POWER ELECTRONICS

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CONTENTS

PREFACE

Why Power Electronics?

Power Electronics is the technology that applies to conversion and control of electric power and serves as the interface between electronic technology and electric power. The term *Power Electronics* originated sometime at the beginning of the 19th century, with the development of mercury-arc rectifiers, but the real revolution started in 1956 with the development of thyristors. The most popular semiconductor device of the thyristor family is the silicon controlled rectifier (SCR), introduced by General Electric in 1957. With the advent of the SCR, power electronics entered a new age.

Continuous evolution of the semiconductor technology ensured furtherance in power electronics with the advent of power semiconductor devices such as the Power BJT, Power MOSFET, TRIAC, GTO, IGBT, LASCR, PUT, LAPUT, RBDT, RCDT, RCTT, SBS, SUS, SIT, SITH, RCT, MCT, etc.,

Presently, power electronics is one of the, if not *the* most active, disciplines for research in electric power engineering.

Prerequisites

A thorough understanding of power electronics can only be attained through an ability to apply fundamental concepts learnt in several disciplines such as semiconductor physics, analog and digital electronics, circuit theory, signals and systems, analog and digital control systems, signal processing, electromagnetics, electrical machines and power systems. Hence, my advice to my dear students and readers would be to equip themselves with a solid background of the prerequisites prior to embarking on the road to master power electronics

Who can Use this Book?

This book caters to undergraduate and polytechnic students, as well as practicing engineers. For all students of Electrical, Instrumentation, Electronics, Electronics & Communication, Electrical & Electronics and Industrial Electronics Engineering, power electronics is now a compulsory subject offering. Those in the automation and process control industries, other than industries related to the disciplines mentioned above, will also find this book extremely useful.

Another group who could benefit immensely from this humble offering are those who are serious about cracking competitive examinations such as IES, UPSC, GATE, etc. When developing this book, a conscious effort has been made in incorporating the curriculum requirements of these important examinations.

Why Should You Choose this Book?

It is because this book provides an interesting and eclectic mix of concepts, examples, applications and various pedagogical features. Highlights of the book have been listed below:

- A dedicated chapter on ac and dc drives
- Equivalent circuits provided for most power circuits
- Elaborate derivations
- A variety of numerical examples designed on semester and competitive examination patterns
- Complete coverage of the various university syllabi on Power Electronics

Pedagogy includes

- Over 700 brilliant illustrations
- Over 230 Solved Examples!
- More than 550 Review Questions
- 650 and more Objective Type Questions

Chapter Organization of the Book

The book contains 12 chapters.

Chapter 1 covers the evolution of power electronics' devices and their applications in different converter circuits in detail.

Chapter 2 deals with the construction, operating principles, ratings, series and parallel operation of power diode in a generalized way. This chapter also covers diode circuits with different types of loads: *R*, *R-L*, *RLC* and *L* and the diode circuit for energy recovery operation.

Chapter 3 describes the construction, operating principles, characteristics, ratings, safe operating area (SOA), switching characteristics, series and parallel operation, gate and base drive circuits of power BJT, power MOSFET, IGBT and SIT elaborately.

Chapter 4 deals with the construction, operating principles, characteristics, ratings, series and parallel operation of thyristors. Two-transistor analogy, transistor model, triggering methods, gate-switching characteristics, protection, snubber circuit and firing circuits of thyristors are discussed elaborately. In this chapter, the commutation of thyristors, both natural and forced, are incorporated in detail. The structure, operating principle, *I-V* characteristics, switching characteristics and drive circuits of DIAC, TRIAC and GTO are explained.

Chapter 5 introduces single-phase uncontrolled rectifiers with *R*, *R-L*, *R-L-E* loads with and without freewheeling diodes. The effect of transformer leakage inductance in performance of single-phase full-wave rectifiers and applications of filters to reduce ripples at output voltage are also incorporated.

Chapter 6 covers single-phase controlled rectifiers (half-wave controlled, full-wave controlled rectifier using centre-tap transformer, bridge converter and semi-converter, etc.) with *R*, *R-L*, *R-L-E* loads with and without freewheeling diodes. The effect of transformer leakage inductance and source inductance in performance of single-phase full-wave controlled rectifiers and the operation of single-phase dual converters are explained in detail.

Chapter 7 presents the operating principle of three-phase uncontrolled rectifiers (half-wave rectifier and bridge rectifier), six-phase half-wave rectifier, multiphase rectifier, six-phase series bridge rectifier, and six-phase parallel bridge rectifier with load.

Chapter 8 covers the operation of different three-phase controlled rectifiers with *R*, *R-L*, and *R-L-E* load. The effect of source inductance in performance of three-phase bridge converters and the operation of three-phase dual converters are discussed elaborately in this chapter.

Chapter 9 deals with ac voltage controllers and cycloconverters. The operating principle of ON and OFF control and phase control of single-phase ac voltage controllers with *R* and *R-L* load, three-phase ac voltage controllers with delta- and star-connected load, and applications of ac voltage controller in tap changer and ac chopper are explained in detail. The principle of operation of single-phase to single- phase, three-phase to single-phase, and three-phase to three-phase cycloconverters are also incorporated.

Chapter 10 describes the classification of chopper, time ratio control and current limit control strategies, step-down choppers, step-up choppers, derivation of load voltage and currents with *R*, *R-L* and *R-L-E* load, Morgan's chopper, Jones chopper and oscillation chopper, non-isolated dc-to-dc converters such as Buck, Boost, Buck-Boost and CUK converters, isolated dc-to-dc converters, namely fly-back, forward, push-pull, half-bridge and full-bridge converters.

Chapter 11 covers single-phase and three-phase voltage source inverters, different control strategies such as PWM techniques, sinusoidal PWM, modified sinusoidal PWM and multiple PWM, voltage and harmonic control, series resonant inverter, current source inverters, McMurray, modified McMurray, McMurray Bedford inverters and their applications.

Chapter 12 deals with thyristor-controlled dc drives, single-phase series dc motor drives, three-phase dc motor drives, dual-converter fed dc motor drives, reversible dc drives, speed regulation of dc series and shunt motor, dc chopper-fed dc drives. Thyristor-controlled ac drives, various speed-control schemes of induction motor and synchronous motor are explained in detail.

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INTRODUCTION TO POWER ELECTRONICS

1

1.1 INTRODUCTION

Power Electronics is the study of conversion and control of electric power. In other words, it is the interface between electronics and power. The primary work of power electronics is to process and control the flow of electric energy by providing voltages and currents in a form which is optimally suited for consumer demands. This conversion of electric power is called *power electronics converter*, as depicted in Fig. 1.1. Power electronics converters are also known as *power converters* or *converters* or *switching converters* or *power electronic circuits*.

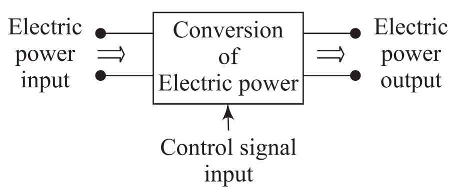


Fig. 1.1 *Conversion of electric power*

Any power electronic system consists of power electronics converter and controller including the corresponding measurement and interface circuits. Power converters are used to convert electric power of one form to another using power semiconductor device, whereas controllers are required to generate control signals for turn on and turn off of switching devices. Subsequently, the required output voltage at specified frequency is available at output terminals. The basic structure of a power electronic-based system is shown in Fig. 1.2.

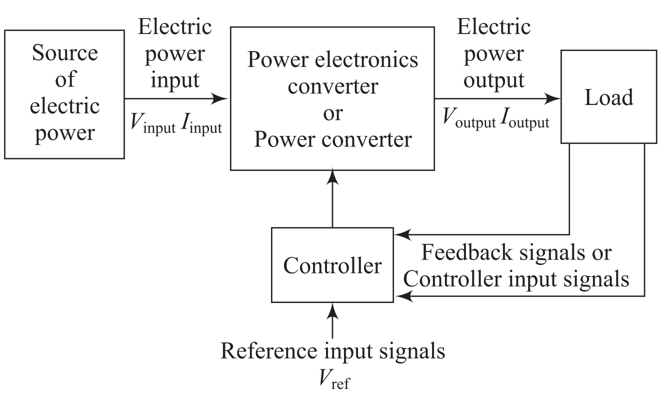


Fig. 1.2 *A power electronics-based system*

The *source of electric power* is dc generator, Photovoltaic (PV) Cell and Battery, i.e., dc power and alternator and induction generator, i.e., ac power. The controlled power flows from ac/dc power source to load through a power electronics converter. The power converter is also called *power semiconductor converter* or *power modulator*.

The output of a power electronics converter may be a variable dc or a variable ac with variable voltage and frequency. Usually, the output of a power converter depends upon the requirement of *load*. When the load is three-phase induction motor, the converter output will be adjustable ac voltage and frequency. If the load is dc motor, the converter output will be adjustable dc voltage.

The *feedback signals* are the measured parameters of the load, i.e., voltage, current, speed and position. These signals are used as input signals of controller. The command signals are also applied to the controller. Then the feedback signals are compared with the reference or command signals and accordingly the control signals are generated by the controller to turn-on the semiconductor switches of power converter. Consequently, the required output at the load is obtained.

The *control circuit* is the heart of the system as it provides triggering pulses to power semiconductor switches of converter. The synchronising circuit is required for dc-to-ac converter and ac-to-ac converters circuits, but the synchronising circuit is not required for dc-to-dc converter. The detail operations of different triggering circuits are explained in Chapter 4.

Presently, power electronics is the most active discipline in electric power engineering and it is related with other disciplines such as semiconductor physics, analog and digital electronics, circuit theory, analog and digital control system, signal processing, electromagnetic, electrical machine and power system. Figure 1.3 shows the relation between power electronics and other disciplines.

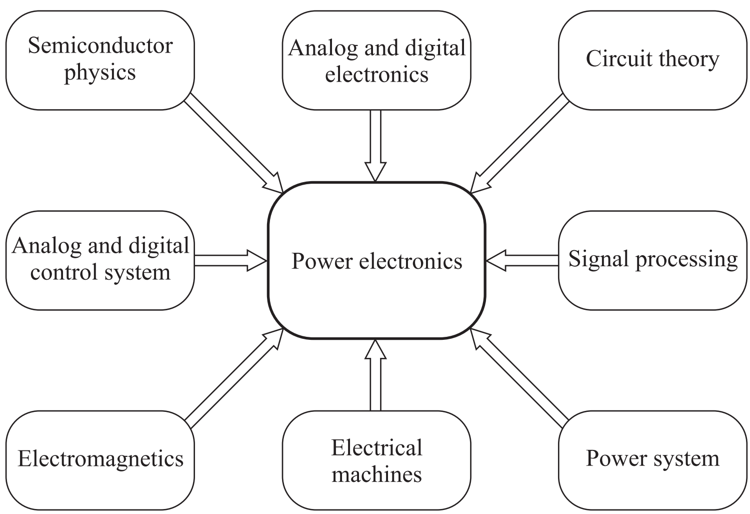


Fig. 1.3 *Relation between power electronics and other disciplines*

Due to very fast development in semiconductor physics, electronics devices and circuits, advanced control theory, signal processing and processor technology, applications of power electronics have improved significantly. At present, the major issues in power electronics are the following:

- To fulfill the requirements of consumer demand.
- To improve the system efficiency for reliable operation of power semiconductor devices as well as energy saving.
- To implement power conversion with less volume, less weight, and less cost.

- To reduce negative influence to other equipments in the electric power system and the electromagnetic environment.

In this chapter, a brief history of power electronics, symbols, characteristics and ratings of power semiconductor devices, classification and applications of solid state power devices, different types of power electronics converter with their applications, advantages, disadvantages and scope of power electronics converter, ideal switch and practical switch, and dynamic performance of switches are discussed elaborately.

1.2 HISTORY OF POWER ELECTRONICS

Power electronics originated at the beginning of 19th century with the development of *mercury-arc rectifiers*. A mercury-arc rectifier or mercury-vapor valve is a type of electrical rectifier which is used to convert high ac voltage into dc voltage. They were very useful to provide power for industrial motors, electric railways and electric locomotives, as well as high voltage direct current (HVDC) power transmission. The mercury-arc rectifiers with glass envelope, grid controlled mercury-arc rectifiers and mercury-arc rectifiers with metal envelope were developed in 1900, 1903 and 1908 respectively. Figure 1.4 shows the glass-bulb envelope mercury-arc rectifier *Thyratrons*, i.e., hot-cathode mercury arc rectifier with grid control was developed by Langmuir in 1914.

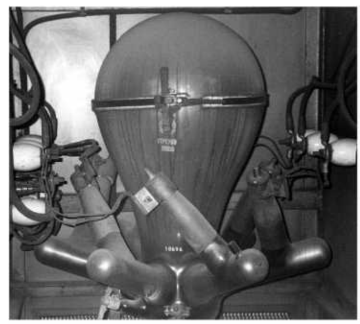


Fig. 1.4 *A glass-bulb envelope mercury-arc rectifier*

Thyratron is a type of gas filled tube which is used as a high voltage electrical switch and a controlled rectifier. Usually, thyratrons are manufactured as triode, tetrode and pentode. Due to the mercury vapour or neon or xenon gas fill, thyratrons can handle much greater currents. Figure 1.5 shows a triode thyraton.

In 1925, the first solid state power device, *selenium rectifier* was developed without a glass tube as depicted in Fig. 1.6. The selenium rectifier is also known as *metal rectifier***.** The selenium rectifier is an early type of semiconductor rectifier in which the semiconductor is copper oxide or selenium. This device is used in phase controlled converters, inverters, battery chargers and cyclo-converters.

Fig. 1.5 *A triode thyratron*

In 1930, *cyclo-converters*, i.e., a variable frequency output voltage from fixed frequency input voltage, were developed by Rissik. During 1933, Lenz developed the ac voltage regulator or controller using solid state power devices. In 1947, the point-contact transistor was developed by W. H. Brattain, J. Bardeen and W. Shockley.

The bipolar junction transistor (BJT) using germanium was invented in 1948 and this was the beginning of the new age of semiconductor electronics. There has been much reduction in size, cost, and power consumption of solid state power devices and simultaneously the research is going on to develop equipments with more complexity and more power handling capability.

A 100A germanium *power diode* was developed in 1953. A new revolution began in 1956 with the development of thyristor, i.e., a four-layer silicon PNPN device. The most popular semiconductor device of the thyristor family is the SCR which was introduced by General Electric in 1957.

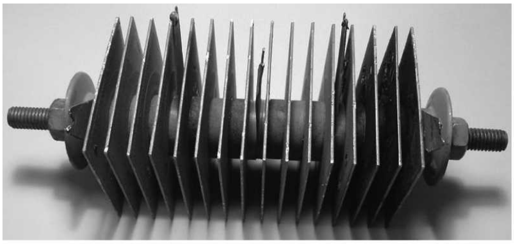


Fig. 1.6 *A selenium rectifier*

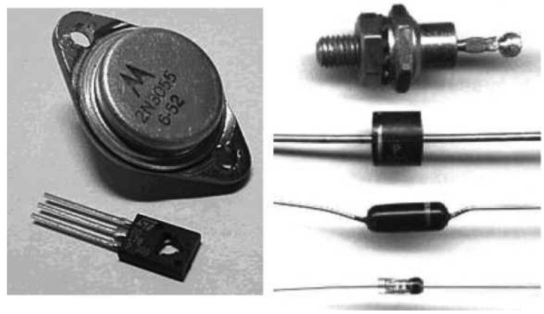


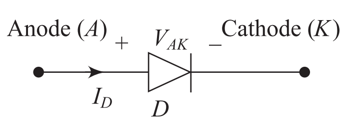
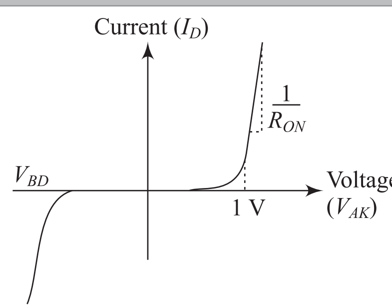
Fig. 1.7 *Power semiconductor devices*

In 1964, the *power FET* was developed Zuleeg and Teszner. The VMOSFET was the first commercially available power FET followed by vertical DMOS (double-diffused MOS) and the HEXFET. The development of power FET has witnessed significant effects on the power semiconductor industry.

Generally, the most commonly used power semiconductor devices are power diodes, power BJT, power MOSFET, thyristors, SCR, GTO, DIAC, TRIAC and IGBT. When these devices are in ON state, switches are closed and these devices behave just like ordinary switches. Current flows from source to load through power semiconductor devices as well as load. In the same way, when these devices are in OFF state, switch is opened and no current flows from source to load. Power semiconductor devices are available in wide range of voltage from few volts to several kV and current from few amperes to several kA. Table 1.1 shows the symbols and characteristics of power semiconductor devices. The voltage and current rating, switching frequency, and switching time of power semiconductor devices are given in Table 1.2. Power semiconductor devices should have the following properties:

- Breakdown voltage should be very high
- On state voltage drop should be low
- On state resistance should be low
- Turn-on and turn-off process should be very fast
- Power dissipation capacity is high

Table 1.1 *Symbols and characteristics of power semiconductor devices*

Device Name	Symbol	I-V Characteristics
Power Diode		

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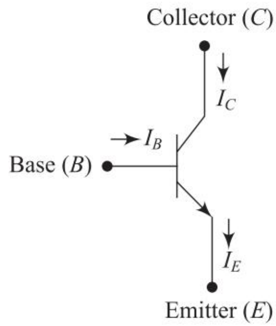
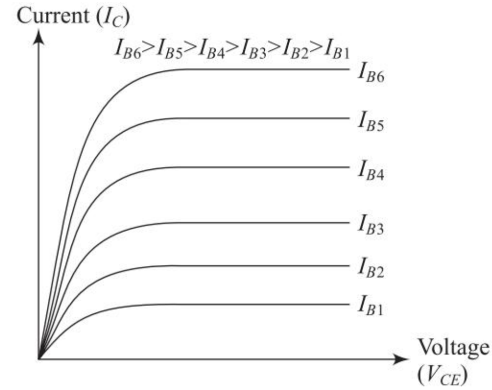
<tr><th>Device Name</th><th>Symbol</th><th>I-V Characteristics</th></tr>
<tr><td>Power BJT NPN</td><td>
</td><td>
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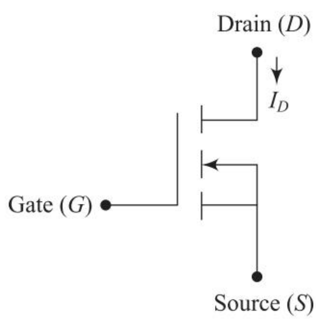
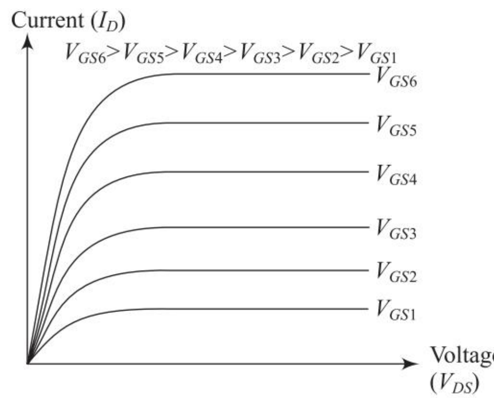
<tr><td>Power MOSFET</td><td>
</td><td>
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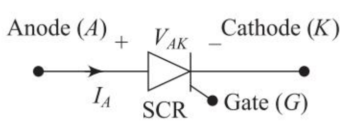
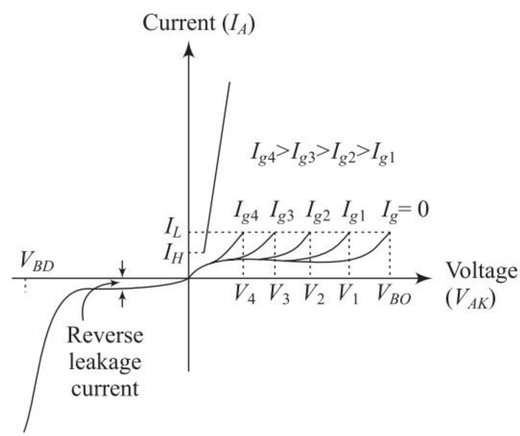
<tr><td>Thyristors (SCR)</td><td>
</td><td>
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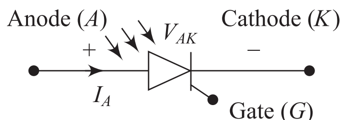
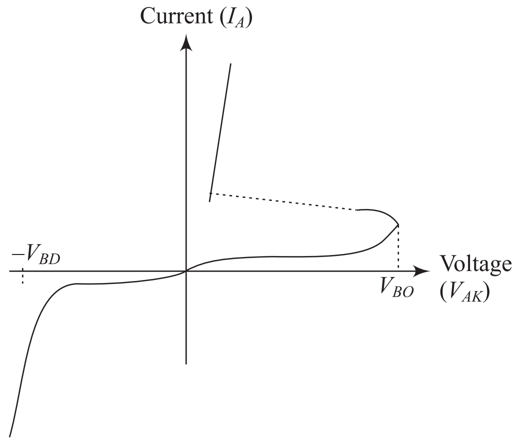
<tr><td>Light Activated SCR</td><td>

LASCR
(Light activated SCR)</td><td>
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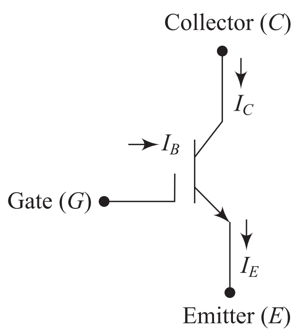
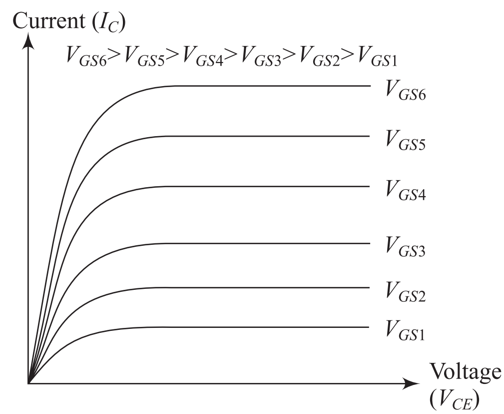
<tr><td>IGBT</td><td>
</td><td>
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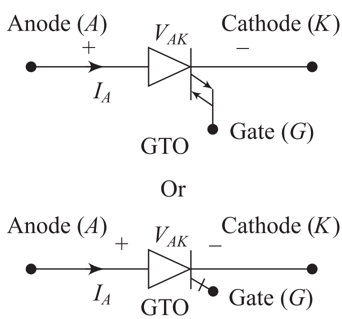
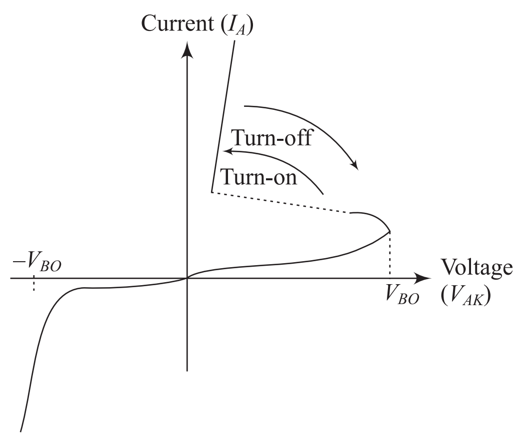
<tr><td>GTO</td><td>
</td><td>
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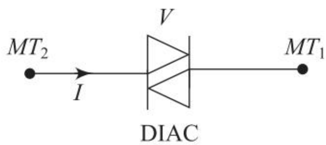
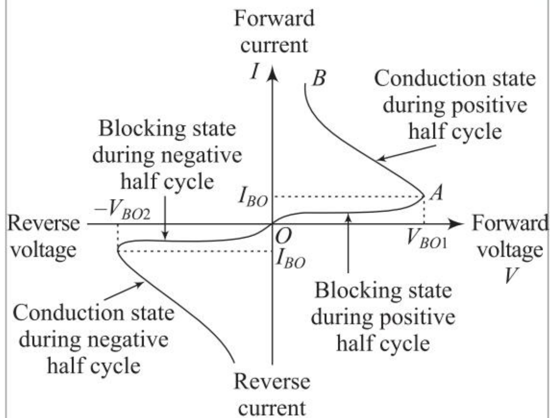
<tr><td>DIAC</td><td>
</td><td>
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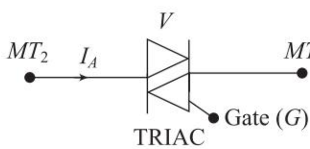
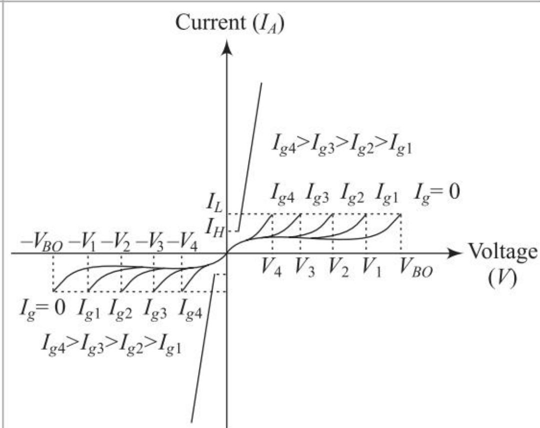
<tr><td>TRIAC</td><td>
</td><td>
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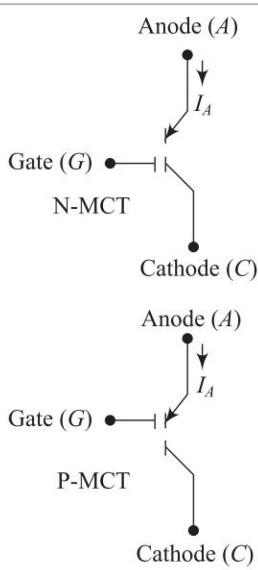
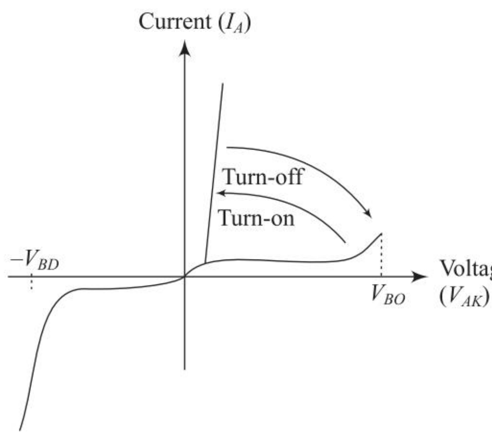
<tr><td>MCTs</td><td>
</td><td>
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Table 1.2 *Voltage and current rating, switching frequency, and switching time of power semiconductor devices*

Device Name	Voltage/ current rating	Maximum operating frequency	Switching on time	Switching off time	On-state resistance
Generalised Power Diode	5 kV/5 kA	1 kHz	50 to 100 μs	50 to 100 μs	0.3 to 0.6 milli-ohm
High Speed Power Diode	3 kV/1 kA	20 kHz	5 to 10 μs	5 to 10 μs	1.2 to 1.96 milli-ohm
Power BJT	1400 V/400 A	10 kHz	2 μs	9 to 30 μs	4 to 10 milli-ohm
Power MOSFET	1 kV/50 A	100 kHz	0.1 μs	1 to 2 μs	1 to 2 milli-ohm
Thyristors	10 kV/5 kA	1 kHz	2 to 5 μs	20 to 100 μs	0.25 to 0.75 milli-ohm
IGBT	3.3 kV/2.500 A	50 kHz	0.2 μs	2–5 μs	2 to 40 milli-ohm
GTO	5 kV/3 kA	2 kHz	3 to 5 μs	10–25 μs	2.5 milli-ohm
TRIAC	1200 V/300 A	0.5 kHz	2 to 5 μs	200 to 400 μs	3.5 milli-ohm
MCTs	1200 V/100 A	20 kHz	0.2 μs	50 to 110 μs	10 to 25 milli-ohm

1.3 CLASSIFICATION OF POWER SEMICONDUCTOR DEVICES

In general, the power semiconductor devices may be classified as the following:

1. Power diodes
2. Power BJTs
3. Power MOSFETs
4. Power IGBT
5. Thyristors (SCR, DIAC, TRIAC)

The power semiconductor devices can be classified based on the following factors:

1. Number of layers of semiconductor materials used in the device
2. Driver circuit used in the device
3. Carrier used in the device
4. Number of terminals in the device
5. Triggering methods

Based on the layers of semiconductor materials used, these devices can also be classified as two layer, three layer, four layers and multilayer device as depicted in Fig. 1.8(a). Power semiconductor devices are also classified based on driver circuit as shown in Fig. 1.8(b).

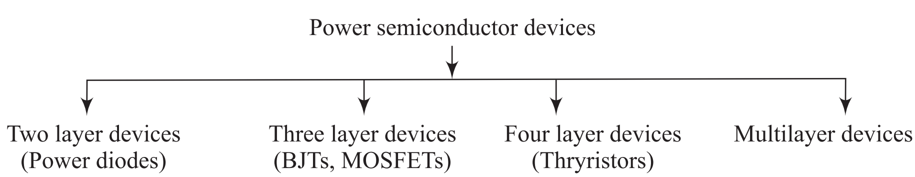


Fig. 1.8(a) *Classification of power semiconductor devices based on number of the layers of semiconductor materials used in the device*

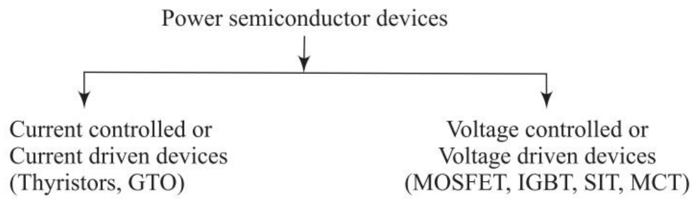


Fig. 1.8(b) *Classification of power semiconductor devices based on driver circuit used in the device*

The power semiconductor devices can also be classified as majority carrier devices and minority carrier devices as depicted in Fig. 1.8(c). The majority carrier devices are called *unipolar devices* and minority carrier devices are known as *bipolar devices*. Power semiconductor devices are also classified based on number of terminals such as two-terminal devices and three-terminal devices as shown in Fig. 1.8(d). These devices can also be classified as triggering methods such as pulse triggering devices and level triggering devices. Figure 1.8(e) shows the classification of power semiconductor devices based on triggering methods.

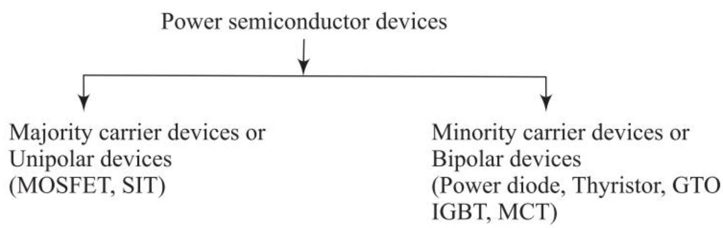


Fig. 1.8(c) *Classification of power semiconductor devices based on carrier used in the device*

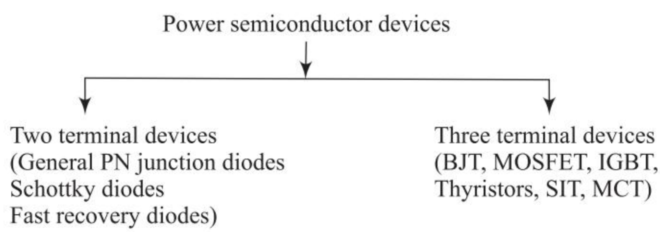


Fig. 1.8(d) *Classification of power semiconductor devices based on number of terminals*

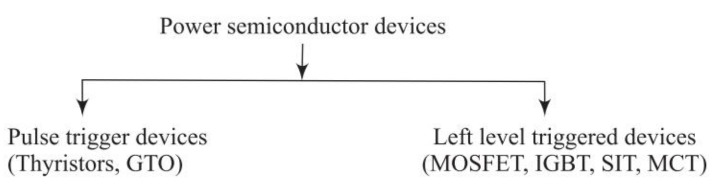


Fig. 1.8(e) *Classification of power semiconductor devices based on triggering methods*

1.4 APPLICATIONS OF POWER ELECTRONICS DEVICES

Power semiconductor devices have a wide spectrum of applications in household products, industrial and commercial applications, transportation and utility systems, Telecommunications, aerospace and defense applications, Electrical power system and space technology. The detail applications of power semiconductor devices are given below.

1. Residential and home appliances

• Lighting control • Cooking • Sewing machines • Entertain equipments	• Heating, electric blankets • Refrigeration, freezers • Washing machines • Games and toys	• Air conditioning • Vacuum cleaners • Grinders and mixers • Dryers and door openers

2. Industrial applications

• dc motor drives • Welding, lighting • Sewing machines • Electroplating • Crane and hoist	• ac motor drives • Refrigeration • Electrolysis • Pumps, compressors • Traffic signal control	• Induction heating • Arc furnaces and ovens • Grinders and mixers • Industrial robot, elevator • Machine tool

3. Commercial applications

• Battery charger • Computer • Office equipments • Photocopiers	• Audio amplifiers • Central refrigeration • Elevator • Electric fans	• Induction heating, Electric dryer • Lighting, Security systems • Air conditioning • Vending machines

4. Transportation applications

• Electric trains, locomotives • Electric vehicles • Street cars	• Trolley buses • Subways • Elevator	• Magnetic levitation • Automotive electronics • Battery chargers

5. Power system applications

• High voltage dc (HVDC) transmission system • Static VAR compensation (SVC) • Thyristor controlled reactor (TCR) • Custom power and power quality control • Non-conventional energy sources • Static circuit breakers	• Flexible ac transmission system (FACTS) • Harmonics suppression • Thyristor switch capacitor (TSC) • Energy storage systems • Alternator excitation system • Fans and boiler-feed pumps

6. Telecommunications Applications

• Battery chargers • VLF transmitters, radars	• DC power supply and UPS • Power supply of wireless communication equipments

7. Aero space Applications

• Aircraft power system • Space vehicle power systems	• Spaceship power systems • Spaceship power system

1.5 ADVANTAGES OF POWER ELECTRONICS CONVERTERS

The advantages of power electronics converters are as follows:

- Increase efficiency of power converter due to less loss in power semiconductor devices.
- High reliability of power converter-based system.
- Life period of power converter circuit is long.
- Less maintenance is required due to absence of moving components in power electronics converters.
- The dynamic response of power electronics converters is very fast compared to electromechanical converters.
- The size and weight of power electronics converters is less. Hence the installation cost is low.

1.6 DISADVANTAGES OF POWER ELECTRONICS CONVERTERS

The disadvantages of power electronics converters are as follows:

- Generate harmonics in the supply system as well as in the load.
- The ac-to-dc and ac-to-ac converters operate at low power factor under certain conditions. Always try to avoid the low power factor operation of power converters.
- The overload capacity of power electronics converters is low.
- The cost of power electronics converters is high.

Example 1.1 A switching waveform of a normal practical switch as shown in Fig. 1.9 has the following parameters:

$V = 250$ V, $V_{\text{on}} = 1.5$ V, $I = 80$ A, $t_d = 0.5\ \mu$s, $t_r = 1.5\ \mu$s, $t_{\text{on}} = 100\ \mu$s, $t_s = 2.5\ \mu$s, $t_f = 3\mu s$, $t_{\text{off}} = 3\ \mu$s

(a) Determine switching frequency.

(b) Find $P_{\text{turn-on}}, P_{\text{turn-off}}, P_{\text{on-state}}$ and P_D.

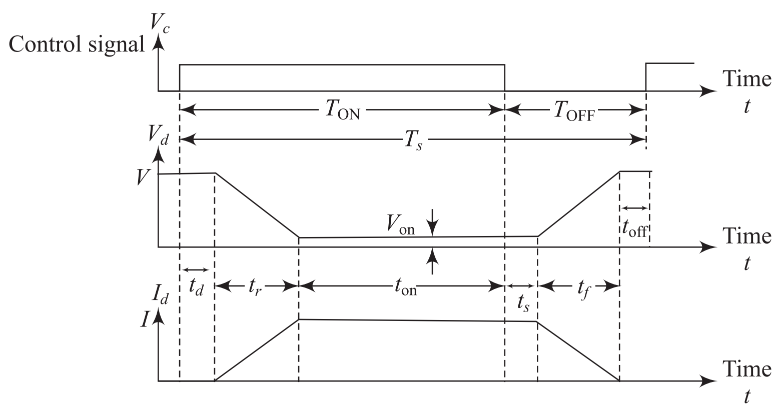


Fig. 1.9

Solution

Given: $V = 250$ V, $I = 80$ A, $V_{on} = 1.5$ V, $t_d = 0.5$ µs, $t_r = 1.5$ µs, $t_{on} = 100$ µs, $t_s = 2.5$ µs, $t_f = 3$ µs, $t_{off} = 30$ µs

(a) The total time period is

$$T_s = T_{ON} + T_{OFF}$$
$$= t_d + t_r + t_{on} + t_s + t_f + t_{off}$$
$$= (0.5 + 1.5 + 100 + 2.5 + 3 + 30)\ \mu s = 137.5\ \mu s$$

The switching frequency is $f_s = \frac{1}{T_s} = \frac{1}{137.5 \times 10^{-6}}$ Hz $= 7.272$ kHz

(b) Average turn-ON loss is equal to $P_{turn\text{-}on} = \frac{1}{6} V I t_r f_s + \frac{1}{3} V_{on} I t_r f_s$

$$= \frac{1}{6} \times 250 \times 80 \times 1.5 \times 10^{-6} \times 7.272 \times 10^3 + \frac{1}{3} \times 1.5 \times 80 \times 1.5 \times 10^{-6} \times 7.272 \times 10^3 \text{ W}$$

= 36.796 W.

Average turn-OFF loss is equal to $P_{turn\text{-}off} = \frac{1}{6} V I t_f f_s + \frac{1}{3} V_{on} I t_f f_s$

$$= \frac{1}{6} \times 250 \times 80 \times 3 \times 10^{-6} \times 7.272 \times 10^3 + \frac{1}{3} \times 1.5 \times 80 \times 3 \times 10^{-6} \times 7.272 \times 10^3 \text{ W}$$

= 73.592 W

Average ON-state loss is equal to $P_{on\text{-}state} = V_{on} I \cdot (t_{on} + t_s) \cdot f_s$

$$= 1.5 \times 80 \times (100 \times 10^{-6} + 2.5 \times 10^{-6}) \times 7.272 \times 10^3 \text{ W} = 89.445 \text{ W}$$

The total power dissipation of a switching device is

$$P_D = P_{on\text{-}state} + P_{turn\text{-}on} + P_{turn\text{-}off} = (89.445 + 36.796 + 73.592) \text{ W} = 199.833 \text{ W}.$$

Example 1.2 If $I = 80$ A, $V = 220$ V, $t_r = 2.5$ µs and $t_f = 4$ µs for a switching waveform of a normal practical as switch as shown in Fig. 1.10, determine the energy loss during switch ON and switch OFF. When the switching frequency is 1 kHz, find the average power loss in the switch.

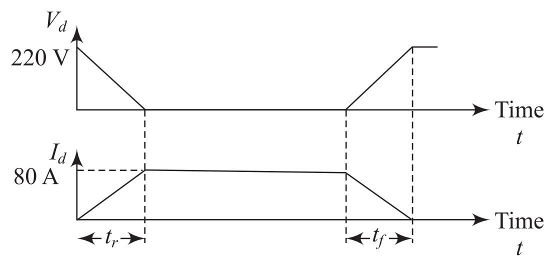


Fig.1.10

Solution

Given: $V = 220$ V, $I = 80$ A, $t_r = 2.5$ µs, $t_f = 4$ µs, and $f_s = 1$ kHz

Energy loss during switch ON is

$$W_{turn\text{-}on} = \frac{1}{6} V I t_r = \frac{1}{6} \times 220 \times 80 \times 2.5 \times 10^{-6} \text{ Ws} = 7.333 \text{ mWs}$$

Energy loss during switch OFF is

$$W_{turn\text{-}off} = \frac{1}{6} V I t_f = \frac{1}{6} \times 220 \times 80 \times 4 \times 10^{-6} \text{ Ws} = 11.733 \text{ mWs}$$

Average turn ON loss is equal to

$$P_{turn\text{-}on} = \frac{1}{6} V I t_r f_s = \frac{1}{6} \times 220 \times 80 \times 2.5 \times 10^{-6} \times 1.0 \times 10^3 \text{ W} = 7.333 \text{ W}$$

Average turn OFF loss is equal to

$$P_{turn\text{-}off} = \frac{1}{6} V I t_f f_s = \frac{1}{6} \times 220 \times 80 \times 4 \times 10^{-6} \times 1.0 \times 10^3 \text{ W} = 11.733 \text{ W}$$

Average ON state loss is equal to $P_{\text{on-state}} = 0$

Average power dissipation of a switching device is

$$P_D = P_{\text{on-state}} + P_{\text{turn-on}} + P_{\text{turn-off}} = (0 + 7.333 + 11.733)\ \text{W} = 19.06\ \text{W}$$

Summary

- Power electronics have revolutionized the concept of power control for power conversion.
- A power electronics based system has been discussed elaborately.
- History of development of power electronics devices and their applications are explained in this chapter.

Multiple-Choice Questions

1.1 Cyclo-converters are used to convert

(a) low frequency ac to high frequency ac (b) high frequency ac to low frequency ac

(c) ac to dc (d) dc to ac

1.2 Controlled rectifier is used to convert

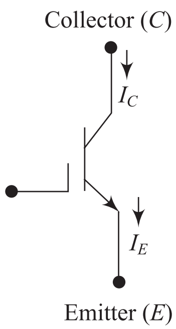
(a) ac to ac (b) ac to dc (c) dc to dc (d) dc to ac

1.3 Inverters are used to convert

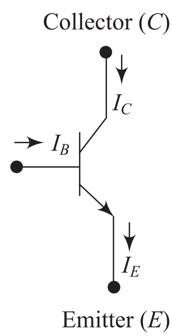
(a) ac to ac (b) ac to dc (c) dc to dc (d) dc to ac

1.4 Match the devices on the left hand with circuit symbols on the right-hand side.

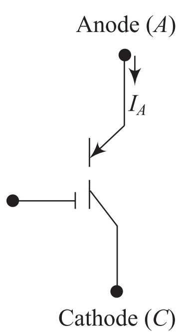
(a) BJT (1) Gate (G)



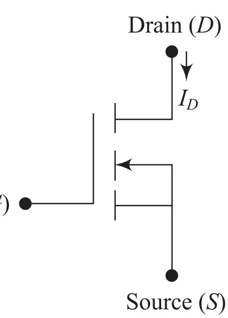
(b) IGBT (2) Base (B)



(c) MOSFET (3) Gate (G)



(d) MCT (4) Gate (G)



(a) $A \to 1, B \to 2, C \to 3, D \to 4$ (b) $A \to 2, B \to 1, C \to 4, D \to 3$

(c) $A \to 2, B \to 1, C \to 3, D \to 4$ (d) $A \to 1, B \to 2, C \to 4, D \to 3$

Fill in the Blanks

1.1 ______ is a unidirectional switching device.
1.2 ______ is a two-terminal three-layer semiconductor device.
1.3 ______ is a bidirectional switching device.
1.4 Bipolar devices are ______.
1.5 ______, ______, ______ and ______ are minority carrier switching device.
1.6 ______ is an uncontrolled turn-ON and turn-OFF device.
1.7 ______ is a majority carrier switching device.
1.8 ______, ______, ______, and ______ are controlled turn-ON and turn-OFF device.
1.9 ______ is a gate turn and turn-OFF device.
1.10 ______ is a two-terminal two-layer semiconductor device.

Review Questions

1.1 What is power electronics? Draw the block diagram of a typical power electronics-based system and explain briefly.
1.2 Write the voltage and current rating, switching frequency, and switching time of power semiconductor devices.
1.3 Draw the symbol and V-I characteristics of the following devices:
(a) SCR (b) IGBT (c) MCT (d) TRIAC
(e) DIAC

Answers to Multiple-Choice Questions

1.1 (b) 1.2 (b) 1.3 (d) 1.4 (b)

Answers to Fill in the Blanks

1.1 SCR
1.2 Power diode
1.3 TRIAC
1.4 BJT, IGBT, and MCT
1.5 Diode, SCR, GTO, IGBT
1.6 Power diode
1.7 MOSFET
1.8 BJT, MOSFET, IGBT, SIT, MCT
1.9 GTO
1.10 Diode

POWER SEMICONDUCTOR DIODES AND CIRCUITS

2

2.1 INTRODUCTION

Low power semiconductor diode is the simplest semiconductor device. These devices consist of two layers; P and N materials. These two layers are formed by epitaxial growth and develop a PN junction. A depletion layer is formed across the junction. The width of depletion layer depends on the doping density of P-type and N-type material, applied voltage across anode to cathode and the junction temperature.

Power diode is more complicated in structure and operating characteristics than low power general-purpose diode. Usually, power diodes have high forward current-carrying capability and high reverse breakdown voltage. The area of *pn*-junction in a power diode is very high compared to low power signal diodes. Therefore, the structure modification of low power general-purpose diode is required to make them suitable for high power applications. There are three types of power diodes:

1. General-purpose diodes
2. Schottky diodes
3. Fast recovery diodes

In this chapter, the structure of power diode, conducting modulation in power diode, I-V characteristics, reverse bias condition in power diode and switching characteristics of power diode are explained elaborately. The operating principle of Schottky diode and fast recovery diode, rating of diode, series and parallel connection of power diode are also incorporated in this chapter. The effect of switching a dc source voltage to a diode circuit with RLC load and application of diodes in freewheeling and stored energy recovery are explained in detail.

2.2 STRUCTURE OF A POWER DIODE

Generally, power diodes should operate in several kA current during conduction in forward biased direction with low power loss and it should block high voltage about several kV in reverse biased direction. For large blocking voltage, the width of depletion layer must be large in order to limit the electric field strength. The lightly doped *pn*-junction power diode should have sufficient width so that the required depletion layer must be accommodating within the device. Then simple lightly doped *pn*-junction power diode provides high resistance during forward biased condition. Consequently, during conduction of forward biased power diode, a large power is dissipated with in the diode but it is not acceptable. To reduce the forward resistance as well as power loss, if the doping level is increased, the reverse breakdown voltage of power diode will be reduced. To sort out this problem, a lightly doped n^- is introduced between two highly doped p^+ and n^+ layers.

Figure 2.1 shows the *pn*-junction power diode. The structure of a *pn*-junction power diode consists of a highly doped *n*-type substrate (n^+) which is grown above the lightly doped n^- epitaxial layer and a highly doped *p*-type region (p^+). Actually, the *pn*-junction is formed by diffusing a highly doped *p*-type region (p^+) in the lightly doped n^- epitaxial layer. The anode terminal of power diode is available from p^+ region and the cathode terminal is taken out from n^+ region. The thickness of n^+ layer is about 250 μm and the doping density is $N_D = 10^{19}$ cm^{-3}. The thickness of n^- epitaxial layer is about 100 μm and the doping density is $N_D = 10^{14}$ cm^{-3} and this region is also called *drift region*. Similarly, the thickness of p^+ layer is about 10 μm and the doping density is $N_A = 10^{19}$ cm^{-3}. The layer thickness and doping density of n^+ cathode substrate, n^- drift region and p^+ anode are depicted in Fig. 2.1(c). The cross-section area of diode depends on current rating of the devices. Power diodes are commercially available from few amperes to several thousand amperes (kA). For 1000 kA rating of power diode, the cross-section area will be several square centimeter and wafer diameter will be about 4 inches.

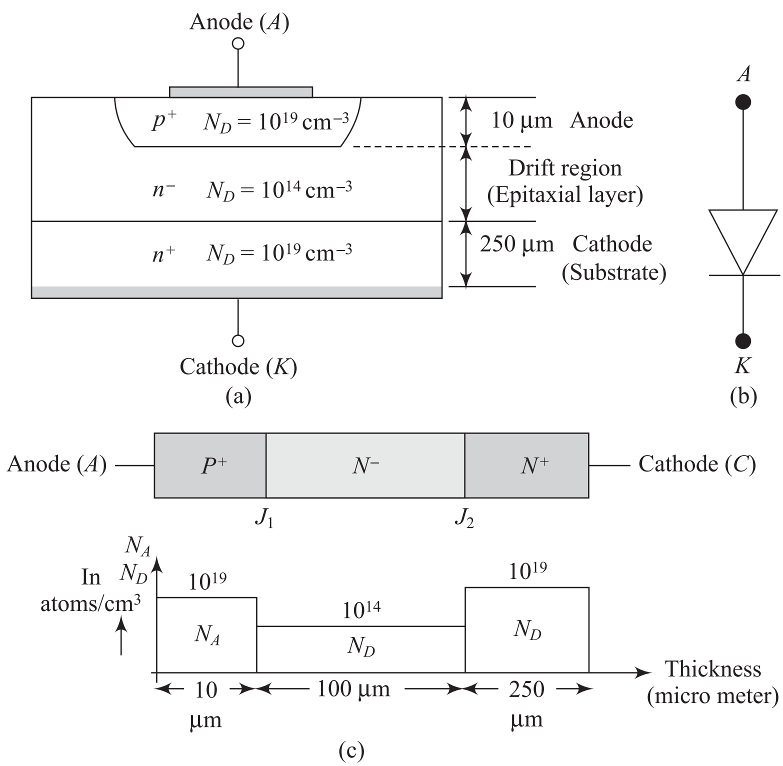


Fig. 2.1 *(a) Structure of power diode, (b) Symbol of power diode and (c) Doping density of semiconductor material and width of each layer*

In the structure of power diode as depicted in Fig. 2.1, the n^- epitaxial layer is known as *drift region*. This region is very lightly doped about $N_D = 10^{14}$ cm^{-3} and sometimes doping density is nearly intrinsic. Therefore, the drift region is also called as *i-layer* and the device is represented by *p-i-n* diode. The drift region or *i*-layer is the most important structure feature in power diode as the breakdown voltage dependent on the thickness of drift region. Actually, the drift region was not found in general purpose low power rating diodes.

In power diode, the n^- layer is used to absorb the depletion layers of the reverse biased p^+ n$^-$ junction. The width of the drift region must be significantly large at large reverse voltages. This lightly doped n^- drift region provides ohmic resistance to the diode during forward biased condition. Consequently, during conduction of forward biased power diode, a large power is dissipated with in the diode.

2.3 *I-V* CHARACTERISTICS OF POWER DIODE

The *I-V* characteristics of power diode are shown in Fig. 2.2. When the diode is *forward biased* and applied voltage is greater than cut-in voltage, diode starts conducting and current flows linearly with forward biased voltage rather than exponentially. The cut-in voltage of a power diode is about 0.7 V. The lightly doped drift region provides resistance. Due to large current, it develops a voltage drop across diode. The ON state voltage drop across resistance of diode is about 1V. The *I-V* characteristics of power diode can be expressed as

$$i = I_o\left(e^{\frac{qV_D}{kT_j}} - 1\right) \tag{2.1}$$

where, i is the current flow through diode in A,

V_D is the voltage across diode with anode is positive with respect to cathode in V,

I_O is the reverse saturation current or leakage current in A,

q is the charge of electron in Coulomb ($q = 1.6022 \times 10^{-19}$ C),

T_j is the junction temperature in Kelvin, and

k is Boltzmann's constant in J/K and $k = 1.3806 \times 10^{-23}$ J/K

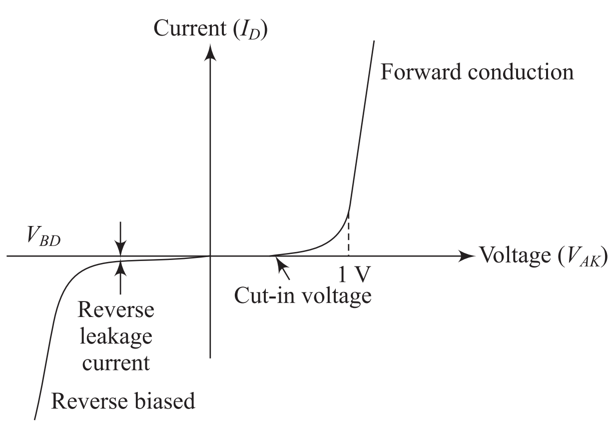


Fig. 2.2 *I-V characteristics of a power diode*

Equation (2.1) can also be expressed as

$$i = I_o\left(e^{\frac{V_D}{V_T}} - 1\right)$$

where, V_T is the thermal voltage and it is expressed by $V_T = \dfrac{kT_j}{q}$

During *reverse bias* condition, a very small leakage current flows from cathode to anode. The amplitude of reverse saturation current does not depend on the applied reverse voltage, but it varies with junction temperature. Therefore, this current is independent of the reverse voltage and it flows until the reverse voltage across the anode to cathode reaches reverse voltage breakdown voltage V_{BD}. For a 1000 A power diode, the value of leakage current is less than 100 mA.

When the voltage reaches at reverse breakdown voltage V_{BD}, voltage across anode to cathode becomes constant but current increases extensively. This current is limited by the external circuit parameters. Due to large voltage at breakdown and large reverse current, the excessive power is dissipated across the power diode and the junction temperature is further increased as the device operates in the reverse breakdown region. Consequently, the power diode may be destroyed quickly due to excessive heat. Therefore, the operation of power diode in breakdown region should be avoided.

During reverse biased condition, the I-V characteristic of a diode is temperature sensitive and the diode current is function of temperature. The diode current can be expressed as

$$i = I_o\left(e^{-\frac{q|V_D|}{kT_j}} - 1\right)$$

where, I_o is the reverse saturation current, and

V_D is the voltage across anode to cathode and T_j is junction temperature.

After substituting the value of $q = 1.6 \times 10^{-19}$ Coulomb and k = Boltzmann's constant = 1.38×10^{-23} Joules/K, we get

$$i = I_o\left(e^{-1.5942\frac{V_D}{T_j}} - 1\right) \text{ where, } T_j \text{ is the junction temperature.}$$

The *I-V* characteristic of a power diode at junction temperature T_{j1} and T_{j2} is depicted in Fig. 2.3.

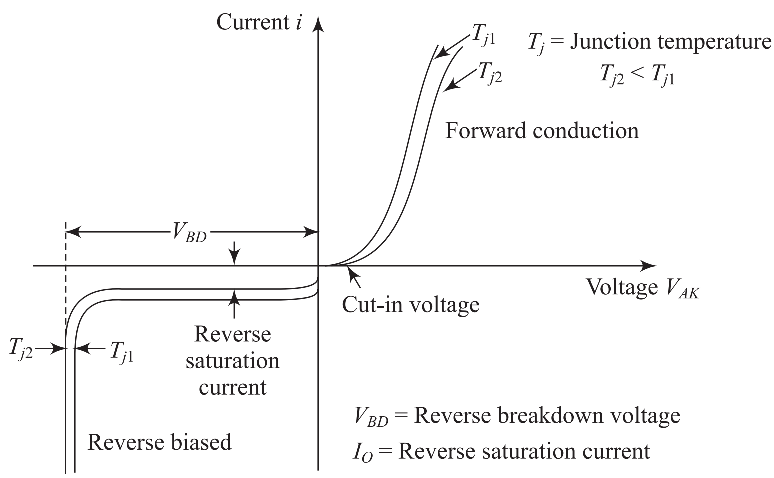


Fig. 2.3 *I-V characteristic of a power diode at junction temperature T_{j1} and T_{j2}*

2.4 CONDUCTIVITY MODULATION IN POWER DIODE

It is clear from Fig. 2.1 that the p^+ and n^+ layers are heavily doped and the n^- layer is lightly doped. Consequently, the depletion layers will be developed at $p^+ n^-$ (J_1) junction and $n^- n^+$ (J_2) junction. During forward bias condition, a large number of holes can be drift from the p^+ region to the n^- drift region and can able to cross the n^- region. In forward bias condition, a large number of electrons from the n^+ region cross to the n^- drift region. Therefore, there is a heavy influx of charge carriers of holes as well as electrons in the middle of n^- drift region as depicted in Fig. 2.4(a). This large influx of holes and electrons is called *high-level injection*. When the influx of holes and electrons is low, it is called *low-level injection*.

In case of *low-level injection,* all excess p-type carriers are recombined with n-type carriers in the n^- region. But in case of *high-level injection,* the excess p-type carriers reach to the $n^- n^+$ (J_2) junction and also attracted by n^+ region and then cross the $n^- n^+$ junction. This injection method is known as *double injection*.

When the influx of holes and electrons is very large, the thermal equilibrium charge densities in the n^- drift region are changed as $\delta p_{n-} >> p_{n0}$ and $\delta n_{n-} >> n_{n0}$. The excess charges from the $p^+ n^-$ (J_1) junction must be equal the excess charges from $n^- n^+$ (J_2) junction must be equal. Therefore, $\delta p_{n-} = \delta n_{n-}$. During reverse bias condition, the carrier density decreases while the excess carriers on the border of the $p^+ n^-$ and $n^- n^+$ junctions are swept away. The resistance of n^- drift region is small during forward bias condition and it provides high conductivity. In reverse bias condition, the resistance of n^- drift region is very large and it provides low conductivity. This phenomenon of n^- drift region is called as the *conductivity modulation*. Due to high conductivity in forward bias condition, the diode is able to carry the high current density. But the large injection of minority carriers in the n^- drift region creates the switching delay problem during turn OFF as there is certain time delay to remove charge during reverse bias.

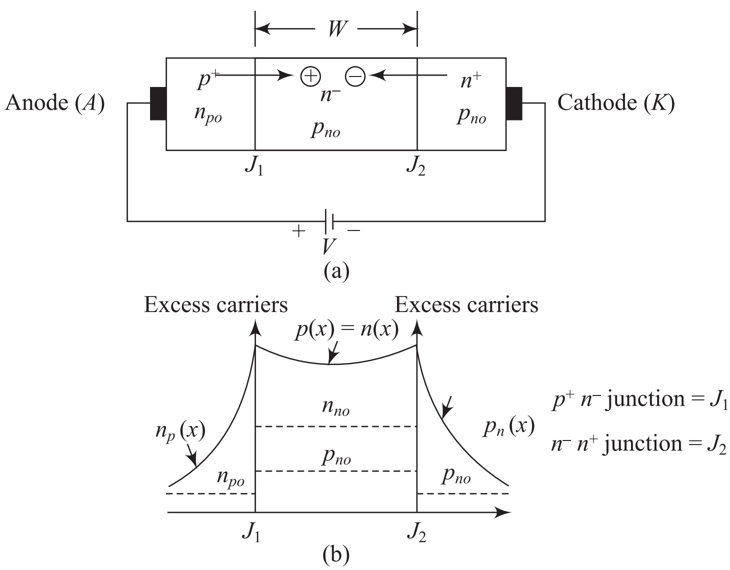


Fig. 2.4 *(a) Forward biased power diode with holes drift from the p^+ region to the n^- region and electrons drift from the n^+ region to the n^- region, (b) Initial carrier distribution (------ line) and carrier distribution at forward bias (—— line)*

The voltage drop across the forward biased power diode (V_D) is sum of the voltage drop across p^+n^- (J_1) junction, V_J and the ohmic drop in the n^--drift region, V_{RD}. The V_D can be expressed by

$$V_D = V_J + V_{RD}$$

The value of V_{RD} depends on forward current density J_F and the width of the drift region W. Due to very high current, J_F is very large and the drift region voltage behaves just like an ohmic drop. If I_F current flows through the forward biased power diode, the voltage drop across diode V_D can be expressed as

$$V_D = V_J + R_{\text{on}} I_F$$

The on state voltage drop across resistance of diode V_D is about 1 V.

2.5 BREAKDOWN VOLTAGE OF POWER DIODE IN REVERSE BIAS CONDITION

When the power diode is reverse biased, the depletion layer is formed at $p^+ n^-$ metallurgical junction. As $N_A >> N_D$, the space charge region extends into the n^- drift region. At reverse breakdown voltage, the physical width of the drift region may be smaller or larger than the depletion layer width. Based on the depletion layer width, there are two types of diodes:

1. Non-punch through type diodes
2. Punch through type diodes

In non-punch through type diodes, the depletion layer boundary cannot reach the end of the n^-drift region. But in punch through diodes, the depletion layer can be extend to the entire n^- drift region and can also be contact with the n^+ cathode. As the doping density of the cathode is very large (10^{+19} cm^{-3}), the penetration of drift region inside the cathode is insignificant.

2.5.1 Breakdown Voltage of Non-punch through Power Diode

Figure 2.5(a) shows the a reverse biased non-punch through diode and the electric field of reverse biased non-punch through diode is depicted in Fig. 2.5(b). Under reverse biased condition of a non-punch through diode, the depletion layer just touch the end of the n^-drift region as shown in Fig. 2.5(c).

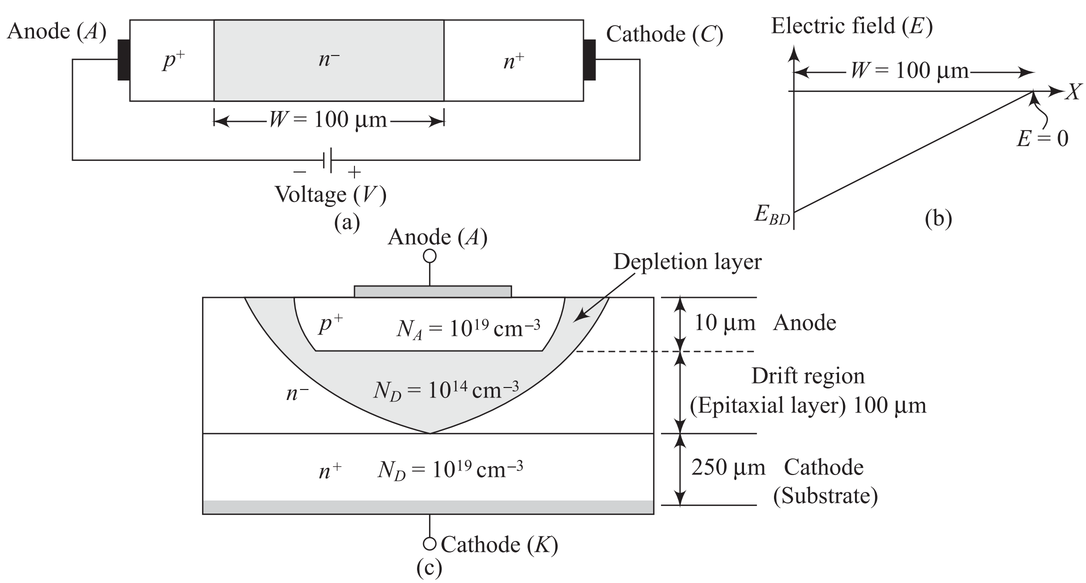


Fig. 2.5 *(a) A reverse biased non-punch through diode, (b) The electric field of reverse biased non-punch through diode, (c) The depletion layer just touch the end of the n^-drift region*

It is clear from Fig. 2.5(b) that the electric field strength is maximum at the $p^+ n^-$ (J_1) junction and its strength decreases towards the depletion region. At the end of the depletion region, the electric field strength is zero. When the depletion layer is almost cover the complete n^- drift region, the breakdown voltage V_{BD} is

$$V_{BD} = \frac{\varepsilon E_{BD}^2}{2qN_D} \tag{2.2}$$

where, E_{BD} is the electric field intensity at avalanche breakdown and its value is about 200 kV/cm, and N_D is the donor density of n^- drift region.

From Eq. (2.2), it is justified that the breakdown voltage is inversely proportional to doping density in per cubic centimeter. For very high break down voltage, the doping density of n^- drift region is low. If the breakdown voltage of a power diode is 1000 V, the doping level of n^- drift region is about $N_D = 10^{14}$ cm^{-3} and the minimum thickness of n^- drift region (W) is about 100 µm.

2.5.2 Breakdown Voltage of Punch through Power Diode

When depletion layer has been extended to n^- drift region and contact with the maximum part of n^+ region as shown in Fig. 2.6(a), punch though occurs in power diode. After that, if the reverse bias voltage is increased, the width of depletion layer cannot increase due to high doping density in n^+ region $N_D = 10^{19}$ cm^{-3}. Actually, the highly doped n^+ region can block the further increase of the depletion layer and the electric field profile starts to flatten as shown in Fig. 2.6. In the punch though power diodes, the electric field strength is almost uniform as the electric field strength is less triangular and moves toward rectangular. When the n^- drift region is very lightly doped, the electric field strength can be almost constant.

It is clear from Fig. 2.6(c) that the electric field has two components such as triangular-shaped component and rectangular component. E_1 is maximum part of triangular shaped component and E_2 is maximum part of rectangular component. The electric field E_{BD} is sum of E_1 and E_2.

The electric field E_1 is expressed as

$$E_1 = \frac{qN_D W}{\varepsilon}$$

where, N_D is the donor density of n^- drift region and W is the width of depletion layer.

The area of the triangular-shaped component is equal to the voltage V_1 and it can be represented by

$$V_1 = \frac{qN_D W^2}{2\varepsilon}$$

The area of the rectangular component represents the voltage V_2 and it can be expressed as

$$V_2 = E_2 W$$

Then, the electric field intensity at avalanche breakdown $E_{BD} = E_1 + E_2$ and the breakdown voltage is equal to

$$V_{BD} = V_1 + V_2$$

After substituting the value of V_1 and V_2, we get

$$V_{BD} = V_1 + V_2 = \frac{qN_D W^2}{2\varepsilon} + E_2 W$$

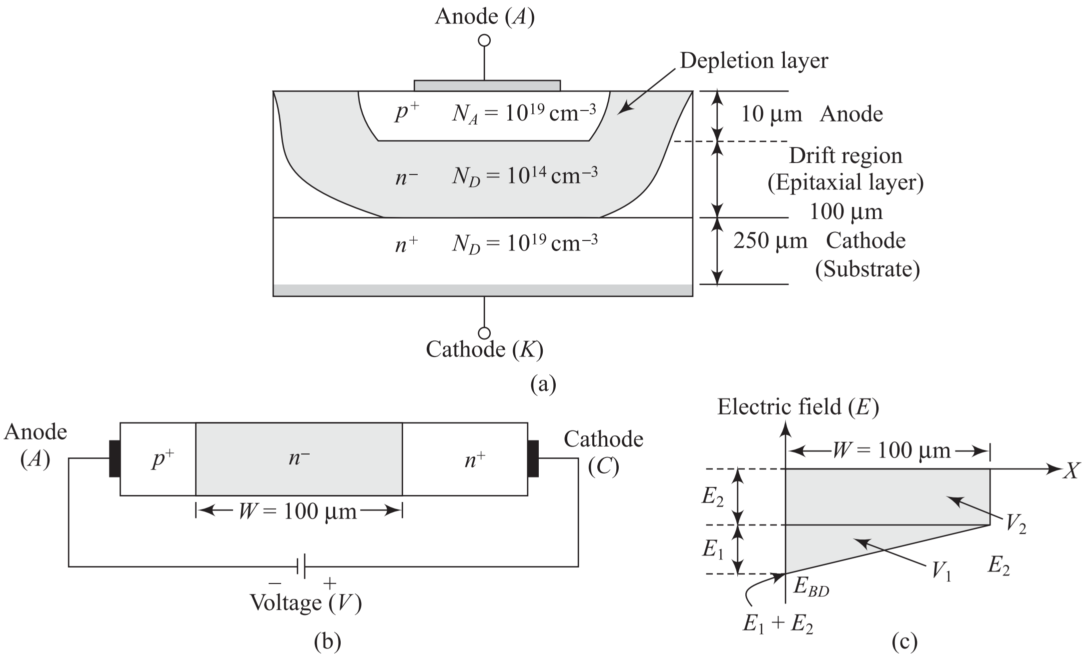


Fig. 2.6 *(a) The depletion layer touch the maximum part of n⁺ region, (b) Reverse biased punch through diode with depletion layer and (c) Electric field intensity of reverse biased punch through diode*

As
$$E_2 = E_{BD} - E_1, \quad V_{BD} = \frac{qN_D W^2}{2\varepsilon} + (E_{BD} - E_1)W$$
$$= \frac{qN_D W^2}{2\varepsilon} + E_{BD}W - E_1 W$$

After substituting $E_1 = \frac{qN_D W}{\varepsilon}$ in the above equation, we get

$$V_{BD} = E_{BD}W + \frac{qN_D W^2}{2\varepsilon} - \frac{qN_D W^2}{\varepsilon}$$
$$= E_{BD}W - \frac{qN_D W^2}{2\varepsilon}$$

When the doping density in the n^- drift region is very small, V_1 will be negligible compared to V_2. Then $V_{BD} = E_{BD}W$.

2.6 REVERSE RECOVERY CHARACTERISTICS OF POWER DIODE

Figure 2.7 shows the reverse recovery characteristics of a power diode. When a diode is switched off, the forward diode current decays to zero and the diode continue to conduct in reverse direction due to storage charge carriers both electrons and holes in the depletion region and semiconductor layers (p^+, n^- and n^+). The reverse current flows through the diode for a time t_{rr} as shown in Fig. 2.7. t_{rr} is

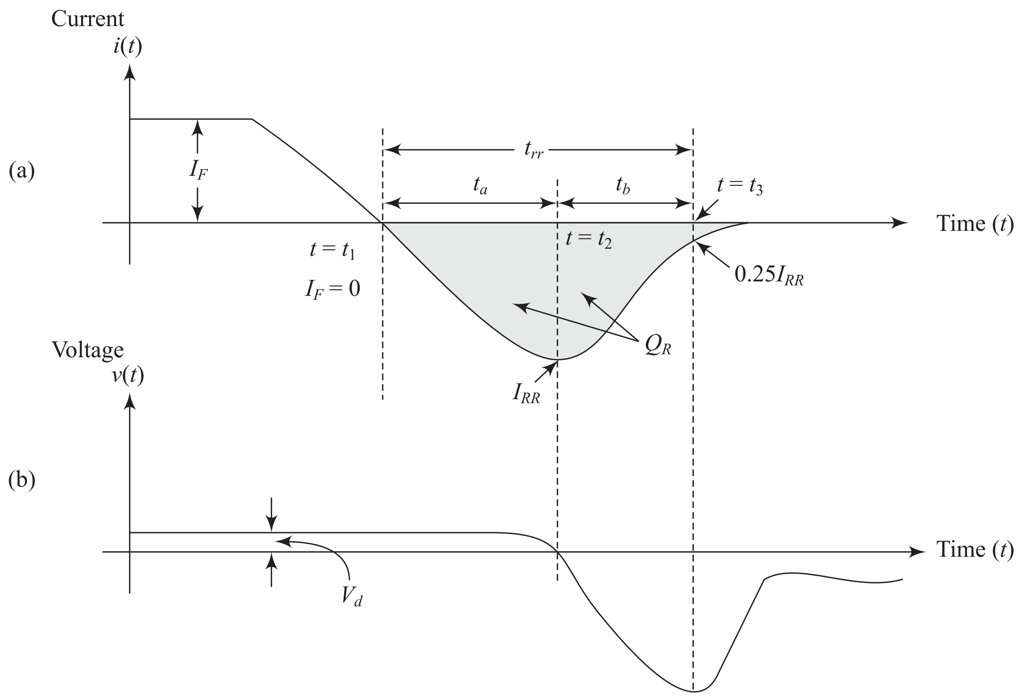


Fig. 2.7 *Reverse recovery characteristics of a power diode (a) Variation of current, (b) Variation of voltage*

known as *reverse recovery time*. Actually, the reverse recovery time t_{rr} is the time interval from the instant $t = t_1$ to the time $t = t_3$. At $t = t_1$, the forward diode current becomes zero and at the instant $t = t_3$ the reverse recovery current reduces to 25% of its peak value. When the reverse recovery current decreases to zero, the diode regains its blocking capability.

The reverse recovery time consists of t_a and t_b, i.e., $t_{rr} = t_a + t_b \cdot t_a$ is the time between $t = t_1$ and $t = t_2$. At time $t = t_1$, the forward diode current becomes zero and at $t = t_2$, current through the diode reaches the peak reverse current I_{RR}. During this time interval t_a, the stored charge in depletion region can be removed completely. t_b is the time interval between $t = t_2$ and $t = t_3$. t_b can be determined from the instant of peak reverse current I_{RM} to the instant reverse recovery current reduces to 25% of its peak value. During the time interval t_b, the storage charge in p^+, n^- and n^+ semiconductor layers is removed.

The shaded area as shown in Fig. 2.7(a) stands for the reverse recovery charge Q_R. During t_{rr} time, the charge Q_R must be removed. The value of Q_R can be expressed as

$$Q_R = \frac{1}{2} I_{RR} t_a + \frac{1}{2} I_{RR} t_b = \frac{1}{2} I_{RR}(t_a + t_b) = \frac{1}{2} I_{RR} t_{rr} \quad \text{as} \quad t_{rr} = t_a + t_b$$

The peak reverse current I_{RR} in terms of Q_R and t_{rr} is equal to

$$I_{RR} = \frac{2Q_R}{t_{rr}} \tag{2.3}$$

The peak reverse current I_{RR} can also be expressed in terms of $\frac{di}{dt}$ and t_a and its value is equal to

$$I_{RR} = t_a \frac{di}{dt} \tag{2.4}$$

From Eqs. (2.3) and (2.4), we obtain

$$I_{RR} = \frac{2Q_R}{t_{rr}} = t_a \frac{di}{dt} \qquad \text{or} \qquad t_{rr} t_a = \frac{2Q_R}{di/dt}$$

As t_b is very small compared to t_a, $t_a \approx t_{rr}$ and $t_{rr}^2 = \dfrac{2Q_R}{di/dt}$.

Then, $t_{rr} = \sqrt{\dfrac{2Q_R}{di/dt}}$ and $I_{RR} = \sqrt{2Q_R \dfrac{di}{dt}}$

The time ratio $\dfrac{t_b}{t_a}$ is called the *softness factor* or *S factor*. The S factor determines the voltage transients when the diode recovers its original state from ON state to OFF state. Usually, its value is equal to unity. When $S = 1$, the reverse recovery process of diode have low oscillatory. If the value of S is less than unity and is very small, the reverse recovery process of diode contains large oscillatory over voltage. According to recovery characteristic, diodes are classified as fast recovery diodes and soft recovery diodes. When S factor is less than 1, diode is called *fast recovery diode*. If S factor is equal to 1, diode is called *soft recovery diode*. The reverse recovery characteristics of general or slow recovery diode and fast recovery diode are depicted in Fig. 2.8(a) and (b) respectively. The reverse recovery time of general-purpose diode is large and its value is about 25 μs but the reverse recovery time of fast recovery diode is small and its value is in the range of 25 ns to 100 ns.

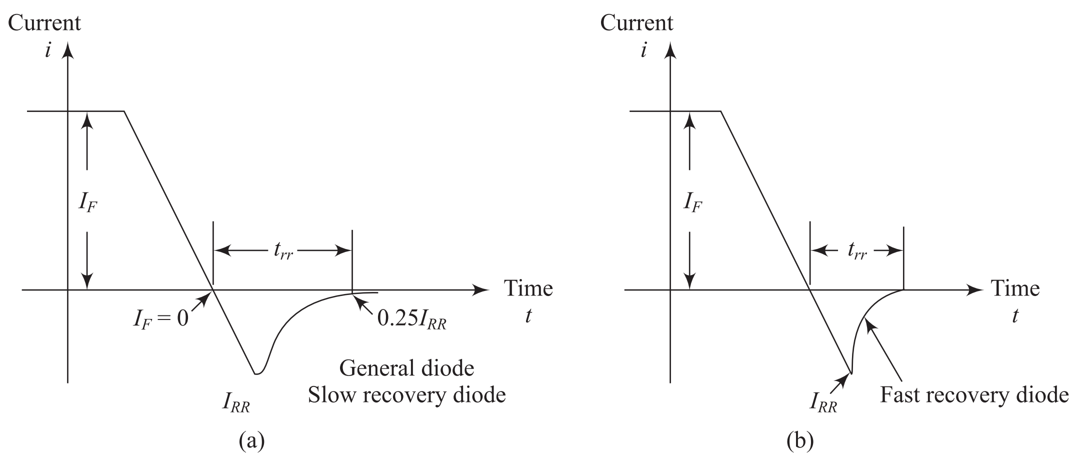


Fig. 2.8 *Reverse recovery characteristics (a) Slow recovery diode and (b) Fast recovery diode*

Example 2.1 The reverse recovery current of a power diode is 5 μs and the rate of fall of current is about 20 A/μs. Calculate (a) the storage charge and (b) the peak reverse current I_{RR}.

Solution

Given: t_{rr} = 5 μs, di/dt = 20 A/μs

(a) The storage charge is $Q_R = \dfrac{1}{2}\dfrac{di}{dt} t_{rr}^2 = \dfrac{1}{2} \times$ 20 A/μs × (5 μs)^2 = 250 μC.

(b) The peak reverse current $I_{RR} = \sqrt{2Q_R \dfrac{di}{dt}} = \sqrt{2 \times 250 \times 10^{-6} \times 20 \times 10^6}$ A = 100 A.

2.7 COMPARISON BETWEEN GENERAL LOW POWER DIODE AND POWER DIODE

The structure and performance of power diodes are different from the small signal diodes. Table 2.1 shows the comparison between general diode and power diode.

Table 2.1 *Comparison between general diode and power diode*

Low power signal diode or general diode	Power diode
When the input forward bias voltage is greater than threshold voltage (V_T = 0.7 V), the diode becomes ON and current increases exponentially.	When the input forward bias voltage is greater than threshold voltage (V_T = 1 V), the diode becomes ON and current increases linearly.
Low power or small signal diodes have low current and low voltage handling capability. They are used in low power circuits and electronics circuits.	Power diodes have high current and high voltage handling capability. They are used in high power circuits.
Low power or small signal diodes are two layer devices.	Power diodes are thee layer device.
Lightly doped n^- epitaxial layer is not present in *pn*-junction diode.	Power diodes has lightly doped n^- epitaxial layer which can absorb the depletion layer during reverse biased. The thickness of n^- layer is adjustable and its width depends upon the reverse blocking voltage
The impurity concentrations of power diodes may not vary layer to layer.	The impurity concentrations of power diodes vary layer to layer.
The low power or small signal diodes operate at high frequency about some hundreds kHz.	Power diodes can able to handle power at frequency 50 Hz and the frequency below 1 kHz.
The general diodes are available from few volts (5 V) to 1000 V (breakdown voltage) and power rating up to 50 W	Power diodes are available from few volts to 10 kV and from few amperes to 5 kA.
The ON state voltage drop is about 0.7 V.	The on state voltage drop is about 1 V.
Reverse recovery time is low, about 1 ns to 1 μs	Reverse recovery time is high, about 25 μs

2.8 SCHOTTKY POWER DIODES

Schottky power diodes operate at high speed and it has low forward voltage drop, i.e., less than 1 V. The ON-state voltage varies between 0.3 V to 0.5 V. It has high reverse leakage current with high temperature withstand capability. This type of diode can able to withstand at high junction temperature (T_j) up to 175°C. This device is build up by placing a thin film of metal, i.e., platinum metal or aluminum metal in homogeneous contact with a semiconductor material. Usually the metal film is placed on an n^--type silicon substrate as shown in Fig. 2.9. The metal film is used as the positive electrode (anode) and the *n*-type silicon substrate is used as cathode.

Electrons in different metals have different absolute potential energy. As electrons have larger average energy in silicon compared to aluminum metal, electrons are diffuse from *n* type material to Al metal as depicted in Fig. 2.10. There is no hole injection in n-type substrate as there is no source of holes in aluminum. Consequently the metal will be negatively charged and the semiconductor material will acquire positive charge. Then a potential barrier is developed with a very thin depletion layers in n^- substrate as shown in Fig. 2.10.

When the Schottky diode is forward biased, electrons from the n^- layer cross the barrier potential and should reach to the positive potential metal. Thus Schottky diode is a majority carrier device and there is no storage charge. In Schottky diode, fast switching is possible due to absence of storage charge.

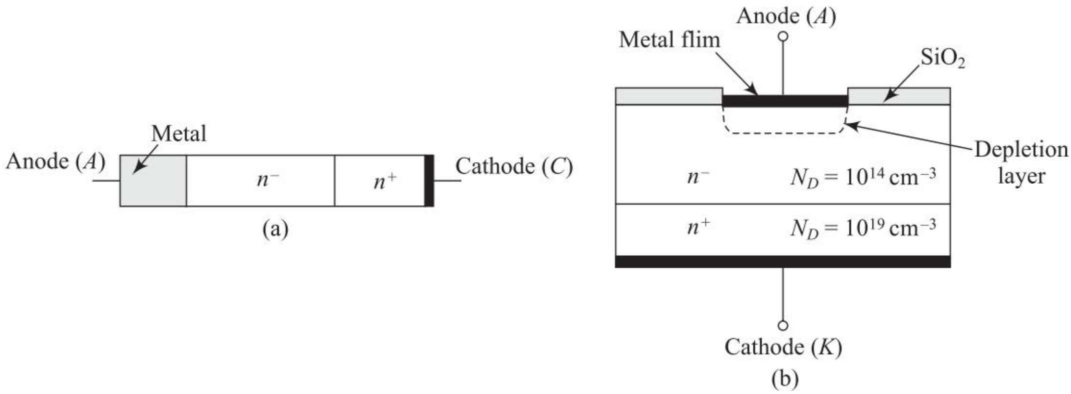


Fig. 2.9 *(a) and (b) Structure of Schottky diode*

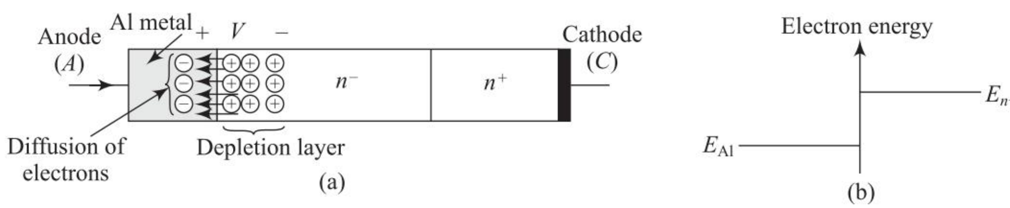


Fig. 2.10 *(a) Diffusion of electrons and depletion layer and (b) Electron energy in metal and n-type silicon substrate*

Actually, the metal film is deposited on an n^--type semiconductor substrate in a Schottky diode as shown in Fig. 2.11. A guard ring structure can be used to improve the breakdown voltage capability of the diode. The metal film is used as positive electrode, i.e., anode and semiconductor is used as cathode. The I-V characteristic of Schottky diodes is similar to the I-V characteristics of a *pn* junction diode and the diode current can be expressed as

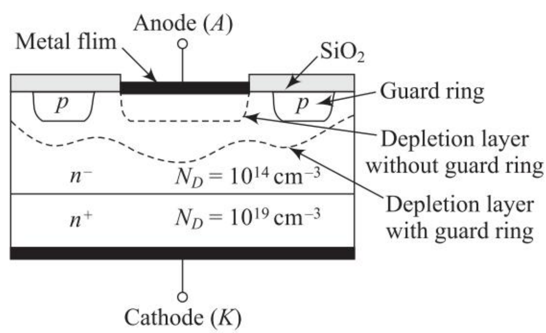


Fig. 2.11 *Structure of Schottky diode with guard ring*

$$i = I_o\left(e^{\frac{qV_D}{kT_j}} - 1\right) \qquad (2.5)$$

where, i is the current flows through diode in A

V_D is the voltage across diode with anode is positive with respect to cathode in V

I_O is the reverse saturation current or leakage current in A

T_j is the junction temperature in Kelvin

Schottky diodes have large reverse leakage current compared to a *pn*-junction diode. The reverse breakdown voltage of Schottky diodes is about 100 V to 200 V. The current rating of Schottky diode is about 1 A to 400 A. This diode is used in high current and low voltage dc power supply. The difference between Schottky diodes and *pn*-junction diodes is given in Table 2.2.

Table 2.2 *Difference between Schottky diode and pn-junction diode*

Schottky diodes	pn-junction diodes
Schottky diode is a majority carrier device and it has high operating frequency.	The *pn*-junction diode is minority carrier device it has low operating frequency.
During reverse biased condition of a Schottky diode, a large leakage current flows from cathode to anode due to minority carrier.	In reverse biased condition of *pn* diode, a small leakage current flows from cathode to anode.
The on state voltage drop across diode is small about 0.3 V to 0.5 V.	The on state voltage drop across diode is about 1 V.
Due to low on state voltage drop, Schottky diodes are less power loss compared to a *pn*-junction diode.	Power loss of *pn*-junction diode is more than Schottky diodes.

2.9 FAST RECOVERY POWER DIODES

The reverse recovery time (t_{rr}) of general-purpose power diodes is greater than 25 μs. Due to large reverse recovery time (t_{rr}), the general-purpose power diodes cannot be used in high frequency power electronics circuits such as choppers, inverters, switch mode power supply (SMPS), resonant converters and uninterrupted power supply (UPS), among others. In the high frequency power electronics circuits, the reverse recovery time must be in the range of 25-100 ns.

The fast recovery diode is formed when a highly doped p^+-type and n^+-type semiconductor layers are sandwitched by a lightly doped n^- semiconductor layer. For very fast switching, reverse recovery time will be minimised when the storage charge is reduced in n^- region and the carrier lifetime is minimised. The recovery time can be controlled by platinum or gold diffusion. The maximum rating of fast recovery power diodes is about 3 kV and 1 kA and the reverse recovery time (t_{rr}) is about 100 ns.

2.10 POWER DIODE RATINGS

The rating of any power semiconductor device is a value which can represent a voltage, current and power handling capability or a limiting condition of the device. The limiting conditions of any power semiconductor device should have a either maximum or minimum value. These values can be determined for a specified environment and operation and can be represented by standard terms. A power diode has different specified voltage and current ratings which are explained below.

2.10.1 Voltage Ratings

1. Non-repetitive peak reverse voltage, V_{RSM} This is the maximum allowable instantaneous reverse voltage across power diode including all non-repetitive transients and its duration must be less than 10 ms. The non-repetitive peak reverse voltage is generated by power line switching such as opening or closing of circuit breaker.

2. Repetitive peak reverse voltage, V_{RRM} It is the maximum allowable instantaneous reverse voltage including transients which occur every cycle and its duration must be less than 10 ms when duty cycle is about ≤ 0.01s. The repetitive peak reverse voltage is applied in converter circuit for commutation of power semiconductor devices such as thyristor and GTO.

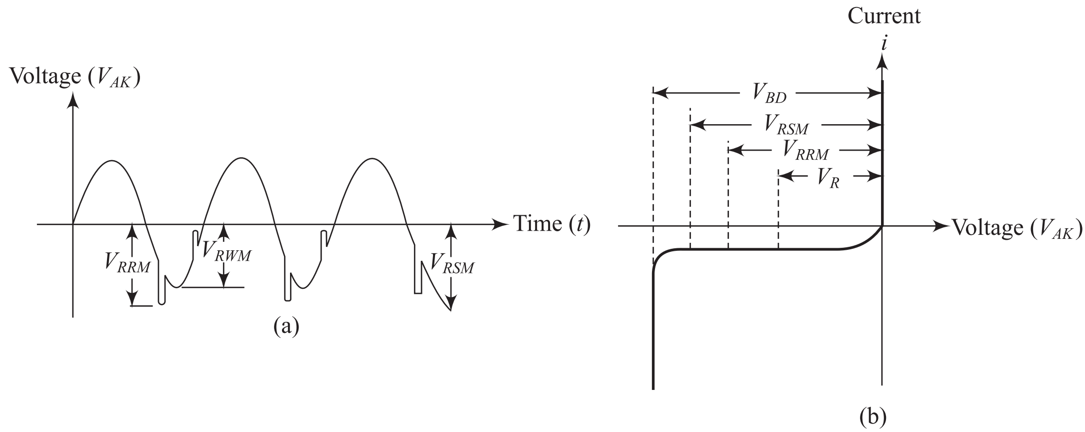


Fig. 2.12 *Voltage rating of power diode (a) Input voltage waveform and (b) Reverse I-V characteristics*

3. Crest working reverse voltage, V_{RWM} This is the maximum allowable instantaneous reverse voltage including transients which may be applied on every cycle excluding all repetitive and non-repetitive transients. The crest working reverse voltage is very useful for selecting diodes for uncontrolled and controlled rectifiers.

4. Continuous reverse voltage, V_R This is the maximum allowable constant reverse voltage. To prevent thermal runaway, junction temperature must be below T_j max and the amplitude of V_R may be limited. This is used for selecting free-wheeling diodes in dc-to-dc converters and dc-to-ac voltage source inverter circuits.

2.10.2 Current Ratings

1. Average forward current, $I_{F(AV)}$ This is the maximum average current which may be passed through the device without exceeding T_j max for either square or sinusoidal current waveforms.

2. Root mean square current, $I_{F(RMS)}$ It is the rms value of a current waveform and it is the value which causes the same power dissipation as the equivalent dc value.

3. Repetitive peak forward current, I_{FRM} This is the maximum allowable peak forward current including transients which occur every cycle. During repetitive current transients, the junction temperature should not be more than T_j max.

4. Non-repetitive forward current, I_{FSM} It is the maximum allowable peak forward current which may be applied no more than 100 times in the life of the device.

5. Repetitive peak reverse current, I_{RRM} This is the maximum allowable peak reverse current including transients which occur every cycle.

6. Non-repetitive reverse current, I_{RSM} It is the maximum allowable peak reverse current which may be applied no more than 100 times in the life of the device.

Summary

- The structure and characteristics of power diode are different from those of ideal diodes. The reverse recovery time is very important for high speed switching applications.
- Diodes are classified as general purpose diodes, Schottky diodes and fast recovery diodes. The operation of power diode, schottky diodes and fast recovery diodes are explained in detail.

Multiple-Choice Questions

2.1 A power diode is a ______ type-three layer device

(a) $p^+ n^- n^+$ (b) $p^+ n^+ n^-$ (a) $n^+ p^- p^+$ (a) $n^+ p^+ n^-$

2.2 The breakdown voltage of non-punch though power diode is

(a) $\dfrac{\varepsilon E_{BD}^2}{2qN_A}$ (b) $\dfrac{\varepsilon E_{BD}^2}{2q(N_A + N_D)}$ (c) $\dfrac{\varepsilon E_{BD}^2}{2q(N_A - N_D)}$ (d) $\dfrac{\varepsilon E_{BD}^2}{2qN_D}$

2.3 The breakdown voltage of punch though power diode is

(a) $E_{BD}W + \dfrac{qN_D W^2}{2\varepsilon}$ (b) $E_{BD}W - \dfrac{qN_D W^2}{2\varepsilon}$

(c) $E_{BD}W + \dfrac{qN_A W^2}{2\varepsilon}$ (d) $E_{BD}W - \dfrac{qN_A W^2}{2\varepsilon}$

2.4 The reverse recovery time of a power diode is defined as

(a) the time between the instant forward diode current $I_F = 0$ and the instant reverse recovery current decays to 25% of its peak value.

(b) the time between the instant forward diode current $I_F = 0$ and the instant reverse recovery current decays to 50% of its peak value.

(c) the time between the instant forward diode current I_F is maximum and the instant reverse recovery current decays to 25% of its peak value.

(d) the time between the instant forward diode current I_F is maximum and the instant reverse recovery current decays to 50% of its peak value

2.5 The peak of reverse recovery current of a power diode depends on

(a) storage charge (b) rate of current flow and storage charge

(c) peak inverse voltage (d) temperature

2.6 Reverse recovery time of a power diode is

(a) $t_{rr} = \sqrt{\dfrac{2Q_R}{di/dt}}$ (b) $t_{rr} = \sqrt{\dfrac{2Q_R}{dv/dt}}$ (c) $t_{rr} = \sqrt{\dfrac{2di/dt}{Q_R}}$ (d) $t_{rr} = \sqrt{\dfrac{2dv/dt}{Q_R}}$

2.7 Compared to general purpose power diode, Schottky diode has

(a) lower cut-in voltage (b) higher operating frequency

(c) higher reverse leakage current (d) All of these

2.8 The softness factor is

(a) $\dfrac{t_b}{t_a}$ (b) $\dfrac{t_a}{t_b}$ (c) $1 + \dfrac{t_a}{t_b}$ (d) $1 + \dfrac{t_b}{t_a}$

2.9 The high frequency operation of a power semiconductor circuit is limited by

(a) on-state loss (b) off-state loss (c) switching loss (d) All of these

2.10 For conduction of Schottky diode

(a) electrons can participate (b) holes can participate

(c) both holes and electrons can participate (d) None of these

2.11 The cut-in voltage of Schottky diode is

(a) greater than the cut-in voltage of *pn*-diode

(b) less than the cut-in voltage of *pn*-diode

(c) equal the cut-in voltage of *pn*-diode

(d) None of these

2.12 The maximum reverse voltage of a power diode is known as

(a) peak inverse voltage (b) breakdown voltage

(c) continuous reverse voltage (d) non-repetitive reverse voltage

Fill in the Blanks

2.1 Power diode is a ______ carrier device.
2.2 A Schottky diode turn on and turn off time is ______ than a *pn*-junction diode.
2.3 Reverse recovery time of a fast recovery diode is ______.
2.4 The softness factor for a soft recovery diode is ______.
2.5 The softness factor for a fast recovery diode is ______.
2.6 The cut-in voltage for power diode is equal to ______.
2.7 The forward voltage drop of a power diode is about ______.
2.8 Schottky diode is a ______ carrier device.
2.9 Power diode is a two terminal ______ layer semiconductor device.
2.10 The I-V characteristics of a power diode at high forward current is ______.
2.11 Power diodes has ______ doped n^--epitaxial layer which can absorb the depletion layer during ______ biased.
2.12 When the width of ______ region in a power diode increases, the reverse voltage blocking capability increases.
2.13 The reverse break down voltage of a power diode is more than ______.
2.14 The doping density in n^--drift region of a power diode is _______.

Review Questions

2.1 What is power diode? Write the difference between general purpose diode and power diode.
2.2 Draw the structure of a power diode and explain its operating principle briefly.
2.3 What are the types of power diodes?
2.4 What is the reverse recovery time of a diode?
2.5 Explain the conductivity modulation in the power diode.
2.6 Draw the V-I characteristics of a power diode. What is the effect of junction temperature on the V-I characteristics of a power diode?
2.7 What is the breakdown voltage of non-punch through diode?
2.8 What is the breakdown voltage of punch through diode?
2.9 What is softness factor of diodes?
2.10 Draw the structure of Schottky diode and explain briefly.
2.11 What are the main differences between general purpose *pn*-junction diode and Schottky power diode?
2.12 Why power diodes are connected in series? What are the different problems of series connected power diodes? How these problems are solved?
2.13 Why power diodes are connected in parallel? What are the different problems of parallel connected power diodes? What are the possible solutions?
2.14 What is fast recovery diode? Draw the reverse recovery characteristic of a diode.
2.15 What is the importance of series parallel operation of power diodes? Explain the series and parallel operation of power diodes with proper diagram.
2.16 The reverse recovery current of a power diode is 4 μs and the rate of fall of current is about 25 A/μs. Calculate (a) the storage charge and (b) the peak reverse current I_{RR}.

Answers to Multiple-Choice Questions

2.1 (a)	2.2 (d)	2.3 (b)	2.4 (a)	2.5 (b)	2.6 (a)	2.7 (d)
2.8 (a)	2.9 (c)	2.10 (a)	2.11 (b)	2.12 (a)		

Answers to Fill in the Blanks

2.1 minority	2.2 faster	2.3 25-100 ns	2.4 1
2.5 <1	2.6 0.7 V	2.7 1 V	2.8 majority
2.9 three	2.10 linear	2.11 lightly, reverse	2.12 n^- drift
2.13 non-repetitive peak reverse voltage V_{RSM}		2.14 10^{14} cm^{-3}	

POWER TRANSISTOR

3

3.1 INTRODUCTION

Power diode is an uncontrolled device as the turn-on and turn-off characteristics of power diodes are not controllable. Power transistors are controlled device as the turn-ON and turn-OFF characteristics of power transistor are controllable. When a current signal is applied to base of the bipolar junction transistor (BJT), the device will operate in ON state as long as the control signal is present. When the control signal is removed from base of BJT, the power transistor will be turned OFF and operate in OFF-state. Similarly, when a voltage signal is applied to gate of MOSFET, the device will be ON and if a voltage signal is removed from gate, MOSFET operates in OFF state. Generally, power transistors are four types namely,

1. Bipolar junction transistor (BJT)
2. Metal oxide semiconductor field-effect transistor (MOSFET)
3. Insulated gate bipolar transistor (IGBT) and
4. Static induction transistor (SIT)

In this chapter, the structure of power BJT, *I-V* characteristics, second breakdown, ON-state loss, safe-operating area and switching characteristics are discussed. The basic operation of *n*-channel and *p*-channel MOSFET and *I-V* characteristics are explained. The structure of power MOSFET, *I-V* characteristics, ON-state loss, safe-operating area and switching characteristics are incorporated in this chapter. The structure, operating principle, and *I-V* characteristics of IGBT and SIT are discussed elaborately. In this chapter, the base drive circuits of BJT and gate drive circuits of MOSFET and IGBT are also explained in detail.

3.2 STRUCTURE OF POWER BIPOLAR JUNCTION TRANSISTOR

Figure 3.1 shows an *n-p-n* power transistor which is a four layer $n^+ p\, n^- n^+$ structure. This transistor has three terminals such as collector (*C*), base (*B*) and emitter (*E*). Base is used as an input terminal, and collector is used as the output terminal. In common emitter configuration, emitter is common between input and output terminals. The width of emitter, n^+ layer is about 10 μm and its doping intensity is $N_A = 10^{19}$ cm^{-3}. Usually, the base thickness or width of p layer is about 5 to 20 μm and the doping density of p-type semiconductor materials is moderate and it is about $N_A = 10^{16}$ cm^{-3}. The width of n^- layer or collector drift region is about 50 μm to 200 μm and the doping density is minimum in drift region and its value is about $N_A = 10^{14}$ cm^{-3}. The width of n^+ collector region is maximum and it is about 250 μm. The doping density of N^+ type semiconductor layer is $N_A = 10^{19}$ cm^{-3}. The doping density of semiconductor materials and width of each layer are depicted in Fig. 3.1(b).

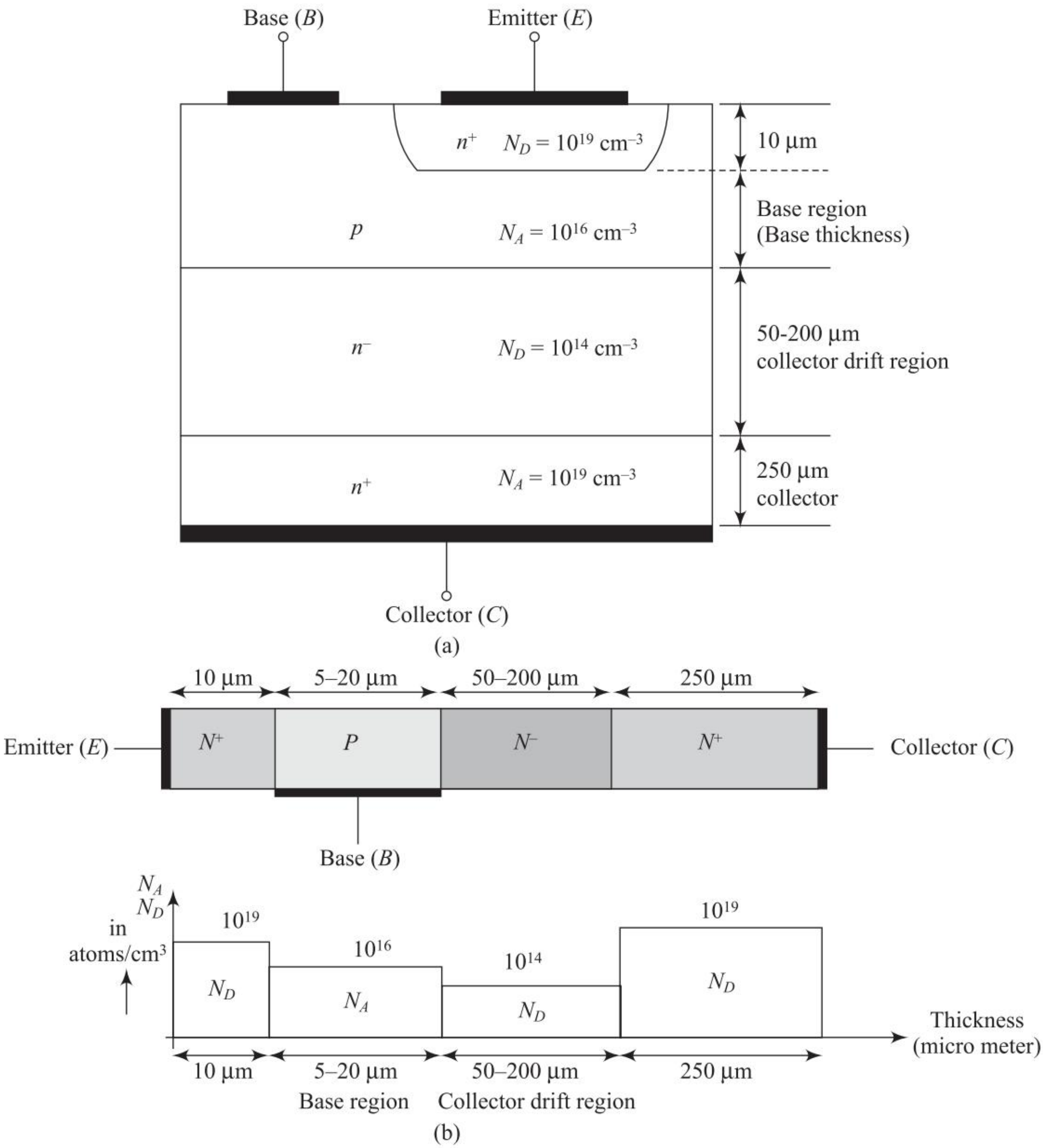
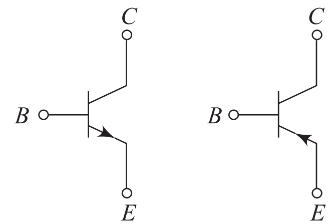


Fig. 3.1 *(a) Structure of n-p-n-type power BJT and (b) Doping density of semiconductor materials*

The base thickness of bipolar junction transistor (BJT) is as small as possible to provide good amplification capability. But due to very small base thickness, the breakdown voltage capability of BJT is reduced. Therefore, the base thickness of power transistors must be more compared with the logic level transistors to increase breakdown voltage capability. In case of logic level BJTs, the base thickness is a small fraction of micrometer. But the base thickness of power transistors should varies with in few tens of micrometers. The breakdown voltage of the power transistors is also depending on the thickness of the collector drift region. As a result, the thickness of collector drift region varies from 50 μm to 200 μm depending upon the break down voltage.

Usually, the vertical structure of power transistors is used in manufacturing as this structure provides the maximum cross-sectional area through which the current flows with in the device and the current density per unit area becomes minimum. Consequently, the on-state resistance as well as the power loss of power transistors will be minimised. Due to large cross-sectional area, the thermal resistance of power transistors becomes low and the power dissipation problems can be reduced.

For a *p-n-p* transistor, the four layer structure of power transistors will be $p+$ n p^- $p+$ and the doping level will be opposite of *n-p-n* transistor. In power electronics circuits, *n-p-n* transistors are most commonly used as power switches. The circuit symbols of *n-p-n* and *p-n-p* transistors are given in Fig. 3.2.



(a) (b)

Fig. 3.2 *(a) Circuit symbol of n-p-n transistor and (b) Circuit symbol of p-n-p transistor*

The current gain of a power transistor is $\beta = \dfrac{I_C}{I_B}$ and its value is small about 5 to 10. To increase the current gain, Darlington pair of BJTs is used. Figure 3.3 shows the monolithic design of a Darlington pair of power BJTs where,

A = auxiliary transistor
M = main transistor
I_{BA} = base current of auxiliary transistor
I_{CA} = collector current of auxiliary transistor
I_{EA} = emitter current of auxiliary transistor
I_{BM} = base current of main transistor
I_{CM} = collector current of main transistor

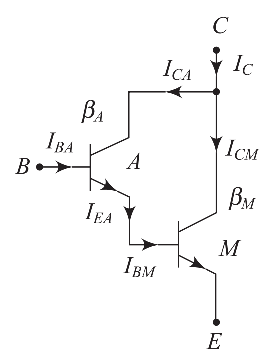


Fig. 3.3 *Darlington pair of power BJTs*

The relationship between collector current I_{CA} and base current I_{BA} of the auxiliary transistor is

$I_{CA} = \beta_A I_{BA}$ where, β_A is current gain of auxiliary transistor.

The emitter current of auxiliary transistor I_{EA} is equal to the base current I_{BM} of the main transistor. Therefore, $I_{BM} = (1 + \beta_A)I_{BA}$

Then the collector current of main transistor is equal to

$I_{CM} = \beta_M I_{BM} = \beta_M(1 + \beta_A)I_{BA}$ where, β_M is current gain of main transistor

The total load current I_C is sum of I_{CA} and I_{CM}.

Then, $I_C = I_{CA} + I_{CM} = \beta_A I_{BA} + \beta_M(1 + \beta_A)I_{BA} = (\beta_A + \beta_M + \beta_M\beta_A)I_{BA}$

Consequently, the new current gain $\beta = \dfrac{I_C}{I_{BA}}$ will be

$$\beta = \beta_M\beta_A + \beta_M + \beta_A$$

where, β_M is current gain of main transistor, β_A is current gain of auxiliary transistor.

Since the values of β_M and β_A are relatively large, the product will be very large compared to β_M and β_A. Subsequently the approximate value of β is about $\beta_M \beta_A$, i.e., the product of the current gains of individual transistors. Therefore, the Darlington pair of power BJTs requires very much smaller current control for its operation compared to single transistor but the switching time of the Darlington pair of power BJTs is more than the switching time of a single transistor. Figure 3.4 shows the vertical cross section of a pair of monolithic Darlington BJTs and its representation is depicted in Fig. 3.5. The diode D_1 is used to fast turn-off of the main transistor and the diode D_2 is used for half-bridge and full-bridge inverter circuit applications.

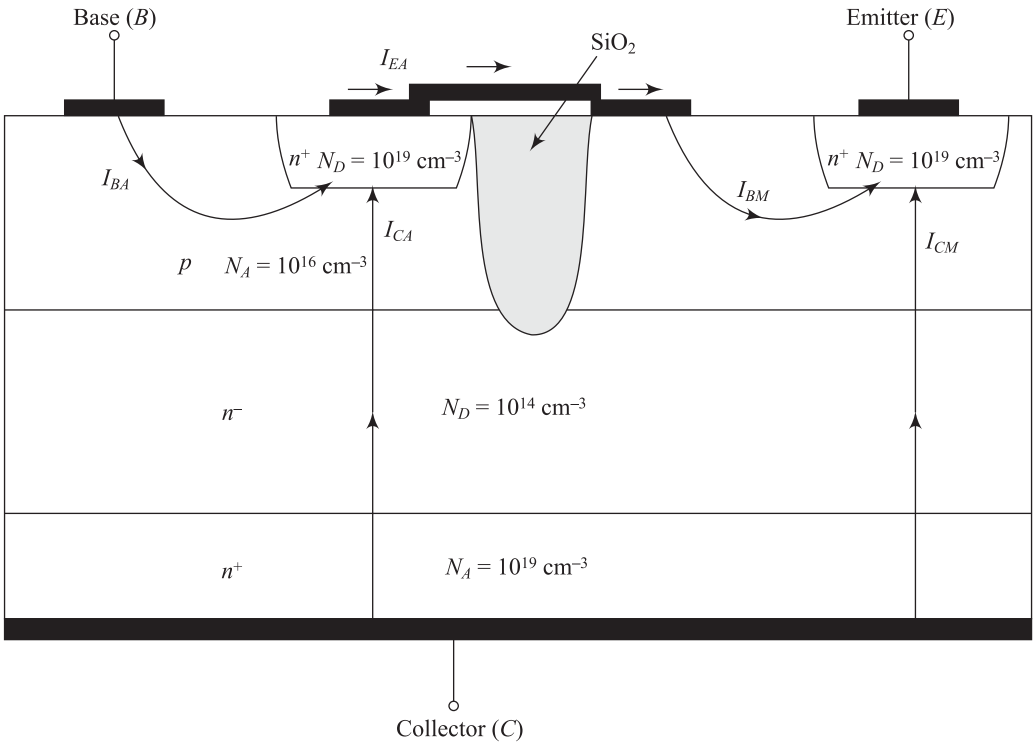


Fig. 3.4 *Structure for Darlington pair of Power BJTs*

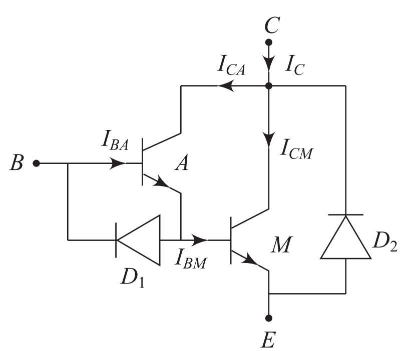


Fig. 3.5 *Darlington pair of power BJTs with diodes D_1 and D_2*

3.3 *I-V* CHARACTERISTICS

The output characteristics of a *n-p-n* power transistor are the plot of i_C versus V_{CE} is shown in Fig. 3.6. Actually, all features of the power transistor characteristics such as cut-off region, active region and saturation region can be found in low level logic devices. But the major difference between I-V characteristics of a power transistor and a low signal or logic level transistor is the *quasi-saturation* which is present in power transistors. The quasi saturation is occurred due to the lightly doped collector drift n^- region in the structure of power transistors. The low level signal transistors have no drift region.

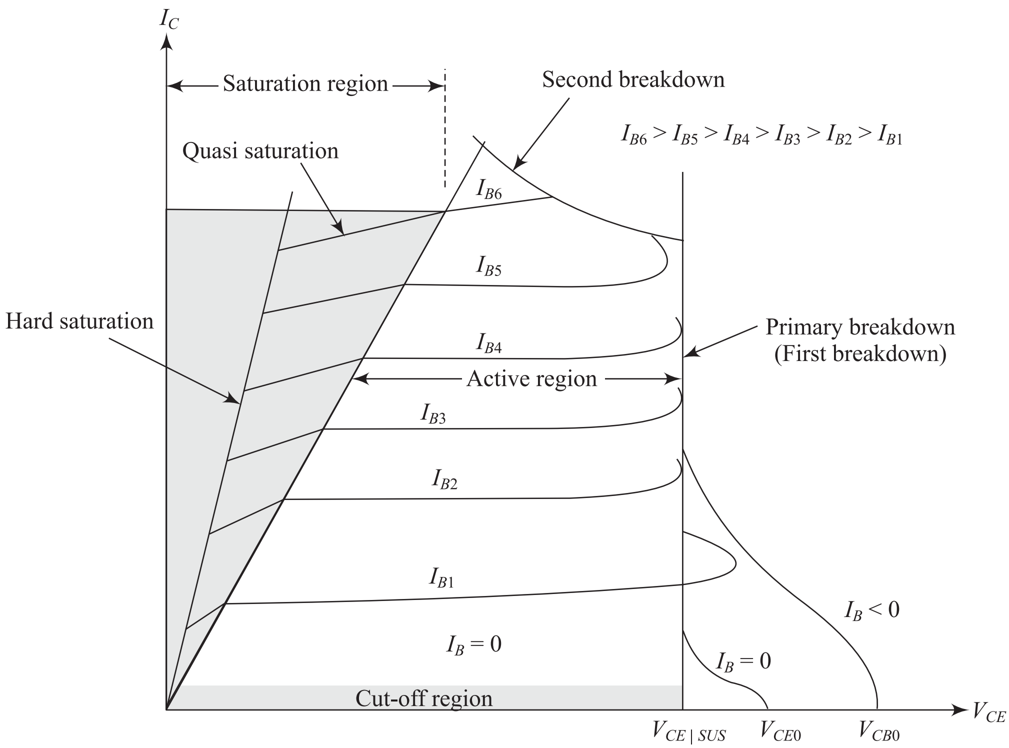


Fig. 3.6 *I-V characteristics of an n-p-n power BJT*

3.3.1 Quasi Saturation

To discuss the quasi-saturation, a collector drift region has been incorporated in the structure of power BJT as shown in Fig. 3.7(a). Initially, the transistor operates in the active region and the base current starts to increase. Due to increase in base current, the collector current will be increased and the collector emitter voltage (V_{CE}) is reduced due to increase voltage drop across the collector load resistance, R_L. Consequently, the voltage drop in the drift region is increased, and the ohmic resistance drop is also increased as the collector current (I_C) is increased. Therefore, the reverse bias voltage across collector base junction, n^-p junction becomes smaller or reduced and the junction may be forward biased at several points.

When n^-p junction is forward biased at several points, holes are injected from the base to the collector n^- drift region. For the space charge neutrality, electrons must be injected into the drift region with the same number of holes. Actually, the large numbers of electrons are supplied to the collector-base junction through injection from the emitter and subsequent diffusion across the base. Then the excess carriers can be build up in the drift region and subsequently the *quasi-saturation region* is developed in I-V characteristics of an *n-p-n* power BJT.

When the *ohmic resistance* of the n^- drift region is R_d , the collector current is equal to

$$I_C = \frac{V_{CE}}{R_d}$$

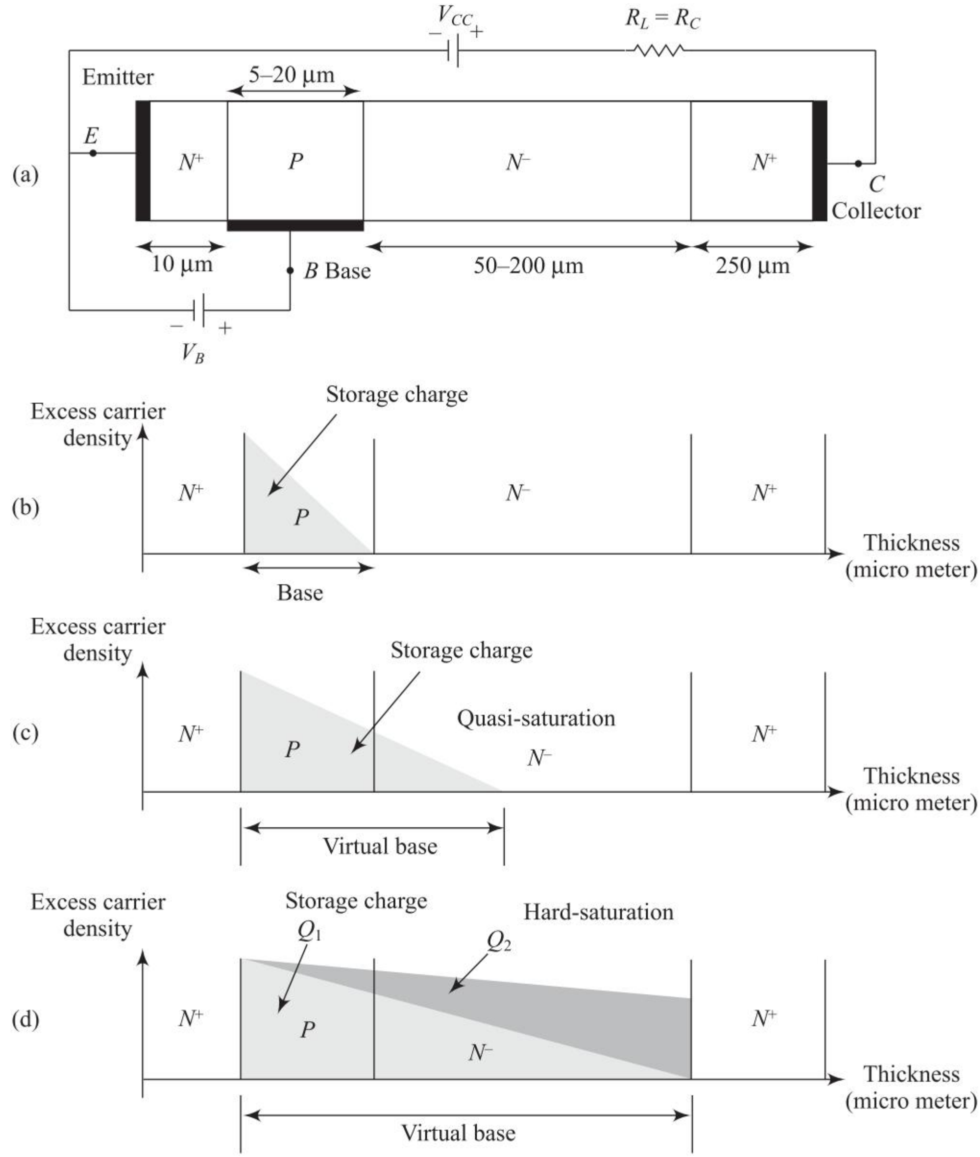


Fig. 3.7 *(a) The biasing of n-p-n power BJT, (b) Storage charge in the base region in active mode, (c) Storage charge in the base region in quasi-saturation and (d) Storage charge in the base region in hard saturation*

In quasi saturation, the *double injection* can take place in the drift region in the same way as in the drift region of a forward biased power diode. The stored charge is accumulated in the n^- drift region from one side of drift region, i.e., the collector-base pn^- junction. The electron injection across the n^-n^+ junction is very less measurable as there are more abundant electrons at the pn^- junction. If the injected carriers increase, the n^- drift region is shorted progressively and the voltage across the drift region is reduced even though the collector current (I_C) is large.

It is clear from Fig. 3.7(c) that the hole injection from the base to the collector-base (pn^-) junction has been started, the thickness of the virtual base is increased. Therefore, the effective value of β decreases and the magnitude of collector current for a specified base current must be decreased. In the quasi-saturation, the n^- drift region can not be completely shorted out through high-level injection. As a result the power dissipation of BJT in the quasi-saturation is greater larger than the power dissipation during hard saturation.

3.3.2 Hard Saturation

The *hard saturation* occurs when the excess carrier density reaches the end of n^+ drift region. For hard saturation, it requires a minimum amount of stored charge Q_1 in the virtual base region as shown in Fig. 3.7(d). Then the effective base thickness is increased and it is roughly the sum of the normal base thickness and the length of the n^- drift region. It is clear from Fig. 3.7(d) that the additional stored charge Q_2 can drive the transistor more into hard saturation. During hard saturation, the voltage drop across the n^- drift region is small and the on state power dissipation is minimized compared to quasi saturation of power transistor.

When the transistor operates in the saturation region, the collector current I_{CS} is equal to

$$I_{CS} = \frac{V_{CC} - V_{CES}}{R_C} \text{ and}$$

the minimum base current required to operate in saturation is $I_{BS} = \frac{I_{CS}}{\beta}$.

When the base current is less than I_{BS}, the transistor operates in active region. While base current I_B is very high ($I_B > I_{BS}$), the collector current $I_{CS} = \frac{V_{CC}}{R_C}$ as $V_{\text{CES}} \approx 0$. With the more base current, transistor operated in *hard saturation*. The ratio of I_B and I_{BS} is known as overdrive factor (ODF) and it can be expressed as

$$ODF = \frac{I_B}{I_{BS}}$$

The value of ODF is about 4 or 5. The forced current gain β_f is the ratio of I_{CS} to I_B. The value of β_f is always less than β.

The total power loss in two junctions of power transistor is

$$P_{\text{Total–loss}} = V_{BE}I_B + V_{CE}I_C$$

The other important features of I-V characteristics of monolithic Darlington pair power transistors are:

1. $V_{CE|\text{sus}}$ When a sustainable collector current flows through power transistor, this device can able to withstand or sustained up to a maximum collector-emitter voltage. This maximum voltage across collector-emitter is represented by $V_{CE|_{\text{SUS}}}$ as depicted in Fig. 3.6.

2. V_{CE0} When the base is open circuit, the base current is zero ($I_B = 0$) and the device operate in cut-off region, the power bipolar junction transistor can able to sustain up to a maximum collector emitter voltage V_{CE0} as shown in Fig. 3.6. The voltage V_{CE0} is called collector emitter breakdown voltage when the base current is zero.

3. V_{CB0} This is collector-base break down voltage when the emitter is open circuit. The value of V_{CB0} is greater than V_{CE0} as depicted in Fig. 3.6.

In the common emitter configuration of power BJT, the breakdown voltage V_{CE0} is smaller than the breakdown voltage V_{CB0}. The relationship between V_{CE0}, V_{CB0} and β can be expressed as

$V_{CE0} = \dfrac{V_{CB0}}{\beta^{\frac{1}{n}}}$ where, $n = 4$ for NPN transistors and $n = 6$ for PNP transistors.

It is clear from the above expression that the transistor with high breakdown voltage has small values of beta. The value of beta for high voltage NPN transistors varies in the range 10 to 20. When $\beta = 15$ and $n = 4$ for a typical NPN transistor, $V_{CE0} = \dfrac{V_{CB0}}{15^{\frac{1}{4}}} = \dfrac{V_{CB0}}{1.967}$ and the value of V_{CE0} is about one-half of V_{CB0}.

4. Primary breakdown The primary breakdown of a power transistor is same as conventional avalanche breakdown of collector-base junction due to large current flow. The primary breakdown should be avoided due to large power dissipation with in the device. This breakdown is also known as first breakdown. Actually, the primary breakdown is the limit of collector emitter voltage at which power BJT can sustain with a high collector current. When the junction temperature within the safe limit, transistor will not be damaged and the device can get its original position if both collector current and collector-emitter voltage are reduced.

5. Second breakdown When the power transistor operates in active region with a large collector current, there is voltage drop across collector-emitter junction. If suddenly collector current increases significantly, there is large power dissipation within the device. This situation is very dangerous as the power dissipation is non-uniformly spread over the entire volume of the power BJT. Consequently, local hot-spots are developed within the device and the transistor's breakdown occurs and the device may be completely damaged. This breakdown is called *second breakdown* as shown in Fig. 3.13. The second breakdown can be avoided by using the *snubber circuit*, maintaining the instantaneous voltage and current with in forward biased safe operating area, and controlling the overall power dissipation within the device.

3.4 ON STATE LOSS OF POWER BJT

When the power bipolar junction transistor operates at low and medium operating frequency but not in high operating frequency, the power dissipation within the device is equal to the power loss during ON state. To determine on state loss, it is assumed that the device operates in hard saturation region. The power dissipation of BJT can be expressed as

$$P = V_{CE} I_C$$

where, V_{CE} is the collector emitter saturation voltage during hard saturation, I_C is the collector current and the collector-emitter saturation voltage V_{CE} increases with increasing collector current I_C.

Figure 3.8 shows the different internal voltage drops in a power transistor. These voltage drops can be used to determine the on state voltage drop across collector emitter of power BJT and the V_{CE} can be expressed as

$$V_{CE} = V_{BE} - V_{BC} + V_D + I_C (R_E + R_C)$$

The V_{BE} is the voltage appearing across the forward biased base-emitter junction and the V_{BC} is the voltage across the forward biased collector-base junction. The V_{BE} and V_{BC} voltages differ from each other by 0.1 V to 0.2 V. The collector base junction has very large area compared to the base-emitter junction but the doping levels across the collector-base junction are very lower than the doping levels across the base-emitter junction. The voltage difference $V_{BE} - V_{BC}$ does not dependent on collector current.

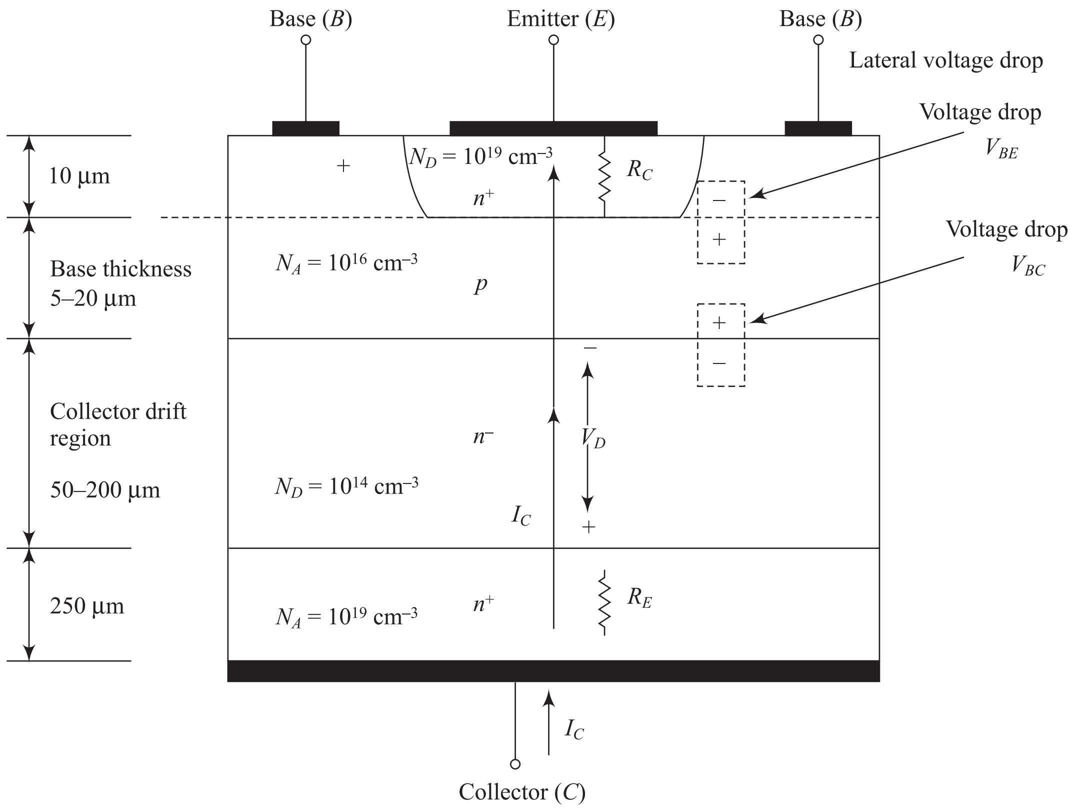


Fig. 3.8 *ON- state collector-emitter voltage of a power transistor*

The resistance R_E is the ohmic resistance of the heavily doped emitter and R_C is the ohmic resistance of the heavily doped collector region. At low and reasonable collector currents, the voltage drops across R_E and R_C are insignificant. When a large current flows though the device, the voltage drops across R_E and R_C are significant. V_D is the voltage drop across the collector drift region. The value of V_D is low due to conductivity modulation and it is independent of collector current.

Example 3.1 A bipolar transistor as shown in Fig. 3.9 has $\beta = 30$ and the load resistance $R_C = 15\ \Omega$. The dc supply voltage $V_{CC} = 200$ V and the input voltage to base is $V_B = 10$ V. When $V_{CE(\text{saturation})} = 1.2$ V and $V_{BE(\text{saturation})} = 1.5$ V, determine (a) the value of resistance R_B so that transistor operates in saturation and (b) power loss in the transistor.

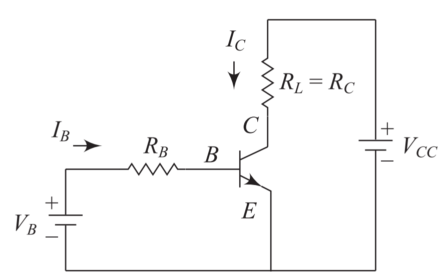


Fig. 3.9

Solution

Given $\beta_f = 30$, $R_C = 15\ \Omega$, $V_{CC} = 200$ V, $V_B = 10$ V, $V_{CE(\text{saturation})} = 1.2$ V, $V_{BE(\text{saturation})} = 1.5$ V

(a) When the transistor operates in saturation, load current is

$$I_{CS} = \frac{V_{CC} - V_{CE(\text{saturation})}}{R_C} = \frac{200 - 1.2}{15} = 13.253 \text{ A}$$

The base current that drives the transistor in saturation is equal to

$$I_{BS} = \frac{I_{CS}}{\beta_f} = \frac{13.253}{30} = 0.4177 \text{ A}$$

The value of resistance R_B so that transistor operates in saturation is equal to

$$R_B = \frac{V_B - V_{BE(\text{saturation})}}{I_{BS}} = \frac{10 - 1.5}{0.4177} = 20.349\ \Omega$$

(b) The power loss in transistor is

$$P_{\text{Total-loss}} = V_{BE}I_B + V_{CE}I_C = V_{BE(\text{saturation})}I_{BS} + V_{CE(\text{saturation})}I_{CS}$$
$$= 1.5 \times 0.4177 + 1.2 \times 13.253 \text{ Watt} = 16.53 \text{ Watt}$$

Example 3.2 A bipolar transistor as depicted in Fig. 3.9 has β in the range 10 to 40 and the load resistance $R_C = 10\ \Omega$. The dc supply voltage $V_{CC} = 150$ V and the input voltage to base is $V_B = 12$ V. When $V_{CE(\text{saturation})} = 1.0$ V and $V_{BE(\text{saturation})} = 1.5$ V, determine (a) the value of resistance R_B so that transistor operates in saturation with $ODF = 4$, (b) forced current gain and (c) power loss in the transistor.

Solution

Given $\beta_{\min} = 10$, $\beta_{\max} = 40$, $R_C = 10\ \Omega$, $V_{CC} = 150$ V,
$V_B = 12$ V, $V_{CE(\text{saturation})} = 1.0$ V, $V_{BE(\text{saturation})} = 1.5$ V

(a) When the transistor operates in saturation, load current is

$$I_{CS} = \frac{V_{CC} - V_{CE(\text{saturation})}}{R_C} = \frac{150 - 1.0}{10} = 14.9 \text{ A}$$

The base current that drives the transistor in saturation is equal to

$$I_{BS} = \frac{I_{CS}}{\beta_{\min}} = \frac{14.9}{10} = 1.49 \text{ A}$$

The base current with $ODF = 4$ is

$$I_B = ODF \times I_{BS} = 4 \times 1.49 \text{ A} = 5.96 \text{ A}$$

The value of resistance R_B so that transistor operates in saturation with $ODF = 4$ is equal to

$$R_B = \frac{V_B - V_{BE(\text{saturation})}}{I_{BS}} = \frac{10 - 1.5}{5.96} = 1.426\ \Omega \quad \text{as } V_{BE(\text{saturation})} = 1.5 \text{ V}$$

(b) Forced current gain is $\beta_{\text{forced}} = \dfrac{I_{CS}}{I_B} = \dfrac{14.9}{5.96} = 2.5$

(c) The power loss in transistor is

$$P_{\text{Total–loss}} = V_{BE}I_B + V_{CE}I_C$$
$$= 1.5 \times 5.96 + 1.0 \times 14.9 \text{ Watt} = 23.84 \text{ Watt}$$

3.5 SAFE OPERATING AREA OF POWER BJT

The safe operating area (SOA) represents the maximum values of current and voltage at which the power BJT can be withstand safely. As per manufacture datasheet related to specification of power BJT, there are two separate SOAs namely

1. Forward bias safe operating area (FBSOA) and
2. Reverse bias safe operating area (RBSOA).

Figure 3.10 shows the forward bias safe operating area (FBSOA) of power transistor where current I_C and voltage V_{CE} are represented by logarithmic scale. The term forward biased states that the base-emitter junction is forward biased and base current flows from source to base.

The boundary of the FBSOA depends on the operation in continuous dc signal and a pulse signal of different frequency. In Fig. 3.10, I_{Cmax} represents the maximum collector current even when a pulse signal is applied to the power transistor. If the device operates with dc signal, the boundary of the FBSOA is A-B-C-D-E. The boundary A-B represents the maximum limit of collector current I_{Cmax} with V_{CE} is less than 100 V. When V_{CE} is greater than 100 V, the collector current has been reduced as per boundary B-C so that the operating junction temperature of power BJT must be less than the maximum operating junction temperature. For high value of V_{CE}, the collector current will be further reduced to avoid the second breakdown of power transistor and the boundary is represented by CD. The boundary D-E represents the maximum voltage capability of a power transistor.

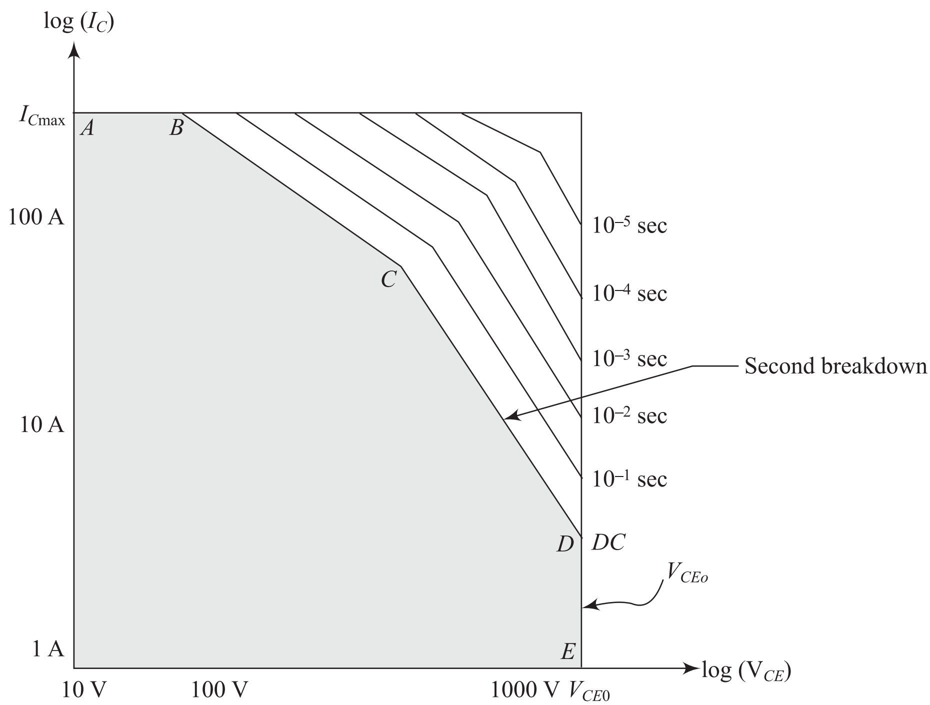


Fig. 3.10 *FBSOA of a power BJT*

When the power transistor operates as a switch and it is driven by a pulse signal, the boundaries of the FBSOA can be extended as shown in Fig. 3.10. The extension of the SOA is only possible for switch mode operation. Actually, the silicon wafer and its packaging of power transistor have a specified thermal capacitance and it has an ability to absorb a limited amount of energy without increasing the junction temperature excessively. When the power transistor turns on for a few microseconds, the device can absorb very small amount of energy and the junction temperature rise will be low and the boundary of FBSOA can be increased. If the pulse width is about 10^{-1} s, the FBSOA is greater than the boundary A-B-C-D-E. If the pulse width is further reduced and its value is about some μs (10^{-6} s), the FBSOA is greater than the boundary with pulse with 10^{-1} s. It is clear from Fig. 3.10 that the FBSOA increases with the decrease of pulse width.

The term reverse biased states that the base-emitter junction is reverse biased and base current flows in reverse direction. During turn-OFF of power transistor, the base-emitter junction is highly reverse biased and the transistor must withstand at high current. The reverse bias safe operating area (RBSOA) is a plot of collector current with respect to collector emitter voltage as depicted in Fig. 3.11. When the base current is zero, transistor can sustained at voltage V_{CE0}. If the base-emitter junction is reverse biased, the power transistor can able to withstand up to collector-base breakdown voltage V_{CB0} with low collector current as depicted in Fig. 3.11.

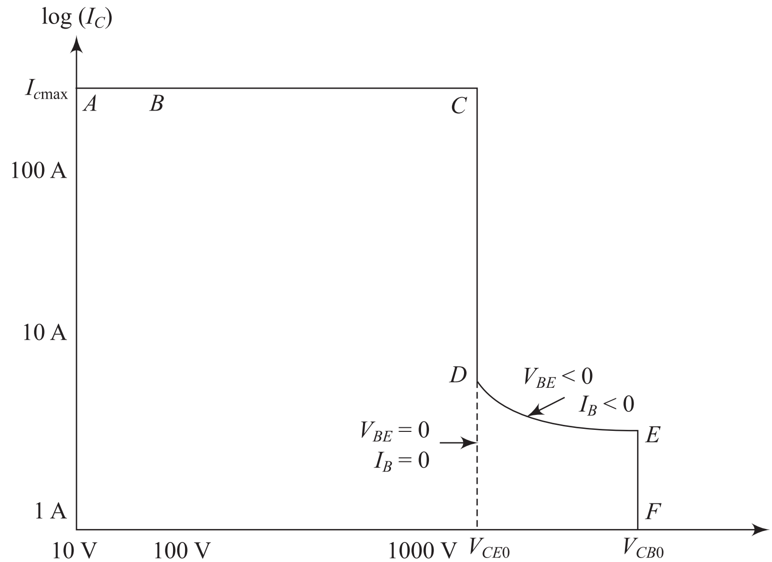


Fig. 3.11 *RBSOA of a power BJT*

3.6 SERIES AND PARALLEL OPERATION OF BJT

Power BJTs can be connected in series to increase the voltage-handling capability. It is extremely important that the series-connected power transistors must be turned on and off simultaneously. If the series-connected transistors are not turned ON and OFF simultaneously, the slowest device will be at turn-ON state but the fastest device will be at turn-OFF state. Subsequently, the turned-OFF transistor must be withstand to the full voltage of the collector-emitter. If the device cannot withstand at high voltage, the device may be completely damaged due to high voltage. Therefore, during series connection, the power BJTs should have same gain, ON-state voltage; turn-ON time, turn-OFF time, and base drive circuit.

When one power transistor cannot handle the load current, transistors may be connected in parallel. For the equal current sharing among transistors, the power BJTs should have same turn-ON time, turn-OFF time, gain, saturation voltage, and transconductance. Figure 3.12 shows the parallel connections of two BJTs. Current I_{E1} flows through transistor T_1 and current I_{E2} flows through transistor T_2. Total current I_T is shared by transistors T_1 and T_2.

Then, $I_T = I_{E1} + I_{E2}$ and $V_{CE1} + I_{E1}R_{E1} = V_{CE2} + I_{E2}R_{E2}$

Therefore, $V_{CE1} + I_{E1}R_{E1} = V_{CE2} + (I_T - I_{E1})R_{E2}$

or $$I_{E1} = \frac{V_{CE2} - V_{CE1} + I_T R_{E2}}{R_{E1} + R_{E2}}$$

About 45% to 55% current can be shared by connecting resistance in series with transistors. Actually, resistances are used for current sharing under steady-state condition. Under dynamic condition, current can be shared by connecting coupled inductors as shown in Fig. 3.12. When the current flow through transistor T_1 increases, the amplitude of $L\frac{di}{dt}$ across L_1 increases. Then a opposite polarity voltage is induced across inductor L_2. Consequently, a low impedance path is provided by transistor T_2, and the current is shifted from transistor T_1 to transistor T_2.

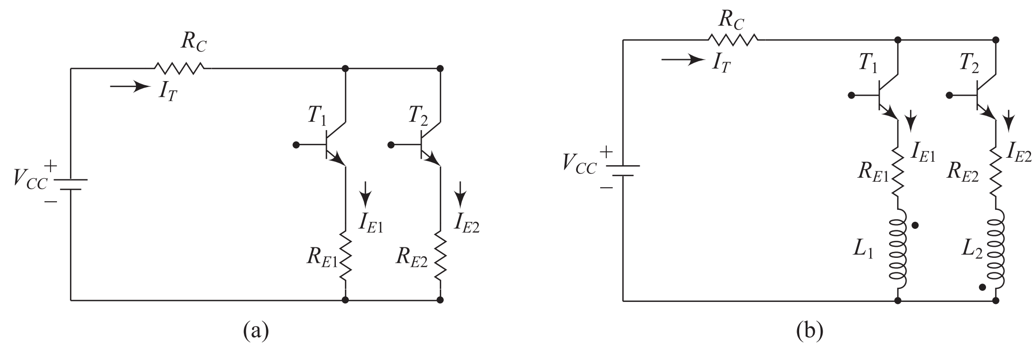


Fig. 3.12 *Parallel connection of transistors (a) Current sharing at steady-state condition (b) Current sharing at dynamic condition*

Example 3.3 Two BJTs are connected in parallel to share the total current 25 A. The collector-to-emitter voltage of T_1 and T_2 are 1.5 V and 1.75 V respectively. Determine the emitter current of each transistors and the difference of current sharing when the current sharing series resistance are (a) $R_{E1} = 0.25\ \Omega$ and $R_{E2} = 0.35\ \Omega$ and (b) $R_{E1} = 0.5\ \Omega$ and $R_{E2} = 0.5\ \Omega$

Solution

Given: $I_T = 25$ A, $V_{CE1} = 1.5$ V, $V_{CE2} = 1.75$ V

(a) When $R_{E1} = 0.25\ \Omega$ and $R_{E2} = 0.35\ \Omega$

$$I_{E1} = \frac{V_{CE2} - V_{CE1} + I_T R_{E2}}{R_{E1} + R_{E2}} = \frac{1.75 - 1.5 + 25 \times 0.35}{0.25 + 0.35} = 15 \text{ A}$$

$I_{E2} = I_T - I_{E1} = 25 - 15 = 10$ A

$\Delta I = I_{E1} - I_{E2} = (15 - 10)$ A $= 5$ A

(b) When $R_{E1} = 0.5\ \Omega$ and $R_{E2} = 0.5\ \Omega$

$$I_{E1} = \frac{V_{CE2} - V_{CE1} + I_T R_{E2}}{R_{E1} + R_{E2}} = \frac{1.75 - 1.5 + 25 \times 0.5}{0.5 + 0.5} = 12.75 \text{ A}$$

$I_{E2} = I_T - I_{E1} = 25 - 12.75 = 12.25$ A

$\Delta I = I_{E1} - I_{E2} = (12.75 - 12.25)$ A $= 0.5$ A

3.7 SWITCHING CHARACTERISTICS OF POWER BJT

Figure 3.13 shows a *n-p-n* power BJT as a switch. When a pulse input voltage is applied to the base of transistor, the base current is $I_B = \dfrac{V_i - V_{BE}}{R_B}$ and the voltage $V_{CE} = V_{CC} - I_C R_C$. The output voltage is $V_o = V_{CE}$. The switching waveforms of a *npn* transistor are depicted in Fig. 3.14 and it is represented in simplified form in Fig. 3.15.

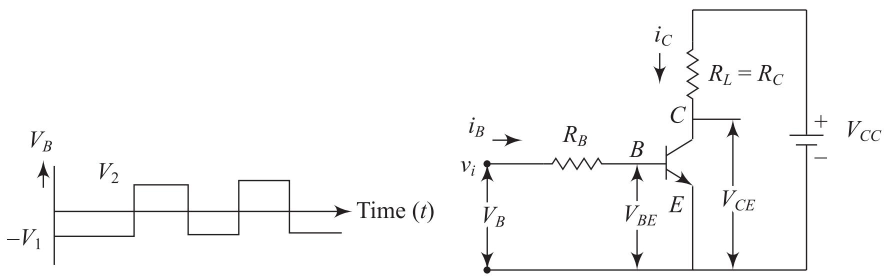


Fig. 3.13 *n-p-n power BJT as a switch*

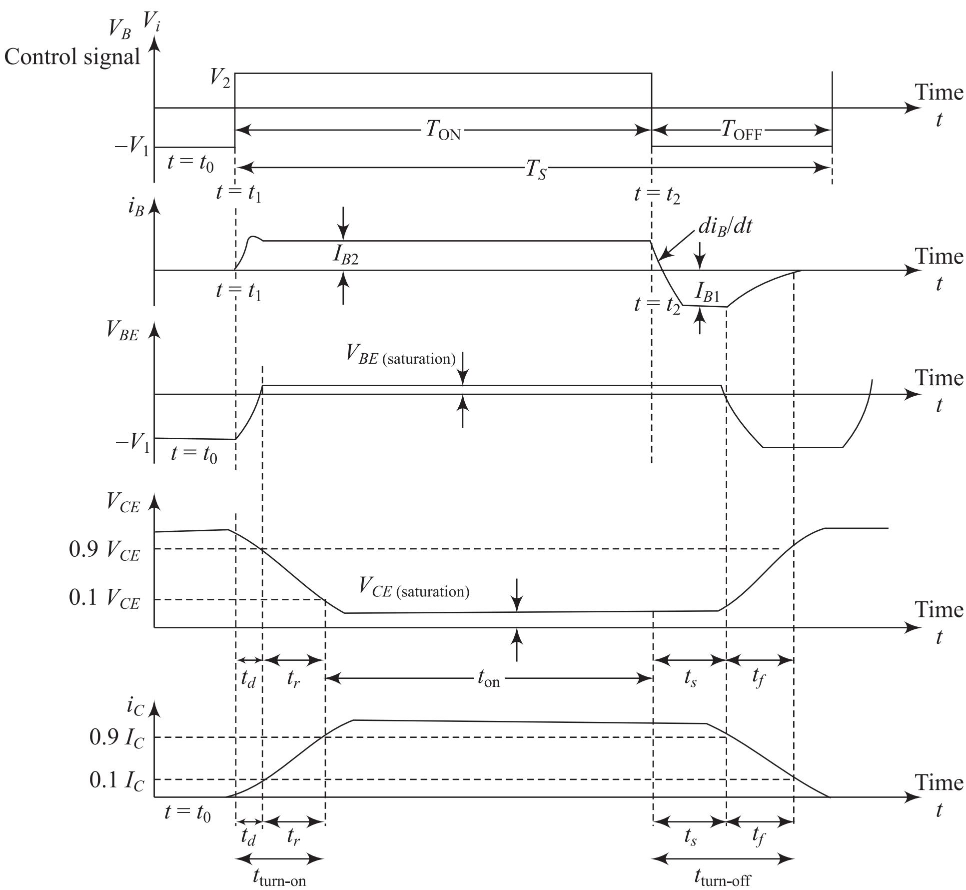


Fig. 3.14 *The switching waveforms of a npn transistor*

Assume the pulsating input voltage V_i varies between $-V_1$ to $+V_2$ as shown in Fig. 3.13. At time $t = t_0$, input voltage at the base of power transistor is $V_i = -V_1$, emitter-base junction is reverse biased. Then transistor operates in OFF state as base current $I_B = 0$ and the collector current I_C is equal to zero and the output voltage is $V_o = V_{CE} = V_{CC}$.

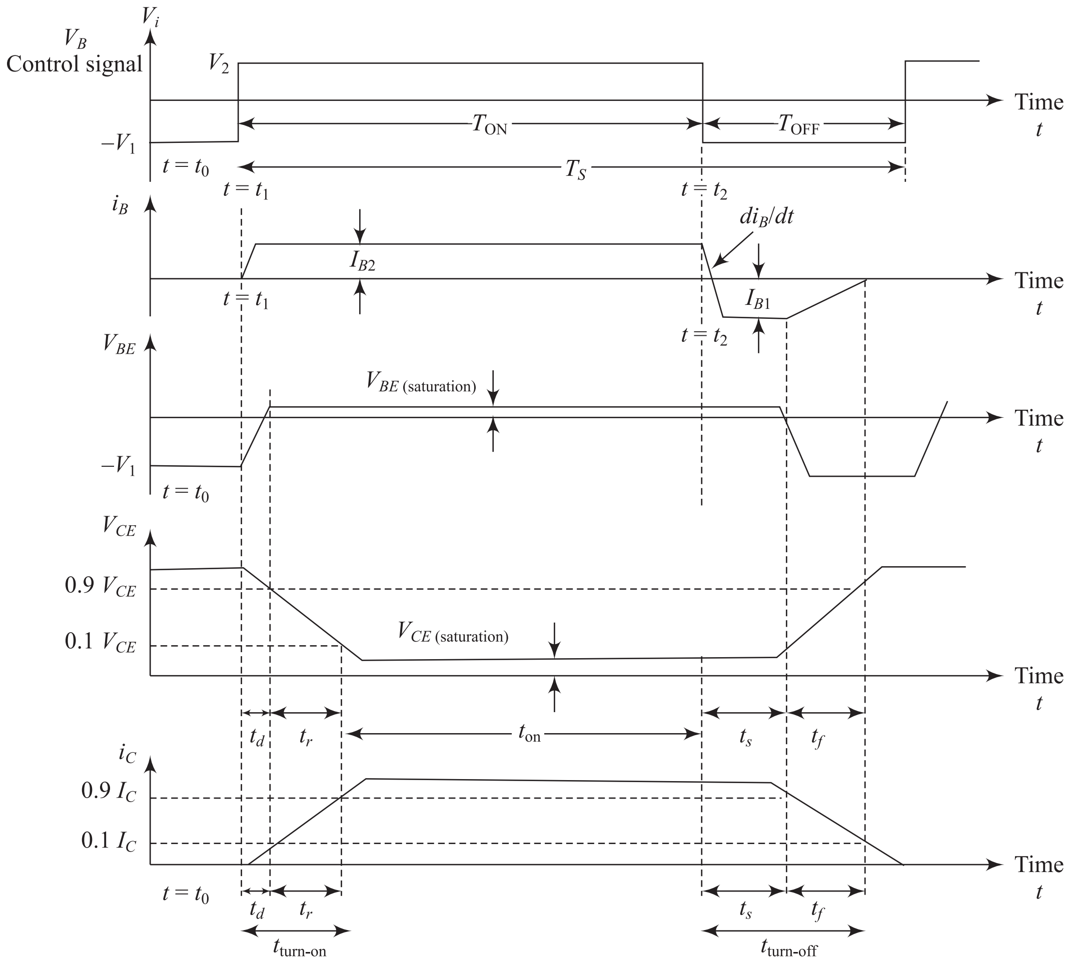


Fig. 3.15 *The switching waveforms of a npn transistor in simplified form*

At time $t = t_1$, voltage starts to increase gradually from $-V_1$ to $+V_2$ and the collector current I_C increases gradually from zero and the collector-emitter voltage V_{CE} starts to fall from V_{CC}. Therefore, the transistor changes its state from cut-off to saturation gradually. The maximum steady state current is $I_C = \dfrac{V_C}{R_C}$.

1. Delay time t_d The *delay time t_d* is the time required for collector current i_C to reach 10% of the steady state current I_C $(0.1I_C)$ and the v_{CE} falls from V_{CC} to 90% of V_{CC} $(0.9V_{CC})$.

2. Rise time t_r The *rise time t_r* is the time required for collector current i_C to increase from 10% of the steady state current I_C $(0.1I_C)$ to 90% of I_C $(0.9I_C)$ and the v_{CE} falls from 90% of V_{CC} $(0.9V_{CC})$ to 10% of V_{CC} $(0.1V_{CC})$.

3. Turn-ON time $t_{\text{turn–on}}$ The *turn-ON time* is the sum of the delay time t_d and the rise time t_r and it can be expressed as $t_{\text{turn–on}} = t_d + t_r$. The turn-ON time of power transistor is typically 30 to 300 ns. When the transistor is ON, it will operate in saturation region as long as input voltage is equal to V_2.

4. Storage time, t_s At time $t = t_2$, the input voltage again changes from $+V_2$ to $-V_1$, then the base current I_B changes from I_{B2} to I_{B1} gradually. The base I_{B1} is negative and excess carriers can be removed from the base. The collector current can not be changed immediately but its value gradually reduced to zero as depicted in Fig. 3.14. The time interval between the instant of the change over of the input voltage from $+V_2$ to $-V_1$ to the instant collector current i_C falls to 90% of the steady state current I_C ($0.9I_C$) is called the *storage time*, t_s. During the storage time t_s, the collect emitter voltage increases from $V_{CE(\text{saturation})}$ to 10% of V_{CC} and all excess carriers will be removed. The applied negative voltage $-V_1$ can increase the rate of removal of excess carriers from base and subsequently the storage time as turn-off time will be reduced.

The *fall time* t_f is the time in which the collector current i_C falls from 90% of steady state current I_C ($0.9I_C$) to 10% of steady state current I_C ($0.1I_C$) and the collector emitter voltage increase from 10% of V_{CC} to 90% of V_{CC}. The *turn-OFF time* is the sum of the storage time (t_s) and fall time (t_f) and it can be represented by $t_{\text{turn-off}} = t_s + t_f$. The turn on time of power transistor is about 30 to 300 ns.

Example 3.4 The switching waveform of a power transistor is shown in Fig. 3.16 where V_{CC} = 200 V, $V_{CE(\text{saturation})}$ = 1.5 V, I_{CS} = 100 A, t_d = 0.25 µs, t_r = 1.25 µs, t_{on} = 40 µs, t_s = 2 µs, t_f = 1.5 µs. Determine (a) energy loss during delay time, (b) energy loss during rise time, (c) energy loss during conduction time t_{on} and (d) average power loss of power transistor during turn-ON when switching frequency is 5 kHz and the emitter leakage current is I_{CE0} = 5 mA.

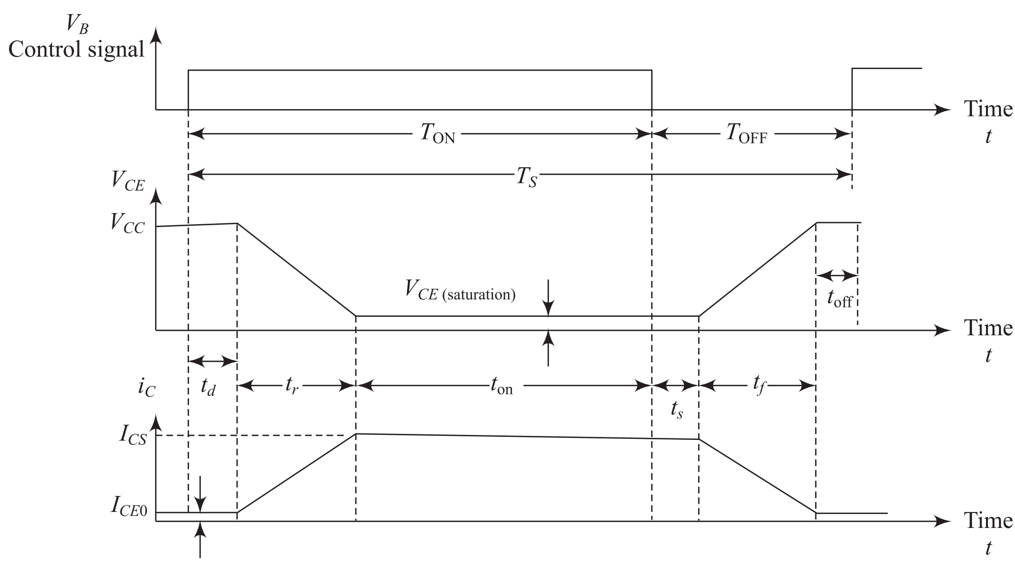


Fig. 3.16

Solution

Given: V_{CC} = 200 V, $V_{CE(\text{saturation})}$ = 1.5 V, I_{CS} = 100 A, t_d = 0.25 µs, t_r = 1.25 µs, t_{on} = 40 µs, t_s = 2 µs, t_f = 1.5 µs, f_s = 5 kHz and I_{CE0} = 5 mA

(a) Energy loss during delay time t_d is equal to

$$W_{\text{delay}} = V_{CC}I_{CEO}t_d = 200 \times 5 \times 10^{-3} \times 0.25 \times 10^{-6} \text{ Watt second} = 0.25 \times 10^{-6} \text{ Watt-second}$$

(b) During rise time t_r the voltage v and current i waveform can be expressed by

$$v_{CE}(t) = \left[V_{CC} - (V_{CC} - V_{CES})\frac{t}{t_r}\right] \text{and } i_C(t) = I_{CS}\frac{t}{t_r} \text{ as } I_{CEO} << I_{CS}$$

Therefore, the instantaneous power is $p = v_{CE}(t)i_C(t) = \left[V_{CC} - (V_{CC} - V_{\text{CES}})\frac{t}{t_r}\right]I_{CS}\frac{t}{t_r}$

The energy loss is $W_{\text{rise}} = \int_0^{t_r} pdt = \int_0^{t_r}\left[V_{CC} - (V_{CC} - V_{CES})\frac{t}{t_r}\right]I_{CS}\frac{t}{t_r}\,dt$

$$= \left[\frac{1}{2}V_{CC} - \frac{1}{3}(V_{CC} - V_{CES})\right]I_{CS}\,t_r$$

$$= \left[\frac{1}{2}\times 200 - \frac{1}{3}(200 - 1.5)\right]\times 100 \times 1.25 \times 10^{-6} \text{ Watt second} = 4229.166 \times 10^{-6} \text{ Watt-second}$$

(c) Energy loss during on time t_{on} is equal to

$$W_{\text{on}} = V_{CES}I_{CS}t_{\text{on}} = 1.5 \times 100 \times 40 \times 10^{-6} \text{ Watt-second} = 6000 \times 10–6 \text{ Watt-second}$$

(d) Average power loss of power transistor during turn-ON is

$$V_{CC}I_{CEO}t_d f_s + \left[\frac{1}{2}V_{CC} - \frac{1}{3}(V_{CC} - V_{\text{CES}})\right]I_{CS}t_r f_s$$

$$= 200 \times 5 \times 10^{-3} \times 0.25 \times 10^{-6} \times 5 \times 10^3 + \left[\frac{1}{2}\times 200 - \frac{1}{3}(200 - 1.5)\right] \times 100 \times 1.25 \times 10^{-6} \times 5 \times 10^3$$

$$= 22.395 \text{ Watt}$$

Example 3.5 The switching waveforms of a power transistor is depicted in Fig. 3.16 where V_{CC} = 220 V, $V_{CE(\text{saturation})}$ = 2 V, I_{CS} = 100 A, t_d = 0.5 µs, t_r = 1.5 µs, t_{on} = 50 µs, t_s = 3.5 µs, t_f = 2.5 µs. When the switching frequency is 2 kHz and the emitter leakage current is I_{CE0} = 2.5 mA. Find

(a) average power loss during delay time,
(b) average power during rise time,
(c) peak instantaneous power loss during rise time,
(d) average power during conduction time t_{on},
(e) average power loss during storage time,
(f) average power during fall time,
(g) peak instantaneous power loss during fall time and

draw the waveform for instantaneous power loss for period T_s.

Solution

Given: V_{CC} = 220 V, $V_{CE(\text{saturation})}$ = 2 V, I_{CS} = 100 A, $t_m = \frac{V_{CC}}{V_{CC} - V_{\text{CES}}}\frac{t_r}{2}$, t_r = 1.5 µs, t_{on} = 50 µs, t_s = 3.5 µs, t_f = 2.5 µs, f_s = 2 kHz and I_{CE0} = 2.5 mA

(a) Average power loss during delay time t_d is equal to

$$P_{\text{delay}} = V_{CC}I_{CE0}t_d f_s = 220 \times 2.5 \times 10^{-3} \times 0.5 \times 10^{-6} \times 2 \times 10^3 \text{ Watt} = 0.55 \times 10^{-3} \text{ Watt} = 0.55 \text{ mW}$$

(b) During rise time t_r the voltage v and current i waveform can be expressed by

$$v_{CE}(t) = \left[V_{CC} - (V_{CC} - V_{\text{CES}})\frac{t}{t_r}\right] \text{and } i_C(t) = I_{CS}\frac{t}{t_r} \text{ as } I_{\text{CEO}} << I_{CS}$$

Therefore, the instantaneous power is $p = v_{CE}(t) i_C(t) = \left[V_{CC} - (V_{CC} - V_{CES}) \frac{t}{t_r} \right] I_{CS} \frac{t}{t_r}$

The energy loss is $P_{\text{rise}} = \frac{1}{T_s} \int_0^{t_r} p dt = \frac{1}{T_s} \int_0^{t_r} \left[V_{CC} - (V_{CC} - V_{\text{CES}}) \frac{t}{t_r} \right] I_{CS} \frac{t}{t_r} dt$

$$= \left[\frac{1}{2} V_{CC} - \frac{1}{3} (V_{CC} - V_{\text{CES}}) \right] I_{CS} t_r f_s \quad \text{as} \quad f_s = \frac{1}{T_s}$$

$$= \left[\frac{1}{2} \times 220 - \frac{1}{3} (220 - 2) \right] \times 100 \times 1.5 \times 10^{-6} \times 2 \times 10^3 \text{ Watt} = 11.2 \text{ Watt}$$

(c) Instantaneous power loss during rise time is

$$p = v_{CE}(t) i_C(t) = \left[V_{CC} - (V_{CC} - V_{\text{CES}}) \frac{t}{t_r} \right] I_{CS} \frac{t}{t_r} = V_{CC} I_{CS} \frac{t}{t_r} - (V_{CC} - V_{\text{CES}}) I_{CS} \frac{t^2}{t_r^{\,2}}$$

At time $t = t_m$ the maximum power loss $\frac{dp}{dt} = 0$,

then $$t_m = \frac{V_{CC}}{V_{CC} - V_{\text{CES}}} \frac{t_r}{2} = \frac{220}{220 - 2} \frac{1.5 \times 10^{-6}}{2} = 0.7568 \ \mu\text{s} \ = 0.7568 \ \mu\text{s}$$

Therefore, the peak instantaneous power loss is

$$p_m|_{\text{at}\cdot t = t_m} = V_{CC} I_{CS} \frac{t_m}{t_r} - (V_{CC} - V_{\text{CES}}) I_{CS} \frac{t_m^2}{t_r^2}$$

$$= 220 \times 100 \times \frac{0.7568}{1.5} - (200 - 2) 100 \frac{0.7568^2}{1.5^2} = 6660.423 \text{ Watt}$$

(d) Average power loss during on time t_{on} is equal to

$$P_{\text{on}} = V_{CES} I_{CS} t_{\text{on}} f_s = 2 \times 100 \times 50 \times 10^{-6} \times 2 \times 10^3 \text{ Watt} = 20 \text{ Watt}$$

(e) Average power loss during storage time t_s is equal to

$$P_{\text{storage}} = V_{CES} I_{CS} t_s f_s = 2 \times 100 \times 3.5 \times 10^{-6} \times 2 \times 10^3 \text{ Watt} = 1.4 \text{ Watt}$$

(f) During fall time t_f the voltage v and current i waveform can be expressed by

$$v_{CE}(t) = (V_{CC} - V_{CES}) \frac{t}{t_f} \text{ and } i_C(t) = I_{CS} \left[1 - \frac{t}{t_r} \right] \text{as } I_{CE0} << I_{CS}$$

Therefore, the instantaneous power is $p = v_{CE}(t) i_C(t) = (V_{CC} - V_{\text{CES}}) \frac{t}{t_f} I_{CS} \left[1 - \frac{t}{t_r} \right]$

The average power loss is $P_{\text{fall}} = \frac{1}{T_s} \int_0^{t_f} p dt = \frac{1}{T_s} \int_0^{t_f} (V_{CC} - V_{\text{CES}}) \frac{t}{t_f} I_{CS} \left[1 - \frac{t}{t_r} \right] dt$

$$= \frac{1}{6} (V_{CC} - V_{\text{CES}}) I_{CS} t_f f_s \quad \text{as} \quad f_s = \frac{1}{T_s}$$

$$= \frac{1}{6} \times (220 - 2) \times 100 \times 2.5 \times 10^{-6} \times 2 \times 10^3 \text{ Watt} = 18.1666 \text{ Watt}$$

(g) Instantaneous power loss during fall time is

$$p_f = v_{CE}(t) i_C(t) = (V_{CC} - V_{CES}) \frac{t}{t_f} I_{CS} \left[1 - \frac{t}{t_r} \right]$$

$$= (V_{CC} - V_{CES}) I_{CS} \frac{t}{t_f} - (V_{CC} - V_{\text{CES}}) I_{CS} \frac{t^2}{t_f^{\,2}}$$

At the maximum power loss $\frac{dp}{dt} = 0$, then $t_m = \frac{t_f}{2}$

Therefore, the peak instantaneous power loss is

$$p_m|_{\text{at}\cdot t=t_m} = (V_{CC} - V_{\text{CES}})I_{CS}\frac{t_m}{t_f} - (V_{CC} - V_{\text{CES}})I_{CS}\frac{t_m^2}{t_f^2}$$

As $\frac{t_m}{t_f} = \frac{1}{2}$, $p_m|_{\text{at}\cdot t=t_m} = \frac{1}{4}(V_{CC} - V_{\text{CES}})I_{CS}$

$$= \frac{1}{4}(220 - 2) \times 100 = 5450 \text{ Watt}$$

Instantaneous power loss during delay time is

$$p = v_{CE}(t)i_C(t) = V_{CC}I_{CEO} = 220 \times 2.5 \times 10^{-3} \text{ Watt} = 0.55 \text{ Watt}$$

Instantaneous power loss during conduction and storage time is

$$p = v_{CE}(t)i_C(t) = V_{CES}I_{CS} = 2 \times 100 \text{ Watt} = 200 \text{ Watt}$$

Instantaneous power loss during rise time is calculated as per equation

$$V_{CC}I_{CS}\frac{t}{t_r} - (V_{CC} - V_{\text{CES}})I_{CS}\frac{t^2}{t_r^2} \text{ and maximum loss is 6660.423 Watt}$$

Instantaneous power loss during fall time is calculated as per equation

$$(V_{CC} - V_{CES})I_{CS}\frac{t}{t_f} - (V_{CC} - V_{\text{CES}})I_{CS}\frac{t^2}{t_f^2} \text{ and maximum loss is 5450 Watt.}$$

Figure 3.17 shows the instantaneous power dissipation with in the power transistor.

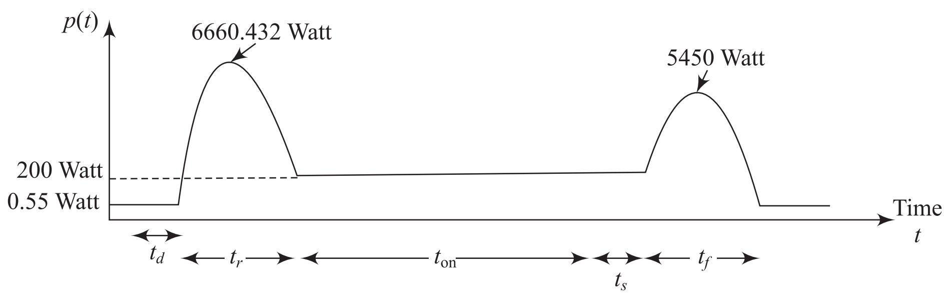


Fig. 3.17 *Instantaneous power loss with in the power transistor*

3.8 POWER MOSFET

A power MOSFET is a four-layer semiconductor structure with alternate *p*-type and *n*-type doping. Figure 3.18 shows a vertically oriented enhancement mode *n*-channel power MOSFET which is a n^+ p n^- n^+ structure. Actually, a power MOSFET consists of a parallel connection of many MOSFET cells in a single IC.

The doping density in the two n^+ end layers of a vertically oriented power MOSFET is same and its value is quite large, typically 10^{19} cm^{-3}. One end is used as source and the other end (n^+ substrate) is used as drain as depicted in Fig. 3.18. The n^- layer is epitaxially grown on the n^+ substrate. Then p-type semiconductor is diffused in the epitaxially grown n^- layer and the p region developed. After that n^+ semiconductor is diffused in the p region and the n^+ region is developed.

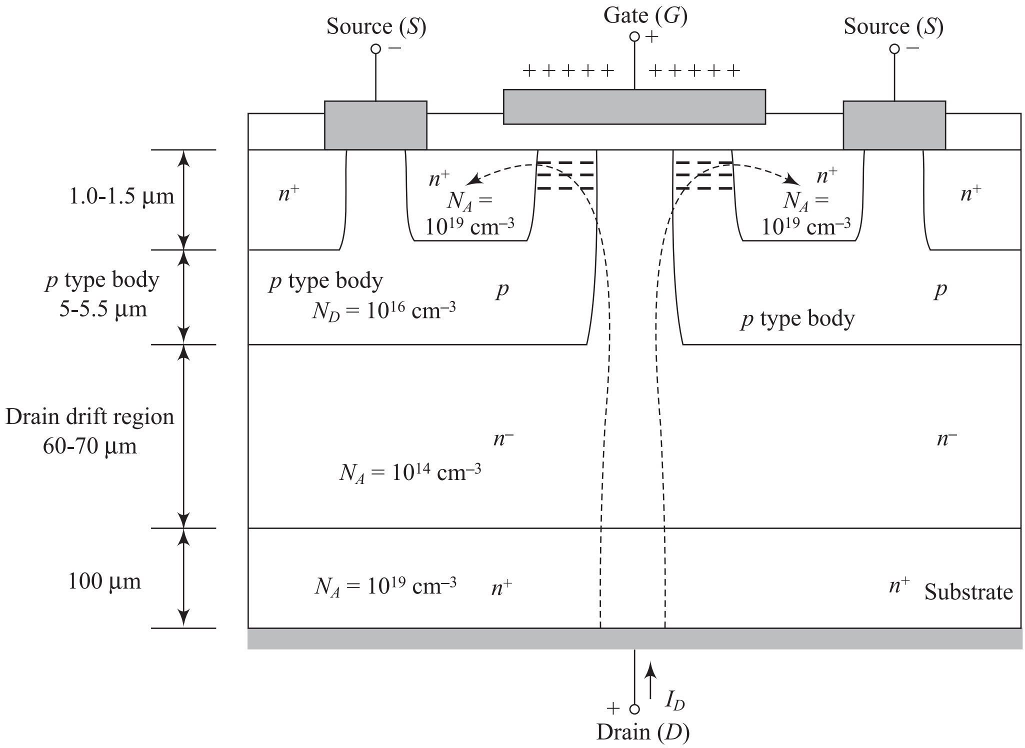


Fig. 3.18 Structure of a *n*-channel power MOSFET

The n^- layer is called the drain drift region and the doping density in the n^- layer is low. The typical value of n^- layer doping density is about 10^{14} cm^{-3} to 10^{15} cm^{-3}. The thickness of n^- drift region determines the breakdown voltage of the device. The p-type semiconductor layer is the region where the channel is established between source and drain. Therefore, p region is called the body of a transistor. The doping density of p region is about 10^{16} cm^{-3}. This structure is known as *vertical diffused MOSFET* (VDMOS). A four-layer semiconductor structure with the opposite doping density of Fig. 3.18 can also be manufactured and then the developed structure is called a p-channel MOSFET.

It is clear from Fig. 3.19 that there are two *pn* junctions such as drain-to-body (n^-p) junction and body-to-source (pn^+) junction. When the gate-to-source voltage is zero ($V_{GS} = 0$), the current I_D can not flow from the drain to source terminals of MOSFET as any one of the *pn* junctions is reverse biased by the input voltage V_{DD}. As the gate is isolated from the body (p region) by a layer of silicon dioxide (SiO_2) which behaves as a very good insulator, the minority carriers can not be injected into the p region through the gate terminal. The thickness of silicon dioxide layer is about 1000 Å.

When the gate to source voltage is greater than zero, the gate is positive with respect to the source, an electric field will be developed and a n channel is formed in the p region as depicted in Fig. 3.19. Then source is connected with drain through n channel and current I_D flows from drain to source. If the gate-to-source voltage is increased, the drain current I_D increases. The value of I_D depend on the thickness of the silicon dioxide or gate oxide, the width of gate and the number of gate and source regions which are connected in parallel.

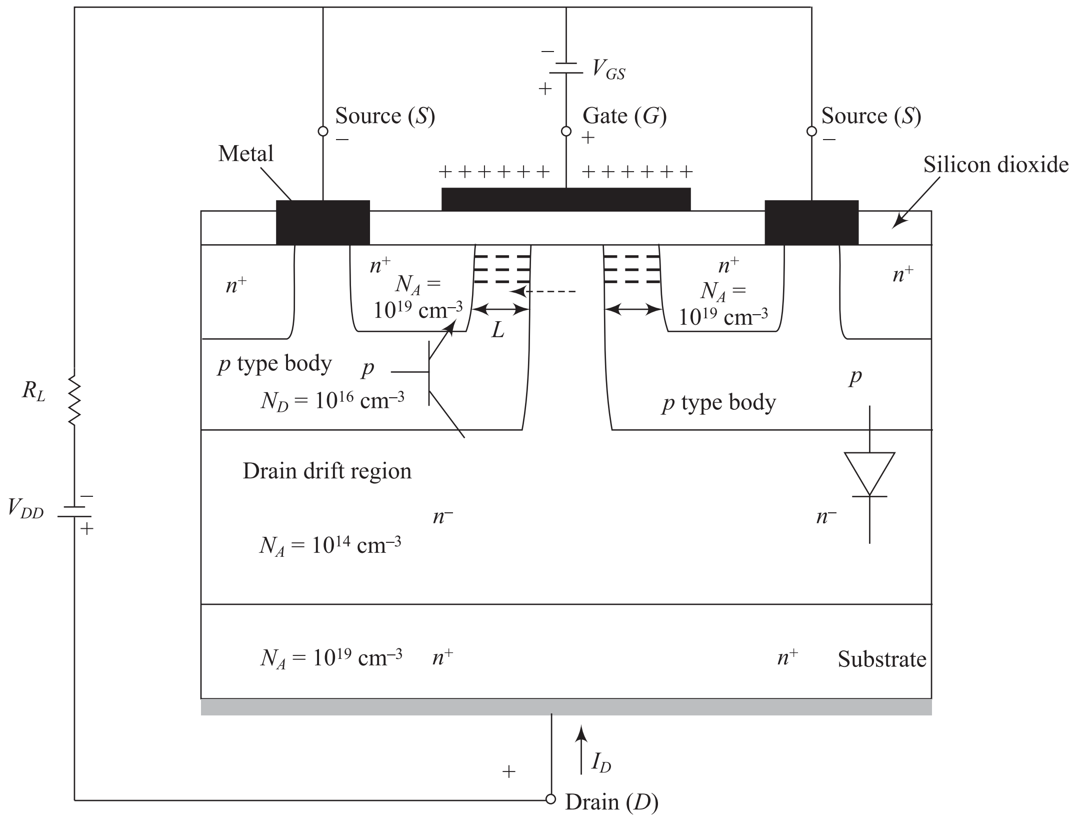


Fig. 3.19 *n channel power MOSFET with parasitic BJT and parasitic diode*

A parasitic *n-p-n* bipolar junction transistor (BJT) is developed between the source and drain contacts as shown in Fig. 3.19. The *p*-type body region acts as the base, n^+ region as emitter and n^- drift region as collector of the parasitic BJT. As the *p*-type body region is shorted to the source region due to overlapping the source metallisation on the *p*-type body region, source is connected with the base and emitter of the parasitic BJT. Therefore, the potential difference between base and emitter of the parasitic BJT is zero and the parasitic BJT always operate in cut-off region. The development of the parasitic diode is depicted in Fig. 3.19. In parasitic diode, source acts as anode and drain acts as cathode. This inbuilt diode is used in half-bridge and full-bridge inverter circuits. The circuit symbol of n channel power MOSFET and *p*-channel power MOSFET are depicted in Fig. 3.20. The direction of the arrow represents the direction of current flow.

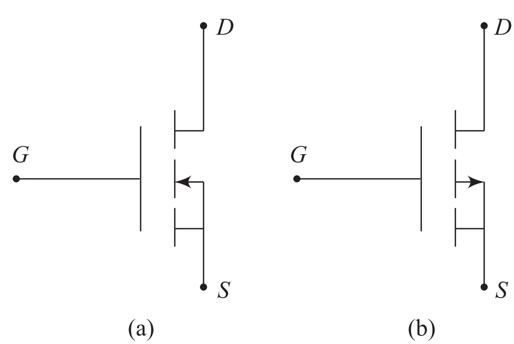


Fig. 3.20 *(a) Symbol of n-channel power MOSFET (b) Symbol of p-channel power MOSFET*

3.9 *I-V* CHARACTERISTICS OF POWER MOSFET

The *n*-channel power MOSFET has three terminals namely gate (G), drain (D) and source (S). Figure 3.21 shows an *n*-channel power MOSFET circuit where the input signal V_{GS} is applied across gate to source and the output signal V_{DS} is obtained from drain. The current flow from drain to source I_D is controlled by gate signal. Usually the source terminal is common between the input and output of a power MOSFET.

The transfer characteristic of an *n*-channel power MOSFET is depicted in Fig. 3.22. The drain current I_D is a function of gate-to-source voltage V_{GS}. When the gate-to-source voltage is less than threshold voltage $V_{GS(\mathrm{th})}$, the current flow from drain to source is zero. The value of $V_{GS(\mathrm{th})}$ for power MOSFET is about few volts (2 to 3 V). Since I_D is equal to zero, the drain to source is open circuit and the device should able to hold the supply voltage V_{DD}.

It is clear from Fig. 3.23 that the drain-to-source breakdown voltage V_{DSS} must be greater than V_{DD} to keep away from the device breakdown. If the drain to source voltage is larger than V_{DSS}, the power MOSFET will be breakdown due to avalanche breakdown of the drain-to-body (n^-p) junction.

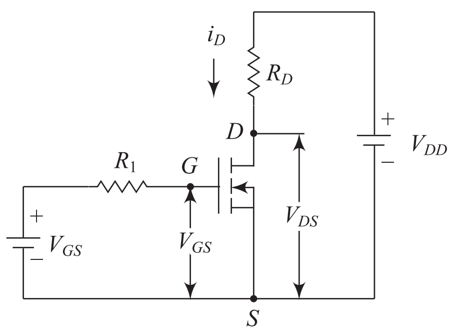


Fig. 3.21 *n-channel power MOSFET circuit*

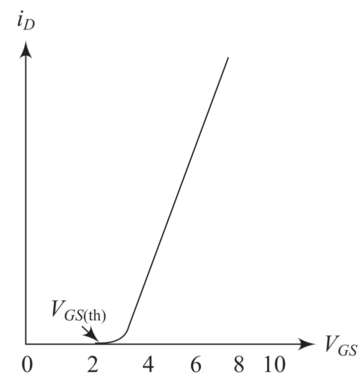


Fig. 3.22 *Transfer characteristics of power MOSFET*

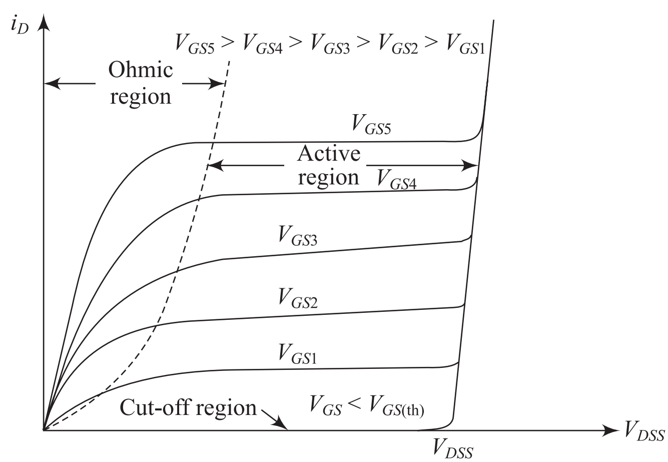


Fig. 3.23 *Output characteristics of power MOSFET*

The output characteristics of a power MOSFET is shown in Fig. 3.23. The drain current I_D is a function of drain-to-source voltage V_{DS} when gate-to-source voltage V_{GS} is constant. It is clear form Fig. 3.23 that the output characteristics consist of three regions such as cut-off, active and ohmic regions.

When the drain-to-source voltage is small, the relation between I_D and V_{DS} is linear and power MOSFET operates in the ohmic region. The device operate in the ohmic region when

$$V_{GS} > V_{GS(\text{th})} \quad \text{and} \quad V_{DS} > 0$$

The power dissipation must be within a limit and its value will be minimum due to small drain to source voltage $V_{DS(\text{on})}$ through large drain current I_D flows from drain to source.

When the power MOSFET is used as a switch to control the flow of power to the load, the $I_D - V_{DS}$ characteristics of the device must be traversed from the cut-off region to the ohmic region through the active region. The device turns OFF in the cut-off region and turns on in the ohmic region.

In the active region the drain current I_D does not depend on the drain-source voltage V_{DS} but it varies with the gate to source voltage V_{GS}. Since the current I_D is known as saturated current, this region is also called the *saturation region*. In this region, the drain current can be expressed by

$$I_D = K[V_{GS} - V_{GS(\text{th})}]^2$$

where, K is a constant which depends on the device parameters device. At the crossing point between the active region and ohmic region, $[V_{GS} - V_{GS(\text{th})}]$ is equal to V_{DS}. Consequently, the above equation can be written as

$$I_D = KV_{DS}^2 \quad \text{as} \quad [V_{GS} - V_{GS(\text{th})}] = V_{DS}$$

The output characteristics for a p-channel power MOSFET will be same as the output characteristics for a n-channel power MOSFET, but the current and voltage polarities are reversed. Therefore the characteristics for the n-channel power MOSFET should be appear in the third quadrant of the $I_D - V_{DS}$ plane.

$V_{GS(\text{max})}$ It is the maximum allowable gate-to-source voltage. When a voltage is applied across gate to source, an electric field will be developed. The $V_{GS(\text{max})}$ voltage can be determined when the gate oxide will not be broken due to large electric field. Usually silicon dioxide can be break at electric field about 5–10 Mega Volt/cm. When the thickness of silicon dioxide is about 1000 Å, the maximum allowable gate-to-source voltage of power MOSFET $V_{GS(\text{max})}$ is about 20–30 V.

V_{DSS} It is the maximum allowable drain to source voltage at which the power MOSFET sustained without avalanche breakdown of the drain to body n^-p junction. To achieve the high breakdown voltage, the n^- drift region must be lightly doped. The value of V_{DSS} is about few hundred volts.

3.10 ON-STATE LOSS POWER MOSFET

The ON-state resistance of a power MOSFET $R_{DS(\text{on})}$ is sum of drain resistance, n^- drift region resistance R_D, accumulation layer resistance, channel resistance and source region resistance as depicted in Fig. 3.24. At low breakdown voltage, all resistance components contribute equally to determine the total ON-state resistance. When the breakdown voltage V_{DSS} is greater than few hundred volts, the n^- drift region resistance R_D contributes maximum value of the total ON-state resistance $R_{DS(\text{on})}$. Actually, the resistance of drift region is a function of breakdown voltage and it can expressed as

$$V_D = IR_D = JAR_D \qquad \text{as } I = JA$$

or

$$\frac{V_D}{J} = AR_D = 3 \times 10^{-7}\, V_{\text{DSS}}^2$$

where, V_D is voltage drop in the n^- drift region, A is the cross-section area through which the drain current I_D flows, J is the current density.

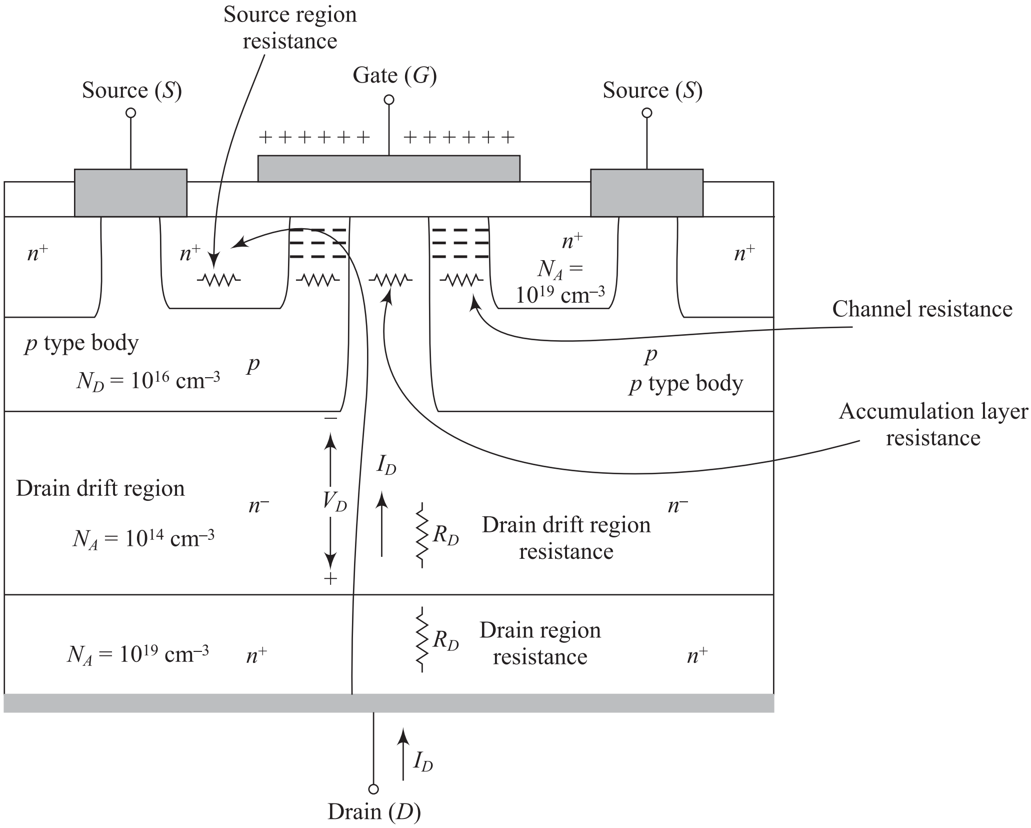


Fig. 3.24 *On-state resistances of n-channel enhancement mode MOSFET*

Since R_D depends on the breakdown voltage V_{DSS}, the value of R_D increases with increasing the breakdown voltage for power MOSFET. The ON-state resistance also depends on the junction temperature. The on-state resistance of power MOSFET increases with increasing junction temperature.

The power dissipation in the device during on-state is equal to

$$P_{on} = I_D R_{DS(on)}$$

where, I_D is the current flow from drain to source and $R_{DS(on)}$ is the total on-state resistance and the typical value of $R_{DS(on)}$ is about 9 to 100 milliohms

3.11 SAFE OPERATING AREA OF POWER MOSFET

The Safe Operating Area (SOA) represents the maximum values of current and voltage at which the power MOSFET can withstand safely. As per manufacture datasheet related to specification of power MOSFET, the safe operating area (SOA) can be determined by the following parameters:

1. Maximum drain current $I_{D(max)}$ or I_{DM}
2. Internal junction temperature T_j

3. Breakdown voltage V_{DSS}
4. Maximum power dissipation $P_{D(\max)}$

Figure 3.25 shows the safe operating area (SOA) of power MOSFET where drain current I_D and drain-to-source voltage V_{DS} are represented in logarithmic scale. At low value of V_{DS}, the maximum drain current $I_{D(\max)}$ is limited by the power dissipation. The maximum power dissipation is also limited by the maximum tolerable junction temperature T_j within the device. The breakdown voltage V_{DSS} at low drain current is limited to avoid the avalanche breakdown of the drain to body n^-p junction.

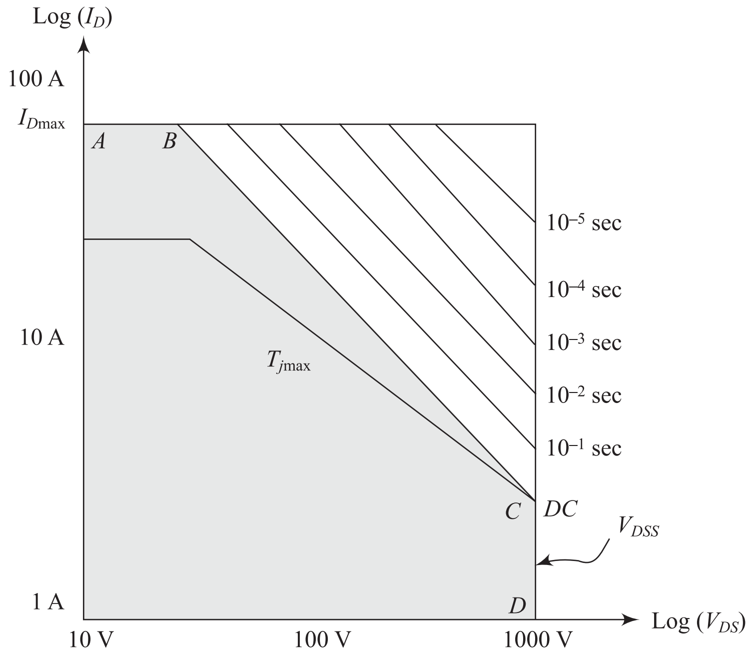


Fig. 3.25 SOA of a power MOSFET

When the device operates with dc signal, the boundary of the SOA is A-B-C-D. During operation with dc signal, a continuous power will be dissipated within device and the junction temperature increases significantly. Boundary A-B represents the maximum limit of drain current I_{DM} with V_{DS} is less than 30 V. When V_{DS} is greater than 30 V, the drain current has been reduced as per boundary B-C so that the operating junction temperature of power MOSFET is less than the maximum operating junction temperature. The boundary C-D represents the maximum voltage capability of a power MOSFET.

When the power MOSFET operates as a switch and it is driven by a pulse signal, the boundaries of the SOA can be extended. The extension of the SOA is only possible for switch mode operation. Actually, the silicon wafer and its packaging of power MOSFET have a specified thermal capacitance and it has an ability to absorb a limited amount of energy without increasing the junction temperature excessively. When the power MOSFET turns ON for a few microseconds, the device can absorb very small amount of energy and the junction temperature rise will be low and the boundary of SOA will be increased. When the pulse width is about 10^{-1} s, the SOA is greater than the boundary A-B-C-D. If the pulse width is further reduced and its value is about some μs (10^{-6} s), the SOA is greater than the boundary with pulse with 10^{-3} s. It is clear from Fig. 3.25 that the SOA increases with the decrease of pulse-width.

3.12 SERIES AND PARALLEL OPERATION OF POWER MOSFET

Power MOSFETs can be connected in series to increase the voltage-handling capability. It is extremely important that the series-connected power MOSFETs must be turned ON and OFF simultaneously. When the series-connected transistors are not turned ON and OFF simultaneously, the slowest device will be at turn-ON state but the fastest device will be at turn-OFF state. Consequently, the turned-off power MOSFETs must be withstand to the full voltage of the drain-source. If the device is not able to withstand at high voltage, the device may be completely destroyed due to high voltage. Therefore, during series connection, the power MOSFETs should have same gain, on-state voltage, transconductance, threshold voltage, turn-ON time, turn-OFF time, and gate drive circuit.

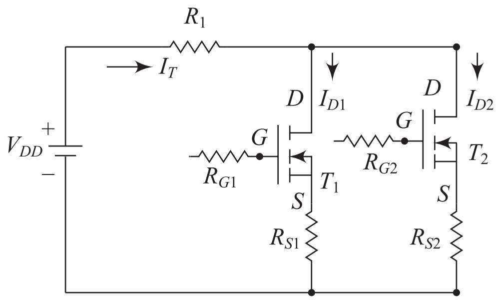
When a power MOSFET is not able to handle the load current, transistors may be connected in parallel. For the equal current sharing among MOSFETs, the power transistors should have same turn-ON time, turn-OFF time, gain, threshold voltage and transconductance. Figure 3.26 shows the parallel connections of two MOSFETs. Current I_{D1} flows through transistor T_1 and current I_{D2} flows through transistor T_2. Total current I_T is shared by transistors T_1 and T_2.

Then, $$I_T = I_{D1} + I_{D2} \quad \text{and} \quad V_{DS1} + I_{D1}R_{S1} = V_{DS2} + I_{D2}R_{S2}$$

Therefore, $$V_{DS1} + I_{D1}R_{S1} = V_{DS2} + (I_T - I_{D1})R_{S2}$$

or $$I_{D1} = \frac{V_{DS2} - V_{DS1} + I_T R_{S2}}{R_{S1} + R_{S2}}$$

About 45% to 55% current can be shared by connecting resistance in series with MOSFETs. Actually, resistances are used for current sharing under steady-state condition. Under dynamic condition, current can be shared by connecting coupled inductors as shown in Fig. 3.26. If the current flow through transistor T_1 increases, the amplitude of $L\frac{di}{dt}$ across L_1 increases. Then a opposite polarity voltage is induced across inductor L_2. Accordingly a low impedance path is provided by transistor T_2, and the current is shifted from transistor T_1 to transistor T_2.



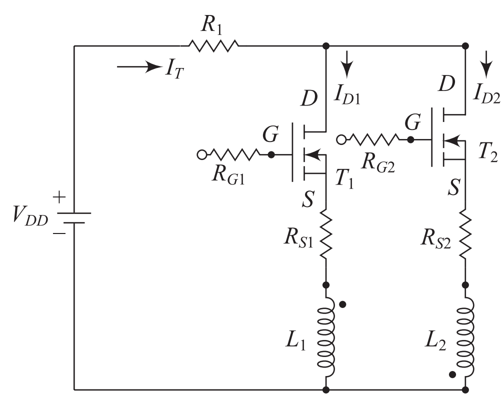


Fig. 3.26 *Parallel connection of power MOSFET (a) Current sharing at steady-state condition (b) Current sharing at dynamic condition*

When n number of power MOSFETs are connected in parallel to share the total current I_T, the average current I_{Dav} is shared by each MOSFET. Then the average current I_{Dav} is equal to

$$I_{Dav} = \frac{I_T}{n}$$

If the maximum current rating of MOSFET is represented by $I_{D\,\max}$, the unbalance factor α is computed from the equation $\alpha = \dfrac{I_{D\max} - I_{Dav}}{I_{Dav}}$.

In ideal case the value of α should be zero but it varies between 0 to 1. In order to protect the device from permanent damage, $I_{D\max}$ must be less than the device current rating $I_{D\text{ rated}}$.

Therefore, $\quad I_{D\,\max} < I_{D.\text{rated}}$

When I_D max = $I_{D\text{ rated}}$, the unbalance factor α is equal to

$$\alpha = \frac{I_{D\max} - I_{Dav}}{I_{Dav}} = \frac{I_{D\cdot\text{rated}} - I_{Dav}}{I_{Dav}} = \frac{I_{D\cdot\text{rated}}}{I_{Dav}} - 1$$

As $$I_{Dav} = \frac{I_T}{n}, \alpha = \frac{I_{D\cdot\text{rated}}}{I_{Dav}} - 1 = \frac{I_{D\cdot\text{rated}}}{I_T/n} - 1 = \frac{nI_{D\cdot\text{rated}}}{I_T} - 1$$

Therefore, the total current $I_T = \dfrac{nI_{D\cdot\text{rated}}}{\alpha + 1}$

For any value of α and n, the total current I_T must be lass than $\dfrac{nI_{D\cdot\text{rated}}}{\alpha + 1}$.

Therefore, $I_T \leq \dfrac{nI_{D\cdot\text{rated}}}{\alpha + 1}$

Example 3.6 Two power MOSFETs are connected in parallel to share the total current 20 A. The drain to source voltage of T_1 and T_2 are 3.5 V and 2.75 V respectively. Determine the drain current of each transistors and the difference of current sharing when the current sharing series resistance are (a) R_{S1} = 0.2 Ω and R_{S2} = 0.3 Ω (b) R_{S1} = 0.45 Ω and R_{S2} = 0.5 Ω

Solution

Given: I_T = 20 A, V_{DS1} = 3.5 V, V_{DS2} = 2.75 V,

(a) When R_{S1} = 0.2 Ω and R_{S2} = 0.3 Ω

$$I_{D1} = \frac{V_{DS2} - V_{DS1} + I_T R_{S2}}{R_{S1} + R_{S2}} = \frac{2.75 - 3.5 + 20 \times 0.3}{0.2 + 0.3} = 10.5 \text{ A}$$

$$I_{D2} = I_T - I_{D1} = 20 - 10.5 = 9.5 \text{ A}$$

$$\Delta I = I_{D1} - I_{D2} = (10.5 - 9.5)A = 1 \text{ A}$$

(b) When R_{S1} = 0.45 Ω and R_{S2} = 0.45 Ω

$$I_{D1} = \frac{V_{DS2} - V_{DS1} + I_T R_{S2}}{R_{S1} + R_{S2}} = \frac{2.75 - 3.5 + 20 \times 0.45}{0.45 + 0.45} = 9.166 \text{ A}$$

$$I_{D2} = I_T - I_{D1} = 20 - 9.166 = 10.834 \text{ A}$$

$$\Delta I = I_{D1} - I_{D2} = (10.834 - 9.166)\text{A} = 1.668 \text{ A}$$

3.13 SWITCHING CHARACTERISTICS OF POWER MOSFET

Figure 3.27 shows a steady state switching circuit of a power MOSFET and the switching model of a power MOSFET is depicted in 3.28. When a pulse input voltage is applied to the gate of power MOSFET, the device will be turn-ON if the gate to source voltage V_{GS} is greater than threshold voltage $V_{GS(\text{th})}$. The switching waveforms of a power MOSFET are illustrated in Fig. 3.29.

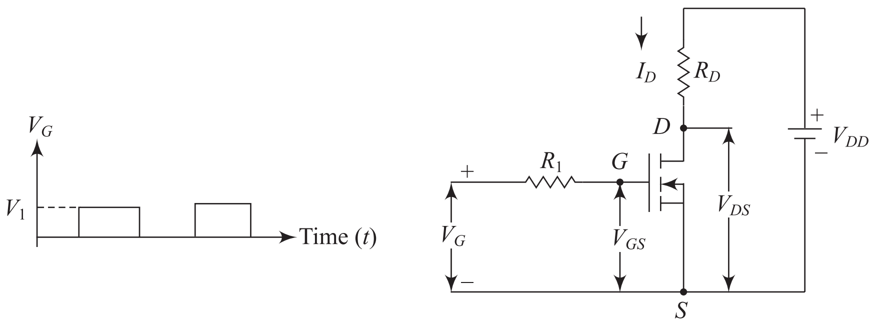


Fig. 3.27 *Power MOSFET as a switch*

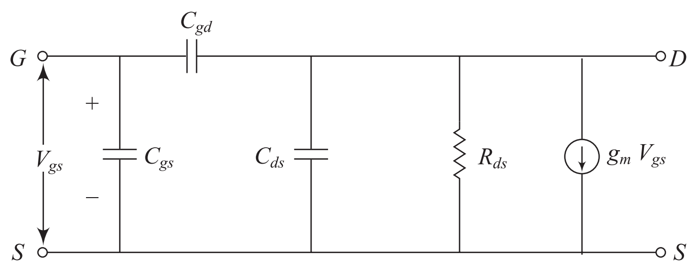


Fig. 3.28 *Switching model of MOSFET*

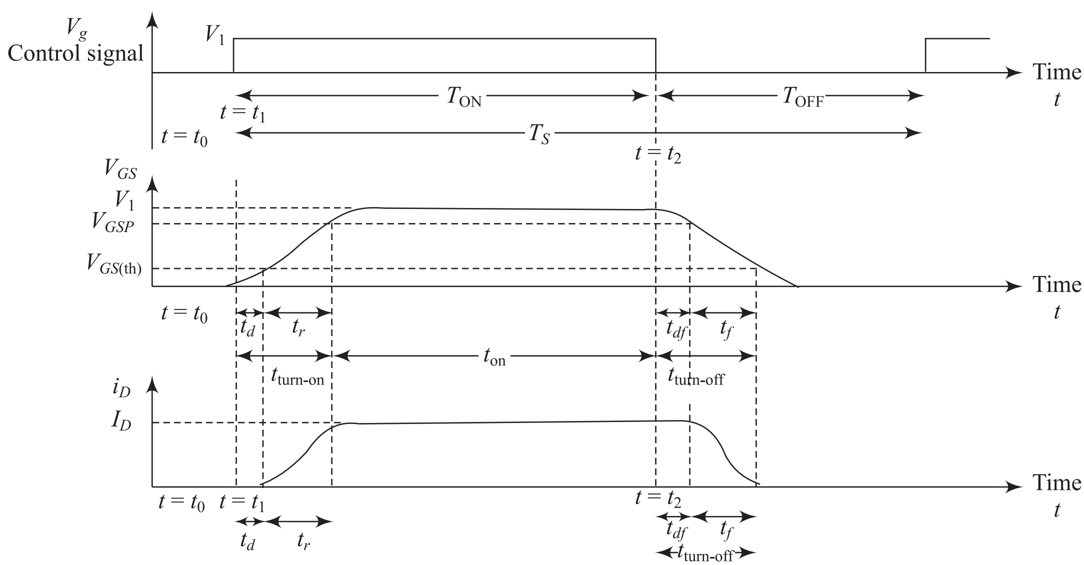


Fig. 3.29 *The switching waveforms of a Power MOSFET*

At time $t = t_0$, input voltage at the gate of power MOSFET is $V_g = 0$ and the gate to source voltage V_{GS} is less than threshold voltage $V_{GS(\text{th})}$. At that moment, the device operates in OFF state and the drain current I_D is equal to zero and the output voltage is $V_o = V_{DS} = V_{DD}$.

At time $t = t_1$, voltage starts to increase from 0 to V_1 and the input capacitance C_{gs} starts to charge as depicted in Fig. 3.29. During the turn on delay time t_d , the capacitance C_{gs} is charged to gate threshold voltage $V_{GS(\text{th})}$. During rise time t_r, the gate to source voltage V_{GS} increases from gate threshold level $V_{GS(\text{th})}$ to the full gate voltage, V_{GSP} to operate the transistor in linear region. In time t_r, the drain current increases from 0 to I_D. The total turn on time of MOSFET is sum of delay time and rise time.

1. Delay time t_d The *delay time* t_d is the time required to charge the input capacitance from its initial value to gate threshold voltage $V_{GS(\text{th})}$.

2. Rise time t_r The *rise time* t_r is the time required to charge the input capacitance C_{gs} from gate threshold level $V_{GS(\text{th})}$ to the full gate voltage, V_{GSP}.

3. Turn-ON time $t_{\text{turn-on}}$ The *turn-ON time* is the sum of the delay time t_d and the rise time t_r and it can be expressed as $t_{\text{turn-on}} = t_d + t_r$. The turn on time of power transistor is typically 1.6 µs.

MOSFET is a majority carrier device. In the turn-OFF process, the gate voltage V_g is removed $t = t_2$, the input capacitance starts to discharge from gate voltage V_1 to V_{GSP}. V_{GS} must be decreased significantly so that V_{DS} starts to increase. The fall time is the time during which input capacitance discharged from V_{GSP} to gate threshold voltage $V_{GS(\text{th})}$. In this time, the drain current decreases from I_D to zero. When $V_{GS} < V_{GS(\text{th})}$, the device completely turn off.

