJ; this is not a quasi-static process)

5. A mass of 8 kg gas expands within a flexible container so that the p–V relationship is of the from $pv^{1.2}$ = constant. The initial pressure is 1,000 kPa and the initial volume is 1 m^3. The final pressure is 5 kPa. If specific internal energy of the gas decreases by 40 kJ/kg, find the heat transfer in magnitude and direction. (**Ans:** 2612.5 kJ) ○●●

6. The properties of a system, during a reversible constant pressure nonflow process at p = 1.8 bar, changed from V_1 = 0.25 m^3/kg, T_1 = 20°C to V_2 = 0.55 m^3/kg, T_2 = 265°C. The specific heat of the fluid is given by $c_p = 1.2 + \frac{72}{T + 42}\,\frac{\text{kJ}}{\text{kg °C}}$, where T is in °C. Determine: (a) Heat added/kg; (b) Work done/kg; (c) Change in internal energy/kg. ○●●
(**Ans:** (a) 414kJ/kg, (b) 54.72 kJ/kg, (c) 359.3 kJ/kg)

7. A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume ($p = a + bV$). The internal energy of the fluid is given by the following equation U = 40 + 3.8 pV where U is in kJ, p in kPa, and V in cubic meter. If the fluid changes from an initial state of 200 kPa, 0.036 m^3 to a final state of 440 kPa, 0.08 m^3, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer. (**Ans:** W = 14.08 kJ, Q_{12} = 120.48 kJ) ●●●

8. A system is supplied with 120 kJ of heat at constant volume. The system rejects 130 kJ of heat at constant pressure and 25 kJ of work is done on it. The system is brought to original state by adiabatic process. Determine: (a) The adiabatic work, (b) The values of internal energy at all end states if initial value is 90 kJ. (**Ans:** (b) Q_{12} = 210 kJ, Q_{23} = 105 kJ, Q_{31} = 15 kJ) ○●●

9. The heat capacity at constant pressure of a certain system is a function of temperature only and may be expressed as $c_p = 2.093 + \frac{41.87}{t + 100}$ J/°C, where t is the temperature of the system in °C. ●●●

The system is heated while it is maintained at a pressure of 1 atmosphere until its volume increases from 2,020 cm^3 to 2,600 cm^3 and its temperature increases from 0°C to 100°C. Find the magnitude of the heat interaction and internal energy change.

(**Ans:** $Q = 238.3$ J, $\Delta U = 179.53$ J)

10. 0.26 m^3 of air at 4.8 bar and 140°C is contained in a system. A reversible adiabatic expansion takes place till the pressure falls to 1.02 bar. The gas is then heated at constant pressure till the heat energy increases by 75 kJ. Calculate: (a) The work done; (b) The index of expansion, if the above processes are replaced by a single reversible polytropic process giving the same work between the same initial and final states. Take $c_p = 1.005$ kJ/kg K, $c_v = 0.714$ kJ/kg K. ●●●

(**Ans:** (a) $W = 140.45$ kJ, (b) 1.18)

MULTIPLE CHOICE QUESTIONS

1. Internal energy of a perfect gas depends on: ○○●
(a) Temperature and pressure (b) Temperature and enthalpy
(c) Temperature and entropy (d) Temperature only

2. The first law of thermodynamics for steady flow: ○●●
(a) Accounts for all energy entering and leaving a control volume
(b) Is an energy balance for a specified mass of fluid
(c) Is an expression of the conservation of linear momentum
(d) Is primarily concerned with heat transfer

3. Which of the following is not a property of the system? ○○●
(a) Temperature (b) Pressure
(c) Enthalpy (d) Heat

4. In a cycle, the net change of internal energy is: ○○●
(a) Negative (b) Zero
(c) Positive (d) Cannot be determined

5. For a gas, a difference of: ○●●
(a) 1°K = 1°F (b) 1°F = 1°C
(c) 1°R = 1°C (d) 1°K = 1°C

6. The average molecular kinetic energy of a gas depends on: ○○●
(a) Pressure (b) Volume
(c) Temperature (d) Number of moles

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (d) 2. (a) 3. (d) 4. (b) 5. (d)
6. (c)

First Law Applied to Flow Processes

4

CHAPTER OUTLINE

- ☞ Assumptions of SFEE
- ☞ Derivation of SFEE
- ☞ Applications of SFEE
- ☞ Throttling Process
- ☞ Comparison of SFEE with Euler's and Bernoulli's Equations
- ☞ Variable Processes

INTRODUCTION

Application of the first law is not restricted to closed systems only. It can be applied to open systems as well. Expectedly, the equation needs to be modified. In case of open systems, the mass flow into and out of the system plays an important role in the analysis. The two rates are usually assumed to be same and constant too. However, in reality, this will not hold good. Primarily, friction plays a major role. Also, the constituents of the fluid may change over a distance and/or over a period of time. Then, for a given control volume, it is useful to express the first law using the rates of heat transfer and work transfer. This article is restricted to steady flow only (properties do not change with respect to time). Unsteady flow is beyond the scope of the present article. In either case, a standard model is required only for the sake of comparison which gives further ideas about intricate analysis. Considering these conditions, certain assumptions are made regarding application of first law to steady flow processes which are listed as follows.

4.1 ASSUMPTIONS OF SFEE

Probe

1. List the assumptions made in arriving at the Steady Flow Energy Equation (SFEE).

1. The mass flow through the system remains constant $\frac{d\dot{m}_{cv}}{dt} = 0$, i.e., $\dot{m}_{\text{out}} = \dot{m}_{\text{in}} = \dot{m}$. This means there is no mass retained by the system. Whatever flows into the system flows out of the system.

2. Fluid is uniform in composition. This means that the fluid will not interact with the system components and change. It is not affected by the variation of pressure or temperature or any other property.
3. The only interactions between the system and surroundings are work and heat.
4. The state of fluid at any point remains constant with time.
5. In the analysis, only potential, kinetic and flow energies are considered along with heat and work energy forms.

4.2 ‖ DERIVATION OF SFEE

Consider an open system. Let some mass of fluid, m_1, flow from left to right in a system with pressure, p_1; velocity, ϑ_1; specific volume, V_1; specific energy, E_1 (usually only U) at a height, z_1. Similarly, let suffix 2 be for respective quantities at the right. Let the heat and work interactions be Q and W respectively (Figure 4.1).

University Questions

1. Derive the Steady Flow Energy Equation (SFEE). **(GTU, 2015)**
2. Define and explain steady flow energy equation.

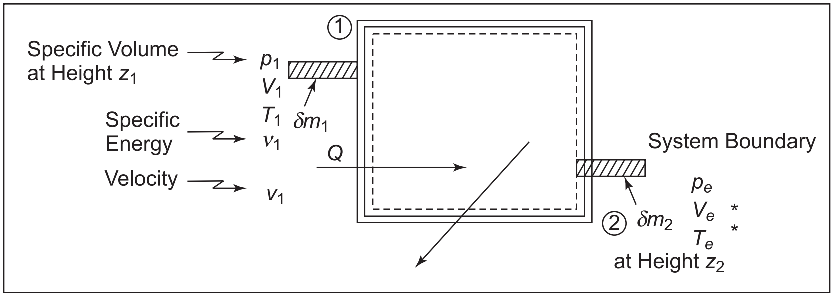


Figure 4.1 *Energy flow in an open system*

Then the energy balance gives,

$$m_1\left(U_1 + \frac{\vartheta_1^2}{2} + z_1 g + p_1 V_1 + Q\right) = m_2\left(U_2 + \frac{\vartheta_1^2}{2} + z_1 g + p_1 V_1 + W\right)$$

$U_1 + \frac{\vartheta_1^2}{2} + z_1 g + p_1 V_1 + Q = U_2 + \frac{\vartheta_2^2}{2} + z_2 g + p_2 V_2 + W$ as the mass flow is assumed to be constant.

$H_1 + \frac{\vartheta_1^2}{2} + z_1 g + Q = H_2 + \frac{\vartheta_2^2}{2} + z_2 g + W$, where H is enthalpy and g is acceleration due to gravity.

It can also be written as:

$$m\left(H_1 + \frac{\vartheta_1^2}{2} + z_1 g\right) + Q = m\left(H_2 + \frac{\vartheta_2^2}{2} + z_2 g\right) + W$$

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) + (z_2g - z_1g)\right] + W$$

Thus, for a steady flow, the heat energy transfer is the sum of changes in internal energy, flow energy, kinetic energy, potential energy and the work transfer.

In a steady flow the rate of mass flow of fluid at any section remains the same throughout. Thus, it can be calculated as, $\dot{m} = \frac{\vartheta A}{V}$, where ϑ is the velocity, A is the cross section and V is the specific volume.

4.3 APPLICATIONS OF SFEE

4.3.1 Hydroturbine

In a hydroturbine water flows from a height and strikes the turbine blades to generate electricity (Figure 4.2). Through the process, the height at exit can be considered zero if the shaft height is considered as datum. Also, the specific volume is not going to change before and after impact. There can be no effective heat transfer either. As there is no change in temperature, the change in internal energy becomes zero. Water does work meaning the work is done by the system and hence, it is positive in the equation.

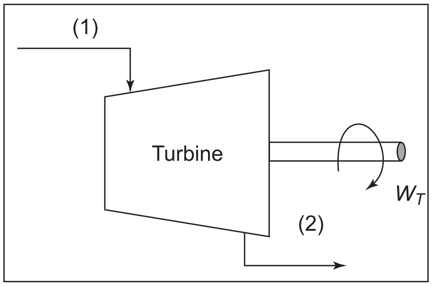


Figure 4. 2 *Hydroturbine system*

Thus, in summary,

$$z_2 = 0;\ V_2 = V_1 = V;\ Q = 0;\ U_2 - U_1 = 0$$

Substituting in the general equation,

$$Q = m\left[\begin{matrix}(U_2 - U_1) + (p_2V_2 - p_1V_1) \\ + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) + (z_2g - z_1g)\end{matrix}\right] + W$$

$$0 = m\left[(p_2 - p_1)V + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) - z_1g\right] + W$$

University Questions

1. Derive the equation for a flow in hydroturbine from the Steady Flow Energy Equation (SFEE).
2. State the Steady Flow Energy Equation and explain how this equation can be applied for (i) Nozzle, (ii) Boiler, and (iii) Steam Turbine. **(GTU, 2015)**
3. Derive the equation for a flow in steam turbine from the Steady Flow Energy Equation (SFEE).
4. Derive the equation for a flow in centrifugal pump from the Steady Flow Energy Equation (SFEE).
5. Derive the equation for a flow in centrifugal compressor from the Steady Flow Energy Equation (SFEE). Hence, derive the equation for the reciprocating pump.
6. Derive the equation for a flow in steam boiler from the Steady Flow Energy Equation (SFEE).
7. Derive the equation for a flow in condenser from the Steady Flow Energy Equation (SFEE).
8. Derive the equation for a flow in an evaporator from the Steady Flow Energy Equation (SFEE).
9. Derive the equation for a flow in nozzle from the Steady Flow Energy Equation (SFEE).
10. Derive the equation for a flow encountering a blunt body such as the nose of an airplane from the Steady Flow Energy Equation (SFEE).

4.3.2 Steam Turbine

In a steam turbine, steam from a steam generator or boiler impacts on the turbine blades (Figure 4.3). Therefore, the heights can be considered the same. Then the general equation reduces to:

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$Q = m\left[(H_2 - H_1) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

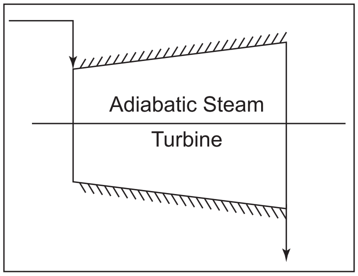


Figure 4.3 *Steam turbine system*

4.3.3 Centrifugal Pump

A centrifugal water pump is used to draw water from a lower level to pump to a higher level as required (Figure 4.4). Work is required to run the pump and this may be supplied from an external source such as an electric motor or a diesel engine. Here, there is no heat transfer. Therefore, $Q = 0$ and the internal energy, $\Delta U = 0$ as there is no change in temperature of water; Further, the inlet and exit volumes are the same $V_1 = V_2 = V$ (Figure 4.5). Applying these to the steady flow equation,

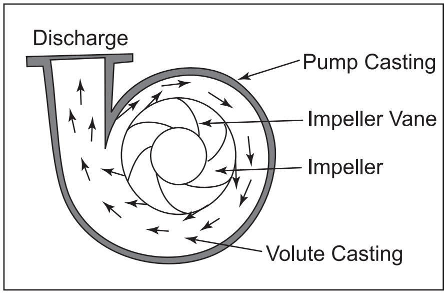


Figure 4.4 *Centrifugal pump*

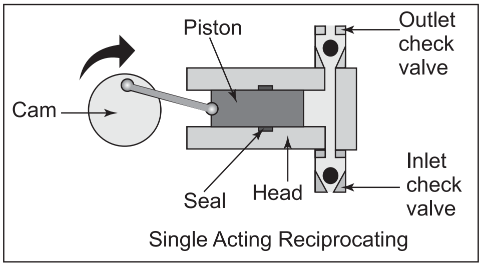


Figure 4.5 *Reciprocating pump*

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$m\left[(p_2 - p_1)V + (z_2 - z_1)g + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W = 0$$

Note: *W* would be negative as the work is done on the system.

4.3.4 Centrifugal Compressor

For an air compressor, the incoming and exit pressures are the same and generally the exit and entry are at the same height (Figure 4.6). The heat is taken out of the system while the work is done on the system (Figure 4.7). Therefore, both are negative. Substituting these the general equation turns out to be:

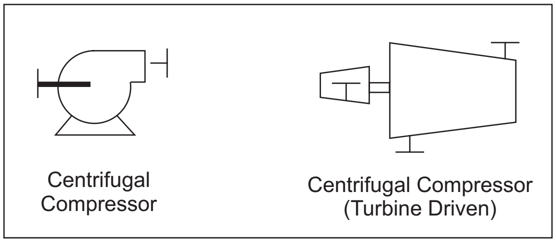


Figure 4.6 *Centrifugal compressor*

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$-Q = m\left[H_2 - H_1 + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] - W$$

For a reciprocating compressor, the velocities are also equal. Therefore, it reduces to:

$$-Q = m[H_2 - H_1] - W$$

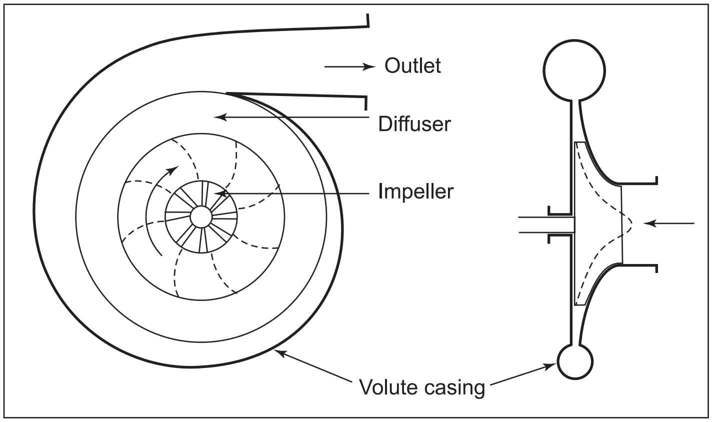


Figure 4.7 *Schematic diagram of centrifugal compressor*

4.3.5 Steam Boiler

For a steam boiler, it generates steam due to heat supplied by heat from below. Therefore, changes in the energy due to elevation and kinetic energy are zero. Also, no work is done on the system or by the system.

In such a case, the general equation reduces to:

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$-Q = m[H_2 - H_1]$$

4.3.6 Condenser

A condenser is used to condense steam or other vapors to their liquid form (Figure 4.8). Both the potential and kinetic energies appear small before enthalpies and hence can be neglected. There is no work done either.

In such a case, the general equation reduces to:

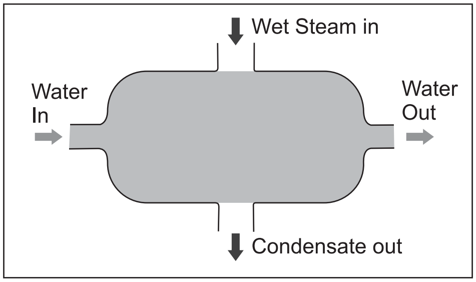


Figure 4.8 *Water-cooled surface type condenser*

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$-Q = m[H_2 - H_1]$$

4.3.7 Evaporator

An evaporator is a useful device used in refrigeration to extract heat from the refrigerator to maintain the required low temperature (Figure 4.9). The refrigerant liquid, as it passes out through the evaporator, comes out as vapor absorbing its latent heat from the surroundings of the evaporator. This is similar to a boiler.

In such a case, the general equation reduces to:

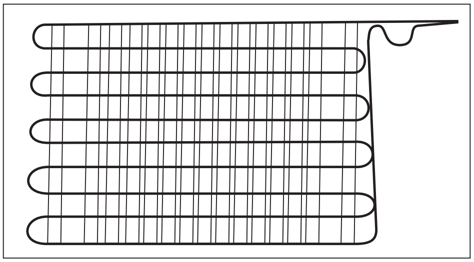


Figure 4.9 *Evaporator*

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$Q = m[H_2 - H_1]$$

4.3.8 Nozzle

In case of a nozzle, the velocity increases due to change in kinetic energy at the cost of potential energy (Figure 4.10). This action also results in the decrease of enthalpy. A pressure drop is also observed. Thus, there is no heat transfer or work done and, therefore, the equation reduces to:

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$H_2 - H_1 + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) = 0$$

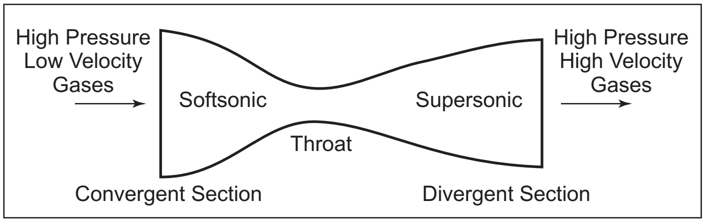


Figure 4.10 *Nozzle*

4.3.9 Blunt Bodies

Consider the flow of streamlines of air around the nose or the cockpit of an airplane (Figure 4.11).

As the streamlines approach the blunt body, they will bend and go on either side of the body maintaining their parallel nature. However, at the point 2, there is stagnation. The streamlines are stationary and, therefore, no external work is involved. The area around the airplane is so vast that it can safely be assumed that there is no heat transfer, i.e., the process is adiabatic. In such a case, the SFEE reduces to:

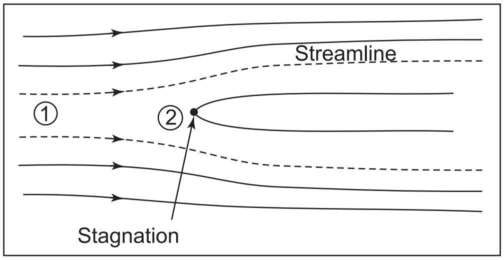


Figure 4.11 *Streamlining of air*

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$H_2 - H_1 + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) = 0$$

Making use of the definition of enthalpy, $H = c_pT$, we get,

$$c_pT_2 - c_pT_1 + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) = 0$$

$$c_pT_2 = c_pT_1 + \frac{1}{2}(\vartheta_1^2)$$

The stagnation temperature is defined as the temperature that the fluid would reach if it were brought to zero speed by a steady, adiabatic process with no external work. Therefore,

$$T_2 = T_1 + \frac{1}{2}\frac{\vartheta^2}{c_p}$$

Further, using the velocity density relation, $\vartheta = \sqrt{\gamma RT}$, we get, $\frac{T_t}{T} = 1 + \frac{(\gamma - 1)}{2}M^2$, where M is the Mach number and the stagnation enthalpy itself is given by:

$$H_t = c_pT + \frac{1}{2}(\vartheta^2)$$

Using the quasi-static adiabatic conditions, $\frac{T_t}{T} = \left(\frac{p_t}{p}\right)^{\frac{\gamma-1}{\gamma}}$ it is possible to bring about a relation between stagnation pressure and static pressure as:

$$\left(\frac{p_t}{p}\right) = \left(1 + \frac{(\gamma-1)}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Example 4.1

At the inlet to a certain nozzle, the enthalpy of fluid passing is measured to be 2,980 kJ/kg, and the velocity is 60 m/s. During discharge the enthalpy is found to have decreased by 280 kJ/kg. Consider the nozzle to be in horizontal position with negligible heat loss, find: (a) the velocity at exit of the nozzle; (b) If the inlet area is 960 cm^2 and the specific volume at inlet is 0.192 m^3/kg, determine the mass flow rate; (c) If the specific volume at the nozzle exit is 0.49 m^3/kg, find the diameter of the exit area of nozzle. ○●●

Solution

(a) The general equation:

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

Reduces to:

$$H_2 - H_1 + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) = 0 \rightarrow \vartheta_2 = \sqrt{2000(H_1 - H_2) + \vartheta_1^2}$$

Plugging in the values as

$$H_2 = 2700\frac{\text{kJ}}{\text{kg}},\ H_1 = 2980\frac{\text{kJ}}{\text{kg}} \text{ or } \left(H_1 - H_2 = 280\frac{\text{kJ}}{\text{kg}}\right) \text{ and } \vartheta_1 = 60\frac{\text{m}}{\text{s}},$$

$$\vartheta_2 = 750.73 \text{ m/s}$$

(b) Mass flow rate is given by:

$$\dot{m} = \rho A\vartheta = \frac{A\vartheta}{V} = \frac{A_1\vartheta_1}{V_1}$$

Plugging in the values $A_1 = 960\ \text{cm}^2$, $\vartheta_1 = 60\frac{\text{m}}{\text{s}}$ and $V_1 = 0.192\frac{\text{m}^3}{\text{kg}}$,

$$\dot{m} = 30 \text{ kg/ s}$$

(c) Using $\dot{m} = \frac{A_2\vartheta_2}{V_2}$ and plugging in $\dot{m} = 30\frac{\text{kg}}{\text{s}}$, $\vartheta_2 = 750.73\frac{\text{m}}{\text{s}}$ and $V_2 = 0.49\frac{\text{m}^3}{\text{kg}}$

$$A_2 = 196 \text{ cm}^2 \rightarrow d = 15.8 \text{ cm}$$

Example 4.2

A turbine operates under steady flow conditions, receiving steam at the following state: Pressure 1.2 MPa, temperature 188°C, enthalpy 2,785 kJ/kg, velocity 33.3 m/s and elevation 3 m. The steam leaves the turbine at the following state: Pressure 20 kPa, enthalpy 2,512 kJ/kg, velocity 100 m/s, and elevation 0 m. Heat is lost to the surroundings at the rate of 0.29 kJ/s. If the rate of steam flow through the turbine is 0.42 kg/s, what is the power output of the turbine in kW? ○○●

Solution The general equation:

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

Substitute the given values:

$$0.29 = 0.42\left[(2512 - 2785) + (3 - 0) + \frac{1}{2000}(100^2 - 33^2)\right] + W \rightarrow$$

$$W = 111.82 \text{ kW}$$

Example 4.3

A blower handles 1.4 kg/s of air at 25°C and consumes a power of 18.6 kW. The inlet and outlet velocities of air are 90 m/s and 160 m/s respectively. Find the exit air temperature, assuming adiabatic conditions. ○●●

Solution The general equation:

$$Q = m\left[(U_2 - U_1) + (p_2V_2 - p_1V_1) + (z_2g - z_1g) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$Q = m\left[(H_2 - H_1) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$$Q = m\left[c_p(T_2 - T_1) + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2)\right] + W$$

$Q = 0$; $m = 1.4$ kg/s, $T_1 = 298$ K, $W = 18.6$ kW, $z_1 - z_2 = 0$.

Substitute and obtain $T_2 = 276$ K.

4.4 || THROTTLING PROCESS

Probe

1. Discuss the throttle flow.

A high-pressure fluid is said to undergo a throttling process if it passes through a narrow constriction. As it passes through, the pressure reduces with a corresponding increase in volume. This process is generally adiabatic as the system is insulated and the temperature remains constant. However, the process is not reversible. Therefore, a throttling process is defined as a process in which a high-pressure fluid is turned into a low-pressure fluid after its transit through the throttling device. Throttling is an isenthalpic process, meaning the constancy of enthalpy. Therefore, for the throttling process,

$$H_2 = H_1$$

This is also called as Joule Thomson Effect. It is an irreversible process (Figure 4.12).

Therefore, the effect obtained by expansion cannot be obtained back by means of compression under adiabatic conditions.

The throttling process is commonly used for: (i) determining the dryness fraction of steam; (ii) controlling the speed of the turbine; (iii) reduction of refrigerant pressure before entry into the evaporator; (iv) air conditioner; (v) heat pump and (vi) liquefiers.

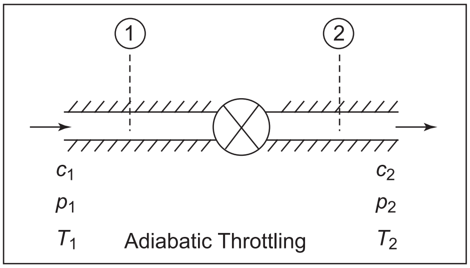


Figure 4.12 *Throttling process*

4.5 COMPARISON OF SFEE WITH EULER'S AND BERNOULLI'S EQUATIONS

Probe

1. Compare SFEE with Euler's and Bernoulli's Equations.

The steady state energy equation is the parental equation for the other two equations. It involves kinetic energy, potential energy, enthalpy, work transfer and heat transfer. In the differential form it becomes,

$$\delta Q = dU + pdV + Vdp + \vartheta d\vartheta + gdz + \delta W$$

Since, $H = U + pV$ and $\delta Q = dU + pdV$ for a quasi-static process which involves only pdV work, for an inviscid frictionless fluid flow in a pipe, the above equation reduces to the Euler's equation.

$$Vdp + \vartheta d\vartheta + gdz = 0$$

Thus, Euler's equation is in differential form but reduced terms. Integration of the above equation between sections 1 and 2 leads to Bernoulli's equation.

$$(p_2 - p_1)V + (z_2 - z_1)g + \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) = \text{constant}$$

$$\frac{p_1}{\rho} + \frac{\vartheta_1^2}{2} + z_1 g = \frac{p_2}{\rho} + \frac{\vartheta_2^2}{2} + z_2 g$$

Bernoulli's equation is in complete form and reflects only changes in potential, kinetic and elevation heads. Thus, both Euler's and Bernoulli's equations are special cases of the SFEE.

Remember to divide the kinetic head in the above equation by 1000 before proceeding with calculations.

Example 4.4

Water is flowing from a garden hose. The gardener places his thumb to affect a jet of high-speed water to emerge. If the hose is held perfectly vertical and the pressure in the hose just upstream of his thumb is 360 kPa, determine the maximum height that the jet can achieve. ○●●

Solution From Bernoulli's equation,

$$\frac{p_1}{\rho}+\frac{\vartheta_1^2}{2}+z_1 g=\frac{p_2}{\rho}+\frac{\vartheta_2^2}{2}+z_2 g$$

z_1 can be taken as zero as that could be the datum. Further, $\vartheta_1 << \vartheta_2$ and hence can be neglected. ϑ_2 at the top is zero. Therefore, the reduced equation is:

$$\frac{p_1}{\rho}=\frac{p_2}{\rho}+z_2 g$$

Substituting,

$$\frac{101.325\times 1000}{1000}=\frac{360\times 1000}{1000}+z_2\times 9.81$$

Solve for $z^2 = 26.37$ m.

Example 4.5

A large rectangular tank is open to the atmosphere at the top and is filled with water to a height of 8 m from the outlet tap. The tap near the bottom of the tank is now opened and water begins to flow out from the smooth and rounded outlet. Determine the maximum water velocity at the outlet. ○●●

Solution Use Bernoulli's equation

$$\frac{p_1}{\rho}+\frac{\vartheta_1^2}{2}+z_1 g=\frac{p_2}{\rho}+\frac{\vartheta_2^2}{2}+z_2 g,$$

$p_1 = p_2$ = atmospheric pressure. ϑ_1 inside the tank can be taken as zero. Therefore, the equation reduces to:

$$z_1 g=\frac{\vartheta_2^2}{2}$$

Substituting, $v_2 = 12.53$ m/s

Use Bernoulli's equation

$$\frac{p_1}{\rho}+\frac{\vartheta_1^2}{2}+z_1 g=\frac{p_2}{\rho}+\frac{\vartheta_2^2}{2}+z_2 g$$

$p_1 = p_2$ = atmospheric pressure. ϑ_1 inside the tank can be taken as zero. Therefore, the equation reduces to

$$z_1 g=\frac{\vartheta_2^2}{2}\rightarrow \vartheta_2=\sqrt{2gz_1}=\sqrt{2\times 9.81\times 8}=12.53\text{ m/s}$$

4.6 VARIABLE PROCESSES

Variable processes refer to unsteady flow or processes which depend on time. These are called unsteady processes or transient processes as they depend on time. It is not necessary that all of the variables should vary with time during the process. Any one variable changing with time will make the entire process unsteady. Such processes are complex in nature and make the analysis difficult. Examples such as evacuation of tank or filling a tank are common ones due to the assumption that the fluid is pure and with no interaction and without any friction. Other such examples are consumption of water, juice, milk and tea. If the container is insulated or has negligible heat transfer, then the equation for filling up a tank can be written as:

Probes

1. Discuss variable process with an example.
2. What are uniform processes?

$$Q - W = m\left[\Delta U + \Delta\left(H + \frac{\vartheta^2}{2} + gz\right)\right] \rightarrow U_2 = H_1 \text{ taking the internal energy before as datum.}$$

Similarly, $U_1 = H_2$ for a fluid leaving the tank.

Uniform processes are processes which depend on time. These processes vary with time at a constant rate and at any particular instance during the process, the state of the control volume is same. Hence, at any given instant, the state of the mass leaving from the exit is the same as the state of the mass in the control volume. The fluid flow at an opening is uniform and steady. That is, the properties do not change with time or position over the cross section of an inlet or exit. But they are different at different openings.

Example 4.6

An air receiver of volume 8 m^3 contains air at 20 bar and 55°C. The receiver is opened accidentally before some air can go out to atmosphere. The pressure of the air in the receiver drops rapidly to 14 bar when the receiver is sealed again. Calculate the mass of air which has left the receiver. ○●●

Solution

$$m_1 = \frac{p_1 V_1}{RT_1} = \frac{20 \times 101.325 \times 8}{0.287 \times 328} = 172 \text{ kg}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} \rightarrow T_2 = 328 \times \left(\frac{14}{20}\right)^{\frac{0.4}{1.4}} = 296.22 \text{ K}$$

$$m_2 = \frac{p_2 V_2}{RT_2} = \frac{14 \times 101.325 \times 8}{0.287 \times 296.22} = 133.49 \text{ kg}$$

$$m_2 - m_1 = 38.51 \text{ kg}$$

This is the mass that has left the receiver.

POINTS TO REMEMBER

☞ First law can be applied to an open system.

☞ First law from a closed system can be extended to an open system with certain assumptions.

☞ Steady flow energy equation can be applied to a variety of applications.

☞ Before the application, the entire equation needs to be considered with all terms.

☞ SFEE can be applied in different contexts.

☞ Flow of fluids such as mercury or blood or even ordinary fluids under extreme gravity need to be considered as special.

☞ Throttling is a constant enthalpy process.

☞ Euler's and Bernoulli's equations are only special cases of the SFEE.

☞ Mass rate flow will be different in case of a variable process.

☞ Remember that only enthalpy and internal energy are to be considered.

PRACTICE PROBLEMS

1. 10 kg of a certain fluid is delivered by a centrifugal air compressor every minute. The inlet and outlet conditions of air are ϑ_1 = 14 m/s, p_1 = 1 bar, V_1 = 0.48 m^3/kg and ϑ_2 = 86 m/s, p_2 = 8 bar, V_2 = 0.16 m^3/kg. The increase in enthalpy of air passing through the compressor is 160 kJ/kg and heat loss to the surroundings is 800 kJ/min. Find: (a) Motor power required to drive the compressor; (b) Ratio of inlet to outlet pipe diameter to the nearest meter. Assume that inlet and discharge lines are at the same level. ○●●

2. A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 16°C, a pressure of 100 kPa, and an enthalpy of 394.2 kJ/kg. The gas leaves the compressor at a temperature of 245°C, a pressure of 0.6 MPa, and an enthalpy of 524.5 kJ/kg. There is no heat transfer to or from the gas as it flows through the compressor. (a) Evaluate the external work done per unit mass of gas assuming the gas velocities at entry and exit to be negligible. (b) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 70 m/s and that at exit is 170 m/s. (**Ans:** (a) $W = -130.3$ kJ/kg; (b) $W = -132.3$ kJ/kg) ○○●

3. A room for six persons has three fans, each consuming 0.22 kW power, and four 120 W lamps. Ventilation air at the rate of 90 kg/h enters with an enthalpy of 88 kJ/kg and leaves with an enthalpy of 56 kJ/kg. If each person puts out heat at the rate of 650 kJ/h determine the rate at which heat is to be removed by a room cooler, so that a steady state is maintained in the room. (**Ans:** $W = 3.02$ kW) ●●●

4. Air at atmospheric pressure and 20°C is suctioned into a gas turbine power plant at a velocity of 120 m/s through an opening of 0.2 m^2 cross-sectional area. The air is compressed heated, expanded through a turbine, and exhausted at 0.18 MPa, 180°C through an opening of 0.10 m^2 cross sectional area. The power output is 380 kW. Calculate the net amount of heat added to the air in kJ/kg. Assume that air obeys the law $pV = 0.287\ (T + 273)$ where p is the pressure in kPa, V is the specific volume in m^3/kg, and T is the temperature in °C. ●●●

(**Ans:** $Q = 4822.17$ kJ, 166.74 kJ/kg)

5. In a steam engine, the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.98 and expansion follows the law $pV^{1.1}$ = constant, down to a pressure of 0.34 bar. Calculate per kg of steam: (a) The work done during expansion; (b) The heat flow to or from the cylinder walls during the expansion. (**Ans:** (a) $W = 449.5$ kJ/kg, (b) 163 kJ/kg) ●●●

6. The pump of a water distribution system for a building is powered by a 25-kW electric motor. The water flow rate through the pump is observed to be 80 L/s. If the diameters of the inlet and outlet pipes are the same and the absolute pressures at the inlet and outlet of the pump are measured to be 100 kPa and 340 kPa, respectively, determine the temperature rise of water as it flows through the pump. (**Ans:** 0.017 K) ●●●

7. A perfect gas is enclosed in a cylinder of volume 12 m^3 at 16 bar and 65°C. The cylinder is opened by means of a valve and some gas escapes to atmosphere. The pressure of the gas in the receiver drops rapidly to 12 bar before the valve is closed again. Calculate the mass of gas which has left the cylinder. (**Ans:** 37.25 kg) ○○●

8. A 2 m^3 tank is filled with air at a pressure of 600 kPa and a temperature of 120°C. The air goes out to the atmosphere by opening a valve. Find the work obtainable by utilising the kinetic energy of the outgoing air to run a frictionless turbine. Assume no heat transfer. (**Ans:** 677 kJ) ●●●

MULTIPLE CHOICE QUESTIONS

1. Steady flow occurs when: ○○●
   - (a) Conditions do not change with time at any point
   - (b) Conditions are the same at adjacent points at any instant
   - (c) Conditions change steadily with the time
   - (d) Temperature and pressure are constants

2. The first law of thermodynamics for steady flow: ○○●
   - (a) Accounts for only heat energy
   - (b) Is an energy balance for a specified mass of fluid
   - (c) Is an expression of the conservation of linear momentum
   - (d) Can be applied even for unsteady flow

3. Work done is the least in the process: ○●●
   - (a) Isenthalpic (b) Isochoric (c) Isothermal (d) Isentropic

4. If a process can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states, it is known as _________ process. ○●●
   - (a) Adiabatic (b) Isothermal (c) Ideal (d) Frictionless

5. For a gas, enthalpy is defined as the sum of product of pressure and: ○○●
   - (a) Volume and internal energy (b) Temperature and internal energy
   - (c) Velocity and internal energy (d) Pressure and density and internal energy

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (a) 2. (d) 3. (c) 4. (d) 5. (a)

Second Law of Thermodynamics

5

CHAPTER OUTLINE

☞ Heat Engine
☞ Heat Pump
☞ Refrigerator
☞ Second Law of Thermodynamics
☞ Equivalence of the two Statements
☞ Irreversibility
☞ Carnot's Theorem
☞ Thermodynamic Scale
☞ Perpetual Motion Machine of the Second Kind (PMM 2)

INTRODUCTION

A reservoir is a vast surrounding that is maintained at a constant temperature. For example, atmosphere is a reservoir for a refrigerator. Any addition or removal of heat from such a reservoir will *not* result in any change of temperature. The terms hot reservoir and cold reservoir are only relative. Thus, a cold reservoir can be maintained at 400 K or a hot reservoir at 300 K. The only thing imperative in a given arrangement is that the temperature of hot reservoir *must* be greater than that of the cold reservoir temperature.

5.1 HEAT ENGINE

Heat engine is defined as a device which operates between two reservoirs, one hot and the other cold. It extracts heat from a higher temperature reservoir (hot reservoir) and gives some useful output while rejecting heat to another reservoir at a lower temperature (cold reservoir).

University Questions

1. Define and explain a heat engine.
2. What is a heat engine? Write characteristic features of a heat engine.

(AU/IP/IEM/ME/AE/PR – 304, 2014)

From Figure 5.1, it is clear that a hot temperature reservoir is available at temperature, T_h. There is also another reservoir available at temperature, T_C. It is assumed that $T_h > T_C$. In between these two reservoirs is a device termed heat engine. This device extracts heat equal to Q_h from the hot reservoir, gives an output work of W and rejects heat equal to Q_C to the cold reservoir. Thus, it is clear that Q_c is less than Q_h and W is also less than Q_h. From the energy balance, it is evident that Q_h is the sum of W and Q_c. It is important to draw the diagram correctly with the indication of arrow heads. If all the directions are reversed, then it is termed as a heat pump or refrigerator.

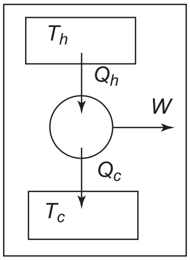


Figure 5.1 *Carnot engine*

A Carnot engine is a perfect engine which can be made reversible, i.e. it can work as a heat pump or refrigerator. The efficiency of a Carnot engine, as can be seen later, is the maximum for a given set of temperatures of the reservoirs. No actual engine can surpass the efficiency of this ideal engine.

The expression for efficiency is $\eta = \dfrac{W}{Q_h} = \dfrac{Q_h - Q_c}{Q_h} = 1 - \dfrac{Q_c}{Q_h}$

5.2 HEAT PUMP

Probe

1. What is a heat pump?

A heat pump is a device which performs between two thermal reservoirs of different temperatures. The primary aim of the heat pump is to extract heat from a lower thermal reservoir and pump it to the higher thermal reservoir. The addition of heat energy keeps the place warmer. This comes at the cost of additional external work and is useful in winter season. Again, the thermal energy balance gives the heat given out as the sum of heat extracted and the work done to accomplish the same. However, in case of heat pumps, the measure of performance is denoted by a factor called Coefficient of Performance (COP). It is defined as the heat given out per unit work supplied.

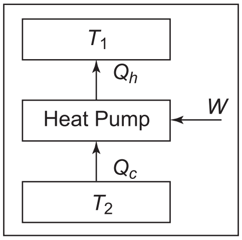


Figure 5.2 *Heat pump*

$\text{COP} = \dfrac{Q_h}{W} = \dfrac{Q_h}{Q_h - Q_c} = \dfrac{T_1}{T_1 - T_2}$ (Figure 5.2). It is obvious that COP is greater than unity.

5.3 REFRIGERATOR

Probe

1. What is a refrigerator?

A refrigerator is a device which works between two thermal reservoirs of different temperatures with the sole aim of cooling the things kept inside (Figure 5.2). Though the working principle is similar to that of a heat pump, the major difference is that the refrigerator focusses on the removal of heat rather than addition of heat. The performance is again based on the heat transfer rates and measured in terms of COP.

Refrigerators are useful throughout the year for various purposes such as retaining the freshness of food and safeguarding medicines. The COP is measured as:

$$\text{COP} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{T_2}{T_1 - T_2}$$

As in the case of a heat pump, the COP is desirably greater than unity.

5.4 || SECOND LAW OF THERMODYNAMICS

University Questions

1. State the Kelvin-Planck statement of the second law of thermodynamics. **(PTU, 2011–2012)**
2. State the Clausius statement of the second law of thermodynamics. **(PTU, 2011–2012)**

Second law of thermodynamics, is a natural law observed and documented over a large period. Hence, there is no mathematical proof as such. The second law attempts to shed more light on energy based on the limitations of the first law.

The second law states that it is impossible to construct a heat engine which can extract heat, Q_h, from a hot reservoir and convert all of it to useful work. This is generally called the Kelvin-Planck statement. This implies that there must be some rejection of heat while producing work.

The same law as stated by Clausius is, “It is not possible for heat energy to flow on its own from a colder reservoir to a hotter reservoir without any external aid”. This implies that there should be work input in order to realise the goal.

Note: In conclusion, no heat engine can interact with only one reservoir. Extraction of work from a cold reservoir is possible with external work only. Note that both statements are negative in nature.

Another version of the second law is—the total entropy (a property to be defined later) of an isolated system can only increase over time. At most, in ideal cases, it can remain the same.

It can also be stated as “the entropy always increases from one equilibrium state to another in any spontaneous change.

Carathéodory formulated the second law of thermodynamics as, “In every neighbourhood of any state *S* of an adiabatically enclosed system there are states inaccessible from *S*”.

Example 5.1

A heat engine absorbs 0.400 kJ of heat from the hot reservoir and does 0.120 kJ of work during each cycle. How much heat is released to the cold reservoir during each cycle? ○○●

Solution

$$\eta = \frac{W}{Q_1} = \frac{0.12}{0.4} = 30\%$$

Also, $Q_2 = Q_1 - W = 0.4 - 0.12 = 0.28$ kJ

Example 5.2

A heat engine absorbs 100 J of heat from the hot reservoir and releases 60 J of heat to the cold reservoir during each cycle. (a) What is its efficiency? (b) If each cycle takes 0.50 s, find the power output of this engine. ○○●

Solution

(a) Efficiency, $$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{100 - 60}{100} = 40\%$$

(b) Power, $$P = \frac{W}{t} = \frac{40}{0.5} = 80 \text{ W}$$

Example 5.3

An inventor claims that he has invented an engine which while working between temperature limits of 627°C and 27°C has an efficiency of 84%. Verify the claim. ○○●

Solution Carnot efficiency is $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = 0.667 \text{ or } 66.67\%$. No engine can surpass the Carnot efficiency. Therefore, the claim is false.

Example 5.4

An inventor claims to have developed a refrigerator that maintains the cold space temperature at –14°C, while operating in a 25°C room with a coefficient of performance of 7.5. Evaluate the claim. ○○●

Solution COP for refrigerator $= \frac{Q_2}{W_1} = \frac{T_2}{T_1 - T_2} = \frac{259}{298 - 259} = 6.64$

The claim is false since the claimed COP is more than that of Carnot COP.

Example 5.5

A heat pump has a coefficient of performance that is 60% of the theoretical maximum. It maintains a house at 24°C, which leaks energy of 0.65 kW per degree temperature difference to the outside. For a maximum of 1 kW power input, find the minimum outside temperature for which the heat pump is a sufficient heat source. ○●●

Solution For the house, $Q_1 = 0.65(297 - T_2)$

COP for the heat pump = 0.6 COP of Carnot = $0.6 \times \frac{Q_1}{W} = \frac{0.6T_1}{T_1 - T_2} = \frac{0.6 \times 297}{297 - T_2}$

$$Q_1 = \text{COP} \times W \rightarrow 0.65\,(297 - T_2) = \frac{0.6 \times 297}{297 - T_2} \times 1$$

Solving, $T_2 = 280.4 \text{ K} = 7.4°\text{C}$

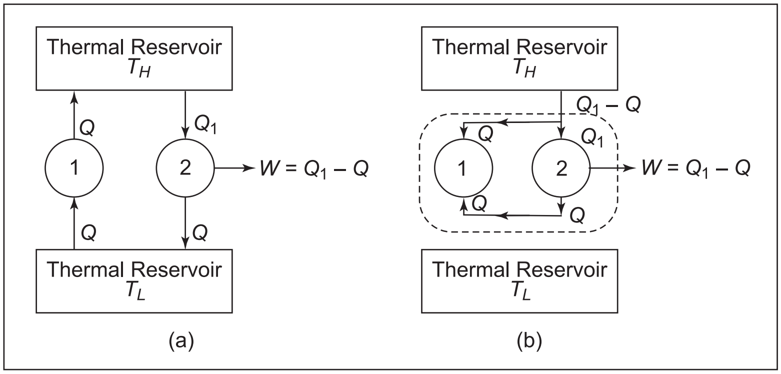
5.5 EQUIVALENCE OF THE TWO STATEMENTS

University Questions

1. Prove that the alternative statements of second law are equivalent.
2. Establish equivalence of Kelvin-Plank and Clausius statements.

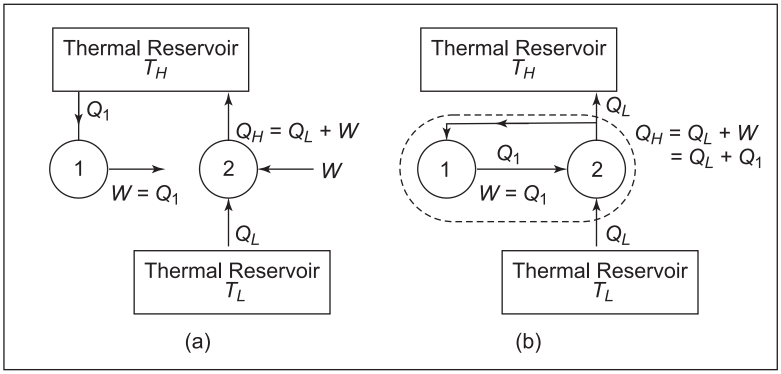
(AU, RTU, 2014–2015)

In order to prove that the two statements are same, the general method adopted is to show that violation of one statement leads to violation of the other (Figures 5.3 a and b). First let us assume that an engine violates by Clausius statement. Therefore, it is possible to transfer heat Q from a cold reservoir to hot reservoir without any work. Let another Carnot engine work between the same temperature limits extracting energy Q_1 from the hot reservoir and rejecting heat Q to the cold reservoir. Combining the performances yields the following results. The heat extracted from the cold reservoir is supplied to the Carnot engine. Thus, the Carnot engine is now extracting only $Q_1 - Q$ from the hot reservoir with a matching work output. Therefore, there is no rejection of heat to the cold reservoir meaning that it is interacting with only one reservoir and converting all of the heat energy to work output. This clearly violates the Kelvin statement. Therefore, the two statements must be the same otherwise violation of one results in violation of another.



Figures 5.3 (a and b) *Diagrammatic representation of equivalence of two statements*

Similarly, let us assume that there is an engine which violates the Kelvin's statement. Let it extract heat energy from a hot reservoir and convert all of it to work output. Let another reversible Carnot engine work between the same temperature limits. Let it operate as a heat pump as shown. Again, a combination of the two engines results in the fact that the Carnot heat pump can be supplied work from Kelvin's engine which results in transfer of heat from a cold reservoir to a hot reservoir without any work being done on the pump. This violates the Clausius statement. Therefore, the two statements must be the same since violation of one results in violation of another (Figures 5.4 a and b).



Figures 5.4 (a and b) *Diagrammatic representation of proof of equivalence of two statements*

Example 5.6

A reversible engine is provided with a power input of 1.2 kW to work as a refrigerator with COP = 3 or as a heat pump with COP = 4. It is used to maintain an office at 20°C through the year and exchanges 0.5 kW per degree temperature difference with the atmosphere. Find the maximum and minimum outside temperature for which this unit is sufficient. ○●●

Solution When it works as a heat pump, $Q = 0.5(T_H - T_{amb})$

$$W = \frac{Q}{\text{COP}} \rightarrow 1.2 = \frac{0.5(293 - T_{amb})}{4} \rightarrow T_{amb} = 283.4\text{ K} = 10.4°\text{C}$$

When it works as a refrigerator,

$$W = \frac{Q}{\text{COP}} \rightarrow 1.2 = \frac{0.5(T_{amb} - T_{house})}{3} \rightarrow T_{amb} = 300.2\text{ K} = 27.2°\text{C}$$

Example 5.7

A refrigerator working on reversed Carnot cycle requires 0.4 kW per kW of cooling to maintain a temperature of –15°C. Determine the following: (a) COP of the refrigerator (b) Temperature at which heat is rejected and (c) Amount of heat rejected to the surroundings per kW of cooling. ○●●

Solution

(a) COP for refrigerator $= \dfrac{Q_2}{W} = \dfrac{1}{0.4} = 2.5$

(b) COP for refrigerator $= \dfrac{Q_2}{W} = \dfrac{T_2}{T_1 - T_2}$

$$\therefore \qquad 2.5 = \frac{T_2}{T_1 - T_2}$$

Substituting,

$$2.5 = \frac{288}{T_1 - 288}$$

$$T_1 = 403.2 \text{ K}$$

(c) COP for refrigerator $= \dfrac{Q_2}{W} = \dfrac{Q_2}{Q_1 - Q_2}$

$$\rightarrow 2.5 = \frac{1}{Q_1 - 1} \rightarrow Q_1 = 1.4 \text{ kW}$$

Example 5.8

A domestic food freezer maintains a temperature of –15°C. The ambient air is at 30°C. If the heat leaks into the freezer at a continuous rate of 1.75 kJ/s, what is the least power necessary to pump the heat out continuously?

○●●

Solution Carnot COP $= \dfrac{T_L}{T_H - T_L} = \dfrac{258}{303 - 258} = 5.73$

For least power, Carnot COP = Actual COP

$$5.73 = \frac{1.75}{W} \rightarrow W = 0.31 \text{ kW}$$

5.6 IRREVERSIBILITY

The second law holds good for any device operating between two temperature limits. The rejection of heat in the performance of an engine or the additional work required in case of a heat pump or refrigerator is because of irreversibility inherent to the system in nature. This irreversibility cannot be overcome. The engineer's sole aim is to minimise this loss. The following factors render the process irreversible and prevent the engine from working reversibly.

University Questions

1. What is irreversibility? State various types of irreversibilities and explain them. **(GTU, 2015)**
2. What are the causes of irreversibility? **(GTU, 2016)**
3. Explain unrestrained expansion. **(GTU, 2011–2012)**
4. Discuss friction, finite temperature difference, unrestrained expansion as factors leading to irreversibility.
5. Discuss mixing of two substances, magnetic hysteresis, electrical resistance, viscosity and inelasticity as factors leading to irreversibility.
6. What are the two types of irreversibility? Explain in brief.

5.6.1 Causes of Irreversibility

Irreversibility is a phenomenon in which the system and surroundings react in such a way that after the interaction the original states of the system and surroundings would have changed. Those original states of the system and surroundings cannot be restored. Any attempt made would involve producing more changes in the surroundings. During irreversibility, it is evident that the system would lose its thermodynamic equilibrium status and starts to dissipate. The loss of thermodynamic equilibrium itself could be due to loss of mechanical or chemical or thermal equilibrium. Any spontaneous reaction would involve dissipation of energy. There are several causes for the irreversibility to take place. Important among these causes are:

Friction

Friction happens in nature in different ways. We cannot walk if there is no friction. Consider bringing up water from a well using a pulley. The work needed is not only to bring the water bucket up but also to overcome the friction between the pulley and rope. Friction produces heat and is dissipated to the surroundings. Though the water bucket can be lowered the heat given to the surroundings cannot be obtained back. Thus, friction has rendered the process irreversible because the surroundings are not restored to their initial state at the conclusion of the reverse process.

Finite Temperature Difference

Consider two systems which are at different temperatures. As the two systems are brought together in close contact, the heat flows naturally from the hotter body to the colder body. If the systems are to be restored to their original state then additional work needs to be done which require work and heat interactions with the surroundings. The process becomes irreversible as the surroundings cannot be brought back to its original state and the lost heat cannot be regained.

Unrestrained Expansion

Consider a balloon filled with helium. Let the gas go out through an opening to the surroundings. If the gas filled balloon is to be restored to its original condition, then there needs to be compression work which involves heat. Work and the removal of heat from the system cause changes in the surroundings. This indicates that an unrestrained expansion results in irreversibility.

Mixing of two Substances

Consider a cup of water. Pour one tablespoon of sugar and one tablespoon of salt into it and stir thoroughly. If the mixture is to be brought back to its original contents in a separated manner then lot of chemical processes need to take place which alter the surroundings. Similarly, milk when mixed with water forms a mixture. From experience, there is no way the two can be separated by themselves or without involving additional work. Therefore, mixing of two substances is irreversible.

Magnetic Hysteresis

Hysteresis is always associated with dissipation. Usually, the area of the cycle represents the dissipated energy during hysteresis. Dissipation energy cannot be claimed back and represents irreversibility. If the thermodynamic change is brought about by a phase change with internal friction then dissipation is bound to occur. At a deeper level, magnetic disturbances can be found due to crystal imperfections or gap between bonding. Also, any process which affects hysteresis by causing disturbance to it will result in dissipation.

Electrical Resistance

When current flows in a circuit, it does so to overcome the resistance offered by the conductor. During this process, irreversible losses occur which result in loss of some portion of the electric power. This loss of electric power gets converted to either internal energy or heat and dissipates. The dissipation takes place irrespective of the direction of current flow. Reversing the direction of energy from internal energy or heat to electrical power is not possible. Hence, current flow during electrical resistance is irreversible.

The rate of energy dissipation is given by the product of the temperature and the rate of entropy production (i.e. $T\sigma$) with: $T\sigma = JX$, where J is a generalised flux of some kind, and X a generalised force.

Viscosity

Viscosity of a fluid varies with temperature which in turn affects the heat transfer that is dissipative in nature. The dissipation cannot be reversed to change the viscosity. This is observed more in convective problems which involve both fluid friction and heat transfer. To determine which of them is more predominant in a given scenario, the irreversibility ratio is used. Hence, viscosity contributes to irreversibility.

Inelasticity

Deformation of a solid will lead to irreversibility if the deformation is outside the elastic limits. Outside the elastic limits, the solid cannot return to its original shape. The deformation involves dissipation energy which cannot be brought back. Therefore, the process becomes irreversible.

Types of Irreversibility

There are two types of irreversibility—internal and external. Internal irreversibility refers to the irreversibility that the system experiences after the process. External irreversibility refers to the irreversibility that the surroundings experience.

Example 5.9

As an engineer, you are designing a heat pump that is capable of delivering heat at the rate of 20 kW to a house. The house is located where, in January, the average outside temperature is –10°C. The temperature of the air in the air handler inside the house is to be 40°C. (a) What is maximum possible COP for a heat pump operating between these temperatures? (b) What must the minimum power be of the electric motor driving the heat pump? c) In reality, the COP of the heat pump will be only 60% of the ideal value. What is the minimum power of the electric motor when the COP is 60% of the ideal value?

●●●

Solution

(a) $$\text{COP}_{HP} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{1 - \frac{Q_2}{Q_1}} = \frac{1}{1 - \frac{T_2}{T_1}} = \frac{T_1}{T_1 - T_2} = \frac{313}{313 - 263} = 6.26$$

(b) $$\text{COP}_{HP} = \frac{P_{\text{out}}}{P_{\text{motor}}} \rightarrow P_{\text{motor}} = \frac{P_{\text{out}}}{\text{COP}} = \frac{20}{6.26} = 3.2 \text{ kW}$$

(c) $\text{COP}_{\text{Act}} = \dfrac{\text{COP}_{HP}}{\in} = \dfrac{3.2}{0.6} = 5.3 \text{ kW}$

5.7 CARNOT'S THEOREM

The efficiency of all reversible engines operating between the same two temperature limits in a cycle is the same, and no irreversible engine operating between these temperature limits can have a greater efficiency than this.

The Carnot cycle consist of an isothermal expansion, an adiabatic expansion, an isothermal compression and an adiabatic compression in the same order. Heat is absorbed during isothermal expansion and rejected during isothermal compression.

Efficiency of the engine is defined as:

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

University Questions

1. State Carnot's theorem.
2. What is a Carnot cycle? **(GTU, 2016)**
3. Explain working of Carnot and reversed Carnot Cycle.
4. Prove Carnot's theorem.
5. State two methods to improve the efficiency of Carnot cycle. **(GTU, 2016)**

5.7.1 Proof

Consider two engines HE_1 and HE_2 operating between the given source at temperature T_1 and the given sink at temperature T_2. Let HE_2 be any irreversible heat engine and HE_1 be any reversible heat engine. The goal is to prove that efficiency of heat engine HE_2 is more than that of heat engine HE_1 (Figure 5.5).

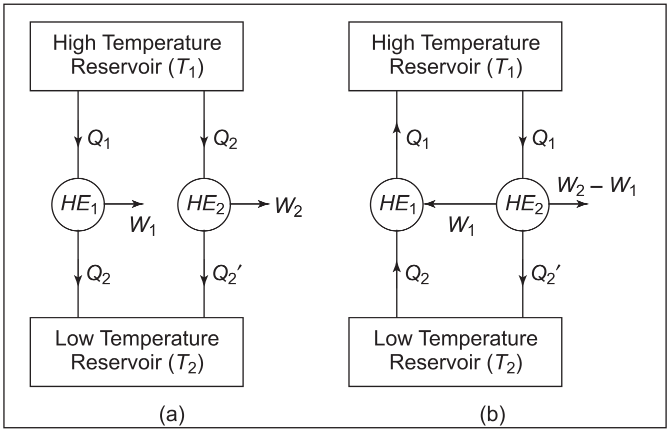


Figure 5.5 *Diagrammatic representation of proof of Carnot's theorem*

Let both the heat engines receive same quantity of heat Q_1 from the source at temperature T_1. Let W_1 and W_2 be the work output from the engines and their corresponding heat rejections be $(Q_1 - W_1)$ and $(Q_1 - W_2)$ respectively.

Assume that the efficiency of the irreversible engine be more than the reversible engine, i.e. $\eta_{\text{irr}} > \eta_{\text{rev}}$. Hence,

$$\frac{W_2}{Q_1} > \frac{W_1}{Q_1}, \text{ leading to } W_2 > W_1$$

Now let us couple both the engines and HE_1 is reversed which will act as a heat pump. It receives Q_2 from sink and W_1 from irreversible engine HE_2 and pumps heat Q_1 to the source at temperature T_1. The net result is that heat $W_2 - W_1$ is taken from sink and equal amount of work is produced. This violates second law of thermodynamics. Hence, the assumption that irreversible engine having higher efficiency than the reversible engine is wrong. Thus, it is concluded that a reversible engine working between any two temperature limits is more efficient than an irreversible engine working between same temperature limits thereby proving Carnot's theorem.

5.7.2 Corollary

All Carnot engines working between the same temperature limits have the same efficiency. Now, we can prove that all Carnot engines working between the same temperature limits have the same efficiency by contradiction method (Figure 5.5).

Assume one engine (HE_2) to be more efficient than the other (HE_1), both working under the same temperature limits. Therefore, the output of HE_2 must be greater than that of HE_1. Reverse the first engine to work as a heat pump. Let the work required for it to run be supplied by HE_2, the greater efficiency engine. Combination of the two engines results in violation of second law (Kelvin–Planck statement) as there would be no need for the reservoir at higher temperature. This cannot be so and hence, the assumption that the second engine was of a greater efficiency is wrong. The same conclusion would have been arrived at if the first was assumed to be of greater efficiency.

Example 5.10

It is required to produce 1,000 kg of ice per hour from water at 0°C with an ambient temperature of 37°C. The heat engine absorbs heat from boiler at 70°C and runs the refrigerator. Calculate (a) the power required for refrigeration, (b) the ratio of heat absorbed by heat engine to the heat extracted from the freezing water and (c) the rate of energy rejection to the ambient. Assume the enthalpy of water at 0°C as 333.43 kJ/kg.

Solution Draw the diagram with relevant details.

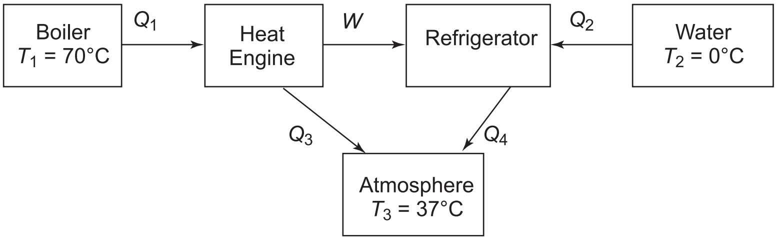


Figure 5.6

(a) $$\text{COP}_{ref} = \frac{Q_2}{W} = \frac{T_2}{T_4 - T_2} = \frac{273}{310 - 273} = 7.38$$

$$\rightarrow \quad W = \frac{Q_2}{7.38} = \frac{1000 \times 333.43}{3600 \times 7.38} = 12.55 \text{ kW}$$

(b) $$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_3}{Q_1} = \frac{T_1 - T_3}{T_1} = \frac{343 - 310}{343} = 0.096$$

$$\frac{Q_1}{Q_2} = \frac{Q_1/W}{Q_2/W} = \frac{1/\eta}{\text{COP}} = \frac{1/0.096}{7.38} = 1.411$$

(c) $$\frac{Q_1}{Q_3} = \frac{T_1}{T_3} = \frac{343}{310}$$

$\rightarrow Q_3 = 0.904\ Q_1 = 1.28\ Q_2 = 92.62$ kW

$$\frac{Q_4}{Q_2} = \frac{T_4}{T_2} = \frac{310}{273}$$

$\rightarrow Q_4 = 1.14\ Q_2 = 105.59$ kW

Total heat rejected is $Q_3 + Q_4 = 92.62 + 105.59 = 198.21$ kW

Example 5.11

A steel bottle $V = 0.1\ \text{m}^3$ contains R-134a at 20°C, 200 kPa. It is placed in a deep freezer where it is cooled to –20°C. The deep freezer sits in a room with ambient temperature of 20°C and has an inside temperature of -20°C. Find the amount of energy the freezer must remove from the R-134a and the extra amount of work input to the freezer to do the process. ●●●

Solution From the first law, energy balance is $m(U_2 - U_1) = Q_{12} - W_{12}$

For a constant volume process, $W_{12} = 0$; From tables,

$V_1 = 0.11436$, $U_1 = 418.145 - 200 \times 0.11436 = 395.273$; $m = \dfrac{V}{v_1} = \dfrac{0.1}{0.11436} = 0.87443$ kg

and, at state 2, dryness fraction needs to be calculated first.

$x_2 = (0.11436 - 0.000738)/0.14576 = 0.77957$ $U_2 = 173.65 + 0.77957 \times 192.85 = 323.99$ kJ/kg

Substituting, $Q_{12} = -62.34$ kJ

$$\text{COP for refrigerator} = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2} = \frac{253}{293 - 253} = 6.33$$

$$W = \frac{Q}{\text{COP}} = \frac{62.334}{6.33} = 9.84 \text{ kJ}$$

Example 5.12

Two Carnot engines work in series between temperature limits of 1,027°C and 27°C. The amount of energy absorbed from the intermediate temperature is the same as that rejected to it. Determine the intermediate temperature if their efficiencies are the same. If the first engine receives 250 kJ from the high temperature reservoir, determine the total work done by the engines. ●●●

Solution Let T be the intermediate temperature. Then,

$$\eta_1 = 1 - \frac{T}{T_1}; \ \eta_2 = 1 - \frac{T_2}{T}$$

$$\eta_1 = \eta_2 \rightarrow \frac{T}{T_1} = \frac{T_2}{T} \rightarrow T = \sqrt{T_1 T_2} = \sqrt{1300 \times 300} = 624.5 \text{ K}$$

$$\therefore \quad W_1 = \eta_1 Q_1 = \left(1 - \frac{624.5}{1300}\right) 250 = 129.9 \text{ kJ}$$

$$Q = Q_1 - W_1 = 120.1 \text{ kJ}$$

$$W_2 = \eta_2 Q = \left(1 - \frac{T_2}{T}\right) Q = \left(1 - \frac{300}{624.5}\right) 120.1 = 62.32 \text{ kJ}$$

$$\therefore \quad \text{Total work} = W_1 + W_2 = 182.42 \text{ kJ}$$

5.8 || THERMODYNAMIC SCALE

This is necessary to show that the temperature is independent of the working medium. Also, the temperature ratio is the same as the heat energy ratio (Figure 5.7).

Probe

1. How do you establish a thermodynamic scale?

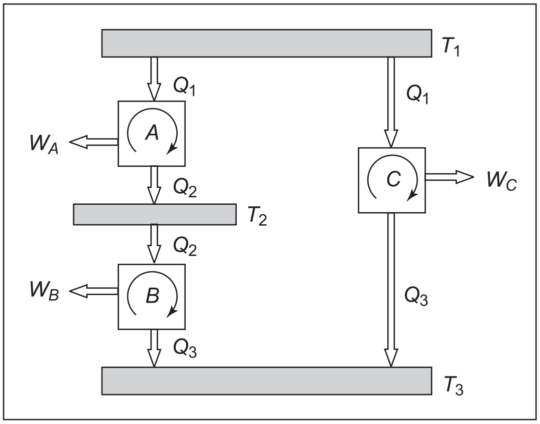


Figure 5.7 *Establishment of the thermodynamic scale of temperature*

Let a reversible engine A work between two reservoirs, a source at T_1 and a sink at T_2. Let another reversible engine B work with source at T_2 and sink T_3. It is obvious that $T_1 > T_2 > T_3$. The reversible engine C works directly between the source T_1 and sink T_3 as shown in Figure 5.7. For the Carnot engine, the efficiency is given by $\eta = 1 - \frac{Q_L}{Q_H}$, where Q_L represents the heat energy rejected to the sink and Q_H is the heat energy absorbed from the source. It is obvious that the efficiency,

$$\eta = 1 - \frac{Q_L}{Q_H} \rightarrow \frac{Q_L}{Q_H} = F(T_L, T_H)$$

Applying this to the three engines above,

$$\eta_A = 1 - \frac{Q_1}{Q_2} \rightarrow \frac{Q_1}{Q_2} = F(T_1, T_2);\ \eta_B = 1 - \frac{Q_2}{Q_3} \rightarrow \frac{Q_2}{Q_3} = F(T_2, T_3);\ \eta_C = 1 - \frac{Q_1}{Q_3} \rightarrow \frac{Q_1}{Q_3} = F(T_1, T_3)$$

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} \rightarrow F(T_1, T_3) = F(T_1, T_2) \times F(T_2, T_3)$$

Since $F(T_1, T_3)$ does not contain T_2, the term T_2 must be in such a way that it would cancel out in the right side. The right expression would be a ratio such as $\frac{Q_1}{Q_3} = F(T_1, T_3) = \frac{f(T_1)}{f(T_3)}$. The simplest relation is line which is $\frac{Q_1}{Q_3} = \frac{T_1}{T_3}$. Thus, the temperature defined in this manner is independent of the medium and depends only on the heat energy transfer.

5.9 PERPETUAL MOTION MACHINE OF THE SECOND KIND (PMM 2)

Probe

1. What is perpetual motion machine of the second kind?

A perpetual motion machine of the second kind (PMM 2) is a machine which spontaneously converts thermal energy into mechanical work. It can be defined as an ideal machine which works with 100% efficiency without any leakage whatsoever. This means, there is no distribution of any heat energy to the sink or the lower temperature reservoir. Since it violates the second law of thermodynamics, such a machine is termed as perpetual motion machine of the second kind.

PMM 2 claims to convert thermal energy into mechanical work, directly, with no exhaust heat being emitted; this violates the principle of entropy that entropy in a system must always increase. These are devices that claim to transform energy by gaining work from a constant temperature. The dictionary definition is, “the hypothetical continuous operation of an isolated mechanical device or other closed system without a sustaining energy source”. PMM 2 tend to work to the maximum without any friction or any other such dissipation.

PMM 2 cannot be constructed in violation of the second law of thermodynamics. This holds good for all machines tried in different sectors including bio-machines.

POINTS TO REMEMBER

- Heat engine always operates between two reservoirs.
- Heat engine has a fixed efficiency—Carnot efficiency which is the maximum.
- A heat pump works in the reverse direction and is useful in warming up the place.
- A refrigerator also works in the reverse direction and is useful for preservation of food items and medicines.
- The fluid medium inside the engine or pump or the refrigerator is assumed to be incompressible.
- The fluid medium will be non-viscous.
- The Second law of thermodynamics is a natural law.
- A heat engine has to interact with two reservoirs while producing the useful work.
- Heat cannot be converted to work in its entirety.
- Heat is regarded as low grade energy while work is regarded as high grade energy.
- Violation of one version of the statement leads to violation of others.
- There is no perfect reversible process in nature.
- Several factors lead to irreversibility.
- Irreversibility can be internal or external.
- Concept of reversible heat engine is necessary.
- When reversed it can act as a refrigerator or a heat pump.
- In case of a heat pump or refrigerator the term efficiency is replaced by Coefficient of Performance (COP).
- Carnot efficiency or COP is the maximum attainable.
- Heat pump refers to the maximum heat that can be pumped.
- Refrigerator refers to the maximum heat that can be extracted.
- Temperature of reservoir is independent of the fluid medium used.
- The ratio of heat energies from reservoirs is the same as the ratio of their temperatures.

PRACTICE PROBLEMS

1. A solar-energy collector produces a maximum temperature of 100°C. New design helps to produce up to 300°C. The energy is used in a cyclic heat engine that operates in a 10°C environment. What is the increase in maximum thermal efficiency? (**Ans:** 26.5%) ○●●
2. An inventor claims to have developed a refrigerator that maintains the cold space temperature at -14°C, while operating in a 25°C room with a coefficient of performance of 7.5. Evaluate the claim. (**Ans:** The claim is false since the claimed COP is more than that of Carnot COP.) ○○●
3. You are installing a heat pump, whose COP is half the COP of a reversible heat pump. You will use the pump on chilly winter nights to increase the air temperature in your bedroom. Your bedroom's dimensions are 5.00 m × 3.50 m × 2.50 m. The air temperature should increase from 63°F to 68°F. The outside temperature is 35°F, and the temperature at the air handler in the room is 112°F. If the pump's electric power consumption is 750 W, how long ●●●

will you have to wait in order for the room's air to warm (take the specific heat of air to be 1.005 kJ/kg°C)? Assume you have good window draperies and good wall insulation so that you can neglect the release of heat through windows, walls, ceilings and floors. Also assume that the heat capacity of the floor, ceiling, walls and furniture are negligible and density of air is 1.293kg/m^3. (**Ans:** 12.15 min)

4. A household freezer operates in a room at 25°C. Heat must be transferred from the cold space at a rate of 2.5 kW to maintain its temperature at –30°C. What is the smallest motor required operating this freezer if the motors are available in steps of 0.25 kW? ○●●
   (**Ans:** Therefore, the smallest motor required is of 0.75 kW capacity.)
5. An ambulance vehicle engine operates with a thermal efficiency of 38%. The air conditioner has a coefficient of performance that is one third of the theoretical maximum. How much fuel energy should be spent extra to remove 1.5 kJ at 12°C when the ambient is at 40°C? ●●●
   (**Ans:** 1.163 kJ)
6. Is decreasing the sink temperature better than increasing the source temperature for greater efficiency? ○●●
   (**Ans:** The denominator is always positive. In the numerator, $(T_2 - T_1) < 0$ and $\Delta T > 0$. Hence, decreasing the sink results in higher efficiency.)
7. An office needs to maintain its rooms at 25°C throughout the year. A heat pump which can also work as a refrigerator is used to combat extreme temperatures of 45°C and 2°C. The energy loss through doors and walls is to be limited to 45 kW. Determine the minimum power required to operate the device and the increase of power in winter. ○●●
   (**Ans:** Minimum power required, summer = 3.36 kW; winter = 3.86; the increase in power is 0.5 kW)
8. Helium has the lowest normal boiling point of any of the elements at –269°C. At this temperature, the enthalpy of evaporation is 83.1 kJ/kmol. A Carnot refrigeration cycle is used to produce 1 kmol of liquid helium at –269°C from saturated vapor at the same temperature. What is the work input needed to the refrigerator and the coefficient of performance for the cycle with an ambient temperature of 24°C? (**Ans:** W = 6087.08 kJ/kmol; COP = 0.0137) ○●●
9. A combination of a heat engine driving a heat pump takes energy from a source at 50°C. The heat engine receives heat equal to Q_1 and rejects heat at 30°C. The remainder Q_2 goes into the heat pump that delivers a Q_3 at 150°C. If the total energy in the source at 50°C is 5 MW find the rate of energy delivered at the high temperature. (**Ans:** 1.09 MW) ○●●
10. A Carnot refrigerator consumes 260 W in summer when the ambient temperature reaches 40°C. Due to constant opening, the leakage through the doors is estimated to be 18 W/°C temperature difference. Determine the temperature inside the refrigerator. (**Ans:** 387.85 K) ○●●

MULTIPLE CHOICE QUESTIONS

1. The following assumptions are made on heat engine except: ○○●
   (a) The source and sink are maintained at constant temperature.
   (b) The source and sink has infinite heat capacity.
   (c) The source is at a higher temperature.
   (d) The sink is at a higher temperature.

2. The Carnot cycle comprises of: ○●●
(a) Two isothermals and two isobars
(b) Two isothermals and two adiabatics
(c) Two isothermals and two reversible adiabatics
(d) Two isothermals and two reversible isobars

3. Mixing of two substances results in: ○○●
(a) Reversible process
(b) Partially reversible process
(c) Partially irreversible process
(d) Irreversible process

4. What are the important characteristics of entropy? ○●●
(a) If the heat is supplied to the system then the entropy will increase.
(b) If the heat is rejected from the system then the entropy will decrease.
(c) The entropy is constant for all adiabatic frictionless process.
(d) All of the above

5. COP of a heat pump as compared to that of a refrigerator is: ○●●
(a) Lesser (b) Greater
(c) The same (d) Cannot be compared

6. In a Carnot cycle: ●●●
(a) For isothermal process the piston moves very slowly and for adiabatic process the piston moves as fast as possible.
(b) For isothermal process the piston moves very fast and for adiabatic process the piston moves as slowly as possible.
(c) For both isothermal process and the adiabatic processes the piston moves very slowly.
(d) For both isothermal process and the adiabatic processes the piston moves very quickly.

7. Modern automobile gasoline engines have efficiencies of about 30%. About what percentage of the heat of combustion is not used for work but released as heat? ○○●
(a) 30% (b) 70%
(c) 65% (d) 130%

8. Carnot's engine is: ○○●
(a) Irreversible (b) Partly reversible
(c) Reversible (d) Hard to say

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (d) 2. (b) 3. (d) 4. (d) 5. (b)
6. (b) 7. (b) 8. (c)

Entropy

6

Chapter Outline

- ☞ Entropy as a Property of System
- ☞ Clausius Theorem
- ☞ Entropy Change in an Irreversible Process
- ☞ Entropy Principle
- ☞ Introduction to the Third Law of Thermodynamics
- ☞ Entropy Generation in Closed and Open Systems

INTRODUCTION

Entropy is a property of the system which measures the randomness in the system. It is also a measure of the energy available for conversion between two forms. If there is any disturbance to ordered or structured form then it is referred to as change in entropy. The symbol is *S*, for specific entropy, and is measured in kJ/kgK. It is difficult to measure absolute entropy and hence, always a change is measured. The change in entropy, *dS*, is defined mathematically as:

$$dS = \frac{\delta Q}{T}$$

6.1 ‖ ENTROPY AS A PROPERTY OF SYSTEM

Probes

1. Define entropy.
2. Prove that entropy is a property.

In statistics, entropy is defined as the lack of orderliness or randomness. It is projected as a probability distribution with an inclination to gradual disorderliness.

In information theory, entropy measures the amount of information that is missing before reception. Thus, it represents the information that is necessary to properly identify the microstate of the system. It is expressed as a discrete set of probabilities.