The *turn-OFF delay time* t_{df} is the time during which the input capacitance discharges from gate voltage V_1 to V_{GSP}.

The *fall time* t_f is the time in which the input capacitance discharges from gate voltage V_{GSP} to $V_{GS(\text{th})}$ and the drain current becomes zero

The *turn-OFF time* is the sum of the turn-OFF delay time (t_{df}) and fall time (t_f) and it can be represented by $t_{\text{turn-off}} = t_{df} + t_f$. The turn-OFF time of power MOSFET is about 30 to 300 ns.

Average power loss when the device in OFF state is

$$P_{\text{off-state}} = V_{DS} I_{DSS} t_{\text{off}} f_s \text{ Watt}$$

Average power loss during turn ON is

$$P_{\text{turn-on}} = \frac{1}{6} V_{DS} I_D t_r f_s \text{ Watt}$$

Average power loss during turn OFF is

$$P_{\text{turn-off}} = \frac{1}{6} V_{DS} I_D t_f f_s \text{ Watt}$$

Average power loss during ON state of device is

$$P_{\text{on-state}} = I_D^2 R_{DS(\text{on})} t_{\text{on}} f_s$$

Example 3.7 The switching waveform of a power transistor has the following parameters: V_{DD} = 150 V, I_{DSS} = 2.5 mA, I_D = 50 A, t_d = 0.1 µs, t_r = 0.45 µs, t_{on} = 10 µs, t_{df} = 0.2 µs, t_f = 0.8 µs, $R_{DS(\text{on})}$ = 0.1 Ω. Determine (a) Average power loss when the device in off state, (b) Average power loss during rise time, (c) Average power loss during conduction time t_{on} and (d) Average power loss during fall time. Assume switching frequency is 50 kHz.

Solution

Given: V_{DD} = 150 V, I_{DSS} = 2.5 mA, I_D = 50 A, t_d = 0.1 µs, t_r = 0.45 µs, t_{on} = 10 µs, t_{df} = 0.2 µs, t_f = 0.8 µs and f_s = 50 kHz

$$T_s = \frac{1}{f_s} = \frac{1}{50 \times 10^3} = 20\ \mu\text{s}$$

$$t_{\text{off}} = T_s - (t_d + t_r + t_{\text{on}} + t_{df} + t_f) = 20\ \mu\text{s} - (0.1 + 0.45 + 10 + 0.2 + 0.8)\ \mu\text{s} = 8.45\ \mu\text{s}$$

(a) Average power loss when the device in OFF state

$$\begin{aligned} P_{\text{off-state}} &= V_{DS} I_{\text{DSS}} t_{\text{off}} f_s \text{ Watt} \\ &= 150 \times 2.5 \times 10^{-3} \times 8.45 \times 10^{-6} \times 50 \times 10^3 \text{ Watt} = 0.1584 \text{ Watt} \end{aligned}$$

(b) Average power loss during rise time

$$\begin{aligned} P_{\text{turn-on}} &= \frac{1}{6} V_{DS} I_D t_r f_s \text{ Watt} \\ &= \frac{1}{6} \times 150 \times 50 \times 0.45 \times 10^{-6} \times 50 \times 10^3 \text{ Watt} = 28.185 \text{ Watt} \end{aligned}$$

(c) Average power loss during conduction time t_{on}

$$\begin{aligned} P_{\text{on-state}} &= I_D^2 R_{DS(\text{on})} t_{\text{on}} f_s \text{ Watt} \\ &= 50^2 \times 0.1 \times 10 \times 10^{-6} \times 50 \times 10^3 \text{ Watt} \\ &= 125 \text{ Watt} \end{aligned}$$

(d) Average power loss during fall time

$$\begin{aligned} P_{\text{turn-off}} &= \frac{1}{6} V_{DS} I_D t_f f_s \text{ Watt} \\ &= \frac{1}{6} \times 150 \times 50 \times 0.8 \times 10^{-6} \times 50 \times 10^3 \text{ Watt} = 50 \text{ Watt} \end{aligned}$$

3.14 COMPARISON BETWEEN POWER MOSFET AND POWER BJT

The comparison between power MOSFET and power BJT is given in Table 3.1.

Table 3.1 *Comparison between power MOSFET and power BJT*

POWER MOSFET	POWER BJT
1. Power MOSFET is a unipolar device.	1. Power BJT is a bipolar device.
2. Power MOSFET is a voltage controlled device.	2. Power BJT is a current controlled device.
3. Power MOSFET is a majority carrier device.	3. Power BJT is a majority as well as minority carrier device.
4. Input impedance of power MOSFET is very high compared to power BJT.	4. Input impedance of power BJT is low compared to power MOSFET.
5. Power MOSFET has positive temperature coefficient. With increase in temperature, resistance increases.	5. Power BJT has negative temperature coefficient. With increase in temperature, resistance decreases.
6. Secondary breakdown does not occur in power MOSFET.	6. Secondary breakdown occurs in power BJT.

(*Contd.*)

POWER MOSFET	POWER BJT
7. The operating frequency of power MOSFET is high about 100 kHz.	7. The operating frequency of Power BJT is low about 10 kHz.
8. Power MOSFET has low switching loss but conduction loss is more due to high on-state resistance.	8. Power BJT has high switching loss but conduction loss is less.
9. The on state voltage of power MOSFETs is high compared to power BJTs.	9. The on state voltage of power BJT is low compared to power MOSFETs.
10. Powers MOSFET are available with low current rating compared to Power BJT. Current rating of power MOSFET is about 140 A.	10. Powers BJT are available with high current rating compared to power MOSFET. Current rating of power BJT is about 800 A.
11. Power MOSFETS are very sensitive to voltage spikes compared to power BJTs.	11. Power BJTs are less sensitive to voltage spikes compared to power MOSFETs.

3.15 INSULATED GATE BIPOLAR TRANSISTOR

An insulated gate bipolar transistor (IGBT) is a multi-layer semiconductor structure with alternate *p*-type and *n*-type doping. Figure 3.30 shows a vertically oriented *n*-channel IGBT which is a $p^+ n^+ n^- p n^+$ structure. To make the *p*-channel IGBT, the structure of IGBT will be implemented by changing the doping type in each layer as depicted in Fig. 3.30. Accordingly, the *p*-channel IGBT will be $n^+ p^+ p^- n p^+$ structure.

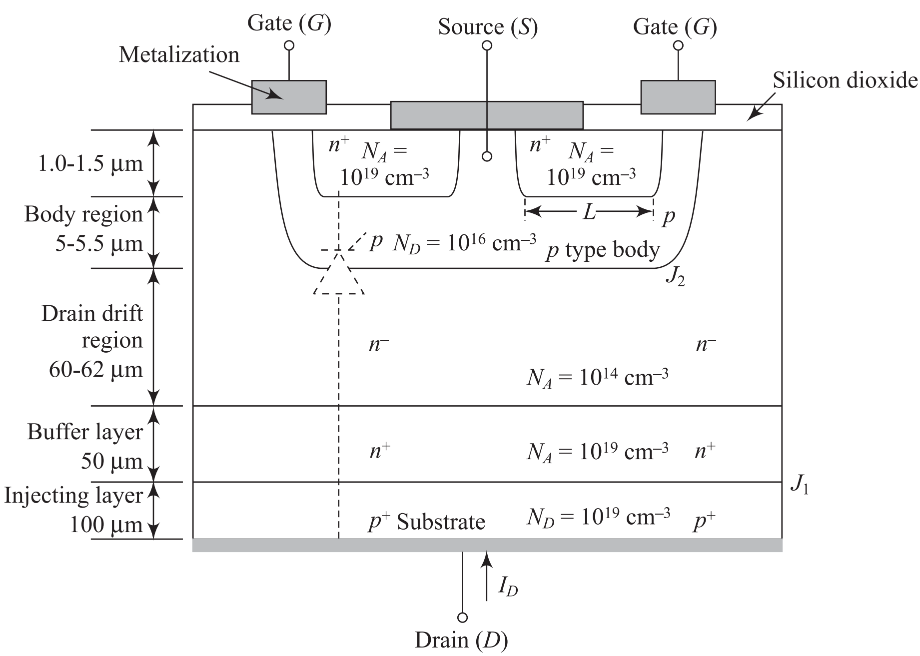


Fig. 3.30 *Structure of a n-channel IGBT*

Initially the n^+ layer is epitaxially grown on the p^+ substrate and a p^+n^+ junction J_1 is formed. The p^+ substrate is used as drain and minority carriers are injected through drain region. Therefore, p^+ is called *injecting layer*. The doping density in the p^+ and n^+ layers is about 10^{19} cm^{-3}. The n^- layer is also epitaxially grown on the n^+ substrate. After that p-type semiconductor is diffused in the epitaxially grown n^- layer and the p region will be developed. Finally n^+ semiconductor is diffused in the p region and the n^+ region is developed.

The n^+ region which is formed above p^+ substrate is known as *buffer layer*. The n^- layer is called the *drain drift region* and the doping density in the n^- layer is low. The typical value of n^- layer doping density is about 10^{14} cm^{-3} to 10^{15} cm^{-3}. The thickness of n^- drift region determines the breakdown voltage of the device. The p-type semiconductor layer is the region where the channel is established between source and drain. For that reason, p region is called the *body* of a IGBT. The doping density of p region is about 10^{16} cm^{-3} to 10^{17} cm^{-3}.

The doping density in the two end layers such as p^+ and n^+ of a vertically oriented IGBT is same and its value is quite large, typically 10^{19} cm^{-3}. n^+ region is used as source and the p^+ region is used as drain as depicted in Fig. 3.30.

It is clear from Fig. 3.30 that there are three p-n junctions such as drain to buffer layer $p^+ n^+$ junction J_1, drain drift region to body n^-p junction J_2 and body to source pn^+ junction J_3.

Actually, the presence of n^+ buffer layer between the p^+ injecting layer or drain contact and the n^- drift region is not important for the operation of the IGBT. Some IGBTs are made without n^+ buffer layer. When n^+ buffer layer is present between p^+ injecting layer and n^- drift layer and the doping density and thickness of n^+ buffer layer are selected properly, the operating performance of the IGBT can be improved significantly.

A parasitic *pnpn* thyristor is developed between the source and drain contacts as shown in Fig. 3.30. The p-type body region acts as the gate, n^+ region as cathode and p^+ drift injecting as anode of the parasitic thyristor. As the p-type body region is shorted to the source region due to overlapping the source metallisation on the p-type body region, source is connected with the gate and cathode of the parasitic thyristor. As the potential difference between gate and cathode of the parasitic thyristor is zero, the parasitic thyristor always operate in cut-OFF region. Actually turn-ON of this thyristor is undesirable. The body and source are shorted in the IGBT to minimise the turn-ON possibility of the parasitic thyristor.

It is clear from Fig. 3.30 and Fig. 3.19 that the IGBT structure is somewhat similar to the structure of power MOSFET except p^+ injecting layer. The n-channel IGBT has three terminals namely gate (G), drain (D) or collector (C) and source (S) or emitter (E). The circuit symbols of n-channel IGBT and p-channel IGBT are depicted in Figs. 3.31(a) and (b) respectively. The direction of the arrow represents the direction of current flow.

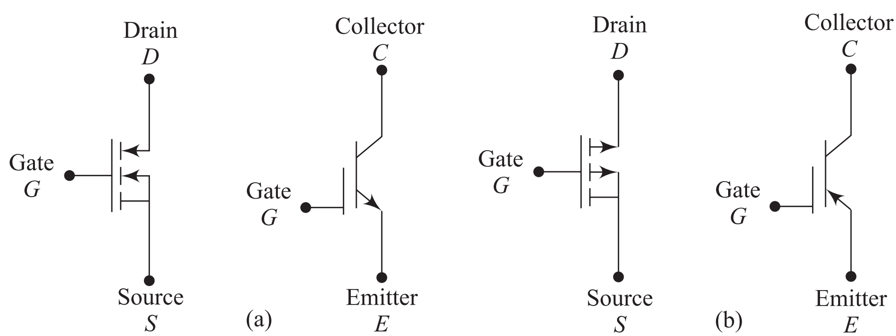


Fig. 3.31 *(a) Circuit symbol of n-channel IGBT (b) Circuit symbol of p-channel IGBT*

3.16 *I-V* CHARACTERISTICS OF IGBT

Figure 3.32(a) shows an *n*-channel IGBT circuit where the input signal V_{GS} is applied across gate to source and the output signal V_{DS} is obtained from drain. The current flow from drain to source I_D is controlled by gate signal. Generally, the source terminal is common between the input and output of a IGBT. The alternative representation of Fig. 3.32(a) is depicted in Fig. 3.32(b) where the emitter terminal is common between the input and output of a IGBT.

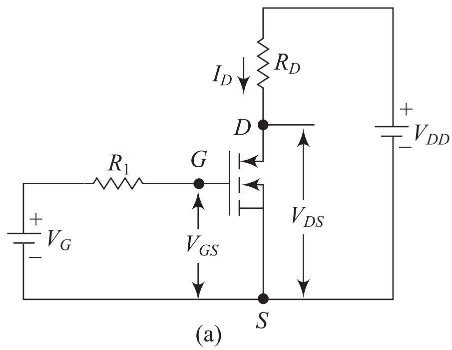


Fig. 3.32 *n-channel IGBT circuit*

The transfer characteristic $I_D - V_{GS}$ of an *n*-channel IGBT is depicted in Fig. 3.33. The drain current I_D is a function of gate-to-source voltage V_{GS} and the curve is reasonably linear in most of the drain current when the gate-to-source voltage is greater than threshold voltage $V_{GS(\text{th})}$. But the curve is non-linear when the drain current is low and the gate-to-source voltage is approaching the threshold voltage $V_{GS(\text{th})}$. If the gate-to-source voltage is less than threshold voltage $V_{GS(\text{th})}$, the current flow from drain to source is about zero. The value of $V_{GS(\text{th})}$ for IGBT is about 2 to 3 volts. While I_D is equal to zero, the drain to source is open circuit and then IGBT operates in off -state and the device should able to hold the supply voltage V_{DD}.

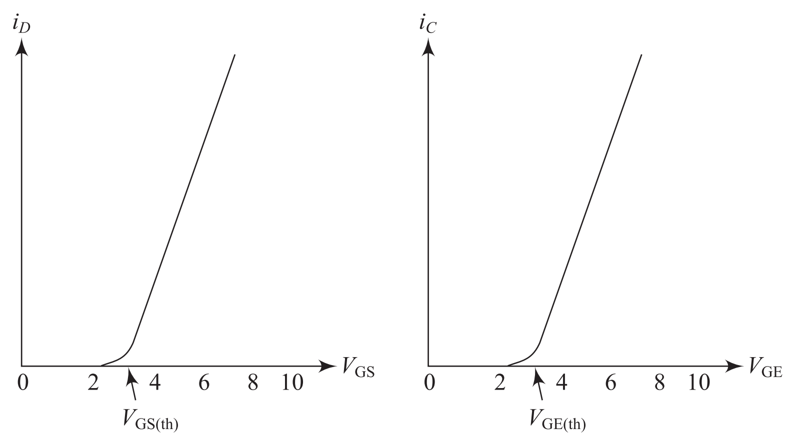


Fig. 3.33 *Transfer characteristic of an n-channel IGBT*

When the gate-to-source voltage is less than threshold voltage $V_{GS(\text{th})}$, there is no inversion layer developed to connect the drain to the source. Consequently, the drain to source is open circuit and the applied voltage is dropped across the junction J_2 and a very small leakage current flows. At that time IGBT operates in off-sate.

It is clear from Fig. 3.34(a) that the drain-to-source breakdown voltage V_{DSS} must be greater than V_{DD} to keep away from the device breakdown. If the drain-to-source voltage is greater than V_{DSS}, the IGBT will be breakdown due to avalanche breakdown of the drain-to-body junction.

The output characteristics of an n channel IGBT is depicted in Fig. 3.34(a). The drain current I_D is a function of drain-to-source voltage V_{DS} when gate-to-source voltage V_{GS} is constant. It is clear form Fig. 3.34(a) that the output characteristics consist of three regions such as cut-off, active and on regions.

The I-V characteristics of IGBT are similar to I-V characteristics of a logic level bipolar junction transistor (BJT). In case of BJT, the controlling parameter is base current where as the controlling parameter of IGBT is gate-to-source voltage V_{GS}. The characteristics of p-channel IGBT will be the same characteristics of n-channel IGBT but the polarities of the voltages and currents will be reversed.

When the IGBT operates in off-state, the junction J_2 blocks the forward voltages. The IGBT has reverse voltage blocking capability. The reverse blocking voltage of IGBT will be equal to the forward blocking voltage when the IGBT is manufactured without n^+ buffer layer. During reverse bias condition, the junction J_1 behaves as reverse blocking junction. When n^+ buffer layer is present within the device, the breakdown voltage of J_1 junction will be reduced significantly and its value is about few tens of volts due to presence of high doping density on both sides of J_1 junction. When the applied reverse voltage is greater than the breakdown voltage of J_1 junction, the IGBT lost his reverse blocking capability. Since IGBT has reverse voltage blocking capability, it can be used in ac circuit applications. In Fig. 3.34(a) V_{BR} is the reverse breakdown voltage.

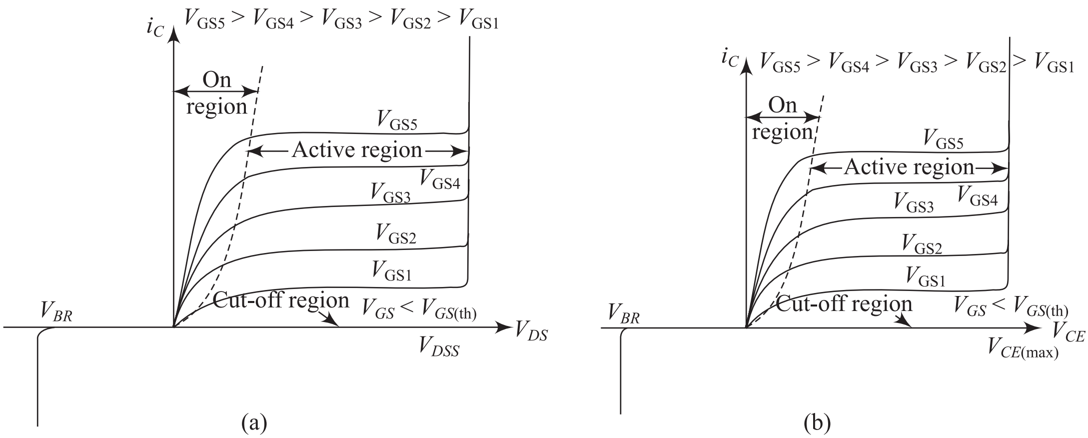


Fig. 3.34 *(a) Output characteristic of an n-channel IGBT (b) Output characteristic of an n-channel IGBT*

3.16.1 Symmetrical IGBT and Anti-symmetrical IGBT

Just like a MOSFET, the gate-to-source voltage V_{GS} controls the state of the IGBT. When V_{GS} is less than the threshold voltage $V_{GS(th)}$, the inversion layer will not be developed to connect the drain to the source. Subsequently, the IGBT operates in the off-state. In this condition, the applied drain-to-source voltage is dropped across the junction J_2 and a very small leakage current flow.

There are two types of IGBT namely

1. Symmetrical IGBT or non-punch through IGBT and
2. Anti-symmetrical IGBT or punch through IGBT

1. Symmetrical IGBT The depletion region of junction J_2 can be extended into the n^-drain drift region as the p-type body region is doped heavily compared to the n^- drain drift region. The thickness of n^- drain drift region will be such that the depletion region of junction J_2 can be accommodated and the depletion layer boundary cannot touch the *p*+ injecting layer. This type of IGBT is called as a *symmetrical IGBT* or *non-punch through IGBT*. Due to absence of n^+ buffer region, the reverse blocking voltage is equal to the forward blocking voltage. The reverse voltage blocking capability of IGBT is useful in ac circuit applications.

2. Anti-symmetrical IGBT In anti-symmetrical IGBT, the thickness of the n^- drift region is reduced by a factor of 2 and the device becomes a punch through structure. In this case, the depletion layer can be extended into the complete n^- drift region when applied voltage is significantly less than the breakdown voltage. Subsequently, the depletion layer boundary can able to touch the *p*+ injecting layer. The reach-through of the depletion layer to the *p*+ injecting layer can be avoided by adding an n^+ buffer region between the n^- drain drift region and the *p*+ injecting layer. This type of IGBT structure is called an anti-*symmetric IGBT* or *punch–through IGBT*. Due to small length of n^- drift region, the on-state loss of IGBT will be less. Since the n^+ buffer region is present with in the device, the reverse blocking capability of anti-*symmetric IGBT* or *punch–through IGBT is* low. Therefore, these types of IGBT are not suitable for ac circuit applications.

3.16.2 On-State Voltage Drop

When a voltage is applied between gate and source and the applied gate-to-source voltage V_{GS} is greater than the threshold voltage $V_{GS(\text{th})}$, an inversion layer is created just below the gate of the IGBT. The developed inversion layer can interconnect the n^- drift region and the n^+ drift region and electron current flows through this inversion layer as shown in Fig. 3.35. In the same time, holes are

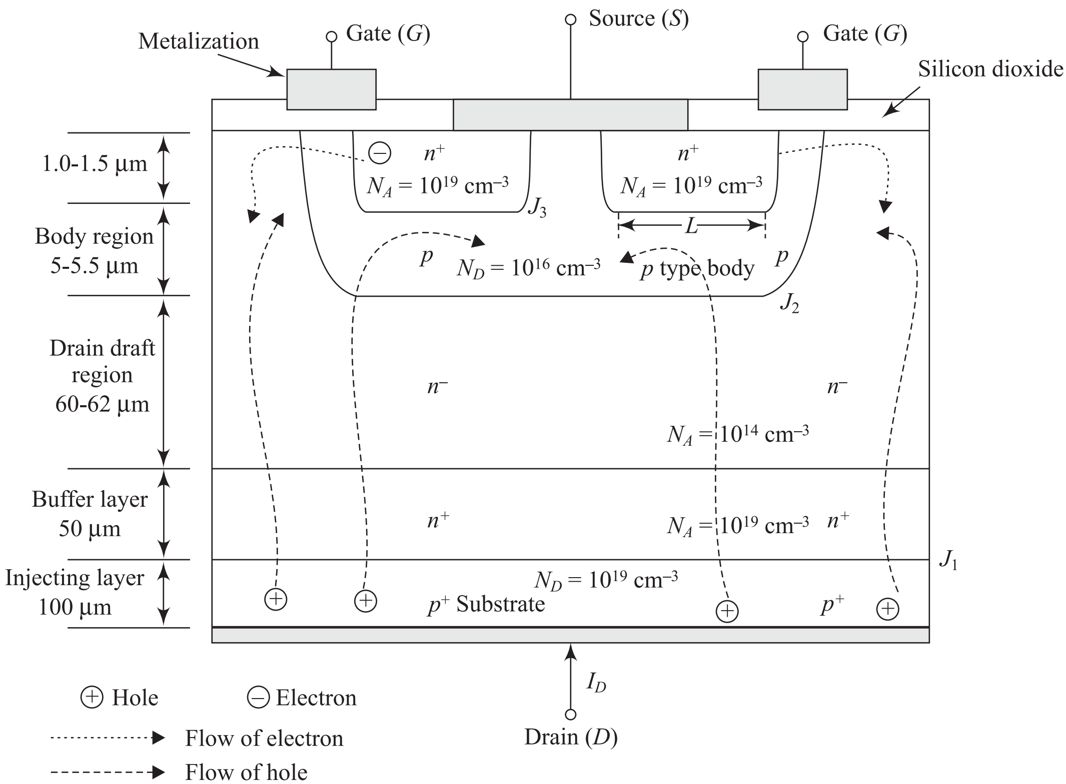


Fig. 3.35 *Flow of electrons and holes during on-state of IGBT*

injected from the $p+$ drain region to the n^- drift region as depicted in Fig. 3.35. The injected holes can come across the n^- drift region using drift and diffusion methods and can able to reach the p-type body region. Whenever the holes reach in the p-type body region, these holes attract electrons from the source and the excess holes recombined with electrons.

The formation of MOSFET, *p-n-p* and *n-p-n* transistors with in the structure of IGBT is depicted in Fig. 3.36. The equivalent circuit of IGBT is shown in Fig. 3.37(a). The IGBT can be represented by a Darlington circuit with the *p-n-p* transistor (main transistor) and the MOSFET as the driver device as given in Fig. 3.37(b). The Fig. 3.37(a) can be represented by Fig. 3.38 as approximate equivalent circuit for normal operating condition represents the resistance of the n^- drift region is the resistance between the *p-n-p* base and the MOSFET drain and it is represented by R_{drift}.

The *ON-state voltage* across drain to source is $V_{DS(\text{on})}$ and it can be expressed as

$$V_{DS(\text{on})} = V_{J1} + V_{\text{drift}} + I_D R_{\text{Channel}}$$

where, V_{J1} is the voltage drop across the junction J_1 and it is the forward biased voltage drop across a PN junction and its value is about 0.7 V to 1.0 V.

V_{drift} is the voltage drop across the drift region. The value of V_{drift} voltage is less in the IGBT than in the MOSFET due to the conductivity modulation of the drift region.

$I_D R_{\text{Channel}}$ is the voltage drop across the channel due to the resistance of the channel.

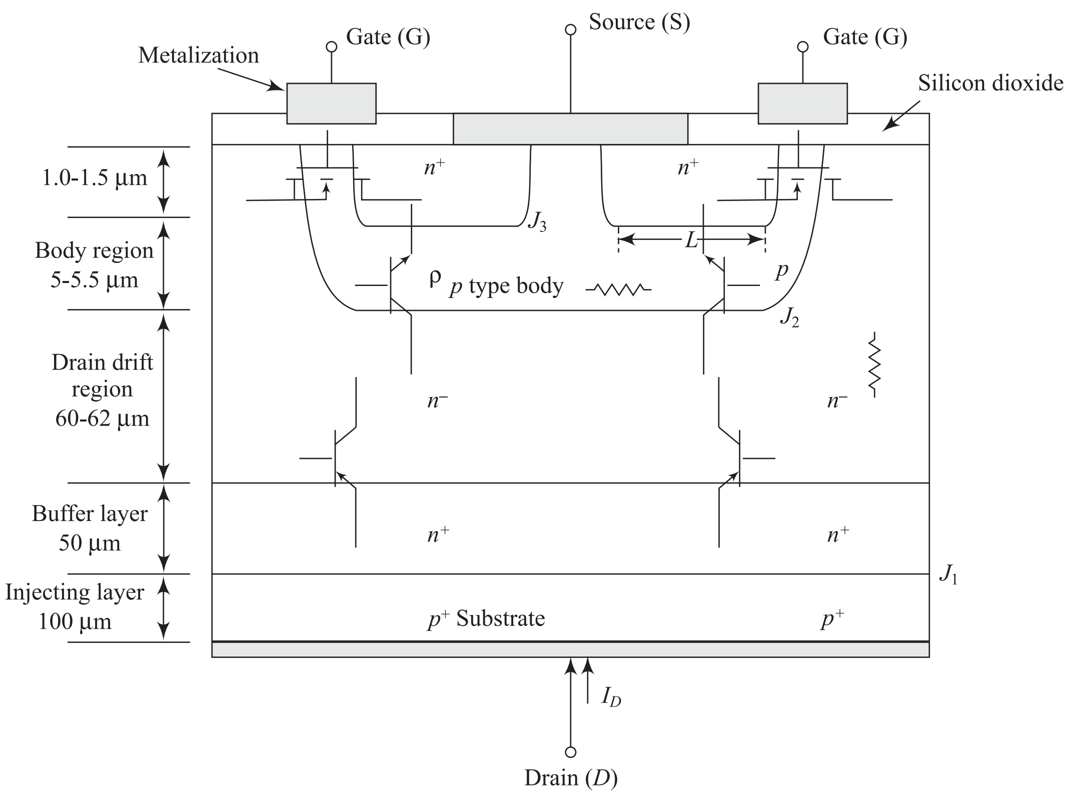


Fig. 3.36 *MOSFET, p-n-p and n-p-n transistors within the structure of IGBT*

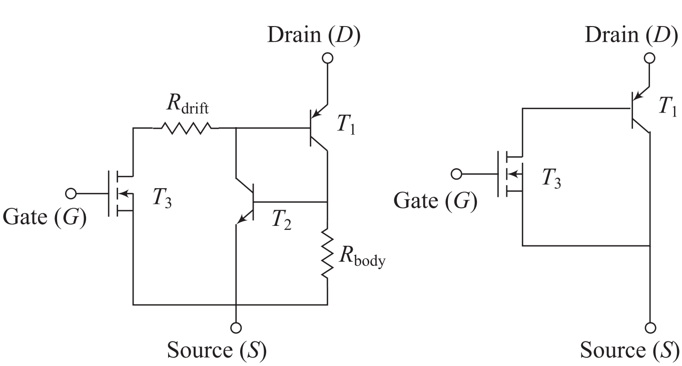


Fig. 3.37 *(a) Equivalent circuit of IGBT*
(b) Darlington circuit of MOSFET and p-n-p transistor

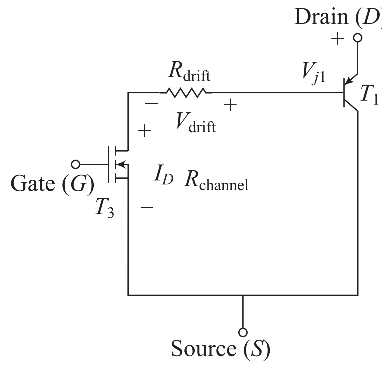


Fig. 3.38 *Approximate equivalent circuit of IGBT*

3.17 SAFE OPERATING AREA OF IGBT

The safe operating area (SOA) represents the maximum values of current and voltage at which the IGBT can withstand safely. As per manufacture datasheet related to specification of IGBT MOSFET, the safe operating area (SOA) can be determined the following parameters:

1. Maximum drain current $I_{D(\max)}$ or maximum collector current $I_{C(\max)}$
2. Maximum permissible internal junction temperature $T_{j(\max)}$
3. Forward blocking voltage in FBSOA V_{DSS} or $V_{CE(\max)}$
4. Reverse breakdown voltage in RBSOA V_{BR}
5. Maximum power dissipation $P_{D(\max)}$
6. Maximum gate voltage $V_{GE(\max)}$

$I_{C(\max)}$ is the maximum permissible collector current of IGBT and its value is about 200 A to 400 A. $V_{GE(\max)}$ is the maximum gate voltage which is determined by the breakdown of the silicon dioxide (SiO_2) layer. $V_{CE(\max)}$ is the maximum collector to emitter voltage which is determined by the breakdown voltage of transistor T_2. The commercially available IGBT has 1700 V blocking capability. $T_{j(\max)}$ is the maximum permissible internal junction temperature and its value is about 150°C.

Figure 3.39 shows the safe operating area (SOA) of IGBT where drain current I_D and drain to source voltage V_{DS} are represented in logarithmic scale. At low value of V_{DS}, the maximum drain current $I_{D(\max)}$ is limited by the power dissipation. The maximum power dissipation is also limited by the maximum tolerable junction temperature $T_{j(\max)}$ within the device. The breakdown voltage V_{DSS} at low drain current is limited to avoid the avalanche breakdown of the drain drift to body n^-p junction.

When the IGBT operates with dc signal, the boundary of the SOA is A-B-C-D. During

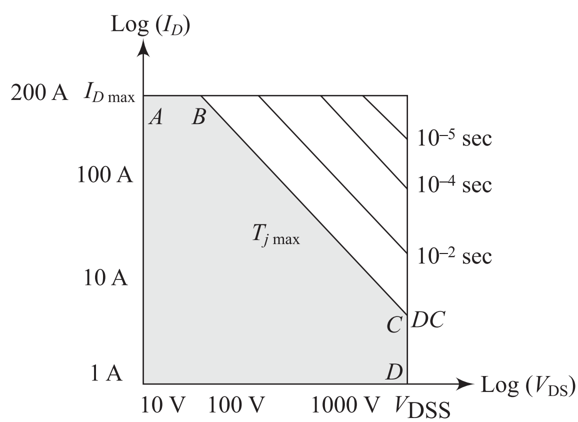


Fig. 3.39 *FBSOA of a IGBT*

operation with dc signal, a continuous power will be dissipated within device and the junction temperature increases significantly. Boundary A-B represents the maximum limit of drain current I_{DM} with V_{DS} is less than 100 V. When V_{DS} is greater than 100 V, the drain current has been reduced as per boundary B-C so that the operating junction temperature of power MOSFET must be less than the maximum operating junction temperature. The boundary C-D represents the maximum voltage capability of a IGBT.

When the IGBT operates as a switch and it is driven by a pulse signal, the boundaries of the SOA can be extended as shown in Fig. 3.39. The extension of the SOA is only possible for switch mode operation. Actually, the silicon wafer and its packaging of IGBT have a specified thermal capacitance and it has an ability to absorb a limited amount of energy without increasing the junction temperature excessively. The forward biased safe operating area (FBSOA) of IGBT is square for short switching times and it is similar to FBSOA of MOSFET. For longer switching times, the FBSOA of IGBT is identical to FBSOA of MOSFET.

When the IGBT turns ON for a few microseconds, the device can absorb very small amount of energy and the junction temperature rise will be low and the boundary of SOA will be increased. When the pulse width is about 10^{-2} s, the SOA is greater than the boundary A-B-C-D. If the pulse width is further reduced and its value is about some 10^{-5} s, the SOA is greater than the boundary with pulse with 10^{-2} s. It is clear from Fig. 3.39 that the SOA increases with the decrease of pulse-width.

The reverse bias safe operating area (RBSOA) of IGBT is different from the FBSOA of IGBT. With the increase of the rate of change of reapplied drain to source voltage $\frac{dV_{DS}}{dt}$, the upper right hand corner of the reverse bias safe operating area (RBSOA) of IGBT is cut out gradually and the RBSOA of IGBT reduces significantly. Figure 3.40 shows the RBSOA of IGBT.

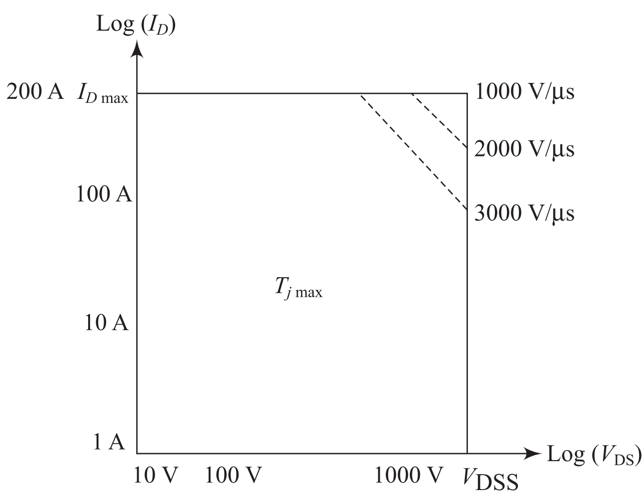


Fig. 3.40 *RBSOA of IGBT*

3.18 SWITCHING CHARACTERISTICS OF IGBT

Figure 3.41 shows the turn on switching characteristics of IGBT. The *delay time* $t_{d(\text{on})}$ is the required time to fall the collector-emitter voltage from V_{CE} to $0.9V_{CE}$ where V_{CE} is the initial collector emitter voltage. The delay time $t_{d(\text{on})}$ is also the time required for collector current to increase from its initial

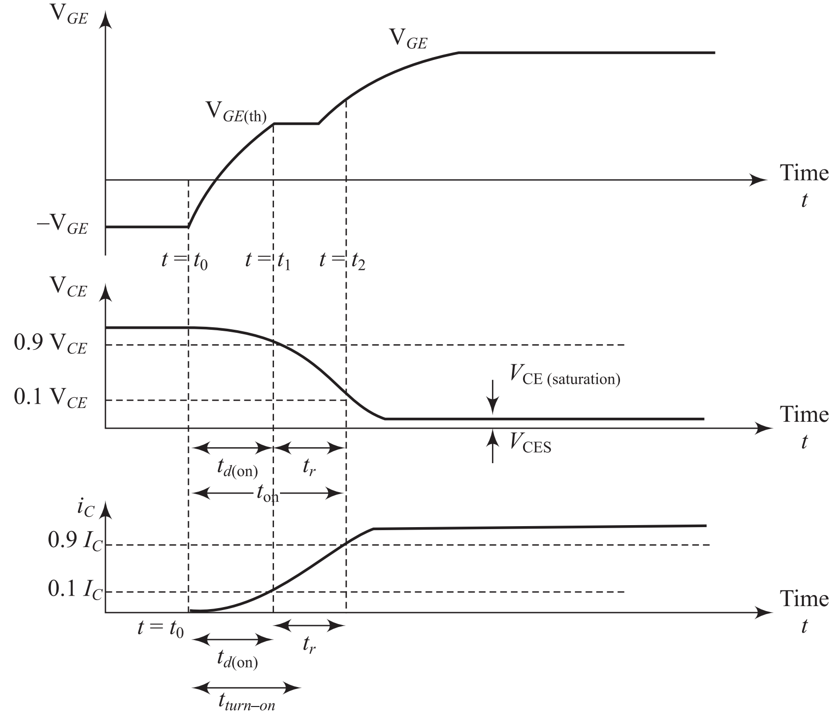


Fig. 3.41 *Turn-ON switching characteristics of IGBT*

value to the 10% of the rated collector current ($0.1I_C$). The *rise time* t_r is the required time to increase the collector current from the 10% of the rated collector current ($0.1I_C$) to its rated value (I_C) and the collector-emitter voltage falls from 90% of $V_{CE}(0.9V_{CE})$ to 10% of $V_{CE}(0.1V_{CE})$. The total *turn-ON time* t_{on} is sum of delay time and rise time and it is expressed as

$$t_{\text{on}} = t_{d(\text{on})} + t_r.$$

After turn on the IGBT, the collector-emitter voltage falls to small value called ON-state voltage drop V_{CES} where S represents the saturated value.

Figure 3.42 shows the turn-OFF switching characteristics of IGBT which is somewhat complex and the bipolar transistor plays an important role to understand the switching characteristics of a IGBT. The total *turn-OFF time* t_{off} consists of delay time $t_{d(\text{off})}$ and rise time, initial fall time t_{f1} and final fall time t_{f2}. The total *turn-off time* t_{off} can be expressed as

$$t_{\text{off}} = t_{d(\text{off})} + t_{f1} + t_{f2}.$$

The *delay time* $t_{d(\text{off})}$ is the required time during which the gate-emitter voltage falls from V_{GE} to the threshold voltage $V_{GE(\text{th})}$. Since the gate-emitter voltage falls to $V_{GE(\text{th})}$ during $t_{d(\text{off})}$, the collector current falls from rated value (I_C) to the 90% of the rated collector current ($0.9I_C$). At the end of delay time $t_{d(\text{off})}$, the collector-emitter voltage starts to rise.

The *first fall time* t_{f1} is the time during which the collector current falls from the 90% of the rated collector current ($0.9I_C$) to the 20% of the rated collector current ($0.2I_C$) or time during which the collector emitter voltage rises from V_{CES} to 10% of $V_{CE}(0.1V_{CE})$.

The *final fall time* t_{f2} is the time during which the collector current falls from the 20% of the rated collector current ($0.2I_C$) to the 10% of the rated collector current ($0.1I_C$) or time during which the collector emitter voltage rises from 10% of V_{CE} ($0.1V_{CE}$) to final value of V_{CE}.

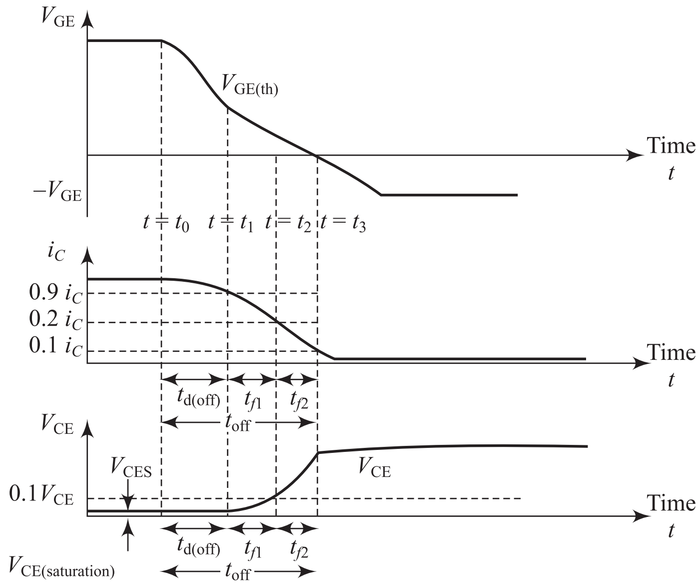


Fig. 3.42 *Turn-OFF switching characteristics of IGBT*

The switching characteristics of an IGBT are very much similar to that of a Power MOSFET but the major difference is that IGBT has a tailing collector current due to the stored charge in the N^- drift region. Actually, the tail current increases the turn-off loss. To reduce switching losses, it is required to switch OFF the gate with a negative voltage of –15 V.

During the turn-ON process of IGBT, the energy loss in the device is

$$W_{on} = \frac{V_{CE_{max}} I_{C_{max}} t_{on}}{6}$$

Then average power loss during the turn-ON process of IGBT is equal to

$$P_{on(average)} = W_{on} f_s = \frac{V_{CE_{max}} I_{C_{max}} t_{on} f_s}{6} \quad \text{where, } f_s \text{ is the switching frequency of IGBT}$$

Similarly, during the turn-OFF process of IGBT, the energy loss in the device is

$$W_{off} = \frac{V_{CE_{max}} I_{C_{max}} t_{off}}{6}$$

Subsequently, the average power loss during the turn-OFF process of IGBT is equal to

$$P_{off(average)} = W_{off} f_s = \frac{V_{CE_{max}} I_{C_{max}} t_{off} f_s}{6} \quad \text{where, } f_s \text{ is the switching frequency of IGBT}$$

Example 3.8 A IGBT switching circuit as shown in Fig. 3.43 has the following parameters:
$t_{on} = 2.5\ \mu s$, $t_{off} = 3.5\ \mu s$, $V_{CE(saturation)} = 2.4$ V, $R_L = 10\ \Omega$, $f_s = 1.5$ kHz, $V_{CC} = 250$ V.

When duty cycle is 50%, determine (a) average load current, (b) average conduction loss, (c) turn-on loss and (d) turn-off loss.

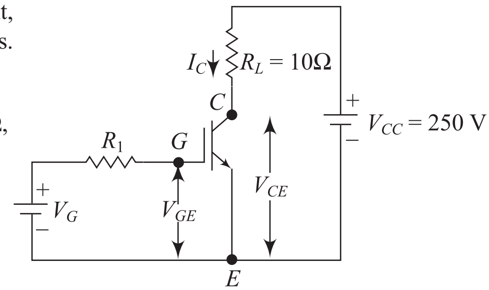


Fig. 3.43

Solution

Given: $t_{on} = 2.5$ μs, $t_{off} = 3.5$ μs, $V_{CE(saturation)} = 2.4$ V, $R_L = 10\ \Omega$, $f_s = 1.5$ kHz, $V_{CC} = 250$ V

(a) $$I_{C_{max}} = \frac{V_{CC} - V_{CE(saturation)}}{R_L} = \frac{250 - 2.4}{10}\text{A} = 24.76 \text{ A}$$

As duty cycle is 50%, the average load (collector) current is

$$I_{C_{average}} = DI_{C_{max}} = 0.5 \times 24.76 = 12.38 \text{ A}$$

(b) Average conduction loss is

$$V_{CE(saturation)} I_{C_{average}} = 2.4 \times 12.38 = 29.712 \text{ Watt}$$

(c) Turn-ON loss is

$$P_{on(average)} = W_{on} f_s = \frac{V_{CE_{max}} I_{C_{max}} t_{on} f_s}{6} = \frac{250 \times 24.76 \times 2.5 \times 10^{-6} \times 1.5 \times 10^3}{6} \text{W} = 3.868 \text{ W}$$

(d) Turn-OFF loss is

$$P_{off(average)} = W_{off} f_s = \frac{V_{CE_{max}} I_{C_{max}} t_{off} f_s}{6} = \frac{250 \times 24.76 \times 3.5 \times 10^{-6} \times 1.5 \times 10^3}{6} \text{W} = 5.416 \text{ W}$$

3.19 COMPARISON BETWEEN POWER MOSFET AND IGBT

The comparison between power MOSFET and IGBT is given in Table 3.2.

Table 3.2 *Comparison between power MOSFET and IGBT*

Power MOSFET	IGBT
1. Power MOSFET is a voltage controlled device.	1. IGBT is a voltage controlled device.
2. Power MOSFET has three terminals namely gate (*G*), drain (*D*) and source (*S*).	2. IGBT has three terminals namely gate (*G*), drain (*D*) or emitter (*E*) and source (*S*) or collector (*C*).
3. Input impedance of power MOSFET is very high.	3. Input impedance of IGBT is very high.
4. Power MOSFET has positive temperature coefficient. With increase in temperature, on-sate resistance increases compared to IGBT.	4. IGBT has positive temperature coefficient. With increase in temperature, on-state resistance increases but rate of increment is less than increase in MOSFET.
5. The on-state voltage drop of MOSFET is large compared to on-state voltage drop of IGBT.	5. IGBT has a very low on-state voltage drop due to conductivity modulation. So smaller chip size is possible and the cost can be reduced.
6. The ON-state voltage drop of MOSFET increases by 3 times for temperature rise from room temperature to 200°C.	6. The increment of ON-state voltage drop of IGBT is very small.

(*Contd.*)

Power MOSFET	IGBT
7. Current sharing between parallel connected MOSFETs is poor compared to IGBTs.	7. Current sharing between parallel connected IGBTs is better compared to MOSFETs.
8. At high ambient temperature, maximum current rating reduces.	8. At high ambient temperature, IGBT is well suited.
9. Switching speed of power MOSFET is superior to that of a IGBT.	9. Switching speed of IGBT is inferior to that of a power MOSFET.
10. Wide FBSOA.	10. Wide FBSOA and RBSOA. It also has excellent forward and reverse blocking capabilities.
11. The power MOSFET has a parasitic BJT as an integral part of its structure.	11. The IGBT has a parasitic thyristor as an integral part of its structure.

3.20 STATIC INDUCTION TRANSISTOR (SIT)

The static induction transistor (SIT) is a high power high frequency semiconductor device. This device was developed by J. Nishizawa in Japan in 1987. The basic structure of an SIT is depicted in Fig. 3.44(a). SIT is the solid state version of the triode vacuum tube. This is a n^+n^-n semiconductor device with p^+ gate. Figure 3.44(a) shows the vertical structure with short n-type multi-channels and the symbol of SIT is illustrated in Fig. 3.44(b). p^+ gate is buried in n^-epitaxial layer, as shown in Fig. 3.44(a). The p^+ buried gate structure provides the low resistance between gate and source, the low capacitance between gate and source and low thermal resistance. Due to low channel resistance, the voltage drop will be less. SIT has high audio frequency power handling capability, low distortion and less noise. The turn ON and turn OFF times (t_{on} and t_{off}) of SIT are very small and these times are about 0.25 μs to 0.35 μs. The operating frequency of SIT is about 100 kHz. The maximum voltage and current rating of SIT is 1200 V and 300 A respectively.

Generally, this device operates in ON state when V_{GS} is zero and V_{DS} is present. The majority carriers, i.e., electrons will flow from source terminal to n-type region and passes though n-type channels to n^- region and finally reach to n^+ region. The drain current I_D can flow from drain to source.

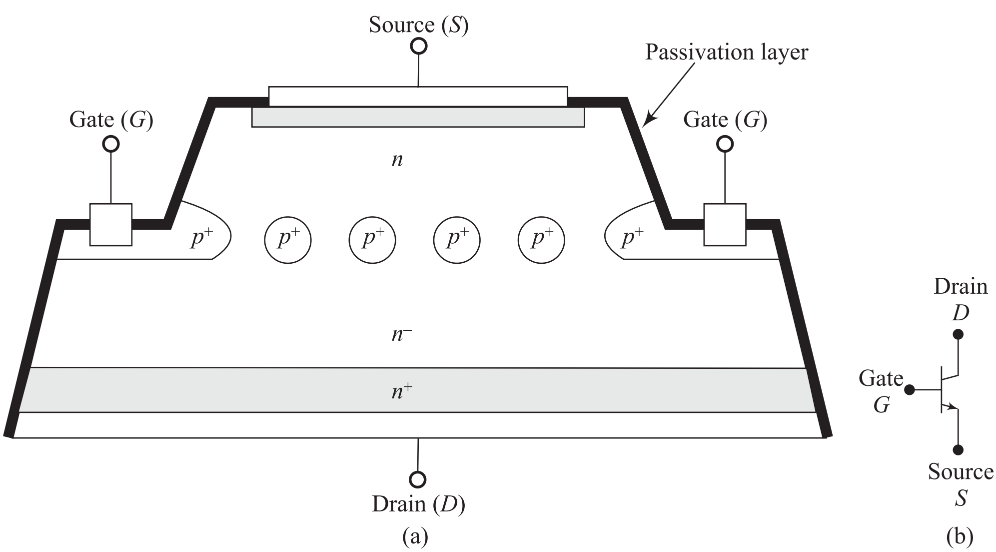


Fig. 3.44 *(a) Basic structure of SIT (b) Symbol of SIT*

When V_{GS} is negative ($V_{GS} < 0$), the p^+n becomes reverse biased and a depletion layer will be developed around p^+ buried gate as depicted in Fig. 3.45. The channel width will be reduced and the current flow from drain to source will be reduced. If the V_{GS} is highly negative, the depletion layer around p^+ buried gate is so high that the channel will be cut-off completely. Consequently, current I_D becomes zero and the device operates in OFF state as shown in Fig. 3.46.

The ON-state voltage drop is typically high. For example, if 180 A current flow through the device, the ON-state voltage drop is about 90 V as the channel resistance of SIT is about 0.5 ohms. The I-V characteristics of SIT are depicted in Fig. 3.47. SIT is most commonly used in high power and high frequency applications such as microwave amplifiers, induction heaters, AM/FM transmitters, VHF/UHF applications, high voltage and low current power supply.

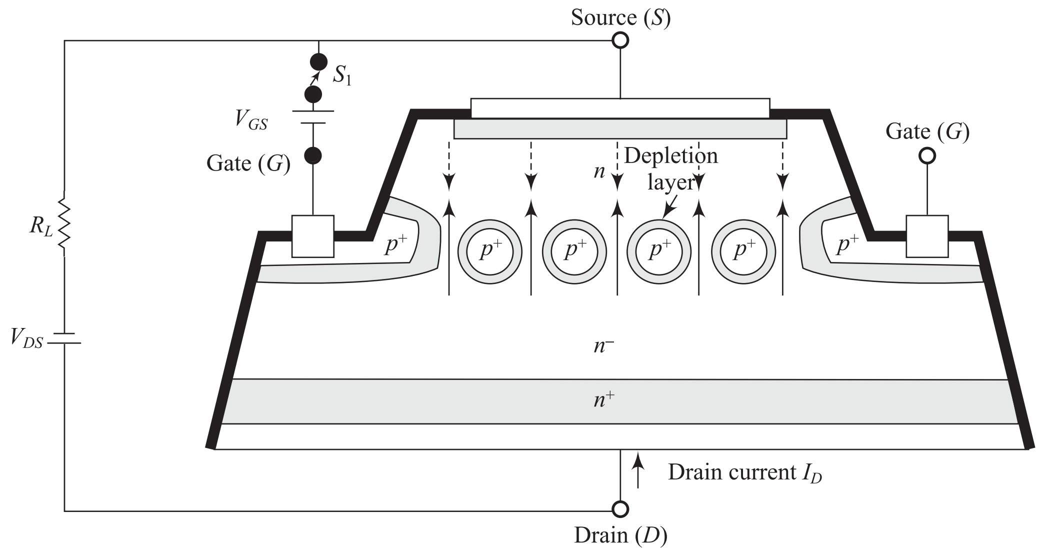


Fig. 3.45 *Drain current I_D reduced due to depletion layer when low reverse voltage is applied*

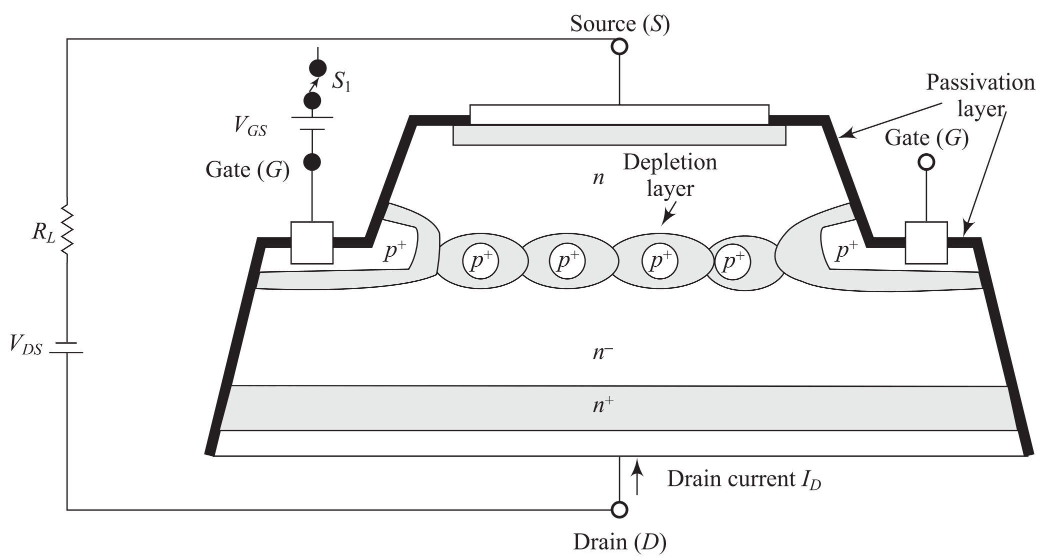


Fig. 3.46 *Drain current I_D reduced to zero due to depletion layer when high reverse voltage is applied*

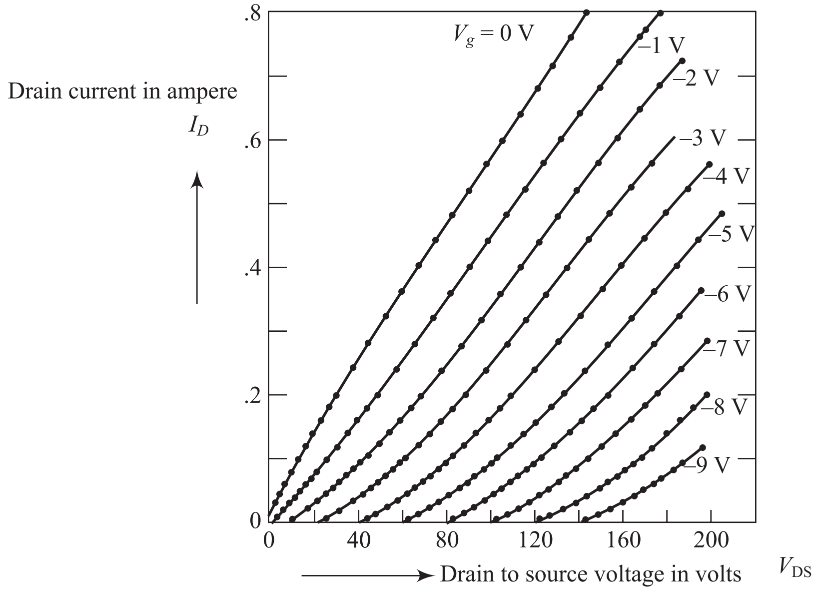


Fig. 3.47 *I-V characteristics of SIT*

3.21 DRIVE CIRCUITS FOR BJT, MOSFET AND IGBT

Power semiconductor devices such as BJTs, MOSFETs and IGBTs are becoming increasingly popular in all applications of power electronics based circuits like the dc-dc converters, ac-to-dc rectifiers with power factor correction, dc-to-ac inverters, dc and ac motor drives, SMPS, choppers and UPS, etc. In almost all power electronics applications, power semiconductor devices operate in switched mode operation to achieve maximum efficiency and the power devices are being used as switches. To achieve proper and efficient operation of the power electronics equipments, the power semiconductor devices should be driven in an appropriate manner so that these devices behave as switches.

The input and output characteristics of transistors (BJT/MOSFET/IGBT) are different from input and output characteristics of thyristors. For example, when a thyristor is forward biased and a triggering pulse is applied in between anode and cathode, it will be turned ON. Once thyristor is triggered and turned ON, it continues to remain in conduction state unless the current passes through it reduces to zero. Actually, if the current flow through thyristor is below the holding current, thyristor will be turned OFF. But in the case of transistors, the device will operate in conduction state when continuous base signal is applied to BJT and continuous gate signal is applied to MOSFET.

Bipolar junction transistors are current controlled devices whereas metal oxide field effect transistors (MOSFETs) are voltage controlled devices. Therefore, separate driver circuits are required for BJTs and MOSFETs. In this section, base drive circuits of BJTs and gate drive circuits of MOSFETs and IGBTs are discussed in detail.

3.21.1 Base Drive Circuit of a Power BJT

Usually, a base drive circuit should provide electrically isolated base signal to turn-ON and turn-OFF a transistor. A power BJT has a current gain about 5 to 10 (β = 5 to 10). The power handling capability of driver circuit should be 20% higher than the power circuit. This circuit should have very high impedance; therefore a very low power driver circuit is carefully designed to drive a BJT.

Figure 3.48 shows the base current I_B, input voltage, voltage across base-emitter and collector current waveforms of the base drive circuit for a power transistor. During the turn-ON period, the base current I_B should be fast rising and should overshoot the steady-on value of I_B. The maximum value of the base current I_B during turn-on is represented by $I_{B\max+}$. During the turn-OFF period, a negative base current I_B should be applied to the base of power BJT to quickly remove the stored charges in the transistor. The maximum negative value of the base current I_B during turn-OFF is represented by $I_{B\max-}$.

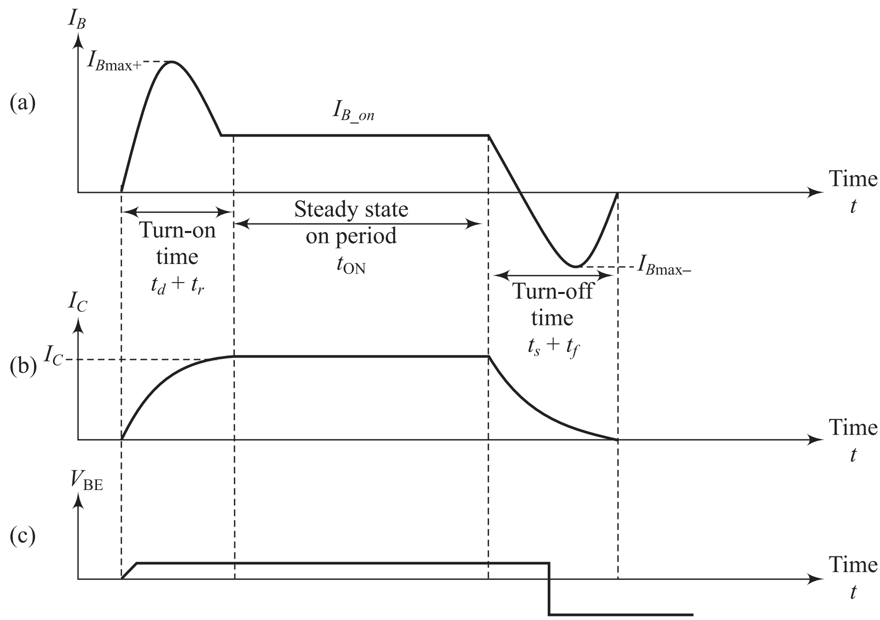


Fig. 3.48 *Waveforms of base current I_B, collector current I_C and voltage across base-emitter V_{BE}*

During the steady-on period of transistor, the base current value is limited by the transistor predictable collector current and the minimum value of β. When the transistor operates in conduction state, the steady-state on state base current is represented by I_{B_on}. For the duration of the steady-off period, the base current value should be zero. Consequently, a base drive circuit of transistor should be designed in such way that the base current wave shape should follow the waveform as depicted in Fig. 3.48(a). If a base drive circuit is properly designed, the circuit reliability improves by minimising the switching time. Then switching losses will be reduced and the device can be operating in robust mode. Usually, the value of I_{B_on}, $I_{B\max+}$ and $I_{B\max-}$ are expressed by

$$I_{B\_on} = 2\frac{I_C}{h_{fe}} = 2\frac{I_C}{\beta}, I_{B\max+} = 1.5 I_{B\_on} \text{ and } I_{B\max-} = 1.5 I_{B\_on}$$

If a high-current pulse is superimposed with the normal turn-ON base current which is supplied by a current source, BJT can be rapidly switched into conduction state from turn-OFF state. Consequently, both the turn-ON switching time and switching losses are reduced. During turn-OFF process of transistor, when a negative base current is applied instead of simply decreasing the input base current, the turn-OFF time of switching time can be reduced.

Usually, the base drive circuits of transistor are placed near to the device. It reduces the length of wire and inductance of the connecting wires. The effects of noise or stray signals are minimised and it clamps the oscillation in the base drive circuit signal.

As a high power current source is required to drive the base of power transistor, the difficulty arises when the emitter is at a fixed voltage level or the emitter is not at ground potential. When a load is connected in between emitter and ground or BJTs are connected in bridge configuration, the emitter terminal will be at floating potential. Subsequently, the emitter potential changes from zero to supply voltage and supply voltage to zero when the BJT is in OFF and ON condition respectively.
A properly designed base drive circuit should be able to

1. Provide adequate positive base current during turn ON
2. Provide adequate base current to maintain the transistor in the steady-on state
3. Supply negative base current during turn OFF of the transistor

For high power applications, the drive circuit should

1. provide isolation between the control circuit which generates control signals for the base drive and the power circuit where power transistors are turned ON and OFF periodically to control power.
2. have proper protection arrangement against high $\frac{dv}{dt}$ or surge voltage and over currents.

3.21.2 Classification of Base Drive Circuits

The base drive circuit can be classified based on presence of isolation and negative voltage source. The following types of base drive circuits are commonly used:

1. Base drive circuit without isolation
2. Base drive circuits without negative voltage source
3. Base drive circuits with negative voltage source
4. Base drive circuit with transformer isolation
5. Base drive circuits with opto-isolation
6. Base drive circuits with various combinations of the above.

In this section, some base drive circuits are explained in detail.

Base drive circuit without isolation and without negative voltage source

Figure 3.49 shows a simplest base drive circuit for a power BJT. Actually, this circuit is the basic building block of any complex base drive circuit. This base drive circuit is suitable only for low power and low frequency applications. When the input signal $V_{control}$ is greater than $V_{reference}$, the comparator output voltage is high. If the input signal $V_{control}$ is less than $V_{reference}$, the comparator output voltage is low. Hence, the comparator output signal V_B is a square wave as depicted in Fig. 3.49.

During the turn-ON time, this circuit does not provide the starting base current $I_{B\max+}$. The resistance R_2 is used to provide the discharge path for removing the base charges in the power transistor T_P.

Generally, the value of I_{B_on} is expressed by $I_{B\_on} = 2\frac{I_C}{h_{fe}} = 2\frac{I_C}{\beta}$ and the current flows through R_2 is $I_{R_2} = I_{B\_on}$.

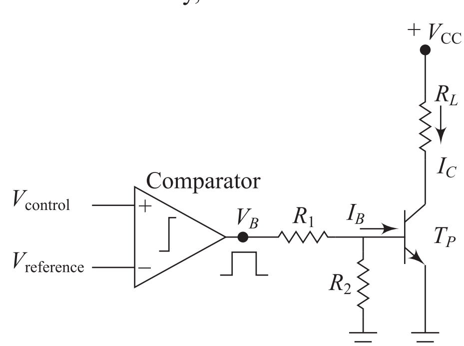


Fig. 3.49 *Simplest base drive circuit for a power BJT*

Then $R_2 = \dfrac{V_{BE}}{I_{R_2}} = \dfrac{V_{BE}}{I_{B\_on}} = \dfrac{V_{BE}h_{fe}}{2I_C}$ as $I_{B\_on} = 2\dfrac{I_C}{h_{fe}}$ and $R_1 = \dfrac{V_B - V_{BE}}{2I_{B\_on}} = \dfrac{(V_B - V_{BE})h_{fe}}{4I_C}$

Figure 3.50 shows a drive circuit of power transistor. In this circuit T_P is the power transistor which will be turned ON and OFF to control power flow and T_1 and T_2 are auxiliary low power transistors. When the current sourcing capability of V_B is low, this circuit is very useful. Actually, transistors T_1 and T_2 provide the required current gain to drive the power transistor T_P.

While the voltage V_B is positive, T_1 turns ON. Consequently, the collector to emitter voltage of transistor T_1 is about zero. Subsequently, resistance R_3 is connected to ground at one end and the emitter-base junction of the *p-n-p* transistor T_2 is forward biased. As a result, T_2 turns ON. Accordingly, the necessary base drive current flows through the resistance R_1 and the power transistor T_P will be turned on.

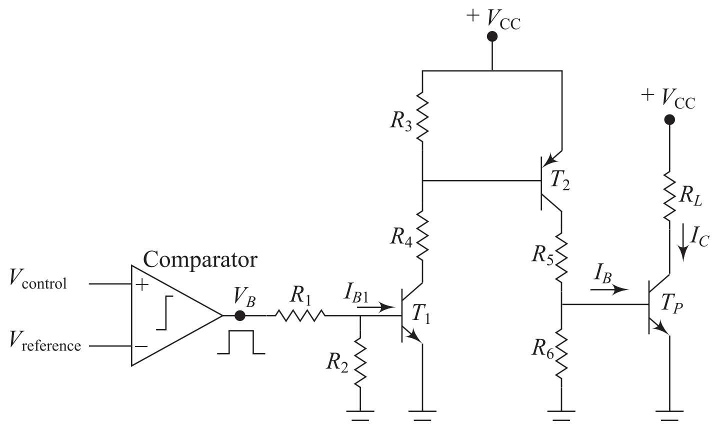


Fig. 3.50 *A drive circuit of power transistor*

When V_B is zero, subsequently transistor T_1 is turned OFF and the voltage at base of transistor T_2 is high, hence T_2 becomes turned OFF. Therefore, there is no base current to drive the power transistor T_P and T_P will operate in off state.

Figure 3.51 shows a drive circuit of power transistor which improved version of Fig. 3.50. In this circuit, the turn-OFF time of the power transistor T_P is reduced by using an inductor L.

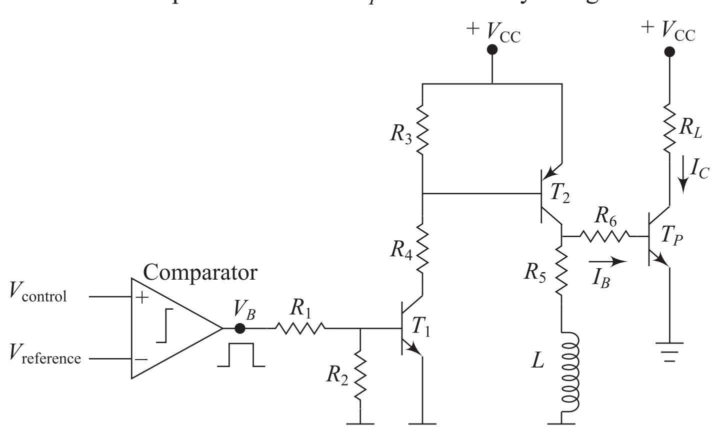


Fig. 3.51 *A drive circuit of power transistor*

When the transistor T_2 operates in conducting state, a current I flows through inductor L and some energy will be stored in the inductor. The current flow through the inductor L is limited by the resistance R_5. Whenever the transistor T_2 operates in cut off, the inductor L does not allow sudden change in its current. Consequently, the voltage polarity across the inductor will be reversed. For this reason, the inductor behaves as a generator and it provides the reverse base current $I_{B\max-}$ for fast turn off of the power transistor T_P. Normally, the value of L is determined by the following equation:

$$L = \frac{(R_5 + R_6)I_{B\max-} - V_{BE}}{\frac{dI_B}{dt}}$$

where, $\frac{dI_B}{dt} = 0.15 I_C A/\mu s$ for high voltage transistors (> 700 V)

$\frac{dI_B}{dt} = 0.5 I_C A/\mu s$ for low voltage transistors (< 200).

Base drive circuit without isolation and with negative voltage source This base drive circuit without isolation and with negative voltage source is shown in Fig. 3.52. This circuit is similar to Fig. 3.50, but the resistance R_6 is connected to a negative supply. The operation of Fig. 3.52 is similar to that of the base drive circuit as shown in Fig. 3.50, but during turn off of the power transistor T_P, a negative base current $I_{B\max-}$ is supplied by the negative voltage source ($-V_{CC}$) through the resistance R_6. The value of resistance R_6 is determined by

$$R_6 = \frac{V_{BE} + V_{CC}}{I_{B\max-}}$$

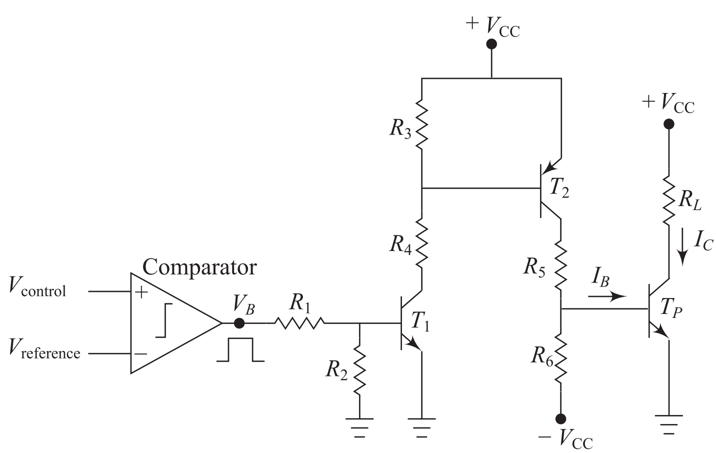


Fig. 3.52 *Base drive circuit without isolation and with negative voltage source*

If $\quad I_{B\_on} = 2\frac{I_C}{h_{fe}} = 2\frac{I_C}{\beta}$, and $I_{B\max-} = 1.5 I_{B\_on}$,

The value of resistance R_6 is equal to $R_6 = \frac{(V_{BE} + V_{CC})h_{fe}}{3I_C}$

Base drive circuit with isolation Actually, the base drive or control circuit operates at low voltage with lower power and the load or the power circuit is connected with high voltage and power circuit is used to control power. If the control circuit is not isolated from power circuit, the control circuit will be damaged. Therefore, the isolation between the base drive or control circuit and the load or the power circuit is required for high power applications. Usually, isolation can be provided either by using opto-couplers or by using transformers.

Base drive circuit using opto-coupler Figure 3.53 shows a base drive circuit which uses an opto-coupler to provide the isolation between the base drive side and the high power collector side circuit.

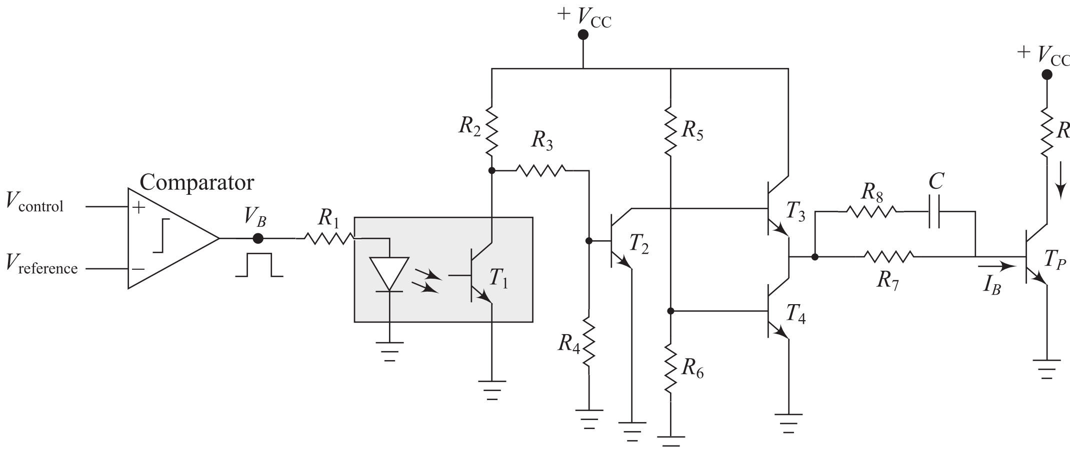


Fig. 3.53 *A base drive circuit using an opto-coupler*

When the base drive pulse V_B is positive, a current flows through the photodiode of the opto-coupler as shown in Fig. 3.53. Consequently, the transistor T_1 of the opto-coupler is to be turned on. As a result the transistor T_2 operates in OFF state and transistor T_3 becomes ON. When T_3 is turned on, a surge current $I_{B\max+}$ is supplied to the base of the power transistor T_P through the path R_8 - C. Due to presence of base surge current, power transistor T_P will be turned on rapidly. When the power transistor T_P is fully on, the capacitor should be charged to V_{CC}.

Similarly, when the base drive pulse V_B is zero, no current flows through the photodiode and the transistor T_1 of the opto-coupler operates in off-state. At this instant, T_2 is turned ON and the power transistor T_P turns OFF with reverse base current flowing through T_4. The selection of the opto-coupler depends on the breakdown voltage between the diode terminals and the transistor terminals, the current transfer ratio of the opto-coupler and the propagation delay.

Base drive circuit using pulse transformer Figure 3.54 shows a base drive circuit which uses a pulse transformer to provide the isolation between the base drive side and the high power collector side circuit.

When a positive base drive pulse voltage V_B is applied to transistor T_1, the transistor T_1 will be turned on and the V_{CC} voltage is applied across the primary of the pulse transformer. Generally, the pulse transformer is a special type of transformer and it is designed to operate at specified frequency range. As a pulse voltage is applied across the primary winding of pulse transformer, it passes the primary pulse to the secondary winding. The output pulse from secondary winding is used to turn on the power transistor T_P.

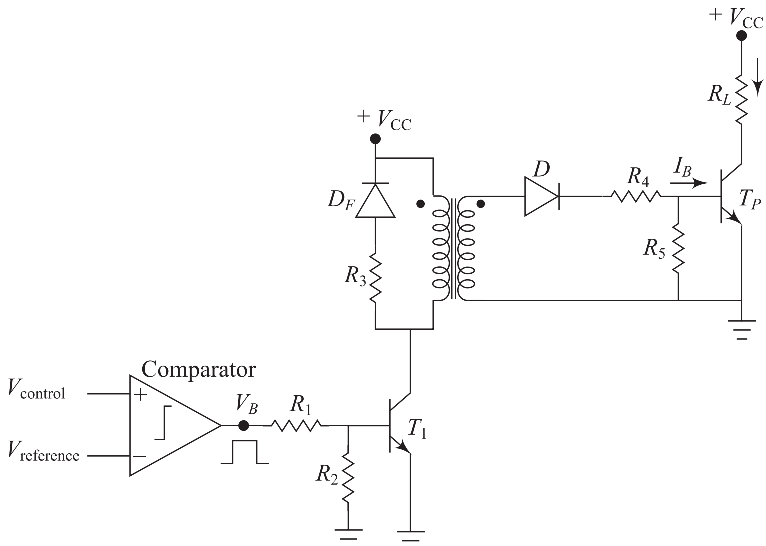


Fig. 3.54 *A base drive circuit using a pulse transformer*

When the base drive pulse V_B is zero volts, the transistor T_1 is turned OFF and suddenly the current in the primary winding will be cut-off and there will be sudden increase in voltage across the primary winding due to $L\frac{di}{dt}$. Since the high voltage appears across transistor T_1 and it can damage the transistor T_1. To save the transistor T_1, the abrupt cut-off of the primary winding current should flow through a freewheeling diode D_F and resistance R_3 when T_1 is turned OFF.

The duty cycle of this circuit is limited as the transistor T_1 must be turned OFF for the time duration when (during which) the core will be reset. When the value of R_3 is large, the core resetting will be faster and subsequently the range of duty cycle will be larger. The voltage drop across R_3 is higher and the collector emitter voltage V_{CE} rating of transistor T_1 is also higher. Practically, the duty ratio D is limited to less than 50%. The collector emitter voltage V_{CE} rating of transistor T_1 should follows the following relationship:

$$V_{CE(T_1)} > V_{CC} + I_m R_3 + V_{D_F}$$

where, I_m is the magnetising current which flows through the primary winding,

V_{D_F} is the voltage drop across freewheeling diode D_F

In Fig. 3.54, the freewheeling path consists of a resistor R_3 and a freewheeling diode D_F and there is power dissipation in resistance R_3. This dissipation can be avoided when a third winding is added in the isolation transformer. Whenever transistor T_1 is turned OFF, the dot poles of the transformer become negative with respect to the other poles. At this instant diode D_F is to be forward biased. Since D_F conducts and the magnetic energy stored in the core freewheel. Accordingly, the core is resetting and it prevents core saturation. Assume winding n_3 is used for demagnetising the core practically, to achieve a very tight coupling between n_1 and n_3, these windings are wound *bifilar*. As the turns ratio n_1:n_3::1:1 is maintained. As a result, a time equal to the on time of T_1 is needed for the core to reset. Hence, the OFF time of transistor T_1 should be equal to the on time of T_1. Consequently, the duty ratio D is limited to 50% and it never exceeds 50%.

3.21.3 Gate Drive Circuit for Power MOSFET

MOSFET is a voltage controlled device and the gate current of MOSFET is independent of the drain current. The input gate characteristics of a power MOSFET is different from power BJT. The gate is electrically isolated from the source by a layer of silicon dioxide. In ideal condition, when a voltage is applied, no current flows into the gate, but a very small leakage current about 10^{-10} A flows to maintain gate voltage. During turn-ON and turn-OFF periods of MOSFET, the small current flow is enough to charge and discharge the capacitances. The capacitances of MOSFET and source impedance of gate drive circuit limit the switching speed.

The driver circuit of power MOSFET is very much similar to driving a very high impedance capacitive network. Therefore, a properly designed very low power driver circuit is required to drive a power MOSFET. Actually, the gate voltage controlled power MOSFET requires a very low power driver circuit.

An n-channel MOSFET has three parasitic capacitances namely C_{gd}, C_{ds} and C_{gs}. These capacitances are non-linear and voltage dependent. The values of capacitances are high for very low value of V_{DS} and are almost constant at higher or rated value of V_{DS}. Figure 3.55 shows an n-channel MOSFET with voltage dependent capacitance.

To turn-OFF a MOSFET, the gate voltage must be reduced to zero and then V_{DS} attains the supply voltage V_{DD}. The potential at drain terminal changes from 0 V to V_{DD} and the capacitance C_{gd} will be charged up to V_{DD} through a low gate-source impedance R_g. When a gate signal is applied to MOSFET and its value is higher than the threshold voltage V_{Th}, MOSFET will be turned ON. When the switch is ON, in ideal case, the voltage across D and S is about zero and the potential of terminal D swings to ground potential. The decreasing of V_{DS} generates a feedback current $i = C_{gd}\dfrac{dV_{DS}}{dt}$ to the gate and these current flows through C_{gd}. This feedback system is known as *Miller Effect*.

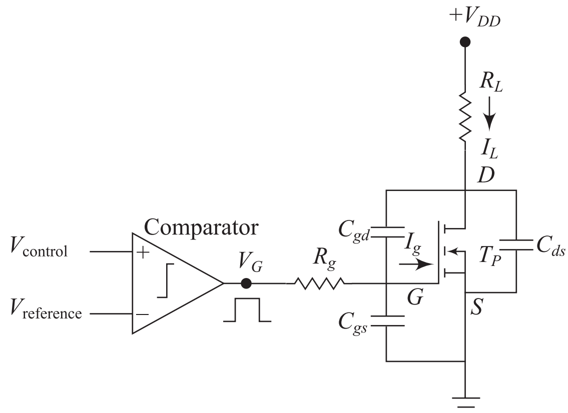


Fig. 3.55 *An n-channel MOSFET with voltage dependent capacitance*

Since the capacitor C_{gs} is charged and discharged and there is a large swing in gate to drain voltage, a gate drive circuit is required with high current source and current sink capability. Though C_{gs} is an important parameter, but the C_{gd} has more effect due to miller effect. Figure 3.56 shows the voltage swing across drain to source of MOSFET during turn ON and turn OFF.

The turn-ON time and turn-OFF time of MOSFET are affected by R_g as R_g is connected in series with the capacitances C_{gs} and C_{gd}. The turn-ON time and turn-OFF time of device (MOSFET) can be reduced by a diode which allows the quick charge and discharge of parasitic capacitors, by shorting R_g. Hence, a bipolar gate drive signal can be used to turn-ON and turn-OFF transistors rapidly. Figure 3.57 shows driver circuits during turn ON and turn OFF.

When the MOSFET is in the ON condition, the gate power requirement is very low. Therefore, the gate current during ON condition of MOSFET is also very low. Figure 3.58 shows the gate current waveform of a gate drive circuit of a MOSFET. It is clear from Fig. 3.58 that the gate current I_g required to maintain the MOSFET in the steady state on condition is about zero. Actually, the energy of gate circuit is only used to turn ON and turn OFF the MOSFET. During turn ON, a maximum positive

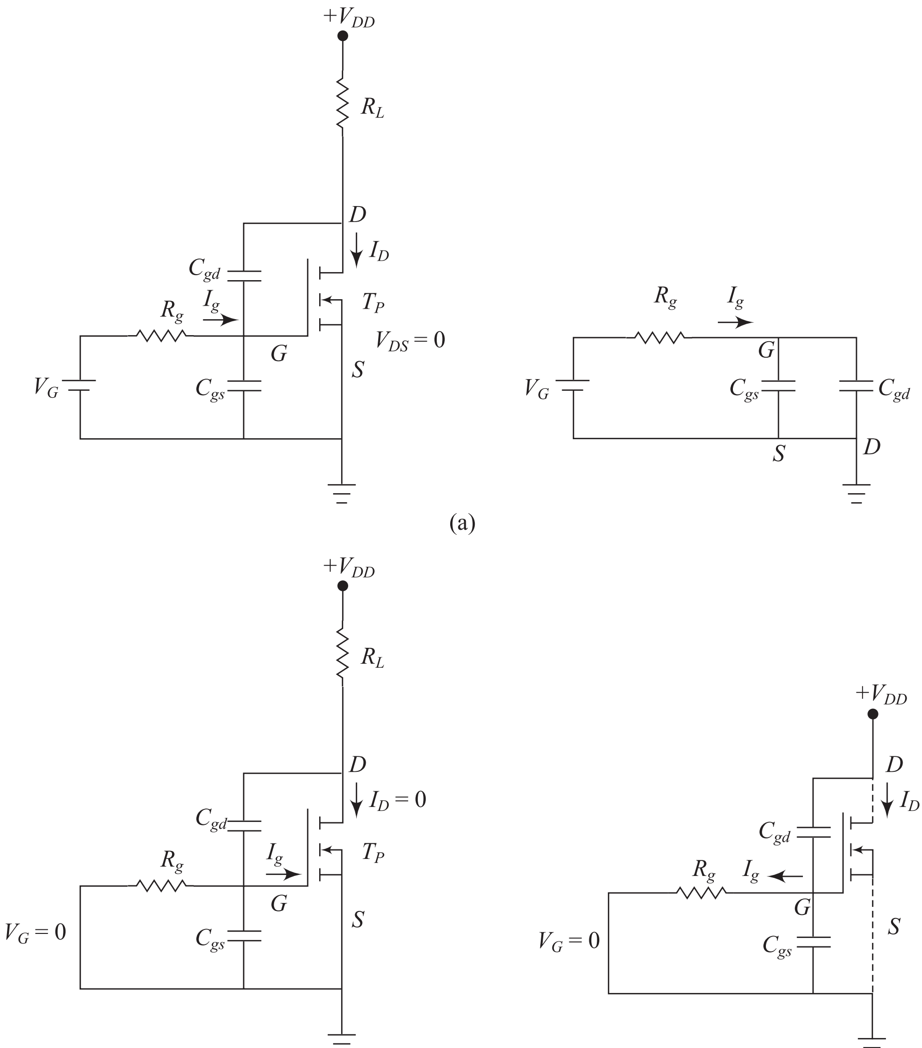


Fig. 3.56 *Voltage swing across drain to source of MOSFET (a) during turn ON and (b) during turn OFF*

gate current of I_{gmax+} is applied to turn ON the MOSFET. During turn OFF a maximum negative gate current of I_{gmax-} is used for fast turn OFF of the MOSFET.

During the turn-ON period of MOSFET, assume a constant gate current of I_{gon} is being applied to the gate of the MOSFET. The turn-ON surge current I_{gmax+} is related with I_{gon} as follows:

$$I_{gon} = \frac{I_{g\max+}}{2}$$

To turn-ON a MOSFET, a specific amount of gate charge Q_G must be supplied to the gate of the MOSFET. As per the manufacturers' data sheets, the amount of gate charge Q_G is equal to

$$Q_G = I_{gon} \cdot t_{on}$$

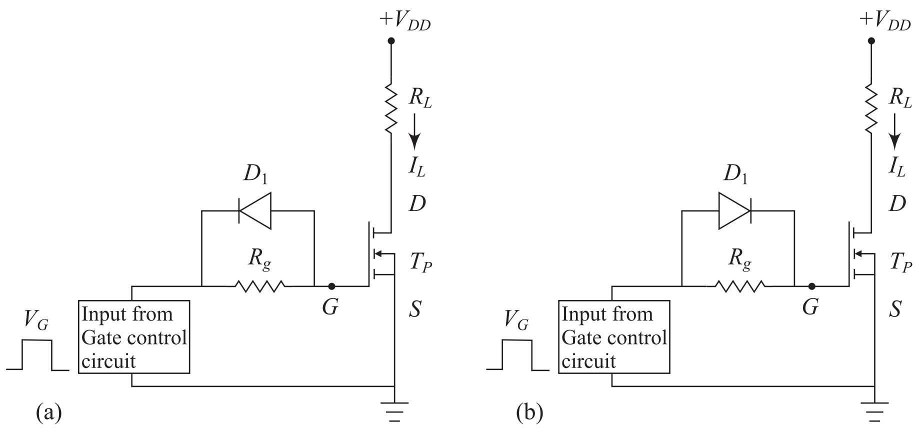


Fig. 3.57 *Driver circuits (a) during turn ON and (b) during turn OFF*

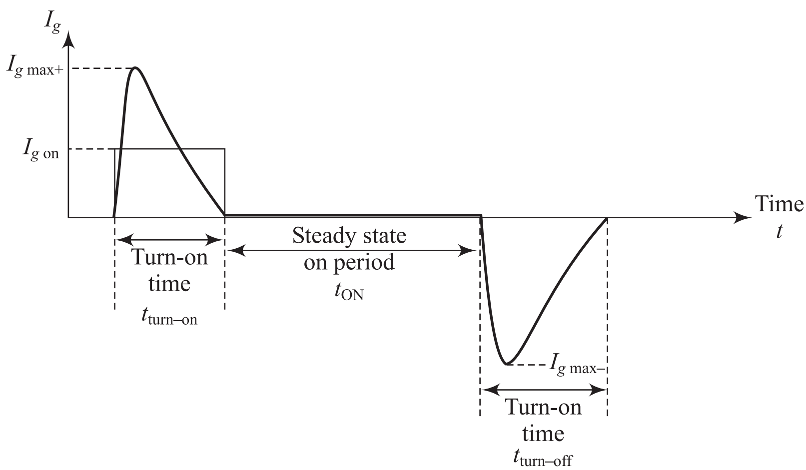


Fig. 3.58 *Gate current waveform of a gate drive circuit of a MOSFET*

For fast turn-ON of the MOSFET, the required I_{gon} must be more. When a smaller I_{gon} is supplied, MOSFET can be switched slower. If about 500 nC of charge is required to turn ON the MOSFET and the device will be turned ON within 1 µs, the required I_{gon} will be about 500 mA and $I_{g\,max+}$ will be 1000 mA. If the required turn-ON time becomes 2 µs, then the required I_{gmax+} will be about 250 mA and the required I_{gmax+} will be about 500 mA. A series resistor R may be connected in series with the gate of the MOSFET to limit the gate current. The value of R will be

$$R = \frac{V_{CC}}{I_{g\,max+}}$$

where, V_{CC} is the gate drive source voltage.

Usually manufacturers do not specify the device parasitic capacitances namely C_{gd}, C_{ds} and C_{gs}, but they provide input, output and reverse common-source capacitances such as C_{iss}, C_{oss} and C_{rss} respectively. All these capacitances are represented by

$$C_{iss} = C_{gs} + C_{gd} \quad \text{when } C_{ds} \text{ is shorted}$$
$$C_{oss} = C_{ds} + C_{gd} \quad \text{when } C_{gs} \text{ is shorted}$$
$$C_{rss} = C_{gd}$$

The switching speed of MOSFET largely depends upon C_{rss} and the gate-drive source impedance. Therefore, the accurate estimation of the switching time (turn-ON and turn-OFF) of MOSFET becomes difficult. Actually, the value of C_{iss} changes with V_{DS} and at low drain-voltage level this variation is large. Subsequently, the switching time constant determined by C_{iss} and gate drive impedance changes during the switching cycle. Figure 3.59 shows the ideal switching characteristics of MOSFET with R load. The ideal switching characteristics of MOSFET with resistive load can be divided into three distinct periods as given below.

$$i_G = \frac{C_{iss} V_{GS(\text{Th})}}{t_1 - t_0} \qquad \text{during} \quad t_0 \le t \le t_1$$

$$i_G = \frac{C_{rss}(V_{DD} - V_{DS(ON)})}{t_2 - t_1} \qquad \text{during} \quad t_1 \le t \le t_2$$

$$i_G = \frac{C_{iss}(V_{GS(\text{ON})} - V_{GS(\text{Th})})}{t_3 - t_2} \qquad \text{during} \quad t_2 \le t \le t_3$$

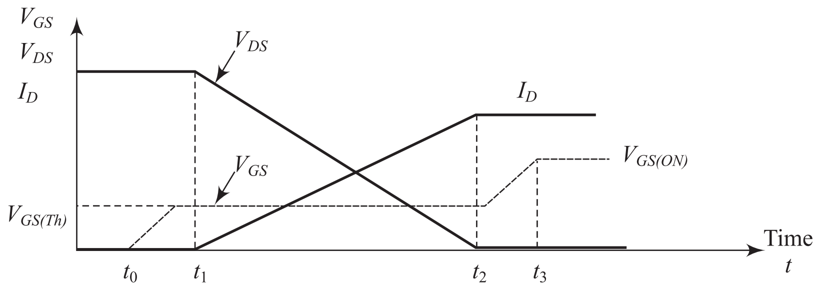


Fig. 3.59 *Ideal switching characteristics of MOSFET with R load*

The value of capacitances may be found from the manufacturers' data sheet. During $t_0 \le t \le t_1$ and $t_1 \le t \le t_2$, C_{iss} and C_{rss} values may be considered corresponding to $V_{DS} = \frac{V_{DD}}{2}$. But, during $t_2 \le t \le t_3$, the value of C_{iss} may be considered corresponding to $V_{DS} = V_{DS(\text{ON})}$.

A MOSFET drive circuit is shown in Fig. 3.60. The resistance R_1 is used for limiting the turn-ON surge current. Resistance R_2 is used to provide a discharge path for the input capacitance during turn-off of the MOSFET. Usually R_2 is about ten times the value of R_1.

When the gate drive supply voltage V_g has sinking current capability, then resistance R_2 is not required. In that case, the output resistance of V_g is low and the input capacitance of MOSFET is charged and discharged through resistance R_1.

MOSFETs can also be driven directly using a CMOS logic IC as depicted in Fig. 3.61. To reduce turn on time, I_{gon} must be increased. This can be achieved by parallel connection of buffers as shown in Fig. 3.61(b). CD 4049 IC is a CMOS hex inverting buffer and CD 4050 IC is the non-inverting hex buffer. Either CD 4049 or CD 4050

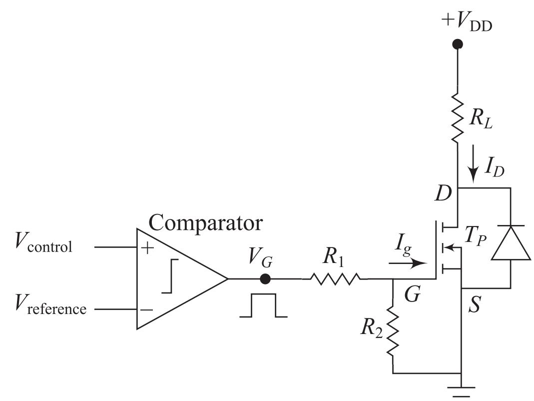


Fig. 3.60 *MOSFET drive circuit*

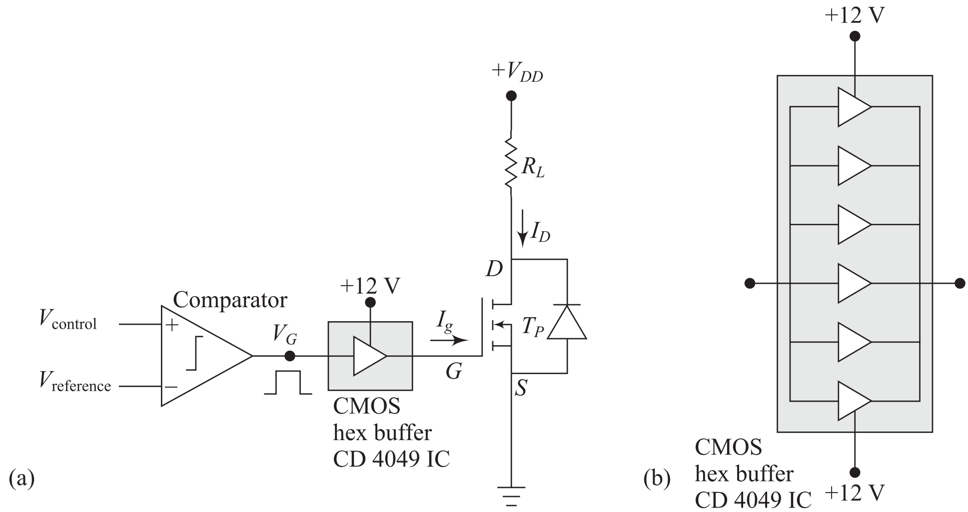


Fig. 3.61 *(a) MOSFET drive circuit using hex inverter CD 4049 IC (b) CMOS hex buffer CD 4049 IC*

can be used to drive MOSFETs. In this circuit, the series resistor R_1 is used to limit the turn-ON gate surge current $I_{g\ max+}$.

Figure 3.62 shows a MOSFET drive circuit by using transistors to increase the turn-on gate drive current I_{gon}. This circuit operates just like the base drive circuit as depicted in Fig. 3.62. The only difference is that the negative power supply is not present.

When the applied voltage V_B is zero, the *p-n-p* transistor T_1 is ON. Subsequently, the base terminals of transistors T_2 and T_3 are positive. Then the base-emitter of transistor T_2 will be forward bias and the base-emitter of transistor T_3 will be reverse bias. Thus transistor T_2 is turned ON and transistor T_3 is turned OFF. When T_2 is turned ON, maximum positive gate current of $I_{g\ max+}$ is supplied to turn ON the power MOSFET T_P. Due to presence of surge gate current, power MOSFET T_P will be turned ON rapidly.

In this circuit operation, the input capacitance of the MOSFET provides the required forward bias to turn on the *p-n-p* transistor T_3 when T_2 is turned OFF. Consequently, the power MOSFET T_P will be turned OFF.

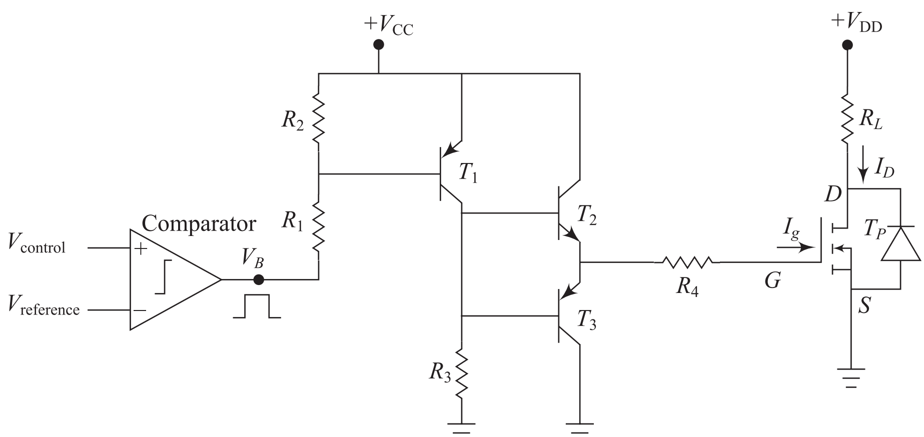


Fig. 3.62 *A MOSFET drive circuit using transistors*

3.21.4 MOSFET Drive Circuit Using Isolation

In some power electronics circuits, the gate and source terminals of a MOSFET are floating with respect of other MOSFETs which exist within the circuit. Isolation can be provided to these MOSFETs by using opto-isolators or by using pulse transformers. Figure 3.63 shows a gate drive circuit using an opto-coupler. This circuit operates just like the base drive circuit of BJT using opto-isolators as depicted in Fig. 3.53. In all opto-isolated drive circuits for BJT and MOSFET, the secondary circuit should have a local power supply and the secondary circuit must be isolated from primary side.

When the base drive pulse V_B is positive, a current flows through the photodiode of the opto-coupler as shown in Fig. 3.63. As a result, the transistor T_1 of the opto-coupler is to be turned on. Subsequently, the transistor T_2 operates in OFF state and transistor T_3 becomes ON. When T_3 is turned on, a surge gate current is supplied to the power MOSFET T_P through resistance R_7. Due to presence of gate surge current, power MOSFET T_P will be turned ON rapidly.

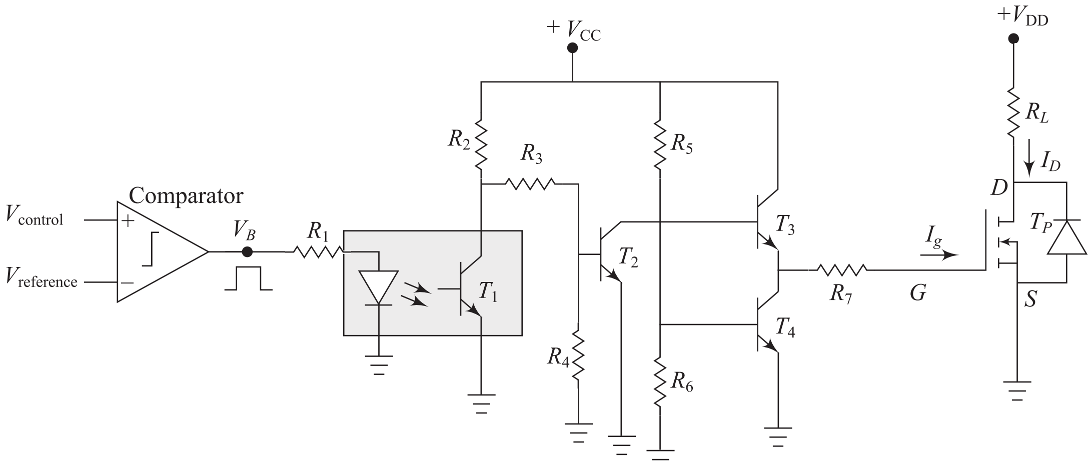


Fig. 3.63 *A gate drive circuit of MOSFET using an opto-coupler*

In this circuit operation, the input capacitance of the MOSFET T_P provides the required forward bias to turn on the *p-n-p* transistor T_3 when T_2 is turned OFF. Consequently, the power MOSFET T_P will be turned OFF.

A gate drive circuit for power MOSFET using an opto-coupler is also depicted in Fig. 3.64. In this figure, the complementary pair transistors T_3 and T_4 are replaced by CMOS hex non-inverting buffer CD 4049 IC. In place of CD 4049 IC, CMOS hex inverting buffer CD 4050 IC can also be used in this circuit.

Figure 3.65 shows a gate drive circuit which uses a pulse-transformer to provide the isolation between the gate drive side circuit and the high power MOSFET side circuit. This circuit operates just like the base drive circuit of BJT using transformer isolation as depicted in Fig. 3.54.

When a positive gate drive pulse voltage V_B is applied to transistor T_1, the transistor T_1 will be turned on and the V_{CC} voltage is applied across the primary of the pulse transformer. Usually, the pulse transformer is a special type of transformer and it is designed to operate at specified frequency range. As a pulse voltage is applied across the primary winding of pulse transformer, it passes the primary pulse to the secondary winding. The output pulse from secondary winding is used to turn on the power MOSFET T_P.

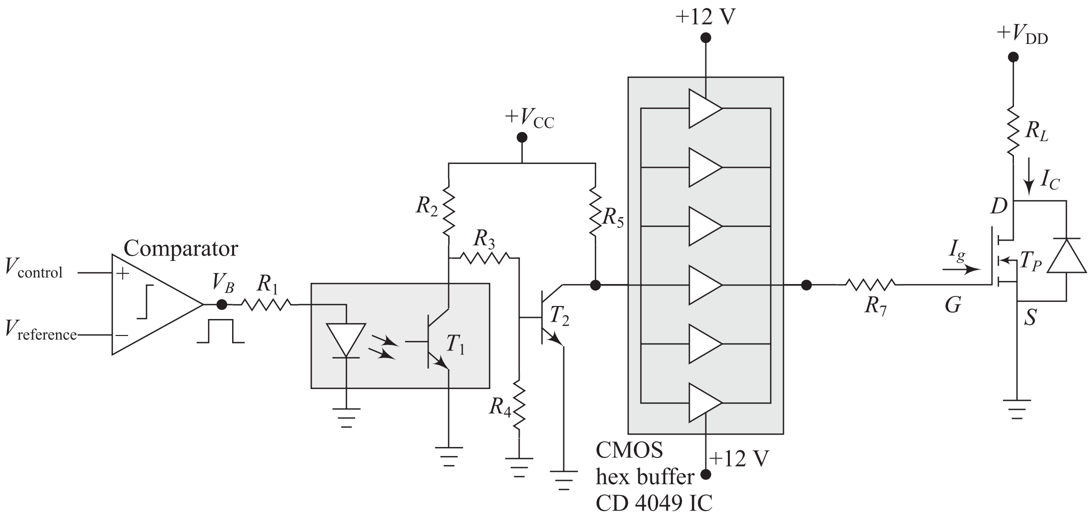


Fig. 3.64 *A gate drive circuit using an opto-coupler and CMOS hex buffer*

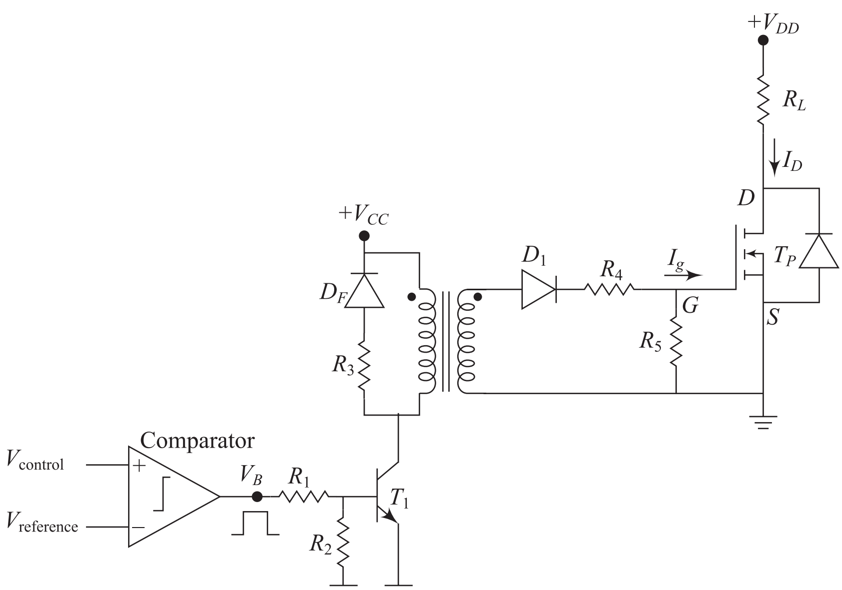


Fig. 3.65 *A gate drive circuit of MOSFET using a pulse transformer*

Whenever the gate drive pulse V_B becomes zero volts, the transistor T_1 is turned OFF and suddenly the current in the primary winding will be cut-off and there will be sudden increase in voltage across the primary winding due to $L\frac{di}{dt}$. Since the high voltage appears across transistor T_1 and it can damage the transistor T_1. To save the transistor T_1, the abrupt cut-off of the primary winding current should flow through a freewheeling diode D_F and resistance R_3 when T_1 is turned off.

The duty cycle of this circuit is limited as the transistor T_1 must be turned OFF for the time duration when (during which) the core will be reset. When the value of R_3 is large, the core resetting will be faster and subsequently the range of duty cycle will be larger. The voltage drop across R_3 is higher and the collector emitter voltage V_{CE} rating of transistor T_1 is also higher. Practically, the duty ratio D is limited to less than 50%. Once again, we can use a demagnetising winding to perform non-dissipate freewheeling as depicted in Fig. 3.66.

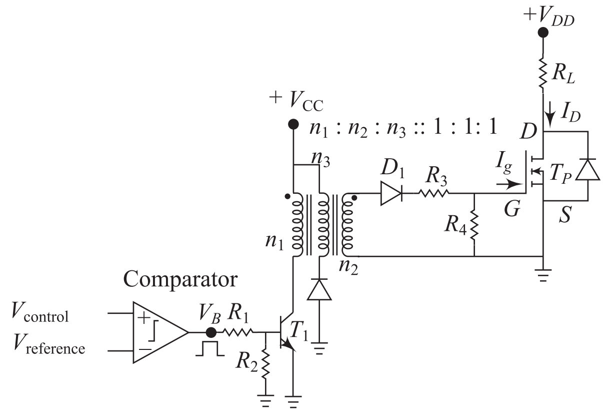


Fig. 3.66 *A gate drive circuit using three winding pulse transformer*

3.21.5 Gate Drive Circuits of IGBT

Insulated gate bipolar transistors (IGBTs) are gaining considerable use in power electronics circuits at high voltage, high current and moderate switching frequencies. Usually, IGBT based circuits are used in motor control, uninterruptible power supply and other similar inverter applications. In comparison to bipolar transistors which were formally used in such power electronics circuits, the IGBT offers a considerable reduction in both size and complexity of the drive circuitry. Present improvements in IGBT switching speed have yielded devices suitable for uninterruptible power supply applications. Hence, in place of MOSFET, IGBTs can also be used for certain high voltage applications. The gate drive circuit requirements of the IGBTs are very much similar to the gate drive circuits of the power MOSFET.

Depending upon the presence of $\frac{dv}{dt}$, IGBT drive circuits can be divided into two basic application categories such as

1. Presence of high dv/dt to the collector/emitter of the IGBT when it is OFF.
2. Absence of high dv/dt to the collector/emitter of the IGBT when it is OFF.

In some power electronics circuits, only one switch is employed or multiple switches are activated synchronously. The high $\frac{dv}{dt}$ is applied to IGBT during the off-state in most bridge circuits when opposing devices are turned on. Therefore, simultaneous conduction of opposing devices in a bridge circuit can occur in, often with catastrophic results when the proper gate drive and layout precautions are not followed. This happens due to the presence of parasitic collector to gate capacitance or miller effect and a capacitive divider with the gate-to-emitter capacitance. Consequently, a gate-to-emitter voltage is induced as shown in Fig. 3.67.

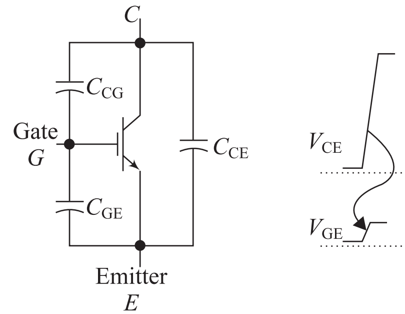


Fig. 3.67 *High dv/dt at the collector couples to the gate through parasitic capacitance in a IGBT*

When high OFF-state dv/dt is not present, all the gate drive circuits of power MOSFET as explained in Section 3.21.4 can be used to drive the IGBT. Generally, 15 volts is applied gate to emitter during the ON-state to minimise saturation voltage. In a gate drive, the gate resistor is used to limit gate current to control IGBT turn ON directly. But turn OFF is partially governed by minority carrier and it is less affected by gate drive.

There are different techniques which can be employed to eliminate simultaneous conduction when high OFF-state dv/dt is present. The most commonly used technique is a Kelvin connection between the IGBT emitter and the driver's ground. If high di/dt is present in the emitter circuit, the substantial transient voltages will be developed in the gate drive circuit when it is not properly referenced. Actually, the Kelvin drive connection minimises the effective driver impedance for maximum attenuation of the dv/dt induced gate voltage.

Design aspects of gate drive circuits for IGBT Since the input characteristics of the IGBT are similar to that of MOSFET, the design considerations of gate drive circuit for an IGBT and the MOSFET are identical. During design of the gate drive circuits for an IGBT, the following design aspects are to be considered:

1. Just like MOSFET IGBT is a voltage controlled device. IGBT has a gate to emitter threshold voltage $V_{GE(\mathrm{Th})}$ and capacitive input impedance. To turn on the IGBT, the input capacitance must be charged up to a voltage which is greater than $V_{GE(\mathrm{Th})}$. The collector-to-emitter saturation voltage decreases with an increase in magnitude of gate-to-emitter voltage $\mathrm{V_{GE}}$. Just after turn on the IGBT, the collector current starts to flow.
2. During turn OFF the IGBT, the gate drive circuit must provide a resistance between gate and emitter ($\mathrm{R_{GE}}$). Actually the gate-to-emitter resistance (R_{GE}) provides the path to discharge the gate capacitance and the device will be turned OFF. The discharge time constant and dv/dt during turn OFF depend on the value of the gate-to-emitter resistance (R_{GE}).
3. IGBTs should have a maximum controllable current which is dependent on the gate to emitter dv/dt. If the gate to emitter dv/dt is high, the controllable collector current will be low. As a result, for a maximum controllable collector current, the gate-to-emitter resistance (R_{GE}) should have a lower limit.
4. The output current of the gate-drive circuit must be sufficient to charge and discharge the gate source capacitance rapidly. Therefore, the turn-ON and turn-OFF time of IGBT can be reduced and the switching losses will also be reduced.
5. The gate drive circuit must be able to source short duration high magnitude gate current pulses for fast turn-ON. The gate drive circuit can also sink short duration high magnitude gate current pulses for fast turn-OFF.
6. The gate drive circuit should be designed with over current protection. Therefore, it can sense the collector current and turn-OFF the device accordingly to protect the device when excessive current flows.
7. The control circuit must be electrically isolated from the power circuit using IGBT. Actually isolation can be provided using either an opto-coupler or pulse transformer. The opto-coupler must be used for isolation with shorter propagation delay and high noise immunity.

The gate drive circuits for MOSFET are also applicable to IGBT. There are different variations in the drive circuit. It is safe to use the gate drive circuits of power MOSFFETs with minor modification for driving to the IGBT. Usually, a negative turn-off voltage is not required between gate and emitter to drive IGBT.

Figure 3.68 shows a typical drive circuit. The value of the gate to emitter resistance (R_{GE}) should be quite large so that the drive current is minimised. Therefore, the turn-ON and turn-OFF times would be slow to about tens of microseconds.

Figure 3.69 shows a high speed asymmetrical gate drive circuit for IGBT. The pulse transformer TR_1 is used to provide isolation between primary and secondary. A diode D is used in combination with R_3 to provide a fast turn-ON time without affecting the turn-OFF time. Hence, R_3 and R_{GE} can be used to determine turn-ON and turn-OFF times respectively.

Figure 3.70 shows the gate drive circuit which satisfies all the design requirements of gate drive for IGBT. The totem pole arrangement of the driver transistors T_3 and T_4 must ensure quick charging and discharging of the input capacitance of the IGBT. Consequently, the turn-ON and turn-OFF times of the IGBT will be reduced and subsequently the switching losses will also be reduced. Whenever transistor T_3 is turned ON, a voltage is applied and the IGBT is to be turned-on. To turn-OFF an IGBT, transistor T_3 is to be turned OFF and T_4 is to be turned ON.

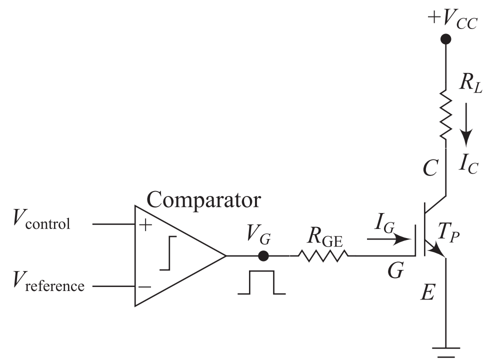


Fig. 3.68 *Simple gate drive circuit of IGBT*

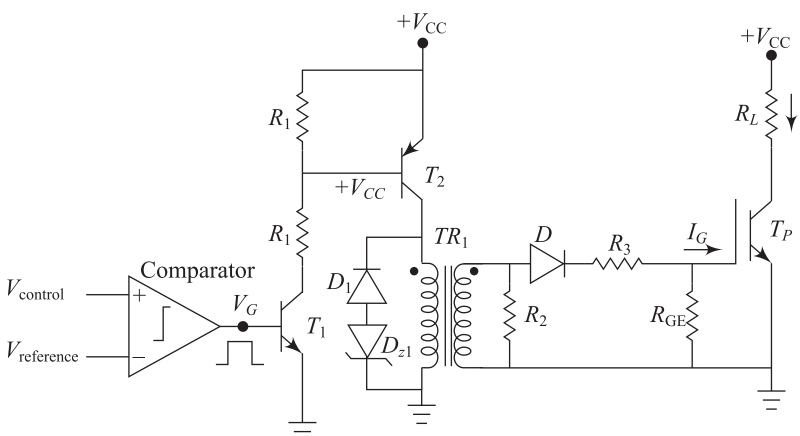


Fig. 3.69 *Gate drive circuit of IGBT using pulse transformer as isolation*

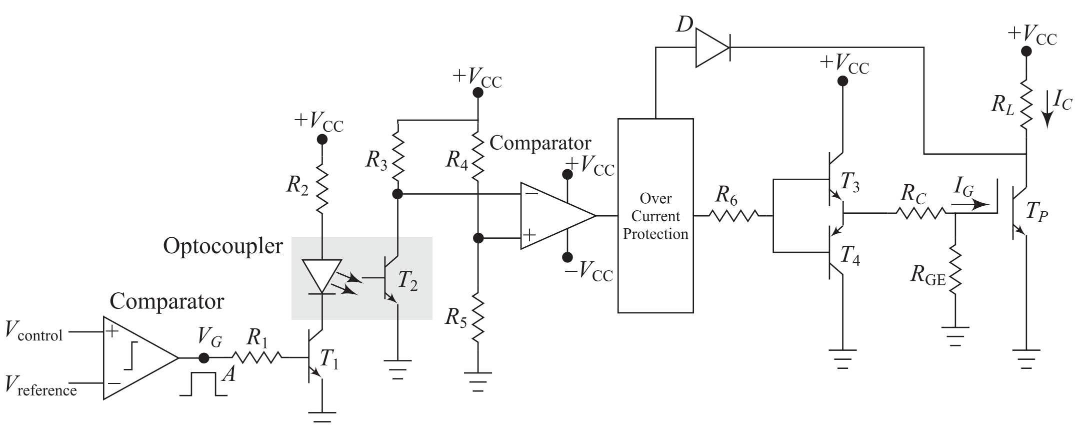


Fig. 3.70 *Gate drive circuit of IGBT using opto-coupler as isolation*

It is clear from Fig. 3.70 that the control circuit is isolated from the power circuit using an optocoupler. When a pulse voltage V_G is applied at point A, transistor T_1 is turned ON and a current flows through the photodiode of the opto-coupler. Consequently, the transistor T_2 of the opto-coupler is to be turned ON. Then the potential at non-inverting terminal of comparator is greater than the potential at inverting terminal of comparator. Hence comparator output is high.

Similarly, when the gate drive pulse V_G is zero, transistor T_1 is turned OFF and no current flows through the photodiode and the transistor T_2 of the opto-coupler operates in OFF state. At this instant the potential at non-inverting terminal of comparator is less than the potential at inverting terminal of comparator. Hence comparator output is low. Actually, the comparator acts as a wave shaping circuit and generates a rectangular waveform with sharp leading and trailing edges. This is necessary to nullify the effect of slow opto-coupler on the shape of the waveform. Whenever the comparator output is high, transistor T_3 is ON, T_4 is OFF and IGBT will be ON. When the comparator output is low, transistor T_3 is OFF, T_4 is ON and IGBT will be OFF.

If a short circuit occurs in the power circuit, the collector current of the IGBT will increase rapidly. Whenever this current exceeds a critical value, the collector-emitter voltage increases quickly. This increased collector-emitter voltage can be used to indicate over current. Due to over current, IGBT may be damaged. Therefore, over current protection feature must be incorporated in the gate drive circuit.

In Fig. 3.70 diode D will conduct at normal operating conditions. Due to short circuit, when the over current condition arises, the collector-emitter voltage increases rapidly and the diode D will be reverse biased. As soon as diode D is turned-off, the comparator output is not allowed to pass through, to the totem pole driver circuit and consequently IGBT quickly turned-OFF. In this way, IGBT can be protected from over current condition.

Example 3.9 Figure 3.71 shows a base drive circuit where a power BJT has to switch a 1 A load. The base drive circuit draws power from +5 V dc voltage source. Determine the values of resistance R_1 and R_2. Assume h_{fe} of BJT is 80 and $V_{BE} = 0.7$ V

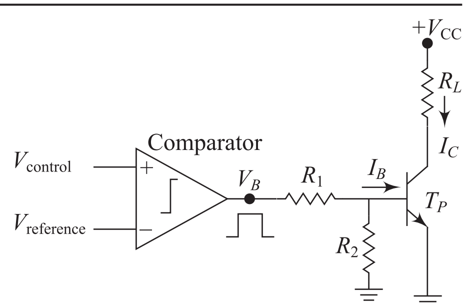


Fig. 3.71

Solution

Given: $I_C = 1$ A, $h_{fe} = 80$, $V_{BE} = 0.7$ V, $V_B = 5$ V

The value of $R_2 = \dfrac{V_{BE}}{I_{R_2}} = \dfrac{V_{BE}}{I_{B\_on}} = \dfrac{V_{BE}h_{fe}}{2I_C}$ as $I_{B\_on} = 2\dfrac{I_C}{h_{fe}}$

$$= \frac{0.7\times 80}{2\times 1} = 28\ \Omega$$

The value of $R_1 = \dfrac{V_B - V_{BE}}{2I_{B\_on}} = \dfrac{(V_B - V_{BE})h_{fe}}{4I_C} = \dfrac{(5-0.7)\times 80}{4\times 1} = 86\ \Omega$

Example 3.10 A base drive circuit is shown in Fig. 3.72 where a BJT has to switch a 10 A load which is connected to a 150 V dc. The base drive circuit draws power from +10 V dc power source. Determine the value of inductance L. Assume h_{fe} of BJT is 100, $V_{BE} = 0.7$ V and $R_5 = R_6 = 100\ \Omega$.

Solution

Given: $I_C = 10$ A, $h_{fe} = 100$, $V_{BE} = 0.7$ V, $V_B = 10$ V, $R_5 = R_6 = 100\ \Omega$.

$$I_{B\_on} = 2\frac{I_C}{h_{fe}} = 2\frac{I_C}{\beta} = 2\times\frac{10}{100} = 0.2\text{ A}$$

$$I_{B\max-} = 1.5 I_{B\_on} = 1.5\times 0.2\text{ A} = 0.3\text{ A}$$

As the voltage of power transistor is less than 200 V,

$$\frac{dI_B}{dt} = 0.5 I_C A/\mu\text{s} = 0.5\times 10\text{ A}/\mu\text{s} = 5\text{ A}/\mu\text{s}$$

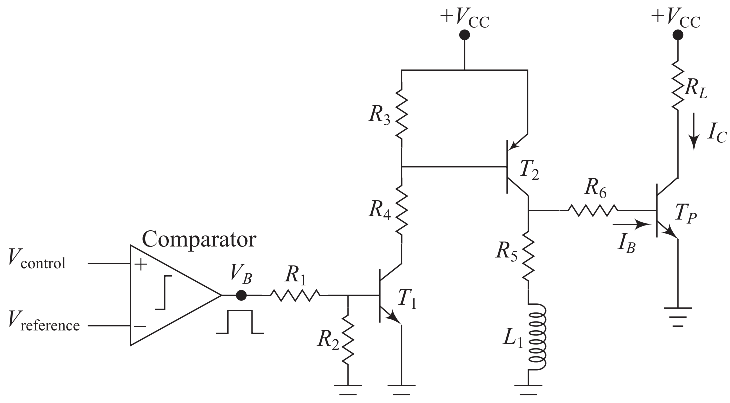


Fig. 3.72

The value of inductance is $L = \dfrac{(R_5 + R_6)I_{B\max-} - V_{BE}}{\dfrac{dI_B}{dt}}$

$$= \frac{(100+100)\times 0.3 - 0.7}{5} = 11.86\ \mu\text{H}$$

Summary

- The structured and operating characteristics of power BJTs are discussed elaborately in this chapter. The power BJTs has a vertically oriented structure and a lightly doped collector drift region. The blocking voltage of power BJT depends on drift region and quasi-saturation region exist in I-V characteristics of power BJT.
- Power BJTs have low current gain. Therefore monolithic Darlington pair power transistors are developed to increase current gain.
- Hard saturation, primary breakdown, second breakdown, on state loss of power BJT are explained in detail. The safe operating area (SOA) of power BJT is limited by second breakdown. The RBSOA is normally the limiting factor.
- The power MOSFETs have a vertically oriented structure with a lightly doped drift region and a highly interdigitated gate-source structure.
- MOSFET is a majority carrier device, it's on state resistance has a positive temperature co-efficient. It is easy to connect MOSFETs in parallel for increasing current handling capability.
- The safe operating area of power MOSFET is large due to the absence of second breakdown.
- Switching characteristics, series and parallel operation of power BJTs, Power MOSFETs are incorporated in this chapter.
- The performance of IGBT is the midway between a MOSFET and BJT. In this chapter structure, *I-V* characteristics, safe operating area, switching characteristics of IGBT are explained.
- The drive circuits of BJT, MOSFET and IGBT are also incorporated in this chapter.

Multiple-Choice Questions

3.1 A power transistor is a ______ layer device.
(a) two (b) three (c) four (d) five

3.2 A *n-p-n* power transistor is a ______ layer device.
(a) $n^+ pn^- n^+$ (b) $p^+ n^+ n^- n^+$ (c) $n^- n^+ p^- p^+$ (d) $n^+ p^- p^+ n^-$

3.3 A *p-n-p* power transistor is a ______ layer device.
(a) $n^+ pn^- n^+$ (b) $p^+ np^- p^+$ (c) $n^- p^+ n^- p^+$ (d) $p^+ n^- p^+ n^-$

3.4 The width of base region is about
(a) 50 to 200 μm (b) 5 to 20 μm (c) 10 to 100 μm (d) 100 to 250 μm

3.5 The droping density of a *n-p-n* transistor collector is
(a) 10^{14} cm^{-3} (b) 10^{15} cm^{-3} (c) 16^{14} cm^{-3} (d) 10^{19} cm^{-3}

3.6 When β_A is gain of auxiliary transistor and β_M is gain of main transistor, the gain of Darlingtor pair BJT is
(a) $\beta = \beta_M\beta_A + \beta_M + \beta_A$ (b) $\beta = \beta_M\beta_A + \beta_M$ (c) $\beta = \beta_M\beta_A + \beta_A$ (d) $\beta = \beta_M\beta_A - \beta_M - \beta_{\mathrm{A}}$

3.7 The overdrive factor (ODF) is equal to
(a) $\frac{I_{BS}}{I_B}$ (b) $\frac{I_B}{I_{BS}}$ (c) $\frac{I_E}{I_{BS}}$ (d) $\frac{I_C}{I_{BS}}$

3.8 The total power loss in two junctions of power transistor is
(a) $V_{BE}I_B + V_{CE}I_C$ (b) $V_{BE}I_B + V_{CE}I_E$ (c) $V_{BC}I_B + V_{CE}I_C$ (d) $V_{BE}I_C + V_{CE}I_B$

3.9 Which of the following statements are true
(a) $V_{CE|_{SUS}} > V_{CE0}$ (b) $V_{CE0} < V_{CB0}$ (c) $V_{CE|_{SUS}} < V_{CE0}$ (d) $V_{CE0} > V_{CB0}$

3.10 Power BJT is a switch when
(a) both base-emitter and collector-base junction is forward biased
(b) both base-emitter and collector-base junction is reverse biased
(c) base-emitter is forward biased and collector-base junction is reverse biased
(d) base-emitter is reverse biased and collector-base junction is forward biased

3.11 Secondary breakdown occurred in
(a) BJT only (b) MOSFET only (c) Both BJT and MOSFET (d) SIT only

3.12 A power BJT is a
(a) *npn* (b) $n^+ pn^- n^+$ (c) $n^+pn^+n^-$ (d) n^+pn^+

3.13 The relation between α and β is
(a) $\alpha = \frac{\beta}{\beta+1}$ (b) $\beta = \frac{\alpha}{1-\alpha}$ (c) $\beta = \frac{\alpha}{1+\alpha}$ (d) $\alpha = \frac{\beta}{\beta-1}$

3.14 The forced current gain β_f is
(a) $\frac{I_{CS}}{I_B}$ (b) $\frac{I_B}{I_{BS}}$ (c) $\frac{I_{BS}}{I_B}$ (d) $\frac{I_B}{I_{CS}}$

3.15 Which is the correct statement for power transistor
(a) FBSOA and RBSOA exist in power transistor (b) RBSOA is less than FBSOA
(c) FBSOA is less than RBSOA (d) only FBSOA exist

3.16 In a *n-p-n* power BJT, the relationship between V_{CE0} and V_{CB0} is
(a) $V_{CE0} = \frac{V_{CB0}}{\beta}$ (b) $V_{CE0} = \frac{V_{CB0}}{\beta^{\frac{1}{2}}}$ (c) $V_{CE0} = \frac{V_{CB0}}{\beta^{\frac{1}{3}}}$ (d) $V_{CE0} = \frac{V_{CB0}}{\beta^{\frac{1}{4}}}$

3.17 ODF of power transistor is
(a) 5 (b) 6 (c) 7 (d) 8

3.18 The n^- drift region has ______ droping density
(a) 10^{14} cm^{-3} (b) 10^{15} cm^{-3} (c) 10^{16} cm^{-3} (d) 10^{19} cm^{-3}

3.19 Power MOSFET has three terminals namely
(a) gate, drain, source (b) gate, emitter, collector (c) gate, source, emitter (d) gate, base, collector

3.20 Which of the following statement is true
(a) BJT is voltage controlled and MOSFET is current controlled
(b) BJT is current controlled and MOSFET is voltage controlled
(c) Both BJT and MOSFET are current controlled
(d) Both BJT and MOSFET are voltage controlled

3.21 Which of the following statement is true?
(a) BJT has negative temperature coefficient and MOSFET has positive temperature coefficient
(b) BJT has positive temperature coefficient and MOSFET has negative temperature coefficient
(c) Both BJT and MOSFET have positive temperature coefficient
(d) Both BJT and MOSFET have negative temperature coefficient

3.22 Compared to BJT, MOSFET has
(a) low switching frequency and low conduction loss
(b) high switching frequency and low conduction loss
(c) high switching frequency and high conduction loss
(d) low switching frequency and high conduction loss

3.23 The high frequency operation of a switching circuit is limited by
(a) turn-ON and turn-OFF loss within the device (b) ON-state conduction loss within the device
(c) OFF-state loss within the device (d) both (a) and (b)

3.24 As compared to BJT, power MOSFET has
(a) high switching loss and low conduction loss (b) high switching loss and high conduction loss
(c) low switching loss and low conduction loss (d) low switching loss and high conduction loss

Fill in the Blanks

3.1 A *n-p-n* power transistor is a_____ layer ______ structure.
3.2 The current gain of Darlingtor pair BJT is______.
3.3 The quasi saturation is occurred due to the _____doped collector drift n^- region in the structure of power transistors.
3.4 The power dissipation of BJT in the quasi saturation is ___ than the power dissipation during____ saturation.
3.5 During quasi saturation of BJT, the double injection can be occurred in the ____region.
3.6 A power transistor (BJT) is a ______.
3.7 Power MOSFET is a ______ device.
3.8 BJTs are ______ controlled devices whereas MOSFETs are ______ controlled devices.
3.9 Power MOSFET has _____ input impedance.
3.10 IGBT is a______ device.
3.11 IGBT can support ______ voltage during OFF state.
3.12 IGBT has ______ overload capability as compared to a MOSFET.
3.13 IGBTs are more like the ______ during turn-OFF and like the ______ during turn on.
3.14 During turn-ON period, the base current must be ______ than the steady state ON current.
3.15 During turn-OFF period, the______ base current must be applied to quickly remove the stored charges in the transistor.
3.16 SIT is a_____ device.
3.17 SIT is a normally _____ device.
3.18 RBSOA of power transistor is ______than FBSOA.
3.19 The safe operating area (SOA) of a MOSFET in switching applications is____ as it is not subject to second breakdown.
3.20 The MOSFET has a ______ BJT in its structure.
3.21 The IGBT structure has ______ thyristor that must not be allowed to turn on.

3.22 The isolation between the high power collector side and the low power base side of power BJT is provided by using ______ and______.
3.23 In case of opto-coupler isolation, the duty cycle can be varied from______.
3.24 In case of transformer isolation, the duty cycle can be varied from______.
3.25 In MOSFET switching, the gate current is ______ on the drain current.
3.26 Compared to the BJT, the MOSFET drive circuit requires ______ power.
3.27 Power BJT is a______ device.

Review Questions

3.1 What is power BJT? What are the types of power BJT? Write the difference between general purpose BJT and power BJT.
3.2 Draw the structure of a power BJT and explain its operating principle briefly.
3.3 What are the difference between *n-p-n* and *p-n-p* transistors?
3.4 Draw the V-I characteristics of a power transistor and explain different operating regions.
3.5 What is beta β and forced beta β_f? What is the difference between β and β_f?
3.6 Explain the operation of transistor as a switch.
3.7 Discuss quasi-saturation and hard saturation of a power BJT.
3.8 What is primary breakdown and secondary breakdown of a power BJT?
3.9 What is the on state loss of a power BJT?
3.10 Explain the safe operating area of a power BJT. Draw the FBSOA and RBSOA of power BJT. Among the FBSOA and RBSOA which area is large?
3.11 Explain the series and parallel operation of power BJTs.
3.12 What are FBSOA and RBSOA of a power BJT?
3.13 Explain the switching characteristics of power BJT. Define delay time, rise time, turn-ON time, storage time, fall time and turn-OFF time.
3.14 What is power MOSFET? What are the types of power MOSFET? Write the difference between general purpose MOSFET and power MOSFET?
3.15 Write the difference between enhancement-type MOSFETs and depletion-type MOSFETs.
3.16 Draw the structure of a power MOSFET and explain its operating principle briefly.
3.17 Draw the V-I characteristics of a power MOSFET and explain different operating regions.
3.18 What is ON-state loss of a power MOSFET?
3.19 Explain the safe operating area (SOA) of a power MOSFET. What is the effect of pulse width on SOA.
3.20 Discuss the series and parallel operation of power MOSFETs.
3.21 Draw the switching characteristics of power MOSFETs. Define turn-ON delay time, rise time, turn-ON time, turn-OFF delay time, fall time and turn-OFF time.
3.22 Write the differences between power BJT and power MOSFET.
3.23 Compare power MOSFET and power BJT.
3.24 What is IGBT? What are advantages of IGBT over power BJT and power MOSFET?
3.25 Draw the structure of IGBT and explain its operating principle briefly.
3.26 Draw the V-I characteristics of a IGBT and explain different operating regions.
3.27 What are symmetrical IGBT and anti-symmetrical IGBT?
3.28 Discuss ON-state voltage drop of IGBT with suitable diagram.
3.29 Explain the safe operating area (SOA) of IGBT. Draw the FBSOA and RBSOA of IGBT. Write the differences between FBSOA and RBSOA of IGBT.
3.30 Draw the switching characteristics of IGBT. Define turn-ON delay time, rise time, turn-ON time, turn-OFF delay time, first fall time, final fall time and turn-OFF time.
3.31 Compare power MOSFET and IGBT.
3.32 What is SIT?

3.33 Draw the structure of a SIT and explain its operating principle briefly.

3.34 Draw the V-I characteristics of a SIT.

3.35 What are the problems of series and parallel operation of power BJTs and power MOSFETs?

3.36 A transistor has an α of 0.95. Determine the value of β.

3.37 A transistor has a β of 60. Find the value of α.

3.38 The collector current of a transistor is 98 mA and its β is 70. Calculate the value of base current and emitter current.

3.39 A transistor has $\beta = 95$ and the emitter current is 85 mA. Determine the value of base current and collector current.

3.40 A transistor has a base current $I_B = 120$ μA and the collector current (I_C) is equal to 6 mA. (a) Determine the value of β, α and emitter current. (b) If the base current changes by 30 μA and the corresponding collector current change is 0.6 mA, calculate the new value of β.

3.41 The common-base dc current gain of a transistor is 0.95. When the emitter current is 145 mA, determine the base current and collector current.

3.42 A bipolar transistor as shown in Fig. 3.73 has $\beta_f = 32$ and the load resistance $R_C = 16\ \Omega$. The dc supply voltage $V_{CC} = 210$ V and the input voltage to base is $V_B = 10$ V. When $V_{CE(\text{saturation})} = 1.25$ V and $V_{BE(\text{saturation})} = 1.6$ V, determine (a) the value of resistance R_B so that transistor operates in saturation and (b) power loss in the transistor.

3.43 A bipolar transistor as depicted in Fig. 3.73 has β in the range 15 to 50 and the load resistance $R_C = 10\ \Omega$. The dc supply voltage $V_{CC} = 160$ V and the input voltage to base is $V_B = 12$ V. When $V_{CE(\text{saturation})} = 1.0$ V and $V_{BE(\text{saturation})} = 1.2$ V, determine (a) the value of resistance R_B so that transistor operates in saturation with $ODF = 4$ (b) forced current gain (c) power loss in the transistor.

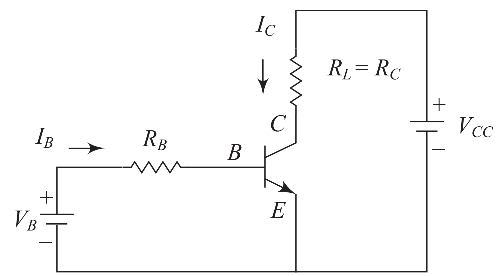


Fig. 3.73

3.44 Two BJTs are connected in parallel to share the total current 30 A. The collector to emitter voltage of T_1 and T_2 are 1.55 V and 1.70 V respectively. Determine the emitter current of each transistors and the difference of current sharing when the current sharing series resistance are (a) $R_{E1} = 0.25\ \Omega$ and $R_{E2} = 0.35\ \Omega$ and (b) $R_{E1} = 0.5\ \Omega$ and $R_{E2} = 0.5\ \Omega$

3.45 The switching waveform of a power transistor is shown in Fig. 3.106 where $V_{CC} = 220$ V, $V_{CE(\text{saturation})} = 2$ V, $I_{CS} = 120$ A, $t_d = 0.25$ μs, $t_r = 1.25$ μs, $t_{\text{on}} = 40$ μs, $t_s = 2$ μs, $t_f = 1.5$ μs. Determine (a) energy loss during delay time, (b) energy loss during rise time, (c) energy loss during conduction time t_{on} and (d) average power loss of power transistor during turn-on when switching frequency is 5 kHz and the emitter leakage current is $I_{EO} = 6$ mA.

3.46 The switching waveforms of a power transistor is depicted in Fig. 3.74 where $V_{CC} = 200$ V, $V_{CE(\text{saturation})} = 1.2$ V, $I_{CS} = 100$ A, $t_d = 0.45$ μs, $t_r = 1.5$ μs, $t_{\text{on}} = 45$ μs, $t_s = 3.5$ μs, $t_f = 2.5$ μs. When the switching frequency is 2 kHz and the emitter leakage current is $I_{EO} = 2.5$ mA, Find

(a) average power loss during delay time,
(b) average power during rise time,
(c) peak instantaneous power loss during rise time,
(d) average power during conduction time t_{on},
(e) average power loss during storage time,
(f) average power during fall time and
(g) peak instantaneous power loss during fall time.

Draw the waveform for instantaneous power loss for period T_s.

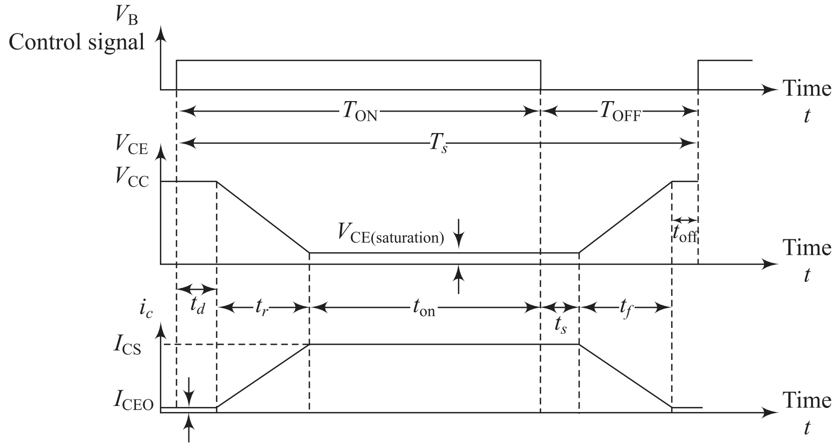


Fig. 3.74

3.47 Two power MOSFETs are connected in parallel to share the total current 25 A. The drain-to-source voltage of T_1 and T_2 are 3.55 V and 2.75 V respectively. Determine the drain current of each transistors and the difference of current sharing when the current sharing series resistance are (a) $R_{S1} = 0.2\ \Omega$ and $R_{S2} = 0.3\ \Omega$ and (b) $R_{S1} = 0.45\ \Omega$ and $R_{S2} = 0.5\ \Omega$.

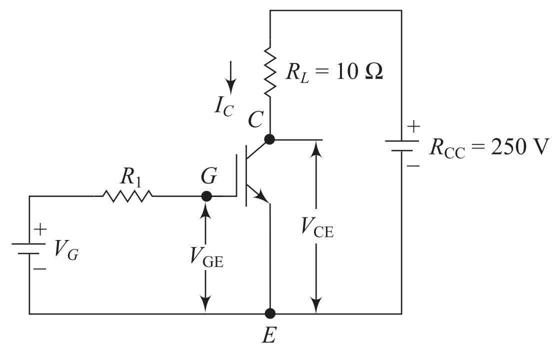


Fig. 3.75

3.48 A IGBT switching circuit as shown in Fig. 3.75 has the following parameters: $t_{on} = 3.5\ \mu s$, $t_{off} = 3.5\ \mu s$, $V_{CE(\text{saturation})} = 2.5$ V, $R_L = 10\ \Omega$, $f_s = 1.5$ kHz, $V_{CC} = 250$ V.
When duty cycle is 50%, determine (a) average load current, (b) average conduction loss, (c) turn-on loss and (d) turn-OFF loss

3.49 What are the requirements of a good base drive circuit? Draw the base current waveform of a BJT during turn-ON and turn-OFF process and explain the effect of base current in turn-ON time and turn-OFF time of a BJT.

3.50 What the types of base drive circuits? Explain any one base drive circuit of BJT.

3.51 Why isolation is required in a base drive circuit? What is the value of duty cycle of transformer isolation base drive circuit? How can the duty cycle range be improved with transformer isolation between the collector side and the base drive side?

3.52 Draw a gate drive circuit of power MOSFET and explain its operation briefly.

3.53 Write the different design aspects of a gate drive circuit for IGBT.

3.54 Draw a gate drive circuit of IGBT and explain its operation briefly.

3.55 Draw and explain a gate drive circuit of IGBT with over current protection.

3.56 Justify the following statements:

(a) IGBT combines the advantages of MOSFET and IGBT.

(b) IGBT is preferred as power switch over power BJT and power MOSFET.

3.57 Figure 3.76 shows a base drive circuit where a power BJT has to switch a 1.5 A load. The base drive circuit draws power from +6 V dc voltage source. Determine the values of resistance R_1 and R_2. Assume h_{fe} of BJT is 100 and $V_{BE} = 0.7$ V.

3.58 A base drive circuit is shown in Fig. 3.77 where a BJT has to switch a 20 A load which is connected to a 100 V dc. The base drive circuit draws power from +10 V dc power source. Determine the value of inductance L. Assume h_{fe} of BJT is 80, $V_{BE} = 0.7$ V, $R_5 = 100\ \Omega$, $R_6 = 120\ \Omega$.

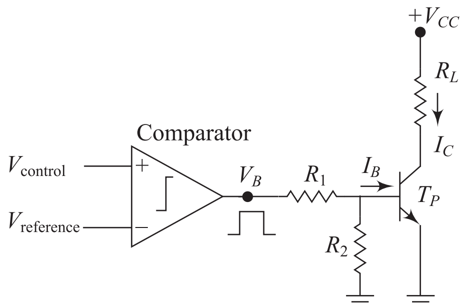


Fig. 3.76

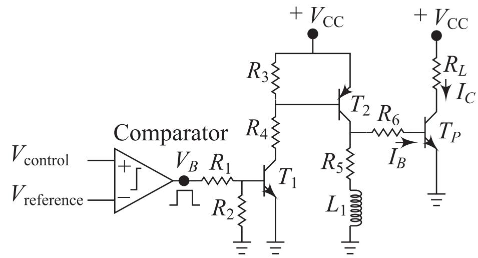


Fig. 3.77

Answers to Multiple-Choice Questions

3.1	(c)	3.2	(a)	3.3	(a)	3.4	(b)	3.5	(d)	3.6	(a)	3.7	(b)
3.8	(a)	3.9	(b) & (c)	3.10	(a)	3.11	(a)	3.12	(b)	3.13	(b) & (a)	3.14	(a)
3.15	(c)	3.16	(d)	3.17	(a)	3.18	(a)	3.19	(a)	3.20	(a)	3.21	(a)
3.22	(c)	3.23	(a)	3.24	(d)								

Answers to Fill in the Blanks

3.1	four, $n^+pn^-n^+$	3.2	$\beta_M\beta_A + \beta_M + \beta_A$	3.3	lightly
3.4	greater, hard	3.5	drift	3.6	current controlled
3.7	voltage controlled	3.8	current, voltage	3.9	high
3.10	voltage controlled	3.11	bipolar	3.12	greater
3.13	BJTs, MOSFETs	3.14	greater	3.15	negative
3.16	$n^+n^-p^+n$	3.17	ON	3.18	larger
3.19	rectangular	3.20	parasitic	3.21	parasitic
3.22	opto-couplers, transformers	3.23	0 to 1	3.24	0 to 0.5
3.25	not dependent	3.26	less	3.27	current controlled

THYRISTORS

4

4.1 INTRODUCTION

Thyristor is a general term and it consists of a family of power semiconductor devices such as silicon controlled rectifier (SCR), light activated SCR (LASCR), gate turn-OFF thyristors (GTO) TRIAC, reverse conducting SCR, asymmetric SCR (ASCR), etc. These devices are used extensively in power electronics circuits. These devices operate in conducting and non-conducting states just like a bistable switch. The conduction of thyristor is possible by providing current to gate terminal. The turn-OFF of thyristor is also possible when the current flows through the device below the holding current. From the above devices, SCR is the most simplest in structure and most commonly used in power electronics circuits. SCR is a unidirectional device, but its turn on process and turn-OFF are controllable. TRIAC is a bidirectional device.

In this chapter, basic structure of silicon controlled rectifiers (thyristors), I-V characteristics, two transistor model of SCR, SCR turn-ON methods, switching characteristics, gate characteristics, SCR ratings, protection of SCR, series and parallel operation of SCR, gate triggering circuits and different commutation techniques are discussed elaborately. The basic structure, operating principle and I-V characteristics of DIAC and TRIAC are also incorporated in this chapter.

GTO is just like a conventional thyristor but the turn-OFF feature is incorporated with in the device. Whenever GTO is forward biased and a positive gate current is applied in between gate and cathode, the device will be turned ON and conducts. When GTO operates in conduction and a negative gate current of required amplitude is applied to gate-cathode terminals of GTO, it will be turned OFF. Due to self-turned OFF capability, GTO is most suitable device for inverters and choppers. In this chapter, the basic structure, operating principle, I-V characteristics and triggering circuits of GTO are also explained in detail.

4.2 SILICON CONTROLLED RECTIFIER (SCR)

SCR is the most commonly used power semiconductor devices of thyristor family and was developed in 1957. SCRs are available from few voltages to several kV and few amperes to several kA.

Figure 4.1(a) shows the basic structure of SCR. It is a four layer PNPN switching device with alternate layers of P and N semiconductor materials. There are three junctions namely J_1, J_2 and J_3 and three external terminals such as anode (A), cathode (K) and gate (G). The symbol of Thyristor is depicted in Fig. 4.1(b). The detail structure of thyristor is given in Fig. 4.2.

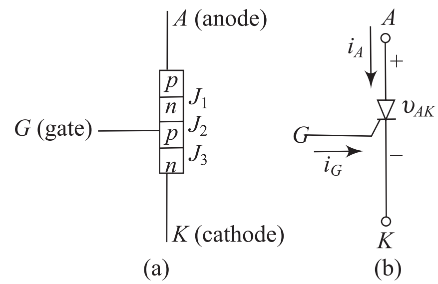


Fig. 4.1 *(a) Structure of thyristor and (b) Symbol of thyristor*

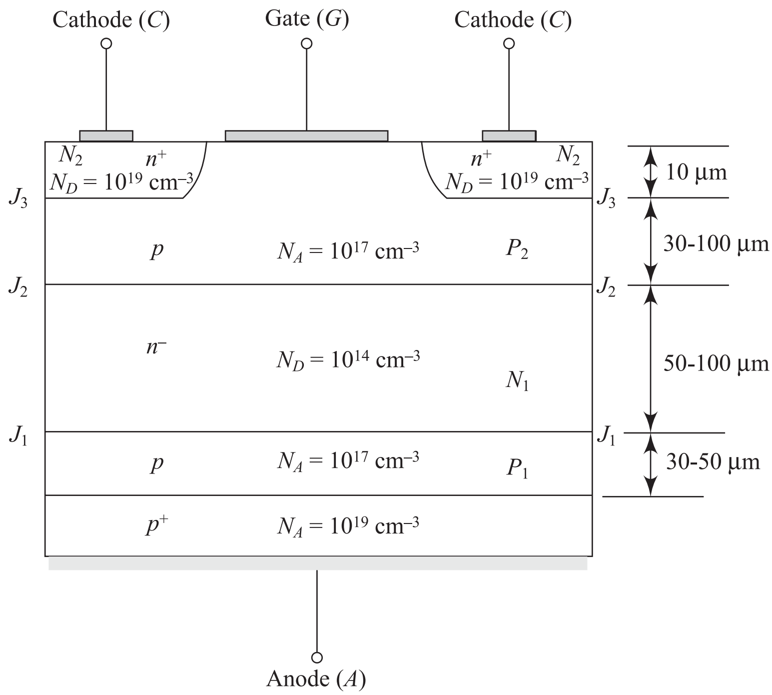


Fig. 4.2 *Detailed structure of thyristor*

The width of each semiconductor layer and doping intensity are depicted in Fig. 4.3. The P_1 and N_2 semiconductor layers are heavily dropped. The width of N_2 is very small about 10 µm and the width of N_1 is large about 50 to 100 µm but most lightly dropped about 10^{14} cm^{-3}. The width of P_1 layer is 30 to 50 µm where as width of P_2 is about 30 to 100 µm.

Due to low doping level, the depletion layer width becomes wide and provides high forward and reverse voltage blocking capability. The turn-OFF time and ON-state device losses are increased with the width of N_1 of the device. Hence the switching speed of SCR is reduced with increasing the width of N_1. For high speed SCR, the width of N_1 layer is reduced and the impurity density of N_1 layer must be increased. But the reverse voltage blocking capability and on-state voltage drops reduced.

Figure 4.4(a) shows the actual V-I characteristics of SCR and ideal V-I characteristics of SCR is depicted in Fig. 4.4(b). It is clear from Fig. 4.4 that SCRs have three operating regions such as *forward conduction state*, *forward blocking state* and *reverse blocking state*.

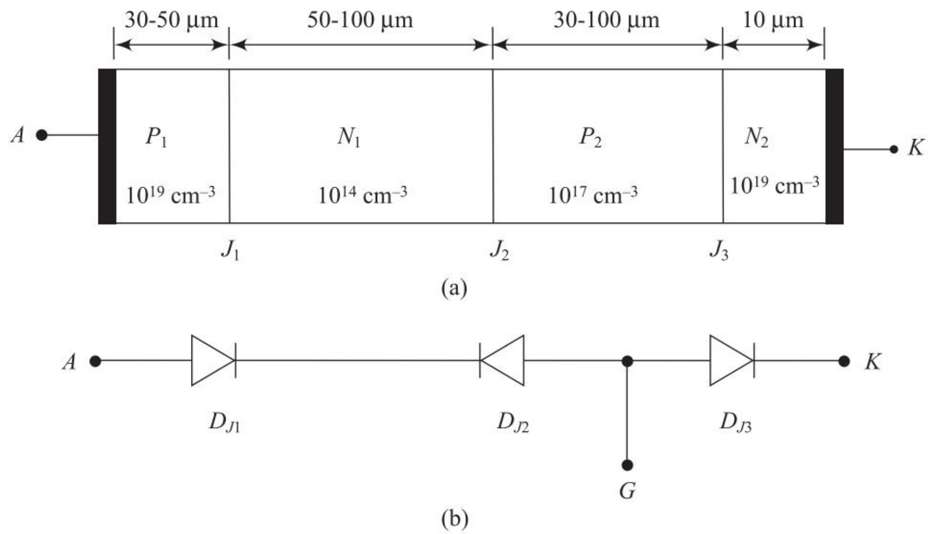


Fig. 4.3 *(a) Doping intensity and width of layers of SCR (b) Diode model of SCR*

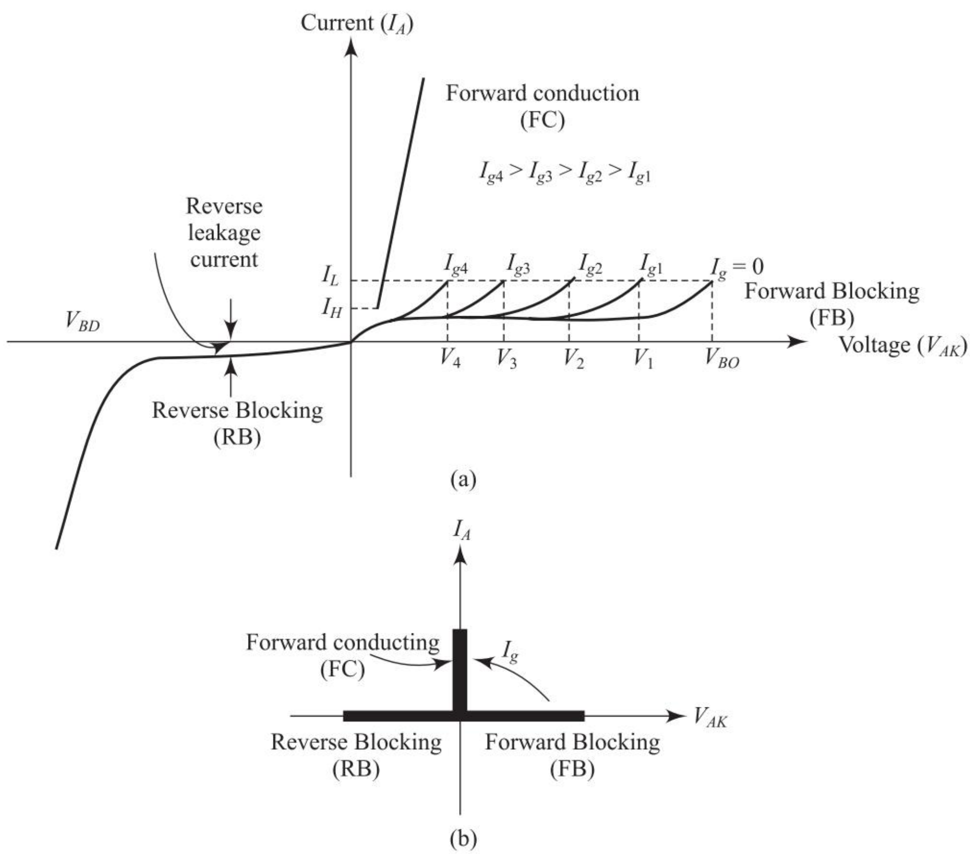


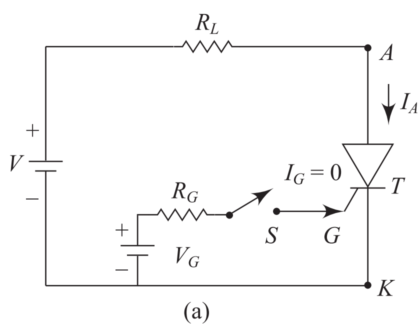
Fig. 4.4 *(a) Actual characteristics of SCR (b) Ideal characteristics of SCR*

4.2.1 Forward Blocking Region

The forward blocking region is the most wide operating region. In this region, anode terminal is positive with respect to cathode. Junction J_1 and J_3 are forward biased (FB), but junction J_2 is reverse biased (RB). The anode current (I_A) and voltage across anode to cathode (V_{AK}) vary in wide range with the amplitude of gate current.

The width of the forward biased junction depletion layer (J_1 and J_3) and the reverse biased junction depletion layer (J_2) depend on the dropping density and biasing voltage. The forward biased junction depletion layer width is thin while the reverse biased junction depletion layer width is thick. When a voltage is applied, as junction J_1 and J_3 are forward biased, the applied voltage should appear across junction J_2 as depicted in Fig. 4.5(b).

Figure 4.5(a) and (b) show the forward blocking of SCR. When the switch(s) is opened, there is no current flow though gate ($i_g = 0$). While the anode-to-cathode voltage increased gradually, the depletion layer across J_2 increased. Due to reverse biased at junction J_2, a reverse current flows through the junction and also from anode (A) to cathode (K). The amplitude of current is in mA range when V_{AK} is continuously increased. Whenever V_{AK} increased to the forward break over voltage, junction J_2 should be break down. The depletion layer of N_1 increased and can reach the opposite end of the layer, i.e., the depletion layer at J_1. Breakdown is possible through punch-through and *avalanche breakdown*.



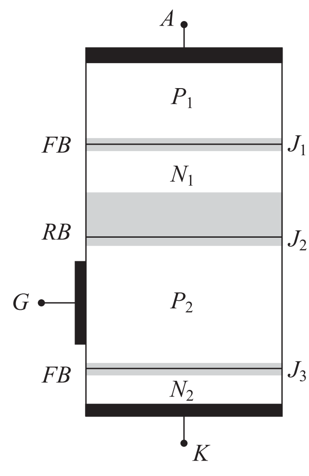


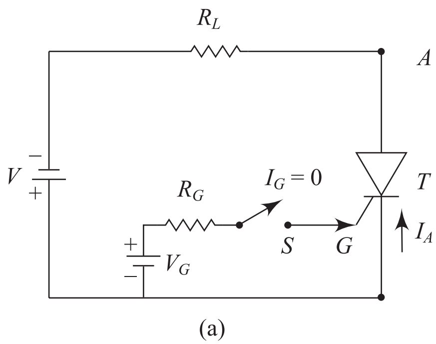
Fig. 4.5 *(a) Forward Blocking of SCR (b) Forward bias of J_1 and J_3 and reverse bias of J_2*

4.2.2 Forward Conduction Region

When the SCR is forward biased and the gate current is applied though the gate terminal (G), the junction J_2 breakdown. Then all the layers are filled with charge carriers. The SCR provides very low impedance between A and K and current flows through anode-to-cathode and amplitude of current is limited by external circuit parameters. The value of gate voltage V_G varies with in few volts. The forward conduction of SCR is depicted in Fig. 4.4(a).

4.2.3 Reverse Blocking Region

During reverse biased, the anode terminal is negative with respect to cathode, junctions J_1 and J_3 are reverse biased and junction J_2 is forward biased as depicted in Fig. 4.6(b). As the N_1 layer is lightly doped and the wide reverse biased depletion layer at J_2 is large. The depletion layer width of junctions J_2 and J_3 are thin. A reverse leakage current flows from cathode (K) to anode (A). When the applied voltage increased to breakdown voltage V_{BD}, the junction breaks. Figures 4.6(a) and (b) show the reverse blocking of SCR.



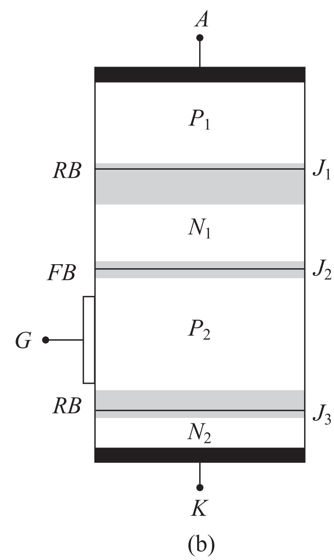


Fig. 4.6 *(a) Reverse blocking of SCR (b) Reverse bias of J_1 and J_3 and forward bias of J_2*

4.3 DIODE MODEL OF THYRISTOR

A thyristor has three PN junctions, J_1, J_2 and J_3 as shown in Fig. 4.3(b). Each PN junction is represented by diodes. When thyristor is forward biased, junctions J_1 and J_3 are forward biased and junction J_2 is reverse biased. Consequently, J_1 and J_3 are represented by forward bias diode D_{J1} and D_{J3} respectively. Similarly, J_2 is represented by reverse biased diode D_{J2}. When the gate current I_g flows, P_2 is flooded by electrons from cathode. Subsequently, P_2 losses its own identity. The electrons cross the junction and accelerated by anode to cathode voltage. The accelerated electrons collide with the covalent bond of silicon and generates more electron-hole pairs or minority carriers in N_1 layer. Hence, both sides of J_2 flooded by the minority carriers and the device become turn ON.

4.4 TWO-TRANSISTOR ANALOGY OF THYRISTORS

The structure of PNPN wafer of thyristor can be represented by two transistors T_1 and T_2. The connection of T_1 and T_2 is depicted in Figs. 4.7 and 4.8. T_1 is a PNP transistor and T_2 is a NPN transistor. The collector of transistor T_1 is connected with the base of T_2 the gate current and collector current I_{C1} provide the base current of T_2. While a gate current flows through the gate terminal, a base current flows through the base of T_2. Then the amplified collector current of T_2 is applied to the base of T_1 which generates the very large collector current I_{C1}. After that the amplified collector current is applied to the base of T_2. Hence a regenerative operation is performed and thyristor becomes ON with in few microseconds.

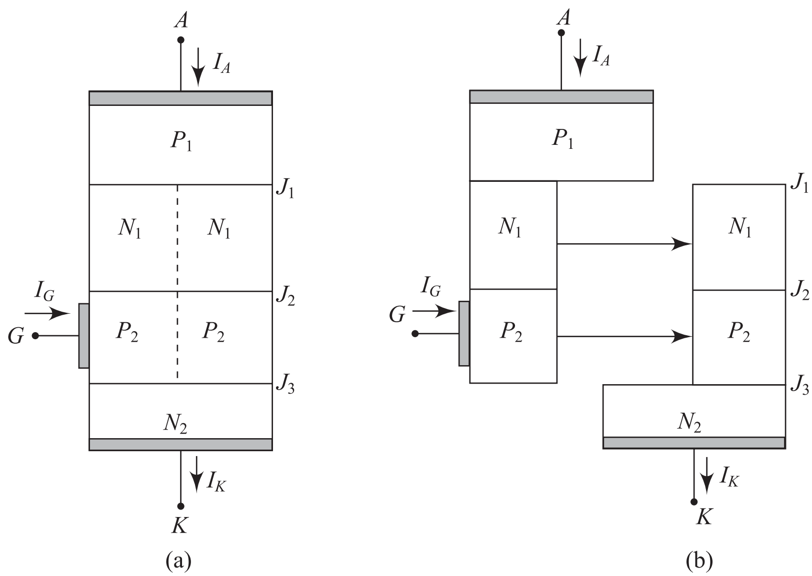


Fig. 4.7 *(a) Schematic diagram of thyristor (b) Two-transistor model of SCR*

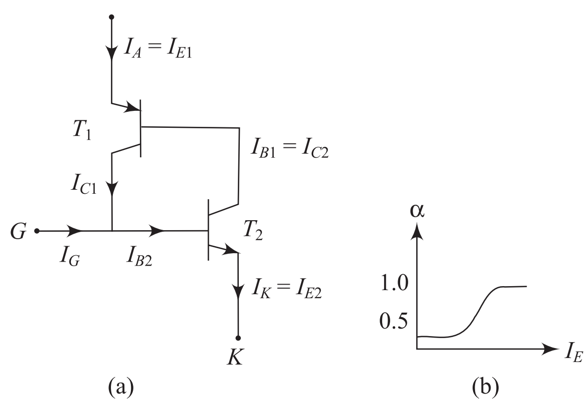


Fig. 4.8 *(a) Two-transistor model of SCR (b) Variation of current gain α with emitter current I_E*

In a BJT, the relationship between collector current I_C and emitter current I_E is

$$I_C = \alpha I_E + I_{CB0}$$

where α is the dc current gain,

I_{CB0} is the collector leakage current when base current is zero.

The base current of transistor T_2 is $I_{B2} = I_{C1} + I_G$

For the transistor T_1, the collector current $I_{C1} = \alpha_1 I_{E1} + I_{CB01}$

For the transistor T_2, the collector current $I_{C2} = \alpha_2 I_{E2} + I_{CB02}$

Assume that $I_A = I_{E1}$, and $I_{E2} = I_K$

Applying the KCL, we obtain

$$I_A = I_{B1} + I_{C1} = I_{C2} + I_{C1}$$

After substituting I_{C1} and I_{C2}, we get

$$I_A = \alpha_2 I_{E2} + I_{CB02} + \alpha_1 I_{E1} + I_{CB01}$$

or $$I_A = \alpha_2 I_K + I_{CB02} + \alpha_1 I_A + I_{CB01} \text{ as } I_A = I_{E1} \text{ and } I_{E2} = I_K$$

or $$I_A = \alpha_1 I_A + I_{CB01} + \alpha_2 I_G + \alpha_2 I_A + I_{CB02} \text{ as } I_K = I_G + I_A$$

or $$I_A = (\alpha_1 + \alpha_2) I_A + \alpha_2 I_G + I_{CB01} + I_{CB02}$$

or $$I_A = \frac{\alpha I_G + I_{CB01} + I_{CB02}}{1 - (\alpha_1 + \alpha_2)}$$

When a small gate current I_G is supplied, I_{E2} increases. Then current gain α_2 increases. Due to increase of α_2, I_{C2} increases. Correspondingly, there will be change in base current as $I_{B1} = I_{C2}$. Then I_{E1} increases as $I_{E1} = I_{C1} + I_{B1}$ and α_1 is also increased. Therefore, the gain $\alpha_1 + \alpha_2$ increases towards 1 (unity). Then one transistor can able to drive another transistor into saturation. Since both the transistors T_1 and T_2 operates in saturation, thyristor operates in conduction state and I_A increases significantly and its amplitude is limited by external circuit parameters though as per equation it is infinite in ideal case.

4.5 TRANSIENT MODEL OF THYRISTOR

Any PN junction can be represented by a resistance (R_J) and a capacitance (C_J) which is connected in parallel with R_J. If a junction is forward biased, R_J will be zero and the junction capacitance is not used to represent forward biased junction. When a PN junction reverse biased, the junction resistance (R_J) will be infinite. Therefore, junction capacitance C_J will be present to represent reverse biased junction.

The equivalent circuit of SCR at transient condition is shown in Fig. 4.9. The current flows though junction capacitance C_{J2} is

$$I_{CJ2} = \frac{dq_{J2}}{dt} = \frac{d(C_{J2}V_{J2})}{dt} = C_{J2}\frac{dV_{J2}}{dt} + V_{J2}\frac{dC_{J2}}{dt}$$

As C_{J2} is constant, $I_{CJ2} = C_{J2}\dfrac{dV_{J2}}{dt}$.

Since $V_{J2} = V_{AK}$, $\quad I_{CJ2} = C_{J2}\dfrac{dV_{AK}}{dt}$

As junctions J_1 and J_3 are forward biased, these junctions provide zero resistance. As junction J_2 is reverse biased, it provides infinite impedance. When $\dfrac{dV}{dt}$ is very large, I_{J2} increased. If the junction current is more than latching current ($I_{J2} > I_L$) for a specified time duration, SCR operates in conduction state.

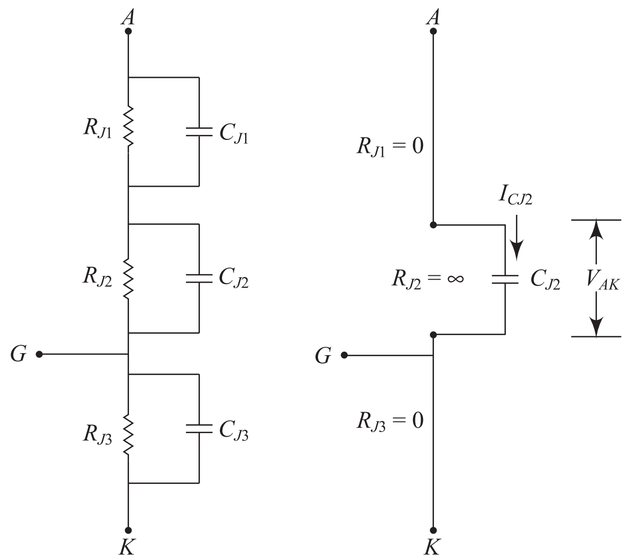


Fig. 4.9 *Dynamic model of thyristor*

4.6 TURN-ON OR TRIGGERING METHODS OF THYRISTOR

Thyristors can be turned on by different methods such as

1. Thermal triggering
2. High forward voltage triggering
3. High $\frac{dV}{dt}$ triggering
4. Light triggering
5. Gate triggering

In this section, the thermal triggering, high forward voltage triggering, high $\frac{dV}{dt}$ triggering, light triggering, and gate triggering of SCRs are discussed elaborately.

4.6.1 Thermal Triggering

While the temperature of thyristor increases, the large numbers of electron-hole pairs are generated. As the minority carriers moves freely and cross the junction J_2, the reverse leakage current increases. Just after crossing the junction, these minority carriers are accelerated and strike the covalent bond and generate more electron hole pairs. This increases the leakage currents and α_1 and α_2 increases. Due to positive feedback or regenerative action, $\alpha_1 + \alpha_2$ increases towards unity (1) and the thyristor will be turned on. Actually, N_1 and P_2 layers have large number of thermally generated carriers and the device is turned on. Usually this method of triggering is not used as it increases junction temperature and the voltage with stand capability is reduced. Sometime thermal runway is developed within the device and this method is normally avoided.

4.6.2 High Forward Voltage Triggering

When the applied voltage across anode to cathode is greater than the break down voltage V_{BD}, a leakage current flow which can generates regenerative action. Then the device is turned on. Actually, this type of turn on method is destructive and must be avoided.

4.6.3 High $\frac{dV}{dt}$ Triggering

If a thyristor is forward biased, the junction J_2 is reverse biased and junction J_1 and J_3 are forward biased. The reverse biased voltage across J_2 blocks the conduction of device. The depletion of junction J_2 provides very large capacitance. When the rate of change of voltage across anode to cathode is high, the charging current $I_{CJ2} = C_{J2}\frac{dV_{AK}}{dt}$ increases which flows though P_2 just like the gate current is supplied from gate terminal. Then the device is turned on. A high value of charging current may destroy the device. Therefore, thyristors must be protected from high $\frac{dV}{dt}$. Actually, the snubber circuit is used for $\frac{dV}{dt}$ protection.

4.6.4 Light Triggering

If a beam of light falls on the junction J_2 of the thyristor, it generates the electron-hole pairs (minority carrier). The generated minority carriers cross the blocking junction J_2 and reach P_2. Due to high electric field, minority carriers are accelerated and strike the covalent bond of silicon wafer and generate more electron-hole pairs. So the regenerative action takes place and the device is turned on. Actually this method of triggering is used in light activated SCR (LASCR). This method is most commonly used for HVDC transmission system, static VAR compensation system where current as well as voltage rating are high.

4.6.5 Gate Triggering

The gate triggering is most commonly used to turn on SCRs. When the thyristor is forward biased and if a positive gate voltage is applied between gate and cathode, a gate current flows through gate and the thyristor will be turn on. For successful triggering of SCR, the applied gate voltage V_g and gate current I_g must be close to their maximum value but always less than maximum value. When the gate current is increased, the forward blocking voltage capability of SCR is decreased. The different gate trigger circuits are explained in Section 4.17.

4.7 SWITCHING CHARACTERISTICS OF SCR

Thyristor is subjected to different voltages across it and different current flows through it during turn-on and turn-OFF process of thyristor. Due to time variation of voltage across thyristor and current through it during turn-ON and turn-OFF process, the switching characteristics of thyristor will be dynamic. There are two types of switching characteristics of thyristor namely

1. Turn-ON characteristics of thyristor
2. Turn-OFF characteristics of thyristor

4.7.1 Turn-ON Characteristics of Thyristor

When the thyristor is forward biased and a positive gate pulse is applied between the gate and cathode, it will be turned ON. But there is a finite transition time to switch from forward OFF-state to forward ON-state for a thyristor. This finite transition time is known as turn-ON time. The turn-ON characteristics of thyristor is shown in Fig. 4.10. The turn-ON time $t_{\text{turn-on}}$ is the sum of the delay time t_d, rise time t_r and spread time t_p and it can be expressed as

$$t_{\text{turn-on}} = t_d + t_r + t_p$$

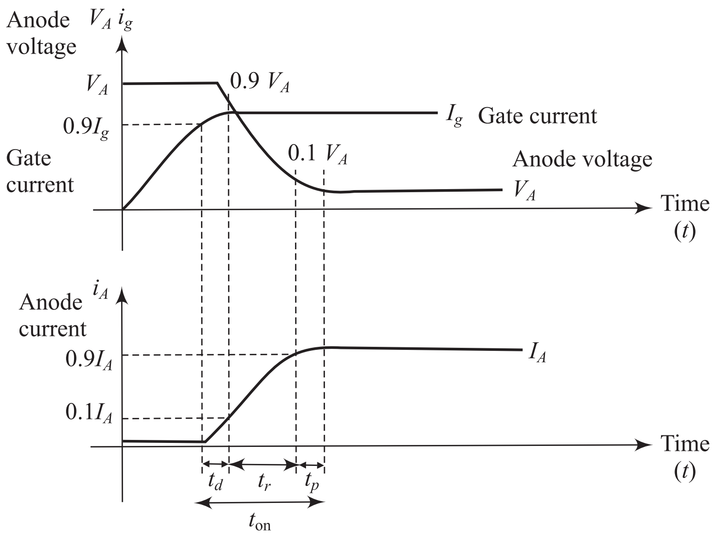


Fig. 4.10 *Turn-ON characteristics of thyristor*

Delay time t_d There is a time delay to turn on a device. The delay time t_d is the required time interval from initial anode current, i.e., forward leakage current to reach 10% of final value of anode current ($0.1I_A$) where I_A is the final value of anode current. The delay time t_d can be measured from the instant of the gate current reaches $0.9I_g$ to the instant at which anode current reaches to $0.1I_A$ where I_g is the final value of gate current and I_A is the final value of anode current. The delay t_d can also be measured by the time interval during which anode voltage falls from V_A to $0.9V_A$ where V_A is the initial value of anode to cathode voltage.

Rise time t_r The rise time is the time interval during which the anode current increases from 10% to 90% of final anode current. During rise time the forward blocking OFF-state voltage decreases, the rise time t_r can be measured from the instant of 90% of forward blocking OFF-state voltage ($0.9V_A$) to the instant at which forward blocking OFF-state voltage reaches to $0.1V_A$. The rise time t_r can also be measured from the instant of the gate current reaches $0.9I_g$ to the instant at which gate current I_g where I_g is the final value of gate current.

The rise time is inversely proportional to the amplitude of gate current and its build up rate. The rise time deceases when high and steep current pulse is applied to gate of SCR. The value of t_r depends on the nature of anode circuit (R-L and R-C circuits). For example, in a series R-L circuit, the rate of rise of anode current is slow due to presence of inductance L and rise time is more. In case of a series R-C circuit, the rate of rise of anode current is high and rise time is less.

Spread time t_p The spread time is the time interval during which the forward blocking voltage falls from 10% of its value ($0.1V_A$) to the ON-state voltage drop about 1 V to 1.5 V. The spread time can be defined as the time taken by the anode current to rise from $0.9I_A$ to I_A. During this time, the conduction spreads over the entire cross section of the SCR cathode. The spreading time depends on the area of cathode and gate structure of SCR.

After spread time, SCR is completely turned on and final steady state anode current flows through the device and the voltage drop across SCR is about 1 V to 1.5 V. The total turn-ON time of SCR is equal to 1 to 4 µs. The total turn-ON time depends on the anode circuit parameters and the amplitude of gate current and its wave shape. With the increase of gate current, the turn-ON time will be decreased significantly. The amplitude of gate current is 3 to 5 times of minimum gate current to trigger thyristor. During design of gate triggering circuit, the following conditions should be maintained:

1. The gate must be removed just after turn ON of thyristor.
2. When the thyristor is reverse biased, the gate signal should not be applied.
3. The pulse width of gate pulse must be sufficient so that the anode current must increases to latching current and it must be grater than turn-ON time.

4.7.2 Turn-OFF Characteristics of Thyristor

When a thyristor is in conducting state and it can be turned OFF if the anode current is reduced slowly below the holding current I_H. The turn-OFF process of thyristor is called *commutation*. There are two different methods of thyristor turn-OFF such as *natural commutation* and *forced commutation*. The turn-OFF time of a SCR t_q can be defined as the time interval between the instant at which anode current through the device becomes zero and the instant at which SCR regain its forward blocking capability. The turn-OFF time of a SCR t_q is sum of the *reverse recovery time* t_{rr} and *gate recovery time* t_{gr} and it is equal to

$$t_q = t_{rr} + t_{gr}$$

The turn-OFF characteristics of thyristor is shown in Fig. 4.11.

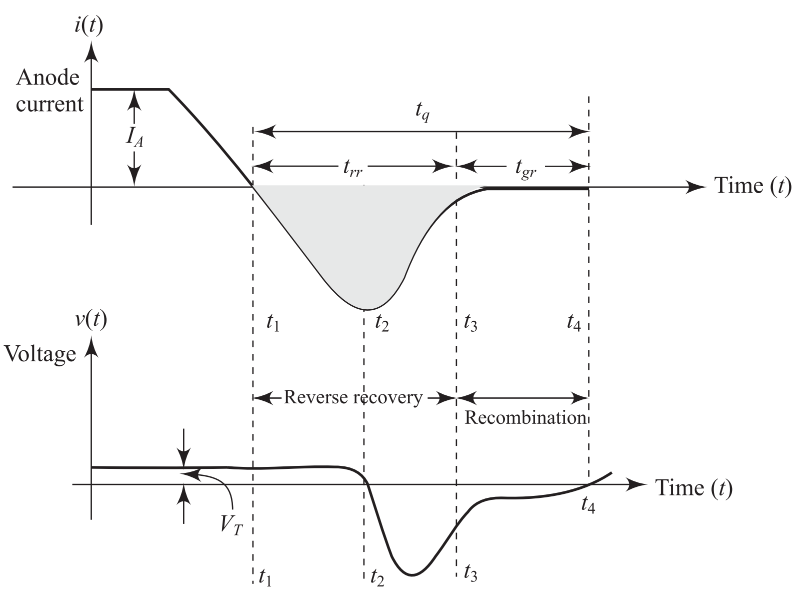


Fig. 4.11 *Turn-OFF characteristics of thyristor*

Reverse recovery time t_{rr} At time $t = t_1$, the anode current becomes zero. After $t = t_1$, anode current starts to build up in the reverse direction due to presence of charge carriers in the four layers of SCR. Actually, the reverse recovery current removes the excess charge carriers from junctions J_1 and J_3 during the time interval between $t = t_1$ and $t = t_3$. Consequently, the holes are sweeping out from top p-layer and the electrons are sweeping out from bottom n-layer. At $t = t_2$, about 60% of the stored charge carriers are removed from outer two layers, i.e., top p-layer and bottom n-layer. As a result, the carrier density in junctions J_1 and J_3 decreases and subsequently the reverse recovery current also decreases. Initially the rate of decrease of reverse recovery current is very fast but it is gradual thereafter. Due to fast decay of reverse recovery current, a reverse surge voltage appears across the SCR and the device may be damaged. This condition may be avoided by using RC snubber circuit across SCR.

Gate recovery time t_{gr} At time $t = t_3$, the reverse recovery current becomes very small and its value is about zero and the thyristor is able to block the reverse voltage. During the time interval between $t = t_1$ and $t = t_3$, all excess charge carriers are removed from outer junctions J_1 and J_3. At $t = t_3$, the middle junction J_2 still consists of charge carriers and it cannot able to block the forward voltage. Since the charge carriers present in the junction J_2 cannot able to flow to the external circuit, these carriers can be removed by recombination only. The recombination is possible when a reverse voltage is applied across SCR for a specified time. The rate of recombination does not depend on the external circuit parameters. The recombination of charge carriers takes place between $t = t_3$ and $t = t_4$. The time interval between $t = t_1$ and $t = t_3$ is called gate recovery time t_{gr}. At $t = t_4$, thyristor operates in OFF state.

When a reverse voltage is applied, all the excess carriers in the four layer sweeps out or recombined. The reverse recovery current flows and its magnitude is more than the rated reverse blocking current. The reverse recover current flows for reverse recovery time t_{rr} and the charge is computed from $I_r \times t_{rr}$ which is called reverse recovery charge Q_r. The excess carrier in N_1 and P_2 will be reduced due to recombination. The time required for this operation is t_{gr} which is called the *gate recovery time*. The thyristor turn-OFF time is in the order of 3 μs to 100 μs. The turn-OFF time depends on the amplitude of forward current, junction temperature and di/dt during commutation process. With the increase of forward current, junction temperature and di/dt, thyristor turn-OFF time increases.

Example 4.1 The turn-ON and turn-OFF time of SCR are 2.5 μs and 7.5 μs respectively, determine the maximum switching frequency of SCR in a converter circuit.

Solution

Given: $t_{\text{turn-on}} = 2.5\ \mu\text{s}$ and $t_{\text{turn-off}} = 7.5\ \mu\text{s}$

The minimum time period is

$$T_{\min} = t_{\text{turn-on}} + t_{\text{turn-off}} = 2.5\ \mu\text{s} + 7.5\ \mu\text{s} = 10\ \mu\text{s}$$

The maximum switching frequency of SCR in a converter circuit is

$$f_{\max} = \frac{1}{T_{\min}} = \frac{1}{10\ \mu\text{s}} = 100\ \text{kHz}$$

Example 4.2 A 230 V, 50 Hz single phase ac supply is connected to a thyristor in series with a load resistance R_L. Under forward blocking condition, the capacitance of junction J_2 is 1.45 nF. At room temperature 30°C, thyristor has the following parameters:

$V_{\text{RRM}} = 1000$ V, $V_{\text{RMS}} = 300$ V, $V_F = 1$ V, $V_{gT} = 2.5$ V, $I_{gT} = 100$ mA, $dV/dt = 75$ V/μs and $di/dt = 50$ V/μs.

Prove that a spike of 300 V for 3 μs duration is sufficient to trigger thyristor.

Solution

The rate of rise of voltage is

$$\frac{dv}{dt} = \frac{300\text{ V}}{3\text{ μs}} = 100\text{ V/μs}$$

This $\frac{dv}{dt}$ value is greater than the maximum limit.

The charging current of capacitor is

$$i_C = C_{j2}\frac{dv}{dt} = 1.45\times10^{-9}\times100\text{ V/μs} = 145\text{ mA}$$

The charging current i_C is grater than the minimum gate current (I_{gT} = 100 mA) required to turn on thyristor. Therefore, thyristor will be falsely triggered.

Example 4.3 When a thyristor acts as a switch, anode current rises linearly from zero to final value of 200 A but the anode voltage falls linearly from 500 V to zero during turn-ON process. The turn-ON time of thyristor is 10 μs. If the switching frequency of thyristor is 250 Hz, determine the average power loss in thyristor.

Solution

Given: V = 500 V, I = 200 A, $t_{\text{turn-on}}$ =10 μs and f = 250 Hz

The average power loss in thyristor is equal to

$$\frac{1}{6}VIt_{\text{turn-on}}f = \frac{1}{6}\times500\times200\times10\times10^{-6}\times250\text{ Watt} = 41.66\text{ Watt}$$

Example 4.4 In a thyristor, the capacitance value of reverse-biased junction J_2 is $C_{J_2} = 30\text{ pF}$ and it is independent of the OFF-state voltage. The limit value of the charging current to turn the thyristor is about 20 mA. Find the critical value of $\frac{dv}{dt}$.

Solution

Given: $C_{J_2} = 30\text{ pF}$ and $i_{J_2} = 20\text{ mA}$

The charging current of capacitor is

$$i_C = C_{j2}\frac{dv}{dt}$$

The critical value of $\frac{dv}{dt}$ is $\frac{dv}{dt} = \frac{i_C}{C_{J2}} = \frac{20\times10^{-3}}{30\times10^{-12}} = 666.67\text{ V/μs}$

Example 4.5 The capacitance value of reverse-biased junction J_2 of a thyristor, C_{J_2} is independent of OFF-state voltage. The limit value of the charging current to turn the thyristor is about 15 mA. If the critical value of $\frac{dv}{dt}$ is 750 V/μs, compute the value of junction capacitance C_{J_2}.

Solution

Given: $\frac{dv}{dt}$ = 750 V/μs and i_{J_2} = 15 mA

The charging current of capacitor is

$$i_C = C_{j2}\frac{dv}{dt}$$

The value of C_{J_2} is $C_{J_2} = \frac{i_C}{\frac{dv}{dt}} = \frac{15\times10^{-3}\times10^{-6}}{750} = 200\text{ pF}$

4.8 GATE CHARACTERISTICS

A SCR can be triggered and turned ON when a positive gate voltage V_g is applied between gate and cathode or a positive gate current i_g flows from gate to cathode. The values of V_g and i_g are not restricted to a specified value but it varies over a wide range and depends upon gate characteristics of SCR. Figure 4.12 shows the forward gate characteristics which is a graph between gate voltage V_g and gate current i_g. Since gate-cathode of a SCR is represented by *pn* junction, the gate characteristic is similar to a *pn*-junction diode. The gate characteristics are widely spread between curve-1 and curve-2 due to inadvertent difference in doping density of P_2 and N_2 layers. Curve-1 represents the lowest gate voltage values which are applied to gate-cathode of SCR for turn-on satisfactorily. Similarly, curve-2 represents the highest possible gate voltage values which are also applied to gate-cathode of SCR for turn-ON adequately.

Always there is a minimum and maximum limit of gate voltages and gate current during triggering a SCR. It is clear from Fig. 4.12 that

ox is the minimum gate current to trigger an SCR
oy is the minimum gate voltage to trigger an SCR
oa is the non-triggering gate voltage of an SCR
I_{gm} is the maximum gate current to trigger an SCR
V_{gm} is the maximum gate voltage to trigger an SCR
P_{gav} is the average gate power dissipation of an SCR

During design of a gate drive circuit, the values of V_{gm}, I_{gm} and P_{gav} should not be exceeded. If these values increase, the junction J_3 will be permanently damage. In Fig. 4.12, the shaded area $b-c-d-e-f-g-h-b$ is the preferred gate drive area where SCR operates safely. *oa* is the non-triggering gate voltage of a thyristor. The value of *oa* is also specified by the manufacturer. When a triggering or firing circuit generates a gate signal with gate voltage *oa* and gate current *ox*, SCR will not be turned ON. Therefore, amplitude of all unwanted signals or noise signals must be less than non-triggering voltage *oa*.

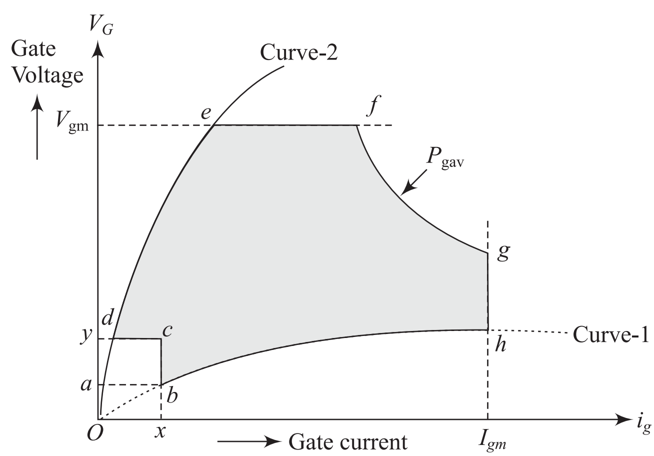


Fig. 4.12 *Gate-characteristics of an SCR*

Figure 4.13 shows a triggering circuit of an SCR where

V_s is the gate source voltage,
R_s is the resistance of source voltage,
V_g is the gate to cathode voltage, and
i_g is the gate current.

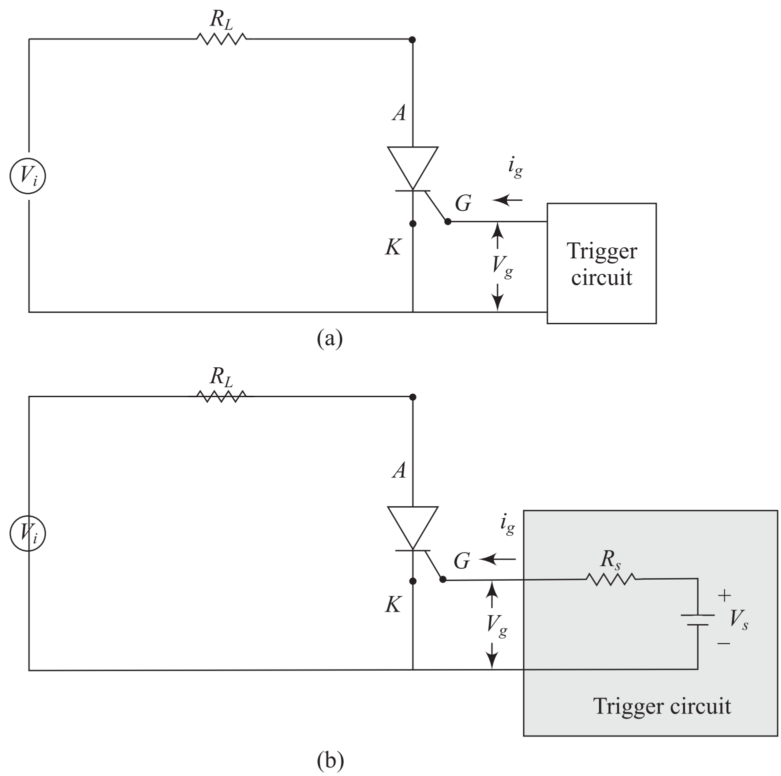


Fig. 4.13 *Triggering circuit of an SCR*

For the above circuit, $V_s = i_g R_s + V_G$. The value of internal resistance R_s should be such that the current $\frac{V_s}{R_s}$ should not be very high so that voltage source as well as gate drive circuit operate safely during turn-on of SCR. If R_s is very small, an external resistance must be connected across gate-cathode terminals of SCR as depicted in Fig. 4.14. When a minimum gate voltage $V_{g\min}$ is applied between gate and cathode, the minimum gate current $I_{g\min}$ flows through gate. Then current flows through resistance R_1 is equal to $\frac{V_{g\min}}{R_1}$ and the source voltage is equal to

$$V_s = V_{g\min} + \left(i_g + \frac{V_{g\min}}{R_1}\right)R_s \quad \text{as} \quad V_G = V_{g\min}$$

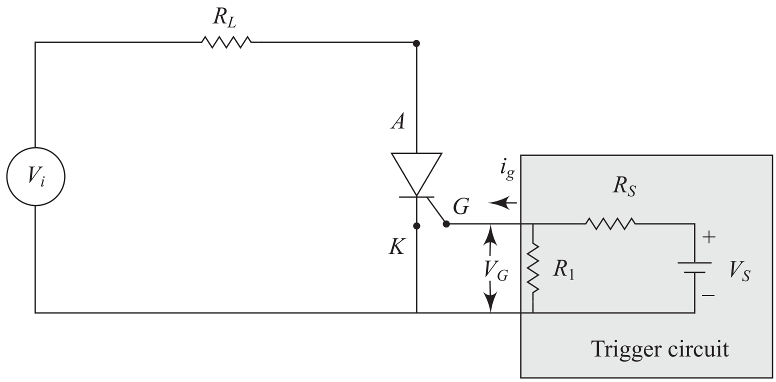


Fig. 4.14 *Triggering circuit of an SCR*

In Fig. 4.15, *A-B* is the load line of gate-cathode circuit. *OB* is the source voltage V_s and *OA* is the current $\frac{V_s}{R_s}$ when triggering circuit is short circuit. Assume that SCR operates in curve -3. The intersection of load line and curve-3 is *Q* which is the operating of SCR. At point *Q*, the gate voltage is *OR* = *QP* and the gate current is equal to *OP*, the intersection between load line and curve-1 and curve-2 are S_1 and S_1 respectively. With the variation of parameters, the SCR operates within the operating points S_1 and S_2 as operating point must lie within the limit curve-1 and curve-2. The gradient of load line *AB* is $\frac{OB}{OA}$ which represents the gate source resistance R_s. The minimum value of gate–source resistance $R_{s\ \min}$ is obtained by drawing a tangent on the P_{gav} curve.

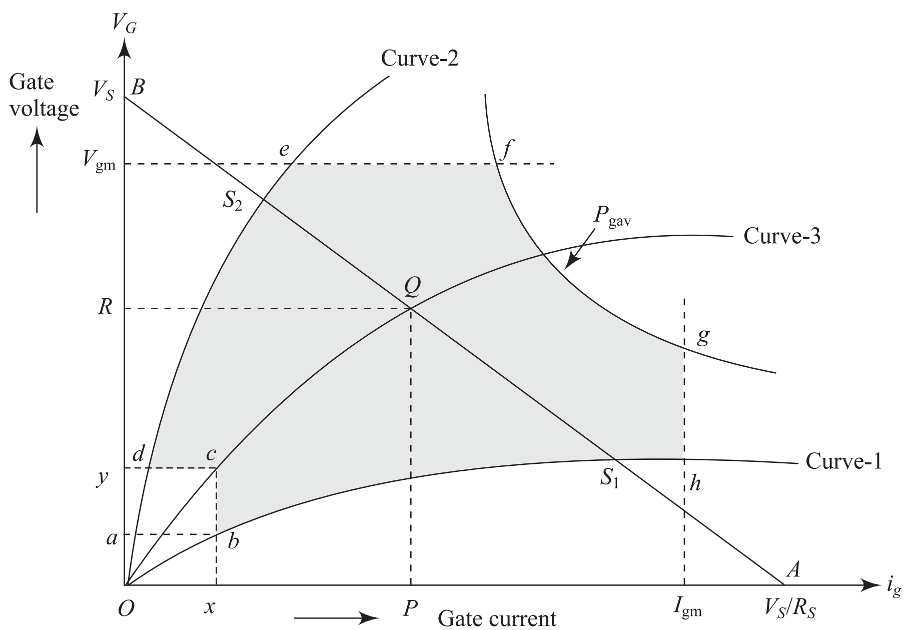


Fig. 4.15 *Gate-characteristics of an SCR*

Since the thyristor is a current or charge controlled device, it must be ensured that if the magnitude of gate current is increased, the time taken to inject the required charge for turn ON of a SCR will be reduced significantly. It can also be ensured that gate pulse width should be sufficient to allow the anode current to exceed the latching current I_L. Therefore the gate pulse width must be greater than to equal to SCR turn-ON time, i.e.,

$$T \geq t_{\text{turn-on}}$$

where, T is the pulse width and

$t_{\text{turn-on}}$ is the turn-ON time of SCR as shown in Fig. 4.16.

When the pulse triggering is used to turn-ON SCR, the greater amount of gate power dissipation may be allowed, but its value must be less than the maximum (peak) instantaneous gate power dissipation P_{gmax}. Generally, P_{gmax} of any SCR is specified by the manufacturers. The frequency of firing pulse to trigger SCR can be obtained from the following equation:

$$\frac{P_{g\max}T}{T_1} \geq P_{gav}$$

where, P_{gmax} is maximum gate power dissipation,

P_{gav} is average gate power dissipation, T is pulse width and T_1 is periodic time.

Since $f = \dfrac{1}{T_1}$ is the frequency of firing pulse in Hz, we can write

$$P_{g\max} fT \geq P_{gav}$$

or

$$P_{g\max} \geq \frac{P_{gav}}{fT} \tag{4.1}$$

For limiting case, $P_{g\max} = \dfrac{P_{gav}}{fT}$ or $f = \dfrac{P_{gav}}{T.P_{g\max}}$

The duty cycle of a pulse is the ratio of pulse on period T to the periodic time of pulse T_1 and it is represented by $\delta = \dfrac{T}{T_1} = fT$ as $f = \dfrac{1}{T_1}$.

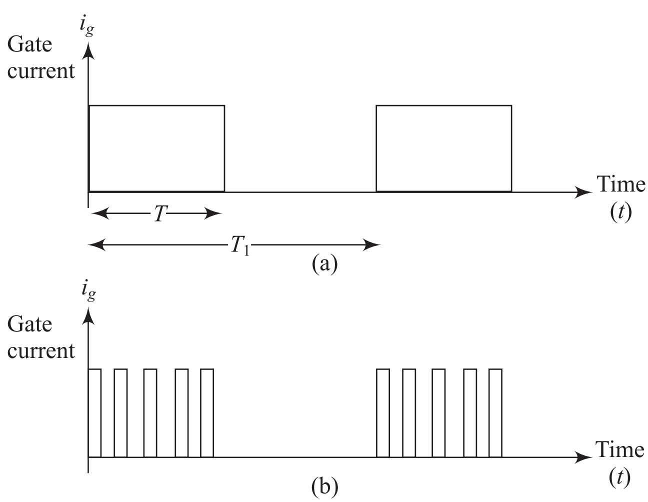


Fig. 4.16 *(a) Pulse triggering of SCR (b) High frequency triggering pulse of SCR*

Then Eq. (4.1) can be written as

$$P_{g\max} \geq \frac{P_{gav}}{\delta} \quad \text{as} \quad \delta = fT$$

For any SCR, V_{gm}, I_{gm} and P_{gmax} are specified in manufacturer data sheet. When V_{gm} and I_{gm} are used for pulse triggering, the power dissipation may be exceed the P_{gmax} and the SCR may be damaged completely. For example, V_{gm} = 10 V and I_{gm} = 1 A for an SCR. Then power dissipation is equal to P_{gmax} = 10 W, but the specified value of P_{gmax} is 5 W. Therefore, it must be ensured that

$$\text{Amplitude of pulse voltage} \times \text{Amplitude of pulse current} < P_{\text{gmax}}$$

Example 4.6 The average gate power dissipation of a thyristor is P_{gav} = 0.45 Watt. If the gate voltage varies from 2.5 V to 10 V, plot the curve where gate voltage is a function of gate current. Assume average gate power dissipation is constant.

Solution

Given: P_{gav} = 0.45 Watt, V_g varies from 2.5 V to 10 V
Assume four voltages V_{g1} = 2.5 V, V_{g2} = 5 V, V_{g3} = 7.5 V and V_{g4} = 10 V
The corresponding currents are

$$I_{g1} = \frac{P_{gav}}{V_{g1}} = \frac{0.45}{2.5} = 0.18A \quad \text{as} \quad V_{g1}I_{g1} = P_{gav}$$

$$I_{g2} = \frac{P_{gav}}{V_{g2}} = \frac{0.45}{5} = 0.09A$$

$$I_{g3} = \frac{P_{gav}}{V_{g3}} = \frac{0.45}{7.5} = 0.06A$$

$$I_{g4} = \frac{P_{gav}}{V_{g4}} = \frac{0.45}{10} = 0.045A$$

The plot between gate voltage and gate current is shown in Fig. 4.17.

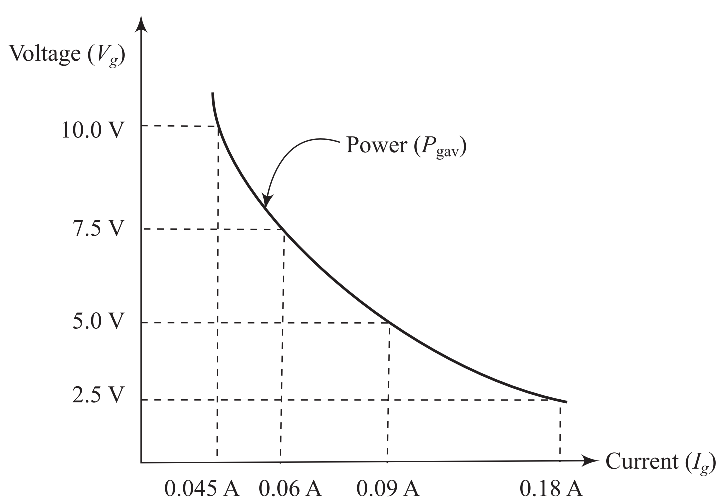


Fig. 4.17 *Gate voltage V_g vs gate current I_g*

Example 4.7 In a gate triggering circuit, the average power dissipation is 0.25 W. The slope of load line is 100 V per ampere, source voltage is 12 V and minimum gate current to turn-ON thyristor is 15 mA. Determine gate current, gate voltage and gate source resistance.

Solution

Given: $V_g I_g = P_{gav} = 0.25$ Watt, $V_s = 12$, $R_s = 100\ \Omega$

The gate voltage is $V_g = \dfrac{0.25}{I_g}$

The gate source voltage is $V_s = I_g R_s + V_g$

or $$12 = 100 I_g + \frac{0.25}{I_g}$$

or $$100 I_g^2 - 12 I_g + 0.25 = 0$$

Then $$I_g = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 100 \times 0.25}}{2 \times 100} = 0.0931\ \text{A}, 0.0268\ \text{A}$$

Then gate voltage $V_g = \dfrac{0.25}{I_g}$ = 2.685 V when I_g = 0.0931 A = 93.1 mA

= 9.328 V when I_g = 0.0268 A = 26.8 mA

Assume the gate current I_g = 93.1 mA as minimum gate current to turn-ON thyristor is 15 mA.
Therefore, the gate voltage is V_g = 2.685 V
The gate source resistance is equal to

$$R_s = \frac{V_s - V_g}{I_g} = \frac{12 - 2.685}{0.0931}\ \Omega = 100.053$$

Example 4.8 The gate characteristics of a thyristor is $V_g = 1 + 5I_g$. When a rectangular pulse of 10 V with 25 μs is applied to gate, determine (a) the series connected resistance in gate, (b) triggering frequency and (c) duty cycle. Assume average power dissipation is 0. 5 W and peak gate drive power is 4 W.

Solution

Given: $V_g I_g = P_{gav} = 0.5$ Watt, $V_s = 10$ V
Then, $V_g I_g = (1 + 5I_g)I_g = 0.5$ as $V_g = 1 + 5I_g$

(a) For pulse triggering of thyristors,

Peak gate voltage × peak gate current = peak gate drive power

Since gate pulse width is 25 μs which is less than 100 μs, the dc data may not be applied. The equation $(1 + 5I_g)I_g = 0.5$ is relevant only for pulse width greater than 100 μs.

Therefore, $(1 + 5I_g)I_g = 4$

or $$5I_g^2 + I_g - 4 = 0$$

Then $$I_g = \frac{-1 \pm \sqrt{1^2 - 4 \times 5 \times (-4)}}{2 \times 5} = 0.8\ \text{A}$$

The gate source voltage $V_s = I_g R_s + V_g = I_g R_s + (1 + 5I_g)$ as $V_g = 1 + 5I_g$

or $$10 = 0.8 R_s + 1 + 5 \times 0.8$$

The gate source resistance is equal to

$$R_s = 6.25\ \Omega$$

(b) We know that $P_{g\max} = \dfrac{P_{gav}}{fT}$

or
$$f = \frac{P_{gav}}{P_{g\max}T} = \frac{0.5}{4 \times 25 \times 10^{-6}}\ \text{Hz} = 5\ \text{kHz}$$

(c) Duty cycle $\delta = fT = 5 \times 10^3 \times 25 \times 10^{-6} = 0.125$

Example 4.9 A 220 V dc voltage is connected to a thyristor in series with R-L load. Assume the latching current of thyristor is 80 mA. When (a) $R = 25\ \Omega$ and $L = 0.1$ H and (b) $R = 25\ \Omega$ and $L = 1$ H, find the minimum width of gate pulse current to turn-ON thyristor. What is the effect of inductance on gate-pulse width?

Solution

Given: $V = 220$ V and Latching current $i_L = 80$ mA

(a) The voltage equation is
$$V = iR + L\frac{di}{dt}$$

or
$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

or
$$80 \times 10^{-3} = \frac{220}{25}\left(1 - e^{-\frac{25}{0.1}t}\right)$$

or
$$\left(1 - e^{-250t}\right) = \frac{2}{220}$$

or $t = 36.529$ μs

Therefore the minimum width of gate pulse current to turn-ON thyristor is 36.529 μs.

(b) We know that $i = \dfrac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

Then or,
$$80 \times 10^{-3} = \frac{220}{25}\left(1 - e^{-\frac{25}{1}t}\right)$$

or
$$\left(1 - e^{-25t}\right) = \frac{2}{220}$$

or $t = 365.29$ μs

Therefore the minimum width of gate pulse current to turn-ON thyristor is 365.29 μs.
With the increase of inductance from 0.1 H to 1 H, the width of gate pulse current increases from 36.529 μs to 365.29 μs.

Example 4.10 The gate-cathode characteristics of SCR are spread by the following equations:

$$I_g = 2.5 \times 10^{-3} V_g^2 \quad \text{and} \quad I_g = 2.5 \times 10^{-3} V_g^{1.5}$$

If the gate source voltage is 16 V and $R_s = 125\ \Omega$, determine the triggering voltage and triggering current. Assume the gate power dissipation is 0.5 W.

Solution

Given: $P_{gav} = 0.5$ Watt, $V_s = 16$ V

We know that $V_g I_g = P_{gav} = 0.5$ Watt

or $$V_g = \frac{0.5}{I_g}$$

The source voltage $V_s = I_g R_s + V_g$

or $$15 = 125 I_g + \frac{0.5}{I_g}$$

or $$125 I_g^2 - 15 I_g + 0.5 = 0$$

Then $$I_g = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \times 125 \times (0.5)}}{2 \times 125} = 73.796 \text{ mA, } 54.20 \text{ mA}$$

According to $I_g - V_g$ characteristics,

$$I_g = 2.5 \times 10^{-3} V_g^2 = 2.5 \times 10^{-3} \frac{0.5^2}{I_g^2}$$

or $$I_g = 85.70 \text{ mA} \text{ and } V_g = \frac{0.5}{I_g} = \frac{0.5}{85.70 \times 10^{-3}} = 5.83 \text{ V}$$

$$I_g = 2.5 \times 10^{-3} V_g^{1.5} = 2.5 \times 10^{-3} \frac{0.5^{1.5}}{I_g^{1.5}}$$

or $$I_g = 60.05 \text{ mA} \text{ and } V_g = \frac{0.5}{I_g} = \frac{0.5}{60.05 \times 10^{-3}} = 8.326 \text{ V}$$

Since the gate current will be in between $I_g = 85.70$ mA and $I_g = 60.05$ mA, the gate current at operating point Q, $I_g = 73.796$ mA

As $$I_g = 73.796 \text{ mA}, \; V_g = \frac{0.5}{73.796 \times 10^{-3}} \text{ V} = 6.775 \text{ V}$$

Hence, the operating point Q is (73.796 mA, 6.775 V).

Example 4.11 A 200 V dc supply voltage is applied to an SCR which is connected in series with RL load as shown in Fig. 4.18. The latching current of a thyristor is 20 mA. When a gate pulse of 30 μs is applied, whether the SCR will be turned ON or not?

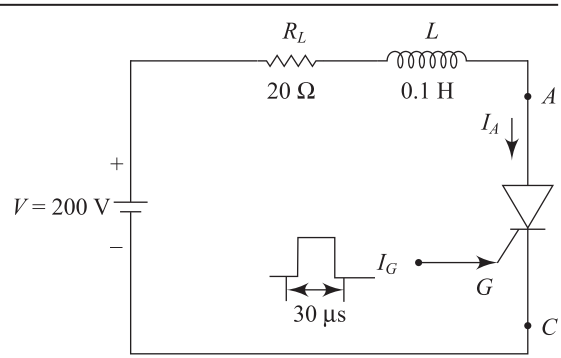


Fig. 4.18

Solution

Given: $V = 200$ V, Latching current $i_L = 20$ mA

The voltage equation is

$$V = iR + L\frac{di}{dt}$$

or $$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

or $$20 \times 10^{-3} = \frac{200}{20}\left(1 - e^{-\frac{20}{0.1}t}\right)$$

or $$\left(1 - e^{-200t}\right) = \frac{400 \times 10^{-3}}{200}$$

or $$t = 10.01 \ \mu\text{s}$$

As the minimum width of gate pulse current to turn-ON thyristor is 10.01 μs and a gate pulse of 30 μs is applied, thyristor will be turned on.

Example 4.12 Figure 4.19 shows that a thyristor is connected in series with R-L load. The latching current is 75 mA. When a firing pulse of 100 μs is applied in between gate and cathode, find the state of thyristor whether it is turned-ON or turned-OFF.

Solution

Given: $R = 10\ \Omega$, $L = 0.25$ H, $V = 200$ V, $I_L = 75$ mA, $t = 100\ \mu$s

When the thyristor is turned ON, the current will increase exponentially due to inductive load. The load current can be expressed by

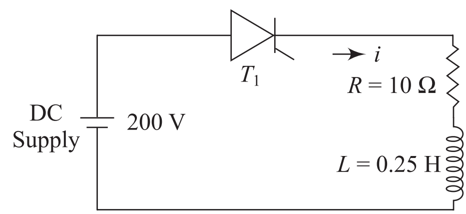


Fig. 4.19

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

Since $$\tau = \frac{L}{R} = \frac{0.25}{10} = 0.025, \quad i(t) = \frac{200}{10}\left(1 - e^{-\frac{t}{0.025}}\right) = 20\left(1 - e^{-\frac{t}{0.025}}\right)$$

At $t = 100\ \mu$s, the value of current is $i(t = 100\,\mu\text{s}) = 20\left(1 - e^{-\frac{100\times10^{-6}}{0.025}}\right) = 79.68$ mA

Since the value of current is greater than latching current, thyristor operates in turned-ON state.

Example 4.13 In Fig. 4.20, the latching current of thyristor is 100 mA. What will be the minimum pulse width of gating pulse to turn-ON thyristor?

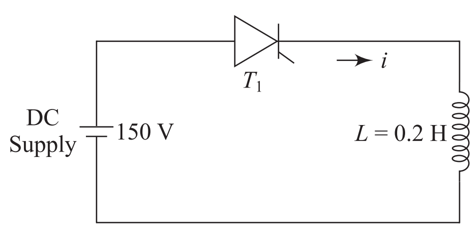


Fig. 4.20

Solution

Given: $L = 0.2$ H, $V = 150$ V, $I_L = 100$ mA

When the thyristor is turned ON, the circuit equation is

$$V = L\frac{di}{dt}$$

or $$dt = \frac{L}{V}di$$

After integrating the above equation, we obtain

$$\int dt = \frac{L}{V}\int di$$

or $$t = \frac{L}{V} i$$

The minimum pulse width of the trigger pulse is $t|_{min} = \frac{L}{V} i = \frac{0.2}{150} \times 100 \times 10^{-3} = 133.33\ \mu s$

Example 4.14 Figure 4.21 shows the $i - v$ the characteristics of a thyristor. Determine the average power loss due to the rectangular current pulse of I_{av} = 80 A for 180° conduction angle of thyristor.

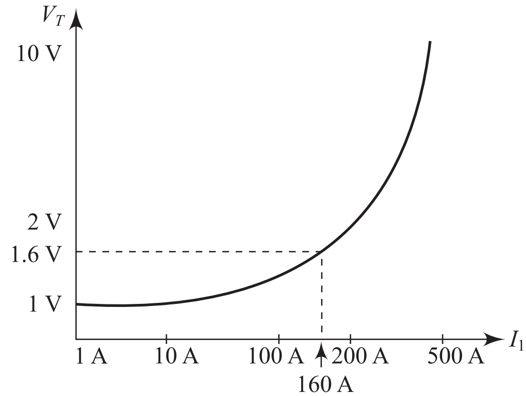


Fig. 4.21

Solution

Given: I_{av} = 80 A, β = 180°

Assume I_m is the current of thyristor during continuous conduction.

If the conduction angle of thyristor is β, the average current is equal to

$$I_{av} = \frac{\beta}{360°} I_m$$

For 180° conduction angle,

$$I_m = \frac{360°}{\beta} I_{av} = \frac{360}{180} \times 80 = 160\ \text{A}$$

According to $i - v$ the characteristics of thyristor as shown in Fig. 4.21, the voltage drop across thyristor is equal to V_T = 1.6 V

Average power loss is $P_{av} = V_T I_m \times \frac{\beta}{360°} = 1.6 \times 160 \times \frac{180°}{360°} = 128$ Watt

Example 4.15 When $i_g - V_g$ characteristics of a thyristor is a straight line passing through origin with a gradient of 2.5×10^3, find the value of gate voltage if P_g = 0.015 Watt

Solution

Given: $\frac{V_g}{i_g} = 2.5 \times 10^3, P_g = 0.015$ Watt

The value of gate voltage is $V_g = 2.5 \times 10^3 i_g$

$$P_g = V_g I_g = 0.015 \text{ Watt}$$

or $$2.5 \times 10^3 I_g I_g = 0.015 \text{ Watt}$$

or $$I_g = 2.449 \text{ mA}$$

The value of gate voltage is $V_g = 2.5 \times 10^3 \times 2.449 \times 10^{-3} = 6.122$ V.

Example 4.16 In a thyristor, the gate-cathode characteristics is a straight line passing through origin with a gradient of $\frac{V_g}{i_g} = 15$ V/A. The maximum turn-on time is 100 μs, the minimum gate current required is 100 mA. When the gate-to-source voltage is 10 V, find the value of gate-source resistance which is connected in series with gate drive circuit as shown in Fig. 4.22 determine the power dissipation. If the average gate power dissipation is 120 mW, determine the maximum triggering frequency of thyristor.

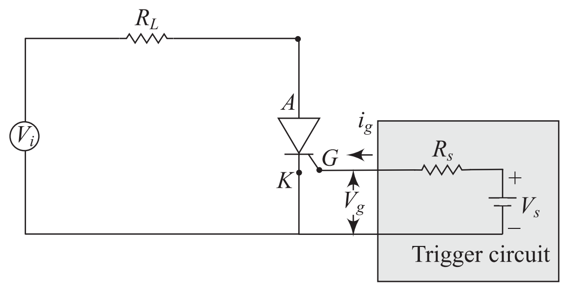


Fig. 4.22

Solution

Given: $\frac{V_g}{i_g} = 15\text{ V/A}$, i_{gmin} = 100 mA, V_s = 10 V

The value of gate voltage is $V_g = 15 i_g$ as $\frac{V_g}{i_g} = 15\text{ V/A}$

or $$V_g = 15 \times 100 \times 10^{-3} = 1.5\text{ V} \text{ as } i_{gmin} = 100\text{ mA}$$

(a) The value of gate-source resistance which is connected in series with gate drive circuit is equal to

$$R_s = \frac{V_s - V_g}{I_g} = \frac{10 - 1.5}{100 \times 10^{-3}} = 85\,\Omega$$

(b) Power dissipation $P_g = V_g I_g = 1.5 \times 100 \times 10^{-3}\text{ Watt} = 150\text{ mW}$

We know that $P_{gmax} = \frac{P_{gav}}{fT}$

The maximum triggering frequency is $f = \frac{P_{gav}}{T.P_{gmax}} = \frac{120}{100 \times 10^{-6} \times 150} = 8\text{ kHz}$

Example 4.17 In a forward biased thyristor, the gate current is gradually increased from zero until the device is turned ON. At the instant of turn ON, the gate current is about 1.2 mA and when the thyristor operates in conduction, gate current decays to about 0.4 mA. Justify the above statement. What will be the gate cathode voltage when V_s is zero?

Solution

When the thyristor is forward biased, anode terminal is positive with respect to cathode, then a small voltage E'_g is generated internally and this voltage appears across the gate cathode terminals as shown in Fig. 4.23(b). The amplitude of E'_g depends on the device geometrical structure of thyristor and applied anode voltage. The gate cathode equivalent circuit between gate cathode of thyristor is depicted in Fig. 4.23(b) where R is the static non-linear gate resistance.

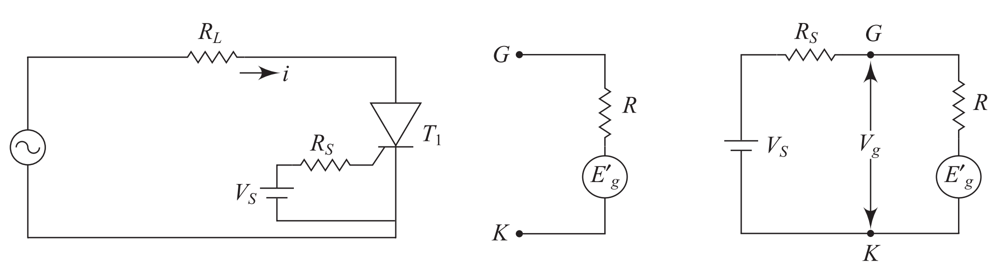


Fig. 4.23 *(a) SCR circuit (b) Equivalent circuit between gate and cathode when thyristor is forward biased (c) Equivalent circuit for the triggering circuit*

When the thyristor is turned ON by applying a positive gate signal, the equivalent circuit for the triggering circuit is depicted in Fig. 4.23(c) where E_g is the internally generated gate voltage due to the flows of anode current. The amplitude of E'_g is less than E_g. For example, the typical value of E'_g = 0.05 V, and E_g = 0.7 V. Just before the thyristor starts to conduct, gate current is equal to

$$I'_g = \frac{V_s - E'_g}{R + R_s}$$

Since E'_g is very small, $I'_g = \frac{V_s}{R + R_s}$ as $V_s >> E'_g$

Just after turned on thyristor, the gate current is equal to

$$I_g = \frac{V_s - E_g}{R + R_s}$$

Since E_g is much greater than E'_g, the gate current is reduced from I'_g to I_g. Hence, the statement "at the instant of turn ON, the gate current is about 1.2 mA and when the thyristor operates in conduction, gate current decays to about 0.4 mA" is justified.

When V_s becomes zero, the gate current is equal to $I''_g = \frac{V_s - E_g}{R + R_s} = -\frac{E_g}{R + R_s}$ as V_s = 0. At this instant, the voltage appears across the gate-cathode is $E_g - I''_g R$.

Example 4.18 In Fig. 4.24, a trigger pulse of 50 μs is applied across gate cathode of thyristor. The latching current of thyristor is 40 mA. When R = 75 Ω and L = 2.5 H, check whether the thyristor will be turned on or turned OFF ? If thyristor is not in turned ON, when the device will be turned ON?

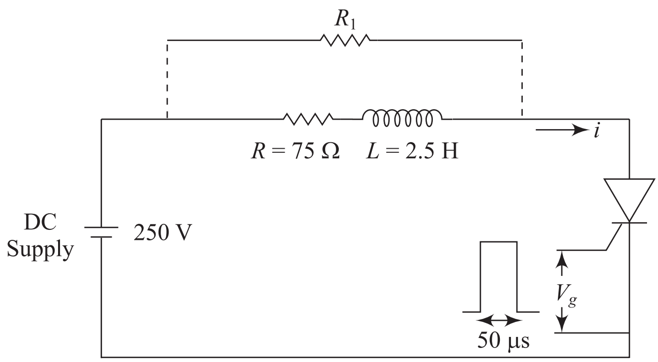


Fig. 4.24

Solution

Given: R = 75 Ω, L = 2.5 H, i_L = 40 mA

When thyristor is conducting, the current flow is

$$i = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{where} \quad \tau = \frac{L}{R} = \frac{2.5}{75} = 0.0333$$

or

$$i = \frac{250}{75}\left(1 - e^{-\frac{50\times10^{-6}}{0.0333}}\right) = 4.992\ \text{mA}$$

Since the pulse width is 50 μs, the anode current is equal to 4.992 mA which is very much less than the latching current, i.e., 40 mA. Hence, thyristor will not be turned ON.

A remedial resistance R_1 is connected in parallel with R-L show that latching current should flow though the device. The value of R_1 may be computed from the following equation:

$$i_L = 40\times10^{-3} = \frac{250}{R_1} + 4.992\times10^{-3}$$

or
$$R_1 = \frac{250}{40\times10^{-3} - 4.992\times10^{-3}} = 7.141\ \text{k}\Omega$$

4.9 RATINGS OF THYRISTOR

Each semiconductor device should have limited power handling capability in terms of voltage and current. Actually, the power handling capability of SCR depends on the temperature withstand capacity of *pn* junction at steady state and dynamic conditions. Thyristor can be used within the safe operating area (SOA) without damage and malfunction when it operates within voltage, current, power and temperature limit. Usually the rating of thyristors are specified in the manufacturer data sheet. Some very useful specification of thyristor voltage and current ratings are discussed in this section.

4.10 VOLTAGE RATINGS

4.10.1 ON-State Voltage Drop (V_T)

This is the voltage drop across anode and cathode with specified forward on state current and junction temperature. Its value is about 1 V to 1.5 V.

4.10.2 Peak Working Forward Blocking Voltage (V_{DWM})

It is the maximum forward blocking voltage at which thyristor can withstand during its working as shown in Fig. 4.25.

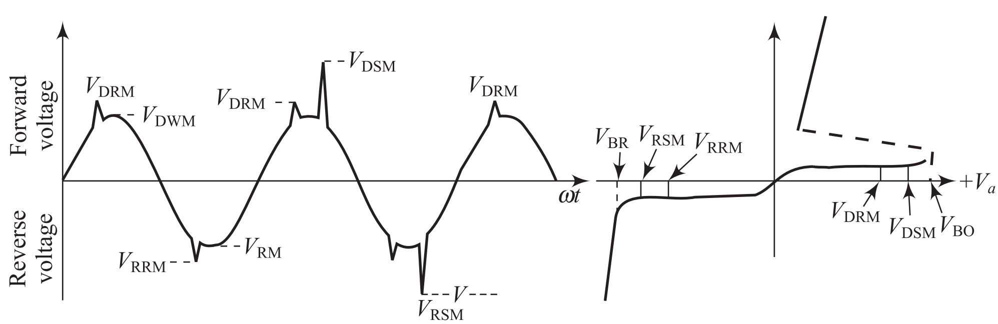


Fig. 4.25 *Voltage rating of thyristor*

4.10.3 Peak Repetitive Forward Blocking Voltage (V_{DRM})

This is the repetitive peak transient voltage at which thyristor can withstand in its forward blocking state. This voltage rating is specified at a maximum allowable junction temperature when gate circuit is open or a specified biasing resistance is present between gate and cathode. V_{DRM} is obtained whenever a thyristor is commutated or turned OFF. Due to abrupt change in reverse recovery current during turn-OFF process of SCR, a spike voltage of amplitude $L\frac{di}{dt}$ is generated and appeared across SCR terminals.

4.10.4 Non-Repetitive Forward Blocking Voltage (V_{DSM})

This is the peak value of surge voltage which is non-repetitive. Its value is about 130% of peak repetitive forward blocking voltage V_{DRM} but its value is less than forward break over voltage V_{B0}.

4.10.5 Peak Working Reverse Voltage (V_{RWM})

It is the maximum reverse voltage at which thyristor can withstand repeatedly. Its value is equal to the peak negative value of a sine voltage wave.

4.10.6 Repetitive Peak Reverse Voltage (V_{RRM})

It is the repetitive maximum reverse voltage at which thyristor can withstand at the allowable maximum junction temperature. This transient voltage lasts for a fraction of the time of a cycle.

4.10.7 Non-repetitive Peak Reverse Voltage (V_{RSM})

This is the maximum allowable instantaneous reverse voltage including all non-repetitive transients. Its value is about 130% of peak repetitive reverse voltage V_{RRM}. But its value is less than reverse break over voltage V_{BR}.

4.10.8 Voltage Safety Factor (V_{SF})

The voltage safety factor is the ratio of the peak repetitive reverse voltage (V_{RRM}) to the maximum value of input voltage and it is represented by

$$V_{SF} = \frac{\text{Peak repetitive reverse voltage } (\text{V}_{\text{RRM}})}{\sqrt{2}\ \text{RMS value of operating voltage}}$$

Usually, voltage safety factor is about 2 to 3.

4.10.9 Finger Voltage

This is the minimum forward bias voltage between anode and cathode for turn ON the SCR using gate triggering. The amplitude of finger is always greater than the normal on state voltage drop across SCR.

4.10.10 Forward *dv*/*dt* Rating

This is the maximum rate of rise of the anode voltage at which thyristor will not be triggered when there is no gate signal and anode to cathode voltage is less than forward breakover voltage.

During the forward blocking mode, the applied voltage appears across the junction J_2 as junctions J_1 and J_3 are forward biased and junction J_2 is reverse biased. The reverse bias junction J_2 behaves as capacitor. When a forward voltage is applied suddenly and dv/dt is very high, a charging current

$i = C_j \dfrac{dv}{dt}$ starts to flow and the thyristor will be turned ON. This type of unwanted triggering of SCR must be avoided. Therefore, if the dv/dt is less than forward dv/dt rating, thyristor must be remain in forward blocking state.

4.11 CURRENT RATINGS

4.11.1 Latching Current (I_L)

The latching current is the minimum value of anode current to trigger or turn ON the thyristor from its OFF state to ON state even after the trigger pulse is removed. To trigger an SCR, the anode current must be build up to the latching current before the gate pulse is removed.

4.11.2 Holding Current (I_H)

The holding current is the minimum value of anode current to hold the thyristor in ON state. During turn OFF, the anode current should be below the holding current. Usually the value of holding current is in milli-amperes.

4.11.3 Gate Current (I_g)

The gate current is required at the gate of the thyristor to turn ON. There are two types of gate current: minimum gate current i_{gmin} and maximum gate current $i_{\text{gmax}} \cdot i_{\text{gmin}}$ is the minimum value of gate current which is required at the gate to trigger the thyristor from its OFF state to ON state and its value depends on the rate of rise of current. i_{gmax} is the maximum value of gate current that can be applied to the device for turn-on without damaging the gate. The turn ON time of thyristor reduces with the increase of gate current.

4.11.4 Average ON-State Current (I_{Tav})

The forward ON-state voltage drop across SCR is low and its value is about 1 V to 1.5 V. Therefore, the average power loss in SCR depends on the forward ON-state average current I_{Tav}. The average power loss is equal to

$$P_{\text{av}} = \text{ON-state voltage drop across thyristor} \times I_{\text{Tav}}$$

When a continuous dc current I_{dc} flows through SCR, the junction temperature increases to $T_j = 125°\text{C}$. If SCR has low thermal time constant, the junction temperature increases to $T_j = 125°\text{C}$ within very short time. When the anode current is a rectangular waveform but the average ON-state current is equal to I_{dc}, the amplitude of rectangular waveform is $2I_{\text{dc}}$. Due to very low thermal time constant of SCR, temperature rise will be more than the allowable junction temperature $T_j = 125°\text{C}$ as shown in Fig. 4.26. In order to maintain the junction temperature at $T_j = 125°\text{C}$, the amplitude of rectangular current waveform may be reduced and better cooling arrangement must be provided with thyristors. Consequently, average ON-state current rating of thyristor will be reduced and thyristor is derated. Usually, manufacturers provide the "forward average current derating characteristics" which shows I_{Tav} as a function of the case temperature T_c with the current conduction angle.

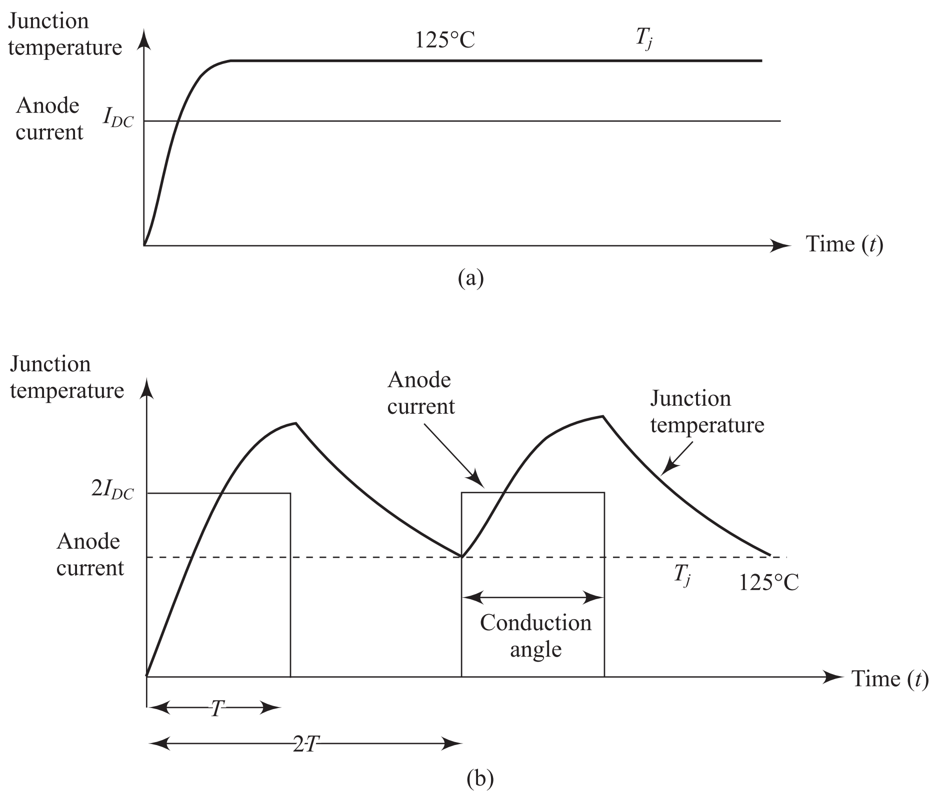


Fig. 4.26 *(a) Constant anode current and junction temperature (b) Rectangular anode current and junction temperature*

Average current rating is affected by different conduction angles. For the same average ON-state current I_{Tav} at different conduction angles, the instantaneous current increases with decrease in conduction angle. Subsequently, the voltage drop across thyristor increases and junction temperature is also increases. Due to over heating, thyristor may be completely damaged. Therefore the maximum allowable average current will be reduced with the decrease in conduction angle.

When current waveform is sine wave, the average on state current of thyristor is equal to

$$I_{Tav} = \frac{I_{\text{rms}}}{FF}$$

where, FF is form factor and I_{rms} is maximum rms ON-state current.

If current waveform is rectangular wave, the average ON-state current of thyristor is equal to

$$I_{Tav} = \frac{I_{\text{dc}}}{FF}$$

where, FF is form factor and I_{dc} is the ON-state current.

4.11.5 RMS Current (I_{RMS})

For dc current, the rms value of current I_{RMS} is equal to the average current I_{av} or dc current I_{dc}. When the SCR is subjected to high peak current and low duty cycle, rms current rating of SCR is most important. The heating of the resistive elements of a SCR such as metallic joints, leads and interfaces depends on the RMS current I_{RMS}. The RMS current rating is specified by manufacturers at maximum junction temperature. This current has an upper limit for dc as well as pulse current waveforms. This limit should not be exceeded on a continuous basis.

4.11.6 Surge Current (I_{TSM})

When the thyristor operates under repetitive rated voltage and current, the junction temperature is never exceeded. If the thyristor operates under abnormal conditions due to some faults or short circuits, the junction temperature may exceed the limit value and the device may be damaged. In order to overcome these unusual working conditions, surge current rating of thyristor must be specified by manufacturers.

Actually, the surge of thyristor specifies the maximum allowable non-repetitive current at which the device can withstand. Surge currents are assumed to be sine waves of power frequency with a minimum duration of ½ cycles. Usually, manufacturers provide at least three different surge current ratings for different durations.

For example, I_{TSM} = 2000 A for ½ cycles

I_{TSM} = 1500 A for 3 cycles

I_{TSM} = 1000 A for 5 cycles

The sub-cycle surge current rating I_{SB} can be determined by equating the energies involved in one cycle surge and one sub-cycle surge as given below:

$$I_{SB}^2 \cdot t = I_{TSM}^2 \cdot T$$

or $$I_{SB} = I_{TSM}\sqrt{\frac{T}{t}} \quad \text{or, } I_{SB} = \frac{I_{TSM}}{10} \cdot \frac{1}{\sqrt{t}} \quad \text{For 50 Hz supply, } T = 10 \text{ ms}$$

where, T is the time for one-half cycle of supply frequency in sec.

t is the duration of subcycle surge in sec.

I_{TSM} is one cycle surge current rating

I_{SB} is sub cycle surge current rating

4.11.7 I^2t Rating

I^2t rating of thyristor is used to select fuse and other protective equipments. This rating is represented in terms of Ampere^2-sec. For example, I^2t rating for 4 A thyristor is about 10 Ampere^2-sec and for 40 A thyristor is about 100 Ampere^2-sec.

This rating specifies the energy that the thyristor can absorb for a short time before the fault is cleared. The I^2t rating is equal to

$$\text{RMS value of one cycle surge current}^2 \times \text{Time for one cycle}$$

4.11.8 *di*/*dt* Rating

The *di*/*dt* rating of thyristor is the maximum rate of rise of current from anode to cathode without any damage to the device. During turn on process, the conduction starts at a place near gate. After that conduction spreads to the whole area of junction. When the rate of rise of anode current is very high compared to the spread velocity of charge carriers across the cathode junction, the local hot spots will be developed near the gate connection. Consequently, the junction temperature may exceed the limit value and thyristor may be damage permanently. I^2t rating of thyristor is also specified by manufacturer and the typical value of I^2t rating is about 20 to 500 A/μ sec.

Example 4.19 The maximum rms ON-state current of thyristor is 50 A. When the thyristor is used in a resistive load circuit and the current waveform is rectangular wave as shown in Fig. 4.27, determine average ON-state current rating for conduction angle (a) 120°, (b) 60°, (c) 30°.

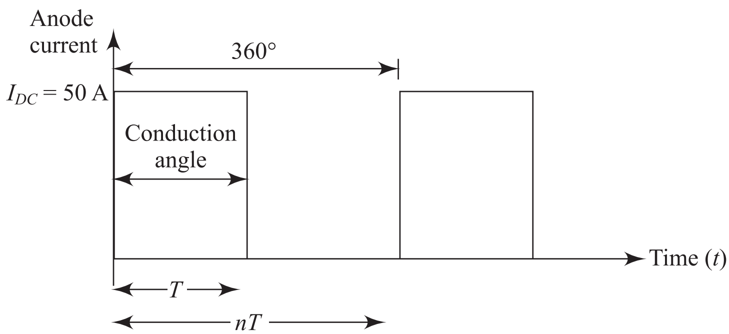


Fig. 4.27 *Rectangular current waveform*

Solution

Given: $I = 50$ A and conduction angle is 120°, 60° and 30°

The conduction angle of thyristor is $= \dfrac{T}{nT} \times 360°$

Therefore, $n = \dfrac{360°}{\text{conduction angle}}$

The average current is $I_{av} = \dfrac{I \times T}{nT} = \dfrac{I}{n}$ and the rms current is $I_{rms} = \left(\dfrac{I^2 T}{nT}\right)^{\frac{1}{2}} = \dfrac{I}{\sqrt{n}}$

(a) When conduction angle is 120°, $n = \dfrac{360°}{\text{conduction angle}} = \dfrac{360°}{120°} = 3$

$$I_{av} = \frac{I}{n} = \frac{50}{3} = 16.666 \text{ A} \text{ and } I_{rms} = \frac{I}{\sqrt{n}} = \frac{50}{\sqrt{3}} = 28.868 \text{ A}$$

Form factor $\text{FF} = \dfrac{I_{rms}}{I_{av}} = \dfrac{28.868}{16.666} = 1.732$

Average ON-state current rating of thyristor is $I_{Tav} = \dfrac{I_{dc}}{\text{FF}} = \dfrac{50}{1.732} = 28.868$ A

(b) When conduction angle is 60°, $n = \dfrac{360°}{\text{conduction angle}} = \dfrac{360°}{60°} = 6$

$$I_{av} = \frac{I}{n} = \frac{50}{6} = 8.333 \text{ A} \text{ and } I_{rms} = \frac{I}{\sqrt{n}} = \frac{50}{\sqrt{6}} = 20.412 \text{ A}$$

Form factor $\text{FF} = \dfrac{I_{rms}}{I_{av}} = \dfrac{20.412}{8.333} = 2.449$

Average on-state current rating of thyristor is $I_{Tav} = \dfrac{I_{dc}}{\text{FF}} = \dfrac{50}{2.449} = 20.416$ A

(c) When conduction angle is 30°, $n = \dfrac{360°}{\text{conduction angle}} = \dfrac{360°}{30°} = 12$

$$I_{av} = \frac{I}{n} = \frac{50}{12} = 4.1666 \text{ A} \text{ and } I_{rms} = \frac{I}{\sqrt{n}} = \frac{50}{\sqrt{12}} = 14.4337 \text{ A}$$

Form factor $FF = \frac{I_{rms}}{I_{av}} = \frac{14.4337}{4.1666} = 3.464$

Average ON-state current rating of thyristor is $I_{Tav} = \frac{I_{dc}}{FF} = \frac{50}{3.464} = 14.434$ A

Example 4.20 The maximum rms ON-state current of thyristor is 40 A. When the thyristor is used in a resistive load circuit and the current waveform is half-sine wave as shown in Fig. 4.28, determine average ON-state current rating for conduction angle (a) 120° and (b) 60°.

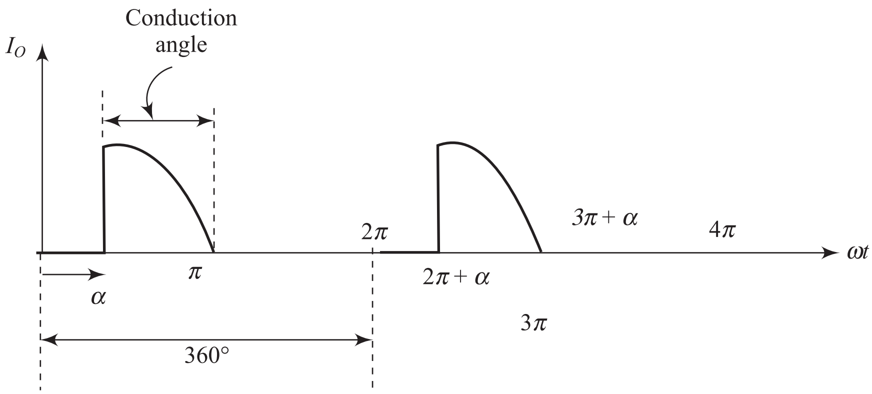


Fig. 4.28 *Current waveform*

Solution

Given: I = 40 A and conduction angle is 120° and 60°

If the thyristor conducts from α to 180°,

the average current is $I_{av} = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t \cdot d(\omega t) = \frac{I_m}{2\pi}(1+\cos\alpha)$ and

the rms current is $$I_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t \cdot d(\omega t)\right]^{\frac{1}{2}} = \left[\frac{I_m^2}{2\pi}\left(\frac{\pi-\alpha}{2} + \frac{1}{4}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

(a) For conduction angle = 120°, α = 180° – 120° = 60° = 1.0476 rad

$$I_{av} = \frac{I_m}{2\pi}(1+\cos\alpha) = \frac{I_m}{2\pi}(1+\cos 60) = 0.2387 I_m$$

$$I_{rms} = \left[\frac{I_m^2}{2\pi}\left(\frac{\pi-\alpha}{2} + \frac{1}{4}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{\pi-1.0476}{2} + \frac{1}{4}\sin(2\times 60)\right)\right]^{\frac{1}{2}} = 0.4484 I_m$$

From factor is $FF = \frac{I_{rms}}{I_{av}} = \frac{0.4484 I_m}{0.2387 I_m} = 1.8785$

Average ON-state current rating of thyristor is $I_{Tav} = \frac{I}{FF} = \frac{40}{FF} = \frac{40}{1.8785} = 21.29$ A

(b) For conduction angle = 60°, α = 180° – 60° = 120° = 2.0952 rad.

$$I_{av} = \frac{I_m}{2\pi}(1+\cos\alpha) = \frac{I_m}{2\pi}(1+\cos 120) = 0.07956 I_m$$

$$I_{rms} = \left[\frac{I_m^2}{2\pi}\left(\frac{\pi-\alpha}{2}+\frac{1}{4}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{2\pi}\left(\frac{\pi-2.0952}{2}+\frac{1}{4}\sin(2\times 120)\right)\right]^{\frac{1}{2}} = 0.2209 I_m$$

From factor is $\text{FF} = \frac{I_{rms}}{I_{av}} = \frac{0.2209 I_m}{0.07956 I_m} = 2.7765$

Average ON-state current rating of thyristor is $I_{Tav} = \frac{I}{FF} = \frac{40}{\text{FF}} = \frac{40}{2.7765} = 14.406 \text{ A}$

Example 4.21 The half cycle surge current rating of thyristor is 2500 A at 50 H supply. Find the one–cycle surge current rating and I^2t rating of thyristor.

Solution

Given: I_{SB} = 2500 A and T = 10 ms for 50 H supply

We know that $I_{SB}^2 t = I_{TSM}^2 \cdot T$

or $I_{TSM}^2 \cdot \frac{1}{100} = I_{SB}^2 \frac{1}{200}$

The one-cycle surge current rating of thyristor is equal to

$$I_{TSM} = \frac{I_{SB}}{\sqrt{2}} = \frac{2500}{\sqrt{2}} = 1767.766 \text{ A}$$

I^2t rating of thyristor is

$$= I_{TSM}^2 \times \frac{1}{2f}$$

$$= 1767.766^2 \times \frac{1}{100} = 31249 \text{ Amp}^2\text{sec}$$

Example 4.22 Figure 4.29 shows a circuit diagram of a thyristor. I^2t rating of thyristor is 50 A^2/s. If the terminal A and B is short circuited, determine fault clearance time so that thyristor is not damaged completely.

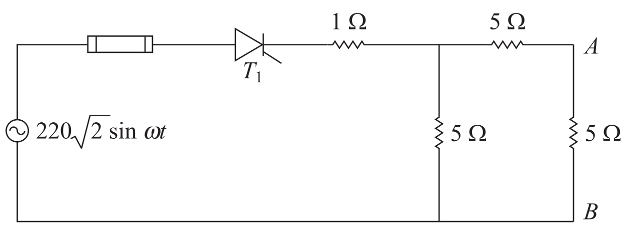


Fig. 4.29

Solution

When the terminals *A* and *B* are short circuited, the equivalent resistance is equal to

$$R_{eq} = 1 + 5 \parallel 5 = 1 + \frac{5 \times 5}{5+5} = 3.5\,\Omega$$

The maximum fault current is

$$\frac{220\sqrt{2}}{3.5}\text{ A} = 88.89\text{ A}$$ which remains constant for a short time t_c, i.e., fault clearance time.

The fault clearance time t_c can be computed from the following expression:

$$\int_0^{t_c} i^2 dt = \int_0^{t_c} 88.89^2\, dt = 50\text{ A}^2/\text{s}$$

or $$(88.89)^2\, t_c = 50\text{ A}^2/\text{s}$$

Therefore, $$t_c = \frac{50}{88.89^2} \times 1000 = 6.32\text{ ms}$$

Example 4.23 The subcycle surge current rating of thyristor is 3000 A for 50 Hz supply. Determine the one cycle surge current rating of thyristor and I^2t rating.

Solution

The subcycle surge current rating I_{SB} can be determined by equating the energies involved in one cycle surge and one subcycle surge as given below.

$$I_{SB}^2 \cdot t = I_{TSM}^2 \cdot T \qquad \text{for 50 Hz supply, } T = 10\text{ ms}$$

where, T is the time for one-half cycle of supply frequency in sec.
t is the duration of subcycle surge in sec.
I_{TSM} is one cycle surge current rating
I_{SB} is sub cycle surge current rating

or $$I_{TSM}^2 \frac{1}{100} = I_{SB}^2 \frac{1}{200}$$

The one-cycle surge current rating of thyristor is equal to

$$I_{TSM} = \frac{I_{SB}}{\sqrt{2}} = \frac{3000}{\sqrt{2}} = 2121.32\text{ A}$$

Therefore, one cycle surge current rating of thyristor is 2121.32 A.
I^2t rating of thyristor is

$$= I_{TSM}^2 \times \frac{1}{2f}$$

$$= 2121.32^2 \times \frac{1}{100} = 49999.98\text{ Amp}^2\text{sec}$$

4.12 PROTECTION OF THYRISTOR

For proper and reliable operation, thyristor must be operating within the specified voltage and current ratings. But in some applications, thyristors may be subjected to over voltage and over current. Due to over voltage and over current, the junction temperature may exceed the maximum allowable temperature 150°C and device may be damaged permanently. Therefore, different efficient cooling methods are used in thyristor based converter circuits to dissipate the excess heat into atmosphere. During turn-ON and turn-OFF process, there is a lot of power loss within the device and temperature increases. In high frequency applications, power loss is significantly high and device may be damaged due to temperature rise.

During turn ON of thyristor, $\frac{di}{dt}$ may be comparatively large and local heating is possible within the device. As a result, thyristor may be destroyed. The false triggering of thyristor is also possible due to presence of high $\frac{dv}{dt}$ and noise signal across gate to cathode terminals. Therefore, thyristor must be always protected from false triggering. For normal operation of thyristors in different converter circuits, the following protections should be taken care:

1. $\frac{di}{dt}$ protection
2. $\frac{dv}{dt}$ protection
3. Over-voltage protection
4. Over-current protection
5. Gate protection

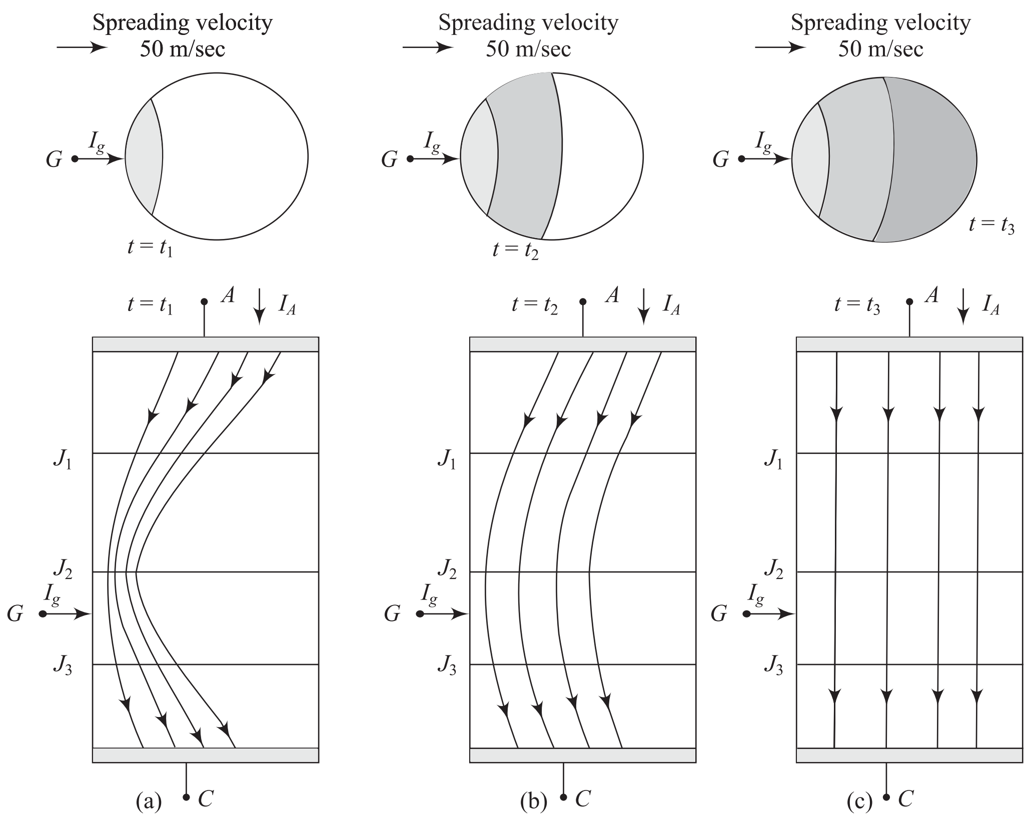


Fig. 4.30 *Anode current through junction J_2 at (a) $t = t_1$, (b) $t = t_2$ and (c) $t = t_3$ where $t_1 < t_2 < t_3$*

4.12.1 $\frac{di}{dt}$ Protection

If a thyristor is forward biased and a gate pulse is applied to gate cathode, anode current starts to flow in the region nearest to gate cathode junction. After that current spreads across the whole area of junction. When the rate of rise of anode current $\frac{di}{dt}$ is large compared to spreading velocity of carries 50 m/s, the local hot spots will be developed near the gate connection due to high current density and thyristor will be damaged permanently. Consequently, the rate of rise anode current must be kept within the specified value during turn on of thyristor. To maintain the $\frac{di}{dt}$ within a specified value, a $\frac{di}{dt}$ inductor may be connected in series with the anode circuit. The $\frac{di}{dt}$ rating varies from few tens of A/µs to 500 A/µs.

4.12.2 $\frac{dv}{dt}$ Protection

When rate of rise of $\frac{dv}{dt}$ across thyristor is high, charging current $i_c = C_j \frac{dV}{dt}$ flows through thyristor and the device will be turned ON without any gate signal. This is called $\frac{dv}{dt}$ triggering of thyristor. Actually, this type of triggering is called *false* or *abnormal triggering*. For proper and reliable operation of thyristor, the rate of rise of $\frac{dv}{dt}$ must be kept within the specified limit. The typical value of $\frac{dv}{dt}$ is about 20 V/µs to 500 V/µs. For $\frac{dv}{dt}$ protection, a snubber circuit which is a series combination of resistance R_s and a capacitance C_s, is connected across thyristor. The design of snubber circuit is explained in Section 4.13.

4.12.3 Over-Voltage Protection

Just like any other semiconductor device, thyristors are very sensitive to over-voltage and the over-voltage is the main cause of failure. The transient over-voltage across thyristor may turn ON the device without any gate signal and the converter perform malfunction. Generally, thyristor should be able to withstand external over-voltage and internal over-voltage.

External over-voltage When the current flow in an inductive circuit is interrupted or the lightning strokes on the lines, external over-voltage is applied across thyristor. If a converter circuit is supplied through transformer, transient over-voltage across thyristor occurs when the transformer is energised or de-energised. Due to transient over-voltage, thyristor will be turn ON abruptly and over-voltage appear across load. Consequently, a large fault current flows though the converter circuit and thyristors may also be damaged partially or completely.

Internal over-voltage Usually internal over voltage is generated during turn-OFF process of thyristor. After the anode current becomes zero, this current starts to flow in reverse direction due to stored charges and reaches the peak reverse recovery current. After that the reverse recovery current starts to fall abruptly with large $\frac{di}{dt}$. As a series inductance L is present in the converter circuit, a large transient $L\frac{di}{dt}$ voltage is generated and thyristor may be damaged due to this transient over-voltage.

Therefore, voltage clamping devices such as varistor may be used to protect thyristors from over-voltage. Actually varistor is a non-linear resistance when the current increases its resistance value decreases. Figure 4.31 shows the V-I characteristics of varistor and action of current limiting fuse is depicted in Fig. 4.32.

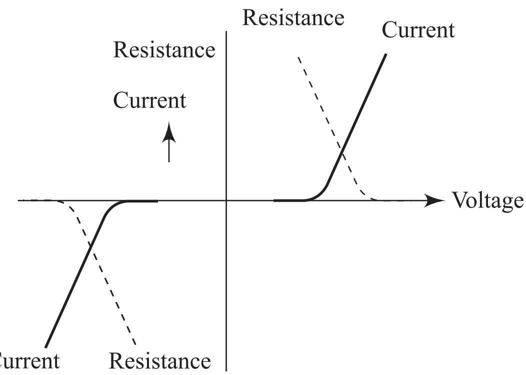


Fig. 4.31 *V-I characteristics of varistor*

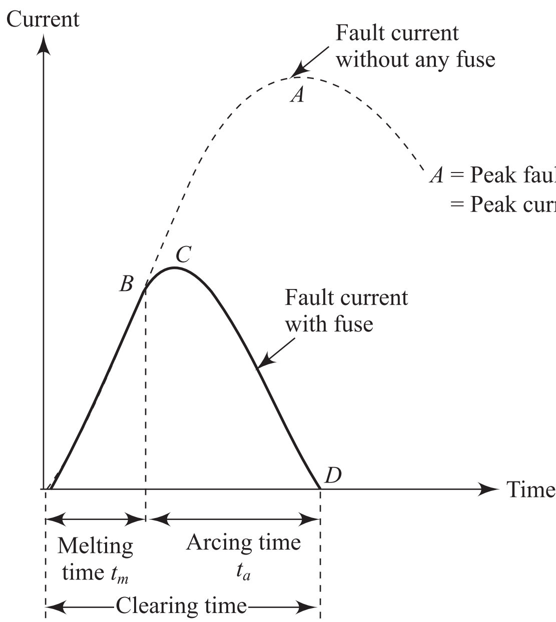


Fig. 4.32 *Function of current limiting switch*

4.12.4 Gate Protection

For proper operation of thyristor-based converter circuits, gate drive should be always protected from over-voltage as well as over-current. Due to sudden over-voltage across gate circuit, thyristor will be triggered. This type of triggering is false or abnormal triggering. A zener diode Z is connected across gate drive circuit to protect from over voltage as shown in Fig. 4.33. Due to over current in the gate circuit, junction temperature may exceed the specified limit and thyristor may be completely damaged. A resistance R_1 is connected in series with the gate drive circuit to protect device from over-current. Sometimes thyristors are also turned ON due to presence of noise signal. Therefore, a resistance R_2 and capacitor C are connected between gate and cathode to bypass the noise signals.

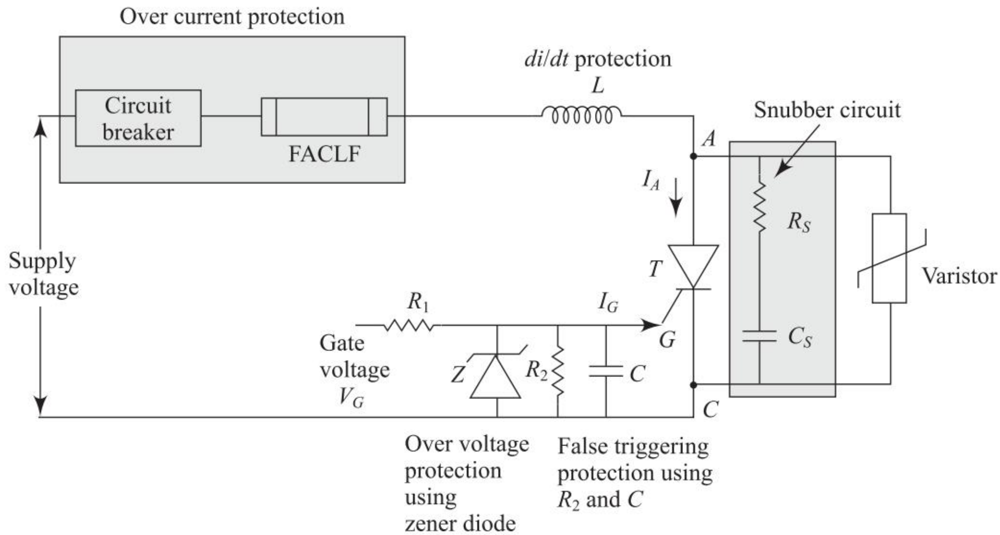


Fig. 4.33 *Thyristor protection*

4.12.5 Over-Current Protection

Figure 4.34 shows the electronic crowbar protection circuit. A crowbar thyristor is connected across dc supply. When over-current flows through converter circuit and current sensing resistance generates voltage which is greater than preset value, triggering circuit generates triggering pulse which is applied to the crowbar thyristor. Subsequently, crowbar thyristor will be turn ON. As thyristor becomes turn ON and dc supply will be short circuited, fuse will be blown due to high current and interrupt the fault current to protect thyristor. Sometimes fuse may be replaced by circuit breaker for thyristor protection.

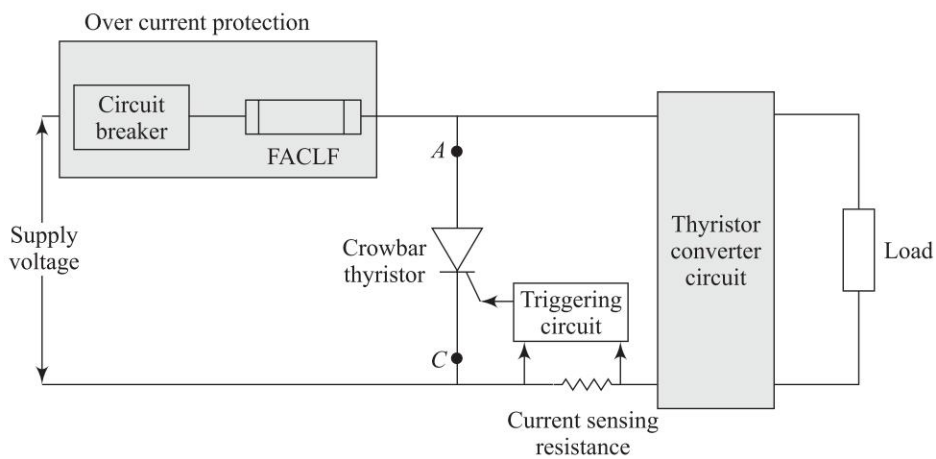


Fig. 4.34 *Electronic crowbar protection circuit*

4.13 DESIGN OF SNUBBER CIRCUIT

When a voltage is applied across anode to cathode of thyristor and anode is positive with respect to cathode, the junctions J_1 and J_3 are forward biased and the junction J_2 is reverse biased. The reverse biased junction J_2 behaves as capacitor. Then holes from p-region of junction J_1 are accumulated at junction J_2 and electrons from n-layer of junction J_3 are also accumulated at the other side of junction J_2. Therefore, space charge carriers are present across the junction J_2. When a large voltage is applied abruptly, charging current i_C flows through thyristor and the device will be turned on without gate signal. This is the unwanted triggering of thyristor.

The charging current $i_c = \dfrac{dq}{dt}$

As $q = C_j V$, we obtain $i_c = \dfrac{d(C_j V)}{dt} = V\dfrac{dC_j}{dt} + C_j\dfrac{dV}{dt}$

or $$i_c = C_j\frac{dV}{dt} \quad \text{as} \quad \frac{dC_j}{dt} = 0$$

Usually, the $\dfrac{dV}{dt}$ rating of thyristor is 20 to 500 V/μ sec. If the value of $\dfrac{dV}{dt}$ is more than its rated value, thyristor will be turned ON and this is the abnormal triggering of thyristor. Subsequently, thyristor-based converter circuit starts to malfunction. Therefore, a snubber circuit should be connected in parallel with thyristor to protect from $\dfrac{dV}{dt}$.

A snubber circuit consists of a series combination of a resistance R_s and a capacitance C_s and it is connected in parallel with thyristor as shown in Fig. 4.35. If a capacitor C_s is connected in parallel with thyristor, the capacitor C_s itself is sufficient to protect from unwanted abnormal triggering of thyristor.

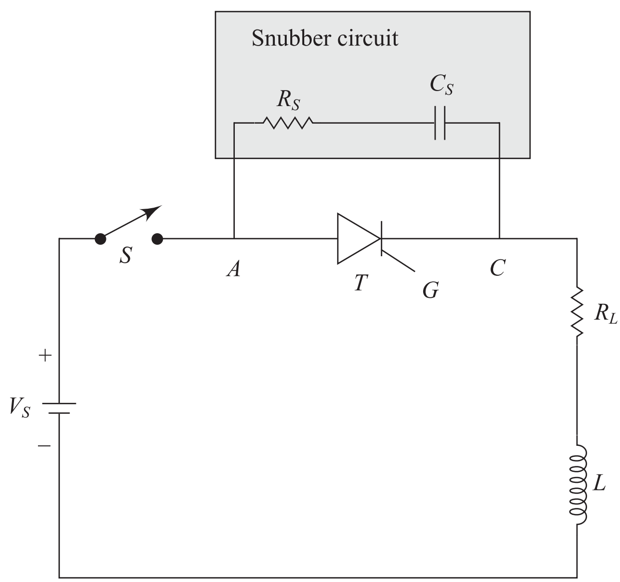


Fig. 4.35 *Snubber circuit across thyristor*

When the switch S is closed at $t = 0$, suddenly the supply voltage appears across the circuit and the capacitor behaves as short circuit. Therefore, the voltage across thyristor is zero. With the progress of time, the voltage across capacitor builds up slowly with a $\frac{dV}{dt}$, which is less than the specified maximum $\frac{dV}{dt}$. Consequently, C_s is sufficient to prevent the thyristor from $\frac{dV}{dt}$ triggering.

When gate pulse is not applied and $\frac{dV}{dt}$ is less than its maximum limit, thyristor will be in OFF state and capacitor is charged to supply voltage V_s. Whenever the thyristor is turned ON by gate pulse, the capacitor starts to discharge through SCR and the current flows through local path formed by capacitor C_s and thyristor is equal to

$$I_S = \frac{V_S}{R_T}$$

where, R_T is the resistance of thyristor during forward conduction state.

Since, the value of R_T is quite low, the $\frac{di}{dt}$ during turn ON will be excessively high and thyristor may be permanently damaged. Then current I_S will be a safe value when a series resistance R_S is connected with the capacitor C_S.

When the switch S is closed, capacitor is short circuit and SCR operates in forward blocking state, the equivalent circuit of Fig. 4.35 is given in Fig. 4.36.

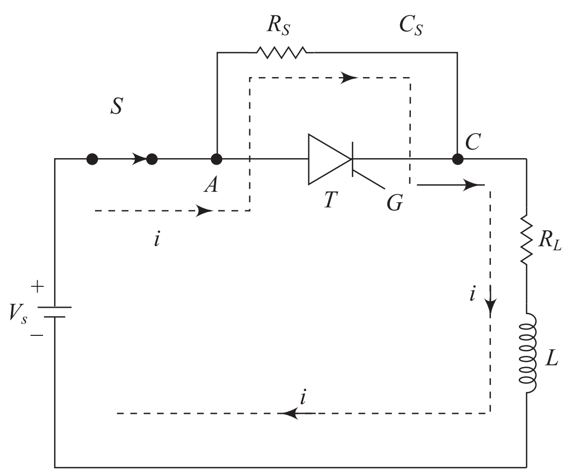


Fig. 4.36 *Equivalent circuit of Fig. 4.35*

Applying the KVL in the circuit, we obtain

$$V_s = i_s(R_s + R_L) + L\frac{di_s}{dt}$$

The solution of above equation is

$$i_s = \frac{V_s}{R_s + R_L}(1 - e^{-\frac{t}{\tau}})$$

or $$i_s = I(1-e^{-\frac{t}{\tau}}) \qquad \text{where, } I = \frac{V_s}{R_s+R_L} \text{ and } \tau = \frac{L}{R_s+R_L}$$

After differentiating the above equation, we get

$$\frac{di}{dt} = Ie^{-\frac{t}{\tau}}\frac{1}{\tau} = \frac{V_s}{R_s+R_L}\frac{R_s+R_L}{L}e^{-\frac{t}{\tau}}$$

$$= \frac{V_s}{L}e^{-\frac{t}{\tau}}$$

At $t = 0$, $\frac{di}{dt}$ is maximum

Then, $$\left.\frac{di}{dt}\right|_{\max} = \frac{V_s}{L}$$

or $$L = \frac{V_s}{\left.\frac{di}{dt}\right|_{\max}}$$

The voltage across SCR is $v = iR_s$
After differentiating, we obtain

$$\frac{dv}{dt} = R_s\frac{di}{dt}$$

Then $$\left.\frac{dv}{dt}\right|_{\max} = R_s\left.\frac{di}{dt}\right|_{\max}$$

or $$\left.\frac{dv}{dt}\right|_{\max} = \frac{R_sV_s}{L}$$

Therefore, $$R_s = \frac{L}{V_s}\left.\frac{dv}{dt}\right|_{\max}$$

The R-L-C circuit should be fully analysed to find the optimum values of snubber circuit parameters R_s and C_s. After analysed, the values of R_s and C_s are

$$R_s = 2\xi\sqrt{\frac{L}{C_s}} \text{ and } C_s = \left(\frac{2\xi}{R_s}\right)^2 L$$

where, ξ is the damping factor and it varies in the range of 0.5 to 1.

Example 4.24 When a SCR is operating with a peak supply voltage of $220\sqrt{2}$ V and it has the following parameters:

$$\text{Repetitive peak current } I_p = 100 \text{ A}, \left.\frac{dv}{dt}\right|_{\max} = 400 \text{ V/μs}, \left.\frac{di}{dt}\right|_{\max} = 80 \text{ A/μs}.$$

Design a snubber circuit for SCR protection. Assume the factor of safety is 2 and the minimum value of resistance is 20 Ω.

Solution

As factor of safety is 2, the permissible value of $I_p = \frac{100}{2}\text{ A} = 50\text{ A}$, $\left.\frac{dv}{dt}\right|_{max} = \frac{400}{2}\text{ V/μs} = 200\text{ V/μs}$, $\left.\frac{di}{dt}\right|_{max} = \frac{80}{2}\text{ A/μs} = 40\text{ A/μs}$

To limit the value of $\left.\frac{di}{dt}\right|_{max}$ within 40 A/μs, a inductance L must be connected in series with thyristor and its value is $L = \frac{V_s}{\left.\frac{di}{dt}\right|_{max}} = \frac{220\sqrt{2}\times 10^{-6}}{40} = 7.778\ \mu\text{H}$

The value of $R_s = \frac{L}{V_s}\left.\frac{dv}{dt}\right|_{max} = \frac{7.778\times 10^{-6}}{220\sqrt{2}} \times \frac{200}{10^{-6}} = 5.5\ \Omega$

When the SCR is in the blocking state, the capacitor will be charged to maximum voltage $220\sqrt{2}$. When the SCR is turned on, the peak current flows through the SCR is

$$\frac{220\sqrt{2}}{20} + \frac{220\sqrt{2}}{5.5} = 72.124\text{ A}$$

The maximum permissible peak current is 50 A. Therefore, the value of R_S should be such that, the peak current is less than 50 A.

Assume R_s = 10 Ω, then the peak current flows through the SCR is

$$\frac{220\sqrt{2}}{20} + \frac{220\sqrt{2}}{10} = 46.668\text{ A}$$

If the damping factor $\xi = 0.65$, the value of capacitance is

$$C_s = \left(\frac{2\xi}{R_s}\right)^2 L = \left(\frac{2\times 0.65}{10}\right)^2 \times 7.778\times 10^{-6} = 0.1314\ \mu\text{F}$$

The parameter of snubber circuit is R_s = 10 Ω and C_s = 0.1314 μF

Example 4.25 Determine the $\frac{di}{dt}$ rating and $\frac{dv}{dt}$ rating of thyristor which is connected in the circuit as shown in Fig. 4.37. Find the average and rms current rating of thyristor at the firing angle $\alpha = 90°$. What is the voltage rating of thyristor?

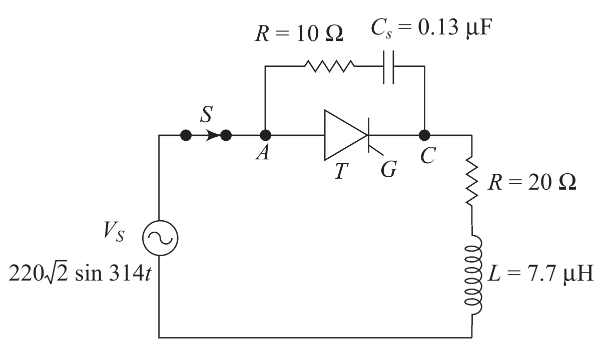


Fig. 4.37

Solution

Given: $V_s = 220\sqrt{2}\sin 314t$, C_s = 0.13 μF, R = 20 Ω and L = 7.7 μH

The $\left.\dfrac{di}{dt}\right|_{\max} = \dfrac{V_s}{L} = \dfrac{220\sqrt{2}}{7.7\times10^{-6}} = 40.406\text{ A/μs}$

The $\left.\dfrac{dv}{dt}\right|_{\max} = R_s \left.\dfrac{di}{dt}\right|_{\max} = 10\times 40.406 = 404.6\text{ A/μs}$

Load reactance is $X_L = \omega L = 314\times 7.7\times10^{-6}\,\Omega = 0.0024\,\Omega$. As the magnitude of X_L is less than R = 20 Ω, the current through thyristor is limited by load resistance.
The maximum load current is equal to

$$\frac{V_s}{R} = \frac{220\sqrt{2}}{20} = 15.556\text{ A}$$

At firing angle $\alpha = 90°, I_{Tav} = \dfrac{\sqrt{2}}{\pi} I_m = \dfrac{\sqrt{2}}{\pi}\times 15.556A = 6.999\text{ A}$

The rms current rating of thyristor is 15.556 A
The voltage rating of thyristor is equal to

$$(2.5 - \text{to} - 3)\times \text{Peak voltage} = 777.817\text{ V to }933.38\text{ V}$$

Therefore, the voltage rating of thyristor is about 900 V.

Example 4.26 When a thyristor is operating with a peak supply voltage of $440\sqrt{2}$ V and it has the following specifications:

$$\text{Repetitive peak current } I_p = 400\text{ A}, \left.\frac{di}{dt}\right|_{\max} = 100\text{ A/μs}, \left.\frac{dv}{dt}\right|_{\max} = 200\text{ V/μs}$$

Assume that the factor of safety is 2.5 for I_p, determine the permissible value of $\left.\dfrac{di}{dt}\right|_{\max}$ and $\left.\dfrac{dv}{dt}\right|_{\max}$. Design a suitable snubber circuit for SCR protection . Assume the factor of safety is 2 and the minimum value of resistance is 10 Ω.

Solution

As the factor of safety is 2.5, the permissible value of $I_p = \dfrac{400}{2.5}\text{ A} = 160\text{ A}$,

The permissible value of $\left.\dfrac{di}{dt}\right|_{\max} = \dfrac{100}{2}\text{ A/μs} = 50\text{ A/μs}$

The permissible value of $\left.\dfrac{dv}{dt}\right|_{\max} = \dfrac{200}{2}\text{ V/μs} = 100\text{ V/μs}$

To limit the value of $\left.\dfrac{di}{dt}\right|_{\max}$ within 50 A/μs, a inductance L must be connected in series with thyristor and its value is $L = \dfrac{V_s}{\left.\dfrac{di}{dt}\right|_{\max}} = \dfrac{440\sqrt{2}\times10^{-6}}{50} = 12.445\text{ μH}$

The value of $R_s = \dfrac{L}{V_s}\left.\dfrac{dv}{dt}\right|_{\max} = \dfrac{12.445\times10^{-6}}{440\sqrt{2}}\times\dfrac{100}{10^{-6}} = 2.0\,\Omega$

When the SCR is in the blocking state, the capacitor will be charged to maximum voltage $440\sqrt{2}$. When the thyristor is turned on, the peak current flows through the thyristor is

$$\frac{440\sqrt{2}}{10}+\frac{440\sqrt{2}}{2.0}=377.35\text{ A}$$

The maximum permissible peak current is 160 A. Therefore, the value of R_S should be such that, the peak current is less than 160 A.

Assume R_s = 70 Ω, then the peak current flows through the SCR is

$$\frac{440\sqrt{2}}{10}+\frac{440\sqrt{2}}{7}=155.11\text{ A}$$

Assume the damping factor ξ = 0.65, then the value of capacitance is

$$C_s=\left(\frac{2\xi}{R_s}\right)^2 L=\left(\frac{2\times 0.65}{7}\right)^2\times 12.445\times 10^{-6}=0.4292\ \mu\text{F}$$

The parameter of snubber circuit is R_s = 7 Ω and C_s = 0.4292 μF.

4.14 SERIES AND PARALLEL CONNECTION OF THYRISTORS

The power handling capability (voltage and current ratings) of thyristors are limited and depends on cooling efficiency, utilisation and encapsulation. Nowadays, voltage rating of 10 kV and current rating of 3.5 kA thyristors are available in market. For high power applications such as HVDC transmission system, a single thyristor cannot able to meet the requirements. Consequently, thyristors are connected in series to increase the voltage handling capability and these devices are connected in parallel to increase the current handling capability. Therefore, the series and parallel connection of thyristors are used to increase voltage as well as current rating.

During series and parallel connections of thyristors, each thyristor must be utilised properly. The term '*string efficiency*' is used to measure the degree of utilisation of thyristors in a string. The *string efficiency* can be expressed as

$$\text{String efficiency}=\frac{\text{Actual voltage or current rating of the string}}{\text{Voltage or current rating of one thyristor}\times\text{Number of thyristor in the string}}$$

Actually, the value of string efficiency is always less than 1. To get maximum string efficiency, thyristors having identical *I-V* characteristics may be connected in series and parallel string. Since same ratings thyristors manufactured by different manufacturers do not have identical I-V characteristics, the voltage and current are shared unequally by series and parallel connected thyristors. As a result, string efficiency can never be equal to 1. But the unequal voltage/current sharing by thyristors can be minimised by using external static equalising circuit.

The reliability of the series and parallel string is measured by the factor called de-rating factor (DRF). The de-rating factor can be expressed as

$$\text{DRF}=1-\text{string efficiency}$$

When a large number of thyristors are used in a string, string efficacy will be reduced and the de-rating factor will be more. Consequently, the reliability of string increases.

For example, if the string voltage is 4000 V and seven 600 V rating thyristors are used in the string, the string efficiency is equal to

$$\text{String efficiency}=\frac{4000}{600\times 7}=0.9523\ \text{ or }\ 95.23\%$$

Then the de-rating factor is

$$\text{DRF} = 1 - \text{string efficiency} = 1 - 0.9523 = 0.0476 \text{ or } 4.76\%$$

When one extra thyristor is connected in the string, the string efficiency is equal to

$$\text{String efficiency} = \frac{4000}{600 \times 8} = 0.8333 \text{ or } 83.33\%$$

and subsequently the de-rating factor is

$$\text{DRF} = 1 - \text{string efficiency} = 1 - 0.8333 = 0.1666 \text{ or } 16.66\%$$

It is clear from above example that if the numbers of thyristors are increased in a string, string efficacy will be reduced, the de-rating factor will be increased and the reliability of string will be high.

4.15 SERIES CONNECTION OF THYRISTORS

During series connection of thyristors in a string, same voltage rating and current rating thyristors must be used and devices must be manufactured by the same company. Even though device ratings are same, the *V-I* characteristics of thyristors are different. Therefore, there is variation in forward breakover voltage, reverse blocking conditions, junction capacitance, ON-state voltage drop, and reverse recovery characteristics of devices. Then the following problems may arise in series connected thyristors:

1. Unequal voltage distribution across thyristors
2. Difference in reverse recovery characteristics

The series connection two thyristors T_1 and T_2 is depicted in Fig. 4.38(a) and the *V-I* characteristics of two thyristors with different forward voltage handling capability is shown in Fig. 4.38(b). When the forward blocking current, I_B i.e., OFF-state leakage current flows through thyristors, thyristor T_1 supports V_1 voltage and thyristor T_2 supports V_2 voltage. Then total blocking voltage is $V_1 + V_2$ which is less than $2V_1$. The switching responses of devices are different due to different characteristics and junction temperature. If there is a large voltage drop across a device, the particular devices will response slowly and turn-ON time is maximum. Therefore, special triggering or gate drive circuits are required to turn-ON all the devices in a string.

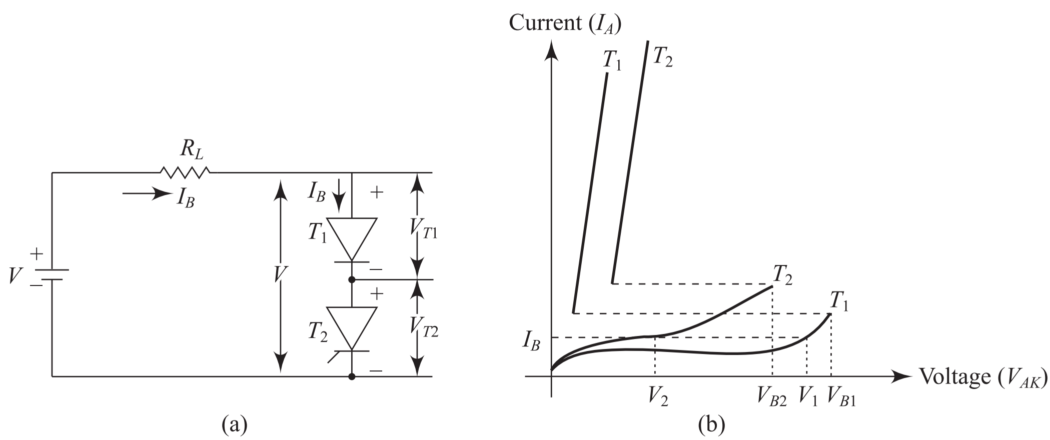


Fig. 4.38 *(a) Series connection of thyristors T_1 and T_2 (b) V-I characteristics of thyristors T_1 and T_2*

The string efficiency of two series connected thyristors is equal to

$$\text{String efficiency} = \frac{V_1 + V_2}{2V_1} = \frac{1}{2}\left(1 + \frac{V_2}{V_1}\right)$$

When the same leakage current I_0 flows through thyristor T_1 and T_2, the leakage resistance of T_1 is $\frac{V_1}{I_0}$ and the leakage resistance of T_2 is $\frac{V_2}{I_0}$. For equal voltage sharing at steady state, a suitable resistance R is connected across each thyristor as depicted in Fig. 4.39. This circuit is known as the *static equalising circuit*.

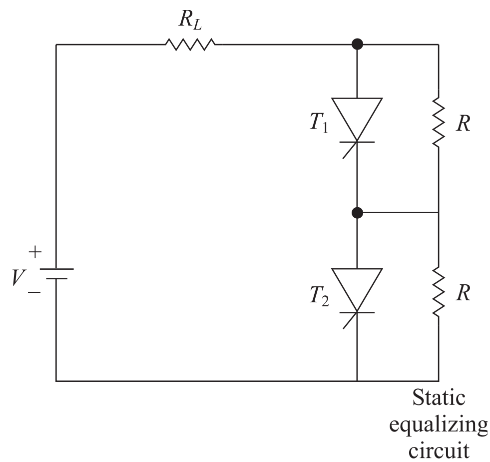


Fig. 4.39 *Static equalising circuit of thyristors T_1 and T_2*

4.15.1 Static Equalizing Circuit

For uniform voltage distribution at steady state condition, resistance R should be connected across each thyristor. Figure 4.40 shows n number of thyristors connected in series. The first thyristor shares the maximum blocking voltage V_{B1} and other *n-1* devices shares the remaining voltage equally

$$V_{B2} = V_{B3} = V_{B4} = V_{B5} = V_{B6} \ldots\ldots\ldots = V_{Bn}$$

The minimum leakage current $I_{B\,(\min)}$ flows through thyristor T_1 and the leakage current $I_{B\,(\max)}$ flows through each of remaining *n-1* thyristors.

Assume $V_{B1} = V_{B(\max)}$ and I_{B2}, I_{B3}, I_{B4}, … I_{Bn} are the current flows through T_2, T_3, T_4, … T_n respectively and I_{R2}, I_{R3}, I_{R4} … I_{Rn} are the current flows through R_2, R_3, R_4 … R_n respectively and

$$R_1 = R_2 = R_3 = R_4 = \ldots\ldots\ldots\ldots R_n$$

$$V_{B2} = V_{B3} = V_{B4} = V_{B5} = V_{B6} \ldots\ldots\ldots = V_{Bn},$$

$$I_{B2} = I_{B3} = I_{B4} = I_{B5} = I_{B6} \ldots\ldots\ldots = I_{Bn}, \text{ and}$$

$$I_{R2} = I_{R3} = I_{R4} = I_{R5} = I_{R6} \ldots\ldots\ldots = I_{Rn}$$

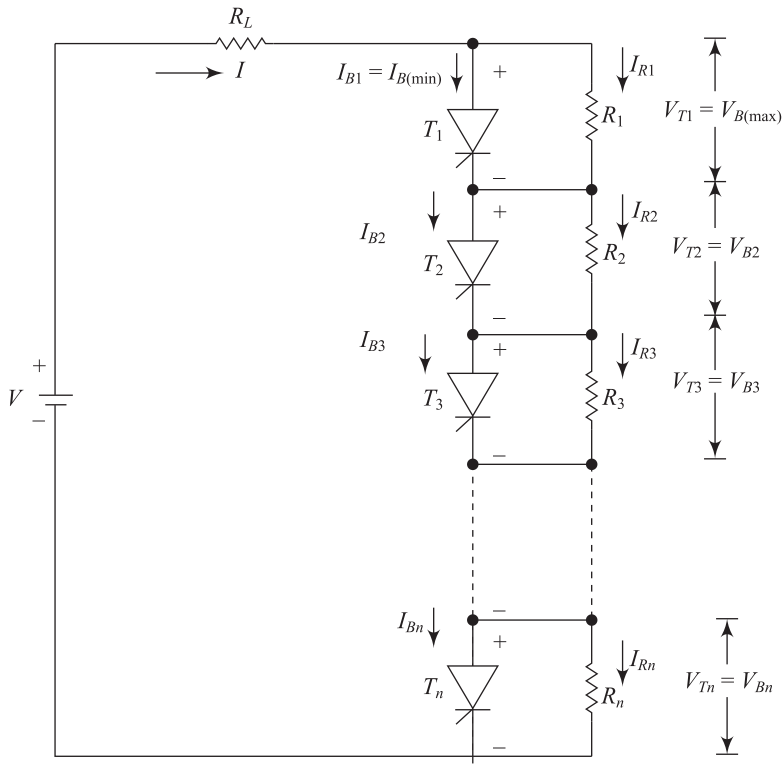


Fig. 4.40 *Series connection of n number thyristors T_1 to T_n*

When the thyristor T_1 has a maximum blocking voltage, it provides maximum internal resistance and the blocking current I_B is minimum.

Consider $I_{B1} = I_{B(\min)}$ and $I_{B2} = I_{B3} = I_{B4} = I_{B5} = I_{B6} \ldots\ldots = I_{Bn} = I_{B(\max)}$

Since the total current is same,

$$I = I_{B1} + I_{R1} = I_{B2} + I_{R2}$$

or

$$I_{B2} - I_{B1} = I_{R1} - I_{R2}$$

or

$$\Delta I_B = I_{B2} - I_{B1} = I_{R1} - I_{R2}$$

Hence, ΔI_B is the difference between the maximum and minimum leakage current of the devices under blocking conditions.

Total voltage drop across n number of thyristors is equal to

$$V = V_{B1} + V_{B2} + V_{B3} + \ldots\ldots\ldots V_{Bn}$$

or

$$V = V_{B(\max)} + (n-1)V_{B2}$$

$$= V_{B1} + (n-1)RI_{R2} \qquad \text{as} \quad V_{B2} = RI_{R2}$$

$$= V_{B1} + (n-1)R(I_{R1} - \Delta I_B) \qquad \text{as} \quad I_{R2} = I_{R1} - \Delta I_B$$

$$= V_{B1} + (n-1)RI_{R1} - (n-1)R\Delta I_B$$

$$= V_{B1} + (n-1)V_{B1} - (n-1)R\Delta I_B \qquad \text{as} \quad V_{B1} = RI_{R1}$$

Therefore, $$R = \frac{nV_{B1} - V}{(n-1)\Delta I_B}$$

Usually manufacturers should specify the maximum value of $I_{B\,(\max)}$ and rarely specify ΔI_B. If $I_{B\,(\min)}$ is neglected, $I_{B\,(\min)} = 0$. and $\Delta I_B = I_{B\,(\max)}$.

Then power dissipation in the resistance R is equal to $P_R = \frac{V_{\text{RMS}}^2}{R}$.

4.15.2 Dynamic Voltage Equalization

During low frequency and steady-state operation, the static voltage equalization circuit is sufficient to protect the thyristor under over-voltage. Since the turn-ON and turn-OFF characteristics of thyristors are varied in wide range, the turn-ON delay time may be decreased by using proper gate drive signal. If a device (thyristor) has maximum turn-ON time delay, device must be capable to withstand at full supply voltage. When the voltage across thyristor is greater than maximum forward blocking voltage, the device may be damaged. During turn-OFF, thyristors with less reverse-recover time (t_{rr}) will be turned OFF firstly and should support the high transient voltage.

For equal voltage distribution during dynamic voltage equalization, capacitors are connected across each thyristor. When one thyristor becomes turned OFF, the reverse current stops to flow through it. Then the capacitor provides a path to flow the reverse current of thyristors. As capacitors are connected in parallel with thyristors, the voltage across the recovered thyristor should not be built up to a high voltage quickly. Therefore, the slower devices in the series string recover slowly and can support the supply voltage. To limit the discharging current of capacitor through thyristor, a resistance R_c is connected in series with capacitor C. The combination of R_c and C is called *dynamic equalizing* circuit. This RC snubber circuit is also used for $\frac{dv}{dt}$ protection. Figure 4.41(a) shows the dynamic equalising circuit for thyristors T_1 and T_2. The reverse recovery characteristics of T_1 and T_2 is depicted in Fig. 4.41(b).

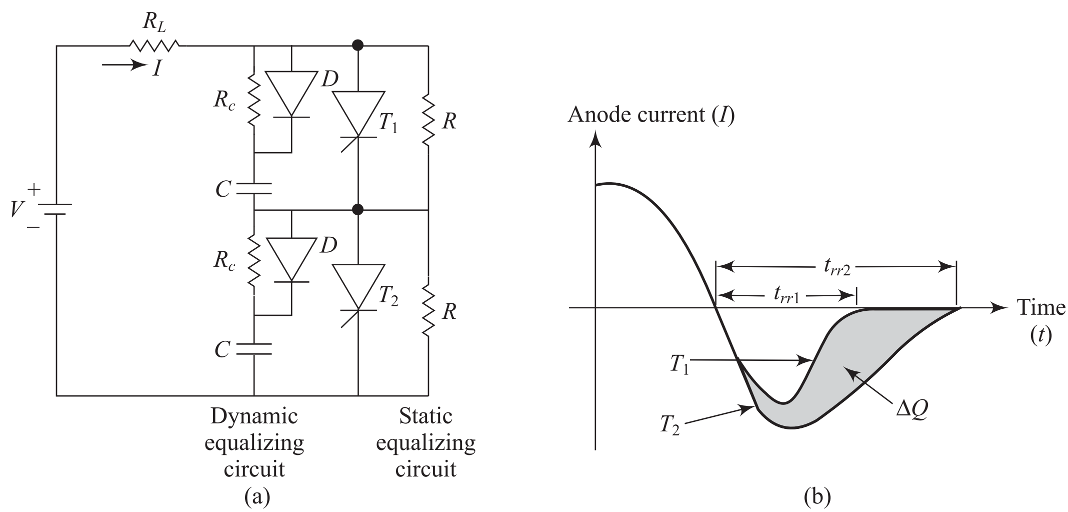


Fig. 4.41 *(a) Dynamic equalizing circuit for thyristors T_1 and T_2 (b) Reverse recovery characteristics of T_1 and T_2*

As the turn-ON process of power semiconductor devices is very fast, the turn-ON time of thyristors is about some microseconds. However, the turn-OFF process of power semiconductor devices is slow compared to the turn-ON process, the turn-OFF of thyristors is about few microseconds to a few hundred microseconds. The voltage unbalance occurs when thyristor T_1 recovers and other thyristors T_2 to T_n recover slowly. Figure 4.42 shows the dynamic equalizing circuit for thyristors T_1, T_2 ……. T_n.

Assume that V_{B1} and V_{B2} are the blocking voltage of fast and slow thyristors

Then the input voltage is equal to

$$V = V_{\text{fast}} + (n-1)V_{\text{slow}}$$

$$V = V_{B1} + (n-1)V_{B2} \qquad \text{as } V_{\text{fast}} = V_{B1} \text{ and } V_{\text{slow}} = V_{B2} = V_{B3} = V_{B4} \cdots = V_{Bn}$$

The unequal voltage distribution of series connected thyristors is more predominant during turn-OFF time rather than during turn-ON time. The choice of capacitor value depends on the reverse recovery characteristics of thyristors. It is clear from Fig. 4.41(b) that the reverse recovery time of thyristor T_1 is short compared to thyristor T_2. The shaded area is equal to ΔQ which is directly proportional to the product of current and time. Actually, ΔQ is the difference between the reverse recovery charges of thyristors T_1 and T_2. Since thyristors T_1 recovers fast and goes to the blocking state, it does not allow passing excess charge ΔQ. Then the excess charge ΔQ should pass through capacitor C. When the thyristor operates in the forward blocking state, the capacitor will be charged to input voltage. In order to bypass the resistance R_C, diode D must be connected in parallel with R_C.

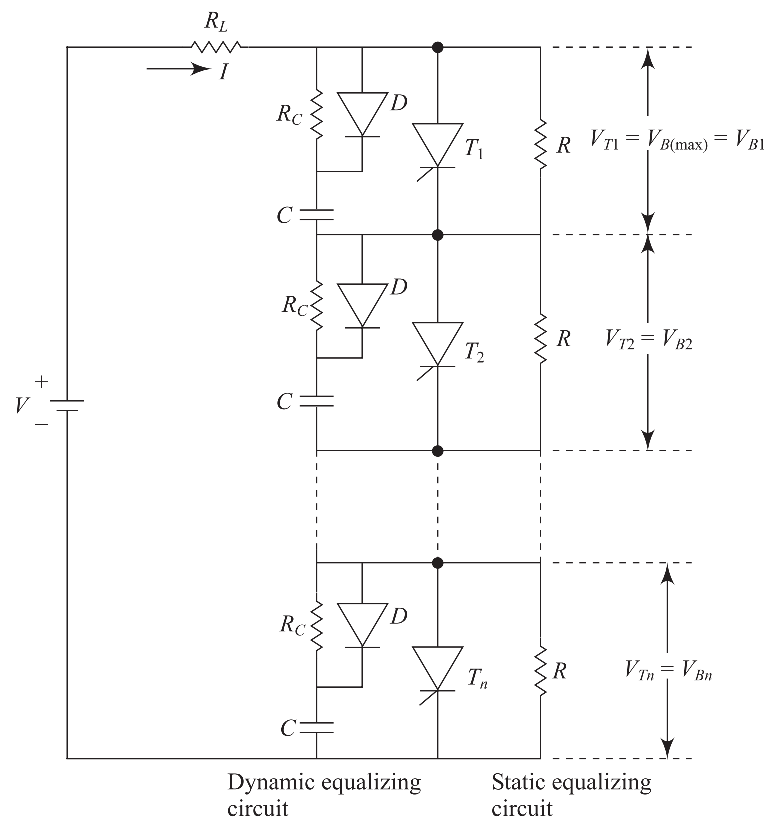


Fig. 4.42 *Dynamic equalizing circuit for thyristors T_1, T_2 ……. T_n.*

Assume that ΔV is the maximum permissible difference between voltages V_{B1} and V_{B2}

Therefore, $$\Delta V = V_{B1} - V_{B2} = \frac{\Delta Q}{C} \qquad \text{as } C = \frac{Q}{V}$$

or $$V_{B2} = V_{B1} - \frac{\Delta Q}{C}$$

The string voltage is equal to

$$V = V_{B1} + (n-1)V_{B2}$$

or $$V = V_{B1} + (n-1)\left(V_{B1} - \frac{\Delta Q}{C}\right) \quad \text{as } V_{B2} = V_{B1} - \frac{\Delta Q}{C}$$

or $$V = V_{B1} + (n-1)V_{B1} - (n-1)\frac{\Delta Q}{C}$$

or $$V = nV_{B1} - (n-1)\frac{\Delta Q}{C}$$

or $$C = \frac{(n-1)\Delta Q}{nV_{B1} - V}$$

4.16 PARALLEL CONNECTION OF THYRISTORS

Thyristors are connected in parallel to increase current rating and reduce the ON-state voltage drop. During parallel operation, ON state voltage drop across each thyristor must be same to share the current equally. Due to difference in *V-I* characteristics, there is a large difference in the current share. The device which share maximum current will be heated and thermal runaway may be developed within the device and the device may be destroyed. The *V-I* characteristics of two thyristors T_1 and T_2 are mismatched and total current *I* flows through T_1 and T_2. Figure 4.43 shows the current sharing of two mismatched *V-I* characteristics parallel connected thyristors.

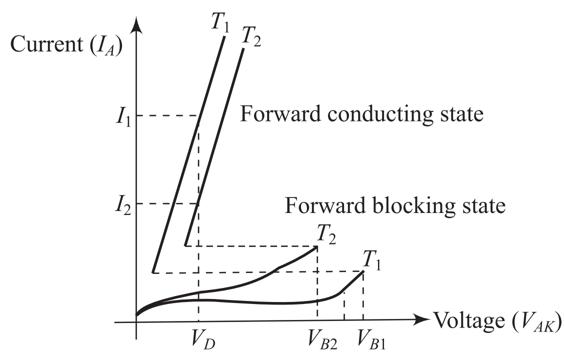


Fig. 4.43 *Current sharing of two mismatched V-I characteristics parallel connected thyristors*

Due to mismatch in *V-I* characteristics, thryristor T_1 shares I_1 current and thryristor T_2 shares I_2 current. The total current $I = I_1 + I_2$ is less than $2I_1$. The string efficiency of two parallel connected thyristors is

$$\eta = \frac{I_1 + I_2}{2I_1}$$

Since I_1 is the rated current of thyristor and $I_1 > I_2$, string efficiency is less than 1 ($\eta < 1$).

Figure 4.44 shows the parallel connection of two thyristors. Due to different *V-I* characteristics, thyristor T_1 shares I_{T1} current and thyristor T_2 shares I_{T2} current and the total current is I_F. Since external resistance *R* is connected with each thyristor, the forward voltage drops across each arm should be equal.

Therefore, $$V_{T1}+I_{T1}R+I_{T1}R_{T1}=V_{T2}+I_{T2}R+I_{T2}R_{T2}$$

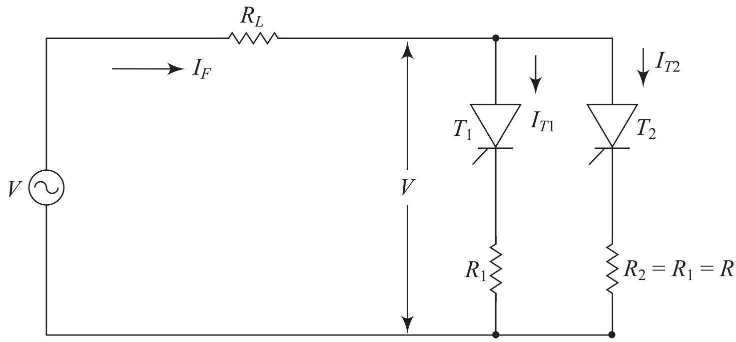
where, V_{T1} is the voltage drop across thyristor T_1

R_{T1} is the junction resistance of thyristor T_1

V_{T2} is the voltage drop across thyristor T_2

R_{T2} is the junction resistance of thyristor T_2

For current equalization, the voltage drop across the resistance varies from 1 V to 2 V.



$R_2 = R_1 = R$

Fig. 4.44 *Current equalization of thyristors T_1 and T_2 using external resistance*

In case of ac circuits, the current equalization can be possible with the help of inductors. Figure 4.45 shows the parallel connection of thyristors with inductive coupling. When the current flows through T_1 and T_2 are same ($I_{T1} = I_{T2}$), the voltage drop across inductance is zero due to mutual cancellation of induced emfs in inductor L_1 and L_2.

If the current flows through T_1 is greater than T_2 ($I_{T1} > I_{T2}$), the induced emfs in the inductive coil is directly proportional to the unbalance currents. Then current flow though the inductance L_1 decreases but the current through the coil L_2 increases. Hence, there is a tendency to share the current equally.

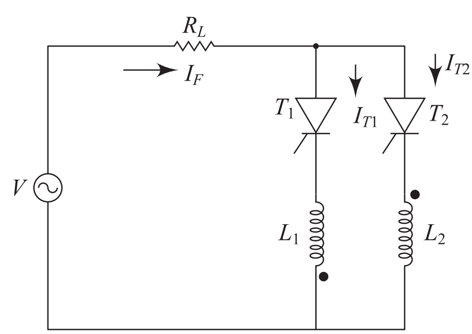


Fig. 4.45 *Current equalization of thyristors T_1 and T_2 using external inductance*

Example 4.27 Thyristors with a rating of 1250 V and 200 A are used in a string to handle 10 kV and 1 kA. Determine the number of series and parallel connected thyristors in case de-rating factor is (a) 0.2, (b) 0.3.

Solution

De-rating factor is $\text{DRF} = 1 - \text{string efficiency}$

$$\text{String efficiency} = \frac{10{,}000}{n_s \times 1250} = \frac{1000}{n_p \times 200}$$

Where, n_s = number of series connected thyristors

n_p = number of parallel connected thyristors

(a) As DRF = 0.2, $$0.2 = 1 - \frac{10,000}{n_s \times 1250} = 1 - \frac{1000}{n_p \times 200}$$

The number of series connected thyristors

$$0.2 = 1 - \frac{10,000}{n_s \times 1250}$$

or $$n_s = \frac{10,000}{(1-0.2)\times 1250} = 10$$

The number of parallel connected thyristors

$$0.2 = 1 - \frac{1000}{n_p \times 200}$$

or $$n_p = \frac{1000}{(1-0.2)\times 200} = 6.25 \cong 7$$

(b) As DRF = 0.3, $$0.3 = 1 - \frac{10,000}{n_s \times 1250} = 1 - \frac{1000}{n_p \times 200}$$

The number of series connected thyristors

$$0.3 = 1 - \frac{10,000}{n_s \times 1250}$$

or $$n_s = \frac{10,000}{(1-0.3)\times 1250} = 11.42 \cong 12$$

The number of parallel connected thyristors

$$0.3 = 1 - \frac{1000}{n_p \times 200}$$

or $$n_p = \frac{1000}{(1-0.3)\times 200} = 7.142 \cong 8$$

Example 4.28 A 175 A SCR is connected in parallel with a 225 A SCR as shown in Fig. 4.46. The ON-state voltage drop across SCRs is 1.85 V and 1.75 V respectively. Determine the series resistance that must be connected in series with each SCR when 400 A current is shared by two SCRs according to their rating.

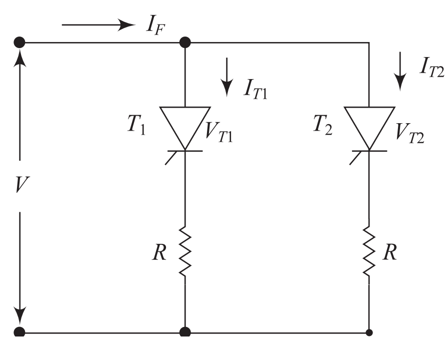


Fig. 4.46

Solution

Given: I_{T1} = 175 A, I_{T2} = 225 A, V_{T1} = 1.85 V, V_{T2} = 1.75 V

Assume that the voltage across each thyristor is same.

Then $$V_{T1} + I_{T1}R = V_{T2} + I_{T2}R$$

or $$R = \frac{V_{T1} - V_{T2}}{I_{T2} - I_{T1}} = \frac{1.85 - 1.75}{225 - 175} = 0.002\ \Omega$$

Example 4.29 When a 250 A thyristor operate in parallel with another 300 A thyristor, the ON-state voltage drops across thyristors is 1.5 V and 1.2 V. Find the value of resistance which will be connected in series with each thyristor. Assume that the total current 550 A is shared by thyristors according to their rating.

Solution

Dynamic resistance of 250 A thyristor $T_1 = \dfrac{1.5}{250}\,\Omega$

Dynamic resistance of 300 A thyristor $T_2 = \dfrac{1.2}{300}\,\Omega$

If external resistance R is connected with each thyristor, the current shared by thyristor T_1 and T_2 are

$$I_{T1} = 550 \times \frac{R + \frac{1.2}{300}}{\text{Total resistance}}$$

$$I_{T2} = 550 \times \frac{R + \frac{1.5}{250}}{\text{Total resistance}}$$

Then $$\frac{I_{T1}}{I_{T2}} = \frac{R + \frac{1.2}{300}}{R + \frac{1.5}{250}}$$

As the total current 550 A is shared by thyristors according to their rating,

$$I_{T1} \propto 250 \quad \text{and} \quad I_{T2} \propto 300$$

Therefore, $$\frac{I_{T1}}{I_{T2}} = \frac{250}{300} = \frac{R + \frac{1.2}{300}}{R + \frac{1.5}{250}}$$

Then the value of R is 0.006 Ω.

Example 4.30 A string of five series connected SCRs has static and dynamic equalizing circuits. The string should withstand at 10 kV. When the static equalizing resistance is 20 kΩ and the dynamic equalizing circuit consists of $R_C = 50\ \Omega$ and $C = 0.05\ \mu\text{F}$, determine the voltage across each SCR and discharge current through SCR T_1. Assume the leakage current of five SCRs are 10 mA, 12 mA, 15 mA, 18 mA and 20 mA respectively.

Solution

When SCRs are in OFF state, current I flows through the string of five series connected SCRs.

The voltage across R is equal to the voltage across each SCR.

Therefore, Voltage across SCR T_1 $V_1 = (I - 0.01) \times 20{,}000$

Voltage across SCR T_2 $V_2 = (I - 0.012) \times 20{,}000$

Voltage across SCR T_3 $V_3 = (I - 0.015) \times 20{,}000$

Voltage across SCR T_4 $V_4 = (I - 0.018) \times 20{,}000$

Voltage across SCR T_5 $V_5 = (I - 0.02) \times 20{,}000$

The sum of voltages V_1, V_2, V_3, V_4, V_5

$$V_1 + V_2 + V_3 + V_4 + V_5 = (5I - 0.075) \times 20{,}000 = \text{string voltage}$$

As string voltage is 10,000 V, $(5I - 0.075) \times 20{,}000 = 10{,}000$

or $$I = 0.085\ \text{A}$$

Then voltages across SCRs are

$$V_1 = (I - 0.01) \times 20{,}000 = (0.085 - 0.01) \times 20{,}000 = 1500\ \text{V}$$

$$V_2 = (I - 0.012) \times 20{,}000 = (0.085 - 0.012) \times 20{,}000 = 1460 \text{ V}$$
$$V_3 = (I - 0.015) \times 20{,}000 = (0.085 - 0.015) \times 20{,}000 = 1400 \text{ V}$$
$$V_4 = (I - 0.018) \times 20{,}000 = (0.085 - 0.018) \times 20{,}000 = 1340 \text{ V}$$
$$V_5 = (I - 0.02) \times 20{,}000 = (0.085 - 0.02) \times 20{,}000 = 1300 \text{ V}$$

The discharge current through SCR T_1 is equal to

$$\frac{V_1}{R_C} = \frac{1500}{50} = 30 \text{ A}$$

Example 4.31 Two thyristors having a difference of 2 mA in latching current are connected in series in a circuit. The voltage across the thyristors are 400 V and 380 V respectively. Determine the required equalizing resistance.

Solution

Given: $\Delta I_B = 2$ mA, $V_{B1} = 400$ V, $V_{B2} = 380$ V

The value of equalizing resistance is

$$R = \frac{nV_{B1} - V}{(n-1)\Delta I_B} \qquad \text{where,} \quad n = 2 \quad \text{and} \quad V = V_{B1} + V_{B2}$$
$$= \frac{2 \times 400 - (400 + 380)}{(2-1) \times 2 \times 10^{-3}} = 10 \text{ k}\Omega$$

Example 4.32 What are the different conditions to turn on a thyristor?

Solution

The following conditions must be satisfied to turn ON a thyristor:

(a) Thyristor must be forward biased and anode potential must be positive with respect to cathode.
(b) The width of gate pulse must be greater than the turn-ON time of thyristor. Anode current must be greater than latching current when the gate signal is removed.
(c) Anode to cathode voltage must be greater than finger voltage.
(d) Amplitude of gate current must be more than the minimum gate current which is required to turn on SCR.
(e) Amplitude of gate current must be less than the maximum permissible gate current so that gate circuit may not be damaged.
(f) The gate triggering must be synchronized with ac supply.

4.17 TRIGGERING CIRCUITS FOR THYRISTORS

A thyristor can be turned on by forward-voltage triggering, $\frac{dv}{dt}$ triggering, temperature triggering, light triggering and gate triggering. From the above these triggering methods, forward-voltage triggering, $\frac{dv}{dt}$ triggering, and temperature triggering are not used to control output of any converter circuit as these methods are abnormal triggering. Light triggering is used in some special applications such as series-parallel connected SCRs. The most commonly used triggering method is gate triggering as this method accurately control the turning on of thyristors output voltage of converter. The gate triggering method is very efficient and most reliable method. To turn ON thyristor from its forward blocking state, a gate signal of proper wave shape and frequency must be applied between gate and cathode.

In gate triggering method, thyristor is forward biased, anode is positive with respect to cathode, a voltage signal is applied at the gate terminal, thyristor will be turned ON. The gate signal may be a slow-rising rectified dc signal (sine wave), a sharp single pulse a constant magnitude dc signal and high frequency pulse signal as depicted in Fig. 4.47. Thyristor will be turned on as soon as the gate voltage V_g is greater than the required critical gate trigger voltage V_{gT}. In Fig. 4.47(a) at $\omega t = \alpha$, voltage V_g is equal to V_{gT} and the thyristor will be triggered. Then the firing angle of thyristor is α.

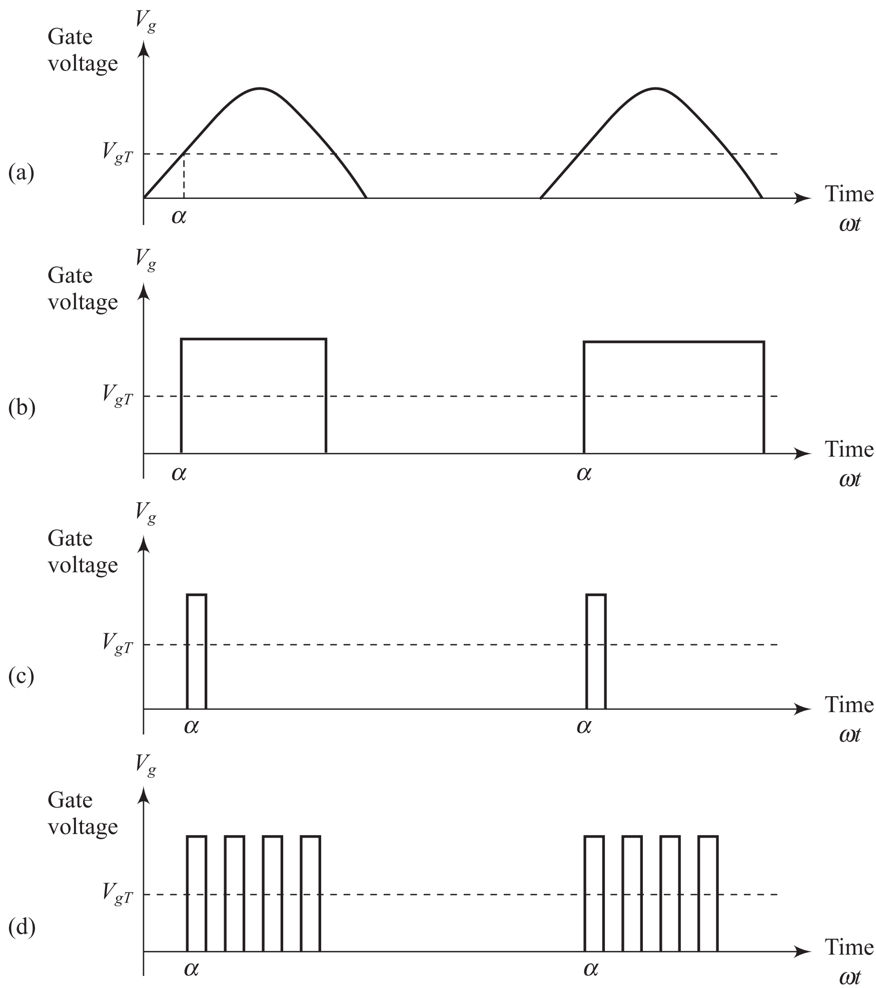


Fig. 4.47 *Triggering signals of a thyristor*

An ideal gate current waveform is shown in Fig. 4.48. The initial high amplitude and fast rise in gate current turn on the device. Usually, a continuous gate is required for thyristor. When thyristor becomes turn-on, there is no requirement of gate signal after successful triggering. The gate voltage signal is generated by a gate-drive which is called *triggering* or *firing* circuit for thyristors. The different triggering circuits are:

1. Resistance (R) triggering circuit
2. Resistance capacitance (*RC*) triggering circuit

3. Uni-junction transistor triggering circuit or UJT relaxation oscillator triggering circuit
4. Half-wave controlled rectifier
5. Full-wave controlled rectifier

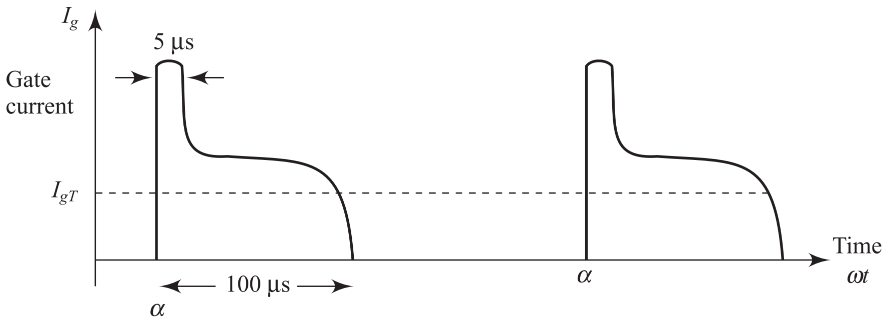


Fig. 4.48 *Gate current waveform*

4.17.1 Resistance (*R*) Triggering Circuit

Figure 4.49 shows the simplest resistance triggering circuit. R_1 is the variable resistance, R_2 is the stabilizing resistance. When R_1 is zero, the gate current flows through R_{min}, D, gate-cathode, load and source. This current should not be greater than maximum permissible gate current I_{gm}.

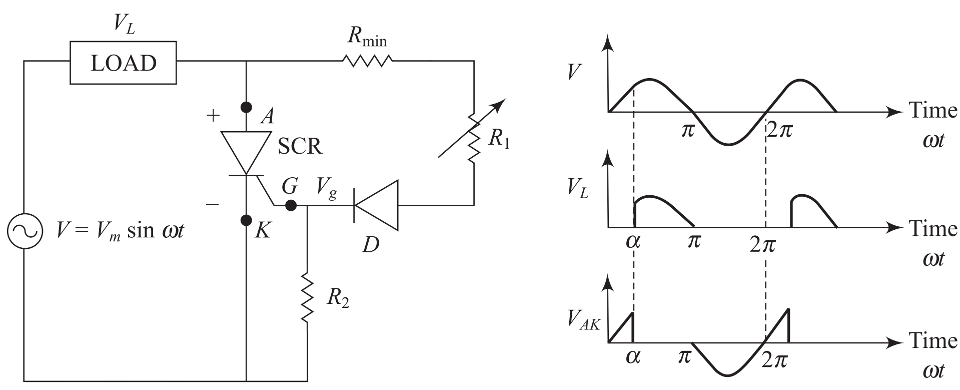
$$\frac{V_m}{R_{\min}} \le I_{gm}$$

or $$R_{\min} \ge \frac{V_m}{I_{gm}}$$ where, V_m is the maximum source voltage.

During positive half-cycle of input voltage at $\omega t = \alpha$, the voltage applied at gate terminal is greater than V_{gT}. Before turn-on of SCR, the input voltage is applied to SCR and operates in forward blocking state.

The value of R_2 should be such that the maximum voltage across it is not greater than gate voltage V_{gm}.

Therefore, $$V_m \frac{R_2}{R_{\min} + R_1 + R_2} \le V_{gm}$$



4.49 *(a) Resistance triggering circuit (b) Voltage waveforms*

Since R_1 and R_2 are very large, the gate trigger circuit draws small current. Due to diode D, the current flows in positive half cycle only. Amplitude of voltage is controlled by V_{gT}. When R_1 is very large and voltage across R_2 is $V_g = iR_2$. If the peak of V_g is less than V_{gT}, thyristor will not be turned-on. Consequently, the output voltage is equal to zero ($V_o = 0$) as shown in Fig. 4.50.

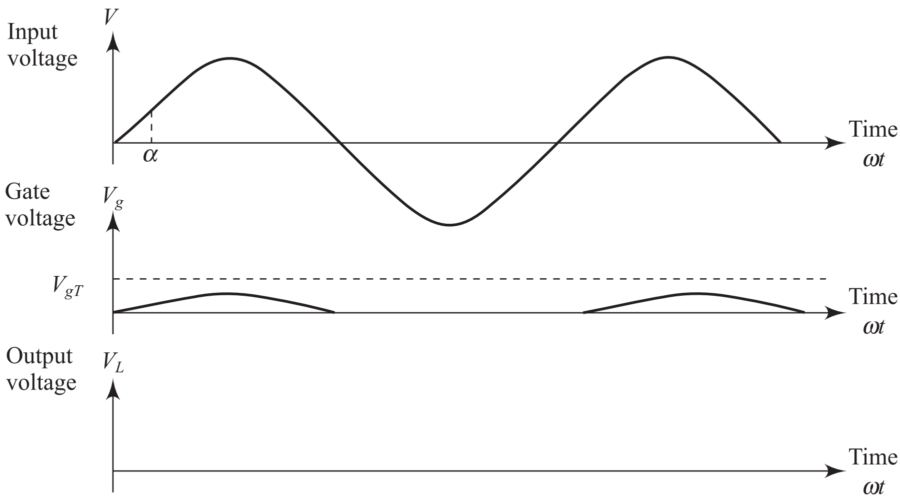


Fig. 4.50 *Output voltage*

When R_1 is reduced, i_g current increases, V_g exceeds the V_{gT} and thyristor will be turn ON.

At $\omega t = \alpha$, V_{gp} sin $\alpha = V_{gT}$ and $\alpha = \sin^{-1}\left(\dfrac{V_{gT}}{V_{gp}}\right)$

When $\quad V_{gp} = V_m \dfrac{R}{R+R_1+R_2} \qquad \alpha = \sin^{-1}\left(\dfrac{V_{gT}}{V_m}\dfrac{R+R_1+R_2}{R}\right)$

As V_{gT}, R_1, R, V_m are constant, $\alpha \propto \sin^{-1}(R_2)$

or $\qquad \alpha \propto R_2$

Hence the firing angle is directly proportional to R_2. As R_2 increases, firing angle α increases. The maximum limit of firing angle is $\alpha = 90°$. Since $\alpha = 0°$ is not possible, the range of firing angle is represented by $90° \geq \alpha > 0°$.

This triggering circuit can be used for TRIAC. In this case, if diode D will be removed and the triggering signal will be available at the gate terminal during positive as well as negative half cycles.

Example 4.33 In resistance triggering circuit as depicted in Fig. 4.52(a), $I_{g\,(min)} = 0.15$ mA and $V_{g\,(min)} = 0.5$ V. When the peak amplitude of input voltage is 100 V, find the trigger angle α for $R_1 = 100$ kΩ and $R_{min} = 10$ kΩ.

Solution

The KVL equation of gate circuit is

$$V = I_g(R_{min} + R_1) + V_D + V_g$$

At the point of trigger,

$$V = I_g(R_{\min} + R_1) + V_D + V_g = 0.15 \times 10^{-3}(10 \times 10^3 + 100 \times 10^3) + 0.7 + 0.5 \text{ V}$$
$$= 17.7 \text{ V}$$

Assume that the firing angle of thyristor is α. Then 17.7 = 100 sin α

$$\alpha = \sin^{-1}\left(\frac{17.7}{100}\right) = 10.195°$$

4.17.2 *RC* Triggering Circuit

Figure 4.51(a) shows a *RC* triggering circuit. In the negative half cycle of supply voltage, the capacitor C charges through diode D_2 to the negative peak value of supply voltage $-V_m$. At $\omega t = -90°$, $V_c = -V_m$. After $\omega t = -90°$, the supply voltage starts to decrease from $-V_m$ to zero at $\omega t = 0°$. In $-90° \le \omega t \le 0°$, the capacitor voltage decreases and finally fall to OA. At $\omega t = 0°$, $V_c = OA$. After $\omega t > 0°$, the supply voltage is positive and capacitor starts to charge through variable resistance R. When the capacitor-voltage reaches B and holds the positive voltage during the positive half cycle of supply voltage, the capacitor voltage V_c is greater than V_{gT} and thyristor will be turned on. Hence, the firing angle can be controlled by varying resistance R.

Diode D_1 is used to flow current in positive direction only. Hence, it prevents the breakdown of cathode to gate junction during negative half cycle.

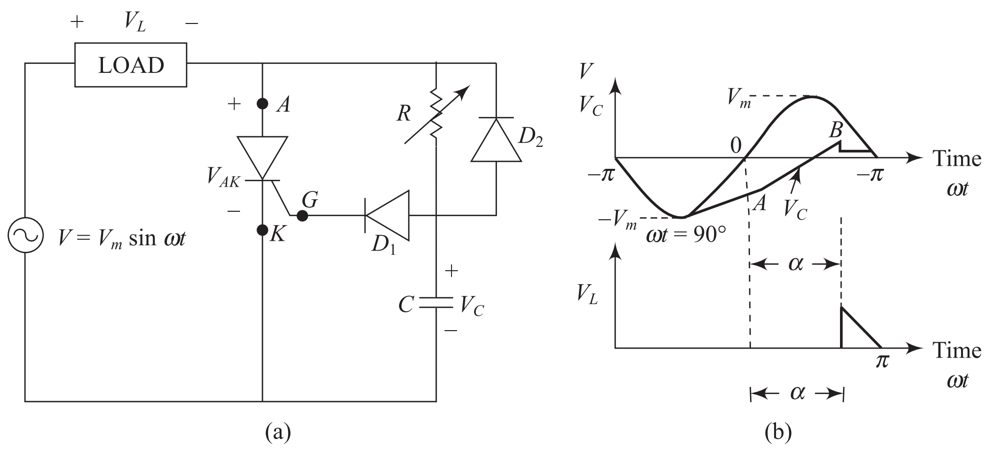


Fig. 4.51 *(a) RC triggering circuit (b) Voltage waveforms*

Using this method, firing angle will be never 0° and 180°. The control range of firing angle is $0° < \alpha < 180°$. In a *RC* firing circuit, the following condition must be satisfied:

$$RC \ge \frac{1.3T}{2} \cong \frac{4}{\omega} \quad \text{where, } T = \frac{1}{f} = \text{time period of ac line frequency in seconds}$$

Thyristor will be turned on when $V_c = V_{gT} + V_d$ where V_d is the voltage drop across diode D_1. If the capacitor voltage drop is constant at the instant of triggering, the gate current I_{gT} must be supplied through R, D_1 and gate-cathode of thyristor.

The maximum value of R is given by

$$V \geq I_{gT}R + V_c$$

or
$$V \geq I_{gT}R + V_{gT} + V_d$$

or
$$R \leq \frac{V - V_{gT} - V_d}{I_{gT}}$$
where, V is the voltage at which thyristor will be turned ON.

When the thyristor is turned ON, the on-state voltage drop across thyristor is about 1 V to 1.5 V. Therefore, the voltage drop across R and C will also be reduced to the value of 1 V to 1.5 V until the negative half cycle voltage appears across C. In the negative half cycle of supply voltage, the capacitor is charged to the maximum voltage of $-V_m$. If the R value is less, firing angle is less and conduction angle is more. When R is increased, firing angle increases and conduction angle decreases.

4.17.3 *RC* Full Wave Triggering Circuit

Figure 4.52 shows the RC full wave triggering circuit. Diodes D_1, D_2, D_3 and D_4 are used as full wave bridge rectifier. Then full wave output voltage is applied across load and thyristor. Initially capacitor is charged to a low positive voltage which is almost zero. Figure 4.52 behaves as clamping circuit. The capacitor upper plate is positively charged and lower plate is negatively charged. Thyristor will be turned on when the capacitor voltage is greater than V_{gT} and the rectified output voltage will be available across load. The value of RC is computed from the following equation:

$$RC \geq 50\frac{T}{2} \cong \frac{157}{\omega}$$

The value of R is given by the relation

$$R << \frac{V - V_{gT}}{I_{gT}}$$

Figure 4.53 shows the voltage and current waveforms of RC full wave triggering circuit.

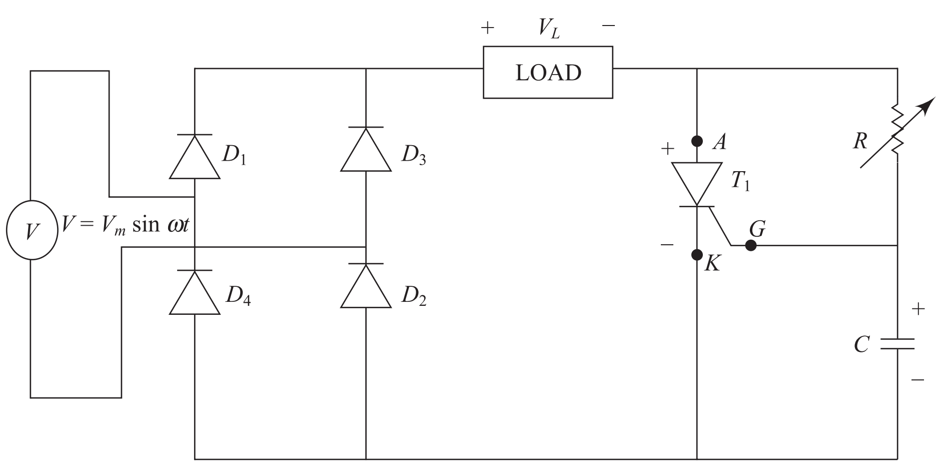


Fig. 4.52 *RC full wave triggering circuit*

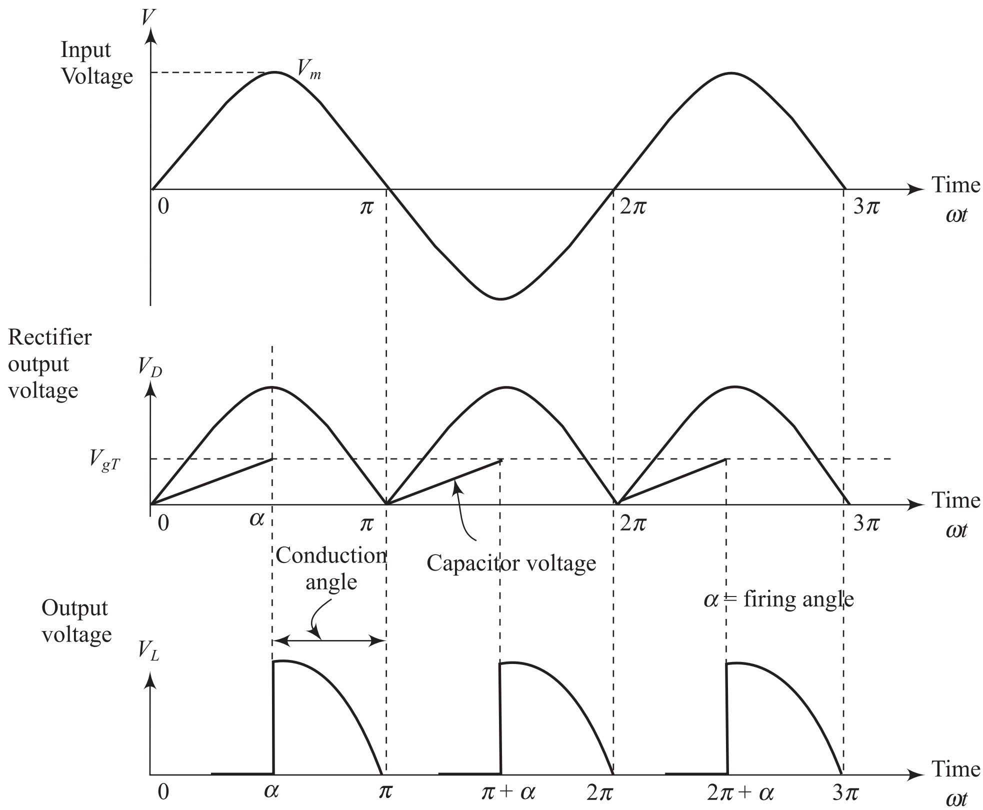


Fig. 4.53 *Voltage waveforms of RC full wave triggering circuit*

Example 4.34 A 230 V, 50 Hz ac supply is connected to a resistance capacitance (RC) triggering circuit as depicted in Fig. 4.51(a). If the resistance R is variable from 2 kΩ to 20 kΩ, V_{gT} = 2 V and C = 0.47 μF, what is the minimum and maximum firing angle?

Solution

The current flow through capacitance C is equal to

$$I = \frac{V}{Z} \quad \text{where, } Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

(a) If R_2 = 2 kΩ, $\quad Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{2000^2 + \left(\frac{1}{2\pi \times 50 \times 0.47 \times 10^{-6}}\right)^2} = 7064.98\ \Omega$

and $\quad \phi = \tan^{-1}\frac{1/\omega C}{R_1} = \tan^{-1}\frac{6775.98}{2000} = 73.55°$

The current I leads V by an angle θ, $I = \frac{V}{Z} = \frac{230\angle 0°}{7064.98\angle -73.55°} = 0.03255\angle 73.55°$

The voltage across capacitor is

$$V_C = IX_C = 0.03255\angle 73.55° \times 6775.98\angle -90° = 220.58\angle -16.45°$$

or $$v_C = \sqrt{2} \times 220.58 \sin(\omega t - 16.45)$$

At $\omega t = \alpha_1$, $$v_C = \sqrt{2} \times 220.58 \sin(\alpha_1 - 16.45) = 2$$

or $$\alpha_1 = \sin^{-1} \frac{2}{\sqrt{2} \times 220.58} + 16.45 = 16.81°$$

(b) If $R_2 = 20\ \text{k}\Omega$, $$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{20000^2 + \left(\frac{1}{2\pi \times 50 \times 0.47 \times 10^{-6}}\right)^2} = 21116.67\ \Omega$$

and $$\phi = \tan^{-1} \frac{1/\omega C}{R_1} = \tan^{-1} \frac{6775.98}{20000} = 18.71°$$

The current I leads V by an angle ϕ, $I = \dfrac{V}{Z} = \dfrac{230\angle 0°}{21116.67\angle -18.71°} = 0.010891\angle 18.71°$

The voltage across capacitor is

$$V_C = IX_C = 0.010891\angle 18.71° \times 6775.98\angle -90° = 73.78\angle -71.29°$$

or $$v_C = \sqrt{2} \times 73.78 \sin(\omega t - 71.29)$$

At $\omega t = \alpha_1$, $$v_C = \sqrt{2} \times 73.78 \sin(\alpha_1 - 71.29) = 2$$

or $$\alpha_1 = \sin^{-1} \frac{2}{\sqrt{2} \times 73.78} + 71.29 = 72.38°$$

4.18 UNIJUNCTION TRANSISTOR (UJT)

A UJT is a *three terminal* semiconductor device (Emitter E, Base B_1 and Base B_2) as shown in Fig. 4.54(a) and (b). It is formed from a lightly doped slab of n-type material which has high resistance. The two base contacts are made at each end of one side of the slab and aluminum rod is inserted on the other side to form a *single pn junction*. Therefore, UJT is called as uni-junction. The aluminum rod is located near to the base terminal 2 (B_2). The B_2 is positive with respect to B_1 by voltage V_{BB}. The symbol of a UJT is shown in Fig. 4.55(a).

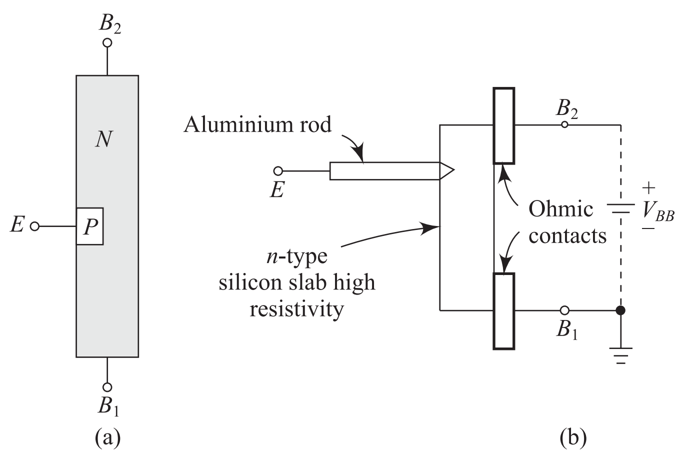


Fig. 4.54 *The basic structure of an UJT*

Figure 4.55(b) shows the equivalent circuit of the UJT. The diode represents the *p-n* junction, R_{B1} is a variable resistance, and R_{B2} is a fixed resistance. The value of R_{B1} decreases when emitter current increases. The R_{B1} varies from 50 kΩ to 50 Ω when emitter-current I_E changes from 0 to 50 mA.

The inter-base resistance between B_1 and B_2 is expressed as

$$R_{BB} = R_{B1} + R_{B2}$$

and it's range is approximately 4 kΩ to 10 kΩ.

When $I_E = 0$, the voltage across R_{B1} is

$$V_{R_{B1}} = \frac{R_{B1}}{R_{B1} + R_{B2}} V_{BB} = \eta V_{BB}$$

$$\eta = \frac{R_{B1}}{R_{B1} + R_{B2}} \text{ is intrinsic stand-off ratio.}$$

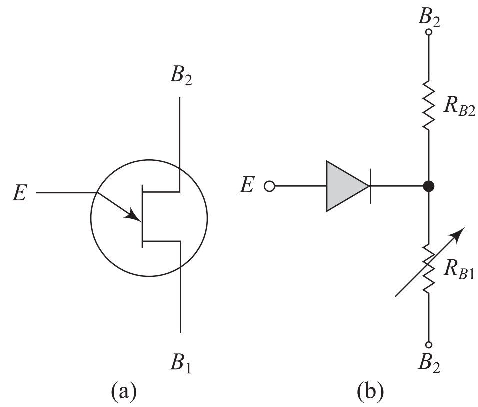


Fig. 4.55 *(a) Symbol of UJT*
(b) Equivalent circuit of the UJT

Actually the intrinsic stand-off ratio is controlled by the location of the aluminum rod.

The emitter threshold potential is

$$V_P = \eta V_{BB} + V_D$$

The *V-I* characteristics of a UJT is depicted in Fig. 4.56. When V_E crosses the threshold potential V_P, the emitter fires and holes are injected into the *n*-type slab through the *p*-type rod. Therefore, the holes content of the *n*-type slab increases. Consequently, the number of free electrons increases and hence conductivity increased. Thus V_E drops off while increasing I_E. The UJT operates in the negative resistance region as depicted in Fig. 4.56. The UJT passes through the valley point (I_V, V_V) and then becomes saturated.

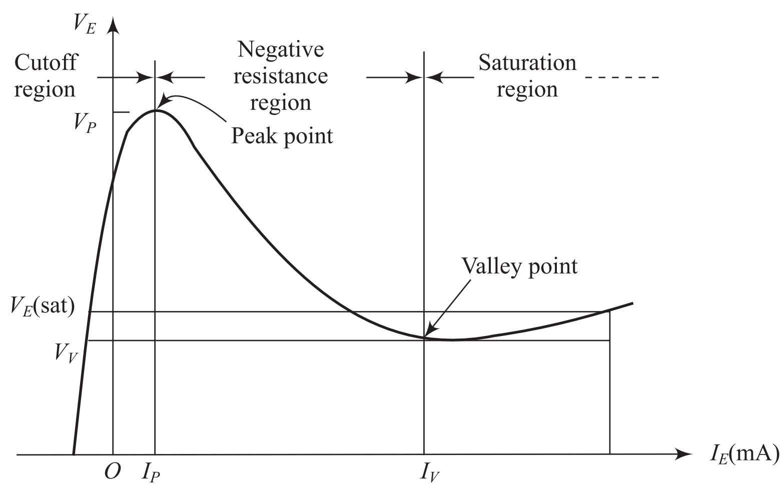


Fig. 4.56 *V-I characteristics of a UJT*

4.19 RELAXATION OSCILLATOR

The *relaxation oscillator* is an oscillator using UJT which can generate saw-tooth waveform. The UJT relaxation oscillators store energy in a capacitor and then dissipate that energy repeatedly to setup the oscillations. For example, the capacitor can be charged toward a positive power supply until it reaches a threshold voltage sufficiently close to the supply. When the capacitor reaches each threshold, capacitor can be quickly discharged due to electrical short. In all such capacitor-based relaxation oscillators, the period of the oscillations is set by the dissipation rates of the capacitor. The UJT relaxation oscillators consist of a UJT, capacitor C which is charged through resistance R_E. Figure 4.57 shows the UJT relaxation oscillator.

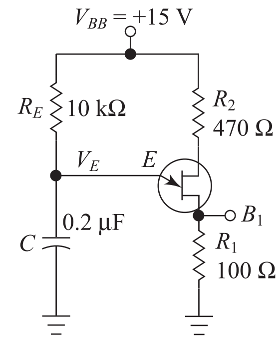


Fig. 4.57 *Relaxation oscillator*

The voltage across the capacitor increases exponentially and when capacitor voltage reaches the peak point voltage V_P, the UJT starts conducting and the capacitor voltage is discharged rapidly through E-B$_1$ and R_1. After the peak point voltage of UJT V_P, it provides negative resistance to the discharge path which is useful in the working of the relaxation oscillator. When the voltage reaches V_V, the capacitor C starts to charge again. This cycle is repeated continuously generating a saw-tooth waveform across C.

The capacitor is charged and discharged cyclically as depicted in Fig. 4.58 where V_C is the voltage across capacitor. The charging time constant of capacitor is RC. At time t_1, $V_C = V_P$, the UJT is turned ON and the capacitor discharges through R_1 for t_2 duration until reaches valley point voltage V_V. When UJT reaches the valley point, it becomes open circuit and capacitor starts to charge again.

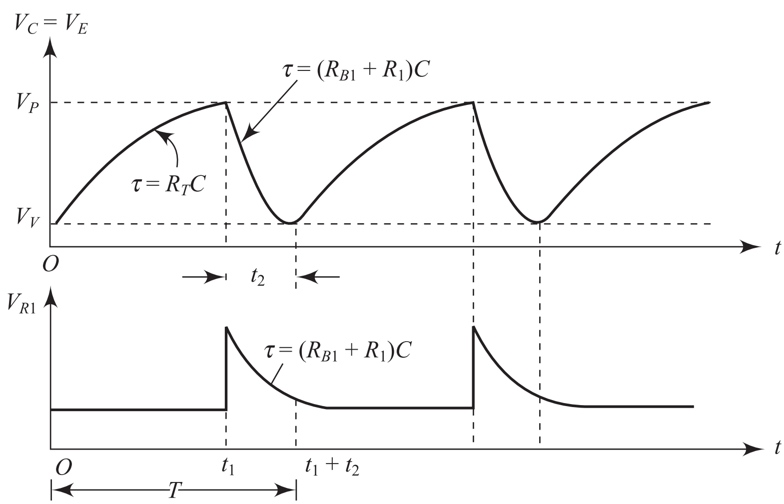


Fig. 4.58 *Waveforms of V_C and V_{R1}*

The total cycle time $T = t_1 + t_2$.

The charging time of capacitor $t_1 = R_E C \ln\left(\dfrac{V - V_V}{V - V_P}\right)$

The discharging time of capacitor $t_2 = (R_{B1} + R_1) C \ln\left(\dfrac{V_P}{V_V}\right)$

Time period $T = t_1 + t_2$

The oscillation frequency is $f = \dfrac{1}{R_E C \ln\left(\dfrac{1}{1-\eta}\right)}$ as $t_1 \gg t_2$

4.20 UJT TRIGGERING CIRCUIT

Figure 4.59(a) shows a UJT triggering circuit. The values of external resistances R_1 and R_2 are smaller than the values of internal resistances R_{B1} and R_{B2} of UJT. The value of charging resistance R should be such that load line must intersect the negative resistance region of UJT characteristics.

When a dc voltage V is applied, the capacitor starts to charge through R. During charging of capacitor, emitter is open circuit. The capacitor voltage V_C can be represented by

$$V_C = V(1 - e^{-\frac{t}{\tau_1}})$$

or $$V_C = V(1 - e^{-\frac{t}{RC}})$$ as $\tau_1 = RC$ is the charging time constant of capacitor.

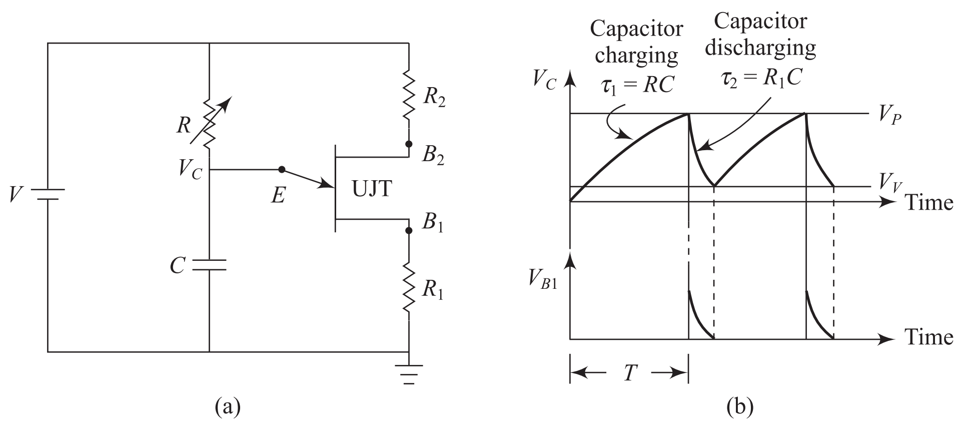


Fig. 4.59 *(a) UJT triggering circuit (b) Waveforms of UJT triggering circuit*

When the capacitor voltage reaches the voltage $V_P (V_P = \eta V + V_D)$, the junction between E-B_1 will be break down. Consequently, UJT will be turned ON and capacitor starts to discharge through resistance R_1. The discharging time constant $\tau_2 = R_1 C$. τ_2 is smaller than τ_1. Accordingly the emitter voltage decreases to valley voltage V_V and emitter current falls below I_V. Subsequently, UJT will be turned OFF.

The voltage $V_P = \eta V + V_D = V_V + V(1 - e^{-\frac{t}{RC}})$ as $V_D = V_V$ and $\eta = (1 - e^{-\frac{t}{RC}})$

At $t = T$, $$\eta = (1 - e^{-\frac{T}{RC}})$$

Therefore, $$T = \frac{1}{f} = RC \ln\left(\frac{1}{1-\eta}\right)$$

The firing angle is equal to

$$\alpha_1 = \omega T = \omega RC \ln\left(\frac{1}{1-\eta}\right) \quad \text{where, } \omega \text{ is the angular frequency of UJT oscillator.}$$

When the output pulse of UJT oscillator is used to trigger thyristor, R_1 should be small so that the normal leakage current drop must be less than V_p and UJT will not be triggered.

Then, $$V\frac{R_1}{R_{BB}+R_1+R_2} < \text{SCR trigger voltage } V_{gT}$$

Where, $R_{BB} = R_{B1} + R_{B2}$

The value of $R_2 = \dfrac{10^4}{\eta V}$ and the width of triggering pulse is equal to R_1C.

The maximum value of R can be determined from the peak voltage V_P and peak current I_P. If the voltage across capacitor is V_P, the voltage across R is equal to $V - V_P$.

Therefore, $$R_{\max} = \frac{V - V_P}{I_P} = \frac{V - (\eta V + V_D)}{I_P}$$

The minimum value of R is computed from valley point voltage and current values V_v and I_v

$$R_{\min} = \frac{V - V_V}{I_V}$$

4.21 SYNCHRONIZED UJT TRIGGERING CIRCUIT

Figure 4.60 shows a synchronized UJT triggering circuit. This circuit has a bridge rectifier which consists of four diodes D_1, D_2, D_3 and D_4 and converts ac to dc. The full wave rectified dc output voltage is obtained from bridge-rectifier. Resistance R_1 and zener diode Z are used to clip the rectified output voltage to specified voltage level V_Z. The output voltage waveforms are depicted in Fig. 4.61.

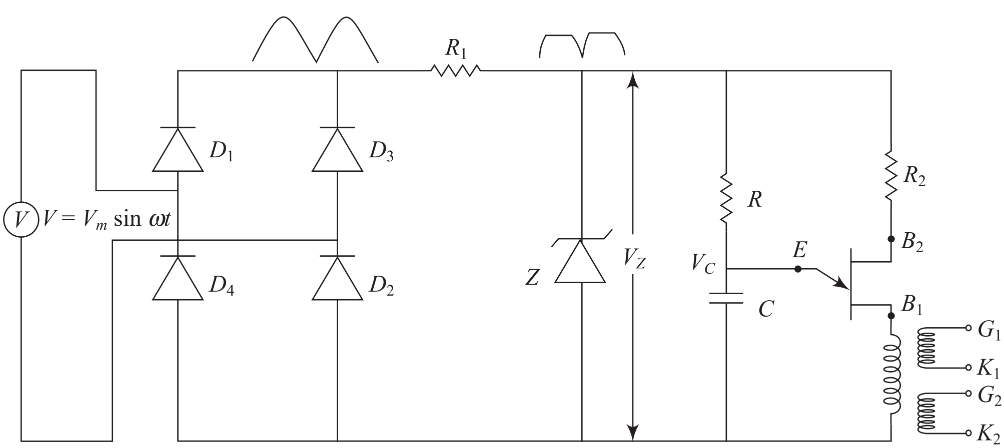


Fig. 4.60 *Synchronized UJT triggering circuit*

When output voltage is applied across RC circuit, the capacitor C is charged and its charging rate depends on the value of resistance R. When the capacitor voltage V_C reaches the threshold voltage of uni-junction transistor (ηV_Z), the emitter-base (E-B_1) junction of UJT will be breakdown and the capacitor will be discharged through primary winding of pulse transformer. As current i flows through primary winding of pulse transformer, a voltage will be induced in the secondary winding which is used as triggering signal of thyristor as depicted in Fig. 4.61.

During discharging of capacitor, when the capacitor voltage is less than valley voltage of UJT, E-B_1 junction becomes open circuit and capacitor starts again to charge through R. The rate of capacitor voltage is controlled by varying the value of resistance R. In this method, the variation of firing angle is $0° < \alpha < 150°$. Figure 4.62 shows the triggering circuit for single phase half-wave controlled rectifier and the generation of triggering pulse and output voltage is depicted in Fig. 4.61.

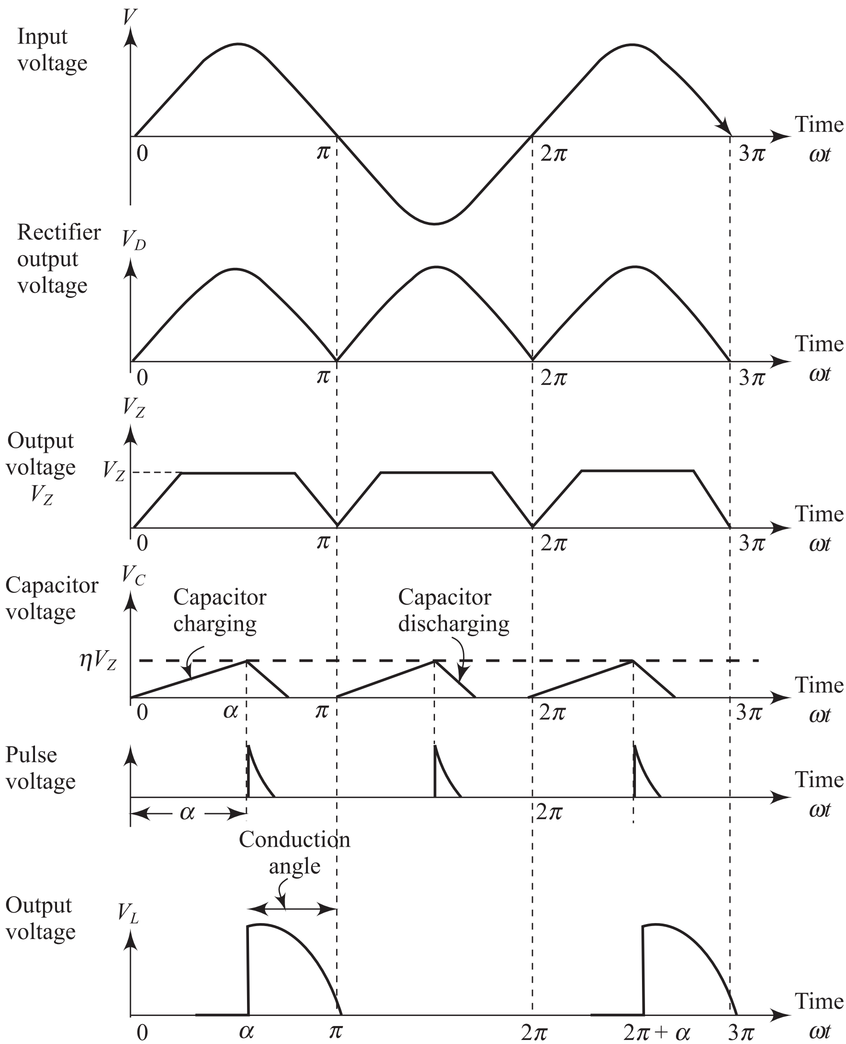


Fig. 4.61 *Generation of pulse voltage and output voltage of single phase half-wave controlled rectifier*

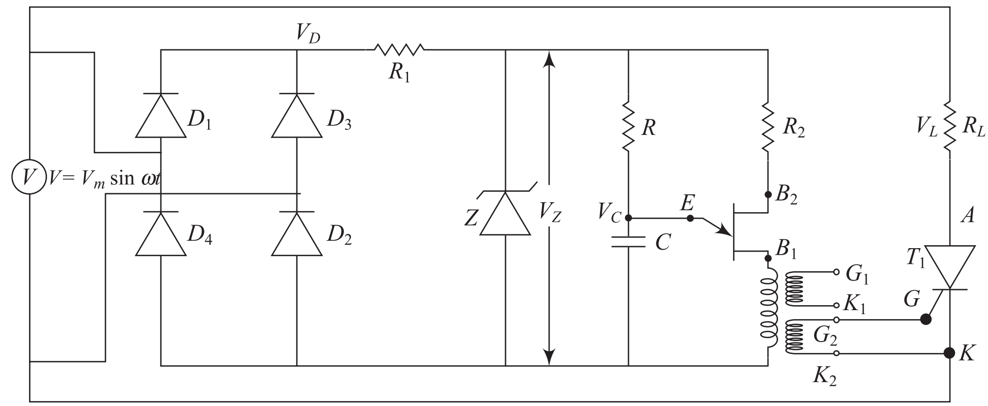


Fig. 4.62 *Triggering circuit for single phase half-wave controlled rectifier*

4.22 RAMP AND PEDESTAL TRIGGERING

Figure 4.63 shows the ramp and pedestal triggering circuit which is the improved synchronised UJT triggering circuit. This triggering circuit can be used for ac voltage controller, single phase semi-converter and full converter. V_Z is the specified output voltage of zener diode. The resistance R_2 is used as potential divider. The pedestal voltage V_{PD} can be controlled by changing the wiper position. The value of V_{PD} is always less than the threshold voltage of UJT (ηV_Z). When wiper voltage V_{PD} is small, capacitor is charged through R. Whenever the capacitor voltage reaches the threshold voltage of UJT ηV_Z, UJT is turned ON and a current flows through primary of pulse transformer and a triggering pulse can be obtained from secondary of pulse transformer. After that capacitor voltage reduces to V_{PD} and then it also reduces to zero at $\omega t = \pi$. Triggering circuit for single phase ac voltage controller using thyristor is depicted in Fig. 4.63 and Fig. 4.64 shows the triggering circuit for single phase ac voltage controller using TRIAC.

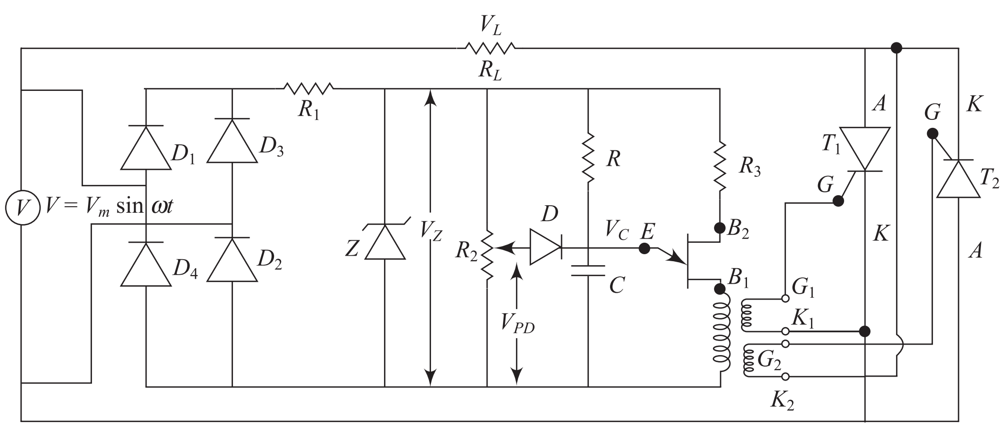


Fig. 4.63 *Triggering circuit for a single phase ac voltage controller using thyristor*

The generation of triggering pulse and output voltage of ac voltage controller is given in Fig. 4.65. During charging of capacitor, V_C is greater than V_{PD}, diode D operates in off-state due to reverse bias. Since the pedestal voltage across C is low, the charging time of capacitor is long and firing angle will be more and output voltage is low. With high value of pedestal voltage across C, the charging time of capacitor is small and firing angle will be less and output voltage is high.

Assume time T is required to charge capacitor from pedestal voltage V_{PD} to the threshold voltage of UJT (ηV_Z). Then we can write

$$\eta V_Z = V_{PD} + (V_Z - V_{PD})(1 - e^{-\frac{T}{RC}})$$

Actually, the ($V_Z - V_{PD}$) is the effective voltage which is used to charge C from V_{PD} to ηV_Z

Then, $$T = RC \ln\left(\frac{V_Z - V_{PD}}{V_Z(1-\eta)}\right)$$ and the firing angle α is equal to

$$\alpha = \omega RC \ln\left(\frac{V_Z - V_{PD}}{V_Z(1-\eta)}\right)$$

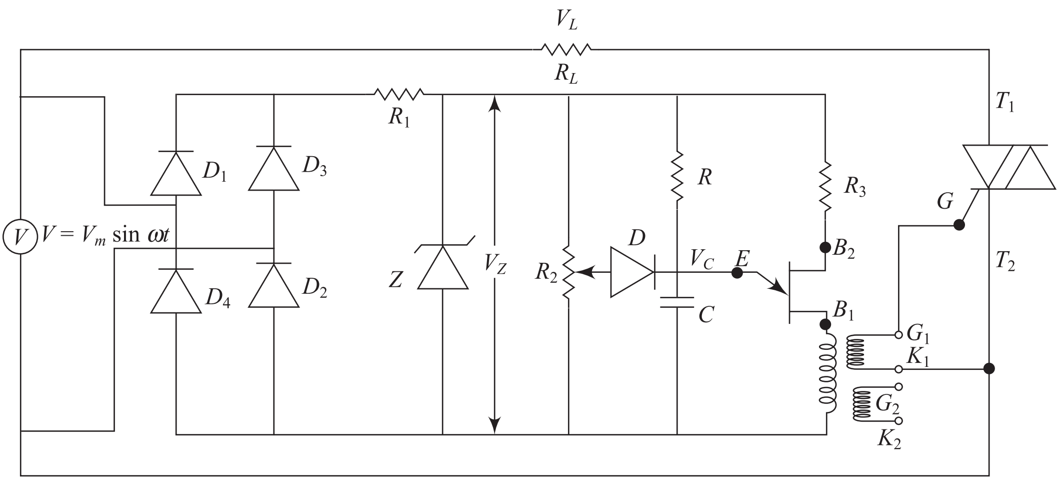


Fig. 4.64 *Triggering circuit for single phase ac voltage controller using TRIAC*

Example 4.35 Design a relaxation oscillator circuit using UJT which has the following specifications:

$$\eta = 0.65,\ I_p = 0.65 \text{ mA},\ V_p = 12 \text{ V},\ I_v = 2.0 \text{ mA},\ V_v = 1.5 \text{ V},\ R_{BB} = 4.5 \text{ k}\Omega$$

and Norman leakage current is 3 mA when emitter open circuit.

The firing frequency is 2.5 kHz. Assume the suitable value of capacitance C.

Solution

$\eta = 0.65$, $I_p = 0.65$ mA, $V_p = 12$ V, $I_v = 2.0$ mA, $V_v = 1.5$ V, $R_{BB} = 4.5$ kΩ, $f = 2.5$ kHz

Choose the value of capacitance $C = 0.047$ μF.

The value of charging resistance of capacitor C is

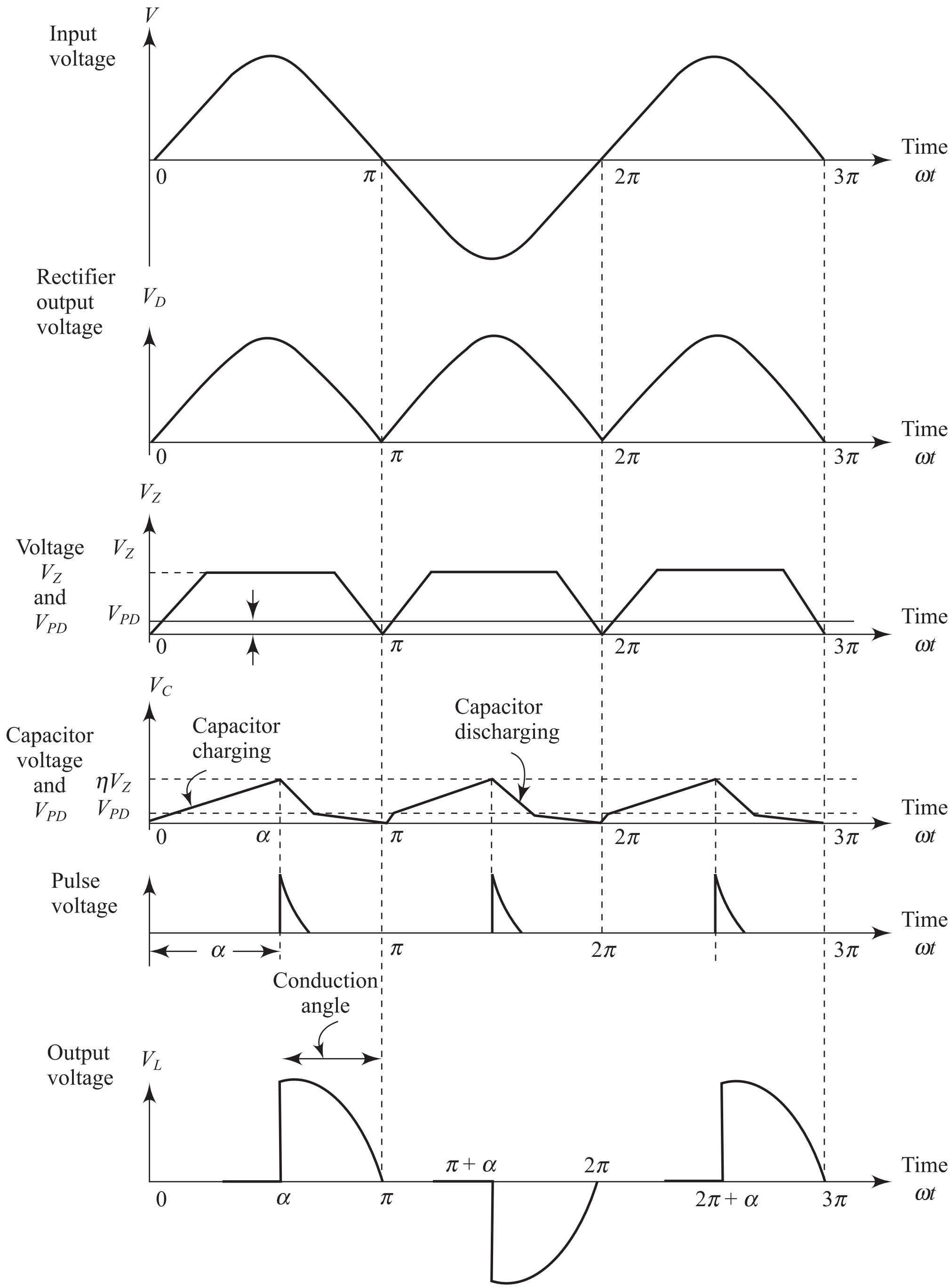


Fig. 4.65 *Generation of pulse voltage and output voltage of single phase ac voltage controller*

$$R = \frac{1}{fC\ln\left(\frac{1}{1-\eta}\right)} \quad \text{as } T = \frac{1}{f} = RC\ln\left(\frac{1}{1-\eta}\right)$$

$$= \frac{1}{2.5\times10^3\times0.047\times10^{-6}\ln\left(\frac{1}{1-0.65}\right)} = 8.1067 \text{ k}\Omega$$

We know that $V_p = \eta V_{BB}$

Therefore, $V_{BB} = \frac{V_p}{\eta} = \frac{12}{0.65} \text{ V} = 18.46 \text{ V} \cong 18.5 \text{ V}$

The value of $R_2 = \frac{10^4}{\eta V} = \frac{10^4}{0.65\times18.5}\,\Omega = 831.60\,\Omega \quad \text{as } V_{BB} = V$

We know that $R_1 + R_2 + R_{BB} = \frac{V_{BB}}{\text{Leakage current}}$

The value of R_1 is

$$R_1 = \frac{V_{BB}}{\text{Leakage current}} - R_2 - R_{BB}$$

$$= \frac{18.5}{4.5\times10^{-3}}\,\Omega - 831.60\,\Omega - 4.5\text{ k}\Omega = 835\,\Omega$$

Example 4.36 The firing frequency of relaxation oscillator is varied by changing the value of charging resistance R. Find the maximum and minimum values of R and their corresponding firing frequencies. Assume $\eta = 0.65$, $I_p = 0.65$ mA, $V_p = 12$ V, $I_v = 2.0$ mA, $V_v = 1.5$ V, $V_{BB} = 20$ V, and $C = 0.047$ µF

Solution

The minimum value of R is computed from valley point voltage and current values V_v and I_v

$$R_{\min} = \frac{V - V_v}{I_v} = \frac{20-1.5}{2.0\times10^{-3}} = 9.25 \text{ k}\Omega$$

The maximum value of R is computed from peak point voltage and current values V_p and I_p

$$R_{\max} = \frac{V - V_p}{I_p} = \frac{20-12}{0.65\times10^{-3}} = 12.3076 \text{ k}\Omega$$

Firing frequency $f = \frac{1}{RC\ln\left(\frac{1}{1-\eta}\right)} \quad \text{as} \quad T = \frac{1}{f} = RC\ln\left(\frac{1}{1-\eta}\right)$

The maximum firing frequency $f_{\max} = \frac{1}{R_{\min}C\ln\left(\frac{1}{1-\eta}\right)} = \frac{1}{9.25\times10^3\times0.047\times10^{-6}\ln\left(\frac{1}{1-0.65}\right)} \text{Hz}$

$= 2191.05$ Hz

The minimum firing frequency $f_{\min} = \frac{1}{R_{\max}C\ln\left(\frac{1}{1-\eta}\right)} = \frac{1}{12.3076\times10^3\times0.047\times10^{-6}\ln\left(\frac{1}{1-0.65}\right)} \text{Hz}$

$= 1646.72$ Hz

4.23 GATE DRIVE CIRCUIT OF THYRISTOR WITH ISOLATION

Actually, the gate drive or control circuit operates at low voltage with lower power and the load or the power circuit is connected with high voltage and power circuit is used to control power. If the control circuit is not isolated from power circuit, the control circuit will be damaged. Therefore, the isolation between the gate drive or control circuit and the load or the power circuit is required for high power applications. Usually, isolation can be provided either by using pulse transformers or by using opto-couplers.

4.23.1 Gate Drive Circuit Using Pulse-Transformer

Pulse transformers are generally used in firing circuits for thyristors. Usually pulse transformer has two secondary windings and one primary. The turn ratio from primary to secondary is 1:1:1 or 2:1:1. The pulse transformer has low winding resistance, low leakage reactance and low inter-winding capacitance. The application of pulse transformer in firing circuit of a thyristor has the following advantages:

1. The isolation between the low voltage gate circuit (control circuit) and high-voltage anode circuit (power circuit).
2. The triggering pulses from same trigger source can be used to turn on two or more number thyristors.

Figure 4.66(a) shows the most simplified pulse transformer triggering circuit for SCR. When a square pulse is applied at the primary terminals of pulse transformer, it will be transmitted at its secondary terminals as a square wave or the transmitted signal will be the derivative of the input voltage signal.

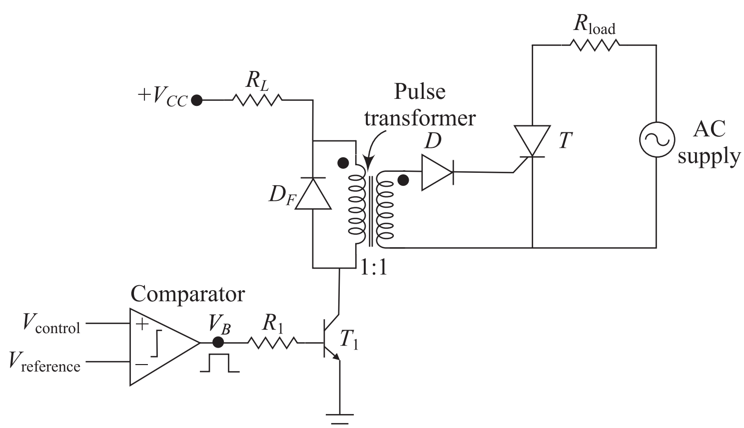


Fig. 4.66 *(a) Simplified pulse transformer triggering circuit for SCR*

Actually a square waveform is applied to base of a transistor T_1 which is act as a switch. When the input pulse is high, transistor T_1 will be turned on and the dc voltage is applied to primary winding of pulse transformer [Fig 4.66 (b)]. The advantage of this circuit is that

1. It is not required a strength pulse generator since the amplitude of pulse is same; the strength of generated pulses may be increased by increasing the amplitude of dc bias voltage.

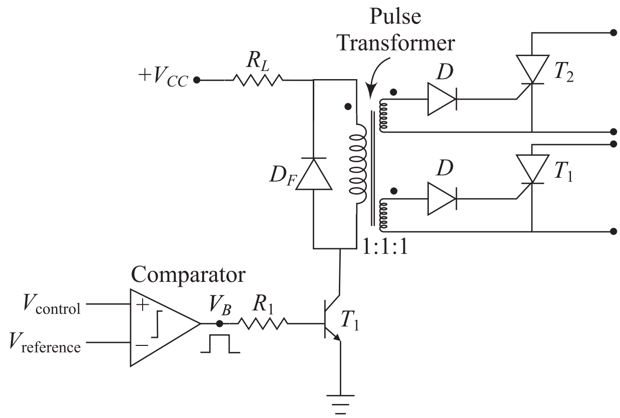


Fig. 4.66 *(b) Simplified pulse transformer triggering circuit for two SCRs*

2. The circuit operation is independent of the pulse characteristics as the pulse is used to turn ON or turn OFF the transistor. The pulse distortion has no effect on the circuit operation. The resistance R_L is connected in series with the primary winding of pulse transformer to limit the current flow.
3. When the amplitude of applied pulse at base of transistor T_1 is low, T_1 will be turned OFF and then diode D starts to operate. Subsequently diode D allows the flow of current through pulse transformer.

Figure 4.67(a) shows the equivalent circuit of pulse transformer trigger circuit as depicted in Fig. 4.66(a) and R_g is the resistance of gate-cathode circuit of thyristor. The modified from of Fig. 4.67(a) is illustrated in Fig. 4.67(b). This circuit can also be represented by Thevenin's theorem in between "1" and "2". Figure 4.67(c) shows the Thevenin's equivalent circuit of Fig. 4.67(b).

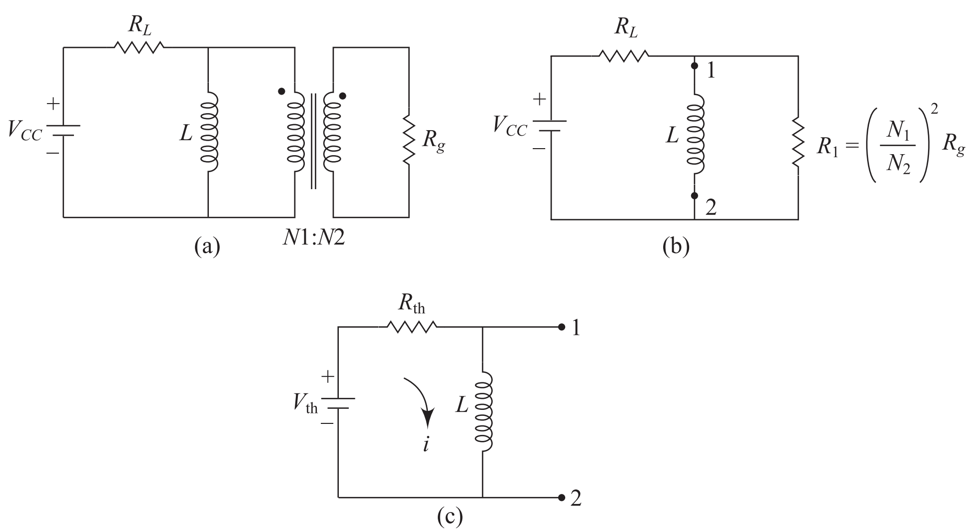


Fig. 4.67 *(a) Equivalent circuit of pulse transformer trigger circuit as shown in Fig. 4.66(a) (b) Modified form of Fig. 4.67(a) (c) Thevenin's equivalent circuit of Fig. 4.67(b)*

Thevenin's voltage is equal to $V_{\text{th}} = V_{CC}\dfrac{R_1}{R_1+R_L}$ and

Thevenin's resistance is $R_{\text{th}} = \dfrac{R_1R_L}{R_1+R_L}$ where, $R_1 = \left(\dfrac{N_1}{N_2}\right)^2 R_g$

KVL equation for Fig. 4.70(c) is

$$V_{\text{th}} = iR_{\text{th}} + L\frac{di}{dt}$$

After substituting the values of V_{th} and R_{th} in the above equation, we obtain

$$V_{CC}\frac{R_1}{R_1+R_L} = \frac{R_1R_L}{R_1+R_L}i + L\frac{di}{dt}$$

or
$$V_{CC} = iR_L + \frac{R_1+R_L}{R_1}L\frac{di}{dt}$$

After solving the above equation, we get

$$i = \frac{V_{CC}}{R_L}\left(1 - e^{-\frac{R_1R_L}{L(R_1+R_L)}t}\right)$$

The amplitude of voltage across pulse transformer is

$$e = L\frac{di}{dt} = V_{CC}\frac{R_1}{R_1+R_L}e^{-\frac{R_1R_L}{L(R_1+R_L)}t}$$

or
$$e = V_{CC}\frac{R_1}{R_1+R_L}e^{-\frac{R_{\text{th}}}{L}t} \quad \text{as} \quad R_{\text{th}} = \frac{R_1R_L}{R_1+R_L}$$

Depending upon the value of R_{th} and L, the pulse transformer operates in two different modes such as

(a) $\dfrac{L}{R_{\text{th}}} < \dfrac{T}{10}$ (b) $\dfrac{L}{R_{\text{th}}} > 10T$ where, T is the pulse width of input signal.

(a) Condition $\dfrac{L}{R_{\text{th}}} < \dfrac{T}{10}$: R_{th} is very large compared with L.

We know that $e = V_{CC}\dfrac{R_1}{R_1+R_L}e^{-\frac{R_{\text{th}}}{L}t}$

At $t = 0$, $e(t=0) = V_{CC}\dfrac{R_1}{R_1+R_L} = e_0$

At $t = T$, $e(t=T) = V_{CC}\dfrac{R_1}{R_1+R_L}e^{-\frac{10}{T}t} = V_{CC}\dfrac{R_1}{R_1+R_L}e^{-\frac{10}{T}T} = e(t=0)\times e^{-10} = 0.0000453e_0$

For step rise in input voltage, the pulse transformer output is a positive pulse. The input signal is transmitted as a derivative of the input waveform for a step rise. For a step fall in input voltage, a negative pulse appears at the pulse transformer output. The operation of the pulse transformer in this mode can be achieved by using a small value of L. It is possible by using an air core pulse transformer.

(b) Condition $\frac{L}{R_{th}} > 10T$, L is very large compared to R_{th}.

Therefore, $e = V_{CC}\frac{R_1}{R_1+R_L}e^{-\frac{R_{th}}{L}t} = V_{CC}\frac{R_1}{R_1+R_L}e^{-\frac{t}{10T}}$

At $t = 0$, $e(t=0) = V_{CC}\frac{R_1}{R_1+R_L} = e_0$

At $t = T$, $e(t=T) = V_{CC}\frac{R_1}{R_1+R_L}e^{-\frac{t}{10T}} = V_{CC}\frac{R_1}{R_1+R_L}e^{-\frac{T}{10T}} = e(t=0)\times e^{-0.1} = 0.9048e_0$

When the pulse transformer has large inductance, the pulses are reproduced. If the inductance is small, the pulses are exponentially decaying pulses. The wave shape of the output pulses from a pulse transformer is depicted in Fig. 4.68. The triggering pulse is exponentially decaying in nature for the following causes:

1. The triggering pulse waveform is suitable for injecting a large current in the gate circuit for reliable operation.
2. The duration of pulse is shorter and there will not be much heating in the gate circuit.
3. For the same gate cathode power, it is permissible to raise V_{CC} to a suitable high value so that a hard drive of thyristor can be obtained. Thyristor with a hard drive can able to withstand high di/dt at the desirable anode circuit.
4. The size of the pulse transformer is reduced. For an extended pulse, a large value of L is required which increases size and cost of the pulse transformer.

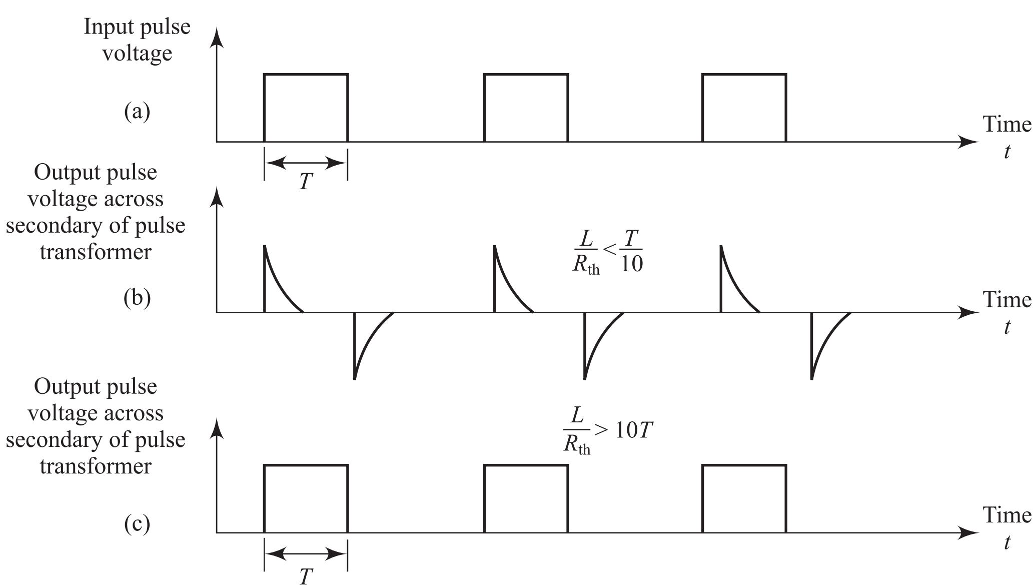


Fig. 4.68 *(a) Input voltage (b) Output pulse voltage across secondary of pulse transformer with* $\frac{L}{R_{th}} < \frac{T}{10}$ *(c) Output pulse voltage across secondary of pulse transformer with* $\frac{L}{R_{th}} > 10T$.

When we know the voltage across the primary winding of pulse transformer, the amplitude of the trigger voltage at the secondary terminals of pulse transformers is equal to

$$V_g = \frac{N_2}{N_1} V_{CC} \frac{R_1}{R_1 + R_L} \quad \text{where, } \frac{N_2}{N_1} \text{ is the turn ratio}$$

The amplitude of V_{CC} must be large enough to produce trigger voltage V_{gT} at the gate circuit of thyristor.

Therefore, $$\frac{N_2}{N_1} V_{CC} \frac{R_1}{R_1 + R_L} \geq V_{gT}$$

or $$V_{CC} \geq V_{gT} \frac{N_1}{N_2}\left(1 + \frac{R_L}{R_1}\right)$$

A gate trigger circuit for thyristors in phase controlled rectifiers and ac voltage controllers should have the following features:

1. A zero crossing detection circuit on the input voltage signal is required.
2. Trigger pulses should be generated as per required wave shape.
3. DC power source is needed for pulse transformer.
4. Gate trigger circuit should be isolated from the power circuit by using pulse transformers or optocouplers.

Figure 4.69 shows a schematic block diagram for gate trigger for single-phase converter. The gating circuit consists of synchronising transformer, diode rectifier, zero crossing detector, firing angle delay generator circuit, pulse transformer, gate pulse isolation transformer and power circuit for converters.

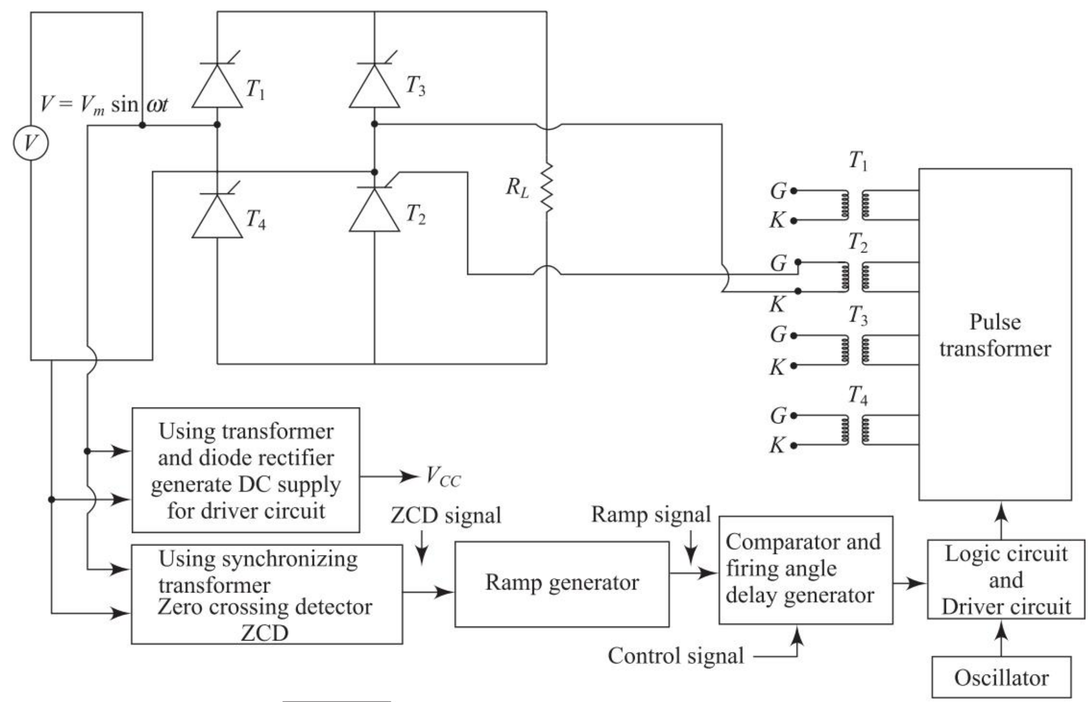


Fig. 4.69 *Block diagram of a thyristor trigger circuit*

The synchronising centre tapped transformer steps down the supply voltage to a suitable low voltage for zero crossing detectors and for generating dc power supply V_{CC}, which is used for the gate trigger circuit and pulse transformer. The zero crossing detector circuit which uses operational amplifier, is used to convert ac synchronising input voltage into a square wave signal as depicted in Fig. 4.70. The negative pulses are eliminated by using diodes at the output stage of each operational amplifier. In Fig. 4.70(a), the first Exclusive-OR gate is used as an inverter and the voltage waveform $\overline{A}$ is generated at the output of the inverter. Subsequently, $\overline{A}$ voltage signal is fed to the delay circuit, i.e., R-C integrator circuit and the voltage waveform B is obtained across the capacitor of the delay circuit.

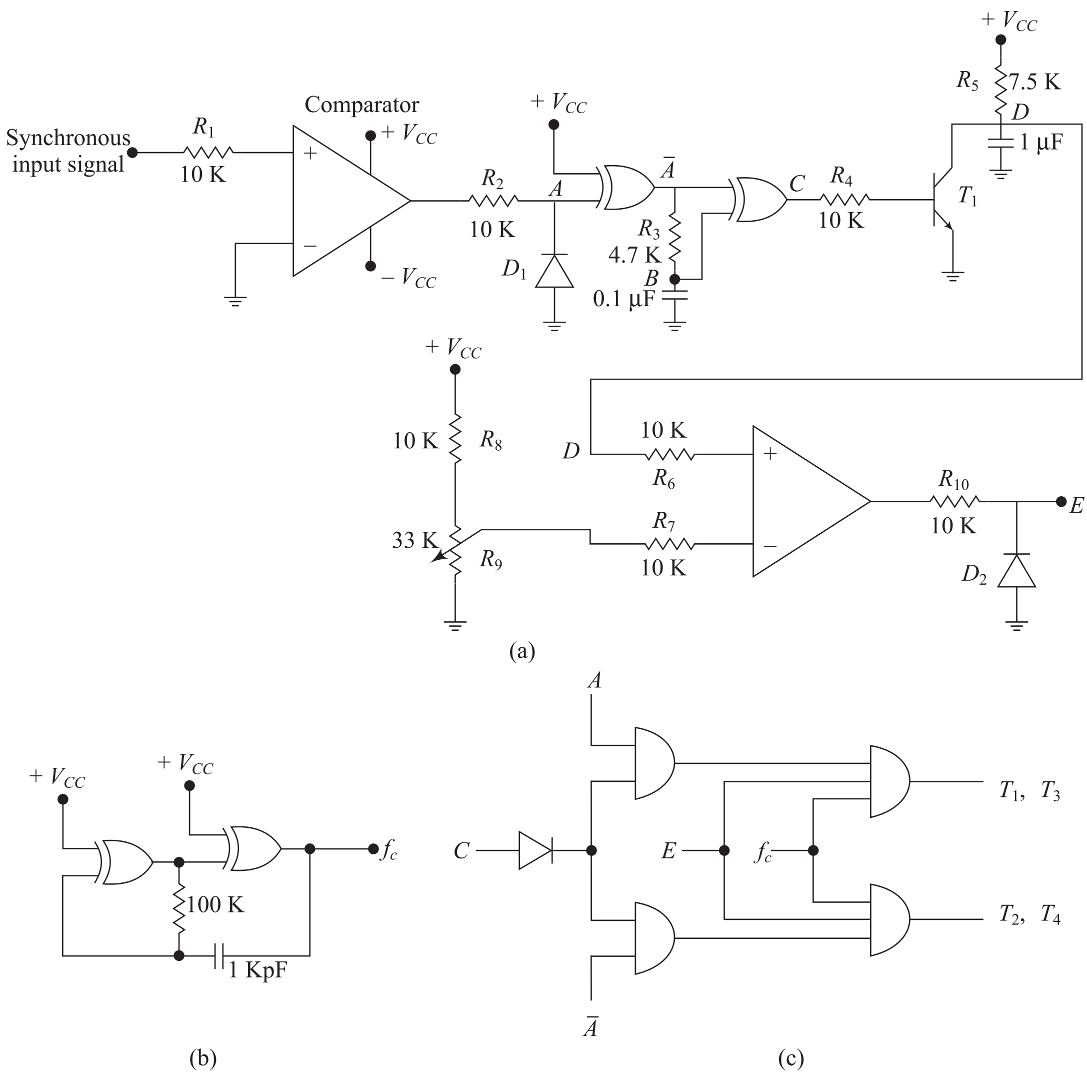


Fig. 4.70 *(a) Hardware implementation of ramp triggering circuit (b) Oscillator circuit (c) Logic circuit to generate trigger pulse*

When the signals $\overline{A}$ and B are fed to the input terminals of second Ex-OR gate and the waveform C is obtained at output of 2nd Ex-OR gate. The C waveform is used for resetting the ramp-generator. The generated ramp voltage is synchronised with the zero crossing of ac input voltage signal. The output signal at point D is a ramp waveform.

In the delay firing angle generation unit, the constant amplitude ramp voltage is compared with the control voltage V_C. At the instant the amplitude of ramp voltage is equal to the control voltage; a pulse signal of controlled duration is generated. Figure 4.71(h) shows the control signals for thyristors 1 and 2 and the control signals for thyristors 3 and 4 are shown in Fig. 4.71(i). Whenever the control voltage V_C is less than the ramp voltage, there is no firing control signal. If the amplitude of dc control voltage decreases, the firing angle α decreases and if V_C is increased, the firing angle α is increased. This states that the firing angle is directly proportional to the control signal. The pulse output from the firing angle generator is fed to a pulse amplifier circuit. After that the amplified pulses are used for triggering thyristors 1, 2, 3, and 4 through gate pulse isolation transformers.

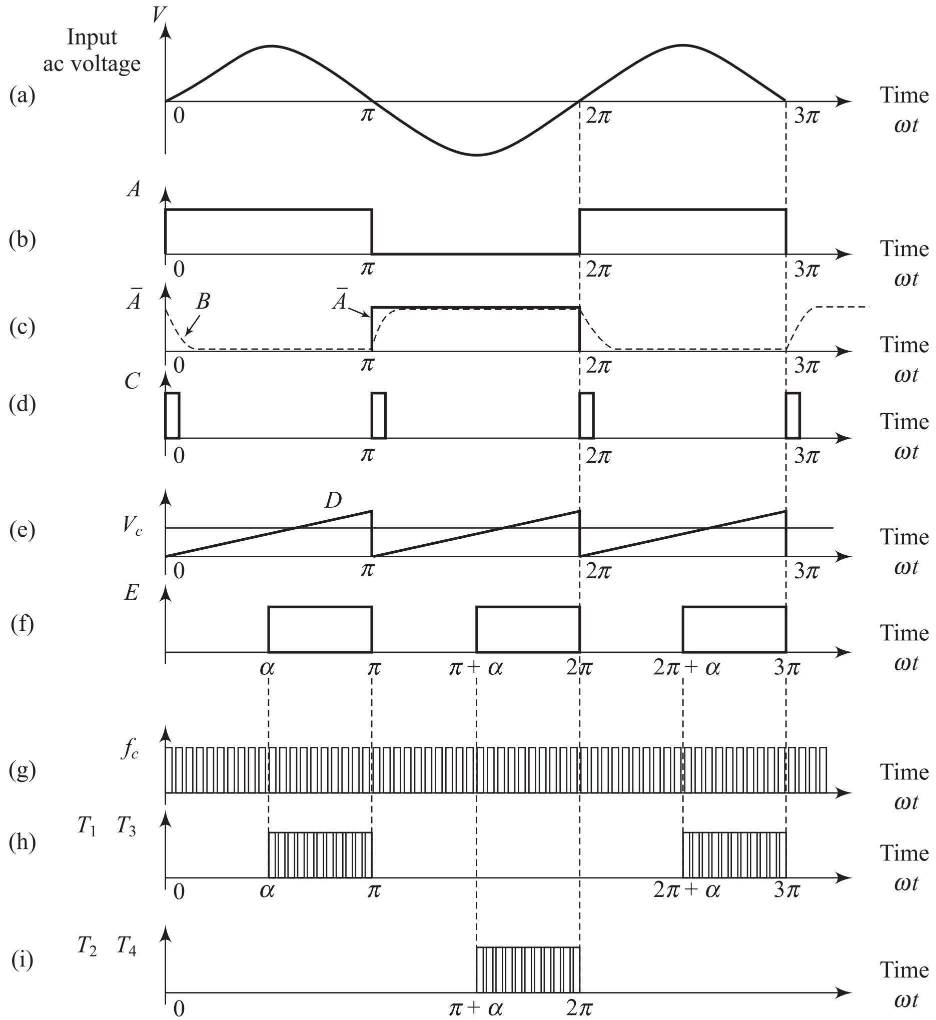
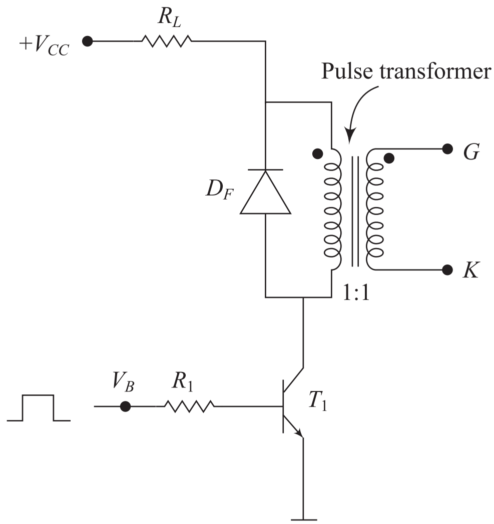


Fig. 4.71 *Ramp triggering circuit for a single phase converter*

The voltage waveform at E is ANDed with A and $\overline{A}$ separately. The oscillator generates pulses at about 7 kHz. The output voltage waveforms are modulated by high frequency pulses. Actually the high frequency switching is required to reduce thyristor gate power dissipation and the size of pulse transformer. The waveforms are fed to separate driver circuits which are also called pulse amplification and isolation stages. Pulse transformer and transistor driver circuit, MOSFET driver circuit and opto-coupler driver circuit are illustrated in Fig. 4.72. The pulse transformer and transistor driver circuit can be handled easily and the operation will be most reliable.



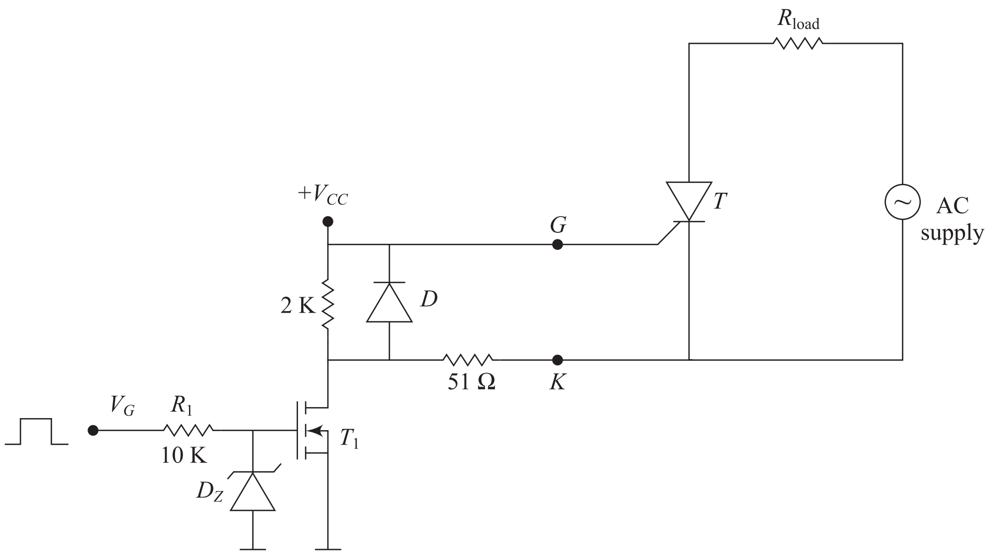


Fig. 4.72 *(a) Gate drive circuit for thyristor using pulse transformer (b) MOSFET driver circuit for thyristor*

4.23.2 Gate Pulse Amplifiers

The pulse output from integrated circuits can be directly fed to gate-cathode terminals of a low power thyristor to turn ON the device. In case of high power thyristors, the high trigger current is required. Therefore, the triggering pulses generated from ICs must be amplified and subsequently fed to thyristor for reliable turn ON-operation. In a thyristor converter circuit, the anode circuit should have the capability to withstand at high voltage but the gate circuit operates at a low voltage. Consequently, isolation is required between a power circuit using thyristor and the control circuit or gate-pulse generator. The isolation is provided by an opto-coupler or a pulse transformer.

Figure 4.73 shows a pulse amplifier circuit for amplifying the input trigger pulses. This circuit consists of a transistor, a pulse transformer for isolation and two diodes D_1 and D_2. Whenever a voltage is applied to the base of transistor or gate of a MOSFET, it will be turned ON. Consequently, the dc voltage V_{CC} appears across the pulse transformer primary winding and accordingly a pulse voltage is induced in the transformer secondary winding. Subsequently, the amplified pulse output at the secondary side of pulse transformer is applied to gate and cathode terminals of a thyristor to turn it ON. When pulse signal is applied to the gate of the MOSFET becomes zero, the MOSFET is turned OFF. Then the current flow thorough primary winding starts to fall as the voltage across secondary winding tends to fall and subsequently flux in core of pulse transformer tends to zero. As a result, a voltage of opposite polarity is induced in both primary and secondary windings of the pulse transformer. Diode D_1 is used in the secondary side of pulse transformer to prevent the negative gate current flow due to the reverse secondary voltage whenever MOSFET is in OFF state. Due to the induced reverse voltage in primary winding of pulse transformer, diode D_2 is forward biased. Subsequently, current starts to flow through primary winding, resistance R_2 and diode D_2. Hence, the stored energy in the transformer magnetic core will be dissipated in R_2 and the core flux will be reset. If the pulse width at the secondary terminals is increased, then a capacitor C is connected across R_2 as depicted in Fig. 4.74.

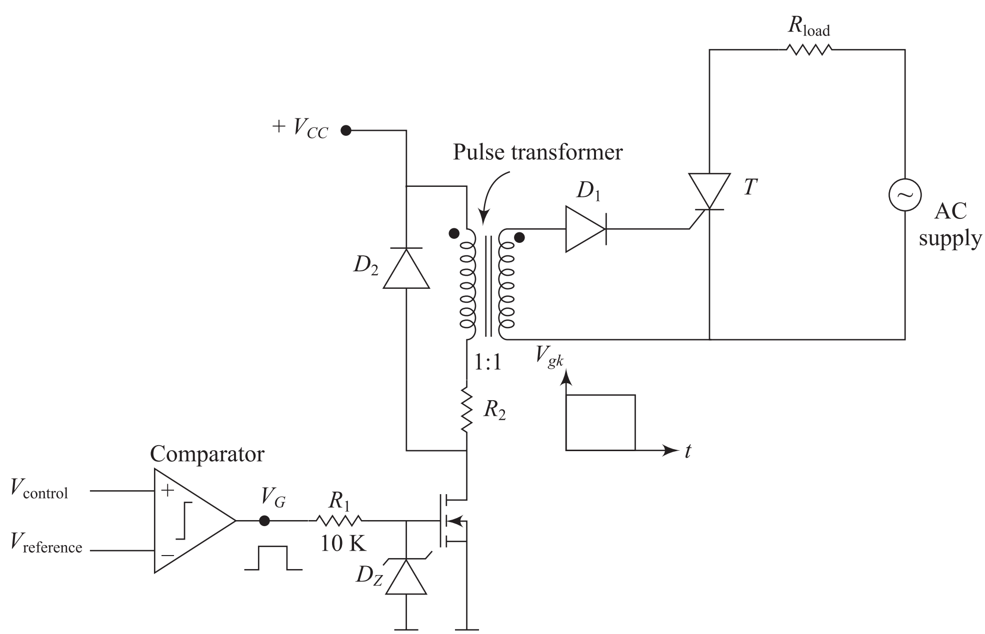


Fig. 4.73 *Pulse transformer driver circuit using MOSFET for short pulse output*

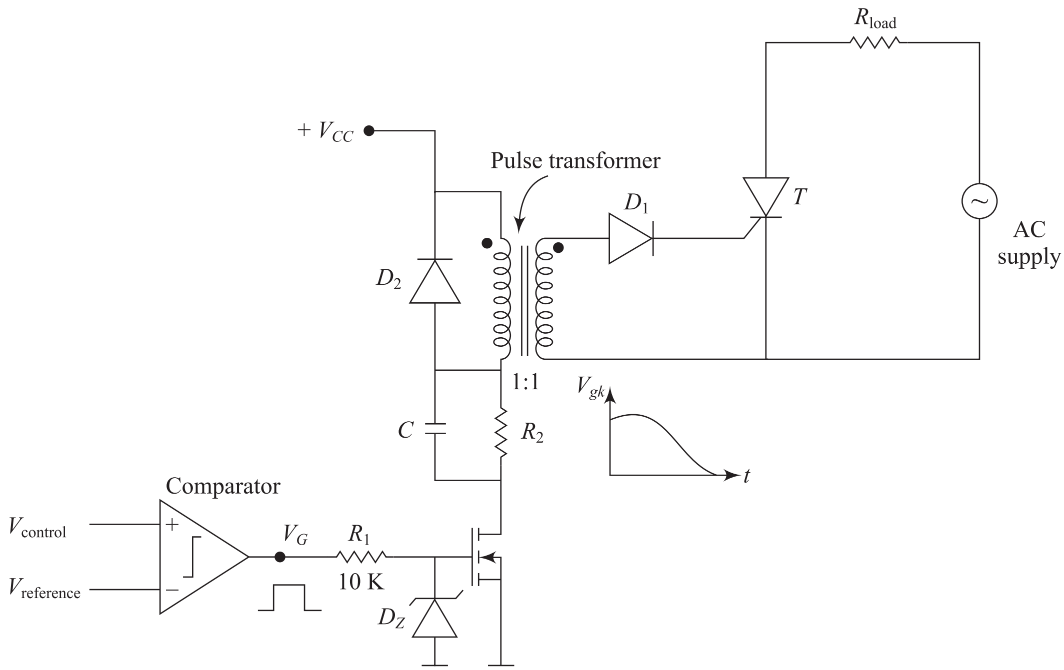


Fig. 4.74 *Pulse transformer driver circuit using MOSFET for long pulse output*

4.23.3 Pulse Train Gating Signal

The gate pulse triggering as explained above is not suitable for inductive load or R-L load as initiation of thyristor conduction is not well defined during inductive load. To overcome this difficulty, a continuous gate triggering signal or a train of firing pulses is used to turn ON a thyristor. When continuous gating is applied to turn on SCRs, the circuit has the following disadvantages:

1. Thyristor losses will be increased.
2. Distortion of output pulse due to saturation in pulse transformer by continuous pulse.

To overcome the problems of continuous gating signal, a train of firing pulses is used to turn ON a thyristor. A pulse train of gating signal is known as high frequency carrier gating signal. A pulse train can be generated by the pulse width modulation technique at frequency about 10 kHz to 20 kHz.

Figure 4.75 shows a circuit for generating a pulse train. This circuit consists of an AND logic gate, 555 timer, MOSFET, isolation pulse transformer and diodes D_1 and D_2. The input pulse signal V_i is obtained from the thyristor trigger circuit and it is fed to an AND gate. The output signal of the 555 timer, V_t is also applied to AND gate. Since, the signals V_i and V_t are processed by AND gate, a pulse train signal will be output from the output terminal of AND gate. Subsequently, the output of AND gate is fed to pulse amplifier circuit to generate pulse with higher amplitude. After that the amplified output voltage waveform is then applied to gate cathode terminals of a thyristor to turn it ON as depicted in Fig. 4.76.

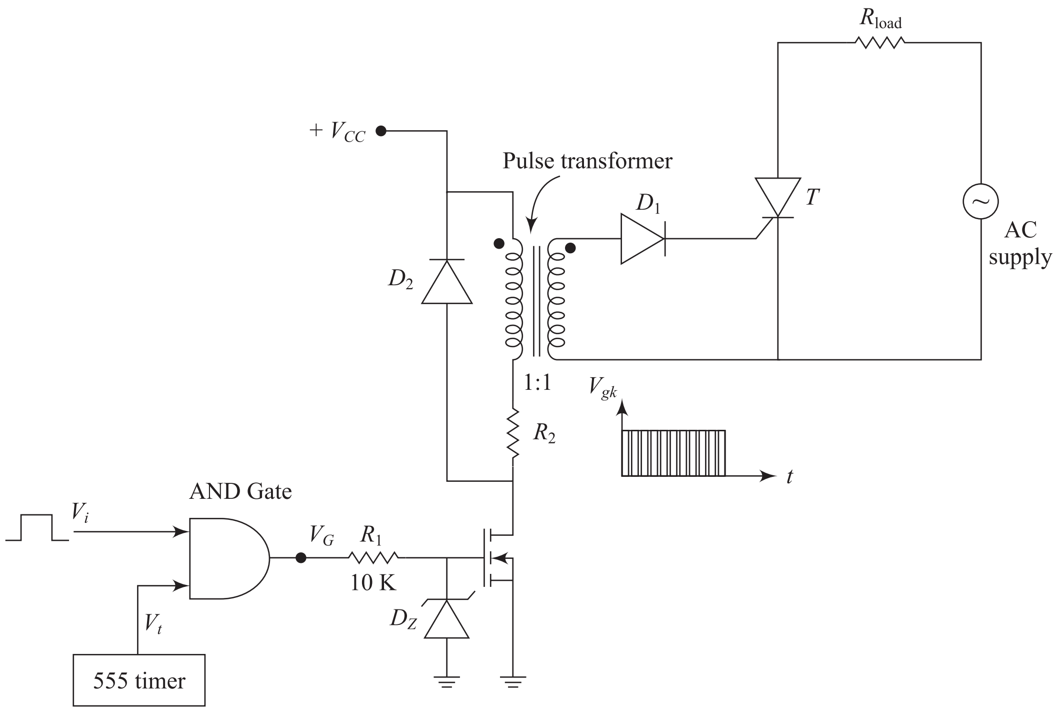


Fig. 4.75 *Pulse train gating circuit for thyristor*

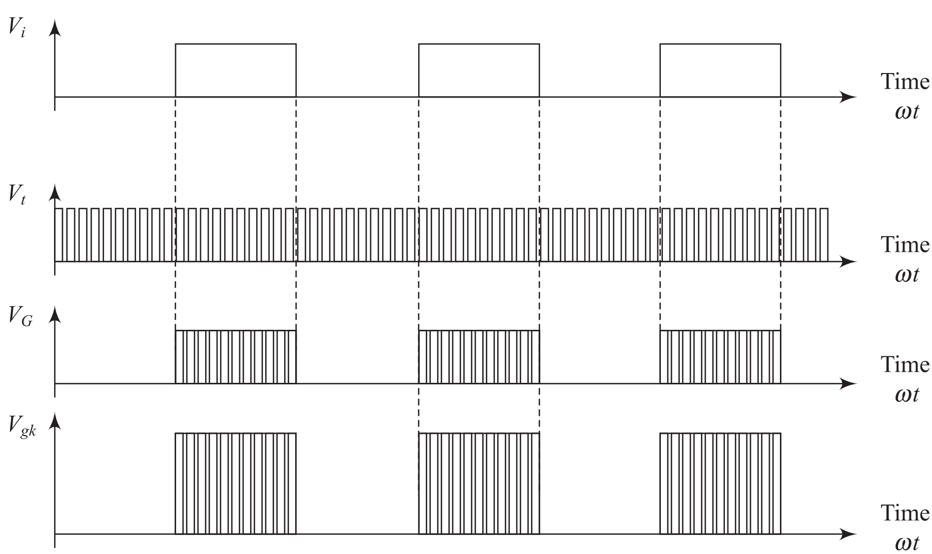


Fig. 4.76 *Waveforms for pulse train gating of thyristor*

4.23.4 Firing Circuit using Cosine Wave Scheme

Figure 4.77 shows the cosine firing scheme for thyristors of a single phase controlled rectifier. This circuit consists of synchronising transformer, non-inverting ZCD, integrator and amplifier, inverting ZCD, inverter and transistorized switch, comparator, oscillator, logic circuit and driver circuit. The synchronising transformer is used to step down the supply voltage to a specified voltage level. The input to synchronising transformer is taken from the same source at which the single phase controlled rectifier circuit is energised. The output voltage of synchronising transformer, V_1 is integrated to obtain cosine-wave signal.

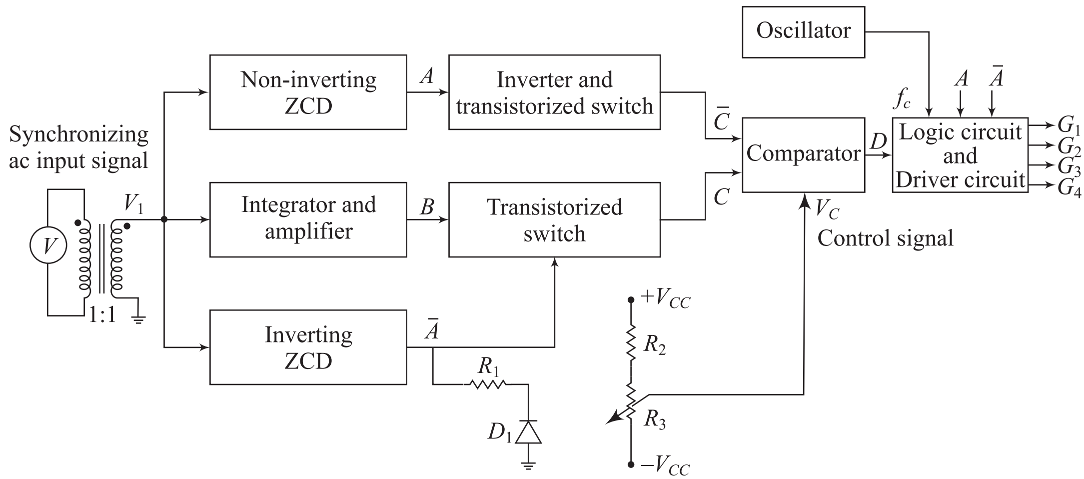


Fig. 4.77 *Schematic block diagram of cosine firing scheme for triggering thyristors*

The low voltage synchronising ac input signal which is obtained from synchronising transformer is fed to non-inverting ZCD, inverting ZCD and an integrator (90° phase shifter). Each ZCD converts the synchronising ac input signal into square wave signal. Hence, A and $\bar{A}$ voltage signals are obtained at the outputs of the ZCD as shown in Fig. 4.78. The signal B is the output of integrator and amplifier (90° phase shifter). The waveform B is also known as *cosine wave* with respect to the input voltage V_1. The signal B is fed to an operational-amplifier (inverting amplifier) and transistorized switch and subsequently $\bar{C}$ is generated. The C and $\bar{C}$ signals are added and are fed to the comparator. The comparator compares the combined waveform ($C + \bar{C}$) with a dc voltage control signal V_C.

The dc voltage control signal V_C varies from the maximum positive $+V_{CC}$ to maximum negative $-V_{CC}$ and actually the amplitude of V_C is controlled by the pot. Since V_C varies from $+V_{CC}$ to $-V_{CC}$, the firing angle can be varied from zero to 180°. The output signal of comparator, D is fed to the logic and driver circuit to generate triggering signals for thyristors T_1, T_2, T_3, and T_4.

The firing angle is governed by the intersection of $C + \bar{C}$ and V_C. When V_C is maximum positive, i.e., $+V_{CC}$, firing angle is zero. If V_C is maximum negative, i.e., $-V_{CC}$, firing angle is 180°. The firing angle in terms of α is expressed by

$$V_{CC} \cos\alpha = V_C$$

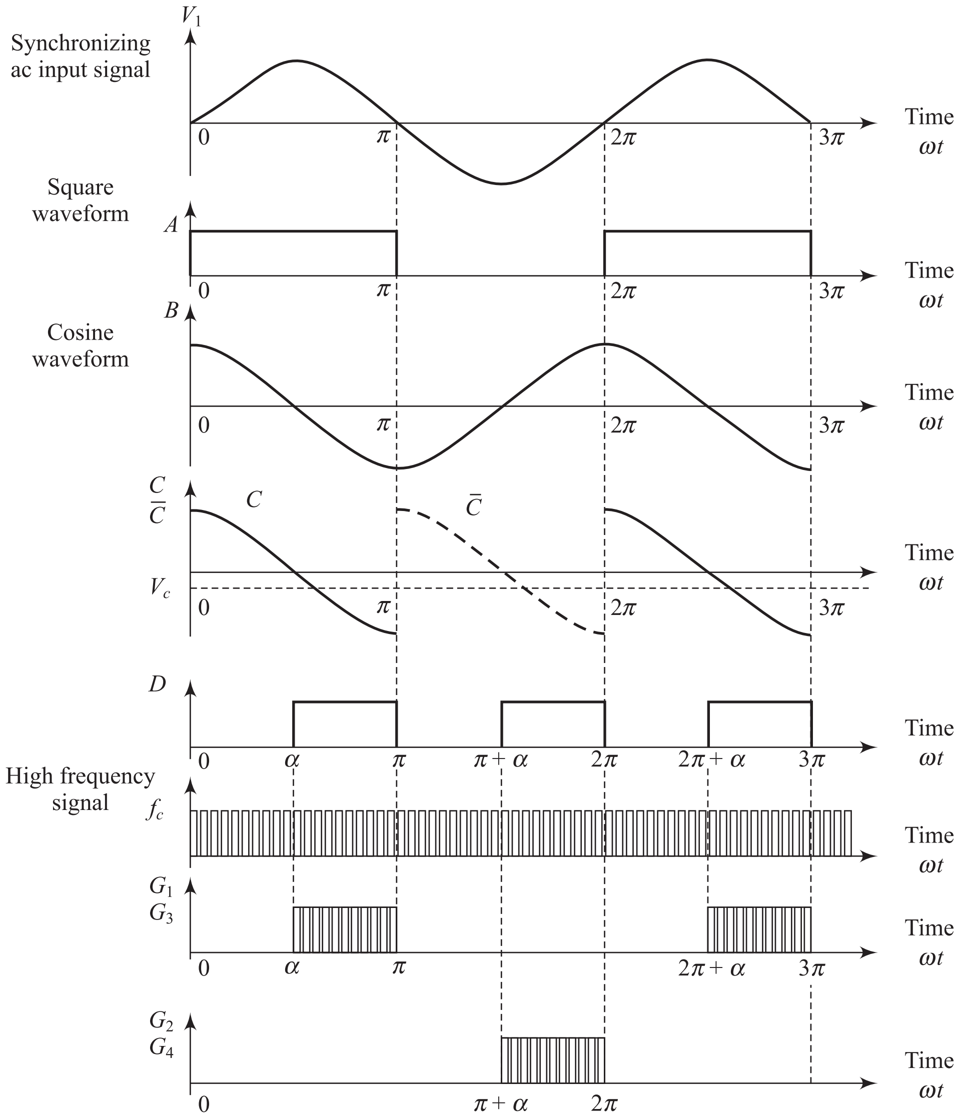


Fig. 4.78 *Waveforms of cosine firing scheme for triggering thyristors*

or $$\alpha = \cos^{-1}\left(\frac{V_C}{V_{CC}}\right)$$

In a single phase full converter, the average output voltage is

$$V_o = \frac{2V_m}{\pi}\cos\alpha$$ where, V_m is the maximum input voltage and α is firing angle

After substituting the value of $\cos\alpha$ in the above equation, we obtain

$$V_o = \frac{2V_m}{\pi}\cos\alpha = \frac{2V_m}{\pi}\frac{V_C}{V_{CC}}$$

Since, V_m and V_{CC} are constant, the output voltage of a single phase full converter is directly proportional to dc control voltage, i.e., $V_o \propto V_C$.