As shown in Figure 6.1, consider a system initially at state 1. Let it reach a state 2 under some process along the path *A*. Let it come back to the original state 1 from state 2 along paths *B* and *C*. Then, for the cycle 1-A-2-B-1,

$$\int \frac{\delta Q}{T} = \int_{1A2} \frac{\delta Q}{T} + \int_{2B1} \frac{\delta Q}{T}$$

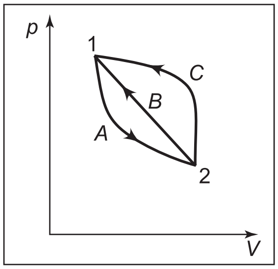


Figure 6.1 *Entropy as a property*

Similarly, for the cycle 1-A-2-C-1,

$$\int \frac{\delta Q}{T} = \int_{1A2} \frac{\delta Q}{T} + \int_{2C1} \frac{\delta Q}{T}$$

Subtracting one from the other,

$$\int_{2B1} \frac{\delta Q}{T} = \int_{2C1} \frac{\delta Q}{T}$$

Therefore, entropy is path independent and hence a property of the system. This indicates that entropy is a perfect or exact differential and is denoted by *dS* and not by *δS*. Entropy is a point function and its value can be determined at a particular state—it can be represented as S_1, S_2 etc. It helps in the description of the state of the system.

6.2 CLAUSIUS THEOREM

Clausius theorem is another version of the second law of thermodynamics. The theorem is expressed as an inequality which expresses the relation of entropy change of any process or different processes over a cycle.

University Questions

1. State and explain the Clausius theorem. **(GTU, 2010–2012, 2014)**
2. What is meant by Clausius inequality? **(GTU, 2010–2011)**

Mathematically, $\int \frac{\delta Q}{T} \leq 0$, i.e., the total entropy change taken over a cycle is always less than or at most equal to zero. The inequality sign holds good for any real change while the equal sign holds good only during reversibility over a cycle. However, it should be noted that a perfect reversible process is not possible and in the extreme it approaches zero. Thus, it is a quantitative representation of the second law of thermodynamics.

6.2.1 Clausius Inequality

Consider two heat Engines *A* and *B* that work between the same two reservoirs (Figure 6.2). Let engine *A* be reversible. Let engine *B* be an irreversible one. Both these engines absorb heat, Q_1, from a common heat source, at a temperature, T_1, and reject heat, Q_{2A} and Q_{2B}, respectively to a common heat sink at temperature, T_2. Then, from energy balance of the first law,

$$W_1 = Q_1 - Q_{2A};$$

$$W_2 = Q_1 - Q_{2B};$$

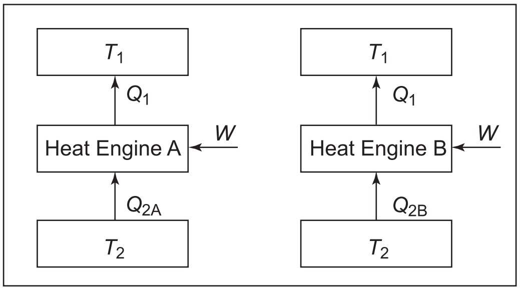


Figure 6.2 *Two engines between same temperature limits*

From earlier chapters, it is known that only a reversible engine has the highest efficiency among the heat engines working between the same source and sink temperatures. Therefore, a reversible heat engine produces more work than an irreversible heat engine for the same heat input, Q_1. However, the amount of heat rejected to the sink is different. As shown below, the reversible engine gives out less heat to the cold reservoir in order to produce more work and maintain the energy balance.

$$W_1 > W_2 \rightarrow$$

$$Q_1 - Q_{2A} > Q_1 - Q_{2B} \rightarrow$$

$$Q_{2A} < Q_{2B}$$

The temperature, T_1 of the reservoir is maintained constant during the heat transfer, Q_1. Similarly, the temperature, T_2 at the sink is held constant during the heat transfers, Q_{2A} and Q_{2B}. That is to say both the processes are isothermal (Figure 6.3).

Then, for the engine *A* over the cycle,

$$\int_A \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_{2A}}{T_2}$$

For the engine *B* over the cycle,

$$\int_B \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_{2B}}{T_2}$$

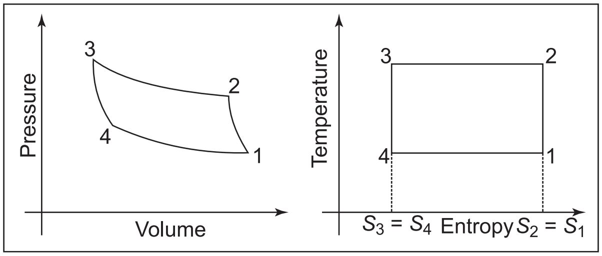


Figure 6.3 *Typical p-V and T-S diagrams of Carnot cycle*

Comparing the above two equations and noting that $Q_{2A} < Q_{2B}$ yields $\int_B \frac{\delta Q}{T} < \int_A \frac{\delta Q}{T}$.

Therefore, the sum entropy change over the cycle for the engine *B* is less than that for the engine *A*, the reversible engine.

For engine *A*, the reversible heat engine, the following relation holds good: $\frac{Q_1}{Q_{2A}} = \frac{T_1}{T_2} \rightarrow \int_A \frac{\delta Q}{T} = 0$

Therefore, $\int \frac{\delta Q}{T} < 0$

In summary, the Clausius inequality states that $\int \frac{\delta Q}{T} \leq 0$ with the equality sign for reversible cycles and inequality sign holding good for irreversible cycles. The reversible cycle is true only for a Carnot cycle or an ideal cycle. Almost all cycles in nature are irreversible. All cycles are compared with the theoretical cycle for the deviation. The amount of deviation from the theoretical cycle is what motivates the engineers to reduce the gap and improvise on the devices. It is also clear that according to Clausius theorem, even the reversible, ideal, theoretical cycle will not have negative entropy. As it applies to any real cycle, the implication is that the entropy transferred from the hot reservoir is much smaller than that to the environment or surroundings. To avoid greater entropy change in the surroundings, it becomes clear that more heat must be given out to the cold reservoir. However, that results in less energy available for useful work. Thus, we are faced with the dilemma and the choice appears to be between less available work and increase of entropy in the surroundings.

Example 6.1

A heat engine receives reversibly 500 kJ/cycle of heat from a source at 427°C, and rejects heat reversibly to a sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of $\delta Q/T$. From these results classify the case as irreversible, reversible and impossible: (a) 125 kJ/cycle rejected, (b) 250 kJ/cycle rejected, (c) 214.29 kJ/cycle rejected. ○○●

Solution Applying Clausius inequality for the cycle,

(a) $$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = \frac{500}{(427+273)} - \frac{125}{(27+273)} > 0 \text{ Impossible}$$

(b) $$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = \frac{500}{(427+273)} - \frac{250}{(27+273)} < 0 \text{ Irreversible but possible}$$

(c) $$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} = \frac{500}{(427+273)} - \frac{214.29}{(27+273)} = 0 \text{ Reversible and possible}$$

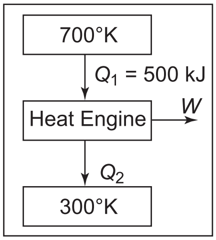


Figure 6.4

Example 6.2

A steam power plant operates between boiler temperature of 160°C and condenser temperature of 50°C. Water enters the boiler as saturated liquid and steam leaves the boiler as saturated vapour. Verify the Clausius inequality for the cycle. Given: Enthalpy of water entering boiler = 687 kJ/kg. Enthalpy of steam leaving boiler = 2,760 kJ/kg, Condenser pressure = 0.124 × 105 N/m^2. Assume isentropic expansion. ○●●

Solution Using steam tables, we get:

Boiler pressure as 6.18 bar along the 160°C line.

Enthalpy of vapour leaving the turbine, H_3 = 2,760 kJ/kg

Enthalpy of water leaving the condenser, H_{f4} = 209 kJ/kg along the 50°C line.

Calculating Q_{boiler},

$$Q_1 = H_2 - H_{f1} = 2{,}760 - 687 = 2{,}073 \text{ kJ/kg}$$

$Q_{\text{condenser}}$,

$$Q_2 = H_{f4} - H_3 = 209 - 2{,}160 = -1{,}951 \text{ kJ/kg}$$

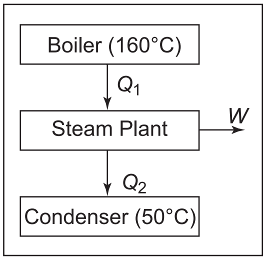


Figure 6.5

$$\sum_{\text{cycle}} \frac{\delta Q}{T} = \frac{Q_{\text{boiler}}}{T_{\text{boiler}}} + \frac{Q_{\text{condenser}}}{T_{\text{condenser}}} = \frac{2073}{433} - \frac{1951}{323} = 4.79 - 6.04 = -1.25 < 0 \text{ irreversible and so verified.}$$

Example 6.3

In a Carnot cycle, heat is supplied at 360°C and rejected at 27°C. The working fluid is water which, while receiving heat, evaporates from liquid at 350°C to steam at 350°C. The associated entropy change is 1.44 kJ/kg K. (a) If the cycle operates on a stationary mass of 1.1 kg of water, how much is the work done per cycle, and how much is the heat supplied? (b) If the cycle operates in steady flow with a power output of 25 kW, what is the steam flow rate?

●●●

Solution

(a) Change in entropy, $dS = \frac{\delta Q}{T}$

$$\delta Q = TdS = (360 + 273) \times 1.44 = 911.52 \frac{\text{kJ}}{\text{kg}}$$

$$Q = 1002.672 \text{ kJ}$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{633} = 0.526$$

$$\therefore \quad W = \eta Q = 0.526 \times 911.52 = 479.46 \text{ kJ/kg}$$

Total work = 479.46 × 1.1 = 527.41 kJ

(b) Output Power, $P = \dot{m}W$

$$\dot{m} = \frac{P}{W} = \frac{25}{527.41} = 0.474 \text{ kg/s}$$

6.3 ENTROPY CHANGE IN AN IRREVERSIBLE PROCESS

Entropy change in any process is calculated as a function of any other two state properties. The procedure is to start with the first law of thermodynamics and converting heat quantity to entropy and using the perfect gas law to get the desired variables.

University Questions

1. Determine the entropy change for an ideal gas undergoing a state change from p_1, V_1, T_1, S_1 to p_2, V_2, T_2, S_2. **(VTU, 2010–2011)**
2. Obtain the entropy change as a function of pressure and volume changes. **(RTU, 2010–11)**

6.3.1 Entropy Change for an Ideal Gas Undergoing a State Change

Consider an ideal gas undergoing a state change from p_1, V_1, T_1, S_1 to p_2, V_2, T_2, S_2 (Figures 6.6 a and b).

From the first law, $\delta Q = dU + \delta W$

$= c_V dT = pdV$, where c_V = heat capacity at constant volume

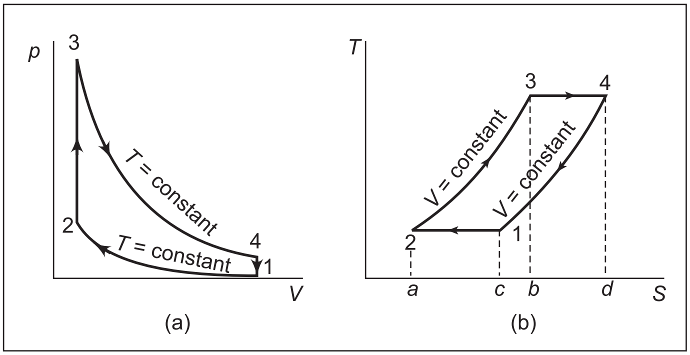


Figure 6.6 (a and b) *Ideal gas change, typical p-V and T-S diagrams*

$$\frac{\delta Q}{T} = \frac{c_V dT}{T} + \frac{pdV}{T} \rightarrow$$

$$dS = \frac{c_V dT}{T} + \frac{RdV}{V}$$

Integrating on either side between the two states,

$$S_2 - S_1 = c_V \ \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$$

6.3.2 Entropy Change as a Function of Pressure and Volume Changes

From Figure 6.6, the above equation is assumed.

$$\frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} \text{ from the gas law}$$

Substituting it in the above general equation,

$$S_2 - S_1 = c_V \ \ln\left(\frac{p_2 V_2}{p_1 V_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$$

$$= c_V \ \ln\left(\frac{p_2}{p_1}\right) + c_V \ \ln\left(\frac{V_2}{V_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$$

$$S_2 - S_1 = c_V \ \ln\left(\frac{p_2}{p_1}\right) + (c_V + R)\ \ln\left(\frac{V_2}{V_1}\right)$$

$$= c_V \ \ln\left(\frac{p_2}{p_1}\right) + c_p \ \ln\left(\frac{V_2}{V_1}\right)$$

Example 6.4

Obtain the entropy change as a function of pressure and temperature changes. ○●●

Solution From the gas law, $\frac{V_2}{V_1} = \frac{p_1 T_2}{p_2 T_1}$

Substitute in the general equation,

$$S_2 - S_1 = c_V \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right) = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

Example 6.5

10 m^3 of air is heated keeping its volume constant from 27°C to 82°C. Determine the change in entropy. Given $c_V = 0.718$. **(JNTU, 2014)** ○○●

Solution From general equation, $S_2 - S_1 = c_V \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$

$$S_2 - S_1 = c_V \ln\left(\frac{T_2}{T_1}\right) \text{ since the volume is constant.}$$

∴ entropy change, $S_2 - S_1 = 0.718 \ln\left(\frac{355}{300}\right) = 0.168 \text{ kJ/kg K}$

Example 6.6

An iron cube at a temperature of 400°C is dropped into an insulated bath containing 10 kg water at 25°C. The water finally reaches a temperature of 50°C at steady state. Given that the specific heat of water is equal to 4,186 J/kg K. Find the entropy changes for the iron cube and the water. Is the process reversible? If so why? ○●●

Solution Heat lost by iron cube = Heat gained by water

$$m_i c_i (673 - 323) = m_w c_w (323 - 298)$$

$$m_i c_i = \frac{10 \times 4.18 \times 25}{350} = 2.99 \text{ kJ/kg}$$

Entropy of iron at 673 K is $S_{i673} = m_i c_i \ln\left(\frac{673}{273}\right) = 2.99 \times (\ln 673 - \ln 273) = 2.698 \text{ kJ/kg K}$

Entropy of iron at 323 K is $S_{i323} = m_i c_i \ln\left(\frac{323}{273}\right) = 2.99 \times (\ln 323 - \ln 273) = 0.5028 \text{ kJ/kg K}$

Change in entropy for iron is 0.5028 – 2.698 = –2.1952 kJ/kg

Entropy of water at 323 K is $S_{w323} = m_w c_w \ \ln\left(\frac{323}{273}\right)$

$$= 10 \times 4.18 \times (\ln 323 - \ln 273) = 7.03 \text{ kJ/kg K}$$

Entropy of water at 298 K is $S_{w298} = m_w c_w \ \ln\left(\frac{298}{273}\right)$

$$S_{w298} = 10 \times 4.18 \times \ (\ln 298 - \ln 273) = 3.663 \text{ kJ/kg K}$$

Change in entropy for water is 7.03 – 3.663 = 3.367 kJ/kg

Net change in entropy is:

Change in entropy for water – change in entropy for iron

i.e., $3.367 - 2.1952 = 1.1718 \frac{\text{kJ}}{\text{kg}} > 0$ and hence, irreversible.

Example 6.7

A rigid cylinder containing 0.004 m^3 of nitrogen at 2 bar and 300 K is heated reversibly until temperature becomes 400 K. Determine the entropy change. Assume nitrogen to be perfect gas (molecular mass = 28) and take γ = 1.4. **(GTU, 2015)** ○●●

Solution Gas constant $R = \frac{\bar{R}}{M} = \frac{8.314}{28} = 0.297 \text{ kJ/kg K}$

(from Avogadro's law, $\bar{R}$ = 8.314 kJ/kg mol K)

Mass, $m = \frac{p_1 V_1}{R T_1} = \frac{2 \times 101.325 \times 0.004}{0.297 \times 300} = 0.0091 \text{ kg}$

$$c_V = \frac{R}{\gamma - 1} = \frac{0.297}{1.4 - 1} = 0.7425$$

$$S_2 - S_1 = m c_V \ \ln\left(\frac{T_2}{T_1}\right) = 0.0091 \times 0.7425 \times \ln\left(\frac{400}{300}\right)$$

Entropy change, $S_2 - S_1 = 0.00193 \text{ kJ/kg K}$

6.4 ENTROPY PRINCIPLE

University Question

1. What is entropy principle? **(RTU, SU, 2015)**

It is observed that any process inherently increases the entropy level. An irreversible process proceeds in such a direction that the entropy level is maximum. The increase in entropy is at the cost of available useful work. The greater the work extracted, greater is the increase in entropy of the surroundings and hence in the universe.

Entropy is associated with probability. It indicates the likelihood of the description of a system through its order or the lack of it. Thus, given the condition of a system, the description of the system is best given by a probability distribution with the highest entropy. Maximum entropy principle is used in both discrete and continuous models. Entropy principle is also used extensively in information theory.

The entropy of the universe always increases. This is known as principle of entropy increase. During a process the system may lose entropy. In such a case the surroundings will have increased entropy and the net change is always positive. This means the entropy of the universe which is the sum of the entropy changes in the system and surroundings will always be positive. Only theoretically for a reversible process it is taken as zero (Figure 6.7).

As shown in the diagram, let a system change from state 1 to state 2 along some arbitrary process. Let it come back to the original state along a reversible path completing the cycle. Then, from Clausius theorem,

$$\int \frac{\delta Q}{T} = \int_1^2 \frac{\delta Q}{T} + \int_2^1 \frac{\delta Q}{T} \le 0$$

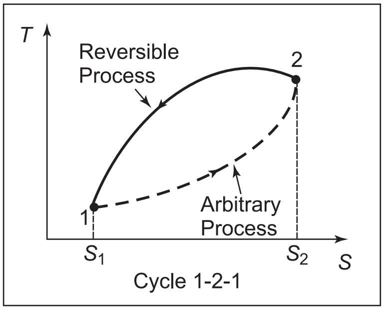


Figure 6.7 *Demonstration of entropy principle*

Now, 2-1 is a reversible path. Therefore,

$$\int_2^1 \frac{\delta Q}{T} = S_1 - S_2$$

$$\therefore \qquad \int_1^2 \frac{\delta Q}{T} + (S_1 - S_2) \le 0$$

$$\int_1^2 \frac{\delta Q}{T} \le \Delta S \rightarrow \Delta S \ge \int_1^2 \frac{\delta Q}{T}$$

Therefore, the change in entropy is always positive with the minimum value of zero when the process is reversible. This is known as principle of increase of entropy.

Example 6.8

Calculate the entropy change of the universe because of the following processes: (a) A copper block of 800 g mass and with Cp of 150 J/K at 120°C is placed in a lake at 6°C; (b) The same block is dropped from a height of 100 m into the lake; (c) Two such blocks, at 120°C and 0°C, are joined together. **(GATE, GTU, PU) ●●●**

Solution

(a) Change in entropy for copper is:

$$(\Delta S)_{cop} = \int_{393}^{279} \frac{C_p dT}{T}$$

$$\therefore \qquad (\Delta S)_{cop} = 150 \ln\left(\frac{279}{393}\right) = -51.39 \text{ J/K}$$

Change in entropy for water is:

$$(\Delta S)_{lake} = \frac{C_p \Delta T}{T}$$

Substituting, $(\Delta S)_{lake} = \dfrac{150(120-6)}{279} = 61.29 \text{ J/K}$

Total change in entropy is:

$$(\Delta S)_{uni} = (\Delta S)_{cop} + (\Delta S)_{lake} = -51.39 + 61.29 = 9.9 \text{ J/K}$$

(b) Work done when it touches water is:

$$mgh = 0.8 \times 9.81 \times 100 = 784.8 \text{ J}$$

This work is dissipated from copper making (ΔS) copper = 0

This work gets converted to heat and is absorbed by water. Change in entropy for the lake is

$$(\Delta S)_{lake} = \frac{Q}{T} = \frac{W}{T} = \frac{784.8}{279} = 2.813 \text{ J/K}$$

$$(\Delta S)_{uni} = (\Delta S)_{cop} + (\Delta S)_{lake} = 0 + 2.813 = 2.813 \text{ J/K}$$

(c) Final temperature is given by:

$$T_f = \frac{T_{b1} + T_{b2}}{2} = \frac{120+0}{2} = 60^\circ\text{C}$$

$$(\Delta S)_{b1} = \int_{393}^{333} \frac{mc_{pdT}}{T} = \int_{393}^{333} \frac{C_p dT}{T} = 150 \ln\left(\frac{333}{393}\right) = -24.85 \text{ J/K}$$

$$(\Delta S)_{b2} = \int_{273}^{333} \frac{mc_{pdT}}{T} = \int_{273}^{333} \frac{C_p dT}{T} = 150 \ln\left(\frac{333}{273}\right) = 29.801 \text{ J/K}$$

$$(\Delta S)_{uni} = (\Delta S)_{b1} + (\Delta S)_{b2} = 4.951 \text{ J/K}$$

Example 6.9

1.4 m^3 of oxygen is heated reversibly at constant pressure from 300 K to 600 K, and is then cooled reversibly at constant volume back to initial temperature. If the initial pressure is 1 bar, calculate: (a) The net heat flow, (b) The overall change in entropy. Take specific heat at constant pressure as 0.9167 kJ/kgK. **(GTU, 2014)** ●●●

Solution $R = \dfrac{\bar{R}}{M} = \dfrac{8.134}{32} = 0.254 \text{ kJ/kg K}$

$$m = \frac{p_1V_1}{RT_1} = \frac{101.325 \times 1.4}{0.254 \times 300} = 1.86 \text{ kg}$$

$$Q = Q_{12} + Q_{23} = mc_p(T_2 - T_1) + mc_V(T_1 - T_2)$$

$$= m(c_p - c_V)(T_2 - T_1) = 1.86 \times 0.254 \times (600 - 300) = 141.732 \text{ kJ}$$

During constant pressure process,

$$S_2 - S_1 = mc_p \ln\left(\frac{T_2}{T_1}\right) = 1.86 \times 0.9167 \times \ln\left(\frac{600}{300}\right) = 1.182 \text{ kJ/K}$$

During constant volume process,

$$S_1 - S_2 = mc_V \ln\left(\frac{T_1}{T_2}\right) = 1.86 \times (0.9167 - 0.254) \times \ln\left(\frac{300}{600}\right) = -0.854 \text{ kJ/K}$$

Total entropy change = 1.182 – 0.854 = 0.328 kJ/K

Example 6.10

20 kg of water at 50°C is mixed with 30 kg of water at 80°C. Determine the change in entropy.

(VTU, 2012–2013) ○●●

Solution The energy balance gives $m_1c_p(T_1 - T) = m_2c_p(T - T_2)$

$$20 \times 4.18\,(T - 323) = 30 \times 4.18 \times (353 - T) \rightarrow T = 341 \text{ K}$$

$$\Delta S = m_1c_p \ln\left(\frac{T}{T_1}\right) + m_2c_p \ln\left(\frac{T}{T_2}\right) = 20 \times 4.18 \times \ln\left(\frac{341}{323}\right) + 30 \times 4.18 \times \ln\left(\frac{341}{353}\right) = 0.197 \frac{\text{kJ}}{\text{kg K}}$$

6.4.1 Application of Entropy Principle

The entropy principle can be applied to a wide variety of situations in physics, engineering and beyond. The fact that entropy can be expressed as a probability distribution, allows us to apply the principle wherever disorder is to be measured and quantified. This applies from a kitchen to a baby room to space stations. However, the analysis of any such situation is beyond the scope of the present article.

Example 6.11

Each of three identical bodies satisfies the equation $U = CT$, where C is the heat capacity of each of the bodies. Their initial temperatures are 280 K, 220 K, and 600 K. If C = 10.2 kJ/K, what is the maximum amount of work that can be extracted in a process in which these bodies are brought to a final common temperature? ○●●

Solution $Q = Q_1 + Q_2$ and $W = W_1 + W_2$

Let T_f be the final temperature. Then,

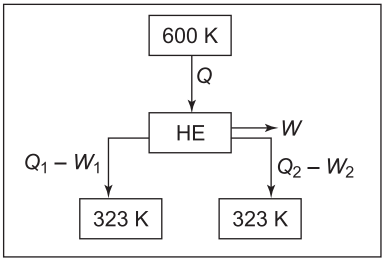


Figure 6.8

$$(\Delta S)_{b1} = C \ln \frac{T_f}{600}$$

$$(\Delta S)_{b2} = C \ln \frac{T_f}{280}$$

$$(\Delta S)_{b3} = C \ln \frac{T_f}{220}$$

$$(\Delta S)_{sur} = 0 = (\Delta S)_{HE}$$

$$(\Delta S)_{uni} = (\Delta S)_{b1} + (\Delta S)_{b2} + (\Delta S)_{b3} + (\Delta S)_{HE} + (\Delta S)_{sur} > 0$$

$$\therefore \qquad C \ln \frac{T_f}{600} + C \ln \frac{T_f}{280} + C \ln \frac{T_f}{220} \geq 0$$

$$T_f = 333.1 \text{ K}$$

$$Q = C\Delta T = 10.2 \times (600 - 333.1) = 2722.38 \text{ kJ}$$

$$Q_1 - W_1 = C\Delta T = 10.2 \times (333.1 - 280) = 541.62 \text{ kJ}$$

$$Q_2 - W_2 = C\Delta T = 10.2 \times (333.1 - 220) = 1153.62 \text{ kJ}$$

$$\therefore \qquad Q_1 + Q_2 - (W_1 + W_2) = 541.62 + 1153.62 = 1695.24 \text{ kJ}$$

$$\therefore \qquad (W_1 + W_2) = Q - 1695.24 = 2722.38 - 1695.24 = 1027.14 \text{ kJ}$$

Example 6.12

A reversible heat engine is in interaction with three reservoirs. The reservoirs are maintained at 480 K, 400 K and 320 K respectively. The engine receives 960 kJ from the source at 480 K while doing work equal to 360 kJ. Determine the nature of interaction with the other two reservoirs. ○●●

Solution Draw the diagram as indicated. Directions of the other two interactions are assumptions only.

From the first law of thermodynamics, the energy balance gives:

$$960 = 360 + Q_2 + Q_3$$

$$Q_2 + Q_3 = 600$$

From the second law of thermodynamics, the entropy balance gives:

$$\frac{960}{480} - \frac{Q_2}{400} - \frac{Q_3}{320} = 0$$

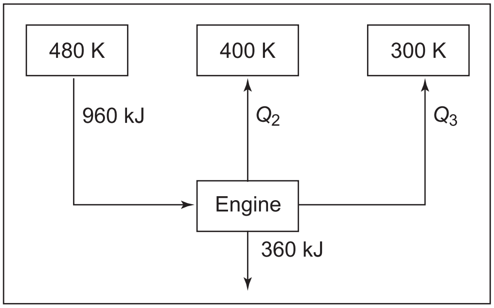


Figure 6.9

$$\frac{Q_2}{400} + \frac{Q_3}{320} = 2$$

Solving,

$$Q_2 = -200 \text{ kJ and } Q_3 = 800 \text{ kJ}.$$

Therefore, the direction of heat interaction with the second reservoir needs to be changed. Thus, the engine absorbs 200 kJ from the 400 K reservoir and rejects 800 kJ to the 320 K reservoir.

Example 6.13

In a foundry, during quenching, a metal weighing 12 kg and experiencing 600°C is suddenly placed in 90 kg of oil at 30°C. If the specific heats of the metal and oil are respectively, 0.6 kJ/kgK and 3.4 kJ/kgK, determine the increase in entropy level. **(GTU, 2014–2015)** ○●●

Solution Making the energy balance, heat lost by the metal = heat gained by oil

$$(mc_p(T_i - T_f))_{\text{metal}} = (mc_p(T_f - T_i))_{\text{oil}}$$

where, T_f is the final temperature.

Substituting the given values,

$$12 \times 0.6 \times (600 - T_f) = 90 \times 3.4 \times (T_f - 30)$$

$$\rightarrow \quad T_f = 316.1 \text{ K}$$

For the metal,

$$\Delta S = mc_p \ln\left(\frac{T_f}{T_i}\right) = 12 \times 0.6 \times \ln\left(\frac{316.1}{873}\right) = -7.314 \text{ kJ/kg K}$$

For the oil,

$$\Delta S = mc_p \ln\left(\frac{T_f}{T_i}\right) = 90 \times 3.4 \times \ln\left(\frac{316.1}{303}\right) = 12.952 \text{ kJ/kg K}$$

$$\Delta S_{\text{uni}} = \Delta S_{\text{metal}} + \Delta S_{\text{oil}} = -7.314 + 12.952 = 5.638 \text{ kJ/kg K}$$

6.6 INTRODUCTION TO THE THIRD LAW OF THERMODYNAMICS

University Question

1. Write briefly about the third law of thermodynamics. **(GTU, 2011–2012)**

The Third Law of Thermodynamics is concerned with the limiting behaviour of systems as the temperature approaches absolute zero. For any real process, we speak only of the entropy differences simply because entropy level cannot be initialised. At absolute zero, all motion will cease and hence entropy tends to zero as no disorder can be observed. This is when zero entropy can be attempted. The third law states, “The entropy of a perfect crystal is zero when the temperature of the crystal is equal to absolute zero (0 K).

It is important to note that the crystal has to be perfect and the temperature must be absolute zero. Absolute zero refers to 0K which is –273.15°C or –459.6°F. It is next to impossible to obtain a perfect crystal. There will always be some kind of defect.

However, in reality, nothing can attain such a temperature as is evident from the second law of thermodynamics. The law implies that it is not possible to transfer heat from a colder body to a hot body on its own. Thus, in order to reach absolute zero, it needs to draw heat energy from neighbouring objects. This is where the difficulty lies—if it draws it cannot reach; but to reach it has to draw. Therefore, in essence, it acts as a mathematical limit for temperature.

6.7 ENTROPY GENERATION IN CLOSED AND OPEN SYSTEMS

Probe

1. Write a note on entropy generation.

An irreversible process always generates entropy. Conversely, entropy generation means an irreversible process. For closed system, during a reversible process, $\Delta S = \dfrac{\delta Q_{\text{rev}}}{T}$ and during an irreversible process, it is given by $\Delta S = \dfrac{\delta Q}{T} + S_{\text{gen}}$. This additional term indicates that entropy is generated due to factors like internal friction, flow which could either be laminar or turbulent, magnetic reversals or any type of gradient.

Therefore, it is better to include that as:

$$\Delta S = \frac{\delta Q_{\text{rev}}}{T} + S_{\text{gen(internal friction)}} + S_{\text{gen(internal heat transfer)}}$$

For an isolated system or an adiabatic system,

$$\Delta S = S_{\text{gen(internal friction)}} + S_{\text{gen(internal heat transfer)}}$$

Thus, in terms of entropy, the second law takes any of the three forms depending upon the type of the system and the heat transfer taking place which are summarised in Table 6.1.

Table 6.1 *System and entropy expression*

System	Entropy
Isolated	S_{gen}
Open	$\int \frac{\delta Q}{T} + (m_{\text{in}} s_{\text{in}} - m_{\text{out}} s_{\text{out}}) + S_{\text{gen}}$
Closed	$\int \frac{\delta Q}{T} + S_{\text{gen}}$

POINTS TO REMEMBER

- Clausius inequality is a quantitative form of second law of thermodynamics. It holds good for any cycle.
- For a perfectly reversible or isolated system, the cyclic integral of entropy is zero, otherwise, it is a negative quantity.
- Entropy change can be shown to be a function of any two properties of the system. Therefore, entropy itself becomes a property.
- Entropy change measured is relative to the entropy level at the first state.
- Entropy of a perfect crystal is zero at absolute zero, otherwise it is only a probability.
- Entropy is always generated internally and externally.

PRACTICE PROBLEMS

1. A heat engine receives reversibly 300 kJ/cycle of heat from a source at 297°C, and rejects heat reversibly to a sink at 27°C. There are no other heat transfers. For each of the three hypothetical amounts of heat rejected, in (a), (b), and (c) below, compute the cyclic integral of $\delta Q/T$. From these results classify the case as irreversible, reversible and impossible: (a) 240 kJ/cycle rejected, (b) 150 kJ/cycle rejected, (c) 65 kJ/cycle rejected. ○○●

(**Ans:** (a) Irreversible but possible, (b) Reversible and possible, (c) Impossible)

2. In a power plant cycle, the temperature range is 164°C to 51°C, the upper temperature being maintained in the boiler where heat is received and the lower temperature being maintained in the condenser where heat is rejected. All other processes in the steady flow cycle are adiabatic. The specific enthalpies at various points are as follows. The enthalpy levels before and after boiler are 690 kJ/kg and 2,760 kJ/kg respectively. The corresponding enthalpies before and after the condenser are 2,360 kJ/kg and 450 kJ/kg respectively. Verify the Clausius Inequality. ○●●

3. 1.2 kg of water at 20°C is converted into ice at –14°C at constant atmospheric pressure. Assuming the specific heat of liquid water to remain constant at 4.18 kJ/kg K and that of ice to be half of this value, and taking the latent heat of fusion of ice at 0°C to be 335 kJ/kg, calculate the total entropy change of the system. **(GTU, 2015)** ○●●

(**Ans:** 0.9954 kJ/kgK)

4. A system maintained at constant volume is initially at temperature, T_1, and a heat reservoir at the lower temperature, T_0 is available. Show that the maximum work recoverable as the system is cooled to T_0 is: ○●●

$$W = c_v \left[(T_1 - T_0) - T_0 \ln\left(\frac{T_1}{T_0}\right) \right]$$

(GTU, 2013–2014)

5. Two bodies of equal heat capacities, C, and temperatures, T_1 and T_2, form an adiabatically closed system. What will the final temperature be if one lets this system come to equilibrium: (a) freely? (b) reversibly? ○●●

6. An ideal gas is heated from temperature, T_1 to T_2, by keeping its volume constant. The gas is expanded back to its initial temperature according to the law pV^n = constant. If the entropy change in the two processes is equal, find the value of n in terms of the adiabatic index γ. ○○●

(GATE, 1996; UPSC, 1997)

7. An automobile research company claims that they have developed an engine which can absorb heat energy equivalent to 275 kJ from a reservoir at 54°C and does work equal to 76 kJ. Further, the engine is also supposed to reject 100 kJ and 99 kJ with two other reservoirs at 30°C and 50°C respectively. Verify the claim. ○●●

8. In a piston cylinder system, under constant pressure, ten moles of methane are held between temperatures of 300 K and 400 K. If the molar heat capacity of methane, in units of J/mol K, is given by $c_p = 17.5 + 60.5 \times 10^{-3}T + 1.117 \times 10^{-6}T^2$, determine the change in entropy. ○○●

(**Ans:** 111.235 J/K)

MULTIPLE CHOICE QUESTIONS

1. Entropy is: ○○●
(a) A process (b) A property (c) A system (d) Temperature

2. The concept of entropy is derived from: ○○●
(a) Zeroth Law (b) First Law (c) Second Law (d) Third Law

3. Change in entropy for a reversible process is: ○●●
(a) Zero (b) Infinite (c) Greater than zero (d) Less than zero

4. What are the important characteristics of entropy? ●●●
(a) If the heat is supplied to the system then the entropy will increase
(b) If the heat is rejected from the system then the entropy will decrease
(c) The entropy is constant for all adiabatic frictionless process
(d) All of the above

5. For a reversible cycle, entropy change: ○●●
(a) Is greater than zero (b) Is less than zero
(c) Increases at first and then decreases (d) Is equal to zero

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (b) 2. (c) 3. (a) 4. (d) 5. (d)

Available Energy, Availability and Irreversibility

7

CHAPTER OUTLINE

- Available and Unavailable Energy
- Availability, Irreversibility and Gouy-Stodola Theorem
- Useful Work
- Second Law Efficiency
- Exergy

INTRODUCTION

First law of thermodynamics provides an important relation of energy transformation. But, it is not possible always for a system to operate at maximum efficiency. This is why the energy conversion alone is not adequate to depict important aspects of energy conversion. The chapter deals with thermodynamic limits of the performance of a system operating in specified end states. Based on second law of thermodynamics, concepts of irreversibility, useful work, exergy are introduced to determine the system performance and overall efficiency more exactly in real world of application.

7.1 AVAILABLE AND UNAVAILABLE ENERGY

Sources of energy can be classified into two categories as follows:

1. ***High grade energy:*** The form of energy like mechanical work, electrical work, wind power, etc., can be fully converted into useful work or into other form of energy without any loss. This form of energy is known as high-grade energy.
2. ***Low grade energy:*** For energy derived from fossil fuels, nuclear fuels, heat, etc., only a certain portion of the total energy can be extracted or converted into useful work. This type of energy is called low-grade energy.

University Question

1. What do you understand by the concept of available work and unavailable work.

(AKTU, 2011–2012)

It can be understood by a simple example that it is possible to generate 10 kJ of heat by operating an electric heater of 1 kW for 10 seconds (in steady state), while on reversal, requirement of heat to generate 10 kW electrical power for 10 seconds will be much more than 10 kJ.

7.1.1 Available Energy and Unavailable Energy

As discussed in previous section, with a low-grade energy it is not possible to convert all of it into useful work, the maximum amount of the energy that can be converted completely into useful work under ideal conditions is known as *available energy.*

The minimum amount of energy which is rejected (discarded to ambient atmosphere) as waste is called *unavailable energy*. Here, it is important to note that the work output is maximum if the process is carried out reversibly.

For a total Q amount of heat supplied at T_1 temperature with ambient temperature as T_0, the maximum amount of work, W, i.e. available energy (AE) can be evaluated by operating a Carnot engine having source and sink temperatures equal to T_1 and T_0 respectively (Figure 7.1).

$$W = Q\left(1 - \frac{T_0}{T_1}\right),$$

$$W = Q\left(1 - \frac{T_0}{T_1}\right) = Q - T_0\frac{Q}{T_1} = Q - T_0\Delta S$$

Here, ΔS is the entropy of the body supplying the heat.

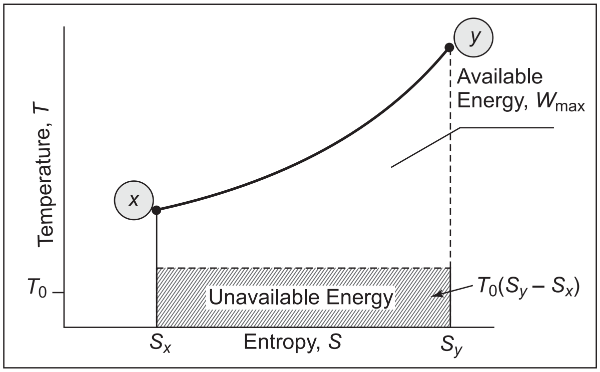


Figure 7.1 *Availability of energy*

7.1.2 Available Energy for a Finite Process

Suppose a finite process a-b where heat is supplied reversibly to the heat engine. In order to calculate the available energy, an elemental cycle with constant temperature T is taken. If heat supplied for this elemental cycle is dQ, then maximum elemental work:

$$dW = dQ\left(1 - \frac{T_0}{T}\right)$$

Total maximum work output is,

$$W_{a-b} = \int_{Ta}^{Tb}\left(1 - \frac{T_0}{T}\right)dQ$$

$$W_{a-b} = \int_{Ta}^{Tb} dQ - T_0\int_{Ta}^{Tb}\frac{dQ}{T} \Rightarrow Q - T_0\int_{a}^{b} dS$$

$$W_{a-b} = Q - T_0(S_b - S_a)$$

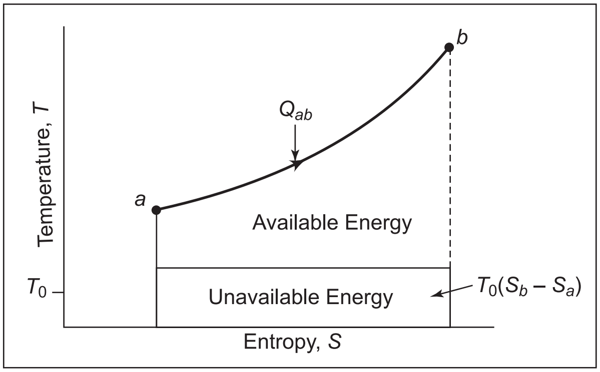


Figure 7.2 *Available energy for a finite process*

7.1.3 Loss of Available Energy due to Heat Transfer through a Finite Temperature Difference

If there is a finite temperature difference between two interacting bodies, the process becomes irreversible and this irreversibility leads to the loss in available energy. Suppose Q amount of heat is being transferred from a body at temperature, T_1, to another body at temperature, T_2. If ambient temperature is T_0 and $T_2 < T_1$,

Initial available energy with body at temperature, $T_1 = Q\left(1 - \frac{T_0}{T_1}\right)$

Final available energy with body at temperature, $T_1 = Q\left(1 - \frac{T_0}{T_2}\right)$

Loss in available energy =

$$Q\left(1 - \frac{T_0}{T_1}\right) - Q\left(1 - \frac{T_0}{T_2}\right) = T_0\left(\frac{T_0}{T_2} - \frac{T_0}{T_1}\right) = T_0\ \Delta S_{\text{universe}}$$

Here, it can be seen that even with same amount of energy available as heat, i.e., Q, available energy is less at, T_2, i.e., lower temperature compared to T_1.

Example 7.1

With keeping source and sink temperature constant at 527°C and 27°C respectively, 5,000 kJ heat is transferred from source to sink. Find out availability and unavailability for the reversible process. ○○●

Solution Given,

$T_0 = 273 + 27 = 300$ K;

$T_1 = 273 + 527 = 800$ K and

$$Q = 5{,}000 \text{ kJ}$$

$$\text{Available energy, } W = Q\left(1 - \frac{T_0}{T_1}\right) = 5{,}000\left(1 - \frac{300}{800}\right) = 3{,}125 \text{ kJ}$$

Unavailable or lost energy = 5,000 – 3,125 = 1,875 kJ

7.2 AVAILABILITY, IRREVERSIBILITY AND GOUY-STODOLA THEOREM

University Questions

1. What do you understand by the concept of availability, irreversibility. **(AKTU, 2011–2012)**
2. What is irreversibility? State its types and causes. **(MU, 2014)**

7.2.1 Availability

Availability is a property, which quantifies how much energy in a system or in a flow stream is potentially available to produce useful work. In this section, expressions of availability for both closed and open system are developed.

For a system, work obtained is maximum if at the end of the process, the system has reached the state where no possibility remains to obtain further work, i.e., system has reached a *dead state* (a system is said to be in the dead state when it is in thermodynamic equilibrium with the environment it is in). For example, an air engine with compressed air inside it (delivered from cylinder) will continue to work till the air pressure inside the cylinder becomes equal to that of the surroundings.

Availability, a thermodynamics property of a system, is defined as maximum theoretical work obtainable as system interacts with environment till it is does not attain dead state. Clearly, the availability of a system depends on the system condition as well as condition of the surroundings.

Availability for Non-flow Process

For a closed system, let p_0 and T_0 be the ambient pressure and temperature respectively. Since system reaches a dead state at the end, its final pressure will be p_0. If initial and final volumes of the system are V_1 and V_0 respectively, change in pressure is $V_0 - V_1$.

Work done for this volume change against the pressure p_0 (to push back the atmosphere) = $p_0(V_0 - V_1)$.

So, availability = Maximum available work = $W_{\max} - P_0\,(V_0 - V_1)$ (7.1)

Recalling from the first law,

$$dW = dQ - dU$$

$$dW = TdS - dU$$

$$W_{\max} = T_0(S_0 - S_1) - (U_0 - U_1)$$

Substituting these expression in equation (7.1) for availability,

$$\text{Availability} = T_0(S_0 - S_1) - (U_0 - U_1) - p_0(V_0 - V_1)$$

Rearranging the terms yields,

$$\text{Availability} = (U_1 + p_0\,V_1 - T_0\,S_1) - (U_0 + p_0\,V_0 - T_0\,S_0) = \phi_1 - \phi_0$$

where ϕ is called availability function, given as: $\phi = U + p_0\,V - T_0\,S$

Note: If a system undergoes a change of state from state 1 (where availability is $\phi_1 - \phi_0$) to the state 2 (where availability is $\phi_2 - \phi_0$), the change in availability will be $\phi_1 - \phi_2$.

Availability for Steady-flow Process

If outlet and inlet quantities of the control volume are denoted by subscript 1 and 2, the energy balance for steady flow can be given as,

$$Q - W_{\max} = \left(H_1 + \frac{1}{2}mV_1^2 + mgZ_1\right) - \left(H_2 + \frac{1}{2}mV_2^2 + mgZ_2\right) \tag{7.2}$$

For reversible process, according to second law,

$$\Delta S_{\text{universe}} = 0$$

$$(S_1 - S_2) - \frac{Q}{T_0} = 0 \Rightarrow Q = T_0(S_1 - S_2)$$

Substituting the value of Q in equation 7.2,

$$W_{\max} = T_0(S_1 - S_2) - \left(H_1 + \frac{1}{2}mV_1^2 + mgZ_1\right) - \left(H_2 + \frac{1}{2}mV_2^2 + mgZ_2\right)$$

$$\Rightarrow \qquad W_{\max} = \left(H_2 - T_0S_2 + \frac{1}{2}mV_2^2 + mgZ_2\right) - \left(H_1 - T_0S_1 + \frac{1}{2}mV_1^2 + mgZ_1\right)$$

For a steady flow, since there is no work done against the environment,

$$\text{Availibility} = W_{\max} = \left(H_2 - T_0S_2 + \frac{1}{2}mV_2^2 + mgZ_2\right) - \left(H_1 - T_0S_1 + \frac{1}{2}mV_1^2 + mgZ_1\right)$$

The expression $H - T_0S$ is known as Keenan function (B).

So, $\text{Availibility} = \left(B_2 + \frac{1}{2}mV_2^2 + mgZ_2\right) - \left(B_1 + \frac{1}{2}mV_1^2 + mgZ_1\right)$

7.2.2 Irreversibility

It has been known now that actual work done is always lesser than the reversible work. The difference between the two is called irreversibility. If *W* is the maximum useful work, invariably the actual work will be less than *W*. This is also termed as degradation.

Irreversibility (*I*) = loss

So, Irreversibility = Maximum useful work – Actual work = $T_0(\Delta S)$universe

There are two reasons of irreversibility:

1. Lack of thermodynamic equilibrium during the process.
2. Involvement of dissipative effect during the process.

Types of Irreversibility

Irreversibility is classified according to its cause. Different types of irreversibilities are:

1. ***External:*** External irreversibility is caused by external physical factors like friction, resistance, viscosity, surface tension, finite temperature difference, etc. Since the energy lost due to a process can never be regained, the direction of the process cannot be reversed without supplying an external work.
2. ***Internal:*** Internal irreversibility occurs due to properties of the working fluid in a process like throttling or free expansion. When a gas expands, work is done by the internal energy. The gas contracts on its own and reverses the process.
3. ***Chemical:*** Irreversibility caused by internal chemical properties of the constituents like atomic structure, bonds, etc., is known as chemical irreversibility. If a chemical reaction occurs in association with consumption or liberation of heat, it cannot be reversed spontaneously.

Calculation for Entropy Change ($S_2 - S_1$)

I. For a closed system

$$TdS = dU + pdV$$

Since, $dU = mc_V dT$ and $pV = mRT$

$$dS = mc_V \frac{dT}{T} + mR\frac{dV}{V}$$

$$S_2 - S_1 = \int_{T_1}^{T_2} mc_V \frac{dT}{T} + \int_{V_1}^{V_2} mR\frac{dV}{V}$$

$$S_2 - S_1 = \left(mc_V \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right)\right)$$

II. For a steady flow

$$TdS = dH - Vdp$$

Since, $dH = mc_p dT$ and $pV = mRT$

$$dS = mc_p \frac{dT}{T} - mR\frac{dp}{p}$$

$$S_2 - S_1 = \int_{T_1}^{T_2} mc_p \frac{dT}{T} - \int_{p_1}^{p_2} mR\frac{dp}{p}$$

$$S_2 - S_1 = \left(mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{p_2}{p_1}\right)\right)$$

7.2.3 Gouy-Stodola Theorem

The theorem gives an important relation for the lost work. According to this theorem, rate of irreversibility is given as:

$$W_{\text{lost}} = I = W_{\max} - W$$

I. For non-flow process: If kinetic and potential energies of the control volume are neglected,

$$I = (\Delta U - T_0 \Delta S) - (\Delta U + Q)$$

$$I = T_0 \Delta S - Q = T_0 (\Delta S)_{\text{Surrounding}}$$

II. For steady-flow process: Assuming a single inlet and single outlet, inlet and outlet availability balance can be written as:

$$I = W_{\max} - W = \left[\left(B_1 + \frac{1}{2} m V_1^2 + mgZ_1\right) - \left(B_2 + \frac{1}{2} m V_2^2 + mgZ_2\right)\right]$$
$$- \left[\left(H_1 + \frac{1}{2} m V_1^2 + mgZ_1\right) - \left(H_2 + \frac{1}{2} m V_2^2 + mgZ_2\right)\right]$$

$$I = I = T_0 \Delta S - Q = T_0 (\Delta S)_{\text{Surrounding}}$$

Hence, rate of irreversibility, i.e., rate of loss of available energy is proportional to rate of entropy generation.

Example 7.2

A 5 ton metal mass is at initial temperature 1,500 K. It is left to cool down to 500 K and heat released is used as source of energy. If ambient temperature is 300 K and specific heat of the metal is 0.5kJ/kg K, find out the available energy. ○●●

Solution Given,

Mass of the metal, $m = 5{,}000$ kg

Specific heat of the metal, $c_p = 0.5$ kJ/kgK

Temperature difference between initial and final state, $\Delta T = 1{,}500 - 500 = 1{,}000$ K; $T_0 = 300$ K

Assumption: Specific heat is constant.

Since heat is released at constant atmosphere pressure,

$$Q = mc_p \Delta T = 5{,}000 \times 0.5 \times 1{,}000 = 2{,}500 \text{ MJ}$$

Since, $\Delta S = T_S \left(mc_p \ln\left(\frac{T_f}{T_{in}}\right) - mR \ln\left(\frac{p_2}{p_1}\right)\right)$ and here, $p_1 = p_2$

So,

$$\Delta S = mc_p \ln\left(\frac{T_f}{T_{in}}\right) = 2{,}746.53$$

Available energy, $AE = Q - T_0\Delta S = 25{,}00{,}000 - 300 \times 2{,}746.53 = 1{,}676.2$ MJ

Example 7.3

Calculate the irreversibility, if one kg of air is compressed polytropically from 0.8 bar pressure and 310 K to 3.2 bar and 350 K. Heat sink temperature is 300 K. Take R = 0.287 k K/kg K. c_p = 1.004 KJ/kg K and c_V = 0.716 kJ/kg K. ○●●

Solution Given,

Initial pressure and temperature: 0.8 bar, 310 K
Final pressure and temperature: 3.2 bar, 350 K
Temperature of heat sink: 300 K
Specific heats: c_p = 1.004 KJ/kg K and c_V = 0.716 kJ/kg K

Assumption: Air is ideal with constant specific heat.

Step 1: Find out work done

$$W = Q - \Delta U = Q - (U_2 - U_1) = T_S(S_2 - S_1) - mc_V(T_1 - T_2)$$

$$W = T_S\left(mc_p \ln\left(\frac{T_f}{T_{in}}\right) - mR\ln\left(\frac{p_2}{p_1}\right)\right) - mc_V(T_1 - T_2)$$

$$= 300\,(1 \times 1.004 \times \ln(350/310) - 1 \times 0.287 \times \ln(3.2/0.8)) - 1 \times 0.716 \times (350 - 310)$$

$$= -111.67 \text{ KJ}$$

Step 2: Find out *n*

For polytropic process, $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$

Solving it, $n = 1.09$

Step 3: Find out actual work and irreversibility

$$W_{\text{actual}} = \frac{mR(T_1 - T_2)}{(n-1)} = \frac{1 \times 0.287(310 - 350)}{(1.09 - 1)} = -127.55 \text{ KJ}$$

$$I = W_m - W_{\text{actual}} = -111.67 - (-127.55) = 15.886 \text{ KJ}$$

Example 7.4

A house is maintained at 27°C temperature by resistance heaters. Rate of heat transfer (from the house to the ambient) is 60,000 kJ/h. If ambient temperature is 7°C, find out the reversible work in the process and irreversibility. ○●●

Solution Given,

Rate of heat transfer, $Q' = 60{,}000$ kJ/h $= 60{,}000/3{,}600 = 16.67$ kW

Room temperature $T_r = 27°\text{C}$

Ambient temperature $T_0 = 7°\text{C}$

Analysis:

Actual power input =power supplied by the heater = $W' = Q' = 16.67$ kW

COP of a reversible heat pump operating between the specified temperature limit,

$$\text{COP rev} = \frac{1}{1-\dfrac{T_0}{T_r}} = \frac{1}{1-\dfrac{280}{300}} = 15$$

So, rate of reversible work, $W'_{\text{rev}} = Q'/\text{COP rev} = 16.67/15 = 1.11$ kW

Rate of irreversibility, $I' = W' - W'_{\text{rev}} = 16.67 - 1.11 = 15.56$ kW

7.3 || USEFUL WORK

We know, from the first law of thermodynamics that actual work done during a process is $W = Q\,(1 - T/T_{\text{sink}})$. Now, if during the process, the volume of the system gets increased. The total work done (W) will have two components (Figure 7.3):

1. Work done against the pressure of the system to increase its volume:

$$W_1 = \int_{V_1}^{V_2} p\,dV$$

2. Work done against the atmospheric pressure to push back the surrounding medium.

$$W_2 = p_{\text{atm}}(V_2 - V_1)$$

Here, useful work is the only one which is done on the system, i.e. W_1.

If surrounding boundaries are fixed than $W_2 = P_{\text{atm}}(V_2 - V_1) = 0$.

Also, if process is cyclic or a steady flow process, $W_2 = 0$.

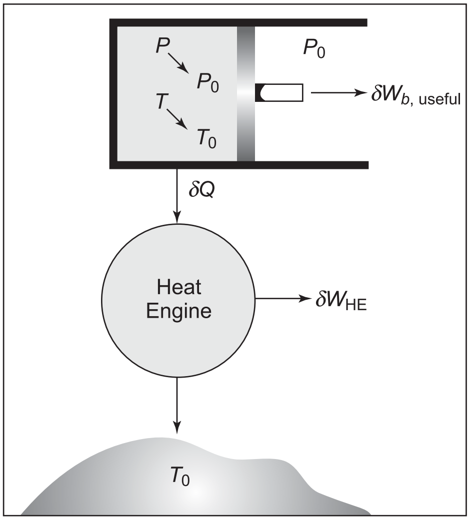


Figure 7.3 *Work done against surrounding and useful work*

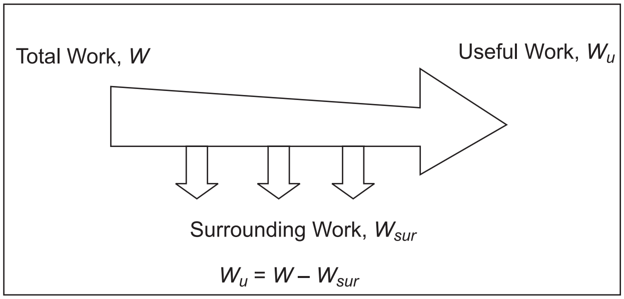


Figure 7.4 *The work flow diagram*

Example 7.5

If pressure and temperature for an ideal gas are during a certain process changed from 9 bar, 400 K to 1.5 bar, and 300 K. Find out work done (per kg) on the surrounding during the process. ○●●

Solution Given,

Initial temperature and pressure: 400 K, 9 bar
Final temperature and pressure: 300 K, 1.5 bar

Assumptions: Process involves no internal irreversibilities such as friction.

Step 1: Find out the change in volume

$$V_1 = \frac{mRT_1}{p_1} = \frac{1 \times 0.287 \times 400}{9 \times 100} = 0.1275 \text{ m}^3$$

$$V_2 = \frac{mRT_2}{P_2} = \frac{1 \times 0.287 \times 300}{1.5 \times 100} = 0.574 \text{ m}^3$$

Step 2: Find work done

Since work done on surrounding is at atmospheric pressure.

Work done on surrounding, $W_s = P_{\text{atm}}(V_2 - V_1) = 1 \times 100(0.574 - 0.1275) = 44.65 \text{ KJ/Kg}$.

Example 7.6

For a piston-cylinder arrangement, find out useful work if air expands from 5 bar, 400 K to 1.25 bar, 300 K. Assume P_{atm} = 1bar and T_{atm} = 288 K. Take m = 1 kg. ●●●

Solution Given,

Initial temperature and pressure: 400 K, 5 bar
Final temperature and pressure: 300 K, 1.25 bar

Assumptions:

1. Process involves no internal irreversibilities such as friction
2. Air is ideal with constant specific heats.

Step 1: Find out $W_{\max}$

Maximum work, $W_{\max} = U_2 - U_1 - T_{\text{atm}}(S_1 - S_1)$ (i)

Here, $U_2 - U_1 = mc_V(T_2 - T_1)$

$$= 0.718 \times (400 - 300) = -71.8 \text{ kJ} \quad \text{(ii)}$$

$$T_{\text{atm}}(S_1 - S_1) = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR\ln\left(\frac{p_2}{p_1}\right)$$

$$= 1.005 \times \ln\left(\frac{300}{400}\right) - 0.287 \times \ln\left(\frac{1.25}{5}\right) = -289 \text{ kJ} \quad \text{(iii)}$$

Substituting (ii) and (iii) in (i):

$$W_{\max} = -71.8 - (-289) = 281.2 \text{ KJ}$$

Step 2: Find out work done on surrounding

$$V_1 = \frac{mRT_1}{p_1} = \frac{1 \times 0.287 \times 400}{5 \times 100} = 0.229 \text{ m}^3$$

$$V_2 = \frac{mRT_2}{p_2} = \frac{1 \times 0.287 \times 300}{1.25 \times 100} = 0.689 \text{ m}^3$$

$$W_s = P_{\text{atm}}(V_2 - V_1)$$

$$= 1 \times 100(0.689 - 0.229) = 45.98 \text{ kJ}$$

$$W_{\text{Useful}} = W_{\max} - W_s$$

$$= 281.2 - 45.98 = 235.22 \text{ kJ}$$

Example 7.7

If 100 kJ heat is extracted from a heat reservoir at 675 K with environment temperature equal to 288 K. Find (a) Useful work, (b) If a temperature drop of 50°C is introduced between heat source and the heat engine and a temperature drop of 50°C is introduced between heat engine and the sink. Find out loss in useful work. Take $m = 1$ kg. ○●●

Solution

(a) Given,

Heat transferred, $Q = 100$ kJ at 675 K

Mass: 1 kg

Step 1: Find out useful work

$$W = Q\left(1 - \frac{T_0}{T_1}\right) = 100\left(1 - \frac{288}{675}\right) = 57.33 \text{ kJ}$$

(b) After introducing temperature differences both sides:

$$T_0' = 675 - 50 = 625 \text{ K} \text{ and } T_1' = 388 - 50 = 338 \text{ K}$$

Change in entropy, $\Delta S' = \frac{Q}{T_1'} = \frac{100}{625} = 0.16 \text{ kJ/kg}$

Useful work now becomes:

$$W' = \Delta S'(T_1' - T_0')$$
$$= 0.16 \times (625 - 338) = 45.92 \text{ kJ}$$

Loss in availability = $W - W' = 57.33 - 45.92 = 11.41$ kJ

Example 7.8

A mass of 6.98 kg of air is in a vessel at 200 kPa, 27°C. Heat is transferred to the air from a reservoir at 727°C. Until the temperature of air rises to 327°C. The environment is at 100 kPa, 17°C. Determine: (a) The initial and final availability of air; (b) The maximum useful work associated with the process. **(JNTU, 2009)** ●●●

Solution Given,

$$P_1 = 200 \text{ kPa}; P_2 = 400 \text{ kPa}; P_0 = 100 \text{ kPa}$$

$$T_1 = 300 \text{ K}; T_2 = 600 \text{ K}; T_0 = 290 \text{ K}$$

$$V_0 = 5.81 \text{ m}^3; m = 6.98 \text{ kg}$$

$$V_1 = \frac{2mRT_1}{p_1} = 3.005 \text{ m}^3;\ V_2 = 3.005 \text{ m}^3$$

(a-1) Initial availability:

$$A_i = U_1 - U_0 - T_0\,(S_1 - S_0) + p_0\,(V_1 - V_0)$$

$$A_i = mc_v(T_1 - T_0)\ -mT_0\left(c_p\ \ln\left(\frac{T_1}{T_1}\right) - R\ln\left(\frac{p_1}{p0}\right)\right) + p_0\,(V_1 - V_0)$$

$$A_i = 103.4 \text{ kJ}$$

(a-2) Final availability:

$$A_i = U_2 - U_0 - T_0\,(S_2 - S_0) + p_0\,(V_2 - V_0)$$

$$A_i = mc_v(T_2 - T_0) \; - mT_0(c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_0}\right)) + p_0 (V_2 - V_0)$$

$$A_i = 599.50 \text{ kJ}$$

(b) Maximum useful work:

$$W_m = U_2 - U_1 - T_0 (S_2 - S_1) + p_0 (V_2 - V_1)$$

$$W_m = mc_v(T_2 - T_1) \; - mT_0\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right] + p_0 (V_2 - V_1)$$

Here, $V_2 = V_1$

$$W_m = 461.35 \text{ kJ}$$

Heat supplied to vessel:

$$Q = \int m\, c_v\, dT = mc_v (T_2 - T_1)$$

$$6.98 \times 0.718 \times (600 - 300) \text{ kJ} = 1{,}503.401 \text{ kJ}$$

Useful work:

$$W = Q\left(1 - \frac{T_0}{T}\right) = 1{,}503.401 \times (1 - 290/1{,}000) = 1{,}067.47 \text{ kJ}$$

7.4 || SECOND LAW EFFICIENCY

University Question

1. What is second law efficiency? **(AKTU, 2013–2014)**

First law of thermodynamics tells us that how much energy we get compared to the energy we put in. Second law expresses how much energy is obtained compared to the maximum possible energy output. Since, it is not always possible for a system to work at the theoretical maximum efficiency, hence, second law efficiency is the ratio of actual work done to the maximum theoretical (Carnot) available energy. Hence, it is the comparison of system's thermal efficiency to the maximum possible energy.

$$\eta_{\text{First}} = \frac{\text{Energy Output}}{\text{Energy Input}} = \frac{W}{Q}$$

While,

$$\eta_{\text{Second}} = \frac{\text{Minumum available energy to perform the work}}{\text{Actual energy in perfroming the work}} = \frac{W}{W_{\max}}$$

Knowing that,

$$\eta_{\text{Carnot}} = \frac{W_{\max}}{Q}$$

$$\eta_{\text{Second}} = \frac{W}{W_{\max}} = \frac{W/Q}{W_{\max}/Q}$$

$$\eta_{\text{Second}} = \frac{\eta_I}{\eta_{\text{Carnot}}}$$

Value of second law efficiency lies between Carnot efficiency and thermal efficiency.

For turbine:

$$\eta_{\text{Second}} = \frac{W_{\text{actual}}}{W_{\text{reversible}}}$$

For heat compressor/heat pump:

$$\eta_{\text{Second}} = \frac{W_{\text{reversible}}}{W_{\text{actual}}}$$

For refrigerator:

$$\eta_{\text{Second}} = \frac{\text{COP}_{\text{actual}}}{\text{COP}_{\text{reversible}}}$$

7.5 EXERGY

University Question

1. Define exergy. **(AU, 2013)**

According to the second law, quality of energy available is not same throughout a process. Suppose, a fuel inside a small container starts to burn. At the last stage of the process, there remains a warm mixture of combustion products and air. Here we see that total energy associated with the *system is constant* but the initial air fuel mixture is intrinsically more useful than the warm mixture of last stage. Hence, system has *greater potential* for use at initial stage than the final stage. Also, this higher initial potential would largely be wasted because the process is irreversible. Hence, availability as potential for use remains the major concern for the heat to work conversion.

The maximum amount of work, that can be extracted from a system at a given state, till the system reaches at thermal equilibrium with the surrounding is called exergy.

A system is said to be in the dead state when it is in thermodynamic equilibrium with the environment it is in. At the dead state, the useful work potential (exergy) of a system is zero because no more useful work can be extracted from the system once it reaches to dead state.

We know that work associated with specified initial and final states is maximum when selected process path is reversible. If the final state is dead state, the work done is maximum possible work the system can deliver for that specified initial state.

This represents the *useful work potential* of the system at the specified state and is called exergy. As an example of exergy, an air with temperature, pressure equal to that of atmosphere has energy but no exergy. In short, exergy of a system in equilibrium with environment is zero.

So, the exergy depends upon atmospheric conditions or reference state and hence selection of actual atmospheric condition (i.e. immediate surroundings to the system) rather than standard atmospheric condition re more useful.

POINTS TO REMEMBER

- Maximum possible work for Q amount of heat supplied from a source at temperature, T is $Q\left(1-\frac{T_0}{T}\right)$, where T_0 is the ambient temperature. This is known as reversible work.
- Irreversibility = Maximum possible work – Actual work
- Availability function for non-flow process:

$$\phi = U + P_0V - T_0\,S$$

- Availability function for steady flow process:

$$\phi = \left(B_2 + \frac{1}{2}mV_2^{\,2} + mgZ_2\right) - \left(B_1 + \frac{1}{2}mV_1^{\,2} + mgZ_1\right)$$

- Irreversibility is given as, $I = W_{\max} - W = T_0\Delta S - Q = T_0(\Delta S)_{\text{Surrounding}}$
- Second law efficiency is given as,

Carnot efficiency > Second law efficiency > Thermal efficiency

$$\eta_{\text{Second}} = \frac{\text{Minumum available energy to perform the work}}{\text{Actual energy in perfroming the work}} = \frac{W}{W_{\max}}$$

$$\eta_{\text{Second}} = \frac{\eta_I}{\eta_{\text{Carnot}}}$$

- Exergy or available energy is the amount of energy that can be converted into useful work whereas the energy that can not be converted into useful work is called anergy or unavailable energy.

PRACTICE PROBLEMS

1. A heat engine receives heat from a furnace at 1,000 K and rejects waste heat to a river at 300 K has a thermal efficiency of 40%. Find out second law efficiency of this power input. (**Ans:** 57%) ○○●

2. A heat engine is receiving heat from a source at 1,200 K at rate of 500 kJ/s. It rejects the waste heat to the surrounding at 310 K. If the power output of the heat engine is 200 kW. Determine (a) the reversible power, (b) the rate of irreversibility, and (c) the second-law efficiency of this heat engine. Environment temperature is 300 K. (**Ans:** (a) 370.83 kJ/s, (b) 129.17 kJ/s, (c) 53.93%) ○●●

3. A 0.04 m^3 tank contains air at ambient conditions of 100 kPa and 22°C. Now, a 15 liter tank of liquid water at 85°C is placed into the tank without causing any escape to the tank air. After some transfer of heat from the water to the air and the surroundings, temperature of both water and air is settled down at 44°C. Determine the exergy destruction during this process. (**Ans:** 308.8 kJ) ●●●

4. 50 kg of water at 123°C is mixed with 20 kg of water at 33°C, while the surrounding temperature remains 15°C. Calculate decrease in available energy due to mixing. (**Ans:** 281 kJ) ○●●

5. In a turbine, air expansion takes place adiabatically from 400 kPa, 400 K and 100 m/s to 80 kPa, 300 K and 50 m/s. The environment is at 100 kPa, 290 K. Calculate per kg of air ○●●
(a) Maximum work output, (b) Actual work output, (c) Irreversibility
(**Ans:** (a) 154.44 kJ, (b) 103.75 kJ, (c) 50.69 kJ)

MULTIPLE CHOICE QUESTIONS

1. For a steam turbine second law efficiency is: ○○●
(a) Higher than Carnot efficiency (b) Lower than Carnot efficiency
(c) Equal to Carnot efficiency (d) 1.5 times of Carnot efficiency

2. A heat transfer process tends to reversibility as the temperature difference between the bodies approaches: ○●●
(a) Infinity (b) Zero (c) +1 (d) –1

3. Availability function for a closed system is written as: ○○●
(a) $U - pV + TS$ (b) $U + pV - TS$ (c) $U + pV - TS$ (d) $U + pV + TS$

4. Useful work is given as: (p is the atmospheric pressure) ○○●
(a) $W_{\text{actual}} + p\,(V_2 - V_1)$ (b) $W_{\text{actual}} - p\,(V_2 - V_1)$
(c) $W_{\text{actual}} - p\,(V_2 + V_1)$ (d) $W_{\text{actual}} + p\,(V_2 + V_1)$

5. For any heat transfer system, as system reaches close to the surroundings, its opportunity to produce more work: ○○●
(a) Decreases (b) Increases (c) Remains constant (d) Nothing can be said

6. If solar energy Q_r is available at a reservoir storage temperature of T_r and if quantity of heat Q_a is transferred by the solar collector at temperature T_a, then which of the following is true? ○●●
(a) First law efficiency = Q_a/Q_r
(b) Second law efficiency = exergy output/exergy input
(c) Second law efficiency = (first law efficiency) × $(1 - T_o/T_a)/(1 - T_o/T_r)$
(d) All of the above

7. Which of the following statement is true? ○●●
(a) KE is entirely available energy
(b) PE is entirely exergy
(c) The exergy of thermal energy of reservoirs is equivalent to the work output of a Carnot engine operating between the reservoir at temperature T and environment T_o.
(d) All of the above

8. The surrounding work is zero for: ○●●
(a) Cyclic devices (b) Steady flow devices
(c) System with fixed boundaries (d) All of the above

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (a)	2. (b)	3. (c)	4. (b)	5. (a)
6. (d)	7. (d)	8. (d)		

Properties of Pure Substances, Gases and Gas Mixtures

8

CHAPTER OUTLINE

- Pure Substance
- Quality or Dryness Fraction
- Use of Steam Tables and Mollier Chart
- Avogadro's Law
- Dalton's Law of Partial Pressure
- Ideal Gas and Real Gas

INTRODUCTION

A substance exists in different forms namely solid, liquid and gas. Each of these forms have different molecular structure. In this chapter, we discuss the properties of a pure substance and explain the effect of different thermodynamic properties on a substance which cause it to change its form.

8.1 PURE SUBSTANCE

University Question

1. What is a pure substance? **(AU, 2014)**

A pure substance is a substance of constant chemical composition throughout its mass. The composition does not change during thermodynamic processes. A pure substance is a one-component system and may exist in one or more phases. Water is an example of a pure substance.

We obtain various phase equilibrium diagrams for water as a pure substance. Water exists in three phases ice, water and vapour with same chemical composition. Water exists in three phases ice, water and vapour with same chemical composition.

8.1.1 *p-V* Diagram for a Pure Substance

We assume a unit mass of ice (solid water) at –10°C and 1 atm contained in a cylinder and piston machine.

Let the ice be heated slowly so that its temperature is always uniform. We monitor the changes in the mass of water corresponding to various temperatures with the pressure as constant.

Table 8.1 lists the distinct regimes and states of above process of heating, which will be utilised in present study.

Table 8.1 *Different states of processes which occur during phase change*

States	Phase	Process/ Regimes
1–2	Solid	The temperature of ice increases from –10°C to 0°C. The volume of ice would increase, as would be the case for any solid upon heating. At state 2, i.e., 0°C, the ice would start melting.
2–3	Combination of solid and liquid	Ice melts into water at a constant temperature of 0°C. At state 3, the melting process ends. There is a decrease in volume, which is a peculiarity of water.
3–4	Liquid	The temperature of water increases, upon heating, from 0°C to 100°C. The volume of water increases because of thermal expansion.
4–5	Combination of liquid and vapour	The water starts boiling at state 4 and boiling ends at state 5. This phase change from liquid to vapour occurs at a constant temperature of 100°C (the pressure being constant at 1 atm). There is a large increase in volume.
5–6	Vapour	The vapour is heated to, say, 250°C (state 6). The volume of vapour increases from V_5 to V_6.

The states 2, 3, 4 and 5 are known as saturation states. A saturation state is a state from which a change of phase may occur without a change of pressure or temperature (Table 8.2).

Table 8.2 *Saturation states during phase change*

States	Description
2: Saturated solid state	State 2 is a saturated solid state because a solid can change into liquid at constant pressure and temperature from state 2
3 and 4: Saturated liquid states	States 3 and 4 are both saturated liquid states. In state 3, the liquid is saturated with respect to solidification, whereas in state 4, the liquid is saturated with respect to vapourisation.
5: Saturated vapour state	State 5 is a saturated vapour state, because from state 5, the vapour can condense into liquid without a change of pressure or temperature.

Now we vary the pressure of the above system and obtain above states and then plot it on *p*-*V* diagram. We obtain the phase equilibrium diagram (Figure 8.1).

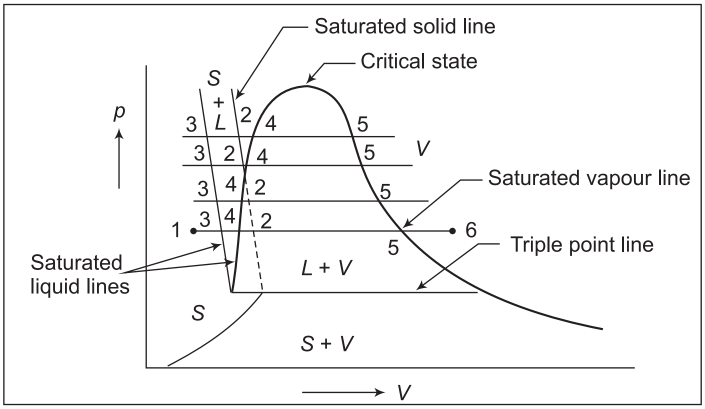


Figure 8.1 *p-V diagram (phase equilibrium diagram)*

Now we define following states lines or regions of the graph shown in Figure 8.1 as follows:

1. ***Saturated solid line:*** The line passing through all the solid states 2 is called the saturated solid line.
2. ***Saturated liquid line:*** The lines passing through all the saturated liquid states 3 and 4 with respect to solidification and vapourisation respectively are known as the saturated liquid lines.
3. ***Saturated vapour line:*** The line passing through all the saturated vapour states 5, is the saturated vapour line
4. ***Saturation or vapour dome:*** The saturated liquid line with respect to vapourisation and the saturated vapour line incline towards each other and form what is known as the saturation or vapour dome. The two lines meet at the critical state.
5. ***Triple point:*** The triple point is a line on the *p-V* diagram, where all the three phases, solid, liquid, and gas, exist in equilibrium. At a pressure below the triple point line, the substance cannot exist in the liquid phase, and the substance, when heated, transforms from solid to vapour (known as sublimation) by absorbing the latent heat of sublimation from the surroundings.

Example 8.1

A rigid vessel contains 1 kg of a mixture of saturated water and saturated steam at a pressure of 0.15 MPa. When the mixture is heated, the state passes through the critical point. Determine:

(a) The volume of the vessel
(b) The mass of liquid and vapour in the vessel initially
(c) The temperature of the mixture when the pressure has risen to 3 MPa
(d) The heat transfer required to produce the final state (c).

○●●

Solution Plotting above process in *p-V* diagram (Figure 8.2).

It is a rigid vessel, so, if we,

(a) Heat this then the process will be constant volume heating. Hence, the volume of the vessel is critical volume of water $= 0.00317\ \text{m}^3$

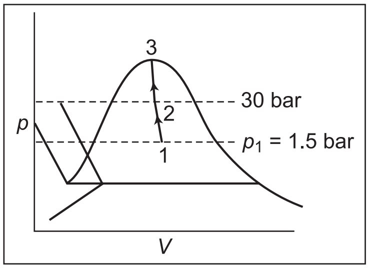


Figure 8.2

(b) Dryness fraction $x = \dfrac{V - V_f}{V_g - V_f} = \dfrac{0.00317 - 0.001053}{1.159 - 0.001053}$

$$= 0.0018282$$

Mass of vapour = 0.0018282 kg

Mass of water = 0.998172 kg

(c) As it passes through critical point then at 3 MPa, i.e., 30 bar also it will be wet steam 50 temperatures will be 233.8°C.

(d) Required heat $Q = U_2 - U_1 = (H_2 - H_1) - (p_2V_2 - p_1V_1)$

Dryness fraction $x = \dfrac{V_2 - V_{f2}}{V_{g2} - V_{f2}} = \dfrac{0.00317 - 0.001216}{1.159 - 0.001216} = 0.029885$

Substituting all values in above expression, we get,

Heat, $Q = 581.806$ kJ/kg

Example 8.2

A sealed rigid tank contains 50 kg of saturated liquid water at 90°C. Determine the pressure in the tank and the volume of liquid water. ○○●

Solution Since the saturation condition exist in a sealed tank, the pressure in the tank is the saturation pressure. Referring steam tables, we get,

$$P_{\text{sat}} = 70.183 \text{ kPa}$$

The specific volume of the saturated liquid at 90°C is $V = 0.001036 \text{ m}^3/\text{kg}$

The total volume of liquid water is, $V = mv = 50 \times 0.001036 = 0.0518 \text{ m}^3$

Example 8.3

A rigid vessel of volume 0.86 m^3 contains 1 kg of steam at a pressure of 2 bar. Find the specific volume and temperature. ○○●

Solution Specific volume $= \dfrac{\text{volume}}{\text{mass}} = \dfrac{0.86}{1} = 0.86 \text{ m}^3/\text{kg}$.

From steam tables,

At 2 bar pressure temperature = 120.2°C

Example 8.4

A mass of 200 g of saturated liquid water is completely vapourised at a constant pressure of 100 kPa. Determine the volume change and amount of energy transferred to the water. ○●●

Solution From steam table, at 100 kPa:

$$V_{fg} = V_g - V_f = 1.6941 - 0.001043 = 1.6931 \text{ m}^3/\text{kg}$$

Change in volume $= mV_{fg} = 0.2 \times 1.6931 = 0.3386 \text{ m}^3$

The amount of energy required to vapourise a unit mass of a substance at a given pressure is the enthalpy of vapourisation at that pressure. Which is $H_{fg} = 2{,}257.5$ kJ/kg for water at 100 kPa.

Thus, the amount of energy needed is $mH_{fg} = 0.2 \times 2257.5 = 451.5$ kJ

8.1.2 *p-T* Diagram for a Pure Substance

The state changes of water, upon slow heating at different constant pressure is now plotted on *p-T* phase equilibrium diagram (Figure 8.3).

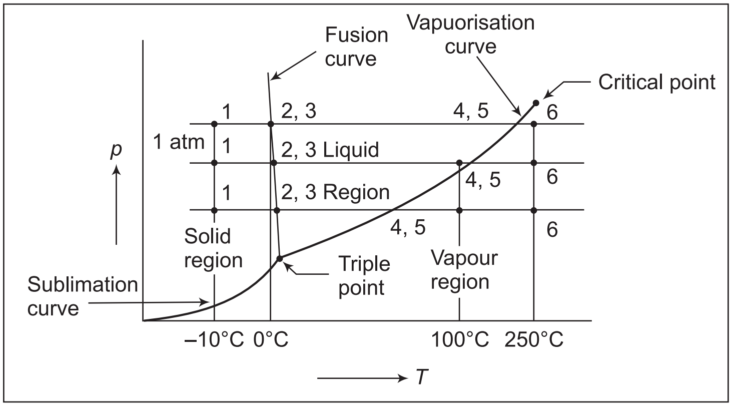


Figure 8.3 *p-T diagram*

We obtain the regions for *p-T* diagram as stated in Table 8.3.

Table 8.3 *Different states of phase change*

States/Regimes		Description
2–3	Fusion curve	The curve passing through the 2, 3 points is called the fusion curve.
4–5	Vapourisation curve	The curve passing through the 4, 5 points (which indicate the vapourisation or condensation at different temperatures and pressures) is called the vapourisation curve.
5	Saturated vapour state	If the vapour pressure of a solid is measured at different temperatures, and these are plotted, the sublimation curve will be obtained.
Triple point		The fusion curve, the vapourisation curve, and the sublimation curve meet at the triple point.

The slopes of the sublimation and vapourisation curves for all substances are positive. The slope of the fusion curve for most substances is positive, but for water, it is negative. The slope of fusion curve is small compared to vapourisation curve indicating that solid melting is not sensitive to temperature as liquid melting. The triple point of water is at 4.58 mm Hg and 273.16 K, whereas that of CO_2 is at 3,885 mm Hg (about 5 atm) and 216.55 K. So, when solid CO_2 ('dry ice') is exposed to 1 atm pressure, it gets transformed into vapour directly, absorbing the latent heat of sublimation from the surroundings, which gets cooled or 'refrigerated'. So, dry ice is often used for low temperature cooling.

Example 8.5

A piston-cylinder contains 3 m^3 of saturated water vapour at 340 kPa pressure, determine the temperature and the mass of the vapour inside the cylinder. ○●●

Solution Since the cylinder contains the saturated vapour at 340 kPa pressure, the temperature inside must be saturation temperature at 340 kPa. However, steam tables do not have this information. Therefore, we use interpolation method.

Steam tables provide the following information,

$$P_{sat} = 325 \text{ kPa}, T_{sat} = 136.27°\text{C}$$

$$P_{sat} = 350 \text{ kPa}, T_{sat} = 138.86°\text{C}$$

Use the interpolation method to get the unknown temperature, $T = 137.54°\text{C}$
Use the similar approach to get: $V_g = 0.539328 \text{ (m}^3\text{/kg)}$

$$m = V/v_g = 3 \text{ (m}^3\text{)}/0.539328 \text{ (m}^3\text{/kg)} = 5.56 \text{ kg}$$

Example 8.6

A rigid vessel contains a mixture of liquid water and steam of total mass of 0.01 kg at a temperature of 80°C. When the mixture is heated at constant volume the state passes through the critical point of water. Calculate (a) the volume of the vessel (b) the initial quality of steam (c) the initial mass of steam and water in the vessel. Sketch the process on the *p-V* and *T-V* diagram for water. ○●●

Solution

(a) Since the mass of steam is fixed and the vessel is rigid, the specific volume is constant during the heating process. From steam table, at the critical point, the vapour specific volume is 0.00317 m^3/kg.

(b) The volume of the vessel is $V_1 = m \times V_1 = 0.01 \times 0.00317 = 3.17 \times 10^{-5}$ m^3

(c) The specific volumes of vapour and liquid are obtained at 80°C from steam tables as

$$V_g = 3.408 \text{ m}^3\text{/kg}$$

$$V_f = 0.001029 \text{ m}^3\text{/kg}$$

Specific volume of steam at state 1 is $V_1 = x_1 V_g + (1 - x_1) V_f$

Above equation gives value of x_1 (initial quality) = 0.0006284

The mass of vapour and liquid at state 1 are

$$mV_1 = mx_1 = 0.000006284 \text{ kg}$$

$$ml_1 = m(1 - x_1) = 0.0099937 \text{ kg}$$

p-V and *T-V* diagram for water is shown in Figure 8.4.

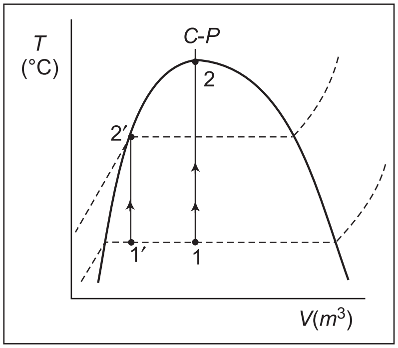
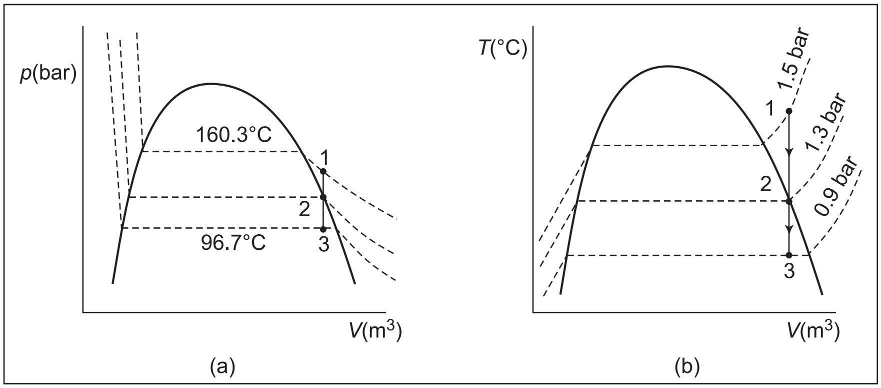


Figure 8.4 *T-V diagram for water*

Example 8.7

A rigid vessel of volume 0.1 m^3 contains 0.0758 kg of steam at an initial pressure of 1.5 bar. The steam is subjected to a constant volume cooling process to a point at which pressure reaches to 0.9 bar. Sketch the *p-V* and *T-V* diagrams for the process. ○●●

Solution Referring steam tables for various phase transitions and obtaining relevant quantities for drawing phase equilibrium diagrams (Figures 8.5 a and b).



Figures 8.5 (a and b) *(a) p-V diagram; (b) T-V diagram*

8.1.3 T-S Diagram for a Pure Substance

The heating of the system of 1 kg of ice at –5°C to steam at 250°C is again considered, the pressure being maintained constant at 1 atm (Table 8.4).

Table 8.4 *States of pure substances at different temperatures*

States	T1	T2	Other Parameters	Formula for Entropy Calculation	Entropy Calculation (kJ/kg K)
1–2	–5°C	0°C	c_p (ice) = 2.093 kJ/kg K	$mc_p \ln\left(\frac{T_2}{T_1}\right)$	0.0398
2–3 (Melting)	0°C	0°C	Latent heat of fusion of ice $Q = 334.96$ kJ/kg	Q/T_2	1.23
3–4	0°C	100°C	c_p (water) = 4.187 kJ/kg K	$mc_p \ln\left(\frac{T_2}{T_1}\right)$	1.305
4–5 (Vapourisation)	100°C	100°C	Latent heat of vapourisation of water $Q = 2{,}257$ kJ/kg	Q/T_2	6.05
5–6	100°C	250°C	c_p (ice) = 2.093kJ/kg K	$mc_p \ln\left(\frac{T_2}{T_1}\right)$	0.706

Now, pressure is maintained constant at 2 atm and same process is repeated. Temperature and entropy values are then plotted as following. The curve 1–2–3–4–5–6 is the isobar of 1 atm and 2 atm (Figure 8.6).

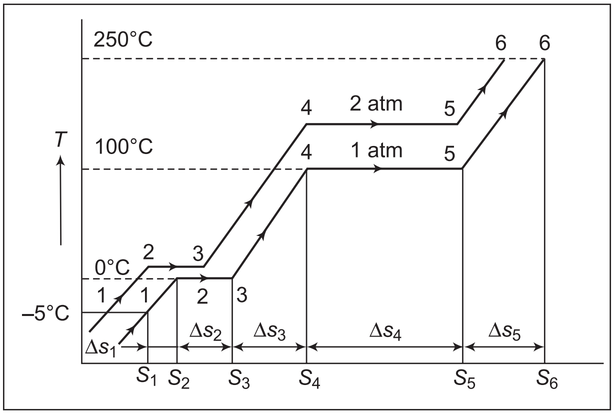


Figure 8.6 *T-S diagram at 1 and 2 atm*

The states 2, 3, 4, and 5 are saturation states. If these states for different pressures are joined, the phase equilibrium diagram of a pure substance on the *T-S* coordinates, would be obtained (Figure 8.7a).

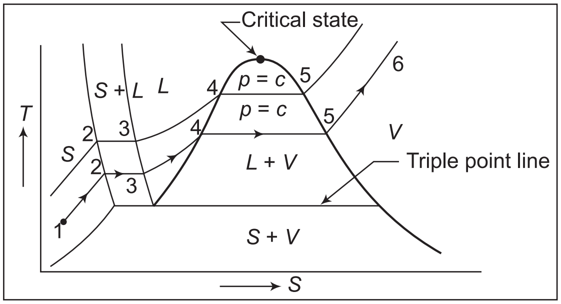


Figure 8.7(a) *T-S diagram*

Liquid-vapour transformations only are most often of interest. Following graph (Figure 8.7b) shows the liquid, the vapour, and the transition zones only.

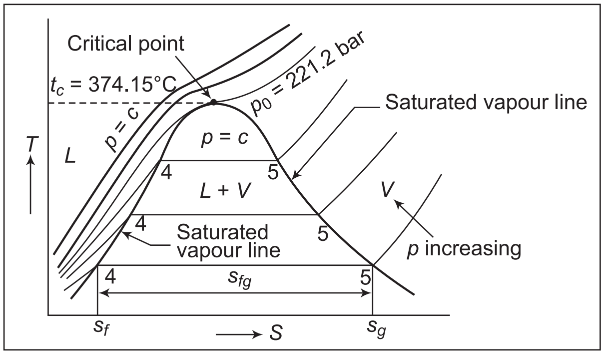


Figure 8.7(b) *Phase equilibrium on T-S plot*

At a particular pressure, S_f is the specific entropy of saturated water, and S_g is that of saturated vapour. The entropy change of the system during the phase change from liquid to vapour at that pressure is $S_{fg} = S_g - S_f$. The value of S_{fg} decreases as the pressure increases, and becomes zero at the critical point.

Example 8.8

A vessel of volume 0.04 m^3 contains a mixture of saturated water and steam at a temperature of 250°C. The mass of the liquid present is 9 kg. Find the pressure, mass, specific volume, enthalpy, entropy and internal energy. **(AU, 2015)** ●●●

Solution Given:

Volume, $V = 0.04\ m^3$

Temperature, $T = 250°C$

Mass of liquid, $m_l = 9$ kg

From the steam tables corresponding to 250°C, $V_f = V_l = 0.001251\ m^3/kg$

$$V_g = V_s = 0.050037\ m^3/kg$$

$$p = 39.776\ \text{bar}$$

Total volume occupied by the liquid, $V_l = m_l \times V_l = 9 \times 0.001251 = 0.0113\ m^3$

Total volume of the vessel,

$$V = \text{Volume of liquid} + \text{Volume of steam} = V_l + V_S$$

$$\Rightarrow \qquad 0.04 = 0.0113 + V_S$$

$$\Rightarrow \qquad V_S = 0.0287\ m^3$$

Mass of steam, $m_s = V_S / V_s = 0.0287/0.050037 = 0.574$ kg

Mass of mixture of liquid and steam, $m = m_l + m_s = 9 + 0.574 = 9.574$ kg

Total specific volume of the mixture, $V = 0.04/9.574 = 0.00418\ m^3/kg$

We know that,

$$V = V_f + x\,V_{fg}$$

$$\Rightarrow \qquad 0.00418 = 0.001251 + x\,(0.050037 - 0.001251)$$

$$\Rightarrow \qquad x = 0.06$$

From steam table corresponding to 250°C,

$$H_f = 1{,}085.8\ \text{KJ/kg}$$

$$H_{fg} = 1{,}714.6\ \text{KJ/kg}$$

$$S_f = 2.794\ \text{KJ/kg K}$$

$$S_{fg} = 3.277\ \text{KJ/kg K}$$

Enthalpy of mixture,

$$H = H_f + x\,H_{fg} = 1{,}085.8 + 0.06 \times 1{,}714.6 = 1{,}188.67\ \text{KJ/kg}$$

Entropy of mixture,

$$S = S_f + x\,S_{fg} = 2.794 + 0.06 \times 3.277 = 2.99\ \text{kJ/kg K.}$$

Internal energy, $U = H - pV = 1{,}188.67 - 39.776 \times 102 \times 0.00418 = 1{,}172$ KJ/kg.

Example 8.9

A mass of wet steam at temperature 165°C is expanded at constant quality 0.8 to pressure 3 bar. It is then heated at constant pressure to a degree of superheat of 66.5°C. Find the enthalpy and entropy changes during expansion and during heating. Draw the *T-S* diagram. ●●●

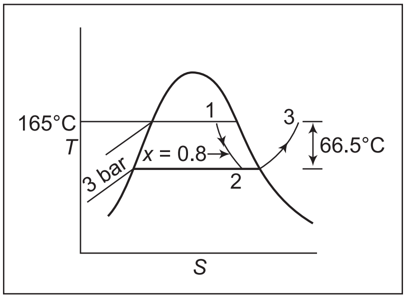


Figure 8.8 *Phase equilibrium on T-S plot*

Solution Plotting above process in *T-S* diagram (Figure 8.8).

From steam table at 7 bar and 165°C,

Enthalpy at state 1, $H_1 = H_f + 0.8\ H_{fg} = 2{,}349$ kJ/kg

Entropy at state 1, $S_1 = S_f + 0.8\ S_{fg} = 5.76252$ kJ/kg K

From steam table at 3 bar, $T_2 = 133.5$°C,

Enthalpy at state 2, $H_2 = H_f + 0.8\ H_{fg} = 561.4 + 0.8 \times 2{,}163.2$
$= 2{,}292$ kJ

Entropy at state 2, $S_2 = S_f + 0.8\ S_{fg} = 1.6716 + 0.8 \times 5.3193$
$= 5.92704$ kJ/kg

With 66.5°C degree of superheat, at state 3,

$T_3 = 200$°C, $H_3 = 2{,}865.6$ kJ/K and $S_3 = 7.311$ kJ/kg K

Enthalpy change in expansion = $H_1 - H_2 = 57$ kJ/kg

Entropy change in expansion = $S_2 - S_1 = 0.16452$ kJ/kg K

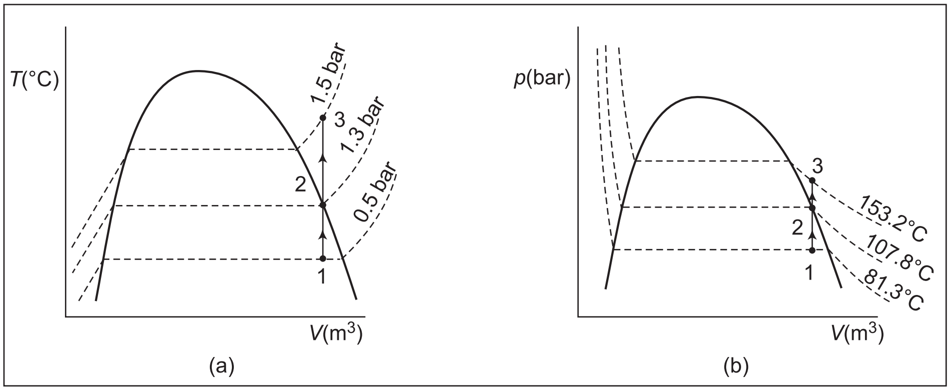
Enthalpy change in heating = $H_3 - H_2 = 573.6$ kJ/kg

Entropy change in heating = $S_3 - S_2 = 1.38396$ kJ/kg K

Example 8.10

A constant mass of steam of quality 0.4 and pressure 0.5 bar is heated at a constant volume of 0.3 m^3 until pressure reaches to 1.5 bar. Sketch the *p-V* and *T-V* diagrams for the process. ○○●

Solution Referring steam tables for various phase transitions and obtaining relevant quantities for drawing phase equilibrium diagrams (Figures 8.9 a and b).



Figures 8.9 (a and b)

8.1.4 H-S Diagram for a Pure Substance

From the first and second laws of thermodynamics, the following property relation can be obtained:

$$\left(\frac{\partial H}{\partial S}\right)_p = T$$

This equation forms the basis of the *H-S* diagram of a pure substance, also called the Mollier diagram.

Consider the heating of a system of ice at –5°C to steam at 250°C, the pressure being maintained constant at 1 atm. Slope of isobar (at 1 atm) is discussed in Table 8.5.

Table 8.5 *State of pure substance on the basis of H-S diagram*

States	Phase	Process/Regimes
1–2	Solid	Slope of isobar first increases as the temperature of the ice increases as temperature increases from 5°C to 0°C
2–3	Combination of solid and liquid	Its slope then remains constant as ice melts into water at the constant temperature of 0°C
3–4	Liquid	The slope of the isobar again increases as the temperature of water rises from 0°C to 100°C
4–5	Combination of liquid and vapour	The slope again remains constant as water vapourises into steam at the constant temperature of 100°C
5–6	Vapour	The slope of the isobar continues to increase as the temperature of steam increases to 250°C (5–6) and beyond

From above, it can be seen that the slope of an isobar on the *H*-S coordinates is equal to the absolute saturation temperature (t_{sat} + 273) at that pressure. If the temperature remains constant the slope will remain constant. If the temperature increases, the slope of the isobar will increase. These are plotted in Figure 8.10.

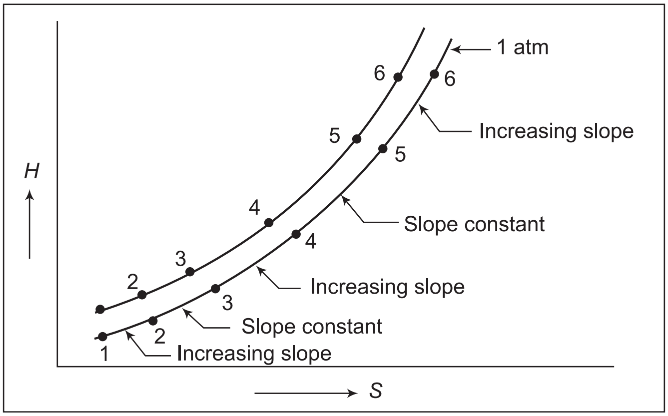


Figure 8.10 *H-S diagram*

Similarly, the isobars of different pressures can be drawn on the *H-S* diagram. States 2, 3, 4 and 5 are saturation states. The diagram shows the phase equilibrium diagram of a pure substance on the

H-S coordinates, indicating the saturated solid line, saturated liquid lines and saturated vapour line, the various phases, and the transition (mixture) zones (Figure 8.11).

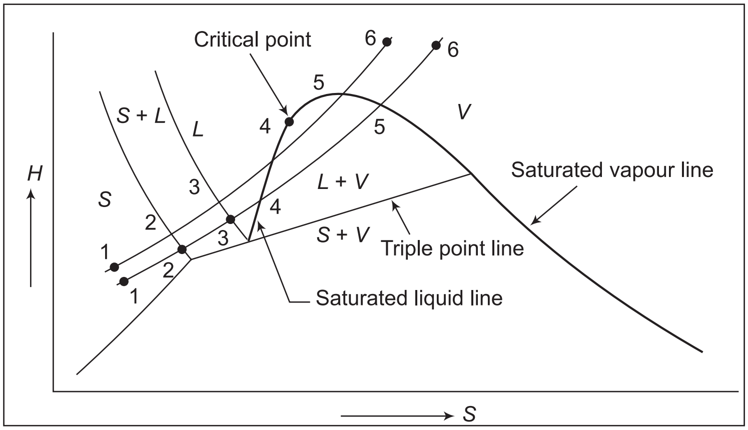


Figure 8.11 *H-S phase-equilibrium diagram of a pure substance*

The *H-S* or the Mollier diagram indicating only the liquid and vapour phases is discussed as follows (Figure 8.12).

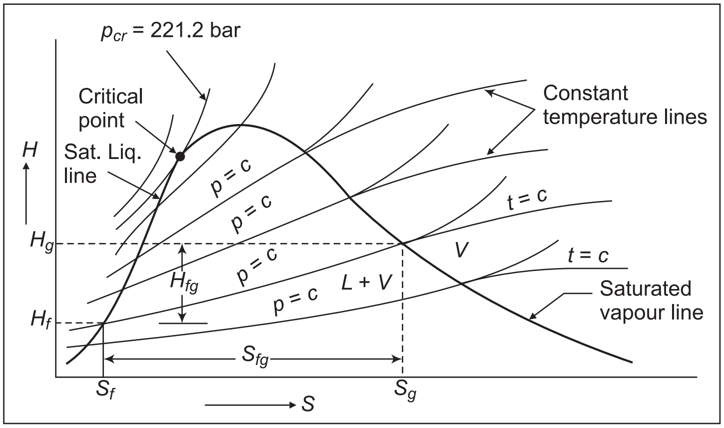


Figure 8.12 *Mollier diagram*

1. ***Critical isobar:*** As the pressure increases, the saturation temperature increases, and so the slope of the isobar also increases. Hence, the constant pressure lines diverge from one another, and the critical isobar is a tangent at the critical point.
2. ***Constant temperature lines**:* In the vapour region, the states of equal slopes at various pressures are joined by lines, which are the constant temperature lines. Although the slope of an isobar remains continuous beyond the saturated vapour line, the isotherm bends towards the right and its slope decreases asymptotically to zero, because in the ideal gas region it becomes horizontal and the constant enthalpy implies constant temperature.

At a particular pressure, H_f is the specific enthalpy of saturated water, H_g is that of saturated vapour, and $H_{fg} = H_g - H_f$ is the latent heat of vapourisation at that pressure. As the pressure increases, H_{fg} decreases, and at the critical pressure, H_{fg} becomes zero.

8.2 || QUALITY OR DRYNESS FRACTION

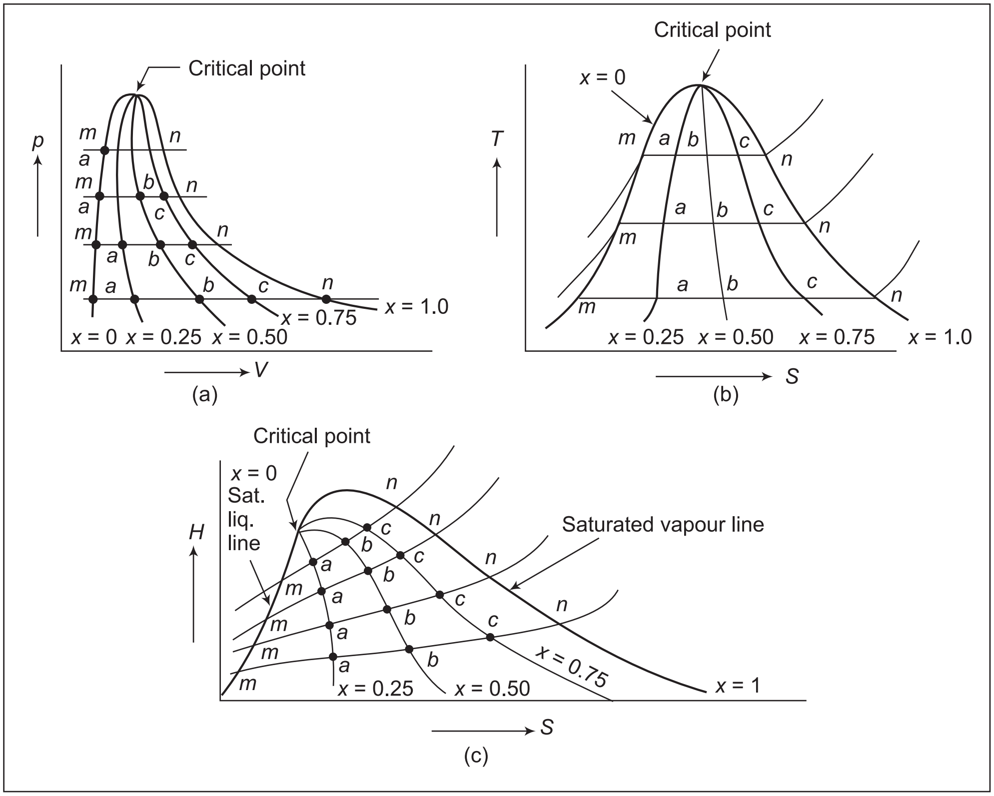
If in 1 kg of liquid-vapour mixture, x kg is the mass of vapour and $(1 - x)$ kg is the mass of liquid, then x is known as the quality or dryness fraction of the liquid vapour mixture. Therefore, quality indicates the mass fraction of vapour in a liquid vapour mixture or:

$$x = \frac{m_v}{m_v + m_l}$$

where, m_v and m_l are the masses of vapour and liquid respectively in the mixture.

The value of x varies between 0 and 1. For saturated water, when water just starts boiling, $x = 0$, and for saturated vapour, when vapourisation is complete, $x = 1$, for which the vapour is said to be dry saturated.

Phase equilibrium diagrams are obtained indicating quality fraction as shown in Figures 8.13 (a to c).



Figures 8.13 (a to c) *(a) p-V diagram; (b) T-S diagram; (c) H-S diagram*

Points *m* indicates the saturated liquid states with $x = 0$,
Points *n* indicates the saturated vapour states with $x = 1$,
the lines *mn* indicating the transition from liquid to vapour

Points *a, b,* and *c* at various pressures indicate the situations when the masses of vapour reached 25%, 50%, and 75% of the total mass. The lines passing through points *a, b* and *c* are the constant quality lines of 0.25, 0.50, and 0.75 respectively.

Let V be the total volume of a liquid vapour mixture of quality x, V_f the volume of the saturated liquid, and V_g the volume of the saturated vapour, the corresponding masses being m, m_f, and m_g respectively. Specific volume (volume per unit mass) can be obtained using following expressions:

$$V = (1 - x)V_f + xV_g$$

$$V = V_f + xV_{fg}$$

Similarly, other quantities of the mixture can be obtained using following expressions:

$$s = (1 - x)s_f + xs_g, \quad s = s_f + xs_{fg}$$

$$h = (1 - x)h_f + xh_g, \quad h = h_f + xh_{fg}$$

$$u = (1 - x)u_f + xu_g, \quad u = u_f + xu_{fg}$$

where *s*, *h*, and *u* refer to a mixture of quality *x*, the suffix *g* indicates the condition of saturated liquid and vapour respectively.

8.3 USE OF STEAM TABLES AND MOLLIER CHART

University Question

1. What information do you get from a Mollier Chart? **(AU, 2014)**

The properties of water are arranged in the steam tables as functions of pressure and temperature. Separate tables are provided to give the properties of water in the saturation states and in the liquid and vapour phases. The internal energy and entropy of saturated water at the triple point (T = 0.01°C) is arbitrarily chosen to be zero while the enthalpy is slightly positive ($H = U + pV$).

Steam tables are used to obtain the following properties:

1. ***Saturation states:*** When a liquid and its vapour are in equilibrium at a certain pressure and temperature, only the pressure or the temperature is sufficient to identify the saturation state. If the pressure is given, the temperature of the mixture gets fixed, which is known as the saturation temperature, or if the temperature is given, the saturation pressure gets fixed. Saturated liquid or the saturated vapour has only one independent variable, i.e., only one property is required to be known to fix up the state.

   Steam tables give the properties of saturated liquid and saturated vapour. First table considers temperature as an independent variable and list other properties while second table takes pressure as an independent variable while listing other properties.

If data are required for intermediate temperatures or pressures, linear interpolation is normally accurate. The reason for the two tables is to reduce the amount of interpolation required.

2. ***Liquid-vapour mixtures:*** Let us consider a mixture of saturated liquid water and water vapour in equilibrium at pressure, *p*, and temperature, *T*. The composition of the mixture by mass will be given by its quality *x*, and its state will be within the vapour dome (Figure 8.14). The properties of the mixture can be evaluated using following formulas:

$$v = v_f + xv_{fg}$$

$$u = u_f + xu_{fg}$$

$$h = h_f + xh_{fg}$$

$$s = s_f + xs_{fg}$$

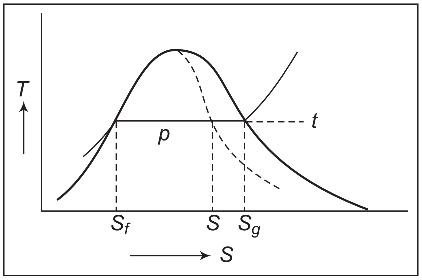


Figure 8.14 *T-S diagram*

3. ***Superheated Vapour and Compressed Liquid:*** When the temperature of the vapour is greater than the saturation temperature corresponding to the given pressure, the vapour is said to be superheated (state 1). The difference between the temperature of the superheated vapour and the saturation temperature at that pressure is called the superheat or the degree of superheat.

   When the temperature of a liquid is less than the saturation temperature at the given pressure, the liquid is called compressed liquid or subcooled liquid (state 2). When a liquid is cooled below its saturation temperature at a certain pressure it is said to be subcooled. The difference in saturation temperature and the actual liquid temperature is known as the degree of subcooling, or simply, subcooling (Figure 8.15).

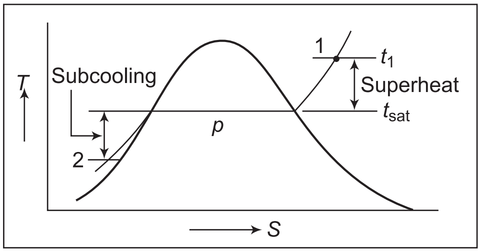


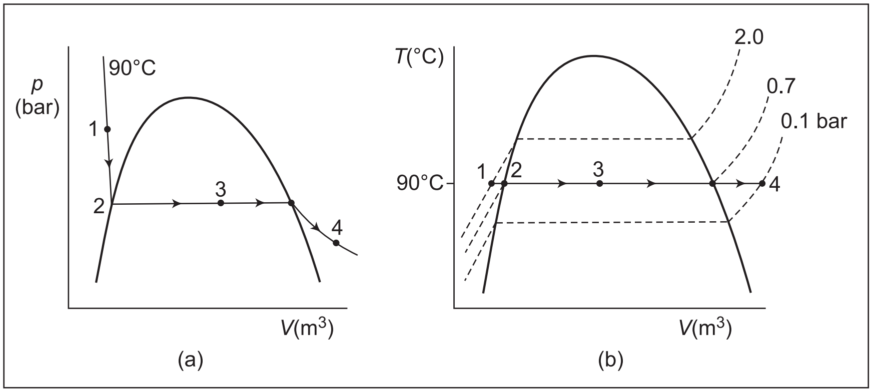
Figure 8.15 *T-S diagram of superheated vapour and compressed liquid*

4. ***Mollier Chart:*** The enthalpy-entropy plot has a larger scale to provide data suitable for many computations as compared to the temperature-entropy chart. This is called Mollier Chart. It contains the same data as does the *T-S* chart.

Example 8.11

A quantity of water of mass 0.05 kg undergoes an isothermal expansion process form an initial state of 2 bar and 90°C to a point at which pressure reaches to 0.1 bar. Sketch the *p-V* and *T-V* diagrams for the process. ○○●

Solution Referring steam tables for various phase transitions and obtaining relevant quantities for drawing phase equilibrium diagrams (Figures 8.16 a and b).



Figures 8.16 (a and b) *(a) p-V diagram; (b) T-V diagram*

Example 8.12

A sample of steam from a boiler drum at 3 MPa is put through a throttling calorimeter in which the pressure and temperature are found to be 0.1 MPa, 120°C. Find the quality of the sample taken from the boiler. ○○●

Solution Plotting above process in *H-S* diagram (Figure 8.17).

Referring steam tables from required quantities:

	State 1	State 2
Pressure	2 MPa = 30 bar	0.1 MPa = 1 bar
Enthalpy	$H_1 = H_2$	$H_2 = 2{,}676.2 + 20/50\ (2{,}776.4\ 2{,}676.2)$

Therefore, dryness fraction is:

$$H_1 = H_{g1} + xH_{fg1}$$

$$\text{Dryness fraction } x = \frac{H_1 - H_{f1}}{H_{fg1}}$$

$$= \frac{2716.3 - 1008.4}{1793.9}$$

$$= 0.952$$

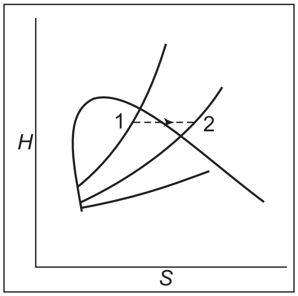


Figure 8.17

Example 8.13

Two kilograms of water at 25°C are placed in a piston cylinder device under 100 kPa (state 1). Heat is added to the water at constant pressure until the piston reaches the stops at a total volume of 0.4 m^3

(state 2). More heat is then added at constant volume until the temperature of the water reaches 300°C (state 3). Determine:

(a) the quality of the fluid and the mass of the vapour at state 2

(b) the pressure of the fluid at state 3.

○●●

Solution Drawing *T*-*V* diagram for above process (Figure 8.18).

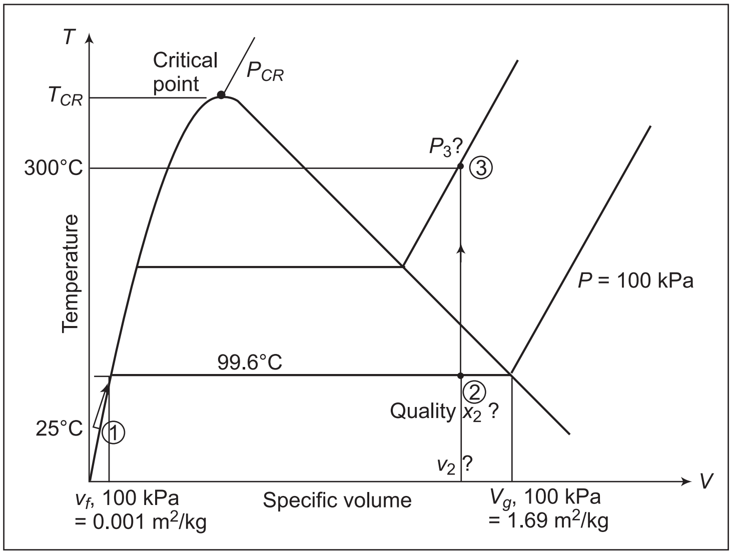


Figure 8.18

(a) Referring steam table for obtaining various volumes in state 2,

$$V_2 = V / m = 0.4 / 2 = 0.2 \text{ m}^3/\text{kg}$$

$$\text{Quality } x_2 = \left[\frac{V_2 - V_f}{V_g - V_f}\right]_{100 \text{ kPa}} = \left[\frac{0.2 - 0.001}{1.694 - 0.001}\right] = 0.118$$

Mass of water vapour in state 2 = 0.118 x 2 = 0.235 kg

(b) Concerning state 3, the problem statement did not specify that it is in the superheat region. We needed to first determine the saturated vapour specific volume vg at 300°C. This value is 0.0216 m^3/kg, which is much less than the specific volume V_3 of 0.2 m^3/kg, thus placing state 3 well into the superheated region. Thus, the two intensive properties which we use to determine the pressure at state 3 are T_3 = 300°C, and V_3 = 0.2 m^3/kg. On scanning the superheat tables, we find that the closest values lie somewhere between 1.2 MPa and 1.4 MPa, thus we use linear interpolation techniques to determine the actual pressure, p_3, as shown:

$$\frac{p_3 - 1.2}{1.4 - 1.2} = \frac{0.2 - 0.2139}{0.1823 - 0.2139} = 0.440$$

This gives $p_3 = 1.29$ MPa

8.4 AVOGADRO'S LAW

Avogadro's law was proposed by Amedeo Avogadro. In 1811, he hypothesized two samples of an ideal gas with the same volume and at the same pressure and temperature, contained the same number of molecules. It is an experimental gas law relating volume of a gas to the amount of substance. Blowing up a balloon is a real-life example of Avogadro's law because it says as you blow more molecules of air into balloon it expands more.

University Question

1. State and explain Dalton's law of partial pressure and Avogadro's law. **(GTU, 2014)**

or

Avogadro's law states that the volume of a mole of any gas at the pressure of 760 mm Hg and temperature of 0°C is the same, and is equal to 22.4 litres. This also implies that for a gas at constant temperature and pressure, the volume is directly proportional to the number of moles of gas.

Volume = Constant × Number of moles (constant p, T)

$$\Rightarrow \quad V = k_n \text{ (constant } p, T)$$

$$\Rightarrow \quad \frac{V_1}{n_1} = \frac{V_2}{n_2}$$

V_1 and V_2 are initial and final volume and n_1 and n_2 are initial and final number of moles.

Figure 8.19 shows the graph for Avogardo's law.

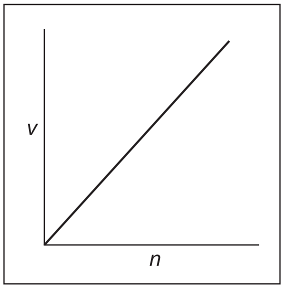


Figure 8.19 *Graph depicting Avogadro's law*

Example 8.14

What volume does 2.25 mole of a gas occupy at NTP? What quantity of gas, in moles, is contained in 2.21 L at NTP? ○○●

Solution

(a) Volume of 1 mole of any gas at NTP is 22.4 L

Therefore, volume of 2.25 mole of the same gas at NTP is 22.4 L × 2.25 = 50.40 L

(b) Number of moles of a gas contained in 2.21 L at NTP is = $\frac{2.21}{22.4}$ = 0.1 mole

Example 8.15

A 2 L sample of gas contains 0.25 moles at 20°C and 1 atm. If 0.1 moles of same gas are added to the sample than find the volume occupied by the gas at the same temperature and pressure. ○○●

Solution According to Avogadro's law if we increase the number of number of moles of a gas, the volume also increases proportionally. Therefore, increase the number of moles will also increase the volume of the gas.

$$\Rightarrow \qquad V_2 = \frac{2}{0.25} \times 0.35 = 2.80 \text{ L}$$

8.5 || DALTON'S LAW OF PARTIAL PRESSURE

Probe

1. Discuss Dalton's law of partial pressure.

Dalton's law of partial pressure states that the total pressure of a mixture of ideal gases is equal to the sum of the partial pressures.

$$p = p_1 + p_2 + \cdots + p_i$$

where p_i is the partial pressure of i^{th} gas of the mixture, which can be evaluated using following expression:

$$p_i = \frac{n_i RT}{V}$$

Dalton's law can also be extended to determine mole fraction of constituent gases:

$$\frac{p_i}{p_T} = \frac{n_i}{n_T}$$

Let us consider a closed vessel of volume V at temperature T, which contains a mixture of perfect gases at a known pressure. If a small part of the mixture is removed, then the pressure would be less than the initial value. If the gas removed were the full amount of one of the constituents then the reduction in pressure would be equal to the contribution of that constituent to the initial total pressure. Each constituent contributes to the total pressure by an amount which is known as the partial pressure of the constituent. The relationship between the partial pressures of the constituents is expressed by Dalton's law, as follows:

1. The pressure of a mixture of gases is equal to the sum of the partial pressures of the constituents.
2. The partial pressure of each constituent is that pressure which the gas would exert if it alone occupied that volume occupied by the mixtures at the same temperature.

Example 8.16

Two gases A and B are contained in a vessel at NTP with partial pressure of 0.5 atm and 1 atm. Gas C is added to this vessel until total pressure becomes 3 atm. Find the mole fraction of Gas C. ○○●

Solution Dalton's law allows to calculate a total pressure of system involving a number of gas. Adding Gas C increases the total pressure to 3 atm. Therefore, the partial pressure of Gas C is 1.5 atm.

Avogadro's law states that 1 mole of a gas exert 1 atm pressure at NTP. Therefore, 1.5 atm partial pressure will be exerted by 1.5 mole of Gas *C* at NTP.

Example 8.17

Two gases *A* and *B* are contained in a vessel with total pressure of 1 atm. Find the mole fraction of Gas *A* if its partial pressure is 300 Torr. ○○●

Solution Using Dalton's law extension to determine mole fraction:

$$\frac{P_i}{P_T} = \frac{n_i}{n_T}$$

Mole fraction of Gas A $n_A = \frac{n_A}{n_T} = \frac{300}{760} = 0.394$

8.6 IDEAL GAS AND REAL GAS

An ideal gas is the one in which the molecular forces of attraction between gas molecules is zero. In addition, the volume of the molecules is negligible compared to total volume for a perfect gas. An ideal (hypothetical) gas obeys the law $pV = n\bar{R}T$ at all pressures and temperatures. While Real gases do not conform to this equation of state with complete accuracy. As $p \to 0$, or $T \to \infty$, the real gas approaches the ideal gas behaviour.

Equation of state of an ideal gas is given by:

$$\Rightarrow \qquad pV = n\bar{R}T$$

where,

m is mass in kg

n is number of moles of the gas

$\bar{R}$ is the characteristic gas constant = 8.3143 kJ/kg mol K

8.6.1 Enthalpy of an Ideal Gas

Enthalpy of a system is the heat content of a system at constant pressure. The heat that is absorbed or released by a reaction at constant pressure is the same as the enthalpy change. It is given by following expression:

$$H = U + pV$$

For an ideal gas, $pV = n\,RT$,

$$H = U + RT = f(T)$$

Enthalpy of an ideal gas is a function of temperature only. Differentiating above equation with respect to T, we get,

$$\Rightarrow \qquad \Delta H = \Delta U + R\Delta T$$

$$\Rightarrow \qquad \Delta H = c_V\Delta T + R\Delta T = (c_V + R)\Delta T = c_p\Delta T$$

The equation $\Delta H = c_p\Delta T$ holds good for an ideal gas, even when pressure changes, but for any other substance, this is true only for a constant pressure change.

8.6.2 Van der Waal's Equation

The ideal gas equation of state is not obeyed by the gases when the pressure increases because the intermolecular forces of attraction and repulsion increase, and also the volume of the molecules becomes appreciable compared to the total gas volume. So, the real gases deviate considerably from the ideal gas equation.

Real gas obeys Van der Waal's equation which includes two correction terms in the equation of ideal gas and is given as follows:

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

The coefficient a was introduced to account for the existence of mutual attraction between the molecules. The term a/V^2 is called the *force of cohesion*. The coefficient b was introduced to account for the volumes of the molecules, and is known as *co-volume.*

Real gases conform more closely with the Van der Waal's equation of state than the ideal gas equation of state, particularly at higher pressures. But this is not obeyed by a real gas in all ranges of pressures and temperatures.

8.6.3 Gibbs Function of a Mixture of Inert Ideal Gases

The enthalpy and the entropy of 1 mole of an ideal gas at temperature, T, and Pressure, p, are:

$$h = h_0 + \int c_p dT,$$

$$s = \int c_p \frac{dT}{T} + s_0 - R\ln p$$

Therefore, the molar Gibbs function, $g = H - TS$ is equal to:

$$g = h_0 + \int c_p dT - T\int c_p \frac{dT}{T} - Ts_0 + RT\ln p$$

Solving above equation using integration by parts, we get:

$$g = h_0 - T\int \frac{\int c_p dT}{T^2} dT - Ts_0 + RT\ln p$$

$$= RT\left(\frac{h_0}{RT} - \frac{1}{R}\int \frac{\int c_p dT}{T^2} dT - \frac{s_0}{R} + \ln p\right)$$

Denoting the first three terms in above equation using ϕ, we get:

$$g = RT(\phi + \ln p)$$

where ϕ is a function of T only and given by:

$$\phi = \frac{h_0}{RT} - \frac{1}{R}\int \frac{\int c_p dT}{T^2} dT - \frac{s_0}{R}$$

Let us consider a number of inert ideal gases separated from one another at the same temperature and pressure,

$$G_i = \sum n_K g_K$$
$$= \bar{R}T \sum n_K (\phi_K + \ln p)$$

After the partitions are removed, the gases will diffuse, and the partial Gibbs function of a particular gas is the value of G, if that gas occupies the same volume at the same temperature exerting a partial pressure p_k. Thus,

$$G_f = \bar{R}T \sum n_K (\phi_K + \ln p_K)$$
$$= \bar{R}T \sum n_K (\phi_K + \ln p + \ln x_K)$$

Therefore,

$$G_f - G_i = \bar{R}T \sum n_K \ln x_K$$

Since $x_K < 1$, $(G_f - G_i)$ is negative because G decreases due to diffusion. Gibbs function of a mixture of ideal gases at T and p is thus:

$$G = \bar{R}T \sum n_K (\phi_K + \ln p + \ln x_K)$$

Example 8.18

0.2 kg of air at 300°C is heated reversibly at constant pressure to 2,066 K. Find the increase in availability. Take T_0 = 30°C and c_p = 1.0047 kJ/kg K. ○○●

Solution From the Gibb's Function

Increase in availability $= H_2 - H_1 - T_0 (S_2 - S_2)$

$= mc (T_2 - T_1) - T_0 \times 0.2577$

$= 1,250.24 - 78.084 = 1,172.2$ kJ

Example 8.19

Find the value of maximum work available if the initial enthalpy and entropy is 2,450 kJ/kg and 0.2504 kJ/kg K at temperature 100°C. The final value of enthalpy and entropy is 3,650 kJ/kg and 0.30 kJ/kg K. ○○●

Solution Maximum work available $= (H_1 - TS_1) - (H_0 - TS_0)$

$= (3,650 - 373 \times 0.30) - (2,450 - 273 \times 0.2504)$

$= 1,156.5$ kJ/kg K

Example 8.20

Consider a mixture of two inert gases *A* (1 mole) and *B* (2 mole). It is noted that change in Gibbs' function of the system after removing the partition 0. Determine if this could ever possible. ○●●

Solution It is given that change in Gibbs' function is 0. Substituting this value in expression:

$$\ln(x) + 2\ln(1-x) = 0$$

Upon solving above expression:

$$\Rightarrow \qquad x = 1$$

This implies that there is no mixture at all, i.e. only one gas is present. Therefore, it can be confirmed that the change in Gibbs's function is always negative and can be never be zero.

POINTS TO REMEMBER

- ☞ A pure substance is a one component system.
- ☞ The amount of heat needed for complete melting is known as heat of fusion.
- ☞ $\left(\frac{\partial H}{\partial S}\right)_p = T$, forms the basis of the H-S diagram of a pure substance.
- ☞ Quality indicates the mass fraction of vapour in a liquid-vapour mixture as $x = \frac{m_v}{m_v + m_l}$.
- ☞ A triple point is a point where all the three phases exist in equilibrium.
- ☞ At critical point only one phase exist.
- ☞ At critical point latent heat of vapourisation is zero.
- ☞ Dalton's law of partial pressure, $p = p_1 + p_2 + \cdots + p_i$.
- ☞ State of an ideal gas, $pV = n\bar{R}T$
- ☞ Enthalpy of an ideal gas, $H = U + pV$
- ☞ Van der Waal's equation, $\left(p + \frac{a}{V^2}\right)(V - b) = RT$
- ☞ Gibbs function of a mixture of ideal gases at *T* and *p* is, $G = \bar{R}T\sum n_K(\phi_K + \ln p + \ln x_K)$

PRACTICE PROBLEMS

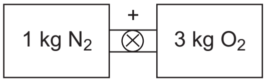
1. A rigid tank contains 10 kg of water at 90°C. If 8 kg of the water is in the liquified and the rest is in the vapour form. Determine the pressure in the tank and the volume of the tank. ○○●

(**Ans:** $p_{\text{sat}} = 70.183$ kPa, $V = 4.73\ \text{m}^3$)

2. Ten kg of water at 45°C is heated at a constant pressure of 10 bar until it becomes superheated vapour at 300°C. Find the change in volume, enthalpy, internal energy and entropy. ○●●

(**Ans:** $V = 2.57$ m^3, $H = 28.628$ MJ, $U = 26.0581$ MJ, $S = 64.3$ kJ/K)

3. Consider following system with Tank *A* and *B* containing gases: ●●●



The valve connecting the tanks is opened. Determine the volume of each tank and the final pressure. (**Ans:** $V_{N_2} = 0.295$ m^3, $V_{O2} = 0.465$ m^3, $p = 422.2$ kPa)

4. A cylinder contains 0.2 kg of water at a pressure of 0.75 bar and temperature 50°C behind a leak-proof piston. Heat is added to the water while maintaining a constant pressure (isobaric process) until temperature reaches 135°C. Sketch the *p-V* and *T-V* diagrams for the process. ○●●

5. 500 kg of water is in a 10 m^3 boiler under 5 MPa. Determining the state of water, temperature, enthalpy and mass of water. ○●●

(**Ans:** Water is in saturated mixture state, $T = 263.99$°C,
$H = 1958$ kJ/kg, $m_g = 245$ kg, $m_l = 255$ kg)

6. A piston-cylinder device contains 0.5 kg saturated liquid water at a pressure of 200 kPa. Heat is added and the steam expands at constant pressure until it reaches 300°C. ●●●

(a) Sketch this process on a T-V (temperature-specific volume) diagram with respect to the saturation lines, critical point, and relevant constant pressure lines, clearly indicating the initial and final states.
(b) Using steam tables determine the initial temperature of the steam prior to heating.
(c) Using steam tables determine the final volume of the steam after heating
(d) Using the ideal gas equation of state determine the final volume of the steam after heating.

(**Ans:** (b) $T_{sat} = 120.2$°C; (c) $V_2 = 0.658$ m^3; (d) $V_2 = 0.661$ m^3)

7. A rigid closed tank of volume 3 m^3 contains 5 kg of wet steam at a pressure of 200 kPa. The tank is heated until the steam becomes dry saturated. Determine the final pressure and the heat transfer to the tank. (**Ans:** $p = 303.16$ kPa, $Q = 3{,}333$ kJ) ●●●

8. Find the mass in grams of 4.2 L of O_2 at NTP. (**Ans:** 6 g) ○○●

9. A 32 ml sampled of argon is collected over water at 20°C and 750 Torr pressure. What is the volume of the dry gas at NTP? It is given that vapour pressure of water at 20°C is 17.5 Torr. ○○●

(**Ans:** $V = 26.88$ ml)

10. Container A (with volume 1.23 m^3) contains a gas under 3 atm of pressure. Container B (with volume 0.93 m^3) contains a gas under 2 atm of pressure. Container *C* (with volume 1.42 m^3) contains a gas under 1 atm of pressure. If all of these gases are put into container *D* (with volume 1.51 m^3), what is the pressure in container *D*? (**Ans:** $P = 2.44$ atm, 1.23 atm, 0.94 atm) ○○●

11. Consider following mixture of gases and associated data in a container. Determine partial pressures of individual gases. ○●●

1 kg CO_2	3 kg CH_4
300 K	200 kPa

(**Ans:** P_{CO_2} = 21.6 kPa, P_{CH_4} = 178.4 kPa, M_m = 19.03 kg/kmol)

12. Consider two inert gases *A* (2 mole) and *B* (3 mole) separated by a partition. Total pressure of the system after the partition is removed is 10 atm. Find the partial pressure of each gas and change in Gibbs' function of the system. (**Ans:** *A* = 4 atm, *B* = 6 atm, *G* = –4.28 RT) ○○●

MULTIPLE CHOICE QUESTIONS

1. A triple point of water on *p-T* and *p-V* diagram is (respectively): ○●●
(a) Point and line (b) Line and point (c) Triangle and line (d) Line and point

2. A triple point of water on *T-S* and *U-V* diagram is (respectively): ○●●
(a) Point and triangle (b) Line and line
(c) Line and triangle (d) Triangle and line

3. How many moles are contained in 2.5 L of CO_2 at NTP? ○○●
(a) 0.114 (b) 0.112 (c) 1.12 (d) 0.0112

4. Which of the following quantity is equal to the slope of an isobar on the *H-S* coordinates? ○○●
(a) Saturation temperature (b) Absolute saturation temperature
(c) Enthalpy (d) Entropy

5. When the temperature of the liquid is less than the saturation temperature at the given pressure, the liquid is called: ○●●
(a) Subheated (b) Superheated (c) Subcooled (d) Superheated

6. A gas exhibits an ideal gas behaviour when temperature approaches to? ●●●
(a) 0 K (b) 0°C (c) 100 K (d) None of these

7. A mixture of three gases *A*, *B* and *C* is at a total pressure of 8.15 atm. The partial pressure of gas *A* is 2.70 atm; that of gas *B* is 2.09 atm. What is the partial pressure of gas *C*? ○○●
(a) 1.36 (b) 2.36 (c) 3.36 (d) 4.36

8. Real gas equation includes correction terms for which of following physical properties? ○●●
(a) Mutual attraction between the molecules
(b) Volume of molecules
(c) Both (a) and (b)
(d) None of above

9. Reduced temperature of a substance is: ○○●
(a) Ratio of actual pressure to the critical pressure
(b) Ratio of critical pressure to the actual pressure
(c) Actual pressure
(d) Critical pressure

10. The generalised compressibility chart is plotted as a function of: ○○●

(a) Reduced pressure and reduced temperature
(b) Reduced pressure
(c) Reduced volume
(d) Reduced temperature

11. The latent heat of vapourisation at critical point is: ○●●

(a) Less than zero (b) Greater than zero (c) Equal to zero (d) None of the above

12. The specific volume of water when heated at 0°C is: ●●●

(a) First increases and then decreases (b) First decreases and then increases
(c) Increases steadily (d) Decreases steadily

13. With increase in pressure: ●●●

(a) Enthalpy of dry saturated steam increases
(b) Enthalpy of dry saturated steam decreases
(c) Enthalpy of dry saturated steam remains same
(d) None of the above

14. With increase in pressure: ●●●

(a) Boiling point of water is increases enthalpy of evapouration increases
(b) Boiling point of water is increases enthalpy of evapouration decreases
(c) Boiling point of water is decreases enthalpy of evapouration increases
(d) None of the above

15. Joule's law states that specific internal energy of a gas depends only on: ○●●

(a) The pressure of the gas (b) The volume of the gas
(c) The temperature of the gas (d) None of the above

16. The equation of state of an ideal gas is a relationship between the variables: ○○●

(a) Pressure and volume (b) Pressure and temperature
(c) Pressure, volume and temperature (d) None of the above

17. Charles law states that if any gas is heated at constant pressure, its volume: ○○●

(a) Changes directly as it absolute temperature
(b) Changes inversely as its absolute temperature
(c) Changes as square of the absolute temperature
(d) Does not change with absolute temperature

18. The equation of the state per kg of a perfect gas is given by: ○○●

(a) $p^2V = RT$ (b) $pV = RT$ (c) $pV^2 = RT$ (d) $p^2V^2 = RT$

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (a) 2. (c) 3. (b) 4. (b) 5. (c)
6. (a) 7. (c) 8. (c) 9. (a) 10. (a)
11. (c) 12. (b) 13. (b) 14. (b) 15. (c)
16. (c) 17. (a) 18. (b)

Thermodynamics Relations, Equilibrium and Stability

9

Chapter Outline

- Important Mathematical Relations
- Gibbs-Helmholtz Functions
- Maxwell's Equations
- TdS Relations
- Joule-Kelvin Effect
- Clausius-Clapeyron Equation
- Gibbs Phase Rule
- General Relations

INTRODUCTION

It is not always possible and convenient to measure all the thermodynamic quantities experimentally. Thermodynamic relations provide important interrelation between these quantities so that all necessary information regarding a system can be collected with minimum number of experiments.

As discussed in previous chapters, there are eight thermodynamic properties of a system—pressure, temperature, volume, internal energy, enthalpy, entropy, Helmholtz function and Gibbs function. Out of these eight, p, V and T are directly measurable quantities. In this chapter, we will introduce combination of other properties which are relatively easier to measure and together with the measurable quantities (p, V and T) enable us to determine more complex properties.

9.1 IMPORTANT MATHEMATICAL RELATIONS

Some of the useful mathematical relations, which are to be frequently used in this chapter, are briefly discussed in this section.

University Question

1. Derive any three of Maxwell's equations. **(JNTU, 2015)**

Theorem 1

If $f(x, y, z) = 0$, then we can write $z = z(x, y)$ and

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$dz = Mdx + Ndy$$

where, $M = \left(\frac{\partial z}{\partial x}\right)_y \quad N = \left(\frac{\partial Z}{\partial y}\right)_x$

Mathematically, dz is an exact differential, which simply means that z is continuous function of the two independent variables x and y. So,

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial^2 z}{\partial x \cdot \partial y}$$

$$\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial^2 z}{\partial x \cdot \partial y}$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

This is an essential condition of exact differential.

Theorem 2

If f is a function of x, y and z, and there exists a relation between x, y and z then f can be written as a function of any of x, y and z. Similarly, any one of x, y and z can be regarded to be a function of f and any one of x, y and z. So, if,

$$x = x(f, y)$$

$$dx = \left(\frac{\partial x}{\partial f}\right)_y df + \left(\frac{\partial x}{\partial y}\right)_f dy \tag{9.1}$$

In same way, if $y = y(f, z)$

$$dy = \left(\frac{\partial y}{\partial f}\right)_z df + \left(\frac{\partial y}{\partial z}\right)_f dz \tag{9.2}$$

Substituting dy in expression (9.1),

$$dx = \left(\frac{\partial x}{\partial f}\right)_y df + \left(\frac{\partial x}{\partial y}\right)_f \left[\left(\frac{\partial y}{\partial f}\right)_z df + \left(\frac{\partial y}{\partial z}\right)_f dz\right]$$

Simplifying the expression results:

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = 1$$

Theorem 3

If $f(x, y, z) = 0$; any of the x, y or z can be considered as function of rest of the two.

So, $x = x(y, z)$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz \tag{9.3}$$

In same way,

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \tag{9.4}$$

Substituting dz in the expression (9.3),

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y \left[\left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy\right]$$

Simplifying the expression results,

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

The relation is known as cyclic relation.

Jacobian

$$J\left[\frac{x, y}{p, r}\right] \text{is defined as}$$

$$J\left[\frac{x, y}{p, r}\right] = \frac{[x, y]}{[p, r]}$$

$$= \begin{bmatrix} \dfrac{\partial x}{\partial p} & \dfrac{\partial x}{\partial r} \\ \dfrac{\partial y}{\partial p} & \dfrac{\partial y}{\partial r} \end{bmatrix} = \left(\frac{\partial x}{\partial p} \cdot \frac{\partial y}{\partial r} - \frac{\partial x}{\partial r} \cdot \frac{\partial y}{\partial p}\right)$$

Jacobians obey the followings properties:

(i) $\dfrac{[x, y]}{[p, r]} \dfrac{[p, r]}{[s, t]} = \dfrac{[x, y]}{[s, t]}$

(ii) $\dfrac{[x, y]}{[p, r]} = \left(\dfrac{\partial x}{\partial y}\right)_z$

(iii) $[x, y] = -[y, x]$

If $f(x, y, z) = 0$, then we can write $z = z(x, y)$ and

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

In Jacobian notation it can be written as,

$$dz = \frac{[z, y]}{[x, y]}dx + \frac{[z, x]}{[y, x]}dy$$

or

$$[x, y]dz + [y, z]dx + [z, x]dy = 0$$

9.2 || GIBBS-HELMHOLTZ FUNCTIONS

In this section, we will discuss four important thermodynamics functions for a reversible process. These four functions are:

- Internal energy (U)
- Enthalpy (H)
- Helmholtz free energy (F)
- Gibbs function (G)

9.2.1 Internal Energy

Recalling the results from combined first and second law of thermodynamics:

$$dU = TdS - pdV \tag{9.5}$$

If internal energy, U, is considered a function of entropy and volume only, i.e. $U = U(S, V)$,

$$dU = \left(\frac{\partial u}{\partial s}\right)_V ds + \left(\frac{\partial u}{\partial v}\right)_S dv \tag{9.6}$$

Comparing (9.5) and (9.6):

$$\left(\frac{\partial u}{\partial s}\right)_V = T$$

$$\left(\frac{\partial u}{\partial v}\right)_S = -p$$

Thus U (V, S) will provide all the thermodynamic information about the system; given V and S one has an expression for U and can evaluate p and T through differentiation.

9.2.2 Enthalpy

Similarly, starting with $dU = TdS - pdV$

The enthalpy H can be differentiated,

$$dH = dU + pdV + Vdp$$

Combining the two results,

$$dH = TdS + Vdp \tag{9.7}$$

If enthalpy considered as function of entropy and pressure only, i.e., $H = H(S, p)$,

$$dH = \left(\frac{\partial H}{\partial p}\right)_S dp + \left(\frac{\partial H}{\partial S}\right)_p dS \tag{9.8}$$

Comparing (9.7) and (9.8):

$$\left(\frac{\partial H}{\partial S}\right)_p = T$$

$$\left(\frac{\partial H}{\partial p}\right)_S = V$$

Since $V > 0$, an isentropic increase in pressure will result increase in enthalpy.

9.2.3 Helmholtz Free Energy

Introducing Helmholtz function as:

$$F = U - TS$$

Using the Gibbs equation with the differential of a,

$$dF = -pdV - SdT$$

Now, if $a = a(V, T)$, $\quad dF = \left(\frac{\partial F}{\partial V}\right)_T dV + \left(\frac{\partial F}{\partial T}\right)_V dT$

$$\left(\frac{\partial F}{\partial V}\right)_T = -p$$

$$\left(\frac{\partial F}{\partial T}\right)_V = -S$$

9.2.4 Gibbs Function

Similarly, using the Gibbs function

$$g = H - TS$$

$$dg = Vdp - SdT$$

Consequently,

$$\left(\frac{\partial g}{\partial P}\right)_T = V$$

$$\left(\frac{\partial g}{\partial T}\right)_P = -S$$

9.3 MAXWELL'S EQUATIONS

Maxwell's equations interrelate the partial derivatives of thermodynamic properties. In this section, we consider the properties: pressure (p), volume (V), temperature (T) and entropy (S) of a compressible substance.

Using first and second laws of thermodynamics, $dU = TdS - pdV$

If E is considered to be function of S and V; $dU(S, V) = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$

For an exact differential, $\frac{\partial}{\partial V}\left(\left(\frac{\partial U}{\partial S}\right)_V\right)_V = \frac{\partial}{\partial S}\left(\left(\frac{\partial U}{\partial V}\right)_S\right)_V \Rightarrow \left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$

Similarly, using the relations:

Enthalpy: $dH = dU + pdV + VdP = TdS + Vdp$;

Helmholtz free energy: $dF = dU - TdS - SdT = -pdV - SdT$

Gibbs free energy: $dG = dH - TdS - SdT = Vdp - SdT$

Total four following relations are developed:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$$

Example 9.1

For cooling by adiabatic expansion, find the expression for temperature change in terms of pressure, initial temperature.

○●●

Solution We know that,

$$c_V = \left(\frac{\partial Q}{\partial T}\right)_V = \frac{T}{n}\left(\frac{\partial S}{\partial T}\right)_V$$

Step 1: Using the Jacobian

$$\frac{T}{n}\left(\frac{\partial S}{\partial T}\right)_V = \frac{T}{n}\left(\frac{\partial(S,V)}{\partial(T,V)}\right)$$

Step 2: Splitting the Jacobean by chain rule

$$\frac{T}{n}\left(\frac{\partial(S,V)\partial(T,p)}{\partial(T,p)\partial(T,V)}\right)$$

Step 3: Since, $\left(\frac{\partial P}{\partial V}\right)_T$ is isothermal compressibility, *k*, substituting it

$$\frac{T}{n}\left(\frac{\partial(S,V)\partial(T,p)}{\partial(T,p)\partial(T,V)}\right) = -\frac{T}{nVk}\left(\frac{\partial(S,V)}{\partial(T,p)}\right)$$

Step 4: Multiply out by the determinant

$$= -\frac{T}{nVk}\left(\left(\frac{\partial S}{\partial T}\right)_p \left(\frac{\partial V}{\partial p}\right)_T - \left(\frac{\partial S}{\partial p}\right)_T \left(\frac{\partial V}{\partial T}\right)_p\right)$$

Step 5: Using the Maxwell equation, $\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T = \alpha$

Here α isobaric expansion coefficient,

$$-\frac{T}{nVk}\left(\left(\frac{\partial S}{\partial T}\right)_p \left(\frac{\partial V}{\partial p}\right)_T - \left(\frac{\partial S}{\partial P}\right)_T \left(\frac{\partial V}{\partial T}\right)_p\right) = -\frac{T}{nVk}\left(\left(\frac{nc_p}{T}\right)\left(\frac{V}{-k}\right) - V\alpha\right)$$

So,

$$c_V = -\frac{T}{nVk}\left(\left(\frac{nc_p}{T}\right)\left(\frac{V}{-k}\right) - V\alpha\right) = c_p - \frac{VT\alpha^2}{nk}$$

Example 9.2

Using the Maxwell relations write down constant volume heat capacity in terms of energy. ●●●

Solution Constant volume heat capacity is given by:

$$c_V = \left(\frac{\partial Q}{\partial T}\right)_V = \frac{T}{n}\left(\frac{\partial S}{\partial T}\right)_V \tag{1}$$

Here,

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{T}{n}\left(\frac{\partial(S,V)}{\partial(T,V)}\right)_V$$

$$= \frac{T}{n}\left(\frac{\partial(S,V)\partial(U,V)}{\partial(U,V)\partial(T,V)}\right)_V$$

$$= \left(\frac{\partial S}{\partial U}\right)_V \left(\frac{\partial U}{\partial T}\right)_V$$

Now since, $\left(\frac{\partial S}{\partial U}\right)_V = \frac{1}{T}$

So, $\left(\frac{\partial S}{\partial U}\right)_V \left(\frac{\partial U}{\partial T}\right)_V = \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V$

Substituting it in equation (1),

$$c_V = \left(\frac{\partial Q}{\partial T}\right)_V = \frac{T}{n}\left(\frac{\partial S}{\partial T}\right)_V = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V$$

Example 9.3

If the volume per unit mass of a particular system near by a state T_1, V_1 is given by the relation:

$$V = V_1 + p(T - V_1) + q(p - p_1)$$

If a small volume change takes place by supplying dQ amount of heat at constant temperature, find out the dQ. ●●●

Solution We know that,

$$dQ = T\left(\frac{\partial S}{\partial V}\right)_T dV \tag{1}$$

Using the Maxwell relation,

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

So, using equation 1:

$$dQ = T\left(\frac{\partial p}{\partial T}\right)_V dV \tag{2}$$

Here, given relation is:

$$V = V_1 + p(T - V_1) + q(p - p_1)$$

Dividing both sides with V as constant:

$$(0 = 0 + pdT + gdp) \text{ at constant } V$$

$$\left(\frac{\partial p}{\partial T}\right)_V = -\frac{p}{q}$$

Using equation 2 now,

$$dQ = T\left(\frac{\partial p}{\partial T}\right)_V dV = -\frac{pT}{q} dV$$

Example 9.4

Verify the validity of the last Maxwell relation using steam table data. ○○●

Solution The last Maxwell's equation is:

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial P}\right)_T$$

Since, we do not have exact property relations for steam, so by replacing the differential quantities with corresponding finite quantities:

$$\left(\frac{\Delta V}{\Delta T}\right)_p = -\left(\frac{\Delta S}{\Delta P}\right)_T$$

$$\left(\frac{V_{900F} - V_{700F}}{900^{\ominus} F - 700^{\ominus} F}\right)_{P=400\ psia} = -\left(\frac{S_{450F} - S_{350F}}{450^{\ominus} F - 350^{\ominus} F}\right)_{800\ F}$$

$$\left(\frac{1.9777 - 1.9507}{900^{\ominus} F - 700^{\ominus} F}\right)_{P=400\ psia} = -\left(\frac{1.6706 - 1.7009}{450^{\ominus} F - 350^{\ominus} F}\right)_{800F}$$

$$-1.639 \times 10^{-3} \approx -1.635 \times 10^{-3}$$

This verifies the desired Maxwell's equation.

9.4 || TdS RELATIONS

University Questions

1. Deduce TdS relations. **(AKTU, 2013–2014)**
2. With the help of Maxwell's equations deduce two TdS equations. **(AKTU, 2011–2012)**

9.4.1 TdS Equations

First TdS Equation

Considering entropy as a function of temperature and volume,

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV \Rightarrow TdS = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV \tag{9.9}$$

Specific heat at constant volume,

$$c_V = T\left(\frac{\partial S}{\partial T}\right)_V \tag{9.10}$$

Using equations (9.9), (9.10) and Maxwell's relation

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$TdS = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$$

Second TdS Equation

Now, instead of temperature and volume, if entropy is considered function of temperature and pressure, then:

$$dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dV \Rightarrow TdS = T\left(\frac{\partial S}{\partial T}\right)_p dT + T\left(\frac{\partial S}{\partial p}\right)_T dp \tag{9.11}$$

Specific heat at constant pressure, $c_p = T\left(\frac{\partial S}{\partial T}\right)_p$ (9.12)

Using (9.11), (9.12) and Maxwell's relation

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

$$TdS = c_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$$

9.4.2 Volume Change Coefficients

- Volume expansion coefficient:

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \left(\frac{\partial \ln V}{\partial T}\right)_p$$

- Isothermal compressibility:

$$k = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\left(\frac{\partial \ln V}{\partial p}\right)_T$$

- Adiabatic compressibility:

$$ks = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s = -\left(\frac{\partial \ln V}{\partial p}\right)_s$$

- Isothermal modulus of elasticity:

$$Et = \frac{1}{k} = -\frac{1}{\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T} = -\frac{1}{\left(\frac{\partial \ln V}{\partial p}\right)_T}$$

- Adiabatic modulus of elasticity:

$$Es = \frac{1}{Ks} = \frac{1}{-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T} = -\frac{1}{\left(\frac{\partial \ln V}{\partial p}\right)_T}$$

Example 9.5

For a reversible isochoric process, temperature of the system changes from 300 K to 400 K, find out change in entropy. Take $c_V = 0.718$ kJ/kg K. ○●●

Solution Given,

Initial temperature, $T_1 = 300$ K
Final temperature, $T_2 = 400$ K
Specific heat, $c_V = 0.718$ kJ/kgK
Assumption: Specific heat is constant throughout the process.
Analysis:
Here, since change in volume is zero (isochoric process), we will use first *TdS* equation:

$$TdS = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV \quad \text{or} \quad dS = \frac{c_V dT}{T} + \frac{T\left(\frac{\partial p}{\partial T}\right)_V dV}{T} \tag{1}$$

Since here, $dV = 0$; (1) becomes:

$$dS = \frac{c_V dT}{T}$$

$$\int_{S_1}^{S_2} dS = \int_{T_1}^{T_2} \frac{c_V dT}{T}$$

$$S_2 - S_1 = 0.718 \int_{300}^{400} \frac{dT}{T}$$

$$S_2 - S_1 = 0.718 \ln\left(\frac{400}{300}\right) = 0.207 \text{ kJ / K}$$

Example 9.6

2 kg of ideal gas undergoes a reversible isothermal process. Pressure of the gas changes from 2.5 bar to 4 bar. Find out the heat absorbed by the gas. Temperature of gas is maintained at 500 K. ○●●

Solution Initial pressure, $P_1 = 2.5$ bar

Final pressure, $P_2 = 4$ bar
Temperature of the gas: 500 K
Mass of the gas: 2 kg

Assumption: Gas is ideal

Process is isothermal, so $dT = 0$. Using second *TdS* equation,

$$\Delta Q = m\Delta Q = mT\Delta S = 0 - m\int_{p_1}^{p_2} T\,V\,\beta dp$$

$$\Delta Q = -m\int_{p1}^{p2} V\,dP = -mRT\int_{p1}^{p2}\frac{dp}{p}$$

$$\Delta Q = -2\times 0.285\times 500\int_{2.5}^{4}\frac{dp}{p}$$

$$\Delta Q = -285\,\ln\left(\frac{4}{2.5}\right) = -133.95\text{ kJ}$$

Negative sign shows the heat rejection.

Example 9.7

Find out the energy released in an isothermal compression of a matter. Assume the process reversible in terms of temperature, change in pressure and coefficient of isochoric expansion. ○○●

Solution Using II equation of *TdS* relation:

$$TdS = c_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$$

Since process is isochoric, $dT = 0$ and the expression gets reduced in the form:

$$TdS = -T\left(\frac{\partial V}{\partial T}\right)_p dp$$

Coefficient of isochoric expansion, $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

So, $TdS = -TV\,\beta\,dp$

Heat released = $TdS = -TV\,\beta\,dp$

9.5 || JOULE-KELVIN EFFECT

University Question

1. With the help of neat sketch explain Joule's experiment. **(AKTU, 2011–2012)**

Sudden pressure change over a valve is found to result in a change in temperature and this is known as Thomson-Kelvin effect. The numerical value of the slope of an isenthalpic on a T-p curve at any point is called the Joule-Kelvin coefficient (μ_J) (Figure 9.1).

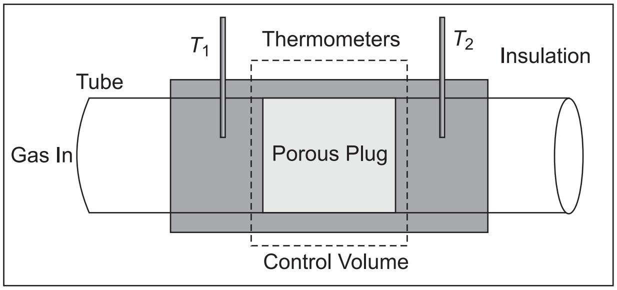


Figure 9.1 *The Joule-Thomson experiment*

Figure 9.1 shows an arrangement for continuous throttling of a gas. System is isolated from the surrounding. Pressure(s) and temperature(s) of the gas at two points (one at inlet and one at outlet) are to be recorded. Gas is throttled to different pressures and temperatures by controlling the valve and measurements are recorded for different equilibrium conditions.