Hence the cosine firing scheme provides a linear transfer characteristics between the average output voltage V_o and control voltage V_C. Consequently, the linear transfer characteristics can able to improve the performance of converter. For this reason, cosine firing angle scheme is popular in the industry.

4.24 COMMUTATION OF THYRISTOR

A thyristor (SCR) can be turned OFF when its forward current I_A is reduced below the holding current I_H and when a reverse voltage is applied across the thyristor for a specified time so that the device recover to the blocking state. After it is turned OFF, the forward voltage can be reapplied after certain time so that the excess carriers in the outer and inner layers of the SCR have to decay adequately. The decay and recombination of excess carriers can be accelerated by applying a reverse voltage across the SCR. The process of turning OFF the thyristor is called *commutation*. There are two types of commutation:

1. Natural or line commutation
2. Forced commutation

4.24.1 Natural or Line Commutation

In ac circuits, when the current in the SCR goes through a natural zero and a reverse voltage appears across the SCR, the SCR will be turned OFF. This type of turned-OFF process of SCR is called *natural commutation*. In natural commutation, there is no requirement of external circuits for turning OFF the SCR. For example, class-F commutation is natural commutation. This type of commutation is used in single phase and three phase controlled rectifiers, ac voltage controllers and cyclo-converters.

4.24.2 Forced Commutation

In dc circuits, the forward current which flows through SCR must be reduced to zero forcefully by an external circuit to turn OFF the SCR. This type of turned-OFF process of SCR is known as *forced commutation*. Since separate commutation circuits are required, forced commutated thyristor converters need more control components. Consequently, the control circuit cost will be more. This type of commutation is used in dc to dc converters, dc to ac inverters.

There are different forced commutation methods. Based on the arrangement of commutation circuit components and the manner in which zero current is obtained in the SCR, forced commutations are classified as the following:

1. Class A Commutation or Resonant Commutation
2. Class B Commutation
3. Class C Commutation
4. Class D Commutation
5. Class E Commutation

Class A commutation or resonant commutation Figure 4.79 shows the Class A commutation circuit where L and C are commutation components and R_L is load resistance. For low value of load resistance, R_L, L and C are connected in series as shown in Fig. 4.79(a). In case of high value of load resistance, C and R_L are connected in parallel and then in series with L as shown in Fig. 4.79(b).

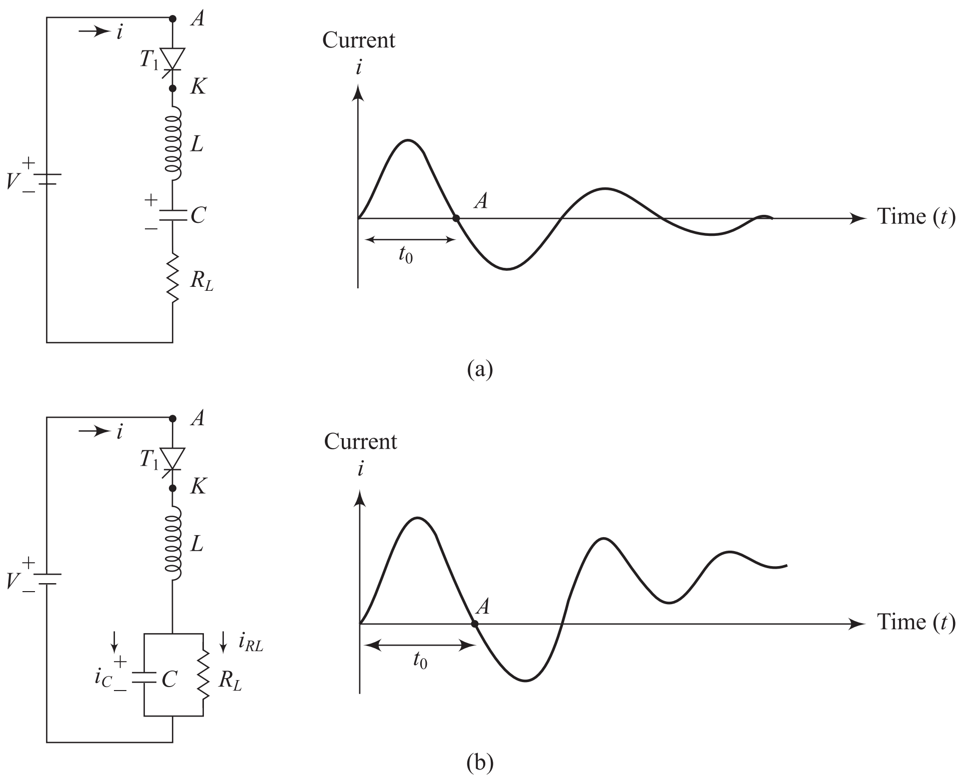


Fig. 4.79 *(a) Class A commutation using series capacitor (b) Class A commutation using shunt capacitor*

When a dc voltage is applied to the circuit and a gate pulse is also applied, thyristor will be turned ON and only the charging current of capacitor flows through thyristor. After certain time, the charging current decays to a value which is less than holding current whenever the capacitor is charged to supply voltage V. The current wave from is shown in Fig. 4.79. At point A, the current becomes zero and the commutation of thyristor occurs at this point. The time taken to turn OFF the thyristor depends on the resonant frequency and the resonant frequency is function of commutating components and load resistance. During high frequency applications above 1 kHz, this type of commutation circuit is used as the LC resonant circuit. Practically, this circuit is widely used in series inverter circuit. This commutation is also known as *resonant commutation* or *self-commutation*.

Design of Class A commutation when Load R_L is connected in series with L and C Assume thyristor T_1 is ON at $t = 0$ and initial capacitor voltage is zero. The voltage and current waveforms of class A commutation is depicted in Fig. 4.80.

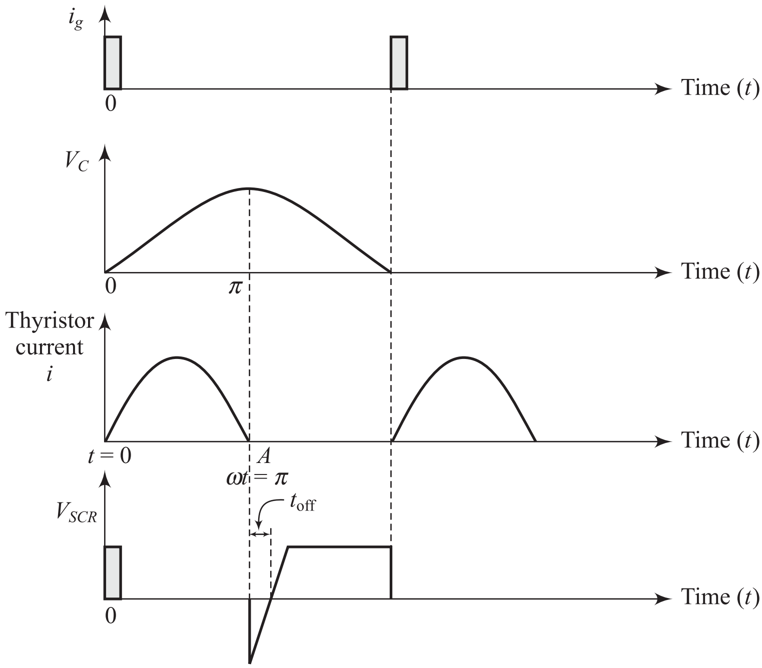


Fig. 4.80 *Voltage and current waveforms of Class A commutation*

Then the circuit equation is

$$V = L\frac{di}{dt} + \frac{1}{C}\int i \cdot dt + i \cdot R_L \tag{4.3}$$

After differentiating the Eq. (4.3) and dividing by L, we obtain

$$\frac{1}{L}\frac{d}{dt}(V) = \frac{d^2 i}{dt^2} + \frac{1}{LC}i + \frac{R_L}{L}\frac{di}{dt}$$

Since V is constant, $\frac{d}{dt}(V) = 0$ and the above equation can be written as

$$\frac{d^2 i}{dt^2} + \frac{1}{LC}i + \frac{R_L}{L}\frac{di}{dt} = 0 \tag{4.4}$$

Equation (4.4) is a second-order equation and the solution of this equation under damped condition is

$$i = e^{-\delta t}(A_1 \cos \omega t + A_2 \sin \omega t)$$

where $\delta = \frac{R_L}{2L}$, and $\omega_o = \frac{1}{\sqrt{LC}}$

$$\omega = \omega_o\sqrt{1-\delta^2} = \omega_o\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

If $i(0^+) = i(0^-) = 0$, $A_1 = 0$ and $A_2 = \frac{V}{L}$

Current $i(t) = e^{-\frac{R_L}{2L}t}\left[\frac{V}{\omega L}\sin\omega t\right]$ (4.5)

It is clear from Fig. 4.83 that thyristor current i becomes zero at $\omega t = \pi$

Then $t = \dfrac{\pi}{\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}}$ and $\dfrac{di}{dt} = -e^{-\frac{\pi R_L}{2\omega L}}\left(\dfrac{V}{L}\right)$

The capacitor voltage at the end of conduction is

$$V_C = V - V_L \text{ where, } V_L = L\frac{di}{dt}$$

Therefore,

$$V_C = V\left[1 + e^{-\frac{\pi R_L}{2\omega L}}\right]$$

When V_O is the initial state voltage of the capacitor, Eq. (4.5) can be written as

$$i(t) = e^{-\frac{R_L}{2L}t}\left[\frac{V - V_o}{\omega L}\sin\omega t\right] \text{ and}$$

$$V_C = V + e^{-\frac{\pi R_L}{2\omega}t}(V - V_o)$$

As $\omega > 0$, the condition for under damped is

$$\frac{1}{LC} - \frac{R^2}{4L^2} > 0$$

or $$\frac{1}{LC} > \frac{R^2}{4L^2}$$

or $$R < \sqrt{\frac{4L}{C}}$$

Design of Class A commutation when load R_L is connected in parallel with C When V is the applied voltage, V_o is the load voltage and i is the load current, the circuit equation is

$$V = L\frac{di}{dt} + V_o \text{ and}$$

$$i = C\frac{dV_o}{dt} + \frac{V_o}{R}$$

After applying the Laplace transform, we obtain

$$V(s) = sL.I(s) + V_o(s) \text{ and}$$

$$I(s) = sCV_o(s) + \frac{V_o(s)}{R} \tag{4.6}$$

Then $V_o(s) = V(s) - sLI(s)$

$$= \frac{V}{s} - sLI(s) \quad \text{as } V(s) = \frac{V}{s}$$

After substituting the value of $V_o(s)$ in equation (4.3), we get

$$I(s) = sC[\frac{V}{s} - sLI(s)] + \frac{V}{sR} - \frac{sLI(s)}{R}$$

or $$I(s)\left[1 + s^2LC + \frac{sL}{R}\right] = \frac{V}{s}\left[\frac{1}{R} + sC\right]$$

or $$I(s) = \frac{V}{s}\left[\frac{1+sRC}{R+sL+s^2RLC}\right]$$

or $$I(s) = \frac{V}{sRLC}\left[\frac{1+sRC}{s^2+\frac{1}{RC}s+\frac{1}{LC}}\right] \tag{4.7}$$

After inverse Laplace transform of Eq. (4.3), we obtain

$$i(t) = \frac{V}{R}\left[1 + \frac{1}{\sqrt{1-\rho^2}}\frac{\omega_n^2}{\rho}e^{-\frac{t}{RC}}\sin(\omega t + \phi)\right]$$

where, $\rho = \frac{1}{2R}\sqrt{\frac{L}{C}}$ = damping ratio,

$$\omega_n = \frac{1}{\sqrt{LC}} \text{ = undamped natural angular frequency}$$

$$\omega = \omega_n\sqrt{1-\rho^2}$$

or $$\omega = \frac{1}{\sqrt{LC}}\sqrt{1-\frac{L}{4R^2C}}$$

or $$\omega = \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$$

$$\phi = \tan^{-1}\frac{2RC\omega}{-\rho} - \tan^{-1}\frac{\sqrt{1-\rho^2}}{-\rho}$$

or $$\phi = \tan^{-1} 2RC\omega$$

At $t = 0$, $i(t) = 0$ and $\phi = -\sin^{-1}\frac{1}{A}$, the current is equal to

$$i(t) = \frac{V}{R}\left[1 + Ae^{-\frac{t}{2RC}}\sin(\omega t - \sin^{-1}\frac{1}{A})\right]$$

The load voltage may be computed from Eq. (4.3) and it is expressed as

$$V_o(s) = \frac{V}{LC\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}$$

After taking the inverse Laplace transform, we obtain

$$V_o(t) = V\frac{\omega_n}{\sqrt{1-\rho^2}}e^{-\frac{t}{2RC}}\sin\omega t + V$$

The condition for class A commutation is that the triggering frequency of thyristor must be less than ω_n so that the conduction cycle of thyristor should be completed properly.

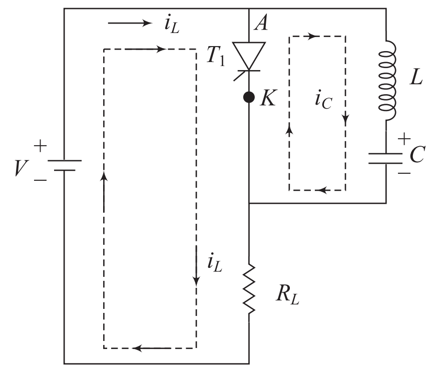


Fig. 4.81 *Class B Commutation*

Class B Commutation Figure 4.81 shows the class B commutation circuit which consist of thyristor T_1 and L-C circuit. Class B commutation is a self-commutation process by an L-C circuit. The L-C resonating circuit is connected across the thyristor. This circuit is also known as *resonant-pulse commutation*.

MODE 1 Initially the supply voltage V is applied to the circuit $t = 0$ and the capacitor starts to charge and it finally charged to voltage V with upper plate positive and lower plate negative. The charging of capacitor is done by the following path:

$$V^+ - L - C - R_L - V^-$$

MODE 2 When the gate pulse is applied to thyristor T_1 at $t = t_1$ and thyristor T_1 will be turned on, a constant current I_L flows through load and capacitor discharging current flows through capacitor. The load current I_L follows though the following path:

$V^+ - T_1 - R_L - V^-$ and the capacitor is discharged through the path

$$C^+ - L - T_1 - C^-$$

MODE 3 When the capacitor is completely discharged, it starts to charge with reverse polarity. Due to reverse polarity of capacitor voltage, the commutating current i_C opposes the load current I_L. Since thyristor is a unidirectional device, the net current flows through is

$$I_{T1} = I_L - i_C$$

As soon as the commutating current i_C is greater than the load current I_L, thyristor becomes turned OFF.

MODE 4 When thyristor is turned OFF, capacitor again starts to charge with upper plate positive and lower plate negative. Whenever capacitor is fully charged, thyristor will operate in the forward blocking state and it will be turned on when a trigger pulse is applied to thyristor.

It is clear from Fig. 4.82 that if the thyristor is turned ON by applying a gate pulse and loads current flows though thyristor and load for certain specified time duration. After the specified time period, thyristor will be turned OFF due to self-commutation.

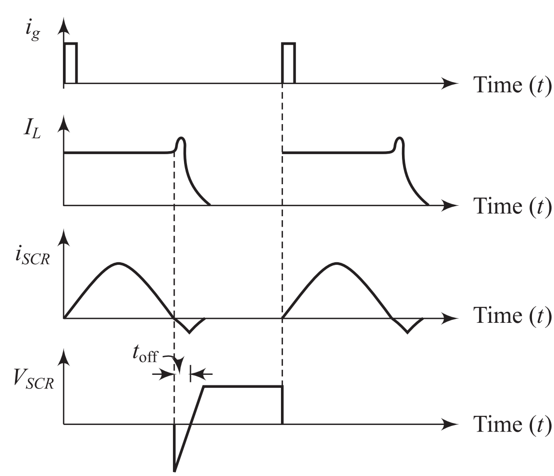


Fig. 4.82 *Voltage and current waveforms of class B commutation*

Design of Class B Commutation The KVL equation for the L-C circuit is

$$L\frac{di}{dt}+\frac{1}{C}\int idt=V$$

After differentiating the above equation, we obtain

$$L\frac{d^2i}{dt^2}+\frac{1}{C}i(t)=\frac{dV}{dt}=0 \text{ as } \frac{dV}{dt}=0$$

or

$$L\frac{d^2i}{dt^2}+\frac{1}{C}i(t)=0$$

After applying Laplace transform on the above equation, we get

$$\left(s^2L+\frac{1}{C}\right)I(s)=0$$

After solving the above equation, we find

$$i(t)=V\sqrt{\frac{C}{L}}\sin\omega_0 t \quad \text{where, } \omega_0=\frac{1}{\sqrt{LC}}$$

The peak commutation current is equal to

$$I_{c(\text{peak})}=V\sqrt{\frac{C}{L}}$$

During class B commutation process, the time taken by thyristor to get into reverse biased is approximately equal to one-quarter period of the resonant circuit. Then turn-OFF time of thyristor is equal to

$$t_{\text{off}}=\frac{\pi}{2}\sqrt{LC}$$

and the peak discharge current is about two times of load current

$$I_{c(\text{peak})}=2I_L=V\sqrt{\frac{C}{L}}.$$

Example 4.37 In a class B resonant pulse commutation circuit, L = 5 μH and C = 25 μF. The initial voltage across capacitor is 220 V, determine (a) resonant frequency, (b) peak value of resonant current and (c) turn-OFF time of thyristor.

Solution

Given: L = 5 μH, C = 25 μF and V = 220 V

(a) Resonant frequency is equal to $\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{5\times10^{-6}\times25\times10^{-6}}}=89.445\text{ kHz}$

(b) Peak value of resonant current is $I_{c(\text{peak})}=V\sqrt{\frac{C}{L}}=220\sqrt{\frac{25\times10^{-6}}{5\times10^{-6}}}=491.93\text{ A}$

(c) Turn-OFF time of thyristor is $t_{\text{off}}=\frac{\pi}{2}\sqrt{LC}=\frac{\pi}{2}\sqrt{5\times10^{-6}\times25\times10^{-6}}=17.553\,\mu\text{s}$

Class C commutation Figure 4.83 shows the class C commutation circuit which consists of two thyristors such as main thyristor T_1 and auxiliary thyristor T_A and a commutating capacitor. The load resistance R_L is connected in series with main thyristor T_1. This commutation technique is also known as *complementary commutation* as the commutation of main thyristor T_1 occurs when the auxiliary thyristor T_A is turned ON. This commutation process is also known as *complementary impulse commutation*.

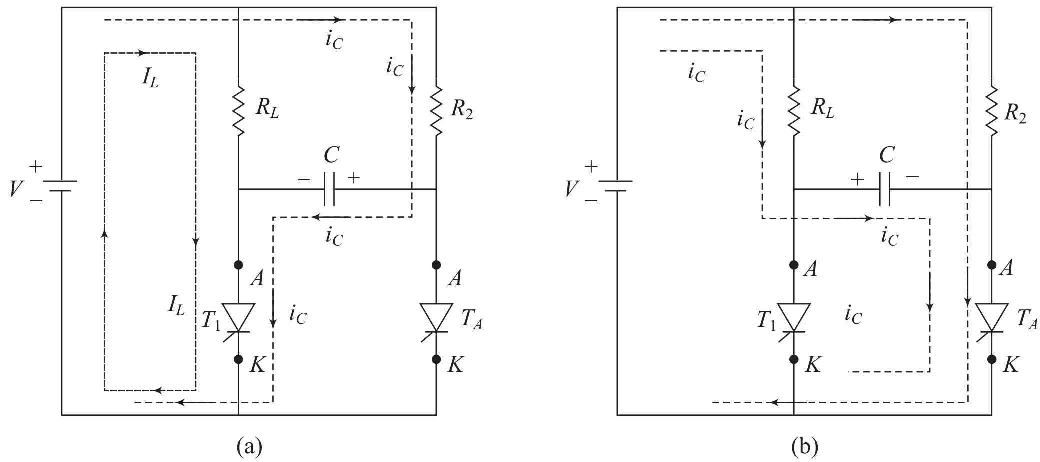


Fig. 4.83 *(a) Class C commutation circuit with T_1 ON and T_A OFF (b) Class C commutation circuit with T_1 OFF and T_A ON*

MODE 1 Initially, both the thyristors T_1 and T_A are in OFF state and the voltage across capacitor is zero. The conditions of T_1 and T_A and capacitor may be represented by

T_1 is in OFF state, T_A is OFF state and $V_C = 0$

MODE 2 When the triggering pulse is applied to main thyristor T_1 at $t = t_1$, thyristor T_1 will be turned on and two currents namely load current I_L and capacitor charging current i_C flows through the circuit.

The load current i_L follows though the following path:

$V^+ - R_L - T_1 - V^-$ and the capacitor charging current flows through the path

$V^+ - R_2 - C^+ - C^- - T_1 - V^-$

At steady state condition, capacitor is fully charged to the supply voltage V with the polarity as shown in Fig. 4.86 and the conditions of T_1 and T_A and capacitor may be represented by

T_1 is in ON state, T_A is OFF state and $V_C = V$

MODE 3 When a triggering pulse is applied to auxiliary thyristor T_A at $t = t_2$, thyristor T_A will be turned on. As soon as thyristor T_A is turned ON and starts conducting, a negative polarity voltage of the capacitor C is applied to cathode of thyristor T_1 with respect to anode. Subsequently, thyristor T_1 will be reverse biased and turned OFF immediately. Therefore, the commutation of main thyristor T_1 is possible by turning on the auxiliary thyristor T_A.

Then the capacitor C is charged through the load and its polarity becomes reverse. The charging path of capacitor is

$$V^+ - R_L - C^+ - C^- - T_A - V^-$$

At the end of this mode of operation, the conditions of T_1 and T_A and capacitor may be represented by

T_1 is in OFF state, T_A is ON state and $V_C = -V$

MODE 4 Again thyristor T_1 is triggered and turned ON at $t = t_2$. Then auxiliary thyristor will be turned OFF immediately as reverse bias voltage is applied across T_A and capacitor starts to charge in reverse direction. At the end of this mode of operation, the conditions of T_1 and T_A and capacitor may be represented by

T_1 is in ON state, T_A is OFF state and $V_C = V$

The voltage and current waveforms of class C commutation is shown in Fig. 4.84. The class C commutation is very useful for operating frequency below 1 kHz and this commutation technique is used in Mc Murray Bedford inverter. The characteristics of class C commutation are sure and most reliable commutation.

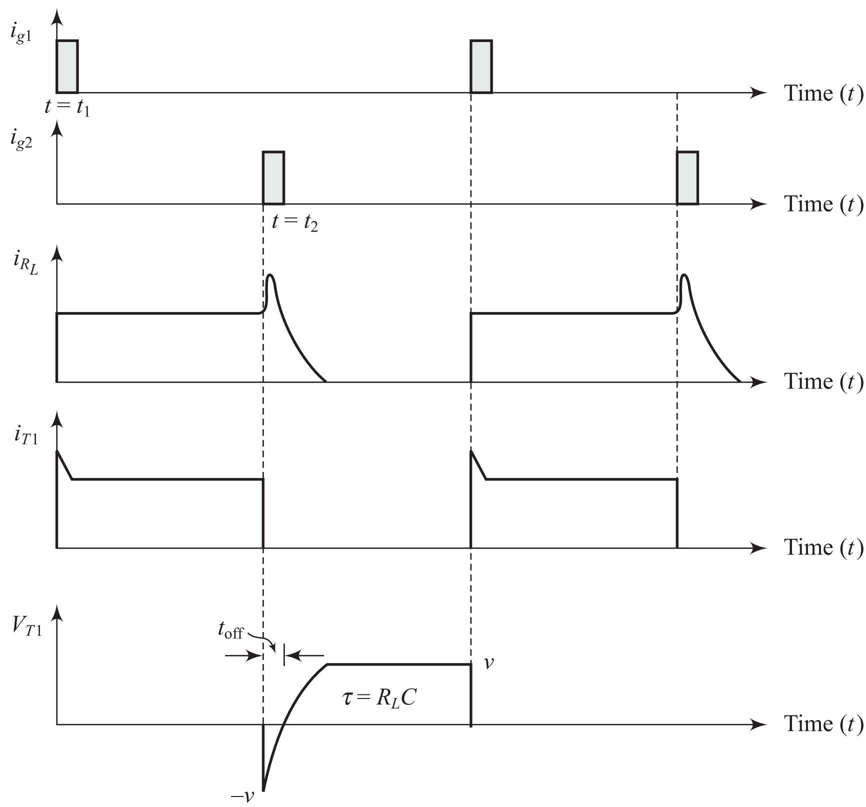


Fig. 4.84 *Voltage and current waveforms of Class C commutation*

Design of Class C commutation When thyristor T_1 conducts and capacitor C is charged to input voltage V through resistance R_L. Whenever a triggering pulse is applied to thyristor T_A and T_A is turned ON, a voltage twice the dc supply voltage is applied to the R_L C series circuit. Then applying the KVL in the closed loop, we can write:

$$iR_L + \frac{1}{C}\int i\,dt + V_{TA} - V = 0$$

or $$iR_L + \frac{1}{C}\int i\,dt - V = 0 \quad \text{as} \quad V_{TA} = 0$$

Applying Laplace transform on the above equation, we get

$$\left(R_L + \frac{1}{sC}\right)I(s) = \frac{V}{s}$$

After applying the inverse Laplace transform of above equation, we obtain

$$i(t) = \frac{2V}{R_L}e^{-\frac{t}{R_L C}}$$

The voltage across thyristor T_1 is equal to

$$V_{T1} = V - iR_L$$

or $$V_{T1} = V - \frac{2V}{R_L}e^{-\frac{t}{R_L C}}R_L$$

or $$V_{T1} = V - 2Ve^{-\frac{t}{R_L C}} = V(1 - 2e^{-\frac{t}{R_L C}})$$

To turn-OFF the main thyristor $T_{1,}$ the capacitor voltage should be equal to the voltage across thyristor $T_{1.}$ Therefore, $V_C = V_{T1} = V(1 - 2e^{-\frac{t}{R_L C}})$

At $t = t_{\text{off}}, V_C = 0$ and $V(1 - 2e^{-\frac{t_{\text{off}}}{R_L C}}) = 0$

or $$(1 - 2e^{-\frac{t_{\text{off}}}{R_L C}}) = 0$$

Therefore, $t_{\text{off}} = 0.6931 R_L C$

or $$C = 1.44\frac{t_{\text{off}}}{R_L}$$

The maximum permissible $\frac{dV}{dt}$ across thyristor T_1 using commutating components is given by

$$\left.\frac{dV}{dt}\right|_{\max} > \frac{2V}{R_L C}$$

Example 4.38 In a Class C commutation, if $V = 220$ V, $R_L = 20\ \Omega$, $R_2 = 100\ \Omega$ calculate (a) the peak value of current through thyristors T_1 and T_2 (b) the value of capacitance C when turn-OFF time of each thyristor is equal to 25 μs. Assume that the factor of safety is 2. Assume $T_2 = T_A$.

Solution

(a) The peak value of current through thyristors T_1 is $V\left(\frac{1}{R_L}+\frac{2}{R_2}\right)$ when thyristor T_1 is turned ON to commutate at the instant $i_{T2} = 0$, $v_{T2} = -V$, $v_{T1} = -0$ and $i_C = \frac{2V}{R_2}$

$$i_{T1} = V\left(\frac{1}{R_L}+\frac{2}{R_2}\right) = 220\left(\frac{1}{20}+\frac{2}{100}\right) = 15.4\text{ A}$$

The peak value of current through T_2 is equal to

$$i_{T2} = V\left(\frac{2}{R_L}+\frac{1}{R_2}\right) = 220\left(\frac{2}{20}+\frac{1}{100}\right) = 44\text{ A}$$

(b) The value of capacitance C when turn-off time of each thyristor is equal to 25 μs is

$$C = 1.44\frac{t_{\text{off}}}{R_L} = 1.44\times\frac{25\times10^{-6}}{20} = 1.8\,\mu\text{F}$$

If factor of safety is 2, the value of C is $C = 1.44\frac{2\times t_{\text{off}}}{R_L} = 1.44\times\frac{2\times25\times10^{-6}}{20} = 3.6\,\mu\text{F}$

Class D commutation Figure 4.85 shows the class D commutation circuit which consists of two thyristors such as main thyristor T_1 and auxiliary thyristor T_A, inductor L, diode D and a commutation capacitor C. The main thyristor T_1 and load resistance R_L act as a power circuit but inductor L, diode D and auxiliary thyristor T_A are used to form the commutation circuit.

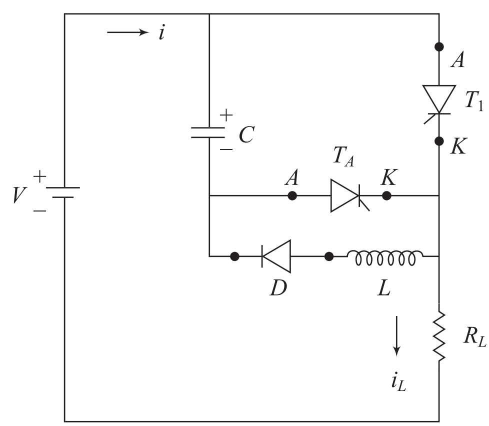


Fig. 4.85 *Class D commutation circuit*

MODE 1 Initially, the dc voltage V is applied to circuit, the thyristor T_1 and T_A are in OFF-state. There is no current flow though dc supply and commutation circuit. The conditions of T_1 and T_A and capacitor may be represented by

T_1 is in OFF state, T_A is OFF state and $V_C = 0$

MODE 2 Firstly the triggering pulse is applied to auxiliary thyristor T_A, thyristor T_A will be turned ON and capacitor C gets charged. The capacitor charging current flows through the path

$$V^+ - C^+ - C^- - T_A - R_L - V^-$$

Since the voltage across the capacitor C increases gradually, the current flow through the thyristor T_A decreases slowly. Whenever the capacitor is fully charged to V, the auxiliary thyristor T_A gets turned OFF.

In this mode, the conditions of T_1 and T_A and capacitor may be represented by
Initially T_A is in ON state, and at steady state condition
T_1 is in OFF state, T_A is OFF state and $V_C = V$

MODE 3 When the triggering pulse is applied to main thyristor T_1, the current flows in the two different paths:

The load current I_L follows though the following path:

$$V^+ - T_1 - R_L - V^-$$

and commutation current (capacitor discharging current) flows through the following path:

$$C^+ - T_1 - L - D - C^-$$

When the capacitor is fully discharged, its polarity will be reversed. The discharging of capacitor C in reverse direction is not possible due to presence of diode D.

At the end of this mode of operation, T_1 is in ON state, T_A is in OFF state and $V_C = -V$.

MODE 4 In this mode, whenever the thyristor T_A is triggered and turned on, capacitor C starts to discharge through the following path:

$$C^+ - T_A - T_1 - C^-$$

When the commutating current (discharging current of capacitor) becomes more than load current I_L, thyristor T_1 gets turned OFF. At the end of mode 3 operation,

T_1 is in OFF state and T_A is OFF state

Capacitor C will again charge to the supply voltage V with the polarity as shown in Fig. 4.86.

Since the commutation energy rapidly transfers to the load, high efficiency is possible in class D commutation. This commutation is most commonly used in Jones chopper circuit.

Design of Class D commutation *Design of Commutation capacitor* (C)
The magnitude of commutating capacitor (C) depends on the following parameters:

1. Maximum load current to be commutated
2. Turn off time of thyristor, t_{off}
3. The input voltage V

The turn-OFF time of thyristor, t_{off} is available from manufacturer data sheet. The capacitor voltage is changed from $-V$ to 0 during turn off time t_{off}. By assuming load current I_L is constant during turn-OFF time t_{off}, we can write

$$CV = I_L t_{\text{off}} \text{ as } CV = q = it \quad \text{where, } i = I_L \text{ and } t = t_{\text{off}}$$

or
$$C = \frac{I_L t_{\text{off}}}{V}$$

Design of Commutation inductor (L)
The magnitude of commutating inductor (L) depends on the following parameters:

1. The maximum permissible value of capacitor current is I_C when the main thyristor T_1 operates in the ON state.
2. During the time interval $t_2 - t_1$, the capacitor voltage must be reset to correct polarity for commutating thyristor T_1.

As the capacitor current I_C is an oscillatory current in nature and flows through main thyristor T_1, L, D and C when thyristor T_1 is triggered and turned ON.

The peak value of current I_C is given by

$$I_{C(\text{peak})} = \frac{V}{\omega_r L} \quad \text{where, } \omega_r = \text{Oscillating frequency} = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

After substituting the value of ω_r, we get

$$I_{C(\text{peak})} = V\sqrt{\frac{C}{L}}$$

The periodic time during oscillation is equal to

$$T_r = \frac{2\pi}{\omega_r} = 2(t_1 - t_2)$$

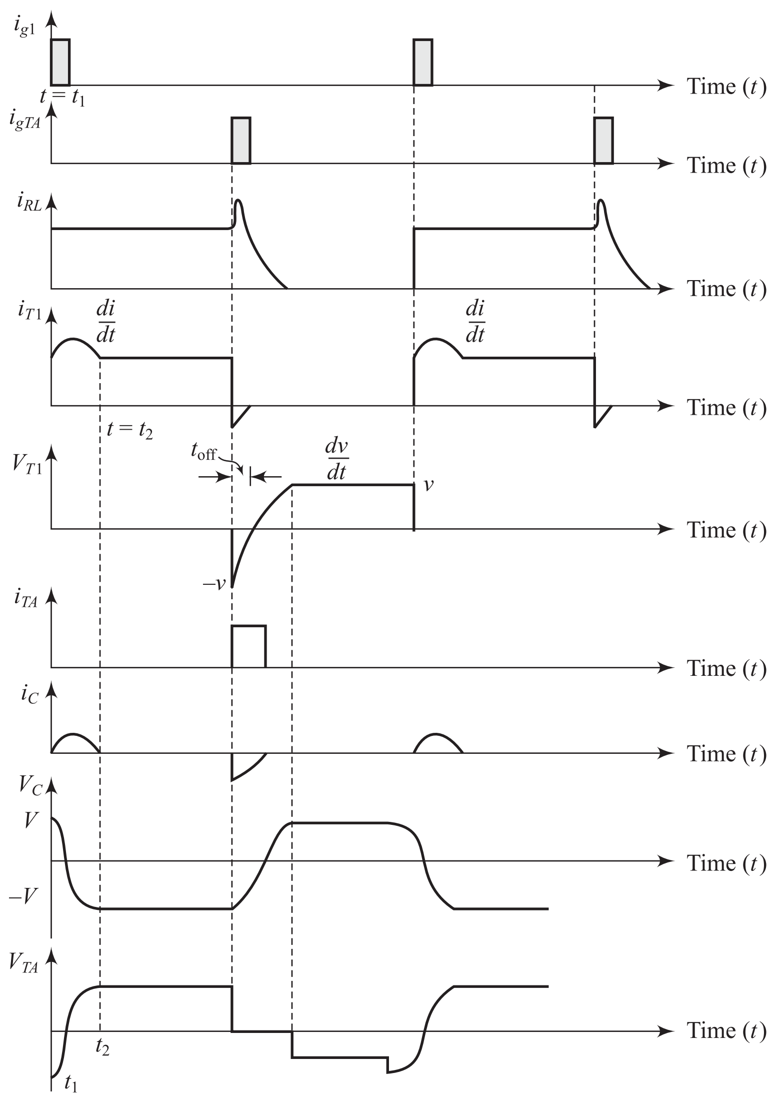


Fig. 4.86 *Voltage and current waveforms of Class D commutation*

When $I_{L(\max)}$ is the maximum current through the main thyristor T_1 for commutation, the capacitor peak current must be equal to the load current.

Therefore, $$V\sqrt{\frac{C}{L}} \le I_{L(\max)}$$

or $$L \ge C\left(\frac{V}{I_{L(\max)}}\right)^2$$

Example 4.39 In a Class D commutation circuit, V = 200 V, L = 25 μH and C = 50 μf. If load current is 100 A, determine (a) peak value of current through capacitance, main thyristor and auxiliary thyristor and (b) turn-OFF time of thyristors.

Solution

Given: V = 200 V, L = 25 μH, C = 50 μF and I_L = 100 A

(a) An oscillatory current flows through C, T_1, and L and it is expressed by

$$i_c(t) = V\sqrt{\frac{C}{L}}\sin\omega_o t$$

Peak value of current through capacitance is $I_p = V\sqrt{\frac{C}{L}} = 200\sqrt{\frac{50\times10^{-6}}{25\times10^{-6}}} = 282.842\text{ A}$

Peak value of current through main thyristor T_1 is $I_p + I_L = 282.842 + 100 = 382.842\text{ A}$

Peak value of current through auxiliary thyristor T_A is $I_L = 100\text{ A}$

(b) As $C = \frac{I_L t_{\text{off}}}{V}$, Turn-OFF time of main thyristor is equal to

$$t_{\text{off}} = C\frac{V}{I_L} = 50\times10^{-6}\times\frac{200}{100} = 100\ \mu\text{s}$$

Turn-OFF time of auxiliary thyristor is equal to

$$t_{\text{off}} = \frac{\pi}{2\omega_o} = \frac{\pi}{2}\sqrt{LC} = \frac{\pi}{2}\sqrt{25\times10^{-6}\times50\times10^{-6}} = 55.50\ \mu\text{s} \quad \text{as } \omega_o = \frac{1}{\sqrt{LC}}$$

Class E commutation Figure 4.87 shows the class E commutation circuit. In this commutation method, the reverse voltage applied to the current carrying thyristor from an external pulse source. This commutation is also known as *external pulse commutation*. In this case, the commutating pulse is applied through a pulse transformer which is design in such a way that there should be tight coupling between the primary and secondary windings of pulse transformer. The pulse transformer is also designed with a small air gap so that there will not be any saturation when a pulse voltage is applied to its primary. Whenever the commutation of thyristor T_1 is required, the pulse duration equal to or slightly greater than the specified turn-OFF time of thyristor T_1 must be applied. The voltage and current waveforms of class E commutation is illustrated in Fig. 4.88.

MODE 1 When the thyristor T_1 is triggered and turned ON, the current starts to flow through the pulse transformer and load resistance R_L. The current flows through the following path:

$$V^+ - T_1 - \text{Primary of pulse transformer} - R_L - V^-$$

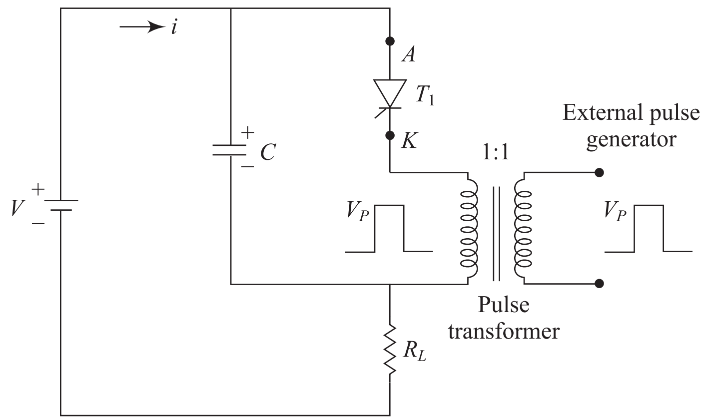


Fig. 4.87 *Class E commutation*

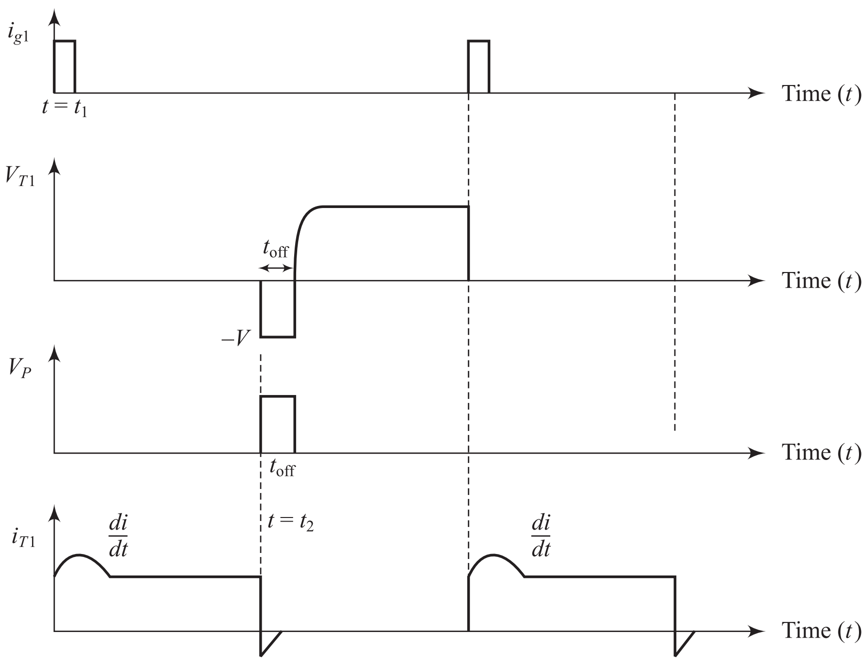


Fig. 4.88 *Voltage and current waveforms of Class E commutation*

MODE 2 When an external pulse voltage V_P is applied across the primary of the pulse transformer, a voltage will be induced in the secondary of the pulse transformer. This induced voltage in the secondary appears as reverse voltage ($-V_P$) across the thyristor T_1. Subsequently, thyristor T_1 gets turned OFF. Since the frequency of induced pulse voltage is very high, the capacitor provides almost zero impedance. When the thyristor T_1 is turned OFF completely, the load current decays to zero. Before the computation process, the capacitor voltage remains at a value of about 1 V.

Since the minimum energy is required for commutation and both the time ratio and pulse width regulations are easily incorporated, this type of commutation method is very efficient.

Class F commutation Figure 4.89 shows the class F commutation and it is voltage and current waveforms are depicted in Fig. 4.90. This type of commutation is also known as *natural commutation* or *ac line commutation*. This type of commutation only occurs when the input voltage is ac. When the thyristor based converter circuit is energised by an ac source voltage and the gate signal is applied to thyristor T_1 during positive half-cycle of supply voltage, thyristor T_1 becomes turn ON and current flows thyristor and load. At the end of each positive half-cycle of supply voltage, current passes through its natural zero and then ac source applies a reverse voltage across thyristor automatically. Consequently, thyristor T_1 will be turned OFF. This commutation technique is called natural commutation as no external circuit is required to turn OFF the thyristor. Usually, this commutation method is commonly used in controlled rectifiers, line commutated inverters, ac voltage controllers and cyclo-converters.

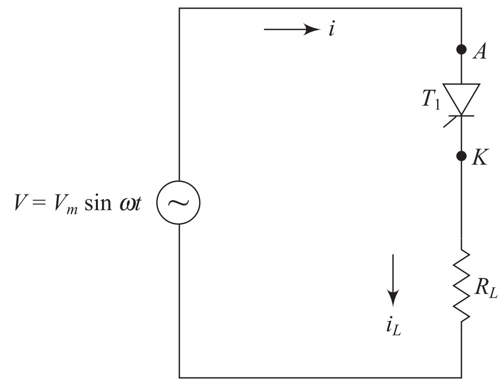


Fig. 4.89 *Class F commutation*

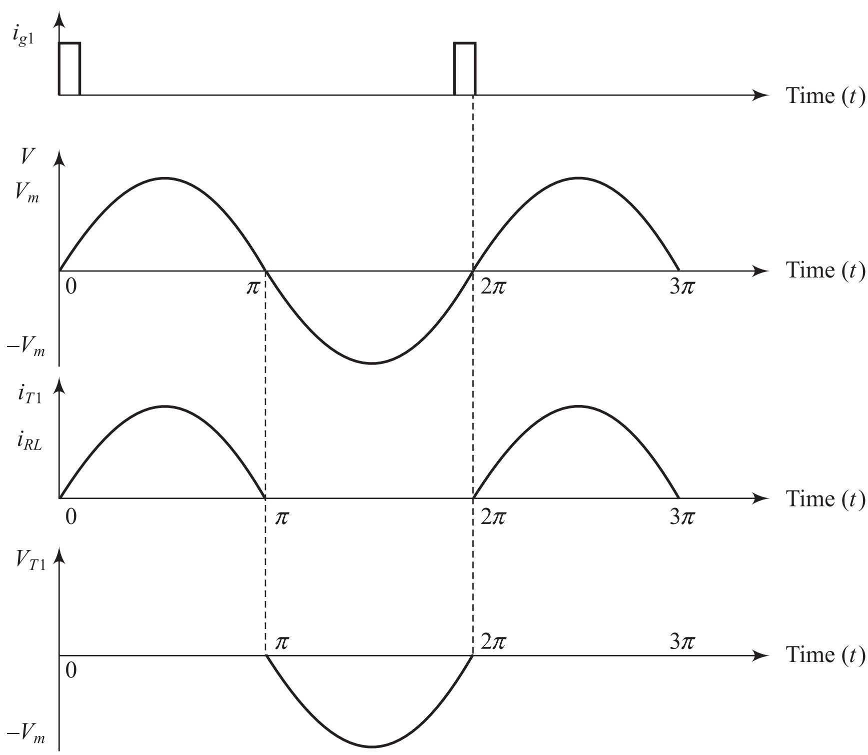


Fig. 4.90 *Voltage and current waveforms of Class F commutation*

Example 4.40 In class A commutation circuit, a thyristor is connected in series with *R-L-C*. When $L = 10$ µH, $C = 20$ µF and $R = 1$ Ω, check whether self-commutation is possible or not. Determine the conduction time of thyristor.

Solution

Given: $L = 10$ µH, $C = 20$ µF and $R = 1$ Ω

The damped frequency of oscillation is

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Since $\omega_d > 0$, $\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 > 0$ or $R < \sqrt{\frac{4L}{C}}$

Then $R < \sqrt{\frac{4 \times 10 \times 10^{-6}}{20 \times 10^{-6}}}$ or $R < \sqrt{2}$

In the circuit, the value of resistance is 1 Ω which is less than $\sqrt{2}$. Therefore, the circuit is underdamped.

Then $\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\frac{1}{10 \times 10^{-6} \times 20 \times 10^{-6}} - \left(\frac{1 \times 10^6}{2 \times 10}\right)^2} = 50{,}000$ rad/sec

The conduction time of thyristor is $t_1 = \frac{\pi}{\omega_r} = \frac{\pi \times 10^6}{50000}\ \mu\text{s} = 62.857\ \mu\text{s}$

Example 4.41 In class A commutation circuit, a thyristor is turned on at $t = 0$. Compute (a) the conduction time of thyristor and (b) voltage across thyristor and capacitor when thyristor is turned OFF. Assume the following parameters $L = 10$ mH, $C = 20$ µF, $R = 0$ Ω and $V = 250$ V.

Solution

Given: $L = 10$ mH, $C = 20$ µF and $R = 0$ Ω

The KVL equation of the circuit is

$$V = L\frac{di}{dt} + \frac{1}{C}\int i dt$$

The solution of above equation is

$$i(t) = V\sqrt{\frac{C}{L}}\sin\omega_o t \quad \text{where, } \omega_o = \sqrt{\frac{1}{LC}} \text{ is the resonant frequency}$$

The voltage across capacitor is equal to

$$V_C(t) = V(1 - \cos\omega_o t)$$

(a) If the thyristor starts conduction at $t = 0$, it conducts up to t_o where, $\omega_o t_o = \pi$ and the conduction time of thyristor is $t_o = \frac{\pi}{\omega_o} = \pi\sqrt{LC} = \pi\sqrt{10 \times 10^{-3} \times 20 \times 10^{-6}} = 0.4472$ ms

(b) The voltage across thyristor at turned-OFF condition is $V_T = -V = -250V$

The voltage across capacitor when thyristor is turned OFF

$$V_C(t_o) = V(1 - \cos\omega_o t_o) = 2V = 2 \times 250 = 500\ V \quad \text{as } \omega_o t_o = \pi$$

Example 4.42 In a class D commutation circuit, determine the value of commutating capacitor C and commutating inductor L with the help of following data:

$$V = 100V, I_{L(\max)} = 50 \text{ A and } t_{\text{off}} \text{ for } T_1 = 20\mu\text{s}$$

Solution

Assuming 50% tolerance given to turn-OFF time of thyristor T_1, the turn-off time is equal to

$$t_{\text{off}} = \left(20 + 20 \times \frac{50}{100}\right)\mu\text{s} = 30\ \mu\text{s}$$

The value of commutating capacitor is

$$C = \frac{I_L t_{\text{off}}}{V} = \frac{50 \times 30 \times 10^{-6}}{100} = 15\ \mu\text{s}$$

The value of commutating inductor is

$$L \geq C\left(\frac{V}{I_{L(\max)}}\right)^2$$

or $$L \geq 15 \times 10^{-6}\left(\frac{100}{50}\right)^2$$

or $$L \geq 60\ \mu\text{H}$$

Example 4.43 Figure 4.91 shows a voltage commutation circuit. Determine the turn off time of main thyristor when C = 15 µF, R = 10 Ω and V = 220 V. Assume the capacitor is charged to voltage V.

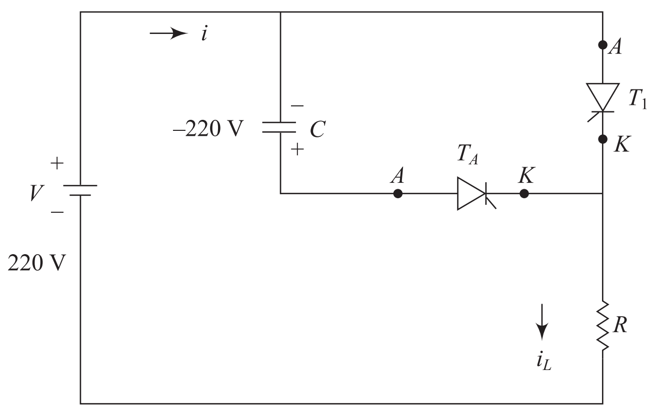


Fig. 4.91

Solution

Given: C = 15 µF, R = 10 Ω and V = 220 V

When T_1 is in off state and T_A in on state, the KVL equation is

$$iR + \frac{1}{C}\int i dt = V$$

After Laplace transform, we get

$$RI(s)+\frac{1}{C}\left(\frac{I(s)}{s}-\frac{CV}{s}\right)=\frac{V}{s}$$

or $$\left(R+\frac{1}{sC}\right)I(s)=\frac{2V}{s}$$

or $$I(s)=\frac{2V}{s}\frac{sC}{sRC+1}=\frac{2V}{R}\frac{1}{s+\frac{1}{RC}}$$

Then $$i(t)=\frac{2V}{R}e^{-\frac{t}{RC}}$$

The voltage across capacitor C is

$$v_c(t)=\frac{1}{C}\int idt+V_c(0) \text{ where, } V_c(0)=-V$$

$$=\frac{1}{C}\int_0^t\frac{2V}{R}e^{-\frac{t}{RC}}dt-V=V\left(1-2e^{-\frac{t}{RC}}\right)$$

The turn-OFF time of auxiliary thyristor t_C is equal to time taken to change the capacitor voltage from $-V$ to zero.

Then, $0=V\left(1-2e^{-\frac{t_c}{RC}}\right)$ or, $t_c=RC\ln(2)=10\times15\times10^{-6}\ln(2)=103.97\ \mu\text{s}$

Example 4.44 Figure 4.92 shows a load commutation circuit. Find the value of commutation time of thyristor and resonant frequency when L = 2 mH and C = 10 μF.

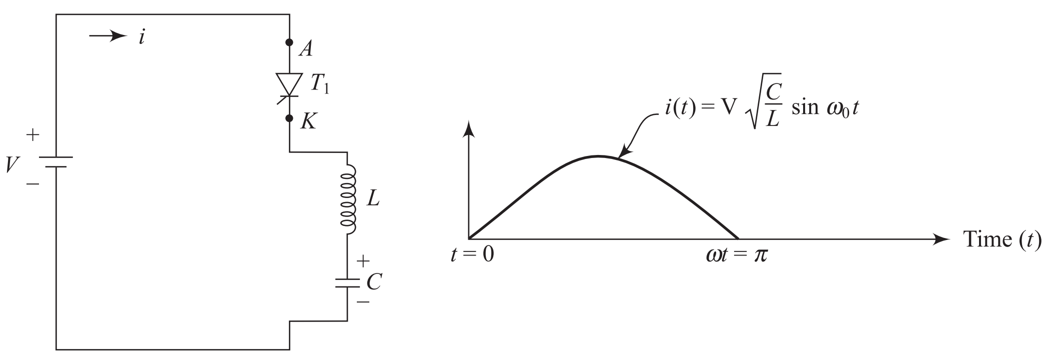


Fig. 4.92

Solution

Given: L = 2 mH and C = 10 μF

When the thyristor is ON, KVL equation of the circuit is

$$L\frac{di}{dt}+\frac{1}{C}\int idt=V$$

The solution of above equation is

$$i(t)=V\sqrt{\frac{C}{L}}\sin\omega_0 t \quad \text{where,} \quad \omega_0=\frac{1}{\sqrt{LC}}$$

At $t = t_c$, thyristor is switched OFF. Then $\omega_0 t_c = \pi$

or conduction time of thyristor is $t_c = \frac{\pi}{\omega_0} = \pi\sqrt{LC} = \pi\sqrt{2\times10^{-3}\times10\times10^{-6}} = 44.446\ \mu\text{s}$

As $\omega_0 = \frac{1}{\sqrt{LC}}$, the resonant frequency is $2\pi f_0 = \frac{1}{\sqrt{LC}}$

or
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2\times10^{-3}\times10\times10^{-6}}} = 11.2494\ \text{kHz}$$

Example 4.45 In a class C commutation circuit, the dc input voltage is 220 V and current through R_L and R_1 is 20 A. If the turn-OFF time of both main and auxiliary thyristor is 25 μs, determine the value of commutating capacitor C.

Solution

$$V = 220\ \text{V},\ I = 20\ \text{A},\ t_{\text{off}} = 25\ \mu\text{s}$$

The value of R_L and R_1 is

$$R_L = R_1 = \frac{V}{I} = \frac{220}{20} = 11\ \Omega$$

The value of commutating capacitor C is

$$C = 1.44\frac{t_{\text{off}}}{R_1} = 1.44\times\frac{25\times10^{-6}}{11} = 3.272\ \mu\text{F}$$

Example 4.46 A class D commutation circuit has the following parameters:

$$V = 230\ \text{V},\ L = 20\ \mu\text{H and } C = 40\ \mu\text{F}$$

If the load current is 120 A, determine the circuit turn-OFF times for main and auxiliary thyristors.

Solution

Given: $V = 230\ \text{V}, L = 20\ \mu\text{H}, C = 40\ \mu\text{F}$ and $I_O = 120$ A

The turn-OFF times for main and auxiliary thyristors is

$$t_{\text{turn-off}} = \frac{CV}{I_o} = \frac{40\times10^{-6}\times230}{120}\text{sec} = 0.07666\ \text{m sec}$$

Example 4.47 For a class C commutation circuit, the dc input voltage is 200 V and current through R_L and R_1 is 10 A. If the turn-OFF time of both main and auxiliary thyristor is 40 μs, calculate the value of commutating capacitor C.

Solution

Given: $V = 200$ V, $I = 10$ A, $t_{\text{off}} = 40$ μs

The value of R_L and R_1 is

$$R_L = R_1 = \frac{V}{I} = \frac{200}{10} = 20\ \Omega$$

The value of commutating capacitor C is

$$C = 1.44\frac{t_{\text{off}}}{R_1} = 1.44\times\frac{40\times10^{-6}}{20} = 2.88\ \mu\text{F}$$

Example 4.48 A class D commutation circuit has the following parameters:

$$V = 220 \text{ V}, L = 12 \text{ μH and } C = 25 \text{ μF}$$

When the load current is 100 A, find the circuit turn off times for main and auxiliary thyristors.

Solution

Given: $V = 220$ V, $L = 12$ μH, $C = 25$ μF and $I_O = 100$ A

The turn off times for main and auxiliary thyristors is

$$t_{\text{turn-off}} = \frac{CV}{I_o} = \frac{25 \times 10^{-6} \times 220}{100} \text{ sec} = 0.055 \text{ m sec}$$

4.25 DIAC

A DIAC is a two-terminal semiconductor device. Figure 4.93 shows the arrangement of semiconductor layers. DIAC is a bidirectional avalanche diode which can be switched from the OFF-state to the ON-state in both positive and negative direction. The terminals are not designated as cathode and anode, but as main terminal-1 (MT-1) and main terminal-2 (MT-2). Sometimes, they are also designated as anode-I and anode-II as the device can conduct and anode current flows in either direction.

Figure 4.93 shows the operating characteristic of a DIAC. It is clear from Fig. 4.94 that it can work for either polarity of ac supply. Therefore, DIAC is also called two-terminal ac (*di* + *ac*) switch. The device operates in I and III quadrants and each direction has a different value of breakover voltage (V_{BO}). DIAC has no gate and the only way to fire a DIAC is to apply its breakover voltage V_{BO}.

When the breakover voltage is reached across two terminals in either direction, DIAC conducts. If the MT-1 is positive with respect to MT-2 and the voltage across two terminals V_{12} is greater than

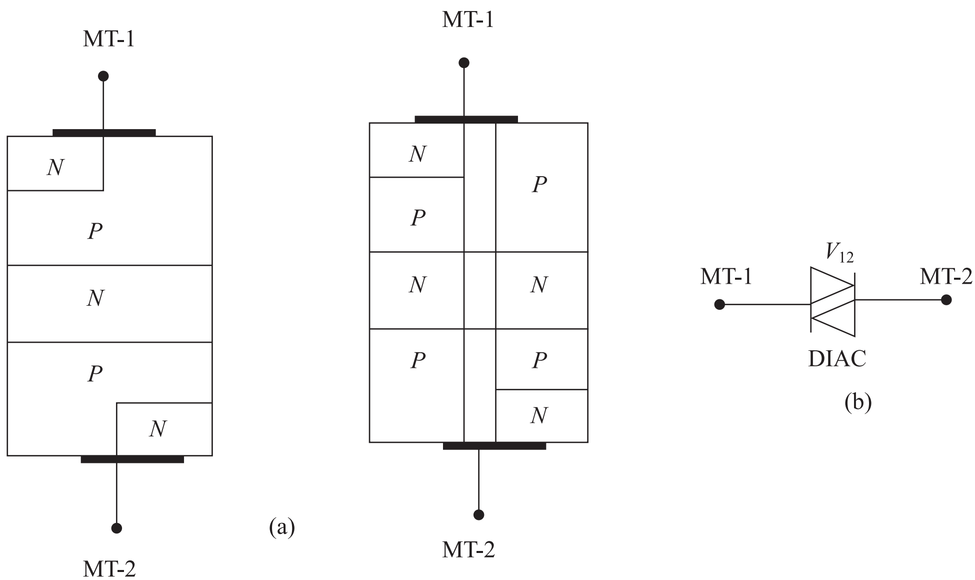


Fig. 4.93 *(a) Construction of DIAC (b) Symbol of DIAC*

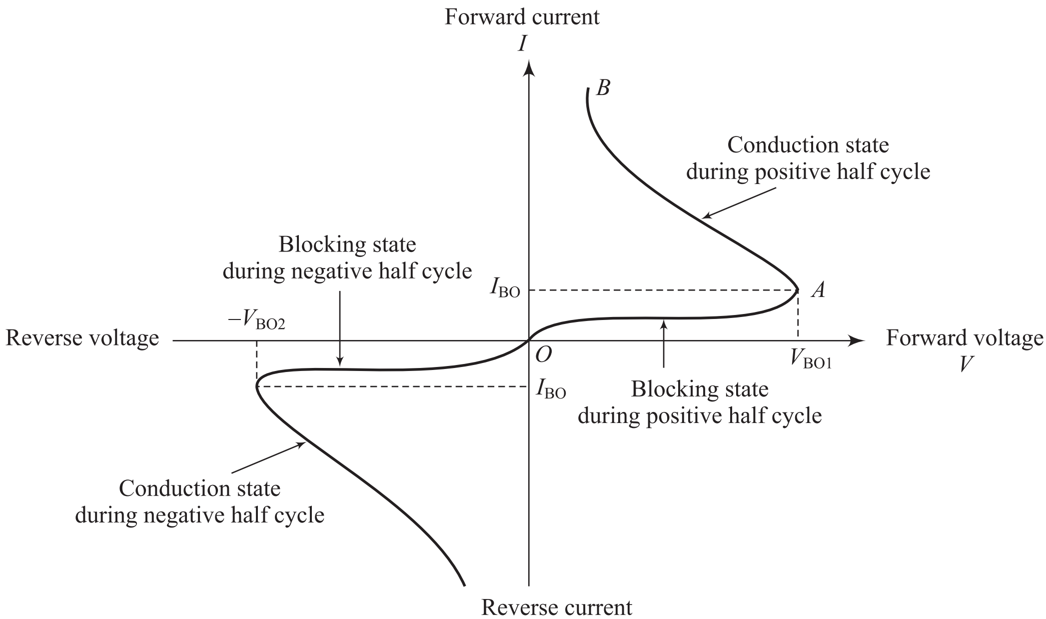


Fig. 4.94 *I-V characteristic of a DIAC*

breakover voltage V_{BO1} the structure PNPN conducts. In the same way when the MT-2 is positive with respect to MT-1 and the voltage across two terminals V_{12} is greater than breakover voltage V_{BO2} the structure PNPN conducts. The breakover voltage of a DIAC is about 30 V. When a DIAC is in conducting state, it acts as a low resistance with about 3 V drop across it.

When the voltage across DIAC is less than the breakover voltage, a leakage current flows through the device. Consequently, the device remains in non-conducting state as shown by region OA in Fig. 4.94. At point A, when the voltage just reaches the breakover voltage, the device starts conducting. During its conduction, DIAC demonstrate the negative resistance characteristics and the current flow through it starts increasing and the voltage across it starts decreasing. AB portion of the characteristic curve as shown in Fig. 4.94 is the conduction state. The magnitude of voltage at point A is called the breakover voltage. Similarly, the characteristic curve can be obtained for negative half-cycle of supply voltage and the device operates in the third quadrant.

Once the device starts conducting, the current flow through the DIAC is high and it is amplitude is limited by external resistance. When a DIAC in conducting state, it stops conducting only when the current is reduced below its holding value.

Figure 4.95 shows the circuit for DIAC operation. DIAC will be ON when the input voltage is greater than its break-over voltage value V_{BO}. During positive half cycle, if the supply is greater the breakover voltage at t_1, the DIAC is closed and starts to conduct from $t = t_1$ to $t = \pi$. In the negative half cycle, when the supply is greater the breakover voltage at t_2, the DIAC is again closed and starts to conduct from $t = t_2$ to $t = 2\pi$. The DIAC can be OFF by reducing the supply voltage when the current through the device falls below its holding value. Since each DIAC has the same value of break over voltage, the firing angle of a DIAC is fixed. Hence, a DIAC is used as a two-terminal ac switch.

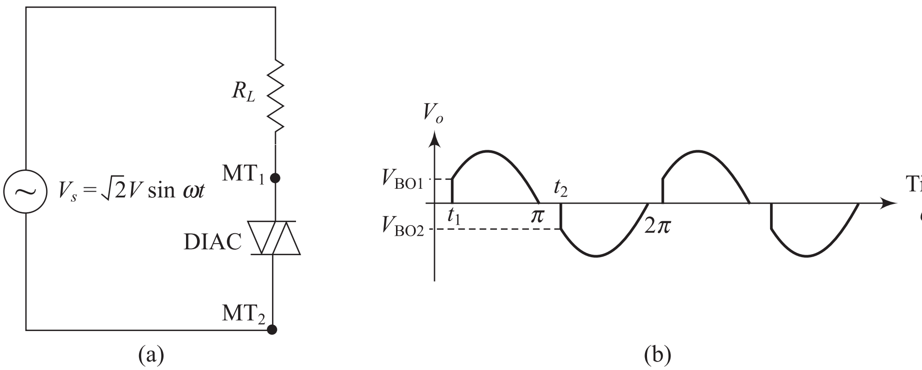


Fig. 4.95 *(a) The circuit for DIAC operation and (b) Output voltage across R_L*

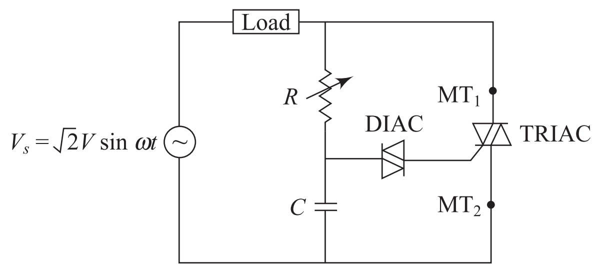


Fig. 4.96 *Application of DIAC to trigger TRIAC*

Usually DIAC is used as triggering device for a TRIAC which require positive or negative trigger pulses to turn ON. It can be used as a lamp dimmer, heater control, fan regulator, ac motor control, etc. All circuits are almost similar and only the load (lamp, heater, fan, motor, etc.) is changed as depicted in Fig. 4.96. During the positive or negative half cycle, the capacitor C is charged depending upon the value of variable resistor R. When the voltage across C is equal to break over voltage of DIAC V_{BO}, the DIAC becomes ON. Since the DIAC is turned ON, the triggering pulse is applied to TRIAC. Then TRIAC becomes turn on and the load circuit (lamp, heater, fan, motor, etc.) is ON. The voltage across the load depends upon the firing angle of the TRIAC. By varying the resistance R, the firing angle of TRIAC will be varied and the voltage across the load will be varied.

4.26 TRIAC

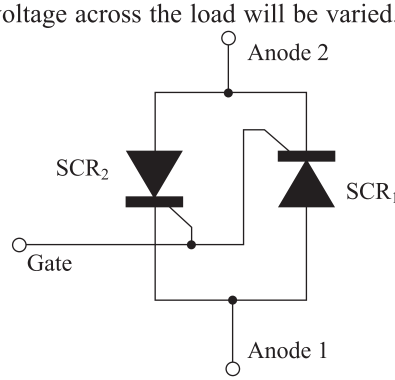


Fig. 4.97 *Equivalent circuit of a TRIAC*

SCR is a unidirectional device as it can conduct from anode to cathode. It has reverse blocking characteristics and current cannot flow from cathode-to-anode direction. But in some applications, particularly in ac circuit, the bidirectional current flow is required. For this, two thyristors are connected back to back or two anti-parallel thyristors can be integrated into a single chip as depicted in Fig. 4.97. This deviced is called a TRIAC (triode ac switch). TRIAC can conduct in both directions. Hence, TRIAC is a bidirectional thyristor and it is extensively used for ac controller circuits.

TRIAC is derived by combining the capital letters from the words TRIode and AC. TRIAC can be conduct in bidirectional and it is equivalent to two SCRs connected in anti-parallel. The anode and cathode terminals are not used to represent a TRIAC. TRIAC has three terminals such as MT_1, MT_2 and gate G. MT_1 is as the reference point to measure voltages and currents at gate terminal and MT_2. The gate G is present near MT_1. The structure of TRIAC is depicted in Fig. 4.98.

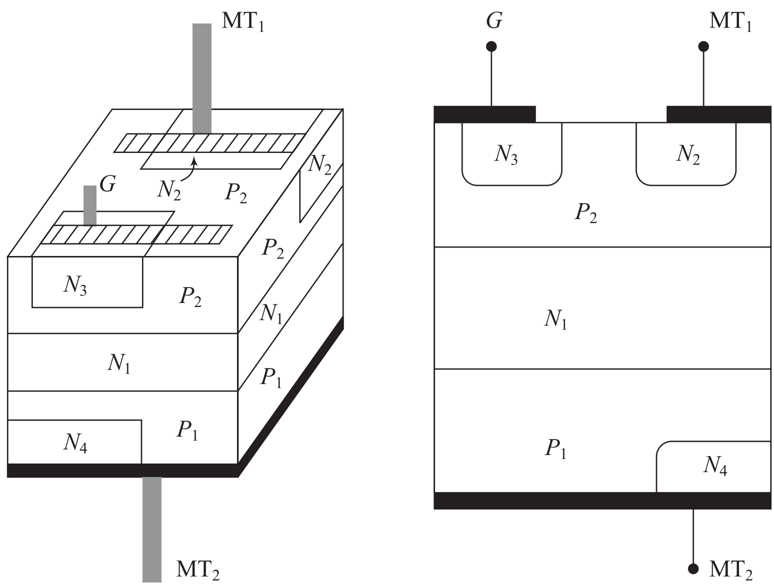


Fig. 4.98 *Structure of a TRIAC*

The circuit symbol of TRIAC is shown in Fig. 4.99(a) and the V-I characteristic of a TRIAC is shown in Fig. 4.99(b). It is clear from Fig. 4.99(b) that TRIAC operates in first quadrant and third quadrant. In the first quadrant, MT_2 is positive with respect to MT_1 and in the third quadrant, MT_1 is positive with respect to MT_2. When no gate pulse is applied, the TRIAC can block the both half cycles of the ac applied voltage and the peak voltage across MT_1 and MT_2 is always less than break over voltage in both directions.

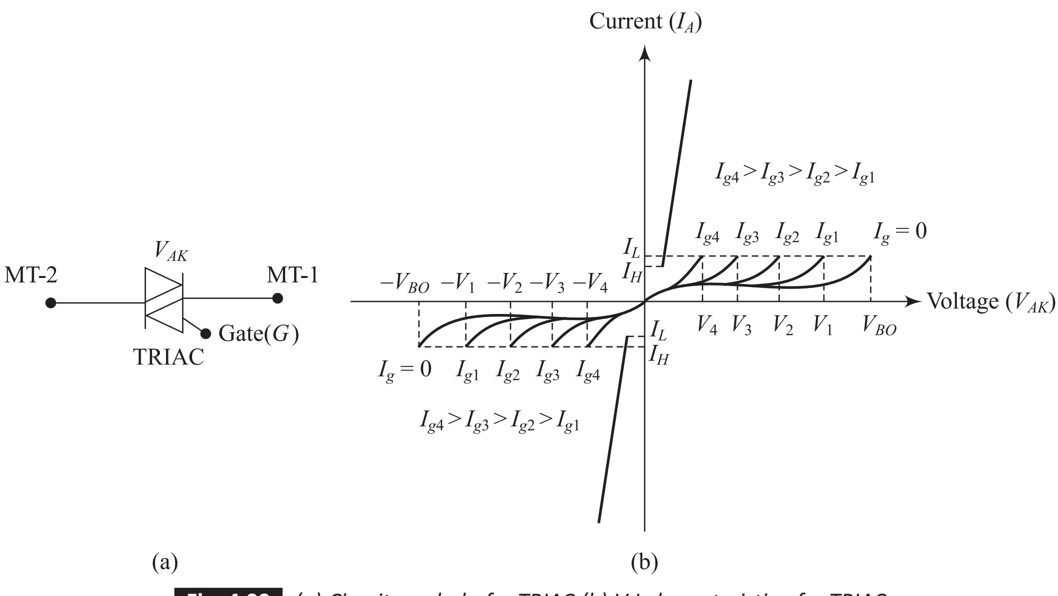


Fig. 4.99 *(a) Circuit symbol of a TRIAC (b) V-I characteristic of a TRIAC*

Assume that the device is in blocking condition before applying the gate signal. When a gate pulse is applied, a gate current flows to trigger the TRIAC either in first or third quadrant. Depending on the gate pulse and biasing condition, TRIAC conducts in four different operating modes as given below:

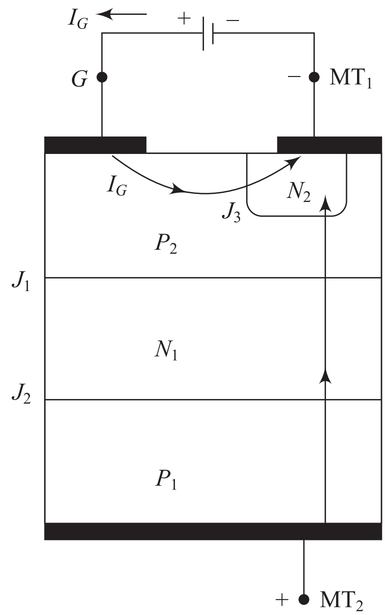


Fig. 4.100 *Mode 1- MT_2 positive and positive gate current*

Mode 1: MT_2 positive and positive gate current When MT_2 is positive with respect to MT_1, junctions P_1 N_1, P_2 N_2 are forward biased and the junction N_1 P_2 is reverse biased. When the gate terminal is positive with respect to MT_1, the gate current is positive and it flows from gate terminal to MT_1 through junction J_3 (P_2 N_2) just like ordinary SCR. This device will be turn on just like an SCR. But in case of TRIAC, the gate current requirement is greater than SCR. Due to the ohmic contacts of gate and MT_1 on P_2 layer, some gate current flows from the gate terminal to MT_1 through the semiconductor layer P_2 without crossing through P_2N_2 junction. P_2 layer is flooded by electrons when gate current flows through P_2N_2 junction. After that these electrons diffuse the junction J_2 and collected at N_1 and the reverse bias junction N_1P_2 break downs just like normal SCR. Then the structure operates as $P_1N_1P_2N_2$ as depicted in Fig. 4.100. Under this condition, the TRIAC operates in first quadrant.

Mode 2: MT_2 positive and gate current negative When MT_2 is positive and gate terminal is negative with respect to MT_1, gate current flows through P_2-N_3 and initially $P_1N_1P_2N_3$ structure start to conduct. With the conduction of $P_1N_1P_2N_3$, the voltage drop across this path falls, and the potential of P_2N_3 junction rises towards the anode potential of MT_2. As the right hand portion of P_2 is clamped at the potential of MT_1, a potential gradient exists across P_2, its left-hand region being at higher potential than its right hand region. A current is established in layer P_2 from left to right which forward biased P_2N_2 junction as depicted in Fig. 4.101 and finally $P_1N_1P_2N_2$ structure starts to conduct. The turn-ON process of TRIAC when MT_2 is positive and gate is negative, is less sensitive compared to the turn-ON process of TRIAC when MT_2 is positive and gate is positive.

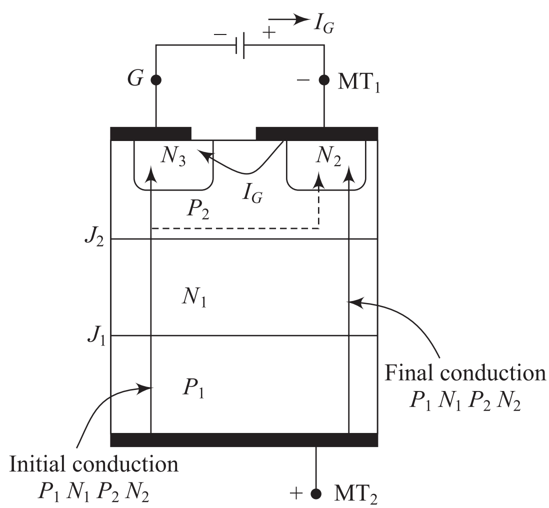


Fig. 4.101 *Mode 2- MT_2 positive and negative gate current*

Mode 3: MT_2 negative, positive gate current When MT_2 terminal in negative with respect to MT_1, the device will be turn on by applying a positive gate voltage across gate and MT_1. The device operates in third quadrant when it is turned ON. The main structure $P_2N_1P_1N_4$ with N_2 acting as a remote gate. The gate current forward biases P_2N_2 junction. Layer N_2 injects electrons into P_2 layer as depicted in Fig. 4.102. The electrons from N_2 are collected by P_2N_1 junction due to increase of current through the junction P_2N_1. The holes are injected from P_2, diffuse through N_1 and reach at P_1. Consequently, a positive space charge builds up in the P_1 region and more electrons from N_4 diffuse into P_1 to neutralise the positive space charge region. These electrons reach at junction J_2. They generate a negative space charge in the N_1 region. Then more holes are injected from P_2 into N_1 and the regenerative process continues until the $P_2N_1P_1N_4$ completely turned ON. After turning ON of TRIAC the current flow is limited by external load. Since the TRIAC is turned ON by the remote gate N_2, this device is less sensitive in the third quadrant with positive gate current compared to MT_2 negative and negative gate current.

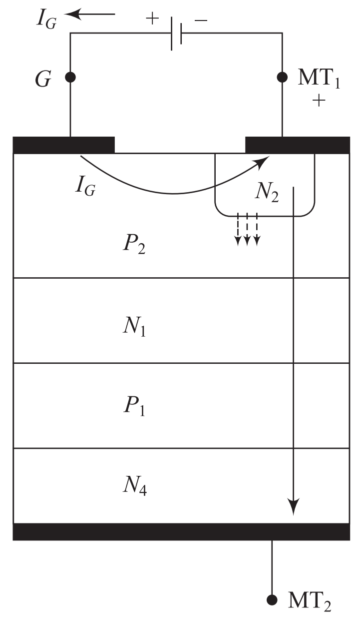


Fig. 4.102 *Mode 2- MT_2 negative and positive gate current*

Mode 4: MT_2 negative and negative gate current In this mode, N_3 acts as a remote gate. The gate current I_G flows through forward biased P_2N_3 junction and electrons are injected from N_3 to P_2 as depicted in Fig. 4.103. The electrons from N_3 are collected by P_2N_1 cause an increase of current across P_1N_1. Then the structure $P_2N_1P_1N_4$ turns ON by the regenerative action. The device turns ON due to the increased current in layer N_1. The device is more sensitive in this mode operation compared to MT_2 negative and positive gate current.

The TRIAC is rarely operated in first quadrant with MT_2 positive and negative gate current and in the third quadrant MT_2 negative and positive gate current. Since there are two conducting paths from MT_2 to MT_1 and MT_1 to MT_2 and semiconductor layers interact with each other in the structure of TRIAC, the voltage, current and frequency ratings are much less than thyristors. Presently TRIACs are available with voltage rating 1200 V and current rating 300 A. TRIACs are extensively used in lamp dimmers, heat control and speed control of single phase ac motors.

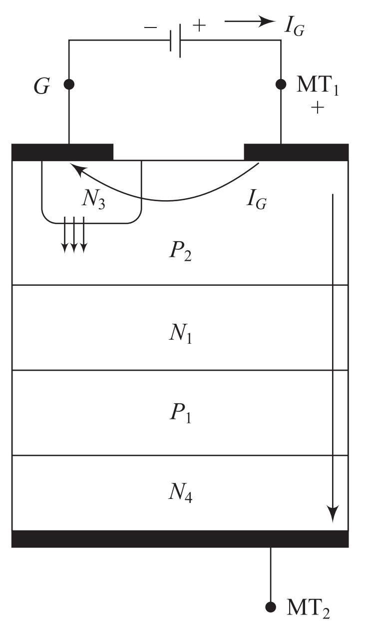


Fig. 4.103 *Mode 2- MT_2 negative and negative gate current*

4.26.1 Firing Circuit of TRIAC

Figure 4.104 shows a firing circuit for TRIAC using a DIAC. R_1 is a constant resistance, R_2 is variable resistance, R_3 is also a constant resistance. When $R_2 = 0$, R_1 is used to protect the DIAC and TRIAC gate circuit from expose to approximately full ac supply voltage. Resistance R_3 is connected in series with DIAC and TRIAC gate circuit to limit the current when the DIAC operates in on-state. The value of R_2 and C are selected in such a way that the firing angle can be varied from 0° to 180°. But practically, the triggering angle range may be varied from 10° to 170°.

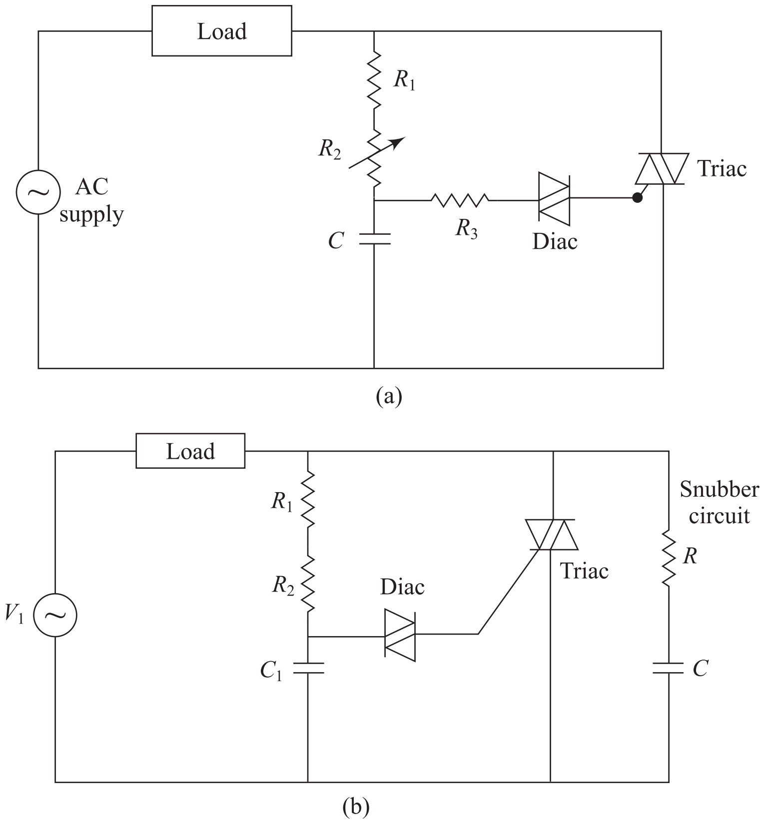


Fig. 4.104 *(a) Firing circuit of TRIAC using a DIAC (b) Firing circuit of TRIAC using a DIAC and snubber circuit*

Actually the value of resistance R_2 is varied to change the charging time of the capacitor C and subsequently the firing angle of TRIAC can be varied. When R_2 is equal to zero, the charging time of capacitor is R_1C which is small but the conduction angle of TRIAC will be high.

If R_2 exists, the charging time constant is $(R_1 + R_2)C$. When the value of R_2 is small, charging time is small and firing angle of TRIAC will be small. If R_2 is high, charging time is high, firing angle is more and the conduction time of TRIAC will be less.

Whenever the capacitor charges to the breakdown voltage V_{DIACT} of DIAC, DIAC will be turned ON. Since the capacitor discharges very rapidly, the capacitor voltage V_C will be applied as a pulse across the TRIAC gate and the device will be turned ON. When the thyristor is turned ON at the firing angle α, the input voltage is applied across load during the positive half-cycle of supply voltage. At $\omega t = \pi$, the input voltage becomes zero and TRIAC will be turned OFF. After $\omega t = \pi$, the input voltage becomes negative, the capacitor C becomes charge with the lower plate positive. At the instant, the capacitor charges to the breakdown voltage V_{DIACT} of DIAC, DIAC will be turned ON. Subsequently, TRIAC will be turned ON and conducts during the positive half-cycle of supply voltage. Then input voltage is applied across load during the negative half-cycle for $\pi - \alpha$ duration. Hence, the turning ON and OFF process will be repeated for both positive and negative half-cycle of supply voltage.

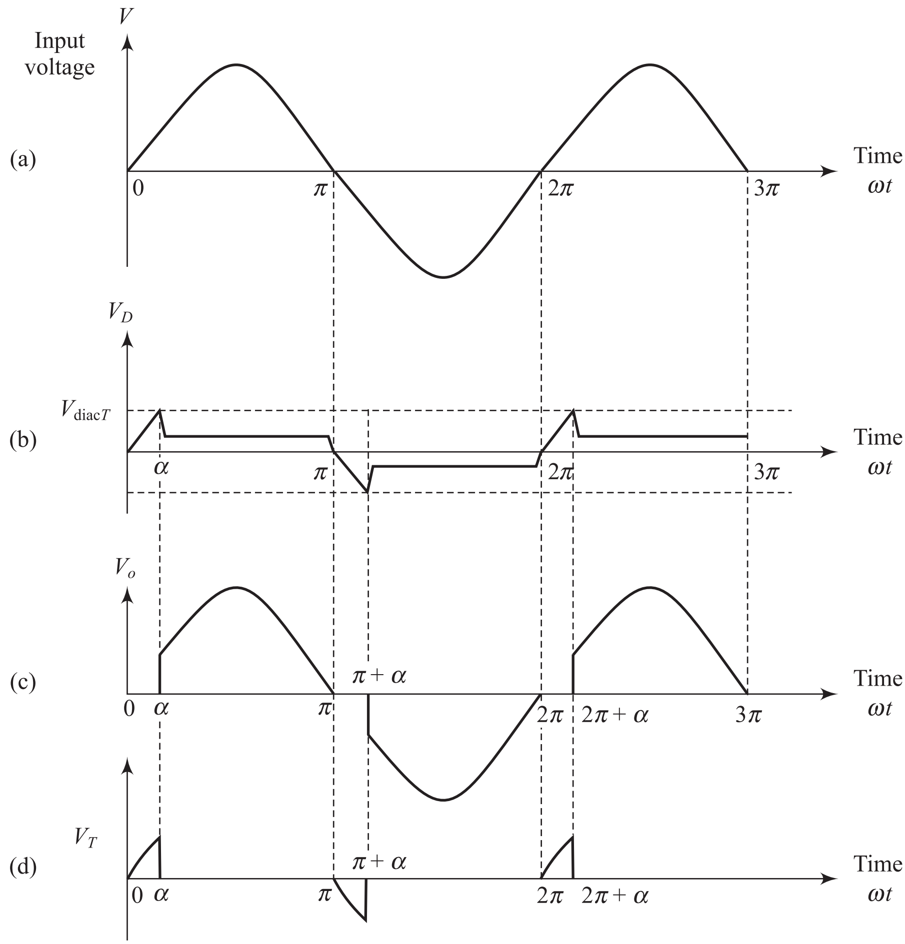


Fig. 4.105 *Waveforms for TRIAC firing circuit with minimum value of R*

The waveforms for input voltage V, capacitor voltage V_C, voltage across TRIAC V_T and output voltage V_o are shown in Fig. 4.105(a), (b), (c) and (d) respectively for minimum value of R. Similarly, Fig. 4.106(a), (b), (c) and (d) show the waveforms for input voltage V, capacitor voltage V_C, voltage across TRIAC V_T and output voltage V_o respectively for maximum value of R.

Figures 4.105 and Fig. 4.106 show the waveform for TRIAC firing circuit with ideal circuit components and these waveforms are unsymmetrical in nature for the positive and negative half-cycles of load voltage. The unsymmetrical is developed due to the asymmetry TRIAC characteristics and hysteresis present in the capacitor. This states that if the input voltage is zero, the capacitor voltage is not zero. We can also state that capacitor retains some charge of the initial voltage applied across its plates when the input voltage becomes zero. The output voltage waveforms for positive and negative half-cycles can be symmetrical when an additional resistance R_3 and capacitor C_1 are connected in the circuit as depicted in Fig. 4.107. This circuit is most commonly used for speed control of single phase induction motor, lamp dimmers, heat converters. This circuit can be operating with inductive load when a R-C snubber circuit is connected across TRIAC.

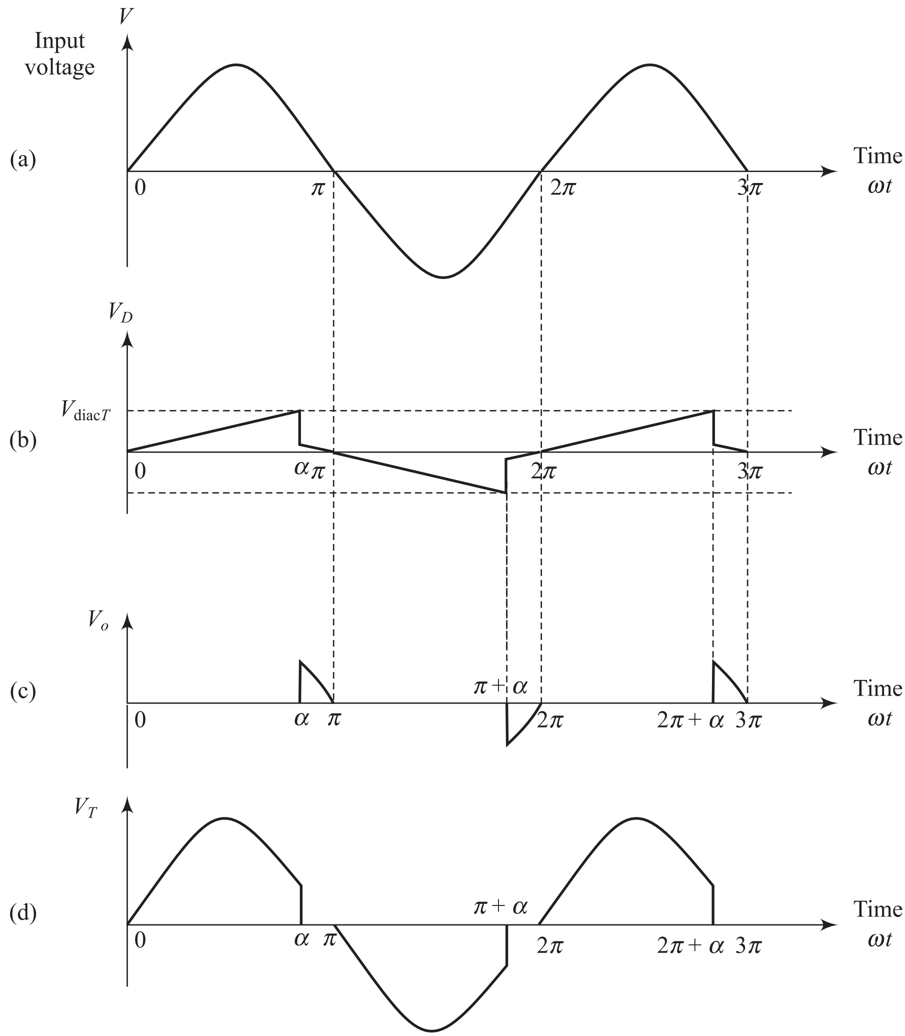


Fig. 4.106 *Waveforms for TRIAC firing circuit with maximum value of R*

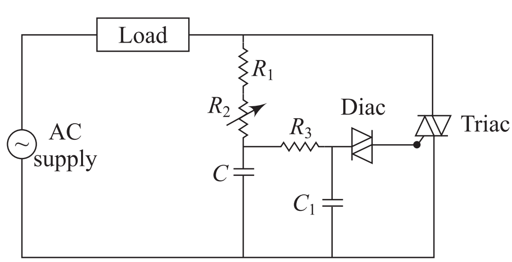


Fig. 4.107 *Commercially available firing circuit of TRIAC using a DIAC*

4.27 GATE TURN-OFF THYRISTOR (GTO)

Thyristors can be used as ideal switches in power electronics circuits. When thyristors are forward biased and a gate pulse is applied between anode to cathode, these devices will be turned ON. Once thyristor is turned ON, it operates in the ON-state. These devices can be turned OFF by natural commutation and forced commutation. The commutation circuit for thyristor is bulky and expensive. Thyristors can be used as switching device up to about 1 kHz. During OFF-state, these devices can block high voltages of about several thousand volts and in the ON-state, a large current about several thousand amperes flows from anode to cathode with only a small ON-state voltage about a few volts drop across thyristor. Due to the disability to turn-OFF the devices by using a control signal at the thyristor gate and cathode terminals, thyristors cannot be used in switch-mode applications such as chopper and inverter circuits. To incorporate the turn-OFF capability in the thyristor, the structure of thyristor must be modified.

GTO is just like a conventional thyristor but the turn-OFF feature is incorporated within the device. When a positive gate current is applied in between gate and cathode, the device will be turned ON and conducts. Whenever a negative gate current of required amplitude is applied to gate-cathode terminals, GTO will be turned OFF. Due to self-turned OFF capability, GTO is most suitable device for inverters and choppers.

4.27.1 Basic Structure of GTO

GTO is a four layer *p-n-p-n* device with three terminals gate (G), cathode (K) and anode (A). The basic structure of GTO is shown in Fig. 4.108. The dropping profile of different layers of GTO will be similar to the dropping profile of different layers of thyristor. In GTO, the thickness of p_2 layer is smaller compared to conventional thyristor.

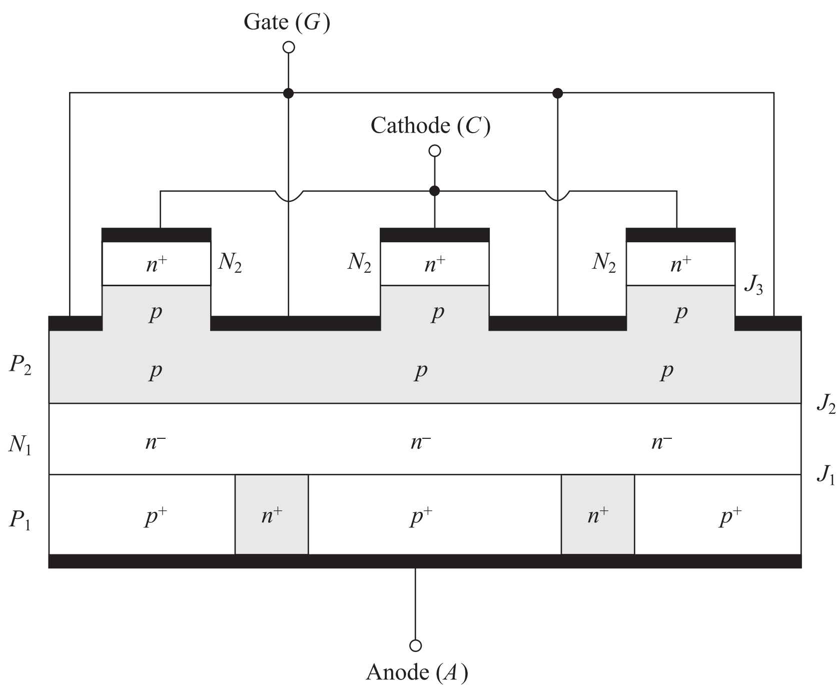
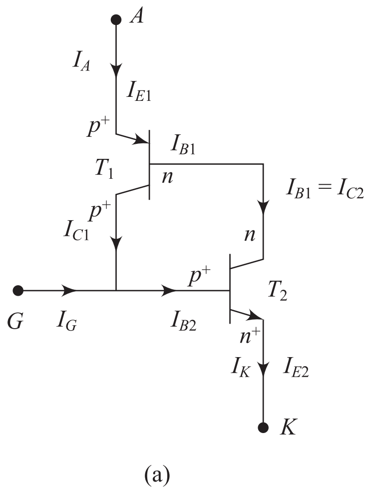


Fig. 4.108 *Basic structure of GTO*

In the anode region of GTO, n^+ region penetrate the p-type anode (p_1 layer) at regular interval so that the n^+ region makes contact with the n^- region (n_1 layer). As the n^+ regions are overlaid with the same metallisation contacts, the p-type anode is called *anode short*. Due to anode short structure, the turn-OFF of GTO will be fast. GTO can be represented by two transistor analogy as shown in Fig. 4.109(a). Transistor T_1 is p^+np^+ type whereas transistor T_2 is np^+n^+ type. The p^+ emitter of transistor T_1 is anode of GTO and n^+ emitter of transistor T_2 is cathode of GTO. Figure 4.109(b) shows the symbol of GTO. The two-way arrow convention on the gate terminal can be used to differentiate GTO from a conventional thyristor.



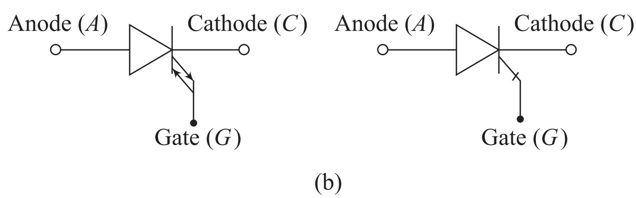


Fig. 4.109 *(a) Two transistor analogy of GTO (b) Circuit symbol of GTO*

4.27.2 *I-V* Characteristic of GTO

The *I-V* characteristic of GTO is identical to a conventional thyristor in the forward direction as depicted in Fig. 4.110. The latching current of GTO is few amperes where as the latching current of conventional thyristor is 100 to 500 mA for same power rating. When the gate current cannot able to turn ON the GTO, it behaves just like a high voltage low gain transistor with sufficient anode current. In this condition, there will be certain power loss. In the reverse direction, the GTO has virtually no blocking capability for anode-short structure. As the junction J_3 operates in reverse direction block and it has low break down voltage about 20–30 V due to large doping densities on both sides of the junction. The difference between GTO and conventional thyristor is given below.

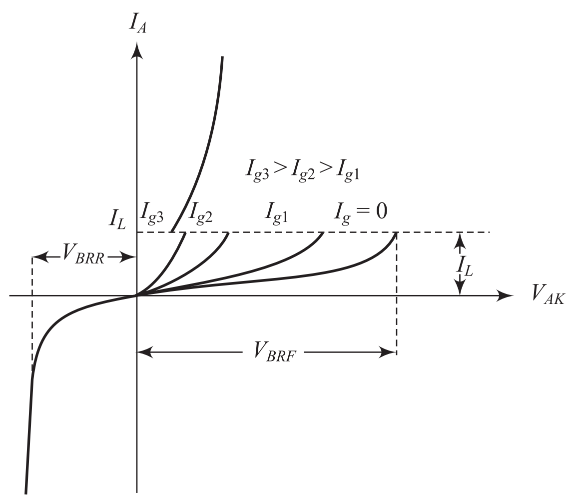


Fig. 4.110 *I-V characteristic of GTO*

Table 4.1 *Differences between GTO and thyristor*

Gate-Turn-OFF thyristor (GTO)	Conventional thyristor
The magnitude of latching and holding current of GTO is more than thyristor.	The magnitude of latching and holding current of thyristor is less than GTO.
The ON-state voltage drop of GTO is more than thyristor.	The ON-state voltage drop of thyristor is less than GTO.
The gate and cathode structure of GTO are highly interdigitated with small cathode and gate widths, the use of anode shorts of GTO.	The gate and cathode structure of GTO are not highly interdigitated
In the drift region, a carrier lifetime is shorter than carrier used in conventional thyristor.	The carrier life time in thyristor is more than the carrier life time of GTO in drift region.
Due to multi-cathode structure of GTO, the required gate current to turn on a GTO is more than the required gate current for a conventional thyristor.	The required gate current to turn on a thyristor is less than the required gate current of a GTO.
Gate drive circuit losses for GTO are more.	Gate drive circuit losses for thyristor are less.
The reverse voltage blocking capability of GTO is less than forward voltage blocking capability.	Reverse voltage blocking capability of thyristor is greater than Reverse voltage blocking capability of GTO.
GTO has faster switching speed compared to thyristor.	Thyristor has less switching speed compared to GTO.
GTO has more *di*/*dt* rating at turn-on compared to thyristor.	Thyristor has less *di*/*dt* rating at turn-on compared to GTO.
GTO circuit has lower size and weight as compared to thyristor circuit.	Thyristor circuit has larger size and weight as compared to GTO circuit.
Due to elimination of commutation choke, GTO circuit has reduced acoustical and electromagnetic noise.	Thyristor circuit has more acoustical and electromagnetic noise compared to GTO circuit.

4.27.3 Operating Principle of GTO

GTO during turn-ON Just like a thyristor, GTO is a monolithic *p-n-p-n* structure and its operating principle can be explained in a similar manner to that of a thyristor. The *p-n-p-n* structure of a GTO can be consist of two transistors where one *p-n-p* and one *n-p-n* transistor are connected in the regenerative configuration as shown in Fig. 4.111.

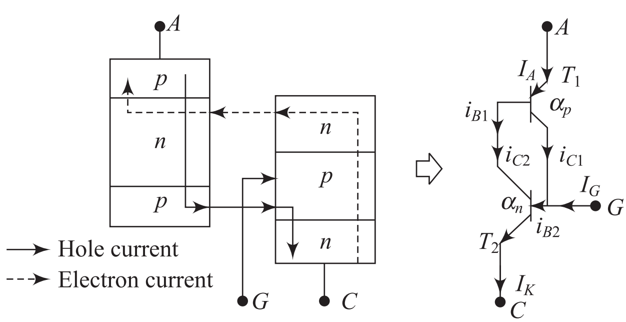


Fig. 4.111 *Current distribution in a GTO during turn ON*

The collector current of T_1 $i_{C1} = \alpha_1 I_A + I_{CB01}$ as $\alpha_1 = \alpha_p$

The base current of T_1 $i_{B1} = i_{C2} = \alpha_2 I_K + I_{CB02}$ as $\alpha_2 = \alpha_n$

We know that $I_K = I_A + I_G$ and $I_A = i_{B1} + i_{C1}$

After combining the above equations, we get

$$I_A = \frac{\alpha_2 I_G + (i_{CB01} + i_{CB02})}{1 - (\alpha_1 + \alpha_2)}$$

When the GTO is forward biased and the applied forward voltage V_{AK} is less than the forward breakover voltage V_{BRF}, both the currents I_{CB01} and I_{CB02} are small. If gate current I_G is zero, anode current I_A is only slightly higher than (I_{CB01} + I_{CB02}). In this instant both the current gain α_1 and α_2 are small and $\alpha_1 + \alpha_2 << 1$. Then GTO operates in off condition and the device is said to be in the forward blocking mode.

To turn ON the device, a gate current is injected through gate terminal. When a positive gate current I_g input to gate and cathode and the GTO is forward biased, the current gain α_1 and α_2 start to increase rapidly as the emitter current increases. As $\alpha_1 + \alpha_2$ approaches to 1 or $\alpha_1 + \alpha_2 \cong 1$, the anode current I_A tends to infinity. When $\alpha_1 + \alpha_2 = 1$, the device starts to regenerate and each transistor operates in saturation. Once transistor T_1 and T_2 operate in saturation, all junctions will be forward biased and the total potential drop across the GTO is equal to that of a single *p-n* diode. Therefore, each transistor reaches saturation level, GTO is turned ON and the anode current begins to flow. Once the device has been turned ON, anode current is limited by load impedance.

GTO during turn-OFF To turn-OFF a GTO, the gate terminal voltage is negative with respect to the cathode terminal so that gate-cathode is negative biased. Actually, the holes injected from the anode are taken out from the *p* base through the gate metallisation into the gate terminal. Hence, the voltage drop across the *p* base and the *n* emitter of transistor T_2 starts reverse biasing the junction J_3 and electron injection stops.

When the electron injection stops completely, depletion layer starts to grow on both junctions J_2 and J_3. Subsequently, the device once again starts blocking forward voltage. The cathode current has stop and the anode to gate current continues to flow as the *n* base excess carriers diffuse towards junction J_1. This current is called '*tail current*'. The amplitude of tail current decays exponentially as the *n* base excess carriers will be reduced by recombination. When the tail current becomes zero, the device regain its steady state blocking characteristics and GTO operates in off-state. Figure 4.112 shows the current distribution in a GTO during turn-OFF.

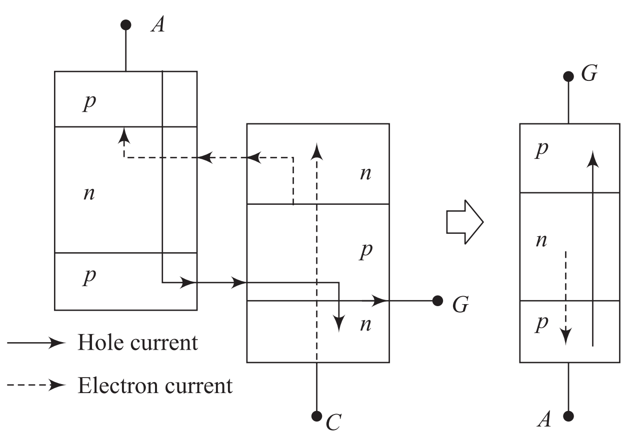


Fig. 4.112 *Current distribution in a GTO during turn-OFF*

The performance of GTO can be analysed using two transistor model of thyristor. In the equivalent circuit, both transistors T_1 and T_2 are saturated when the GTO is in ON state. When the base current to transistor T_2 is less than the value needed to maintain saturation, i.e., $I_{B2} < \frac{I_{C2}}{\beta_2}$, subsequently transistor T_2 operates in active mode and the gate turn-OFF thyristor starts to turn-OFF as the regenerative action present in the circuit when both transistors T_1 and T_2 operate in active mode or any one transistor operates in active mode.

In the equivalent circuit of GTO as shown in Fig. 4.113,

$$I_{C2} = \beta_2 I_{B2} \quad \text{and} \quad I_{C1} = \beta_1 I_{B1}$$
$$I_{C2} = \alpha_2 I_{E2} \quad \text{and} \quad I_{C1} = \alpha_1 I_{B1}$$

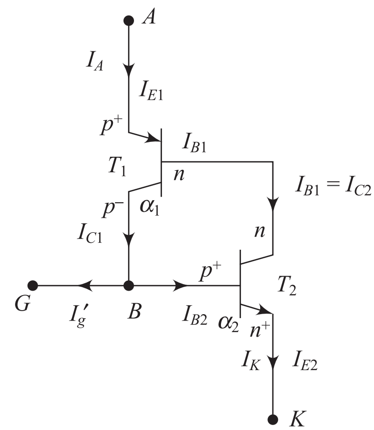


Fig. 4.113 *Two transistor analogy of GTO during turn-OFF*

When a negative gate current I_g' is applied across gate-cathode terminals, the KCL equation at node B is equal to

$$I_{C1} - I_g' - I_{B2} = 0$$

or $$I_{C1} - I_g' = I_{B2}$$

or $$\alpha_1 I_A - I_g' = I_{B2} \quad \text{as} \quad I_{C1} = \alpha_1 I_A$$

Since $$I_A = I_{C1} + I_{C2}, \quad I_A - I_{C1} = I_{C2}$$

or $$I_{C2} = I_A - I_{C1} = (1 - \alpha_1) I_A$$

When transistor T_2 is saturated, $I_{B2} = \frac{I_{C2}}{\beta_2}$. For turning OFF GTO, transistor T_2 must be out of saturation. This is possible when the base current of transistor T_2, I_{B2} is less than $\frac{I_{C2}}{\beta_2}$, i.e., $I_{B2} < \frac{I_{C2}}{\beta_2}$.

Consequently, the transistor T_2 is shifted from saturation region to active region, then the regenerative action starts and GTO will be turned OFF. Therefore, the condition for turn-OFF of GTO is

$$I_{B2} < \frac{I_{C2}}{\beta_2} \tag{4.8}$$

Substituting the values of I_{B2} and I_{C2} in Eq. (4.8), we obtain

$$\alpha_1 I_A - I_g' < \frac{(1-\alpha_1) I_A}{\beta_2} \quad \text{as} \quad I_{B2} = \alpha_1 I_A - I_g' \quad \text{and} \quad I_{C2} = (1-\alpha_1) I_A$$

or $$-I_g' < \frac{(1-\alpha_1) I_A}{\beta_2} - \alpha_1 I_A$$

or $$-I_g' < \frac{I_A}{\beta_2} - \alpha_1 I_A \left(1 + \frac{1}{\beta_2}\right)$$

As $\beta_2 = \frac{\alpha_2}{1-\alpha_2}$, $-I_g' < \frac{I_A}{\alpha_2}(1-\alpha_2) - \alpha_1 I_A \left(1 + \frac{1-\alpha_2}{\alpha_2}\right)$

or $$-I'_g < I_A\left(\frac{1}{\alpha_2} - 1\right) - \frac{\alpha_1}{\alpha_2} I_A$$

or $$-I'_g < I_A\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)$$

or $$I'_g > I_A\left(\frac{\alpha_1+\alpha_2-1}{\alpha_2}\right)$$

The gate current I'_g for turning-OFF GTO is low, α_2 is near to unity whereas α_1 should be small. The turn-OFF gain of GTO is the ratio of anode current I_A to gate current I'_g needed to turn-OFF the GTO.

The turn-OFF gain is equal to $\beta_{\text{off}} = \frac{I_A}{I'_g} = \frac{\alpha_2}{\alpha_1+\alpha_2-1}$

When a negative gate current I'_g flows between gate-cathode terminals, the total base current $I_{B2} - I'_g$ is reversed, excess carriers are drawn from p^+ region of T_2 and collector current I_{C1} of Q_1 is diverted into the external gate circuit. Hence, the base drive of transistor T_2 is removed and the base current I_{B1} of transistor T_1 is further removed. As a result, GTO will be turned OFF.

A low value of negative gate current requires low value of α_1 and high value of α_2. Low value of current gain α_1 of transistor T_1 can be possible by diffusing gold or heavy metal into *n* base of T_1 transistor or by short circuiting *n*+ fingers in the anode p^+ layer or by combination of both techniques.

4.27.4 GTO Turn-ON Characteristics

The turn-ON process of GTO is similar to that of a conventional thyristor. Turn-ON is initiated by a gate pulse. The sequence of events during turn-ON of a thyristor is same as GTO. During turn-ON of GTO, the rate of gate current $\frac{di_g}{dt}$ increases and the peak gate current I_{gm} must be large enough so that all cathode islands start to conduct and there should be good dynamic sharing of the anode current. If all cathode islands do not conduct simultaneously, a small number of islands may be carrying the total current and local thermal runway could occur and subsequently there is a possibility of destruction of GTO.

The gate current I_{gm} must be supplied for long time about 10 μs so that the turn-ON process of GTO is complete properly. After turn-ON of a GTO, a minimum gate current $I_{g\min}$ must be flow during the entire ON-state period to prevent unwanted turn OFF. The gate current I_{gT} is known as backporch current as shown in Fig. 4.114. When the gate current is zero and the anode current is too low, some cathode islands stop conducting. If the anode current increases, the remaining cathode islands will not be conducting and cannot share anode current. Consequently, GTO can be destroyed due to localised thermal runway.

Figure 4.115 shows a gate drive circuit of a GTO. A large pulse of gate current is provided by the gate drive circuit when both transistors T_1 and T_2 are ON. The stay inductance in the positive gate drive circuit should be minimum so that there is a large $\frac{di_g}{dt}$ at turn ON. The gate current $I_{g\max}$ is maintained for t_W duration and after that the gate current is reduced to a minimum value I_{gT} by turning OFF of transistor T_1.

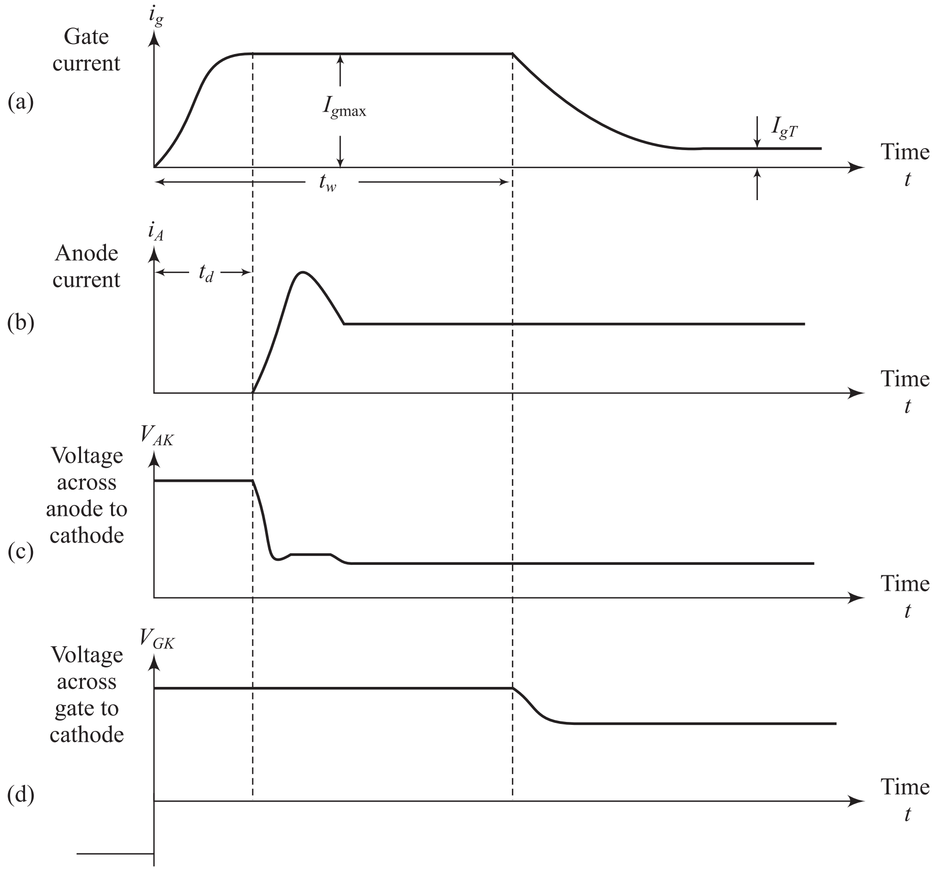


Fig. 4.114 *(a) Gate current i_g (b) Anode current i_A (c) Anode-to-cathode voltage V_{AK} (d) Gate-to-cathode voltage V_{GK} during turn-ON*

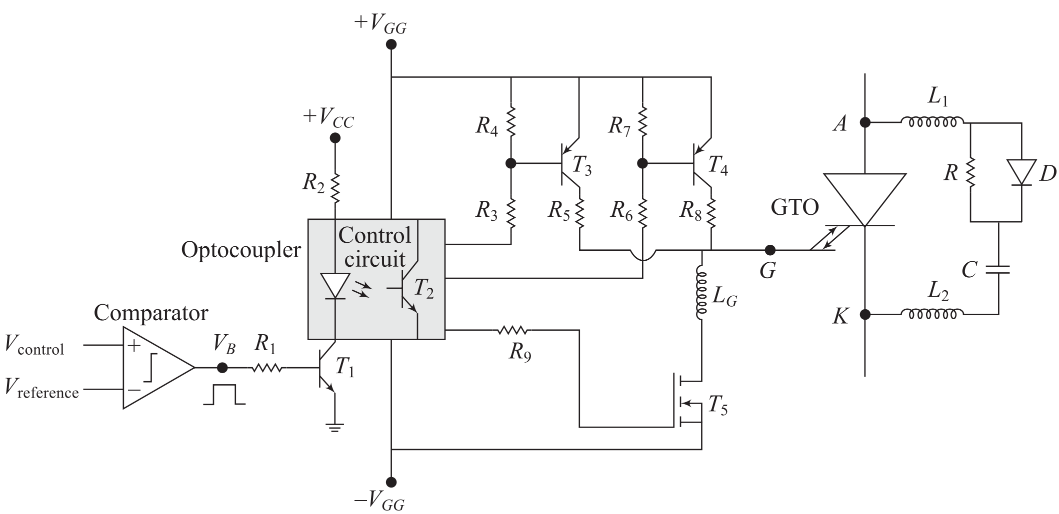


Fig. 4.115 *Gate drive circuit of a GTO*

4.27.5 GTO Turn-OFF Characteristics

Figure 4.116 shows the GTO turn-OFF characteristics when a large negative gate current is applied. Figure 4.115 shows the gate drive circuit which supplies the negative gate current by turning on transistor T_3. The gate current should be very large about 1/5 to 1/3 of the anode current. Due to large negative gate current, GTO will be turned OFF within a very short time. Transistor T_3 will be a low-voltage MOSFET. The $\frac{di_g}{dt}$ must be negative large in order to have a short storage time t_s and

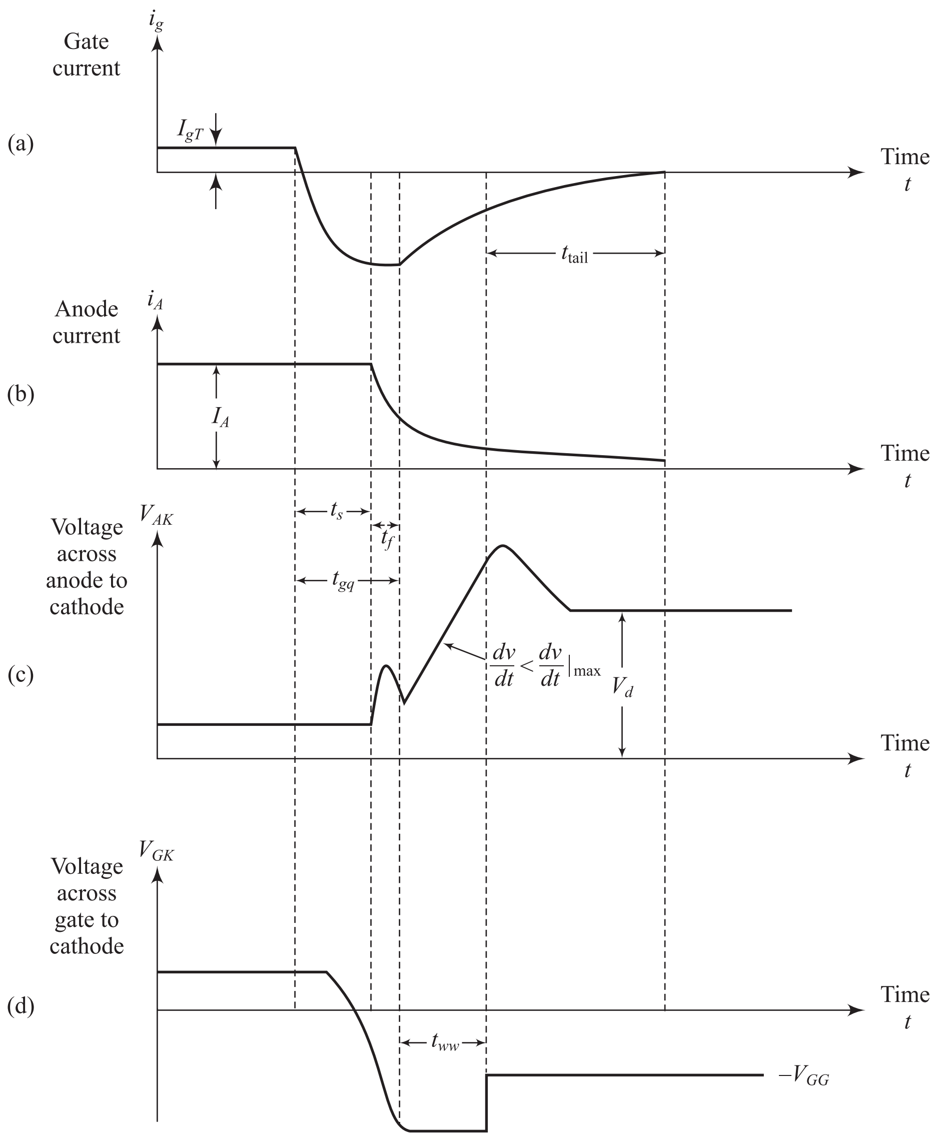


Fig. 4.116 *(a) Gate current i_g, (b) Anode current i_A, (c) Anode-to-cathode voltage V_{AK}, (d) Gate to cathode voltage V_{GK} during turn-OFF*

a short anode current fall time t_f. Hence, the gate power dissipation will be reduced. The value of $\frac{di_g}{dt}$ must be as per manufacturer data sheet.

The negative $\frac{di_g}{dt}$ can be controlled by $-V_{GG}$ and L_G of the drive circuit as shown in Fig. 4.118. The value of $-V_{GG}$ must be less than the gate-cathode junction breakdown voltage. When the value of $-V_{GG}$ is known, the value of L_G can be selected properly to provide the specified $\frac{di_g}{dt}$. If the current rating of GTO is large, the stay inductance in the negative gate drive circuit may be equal to L_G.

During the storage time t_s, the negative gate current is removing the stored charge in the p_2 and n_2 layers at the periphery of the cathode islands. Since the stored charge is continuously removed from the periphery, the size of the plasma-free region grows as it expands in the lateral direction towards the centres of the cathode islands with a squeezing velocity. The removal of plasma is the inverse of establishment during turn ON. After removal of sufficient amount of stored charge, the regenerative action in the GTO is stopped and the anode current starts to fall. This represents the end of the storage charge interval. Figure 4.117 shows turn-off phenomena of GTO.

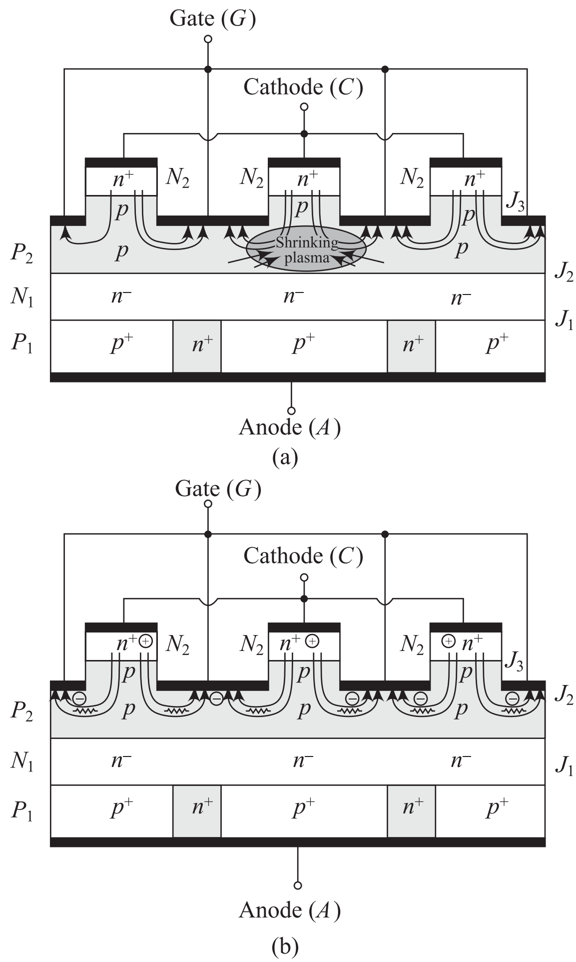


Fig. 4.117 *Turn-off phenomena of GTO (a) Negative gate current squeezing the excess carrier plasma (b) Lateral ohmic resistance in p_2 layer*

4.27.6 Rating of GTO

Usually the rating of GTOs is specified in the manufacturer data sheet. Some very useful specification of GTOs voltage and current ratings are discussed in this section.

1. Peak repetitive forward blocking voltage (V_{DRM}) This is the repetitive peak transient voltage at which GTO can withstand in its forward blocking state. GTO can block rated voltage only if the gate is reverse biased or at least connected to the cathode through a low value resistance. The forward voltage withstanding capacity of GTO is a function of the gate cathode reverse voltage for a specified forward $\frac{dv}{dt}$.

2. Repetitive peak reverse voltage (V_{RRM}) It is the maximum repetitive reverse voltage at which GTO can able to withstand. The typical value of V_{RRM} is in the range of 20–30 V.

3. V_{DC} It is the maximum continuous dc voltage at which the GTO can withstand. If the applied dc voltage is exceeding V_{DC} voltage, there is the possibility of a cosmic radiation failure.

4. I_{FRMS} It is the maximum RMS on state current of GTO. The value of I_{FRMS} is specified at a given case temperature assuming half wave sinusoidal on state current at power frequency.

5. I_{FAVM} This is the maximum average on state current of GTO. The value of I_{FRMS} is specified at a given case temperature assuming half wave sinusoidal on state current at power frequency

6. I_{FSM} It is the maximum allowable peak value of non-repetitive surge current assuming power frequency half-wave sinusoidal.

7. $\int i^2 dt$ It is the limiting value of the surge current integral assuming half-cycle sine wave surge current. For this rating, the junction temperature is to be at the maximum value before the surge current and the voltage across the GTO following the surge is to be zero. Fuse rating must be less than the $\int i^2 dt$ rating of GTO to protect the device.

8. I_H It is the holding current of the GTO. The value of holding current of a GTO is significantly higher compared to a similarly rated thyristor.

4.27.7 Advantages of GTOs over Bipolar Junction Transistors

1. GTO has higher blocking voltage capability.
2. The ratio of maximum (peak) current rating to average current is high.
3. The ratio of maximum (peak) surge current rating to average current is high and it is about 10:1.
4. High ON-state current gain and it value is about 600.
5. The pulse width of gate signal is short.
6. Under surge conditions, GTO goes into deeper saturation due to regenerative action.

4.27.8 Advantages of GTOs over Thyristors

1. GTO has faster switching speed compared to thyristor.
2. GTO has more *di*/*dt* rating at turn-ON compared to thyristor.
3. The surge current capability is analogous with an SCR.
4. GTO circuit has lower size and weight as compared to thyristor circuit.
5. Due to elimination of commutation choke, GTO circuit has reduced acoustical and electromagnetic noise.
6. Elimination of commutating circuit components in forced commutation.
7. Improved efficiency of converter circuits.

4.27.9 Applications of GTO

1. High performance DC drives.
2. High performance AC drives where the field-oriented control or vector control are used specially in rolling mills, process control industry, machine tools control, and robotics.
3. Electric traction drive system.
4. Variable voltage variable frequency inverter fed ac drives.

Summary

- The structure, operating principles, *I-V* characteristics of thyristor are explained in detail.
- Thyristor has four layer construction of alternating *p*-type and *n*-type regions. The *I-V* characteristics of thyristor has two stable states, one is on-state and other is off-state or forward blocking state.
- Thyristor is a minority carrier device and it has highest blocking voltage capability and current carrying capability with respect to any of semiconductor switches.
- Switching characteristics, series and parallel operation, triggering methods, ratings and protection of thyristors are discussed elaborately.
- Different triggering circuits and commutation of thyristors are also incorporated in this chapter.
- The structure, operating principle, *I-V* characteristic of DIAC, TRIAC and GTO are explained in detail.
- GTO has four layer structure just like thyristor but there is some modifications in the structure to enable the gate to turn-off the device.
- Switching characteristics and gate drive circuits of GTO are also discussed in this chapter.

Multiple-Choice Questions

4.1 A thyristor is a ______ layer device.
(a) two (b) three (c) four (d) five

4.2 The number of *pn* junctions in a thyristor is
(a) 4 (b) 3 (c) 2 (d) 1

4.3 A thyristor is a ______ layer device.
(a) *pnpn* (b) $p^+n^+n^-n^+$ (a) $n^-n^+p^-p^+$ (a) $n^+p^-p^+n^-$

4.4 If anode terminal of thyristor is positive with respect to cathode, the number of forward biased junctions are
(a) 1 (b) 2 (c) 3 (d) 4

4.5 If thyristor is forward biased, the number of reverse biased junctions are
(a) 1 (b) 2 (c) 3 (d) 4

4.6 The width of *p* region near anode is about
(a) 50 to 200 cm^{-3} (b) 30 to 50 μm (c) 10 to 100 μm (d) 100 to 250 μm

4.7 If anode terminal of thyristor is negative with respect to cathode, the number of blocked *p-n* junctions are
(a) 1 (b) 2 (c) 3 (d) 4

4.8 The droping density of *n*-region near cathode is
(a) 10^{14} cm^{-3} (b) 10^{15} cm^{-3} (c) 10^{16} cm^{-3} (d) 10^{19} cm^{-3}

4.9 The anode current in a thyristor is made up of
(a) only holes (b) only electrons (c) electrons or holes (d) electrons and holes

4.10 When the gate circuit is open, thyristor operating state can be changed from forward blocking state to forward conducting state when
(a) The applied voltage is greater than forward break over voltage
(b) The applied voltage is greater than reverse break down voltage
(c) 1.2 V
(d) The applied voltage is greater than peak repetitive reverse voltage

4.11 In a thyristor, the latching is
(a) equal to holding current (b) greater than holding current
(c) less than holding current (d) 2.5 time of holding current.

4.12 In a thyristor, the holding is
(a) equal to latching current (b) greater than latching current
(c) less than latching current (d) 0.4 time of latching current.

4.13 After turn-ON, a thyristor,
(a) the gate pulse should not be removed
(b) the gate pulse should be removed to reduce losses and junction temperature
(c) the gate pulse nay or may not be removed
(d) the gate pulse has no relation with thyristor turn on

4.14 The turn-ON time of thyristor is equal to
(a) delay time (b) rise time
(c) spread time (d) sum of delay time, rise time and spread time

4.15 In a thyristor, turn-ON time is
(a) equal to turn-OFF time (b) greater than turn-OFF time
(c) less than turn-OFF time

4.16 In the forward blocking state of thyristor,
(a) large current flows through thyristor
(b) voltage across thyristor is 1.2 V
(c) voltage across thyristor is high and very small current flows through thyristor
(d) voltage across thyristor is low and high current flows through thyristor

4.17 The ratio of latching current to holding current is
(a) 1 (b) 1.5 (c) 2 (d) 2.5

4.18 The ON-state voltage drop of a thyristor is
(a) 0.7 V (b) 1 V to 1.5 V (c) 50 V (d) 100 V

4.19 The forward voltage drop across thyristor
(a) remain constant with load current change
(b) increases slightly with load current
(c) decreases with load current

4.20 Which of the following is the normal trigger of thyristor?
(a) thermal triggering (b) high forward voltage triggering
(c) $\frac{dv}{dt}$ triggering (d) gate triggering

4.21 In gate triggering turn on method of thyristor,
(a) the gate pulse width must be greater than or equal to turn-ON time.
(b) the gate pulse width must be less than or equal to turn-ON time.
(c) the gate pulse width must be equal to turn-ON time.
(d) the gate pulse width must be less than turn-ON time.

4.22 The typical $\frac{di}{dt}$ rating of thyristor is
(a) 0.1 μA to 1 μA (b) 0.5 μA to 5 μA (c) 20 μA to 500 μA (d) 1000 μA

4.23 The typical $\frac{dv}{dt}$ rating of thyristor is
(a) 0.1 μV to 1 μV (b) 0.5 μV to 5 μV (c) 20 μV to 500 μV (d) 1000 μV

4.24 Snubber circuit is used for ______ protection of thyristor.
(a) $\frac{di}{dt}$ (b) $\frac{dv}{dt}$ (c) over current (d) gate

4.25 Thyristor can be used as
(a) ac switch (b) dc switch (c) either ac or dc switch (d) square wave switch

4.26 The static voltage equalisation in series connected thyristors is possible by using
(a) resistors of same value across each thyristor
(b) resistors of different values across each thyristor
(c) one resistor in series with the string of thyristors
(d) one resistor across the string of thyristors

4.27 The dynamic voltage equalization in series connected thyristors is possible by using
(a) resistor R and capacitance C in series and diode D across R
(b) resistor R and capacitance C in series and diode D across C
(c) resistor R and diode D in series and capacitance C across D
(d) capacitance C and diode D in series and resistor R across C

4.28 The snubber circuit parameters are

(a) $R_s = 2\xi\sqrt{\frac{C_s}{L}}$ and $C_s = \left(\frac{2\xi}{R_s}\right)^2 L$ (b) $R_s = 2\xi\sqrt{\frac{L}{C_s}}$ and $C_s = \left(\frac{2\xi}{L}\right)^2 R_s$

(c) $R_s = 2L\sqrt{\frac{\xi}{C_s}}$ and $C_s = \left(\frac{2\xi}{R_s}\right)^2 L$ (d) $R_s = 2\xi\sqrt{\frac{L}{C_s}}$ and $C_s = \left(\frac{2\xi}{R_s}\right)^2 L$

4.29 In a RC triggering circuit of thyristor, the value of R will be

(a) $R \le \frac{V - V_{gT} - V_d}{I_{gT}}$ (b) $R \le \frac{V + V_{gT} - V_d}{I_{gT}}$ (c) $R \le \frac{V - V_{gT} + V_d}{I_{gT}}$ (d) $R \le \frac{V + V_{gT} + V_d}{I_{gT}}$

4.30 The oscillation frequency of UJT oscillator is

(a) $f = \frac{1}{R_E C \ln\left(\frac{1}{1+\eta}\right)}$ (b) $f = \frac{1}{R_E C \ln\left(\frac{1}{1-\eta}\right)}$ (c) $f = \frac{1}{R_E \ln\left(\frac{1}{1-\eta}\right)}$ (d) $f = \frac{1}{C \ln\left(\frac{1}{1-\eta}\right)}$

4.31 In a UJT with V_{BB} voltage across two base terminals, the emitter potential at the peak point is given by

(a) $V_P = V_{BB} + \eta V_D$ (b) $V_P = \eta V_{BB}$ (c) $V_P = \eta V_{BB} - V_D$ (d) $V_P = \eta V_{BB} + V_D$

4.32 A DIAC can be used
(a) in triggering circuit of a TRIAC (b) for protection of a TRIAC
(c) to increase efficiency of a TRIAC (d) to decrease efficiency of a TRIAC

4.33 The latching current of a thyristor is involved with
(a) turn-OFF process (b) turn-ON process (c) turn-ON and turn-OFF process

4.34 The snubber circuit is used in a thyristor

(a) to limit the value of $\frac{di}{dt}$ (b) to limit the value of $\frac{dv}{dt}$

(c) to limit both the value of $\frac{di}{dt}$ and $\frac{dv}{dt}$ (d) to turn on

4.35 In the forward blocking state, a thyristor is associated with
(a) large voltage, large current (b) large voltage, low current
(c) low voltage, large current (d) low voltage, low current

4.36 In the forward conducting state, a thyristor is associated with
(a) large voltage, large current (b) large voltage, low current
(c) low voltage, large current (d) low voltage, low current

4.37 The three terminals of TRIAC are
(a) MT1, MT2 and gate (b) Base 1, Base 2 and emitter
(c) Anode, cathode and gate (d) Base, emitter, collector

4.38 Thyristor is equivalent to
(a) two transistors (b) two diodes (c) two DIAC (d) two MOSFET

4.39 TRIAC is equivalent to
(a) two transistors (b) two Thyristors (c) two DIAC (d) two MOSFET

4.40 The turn-off time of thyristor with a series connected R-L circuit can be reduced by
(a) increasing L (b) increasing R (c) decreasing L (d) decreasing R

4.41 The minimum value of current required to maintain conduction of thyristor is known as

(a) latching current (b) holding current (c) gate current (d) surge current

4.42 Thyristor is a

(a) voltage controlled
(b) current controlled
(c) both voltage and current controlled

4.43 Thyristor is a

(a) bi-directional device
(b) uni-directional device
(c) bipolar device
(d) unipolar device

4.44 The commutation of thyristor means

(a) turn ON
(b) turn OFF
(c) both turn ON and turn OFF
(d) breakdown

4.45 Figure 4.118 is a ______ commutation circuit.

(a) class A
(b) class B
(c) class C
(d) class D

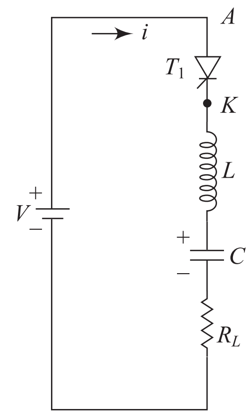
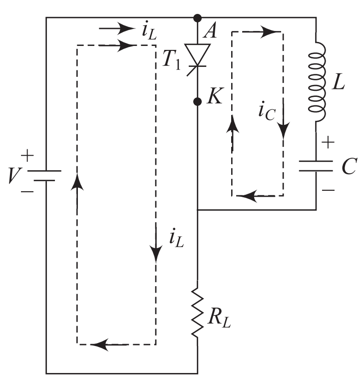


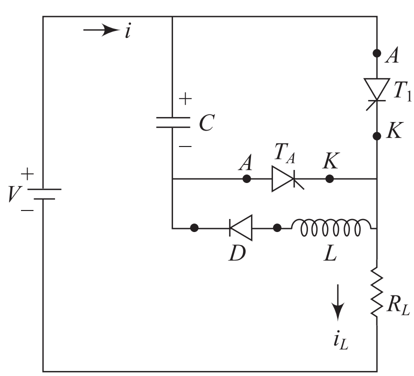
Fig. 4.118

4.46 Match List I and List II and select the correct answer

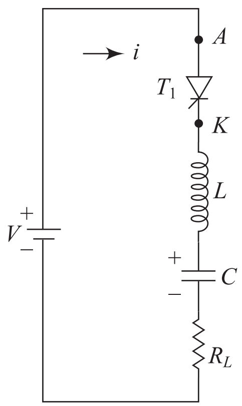
(a) A-1, B-2, C-3, D-4 (b) A-1, B-2, C-4, D-3
(c) A -3, B-4, C-1, D-2 (d) A-3, B-4, C-2, D-1

(A)


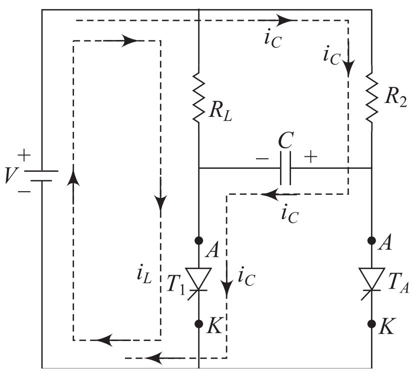
(1) class C commutation

(B)


(2) class A commutation

(C) 

(3) class B commutation

(D) 

(4) class D commutation

4.47 Match List I and List II and select the correct answer

(a) A-2, B-4, C-1, D-3 (b) A-1, B-2, C-4, D-3 (c) A-3, B-4, C-1, D-2 (d) A-3, B-4, C-2, D-1

(A) Resonant commutation	(1) Class E
(B) Complementary commutation	(2) Class A
(C) External pulse commutation	(3) Class E
(D) Line commutation	(4) Class C

4.48 In a commutation circuit of a thyristor, for satisfactory turn of thyristor the following condition must be satisfied:

(a) circuit time constant is less than device turn-OFF time.
(b) circuit time constant is greater than device turn-OFF time.
(c) circuit time constant is equal or less than device turn-OFF time.
(d) circuit time constant is equal to device turn-OFF time.

4.49 Match List I and List II and select the correct answer.

(a) A-1, B-4, C-2, D-3 (b) A-1, B-2, C-4, D-3 (c) A-3, B-4, C-1, D-2 (d) A-3, B-4, C-2, D-1

(A) Resonant commutation	(1) Load commutation
(B) Complementary impulse commutation	(2) Current commutation
(C) Resonant pulse commutation	(3) Natural commutation
(D) Line commutation	(4) Parallel capacitor commutation

4.50 When the amplitude of the gate pulse to thyristor is increased,
(a) the delay time would decrease but the rise time would increase
(b) both delay time and rise time would increase
(c) the delay time would decrease but the rise time remains unaffected
(d) the delay time would increase but the rise time would decrease

4.51 TRIACs are most commonly used when the supply voltage is
(a) low frequency ac voltage (b) high frequency ac voltage
(c) dc voltage (d) full-wave rectified output voltage

4.52 Which of the following statements are correct?
(a) TRIAC is a 2 terminal switch (b) TRIAC is a 2 terminal bilateral switch
(c) TRIAC is a 3 terminal bilateral switch (d) TRIAC is a 3 terminal unilateral switch

4.53 Which of the following characteristic of a silicon p-n junction diode makes an ideal diode?
(a) It has low saturation current,
(b) It has a high value of forward cut-in voltage,
(c) It can withstand large reverse voltage
(d) It has low saturation current and it can withstand large reverse voltage

4.54 Which of the following statements are correct in a two–transistor model of the *p-n-p-n* four layer device?
(a) It is used to explain the turn-on process of the *p-n-p-n* device
(b) It is used to explain the turn-off process of the *p-n-p*-n device
(c) It explains the device characteristics in reverse biased region
(d) It explains all the regions of the device characteristics

4.55 Thyristors can be turned OFF by
(i) Reducing the anode current below the holding current
(ii) Applying a reverse voltage between anode and cathode of the device
(iii) Reducing the gate current
(a) i (b) i, ii (c) iii (d) i, ii, iii

4.56 The turn-ON time for a thyristor is 40 μs. If the pulse train is applied at the gate having frequency of 2.5 kHz with duty ratio of 0.25, the thyristor will be
(a) turn ON (b) not turn ON
(c) turn ON if pulse frequency is increased
(d) turn ON if pulse frequency is decreased

4.57 In turn-ON process of a thyristor, the maximum power losses occur during
(a) t_r (b) t_d (c) t_s (d) None of these

4.58 A pulse transformer is used in a thyristor driver circuit
(a) to generate high frequency pulses (b) to save from dc triggering
(c) to shape the trigger signal (d) to provide isolation

4.59 Unequal current distribution in the parallel-connected thyristors is developed due to the non-uniformity in the
(a) forward characteristics (b) reverse characteristics
(c) di/dt withstand capability (d) dv/dt withstand capability

4.60 di/dt protection in a device is needed because
(a) it destroys the device (b) it interferes with control electronics
(c) it introduces voltage surges (d) None of these

4.61 The over-current protection of thyristor is provided by using
(a) circuit breaker and fuse (b) saturable di/dt coils
(c) snubber circuit (d) heat sinking

4.62 The capacitance of a reverse biased junction of a thyristor is 20 pico-farad. The charging current of the thyristor is 4 mA. What is the limiting value of dv/dt in V/μs?
(a) 100 V/μs (b) 200 V/μs (c) 300 V/μs (d) 400 V/μs

4.63 The cosine triggering circuit is used to linearise the relation between
(a) V_C, V_o (b) α, V_o (c) V_C, α (d) I_o, V_o

4.64 To generate triggering signal for thyristors, 555 time IC is used as
(a) astable multivibrator (b) mono-stable multivibrator
(c) bistable multivibrator (d) None of these

4.65 In synchronised UJT triggering of a thyristor, the voltage across capacitor reaches threshold voltage of UJT thrice in each half cycle. Therefore there are three firing pulses during each half cycle. The firing angle of the thyristor can be controlled
(a) thrice in each half cycle (b) twice in each half cycle
(c) Once in each half cycle (d) None of these

4.66 The function of a zener diode which is connected in an UJT trigger circuit for thyristor is,
(a) to provide a constant voltage to UJT to prevent unreliable firing
(b) to expedite the generation of triggering pulses
(c) to provide a variable voltage to UJT as the source voltage changes
(d) to provide delay the generation of triggering pulses

4.67 A relaxation oscillator circuit is shown in Fig. 4.119. What will be the change in voltage across capacitor, if the input voltage V is doubled.
(a) The frequency of V_C will reduce to half its value.
(b) The frequency of V_C will get doubled
(c) The amplitude will get doubled, but the frequency will remain unchanged
(d) The amplitude will remain unchanged, but the frequency will get doubled?

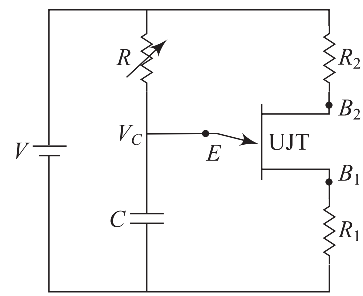


Fig. 4.119

4.68 In a pulse transformer, the ferrite material used for its core and the possible turn-ratio from primary to secondary is
(a) 10:1 (b) 20:1 (c) 1:1 (d) 5:1

4.69 GTOs with anode fingers have
(a) high turn-OFF time (b) no reverse blocking capability
(c) reduced turn-OFF gain (d) reduced tail current

4.70 Which of the following statements are correct?
(i) Anode fingers in GTO increase its turn-OFF gain
(ii) Anode-shorting reduces its reverse-voltage blocking capability to zero
(iii) GTO has n^+-type fingers in the anode
(a) i (b) i, ii (c) i, iii (d) i, ii, iii

4.71 A gold-doped GTO has
(a) low turn-OFF time (b) high reverse blocking capability
(c) low on-state voltage drop (d) increased tail current

4.72 In a GTO, anode current begins to fall when
(a) gate current is negative peak at time $t = 0$
(b) gate current is negative peak at t = storage period t_s
(c) gate current Is negative peak at $t = t_s$ + fall time

4.73 A reverse conducting thyristor is used in place of anti-parallel combination of thyristor and feedback diode in an inverter circuit
(a) decreases the operating frequency of operation
(b) effectively minimises the peak commutating current
(c) weakening in the commutation performance
(d) minimises the effect of lead inductance on the commutation performance

4.74 In a normal SCR, turn-ON time is
(a) equal to turn-OFF time t_q (b) less than turn-OFF time t_q
(c) more than turn-OFF time t_q (d) about half of turn-OFF time t_q

4.75 The average current rating of a thyristor, as specified by the manufacturer data sheet, corresponds to
(a) resistive and inductive current (b) resistive current
(c) inductive current (d) capacitive current

4.76 In reliable gate triggering of thyristors, usually we use
(a) slight over triggering (b) very hard triggering
(c) very soft triggering (d) none of these

4.77 The most commonly used gate triggering signal for thyristors is
(a) a short duration pulse (b) a steady dc signal
(c) a high-frequency pulse train (d) a low frequency pulse train

4.78 When thyristor starts conducting a forward current, its gate losses control over
(a) anode circuit voltage and current
(b) anode circuit voltage only
(c) anode circuit voltage and current and time
(d) anode circuit current only

4.79 The thyristor ratings such as *di*/*dt* in A/µs and *dv*/*dt* in V/µs, can vary, respectively between
(a) 20 to 500 A/µs, 10 to 100 V/µs (b) 20 to 500 A/µs, 20 to 500 V/µs
(c) 10 to 100 A/µs, 10 to 100 V/µs (d) 50 to 300 A/µs, 20 to 500 V/µs

4.80 A driver circuit is mainly needed between the controller and the power circuit for
(a) providing necessary drive power (b) voltage level change
(c) polarity change (d) isolation between power circuit and control circuit

4.81 Turn-ON time of an SCR can be decreased by using a
(a) rectangular pulse of high amplitude and narrow width
(b) triangular pulse
(c) rectangular pulse of low amplitude and wide width
(d) trapezoidal pulse

4.82 If the turn-ON time of an SCR is about 5 µs, an ideal trigger pulse should have
(a) short rise time with pulse width of 6 µs
(b) long rise time with pulse width of 6 µs
(c) short rise time with pulse width of 3 µs
(d) long rise time with pulse width of 3 µs

4.83 Turn-ON time of a thyristor in series with R-L load can be reduced by
(a) increasing inductance L (b) decreasing L
(c) increasing resistance R (d) decreasing R

4.84 A forward voltage can be applied to a thyristor after its
(a) reverse recovery time (b) gate recovery time
(c) anode voltage reduces to zero (d) anode current reduces to zero

4.85 The turn-ON time for a thyristor is 15 µs. If an inductance *L* is inserted in the anode circuit then the turn-ON time will be
(a) 15 µs (b) less than 15 µs (c) more than 15 µs (d) about 15 µs

4.86 Turn-OFF time of a thyristor is measured from the instant
(a) gate current becomes zero (b) anode current becomes zero
(c) anode voltage becomes zero (d) both anode voltage and anode current become zero

4.87 In a thyristor, anode current flows over a narrow region near the gate during
(a) delay time t_d and rise time t_r (b) rise time t_r and spread time t_p
(c) delay time t_d and spread time t_p (d) delay time t_d

4.88 The average on-state current for a thyristor is 25 A for a conduction angle of 120°. What will be the average ON-state current for 60° conduction angle?
(a) less than 25 A (b) 10 A (c) greater than 20 A (d) 50 A

4.89 Surge current rating of a thyristor specifies the maximum
(a) repetitive current with sine wave
(b) non-repetitive current with sine wave
(c) repetitive current with rectangular wave
(d) non-repetitive current with rectangular wave

4.90 Gate characteristic of a thyristor
(a) is $V_g = a + bI_g$ (b) is a straight line passing through origin
(c) is a curve between V_g and I_g (d) is spread between two curves of $V_g - I_g$

4.91 The *di*/*dt* rating of a thyristor is specified for its
(a) rising anode current (b) decaying anode current
(c) decaying gate current (c) rising gate current

4.92 The function of snubber circuit which is connected across a thyristor is to
(a) suppress *dv*/*dt* (b) increase *dv*/*dt*
(c) decrease *dv*/*dt* (d) remain transient overvoltage at a constant value

4.93 In a thyristor based converter circuit, *dv*/*dt* protection is achieved through the use of
(a) R-L in series with thyristor (b) R-C snubber across thyristor
(c) *L* in series with thyristor (d) RC in series with thyristor thyristor

4.94 In a thyristor based converter circuit, *di*/*dt* protection is achieved through the use of
(a) *R* in series with thyristor (b) RL in series with thyristor
(c) *L* in series with thyristor (d) *L* across

4.95 The maximum *di*/*dt* in a thyristor circuit is
(a) directly proportional to maximum value of supply voltage
(b) inversely proportional to maximum value of supply voltage
(c) directly proportional to maximum value of supply voltage and inversely proportional to circuit inductance *L*
(d) directly proportional to L

4.96 The resistance and capacitance are connected across gate circuit to protect the thyristor gate against
(a) overvoltages (b) *dv*/*dt* (c) over currents (d) noise signals

4.97 During the turn-OFF process in a thyristor, the current flow will not stop at the instant current becomes zero but continues to flow to a peak value in the reverse direction. This is happened due to
(a) hole-storage effect
(b) commutation failure
(c) presence of reverse voltage across the thyristor
(d) protective inductance in series with the thyristor

4.98 UJT has negative resistance region
(a) between peak and valley points (b) before the peak point
(c) after the valley point (d) Both (a) and (b)

4.99 When an UJT relaxation oscillator circuit is used for triggering a thyristor, the wave shape of the voltage which is generated by UJT circuit is a
(a) saw-tooth wave (b) sine wave (c) square wave (d) trapezoidal wave

4.100 In an UJT relaxation oscillator, the maximum value of charging resistance is associated with
(a) valley point (b) peak point
(c) any point between peak point and valley point
(d) any point after the valley point

4.101 If an UJT relaxation oscillator circuit is used for triggering of a thyristor, stand-off ratio $\eta = 0.64$ and dc source voltage V_{BB} is 20V. The UJT would trigger at the instant when the emitter voltage is
(a) 5 (b) 10 (c) 12.8 V (d) 13.5 V

4.102 UJT is used for relaxation oscillator due to
(a) positive resistance part of V-I characteristics (b) negative resistance part of V-I characteristics
(c) peak-point potential (d) valley-point potential

4.103 UJT is used to make a relaxation oscillator. Whenever it is energised, but it fails to oscillate due to
(a) large value of capacitor (b) low value of charging resistor
(c) high base terminal voltage V_{BB} (d) Both (b) and (c)

4.104 In dynamic equalising circuit for series connected thyristors, the value of *C* is depends on
(a) reverse recovery characteristics (b) turn-OFF characteristics
(c) turn-ON characteristics (d) rise time characteristics

4.105 A power semiconductor device may be damaged due to
(a) high *di*/*dt* (b) low *di*/*dt* (c) high *dv*/*dt* (d) low *dv*/*dt*

4.106 A TRIACe is corresponding to
(a) two thyristors connected in anti-parallel
(b) two diodes connected in anti-parallel
(c) one thyristor and one diode connected in parallel
(d) two thyristors connected in parallel

4.107 Which of the following statements are correct?
(i) TRIAC is a five layer devices
(ii) TRIAC consists of two parallel sections $p_1n_1p_2n_2$ and $p_2n_1p_1n_4$
(iii) TRIAC is a double ended SCR
(a) i (b) ii (c) iii (d) i, ii, iii

4.108 For the semiconductor devices TRIAC and thyristor
(a) TRIAC requires more current for turn-on than SCR at a particular voltage
(b) a TRIAC has less time for turn-off than hyristor
(c) both are unidirectional
(d) both are bidirectional

4.109 In a reverse blocking thyristor
(a) external layers are heavily doped and internal layers are lightly doped
(b) external layers are lightly doped and internal layers are heavily doped
(c) *p*-layers are lightly doped and *n*-layers are heavily doped
(d) *p*-layers are heavily doped and *n*-layers are lightly doped

Fill in the Blanks

4.1 A thyristor is a ______ layer ______ structure.
4.2 Thyristor is a ________ device.
4.3 When gate is open, thyristor may turn ON due to large forward ______.
4.4 To turn-OFF, a thyristor the anode current must be bring below ______current.
4.5 A thyristor can be turned on when a forward voltage greater than _______ voltage is applied.
4.6 A forward biased thyristor can be turned ON when a positive ______ pulse is applied.
4.7 The forward break over voltage of thyristor decreases with increase in ______ current.
4.8 A ______ biased thyristor can be turned ON by applying a ______ gate current pulse.
4.9 The reverse breakdown voltage of a thyristor is ______of gate current.
4.10 The total turn-ON time of a thyristor is sum of ______ time______ time and ______ time.
4.11 In the gate triggering of a thyristor, the pulse width of gate current must be larger than the ________ time of the device.
4.12 The total turn-OFF time of a thyristor is sum of ______ time and _______ time.
4.13 During ______ time the rate of rise of anode current must restricted to avoid local ______.
4.14 The reverse recovery charge of a thyristor depends on the ______current and ______time. .
4.15 ______ grade thyristors have _______ turn-OFF time compared to a converter grade thyristor.
4.16 SCR is a ______device.
4.17 TRIAC is ______device.
4.18 TRIAC is a ______and ______carrier device.
4.19 TRIAC is just like two _______ connected thyristors.
4.20 TRIAC operates either in the ______ quadrant or the ______ quadrant of the V-I characteristics.
4.21 Thyristor is a ______ layer, ______ terminal, ______ carrier semi-controlled device
4.22 Three terminals of a thyristor are called ______, ______ and ______.
4.23 A thyristor can _____voltage of both polarity but conducts current only from anode to cathode.
4.24 After turn on the voltage across the thyristor drops to a very low value about ____Volt.
4.25 A TRIAC is functionally equivalent to ______ anti parallel connected thyristors. It can ______ voltages in both directions and ______ current in both directions.
4.26 TRIACs are extensively used at power frequency ______ load control applications.

4.27 In the ______ quadrant the TRIAC is fired with ______ gate current while in the _____ quadrant the gate current must be ______.
4.28 To avoid unwanted turn ON of a TRIAC due to large dv/dt, ______ circuits are used across TRIACs.
4.29 For "proper turn ON" of a TRIAC, the ______ time of the gate current pulse should be as ______ as possible.
4.30 A thyristor can be turned ON by applying a ______ gate current pulse when it is ______ biased.
4.31 A thyristor can be turned OFF by decreasing its anode current below ______ current and applying a reverse voltage across the device for duration ______ than the turn OFF time of the device.
4.32 The reverse recovery charge of a thyristor depends on the ______ of the forward current just before turn OFF and its rate of ______.
4.33 Inverter grade thyristors have______ turn-OFF time compared to a converter grade thyristor.
4.34 V_{RSM} rating of a thyristor is ______ than the V_{RRM} rating but ______ than the V_{BRR} rating.
4.35 The maximum average current rating of a thyristor depends on the______ temperature of the thyristor and the ______ of the current wave form.
4.36 The gate non-trigger voltage specification of a thyristor is useful for avoiding unwanted turn on of the thyristor due to ______ voltage signals at the gate.
4.37 The reverse saturation current of a thyristor ______ with gate current.
4.38 To prevent unwanted turn ON of a thyristor all ______signals between the gate and the cathode must be less than the______.
4.39 A thyristor can conduct current in ______ direction and can block voltage in ______ direction.
4.40 Thyristor can be turned ON due to large forward ______.
4.41 GTO is a ______layer, ______terminal current controlled minority carrier device.
4.42 A GTO can be turned ON by applying a ______gate current pulse when it is forward biased and turned OFF by applying a ______gate current.
4.43 GTOs have relatively ______ turn-OFF current gain.
4.44 GTO can block rated forward voltage only when the gate cathode junction is______.
4.45 GTO is a____ controlled _____ carrier device.
4.46 The anode shorts of GTO improve the _____ performance but degrade the ______ performance.
4.47 To turn OFF a conducting GTO the gate terminal is ______biased with respect to the ______.
4.48 The reverse voltage blocking capacity of a GTO is ______due to the presence of anode shorts.
4.49 A conducting GTO reverts back to the blocking mode when the anode current falls below ______ current.
4.50 Anode shorts help to reduce the ______ current in a GTO.
4.51 In the gate drive circuit of GTO, a low value resistance is connected between the gate and the cathode terminals for minimum______ voltage.
4.52 The gate drive circuit of a GTO must provide continuous positive gate ______ during ON period and continuous negative gate______ during OFF period.
4.53 The holding current of a GTO is______ compared to a thyristor.
4.54 During turn-ON process, the turn on delay time and current rise time of a GTO can be reduced by increasing the gate current______ and______.
4.55 GTO can operate safely in the ______ region for a short time provided the gate cathode junction is reverse biased.
4.56 The switching delay times and energy loss of a GTO can be reduced by ______the gate current magnitude and its rate of rise.
4.57 The maximum turn-OFF anode current of a GTO can be increased by increasing the______.

Review Questions

4.1 What is SCR? What is thyristor? What are the different names of thyristors? Write the difference between thyristor and power BJT.

4.2 Draw the structure of a thyristor and explain its operating principle briefly.
4.3 Draw the constructional details of thyristor with doping intensity and width of layers and explain in detail. What is the symbol of thyristor?
4.4 What is the diode model of thyristor?
4.5 Describe the basic behaviour of thyristor using a two-transistor model.
4.6 What are the different modes of operation of a thyristor? Explain each mode briefly.
4.7 What is two transistor model of thyristor? Prove that anode current of thyristor is given by

$$I_A = \frac{\alpha I_G + I_{CB01} + I_{CB02}}{1-(\alpha_1+\alpha_2)}$$

4.8 Draw the V-I characteristics of a thyristor and explain different operating regions. What is the effect of gate current on the V-I characteristics of a thyristor?
4.9 Explain the transient model of thyristor.
4.10 What are the necessary conditions for turning ON of a thyristor?
4.11 What are the different turning-ON methods of a thyristor? Explain each method.
4.12 Describe the different methods of triggering of SCR.
4.13 What is gate triggering of thyristor? What is latching current of thyristor? What is holding current of thyristor?
4.14 What are the types of switching characteristics of SCR? Draw the turn-ON characteristics of SCR and explain briefly?
4.15 Draw the turn-OFF characteristics of SCR and explain briefly?
4.16 Define (a) delay time (b) rise time (c) spread time (d) reverse recovery time (e) gate recovery time
4.18 Explain the turn-ON and turn-OFF time of SCR.
4.19 Draw the gate characteristics of a thyristor and explain its importance in design of gate drive(triggering) circuit.
4.20 Discuss following ratings of thyristor
(a) On state voltage drop (b) Finger voltage (c) Voltage safety factor
(d) Latching current (e) Holding current (f) Surge current (g) I^2t rating
4.21 Why turn-ON time is less than turn-OFF time in a thyristor?
4.22 Why protection circuit is required during operation of thyristor? What are the different protection schemes of thyristor?
4.23 What is the purpose of $\frac{di}{dt}$ and $\frac{dv}{dt}$ protection?
4.24 Write short notes on the following:
(a) Over voltage protection (b) Over current protection (c) Gate protection
4.25 What is snubber circuit? Explain the design method of a snubber circuit.
4.26 Explain the series and parallel operation of thyristors.
4.27 Define string efficiency. What is the derating factor of a series connected thyristors? Prove that string efficiency of two series connected SCRs is less than unity or 1.
4.28 Discuss the common technique for voltage sharing of series connected thyristors. Derive the expression for the resistance (R) used for static voltage equalisation for a series connected string.
4.29 Explain the common technique for current sharing of parallel connected thyristors. Prove that string efficiency of two parallel connected SCRs is less than unity or 1.
4.30 What is the effect of reverse recovery time on the dynamic (transisent) voltage sharing of series connected thyristors? Derive an expression for the capacitance (C) used for dynamic voltage equalisation for a series connected string.
4.31 Describe the crowbar protection circuit for the over current protection of thyristors.
4.32 Explain the different triggering signals of a thyristor. What is the nature of gate current waveform?
4.33 What are the different triggering circuits of a SCR? Draw a resistance triggering circuit and explain its operation. What is the limitation of a resistance triggering circuit?

4.34 Draw a RC triggering circuit and explain its operation. What are the advantages of RC triggering over R triggering circuit?

4.35 What do you mean by the following terms in a UJT:
(a) Peak voltage (b) Valley voltage (c) Stand-off ratio

4.36 What is relaxation oscillator?

4.37 Write short notes on the following:
(a) UJT triggering circuit (b) Synchronised UJT triggering circuit
(c) Ramp and pedestal triggering circuit.

4.38 Compare UJT triggering circuit with R and RC firing circuit.

4.39 If the turn-ON and turn-OFF times of a thyristor are not constant, list the factors briefly.

4.40 Justify the statement "lower the gate current, the forward break-over voltage is higher".

4.41 What is commutation? What are the types of commutation? Explain any one commutation circuit with a diagram and waveforms.

4.42 Distinguish between the following:
(a) Natural commutation and forced commutation
(b) Voltage commutation and current commutation

4.43 Draw the self-commutation circuit and discuss in detail. What are the advantages and disadvantages of self-commutation with respect to other commutations?

4.44 Write short notes on the following:
(a) Resonant pulse commutations (b) Complementary commutations (c) Class D commutations
(d) Class E commutations (f) Class F commutations

4.45 What is DIAC? Explain the operating principle of DIAC with proper diagram.

4.46 Draw the V-I characteristics of a DIAC and explain different operating regions. What are the applications of DIAC?

4.47 What is TRIAC? Explain the operating principle of TRIAC with proper diagram.

4.48 Draw the V-I characteristics of a TRIAC and explain different operating regions. What are the applications of TRIAC?

4.49 What are the problems of series and parallel operation of thyristors?

4.50 The turn-ON and turn-OFF time of SCR are 3.5 μs and 6.5 μs respectively, determine the maximum switching frequency of SCR in a converter circuit.

4.51 A 220 V, 50 Hz single phase ac supply is connected to a thyristor in series with a load resistance R_L. Under forward blocking condition, the capacitance of junction J_2 is 1.35 nF. At room temperature 30°C, thyristor has the following parameters:

$V_{\text{RRM}} = 1000$ V, $V_{\text{RMS}} = 300$ V, $V_F = 1$ V, $V_{gT} = 2.5$ V, $I_{gT} = 100$ mA, $dV/dt = 75$ V/μs and $di/dt = 50$ V/μs.

Prove that a spike of 300 V for 3 μs duration is sufficient to trigger thyristor.

4.52 When a thyristor acts as a switch, anode current rises linearly from zero to final value of 210 A but the anode voltage falls linearly from 520 V to zero during turn-ON process. The turn-ON time of thyristor is 9.5 μs. If the switching frequency of thyristor is 260 Hz, determine the average power loss in thyristor.

4.53 The average gate power dissipation of a thyristor is $P_{\text{gav}} = 0.5$ Watt. If the gate voltage varies from 2.45 V to 9.8 V, plot the curve where gate voltage is a function of gate current. Assume average gate power dissipation is constant.

4.54 Assume the slope of the gate cathode characteristics of thyristor is 160 and average power dissipation is 0.85 W. When the gate source voltage is 16 V, determine the gate source resistance.

4.55 In a gate triggering circuit, the average power dissipation is 0.35 W. The slope of load line is 110 V per ampere, source voltage is 14 V and minimum gate current to turn-ON thyristor is 16 mA. Determine gate current, gate voltage and gate source resistance.

4.56 The gate characteristics of a thyristor is $V_g = 1 + 4.5I_g$. When a rectangular pulse of 12 V with 25 μs is applied to gate, determine (a) the series connected resistance in gate, (b) triggering frequency and (c) duty cycle. Assume average power dissipation is 0. 45 W and peak gate drive power is 4 W.

4.57 A 230 V dc voltage is connected to a thyristor in series with R-L load. Assume the latching current of thyristor is 80 mA. When (a) $R = 20\ \Omega$ and $L = 0.1$ H and (b) $R = 20\ \Omega$ and $L = 1$ H, find the minimum width of gate pulse current to turn-ON thyristor. What is the effect of inductance on gate-pulse width?

4.58 The gate-cathode characteristics of SCR are spread by the following equations:

$$I_g = 2.45 \times 10^{-3} V_g^2 \quad \text{and} \quad I_g = 2.45 \times 10^{-3} V_g^{1.5}$$

If the gate source voltage is 15 V and $R_s = 120\ \Omega$, determine the triggering voltage and triggering current. Assume the gate power dissipation is 0.45 W.

4.59 A 220 V dc supply voltage is applied to an SCR which is connected in series with RL load as shown in Fig. 4.120. The latching current of a thyristor is 25mA. When a gate pulse of 30 μs is applied, whether the SCR will be turned-on or not?

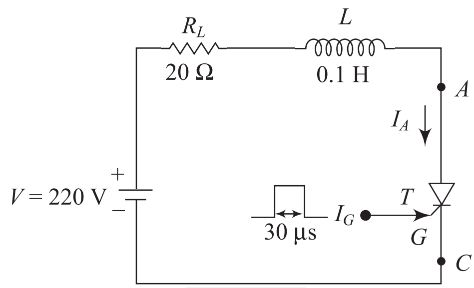


Fig. 4.120

4.60 The maximum rms ON-state current of thyristor is 60 A. When the thyristor is used in a resistive load circuit and the current waveform is rectangular wave as shown in Fig. 4.121, determine average ON-state current rating for conduction angle (a) 90° and (b) 60°.

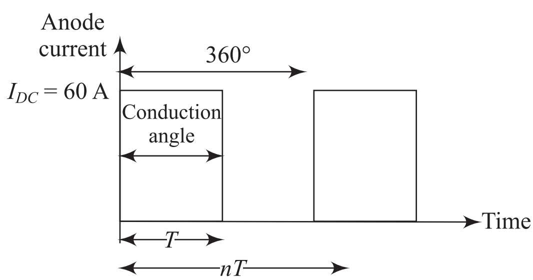


Fig. 4.121

4.61 The maximum rms ON-state current of thyristor is 50 A. When the thyristor is used in a resistive load circuit and the current waveform is half-sine wave as shown in Fig. 4.122, determine average ON-state current rating for conduction angle (a) 120° (b) 90° (c) 60°.

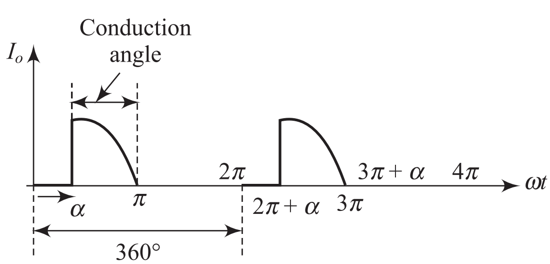


Fig. 4.122

4.62 The half-cycle surge current rating of thyristor is 3000 A at 50 H supply. Find the one-cycle surge current rating and I^2t rating of thyristor.

4.63 When a SCR is operating with a peak supply voltage of $230\sqrt{2}$ V and it has the following parameters:

$$\text{Repetitive peak current } I_p = 100 \text{ A}, \left.\frac{dv}{dt}\right|_{\max} = 400 \text{ V/μs}, \left.\frac{di}{dt}\right|_{\max} = 80 \text{ A/μs} \cdot$$

Design a snubber circuit for SCR protection. Assume the factor of safety is 2.5 and the minimum value of resistance is 25 Ω.

4.64 Determine the $\frac{di}{dt}$ rating and $\frac{dv}{dt}$ rating of thyristor which is connected in the circuit as shown in Fig. 4.123. Find the average and rms current rating of thyristor at the firing angle $\alpha = 60°$. What is the voltage rating of thyristor?

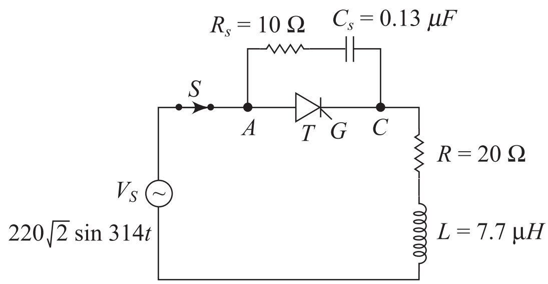


Fig. 4.123

4.65 Thyristors with a rating of 1200 V and 200 A are used in a string to handle 15 kV and 1.5 kA. Determine the number of series and parallel connected thyristors in case derating factor is (a) 0.2 (b) 0.3.

4.66 A 180 A SCR is connected in parallel with a 220 A SCR as shown in Fig. 4.124. The on-state voltage drop across SCRs is 1.8 V and 1.7 V respectively. Determine the series resistance that must be connected in series with each SCR when 400 A current is shared by two SCRs according to their rating.

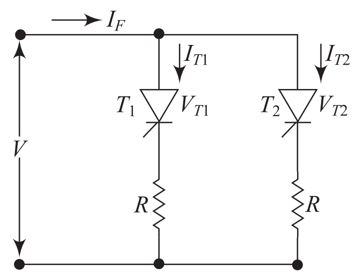


Fig. 4.124

4.67 When a 280 A thyristor operate in parallel with another 320 A thyristor, the ON-state voltage drops across thyristors is 1.5 V and 1.2 V. Find the value of resistance which will be connected in series with each thyristor. Assume that the total current 600 A is shared by thyristors according to their rating.

4.68 A string of five series connected SCRs has static and dynamic equalising circuits. The string should withstand at 11 kV. When the static equalizing resistance is 20 kΩ and the dynamic equalizing circuit consists of $R_C = 50\ \Omega$ and $C = 0.04\ \mu F$, determine the voltage across each SCR and discharge current through SCR T_1. Assume the leakage current of five SCRs are 11 mA, 12 mA, 14 mA, 18 mA and 20 mA respectively.

4.69 Two thyristors having a difference of 2.5 mA in latching current are connected in series in a circuit. The voltage across the thyristors are 410 V and 370 V respectively. Determine the required equalising resistance.

4.70 Design a relaxation oscillator circuit using UJT which has the following specification:

$\eta = 0.7$, $I_p = 0.65$ mA, $V_p = 12$ V, $I_v = 2.0$ mA, $V_v = 1.5$ V, $R_{BB} = 4.5$ kΩ

and Norman leakage current is 4 mA when emitter open circuit.

The firing frequency is 2.5 kHz. Assume the suitable value of capacitance C.

4.71 The firing frequency of relaxation oscillator is varied by changing the value of charging resistance R. Find the maximum and minimum values of R and their corresponding firing frequencies.

Assume $\eta = 0.7$, $I_p = 0.65$ mA, $V_p = 12.5$ V, $I_v = 2.0$ mA, $V_v = 1.5$ V, $V_{BB} = 20$ V, and $C = 0.047\ \mu F$.

4.72 In class A commutation circuit, A thyristor is connected in series with R-LC. When $L = 11\ \mu H$, $C = 21\ \mu F$ and $R = 1\ \Omega$, check whether self-commutation is possible or not. Determine the conduction time of thyristor.

4.73 In a class D commutation circuit, determine the value of commutating capacitor C and commutating inductor L with the help of following data:

$V = 110$ V, $I_{L(\max)} = 55$ A and t_{off} for $T_1 = 20$ μs

4.74 Figure 4.125 shows a voltage commutation circuit. Determine the turn-OFF time of main thyristor when $C = 16$ μF, $R = 11$ Ω and $V = 220$ V. Assume the capacitor is charged to voltage V.

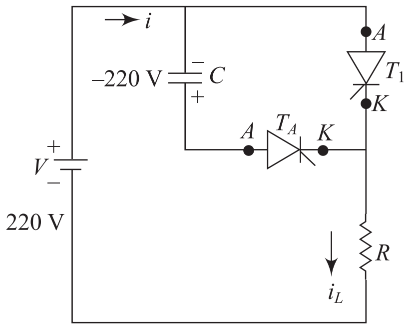


Fig. 4.125

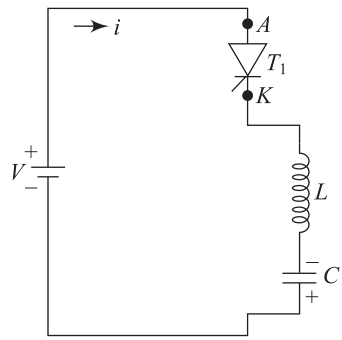


Fig. 4.126

4.75 Figure 4.126 shows a load commutation circuit. Find the value of commutation time of thyristor and resonant frequency when $L = 2.5$ H and $C = 12$ μF.

4.76 In a class C commutation circuit, the dc input voltage is 230 V and current through R_L and R_1 is 25 A. If the turn-OFF time of both main and auxiliary thyristor is 24 μs, determine the value of commutating capacitor C.

4.77 A class D commutation circuit has the following parameters:

$V = 220$ V, $L = 20$ μH and $C = 40$ μF

If the load current is 125 A, determine the circuits turn-OFF times for main and auxiliary thyristors.

4.78 Explain the different features of a thyristor firing circuit. Draw a schematic block diagram of thyristor firing circuit and explain the function of each block elaborately.

4.79 Draw the circuit diagram of a resistance firing circuit for thyristor. Is it possible to obtain a firing angle greater than 90° using R triggering method?

4.80 In resistance firing circuit, prove that firing angle is proportional to the variable resistance.

4.81 Compare the UJT firing circuit with R and RC firing circuits.

4.82 Discuss the working principle of UJT relaxation Oscillator. Derive the expressions for the frequency of triggering and firing angle delay in terms of eta and charging resistance.

4.83 Draw the circuit diagram for the ramp and pedestal trigger circuit which is used for a single phase semi-converter. Explain circuit operation with waveforms.

4.84 Explain the application of pulse transformer in the triggering circuit of thyristors and GTOs.

4.85 Draw the trigger circuit for a TRIAC using a DIAC and explain its operation with waveforms.

4.86 Draw a gate trigger circuit for a single phase full converter. Explain how the adjustment of control voltage varies the firing delay angle.

4.86 Describe the gate-pulse amplifier using a MOSFET.

4.87 Draw a cosine firing scheme for the triggering of thyristors and explains its operation. Why is the cosine-firing scheme so popular?

4.88 Why pulse train gating is preferred over pulse gating? Explain the pulse-train gating of thyristors with relevant circuit waveforms.

4.89 What is GTO? Draw the structure of a GTO.

4.90 Explain the turn-OFF process of GTO with its two-transistor model.

4.91 Describe switching performance of a GTO with proper voltage and current waveforms.

4.92 Write the merits and demerits of GTOs as compared to conventional thyristors. Write some applications of GTOs.

4.93 In a thyristor the capacitance value of reverse-biased junction J_2 is $C_{J_2} = 40$ pF and it is independent of the OFF-state voltage. The limit value of the charging current to turn the thyristor is about 25 mA. Find the critical value of $\frac{dv}{dt}$.

4.94 The capacitance value of reverse-biased junction J_2 of a thyristor, C_{J_2} is independent of OFF-state voltage. The limit value of the charging current to turn the thyristor is about 20 mA. If the critical value of $\frac{dv}{dt}$ is 500 V/μs, compute the value of junction capacitance C_{J_2}.

4.95 Figure 4.127 shows that a thyristor is connected in series with R-L load. The latching current is 80 mA. When a firing pulse of 90 μs is applied in between gate and cathode, find the state of thyristor whether it is turned ON or turned OFF.

4.96 In Fig. 4.128, the latching current of thyristor is 80 mA. What will be the minimum pulse width of gating pulse to turn-ON thyristor?

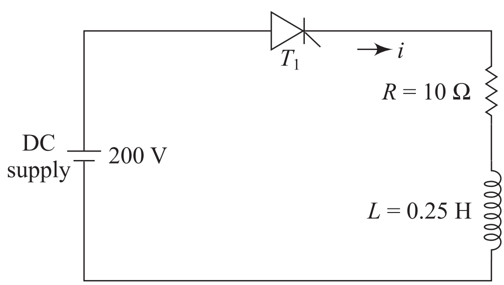


Fig. 4.127

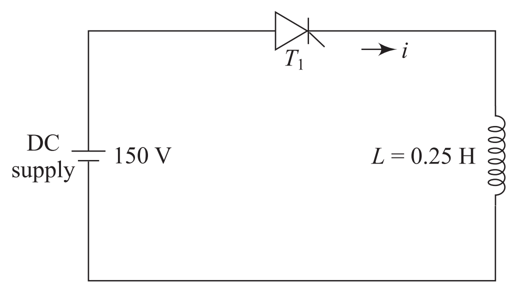


Fig. 4.128

4.97 When $i_g - V_g$ characteristics of a thyristor is a straight line passing through origin with a gradient of 2.5 × 10^3, find the value of gate voltage if $P_g = 0.025$ Watt.

4.98 In a thyristor, the gate-cathode characteristics is a straight line passing through origin with a gradient of $\frac{V_g}{i_g} = 14$ V/A. The maximum turn-ON time is 100 μs, the minimum gate current required is 100 mA. When the gate to source voltage is 10 V, (a) find the value of gate-source resistance which is connected in series with gate drive circuit as shown in Fig. 4.129 (b) determine the power dissipation. If the average gate power dissipation is 150 mW, determine the maximum triggering frequency of thyristor.

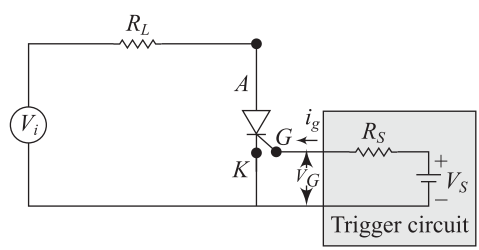


Fig. 4.129

4.99 In a forward biased thyristor, the gate current is gradually increased from zero until the device is turned ON. At the instant of turn-ON, the gate current is about 1.2 mA and when the thyristor operates in conduction, gate current decays to about 0.4 mA. Justify the above statement. What will be the gate cathode voltage when *V*s is zero?

4.100 In Fig. 4.130, a trigger pulse of 50 μs is applied across gate cathode of thyristor. The latching current of thyristor is 50 mA. When $R = 75\ \Omega$ and $L = 2.5$ H, check whether the thyristor will be turned on or turned OFF? If thyristor is not in turned ON, when the device will be turned ON.

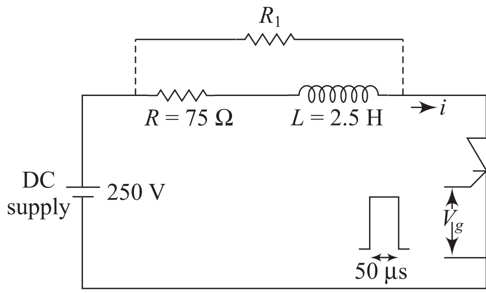


Fig. 4.130

4.101 I-V characteristic of a thyristor is a straight line as shown in Fig. 4.131. Calculate the average power loss in the thyristor and rms current rating of thyristor for following conditions:

(i) A constant current of 150 A for one half cycle.

(ii) A half cycle sine wave current with peak value of 150 A.

Draw the voltage drop across thyristor for the above conditions.

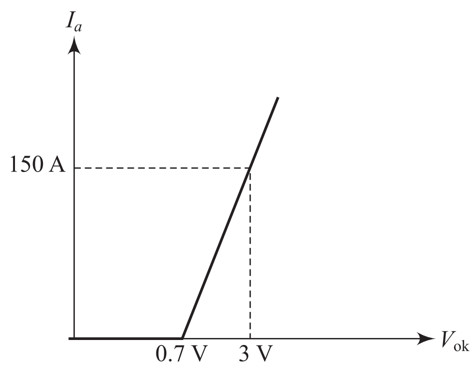


Fig. 4.131

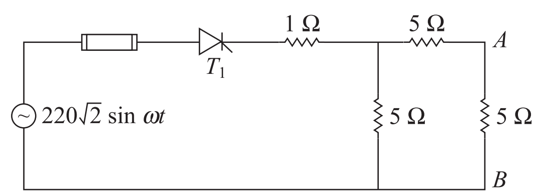


Fig. 4.132

4.102 Fig. 4.132 shows a circuit diagram of a thyristor. I^2t rating of thyristor is 60 A^2/s. If the terminal A and B is short circuited, determine fault clearance time so that thyristor is not damaged completely.

4.103 The sub-cycle surge current rating of thyristor is 3500 A for 50 Hz supply. Determine the one cycle surge current rating of thyristor and I^2t rating.

4.104 When a SCR is operating with a peak supply voltage of $230\sqrt{2}$ V and it has the following parameters:

$$\text{Repetitive peak current } I_p = 120 \text{ A}, \left.\frac{dv}{dt}\right|_{\max} = 400 \text{ V/µs}, \left.\frac{di}{dt}\right|_{\max} = 90 \text{ A/µs}$$

Design a snubber circuit for SCR protection. Assume the factor of safety is 2 and the minimum value of resistance is 20 Ω.

4.105 When a thyristor is operating with a peak supply voltage of $400\sqrt{2}$ V and it has the following specifications:

$$\text{Repetitive peak current } I_p = 450\text{A}, \left.\frac{di}{dt}\right|_{\max} = 100 \text{ A/µs}, \left.\frac{dv}{dt}\right|_{\max} = 210 \text{ V/µs}$$

Assume that the factor of safety is 2.5 for $I_p, \left.\frac{di}{dt}\right|_{\max}$ and $\left.\frac{dv}{dt}\right|_{\max}$ Design a suitable snubber circuit for SCR protection . Assume the factor of safety is 2 and the minimum value of resistance is 10 Ω.

4.106 In resistance triggering circuit, $I_{g(min)} = 0.16$ mA and $V_{g(min)} = 0.6$ V. When the peak amplitude of input voltage is 100 V, find the trigger angle α for $R_1 = 100$ kΩ and $R_{min} = 12$ kΩ.

4.107 A 230 V, 50 Hz ac supply is connected to a resistance capacitance (RC) triggering circuit. If the resistance R is variable from 5 kΩ to 20 kΩ, $V_{gT} = 2.5$ V and $C = 0.47$ μF, what is the minimum and maximum firing angle?

4.108 In a class B resonant pulse commutation circuit, $L = 10$ μH and $C = 25$ μF. The initial voltage across capacitor is 230 V, determine (a) resonant frequency (b) peak value of resonant current (c) turn-OFF time of thyristor.

4.109 In a Class C commutation, if $V = 200$ V, $R_L = 20$ Ω, $R_2 = 100$ Ω calculate (a) the peak value of current through thyristors T_1 and T_2 (b) the value of capacitance C when turn-OFF time of each thyristor is equal to 20 μs. Assume that the factor of safety is 2.

4.110 In a Class D commutation circuit, $V = 220$ V, $L = 20$ μH and C = 60 μF. If load current is 100 A, determine (a) peak value of current through capacitance, main thyristor and auxiliary thyristor (b) turn-OFF time of thyristors.

4.111 In class A commutation circuit, a thyristor is turned on at $t = 0$. Compute (a) the conduction time of thyristor (b) voltage across thyristor and capacitor when thyristor is turned OFF. Assume the following parameters $L = 12$ mH, $C = 25$ μF, $R = 0$ Ω and V = 200 V.

4.112 A class D commutation circuit has the following parameters:

$$V = 220 \text{ V}, L = 20\ \mu\text{H and } C = 40\ \mu\text{F}$$

If the load current is 100 A, determine the circuit turns OFF times for main and auxiliary thyristors.

4.113 For a class C commutation circuit, the dc input voltage is 220 V and current through R_L and R_1 is 12 A. If the turn-OFF time of both main and auxiliary thyristor is 40 μs, calculate the value of commutating capacitor C.

4.114 A class D commutation circuit has the following parameters:

$$V = 230 \text{ V}, L = 12\ \mu\text{H and } C = 25\ \mu\text{F}$$

When the load current is 100 A, find the circuit turn-OFF times for main and auxiliary thyristors.

4.115 The firing circuit of a TRIAC using a DIAC is shown in shown in Fig. 4.135 and this circuit has the following parameters:
$R_1 = 1000$ Ω, $R_2 = 0$ Ω to 25000 Ω, $C = 0.47$ μF, $V = 230$ V, $f = 50$ Hz and DIAC breakdown voltage is 25 V. Determine the minimum and maximum value of firing angle of TRIAC if the effect of load impedance is neglected.

Answers to Multiple-Choice Questions

4.1 (c)	4.2 (b)	4.3 (a)	4.4 (b)	4.5 (a)	4.6 (b)	4.7 (b)
4.8 (d)	4.9 (d)	4.10 (a)	4.11 (b) & (d)	4.12 (d)	4.13 (b)	4.14 (d)
4.15 (c)	4.16 (c)	4.17 (d)	4.18 (b)	4.19 (b)	4.20 (d)	4.21 (a)
4.22 (c)	4.23 (c)	4.24 (b)	4.25 (b)	4.26 (a)	4.27 (a)	4.28 (d)
4.29 (a)	4.30 (b)	4.31 (d)	4.32 (a)	4.33 (b)	4.34 (b)	4.35 (b)
4.36 (c)	4.37 (a)	4.38 (a)	4.39 (b)	4.40 (c)	4.41 (b)	4.42 (b)
4.43 (b)	4.44 (b)	4.45 (a)	4.46 (d)	4.47 (a)	4.48 (b)	4.49 (a)
4.50 (c)	4.51 (a)	4.52 (c)	4.53 (d)	4.54 (a)	4.55 (b)	4.56 (a)
4.57 (a)	4.58 (d)	4.59 (a)	4.60 (a)	4.61 (a)	4.62 (a)	4.63 (a)
4.64 (a)	4.65 (c)	4.66 (a)	4.67 (a)	4.68 (c)	4.69 (a)	4.70 (c)
4.71 (c)	4.72 (b)	4.73 (d)	4.74 (b)	4.75 (b)	4.76 (a)	4.77 (c)
4.78 (a)	4.79 (b)	4.80 (a)	4.81 (a)	4.82 (a)	4.83 (b)	4.84 (b)
4.85 (c)	4.86 (b)	4.87 (a)	4.88 (a)	4.89 (b)	4.90 (d)	4.91 (a)
4.92 (a)	4.93 (b)	4.94 (c)	4.95 (c)	4.96 (d)	4.97 (a)	4.98 (a)
4.99 (a)	4.100 (b)	4.101 (d)	4.102 (b)	4.103 (d)	4.104 (a)	4.105 (a)
4.106 (a)	4.107 (d)	4.108 (a)	4.109 (a)			

Answers to Fill in the Blanks

4.1 four, p-n-p-n
4.2 minority carrier
4.3 $\left(\frac{dv}{dt}\right)$
4.4 holding
4.5 forward break over
4.6 gate
4.7 gate
4.8 forward, positive
4.9 independent
4.10 delay time, rise time, spread time
4.11 turn ON
4.12 reverse recovery time, gate recovery time
4.13 rise, hot spots
4.14 I_{RR}, reverse recovery
4.15 inverter, faster
4.16 unidirectional
4.17 bidirectional
4.18 bidirectional, minority
4.19 anti-parallel
4.20 first, third
4.21 four, three, minority
4.22 anode, cathode, gate
4.23 block
4.24 1
4.25 two, block, conduct
4.26 AC
4.27 first, positive, third, negative
4.28 R-C snubber
4.29 rise, small.
4.30 positive, forward
4.31 holding, larger
4.32 Amplitude, decrease
4.33 less
4.34 greater, less
4.35 case, conduction angle
4.36 noise or surge
4.37 increases
4.38 spurious noise, gate non-trigger voltage
4.39 one, both
4.40 dv/dt
4.41 four, three
4.42 positive, negative
4.43 low
4.44 reverse biased
4.45 current, minority
4.46 turn OFF, turn ON)
4.47 reverse (negative), cathode
4.48 small
4.49 holding
4.50 tail
4.51 forward blocking
4.52 current, voltage
4.53 large
4.54 Amplitude, di_g/dt
4.55 reverse avalanche
4.56 increasing
4.57 turn-OFF snubber capacitance

SINGLE-PHASE UNCONTROLLED RECTIFIERS

5

5.1 INTRODUCTION

The rectifier circuit is used to convert ac input voltage into fixed dc voltage. Figure 5.1 shows the block diagram of a rectifier with its input voltage and output voltage. This circuit is also called ac-to-dc converter (uncontrolled). Power diodes are used in rectifier circuit. When a power semiconductor diode is forward biased, it conducts if input voltage is greater than cut-off voltage of diode. As soon as power diode is reverse biased, it turns OFF and stop conducting. Usually single-phase and three-phase rectifier circuits are used as dc power supply in different applications depending upon the voltage and current rating. Generally, dc power supply is used in dc motor drives, battery charging system, regulated dc power supplies, electronics equipments and different home appliances.

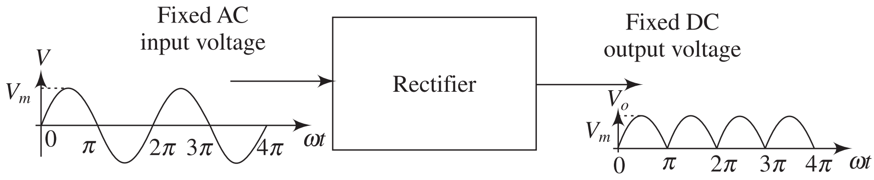


Fig. 5.1 *Block diagram of a rectifier*

In this chapter, the classification of rectifiers based on their number of phases and the type of devices used, are discussed. The working principle and analysis of single-phase uncontrolled half wave and full wave rectifiers with resistive, inductive, capacitive and back emf-type loads are also incorporated in this chapter.

5.2 CLASSIFICATION OF RECTIFIERS

The phase controlled converters or rectifiers are used to convert ac power into dc power. These converter circuits are divided into two types such as uncontrolled converters and controlled converters. When diodes are used in ac-to-dc converter circuits, the circuit is known as uncontrolled ac-to-dc converters or uncontrolled rectifiers. In these circuits, diodes are turned ON and OFF depending upon the supply voltage and circuit parameters such as resistive, inductive and capacitive load. Converters are also divided based on positive and negative half-cycle voltage. When either positive or negative half-cycle voltage is applied to load, it is known as *half wave converter*. But, if both positive as well as negative half-cycle voltages are applied across load, it is called as *full-wave converter*.

Usually phase control converters are classified as single-phase converter and three-phase converter. Then both single-phase and three-phase converters are also classified depending upon the number of pulses in cycle. The single-phase uncontrolled and controlled converters are classified as single pulse and two pulse converters. Different types of three-phase converters are three pulse, six pulse and twelve pulse converters. The detail classification of rectifiers are given in Figs. 5.2(a), (b) and (c).

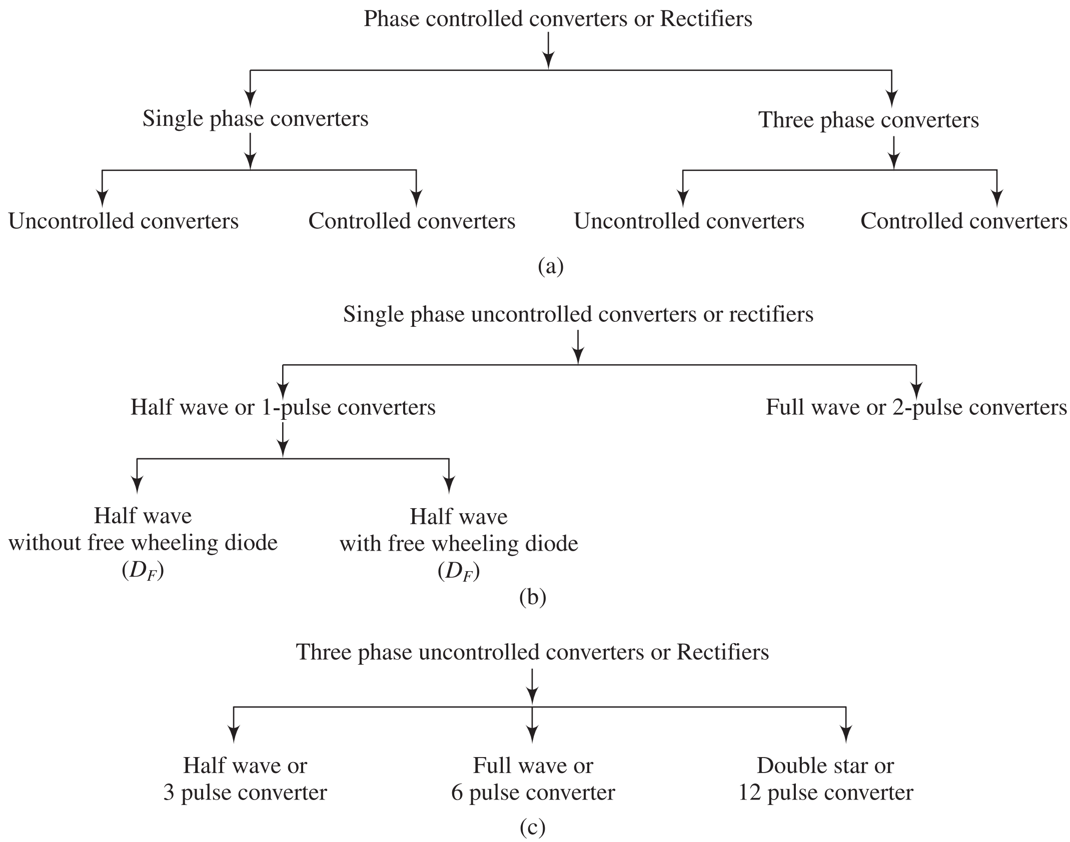


Fig. 5.2 *(a), (b) and (c) Classification of rectifiers*

5.3 SINGLE-PHASE HALF-WAVE UNCONTROLLED CONVERTER WITH RESISTIVE LOAD

Figure 5.3 shows the single-phase half-wave uncontrolled rectifier with resistive (R) load and it is the most simple rectifier circuit or ac-to-dc converter with fixed output voltage. In this circuit, diode conducts when it is forward biased. When the diode (D) is ON, switch is closed and current flows through diode and load resistance R_L. When the diode is reverse biased, diode stops conduction and switch becomes opened and current does not flows through diode as well as load.

A single-phase half-wave rectifier circuit consists of a diode (D) in series with a load resistance R_L, as depicted in Fig. 5.3. AC input voltage (v) is applied as input of the half-wave rectifier and this voltage can be expressed as

$$v = V_m \sin \omega t = V_m \sin \theta$$
$$= \sqrt{2}V \sin \omega t = \sqrt{2}V \sin \theta$$

where v is the instantaneous voltage

V_m is the maximum voltage,

V is the rms voltage

$\omega = 2\pi f$ is the angular frequency

$\omega t = \theta$ is the angle

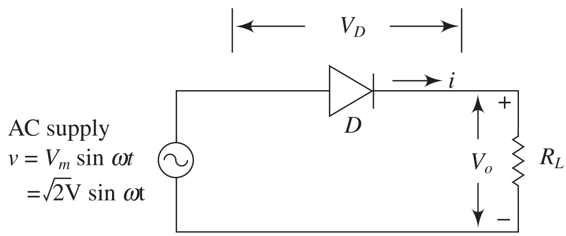


Fig. 5.3 *Single-phase half-wave rectifier circuit with resistive (R) load*

During the positive half cycle of the ac input voltage, the diode is forward biased and conducts when the instantaneous voltage is greater than the knee voltage or cut-in voltage of diode V_γ about 1 V for power diodes. The peak value or the maximum voltage V_m is very large compared to the cut-in voltage V_γ, it is assumed $V_\gamma = 0$ for rectifier circuit analysis. When the diode conducts, the diode acts as a closed switch and the current flows through load and diode. Therefore, there is a voltage drop across the load resistance R_L. The output voltage at load R_L will be same as the positive half cycle ac input voltage. The waveform of load current must be same as output voltage wave shape, but its amplitude depends on the load resistance R_L.

In the negative half cycle of ac input voltage the diode is reverse biased and it will not conduct. Consequently, there is no current flow through the diode and the voltage drop across the load is zero. Hence, the output voltage is zero in the negative half cycle of input voltage. It is clear from Fig. 5.4 that

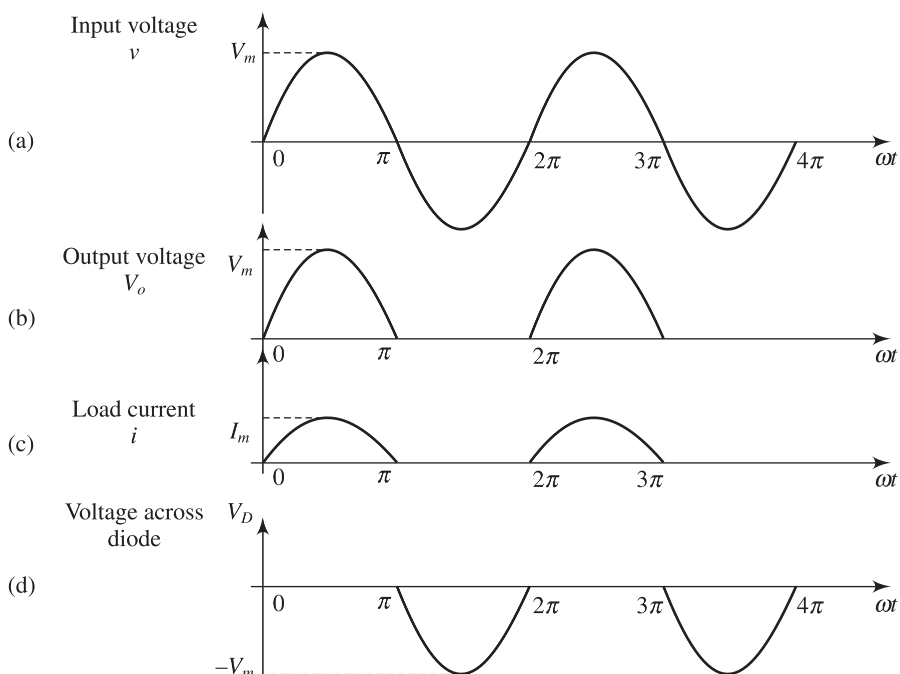


Fig. 5.4 *(a) Input voltage (b) Output voltage (c) Load current (d) Diode voltage*

only the positive half cycle of ac input voltage will be output across R_L. Therefore, only the positive half cycle of the ac input voltage can be utilised to deliver power to the circuit.

The output voltage is not a steady dc voltage. It is a pulsating dc voltage with a ripple frequency equal to the input voltage frequency. The output voltage can be measured by a dc voltmeter and output current can be measured by a dc ammeter. The output voltage and output current waveforms can be displayed on cathode ray oscilloscope (CRO). As the circuit uses only the positive half cycle of the ac input voltage, it is known as *a half-wave rectifier*.

5.3.1 Average (dc) Output Voltage

Assume that a sinusoidal ac input voltage is applied to a half-wave rectifier. The instantaneous value of sinusoidal ac input voltage is

$$v = V_m \sin \omega t = V_m \sin \theta$$

where V_m is the maximum voltage

$\theta = \omega t$ is the angle

The output voltage across the load can be expressed as

$$v_o = V_m \sin \omega t = V_m \sin \theta \qquad 0 \le \theta \le \pi$$

$$= 0 \qquad \pi \le \theta \le 2\pi$$

$$V_{av} = V_{dc} = V_O = \frac{\text{Area under the curve over the full cycle}}{\text{base}}$$

where, base = full cycle time = T

$$= \frac{\int_0^T v d\theta}{T} = \frac{\int_0^\pi V_m \sin \theta \, d\theta + \int_\pi^{2\pi} 0 \cdot d\theta}{2\pi}$$

$$= \frac{\int_0^\pi V_m \sin \theta \, d\theta}{2\pi} = \frac{V_m}{2\pi} |-\cos \theta|_0^\pi = \frac{V_m}{2\pi}[+1-(-1)] = \frac{V_m}{\pi} = 0.318\, V_m$$

Form the above expression we can say that the average or dc output voltage is 31.8% of the maximum ac input voltage.

5.3.2 Average (dc) Load Current

The instantaneous current i flows though the diode D and the load resistance R_L and it can be expressed as

$$i = I_m \sin \omega t = I_m \sin \theta \qquad 0 \le \theta \le \pi$$

$$= 0 \qquad \pi \le \theta \le 2\pi$$

where $I_m = \dfrac{V_m}{R_f + R_L}$ as R_f is the ON state resistance of diode and R_L is load resistance

As $R_L >> R_f$, $I_m = \dfrac{V_m}{R_L}$

$$I_{av} = I_{dc} = \frac{\text{Area under the curve } i \text{ over the full cycle}}{\text{base}}$$

where, base = full cycle time = T

$$= \frac{\int_0^T i d\theta}{T} = \frac{\int_0^\pi I_m \sin\theta d\theta + \int_\pi^{2\pi} 0 \cdot d\theta}{2\pi}$$

$$= \frac{\int_0^\pi I_m \sin\theta d\theta}{2\pi} = \frac{I_m}{2\pi} |-\cos\theta|_0^\pi$$

$$= \frac{I_m}{2\pi}[+1-(-1)] = \frac{I_m}{\pi} = 0.318 I_m \text{ where, } I_m = \frac{V_m}{R_L}$$

From the above equation, it is justified that the average or dc load current is 31.8% of the maximum load current.

5.3.3 Diode Voltage

When a voltmeter is connected across the diode to measure the dc voltage, the reading of voltmeter is not equal to $I_{dc}R_f$ as the diode can not be modelled as a constant resistance. Actually diode has two resistance values such as R_f is the resistance during ON state of diode and its value is low, R_r is the resistance during OFF state of diode and its value is very high (infinite).