Since system is isolated, enthalpy of the gas during the throttling does not change. So, when plotted, these measurements of pressure and temperature form isenthalpic curve or an isenthalpic as shown in Figure 9.2.

Now, the same process is repeated with different initial (or inlet) temperatures and pressures. Such iteration will result a family of isenthalpic curves (one curve for a particular initial condition). The curve passing through the maxima of this family of isenthalpic curves is called inversion curve.

This inversion curve provides information about the cooling, which can be achieved by the throttling process. Region on the left side of the inversion curve on Temperature-Pressure plane is called cooling region and since the slope of the curve is positive, throttling

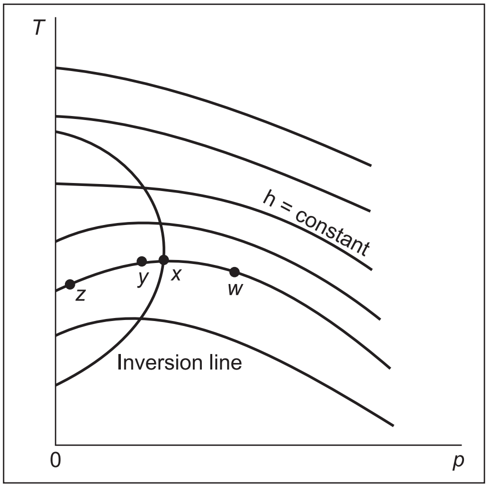


Figure 9.2 *Isenthalpic curves and inversion curve*

(i.e. decrease in pressure) will always produce cooling effect. Maximum cooling will be achieved if the initial state lies on the inversion curve.

Since during throttling, enthalpy of gas remains constant,

$$dH = dU + pdV + Vdp = 0$$

Using, first and second laws of thermodynamics, $dQ = TdS + Vdp = 0$ (9.13)

Writing S as f (P,T), $dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$. Substituting it in equation (9.13);

$$dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left[T\left(\frac{\partial S}{\partial p}\right)_T + V\right]dp = 0; \tag{9.14}$$

Since, $c_p = T\left(\frac{\partial S}{\partial T}\right)_p$ and $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$, substituting these in equation (9.14):

$$c_p dT = -\left[T\left(\frac{\partial S}{\partial p}\right)_T + V\right]dp \Rightarrow c_p dT = \left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]dp$$

$$\Rightarrow \qquad dT = \frac{1}{c_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]dp \tag{9.15}$$

Joule-Thomson coefficient, $\mu_j = \left(\frac{\partial T}{\partial p}\right)_H$

Using relation (9.15); $\mu_j = \left(\frac{\partial T}{\partial p}\right)_H = \frac{1}{c_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]$ (9.16)

If α is the isobaric coefficient of volume expansion, i.e., $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ $dT \Rightarrow \frac{V}{c_p}[\alpha T - 1]dp$

For a perfect gas:

$$pV = RT \Rightarrow \left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p} \Rightarrow \left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] = 0$$

Since for a perfect gas, $dT = 0$, $\mu_j = \left(\frac{\partial T}{\partial p}\right)_H = \left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right] = 0$

Example 9.8

One mole gas undergoes a throttling process at 100 atm pressure and 29°C temperature. If the final pressure is found to be 1 atm, calculate final temperature of the gas. Given that joule kelvin coefficient of throttling is 0.150 K/atm. ○○●

Solution Inlet pressure and temperature, P_1, T_1 = 100 atm, 29°C

Outlet pressure, P_2, = 1 atm

Joule kelvin coefficient of throttling μj = 0.150 K/atm.

Outlet temperature T_2 =?

Assumptions: 1. Steady flow process so there is no change with time; 2. Kinetic and potential energies are negligible; 3. Heat transfer to and from the gas is negligible; 4. There is no work interaction.

Analysis: Since during throttling enthalpy remains constant, so:

$$\mu_\mathrm{j} = \left(\frac{\partial T}{\partial p}\right)_H = \frac{\Delta T}{\Delta p}$$

$$\Delta T = \mu_\mathrm{j}\,\Delta p$$

$$\Delta T = 0.150 \times (100 - 1) = -14.85 \text{ K}$$

$$T_f = T_{in} + \Delta T = (29 + 273) - 14.85 = 287.15 \text{ K} = 14.15°\text{C}$$

Example 9.9

Incompressible fluids cannot be used for cooling by throttling process. Justify the statement. ○●●

Solution Joule-Kelvin coefficient for a coolant is given as:

$$\mu_j = \left(\frac{\partial T}{\partial p}\right)_H$$ which can be written as:

$$\mu_j = \frac{1}{c_p}\left(\left(\frac{\partial V}{\partial T}\right)_p - V\right)$$

For an incompressible solid or gas:

$$\left(\frac{\partial V}{\partial T}\right)_p = 0$$

$$\mu_j = \frac{1}{c_p}\left(\left(\frac{\partial V}{\partial T}\right)_p - V\right) = \frac{1}{c_p}(0 - V) = -\frac{V}{c_p}$$

Both c_p and V are positive, so μj is negative for incompressible liquid or solid. This means that if pressure is decreased temperature will increase, so if such gas/liquid when undergoes the throttling will generate heat, not cooling.

Example 9.10

State equation of a gas is given as: $p(V - \alpha) = RT$. Find out the locus p–V for inversion temperature.

●●●

Solution **Step 1:** Write expression for joule coefficient

$$\mu_j = \frac{1}{cp}\left(\left(\frac{\partial V}{\partial T}\right)_p - V\right)$$

For the given here, state equation is: $p(V - \alpha) = RT$

Step 2: Find out $\left(\frac{\partial V}{\partial T}\right)_p$

Differentiating the state equation with p as constant

$$pdV - 0 = RdT$$

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p}$$

Step 3: Find inversion temperature

$$\mu_j = \frac{1}{cp}\left(\left(\frac{\partial V}{\partial T}\right)_p - V\right)$$

$$\mu_j = \frac{1}{c_p}\left(\frac{R}{p} - V\right)$$

At inversion temperature $\mu j = 0$.

So $$\frac{1}{c_p}\left(\frac{R}{p} - V\right) = 0 \quad \Rightarrow \quad \frac{R}{p} - V = 0 \text{ or } R = pV \qquad (1)$$

From state equation: $R/p = (V - \alpha)/T$, So, substituting it in expression (1):

$$T = 1 - \frac{p\alpha}{R}$$

9.6 || CLAUSIUS-CLAPEYRON EQUATION

Clausius-Clapeyron equation is used as a base to calculate fluid properties for a two-phase flow. It provides an important relation between saturation pressure, temperature, enthalpy of evapouration and specific volume of two phases involved.

9.6.1 Equation

Using Maxwell's equation- $\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$

If, phase transition occurs from pure saturated liquid to pure saturated vapour (which occurs at constant temperature and pressure).

$$\frac{dp}{dT} = \frac{S_g - S_f}{V_g - V_f}$$

$$S_g - S_f = \frac{H_g - H_f}{T}$$

$$\Rightarrow \qquad \frac{dp}{dT} = \frac{S_g - S_f}{V_g - V_f} = \frac{H_g - H_f}{T(V_g - V_f)} \tag{9.17}$$

The equation is known as Clausius-Clapeyron equation. With using, $\frac{dp}{dT}$ *i.e.* slope of vapour pressure *v*/*s* temperature curve, specific volumes (V_g, V_f), enthalpy of evapouration ($h_g - h_f$) can be determined experimentally.

At low pressure, $V_g - V_f \approx V_g$, equation (9.17) can be written as:

$$\frac{dp}{dT} = \frac{H_g - H_f}{TV_g}$$

Assuming vapour as an ideal gas $\left(V_g = \frac{RT}{p}\right)$, $\frac{dp}{dT} = \frac{(H_g - H_f)p}{RT^2}$

$$\frac{dp}{dT} = \frac{H_{fg}p}{RT^2} \Rightarrow \frac{dp}{p} = \frac{H_{fg}}{R}\frac{dT}{T^2} \tag{9.18}$$

Integrating equation (9.18) both sides, $\int_{p_1}^{p_2} \frac{dp}{p} = \frac{H_{fg}}{R}\int_{T_1}^{T_2} \frac{dT}{T^2}$

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{H_{fg}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \tag{9.19}$$

Enthalpy of vapourisation can be found using equation (9.18) or (9.19).

Note: $V_g - V_f > 0$ and $H_g - H_f > 0$, so, $\left(\frac{dp}{dT}\right)_{\text{vapour pressure}}$ is always positive.

Slope of melting curve for substances that contract on freezing is negative.

Example 9.11

Evaluate the change in specific enthalpy and specific internal energy for water at vapourisation. Given that slope of *p*-*T* curve for saturation at 100°C is equal to 3,571 N/m^2 K and specific volumes of vapour and water are 1.673m^3 and 1.0435m^3 respectively. ○○●

Solution Given,

At 100°C: $V_g = 1.673\text{m}^3$; $V_f = 1.0435\text{m}^3$ and $\dfrac{dp}{dT} = 3{,}571$ N/m^2 K

From Clausius-Clapeyron equation, at vapourisation,

$$\frac{dp}{dT} = \frac{S_g - S_f}{V_g - V_f}$$

$$S_g - S_f = (V_g - V_f)\frac{dp}{dT}$$

$$S_g - S_f = 3{,}571(1.673 - 1.0435)$$

$$S_g - S_f = 5{,}971\frac{\text{J}}{\text{KgK}} = 5.971\frac{\text{kJ}}{\text{KgK}}$$

Example 9.12

For water, at 100°C, find out change in specific entropy at vapourisation. Given that slope of *p*-*T* curve for saturation at 100°C is equal to 3,571 N/m^2 K, specific volumes of vapour and water are 1.673m^3 and 1.0435m^3 respectively $P_{\text{saturation}} = 1.014$ bar. ○●●

Solution Given,

At 100°C: $V_g = 1.673\text{m}^3$; $V_f = 1.0435\text{m}^3$; $\dfrac{dp}{dT} = 3{,}571$ N/m^2 K and $P_{\text{saturation}} = 1.014$ bar.

(a) From Clausius-Clapeyron equation, at vapourisation,

$$\frac{dp}{dT} = \frac{H_g - H_f}{T(V_g - V_f)}$$

$$H_g - H_f = T(V_g - V_f)\frac{dp}{dT}$$

$$H_g - H_f = (273 + 100)(3{,}571)(1.673 - 1.0435)$$

$$H_g - H_f = 2{,}727 \text{ kJ / kg}$$

(b) From Clausius-Clapeyron equation, at vapourisation,

$$U_g - U_f = (H_g - H_f) - P_{\text{Saturation}}(V_g - V_f)$$

$$U_g - U_f = (2727) - (1.014 \times 1000)(1.673 - 1.0435)$$

$$U_g - U_f = 2{,}057.5 \,\frac{\text{kJ}}{\text{kg}}$$

Example 9.13

Find out the $\Delta H_{\text{vaporisation}}$ if pressure and temperature during the vapourisation changes from 5 bar, 18°C to 1 bar, –23°C. Take $R = 8.314$ J/K. ○○●

Solution Given,

$$p_1 = 5 \text{ bar};\ p_2 = 1 \text{ bar};\ T_1 = 18°\text{C} = 296 \text{ K};\ T_2 = -23°\text{C} = 248 \text{ K}$$

Change in pressure and temperature at vapourisation is related as:

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{H_{fg}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \quad \text{Here } \Delta H_{\text{vaporisation}} = H_{fg}$$

$$\Delta H_{\text{vaporisation}} = R \ln \frac{\left(\dfrac{p_2}{p_1}\right)}{\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)}$$

$$\Delta H_{\text{vaporisation}} = 0.008314 \ln \frac{\left(\dfrac{1}{5}\right)}{\left(\dfrac{1}{296} - \dfrac{1}{248}\right)} = 20.4 \text{ kJ}$$

Example 9.14

If during vapourisation initial and final pressure are found to be 0.8 bar and 0.1 bar respectively with $\Delta H_{\text{vaporisation}}$ is evaluated to be equal to 35,255.61 J. Find out final temperature, if initial temperature is 298 K. Take $R = 8.314$ J/K. ○●●

Solution Given,

$$p_1 = 0.8 \text{ bar};\ p_2 = 0.1 \text{ bar};\ T_1 = 298 \text{ K}\ \Delta H_{\text{vaporisation}} = 35{,}255.61 \text{ J}$$

Change in pressure and temperature at vapourisation is related as:

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{H_{fg}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

Here, $\Delta H_{\text{vaporisation}} = h_{fg}$

$$\ln\left(\frac{0.8}{0.1}\right) = \frac{35{,}255.61}{8.314}\left(\frac{1}{T_2} - \frac{1}{298}\right)$$

$$2.079 = \frac{36{,}255.61}{8.31}\left(\frac{1}{T_2} - 0.003355\right)$$

Solving the equation for T_2,

$T_2 = 349$ K

9.7 || GIBBS PHASE RULE

Number of degrees of freedom (F) or variance of a chemical system is determined by Gibbs phase rule. It is useful for interpreting the phase diagrams.

According to this rule, for a non-reactive multi-component heterogeneous systems in thermodynamic equilibrium, degree of freedom (F), number of components (C) and number of phases (P) are related as:

$$F = 2 + C - P$$

Here, number of degrees of freedom is the number of independent intensive variables, i.e. the largest number of thermodynamic parameters such as T, P which may vary simultaneously and arbitrarily without having any impact on another one.

Example 9.15

A mixture contains water, liquid benzene and oxygen, what is the degree of freedom? ○○●

Solution Here, number of components $C = 3$ (water, liquid benzene and oxygen)

Number of phase = 2 (Liquid and gas)

According to Gibbs phase rule,

Degree of freedom, $F = 2 + C - P = 2 + 3 - 2 = 3$

9.8 || GENERAL RELATIONS

The variation in internal energy, enthalpy, and entropy of a simple, compressible substance can be expressed in terms of pressure, specific volume, temperature, and specific heats alone.

University Question

1. Explain the phenomena of resonance in AC parallel circuit. Derive the mathematical expression of resonant frequency. **(GTU, 2011)**

9.8.1 Internal Energy

Using first and second laws of thermodynamics, $dU = TdS - pdV$

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

Since, $c_V = \left(\frac{\partial U}{\partial T}\right)_V \Rightarrow dU = c_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \qquad (9.20)$

Now, writing

$$U = f(S, V) \Rightarrow dU = \left(\frac{\partial U}{\partial S}\right)_V dV + \left(\frac{\partial U}{\partial V}\right)_S dV \Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = \left(\frac{\partial U}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T + \left(\frac{\partial U}{\partial V}\right)_S \qquad (9.21)$$

$$\left(\frac{\partial U}{\partial S}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T ; \left(\frac{\partial U}{\partial S}\right)_V = T \text{ and } \left(\frac{\partial U}{\partial V}\right)_S = -p$$

Substituting these relations in equation (9.21);

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - p \qquad (9.22)$$

Putting (9.22) in equation (9.20):

$$dU = c_V dT + \left(T\left(\frac{\partial p}{\partial T}\right)_V - p\right) dV$$

9.8.2 Enthalpy

Similarly, considering $H = f(T, p)$.

$$dH = c_p\, dT + \left(\frac{\partial H}{\partial p}\right)_T dp$$

$$H = f(S, p); dH = \left(\frac{\partial H}{\partial S}\right)_p dS + \left(\frac{\partial H}{\partial p}\right)_S dp \Rightarrow \left(\frac{\partial H}{\partial p}\right)_T = \left(\frac{\partial H}{\partial S}\right)_p \left(\frac{\partial S}{\partial p}\right)_T + \left(\frac{\partial H}{\partial p}\right)_S \qquad (9.23)$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial p}\right)_p ; \left(\frac{\partial H}{\partial S}\right)_p = T \text{ and } \left(\frac{\partial H}{\partial p}\right)_S = V$$

Substituting these relations in equation (9.23);

$$\left(\frac{\partial S}{\partial p}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_p \qquad (9.24)$$

Putting (9.24) in equation (9.23):

$$dH = c_p\, dT + \left(V - T\left(\frac{\partial V}{\partial T}\right)_p\right) dp$$

9.8.3 Entropy

From the *TdS* equations, change in entropy,

$$dS = c_V \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_V dV$$

$$dS = c_p \frac{dT}{T} - T\left(\frac{\partial V}{\partial T}\right)_p dp$$

POINTS TO REMEMBER

☞ Maxwell relations depict relationship between measurable quantities and entropy.

Total four following relations that are developed are:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V ;$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P ;$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T ;\text{ and}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

☞ Joule-Kelvin coefficient (μ_j) is the numerical value of the slope of *T-p* curve at any point for an isenthalpic process.

$$\mu_j = \left(\frac{\partial T}{\partial p}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]$$

For an ideal gas: $\mu_j = 0$.

☞ First TdS equation is given as $TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$

☞ Second TdS equation is given as $TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$

☞ Clausius-Clapeyron equation: The equation is used to find out enthalpy of vapourisation using measureable quantities, i.e. pressure, volume and temperature.

$$\frac{dp}{dT} = \frac{H_g - H_f}{T(V_g - V_f)}$$

PRACTICE PROBLEMS

1. A liquid is compressed adiabatically (and reversibly) from specific volume 1.073 m^3 to 1.0435 m^3 at temperature 90°C. if coefficients of isochoric and isobaric expansion are 16 and 37 respectively. Find out change in volume. $c_V = 0.140$ kJ/kg K ○●●

2. Show that Joule-Thomson coefficient is given by: $\mu_j = \frac{T^2}{cp}\left(\frac{\partial\left(\frac{V}{T}\right)}{\partial T}\right)_p$ ○●●

3. If state equation of a gas is given as $pV = RT + \alpha p$. Find out expression for relation between inversion temperature and pressure. (**Ans:** $\alpha P^2 + RPT - R = 0$) ●●●

4. Find out the maximum inversion temperature for the situation given in question 3. (**Ans:** $T = \sqrt{(\alpha / R)} + \sqrt{(\alpha)}R^{3/2}$) ●●●

5. If $c_p - c_V$ is 3 kJ/K at 1,000 K temperature. Find out specific volume. Given that coefficients of isochoric and isobaric expansion are 6.5×10^{-5} and 9.5×10^{-12} respectively. (**Ans:** $V = 7.5 \times 10^{-3}$ m^3) ○○●

6. If during vapourisation initial and final temperature are found to be 298 K and 349 K respectively with $\Delta H_{\text{vaporisation}}$ is evaluated to be equal to 35,255.61 J. Find out final pressure, if initial pressure is 0.8 bar. Take $R = 8.314$ J/K. (**Ans:** $P = 0.1$ bar) ○○●

7. Find degree of freedom for: (a) a mixture of ice and water, (b) a system with pure water. (**Ans:** (a) 1, (b) 2) ○○●

MULTIPLE CHOICE QUESTIONS

1. Joule-Kelvin coefficient for non-compressible substance is: ○●●

(a) Zero (b) Positive (c) Negative (d) Infinite

2. At inversion temperature, slope of T-p curve is: ○●●

(a) 1 (b) 0 (c) Infinite (d) 1.732

3. Value of specific heat c_p is given by: ○●●

(a) $T\left(\frac{\partial V}{\partial S}\right)_p$ (b) $T\left(\frac{\partial S}{\partial T}\right)_p$ (c) $T\left(\frac{\partial s}{\partial v}\right)_p$ (d) $T\left(\frac{\partial T}{\partial V}\right)_p$

4. Degrees of freedom of a system with 5 phases and 5 components is: ○●●

(a) 4 (b) 3 (c) 2 (d) 1

5. Read following statements regarding resonance in an electric circuit: ○●●

I. $TdS = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$

II. $TdS = c_p dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$

III. $TdS = c_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp$

IV. $TdS = c_p dT + T\left(\frac{\partial V}{\partial T}\right)_p dp$

Which one of above is correct:
(a) I only (b) I, II (c) I and III (d) II and IV

6. For getting *TdS* equations, we assume entropy to be a function of T and V and also of T and p. ○●●
(a) True (b) False

7. The equation $dU = c_V \times dT$ holds good for: ○●●
(a) Any process for an ideal gas, even when the volume changes
(b) For other substances, it is true only when the volume is constant
(c) Both of the mentioned
(d) None of the mentioned

8. The energy equation is given by: ○●●
(a) $(\partial U/\partial V) = T \times (\partial p/\partial T) + p$ (b) $(\partial U/\partial V) = T \times (\partial p/\partial T) - p$
(c) $(\partial U/\partial V) = -T \times (\partial p/\partial T) - p$ (d) $(\partial U/\partial V) = p - T \times (\partial p/\partial T)$

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (c) 2. (b) 3. (b) 4. (c) 5. (c)
6. (a) 7. (c) 8. (b)

Vapour Power Cycle

10

Chapter Outline

- ☞ Vapour Power Cycles
- ☞ Rankine Cycle
- ☞ Actual Vapour Cycle Processes
- ☞ Piping Losses
- ☞ Turbine Losses
- ☞ Pump Losses
- ☞ Condenser Losses
- ☞ Reheat Cycle
- ☞ Ideal Regenerative Cycle
- ☞ Regenerative Cycle
- ☞ Reheat-Regenerative Cycle
- ☞ Binary Vapour Cycle

INTRODUCTION

Power cycles are those cycles which continuously convert heat into work. There are two types of power cycles namely vapour power cycle and gas power cycle. This chapter contains details about vapour power cycles, where the working fluid is alternatively vapourised and condensed. Due to the continued demand for higher thermal efficiencies some innovative modifications to the basic vapour power cycle are required. Among these, *reheat* and *regenerative cycles,* as well as *combined gas-vapour power cycles* are important to consider. As steam has many desirable characteristics such as low cost, availability, and high enthalpy of vapourisation, hence, it is the most common working fluid used in vapour power cycles. Coal plants, nuclear plants, or natural gas plants are examples of steam power plants.

10.1 VAPOUR POWER CYCLES

Heat is transferred to water in the boiler from an external source (furnace, where fuel is continuously burnt) to produce steam. This high pressure, high temperature steam leaves the boiler and expands in the turbine to produce shaft work. The steam leaving the turbine condenses into water in the condenser (where cooling water circulates), rejecting heat, and then the water is pumped back to the boiler. A unit mass of the working fluid, sometimes in the liquid phase and sometimes in the vapour phase, undergoes various external heat and work interactions in executing a power cycle. Since the fluid is undergoing a cyclic process, there will be no net change in its internal energy over the cycle, and consequently the net energy transferred to the unit mass of the fluid as heat during the cycle must be equal to the net energy transfer as work from the fluid (Figure 10.1).

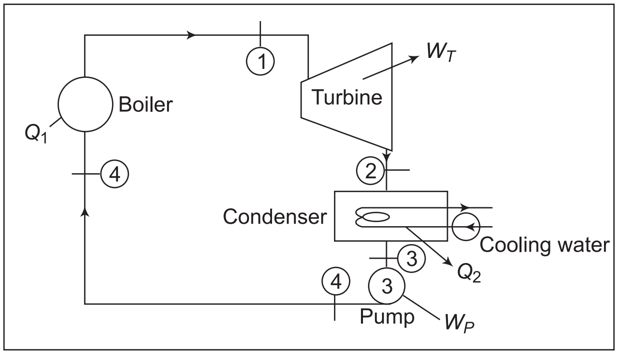


Figure 10.1 *A simple steam plant*

10.2 || RANKINE CYCLE

University Question

1. Why study of Rankine cycle is important? Describe. **(AKTU, 2008–2009)**

Two important areas of application for thermodynamics are power generation and refrigeration. Thermodynamic cycles can also be categorised as gas cycles and vapour cycles, depending on the phase of the working fluid. Thermodynamic cycles can also be categorised as power producing cycles and power absorbing cycles. Gas power cycles are those cycles in which working fluid is gas which remains in gaseous phase throughout the cycle.

Vapour power cycles are those cycles in which working fluid exists in both liquid and vapour phases. For the first half of the cycle, the working fluid is in liquid phase while for the next half it is in vapour phase. In a gas cycle, air is taken as standard working fluid while for vapour cycle water is taken as standard working fluid because both are easily available and fulfil almost all of the desirable characteristics of the cycle.

Rankine cycle is an ideal cycle for all vapour power cycles and is used mostly for comparison with other cycles. It is a reversible cycle. Figure 10.2 shows the p-V, T-S and H-S diagram of the Rankine cycle and Figure 10.3 shows the cycle being plotted on the T-S plane. The numbers on the plots correspond to the numbers on the flow diagram. For any given pressure, the steam approaching the turbine may be dry saturated (state 1), wet (state $1'$), or superheated (state $1''$), but the fluid approaching the pump is, in each case, saturated liquid (state 3). Steam expands reversibly and adiabatically in the turbine from state 1 to state 2 (or $1'$ to $2'$, or $1''$ to $2''$); the steam leaving the turbine condenses to water in the condenser reversibly at constant pressure from state 2 (or $2'$, or $2''$) to state 3; the water at state 3 is then pumped to the boiler at state 4 reversibly and adiabatically, and the water is heated in the boiler to form steam reversibly at constant pressure from state 4 to state 1 (or $1'$ or $1''$).

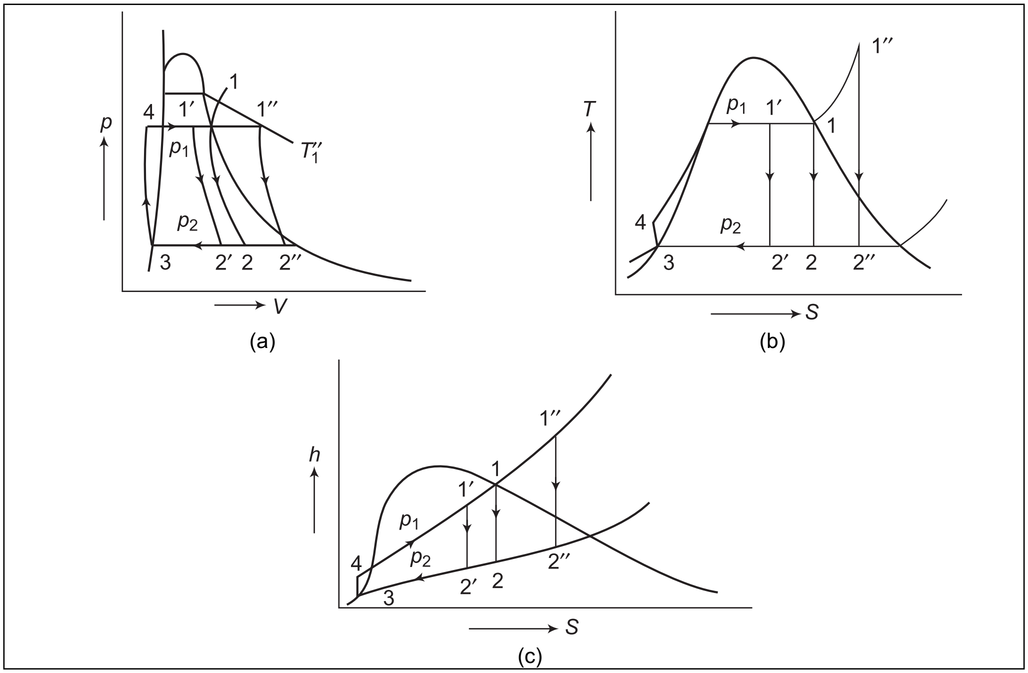


Figure 10.2 *Rankine cycle on p-V, T-S and H-S diagrams*

Rankine cycle is an ideal cycle in which all the four processes are ideal (i.e. all the four processes are reversible). The four processes which constitute the cycle are as follow:

- Process 4-1: Constant pressure heat addition in the boiler (phase changes from liquid to vapour)
- Process 1-2: Reversible adiabatic or isentropic expansion of steam in the turbine
- Process 2-3: Constant pressure heat rejection in the condenser (phase changes from vapour to liquid)
- Process 3-4: Reversible adiabatic or isentropic compression of liquid in the pump

For purposes of analysis the Rankine cycle is assumed to be carried out in a steady flow operation. Applying the steady flow energy equation to each of the processes on the basis of unit mass of fluid, and neglecting changes in kinetic and potential energy, the work and heat quantities can be evaluated in terms of the properties of the fluid. For 1 kg of working fluid.

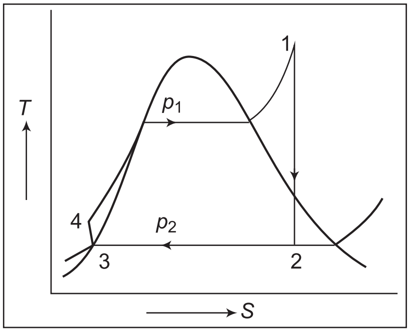


Figure 10.3 *Rankine cycle on T-S diagrams*

The SFEE for the boiler (control volume) gives:

$$H_4 + Q_1 = H_1$$

$$Q_1 = H_1 - H_4$$

The SFEE for the turbine as the control volume gives:

$$H_1 = W_T + \mathrm{H}_2$$

$$W_T = H_1 - H_2$$

Similarly, the SFEE for the condenser is:

$$H_2 = Q_2 + H_3$$

$$Q_2 = H_2 - H_3$$

and the SFEE for the pump gives:

$$H_3 + W_P = H_4$$

$$W_P = H_4 - H_3$$

The efficiency of the Rankine cycle is then given by:

$$\eta = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1} = \frac{(H_1 - H_2) - (H_4 - H_3)}{(H_1 - H_4)}$$

The pump handles liquid water which is incompressible, i.e., its density or specific volume undergoes little change with an increase in pressure. For reversible adiabatic compression, by the use of the general property relation:

$$TdS = dH - Vdp;\ dS = 0$$

$$dH = Vdp$$

Since change in specific volume is negligible:

$$\Delta H = V\Delta p$$

$$H_4 - H_3 = V_3(p_1 - p_2)$$

$$H_4 - H_3 = V_3(p_1 - p_2) \times 10^5 \text{ J/kg}$$

Usually, the pump work is quite small compared to the turbine work and is sometimes neglected. Then $H_4 = H_3$, and the cycle efficiency approximately becomes:

$$\eta = \frac{(H_1 - H_2)}{(H_1 - H_4)}$$

The capacity of a steam plant is often expressed in terms of *steam rate*, which is defined as the rate of steam flow (kg/h) required to produce unit shaft output (1 kW). Therefore,

$$\text{Steam rate} = \frac{1}{W_T - W_P} \frac{\text{kg}}{\text{kJ}} \frac{1\text{ kJ/s}}{1\text{ kW}}$$

$$= \frac{1}{W_T - W_P} \frac{\text{kg}}{\text{kWs}}$$

$$= \frac{3600}{W_T - W_P} \frac{\text{kg}}{\text{kWh}}$$

The cycle efficiency is sometimes expressed alternatively as *heat rate* which is the rate of heat input (Q_1) required to produce unit work output (1 kW)

$$\text{Heat rate} = \frac{3600\ Q_1}{W_T - W_P} = \frac{3600}{\eta_{\text{cycle}}} \frac{\text{kJ}}{\text{kWh}}$$

From the equation $W_{rev} = -\int_3^4 Vdp$, it is obvious that the reversible steady-flow work is closely associated with the specific volume of fluid flowing through the device. The larger the specific volume, the larger the reversible work produced or consumed by the steady-flow device. Therefore, it is required to keep the specific volume very low during the compression process so that less work is consumed during the compression process and large work is consumed during an expansion process to obtain the maximum work. Doing so the sufficiency of the cycle increases.

10.3 ACTUAL VAPOUR CYCLE PROCESSES

The processes of an actual cycle differ from those of an ideal cycle. In an actual cycle, all the processes are irreversible. The main reason for irreversibility of the cycle is friction. The thermal efficiency of an actual vapour power cycle is much lower compared to an ideal cycle because in an ideal cycle all the losses are neglected. Figures 10.4 and 10.5 depict all the losses taking place in an actual vapour cycle.

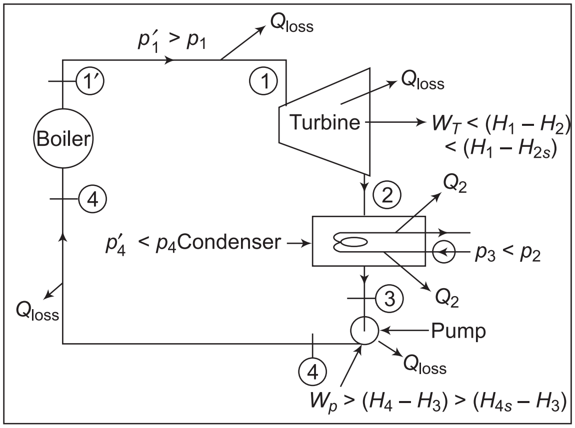


Figure 10.4 *Various losses in a steam plan*

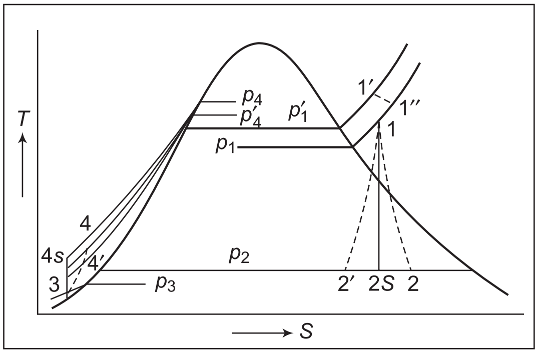


Figure 10.5 *Various losses on T-S plot*

10.4 PIPING LOSSES

Pressure drop due to friction and heat loss to the surroundings are the most important piping losses. A similar loss is the pressure drop in the boiler and also in the pipeline from the pump to the boiler. Due to this pressure drop, the water entering the boiler must be pumped to a much higher pressure than the desired steam pressure leaving the boiler, and this requires additional pump work.

10.5 TURBINE LOSSES

The losses in the turbine are those associated with frictional effects and heat loss to the surroundings. The steady flow energy equation for the turbine gives:

$$H_1 = W_T + H_2 + Q_{\text{loss}}$$

$$W_T = H_1 - H_2 - Q_{\text{loss}}$$

For the reversible adiabatic expansion, the path will be 1–2*s*. For an ordinary real turbine, the heat loss is small, and W_T is $H_1 - H_2$, with Q_2 equal to zero. Since actual turbine work is less than the reversible ideal work output, H_2 is greater than H_2S. However, if there is heat loss to the surroundings, H_2 will decrease, accompanied by a decrease in entropy. The isentropic efficiency of the turbine is defined as follows:

$$\eta_T = \frac{W_T}{H_1 - H_{2S}} = \frac{H_1 - H_2}{H_1 - H_{2S}}$$

where, W_T is the actual turbine work, and $(H_1 - H_{2S})$ is the isentropic enthalpy drop in the turbine (i.e. the ideal output).

10.6 PUMP LOSSES

The losses in the pump are similar to those of the turbine, and are primarily due to the irreversibilities associated with fluid friction. Heat transfer is usually negligible. The pump efficiency is defined as:

$$\eta_P = \frac{H_{4S} - H_3}{W_P} = \frac{H_{4S} - H_3}{H_4 - H_3}$$

where, W_P is the actual pump work.

10.7 CONDENSER LOSSES

The losses in the condenser are usually small. These include the loss of pressure and the cooling of condensate below the saturation temperature.

Example 10.1

Steam is the working fluid in an ideal Rankine cycle. Saturated vapour enters the turbine at 8.0 MPa and saturated liquid exits the condenser at a pressure of 0.008 MPa. The net power output of the cycle is 100 MW. Determine for the cycle (a) the thermal efficiency, (b) the back-work ratio, (c) the mass flow rate of the steam in kg/h, (d) the rate of heat transfer into the working fluid as it passes through the boiler in MW, (e) the rate of heat transfer from the condensing steam as it passes through the condenser in MW, (f) the mass flow rate of the condenser cooling water, in kg/ h, if cooling water enters the condenser at 15°C and exits at 35°C. ○●●

Solution To begin the analysis, we fix each of the principal states located on the accompanying schematic and *T-S* diagrams. Starting at the inlet to the turbine, the pressure is 8.0 MPa and the steam is a saturated vapour, so from steam table:

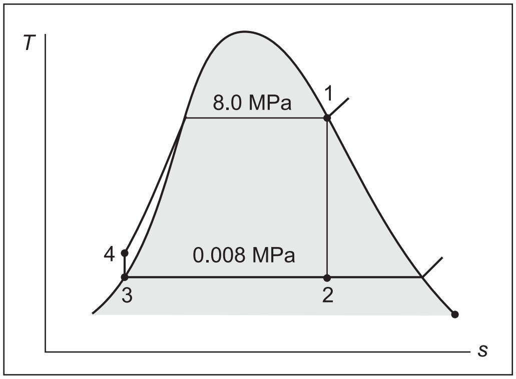


Figure 10.6

$$H_1 = 2758.0\,\frac{\text{kJ}}{\text{kg}} \text{ and } S_1 = 5.7432\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

State 2 is fixed by p_2 = 0.008 MPa and the fact that the specific entropy is constant for the adiabatic, internally reversible expansion through the turbine. Using saturated liquid and saturated vapour data from steam table, we find that the quality at state 2 is:

$$x_2 = \frac{S_2 - S_f}{S_g - S_f} = \frac{5.7432 - 0.5926}{7.6361} = 0.6745$$

The enthalpy is then:

$$H_2 = H_f + x_2 H_{fg} = 173.88 + 0.6745 \times 2403.1 = 1794.8 \text{ kJ / kg}$$

State 3 is saturated liquid at 0.008 MPa, so H_3 = 173.88 kJ/kg.

State 4 is fixed by the boiler pressure p_4 and the specific entropy $S_4 = S_3$. The specific enthalpy H_4 can be found:

$$H_4 = H_3 + V_3(p_4 - p_3)$$

By using steam tables, we get:

$$H_4 = 181.94 \text{ kJ / kg}$$

(a) The *net* power developed by the cycle is:

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p$$

$$\frac{\dot{W}_t}{m} = H_1 - H_2 \quad \text{and} \quad \frac{\dot{W}_p}{m} = H_4 - H_3$$

$$\frac{\dot{Q}_{\text{in}}}{m} = H_1 - H_4$$

The thermal efficiency is then:

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{(H_1 - H_2) - (H_4 - H_3)}{H_1 - H_4} = \frac{(2758.0 - 1794.8) - (181.94 - 173.88)}{2758.0 - 181.94}$$

$$\eta = 0.371(37.1\ \%)$$

(b) The back-work ratio is calculated by:

$$\text{Back–work ration } = \frac{\dot{W}_p}{\dot{W}_t} = \frac{(H_4 - H_3)}{(H_1 - H_2)} = \frac{8.06}{963.2} = 8.37 \times 10^{-3} \ (0.84\%)$$

(c) The mass flow rate of the steam can be obtained from the expression for the net power given in part a):

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{(H_1 - H_2) - (H_4 - H_3)} = 3.77 \times 10^5 \text{ kg / h}$$

(d) The rate of heat transfer into the working fluid as it passes through the boiler in MW is:

$$\dot{Q}_{in} = \dot{m}(H_1 - H_4) = 269.77 \text{ MW}$$

(e) The rate of heat transfer, from the condensing steam as it passes through the condenser in MW is:

$$\dot{Q}_{\text{out}} = \dot{m}(H_2 - H_3) = 169.75 \text{ MW}$$

(f) If $\dot{m}_w$ is the mass flow rate of the cooling water. Then from energy balance:

$$\dot{m}_w(H_{cw,\text{out}} - H_{cw,\text{in}}) = \dot{m}(H_2 - H_3)$$

$$\dot{m}_w = 7.3 \times 10^6 \text{ kg/s}$$

Example 10.2

Reconsider the vapour power cycle, but include in the analysis that the turbine and the pump each have an isentropic efficiency of 85%. Determine for the modified cycle (a) the thermal efficiency, (b) the mass flow rate of steam, in kg/h, for a net power output of 100 MW, (c) the rate of heat transfer into the working fluid as it passes through the boiler in MW, (d) the rate of heat transfer from the condensing steam as it passes through the condenser in MW, (e) the mass flow rate of the condenser cooling water in kg/h, if cooling water enters the condenser at 15°C and exits as 35°C. Discuss the effects on the vapour cycle of irreversibilities within the turbine and pump. ○●●

Solution Owing to the presence of irreversibility during the expansion of the steam through the turbine, there is an increase in specific entropy from turbine inlet to exit, as shown on the accompanying *T-S* diagram. Similarly, there is an increase in specific entropy from pump inlet to exit. Let us begin the analysis by fixing each of the principal states. State 1 is the same as in Example 10.1, so:

$$H_1 = 2758.0 \frac{\text{kJ}}{\text{kg}} \text{ and } S_1 = 5.7432 \frac{\text{kJ}}{\text{kg.K}}$$

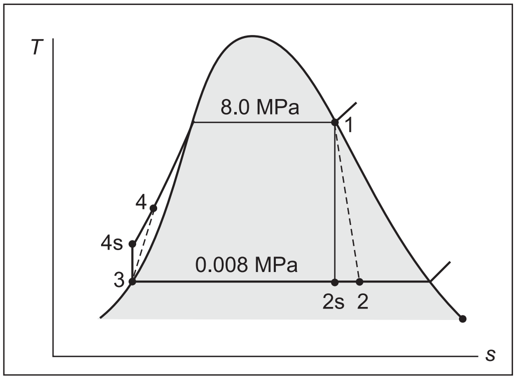


Figure 10.7

The specific enthalpy at the turbine exit, state 2, can be determined using the isentropic turbine efficiency,

$$\eta_t = \frac{(H_1 - H_2)}{(H_1 - H_{2S})}$$

where, H_{2S} is the specific enthalpy at state 2S on the accompanying *T-S* diagram. From the solution to Example 10.1, 5 H_{2S} = 1,794.8 kJ/kg. Solving for H_2 and inserting known values:

$$H_2 = H_1 - \eta_t(H_1 - H_{2S})$$

$$H_2 = 2{,}758 - 0.85(2{,}758 - 1{,}794.8) = 1{,}939.3 \text{ kJ / kg}$$

State 3 is the same as in Example 10.1, so H_3 = 173.88 kJ/kg

To determine H_4 requires the pump work. Pump work can be evaluated using the isentropic pump efficiency.

$$H_4 = H_3 + \frac{\dot{W}_p}{\eta_t \dot{m}}$$

$$H_4 = H_3 + \frac{V_3(p_4 - p_3)}{\eta_t} = 173.88 + \frac{8.06}{0.85} = 183.36 \text{ kJ / kg}$$

(a) The net power developed by the cycle is:

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{(H_1 - H_2) - (H_4 - H_3)}{h_1 - h_4}$$

$$\eta_t = \frac{(2{,}758 - 1{,}939.3) - (183.36 - 173.88)}{2{,}758 - 183.36} = 0.314(31.4\%)$$

(b) The mass flow rate of the steam can be obtained from:

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{(H_1 - H_2) - (H_4 - H_3)} = 4.449 \times 10^5 \text{ kg / h}$$

(c) The rate of heat transfer into the working fluid as it passes through the boiler in MW:

$$\dot{Q}_{\text{in}} = \dot{m}(H_1 - H_4) = 318.2 \text{ MW}$$

(d) The rate of heat transfer, from the condensing steam as it passes through the condenser in MW:

$$\dot{Q}_{\text{out}} = \dot{m}(H_2 - H_3) = 218.2 \text{ MW}$$

(e) If $\dot{m}_w$ is the mass flow rate of the cooling water. Then from energy balance:

$$\dot{m}_w(h_{cw,\text{out}} - h_{cw,\text{in}}) = \dot{m}(h_2 - h_3)$$

$$\dot{m}_w = 9.39 \times 10^6 \text{ kg / s}$$

Example 10.3

Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of this cycle. ○●●

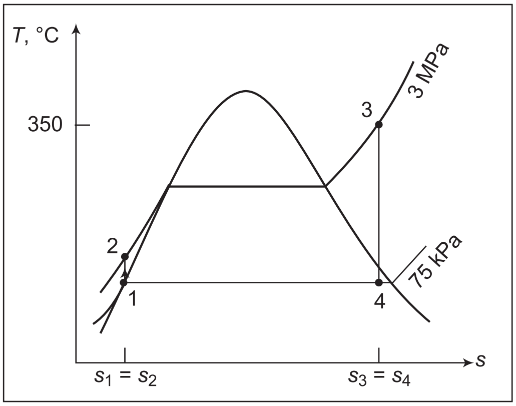


Figure 10.8

Solution The schematic of the T-S diagram of the cycle is shown in Figure 10.8. First, we determine the enthalpies at various points in the cycle.

At state 1:

Since $p_1 = 75$ kPa and it is saturated state. From steam table, we can get:

$$H_1 = 384.44 \text{ kJ / kg}$$

$$V_1 = 0.001037 \text{ m}^3 \text{/kg}$$

At state 2: $p_2 = 3$ MPa and $S_2 = S_1$

$$\begin{aligned} W_{\text{pump,in}} &= V_1(p_2 - p_1) \\ &= 0.001037(3{,}000 - 75) = 3.03 \text{ kJ / kg} \end{aligned}$$

$$H_2 = H + W_{\text{pump,in}} = 384.44 + 3.03 = 387.47 \text{ kJ / kg}$$

At state 3: $p_3 = 3\text{MPa}$ and $T_3 = 350°\text{C}$

$$H_3 = 3{,}116.1 \text{ kJ / kg}$$

$$S_3 = 6.7450 \text{ m}^3 \text{/kg}$$

At state 4: $p_4 = 75$ kPa and $S_4 = S_3$

$$x_4 = \frac{S_4 - S_f}{S_{fg}} = 0.8861$$

$$H_4 = H_f + x_4 H_{fg} = 2{,}403.0 \text{ kJ / kg}$$

Thus, efficiency is given by:

$$\eta = \frac{(H_3 - H_4) - (H_2 - H_1)}{(H_3 - H_2)} = 0.26(26\%)$$

Example 10.4

A steam power plant operates on the cycle shown in Figure 10.9. If the isentropic efficiency of the turbine is 87% and the isentropic efficiency of the pump is 85 percent, determine (a) the thermal efficiency of the cycle and (b) the net power output of the plant for a mass flow rate of 15 kg/s. ○○●

Solution Pump work input:

$$W_{\text{pump, in}} = \frac{V_1(p_2 - p_1)}{\eta_p} = 19.0 \text{ kJ / kg}$$

Turbine work output:

$$W_{\text{turb, out}} = \eta_t (H_5 - hH_{6S})$$

Boiler heat input:

$$Q_{in} = H_4 - H_3 = 3{,}647.6 - 160.1 = 3{,}487.5 \text{ kJ / kg}$$

(a) Thus, thermal efficiency:

$$\eta = \frac{W_{\text{turb, out}} - W_{\text{pump, in}}}{Q_{in}} = 0.361 \ (36.1\%)$$

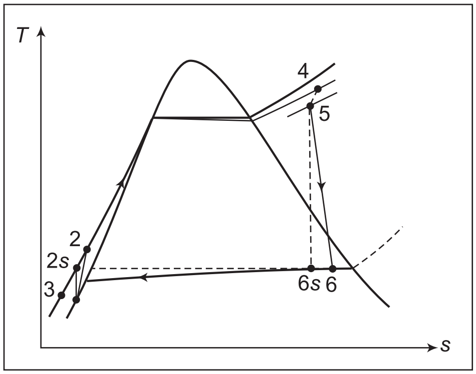


Figure 10.9

(b) The power produced by this power plant is:

$$W_{\text{net}} = \dot{m}(W_{\text{turb, out}} - W_{\text{pump, in}}) = 18.9 \text{ MW}$$

10.8 REHEAT CYCLE

University Question

1. How can we take advantage of the increased efficiencies at higher boiler pressures without facing the problem of excessive moisture at the final stages of the turbine? **(PTU, 2012–2013)**

Two possibilities are:

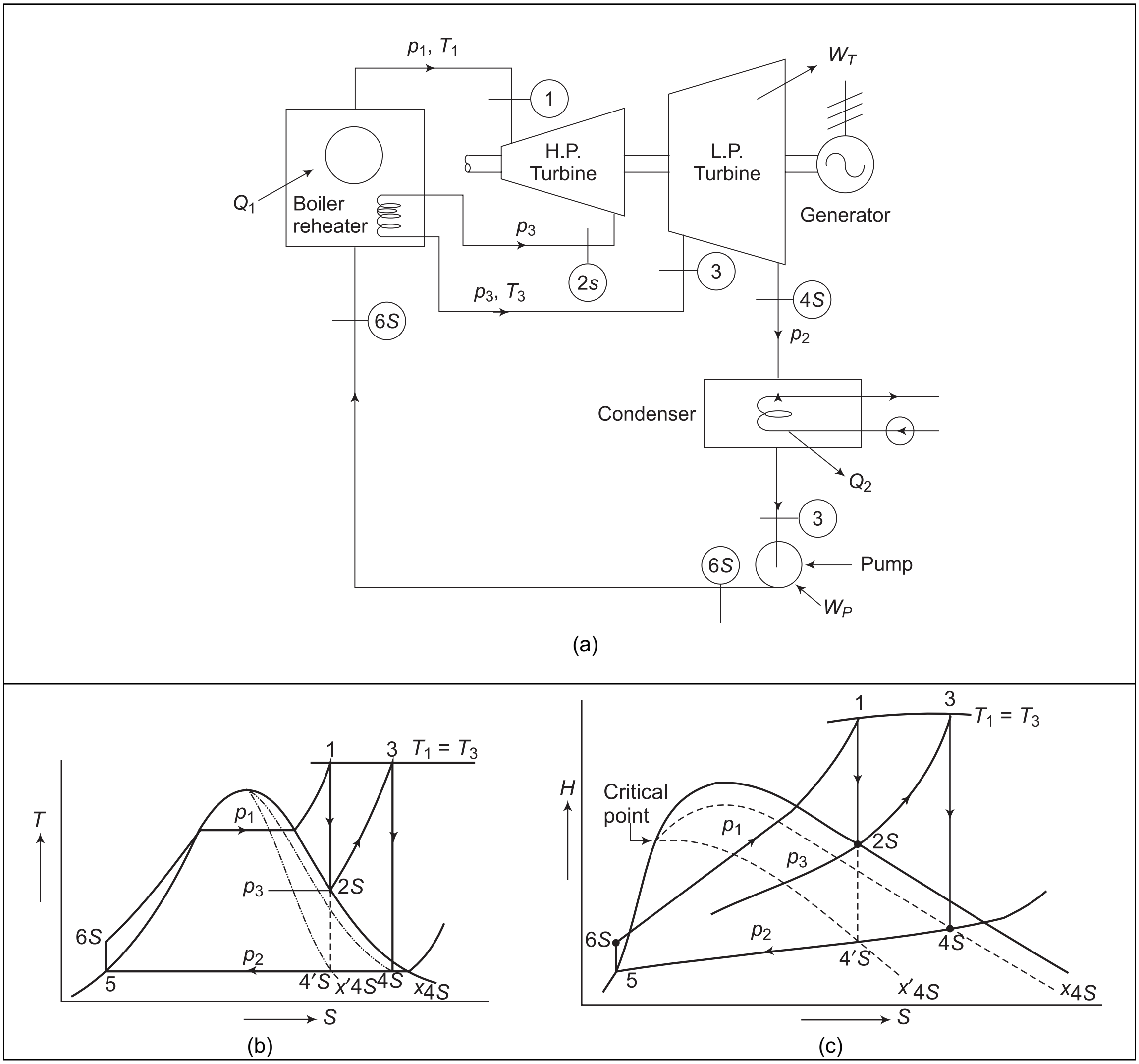
1. Superheat the steam to very high temperatures before it enters the turbine. This would be the desirable solution since the average temperature at which heat is added would also increase, thus increasing the cycle efficiency. This is not a viable solution, however, since it requires raising the steam temperature to metallurgically unsafe levels.
2. Expand the steam in the turbine in two stages, and reheat it in between. In other words, modify the simple ideal Rankine cycle with a *reheat* process. Reheating is a practical solution to the excessive moisture problem in turbines, and it is commonly used in modern steam power plants. Figure 10.10 a shows the reheat cycle power plant while Figures 10.10 b and 10.10 c shows the *T-S* and *p-V* diagram of the reheat cycle respectively.

$$Q_1 = H_1 - H_{6S} + H_3 - H_{2S}$$

$$Q_2 = H_{4S} - H_5$$

$$W_T = H_1 - H_{2S} + H_3 - H_{4S}$$

$$W_P = H_{6S} - H_5$$



Figures 10.10 (a to c) *(a) Reheat cycle power plant; (b) T-S diagram of reheat cycle; (c) H-S diagram of reheat cycle*

$$\eta = \frac{W_T - W_P}{Q_1} = \frac{(H_1 - H_{2S} + H_3 - H_{4S}) - (H_{6S} - H_5)}{(H_1 - H_{6S} + H_3 - H_{2S})}$$

$$\text{Steam rate } = \frac{3{,}600}{(H_1 - H_{2S} + H_3 - H_{4S}) - (H_{6S} - H_5)} \text{ kg/kWh}$$

Example 10.5

Steam is the working fluid in an ideal Rankine cycle with superheat and reheat. Steam enters the first-stage turbine at 8.0 MPa, 480°C, and expands to 0.7 MPa. It is then reheated to 440°C before entering the second-stage turbine, where it expands to the condenser pressure of 0.008 MPa. The net power output is 100 MW. Determine (a) the thermal efficiency of the cycle, (b) the mass flow rate of steam, in kg/h, (c) the rate of heat transfer from the condensing steam as it passes through the condenser in MW. Discuss the effects of reheat on the vapour power cycle. ○●●

Solution To begin, we fix each of the principal states. Starting at the inlet to the first turbine stage, the pressure is 8.0 MPa and the temperature is 480°C, so the steam is a superheated vapour. From steam tables,

$$H_1 = 3{,}348.4 \text{ kJ/kg}$$

$$S_1 = 6.6586 \frac{\text{kJ}}{\text{kg K}}$$

State 2 is fixed by $p_2 = 0.7$ MPa and $S_2 = S_1$ for the isentropic expansion through the first-stage turbine. Using saturated liquid and saturated vapour data from steam tables, the quality at state 2 is:

$$x_2 = \frac{S_2 - S_f}{S_g - S_f} = 0.9895$$

$$H_2 = H_f + x_2 H_{fg} = 2{,}428.5 \text{ kJ/kg}$$

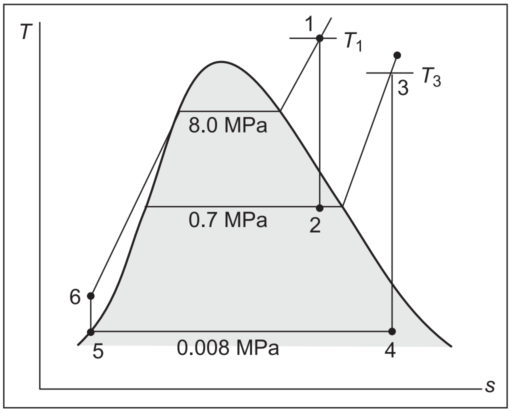


Figure 10.11

State 5 is saturated liquid at 0.008 MPa, so $H_5 = 173.88$ kJ/kg. Finally, the state at the pump exit is the same as in Example 10.1, so $H_6 = 181.94$ kJ/kg.

(a) The thermal efficiency of the cycle is:

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{(H_1 - H_2) + (H_3 - H_4) - (H_6 - H_5)}{(H_1 - H_6) + (H_3 - H_2)} = 0.403$$

(b) The mass flow rate of the steam can be obtained:

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{(H_1 - H_2) + (H_3 - H_4) - (H_6 - H_5)} = 2.363 \times 10^5 \text{ kg/h}$$

(c) The rate of heat transfer from the condensing steam to the cooling water is

$$\dot{Q}_{\text{out}} = \dot{m}(H_4 - H_5) = 148 \text{ MW}$$

Example 10.6

A reheat cycle has steam generated at 40 bar, 500°C for being sent to high pressure turbine and expanded upto 5 bar before supplied to low pressure turbine. Steam enters at 5 bar 400°C into low pressure turbine after being reheat boiler. Steam finally enters condenser at 0.04 bar and subsequently feed water is sent to boiler. Determine cycle efficiency, specific steam consumption. ○●●

Solution From steam table:

At 40 bar, 500°C

$$H_2 = 3{,}445.00 \text{ kJ/kg}$$

$$S_2 = 7.091 \text{ kJ/kg K}$$

$$T_3 = 400°\text{C at 5 bar}$$

$$H_3 = 2{,}818.03 \text{ kJ/kg}$$

At 40 bar, 500°C

$$H_4 = 3{,}271.9 \text{ kJ/kg}$$

$$S_4 = 7.7938 \text{ kJ/kg K}$$

At 0.04 bar

$$S_f = 0.423 \text{ kJ/kg K}$$

$$S_{fg} = 8.053 \text{ kJ/kg K}$$

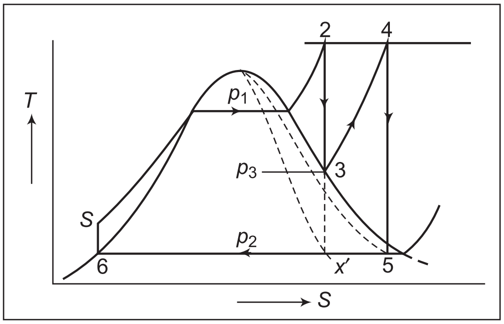


Figure 10.12

For expansion process 4-5

$$S_4 = S_5$$

$$7.7938 = S_{f5} + x_5 S_{fg5} = 0.423 + x_5\, 8.053$$

$$X_5 = 0.9152$$

$$H_5 = H_{f5} + x_5 H_{fg5} = 121.4 + 0.9152 \times 2{,}433.1 = 2{,}348.38 \text{ kJ/kg}$$

For process 6-1 in feed pump

$$H_1 = H_6 + v_{f6}\,(p_1 - p_6) = 142.84 \text{ kJ/kg}$$

Turbine work

$$W_T = (H_2 - H_3) + (H_4 - H_5) = 1{,}550.49 \text{ kJ/kg}$$

Pump work

$$W = (H_1 - H_6) = 21.44 \text{ kJ/kg}$$

Net Work

$$W_{\text{net}} = W_T - W_P = 1{,}550.49 - 21.44 = 1{,}529.05 \text{ kJ/kg}$$

Cycle efficiency $\eta = W_{\text{net}}/Q_1 = W_{\text{net}}/(H_2 - H_1)$

$$= 1529.05/(3{,}445.00 - 142.84)$$

$$= 0.4630 \text{ or } 46.30\ \%$$

Specific steam consumption $= 3600/W_{\text{net}}$

$$= 3{,}600/1{,}529.05 = 2.354 \text{ kJ/kg Wh}$$

Example 10.7

Consider a steam power plant operating on the ideal reheat Rankine cycle. Steam enters the high-pressure turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10.4%, determine (a) the pressure at which the steam should be reheated and (b) the thermal efficiency of the cycle. Assume the steam is reheated to the inlet temperature of the high-pressure turbine. ○●●

Solution The schematic of the power plant and the *T-S* diagram of the cycle are shown in Figure 10.13.

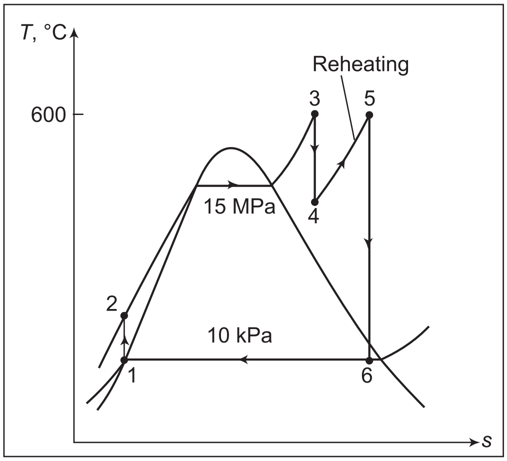


Figure 10.13

(a) The reheat pressure is determined from the requirement that the entropies at states 5 and 6 be the same:

$p_6 = 10$ kPa

$x_6 = 0.896$

$$S_6 = S_f + x_6 S_{fg} = 7.3688 \frac{\text{kJ}}{\text{kg. K}}$$

$$H_6 = H_f + x_6 H_{fg} = 2{,}335.1 \text{ kJ/kg}$$

Therefore, from above data:

$p_5 = 4.0$ MPa

$h_5 = 3{,}674.9$ kJ/kg

Therefore, steam should be reheated at a pressure of 4 MPa or lower to prevent a moisture content above 10.4%.

(b) To determine the thermal efficiency, we need to know the enthalpies at all other states:

At state 1: pressure is 10 kPa, therefore:

$H_1 = 191.81$ kJ/kg

$V_1 = 0.00101 \text{m}^3/\text{kg}$

At state 2: pressure is $p_2 = 15$ MPa and $S_2 = S_1$

$$W_{\text{pump, in}} = V_1(p_2 - p_1) = 15.14 \text{ kJ/kg}$$

$$H_2 = H_1 + W_{\text{pump, in}} = 206.95 \text{ kJ/kg}$$

At state 3: $p_3 = 15$ MPa and $T_3 = 600$°C

$$H_3 = 3{,}583.1 \text{ kJ/kg}$$

$$S_3 = 6.6796 \frac{\text{kJ}}{\text{kg. K}}$$

At state 4: $p_4 = 4$ MPa and $S_4 = S_3$

$$H_4 = 3{,}155.0\ \text{kJ/kg}$$

Thus

$$Q_{\text{in}} = (H_3 - H_2) + (H_5 - H_4) = 3{,}896.1\ \text{kJ/kg}$$

$$Q_{\text{out}} = (H_6 - H_1) = 2{,}143.3\ \text{kJ/kg}$$

$$\eta = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 0.450(45\%)$$

10.9 IDEAL REGENERATIVE CYCLE

University Question

1. What is ideal regeneration cycle? **(BPUT, 2009–2010)**

Regeneration process in steam power plants is accomplished by *extracting or bleeding* steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feedwater instead. The device where the feedwater is heated by regeneration is called a regenerator. Figure 10.14 shows the T-S diagram of ideal Rankine cycle.

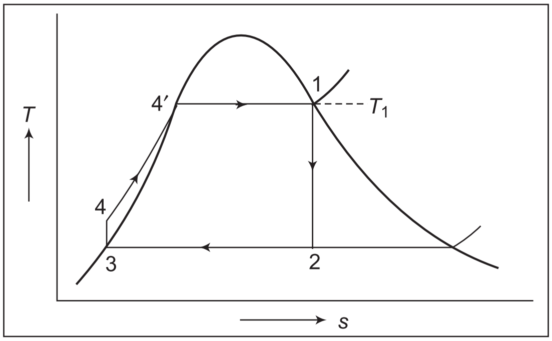


Figure 10.14 *Simple Rankine cycle*

The net work output of the ideal regenerative cycle is thus less, and hence its steam rate will be more, although it is more efficient, when compared with the Rankine cycle. However, the cycle is not practicable for the following reasons:

1. Reversible heat transfer cannot be obtained in finite time.
2. Heat exchanger in the turbine is mechanically impracticable.
3. The moisture content of the steam in the turbine will be high.

10.10 || REGENERATIVE CYCLE

$$W_T = 1(H_1 - H_2) + (1 - m_1)(H_2 - H_3) + (1 - m_1 - m_2)(H_3 - H_4)\,\text{kJ/kg}$$

$$W_P = W_{P1} + W_{P2} + W_{P3}$$

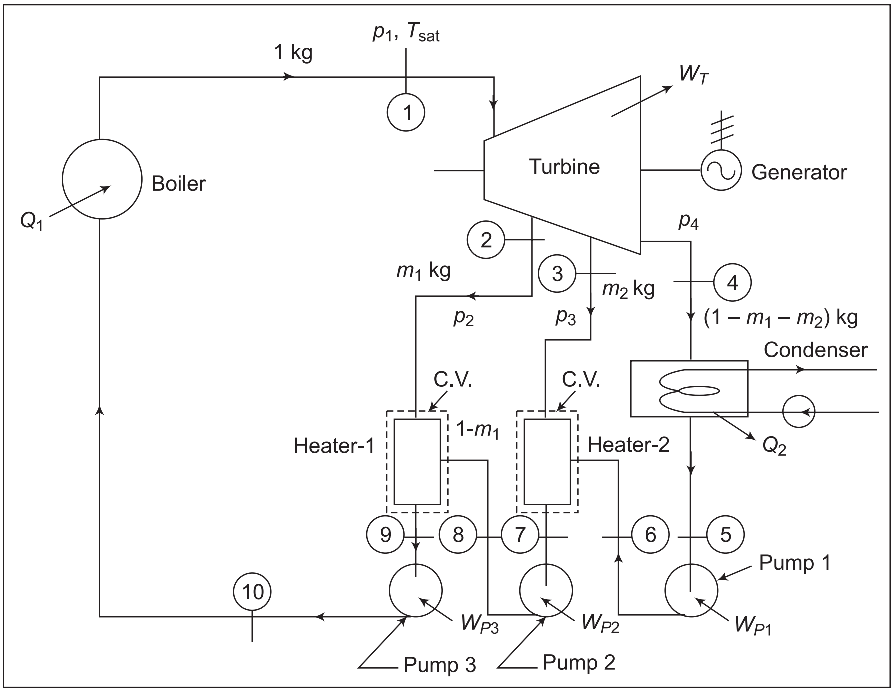
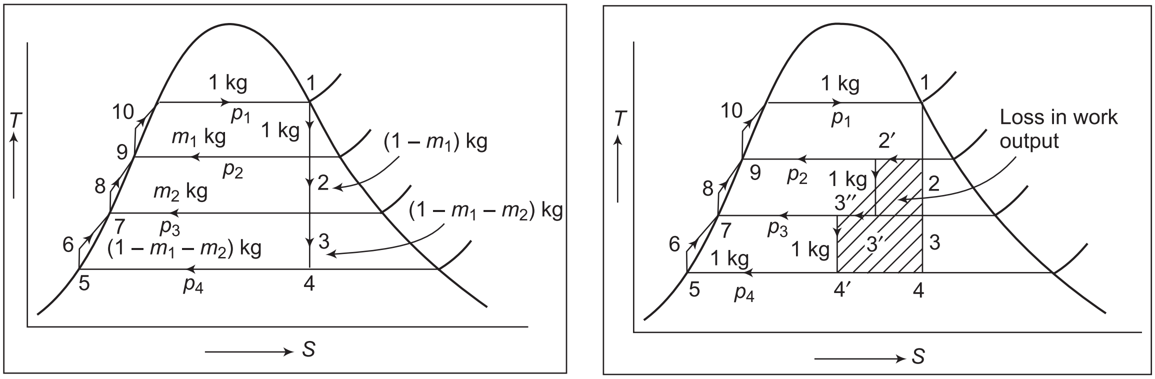


Figure 10.15 *Regenerative cycle flow diagram with two feedwater heaters*



(a) (b)

Figures 10.16 (a and b) *(a) Regenerative cycle on T-S plot with decreasing mass of fluid; (b) Regenerative cycle on T-S plot for unit mass of fluid*

$$W_P = (1 - m_1 - m_2)(H_6 - H_5) + (1 - m_1)(H_8 - H_7) + 1(H_{10} - H_9)\text{kJ / kg}$$

$$Q_1 = 1(H_1 - H_{10})\text{kJ / kg}$$

$$Q_2 = (1 - m_1 - m_2)(H_4 - H_5)\text{kJ / kg}$$

Cycle efficiency, $\eta = \dfrac{Q_1 - Q_2}{Q_1} = \dfrac{W_T - W_P}{Q_1}$

Steam rate $= \dfrac{3600}{W_T - W_P} \dfrac{\text{kg}}{\text{kWh}}$

$$(T_{m_1})_{\text{with regeneration}} = \frac{H_1 - H_{10}}{S_1 - S_{10}}$$

$$(T_{m_1})_{\text{without regeneration}} = \frac{H_1 - H_6}{S_1 - S_6}$$

$$(T_{m_1})_{\text{with regeneration}} > (T_{m_1})_{\text{without regeneration}}$$

The efficiency of the regenerative cycle will be higher than that of the Rankine cycle.

The energy balance for heater 2 gives:

$$m_1 h_2 + (1 - m_1)h_8 = 1h_9$$

$$m_1 = \frac{h_9 - h_8}{h_2 - h_8}$$

The energy balance for heater 1 gives:

$$m_2 H_3 + (1 - m_1 - m_2)H_6 = (1 - m_1)H_7$$

$$m_2 = (1 - m_1)\frac{H_7 - H_6}{H_3 - H_6}$$

$$(1 - H_1)(H_9 - H_8) = m_1(H_2 - H_9)$$

$$(1 - m_1 - m_2)(H_7 - H_6) = m_2(H_3 - H_7)$$

Energy gain of feedwater = Energy given off by vapour in condensation

Heaters have been assumed to be adequately insulated, and there is no heat gain from, or heat loss to, the surroundings.

$$W_T = (H_1 - H_2) + (1 - m_1)(H_2 - H_3) + (1 - m_1 - m_2)(H_3 - H_4)$$

$$W_T = (H_1 - H_2) + (H_{2'} - H_{3'}) + (H_{3'} - H_{4'})$$

$$(1-m_1)(H_2-H_3)=1(H_{2'}-H_{3'})$$

$$(1-m_1-m_2)(H_3-H_4)=1(H_{3''}-H_{4'})$$

$$(1-m_1-m_2)(H_3-H_4)=1(H_{3''}-H_{4'})$$

$$1(H_2-H_{2'})=1(H_9-H_8)$$

$$1(H_{3'}-H_{3''})=1(H_7-H_6)$$

$$W_T=(H_1-H_{4'})-(H_2-H_{2'})-(H_{3'}-H_{3''})$$

$$W_T=(H_1-H_{4'})-(H_9-H_8)-(H_7-H_6)$$

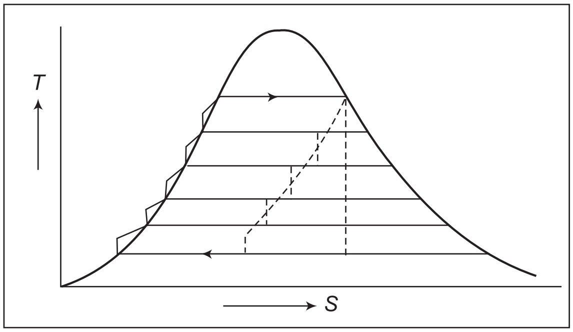


Figure 10.17 *T-S diagram of regenerative cycle with many stages of feedwater heating*

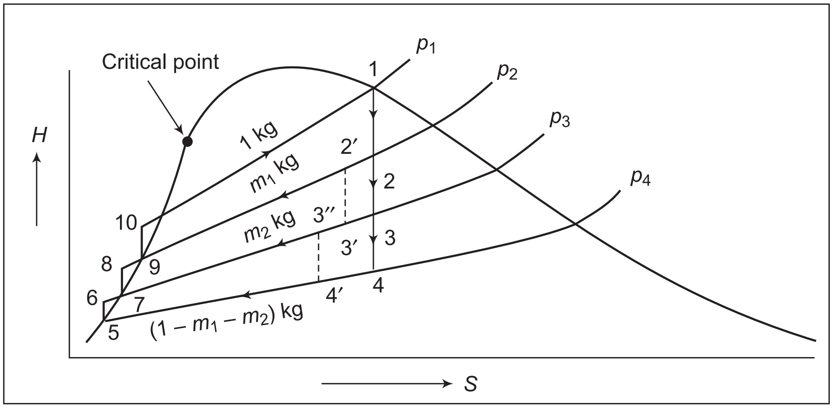


Figure 10.18 *Regenerative cycle on H-S diagram*

10.11 REHEAT-REGENERATIVE CYCLE

$$W_T = (H_1 - H_2) + (1 - m_1)(H_2 - H_3) + (1 - m_1)$$
$$(H_4 - H_5) + (1 - m_1 - m_2)(H_5 - H_6) + (1 - m_1 - m_2 - m_3)(H_6 - H_7)$$

$$W_P = (1 - m_1 - m_2 - m_3)(H_9 - H_8) + (1 - m_1 - m_2)(H_{11} - H_{10}) + (1 - m_1)(H_{13} - H_{12}) + 1(H_{15} - H_{14})\text{kJ/kg}$$

$$Q_1 = (H_1 - H_{15}) + (1 - m_1)(H_4 - H_3)\text{kJ/kg}$$

$$Q_2 = (1 - m_1 - m_2 - m_3)(H_7 - H_8)\text{kJ/kg}$$

The energy balance of heaters 1, 2, and 3 give,

$$m_1 H_2 + (1 - m_1)H_{13} = 1 \times H_{14}$$

$$m_2 H_5 + (1 - m_1 - m_2)H_{11} = (1 - m_1)H_{12}$$

$$m_3 H_6 + (1 - m_1 - m_2 - m_3)H_9 = (1 - m_1 - m_2)H_{10}$$

from which m_1, m_2, and m_3 can be evaluated.

Flow diagram for combined reheat and regenerative cycle is shown in Figure 10.19.

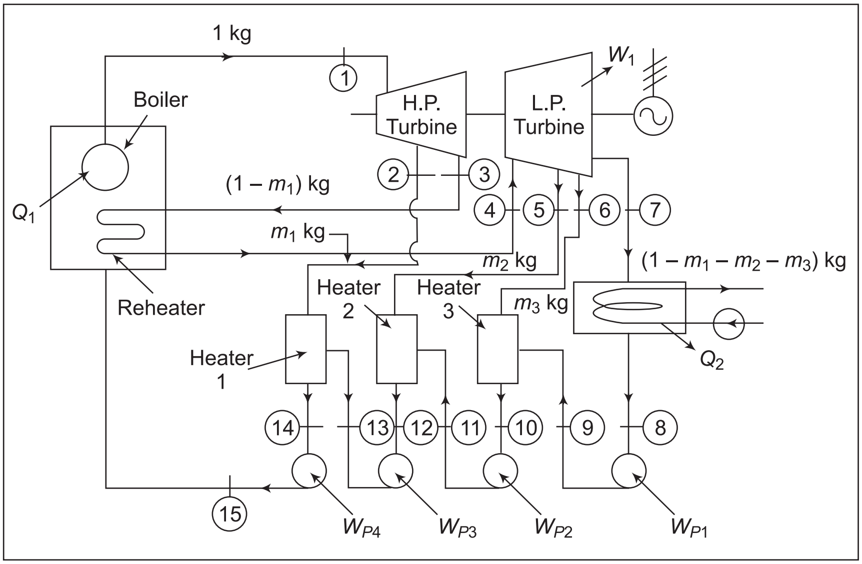


Figure 10.19 *Reheat-Regenerative cycle flow diagram*

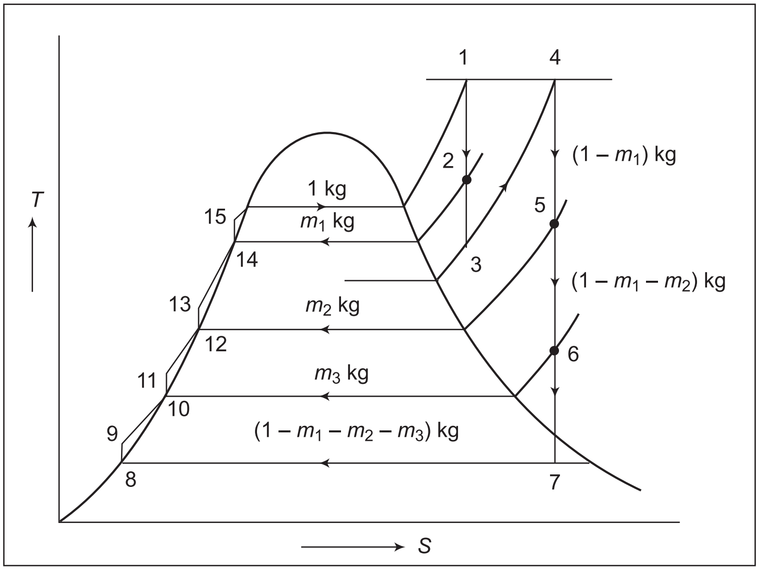


Figure 10.20 *T-S diagram of reheat-regenerative cycle*

Example 10.8

Steam at 70 bar and 450°C is supplied to a steam turbine. After expanding to 25 bar in high pressure stages, it is reheated to 420°C at the constant pressure. Next, it is expanded in intermediate pressure stages to an appropriate minimum pressure such that part of the steam bled at this pressure heats the feed water to a temperature of 180°C. The remaining steam expands from this pressure to a condenser pressure of 0.07 bar in the low-pressure stage. The isentropic efficiency of H.P. stage is 78.5%, while that of the intermediate and L.P. stages is 83% each. From the above data, determine:

(a) The minimum pressure at which bleeding is necessary.
(b) The quantity of steam bled per kg of flow at the turbine inlet.
(c) The cycle efficiency.