The dc voltmeter measures the average value of the voltage across the terminals of diode.

$$V'_{dc} = \frac{1}{2\pi}\left[\int_0^\pi I_m R_f \sin\theta d\theta + \int_\pi^{2\pi} V_m \sin\theta d\theta\right]$$

$$= \frac{I_m}{\pi}R_f - \frac{V_m}{\pi} = \frac{1}{\pi}[I_m R_f - I_m(R_f + R_L)] = -\frac{I_m R_L}{\pi}$$

The negative result means that positive terminal of voltmeter must be connected to cathode and the negative terminal is connected to the anode.

The dc voltage across diode is equal to the voltage across load resistance R_L. This result is correct as the sum of the dc voltage around the complete circuit must be zero.

5.3.4 Peak Inverse Voltage (PIV) of Diode

During the negative half cycle of ac input voltage, the diode is reverse biased. As diode will not conduct, no current flows through diode and load resistance R_L, the voltage drop across R_L is zero. When KVL is used in the circuit, we find that the negative voltage appears as reverse voltage across the diode. The maximum value of reverse voltage is the peak of the negative voltage of ac input voltage which is equal to V_m. Thus the maximum reverse voltage is called the peak inverse voltage (PIV) Therefore the peak inverse voltage (PIV) of a diode in a half wave rectifier is V_m.

5.3.5 RMS value of Load Current

A root-mean-square (RMS) ammeter is used to measure RMS current. As per definition of the RMS value, squared of a periodic function time is given by the area of one cycle of the curve, which represents the square of the function, divide by the time period.

$$I_{\text{rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi}\left(\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{2\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2} d\theta\right]^{\frac{1}{2}} = \frac{I_m}{2}$$

The pulsating load current is sum of the dc load current and ripple (ac) current. The instantaneous value of the ripple (ac) current is i_{ripple} is the difference between the instantaneous value of current i and the dc current I_{dc}. The instantaneous value of the ripple current can be expressed as

$$i_{\text{ripple}} = i - I_{\text{dc}}$$

The rms value of the ripple current is

$$I_{\text{ripple,rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} (i - I_{\text{dc}})^2 d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{2\pi}\int_0^{2\pi} (i^2 + I_{\text{dc}}^2 - 2iI_{\text{dc}}) d\theta\right]^{\frac{1}{2}}$$

As $\int_0^{2\pi} (i^2 + I_{\text{dc}}^2 - 2iI_{\text{dc}})\, d\theta = 2\pi(I_{\text{rms}}^2 - I_{\text{dc}}^2)$, we get $I_{\text{ripple,rms}} = [I_{\text{rms}}^2 - I_{\text{dc}}^2]^{\frac{1}{2}}$

5.3.6 RMS value of Output Voltage

A root-mean-square (RMS) voltmeter is used to measure RMS voltage. According to the definition of the RMS value, squared of a periodic function time is given by the area of one cycle of the curve, which represents the square of the function, divide by the time period. The RMS output voltage is equal to

$$V_{\text{rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} v^2 d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi}\left(\int_0^{\pi} V_m^2 \sin^2\theta d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{2\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2} d\theta\right]^{\frac{1}{2}} = \frac{V_m}{2}$$

The pulsating output voltage is sum of the dc output voltage and ripple component (ac) output voltage. The instantaneous value of the ripple (ac) output voltage is v_{ripple} is the difference between the instantaneous value of voltage v and the dc output voltage V_{dc}. The instantaneous value of the ripple output voltage can be expressed as

$$v_{\text{ripple}} = v - V_{\text{dc}}$$

The rms value of the ripple output voltage is

$$v_{\text{ripple,rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} v_{\text{ripple}}^2\, d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{2\pi}\int_0^{2\pi} (v - V_{\text{dc}})^2 d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi}\int_0^{2\pi} (v^2 + V_{\text{dc}}^2 - 2vV_{\text{dc}}) d\theta\right]^{\frac{1}{2}}$$

As $\int_0^{2\pi} (v^2 + V_{\text{dc}}^2 - 2vV_{dc})\,d\theta = 2\pi(V_{\text{rms}}^2 - V_{\text{dc}}^2)$, we find

$$V_{\text{ripple,rms}} = [V_{\text{rms}}^2 - V_{\text{dc}}^2]^{\frac{1}{2}}$$

5.3.7 Ripple Factor

The output voltage and output current have two components such as dc component and ac component. The ac component part of the output voltage and current is called a ripple. The ripple factor can be expressed as

$$\text{Ripple factor } (\gamma) = \frac{\text{RMS value of the ac component of pulsating output voltage}}{\text{dc component of pulsating output voltage}}$$

$$= \frac{V_{\text{ripple,rms}}}{V_{\text{dc}}}$$

We know that $V_{\text{ripple,rms}} = [V_{\text{rms}}^2 - V_{\text{dc}}^2]^{\frac{1}{2}}$

Therefore, the ripple factor is

$$\gamma = \frac{[V_{\text{rms}}^2 - V_{\text{dc}}^2]^{\frac{1}{2}}}{V_{\text{dc}}} = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1} = \sqrt{FF^2 - 1} \text{ as Form factor } FF = \frac{V_{\text{rms}}}{V_{\text{dc}}}$$

In a half-wave rectifier, $V_{\text{rms}} = \frac{V_m}{2}$ and $V_{\text{dc}} = \frac{V_m}{\pi}$,

$$\text{then the ratio } \frac{V_{\text{rms}}}{V_{\text{dc}}} = \frac{V_m}{2} \times \frac{\pi}{V_m} = \frac{\pi}{2}$$

So ripple factor is

$$\gamma = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$$

The ripple factor in terms of current is

$$\text{Ripple factor } (\gamma) = \frac{I_{\text{ripple,rms}}}{I_{\text{dc}}}$$

We know that $I_{\text{ripple,rms}} = [I_{\text{rms}}^2 - I_{\text{dc}}^2]^{\frac{1}{2}}$

So the ripple factor is

$$\gamma = \frac{[I_{\text{rms}}^2 - I_{\text{dc}}^2]^{\frac{1}{2}}}{I_{\text{dc}}} = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1} = \sqrt{FF^2 - 1} \text{ as Form factor } FF = \frac{I_{\text{rms}}}{I_{\text{dc}}}$$

In a half-wave rectifier, $I_{\text{rms}} = \frac{I_m}{2}$ and $I_{\text{dc}} = \frac{I_m}{\pi}$,

$$\text{then the ratio } \frac{I_{\text{rms}}}{I_{\text{dc}}} = \frac{I_m}{2} \times \frac{\pi}{I_m} = \frac{\pi}{2}$$

So ripple factor is

$$\gamma = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$$

Therefore, AC component is 121% of DC output voltage is present in the output of a half-wave rectifier. Consequently, the half-wave rectifier is not a very good ac to dc converter.

5.3.8 Rectifier Efficiency

The efficiency of a single-phase half-wave rectifier is defined as the ratio of dc power output to the load to the ac power input from the secondary winding of transformer. The efficiency can be expressed as

$$\eta = \frac{\text{dc power output to load}}{\text{ac power input}} = \frac{P_{\text{dc}}}{P_{\text{ac}}}$$

The dc power output to the load is

$$P_{\text{dc}} = I_{\text{dc}}^2 R_L = I_{\text{av}}^2 R_L$$

The ac power input to the rectifier is $P_{\text{ac}} = I_{\text{rms}}^2 (R_f + R_L)$

Then $\eta = \dfrac{I_{\text{dc}}^2 R_L}{I_{\text{rms}}^2 (R_f + R_L)} = \left(\dfrac{I_{\text{dc}}}{I_{\text{rms}}}\right)^2 \dfrac{R_L}{(R_f + R_L)}$

In a half-wave rectifier, $I_{\text{rms}} = \dfrac{I_m}{2}$ and $I_{\text{dc}} = \dfrac{I_m}{\pi}$, then the ratio $\dfrac{I_{\text{dc}}}{I_{\text{rms}}} = \dfrac{I_m}{\pi} \times \dfrac{2}{I_m} = \dfrac{2}{\pi}$

Therefore, efficiency $\eta = \left(\dfrac{2}{\pi}\right)^2 \dfrac{R_L}{(R_f + R_L)} = \dfrac{4}{\pi^2} \dfrac{R_L}{(R_f + R_L)} = \dfrac{0.406}{1 + \dfrac{R_f}{R_L}}$

As $R_L >> R_f$, $\eta_{\max} = 0.406 = 40.6\%$

Consequently, it is proved that the maximum efficiency of a half-wave rectifier is 40.6% when the load resistance is very large compared to the forward resistance of a diode.

5.4 SINGLE-PHASE HALF-WAVE RECTIFIER CIRCUIT WITH TRANSFORMER COUPLED INPUT

Figure 5.5 shows a single-phase half-wave rectifier circuit with transformer coupled input. The transformer coupling has the following advantages:

1. The applied ac voltage across the rectifier can be stepped up or stepped down from the ac supply as per requirement
2. AC power supply is electrically isolated from the rectifier circuit.

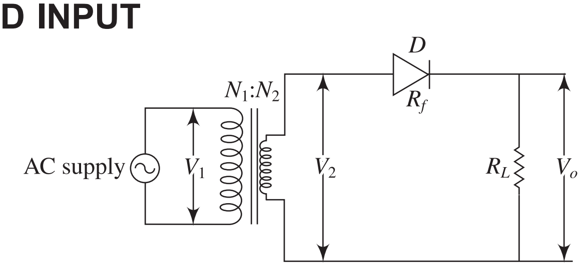


Fig. 5.5 *Single-phase half-wave rectifier circuit with transformer coupled input*

3. The output voltage of transformer V_2 is obtained from the expression

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

where, V_1 = rms voltage across the primary winding of transformer
V_2 = rms voltage across the secondary winding of transformer
N_1 = number of turns in the primary winding of transformer
N_2 = number of turns in the secondary winding of transformer

5.4.1 Transformer Utilization Factor (TUF)

It is defined as the ratio of dc power output to the load and the ac power rating of transformer. The transformer utilisation factor (TUF) can be expressed as

$$\text{TUF} = \frac{\text{dc power output to load}}{\text{ac power rating of transformer}} = \frac{P_{\text{dc}}}{P_{\text{ac rating of transformer}}}$$

The dc power output to the load is $P_{\text{dc}} = I_{\text{dc}}^2 R$ where, $I_{\text{dc}} = \frac{I_m}{\pi}$

The ac power rating of transformer is

$$P_{\text{ac rating of transformer}} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{2} \quad \text{where, } V_m = I_m(R_f + R_L)$$

So the transformer utilisation factor (TUF) of single-phase half-wave rectifier with transformer coupled input is

$$\text{TUF} = \frac{\left(\frac{I_m}{\pi}\right)^2 R_L}{\frac{I_m(R_f + R_L)}{\sqrt{2}} \times \frac{I_m}{2}} = \frac{2\sqrt{2}}{\pi^2} \frac{R_L}{R_f + R_L}$$

If $R_L >> R_f, \text{TUF}_{\max} = 0.287$. Therefore in a half-wave rectifier, the maximum TUF is 0.287.

5.5 FOURIER SERIES OF OUTPUT VOLTAGE OF A HALF-WAVE RECTIFIER

The output voltage can be expressed as

$$v(t) = V_o + \sum_{n=1,2,3.....}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$V_o = \frac{1}{2\pi}\int_0^{2\pi} v \cdot d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}V \sin \omega t \cdot d(\omega t) = \frac{\sqrt{2}V}{\pi} = \frac{V_m}{\pi}$$

where, V is the rms voltage and maximum voltage $V_m = \sqrt{2}V$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v \cdot \cos n\omega t \cdot d(\omega t) = \frac{1}{\pi}\int_0^{\pi} \sqrt{2}V \sin \omega t \cdot \cos n\omega t \cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi} \frac{1+(-1)^n}{1-n^2} = \frac{V_m}{\pi} \frac{1+(-1)^n}{1-n^2} \qquad \text{for } n = 2, 4, 6......$$

$$= 0 \qquad \text{for } n = 1, 3, 5,......$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v \cdot \sin n\omega t d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}V \sin \omega t \cdot \sin n\omega t \cdot d(\omega t)$$

$$= \frac{V}{\sqrt{2}} = \frac{V_m}{2} \qquad \text{for } n = 1$$

$$= 0 \qquad \text{for } n = 2, 3, \ldots\ldots$$

After substituting the V_o, a_n and b_n in the output voltage equation, we get

$$v = \frac{\sqrt{2}V}{\pi} + \frac{V}{\sqrt{2}}\sin \omega t - \frac{2\sqrt{2}V}{3\pi}\cos 2\omega t - \frac{2\sqrt{2}V}{15\pi}\cos 4\omega t - \frac{2\sqrt{2}V}{35\pi}\cos 6\omega t \ldots\ldots$$

It is clear from the above equation, the output voltage contains dc voltage and harmonics at multiples of fundamental frequency as depicted in Fig. 5.6.

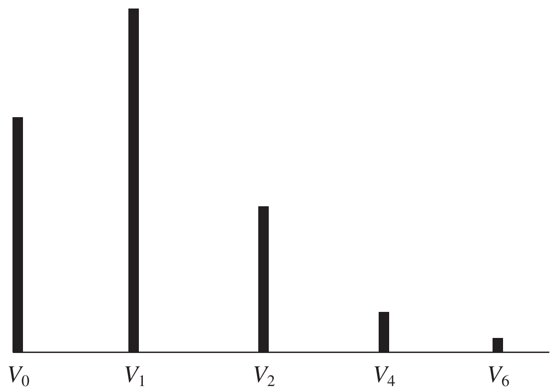


Fig. 5.6 *DC output voltage and harmonics*

5.6 SINGLE-PHASE HALF-WAVE RECTIFIER CIRCUIT WITH *L* LOAD

Figure 5.7 shows a single-phase half-wave rectifier circuit with inductive load. If the switch is closed at $\omega t = 0$, the diode is forward biased and starts conducting. Figure 5.8 shows the output voltage and current waveforms. The output voltage across inductance (L) is equal to input voltage and it can be expressed as

$$v = \sqrt{2}V \sin \omega t = V_m \sin \omega t$$

$$= v_o = L\frac{di}{dt}$$

Therefore, $L\dfrac{di}{dt} = V_m \sin \omega t$

or $\qquad i = \dfrac{V_m}{L}\int \sin \omega t dt = -\dfrac{V_m}{\omega L}\cos \omega t + C \qquad (5.1)$

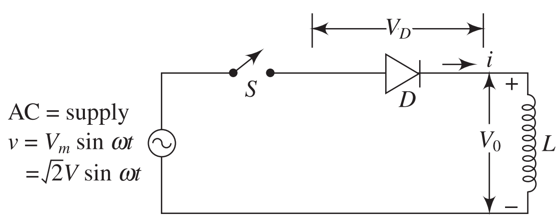


Fig. 5.7 *Single-phase half-wave rectifier circuit with inductive load*

Since $i = 0$ at $\omega t = 0$, we can write that $0 = -\dfrac{V_m}{\omega L} + C$

Then, $C = \dfrac{V_m}{\omega L}$

After substituting the value C in Eq. (5.1), the current flow through load can be expressed as

$$i = -\frac{V_m}{\omega L}\cos \omega t + \frac{V_m}{\omega L} = \frac{V_m}{\omega L}(1 - \cos \omega t)$$

The output voltage is $v_o = L\dfrac{di}{dt} = V_m \sin \omega t$

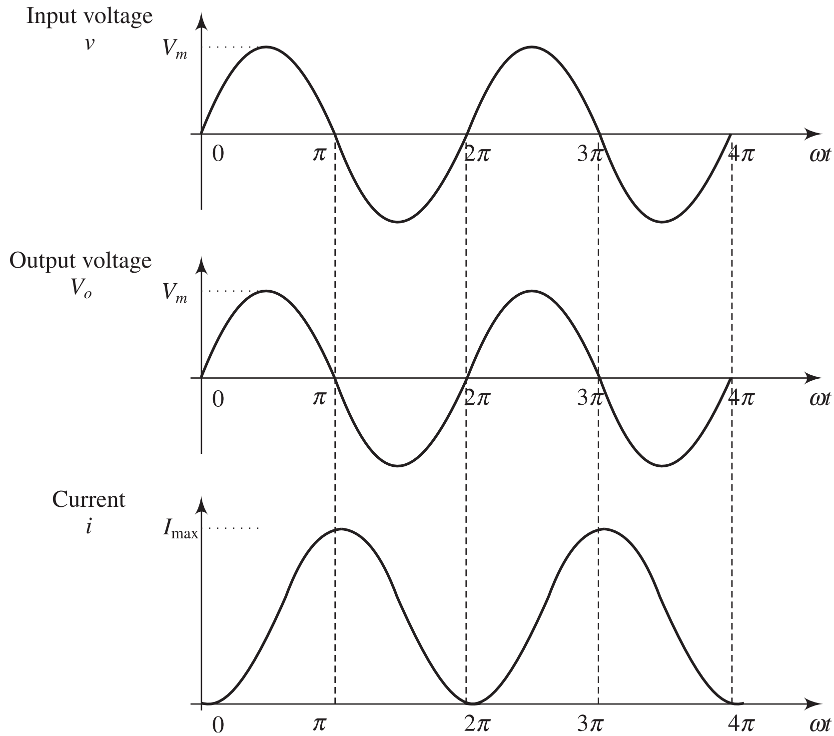


Fig. 5.8 *Input voltage, output voltage and current waveform*

The average value of output voltage is

$$V_0 = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin \omega t \cdot d\omega t = 0$$

At $\omega t = \pi$, the current is maximum. The maximum value of current is

$$I_{\max} = \frac{V_m}{\omega L}(1 - \cos \omega t) = \frac{V_m}{\omega L}(1+1) = \frac{2V_m}{\omega L}$$

The current waveform consists of dc or average current I_{av} and fundamental current I_1.
Average value of current is

$$I_{av} = I_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m}{\omega L}(1 - \cos \omega t) \cdot d\omega t$$

$$= \frac{V_m}{\omega L} = \frac{I_{\max}}{2}$$

The rms value of fundamental component of current is

$$I_1 = \left[\frac{1}{2\pi}\int_0^{2\pi} \left(\frac{V_m}{\omega L}\right)^2 (1 - \cos \omega t)^2 \, d\omega t\right]^{\frac{1}{2}}$$

$$= \frac{V_m}{\sqrt{2}\omega L} = \frac{I_{av}}{\sqrt{2}}$$

The voltage across diode is zero.

5.7 SINGLE-PHASE HALF-WAVE RECTIFIER CIRCUIT WITH *C* LOAD

Figure 5.9 shows a single-phase half-wave rectifier circuit with capacitive (C) load. When the switch(s) is closed at $\omega t = 0$, the diode is forward biased and starts conducting. The input voltage, current flow through capacitor, voltage across capacitor and diode are depicted in Fig. 5.10.

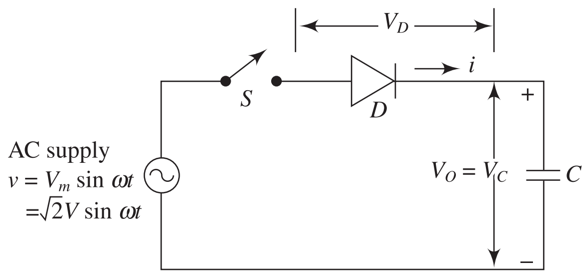


Fig. 5.9 *Single-phase half-wave rectifier circuit with capacitive load*

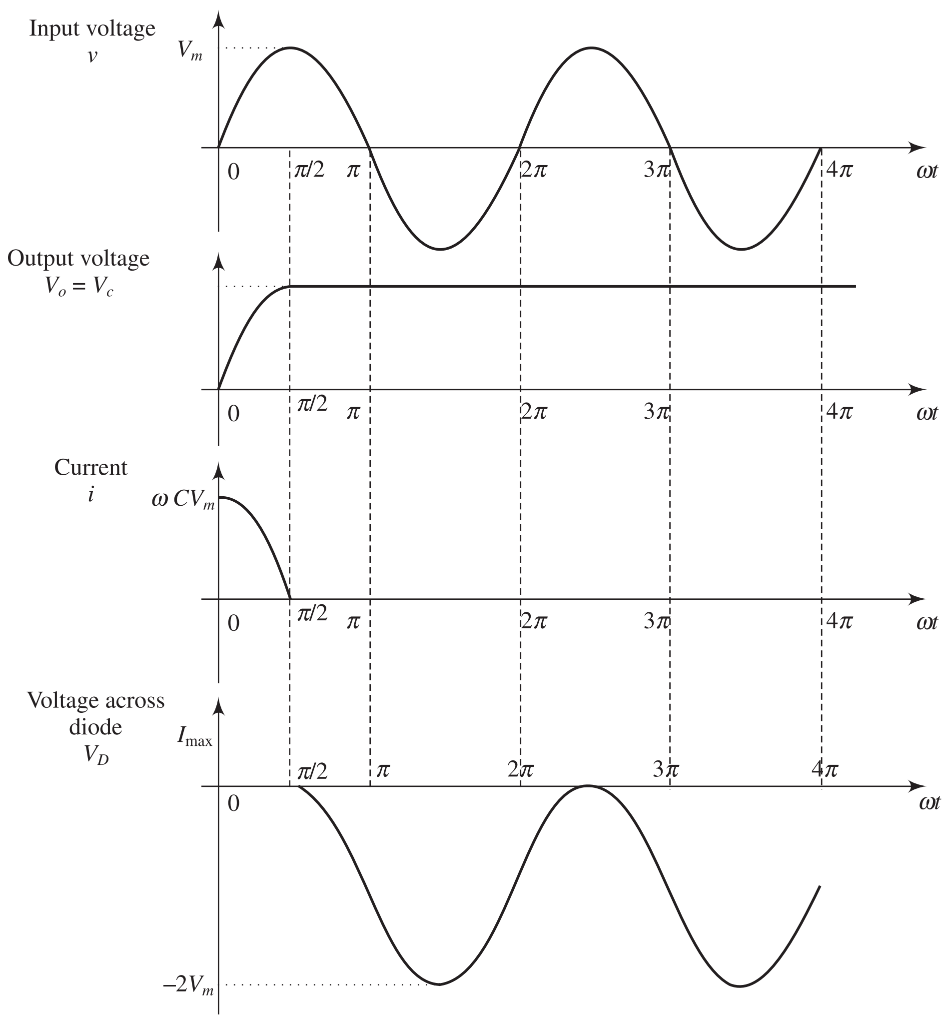


Fig. 5.10 *Input voltage, output voltage, current waveform and voltage across diode*

The voltage across capacitance (C) is equal to input voltage and the current flow through capacitance is

$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t$$

At $\omega t = 0$, the current flows through capacitance C is maximum. The value of capacitor current $I_{\max}$ at $\omega t = 0$ is

$$I_{\max} = \omega C V_m \cos(\cos 0) = \omega C V_m$$

The output voltage across capacitance is

$$v_c = \frac{1}{C}\int i \cdot d\omega t = V_m \sin \omega t$$

At $\omega t = \frac{\pi}{2}$, the capacitor is charged to maximum voltage V_m and after $\omega t = \frac{\pi}{2}$, the voltage across capacitance remains constant. After $\omega t = \frac{\pi}{2}$, the voltage across diode is

$$v_D = v - v_C = V_m \sin \omega t - V_m = V_m(\sin \omega t - 1)$$

5.8 SINGLE-PHASE HALF-WAVE RECTIFIER CIRCUIT WITH *RL* LOAD

A single-phase half-wave rectifier with resistive and inductive (RL) load is depicted in Fig. 5.11. During the positive half-cycle of supply voltage, inductance (L) stores energy and it continue in conduction state during negative half-cycle and release the stored energy for a certain period. The extension of conduction of diode depends upon the value of inductance (L). The output voltage is sum of the voltage across R and L. As inductance (L) is a energy storage device, the energy storage over a cycle is zero. Therefore, the energy storage must be equal to the energy release over a cycle.

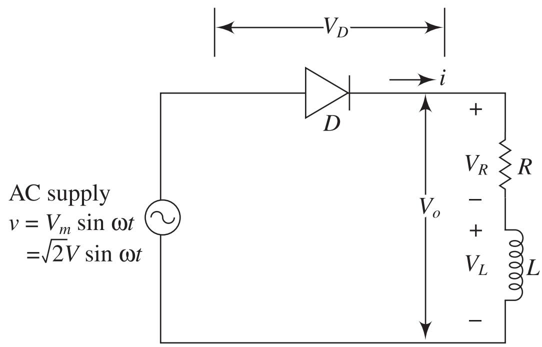


Fig. 5.11 *Single-phase half-wave rectifier circuit with RL load*

The voltage and current waveforms of a single-phase half-wave uncontrolled rectifier with RL load are shown in Fig. 5.12. The voltage across resistance is $V_R = iR$. The waveform of V_R will be same as current waveform. The voltage across inductance (L) is $v_L = v - v_R$ as depicted in Fig. 5.11. When $v > v_R$, the energy is stored in L and it is represented by area A. While $v < v_R$, the energy is released by L and it is represented by area B. The area A must be equal to area B as shown in Fig. 5.12.

At $\omega t = 0$, diode starts to conduct and current flows through it. At $\omega t = \pi$, diode conducts and current flow through it continuously. After $\omega t > \pi$, inductor always provides a negative voltage at cathode of diode and the cathode voltage is more negative than anode. Consequently anode is positive with respect to cathode. The diode continues its conduction though the supply voltage becomes negative and it conducts till the current through it reduces to zero. It is clear from Fig. 5.12 that the diode conducts up to $\omega t = \beta$ which is known as *extinction angle* or *cut-off angle*. Therefore, output voltage across the RL load is available from 0 to β and the supply voltage is available across diode during the angle β to 2π.

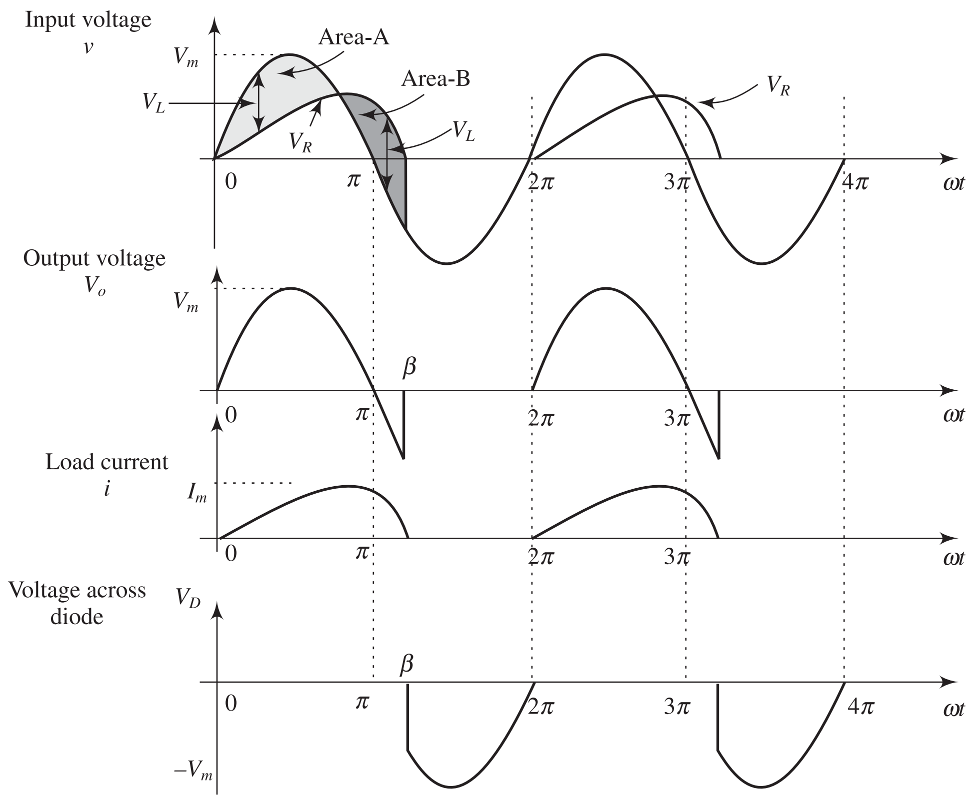


Fig. 5.12 *Input voltage, output voltage, load current, and voltage across diode of a single-phase half-wave rectifier circuit with RL load*

When the diode conducts, the voltage equation is

$$\sqrt{2}V \sin \omega t = L\frac{di}{dt} + Ri \qquad \text{at } 0 \le \omega t \le \beta$$

where, V is the rms voltage and input current is i

After solving the above differential equation, the output current can be expressed as

$$i(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) + \sin \phi \cdot e^{-\frac{R}{L}t}\right] \qquad 0 \le \omega t \le \beta$$

or
$$i(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) + \sin \phi \cdot e^{-\frac{\omega t}{\tan\phi}}\right] \qquad 0 \le \omega t \le \beta \tag{5.2}$$

$$\text{where, } Z = \sqrt{R^2 + (\omega L)^2} \text{ and } \tan \phi = \frac{\omega L}{R}$$

and current $i(t) = 0$ for $\beta \le \omega t \le 2\pi$

The first term of the above equation is sinusoidal and it becomes zero at certain value of ωt. The second term is exponential component which is always positive. When L is large and R is small, the time constant is large and the amplitude of exponential term decreases very slowly. Therefore, the current flows after the input voltage becomes zero at $\omega t = \pi$. Due to the induced voltage $L\frac{di}{dt}$

across inductance by the decreasing current, diode anode terminal is maintained positive with respect to cathode for the period $\pi \le \omega t \le \beta$. If $\frac{\omega L}{R}$ is very high or infinite, the cut-off angle β will be about 2π or 360°. At $\omega t = \beta$, current $i = 0$.

After substituting $\omega t = \beta$ in Eq. (5.2), the extinction or cut-off angle can be computed from

$$i(t) = 0 = \frac{\sqrt{2}V}{Z}\left[\sin(\beta - \phi) + \sin\phi \cdot e^{-\frac{\beta}{\tan\phi}}\right]$$

or $$\sin(\beta - \phi) + \sin\varphi \cdot e^{-\frac{R\beta}{\omega L}} = 0 \quad (5.3)$$

or $$\frac{R}{\omega L} = \frac{\cos\beta - e^{-\frac{R\beta}{\omega L}}}{\sin\beta}$$

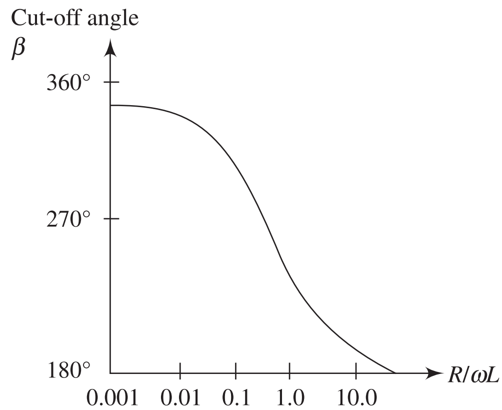


Fig. 5.13 *Variation of β with $\frac{R}{\omega L}$*

The variation of cut-off angle β with respect to $\frac{R}{\omega L}$ for single-phase half-wave rectifier is shown in Fig. 5.13. It is clear from Fig. 5.13 that the conduction angle β is increased with increasing inductance and decreasing $\frac{R}{\omega L}$.

Equation (5.3) can be written as $\sin\phi \cdot e^{-\frac{\beta}{\tan\phi}} = \sin(\phi - \beta)$. Therefore, β is function of ϕ and the graphical solution of $\sin\phi \cdot e^{-\frac{\beta}{\tan\phi}} = \sin(\phi - \beta)$ is depicted in Fig. 5.14.

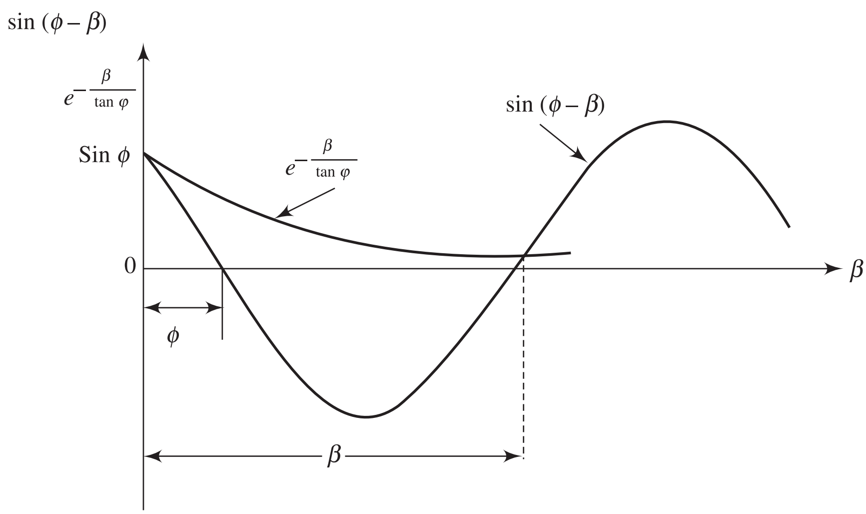


Fig. 5.14 *Graphical solution of $\sin\phi \cdot e^{-\frac{\beta}{\tan\phi}} = \sin(\phi - \beta)$*

The average voltage drop across a pure inductance is zero in steady state condition. Consequently average output voltage across *R* is equal to the voltage drop across *RL*. Then the dc output voltage is given by

$$V_o = \frac{1}{2\pi}\int_0^{\beta} V_m \sin \omega t \cdot d(\omega t) = \frac{1}{2\pi}\int_0^{\beta} V_m \sin\theta \cdot d\theta = \frac{V_m}{2\pi}[-\cos\theta]_0^{\beta} = \frac{V_m}{2\pi}(1-\cos\beta)$$

The rms value of output voltage is

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{\beta} (V_m \sin\omega t)^2 \cdot d(\omega t) = \frac{1}{2\pi}\int_0^{\beta} V_m^2 \sin^2 \omega t \cdot d(\omega t) = \frac{V_m{}^2}{4\pi}\left(\beta - \frac{\sin 2\beta}{2}\right)$$

or $$V_{\text{rms}} = \frac{V_m}{2\sqrt{\pi}}\left(\beta - \frac{\sin 2\beta}{2}\right)^{\frac{1}{2}}$$

The average output current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} \frac{V_m}{Z}\left[\sin(\omega t - \phi) + \sin\phi \cdot e^{-\frac{Rt}{L}}\right] \cdot d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{\beta} \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left[\sin(\omega t - \varphi) + \sin\varphi \cdot e^{-\frac{R}{\omega L}\omega t}\right] \cdot d(\omega t)$$

For large value of time constant, $\frac{\omega L}{R}$ tends to infinity, $e^{-\frac{R}{\omega L}\omega t} \to 0$ and $\sqrt{R^2+\omega^2L^2} \to R$. As $\frac{\omega L}{R} << 1$, $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = 0$. Therefore the average output current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} \frac{V_m}{R} \sin \omega t \cdot d(\omega t)$$

$$= \frac{V_m}{2\pi R}[-\cos\theta]_0^{\beta} = \frac{V_m}{2\pi R}(1-\cos\beta)$$

5.9 SINGLE-PHASE HALF-WAVE UNCONTROLLED RECTIFIER WITH *RL* LOAD AND FREE WHEELING DIODE

In single-phase half-wave uncontrolled rectifier with RL load, the load current i is discontinuous and it is same as line current. When a free wheeling diode is connected across load, only positive half cycle supply voltage is applied across load as diode D will conduct from 0 to π duration and line current flows during 0 to π duration. Single-phase half-wave uncontrolled rectifier with RL load and free wheeling diode is shown in Fig. 5.15. At $\omega t = \pi$, conduction of diode D stops and free wheeling diode D_F starts conduction. If D is OFF and D_F starts conduction, load current flows through RL load and diode D_F, but the magnitude of load current decreases. The load current may be continuous or discontinuous depending upon β or the value of inductance L.

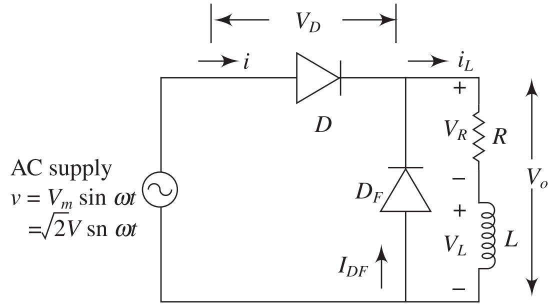


Fig. 5.15 *Single-phase half-wave uncontrolled rectifier with RL load and free wheeling diode*

Figure 5.16 shows the voltage and current waveforms. The output voltage is same as resistive load, but the load current waveform is different. Due to presence of free wheeling diode in Fig. 5.15, the output voltage is always positive. The energy stored in inductance L during positive half cycle of input voltage is supplied to load resistance R through free wheeling diode. As a result the system efficiency will be improved.

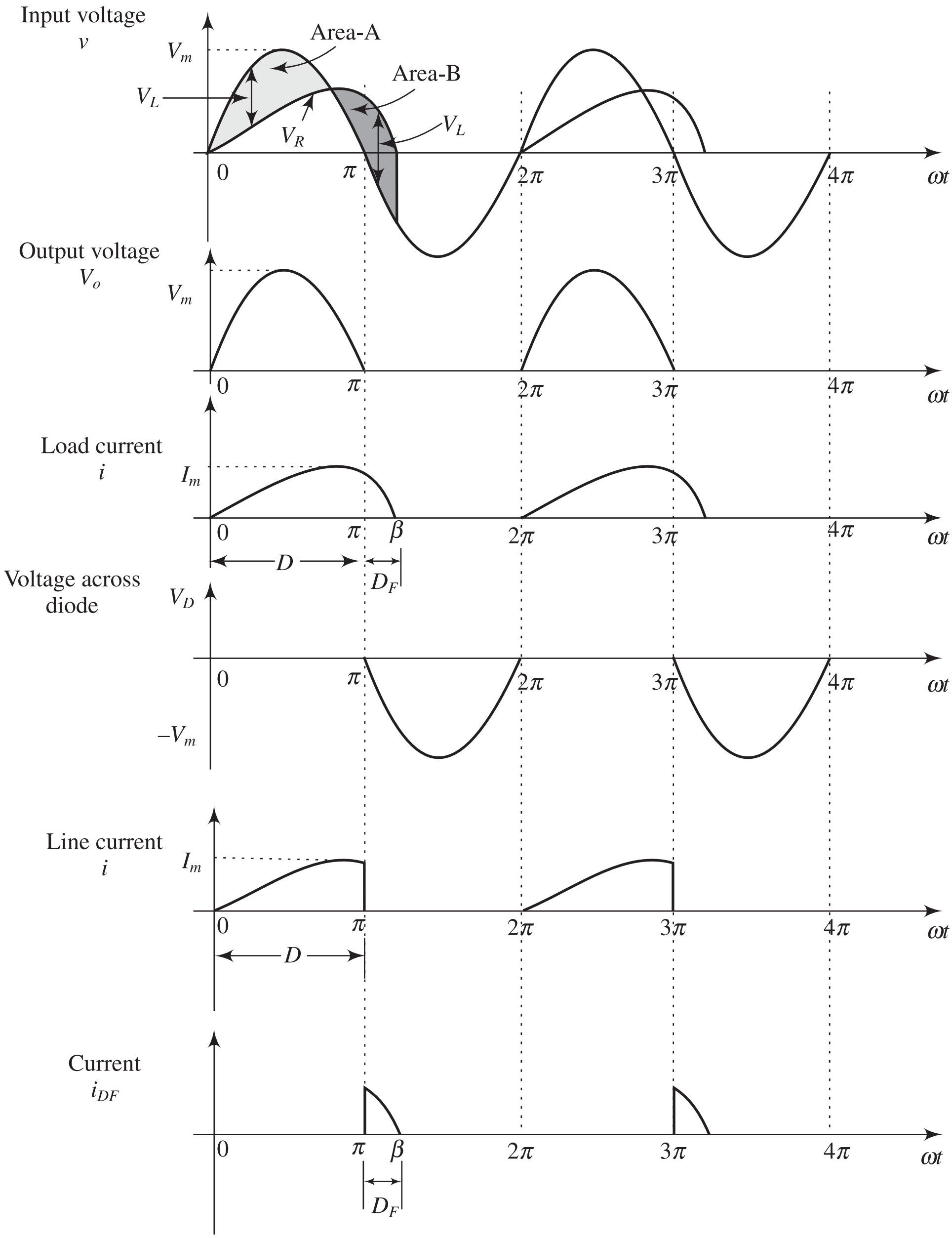


Fig. 5.16 *Input voltage, output voltage, load current, and voltage across diode of a single-phase half-wave rectifier circuit with RL load and free wheeling diode*

The average dc output voltage is

$$V_O = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}V \sin \omega t d(\omega t) = \frac{\sqrt{2}V}{\pi}$$

The average load current is $I_O = \frac{V_o}{R} = \frac{\sqrt{2}V}{\pi R}$

Example 5.1 The dc output voltage of a single-phase uncontrolled rectifier with *RL* load is 75 V. If the input voltage is 220 V, determine (a) the cut-off angle β and (b) rms value of output voltage.

Solution

Given: $V_o = 75$ V, $V = 220$ V

The maximum voltage $V_m = \sqrt{2}\text{V} = \sqrt{2} \times 220 \text{ V} = 311.12$

(a) The dc output voltage is $V_o = \frac{1}{2\pi}\int_0^{\beta} V_m \sin \omega t \cdot d(\omega t) = \frac{V_m}{2\pi}(1 - \cos\beta)$

or $$75 = \frac{311.12}{2\pi}(1 - \cos\beta)$$

or $$1.513 = (1 - \cos\beta)$$

or $$\cos\beta = -0.513$$

The cut-off angle $\beta = 239° = 4.169$ rad

(b) The rms value of output voltage is

$$V_{\text{rms}} = \frac{V_m}{2\sqrt{\pi}}\left(\beta - \frac{\sin 2\beta}{2}\right)^{\frac{1}{2}} = \frac{311.12}{2\sqrt{\pi}}\left(4.149 - \frac{\sin(2 \times 239)}{2}\right)^{\frac{1}{2}} = 169 \text{ V}$$

5.10 BATTERY CHARGER

When the output of a half-wave rectifier circuit is connected to a battery, the rectifier can be used as a battery charger. Figure 5.17 shows a battery charger circuit. When input voltage v is greater than battery voltage (E), diode D starts conduction. When the input voltage v is less than battery voltage, diode is reverse biased and it turns OFF. The input voltage, output voltage, load current and v-E waveforms are illustrated in Fig. 5.18.

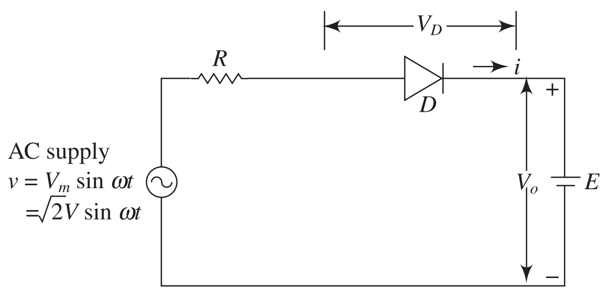


Fig. 5.17 *Battery charger circuit*

The angle α at which diode starts conduction can be computed from the equation

$$\sqrt{2}V \sin\alpha = E \qquad \text{at } \omega t = \alpha$$

or $$\alpha = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right)$$

When the input voltage v is less than battery voltage, diode is reverse biased at $\beta = \pi - \alpha$ and it is turned OFF. Therefore, the charging current flows though load during $\alpha \le \omega t \le \beta$ and it can be expressed as

$$i = \frac{v - E}{R} = \frac{V_m \sin\omega t - E}{R} = \frac{\sqrt{2}V \sin\omega t - E}{R}$$

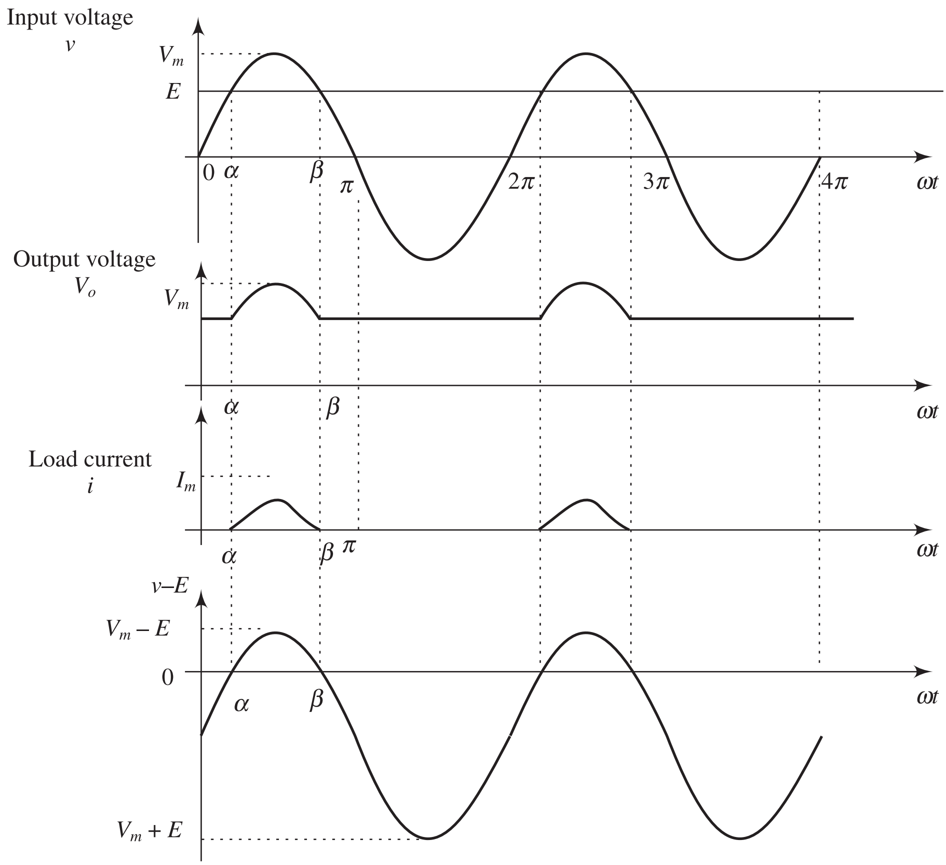


Fig. 5.18 *Input voltage, output voltage, load current and v-E waveforms*

The average charging current is

$$I_{\text{av}} = \frac{1}{2\pi}\int_{\alpha}^{\beta} \frac{V_m \sin \omega t - E}{R} d(\omega t)$$

$$= \frac{1}{2\pi R}(2V_m \cos \alpha + 2E\alpha - \pi E) \quad \text{as } \beta = \pi - \alpha$$

When the charging current (I_{av}) is known, the resistance value can determined from

$$R = \frac{1}{2\pi I_{\text{av}}}(2V_m \cos \alpha + 2E\alpha - \pi E)$$

The rms current flows through battery is I_{rms} and it is given by

$$I_{\text{rms}}^2 = \frac{1}{2\pi}\int_{\alpha}^{\beta} \frac{(V_m \sin \omega t - E)^2}{R^2} d(\omega t)$$

$$= \frac{1}{2\pi R^2}\left[\left(\frac{V_m^2}{2} + E^2\right)(\pi - 2\alpha) + \frac{V_m^2}{2}\sin 2\alpha - 4V_m E \cos \alpha\right]$$

During charging power loss across resistance is $I_{\text{rms}}^2 R$

Power delivered to battery is $P_{\text{dc}} = EI_{\text{dc}} = EI_{\text{av}}$ as $I_{\text{dc}} = I_{\text{av}}$

The rectifier efficiency is

$$\eta = \frac{\text{Power delivered to the battery}}{\text{Power input to the rectifier}} = \frac{P_{\text{dc}}}{P_{\text{dc}} + I_{\text{rms}}^2 R}$$

The peak inverse voltage of diode is $PIV = V_m + E$

Example 5.2 A 24 V, 500 W-Hr is charged using single-phase half-wave rectifier as depicted in Fig. 5.17. The input ac voltage is 220 V, 50 Hz. The average charging current is 5 A. Determine (a) conduction angle of diode, (b) the value of current limiting resistance R, (c) rms battery current, (d) charging time, (e) rectification efficiency and (f) peak inverse voltage of diode.

Solution

Given: $V = 220$ V, $E = 24$ V, $I_{av} = 5$ A

The maximum voltage $V_m = \sqrt{2}\text{V} = \sqrt{2} \times 220 \text{ V} = 311.12$

(a) The angle α is equal to

$$\alpha = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right) = \sin^{-1}\left(\frac{24}{\sqrt{2} \times 220}\right) = 3.682° = 0.064 \text{ rad}$$

Then $\beta = \pi - \alpha = 180° - \alpha = 180° - 3.682° = 176.318°$

The conduction angle of diode is $\beta - \alpha = 176.318° - 3.682° = 172.636°$

(b) The value of current limiting resistance R is

$$R = \frac{1}{2\pi I_{\text{av}}}(2V_m \cos\alpha + 2E\alpha - \pi E)$$

$$= \frac{1}{2\pi \times 5}(2 \times \sqrt{2} \times 220 \cos 3.682 + 2 \times 24 \times 0.064 - \pi \times 24) = 17.47\ \Omega$$

(c) The rms current flows through battery is I_{rms}

$$I_{\text{rms}}^2 = \frac{1}{2\pi R^2}\left[\left(\frac{V_m^2}{2} + E^2\right)(\pi - 2\alpha) + \frac{V_m^2}{2}\sin 2\alpha - 4V_m E\cos\alpha\right]$$

$$= \frac{1}{2\pi \times 17.47^2}\left[\left(\frac{311.12^2}{2} + 24^2\right)(\pi - 2 \times 0.064)\right.$$

$$\left. + \frac{311.12^2}{2}\sin(2 \times 3.682) - 4 \times 311.12 \times 24 \times \cos 3.682\right]$$

$$= \frac{1}{2\pi \times 17.47^2}[147509.14 + 6203.26 - 29805.86] = 64.64$$

$I_{\text{rms}} = 8.04$A

(d) Power delivered to battery $P_{\text{dc}} = EI_{\text{dc}} = 24 \times 5 = 120$ W

500 W-Hr $= P_{\text{dc}} \times h = 120 \times h$

Then $h = 4.166$ Hr

(e) The rectifier efficiency is

$$\eta = \frac{\text{Power delivered to the battery}}{\text{Power input to the rectifier}} = \frac{P_{\text{dc}}}{P_{\text{dc}} + I_{\text{rms}}^2 R} = \frac{120}{120 + 64.64 \times 17.47} = 9.60\%$$

(f) The peak inverse voltage of diode is $PIV = V_m + E = \sqrt{2} \times 220 + 24 = 335.12$ V

5.11 SINGLE-PHASE FULL-WAVE RECTIFIER

Single-phase full-wave rectifier is a circuit through which a current flows through load in one direction during the complete cycle (positive half cycle as well as negative half cycle) of input voltage. Figure 5.19 shows the block diagram representation of a single-phase full-wave rectifier.

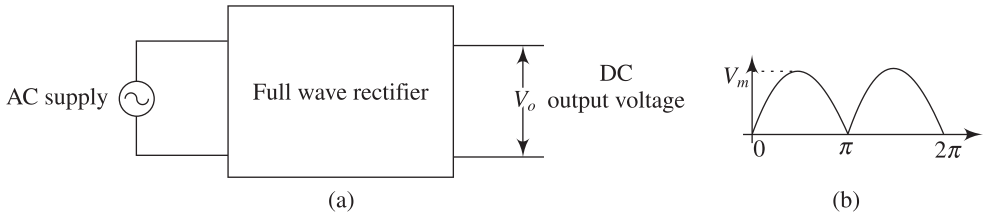


Fig. 5.19 *(a) The block diagram of a single-phase full-wave rectifier (b) The output voltage waveform*

There are two types of single-phase full-wave rectifier such as

1. Centre-tapped single-phase full wave rectifier
2. Single-phase bridge rectifier

In this section, the operation of centre-tapped single-phase full-wave rectifier and single-phase bridge rectifier are discussed elaborately.

5.12 CENTRE-TAPPED SINGLE-PHASE FULL-WAVE RECTIFIER

The circuit diagram of a centre-tapped single-phase full-wave rectifier is shown in Fig. 5.20. This circuit consists of two diodes D_1 and D_2 which are connected to the centre tapped secondary winding of a transformer. The input ac supply voltage is applied across the primary winding of the transformer. The centre tap of secondary winding is used as ground or zero reference voltage.

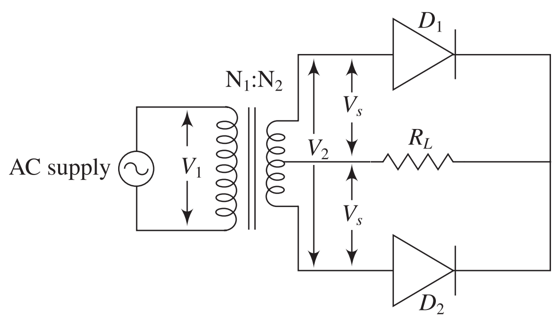


Fig. 5.20 *Centre-tapped single-phase full-wave rectifier*

The voltage between the centre tap of transformer and either end of the secondary winding is half of the secondary winding voltage, $V_S = \frac{V_2}{2}$. The operating principle of centre-tapped single-phase full wave rectifier is explained below.

During the positive half cycle of supply voltage, the polarities of secondary winding are depicted in Fig. 5.21. The diode D_1 is forward biased and the diode D_2 is reverse bias. As a result, the diode D_1 conducts as it is in ON state and the diode D_2 is in OFF state and current flows through load resistance R_L and diode D_1 as shown in Fig. 5.21.

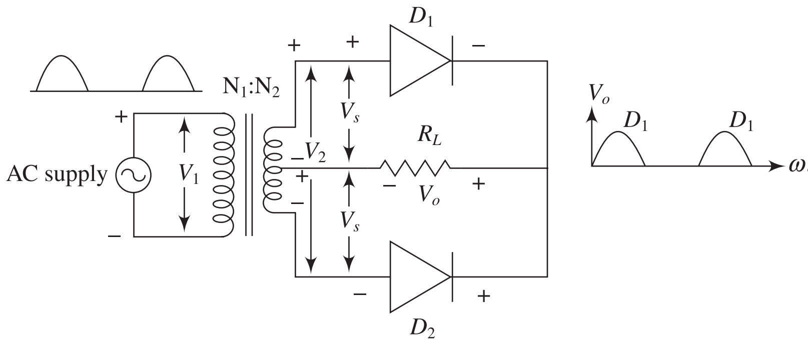


Fig. 5.21 *Centre-tapped single-phase full-wave rectifier during positive half cycle of supply voltage*

In the negative half cycle of supply voltage, the polarities of secondary winding are depicted in Fig. 5.22. The diode D_2 is forward biased and the diode D_1 is reverse bias. As a result, the diode D_2 conducts as it is in ON state and the diode D_1 is in OFF state and current flows through load resistance R_L and diode D_2 as shown in Fig. 5.22.

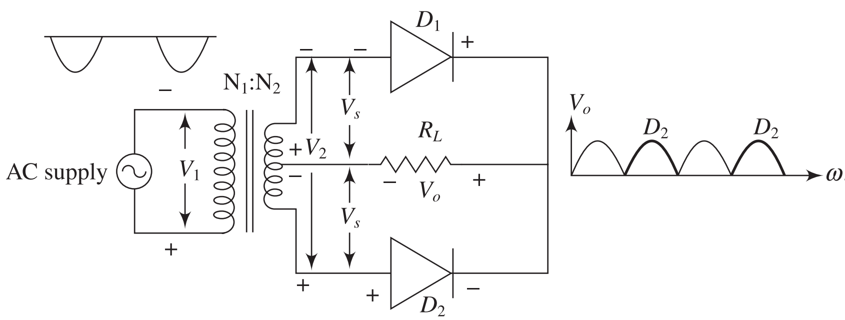


Fig. 5.22 *Centre-tapped single-phase full-wave rectifier during positive half cycle of supply voltage*

Hence the current flows through load in the same direction for positive as well as negative half cycle of supply voltage. Consequently, the output voltage across the load resistance is full-wave rectified dc voltage.

The centre-tapped single-phase full-wave rectifier can also be represented by Figs. 5.23 and 5.24. In Fig. 5.23, the load resistance R_L is connected to the centre tap of transformer and cathodes of diodes D_1 and D_2. In Fig. 5.24, one terminal of the load resistance R_L is connected to the cathodes of diodes D_1 and D_2 and other terminal of R_L is grounded. The centre tap of transformer is also grounded.

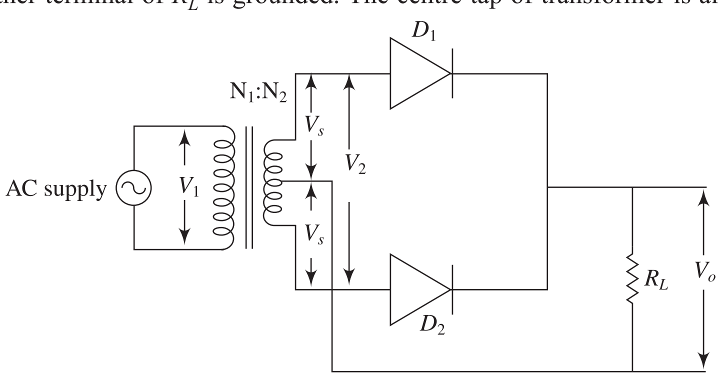


Fig. 5.23 *Centre-tapped single-phase full-wave rectifier*

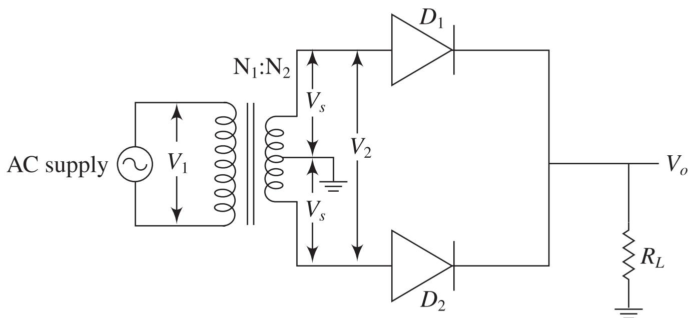


Fig. 5.24 *Another representation of Fig. 5.23*

In a centre-tapped single-phase full-wave rectifier, the voltage between the centre tap and either end of secondary winding is half the secondary winding voltage. When V_1 is the rms voltage across the primary winding of transformer, V_2 is the rms voltage across the secondary winding of transformer, N_1 is the number of turns in the primary winding of transformer and N_2 is the number of turns in the secondary winding of transformer, the transformer secondary voltage is computed from

$$V_2 = \frac{N_2}{N_1} V_1$$

If the turn ratio $\frac{N_2}{N_1}$ is 1, the secondary winding voltage is equal to the primary winding voltage ($V_2 = V_1$). Therefore, the voltage between the centre-tap of transformer and either end of the secondary winding of transformer is $V_S = \frac{V_2}{2} = \frac{V_1}{2}$

5.12.1 Average or dc Output Voltage

Figure 5.25 shows the output voltage across the load and it can be expressed as

$$v_o = V_m \sin \omega t = V_m \sin \theta \qquad 0 \le \theta \le \pi$$
$$= -V_m \sin \omega t = -V_m \sin \theta \qquad \pi \le \theta \le 2\pi$$

where, V_m is the maximum voltage across each half of the secondary winding

$$V_{av} = V_{dc} = V_O = \frac{\text{Area under the curve over the half cycle}}{\text{base}}$$

where, base = half cycle time

$$= \frac{\int_0^{\pi} v d\theta}{\pi} = \frac{\int_0^{\pi} V_m \sin\theta d\theta}{\pi}$$

$$= \frac{V_m}{\pi} \left| -\cos\theta \right|_0^{\pi} = \frac{V_m}{2\pi}[+1-(-1)]$$

$$= \frac{2V_m}{\pi} = 0.636 V_m$$

From the above expression we can say that the average or DC output voltage is 63.6% of the maximum AC input voltage and it is double that of a half-wave rectifier. Hence a full-wave rectifier is two times effective than a half-wave rectifier.

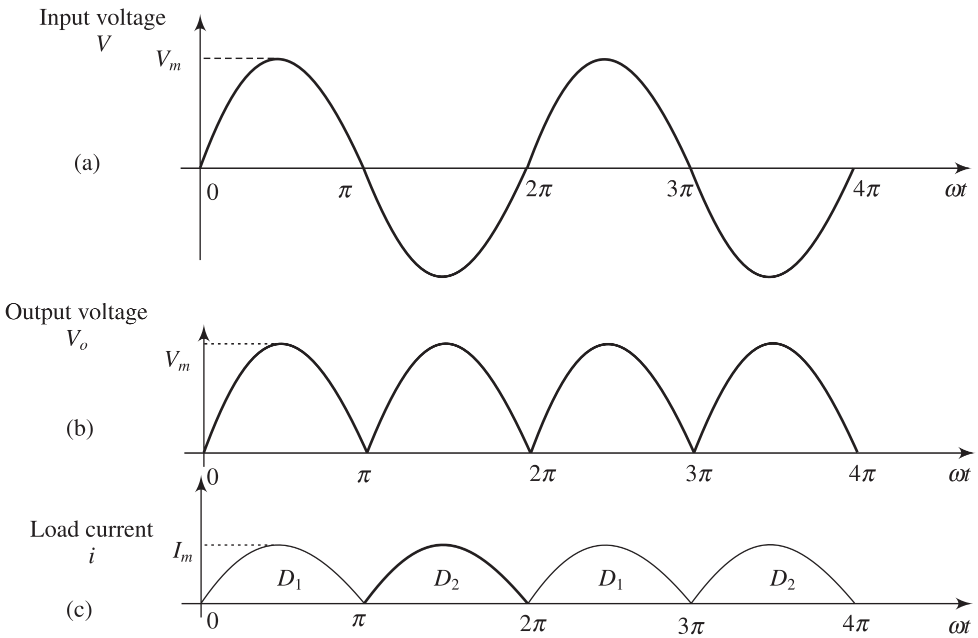


Fig. 5.25 *(a) Input voltage (b) Output voltage (c) Load current waveforms*

5.12.2 Average (dc) Load Current

The instantaneous current i flows though the diode D and the load resistance R_L and it can be expressed as

$$i = I_m \sin \omega t = I_m \sin \theta \qquad 0 \le \theta \le \pi$$

$$= -I_m \sin \omega t = -I_m \sin \theta \qquad \pi \le \theta \le 2\pi$$

where, $I_m = \dfrac{V_m}{R_L}$

$$I_{av} = I_{dc} = \frac{\text{Area under the curve } i \text{ over the half cycle}}{\text{base}}$$

where, base = half cycle time

$$= \frac{\int_0^\pi i d\theta}{\pi} = \frac{\int_0^\pi I_m \sin\theta d\theta}{\pi}$$

$$= \frac{I_m}{\pi} \left| -\cos\theta \right|_0^\pi$$

$$= \frac{I_m}{\pi}[+1-(-1)] = \frac{2I_m}{\pi} = 0.636 I_m \text{ where, } I_m = \frac{V_m}{R_L}$$

From the above equation, it is justified that the average or DC load current is 63.6% of the maximum load current.

5.12.3 Peak Inverse Voltage (PIV)

In a centre-tapped single-phase full-wave rectifier, each diode is alternately forward biased and reverse biased. If the secondary voltage V_S has the polarity, the anode of the diode D_1 is at V_m and anode of D_2 is at $-V_m$. As the diode D_1 is forward biased, its cathodes is at same potential as its anode and its voltage is V_m. Therefore, the total reverse voltage across the diode D_2 is

$$\text{PIV} = V_m - (-V_m) = 2V_m$$

The maximum reverse voltage at which each diode must be with stand is equal to the maximum secondary voltage or two times of the maximum voltage across each half of the secondary voltage ($2V_m$).

5.12.4 RMS Value of Load Current

The rms value of load current is given by

$$I_{\text{rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi}\int_0^{\pi}(I_m \sin\theta)^2 d\theta + \frac{1}{2\pi}\int_{\pi}^{2\pi}(-I_m \sin\theta)^2 d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2}d\theta\right]^{\frac{1}{2}} = \frac{I_m}{\sqrt{2}}$$

The pulsating load current is sum of the dc load current and ripple (ac) current. The instantaneous value of the ripple (ac) current is i_{ripple} is the difference between the instantaneous value of current i and the dc current I_{dc}. The instantaneous value of the ripple current can be expressed as

$$i_{\text{ripple}} = i - I_{\text{dc}}$$

The rms value of the ripple current is

$$I_{\text{ripple,rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} i_{\text{ripple}}^2 d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{2\pi}\int_0^{2\pi}(i - I_{\text{dc}})^2 d\theta\right]^{\frac{1}{2}}$$

$$= [I_{\text{rms}}^2 - I_{\text{dc}}^2]^{\frac{1}{2}}$$

5.12.5 RMS Value of Output Voltage

The rms value of output voltage is given by

$$V_{\text{rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} v^2 d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi}\int_0^{\pi}(V_m^2 \sin\theta)^2 d\theta + \frac{1}{2\pi}\int_{\pi}^{2\pi}(-V_m \sin\theta)^2 d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta d\theta\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2}d\theta\right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}}$$

The pulsating output voltage is sum of the dc output voltage and ripple component (ac) output voltage. The instantaneous value of the ripple (ac) output voltage is v_{ripple} which is the difference between the instantaneous value of AC input voltage v and the dc output voltage V_{dc}. The instantaneous value of the ripple output voltage can be expressed as

$$V_{\text{ripple}} = v - V_{\text{dc}}$$

The rms value of the ripple output voltage is

$$V_{\text{ripple,rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi} v_{\text{ripple}}^2 d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{2\pi}\int_0^{2\pi} (v - V_{\text{dc}})^2 d\theta\right]^{\frac{1}{2}}$$

$$= \left[V_{\text{rms}}^2 - V_{\text{dc}}^2\right]^{\frac{1}{2}}$$

5.12.6 Ripple Factor

The ripple factor can be expressed as

$$\text{Ripple factor } (\gamma) = \frac{V_{\text{ripple,rms}}}{V_{\text{dc}}}$$

Therefore, the ripple factor can be expressed as

$$\gamma = \frac{\left[V_{\text{rms}}^2 - V_{\text{dc}}^2\right]^{\frac{1}{2}}}{V_{\text{dc}}} = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1} \quad \text{As } V_{\text{ripple.rms}} = [V_{\text{rms}}^2 - V_{\text{dc}}^2]^{\frac{1}{2}}$$

In a full-wave rectifier, $V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$ and $V_{\text{dc}} = \frac{2V_m}{\pi}$,

then the ratio $\frac{V_{\text{rms}}}{V_{\text{dc}}} = \frac{V_m}{\sqrt{2}} \times \frac{\pi}{2V_m} = \frac{\pi}{2\sqrt{2}}$

So ripple factor is

$$\gamma = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1} = \sqrt{FF^2 - 1} \quad \text{as form factor } FF = \frac{V_{\text{rms}}}{V_{\text{dc}}}$$

$$= \sqrt{\left(\frac{\pi}{2\sqrt{2}}\right)^2 - 1} = 0.482$$

The ripple factor in terms of current is

$$\text{Ripple factor } (\gamma) = \frac{I_{\text{ripple,rms}}}{I_{\text{dc}}}$$

The ripple factor can be given by

$$\gamma = \frac{[I_{\text{rms}}^2 - I_{\text{dc}}^2]^{\frac{1}{2}}}{I_{\text{dc}}} = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1} = \sqrt{FF^2 - 1} \quad \text{as } I_{\text{ripple,rms}} = [I_{\text{rms}}^2 - I_{\text{dc}}^2]^{\frac{1}{2}} \text{ and}$$

$$\text{form factor } FF = \frac{I_{\text{rms}}}{I_{\text{dc}}}$$

In a full-wave rectifier, $I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$ and $I_{\text{dc}} = \frac{2I_m}{\pi}$, and the ratio $\frac{I_{\text{rms}}}{I_{\text{dc}}} = \frac{I_m}{\sqrt{2}} \times \frac{\pi}{2I_m} = \frac{\pi}{2\sqrt{2}}$

So ripple factor is

$$\gamma = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2\sqrt{2}}\right)^2 - 1} = 0.482$$

As a result, AC component is 48.2% of DC output voltage which is present in the output of a full wave rectifier. Therefore, the full wave rectifier is a good ac to dc converter.

5.12.7 Efficiency

The efficiency of a single-phase full-wave rectifier can be defined as the ratio of dc power output to the load to the ac power input from the secondary winding of transformer. The efficiency of rectifier is

$$\eta = \frac{\text{DC power output to load}}{\text{AC power input}} = \frac{P_{\text{dc}}}{P_{\text{ac}}}$$

The dc power output to the load is

$$P_{\text{dc}} = I_{\text{dc}}^2 R_L$$

The ac power input to the rectifier is

$$P_{\text{ac}} = I_{\text{rms}}^2 (R_f + R_L)$$

Then $\eta = \frac{I_{\text{dc}}^2 R_L}{I_{\text{rms}}^2 (R_f + R_L)} = \left(\frac{I_{\text{dc}}}{I_{\text{rms}}}\right)^2 \frac{R_L}{(R_f + R_L)} = \left(\frac{I_{\text{dc}}}{I_{\text{rms}}}\right)^2 \frac{R_L}{R_L} \qquad \text{as } R_f \to 0$

In a full-wave rectifier, $I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$ and $I_{\text{dc}} = \frac{2I_m}{\pi}$,

then the ratio $\frac{I_{\text{dc}}}{I_{\text{rms}}} = \frac{2I_m}{\pi} \times \frac{\sqrt{2}}{I_m} = \frac{2\sqrt{2}}{\pi}$

Thus, the efficiency is equal to $\eta = \left(\frac{2\sqrt{2}}{\pi}\right)^2 \frac{R_L}{(R_f + R_L)} = \frac{8}{\pi^2} \frac{R_L}{(R_f + R_L)} = \frac{0.812}{1 + \frac{R_f}{R_L}}$

If $R_L >> R_f$, $\eta_{\max} = 0.812 = 81.2\%$

Therefore, the maximum efficiency of a full-wave rectifier is 81.2% while the load resistance is very large compared to the forward resistance of a diode. However, the efficiency is always less than 81.2% in a full-wave rectifier circuit.

The advantages and disadvantages of a centre-tapped single-phase full-wave rectifier are given below:

Advantages

1. The average or dc output voltage of full-wave rectifier is twice of the average or dc output voltage of a half-wave rectifier.
2. The average or dc load current of full-wave rectifier is twice of the average or dc load current of a half-wave rectifier.
3. The ripple factor is comparatively less than that of half-wave rectifier.
4. The efficiency of full-wave rectifier is about 81.2%, but the efficiency of half wave rectifier is 40.6%. Therefore, the efficiency is twice that of half-wave rectifier.

Disadvantages

1. The output voltage is half of the secondary winding.
2. The peak inverse voltage of diode is two times of the half-wave rectifier.
3. This is expensive due to the centre-tapped transformer, which generates equal voltages on either half of the secondary winding.

5.13 SINGLE-PHASE BRIDGE RECTIFIER

Figure 5.26 shows a single-phase bridge rectifier which consists of four diodes (D_1, D_2, D_3 and D_4) and a load resistance R_L.

During the positive half cycle of input voltage, the diodes D_1 and D_2 are forward biased and conduct as they are in the ON state. The diodes D_3 and D_4 are reverse biased and they are in the OFF state. The current flows through load R_L and diodes D_1 and D_2 as shown in Fig. 5.27. The output across load R_L is the positive half cycle of input voltage.

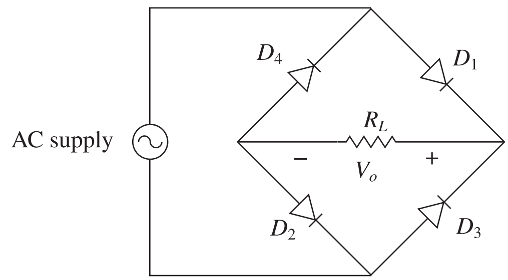


Fig. 5.26 *Single-phase bridge rectifier circuit*

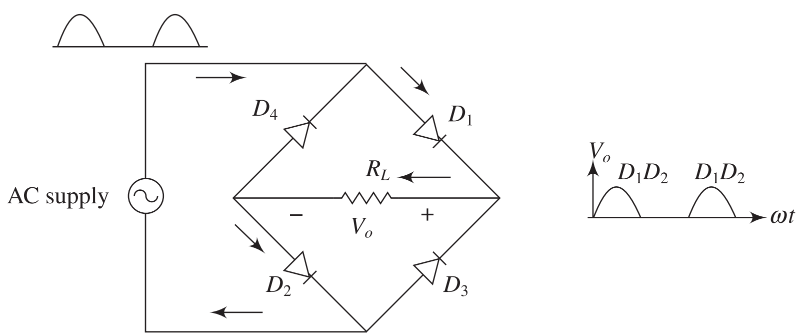


Fig. 5.27 *Operation during positive half cycle of supply voltage*

In the negative half cycle of input voltage, the diodes D_3 and D_4 are forward biased and conduct as they are in the ON state. The diodes D_1 and D_2 are reverse biased and they are in the OFF state. The current flows through load and diodes D_3 and D_4 as shown in Fig. 5.28. The output across load R_L is the negative half cycle of input voltage. As current flows in the same direction through load, a full wave rectified output voltage is developed across the resistance R_L.

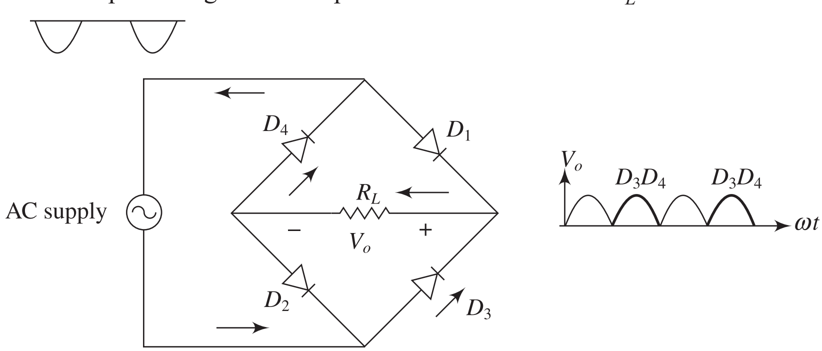


Fig. 5.28 *Operation during negative half cycle of supply voltage*

Figure 5.29 shows the alternative arrangement of full-wave rectifier circuit. In Fig. 5.31(a), the load resistance connected between A and B. In Fig. 5.29(b), one end of the load resistance is connected to A and other end is connected to ground. The B point is also at ground potential.

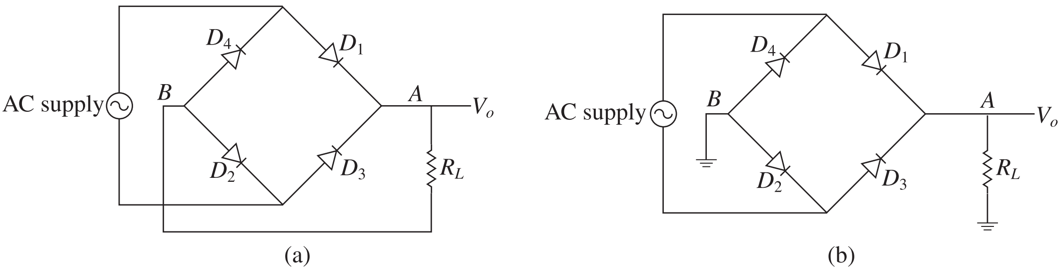


Fig. 5.29 *Alternative arrangement of single-phase bridge rectifier*

5.13.1 Average or dc Output Voltage

Figure 5.30 shows the input voltage, output voltage and load current waveforms. The output voltage across the load can be expressed as

$$v_o = v_m \sin \omega t = V \sin \theta \qquad 0 \le \theta \le \pi$$
$$= -V_m \sin \omega t = -V_m \sin \theta \qquad \pi \le \theta \le 2\pi$$

where, V_m is the maximum voltage across each half of the secondary winding

$$V_{\text{av}} = V_{\text{dc}} = V_o = \frac{\text{Area under the curve over the half cycle}}{\text{base}}$$

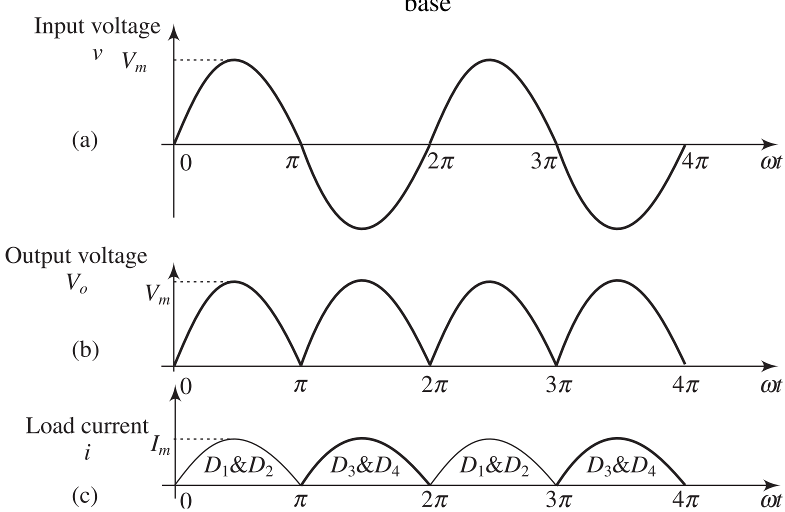


Fig. 5.30 *(a) Input voltage (b) Output voltage (c) Load current waveforms*

where, base = half cycle time

$$= \frac{\int_0^{\pi} v d\theta}{\pi} = \frac{\int_0^{\pi} V_m \sin \theta d\theta}{\pi}$$

$$= \frac{V_m}{\pi} \left| -\cos\theta \right|_0^{\pi} = \frac{V_m}{2\pi}[+1-(-1)]$$
$$= \frac{2V_m}{\pi} = 0.636V_m$$

From the above expression, we can say that the average or dc output voltage is 63.6% of the maximum ac input voltage and it is double that of a half-wave rectifier. Hence, a full-wave rectifier is two times effective than a half-wave rectifier.

5.13.2 Average (dc) Load Current

The instantaneous current i flows though the diode D and the load resistance R_L and it can be expressed as

$$i = I_m \sin \omega t = I_m \sin \theta \qquad 0 \le \theta \le \pi$$
$$= -I_m \sin \omega t = -I_m \sin \theta \qquad \pi \le \theta \le 2\pi$$

where, $I_m = \frac{V_m}{R_L}$

$$I_{av} = I_{dc} = \frac{\text{Area under the curve } i \text{ over the half cycle}}{\text{base}}$$

where, base = half cycle time

$$= \frac{\int_0^{\pi} i d\theta}{\pi} = \frac{\int_0^{\pi} I_m \sin\theta d\theta}{\pi}$$
$$= \frac{I_m}{\pi} \left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi}[+1-(-1)] = \frac{2I_m}{\pi} = 0.636I_m \text{ where, } I_m = \frac{V_m}{R_L}$$

From the above equation, it is justified that the average or DC load current is 63.6% of the maximum load current.

5.13.3 Peak Inverse Voltage

In a single-phase full-wave bridge rectifier, the polarities of secondary windings corresponding to the positive half cycle of input voltage is shown in Fig. 5.31. The diode D_1 and D_2 are forward biased and they are in ON state. The diodes D_3 and D_4 are reverse biased and have a maximum voltage equal to V_m. Therefore, peak inverse voltage of each diode is V_m.

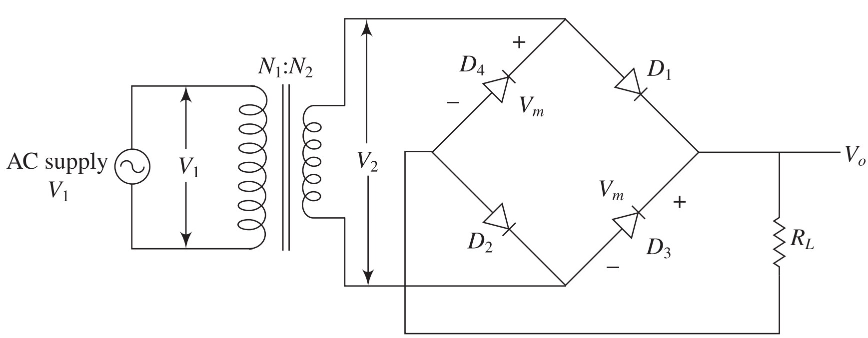


Fig. 5.31 *Polarities of secondary windings w.r.t. positive half cycle of input voltage*

5.14 ADVANTAGES AND DISADVANTAGES OF SINGLE-PHASE FULL-WAVE BRIDGE RECTIFIER

The advantages and disadvantages of a single-phase full-wave bridge rectifier are given below.

Advantages

1. No centre tap is required on the transformer.
2. This circuit allow the floating point output terminals and no terminal is grounded.
3. This transformer is less costly compared to centre tap transformer.
4. The average or dc output voltage and average or dc load current is twice of a half-wave rectifier.
5. The ripple factor is comparatively less than that of half-wave rectifier.
6. The efficiency of full-wave rectifier is about 81.2%, but the efficiency of half-wave rectifier is 40.6%. Therefore, the efficiency is twice that of half-wave rectifier.

Disadvantages

1. This circuit requires four diodes as compared to the centre-tapped full-wave rectifier. As the silicon diodes are less cost, this circuit is economical
2. The PIV rating of diodes is half of the centre-tapped full-wave rectifier.
3. The output voltage is half of the secondary winding.
4. The peak inverse voltage of diode is two times of the half-wave rectifier.
5. This is expensive due to the centre-tapped transformer, which generates equal voltages on either half of the secondary winding.

5.15 COMPARISON OF RECTIFIERS

The comparison between half-wave rectifier, centre tapped full-wave rectifier and full-wave bridge rectifier is given in Table 5.1

Table 5.1 *Comparison of half-wave, centre-tapped full wave and full-wave bridge rectifier*

Parameter	Half-wave rectifier	Centre-tapped full-wave rectifier	Full-wave bridge rectifier
Number of diodes	1	2	4
Peak inverse voltage	V_m	$2V_m$	V_m
Average of DC output voltage	$0.318V_m$	$0.636V_m$	$0.636V_m$
RMS value of output voltage	$\frac{V_m}{2}$	$\frac{V_m}{\sqrt{2}}$	$\frac{V_m}{\sqrt{2}}$
Ripple factor	1.21	0.482	0.482
Ripple frequency	f	$2f$	$2f$
Percentage efficiency	40.6	81.2	81.2
Transformer utilization factor	0.287	0.693	0.812

5.16 FOURIER SERIES OF THE OUTPUT VOLTAGE OF A FULL-WAVE RECTIFIER

The output voltage of a full-wave rectifier can be expressed by Fourier series as given below:

$$v(t) = V_o + \sum_{n=1,2,3.....}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

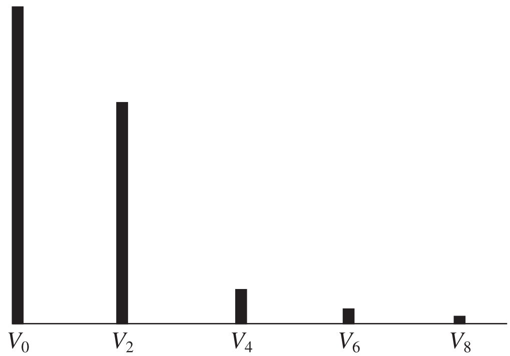


Fig. 5.32 *DC output voltage and harmonics*

The dc component of output voltage is V_o and its value is

$$V_o = \frac{1}{\pi}\int_0^{\pi} v d(\omega t) = \frac{1}{\pi}\int_0^{\pi} \sqrt{2}V \sin \omega t \cdot d(\omega t)$$

$$= \frac{2\sqrt{2}V}{\pi} = \frac{2V_m}{\pi}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v \cdot \cos n\omega t \cdot d(\omega t)$$

$$= \frac{2}{\pi}\int_0^{\pi} \sqrt{2}V \sin \omega t \cdot \cos n\omega t \cdot d(\omega t)$$

$$= \frac{4\sqrt{2}V}{\pi} \sum_{n=2,4,6...}^{\infty} \frac{-1}{(n-1)(n+1)} \qquad \text{for } n = 2, 4, 6, 8......$$

$$= 0 \qquad \text{for } n = 1, 3, 5, 7.....$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v \cdot \sin n\omega t \cdot d(\omega t) = \frac{2}{\pi}\int_0^{\pi} \sqrt{2}V \sin \omega t \cdot \sin n\omega t \cdot d(\omega t) = 0$$

After substituting the V_0, a_n and b_n in the output voltage equation, we get

$$v = \frac{2\sqrt{2}V}{\pi} - \frac{4\sqrt{2}V}{3\pi}\cos 2\omega t - \frac{4\sqrt{2}V}{15\pi}\cos 4\omega t - \frac{4\sqrt{2}V}{35\pi}\cos 6\omega t.....$$

It is clear from the above equation, that the output voltage contains dc voltage and harmonics at multiples of fundamental frequency as depicted in Fig. 5.32.

5.17 SINGLE-PHASE FULL-WAVE UNCONTROLLED CONVERTER WITH *RL* LOAD

Figure 5.33 shows the single-phase full-wave uncontrolled converters with *RL* load. The load current and line current depends on the nature of load impedance and phase angle ϕ. For *RL* load, the current flows *RL* from 0 to π duration and also from π to 2π.

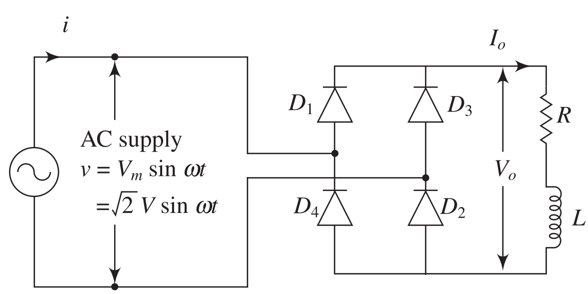


Fig. 5.33 *Single-phase full-wave rectifier with RL load*

When diode is forward biased, it conducts. During the positive half cycle of supply voltage, diode D_1 and D_2 are forward biased and conduct. Similarly, during the negative half cycle of supply voltage, diodes D_3 and D_4 are forward biased and conduct. When diodes D_1 and D_2 conduct, current i_o flows through load from $\omega t = 0$ to $\omega t = \pi$. After $\omega t = \pi$, D_3 and D_4 become forward biased and current flows through load from $\omega t = \pi$ to $\omega t = 2\pi$. The input voltage, output voltage, load current, voltage across V_{D1} and V_{D2} for a single-phase full-wave rectifier with *RL* load are depicted in Fig. 5.34 when $\phi < 90°$. Figure 5.35 shows the input voltage, output voltage, load current, current through D_1, D_2, D_3 and D_4, line current *i* for a single-phase full-wave rectifier with *RL* load where $\phi \cong 90°$.

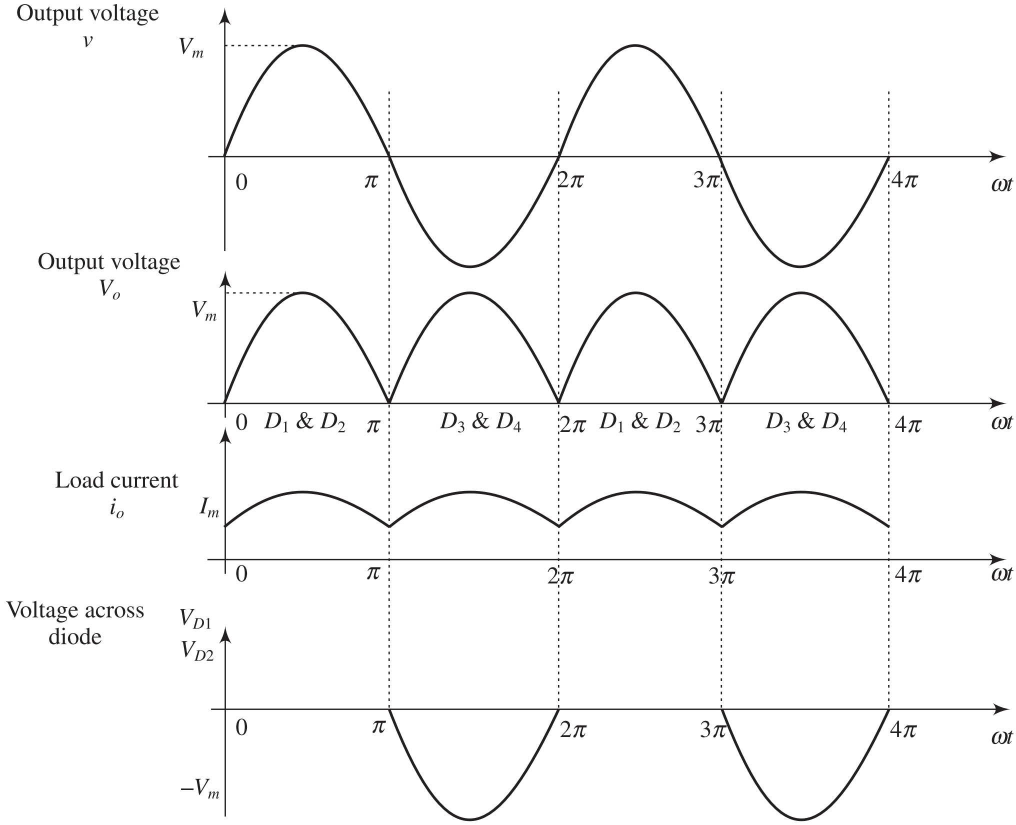


Fig. 5.34 *Input voltage, output voltage, load current, voltage across V_{D1} and V_{D2} for a single-phase full-wave rectifier with RL load where $\phi < 90°$*

The input voltage and output current can be expressed as

$$\sqrt{2}V \sin \omega t = L\frac{di_o}{dt} + Ri_o \tag{5.4}$$

where, V is the rms voltage and output current is i_o

After solving the above differential Eq. (5.4), the output current can be expressed as

$$i_o(t) = \frac{\sqrt{2}\,\text{V}}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} \quad \text{where, } \phi = \tan^{-1}\frac{\omega L}{R} \tag{5.5}$$

At $\omega t = 0$, the output current $i_o = 0$.

Therefore, $i_o(\omega t = 0) = 0 = \dfrac{\sqrt{2}V}{Z}\sin(-\varphi) + Ae^{-\frac{R}{L}\times 0}$

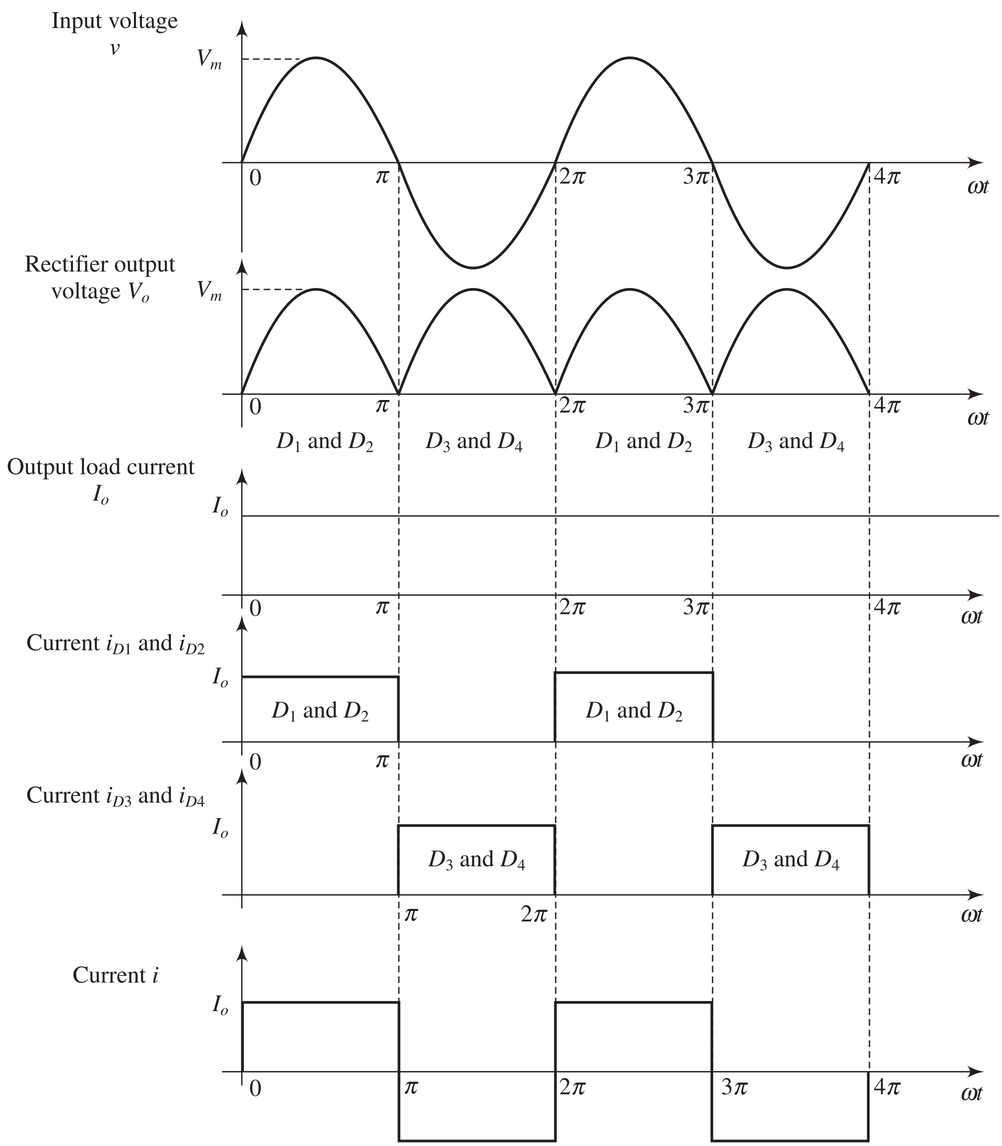


Fig. 5.35 *Input voltage, output voltage, load current, current through D_1, D_2, D_3, and D_4, line current i for a Single-phase full wave rectifier with RL load where $\phi \cong 90°$*

or
$$A = \frac{\sqrt{2}\,\mathrm{V}}{Z}\sin\phi \tag{5.5}$$

After substituting the value of A in Eq. (5.5), we obtain

$$i_o(t) = \frac{\sqrt{2}V}{Z}[\sin(\omega t - \phi) + \sin\phi \cdot e^{-\frac{R}{L}t}]$$

or $$i_o(t) = \frac{\sqrt{2}V}{Z}[\sin(\omega t - \phi) + \sin\phi \cdot e^{-\frac{\omega t}{\tan\phi}}]$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$ and $\tan\phi = \dfrac{\omega L}{R}$

The average output voltage can be expressed as

$$V_O = \frac{1}{\pi}\int_0^{\pi} \sqrt{2}V \sin\omega t d(\omega t) = \frac{2\sqrt{2}V}{\pi}$$

Example 5.3 A single-phase full-wave bridge rectifier circuit is fed from a 220 V, 50 Hz supply. It consists of four diodes, a load resistance 20 Ω and a very large inductance so that the load current is constant. Determine (a) the average or dc output voltage, (b) average load current, (c) average value of diode current, (d) rms value of diode current, (e) the rms value of input current, (f) the dc output power and (g) the input power factor. Assume all diodes are ideal.

Solution

The maximum input voltage is $V_m = \sqrt{2}V = \sqrt{2} \times 220\text{ V} = 311.125\text{ V}$

(a) The dc output voltage is

$$V_{av} = V_o = \frac{2V_m}{\pi} = \frac{2 \times 311.12}{\pi} = 198.165\text{ V}$$

(b) Average load current is

$$I_{av} = I_o = \frac{V_{av}}{R} = \frac{198.165}{20} = 9.9\text{ A}$$

(c) Average value of diode current is

$$I_{\text{av of diode}} = \frac{I_{av} \times \pi}{2\pi} = \frac{I_{av}}{2} = \frac{9.9}{2} = 4.95\text{ A}$$

(d) The rms value of diode current is

$$I_{\text{rms of diode}} = \sqrt{\frac{I_{av}^2 \times \pi}{2\pi}} = \frac{I_{av}}{\sqrt{2}} = \frac{9.9}{\sqrt{2}} = 7.0\text{ A}$$

(e) The rms value of input current is

$$I_{\text{rms of input current}} = \sqrt{\frac{I_{av}^2 \times \pi}{\pi}} = I_{av} = 9.9\text{ A}$$

(f) The dc output power is $P_{dc} = V_{av}I_{av} = V_oI_o = 198.165 \times 9.9\text{ W} = 1961.83\text{ W}$

(g) We know that $V \times I_{\text{rms of input current}} \times \cos\phi = P_{dc}$

The input power factor is

$$\cos\phi = \frac{P_{dc}}{V \times I_{\text{rms of input current}}} = \frac{1961.83}{220 \times 9.9} = 0.9$$

5.18 SINGLE-PHASE FULL-WAVE UNCONTROLLED CONVERTER WITH *R-L-E* LOAD

Figure 5.36 shows a single-phase full-wave rectifier with *R-L-E* load. When the input voltage v is greater than E, the load current flows. The relation between input voltage and load current can be expressed as

$$\sqrt{2}V \sin\omega t = L\frac{di_o}{dt} + Ri_o + E \quad \text{for} \quad i_o \geq 0$$

where, V is the rms voltage and load current is i_o, and E is battery voltage

After solving the above differential equation, the output current can be expressed as

$$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} - \frac{E}{R} \quad (5.6)$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$ and $\tan\phi = \frac{\omega L}{R}$

The single-phase full-wave rectifier with R-L-E load can be operating in continuous current mode and discontinuous current mode.

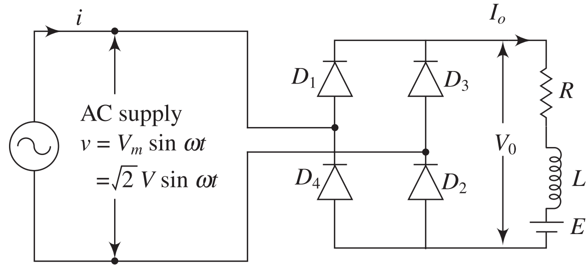


Fig. 5.36 *Single-phase full-wave rectifier with R-L-E load*

5.18.1 Continuous Current Mode Operation

Input voltage, rectifier output voltage, battery voltage, output load current and input line current waveforms of single-phase full-wave rectifier with *R-L-E* load during continuous mode operation is depicted in Fig. 5.37. For continuous load current, at $\omega t = \pi$, $i_o = I_o$, then the value of A is

$$A = \left(I_o + \frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin\phi\right)e^{\frac{R}{L}\frac{\pi}{\omega}}$$

After substituting the value of A in Eq. (5.6), we get

$$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + \left(I_o + \frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin\phi\right)e^{\frac{R}{L}\left(\frac{\pi}{\omega} - 1\right)} - \frac{E}{R} \quad (5.7)$$

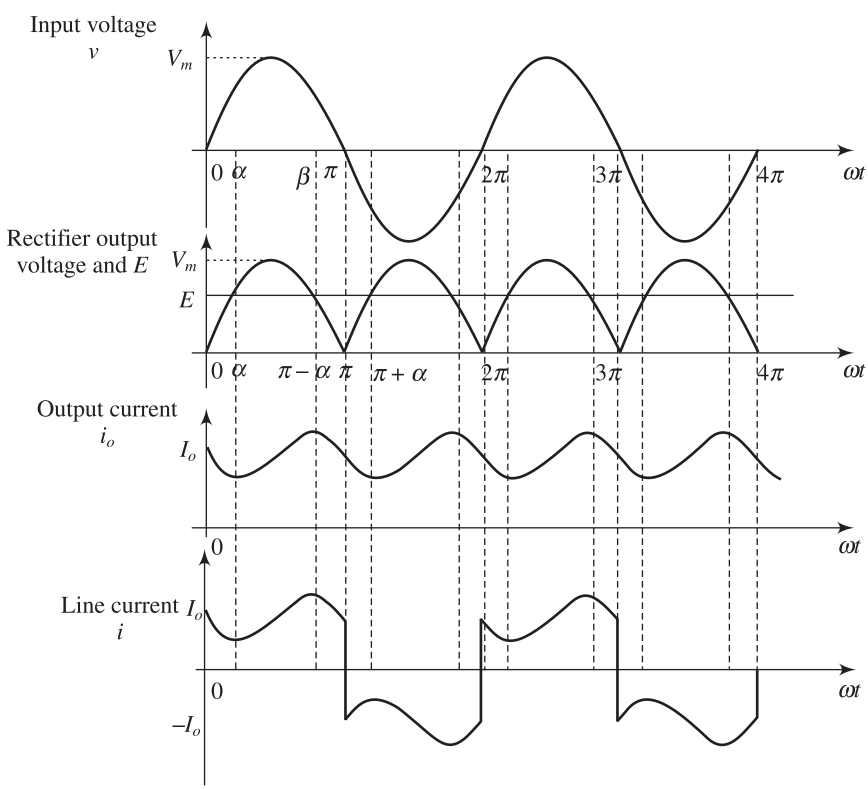


Fig. 5.37 *Continuous mode operation*

At $\omega t = 0$, $i_o = I_o$, then we can write

$$I_o = \frac{\sqrt{2}V}{Z}\sin(-\phi) + \left(I_o + \frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin\phi\right)e^{\frac{R}{L}\frac{\pi}{\omega}} - \frac{E}{R} \tag{5.8}$$

At $\omega t = \pi$, $i_o = I_o$, then we obtain

$$I_o = \frac{\sqrt{2}V}{Z}\sin(\pi - \phi) + \left(I_o + \frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin\varphi\right)e^{\frac{R}{L}\left(\frac{\pi}{\omega} - \frac{\pi}{\omega}\right)} - \frac{E}{R} \tag{5.9}$$

Using the above equations, we get the current I_o is equal to

$$I_o = \frac{\sqrt{2}V}{Z}\sin\phi \frac{1 + e^{-\frac{R}{L}\frac{\pi}{\omega}}}{1 - e^{-\frac{R}{L}\frac{\pi}{\omega}}} - \frac{E}{R}$$

After substituting the value of I_o in Eq. (5.7), we obtain

$$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + \frac{2\sqrt{2}V}{Z}\sin\phi \frac{e^{-\frac{R}{L}t}}{1 - e^{-\frac{R}{L}\frac{\pi}{\omega}}} - \frac{E}{R}$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$ and $\tan\varphi = \dfrac{\omega L}{R}$

5.18.2 Discontinuous Current Mode Operation

Figure 5.38 shows input voltage, rectifier output voltage, battery voltage, and output load current waveforms of single-phase full-wave rectifier with *R-L-E* load during discontinuous mode operation.

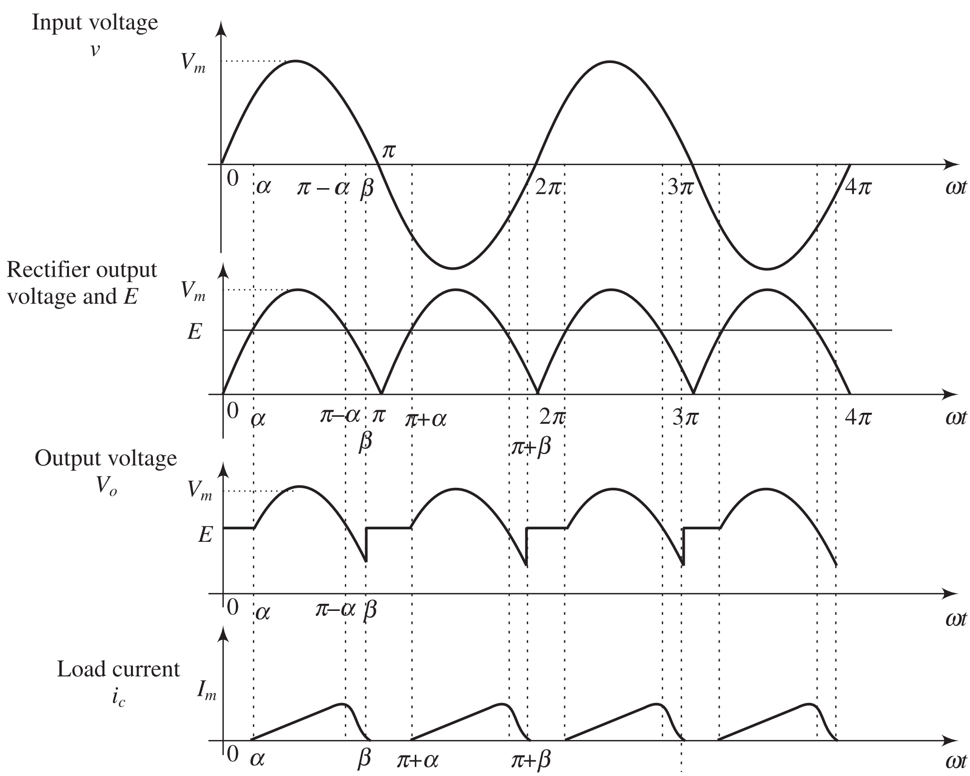


Fig. 5.38 *Discontinuous mode operation*

For discontinuous current, the load current flows from $\omega t = \alpha$ to $\omega t = \beta$.

Then $$V_m \sin \alpha = E$$

or $$\alpha = \sin^{-1}\frac{E}{V_m} = \sin^{-1}\frac{E}{\sqrt{2}V} = \sin^{-1} x \quad \text{where, } x = \frac{E}{\sqrt{2}V}$$

If v is greater than E, the load current flows. The relation between input voltage and load current i_o can be expressed as

$$\sqrt{2}V \sin \omega t = L\frac{di_o}{dt} + Ri_o + E \tag{5.10}$$

where, V is the rms voltage and output current is i_o, and E is battery voltage.

After solving the above differential Eq. (5.10), the output current can be expressed as

$$i(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} - \frac{E}{R} \tag{5.11}$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$ and $\phi = \tan^{-1}\frac{\omega L}{R}$

For discontinuous load current, at $\omega t = \alpha$, $i_o = 0$ then the value of A is

$$A = \left(\frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)\right)e^{\frac{R}{L}\frac{\pi}{\omega}}$$

After substituting the value of A in Eq. (5.11), we obtain

$$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + \left(\frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)e^{\frac{R}{L}\left(\frac{\alpha}{\omega} - t\right)}\right) - \frac{E}{R}$$

At $\omega t = \beta$, $i_o = 0$, then we get

$$i_o(t) = 0 = \frac{\sqrt{2}V}{Z}\sin(\beta - \phi) + \left(\frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)e^{\frac{R}{L}\left(\frac{\alpha - \beta}{\omega}\right)}\right) - \frac{E}{R}$$

The value of β can be determined by numerical analysis.

5.19 EFFECT OF TRANSFORMER LEAKAGE INDUCTANCE IN FULL WAVE RECTIFIER WITH *R-L* LOAD

Figure 5.39 shows a single-phase full-wave rectifier with inductive load where L is load inductance and L_c is transformer leakage inductance or source inductance. Due to the presence of source inductance

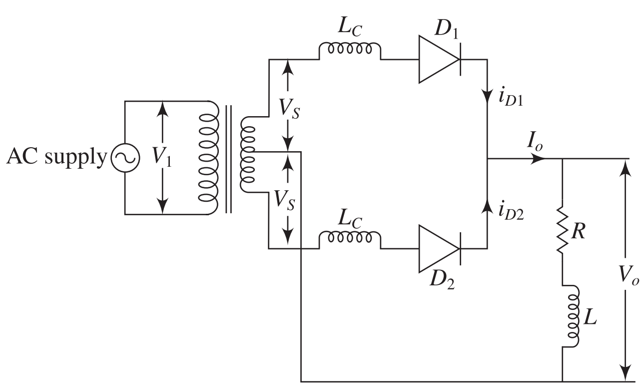


Fig. 5.39 *Single-phase full-wave rectifier with inductive load and transformer leakage inductance*

in series with diode, the instantaneous transfer of load current from one diode to other diode is not possible. Therefore current will flows through a diode though input voltage polarity becomes reverse. As the input voltage becomes zero, the load current can be transferred gradually from one diode to other diode as shown in Fig. 5.40.

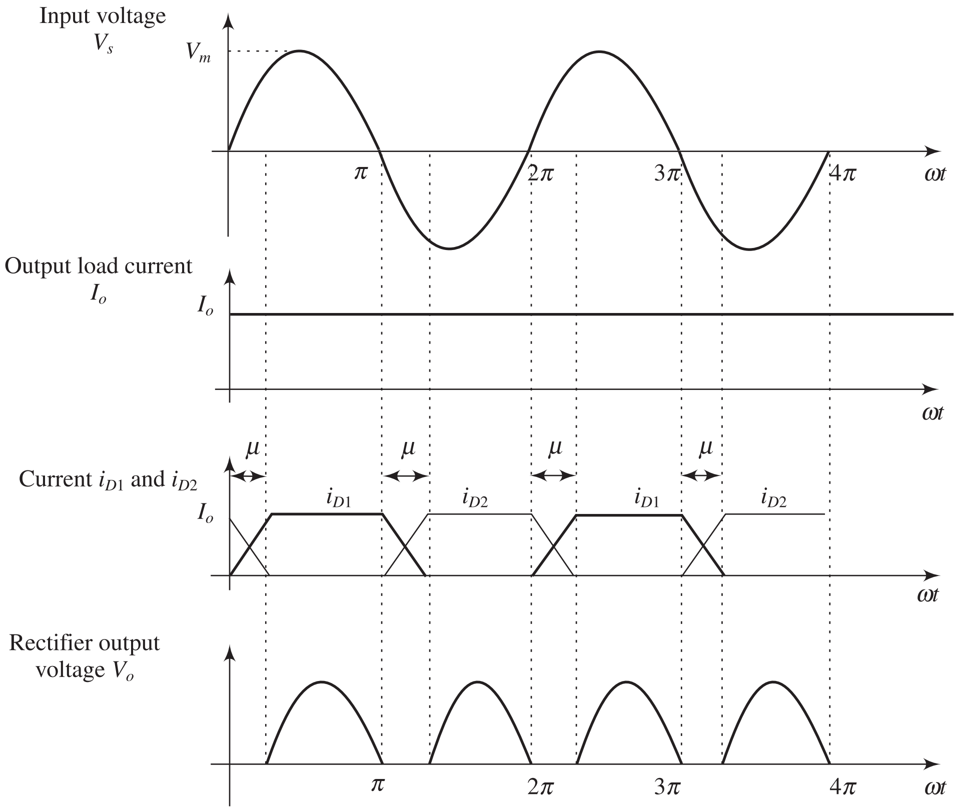


Fig. 5.40 *The effect of transformer leakage inductance in full-wave rectifier with inductive load*

The current flow through diode D_1 is i_{D1} and the current flow through diode D_2 is i_{D2}. During transition, i_{D1} slowly decreases to zero and i_{D2} slowly increases to I_o. At any instant, sum of two diode currents $i_{D1} + i_{D2}$ is equal to I_o. During the dual conduction period, both diodes conduct and the transformer secondary winding will be shorted though transformer leakage inductance (L_c). Since the potential difference between the transformer centre tap point and the common cathode terminal is zero, the output voltage across load during the overlap period (μ) is zero.

The angle of overlap period (μ) is function of transformer leakage inductance (L_c), applied voltage and load current (I_o). While the diode D_1 is in the conducting state, the input voltage phase is reversed at $\omega t = \pi$ and the current flows through the diode D_1 continuously.

The voltage across leakage inductance is

$$v_{Lc} = L_C \frac{di_{D1}}{dt}$$

or $$v_{Lc} dt = L_C di_{D1}$$

Therefore, $$\frac{2}{T}\int_{o}^{T/2} v_{Lc}dt = \frac{2}{T}\int_{0}^{I_o} L_C di_{D1}$$

or $$V_{L_C} = \frac{2}{T}L_C I_o = 2fL_C I_o$$

The average voltage across the leakage inductance is

$$V_{L_C} = 2fL_C I_o = \frac{\omega L_C}{\pi} I_o$$

The average output voltage of a single-phase full-wave rectifier is

$$V_{\text{av}} = \frac{2\sqrt{2}V}{\pi} - \frac{\omega L_C I_o}{\pi}$$

This output voltage is available from μ to π.

The average output voltage of a single-phase full-wave rectifier in terms of overlap angle μ is

$$V_{\text{av}} = \frac{1}{\pi}\int_{\mu}^{\pi} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi}(1+\cos\mu) \tag{5.12}$$

The load current is $I_o = \dfrac{1}{\omega L_C}\displaystyle\int_{0}^{\mu} \sqrt{2}V \sin \omega t \cdot d\omega t = \dfrac{\sqrt{2}V}{\omega L_C}(1-\cos\mu)$

Therefore, $\cos\mu = 1 - \dfrac{\omega L_C I_o}{\sqrt{2}V}$

After substituting the value of μ in Eq. (5.12), we get

$$V_{\text{av}} = \frac{\sqrt{2}V}{\pi}\left\{1+\left(1-\frac{\omega L_C I_o}{\sqrt{2}V}\right)\right\} = \frac{2\sqrt{2}V}{\pi} - \frac{\omega L_C I_o}{\pi}$$

Summary

- The rectifier circuit is used to convert ac input voltage into fixed dc voltage and electrical power flows from the ac input to the dc output.
- Classification of rectifiers based on their number of phases, the type of devices used, circuit topology, and the control mechanism are discussed in this chapter.
- The operation of single-phase uncontrolled half-wave and full-wave rectifiers with resistive (R), inductive (L), and back emf (E) type loads explained in detail.
- Determine the characteristic parameters of single-phase uncontrolled rectifiers from the input voltage, output voltage and current waveforms.
- The effect of transformer leakage inductance (L_C) in performance of single-phase full-wave rectifier is also incorporated in this chapter.
- Applications of filters to reduce ripples at output voltage are explained briefly.

Multiple-Choice Questions

5.1 A rectifier is used to

(a) convert ac to dc (b) convert dc to ac

(c) convert high voltage to low voltage (d) convert low voltage to high voltage

5.2 In a half-wave rectifier, the load current flows
(a) during complete cycle of the input voltage signal
(b) during positive half-cycle of the input voltage signal
(c) during negative half-cycle of the input voltage signal
(d) in either positive half cycle or negative half cycle of the input voltage signal

5.3 In a full-wave bridge rectifier, the current in each of the diodes flows for
(a) a complete cycle of the input signal. (b) a half cycle of the input signal
(c) less than half cycle of the input signal (d) more than half cycle of the input signal

5.4 The ripple factor of a half wave rectifier is
(a) 0.482 (b) 0.284 (c) 1.12 (d) 1.21

5.5 The ripple factor of a full wave bridge rectifier is
(a) 0.482 (b) 0.842 (c) 1.12 (d) 1.21

5.6 A bridge rectifier is preferable to a full-wave rectifier using centre tap transformer as
(a) it uses four diodes (b) its transformer does not require centre tap
(c) it requires much smaller transformer for the same output
(d) All of these

5.7 The function of a filter in a dc power supply is to
(a) remove ripples from the rectified output signal
(b) minimise voltage variations in ac input signal
(c) reduce harmonics in rectified output signal
(d) introduce more ripples into the rectified output signal

5.8 If a 50 Hz ac signal is fed to a rectifier, the ripple frequency of output voltage waveform for full bridge rectifier is
(a) 25 Hz (b) 50 Hz (c) 100 Hz (d) 150 Hz

5.9 If the peak value of an applied voltage in a half-wave rectifier is V_m, the peak-inverse voltage of diode is
(a) V_m (b) $2V_m$ (c) $\sqrt{2}V_m$ (d) $2\sqrt{2}V_m$

5.10 A full-wave rectifier has twice the efficiency of a half-wave rectifier as
(a) full-wave rectifier use a transformer
(b) ripple factor of full-wave rectifier is less
(c) full-wave rectifier utilizes both half cycle of the input signal
(d) output frequency is equal to the line frequency.

5.11 If the peak value of an applied voltage in a full wave rectifier is V_m, the peak-inverse voltage of diode is
(a) V_m (b) $2V_m$ (c) $\sqrt{2}V_m$ (d) $2\sqrt{2}V_m$

5.12 If the peak value of an applied voltage of each half of secondary winding of centre tap transformer is V_m, the peak-inverse voltage of each diode in a full wave rectifier using centre tap transformer is
(a) V_m (b) $2V_m$ (c) $\sqrt{2}V_m$ (d) $2\sqrt{2}V_m$

5.13 Which rectifier requires four diodes?
(a) half-wave rectifier
(b) full-wave bridge circuit full-wave voltage
(c) full-wave rectifier circuit using centre tap transformer
(d) voltage clipping circuit

5.14 Which rectifier requires two diodes?
(a) half-wave rectifier
(b) full-wave bridge circuit full-wave voltage
(c) full-wave rectifier circuit using centre tap transformer
(d) voltage clipping circuit

5.15 Which circuit requires one diode?
(a) half-wave rectifier (b) full-wave bridge circuit full-wave voltage

(c) full-wave rectifier circuit using center tap transformer
(d) None of the above

5.16 When 220 V dc voltage is connected to a bridge rectifier in place of ac source, the bridge rectifier will be damaged due to short circuit of
(a) one diode (b) two diodes (c) three diodes (d) four diodes

5.17 The polarity of dc output voltage of a half-wave rectifier can be reversed by reversing
(a) transformer primary (b) the diode (c) transformer secondary (d) None of these

5.18 In a LC filter, the ripple factor
(a) increases with the load current (b) increases with the load resistance
(c) remains constant with the load current (d) decreases with the load current

5.19 The bleeder resistor is used in filter circuit of a dc power supply to
(a) reduce voltage regulation (b) improve voltage regulation
(c) improve filtering action (d) improve voltage regulation and filtering action

5.20 A capacitor filter rectifier provides
(a) poor voltage regulation (b) good voltage regulation
(c) voltage regulation remain constant (d) None of these

5.21 In a dc regulated power supply, the ripple factor is a measure of
(a) voltage regulation (b) filter efficiency
(c) diode rating (d) quality of power output

5.22 The use of a capacitor filter in a rectifier circuit provides satisfactory performance only while
(a) the load current is high (b) the load voltage is high
(c) the load current is low (d) the load voltage is low

Fill in the Blanks

5.1 In a single-phase rectifier, power flows from the _________ side to the ________ side.

5.2 A rectifier is a power electronic converter which converts ________sources to dc voltage and current.

5.3 In an uncontrolled rectifiers, semiconductor device_______ is used where as in controlled rectifiers semiconductor device_______ is used.

5.4 The ripple factor of the output voltage and current waveforms of a single-phase uncontrolled half wave rectifier is __________.

5.5 Single-phase uncontrolled full-wave rectifier have ______ average output voltage and improved ripple factor compared to a half-wave rectifier with resistive and inductive load.

5.6 In case of an inductive load, the ripple factor of the output current of the half-wave rectifier ________but that of the output voltage becomes ________.

5.7 The output voltage of a single-phase full-wave rectifier with an inductive load is _______ of the load parameters.

5.8 In continuous conduction, the load of a single-phase bridge rectifier must be ______.

5.9 In the _____ conduction mode, the output voltage of a single-phase bridge rectifier is _______ of load parameters.

5.10 With highly _____ load the output voltage waveform of a full-wave rectifier may be independent of the load parameters.

Review Questions

5.1 Draw the circuit diagram of half-wave rectifier with R load. Explain its working principle. What is the peak-inverse voltage of a diode? Determine the following parameters:
(a) dc output voltage (b) Average dc load current (c) rms output voltage
(d) rms load current (e) Ripple factor (f) Regulation (g) Efficiency

5.2 Draw the circuit diagram of a full-wave rectifier using center-tap transformer and R load. Explain its working principle. What is the peak-inverse voltage of a diode?. Determine the following parameters
(a) dc output voltage (b) Average d.c. load current (c) rms output voltage
(d) rms load current (e) Ripple factor (f) regulation (g) efficiency

5.3 Draw the circuit diagram of a full-wave bridge rectifier circuit R load. Explain its working principle. What is the peak-inverse voltage of a diode? Determine the following parameters
(a) dc output voltage (b) Average dc load current (c) rms output voltage
(d) rms load current (e) Ripple factor (f) Regulation (g) Efficiency

5.4 What are the advantages of a full wave bridge rectifier as compared to a full-wave center-tapped rectifier?

5.5 Give a list of comparison between a half-wave and full-wave rectifiers.

5.6 What is ripple factor? Why ripple factor is so important is power supply? Derive the expression of ripple factor of a half-wave rectifier and full-wave rectifier using centre-tap transformer.

5.7 What is the transformer utilisation factor (TUF)? Derive the expression of transformer utilisation factor (TUF) of a half-wave rectifier and full-wave rectifier using centre-tap transformer.

5.8 What is the efficiency of a rectifier? Derive the expression of efficiency of a half-wave rectifier and full-wave rectifier using centre-tap transformer. Discuss the difference between half wave rectifier and full-wave rectifier.

5.9 Discuss the effect of transformer leakage inductance on full-wave rectifier circuit with voltage and current waveforms. Prove that the average output voltage of a single-phase full-wave rectifier is $V_{av} = \dfrac{2\sqrt{2}V}{\pi} - \dfrac{\omega L_C I_o}{\pi}$.

5.10 Define filter. Why filters are used in a dc power supply?

5.11 What are the types of filters? Explain any two types with circuit diagram and waveform. What is the function of bleeder resistance in filter?

5.12 Explain LC filter and Π-filter with suitable diagram.

5.13 In a half-wave rectifier circuit (Fig. 5.41), a diode is connected in series with a load resistance about 500 ohms. The forward resistance of the diode is 25 ohms and the reverse resistance is infinite. The rectifier circuit is fed from a rms supply of 220 V. Calculate (a) the dc output voltage at no load (S is opened), (b) the dc load current (S is closed), (c) the dc output voltage at full load, (d) the percentage of voltage regulation, (e) reading of voltmeter connected across the diode, (f) the dc power delivered to the load and (g) efficiency at full load.

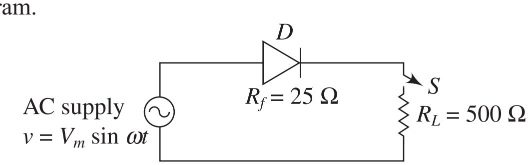


Fig. 5.41

5.14 In a half-wave rectifier circuit with transformer coupled input as shown in Fig. 5.42, ac input is fed from a rms supply of V_1 = 100 V. Calculate (a) the dc load current, (b) the dc output voltage, (c) rectifier efficiency and (d) transformer utilisation factor. Assume the diode is an ideal one.

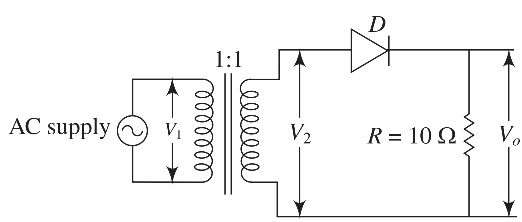


Fig. 5.42

5.15 In a half-wave rectifier circuit with transformer coupled input, ac input is fed from a rms supply of 220 V. Determine (a) the dc load current, (b) the dc output voltage and (c) the peak inverse voltage of diode.

5.16 In a half-wave rectifier circuit with transformer coupled input, the rms output voltage at secondary of transformer is 110 V. Determine (a) turn ratio of transformer if ac input is fed from a rms supply of 220 V, (b) the dc load current, (c) the dc output voltage and (d) the dc power delivered to the load and (e) efficiency and (f) transformer utilisation factor. Assume the forward resistance of the diode is zero ohms and the reverse resistance is infinite.

5.17 The dc output voltage of a single-phase uncontrolled rectifier with RL load is 70 V. If the input voltage is 200 V, compute (a) the cut-off angle β and (b) rms value of output voltage.

5.18 A centre-tapped single-phase full-wave rectifier has two diodes and the transformer secondary voltage from centre to each half of the secondary winding is $100\sqrt{2}\sin\omega t$ and the load resistance is 100 ohms. Determine (a) the average value of load current, (b) the rms value of load current and (c) the peak inverse voltage of each diode.

5.19 In a centre-tapped single-phase full-wave rectifier, the voltage across each half of the secondary winding is 141.4 sin ωt and the load resistance is 50 ohms. Determine (a) the average value of load current, (b) the rms value of load current, (c) the ripple factor, (d) the dc power, (e) the ac power and (f) efficiency of rectifier.

5.20 A 12 V, 250 Watt-Hour battery is charged using single-phase half-wave rectifier as depicted in Fig. 5.43. The input ac voltage is 220 V, 50 Hz. The average charging current is 4 A. Determine (a) conduction angle of diode, (b) the value of current limiting resistance R, (c) rms battery current, (d) charging time, (e) rectification efficiency and (f) peak inverse voltage of diode.

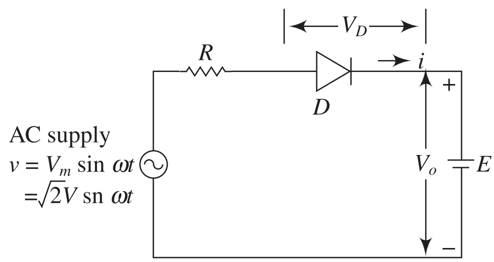


Fig. 5.43

5.21 The primary winding to secondary winding turns ratio of a centre-tapped transformer is 4:1. The primary winding of transformer is fed from 220 V ac supply and the secondary voltage is connected to a single-phase full-wave rectifier which consists of two diodes. The load resistance is 25 ohms. Determine (a) the average value of load current, (b) the average value of current in each diode, (c) the rms value of load current, (d) the ripple factor, (e) the dc power, (f) the ac power, (g) regulation and (h) efficiency of rectifier.

5.22 A single-phase full-wave rectifier using centre-tapped transformer of 100-0-100 V has two diodes. The dc output current is I_{av} = 15 A. Determine (a) the value of load resistance that can be connected across load terminals, (b) the average value of output voltage and (c) the peak inverse voltage of each diode. Assume all diodes are ideal.

5.23 A single-phase full-wave bridge rectifier circuit is fed from a 230 V, 50 Hz supply. It consists of four diodes, a load resistance 25 Ω and a very large inductance so that the load current is constant. Determine (a) the average or dc output voltage, (b) average load current, (c) average value of diode current, (d) rms value of diode current, (e) the rms value of input current, (f) the dc output power and (g) the input power factor. Assume all diodes are ideal.

5.24 The turn ratio of the transformer used in a single-phase full-wave rectifier circuit is 2:1. The primary of transformer is connected to 220 V, 50 Hz ac supply. Determine (a) the dc output voltage, (b) peak inverse voltage of each diode and (c) output frequency. Assume all diodes are ideal.

5.25 A single-phase full-wave bridge rectifier circuit is fed from a 230 V, 50 Hz supply. It consists of four diodes, a load resistance 25 Ω and a very large inductance so that the load current is constant. Determine (a) the average or dc output voltage, (b) average load current, (c) average value of diode current, (d) rms value of diode current, (e) the rms value of input current, (f) the dc output power and (g) the input power factor. Assume all diodes are ideal.

5.26 The output voltage of a full-wave rectifier circuit is fed to an inductor filter. Design the inductor filter if the ripple factor of filter output voltage is 4% for a load resistance 150 Ω and the supply frequency is 50 Hz.

5.27 The output voltage of a half-wave rectifier circuit is fed to a capacitor filter. Design the capacitor filter if the ripple factor of filter output voltage is 1% for a load resistance 1000 Ω and the supply frequency is 50 Hz.

5.28 Design a LC filter when the output voltage of a full-wave rectifier circuit is fed to filter circuit and the ripple factor is 0.01 and the supply frequency is 50 Hz. Assume L = 1.1 H.

Answers to Multiple-Choice Questions

5.1 (a)	5.2 (d)	5.3 (b)	5.4 (d)	5.5 (a)	5.6 (c)	5.7 (a)
5.8 (c)	5.9 (a)	5.10 (c)	5.11 (a)	5.12 (b)	5.13 (b)	5.14 (c)
5.15 (a)	5.16 (d)	5.17 (b)	5.18 (c)	5.19 (d)	5.20 (b)	5.21 (d)
5.22 (c)						

Answers to Fill in the Blanks

5.1 ac, dc	5.2 ac voltage or current	5.3 diode, thyristor	5.4 1.21
5.5 higher	5.6 improves, poorer	5.7 independent	5.8 inductive
5.9 continuous, independent		5.10 inductive	

SINGLE-PHASE CONTROLLED RECTIFIERS

6

6.1 INTRODUCTION

In uncontrolled converter or rectifier, the average output voltage is constant for a given load and for fixed input voltage. When a variable input voltage is applied to uncontrolled converter through an auto-transformer or VARIAC or tap changing transformer, the output voltage will be varied. AC-to-dc converters or rectifiers using tap changing transformer are shown in Fig. 6.1 and Fig. 6.2. Figure 6.3 shows a full-wave rectifier circuit using auto-transformer.

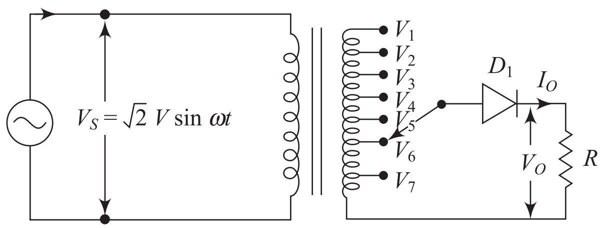


Fig. 6.1 *ac-to-dc converter (half-wave rectifier) using tap changing transformer*

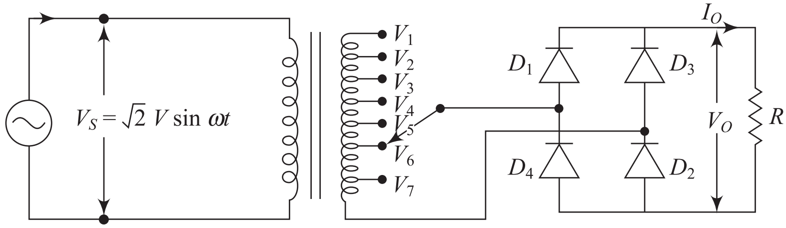


Fig. 6.2 *ac-to-dc converter (full-wave rectifier) using tap changing transformer*

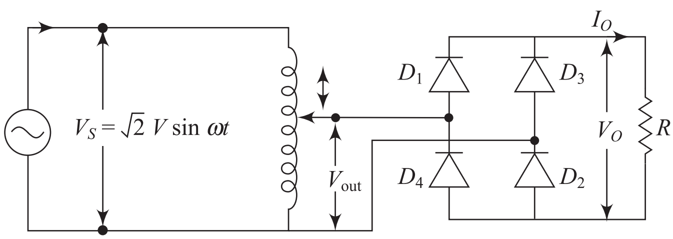


Fig. 6.3 *ac-to-dc converter (full-wave rectifier) using auto-transformer*

Using the above circuits, the variable output voltage can be obtained very simply, but these circuits have some demerits such as large size, heavy weight and high cost of transformers. Therefore, phase controlled converters are used in place of half-wave and full-wave rectifier using auto transformer or tap changing transformer. Single-phase controlled rectifiers are extensively used in many industrial applications such as electric traction systems, steel rolling mills, paper mills, textile mills, magnet power supply and electro-mechanical devices, etc.

Power semiconductor devices such as power diodes and SCRs are used in phase controlled converter as switches. These devices are turned ON and OFF sequentially and repetitively. Hence, the required ac-to-dc power conversion is possible. When a power semiconductor diode is forward biased, it will conduct if input voltage is greater than cut-off voltage of diode. As soon as power diode is reverse biased, it becomes turned OFF and stop conducting. In case of SCRs, when SCRs are forward biased and a triggering pulse is applied, it becomes turned ON and starts conducting. The conducting SCR will be turned OFF while a reverse bias or commutation voltages appears across it. The turned OFF process of SCR is known as *commutation*. The commutations of SCRs are two types such as *natural commutation* (*line commutation*) and *forced commutation*.

In forced commutation, thyristor is turned OFF when a specially designed circuit is used to apply a reverse voltage across SCR for short duration and SCR is forced to turn OFF. This commutation method is also known as *artificial commutation*.

In natural commutation, the SCR will be turned OFF whenever the applied ac voltage of the circuit becomes zero. This commutation method is called as line commutation. This technique is commonly used in controlled rectifier circuit. In this chapter, all types of single phase controlled converters with *R*, *R-L* and *R-L-E* load are discussed.

6.2 CLASSIFICATION OF SINGLE-PHASE CONTROLLED RECTIFIERS

When diodes are replaced by SCRs in all the figures of Chapter 5, a controllable output voltage can be obtained by controlling the delay angle or firing angle or triggering angle α. Hence, a variable voltage can be available at output terminals of controlled rectifiers. Similar to uncontrolled converters, single phase controlled converters are classified as single-phase half-wave (1-pulse) controlled converter, single-phase full-wave (2-pulse) controlled converter. Figure 6.4 shows classification of single-phase controlled converters.

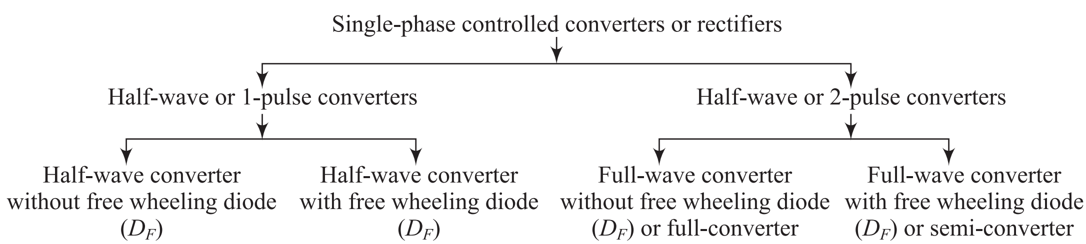


Fig. 6.4 *Classification of single-phase controlled converters*

6.3 SINGLE-PHASE HALF-CONTROLLED CONVERTERS WITH *R* LOAD

The single-phase half-controlled converter with resistive (R) load is shown in Fig. 6.5. The thyristor T_1 will conduct only when the anode is positive with respect to cathode and a positive gate pulse is

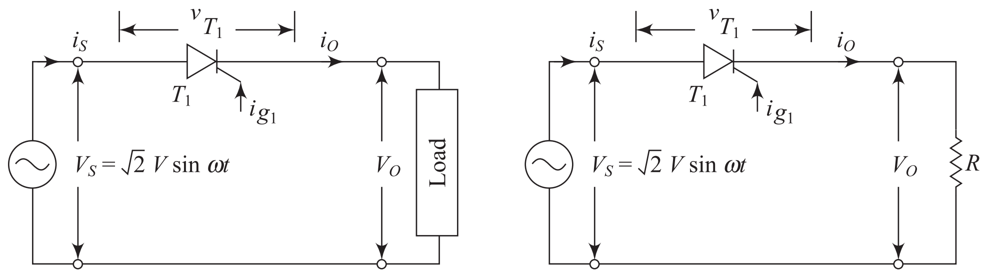


Fig. 6.5 *Single-phase half controlled converter with R load*

applied, otherwise it can operate in forward blocking state and the load current can not flow though thyristor. During the positive half cycle of input voltage, thyristor T_1 is forward biased. When the SCR T_1 is fired at $\omega t = \alpha$, it starts conduction at $\omega t = \alpha$ and the load current flows through T_1 and load. After that the SCR T_1 continue it's conduction up to $\omega t = \pi$.

During the negative half cycle of input voltage, thyristor T_1 is reverse biased. Therefore, SCR T_1 will be turned OFF due to natural commutation at $\omega t = \pi$ and the current flow though load becomes zero. Consequently, the average or dc output voltage is controlled by varying the firing angle α. Figure 6.6 shows the input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for half-wave controlled rectifier with R load. It is clear from Fig. 6.6 that the output voltage waveform is same as output or load current waveform due to zero phase difference in resistive load. In this converter, the dc or average output voltage is always positive and the current is also positive. As a result single phase half-wave controlled rectifier with R load operates in first quadrant and this converter is known as *half-wave semi-converter*.

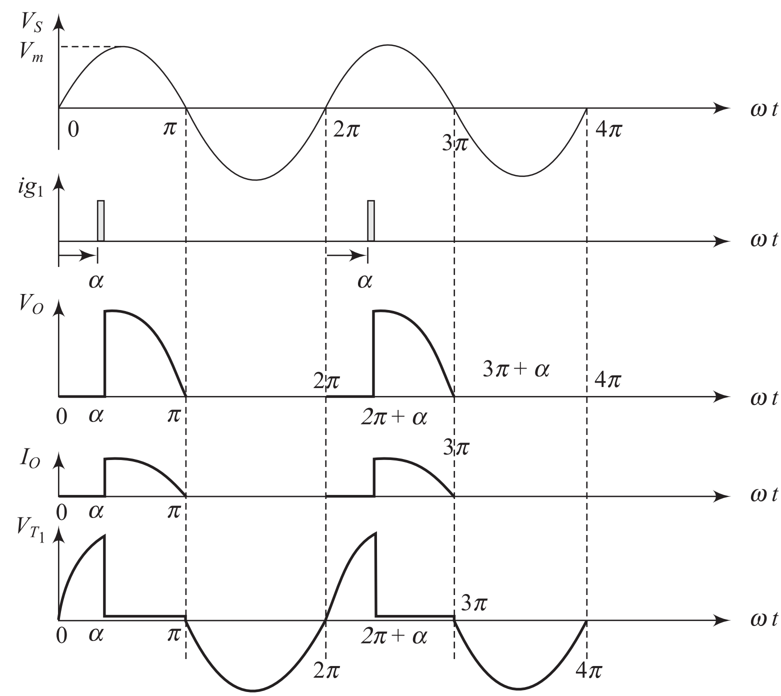


Fig. 6.6 *Input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for half wave controlled rectifier with R load*

Since α is the delay or firing angle of thyristor T_1, T_1 will be OFF from $\omega t = 0$ to $\omega t = \alpha$ and $\omega t = \pi$ to $\omega t = 2\pi$. Thyristor T_1 will be ON from $\omega t = \alpha$ to $\omega t = \pi$. Therefore the output voltage across load is zero from $\omega t = 0$ to $\omega t = \alpha$ and $\omega t = \pi$ to $\omega t = 2\pi$ and the output voltage across load is equal to input voltage from $\omega t = \alpha$ to $\omega t = \pi$. Consequently, in a complete cycle, the output voltage across load is available during positive half cycle only.

From the above discussion, the output voltage across the load can be expressed as

$$\begin{aligned} v_O &= V_m \sin \omega t \quad \text{for } \alpha \le \omega t \le \pi \\ &= 0 \qquad\qquad\; \text{for } 0 \le \omega t \langle \alpha \quad \text{and} \quad \pi \le \omega t \le 2\pi \end{aligned}$$

The value of average output voltage is equal to

$$V_O = \frac{1}{2\pi}\int_\alpha^\pi \sqrt{2}V \sin \omega t d(\omega t) = \frac{\sqrt{2}V}{2\pi}(1+\cos\alpha)$$

When $\alpha = 0$, the average output voltage is

$$\frac{\sqrt{2}V}{2\pi}(1+\cos 0) = \frac{\sqrt{2}V}{\pi}$$

If $\alpha = \pi$, the average output voltage is

$$\frac{\sqrt{2}V}{2\pi}(1+\cos\pi) = 0$$

Hence the dc output voltage can be varied from $\dfrac{\sqrt{2}V}{\pi}$ to 0 when firing angle varies from $\alpha = 0$ to $\alpha = \pi$ as depicted in Fig. 6.7.

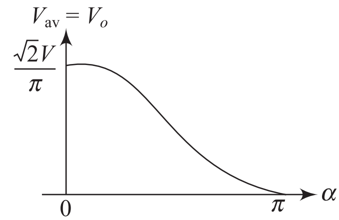


Fig. 6.7 *Firing angle vs output voltage*

The average current is $I_{av} = I_{dc} = I_o = \dfrac{V_O}{R} = \dfrac{\sqrt{2}V}{2R\pi}(1+\cos\alpha)$.

The current waveform is similar to voltage waveform. Hence, the converter operates in first quadrant only as shown in Fig. 6.8.

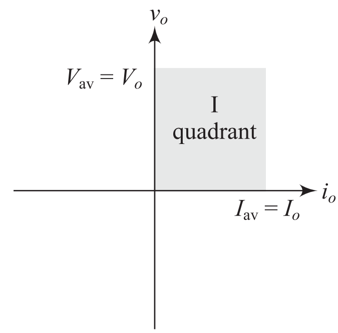


Fig. 6.8 *Quadrant operation of single half-wave controlled rectifier with R load*

The rms value of output voltage across load is

$$\begin{aligned} V_{rms} &= \left[\frac{1}{2\pi}\int_\alpha^\pi (\sqrt{2}V \sin\omega t)^2 d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{2\pi}\int_\alpha^\pi 2V^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} \\ &= \left[\frac{V^2}{2\pi}\int_\alpha^\pi (1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}} = \frac{V}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} \end{aligned}$$

Then the rms value of load current is

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V}{\sqrt{2}R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

The dc output power is equal to

$$\begin{aligned} P_{dc} &= V_{dc} \times I_{dc} = V_o \times I_o \\ &= \frac{\sqrt{2}V}{2\pi}(1+\cos\alpha) \times \frac{\sqrt{2}V}{2R\pi}(1+\cos\alpha) \\ &= \frac{V^2}{2\pi^2 R}(1+\cos\alpha)^2 \end{aligned}$$

The ac output power is equal to

$$P_{ac} = V_{rms} \times I_{rms}$$

$$= \frac{V}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} \times \frac{V}{\sqrt{2}R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

$$= \frac{V^2}{2R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]$$

The from factor of load current is

$$FF = \frac{I_{rms}}{I_{av}} = \frac{V}{\sqrt{2}R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} \Big/ \frac{\sqrt{2}V}{2R\pi}(1 + \cos\alpha)$$

$$= \frac{\pi}{(1 + \cos\alpha)}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} = \frac{1}{(1 + \cos\alpha)}\left[\pi\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

The ripple factor of load current is

$$RF = \sqrt{FF^2 - 1} = \frac{1}{(1 + \cos\alpha)}\left[\pi\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right) - (1 + \cos\alpha)\right]^{\frac{1}{2}}$$

The volt ampere rating of transformer is

$$VA = VI_{rms} = \frac{V^2}{\sqrt{2}R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

Transformer utilisation factor is

$$TUF = \frac{\text{Power delivered to load}}{\text{Input } VA} = \frac{P_{dc}}{VA}$$

$$= \frac{V_{dc}I_{dc}}{VI_{rms}} = \frac{V_o I_o}{VI_{rms}} = \frac{(1 + \cos\alpha)^2}{\pi\left[2\pi\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}}$$

Example 6.1 A single-phase half-wave controlled rectifier with R load is shown in Fig. 6.5 and it is fed from a 220 V, 50 Hz ac supply. When $R = 10\ \Omega$ and $\alpha = 45°$, determine (a) average dc output voltage, (b) rms output voltage, (c) form factor, (d) ripple factor, (e) rectification efficiency, (f) TUF and (g) peak inverse voltage of thyristor.

Solution

Given: $V = 220$ V, $R = 10\ \Omega$ and $\alpha = 45° = 0.785$ rad

(a) The average output voltage is equal to

$$V_O = \frac{\sqrt{2}V}{2\pi}(1 + \cos\alpha) = \frac{\sqrt{2} \times 220}{2\pi}(1 + \cos 45) = 84.56 \text{ V}$$

(b) The rms value of output voltage is

$$V_{rms} = \frac{V}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} = \frac{220}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - 0.785 + \frac{1}{2}\sin(2 \times 45)\right)\right]^{\frac{1}{2}} = 148.35 \text{ V}$$

(c) The form factor is equal to

$$FF = \frac{V_{\text{rms}}}{V_{\text{av}}} = \frac{148.35}{84.56} = 1.754$$

(d) The ripple factor of load current is

$$RF = \sqrt{FF^2 - 1} = \sqrt{(1.754)^2 - 1} = 1.441$$

(e) The dc output current is $I_o = \frac{V_o}{R} = \frac{84.56}{10} = 8.456$ A

The dc output power is equal to

$$P_{\text{dc}} = V_{\text{dc}} \times I_{\text{dc}} = V_o \times I_o = 84.56 \times 8.456 \text{ Watt} = 715.039 \text{ Watt}$$

The rms output current is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{148.35}{10} = 14.835$ A

The ac output power is equal to

$$P_{\text{ac}} = V_{\text{rms}} \times I_{\text{rms}} = 148.35 \times 14.835 = 2200.77 \text{ Watt}$$

Rectification efficiency = $\frac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\% = \frac{715.039}{2200.77} \times 100\% = 32.49\%$

(f) The volt ampere rating of transformer is

$$VA = VI_{\text{rms}} = 220 \times 14.835 \text{ VA} = 3263.7 \text{ VA}$$

Transformer utilisation factor is

$$TUF = \frac{\text{Power delivered to load}}{\text{Input VA}} = \frac{P_{\text{dc}}}{VA} = \frac{715.039}{220 \times 14.835} = 0.219$$

(g) Peak inverse voltage of thyristor is

$$\text{PIV} = \sqrt{2}V = \sqrt{2} \times 220 \text{ V} = 311.08 \text{ V}$$

Example 6.2 A single phase 220 V, 1 kW heater is connected a half-wave controlled rectifier and it fed from a 220 V, 50 Hz ac supply. Determine the power absorbed by the heater when the firing angle is (a) $\alpha = 30°$ and (b) $\alpha = 90°$.

Solution

Given: $V = 220$ V, $\alpha = 30° = 0.523$ rad and $\alpha = 90° = 1.57$ rad

The resistance of heater is $R = \frac{V^2}{W} = \frac{220^2}{1000} = 48.4\ \Omega$ as $W = 1$ kW = 1000 Watt

(a) The rms value of output voltage at $\alpha = 30°$ is

$$V_{\text{rms}} = \frac{V}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} = \frac{220}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - 0.523 + \frac{1}{2}\sin(2 \times 30)\right)\right]^{\frac{1}{2}} = 153.34 \text{ V}$$

The power absorbed by the heater at firing angle $\alpha = 30°$ is

$$P_{at\cdot\alpha=30°} = \frac{V_{\text{rms}}^2}{R} = \frac{(153.34)^2}{48.4} \text{Watt} = 485.809 \text{ Watt}$$

(b) The rms value of output voltage at $\alpha = 90°$ is

$$V_{\text{rms}} = \frac{V}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} = \frac{220}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - 1.57 + \frac{1}{2}\sin(2 \times 90)\right)\right]^{\frac{1}{2}} = 110.01 \text{ V}$$

The power absorbed by the heater at firing angle $\alpha = 90°$ is

$$P_{at \cdot \alpha = 90°} = \frac{V_{\text{rms}}^2}{R} = \frac{(110.01)^2}{48.4}\text{Watt} = 250.04 \text{ Watt}$$

Example 6.3 A single-phase half-wave controlled rectifier with R load is supplied from a 230 V, 50 Hz ac source. When average dc output voltage is 50% of maximum possible average dc output voltage, determine (a) firing angle of thyristor, (b) average dc output voltage, (c) rms output voltage, (d) average and rms output current and (e) average and rms current of thyristor. Assume $R = 20\ \Omega$

Solution

Given: $V = 230$ V, $R = 20\ \Omega$

(a) When $\alpha = 0°$, the maximum possible average output voltage is available across load.

$$V_O = \frac{\sqrt{2}V}{2\pi}(1+\cos\alpha) = \frac{\sqrt{2}V}{2\pi}(1+\cos 0) = \frac{\sqrt{2}V}{\pi}$$

Therefore $V_{O\max} = \dfrac{\sqrt{2}V}{\pi}$

At firing angle α,

$$V_{O\,\text{at}\cdot\alpha} = \frac{\sqrt{2}V}{2\pi}(1+\cos\alpha) \text{ = 50\% of maximum possible average dc output voltage}$$

$$= 0.5V_{O\max} = 0.5 \times \frac{\sqrt{2}V}{\pi}$$

or $(1 + \cos\alpha) = 1$

Therefore, firing angle of thyristor is $\alpha = 90°$

(b) Average dc output voltage is $V_O = \dfrac{\sqrt{2}V}{2\pi}(1+\cos\alpha) = \dfrac{\sqrt{2}\times 230}{2\pi}(1+\cos 90) = 51.78 \text{ V}$

(c) the rms value of output voltage is

$$V_{\text{rms}} = \frac{V}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} = \frac{230}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - 1.57 + \frac{1}{2}\sin(2\times 90)\right)\right]^{\frac{1}{2}} = 115.01 \text{ V}$$

as $\alpha = 90° = 1.57$ rad

(d) The dc output current is $I_o = \dfrac{V_o}{R} = \dfrac{51.78}{20} = 2.589$ A

The rms output current is $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{115.01}{20} = 5.755$ A

(e) Since the thyristor current waveform is same as output current waveform,
average current of thyristor is $I_o = 2.589$ A
rms current of thyristor is $I_{\text{rms}} = 5.755$ A

6.4 SINGLE-PHASE HALF-CONTROLLED CONVERTERS WITH *RL* LOAD

A single-phase half-wave rectifier with *RL* load is depicted in Fig. 6.9. During the positive half-cycle, thyristor T_1 is forward biased and a gate pulse is applied at $\omega t = \alpha$. Thyristor T_1 becomes turn ON and input voltage is applied across the load. Due to inductive load, the output current increases gradually from zero to maximum value and then it starts decreasing. At $\omega t = \pi$, the polarity of input voltage has been changed from positive to negative and the load current flows through thyristor T_1 due to inductive load.

After $\omega t = \pi$, thyristor T_1 is subjected to reverse anode voltage but it will not be turned OFF as load current i_o is greater than the holding current. Therefore, thyristor T_1 remains in conduction. At $\omega t = \beta$, load current i_o becomes zero and thyristor T_1 is turned OFF as it was already reverse biased. After $\omega t = \beta$, the output voltage across load is zero and current $i_o = 0$. As the thyristor T_1 starts conduction at $\omega t = \alpha$ and it is turned OFF $\omega t = \beta$, the conduction period of SCR is $\beta - \alpha$. β is known as *extinction angle* and $\beta - \alpha$ is called the *conduction angle*.

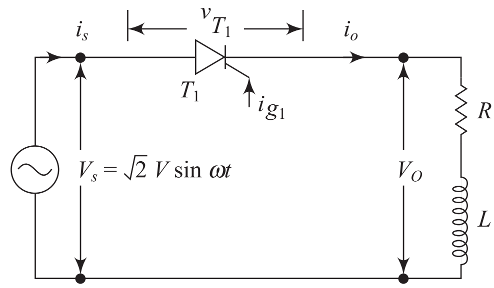


Fig. 6.9 *Single-phase half controlled converter with RL load*

From the above discussion, the output voltage across the load can be expressed as

$$
\begin{aligned}
v_O &= V_m \sin \omega t \quad \text{for} \quad \alpha \le \omega t \le \beta \\
&= 0 \qquad\qquad\quad \text{for} \quad 0 \le \omega t < \alpha \quad \text{and} \quad \beta \le \omega t \le 2\pi
\end{aligned}
$$

Figure 6.10 shows the waveforms of input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for half-wave controlled rectifier with *RL* load. During the time from $\omega t = \alpha$ to $\omega t = \pi$, the input voltage and output current are positive. Hence, power flows from the supply

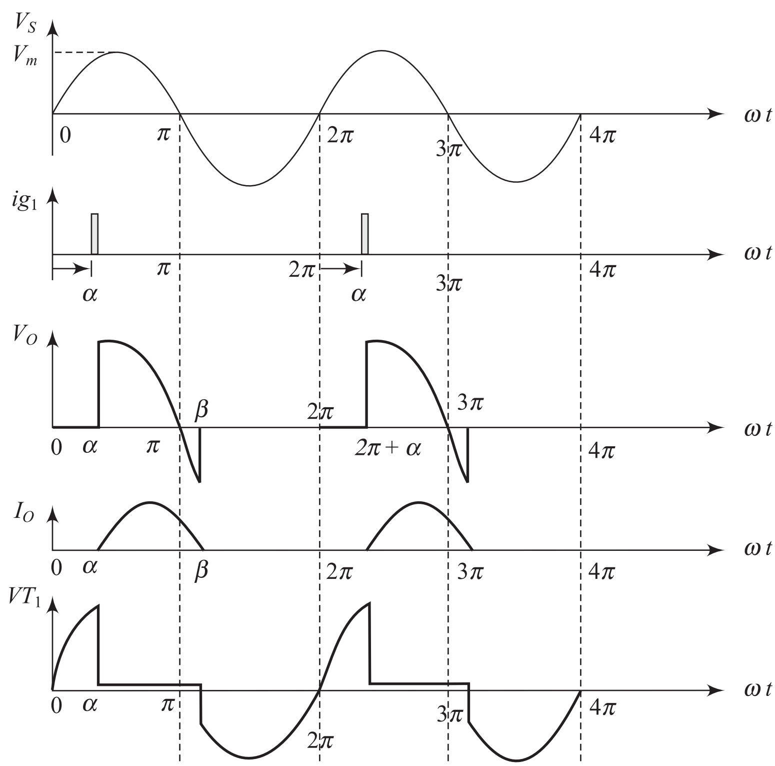


Fig. 6.10 *Input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for half wave controlled rectifier with R L load*

to load. This mode of operation is called *rectification mode* and single-phase half wave controlled rectifier with RL load operates in I quadrant. During the time from $\omega t = \pi$ to $\omega t = \beta$, the input voltage is negative but output current is positive and power flows in reverse direction from the load to supply. This mode of operation is known as *inversion mode* and single phase half wave controlled rectifier with RL load operates in IV quadrant. Depending upon the firing or delay angle α, the average or dc output voltage will be either positive or negative. Therefore, the single phase half wave controlled rectifier with RL load operates in two quadrants as depicted in Fig. 6.11 and it is called *full converter*.

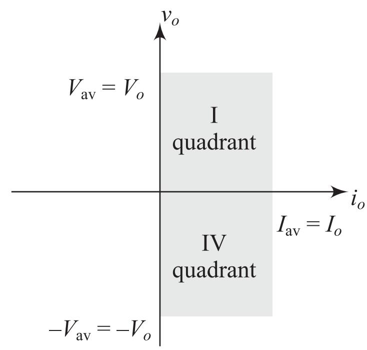


Fig. 6.11 *Quadrant operation of single half-wave controlled rectifier with RL load*

The voltage equation for the circuit as shown in Fig. 6.9 when thyristor T_1 is ON, is

$$\sqrt{2}V \sin \omega t = L\frac{di_o}{dt} + Ri_o \quad \text{at} \quad \alpha \le \omega t \le \beta$$

where, V is the rms input voltage and output current is i_o

The output current i_o can be expressed as

$$i_o(t) = i_s + i_t = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} \tag{6.1}$$

The current i_o has two components such as steady state component (i_s) and transient component (i_t). The steady state component current is $i_s(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi)$ and the transient component current is $i_t(t) = Ae^{-\frac{R}{L}t}$

At $\omega t = \alpha$, current $i_o = 0$, Eq. (6.1) can be written as

$$0 = \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi) + Ae^{-\frac{R}{L}\frac{\alpha}{\omega}}$$

or
$$A = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \phi) \times e^{\frac{R}{L}\frac{\alpha}{\omega}}$$

After substituting the value of A in Eq. (6.1), we get

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{R\alpha}{L\omega}}e^{-\frac{R}{L}t}\right]$$

or
$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-\frac{R}{\omega L}(\omega t - \alpha)}\right]$$

or
$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-\frac{\omega t - \alpha}{\tan\phi}}\right] \quad \text{for } \alpha \le \omega t \le \beta$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$, and $\tan\phi = \frac{\omega L}{R}$

It is clear from output current waveform that $i_o = 0$ at $\omega t = \beta$.

Therefore, $0 = \frac{\sqrt{2}V}{Z}\left[\sin(\beta - \phi) - \sin(\alpha - \phi)e^{-\frac{\beta - \alpha}{\tan\phi}}\right]$

or
$$\sin(\beta - \phi) = \sin(\alpha - \phi)e^{-\frac{\beta - \alpha}{\tan\phi}} \tag{6.2}$$

Equation (6.2) can be solved to find out the value of β.

Since the average output voltage across the inductor (L) is zero, the average output voltage across RL is equal to the average voltage across R. The average output voltage can be determined from

$$V_{av} = V_O = \frac{1}{2\pi}\int_{\alpha}^{\beta} \sqrt{2}V \sin \omega t d(\omega t) = \frac{\sqrt{2}V}{2\pi}(\cos \alpha - \cos \beta)$$

When $\beta = \pi$, the average output voltage is maximum. With increasing the value of β, the average output voltage decreases.

The rms value of output voltage is

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\beta} (\sqrt{2}V \sin \omega t)^2 d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{2\pi}\int_{\alpha}^{\beta} 2V^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V^2}{2\pi}\int_{\alpha}^{\beta} (1 - \cos 2\omega t) \cdot d\omega t\right]^{\frac{1}{2}} = \frac{V}{\sqrt{2}\sqrt{\pi}}\left[(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{\frac{1}{2}}$$

The form factor is

$$FF = \frac{V_{rms}}{V_{av}}$$

$$= \frac{V}{\sqrt{2}\sqrt{\pi}}\left[(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{\frac{1}{2}} \Bigg/ \frac{\sqrt{2}V}{2\pi}(\cos \alpha - \cos \beta)$$

$$= \frac{\sqrt{\pi}\left[(\beta - \alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right]^{\frac{1}{2}}}{\cos \alpha - \cos \beta}$$

Example 6.4 A single-phase half-wave controlled rectifier is connected across a RL load as shown in Fig. 6.9 and feeds from a 230 V, 50 Hz ac supply. When $R = 10\ \Omega$ and $L = 0.05$ H, determine (a) the firing angle to ensure no transient current and (b) the firing angle for the maximum transient.

Solution

Given: $V = 230$ V, $f = 50$ Hz, $R = 10\ \Omega$ and $L = 0.05$ H

The current flows through load is

$$i(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-\frac{\omega t - \alpha}{\tan \phi}}\right] \quad \text{for } \alpha \le \omega t \le \beta$$

Where, $Z = \sqrt{R^2 + (\omega L)^2}$, and $\tan \phi = \dfrac{\omega L}{R}$

The transient current is $i_t(t) = -\dfrac{\sqrt{2}V}{Z}\left[\sin(\alpha - \phi)e^{-\frac{\omega t - \alpha}{\tan \phi}}\right]$

(a) When there is no transient current

$$i_t(t) = 0 = -\frac{\sqrt{2}V}{Z}\left[\sin(\alpha - \phi)e^{-\frac{\omega t - \alpha}{\tan \phi}}\right]$$

or $\qquad \sin(\alpha - \phi) = 0$

Then the firing angle to ensure no transient current is

$$\alpha = \phi = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{2\pi fL}{R} = \tan^{-1}\frac{2\pi \times 50 \times 0.05}{10} = 57.50°$$

(b) For maximum transient current,

$$\sin(\alpha - \phi) = 1 = \sin 90°$$

or $$\alpha = 90° + \phi = 90° + 57.50° = 147.50°$$

Therefore, the maximum transient occurs at firing angle α = 147.50°

6.5 SINGLE-PHASE HALF-CONTROLLED CONVERTERS WITH *RL* LOAD AND FREE WHEELING DIODE

In single-phase half-wave controlled converter with *RL* load, a negative voltage will be output during $\omega t = \pi$ to $\omega t = \beta$. Due to negative voltage, the average output voltage reduces with increasing β and the performances of converter are reduced. To improve converter performance, a freewheeling diode can be connected across *RL* load. Figure 6.12 shows the circuit diagram of single phase half-wave controlled converter with *RL* load and freewheeling diode D_F. As free wheeling diode is connected across load, the converter operates as half-controlled converter. This converter operates in continuous mode and discontinuous mode.

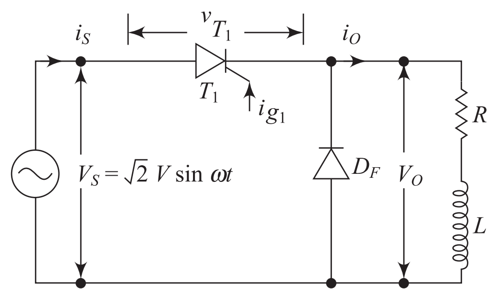


Fig. 6.12 *Single-phase half-controlled converter with RL load and freewheeling diode D_F*

6.5.1 Discontinuous Mode Operation

During the positive half-cycle of input voltage, thyristor T_1 is forward biased and triggering pulse is applied at $\omega t = \alpha$. Just after application of triggering pulse, thyristor T_1 becomes ON and input voltage is applied across load and the current starts to flow through load. Then thyristor T_1 conducts from $\omega t = \alpha$ to $\omega t = \pi$. After $\omega t = \pi$, the polarity of input is reversed, but the load current still flows due to inductance (L). This current decreases with time. At $\omega t = \pi$, diode D_F is forward biased and starts conduction. Then the decaying inductive current flows though load and diode D_F up to β for discontinuous mode operation. As the thyristor T_1 is reverse biased, it turns OFF at $\omega t = \pi$. Figure 6.13 shows the waveforms of input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for half-wave controlled rectifier with *RL* load and freewheeling diode during *discontinuous mode operation*.

The dc or average output voltage will be

$$V_{\text{av}} = V_O = \frac{1}{2\pi}\int_{\alpha}^{\pi}\sqrt{2}V \sin \omega t d(\omega t) = \frac{\sqrt{2}V}{2\pi}(1 + \cos \alpha)$$

The dc or average output current will be

$$I_{\text{av}} = \frac{V_O}{R} = \frac{\sqrt{2}V}{2\pi R}(1 + \cos \alpha)$$

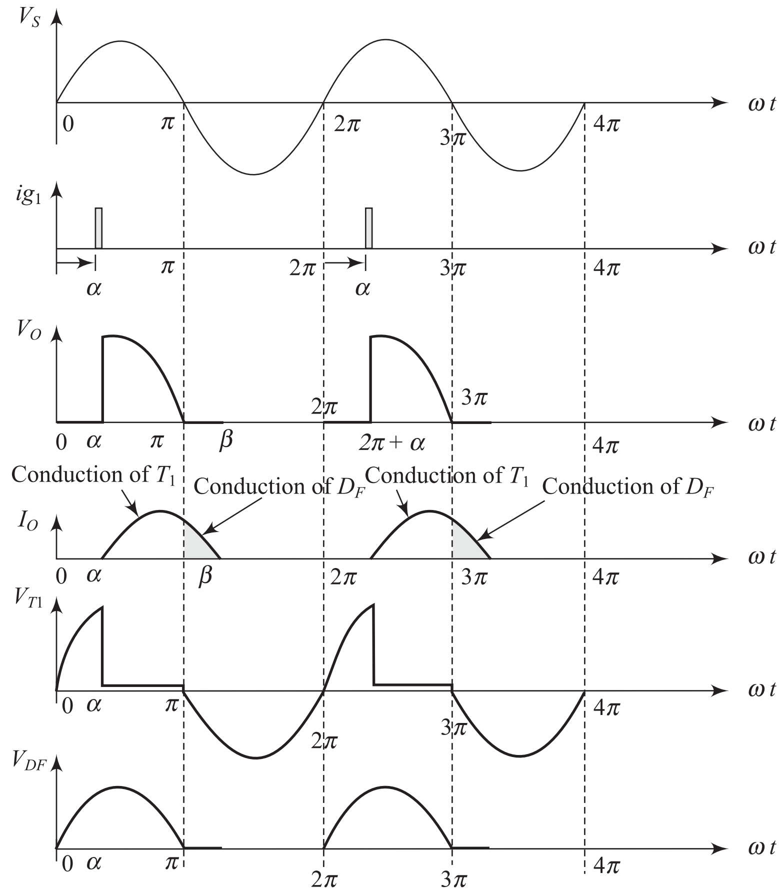


Fig. 6.13 *Input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for single phase half wave controlled rectifier with R-L load and freewheeling diode (discontinuous mode)*

6.5.2 Continuous Mode Operation

The continuous mode operation of single-phase half-wave controlled rectifier with *RL* load and freewheeling diode has two subparts such as (i) *MODE I: Conduction of T_1* (ii) *MODE II : Conduction of D_F*. Figure 6.14 shows input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for half-wave controlled rectifier with *RL* load and freewheeling diode.

MODE I: Conduction of T_1 For continuous mode operation, the voltage equation for the circuit as shown in Fig. 6.12, is

$$\sqrt{2}V \sin \omega t = L\frac{di_o}{dt} + Ri_o \quad \text{for } \alpha \le \omega t \le \pi$$

where, V is the rms input voltage and output current is i_o

The output current i_o can be expressed as

$$i_o(t) = i_s + i_t = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} \tag{6.3}$$

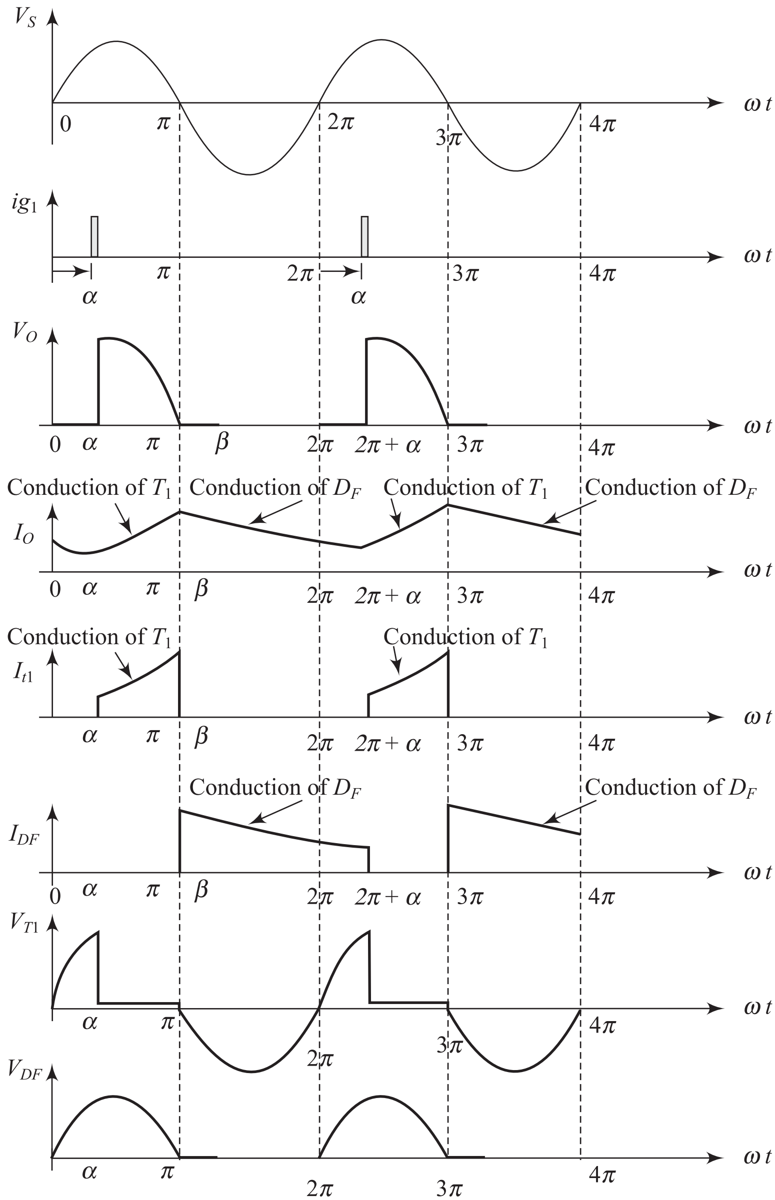


Fig. 6.14 *Input voltage, triggering pulse of T_1, output voltage, output current and voltage across T_1 for single phase half wave controlled rectifier with R-L load and freewheeling diode (continuous mode)*

The current i_o has two components such as steady state component (i_s) and transient component (i_t). The steady state component current is $i_s(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi)$ and the transient component current is $i_t(t) = Ae^{-\frac{R}{L}t}$

At $\omega t = \alpha$, current $i_o = I_o$, Eq. (6.3) can be written as

$$I_o = \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi) + Ae^{-\frac{R}{L}\frac{\alpha}{\omega}}$$

or $$A = \left\{I_o - \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)\right\} \times e^{\frac{R}{L}\frac{\alpha}{\omega}}$$

After substituting the value of A in Eq. (6.3), we get

$$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + \left[I_o - \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)\right] \times e^{\frac{R\alpha}{L\omega}} \times e^{-\frac{R}{L}t}$$

or $$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + \left[I_o - \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)\right] \times e^{-\frac{R}{L\omega}(\omega t - \alpha)} \quad \text{for } \alpha \le \omega t \le \pi$$

MODE II : Conduction of D_F The voltage equation during conduction of D_F, is

$$0 = L\frac{di_o}{dt} + Ri_o \quad \text{for } \pi \le \omega t \le 2\pi + \alpha$$

The solution of the above equation is

$$i_o(t) = Ae^{-\frac{R}{L}t} \tag{6.4}$$

At $\omega t = \pi$, $t = \dfrac{\pi}{\omega}$ and current $i_o = I_{o1}$, Eq. (6.4) can be written as

$$I_{o1} = Ae^{-\frac{R\pi}{L\omega}}$$

After substituting the value of A in Eq. (6.4), we obtain

$$i_o(t) = Ae^{-\frac{R}{L}t}$$

or $$i_o(t) = I_{o1}e^{\frac{R\pi}{L\omega}}e^{-\frac{R}{L}t} = I_{o1}e^{-\frac{R}{L}(\omega t - \pi)} \quad \text{for } \pi \le \omega t \le 2\pi + \alpha$$

The advantages of free wheeling diode in single-phase half-wave controlled rectifier with RL load are given below:

1. Output voltage is increased
2. Input power factor can be improved
3. Load current wave form is improved
4. Performance of controlled rectifier is better
5. Since the stored energy in inductance L is transferred to R when the free-wheeling diode conducts, the converter efficiency is improved.

6.6 SINGLE-PHASE HALF-WAVE CONTROLLED CONVERTERS WITH *RE* LOAD

Figure 6.15 shows a single-phase half-wave controlled rectifier with RE load. Actually the output voltage of a half-wave rectifier circuit is connected to a battery (E) in series with a resistance R. This circuit can be used as a battery charger.

When input voltage V_s is greater than battery voltage (E), thyristor is forward biased. As soon as a gate pulse is applied between gate and cathode, thyristor T_1 will be turned ON. When the input voltage V_s is less than battery voltage, thyristor T_1 is reverse biased and it becomes turned OFF. The input voltages, gate pulse, output voltage, voltage across thyristor and load current waveforms are illustrated in Fig. 6.16.

It is clear from Fig. 6.16 that at $\omega t = \theta_1$ thyristor T_1 is forward bias and starts conduction as gate pulse is applied at $\omega t = \theta_1$. The angle $\alpha = \theta_1$ can be computed from the equation

$$\sqrt{2}V \sin\theta_1 = E \quad \text{at } \omega t = \theta_1$$

or
$$\theta_1 = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right)$$

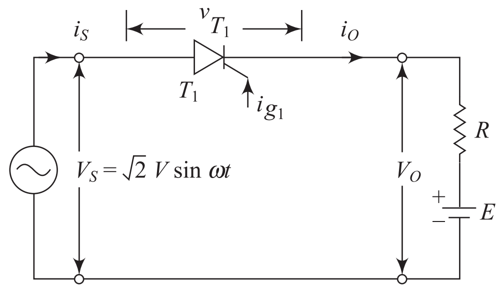


Fig. 6.15 *Single-phase half-wave controlled rectifier with RE load*

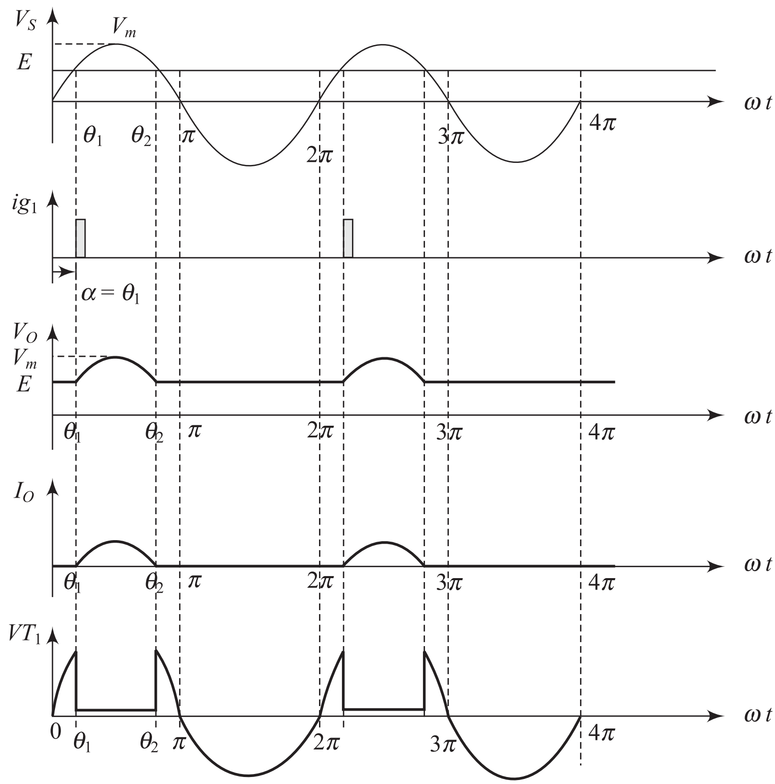


Fig. 6.16 *Input voltages, gate pulse, output voltage, voltage across thyristor and load current waveforms $\omega t = \alpha = \theta_1$*

At $\omega t = \theta_2$, the input voltage V_s is less than battery voltage (E), thyristor T_1 is reverse biased at $\theta_2 = \pi - \theta_1$ and it is turned OFF. Therefore the charging current flows though load during $\theta_1 \le \omega t \le \theta_2$ and it can be expressed as

$$i_o = \frac{v_s - E}{R} = \frac{V_m \sin\omega t - E}{R} = \frac{\sqrt{2}V \sin\omega t - E}{R}$$

The average charging current is

$$I_{\text{av}} = \frac{1}{2\pi}\int_{\theta_1}^{\theta_2}\frac{V_m \sin\omega t - E}{R}d(\omega t)$$

$$= \frac{1}{2\pi R}(2V_m \cos\theta_1 - 2E(\pi - 2\theta_1)) \quad \text{as } \theta_2 = \pi - \theta_1$$

When the charging current (I_{av}) is known, the resistance value can determined from

$$R = \frac{1}{2\pi I_{av}}[2V_m \cos\theta_1 - 2E(\pi - 2\theta_1)]$$

The rms current flows through battery is I_{rms} and it is given by

$$I_{\text{rms}}^2 = \frac{1}{2\pi}\int_{\theta_1}^{\theta_2}\frac{(V_m \sin\omega t - E)^2}{R^2}d(\omega t)$$

$$= \frac{1}{2\pi R^2}\int_{\theta_1}^{\theta_2}(V_m^2 \sin^2\omega t + E^2 - 2V_m E \sin\omega t)d(\omega t)$$

$$= \frac{1}{2\pi R^2}[(V_m^2 + E^2)(\pi - 2\theta_1) + V_m^2 \sin^2\theta_1 - 4V_m E \cos\theta_1]$$

During charging power loss across resistance is $I_{\text{rms}}^2 R$

Power delivered to battery is $P_{\text{dc}} = EI_{\text{dc}} = EI_{\text{av}}$ as $I_{\text{dc}} = I_{\text{av}}$

The rectifier efficiency is

$$\eta = \frac{\text{Power delivered to the battery}}{\text{Power input to the rectifier}} = \frac{P_{\text{dc}}}{P_{\text{dc}} + I_{\text{rms}}^2 R}$$

When the triggering pulse is applied at $\omega t = \alpha > \theta_1$, thyristor T_1 will conducts from $\omega t = \alpha$ to $\omega t = \theta_2$. Then input voltages, gate pulse, output voltage, voltage across thyristor and load current waveforms at $\omega t = \alpha > \theta_1$ are depicted in Fig. 6.17. Then the average charging current can be determined from

$$I_{\text{av}} = \frac{1}{2\pi}\int_{\alpha}^{\theta_2}\frac{V_m \sin\omega t - E}{R}d(\omega t) = \frac{1}{2\pi R}[V_m(\cos\alpha - \cos\theta_2) - E(\theta_2 - \alpha)]$$

The rms current flows through battery is I_{rms} and it can be determined from

$$I_{\text{rms}}^2 = \frac{1}{2\pi}\int_{\alpha}^{\theta_2}\frac{(V_m \sin\omega t - E)^2}{R^2}d(\omega t)$$

$$= \frac{1}{2\pi R^2}\int_{\alpha}^{\theta_2}(V_m^2 \sin^2\omega t + E^2 - 2V_m E \sin\omega t)d(\omega t)$$

$$= \frac{1}{2\pi R^2}\left[(V^2 + E^2)(\theta_2 - \alpha) - \frac{1}{2}V^2(\sin 2\theta_2 - 2\sin 2\alpha) - 2V_m E(\cos\alpha - \cos\theta_2)\right]$$

Therefore,

$$I_{\text{rms}} = \frac{1}{\sqrt{2\pi}R}\left[(V^2 + E^2)(\theta_2 - \alpha) - \frac{1}{2}V^2(\sin 2\theta_2 - 2\sin 2\alpha) - 2V_m E(\cos\alpha - \cos\theta_2)\right]^{\frac{1}{2}}$$

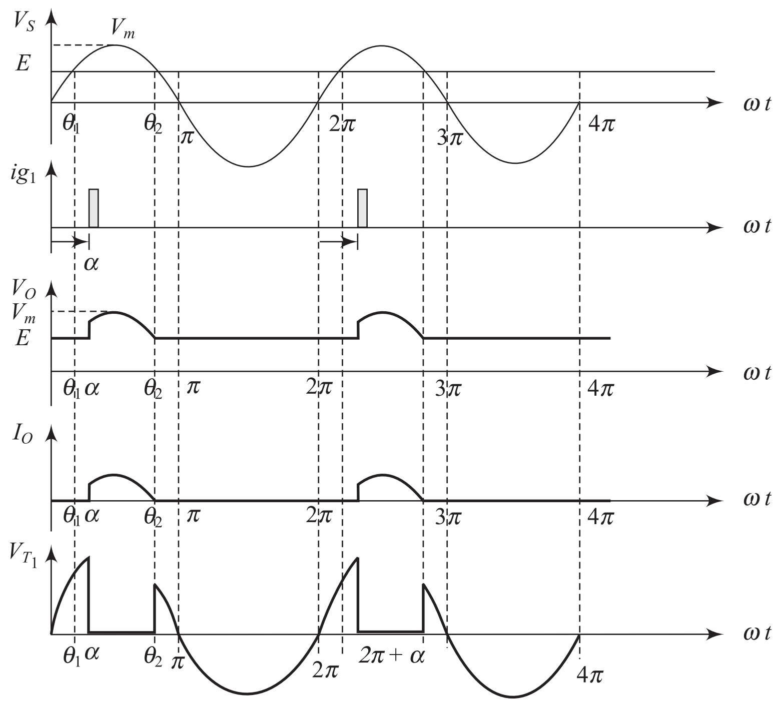


Fig. 6.17 *Input voltages, gate pulse, output voltage, voltage across thyristor and load current waveforms at $\omega t = \alpha > \theta_1$*

Power loss across resistance R is $I_{\text{rms}}^2 R$

Power delivered to battery is $P_{\text{dc}} = EI_{\text{dc}} = EI_{\text{av}}$ as $I_{\text{dc}} = I_{\text{av}}$

Input power factor is $= \dfrac{P_{\text{dc}} + I_{\text{rms}}^2 R}{VI_{\text{rms}}}$

Example 6.5 A 100 V battery is charged through a resistor R as depicted in Fig. 6.15. When the charger circuit is fed from 220 V, 50 Hz ac supply and R is 5 Ω, compute (a) the minimum angle at which thyristor will be turned ON, (b) the angle at which thyristor will be turned OFF, (c) maximum conduction period of thyristor, (d) average charging current when $\alpha = 45°$, (e) Power supplied to battery, (f) Power dissipated in resistor R and (g) input power factor.

Solution

Given: $V = 220$ V, $f = 50$ Hz, $E = 100$ V

(a) The minimum angle at which thyristor will be turned ON is $\alpha = \theta_1$ and it can be computed from the equation

$$\theta_1 = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right) = \sin^{-1}\left(\frac{100}{\sqrt{2} \times 220}\right) = 20.70° = 0.3611 \text{ rad}$$

(b) The angle at which thyristor will be turned OFF is

$$\theta_2 = \pi - \theta_1 = 180° - 20.70° = 159.29° = 2.778 \text{ rad.}$$

(c) Maximum conduction period of thyristor is $\theta_2 - \theta_1 = 159.20° - 20.70° = 138.59°$

(d) The average charging current at firing angle α is

$$I_{av} = \frac{1}{2\pi}\int_{\alpha}^{\theta_2} \frac{V_m \sin \omega t - E}{R} d(\omega t) \quad \text{where,} \quad V_m = \sqrt{2}V = \sqrt{2} \times 220 = 310.2 \text{ V}$$

$$= \frac{1}{2\pi R}[V_m(\cos\alpha - \cos\theta_2) - E(\theta_2 - \alpha)] \quad \text{where,} \quad \alpha = 45° = 0.785 \text{ rad}$$

$$= \frac{1}{2\pi \times 5}[\sqrt{2} \times 220(\cos 45 - \cos 159.29) - 100(2.778 - 0.785)] \text{ A} = 9.923 \text{ A}$$

(e) Power supplied to battery is $P_{dc} = EI_{dc} = EI_{av} = 100 \times 9.923$ Watt = 992.3 Watt

(f) The rms current flows through battery is I_{rms} and it is given by

$$I_{rms} = \frac{1}{\sqrt{2\pi}R}\left[(V^2 + E^2)(\theta_2 - \alpha) - \frac{1}{2}V^2(\sin 2\theta_2 - 2\sin 2\alpha) - 2V_m E(\cos\alpha - \cos\theta_2)\right]^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2\pi} \times 5}\left[(220^2 + 100^2)(2.778 - 0.785) - \frac{1}{2}220^2(\sin(2 \times 159.29) - 2\sin(2 \times 45)) \right.$$

$$\left. - 2 \times 310.2 \times 100(\cos 45 - \cos 159.29)\right]^{\frac{1}{2}}$$

= 22.419 A

(g) Power dissipated in resistor $I_{rms}^2 R = 22.419^2 \times 5$ = 2513.05 Watt

(e) Input power factor is $= \dfrac{P_{dc} + I_{rms}^2 R}{VI_{rms}} = \dfrac{992.3 + 2513.05}{220 \times 22.419} = 0.7107$ lagging

Example 6.6 A 100 V battery is charged through a resistor R as depicted in Fig. 6.15. When the charger circuit is fed from 220 V, 50 Hz ac supply and R is 5 Ω. If thyristor is triggered at $\alpha = 35°$ in every positive cycle., compute (a) average charging current, (b) power supplied to battery, (c) rms value of load current, (d) Power dissipated in resistor R and (e) input power factor.

Solution

Given: $V = 220$ V, $f = 50$ Hz, $E = 100$ V, $\alpha = 35°$

As per calculation in example 6.5, $\theta_1 = 20.70°$

Therefore, $\theta_2 = 180° - 20.70° = 159.3°$

(a) The average charging current at firing angle α is

$$I_{av} = \frac{1}{2\pi}\int_{\alpha}^{\theta_2} \frac{V_m \sin \omega t - E}{R} d(\omega t) \quad \text{where,} \quad V_m = \sqrt{2}V = \sqrt{2} \times 220 = 310.2 \text{ V}$$

$$= \frac{1}{2\pi R}[V_m(\cos\alpha - \cos\theta_2) - E(\theta_2 - \alpha)] \quad \text{where,} \quad \alpha = 35° = 0.61055 \text{ rad}$$

$$= \frac{1}{2\pi \times 5}[\sqrt{2} \times 220(\cos 35 - \cos 159.29) - 100(2.778 - 0.61055)] \text{ A} = 10.428 \text{ A}$$

(b) Power supplied to battery is $P_{dc} = EI_{dc} = EI_{av} = 100 \times 10.428$ Watt = 1042.8 Watt

(c) The rms current flows through battery is I_{rms} and it is given by

$$I_{rms} = \frac{1}{\sqrt{2\pi}R}\left[(V^2 + E^2)(\theta_2 - \alpha) - \frac{1}{2}V^2(\sin 2\theta_2 - 2\sin 2\alpha) - 2V_m E(\cos\alpha - \cos\theta_2)\right]^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2\pi} \times 5}\left[(220^2 + 100^2)(2.778 - 0.61055) - \frac{1}{2}220^2(\sin(2 \times 159.29) - 2\sin(2 \times 35)) \right.$$

$$\left. - 2 \times 310.2 \times 100(\cos 35 - \cos 159.29)\right]^{\frac{1}{2}}$$

= 22.4628 A

(d) Power dissipated in resistor $I_{\text{rms}}^2 R = 22.4628^2 \times 5 = 2522.88$ Watt

(e) Input power factor is $= \dfrac{P_{\text{dc}} + I_{\text{rms}}^2 R}{VI_{\text{rms}}} = \dfrac{1042.8 + 2522.88}{220 \times 22.4628} = 0.7215$ lagging

Example 6.7 The voltage across secondary winding of a single phase transformer is 200 V, 50 Hz and the transformer deliver power to resistive load of $R = 2\ \Omega$ through a single-phase half-wave controlled rectifier. If the firing angle of thyristor is 90°, calculate (a) average dc output voltage, (b) average dc output current, (c) rms output voltage, (d) rms output current, (e) form factor, (f) voltage ripple factor, (g) rectification efficiency, (h) transformer utilization factor and (i) PIV of thyristor.

Solution

Given: $V = 200$, $f = 50$ Hz, $R = 2\ \Omega$, $\alpha = 90°$

(a) The average output voltage is equal to

$$V_O = \frac{\sqrt{2}V}{2\pi}(1 + \cos\alpha) = \frac{\sqrt{2} \times 200}{2\pi}(1 + \cos 90) = 45.038 \text{ V}$$

(b) Average dc output current is $I_o = \dfrac{V_o}{R} = \dfrac{45.038}{2} \text{A} = 22.519 \text{ A}$

(c) The rms value of output voltage is

$$V_{\text{rms}} = \frac{V}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} = \frac{200}{\sqrt{2}}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{1}{2}\sin(2 \times 90)\right)\right]^{\frac{1}{2}} = 100 \text{ V}$$

(d) rms output current $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{100}{2} = 50 \text{ A}$

(e) The form factor is equal to

$$FF = \frac{V_{\text{rms}}}{V_{\text{av}}} = \frac{100}{45.038} = 2.22$$

(f) The voltage ripple factor of load current is

$$RF = \sqrt{FF^2 - 1} = \sqrt{(2.22)^2 - 1} = 1.982$$

(e) The dc output power is equal to

$$P_{\text{dc}} = V_{\text{dc}} \times I_{\text{dc}} = V_o \times I_o = 45.038 \times 22.519 \text{ Watt} = 1014.21 \text{ Watt}$$

The ac output power is equal to

$$P_{\text{ac}} = V_{\text{rms}} \times I_{\text{rms}} = 100 \times 50 = 5000 \text{ Watt}$$

Rectification efficiency $\dfrac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\% = \dfrac{1014.21}{5000} \times 100\% = 20.28\%$

(f) The volt ampere rating of transformer is

$$\text{VA} = VI_{\text{rms}} = 200 \times 50 \text{ VA}$$

Transformer utilization factor is

$$TUF = \frac{\text{Power delivered to load}}{\text{Input } VA} = \frac{P_{\text{dc}}}{VA} = \frac{1014.21}{200 \times 50} = 0.1014$$

(g) The peak inverse voltage of thyristor is

$$\text{PIV} = \sqrt{2}V = \sqrt{2} \times 200 \text{ V} = 282.84 \text{ V}$$

6.7 SINGLE-PHASE HALF-WAVE CONTROLLED CONVERTERS WITH *RLE* LOAD

A single-phase half-wave controlled rectifier with *RLE* load is depicted in Fig. 6.18. Actually the output voltage of a half-wave rectifier circuit is connected to dc motor as armature of dc motor is represented by resistance R, inductance L and back emf (E). This circuit operation is similar to Fig. 6.15.

If the input voltage V_s is greater than back emf or battery voltage (E), thyristor is forward biased. When a gate pulse is applied between gate and cathode, thyristor T_1 will be turned ON. The minimum value of firing angle is $\alpha = \theta_1$. The angle $\alpha = \theta_1$ can be computed from the equation

$$\sqrt{2}V \sin \theta_1 = E$$

or

$$\theta_1 = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right)$$

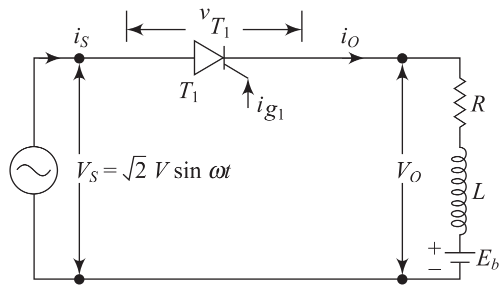


Fig. 6.18 *Single-phase half-wave controlled rectifier with RE load*

When the firing angle is $\alpha < \theta_1$, the voltage E is greater than input voltage and thyristor T_1 is reverse biased and it will not be turned ON. In the same way maximum firing angle of thyristor T_1 is $\theta_2 = \pi - \theta_1$. The input voltages, gate pulse, output voltage, voltage across thyristor and load current waveforms are shown in Fig. 6.19.

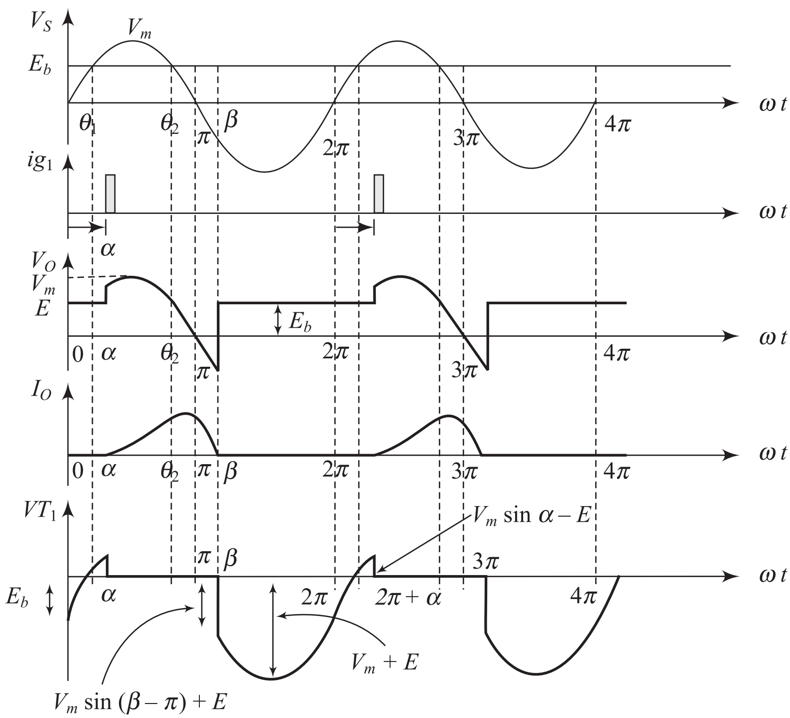


Fig. 6.19 *Input voltage, gate pulse, output voltage, voltage across thyristor and load current waveforms at $\omega t = \alpha > \theta_1$*

The voltage equation for the circuit as shown in Fig. 6.18 when thyristor T_1 is ON is

$$\sqrt{2}V \sin \omega t = L\frac{di_o}{dt} + Ri_o + E \quad \text{at } \alpha \le \omega t \le \beta$$

where, V is the rms input voltage and output current is i_o

The solution of the above equation consists of steady state component current and transient component current. The output current i_o can be expressed as

$$i_o(t) = i_s + i_t = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) - \frac{E}{R} + Ae^{-\frac{R}{L}t} \tag{6.5}$$

The steady state component current is $i_s(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) - \frac{E}{R}$ and the transient component current is $i_t(t) = Ae^{-\frac{R}{L}t}$

At $\omega t = \alpha$, $t = \frac{\alpha}{\omega}$ and current $i_o = 0$, Eq. (6.5) can be written as

$$0 = \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi) - \frac{E}{R} + Ae^{-\frac{R}{L}\frac{\alpha}{\omega}}$$

or

$$A = \left[\frac{E}{R} - \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)\right] \times e^{\frac{R}{L}\frac{\alpha}{\omega}}$$

After substituting the value of A in Eq. (6.5), we get

$$i(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{R\alpha}{L\omega}}e^{-\frac{R}{L}t}\right] - \frac{E}{R}\left[1 - e^{\frac{R\alpha}{L\omega}} \cdot e^{-\frac{R}{L}t}\right]$$

or

$$i(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-\frac{R}{\omega L}(\omega t - \alpha)}\right] - \frac{E}{R}\left[1 - e^{-\frac{R}{\omega L}(\omega t - \alpha)}\right]$$

or

$$i(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-\frac{\omega t - \alpha}{\tan\phi}}\right] - \frac{E}{R}\left[1 - e^{-\frac{\omega t - \alpha}{\tan\phi}}\right] \quad \text{for } \alpha \le \omega t \le \beta$$

Where, $Z = \sqrt{R^2 + (\omega L)^2}$, and $\tan\phi = \frac{\omega L}{R}$

The average charging current is

$$I_{\text{av}} = \frac{1}{2\pi}\int_{\alpha}^{\beta}\frac{V_m \sin \omega t - E}{R}d(\omega t)$$

$$= \frac{1}{2\pi R}[V_m(\cos\alpha - \cos\beta) - E(\beta - \alpha)]$$

The average output voltage across load is

$$V_{\text{av}} = V_O = I_o R + E = \frac{1}{2\pi}[V_m(\cos\alpha - \cos\beta) - E(\beta - \alpha)] + E$$

Example 6.8 A single-phase half-wave converter with *RLE* load is connected to 230 V, 50 Hz ac supply. When R is 5 Ω, L is 2.5 mH, E = 125 V and firing angle of SCR is $\alpha = 45°$, determine (a) the circuit turn off time if load current is zero at 210°, (b) average charging current when $\alpha = 45°$ and (c) average output voltage

Solution

Given: V = 230 V, f = 50 Hz, E = 125 V, R = 5 Ω, L = 2.5 mH and $\alpha = 45°$

The minimum angle at which thyristor will be turned ON is $\alpha = \theta_1$

$$\theta_1 = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right) = \sin^{-1}\left(\frac{125}{\sqrt{2} \times 230}\right) = 22.60°$$

The angle at which thyristor will be turned OFF is $\beta = 210°$.

(a) As the firing angle of SCR is $\alpha = 45°$,
SCR will conducts for $\beta - \alpha = 210° - 45° = 165° = 2.878$ rad

Then the circuit turn OFF time is $t_c = \dfrac{2\pi - (\beta - \alpha)}{\omega} = \dfrac{2\pi - 2.878}{2\pi \times 50} = 10.83 \text{ ms}$

(b) The average charging current when $\alpha = 45°$

$$I_{av} = \frac{1}{2\pi R}[V_m(\cos\alpha - \cos\beta) - E(\beta - \alpha)]$$

$$= \frac{1}{2\pi \times 5}[\sqrt{2} \times 230(\cos 45 - \cos 210) - 125 \times 2.878] = 4.82 \text{ A}$$

(c) The average output voltage across load is

$$V_{av} = V_O = I_o R + E = 4.82 \times 5 + 125 = 149.1 \text{ V}$$

6.8 SINGLE-PHASE FULL-WAVE-CONTROLLED RECTIFIERS USING CENTRE TAP TRANSFORMER

Figure 6.20 shows the single-phase full-wave controlled rectifier using centre tap transformer. Due to use of center tap transformer, the circuit configuration is same as the single-phase full-wave uncontrolled rectifier, but SCRs are used in place of diodes.

As SCR cathodes are commonly connected, single gate drive circuit is required to turn ON SCRs. The gate drives circuit can be operated from single dc power supply and no isolation circuit is required, but the SCRs have twice of the peak supply voltage $2\sqrt{2}V$.

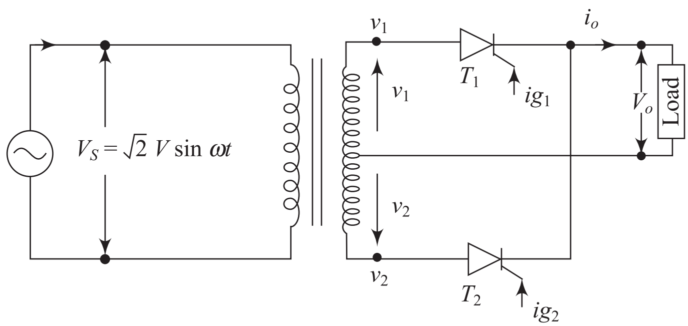
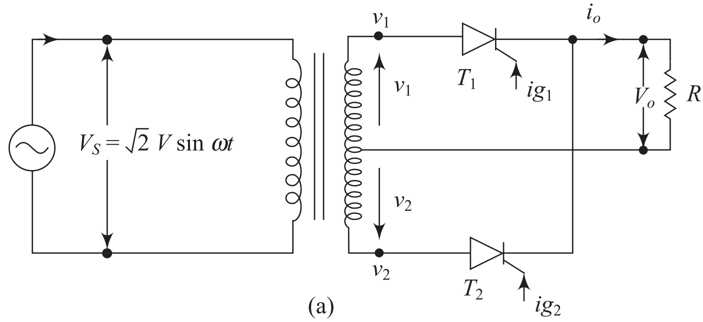


Fig. 6.20 *Single-phase full-wave controlled rectifier using centre tap transformer*

The input ac supply voltage is applied across the primary winding of the transformer. The centre tap of secondary winding is used as ground or zero reference voltage. The voltage between the centre tap of transformer and either end of the secondary winding is half of the secondary winding voltage.

6.9 SINGLE-PHASE FULL-WAVE CONTROLLED RECTIFIER USING CENTRE TAP TRANSFORMER WITH R LOAD

During the positive half cycle of supply voltage, the thyristor T_1 is forward biased and the thyristor T_2 is reverse bias. As soon as a gate pulse is applied to the thyristor T_1 at $\omega t = \alpha$, T_1 conducts as it is in ON state and the thyristor T_2 is in OFF state and current flows through load resistance R and T_1 as shown in Fig. 6.21.



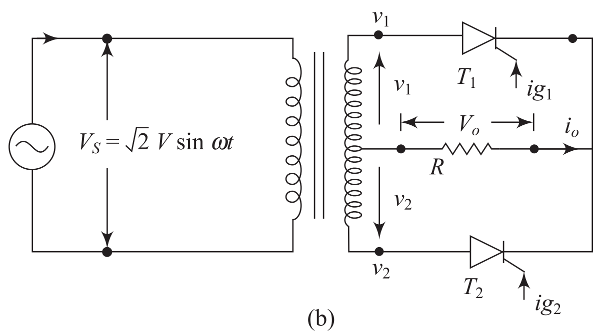


Fig. 6.21 *(a) and (b) Single-phase full-wave controlled rectifier using centre tap transformer with R load*

In the negative half cycle of supply voltage, the polarities of secondary winding have been reversed. The thyristor T_2 is forward biased and the thyristor T_1 is reverse biased. As trigger pulse is applied at $\omega t = \pi + \alpha$, the thyristor T_2 conducts as it is in ON state and the thyristor T_1 is in OFF state and current flows through load resistance R and thyristor T_2 as shown in Fig. 6.22.

Hence the current flows through load in the same direction for positive as well as negative half cycle of supply voltage. Consequently, the output voltage across the load resistance is full-wave rectified dc voltage. Thyristor T_1 conducts from $\omega t = \alpha$ to $\omega t = \pi$ and thyristor T_2 conducts from $\omega t = \pi + \alpha$ to $\omega t = 2\pi$. The input voltage, triggering pulses, output voltage, output current, voltage across thyristors are depicted in Fig. 6.22.