Neglect pump work. **(Roorkee University)** ●●●

Solution The schematic arrangement of the plant and the processes are represented on T-S and H-S diagrams as shown in Figures 10.22 a and b.

The minimum pressure at which bleeding is necessary:

It would be assumed that the feed water heater is an open heater. Feed water is heated to 180°C. So psat at 180°C ~ 10 bar is the pressure at which the heater operates.

Thus, the pressure at which bleeding is necessary is 10 bar.

From the *H-S* chart (Mollier chart), we have:

$$H_1 = 3{,}285 \text{ kJ/kg}; H_2 = 2{,}980 \text{ kJ/kg}; H_3 = 3{,}280 \text{ kJ/kg}; H_4 = 3{,}030 \text{ kJ/kg}$$

$$H_3 - \mathrm{H}_4' = 0.83(H_3 - H_4) = 0.83\ (3{,}280 - 3{,}030) = 207.5 \text{ kJ/kg}$$

$$\therefore \quad H_4' = H_3 - 207.5 = 3280 - 207.5 = 3{,}072.5 \text{ kJ/kg}$$

$$H_5 = 2{,}210 \text{ kJ/kg}$$

$$H_4' - H_5' = 0.83(H_4' - H_5) = 0.83\ (3{,}072.5 - 2{,}210) \sim 715.9 \text{ kJ/kg}$$

$$H_5' = H_4' - 715.9 = 3072.5 - 715.9 = 2{,}356.6 \text{ kJ/kg}$$

From steam tables, we have:

$$H_{f6} = 163.4 \text{ kJ/kg};\ H_{f8} = 762.6 \text{ kJ/kg}$$

$$H_1 - H_2' = 0.785(H_1 - H_2) = 0.785(3{,}285 - 2{,}980) = 239.4 \text{ kJ/kg}$$

$$\therefore \quad H_2' = H_1 - 239.4 = 3{,}285 - 239.4 = 3{,}045.6 \text{ kJ/kg}$$

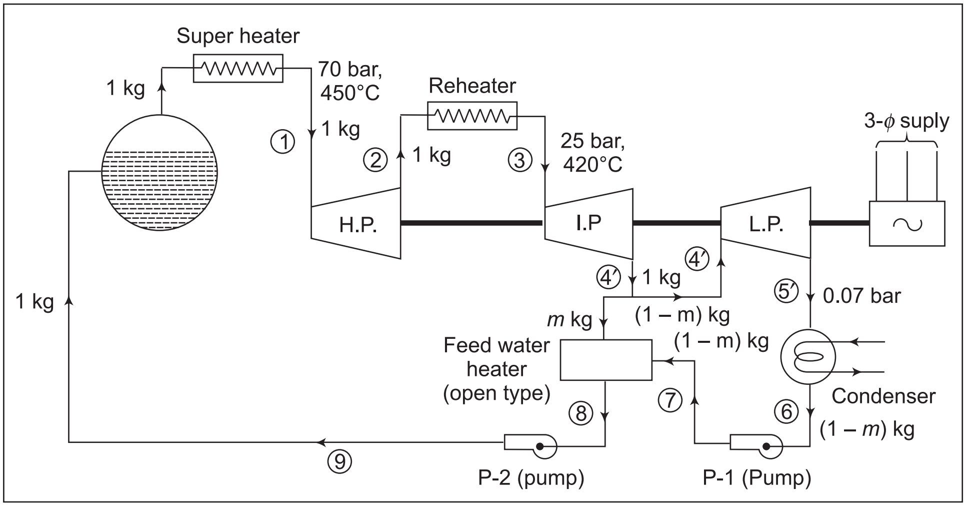
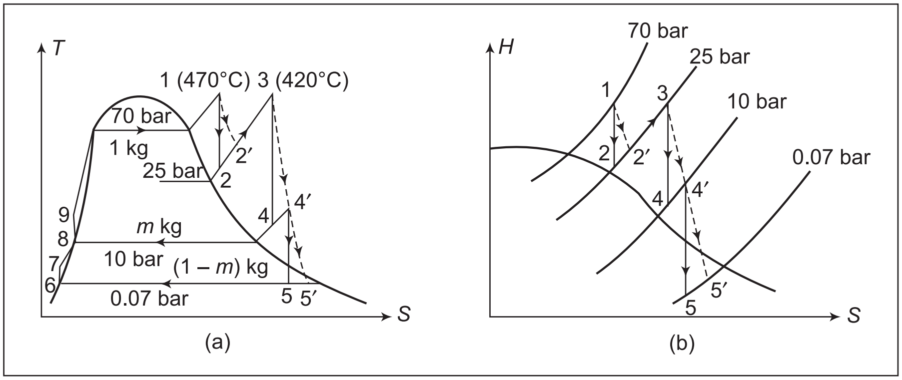


Figure 10.21 *Schematic diagram of plant*



Figures 10.22 (a and b) *(a) T-S diagram, (b) H-S diagram (pump work not shown)*

(b) The quantity of steam bled per kg of flow at the turbine inlet, m:

Considering energy balance for the feed water heater, we have:

$$m \times H_4' + (1 - m)\, H_{f7} = 1 \times H_{f8}$$

$$m \times 3{,}072.5 + (1 - m) \times 163.4 = 1 \times 762.6 \text{ (As } H_{f7} = H_{f6}\text{)}$$

$$3{,}072.5\, m + 163.4 - 163.4\, m = 762.6$$

$$m = \frac{762.6 - 163.4}{3072.5 - 163.4} = 0.206 \text{ kg of steam flow at turbine inlet.}$$

c) Cycle efficiency, η_{cycle}

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{1(H_1 - H_2) + 1(H_3 - H_4) + (1 - m)(H_4' - H_5')}{(H_1 - H_{f8}) + (H_3' - H_2')} = 0.3683 \text{ or } 36.83\%.$$

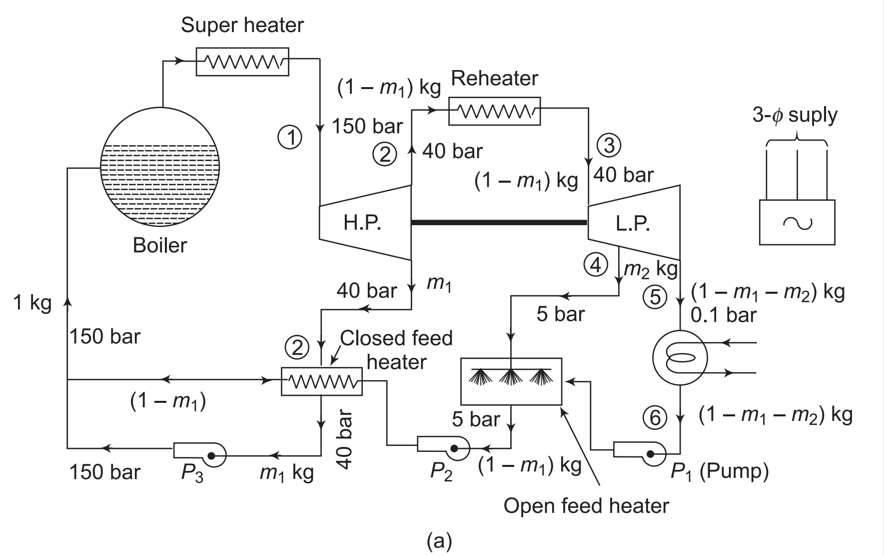
Example 10.9

A steam power plant operates on ideal Rankine cycle using reheater and regenerative feed water heaters. It has one open feed heater. Steam is supplied at 150 bar and 600°C. The condenser pressure is 0.1 bar. Some steam is extracted from the turbine at 40 bar for closed feed water heater and remaining steam is reduced at 40 bar to 600°C. Extracted steam is completely condensed in this closed feed water heater and is pumped to 150 bar before mixing with the feed water heater. Steam for the open feed water heater is bled from L.P. turbine at 5 bar. Determine:

(a) Fraction of steam extracted from the turbines at each bled heater, and

(b) Thermal efficiency of the system. **(AMIE Summer, 1999)** ●●●

Solution The arrangement of the components and the processes are represented on H-S diagram as shown in Figures 10.23 a and b.



(a)

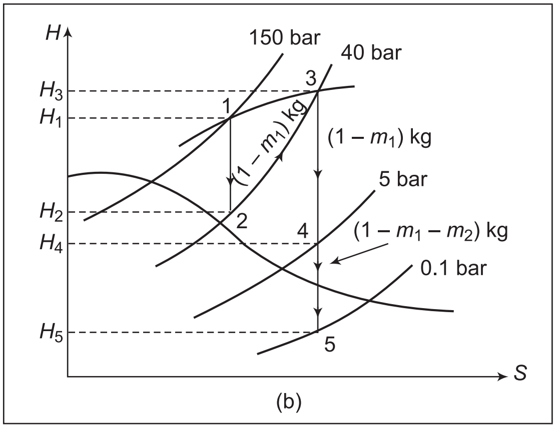


Figure 10.23 *(a) Schematic diagram of power plant, (b) H-S diagram*

From H-S chart and steam tables we have enthalpies at different points as follows:

$$H_1 = 3{,}578 \text{ kJ/kg}; \ H_2 = 3{,}140 \text{ kJ/kg};$$

$$H_3 = 3{,}678 \text{ kJ/kg}; \ H_4 = 3{,}000 \text{ kJ/kg};$$

$$H_5 = 2{,}330 \text{ kJ/kg}$$

$$H_{f1} \text{ (at 150 bar)} = 1{,}611 \text{ kJ/kg}$$

$$H_{f2} \text{ (at 40 bar)} = 1{,}087.4 \text{ kJ/kg}; \ H_{f4} \text{ (at 5 bar)} = 640.1 \text{ kJ/kg}$$

$$H_{f5} = H_{f6} \text{ (at 0.1 bar)} = 191.8 \text{ kJ/kg; Steam tables}$$

(a) Fraction of steam extracted from the turbines at each bled heater m_1, m_2:

Considering energy balance for closed feed heater, we have:

$$m_1(H_2 - H_{f2}) = (1 - m_1)\,(H_{f2} - H_{f4})$$

$$m_1\,(3{,}140 - 1{,}087.4) = (1 - m_1)(1{,}087.4 - 640.1)$$

or $$2{,}052.6\, m_1 = (1 - m_1) \times 447.3$$

$$m_1 = 0.179 \text{ kg/kg of steam supplied by the boiler.}$$

Considering energy balance for open feed heater, we have:

$$m_2(H_4 - H_{f4}) = (1 - m_1 - m_2)(H_{f4} - H_{f6})$$

or $$m_2(H_4 - H_{f4}) = (1 - m_1 - m_2)(H_{f4} - H_{f5}) \ (H_{f6} = H_{f5})$$

or $$m_2(3000 - 640.1) = (1 - 0.179 - m_2)\,(640.1 - 191.8)$$

or $$2359.9\, m_2 = (0.821 - m_2) \times 448.3 = 368.05 - 448.3\, m_2$$

∴ $$m_2 = 0.131 \text{ kg/kg of steam supplied by boiler.}$$

(b) Thermal efficiency of the system, η_{thermal}:

Total work done per kg of steam supplied by the boiler

$$= 1 \times (H_1 - \mathrm{H}_2) + (1 - m_1)\,(\mathrm{H}_3 - \mathrm{H}_4) + (1 - m_1 - m_2)\,(H_4 - H_5)$$

$$= (3578 - 3140) + (1 - 0.179)\,(3678 - 3000) + (1 - 0.179 - 0.131)(3000 - 2330)$$

$$= 438 + 556.64 + 462.3 = 1456.94 \text{ kJ/kg}$$

Work done by the pump P_1

$$W_{P1} = V_{W1}\,(1 - m_1 - m_2)(5 - 0.1) \times 10^5 \times 10^{-3} \text{ kJ/kg}$$

$$= 0.001 \times (1 - 0.179 - 0.131)(5 - 0.1) \times 10^5 \times 10^{-3} = 0.338 \text{ kJ/kg}$$

Work done by the pump $P2$,

$$W_{P2} = V_{W2}\,(1 - m_1)(150 - 5) \times 10^5 \times 10^{-3} \text{ kJ/kg} = 11.9 \text{ kJ/kg}$$

Work done by pump P_3,

$$W_{P3} = V_{W3} \times m_1 \times (150 - 40) \times 10^5 \times 10^{-3} = 1.97 \text{ kJ/kg}$$

Total pump work = $W_{P1} + W_{P2} + W_{P3} = 0.338 + 11.9 + 1.97 = 14.21$ kJ/kg of steam supplied by boiler

Net work done by the turbine per kg of steam supplied by the boiler,

$$W_{\text{net}} = 1{,}456.94 - 14.21 = 1{,}442.73 \text{ kJ/kg}$$

Heat of feed water entering the boiler = $(1 - m_1) \times 1{,}611 + m_1 \times 1{,}611 = 1{,}611$ kJ/kg

Heat supplied by the boiler per kg of steam,

$$QS_1 = H_1 - 1{,}610 = 3{,}578 - 1{,}610 = 1{,}968 \text{ kJ/kg}$$

$$QS_2 = \text{Heat supplied in the reheater} = (1 - m_1)\,(H_3 - H_2) = (1 - 0.179)(3{,}678 - 3{,}140)$$

$$= 441.7 \text{ kJ/kg of steam supplied by the boiler}$$

QS_T (Total heat supplied) = $QS_1 + QS_2 = 1{,}968 + 441.7 = 2{,}409.7$ kJ/kg

$$\eta_{\text{thermal}} = \frac{W_{net}}{QS_t} = 0.5987 \quad \text{or} \quad 59.87\%.$$

Example 10.10

A steam power plant equipped with regenerative as well as reheat arrangement is supplied with steam to the H.P. turbine at 80 bar 470°C. For feed heating, a part of steam is extracted at 7 bar and remainder of the steam is reheated to 350°C in a reheater and then expanded in L.P. turbine down to 0.035 bar. Determine:

(a) Amount of steam bled-off for feed heating,
(b) Amount of steam supplied to L.P. turbine,
(c) Heat supplied in the boiler and reheater
(d) Cycle efficiency, and
(e) Power developed by the system.

The steam supplied by the boiler is 50 kg/s. **(BU, 2000)**●●●

Solution The schematic arrangement of the steam power plant and the processes are represented on *H-S* diagram as shown in Figure 10.24.

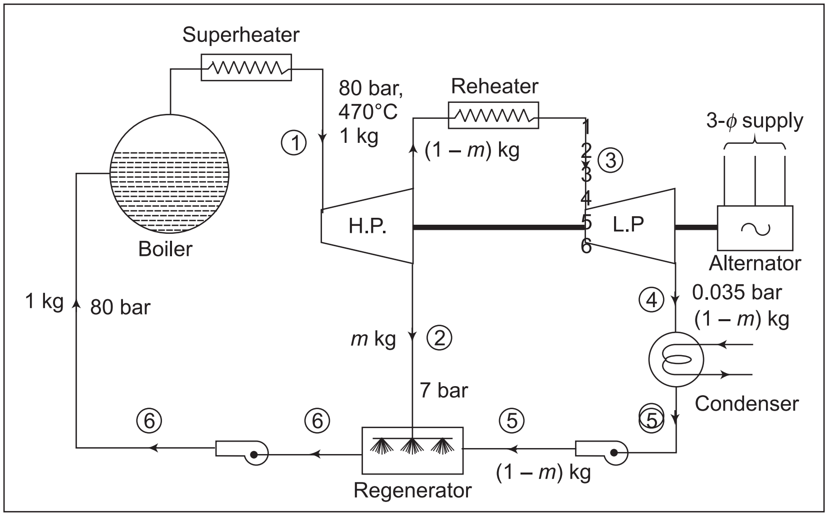


Figure 10.24

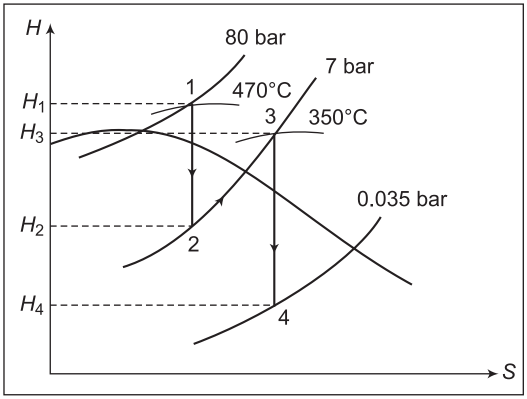


Figure 10.25 *H-S diagram*

From *H-S* chart and steam tables, we have enthalpies at different points as follows:

$$H_1 = 3{,}315 \text{ kJ/kg}; H_2 = 2{,}716 \text{ kJ/kg}$$

From *H-S* chart, $H_3 = 3{,}165$ kJ/kg; $H_4 = 2{,}236$ kJ/kg

$$H_{f6} = H_{f2} = 697.1 \text{ kJ/kg}; H_{f5} = H_{f4} = 101.9 \text{ kJ/kg}$$

(a) Amount of steam bled off for feed heating:

Considering energy balance at regenerator, we have:

Heat lost by steam = Heat gained by water

$$m(H_2 - H_{f6}) = (1 - m)\,(H_{f6} - H_{f5})$$

or $$m(H_2 - H_{f2}) = (1 - m)\,(H_{f2} - H_{f4})\ [H_{f6} = H_{f2};\ H_{f5} = H_{f4}]$$

or $$m(2{,}716 - 697.1) = (1 - m)\,(697.1 - 111.9)$$

or $$2{,}018.9\,m = 585.2\,(1 - m)$$

$$m = 0.225 \text{ g of steam supplied}$$

Hence, amount of steam bled off is 22.5% of steam generated by the boiler.

(b) Amount of steam supplied to L.P. turbine:

Amount of steam supplied to L.P. turbine = 100 – 22.5 = 77.5% of the steam generated by the boiler.

(c) Heat supplied in the boiler and reheater

Heat supplied in the boiler per kg of steam generated = $H_1 - H_{f6}$ = 3,315 – 697.1 = 2,617.9 kJ/kg.

$$(H_{f6} = H_{f2})$$

Heat supplied in the *reheater* per kg of steam generated = $(1 - m)\,(H_3 - H_2)$

$$= (1 - 0.225)\,(3{,}165 - 2{,}716) = 347.97 \text{ kJ/kg.}$$

Total amount of heat supplied by the boiler and reheater per kg of steam generated,

$$Qs = 2{,}617.9 + 347.97 = 2{,}965.87 \text{ kJ/kg}$$

(d) Cycle efficiency, η_{cycle}: Amount of work done by per kg of steam generated by the boiler,

$$W = 1(H_1 - \mathrm{H}_2) + (1 - m)\,(H_3 - H_4), \text{ Neglecting pump work}$$

$$= (3{,}315 - 2{,}716) + (1 - 0.225)\,(3{,}165 - 2{,}236) \sim 1{,}319 \text{ kJ/kg}$$

$$\eta_{\text{cycle}} = W/Qs = 0.4447 \text{ or } 44.47\%$$

(e) Power developed by the system:

Power developed by the system = $ms \times W = 50 \times 1{,}319$ kJ/s = 65.95 MW

Example 10.11

A steam power plant operates on a theoretical reheat cycle. Steam at boiler at 150 bar, 550°C expands through the high-pressure turbine. It is reheated at a constant pressure of 40 bar to 550°C and expands through the low-pressure turbine to a condenser at 0.1 bar. Draw *T-S* and *H-S* diagrams. Find:

(a) Quality of steam at turbine exhaust

(b) Cycle efficiency

(c) Steam rate in kg/kWh. **(AMIE Summer, 1999)** ●●●

Solution From Mollier diagram [*H-S* diagram (Figure 10.27)]:

$$H_1 = 3{,}450 \text{ kJ/kg};\ H_2 = 3{,}050 \text{ kJ/kg};\ H_3 = 3{,}560 \text{ kJ/kg};\ H_4 = 2{,}300 \text{ kJ/kg}$$

H_{f4} (from steam tables, at 0.1 bar) = 191.8 kJ/kg

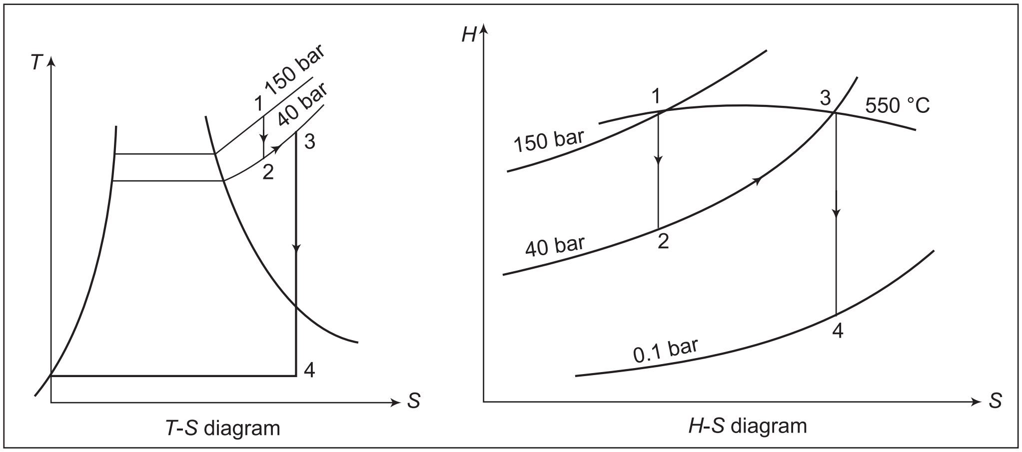


Figure 10.26

(a) Quality of steam at turbine exhaust, x_4:

$$x_4 = 0.88 \text{ (From Mollier diagram)}$$

(b) Cycle efficiency, η_{cycle}:

$$\eta_{\text{cycle}} = \frac{H_1 - H_2 + H_3 - H_4}{H_1 - H_{f4} + H_3 - H_2} = 0.4405 \quad \text{or} \quad 44.05\%$$

(c) Steam rate in kg/kWh:

$$\text{Steam rate} = \frac{3{,}600}{H_1 - H_2 + H_3 - H_4} = 2.17 \text{ kg/kWH}$$

10.12 || BINARY VAPOUR CYCLE

Thermal efficiency of Rankine cycle can be increased by:

1. Increasing the average temperature of heat addition.
2. Decreasing the average temperature of heat rejection.

Maximum temperature of the cycle is limited by practical considerations.

10.12.1 Characteristics of an Ideal Working Fluid in Vapour Power Cycles

For steam as a working fluid, the following difficulties arise at maximum temperature.

1. Critical temperature of steam is equal to 374.15°C and critical pressure is 225.65 bar. It is not possible to work at this pressure.
2. Latent heat of vapourisation decreases as the pressure increases.
3. If high pressure steam is expanded, high degree of moisture content will be present at the end of process.

The minimum temperature of the cycle is usually limited to natural water temperature of 25°C. At this temperature, the saturation pressure of water will be 0.0318 bar. It means that the condenser has to work at vacuum which is very difficult. So, ideal working fluid for Rankine cycle should fulfil the following requirements:

1. Reasonable saturation pressure at maximum temperature.
2. The fluid should have high critical temperature so that the saturation pressure is minimum.
3. The saturation pressure at the temperature of heat rejection should be above atmospheric pressure so as to avoid the necessity of maintaining vacuum in the condenser.
4. The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to boiling point.
5. The working fluid should be non-toxic and should be chemically stable.

However, in the high temperature range, there are a few better fluids, and notable amongst them are: (a) diphenyl ether, $(C_6H_5)_2O$, (b) aluminium bromide, Al_2Br_6, and (c) mercury and other liquid metals like sodium or potassium. From among these, only mercury has actually been used in practise. Diphenyl ether could be considered, but it has not yet been used because, like most organic substances, it decomposes gradually at high temperatures. Aluminium bromide is a possibility and yet to be considered. When $p = 12$ bar, the saturation temperature for water, aluminium bromide, and mercury are 187°C, 482.5°C, and 560°C respectively. Mercury is thus a better fluid in the high temperature range, because at high temperature, its vapourisation pressure is relatively low. Its critical pressure and temperature are 1,080 bar and 1,460°C respectively. But in the low temperature range, mercury is unsuitable, because its saturation pressure becomes exceedingly low and it would be impractical to maintain such a high vacuum in the con denser.

Mercury and steam are most commonly used working fluids. Saturation pressure and saturation temperature of mercury is 20.6 bar and 5,400°C at critical point.

The flow diagram of mercury-steam binary cycle and the corresponding *T-S* diagram are given in Figures 10.26 and 10.27 respectively. The mercury cycle, *a-b-c-d*, is a simple Rankine type of cycle using saturated vapour. Heat is supplied to the mercury in process *d-a*. The mercury expands in a turbine (process *a-b*) and is then condensed in process *b-c*. The feed pump process, *c-d*, completes the cycle. The heat rejected by mercury during condensation is transferred to boil water and form saturated vapour (process 5-6). The saturated vapour is heated from the external source (furnace) in the superheater (process 6-1). Superheated steam expands in the turbine (process 1-2) and is then condensed (process 2-3). The feed water (condensate) is then pumped (process 3-4), heated till it is saturated liquid in the economiser (process 4-5) before going to the mercury condenser-steam boiler, where the latent heat is absorbed. In an actual plant, the steam cycle is always a regenerative cycle, but for the sake of simplicity, this complication has been omitted. Let *m* represent the flow rate of mercury in the mercury cycle per kilogram of steam circulating in the steam cycle. Then for 1 kg of steam,

$$Q_1 = m(H_a - H_d) + (H_1 - H_6) + (H_5 - H_4)$$

$$Q_2 = H_2 - H_3$$

$$W_T = m(H_a - H_b) + (H_1 - H_2)$$

$$W_P = m(H_d - H_c) + (H_4 - H_3)$$

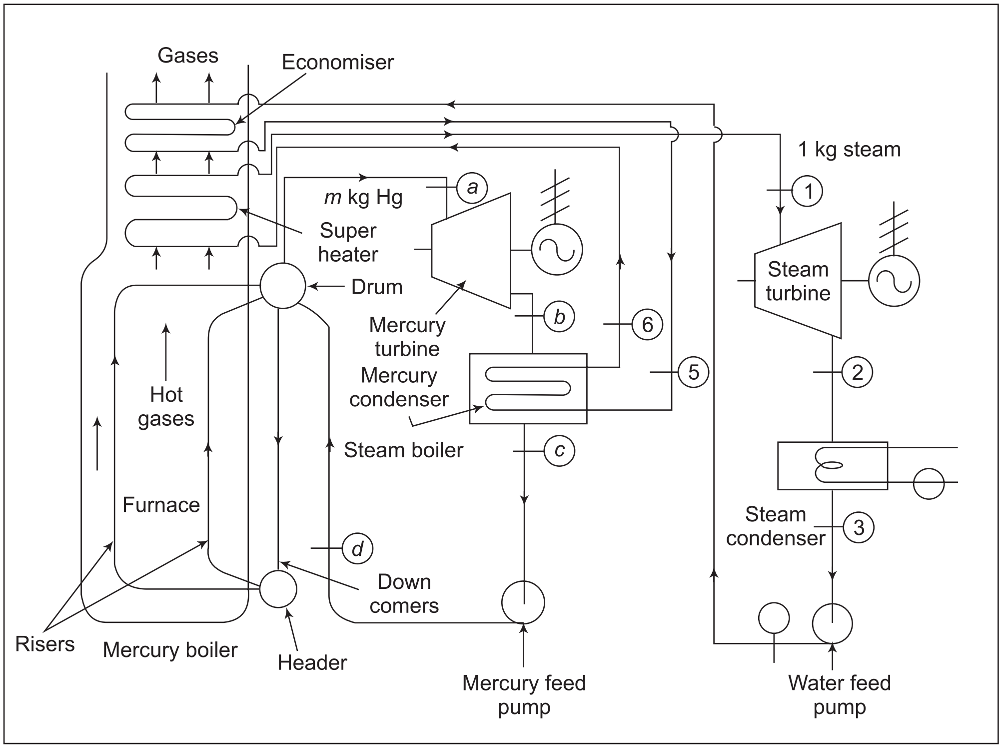


Figure 10.27 *Mercury-steam binary cycle power plant flow diagram*

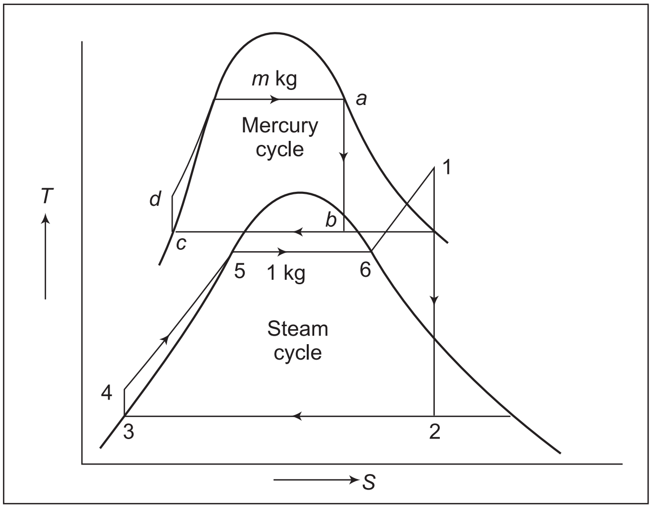


Figure 10.28 *T-S diagram of mercury-steam binary cycle*

$$\eta_{cycle} = \frac{Q_1 - Q_2}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$\text{Steam rate } = \frac{3{,}600}{W_T - W_P} \text{ kg / kWh}$$

The energy balance of the mercury condenser-steam boiler gives,

$$m(H_b - H_c) = (H_6 - H_5)$$

$$m = \frac{(H_6 - H_5)}{(H_b - H_c)} \text{ kg Hg / kg H}_2\text{O}$$

Example 10.12

A turbine is supplied with steam at a pressure of 32 bar and a temperature of 410°C. The steam then expands isentropically to a pressure of 0.08 bar. Find the dryness fraction at the end of expansion and thermal efficiency of the cycle. ○○●

Solution

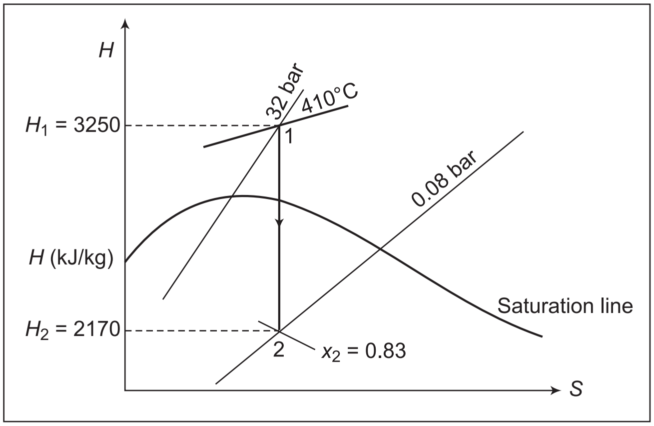


Figure 10.29

From Mollier chart,

$$H_1 = 3{,}250 \text{ kJ/kg}$$

$$H_2 = 2{,}170 \text{ kJ/kg}$$

Heat drop (or work done) $= H_1 - H_2 = 3{,}250 - 2{,}170 = 1{,}080$ kJ/kg

Heat supplied $= H_1 - H_{f2}$

$= 3{,}250 - 173.9$ ($H_{f2} = 173.9$ kJ/kg at 0.08 bar)

$= 3{,}076.1$ kJ/kg

$$\text{Thermal efficiency} = \frac{\text{Work done}}{\text{Heat supplied}} = 0.351 \text{ or } 35.1\%$$

Example 10.13

A Rankine cycle operates between pressures of 80 bar and 0.1 bar. The maximum cycle temperature is 600°C. If the steam turbine and condensate pump efficiencies are 0.9 and 0.8 respectively, calculate the specific work and thermal efficiency. Relevant steam table extract is given below.

p(bar)	*t*(°C)	Specific volume (m^3/kg)		Specific enthalpy (kJ/kg)			Specific entropy (kJ/kg K)		
		V_f	V_g	H_f	H_{fg}	H_g	S_f	S_{fg}	S_g
0.1	45.84	0.0010103	14.68	191.9	2392	2584.2	0.6488	7.5006	8.1494
80	295.1	0.001385	0.0235	1317	1440.5	2757.5	3.2073	2.5351	5.7424

80 bar, 600°C	*V*	0.486m^3/kg
Superheat	*H*	3642 kJ/kg
table	*3*	7.0206 kJ/kgK

○●●

Solution At 80 bar, 600°C,

$$H_1 = 3{,}642 \text{ kJ/kg}$$

$$S_1 = 7.0206 \text{ kJ/kg K}$$

Since, $S_1 = S_2$,

$$\therefore \quad S_2 = S_{f2} + x_2 S_{fg2}$$

$$7.0206 = 0.6488 + x_2 \times 7.5006$$

$$x_2 = \frac{7.0206 - 0.6488}{7.5006} = 0.85$$

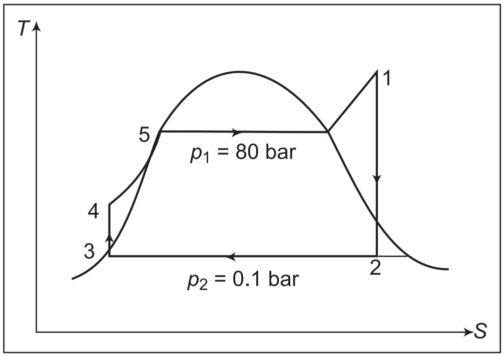


Figure 10.30

Now,
$$H_2 = H_{f2} + x_2 H_{fg2}$$
$$= 191.9 + 0.85 \times 2{,}392.3 = 2{,}225.36 \text{ kJ/kg}$$

$$\text{Actual turbine work} = \eta_{\text{turbine}} \times (H_1 - H_2) = 0.9\,(3{,}642 - 2{,}225.36) = 1{,}275 \text{ kJ/kg}$$

$$\text{Pump work} = V_f\,(p_2)(p_1 - p_2) = 8.072 \text{ kJ/kg}$$

$$\text{Actual pump work} = \frac{8.072}{\eta_{\text{pump}}} = \frac{8.072}{0.8} = 10.09 \text{ kJ / kg}$$

$$\text{Specific work } (W_{\text{net}}) = \text{Actual turbine work} - \text{Actual pump work}$$
$$= 1{,}275 - 10.09 = 1{,}264.91 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{W_{\text{net}}}{Q_1}$$

$$Q_1 = H_1 - H_{f4}$$

But
$$H_{f4} = H_{f3} + \text{pump work} = 191.9 + 10.09 = 202 \text{ kJ/kg}$$

Therefore, $Q_1 = 3{,}642 - 202 = 3{,}440$ kJ/kg

Thermal efficiency = 0.368 or 36.8%

POINTS TO REMEMBER

- ☞ Pressure drop due to friction and heat loss to the surroundings are the most important piping losses.
- ☞ Pump losses occur primarily due to irreversibilities associated with fluid friction.
- ☞ Condenser losses include the loss of pressure and the cooling of condensate below the saturation temperature.
- ☞ Reheating is a practical solution to the excessive moisture problem in turbines, and it is commonly used in modern steam power plants.
- ☞ Super heating the steam before it enters the turbine is a way of increasing cycle efficiency.
- ☞ Regeneration process in steam power plants is accomplished by *extracting* or *bleeding* steam from the turbine at various points.
- ☞ The device where the feedwater is heated by regeneration is called a regenerator.
- ☞ The net work output of the ideal regenerative cycle is thus less, and hence its steam rate will be more, although it is more efficient, when compared with the Rankine cycle.
- ☞ Thermal efficiency of Rankine cycle can be increased by either increasing the average temperature of heat addition or decreasing the average temperature of heat rejection.
- ☞ Mercury and steam are most commonly used working fluids.
- ☞ Mercury is a better fluid in the high temperature range, because at high temperature, its vapourisation pressure is relatively low.

PRACTICE PROBLEMS

1. In a steam power cycle, the steam supply is at 15 bar and dry and saturated. The condenser pressure is 0.4 bar. Calculate Rankine efficiency of the cycle. (**Ans:** 23.54%)
2. A simple Rankine cycle works between pressure 25 bar and 0.05 bar, the initial condition of steam being dry saturated. Calculate the (a) cycle efficiency, (b) work ratio and (c) specific heat consumption. (**Ans:** (a) 34.77%, (b) 0.9933, (c) 3.861 kJ/kg Wh)
3. Steam is supplied to a turbine at a pressure of 30 bar and a temperature of 400°C and is expanded adiabatically to a pressure of 0.04 bar. At a stage of turbine where the pressure is 3 bar a connection is made to a surface heater in which the feed water is heated by bled steam to a temperature of 130°C. The condensed steam from the feed heater is cooled in a drain cooler to 27°C. The feed water passes through the drain cooler before entering the feed heater. The cooled drain water combines with the condensate in the well of the condenser. Assuming no heat losses in the steam, calculate (a) mass of steam used for feed heating per kg of steam entering the turbine and (b) thermal efficiency of the cycle. (**Ans:** (a) 0.1672 kg, (b) 38.82%)
4. Steam at a pressure of 15 bar 250°C is expanded through a turbine at first to a pressure of 4 bar. It is then reheated at constant pressure to the initial temperature of 250°C and is finally

expanded to 0.1 bar. Using Mollier chart, estimate (a) the work done per kg of steam flowing through the turbine and (b) amount of heat supplied during the processes of reheat.

(**Ans:** (a) 885 kJ/kg, (b) 300 kJ/kg)

5. Consider a steam power plant operating on a regenerative vapour power cycle with one open feedwater heater. Steam enters the turbine at 8.0 MPa, 480°C and expands to 0.7 MPa. During expansion, some of the steam is extracted and diverted to the open feedwater heater operating at a pressure of 0.7 MPa. The remaining steam expands through the second-stage turbine to the condenser pressure of 0.008 MPa. Saturated liquid exits the open feedwater heater at a pressure of 0.7 MPa. The isentropic efficiency of each turbine stage is 85% and each pump operates isentropically. If the net power output of the cycle is 100 MW, determine: ●●●
   (a) the thermal efficiency
   (b) the mass flow rate of steam entering the first turbine stage, in kg/h.

   (**Ans:** (a) 36.9 %, (b) 3.69×10^5 kg/h)

6. Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open feedwater heater. Determine (a) the fraction of steam extracted from the turbine and (b) the thermal efficiency of the cycle. (**Ans:** (a) 1,486.89 kJ/kg, (b) 46.3%) ●●●

7. In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Find: (a) the efficiency and the steam rate of the cycle, (b) the increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration). Pump work may be neglected. ●●●

   (**Ans:** (a) 36.08%, 3.85 kg/kWh, (b) 238.9°C, 34.18%, 3.46 kg/kWh)

8. A binary cycle operates on mercury and steam. Standard mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhaust at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar. Determine the overall efficiency of the cycle. (**Ans:** 53%)

MULTIPLE CHOICE QUESTIONS

1. A Rankine cycle efficiency of a good steam power plant may be in the range of: ○○●
   (a) 10 to 20% (b) 35 to 45% (c) 70 to 80% (d) 85 to 95%

2. A Rankine cycle operating on low pressure limit of p_1 and high pressure limit of p_2: ○●●
   (a) Has higher thermal efficiency than the Carnot cycle operating between same pressure limits
   (b) Has lower thermal efficiency than Carnot cycle operating between same pressure limits
   (c) Has same thermal efficiency as Carnot cycle operating between same pressure limits
   (d) May be more or less depending upon the magnitudes of p_1 and p_2

3. In Rankine cycle the work output from the turbine is given by: ●●●
   (a) Change of internal energy between inlet and outlet
   (b) Change of enthalpy between inlet and outlet

(c) Change of entropy between inlet and outlet
(d) Change of temperature between inlet and outlet

4. Consider in a regenerative feed heating cycle, the greatest economy is affected: ●●●
(a) When steam is extracted from only one suitable point of steam turbine
(b) When steam is extracted from several places in different stages of steam turbine
(c) When steam is extracted only from the last stage of steam turbine
(d) When steam is extracted only from the first stage of steam turbine.

5. Mercury is choice with steam in binary vapour cycle because: ○●●
(a) Relative low vapourisation pressure
(b) Higher critical temperature and pressure
(c) Higher saturation temperature than other fluids
(d) All of above

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (b) 2. (a) 3. (b) 4. (b) 5. (a)

Refrigeration Cycle

12

CHAPTER OUTLINE

- Vapour Compression Refrigeration Cycle
- Vapour Absorption Cycle
- Gas Cycle of Refrigeration

INTRODUCTION

Refrigeration is an important concept which is useful in various sectors of our present day life. It is useful in preserving food, fruits and vegetables, medicines, vaccines and blood, science research in keeping DNA or other vital clues from the species.

12.1 VAPOUR COMPRESSION REFRIGERATION CYCLE

Probes

1. What is vapour compression refrigeration cycle?
2. What are the features of the different processes?
3. What are the deviations of the cycle from Carnot cycle?
4. How is the analysis of the cycle done?
5. How do you rate a refrigerator?

The vapour compression refrigeration cycle is the same as Rankine cycle taken in reverse. As in any other kind of refrigeration, work input is required. Basically, a fluid called refrigerant cycles through the main parts effectively taking heat off the required substance and rejecting it to the atmosphere or surroundings. The principle of refrigeration is that liquids absorb heat when changed from liquid to gas and gases give off heat when changed from gas to liquid.

The essential parts involved are compressor—to compress the refrigerant; condenser—to act as a heat exchanger and heat is given out to the surroundings; an expansion valve—where the refrigerant undergoes cooling; and lastly, an evaporator—which helps the refrigerant exchange heat from the required substance (Figure 12.1). The purpose is to extract heat more at the evaporator and reject it efficiently at the condenser. The performance is measured by the coefficient of performance which is desirably greater than unity. Quantitatively, it is the ratio of Q_{in} to W_{in} (Figures 12.2 a and b).

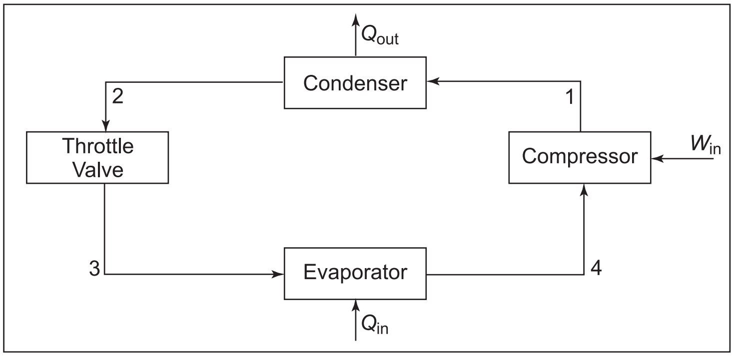
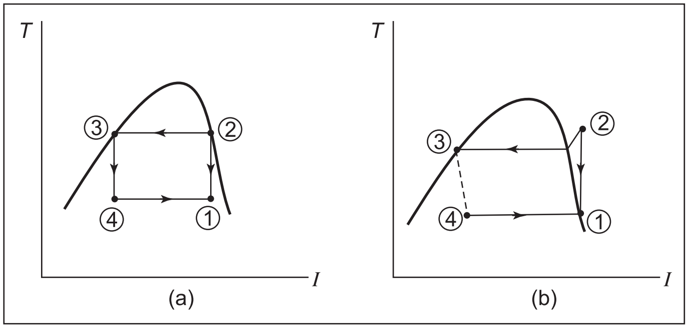


Figure 12.1 *Vapour compression cycle*



Figures 12.2 (a and b) *Ideal and typical cycles*

12.1.1 Features of Different Processes

Features of different processes are:

1. 1–2 Compression process—High pressure, high temperature gas
2. 2–3 Heat exchange—Same temperature and pressure, saturated liquid
3. 3–4 Expansion process—Reduced temperature and pressure liquid
4. 4–1 Heat exchange—Same temperature reduced pressure gas

12.1.2 Deviations of Carnot Cycle

Carnot cycle is the best but in practicality, it is not possible. There are certain deviations as listed:

1. Pressure drops due to friction in connecting pipes.
2. Heat transfer occurs from or to the refrigerant through the pipes connecting the components.

3. Pressure drops occur through the condenser and evaporator tubes.
4. Heat transfer occurs from the compressor.
5. Frictional effects and flow separation occur on the compressor blades.
6. The vapour entering the compressor may be slightly superheated.
7. The temperature of the liquid exiting the condenser may be below the saturation temperature.
8. Effectively, the condenser temperature is higher than that predicted by the Carnot cycle.
9. Similarly, the evaporator temperature is lower than that predicted by the Carnot cycle.
10. This results in a lower efficiency.
11. Wet compression—a mixture of vapour and liquid—needs to be avoided because it may damage the compressor in the long run with efficiency coming down.
12. Similar argument is forwarded with respect to the evaporator.

12.1.3 Analysis of Vapour Compression Refrigeration Cycle

Referring to Figures 12.1 and 12.2, at the evaporator, the heat flux is given by,

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = H_1 - H_4$$

where $\dot{Q}_{\text{in}}$ is the heat input called as the refrigeration capacity, $\dot{m}$ is the mass rate flow and H_1, H_4 are the enthalpies .

At the compressor, the work input is given by,

$$\frac{\dot{W}_{\text{in}}}{\dot{m}} = H_2 - H_1$$

In the condenser, the heat rejected is given by,

$$\frac{\dot{Q}_{\text{out}}}{\dot{m}} = H_2 - H_3$$

During throttling process, $H_4 = H_3$ as the expansion takes place.

Finally, the COP is given by $\text{COP} = \dfrac{\dfrac{\dot{Q}_{\text{in}}}{\dot{m}}}{\dfrac{\dot{W}_{\text{in}}}{\dot{m}}} = \dfrac{H_1 - H_4}{H_2 - H_1}$

12.1.4 Rating of a Refrigerator

Refrigerators are rated with a SEER (Seasonal Adjusted Energy Efficiency) number. It is defined as the heat transferred per watt of work input. Higher the SEER number, greater are the savings in money spent on electricity because of better efficiency. A SEER number of 10 corresponds to a COP of 2.9. However, it is important to note whether the cooling unit is placed indoors or outdoors, typical COP value ranges between 2 and 4. Refrigeration is also measured in terms of tons of refrigeration. One ton of refrigeration is equivalent to the heat rate necessary to melt a ton of ice in 24 hours 3.52 kW or 211 kJ/min.

The label on the refrigerator indicates the amount of energy the refrigerator uses in kWh. The annual cost of running the refrigerator is obtained by the product of this kWh and the cost of electricity by the provider. Typically, a normal refrigerator can work up to 20 years. Sometimes, the cost of running can exceed the cost of buying it.

Example 12.1

Freon 12 is compressed from 200 kPa to 1.0 MPa in an 80% efficient compressor as shown in Figure 12.3. The condenser exiting temperature is 40°C. Calculate the COP and the refrigerant mass flux for 100 tons of refrigeration. ○●●

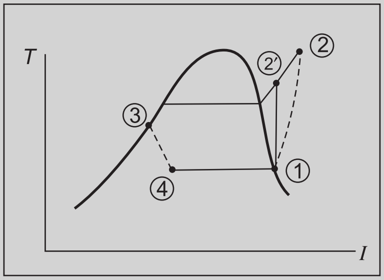


Figure 12.3

Solution Use refrigeration tables/hand book to locate the values of required enthalpies and entropy at the specified points. Thus, at 200 kPa, for Freon 12,

$$H_1 = 182.07\frac{\text{kJ}}{\text{kg}},\ H_3 = H_4 = 74.53\frac{\text{kJ}}{\text{kg}},\ S_1 = 0.7035\ \text{kJ/kgK}$$

$$S_{2'} = S_1 = 0.7035\,\frac{\text{kJ}}{\text{kgK}}\ \text{and}\ p_2 = 1.0\ \text{MPa} \rightarrow H_{2'} = 210.6\ \text{kJ/kg}$$

$$\text{Compressor work, } W_a = \frac{W_s}{\eta} = \frac{H_{2'} - H_1}{\eta} = \frac{210.6 - 182.07}{0.8} = 35.7\ \text{kJ/kg}$$

$$\text{COP} = \frac{H_1 - H_4}{w_a} = \frac{182.07 - 74.53}{35.7} = 3.01$$

$$Q = \dot{m}(H_1 - H_4) \rightarrow 352 = \dot{m}(182.07 - 74.53) \rightarrow \dot{m} = 3.27\ \text{kg/s}$$

Therefore, the mass flux rate is 3.27 kg/s.

Example 12.2

Refrigerant 134a is the working fluid in an ideal vapour-compression refrigeration cycle that communicates thermally with a cold region at –8°C and a warm region at 26°C. Saturated vapour enters the compressor at 0°C and saturated liquid leaves the condenser at 26°C. The mass flow rate of the refrigerant is 0.08 kg/s. Determine (a) the compressor power, in kW, (b) the refrigeration capacity, in tons, (c) the coefficient of performance, and (d) the coefficient of performance of a Carnot refrigeration cycle operating between warm and cold regions at 26°C and –8°C, respectively. ○●●

Solution From tables, corresponding to 0°C, $H_1 = 247.23$ kJ/kg and $S_1 = 0.9190$ kJ/kg K.

Pressure at 2s corresponds to 26°C and 6.853 bar. Therefore, $H_{2s} = 264.7$ kJ/kg, $H_3 = 85.75$ kJ/kg
Through the throttling process, the enthalpy remains constant. Therefore, $H_4 = H_3$
The compressor work input is given by:

$$\dot{W}_c = \dot{m}(H_{2S} - H_1)$$

Substituting, $\dot{W}_c = 0.08 \times (264.7 - 247.23) = 1.4 \text{ kW}$

The refrigeration capacity is given by:

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = H_1 - H_4$$

$$\dot{Q}_{\text{in}} = 0.08 \times 60 \times (247.23 - 85.75) \times \frac{1}{211} = 3.67 \text{ ton}$$

The coefficient of performance is given by:

$$\text{COP} = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{H_1 - H_4}{(H_{2s} - H_1)} = \frac{(247.23 - 85.75)}{(264.7 - 247.23)} = 9.24$$

For the Carnot cycle the COP would be given by:

$$\text{COP}_{\text{max}} = \frac{T_2}{T_1 - T_2} = \frac{273}{299 - 273} = 10.5$$

Example 12.3

If, in Example 12.2 all given data are retained the same, determine the mass flow rate of the refrigerant for 20 ton refrigeration capacity. ○○●

Solution From the previous problem,

$$\dot{Q}_{\text{in}} = \dot{m} \times 60 \times (247.23 - 85.75) \times \frac{1}{211} = 20 \text{ ton} \rightarrow \dot{m} = 0.436 \text{ kg/s}$$

12.2 VAPOUR ABSORPTION CYCLE

Probe

1. Describe the function of a vapour absorption cycle.

As can be seen from Figure 12.4, all the parts of a vapour compression refrigeration cycle remain the same except for the compressor. The compressor is replaced by an absorbing system which consists of a pump, an absorber, a heat exchanger and a vapour generator unit. Heat exchanger is the place where strong and weak solutions interact.

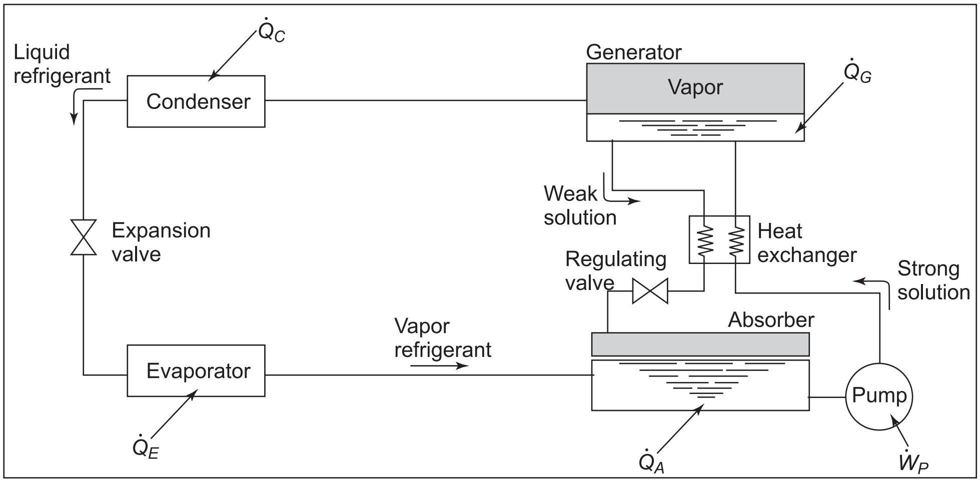


Figure 12.4 *Vapour absorption cycle*

From the evaporator, saturated, low-pressure refrigerant vapour enters the absorber where it is absorbed into the weak carrier solution. Heat is released in this absorption process. The strong solution from the absorber gets pumped to a greater pressure to reach the heat exchanger. This strong solution gains further temperature in the heat exchanger and proceeds to the generator. The weak solution from the exchanger reaches the absorber to become strong after interacting with the refrigerant from the evaporator.

The availability of a relatively high-temperature energy source to supply the heat transfer proves to be a disadvantage of the absorption cycle. Though this could be supplied by a power plant which otherwise would go as a waste, the additional heat generated must be inexpensive.

Example 12.4

Estimate the overall COP of an ammonia absorption cycle used as a chiller to cool water to 5°C in a 25°C environment. The small pump work may be neglected. If a heat source is available at 100°C, find the efficiency of the heat engine and the COP of the refrigeration cycle part. ○●●

Solution Efficiency of the heat engine is given by:

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{298}{373} = 0.202$$

From tables, $P_{\text{sat25}} = 1{,}003$ kPa, $H_3 = H_2 = 298.25$ kJ/kg, $h_4 = 1{,}447.3$ kJ/kg

At $T_1 = 100°\text{C}$, $p_1 = p_2$ and $H_1 = 1{,}664.3$ kJ/kg

$$\text{COP} = \frac{Q_2}{W} = \frac{H_4 - H_2}{H_1 - H_4} = \frac{1{,}447.3 - 298.25}{1{,}664.3 - 1{,}447.3} = 5.3$$

$$\text{COP}_{\text{overall}} = COP \times \eta = 5.3 \times 0.202 = 1.071$$

Example 12.5

In a small ammonia absorption refrigeration cycle that is powered by solar energy and used as an air conditioner, saturated vapour ammonia leaves the generator at 50°C, and saturated vapour leaves the evaporator at 10°C. If 2,400 kJ of heat is required in the generator per kilogram of ammonia vapour generated, determine the overall performance of this system. ○●●

Solution $Q_1 = Q_{\text{gen}} = 2{,}400$ kJ

$$Q_2 = H_2 - H_1 = H_{g10} - H_{f50} = 1{,}452.2 - 421.6 = 1{,}030.6 \text{ kJ/kg}$$

$$\text{COP} = \frac{Q_2}{Q_1} = \frac{1{,}030.6}{2{,}400} = 0.43$$

12.3 || GAS CYCLE OF REFRIGERATION

Probes

1. What are the different components of the gas refrigeration cycle?
2. Describe the gas refrigeration cycle.
3. Derive an expression for the COP of an ideal gas refrigeration cycle with a regenerative heat exchanger. Express the result in terms of the minimum gas temperature during heat rejection (T_h) maximum gas temperature during heat absorption (T_1) and pressure ratio for the cycle (p_2/p_1).

In a gas refrigeration cycle, the gas is throttled from a very high pressure to low pressure. In passing through the throttling valve, its temperature reduces suddenly while its enthalpy remains constant. The greatest advantage of a gas cycle is that there is no phase change as the refrigerant is gas before, during and after the cycle. Thus, air is one of the most common refrigerants available and also being used. They are used to achieve very low temperature levels for liquefaction of air and other gases.

The different components of the cycle are: compressor, heat exchanger, throttling valve (expander) and (another) heat exchanger in the same order (Figure 12.5).

The processes can be defined as:

1. 1–2 Process: Isobaric cooling by rejecting heat to the hot reservoir.
2. 2–3 Process: Isentropic expansion in the expander from the high pressure of heat exchanger 1 down to the low pressure of heat exchanger 2.
3. 3–4 Process: Isobaric heating by absorbing heat from the cold reservoir
4. 4–1 Process: Isentropic compression from the low pressure of heat exchanger 2 to the high pressure of heat exchanger 1.

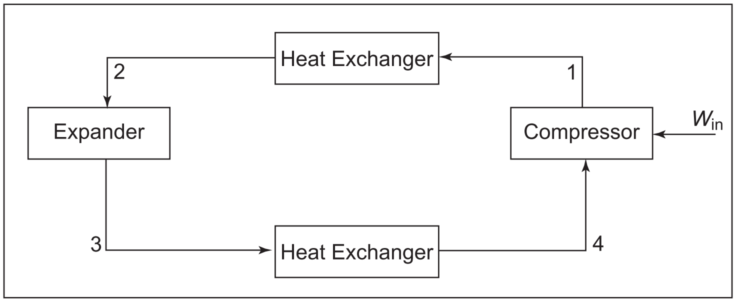


Figure 12.5 *Gas refrigeration cycle*

12.3.1 Description of Gas Refrigeration Cycle

From Figure 12.6, it can be seen that the refrigerant gas flows through the compressor which increases its pressure and temperature to a very high level. The gas then flows into the heat exchanger, which performs the function similar to the condenser in the vapour compression cycle. In this case, however, there is no change in the phase of gas. In the heat exchanger, the gas gives up heat, but its pressure remains constant. The high pressure and medium temperature air then enters the throttling valve (also called expander), where its pressure is reduced suddenly. Sometimes, a turbine is also used to drop the pressure. The sudden expansion also results in low temperature. The low temperature and low pressure gas then enters the other heat exchanger (also called refrigerator) which performs the function similar to the evaporator in vapour compression cycle. The gas absorbs heat from the substance to be cooled

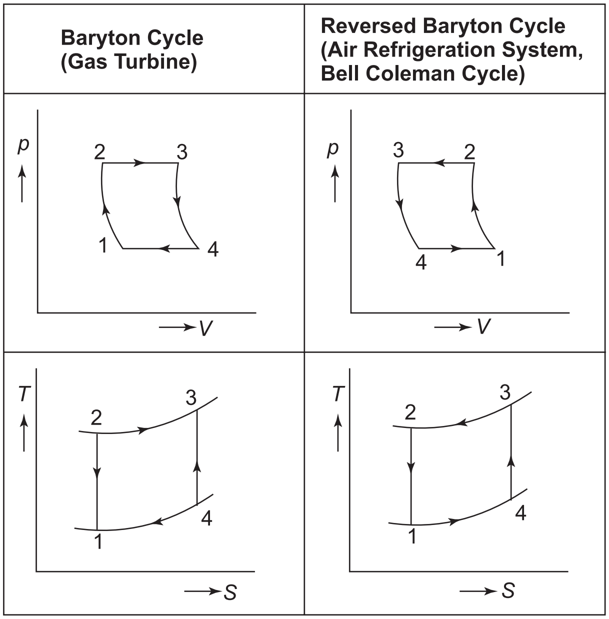


Figure 12.6 *Brayton versus Bell Coleman cycle*

and becomes hotter, while the substance becomes cooler. There is no change in phase of the gas in this heat exchanger. The low pressure and higher temperature gas then enters the compressor from where the cycle repeats.

When air is used as the refrigerant in the gas cycle, reverse Carnot cycle can be followed to achieve the refrigeration effect. However, the reverse Carnot cycle is only an ideal cycle and cannot be implemented in practical applications. Instead, for all real cycles, Bell-Coleman cycle or Brayton cycle is used in which the isothermal processes are replaced by the constant pressure processes. Thus, the cycle uses two isobars and two adiabatics. Interestingly, this was one of the earliest types of refrigerators used in ships for transport of food items.

However, it should be noted that the efficiency of the gas cycles is less compared to the vapour compression cycle. The amount of gas required is very high compared to the amount of the liquid refrigerant required for absorbing the same amount of heat or producing the same refrigerating effect. Due to this reason, the refrigeration systems with gas cycles tend to be very large and bulky. Though bulky, they are of light weight which makes them suitable for mobile air-conditioning and refrigeration applications, such as in airplanes.

12.3.2 Expression for COP of an Ideal Gas Refrigeration Cycle with a Regenerative Heat Exchanger

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1(r_p)^{\frac{\gamma-1}{\gamma}};$$

Similarly, $$T_4 = T_5\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = T_5(r_p)^{\frac{\gamma-1}{\gamma}}$$

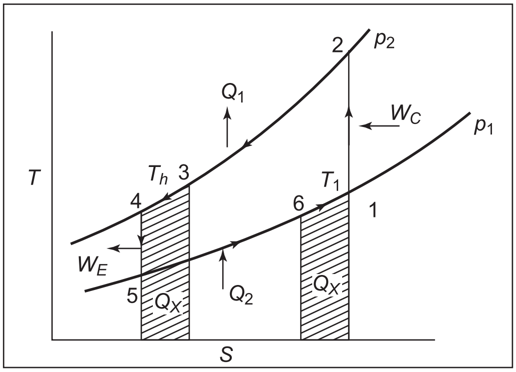


Figure 12.7 *Gas refrigeration cycle with heat regeneration*

For regeneration ideal, $c_p(T_3 - T_4) = c_p(T_1 - T_6)$

Work input, $W = (H_2 - H_1) - (H_4 - H_5)$

$= c_p[(T_2 - T_1) - (T_4 - T_5)]$

Heat rejection $= c_p[(T_2 - T_3)]$

Heat absorption $= c_p[(T_6 - T_5)]$

$$\text{COP} = \frac{\text{Heat absorbed}}{\text{Heat rejected} - \text{Heat absorbed}} = \frac{(T_6 - T_5)}{(T_2 - T_3) - (T_6 - T_5)}$$

$$(T_2 - T_3) = T_1 r_p^{\left(\frac{\gamma-1}{\gamma}\right)} - T_1 \text{ and}$$

$$(T_6 - T_5) = T_H - \frac{T_H}{r_p^{\left(\frac{\gamma-1}{\gamma}\right)}}$$

Substituting, COP $= \dfrac{T_H}{T_1 r_p^{\left(\frac{\gamma-1}{\gamma}\right)} - T_H}$

Example 12.6

An air standard refrigeration cycle leads air to the compressor at 120 kPa, 268K with a compression ratio of 4.2:1. The temperature after heat rejection is 301 K. Find the COP and the highest temperature in the cycle.

○●●

Solution

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 4.2^{0.2857} = 1.507 = \frac{T_3}{T_4}$$

$$T_2 = 1.507 \times 268 = 403.9 \text{ K}$$

$$\text{COP} = \frac{T_1}{T_2 - T_1} = \frac{268}{403.9 - 268} = 1.972$$

Example 12.7

A standard air refrigeration cycle has –15°C, 110 kPa into the compressor, and the ambient cools the air down to 30°C at 400 kPa. Find the lowest temperature in the cycle, the low T specific heat transfer and the specific compressor work.

○●●

Solution State 3: 30°C = 303 K, 400 kPa

The lowest T is at state 4 which can be related to state 3 constant as

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = 303 \cdot \left(\frac{110}{400}\right)^{0.2857} = 209.54 \text{ K}$$

Heat rejection $= Q_{41} = H_1 - H_4 = c_p(T_1 - T_4) = 1.005 \times (258 - 209.54) = 48.7 \text{ kJ/ kg}$

The isentropic compression, $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 258 \times \left(\frac{400}{110}\right)^{0.2857} = 373.1 \text{ K}$

Therefore, compressor work $= H_1 - H_2 = c_p(T_1 - T_2) = 1.005 \times (258 - 373.1) = -115.68 \text{ kJ/ kg}$

Example 12.8

Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kPa, 27°C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume $W_T = 2.5\ W_C$, where W_T and W_C are the turbine and the compressor work respectively. Take $\gamma = 1.4$. **(PU, 2012)** ○●●

Solution The isentropic compression, $T_2 = T_1\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times (6)^{0.2857} = 500.54\text{ K}$

Similarly, $T_3 = T_4\left(\dfrac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = T_4(6)^{0.2857} = 1.668$

Since $W_T = 2.5W_C,\ mc_p(T_3 - T_4) = 2.5mc_p(T_2 - T_1)$

$$\therefore \quad T_3 - \frac{T_3}{1.668} = 2.5(500.54 - 300)$$

$$\rightarrow \quad T_3 = 1251.87\text{ K and } T_4 = 750.5\text{ K}$$

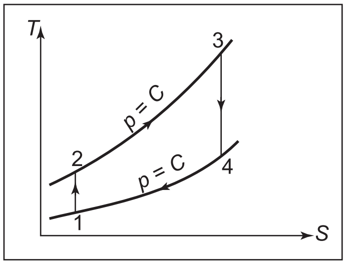


Figure 12.8

Cycle efficiency, $\eta = \dfrac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)} = 40.04\%$

POINTS TO REMEMBER

- Refrigerators work with exchange of heat.
- Primary aim of a refrigerator is to cool things kept inside a refrigerator.
- The performance is measured by COP, SEER or tons of refrigeration.
- Carnot cycle is ideal and real cycle differs from it.
- Multi stage compression can be obtained by using more than one compressor.
- All processes are internally reversible except for the throttling process.
- For applications in which the refrigerated space is maintained at temperatures below 0°C, the refrigerant is normally ammonia and the carrier is water.
- The refrigerant is always a mixture—analysis is not straightforward.
- Concentration fraction and enthalpy of the mixture need to be found out.
- Mass balance for the absorber is crucial.
- In a gas refrigeration system, the greatest advantage is that there is no phase change.
- There are two heat exchangers, an expander and a compressor.
- Even air can be used as a refrigerator.

PRACTICE PROBLEMS

1. Refrigerant 134a is the working fluid in an ideal vapour-compression refrigeration cycle that communicates thermally with a cold region at –8°C and a warm region at 26°C. Saturated vapour enters the compressor at -10°C and saturated liquid leaves the condenser at 9 bar. The mass flow rate of the refrigerant is 0.08 kg/s. Determine (a) the compressor power, in kW, (b) the refrigeration capacity, in tons, (c) the coefficient of performance, and (d) the coefficient of performance of a Carnot refrigeration cycle operating between warm and cold regions at 26°C and –10°C, respectively. (**Ans:** (a) 2.48 kW; (b) 3.23 ton; (c) 4.57; (d) 10.5) ○●●

2. Refrigerant 134a is the working fluid in a vapour-compression refrigeration cycle that communicates thermally with a cold region at -8°C and a warm region at 30°C. Saturated vapour enters the compressor at –10°C and saturated liquid leaves the condenser at 9 bar. The efficiency of the compressor is 80%. The mass flow rate of the refrigerant is 0.08 kg/s. Determine (a) the compressor power, in kW, (b) the refrigeration capacity, in tons, (c) the coefficient of performance, and (d) the coefficient of performance of a Carnot refrigeration cycle operating between warm and cold regions at 26°C and –10°C, respectively. ●●●
(**Ans:** (a) $W_c = 3.1$ kW; (b) $\dot{Q}_{\text{in}} = 3.41$ ton; (c) 3.86; (d) 7.30)

3. Determine the ideal COP of an absorption refrigerating system in which the heating, cooling, and refrigeration take place at 227°C, 17°C, and –10°C respectively. (**Ans:** 4.09) ○●●

4. A gas turbine is supplied with gas at 5 bar and 1000 K and expands it adiabatically to 1 bar. The mean specific heat at constant pressure and constant volume are 1.0425 kJ/kg K and 0.7662 kJ/kg K respectively. (a) Draw the temperature-entropy diagram to represent the processes of the simple gas turbine system. (b) Calculate the power developed in kW per kg of gas per second and the exhaust gas temperature. **(GATE, 1995)** (**Ans:** (b) 361.64 kW/kg) ○●●

5. An isentropic air turbine is used to supply 0.1 kg/s of air at 0.1 MN/m^2 and at 285 K to a cabin. The pressure at inlet to the turbine is 0.4 MN/m^2. Determine the temperature at turbine inlet and the power developed by the turbine. Assume $c_p = 1.0$ kJ/kg K. **(GATE, 1999)** ○●●
(**Ans:** $T = 425.3$ K, $P = 13.85$ kW)

MULTIPLE CHOICE QUESTIONS

1. _______________ provides a heat transfer surface through which heat can pass from the refrigerated space or product into the vapourising refrigerant. ○○●
   (a) Condenser (b) Evaporator (c) Absorber (d) Compressor

2. The function of a _______________ is to remove the vapour from the evaporator and to raise its temperature and pressure to a point such that it (vapour) can be condensed with normally available condensing media. ○●●
   (a) Evaporator (b) Condenser (c) Compressor (d) Throttle valve

3. The COP of an air refrigerator system is _____________ a vapour compression system. ○○●
   (a) Less than (b) Greater than
   (c) Equal to (d) Greater than (but only sometimes)

4. The ideal gas-refrigeration cycle is similar to: ●●●

(a) Brayton cycle (b) Rankine cycle
(c) Reversed Rankine cycle (d) Reversed Brayton cycle

5. In gas cycle refrigeration system, the throttle valve of a vapour compression refrigerant system is replaced by: ○○●

(a) Capillary tube (b) Expander
(c) Reverse throttle valve (d) None of the above

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (b) 2. (c) 3. (a) 4. (d) 5. (b)

Psychrometrics

13

CHAPTER OUTLINE

- Properties of Atmospheric Air
- Psychrometric Chart
- Psychrometric Process

INTRODUCTION

Psychrometrics is the study of the properties of air and water vapour mixtures. The atmospheric air is composed of mixture of dry air and water vapour.

13.1 PROPERTIES OF ATMOSPHERIC AIR

University Questions

1. State Dalton's law of partial pressure. **(JNTU, 2013)**
2. Mention some gases present in dry air.
3. Define relative humidity. **(AU, 2015)**
4. Define specific humidity. **(AU, 2015)**
5. Define degree of saturation. **(AKTU, 2015)**
6. What is adiabatic saturation temperature? **(AU, 2015)**

By Dalton's law of partial pressure if p_a and p_w are partial pressure of dry air and water vapour then:

$$p_a + p_w = p$$

where, p is the atmospheric pressure of the air.

Mole fraction of dry air is given by $x_a = \frac{p_a}{p} = p_a$ ($\because$ $p = 1\text{atm}$)

Mole reaction of water vapour is given by $x_w = \frac{p_w}{p} = p_w$

In dry air is the mixture of gases like oxygen, nitrogen, carbon dioxide, hydrogen, argon, neon, helium, ozone, and xenon.

Relative humidity (RH) is defined as the ratio of partial pressure of water vapour, p_w, to the saturation pressure, p_s, of pure water at the same temperature of the mixture.

Relative humidity, $RH = \dfrac{p_w}{p_s}$

Specific humidity or humidity ratio, W, is defined as the mass of water vapour per unit mass of dry air in a mixture of air and water vapour.

Specific humidity, $W = \dfrac{m}{G}$.

where, G = mass of dry air and m = mass of water vapour.

If dry air and water vapour behave as ideal gases then, $W = 0.622\left(\dfrac{p_s}{p - p_s}\right)$, where p = atmospheric pressure, p_s = saturation pressure of water at temperature T.

The degree of saturation, μ, is defined as the ratio of actual specific humidity and saturated specific humidity, in which both should be at same temperature.

$$\mu = \frac{W}{W_s}$$

Adiabatic saturation temperature is the temperature at which the thermal equilibrium exists between the air, water vapour, and so because of which the air is being saturated.

Example 13.1

Atmospheric air at 1.0132 bar has a dry bulb temperature (DBT) of 34°C and a wet bulb temperature (WBT) of 28°C. Compute: (a) the partial pressure of water vapour, (b) the specific humidity, (c) the relative humidity, (d) the degree of saturation. **(JNU, 2014)** ●●●

Solution **Step 1:** Determine the specific humidity at exit

$$W_2 = \frac{0.622 p_s}{p - p_s}$$

The saturation pressure at wbt is 28°C is calculated using,

$$P = 0.61078 \exp\left(\frac{17.27T}{T + 237.3}\right)$$

$$p_s = 0.037798 \text{ bar}$$

$$W_2 = \frac{0.622 \times 0.03779}{1.0132 - 0.037798}$$

$$W_2 = 0.0240 \text{ kg vapour/kg dry air}$$

Step 2: Determine the specific humidity

$$W_1 = \frac{c_{pa}(T_2 - T_1) + W_2 \cdot H_{fg_2}}{H_{w_1} - H_{f_2}}$$

The values of latent heat of steam H_{fg_2}, specific enthalpy of saturated air H_{f_2}, specific enthalpy of saturated steam H_{w_1} are determined from the steam table.

From the temperature 28°C the values of:

$$H_{fg_2} = 2434.59 \text{ kJ/kg}$$

$$H_{f_2} = 117.38 \text{ kJ/kg}$$

From the temperature 34°C:

$$H_{w_1} = 2562.79 \text{ kJ/kg}$$

$$W_1 = \frac{1.005(28-34)+(0.0240\times 2434.59)}{2562.79-117.38}$$

$$W_1 = \frac{-6.03+58.4301}{2445.41}$$

$$W_1 = 0.0214 \text{ kg vapour/kg dry air}$$

Step 3: Partial pressure of water vapour

$$W_1 = \frac{0.622 p_w}{p - p_w}$$

$$0.0214 = \frac{0.622 \times p_w}{1.0132 - p_w}$$

$$\frac{0.0214}{0.622} = \frac{p_w}{1.0132 - p_w}$$

$$\frac{1.0132 - p_w}{p_w} = 29.065$$

$$1.0132 - p_w = 29.065\, p_w$$

$$p_w = 0.033 \text{ bar}$$

Step 4: Relative humidity RH

$$RH = \frac{p_w}{p_s}$$

Saturation pressure at 34°C is determine by:

$$P = 0.61078 \exp\left(\frac{17.27T}{T+237.3}\right)$$

$$p_s = 0.0531$$

$$RH = \frac{p_w}{p_s}$$

$$RH = \frac{0.033}{0.0531}$$

$$RH = 0.6214$$

Step 5: Degree of saturation

$$\mu = \frac{W}{W_s} = \frac{p_w}{p_s}\,\frac{p - p_s}{p - p_w}$$

$$\mu = \frac{0.033}{0.0531}\,\frac{1.0132 - 0.0531}{1.0132 - 0.033}$$

$$\underline{\mu} = 0.608$$

13.2 PSYCHROMETRIC CHART

The psychrometric chart is a graphical plot with specific humidity and partial pressure of water vapour in *Y* coordinate, and dry bulb temperature in *X* coordinate.

The parameters of psychrometric chart are volume of the mixture, wet bulb temperature, relative humidity and enthalpy of the mixture.

Probes

1. What is psychrometric chart?
2. What are the parameters used in psychrometric chart?

Example 13.2

Determine the relative humidity when the temperature measured using dry bulb thermometer is 50°F and 40°F measured by wet bulb thermometer. **(AU, 2015)** ○○●

Solution

Step 1: By using psychrometric chart, take a look at horizontal axis to find 50°F.

Step 2: Move a pencil up this line to meet the intersection with the diagonal line for 40°F.

Step 3: Identify that this point falls just over half way between the lines of relative humidly for 60% and 70%. So, the relative humidity is 68%.

Example 13.3

An air conditioning system was not working properly. The temperature of the evaporator coil is 40°C. The air in the room is at 45°C and relative humidity is at 60%. Will the air conditioner remove moisture from this air? ●●●

Solution

Step 1: By using psychrometric chart, take a look at horizontal axis to find 45°C.

Step 2: Move a pencil up this line to meet the intersection with the exponential line 40% relative humidity.

Step 3: Identify the dew point which is 35°C. From this evaporator coil is warmer than the dew point so it will condense water from air.