The average or dc output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot d\omega t$$

$$= \frac{\sqrt{2}V}{\pi}(1 + \cos \alpha)$$

If $\alpha = 0$, the average output voltage is

$$\frac{\sqrt{2}V}{\pi}(1 + \cos 0) = \frac{2\sqrt{2}V}{\pi}$$

If $\alpha = \pi$, the average output voltage is

$$\frac{\sqrt{2}V}{2\pi}(1 + \cos \pi) = 0$$

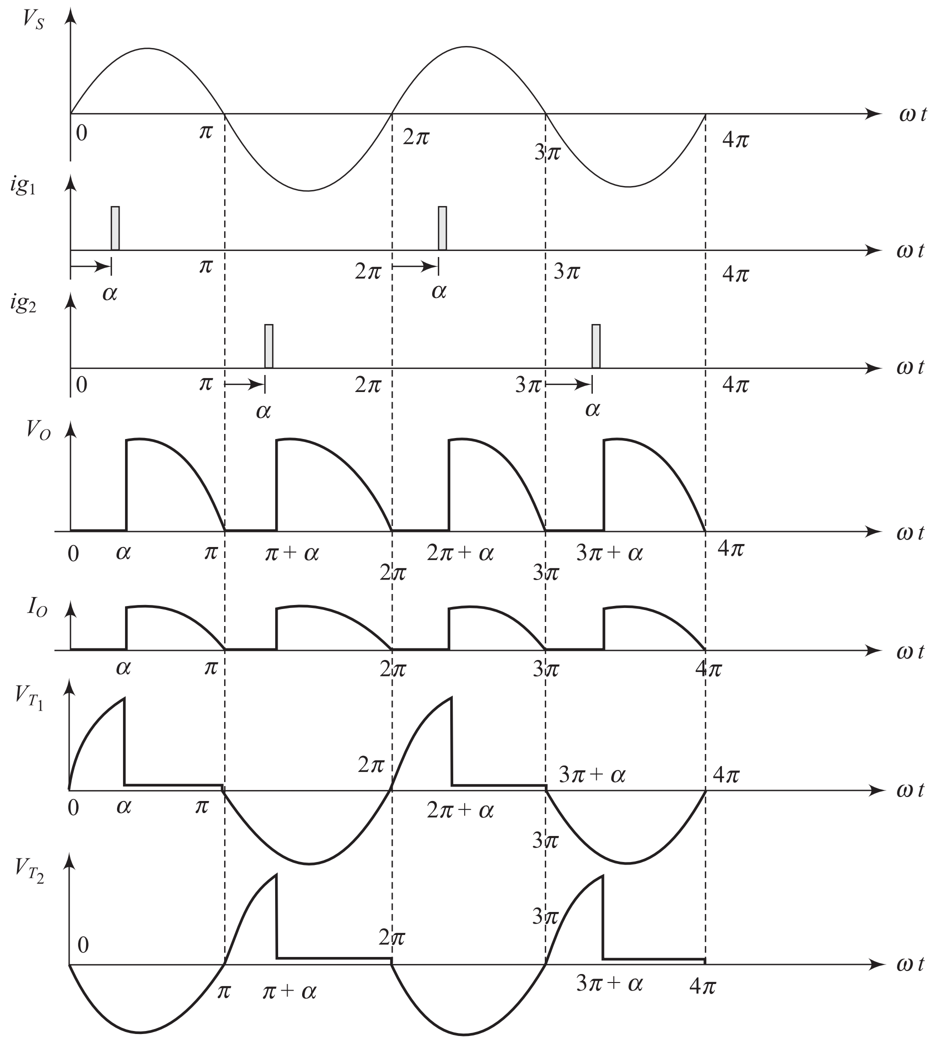


Fig. 6.22 *Input voltage, gate pulse, output voltage, voltage across thyristor and load current waveforms at firing angle α for single-phase full-wave controlled rectifier using centre tap transformer with R load*

Therefore, the dc output voltage can be varied from $\frac{2\sqrt{2}V}{\pi}$ to 0 when firing angle varies from $\alpha = 0$ to $\alpha = \pi$ as depicted in Fig. 6.23. The average or dc load current is

$$I_o = I_{av} = \frac{V_o}{R} = \frac{\sqrt{2}V}{\pi R}(1 + \cos\alpha)$$

Since the current waveform is similar to the output voltage waveform, this converter operates in first quadrant only as shown in Fig. 6.24.

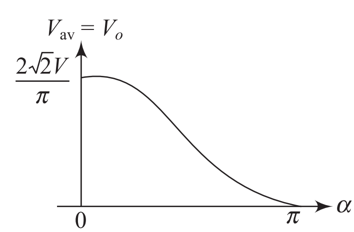


Fig. 6.23 *Firing angle vs output voltage*

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2\, d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} 2V^2\sin^2\omega t\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V^2}{2\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}} = \sqrt{2}V\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

The rms output current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{\sqrt{2}V}{R}\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

The form factor is

$$FF = \frac{V_{\text{rms}}}{V_{\text{av}}} = \sqrt{2}V\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} \Bigg/ \frac{\sqrt{2}V}{\pi}(1+\cos\alpha)$$

The load ripple factor is $RF = \sqrt{FF^2 - 1}$

The dc output power is $P_{\text{dc}} = V_{\text{av}} I_{\text{av}} = V_o I_o$

$$= \frac{\sqrt{2}V}{\pi}(1+\cos\alpha)\times\frac{\sqrt{2}V}{\pi R}(1+\cos\alpha) = \frac{2V^2}{\pi^2 R}(1+\cos\alpha)^2$$

The ac output power is $P_{\text{dc}} = V_{\text{rms}} I_{\text{rms}}$

$$= \sqrt{2}V\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} \times \frac{\sqrt{2}V}{R}\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

$$= \frac{2V^2}{R}\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right] = \frac{V^2}{\pi R}\left[\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]$$

The rms value of transformer secondary current is

$$I_{\text{rms-transformer}} = \frac{I_{\text{rms}}}{2} = \frac{\sqrt{2}V}{2R}\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

The VA rating of transformer is

$$\text{VA} = 2\text{V}_{\text{rms}}\, I_{\text{rms-transformer}}$$

$$= 2\times V\times\frac{\sqrt{2}V}{2R}\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}V^2}{R}\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

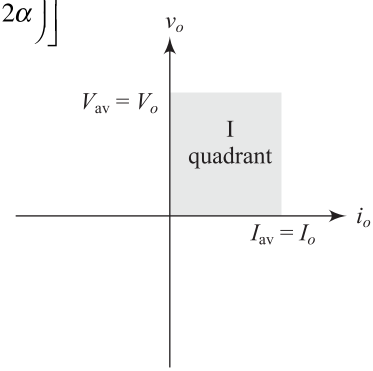


Fig. 6.24 *Quadrant operation of single phase full wave controlled rectifier using centre tap transformer with R load*

The transformer utilisation factor (TUF) is

$$\text{TUF} = \frac{P_{dc}}{VA} = \frac{2V^2}{\pi^2 R}(1+\cos\alpha)^2 \Bigg/ \frac{\sqrt{2}V^2}{R}\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

$$= \frac{2(1+\cos\alpha)^2}{\pi\left[\pi\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}}$$

6.10 SINGLE-PHASE FULL-WAVE CONTROLLED RECTIFIER USING CENTRE TAP TRANSFORMER WITH *RL* LOAD

Figure 6.25 shows the single-phase full-wave controlled rectifier using centre tap transformer at *RL* load. In the positive half cycle of supply voltage, the thyristor T_1 is forward biased and the thyristor T_2 is reverse biased. When a gate pulse is applied to the thyristor T_1 at $\omega t = \alpha$, T_1 becomes ON and the thyristor T_2 is in OFF state and current flows through *RL* load and T_1 as depicted in Fig. 6.26.

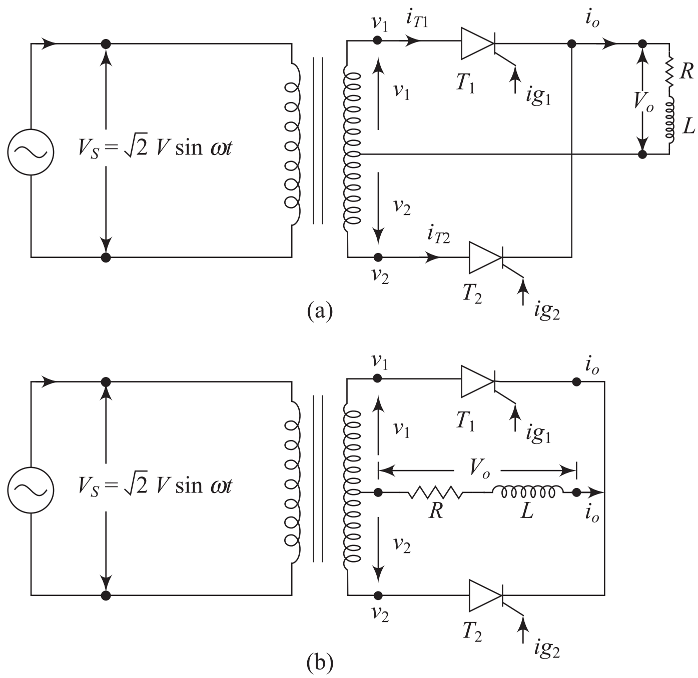


Fig. 6.25 *Single-phase full-wave controlled rectifier using centre tap transformer with RL load*

During the negative half cycle of supply voltage, the polarities of secondary winding have been reversed. Though the voltage across thyristor T_1 is negative, thyristor T_1 will not be OFF due to inductive load. When a trigger pulse is applied to the thyristor T_2 at $\omega t = \pi + \alpha$, the thyristor T_2 conducts and the thyristor T_1 will be in OFF state and current flows through *RL* load and T_2 as shown in Fig. 6.26.

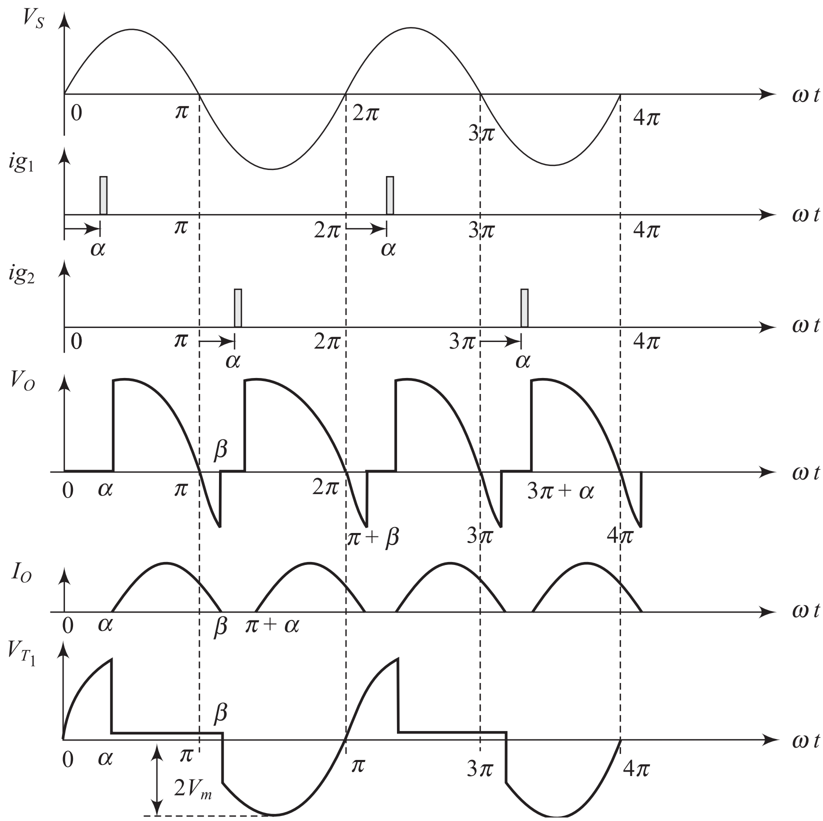


Fig. 6.26 *Voltage and current waveforms in discontinuous load current*

The load current may be continuous or discontinuous depending upon the inductive load. When the inductance value is very large, the load current will be continuous and each thyristor conducts for 180° duration. For a complete cycle, thyristor T_1 conducts from $\omega t = \alpha$ to $\omega t = \pi + \alpha$ and thyristor T_2 conducts from $\omega t = \pi + \alpha$ to $\omega t = 2\pi + \alpha$. Voltage and current waveforms for continuous mode operation are depicted in Fig. 6.27.

If the inductance value is low, the load current will be discontinuous and each thyristor conducts for less than 180° duration. In a complete cycle, thyristor T_1 conducts from $\omega t = \alpha$ to $\omega t = \beta$ and thyristor T_2 conducts from $\omega t = \pi + \alpha$ to $\omega t = \pi + \beta$. Voltage and current waveforms for discontinuous mode operation are shown in Fig. 6.26.

6.10.1 Discontinuous Load Current

For discontinuous load current, each thyristor conducts for less than 180° duration and the average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\beta} \sqrt{2}V \sin \omega t \cdot d\omega t$$

$$= \frac{\sqrt{2}V}{\pi}(\cos \alpha - \cos \beta)$$

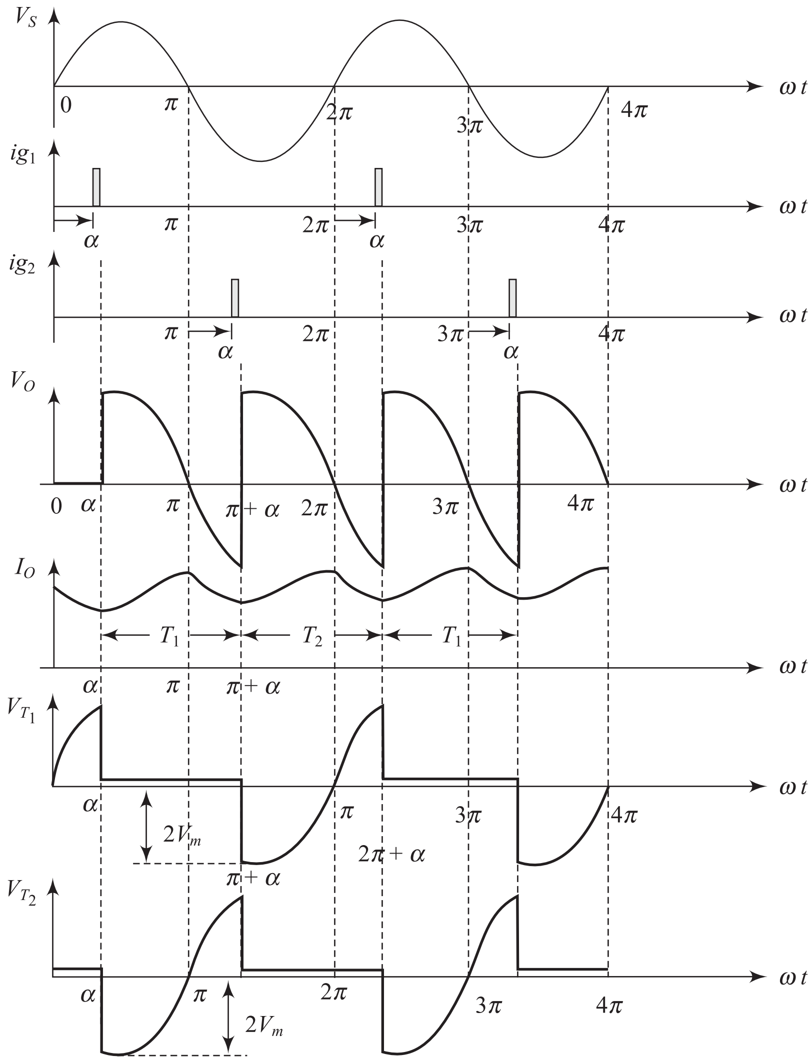


Fig. 6.27 *Voltage and current waveforms in continuous load current*

The average or dc load current is

$$I_o = I_{av} = \frac{V_o}{R}$$

$$= \frac{\sqrt{2}V}{\pi R}(\cos\alpha - \cos\beta)$$

The rms output voltage is

$$V_{rms} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta}(\sqrt{2}V\sin\omega t)^2\,d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta}2V^2\sin^2\omega t\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V^2}{\pi} \int_{\alpha}^{\beta} (1 - \cos 2\omega t) \cdot d\omega t \right]^{\frac{1}{2}} = \left[\frac{V^2}{\pi} \left| \omega t - \frac{1}{2} \sin 2\omega t \right|_{\alpha}^{\beta} \right]^{\frac{1}{2}}$$

$$= V \left[\frac{1}{\pi} \left\{ (\beta - \alpha) + \frac{1}{2} (\sin 2\alpha - \sin 2\beta) \right\} \right) \right]^{\frac{1}{2}}$$

The rms output current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{V}{R} \left[\frac{1}{\pi} \left\{ (\beta - \alpha) + \frac{1}{2} (\sin 2\alpha - \sin 2\beta) \right\} \right) \right]^{\frac{1}{2}}$$

The form factor is $FF = \dfrac{V_{\text{rms}}}{V_{\text{av}}}$

The load ripple factor is $RF = \sqrt{FF^2 - 1}$

The dc output power is $P_{\text{dc}} = V_{\text{av}} I_{\text{av}} = V_o I_o$

$$= \frac{\sqrt{2}V}{\pi} (\cos \alpha - \cos \beta) \times \frac{\sqrt{2}V}{\pi R} (\cos \alpha - \cos \beta) = \frac{2V^2}{\pi^2 R} (\cos \alpha - \cos \beta)^2$$

The ac output power is $P_{\text{ac}} = V_{\text{rms}} I_{\text{rms}}$

The rms value of transformer secondary current is $I_{\text{rms-transformer}} = \dfrac{I_{\text{rms}}}{2}$

The VA rating of transformer is VA=2V $I_{\text{rms-transformer}}$

The transformer utilisation factor (TUF) is $\text{TUF} = \dfrac{P_{\text{dc}}}{VA}$

6.10.2 Continuous Load Current

For continuous mode operation, each thyristor conducts for 180° duration and the average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} \sqrt{2}V \sin \omega t \cdot d\omega t$$

$$= \frac{2\sqrt{2}V}{\pi} \cos \alpha$$

When $\alpha = 0$, the average output voltage is $\dfrac{2\sqrt{2}V}{\pi}$. If $\alpha = \pi/2$, the average output voltage is 0 and if $\alpha = \pi$, the average output voltage is $-\dfrac{2\sqrt{2}V}{\pi}$. Therefore, the dc output voltage can be varied from $\dfrac{2\sqrt{2}V}{\pi}$ to $-\dfrac{2\sqrt{2}V}{\pi}$ when firing angle varies from $\alpha = 0$ to $\alpha = \pi$ as depicted in Fig. 6.28.

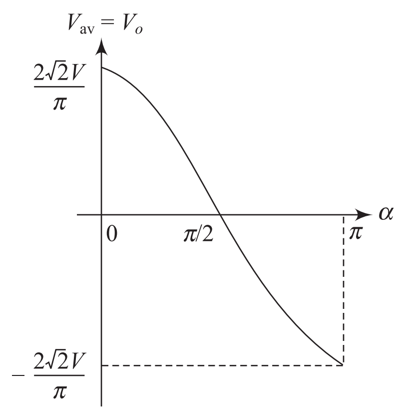


Fig. 6.28 *Firing angle vs output voltage*

Since the load current is always positive and output voltage can be varied from $\dfrac{2\sqrt{2}V}{\pi}$ to $-\dfrac{2\sqrt{2}V}{\pi}$, this converter operates in first and fourth quadrant only as shown in Fig. 6.29.

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} (\sqrt{2}V \sin \omega t)^2 d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} 2V^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V^2}{\pi}\int_{\alpha}^{\pi+\alpha} (1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V^2}{\pi}\left|\omega t - \frac{1}{2}\sin 2\omega t\right|_{\alpha}^{\pi+\alpha}\right]^{\frac{1}{2}}$$

$$= V$$

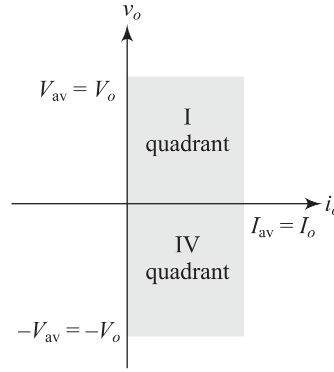


Fig. 6.29 *Quadrant operation of single-phase full-wave controlled rectifier using centre tap transformer with RL load*

The voltage equation for the circuit as shown in Fig. 6.25, is

$$\sqrt{2}V \sin \omega t = L\frac{di_o}{dt} + Ri_o \quad \text{for } \alpha \le \omega t \le \pi + \alpha$$

where V is the rms input voltage and output current is i_o.
The output current i_o can be expressed as

$$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} \tag{6.6}$$

At $\omega t = \alpha$ and $\omega t = \pi + \alpha$, current $i_o = I_o$, Eq. (6.6) can be written as

$$I_o = \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi) + Ae^{-\frac{R}{L}\frac{\alpha}{\omega}} \text{ and } I_o = \frac{\sqrt{2}V}{Z}\sin(\pi + \alpha - \phi) + Ae^{-\frac{R}{L}\frac{(\pi+\alpha)}{\omega}}$$

After solving above equation, we get

$$i_o(t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + \frac{\sqrt{2}V}{Z}\frac{2\sin(\phi - \alpha)}{1 - e^{-\frac{\pi}{\tan\phi}}}e^{-\frac{(\omega t - \alpha)}{\tan\phi}}$$

6.11 SINGLE-PHASE FULL-WAVE CONTROLLED RECTIFIER USING CENTRE TAP TRANSFORMER WITH *RL* LOAD AND FREE WHEELING DIODE

Figure 6.30 shows a single-phase full-wave controlled rectifier using centre tap transformer with *RL* load and free wheeling diode. In single-phase full-wave controlled rectifier, the conduction period of SCR depends upon firing angle α and the phase angle ϕ. Due to *RL* load, the conduction of SCR continues in the negative half cycle of supply voltage. The load current may be continuous or discontinuous depending upon the load impedance.

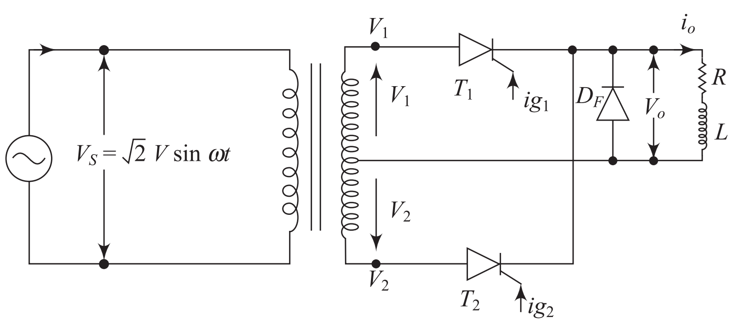


Fig. 6.30 *Single-phase full-wave controlled rectifier using centre tap transformer with RL load and free wheeling diode*

When a free wheeling diode is connected across *RL* load of single-phase full-wave controlled rectifier using centre tap transformer, this converter works as a *semi-converter* or *half-controlled converter*. The switching of thyristors T_1 and T_2 and free wheeling diode D_F for discontinuous load current are depicted in Fig. 6.31. During the positive half cycle of input voltage, thyristor T_1 is forward biased and triggering pulse is applied at $\omega t = \alpha$. Then thyristor T_1 is turned ON at $\omega t = \alpha$ and conducts upto $\omega t = \pi$. At $\omega t = \pi$, the diode D_F becomes forward biased as cathode is negative with respect to anode and conducts. As a result, the load current flows through diode D_F and *RL* load from $\omega t = \pi$ to $\omega t = \beta$ for discontinuous load current. At $\omega t = \beta$, diode D_F becomes OFF as load current reduces to zero.

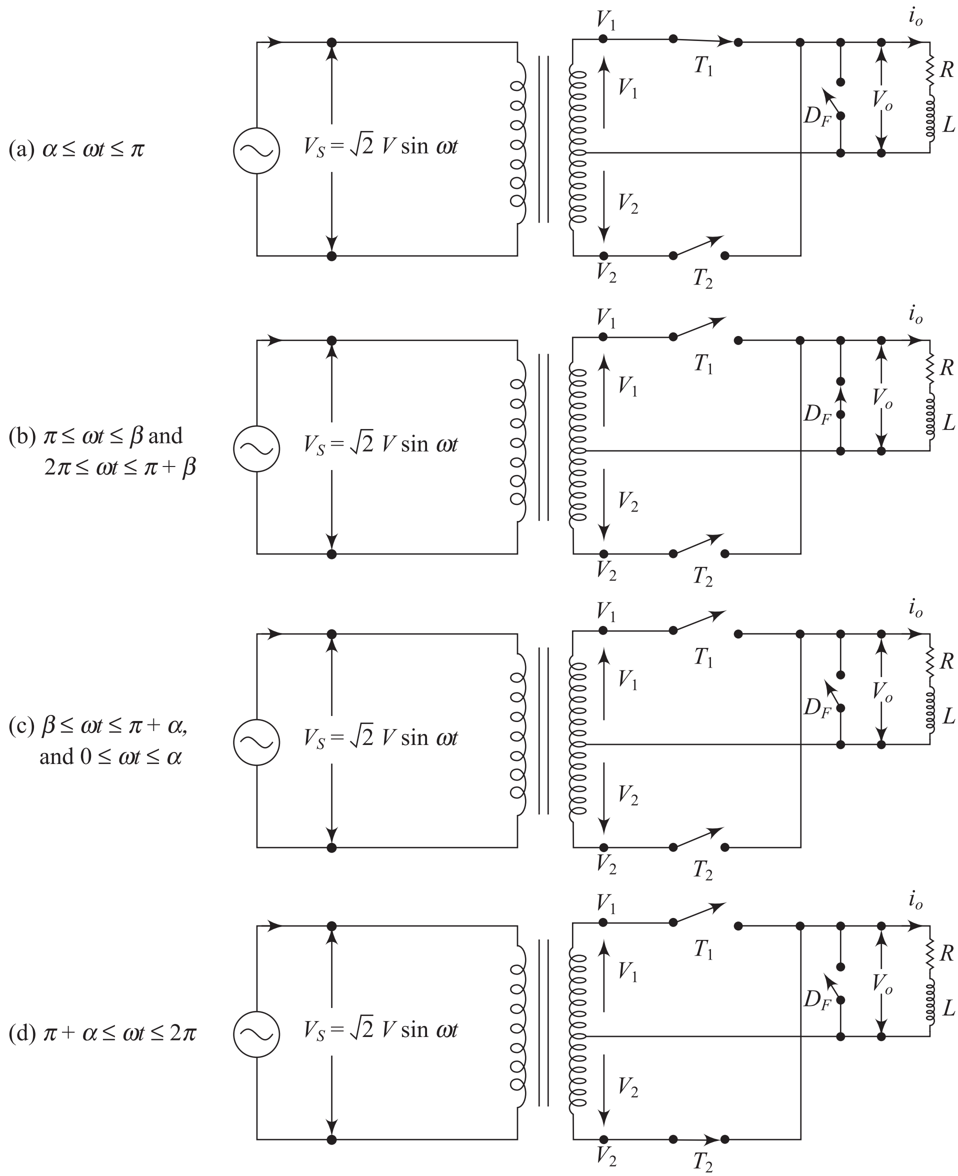


Fig. 6.31 *Switching of thyristors T_1 and T_2 and free wheeling diode for discontinuous load current*

In the negative half cycle of input voltage, thyristor T_2 is forward biased and triggering pulse is applied at $\omega t = \pi + \alpha$. Then thyristor T_2 becomes turned on at $\omega t = \pi + \alpha$ and conducts up to $\omega t = 2\pi$. At $\omega t = 2\pi$, the diode D_F becomes forward biased and conducts. Consequently, the load current flows through diode D_F and RL load from $\omega t = 2\pi$ to $\omega t = \pi + \beta$ for discontinuous load current. At $\omega t = \pi + \beta$, diode D_F becomes OFF as load current reduces to zero. The voltage and current waveforms in discontinuous load current is depicted in Fig. 6.32. Similarly, Fig. 6.33 shows the voltage and current waveforms in continuous load current.

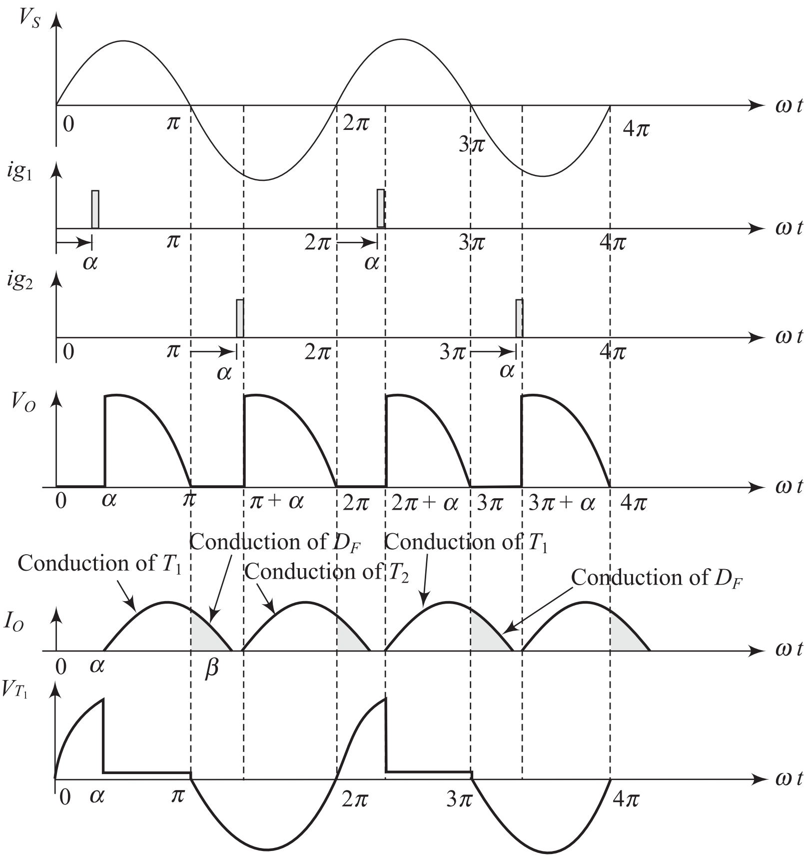


Fig. 6.32 *Voltage and current waveforms in discontinuous load current*

The average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi}(1 + \cos \alpha)$$

The average or dc load current is

$$I_o = I_{\text{av}} = \frac{V_o}{R} = \frac{\sqrt{2}V}{\pi R}(1 + \cos \alpha)$$

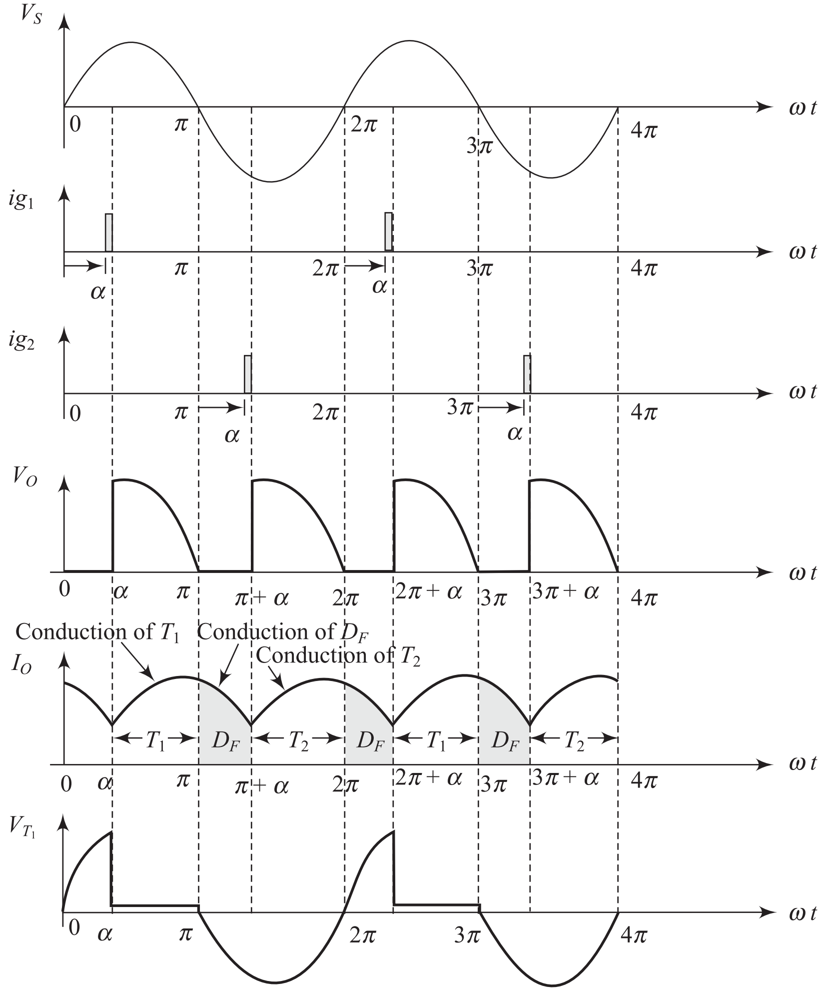


Fig. 6.33 *Voltage and current waveforms in continuous load current*

6.12 SINGLE-PHASE FULL-WAVE CONTROLLED RECTIFIER USING CENTRE TAP TRANSFORMER WITH *RLE* LOAD

Figure 6.34 shows a single-phase full-wave controlled rectifier with *RLE* load. This circuit operates just like half-wave controlled rectifier with *RLE* load, but the only difference is that the load current flows in both positive as well as negative half cycle of the supply voltage. The load current depends upon the firing angle of thyristors, inductive load and the ratio of battery voltage to transformer secondary voltage. The input voltage, gate pulse, and load current waveforms $\omega t = \alpha$ are illustrated in Fig. 6.35.

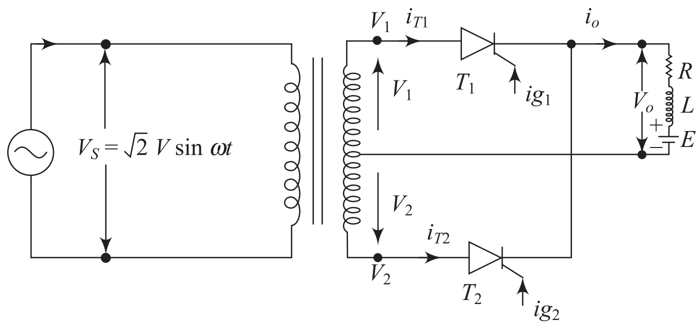


Fig. 6.34 *Single-phase full-wave controlled rectifier using centre tap transformer with RLE load*

The minimum value of firing angle of thyristor is α_{min}, then

$$\sqrt{2}V \sin \alpha_{min} = E \quad \text{at } \omega t = \alpha_{min}$$

or

$$\alpha_{min} = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right)$$

The maximum firing angle is $\alpha_{max} = \pi - \alpha_{min}$

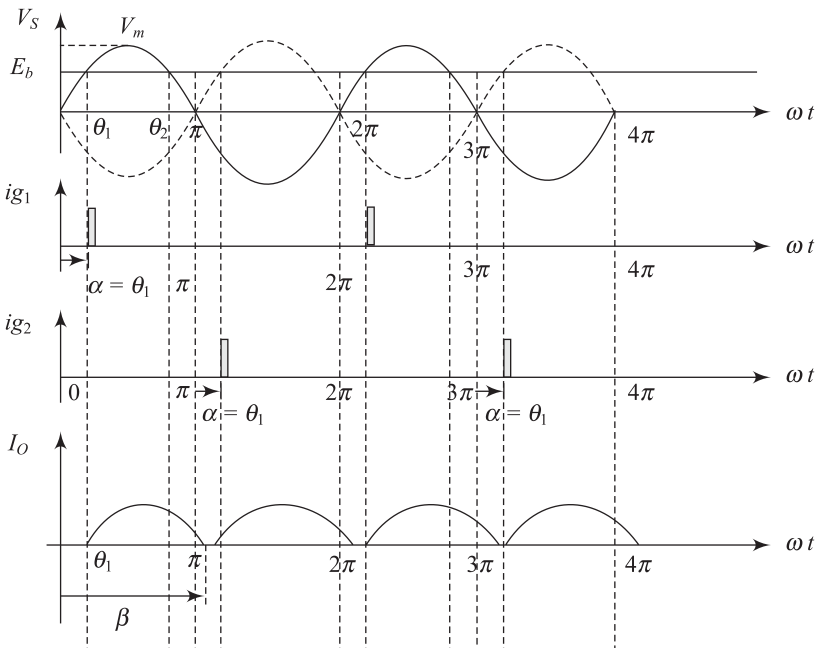


Fig. 6.35 *Input voltage, gate pulse, and load current waveforms $\omega t = \alpha$ where $\theta_1 = \alpha_{min}$ and $\theta_2 = \alpha_{max}$*

Example 6.9 A single-phase full-wave controlled rectifier using centre tap transformer is fed from a 230 V, 50 Hz ac source and it is connected a resistive load $R = 10\ \Omega$. Determine (a) the output voltage, (b) form factor, (c) ripple factor, (d) efficiency, (e) transformer utilisation factor at $\alpha = 30°$. Turn ratio of transformer is 1:1.

Solution

Given: $\alpha = 30° = 30 \times \dfrac{\pi}{180} = 0.523$ rad

As turn ratio of transformer is 1:1, the rms voltage between centre tap and other terminal of transformer secondary is $V = 230/2 = 115$ V.

(a) The average or dc output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 115}{\pi}(1+\cos 30°) = 96.63 \text{ V}$$

(b) The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2 d\omega t\right]^{\frac{1}{2}} = \sqrt{2}V\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

$$= \sqrt{2}\times 115\left[\frac{1}{2\pi}\left(\pi - 0.523 + \frac{1}{2}\sin(2\times 30)\right)\right]^{\frac{1}{2}} = 113.32 \text{ V}$$

The form factor is

$$FF = \frac{V_{\text{rms}}}{V_{\text{av}}} = \frac{113.32}{96.63} = 1.172$$

(c) The load ripple factor is $RF = \sqrt{FF^2 - 1} = \sqrt{(1.172)^2 - 1} = 0.6112$

(d) The dc output current is $I_o = \dfrac{V_o}{R} = \dfrac{96.63}{10} = 9.663$ A

rms output current $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{113.32}{10} = 11.332$ A

The dc output power is $P_{\text{dc}} = V_o I_o = 96.63 \times 9.663$ Watt = 933.73 Watt

The ac output power is $P_{\text{ac}} = V_{\text{rms}} I_{\text{rms}} = 113.32 \times 11.332$ Watt = 1284.142 Watt

Rectifier efficiency is $\eta = \dfrac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\% = \dfrac{933.73}{1284.142} \times 100\% = 72.71\%$

(e) The rms value of transformer secondary current is

$$I_{\text{rms-transformer}} = \frac{I_{\text{rms}}}{2} = \frac{11.332}{2} \text{ A} = 5.666 \text{ A}$$

The VA rating of transformer is

$$\text{VA} = 2\text{VI}_{\text{rms-transformer}} = 2 \times 115 \times 5.666 = 1303.18 \text{ VA}$$

The transformer utilisation factor (TUF) is

$$TUF = \frac{P_{\text{dc}}}{VA} = \frac{933.73}{1303.18} = 0.7165$$

Example 6.10 The peak forward voltage rating of thyristors is 1200 V and average ON-state current rating is 50 A in a single-phase mid-point full converter and single-phase bridge converter. Determine the power rating of converters which can handle properly. The factor of safety is 2.5.

Solution

The maximum voltage across thyristor which is used in a single phase mid-point full converter is $2V_m$

Therefore this converter can be designed for a maximum voltage of $\dfrac{1200}{2 \times 2.5} = 240$ V

The maximum power output of single phase mid-point full converter is

$$\frac{2V_m}{\pi}\cos\alpha \times I_{Tav} = \frac{2\times 240}{\pi}\cos 0\times 50 = 7643.31 \text{ Watt as } \alpha = 0° \text{ and } I_{Tav} = 50 \text{ A}$$

In a single-phase bridge converter, each thyristor can able to withstand at the maximum voltage of V_m

Therefore this converter can be designed for a maximum voltage of $\frac{1200}{2.5} = 480$ V

The maximum power output of a single phase bridge converter is

$$\frac{2V_m}{\pi}\cos\alpha \times I_{Tav} = \frac{2\times 480}{\pi}\cos 0\times 50 = 15286.624 \text{ Watt} \quad \text{as } \alpha = 0° \text{ and } I_{Tav} = 50 \text{ A}$$

6.13 SINGLE-PHASE FULL-WAVE CONTROLLED BRIDGE RECTIFIER

Single-phase full-wave controlled bridge rectifier is extensively used in various industrial applications. This converter circuit provide unidirectional current with both positive and negative voltage polarity depending upon the firing angle of SCRs. Hence this converter can operate as a controlled rectifier or an inverter. Since a single-phase full-wave controlled bridge rectifier consists of four SCRs, the gate drive circuit will be more complex and expensive. This circuit has relatively poor form factor and input power factor. To improve the output voltage and current form factor, a free wheeling diode is connected across diode D_F. Whenever the output voltage tries to become negative, the diode D_F across load becomes forward bias and clamp the load voltage to zero. Consequently, this circuit can not be able to operate in the inverter mode. When two SCRs of a single-phase fully controlled converter are replaced by two diodes, the fully converter becomes half-controlled converters. The classification of single-phase controlled bridge rectifier is depicted in Fig. 6.36.

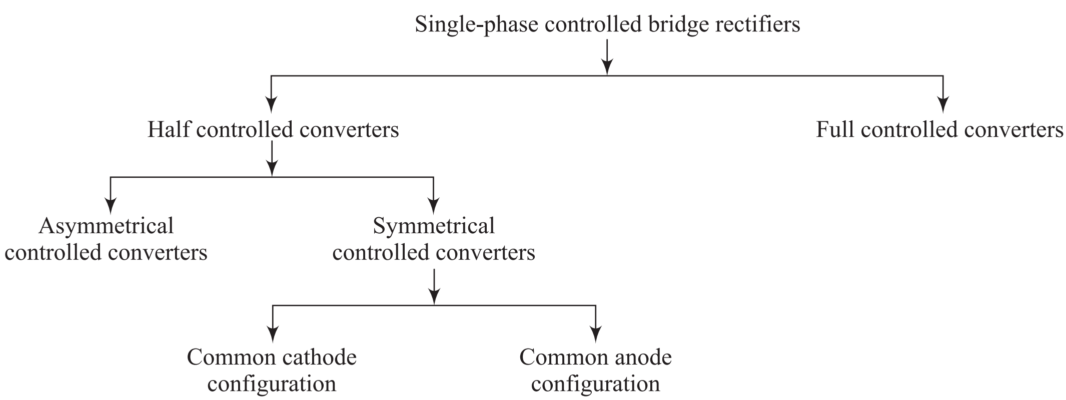


Fig. 6.36 *Classification of single-phase fully controlled bridge rectifier*

6.14 SINGLE-PHASE FULLY CONTROLLED BRIDGE RECTIFIER WITH *R* LOAD

Figure 6.37 shows a single-phase full-wave controlled bridge rectifier which consists of four thyristors and *R* load. The operation of this circuit is similar to a single-phase full-wave controlled rectifier using centre tap transformer and *R* load. In the bridge circuit, diagonally pair of SCRs (T_1 and T_2 or T_3 and T_4) are conduct and line commutated simultaneously.

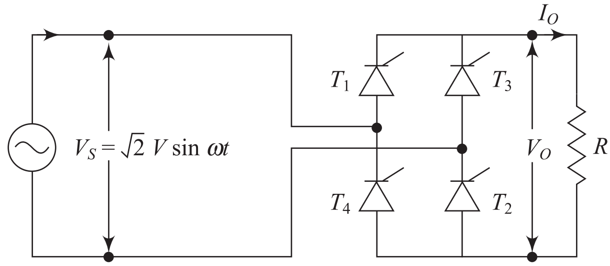


Fig. 6.37 *Single-phase full-wave controlled bridge rectifier with R load*

In the positive half cycle of supply voltage, SCRs T_1 and T_2 are forward biased. As soon as any triggering pulse is applied to T_1 and T_2 simultaneously at $\omega t = \alpha$, SCRs T_1 and T_2 will be turned ON and conduct during $\alpha \leq \leq \omega t \leq 2\pi$. Then current flows through T_1, T_2 and R as depicted in Fig. 6.38.

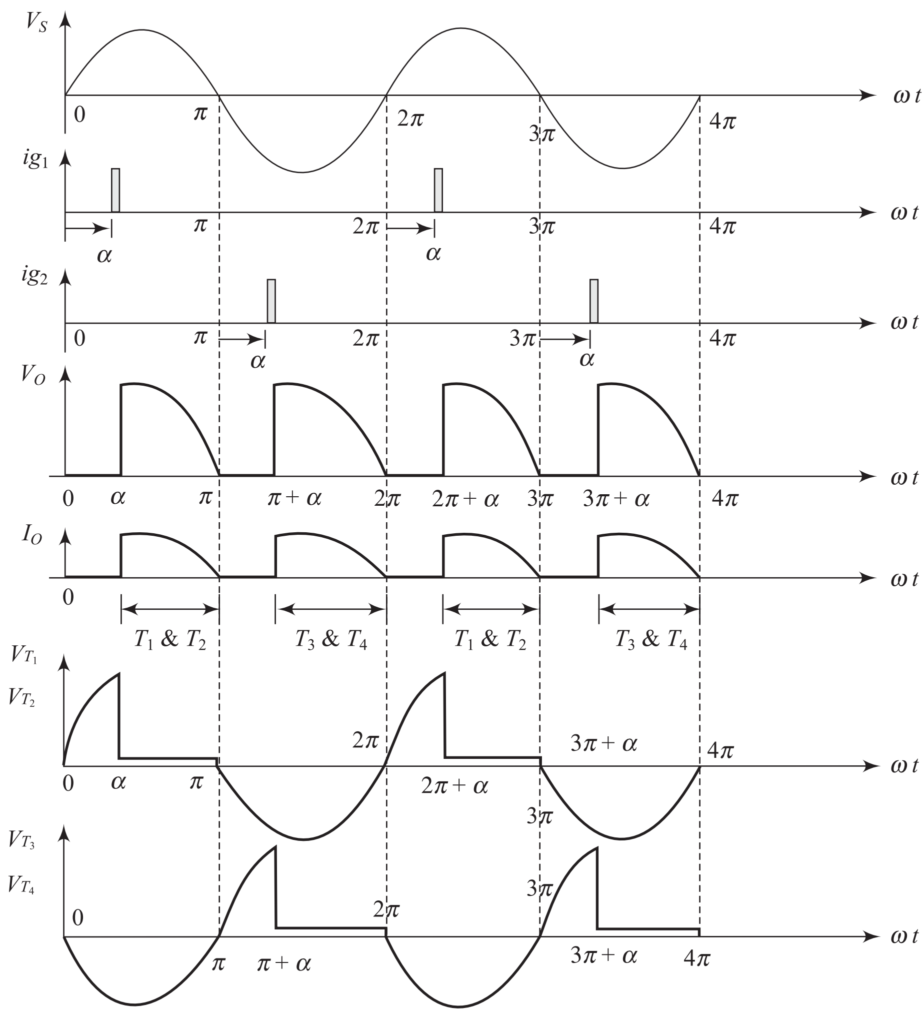


Fig. 6.38 *Voltage and current waveforms of single-phase full-wave controlled bridge rectifier with R load*

During the negative half cycle of input voltage, SCRs T_3 and T_4 are forward biased. If any triggering pulse is applied to T_3 and T_4 simultaneously at $\omega t = \pi + \alpha$, SCRs T_3 and T_4 can be turned ON and conduct during $\pi + \alpha \le \omega t \le 2\pi$. Subsequently current flows through T_3 and T_4 and R as shown in Fig. 6.38.

6.15 SINGLE-PHASE FULLY CONTROLLED BRIDGE RECTIFIER WITH *RL* LOAD

A single-phase full-wave controlled bridge rectifier which consists of four thyristors and *RL* load is depicted in Fig. 6.39. In the positive half-cycle of supply voltage, thyristors T_1 and T_2 are forward biased and operate in the forward blocking sate. When the firing pulse is applied to T_1 and T_2 at $\omega t = \alpha$, both thyristors are turned ON simultaneously and input or supply voltage is applied across *RL* load. Subsequently, the load current flows though T_1, T_2 and *RL* load. Due to inductive load thyristor T_1 and T_2 will conduct beyond $\omega t = \pi$.

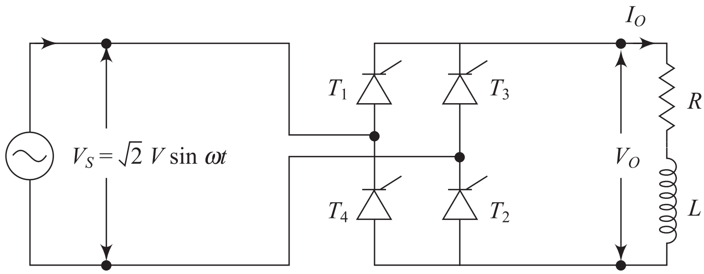


Fig. 6.39 *Single-phase full-wave controlled bridge rectifier with RL load*

During the negative half-cycle of supply voltage, thyristors T_3 and T_4 are forward biased and operate in the forward blocking sate. As soon as the firing pulse is applied to T_3 and T_4 at $\omega t = \pi + \alpha$, both thyristors are turned ON simultaneously and input or supply voltage is applied across *RL* load. As a result, the load current flows though T_3, T_4 and *RL* load. Since the load is inductive, thyristor T_3 and T_4 will conduct beyond $\omega t = 2\pi$. This converter circuit operates in continuous mode and discontinuous mode. The voltage and current waveforms of single phase full wave controlled bridge rectifier with *RL* load are depicted in Fig. 6.40.

6.15.1 Continuous Load Current

For continuous load current, the average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\cos \alpha$$

The average or dc load current is

$$I_o = I_{\text{av}} = \frac{V_o}{R} = \frac{2\sqrt{2}V}{\pi R}\cos \alpha$$

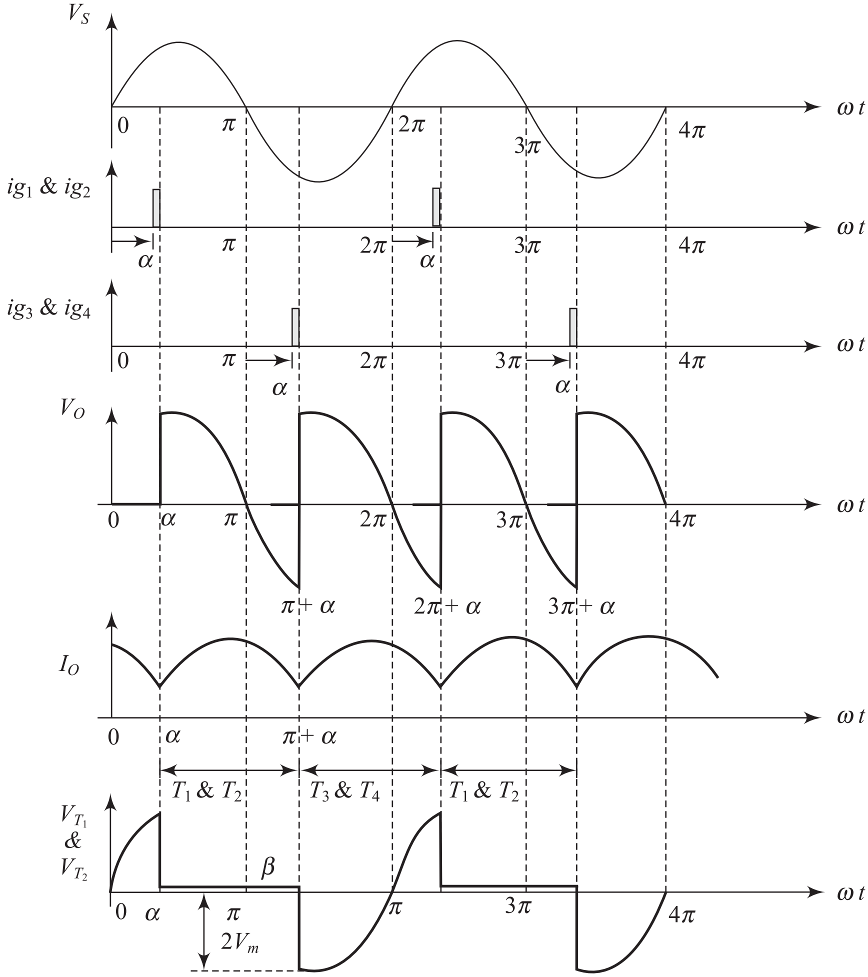


Fig. 6.40 *The voltage and current waveforms for continuous load current*

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} (\sqrt{2}V\sin\omega t)^2\, d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} 2V^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V^2}{2\pi}\int_{\alpha}^{\pi+\alpha} (1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}} = \text{V}$$

The form factor is

$$FF = \frac{V_{\text{rms}}}{V_{\text{av}}} = V \Bigg/ \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{\pi}{2\sqrt{2}\cos\alpha}$$

The load ripple factor is $RF = \sqrt{FF^2 - 1} = \left[\dfrac{\pi^2}{8\cos^2\alpha} - 1\right]^{1/2}$

6.15.2 Discontinuous Load Current

For discontinuous load current, the voltage and current waveforms are shown in Fig. 6.41. The average output voltage is

$$V_o = V_{av} = \frac{1}{\pi}\int_{\alpha}^{\beta} \sqrt{2}V \sin \omega t \cdot d\omega t$$

$$= \frac{\sqrt{2}V}{\pi}(\cos \alpha - \cos \beta)$$

The average or dc load current is

$$I_o = I_{av} = \frac{V_o}{R}$$

$$= \frac{\sqrt{2}V}{\pi R}(\cos \alpha - \cos \beta)$$

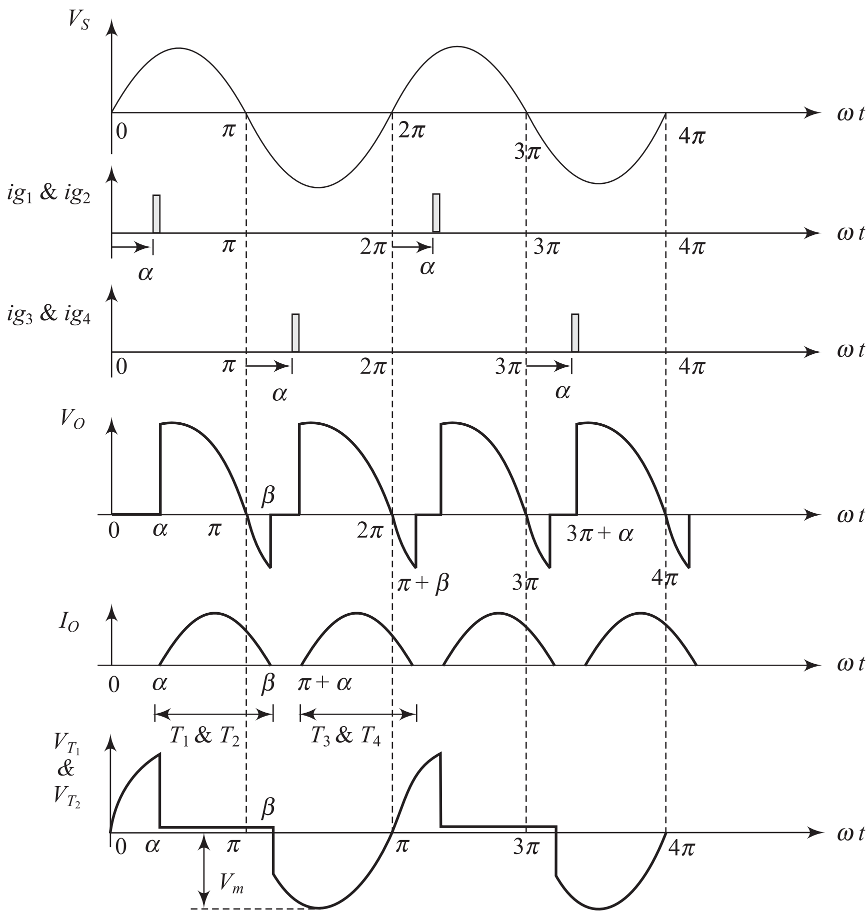


Fig. 6.41 *The voltage and current waveforms for discontinuous load current*

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta}(\sqrt{2}V\sin\omega t)^2\,d\omega t\right]^{\frac{1}{2}} = \sqrt{2}V\left[\frac{1}{\pi}\left\{(\beta-\alpha)+\frac{1}{2}(\sin 2\alpha - \sin 2\beta)\right\}\right]^{\frac{1}{2}}$$

The rms output current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{\sqrt{2}V}{R}\left[\frac{1}{\pi}\left\{(\beta-\alpha)+\frac{1}{2}(\sin 2\alpha - \sin 2\beta)\right\}\right]^{\frac{1}{2}}$$

6.16 SINGLE-PHASE FULLY CONTROLLED BRIDGE RECTIFIER WITH FREE WHEELING DIODE D_F AND *RL* LOAD

Figure 6.42 shows a single-phase full-wave controlled bridge rectifier which consists of four thyristors, *RL* load and free wheeling diode D_F. During the positive half-cycle of supply voltage, thyristors T_1 and T_2 are forward biased and the firing pulse is applied to T_1 and T_2 at $\omega t = \alpha$. Then both thyristors are turned ON at the same time and input or supply voltage is applied across *RL* load. Afterward, the load current flows though T_1, T_2 and *RL* load. As load is inductive, the low current continuously flows after $\omega t = \pi$. This load current flows through D_F as thyristors T_1 and T_2 are turned OFF due to reverse bias after $\omega t = \pi$ and thyristor current is below the holding current.

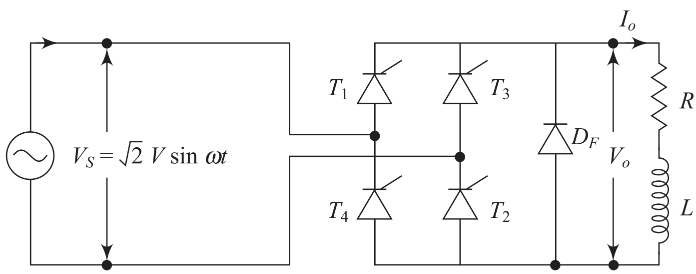


Fig. 6.42 *Single-phase full-wave controlled bridge rectifier with RL load and free wheeling diode D_F*

In the negative half-cycle of supply voltage, thyristors T_3 and T_4 are forward biased and operate in the forward blocking sate. When the firing pulse is applied to T_3 and T_4 at $\omega t = \pi + \alpha$, both thyristors are turned ON simultaneously and input voltage is applied across *RL* load. Consequently, the load current flows though T_3, T_4 and *RL* load. Due to inductive load, the load current continuously flows after $\omega t = 2\pi$. Again this load current flows through D_F as thyristors T_3 and T_4 are turned OFF due to reverse bias after $\omega t = 2\pi$ and thyristor current is less than the holding current. This converter circuit operates in continuous mode and discontinuous mode. The voltage and current waveforms are depicted in Figs. 6.43 and 6.44.

6.17 SINGLE-PHASE FULL-WAVE CONTROLLED BRIDGE RECTIFIER WITH *RLE*

Figure 6.45 shows the circuit configuration of a single-phase fully controlled bridge rectifier with resistance inductance and a back emf. This circuit extensively used in speed control of separately excited dc motor. Thyristors T_1 and T_2 are turned on together while T_3 and T_4 are turned on 180° after

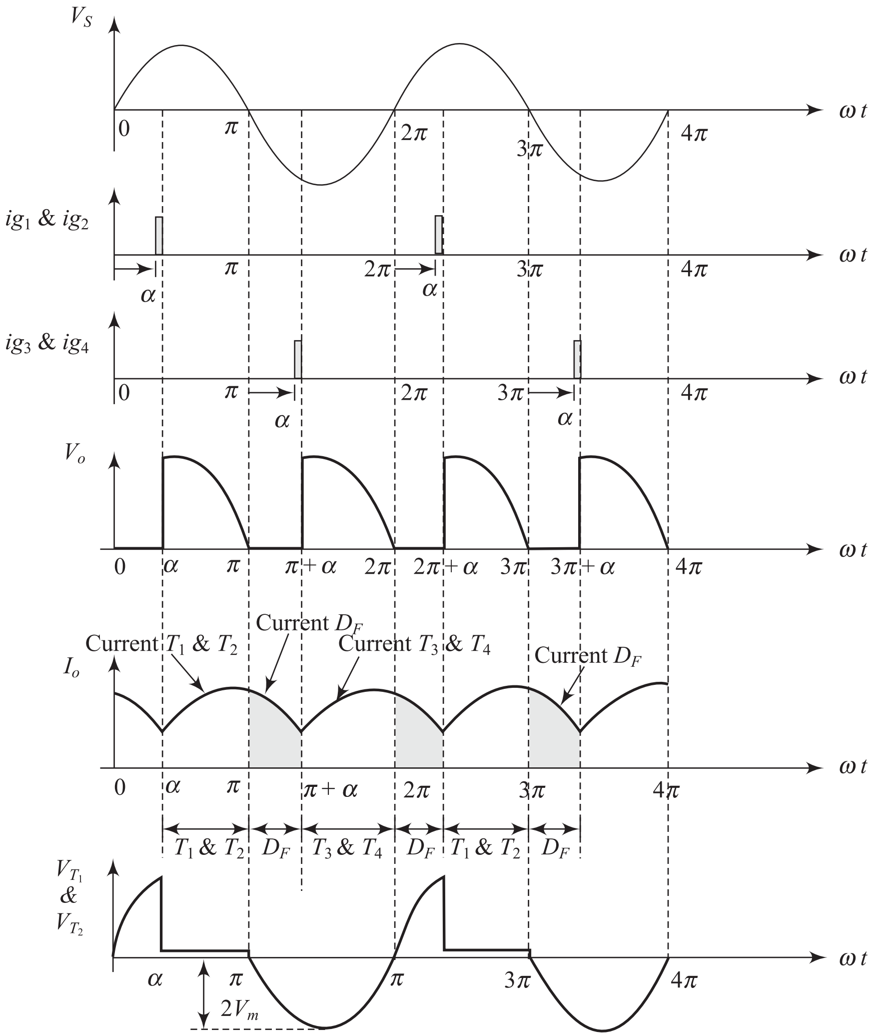


Fig. 6.43 *The voltage and current waveforms for continuous load current*

T_1 and T_2. Hence the load current flows through either T_1 and T_2 or T_3 and T_4. It is never possible for this converter that neither T_1 and T_3 nor T_2 and T_4 conduct simultaneously. When T_1 and T_2 are forward biased during positive half cycle of input voltage and a gate pulse is applied to them, T_1 and T_2 will be turned ON. At the same time, a negative voltage is applied across T_3 and T_4 and they are in OFF state. In the negative half cycle of supply voltage, T_3 and T_4 are forward biased and a gate pulse is applied to them. Then T_3 and T_4 will be turned on and T_1 and T_2 will be turned OFF immediately due to reverse biased.

This circuit can operate in either continuous or discontinuous mode. The load current flows i_o through T_1 and T_2 or T_3 and T_4. Sometime the load current i_o becomes zero in between the firing of T_1 and T_3 and T_2 and T_4. While the load current is zero, all four thyristors must be in OFF state. Consequently, this converter operates in the discontinuous conduction mode.

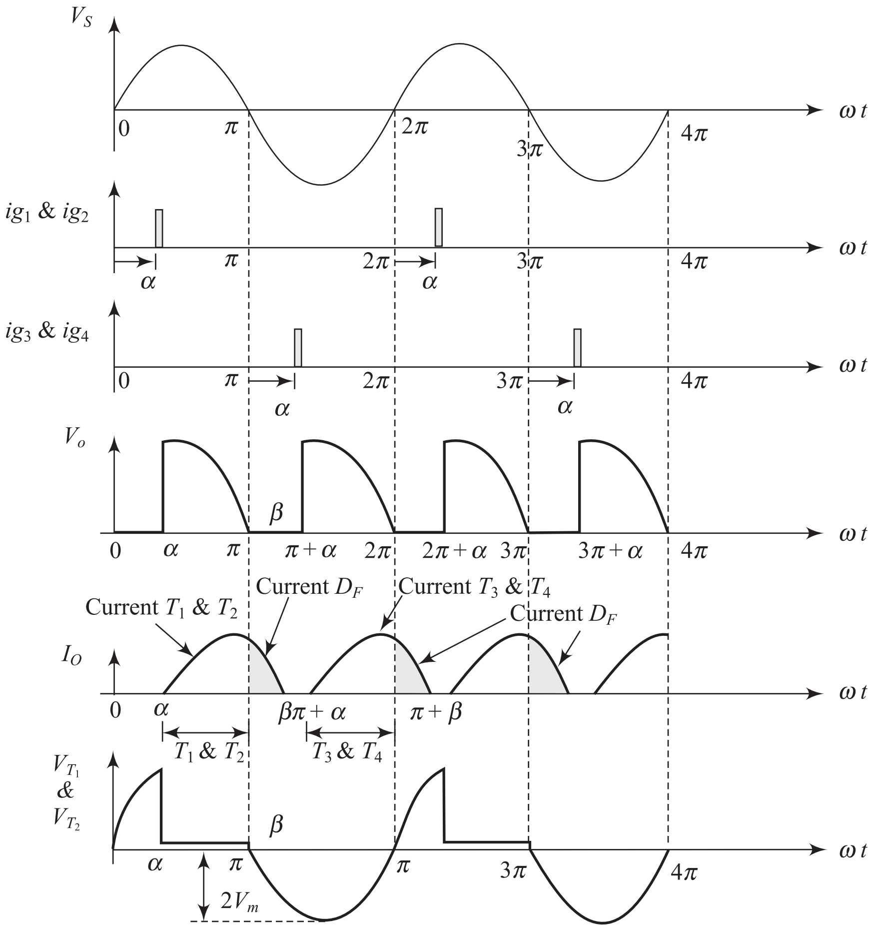


Fig. 6.44 *The voltage and current waveforms for discontinuous load current*

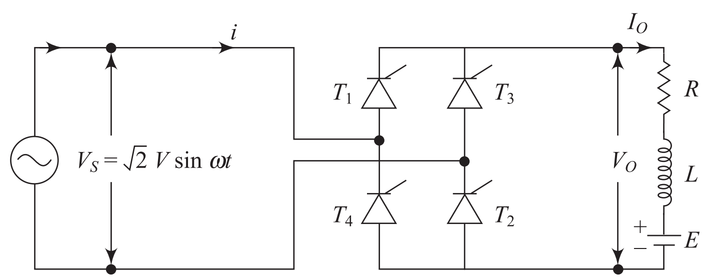


Fig. 6.45 *Single-phase full-wave controlled bridge rectifier with RLE load*

6.17.1 Discontinuous Load Current

The voltage and current wave forms for discontinuous current mode operation are shown in Fig. 6.46. In discontinuous current mode operation, the load current i_o is zero for certain time interval. During

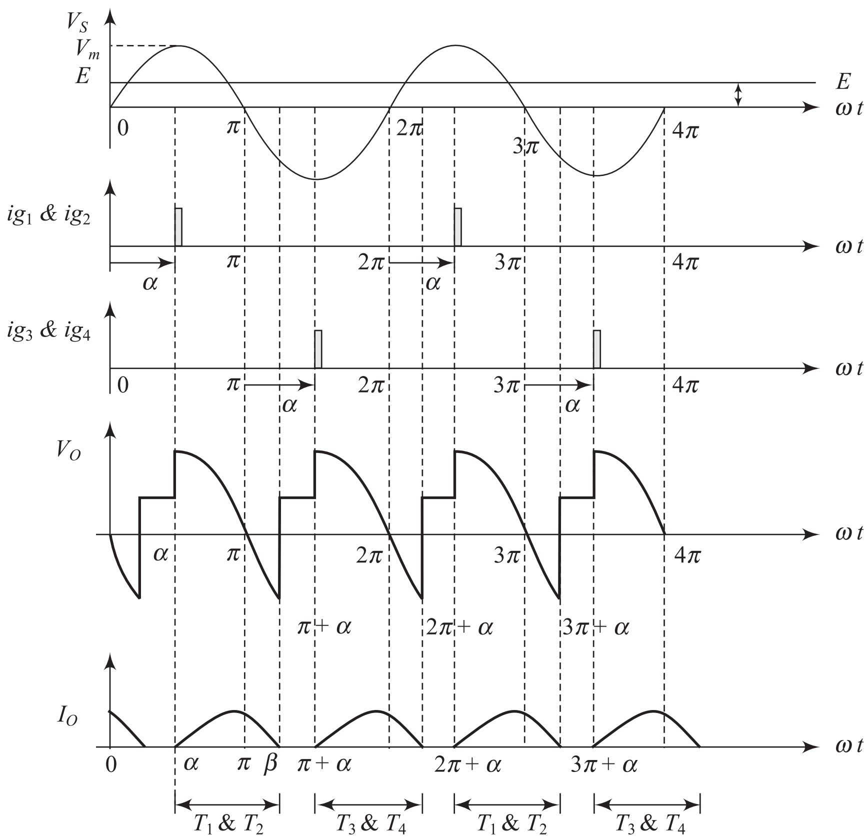


Fig. 6.46 *Voltage and current waveforms for discontinuous mode operation*

this time interval, none of T_1, T_2, T_3 and T_4 conducts. SCRs T_1 and T_2 are triggered when input voltage is greater than E. Therefore SCR conducts from $\alpha_{\min}$ and the load current i_o starts rising from zero and continues to increase till $\omega t = \pi - \alpha_{\min}$. After $\omega t = \pi - \alpha_{\min}$, the output voltage v_o falls below the back emf E and i_o starts to decrease. At $\omega t = \beta$, the current i_o becomes zero. Then i_o remains zero till $\omega t = \pi + \alpha_{\min}$ where T_3 and T_4 are triggered simultaneously. Therefore, none of the thyristors conduct during $\beta \le \omega t \le \pi + \alpha_{\min}$ and the output voltage is $v_o = E$ as depicted in Fig. 6.46.

The minimum firing angle can be determined

$$\sqrt{2}V \sin \alpha_{\min} = E$$

or
$$\alpha_{\min} = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right)$$

The voltage equation of the circuit as shown in Fig. 6.46 is

$$\sqrt{2}V \sin \omega t = L\frac{di_0}{dt} + Ri_0 + E \quad \text{at } \alpha \le \omega t \le \beta$$

and $\quad i_o = 0$ at $\omega t = 0$

where, V is the rms voltage and output current is i_o

The output current can be expressed as

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-\frac{\alpha - \omega t}{\tan\phi}}\right] - \frac{E}{R}\left[1 - e^{\frac{\alpha - \omega t}{\tan\phi}}\right] \quad \text{at } \alpha \le \omega t \le \beta$$

Where, $Z = \sqrt{R^2 + (\omega L)^2}$ and $\tan\phi = \dfrac{\omega L}{R}$

or

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{-\frac{\alpha - \omega t}{\tan\phi}} - \frac{\sin\theta}{\cos\phi}(1 - e^{-\frac{\omega t - \alpha}{\tan\phi}})\right]$$

where, $\sqrt{2}V\sin\theta = E$, and $R = Z\cos\phi$

At $\omega t = \beta$, $i_o = 0$. Then we can write

$$\left[\sin(\beta - \phi) - \sin(\alpha - \phi)e^{-\frac{\alpha - \beta}{\tan\phi}} - \frac{\sin\theta}{\cos\phi}(1 - e^{-\frac{\beta - \alpha}{\tan\phi}})\right] = 0 \tag{6.7}$$

For given value of ϕ, α and θ, we can determine the value of β by solving Eq. (6.7).

The average output voltage V_o is

$$V_o = \frac{1}{\pi}\left[\int_\alpha^\beta \sqrt{2}V\sin\omega t \cdot d(\omega t) + \int_\beta^{\pi+\alpha} E \cdot d(\omega t)\right] \quad \text{where, } \pi \le \beta \le \pi + \alpha$$

$$V_o = \frac{1}{\pi}\left[\int_\alpha^\beta \sqrt{2}V\sin\omega t \cdot d(\omega t) + \int_\beta^{\pi+\alpha} \sqrt{2}V\sin\theta \cdot d(\omega t)\right] \text{where, } \sqrt{2}V\sin\theta = E$$

$$= \frac{\sqrt{2}V}{\pi}[(\cos\alpha - \cos\beta) + \sin\theta(\pi - \beta + \alpha)]$$

The average current is

$$I_O = \frac{V_o - E}{R} = \frac{V_o - \sqrt{2}V\sin\theta}{Z\cos\phi}$$

$$= \frac{\sqrt{2}V}{\pi Z\cos\phi}[(\cos\alpha - \cos\beta) + \sin\theta(\pi - \beta + \alpha)] - \frac{\sqrt{2}V\sin\theta}{Z\cos\phi}$$

$$= \frac{\sqrt{2}V}{\pi Z\cos\phi}[(\cos\alpha - \cos\beta) + \sin\theta(\alpha - \beta)]$$

As the average output voltage and current depends on the value of β, the performance of the single-phase fully controlled bridge rectifier with *RLE* load is affected by β. Figure 6.47 shows the boundary between continuous and discontinuous conduction.

6.17.2 Continuous Load Current

In continuous load current mode operation, the load current i_o is always greater than zero and the converter is said to operate in the continuous conduction mode. In this mode of operation, T_1 and T_2 and T_3 and T_4 conducts for alternate half cycle of the input voltage (180° duration). The minimum firing angle converter is $\alpha_{\min} = \sin^{-1}\left(\dfrac{E}{\sqrt{2}V}\right)$.

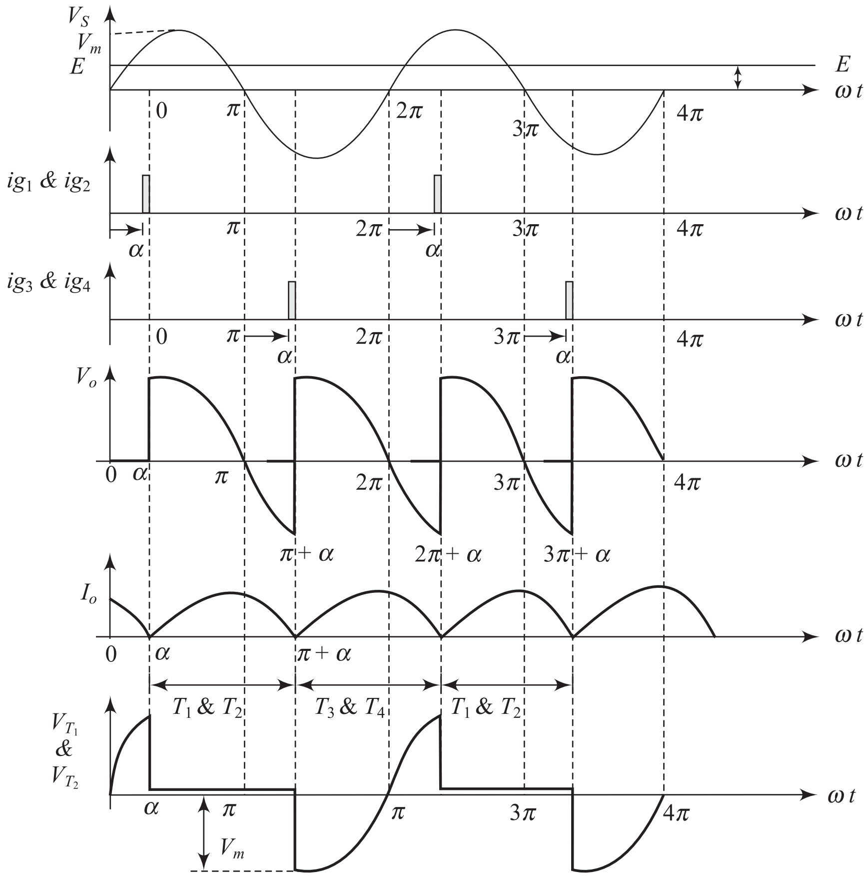


Fig. 6.47 *Boundary between continuous and discontinuous conduction*

Assume that at $\omega t = 0$, T_3 and T_4 was conducting. At $\omega t = \alpha$, thyristors T_1 and T_2 are triggered and turned on commutating T_3 and T_4 immediately. Then T_1 and T_2 conduct for $\alpha \le \beta \le \pi + \alpha$. At $\omega t = \pi + \alpha$, thyristors T_3 and T_4 are triggered and turned on commutating T_1 and T_2 immediately. Therefore, T_3 and T_4 will conduct for $\pi + \alpha \le \beta \le 2\pi + \alpha$. The conduction period of SCRs is depicted in Fig. 6.48. The voltage and current waveforms of single-phase fully controlled bridge rectifier with *RLE* load is shown in Fig. 6.48.

The voltage equation of the circuit as shown in Fig. 6.45 is

$$\sqrt{2}V \sin \omega t = L\frac{di_0}{dt} + Ri_0 + E \quad \text{at } \alpha \le \omega t \le \pi + \alpha$$

where, V is the rms voltage and output current is i_o

The output current can be expressed as

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \frac{\sin\theta}{\cos\phi}\right] - Ie^{-\frac{\omega t - \alpha}{\tan\phi}} \quad \text{at } \alpha \le \omega t \le \pi + \alpha$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$, $\tan\phi = \dfrac{\omega L}{R}$, $\sqrt{2}V \sin\theta = E$, and $R = Z\cos\phi$

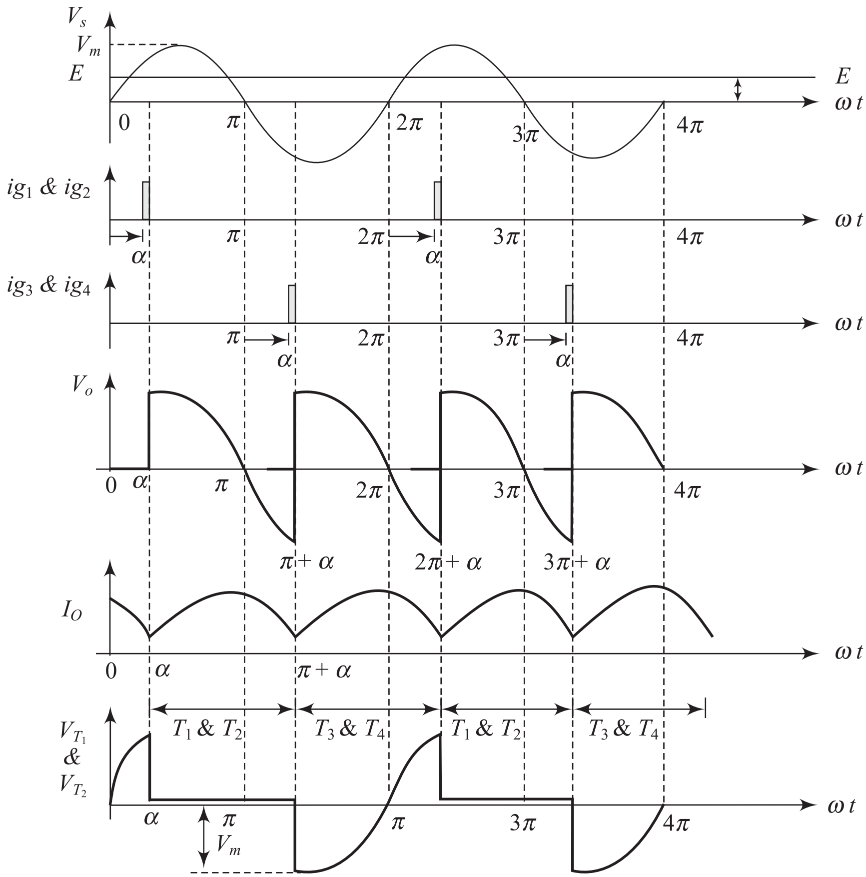


Fig. 6.48 *Voltage and current waveforms for continuous mode operation*

At steady state condition, $i_o|_{\omega t = \alpha} = i_o|_{\omega t = \pi + \alpha} = I_o$. Using the boundary condition, we get

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \frac{\sin\theta}{\cos\phi} + \frac{2\sin(\phi - \alpha)}{1 - e^{-\frac{\pi}{\tan\phi}}} e^{-\frac{\omega t - \alpha}{\tan\phi}}\right]$$

The average output voltage V_o is

$$V_o = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin\omega t \cdot d(\omega t)\right] \quad \text{where,} \quad \pi \le \omega t \le \pi + \alpha$$

$$= \frac{2\sqrt{2}V}{\pi}\cos\alpha$$

The average current is $I_O = \dfrac{V_o - E}{R} = \dfrac{V_o - \sqrt{2}V\sin\theta}{Z\cos\phi}$

6.17.3 Performance Analysis of Single-Phase Full Converter

When the inductive load is very high, the load current is continuous and its ripple content is negligible as depicted Fig. 6.49. Then the input current i can be expressed as

$i = I_o$ for $\pi \le \omega t \le \pi + \alpha$
$i = -I_o$ for $\pi + \alpha \le \omega t \le 2\pi + \alpha$

The above input current can be expressed in Fourier series form as given below.

$$i(t) = a_o + \sum_{n=1,2,3\ldots}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_o = \frac{1}{2\pi}\int_{\alpha}^{2\pi+\alpha} i \cdot d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} I_o \cdot d(\omega t) - \frac{1}{2\pi}\int_{\pi+\alpha}^{2\pi+\alpha} I_o \cdot d(\omega t) = 0$$

$$a_n = \frac{1}{\pi}\int_{\alpha}^{2\pi+\alpha} I_o \cdot \cos n\omega t \cdot d(\omega t) = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} I_o \cdot \cos n\omega t \cdot d(\omega t) - \frac{1}{\pi}\int_{\pi+\alpha}^{2\pi+\alpha} I_o \cdot \cos n\omega t \cdot d(\omega t)$$

$$= -\frac{4I_o}{n\pi}\sin n\alpha \quad \text{for } n = 1, 3, 5, \ldots$$

$$= 0 \qquad \text{for } n = 2, 4, 6 \ldots$$

$$b_n = \frac{1}{\pi}\int_{\alpha}^{2\pi+\alpha} I_o \cdot \sin n\omega t \cdot d(\omega t) = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} I_o \cdot \sin n\omega t \cdot d(\omega t) - \frac{1}{\pi}\int_{\pi+\alpha}^{2\pi+\alpha} I_o \cdot \sin n\omega t \cdot d(\omega t)$$

$$= \frac{4I_o}{n\pi}\cos n\alpha \quad \text{for } n = 1, 3, 5, \ldots$$

$$= 0 \qquad \text{for } n = 2, 4, 6 \ldots$$

The input current can be expressed as

$$i(t) = \sum_{n=1,3,5,7\ldots}^{\infty} \sqrt{2} I_n \sin(n\omega t + \phi_n)$$

where, $I_n = \frac{1}{\sqrt{2}}[a_n^2 + b_n^2]^{\frac{1}{2}}$ and $\phi_n = \tan^{-1}\frac{a_n}{b_n}$

Therefore, $I_n = \frac{1}{\sqrt{2}}[a_n^2 + b_n^2]^{\frac{1}{2}} = \frac{1}{\sqrt{2}}\left[\left(-\frac{4I_o}{n\pi}\sin n\alpha\right)^2 + \left(\frac{4I_o}{n\pi}\cos n\alpha\right)^2\right]^{\frac{1}{2}} = \frac{2\sqrt{2}I_o}{n\pi}$

and $\qquad \phi_n = \tan^{-1}\frac{a_n}{b_n} = -n\alpha$

The rms value of the fundamental current is $I_1 = \frac{2\sqrt{2}I_o}{\pi}$
The rms value of input current is

$$I_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} I_o^2\, d\omega t\right]^{\frac{1}{2}} = I_o$$

Displacement factor is $DF = \cos \phi_1 = \cos(-\alpha) = \cos \alpha$

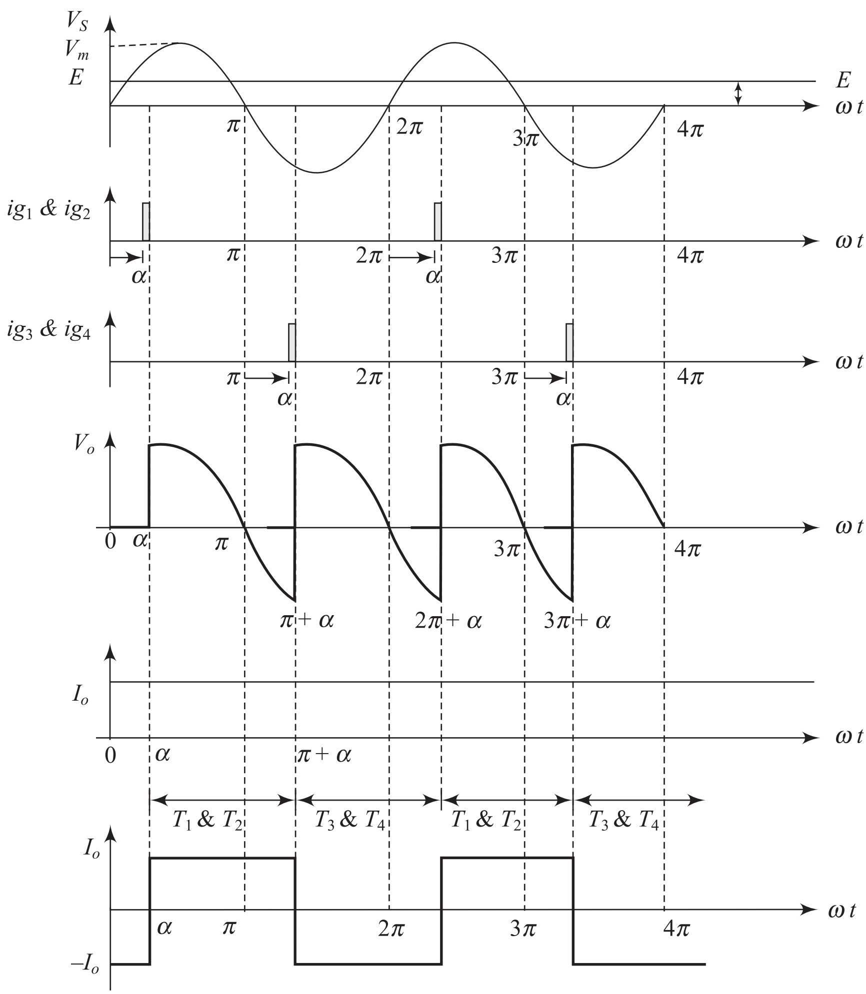


Fig. 6.49 *Voltage and current waveforms for single-phase full-wave controlled bridge rectifier at high inductive load*

Current distortion factor $CDF = \dfrac{I_1}{I_{\text{rms}}} = \dfrac{2\sqrt{2}}{\pi} = 0.90032$

Power factor $PF = CDF \times DF = \dfrac{2\sqrt{2}}{\pi}\cos\alpha$

Harmonic factor is $HF = \left[\left(\dfrac{I_{\text{rms}}}{I_1}\right)^2 - 1\right]^{\frac{1}{2}} = \left[\left(\dfrac{\pi}{2\sqrt{2}}\right)^2 - 1\right]^{\frac{1}{2}} = 0.483$

Active input power is

P_i = rms value of input voltage × rms fundamental component of input current × displacement factor

$$= V \times I_1 \times \cos\alpha = V \times \frac{2\sqrt{2}I_o}{\pi} \times \cos\alpha = \frac{2\sqrt{2}VI_o}{\pi}\cos\alpha$$

Reactive power input is

$$Q_i = V \times I_1 \times \sin\alpha = V \times \frac{2\sqrt{2}I_o}{\pi} \times \sin\alpha = \frac{2\sqrt{2}VI_o}{\pi}\sin\alpha$$

6.17.4 Inverter Mode Operation of Single-Phase full Converter

The average dc output voltage of a single-phase full converter in continuous conduction mode is

$$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha$$

If the firing angle is $\alpha < \frac{\pi}{2}$, the dc output voltage is positive. As the current flows in one direction and it's magnitude is always positive, power $P = V_O I_O > 0$ for $\alpha < \frac{\pi}{2}$. When the firing angle is $\alpha > \frac{\pi}{2}$, the dc output voltage is negative. Hence the power flow is negative or $P = V_O I_O < 0$. Then power back to ac supply from load and the converter can be operated as an inverter by increasing the firing angle α beyond $\frac{\pi}{2}$ ($\alpha > \pi/2$). Figure 6.50 shows the waveforms of the inverter operating mode.

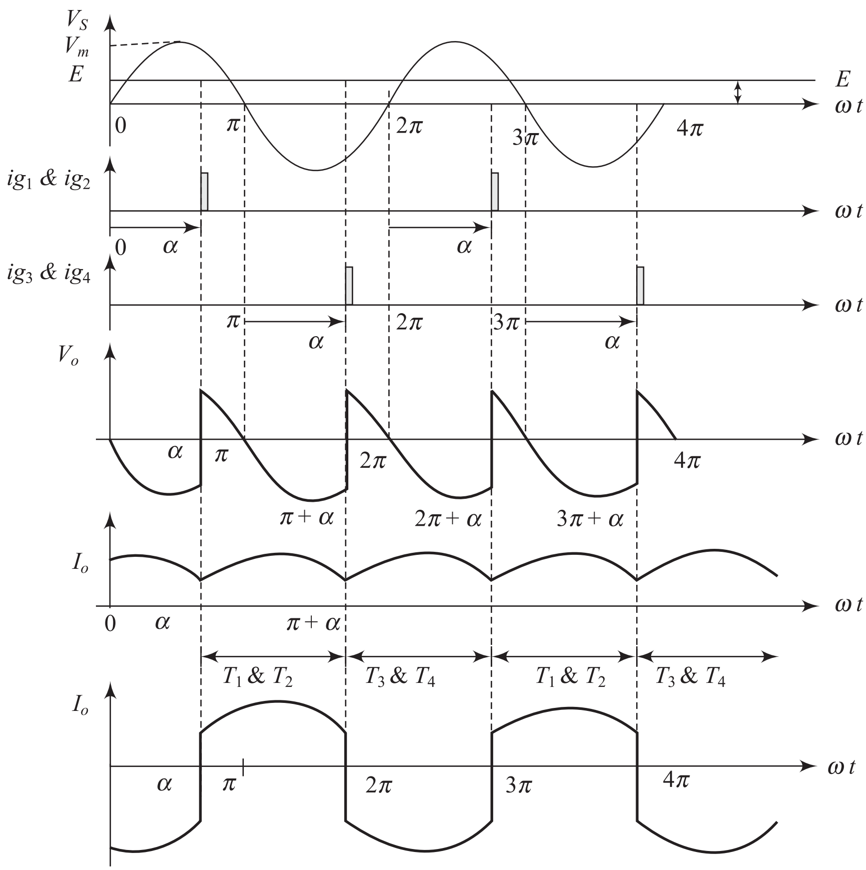


Fig. 6.50 *Voltage and current waveforms in inverter mode operation with continuous conduction*

Example 6.11 A single-phase fully controlled bridge converter with *RL* load is supplied from 220 V, 50 Hz ac supply. If the firing angle is 45°, determine (a) average output voltage, (b) displacement factor, (c) input power factor and (d) harmonic factor.

Solution

Given: $V = 220$ V, $\alpha = 45°$

(a) The average output voltage is

$$V_o = V_{av} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 220}{\pi}\cos 45 = 140.10 \text{ V}$$

(b) Displacement factor is $DF = \cos\phi_1 = \cos(-\alpha) = \cos\alpha = \cos 45 = 0.707$

(c) Current distortion factor $CDF = \frac{I_1}{I_{rms}} = \frac{2\sqrt{2}}{\pi} = 0.90032$

Power factor $PF = CDF \times DF = \frac{2\sqrt{2}}{\pi}\cos\alpha = 0.90032 \times 0.707 = 0.6365$

(d) Harmonic factor is $HF = \left[\left(\frac{I_{rms}}{I_1}\right)^2 - 1\right]^{\frac{1}{2}} = \left[\left(\frac{\pi}{2\sqrt{2}}\right)^2 - 1\right]^{\frac{1}{2}} = 0.483$

Example 6.12 A single-phase fully controlled bridge converter with *RLE* load is supplied from 230 V, 50 Hz ac supply. The average load current is 5 A which is constant over the working range. Determine the firing angle for (a) $E = 100$ V (b) $E = -100$ V. Assume $R = 4\ \Omega$ and $L = 5$ mL.

Solution

Given: $V = 230$ V, $f = 50$ Hz, $I_o = 5$ A, and $E = 100$ V

At firing angle α the average output voltage is

$$V_o = V_{av} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\cos\alpha$$

(a) When $E = 100$ V, The average current is $I_o = \frac{V_o - E}{R}$

or $$5 = \frac{V_o - 100}{4}$$

or $$V_o = 120 = \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 230}{\pi}\cos\alpha$$

Then $$\cos\alpha = \frac{120\pi}{2\sqrt{2}\times 230} = 0.579$$

or $$\alpha = 54.59°$$

(b) If $E = -100$ V, $V_o = I_oR + E = 5 \times 4 - 100 = -80$ V

or $$V_o = I_oR + E = \frac{2\sqrt{2}V}{\pi}\cos\alpha = -80 \text{ V}$$

Therefore, $$\frac{2\sqrt{2}\times 230}{\pi}\cos\alpha = -80$$

or $$\cos\alpha = \frac{-80\pi}{2\sqrt{2}\times 230} = -0.386$$

Then the firing angle is $\alpha = 112.718°$

Example 6.13 A single phase fully controlled bridge converter is connected with *RLE* load where $R = 5\ \Omega$, $L = 4$ mL and $E = 120$ V. This converter circuit is supplied from 220 V, 50 Hz ac supply. Calculate the average value of load current when the firing angle $\alpha = 60°$.

Solution

Given: $V = 230$ V, $f = 50$ Hz, and $E = 120$ V

At firing angle α the average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 220}{\pi}\cos 60 = 99.07 \text{ V}$$

The average current is $I_o = \dfrac{V_o - E}{R} = \dfrac{99.07 - 50}{5}$ A = 9.81 A

Example 6.14 A single-phase fully controlled bridge converter is connected to *RLE* load with $R = 5\ \Omega$, $L = 6$ mL and $E = 60$ V. This converter is supplied from 220 V, 50 Hz ac supply. (a) Determine the average load current at $\alpha = 45°$. (b) In the bridge circuit, one thyristor is open circuited due to fault. At this condition what will be the average load current.

Solution

Given: $V = 220$ V, $f = 50$ Hz, $\alpha = 45°$, $R = 5\ \Omega$, $L = 6$ mL and $E = 60$ V

(a) When all four thyristors are healthy, the average output voltage at firing angle α is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\cos\alpha$$

$$= \frac{2\sqrt{2}\times 220}{\pi}\cos 45 = 140.10 \text{ V}$$

As $E = 60$ V, the average current is $I_o = \dfrac{V_o - E}{R} = \dfrac{140.10 - 60}{5}\text{A} = 16.02 \text{ A}$

(b) When one thyristor is open circuited due to fault in the bridge circuit, the average output voltage at firing angle α is

$$V_o = V_{\text{av}} = \frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi}\cos\alpha$$

$$= \frac{\sqrt{2}\times 220}{\pi}\cos 45 = 70.05 \text{ V}$$

Since $E = 60$ V, the average current is $I_o = \dfrac{V_o - E}{R} = \dfrac{70.05 - 60}{5}\text{A} = 2.01 \text{ A}$

Example 6.15 A single-phase fully controlled bridge converter is connected to *RLE* load and it is supplied from 220 V, 50 Hz ac supply. The average load current is 7.5 A which is constant over the working range. Determine the firing angle for (a) $E = 110$ V (b) $E = -110$ V. Assume $R = 5\ \Omega$ and $L = 5$ mL. Specify which source is delivering power to load in case (a) and (b). When the output current is constant, determine the input power factor for both cases (a) and (b).

Solution

Given: $V = 220$ V, $f = 50$ Hz, $I_o = 7.5$ A, and $E = 110$ V

At firing angle α the average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\cos\alpha$$

(a) When E = 110 V, The average current is $I_o = \frac{V_o - E}{R}$

or $$7.5 = \frac{V_o - 110}{5}$$

or $$V_o = 147.5 = \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 220}{\pi}\cos\alpha$$

Then $\cos\alpha = \frac{147.5\pi}{2\sqrt{2}\times 220} = 0.744$

or $\alpha = 41.89°$

Since α is less than 90°, power flows from ac source to dc load

(b) If E = –110 V, $V_o = I_oR + E$ = 7.5 × 5 – 110 = –72.5 V

or $$V_o = I_oR + E = \frac{2\sqrt{2}V}{\pi}\cos\alpha = -72.5 \text{ V}$$

Therefore, $\frac{2\sqrt{2}\times 220}{\pi}\cos\alpha = -72.5$

or $$\cos\alpha = \frac{-72.5\pi}{2\sqrt{2}\times 220} = -0.3659$$

Then the firing angle is α = 111.463°

Since α is greater than 90°, power flows from dc load to ac source.

(c) Since the load current is constant, the rms value of load current is $I_{rms} = I_o$ = 7.5 A

We know that $VI_{rms}\cos\phi = EI_o + I_o^2R$

At $\alpha = 41.89°, \cos\phi = \frac{EI_o + I_o^2R}{VI_{rms}} = \frac{110\times 7.5 + 7.5^2\times 5}{220\times 7.5} = 0.67$ lag

At $\alpha = 111.463°, \cos\phi = \frac{EI_o - I_o^2R}{VI_{rms}} = \frac{110\times 7.5 - 7.5^2\times 5}{220\times 7.5} = 0.329$ lag

Example 6.16 A single-phase bridge controlled rectifier consists of a thyristor and three diodes and it is supplied by 220 V, 50 Hz ac supply. If the firing angle of thyristor is 30°, determine the average output current and power delivered to battery when *RLE* load consists of R = 4 Ω, L = 5 mH, and E = 120 V. Assume current is constant.

Solution

If the firing angle of thyristor is α, during the positive half cycle of supply voltage thyristor conducts from α to π only. During the negative half cycle of supply voltage two diodes conduct from π to 2π.

The average output voltage across load is equal to

$$V_o = \frac{1}{2\pi}\left[\int_\alpha^\pi \sqrt{2}V\sin\omega t\cdot d(\omega t) + \int_\pi^{2\pi}\sqrt{2}V\sin\omega t\cdot d(\omega t)\right]$$

$$= \frac{\sqrt{2}V}{2\pi}(3+\cos\alpha) = \frac{\sqrt{2}\times 220}{2\pi}(3+\cos 30°)\text{V} = 191.50 \text{ V}$$

The average output load current is $I_o = \frac{V_o - E}{R} = \frac{191.50 - 120}{4}\text{A} = 17.875 \text{ A}$

Power delivered to battery = EI_o = 120 × 17.875 = 2145 Watt

Example 6.17 A single-phase fully controlled bridge converter is supplied from 220 V, 50 Hz ac supply and fed to a load which consists of R = 12 Ω and large inductance so that the load current is constant. If the firing angle is 45°, calculate (a) average output voltage, (b) average output current, (c) average current of thyristor, (d) rms current of thyristor and (e) power factor.

Solution

Given: V = 220 V, f = 50 Hz, R = 12 ΩA and α = 45°

(a) At firing angle α the average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} \sqrt{2}V \sin\omega t \cdot d\omega t \qquad = \frac{2\sqrt{2}V}{\pi}\cos\alpha$$

$$= \frac{2\sqrt{2}\times 220}{\pi}\cos 45 = 140.1 \text{ V}$$

(b) The average current is $I_o = \dfrac{V_o}{R} = \dfrac{140.1}{12}$ = 11.675 A

(c) The average current of thyristor is $I_{T\text{av}} = \dfrac{I_o}{2} = \dfrac{11.675}{2} = 5.8375 \text{ A}$

(d) The rms current of thyristor is $I_{T\text{rms}} = \dfrac{I_o}{\sqrt{2}} = \dfrac{11.675}{\sqrt{2}} = 8.256 \text{ A}$

(e) rms value of source current $I_{\text{rms}} = I_o$ = 11.675 A

DC output is $P_{\text{dc}} = V_o I_o = 140.1 \times 11.675 = 1635.66$ Watt

Power factor $\cos\phi = \dfrac{V_o I_o}{VI_{\text{rms}}} = \dfrac{140.1\times 11.675}{220\times 11.675} = 0.641$ lag

Example 6.18 In the above example, if the source inductance is about 1.4 mH, find the average output voltage, overlap angle and power factor.

Solution

The average output voltage is equal to

$$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha - \frac{\omega_o L_s}{\pi} I_o$$

$$= \frac{2\sqrt{2}\times 220}{\pi}\cos 45 - \frac{2\pi\times 50\times 1.4\times 10^{-3}}{\pi}\times 11.675 = 138.4655 \text{ V}$$

Average load current $I_o = \dfrac{\sqrt{2}V}{\omega L_s}[\cos\alpha - \cos(\alpha+\mu)]$

or
$$11.675 = \frac{\sqrt{2}\times 220}{2\pi\times 50\times 1.4\times 10^{-3}}[\cos 45 - \cos(45+\mu)]$$

or
$$11.675 = 707.64[\cos 45 - \cos(45+\mu)]$$

The overlap angle μ = 1.32°

The power factor $\cos\phi = \dfrac{V_o I_o}{VI_{\text{rms}}} = \dfrac{138.4655\times 11.675}{220\times 11.675} = 0.629$ lag

Example 6.19 A single-phase fully controlled bridge converter is supplied from a 220 V, 50 Hz single-phase supply and operates in the continuous conduction mode at a firing angle α = 45°. If the load resistance and inductances are 5 Ω and 20 mH respectively, determine (a) average dc output voltage, (b) average output load current, (c) third harmonic load current as a percentage of the average load current.

Solution

(a) The average dc output voltage is

$$V_{oav} = \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 220}{\pi}\cos 45 = 140.106 \text{ V}$$

(b) Average output load current is

$$I_{oav} = \frac{V_{oav}}{R} = \frac{140.106}{5} = 28.08 \text{ A}$$

(c) The dc voltage waveform is periodic over half the input cycle. From the Fourier series analysis of output voltage, we obtain

$$v_o = V_{oav} + \Sigma V_{an}\cos 2n\omega t + V_{bn}\sin 2\omega t$$

and
$$V_{an} = \frac{2}{\pi}\int_0^{\pi} v_o \cos 2n\omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\left[\frac{\cos(2n+1)\alpha}{2n+1} - \frac{\cos(2n-1)\alpha}{2n-1}\right]$$

$$V_{bn} = \frac{2}{\pi}\int_0^{\pi} v_o \sin 2n\omega t \cdot d\omega t = \frac{2\sqrt{2}V}{\pi}\left[\frac{\sin(2n+1)\alpha}{2n+1} - \frac{\sin(2n-1)\alpha}{2n-1}\right]$$

$$V_{a3} = \frac{2\sqrt{2}\times 220}{\pi}\left[\frac{\cos(2\times 3+1)\times 45}{2\times 3+1} - \frac{\cos(2\times 3-1)\times 45}{2\times 3-1}\right] = 48.036 \text{ V}$$

$$V_{b3} = \frac{2\sqrt{2}\times 220}{\pi}\left[\frac{\sin(2\times 3+1)\times 45}{2\times 3+1} - \frac{\sin(2\times 3-1)\times 45}{2\times 3-1}\right] = 8.004 \text{ V}$$

Third harmonic impedance is

$$Z_3 = \sqrt{R^2 + (2\pi fL)^2} = \sqrt{5^2 + (2\pi\times 50\times 3\times 20\times 10^{-3})^2} = 19.49\ \Omega$$

Third harmonic rms voltage is

$$V_{3\text{rms}} = \frac{\sqrt{V_{a3}^2 + V_{b3}^2}}{\sqrt{2}} = \frac{\sqrt{48.036^2 + 8.004^2}}{\sqrt{2}} = 34.44 \text{ V}$$

Third harmonic current

$$I_{3\text{rms}} = \frac{V_{3\text{rms}}}{Z_3} = \frac{34.44}{19.49} = 1.767 \text{ A}$$

Example 6.20 A single-phase fully controlled bridge converter is connected to 220 V, 50 Hz. A load of R = 10 Ω is connected in series with a large inductance and load current is ripple free. If the firing angle of converter is 60°, determine different performance parameters of the converter.

Solution

The output voltage $V_o = \dfrac{2\sqrt{2}V}{\pi}\cos\alpha = \dfrac{2\sqrt{2}\times 220}{\pi}\cos 60 \text{ V} = 99.08 \text{ V}$

Load current $I_o = \dfrac{V_o}{R} = \dfrac{99.08}{10} = 9.908 \text{ A}$

DC power output $P_o = V_o I_o = 99.08 \times 9.908 = 981.68$ Watt

RMS value of load current $I_{\text{rms}} = I_o = 9.908$ A

AC power = $P_{\text{ac}} = VI_{\text{rms}} = 220 \times 9.908 = 2179.76$ Watt

Rectification efficiency = $\eta = \dfrac{P_{\text{dc}}}{P_{\text{ac}}} = \dfrac{P_o}{P_{\text{ac}}} = \dfrac{981.68}{2179.76} = 45.03\%$

Form factor (FF) = $\dfrac{V}{V_o} = \dfrac{220}{99.08} = 2.22$

Voltage ripple factor is RF = $\sqrt{FF^2 - 1} = \sqrt{2.22^2 - 1} = 1.982$

Since the load current is ripple free, current ripple factor CRF = 0

RMS value of fundamental current = $I_1 = \frac{2\sqrt{2}}{\pi} I_o = \frac{2\sqrt{2}}{\pi} \times 9.908 = 8.924$ A

$\phi_1 = -\alpha = -60°$

Displacement factor $DF = \cos\phi_1 = \cos(-\alpha) = \cos\alpha = \cos 60 = 0.5$

Current distortion factor $CDF = \frac{I_1}{I_{\text{rms}}} = \frac{8.924}{9.908} = 0.9006$

Power factor = $PF = CDF \times DF = 0.9006 \times 0.5 = 0.4503$

Active power input = $VI_1 \cos\alpha = 220 \times 8.924 \times \cos 60 = 981.64$ Watt

Reactive power input = $VI_1 \sin\alpha = 220 \times 8.924 \times \sin 60 = 1700.25$ VAR

6.18 SINGLE-PHASE HALF-CONTROLLED BRIDGE RECTIFIER WITH *R* LOAD

When two thyristors of full converter are replaced by diodes as shown in Fig. 6.51, the new converter circuit is known as half-controlled bridge rectifier. There are different types of half controlled bridge rectifier such as asymmetrical configurations (common cathode, common anode) and asymmetrical configurations. But the common cathode symmetrical type half bridge converter is most commonly used as a single triggering circuit can be used to turn ON both thyristors T_1 and T_3 as depicted in Fig. 6.51(a).

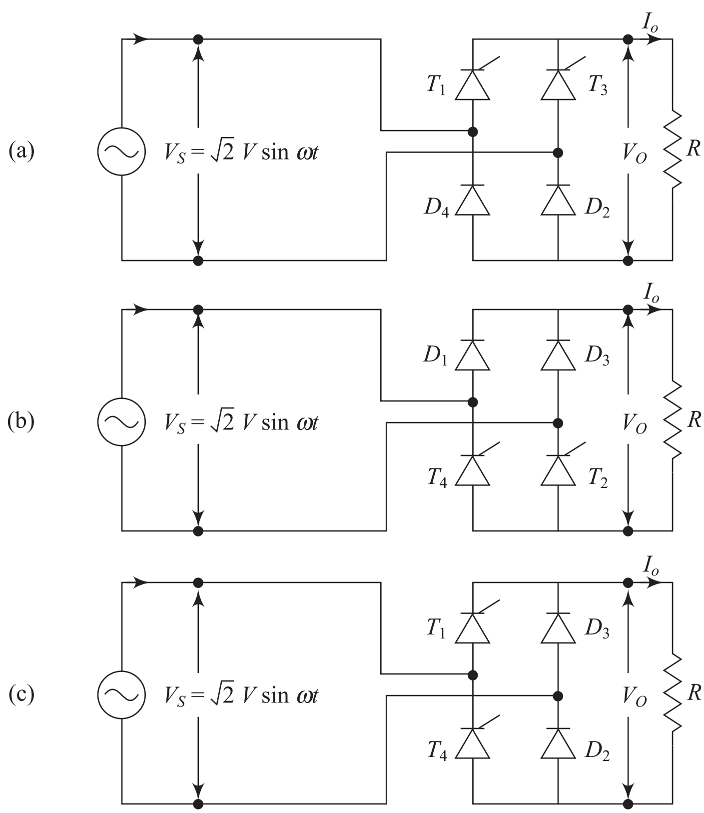


Fig. 6.51 *Single-phase half controlled bridge rectifier (a) common-cathode, (b) common anode and (c) asymmetrical*

In the positive half cycle of input voltage, thyristor T_1 is forward biased and operates in forward blocking state. As soon as a triggering pulse is applied at $\omega t = \alpha$, the thyristor T_1 becomes ON and supply voltage is connected across resistance. Then current flows though T_1 and D_2. At $\omega t = \pi$, the thyristor T_1 is reverse biased and it is turned OFF.

During the negative half cycle of input voltage, thyristor T_3 is forward biased. When a triggering pulse is applied at $\omega t = \pi + \alpha$, the thyristor T_3 becomes ON and supply voltage is applied across resistance. Subsequently current flows though T_3 and D_4. At $\omega t = 2\pi$, the thyristor T_3 is reverse biased and it is turned OFF. The voltage and current wave forms are depicted in Fig. 6.52.

The average output voltage is

$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi}(1 + \cos \alpha)$$

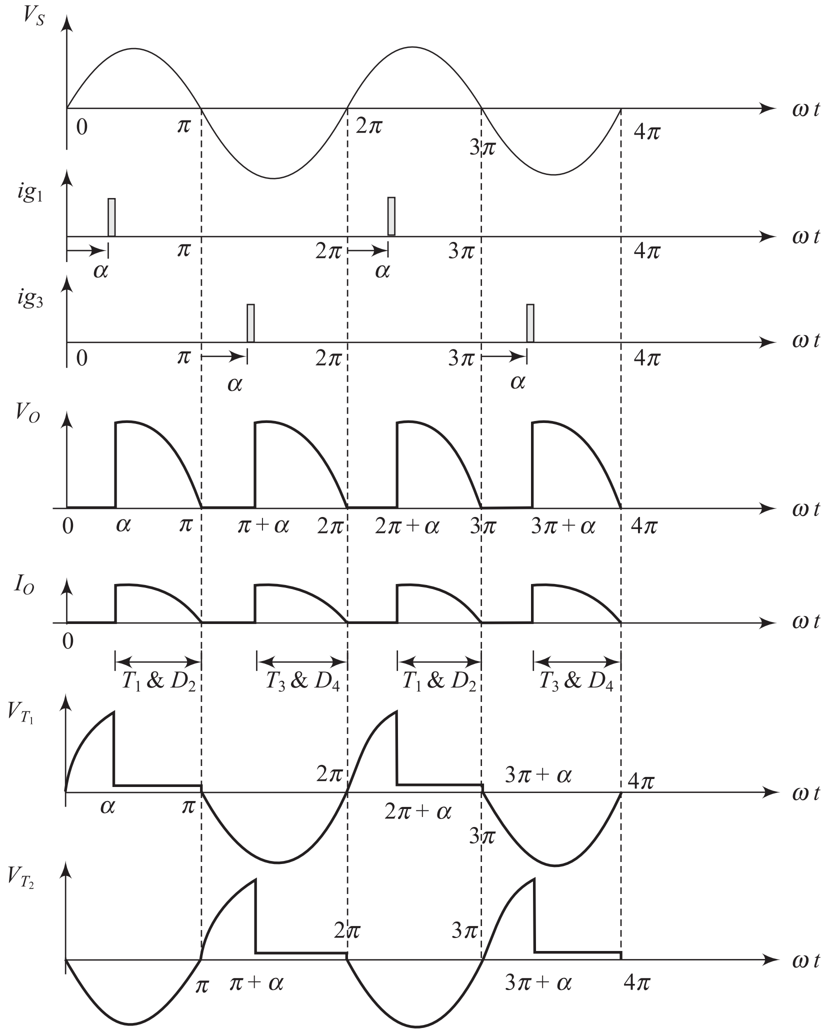


Fig. 6.52 *Voltage and current waveforms of single-phase half controlled bridge rectifier with R load*

The average or dc load current is

$$I_o = I_{\text{av}} = \frac{V_o}{R} = \frac{\sqrt{2}V}{\pi R}(1+\cos\alpha)$$

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2\,d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}2V^2\sin^2\omega t\cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V^2}{\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= V\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

The rms output current is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{V}{R}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$

The form factor is $FF = \frac{V_{\text{rms}}}{V_{\text{av}}} = V\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}} \bigg/ \frac{\sqrt{2}V}{\pi}(1+\cos\alpha)$

$$= \frac{\left[\pi\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}}{\sqrt{2}(1+\cos\alpha)}$$

The load ripple factor is $RF = \sqrt{FF^2-1} = \left[\frac{\pi\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)}{2(1+\cos\alpha)^2}-1\right]^{\frac{1}{2}}$

The dc output power is $P_{\text{dc}} = V_{\text{av}}I_{\text{av}} = V_oI_o$

$$= \frac{\sqrt{2}V}{\pi}(1+\cos\alpha)\times\frac{\sqrt{2}V}{\pi R}(1+\cos\alpha) = \frac{2V^2}{\pi^2 R}(1+\cos\alpha)^2$$

The ac output power is $P_{\text{ac}} = V_{\text{rms}}\,I_{\text{rms}}$

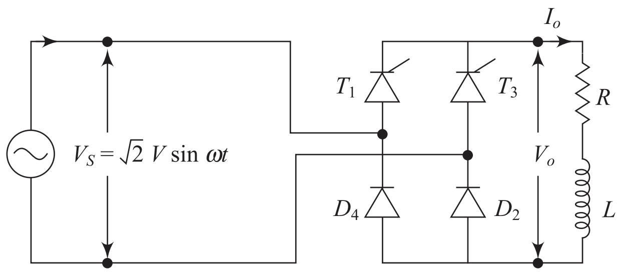
$$= V\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}\times\frac{V}{R}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

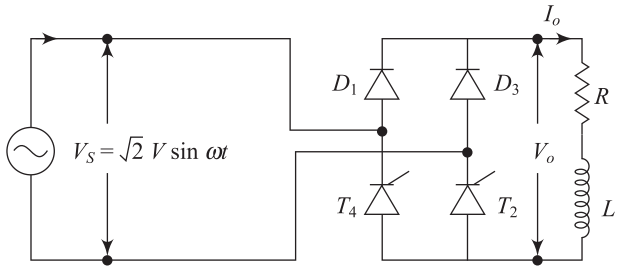
$$= \frac{V^2}{R}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right] = \frac{V^2}{\pi R}\left[\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]$$

6.19 SINGLE-PHASE HALF-CONTROLLED BRIDGE RECTIFIER WITH *RL* LOAD

The common cathode, common anode and asymmetrical configurations of single-phase half-wave controlled bridge rectifier with *RL* are depicted in Fig. 6.53(a), (b) and (c) respectively. Due to presence of inductance, the load current will flows at the end of each half cycle of input voltage.

During the positive half cycle of input voltage, thyristor T_1 conducts and load current flows through T_1 and D_2 from $\omega t = \alpha$ to $\omega t = \pi$ in Fig. 6.53(a). After $\omega t = \pi$, the input voltage becomes negative passes through zero crossing and diode D_4 comes into conduction commutating diode D_2. Then load current flows though T_1 and diode D_4 and it decays exponentially. As soon as thyristor T_3 is triggered





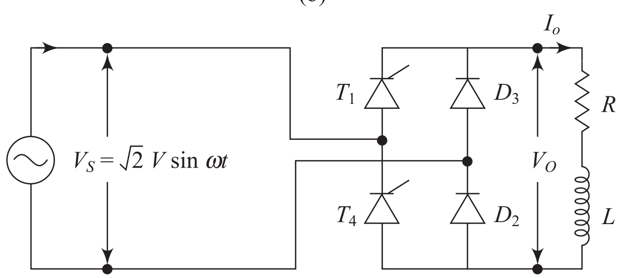


Fig. 6.53 *Single-phase half-controlled bridge rectifier with RL load (a) common-cathode, (b) common anode and (c) asymmetrical configuration*

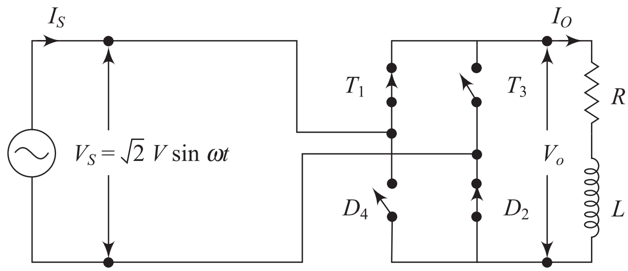
in the negative half cycle of supply voltage, thyristor T_1 will be turned OFF. Then load current flows though T_3 and D_4 from $\omega t = \pi + \alpha$ to $\omega t = 2\pi$. After $\omega t = 2\pi$, the input voltage becomes positive passes through zero crossing and diode D_2 comes into conduction commutating diode D_4. Subsequently, the load current flows though T_3 and D_2 and it again decays exponentially. After that the cycle operation will be repetitive after triggering T_1. Figure 6.54 shows the switching of thyristors T_1, T_3, and diodes D_2 and D_4. This circuit can able to operate in continuous and discontinuous mode. Voltage and current waveforms for continuous load current mode operation is depicted in Figs. 6.55 and 6.56 shows the voltage and current waveforms for discontinuous load current.

The average output voltage is

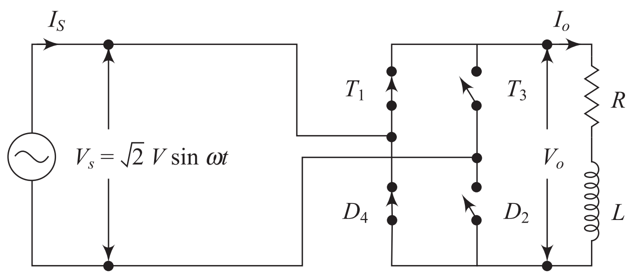
$$V_o = V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi}(1 + \cos \alpha)$$

The average or dc load current is

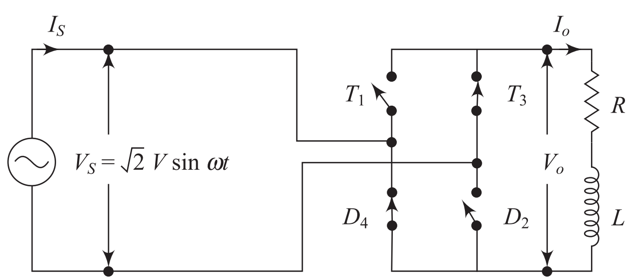
$$I_o = I_{\text{av}} = \frac{V_o}{R} = \frac{\sqrt{2}V}{\pi R}(1 + \cos \alpha)$$



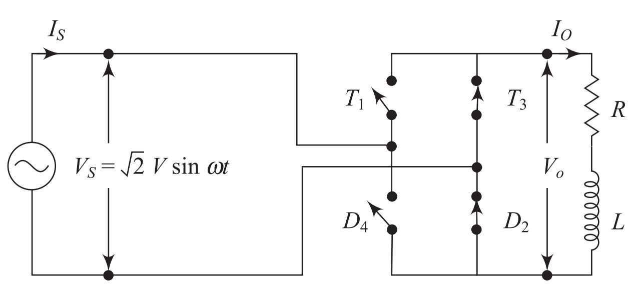
(a) T_1 and D_1 ON during α to π



(b) T_1 and D_4 ON during π to $\pi + \alpha$



(c) T_3 and D_4 ON during $\pi + \alpha$ to 2π



(d) T_3 and D_2 ON during 2π to $2\pi + \alpha$

Fig. 6.54 *Switching of thyristors T_1, T_3, and diodes D_2 and D_4 (a) $\alpha \le \omega t \le \pi$ (b) $\pi \le \omega t \le \pi + \alpha$ (c) $\pi + \alpha \le \omega t \le 2\pi$ and (d) $2\pi \le \omega t \le 2\pi + \alpha$*

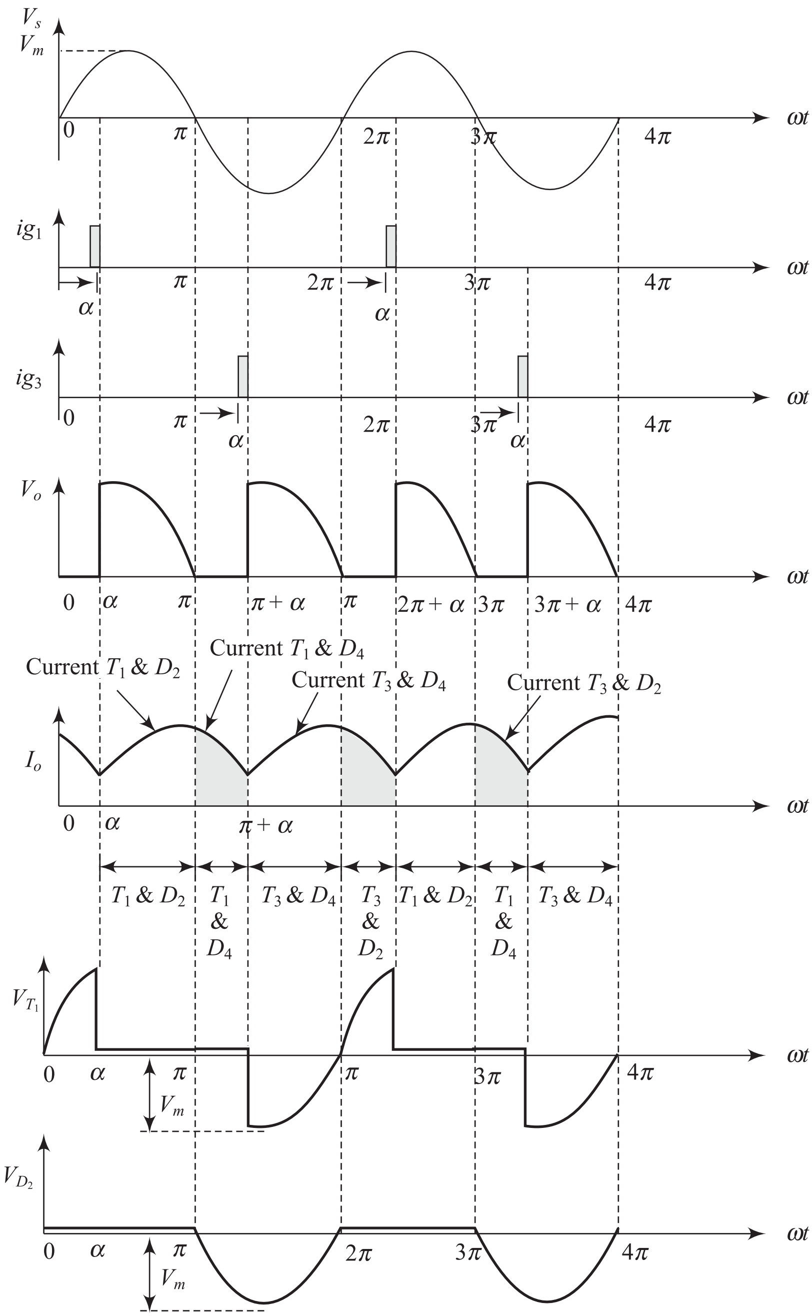


Fig. 6.55 *Voltage and current waveforms for continuous load current*

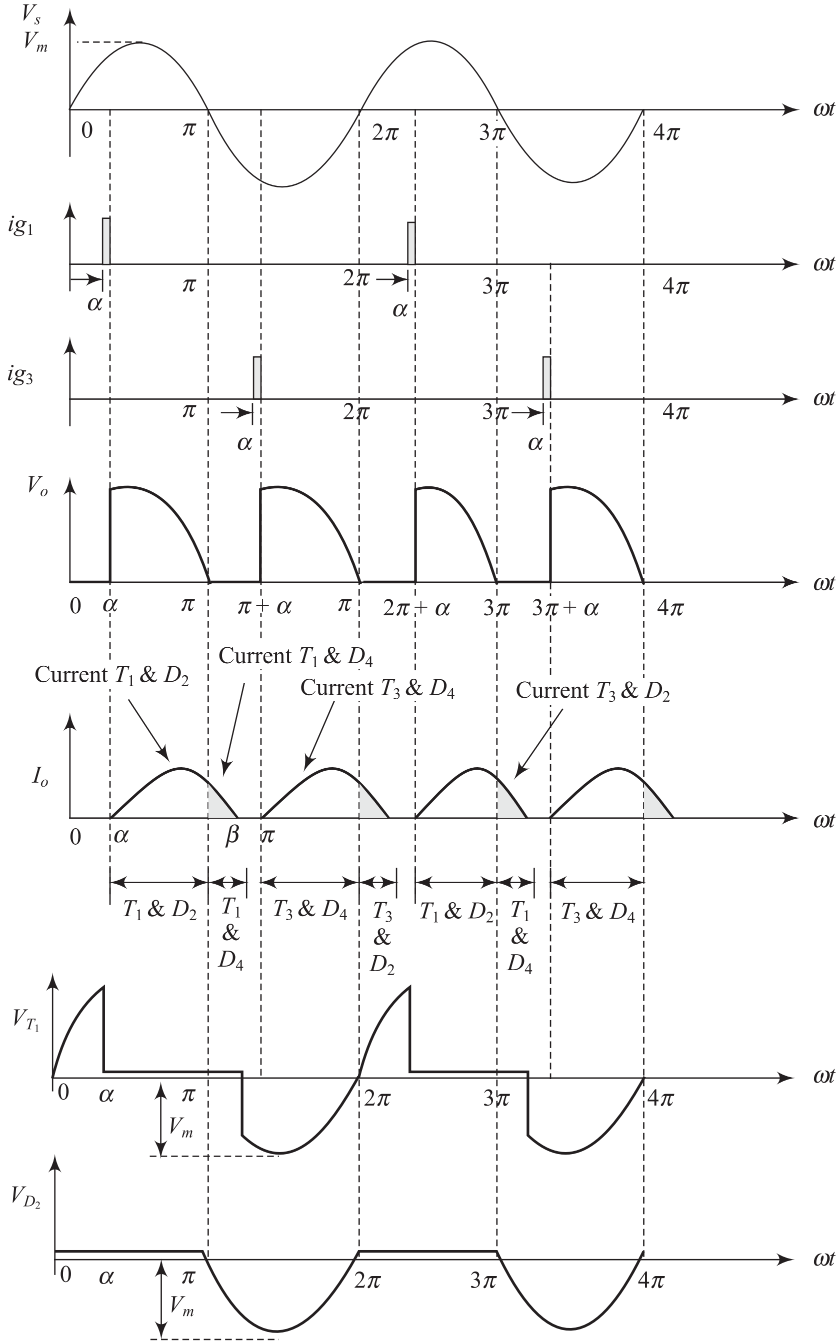


Fig. 6.56 *Voltage and current waveforms for discontinuous load current*

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2\,d\omega t\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}2V^2\sin^2\omega t\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V^2}{2\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \sqrt{2}V\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

The rms output current is $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{\sqrt{2}V}{R}\left[\dfrac{1}{2\pi}\left(\pi-\alpha+\dfrac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$

6.20 SINGLE-PHASE HALF-CONTROLLED BRIDGE RECTIFIER WITH *RLE*

Figure 6.57 shows the circuit configuration of a single-phase half controlled bridge rectifier with resistance (R), inductance (L) and a back emf (E). In this circuit, load current flows through either T_1 or T_3 and any one diode of D_2 and D_4. If the firing of SCRs is α, in the positive half cycle of input voltage T_1 and D_2 conduct during $\alpha \le \omega t \le \pi$. Similarly, in the negative half cycle of input voltage T_3 and D_4 conduct during $\pi + \alpha \le \omega t \le 2\pi$. Whenever T_1 or T_3 is in ON state and the output voltages tend to negative, T_1 and D_4 and T_3 and D_2 conduct. T_1 and D_4 conducts during $\pi \le \omega t \le \pi + \alpha$ and T_3 and D_2 conducts during $2\pi \le \omega t \le 2\pi + \alpha$. Figure 6.58 shows the conduction of thyristors and diodes. This circuit can be operating in continuous conduction mode and discontinuous conduction mode. In continuous conduction mode, the load current is always positive and two devices must be conduct. During discontinuous conduction mode, the load current will be zero for certain time and none of the four devices conduct.

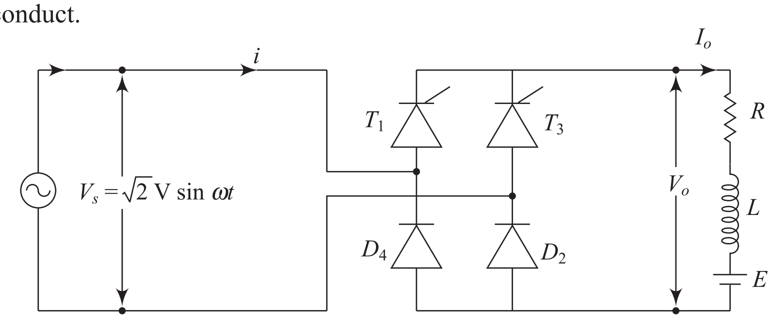


Fig. 6.57 *Single-phase full-wave controlled bridge rectifier with RLE load*

6.20.1 Continuous Load Current

During continuous load current mode operation, the load current i_o is always greater than zero and the converter is said to operate in the continuous conduction mode. In this mode of operation, diode D_2 and D_4 conduct for the positive and negative half cycle of input voltage respectively. T_1 starts conduction when a firing pulse is applied in positive half cycle of input voltage and conducts till T_3 is fired in the negative half cycle. The voltage and current waveforms of a single-phase half-controlled converter with *RLE* load are depicted in Fig. 6.58.

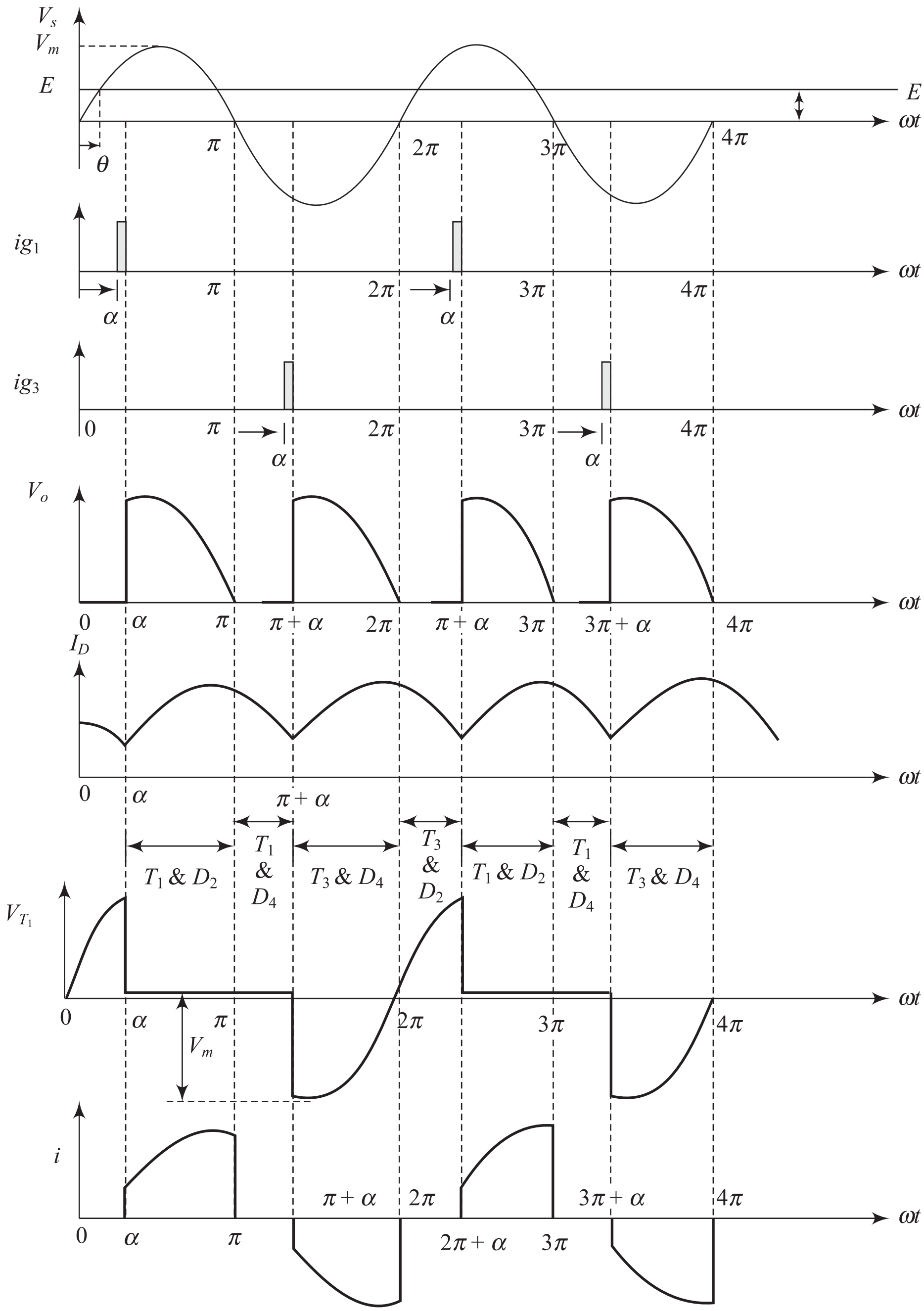


Fig. 6.58 *Voltage and current waveforms for continuous mode operation*

The average output voltage V_o is

$$V_o = \frac{1}{\pi}\left[\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot d(\omega t)\right] \quad \text{where, } \alpha \le \omega t \le \pi$$

$$= \frac{\sqrt{2}V}{\pi}(1 + \cos \alpha)$$

The average current is $I_O = \dfrac{V_o - E}{R} = \dfrac{V_o - \sqrt{2}V \sin \theta}{R}$ as $\sqrt{2}V \sin \theta = E$

$$= \frac{\sqrt{2}V}{\pi R}(1 + \cos \alpha) - \frac{\sqrt{2}V \sin \theta}{R}$$

$$= \frac{\sqrt{2}V}{\pi R}(1 + \cos \alpha - \pi \sin \theta)$$

The voltage equation of the circuit as shown in Fig. 6.58 can be expressed as

$$\sqrt{2}V \sin \omega t = L\frac{di_0}{dt} + Ri_0 + E \quad \text{at } \alpha \le \omega t \le \pi$$

where, V is the rms voltage and output current is i_o.
The output current can be expressed as

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \frac{\sin \theta}{\cos \phi}\right] - Ie^{-\frac{\omega t - \alpha}{\tan \phi}} \quad \text{at } \alpha \le \omega t \le \pi$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$, $\tan \phi = \dfrac{\omega L}{R}$, $\sqrt{2}V \sin \theta = E$, and $R = Z \cos \phi$

At $\omega t = \alpha$, current $i_o|_{\omega t = \alpha} = \dfrac{\sqrt{2}V}{Z}\left[\sin(\alpha - \phi) - \dfrac{\sin \theta}{\cos \phi}\right] - I$

At $\omega t = \pi$, current $i_o|_{\omega t = \pi} = \dfrac{\sqrt{2}V}{Z}\left[\sin(\pi - \phi) - \dfrac{\sin \theta}{\cos \phi}\right] - Ie^{-\frac{\pi - \alpha}{\tan \phi}}$

During $\pi \le \omega t \le \pi + \alpha$, the voltage equation is

$$0 = L\frac{di_0}{dt} + Ri_0 + E \quad \text{at } \pi \le \omega t \le \pi + \alpha$$

The current can be expressed as

$$i_o(t) = -\frac{\sqrt{2}V}{Z}\frac{\sin \theta}{\cos \phi}\left[1 - e^{-\frac{\omega t - \pi}{\tan \phi}}\right] - i_o|_{\omega t = \pi}\, e^{-\frac{\omega t - \alpha}{\tan \phi}}$$

After substituting the value of $i_o|_{\omega t = \pi}$ in the above equation, we get

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin \phi \cdot e^{-\frac{\omega t - \pi}{\tan \phi}} - \frac{\sin \theta}{\cos \phi}\right] + I_1 e^{-\frac{\omega t - \alpha}{\tan \phi}}$$

At $\omega t = \pi + \alpha$, $i_o|_{\omega t = \pi + \alpha} = \dfrac{\sqrt{2}V}{Z}\left[\sin \phi \cdot e^{-\frac{\alpha}{\tan \phi}} - \dfrac{\sin \theta}{\cos \phi}\right] + I_1 e^{-\frac{\pi}{\tan \phi}}$

At steady state condition, $i_o|_{\omega t=\alpha} = i_o|_{\omega t=\pi+\alpha}$, $I_1 = \frac{\sqrt{2}V}{Z}\left[\frac{\sin(\phi-\alpha)+\sin\phi\cdot e^{-\frac{\alpha}{\tan\phi}}}{1-e^{-\frac{\pi}{\tan\phi}}}\right]$

Using the boundary condition, we get

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\{\sin(\phi-\alpha)+\sin\phi e^{-\frac{\alpha}{\tan\phi}}\}\frac{e^{-\frac{\omega t-\alpha}{\tan\phi}}}{1-e^{-\frac{\pi}{\tan\phi}}}+\sin(\omega t-\phi)-\frac{\sin\theta}{\cos\phi}\right] \quad \text{for } \alpha \le \omega t \le \pi$$

and

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\{\sin(\phi-\alpha)+\sin\phi e^{-\frac{\alpha}{\tan\phi}}\}\frac{e^{-\frac{\omega t-\alpha}{\tan\phi}}}{1-e^{-\frac{\pi}{\tan\phi}}}+\sin\phi e^{-\frac{\omega t-\pi}{\tan\phi}}-\frac{\sin\theta}{\cos\phi}\right] \quad \text{for } \pi \le \omega t \le \pi+\alpha$$

6.20.2 Discontinuous Load Current

The voltage and current wave forms for discontinuous current mode operation are shown in Fig. 6.59. In discontinuous current mode operation, the load current i_o is zero for certain time period. During this time interval, none of thyristors and diodes conducts. When SCR T_1 is triggered at $\omega t = \alpha$ and output voltage is larger than E, the load current i_o starts from zero and continues to increase till $\omega t = \pi - \theta$ where output voltage is equal to E. After $\omega t = \pi - \theta$, the output voltage v_o falls below the back emf E and i_o starts to decrease. At $\omega t = \beta$, i_o becomes zero before T_3 is triggered at $\omega t = \pi + \alpha$. Therefore, none of the devices conduct during $\beta \le \omega t \le \pi + \alpha$ and the output voltage is $v_o = E$ as shown in Fig. 6.59.

The output voltage is

$$v_o = \sqrt{2}V\sin\omega t \qquad \textit{for } \alpha \le \omega t \le \pi$$
$$v_o = 0 \qquad \textit{for } \pi \le \omega t \le \beta$$
$$v_o = E \qquad \text{for } \beta \le \omega t \le \pi + \alpha$$

The average output voltage V_o is

$$V_o = \frac{1}{\pi}\left[\int_\alpha^\pi \sqrt{2}V\sin\omega t\cdot d(\omega t)+\int_\beta^{\pi+\alpha} E d\omega t\right] = \frac{1}{\pi}\left[\int_\alpha^\pi \sqrt{2}V\sin\omega t\cdot d(\omega t)+\int_\beta^{\pi+\alpha}\sqrt{2}V\sin\theta d\omega t\right]$$

$$= \frac{\sqrt{2}V}{\pi}[1+\cos\alpha+(\pi+\alpha-\beta)\sin\theta] \quad \text{as } \sqrt{2}V\sin\theta = E$$

The average current is $I_O = \frac{V_o - E}{R} = \frac{V_o - \sqrt{2}V\sin\theta}{R}$

The voltage equation of the circuit as shown in Fig. 6.59 is

$$\sqrt{2}V\sin\omega t = L\frac{di_0}{dt} + Ri_0 + E \quad \text{at } \alpha \le \omega t \le \pi$$

and $i_o = 0$ at $\omega t = \alpha$

where, V is the rms voltage and output current is i_o

The output current can be expressed as

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t-\phi)-\frac{\sin\theta}{\cos\phi}\right]+I_o e^{-\frac{\omega t-\alpha}{\tan\phi}} \quad \text{at } \alpha \le \omega t \le \pi$$

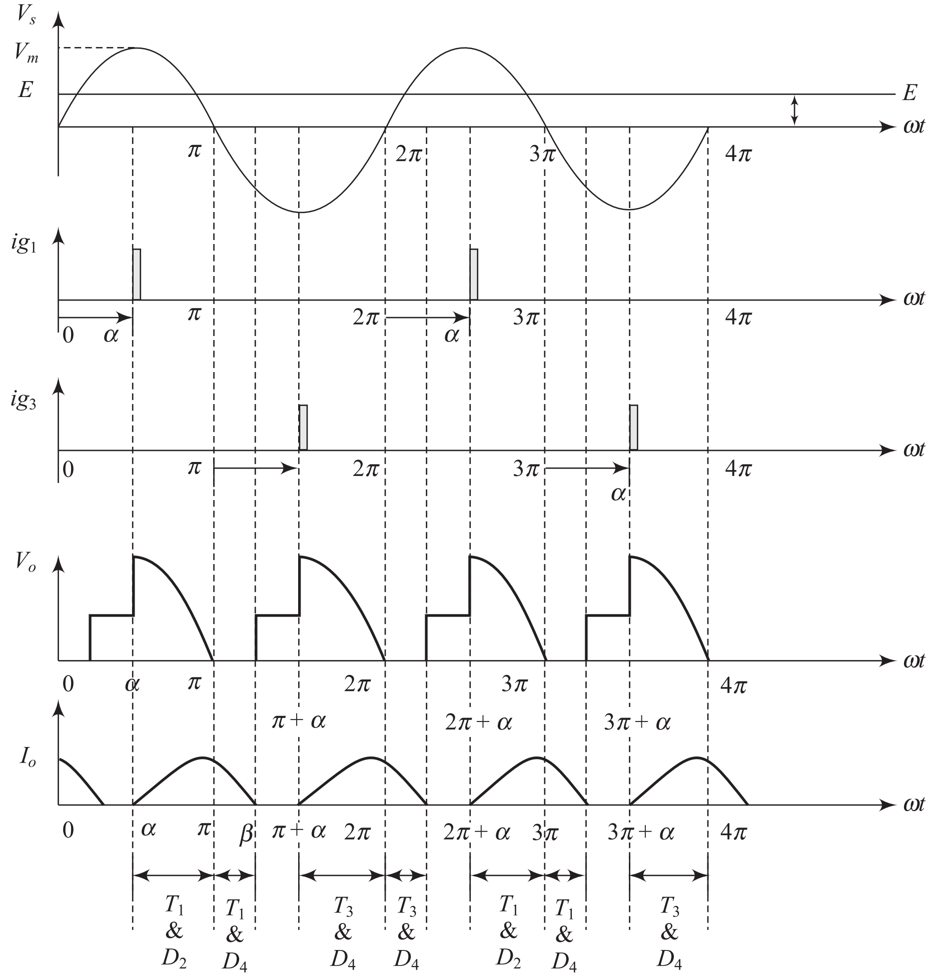


Fig. 6.59 *Voltage and current waveforms for discontinuous mode operation*

where, $Z = \sqrt{R^2 + (\omega L)^2}$, $\tan \phi = \dfrac{\omega L}{R}$, $\sqrt{2}V \sin \theta = E$, and $R = Z \cos \phi$

At $\omega t = \alpha$, $i_o = 0$. Then we can write

$$0 = \frac{\sqrt{2}V}{Z}\left[\sin(\alpha - \phi) - \frac{\sin \theta}{\cos \phi}\right] + I_o e^{-\frac{\alpha - \alpha}{\tan \phi}}$$

or

$$I_o = \frac{\sqrt{2}V}{Z}\left[\sin(\phi - \alpha) + \frac{\sin \theta}{\cos \phi}\right]$$

After substituting the value of I_o, we obtain

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \frac{\sin \theta}{\cos \phi}\right] + \frac{\sqrt{2}V}{Z}\left[\sin(\phi - \alpha) + \frac{\sin \theta}{\cos \phi}\right] e^{-\frac{\omega t - \alpha}{\tan \phi}}$$

The voltage equation during $\pi \le \omega t \le \beta$ is

$$0 = L\frac{di_0}{dt} + Ri_0 + E$$

After solving the equation, we get

$$i_o = -\frac{\sqrt{2}V}{Z}\frac{\sin\theta}{\cos\phi} + I_1 e^{-\frac{\omega t - \pi}{\tan\phi}}$$

At $\quad \omega t = \pi, i_o|_{\omega t=\pi} = -\frac{\sqrt{2}V}{Z}\frac{\sin\theta}{\cos\phi} + I_1 e^{-\frac{\pi-\pi}{\tan\phi}}$

$$= \frac{\sqrt{2}V}{Z}\left[\sin(\pi-\phi) - \frac{\sin\theta}{\cos\phi}\right] + \frac{\sqrt{2}V}{Z}\left[\sin(\phi-\alpha) + \frac{\sin\theta}{\cos\phi}\right]e^{-\frac{\pi-\alpha}{\tan\phi}}$$

Then current I_1 is equal to

$$I_1 = \frac{\sqrt{2}V}{Z}\sin\phi + \frac{\sqrt{2}V}{Z}\left[\sin(\phi-\alpha) + \frac{\sin\theta}{\cos\phi}\right]e^{-\frac{\pi-\alpha}{\tan\phi}}$$

The output current can be expressed as

$$i_o(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\phi-\alpha) + \frac{\sin\theta}{\cos\phi}\right]e^{-\frac{\omega t-\alpha}{\tan\phi}} + \frac{\sqrt{2}V}{Z}\sin\phi e^{-\frac{\omega t-\pi}{\tan\phi}} - \frac{\sqrt{2}V}{Z}\frac{\sin\theta}{\cos\phi}$$

At $\omega t = \beta$, $i_o = 0$ and then we can write

$$\left[\sin(\phi-\alpha) + \frac{\sin\theta}{\cos\phi}\right]e^{-\frac{\beta-\alpha}{\tan\phi}} + \frac{\sqrt{2}V}{Z}\sin\phi e^{-\frac{\beta-\pi}{\tan\phi}} - \frac{\sqrt{2}V}{Z}\frac{\sin\theta}{\cos\phi} = 0 \tag{6.7}$$

For given value of ϕ, α and θ, we can determine the value of β by solving Eq. (6.7).

6.20.3 Performance Analysis of Single-Phase Half Bridge or Semi-converter Converter

While the inductive load is very high, the load current is continuous and its ripple content is negligible as depicted Fig. 6.60. Then the input current i can be expressed as

$i = I_o \quad$ for $\alpha \le \omega t \le \pi$
$i = -I_o \quad$ for $\pi + \alpha \le \omega t \le 2\pi$
$i = 0 \quad$ otherwise

The above input current can be expressed in Fourier series form as given below.

$$i(t) = a_o + \sum_{n=1,2,3\ldots}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_o = \frac{1}{2\pi}\int_{\alpha}^{2\pi+\alpha} i\cdot d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_o\cdot d(\omega t) - \frac{1}{2\pi}\int_{\pi+\alpha}^{2\pi} I_o\cdot d(\omega t) = 0$$

$$a_n = \frac{1}{\pi}\int_{\alpha}^{2\pi} I_o\cdot\cos n\omega t\cdot d(\omega t) = \frac{1}{\pi}\int_{\alpha}^{\pi} I_o\cdot\cos n\omega t\cdot d(\omega t) - \frac{1}{\pi}\int_{\pi+\alpha}^{2\pi} I_o\cdot\cos n\omega t\cdot d(\omega t)$$

$$= -\frac{2I_o}{n\pi}\sin n\alpha \quad \text{for } n = 1, 3, 5,$$

$$= 0 \quad \text{for } n = 2, 4, 6$$

$$b_n = \frac{1}{\pi}\int_{\alpha}^{2\pi} I_o \cdot \sin n\omega t \cdot d(\omega t) = \frac{1}{\pi}\int_{\alpha}^{\pi} I_o \cdot \sin n\omega t \cdot d(\omega t) - \frac{1}{\pi}\int_{\pi+\alpha}^{2\pi} I_o \cdot \sin n\omega t \cdot d(\omega t)$$

$$= \frac{2I_o}{n\pi}(1+\cos n\alpha) \quad \text{for } n = 1, 3, 5,$$

$$= 0 \qquad\qquad \text{for } n = 2, 4, 6$$

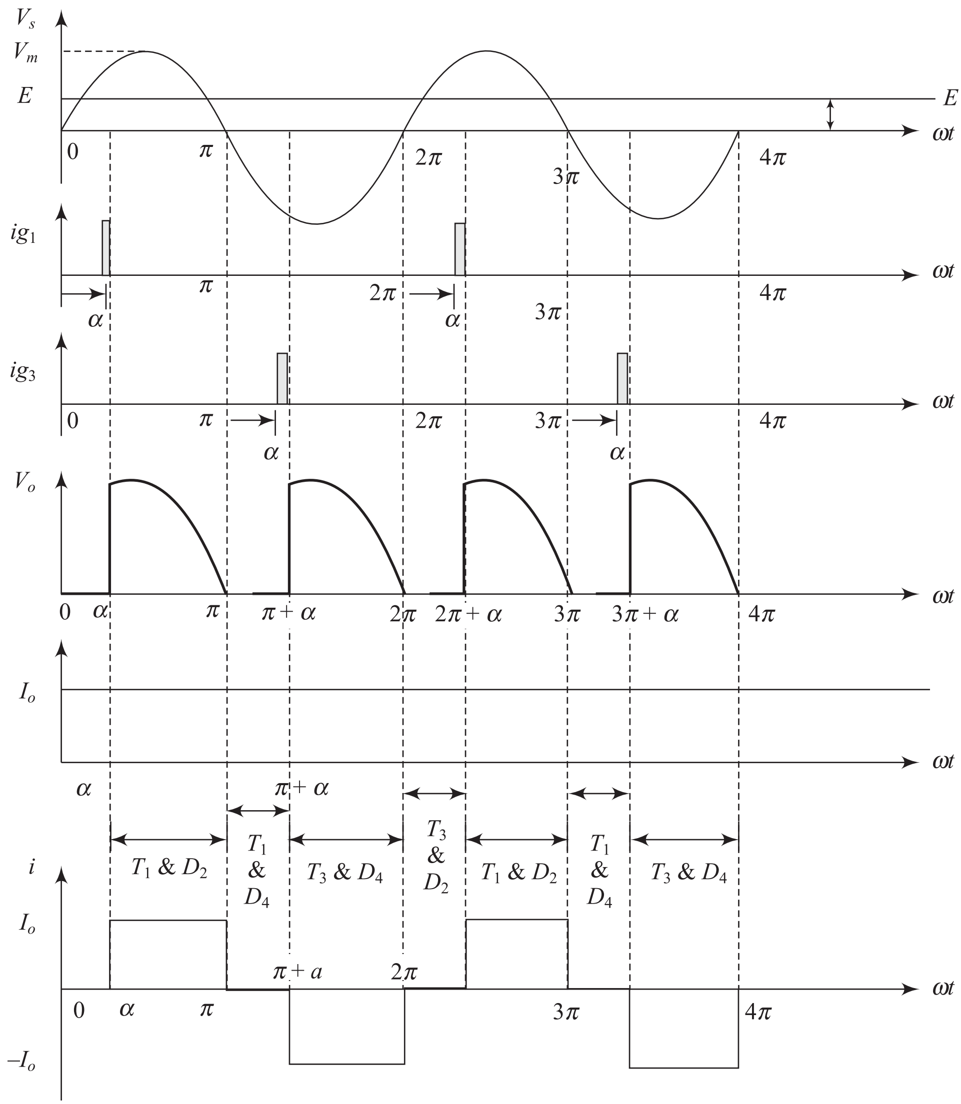


Fig. 6.60 *Voltage and current waveforms for Single phase full wave controlled bridge rectifier at high inductive load*

The input current can be expressed as

$$i(t) = \sum_{n=1,3,5,7\ldots}^{\infty} \sqrt{2} I_n \sin(n\omega t + \phi_n)$$

where $$I_n = \frac{1}{\sqrt{2}}[a_n^2 + b_n^2]^{\frac{1}{2}} \text{ and } \phi_n = \tan^{-1}\frac{a_n}{b_n}$$

Therefore,

$$I_n = \frac{1}{\sqrt{2}}[a_n^2 + b_n^2]^{\frac{1}{2}} = \frac{1}{\sqrt{2}}\left[\left(-\frac{2I_o}{n\pi}\sin n\alpha\right)^2 + \left(\frac{2I_o}{n\pi}(1+\cos n\alpha)\right)^2\right]^{\frac{1}{2}} = \frac{2I_o}{n\pi}(1+\cos n\alpha)^{\frac{1}{2}}$$

Since $1 + \cos n\alpha = 2\cos^2\frac{n\alpha}{2}$, $I_n = \frac{2\sqrt{2}I_o}{n\pi}\cos\frac{n\alpha}{2}$

and $$\phi_n = \tan^{-1}\frac{a_n}{b_n} = \tan^{-1}\left(-\frac{\sin n\alpha}{1+\cos n\alpha}\right) = \tan^{-1}\left(-\frac{2\sin\frac{n\alpha}{2}\cos\frac{n\alpha}{2}}{2\cos^2\frac{n\alpha}{2}}\right) = -\frac{n\alpha}{2}$$

After substituting the value of I_n and ϕ_n, the input current is

$$i(t) = \sum_{n=1,3,5,7\ldots}^{\infty} \sqrt{2} I_n \sin(n\omega t + \phi_n) = \sum_{n=1,3,5,7\ldots}^{\infty} \sqrt{2}\frac{2\sqrt{2}I_o}{n\pi}\cos\frac{n\alpha}{2}\sin\left(n\omega t - \frac{n\alpha}{2}\right)$$

$$= \sum_{n=1,3,5,7\ldots}^{\infty} \frac{4I_o}{n\pi}\cos\frac{n\alpha}{2}\sin\left(n\omega t - \frac{n\alpha}{2}\right)$$

The rms value of the fundamental current is $I_1 = \frac{2I_o}{\pi}(1+\cos\alpha)^{\frac{1}{2}}$

Since $1 + \cos n\alpha = 2\cos^2\frac{n\alpha}{2}$, $I_1 = \frac{2I_o}{\pi}(1+\cos\alpha)^{\frac{1}{2}} = \frac{2I_o}{\pi}\left(2\cos^2\frac{\alpha}{2}\right)^{\frac{1}{2}} = \frac{2\sqrt{2}I_o}{\pi}\cos\frac{\alpha}{2}$

The rms value of input current is

$$I_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} I_o^2\, d\omega t\right]^{\frac{1}{2}} = I_o\left[\frac{\pi - \alpha}{\pi}\right]^{\frac{1}{2}}$$

Displacement factor is $DF = \cos\phi_1 = \cos\left(-\frac{\alpha}{2}\right) = \cos\frac{\alpha}{2}$

Current distortion factor $CDF = \frac{I_1}{I_{\text{rms}}} = \left.\frac{2\sqrt{2}I_o}{\pi}\cos\frac{\alpha}{2}\middle/ I_o\left[\frac{\pi-\alpha}{\pi}\right]^{\frac{1}{2}}\right. = \frac{2\sqrt{2}\cos\frac{\alpha}{2}}{\sqrt{\pi(\pi-\alpha)}}$

Power factor

$$PF = CDF \times DF = \frac{2\sqrt{2}\cos\frac{\alpha}{2}}{\sqrt{\pi(\pi-\alpha)}} \times \cos\frac{\alpha}{2} = \frac{2\sqrt{2}\cos^2\frac{\alpha}{2}}{\sqrt{\pi(\pi-\alpha)}} = \frac{\sqrt{2}}{\sqrt{\pi(\pi-\alpha)}}(1+\cos\alpha)$$

Harmonic factor is $HF = \left[\left(\frac{I_{\text{rms}}}{I_1}\right)^2 - 1\right]^{\frac{1}{2}} = \left[\left(\frac{1}{CDF}\right)^2 - 1\right]^{\frac{1}{2}} = \left[\frac{\pi(\pi-\alpha)}{8\cos^2\frac{\alpha}{2}} - 1\right]^{\frac{1}{2}}$

$$= \left[\frac{\pi(\pi-\alpha)}{4(1+\cos\alpha)} - 1\right]^{\frac{1}{2}}$$

Active input power is

P_i = rms value of input voltage × rms fundamental component of input current × displacement factor

$$= V \times I_1 \times \cos\frac{\alpha}{2} = V \times \frac{2\sqrt{2}I_o}{\pi}\cos\frac{\alpha}{2} \times \cos\frac{\alpha}{2} = V \times \frac{\sqrt{2}I_o}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}VI_o}{\pi}(1+\cos\alpha)$$

Reactive power input is

$$Q_i = V \times I_1 \times \sin\frac{\alpha}{2} = V \times \frac{2\sqrt{2}I_o}{\pi}\sin\frac{\alpha}{2} \times \cos\frac{\alpha}{2} = \frac{\sqrt{2}VI_o}{\pi}\sin\alpha$$

Example 6.21 A single-phase semi-converter is supplied by 200 V, 50 Hz and it is connected with a *RLE* load where $R = 15\ \Omega$, $E = 80$ V and L is very large so that the load current is ripple free. Determine (a) average output voltage, (b) average output current, (c) average and rms value of thyristor current, (d) average and rms value of diode current, (e) circuit turn-OFF time at $\alpha = 35°$.

Solution

(a) The average output voltage of single-phase semi-converter is

$$V_o = \frac{\sqrt{2}V}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 200}{\pi}(1+\cos 35) = 163.83 \text{ V}$$

(b) We know that $V_o = E + I_oR$

Average output current $I_o = \frac{V_o - E}{R} = \frac{163.83-80}{15} = 5.588$ A

(c) Average value of thyristor current is $I_{Tav} = \frac{1}{2}I_o = 0.5 \times 5.588 = 2.794$ A

rms value of thyristor current is $I_{Trms} = \frac{I_o}{\sqrt{2}} = \frac{5.588}{\sqrt{2}} = 3.951$ A

(d) Average value of diode current is equal to average value of thyristor current and rms value of diode current is equal to rms value of thyristor current.

$I_{Dav} = I_{Tav} = 2.794$ A

$I_{Drms} = I_{Trms} = 3.951$ A

(e) Circuit turn-off time is $t_c = \frac{\pi-\alpha}{\omega} = \frac{\pi - \frac{\pi}{180}\times 35}{2\pi\times 50} = 8.055$ ms

Example 6.22 A single-phase semi-converter is connected to 220 V, 50 Hz. A load of $R = 10\ \Omega$ is connected in series with a large inductance and load current is ripple free. If the firing angle of converter is 60°, determine different performance parameters of the converter.

Solution

The output voltage $V_o = \frac{\sqrt{2}V}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 220}{\pi}(1+\cos 60)$ V = 148.62 V

Load current $I_o = \frac{V_o}{R} = \frac{148.62}{10} = 14.862$ A

DC power output $P_o = V_oI_o = 148.62 \times 14.862 = 2208.79$ Watt

As the load current is ripple free, RMS value of load current $I_{rms} = I_o = 14.862$ A

RMS output voltage $V_{rms} = V\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]$

As $\alpha = 60° = \frac{\pi}{3}$, $V_{rms} = V\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{3} + \frac{\sin 2\times 60}{2}\right)\right] = 177$ V

AC power = $P_{ac} = V_{rms}I_{rms} = 177 \times 14.862 = 2630.57$ Watt

Rectification efficiency = $\eta = \frac{P_{dc}}{P_{ac}} = \frac{P_o}{P_{ac}} = \frac{2208.79}{2630.57} = 83.96\%$

Form factor FF = $\frac{V_{rms}}{V_o} = \frac{177}{148.62} = 1.1909$

Voltage ripple factor is RF = $\sqrt{FF^2 - 1} = \sqrt{1.1909^2 - 1} = 0.646$

Since the load current is ripple free, current ripple factor $CRF = 0$

RMS value of fundamental current = $I_1 = \frac{2I_o}{\pi}\sqrt{1+\cos\alpha}_o$

$$= \frac{2 \times 14.862}{\pi}\sqrt{1+\cos 60} = 11.593 \text{ A}$$

Displacement factor $DF = \cos\phi_1 = \cos\left(-\frac{\alpha}{2}\right) = \cos\frac{\alpha}{2} = \cos\frac{60}{2} = 0.866$

Current distortion factor $CDF = \frac{2\sqrt{2}\cos\frac{\alpha}{2}}{\sqrt{\pi(\pi-\alpha)}} = \frac{2\sqrt{2}\cos\frac{60}{2}}{\sqrt{\pi\left(\pi - \frac{\pi}{3}\right)}} = 0.9554$

Power factor = $PF = CDF \times DF = 0.9554 \times 0.866 = 0.8273$

Active power input = $VI_1 \cos\frac{\alpha}{2} = 220 \times 11.593 \times \cos\frac{60}{2} = 2208.76$ Watt

Reactive power input = $VI_1 \sin\frac{\alpha}{2} = 220 \times 11.593 \times \sin\frac{60}{2} = 1275.23$ VAR

6.21 SINGLE-PHASE FULL-WAVE CONTROLLED RECTIFIER USING CENTRE TAP TRANSFORMER WITH TRANSFORMER LEAKAGE INDUCTANCE AND *R-L* LOAD

Figure 6.61 shows a single-phase full-wave controlled rectifier using centre tap transformer with *RL* load and leakage inductance. Due to presence of transformer leakage inductance, current can not be transferred from one thyristor to other thyristor instantaneously, but a finite time is required for commutation process. During the commutation interval (μ), current in one thyristor decreases gradually and current in other thyristor increases simultaneously. Hence, a short circuit occurs in the transformer secondary winding and leakage inductance.

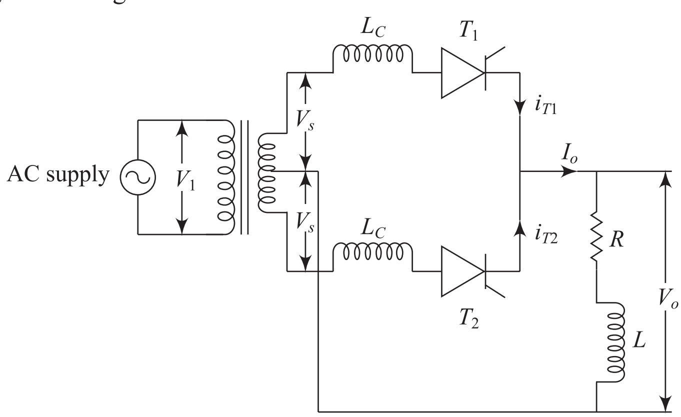


Fig. 6.61 *Single-phase full-wave controlled rectifier using centre tap transformer with RL load and leakage inductance*

The leakage inductance is represented by L_C and the loads current is constant and ripple free due to high inductive load. Initially SCR T_2 was in conduction state and current i_2 flows though it. At $\omega t = \alpha$, T_1 is triggered. When the transformer leakage inductance is zero, SCRs T_2 can be commutated as soon as T_1 is turned ON and input current polarity can be changed immediately. Since leakage inductance has a finite value, the commutation of T_2 can not be done instantaneously and thus input current polarity can not be changed immediately. Therefore, for certain time interval SCRs T_1 and T_2 continue to conduct. This time interval is called *commutation overlap* (μ). During this overlap period current of T_2 decreases gradually, but current of T_1 increases progressively. In this time period, the load current freewheels through SCRs and output becomes zero. Hence the input current starts changing polarity as current of T_2 decreases slowly and current of T_1 increases gradually. At the end of overlap period, current of T_2 is zero and T_2 will be turned OFF. Then T_1 conduct and load current flows through T_1 and RL load. Again the above operation will be repeated at $\omega t = \pi + \alpha$ where T_1 will be commutated and T_2 starts to conduct. The voltage and current waveforms are shown in Fig. 6.62.

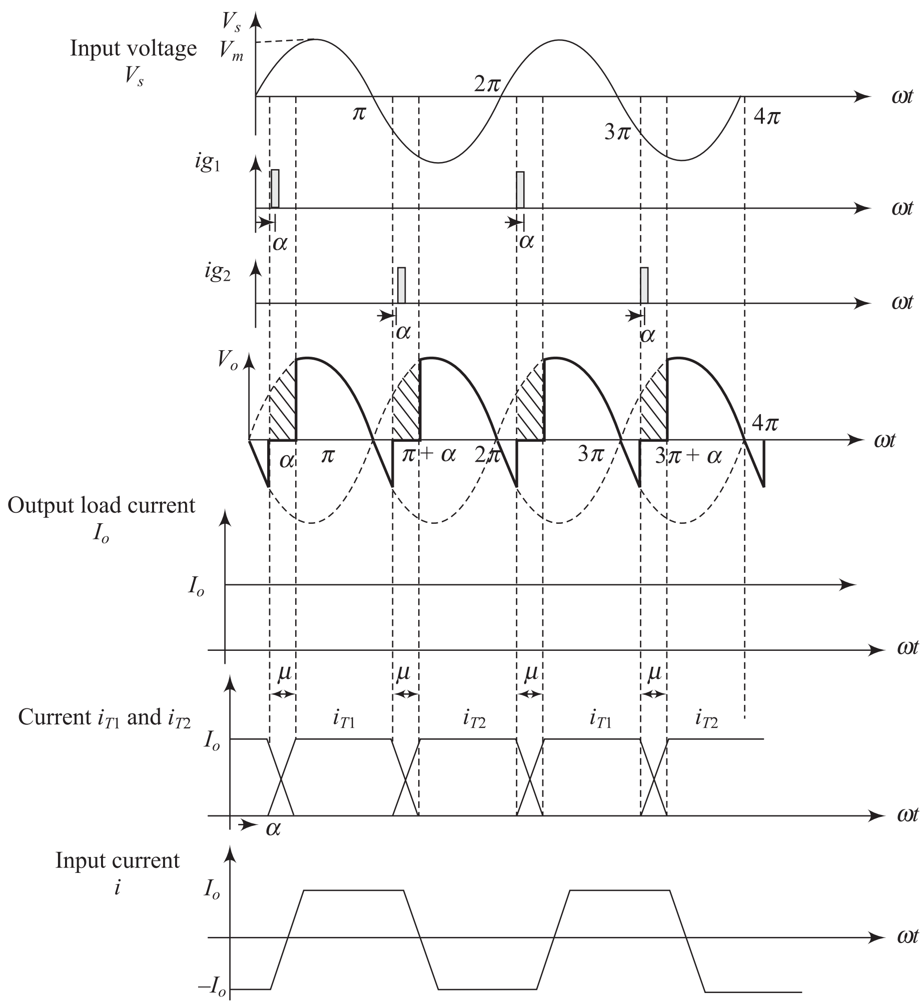


Fig. 6.62 *Voltage and current waveforms of single-phase full wave controlled rectifier using center tap transformer with leakage inductance*

Average voltage drop across leakage inductance is

$$V_{L_C} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\mu} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi}[\cos\alpha - \cos(\alpha+\mu)] = 2fL_C I_o$$

Then $$\cos(\alpha+\mu) = \cos\alpha - \frac{2\pi f L_C I_o}{\sqrt{2}V} = \cos\alpha - \frac{\omega L_C I_o}{\sqrt{2}V} \quad \text{as } \omega = 2\pi f$$

At no load average output voltage is $V_o = \dfrac{\sqrt{2}V}{\pi}\cos\alpha$

Due to load current I_o, the average output voltage is equal to

$$V_o = \frac{\sqrt{2}V}{\pi}\cos\alpha - V_{L_C} = \frac{2\sqrt{2}V}{\pi}\cos\alpha - \frac{\sqrt{2}V}{\pi}[\cos\alpha - \cos(\alpha+\mu)]$$

$$= \frac{\sqrt{2}V}{\pi}[\cos\alpha + \cos(\alpha+\mu)]$$

After substituting the value of $\cos(\alpha+\mu) = \cos\alpha - \dfrac{\omega L_C I_o}{\sqrt{2}V}$ in the above equation, we obtain

$$V_o = \frac{\sqrt{2}V}{\pi}[\cos\alpha + \cos(\alpha+\mu)] = \frac{2\sqrt{2}V}{\pi}\cos\alpha - \frac{\omega L_C}{\pi} I_o$$

6.22 EFFECT OF SOURCE INDUCTANCE IN PERFORMANCE OF SINGLE-PHASE FULL WAVE CONTROLLED BRIDGE RECTIFIER WITH *RL* LOAD

In Section 6.15, the analysis of single-phase full-wave controlled bridge rectifier with *RL* load was done assuming negligible source inductance. Actually all ac-to-dc converters are supplied from transformers. Usually the series impedance of transformer can not be neglected. Therefore series impedance must be present in any converter circuits. Generally, this impedance is inductive with negligible resistive component. Due to presence of source inductance, the output voltage of a converter will not be remaining constant and input current waveform will be changed significantly.

Figure 6.63 shows a single-phase full-wave controlled bridge rectifier with source inductance and *RL* load. Assume that the converter operates in the continuous conduction mode and the load current is constant and ripple free.

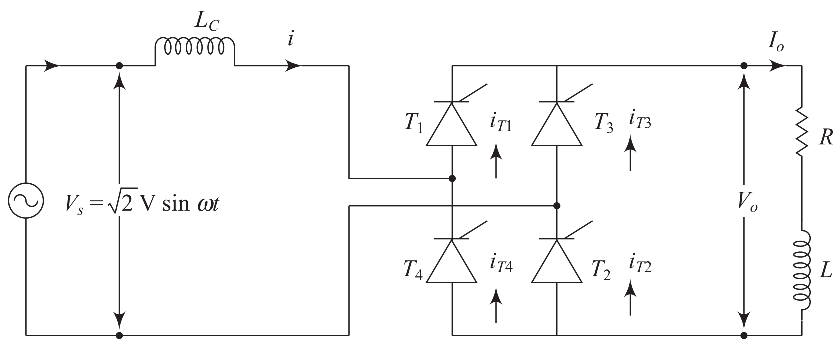


Fig. 6.63 *Single-phase full-wave controlled bridge rectifier with source inductance and RL load*

Initially SCRs T_3 and T_4 are in conduction state and load current flows through T_3, T_4 and RL load. At $\omega t = \alpha$, T_1 and T_2 are triggered. If the source inductance is zero, SCRs T_3 and T_4 can be commutated as soon as T_1 and T_2 are turned ON and input current polarity can be changed instantly. As source inductance is present, the commutation of T_3 and T_4 will not be done immediately and hence input current polarity can not be changed instantly. Consequently, for certain time interval all four SCRs continue to conduct. This time interval is called *commutation overlap (μ)*. During this overlap period current of T_3 and T_4 will be decreased gradually, but current of T_1 and T_2 will be increased progressively. In this time period, the load current freewheels through all four SCRs and output becomes zero. Correspondingly, the input current starts changing polarity as current of T_3 and T_4 decreases slowly and current of T_1 and T_2 increases gradually. At the end of overlap period, current of T_3 and T_4 becomes zero and SCRs T_3 and T_4 will be turned OFF. Then T_1 and T_2 conduct and load current flows through T_1, T_2 and RL load. The above process will be repeated at $\omega t = \pi + \alpha$ where T_1 and T_2 will be commutated and T_3 and T_4 start to conduct. Figure 6.64 shows the voltage and current waveforms.

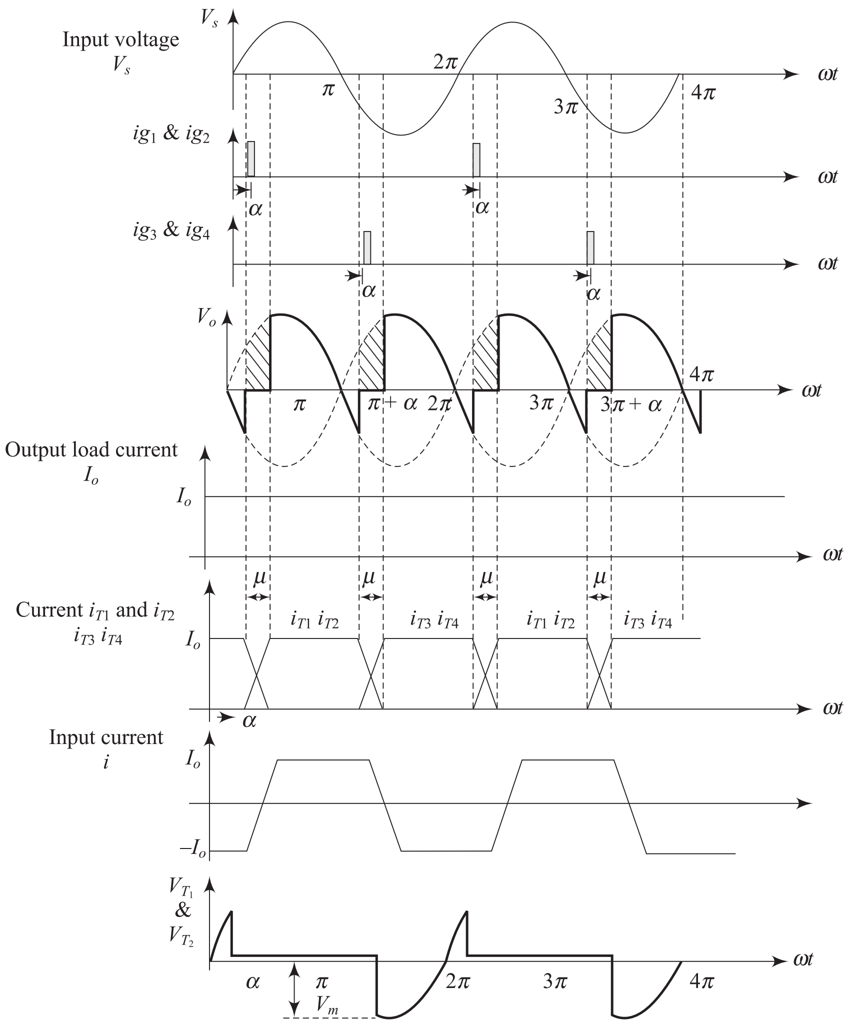


Fig. 6.64 *Voltage and current waveforms of single-phase full-wave controlled bridge rectifier with source inductance*

Figure 6.65 shows the equivalent circuit of converter during overlap period. The voltage equation is

$$v_i = L_C \frac{di}{dt} \quad \text{for } \alpha \le \omega t \le \alpha + \mu$$

Where, $v_i = \sqrt{2}V \sin \omega t$ and i is the input current

or $$di = \frac{\sqrt{2}V}{L} \sin \omega t \cdot dt$$

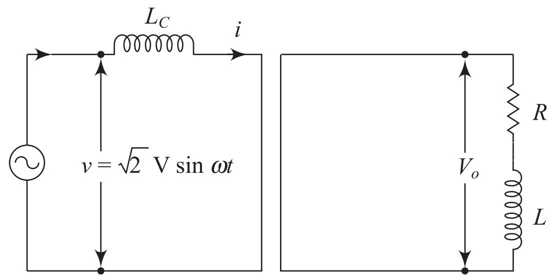


Fig. 6.65 *Equivalent circuit during overlap period*

After integrating the above equation, we obtain

$$i = -\frac{\sqrt{2}V}{\omega L_C} \cos \omega t + C$$

At $\omega t = \alpha, i_{at \cdot \omega t = \alpha} = -I_o$. Then $-I_o = -\frac{\sqrt{2}V}{\omega L_C} \cos \alpha + C$ or $C = \frac{\sqrt{2}V}{\omega L_C} \cos \alpha - I_o$

After substituting the value of C, we get

$$i = \frac{\sqrt{2}V}{\omega L_C} (\cos \alpha - \cos \omega t) - I_o$$

At $\omega t = \alpha + \mu$, $i_{at \cdot \omega t = \alpha + \mu} = I_o$. Then we can write

$$I_o = \frac{\sqrt{2}V}{\omega L_C} [\cos \alpha - \cos(\alpha + \mu)] - I_o$$

or $$[\cos \alpha - \cos(\alpha + \mu)] = \frac{\sqrt{2}\omega L_C}{V} I_o$$

or $$\cos(\alpha + \mu) = \cos \alpha - \frac{\sqrt{2}\omega L_C}{V} I_o$$

Average output voltage is

$$V_o = \frac{1}{\pi} \int_{\alpha+\mu}^{\alpha+\pi} \sqrt{2}V \sin \omega t \cdot d\omega t = \frac{\sqrt{2}V}{\pi} [-\cos(\alpha + \pi) + \cos(\alpha + \mu)]$$

$$= \frac{\sqrt{2}V}{\pi} [\cos \alpha + \cos(\alpha + \mu)]$$

After substituting the value of $\cos(\alpha + \mu) = \cos \alpha - \frac{\sqrt{2}\omega L_C}{V} I_o$ in the above equation, we get

$$V_o = \frac{\sqrt{2}V}{\pi} [\cos \alpha + \cos(\alpha + \mu)] = \frac{2\sqrt{2}V}{\pi} \cos \alpha - \frac{2\omega L_C}{\pi} I_o$$

The most simplified equivalent circuit of Fig. 6.63 is a dc voltage source in series with commutation resistance (R_C) as depicted in Fig. 6.66. The open-circuit voltage of the circuit is the average dc output voltage of single-phase full waved controlled rectifier. The magnitude of R_C is $R_C = \frac{2\omega L_C}{\pi}$. This resistance is used to represent the voltage drop across commutation resistance.

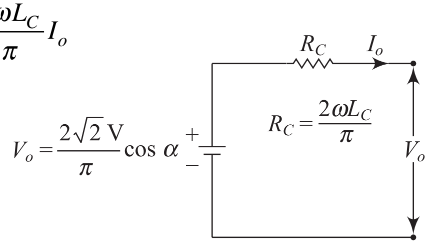


Fig. 6.66 *Equivalent circuit of single-phase full-wave controlled rectifier with source inductance*

Example 6.23 A single-phase full-converter is used to deliver a constant load current. If the overlap angle is 10° for zero degree firing angle of converter, determine the overlap angle when firing angle is (i) α = 30°, (ii) 45° and (iii) 60°.

Solution

The dc load current is $I_o = \frac{\sqrt{2}V}{\omega L_C}[\cos\alpha - \cos(\alpha+\mu)]$

For firing angle α_1, the overlap angle is μ_1

Therefore, $I_o = \frac{\sqrt{2}V}{\omega L_C}[\cos\alpha - \cos(\alpha+\mu)] = \frac{\sqrt{2}V}{\omega L_C}[\cos\alpha_1 - \cos(\alpha_1+\mu_1)]$

For α = 0°, μ = 10°

$$[\cos\alpha_1 - \cos(\alpha_1+\mu_1)] = [\cos\alpha - \cos(\alpha+\mu)] = [\cos 0 - \cos(0+10)] = 0.01519$$

or $$\cos(\alpha_1+\mu_1) = \cos\alpha_1 - 0.01519$$

(i) For $\alpha = 30°, \cos(\alpha_1+\mu_1) = \cos\alpha_1 - 0.01519$ as $\alpha_1 = \alpha = 30°$

or $$\cos(30+\mu_1) = \cos 30 - 0.01519 = 0.8508$$

or $$\mu_1 = \cos^{-1}(0.8508) - 30 = 1.6973°$$

(ii) For $\alpha = 45°, \cos(\alpha_2+\mu_2) = \cos\alpha_2 - 0.01519$ as $\alpha_2 = \alpha = 45°$

or $$\cos(45+\mu_2) = \cos 45 - 0.01519 = 0.6919$$

or $$\mu_2 = \cos^{-1}(0.6919) - 45 = 1.2179°$$

(iii) For $\alpha = 60°, \cos(\alpha_3+\mu_3) = \cos\alpha_3 - 0.01519$ as $\alpha_3 = \alpha = 60°$

or $$\cos(60+\mu_3) = \cos 60 - 0.01519 = 0.4848$$

or $$\mu_3 = \cos^{-1}(0.4848) - 60 = 1.0006°$$

6.23 DUAL CONVERTERS

One quadrant converters are those converters in which the output voltage and output current have same polarity during the entire firing angle control range. One quadrant converters are known as first quadrant converters. The power flow in first quadrant converters is from source to load.

Two quadrant converters are those converters in which converter acts as controlled rectifier when the output voltage and output current have same polarity during the firing angle control range $0 \le \alpha \le \frac{\pi}{2}$ and converter acts as an inverter when the output voltage and output current have opposite polarity during the firing angle control range, $\frac{\pi}{2} \le \alpha \le \pi$. Two quadrant converters operate in the first quadrant and fourth quadrant. In the first quadrant, power flows from source to load and in the fourth quadrant power flows from load to source.

Semi-converters are single quadrant (I-quadrant) converters. It means that the output voltage across load and load current have same polarity over the entire firing angle range as depicted in Fig. 6.67. In this figure, V_o is the average dc output voltage and I_o is the average dc output current. In semi-converter, V_o and I_o are positive and this converter operates in rectifier mode and power flows from ac source to dc load.

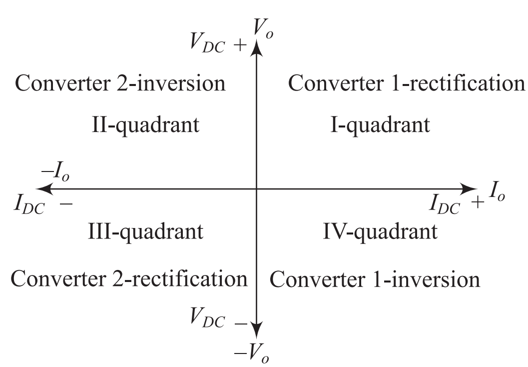


Fig.6.67 *Four quadrant operation of converters*

Full converters are two quadrant (I-quadrant and IV-quadrant) converters. In a full converter, the direction of current can not be reverse due to unidirectional property of thyristor but the polarity of output voltage can be reversed as depicted in Fig. 6.67. A full converter can operate in the first quadrant as a controlled rectifier where V_o (V_{DC}) and I_o (I_{DC}) are positive and firing angle varies in the control range $0 \le \alpha \le \frac{\pi}{2}$. It can also be operated in the fourth quadrant as a controlled rectifier where V_o is negative and I_o is positive and firing angle varies in the control range $\frac{\pi}{2} \le \alpha \le \pi$. In the first quadrant, power flows from ac supply to dc load and in the fourth quadrant power flows from load to ac source.

dc motors are operated in four quadrants such as forward motoring, forward regeneration, reverse motoring and reverse regeneration. For such operation, four quadrant converters are required. Dual converters are used as four quadrant converters. Dual converters are those converters in which two fully controlled converters are connected in back to back to the load circuit as depicted in Fig. 6.68(a) and four quadrant operation of dual converter is given in Fig. 6.68(b).

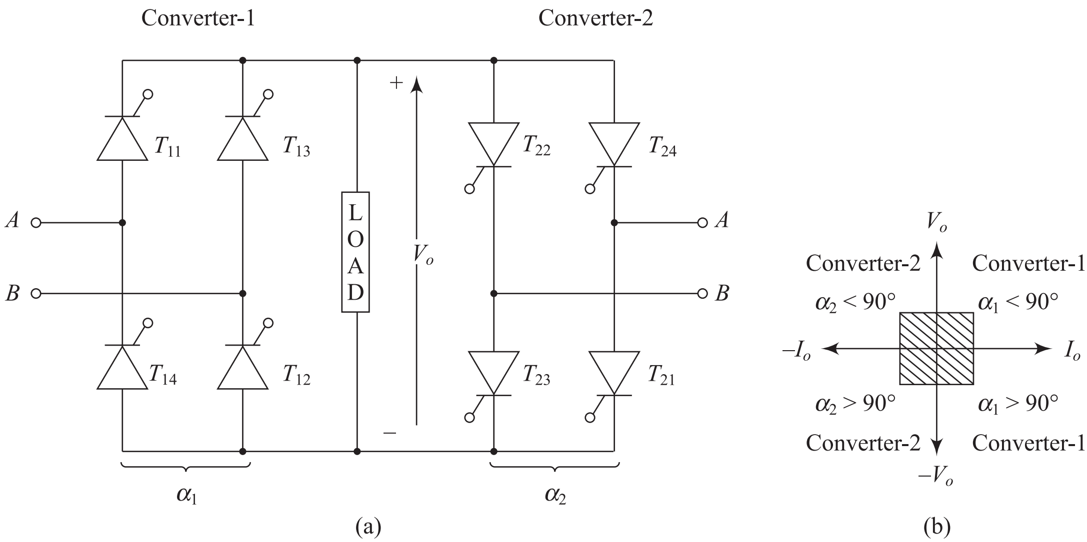


Fig. 6.68 *(a) Dual converter and (b) Four quadrant operation of dual converter*

6.23.1 Operating Principle of Ideal Dual Converter

The basic operating principle of dual converter can be explained using the most simplified equivalent circuit diagram of the dc circuit as illustrated in Fig. 6.69. This circuit consists of two ideal two quadrant converters such as converter-1 and converter-2, two diodes such as D_1 and D_2 and load. It is clear from Fig. 6.69 that two ideal two quadrant converters are assumed to be controllable direct voltage sources connected in series with the diodes.

For analysis of dual converter, it is assumed that dual converters are made by ideal full converters and there is no ripple in the output voltage. Therefore, these converters generate pure dc output voltage without any ac ripple at the dc terminals. The current can flow in either direction through diodes D_1 and D_2 which represents the unidirectional current flow of converters. The firing angle of converters are controlled by the control voltage V_C.

V_{o1} and V_{o2} are the average output voltages of converter-1 and converter-2 respectively. These output voltages are equal in magnitude but they are of opposite polarity. These can drive the current

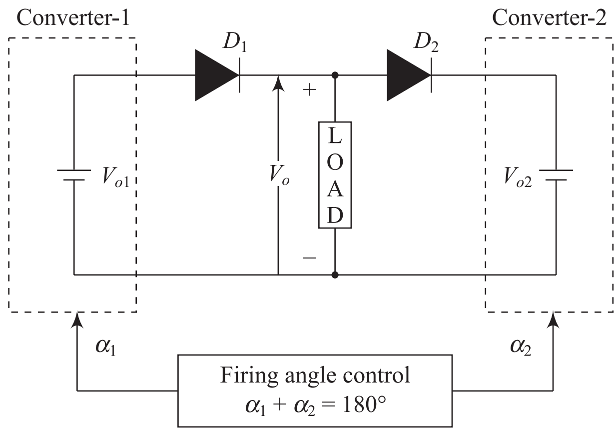


Fig. 6.69 *Equivalent circuit of an ideal dual converter*

in opposite directions through load. Consequently, whenever one converter operates as a controlled rectifier, the other converter operates as an inverter. The converter working as a rectifier is called as *positive group converter* and the converter working as an inverter is known as *negative group converter*.

The average output voltage of converter-1 is $V_{o1} = V_{max} \cos \alpha_1$

The average output voltage of converter-1 is $V_{o2} = V_{max} \cos \alpha_2$

For a single full converter, $V_{max} = \dfrac{2\sqrt{2}V}{\pi}$

For an ideal converter $V_o = V_{o1} = -V_{o2}$

Therefore, $\quad V_{max} \cos \alpha_1 = -V_{max} \cos \alpha_2$

or $\quad \cos \alpha_1 = -\cos \alpha_2 = \cos(180 - \alpha_2)$

or $\quad \alpha_1 = 180 - \alpha_2 \quad$ or $\quad \alpha_1 + \alpha_2 = 180$

The variation of output voltage with firing angle for the two converters is depicted in Fig. 6.70. The firing angles α_1 and α_2 are varied in such a way that always $\alpha_1 + \alpha_2 = 180$.

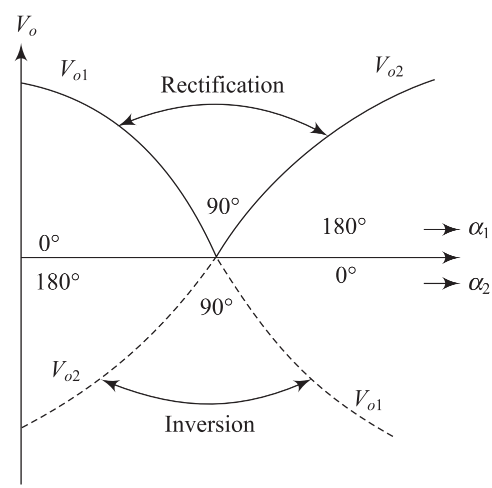


Fig. 6.70 *Variation of terminal voltage for an ideal dual converter with firing angle*

6.23.2 Practical Dual Converter

The firing angles α_1 and α_2 are controlled in such a way that always $\alpha_1 + \alpha_2 = 180$ and the average output voltage of converter-1 and converter-2 are equal but opposite polarity. One converter operates as a rectifier with a firing angle α_1 and the other converter operates as an inverter with firing angle $\alpha_2 = 180 - \alpha_1$. Subsequently, the output voltage of converter-1 is equal to the output voltage of converter-2 though instantaneous output voltages V_{o1} and V_{o2} are out of phase in a practical dual converter. Therefore, there is a voltage difference when two converters are interconnected, a large circulating current flow between two converters but not through load. This circulating current can be limited to a specified value by inserting a reactor between two converters. The circulating current can be avoided by providing the trigger pulses. There are two operating modes of a practical dual converter such as non-circulating current mode and circulating current mode.

6.23.3 Non-circulating Current Mode Dual Converter

In non-circulating current mode dual converter, only one converter operates at a time which alone carries the entire load current. Only this converter receives the firing pulses from the triggering circuit. The other converter remains blocked from conduction by removing the triggering pulses to that converter. Hence only one converter is in operation at a time and the other converter is idle. As there is no circulating current flow, reactor is not required. The circuit diagram of non-circulating current dual converter is shown in Fig. 6.71.

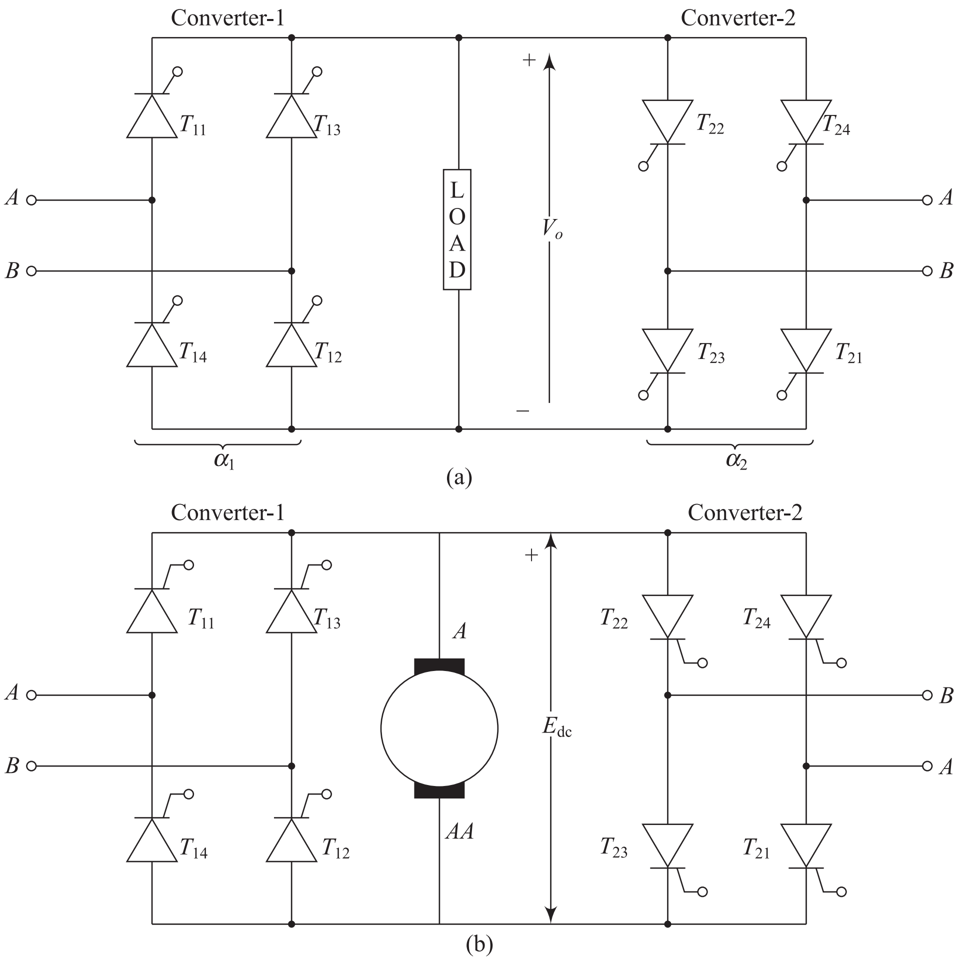


Fig. 6.71 *(a) Non-circulating current mode dual converter with a load (b) Non-circulating current mode dual converter with DC motor*

If converter-1 is operating and is supplying the load current, converter-2 will be in the blocking state. In order to block the converter-1 from conduction and switch on the converter-2, it is required to commutate the thyristors of converter-1 either by removing the firing pulses to its thyristors or by increasing the firing angle of converter-1 to the maximum value such that its firing pulses are being blocked. Subsequently, the load current would decays to zero. As the triggering pulses are applied to the thyristors of converter 2, converter-2 gets switched on. Thus the converter-2 is in the operating mode and converter-1 remains idle. Now the load current builds up in the opposite direction. So long as converter-2 is in operation, converter-1 will be in idle since firing pulses are withdrawn from this converter. During changeover from one converter to other converter, a delay time of about 10 to 20 ms must be ensured between the instants at which converter-1 is in the OFF state and converter-2 is in the ON state. This delay time ensures the reliable commutation of thyristors present in the converter-1. If the thyristors of converter-2 are triggered before turn off of the thyristors of converter-1, a large circulating current would flow between the two converters which is a undesirable condition.

In non-circulating current dual converter, the load current may be continuous or discontinuous. The control circuit of dual converter is designed in such a way that the performance of non-circulating current dual converter is satisfactory during continuous or discontinuous load current.

6.23.4 Circulating Current Mode Dual Converter

In circulating current mode dual converter, two converters are in the operating condition when one converter operates in the controlled rectifier mode and other operates in the inverting mode. In this converter, a rector is inserted in between converter-1 and converter-2. The rector is used to limit the amplitude of circulating current to a specified value. Figure 6.72 shows the circulating current mode dual converter. The voltage waveforms of a single-phase dual converter is illustrated in Fig. 6.73.

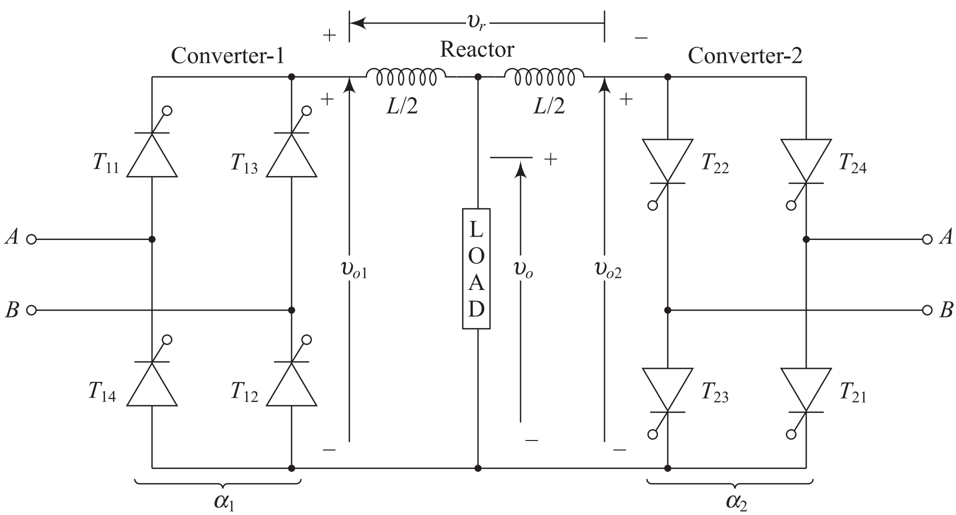


Fig. 6.72 *Circulating current mode dual converter*

The firing angles of two converters are adjusted in such a manner that $\alpha_1 + \alpha_2 = 180°$ is always satisfied. For example, if the firing angle of converter-1 is $\alpha_1 = 45°$ then the firing angle of converter-2 is $\alpha_2 = 135°$. In this case converter-1 operates as a controlled rectifier but the converter-2 operates as an inverter. The output voltage at the terminals of both converters has the same average output voltage. The instantaneous value of output voltages V_{o1} and V_{o2} are not similar as depicted in Fig. 6.74. Consequently, a large circulating current flows between the two converters. To limit the circulating current, a reactor must be introduced between converter-1 and converter-2.

The load current can be reversed by interchanging the role of two converters. Then converter-1 should operate as an inverter and converter-2 should operate as rectifier. Therefore, the firing angle of converter-1 is greater than 90° and the firing angle of converter-2 is less than 90°. However the

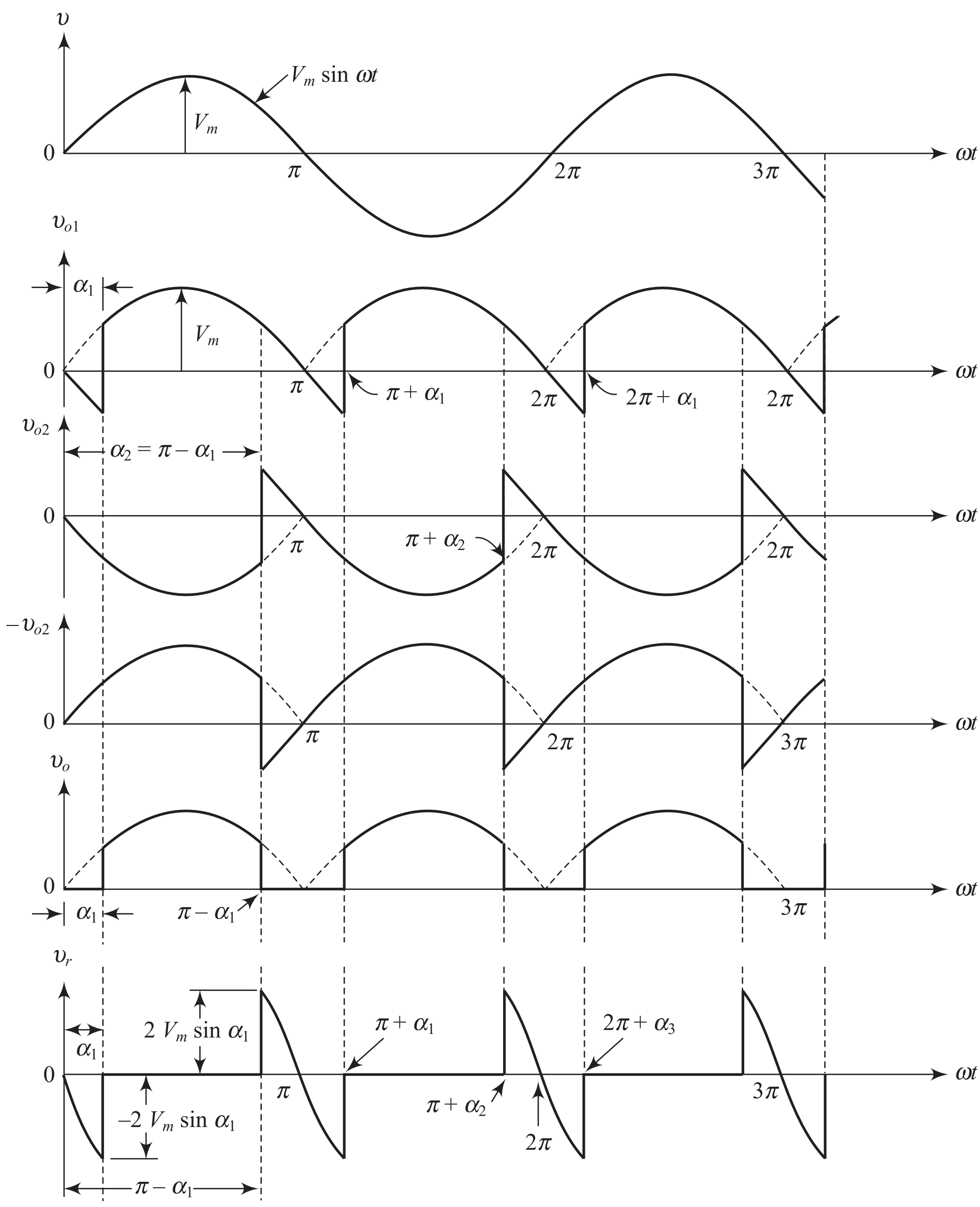


Fig. 6.73 *Voltage waveforms of a single-phase dual converter*

equation $\alpha_1 + \alpha_2 = 180°$ must be satisfied. The normal time delay 10 to 20 ms is not required in this converter. Consequently the operation of this type of dual converter is faster.

The disadvantages of dual converters are given below:

1. A reactor is used to limit the circulating current and the size and cost of this reactor is significantly high at high power levels.

2. The efficiency and power factor of dual converters are low due to the losses occurred by the circulating current.
3. Circulating current gives rise to more losses in the dual converters.
4. Since the converters have to handle load and circulating current, the current rating of thyristors used in dual converters must be high.

Though the dual converters have the above disadvantages, a dual converter with circulating current mode dual converter is preferred when the load current is to be reversed frequently and whenever a fast response four-quadrant operation is required.

Example 6.24 A single-phase dual converter is supplied from a 220 V, 50 Hz ac source and it is connected to a load of $R = 20\ \Omega$. The value of current limiting reactor is $L = 0.08H$. If the firing angle of converter-1 $\alpha_1 = 45°$, determine (a) firing angle of converter-2 (b) Average value of load voltage (b) peak value of circulating current and (c) peak currents of both converters.

Solution

Given: $V = 220$ V, $f = 50$ Hz, $R = 20\ \Omega$, $L = 0.08H$ and $\alpha_1 = 45°$

We know that $\alpha_1 + \alpha_2 = 180°$

The firing angle of converter-2 is $\alpha_2 = 180° - \alpha_1 = 180° - 45° = 135°$

Average value of load voltage is $$V_o = \frac{1}{\pi}\int_{\alpha_1}^{\pi-\alpha_1} \sqrt{2}V \sin \omega t \cdot d(\omega t) = \frac{2\sqrt{2}V}{\pi}\cos \alpha_1$$

$$= \frac{2\sqrt{2} \times 220}{\pi}\cos 45 = 140.106 \text{ V}$$

The value of circulating current is

$$i_c = \frac{1}{L}\int_t^{\alpha_1/\omega} 2V_m \sin \omega t \cdot dt = \frac{2V_m}{\omega L}[\cos \omega t - \cos \alpha_1]$$

The maximum value of circulating current occurs at cos $\omega t = 1$

$$i_{cp} = \frac{2V_m}{\omega L}[1 - \cos \alpha_1] = \frac{2\sqrt{2} \times 220}{2\pi \times 50 \times 0.08}[1 - \cos 45] = 7.254 \text{ A}$$

The peak value of load current $i_{op} = \frac{V_m}{R} = \frac{\sqrt{2}V}{R} = \frac{\sqrt{2} \times 220}{20} = 15.556$ A

Peak value of current in converter-1 = $i_{cp} + i_{op}$ = 7.254 + 15.556 = 22.81 A

Peak value of current in converter-2 = i_{cp} = 7.254 A

Summary

- The controlled rectifier circuit is used to convert ac input voltage into variable dc voltage. Electrical power flows from the ac input to the dc output during rectifier mode operation and power flows from load to ac input in inverter mode operation.
- Classification of controlled rectifiers based on their number of phases, the type of devices used, and circuit topology are incorporated in this chapter.
- The operation of different single-phase controlled rectifiers with resistive (R), inductive (L), and back emf (E) type loads explained in detail.
- The mode of operation (continuous or discontinuous) of the converter depend on load parameters and firing angle. In the continuous conduction mode, the load voltage depends only on the firing angle.