13.3 PSYCHROMETRIC PROCESS

University Questions

1. What are the types of Psychrometric process?
2. How the air can be cooled and dehumidified? **(AU, 2013)**
3. What is adiabatic mixing of two streams?
4. What is chemical humidification? **(AU, 2015)**
5. What is the process happens in cooling tower? **(AU, 2013)**

Psychometric process involves:

- Sensible heating or cooling
- Cooling and dehumidification
- Heating and humidification
- Chemical dehumidification
- Adiabatic evaporative cooling
- Adiabatic mixing of two streams

Air can be cooled and dehumidified by:

- Placing the evaporator coil across the air flow
- Spraying chilled water to air in the form of fine mist
- Circulating chilled water using tube placed across the air flow
- By exposing to large area

Adiabatic mixing of two streams is a problem that occurs in air conditioning when the ventilation air and the room air are mixed before the desired state and supplying to the conditioned space**.**

Chemical humidification is a process in which air is passed through substances like silica gel, the water vapour gets adsorbed on the surface of the silica gel by which the dbt of air increases.

The warm water is sprayed from top of the tower to expose the large area for evaporation. When the water gets evaporated more, then the cooling is very effective.

Example 13.4

Air at 25°C, 40% RH is mixed adiabatically with air at 45ºC, 40% RH in the ratio of 1 kg of the former with 2 kg of the latter. Find the final condition of air. **(AU, 2013)** ○○●

Solution

Step 1: Draw the psychrometric chart from the chart we can determined as it is adiabatic mixing two stream.

Step 2: By using psychrometric chart, take a look at horizontal axis to find DBT of 25°C. Move a pencil up this line to meet the intersection with the exponential line 40% relative humidity. Mark it as 1.

Step 3: Similarly take a look at horizontal axis to find DBT of 45°C. Move a pencil up this line to meet the intersection with the exponential line 40% relative humidity. Mark it as 2.

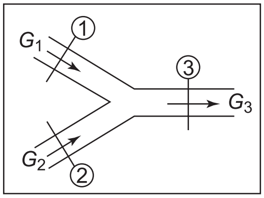


Figure 13.1

Step 4: From the psychrometric chart we can calculate the terms like enthalpy H_1, H_2 specific humidity, W_1, W_2

Step 5: Join the line 1–2. By using the formula of adiabatic equation we can calculate enthalpy H_3 and specific humidity W_3.

By this way the point 3 is located and the psychrometric chart is drawn.

Step 6: From the psychrometric chart we got the values like

$$H_1 = 45 \text{ kJ/kg}, H_2 = 109 \text{ kJ/kg},$$

$$W_1 = 0.007 \text{ kg vapour/kg dry air}, W_2 = 0.024 \text{ kg vapour/kg dry air},$$

$$G_1 = 1 \text{ and } G_2 = 2.$$

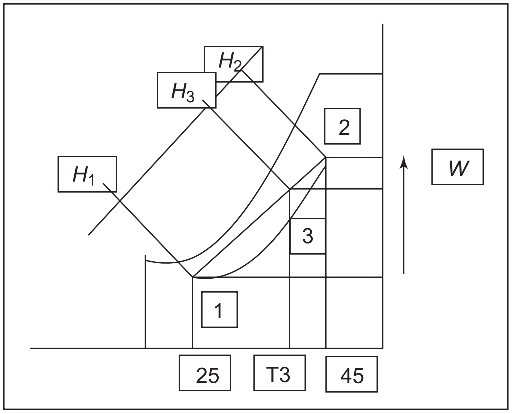


Figure 13.2

Step 7: The adiabatic equation of mixed two stream are

$$G_1 + G_2 = G_3$$

$$G_1W_1 + G_2W_2 = G_3W_3$$

$$G_1H_1 + G_2H_2 = G_3H_3$$

By simplifying the above equations we get the result as,

$$\frac{W_2 - W_3}{W_3 - W_1} = \frac{H_2 - H_3}{H_3 - H_1} = \frac{G_1}{G_2}$$

$$\frac{W_2 - W_3}{W_3 - W_1} = \frac{G_1}{G_2}$$

$$\frac{0.0024 - W_3}{W_3 - 0.007} = \frac{1}{2}$$

$$(0.0024 - W_3)2 = W_3 - 0.007$$

$$0.0055 = 3W_3$$

$$W_3 = 0.0018 \text{ kg vapour/kg dry air.}$$

$$\frac{H_2 - H_3}{H_3 - H_1} = \frac{G_1}{G_2}$$

$$\frac{108.7 - H_3}{H_3 - 45.29} = \frac{1}{2}$$

$$(108.7 - H_3)2 = H_3 - 45.29$$

$$262.61 = 3H_3$$

$$H_3 = 87.5 \text{ kJ/kg dry air.}$$

Therefore, the final condition of air is:

$$W_3 = 0.0018 \text{ kg vapour/kg dry air}$$

$$H_3 = 87.5 \text{ kJ/kg dry air}$$

Example 13.5

It is required to design an air-conditioning system for an industrial process for the following hot and wet summer conditions:

- Outdoor conditions—34°C DBT and 70% R.H.
- Required air inlet conditions—25°C DBT and 65% R.H.
- Amount of free air circulated—5 m^3/s
- Coil dew temperature—15°C.

The required condition is achieved by first cooling and dehumidifying and then by heating. Calculate the following:

(a) The cooling capacity of the cooling coil in tonnes and its by-pass factor.

(b) Heating capacity of the heating coil in kW

(c) The mass of water vapour removed. Solve this problem with the use of psychrometric chart.

(AU, 2014) ○○●

Solution

Step 1: Draw the psychrometric chart.

Step 2: By using psychrometric chart, take a look at horizontal axis to find DBT of 34°C. Move a pencil up this line to meet the intersection with the exponential line 70% relative humidity. Mark it as 1.

Step 3: Similarly take a look at horizontal axis to find DBT of 25°C. Move a pencil up this line to meet the intersection with the exponential line 65% relative humidity. Mark it as 2.

Step 4: Similarly take a look at horizontal axis to find dew point of 15°C. Move a pencil up this line to meet the intersection with the exponential line 100% relative humidity. Mark it as 4. The relative humidity is 100% as the dew point is in saturation.

Step 5: Join the line 1–4. Draw the constant specific humidity line through 2 which cuts the line 1–4 at point 3.

By this way the point 3 is located for which we can determine the enthalpy.

The enthalpy and specific humidity from each point is determined from the psychrometric chart.

From psychrometric chart we calculate the values:

$$H_1 = 95.3 \text{ kJ/kg}, H_3 = 48.27 \text{ kJ/kg}, H_2 = 58.143 \text{ kJ/kg}, H_4 = 42.11 \text{ kJ/kg}$$

$$W_1 = 0.023 \text{ kg/kg dry air}, W_2 = 0.012 \text{ kg/kg dry air}, V_1 = 0.9022 \text{ m}^3\text{/kg}.$$

Step 6: Determine the mass of air per minute

$$G = \frac{m}{W} = \frac{5}{0.902} = 5.54 \text{ kg/ sec}$$

Step 7: Determine the cooling coil capacity

Cooling coil capacity = $G (H_1 - H_3) = 95.3 - 48.27 = 47$ kJ/s

$$\frac{5.54 \times 47 \times 3600}{14000} = 66.95 \text{ ton}$$

Step 8: Determine the capacity heating coil

Cooling coil capacity = $G (H_2 - H_3) = 58.143 - 48.27 = 9.877$ kJ/s

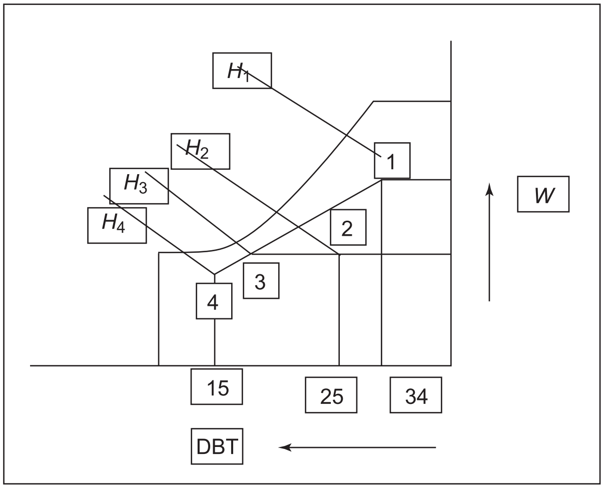


Figure 13.3

$9.877 \times 5.44 = 54.71$ kW.

Step 9: Determine the mass of water vapour removed

Cooling coil capacity = $G(W_1 - W_3) = 0.023 - 0.012 = 0.011$

$0.011 \times 5.54 = 0.06$ kg/s.

Example 13.6

It is required to design an air-conditioning system for an industrial process for the following hot and wet summer conditions:

- Outdoor conditions—32°C DBT and 65% RH.
- Required air inlet conditions—25°C DBT and 60% RH.
- Amount of free air circulated—250 m^3/min
- Coil dew temperature—13°C.

The required condition is achieved by first cooling and dehumidifying and then by heating. Calculate the following:

(a) The cooling capacity of the cooling coil in tonnes and its by-pass factor.

(b) Heating capacity of the heating coil in kW

(c) The mass of water vapour removed per hour. Solve this problem with the use of psychrometric chart. **(AU, 2014)** ●●●

Solution

Step 1: Draw the psychrometric chart.

Step 2: By using psychrometric chart, take a look at horizontal axis to find DBT of 32°C. Move a pencil up this line to meet the intersection with the exponential line 65% relative humidity. Mark it as 1.

Step 3: Similarly take a look at horizontal axis to find DBT of 25°C. Move a pencil up this line to meet the intersection with the exponential line 65% relative humidity. Mark it as 2.

Step 4: Similarly take a look at horizontal axis to find dew point of 13°C. Move a pencil up this line to meet the intersection with the exponential line 100% relative humidity. Mark it as 4. The relative humidity is 100% as the dew point is in saturation.

Step 5: Join the line 1–4. Draw the constant specific humidity line through 2 which cuts the line 1–4 at point 3.

By this way the point 3 is located for which we can determine the enthalpy.

The enthalpy and specific humidity from each point is determined from the psychrometric chart.

From psychrometric chart, we calculate the values:

$$H_1 = 82.5 \text{ kJ/kg}, H_3 = 47.5 \text{ kJ/kg}, H_2 = 55.7 \text{ kJ/kg}, H_4 = 36.6 \text{ kJ/kg}$$

$$W_1 = 19.6 \text{ g/kg}, W_2 = 11.8 \text{ g/kg}, V_1 = 0.892 \text{ m}^3\text{/kg}$$

Step 6: Determine the mass of air per minute

$$G = \frac{m}{W} = \frac{250}{0.892} = 280.26 \text{ kg/ min}$$

Step 7: Determine the cooling coil capacity

Cooling coil capacity = $G(H_1 - H_3) = 82.5 - 47.5 = 35$ kJ/s

$$\frac{280.26 \times 35 \times 3600}{14000} = 42.039 \text{ ton.}$$

Step 8: Determine the capacity heating coil

Cooling coil capacity = $G\,(h_2 - h_3) = 55.7 - 47.5 = 8.2$ kJ/s

$8.2 \times 280.26 = 2{,}298.132$ kJ/min

$$\frac{2{,}298.132}{60} = 38.3 \text{ kW}$$

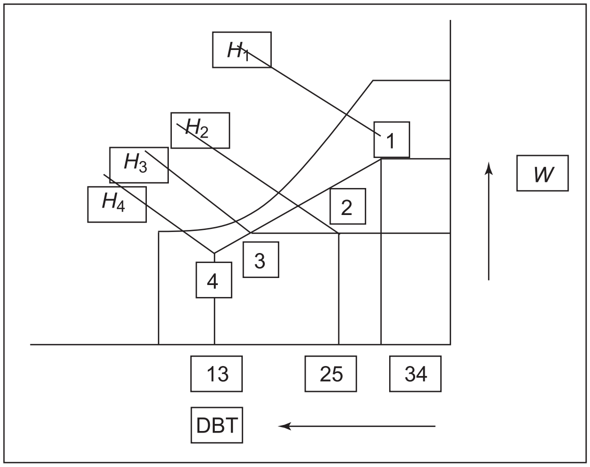


Figure 13.4

Step 9: Determine the mass of water vapour removed

Cooling coil capacity = $G\,(W_1 - W_3) = 19.6 - 11.8 = 7.8$

$$\frac{280.26 \times 7.8 \times 60}{1000} = 131.16 \text{ kg / h}$$

POINTS TO REMEMBER

- ☞ Psychrometics is the study of properties of air-water vapour mixtures.
- ☞ Dry air has 79% nitrogen and 21% oxygen by volume and 77% nitrogen and 23% oxygen by mass.
- ☞ A psychrometer is an instrument which measures both the dry bulb and the wet bulb temperatures.
- ☞ In psychrometric chart:
  - Dry bulb temperature—horizontal axis
  - Wet bulb temperature—diagonal axis
  - Relative humidity—exponential line

  - Dew point—vertical axis
- ☞ In psychrometric process
  - Sensible heating or cooling, $G_1 = G_2$
  - Cooling and dehumidification, $G_1 = G_2 = G$
  - Heating and humidification, $G_1 = G_2 = G$
  - Adiabatic mixing, $G_1 + G_2 = G_3$
- ☞ Chemical dehumidification uses silica gel as an adsorbent
- ☞ Adiabatic evaporative cooling, $G_1 = G_2 = G$

PRACTICE PROBLEMS

1. An air-water vapour mixture enters an adiabatic saturator at 35ºC and leaves at 25ºC, which is the adiabatic saturation temperature. The atmospheric pressure remains constant at 1.0132. Determine the relative humidity and the humidity ratio of the inlet mixture. ●●●
   (**Ans:** Relative humidity = 0.78623, Humidity ratio = 0.0157)
2. The atmospheric condition of the dry bulb temperature is 20°C and specific humidity, W 0.0095 kg/kg of dry air and the atmospheric pressure is at 1.0132 bar. Calculate the following: (a) Partial pressure of vapour, (b) Relative humidity. (**Ans:** (a) 0.01524 bar, (b) 65%) ○○●
3. Determine the relative humidity using psychrometric chart where dry bulb thermometer reads 60°C and wet bulb thermometer reads the reading as 45°C. (**Ans:** 47%) ○○●
4. Determine the dew point using psychrometric chart where the relative humidity is 60% and the temperature measured using dry bulb thermometer is 40°C. ●●●
5. Air at 30°C, 45% RH is mixed adiabatically with air at 45°C, 45% RH in the ratio of 2 kg of the former with 3 kg of the latter. Find the final condition of air. ○○●
   (**Ans:** $W = 0.021$ kg vapour/kg dry air, $H = 94.5$ kJ/kg dry air)
6. Design an air-conditioning plant for a small office room for following winter conditions: ○●●
   - Outdoor conditions—14°C DBT and 10°C WBT
   - Required conditions—20°C DBT and 60% RH
   - Amount of air circulation—0.30 m^3/min/person
   - Seating capacity of office—60

   The required condition is achieved first by heating and then by adiabatic humidifying. Determine the following: (a) Heating capacity of the coil in kW (b) The capacity of the humidifier. Solve the problem by using psychrometric chart.
   (**Ans:** (a) 4.76 kW, (b) 2.3376 kg/h)
7. For a hall to be air-conditioned, the following conditions are given: ●●●
   - Outdoor condition—30°C dbt, 10°C WBT
   - Required comfort condition—10°C DBT, 50% RH
   - Seating capacity of hall—1500
   - Amount of outdoor air supplied—0.3 m^3/min per person

If the required condition is achieved first by adiabatic humidification and then by cooling, estimate:

(a) the capacity of the cooling coil in tonnes and
(b) the capacity of the humidifier in kg/h.

(**Ans:** (a) 81.82 tonnes, (b) 191.9 kg/h)

MULTIPLE CHOICE QUESTIONS

1. The temperature at which water vapour starts condensing is called _________ of the mixture. ○○●
(a) Dew point temperature (b) Saturated temperature
(c) Adiabatic temperature (d) Unsaturated temperature

2. _________ is used to measure the dry bulb and wet bulb temperatures of air ○○●
(a) Psychrometer (b) Galvanometer
(c) Temperature sensor (d) Resistance heating

3. _____________ occurs in summer air conditioning. ○●●
(a) Sensible heating or cooling (b) Cooling and dehumidification
(c) Heating and humidification (d) Chemical dehumidification

4. ____________ problem occurs in winter air conditioning. ○●●
(a) Sensible heating or cooling (b) Cooling and dehumidification
(c) Heating and humidification (d) Chemical dehumidification

5. Cooling tower utilises the phenomenon of _________. ●●●
(a) Evaporative cooling (b) Cooling and dehumidification
(c) Heating and humidification (d) Chemical dehumidification

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (a) 2. (a) 3. (b) 4. (c) 5. (a)

Reactive Systems

14

Chapter Outline

- ☞ Law of Mass Action
- ☞ Enthalpy of Formation and Enthalpy of Combustion
- ☞ Combustion and Fuel
- ☞ First Law for Reactive System
- ☞ Adiabatic Flame Temperature
- ☞ Second Law Analysis of Reactive Systems

INTRODUCTION

For many thermodynamics processes like combustion, the chemical composition of the system does not remain same throughout the process. This is because of breaking down of intermolecular bonds and hence formation of new molecules as the process advances. Such systems are called reactive systems.

For a chemically reactive system, the thermodynamics analysis requires the chemical energies associated with the process to be taken into account along with sensible energy and latent internal energies. In this module, thermodynamic analysis of such reactive system is discussed with combustion being specifically emphasized.

14.1 || LAW OF MASS ACTION

University Question

1. Explain law of mass action. **(JNTU, 2015)**

For a chemical reaction,

$$uA + vB \longrightarrow xC + yD \qquad (14.1)$$

the driving force of a reaction depends upon the concentrations of reactants and products.

The ratio of products to reactants is expressed as:

$$\frac{[C]^x[D]^y}{[A]^u[B]^v} = K$$

If the system is at equilibrium at a given temperature, this ratio is constant. This is called *law of mass action*. The ratio at equilibrium is called *equilibrium constant*.

14.1.1 Standard Derivation using Chemical Potential

Gibbs function for a chemical reaction [here, Equation (14.1)], at any stage is written in terms of chemical potentials of reactants (μ) as:

$$G = \sum a_i \mu_i = a_1\mu_1 + b\mu_1 + c_1\mu_1 + d_1\mu_1$$

a_1 is the number of moles of reactant A at that instant and μ_1 is the chemical potential of A.

At equilibrium G is minimum and this gives,

$$(u\mu_1 + v\mu_1) - (x\mu_1 + y\mu_1) = 0$$

$$u\mu_1 + v\mu_1 = x\mu_1 + y\mu_1 \tag{14.2}$$

Chemical potential for an ideal gas, $\mu = RT(\phi + \ln P + \ln x)$

Here, ϕ is function of temperature and x denotes the degree of mole fractions.

Substituting expressions of ϕ in the equation (14.2),

$$u\mu_1 + v\mu_1 = x\mu_1 + y\mu_1$$

$$\Rightarrow \quad u\,(\phi_1 + \ln P + \ln x_1) + v(\phi_2 + \ln P + \ln x_2) = x(\phi_3 + \ln P + \ln x_3) + y(\phi_4 + \ln P + \ln x_4)$$

Rearranging the terms and taking logarithm both sides,

$$\Rightarrow \quad \ln\left(\frac{(x_3{}^{x} x_4{}^{y})}{(x_1{}^{u} x_2{}^{v})}\right)P^{(x+y-u-v)} = u\phi_1 + v\phi_2 - x\phi_3 - y\phi_4$$

$$\Rightarrow \quad \frac{(x_3{}^{x}\, x_4{}^{y})}{(x_1{}^{u}\, x_2{}^{v})}\,P^{(x+y-u-v)} = e^{(u\phi_1 + v\phi_2 - x\phi_3 - y\phi_4)} = K$$

The equation is known as law of mass action and K is called equilibrium constant.

Example 14.1

For a chemical reaction $H_2 + I_2 \longrightarrow 2HI$, Find out the equilibrium constant at 426°C, if concentrations of H_2, I_2, HI are respectively 0.0037 mol/L, 0.0037 mol/L and 0.0276 mol/L at equilibrium (at 426°C). Also, find the equilibrium constant for the reverse equation. ○○●

Solution Given, [HI] = 0.0276 mol/L, [H_2] = 0.0037 mol/L and [I_2] = 0.0037 mol/L.

(a) Equilibrium constant from the mass action law is given by:

$$K = \frac{[\mathrm{HI}]^2}{[\mathrm{H_2}]^1[\mathrm{I_2}]^1} = \frac{0.0276^2}{0.0037 \times 0.0037} = 55.6$$

(b) For the reverse equation:

$$2\mathrm{HI} \longrightarrow \mathrm{H_2} + \mathrm{I_2}$$

$$K' = \frac{[\mathrm{H_2}]^1[\mathrm{I_2}]^1}{[\mathrm{HI}]} = \frac{0.0037 \times 0.0037}{0.0276^2} = \frac{1}{55.6}$$

14.2 ENTHALPY OF FORMATION AND ENTHALPY OF COMBUSTION

University Question

1. Define the following terms: (a) enthalpy of formation (b) enthalpy of combustion **(AKTU, 2011–2012)**

14.2.1 Heat Generated by Combustion

So far, we have worked with a thermodynamic process with a fixed chemical composition. And as a result, we could use thermodynamic properties relative to an arbitrary base value, since all comparisons could be made with respect to the chosen base.

For example, the specific energy $u_f(0.01^\circ\text{C}) = 0.0$ for steam. If there are no changes in composition, and only changes in properties of given substances, this is adequate. But, in a process like combustion, the composition does not remain same; and this arises the need to have a reference state so there is consistency for different substances.

The convention used for this is that the reference state is a temperature of $(01^\circ\text{C or } 298\text{ K})$ and a pressure of 0.1 MPa or 1 bar (these are the room conditions.) At these reference conditions, the enthalpy of the elements (oxygen, hydrogen, nitrogen, carbon, etc.) is taken as zero.

A combustion process is shown in Figure 14.1. The reactants enter at standard conditions; the combustion (reaction) takes place in the volume indicated. Downstream of the reaction zone there is an appropriate amount of heat transfer with the surroundings so that the products leave at the standard conditions. For the reaction of carbon and oxygen to produce CO_2, the heat that has to be extracted is $Q_{CV} = -3,93,522$ kJ; this is heat that comes out of the control volume or the heat generated by combustion.

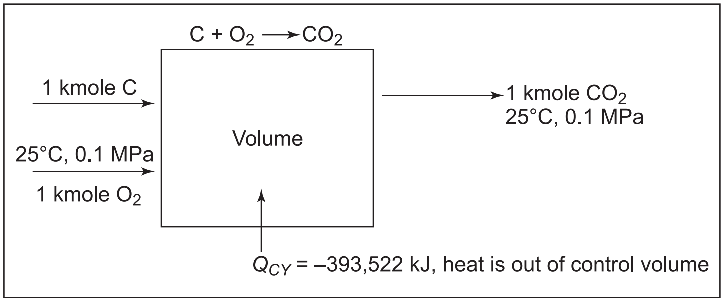


Figure 14.1 *Combustion at constant pressure (negative sign shows direction of net heat transfer is opposite to the direction of arrow)*

14.2.2 Enthalpy of formation (ΔH_f^0)

The amount of heat absorbed when one mole of a substance is formed at 25°C temperature and 1 atm pressure is called enthalpy of formation (Figure 14.2). It is measured in kJ/mol.

Since the value depends upon the temperature, pressure and the chemical state of the reactants and products, state of the system, *i.e.* generally the temperature and pressure is to be specified while mentioning the heat of formation.

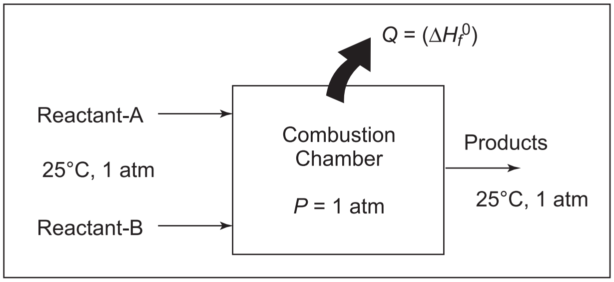


Figure 14.2 *Enthalpy of formation*

The heat of formation of a compound from its elements in their standard states is called the standard heat of formation.

Generally, 25°C temperature and 1 atm pressure is chosen as the standard condition and enthalpy change in this state is written as ΔH_0.

As a convention, a value of zero is assigned to the standard heat content of a pure element at 25°C and 1 atm pressure.

14.2.3 Enthalpy of Combustion

Enthalpy of combustion is the enthalpy change associated with complete combustion of a substances at a given temperature and pressure.

$$H_{RP} = H_R - H_P$$

Here, $$H = (H_f^0 + \Delta H)$$

H_f^0 : Enthalpy at reference state i.e. 1 atm pressure and 273.15°C temperature.

ΔH: Enthalpy change between the reference state and the state of interest.

Enthalpy of combustion = Sum of enthalpies of all reactants – Sum of enthalpies of all products

or $$H_{RP} = \sum n_P (H_f^0 + \Delta H) - \sum n_R (H_f^0 + \Delta H)$$

Example 14.2

Chemical reaction, $C(s) + O_2(g) \longrightarrow CO_2\ (g)$ at 25°C and 1 atm releases –17.9 kcal of heat, if temperature of outlet, i.e. CO_2 is kept constant at 25°C and 1 atm respectively.

Calculate the enthalpy of formation for CO_2.

Using the value of enthalpy of formation for CO_2, find out the enthalpy of combustion of Methane, which is burnt in presence of pure oxygen. Given here:

$\Delta H^0_{CH_4} = -94.0$ kCal ; $\Delta H^0_{O_2} = -26.3$ kcal ; $\Delta H^0_{CO_2} = ?$; $\Delta H^0_{H_2O} = 0.0$ kcal ○●●

Solution Given,

1. For CO_2 formation, heat released, $Q = -17.9$ kcal
2. Enthalpy change data

Find out,

1. Enthalpy of formation for CO_2
2. Enthalpy of combustion for methane.

Analysis,

(a) From the definition of enthalpy of formation, $\Delta H^0_{CO_2}$ for $CO_2 = Q = -17.9$ kcal.

(b) Chemical reaction for the burning of Methane is:

$$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$$

Enthalpy of combustion = Sum of enthalpies of all reactants – Sum of enthalpies of all products

or
$$H_{RP} = \sum n_P (H_f^0 + \Delta H) - \sum n_R (H_f^0 + \Delta H)$$

$$H_{RP} = [n_{CO2} \times (H_f^0 + \Delta H)_{\mathrm{CO_2}}) + n_{\mathrm{H_2O}} \times (H_f^0 + \Delta H)_{\mathrm{H_2O}}]$$
$$- [n_{CH4} \times ((H_f^0 + \Delta H)_{\mathrm{CH_4}} + n_{\mathrm{O_2}} \times (H_f^0 + \Delta H)_{\mathrm{O_2}})]$$

Since reaction takes place at reference temperature (for both reactants and products), terms $\Delta H = 0$.

$$H_{RP} = [(1x - 94.0) + (2x - 26.3)] - [(1x - 17.19) + (2 \times 0)] = -212.70 \text{ kCal}$$

14.3 || COMBUSTION AND FUEL

University Question

1. Discuss various factors affecting the process of combustion. **(AKTU, 2011–2012)**

A fuel is a combustible substance having carbon as its main constituent which on proper burning gives large amount of heat which can be used economically for various domestic and industrial purposes. The calorific value of a fuel depends mainly on the two elements.

Combustion is an exothermic redox chemical reaction between a fuel and an oxidant (which is generally the air oxygen), that releases energy in the form of heat and/or light. It releases water/vapour and other products in gaseous form called smoke. So, during the process of combustion of a fuel, the atoms of carbon, hydrogen, etc., combine with oxygen with simultaneous liberation of heat.

Fuel + Oxidant → Products of Combustion + Energy

A complete combustion means entire mass of fuel is oxidised and no further air is required. A complete combustion requires a minimum amount of air which supplies the required amount of oxygen for the complete combustion of a fuel. This minimum amount of air is called the *stoichiometric or theoretical air*.

The air in excess of the stoichiometric air is called *excess air*. This is generally expressed in terms of the stoichiometric air as percent excess air. Amount of air less than the stoichiometric are is called *deficiency of air*.

A complete oxidation of simple hydrocarbon fuels results formation of carbon dioxide (CO_2) from all of the carbon and water (H_2O) from the hydrogen (Figure 14.3).

For example, for a typical hydrocarbon fuel with the general composition C_xH_y,

$$\mathrm{C}_x\mathrm{H}_y + \left(x + \frac{y}{4}\right)\mathrm{O}_2 \longrightarrow x\mathrm{CO}_2 + \frac{y}{2}\mathrm{H}_2\mathrm{O} \qquad (14.3)$$

Since, the combustion takes place in presence of natural air rather than pure oxygen, which contains 79.1% of nitrogen (which means for 1 mole of O_2, 3.78 moles of N_2 will also be naturally present), the stoichiometric equation (14.3) gets modified as:

$$\mathrm{C}_x\mathrm{H}_y + \left(x + \frac{y}{4}\right)(\mathrm{O}_2 + 3.78\,\mathrm{N}_2) \longrightarrow x\mathrm{CO}_2 + \frac{y}{2}\mathrm{H}_2\mathrm{O} + \left(x + \frac{y}{4}\right)(3.78\,\mathrm{N}_2) \qquad (14.3)$$

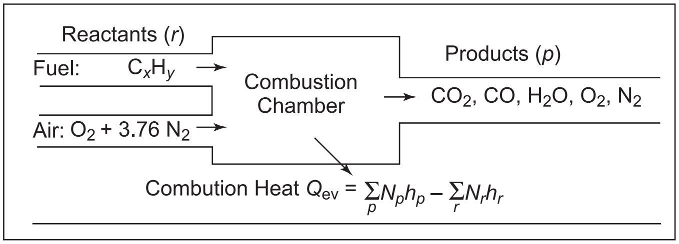


Figure 14.3 *Combustion of a hydrocarbon in natural air*

14.3.1 Air-Fuel Ratio

For a combustion process, air-fuel ratio (AFR) is the mass ratio of air to a solid, liquid, or gaseous fuel present in the process.

$$\text{AFR} = \frac{\text{Mass of air}}{\text{Mass of fuel}}$$

For gasoline fuel, the stoichiometric AFR is about 15, i.e. for every 1 gram of fuel, 15 grams of air is required.

AFR for the stoichiometric equation (14.4) can be calculated as:

$$\frac{\text{Mass of air}}{\text{Mass of fuel}} = \frac{\text{Mass of oxygen} + \text{Mass of nitrogen}}{\text{Mass of fuel}}$$

$$= \left[\frac{\left\{ \left(x + \frac{y}{4} \right) \times 32 + 3.78 \times \left(x + \frac{y}{4} \right) \times 34 \right\}}{(12x + 1y)} \right]$$

14.3.2 Factors Affecting the Process of Combustion

1. ***Molecular structure of fuel:*** If molecular structure is compact, tendency to detonate is lower. For example, products like Benzene, Toluene and Xylenes have least tendency for detonation. On other hand, alcohol is definitely one of the best fuels if it contains no water.
2. ***Temperature of self-ignition:*** Generally, fuels with higher self-ignition temperature are less detonating, even though there exists no strict relation between the self-ignition temperature and detonation.
3. ***Effect of high temperature and pressure after compression:*** The velocity of the flame decreases the tendency to detonate, though the pressure and temperature increase at the end of compression. In other words, increase in these two factors leads to decrease the delay period of the initial reactions there by increasing the tendency to detonate. This will also result in predomination and the tendency increases due to rise in both temperature and pressure.

4. ***Temperature of combustion wall chamber:*** Wall temperature gives a profound influence indicating the liability of an air-cooled engine for detonating more readily than water cooled engine for the same combustion chamber. The exhaust valves and effective cooling lead to reduce the detonation.
5. ***Rate of burning:*** The huge rate of burning will give less time for flow of heat to the wall of chamber and hence results in high temperature that normally increases the tendency to detonate.
6. ***Spark timing:*** Spark timing also called as ignition timing is one of the important factors controlling the combustion efficiency. Generally, advanced ignition timing makes the engine power to be maximum. In other words, too little spark advance known as spark retard will be wasting the energy allowing the combustion heat to leave the engine without being used. Large combustion heat will exit out past the open exhaust valve. If retard is too much, it can overheat and make some cracks on the engine manifold. By having late timing of spark, most of heat is passed to the engine block, cylinder head, and exhaust system instead of its use to power the engine. Spark timing has to be set to manufacturers' specifications to burn the fuel as fast as possible.
7. ***Air-fuel ratio:*** It is defined as the amount of fuel compared to air entering an engine during the suction stroke. This ratio gap be adjusted for meeting the changing requirements of the engine and also affect the speed of flame during travel.
8. ***Charge distribution:*** There are different detonating characteristics for different fractions of a commercial gasoline. The more volatile fractions having higher octane number; are evaporated completely in the inlet manifold and therefore they are distributed more or less constantly among all the cylinders while carburettor is used for supplying fuel for a multi-cylinder engine. If fraction is less, then they are distributed uniformly less. This will result in different knocking tendencies in various cylinders.
9. ***Charge temperature:*** When the charge temperature increases, the evaporation of the fuel also increases and gives better distribution of the heavier fuel fractions which reduces the tendency to detonate. Also, there is a reduction of the density of the charge with an increase in mixture temperature, which further decreases the combustion pressure reducing the tendency to detonate.

Example 14.3

Determine the air-fuel ratio for the complete combustion of C_8H_{18} with theoretical amount of air.

○●●

Solution Given: Reactants—Air and C_8H_{18}

Find out: Air fuel ratio for the combustion of C_8H_{18}.

Properties: Molar mass of C, H_2 and O_2 and air are: 12 kg/kmol, 2 kg/kmol , 32 kg/kmol and 29 kg/kmol respectively.

Assumption: (1). Combustion is complete and (2) Combustion products are H_2O, CO_2 and N_2 only.

Chemical reaction taking place for the combustion of a hydrocarbon can be written as:

$$C_xH_y + \left(x + \frac{y}{4}\right)(O_2 + 3.78\,N_2) \longrightarrow xCO_2 + \frac{y}{2}H_2O + \left(x + \frac{y}{4}\right)(3.78\,N_2)$$

Here for C_8H_{18}, $x = 8$ and $y = 18$.
Putting these values in the chemical reaction gives:

$$C_8H_{18} + \left(8 + \frac{18}{4}\right)(O_2 + 3.78\,N_2) \longrightarrow 8\,CO_2 + \frac{18}{2}H_2O + \left(8 + \frac{18}{4}\right)(3.78\,N_2)$$

Simplifying further:

$$C_8H_{18} + 12.5(O_2 + 3.78\,N_2) \longrightarrow 8\,CO_2 + 9H_2O + 12.5(3.78\,N_2)$$

The molar air-fuel ratio:
Mole of air = 12.5 moles of oxygen + 12.5 × 3.78 moles of nitrogen = 59.5

$$\text{AFR mole} = \frac{\text{Mole of air}}{\text{Mole of fuel}} = \frac{59.5}{1} = 59.5$$

Mass air-fuel ratio:

$$\text{AFR mass} = \frac{\text{Mass of oxygen + Mass of nitrogen}}{\text{Mass of fuel}} = \frac{[12.5 \times 32 + 3.78 \times 12.5 \times 34]}{[12 \times 8 + 1 \times 8]} = 15.1$$

which means 15.1 kg of air is required for a complete combustion of 1 kg of C_8H_{18}.

Example 14.4

Determine the mass and mole fraction of CO_2 and water-vapour if C_3H_8 is burned with theoretical amount of air. ○●●

Solution Given: Combustion reactants: Air and C_3H_8
Find out: Mass and mole fraction of CO_2.

Properties: Molar mass of C, H_2 and O_2 and air are: 12 kg/kmol, 2 kg/kmol, 32 kg/kmol and 29 kg/kmol respectively.

Assumption: (1) Combustion is complete and (2) Combustion products are H_2O, CO_2 and N_2 only.

Chemical reaction taking place for the combustion of a hydrocarbon can be written as-

$$C_3H_8 + a(O_2 + 3.78\,N_2) \longrightarrow bCO_2 + cH_2O + d(3.78\,N_2)$$

Applying conservation of mass (atoms) both sides for Hydrogen, Carbon, Oxygen and Nitrogen gives:

H balance: $2\,c = 8$ or $c = 4$.
C balance: $b = 3$.
O balance: $2\,a = z\,b + c$ or $a = 5$.
N balance: $2 \times 3.78\,a = 2d$ or $d = 18.8$

Balanced equation for the combustion will be:

$$C_3H_8 + 5(O_2 + 3.78\ N_2) \longrightarrow 3\ CO_2 + 4H_2O + 5(3.78\ N_2)$$

$$C_3H_8 + (5O_2 + 18.8\ N_2) \longrightarrow 3\ CO_2 + 4H_2O + (18.8\ N_2)$$

Mass fraction

$$\text{for } CO_2 = \frac{\text{Mass of } CO_2}{\text{Mass of products}} = \frac{\text{Mass of } CO_2}{\text{Mass of } CO_2 + \text{Mass of } H_2O + \text{Mass of } N_2}$$

$$= \frac{[3 \times 44]}{[3 \times 44 + 4 \times 18 + 18.8 \times 28]} = 0.181$$

Mass fraction

$$\text{for } H_2O = \frac{\text{Mass of } H_2O}{\text{Mass of products}} = \frac{\text{Mass of } H_2O}{\text{Mass of } CO_2 + \text{Mass of } H_2O + \text{Mass of } N_2}$$

$$= \frac{[4 \times 18]}{[3 \times 44 + 4 \times 18 + 18.8 \times 28]} = 0.155$$

Example 14.5

Repeat Example 14.4 if the burning takes place with 80% excess air. ●●●

Solution The excess air, i.e. 80% or 0.8 will be the surplus air and will reflect as product, hence the chemical reaction will become:

$$C_3H_8 + 1.8\,a(O_2 + 3.78\ N_2) \rightarrow bCO_2 + cH_2O + (0.8a \times d)(3.78\ N_2) + 0.8aO_2$$

Both side atom conservation gives:
H balance: $2\,c = 8$ or $c = 4$.
C balance: $b = 3$.
O balance: $2\,a = z\,b + c$ or $a = 5$.
N balance: $2 \times 3.78\,a = 2d$ or $d = 18.8$
Reaction equation becomes:

$$C_3H_8 + (9\ O_2 + 34.02\ N_2) \longrightarrow 3\ CO_2 + 4H_2O + 34.02\ N_2 + 4\ O_2$$

Mass fraction

$$\text{for } CO_2 = \frac{\text{Mass of } CO_2}{\text{Mass of products}} = \frac{\text{Mass of } CO_2}{\text{Mass of } CO_2 + \text{Mass of } H_2O + \text{Mass of } N_2 + \text{Mass of } O_2}$$

$$= \frac{[3 \times 44]}{[3 \times 44 + 4 \times 18 + 34.02 \times 28 + 4 \times 16]} = 0.108$$

Mass fraction

$$\text{for } H_2O = \frac{\text{Mass of } H_2O}{\text{Mass of products}} = \frac{\text{Mass of } H_2O}{\text{Mass of } CO_2 + \text{Mass of } H_2O + \text{Mass of } N_2 + \text{Mass of } O_2}$$

$$= \frac{[4 \times 18]}{[3 \times 44 + 4 \times 18 \times 34.02 \times 28 + 4 \times 16]} = 0.059$$

14.4 FIRST LAW FOR REACTIVE SYSTEM

So far, we have applied the first law of thermodynamics for non-reacting systems. The law can also be applied to the reacting systems by re-wringing the energy balance equation so that change in chemical energies can also be included.

14.4.1 For Steady Flow Systems

Assuming the changes in kinetic and potential energies to be negligible, the first law of thermodynamics for a chemically reacting steady flow system (for total N_R number of moles of reactants and N_P number of moles of products) can be written as (Figure 14.4):

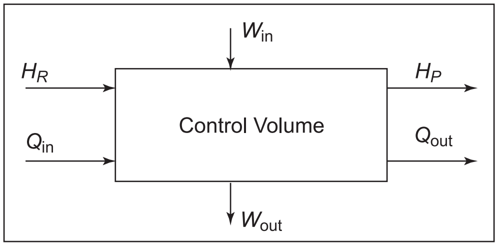


Figure 14.4 *Reactive system: energy transfer*

$$Q_{in} + W_{in} + \sum N_R(H_f^0 + H - H_0)_R = Q_{out} + W_{out} + \sum N_P(H_f^0 + H - H_0)_P \tag{14.5}$$

Net heat supplied, $Q = Q_{in} - Q_{out}$

Net work done on the system, $W = W_{in} - W_{out}$

Enthalpy of reactants, $H_R = \sum N_R(H_f^0 + H - H_0)_R$

Enthalpy of products, $H_P = \sum N_R(H_f^0 + H - H_0)_P$

Here, H_f^0, is chemical enthalpy at standard reference state.

$H - H_0$, relative enthalpy to the reference state

Substituting these terms in (14.5),

$$Q = W + H_P - H_R$$

or

$$Q = W + \sum N_P(H_f^0 + H - H_0)_P - \sum N_R(H_f^0 + H - H_0)_R$$

14.4.2 For Closed Systems

For a closed system, energy balance equation can be written as:

$$E_{in} - E_{out} = \Delta E_{system}$$

$$Q_{\text{in}} + W_{\text{in}} + U_R = Q_{\text{out}} + W_{\text{out}} + U_P \tag{14.6}$$

Since, $U = H - pV$ or $U = N(H_f^0 + H - H_0 - pV) - pV$ expression (14.6) becomes:

$$Q_{\text{in}} + W_{\text{in}} + \sum N_R(H_f^0 + H - H_0 - pV)_R = Q_{\text{out}} + W_{\text{out}} + \sum N_P(H_f^0 + H - H_0 - pV)_p$$

$$Q - W = \sum N_R(H_f^0 + H - H_0 - pV)_R - \sum N_P(H_f^0 + H - H_0 - pV)_p$$

Example 14.6

Find out the heat generated by burning of *n*-Octane for a product temperature of 257°C (530 K) without excess air supply.

Substance	$\bar{h}_f^o$ kJ/ kmol	$\bar{h}_{298\,\text{K}}$ kJ/ kmol	$\bar{h}_{530\,\text{K}}$ kJ/ kmol
$C_8H_{18}(g)$	–208,450	– – –	– – –
O_2	0	8682	15,708
N_2	0	8669	15,469
H_2O(g)	–241,820	9904	17,889
CO_2	–393,520	9364	19,029

○●●

Solution

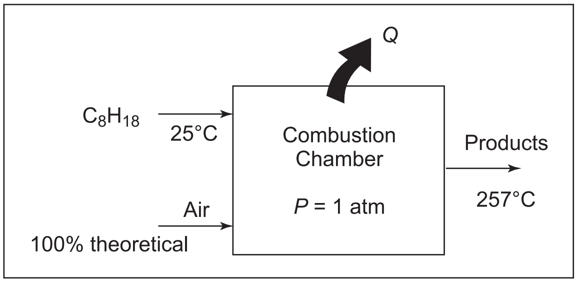


Figure 14.5

Given: Combustion reactants: Air and *n*-octane
Find out: Heat generated per kg of *n*-octane.

Properties: Molar mass of C, H_2 and O_2 and air are: 12 kg/kmol, 2 kg/kmol, 32 kg/kmol and 29 kg/kmol respectively.

Assumption: (1) Combustion is complete, (2) Air and combustion gases are ideal, (3) Kinetic and potential energies are negligible, (4). Combustion products are H_2O, CO_2 and N_2 only.

Molar masses of propane and air are 44 kg/kmol and 29 kg/kmol, respectively.

Combustion reaction with no-excess air will be:

$$C_8H_{18} + (12.5\,O_2 + 47.25\,N_2) \longrightarrow 8\,CO_2 + 9H_2O + (47.25\,N_2)$$

From the first law:

$$Q = W + \sum N_P(H_f^0 + H - H_0)_P - \sum N_R(H_f^0 + H - H_0)_R$$

With $W = 0$,

$$Q = \sum N_P(H_f^0 + H - H_0)_P - \sum N_R(H_f^0 + H - H_0)_R$$

From the given table data:

$$\sum N_P(H_f^0 + H - H_0)_P = 8 \times (-3{,}93{,}520 + 19{,}029 - 9{,}364) + 9 \times (-2{,}41{,}820 + 17{,}699 - 9{,}904) + 47.25 \times (0 + 15{,}469 - 8{,}669)$$

$$= -4{,}85{,}55{,}765 \text{ KJ/kmol}$$

and

$$\sum N_R(H_f^0 + H - H_0)_R = 1 \times (-2{,}08{,}450) + 12.5 \times 0 + 47.25 \times 0 = -2{,}08{,}450 \text{ kJ/kmol}$$

So, $Q = -48{,}555{,}765 - (-2{,}08{,}450) = -46{,}47{,}315$ kJ/kmol.

Negative sign shows indicates heat outgoing.

Heat generated per kg:

= Heat generated / Mass of fuel = 46,47,315 (kJ/kmol)/114 (kg/kmol) = 40,765.92 kJ/kg

---

Example 14.7

A mixture of benzene gas with 60 % excess air contained in a constant-volume tank undergoes the combustion. Determine the net heat transfer. Product-temperature is 1,000 K. Use the data given below:

Substance	$\bar{h}_f^o$ kJ/ kmol	$\bar{h}_{298\,K}$ kJ/ kmol	$\bar{h}_{1000\,K}$ kJ/ kmol
$C_6H_6(g)$	82,930	– – –	– – –
O_2	0	8682	31,389
N_2	0	8669	30,129
H_2O(g)	–241,820	9904	35,882
CO	–110,530	8669	30,355
CO_2	–393,520	9364	42,769

●●●

Solution Given: (1) Combustion reactants: Air and benzene, (2) Excess air: 60%

Find out: heat transferred per kmol of benzene.

Properties: Molar mass of C, H_2 and O_2 and air are: 12 kg/kmol, 2 kg/kmol, 32 kg/kmol and 29 kg/kmol respectively.

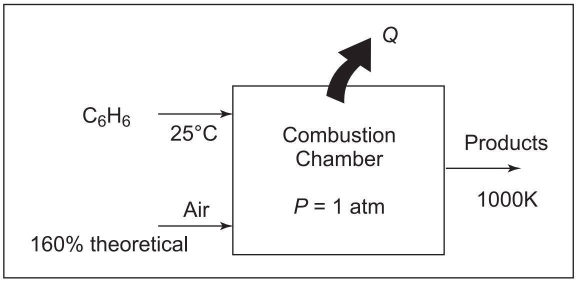


Figure 14.6

Assumption: (1) Combustion is complete, (2) Air and combustion gases are ideal (3) Kinetic and potential energies are negligible (4) Combustion products are H_2O, CO_2 and N_2 only.

Combustion reaction with no-excess air will be:

$$C_6H_6 + (7.5\,O_2 + 28.2\,N_2) \rightarrow 6\,CO_2 + 3\,H_2O(28.2\,N_2)$$

From the first law:

For constant volume,

$$Q - W = \sum N_R(H_f^0 + H - H_0 - pV)_R - \sum N_P(H_f^0 + H - H_0 - pV)_p$$

With $W = 0$,

$$Q = \sum N_R(H_f^0 + H - H_0 - pV)_R - \sum N_P(H_f^0 + H - H_0 - pV)_p$$

Assuming ideal gas condition:

$$Q = \sum N_R(H_f^0 + H - H_0 - RT)_R - \sum N_P(H_f^0 + H - H_0 - RT)_p$$

From the given table data:

$$\begin{aligned}\sum N_P(H_f^0 + H - H_0 - RT)_p = {} & 6 \times (-3{,}93{,}520 + 42{,}769 - 9{,}364 - 8.314 \times 1{,}000) \\ & + 3 \times (-2{,}41{,}820 + 35{,}882 - 9{,}904 - 8.314 \times 1{,}000) \\ & + 28.2 \times (0 + 30{,}159 - 8{,}669 - 8.314 \times 1{,}000) \\ = {} & -24{,}81{,}766 \text{ kJ/kmol}\end{aligned}$$

$$\begin{aligned}\sum N_R(H_f^0 + H - H_0 - RT)_R = {} & 1 \times (82{,}930 - 8.314 \times 1{,}000) + 7.5 \times (-8.314 \times 1{,}000) \\ & + 28.2 \times (-8.314 \times 1{,}000) \\ = {} & -7{,}996.89 \text{ kJ/kmol.}\end{aligned}$$

So, $Q = -24{,}81{,}766 - (-7{,}996.89) = -24{,}73{,}769.11$ kJ/kmol.

Negative sign indicates heat outgoing.

14.5 ADIABATIC FLAME TEMPERATURE

If the combustion is done in an adiabatic system without applying any shaft work, the temperature of the products is called the *adiabatic flame temperature* (Figure 14.7).

In the adiabatic process, since heat dissipation to the surrounding does not take place, entire heat released in the combustion will suddenly increase the temperature of the combustion products and it produces a flame. This will heat up the gaseous products in flame region and the temperature rise will be maximum. The highest temperature is known as *adiabatic flame temperature*.

The temperature rise depends upon:

1. Amount of excess air used or the air-fuel ratio.
2. Oxygen content.

There are two types of adiabatic flame temperature (Figure 14.8):

1. Adiabatic flame temperature at constant volume
2. Adiabatic flame temperature at constant volume

University Question

1. What is adiabatic flame temperature? Explain how adiabatic flame temperature for constant volume is evaluated.

(AU, 2012)

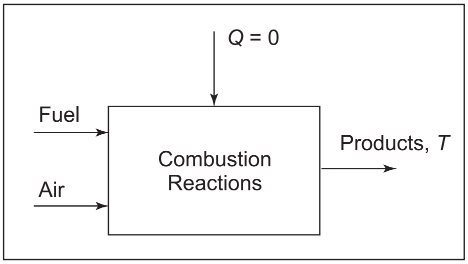


Figure 14.7 *Temperature of the chamber reaches to the maximum when combustion completes*

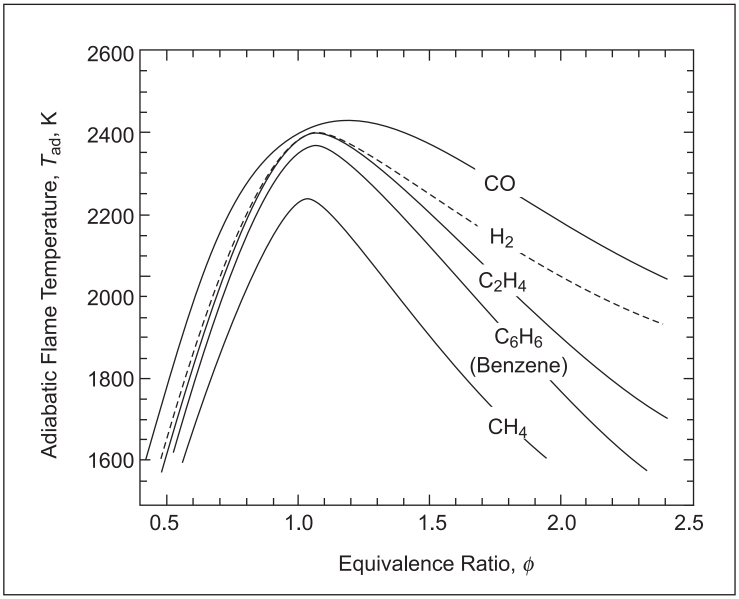


Figure 14.8 *Equivalence ratio and adiabatic flame temperature*

14.5.1 Adiabatic Flame Temperature at Constant Volume

For the constant volume system, from the first law of thermodynamics:

$$Q_{\text{Supplied}} = \Delta U$$

Since for the adiabatic process $Q_{\text{Supplied}} = 0$ so, $\Delta U = 0$

$$U_{\text{products}} = U_{\text{reactants}}$$

Since,

$$H_R(T_{\text{in}}, P_{\text{in}}) - H_P(T_{adi}, P_{\text{out}}) - R(N_R(T_{in}, P_{in})T_i - N_P(T_{adi}, P_{\text{out}})T_{adi}) = 0$$

Here, T_{in} is the inlet temperature, T_{adi} is the adiabatic flame temperature. Same notation is used for the pressure.

14.5.2 Adiabatic Flame Temperature at Constant Pressure

From the 1st law of thermodynamics,

$$Q = dU + pdV$$

For the constant pressure system, $pdV = 0$

So, $$H = U + pV \Rightarrow dH = dU + pdV$$

Since process is adiabatic, $Q = 0,\ dH = 0$

$$H_{\text{products}} = H_{\text{reactants}}$$

$$\Rightarrow \qquad \sum H_R(T_{in}, p) = \sum H_p(T_i, p)$$

$$\Rightarrow \qquad \sum H_p(T_i, p) = \sum N_i \int_{T_{ref}}^{T} C_{p,i}\, dT + Q_L$$

Here, T_{ref} is the reference temperature 0°C or 298K.

Q_L is the latent heat of vapourisation as temperature increases to reference temperature to T.

Example 14.8

With initial temperature and pressure being 1 atm and 298K, find out the constant pressure adiabatic flame temperature for Methane, when it is burned with stoichiometric combustion. *Assumption- Product mixture enthalpy can be calculated assuming c_p values constant as evaluated at mean temperature 1,200 K.*

Enthalpy data are given below:

Species	Enthalpy of Formation @ 298 K $\bar{h}^o_{f,i}$ (kJ/ kmol)	Specific Heat @ 1200 K $\bar{c}_{p,i}$(kJ/ kmol– K)
CH_4	–74,831	– – –
CO_2	–393,546	56.21
H_2O	–241,845	43.87
N_2	0	33.71
O_2	0	– – –

○●●

Solution Given: (1) Combustion reactants: Air and Methane, (2) Initial condition: 1 atm pressure and 298 K temperature.

Find out: Adiabatic flame temperature at constant pressure.

Properties: Molar mass of C, H_2 and O_2 and air are: 12 kg/kmol, 2 kg/kmol, 32 kg/kmol and 29 kg/kmol respectively.

Assumption: (1) Combustion is complete, (2) Air and combustion gases are ideal (3) Kinetic and potential energies are negligible, (4) There is no work interaction, (5) Combustion chamber is isolated.

Chemical reaction for the combustion is:

$$CH_4 + 2(O_2 + 3.78\,N_2) \longrightarrow 1\,CO_2 + 2H_2O + 7.56\,N_2$$

Here, $N_{CH_4} = 1$, $N_{O_2} = 2$, $N_{N_2} = 7.56$, $N_{CO_2} = 1$, $N_{H_2O} = 2$.

Constant pressure adiabatic flame temperature is given by the relation:

$$H_{\text{products}} = H_{\text{reactants}}$$

$$\Rightarrow \qquad \sum H_R(T_{\text{in}}, p) = \sum H_p(T, p)$$

$$\Rightarrow \qquad \sum H_p(T_i, p) = \sum N_i \int_{T_{ref}}^{T} C_{p,i}\, dT + Q_L$$

$$\begin{aligned} H_{\text{reactants}} &= \sum N_p (H^0{}_f + H - H_0)_p = N_{CH_4}(H^0{}_f + H - H_0)_{CH_4} + N_{O_2}(H^0{}_f + H - H_0)_{O_2} \\ &\quad + N_{N_2}(H^0{}_f + H - H_0)_{N_2} \\ &= 1 \times (-74{,}831) + 2 \times 0 + 7.56 \times 0 = -\,74{,}831 \text{ kJ/kmol} \end{aligned}$$

$$H_{\text{products}} = \sum N_i \int_{T_{ref}}^{T} C_{p,i}\, dT + Q_L$$

Assuming c_p to be constant,

$$H_{\text{products}} = \sum N_i (H_f^0 + C_{p,i}(T_{adi} - 298))_{N_2}$$

$$= 1 \times (-3{,}93{,}546 + 56.21(T_{adi} - 298)) + 2x\ (-2{,}41{,}845 + 43.87(T_{adi} - 298)) + 7.52\,x(0 + 33.71(T_{adi} - 298))$$

$$H_{\text{products}} = H_{\text{reactants}}$$

$$1 \times (-3{,}93{,}546 + 56.21(T_{adi} - 298)) + 2 \times (-2{,}41{,}845 + 43.87(T_{adi} - 298)) + 7.52 \times (0 + 33.71(T_{adi} - 298)) = -74{,}831$$

Solving it for T_{adi},

$$T_{adi} = 2{,}318 \text{ K}$$

Example 14.9

With initial temperature and pressure being 1 atm and 298 K, find out the constant volume adiabatic flame temperature for Methane, when it is burned with stoichiometric combustion. Enthalpy data are given below:

Species	Enthalpy of Formation @ 298 K $\bar{h}_{f,i}^{o}$ (kJ/ kmol)	Specific Heat @ 1200 K $\bar{c}_{p,i}$(kJ/ kmol- K)
CH_4	–74,831	– – –
CO_2	–393,546	56.21
H_2O	–241,845	43.87
N_2	0	33.71
O_2	0	– – –

○●●

Solution Given: (1) Combustion reactants: Air and Methane, (2) Initial condition: 1 atm pressure and 298 K temperature.

Find out: Adiabatic flame temperature at constant volume.

Properties: Molar mass of C, H_2 and O_2 and air are: 12 kg/kmol, 2 kg/kmol, 32 kg/kmol and 29 kg/kmol respectively.

Assumption: (1) Combustion is complete, (2) Air and combustion gases are ideal (3) Kinetic and potential energies are negligible (4) There is no work interaction (5) Combustion chamber is isolated.

Chemical reaction for the combustion is:

$$CH_4 + 2(O_2 + 3.78\ N_2) \longrightarrow 1\,CO_2 + 2H_2O + 7.56\ N_2$$

Here, $N_{CH_4} = 1$, $N_{O_2} = 2$, $N_{N_2} = 7.56$, $N_{CO_2} = 1$, $N_{H_2O} = 2$.

Adiabatic flame temperature for constant volume given by the relation:

$$\Delta U = 0$$

$$U_{\text{products}} = U_{\text{reactants}}$$

Since,

$$H_R\,(T_{\text{in}}, p_{\text{in}}) - H_p(T_{adi}, p_{\text{out}}) - R(N_R(T_{\text{in}}, p_{\text{in}})T_i - N_p(T_{adi}, p_{\text{out}})T_{adi}) = 0 \tag{14.7}$$

$$\begin{aligned} H_R(T_{\text{in}}, p_{\text{in}}) &= \sum N_p(H_f^0 + H - H_0)_p \\ &= N_{CH_4}(H_f^0 + H - H_0)_{CH_4} + N_{O_2}(H_f^0 + H - H_0)_{O_2} + N_{N_2}(H_f^0 + H - H_0)_{N_2} \\ &= 1 \times (-74{,}831) + 2 \times 0 + 7.56 \times 0 = -\,74{,}831 \text{ kJ/kmol} \end{aligned} \tag{14.8}$$

$$H_p(T_{adi}, p_{\text{out}}) = \sum N_i \int_{T_{ref}}^{T} C_{p,i}\, dT + Q_L$$

Assuming c_p to be constant,

$$H_{\text{products}} = \sum Ni(H_f^0 + C_{p,i}(T_{adi} - 298))_{N_2}$$

$$\begin{aligned} &= 1 \times (-3{,}93{,}546 + 56.21(T_{adi} - 298)) + 2 \times (-2{,}41{,}845 + 43.87(T_{adi} - 298)) \\ &\quad + 7.52 \times (0 + 33.71)(T_{adi} - 298)) \text{ kJ/kmol} \end{aligned} \tag{14.9}$$

Now,

$$R(N_R(T_{\text{in}}, p_{\text{in}})T_i - N_p(T_{adi}, p_{\text{out}})T_{adi})$$

$$N_R(T_{\text{in}}, p_{\text{in}})T_i = 1 + 2 + 3.78 = 10.78 \text{ kmol}\ ,\ N_p(T_{adi}, p_{\text{out}}) = 1 + 2 + 7.56 = 10.56 \text{ kmol}$$

$$R(N_R(T_{\text{in}}, p_{\text{in}})T_i - N_p(T_{adi}, p_{\text{out}})T_{adi}) = 8.315(10.56)(298 - T_{adi}) \text{ kJ/kmol} \tag{14.10}$$

Substituting (14.8), (14.9) and (14.10) in expression (14.7):

$$\begin{aligned} &-74{,}831 - [1 \times (-3{,}93{,}546 + 56.21(T_{adi} - 298)) + 2 \times (-2{,}41{,}845 + 43.87(T_{adi} - 298)) \\ &+ 7.52 \times (0 + 33.71(T_{adi} - 298))] + 8.315(10.56)(298 - T_{adi}) = 0 \end{aligned}$$

Solving it for T_{adi}, T_{adi} = 2,889 K

14.6 SECOND LAW ANALYSIS OF REACTIVE SYSTEMS

Second law analysis for the reactive system determines the irreversibility and the reversible work. As it has been

University Question

1. Briefly discuss the second law analysis for reactive systems. **(AMU, 2011)**

explained in previous chapters that irreversibility is the product of absolute temperature of surrounding and the entropy generated in the process, which represents the losses such as those to friction, etc.

The entropy generated during a process = Entropy change of the system + Entropy change of the surroundings

$$S_{\text{gen}} = \Delta S_{\text{sys}} + \Delta S_{\text{sur}} \tag{14.11}$$

$$S_{\text{gen}} \geq 0$$

For a reactive system,

$$\Delta S_{\text{sys}} = \sum N_P S_P - \sum N_R S_R$$

where S_P and S_R are the molar entropies of the products and reactants.

$$\Delta S_{\text{sur}} = \frac{Q_{\text{sur}}}{T_o}$$

Here, Q_{sur} is the heat transferred to the surrounding and T_0 is the absolute temperature.

Putting these values in equation 14.11,

$$S_{\text{gen}} = \Delta S_{\text{sys}} + \frac{Q_{\text{sur}}}{T_o}$$

$$S_{\text{gen}} = \left(\sum N_P S_P - \sum N_R S_R\right) + \frac{Q_{\text{sur}}}{T_o} \tag{14.12}$$

Irreversibility, $I = T_{\text{o}} S_{\text{gen}}$

14.6.1 Determination of Reversible Work

From the first law analysis of reactive systems:

$$Q = W + \sum N_p (H_f^0 + H - H_0)_p - \sum N_R (H_f^0 + H - H_0)_R$$

$$W = Q - \sum N_p (H_f^0 + H - H_0)_p + \sum N_R (H_f^0 + H - H_0)_R \tag{14.13}$$

Since, $Q_{\text{sur}} = Q$, from the equation 14.12:

$$Q = \left(\sum T_o N_p S_p - \sum T_o N_R S_R\right) - T_o S_{\text{gen}} \tag{14.14}$$

Substituting (14.14) in (14.13):

$$W = -\sum N_p (H_f^0 + H - H_0 - T_o S_p)_p + \sum N_R (H_f^0 + H - H_0 - T_o S_R)_R - T_o S_{\text{gen}}$$

When $S_{\text{gen}} = 0$, W is the reversible work,

$$W_{\text{rev}} = -\sum N_P (H_f^0 + H - H_0 - T_o S_P)_P + \sum N_R (H_f^0 + H - H_0 - T_o S_R)_R$$

Example 14.10

Burning of a hydrocarbon at 25°C and 1 atm pressure takes place adiabatically. Temperature of the products is 980 K. Find out the irreversibility if entropies of the reactants and products are given as:

Sum of entropies of reactants at 25°C = 18,782.01 kJ/kg mol K

Sum of entropies of products at 980°C = 22,481.68 kJ/kg mol K ○●●

Solution Given: (1) Combustion reactants: Air and a hydrocarbon, (2) Initial condition: 1 atm pressure and 298K temperature. 3). Sum of entropies of reactants at 25°C = 18,782.01 kJ/kg mol K and 4). Sum of entropies of products at 980°C = 22,481.68 kJ/kg mol K

Find out: irreversibility

Properties: Molar mass of C, H_2 and O_2 and air are: 12 kg/kmol, 2 kg/kmol, 32 kg/kmol and 29 kg/kmol respectively.

Assumption: (1) Combustion is complete, (2) Air and combustion gases are ideal (3) Kinetic and potential energies are negligible

Analysis:

$\sum N_p S_p$ = 22,481.68 kJ/kg mol K and $\sum N_R S_R$ = 18782.01 kJ/kg mol K.

Knowing that- $I = T_o S_{\text{gen}}$, $S_{\text{gen}} = \Delta S_{\text{sys}} + \dfrac{Q_{\text{sur}}}{T_o}$

Since, process is adiabatic, $Q_{\text{sur}} = 0$

So, $S_{\text{gen}} = \Delta S_{\text{sys}} \Rightarrow S_{\text{gen}} = (\sum N_p S_p - \sum N_R S_R)$

$$\Rightarrow \qquad S_{\text{gen}} = 22{,}481.68 - 18{,}782.01 = 3{,}699.67 \text{ kJ/kg mol K.}$$

$$I = T_o S_{\text{gen}} = 298 \times 3{,}699.67 = 1{,}102{,}501.66 \text{ kJ/kg mol}$$

Example 14.11

If difference between enthalpies (per mole) of products and reactants at 25°C is 20,57,125.55 kJ/k mol. If the combustion undergoes adiabatically with product temperature equal to 980 K. Find out the availability of combustion products. Given that:

Sum of entropies of reactants at 25°C = 17,712.12 kJ/kg mol K

Sum of entropies of products at 980°C = 21,491.57 kJ/kg mol K ○●●

Solution Given: $\sum N_p (H_f^0 + H - H_0)_p - \sum N_R (H_f^0 + H - H_0)_R =$ 20,57,125.55 kJ/k mol.

Step 1: Determine reversible work

$$W = \sum N_p (H_f^0 + H - H_0)_p - \sum N_R (H_f^0 + H - H_0)_R = 20{,}57{,}125.55 \text{ kJ/kmol.}$$

Step 2: Calculate irreversibility

$$\sum N_p S_p = 21{,}491.57 \text{ kJ/kg mol K and } \sum N_R S_R = 17{,}712.12 \text{ kJ/kg mol K}$$

Since, process is adiabatic, $Q_{\text{sur}} = 0$

So, $S_{\text{gen}} = \Delta S_{\text{sys}} \Rightarrow S_{\text{gen}} = (\sum N_p S_p - \sum N_R S_R)$

$\Rightarrow \qquad S_{\text{gen}} = 21{,}491.57 - 17{,}712.12 = 3{,}779.45$ kJ/kg mol K.

$I = T_o S_{\text{gen}} = 298 \times 3{,}779.45 \quad = 11{,}26{,}276.10$ kJ/kg mol

Step 3: Calculate availability

Availability = Reversible work – Irreversibility

Availability = 20,57,125.55 – 11,26,276.10 = 9,30,849.45 kJ/kg mol

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POINTS TO REMEMBER

- Air fuel ratio for combustion process is given as:

$$AFR = \frac{\text{Mass of air}}{\text{Mass of fuel}}$$

- Reference state for calculation of enthalpy of formation: Temperature of (01C or 298K) and a pressure of 0.1 MPa or 1 bar. (These are the room conditions.)
- At these reference conditions, the enthalpy of the elements (oxygen, hydrogen, nitrogen, carbon, etc.) is taken as zero.
- Enthalpy of combustion = Sum of enthalpies of all reactants – Sum of enthalpies of all products

$$H_{RP} = \sum n_P (H_f^0 + H) - \sum n_R (H_f^0 + H)$$

- Stoichiometric equation for combustion of a hydrocarbon in presence of oxygen:

$$\mathrm{C}_x\mathrm{H}_y + \left(x + \frac{y}{4}\right)\mathrm{O}_2 \longrightarrow x\mathrm{CO}_2 + \frac{y}{2}\mathrm{H}_2\mathrm{O}$$

- Stoichiometric equation for combustion of a hydrocarbon in presence of air:

$$\mathrm{C}_x\mathrm{H}_y + \left(x + \frac{y}{4}\right)(\mathrm{O}_2 + 3.78\mathrm{N}_2) \longrightarrow x\mathrm{CO}_2 + \frac{y}{2}\mathrm{H}_2\mathrm{O} + \left(x + \frac{y}{4}\right)(3.78\mathrm{N}_2)$$

- First law for a reactive system:

  (a) For steady flow process:

$$Q_{\text{in}} + W_{\text{in}} + \sum N_R (H_f^0 + H - H_0)_R = Q_{\text{out}} + W_{\text{out}} + \sum N_P (H_f^0 + H - H_0)_p$$

  (b) For closed system:

$$Q - W = \sum N_R (H_f^0 + H - H_0 - pV)_R - \sum N_p (H_f^0 + H - H_0 - pV)_p$$

- Adiabatic flame temperature: It is the maximum temperature of the reaction products if the combustion is done in an adiabatic system without applying any shaft work.

PRACTICE PROBLEMS

1. For the chemical reaction, $2SO_2 + O_2 \Leftrightarrow 2\,SO_3$ ○○●
Evaluate the equilibrium constant at pressure, *p* atm.
If mole fractions at equilibrium are: $[SO_3]^2 = 1.1, [SO_2]^2 = 0.9, [O_2] = 0.35$
(**Ans:** 4.3)

2. Determine the enthalpy of combustion for the complete combustion of C_8H_{18} with theoretical amount of air 25°C. If the enthalpy of formation of following molecules are: ○●●
For CO_2 = 3,93,520 kJ/kmol
For H_2O = 285830 kJ/kmol
For C_8H_{18} = 249950 kJ/kmol
(**Ans:** 54,71,000 kJ/kmol)

3. A mixture of air and liquid propane enters a combustion chamber at 25°C. If the consumption rate of the liquid propane is 0.06 kg/min, calculate the air flow rate if combustion occurs at 50% excess air. (**Ans:** 2.124 kg/min) ○●●

4. Combustion of liquid octane in a combustion chamber takes place as: ●●●

$$C_8H_{18} + 12.58\,(O_2 + 3.76\,N_2) \rightarrow 55.9\,[0.114\,CO_2 + 0.029\,CO + 0.016\,O_2 + 0.841\,N_2] + 9\,H_2O$$

If the mass flow rate of fuel is 0.01 kg/min and engine develops the 0.8 kW power, determine the heat developed. (Inlet temperature for fuel- 25°C, Product temperature: 670 K). Enthalpy of formation or CO_2 = 3,93,520 kJ/kmol, for H_2O = 2,85,830 kJ/kmol, for C_8H_{18} = 2,49,950 kJ/kmol, Molar weight of C_8H_{18} = 114.22 kg/kmol. (**Ans:** 4.17 kW)

5. For the combustion reaction: ●●●

$$C_8H_{18} + (12.5\,O_2 + 47.25\,N_2) \longrightarrow 8\,CO_2 + 9H_2O + (47.25\,N_2)$$

Find out the dew point temperature. (**Ans:** 53.5663°C)

MULTIPLE CHOICE QUESTIONS

1. Minimum air required for a complete combustion is called: ○●●
(a) Theoretical air (b) Positive (c) Negative (d) Infinite

2. Enthalpy of formation of O_2: ○●●
(a) Is zero (b) Is equal to –14,271 kJ/kmol
(c) Is infinite (d) Varies with temperature.

3. Adiabatic flame temperature for constant volume is: ○●●
(a) Greater than adiabatic flame temperature for constant pressure.
(b) Smaller than adiabatic flame temperature for constant pressure.
(c) Equal to the adiabatic flame temperature for constant pressure.
(d) May or may not be greater than adiabatic flame temperature for constant pressure.

4. The internal energy of the combustion products is __________ compared to that of reactants. ○●●

(a) Higher (b) Lower (c) Same (d) Not always lower

5. According to the first law of thermodynamics, the difference between the heat generated and work done for a reactive system is: ○●●

I. Enthalpy of formation
II. Enthalpy of combustion
III. Loss
IV. Irreversibility

Which one of above is correct:

(a) I only (b) I, II (c) I and III (d) II and IV

6. An incomplete combustion gives as product: ○●●

(a) CO_2 (b) CO (c) Carbon (d) None of these

7. The ratio of the maximum adiabatic flame temperature in air to the maximum adiabatic temperature in pure oxygen will be always: ○●●

(a) Higher than unity (b) Unity
(c) Less than unity (d) Depends upon the fuel

8. The adiabatic flame temperatures of pentane and hydrogen are T_p and T_h °C. The difference $T_p - T_h$ is always: ○●●

(a) Zero (b) Negative (c) Positive (d) Unity

9. Production of flue gas as kg/kg of furnace oil at about 40% excess air will approximately: ○●●

(a) 1.5 times (b) 1 time (c) 20 times (d) 50 times

10. Which one of the following is incorrect regarding the too much excess air supply? ○●●

(a) High percentages of O_2 in the flue gas
(b) High stack temperature
(c) Increased fuel consumption for the same heating requirement
(d) Decreased fuel consumption for the same heating requirement

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (a) 2. (a) 3. (a) 4. (b) 5. (b)
6. (b) 7. (c) 8. (b) 9. (c) 10. (d)

ANNEXURE 1

Enthalpy of Formation (at 298 K) for Commonly Used Fuels or Products

Substance	Formula	ΔH_f^0 (kJ/mol)	Substance	Formula	ΔH_f^0 (kJ/mol)
Acetylene	$C_2H_2(g)$	226.7	Hydrogen chloride	$HCl(g)$	–92.30
Ammonia	$NH_2(g)$	–46.19	Hydrogen fluoride	$HF(g)$	–268.6
Benzene	$C_6H_6(l)$	49.0	Hydrogen iodide	$HI(g)$	25.9
Calcium carbonate	$CaCO_3(s)$	–1207.1	Methane	$CH_4(g)$	–74.8
Calcium oxide	$CaO(s)$	–635.5	Methanol	$CH_3OH(l)$	–238.6
Carbon dioxide	$CO_2(g)$	–393.5	Propane	$C_2H_8(g)$	–103.85
Carbon monoxide	$CO(g)$	–110.5	Silver chloride	$AgCl(s)$	–127.0
Diamond	$C(s)$	1.88	Sodium bicarbonate	$NaHCO_3(s)$	–947.7
Ethane	$C_2H_6(g)$	–84.68	Sodium carbonate	$Na_2CO_3(s)$	–1130.9
Ethanol	$C_2H_5OH(l)$	–277.7	Sodium chloride	$NaCl(s)$	–410.9
Ethylene	$C_2H_4(g)$	52.30	Sucrose	$C_{12}H_{22}O_{11}(\mathrm{s})$	–2221
Glucose	$C_6H_{12}O_6(s)$	–1273	Water	$H_2O(l)$	–285.8
Hydrogen bromide	$HBr(g)$	–36.23	Water vapor	$H_2O(\mathrm{g})$	–241.8

Gas Compressors

15

CHAPTER OUTLINE

INTRODUCTION

In industrial applications, mechanical energy devices play a vital role. Most energy devices use fluids to serve their purpose, some devices use liquid fuels and some use gases/air. One such device which is used to deliver pressurised air/gas is a gas compressor. A gas compressor is an energy consuming device.

15.1 MECHANICAL ENERGY

For mechanical energy devices that run on gases/air, atmospheric air/gases cannot be used directly at standard conditions. Either the gases/air have to be pressurised or the pressure must be reduced below atmospheric conditions in order to use it for industrial applications. One such device which uses air to deliver pressurised air/gas is a gas compressor. The gas compressor is an energy consuming device.

Mechanical energy devices are of two types, energy producing devices and energy consuming devices.

University Questions

1. Discuss the merits and demerits of centrifugal compressor over the axial flow compressor? **(IES-ME, 1988)**
2. Which of the following techniques are employed for control of reciprocating compressors?
   i. Throttle control
   ii. Clearance control
   iii. Blowing air to waste **(ESE, 2007)**

1. ***Energy producing devices:*** When work is done by the system (devices), then the device is called energy producing device. These devices use heat energy to generate mechanical work. The examples of such devices are heat engines and turbines. The heat is supplied to these systems using combustion of fuels in most case.

2. ***Energy consuming devices:*** In energy consuming devices, work is done on the system (devices). Generally electrical energy is used for doing work on these devices. Examples of these devices are compressors, refrigerators, air conditioners, pumps, etc.

15.1.1 Compressor

A compressor is an energy consuming device, which takes in atmospheric air, compresses it and delivers it at high pressure.

Uses of Compressed Air

- Used in borewells for pushing out air by applying high pressure air.
- Robots are actuated using compressed air.
- Compressed air is used in cleaning applications in industries.
- Compressed air is used in boiler system.
- In air-conditioning and refrigeration system compression of fluid is very important where compressor is used to pressurise the refrigerant.
- Some gases like nitrogen, hydrogen, etc. are stored in a container in a compressed state in order to reduce the capacity of storage.

Classification of Compressors

Compressors can be classified into different types as follows (Figure 15.1):

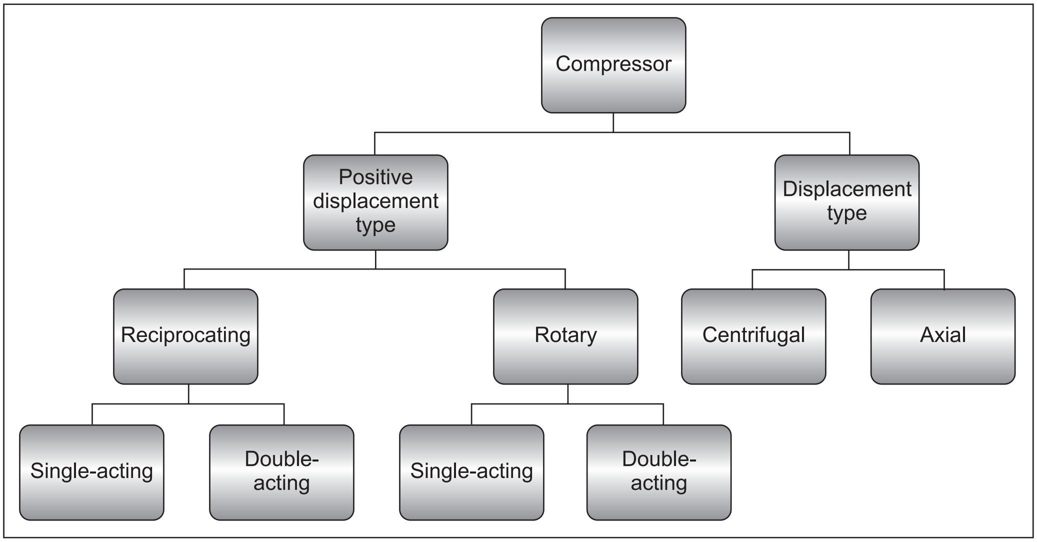

Figure 15.1 *Different types of compressors*

1. ***Positive displacement compressor:*** Positive displacement type compressors draws in air and compresses it by reducing the volume of the chamber. All reciprocating compressors and

some rotary compressors fall in this category. There is no slip in this type of compressor while compressing.

2. ***Dynamic type compressor:*** In dynamic type compressors the air is pressurised by physical volume reduction of chamber. In the case of dynamic type, the pressure applied on the fluid is through variation in the geometry of chamber, i.e. by providing diffuser section at the exit, which causes high velocity fluid to get converted to high pressure fluid.

15.2 || RECIPROCATING AIR COMPRESSOR

15.2.1 Theory

Reciprocating air compressor uses piston-cylinder arrangement where air is sucked at low pressure during suction stroke and is delivered at high pressure by piston-cylinder arrangement.

University Question

1. Which of the following can be the cause/causes of an air-cooled compressor getting overheated during operation?
   i. Insufficient lubricating oil
   ii. Broken valve tip
   iii. Clogging intake filter. **(ESE, 2006)**

15.2.2 Construction

Figure 15.2 shows the schematic of a single stage air compressor with clearance. It consists of a piston-cylinder arrangement, cooling system, inlet and delivery valves, connecting rod and a crank.

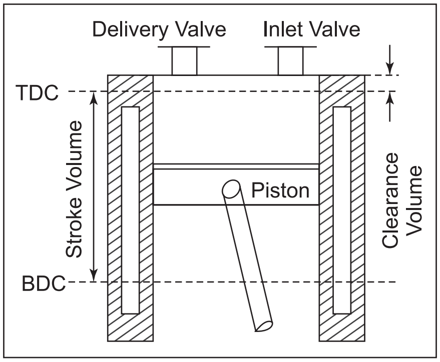

Figure 15.2 *Single stage reciprocating air compressor*

15.2.3 Working

When the piston moves from top-dead centre (TDC) to bottom-dead centre (BDC) the leftover air from previous cycle in the clearance space expands first and then the fresh air is taken in through the inlet valve for the remaining portion of the stroke. Once the piston reaches the BDC, inlet valve is closed and now the piston returns back from BDC to TDC. In the process the air is compressed. The delivery valve opens when the pressure inside is more than the pressure of the tanks connected to the delivery valve. This process repeats for the entire cycle.

Different processes involved are:

1. Suction
2. Compression
3. Delivery

15.2.4 Important Terminology of Reciprocating Compressor

- Compressor displacement volume: $V = (\pi/4) \times d^2 \times L$
- Volume flow rate: $V = (\pi/4) \times d^2 \times L \times (N/60) \times n$
- Free Air Delivery (FAD): Discharge volume of the compressor corresponding to ambient conditions
- Piston Speed: (m/min)$V_{\text{piston}} = 2 \times L \times N$

where, d = diameter of the cylinder/bore

L = stroke length of piston

$n = 1$ for single acting and $n = 2$ for double acting

N = crank speed

Example 15.1

An air compressor is used to rapidly fill a 3 m^3 tank at 20°C and 1 atm. The filling process is governed by the law $pV^{1.4} = C$. The kinetic and potential energy effects are negligible. The ratio of the final to initial mass of air in the tank is 4. Work out the work input to the compressor.

(IES-ME, 2007) ○●●

Solution Assumptions:

- Kinetic and potential energy changes are neglected
- No heat interaction
- Air as ideal gas
- Inlet air 1 atm is 20°C

(a) Initial conditions:

$$p_1 = 101.3\ k\text{Pa},\ T_1 = 20°\text{C} = 293\ \text{K},\ V = 3\ \text{m}^3$$

Equation of state:

$$m_1 = p_1V_1/RT_1 = 101.3 \times 3/\ (0.287 \times 293) = 3.61\ \text{kg}$$

Thus,

$$m_2 = 4m_1 = 14.46\ \text{kg}$$

Applying energy balance with assumptions made,

$$\begin{aligned} dU/dT &= -dW/dt + \Sigma m_i h_i \\ \Delta U &= -\Delta W + \Delta m H_1 \\ m_2U_2 - m_1U_1 &= -W + (m_2 - m_1)H_1 \\ W &= m_1U_1 - m_2U_2 + (m_2 - m_1)\ c_pT_1 \end{aligned}$$

For adiabatic process,

$$pV\gamma = c,\ (V_1 = V_2)$$

$$T_2/T_1 = (v_1/v_2)^{(\gamma-1)} = (m_2V_1/m_1V_2)^{(\gamma-1)}$$

$$T_2 = 4^{(\gamma-1)} \times T_1 = 4^{(1.4-1)} \times 293 = 510.14 \text{ K}$$

Thus,

$$W = 3.61 \times 0.71 \times 293 - 14.46 \times 0.71 \times 510.14 + (14.46 - 3.61) \times 1.005 \times 23$$

$$= -1291.47 \text{ kJ}$$

15.3 WORK OF COMPRESSION IN SINGLE STAGE COMPRESSOR

University Question

1. In a single stage reciprocating air compressor the work done on the air to compress it from suction pressure to delivery pressure will be minimum in which process? **(ESE, 2011)**

15.3.1 Indicated Work for a Single Acting Compressor Without Clearance

Indicated work is given by (Figure 15.3):

$$IW = \text{Area behind } (c - 1 - 2 - d - c)$$

$$IW_{in} = p_2V_2 + (p_1V_1 - p_2V_2)/(n-1) - p_1V_1$$

$$= [p_2V_2(n-1) + (p_1V_1 - p_2V_2) - p_1V_1(n-1)]/(n-1)$$

$$= p_2V_2n - p_2V_2 + p_1V_1 - p_2V_2 - p_1V_1n - p_1V_1$$

$$= n(p_1V_1 - p_2V_2)/(n-1)$$

Further,

$$W_{in} = [(n/(n-1))\,(p_2V_2/p_1V_1 - 1)\,[p_1V_1^{\,n} - p_2V_2^{\,n}]$$

$$= (n/(n-1))m_aRT_1[(p_2/p_1)^{(1-1/n)} - 1\,]$$

$$= n \times m_aRT_1[(p_2/p_1)^{((n-1)/n)} - 1\,]/(n-1)$$

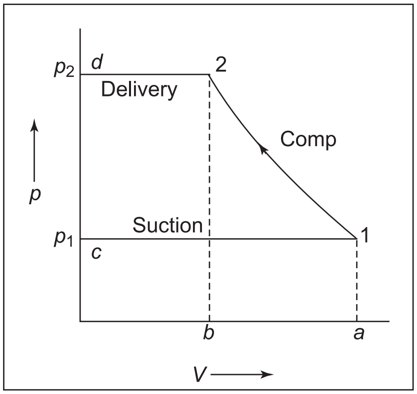

Figure 15.3 *p-V diagram showing work for a single-acting compressor without clearance*

Example 15.2

What is the work done on a single acting compressor when the air is compressed from 1 bar with volume of 5 m^3 to the pressure of 8 bar with index of compression $n = 1.3$. ○○●

Solution Given: $p_1 = 1$ bar, $p_2 = 8$ bar, $V = 5\text{m}^3$.

$$IW_{in} = n\,[p_1V_1 - p_2V_2]/(n-1)$$

$$V_2 = (p_1/p_2)^n \times V_1 = (1/8)^{1.3} \times 5 = 0.335 \text{ m}^3$$

$$IW_{in} = 1.3\,(1 \times 5 - 8 \times 0.335)/(1.3 - 1) = 10.05 \text{ kJ/kg}$$

15.3.2 Minimising Compression Work

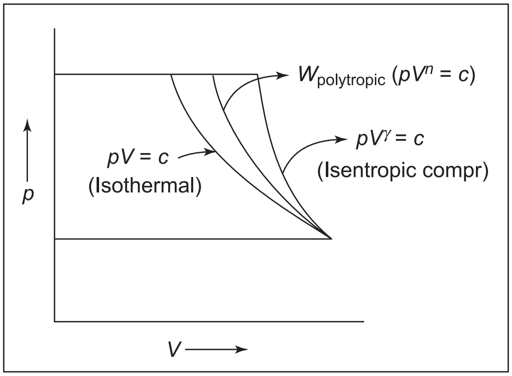

Figure 15.4 *p-V diagram showing indicated work for different processes*

Polytropic Compression

$$W_{poly} = n \times p_1 V_1 [(p_2/p_1)^{((n-1)/n)} - 1]/(n-1)$$

Isentropic Compression

$$W_{isent} = \gamma \times p_1 V_1 [(p_2/p_1)^{((\gamma-1)/\gamma)} - 1]/(\gamma - 1)$$

Isothermal Process

$$W_{IT} = p_2 V_2 + p_1 V_1 \ln(V_2/V_1) - p_1 V_1$$

$$W_{IT} = p_2 V_2 + p_1 V_1 \ln(V_2/V_1) \qquad [pV = \text{const}]$$

$$K_{comp} = W_{IT}/\text{Indicator work}$$

15.3.3 Methods for Improving Isothermal Efficiency

Increase in surface/volume ratio increases the surface area which eventually makes more surface to be available to cool the air, thereby temperature decreases resulting in increase of isothermal efficiency.

15.3.4 Advantages of Compressor with Clearance Volume

1. Avoid striking of piston on cylinder head
2. Accommodate valve's activation

15.3.5 Disadvantages of Compressor with Clearance Volume

1. *Volume decreases:* Some air remains in the clearance volume at the end of compression. During next, suction of some air in the clearance volume expands before fresh air enters into the cylinder, hence air intake.

2. Clearance volume controls the compression of air, [more V_c leads to decrease in the pressure (back pressure)]
3. More power required, since it handles more volume.

15.3.6 Indicated Compression Work with Clearance

Assume $n_c = n_e = n$

n_c = compression index

n_e = expansion index

$$W_{\text{comp}} = n \times p_1 V_1 [(p_2/p_1)^{((n-1)/n)} - 1]/(n-1)$$

$$W_{\text{exp}} = n \times p_1 V_4 [(p_2/p_1)^{((n-1)/n)} - 1]/(n-1)$$

$$W_{\text{in}} = W_{\text{comp}} - W_{\text{exp}}$$

$$= n \times p_1 (V_1 - V_4)[(p_2/p_1)^{((n-1)/n)} - 1]/(n-1)$$

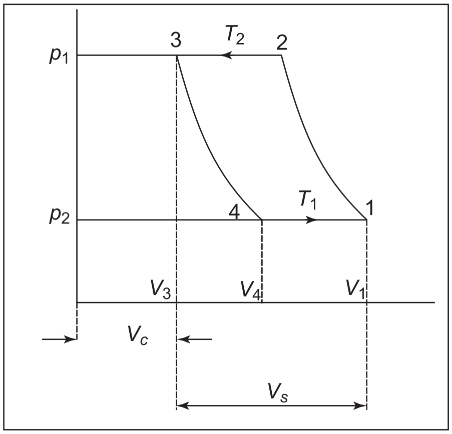

Figure 15.5

Example 15.3

The clearance volume of a single cylinder air compressor is 10% of the swept volume. The compressor has a bore of 120 mm and a stroke of 152 mm and runs at 400 rpm. Suction conditions are 1 bar and 24°C and the delivery pressure is 10 bar. Compression and re-expansion curves follow the law $pV^{1.3} = C$. Calculate the diagram volumetric efficiency, the rate of air delivery and power input required to drive the compressor assuming a mechanical efficiency of 74%. Take $R = 0.287$ kJ/kg K.

(IES-ME, 2006) ○●●

Solution Given:

Clearance volume = 10% of the swept volume

Bore $D = 120$ mm

Stroke $L = 152$ mm

Speed of compressor $N = 400$ rpm

Suction condition, $p_1 = 1$ bar, $T_1 = 24°\text{C}$

Delivery pressure, $p_d = 10$ bar

Mechanical efficiency $\eta_m = 74\%$

$R = 0.287$kJ/kgK

Swept volume $V_s = (\pi/4)D^2 L \text{ m}^3$

$$= (\pi/4) \times (0.12)^2 \times 0.152 = 1.719 \times 10^{-3} \text{ m}^3$$

Clearance volume $V_c = 10\%\ V_s = 1.719 \times 10^{-4}\ \text{m}^3$

i. Volumetric efficiency η_v:

$$\eta_v = 1 + C - C(p_d/p_1)^{(1/n)}\ C = \text{clearance ratio},\ n = \text{index of expansion}$$

As re-expansion curves follow the law $pV1.3 = C$, $n = 1.3$

Thus

$$\eta_v = 1 + 0.1 - 0.1(10)^{(1/1.3)}$$

ii. Rate of air delivery:

$$V_1 = V_s + V_c = 1.89 \times 10^{-3}$$

For process 3–4

$$p_3V_3^n = p_4V_4^n$$

$$10 \times (1.719 \times 10^{-3})^{1.3} = 1 \times V_4^{1.3}$$

$$V_4 = 1.01 \times 10^{-3}\ \text{m}^3$$

Actual volume of air sucked

$$V_a = V_1 - V_4 = (1.89 - 1.01) \times 10^{-3}\ \text{m}^3 = 0.88 \times 10^{-3}\ \text{m}^3$$

From ideal gas equation,

$$pV = mRT$$

Suction condition: $p_1 = 100\ k\text{Pa}$

$$T_1 = 24 + 273 = 297\ \text{K}$$

$$p = \rho RT$$

$$\rho_1 = p/RT = 100/(0.287 \times 297) = 1.173\ \text{kg/m}^3$$

Mass of air compressed per cycle:

$$m = \rho_1 \times \text{actual volume} = 1.173 \times 0.88 \times 10^{-3} = 1.03 \times 10^{-3}$$

Mass delivered per second = $m \times N/60 = 1.03 \times 400 \times 10^{-3}/60 = 6.88 \times 10^{-3}$ kg/s

iii. Power input to drive the compressor work done/cycle:

$$= n \times p_1(V_1 - V_4)[(p_2/p_1)^{((n-1)/n)} - 1]/(n - 1)$$

$$= 1.3 \times 100(0.88 \times 10^{-3})[(10)^{((1.3-1)/1.3)} - 1]/(1.3 - 1)$$

$$= 267.41\ \text{kJ/cycle}$$

Power input = 267.41 × 400 = 106.96 kJ/min

Mechanical efficiency $\eta_n = 0.74$

Thus,

Actual power input = 106.96/0.74 = 144.55 kJ/min

Thus,

$$p = 2.41\ \text{kW}$$

15.3.7 Limitations of Single Stage Compression

1. Increase in delivery pressure, increases temperature and hence volume is increased [T proportional to V], hence more work is required
2. η_{vol} decreases because more clearance air is expanded, reducing fresh air induction
3. Cylinder size is more, heat involved is high and materials should be very strong

15.4 VOLUMETRIC EFFICIENCY

University Question

1. How can the volumetric efficiency of reciprocating compressor be enhanced? **(ESE, 2000)**

15.4.1 Definition

Volumetric efficiency is the ratio of actual mass sucked by the compressor to the mass corresponding to stroke volume

$$\eta_{vol} = \text{effective swept volume/piston displacement volume}$$
$$= \text{Actual mass sucked / Mass corresponding to } V_s$$
$$= (V_1 - V_4)/(V_1 - V_3)$$
$$= (V_s + V_c - V_4)/(V_s)$$
$$= 1 + V_c/V_s - (V_4/V_s)*(V_c/V_c) \qquad [K = V_c/V_s]$$
$$= 1 + k - k(V_4/V_c)$$
$$= 1 + k - k\,(V_4/V_3)$$

Since $n_e = n_c$

$$p_1V_1^n = p_2V_2^n \text{ (compression); } p_1V_4^n = p_2V_3^n \text{ (expansion)}$$

Hence, $V_3/V_4 = V_2/V_1$

Thus, $\eta_{vol} = 1 + k - k\,(V_1/V_3)$

15.4.2 Free Air Delivery (FAD)

The volume of compressed air corresponding to atmospheric conditions is known as Free Air Delivery (FAD),

$$p_fV_f/T_f = p_1(V_1 - V_4)/T_1$$
$$V_f = pT_f(V_1 - V_4)/p_fT_1$$
$$\eta_{vol} = V_f/(V_1 - V_2)$$

Suffix 1 denotes suction conditions, suffix 2 denotes ambient conditions

Example 15.4

A double acting single cylinder reciprocating air compressor has a piston displacement of 0.015 m^3/rev, operates at 500 rpm and has 5% clearance. Index of compression and expansion is 1.3. Determine volumetric efficiency. Air is received at 1 bar and delivered at 6 bar. **(UPSC, 1998)** ○●●

Solution Given: $V = 0.015\ \text{m}^3/\text{rev}$, $N = 500$ rpm, $C = 5\% = 0.05$, $n = 1$, $p_1 = 1$ bar, $p_2 = 6$ bar, $T_1 = 20°\text{C}$

$$\text{Volumetric efficiency} = \eta_{\text{vol}} = 1 + C - C\,(P_2/P_1)^{(1/n)}$$
$$= 1 + 0.05 - 0.05\,(6)^{(1/1.3)}$$
$$= 0.8516$$
$$\eta_{\text{vol}} = 85.16\%$$

15.5 MULTISTAGE COMPRESSION

University Question

1. If n is the polytropic compression and p_2/p_1 is the pressure ratio for a three-stage compressor with ideal intercooling the expression for total work of three stages is? **(ESE, 2001)**

15.5.1 Advantages

1. Inter cooling, hence volume decreases, hence η_{vol} increases
2. Power required decreases since intercooling makes the process isothermal
3. Due to phasing of operation in stages, fluctuation is reduced and hence size of flywheel is small for uniform torque.

15.5.2 Disadvantages

1. Complex compressor body
2. Consumed in increased piping, equipment, valves in the resistance loss
3. Increase in manufacturing costs and operating costs

15.5.3 Work Done in Multistage Compression

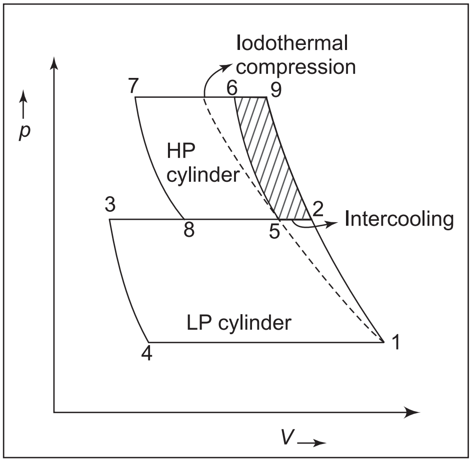

Figure 15.6 *p-V diagram showing indicated work of multistage compression*

LP cylinder compression work = 1 – 2 – 3 – 4 – 1
HP cylinder compression work = 5 – 6 – 7 – 8 – 5

$$W_{in} = W_{LP} + W_{HP}$$

$$= n \times p_1(V_1 - V_4)[(p_2/p_1)^{((n-1)/n)} - 1]/(n-1) + n \times p_2(V_5 - V_8)[(p_3/p_2)^{((n-1)/n)} - 1]/(n-1)$$

$$= n \times m_a RT_1[(p_2/p_1)^{((n-1)/n)} - 1\ ]/(n-1) + n \times m_a RT_3[(p_3/p_2)^{((n-1)/n)} - 1\ ]/(n-1)$$

Here, $T_1 = T_3$, perfect intercooling,

$$W_{in} = n \times m_a RT_1[(p_2/p_1)^{((n-1)/n)} + (p_3/p_2)^{((n-1)/n)} - 2\ ]/(n-1)\ \text{(kW)}$$

15.5.4 Heat Rejected Per Stage of Compression

$$Q_{stage} = Q_{comp} + Q_{cooling}$$

Polytropic Compression

$$Q_{comp} = m_a C_n(T_2 - T_1) \qquad \text{[kJ/s]}$$

$$C_n = C_p - nC_v/(n-1)$$

Constant Pressure Cooling

$$Q_{cooling} = m_a C_p(T_2 - T_1) \qquad \text{[kJ/s]}$$

15.5.5 Condition for Minimum Work

Input pressure, $p_{1,}$ is constant, similarly, output pressure, $p_{3,}$ is constant. Hence, variable parameter is p_2.

$$IP = n \times m_a RT_1[(p_2/p_1)^{((n-1)/n)} + (p_3/p_2)^{((n-1)/n)} - 2\ ]/(n-1)$$

$$d(W_{in})/dp_2 = 0;$$

$$d/dp_2[(p_2/p_1)^{((n-1)/n)} + (p_3/p_2)^{((n-1)/n)} - 2] = 0$$

$$(1/p_1)^{((n-1)/n)} \times ((n-1)/n)(p_2)^{((n-1-n)/n)} + (p_3)^{((n-1)/n)} \times (p_2)^{(((1-n)/n)-1)} \times (1-n)/n = 0$$

$$(1/p_1)^{((n-1)/n)} \times ((n-1)/n)(p_2)^{(-1/n)} = ((n-1)/n) \times (p_3)^{((n-1)/n)} \times (P_2)^{((1-2n)/n)}$$

$$(p_1)^{((n-1)/n)} \times (p_3)^{((n-1)/n)} = (p_2)^{(-1/n)} \times (p_2)^{((-1+2n)/n)}$$

$$(p_1p_3)^{((n-1)/n)} = p_2^{2((n-1)/n)}$$

$$p_2^2 = p_1p_3$$

Hence, $p_2/p_1 = p_3/p_2$

15.5.6 Minimum Compressor Work

$$W_{in} = 2 \times n \times m_a RT_1[(p_2/p_1)^{((n-1)/n)} - 1]/(n-1)$$

$$p_2/p_1 = \frac{\sqrt{p_1 p_3}}{p_1} = \sqrt{\frac{p_3}{p_1}} = (p_3/p_1)^{1/2}$$

$$W_{\text{in}} = 2 \times n \times m_a R T_1[(p_3/p_1)^{((n-1)/2n)} - 1\,]/(n-1)$$

For 'Z' stages,

$$W_{\text{in}} = Z \times n \times m_a R T_1[(p_{Z+1}/p_1)^{((n-1)/Zn)} - 1\,]/(n-1)$$

Pressure ratio in each stage: $(p_{z+1}/p_1)^{1/2}$

Example 15.5

A single acting two-stage compressor with complete intercooling, delivers air at 16 bar. Assuming an intake of 1 bar at 15°C, the pressure ratio per state is? **(Gate, 2001)** ○○●

Solution Given: $p_1 = 1$ bar, $p_2 = 16$ bar

Evaluation: $p_2/p_1 = p_3/p_2 => p_2 = p_3 p_1$

$$p_2 = \sqrt{p_3 p_1} = \sqrt{1 \times 16} = 4 \text{ bar}$$

Pressure ratio per state = $p_2/p_1 = 4$

Example 15.6

Derive the condition for minimum work for a unit mass of air delivered by a two-stage reciprocating air compressor with a perfect intercooler. **(IES-ME, 1980)** ○○●

Solution Let air compressed in LP compressor from initial pressure p_1 to intermediate pressure p_2 and undergone perfect cooling ($T_3 = T_1$) followed by second stage compression in HP compressor to p_4.

Work done in LP compressor is:

$$\begin{aligned} W_1 &= -n(p_1V_1 - p_2V_2)/(n-1) \\ &= np_1V_1(p_2V_2/(p_1V_1) - 1)/(n-1) \\ &= np_1\,V_1[(p_2/p_1)(p_2/p_1)^{(-1/n)} - 1] \qquad \{\text{since } V_2/V_1 = (p_1/p_2)^{(1/n)}\} \\ &= n \times p_1V_1[(p_2/p_1)^{((n-1)/n)} - 1]/(n-1) \end{aligned}$$

End work done in HP compressor is:

$$W_2 = n \times p_3V_3[(p_4/p_2)^{((n-1)/n)} - 1]/(n-1)$$

Neglecting clearance volume, Total work done:

$$\begin{aligned} W &= W_1 + W_2 \\ &= n[p_1V_1 \times (p_2/p_1)^{((n-1)/n)} - p_1V_1 + p_3V_3 \times (p_4/p_3)^{((n-1)/n)} - p_3V_3]/(n-1) \end{aligned}$$

For minimum work done,

$$dW/dp_2 = 0$$

Thus,

$$p_1V_1\ ((n-1)/n)[p_2^{(-1/n)}/p_1^{((n-1)/n)}] - p_3V_3((n-1)/\mathrm{n})[[p_4^{((n-1)/n)}/p_2^{((2n-1)/n)}] = 0$$

$$p_1V_1\ [p_2^{(-1/n)}/p_1^{((n-1)/n)}] = p_3V_3[p_4^{((n-1)/n)}/p_1^{((2n-1)/n)}]$$

$$p_2^{((-1+2n-1)/n)} = p_1V_4^{\ ((n-1)/n)}\ p_3V_3/p_1V_1$$

$$p_2(2(n-1)/n) = p_1V_4((n-1)/n)p_3V_3/p_1V_1$$

$$p_2 = (p_1V_4)^{(1/2)}[p_3V_3/p_1V_1]^{(n/(2(n-1))}$$

Since $T_1 = T_3$,

Thus, $p_1V_1/mR = p_3V_3/mR$,

Thus $p_1V_1 = p_3V_3$

Thus,

Total work:

$$W = np_1V_1[((p_1p_4)^{(1/2)}/p_1)^{((n-1)/n)} + (p_4/(p_1p_4)^{(1/2)})^{((n-1)/2n)} - 2]/(n-1)$$

$$= np_1V_1[(p_4/p_1)^{((n-1)/2n)} + (p_4/p_1)^{((n-1)/2n)} - 2]/(n-1)$$

$$= 2\mathrm{n}p_1V_1[(p_4/p_1)^{((n-1)/2n)} - 1]/(n-1)$$

15.6 ROTARY COMPRESSOR

15.6.1 Introduction

Rotary compressors are used in places where continuous supply of air is to be maintained. In the place of piston-cylinder arrangement, rotors and casing are used. Rotary compressors are classified into two types:

1. Position displacement type
2. Dynamic action steady flow type

University Question

1. What does the application of centrifugal air compressor lead to? **(ESE, 2000)**

15.6.2 Advantages

1. Compressor is compact in size
2. Noisy less operation.
3. Output air is free from oil.
4. When the compressor operates with deliver valve closed, there won't be any damage to the compressor due to presence of slip during operation.

15.6.3 Disadvantages

1. Output capacity is low.
2. The temperature of compressed air is too much high.

15.6.4 Root Blower Type Rotary Compressor

Root blower type rotary compressor is a positive displacement type, which is also called lobe compressor. Schematic of lobe compressor are shown in Figure 15.7.

Construction

Root blower type of rotary compressor consists of a casing within which two lobed motors are there. There is also inlet and outlet passage for air.

Working

Root blower type of rotary compressor traps atmospheric air and pushes it through the outlet. The trapped air is not compressed. The compression takes place in the deliver side, which increases the pressure. This pressurised air is delivered or discharged to the tank connected to the outlet.

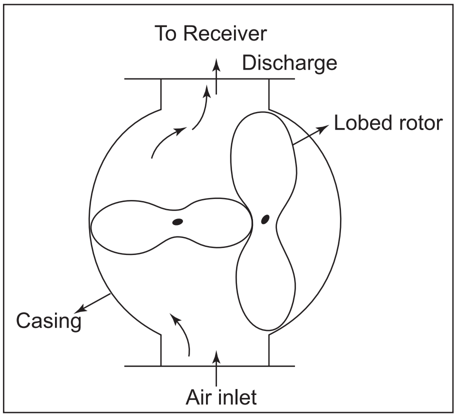

Figure 15.7 *Root blower type compressor*

15.7 || CENTRIFUGAL COMPRESSOR

15.7.1 Introduction

Centrifugal compressors are dynamic action compressor. It is used for larger volumetric flow rate and for mediocre pressure ratio. It is also an oil free compressor.

15.7.2 Velocity Diagram

Velocity diagram of impeller blade, notation indicates (Figure 15.8):

- u_1 = blade velocity at inlet
- V_{r1} = relative velocity of blade at inlet
- V_1 = absolute velocity of inlet air
- V_{f1} = inlet flow velocity
- α = air inlet angle
- β = blade angle at inlet

Similarly, u_2, u_{r2}, V_2, V_{f2}, θ and ϕ are the corresponding values at the outlet

Work input per kg is,

$$W = u_1 V_{w1} + u_2 V_{w2}$$

Here, $\alpha = 90°$, $V_{w1} = 0$,

$$W = u_2 V_{w2}$$

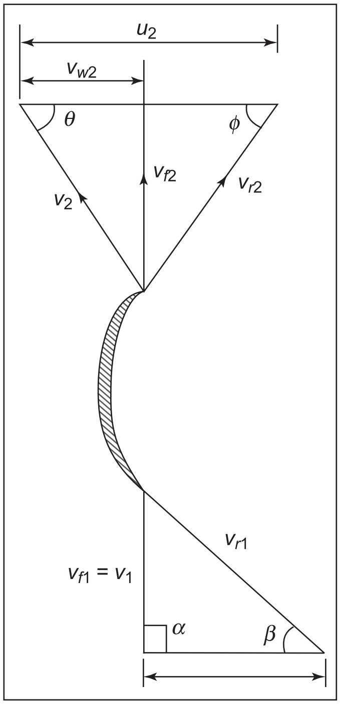

Figure 15.8 *Velocity diagram*

Power is given as:

$$\text{Power} = mu_2V_{w2}$$

m = mass flow rate of air in kg/s = AV_1/v_1

v_1 = specific volume of air at inlet

$$A = \pi D_1B_1$$

D_1 = diameter of impeller

B_1 = width of impeller

Thus, $m = \pi D_1B_1V_1/v_1$

Example 15.7

Explain the concept of polytropic efficiency as applied to a multistage rotary compressor.

(IES-ME, 1998) ○○●

Solution Polytropic efficiency is nothing but the isentropic efficiency of one very small stage of a multistage compressor. This small stage efficiency is supposed to be constant for all the stages of a compressor with infinite number of stages. The overall isentropic efficiency which is a very good measure of overall performance of a machine is not suitable to compare two compressors having different pressure ratio or stages. The concept of polytropic efficiency is used for such a comparison among compressors.

Example 15.8

Sketch the inlet and outlet velocity diagrams of a centrifugal air compressor with reference to the impeller vane configuration. Develop and expression for the work done by the impeller for compressing a unit mass of air. **(IES-ME, 1980)** ○●●

Solution For velocity diagram—Refer Figure 15.8.

State 1 refers to inlet to impeller and state 2 to outlet. Work done by impeller.

The work supplied to fluid in a stage of compressor may be found by applying moment of momentum theorem. Considering 1 kg of working fluid passing through the impeller, theoretical torque which must be applied to the impeller will be equal to the rate of change of momentum experienced by the fluid.

Theoretical torque, $\tau = (V_{w2}.r_2 - V_{w1}.r_1)$ J kg

If ω is angular velocity in radian per sec then the work done per kg of fluid will be:

$$W\tau\omega = (Vw_2.r_2 - Vw_1.r_1)\omega = (Vw_2.r_2 - Vw_1.r_1)$$

The above equation is known as Euler's equation. If the fluid enters radially, that is, Vw1 = 0, Thus,

$$W = (Vw_2.r_2 - Vw_1.r_1) \text{ J/kg}$$

Example 15.9

Comparison of centrifugal and axial flow compressors. **(IES-ME, 1987)** ○●●

Solution The following main features give the comparison between centrifugal and axial flow compressors. The comparison helps us to form some criterion for the choice between these two compressors.

	Centrifugal	Axial
1	The flow is radial.	The flow is parallel to the axis of the machine, i.e., axial.
2	The pressure ratio per stage is high, about 4.5:1. Thus, the unit is compact. In supersonic compressors, the pressure ratio per stage is about 10 but at the cost of efficiency. Operation is not so difficult and risky.	The pressure ratio per stage is low, about 1.2:1. This is due to the absence of centrifugal above 10 stages are required. The unit is less compact and less rugged. The pressure ratio per stage in supersonic compressors is about 10, but the efficiency drops rapidly. The compression is achieved through shock wave but operation is risky.
3	Isentropic efficiency of centrifugal compressor is about 80–84%.	Isentropic efficiency of axial flow compressor is about 86–88%.
4	Centrifugal compressors have a wide range of operation between surging and choking limit. The head capacity curve is flat. The part load performance is better.	Axial compressors have narrow range of operations between surging and choking limit. The part load performance is poor.

POINTS TO REMEMBER

- Reciprocating compressors are always positive displacement kind whereas the rotary type can be either of the two. Reciprocating air compressors can be of single-stage and multi-stage type.
- While solving problems the effect of clearance if given should be considered carefully and also it is must be noted whether the compressor is single acting or double acting.
- Volumetric efficiency is low in case of single-stage compressor compared with multi-stage compressor since clearance air is expanded reducing fresh air induction.
- Multi-stage compressor requires less power as it incorporates intercooler to make the process isothermal. Also, the output pressure achieved in the case of multi-stage compressor is very high.
- Surging is a very common phenomenon in the case of rotary compressor. Also, compared with reciprocating compressor slip factor is an issue dynamic type rotary compressor like centrifugal compressor.

PRACTICE PROBLEMS

1. Calculate the compressor work per kg, when air at 17°C and 1 bar is compressed to 6 bar. Assume that compression follows the law: $PV^{1.3} = C$. (**Ans:** 202 kJ/kg) ○○●
2. A single acting, single cylinder reciprocating compressor is compressing 5 kg/min of air from 1 bar 25°C to 7 bar. Assume mechanical efficiency to be 85%. Find the power input to the compressor neglecting clearance. Law of compression is: $PV^{1.25} = C$. (**Ans:** 19.9465 kW) ○●●
3. A single cylinder, double acting reciprocating air compressor receives air at 1 bar, 20°C and compresses it to 6 bar according to the law $PV^{1.35} = C$. The cylinder diameter is 250 mm. Average piston speed is 100 m/min at 100 rpm. Calculate the power required in kW for driving the compressor. Neglect clearance. (**Ans:** 18.62 kW) ●●●
4. An ideal single stage acting compressor with displacement volume of 16 *L* and a clearance volume of 0.8 *L* receives air at pressure 1 bar and delivers at pressure 7 bar. Compression index is 1.3 and expansion index is 1.4. Calculate the net indicated work of a cycle. ●●●

   (**Ans:** 3.285 kJ/cycle)
5. A single stage single acting reciprocating air compressor receives air at 1.013 bar, 30 deg C, and delivers it at 10 bar. The compressor has a bore = 200 mm and a stroke = 250 mm and it runs at 200 rpm. The mass flow rate of air is 100 kg/h. Find the volumetric efficiency of the compressor. (**Ans:** 91.1%) ○○●
6. The ratio of clearance volume to the displacement volume of a reciprocating compressor is 0.06, Specific volume at the inlet and outlet of compressor are 0.04 and 0.02 m^3/kg respectively. What is the volumetric efficiency of compressor? (**Ans:** 94%) ○○●
7. The low-pressure cylinder of a two-stage single acting reciprocating air compressor running at 120 rpm has a diameter of 7.5 cm, and stroke of 100 cm. It draws air at 1 bar pressure and 20°C and compresses it adiabatically to 3 bar. The air is then delivered to the intercooler where it is cooled at constant pressure to 30°C and then further compressed to 10 bar in high pressure cylinder, with poly-tropic index 1.3. Determine theoretical power required to run the compressor. (**Ans:** 240 kW) ●●●

8. Derive the relationship between isentropic efficiency and polytropic efficiency. ●●●

$\eta_c = [r^{(\gamma-1)}/(\gamma-1)]/[r^{(\gamma-1)}/(\gamma\eta_c-1)]$, where r is the pressure ratio of the compressor.

MULTIPLE CHOICE QUESTIONS

1. In a centrifugal compressor, how can the pressure ratio be increased? ○○●

(a) By increasing the tip speed
(b) By decreasing the inlet temperature
(c) By both (a) & (b)
(d) Only by increasing the inlet temperature

2. Which one of the following compressor is mostly used for supercharging of IC engine? ○●●

(a) Radial flow compressor
(b) Axial flow compressor
(c) Root blower
(d) Reciprocating compressor

3. Consider the following factors: ●●●

1. Cylinder size
2. Clearance ratio
3. Delivery pressure
4. Compressor shaft power

The factors which affect the volumetric efficiency of a single stage reciprocating air compressor would include

(a) 1 and 2 (b) 3 and 4 (c) 2 and 3 (d) 1 and 4

4. Cooling of reciprocating compressor cylinder: ○●●

1. Increases the volumetric efficiency
2. Increases the work input
3. Decreases the volumetric efficiency
4. Decreases the work input

Which of the above statements are correct?

(a) 1 and 2 (b) 2 and 3 (c) 3 and 4 (d) 1 and 4

5. For minimum work input in a two-stage compression process the intermediate pressure is the: ○○●

(a) Geometric mean of suction and discharge pressure
(b) Logarithmic mean of suction and discharge pressure
(c) Arithmetic mean of suction and discharge pressure
(d) Hyperbolic mean of suction and discharge pressure

6. Compressors used in gas turbines are of: ○○●

(a) Reciprocating type
(b) Centrifugal type
(c) Axial flow type
(d) All of the above

7. Losses in centrifugal compressors are due to: ○●●

(a) Impeller channel losses only
(b) Inlet losses only
(c) Both a and b
(d) None

8. Multi-stage compression of air as compared to single stage compression: ○●●

(a) Improves volumetric efficiency for the given pressure ratio

(b) Reduces work done per kg of air
(c) Gives more uniform torque
(d) All of the above

9. Surging is the phenomenon of: ●●●
(a) Steady periodic and reverse flow
(b) Un-steady, periodic and reverse flow
(c) Un-steady, periodic and uniform flow
(d) One dimensional steady and uniform flow

10. In a multi stage axial flow compressor, with equal temperature rise in all stages, the pressure ratio in the subsequent stages: ○○●
(a) Remains constant
(b) Increases gradually
(c) Decreases
(d) Increases rapidly

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. (c)	2. (c)	3. (c)	4. (b)	5. (a)
6. (c)	7. (c)	8. (d)	9. (b)	10. (b)

Appendix A

A.1 STEAM TABLES*

Table A.1.1 *Saturates steam : Temperature table*

	Specific Volume, m^3/kg			*Internal Energy, kJ/kg*				*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*		
Temp. °C T	*Pressure kPa, MPa* P	*Sat. Liquid* v_f	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Evap.* u_{fg}	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Evap.* s_{fg}	*Sat Vapour* s_g
0.01	0.6113	0.001000	206.132	0.00	2375.3	2375.3	0.00	2501.3	2501.3	0.0000	9.1562	9.1562
5	0.8721	0.001000	147.118	20.97	2361.3	2382.2	20.98	2489.6	2510.5	0.0761	8.9496	9.0257
10	1.2276	0.001000	106.377	41.99	2347.2	2389.2	41.99	2477.7	2519.7	0.1510	8.7498	8.9007
15	1.7051	0.001001	77.925	62.98	2333.1	2396.0	62.98	2465.9	2528.9	0.2245	8.5569	8.7813
20	2.3385	0.001002	57.790	83.94	2319.0	2402.9	83.94	2454.1	2538.1	0.2966	8.3706	8.6671
25	3.1691	0.001003	43.359	104.86	2304.9	2409.8	104.87	2442.3	2547.2	0.3673	8.1905	8.5579
30	4.2461	0.001004	32.893	125.77	2290.8	2416.6	125.77	2430.5	2556.2	0.4369	8.0164	8.4533
35	5.6280	0.001006	25.216	146.65	2276.7	2423.4	146.66	2418.6	2565.3	0.5052	7.8478	8.3530
40	7.3837	0.001008	19.523	167.53	2262.6	2430.1	167.54	2406.7	2574.3	0.5724	7.6845	8.2569
45	9.5934	0.001010	15.258	188.41	2248.4	2436.8	188.42	2394.8	2583.2	0.6386	7.5261	8.1647
50	12.350	0.001012	12.032	209.30	2234.2	2443.5	209.31	2382.7	2592.1	0.7037	7.3725	8.0762
55	15.758	0.001015	9.568	230.19	2219.9	2450.1	230.20	2370.7	2600.9	0.7679	7.2234	7.9912
60	19.941	0.001017	7.671	251.09	2205.5	2456.6	251.11	2358.5	2609.6	0.8311	7.0784	7.9095
65	25.033	0.001020	6.197	272.00	2191.1	2463.1	272.03	2346.2	2618.2	0.8934	6.9375	7.8309
70	31.188	0.001023	5.042	292.93	2176.6	2469.5	292.96	2333.8	2626.8	0.9548	6.8004	7.7552
75	38.578	0.001026	4.131	313.87	2162.0	2475.9	313.91	2321.4	2635.3	1.0154	6.6670	7.6824
80	47.390	0.001029	3.407	334.84	2147.4	2482.2	334.88	2308.8	2643.7	1.0752	6.5369	7.6121
85	57.834	0.001032	2.828	355.82	2132.6	2488.4	355.88	2296.0	2651.9	1.1342	6.4102	7.5444
90	70.139	0.001036	2.361	376.82	2117.7	2494.5	376.90	2283.2	2660.1	1.1924	6.2866	7.4790
95	84.554	0.001040	1.982	397.86	2102.7	2500.6	397.94	2270.2	2668.1	1.2500	6.1659	7.4158
100	0.10135	0.001044	1.6729	418.91	2087.6	2506.5	419.02	2257.0	2676.0	1.3068	6.0480	7.3548

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* Adapted from Joseph H. Keenan, Frederick G.Keyes, Philip G. Hill, and Joan G. Moore, *Steam Tables*, John Wiley and Sons, New York, 1969.

(Contd.)

Table A.1.1 *(Contd.)*

Temp. °C T	Specific Volume, m³/kg Pressure kPa, MPa P	Sat Liquid v_f	Sat. Liquid v_g	Internal Energy, kJ/kg Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Enthalpy, kJ/kg Sat. Liquid h_f	Evap. u_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
105	0.12082	0.001047	1.4194	440.00	2072.3	2512.3	440.13	2243.7	2683.8	1.3629	5.9328	7.2958
110	0.14328	0.001052	1.2102	461.12	2057.0	2518.1	461.27	2230.2	2691.5	1.4184	5.8202	7.2386
115	0.16906	0.001056	1.0366	482.28	2041.4	2523.7	482.46	2216.5	2699.0	1.4733	5.7100	7.1832
120	0.19853	0.001060	0.8919	503.48	2025.8	2529.2	503.69	2202.6	2706.3	1.5275	5.6020	7.1295
125	0.2321	0.001065	0.77059	524.72	2009.9	2534.6	524.96	2188.5	2713.5	1.5812	5.4962	7.0774
130	0.2701	0.001070	0.66850	546.00	1993.9	2539.9	546.29	2174.2	2720.5	1.6343	5.3925	7.0269
135	0.3130	0.001075	0.58217	567.34	1977.7	2545.0	567.67	2159.6	2727.3	1.6869	5.2907	6.9777
140	0.3613	0.001080	0.50885	588.72	1961.3	2550.0	589.11	2144.8	2733.9	1.7390	5.1908	6.9298
145	0.4154	0.001085	0.44632	610.16	1944.7	2554.9	610.61	2129.6	2740.3	1.7906	5.0926	6.8832
150	0.4759	0.001090	0.39278	631.66	1927.9	2559.5	632.18	2114.3	2746.4	1.8417	4.9960	6.8378
155	0.5431	0.001096	0.34676	653.23	1910.8	2564.0	653.82	2098.6	2752.4	1.8924	4.9010	6.7934
160	0.6178	0.001102	0.30706	674.85	1893.5	2568.4	675.53	2082.6	2758.1	1.9426	4.8075	6.7501
165	0.7005	0.001108	0.27269	696.55	1876.0	2572.5	697.32	2066.2	2763.5	1.9924	4.7153	6.7078
170	0.7917	0.001114	0.24283	718.31	1858.1	2576.5	719.20	2049.5	2768.7	2.0418	4.6244	6.6663
175	0.8920	0.001121	0.21680	740.16	1840.0	2580.2	741.16	2032.4	2773.6	2.0909	4.5347	6.6256
180	1.0022	0.001127	0.19405	762.08	1821.6	2583.7	763.21	2015.0	2778.2	2.1395	4.4461	6.5857
185	1.1227	0.001134	0.17409	784.08	1802.9	2587.0	785.36	1997.1	2782.4	2.1878	4.3586	6.5464
190	1.2544	0.001141	0.15654	806.17	1783.8	2590.0	807.61	1978.8	2786.4	2.2358	4.2720	6.5078
195	1.3978	0.001149	0.14105	828.36	1764.4	2592.8	829.96	1960.0	2790.0	2.2835	4.1863	6.4697
200	1.5538	0.001156	0.12736	850.64	1744.7	2595.3	852.43	1940.7	2793.2	2.3308	4.1014	6.4322
205	1.7230	0.001164	0.11521	873.02	1724.5	2597.5	875.03	1921.0	2796.0	2.3779	4.0172	6.3951
210	1.9063	0.001173	0.10441	895.51	1703.9	2599.4	897.75	1900.7	2798.5	2.4247	3.9337	6.3584
215	2.1042	0.001181	0.09479	918.12	1682.9	2601.1	920.61	1879.9	2800.5	2.4713	3.8507	6.3221
220	2.3178	0.001190	0.08619	940.85	1661.5	2602.3	943.61	1858.5	2802.1	2.5177	3.7683	6.2860

(Contd.)

Table A.1.1 *(Contd.)*

Temp. °C T	*Specific Volume, m^3/kg*			*Internal Energy, kJ/kg*			*Enthalpy, kJ/kg*			*Entropy, kJ/kg K*		
	Pressure kPa, Mpa P	*Sat. Liquid v_f*	*Sat. Vapour v_g*	*Sat. Liquid u_f*	*Evap. u_{fg}*	*Sat. Vapour u_g*	*Sat. Liquid h_f*	*Evap. u_{fg}*	*Sat. Vapour h_g*	*Sat. Liquid s_f*	*Evap. s_{fg}*	*Sat. Vapour s_g*
225	2.5477	0.001199	0.7849	963.72	1639.6	2603.3	966.77	1836.5	2803.3	2.5639	3.6863	6.2502
230	2.7949	0.001209	0.07158	986.72	1617.2	2603.9	990.10	1813.8	2803.9	2.6099	3.6047	6.2146
235	3.0601	0.001219	0.06536	1009.88	1594.2	2604.1	1013.61	1790.5	2804.1	2.6557	3.5233	6.1791
240	3.3442	0.001229	0.05976	1033.19	1570.8	2603.9	1037.31	1766.5	2803.8	2.7015	3.4422	6.1436
245	3.6482	0.001240	0.05470	1056.69	1546.7	2603.4	1061.21	1741.7	2802.9	2.7471	3.3612	6.1083
250	3.9730	0.001251	0.05013	1080.37	1522.0	2602.4	1085.34	1716.2	2801.5	2.7927	3.2802	6.0729
255	4.3195	0.001263	0.04598	1104.26	1496.7	2600.9	1109.72	1689.8	2799.5	2.8382	3.1992	6.0374
260	4.6886	0.001276	0.04220	1128.37	1470.6	2599.0	1134.35	1662.5	2796.9	2.8837	3.1181	6.0018
265	5.0813	0.001289	0.03877	1152.72	1443.9	2596.6	1159.27	1634.3	2793.6	2.9293	3.0368	5.9661
270	5.4987	0.001302	0.03564	1177.33	1416.3	2593.7	1184.49	1605.2	2789.7	2.9750	2.9551	5.9301
275	5.9418	0.001317	0.03279	1202.23	1387.9	2590.2	1210.05	1574.9	2785.0	3.0208	2.8730	5.8937
280	6.4117	0.001332	0.03017	1227.43	1358.7	2586.1	1235.97	1543.6	2779.5	3.0667	2.7903	5.8570
285	6.9094	0.001348	0.02777	1252.98	1328.4	2581.4	1262.29	1511.0	2773.3	3.1129	2.7069	5.8198
290	7.4360	0.001366	0.02557	1278.89	1297.1	2576.0	1289.04	1477.1	2766.1	3.1593	2.6227	5.7821
295	7.9928	0.001384	0.02354	1305.21	1264.7	2569.9	1316.27	1441.8	2758.0	3.2061	2.5375	5.7436
300	8.5810	0.001404	0.02167	1331.97	1231.0	2563.0	1344.01	1404.9	2748.9	3.2533	2.4511	5.7044
305	9.2018	0.001425	0.01995	1359.22	1195.9	2555.2	1372.33	1366.4	2738.7	3.3009	2.3633	5.6642
310	9.8566	0.001447	0.01835	1387.03	1159.4	2546.4	1401.29	1326.0	2727.3	3.3492	2.2737	5.6229
315	10.547	0.001472	0.01687	1415.44	1121.1	2536.6	1430.97	1283.5	2714.4	3.3981	2.1821	5.5803
320	11.274	0.001499	0.01549	1444.55	1080.9	2525.5	1461.45	1238.6	2700.1	3.4479	2.0882	5.5361
330	12.845	0.001561	0.012996	1505.24	993.7	2498.9	1525.29	1140.6	2665.8	3.5506	1.8909	5.4416
340	14.586	0.001638	0.010797	1570.26	894.3	2464.5	1594.15	1027.9	2622.0	3.6593	1.6763	5.3356
350	16.514	0.001740	0.008813	1641.81	776.6	2418.4	1670.54	893.4	2563.9	3.7776	1.4336	5.2111
360	18.651	0.001892	0.006945	1725.19	626.3	2351.5	1760.48	720.5	2481.0	3.9146	1.1379	5.0525
370	21.028	0.002213	0.004926	1843.84	384.7	2228.5	1890.37	441.8	2332.1	4.1104	0.6868	4.7972
374.14	22.089	0.003155	0.003155	2029.58	0	2029.6	2099.26	0	2099.3	4.4297	0	4.4297

(Contd.)

Table. A.1.2 *Saturated water: Pressure table*

Pressure mPa P	Tem. °C T	*Specific Volume, m^3/kg* Sat. Liquid v_f	Sat. Vapour v_g	*Internal Energy, kJ/kg* Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	*Entropy, kJ/kg* Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	*Entropy, kJ/kg K* Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
0.6113	0.01	0.001000	206.132	0	2375.3	2375.3	0.00	2501.3	2501.3	0	9.1562	9.1562
1.0	6.98	0.001000	129.208	29.29	2355.7	2385.0	29.29	2484.9	2514.2	0.1059	8.8697	8.9756
1.5	13.03	0.001001	87.980	54.70	2338.6	2393.3	54.70	2470.6	2525.3	0.1956	8.6322	8.8278
2.0	17.50	0.001001	67.004	73.47	2326.0	2399.5	73.47	2460.0	2533.5	0.2607	8.4629	8.7236
2.5	21.08	0.001002	54.254	88.47	2315.9	2404.4	88.47	2451.6	2540.0	0.3120	8.3311	8.6431
3.0	24.08	0.001003	45.665	101.03	2307.5	2408.5	101.03	2444.5	2545.5	0.3545	8.2231	8.5775
4.0	28.96	0.001004	34.800	121.44	2293.7	2415.2	121.44	2432.9	2554.4	0.4226	8.0520	8.4746
5.0	32.88	0.001005	28.193	137.79	2282.7	2420.5	137.79	2423.7	2561.4	0.4763	7.9187	8.3950
7.5	40.29	0.001008	19.238	168.76	2261.7	2430.5	168.77	2406.0	2574.8	0.5763	7.6751	8.2514
10.0	45.81	0.001010	14.674	191.79	2246.1	2437.9	191.81	2392.8	2584.6	0.6492	7.5010	8.1501
15.0	53.97	0.001014	10.022	225.90	2222.8	2448.7	225.91	2373.1	2599.1	0.7548	7.2536	8.0084
20.0	60.06	0.001017	7.649	251.35	2205.4	2456.7	251.38	2358.3	2609.7	0.8319	7.0766	7.9085
25.0	64.97	0.001020	6.204	271.88	2191.2	2463.1	271.90	2346.3	2618.2	0.8930	6.9383	7.8313
30.0	69.10	0.001022	5.229	289.18	2179.2	2468.4	289.21	2336.1	2626.3	0.9439	6.8247	7.7686
40.0	75.87	0.001026	3.993	317.51	2159.5	2477.0	317.55	2319.2	2636.7	1.0258	6.6441	7.6700
50.0	81.33	0.001030	3.240	340.42	2143.4	2483.8	340.47	2305.4	2645.9	1.0910	6.5029	7.5939
75.0	91.77	0.001037	2.217	384.29	2112.4	2496.7	384.36	2278.6	2663.0	1.2129	6.2434	7.4563
MPa												
0.100	99.62	0.001043	1.6940	417.33	2088.7	2506.1	417.44	2258.0	2675.5	1.3025	6.0568	7.3593
0.125	105.99	0.001048	1.3749	444.16	2069.3	2513.5	444.30	2241.1	2685.3	1.3739	5.9104	7.2843
0.150	111.37	0.001053	1.1593	466.92	2052.7	2519.6	467.08	2226.5	2693.5	1.4335	5.7897	7.2232
0.175	116.06	0.001057	1.0036	486.78	2038.1	2524.9	486.97	2213.6	2700.5	1.4848	5.6868	7.1717
0.200	120.23	0.001061	0.8857	504.47	2025.0	2529.5	504.68	2202.0	2706.6	1.5300	5.5970	7.1271
0.225	124.00	0.001064	0.7933	520.45	2013.1	2533.6	520.69	2191.3	2712.0	1.5705	5.5173	7.0878
0.250	127.43	0.001067	0.7187	535.08	2002.1	2537.2	535.34	2181.5	2716.9	1.6072	5.4455	7.0526

(Contd.)

Table A.1.2 *(Contd.)*

Pressure MPa P	Temp. °C T	Specific Volume, m^3/kg Sat. Liquid v_f	 Sat. Vapour v_g	Internal Energy, kJ/kg Sat. Liquid u_f	 Evap. u_{fg}	 Sat. Vapour v_g	Enthalpy, kJ/kg Sat. Liquid h_f	 Evap. h_{fg}	 Sat. Vapour h_g	Entropy, kJ/kg K Sat. Liquid s_f	 Evap. s_{fg}	 Sat. Vapour s_g
0.275	130.60	0.001070	0.6573	548.57	1992.0	2540.5	548.87	2172.4	2721.3	1.6407	5.3801	7.0208
0.300	133.55	0.001073	0.6058	561.13	1982.4	2543.6	561.45	2163.9	2725.3	1.6717	5.3201	6.9918
0.350	138.88	0.001079	0.5243	583.93	1965.0	2548.9	584.31	2148.1	2732.4	1.7274	5.2130	6.9404
0.375	141.32	0.001081	0.4914	594.38	1956.9	2255.13	594.79	2140.8	2735.6	1.7527	5.1647	6.9174
0.40	143.63	0.001084	0.4625	604.29	1949.3	2553.6	604.73	2133.8	2738.5	1.7766	5.1193	6.8958
0.45	147.93	0.001088	0.4140	622.75	1934.9	2557.6	623.24	2120.7	2743.9	1.8206	5.0359	6.8565
0.50	151.86	0.001093	0.3749	639.66	1921.6	2561.2	640.21	2108.5	2748.7	1.8606	4.9606	6.8212
0.55	155.48	0.001097	0.3427	655.30	1909.2	2564.5	655.91	2097.0	2752.9	1.8972	4.8920	6.7892
0.60	158.85	0.001101	0.3157	669.88	1897.5	2567.4	670.54	2086.3	2756.8	1.9311	4.8289	6.7600
0.65	162.01	0.001104	0.2927	683.55	1886.5	2570.1	684.26	2076.0	2760.3	1.9627	4.7704	6.7330
0.70	164.97	0.001108	0.2729	696.43	1876.1	2572.5	697.20	2066.3	2763.5	1.9922	4.7158	6.7080
0.75	167.77	0.001111	0.2556	708.62	1866.1	2574.7	709.45	2057.0	2766.4	2.0199	4.6647	6.6846
0.80	170.43	0.001115	0.2404	720.20	1856.6	2576.8	721.10	2048.0	2769.1	2.0461	4.6166	6.6627
0.85	172.96	0.001118	0.2270	731.25	1847.4	2578.7	732.20	2039.4	2771.6	2.0709	4.5711	6.6421
0.90	175.38	0.001121	0.2150	741.81	1838.7	2580.5	742.82	2031.1	2773.9	2.0946	4.5280	6.6225
0.95	177.69	0.001124	0.2042	751.94	1830.2	2582.1	753.00	2023.1	2776.1	2.1171	4.4869	6.6040
1.00	179.91	0.001127	0.19444	761.67	1822.0	2583.6	762.79	2015.3	2778.1	2.1386	4.4478	6.5864
1.10	184.09	0.001133	0.17753	780.08	1806.3	2586.4	781.32	2000.4	2781.7	2.1791	4.3744	6.5535
1.20	187.99	0.001139	0.16333	797.27	1791.6	2588.8	798.64	1986.2	2784.8	2.2165	4.3067	6.5233
1.30	191.64	0.001144	0.15125	813.42	1777.5	2590.9	814.91	1972.7	2787.6	2.2514	4.2438	6.4953
1.40	195.07	0.001149	0.14084	828.68	1764.1	2592.8	830.29	1959.7	2790.0	2.2842	4.1850	6.4692
1.50	198.32	0.001154	0.13177	843.14	1751.3	2594.5	844.87	1947.3	2792.1	2.3150	4.2198	6.4448
1.75	205.76	0.001166	0.11349	876.44	1721.4	2597.8	878.48	1918.0	2796.4	2.3851	4.0044	6.3895
2.00	212.42	0.001177	0.09963	906.42	1693.8	2600.3	908.77	1890.7	2799.5	2.4473	3.8935	6.3408
2.25	218.45	0.001187	0.08875	933.81	1668.2	2602.0	936.48	1865.2	2801.7	2.5034	3.7938	6.2971

(Contd.)

Table A.1.2 *(Contd.)*

Pressure MPa P	Temp. °C T	Specific Volume, m³/kg Sat. Liquid v_f	Sat. Vapour v_g	Internal Energy, kJ/kg Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Enthalpy, kJ/kg Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Entropy, kJ/kg K Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
2.50	223.99	0.001197	0.07998	959.09	1644.0	2603.1	962.09	1841.0	2803.1	2.5546	3.7028	6.2574
2.75	229.12	0.001207	0.07275	982.65	1621.2	2603.8	985.97	1817.9	2803.9	2.6018	3.6190	6.2208
3.00	233.90	0.001216	0.06668	1004.76	1599.3	2604.1	1008.41	1795.7	2804.1	2.6456	3.5412	6.1869
3.25	238.38	0.001226	0.06152	1025.62	1578.4	2604.0	1029.60	1774.4	2804.0	2.6866	3.4685	6.1551
3.50	242.60	0.001235	0.05707	1045.41	1558.3	2603.7	1049.73	1753.7	2803.4	2.7252	3.4000	6.1252
4.0	250.40	0.001252	0.049778	1082.28	1520.0	2602.3	1087.29	1714.1	2801.4	2.7963	3.2737	6.0700
5.0	263.99	0.001286	0.039441	1147.78	1449.3	2597.1	1154.21	1640.1	2794.3	2.9201	3.0532	5.9733
6.0	275.64	0.001319	0.032440	1205.41	1384.3	2589.7	1213.32	1571.0	2784.3	3.0266	2.8625	5.8891
7.0	285.88	0.001351	0.027370	1257.51	1323.0	2580.5	1266.97	1505.1	2772.1	3.1210	2.6922	5.8132
8.0	295.06	0.001384	0.023518	1305.54	1264.3	2569.8	1316.61	1441.3	2757.9	3.2067	2.5365	5.7431
9.0	303.40	0.001418	0.020484	1350.47	1207.3	2557.8	1363.23	1378.9	2742.1	3.2857	2.3915	5.6771
10.0	311.06	0.001452	0.018026	1393.00	1151.4	2544.4	1407.53	1317.1	2724.7	3.3595	2.2545	5.6140
11.0	318.15	0.001489	0.015987	1433.68	1096.1	2529.7	1450.05	1255.5	2705.6	3.4294	2.1233	5.5527
12.0	324.75	0.001527	0.014263	1472.92	1040.8	2513.7	1491.24	1193.6	2684.8	3.4961	1.9962	5.4923
13.0	330.93	0.001567	0.012780	1511.09	985.0	2496.1	1531.46	1130.8	2662.2	3.5604	1.8718	5.4323
14.0	336.75	0.001611	0.011485	1548.53	928.2	2476.8	1571.08	1066.5	2637.5	3.6231	1.7485	5.3716
15.0	342.24	0.001658	0.010338	1585.58	869.8	2455.4	1610.45	1000.0	2610.5	3.6847	1.6250	5.3097
16.0	347.43	0.001711	0.009306	1622.63	809.1	2431.7	1650.00	930.6	2580.6	3.4760	1.4995	5.2454
17.0	352.37	0.001770	0.008365	1660.16	744.8	2405.0	1690.25	856.9	2547.2	3.8078	1.3698	5.1776
18.0	357.06	0.001840	0.007490	1698.86	675.4	2374.3	1731.97	777.1	2509.1	3.8713	1.2330	5.1044
19.0	361.54	0.001924	0.006657	1739.87	598.2	2338.1	1776.43	688.1	2464.5	3.9387	1.0841	5.0227
20.0	365.81	0.002035	0.005834	1785.47	507.6	2293.1	1826.18	583.6	2409.7	4.0137	0.9132	4.9269
21.0	369.89	0.002206	0.004953	1841.97	388.7	2230.7	1888.30	446.4	2334.7	4.1073	0.6942	4.8015
22.0	373.80	0.002808	0.003526	1973.16	108.2	2081.4	2034.92	124.0	2159.0	4.3307	0.1917	4.5224
22.09	374.14	0.003155	0.003155	2029.58	0	2029.6	2099.26	0	2099.3	4.4297	0	4.4297

Table A.1.3 *Superheated vapour*

	P = 10 kPa (45.81)				*P* = 50 kPa, (81.33)				*P* = 100 kPa (99.62)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
Sat.	14.674	2437.9	2584.6	8.1501	3.240	2483.8	2645.9	7.5939	1.6940	2506.1	2675.5	7.3593
50	14.869	2443.9	2592.6	8.1749	—	—	—	—	—	—	—	—
100	17.196	2515.5	2687.5	8.4479	3.418	2511.6	2682.5	7.6947	1.6958	2506.6	2676.2	7.3614
150	19.513	2587.9	2783.0	8.6881	3.889	2585.6	2780.1	7.9400	1.9364	2582.7	2776.4	7.6133
200	21.825	2661.3	2879.5	8.9037	4.356	2659.8	2877.6	8.1579	2.1723	2658.0	2875.3	7.8342
250	24.136	2736.0	2977.3	9.1002	4.821	2735.0	2976.0	8.3555	2.4060	2733.7	2974.3	8.0332
300	26.445	2812.1	3076.5	9.2812	5.284	2811.3	3075.5	8.5372	2.6388	2810.4	3074.3	8.2157
400	31.063	2968.9	3279.5	9.6076	6.209	2968.4	3278.9	8.8641	3.1026	2967.8	3278.1	8.5434
500	35.679	3132.3	3489.0	9.8977	7.134	3131.9	3488.6	9.1545	3.5655	3131.5	3488.1	8.8341
600	40.295	3302.5	3705.4	10.1608	8.058	3302.2	3705.1	9.4177	4.0278	3301.9	3704.7	9.0975
700	44.911	3479.6	3928.7	10.4028	8.981	3479.5	3928.5	9.6599	4.4899	3479.2	3928.2	9.3398
800	49.526	3663.8	4159.1	10.6281	9.904	3663.7	4158.9	9.8852	4.9517	3663.5	4158.7	9.5652
900	54.141	3855.0	4396.4	10.8395	10.828	3854.9	4396.3	10.0967	5.4135	3854.8	4396.1	9.7767
1000	58.757	4053.0	4640.6	11.0392	11.751	4052.9	4640.5	10.2964	5.8753	4052.8	4640.3	9.9764
1100	63.372	4257.5	4891.2	11.2287	12.674	4257.4	4891.1	10.4858	6.3370	4257.3	4890.9	10.1658
1200	67.987	4467.9	5147.8	11.4090	13.597	4467.8	5147.7	10.6662	6.7986	4467.7	5147.6	10.3462
1300	72.603	4683.7	4409.7	11.5810	14.521	4683.6	5409.6	10.8382	7.2603	4683.5	5409.5	10.5182
	P = 200 kPa (120.23)				*P* = 300 kPa (133.55)				*P* = 400 kPa (143.65)			
Sat.	0.88573	2529.5	2706.6	7.1271	0.60582	2543.6	2725.3	6.9918	0.46246	2553.6	2738.5	6.8958
150	0.95964	2576.9	2768.8	7.2795	0.63388	2570.8	2761.0	7.0778	0.47084	2564.5	2752.8	6.9299
200	1.08034	2654.4	2870.5	7.5066	0.71629	2650.7	2865.5	7.3115	0.53422	2646.8	2860.5	7.1706
250	1.19880	2731.2	2971.0	7.7085	0.79636	2728.7	2967.6	7.5165	0.59512	2726.1	2964.2	7.3788
300	1.31616	2808.6	3071.8	7.8926	0.87529	2806.7	3069.3	7.7022	0.65484	2804.8	3066.7	7.5661
400	1.54930	2966.7	3276.5	8.2217	1.03151	2965.5	3275.0	8.0329	0.77262	2964.4	3273.4	7.8984
500	1.78139	3130.7	3487.0	8.5132	1.18669	3130.0	3486.0	8.3250	0.88934	3129.2	3484.9	8.1912
600	2.01297	3301.4	3704.0	8.7769	1.34136	3300.8	3703.2	8.5892	1.00555	3300.2	3702.4	8.4557
700	2.24426	3478.8	3927.7	9.0194	1.49573	3478.4	3927.1	8.8319	1.12147	3477.9	3926.5	8.6987
800	2.47539	3663.2	4158.3	9.2450	1.64994	3662.9	4157.8	9.0575	1.23722	3662.5	4157.4	8.9244

(Contd.)

Table A.1.3 *(Contd.)*

	P = 200 kPa (120.23)				*P* = 300 kPa (133.55)				*P* = 400 kPa (143.63)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
900	2.70643	3854.5	4395.8	9.4565	1.80406	3854.2	4395.4	9.2691	1.35288	3853.9	4395.1	9.1361
1000	2.93740	4052.5	4640.0	9.6563	1.95812	4052.3	4639.7	9.4689	1.46847	4052.0	4639.4	9.3360
1100	3.16834	4257.0	4890.7	9.8458	2.11214	4256.8	4890.4	9.6585	1.58404	4256.5	4890.1	9.5255
1200	3.39927	4467.5	5147.3	10.0262	2.26614	4467.2	5147.1	9.8389	1.69958	4467.0	5146.8	9.7059
1300	3.63018	4683.2	5409.3	10.1982	2.42013	4683.0	5409.0	10.0109	1.81511	4682.8	5408.8	9.8780
	P = 500 kPa (151.86)				*P* = 600 kPa (158.85)				*P* = 800 kPa (170.43)			
Sat.	0.37489	2561.2	2748.7	6.8212	0.31567	2567.4	2756.8	6.7600	0.24043	2576.8	2769.1	6.6627
200	0.42492	2642.9	2855.4	7.0592	0.35202	2638.9	2850.1	6.9665	0.26080	2630.6	2839.2	6.8158
250	0.47436	2723.5	2960.7	7.2708	0.39383	2720.9	2957.2	7.1816	0.29314	2715.5	2950.0	7.0384
300	0.52256	2802.9	3064.2	7.4598	0.43437	2801.0	3061.6	7.3723	0.32411	2797.1	3056.4	7.2372
350	0.57012	2882.6	3167.6	7.6328	0.47424	2881.1	3165.7	7.5463	0.35439	2878.2	3161.7	7.4088
400	0.61728	2963.2	3271.8	7.7937	0.51372	2962.0	3270.2	7.7078	0.38426	2959.7	3267.1	7.5715
500	0.71093	3128.4	3483.8	8.0872	0.59199	3127.6	3482.7	8.0020	0.44331	3125.9	3480.6	7.8672
600	0.80406	3299.6	3701.7	8.3521	0.66974	3299.1	3700.9	8.2673	0.50184	3297.9	3699.4	8.1332
700	0.89691	3477.5	3926.0	8.5952	0.74720	3477.1	3925.4	8.5107	0.56007	3476.2	3924.3	8.3770
800	0.98959	3662.2	4157.0	8.8211	0.82450	3661.8	4156.5	8.7367	0.61813	3661.1	4155.7	8.6033
900	1.08217	3853.6	4394.7	9.0329	0.90169	3853.3	4394.4	8.9485	0.67610	3852.8	4393.6	8.8153
1000	1.17469	4051.8	4639.1	9.2328	0.97883	4051.5	4638.8	9.1484	0.73401	4051.0	4638.2	9.0153
1100	1.26718	4256.3	4889.9	9.4224	1.05594	4256.1	4889.6	9.3381	0.79188	4255.6	4889.1	9.2049
1200	1.35964	4466.8	5146.6	9.6028	1.13302	4466.5	5146.3	9.5185	0.84974	4466.1	5145.8	9.3854
1300	1.45210	4682.5	5408.6	9.7749	1.21009	4682.3	5408.3	9.6906	0.90758	4681.8	5407.9	9.5575
	P = 1.00 MPa (179.91)				*P* = 1.20 MPa (187.99)				*P* = 1.40 MPa (195.07)			
Sat.	0.19444	2583.6	2778.1	6.5864	0.16333	2588.8	2784.8	6.5233	0.14084	2592.8	2790.0	6.4692
200	0.20596	2621.9	2827.9	6.6939	0.16930	2612.7	2815.9	6.5898	0.14302	2603.1	2803.3	6.4975
250	0.23268	2709.9	2942.6	6.9246	0.19235	2704.2	2935.0	6.8293	0.16350	2698.3	2927.2	6.7467
300	0.25794	2793.2	3051.2	7.1228	0.21382	2789.2	3045.8	7.0316	0.18228	2785.2	3040.4	6.9533
350	0.28247	2875.2	3157.7	7.3010	0.23452	2872.2	3153.6	7.2120	0.20026	2869.1	3149.5	7.1359
400	0.30659	2957.3	3263.9	7.4650	0.25480	2954.9	3260.7	7.3773	0.21780	2952.5	3257.4	7.3025
500	0.35411	3124.3	3478.4	7.7621	0.29463	3122.7	3476.3	7.6758	0.25215	3121.1	3474.1	7.6026

(Contd.)

Table A.1.3 *(Contd.)*

	P = 1.00 MPa (179.91)				*P* = 1.20 MPa (187.99)				*P* = 1.40 MPa (195.07)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
600	0.40109	3296.8	3697.9	8.0289	0.33393	3295.6	3696.3	7.9434	0.28596	3294.4	3694.8	7.8710
700	0.44779	3475.4	3923.1	8.2731	0.37294	3474.5	3922.0	8.1881	0.31947	3473.6	3920.9	8.1160
800	0.49432	3660.5	4154.8	8.4996	0.41177	3659.8	4153.9	8.4149	0.35281	3659.1	4153.0	8.3431
900	0.54075	3852.5	4392.9	8.7118	0.45051	3851.6	4392.2	8.6272	0.38606	3851.0	4391.5	8.5555
1000	0.58712	4050.5	4637.6	8.9119	0.48919	4050.0	4637.0	8.8274	0.41924	4049.5	4636.4	8.7558
1100	0.63345	4255.1	4888.5	9.1016	0.52783	4254.6	4888.0	9.0171	0.45239	4254.1	4887.5	8.9456
1200	0.67977	4465.6	5145.4	9.2821	0.56646	4465.1	5144.9	9.1977	0.48552	4464.6	5144.4	9.1262
1300	0.72608	4681.3	5407.4	9.4542	0.60507	4680.9	5406.9	9.3698	0.51864	4680.4	5406.5	9.2983
	P = 1.60 MPa (201.40)				*P* = 1.80 MPa (207.15)				*P* = 2.00 MPa (212.42)			
Sat.	0.12380	2595.9	2794.0	6.4217	0.11042	2598.4	2797.1	6.3793	0.09963	2600.3	2799.5	6.3408
225	0.13287	2644.6	2857.2	6.5518	0.11673	2636.6	2846.7	6.4807	0.10377	2628.3	2835.8	6.4146
250	0.14184	2692.3	2919.2	6.6732	0.12497	2686.0	2911.0	6.6066	0.11144	2679.6	2902.5	6.5452
300	0.15862	2781.0	3034.8	6.8844	0.14021	2776.8	3029.2	6.8226	0.12547	2772.6	3023.5	6.7663
350	0.17456	2866.0	3145.4	7.0693	0.15457	2862.9	3141.2	7.0099	0.13857	2859.8	3137.0	6.9562
400	0.19005	2950.1	3254.2	7.2373	0.16847	2947.7	3250.9	7.1793	0.15120	2945.2	3247.6	7.1270
500	0.22029	3119.5	3471.9	7.5389	0.19550	3117.8	3469.7	7.4824	0.17568	3116.2	3467.6	7.4316
600	0.24998	3293.3	3693.2	7.8080	0.22199	3292.1	3691.7	7.7523	0.19960	3290.9	3690.1	7.7023
700	0.27937	3472.7	3919.7	8.0535	0.24818	3471.9	3918.6	7.9983	0.22323	3471.0	3917.5	7.9487
800	0.30859	3658.4	4152.1	8.2808	0.27420	3657.7	4151.3	8.2258	0.24668	3657.0	4150.4	8.1766
900	0.33772	3850.5	4390.8	8.4934	0.30012	3849.9	4390.1	8.4386	0.27004	3849.3	4389.4	8.3895
1000	0.36678	4049.0	4635.8	8.6938	0.32598	4048.4	4635.2	8.6390	0.29333	4047.9	4634.6	8.5900
1100	0.39581	4253.7	4887.0	8.8837	0.35180	4253.2	4886.4	8.8290	0.31659	4252.7	4885.9	8.7800
1200	0.42482	4464.2	5143.9	9.0642	0.37761	4663.7	5143.4	9.0096	0.33984	4463.2	5142.9	8.9606
1300	0.45382	4679.9	5406.0	9.2364	0.40340	4679.4	5405.6	9.1817	0.36306	4679.0	5405.1	9.1382
	P = 2.50 MPa (223.99)				*P* = 3.00 MPa (233.90)				*P* = 3.50 MPa (242.66)			
Sat.	0.07998	2603.1	2803.1	6.2574	0.06668	2604.1	2804.1	6.1869	0.05707	2603.7	2803.4	6.1252
225	0.08027	2605.6	2806.3	6.2638	—	—	—	—	—	—	—	—
250	0.08700	2662.5	2880.1	6.4084	0.07058	2644.0	2855.8	6.2871	0.05873	2623.7	2829.2	6.1748

(Contd.)