- The operation of converters in the discontinuous and continuous conduction mode of operation is discussed in this chapter.
- Single-phase half-wave fully controlled converters always operate in the discontinuous conduction mode. Single phase fully controlled bridge converters are extensively used for dc motor drives.
- Determination of the performance parameters of single-phase converters from the input voltage, output voltage and current waveforms has been discussed.
- The effect of transformer leakage inductance (L_C) in performance of single-phase full wave controlled rectifier using center tap transformer is explained briefly. The effect of source inductance in performance of single-phase full wave controlled rectifier also incorporated in this chapter.
- The operation of singlephase dual converter is explained in detail.

Multiple-Choice Questions

6.1 A single-phase one pulse controlled rectifier with *RE* load is supplied through 200 sin 314*t*. If the back emf E = 100 V, the range of firing angle is

(a) 30° to 150° (b) 40° to 140° (c) 30° to 180° (d) 60° to 120°

6.2 In a half-wave rectifier circuit with R load, the average output voltage at firing angle α is ______ when input voltage is $\sqrt{2}V \sin \omega t$.

(a) $V_O = \dfrac{\sqrt{2}V}{2\pi}(1-\cos\alpha)$ (b) $V_O = \dfrac{\sqrt{2}V}{2\pi}(1+\cos\alpha)$

(c) $V_O = \dfrac{\sqrt{2}V}{\pi}(1+\cos\alpha)$ (d) $V_O = \dfrac{\sqrt{2}V}{\pi}\cos\alpha$

6.3 In a single-phase half-wave controller rectifier with *RL* load, the free wheeling diode is connected across load. The extinction angle β is greater than π. When the firing angle is α, the conduction period of thyristor and free wheeling diode are

(a) $\pi \le \omega t \le \beta$ and $\alpha \le \omega t \le \pi$ (b) $0 \le \omega t \le \pi$ and $\pi \le \omega t \le \beta$

(c) $\alpha \le \omega t \le \pi$ and $\pi \le \omega t \le \beta$ (d) $\alpha \le \omega t \le \pi$ and $2\pi \le \omega t \le \beta$

6.4 In a single-phase full bridge converter, the average dc output voltage is equal to

(a) $V_o = \dfrac{1}{\pi}\int_{\alpha}^{\pi}\sqrt{2}V \sin \omega t \cdot d\omega t$ (b) $V_o = \dfrac{1}{\pi}\int_{0}^{\pi+\alpha}\sqrt{2}V \sin \omega t \cdot d\omega t$

(c) $V_o = \dfrac{1}{\pi}\int_{\alpha}^{2\pi+\alpha}\sqrt{2}V \sin \omega t \cdot d\omega t$ (d) $V_o = \dfrac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\sqrt{2}V \sin \omega t \cdot d\omega t$

6.5 In a single-phase semi-converter, the average dc output voltage is equal to

(a) $V_o = \dfrac{1}{\pi}\int_{\alpha}^{\pi}\sqrt{2}V \sin \omega t \cdot d\omega t$ (b) $V_o = \dfrac{1}{\pi}\int_{0}^{\pi+\alpha}\sqrt{2}V \sin \omega t \cdot d\omega t$

(c) $V_o = \dfrac{1}{\pi}\int_{\alpha}^{2\pi+\alpha}\sqrt{2}V \sin \omega t \cdot d\omega t$ (d) $V_o = \dfrac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\sqrt{2}V \sin \omega t \cdot d\omega t$

6.6 When a single-phase full converter operates in continuous conduction mode, each thyristor conducts for ______ duration

(a) π (b) 2π (c) $\pi - \alpha$ (d) $\pi + \alpha$

6.7 If a single-phase full converter operates in discontinuous conduction mode and extinction angle β is greater than π, each thyristor conducts for ______ duration

(a) π (b) $\beta - \alpha$ (c) $\pi - \alpha$ (d) $\pi + \alpha$

6.8 In a single-phase full converter with R load, peak output voltage is 300 V and average output voltage is 100 V. Then the firing angle of the converter is

(a) 57.3° (b) 67.3° (c) 87.3° (d) 97.3°

6.9 If a single-phase semi converter operates in continuous conduction mode, each thyristor conducts for ______duration

(a) π (b) α (c) $\pi - \alpha$ (d) $\pi + \alpha$

6.10 A freewheeling diode is connected across *RL* load to provide

(a) power factor improvement (b) reduce utilization factor
(c) first turn on (d) slow turn off

6.11 A single-phase full-wave bridge rectifier operates as an inverter when the firing angle in the range ______

(a) $0° \le \alpha \le 90°$ (b) $90° < \alpha \le 180°$ (c) $30° \le \alpha \le 90°$ (d) $60° < \alpha \le 180°$

6.12 In a single-phase full converter, if the load current is I_o which is ripple free, the average thyristor current is

(a) I_o (b) $\frac{I_o}{2}$ (c) $\frac{I_o}{3}$ (d) $\frac{I_o}{4}$

6.13 In a single-phase full converter with source inductance and inductive load, the output voltage during overlap is equal

(a) source voltage (b) source voltage –voltage drop in inductance
(c) zero (d) source voltage +voltage drop in inductance

6.14 Which rectifier requires two diodes and two SCRs?

(a) Half-wave controlled rectifier (b) Full-wave controlled bridge rectifier
(c) Half controlled bridge rectifier (d) Semi-converter

6.15 Which rectifier requires four diodes?

(a) Half-wave controlled rectifier (b) Full-wave controlled bridge rectifier
(c) Half controlled bridge rectifier (d) Semi-converter

6.16 Which circuit requires one SCR?

(a) Half-wave controlled rectifier
(b) Full-wave controlled rectifier circuit using center tap transformer
(c) Half controlled bridge rectifier
(d) Semi-converter

6.17 In a controlled rectifier, the load current depends on

(a) firing angle and type of load (b) firing angle only (c) type of load only

6.18 In a single-phase full converter, α is the firing angle and β is extinction angle and load current is discontinuous when

(a) $(\beta - \alpha) < \pi$ (b) $(\beta - \alpha) > \pi$ (c) $(\beta - \alpha) + \pi$

6.19 Active input power of single-phase full converter at firing angle α is

(a) $\frac{2\sqrt{2}VI_o}{\pi}\cos\alpha$ (b) $\frac{2\sqrt{2}VI_o}{\pi}\sin\alpha$ (c) $\frac{\sqrt{2}VI_o}{\pi}\cos\alpha$ (d) $\frac{\sqrt{2}VI_o}{\pi}\sin\alpha$

6.20 Reactive power input of single-phase full converter at firing angle α is

(a) $\frac{2\sqrt{2}VI_o}{\pi}\cos\alpha$ (b) $\frac{2\sqrt{2}VI_o}{\pi}\sin\alpha$ (c) $\frac{\sqrt{2}VI_o}{\pi}\cos\alpha$ (d) $\frac{\sqrt{2}VI_o}{\pi}\sin\alpha$

6.21 At firing angle α, the average dc output voltage of a single-phase full wave controlled rectifier with transformer leakage inductance is

(a) $\frac{2\sqrt{2}V}{\pi}\cos\alpha - \frac{2\omega L_C}{\pi}I_o$ (b) $\frac{2\sqrt{2}V}{\pi}\cos\alpha - \frac{\omega L_C}{\pi}I_o$

(c) $\frac{2\sqrt{2}V}{\pi}\cos\alpha + \frac{2\omega L_C}{\pi}I_o$ (d) $\frac{2\sqrt{2}V}{\pi}\cos\alpha + \frac{\omega L_C}{\pi}I_o$

6.22 At firing angle α, the average dc output voltage of a single-phase full controlled bridge rectifier with source inductance is

(a) $\dfrac{2\sqrt{2}V}{\pi}\cos\alpha - \dfrac{2\omega L_C}{\pi}I_o$ (b) $\dfrac{2\sqrt{2}V}{\pi}\cos\alpha - \dfrac{\omega L_C}{\pi}I_o$

(c) $\dfrac{2\sqrt{2}V}{\pi}\cos\alpha + \dfrac{2\omega L_C}{\pi}I_o$ (d) $\dfrac{2\sqrt{2}V}{\pi}\cos\alpha + \dfrac{\omega L_C}{\pi}I_o$

6.23 In a single-phase half wave rectifier circuit with *RL* load and a free wheeling diode across the load and extinction angle β is greater than π. If the firing angle of thyristor is α, then thyristor conducts for ______ duration and free wheeling diode conducts for ______ duration.

(a) $\pi - \alpha, \beta - \pi$ (b) $\pi - \alpha, \beta$ (c) $\alpha - \pi, \pi - \beta$ (d) $\pi - \alpha, \pi - \beta$

6.24 In discontinuous conduction of semi-converter and extinction angle $\beta > \pi$, in a single-phase semi-converter, each thyristor conducts for ______

(a) 0° (b) $\pi - \alpha$ (c) α (d) $\alpha + \pi$

6.25 In a single-phase half wave rectifier circuit with *RL* load and a free wheeling diode across the load and extinction angle β is less than π. If the firing angle of thyristor is α, then thyristor conducts for ______ duration and free wheeling diode conducts for ______ duration.

(a) $\pi - \alpha, \beta - \pi$ (b) $\beta - \alpha, 0$ (c) $\alpha - \pi, \pi - \beta$ (d) $\pi - \alpha, \pi - \beta$

6.26 In discontinuous conduction of semi-converter and extinction angle $\beta > \pi$, in a single-phase semi-converter, freewheeling diode conducts for ______

(a) 0° (b) $\beta - \pi$ (c) α (d) $\alpha + \pi$

6.27 In a single-phase full converter with resistive load and firing angle α, the load current is zero for ______ duration and non-zero for ______ duration.

(a) $\pi - \alpha, \alpha$ (b) $\alpha, \pi - \alpha$ (c) $\pi + \alpha, \alpha$ (d) $\alpha, \pi + \alpha$

6.28 In continuous conduction of single-phase semi-converter, freewheeling diode conducts for ______

(a) 0° (b) $\beta - \pi$ (c) α (d) $\alpha + \pi$

6.29 A single-phase half wave rectifier circuit generates ______ number of pulses of load current and ______ number of pulses of output voltage during one cycle of supply voltage.

(a) one, one (b) one, two (c) two, one (d) two, two

6.30 In discontinuous conduction of semi-converter and extinction angle $\beta < \pi$, in a single-phase semi-converter, each thyristor conducts for ______

(a) β (b) $\beta - \pi$ (c) α (d) $\alpha + \pi$

6.31 In continuous conduction of a single-phase semi-converter, each thyristor conducts for

(a) $\pi - \alpha$ (b) $\pi + \alpha$ (c) β (d) α

6.32 In discontinuous conduction of semi-converter and extinction angle $\beta < \pi$, in a single-phase semiconverter, freewheeling diode conducts for ______

(a) 0° (b) $\beta - \pi$ (c) α (d) $\alpha + \pi$

Fill in the Blanks

6.1 Single-phase fully controlled converters are obtained by replacing the diodes of an uncontrolled converter with ______.

6.2 In a single-phase fully controlled converter the output voltage can be controlled by controlling the ______ of the thyristors.

6.3 Single-phase fully controlled half wave converters always operate in the ______ conduction mode.

6.4 Depending on the load condition and the firing angle a fully controlled bridge converter can operate either in the ______ conduction mode or in the ______ conduction mode.

6.5 In the continuous conduction mode the load voltage depends only on the ______ and not on load parameters.

6.6 The fully controlled bridge converter can operate as an inverter if firing angle ______.

6.7 In the continuous conduction mode at least ______ thyristors conduct always.

6.8 The input displacement factor of a single-phase fully controlled bridge converter during continuous conduction mode operation is equal to the cosine of the ______.

6.9 In the inverter mode of operation of full controlled bridge converter power flows from the ______ to the ______.

6.10 In the rectifier mode of operation of full controlled bridge converter power flows from the ______ to the ______.

6.11 If the firing angle of a single-phase fully controlled bridge converter is same, the load voltage in the discontinuous conduction mode is ______ than the load voltage in continuous conduction mode of operation.

6.12 In the ______ conduction mode the load current remains zero for a part of the input cycle.

Review Questions

6.1 (a) Draw the circuit diagram of a single-phase half-wave controlled rectifier with *R* load. Discuss its working principle.

(b) Draw the voltage and current waveforms. Determine the following parameters

(i) dc output voltage (ii) Average dc load current (iii) rms output voltage
(iv) rms load current (v) Ripple factor (vi) Regulation
(vii) Efficiency

6.2 (a) What is phase angle controlled technique? Explain the operation of single-phase angle controlled rectifier. Derive the expression for average dc output voltage.

(b) Write the applications of phase controlled rectifiers.

6.3 A single-phase full wave controlled rectifier circuit feeds power to a resistive load. Draw the waveforms for input voltage, output voltage, load current and voltage across a thyristor for a specified firing angle α.

6.4 (a) Draw the circuit diagram of a single-phase half-wave controlled rectifier with *RL* load.

(b) Prove that average dc output voltage is $V_O = \dfrac{\sqrt{2}V}{2\pi}(\cos\alpha - \cos\beta)$ where, V is rms input voltage, α is firing angle and β is extinction angle.

(c) Find the expression for rms output voltage.

(d) Determine (i) the firing angle to ensure no transient current and (ii) the firing angle for the maximum transient.

6.5 (a) Determine the output voltage of a single-phase half-wave controlled converter with *RL* load and free wheeling diode at the firing angle α.

(b) Write the advantages of free wheeling diode in single-phase half-wave controlled rectifier with *RL* load.

6.6 (a) A dc battery is charged through a resistance *R* from a single-phase half-wave controlled rectifier. Derive an expression for the average value of charging current in terms of V_m, E, R and firing angle α.

(b) Derive an expression for the rms value of charging current in terms of V_m, E, R and firing angle α.

(c) Determine (i) the power delivered to battery, (ii) rectifier efficiency and (iii) input power factor.

6.7 (a) Draw the circuit diagram of a single-phase half-wave controlled rectifier with *RLE* load. Discuss its working principle.

(b) Prove that the average charging current is $I_{av} = \dfrac{1}{2\pi R}\,[V_m(\cos\alpha - \cos\beta) - E(\beta - \alpha)]$

6.8 Draw the circuit diagram of a single-phase full wave controlled rectifier using centre tap transformer with R load and find

(a) dc output voltage (b) average d.c. load current (c) rms output voltage
(d) rms load current (e) Ripple factor (g) efficiency

6.9 What are the different modes of operation of single-phase full wave controlled rectifier using centre tap transformer with *RL* load? Draw voltage and current waveform in each operating mode.

6.10 (a) Draw the circuit diagram of single-phase full wave controlled rectifier using centre tap transformer with *RL* load and free wheeling diode.

(b) Prove that the average output voltage at firing angle α is $V_o = \frac{\sqrt{2}V}{\pi}(1+\cos\alpha)$

6.11 Explain how a free-wheeling diode improves power factor in a converter.

6.12 Assume that a single-phase full controlled rectifier is connected to *RLE* load. If the converter operates in discontinuous mode, draw the input voltage, output voltage, load current and supply current wave-from when (i) extinction angle $\beta < \pi$ and (ii) extinction angle $\beta > \pi$

6.13 A single-phase semi-converter is connected to *RLE* load. If the converter operates with discontinuous load current, draw the input voltage, output voltage, load current, supply current and freewheeling diode current wave-from when (a) extinction angle $\beta < \pi$ and (b) extinction angle $\beta > \pi$.

6.14 (a) Draw the circuit diagram of single-phase bridge converter with *R* load. Discuss its working principle.

(b) Draw the voltage and current waveforms. Determine the following parameters:

(i) dc output voltage (ii) average dc load current (iii) rms output voltage

6.15 Assume that a single-phase semi converter provides a constant load current *I*. Determine the following performance parameters:

(a) Displacement factor, (b) Distortion factor, (c) Input power factor,
(d) Ripple factor, (e) Active and reactive power inputs.

6.16 (a) Compare between semi converter and full converter.

(b) Compare between half-controlled converter and full controlled converter.

6.17 How a single-phase full converter operates as an inverter.

6.18 (a) Draw the circuit diagram of single-phase half-controlled bridge rectifier with *R* load. Discuss its working principle.

(b) Draw the voltage and current waveforms for single-phase half-controlled bridge rectifier with R Load. Determine the following parameters:

(i) dc output voltage (ii) Average dc load current (iii) rms output voltage
(iv) rms load current (v) Ripple factor

6.19 (a) What is the effect of leakage inductance in single-phase full-wave controlled rectifier using center tap transformer with *RL* load?

(b) Draw the voltage and current waveforms

(c) Prove that the average dc output voltage is $V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha - \frac{\omega L_C}{\pi} I_o$

6.20 (a) What is the effect of source inductance in single-phase full-wave controlled bridge rectifier with *RL* load?

(b) Draw the voltage and current waveforms

(c) Prove that the average dc output voltage is

$$V_o = \frac{\sqrt{2}V}{\pi}[\cos\alpha + \cos(\alpha+\mu)] = \frac{2\sqrt{2}V}{\pi}\cos\alpha - \frac{2\omega L_C}{\pi} I_o$$

6.21 A single-phase half-wave controlled rectifier with *R* load is shown in Fig. 6.5 and it is fed from a 230 V, 50 Hz ac supply. When $R = 15\ \Omega$ and $\alpha = 30°$, determine (a) average dc output voltage, (b) rms output voltage, (c) form factor, (d) ripple factor, (e) rectification efficiency, (f) TUF and (g) peak inverse voltage of thyristor.

6.22 A single-phase 220 V, 1.5 kW heater is connected a half-wave controlled rectifier and it fed from a 220 V, 50 Hz ac supply. Determine the power absorbed by the heater when the firing angle is (a) $\alpha = 45°$ and (b) $\alpha = 60°$.

6.23 A single-phase half-wave controlled rectifier with *R* load is supplied from a 230 V, 50 Hz ac source. When average dc output voltage is 50% of maximum possible average dc output voltage, determine (a) firing angle of thyristor, (b) average dc output voltage, (c) rms output voltage, (d) average and rms output current and (e) average and rms current of thyristor. Assume $R = 10\ \Omega$.

6.24 A single-phase half-wave controlled rectifier is connected across a *RL* load and feeds from a 220 V, 50 Hz ac supply. When $R = 10\ \Omega$ and $L = 0.025$ H, determine (a) the firing angle to ensure no transient current and (b) the firing angle for the maximum transient.

6.25 A 80 V battery is charged through a resistor R as depicted in Fig. 6.15. When the charger circuit is fed from 220 V, 50 Hz ac supply and R is 10 Ω, compute (a) the minimum angle at which thyristor will be turned ON, (b) the angle at which thyristor will be turned OFF, (c) maximum conduction period of thyristor, (d) average charging current when $\alpha = 45°$, (e) power supplied to battery , (f) power dissipated in resistor R and (g) input power factor.

6.26 A single-phase half-wave converter with *RLE* load is connected to 220 V, 50 Hz ac supply. When R is 5 Ω, L is 3 mH, E = 110 V and firing angle of SCR is $\alpha = 60°$, determine (a) the circuit turn-OFF time if load current is zero at 210°, (b) average charging current when $\alpha = 40°$ and (c) average output voltage.

6.27 A single-phase full-wave controlled rectifier using centre tap transformer is fed from a 220 V, 50 Hz ac source and it is connected a resistive load R = 20 Ω. Determine (a) the output voltage, (b) form factor, (c) ripple factor, (d) efficiency, (e) transformer utilisation factor at $\alpha = 40°$. Turn ratio of transformer is 1:1.

6.28 A single-phase fully controlled bridge converter with *RL* load is supplied from 220 V, 50 Hz ac supply. If the firing angle is 60°, determine (a) average output voltage, (b) displacement factor , (c) input power factor and (d) harmonic factor.

6.29 A single-phase fully controlled converter with *RLE* load is supplied from 220 V, 50 Hz ac supply. The average load current is 6 which is constant over the working range. Determine the firing angle for (a) E = 100 V (b) E = –100 V. Assume R = 5 Ω and L = 4 mL.

6.30 A single-phase fully controlled bridge converter is connected with *RLE* load where R = 10 Ω, L = 5 mL and E = 100 V. This converter circuit is supplied from 230 V, 5 0 Hz ac supply. Calculate the average value of load current when the firing angle $\alpha = 45°$.

6.31 A 120 V battery is charged through a resistor R as depicted in Fig. 6.34. When the charger circuit is fed from 230 V, 50 Hz ac supply and R is 5 Ω. If thyristor is triggered at $\alpha = 30°$ in every positive cycle, compute (a) average charging current, (b) power supplied to battery, (c) rms value of load current, (d) power dissipated in resistor R, (e) input power factor.

6.32 The voltage across secondary winding of a single-phase transformer is 210 V, 50 Hz and the transformer deliver power to resistive load of R = 2.5 Ω through a single-phase half-wave controlled rectifier. If the firing angle of thyristor is 60°, calculate (a) average dc output voltage, (b) average dc output current, (c) rms output voltage, (d) rms output current, (e) form factor, (f) voltage ripple factor, (g) rectification efficiency, (h) transformer utilisation factor (i) PIV of thyristor.

6.33 The peak forward voltage rating of thyristors is 1000 V and average on-state current rating is 50 A in a single-phase mid-point full converter and single-phase bridge converter. Determine the power rating of converters which can handle properly. The factor of safety is 2.

6.34 A single-phase fully controlled bridge converter with *RL* load is supplied from 230 V, 50 Hz ac supply. If the firing angle is 30°, determine (a) average output voltage, (b) displacement factor, (c) input power factor and (d) harmonic factor.

6.35 A single-phase fully controlled bridge converter with *RLE* load is supplied from 210 V, 50 Hz ac supply. The average load current is 6 A which is constant over the working range. Determine the firing angle for (a) E = 100 V and (b) E = –100 V. Assume R = 5 Ω and L = 5 mL.

6.36 A single-phase fully controlled bridge converter is connected with *RLE* load where R = 6 Ω, L = 4 mL and E = 100 V. This converter circuit is supplied from 220 V, 50 Hz ac supply. Calculate the average value of load current when the firing angle $\alpha = 45°$.

6.37 A single-phase fully controlled bridge converter is connected to *RLE* load with R = 5 Ω, L = 5 mL and E = 80 V. This converter is supplied from 230 V, 50 Hz ac supply. (a) Determine the average load current at $\alpha = 45°$. (b) In the bridge circuit, one thyristor is open circuited due to fault. At this condition what will be the average load current.

6.38 A single-phase fully controlled bridge converter is connected to *RLE* load and it is supplied from 230 V, 50 Hz ac supply. The average load current is 7.5 A which is constant over the working range. Determine the firing angle for (a) E = 110 V and (b) E = –110 V. Assume R = 5 Ω and L = 3 mL. Specify which source is delivering power to load in cases (a) and (b). When the output current is constant, determine the input power factor for both cases (a) and (b).

6.39 A single-phase bridge controlled rectifier consists of a thyristor and three diodes and it is supplied by 220 V, 50 Hz ac supply. If the firing angle of thyristor is 30°, determine the average output current and power delivered to battery when *RLE* load consists of $R = 4\ \Omega$, $L = 4$ mH, and $E = 100$ V. Assume current is constant.

6.40 A single-phase fully controlled bridge converter is supplied from 200 V, 50 Hz ac supply and fed to a load which consists of $R = 10\ \Omega$ and large inductance so that the load current is constant. If the firing angle is 45°, calculate (a) average output voltage, (b) average output current, (c) average current of thyristor, (d) rms current of thyristor and power factor.

6.41 In the above question, if the source inductance is about 1.5 mH, find the average output voltage, overlap angle and power factor.

6.42 A single-phase fully controlled bridge converter is supplied from a 210 V, 50 Hz single-phase supply and operates in the continuous conduction mode at a firing angle $\alpha = 35°$. If the load resistance and inductances are 5 Ω and 25 mH respectively, determine (a) average dc output voltage, (b) average output load current, (c) third harmonic load current as a percentage of the average load current.

6.43 A single-phase fully controlled bridge converter is connected to 230 V, 50 Hz. A load of $R = 12\ \Omega$ is connected in series with a large inductance and load current is ripple free. If the firing angle of converter is 60°, determine different performance parameters of the converter.

6.44 A single-phase semi-converter is supplied by 210 V, 50 Hz and it is connected with a *RLE* load where $R = 15\ \Omega$, $E = 90$ V and L is very large so that the load current is ripple free. Determine (a) average output voltage, (b) average output current, (c) average and rms value of thyristor current, (d) average and rms value of diode current and (e) circuit turn-OFF time at $\alpha = 35°$.

6.45 A single-phase semi-converter is connected to 230 V, 50 Hz. A load of $R = 10\ \Omega$ is connected in series with a large inductance and load current is ripple free. If the firing angle of converter is 45°, determine different performance parameters of the converter.

6.46 A single-phase full-converter is used to deliver a constant load current. If the overlap angle is 15° for zero degree firing angle of converter, determine the overlap angle when firing angle is (a) $\alpha = 30°$, (b) 45°, (c) 60°.

6.47 A single-phase dual converter is supplied from a 200 V, 50 Hz ac source and it is connected to a load of $R = 10\ \Omega$. The value of current limiting reactor is $L = 0.08H$. If the firing angle of converter-1 $\alpha_1 = 45°$, determine (a) firing angle of converter-2, (b) average value of load voltage, (c) peak value of circulating current and (b) peak currents of both converters.

6.48 Draw the circuit diagram of a dual converter and explain the basic operating principle of a dual converter.

6.49 What are the types of dual converter? Explain any one type dual converter in detail.

6.50 With a neat circuit diagram describe the circulating mode dual converter. Draw the voltage waveforms of dual converter.

6.51 Derive the expression for the peak value of circulating current. Write the disadvantage of circulating mode dual converter.

6.52 Compare non-circulating mode dual converter and circulating mode dual converter.

Answers to Multiple-Choice Questions

6.1 (a)	6.2 (b)	6.3 (c)	6.4 (d)	6.5 (a)	6.6 (a)	6.7 (b)
6.8 (c)	6.9 (c)	6.10 (a)	6.11 (b)	6.12 (b)	6.13 (c)	6.14 (c)
6.15 (b)	6.16 (a)	6.17 (a)	6.18 (a)	6.19 (a)	6.20 (b)	6.21 (b)
6.22 (b)	6.23 (a)	6.24 (b)	6.25 (b)	6.26 (b)	6.27 (a)	6.28 (c)
6.29 (a)	6.30 (b)	6.31 (a)	6.32 (a)			

Answers to Fill in the Blanks

6.1 thyristors	6.2 firing delay angle α	6.3 Discontinuous	6.4 continuous, discontinuous
6.5 firing angle	6.6 $\left(\alpha > \dfrac{\pi}{2}\right)$	6.7 Two	6.8 firing angle
6.9 dc side, ac side	6.10 ac source, dc side	6.11 greater	6.12 discontinuous

THREE-PHASE UNCONTROLLED RECTIFIERS

7

7.1 INTRODUCTION

Single-phase uncontrolled rectifiers are extensively used in low to medium power applications as dc power supply in different electronics equipments. The single-phase uncontrolled rectifiers can able to handle up to 15 KW as high KVA transformers are required for a specified dc output power. Where single-phase rectifiers are not suitable, three-phase uncontrolled rectifiers are used for above 15 KW and high power applications such as

1. Power supply of electrical machines
2. High voltage dc transmission
3. dc motor drives
4. Radio transmitters
5. TV transmitters
6. Power supply of telephone exchange
7. Electronics equipments

Three-phase uncontrolled rectifiers are known as *polyphase rectifiers*. Harmonics and ripple in output voltage are more in single-phase rectifiers. Since less harmonics and less ripple voltage exist in three phase rectifier, three-phase and multiphase (polyphase) uncontrolled rectifiers can be used for high power applications with high voltage and current rating. In high power applications, three-phase rectifiers are preferred over single phase rectifier due to the following advantages:

1. High dc output voltage
2. Less ripple in output current
3. High input power factor
4. High transformer utilization factor (TUF)
5. Size of filter is low due to high ripple frequency

Three-phase rectifiers can convert a three phase ac supply into a fixed dc voltage as shown in Fig. 7.1. There are two types of three phase uncontrolled rectifiers such as half-wave uncontrolled rectifiers and full-wave bridge rectifiers. The classification of three phase uncontrolled rectifier is depicted in Fig. 7.2.

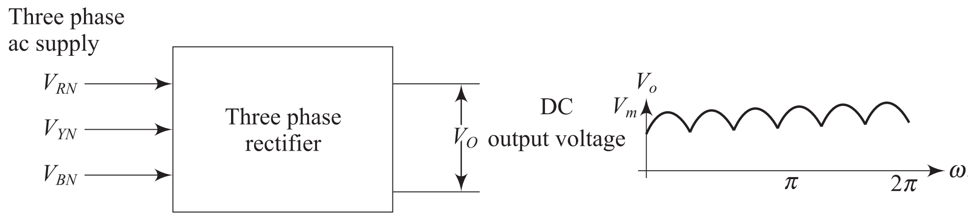


Fig. 7.1 *Block diagram of three-phase rectifier*

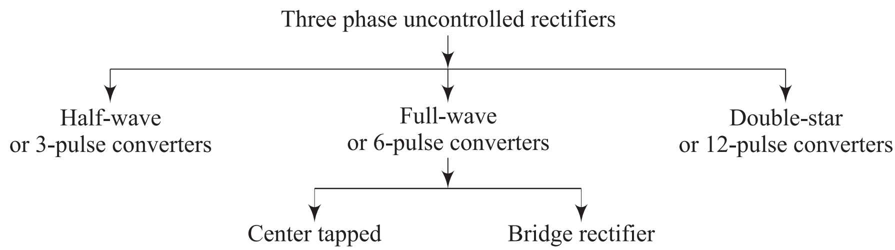
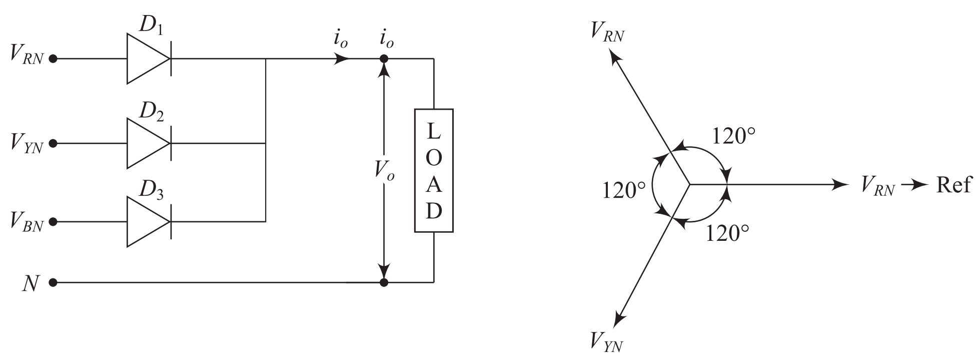


Fig. 7.2 *Classification of three-phase rectifiers*

In this chapter, operating principle and analysis of three-phase rectifiers are discussed in detail. The comparative performance of different three phase rectifiers is given in Section 7.9. During analysis of rectifier circuits, assume transformer and diodes are ideal one. The forward voltage drop across diode and reverse diode current are zero. In the analysis of three-phase rectifier circuits, it is assumed that transformer has zero resistance and zero leakage inductance.

7.2 THREE-PHASE HALF-WAVE RECTIFIER

Figure 7.3 shows a three-phase half-wave rectifier with resistive load. Actually this circuit consists of three single-phase half-wave rectifier. Therefore this circuit is also known as three-phase Star rectifier. The diode in R or Y or B phase conducts when the voltage on the particular phase is higher than that on the other two phases.



(a) (b)

Fig. 7.3 *Three-phase half-wave rectifier circuit with load*

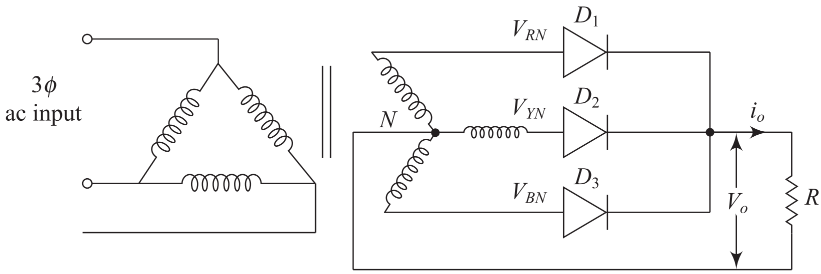


Fig. 7.4 *Three-phase half-wave rectifier circuit with R load*

Three phase voltages are

$$V_{RN} = \sqrt{2}V \sin \omega t = V_m \sin \omega t$$

$$V_{YN} = \sqrt{2}V \sin(\omega t - 2\pi/3) = V_m \sin(\omega t - 2\pi/3)$$

$$V_{BN} = \sqrt{2}V \sin(\omega t + 2\pi/3) = V_m \sin(\omega t + 2\pi/3) = V_m \sin(\omega t - 4\pi/3)$$

The vector diagram of V_{RN}, V_{YN} and V_{BN} is depicted in Fig. 7.3(b). Three-phase half-wave rectifier circuit with R load is depicted in Fig. 7.4. The voltage and current waveforms of a three phase half-wave rectifiers are depicted in Fig. 7.5. For diode D_1 in R phase, V_{RN} is greater than V_{YN} and V_{BN} from $\omega t = \frac{\pi}{6}$ to $\omega t = \frac{5\pi}{6}$. Hence diode D_1 is forward biased during $\frac{\pi}{6}$ to $\frac{5\pi}{6}$ and conducts for $\frac{2\pi}{3}$ duration. Similarly D_2 and D_3 are also conducts for $\frac{2\pi}{3}$ duration as depicted in Fig. 7.5. Hence, diode D_2 is forward biased during $\omega t = \frac{5\pi}{6}$ to $\omega t = \frac{3\pi}{2}$ and conducts for $\frac{2\pi}{3}$ duration. Subsequently diode D_3 is also forward biased during $\omega t = \frac{3\pi}{2}$ to $\omega t = \frac{13\pi}{6}$ and conducts for $\frac{2\pi}{3}$ duration.

The three-phase centre-tap rectifier uses the neutral connection of the supply as the return path for the load.

For this circuit, the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_m \sin \omega t \cdot d(\omega t) = \frac{V_m}{2\pi/3}[-\cos \omega t]_{\pi/6}^{5\pi/6}$$

$$= \frac{V_m}{2\pi/3}\left[-\cos\frac{5\pi}{6} + \cos\frac{\pi}{6}\right]_{\pi/6}^{5\pi/6} = V_m \frac{3}{\pi}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi}V_m = 0.827V_m$$

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/3}\int_{\pi/6}^{5\pi/6} \frac{1}{2}(1 - \cos 2\omega t) \cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^2}{4\pi}\left(\omega t - \frac{1}{2}\sin 2\omega t\right)_{\pi/6}^{5\pi/6}\right]^{\frac{1}{2}} = \left[\frac{3V_m^2}{4\pi}\left(\frac{5\pi}{6} - \frac{\pi}{6} - \frac{1}{2}\sin\frac{10\pi}{6} + \frac{1}{2}\sin\frac{2\pi}{6}\right)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{3}{2\pi}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)\right]^{\frac{1}{2}} = 0.8406V_m$$

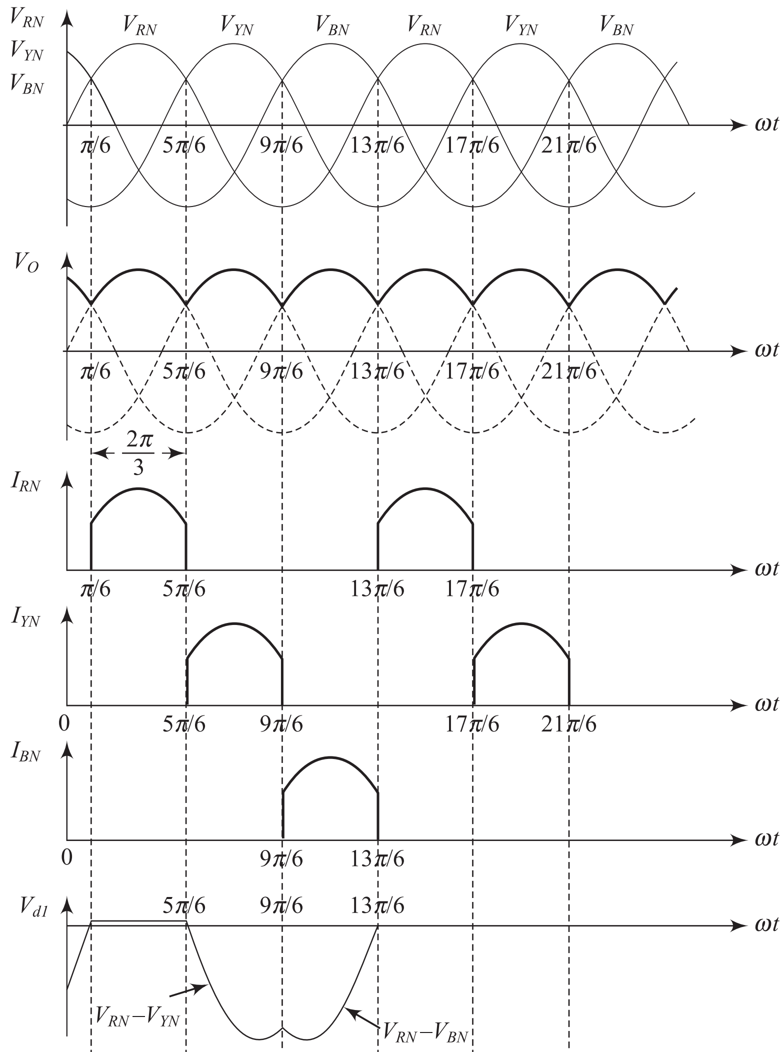


Fig. 7.5 *Voltage and current waveforms of three phase half-wave rectifier with R load*

The average output current is

$$I_{av} = I_o = \frac{V_o}{R} = \frac{0.827V_m}{R}$$

The dc output power is

$$P_{dc} = V_{dc}I_{dc} = V_oI_o = \frac{(0.827V_m)^2}{R} = \frac{0.683V_m^2}{R}$$

The rms value of load current is $I_{rms} = \dfrac{V_{rms}}{R} = \dfrac{0.8406V_m}{R}$

The ac power supplied by the transformer secondary is

$$P_{ac} = I_{rms}^2\, R = \frac{(0.8406V_m)^2}{R^2} = \frac{0.7066V_m^2}{R}$$

Rectification efficiency is

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\% = \frac{0.683V_m^2/R}{0.7066V_m^2/R} \times 100\% = 96.66\%$$

Form factor is $FF = \frac{V_{rms}}{V_{av}} = \frac{0.8406V_m}{0.827V_m} = 1.01$

Ripple factor is $RF = \sqrt{FF^2 - 1}$ = 0.1821

The rms current in each phase (winding) of the transformer secondary is

$$I_{rms(transformer)} = \left[\frac{1}{2\pi}\int_{\pi/6}^{5\pi/6} I_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} = 0.4853\, I_m \quad \text{where, } I_m = \frac{V_m}{R}$$

The VA rating of transformer is

$$\text{VA} = 3V_{rms} \times I_{rms(transformer)} = 3 \times \frac{V_m}{\sqrt{2}} \times 0.4853 I_m$$

$$= 1.029 V_m I_m = 1.029 \frac{V_m^2}{R} \text{ as } I_m = \frac{V_m}{R}$$

Transformer utilisation factor is

$$\text{TUF} = \frac{P_{dc}}{\text{VA}} = \frac{0.683V_m^2/R}{1.029V_m^2/R} = 0.663$$

As there are three pulses in a complete cycle of supply voltage, the pulse frequency of output is $f_o = 3f$. The peak inverse voltage diode is available when diode is in the non-conducting state. When D_1 is in non-conducting state, the voltage across diode D_1 is $V_R - V_Y$ or $V_R - V_B$. The $V_R - V_Y = V_{RY}$ is the line voltage which is equal to $\sqrt{3}$ times phase voltage. Therefore, peak inverse voltage of each diode is $\sqrt{3}V_m$. The disadvantages of this rectifier are (i) high ripple factor and (ii) dc magnetisation current in transformer secondary. From the above analysis, the following observations are made:

1. Each diode conducts for 120° duration.
2. During one cycle of input voltage, there are three pulses of output voltage.
3. Current flow in transformer secondary is unidirectional. dc current exists in the secondary winding. Therefore, transformer core gets saturated leading to more iron losses and the efficiency is reduced.

Example 7.1 A three-phase half-wave rectifier is fed from a 3ϕ, 400 V, 50 Hz ac supply and it is connected with a R load of 10 Ω. Calculate (a) dc output voltage, (b) rms value of output voltage, (c) average output current, (d) dc output power, (e) rms load current, (f) ac power supplied and (g) rectification efficiency.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$ and R = 10 Ω

Maximum phase voltage is $V_m = \sqrt{2}V = \sqrt{2} \times 230.94$ = 326.54 V

(a) The average dc output voltage is

$$V_{av} = V_o = \frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_m \sin \omega t \cdot d(\omega t) = \frac{V_m}{2\pi/3}[-\cos \omega t]_{\pi/6}^{5\pi/6}$$

$$= \frac{3\sqrt{3}}{2\pi}V_m = 0.827V_m = 0.827 \times 326.54 = 270.05 \text{ V}$$

(b) The rms value of output voltage is

$$V_{rms} = \left[\frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{3}{2\pi}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)\right]^{\frac{1}{2}} = 0.8406 \times 326.54 = 274.489 \text{ V}$$

(c) The average output current is

$$I_{av} = I_o = \frac{V_o}{R} = \frac{270.05}{10} = 27.005 \text{ A}$$

(d) The dc output power is

$$P_{dc} = V_{dc}I_{dc} = V_oI_o = 270.05 \times 27.005 \text{ Watt} = 7292.7 \text{ Watt}$$

(e) The rms value of load current is $I_{rms} = \frac{V_{rms}}{R} = \frac{274.489}{10} = 27.4489$

(f) The ac power supplied by the transformer secondary is

$$P_{ac} = I_{rms}^2 R = 27.4489^2 \times 10 = 7534.42 \text{ Watt}$$

(g) Rectification efficiency is

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\% = \frac{7292.7}{7534.42} \times 100\% = 96.79\%$$

Example 7.2 A three-phase half-wave rectifier is fed from a 3ϕ ac supply and it is connected with a R load. Determine (a) form factor, (b) ripple factor and (c) TUF.

Solution

Assume that the maximum phase voltage is V_m.
The average dc output voltage is $V_o = 0.827V_m$
The rms value of output voltage is $V_{rms} = 0.8406V_m$

(a) Form factor is $FF = \frac{V_{rms}}{V_{av}} = \frac{0.8406V_m}{0.827V_m} = 1.01$

(b) Ripple factor is $RF = \sqrt{FF^2 - 1} = 0.1821$

(c) The rms current in each phase of the transformer secondary is

$$I_{rms(transformer)} = \left[\frac{1}{2\pi}\int_{\pi/6}^{5\pi/6} I_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} = 0.4853\, I_m = 0.4853\frac{V_m}{R} \quad \text{when } I_m = \frac{V_m}{R}$$

The dc output power is

$$P_{dc} = V_oI_o = 0.827V_m \times \frac{0.827V_m}{R} = \frac{0.683V_m^2}{R}$$

The VA rating of transformer is

$$\text{VA} = 3V_{\text{rms}} \times I_{\text{rms(transformer)}} = 3 \times \frac{V_m}{\sqrt{2}} \times 0.4853 I_m = 3 \times \frac{V_m}{\sqrt{2}} \times 0.4853 \frac{V_m}{R} = 1.029 \frac{V_m^2}{R}$$

Transformer utilisation factor is

$$\text{TUF} = \frac{P_{\text{dc}}}{\text{VA}} = \frac{0.683 V_m^2/R}{1.029 V_m^2/R} = 0.663$$

7.3 SIX-PHASE HALF-WAVE RECTIFIER

The problems of three-phase half-wave rectifier can be removed up to certain extend by six-phase full-wave rectifier. Figure 7.6 shows a six-phase rectifier circuit. The six-phase voltages can be obtained in the secondary by using a canter-tapped arrangement on a Star connected three-phase winding and the vector diagram of six phase voltages is shown in Fig. 7.7. There are six diodes in a six-phase rectifier. When a particular phase voltage is higher than other phases, diodes on the particular phase conducts. This rectifier is also known as *three-phase full-wave rectifier*. Figure 7.8 shows the output voltage and current waveforms. This rectifier circuit is also called as *three-phase mid point six-pulse rectifier.*

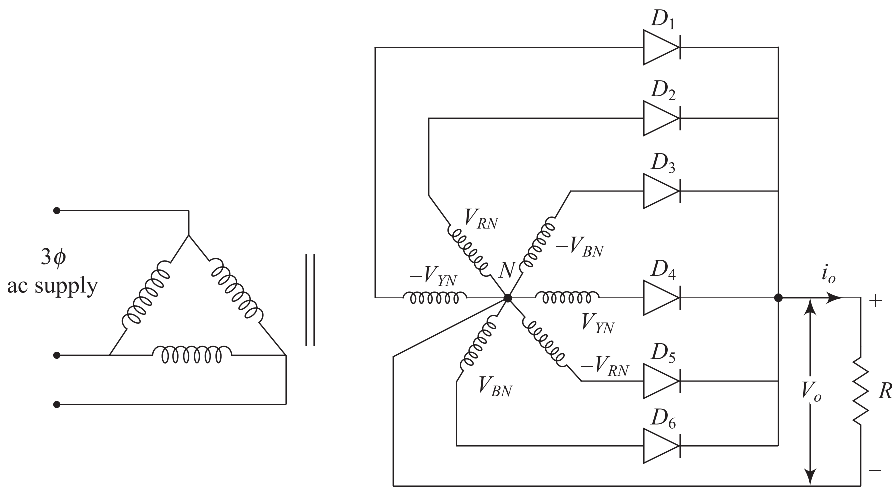


Fig. 7.6 *Six-phase half-wave rectifier*

It is clear from Fig. 7.8 that each diode conducts for $\pi/3$ or 60° duration. Current flows through one diode at a time. Therefore, average current is low but the ratio between maximum current to average current in the diodes is high. Hence the utilisation of transformer secondary is poor. The dc currents in the secondary of the six-phase Star rectifier can be cancelled in the secondary windings and core saturation is not encountered.

For this circuit, the average dc output voltage is

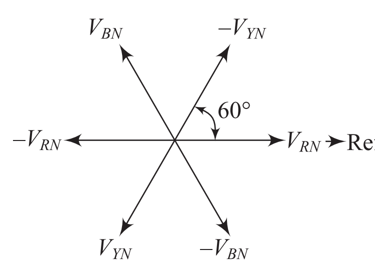


Fig. 7.7 *Vector diagram of six-phase voltages*

$$V_{\text{av}} = V_o = \frac{1}{2\pi/6} \int_{\pi/3}^{2\pi/3} V_m \sin \omega t \cdot d(\omega t) = \frac{V_m}{2\pi/6} [-\cos \omega t]_{\pi/3}^{2\pi/3}$$

$$= \frac{V_m}{2\pi/6} \left[-\cos\frac{\pi}{3} + \cos\frac{2\pi}{3} \right]_{\pi/3}^{2\pi/3} = V_m \frac{6}{\pi} \frac{1}{2} = 0.955 V_m$$

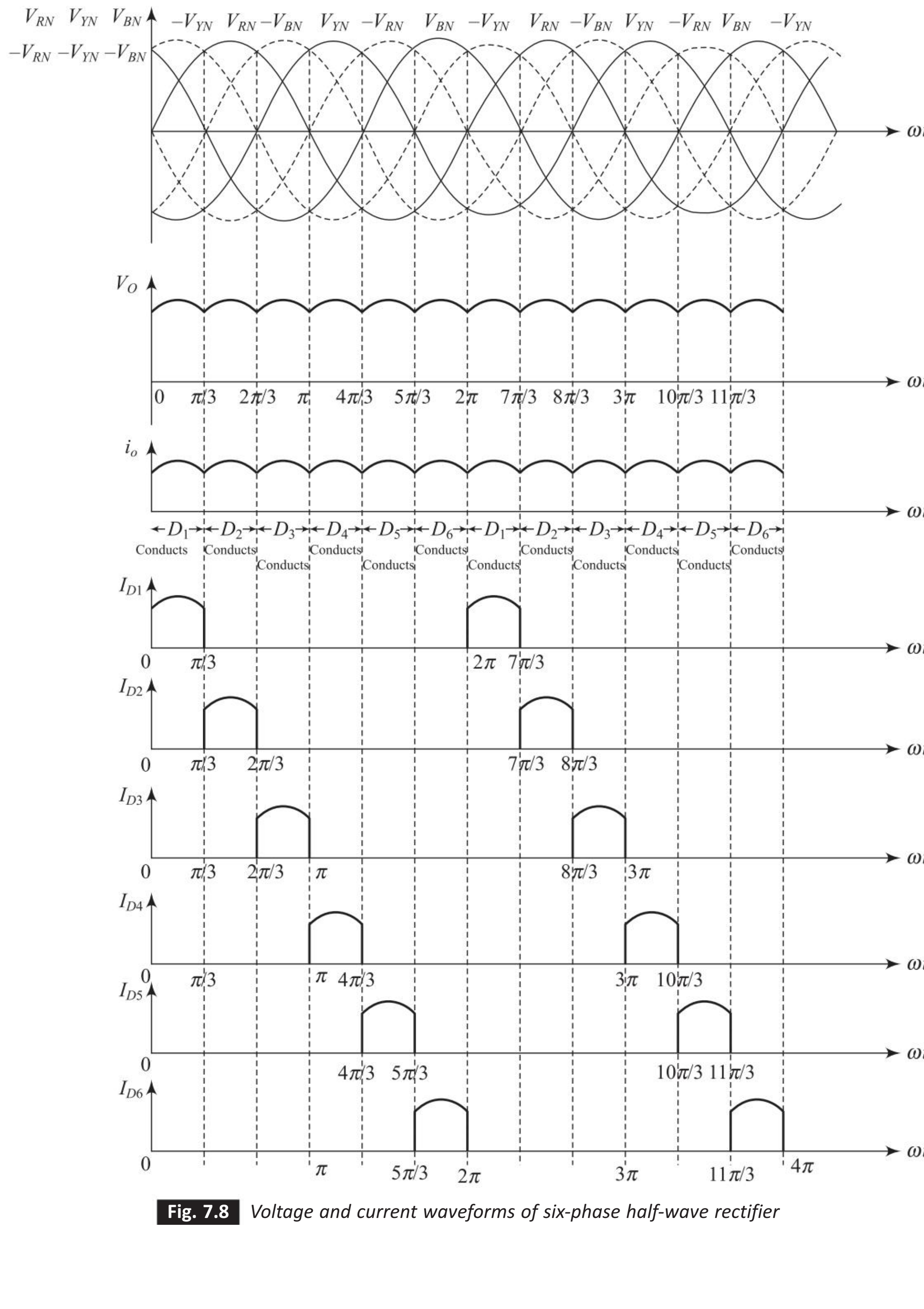


Fig. 7.8 *Voltage and current waveforms of six-phase half-wave rectifier*

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} V_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/6}\int_{\pi/3}^{2\pi/3} \frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{2\pi/6}\left(\frac{1}{2}\omega t - \frac{1}{2}\sin 2\omega t\right)_{\pi/6}^{5\pi/6}\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/6}\left(\frac{2\pi}{6} - \frac{\pi}{6} - \frac{1}{2}\sin\frac{2\pi}{6} + \frac{1}{2}\sin\frac{\pi}{6}\right)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{6}{2\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)\right]^{\frac{1}{2}} = 0.956V_m$$

Form factor is $FF = \dfrac{V_{\text{rms}}}{V_{\text{dc}}} = \dfrac{0.956V_m}{0.955V_m} = 1.001$

The ripple factor is $RF = \sqrt{FF^2 - 1} = \sqrt{(1.001)^2 - 1} = 0.045$

The average output current is

$$I_{\text{av}} = I_o = \frac{V_o}{R} = \frac{0.955V_m}{R}$$

The rms output current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{0.956V_m}{R}$$

The rms current in each winding of the transformer secondary is

$$I_{\text{rms(transformer)}} = \left[\frac{1}{2\pi}\int_{\pi/3}^{2\pi/3} I_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} = I_m\sqrt{\frac{1}{2\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} = 0.39I_m$$

Where, $I_m = \dfrac{V_m}{R}$

The output power of rectifier is

$$P_{\text{dc}} = V_{\text{av}}I_{\text{av}} = V_oI_o = \frac{(0.955V_m)^2}{R} = 0.912\frac{V_m^2}{R}$$

The input ac power to transformer secondary is

$$P_{\text{ac}} = I_{\text{rms}}^2 R = \left(\frac{0.956V_m}{R}\right)^2 R = 0.913\frac{V_m^2}{R}$$

Efficiency of rectifier is $\eta = \dfrac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\% = \dfrac{0.912V_m^2/R}{0.913V_m^2/R} \times 100 = 99.79\%$

In is clear from Fig. 7.8 that there are six pulses in the output voltage for a complete cycle of input voltage. Therefore, ripple frequency is six times of fundamental frequency ($f_o = 6f$)

The peak inverse voltage (PIV) of six-phase half-wave rectifier is same as the PIV of three-phase half-wave rectifier. $\text{PIV} = \sqrt{3}V_m$

The VA rating of transformer is $\mathrm{VA} = 6V_{\mathrm{rms(transformer)}} I_{\mathrm{rms(transformer)}}$

$$= 6\frac{V_m}{\sqrt{2}} \times \frac{0.39V_m}{R} = 1.654\frac{V_m^2}{R}$$

The transformer utilisation factor is

$$\mathrm{TUF} = \frac{P_{\mathrm{dc}}}{\mathrm{VA}} = \frac{0.912V_m^2/R}{1.654V_m^2/R} = 0.551$$

From the above analysis,

1. It is clear that the quality of output voltage is better as compared to three-phase half-wave rectifier due to low ripple factor ($RF = 0.045 = 4.5\%$) and form factor is close to unity ($FF = 1.001$).
2. As output frequency is equal to $6f$, the filter size will be reduced.
3. The disadvantages of six-phase half-wave rectifier is low transformer utilisation factor compared to three phase half-wave rectifier.

Example 7.3 A six-phase half-wave rectifier is fed from a 3ϕ, 440 V, 50 Hz ac supply and it is connected with a R load of 5 Ω. Calculate (a) dc output voltage, (b) rms value of output voltage, (c) average output current, (d) rms load current, (e) dc output power, (f) ac power supplied and (g) rectification efficiency.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.04$ and $R = 5\ \Omega$

Maximum phase voltage is $V_m = \sqrt{2}V = \sqrt{2} \times 254.04 = 359.21$ V

(a) The average dc output voltage is

$$V_o = \frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi/6}[-\cos\omega t]_{\pi/3}^{2\pi/3}$$

$$= 0.955V_m = 0.955 \times 359.21\ \mathrm{V} = 343.05\ \mathrm{V}$$

(b) The rms value of output voltage is

$$V_{\mathrm{rms}} = \left[\frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} V_m^2 \sin^2\omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/6}\int_{\pi/3}^{2\pi/3}\frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{6}{2\pi}\left(\frac{\pi}{6}+\frac{\sqrt{3}}{4}\right)\right]^{\frac{1}{2}} = 0.956V_m = 0.956 \times 359.21\ \mathrm{V} = 343.40\ \mathrm{V}$$

(c) The average output current is

$$I_{\mathrm{av}} = I_o = \frac{V_o}{R} = \frac{343.05}{5} = 68.61\ \mathrm{A}$$

(d) The rms output current is

$$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{R} = \frac{343.40}{5} = 68.68\ \mathrm{A}$$

(e) Output power is

$$P_{\mathrm{dc}} = V_o I_o = 343.05 \times 68.61 = 23.536\ \mathrm{kW}$$

(f) The input ac power to transformer secondary is

$$P_{\mathrm{ac}} = I_{\mathrm{rms}}^2 R = 68.68^2 \times 5 = 23.584\ \mathrm{kw}$$

(g) Efficiency of rectifier is $\eta = \frac{P_{\mathrm{dc}}}{P_{\mathrm{ac}}} \times 100\% = \frac{23.536}{23.584} \times 100 = 99.79\%$

7.4 MULTIPHASE RECTIFIER

In a three-phase half-wave rectifier, each phase conducts for 120° or $\frac{2\pi}{3}$ radian of a cycle of 360° (2π radians). For a six-phase half-wave rectifier, each phase conducts for 60° or $\frac{2\pi}{6}$ radian of a cycle of 360° (2π radians). Therefore in a m phase half-wave rectifier, each phase as well as each diode would conduct for $\frac{2\pi}{m}$ radian and the number of output voltage pulses p will be equal to number of phases m. Figure 7.9 shows the output voltage wave from for m-phase half-wave diode rectifier where a diode conducts from $-\frac{\pi}{m}$ to $\frac{\pi}{m}$ or from $-\frac{\pi}{p}$ to $\frac{\pi}{p}$.

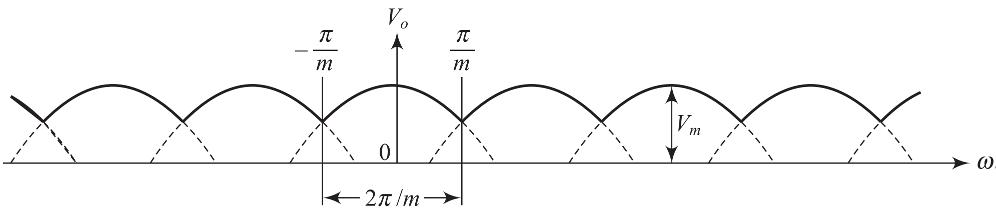


Fig. 7.9 *Output voltage wave from for m-phase half-wave diode rectifier*

Assume that the instantaneous phase voltage is $v = V_m \cos \omega t = \sqrt{2}V \cos \omega t$ where, V_m is the maximum phase voltage and V is rms phase voltage.

Since each diode conducts from $-\frac{\pi}{m}$ to $\frac{\pi}{m}$ as depicted in Fig. 7.9, the average output voltage is equal to

$$V_o = \frac{1}{2\pi/m} \int_{-\pi/m}^{\pi/m} V_m \cos \omega t \cdot d(\omega t)$$

$$= V_m \frac{m}{\pi} \sin \frac{\pi}{m}$$

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/m} \int_{-\pi/m}^{\pi/m} (V_m \cos \omega t)^2 d(\omega t) \right]^{\frac{1}{2}}$$

$$= \left[\frac{m}{2\pi} \frac{V_m^2}{2} \int_{-\pi/m}^{\pi/m} (1 + \cos 2\omega t) d(\omega t) \right]^{\frac{1}{2}}$$

$$= V_m \left[\frac{m}{2\pi} \left(\frac{\pi}{m} + \frac{1}{2} \sin \frac{2\pi}{m} \right) \right]^{\frac{1}{2}}$$

Maximum value of load current is $I_m = \frac{V_m}{R}$

Average value of diode current is

$$I_D = \frac{1}{2\pi} \int_{-\pi/m}^{\pi/m} I_m \cos \omega t \cdot d(\omega t) = \frac{I_m}{\pi} \sin \frac{\pi}{m}$$

The rms value of diode current is

$$I_{\text{Drms}} = \left[\frac{1}{2\pi}\int_{-\pi/m}^{\pi/m}(I_m \cos\omega t)d(\omega t)\right]^{\frac{1}{2}} = I_m\left[\frac{1}{2\pi}\left(\frac{\pi}{m}+\frac{1}{2}\sin\frac{2\pi}{m}\right)\right]^{\frac{1}{2}}$$

The output voltage of a m phase rectifier can be represented by Fourier series. To find the constants of the Fourier series, we have to integrate from $-\frac{\pi}{m}$ to $\frac{\pi}{m}$. Then constants a_n and b_n are computed by the following expressions:

$$a_n = \frac{1}{\pi/m}\int_{-\pi/m}^{\pi/m} V_m \cos\omega t \cdot \cos n\omega t \cdot d\omega t$$

$$= \frac{mV_m}{\pi}\left[\frac{\sin\left(\frac{(n-1)\pi}{m}\right)}{n-1} + \frac{\sin\left(\frac{(n+1)\pi}{m}\right)}{n+1}\right]$$

$$= \frac{mV_m}{\pi}\left[\frac{(n+1)\sin\left(\frac{(n-1)\pi}{m}\right) + (n-1)\sin\left(\frac{(n+1)\pi}{m}\right)}{n^2-1}\right]$$

After simplification, the above equation can be written as

$$a_n = \frac{2mV_m}{\pi(n^2-1)}\left[n\sin\frac{n\pi}{m}\cos\frac{\pi}{m} - \cos\frac{n\pi}{m}\sin\frac{\pi}{m}\right]$$

$$b_n = 0$$

The dc component is $V_{\text{dc}} = \frac{a_o}{2} = V_m\frac{m}{\pi}\sin\frac{\pi}{m} = \frac{m}{\pi}V_m\sin\frac{\pi}{m}$

The Fourier series of output voltage is equal to

$$V_o(t) = \frac{1}{2}a_0 + \sum_{n=6,12,18\ldots}^{\infty} a_n\cos n\omega t$$

After substituting the value of a_n, the output voltage can be expressed by

$$V_o(t) = \frac{m}{\pi}V_m\sin\frac{\pi}{m}\left[1 - \sum_{n=m,2m,3m\ldots}^{\infty}\frac{2}{n^2-1}\cos\frac{n\pi}{m}\cos n\omega t\right]$$

Example 7.4 A three phase mid-point six-pulse rectifier is connected to a load of 10 Ω at dc voltage 200 V. Determine the ratings of diodes and the three-phase transformer.

Solution

The average dc output voltage is $V_o = 0.955V_m$ where the maximum phase voltage is V_m.

$V_o = 0.955V_m = 200$, therefore $V_m = \frac{200}{0.955} = 209.424$ V

Maximum value of load current is $I_m = \frac{V_m}{R} = \frac{209.424}{10} = 20.9424$ A

Average value of diode current is

$$I_{D\_av} = \frac{1}{2\pi}\int_{-\pi/m}^{\pi/m} I_m \cos\omega t \cdot d(\omega t) = \frac{I_m}{\pi}\sin\frac{\pi}{m} = \frac{I_m}{\pi}\sin\frac{\pi}{6} \quad \text{as } m = 6$$

$$= \frac{20.9424}{\pi}\sin\frac{\pi}{6} = 3.3347 \text{ A}$$

The rms value of diode current is

$$I_{D\_rms} = \left[\frac{1}{2\pi}\int_{-\pi/m}^{\pi/m} (I_m \cos\omega t) d(\omega t)\right]^{\frac{1}{2}} = I_m\left[\frac{1}{2\pi}\left(\frac{\pi}{m} + \frac{1}{2}\sin\frac{2\pi}{m}\right)\right]^{\frac{1}{2}}$$

$$= I_m\left[\frac{1}{2\pi}\left(\frac{\pi}{6} + \frac{1}{2}\sin\frac{2\pi}{6}\right)\right]^{\frac{1}{2}} = 0.3902 I_m = 0.3902 \times 20.9424 = 8.172 \text{ A}$$

PIV of each diode is $2V_m = 2 \times 209.424 = 418.848$ V

The rms current in each phase of the transformer secondary is

$$I_{\text{rms(transformer)}} = \left[\frac{1}{2\pi}\int_{\pi/3}^{2\pi/3} I_m^2 \sin^2\omega t \cdot d\omega t\right]^{\frac{1}{2}} = I_m\sqrt{\frac{1}{2\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} = 0.39 I_m \quad \text{where, } I_m = \frac{V_m}{R}$$

The VA rating of transformer is

$$\text{VA} = 6V_{\text{rms(transformer)}} I_{\text{rms(transformer)}} = 6\frac{V_m}{\sqrt{2}} \times \frac{0.39V_m}{R} = 1.654\frac{V_m^2}{R}$$

$$= 1.654 \times \frac{209.424^2}{10} = 7254.18 \text{ VA}$$

7.5 THREE-PHASE DOUBLE STAR RECTIFIER WITH INTER-PHASE TRANSFORMER

Figure 7.10 shows a three-phase double Star rectifier with inter-phase transformer *R* load. This circuit consists of two three-phase Star rectifier with neutral points interconnected through an inter-phase reactor (IPT). The functions of IPT are:

1. The inter-phase reactor can be used to isolate two distinct three-phase rectifying system and maintain a potential difference at a minimum excitation current.
2. A single load can be connected to two distinct three-phase rectifying system.

The polarities of secondary windings in the inter-connected system are reversed as depicted in Fig. 7.10. When the output voltage of one three-phase rectifier unit is minimum, the output voltage of other rectifier unit is maximum. Therefore, the output voltage is average of V_{o1} and V_{o2}. The ripple frequency of output voltage is six times of fundamental frequency. Hence a filter size is reduced.

In a balanced circuit, the output currents of two three-phase Star rectifier units flowing in the opposite directions in the inter-phase transformer winding can not generate dc magnetization current. In the same way, the dc magnetization currents in the secondary windings of two three phase Star rectifier units can cancel each other. Due to symmetry of the secondary circuits, the sum of three primary current will be equal to zero at all times.

The advantages of three-phase double Star rectifier with inter-phase transformer and *R* load are:

1. Each of the six phase conducts for one third of the time period (120°) instead of one sixth of the time period (60°). Hence the utilization factor of transformer will be high.
2. This circuit has low ripple voltage, low peak inverse voltage and high rectifier efficiency.

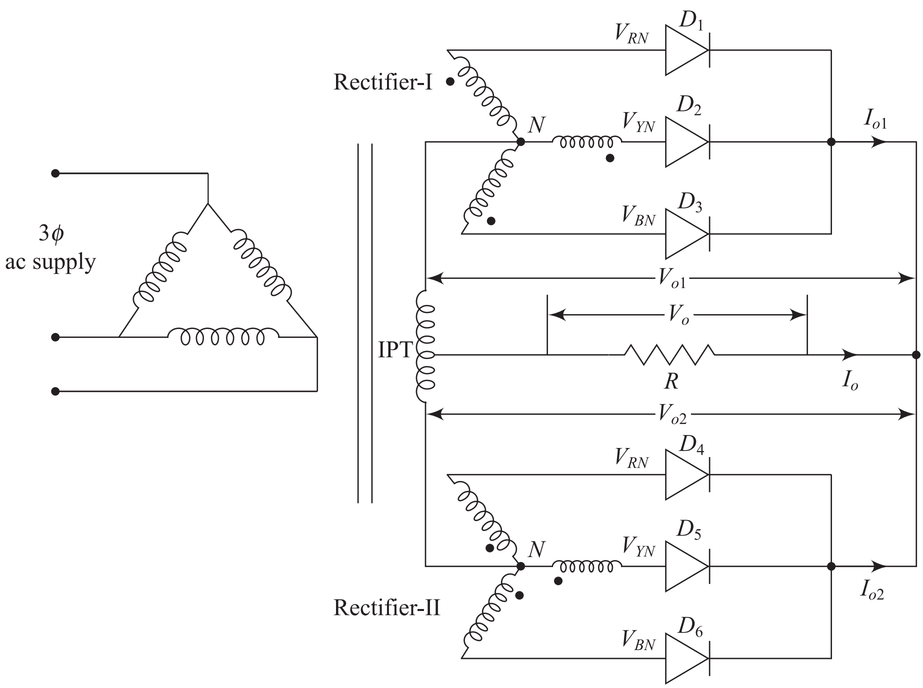


Fig. 7.10 *Three-phase double Star rectifier with inter-phase transformer*

The behaviour of circuit can be explained with the help of vector diagram as depicted in Fig. 7.11. There are two independent systems which can operate separately. There is 60° phase shift between rectifier-I and rectifier-II. When neutrals N and N' are connected directly, there is no potential difference between N and N' and the complete system works as a six-phase half-wave rectifier and each phase conducts for 60° duration. The output voltage wave from is depicted in Fig. 7.12.

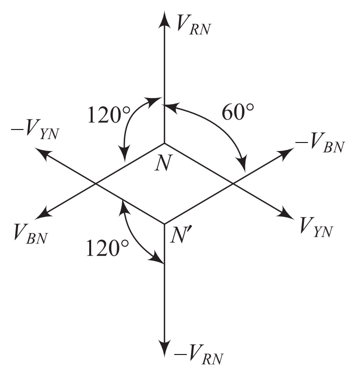


Fig. 7.11 *Vector diagram three-phase double Star rectifier with inter-phase reactor (IPT)*

7.6 THREE-PHASE BRIDGE RECTIFIER

Three-phase bridge rectifier are extensively used in high power applications since transformer utilization factor is high. Figure 7.13 shows a three-phase bridge rectifier circuit with R load. In this circuit, diodes are numbered in order of conduction sequence and each diode conducts for $2\pi/3$ duration. The conduction sequences of diodes are D_1 D_2, D_2 D_3, D_3 D_4, D_4 D_5, D_5, D_6, D_6 D_1. The vector diagram three-phase voltages and line voltages are shown in Fig. 7.14. The voltage and current waveforms of a three-phase bridge rectifier circuit are depicted in Fig. 7.15. In this circuit, the combination of Star or delta connected primary and secondary windings are symmetrical.

Assume V_m is the peak value of the phase voltage. The instantaneous phase voltages are

$$V_{RN} = V_m \sin \omega t, V_{YN} = V_m \sin(\omega t - 120°) \text{ and } V_{BN} = V_m \sin(\omega t - 240°).$$

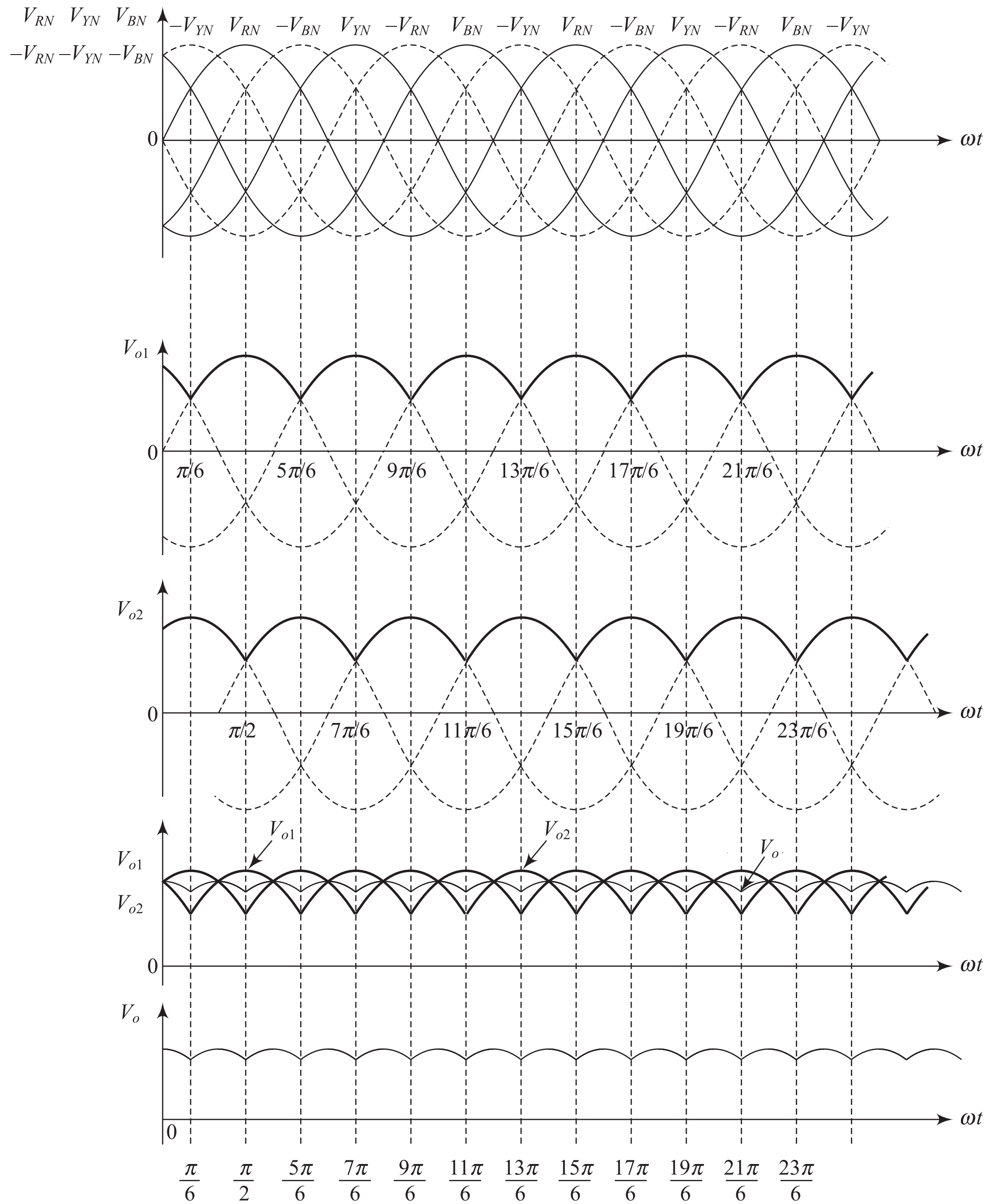


Fig. 7.12 *Output voltage waveform of three-phase double Star rectifier with inter-phase transformer and R load*

Then the line-to-line voltages lead the phase voltage by 30° and the instantaneous line voltages are as follows:

$$V_{RY} = \sqrt{3}V_m \sin(\omega t + 30°), V_{YB} = \sqrt{3}V_m \sin(\omega t - 90°) \text{ and } V_{BR} = \sqrt{3}V_m \sin(\omega t - 210°)$$

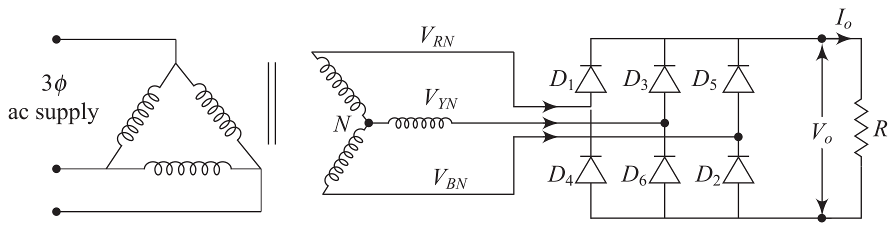


Fig. 7.13 *Three-phase bridge rectifier*

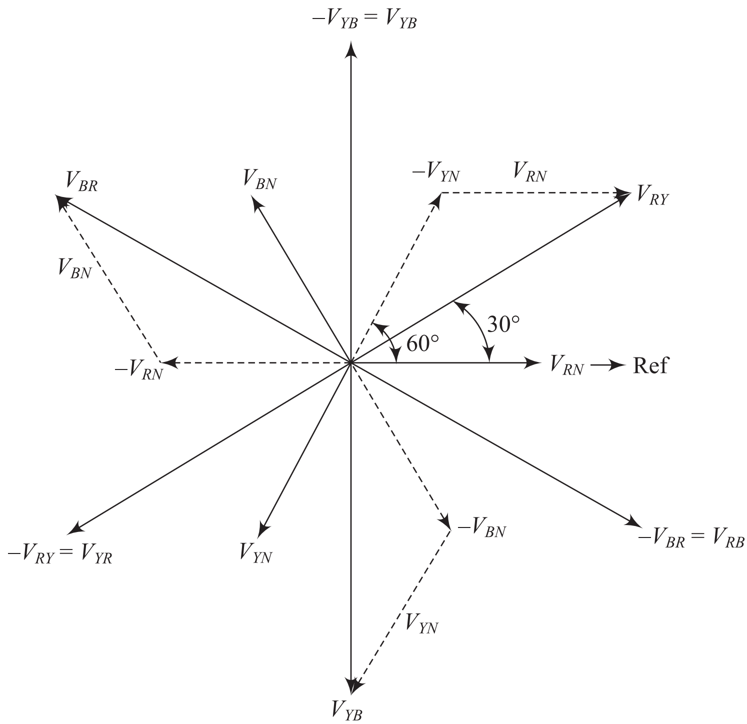


Fig. 7.14 *Vector diagram of phase and line voltages*

For this circuit, the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{1}{2\pi/6}\int_{\pi/6}^{\pi/2} \sqrt{3}V_m \sin(\omega t + 30°)\cdot d(\omega t + 30°)$$

Assuming $\theta = \omega t + 30°$, we can write $V_o = \dfrac{1}{2\pi/6}\displaystyle\int_{\pi/6}^{\pi/2} \sqrt{3}V_m \sin\theta \cdot d\theta$

$$= V_m \frac{3\sqrt{3}}{\pi} = \frac{3\sqrt{3}}{\pi}V_m = 1.654V_m$$

The average voltage of output voltage can also be determined by the following ways:

1. Consider any sinusoidal voltage waveform and integrate it from 60° to 120°.

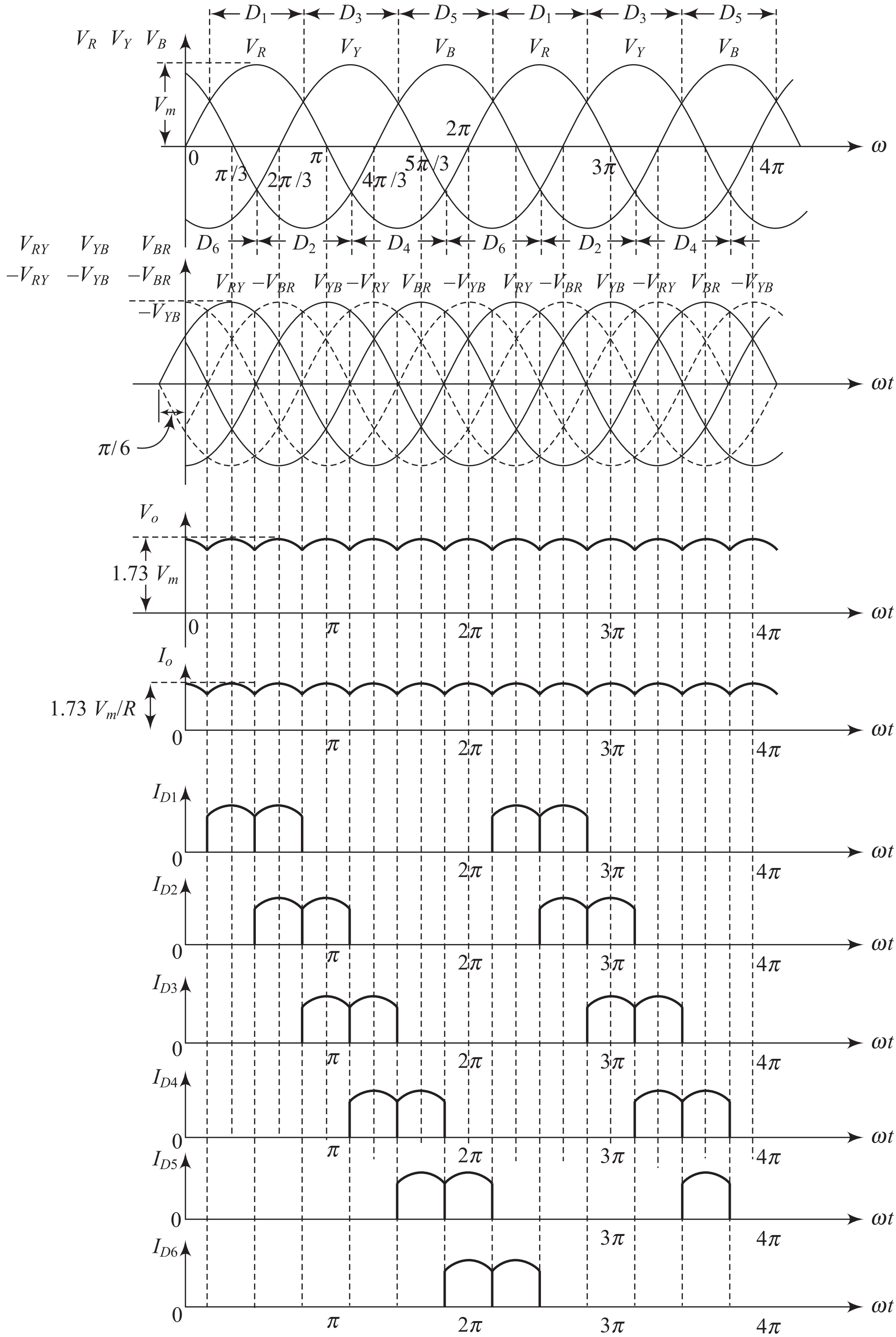


Fig. 7.15 *Voltage and current waveforms of three-phase bridge rectifier*

Then the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} \sqrt{3}V_m \sin\omega t \cdot d(\omega t)$$

$$= \frac{\sqrt{3}V_m}{2\pi/6}[-\cos\omega t]_{\pi/3}^{2\pi/3}$$

$$= \frac{\sqrt{3}V_m}{2\pi/6}\left[-\cos\frac{\pi}{3} + \cos\frac{2\pi}{3}\right]_{\pi/3}^{2\pi/3} = V_m\frac{3\sqrt{3}}{\pi} = \frac{3\sqrt{3}}{\pi}V_m = 1.654V_m$$

2. Consider any cosine voltage waveform and integrate it from –30° left of its peak value to 30° right of its peak value.

   Then the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{1}{2\pi/6}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sqrt{3}V_m \cos\omega t \cdot d(\omega t)$$

$$= V_m\frac{3\sqrt{3}}{\pi} = \frac{3\sqrt{3}}{\pi}V_m = 1.654V_m$$

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} 3V_m^2 \sin^2\omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{\pi/9}\int_{\pi/3}^{2\pi/3}\frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{\pi/9}\left(\frac{1}{2}\omega t - \frac{1}{2}\sin 2\omega t\right)_{\pi/6}^{5\pi/6}\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{\pi/9}\left(\frac{2\pi}{6} - \frac{\pi}{6} - \frac{1}{2}\sin\frac{2\pi}{6} + \frac{1}{2}\sin\frac{\pi}{6}\right)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}\right)\Bigg]^{\frac{1}{2}} = 1.655V_m$$

The average output current is

$$I_{\text{av}} = I_o = \frac{V_o}{R} = \frac{1.654V_m}{R}$$

The dc output power is

$$P_{\text{dc}} = V_{\text{dc}}I_{\text{dc}} = V_oI_o = \frac{(1.654V_m)^2}{R} = \frac{2.735V_m^2}{R}$$

The rms value of load current is $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{1.655V_m}{R}$

The ac power output of the transformer secondary is

$$P_{\text{ac}} = I_{\text{rms}}^2 R = \frac{(1.655V_m)^2}{R^2} = \frac{2.739V_m^2}{R}$$

Rectification efficiency is

$$\eta = \frac{P_{\text{dc}}}{P_{\text{ac}}}\times 100\% = \frac{2.735V_m^2/R}{2.739V_m^2/R}\times 100\% = 99.85\%$$

Form factor is $FF = \frac{V_{\text{rms}}}{V_{\text{av}}} = \frac{1.655V_m}{1.654V_m} = 1.0006$

Ripple factor is $RF = \sqrt{FF^2 - 1} = 0.034$

In this circuit, each secondary of the transformer conducts for 240° duration. The current for forward path duration is 120° and return path duration is 120°. Then the rms current of each transformer secondary winding is

$$I_{\text{rms(transformer)}} = \left[\frac{1}{2\pi/4}\int_{\pi/3}^{2\pi/3} I_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi/4}\int_{\pi/3}^{2\pi/3}\left(\frac{\sqrt{3}V_m}{R}\right)^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} \quad \text{as } I_m = \frac{\sqrt{3}V_m}{R}$$

$$= \left[\frac{6V_m^2}{\pi R^2}\int_{\pi/3}^{2\pi/3}\frac{1}{2}(1 - \cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}} = \frac{\sqrt{3}V_m}{R}\sqrt{\frac{2}{\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)}$$

$$= I_m\sqrt{\frac{2}{\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} = 0.78\, I_m = \frac{1.351V_m}{R}$$

The VA rating of transformer is

$$\text{VA} = 3V_{\text{rms(trasformer)}}I_{\text{rms(transformer)}} = 3\frac{V_m}{\sqrt{2}} \times \frac{1.351V_m}{R} = \frac{2.866V_m^2}{R} \quad \text{as } V_{\text{rms(trasformer)}} = \frac{V_m}{\sqrt{2}}$$

Transformer utilisation factor is $\text{TUF} = \frac{P_{\text{dc}}}{\text{VA}} = \frac{2.735V_m^2/R}{2.866V_m^2/R} = 0.954$

Since each diode conducts for 120° duration, the rms current flows though each diode is

$$I_D = I_m\sqrt{\frac{1}{\pi}\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)} = 0.552I_m \quad \text{where, } I_m = \frac{1.732V_m}{R}$$

The output voltage of a three phase bridge rectifier can be represented by Fourier series. The instantaneous output voltage can be expressed as

$$V_o(t) = \frac{m}{\pi}V_m \sin\frac{\pi}{m}\left[1 - \sum_{n=m,2m,3m\ldots}^{\infty}\frac{2}{n^2 - 1}\cos\frac{n\pi}{q}\cos n\omega t\right] \quad \text{where, } m = 6$$

or $$V_o(t) = 0.9549V_m\left[1 + \frac{2}{35}\cos 6\omega t - \frac{2}{143}\cos 12\omega t + \cdots\right]$$

The dc output voltage is slightly lower than the peak line voltage, but it is about 2.34 times the rms phase voltage. The peak repetitive reverse voltage (V_{RRM}) rating of each diode is 1.05 times the dc output voltage. The peak repetitive forward current (I_{FRM}) rating of each diode is 0.579 times the dc output current. Consequently, three phase bridge rectifier is very efficient and extensively used where dc voltage and current requirements are high. Usually, any additional filter circuit is not required as the output ripple voltage is only about 4%. If filter is required in some applications, the size of the filter is relatively small as the ripple frequency is increased to six times the input frequency.

Example 7.5 A three-phase bridge rectifier is used to charge a 200 V battery when the input voltage of rectifier is three phase, 220 V, 50 Hz. When the current limiting resistance is 10 Ω and the load is free from ripples. Calculate (a) power input to battery, (b) power loss across limiting resistance, (c) displacement factor, (d) current distortion factor, (e) input power factor, (f) harmonic factor and (g) VA rating of transformer. Assume transformer utilisation factor is 0.9541.

Solution

Line voltage $V_L = 200$ V and phase voltage is $V_{Ph} = \dfrac{220}{\sqrt{3}} = 127.02$

The maximum value of phase voltage is $V_m = \sqrt{2}V_{Ph} = \sqrt{2} \times 127.02 = 179.63$ V

The average dc output voltage is

$$V_o = \frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} \sqrt{3}V_m \sin\omega t \cdot d(\omega t) = \frac{\sqrt{3}V_m}{2\pi/6}[-\cos\omega t]_{\pi/3}^{2\pi/3}$$

$$= \frac{3\sqrt{3}}{\pi}V_m = 1.654V_m = 1.654 \times 179.63 \text{ V} = 297.11 \text{ V}$$

Average value of battery charging current $I_o = \dfrac{V_o - 200}{R} = \dfrac{297.11 - 200}{10} = 9.711$ A

(a) Power input to battery $EI_o = 200 \times 9.711 = 1942.2$ Watt

(b) Power loss across limiting resistance $I_o^2 R = 9.711^2 \times 10 = 943.03$ Watt

Assume that the load current is ripple free. Then transformer secondary current i_s is constant. Since the positive and negative half cycle of transfer secondary current i_s is identical, average value of i_s current is zero. The fundamental component of i_s can be determined by Fourier series as given below.

$$a_n = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_o \cos n\omega t \cdot d(\omega t) \text{ and } b_n = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_o \sin n\omega t \cdot d(\omega t)$$

Then
$$a_1 = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_o \cos\omega t \cdot d(\omega t) = \frac{2I_o}{\pi}\left(\sin\frac{5\pi}{6} - \sin\frac{\pi}{6}\right) = 0$$

$$b_1 = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_o \sin\omega t \cdot d(\omega t) = \frac{2I_o}{\pi}\left(-\cos\frac{5\pi}{6} + \cos\frac{\pi}{6}\right) = \frac{2\sqrt{3}I_o}{\pi}$$

The fundamental component of transformer secondary current is

$$i_{s1} = \sqrt{a_1^2 + b_1^2}\,\angle\tan^{-1}\frac{a_1}{b_1} = \frac{2\sqrt{3}I_o}{\pi}\angle 0° = \frac{2\sqrt{3}I_o}{\pi}\sin\omega t \text{ as } \phi_1 = 0$$

(c) Displacement factor is $DF = \cos\phi_1 = \cos 0 = 1$

(d) The rms value of I_{s1} is $= \dfrac{1}{\sqrt{2}} \times \dfrac{2\sqrt{3}I_o}{\pi}$

The rms value of source current is $I_s = \left[\dfrac{I_o^2 \times 2\pi}{\pi \times 3}\right]^{\frac{1}{2}} = \sqrt{\dfrac{2}{3}}I_o$

Current distortion factor is $\text{CDF} = \dfrac{I_{s1}}{I_s} = \dfrac{2\sqrt{3}I_o}{\sqrt{2}\pi} \times \dfrac{\sqrt{3}}{\sqrt{2}I_o} = \dfrac{3}{\pi} = 0.9954$

(e) Input power factor is $pf = \text{CDF} \times DF = 0.9554 \times 1 = 0.9554$

(f) Harmonic factor is $\text{HF} = \sqrt{\left(\dfrac{I_s}{I_{s1}}\right)^2 - 1} = \sqrt{\left(\dfrac{1}{0.9954}\right)^2 - 1} = 0.309$

(g) VA rating of transformer is

$$\text{VA} = \frac{P_{dc}}{\text{TUF}} \quad \text{as} \quad \text{TUF} = \frac{P_{dc}}{\text{VA}}$$

Power delivered to load is

$P_{dc} = EI_o + I_o^2 R = 1942.2 + 943.03 = 2885.23$ Watt and assume that TUF is equal to 0.9541.

Then $\text{VA} = \frac{P_d}{\text{TUF}} = \frac{2885.23}{0.9541} = 3024 \text{ VA}$

Example 7.6 A three-phase full-wave rectifier delivers power to a highly inductive load with ripple free current of 90 A. If the supply voltage is three phase, 440 V, 50 Hz, determine the ratings of diodes.

Solution

Average current flow through each diode is $I_{D-\text{av}} = \frac{I_o}{3} = \frac{90}{3} = 30 \text{ A}$
rms current rating of diode is

$$I_{D-\text{rms}} = \left(\frac{2}{2\pi} \int_{\pi/3}^{2\pi/3} I_m^2 \sin^2 \omega t \cdot d\omega t \right)^{1/2} = 0.582 I_m = 0.582 \times 90 = 52.38 \text{ A}$$

Peak inverse voltage of each diode is $\text{PIV} = V_m = \sqrt{2} \times 440 = 622.25 \text{ V}$

Example 7.7 A three-phase bridge rectifier supplies a highly inductive load and the average load current is 120 A and the ripple content is negligible. Calculate the rating each diode if the line to neutral voltage of a *Y* connected supply is 130 V at 50 Hz.

Solution

The average dc current flows through each diode is $I_{D\_\text{av}} = \frac{I_o}{3} = \frac{120}{3} = 40 \text{ A}$
The rms current flows through each diode is

$$I_{D\_\text{rms}} = \left(\frac{1}{2\pi} \int_{\pi/3}^{\pi} I_o^2 \, d \cdot \omega t \right)^{1/2} = \frac{I_o}{\sqrt{3}} = \frac{120}{\sqrt{3}} = 69.284 \text{ A}$$

The PIV of each diode is $\text{PIV} = \sqrt{3} V_m = \sqrt{3} \times \sqrt{2} \times 130 = 318.42 \text{ V}$

7.7 SIX-PHASE SERIES BRIDGE RECTIFIERS

Figure 7.16 shows a six-phase series bridge rectifier. In this circuit, the Star and Delta connected secondary windings have $\pi/6$ (30°) displacement between their output voltages. If a Star-connected Star and Delta connected bridge rectifier are connected in series, the output voltage ripple frequency is twelve times of the fundamental frequency ($f_o = 12f$). The ripple of combined output voltage is about 1% whereas the ripple of each individual bridge rectifier is about 4%. The voltage waveforms of (a) voltage across Star connected secondary (phase voltage) (b) line voltages of Star connected secondary (c) V_{o1} six pulse output voltage of bridge – I (d) line voltages of Delta connected secondary (e) V_{o2} six pulse output voltage of bridge – II and (f) twelve pulse output voltage of six-phase series bridge rectifier are depicted in Fig. 7.17.

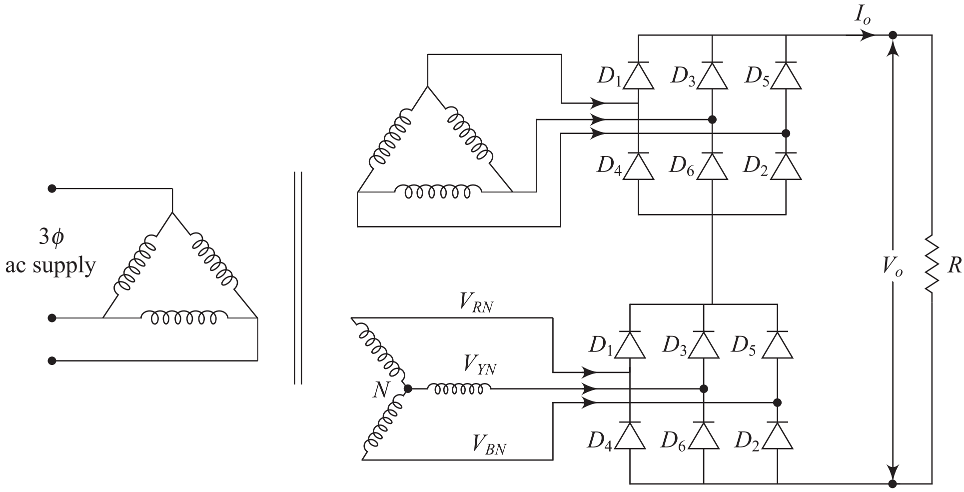


Fig. 7.16 *Six-phase series bridge rectifier or three-phase twelve-pulse full bridge rectifier*

Assume that V_m^* is the peak voltage of the Delta connected secondary. Then the peak voltage between the lines of Star connected secondary is V_m^*. The peak voltage across the load is V_p which is equal to

$V_p = 2V_m^* \times \cos(\pi/12) = 1.932V_m^*$ as there is $\pi/6$ phase shift between the delta and Star secondary.

Assume that $V_m^* = 1.73V_m$ and V_m is the maximum phase voltage of Star connected secondary.
In this circuit, the average dc output voltage is equal to

$$V_{\text{av}} = V_o = \frac{1}{\pi/12}\int_{5\pi/12}^{7\pi/12} V_p \sin \omega t \cdot d(\omega t) = \frac{V_p}{\pi/12}[-\cos \omega t]_{5\pi/12}^{7\pi/12}$$

$$= \frac{V_p}{\pi/12}\left[-\cos\frac{7\pi}{12} + \cos\frac{5\pi}{12}\right] = V_p\,\frac{12}{\pi}\,\frac{\sqrt{3}-1}{2\sqrt{2}} = 0.98862V_p$$

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/12}\int_{5\pi/12}^{7\pi/12} V_p^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_p^2}{2\pi/12}\int_{5\pi/12}^{7\pi/12}\frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_p^2}{2\pi/12}\left(\frac{1}{2}\omega t - \frac{1}{2}\sin 2\omega t\right)_{5\pi/12}^{7\pi/12}\right]^{\frac{1}{2}} = \left[\frac{V_p^2}{2\pi/12}\left(\frac{7\pi}{24} - \frac{5\pi}{12} - \frac{1}{2}\sin\frac{14\pi}{12} + \frac{1}{2}\sin\frac{10\pi}{12}\right)\right]^{\frac{1}{2}}$$

$$= V_p\left[\frac{12}{2\pi}\left(\frac{\pi}{12} + \frac{1}{4}\right)\right]^{\frac{1}{2}} = 0.98867V_p$$

The average output current is

$$I_{\text{av}} = I_o = \frac{V_o}{R} = \frac{0.98862V_p}{R}$$

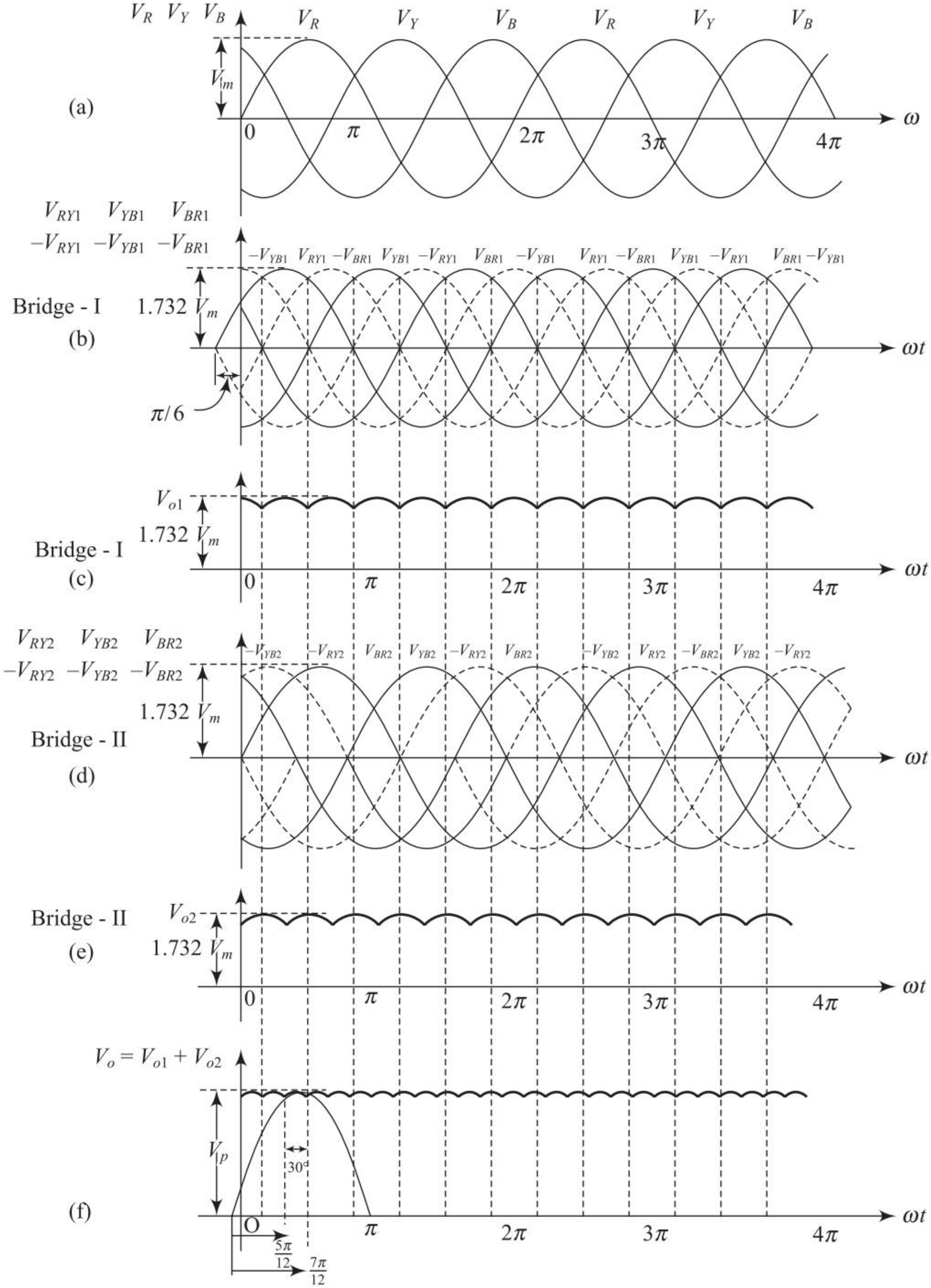


Fig. 7.17 *Waveforms of (a) voltage across Star connected secondary (phase voltage), (b) line voltages of Star connected secondary, (c) V_{o1} six pulse output voltage of bridge-I, (d) line voltages of delta connected secondary, (e) V_{o2} six pulse output voltage of bridge-II, and (f) twelve pulse output voltage of six-phase series bridge rectifier*

The dc output power is

$$P_{\text{dc}} = V_{\text{dc}} I_{\text{dc}} = V_o I_o = \frac{(0.98862V_p)^2}{R} = \frac{0.97736V_p^2}{R}$$

The rms value of lad current is $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{0.98867V_p}{R}$

The ac power output is the transformer secondary is

$$P_{\text{ac}} = I_{\text{rms}}^2 R = \frac{(0.98867V_p)^2}{R^2} = \frac{0.97746V_p^2}{R}$$

Rectification efficiency is

$$\eta = \frac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\% = \frac{0.97736V_p^2/R}{0.97746V_p^2/R} \times 100\% = 99.98\%$$

Form factor is $FF = \dfrac{V_{\text{rms}}}{V_{\text{av}}} = \dfrac{0.98867V_p}{0.98862V_p} = 1.00005$

Ripple factor is $RF = \sqrt{FF^2 - 1} = 0.01$

The rms current in each transformer secondary winding is

$$I_{\text{rms(transformer)}} = I_m \sqrt{\frac{4}{\pi}\left(\frac{\pi}{12} + \frac{1}{4}\right)} = 0.807 I_m \quad \text{where, } I_m = V_p / R$$

The rms current through each diode is

$$I_{\text{rms(transformer)}} = I_m \sqrt{\frac{2}{\pi}\left(\frac{\pi}{12} + \frac{1}{4}\right)} = 0.57 I_m$$

Example 7.8 A six-phase series bridge rectifier is depicted in Fig. 7.16 where the dc output voltage is 400 V and $R = 10\ \Omega$. Determine (a) V_p (b) V_{rms} (c) I_{av} (d) P_{dc} (e) P_{ac} (f) η.

Solution

(a) $V_{\text{av}} = V_0 = 400\text{ V} = 0.98862V_p$

$$V_p = \frac{400}{0.98862}\text{ V} = 404.604\text{ V}$$

(b) $V_{\text{rms}} = 0.98867V_p = 0.98867 \times 404.604 = 400.02$

(c) $I_{\text{av}} = \dfrac{V_{\text{av}}}{R} = \dfrac{400}{10} = 40\text{ A}$

(d) $P_{\text{dc}} = V_{\text{dc}} I_{\text{dc}} = 400 \times 40 = 16000\text{ Watt}$

(e) $P_{\text{ac}} = \dfrac{(0.98867V_p)^2}{R} = \dfrac{(0.98867 \times 404.604)^2}{10} = 16001.58\text{ Watt}$

(f) $\eta = \dfrac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\% = \dfrac{16000}{16001.58} \times 100 = 99.99\%$

7.8 SIX-PHASE PARALLEL BRIDGE RECTIFIER

Usually six-phase series bridge rectifiers are used for high output voltage applications. But for high output current applications, six-phase parallel bridge rectifiers are used. Figure 7.18 shows a six-phase parallel bridge rectifier with an inter-phase transformer. The output voltage of bridge-I is V_{o1} and the output voltage of bridge-II is V_{o2}. The output voltage across load is the average of the rectified output voltages V_{o1} and V_{o2}. The output ripple frequency of six-phase parallel bridge rectifiers is 12 times of fundamental frequency as depicted in Fig. 7.19. Normally filter circuit is not required. When the circuit is a balanced one, the output current of two three phase units will not generate dc magnetisation current.

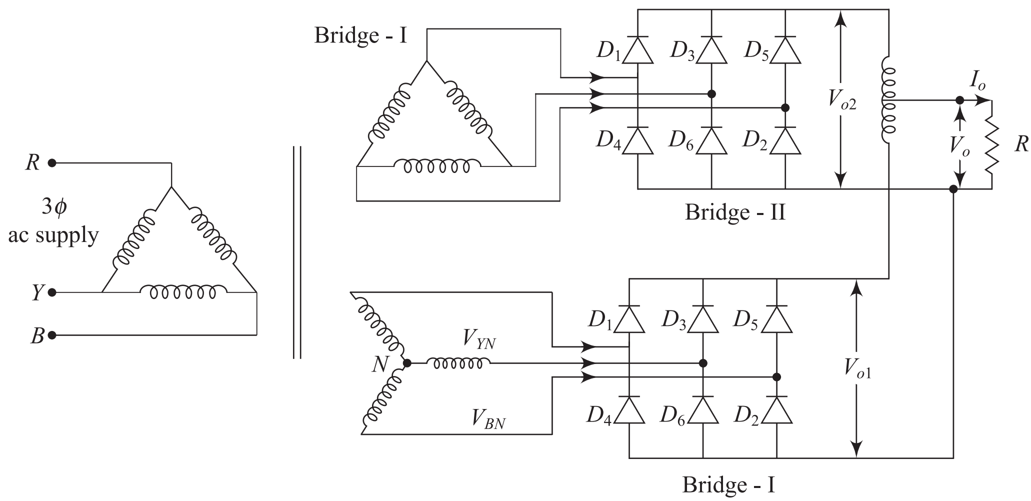


Fig. 7.18 *Six-phase parallel bridge rectifier or three-phase twelve-pulse full-bridge rectifier*

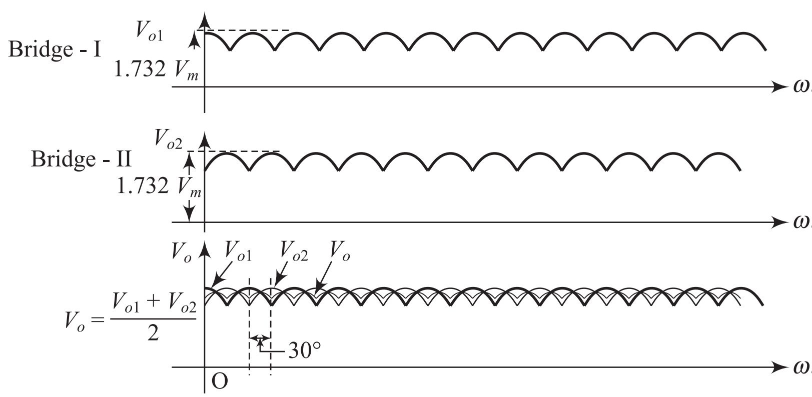


Fig. 7.19 *Voltage waveforms of the six-phase bridge rectifier with inter-phase transformer*

7.9 COMPARATIVE PERFORMANCE OF DIFFERENT THREE PHASE RECTIFIERS

The comparative performance of different three phase rectifiers is given in Table 7.1.

Parameters	Three-phase half wave or three-pulse rectifier	Six-phase half-wave or six-pulse rectifier	Three-phase bridge rectifier (six-pulse rectifier)	12 pulse rectifier
Average dc output voltage V_{av}	$\frac{3\sqrt{3}}{2\pi}V_m$	$0.955V_m$	$1.654V_m$	$0.98862V_p$ $V_p = 3.342V_m$
Rms output voltage V_{rms}	$0.8406V_m$	$0.956V_m$	$1.655V_m$	$0.98867V_p$
Form factor (FF)	1.01	1.001	1.0006	1.00005
Voltage ripple factor (RF)	0.1821 = 18.21%	0.045 = 4.5%	0.034 = 3.4%	0.01= 1%
Efficiency (η)	96.66%	99.79%	99.85%	99.98%
TUF	0.663	0.551	0.954	
PIV of Diode	$\sqrt{3}V_m$	$2V_m$	$\sqrt{3}V_m$	

Summary

- Three phase uncontrolled rectifiers are used for higher power applications with high voltage and high current rating.
- The operation of three phase half wave rectifier, six-phase half wave rectifier, multiphase rectifier, three-phase double Star rectifier, three phase bridge rectifier, six phase series bridge rectifiers and six phase parallel bridge rectifiers are discussed elaborately.
- The comparative performance of different three phase rectifiers is incorporated in this chapter.

Multiple-Choice Questions

7.1 In a three-phase half-wave uncontrolled rectifier each diode conducts for ______ duration.
(a) 180° (b) 150° (c) 120° (d) 60°

7.2 The dc output voltage of a three-phase half-wave uncontrolled rectifier is

(a) $\frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_m \sin\omega t \cdot d(\omega t)$ (b) $\frac{1}{2\pi/6}\int_{\pi/6}^{5\pi/6} V_m \sin\omega t \cdot d(\omega t)$

(c) $\frac{1}{2\pi/3}\int_{\pi/3}^{5\pi/3} V_m \sin\omega t \cdot d(\omega t)$ (d) $\frac{1}{2\pi/6}\int_{\pi/3}^{5\pi/3} V_m \sin\omega t \cdot d(\omega t)$

7.3 The frequency of output voltage of a three phase half wave uncontrolled rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

7.4 The peak inverse voltage of each diode of a three-phase half-wave uncontrolled rectifier is
(a) $\sqrt{3}E_m$ (b) $\sqrt{2}E_m$ (c) E_m (d) E

7.5 In a six-phase half-wave uncontrolled rectifier each diode conducts for ______ duration.
(a) 180° (b) 150° (c) 120° (d) 60°

7.6 The dc output voltage of a six-phase half-wave uncontrolled rectifier is

(a) $\frac{1}{2\pi/3}\int_{\pi/6}^{5\pi/6} V_m \sin\omega t \cdot d(\omega t)$ (b) $\frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} V_m \sin\omega t \cdot d(\omega t)$

(c) $\frac{1}{2\pi/6}\int_{\pi/6}^{5\pi/6} V_m \sin \omega t \cdot d(\omega t)$ (d) $\frac{1}{2\pi/3}\int_{\pi/3}^{2\pi/3} V_m \sin \omega t \cdot d(\omega t)$

7.7 The frequency of output voltage of a six-phase half-wave uncontrolled rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

7.8 The peak inverse voltage of each diode of a six-phase half-wave uncontrolled rectifier is
(a) $\sqrt{3}E_m$ (b) $\sqrt{2}E_m$ (c) $2E_m$ (d) E

7.9 In a three-phase bridge rectifier each diode conducts for ______ duration.
(a) 180° (b) 150° (c) 120° (d) 60°

7.10 The dc output voltage of a three-phase bridge rectifier is

(a) $\frac{1}{2\pi/6}\int_{\pi/3}^{2\pi/3} \sqrt{3}V_m \sin \omega t \cdot d(\omega t)$ (b) $\frac{1}{2\pi/6}\int_{\pi/6}^{5\pi/6} V_m \sin \omega t \cdot d(\omega t)$

(c) $\frac{1}{2\pi/3}\int_{\pi/3}^{5\pi/3} V_m \sin \omega t \cdot d(\omega t)$ (d) $\frac{1}{2\pi/6}\int_{\pi/3}^{5\pi/3} V_m \sin \omega t \cdot d(\omega t)$

7.11 The frequency of output voltage of a three-phase half-wave uncontrolled rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

7.12 In a three-phase double Star rectifier, each diode conducts for ______ duration.
(a) 180° (b) 150° (c) 120° (d) 60°

7.13 The frequency of output voltage of a three-phase double Star rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

7.14 An inter-phase transformer is used in three-phase double Start rectifier
(a) to provide a common load to both rectifiers (b) to isolate both systems
(c) for both (a) and (b) (d) None of these

7.15 The dc output voltage of three-phase half-wave rectifier and three-phase six-pulse rectifier are ______ and ______ respectively.

(a) $\frac{3\sqrt{3}V_m}{2\pi}, \frac{6V_m}{2\pi}$ (b) $\frac{3\sqrt{3}V_m}{2\pi}, \frac{3\sqrt{3}V_m}{2\pi}$ (c) $\frac{\sqrt{3}V_m}{2\pi}, \frac{3V_m}{2\pi}$ (d) $\frac{6V_m}{2\pi}, \frac{3\sqrt{3}V_m}{2\pi}$

7.16 The frequency of output voltage of a three-phase 12 pulse rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

7.17 The number of pulse of a three phase 12 pulse rectifier is
(a) 3 (b) 6 (c) 9 (d) 12

7.18 Which of the following statements are correct?
(a) RF of three-pulse rectifier is 4.3% (b) η of three-pulse rectifier is 99.82%
(c) TUF of three-pulse rectifier is 0.6644 (d) FF is 1.001

7.19 The voltage ripple factor of a twelve-pulse rectifier is
(a) 1.023% (b) 4.27% (c) 4.3% (d) 18.26%

7.20 Form factor of a three-phase half-wave rectifier is
(a) 1.0165 (b) 1.001 (c) 1.0005 (d) 1.00005

7.21 Ripple factor of a twelve-pulse rectifier is
(a) 1.0165 (b) 1.001 (c) 1.0005 (d) 1.00005

Fill in the Blanks

7.1 In a three-phase three-pulse diode rectifier, each diode conducts for ______ duration.

7.2 In a three-phase half-wave rectifier, peak inverse voltage across each diode is PIV = ______.

7.3 TUF of a three-phase three-pulse rectifier is ______.

7.4 For a three-phase half-wave rectifier, current in the transformer secondary is ______.

7.5 In three-phase mid-point six-pulse diode rectifier, each diode conducts for ______ duration.

7.6 In a six-phase diode rectifier, the peak inverse voltage across each diode is ______.

7.7 Ripple factor of a three-phase three-pulse rectifier is ______ and ripple factor of 6 phase 6 pulse rectifier is ______.

7.8 TUF of six-phase six-pulse rectifier is ______ as compared to TUF of a three-phase three-pulse rectifier.

7.9 The average output voltage of a m-phase rectifier is ______ and each diode conducts from ______.

7.10 In three-phase bridge rectifier, the positive group of diodes are ______ and the negative group diodes are ______.

7.11 In three-phase bridge rectifier, each diode conducts for ______ duration.

7.12 Form factor of a three-phase bridge rectifier is ______.

7.13 Peak inverse voltage across each diode in three-phase bridge rectifier is _______.

7.14 A three-phase 12-pulse rectifier consists of ______ diodes.

7.15 The quality of dc output voltage ______ significantly with three-phase twelve pulse rectifier.

Review Questions

7.1 Draw the circuit diagram of a three-phase half-wave uncontrolled rectifier and explain its operating principle with voltage and current waveforms. Determine the following parameters:
(a) dc output voltage (b) Average dc load current (c) rms output voltage
(d) rms load current (e) Ripple factor (f) TUF (g) Efficiency

7.2 (a) Why are three-phase rectifiers preferred over single-phase rectifiers?
(b) Give a list of applications of three phase rectifiers.

7.3 (a) Draw the circuit diagram of a three-phase mid point six-pulse diode rectifier with R load. Discuss its working principle.
(b) Draw the voltage and current waveforms. Determine the following parameters:
(i) dc output voltage (ii) Average dc load current (iii) rms output voltage (iv) rms load current
(v) Form factor (v) Ripple factor (vi) Efficiency (vii) TUF

7.4 (a) Draw the circuit diagram of a three-phase bridge with R load. Discuss its working principle.
(b) Draw the voltage and current waveforms. Determine the following parameters:
(i) dc output voltage (ii) Average dc load current (iii) rms output voltage (iv) rms load current
(v) Form factor (vi) Ripple factor (vii) Efficiency (viii) TUF

7.5 (a) What is multiphase rectifier? What are the advantages of multiphase rectifiers?
(b) Derive the average output voltage and rms output voltage of a multiphase rectifier.

7.6 (a) What are the advantages of three-phase bridge rectifier over three-phase mid point six-pulse rectifier?
(b) For a three-phase q pulse diode rectifier, prove the following expressions:

Average dc output voltage is $V_{av} = V_m \frac{q}{\pi}\sin\frac{\pi}{q}$ and

rms output voltage $V_{rms} = V_m\left(\frac{q}{2\pi}\left(\frac{\pi}{q}+\frac{1}{2}\sin\frac{2\pi}{q}\right)\right)$ where, V_m is maximum phase voltage.

7.7 (a) Draw the circuit diagram of a three-phase double Star rectifier with inter-phase transformer. Discuss its working principle.
(b) Draw the voltage and current waveforms.

7.8 (a) Describe a three-phase 12-pulse diode rectifier with circuit diagram and waveforms.
(b) Derive expressions for average and rms values of output voltage. Determine form factor and voltage ripple factor.

7.9 A three-phase half-wave rectifier is fed from a 3ϕ, 440 V, 50 Hz ac supply and it is connected with a R load of 5 Ω. Calculate (a) dc output voltage, (b) rms value of output voltage, (c) Average output current, (d) rms load current, (e) dc output power, (f) ac power supplied and (g) Rectification efficiency.

7.10 A three-phase half-wave rectifier is fed from a 3ϕ ac supply and it is connected with a R load. Determine (a) form factor, (b) ripple factor and (c) TUF

7.11 A three-phase step-down Delta-Star transformer with per phase turn ratio 100 is fed from a 3ϕ, 11 kV, 50 Hz ac supply and it is connected with three-phase half-wave rectifier. When R is equal to 10 Ω, determine (a) dc output voltage, (b) rms value of output voltage, (c) average output current, (d) rms load current, (e) power delivered to load, (f) maximum value of load current, (g) average and (h) rms value of diode current.

7.12 A three-phase Star rectifier is connected with a purely R load. Calculate (a) dc output voltage, (b) rms value of output voltage, (c) form factor, (FF) (d) ripple factor (RF), (e) efficiency, (f) transformer utilisation factor, (g) PIV of each diode and (h) peak current through diode. Assume the current flow through R is 30 A and output dc voltage 120 V.

7.13 The output voltage waveform of a three-phase half-wave rectifier is shown in Fig. 7.5. Determine the rms value of the dominant harmonics and their frequencies if $V_m = 220$ V and fundamental frequency $f = 50$ Hz.

7.14 A six-phase half-wave rectifier is fed from a 3ϕ, 400 V, 50 Hz ac supply and it is connected with a R load of 10 Ω. Calculate (a) dc output voltage, (b) rms value of output voltage, (c) average output current, (d) rms load current, (e) dc output power, (f) ac power supplied and (g) rectification efficiency.

7.15 A six-phase half-wave rectifier is fed from a 3ϕ ac supply and it is connected with a R load as depicted in Fig. 7.6. Determine (a) form factor, (b) ripple factor and (c) TUF.

7.16 A three-phase step-down delta-Star transformer with per phase turn ratio 10 is fed from a 3ϕ, 1000 V, 50 Hz ac supply and it is connected with six-phase half-wave rectifier. When R is equal to 5 Ω, determine (a) maximum load current, (b) dc output voltage, (c) rms value of output voltage, (d) average output current, (e) rms load current, (f) power delivered to load, (g) maximum value of load current, (h) average and rms value of diode current.

7.17 The output voltage waveform of a six-phase rectifier is shown in Fig. 7.8. Determine the rms value of the dominant harmonics and their frequencies if $V_m = 230$ V and fundamental frequency $f = 50$ Hz.

7.18 A three-phase mid-point six-pulse rectifier is connected to a load of 5 Ω at dc voltage 220 V. Determine the ratings of diodes and the three-phase transformer.

7.19 A three-phase bridge rectifier is shown in Fig. 7.13 and it provides 230 V dc output voltage across load resistance $R = 10$ Ω. Find the rating each diode and delta-Star transformer. Assume transformer utilisation factor is 0.9541.

7.20 A three-phase bridge rectifier is used to charge a 220 V battery when the input voltage of rectifier is three phase, 240 V, 50 Hz. When the current limiting resistance is 10 Ω and the load is free from ripples. Calculate (a) power input to battery, (b) power loss across limiting resistance (c) displacement factor, (d) current distortion factor, (e) input power factor, (f) harmonic factor and (g) VA rating of transformer. Assume transformer utilisation factor is 0.9541.

7.21 A three-phase bridge rectifier is connected to a purely resistive load $R = 15$ Ω. Compute (a) efficiency, (b) form factor, (c) ripple factor, (d) transformer utilisation factor, (f) peak inverse voltage of each diode, (g) average current through diode and (h) peak current through diode. Assume dc output voltage is 220 V.

7.22 A three-phase full-wave rectifier delivers power to a highly inductive load with ripple free current of 120 A. If the supply voltage is three phase, 400 V, 50 Hz, determine the ratings of diodes.

7.23 A three-phase bridge rectifier supplies a highly inductive load and the average load current is 90 A and the ripple content is negligible. Calculate the rating each diode if the line to neutral voltage of a Y connected supply is 120 V at 50 Hz.

7.24 A six-phase series bridge rectifier is depicted in Fig. 7.16 where the dc output voltage is 440 V and $R = 10$ Ω. Determine (a) V_p, (b) V_{rms}, (c) I_{av}, (d) P_{dc}, (e) P_{ac} and (f) η.

Answers to Multiple-Choice Questions

7.1	(c)	7.2	(a)	7.3	(b)	7.4	(a)	7.5	(d)	7.6	(b)	7.7	(c)
7.8	(c)	7.9	(c)	7.10	(a)	7.11	(c)	7.12	(c)	7.13	(c)	7.14	(c)
7.15	(a)	7.16	(d)	7.17	(d)	7.18	(c)	7.19	(a)	7.20	(a)	7.21	(d)

Answers to Fill in the Blanks

7.1 120°

7.2 $\sqrt{3}V_m$

7.3 0.6644

7.4 unidirectional

7.5 60°

7.6 $2V_m$

7.7 18.26% and 4.3%

7.8 poor

7.9 $V_m \dfrac{m}{\pi} \sin \dfrac{\pi}{m}, -\dfrac{\pi}{m} \text{ to } \dfrac{\pi}{m}$

7.10 $(D_1, D_3, D_5), (D_2, D_4, D_6)$

7.11 120°

7.12 1.0009

7.13 $\sqrt{3}V_m$

7.14 12

7.15 Improved

THREE-PHASE CONTROLLED RECTIFIERS

8

8.1 INTRODUCTION

In a single-phase controlled rectifier, output voltage can be varied from maximum to zero when the firing angle increases progressively. The harmonics in the output voltage increases significantly with increasing firing angle. Hence filter circuit is required. The amplitude of harmonics in three-phase controlled rectifiers is comparatively low with respect to single-phase rectifier. The ripple voltage at output decreases with increasing number of pulses in a complete cycle. Actually three pulses, six-pulses, 12-pulses and 24 pulses rectifiers are used depending upon the requirement. When the firing angle of three-phase rectifier is controlled, the output voltage will be variable. Then the three-phase controlled rectifier can be used to provide variable dc voltage to

1. Electrical dc machines,
2. Electronics equipments,
3. HVDC power system,
4. DC motor drives, etc.

Essentially, the three-phase controlled rectifiers are extensively used in high power applications. The advantages of three-phase controlled rectifiers compared to single-phase controlled rectifiers are as follows:

1. The output voltage has less ripple and is more smoother than single-phase controlled rectifier output voltage.
2. The size of filter used in three-phase controlled rectifier is reduced.
3. Rectification efficiency is high.
4. Transformer utilisation factor is high.

There are different types of three-phase controlled rectifier as depicted in Fig. 8.1
In this chapter, the operating principle and analysis of three-phase controlled rectifiers are discussed in detail. In the analysis of controlled rectifier circuits, we assume that transformer, thyristors and diodes are ideal one. The forward voltage drop across diode and thyristors are zero and the transformer has zero resistance and zero leakage inductance.

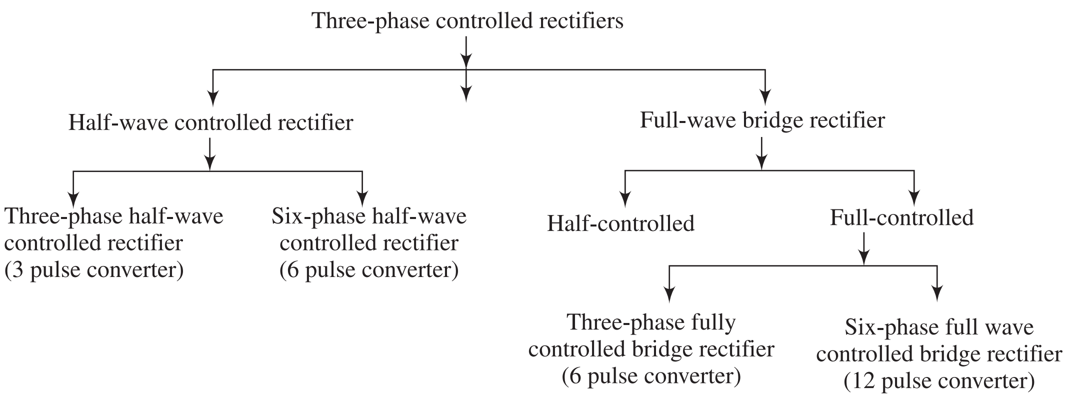


Fig. 8.1 *Classification of three-phase controlled rectifiers*

8.2 THREE-PHASE HALF-WAVE CONTROLLED RECTIFIER

Figure 8.2 shows a three-phase half-wave controlled rectifier which consists of a three-phase transformer with delta connected primary and star connected secondary, three SCRs and resistive *R* load. During analysis of this circuit, we assume that inductance of transformer is negligible and on state voltage across each SCR is also negligible.

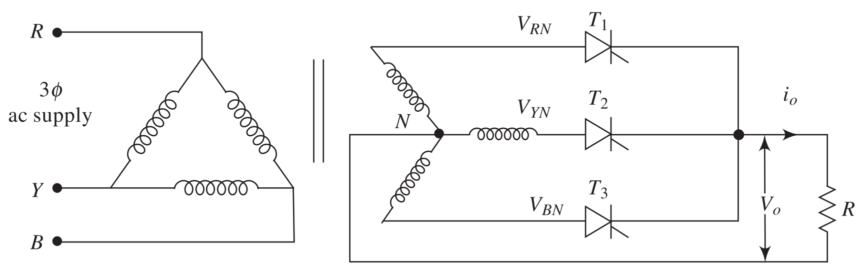


Fig. 8.2 *Three-phase half-wave controlled rectifier circuit with R load*

Actually this circuit consists of three single-phase half-wave controlled rectifier. Whenever a particular phase voltage is higher than other two phase voltages, the SCR of the corresponding phase will be forward biased and conducts as soon as the triggering pulse is applied. The simplified representation of three single-phase half-wave controlled rectifier is depicted in Fig. 8.3. This converter is also known as *three phase three-pulse converter* or *three-phase M-3 converter.*

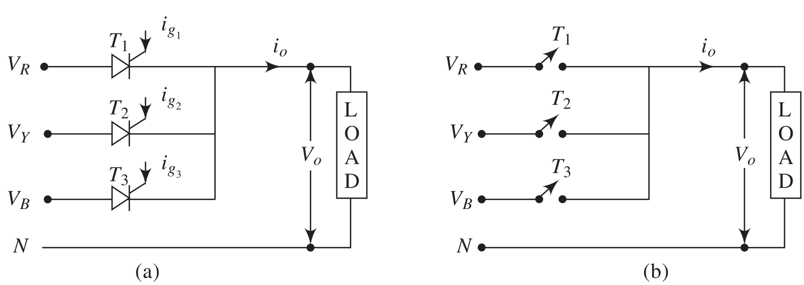


Fig. 8.3 *(a) Three-phase half-wave controlled rectifier (b) Equivalent representation of three phase half-wave controlled rectifier*

Three phase voltages are

$$V_{RN} = \sqrt{2}V \sin \omega t = V_m \sin \omega t$$
$$V_{YN} = \sqrt{2}V \sin(\omega t - 2\pi/3) = V_m \sin(\omega t - 2\pi/3)$$
$$V_{BN} = \sqrt{2}V \sin(\omega t + 2\pi/3) = V_m \sin(\omega t + 2\pi/3) = V_m \sin(\omega t - 4\pi/3)$$

where, V_m is maximum phase voltage and V is rms phase voltage.

The vector diagram of V_{RN}, V_{YN} and V_{BN} is depicted in Fig. 8.4.

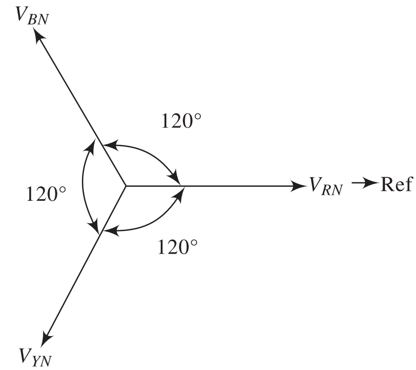


Fig. 8.4 *Vector diagram V_{RN}, V_{YN} and V_{BN}*

With resistive load, three-phase half-wave controlled rectifier operates in two different modes of conduction such as

1. Continuous conduction mode when firing angle α is less than 30°.
2. Discontinuous conduction mode when firing angle α is greater than 30°.

Figure 8.5 shows the voltage and current waveforms of three-phase half-wave controlled rectifier when SCR is fired at $\omega t = \frac{\pi}{6}$ with firing angle α = 0. If any firing pulse is applied in between ωt = 0 and $\omega t = \frac{\pi}{6}$, thyristor T_1 will not be conduct as it is reverse biased.

When thyristor T_1 is turned on at $\omega t = \frac{\pi}{6} + \alpha$, the R phase voltage V_{RN} is applied across the load until the thyristor T_2 is fired at $\omega t = \frac{5\pi}{6} + \alpha$. Since thyristor T_2 is turned ON at $\omega t = \frac{5\pi}{6} + \alpha$, thyristor T_1 will be reverse biased and turned OFF and the Y phase voltage V_{YN} is applied across the load until the thyristor T_3 is fired at $\omega t = \frac{9\pi}{6} + \alpha$. As thyristor T_3 is turned ON at $\omega t = \frac{9\pi}{6} + \alpha$, thyristor T_2 will be reverse biased and turned OFF and then B phase voltage V_{BN} is applied across the load until the thyristor T_1 is fired at $\omega t = \frac{13\pi}{6} + \alpha$. Subsequently, the cycle will be repeated and each thyristor conducts for $\frac{2\pi}{3}$ duration when the firing angle α is less than 30°(α < 30°). The equivalent switching representation of three-phase half-wave controlled rectifier during

(a) $\frac{5\pi}{6} + \alpha \geq \omega t \geq \frac{\pi}{6} + \alpha$

(b) $\frac{9\pi}{6} + \alpha \geq \omega t \geq \frac{5\pi}{6} + \alpha$ and

(c) $\frac{13\pi}{6} + \alpha \geq \omega t \geq \frac{9\pi}{6} + \alpha$

are depicted in Fig. 8.6(a), (b) and (c) respectively. The sequence of triggering pulse of thyristors T_1, T_2 and T_2 at firing angle α = 30° and the output voltage are depicted in Fig. 8.7.

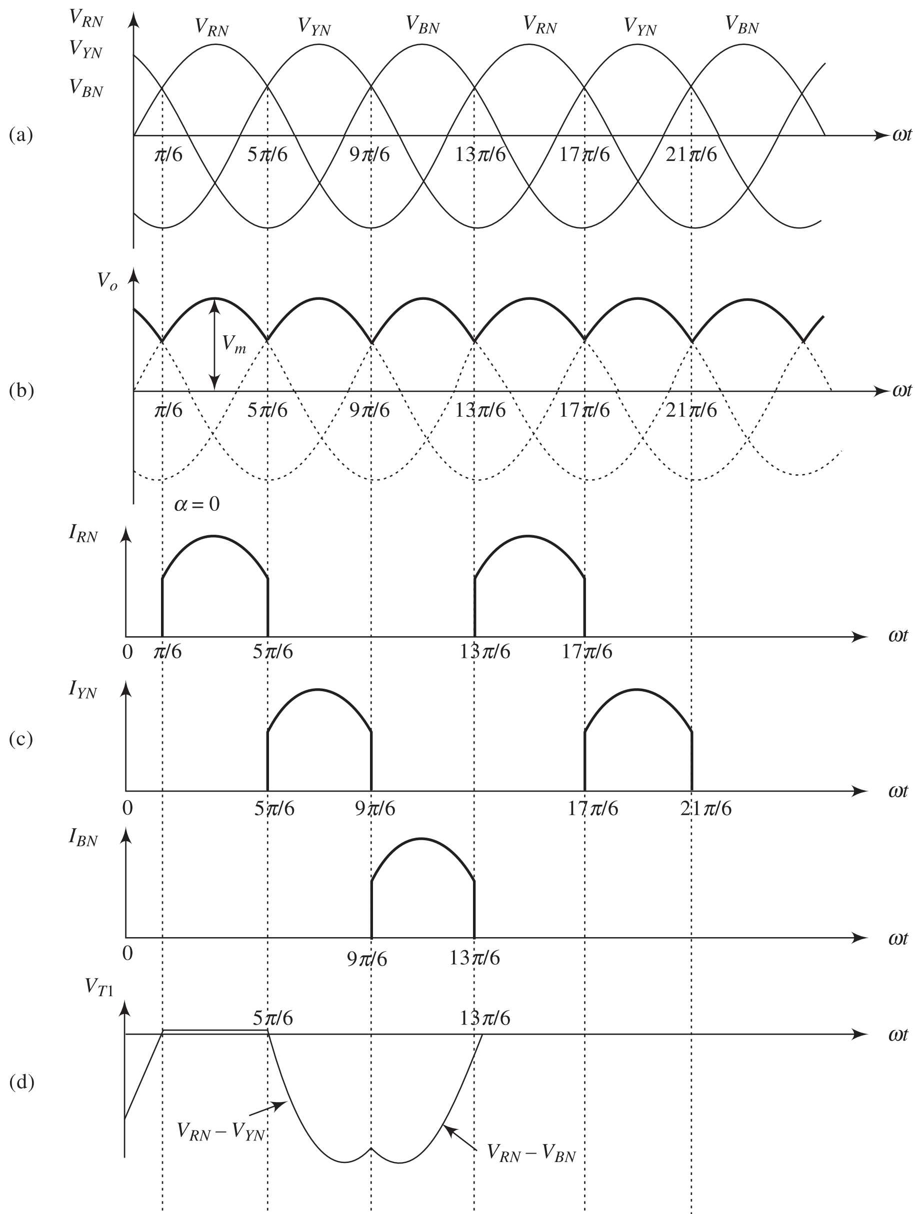


Fig. 8.5 *Voltage and current waveforms of three-phase half-wave controlled rectifier circuit with R load (a) Phase voltages V_{RN}, V_{YN}, V_{BN}, (b) Output voltage at firing angle $\alpha = 0$, (c) Phase currents I_{RN}, I_{YN}, I_{BN} (d) Voltage across thyristor T_1*

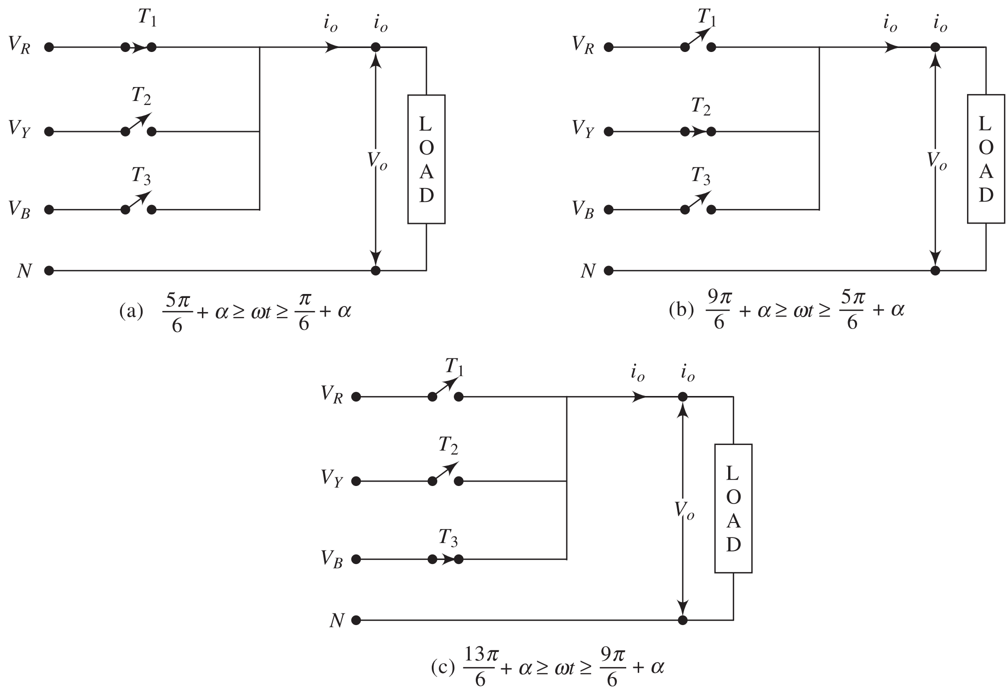


Fig. 8.6 *Equivalent switching representation of three phase half-wave controlled rectifier during (a) $\frac{5\pi}{6}+\alpha \geq \omega t \geq \frac{\pi}{6}+\alpha$, (b) $\frac{9\pi}{6}+\alpha \geq \omega t \geq \frac{5\pi}{6}+\alpha$ and (c) $\frac{13\pi}{6}+\alpha \geq \omega t \geq \frac{9\pi}{6}+\alpha$*

8.2.1 Continuous Load Current with *R* load when Firing Angle α < 30°

In continuous conduction mode, the load current is continuous in nature. When firing angle α is less than 30° and each thyristor conducts for 120° duration, the output V_o is shown in Fig. 8.8.

The average dc output voltage is

$$V_{av} = V_{dc} = V_o = \frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t . d(\omega t) = \frac{V_m}{2\pi/3}[-\cos\omega t]_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}$$

$$= \frac{V_m}{2\pi/3}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

$$= V_m \frac{3}{\pi}\frac{\sqrt{3}}{2}\cos\alpha = \frac{3\sqrt{3}}{2\pi}V_m \cos\alpha = 0.827V_m \cos\alpha \tag{8.1}$$

where, V_m is the maximum phase voltage and α is the firing angle.

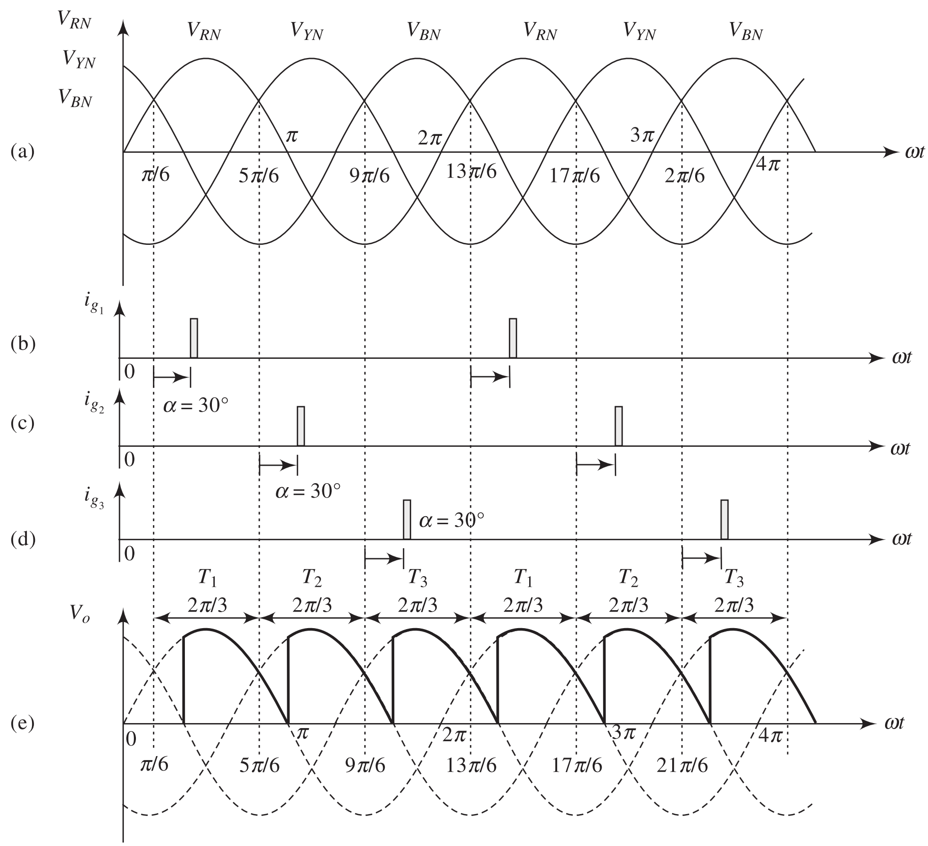


Fig. 8.7 *Waveforms of three phase half-wave controlled rectifier circuit with R load (a) Phase voltages V_{RN}, V_{YN}, V_{BN}, (b) Gate pulse of thyristor T_1, (c) Gate pulse of thyristor T_2, (d) Gate pulse of thyristor T_3, (e) Output voltage at firing angle $\alpha = 30°$*

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^{\ 2}\sin^2\omega t.d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^{\ 2}}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\frac{1}{2}(1-\cos 2\omega t).d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^{\ 2}}{4\pi}\left(\omega t - \frac{1}{2}\sin 2\omega t\right)_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^{\ 2}}{4\pi}\left(\frac{5\pi}{6}+\alpha-\frac{\pi}{6}-\alpha-\frac{1}{2}\sin\left(\frac{10\pi}{6}+2\alpha\right)+\frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{3}{2\pi}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\cos 2\alpha\right)\right]^{\frac{1}{2}} \tag{8.2}$$

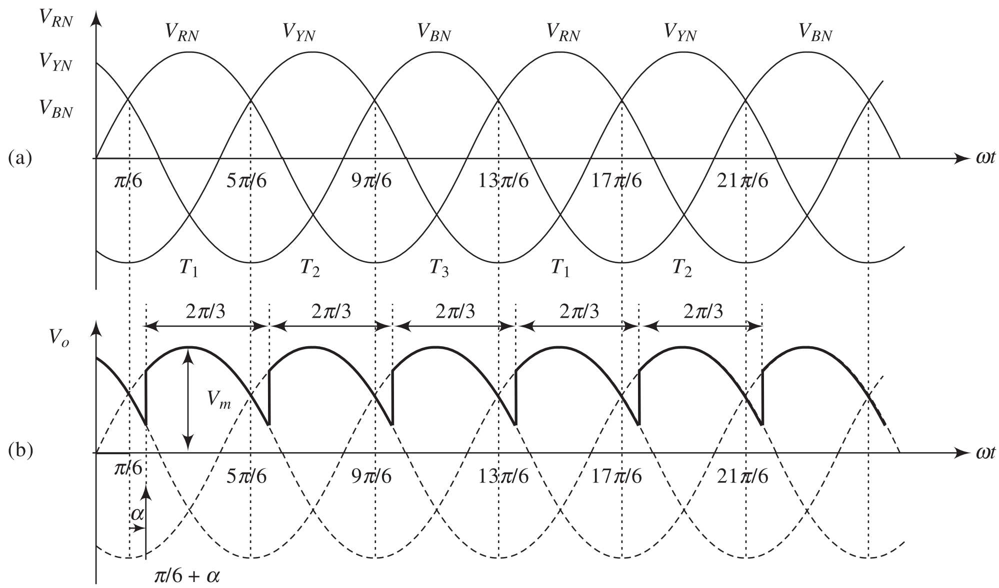


Fig. 8.8 *(a) Phase voltages V_{RN}, V_{YN}, V_{BN} (b) Output voltage of three-phase half-wave converter at firing angle $\alpha < 30°$*

The output voltage form factor is $FF = \dfrac{V_{\text{rms}}}{V_{dc}}$

The ripple factor is $RF = \sqrt{FF^2 - 1}$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{3\sqrt{3}}{2\pi}\frac{V_m}{R}\cos\alpha = 0.827\frac{V_m}{R}\cos\alpha$$

The dc output power is

$$P_{dc} = V_{dc}I_{dc} = V_oI_o = \frac{(0.827V_m)^2}{R}\cos\alpha = \frac{0.683V_m^2}{R}\cos\alpha$$

The rms value of load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{V_m}{R}\left[\frac{3}{2\pi}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\cos 2\alpha\right)\right]^{\frac{1}{2}}$$

The ac power supplied by the transformer secondary is $P_{ac} = I_{\text{rms}}^2 R$

Rectification efficiency is $\eta = \dfrac{P_{dc}}{P_{ac}} \times 100\%$

The current through the transformer secondary is $I_{\text{rms-transfor-secondary}} = \dfrac{I_{\text{rms}}}{\sqrt{3}}$

The VA rating of the transformer secondary is $VA = 3VI_{\text{rms-transformer-secondary}}$

Transformer utilisation factor is $TUF = \dfrac{P_{dc}}{VA}$

8.2.2 Discontinuous Load Current with *R* load when Firing Angle α > 30°

If the firing angle $\alpha > 30°$, the output voltage wave from will be discontinuous and the conduction angle of each thyristor is less than 120°. Therefore, the load current will be discontinuous and its value will be zero for certain duration. Figure 8.9 shows the output voltage waveform during discontinuous mode of operation at $\alpha = 60° = \dfrac{\pi}{3}$.

When the firing pulse is applied to SCR T_1 at $\omega t = \dfrac{\pi}{6} + \alpha = \dfrac{\pi}{6} + \dfrac{\pi}{3} = \dfrac{\pi}{2}$, thyristor T_1 is turned ON at $\omega t = \dfrac{\pi}{6} + \alpha = \dfrac{\pi}{2}$ and the *R* phase voltage V_{RN} is applied across the load up to $\omega t = \pi$. Consequently SCR T_1 conducts from $\omega t = \dfrac{\pi}{2}$ to $\omega t = \pi$ and the conduction period of T_1 is equal to $90° = \dfrac{\pi}{2}$. Since no thyristors conduct from $\omega t = \pi$ to $\omega t = \dfrac{5\pi}{6} + \alpha$, the output voltage across *R* load is zero. As triggering pulse is applied to thyristor T_2 at $\omega t = \dfrac{5\pi}{6} + \alpha = \dfrac{5\pi}{6} + \dfrac{\pi}{3} = \dfrac{7\pi}{6}$, thyristor T_2 will be turned ON at $\omega t = \dfrac{5\pi}{6} + \alpha = \dfrac{7\pi}{6} = 210°$ and continuously conducts up to 300°. Since thyristor T_2 is turned OFF from 300° to $\omega t = \dfrac{9\pi}{6} + \alpha = \dfrac{9\pi}{6} + \dfrac{\pi}{3} = \dfrac{11\pi}{6}$, the output voltage during this period is also zero. After application of triggering pulse to thyristor T_3 at $\omega t = \dfrac{9\pi}{6} + \alpha = \dfrac{11\pi}{6} = 330°$, thyristor T_3 is turned on at $\omega t = \dfrac{9\pi}{6} + \alpha = 330°$, and then B phase voltage V_{BN} is applied across the load up to 420°. Again output voltage is zero from 420° to 450°. After that the above cycle will be repeated and each thyristor

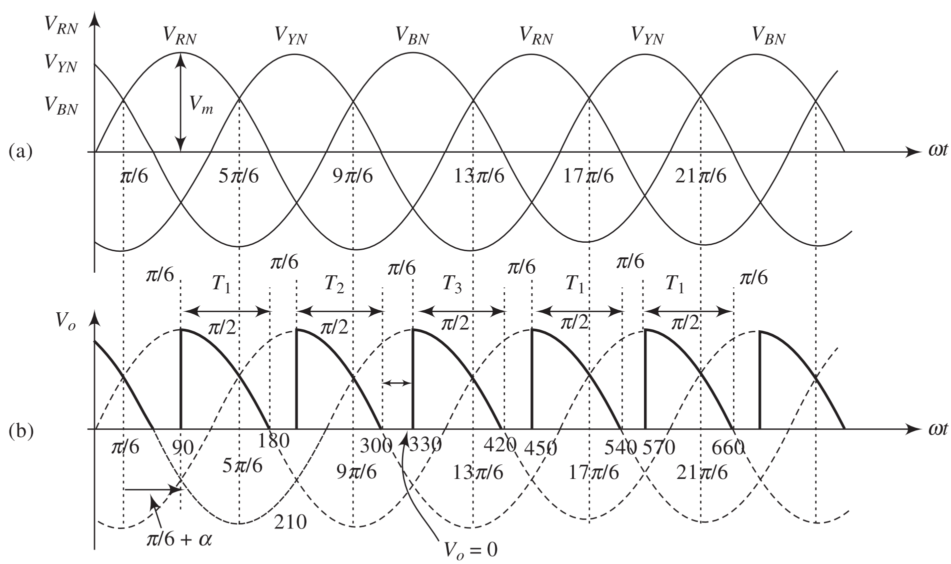


Fig. 8.9 *(a) Phase voltages V_{RN}, V_{YN}, V_{BN} (b) Output voltage of three-phase half-wave converter at firing angle $\alpha > 30°$ ($\alpha = 60°$)*

conducts for 90° duration. Hence, when the firing angle is $\alpha > 30°$, conduction angle of each thyristor is less than $\frac{2\pi}{3}$ duration.

The average dc output voltage is

$$V_{av} = V_o \;= \frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \sin\omega t.d(\omega t) = \frac{V_m}{2\pi/3}\left[-\cos\omega t\right]_{\frac{\pi}{6}+\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi/3}\left[-\cos\pi + \cos\left(\frac{\pi}{6}+\alpha\right)\right] = \frac{3V_m}{2\pi}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

where, V_m is the maximum phase voltage and α is the firing angle .

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m^2 \sin^2\omega t.d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} \frac{1}{2}(1-\cos 2\omega t).d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^2}{4\pi}\left(\omega t - \frac{1}{2}\sin 2\omega t\right)_{\frac{\pi}{6}+\alpha}^{\pi}\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^2}{4\pi}\left(\pi - \frac{\pi}{6} - \alpha - \frac{1}{2}\sin 2\pi + \frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^2}{4\pi}\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}} = \frac{\sqrt{3}V_m}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{3V_m}{2\pi R}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

The dc output power is

$$P_{dc} = V_{dc}I_{dc} = V_o I_o$$

The rms value of load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{\sqrt{3}V_m}{2\sqrt{\pi}R}\left[\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

The ac power supplied by the transformer secondary is $P_{ac} = I_{\text{rms}}^2 R$

Rectification efficiency is $\eta = \frac{P_{dc}}{P_{ac}} \times 100\%$

Example 8.1 A three-phase half-wave controlled rectifier is fed from a 3ϕ, 440 V, 50 Hz ac supply and it is connected with a R load of 10 Ω. When the firing angle of thyristor is 20°, calculate (a) dc output voltage, (b) r.m.s value of output voltage, (c) output voltage form factor, (d) ripple factor, (e) average output current, (f) rms load current, (g) dc output power, (h) ac power supplied and (i) rectification efficiency and TUF.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.04$ V, $R = 10\ \Omega$ and firing angle $\alpha = 20°$

$$\text{Maximum phase voltage } V_m = \sqrt{2}V = \sqrt{2} \times 254.04 \text{ V} = 359.21 \text{ V}$$

Since firing angle α is less than 30°, the output voltage as well as load current will be continuous.

(a) Then the average dc output voltage is equal to

$$V_{dc} = V_o = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t . d(\omega t)$$

$$= \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha = 0.827 V_m \cos\alpha = 0.827 \times 359.21 \times \cos 20 = 279.15 \text{ V}$$

(b) The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t . d\omega t \right]^{\frac{1}{2}}$$

$$= V_m \left[\frac{3}{2\pi} \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4} \cos 2\alpha \right) \right]^{\frac{1}{2}} = 359.21 \times \left[\frac{3}{2\pi} \left\{ \frac{\pi}{3} + \frac{\sqrt{3}}{4} \cos(2 \times 20) \right\} \right]^{\frac{1}{2}} = 291.48 \text{ V}$$

(c) The output voltage form factor is

$$FF = \frac{V_{\text{rms}}}{V_{dc}} = \frac{291.48}{279.15} = 1.0441$$

(d) The ripple factor is

$$RF = \sqrt{FF^2 - 1} = \sqrt{1.0441^2 - 1} = 0.30$$

(e) Average load current is

$$I_o = \frac{V_o}{R} = \frac{279.15}{10} = 27.915 \text{ A}$$

(f) The rms value of load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{291.48}{10} = 29.148 \text{ A}$$

(g) The dc output power is

$$P_{dc} = V_{dc} I_{dc} = V_o I_o = 279.15 \times 27.915 = 7792.47 \text{ Watt}$$

(h) The ac power supplied by the transformer secondary is

$$P_{ac} = I_{\text{rms}}^2 R = 29.148^2 \times 10 = 8696.05 \text{ Watt}$$

(i) Rectification efficiency is

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\% = \frac{7792.47}{8696.05} \times 100\% = 89.60\%$$

The current through the transformer secondary is

$$I_{\text{rms-transfor-secondary}} = \frac{I_{\text{rms}}}{\sqrt{3}} = \frac{29.148}{\sqrt{3}} = 16.82 \text{ A}$$

The VA rating of the transformer secondary is

$$VA = 3VI_{\text{rms-transformer-secondary}} = 3 \times 254.04 \times 16.82 = 12825.79 \text{ VA}$$

Transformer utilisation factor is $TUF = \dfrac{P_{dc}}{VA} = \dfrac{7792.47}{12825.79} = 0.6075$

Example 8.2 A three-phase half-wave controlled rectifier is connected to a 3ϕ, 220 V, 50 Hz ac supply and it is also connected with a resistive load of 5 Ω. If the firing angle of thyristor is 60°, calculate (a) dc output voltage, (b) rms value of output voltage, (c) average output current, (d) rms load current and (e) TUF.

Solution

Given: Phase voltage is $V = \dfrac{V_L}{\sqrt{3}} = \dfrac{220}{\sqrt{3}} = 127.02$ V , R = 5 Ω and firing angle α = 60°

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2} \times 127.02 \text{ V} = 179.60 \text{ V}$

Since firing angle α is greater than 30°, the output voltage as well as load current will be discontinuous.

(a) Then the average dc output voltage is

$$\begin{aligned} V_{av} = V_o &= \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \sin \omega t . d(\omega t) \\ &= \frac{3V_m}{2\pi} \left[1 + \cos\left(\frac{\pi}{6} + \alpha\right) \right] \\ &= \frac{3 \times 179.60}{2\pi} [1 + \cos(30 + 60)] \qquad \text{as } \frac{\pi}{6} = 30° \text{ and } \alpha = 60° \\ &= 85.796 \text{ V} \end{aligned}$$

(b) The rms value of output voltage is

$$\begin{aligned} V_{\text{rms}} &= \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\pi} V_m^2 \sin^2 \omega t . d\omega t \right]^{\frac{1}{2}} = \left[\frac{3V_m^2}{4\pi} \left(\frac{5\pi}{6} - \alpha + \frac{1}{2} \sin\left(\frac{2\pi}{6} + 2\alpha \right) \right) \right]^{\frac{1}{2}} \\ &= \frac{\sqrt{3}V_m}{2\sqrt{\pi}} \left[\left(\frac{5\pi}{6} - \alpha + \frac{1}{2} \sin\left(\frac{2\pi}{6} + 2\alpha \right) \right) \right]^{\frac{1}{2}} \\ &= \frac{\sqrt{3} \times 179.60}{2\sqrt{\pi}} \left[\left(\frac{5\pi}{6} - \frac{2\pi}{6} + \frac{1}{2} \sin(60 + 2 \times 60) \right) \right]^{\frac{1}{2}} \qquad \text{as } \frac{2\pi}{6} = 60° \text{ and } \alpha = 60° = \frac{2\pi}{6} \\ &= 137.807 \text{ V} \end{aligned}$$

(c) Average load current is

$$I_o = \frac{V_o}{R} = \frac{85.796}{5} = 17.1592 \text{ A}$$

(d) The rms value of load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{137.807}{5} = 27.5614 \text{ A}$$

(e) The dc output power is

$$P_{dc} = V_{dc} I_{dc} = V_o I_o = 85.796 \times 17.1592 = 1472.19 \text{ Watt}$$

The current through the transformer secondary is

$$I_{\text{rms-transfor-secondary}} = \frac{I_{\text{rms}}}{\sqrt{3}} = \frac{27.5614}{\sqrt{3}} = 15.913 \text{ A}$$

The *VA* rating of the transformer secondary is

$$VA = 3VI_{\text{rms-transformer-secondary}} = 3 \times 127.02 \times 15.913 = 6063.80 \text{ VA}$$

Transformer utilisation factor is

$$TUF = \frac{P_{dc}}{VA} = \frac{1472.19}{6063.80} = 0.2427$$

Example 8.3 A three-phase half-wave controlled rectifier is fed from a 3ϕ, 400 V, 50 Hz ac supply and it is connected with a R load of 10 Ω. When the average dc output voltage is equal to 60% of the maximum dc output voltage, determine (a) the firing angle of thyristor, (b) dc output voltage and (c) rms value of output voltage.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$, and R = 10 Ω

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2} \times 230.94 \text{ V} = 326.54 \text{ V}$

(a) At firing angle α, the average dc output voltage is

$$V_{\text{dc}} = V_o = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \omega t \cdot d(\omega t)$$

$$= \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha = 0.827 V_m \cos\alpha$$

When α = 0°, maximum dc output voltage is $V_{\text{dc|max}} = 0.827 V_m$

Since the average dc output voltage is equal to 60% of the maximum dc output voltage,

$$V_{\text{dc}} = 0.827 V_m \cos\alpha = 0.6 V_{\text{dc|max}} = 0.6 \times 0.827 V_m$$

or, cos α = 0.6

The firing angle α is $\cos^{-1}$ 0.6 = 53.13°

Since the firing angle α is greater than 30°, the output voltage can not be computed from above equations.

Since α > 30°, voltage as well as load current will be discontinuous.

Then the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \sin \omega t \cdot d(\omega t)$$

$$= \frac{3V_m}{2\pi}\left[1 + \cos\left(\frac{\pi}{6} + \alpha\right)\right] = 0.6 \times 0.827 V_m$$

or $1 + \cos(30 + \alpha) = \frac{0.6 \times 0.827 \times 2\pi}{3} = 1.0387$

or $30+\alpha=\cos^{-1}(0.0387)=87.78$

Then the firing angle is $\alpha = 57.78°$

(b) Subsequently, the average dc output voltage is

$$\begin{aligned}V_{\text{av}}=V_o &= \frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \sin\omega t\cdot d(\omega t)\\ &= \frac{3V_m}{2\pi}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right]\\ &= \frac{3\times 326.54}{2\pi}\left[1+\cos(30+57.78)\right] \qquad \text{as } \frac{\pi}{6}=30° \text{ and } \alpha=57.78°\\ &= 162.02 \text{ V}\end{aligned}$$

(c) The rms value of output voltage is

$$V_{\text{rms}}=\left[\frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m^2\sin^2\omega t\cdot d\omega t\right]^{\frac{1}{2}}=\left[\frac{3V_m^2}{4\pi}\left(\frac{5\pi}{6}-\alpha+\frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$=\frac{\sqrt{3}V_m}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6}-\alpha+\frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$=\frac{\sqrt{3}\times 326.54}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6}-\frac{57.78\times\pi}{180}+\frac{1}{2}\sin(60+2\times 57.78)\right)\right]^{\frac{1}{2}}$$

$$\text{as } \frac{2\pi}{6}=60° \text{ and } \alpha=53.13°=\frac{53.13\times\pi}{180}$$

$$= 204.825 \text{ V}$$

8.3 THREE-PHASE HALF-WAVE CONTROLLED RECTIFIER WITH *RL* LOAD

Figure 8.10 shows a three-phase half-wave controlled rectifier with *RL* load. Assume that the load inductance L is very large so that the load current I_o is continuous and constant. When the firing angle $\alpha < 30°$, the average dc output voltage and rms output voltage will be same as Eqs. (8.1) and (8.2) respectively. If the firing angle α varies with in the range of $30° < \alpha < 90°$ and $90° < \alpha < 180°$, the three-phase half-wave controlled rectifier behaves differently as explained below:

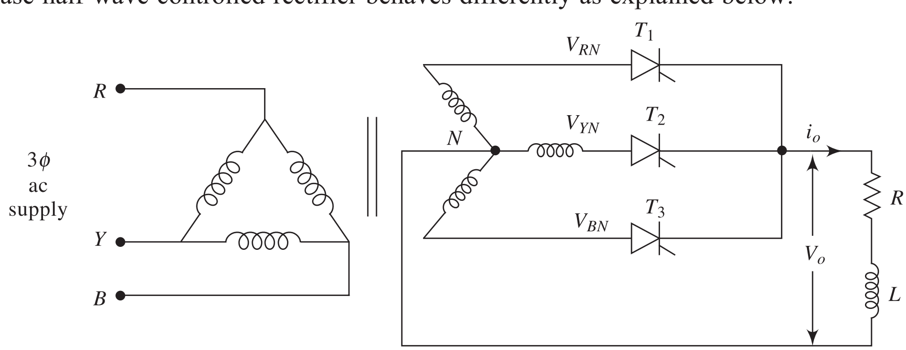


Fig. 8.10 *Three-phase half-wave controlled rectifier with RL load*

8.3.1 Operation of Converter When α Varies Within 30° and 90° (30° < α < 90°)

Operation of converter when α = 45° Assume that the firing angle of thyristors is $\alpha = 45°$. The voltage and current waveforms for this converter is depicted in Fig. 8.11. It is clear from Fig. 8.11 that each thyristor conducts for 120° duration. Hence thyristor T_1 conducts from 30° + α (75°) to 150° + α (195°), T_2 conducts from 150° + α (195°) to 270° + α (315°) and T_3 conducts from 270° + α (315°) to 390° + α (435°) as α = 45°.

At ωt = 180° or π, the R phase voltage V_{RN} becomes zero, but the current I_{RN} is not zero as thyristor T_1 continuously conduct until the firing pulse is applied to T_2 at 150° + α (195°). Hence the output voltage is negative during 150° + α > ωt > 180°. As soon as thyristor T_2 is turned on at ωt = 150° + α (195°), the output voltage will be equal to the Y phase voltage V_{YN} and the load current shifts from T_1 to T_2. When thyristor T_1 conducts, the forward voltage drop across T_1 is about zero. At ωt = 150° + α, thyristor T_1 is reverse biased and the amplitude of reverse bias voltage is equal to $V_{RN} - V_{YN} = V_m \sin(150+\alpha) - V_m \sin(150+\alpha-120)$ as depicted in Fig. 8.11. Similarly, the reverse-biased voltage and forward blocking voltage across thyristor T_1 can be computed at different ωt in the range of 435° ≥ ωt ≥ 195° as given below.

1. T_1 is ON from ωt = 30° + α (75°) to ωt = 150° + α (195°), voltage across T_1 is 0 V.
2. T_2 is ON from ωt = 150° + α (195°) to ωt = 270° + α (315°), voltage across T_1 is $V_{RN} - V_{YN}$.

ωt	Voltage across T_1
$\omega t = 195°$	$V_{RN} - V_{YN} = V_m \sin 195 - V_m \sin(195-120) = V_m \sin 195 - V_m \sin 75 = -1.2247V_m$
$\omega t = 210°$	$V_{RN} - V_{YN} = V_m \sin 210 - V_m \sin(210-120) = V_m \sin 210 - V_m \sin 90 = -1.5V_m$
$\omega t = 240°$	$V_{RN} - V_{YN} = V_m \sin 240 - V_m \sin(240-120) = V_m \sin 240 - V_m \sin 120 = -1.732V_m$
$\omega t = 270°$	$V_{RN} - V_{YN} = V_m \sin 270 - V_m \sin(270-120) = V_m \sin 270 - V_m \sin 150 = -1.5V_m$
$\omega t = 300°$	$V_{RN} - V_{YN} = V_m \sin 300 - V_m \sin(300-120) = V_m \sin 300 - V_m \sin 180 = -0.866V_m$
$\omega t = 315°$	$V_{RN} - V_{YN} = V_m \sin 315 - V_m \sin(315-120) = V_m \sin 315 - V_m \sin 195 = -0.4482V_m$

3. T_3 is ON from ωt = 270° + α (315°) to ωt = 390° + α (435°), voltage across T_1 is $V_{RN} - V_{BN}$

ωt	**Voltage across T_1**
$\omega t = 315°$	$V_{RN} - V_{BN} = V_m \sin 315 - V_m \sin(315-240) = V_m \sin 315 - V_m \sin 75 = -1.673V_m$
$\omega t = 330°$	$V_{RN} - V_{BN} = V_m \sin 330 - V_m \sin(330-240) = V_m \sin 330 - V_m \sin 90 = -1.5V_m$
$\omega t = 360°$	$V_{RN} - V_{BN} = V_m \sin 360 - V_m \sin(360-240) = V_m \sin 360 - V_m \sin 120 = -0.866V_m$
$\omega t = 390°$	$V_{RN} - V_{BN} = V_m \sin 390 - V_m \sin(390-240) = V_m \sin 390 - V_m \sin 150 = 0$
$\omega t = 420°$	$V_{RN} - V_{BN} = V_m \sin 420 - V_m \sin(420-240) = V_m \sin 420 - V_m \sin 180 = 0.866V_m$
$\omega t = 435°$	$V_{RN} - V_{BN} = V_m \sin 435 - V_m \sin(435-240) = V_m \sin 435 - V_m \sin 195 = -1.2247V_m$

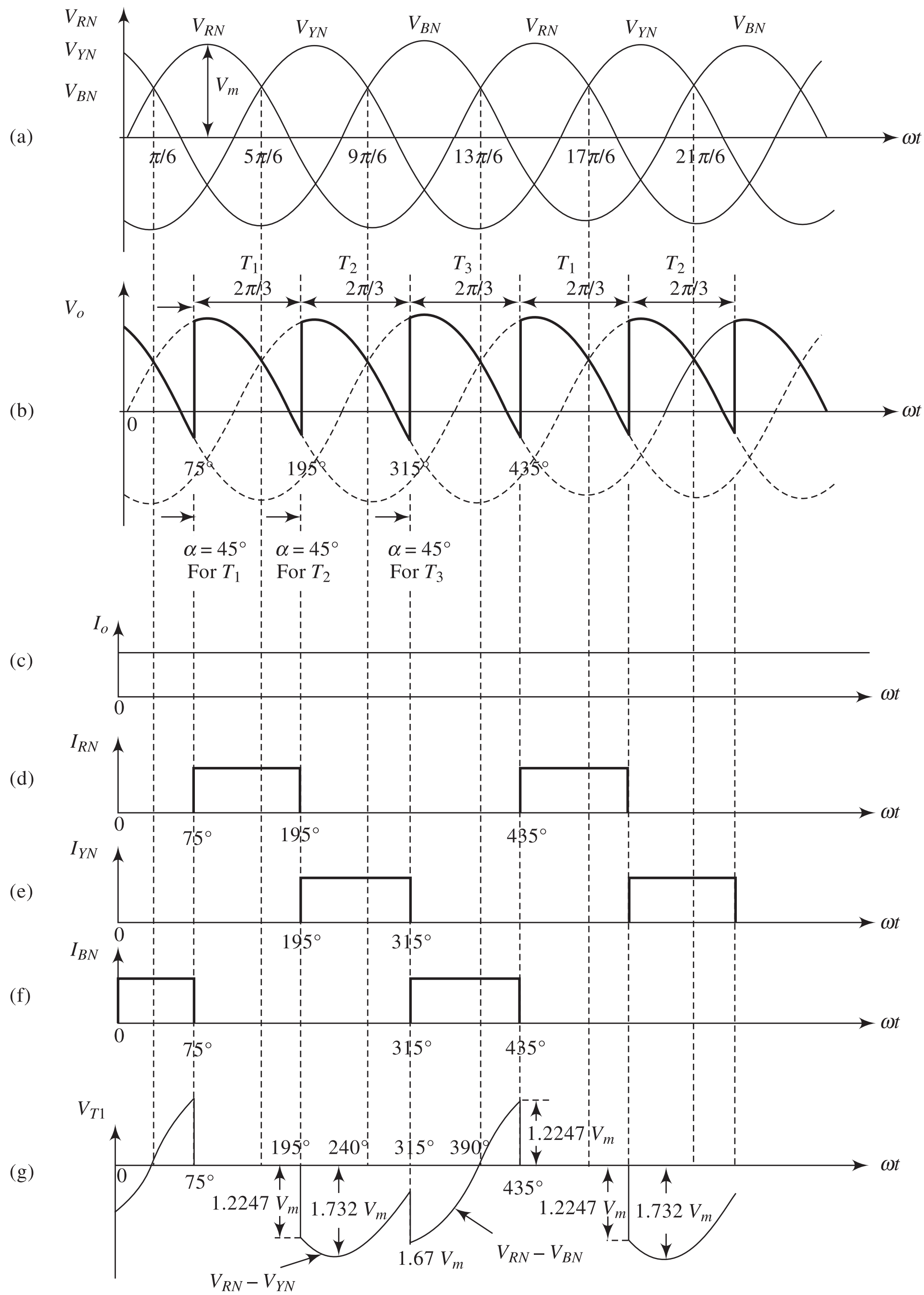


Fig. 8.11 *Voltage and current waveforms of three-phase half-controlled converter with RL load at α = 45°*

The average and rms values of output voltages of three-phase half-wave converter with highly inductive load and firing angle varies in the range 30° < α < 90°, will be as follows

The average dc output voltage is

$$V_{\text{av}} = V_{\text{dc}} = V_o = \frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi/3}\left[-\cos\omega t\right]_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}$$

$$= \frac{3\sqrt{3}}{2\pi}V_m \cos\alpha = 0.827V_m \cos\alpha$$

where, V_m is the maximum phase voltage and α is the firing angle.

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2\omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{3}{2\pi}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\cos 2\alpha\right)\right]^{\frac{1}{2}}$$

Operation of converter when α = 60° When the firing angle of thyristors is α = 60°, the voltage and current waveforms for this converter are depicted in Fig. 8.12. It is clear from Fig. 8.12 that each thyristor conducts for 120° duration. Hence thyristor T_1 conducts from 30° + α (90°) to 150° + α (210°), T_2 conducts from 150° + α (210°) to 270° + α (330°) and T_3 conducts from 270° + α (330°) to 390° + α (450°).

At ωt = 180° or π, the R phase voltage V_{RN} becomes zero, but the current I_{RN} is not zero as thyristor T_1 continuously conduct until the firing pulse is applied to T_2 at 150° + α. Therefore, the output voltage is negative during 150° + α > ωt > 180°. As soon as thyristor T_2 is turned ON at ωt = 150° + α (210°), the output voltage will be equal to the Y phase voltage V_{YN} and the load current shifts from T_1 to T_2. When thyristor T_1 conducts, the forward voltage drop across T_1 is about zero. At ωt = 150° + α, thyristor T_1 is reverse biased and the amplitude of reverse bias voltage is equal to $V_{RN} - V_{YN} = V_m \sin(150+\alpha) - V_m \sin(150+\alpha-120) = V_m \sin 210 - V_m \sin 90 = -1.5V_m$. Similarly, the reverse-biased voltage across at different ωt in the range of $390°+\alpha > \omega t > 150°+\alpha$ can be computed.

T_1 is ON from ωt = 30° + α (90°) to ωt = 150° + α (210°), voltage across T_1 is 0 V

T_2 is ON from ωt = 150° + α (210°) to ωt = 270° + α (330°), voltage across T_1 is $V_{RN} - V_{YN}$

At ωt = 150° + α = 210°,

$$V_{RN} - V_{YN} = V_m \sin(150+\alpha) - V_m \sin(150+\alpha-120) = V_m \sin 210 - V_m \sin 90 = -1.5V_m$$

At $\omega t = 240°$, $V_{RN} - V_{YN} = V_m \sin 240 - V_m \sin(240-120) = -1.732V_m$

At $\omega t = 270°$, $V_{RN} - V_{YN} = V_m \sin 270 - V_m \sin(270-120) = -1.5V_m$

At ωt = 330°, the amplitude of reverse bias voltage across T_1 is equal to

$$V_{RN} - V_{YN} = V_m \sin 330 - V_m \sin(330-120) = V_m \sin 330 - V_m \sin 210 = 0$$

T_3 is ON from ωt = 270° + α (330°) to ωt = 390° + α (450°), voltage across T_1 is $V_{RN} - V_{BN}$

and $V_{RN} - V_{BN} = V_m \sin 330 - V_m \sin(330-240) = V_m \sin 330 - V_m \sin 90 = -1.5V_m$

At ωt = 390°, the amplitude of reverse bias voltage across T_1 is equal to

$$V_{RN} - V_{BN} = V_m \sin 390 - V_m \sin(390-240) = V_m \sin 390 - V_m \sin 150 = 0$$

At ωt = 450°, the amplitude of reverse bias voltage across T_1 is equal to

$$V_{RN} - V_{BN} = V_m \sin 450 - V_m \sin(450-240) = V_m \sin 450 - V_m \sin 330 = 1.5V_m$$

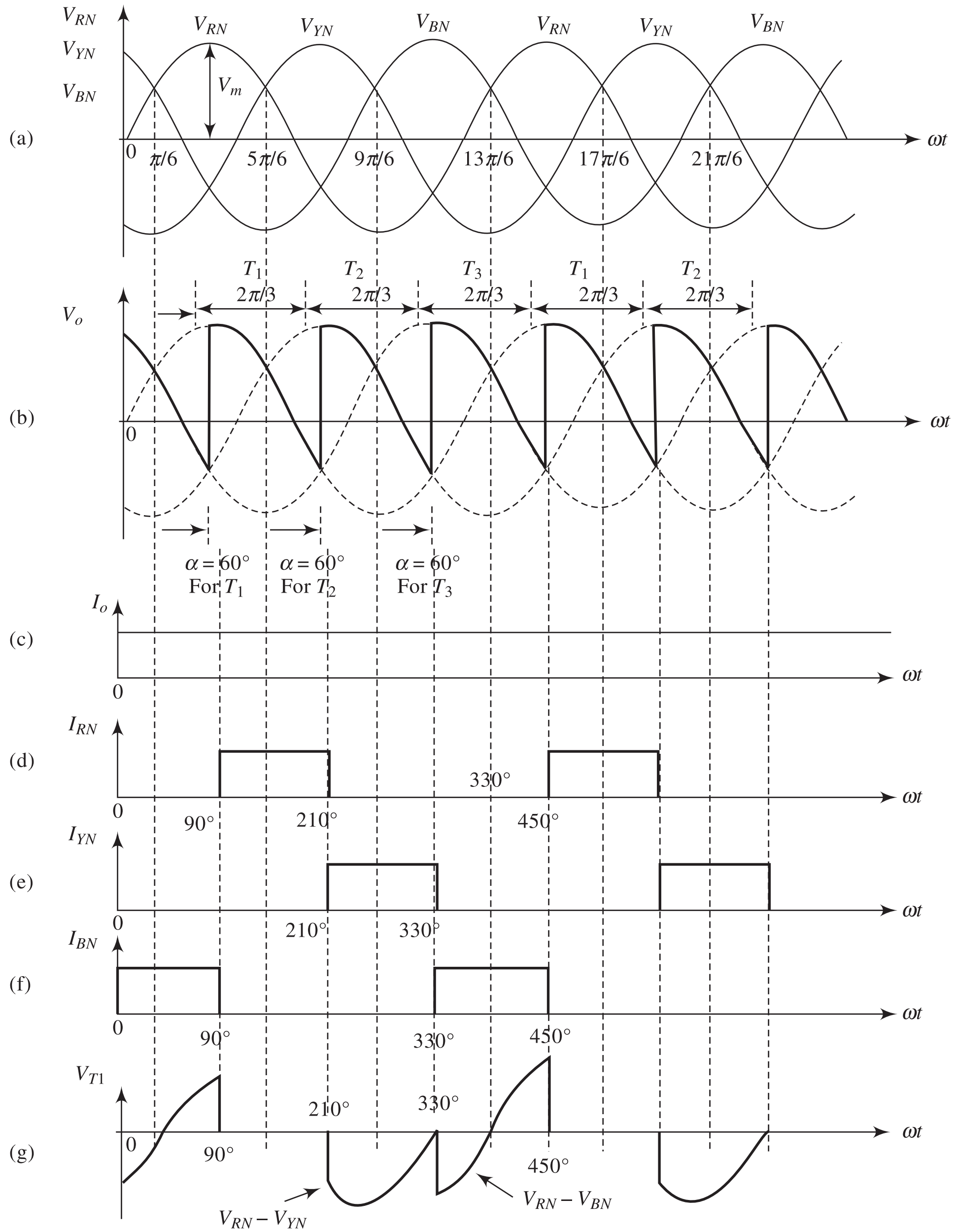


Fig. 8.12 *Voltage and current waveforms of three-phase half-wave converter with RL load at α = 60°*

8.3.2 Operation of converter when α varies within $90° < \alpha < 180°$

Assume that the firing angle of thyristors is $\alpha = 120°$ and the load I_o is continuous and ripple free due to highly inductive load. The voltage and current waveforms for this converter at $\alpha = 120°$ is shown in Fig. 8.13. It is clear from Fig. 8.13 that each thyristor conducts for 120° duration and the output voltage is positive for small duration and it is negative for large duration. Consequently, the average dc output voltage is negative.

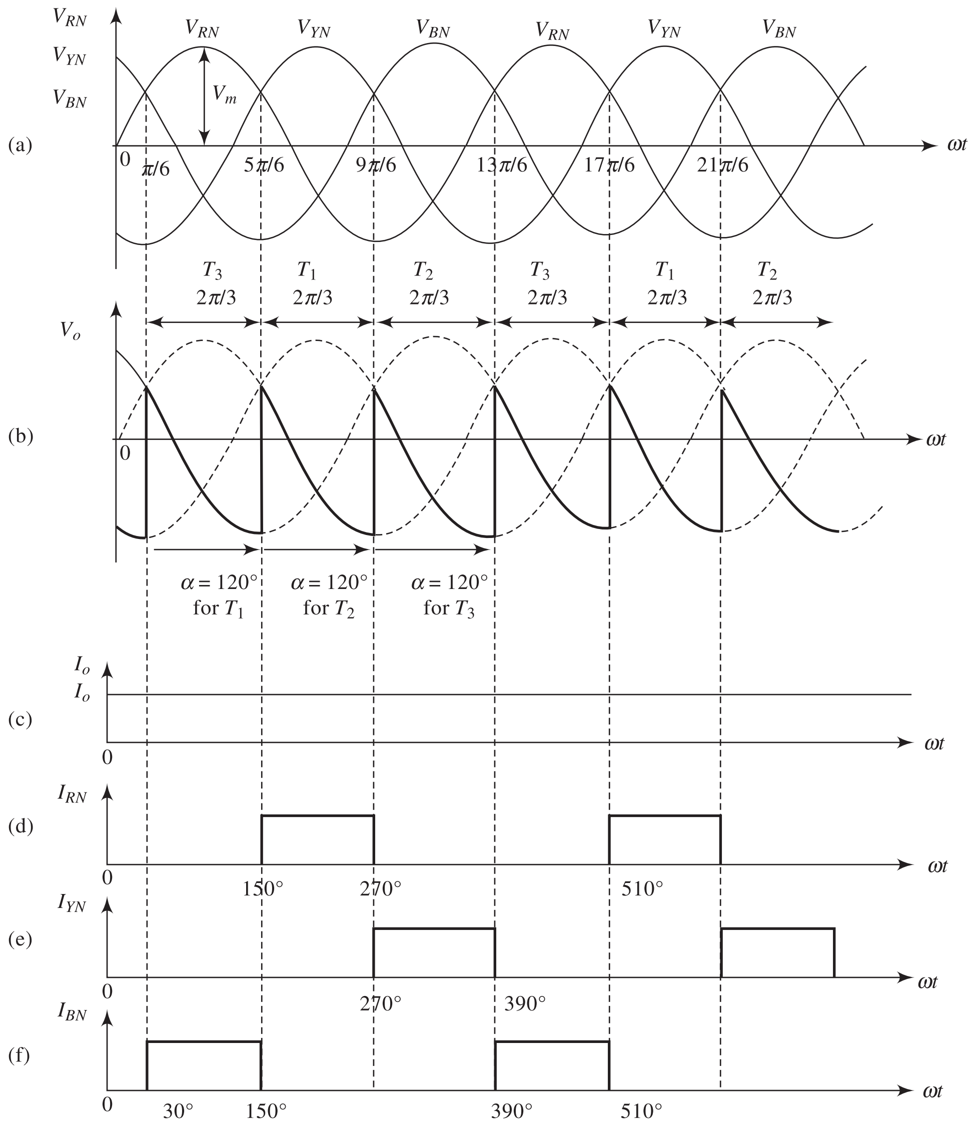


Fig. 8.13 *Voltage and current waveforms of three-phase half-controlled converter with RL load at α = 120°*

In this converter, thyristor T_1 conducts from 30° + α to 150° + α, T_2 conducts from 150° + α to 270° + α and T_3 conducts from 270° + α to 390° + α. Since α = 120°, the conduction period for T_1 is from 150° to 270°, T_2 conducts from 270° to 390° and the conduction period for T_3 is from 390° to 510°.

As the average dc output voltage is $V_o = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha$ and the firing angle α is greater than 90°, V_o is negative. Since each thyristor conducts for 120°, average thyristor current is

$$I_{T\text{-average}} = \frac{120°}{360°} \times I_o = \frac{I_o}{3} \text{ and}$$

rms value of thyristor current is

$$I_{T\text{-rms}} = \left[\frac{120°}{360°} \times I_o^2\right]^{\frac{1}{2}} = \frac{I_o}{\sqrt{3}}$$

Figure 8.14 shows the voltage and current waveforms of three-phase half-controlled converter with *RL* load at α = 165°.

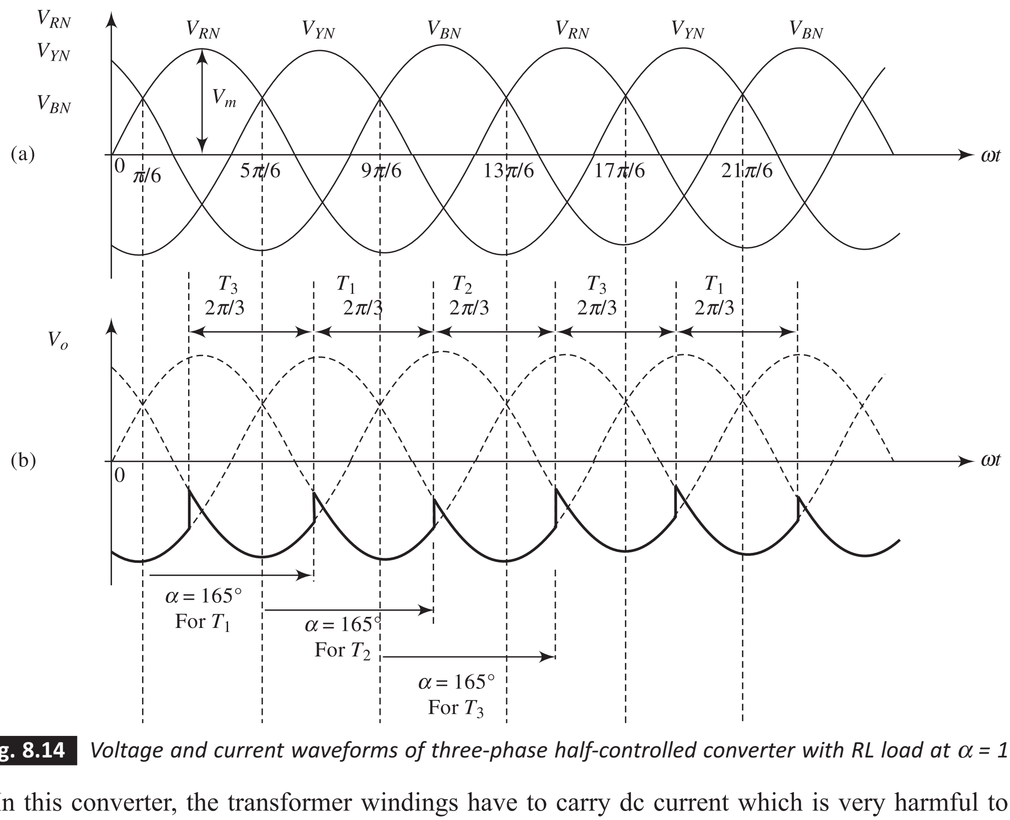


Fig. 8.14 *Voltage and current waveforms of three-phase half-controlled converter with RL load at α = 165°*

In this converter, the transformer windings have to carry dc current which is very harmful to the transformer. This problem can be solved using delta-zigzag connection as depicted in Fig. 8.15. It is clear from Fig. 8.15 that load current I_o enters the neutral N of the secondary zigzag which divides equally in the three half-windings. Each half-winding *a*, *b*, *c* carries $\frac{I_o}{3}$ load current. The current $\frac{I_o}{3}$ flows through other half windings b_1, c_1, a_1 and thyristors T_1, T_2, T_3 and *RL* load.

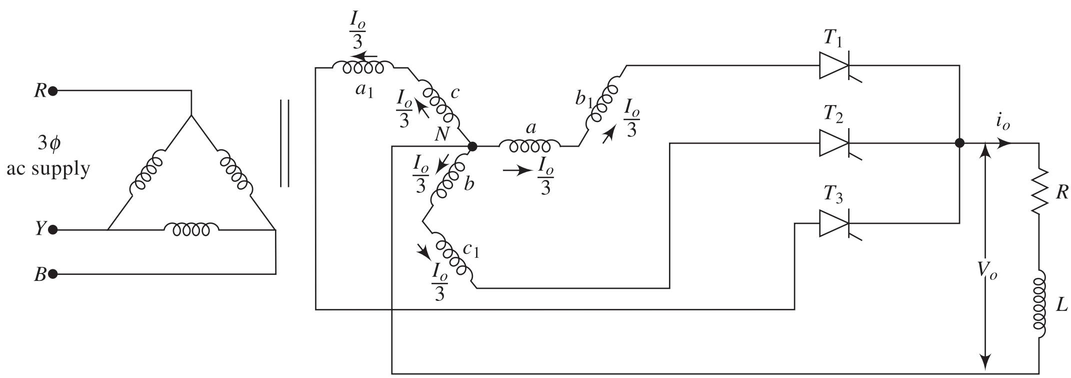


Fig. 8.15 *Three-phase half-controlled converter with RL load is supplied from a delta-zigzag transformer*

In zigzag winding, each phase winding is divided into two halves such as a, a_1; b, b_1; c, c_1 and $\frac{I_o}{3}$ current flows through each half in opposite direction as depicted in Fig. 8.15. As each half of the secondary windings caries dc current in opposite direction, the magnetic effects due to dc currents cancel each other. Consequently, the core flux, core loss and the temperature rise of transformer are not affected. Therefore, three-phase half-controlled converter can be used for energizing a dc load through delta-zigzag transformer.

Example 8.4 A three-phase three pulse controlled rectifier is fed from a 3ϕ, 220 V, 50 Hz ac supply and it is connected with a R load of 10 Ω. If the average dc output voltage is equal to 90% of the maximum dc output voltage, determine (a) the firing angle of thyristor, (b) dc output voltage, (c) rms value of output voltage and (d) rectification efficiency.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.02$ V, and $R = 10\ \Omega$

$$\text{Maximum phase voltage } V_m = \sqrt{2}V = \sqrt{2}\times 127.02 \text{ V} = 179.63 \text{ V}$$

(a) At firing angle α, the average dc output voltage is

$$V_{\text{dc}} = V_o = \frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t \cdot d(\omega t)$$

$$= \frac{3\sqrt{3}}{2\pi}V_m \cos\alpha = 0.827V_m \cos\alpha$$

When $\alpha = 0°$, maximum dc output voltage is $V_{\text{dc|max}} = 0.827V_m$

Since the average dc output voltage is equal to 90% of the maximum dc output voltage,

$$V_{\text{dc}} = 0.827V_m \cos\alpha = 0.9V_{\text{dc|max}} = 0.9\times 0.827V_m$$

or $\cos\alpha = 0.9$

The firing angle α is $\cos^{-1}0.9 = 25.84°$

Since the firing angle α is less than 30°, the output voltage can be computed from above equations.

(b) Therefore, the average dc output voltage is

$$V_{\text{av}} = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha = 0.827 V_m \cos\alpha = 0.827 \times 179.63 \times 0.9 = 133.69 \text{ V}$$

(c) The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t \cdot d\omega t \right]^{\frac{1}{2}} = V_m \left[\frac{3}{2\pi} \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4} \cos 2\alpha \right) \right]^{\frac{1}{2}} \quad \text{as } \alpha = 25.84^\circ$$

$$= 179.63 \times \left[\frac{3}{2\pi} \left\{ \frac{\pi}{3} + \frac{\sqrt{3}}{4} \cos(2 \times 25.84) \right\} \right]^{\frac{1}{2}} = 142.37 \text{ V}$$

(d) Average load current is

$$I_o = \frac{V_o}{R} = \frac{133.69}{10} = 13.369 \text{ A}$$

The rms value of load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{142.37}{10} = 14.237 \text{ A}$$

The dc output power is

$$P_{\text{dc}} = V_{\text{dc}} I_{\text{dc}} = V_o I_o = 133.69 \times 13.369 = 1787.30 \text{ Watt}$$

The ac power supplied by the transformer secondary is

$$P_{\text{ac}} = I_{\text{rms}}^2 R = 14.237^2 \times 10 = 2026.92 \text{ Watt}$$

Rectification efficiency is

$$\eta = \frac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\% = \frac{1787.30}{2026.92} \times 100\% = 88.17\%$$

Example 8.5 A three-phase three pulse controlled rectifier is fed from a 3ϕ, 440 V, 50 Hz ac supply and it is connected with a constant current load of 40 A. The voltage drop across each thyristor is 1.5 V. (a) Determine the dc output voltage at firing angle of 60° and 45°, (b) Calculate the average and rms current rating and PIV of thyristors and (c) Determine the average power dissipated in each thyristor.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.04 \text{ V}$, V_T = 1.5 V and R = 10 Ω

Maximum phase voltage $V_m = \sqrt{2} V = \sqrt{2} \times 254.04 = 359.21 \text{ V}$

(a) At firing angle α, the average dc output voltage is

$$V_{\text{dc}} = V_o = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \omega t \cdot d(\omega t) - V_T$$

$$= \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha - V_T = 0.827 V_m \cos\alpha - V_T$$

At $\alpha = 60^\circ$, $V_o = 0.827 V_m \cos\alpha - V_T = 0.827 \times 359.21 \cos 60 - 1.5 \text{ V} = 147.03 \text{ V}$

At $\alpha = 45^\circ$, $V_o = 0.827 V_m \cos\alpha - V_T = 0.827 \times 359.21 \cos 45 - 1.5 \text{ V} = 208.557 \text{ V}$

(b) Average current rating of SCR is $I_{T\text{av}} = \dfrac{I_o}{3} = \dfrac{40}{3} = 13.333$ A

rms current rating of SCR is $I_{T\text{-rms}} = \dfrac{I_o}{\sqrt{3}} = \dfrac{40}{\sqrt{3}} = 23.094$ A

PIV of thyristor is $\sqrt{3}V_m = \sqrt{3} \times 359.21 = 622.15$ V

(c) Average power dissipated in each SCR is $I_{T\text{av}} \times V_T = 13.333 \times 1.5 = 19.9995$ Watt

Example 8.6 A 150 V battery is charged using three phases half-wave rectifier as depicted in Fig. 8.16. The input phase voltage is 220 V, 50 Hz and the firing angle of thyristors is 30°. Determine the average current flows through the battery. Draw the charging voltage and charging current waveforms.

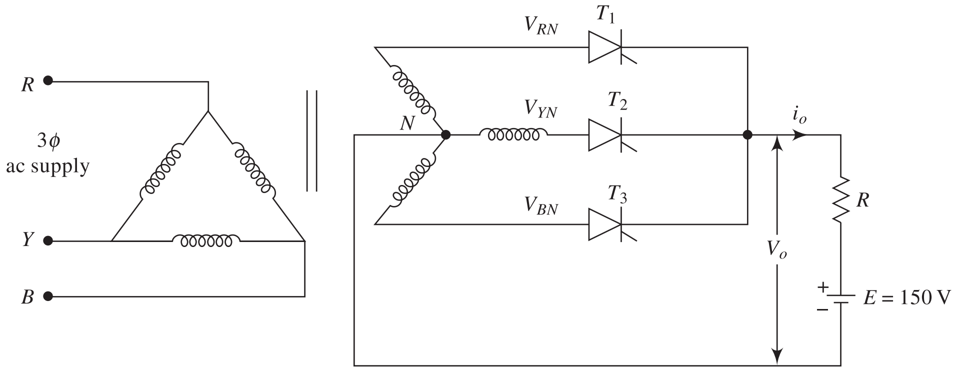


Fig. 8.16

Solution

Phase voltage $V = 220$ V and maximum phase voltage is $V_m = \sqrt{2}V = \sqrt{2} \times 220 = 311.1269$ V

At $\omega t = 30° + \alpha = 30° + 30° = 60°$, thyristor T_1 starts to conduct and input voltage is applied to the battery. After that thyristor T_1 will conduct up to a voltage 150 V and it will turned OFF when

$$V_m \sin\beta = 150$$

or

$$311.1269 \times \sin\beta = 150 \quad \text{or} \quad \beta = \sin^{-1}\left(\frac{150}{311.1269}\right) = 90 + (90 - 28.82) = 151.13°$$

As β must be greater than 90°.

The KVL equation of the circuit during conduction of thyristor is

$$V_m \sin\omega t - E = i_o R$$

The average current flow through thyristor is

$$I_o = \frac{3}{2\pi}\int_{\alpha+\frac{\pi}{6}}^{\beta} \frac{V_m \sin\omega t - E}{R} \cdot d\omega t = \frac{3V_m}{2\pi R}[-\cos\omega t]_{\alpha+\frac{\pi}{6}}^{\beta} - \frac{3E}{2\pi R}(\beta - \alpha - 30°)$$

$$= \frac{3V_m}{2\pi R}\left[\cos(\alpha + 30°) - \cos\beta\right] - \frac{3E}{2\pi R}(\beta - \alpha - 30°)$$

After substituting the value of V_m, β and α in the above equation, we obtain

$$I_o = \frac{3V_m}{2\pi R}\left[\cos(\alpha + 30°) - \cos\beta\right] - \frac{3E}{2\pi R}(\beta - \alpha - 30°)$$

$$= \frac{3 \times 311.1269}{2\pi \times 10}\left[\cos(30° + 30°) - \cos(151.13°)\right] - \frac{3 \times 150}{2\pi \times 10}(151.13° - 30° - 30°)$$

$$= 9.0554 \text{ A}$$

The charging voltage and charging current waveforms are depicted in Fig. 8.17.

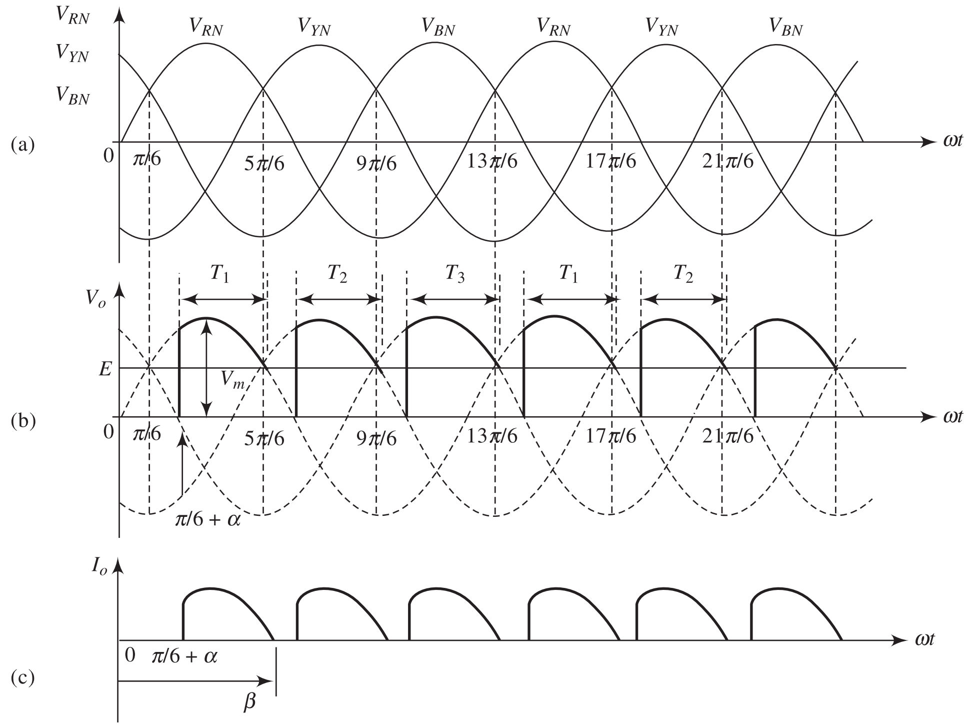


Fig. 8.17 *Charging voltage and charging current waveforms*

8.3.3 Three-Phase Half-Wave Controlled Rectifier With *RL* Load and Free Wheeling Diode

Figure 8.18 shows a three-phase half-wave controlled rectifier with *RL* load and free wheeling diode D_F. When the load inductance L is very large, the load current I_o is continuous and constant. If the firing angle $\alpha < 30°$, each thyristor conducts for 120° duration and free wheeling diode does not come in the conduction. When the firing angle $\alpha > 30°$, each thyristor conducts for less than 120° duration and free wheeling diode comes into conduction.

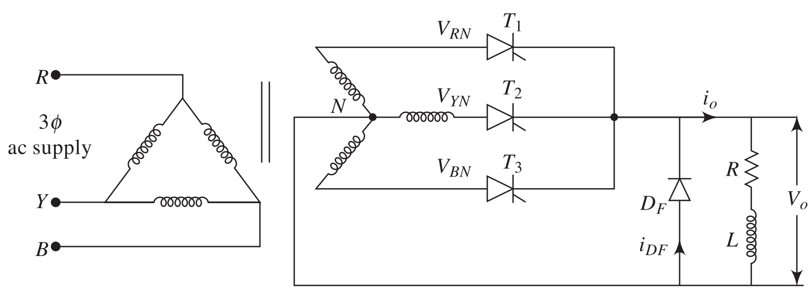


Fig. 8.18 *Three-phase half-wave controlled rectifier with RL load and free wheeling diode*

Assume the firing angle is $\alpha = 60°$. When the firing pulse is applied to thyristor T_1, thyristor T_1 starts conduction at $\omega t = 30° + \alpha = 90°$ and continuously conducts up to $\omega t = \pi = 180°$. At $\omega t = \pi$, the R phase voltage V_{RN} becomes zero and tends to negative. Subsequently thyristor T_1 will be turned OFF and freewheeling diode D_F will be forward biased and starts conducting. Therefore, the freewheeling diode conducts from $\omega t = \pi$ to $\omega t = 150° + \alpha$ (210°) at which thyristor T_2 is turned ON. In the same way, when V_{YN} become negative at $\omega t = \pi + \frac{2\pi}{3} = \frac{5\pi}{3}$, the freewheeling diode again conducts. Figure 8.19 shows the voltage and current waveforms of a three-phase half-wave controlled rectifier with RL load and free wheeling diode D_F. It is clear from Fig. 8.19 that each thyristor conducts for $150° - \alpha$ and the free wheeling diode conducts for $\alpha - 30°$ duration.