Table A.1.3 *(Contd.)*

	P = 2.50 MPa (223.99)				*P* = 3.00 MPa (233.90)				*P* = 3.50 MPa (242.60)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
300	0.09890	2761.6	3008.8	6.6437	0.08114	2750.0	2993.5	6.5389	0.06842	2738.0	2977.5	6.4460
350	0.10976	2851.8	3126.2	6.8402	0.09053	2843.7	3115.3	6.7427	0.07678	2835.3	3104.0	6.6578
400	0.12010	2939.0	3239.3	7.0147	0.09936	2932.7	3230.8	6.9211	0.08453	2926.4	3222.2	6.8404
450	0.13014	3025.4	3350.8	7.1745	0.10787	3020.4	3344.0	7.0833	0.09196	3015.3	3337.2	7.0051
500	0.13998	3112.1	3462.0	7.3233	0.11619	3107.9	3456.5	7.2337	0.09918	3103.7	3450.9	7.1571
600	0.15930	3288.0	3686.2	7.5960	0.13243	3285.0	3682.3	7.5084	0.11324	3282.1	3678.4	7.4338
700	0.17832	3468.8	3914.6	7.8435	0.14838	3466.0	3911.7	7.7571	0.12699	3464.4	3908.8	7.6837
800	0.19716	3655.3	4148.2	8.0720	0.16414	3653.6	4146.0	7.9862	0.14056	3651.8	4143.8	7.9135
900	0.21590	3847.9	4387.6	8.2853	0.17980	3846.5	4385.9	8.1999	0.15402	3845.0	4384.1	8.1275
1000	0.23458	4046.7	4633.1	8.4860	0.19541	4045.4	4631.6	8.4009	0.16743	4044.1	4630.1	8.3288
1100	0.25322	4251.5	4884.6	8.6761	0.21098	4250.3	4883.3	8.5911	0.18080	4249.1	4881.9	8.5191
1200	0.27185	4462.1	5141.7	8.8569	0.22652	4460.9	5140.5	8.7719	0.19415	4459.8	5139.3	8.7000
1300	0.29046	4677.8	5404.0	9.0291	0.24206	4676.6	5402.8	8.9442	0.20749	4675.5	5401.7	8.8723
	P = 4.00 MPa (250.40)				*P* = 4.50 MPa (257.48)				*P* = 5.00 MPa (263.99)			
Sat.	0.04978	2602.3	2801.4	6.0700	0.04406	2600.0	2798.3	6.0198	0.03944	2597.1	2794.3	5.9733
275	0.05457	2667.9	2886.2	6.2284	0.04730	2650.3	2863.1	6.1401	0.04141	2631.2	2838.3	6.0543
300	0.05884	2725.3	2960.7	6.3614	0.05135	2712.0	2943.1	6.2827	0.04532	2697.9	2924.5	6.2083
350	0.06645	2826.6	3092.4	6.5820	0.05840	2817.8	3080.6	6.5130	0.05194	2808.7	3068.4	6.4492
400	0.07341	2919.9	3213.5	6.7689	0.06475	2913.3	3204.7	6.7046	0.05781	2906.6	3195.6	6.6458
450	0.08003	3010.1	3330.2	6.9362	0.07074	3004.9	3323.2	6.8745	0.06330	2999.6	3316.1	6.8185
500	0.08643	3099.5	3445.2	7.0900	0.07651	3095.2	3439.5	7.0300	0.06857	3090.9	3433.8	6.9758
600	0.09885	3279.1	3674.4	7.3688	0.08765	3276.0	3670.5	7.3109	0.07869	3273.0	3666.5	7.2588
700	0.11095	3462.1	3905.9	7.6198	0.09847	3459.9	3903.0	7.5631	0.08849	3457.7	3900.1	7.5122
800	0.12287	3650.1	4141.6	7.8502	0.10911	3648.4	4139.4	7.7942	0.09811	3646.6	4137.2	7.7440
900	0.13469	3843.6	4382.3	8.0647	0.11965	3842.1	4380.6	8.0091	0.10762	3840.7	4378.8	7.9593
1000	0.14645	4042.9	4628.7	8.2661	0.13013	4041.6	4627.2	8.2108	0.11707	4040.3	4625.7	8.1612
1100	0.15817	4248.0	4880.6	8.4566	0.14056	4246.8	4879.3	8.4014	0.12648	4245.6	4878.0	8.3519
1200	0.16987	4458.6	5138.1	8.6376	0.15098	4457.4	5136.9	8.5824	0.13587	4456.3	5135.7	8.5330
1300	0.18156	4674.3	5400.5	8.8099	0.16139	4673.1	5399.4	8.7548	0.14526	4672.0	5398.2	8.7055

(Contd.)

Table A.1.3 *(Contd.)*

	P = 6.00 MPa (275.64)				P = 7.00 MPa (285.88)				P = 8.00 MPa (295.06)			
T	v	u	h	s	v	u	h	s	v	u	h	s
Sat.	0.03244	2589.7	2784.3	5.8891	0.02737	2580.5	2772.1	5.8132	0.02352	2569.8	2757.9	5.7431
300	0.03616	2667.2	2884.2	6.0673	0.02947	2632.1	2838.4	5.9304	0.02426	2590.9	2785.0	5.7905
350	0.04223	2789.6	3043.0	6.3334	0.03524	2769.3	3016.0	6.2282	0.02995	2747.7	2987.3	6.1300
400	0.04739	2892.8	3177.2	6.5407	0.03993	2878.6	3158.1	6.4477	0.03432	2863.8	3138.3	6.3633
450	0.05214	2988.9	3301.8	6.7192	0.04416	2977.9	3287.0	6.6326	0.03817	2966.7	3272.0	6.5550
500	0.05665	3082.2	3422.1	6.8802	0.04814	3073.3	3410.3	6.7974	0.04175	3064.3	3398.3	6.7239
550	0.06101	3174.6	3540.6	7.0287	0.05195	3167.2	3530.9	6.9486	0.04516	3159.8	3521.0	6.8778
600	0.06525	3266.9	3658.4	7.1676	0.05565	3260.7	3650.3	7.0894	0.04845	3254.4	3642.0	7.0205
700	0.07352	3453.2	3894.3	7.4234	0.06283	3448.6	3888.4	7.3476	0.05481	3444.0	3882.5	7.2812
800	0.08160	3643.1	4132.7	7.6566	0.06981	3639.6	4128.3	7.5822	0.06097	3636.1	4123.8	7.5173
900	0.08958	3837.8	4375.3	7.8727	0.07669	3835.0	4371.8	7.7991	0.06702	3832.1	4368.3	7.7350
1000	0.09749	4037.8	4622.7	8.0751	0.08350	4035.3	4619.8	8.0020	0.07301	4032.8	4616.9	7.9384
1100	0.10536	4243.3	4875.4	8.2661	0.09027	4240.9	4872.8	8.1933	0.07896	4238.6	4870.3	8.1299
1200	0.11321	4454.0	5133.3	8.4473	0.09703	4451.7	5130.9	8.3747	0.08489	4449.4	5128.5	8.3115
1300	0.12106	4669.6	5396.0	8.6199	0.10377	4667.3	5393.7	8.5472	0.09080	4665.0	5391.5	8.4842
	P = 9.00 MPa (303.40)				P = 10.00 MPa (311.06)				P = 12.50 MPa (327.89)			
Sat.	0.02048	2557.8	2742.1	5.6771	0.01803	2544.4	2724.7	5.6140	0.01350	2505.1	2673.8	5.4623
325	0.02327	2646.5	2855.9	5.8711	0.01986	2610.4	2809.0	5.7568	—	—	—	—
350	0.02580	2724.4	2956.5	6.0361	0.02242	2699.2	2923.4	5.9442	0.01613	2624.6	2826.2	5.7117
400	0.02993	2848.4	3117.8	6.2853	0.02641	2832.4	3096.5	6.2119	0.02000	2789.3	3039.3	6.0416
450	0.03350	2955.1	3256.6	6.4843	0.02975	2943.3	3240.8	6.4189	0.02299	3912.4	3199.8	6.2718
500	0.03677	3055.1	3386.1	6.6575	0.03279	3045.8	3373.6	6.5965	0.02560	3021.7	3341.7	6.4617
550	0.03987	3152.2	3511.0	6.8141	0.03564	3144.5	3500.9	6.7561	0.02801	3124.9	3475.1	6.6289
600	0.04285	3248.1	3633.7	6.9588	0.03837	3241.7	3625.3	6.9028	0.03029	3225.4	3604.0	6.7810
650	0.04574	3343.7	3755.3	7.0943	0.04101	3338.2	3748.3	7.0397	0.03248	3324.4	3730.4	6.9218
700	0.04857	3439.4	3876.5	7.2221	0.04358	3434.7	3870.5	7.1687	0.03460	3422.9	3855.4	7.0536

(Contd.)

Table A.1.3 *(Contd.)*

	P = 9.00 MPa (303.40)				*P* = 10.00 MPa (311.06)				*P* = 12.50 MPa (327.89)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
800	0.05409	3632.5	4199.4	7.4597	0.04859	3629.0	4114.9	7.4077	0.03869	3620.0	4103.7	7.2965
900	0.5950	3829.2	4364.7	7.6782	0.05349	3826.3	4361.2	7.6272	0.04267	3819.1	4352.5	7.5181
1000	0.06485	4030.3	4613.9	7.8821	0.05832	4027.8	4611.0	7.8315	0.04658	4021.6	4603.8	7.7237
1100	0.07016	4236.3	4867.7	8.0739	0.06312	4234.0	4865.1	8.0236	0.05045	4228.2	4858.8	7.9165
1200	0.07544	4447.2	5126.2	8.2556	0.06789	4444.9	5123.8	8.2054	0.05430	4439.3	5118.0	8.0987
1300	0.08072	4662.7	5389.2	8.4283	0.07265	4660.4	5387.0	8.3783	0.05813	4654.8	5381.4	8.2717
	P = 15 MPa (342.24)				*P* = 17.5 MPa (354.75)				*P* = 20 MPa (365.81)			
Sat.	.010338	2455.4	2610.5	5.3097	.0079204	2390.2	2528.8	5.1418	.0058342	2293.1	2409.7	4.9269
350	.011470	2520.4	2692.4	5.4420	.0017139	1632.0	1662.0	3.7612	.0016640	1612.3	1645.6	3.7280
400	.015649	2740.7	2975.4	5.8810	.0124477	2685.0	2902.8	5.7212	.0099423	2619.2	2818.1	5.5539
450	.018446	2879.5	3156.2	6.1403	.0151740	2844.2	3109.7	6.0182	.0126953	2806.2	3060.1	5.9016
500	.020800	2996.5	3308.5	6.3442	.0173585	2970.3	3274.0	6.2382	.0147683	2942.8	3238.2	6.1400
550	.022927	3104.7	3448.6	6.5198	.0192877	3083.8	3421.4	6.4229	.0165553	3062.3	3393.5	6.3347
600	.024911	3208.6	3582.3	6.6775	.0210640	3191.5	3560.1	6.5866	.0181781	3174.0	3537.6	6.5048
650	.026797	3310.4	3712.3	6.8223	.0227372	3296.0	3693.9	6.7356	.0196929	3281.5	3675.3	6.6582
700	.028612	3410.9	3840.1	6.9572	.0243365	3398.8	3824.7	6.8736	.0211311	3386.5	3809.1	6.7993
800	.032096	3611.0	4092.4	7.2040	.0273849	3601.9	4081.1	7.1245	.0238532	3592.7	4069.8	7.0544
900	.035457	3811.9	4343.8	7.4279	.0303071	3804.7	4335.1	7.3507	.0264463	3797.4	4326.4	7.2830
1000	.038748	4015.4	4596.6	7.6347	.0331580	4009.3	4589.5	7.5588	.0289666	4003.1	4582.5	7.4925
1100	.042001	4222.6	4852.6	7.8282	.0359695	4216.9	4846.4	7.7530	.0314471	4211.3	4840.2	7.6874
1200	.045233	4433.8	5112.3	8.0108	.0387605	4428.3	5106.6	7.9359	.0339071	4422.8	5101.0	7.8706
1300	.048455	4649.1	5375.9	8.1839	.0415417	4643.5	5370.5	8.1093	.0363574	4638.0	5365.1	8.0441

(Contd.)

Table A.1.3 *(Contd.)*

	P =25 *MPa*				*P* = 30 *MPa*				*P* = 35 *MPa*			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
375	.001973	1798.6	1847.9	4.0319	.001789	1737.8	1791.4	3.9303	.001700	1702.9	1762.4	3.8721
400	.006004	2430.1	2580.2	5.1418	.002790	2067.3	2151.0	4.4728	.002100	1914.0	1987.5	4.2124
425	.007882	2609.2	2806.3	5.4722	.005304	2455.1	2614.2	5.1503	.003428	2253.4	2373.4	4.7747
450	.009162	2720.7	2949.7	5.6743	.006735	2619.3	2821.4	5.4423	.004962	2498.7	2672.4	5.1962
500	.011124	2884.3	3162.4	5.9592	.008679	2820.7	3081.0	5.7904	.006927	2751.9	2994.3	5.6281
550	.012724	3017.5	3335.6	6.1764	.010168	2970.3	3275.4	6.0342	.008345	2920.9	3213.0	5.9025
600	.014138	3137.9	3491.4	6.3602	.011446	3100.5	3443.9	6.2330	.009527	3062.0	3395.5	6.1178
650	.015433	3251.6	3637.5	6.5229	.012596	3221.0	3598.9	6.4057	.010575	3189.8	3559.9	6.3010
700	.016647	3361.4	3777.6	6.6707	.013661	3335.8	3745.7	6.5606	.011533	3309.9	3713.5	6.4631
800	.018913	3574.3	4047.1	6.9345	.015623	3555.6	4024.3	6.8332	.013278	3536.8	4001.5	6.7450
900	.021045	3783.0	4309.1	7.1679	.017448	3768.5	4291.9	7.0717	.014883	3754.0	4274.9	6.9886
1000	.023102	3990.9	4568.5	7.3801	.019196	3978.8	4554.7	7.2867	.016410	3966.7	4541.1	7.2063
1100	.025119	4200.2	4828.2	7.5765	.020903	4189.2	4816.3	7.4845	.017895	4178.3	4804.6	7.4056
1200	.027115	4412.0	5089.9	7.7604	.022589	4410.3	5079.0	7.6691	.019360	4390.7	5068.4	7.5910
1300	.029101	4626.9	5354.4	7.9342	.024266	4616.0	5344.0	7.8432	.020815	4605.1	5333.6	7.7652
	P = 40 *MPa*				*P* = 50 *MPa*				*P* = 60 *MPa*			
375	.0016406	1677.1	1742.7	3.8289	.0015593	1638.6	1716.5	3.7638	.0015027	1609.3	1699.5	3.1740
400	.0019077	1854.5	1930.8	4.1134	.0017309	1788.0	1874.6	4.0030	.0016335	1745.3	1843.4	3.9317
425	.0025319	2096.8	2198.1	4.5028	.0020071	1959.6	2060.0	4.2733	.0018165	1892.7	2001.7	4.1625
450	.0036931	2365.1	2512.8	4.9459	.0024862	2159.6	2283.9	4.5883	.0020850	2053.9	2179.0	4.4119
500	.0056225	2678.4	2903.3	5.4699	.0038924	2525.5	2720.1	5.1725	.0029557	2390.5	2567.9	4.9320
600	.0080943	3022.6	3346.4	6.0113	.0061123	2942.0	3247.6	5.8177	.0048345	2861.1	3151.2	5.6451
650	.0090636	3158.0	3520.6	6.2054	.0069657	3093.6	3441.8	6.0342	.0055953	3028.8	3364.6	5.8829
700	.0099415	3283.6	3681.3	6.3750	.0077274	3230.5	3616.9	6.2189	.0062719	3177.3	3553.6	6.0824
800	.0115228	3517.9	3978.8	6.6662	.0090761	3479.8	3933.6	6.5290	.0074588	3441.6	3889.1	6.4110
900	.0129626	3739.4	4257.9	6.9150	.0102831	3710.3	4224.4	6.7882	.0085083	3681.0	4191.5	6.6805
1000	.0143238	3954.6	4527.6	7.1356	.0114113	3930.5	4501.1	7.0146	.0094800	3906.4	4475.2	6.9126
1100	.0156426	4167.4	4793.1	7.3364	.0124966	4145.7	4770.6	7.2183	.0104091	4124.1	4748.6	7.1194
1200	.0169403	4380.1	5057.7	7.5224	.0135606	4359.1	5037.2	7.4058	.0113167	4338.2	5017.2	7.3082
1300	.0182292	4594.3	5323.5	7.6969	.0146159	4572.8	5303.6	7.5807	.0122155	4551.4	5284.3	7.4837

A.2 THERMODYNAMIC PROPERTIES OF REFRIGERANT-12* (DICHLORODIFLUOROMETHANE)

Table A.2.1 *Saturated refrigerant-12*

		Specific Volume		*Enthalpy*			*Entropy*	
Temperature °C t	*Presssure* MPa p	*Sat. Liquid* v_f Cm³/g	*Sat. Vapour* v_g m³/kg	*Sat. Liquid* h_f	*Evap.* h_{fg} kJ/kg	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g kJ/kg K
– 90	0.0028	0.608	4.415545	– 43.243	189.618	146.375	– 0.2084	0.8268
– 85	0.0042	0.612	3.037316	– 38.968	187.608	148.640	– 0.1854	0.8116
– 80	0.0062	0.617	2.038345	– 34.688	185.612	150.924	– 0.1630	0.7979
– 75	0.0088	0.622	1.537651	– 30.401	183.625	153.224	– 0.1411	0.7855
– 70	0.0123	0.627	1.127280	– 26.103	181.640	155.536	– 0.1197	0.7744
– 65	0.0168	0.632	0.841166	– 21.793	179.651	157.857	– 0.0987	0.7643
– 60	0.0226	0.637	0.637910	– 17.469	177.653	160.184	– 0.0782	0.7552
– 55	0.0300	0.642	0.491000	– 13.129	175.641	162.512	– 0.0581	0.7470
– 50	0.0391	0.648	0.383105	– 8.772	173.611	164.840	– 0.0384	0.7396
– 45	0.0504	0.654	0.302682	– 4.396	171.558	167.163	– 0.0190	0.7329
– 40	0.0642	0.659	0.241910	– 0.000	169.479	169.479	– 0.0000	0.7269

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* Repeated from *Fundamentals of Classical Thermodynamics* by G.J. Van Wylen and R. Sonntag. John Wiley, New York 1976, P. 667–673 (with kind permission of the publishers, John Wiley & Sons, Inc, New York).

(Contd.)

Table A.2.1 *(Contd.)*

T	P	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_g
– 35	0.0807	0.666	0.195398	4.416	167.368	171.784	0.0187	0.7214
– 30	0.1004	0.672	0.159375	8.854	165.222	174.076	0.0371	0.7165
– 25	0.1237	0.679	0.131166	13.315	163.037	176.352	0.0552	0.7121
– 20	0.1509	0.685	0.108847	17.800	160.810	178.610	0.0730	0.7082
– 15	0.1826	0.693	0.091018	22.312	158.534	180.846	0.0906	0.7046
– 10	0.2191	0.700	0.076646	26.851	156.207	183.058	0.1079	0.7014
– 5	0.2610	0.708	0.064963	31.420	153.823	185.243	0.1250	0.6986
0	0.3086	0.716	0.055389	36.022	151.376	187.397	0.1418	0.6960
5	0.3626	0.724	0.047485	40.659	148.859	189.518	0.1585	0.6937
10	0.4233	0.733	0.040914	45.337	146.265	191.602	0.1750	0.6916
15	0.4914	0.743	0.035413	50.058	143.586	193.644	0.1914	0.6897
20	0.5673	0.752	0.030780	54.828	140.812	195.641	0.2076	0.6879
25	0.6516	0.763	0.026854	59.653	137.933	197.586	0.2237	0.6863
30	0.7449	0.774	0.023508	64.539	134.936	199.475	0.2397	0.6848
35	0.8477	0.786	0.020641	69.494	131.805	201.299	0.2557	0.6834
40	0.9607	0.798	0.018171	74.527	128.525	203.051	0.2716	0.6820
45	1.0843	0.811	0.016032	79.647	125.074	204.722	0.2875	0.6806
50	1.2193	0.826	0.014170	84.868	121.430	206.298	0.3034	0.6792
55	1.3663	0.841	0.012542	90.201	117.565	207.766	0.3194	0.6777
60	1.5259	0.858	0.01111	95.665	113.443	209.109	0.3355	0.6760
65	1.6988	0.877	0.009847	101.279	109.024	210.303	0.3518	0.6742
70	1.8858	0.897	0.008725	107.067	104.255	211.321	0.3683	0.6721
75	2.0874	0.920	0.007723	113.058	99.068	212.126	0.3851	0.6697
80	2.3046	0.946	0.006821	119.291	93.373	212.665	0.4023	0.6667
85	2.5380	0.976	0.006005	125.818	87.047	212.865	0.4201	0.6631
90	2.7885	1.012	0.005258	132.708	79.907	212.614	0.4385	0.6585
95	3.0569	1.056	0.004563	140.068	71.658	211.726	0.4579	0.6526
100	3.3440	1.113	0.003903	148.076	61.768	209.843	0.4788	0.6444
105	3.6509	1.197	0.003242	157.085	49.014	206.099	0.5023	0.6319
110	3.9784	1.364	0.002462	168.059	28.425	196.484	0.5322	0.6064

Table A.2.2 *Saturated refrigerant-12*

t °C	*v* m^3kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m/kg	*h* kJ/kg	*s* kJ/kg K
		0.05 MPa			*0.10 MPa*			*0.15 MPa*	
– 20.00	0.341857	181.042	0.7912	0.167701	179.861	0.7401			
– 10.00	0.356228	186.757	0.8133	0.175222	185.707	0.7628	0.114716	184.619	0.7318
0.00	0.370508	192.567	0.8350	0.182647	191.628	0.7849	0.119866	190.660	0.7543
10.00	0.384716	198.471	0.8562	0.189994	197.628	0.8064	0.124932	196.762	0.7763
20.00	0.398863	204.469	0.8770	0.197277	203.707	0.8275	0.129930	202.927	0.7977
30.00	0.412959	210.557	0.8974	0.204506	209.866	0.8482	0.134873	209.160	0.8186
40.00	0.427012	216.733	0.9175	0.211691	216.104	0.8684	0.139768	215.463	0.8390
50.00	0.441030	222.997	0.9372	0.218839	222.421	0.8883	0.144625	221.835	0.8591
60.00	0.455017	229.344	0.9565	0.225955	228.815	0.9078	0.149450	228.277	0.8787
70.00	0.458978	235.774	0.9755	0.233044	235.285	0.9269	0.154247	234.789	0.8980
80.00	0.482917	242.282	0.9942	0.240111	241.829	0.9457	0.159020	241.371	0.9169
90.00	0.496838	248.868	1.0126	0.247159	248.446	0.9642	0.163774	248.020	0.9354
		0.20 MPa			*0.25 MPa*			*0.30 MPa*	
00.0	0.088608	189.669	0.7320	0.069752	188.644	0.7139	0.057150	187.583	0.6984
10.00	0.092550	195.878	0.7543	0.073024	194.969	0.7366	0.059984	194.034	0.7216
20.00	0.096418	202.135	0.7760	0.076218	201.322	0.7587	0.062734	200.490	0.7440
30.00	0.100228	208.446	0.7972	0.079350	207.715	0.7801	0.065418	206.969	0.7658
40.00	0.103989	214.814	0.8178	0.082431	214.153	0.8010	0.068049	213.480	0.7869
50.00	0.107710	221.243	0.8381	0.085470	220.642	0.8214	0.070635	220.030	0.8075
60.00	0.111397	227.735	0.8578	0.088474	227.185	0.8413	0.073185	226.627	0.8276
70.00	0.115055	134.291	0.8772	0.091449	233.785	0.8608	0.075750	233.273	0.8473
80.00	0.118690	240.910	0.8962	0.094398	240.443	0.8800	0.078200	239.271	0.8665
90.00	0.122304	247.593	0.9149	0.097327	247.160	0.8987	0.080673	246.723	0.8853
100.0	0.125901	254.339	0.9332	0.100238	253.936	0.9171	0.83127	253.530	0.9038
110.0	0.129483	261.147	0.9512	0.103134	260.770	0.9352	0.085566	260.391	0.9220

(Contd.)

Table A.2.2 *(Contd.)*

t	v	h	s	v	h	s	v	h	s
	0.40 MPa			*0.50 MPa*			*0.60 MPa*		
20.0	0.045836	198.762	0.7199	0.035646	196.935	0.6999			
30.0	0.047971	205.428	0.7423	0.037464	203.814	0.7230	0.030422	202.116	0.7063
40.0	0.050046	212.095	0.7639	0.039214	210.656	0.7452	0.031966	209.154	0.7291
50.0	0.052072	218.779	0.7849	0.040911	217.484	0.7667	0.033450	216.141	0.7511
60.0	0.054059	225.488	0.8054	0.042565	224.315	0.7875	0.034887	223.104	0.7723
70.0	0.056014	232.230	0.8253	0.044184	232.161	0.8077	0.036285	230.062	0.7929
80.0	0.057941	239.012	0.8448	0.045774	238.031	0.8275	0.037653	237.027	0.8129
90.0	0.059846	245.837	0.8638	0.047340	244.932	0.8467	0.038995	244.009	0.8324
100.0	0.061731	252.707	0.8825	0.048886	251.869	0.8656	0.040316	251.016	0.8514
110.0	0.063600	259.624	0.9008	0.050415	258.845	0.8840	0.041619	258.053	0.8700
	0.70 MPa			*0.80 MPa*			*0.90 MPa*		
40.0	0.026761	207.580	0.7148	0.022830	205.924	0.7016	0.019744	204.170	0.6982
50.0	0.028100	214.745	0.7373	0.024068	213.290	0.7248	0.020912	211.765	0.7131
60.0	0.029387	221.854	0.7590	0.025247	220.558	0.7469	0.022012	218.212	0.7358
70.0	0.030632	228.931	0.7799	0.026380	227.766	0.7682	0.023062	226.564	0.7575
80.0	0.031843	235.997	0.8002	0.027477	234.941	0.7888	0.024072	233.856	0.7785
90.0	0.033027	243.066	0.8199	0.028545	242.101	0.8088	0.025051	141.113	0.7987
100.0	0.034189	250.146	0.8392	0.029588	249.260	0.8283	0.026005	248.355	0.8184
110.0	0.035332	257.247	0.8579	0.030612	256.428	0.8472	0.026937	255.593	0.8376
	1.00 MPa			*1.20 MPa*			*1.40 MPa*		
50.0	0.018366	210.162	0.7021	0.014483	206.661	0.6812			
60.0	0.019410	217.810	0.7254	0.015463	214.805	0.7060	0.012579	211.457	0.6876
70.0	0.020397	225.319	0.7476	0.016368	222.687	0.7293	0.013448	219.822	0.7123
80.0	0.021341	232.739	0.7689	0.017221	230.398	0.7514	0.014247	227.891	0.7355
90.0	0.022251	240.101	0.7895	0.018032	237.995	0.7727	0.014997	235.766	0.7575
100.0	0.023133	247.430	0.8094	0.018812	245.518	0.7931	0.015710	243.512	0.7785
110.10	0.023993	254.743	0.8287	0.019567	252.993	0.8129	0.016393	251.170	0.7988
	1.60 MPa			*1.80 MPa*			*2.00 MPa*		
70.0	0.011208	216.650	0.6959	0.009406	213.049	0.6794			
80.0	0.011984	225.177	0.7204	0.010187	222.198	0.7057	0.008704	218.859	0.6909
90.0	0.012698	233.390	0.7433	0.010884	230.835	0.7298	0.009406	228.056	0.7166
100.0	0.013366	241.397	0.7651	0.011526	239.155	0.7524	0.010035	263.760	0.7402
110.0	0.014000	249.264	0.7859	0.012126	2467.264	0.7739	0.010615	245.154	0.7624

A.3 THERMODYNAMIC PROPERTIES OF REFRIGERENT-12 (MONOCHLORODIFLUOROMETHANE)

Table A.3.1 *Saturated refrigerant-22*

		Specific volume (m^3/kg)			Enthalpy (kJ/kg)			Entropy (kJ/kg K)		
Temp. °C	Abs. Press. MPa P	Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapour v_g	Sat. Liquid h_f	Evap. h_{fg}	Vapour h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
70	0.0205	0.000670	0.940268	0.94093	– 30.607	249.425	218.180	– 0.1401	1.2277	1.0876
65	0.0280	0.000676	0.704796	0.705478	– 25.658	246.925	221.267	– 0.1161	1.1862	1.0701
60	0.0375	0.000682	0.536470	0.537152	– 20.652	244.354	223.702	– 0.0924	1.1463	1.0540
55	0.0495	0.000689	0.414138	0.414827	– 15.585	241.703	226.117	– 0.0689	1.1079	1.0390
50	0.0644	0.000695	0.323862	0.324557	– 10.456	238.965	228.509	– 0.0457	1.0708	1.0251
45	0.0827	0.000702	0.256288	0.256990	– 5.262	236.132	230.870	– 0.0227	1.0349	1.0122
40	0.1049	0.000709	0.205036	0.205745	0	233.198	233.197	0	1.0002	1.0002
35	0.1317	0.000717	0.165583	0.166400	5.328	230.156	235.484	0.0225	0.9664	0.9889
30	0.1635	0.000725	0.135120	0.135844	10.725	227.001	237.726	0.0449	0.9335	0.9784
25	0.2010	0.000733	0.111126	0.111859	16.191	223.727	239.918	0.0670	0.9015	0.9685
20	0.2448	0.000741	0.092102	0.092843	21.728	220.327	242.055	0.0890	0.8703	0.9593
15	0.2957	0.000750	0.076876	0.077625	27.334	216.798	244.132	0.1107	0.8398	0.9505
10	0.3543	0.000759	0.064581	0.065340	33.012	213.132	246.144	0.1324	0.8099	0.9422
5	0.4213	0.000768	0.054571	0.055339	38.762	209.323	248.085	0.1538	0.7806	0.9344
0	0.4976	0.000778	0.046357	0.047135	44.586	205.364	249.949	0.1751	0.7518	0.9269
5	0.5838	0.000789	0.039567	0.040356	50.485	201.246	251.731	0.1963	0.7235	0.9197
10	0.6807	0.000800	0.033914	0.034714	56.463	196.960	253.423	0.2173	0.6956	0.9129
15	0.7891	0.000812	0.029176	0.029987	62.523	192.495	255.018	0.2382	0.6680	0.9062
20	0.9099	0.000824	0.025179	0.026003	68.670	187.836	256.506	0.2590	0.6407	0.8997
25	1.0439	0.000838	0.021787	0.022624	74.910	182.968	257.877	0.2797	0.6137	0.8934
30	1.1919	0.000852	0.018890	0.019742	81.250	177.869	259.119	0.3004	0.5867	0.8871

(*Contd.*)

Table A.3.1 *(Contd.)*

		Specific volume (m^3/kg)			Enthalpy (kJ/kg)			Entropy (kJ/kg K)		
Temp. °C	Abs. Press. MPa P	Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapour v_g	Sat. Liquid h_f	Evap. h_{fg}	Vapour h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
35	1.3548	0.000867	0.016401	0.017269	87.700	172.516	260.216	0.3210	0.5598	0.8809
40	1.5335	0.000884	0.014251	0.015135	94.272	166.877	261.149	0.3417	0.5329	0.8746
45	1.7290	0.000902	0.012382	0.013284	100.982	160.914	261.896	0.3624	0.5058	0.8682
50	1.9423	0.000922	0.010747	0.011669	107.851	154.576	262.428	0.3832	0.4783	0.8615
55	2.1744	0.000944	0.009308	0.010252	114.905	147.800	262.705	0.4042	0.4504	0.8546
60	2.4266	0.000969	0.008032	0.009001	122.180	140.497	262.678	0.4255	0.4217	0.8472
65	2.6999	0.000997	0.006890	0.007887	129.729	132.547	262.276	0.4472	0.3920	0.8391
70	2.9959	0.001030	0.005859	0.006889	137.625	123.772	261.397	0.4695	0.3607	0.8302
75	3.3161	0.001069	0.004914	0.005983	145.986	113.902	259.888	0.4927	0.3272	0.8198
80	3.6623	0.001118	0.004031	0.005149	155.011	102.475	257.486	0.5173	0.2902	0.8075
85	4.0368	0.001183	0.003175	0.004358	165.092	88.598	253.690	0.5445	0.2474	0.7918
90	4.4425	0.001282	0.002282	0.003564	177.204	70.037	247.241	0.5767	0.1929	0.7695
95	4.8835	0.001521	0.001030	0.002551	196.359	34.925	231.284	0.6273	0.0949	0.7222
96.006	4.9773	0.001906	0	0.001906	212.546	0	212.546	0.6708	0	0.6708

Table A.3.2 *Superheated refrigerant-22*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
		0.05 MPa			*0.10 MPa*			*0.15 MPa*	
– 40	0.440633	234.724	1.07616	0.216331	233.337	1.00523	—	—	—
– 30	0.460641	240.602	1.10084	0.226754	239.359	1.03052	0.148723	238.078	0.98773
– 20	0.480543	246.586	1.12495	0.237064	245.466	1.05513	0.155851	244.319	1.01288
– 10	0.500357	252.676	1.14855	0.247279	251.665	1.07914	0.162879	250.631	1.03733
0	0.520095	258.874	1.17166	0.257415	257.956	1.10261	0.169823	257.022	1.06116
10	0.539771	265.180	1.19433	0.267485	264.345	1.12558	0.176699	263.496	1.08444
20	0.559393	271.594	1.21659	0.277500	270.831	1.14809	0.183516	270.057	1.10721
30	0.578970	278.115	1.23846	0.287467	277.416	1.17017	0.190284	276.709	1.12952
40	0.598507	284.743	1.25998	0.297394	284.101	1.19187	0.197011	283.452	1.15140
50	0.618011	291.478	1.28114	0.307287	290.887	1.21320	0.203702	290.289	1.17289
60	0.637485	298.319	1.30199	0.317149	297.772	1.23418	0.210362	297.220	1.19402
70	0.656935	305.265	1.32253	0.326986	304.757	1.25484	0.216997	304.246	1.21479
80	0.676362	312.314	1.34278	0.336801	311.842	1.27519	0.223608	311.368	1.23525
90	0.695771	319.465	1.36275	0.346596	319.026	1.29524	0.230200	318.584	1.25540
		0.20 MPa			*0.25 MPa*			*0.30 MPa*	
– 20	0.115203	243.140	0.98184	—	—	—	—	—	—
– 10	0.120647	249.574	1.00676	0.95280	248.492	0.98231	0.078344	247.382	0.96170
0	0.126003	256.069	1.03098	0.099689	255.097	1.00695	0.082128	254.104	0.98677
10	0.131286	262.633	1.05458	0.104022	261.755	1.03089	0.085832	260.861	1.01106

(Contd.)

Table A.3.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* *kJ/kg*	*s.* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*
		0.20 MPa			*0.25 MPa*			*0.30 MPa*	
20	0.136509	269.273	1.07763	0.108292	268.476	1.05421	0.089469	267.667	1.03468
30	0.141681	275.992	1.10016	0.112508	275.267	1.07699	0.093051	274.531	1.05771
40	0.146809	282.796	1.12224	0.116681	282.132	1.09927	0.096588	281.460	1.08019
50	0.151902	289.686	1.14390	0.120815	289.076	1.12109	0.100085	288.460	1.10220
60	0.156963	296.664	1.16516	0.124918	296.102	1.14250	0.103550	295.535	1.12376
70	0.161997	303.731	1.18607	0.128993	303.212	1.16353	0.106986	302.689	1.14491
80	0.167008	310.890	1.20663	0.133044	310.409	1.18420	0.110399	309.924	1.16569
90	0.171999	318.139	1.22687	0.137075	317.692	1.20454	0.113790	317.241	1.18612
100	0.176972	325.480	1.24681	0.141089	325.063	1.22456	0.117164	324.643	1.20623
110	0.181931	332.912	1.26646	0.145086	332.522	1.24428	0.120522	332.129	1.22603
		0.40 MPa			*0.50 MPa*			*0.60 MPa*	
0	0.060131	252.051	0.95359	—	—	—	—	—	—
10	0.063060	259.023	0.97866	0.049355	257.108	0.95223	0.040180	255.109	0.92945
20	0.065915	266.010	1.00291	0.051751	264.295	0.97717	0.042280	262.517	0.95517
30	0.068710	273.029	1.02646	0.054081	271.483	1.00128	0.044307	269.888	0.97989
40	0.071455	280.092	1.04938	0.056358	278.690	1.02467	0.046276	277.250	1.00378
50	0.074160	287.209	1.07175	0.058590	285.930	1.04743	0.048198	284.622	1.02695
60	0.076830	294.386	1.09362	0.060786	293.215	1.06963	0.050081	292.020	1.04950
70	0.079470	301.630	1.11504	0.062951	300.552	1.09133	0.051931	299.456	1.07149
80	0.082085	308.944	1.13605	0.065090	307.949	1.11257	0.053754	306.938	1.09298
90	0.084679	316.332	1.15668	0.067206	315.410	1.13340	0.055553	314.475	1.11403
100	0.087254	323.796	1.17695	0.069303	322.939	1.15386	0.057332	322.071	1.13466
110	0.089813	331.339	1.19690	0.071384	330.539	1.17395	0.059094	329.731	1.15492
120	0.092358	338.961	1.21654	0.073450	338.213	1.19373	0.060842	337.458	1.17482
130	0.094890	346.664	1.23588	0.075503	345.963	1.21319	0.062576	345.255	1.19441

(Contd.)

Table A.3.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
		0.70 MPa			*0.80 MPa*			*0.90 MPa*	
20	0.035487	260.667	0.93565	0.030366	258.737	0.91787	0.026355	256.713	0.90132
30	0.037305	268.240	0.96105	0.032034	266.533	0.94402	0.027915	264.760	0.92831
40	0.039059	275.769	0.98549	0.033632	274.243	0.96905	0.029397	272.670	0.95398
50	0.040763	283.282	1.00910	0.035175	281.907	0.99314	0.030819	280.497	0.97859
60	0.042424	290.800	1.03201	0.036674	289.553	1.01644	0.032193	288.278	1.00230
70	0.044052	298.339	1.05431	0.038136	297.202	1.03906	0.033528	296.042	1.02526
80	0.045650	305.912	1.07606	0.039568	304.868	1.06108	0.034832	303.807	1.04757
90	0.047224	313.527	1.09732	0.040974	312.565	1.08257	0.036108	311.590	1.06930
100	0.048778	321.192	1.11815	0.042359	320.303	1.10359	0.037363	319.401	1.09052
110	0.050313	328.914	1.13856	0.043725	328.087	1.12417	0.038598	327.251	1.11128
120	0.051834	336.696	1.15861	0.045076	335.925	1.14437	0.039817	335.147	1.13162
130	0.053341	344.541	1.17832	0.046413	343.821	1.16420	0.041022	343.094	1.15158
140	0.054836	352.454	1.19770	0.047738	351.778	1.18369	0.042215	351.097	1.17119
150	0.056321	360.435	1.21679	0.049052	359.799	1.20288	0.043398	359.159	1.19047
		1.00 MPa			*1.20 MPa*			*1.40 MPa*	
30	0.024600	262.912	0.91358	—	—	—	—	—	—
40	0.025995	271.042	0.93996	0.020851	267.602	0.91411	0.017120	263.861	0.89010
50	0.027323	279.046	0.96512	0.022051	276.011	0.94055	0.018247	272.766	0.91809
60	0.028601	286.973	0.98928	0.023191	284.263	0.96570	0.019299	281.401	0.94441
70	0.029836	294.859	1.01260	0.024282	292.413	0.98981	0.020295	289.858	0.96942
80	0.031038	302.727	1.03520	0.025336	330.508	1.01305	0.021248	298.202	0.99339
90	0.032213	310.599	1.05718	0.026359	308.570	1.03556	0.022167	306.473	1.01649
100	0.033364	318.488	1.07861	0.027357	316.623	1.05744	0.023058	314.703	1.03884

(Contd.)

Table A.3.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/k g	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
	1.00 MPa			*1.20 MPa*			*1.40 MPa*		
110	0.034495	326.405	1.09955	0.028334	324.682	1.07875	0.023926	322.916	1.06056
120	0.035609	334.360	1.12004	0.029292	332.762	1.09957	0.024775	331.128	1.08172
130	0.036709	342.360	1.14014	0.030236	340.871	1.11994	0.025608	339.354	1.10238
140	0.037797	350.410	1.15986	0.031166	349.019	1.13990	0.026426	347.603	1.12259
150	0.038873	358.514	1.17924	0.032084	357.210	1.15949	0.027233	355.885	1.14240
160	0.039940	366.677	1.19831	0.032993	365.450	1.17873	0.028029	364.206	1.16183
	1.60 MPa			*1.80 MPa*			*2.00 MPa*		
50	0.015351	269.262	0.89689	0.013052	265.423	0.87625	—	—	—
60	0.016351	278.358	0.92461	0.014028	275.097	0.90573	0.012135	271.563	0.88729
70	0.017284	287.171	0.95068	0.014921	284.331	0.93304	0.013008	281.310	0.91612
80	0.018167	295.797	0.97546	0.015755	293.282	0.95876	0.013811	290.640	0.94292
90	0.019011	304.301	0.99920	0.016546	302.046	0.98323	0.014563	299.697	0.96821
100	0.019825	312.725	1.02209	0.017303	310.683	1.00669	0.015277	308.571	0.99232
110	0.020614	321.103	1.04424	0.018032	319.239	1.02932	0.015960	317.322	1.01546
120	0.021382	329.457	1.06576	0.018738	327.745	1.05123	0.016619	325.991	1.03780
130	0.022133	337.805	1.08673	0.019427	336.224	1.07253	0.017258	334.610	1.05944
140	0.022869	346.162	1.10721	0.020099	344.695	1.09329	0.017881	343.201	1.08049
150	0.023592	354.540	1.12724	0.020759	353.172	1.11356	0.018490	351.783	1.10102
160	0.024305	362.945	1.14688	0.021407	361.666	1.13340	0.019087	360.369	1.12107
170	0.025008	371.386	1.16614	0.022045	370.186	1.15284	0.019673	368.970	1.14070
180	0.25703	379.869	1.18507	0.022675	378.738	1.17193	0.020251	377.595	1.15995
	2.50 MPa			*3.00 MPa*			*3.50 MPa*		
70	0.009459	272.677	0.87476	—	—	—	—	—	—
80	0.010243	283.332	0.90537	0.007747	274.530	0.86780	0.005765	262.739	0.82489
90	0.010948	293.338	0.93332	0.008465	286.042	0.89995	0.006597	277.268	0.86548

(Contd.)

Table A.3.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
		2.50 MPa			*3.00 MPa*			*3.50 MPa*	
100	0.011598	302.935	0.95939	0.009098	296.663	0.92881	0.007257	289.504	0.89872
110	0.012208	312.261	0.98405	0.009674	306.744	0.95547	0.007829	300.640	0.92818
120	0.012788	321.400	1.00760	0.010211	316.470	0.98053	0.008346	311.129	0.95520
130	0.013343	330.412	1.03023	0.010717	325.955	1.00435	0.008825	321.196	0.98047
140	0.013880	339.336	1.05210	0.011200	335.270	1.02718	0.009276	330.976	1.00445
150	0.014400	348.205	1.07331	0.011665	344.467	1.04918	0.009704	340.554	1.02736
160	0.014907	357.040	1.09395	0.012114	353.584	1.07047	0.010114	349.989	1.04940
170	0.015402	365.860	1.11408	0.012550	362.647	1.09116	0.010510	359.324	1.07071
180	0.015887	374.679	1.13376	0.012976	371.679	1.11131	0.010894	368.590	1.09138
190	0.016364	383.508	1.15303	0.013392	380.695	1.13099	0.011268	377.810	1.11151
200	0.016834	392.354	1.17912	0.013801	389.708	1.15024	0.011634	387.004	1.13115
		4.00 MPa			*5.00 MPa*			*6.00 MPa*	
90	0.005037	265.629	0.82544	—	—	—	—	—	—
100	0.005804	280.997	0.86721	0.003334	253.042	0.78005	—	—	—
110	0.006405	293.748	0.90094	0.004255	275.919	0.84064	0.002432	243.278	0.74674
120	0.006924	305.273	0.93064	0.004851	291.362	0.88045	0.003333	272.385	0.82185
130	0.007391	316.080	0.95778	0.005335	304.469	0.91337	0.003899	290.253	0.86675
140	0.007822	326.422	0.98312	0.005757	316.379	0.94256	0.004345	304.757	0.90230
150	0.008226	336.446	1.00710	0.006139	327.563	0.96931	0.004728	317.633	0.93310
160	0.008610	346.246	1.02999	0.006493	338.266	0.99431	0.005071	329.553	0.96094
170	0.008978	355.885	1.05199	0.006826	348.633	1.01797	0.005386	340.849	0.98673
180	0.009332	365.409	1.07324	0.007142	358.760	1.04057	0.005680	351.715	1.01098
190	0.009675	374.853	1.09386	0.007444	368.713	1.06230	0.005958	362.271	1.03402
200	0.010009	384.240	1.11391	0.007735	378.537	1.08328	0.006222	372.602	1.05609
210	0.010335	393.593	1.13347	0.008018	388.268	1.10363	0.006477	382.764	1.07734
220	0.010654	402.925	1.15259	0.008292	397.932	1.12343	0.006722	392.801	1.09790

(Contd.)

A.4 THERMODYNAMIC PROPERTIES OF REFRIGERANT-134A (1,1,1,1–TETRAFLUOROETHANE)

Table A.4.1 *Saturated R-134a*

		Specific Volume, m^3/kg			*Enthalpy, kJ/kg*			*Entropy, kJ/kg K*		
Temp. °C	*Abs. Press. MPa* P	*Sat. Liquid* v_f	*Evap.* v_{fg}	*Sat. Vapour* v_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Evap.* s_{fg}	*Sat. Vapour* s_g
– 33	0.0737	0.000718	0.25574	0.25646	157.417	220.491	377.908	0.8346	0.9181	1.7528
– 30	0.0851	0.000722	0.22330	0.22402	161.118	218.683	379.802	0.8499	0.8994	1.7493
26.25	0.1013	0.000728	0.18947	0.19020	165.802	216.360	382.162	0.8690	0.8763	1.7453
– 25	0.0173	0.000730	0.17956	0.18029	167.381	215.569	382.950	0.8754	0.8687	1.7441
– 20	0.1337	0.000738	0.14575	0.14649	173.744	212.340	386.083	0.9007	0.8388	1.7395
– 15	0.1650	0.000746	0.11932	0.12007	180.193	209.004	389.197	0.9258	0.8096	1.7354
– 10	0.2017	0.000755	0.098454	0.099209	186.721	205.564	392.285	0.9507	0.7812	1.7319
– 5	0.2445	0.000764	0.081812	0.082576	193.324	202.016	395.340	0.9755	0.7534	1.7288
0	0.2940	0.000773	0.068420	0.069193	200.000	198.356	398.356	1.0000	0.7262	1.7262
5	0.3509	0.000783	0.057551	0.058334	206.751	194.572	401.323	1.0243	0.6995	1.7239
10	0.4158	0.000794	0.048658	0.049451	213.580	190.652	404.233	1.0485	0.6733	1.7218
15	0.4895	0.000805	0.041326	0.042131	220.492	186.582	407.075	1.0725	0.6475	1.7200
20	0.5728	0.000817	0.035238	0.036055	227.493	182.345	409.838	1.0963	0.6220	1.7183
25	0.6663	0.000829	0.030148	0.030977	234.590	177.920	412.509	1.1201	0.5967	1.7168
30	0.7710	0.000843	0.025865	0.026707	241.790	173.285	415.075	1.1437	0.5716	1.7153
35	0.8876	0.000857	0.022237	0.023094	249.103	168.415	417.518	1.1673	0.5465	1.7139
40	1.0171	0.000873	0.019147	0.020020	256.539	163.282	419.821	1.1909	0.5214	1.7123
45	1.1602	0.000890	0.016499	0.017389	264.110	157.852	421.962	1.2145	0.4962	1.7106
50	1.3180	0.000908	0.014217	0.015124	271.830	152.085	423.915	1.2381	0.4706	1.7088
55	1.4915	0.000928	0.012237	0.013166	279.718	145.933	425.650	1.2619	0.4447	1.7066
60	1.6818	0.000951	0.010511	0.011462	287.794	139.336	427.130	1.2857	0.4182	1.7040

(Contd.)

Table A.4.1 *(Contd.)*

		Specific Volume, m³/kg			*Enthalpy, kJ/kg*			*Entropy, kJ/kg K*		
Temp. °C	*Abs. Press. MPa* P	*Sat. Liquid* v_f	*Evap.* v_{fg}	*Sat. Vapour* v_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Evap.* s_{fg}	*Sat. Vapour* s_g
65	1.8898	0.000976	0.008995	0.009970	296.088	132.216	428.305	1.3099	0.3910	1.7009
70	2.1169	0.001005	0.007653	0.008657	304.642	124.468	429.110	1.3343	0.3627	1.6970
75	2.3644	0.001038	0.006453	0.007491	313.513	115.939	429.451	1.3592	0.3330	1.6923
80	2.6337	0.001078	0.005368	0.006446	322.794	106.395	429.189	1.3849	0.3013	1.6862
85	2.9265	0.001128	0.004367	0.005495	332.644	95.440	428.084	1.4117	0.2665	1.6782
90	3.2448	0.001995	0.003412	0.004606	343.380	82.295	425.676	1.4404	0.2266	1.6670
95	3.5914	0.001297	0.002432	0.003729	355.834	64.984	420.818	1.4733	0.1765	1.6498
101.15	4.0640	0.001969	0	0.001969	390.977	0	390.977	1.5658	0	1.5658

Table A.4.2 *Superheated R-134a*

Temp. °C	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*
		0.10 MPa			*0.15 MPa*			*0.20 MPa*	
– 25	0.19400	383.212	1.75058	—	—	—	—	—	—
– 20	0.19860	387.215	1.76655	—	—	—	—	—	—
– 10	0.20765	395.270	1.79775	0.13603	393.839	1.76058	0.10013	392.338	1.73276
0	0.21652	403.413	1.82813	0.14222	402.187	1.79171	0.10501	400.911	1.76474
10	0.22527	411.668	1.85780	0.14828	410.602	1.82197	0.10974	409.500	1.79562
20	0.23393	420.048	1.88689	0.15424	419.111	1.85150	0.11436	418.145	1.82563
30	0.24250	428.564	1.91545	0.16011	427.730	1.88041	0.11889	426.875	1.85491
40	0.25102	437.223	1.94355	0.16592	436.473	1.90879	0.12335	435.708	1.88357
50	0.25948	446.029	1.97123	0.17168	445.350	1.93669	0.12776	444.658	1.91171
60	0.26791	454.986	1.99853	0.17740	454.366	1.96416	0.13213	453.735	1.93937
70	0.27631	464.096	2.02547	0.18308	463.525	1.99125	0.13646	462.946	1.96661
80	0.28468	473.359	2.05208	0.18874	472.831	2.01798	0.14076	472.296	1.99346
90	0.29303	482.777	2.07837	0.19437	482.285	2.04438	0.14504	481.788	2.01997
100	0.30136	492.349	2.10437	0.19999	491.888	2.07046	0.14930	491.424	2.04614
		0.25 MPa			*0.30 MPa*			*0.40 MPa*	
0	0.082637	399.579	1.74284	—	—	—	—	—	—
10	0.086584	408.357	1.77440	0.071110	407.171	1.75637	0.051681	404.651	1.72611
20	0.090408	417.151	1.80492	0.074415	416.124	1.78744	0.054362	413.965	1.75844
30	0.090139	425.997	1.83460	0.077620	425.096	1.81754	0.056926	423.216	1.78947
40	0.097798	434.925	1.86357	0.080748	434.124	1.84684	0.059402	432.465	1.81949
50	0.101401	443.953	1.89195	0.083816	443.234	1.87547	0.061812	441.751	1.84868
60	0.104958	453.094	1.91980	0.086838	452.442	1.90354	0.064169	451.104	1.87718
70	0.108480	462.359	1.94720	0.089821	461.763	1.93110	0.066484	460.545	1.90510
80	0.111972	471.754	1.97419	0.092774	471.206	1.95823	0.068767	470.088	1.93252
90	0.115440	481.285	2.00080	0.095702	480.777	1.98495	0.071022	479.745	1.95948
100	0.118888	490.955	2.02707	0.098609	490.482	2.01131	0.073254	489.523	1.98604
110	0.122318	500.766	2.05302	0.101498	500.324	2.03734	0.075468	499.428	2.01223
120	0.125734	510.720	2.07866	0.104371	510.304	2.06305	0.077665	509.464	2.03809

(Contd.)

Table A.4.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
	0.50 MPa			*0.60 MPa*			*0.70 MPa*		
20	0.042256	411.645	1.73420	—	—	—	—	—	—
30	0.044457	421.221	1.76632	0.036094	419.093	1.74610	0.030069	416.809	1.72770
40	0.046557	430.720	1.79715	0.037958	428.881	1.77786	0.031781	426.933	1.76056
50	0.048581	440.205	1.82696	0.039735	438.589	1.80838	0.033392	436.895	1.79187
60	0.050547	449.718	1.85596	0.041447	448.279	1.83791	0.034929	446.782	1.82201
70	0.052467	459.290	1.88426	0.043108	457.994	1.86664	0.036410	456.655	1.85121
80	0.054351	468.942	1.91199	0.044730	467.764	1.89471	0.037848	466.554	1.87964
90	0.056205	478.690	1.93921	0.046319	477.611	1.92220	0.039251	476.507	1.90743
100	0.058035	488.546	1.96598	0.047883	487.550	1.94920	0.040627	486.535	1.93467
110	0.059845	498.518	1.99235	0.049426	497.594	1.97576	0.041980	496.654	1.96143
120	0.061639	508.613	2.01836	0.050951	507.750	2.00193	0.043314	506.875	1.98777
130	0.063418	518.835	2.04403	0.052461	518.026	2.02774	0.044633	517.207	2.01372
140	0.065184	529.187	2.06940	0.053958	528.425	2.05322	0.045938	527.656	2.03932
	0.80 MPa			*0.90 MPa*			*1.00 MPa*		
40	0.027113	424.860	1.74457	0.023446	422.642	1.72943	0.020473	420.249	1.71479
50	0.028611	435.114	1.77680	0.024868	433.235	1.76273	0.021849	431.243	1.74936
60	0.030024	445.223	1.80761	0.026192	443.595	1.79431	0.023110	441.890	1.78181
70	0.031375	455.270	1.83732	0.027447	453.835	1.82459	0.024293	452.345	1.81273
80	0.032678	465.308	1.86616	0.028649	464.025	1.85387	0.025417	462.703	1.84248
90	0.033944	475.375	1.89427	0.029810	474.216	1.88232	0.026497	473.027	1.87131
100	0.035180	485.499	1.92177	0.030940	484.441	1.91010	0.027543	483.361	1.89938
110	0.036392	495.698	1.94874	0.032043	494.726	1.93730	0.028561	493.736	1.92682
120	0.037584	505.988	1.97525	0.033126	505.088	1.96399	0.029556	504.175	1.95371
130	0.038760	516.379	2.00135	0.034190	515.542	1.99025	0.030533	514.694	1.98013
140	0.039921	526.880	2.02708	0.035241	526.096	2.01611	0.031495	525.305	2.00613
150	0.041071	537.496	2.05247	0.036278	536.760	2.04161	0.032444	536.017	2.03175

(Contd.)

Table A.4.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
		1.20 MPa			*1.40 MPa*			*1.60 MPa*	
50	0.017243	426.485	1.72373	—	—	—	—	—	—
60	0.018439	438.210	1.75837	0.015032	434.079	1.73597	0.012392	429.322	1.71349
70	0.019530	449.179	1.79081	0.016083	445.720	1.77040	0.013449	441.888	1.75066
80	0.020548	459.925	1.82168	0.017040	456.944	1.80265	0.014378	453.722	1.78466
90	0.021512	470.551	1.85135	0.017931	467.931	1.83333	0.015225	465.145	1.81656
100	0.022436	481.128	1.88009	0.018775	478.790	1.86282	0.016015	476.333	1.84695
110	0.023329	491.702	1.90805	0.019583	489.589	1.89139	0.016763	487.390	1.87619
120	0.024197	502.307	1.93537	0.020362	500.379	1.91918	0.017479	498.387	1.90452
130	0.025044	512.965	1.96214	0.021118	511.192	1.94634	0.018169	509.371	1.93211
140	0.025874	523.697	1.98844	0.021856	522.054	1.97296	0.018840	520.376	1.95908
150	0.026691	534.514	2.01431	0.022579	532.984	1.99910	0.019493	531.427	1.98551
160	0.027495	545.426	2.03980	0.023289	543.994	2.02481	0.020133	542.542	2.01147
170	0.028289	556.443	2.06494	0.023988	555.097	2.05015	0.020761	553.735	2.03702
		1.80 MPa			*2.0 MPa*			*2.50 MPa*	
70	0.011341	437.562	1.73085	0.009581	432.531	1.71011	—	—	—
80	0.012273	450.202	1.76717	0.010550	446.304	1.74968	0.007221	433.797	1.70180
90	0.013099	462.164	1.80057	0.011374	458.951	1.78500	0.008157	449.499	1.74567
100	0.013854	473.741	1.83202	0.012111	470.996	1.81772	0.008907	463.279	1.78311
110	0.014560	485.095	1.86205	0.012789	482.693	1.84866	0.009558	476.129	1.81709
120	0.015230	496.325	1.89098	0.013424	494.187	1.87827	0.010148	488.457	1.84886
130	0.015871	507.498	1.91905	0.014028	505.569	1.90686	0.010694	500.474	1.87904
140	0.016490	518.659	1.94639	0.014608	516.900	1.93463	0.011208	512.307	1.90804
150	0.017091	529.841	1.97314	0.015168	528.224	1.96171	0.011698	524.037	1.93609
160	0.017677	541.068	1.99936	0.015712	539.571	1.98821	0.012169	535.722	1.96338
170	0.018251	552.357	2.02513	0.016242	550.963	2.01421	0.012624	547.399	1.99004
180	0.018814	563.724	2.05049	0.016762	562.418	2.03977	0.013066	559.098	2.01614
190	0.019369	575.177	2.07549	0.017272	573.950	2.06494	0.013498	570.841	2.04177

Table A.4.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
		3.0 MPa			*3.50 MPa*			*4.0 MPa*	
90	0.005755	436.193	1.69950	—	—	—	—	—	—
100	0.006653	453.731	1.74717	0.004839	440.433	1.70386	—	—	—
110	0.007339	468.500	1.78623	0.005667	459.211	1.75355	0.004277	446.844	1.71840
120	0.007924	482.043	1.82113	0.006289	474.697	1.79346	0.005005	465.987	1.76415
130	0.008446	494.915	1.85347	0.006813	488.771	1.82881	0.005559	481.865	1.80404
140	0.008926	507.388	1.88403	0.007279	502.079	1.86142	0.006027	496.295	1.83940
150	0.009375	519.618	1.91328	0.007706	514.928	1.89216	0.006444	509.925	1.87200
160	0.009801	531.704	1.94151	0.008103	527.496	1.92151	0.006825	523.072	1.90271
170	0.010208	543.713	1.96892	0.008480	539.890	1.94980	0.007181	535.917	1.93203
180	0.010601	555.690	1.99565	0.008839	552.185	1.97724	0.007517	548.573	1.96028
190	0.010982	567.670	2.02180	0.009185	564.430	2.00397	0.007837	561.117	1.98766
200	0.011353	579.678	2.04745	0.009519	576.665	2.03010	0.008145	573.601	2.01432

A.5 THERMODYNAMIC PROPERTIES OF AMMONIA

Table A.5.1 *Saturated ammonia table*

		Specific volume (m^3/kg)			*Enthalpy (kJ/kg)*			*Entropy (kJ/kg K)*	
T (°C)	*P* (kPa)	*Sat. liquid* *vt*	*t*	*Sat. vapour,* *vg*	*Sat. liquid,* *ht*	*Evap.,* *htg*	*Sat. vapour,* *hg*	*Sat. liquid,* *st*	*Sat. vapour* *sg*
– 50	40.88	0.001424		2.6254	– 44.3	1416.7	1372.4	– 0.1942	6.1561
– 48	45.96	0.001429		2.3533	– 35.5	1411.3	1375.8	– 0.1547	6.1149
– 46	51.55	0.001434		2.1140	– 26.6	1405.8	1379.2	– 0.1156	6.0746
– 44	57.69	0.001439		1.9032	– 17.8	1400.3	1382.5	– 0.0768	6.0352
– 42	64.42	0.001444		1.7170	– 8.9	1394.7	1385.8	– 0.0382	5.9967
– 40	71.77	0.001449		1.5521	0.0	1389.0	1389.0	0.0000	5.9589
– 38	79.80	0.001454		1.4058	8.9	1383.3	1392.2	0.0380	5.9220
– 36	88.54	0.001460		1.2757	17.8	1377.6	1395.4	0.0757	5.8858
– 34	98.05	0.001465		1.1597	26.8	1371.8	1398.5	0.1132	5.8504
– 32	108.37	0.001470		1.0562	35.7	1365.9	1401.6	0.1504	5.8156
– 30	119.55	0.001476		0.9635	44.7	1360.0	1404.6	0.1873	5.7815
– 28	131.64	0.001481		0.8805	53.6	1354.0	1407.6	0.2240	5.7481
– 26	144.70	0.001487		0.8059	62.6	1347.9	1410.5	0.2605	5.7153
– 24	158.78	0.001492		0.7388	71.6	1341.8	1413.4	0.2967	5.6831
– 22	173.93	0.001498		0.6783	80.7	1335.6	1416.2	0.3327	5.6515
– 20	190.22	0.001504		0.6237	89.7	1329.3	1419.0	0.3684	5.6205
– 18	207.71	0.001510		0.5743	98.8	1322.9	1421.7	0.4040	5.5900

(Contd.)

Table A.5.1 *(Contd.)*

T (°C)	P (kPa)	Specific volume (m^3/kg)			Enthalpy (kJ/kg)			Entropy (kJ/kg K)	
		Sat. liquid vt	t	Sat. vapour, vg	Sat. liquid, ht	Evap., htg	Sat. vapour, hg	Sat. liquid, st	Sat. vapour sg
– 16	226.45	0.001515		0.5296	107.8	1316.5	1424.4	0.4393	5.5600
– 14	246.51	0.001521		0.4889	116.9	1310.0	1427.0	0.4744	5.5305
– 12	267.95	0.001528		0.4520	126.0	1303.5	1429.5	0.5093	5.5015
– 10	290.95	0.001534		0.4185	135.2	1296.8	1432.0	0.5440	5.4730
– 8	315.25	0.001540		0.3878	144.3	1290.1	1434.4	0.5785	5.4449
– 6	341.25	0.001546		0.3599	153.5	1283.3	1436.8	0.6128	5.4173
– 4	368.90	0.001553		0.3343	162.7	1276.4	1439.1	0.6469	5.3901
– 2	398.27	0.001559		0.3109	171.9	1269.4	1441.3	0.6808	5.3633
0	429.44	0.001566		0.2895	181.1	1262.4	1443.5	0.7145	5.3369
2	462.49	0.001573		0.2698	190.4	1255.2	1445.6	0.7481	5.3108
4	497.49	0.001580		0.2517	199.6	1243.0	1447.6	0.7815	5.2852
6	531.51	0.001587		0.2351	208.9	1240.6	1449.6	0.8148	5.2599
8	573.64	0.001594		0.2198	218.3	1233.2	1451.5	0.8479	5.2350
10	614.95	0.001601		0.2056	227.6	1225.7	1453.3	0.8808	5.2104
12	658.52	0.001608		0.1926	237.0	1218.1	1455.1	0.9136	5.1861
14	704.44	0.001616		0.1805	246.4	1210.4	1456.8	0.9463	5.1621
16	752.79	0.001623		0.1693	255.9	1202.6	1458.5	0.9788	5.1385
18	803.66	0.001631		0.1590	265.4	1194.7	1460.0	1.0112	5.1151
20	857.12	0.001639		0.1494	274.9	1186.7	1461.5	1.0434	5.0920
22	913.27	0.001647		0.1405	284.4	1178.5	1462.9	1.0755	5.0692
24	972.19	0.001655		0.1322	294.0	1170.3	1464.3	1.1075	5.0467
26	1033.97	0.001663		0.1245	303.6	1162.0	1465.6	1.1394	5.0244
28	1098.71	0.001671		0.1173	313.2	1153.6	1466.8	1.1711	5.0023
30	1166.49	0.001680		0.1106	322.9	1145.0	1467.9	1.2028	4.9805
32	1237.41	0.001689		0.1044	332.6	1136.4	1469.0	1.2343	4.9589
34	1311.55	0.001698		0.0986	342.3	1127.6	1469.9	1.2656	4.9374

(Contd.)

Table A.5.1 *(Contd.)*

T (°C)	P (kPa)	Specific volume (m^3/kg)			Enthalpy (kJ/kg)			Entropy (kJ/kg K)	
		Sat. liquid vt	t	Sat. vapour, vg	Sat. liquid, ht	Evap., htg	Sat. vapour, hg	Sat. liquid, st	Sat. vapour sg
36	1369.03	0.001707		0.0931	352.1	1118.7	1470.8	1.2969	4.9161
38	1469.92	0.001716		0.0880	361.9	1109.7	1471.5	1.3281	4.8950
40	1554.33	0.001726		0.0833	371.7	1100.5	1472.2	1.3591	4.8740
42	1642.35	0.001735		0.0788	381.6	1091.2	1472.8	1.3901	4.8530
44	1734.09	0.001745		0.0746	391.5	1081.7	1473.2	1.4209	4.8322
46	1829.65	0.001756		0.0707	401.5	1072.0	1473.5	1.4518	4.8113
48	1929.13	0.001766		0.0669	411.5	1062.2	1473.7	1.4826	4.7905
50	2032.62	0.001777		0.0635	421.7	1052.0	1473.7	1.5135	4.7696

Table A.5.2 *Superheated ammonia table*

Abs. Press. (kPa) (Sat. temp., °C)		*Temperature (°C)*											
		– 20	*– 10*	*0*	*10*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*100*
	v	2.4474	2.5481	2.6482	2.7479	2.8479	2.9464	3.0453	3.1441	3.2427	3.3413	3.4397	
50	*h*	1435.8	1457.0	1478.1	1499.2	1520.4	1541.7	1563.0	1584.5	1606.1	1627.8	1649.7	
(– 46.54)	*s*	6.3256	6.4077	6.4865	6.5625	6.6360	6.7073	6.7766	6.8441	6.9099	6.9743	7.0372	
	v	1.6233	1.6915	1.7591	1.8263	1.8932	1.9597	2.0261	2.0933	2.1584	2.2244	2.2903	
75	*h*	1433.0	1454.7	1476.1	1497.5	1518.9	1540.3	1561.8	1583.4	1605.1	1626.9	1648.9	
(– 39.18)	*s*	6.1190	6.2028	6.2828	6.3597	6.4339	6.5058	6.5756	6.6434	6.7096	6.7742	6.8373	
	v	1.2110	1.2631	1.3145	1.3654	1.4160	1.4664	1.5165	1.5664	1.6163	1.6659	1.7155	1.8145
100	*h*	1430.1	1452.2	1474.1	1495.7	1517.3	1538.9	1560.5	1582.2	1604.1	1626.0	1648.0	1692.6
(– 33.01)	*s*	5.9695	6.0552	6.1366	6.2144	6.2894	6.3618	6.4321	6.5003	0.5668	6.6316	6.6950	6.8177
	v	0.9635	1.0059	1.0476	1.0889	1.1297	1.1703	1.2107	1.2509	1.2909	1.3309	1.3707	1.4501
125	*h*	1427.2	1449.8	1472.0	1493.9	1515.7	1537.5	1559.3	1581.1	1603.0	1625.0	1647.2	1691.8
(– 29.08)	*s*	5.8512	5.9389	6.0217	6.1006	6.1763	6.2494	6.3201	6.3887	6.4555	6.5206	6.5842	6.7072
	v	0.7984	0.8344	0.8697	0.9045	6.9388	0.9729	1.0068	1.0405	1.0740	1.1074	1.1408	1.2072
150	*h*	1424.1	1447.3	1469.8	1492.1	1514.1	1536.1	1558.0	1580.0	1602.0	1624.1	1646.3	1691.1
(– 25.23)	*s*	5.7526	5.8424	5.9266	6.0066	6.0831	6.1568	6.2280	6.2970	6.3641	6.4295	6.4933	6.6167
	v		0.6199	0.6471	0.6738	0.7001	0.7261	0.7519	0.7774	0.8029	0.8282	0.8533	0.9035
200	*h*		1442.0	1465.5	1448.1	1510.9	1533.2	1555.5	1577.7	1599.9	1622.2	1644.6	1689.6
(– 18.86)	*s*		5.6863	5.7737	5.8559	5.9342	6.0091	6.0813	6.1512	6.2189	6.2849	6.3491	6.4732
	v		0.4910	0.5135	0.5354	0.5568	0.5780	0.5989	0.6196	0.6401	0.6605	0.6809	0.7212
250	*h*		1436.6	1461.0	1484.5	1507.6	1530.3	1552.9	1575.4	1597.8	1620.3	1642.8	1688.2
(– 13.67)	*s*		5.5609	5.6517	5.7465	5.8165	5.8928	5.9661	6.0368	6.1052	6.1717	6.2365	6.3613
	v			0.4243	0.4430	0.4613	0.4792	0.4968	0.5113	0.5316	0.5488	0.5658	0.5997
300	*h*			1456.3	1480.6	1504.2	1527.4	1550.3	1573.0	1595.7	1618.4	1641.1	1686.7
(– 9.23)	*s*			5.5193	5.6366	5.7186	5.7963	5.8707	5.9423	6.0114	6.0785	6.1437	6.2693
	v			0.3605	0.3770	0.3929	0.4086	0.4239	0.4391	0.4541	0.4689	0.4837	0.5129

(Contd.)

Table A.5.2 *(Contd.)*

Abs. Press. (kPa) (Sat. temp., °C)		*Temperature (°C)*											
		– 20	*– 10*	*0*	*10*	*20*	*30*	*40*	*50*	*60*	*70*	*80*	*100*
350	*h*			1451.5	1478.5	1590.7	1521.4	1547.6	1570.7	1593.6	1616.5	1639.3	1685.2
(– 5.35)	*s*			5.4600	5.5502	5.6312	5.7135	5.7800	5.8615	5.9314	5.9990	6.0647	6.1910
	v			0.3125	0.3274	0.3417	0.3556	0.3692	0.3826	0.3959	0.4090	0.4220	0.4478
400	*h*			1446.5	1472.4	1497.2	1521.3	1544.9	1568.3	1591.5	1614.5	1637.6	1683.7
(– 1.89)	*s*			5.3803	5.4735	5.5597	5.6405	5.7173	5.7907	5.8613	5.9296	5.9957	6.1228
	v			0.2752	0.2887	0.3017	0.3143	0.3266	0.3387	0.3506	0.3624	0.3740	0.3971
450	*h*			1441.3	1468.1	1493.6	1518.2	1542.2	1565.9	1589.3	1612.6	1635.8	1682.2
(1.26)	*s*			5.3078	5.4042	5492.6	5.5752	5.6532	5.7275	5.7989	5.8678	5.9345	6.0623
		20	*30*	*40*	*50*	*60*	*70*	*80*	*100*	*120*	*140*	*160*	*180*
	v	0.2698	0.2813	0.2926	0.3036	0.3144	0.3251	0.3357	0.3565	0.3771	0.3975		
500	*h*	1489.9	1515.0	1539.5	1563.4	1587.1	1610.6	1634.0	1680.7	1727.5	1774.7		
(4.14)	*s*	5.4314	5.5157	5.5950	5.6704	5.7425	5.8120	5.8793	6.0079	6.1301	6.2472		
	v	0.2217	0.2317	0.2414	0.2508	0.2600	0.2691	0.2781	0.2957	0.3130	0.3302		
600	*h*	1482.4	1508.6	1533.8	1558.5	1582.7	1606.6	1630.4	1677.7	1724.9	1772.4		
(9.29)	*s*	5.3222	5.4102	5.4923	5.5697	5.6436	5.7144	5.7826	5.9129	6.0363	6.1541		
	v	0.1874	0.1963	0.2048	0.2131	0.2212	0.2291	0.2369	0.2522	0.2672	0.2821		
700	*h*	1474.5	1501.9	1528.1	1553.4	1578.2	1602.6	1626.8	1674.6	1722.4	1770.2		
(13.81)	*s*	5.2259	5.3179	5.4029	5.4826	5.5582	5.6303	5.6997	5.8316	5.9562	6.0749		
	v	0.1615	0.1696	0.1773	0.1848	0.1920	0.1991	0.2060	0.2196	0.2329	0.2459	0.2589	
800	*h*	1466.3	1495.0	1522.2	1548.3	1573.7	1598.6	1623.1	1671.6	1719.8	1768.0	1816.4	
(17.86)	*s*	5.1387	5.2351	5.3232	5.4053	5.4827	5.5562	5.6268	5.7603	5.8861	6.0057	6.1202	
	v		0.1488	0.1559	0.1627	0.1693	0.1757	0.1820	0.1942	0.2061	0.2178	0.2294	
900	*h*		1488.0	1516.2	1543.0	1569.1	1594.4	1619.4	1668.5	1717.1	1765.7	1814.4	
(21.54)	*s*		5.1593	5.2508	5.3354	5.4147	5.4897	5.5614	5.6968	5.8237	5.9442	6.0594	
	v		0.1321	0.1388	0.1450	0.1511	0.1570	0.1627	0.1739	0.1847	0.1954	0.2058	0.2162

(Contd.)

Table A.5.2 *(Contd.)*

		20	*30*	*40*	*50*	*60*	*70*	*80*	*100*	*120*	*140*	*160*	*180*
1000	*h*		1480.6	1510.0	1537.7	1564.4	1590.3	1615.6	1665.4	1714.5	1763.4	1812.4	1861.7
(24.91)	*s*		5.0889	5.1840	5.2713	5.3525	5.4292	5.5021	5.6392	5.7674	5.8888	6.0047	6.1159
	v			0.1129	0.1185	0.1238	0.1289	0.1338	0.1434	0.1526	0.1616	0.1705	0.1792
1200	*h*			1497.1	1526.6	1554.7	1581.7	1608.0	1659.2	1709.2	1758.9	1808.5	1858.2
(30.96)	*s*			5.0629	5.1560	5.2416	5.3215	5.3970	5.5379	5.6687	5.7919	5.9091	6.0214
	v			0.0944	0.0995	0.1042	0.1088	0.1132	0.1216	0.1297	0.1376	0.1452	0.1528
1400	*h*			1483.4	1515.1	1544.7	1573.0	1600.2	1652.8	1703.9	1754.3	1804.5	1854.7
(36.28)	*s*			4.9534	5.0530	5.1434	5.2270	5.3053	5.4501	5.5836	5.7087	5.8273	5.9406
	v				0.0851	0.0895	0.0937	0.0977	0.1053	0.1125	0.1195	0.1263	0.1330
1600	*h*				1502.9	1534.4	1564.0	1592.3	1646.4	1698.5	1749.7	1800.5	1851.2
(14.05)	*s*				4.9584	5.0543	5.1419	5.2232	5.3722	5.5084	5.6355	5.7555	5.8699