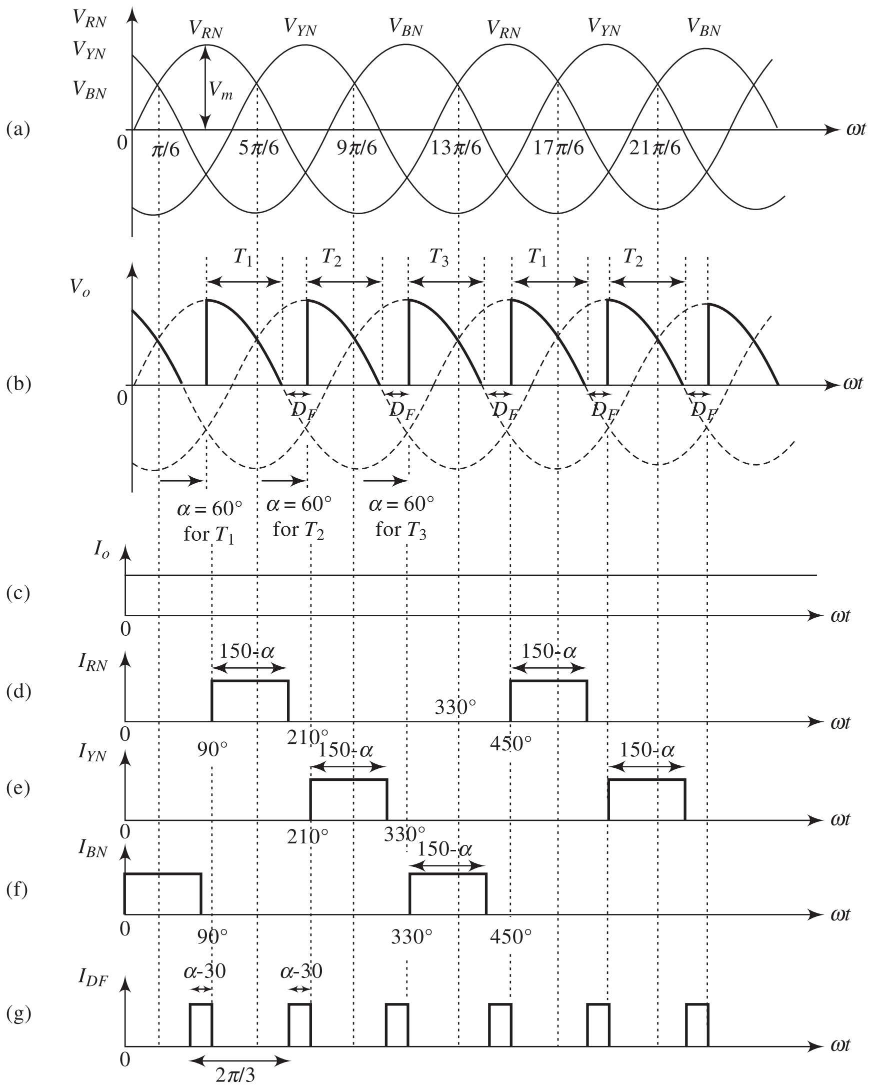


Fig. 8.19 *Voltage and current waveforms of three-phase half-controlled converter with RL load free wheeling diode at $\alpha = 60°$*

The average dc output voltage is

$$V_{\text{av}} = V_o = \frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi/3}\left[-\cos\omega t\right]_{\frac{\pi}{6}+\alpha}^{\pi}$$

$$= \frac{3V_m}{2\pi}\left[1+\cos\left(\alpha+\frac{\pi}{6}\right)\right]$$

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} V_m^2 \sin^2\omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\pi} \frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^2}{4\pi}\left(\omega t - \frac{1}{2}\sin 2\omega t\right)_{\frac{\pi}{6}+\alpha}^{\pi}\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^2}{4\pi}\left(\pi - \frac{\pi}{6} - \alpha - \frac{1}{2}\sin 2\pi + \frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{3V_m^2}{4\pi}\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}} = \frac{\sqrt{3}V_m}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{2\pi}{6}+2\alpha\right)\right)\right]^{\frac{1}{2}}$$

Thyristor average current is

$$I_{T\text{-average}} = \frac{\frac{5\pi}{6}-\alpha}{2\pi} I_o = \frac{I_o}{2\pi}\left[\frac{5\pi}{6}-\alpha\right]$$

Thyristor rms current is

$$I_{T\text{-rms}} = \left[\frac{\frac{5\pi}{6}-\alpha}{2\pi} I_o^2\right]^{\frac{1}{2}} = I_o\left[\frac{1}{2\pi}\left(\frac{5\pi}{6}-\alpha\right)\right]^{\frac{1}{2}}$$

Freewheeling diode average current is

$$I_{\text{DF-average}} = \frac{\alpha-\frac{\pi}{6}}{2\pi/3} I_o = \frac{3I_o}{2\pi}\left[\alpha-\frac{\pi}{6}\right]$$

Freewheeling diode rms current is

$$I_{\text{DF-rms}} = \left[\frac{\alpha-\frac{\pi}{6}}{2\pi/3} I_o^2\right]^{\frac{1}{2}} = I_o\left[\frac{3}{2\pi}\left(\alpha-\frac{\pi}{6}\right)\right]^{\frac{1}{2}}$$

Operation of three-phase half-wave controlled rectifier with RL load and free wheeling diode when α = 60° Assume that the firing angle of thyristors is $\alpha = 60°$. The voltage and current waveforms for this converter is depicted in Fig. 8.19. It is clear from Fig. 8.19 that each thyristor conducts for $150° - \alpha$ duration. Hence thyristor T_1 conducts from $\omega t = 30° + \alpha$ (90°) to $\omega t = 90° + (150° - \alpha) = 240° - 60° = 180°$, free wheeling diode conducts for $\alpha - 30 = 60 - 30 = 30°$ duration from $\omega t = 180°$ to $\omega t = 210°$. Thyristor T_2 starts conduction from $\omega t = 150° + \alpha$ (210°) to $\omega t = 300°$. Again free wheeling diode conducts for $\alpha - 30 = 60 - 30 = 30°$ duration from $\omega t = 300°$ to $\omega t = 330°$. After that T_3 conducts from $\omega t = 270° + \alpha$ (330°) to $\omega t = 410°$.

Example 8.7 A three-phase three pulse controlled rectifier with free wheeling diode D_F is fed from a 3ϕ, 400 V, 50 Hz ac supply and it is connected with a constant current load of 90 A at firing angle of 45°. Determine (a) the dc output voltage, (b) rms output voltage, (c) the average and rms current of thyristors and (d) the average and rms current of free-wheeling diode.

Solution

Given: Phase voltage is $V = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94$ V

$$\text{Maximum phase voltage } V_m = \sqrt{2}V = \sqrt{2} \times 230.94 = 326.598 \text{ V}$$

(a) At firing angle α, the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\pi} V_m \sin \omega t \cdot d(\omega t) = \frac{V_m}{2\pi/3}\left[-\cos \omega t\right]_{\frac{\pi}{6}+\alpha}^{\pi} \quad \text{as } \alpha = 45°$$

$$= \frac{3V_m}{2\pi}\left[1 + \cos\left(\alpha + \frac{\pi}{6}\right)\right] = \frac{3 \times 326.598}{2\pi}\left[1 + \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)\right] = 196.398 \text{ V}$$

(b) The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/3} \int_{\frac{\pi}{6}+\alpha}^{\pi} V_m^2 \sin^2 \omega t \cdot d\omega t\right]^{\frac{1}{2}} = \frac{\sqrt{3}V_m}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - \alpha + \frac{1}{2}\sin\left(\frac{2\pi}{6} + 2\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{3} \times 326.598}{2\sqrt{\pi}}\left[\left(\frac{5\pi}{6} - \frac{\pi}{4} + \frac{1}{2}\sin\left(\frac{2\pi}{6} + 2 \times \frac{\pi}{4}\right)\right)\right]^{\frac{1}{2}} = 230.285 \text{ V}$$

(c) Thyristor average current is

$$I_{T\text{-average}} = \frac{\frac{5\pi}{6} - \alpha}{2\pi} I_o = \frac{I_o}{2\pi}\left[\frac{5\pi}{6} - \alpha\right] = \frac{90}{2\pi}\left[\frac{5\pi}{6} - \frac{\pi}{4}\right] = 26.25 \text{ A}$$

Thyristor rms current is

$$I_{T\text{-rms}} = \left[\frac{\frac{5\pi}{6} - \alpha}{2\pi} I_o^2\right]^{\frac{1}{2}} = I_o\left[\frac{1}{2\pi}\left(\frac{5\pi}{6} - \alpha\right)\right]^{\frac{1}{2}} = 90\left[\frac{1}{2\pi}\left(\frac{5\pi}{6} - \frac{\pi}{4}\right)\right]^{\frac{1}{2}} = 48.60 \text{ A}$$

(d) Freewheeling diode average current is

$$I_{\text{DF-average}} = \frac{\alpha - \frac{\pi}{6}}{2\pi/3} I_o = \frac{3I_o}{2\pi}\left[\alpha - \frac{\pi}{6}\right] = \frac{3 \times 90}{2\pi}\left[\frac{\pi}{4} - \frac{\pi}{6}\right] = 11.259 \text{ A}$$

Freewheeling diode rms current is

$$I_{\text{DF-rms}} = \left[\frac{\alpha - \frac{\pi}{6}}{2\pi/3} I_o^2\right]^{\frac{1}{2}} = I_o\left[\frac{3}{2\pi}\left(\alpha - \frac{\pi}{6}\right)\right]^{\frac{1}{2}} = 90\left[\frac{3}{2\pi}\left(\frac{\pi}{4} - \frac{\pi}{6}\right)\right]^{\frac{1}{2}} = 31.83 \text{ A}$$

8.4 SIX-PULSE OR SIX-PHASE HALF-WAVE CONTROLLED RECTIFIER

A three-phase converter with higher n umber of pulses can generate large output voltage with least ripple current. Therefore, six-pulse and 12-pulse converters are developed. It is also possible to develop 18 pulse and twenty four pulse ac-to-dc converter. But six-pulse converters are most widely used for different industrial applications as the six-pulse converters have the following advantages over three pulse converters:

1. In six-pulse converter, commutation is very easy.
2. Distortion on the ac supply is reduced drastically due to the reduction in lower order harmonics.
3. The value of inductance which is connected in series is considerably low.

The six-pulse converters can be implemented in the following ways:

1. Six-phase half-wave controlled rectifier circuit.
2. Six-pulse mid-point converter with interphase transformers.
3. Six-pulse bridge converter.

Figure 8.20 shows a six-pulse or six-phase half-wave controlled rectifier circuit. The six-phase voltages can be obtained in the secondary by using a centre-tapped arrangement on a star connected

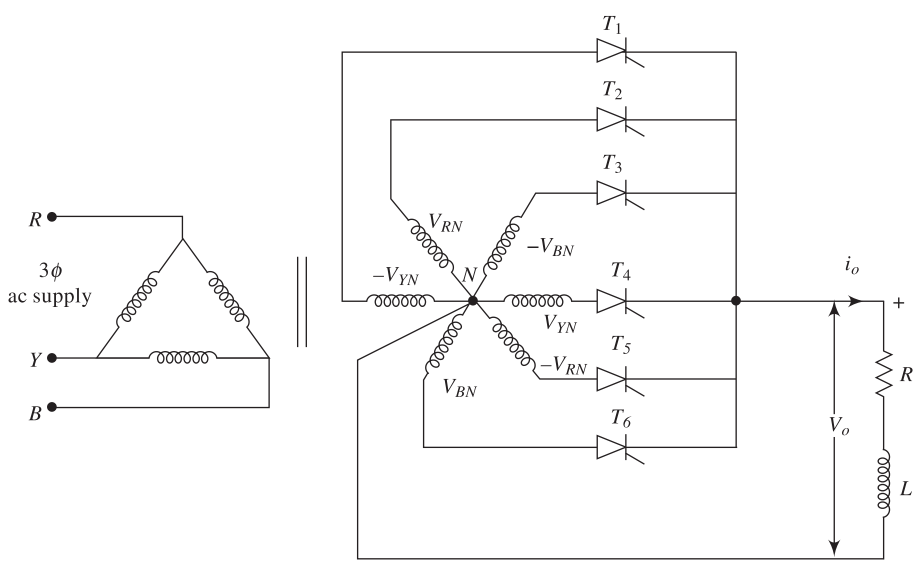


Fig. 8.20 *Six-phase half-wave controlled rectifier*

three-phase winding and the vector diagram of six-phase voltages is shown in Fig. 8.21. There are six thyristors in a six-phase controlled rectifier. When a particular phase voltage is higher than other phases, thyristor on the particular phase conducts. Figure 8.22 shows the output voltage and current waveforms.

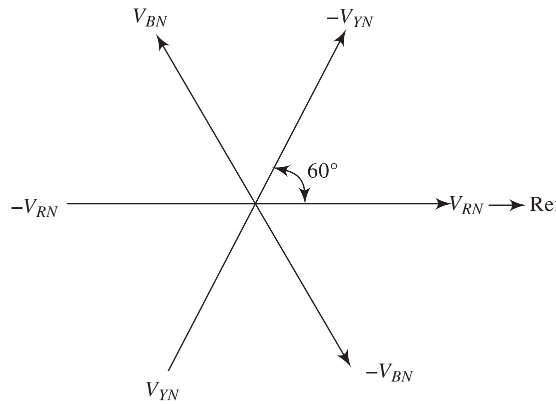


Fig. 8.21 *Vector diagram of six-phase voltages*

When the inductance is very large, the load current I_o is continuous and constant. It is clear from Fig. 8.22 that each thyristor conducts for $\pi/3$ or 60° duration and current flows through one thyristor at a time. As a result average current is low but the ratio between maximum current to average current in the thyristors is high. Consequently the utilisation of transformer secondary is poor. The dc currents in the secondary of the six-phase star rectifier can be cancelled in the secondary windings and core saturation is not encountered.

Figure 8.23 shows the output voltage waveforms of six-phase half-wave controlled rectifier at α = 30° and α = 60°.

For this circuit, the average dc output voltage is

$$V_{\text{av}} = V_{\text{dc}} = V_o = \frac{1}{2\pi/6}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi/6}\left[-\cos\omega t\right]_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}$$

$$= \frac{V_m}{2\pi/6}\left[-\cos\left(\frac{2\pi}{3}+\alpha\right)+\cos\left(\frac{\pi}{3}+\alpha\right)\right] = \frac{3V_m}{\pi}\cos\alpha$$

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{1}{2\pi/6}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_m^2 \sin^2\omega t \cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi/6}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} \frac{1}{2}(1-\cos 2\omega t)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{2\pi/6}\left(\frac{1}{2}\omega t - \frac{1}{2}\sin 2\omega t\right)_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}\right]^{\frac{1}{2}}$$

$$= \left[\frac{V_m^2}{2\pi/6}\left(\frac{2\pi}{6} - \frac{\pi}{6} - \frac{1}{2}\sin 2\left(\frac{2\pi}{3}+\alpha\right) + \frac{1}{2}\sin 2\left(\frac{\pi}{3}+\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{6}{2\pi}\left(\frac{\pi}{6} - \frac{1}{2}\sin 2\left(\frac{2\pi}{3}+\alpha\right) + \frac{1}{2}\sin 2\left(\frac{\pi}{3}+\alpha\right)\right)\right]^{\frac{1}{2}}$$

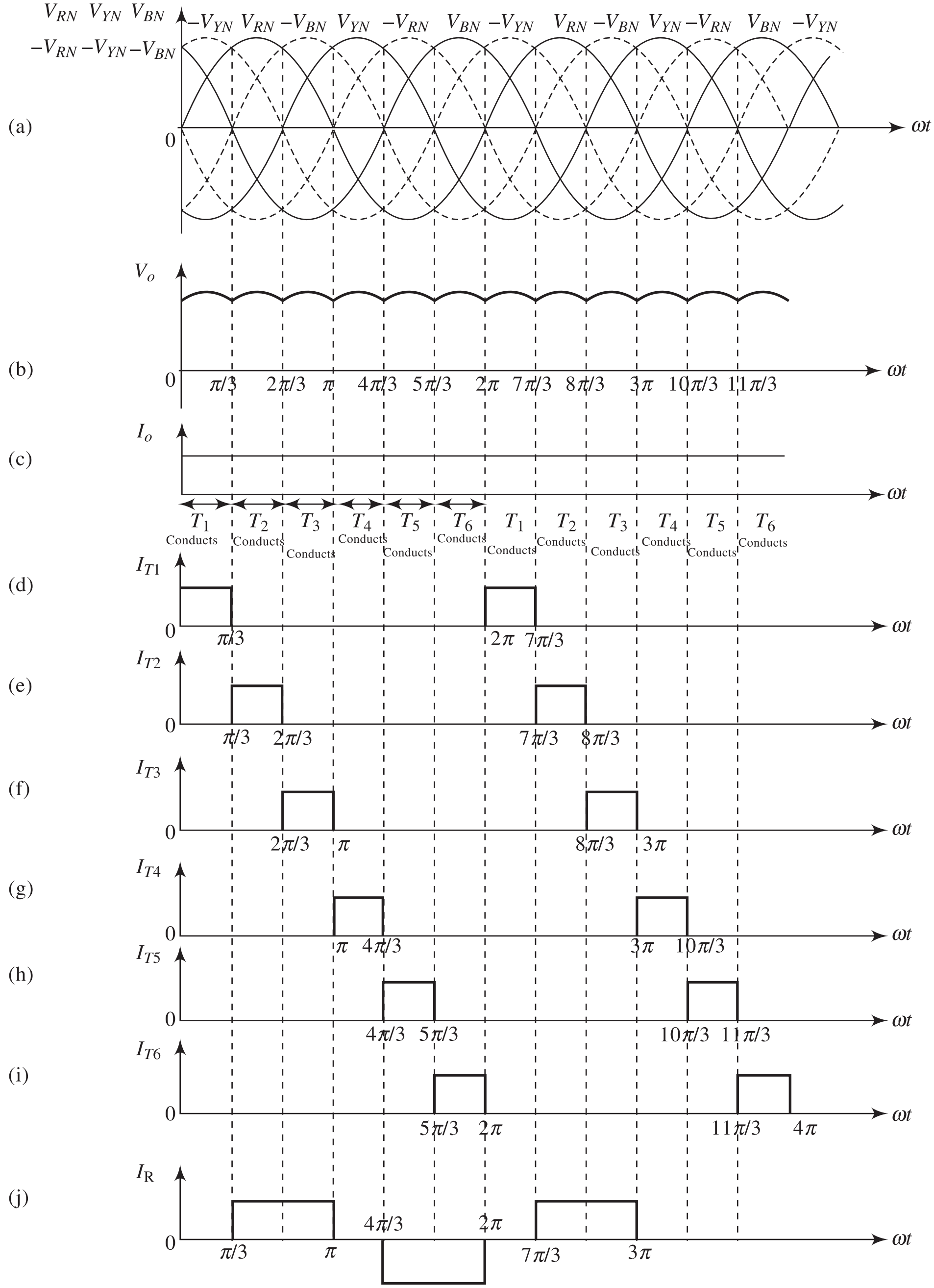


Fig. 8.22 *Voltage and current waveforms of six-phase half-wave controlled rectifier at $\alpha = 0°$*

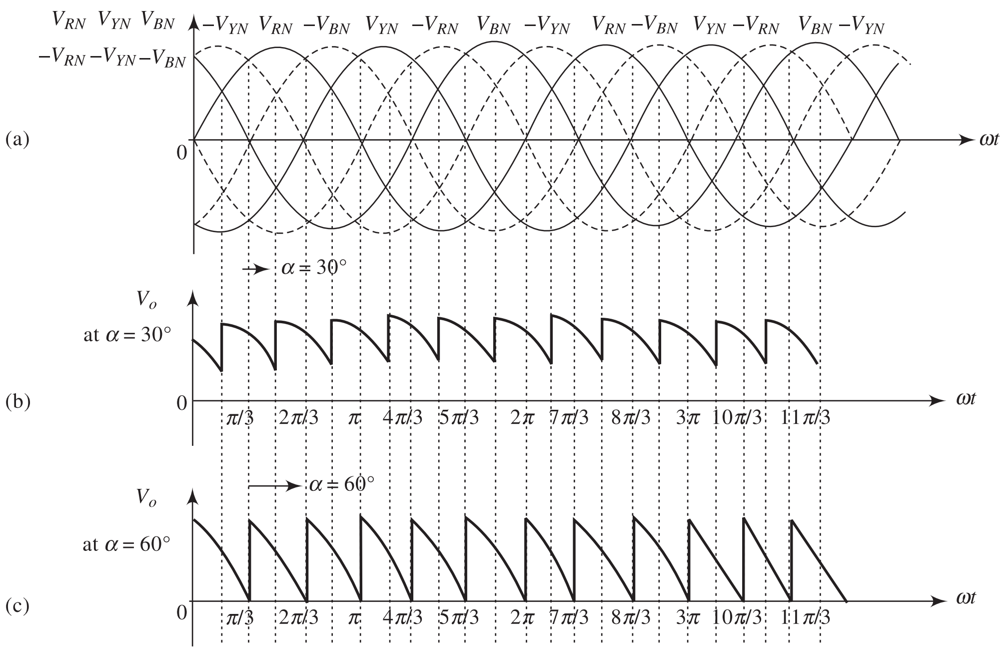


Fig. 8.23 *Output voltage waveforms of six-phase half-wave controlled rectifier at α = 30° and α = 60°*

Form factor is $FF = \dfrac{V_{\text{rms}}}{V_{\text{dc}}}$

The ripple factor is $RF = \sqrt{FF^2 - 1}$

The average output current is

$$I_{\text{av}} = I_o = \frac{V_o}{R} = \frac{3V_m}{\pi R}\cos\alpha$$

The rms output current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{V_m}{R}\left[\frac{6}{2\pi}\left(\frac{\pi}{6} - \frac{1}{2}\sin 2\left(\frac{2\pi}{3} + \alpha\right) + \frac{1}{2}\sin 2\left(\frac{\pi}{3} + \alpha\right)\right)\right]^{\frac{1}{2}}$$

The output power of rectifier is $P_{\text{dc}} = V_{\text{av}} I_{\text{av}} = V_o I_o$

The input ac power to transformer secondary is $P_{\text{ac}} = I_{\text{rms}}^2 R$

Efficiency of rectifier is $\eta = \dfrac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\%$

It is clear from Figs. 8.22 and 8.23 that there are six-pulses in the output voltage for a complete cycle of input voltage. Therefore, the ripple frequency is six times of fundamental frequency (f_o = 6f). The minimum order of the harmonics present is six times the supply frequency. Subsequently the value of required smoothing inductance is smaller compared to the required inductance of three-pulse converter. When the load is resistive, this converter operates in two different modes namely continuous conduction mode and discontinuous conduction mode depending on the value of firing angle α. The ranges of firing angle for different modes are

1. Continuous conduction mode $0 \le \alpha \le \frac{\pi}{3}$
2. Discontinuous conduction mode $\frac{\pi}{3} \le \alpha \le \frac{2\pi}{3}$

Example 8.8 A six-phase half-wave controlled rectifier is fed from a 3ϕ, 440 V, 50 Hz ac supply and it is connected with a R load of 5 Ω. If the firing angle of thyristor is 30°, determine (a) dc output voltage, (b) rms value of output voltage, (c) form factor, (d) ripple factor, (e) average output current, (f) rms load current, (g) dc output power, (h) ac power supplied and (i) rectification efficiency.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.04$ and $R = 5\ \Omega$

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2} \times 254.04\text{ V} = 359.21\text{ V}$

(a) The average dc output voltage is

$$V_{av} = V_{dc} = V_o = \frac{1}{2\pi/6}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_m \sin\omega t \cdot d(\omega t) = \frac{V_m}{2\pi/6}\left[-\cos\omega t\right]_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}$$

$$= \frac{3V_m}{\pi}\cos\alpha = \frac{3 \times 359.21}{\pi}\cos 30 = 297.21\text{ V}$$

(b) The rms value of output voltage is

$$V_{rms} = \left[\frac{1}{2\pi/6}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_m^2 \sin^2\omega t \cdot d\omega t\right]^{\frac{1}{2}} = V_m\left[\frac{6}{2\pi}\left(\frac{\pi}{6} - \frac{1}{2}\sin 2\left(\frac{2\pi}{3}+\alpha\right) + \frac{1}{2}\sin 2\left(\frac{\pi}{3}+\alpha\right)\right)\right]^{\frac{1}{2}}$$

$$= 359.21\left[\frac{6}{2\pi}\left(\frac{\pi}{6} - \frac{1}{2}\sin 2\left(\frac{2\pi}{3}+\frac{\pi}{6}\right) + \frac{1}{2}\sin 2\left(\frac{\pi}{3}+\frac{\pi}{6}\right)\right)\right]^{\frac{1}{2}} = 343.359\text{ V}$$

(c) Form factor is $FF = \frac{V_{rms}}{V_{dc}} = \frac{343.359}{297.21} = 1.1552$

(d) The ripple factor is $RF = \sqrt{FF^2 - 1} = \sqrt{1.1552^2 - 1} = 0.5873$

(e) The average output current is

$$I_{av} = I_o = \frac{V_o}{R} = \frac{297.21}{5} = 59.442\text{ A}$$

(f) The rms output current is

$$I_{rms} = \frac{V_{rms}}{R} = \frac{343.359}{5} = 68.67\text{ A}$$

(g) The output power of rectifier is $P_{dc} = V_{av}I_{av} = V_oI_o = 297.21 \times 59.442 = 17666.75$ Watt

(h) The input ac power to transformer secondary is $P_{ac} = I_{rms}^2 R = 68.67^2 \times 5 = 23577.84$ Watt

(i) Efficiency of rectifier is $\eta = \dfrac{P_{dc}}{P_{ac}} \times 100\% = \dfrac{17666.75}{23577.84} \times 100\% = 74.929\%$

Example 8.9 A six-phase half-wave controlled rectifier is fed from a 3ϕ ac supply and it is connected with a R load with firing angle 45°. Determine (a) form factor and (b) voltage ripple factor.

Solution

Assume maximum phase voltage is V_m.
The average dc output voltage is

$$V_{av} = V_{dc} = V_o = \frac{1}{2\pi/6} \int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_m \sin \omega t \cdot d(\omega t) = \frac{V_m}{2\pi/6} [-\cos \omega t]_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}$$

$$= \frac{3V_m}{\pi} \cos \alpha = \frac{3 \times V_m}{\pi} \cos 45 = 0.6755 V_m$$

The rms value of output voltage is

$$V_{rms} = \left[\frac{1}{2\pi/6} \int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha} V_m^2 \sin^2 \omega t \cdot d\omega t \right]^{\frac{1}{2}} = V_m \left[\frac{6}{2\pi} \left(\frac{\pi}{6} - \frac{1}{2} \sin 2\left(\frac{2\pi}{3} + \alpha \right) + \frac{1}{2} \sin 2\left(\frac{\pi}{3} + \alpha \right) \right) \right]^{\frac{1}{2}}$$

$$= V_m \left[\frac{6}{2\pi} \left(\frac{\pi}{6} - \frac{1}{2} \sin 2\left(\frac{2\pi}{3} + \frac{\pi}{4} \right) + \frac{1}{2} \sin 2\left(\frac{\pi}{3} + \frac{\pi}{4} \right) \right) \right]^{\frac{1}{2}} = 0.7072 V_m$$

(a) Form factor is $FF = \dfrac{V_{rms}}{V_{dc}} = \dfrac{0.7072 V_m}{0.6755 V_m} = 1.0469$

(b) The ripple factor is $RF = \sqrt{FF^2 - 1} = \sqrt{1.0469^2 - 1} = 0.3098$

8.5 MULTI-PHASE CONTROLLED RECTIFIER

In a three-phase half-wave controlled rectifier, each phase conducts for 120° or $\dfrac{2\pi}{3}$ radian of a cycle of 360°(2π radians). But in a six-phase half-wave controlled rectifier, each phase conducts for 60° or $\dfrac{2\pi}{6}$ radian of a cycle of 360°(2π radians). Consequently in a m-phase half-wave controlled rectifier, each phase as well as each thyristor would conduct for $\dfrac{2\pi}{m}$ radian and the number of output voltage pulses p will be equal to number of phases m. Figure 8.24 shows the output voltage wave from for m-phase half-wave controlled rectifier where a thyristor conducts from $-\left(\dfrac{\pi}{m} - \alpha\right)$ to $\dfrac{\pi}{m} + \alpha$ or from $-\left(\dfrac{\pi}{p} - \alpha\right)$ to $\dfrac{\pi}{p} + \alpha$.

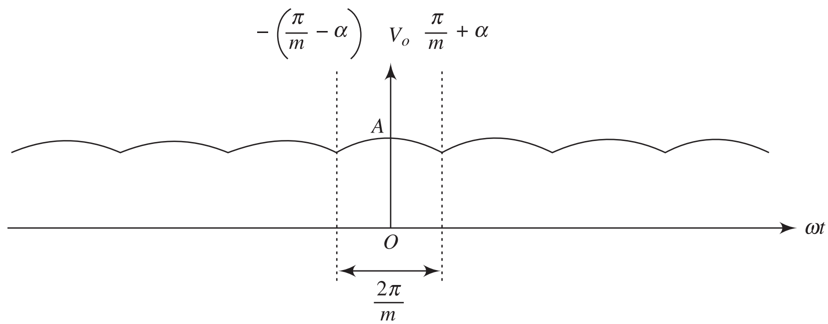


Fig. 8.24 *Output voltage wave from for m-phase half-wave diode rectifier*

Assume that the instantaneous phase voltage is $v = V_m \cos\omega t = \sqrt{2}V\cos\omega t$
Average output voltage is

$$V_o = \frac{1}{2\pi/m}\int_{-\left(\frac{\pi}{m}-\alpha\right)}^{\left(\frac{\pi}{m}+\alpha\right)} V_m \cos\omega t \cdot d(\omega t)$$

$$= V_m \frac{m}{\pi}\sin\frac{\pi}{m}\cos\alpha$$

For a six-pulse converter, average output voltage is

$$V_{6\_\mathrm{av}} = V_m \frac{6}{\pi}\sin\frac{\pi}{6}\cos\alpha = \frac{3V_m}{\pi}\cos\alpha$$

In case of a twelve-pulse converter, average output voltage is

$$V_{12\_\mathrm{av}} = V_m \frac{12}{\pi}\sin\frac{\pi}{12}\cos\alpha = 0.98816V_m\cos\alpha$$

The rms output voltage is

$$V_{\mathrm{rms}} = \left[\frac{1}{2\pi/m}\int_{-\left(\frac{\pi}{m}-\alpha\right)}^{\frac{\pi}{m}+\alpha} (V_m\cos\omega t)d(\omega t)\right]^{\frac{1}{2}}$$

$$= \left[\frac{m}{2\pi}\frac{V_m^2}{2}\int_{-\left(\frac{\pi}{m}-\alpha\right)}^{\frac{\pi}{m}+\alpha} (1+\cos 2\omega t)d(\omega t)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{m}{2\pi}\left(\frac{\pi}{m}+\frac{1}{2}\sin\frac{2\pi}{m}\cos 2\alpha\right)\right]^{\frac{1}{2}}$$

For a six-pulse converter, rms output voltage is

$$V_{6\_\text{rms}} = V_m\left[\frac{6}{2\pi}\left(\frac{\pi}{6}+\frac{1}{2}\sin\frac{2\pi}{6}\cos 2\alpha\right)\right]^{\frac{1}{2}}$$

In case of a twelve-pulse converter, rms output voltage is

$$V_{12\_\text{rms}} = V_m\left[\frac{12}{2\pi}\left(\frac{\pi}{12}+\frac{1}{2}\sin\frac{2\pi}{12}\cos 2\alpha\right)\right]^{\frac{1}{2}}$$

Maximum value of load current is

$$I_m = \frac{V_m}{R}$$

Average value of thyristor current is

$$I_{T\text{av}} = \frac{1}{2\pi}\int_{-\left(\frac{\pi}{m}-\alpha\right)}^{\frac{\pi}{m}+\alpha} I_m \cos\omega t \cdot d(\omega t) = \frac{I_m}{\pi}\sin\frac{\pi}{m}\cos\alpha$$

The rms value of thyristor current is

$$I_{T\text{rms}} = \left[\frac{1}{2\pi}\int_{-\left(\frac{\pi}{m}-\alpha\right)}^{\frac{\pi}{m}+\alpha}(I_m\cos\omega t)d(\omega t)\right]^{\frac{1}{2}} = I_m\left[\frac{1}{2\pi}\left(\frac{\pi}{m}+\frac{1}{2}\sin\frac{2\pi}{m}\cos 2\alpha\right)\right]^{\frac{1}{2}}$$

Example 8.10 A three-phase step-down delta-star transformer with per phase turn ratio 5 is fed from a 3ϕ, 1000 V, 50 Hz ac supply and it is connected with six-phase half-wave controlled rectifier . When R is equal to 10 Ω and $\alpha = 30°$, determine (a) maximum load current, (b) dc output voltage, (c) rms value of output voltage, (d) average output current, (e) rms load current, (f) power delivered to load, (g) average and rms value of thyristor current.

Solution

Given: Phase voltage secondary voltage is $V = \frac{1000}{5} = 200$ V and $R = 10\ \Omega$ and $\alpha = 30°$

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2}\times 200\text{ V} = 282.84\text{ V}$

(a) Maximum value of load current $I_m = \frac{V_m}{R} = \frac{282.84}{10} = 28.284\text{ A}$

(b) For a six-pulse converter, average output voltage is

$$V_{6\_\text{av}} = V_m\frac{6}{\pi}\sin\frac{\pi}{6}\cos\alpha = \frac{3V_m}{\pi}\cos\alpha = \frac{3\times 282.84}{\pi}\cos 30 = 234.025\text{ V}$$

(c) rms output voltage is

$$V_{6\_\text{rms}} = V_m\left[\frac{6}{2\pi}\left(\frac{\pi}{6}+\frac{1}{2}\sin\frac{2\pi}{6}\cos 2\alpha\right)\right]^{\frac{1}{2}} = 282.84\left[\frac{6}{2\pi}\left(\frac{\pi}{6}+\frac{1}{2}\sin\frac{2\pi}{6}\cos(2\times 30)\right)\right]^{\frac{1}{2}}$$

$$= 237.795\text{ V}$$

(d) The average output current is

$$I_{\text{av}} = I_o = \frac{V_o}{R} = \frac{V_{6\_\text{av}}}{R} = \frac{234.025}{10} = 23.4025 \text{ A}$$

(e) The rms output current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{V_{6\_\text{rms}}}{R} = \frac{237.795}{10} = 23.7795 \text{ A}$$

(f) Power delivered to load is

$$P_{\text{dc}} = V_o I_o = 234.025 \times 23.4025 = 5076.77 \text{ W}$$

(g) Average value of thyristor current is

$$I_{T\text{av}} = \frac{1}{2\pi} \int_{-\left(\frac{\pi}{m}-\alpha\right)}^{\frac{\pi}{m}+\alpha} I_m \cos\omega t \cdot d(\omega t) = \frac{I_m}{\pi} \sin\frac{\pi}{m} \cos\alpha = \frac{I_m}{\pi} \sin\frac{\pi}{6} \cos\alpha \quad \text{as } m = 6$$

$$= \frac{28.284}{\pi} \sin\frac{\pi}{6} \cos 30 \text{ A} = 3.90 \text{ A}$$

The rms value of thyristor current is

$$I_{T\text{rms}} = \left[\frac{1}{2\pi} \int_{-\left(\frac{\pi}{m}-\alpha\right)}^{\frac{\pi}{m}+\alpha} (I_m \cos\omega t) d(\omega t) \right]^{\frac{1}{2}} = I_m \left[\frac{1}{2\pi} \left(\frac{\pi}{m} + \frac{1}{2} \sin\frac{2\pi}{m} \cos 2\alpha \right) \right]^{\frac{1}{2}} \quad \text{as } m = 6$$

$$= I_m \left[\frac{1}{2\pi} \left(\frac{\pi}{6} + \frac{1}{2} \sin\frac{2\pi}{6} \cos 2\alpha \right) \right]^{\frac{1}{2}} = 28.284 \left[\frac{1}{2\pi} \left(\frac{\pi}{6} + \frac{1}{2} \sin\frac{2\pi}{6} \cos(2 \times 30) \right) \right]^{\frac{1}{2}} = 9.707 \text{ A}$$

8.6 THREE-PHASE FULL-CONTROLLED BRIDGE RECTIFIER

The three-phase fully controlled bridge converter is most extensively used power electronic converter in the medium to high power applications. The three-phase converters are preferred whenever high power is required in dc power supply of electrical drives as well as power system. The three-phase fully controlled rectifier can provide controllable dc output voltage in a single unit. Actually the dc output voltage is controlled by controlling the conduction period of each thyristor. Since phase controlled technique is used to control the firing angle of thyristor, this converter is also known as *phase controlled converters* or *line commutated converters*. As thyristors can block voltage in positive as well as negative directions, it is possible to get the reverse polarity dc output voltage and hence the power can feed back to ac supply from dc side. In this operating condition, the converter can be operating in inverter mode.

8.6.1 Three-Phase Full-Controlled Bridge Rectifier With *RL* Load

Figure 8.25 shows a three-phase fully controlled bridge rectifier circuit with *RL* load. In this circuit, thyristors are numbered in order of conduction sequence and each thyristor conducts for $2\pi/3$ (120°) duration. The conduction sequences of thyristors are T_1 T_2, T_2 T_3, T_3 T_4, T_4 T_5, T_5 T_6, and T_6 T_1. The

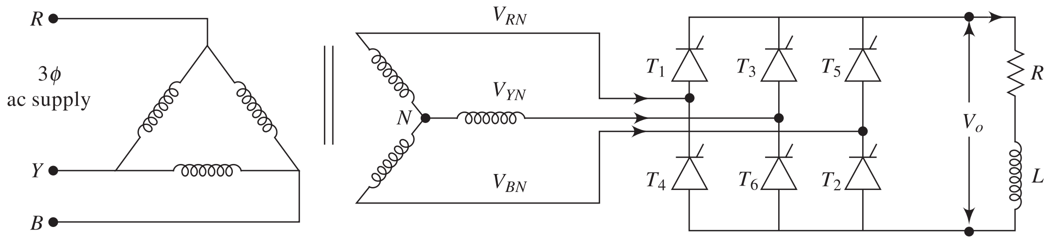


Fig. 8.25 *Three-phase controlled bridge rectifier*

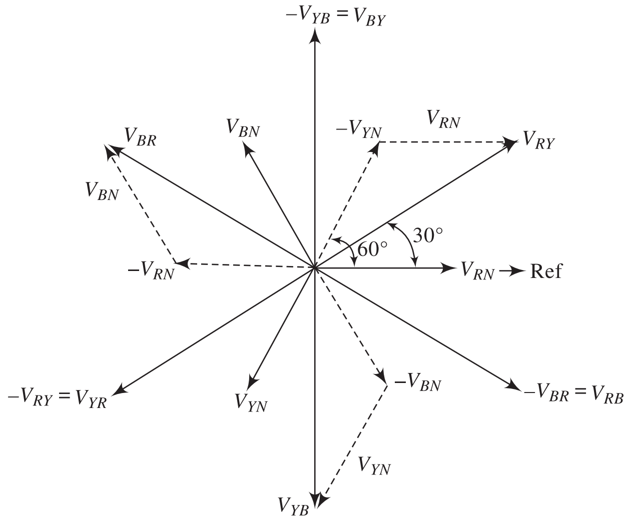


Fig. 8.26 *Vector diagram of phase and line voltages*

vector diagram three-phase voltages and line voltages are shown in Fig. 8.26. The voltage and current waveforms of a three-phase fully controlled bridge rectifier circuit at $\alpha = 0°$ are depicted in Fig. 8.27. During analysis of three-phase full converter, we assume that the combination of star or delta connected primary and secondary windings are symmetrical. Three-phase fully controlled bridge rectifier are extensively used high power applications since transformer utilisation factor is high.

At any operating condition, at least one thyristor from the upper group (T_1, T_3, T_5) and one thyristor from the lower group (T_2, T_4, T_6) must be conduct. It is clear from Fig. 8.27 each thyristor conducts for 120° and the thyristors are fired in the sequence T_1, T_2, T_3, T_4, T_5, T_6, T_1 with 60° interval between each firing. As a result, thyristors on the same phase leg are fired at 180° interval and thyristors on the same phase leg can not conduct simultaneously. In continuous conduction mode of operation, there are six possible conduction mode of operations such as T_1 T_2, T_2 T_3, T_3 T_4, T_4 T_5, T_5 T_6, and T_6 T_1. Each conduction mode has 60° conduction period. Table 8.1 shows the conduction table indicating voltage across different devices. The phasor diagram of line voltages are depicted in Fig. 8.28.

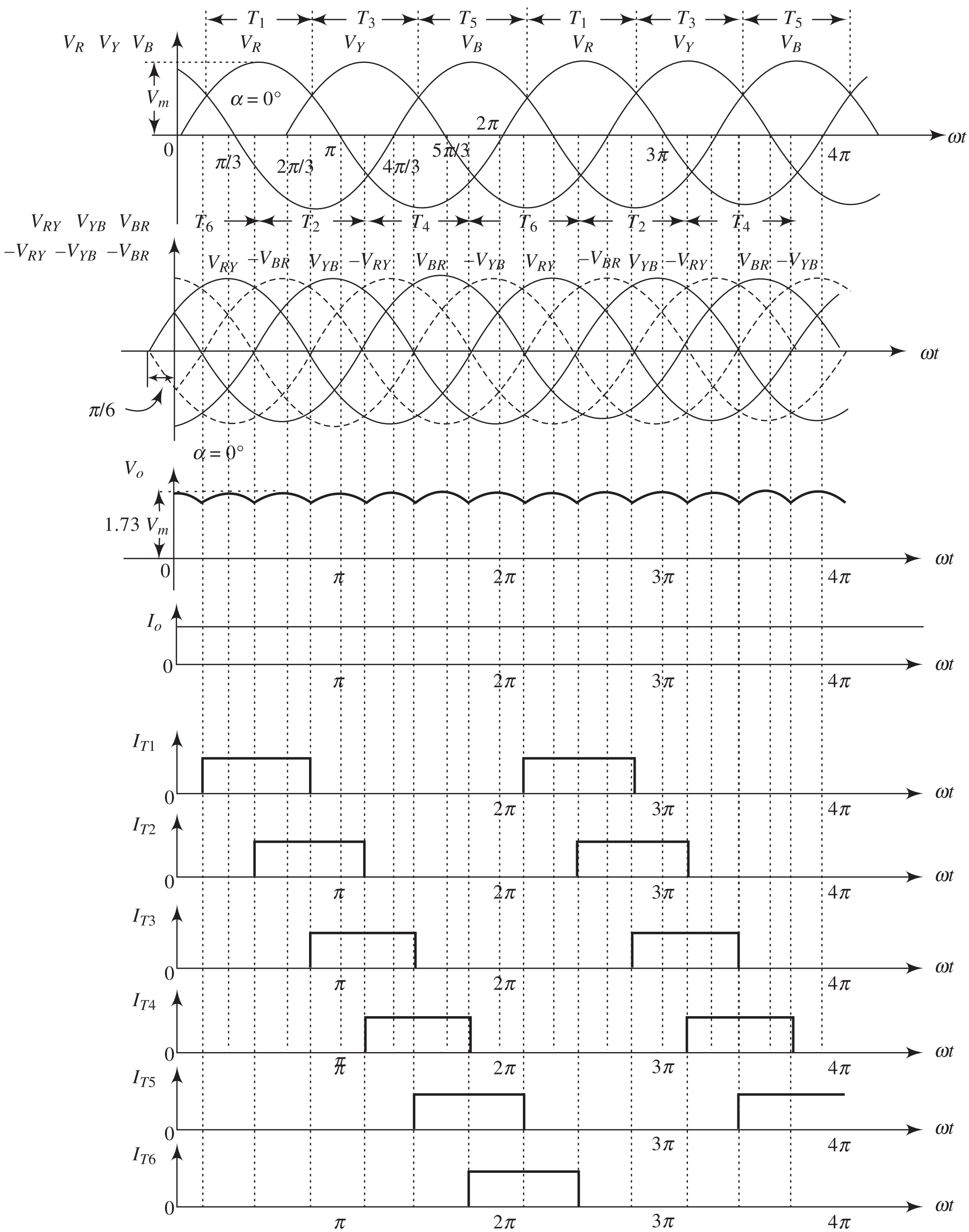


Fig. 8.27 *Voltage and current waveforms of three-phase fully controlled bridge rectifier at firing angle $\alpha = 0°$*

Table 8.1 *Conduction mode of operation of thyristors and voltage across devices*

Voltage across devices	Conduction mode of operation					
	T_1T_2	T_2T_3	T_3T_4	T_4T_5	T_5T_6	T_6T_1
V_{T1}	0	V_{ab}	V_{ab}	V_{ac}	V_{ac}	0
V_{T2}	0	0	V_{ac}	V_{ac}	V_{bc}	V_{bc}
V_{T3}	V_{ba}	0	0	V_{bc}	V_{bc}	V_{ba}
V_{T4}	V_{ca}	V_{ca}	0	0	V_{ba}	V_{ba}
V_{T5}	V_{ca}	V_{cb}	V_{cb}	0	0	V_{ca}
V_{T6}	V_{cb}	V_{cb}	V_{ab}	V_{ab}	0	0
V_0	V_{ac}	V_{bc}	V_{ba}	V_{ca}	V_{cb}	V_{ab}

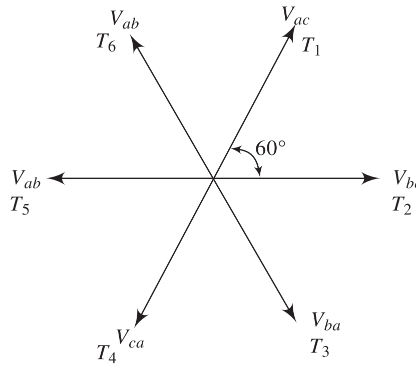


Fig. 8.28 *Phasor diagram of line voltages*

At the end of conduction of T_5 and T_6 thyristors, thyristor T_1 must be fired. When line voltage V_{RY} is greater than other line voltages and firing pulse is applied to thyristor T_1, T_1 will be turned ON and it conducts with the combination thyristor T_6 and it also conducts until T_3 is fired.

Assume that V_m is the peak value of the phase voltage. The instantaneous phase voltages are

$$V_{RN} = V_m \sin \omega t,$$
$$V_{YN} = V_m \sin(\omega t - 120°) \text{ and}$$
$$V_{BN} = V_m \sin(\omega t - 240°).$$

Then the line to line voltages lead the phase voltage by 30° and the instantaneous line voltages are as follows:

$$V_{RY} = \sqrt{3}V_m \sin(\omega t + 30°),$$
$$V_{YB} = \sqrt{3}V_m \sin(\omega t - 90°) \text{ and}$$
$$V_{BR} = \sqrt{3}V_m \sin(\omega t - 210°)$$

The average dc output voltage is two times of the average of the upper part or lower part of the curve. Considering the upper part of the curve, the average dc output voltage for this circuit, is

$$V_{\text{av}} = V_o = 2\frac{1}{2\pi/3}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin\omega t \cdot d(\omega t) = \frac{3V_m}{\pi}\left[-\cos\omega t\right]_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}$$

$$= -\frac{3V_m}{\pi}\left[\cos\left(\frac{5\pi}{6}+\alpha\right) - \cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

$$= -\frac{3V_m}{\pi}\left[2\sin\left(\frac{\frac{5\pi}{6}+\alpha+\frac{\pi}{6}+\alpha}{2}\right)\sin\left(\frac{\frac{5\pi}{6}+\alpha-\frac{\pi}{6}-\alpha}{2}\right)\right]$$

$$= -\frac{3V_m}{\pi}\left[2\sin\left(\frac{\pi}{2}+\alpha\right)\sin\left(\frac{2\pi}{6}\right)\right] = -\frac{3V_m}{\pi}(-2\cos\alpha)\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{\pi}V_m\cos\alpha$$

or $$V_{\text{av}} = V_o = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{\text{RY}}\cdot d(\omega t) = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}V_m \sin\left(\omega t+\frac{\pi}{6}\right)\cdot d(\omega t) = \frac{3\sqrt{3}}{\pi}V_m\cos\alpha$$

The rms value of output voltage is

$$V_{\text{rms}} = \left[\frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{\text{RY}}^2\cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3V_m^2\sin^2\left(\omega t+\frac{\pi}{6}\right)\cdot d\omega t\right]^{\frac{1}{2}}$$

$$= \left[\frac{9V_m^2}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha}\frac{1}{2}\left\{1-\cos 2\left(\omega t+\frac{\pi}{6}\right)\right\}\cdot d\omega t\right]^{\frac{1}{2}} = \left[\frac{9V_m^2}{\pi}\left\{\frac{1}{2}\omega t-\frac{1}{4}\sin 2\left(\omega t+\frac{\pi}{6}\right)\right\}_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha}\right]^{\frac{1}{2}}$$

$$= \left[\frac{9V_m^2}{\pi}\frac{1}{2}\left(\frac{\pi}{2}+\alpha-\frac{\pi}{6}-\alpha\right)-\frac{9V_m^2}{\pi}\frac{1}{4}\left(\sin 2\left(\frac{\pi}{2}+\alpha+\frac{\pi}{6}\right)-\sin 2\left(\frac{\pi}{6}+\alpha+\frac{\pi}{6}\right)\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{9V_m^2}{\pi}\frac{1}{2}\frac{\pi}{3}-\frac{9V_m^2}{\pi}\frac{1}{4}\left(\sin(240+2\alpha)-\sin(120+2\alpha)\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{9V_m^2}{\pi}\frac{1}{2}\frac{\pi}{3}-\frac{9V_m^2}{\pi}\frac{1}{4}\left(2\cos\left(\frac{240+2\alpha+120+2\alpha}{2}\right)\sin\left(\frac{240+2\alpha-120-2\alpha}{2}\right)\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{9V_m^2}{\pi}\frac{1}{2}\frac{\pi}{3}-\frac{9V_m^2}{\pi}\frac{1}{4}\left(2\cos(180+2\alpha)\sin 60\right)\right]^{\frac{1}{2}}$$

$$= \left[\frac{9V_m^2}{\pi}\frac{1}{2}\frac{\pi}{3}+\frac{9V_m^2}{\pi}\frac{1}{4}2\cos 2\alpha\frac{\sqrt{3}}{2}\right]^{\frac{1}{2}}$$

$$= \sqrt{3}V_m\left[\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

Form factor is $FF = \dfrac{V_{\text{rms}}}{V_{\text{av}}}$

Ripple factor is $RF = \sqrt{FF^2-1}$

The average output current is

$$I_{\text{av}} = I_o = \frac{V_o}{R}$$

The dc output power is

$$P_{\text{dc}} = V_{\text{dc}}I_{\text{dc}} = V_oI_o$$

The rms value of load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

The ac power output is the transformer secondary is

$$P_{\text{ac}} = I_{\text{rms}}^2 R$$

Rectification efficiency is

$$\eta = \frac{P_{\text{dc}}}{P_{\text{ac}}} \times 100\%$$

It is clear from Fig. 8.27 that the source current for phase A is I_o for 120° for every 180° duration. Since the current I_o is constant, the rms value of source current is

$$I_R = \left[\frac{120°}{180°} \times I_o^2\right]^{\frac{1}{2}} = \sqrt{\frac{2}{3}} I_o$$

As each thyristor for 120° for every 360° duration, the rms value of thyristor current is

$$I_{\text{Thyristor}} = \left[\frac{120°}{360°} \times I_o^2\right]^{\frac{1}{2}} = \sqrt{\frac{1}{3}} I_o$$

The output voltage waveform of a three-phase bridge converter at firing angle $\alpha = 60°$ and $\alpha = 90°$ are depicted in Figs. 8.29 and 8.30 respectively. The output voltage waveforms of three-phase fully controlled bridge rectifier with *RL* load at firing angle $\alpha = 120°$ is shown in Fig. 8.31.

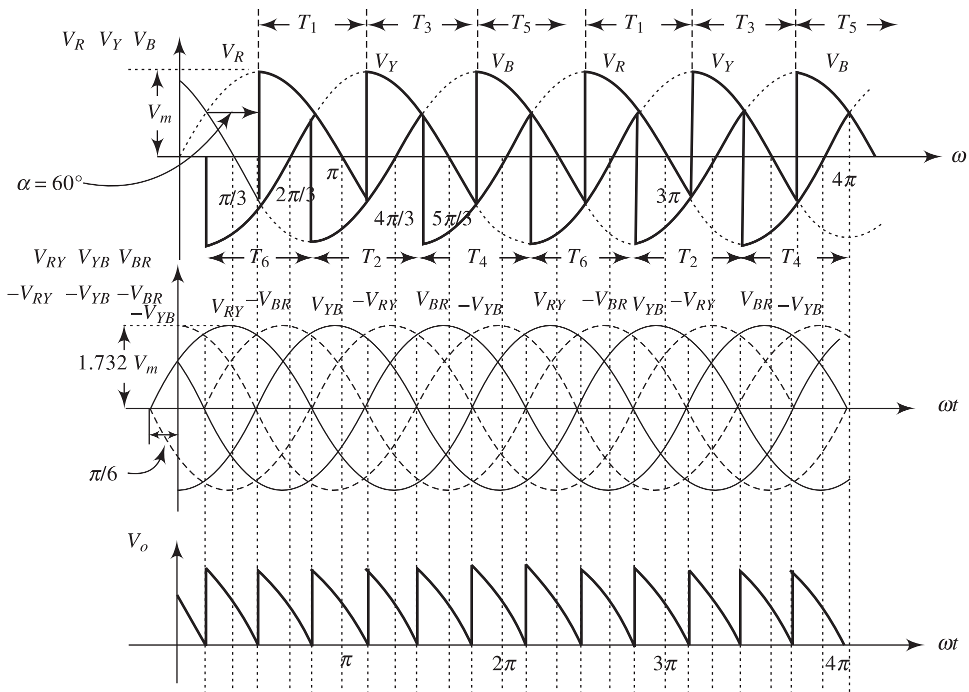


Fig. 8.29 *Output voltage waveforms of three-phase fully controlled bridge rectifier with RL load at firing angle α = 60°*

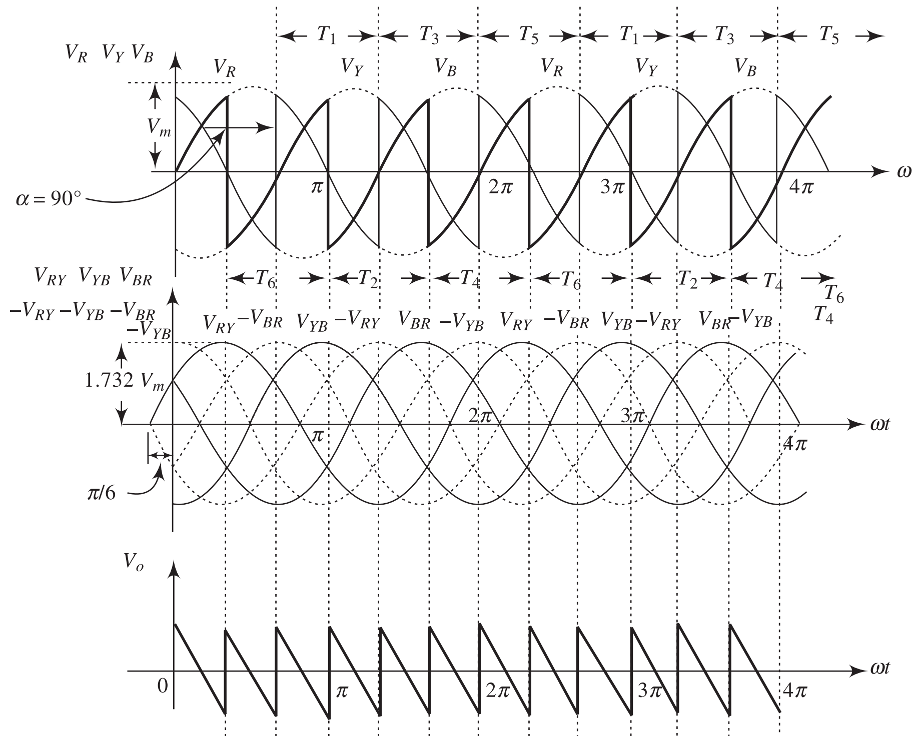


Fig. 8.30 *Output voltage waveforms of three-phase fully controlled bridge rectifier with RL load at firing angle α = 90°*

From the above waveforms, can state that

1. The output voltage waveforms for $\alpha = 0°$, $\alpha = 30°$ and $\alpha = 60°$ of three-phase fully controlled bridge rectifier with *RL* load will be same as the waveforms for $\alpha = 0°$, $\alpha = 30°$ and $\alpha = 60°$ of three-phase fully controlled bridge rectifier with *R* load.
2. If firing angle $\alpha > 60°$, the output voltage waveforms of three-phase fully controlled bridge rectifier with *RL* load will be different from the output voltage waveforms of three-phase fully controlled bridge rectifier with *R* load.
3. At $\alpha = 90°$, the average output voltage is equal to zero as the area under the positive and the negative cycle are equal.
4. If firing angle $\alpha < 90°$, the average output voltage is positive. When firing angle $\alpha > 90°$, the average output voltage is negative.
5. The maximum value of α is 180°.
6. Since the frequency of input voltage is 50 Hz and the output voltage waveforms have six-pulse, the ripple frequency is 300 Hz at any value of α.

Three-phase fully controlled bridge rectifier can be able to operate in two modes such as rectifier mode and inverter mode of operation.

Rectifier mode of operation (α < 90°) During the rectifier mode of operation,

1. the average output voltage is positive.

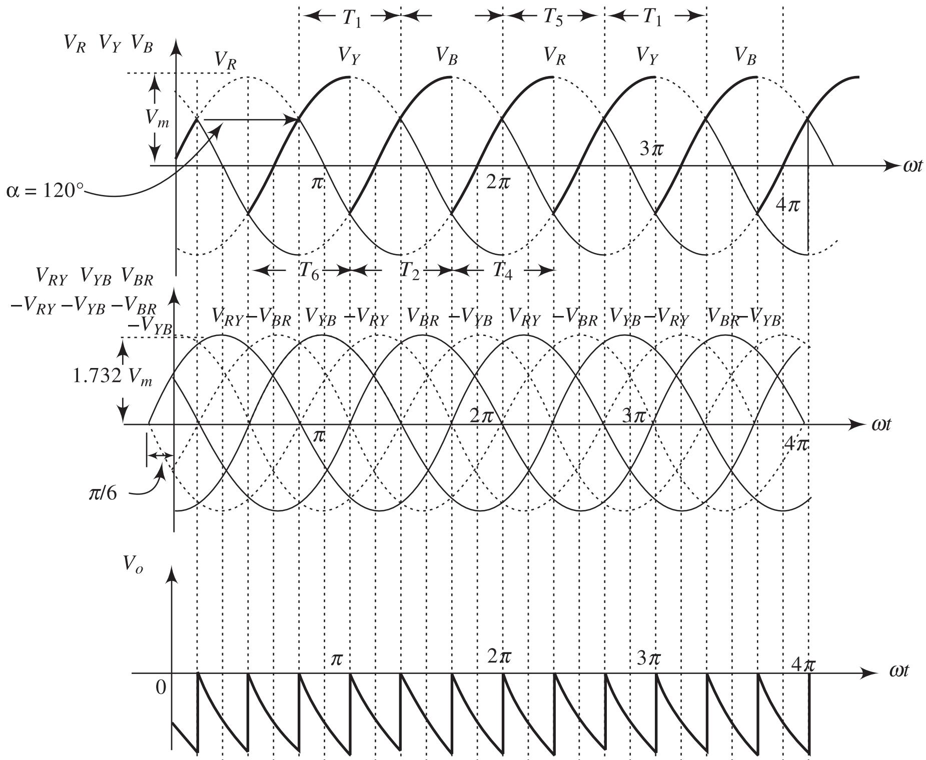


Fig. 8.31 *Output voltage waveforms of three-phase fully controlled bridge rectifier with RL load at firing angle $\alpha = 120°$*

2. the thyristor current waveforms are rectangular of 120° duration.
3. PIV rating of thyristor is $\sqrt{3}V_m$.
4. the three-phase supply currents are phase shifted by 120° and the thyristor currents are phase shifted by 60°.
5. the load current waveform is continuous. For highly inductive load, the load current is ripple free. Hence the average, rms and peak values of load current are equal to I_o.
6. since the values of V_o and I_o are positive, power output is positive and power can flow from supply to load.

Inverter mode of operation (α > 90°) During the inverter mode of operation,

1. the average output voltage is negative.
2. the thyristor current waveforms are rectangular of 120° duration.
3. PIV rating of thyristor is $\sqrt{3}V_m$.
4. the three-phase supply currents are phase shifted by 120° and the thyristor currents are phase shifted by 60°.
5. the load current waveform is continuous. For highly inductive load, the load current is ripple free. Hence the average, rms and peak values of load current are equal to I_o.
6. as the value of V_o is negative but the value of I_o is positive, power output is negative and power can flow from dc load side to supply.

Analysis of output voltage The output voltage waveform can be written as

$$v_o = V_o + \sum_{n=1,2,3....}^{\infty} V_{\text{an}} \cos 6n\omega t + \sum_{n=1,2,3....}^{\infty} V_{\text{bn}} \sin 6n\omega t$$

At firing angle α, the average dc output voltage is

$$V_o = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha$$

$$V_{\text{an}} = \frac{6}{\pi} \int_{\alpha}^{\frac{\pi}{3}+\alpha} v_o \cos 6n\omega t \cdot d\omega t = \frac{6}{\pi} \int_{\alpha}^{\frac{\pi}{3}+\alpha} \sqrt{3} V_m \sin(\omega t + 30°) \cos 6n\omega t \cdot d\omega t$$

$$= \frac{3\sqrt{3}}{\pi} V_m \left[\frac{\cos(6n+1)\alpha}{6n+1} - \frac{\cos(6n-1)\alpha}{6n-1} \right]$$

$$V_{\text{bn}} = \frac{6}{\pi} \int_{\alpha}^{\frac{\pi}{3}+\alpha} v_o \sin 6n\omega t \cdot d\omega t = \frac{6}{\pi} \int_{\alpha}^{\frac{\pi}{3}+\alpha} \sqrt{3} V_m \sin(\omega t + 30°) \sin 6n\omega t \cdot d\omega t$$

$$= \frac{3\sqrt{3}}{\pi} V_m \left[\frac{\sin(6n+1)\alpha}{6n+1} - \frac{\sin(6n-1)\alpha}{6n-1} \right]$$

Performance analysis of three phase full converter The output voltage, load current and input current waveforms are shown in Fig. 8.32. During analysis of converter, we assume that load current is continuous and constant. The instantaneous input current of a phase can be expressed by Fourier series as given below.

$$i_s(t) = a_0 + \sum_{n=1,2,3,4.....}^{\infty} a_n \cos n\omega t + \sum_{n=1,2,3,4....}^{\infty} b_n \sin n\omega t$$

where, $a_o = \dfrac{1}{2\pi} \displaystyle\int_0^{2\pi} i_s(t)\, d\omega t = \dfrac{1}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_o d\omega t - \dfrac{1}{2\pi} \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} I_o dt = 0$

Since the source current i_s for phase R is positive from $\dfrac{\pi}{6}+\alpha$ to $\dfrac{5\pi}{6}+\alpha$ and is negative from $\dfrac{7\pi}{6}+\alpha$ to $\dfrac{11\pi}{6}+\alpha$ and average value $a_o = 0$.

$$a_n = \frac{1}{\pi} \int_0^{2\pi} i_s(t) \cos n\omega t dt$$

$$= \frac{1}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_o \cos n\omega t dt - \frac{1}{\pi} \int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} I_o \cos n\omega t dt$$

$$= \frac{I_o}{n\pi}\left[\sin n\omega t\Big|_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} - \sin n\omega t\Big|_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha}\right]$$

$$= -\frac{4I_o}{n\pi}\sin\frac{n\pi}{3}\sin n\alpha \qquad \text{for } n = 1, 3, 5, 7 \ldots$$

$$= 0 \qquad \text{for } n = 2, 4, 6, 8 \ldots$$

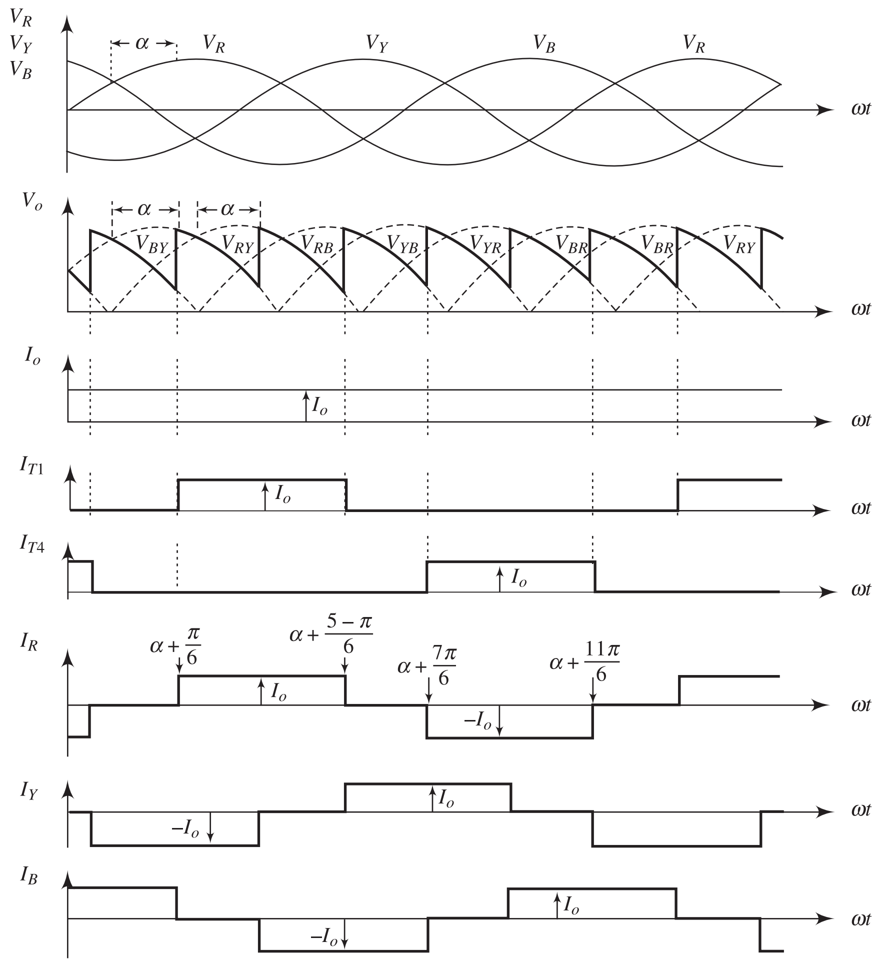


Fig. 8.32 *Output voltage, load current and input current waveforms*

At $n = 1$, $a_1 = -\frac{4I_o}{\pi}\sin\frac{\pi}{3}\sin\alpha = -\frac{4I_o}{\pi}\frac{\sqrt{3}}{2}\sin\alpha$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\sin n\omega t d\omega t$$

$$= \frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_o \sin n\omega t d\omega t - \frac{1}{\pi}\int_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha} I_o \sin n\omega t d\omega t$$

$$= \frac{I_o}{n\pi}\left[(-\cos n\omega t)\Big|_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} - (-\cos n\omega t)\Big|_{\frac{7\pi}{6}+\alpha}^{\frac{11\pi}{6}+\alpha}\right]$$

$$= \frac{4I_o}{n\pi}\sin\frac{n\pi}{3}\cos n\alpha \qquad \text{for } n = 1,\ 3,\ 5,\ 7\ \ldots$$

$$= 0 \qquad \text{for } n = 2,\ 4,\ 6,\ 8\ \ldots$$

At $n = 1$, $b_1 = \dfrac{4I_o}{\pi}\sin\dfrac{\pi}{3}\cos\alpha = \dfrac{4I_o}{\pi}\dfrac{\sqrt{3}}{2}\cos\alpha$

$$c_n = \sqrt{a_n^2 + b_n^2}$$

$$= \left[\left(-\frac{4I_o}{n\pi}\sin\frac{n\pi}{3}\sin n\alpha\right)^2 + \left(\frac{4I_o}{n\pi}\sin\frac{n\pi}{3}\cos n\alpha\right)^2\right]^{1/2}$$

$$= \frac{4I_o}{n\pi}\sin\frac{n\pi}{3} \quad \text{and} \quad \theta_n = \tan^{-1}\left[\frac{a_n}{b_n}\right] = \tan^{-1}(-\tan n\alpha) = -n\alpha$$

The source current can be expressed as

$$i_s(t) = \sum_{n=1,3,5,7\ldots}^{\infty} c_n \sin(n\omega t + \theta_n)$$

$$= \sum_{n=1,3,5,7\ldots}^{\infty} \frac{4I_o}{n\pi}\sin\frac{n\pi}{3}\sin(n\omega t - n\alpha)$$

The rms value of n^{th} order harmonic is

$$I_{sn} = \frac{4I_o}{\sqrt{2}n\pi}\sin\frac{n\pi}{3}$$

The rms value of first-order harmonic is

$$I_{s1} = \frac{4I_o}{\sqrt{2}\pi}\sin\frac{\pi}{3} = \frac{4I_o}{\sqrt{2}\pi}\frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{\pi}I_o$$

The rms value of input current is

$$I_{s\text{rms}} = \left[\frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_o^2 d\omega t\right]^{1/2} = I_o\sqrt{\frac{2}{3}}$$

$$DF = \cos\phi_1 = \cos(-\alpha) = \cos\alpha$$

$$CDF = \frac{I_{s1}}{I_s} = \frac{\sqrt{6}}{\pi}I_o \times \frac{1}{I_o}\sqrt{\frac{3}{2}} = \frac{3}{\pi}$$

Power factor $PF = DF \times CDF = \cos\alpha \times \frac{3}{\pi} = \frac{3}{\pi}\cos\alpha$

$$HF = \left[\frac{1}{CDF^2} - 1\right]^{1/2} = \left[\left(\frac{\pi}{3}\right)^2 - 1\right]$$

Active power input is

$$P_i = 3V_s I_{s1}\cos\phi_1 = 3\frac{V_L}{\sqrt{3}}\frac{\sqrt{6}}{\pi}I_o\cos\alpha$$

Reactive power input is

$$Q_i = 3V_s I_{s1}\sin\phi_1 = 3\frac{V_L}{\sqrt{3}}\frac{\sqrt{6}}{\pi}I_o\sin\alpha$$

Example 8.11 A three-phase full converter is used to charge a 200 V battery from a 220 V, 50 H ac supply as depicted in Fig. 8.33. Assume the internal resistance of battery is 0.5 Ω and a inductance is connected in series with a battery so that the 10 A constant charging current flows. (a) Determine the firing angle of converter and the input power factor. (b) If power flows from battery to ac side, what will be the firing angle of converter?

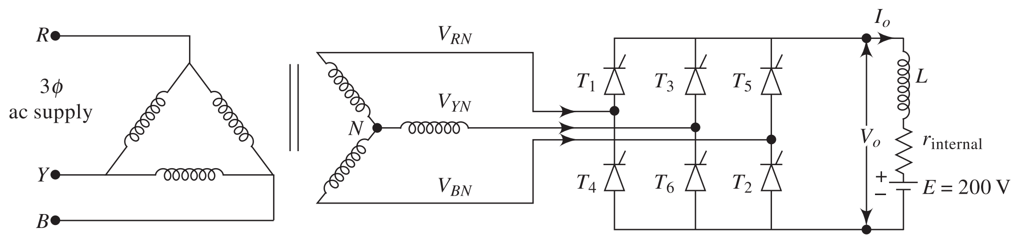


Fig. 8.33

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.02$,

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2}\times 127.02\text{ V} = 179.63\text{ V}$

Battery voltage E = 200 V and internal resistance of battery r_{internal} = 0.5 Ω

The charging current I_o = 10 A

(a) The output voltage of the converter when it is charging the 200 V battery is equal to

$$V_o = E + I_o r_{\text{internal}} = 200 + 10\times 0.5 = 205\text{ V}$$

At firing angle α, the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{\text{RY}}\cdot d(\omega t) = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha}\sqrt{3}V_m\sin\left(\omega t + \frac{\pi}{6}\right)\cdot d(\omega t) = \frac{3\sqrt{3}}{\pi}V_m\cos\alpha$$

$$= 1.6548V_m\cos\alpha = 1.6548\times 179.63\cos\alpha = 205\text{ V}$$

Therefore firing angle is $\alpha = \cos^{-1}\left(\dfrac{205}{1.6548 \times 179.63}\right) = 46.39°$

rms value of supply current is

$$I_s = \left[\frac{1}{\pi} I_o^2 \frac{2\pi}{3}\right]^{1/2} = \left[\frac{1}{\pi} 10^2 \frac{2\pi}{3}\right]^{1/2} = 8.164 \text{ A}$$

Power delivered to load is

$$P_o = EI_o + I_o^2 r_{\text{internal}} = 200 \times 10 + 10^2 \times 0.5 = 2050 \text{ Watt}$$

We know that

$$\sqrt{3} VI \cos\phi = P_o$$

Then power factor is $\cos\phi = \dfrac{P_o}{\sqrt{3}VI} = \dfrac{2050}{\sqrt{3} \times 220 \times 8.164} = 0.6589$ lag

(b) If the battery is used to deliver power to ac side, the output voltage converter is

$$V_o = E - I_o r_{\text{internal}} = 200 - 10 \times 0.5 = 195 \text{ V}$$

The output voltage will be negative.
At firing angle α, the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{\text{RY}} \cdot d(\omega t) = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha$$

$$= 1.6548 V_m \cos\alpha = 1.6548 \times 179.63 \cos\alpha = -195 \text{ V}$$

Therefore, firing angle is $\alpha = \cos^{-1}\left(-\dfrac{195}{1.6548 \times 179.63}\right) = 130.99°$

Example 8.12 A three-phase full bridge converter is fed from a delta-star transformer and it is connected to a *RL* load with ripple free current 15 A at firing angle 45°. It is fed from 440 V, 50 Hz ac supply, determine rectification efficiency, transformer utilisation factor and input power factor.

Solution

Given: Phase voltage is $V = \dfrac{V_L}{\sqrt{3}} = \dfrac{440}{\sqrt{3}} = 254.04$ V ,

Maximum phase voltage $V_m = \sqrt{2} V = \sqrt{2} \times 254.04 \text{ V} = 359.26 \text{ V}$

The current I_o = 15 A ripple free
At firing angle α, the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{\text{RY}} \cdot d(\omega t) = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha$$

$$= 1.6548 V_m \cos\alpha = 1.6548 \times 359.26 \times \cos 45 = 420.37 \text{ V as } \alpha = 45°$$

At firing angle α, the rms value of output voltage is equal to

$$V_{\text{rms}} = \sqrt{3} V_m \left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha\right]^{\frac{1}{2}} = \sqrt{3} \times 359.26 \left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos(2 \times 45)\right]^{\frac{1}{2}} = 440.05 \text{ V}$$

DC power output $P_{dc} = V_o I_o = 420.37 \times 15 \text{ Watt} = 6305.55 \text{ Watt}$

AC power $P_{ac} = V_{rms} I_o = 440.05 \times 15 \text{ Watt} = 6600.75 \text{ Watt}$

Rectification efficiency is $\eta = \dfrac{P_{dc}}{P_{ac}} \times 100\% = \dfrac{6305.55}{6600.75} \times 100 = 95.52\%$

rms value of source current $I_{s\text{rms}} = I_o \sqrt{\dfrac{2}{3}} = 15\sqrt{\dfrac{2}{3}} = 12.245 \text{ A}$

Input VA = 3VI$_{\text{srms}}$ = $3 \times \dfrac{440}{\sqrt{3}} \times 12.245 = 9332.15 \text{ VA}$

$$TUF = \frac{P_{dc}}{\text{Input VA}} = \frac{6305.55}{9332.15} = 0.6756$$

Input power factor is $PF = \dfrac{P_{ac}}{\text{Input VA}} = \dfrac{6600.75}{9332.15} = 0.7073 \text{ lag}$

Example 8.13 A three-phase full bridge converter is fed from 440 V, 50 Hz ac supply and is connected to a *RLE* load with ripple free load current. If $R = 10\ \Omega$, $E = 250$ V and inductance is very large, determine the power delivered to load and input power factor at firing angle (a) $\alpha = 45°$, (b) $\alpha = 60°$ and (c) firing angle advance of 75°.

Solution

Given: Phase voltage is $V = \dfrac{V_L}{\sqrt{3}} = \dfrac{440}{\sqrt{3}} = 254.04 \text{ V}$,

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2} \times 254.04 \text{ V} = 359.26 \text{ V}$

At firing angle α, the average dc output voltage is

$$V_{av} = V_o = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{RY}\cdot d(\omega t) = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)\cdot d(\omega t) = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha$$

$$= 1.6548 V_m \cos\alpha$$

(a) At $\alpha = 45°$, the average dc output voltage is

$$V_o = 1.6548 V_m \cos\alpha = 1.6548 \times 359.26 \times \cos 45 = 420.377 \text{ V}$$

The output voltage $V_o = E + I_o R$

The load current is $I_o = \dfrac{V_o - E}{R} = \dfrac{420.377 - 250}{10} \text{ A} = 17.037 \text{ A}$

The power delivered to load is

$$P_{dc} = EI_o + I_o^2 R = 250 \times 17.037 + 17.037^2 \times 10 = 7161.84 \text{ Watt}$$

rms value of source current $I_{s\text{rms}} = I_o\sqrt{\dfrac{2}{3}} = 17.037\sqrt{\dfrac{2}{3}} = 13.91 \text{ A}$

Input VA = 3VI$_{\text{srms}}$ = $3 \times \dfrac{440}{\sqrt{3}} \times 13.91 = 10601.089 \text{ VA}$

At firing angle α, the rms value of output voltage is equal to

$$V_{rms} = \sqrt{3}V_m \left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right]^{\frac{1}{2}} = \sqrt{3} \times 359.26 \left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos(2 \times 45)\right]^{\frac{1}{2}} = 440.05 \text{ V}$$

AC power $P_{ac} = V_{rms} I_o = 440.05 \times 13.91 \text{ Watt} = 6121.09 \text{ Watt}$

Input power factor is $PF = \dfrac{P_{ac}}{\text{Input VA}} = \dfrac{6121.09}{10601.089} = 0.5774 \text{ lag}$

(b) At $\alpha = 60°$, the average dc output voltage is

$$V_o = 1.6548 V_m \cos\alpha = 1.6548 \times 359.26 \times \cos 60 = 297.25 \text{ V}$$

The output voltage $V_o = E + I_o R$

The load current is $I_o = \dfrac{V_o - E}{R} = \dfrac{297.25 - 250}{10} \text{ A} = 4.725 \text{ A}$

The power delivered to load is

$$P_{dc} = EI_o + I_o^2 R = 250 \times 4.725 + 4.725^2 \times 10 = 1404.50 \text{ Watt}$$

Rms value of source current $I_{srms} = I_o \sqrt{\dfrac{2}{3}} = 4.725\sqrt{\dfrac{2}{3}} = 3.8579 \text{ A}$

Input VA = 3VI$_{srms}$ = $3 \times \dfrac{440}{\sqrt{3}} \times 3.8579 = 2940.18 \text{ VA}$

At firing angle α, the rms value of output voltage is equal to

$$V_{rms} = \sqrt{3} V_m \left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right]^{\frac{1}{2}} = \sqrt{3} \times 359.26 \left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos(2\times 60)\right]^{\frac{1}{2}} = 336.90 \text{ V}$$

Ac power $P_{ac} = V_{rms} I_o = 336.90 \times 3.8579 \text{ Watt} = 1299.72 \text{ Watt}$

Input power factor is $PF = \dfrac{P_{ac}}{\text{Input VA}} = \dfrac{1299.72}{2940.18} = 0.4420 \text{ lag}$

(c) Firing angle advance of 75° means α = 180 – 75 = 105°
The dc output voltage

$$V_o = 1.6548 \times 359.26 \times \cos 105 = -153.86 \text{ V}$$

As output voltage is negative, the converter operates as inverter. Therefore, polarity of emf E is reversed. The KVL equation is

$$V_o = -E + I_o R$$

or $-153.86 = -250 + I_o \times 10$

Load current $I_o = \dfrac{250 - 153.86}{10} = 9.614 \text{ A}$

Power delivered by battery from dc side to ac source

$$P = EI_o - I_o^2 R = 250 \times 9.614 - 9.614^2 \times 10 = 1479.21 \text{ Watt}$$

rms value of source current $I_{srms} = I_o \sqrt{\dfrac{2}{3}} = 9.614\sqrt{\dfrac{2}{3}} = 7.849 \text{ A}$

Input VA = 3 VI$_{srms}$ = $3 \times \dfrac{440}{\sqrt{3}} \times 7.849 = 5981.91 \text{ VA}$

Input power factor is $PF = \dfrac{\text{Power delivered to ac source from battery}}{\text{Input VA}} = \dfrac{1479.21}{5981.91} = 0.2472 \text{ lag}$

Example 8.14 A three-phase full bridge converter provides a ripple free load current of 15 A with a firing angle of 30°. The input voltage of the converter is three-phase, 400 V, 50 Hz.

(a) Derive the expression of source current by Fourier series
(b) Determine DF, CDF, HF and PF
(c) Find the active and reactive input power

Solution

(a) The source current can be expressed by Fourier series as

$$i_s(t) = \sum_{n=1,3,5,7...}^{\infty} c_n \sin(n\omega t + \theta_n)$$

$$= \sum_{n=1,3,5,7...}^{\infty} \frac{4I_o}{n\pi} \sin\frac{n\pi}{3} \sin(n\omega t - n\alpha)$$

$$= \sum_{n=1,3,5,7...}^{\infty} \frac{4\times 15}{n\pi} \sin\frac{n\pi}{3} \sin(n\omega t - n\times 30) \quad \text{as } I_o = 15 \text{ and } \alpha = 30°$$

$$I_{s1} = \frac{60}{\pi}\sin\frac{\pi}{3}\sin(\omega t - 30) = 16.548\sin(\omega t - 30)$$

$$I_{s3} = \frac{60}{3\pi}\sin\frac{3\pi}{3}\sin(3\omega t - 90) = 0$$

$$I_{s5} = \frac{60}{5\pi}\sin\frac{5\pi}{3}\sin(5\omega t - 150) = -3.309\sin(5\omega t - 150)$$

$$I_{s7} = \frac{60}{7\pi}\sin\frac{7\pi}{3}\sin(7\omega t - 310) = 2.364\sin(7\omega t - 310)$$

$$I_s = 16.548\sin(\omega t - 30) - 3.309\sin(5\omega t - 150) + 2.364\sin(7\omega t - 310) + \cdots\cdots$$

(b) $DF = \cos\phi_1 = \cos(-\alpha) = \cos\alpha = \cos 30 = 0.866$

$$CDF = \frac{I_{s1}}{I_s} = \frac{\sqrt{6}}{\pi} I_o \times \frac{1}{I_o}\sqrt{\frac{3}{2}} = \frac{3}{\pi} = 0.9554$$

$$HF = \left[\frac{1}{CDF^2} - 1\right]^{1/2} = \left[\left(\frac{\pi}{3}\right)^2 - 1\right]^{1/2} = 0.309$$

Power factor $PF = DF \times CDF = \cos\alpha \times \frac{3}{\pi} = \frac{3}{\pi}\cos\alpha = 0.9554 \times 0.866 = 0.8273$ lag

(c) Active power input is

$$P_i = 3V_s I_{s1}\cos\phi_1 = 3\frac{V_L}{\sqrt{3}}\frac{\sqrt{6}}{\pi} I_o \cos\alpha = 3\frac{400}{\sqrt{3}}\frac{\sqrt{6}}{\pi} 15\cos 30 = 7019.77 \text{ Watt}$$

Reactive power input is

$$Q_i = 3V_s I_{s1}\sin\phi_1 = 3\frac{V_L}{\sqrt{3}}\frac{\sqrt{6}}{\pi} I_o \sin\alpha = 3\frac{400}{\sqrt{3}}\frac{\sqrt{6}}{\pi} 15\sin 30 = 4052.866 \text{ VAR}$$

8.6.2 Three-Phase Full-Controlled Bridge Rectifier with *R* Load

Figure 8.34 shows a three-phase fully controlled bridge rectifier circuit with *R* load. In this circuit, thyristors are numbered in order of conduction sequence and each thyristor conducts for $2\pi/3$ (120°)

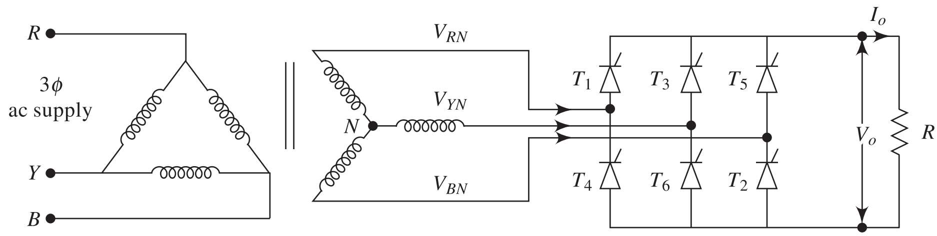


Fig. 8.34 *Three-phase fully controlled bridge rectifier with resistive load*

duration. The conduction sequences of thyristors are T_1 T_2, T_2 T_3, T_3 T_4, T_4 T_5, T_5 T_6, and T_6 T_1. The vector diagram three phase voltages and line voltages are shown in Fig. 8.26. The voltage and current waveforms of a three-phase fully controlled bridge rectifier circuit at $\alpha = 0°$ are depicted in Fig. 8.35. During analysis of three-phase full converter, we assume that the combination of star or delta connected primary and secondary windings are symmetrical. Three-phase fully controlled bridge rectifier are extensively used high power applications since transformer utilisation factor is high.

To explain the operating principal of a three-phase fully controlled bridge rectifier circuit with R load, we should assume the following points:

1. Each thyristor can be triggered at a firing angle α.
2. Each switching device conducts for 120° duration.
3. Thyristors must be trigger in the sequence T_1, T_2, T_3, T_4, T_5, $T_{6.}$
4. The phase shift between the triggering of two adjacent thyristors is 60°.
5. There are six pairs of conduction of thyristors such as T_1 T_2, T_2 T_3, T_3 T_4, T_4 T_5, T_5 T_6, and T_6 T_1.
6. Each thyristor conducts in pair of two thyristors and the conduction period of each thyristor is 60°.
7. The peak inverse voltage across each thyristor is $\sqrt{3}V_m$.

The waveforms of input voltage and output voltage of three-phase fully controlled rectifier with R load are depicted in Fig. 8.36. From Fig. 8.36, it is clear that the converter operates in two modes such as continuous conduction mode and discontinuous conduction mode.

Continuous conduction mode $\left(0 \le \alpha \le \dfrac{\pi}{3}\right)$ When the firing angle α is within the range $0 \le \alpha \le \dfrac{\pi}{3}$, the line voltage V_{RY} is applied across load as thyristors T_6 T_1 conduct for 60°duration. After that V_{RB} is applied across load as T_6 becomes turned OFF due to natural commutation and T_1 T_2 conduct for 60°duration. Subsequently, other thyristors will be turned OFF and turned ON sequentially. We get a continuous output voltage across load.

Discontinuous conduction mode $\left(\dfrac{\pi}{3} \le \alpha \le \dfrac{2\pi}{3}\right)$ If the firing angle α is within the range $\dfrac{\pi}{3} \le \alpha \le \dfrac{2\pi}{3}$, the line voltage V_{RY} is applied across load up to π as thyristors T_6 T_1 conducts from $\omega t = \dfrac{\pi}{6} + \alpha$ to $\omega t = \pi$. At $\omega t = \pi$, both thyristors will be turned OFF. Subsequently the output voltage across load is zero for certain period. After that V_{RB} is applied across load as T_1 T_2 are turned ON by applying triggering pulses and conduct for certain duration as depicted in Fig. 8.36. Hence, the output voltage across load is discontinuous.

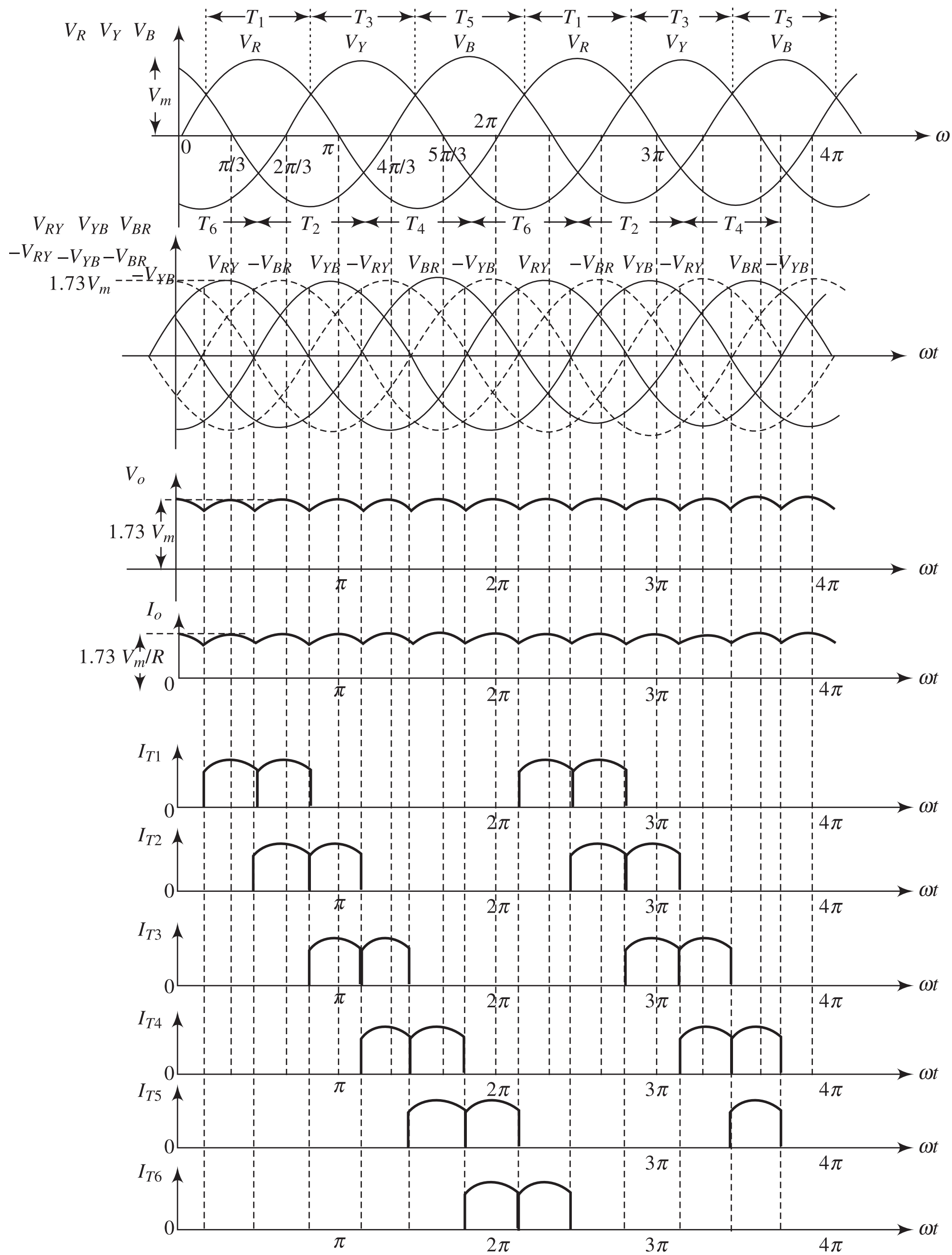


Fig. 8.35 *Voltage and current waveforms of three-phase fully controlled bridge rectifier with resistive load $\alpha = 0°$*

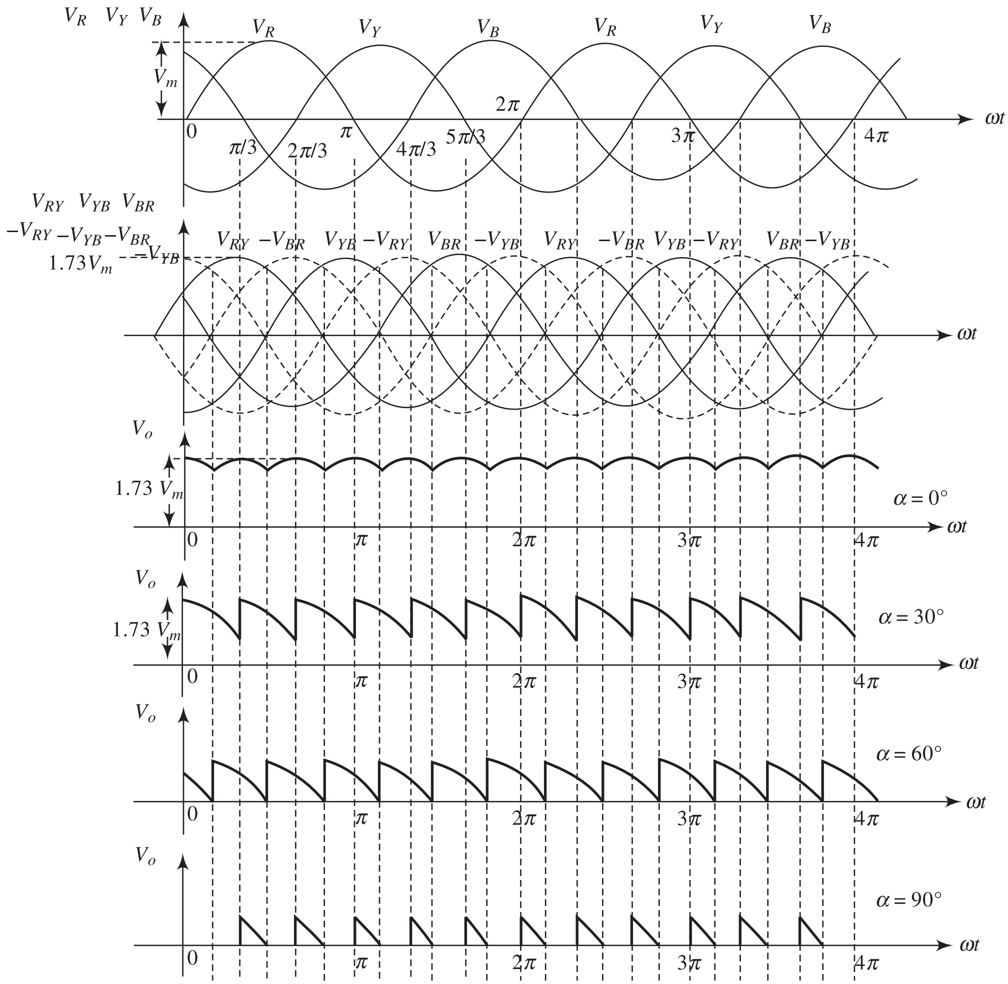


Fig. 8.36 *Waveforms for input voltage and output voltage of three-phase fully controlled bridge rectifier with resistive load at α = 0°, α = 30°, α = 60° and α = 90°*

Continuous conduction mode For $0 \le \alpha \le \dfrac{\pi}{3}$, the average dc output voltage for this circuit is

$$V_{\text{av}} = V_o = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{\text{RY}} \cdot d(\omega t)$$

$$= \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) = \frac{3\sqrt{3}}{\pi}V_m\left[-\cos\left(\omega t + \frac{\pi}{6}\right)\right]_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha}$$

$$= \frac{3\sqrt{3}}{\pi}V_m \cos\alpha$$

Average load current is equal to

$$I_{\text{av}} = \frac{V_{\text{av}}}{R}$$

Discontinuous conduction mode For $\frac{\pi}{3} \le \alpha \le \frac{2\pi}{3}$, the average dc output voltage for this circuit is

$$\begin{aligned}
V_{\text{av}} = V_o &= \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}} V_{\text{RY}} \cdot d(\omega t) \\
&= \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) = \frac{3\sqrt{3}}{\pi} V_m \left[-\cos\left(\omega t + \frac{\pi}{6}\right)\right]_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}} \\
&= \frac{3\sqrt{3}}{\pi} V_m \left[1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right]
\end{aligned}$$

At $\alpha = 120°$, $V_{av} = 0$. Therefore, maximum firing angle $\alpha_{\max} = 120°$
Average load current is equal to

$$I_{\text{av}} = \frac{V_{\text{av}}}{R}$$

Example 8.15 A three-phase full converter is connected to a load resistance of 5 Ω and it is supplied from a 220 V, 50 H ac supply. If the firing angle of thyristor is $\alpha = 30°$, determine (a) average output voltage, (b) average output current, (c) rms output voltage and (d) rms output current.

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.02$,

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2} \times 127.02 \text{ V} = 179.63 \text{ V}$

(a) At firing angle $\alpha\left(0 \le \alpha \le \frac{\pi}{3}\right)$, the average dc output voltage is

$$\begin{aligned}
V_{\text{av}} = V_o &= \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} V_{\text{RY}} \cdot d(\omega t) = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) \\
&= \frac{3\sqrt{3}}{\pi} V_m \cos\alpha = \frac{3\sqrt{3}}{\pi} \times 179.63 \times \cos 30 \text{ V} = 257.4239 \text{ V} \quad \text{as } \alpha = 30°
\end{aligned}$$

(b) Average output current is

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = \frac{257.4239}{5} = 51.48 \text{ A}$$

(c) At firing angle α, the rms output voltage is

$$V_{\text{rms}} = \sqrt{3}V_m\left[\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

$$= \sqrt{3}\times 179.63\left[\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos(2\times 30)\right]^{\frac{1}{2}} = 261.57 \text{ V}$$

(d) rms output current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{261.57}{5} = 52.31 \text{ A}$$

Example 8.16 A three-phase full converter is connected to a load resistance of 10 Ω. If the firing angle of thyristor is α = 30°, three phase converter feeds 4 kW power to a resistive load. Determine the amplitude of maximum per phase input voltage.

Solution

(a) At firing angle $\alpha\left(0 \le \alpha \le \frac{\pi}{3}\right)$, the rms output voltage is

$$V_{\text{rms}} = \sqrt{3}V_m\left[\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

$$= \sqrt{3}\times V_m\left[\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos(2\times 30)\right]^{\frac{1}{2}} = 1.456V_m \quad \text{as } \alpha = 30°$$

rms output current is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{1.456V_m}{R}$

Power output is $I_{\text{rms}}^2 R = \left(\frac{1.456V_m}{R}\right)^2 R = \frac{2.1199}{R}V_m^2 = 4000 \text{ Watt}$

or $\frac{2.1199}{10}V_m^2 = 4000 \quad \text{as } R = 10\,\Omega$

The amplitude of maximum per phase input voltage is $V_m = \sqrt{\frac{4000\times 10}{2.1199}} \text{ V} = 137.363 \text{ V}$

8.7 THREE-PHASE SEMICONVERTER

Three-phase fully controlled converters are extensively used in many industrial applications where power regeneration from the dc side is required. This converter can able to handle high power with in a certain limit of harmonics. But the cost of three-phase fully controlled converters is high due to the use of six thyristors and their associated complex control circuit. The complexity of control circuit can be reduced when the top group or the bottom group thyristors of three-phase converters are replaced by diodes. If three thyristors are replaced by three diodes, the circuit complexity is drastically reduced and at the same time it prevents negative voltage appearing at the output at any time. Hence the converter can not able to operate in regeneration or inverter mode. This converter is known as *three-phase half controlled converter* or *three-phase semiconverter or three-phase half controlled bridge converter*.

The three-phase half controlled converter has the following advantages over a three-phase fully controlled converter:

1. For the same firing angle, the three-phase half-controlled converter has lower input displacement factor compared to a fully controlled converter.
2. The range of continuous conduction mode operation of converter is increased.

But the three-phase half controlled converter has one disadvantage, the output voltage is periodic over one third of the input cycle where the output voltage of three-phase fully controlled converter is periodic over one sixth of the input cycle. Subsequently, both input and output harmonics are of lower frequency and the size of filter is large. Though the circuit configuration of three phase half controlled converter is simpler compared to three-phase full converter, the circuit analysis is reasonably more difficult. In this section, the operating principle and analysis of a three-phase half controlled converter are discussed elaborately.

Figure 8.37 shows the three-phase half-controlled converter which is connected to a *RL* load. During continuous conduction mode, only one thyristor from the top group and only one diode from the bottom group are conducted at a time. Just like three-phase full converter, both devices from the same leg cannot conduct at the same instant. Table 8.2 shows the conduction mode of operation and voltage across devices.

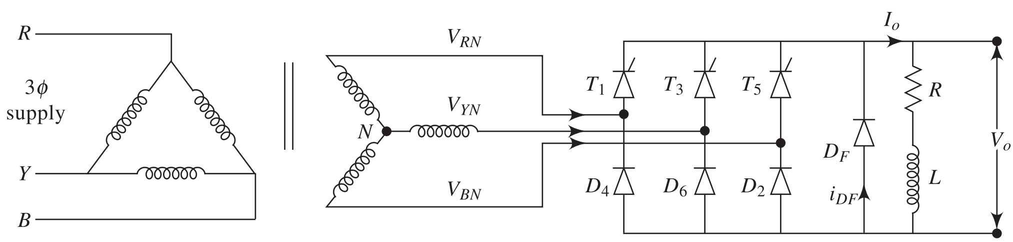


Fig. 8.37 *Three-phase half-controlled converter with a RL load*

Table 8.2 *Conduction mode of operation of thyristors and voltage across devices*

Voltage across devices	Conduction mode of operation					
	T_1D_2	D_2T_3	T_3D_4	D_4T_5	T_5D_6	D_6T_1
V_{T1}	0	V_{ab}	V_{ab}	V_{ac}	V_{ac}	0
V_{D2}	0	0	V_{ac}	V_{ac}	V_{bc}	V_{bc}
V_{T3}	V_{ba}	0	0	V_{bc}	V_{bc}	V_{ba}
V_{D4}	V_{ca}	V_{ca}	0	0	V_{ba}	V_{ba}
V_{T5}	V_{ca}	V_{cb}	V_{cb}	0	0	V_{ca}
V_{D6}	V_{cb}	V_{cb}	V_{ab}	V_{ab}	0	0
V_0	V_{ac}	V_{bc}	V_{ba}	V_{ca}	V_{cb}	V_{ab}

Assume that V_m is the peak value of the phase voltage. The instantaneous phase voltages are:

$$V_{RN} = V_m \sin\omega t,$$
$$V_{YN} = V_m \sin(\omega t - 120°) \text{ and}$$
$$V_{BN} = V_m \sin(\omega t - 240°).$$

Then the line to line voltages lead the phase voltage by 30° and the instantaneous line voltages are as follows:

$$V_{RY} = \sqrt{3}V_m \sin(\omega t + 30°), \qquad V_{\text{RY}} = V_{ab}$$
$$V_{YB} = \sqrt{3}V_m \sin(\omega t - 90°), \qquad V_{\text{YB}} = V_{bc} \text{ and}$$
$$V_{BR} = \sqrt{3}V_m \sin(\omega t - 210°) \qquad V_{\text{BR}} = V_{ca}$$

When diode D_2 is forward biased and conducting, the voltage across diode D_2 is zero. While diode D_2 is reverse biased and not conducting, the voltage across diode D_2 is either V_{RB} or V_{YB}. The firing sequence of thyristors is T_1, T_3 and T_5. Figure 8.38 shows the waveforms for input voltage, output voltage, and output current at different firing angle. The firing angle α can be varied in the range $0 \leq \alpha \leq \pi$. Thyristor T_1 is forward biased during the interval $\frac{\pi}{6} \leq \omega t \leq \frac{7\pi}{6}$. When the thyristor T_1 is fired at $\omega t = \frac{\pi}{6} + \alpha$ with firing angle α, thyristor T_1 and diode D_2 conduct and the line to line voltage V_{RB} is applied across the load. thyristor T_1 continuously conduct until thyristor T_2 is fired at $\omega t = \frac{5\pi}{6} + \alpha$.

If firing angle $\alpha \leq \frac{\pi}{3}$, the output voltage is continuous and free wheeling diode D_F does not conduct. When firing angle $\alpha \geq \frac{\pi}{3}$, the output voltage is discontinuous and free wheeling diode conducts. For $\alpha \leq \frac{\pi}{3}$, the average output voltage is equal to

$$V_{\text{av}} = V_o = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} V_{\text{RY}} \cdot d(\omega t) + \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} V_{\text{RB}} \cdot d(\omega t)$$

$$= \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) - \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} \sqrt{3}V_m \sin\left(\omega t - \frac{7\pi}{6}\right) \cdot d(\omega t)$$

$$= \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) + \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right) \cdot d(\omega t)$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[\left\{-\cos\left(\omega t + \frac{\pi}{6}\right)\right\}_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} + \left\{-\cos\left(\omega t - \frac{\pi}{6}\right)\right\}_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha}\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[\cos\left(\frac{\pi}{3} + \alpha\right) - \cos\frac{2\pi}{3} + \cos\frac{\pi}{3} - \cos\left(\frac{2\pi}{3} + \alpha\right)\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[1 + \cos\left(\frac{\pi}{3} + \alpha\right) - \cos\left(\frac{2\pi}{3} + \alpha\right)\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos\alpha)$$

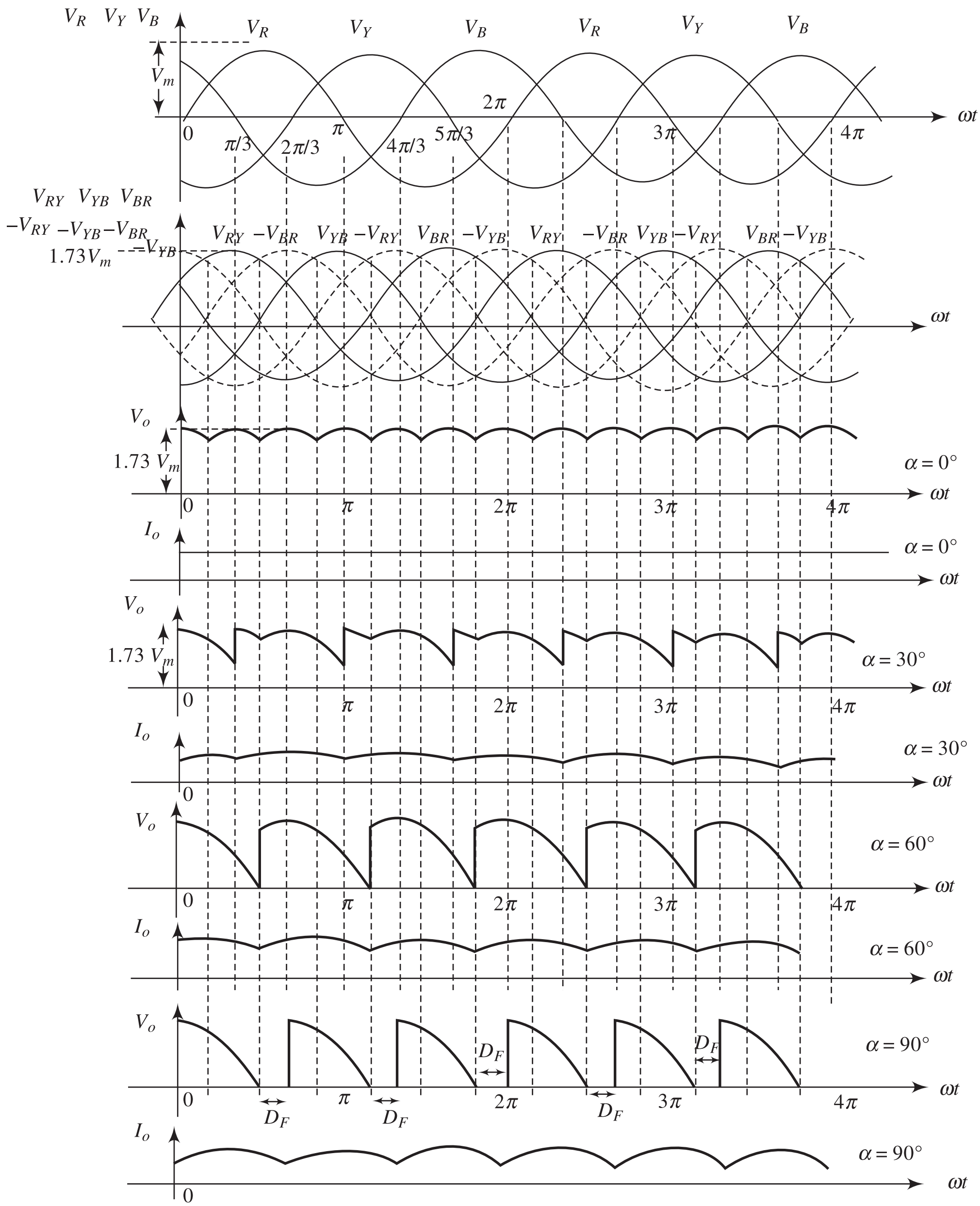


Fig. 8.38 *Input voltage, output voltage and output current waveforms of a three-phase semi-converter at different firing angle with RL load*

For $\alpha \geq \dfrac{\pi}{3}$, the average output voltage is equal to

$$V_{\text{av}} = V_o = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} V_{\text{RB}}\cdot d(\omega t) = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \sqrt{3}V_m \sin\left(\omega t-\frac{\pi}{6}\right)\cdot d(\omega t)$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[-\cos\left(\omega t-\frac{\pi}{6}\right)\right]_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} = \frac{3\sqrt{3}V_m}{2\pi}[\cos\alpha-\cos 180]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}[1+\cos\alpha]$$

In this way, for both cases, the average output voltage at firing angle α is same

For $\alpha \leq \dfrac{\pi}{3}$, the rms output voltage is equal to

$$V_{\text{rms}} = V_{o\text{rms}} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} V_{\text{RY}}^2\cdot d(\omega t) + \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} V_{\text{RB}}^2\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}}\left(\sqrt{3}V_m\sin\left(\omega t+\frac{\pi}{6}\right)\right)^2\cdot d(\omega t) - \frac{3}{2\pi}\left\{\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha}\sqrt{3}V_m\sin\left(\omega t-\frac{7\pi}{6}\right)\right\}^2\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} 3V_m^2\sin^2\left(\omega t+\frac{\pi}{6}\right)\cdot d(\omega t) + \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} 3V_m^2\sin^2\left(\omega t-\frac{\pi}{6}\right)\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}}\left\{1-\cos 2\left(\omega t+\frac{\pi}{6}\right)\right\}\cdot d(\omega t) + \frac{9V_m^2}{4\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha}\{1-\cos 2\left(\omega t-\frac{\pi}{6}\right)\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi}\left\{\omega t-\frac{1}{2}\sin 2\left(\omega t+\frac{\pi}{6}\right)\right\}_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} + \frac{9V_m^2}{4\pi}\left\{\omega t-\frac{1}{2}\sin 2\left(\omega t-\frac{\pi}{6}\right)\right\}_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha}\right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi}\left\{\frac{2\pi}{3}+\frac{\sqrt{3}}{2}(1+\cos 2\alpha)\right\}\right]^{1/2} = \frac{3V_m}{2}\left[\left\{\frac{2}{3}+\frac{\sqrt{3}}{2\pi}(1+\cos 2\alpha)\right\}\right]^{1/2}$$

For $\alpha \geq \dfrac{\pi}{3}$, the rms output voltage is equal to

$$V_{\text{rms}} = V_{o\text{rms}} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} V_{\text{RB}}^2 . d(\omega t)\right]^{1/2} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \left(\sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)\right)^2 \cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right)\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \left\{1 - \cos 2\left(\omega t - \frac{\pi}{6}\right)\right\}\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi}\left\{\omega t - \frac{1}{2}\sin 2\left(\omega t - \frac{\pi}{6}\right)\right\}_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}}\right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi}\left\{\frac{7\pi}{6} - \frac{\pi}{6} - \alpha - \frac{1}{2}\sin 2\left(\frac{7\pi}{6} - \frac{\pi}{6}\right) + \frac{1}{2}\sin 2\left(\frac{\pi}{6} + \alpha - \frac{\pi}{6}\right)\right\}\right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi}\left\{\pi - \alpha + \frac{1}{2}\sin 2\alpha\right\}\right]^{1/2} = \frac{3V_m}{2}\left[\frac{1}{\pi}\left\{\pi - \alpha + \frac{1}{2}\sin 2\alpha\right\}\right]^{1/2}$$

If firing angle $\alpha \leq 60°$, thyristors and diodes conduct for 120°duration. When the firing angle $\alpha \geq 60°$, thyristors and diodes conduct for $180° - \alpha$ duration.

For $\alpha \leq 60°$: Rms current ratings of thyristor and diode are:

$$I_{T\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}} \quad \text{and} \quad I_{D\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}}$$

Average current ratings of thyristor and diode are

$$I_{T\_\text{av}} = \frac{I_{o\_\text{av}}}{3} \quad \text{and} \quad I_{D\_\text{av}} = \frac{I_{o\_\text{av}}}{3}$$

For $\alpha \geq 60°$: Rms current ratings of thyristor and diode are:

$$I_{T\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}} \quad \text{and} \quad I_{D\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}}$$

Average current ratings of thyristor and diode are:

$$I_{T\_\text{av}} = \frac{I_{o\_\text{av}}}{3} \quad \text{and} \quad I_{D\_\text{av}} = \frac{I_{o\_\text{av}}}{3}$$

Example 8.17 A three-phase semiconverter is connected to a *RL* load with *R* = 10 Ω. If the firing angle of thyristors is 45° and it fed power of 5 kW. Determine the maximum amplitude of per phase input voltage.

Solution

If firing angle α is less than 60°, for any value of α the rms output voltage is

$$V_{\text{rms}} = \frac{3V_m}{2}\left[\left\{\frac{2}{3}+\frac{\sqrt{3}}{2\pi}(1+\cos 2\alpha)\right\}\right]^{1/2} = \frac{3V_m}{2}\left[\left\{\frac{2}{3}+\frac{\sqrt{3}}{2\pi}\big(1+\cos(2\times 45)\big)\right\}\right]^{1/2}$$

$$= 1.4562V_m \qquad \text{as } \alpha = 45°$$

rms output current is $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{1.4562V_m}{R}$

Power output is $I_{\text{rms}}^2 R = \left(\dfrac{1.4562V_m}{R}\right)^2 R = \dfrac{2.1205}{R}V_m^2 = 5000$ Watt

or $\dfrac{2.1205}{10}V_m^2 = 5000 \quad \text{as } R = 10\,\Omega$

The amplitude of maximum per phase input voltage is $V_m = \sqrt{\dfrac{5000\times 10}{2.1205}}\ \text{V} = 153.55\ \text{V}$

Example 8.18 A three-phase semiconverter is connected to a *RL* load with of 10 Ω. If the firing angle of thyristors is 75° and it fed power of 5 kW. Determine the maximum amplitude of per phase input voltage.

Solution

If firing angle α is greater than 60°, for any value of α the rms output voltage is

$$V_{\text{rms}} = \frac{3V_m}{2}\left[\frac{1}{\pi}\left\{\pi-\alpha+\frac{1}{2}\sin 2\alpha\right\}\right]^{1/2} = \frac{3V_m}{2}\left[\frac{1}{\pi}\left\{\pi-\frac{75\times\pi}{180}+\frac{1}{2}\sin(2\times 75)\right\}\right]^{1/2}$$

$$= 1.2213V_m \qquad \text{as } \alpha = 75°$$

rms output current is $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{1.2213V_m}{R}$

Power output is $I_{\text{rms}}^2 R = \left(\dfrac{1.2213V_m}{R}\right)^2 R = \dfrac{1.4915}{R}V_m^2 = 5000$ Watt

or $\dfrac{1.4951}{10}V_m^2 = 5000 \quad \text{as } R = 10\,\Omega$

The amplitude of maximum per phase input voltage is $V_m = \sqrt{\dfrac{5000\times 10}{1.4951}}\ \text{V} = 182.87\ \text{V}$

Example 8.19 A three-phase semi converter is used to charge a 200 V battery from a 220 V, 50 H ac supply as depicted in Fig. 8.39. Assume the internal resistance of battery is 0.5 Ω and an inductance is connected in series with a battery so that the 10 A constant charging current flows. Determine the firing angle of converter, conduction period of each thyristor and the input power factor.

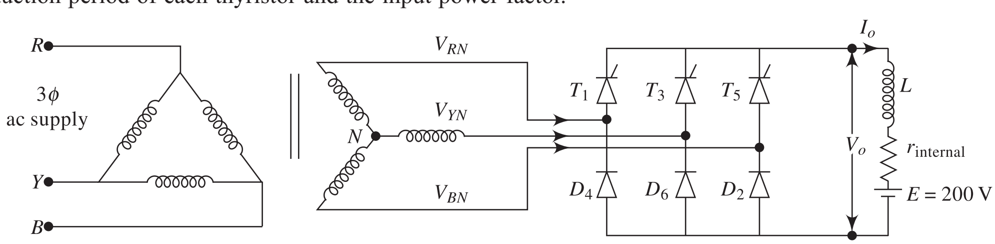


Fig. 8.39

Solution

Given: Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.02\text{ V},$

$$\text{Maximum phase voltage } V_m = \sqrt{2}V = \sqrt{2} \times 127.02\text{ V} = 179.63\text{ V}$$

Battery voltage E = 200 V and internal resistance of battery r_{internal} = 0.5 Ω

The charging current I_o = 10 A

The output voltage of the converter when it is charging the 200 V battery is equal to

$$V_o = E + I_o r_{\text{internal}} = 200 + 10 \times 0.5 = 205\text{ V}$$

At firing angle α, the average dc output voltage is

$$V_{\text{av}} = V_o = \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos\alpha) = \frac{3\sqrt{3} \times 179.63}{2\pi}(1 + \cos\alpha) = 205\text{ V}$$

or $148.628(1 + \cos\alpha) = 205$

or $\cos\alpha = \frac{205}{148.628} - 1 = 0.379$

Therefore, firing angle is $\alpha = \cos^{-1}(0.379) = 67.71°$

For $\alpha \geq 60°$, the conduction period of each thyristor is 180° – α duration. As α = 67.71°, the conduction period of each thyristor is equal to $180° - \alpha = 180° - 67.71° = 112.29°$ duration.

For constant load current of 10 A, the supply current is of square wave of amplitude 10 A. Since i_R flows for 112.29° over every half cycle of 180°. The rms value of supply current is equal to

$$I_{R\text{rms}} = \left[\frac{1}{\pi} I_o^2 \frac{112.29 \times \pi}{180}\right]^{1/2} = \left[\frac{1}{\pi} 10^2 \frac{112.29 \times \pi}{180}\right]^{1/2} = 7.898\text{ A}$$

Power delivered to load is

$$P_o = EI_o + I_o^2 r_{\text{internal}} = 200 \times 10 + 10^2 \times 0.5 = 2050\text{ Watt}$$

We know that

$$\sqrt{3}VI\cos\phi = P_o$$

Then power factor is $\cos\phi = \frac{P_o}{\sqrt{3}VI} = \frac{2050}{\sqrt{3} \times 220 \times 7.898} = 0.681\text{ lag}$

8.7.1 Operation with R Load

Figure 8.40 shows the voltage and current waveforms of a three phase semiconverter. The output voltage waveforms at firing angle α = 0°, α = 30° and α = 90° are depicted in Fig. 8.40. It is clear from Fig. 8.40 that

1. If firing angle α = 0°, the output voltage waveform consists of six-pulse within a cycle of input voltage.
2. If firing angle α = 30°, the output voltage waveform has only three pulse within a cycle of input voltage. Subsequently, this converter is known as three-pulse converter.
3. If firing angle α > 60°, the output voltage waveform becomes zero after every pulse for α = 60° and α > 60°, it will remain zero for a finite time. Consequently, the output voltage waveform will be discontinuous.
4. If firing angle $\alpha \leq 60°$, each device conducts for 120° duration. When firing angle $\alpha \geq 60°$, each device conducts for 180° – α duration.

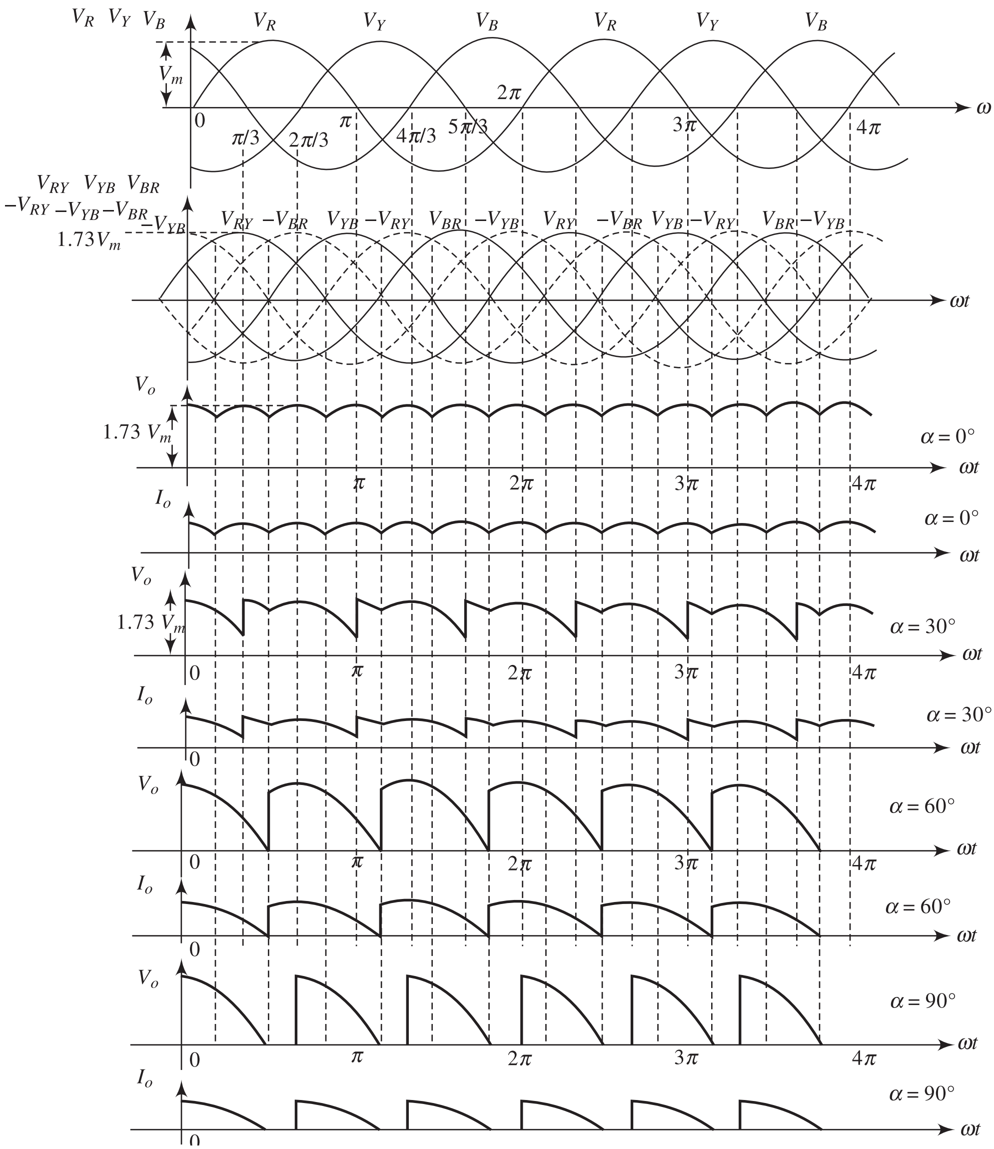


Fig. 8.40 *Input voltage, output voltage and output current waveforms of a three-phase semi-converter at different firing angle with R load*

8.7.2 Expression of Output Voltage of Semi-converter With *R* load

Average output voltage For $\alpha \le \dfrac{\pi}{3}$, the average output voltage is equal to

$$V_{\text{av}} = V_o = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} V_{\text{RY}}\cdot d(\omega t) + \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} V_{\text{RB}}\cdot d(\omega t)$$

$$= \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)\cdot d(\omega t) - \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} \sqrt{3}V_m \sin\left(\omega t - \frac{7\pi}{6}\right)\cdot d(\omega t)$$

$$= \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)\cdot d(\omega t) + \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)\cdot d(\omega t)$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[\left\{-\cos\left(\omega t + \frac{\pi}{6}\right)\right\}_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} + \left\{-\cos\left(\omega t - \frac{\pi}{6}\right)\right\}_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha}\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[\cos\left(\frac{\pi}{3}+\alpha\right) - \cos\frac{2\pi}{3} + \cos\frac{\pi}{3} - \cos\left(\frac{2\pi}{3}+\alpha\right)\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[1 + \cos\left(\frac{\pi}{3}+\alpha\right) - \cos\left(\frac{2\pi}{3}+\alpha\right)\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

For $\alpha \geq \frac{\pi}{3}$, the average output voltage is equal to

$$V_{\text{av}} = V_o = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} V_{\text{RB}}\cdot d(\omega t) = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)\cdot d(\omega t)$$

$$= \frac{3\sqrt{3}V_m}{2\pi}\left[-\cos\left(\omega t - \frac{\pi}{6}\right)\right]_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} = \frac{3\sqrt{3}V_m}{2\pi}[\cos\alpha - \cos 180]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}[1+\cos\alpha]$$

In this way, for both the cases, the average output voltage at firing angle α is same.

RMS output voltage For $\alpha \leq \frac{\pi}{3}$, the rms output voltage is equal to

$$V_{\text{rms}} = V_{o\,\text{rms}} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} V_{\text{RY}}^2\cdot d(\omega t) + \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} V_{\text{RB}}^2\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} \left(\sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)\right)^2\cdot d(\omega t) - \frac{3}{2\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} \left\{\sqrt{3}V_m \sin\left(\omega t - \frac{7\pi}{6}\right)\right\}^2\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6}\right) \cdot d(\omega t) + \frac{3}{2\pi} \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right) \cdot d(\omega t) \right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} \left\{1 - \cos 2\left(\omega t + \frac{\pi}{6}\right)\right\} \cdot d(\omega t) + \frac{9V_m^2}{4\pi} \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} \left\{1 - \cos 2\left(\omega t - \frac{\pi}{6}\right)\right\} \cdot d(\omega t) \right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi} \left\{\omega t - \frac{1}{2}\sin 2\left(\omega t + \frac{\pi}{6}\right)\right\}_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}} + \frac{9V_m^2}{4\pi} \left\{\omega t - \frac{1}{2}\sin 2\left(\omega t - \frac{\pi}{6}\right)\right\}_{\frac{\pi}{2}}^{\frac{5\pi}{6}+\alpha} \right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi} \left\{\frac{2\pi}{3} + \frac{\sqrt{3}}{2}(1 + \cos 2\alpha)\right\} \right]^{1/2} = \frac{3V_m}{2} \left[\left\{\frac{2}{3} + \frac{\sqrt{3}}{2\pi}(1 + \cos 2\alpha)\right\} \right]^{1/2}$$

For $\alpha \geq \frac{\pi}{3}$, the rms output voltage is equal to

$$V_{\text{rms}} = V_{o\text{rms}} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} V_{\text{RB}}^2 \cdot d(\omega t) \right]^{1/2} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \left(\sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)\right)^2 \cdot d(\omega t) \right]^{1/2}$$

$$= \left[\frac{3}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right) \cdot d(\omega t) \right]^{1/2} = \left[\frac{9V_m^2}{4\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \left\{1 - \cos 2\left(\omega t - \frac{\pi}{6}\right)\right\} \cdot d(\omega t) \right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi} \left\{\omega t - \frac{1}{2}\sin 2\left(\omega t - \frac{\pi}{6}\right)\right\}_{\frac{\pi}{6}+\alpha}^{\frac{7\pi}{6}} \right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi} \left\{\frac{7\pi}{6} - \frac{\pi}{6} - \alpha - \frac{1}{2}\sin 2\left(\frac{7\pi}{6} - \frac{\pi}{6}\right) + \frac{1}{2}\sin 2\left(\frac{\pi}{6} + \alpha - \frac{\pi}{6}\right)\right\} \right]^{1/2}$$

$$= \left[\frac{9V_m^2}{4\pi} \left\{\pi - \alpha + \frac{1}{2}\sin 2\alpha\right\} \right]^{1/2} = \frac{3V_m}{2} \left[\frac{1}{\pi} \left\{\pi - \alpha + \frac{1}{2}\sin 2\alpha\right\} \right]^{1/2}$$

If firing angle $\alpha \leq 60°$, thyristors and diodes conduct for 120° duration. When the firing angle $\alpha \geq 60°$, thyristors and diodes conduct for $180° - \alpha$ duration.
For $\alpha \leq 60°$*:* Rms current ratings of thyristor and diode are:

$$I_{T\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}} \quad \text{and} \quad I_{D\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}}$$

Average current ratings of thyristor and diode are:

$$I_{T\_\text{av}} = \frac{I_{o\_\text{av}}}{3} \quad \text{and} \quad I_{D\_\text{av}} = \frac{I_{o\_\text{av}}}{3}$$

For $\alpha \geq 60°$*:* Rms current ratings of thyristor and diode are:

$$I_{T\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}} \quad \text{and} \quad I_{D\_\text{rms}} = \frac{I_{o\_\text{rms}}}{\sqrt{3}}$$

Average current ratings of thyristor and diode are:

$$I_{T\_\text{av}} = \frac{I_{o\_\text{av}}}{3} \quad \text{and} \quad I_{D\_\text{av}} = \frac{I_{o\_\text{av}}}{3}$$

8.8 EFFECT OF SOURCE IMPEDANCE ON THE PERFORMANCE OF THREE-PHASE FULL BRIDGE CONVERTERS

Figure 8.41 shows a three-phase full bridge converter with a source inductance L_s in each line. During analysis of this converter, we assume that the load current I_o is constant and ripple free.

The conduction of thyristors at firing angle $\alpha = 0°$ and overlap angle $\mu = 0°$ are depicted in Fig. 8.41. It is clear from Fig. 8.41 that thyristors, T_5, T_6 conducts up to $\omega t = 30°$. Thyristors T_1 and T_6 conduct from $\omega t = 30°$ to $\omega t = 90°$ (60° duration). After that T_1 and T_2 conduct from $\omega t = 90°$ to $\omega t = 150°$. Similarly, other thyristor pairs conduct sequentially and each pair conducts for 60° duration. It is also revealed from Fig. 8.41 that only two thyristors conduct at a time, one from the positive (upper) group and other from the negative (lower) group.

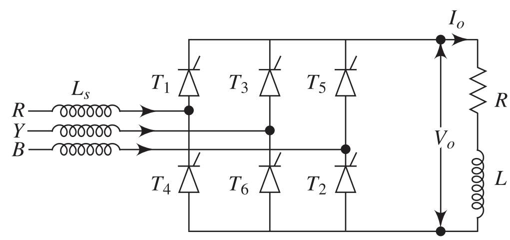


Fig. 8.41 *Three-phase full bridge converter with a source inductance L_s in each line*

Due to presence of source inductance L_s, there will be overlap of conduction of thyristors as load current I_o will be shared by two thyristors and one conduction thyristor is going towards commutation process and one incoming thyristor is going towards conduction process. Assume thyristors, T_5 and T_6 conduct up to $\omega t = 30°$. At $\omega t = 30°$, incoming thyristors T_1 is going towards conduction process and outgoing thyristor T_5 is going towards commutation (turn-OFF) process. Both thyristors T_1 and T_5 are belonging to positive group. Since thyristor T_1 is triggered at $\omega t = 30°$, current through T_5 starts decaying but the current through T_1 starts to build up. Hence, during the overlap angle μ, load current is shared by thyristors T_1 and T_5. At $\omega t = 30° + \mu$, current through thyristor T_5 becomes zero and full load current flows through thyristor T_1. Consequently, during $30° \leq \omega t \leq 30° + \mu$ three thyristors T_5, T_6 and T_1 conduct. At $\omega t = 30° + \mu$, only T_1 from upper group conduct with T_6. Thus, when a positive group thyristor is going towards commutation, three thyristors conduct with two from the positive group and one from negative group.

Similarly at $\omega t = 90°$, incoming thyristors T_2 is going towards conduction process and outgoing thyristor T_6 is going towards commutation (turn OFF) process. Both thyristors T_2 and T_6 are belong to negative group. As thyristor T_2 is triggered at $\omega t = 90°$, current through T_6 starts decaying but the current through T_2 starts to build up. Therefore during the overlap angle μ, load current is shared by thyristors T_2 and T_6. At $\omega t = 90° + \mu$, current through thyristor T_6 becomes zero and full load current flows through thyristor T_2. As a result during $90° \leq \omega t \leq 90° + \mu$, three thyristors T_1, T_6 and T_2 conduct. At $\omega t = 90° + \mu$, only T_2 from lower group conduct with T_1. Hence, when a negative group thyristor is going towards commutation, three thyristors conduct with two from the negative group and one from positive group. The conduction of thyristors due to overlap will be as follows:

$$T_5\text{-}T_6,\ T_5\text{-}T_6\text{-}T_1,\ T_1\text{-}T_6,\ T_1\text{-}T_6\text{-}T_2,\ T_1\text{-}T_2,\ T_1\text{-}T_2\text{-}T_3,\ T_2\text{-}T_3,\ T_2\text{-}T_3\text{-}T_4,\ T_3\text{-}T_4,\ T_3\text{-}T_4\text{-}T_5,\ T_4\text{-}T_5,\ T_4\text{-}T_5\text{-}T_6,\ T_5\text{-}T_6$$

It is clear from above sequence that three thyristors and two thyristors conduct alternatively. In Fig. 8.42, there are six shaded areas with six commutations per cycle of input voltage.

When thyristor T_5 is going towards commutation and T_1 starts conduction, current will be transferred from T_5 to T_1, the output voltage is equal the average of corresponding phase voltage V_Y and V_B of the positive group. Therefore, the output voltage follows the curve $\frac{V_R + V_B}{2}$ for positive group during $30° \le \omega t \le 30° + \mu$. During commutation of T_6 and T_2 starts conduction, the output voltage follows the curve $\frac{V_Y + V_B}{2}$ for negative group during $90° \le \omega t \le 90° + \mu$. Similarly, when thyristor T_1 is going towards commutation and T_3 starts conduction, current will be transferred from T_1 to T_3 and the output voltage follows the curve $\frac{V_R + V_Y}{2}$ for positive group during $150° \le \omega t \le 150° + \mu$.

Due to the presence of source inductance L_s, the average dc output voltage can be reduced. The amplitude of fall in reduced output voltage is directly proportional to the triangular area *a-b-c* as depicted in Fig. 8.42.

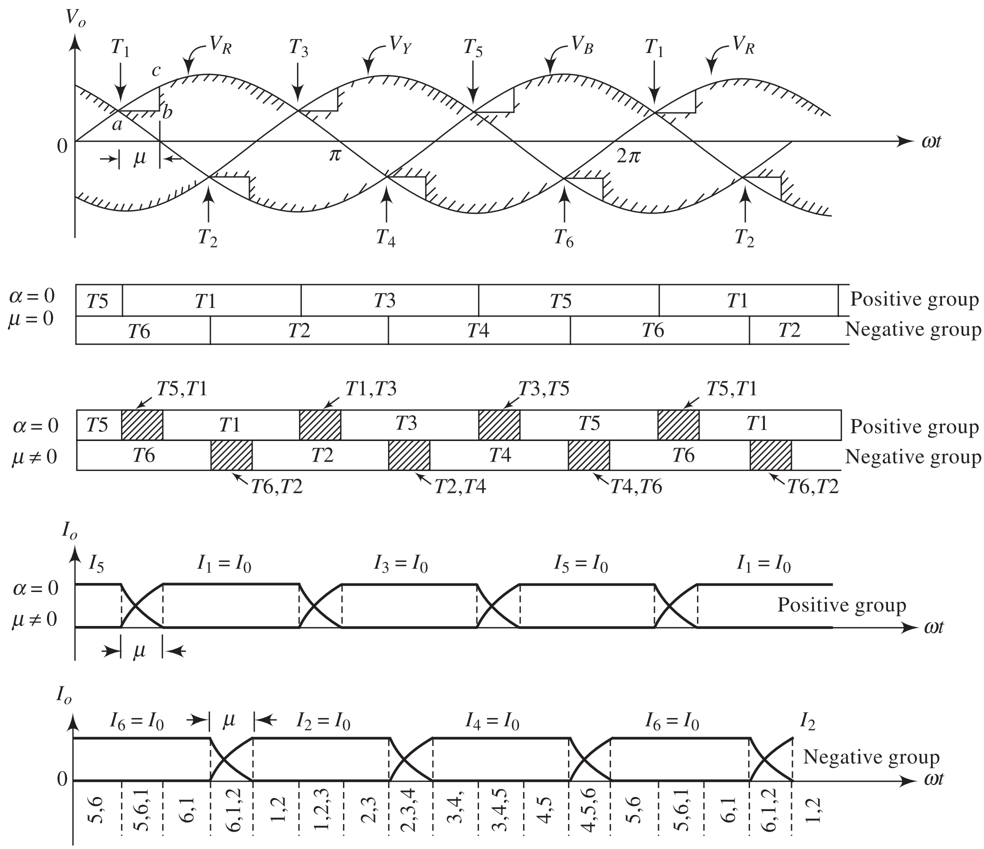


Fig. 8.42 *Voltage and current waveforms of a three phase full bridge converter with overlap*

The amplitude of fall in reduced output voltage is equal to

$$\frac{3}{\pi}\int_0^{\mu} v_L \cdot d(\omega t) = \frac{3}{\pi}\int_0^{\mu} L_s \frac{di}{dt}\cdot d(\omega t)$$

$$= \frac{3L_s}{\pi}\int_0^{\frac{\mu}{\omega}} \omega \cdot \frac{di}{dt} dt = \frac{3\omega L_s}{\pi}\int_0^{I_o} di = \frac{3\omega L_s}{\pi} I_o$$

The output voltage of three-phase fully controlled bridge converter with no overlap ($\mu = 0$) is

$$V_{\text{av}} = V_o = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha$$

The output voltage of three-phase fully controlled bridge converter with over lap angle μ is

$$V_{\text{av—with}\,\mu} = V_{o\_\text{with}\,\mu} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha - \frac{3\omega L_s}{\pi} I_o = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha - R_C I_o$$

where, $R_C = \frac{3\omega L_s}{\pi}$ is commutation resistance

The current I_o for a three-phase fully controlled bridge converter is

$$I_o = \frac{\sqrt{3}V_m}{2\omega L_s}\left[\cos\alpha - \cos(\alpha+\mu)\right]$$

After substituting the value of I_o in the output voltage equation with overlap angle μ is equal to

$$V_{\text{av—with}\,\mu} = V_{o\_\text{with}\,\mu} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha - \frac{3\omega L_s}{\pi}\frac{\sqrt{3}V_m}{2\omega L_s}\left[\cos\alpha - \cos(\alpha+\mu)\right]$$

$$= \frac{3\sqrt{3}V_m}{\pi}\cos\alpha - \frac{3\sqrt{3}V_m}{2\pi}\left[\cos\alpha - \cos(\alpha+\mu)\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi}[\cos\alpha + \cos(\alpha+\mu)]$$

If the overlap angle $\mu = 0$, the output voltage is $V_{\text{av}} = V_o = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha$

The output voltage for a three-phase full converter for similar way of representation of single phase full converter is

$$V_o = \frac{3\sqrt{3}V_m}{\pi}\cos(\alpha+\mu) + \frac{3\omega L_s}{\pi} I_o$$

The voltage regulation of a three-phase full converter due to source inductance is equal to

$$\frac{3\omega L_s I_o}{\pi} \times \frac{1}{V_{o\text{-at-no-load}}} = \frac{2\pi f L_s I_o}{\sqrt{3}V_m \cos\alpha}$$

Example 8.20 A three-phase full bridge converter is connected to a load with ripple free current I_o. When source inductance L_s is connected in each line, prove that the load is equal to

$$I_o = \frac{\sqrt{3}V_m}{2\omega L_s}[\cos\alpha - \cos(\alpha+\mu)]$$

Solution

Figure 8.43 shows a equivalent circuit of a three-phase full converter when thyristors T_1, T_5 and T_6 are conducting. Thyristor T_1 is turned on at $\alpha = 0°$, current flow in T_5 starts to decay where as current in T_1 starts to increase from zero. The overlap angle is μ.

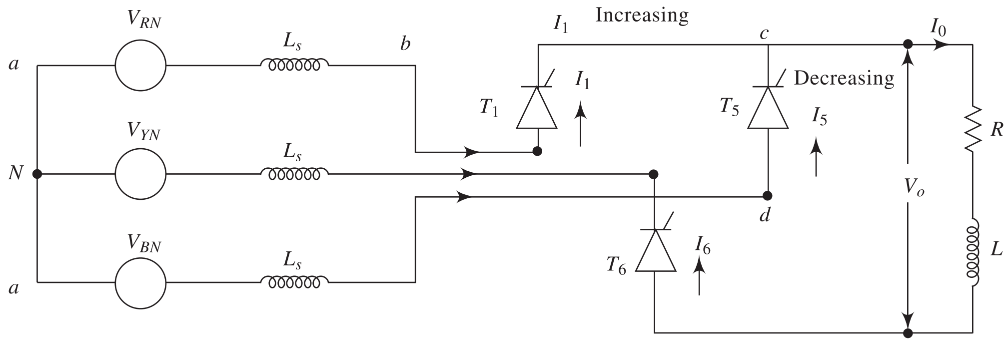


Fig. 8.43 *Equivalent circuit of a three-phase full converter when thyristors T_1, T_5 and T_6 are conducting*

KVL equation in the loop a-b-c-d-a is expressed by

$$V_R - L_s\frac{di_1}{dt} = V_B - L_s\frac{di_5}{dt}$$

or $$V_R - V_B = L_s\left[\frac{di_1}{dt} - \frac{di_5}{dt}\right] \tag{8.1}$$

$$V_R = V_m \sin\omega t \text{ and } V_B = V_m\sin(\omega t - 240°) = V_m\sin(\omega t + 120°)$$

$$V_R - V_Y = V_m\sin\omega t - V_m\sin(\omega t + 120°)$$

$$= V_m[\sin\omega t - \sin(\omega t + 120°)] = \sqrt{3}V_m\sin(\omega t - 30°)$$

After substituting the value of $(V_R - V_Y)$ in Eq. (8.1), we obtain

$$\left[\frac{di_1}{dt} - \frac{di_5}{dt}\right] = \frac{\sqrt{3}V_m}{L_s}\sin(\omega t - 30°) \tag{8.2}$$

During the overlap period, the sum of current flows through T_1 and T_5 is equal to zero

$$i_1 + i_5 = I_o$$

or $$\frac{di_1}{dt} + \frac{di_5}{dt} = \frac{dI_o}{dt} = 0 \quad \text{as} \quad I_o \text{ is constant}$$

or $$\frac{di_1}{dt} + \frac{di_5}{dt} = 0 \tag{8.3}$$

After adding Eqs. (8.2) and (8.3), we get

$$\frac{di_1}{dt} = \frac{\sqrt{3}V_m}{2L_s}\sin(\omega t - 30°)$$

At $\omega t = 30° + \alpha$, $i_1 = 0$. At $\omega t = 30° + \alpha + \mu$, $i_1 = I_1 = I_o$.

$$\int_0^{I_o} di = \int_{\left(\frac{\pi}{6}+\alpha\right)/\omega}^{\left(\frac{\pi}{6}+\alpha+\mu\right)/\omega} \frac{\sqrt{3}V_m}{2L_s}\sin\left(\omega t - 30°\right)\cdot dt$$

$$= \frac{\sqrt{3}V_m}{2\omega L_s}\left[-\sin\left(\omega t - \frac{\pi}{6}\right)\right]_{\left(\frac{\pi}{6}+\alpha\right)/\varpi}^{\left(\frac{\pi}{6}+\alpha+\mu\right)/\omega} = \frac{\sqrt{3}V_m}{2\omega L_s}\left[\cos\alpha - \cos(\alpha+\mu)\right]$$

Hence, it is proved that $I_o = \frac{\sqrt{3}V_m}{2\omega L_s}[\cos\alpha - \cos(\alpha+\mu)]$ when source inductance L_s is connected in each line of a three-phase full bridge converter and load current I_o is ripple free.

Example 8.21 A three-phase M-3 converter is supplied from a three phase, 440 V, 50 Hz ac supply and it is connected to a load *RLE* with ripple free current of 20 A. If $R = 2\ \Omega$, $E = 250$ V, (a) determine the firing angle for inverter mode operation and (b) when a source inductance of 5 mH is connected in each line, determine the firing angle and overlap angle of inverter.

Solution

(a) Phase voltage is $V = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.04\text{ V}$,

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2}\times 254.04\text{ V} = 359.26\text{ V}$

When the converter working as an inverter, the average output voltage of converter is

$$V_{\text{av}} = V_o = -E + I_oR$$

We know $V_{\text{av}} = V_o = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha = \frac{3\sqrt{3}\times 359.26}{2\pi}\cos\alpha$

Therefore, $\frac{3\sqrt{3}\times 359.26}{2\pi}\cos\alpha = -250 + 20\times 2 = -210$

The firing angle for inverter mode operation is $\alpha = \cos^{-1}\left(-\frac{210\times 2\pi}{3\sqrt{3}\times 359.26}\right) = 134.94°$

(b) The voltage drop in average output voltage due to source inductance is

$$\frac{3\omega L_s}{2\pi}I_o = \frac{3\times 2\pi f\times L_s}{2\pi}I_o = 3fL_sI_o = 3\times 50\times 5\times 10^{-3}\times 20 = 15\text{ V}$$

Then $\frac{3\sqrt{3}\times 359.26}{2\pi}\cos\alpha - 15 = -250 + 20\times 2 = -210\text{ V}$

or $\frac{3\sqrt{3}\times 359.26}{2\pi}\cos\alpha = -195\text{ V}$

The firing angle for inverter mode operation is $\alpha = \cos^{-1}\left(-\frac{195\times 2\pi}{3\sqrt{3}\times 359.26}\right) = 130.99°$

For a three-phase converter

$$\cos(\alpha+\mu) = \cos\alpha - \frac{2\omega L_s}{V_{ml}}I_o$$

$$= \cos 130.99 - \frac{2\times 2\pi\times 50\times 5\times 10^{-3}}{\sqrt{3}\times 359.26}20 = -0.7568 = \cos 139.18$$

Therefore, $130.99 + \mu = 139.18$ or $\mu = 8.19°$

Example 8.22 A three-phase full bridge converter is fed from a 220 V per phase, 50 Hz ac supply. It is connected to a *RLE* load with ripple free 15 A dc current. When $E = 250$ V, $R = 2\ \Omega$, determine (a) firing angle of converter and (b) overlap angle of converter. Assume $L_s = 5$ mH

Solution

(a) Phase voltage is $V = 200$ V,

Maximum phase voltage $V_m = \sqrt{2}V = \sqrt{2} \times 220\text{ V} = 311.12\text{ V}$

When the converter working as controlled rectifier, the average output voltage of converter is

$$V_{\text{av}} = V_o = E + I_o R \quad 250 + 15 \times 2 = 280\text{ V}$$

We know $V_{\text{av}} = V_o = \dfrac{3\sqrt{3}V_m}{\pi}\cos\alpha - \dfrac{3\omega L_s}{\pi}I_o = \dfrac{3\sqrt{3} \times 311.12}{\pi}\cos\alpha - \dfrac{3 \times 2\pi f L_s}{\pi}I_o$

Therefore, $\dfrac{3\sqrt{3} \times 311.12}{\pi}\cos\alpha - \dfrac{3 \times 2\pi \times 50 \times 5 \times 10^{-3}}{\pi} \times 15 = 250 + 15 \times 2$

$$\frac{3\sqrt{3} \times 311.12}{\pi}\cos\alpha = 250 + 15 \times 2 + 22.5 = 302.5$$

The firing angle for rectifier mode operation is $\alpha = \cos^{-1}\left(\dfrac{302.5 \times \pi}{3\sqrt{3} \times 311.12}\right) = 54.015°$

(b) The output voltage of converter is

$$V_o = \frac{3\sqrt{3}V_m}{\pi}\cos(\alpha + \mu) + \frac{3\omega L_s}{\pi}I_o$$

$$280 = \frac{3\sqrt{3} \times 311.12}{\pi}\cos(54.015 + \mu) + \frac{3 \times 2\pi \times 50 \times 5 \times 10^{-3}}{\pi} \times 15$$

or $\cos(54.015 + \mu) = \cos(59.88)$

The overlap angle of converter $\mu = 59.88 - 54.015 = 5.865°$

8.9 TWELVE-PULSE CONVERTERS

The three-phase fully controlled bridge converters are extensively used in the medium to moderately high power applications. Usually low frequency harmonics (sixth in dc side, fifth and seventh in ac side) voltage and currents are generated by the 6 pulse bridge converter. Therefore, these converters may be connected in series and parallel combination in order to increase the voltage and current rating of converter respectively. When the converters are controlled properly, the lower order harmonics can be eliminated from both input and output side. Hence a higher number pulse converter is developed. In very high power applications such as high voltage dc (HVDC) transmission systems, higher number pulse converters are used.

Figure 8.44 shows the series connection of two six-pulse converters. In this circuit, the star and delta connected secondary windings have $\pi/6(30°)$ displacement between their output voltages. If a star connected and a delta connected bridge rectifier are connected in series, the output voltage ripple frequency is twelve times of the fundamental frequency ($f_o = 12f$). Converter-I and converter-II are identical in construction and they have same firing angle α. The input ac supplies have same amplitude but they are phase displaced by an angle ϕ. The output voltage of converter-I is

$$V_{o1} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha + \sum_{n=1,2,3,4,5...}^{\infty} V_{an}\cos 6n\omega t + \sum_{n=1,2,3,4,5,...}^{\infty} V_{bn}\sin 6n\omega t$$

The output voltage of converter-II is

$$V_{o2} = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha + \sum_{n=1,2,3,4,5...}^{\infty} V_{an}\cos 6n(\omega t - \phi) + \sum_{n=1,2,3,4,5,...}^{\infty} V_{bn}\sin 6n(\omega t - \phi)$$

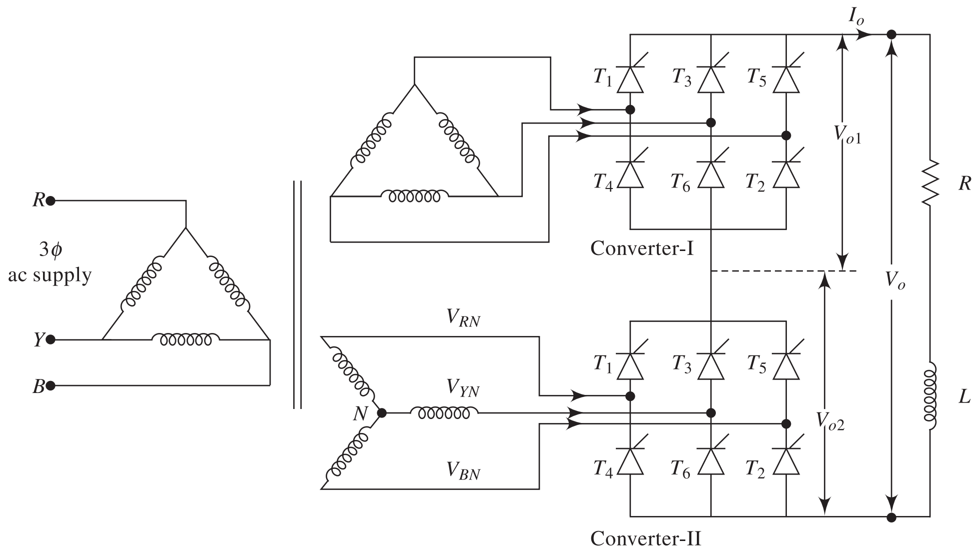


Fig. 8.44 *Series connection of six-pulse converters*

The output voltage of twelve-pulse converter is equal to

$$V_o = V_{o1} + V_{o2}$$

$$= \frac{6\sqrt{3}V_m}{\pi}\cos\alpha + 2\sum_{n=1,2,3,4,5...}^{\infty} \cos 3n\phi[V_{an}\cos 3n(2\omega t - \phi) + V_{bn}\sin 3n(2\omega t - \phi)]$$

If cos $3n\phi = 0$, then the corresponding n^{th} harmonic may disappear.
If $\phi = 30°$, cos $3n\phi = 0$ where n = 1, 2, 3, 4

Then $V_o = \frac{6\sqrt{3}V_m}{\pi}\cos\alpha + 2\sum_{m=1,2,3,4,5...}^{\infty} [V_{am}\cos 12m\omega t + V_{bm}\sin 12m\omega t]$

Hence, the frequency of harmonics present in output voltage of a 12-pulse converter are:

$12f$, $24f$, $36f$ … and the ac input current contains the harmonic frequency $11f$, $13f$, $23f$, $35f$, …

Usually six-phase series bridge controlled rectifiers are used for high output voltage applications. But for high output current applications, six-phase parallel bridge controlled rectifiers are used. Figure 8.45 shows the parallel connections of two six-pulse bridge controlled rectifiers with an inter-phase transformer. The output voltage of converter-I is V_{o1} and The output voltage of converter-II is V_{o2}. The output voltage across load is the average of the rectified output voltages V_{o1} and V_{o2}. The output ripple frequency of six-phase parallel bridge controlled rectifiers is 12 times of fundamental frequency. Normally filter circuit is not required .When the circuit is a balanced one, the output current of two three phase units will not generate dc magnetisation current.

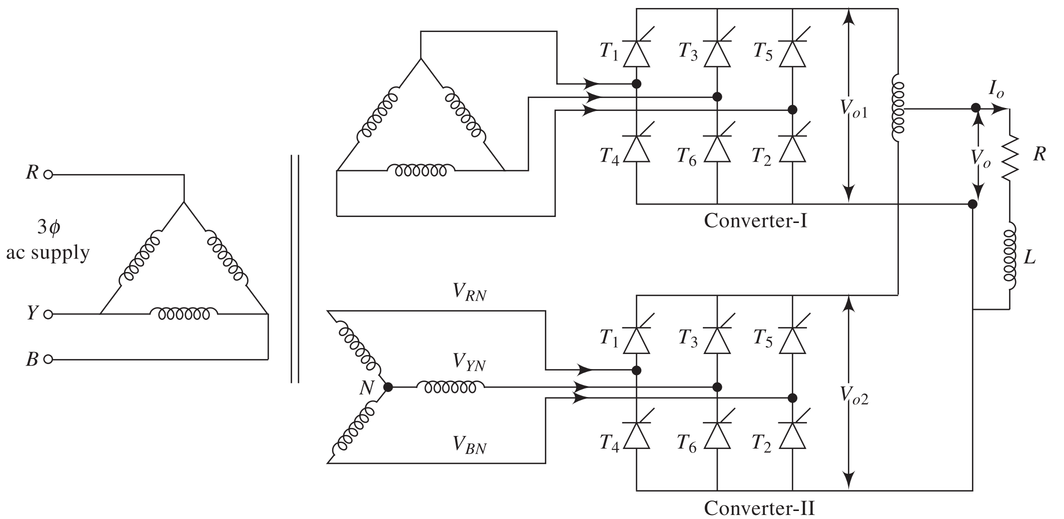


Fig. 8.45 *Parallel connection of two six-pulse converters*

8.10 THREE-PHASE DUAL CONVERTERS

A three-phase fully controlled converter can provide both positive and negative voltage but it cannot supply current in both directions. In some applications such as four quadrant operation of dc motor, only a three-phase fully controlled converter can provide required control features. This problem can be solved by connecting two three-phase fully controlled converters in antiparallel. In Fig. 8.46, the converter-1 supplies positive load current whereas the converter-2 supplies the negative load current. The converter-1 operates in the first and fourth quadrant but the converter-2 operates in second and third quadrant. In this way, two converters can operate together in all four quadrants. This combined converter is known as *dual converter*.

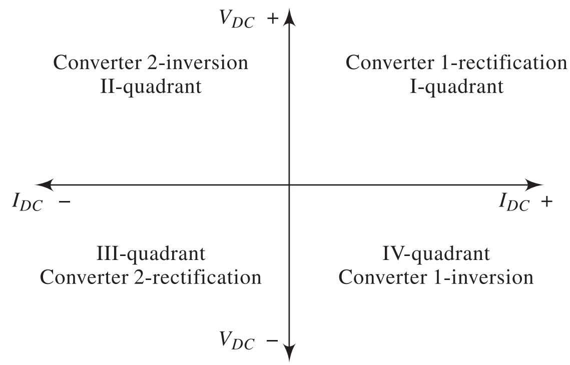
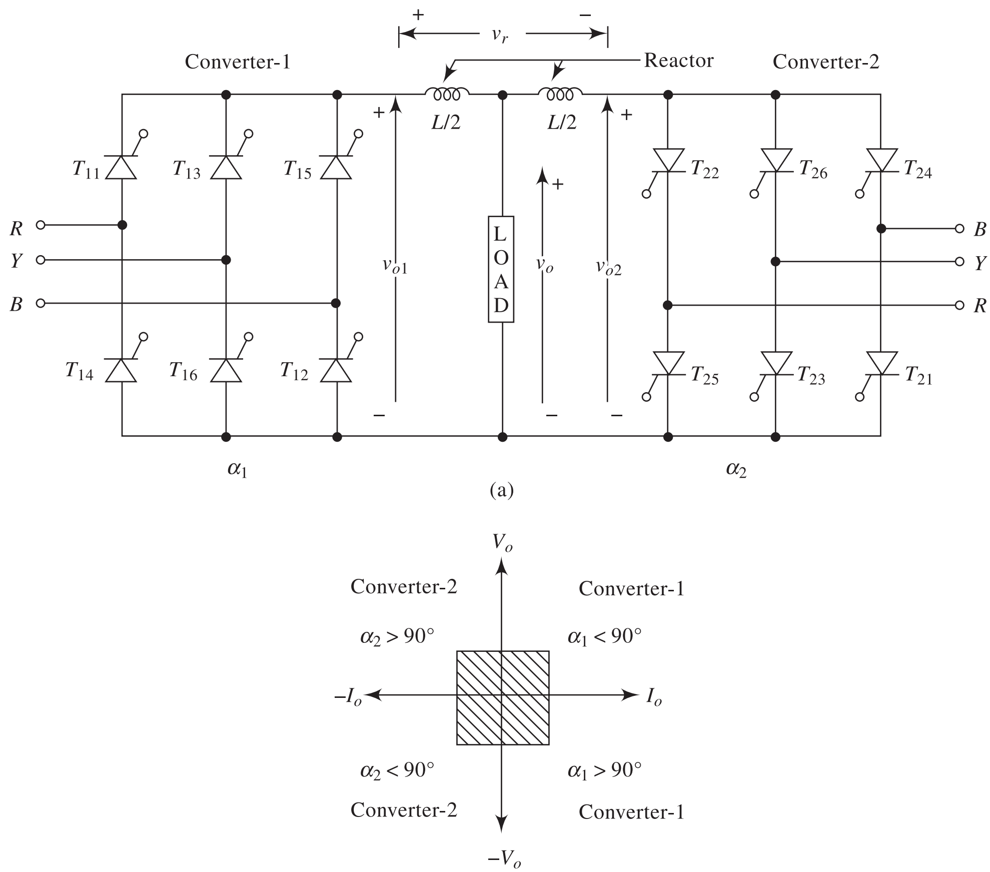


Fig. 8.46 *Four quadrant operation of converters*

Full converters operate as two quadrant (I-quadrant and IV-quadrant) converters. In a full converter, the direction of current can not be reverse due to unidirectional property of thyristor but the polarity of output voltage can be reversed as depicted in Fig. 8.46. A full converter can operate in the first quadrant as a controlled rectifier where V_o (V_{DC}) and I_o (I_{DC}) are positive and firing angle varies in the control range $0 \le \alpha \le \frac{\pi}{2}$. It can also operate in the fourth quadrant as a controlled rectifier where V_o is negative and I_o is positive and firing angle varies in the control range $\frac{\pi}{2} \le \alpha \le \pi$. In the first quadrant, power flows from ac supply to dc load and in the fourth quadrant power flows from load to ac source.

DC motors are operated in four quadrants such as forward motoring, forward regeneration, reverse motoring and reverse regeneration. For such operation, four quadrant converters are required. Dual converters are used as four quadrant converters. Dual converters are those converters in which two fully controlled converters are connected in back to back to the load circuit as depicted in Figure 8.47(a) and four quadrant operation of dual converter is given in Fig. 8.47(b).



(b)

Fig. 8.47 *(a) Dual converter using three phase full converters and (b) Four quadrant operation of dual converter*

8.10.1 Operating Principle of Ideal Dual Converter

The basic operating principle of dual converter can be explained using the most simplified equivalent circuit diagram of the dc circuit as illustrated in Fig. 8.48. This circuit consists of two ideal two quadrant converters such as converter-1 and converter-2, two diodes such as D_1 and D_2 and load. It is clear from Fig. 8.48 that two ideal two quadrant converters are assumed to be controllable direct voltage sources connected in series with the diodes.

For analysis of dual converter, it is assumed that dual converters are made by ideal full converters and there is no ripple in the output voltage. Therefore, these converters generate pure dc output voltage without any ac ripple at the dc terminals. The current can flow in either direction through diodes D_1 and D_2 which represents the unidirectional current flow of converters. The firing angle of converters are controlled by the control voltage V_C.

V_{o1} and V_{o2} are the average output voltages of converter-1 and converter-2 respectively. These output voltages are equal in magnitude but they are of opposite polarity. These can drive the current in opposite directions through load. Subsequently, whenever one converter operates as a controlled

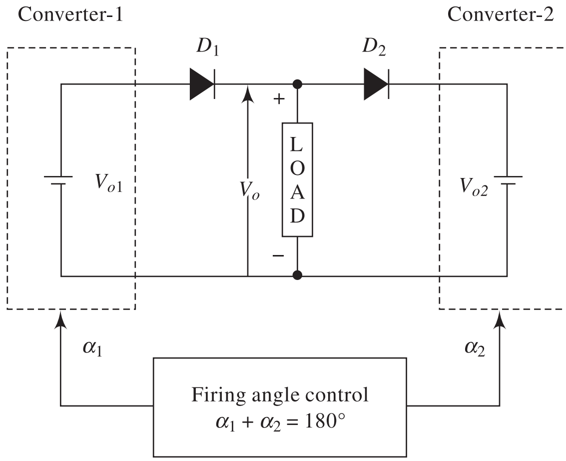


Fig. 8.48 *The equivalent circuit of an ideal dual converter*

rectifier, the other converter operates as an inverter. The converter working as a rectifier is called as *positive group converter* and the converter working as an inverter is known as *negative group converter*. The average output voltage of converter-1 is

$$V_{o1} = V_{\max} \cos \alpha_1$$

The average output voltage of converter-2 is

$$V_{o2} = V_{\max} \cos \alpha_2$$

For a three-phase full converter, $V_{\max} = \dfrac{3\sqrt{3}V_m}{\pi}$

For an ideal converter $V_o = V_{o1} = -V_{o2}$

Therefore, $\quad V_{\max} \cos\alpha_1 = -V_{\max} \cos\alpha_2$

or $\quad \cos\alpha_1 = -\cos\alpha_2 = \cos(180 - \alpha_2)$

or $\quad \alpha_1 = 180 - \alpha_2$ or, $\alpha_1 + \alpha_2 = 180$

The variation of output voltage with firing angle for the two converters is depicted in Fig. 8.49. The firing angles α_1 and α_2 are varied in such a way that $\alpha_1 + \alpha_2 = 180$ always.

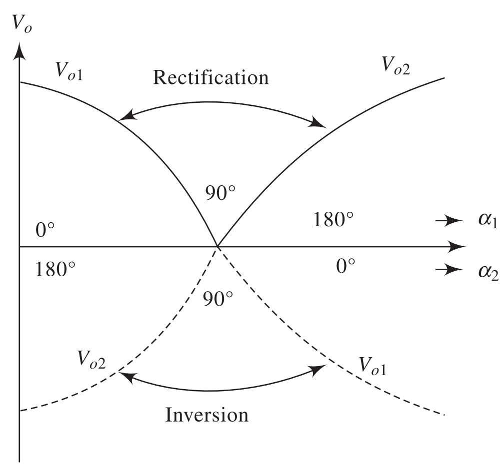


Fig. 8.49 *Variation of terminal voltage for an ideal dual converter with firing angle*

8.10.2 Practical Dual Converter

The firing angles α_1 and α_2 are controlled in such a way that $\alpha_1 + \alpha_2 = 180°$ always and the average output voltage of converter-1 and converter-2 are equal but opposite polarity. One converter operates as a rectifier with a firing angle α_1 and the other converter operates as an inverter with firing angle $\alpha_2 = 180° - \alpha_1$. Subsequently, the output voltage of converter-1 is equal to the output voltage of converter-2, though instantaneous output voltages V_{o1} and V_{o2} are out of phase in a practical dual converter. Therefore, there is a voltage difference when two converters are interconnected, a large circulating current flows between two converters but not through load. This circulating current can be limited to a specified value by inserting a reactor between two converters. The circulating current can be avoided by providing the trigger pulses. There are two operating modes of a practical dual converter such as non-circulating current mode and circulating current mode.

Non-circulating Current Mode Dual Converter In non-circulating current mode dual converter, only one converter operates at a time which alone carries the entire load current. Only this converter receives the firing pulses from the triggering circuit. The other converter remains blocked from conduction by removing the triggering pulses to that converter. Hence, only one converter is in operation at a time and the other converter is idle. As there is no circulating current flow, reactor is not required. The circuit diagram of non-circulating current dual converter is shown in Fig. 8.50.

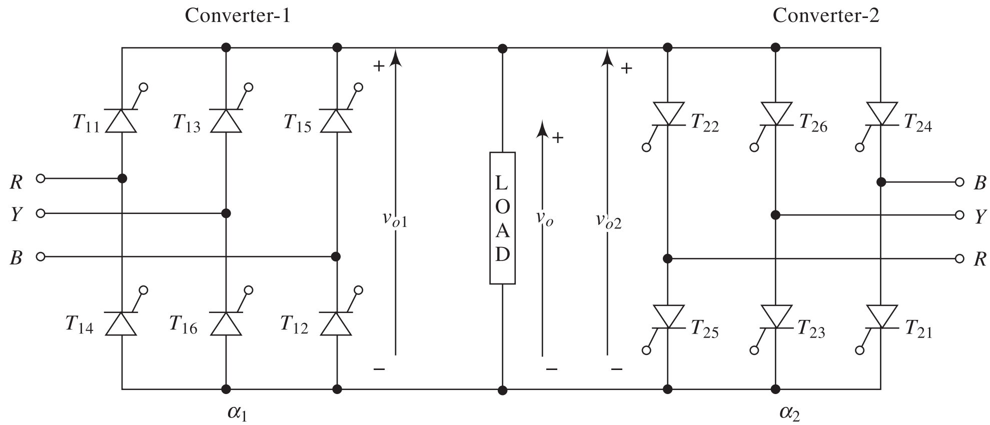


Fig. 8.50 *Non-circulating current mode dual converter*

If converter-1 is operating and is supplying the load current, converter-2 will be in the blocking state. In order to block the converter-1 from conduction and switch on the converter-2, it is required to commutate the thyristors of converter-1 either by removing the firing pulses to its thyristors or by increasing the firing angle of converter-1 to the maximum value such that its firing pulses are being blocked. Afterwards, the load current would decays to zero. As the triggering pulses are applied to the thyristors of converter 2, converter-2 gets switched on. Thus the converter-2 is in the operating mode and converter-1 remains idle. Now the load current builds up in the opposite direction. So long as converter-2 is in operation, converter-1 will be in idle since firing pulses are withdrawn from this converter. During changeover from one converter to other converter, a delay time of about 10 to 20 ms must be ensured between the instants at which converter-1 is in the OFF state and converter-2 is in the ON state. This delay time ensures the reliable commutation of thyristors present in the converter-1. If the thyristors of converter-2 are triggered before turn OFF of the thyristors of converter-1, a large circulating current would flow between the two converters which is a undesirable condition.

In non-circulating current dual converter, the load current may be continuous or discontinuous. The control circuit of dual converter is designed in such a way that the performance of non-circulating current dual converter is satisfactory during continuous or discontinuous load current.

Circulating Current Mode Dual Converter In circulating current mode dual converter, two converters are in the operating condition when one converter operates in the controlled rectifier mode and other operates in the inverting mode. In this converter, a rector is inserted in between converter-1 and converter-2. The rector is used to limit the amplitude of circulating current to a specified value. Figure 8.51 shows the circulating current mode dual converter. The voltage waveforms of a three-phase dual converter is illustrated in Fig. 8.52 at $\alpha_1 = 60°$ and $\alpha_2 = 120°$.

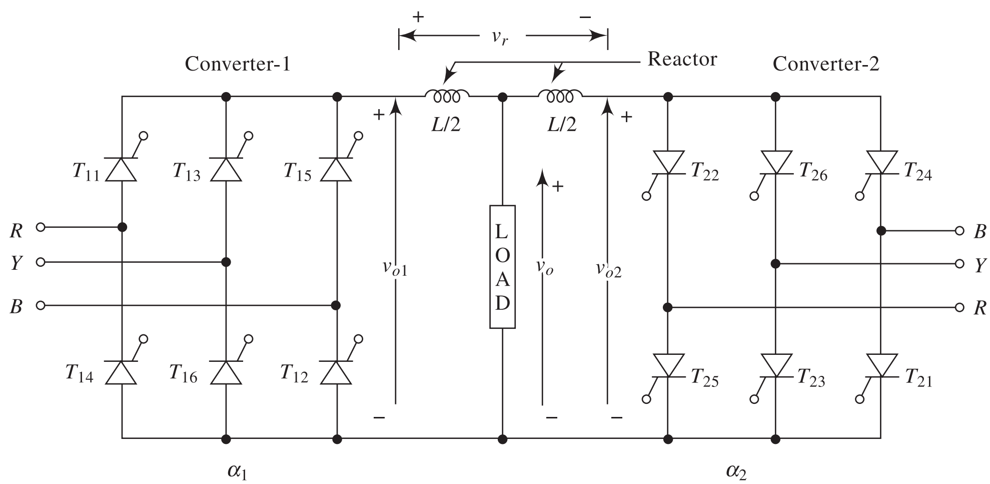


Fig. 8.51 *The circulating current mode dual converter*

The firing angles of two converters are adjusted in such a manner that $\alpha_1 + \alpha_2 = 180°$ is always satisfied. For example, if the firing angle of converter-1 is $\alpha_1 = 60°$ then the firing angle of converter-2 is $\alpha_2 = 120°$. In this case converter-1 operates as a controlled rectifier but the converter-2 operates as an inverter. The output voltage at the terminals of both converters has the same average output voltage. The instantaneous value of output voltages V_{o1} and V_{o2} are not similar as depicted in Fig. 8.52. Consequently, a large circulating current flows between the two converters. To limit the circulating current, a reactor must be introduced between converter-1 and converter-2.

The load current can be reversed by interchanging the role of two converters. Then converter-1 should operate as an inverter and converter-2 should operate as rectifier. Therefore, the firing angle of converter-1 is greater than 90° and the firing angle of converter-2 is less than 90°. However the equation $\alpha_1 + \alpha_2 = 180°$ must be satisfied. The normal time delay 10 to 20 ms is not required in this converter. Consequently, the operation of this type of dual converter is faster.

Circulating current of dual converter When the firing angle $\alpha_1 < 90°$, converter-1 operates as a rectifier and it carries circulating current and load current. Since the load current is I_o and the circulating current is i_c, the current flows from converter–1 is $I_1 = I_o + i_c$.

As the firing angle $\alpha_2 > 90°$, converter-2 operates as an inverter and it carries circulating current i_c only. Then current flows from converter –2 is $I_2 = i_c$.

During $\frac{\pi}{3} + \alpha_1 < \omega t \le \frac{2\pi}{3} + \alpha_1$, the output voltages of converters are as follows:

Output voltage of converter-1 is $V_{o1} = V_{RY}$ and output voltage of converter-2 is $V_{o2} = V_{RB}$.

The voltage across reactor is $V_r = V_{o1} - V_{o2} = V_{\text{RY}} - V_{\text{YB}}$

We know that $V_{RY} = \sqrt{3}V_m \sin \omega t$ and $V_{YB} = \sqrt{3}V_m \sin(\omega t - 120)$

Therefore, $V_r = V_{o1} - V_{o2} = V_{RY} - V_{YB} = 3V_m \sin(\omega t + 30)$

The amplitude of circulating current i_c is the time integral of reactor voltage V_r and it is expressed by

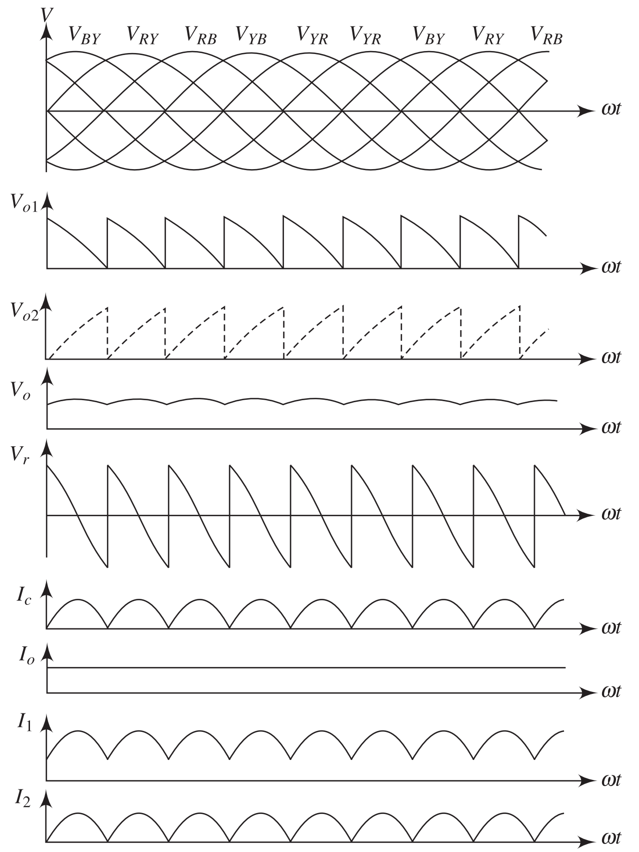


Fig. 8.52 *Voltage waveforms of a three-phase dual converter with α_1 = 60° and α_2 = 120°*

$$i_c = \frac{1}{L}\int_{\left(\frac{\pi}{3}+\alpha_1\right)/\omega}^{t} V_r \cdot dt = \frac{3V_m}{L}\int_{\left(\frac{\pi}{3}+\alpha_1\right)/\omega}^{t} \sin(\omega t + 30)\cdot dt$$

$$= \frac{3V_m}{\omega L}\int_{\left(\frac{\pi}{3}+\alpha_1\right)}^{\omega t} \sin(\omega t + 30)\cdot d\omega t$$

or
$$i_c = \frac{3V_m}{\omega L}\left[-\sin\alpha_1 - \cos\left(\omega t + \frac{\pi}{6}\right)\right]$$

It is clear from above equation that the magnitude of circulating current depends upon the firing angle α_1 and ωt. At any value of firing angle, the peak value of circulating current occurs at $\omega t = \frac{5\pi}{6}$. The peak value of circulating current is

$$i_{c\max} = \frac{3V_m}{\omega L}[1 - \sin\alpha_1]$$

If $\alpha_1 = 0$, the peak value of current is $i_{c\max} = \frac{3V_m}{\omega L}$ where, V_m is the maximum phase voltage.

The disadvantages of dual converters are given below:

1. A reactor is used to limit the circulating current and the size and cost of this reactor is significantly high at high power levels.
2. The efficiency and power factor of dual converters are low due to the losses occurred by the circulating current.
3. Circulating current gives rise to more losses in the dual converters.
4. Since the converters have to handle load and circulating current, the current rating of thyristors used in dual converters must be high.

Though the dual converters have the above disadvantages, a dual converter with circulating current mode dual converter is preferred when the load current is to be reversed frequently and whenever a fast response four-quadrant operation is required.

Example 8.23 A three-phase dual converter operates in the circulating current mode when per phase rms voltage is 220 V, 50 Hz and L = 20 mH with firing angle $\alpha_1 = 45°$. (a) Find the expression of circulating current. (b) Determine the peak value of circulating current.

Solution

Given $V_m = \sqrt{2}\, V = \sqrt{2} \times 220$ V, f = 50 Hz, L = 20 mH

(a) The circulating current is equal to

$$i_c = \frac{3V_m}{\omega L}\left[-\sin\alpha_1 - \cos\left(\omega t + \frac{\pi}{6}\right)\right]$$

$$= \frac{3\sqrt{2} \times 220}{2\pi \times 50 \times 20 \times 10^{-3}}\left[-\sin 45 - \cos\left(\omega t + \frac{\pi}{6}\right)\right] \quad \text{as } \alpha_1 = 45°$$

$$= \frac{3\sqrt{2} \times 220}{2\pi \times 50 \times 20 \times 10^{-3}}\left[-\sin 45 - \cos\left(\omega t + \frac{\pi}{6}\right)\right]$$

$$= 148.62\left[-\sin 45 - \cos\left(\omega t + \frac{\pi}{6}\right)\right]$$

(b) The peak value of circulating current is

$$i_{c\max} = \frac{3V_m}{\omega L}[1 - \sin\alpha_1] = \frac{3\sqrt{2} \times 220}{2\pi \times 50 \times 20 \times 10^{-3}}[1 - \sin 45] = 43.529 \text{ A}$$

Summary

- The operation of different three phase controlled rectifiers with resistive (R), inductive (L) and back emf (E) type loads are explained in detail.
- The continuous and discontinuous modes of operation of three phase converters depend on load parameters and firing angle. The operation of converters in continuous and discontinuous mode of operation is discussed in this chapter.
- The effect of source impedance on the performance of three phase full bridge converters is explained briefly.
- The operation of three phase dual converters is explained in detail.

Multiple-Choice Questions

8.1 In a three-phase half-wave controlled rectifier each thyristor conducts for ______ duration
(a) 180° (b) 150° (c) 120° (d) 60°

8.2 The dc output voltage of a three-phase half-wave controlled rectifier at firing angle α is

(a) $\frac{1}{2\pi/3}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin\omega t \cdot d(\omega t)$ (b) $\frac{1}{2\pi/6}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin\omega t \cdot d(\omega t)$

(c) $\frac{1}{2\pi/3}\int_{\pi/3+\alpha}^{5\pi/3+\alpha} V_m \sin\omega t \cdot d(\omega t)$ (d) $\frac{1}{2\pi/6}\int_{\pi/3+\alpha}^{5\pi/3+\alpha} V_m \sin\omega t \cdot d(\omega t)$

8.3 The frequency of output voltage of a three-phase half-wave controlled rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

8.4 The peak inverse voltage of each diode of a three-phase half-wave controlled rectifier is
(a) $\sqrt{3}E_m$ (b) $\sqrt{2}E_m$ (c) E_m (d) E

8.5 In a six-phase half-wave controlled rectifier each thyristor conducts for ______ duration.
(a) 180° (b) 150° (c) 120° (d) 60°

8.6 The dc output voltage of a six-phase half-wave controlled rectifier is

(a) $\frac{1}{2\pi/3}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin\omega t \cdot d(\omega t)$ (b) $\frac{1}{2\pi/6}\int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_m \sin\omega t \cdot d(\omega t)$

(c) $\frac{1}{2\pi/6}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin\omega t \cdot d(\omega t)$ (d) $\frac{1}{2\pi/3}\int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_m \sin\omega t \cdot d(\omega t)$

8.7 The frequency of output voltage of a six-phase half-wave controlled rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

8.8 The peak inverse voltage of each diode of a six-phase half-wave controlled rectifier is
(a) $\sqrt{3}E_m$ (b) $\sqrt{2}E_m$ (c) E_m (d) E

8.9 In a three-phase bridge converter each thyristor conducts for ______ duration.
(a) 180° (b) 150° (c) 120° (d) 60°

8.10 The dc output voltage of a three-phase full-wave bridge converter is

(a) $\frac{1}{2\pi/6}\int_{\pi/3+\alpha}^{2\pi/3+\alpha} \sqrt{3}V_m \sin\omega t \cdot d(\omega t)$ (b) $\frac{1}{2\pi/6}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} V_m \sin\omega t \cdot d(\omega t)$

(c) $\frac{1}{2\pi/3}\int_{\pi/3+\alpha}^{5\pi/3+\alpha} V_m \sin\omega t \cdot d(\omega t)$ (d) $\frac{1}{2\pi/6}\int_{\pi/3+\alpha}^{5\pi/3+\alpha} V_m \sin\omega t \cdot d(\omega t)$

8.11 The frequency of output voltage of a three-phase fully controlled bridge converter is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

8.12 At firing angle α, the dc output voltage of three-phase half-wave controlled rectifier and three-phase six-pulse controlled rectifier are __________ and _________ respectively.

(a) $\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha, \frac{3V_m}{\pi}\cos\alpha$ (b) $\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha, \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$

(c) $\frac{\sqrt{3}V_m}{2\pi}\cos\alpha, \frac{3V_m}{2\pi}\cos\alpha$ (d) $\frac{6V_m}{2\pi}\cos\alpha, \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$

8.13 The frequency of output voltage of a three-phase 12-pulse controlled rectifier is
(a) $f_o = f$ (b) $f_o = 3f$ (c) $f_o = 6f$ (d) $f_o = 12f$

8.14 The number of pulse of a three-phase 12-pulse controlled rectifier is
(a) 3 (b) 6 (c) 9 (d) 12

8.15 At firing angle α ($\alpha > 30°$), the dc output voltage of three-phase half-wave controlled rectifier is
(a) $\frac{3\sqrt{3}V_m}{\pi}\cos\alpha$ (b) $\frac{\sqrt{3}V_m}{2\pi}\cos\alpha$ (c) $\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$ (d) $\frac{3\sqrt{3}V_m}{2\pi}\cos 2\alpha$

8.16 At firing angle α ($\alpha > 30°$), the dc output voltage of three phase half-wave controlled rectifier is
(a) $\frac{3V_m}{2\pi}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$ (b) $\frac{3V_m}{2\pi}\left[1+\cos\left(\frac{\pi}{6}-\alpha\right)\right]$
(c) $\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$ (d) $\frac{3\sqrt{3}V_m}{2\pi}\cos 2\alpha$

8.17 At firing angle α, the dc output voltage of six-phase half-wave controlled rectifier is
(a) $\frac{3\sqrt{3}V_m}{\pi}\cos\alpha$ (b) $\frac{\sqrt{3}V_m}{2\pi}\cos\alpha$ (c) $\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$ (d) $\frac{3V_m}{\pi}\cos\alpha$

8.18 A three-phase full bridge converter operates as an inverter, if firing angle
(a) $\alpha > 30°$ (b) $\alpha > 45°$ (c) $\alpha > 60°$ (d) $\alpha > 90°$

8.19 A three-phase full bridge converter operates as a rectifier, if firing angle
(a) $\alpha > 90°$ (b) $\alpha > 120°$ (c) $\alpha > 150°$ (d) $\alpha < 90°$

8.20 At firing angle α, the dc output voltage of three-phase full bridge converter is
(a) $\frac{3\sqrt{3}V_m}{\pi}\cos\alpha$ (b) $\frac{\sqrt{3}V_m}{2\pi}\cos\alpha$ (c) $\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$ (d) $\frac{3V_m}{\pi}\cos\alpha$

8.21 At firing angle α, the dc output voltage of three-phase semi-converter is
(a) $\frac{3\sqrt{3}V_m}{\pi}\cos\alpha$ (b) $\frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$ (c) $\frac{3\sqrt{3}V_m}{2\pi}\cos\alpha$ (d) $\frac{3V_m}{\pi}\cos\alpha$

8.22 A converter can operate in both three-pulse and six-pulse mode
(a) three-phase semi-converter (b) three-phase full bridge converter
(c) one-phase full converter (d) three-phase half-wave converter

8.23 The effect of source inductance on the performance of three-phase full converter is to
(a) reduce the average dc output voltage (b) reduce the ripples in the load current
(c) obtain discontinuous current (d) increase the load current

8.24 For four quadrant operation,
(a) two full converters are connected in parallel
(b) two full converters are connected in series
(c) two full converters are connected back to back
(d) two semi-converters are connected back to back

8.25 In a three-phase full converter, the output voltage pulsates at a frequency of
(a) $6f$ (b) $3f$ (c) $2f$ (d) f

8.26 In a circulating current type of dual converter, the voltage across reactor is
(a) triangular wave (b) sine wave
(c) pulsating wave (d) alternating triangular wave (approx)

Fill in the Blanks

8.1 In a three-phase three-pulse half-controlled rectifier with *RL* load, each thyristor conducts for _____________ duration.
8.2 A three-phase half-controlled converter (semiconverter) cannot able to operate in _____ mode.
8.3 A three-phase half-controlled converter (semiconverter), thyristors conduct just like a __________ rectifier and diode conduct just like _________ rectifier.
8.4 In a three-phase half-wave controlled rectifier, peak inverse voltage across each thyristor is PIV = _______.
8.5 For a three-phase half-wave controlled rectifier, current in the transformer secondary is __________.
8.6 In three-phase mid-point six-pulse controlled rectifier, each thyristor conducts for _______ duration.
8.7 A three-phase fully controlled bridge converter has_______ number of pulses.
8.8 In a three-phase fully controlled bridge converter each thyristor conducts for _______ duration.
8.9 In a six-phase controlled rectifier, the peak inverse voltage across each thyristor is ______.
8.10 The input ac current of a three-phase fully controlled converter consists of _______harmonics only.
8.11 A three-phase fully controlled converter operates in rectifier mode during firing angle_____.
8.12 A three-phase half-controlled converter (semiconverter) consists of three thyristors and ___________.
8.13 A three-phase 12-pulse controlled rectifier consists of _________________ thyristor.
8.14 The quality of dc output voltage _____________ significantly with three-phase 12-pulse controlled rectifier.
8.15 Due to the presence of source inductance in each line of the three-phase full converter, commutation is not ____________________.
8.16 The duration of overlap angle depends on the value of _______ and ______.
8.17 Three-phase fully controlled bridge converter can develop by replacing six diodes of an uncontrolled rectifier by _______.
8.18 A three-phase fully controlled converter operates in inverter mode during firing angle_____.
8.19 If a three-phase fully bridge converter operates in continuous conduction, there will be ________ different conduction modes.
8.20 The PIV across each thyristor of a three-phase fully controlled bridge converter is equal to ________.
8.21 A three-phase half-controlled converter (semiconverter) has ___________ conduction modes.
8.22 The three-phase half-controlled converter (semiconverter) operate in ______ mode only.
8.23 In a three-phase half-controlled converter (semiconverter), each thyristor conduct for ______duration and each diode conduct for _______ duration.
8.24 The output voltage and current of a three-phase half controlled converter consists of ________harmonics of fundamental frequency ac supply.
8.25 The input current of three phase half-controlled converter contain ______harmonics only.
8.26 The output voltage of three-phase converter with firing angle α and commutation angle μ is ___________.
8.27 In a three-phase fully controlled converter ______ thyristors conduct during the overlap period and the overlap angle is about ______.
8.28 The average output voltage of a ac-to-dc converter decreases due to _______.
8.29 In a three-phase converter, the commutation resistance is _______.
8.30 In the dc equivalent circuit representation of a converter, ac source inductance is represented as ____________.
8.31 The commutation overlap angle generates _______ in the input ac supply voltage.
8.32 In a three-phase half-controlled converter (semiconverter), the output voltage waveform is periodic over ____________ of input ac supply voltage cycle.
8.33 The output voltage waveform of a three-phase half-controlled converter (semiconverter), for $\alpha < 60°$ and for $\alpha > 60°$ are __________ but the value of average output voltage is ___________.

Review Questions

8.1 (a) What are the advantages of three-phase controlled rectifier over single-phase controlled rectifier?
(b) Give a list of applications of a three-phase controller rectifiers.

8.2 What are the types of three-phase controlled rectifiers? Write the detail classification of three-phase controlled converters.

8.3 Draw the circuit diagram of three-phase half-wave controlled rectifier with *RL* load and explain its operating principle with voltage and current waveforms. Determine the following parameters for *RL* load with firing angle $\alpha = 30°$:
(a) dc output voltage (b) Average dc load current (c) rms output voltage (d) rms load current
(e) Ripple factor (f) TUF (g) Efficiency

8.4 Draw the circuit diagram of three-phase half-wave controlled rectifier with *R* load and explain its operating principle with voltage and current waveforms. Determine the following parameters for R load with firing angle $\alpha = 60°$:
(a) dc output voltage (b) Average dc load current (c) rms output voltage (d) rms load current
(e) Ripple factor (f) TUF (g) Efficiency

8.5 A three-phase converter is fed to a *RLE* load and draw the output voltage and current waveforms. Derive the expression of current flow through load.

8.6 (a) Why are three-phase controlled rectifiers preferred over single-phase controlled rectifiers?
(b) Give a list of applications of three-phase controlled rectifiers.

8.7 (a) Draw the circuit diagram of a three-phase mid point six-pulse controlled rectifier with *R* load. Discuss its working principle.
(b) Draw the voltage and current waveforms. Determine the following parameters:
(i) dc output voltage (ii) Average dc load current (iii) rms output voltage
(iv) rms load current (v) Form factor (vi) Ripple factor
(vii) Efficiency (vii) TUF

8.8 (a) Draw the circuit diagram of a three-phase bridge converter with *R* load. Discuss its working principle.
(b) Draw the voltage and current waveforms. Determine the following parameters:
(i) dc output voltage (ii) Average dc load current
(iii) rms output voltage (iv) rms load current

8.9 Draw the circuit diagram of a three phase bridge converter with *RL* load. Explain its working principle at $\alpha = 30°$.

8.10 (a) What is multiphase controlled rectifier? What are the advantages of multiphase controlled rectifiers?
(b) Derive the average output voltage and rms output voltage of a multiphase controlled rectifier.

8.11 What are the advantages of three-phase bridge controlled rectifier over three-phase mid point six-pulse controlled rectifier?

8.12 A three-phase full converter is connected to a *R* load. Prove that the average output voltage across load is

$$V_o = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha \quad \text{for } 0 \le \alpha \le \frac{\pi}{3}$$

and
$$V_o = \frac{3\sqrt{3}V_m}{\pi}\left[1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right] \quad \text{for } \frac{\pi}{3} \le \alpha \le \frac{2\pi}{3}$$

where, V_m is the maximum phase voltage.

8.13 (a) Draw the circuit diagram of a three-phase semiconverter with *RL* load. Explain its working principle with voltage and current waveforms at $\alpha = 30°$.
(b) Explain whether freewheeling diode comes into play or not.
(c) Derive the expression for average output voltage.

8.14 Prove that the performance of three-phase full converter is effected by the source inductance L_s. Prove that $\cos(\alpha + \mu) = \cos\alpha - \frac{2\omega L_s}{V_{ml}} I_o$

8.15 Discuss how two three-phase full converters can be connected back to back to make a circulating current type dual converter. Discuss its operation with the help of voltage waveforms across (a) converter-1 and converter-2, (b) load and (c) reactor. Assume $\alpha_1 = 0°$.

8.16 Derive an expression for the circulating current in terms of input voltage, reactor inductance, and firing angle for three-phase dual converter.

8.17 A three-phase half-wave controlled rectifier is fed from a 3ϕ, 400 V, 50 Hz ac supply and it is connected with a R load of 10 Ω.When the firing angle of thyristor is 30°, calculate (a) dc output voltage, (b) rms value of output voltage, (c) output voltage form factor, (d) ripple factor, (e) average output current, (f) rms load current, (g) dc output power, (h) ac power supplied and (i) rectification efficiency and TUF.

8.18 A three-phase half-wave controlled rectifier is connected to a 3ϕ, 230 V, 50 Hz ac supply and it is also connected with a resistive load of 5 Ω. If the firing angle of thyristor is 45°, calculate (a) dc output voltage, (b) rms value of output voltage, (c) average output current, (d) rms load current and (e) TUF.

8.19 A three-phase step-down delta-star transformer with per phase turn ratio 100 is fed from a 3ϕ, 11 kV, 50 Hz ac supply and it is connected with three-phase half-wave controlled rectifier. When R is equal to 10 Ω and firing angle is 15°, determine (a) dc output voltage, (b) rms value of output voltage, (c) average output current, (d) rms load current, (e) power delivered to load and (f) maximum value of load current.

8.20 A three-phase half-wave controlled rectifier is fed from a 3ϕ, 440 V, 50 Hz ac supply and it is connected with a R load of 10 Ω.When the average dc output voltage is equal to 65% of the maximum dc output voltage, determine (a) the firing angle of thyristor, (b) dc output voltage and (c) rms value of output voltage.

8.21 A three-phase three-pulse controlled rectifier is fed from a 3ϕ, 230 V, 50 Hz ac supply and it is connected with a R load of 7.5 Ω. If the average dc output voltage is equal to 90% of the maximum dc output voltage, determine (a) the firing angle of thyristor, (b) dc output voltage, (c) rms value of output voltage and (d) rectification efficiency.

8.22 A three-phase three-pulse controlled rectifier is fed from a 3ϕ, 400 V, 50 Hz ac supply and it is connected with a constant current load of 40 A. The voltage drop across each thyristor is 2 V. (a) Determine the dc output voltage at firing angle of 60° and 45°. (b) Calculate the average and rms current rating and PIV of thyristors. (c) Determine the average power dissipated in each thyristor.

8.23 A 150 V battery is charged using three phases half-wave rectifier as depicted in Fig. 8.53. The input phase voltage is 230 V, 50 Hz and the firing angle of thyristors is 30°. Determine the average current flows through the battery. Draw the charging voltage and charging current waveforms.

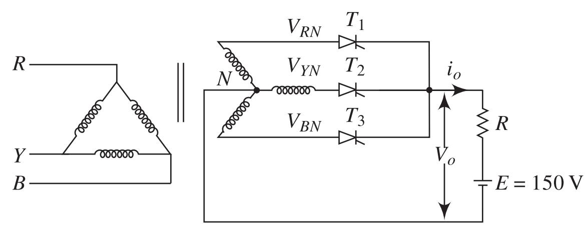


Fig. 8.53

8.24 A three-phase three-pulse controlled rectifier with free wheeling diode D_F is fed from a 3ϕ, 420 V, 50 Hz ac supply and it is connected with a constant current load of 90 A at firing angle of 45°. Determine (a) the dc output voltage, (b) rms output voltage, (c) the average and rms current of thyristors, (d) the average and rms current of free-wheeling diode.

8.25 A six-phase half-wave controlled rectifier is fed from a 3ϕ, 400 V, 50 Hz ac supply and it is connected with a R load of 5 Ω. If the firing angle of thyristor is 30°, determine (a) dc output voltage, (b) rms value of output voltage, (c) form factor, (d) ripple factor, (e) average output current, (f) rms load current, (g) dc output power, (h) ac power supplied and (i) rectification efficiency.

8.26 A six-phase half-wave controlled rectifier is fed from a 3ϕ ac supply and it is connected with a R load with firing angle 60°. Determine (a) form factor and (b) voltage ripple factor.

8.27 A three-phase step-down delta-star transformer with per phase turn ratio 5 is fed from a 3ϕ, 1000 V, 50 Hz ac supply and it is connected with six-phase half-wave controlled rectifier . When R is equal to 8 Ω and $\alpha = 30°$, determine (a) maximum load current, (b) dc output voltage, (c) rms value of output voltage, (d) average output current, (e) rms load current, (f) power delivered to load and (g) average and rms value of thyristor current.

8.28 A three-phase full converter is used to charge a 200 V battery from a 230 V, 50 H ac supply as depicted in Fig. 8.54. Assume the internal resistance of battery is 0.5 Ω and an inductance is connected in series with a battery so that the 10 A constant charging current flows. (a) Determine the firing angle of converter and the input power factor. (b) If power flows from battery to ac side, what will be the firing angle of converter?

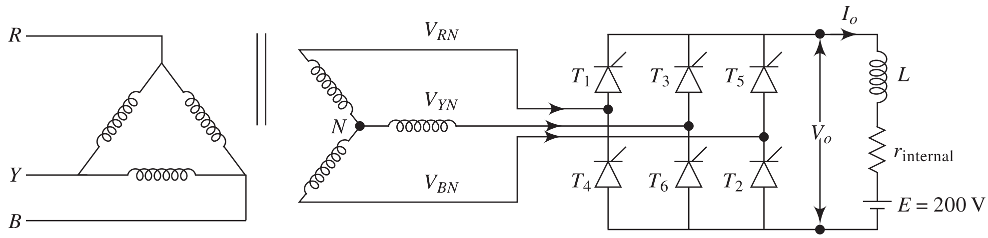


Fig. 8.54

8.29 A three-phase full bridge converter is fed from a delta-star transformer and it is connected to a *RL* load with ripple free current 18 A at firing angle 45°. It is fed from 440 V, 50 Hz ac supply, determine rectification efficiency, transformer utilisation factor and input power factor.

8.30 A three-phase full bridge converter is fed from 400 V, 50 Hz ac supply and is connected to a *RLE* load with ripple free load current. If R = 10 Ω, E = 250 V and inductance is very large, determine the power delivered to load input power factor at firing angle (a) α = 45° and (b) α = 60° and (c) firing angle advance of 75°.

8.31 A three-phase full bridge converter provides a ripple free load current of 17 A with a firing angle of 30°. The input voltage of the converter is three-phase, 410 V, 50 Hz.
(a) Derive the expression of source current by Fourier series. (b) Determine DF, CDF, HF and PF.
(c) Find the active and reactive input power.

8.32 A three-phase full converter is connected to a load resistance of 5 Ω and it is supplied from a 230 V, 50 H ac supply. If the firing angle of thyristor is α = 30°, determine (a) average output voltage, (b) average output current, (c) rms output voltage and (d) rms output current.

8.33 A three-phase full converter is connected to a load resistance of 7.5 Ω. If the firing angle of thyristor is α = 30°, three phase converter feeds 4 kW power to a resistive load. Determine the amplitude of maximum per phase input voltage.

8.34 A three-phase semiconverter is connected to a *RL* load with R = 10 Ω. If the firing angle of thyristors is 60° and it fed power of 5 kW. Determine the maximum amplitude of per phase input voltage.

8.35 A three-phase semiconverter is connected to a *RL* load with of 10 Ω. If the firing angle of thyristors is 70° and it fed power of 5 kW. Determine the maximum amplitude of per phase input voltage.

8.36 A three phase semi converter is used to charge a 200 V battery from a 230 V, 50 H ac supply as depicted in Fig. 8.55. Assume the internal resistance of battery is 0.5 Ω and an inductance is connected in series with a battery so that the 10 A constant charging current flows. Determine the firing angle of converter, conduction period of each thyristor and the input power factor.

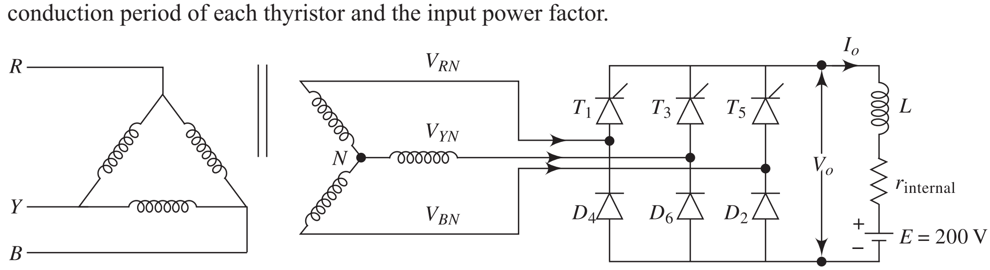


Fig. 8.55

8.37 A three-phase full bridge converter is connected to a load with ripple free current L_o. When source inductance L_s is connected in each line, prove that the load is equal to

$$I_o = \frac{\sqrt{3}V_m}{2\omega L_s}\left[\cos\alpha - \cos(\alpha+\mu)\right]$$

8.38 A three-phase M-3 converter is supplied from a three-phase, 400 V, 50 Hz ac supply and it is connected to a load *RLE* with ripple free current of 20 A. If $R = 2\ \Omega$, $E = 250$ V, (a) determine the firing angle for inverter mode operation. (b) When a source inductance of 5 mH is connected in each line, determine the firing angle and overlap angle of inverter.

8.39 A three-phase full bridge converter is fed from a 230 V per phase, 50 Hz ac supply. It is connected to a *RLE* load with ripple free 15 A dc current. When $E = 250$ V, $R = 2\ \Omega$, determine (a) firing angle of converter and (b) overlap angle of converter.

8.40 A three-phase dual converter operates in the circulating current mode when per phase rms voltage is 230 V, 50 Hz and $L = 20$ mH with firing angle $\alpha_1 = 45°$. (a) Find the expression of circulating current. (b) Determine the peak value of circulating current.

Answers to Multiple-Choice Questions

8.1 (c)	8.2 (a)	8.3 (b)	8.4 (a)	8.5 (d)	8.6 (b)	8.7 (c)
8.8 (c)	8.9 (c)	8.10 (b)	8.11 (c)	8.12 (a)	8.13 (d)	8.14 (d)
8.15 (c)	8.16 (a)	8.17 (d)	8.18 (d)	8.19 (d)	8.20 (a)	8.21 (b)
8.22 (a)	8.23 (a)	8.24 (c)	8.25 (a)	8.26 (d)		

Answers to Fill in the Blanks

8.1 120°
8.2 inverter
8.3 controlled, uncontrolled
8.4 $\sqrt{3}V_m$
8.5 unidirectional
8.6 60°
8.7 six
8.8 120°
8.9 $2V_m$
8.10 odd
8.11 $\alpha \le 90°$
8.12 three diodes
8.13 12
8.14 improved
8.15 instantaneous
8.16 source inductance, load current
8.17 six thyristors
8.18 $\alpha > 90°$
8.19 six
8.20 $\sqrt{3}V_m$
8.21 six
8.22 rectifier
8.23 120°, 120°
8.24 triplen
8.25 even
8.26 $= \dfrac{3\sqrt{3}V_m}{\pi}\cos\alpha - R_C I_o$
8.27 three, 6 to 10 degree
8.28 commutation overlap
8.29 $R_C = \dfrac{3\omega L_s}{\pi}$
8.30 commutation resistance
8.31 notches
8.32 one third
8.33 (different, same)

AC VOLTAGE CONTROLLERS AND CYCLOCONVERTERS

9

9.1 INTRODUCTION

Conventionally power flow through ac system can be controlled by adding series impedance in system. Due to additional series impedance, there will be the unwanted voltage drop across the impedance and some power is also dissipated in this impedance. Usually, auto-transformers or variacs are used to control the voltage across load but this method is inefficient. Nowadays silicon controlled rectifiers (SCRs) are used in *ac voltage controller*. ac voltage controller using SCRs is also equivalent to an auto transformer. This method is very efficient than the previous one. Basically, ac voltage controller's operating principle is that it can block the unwanted power flow rather than dissipate at the control device.

Silicon controlled rectifiers (SCRs) have capability to flow current in one direction only. When two SCRs are connected back to back, it is possible to flow current in bi-directional. Hence combination of two SCRs can be used as bi-directional switch in ac circuits. The ac-to-ac converters receive electric power from fixed voltage ac utility system and convert it into variable voltage ac system. Actually, ac-to-ac converters are used to vary the RMS output voltage at load at constant frequency and these converters are called as *ac voltage controllers* or *ac voltage regulators*.

When ac-to-ac converters receive power at fixed frequency voltage and converts into another ac system at different frequency with variable voltage, these converters are known as *cycloconverters*. In cycloconverter, there is no intermediate converter stage.

The control strategies of ac voltage controllers are ON-OFF control or Integral Cycle Control and Phase control. The detail classification of ac voltage controllers is shown in Fig. 9.1. Semiconductor switches such as thyristors or triacs are used in ac

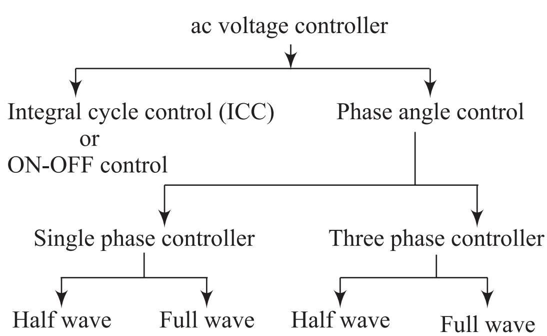


Fig. 9.1 *Classification of ac voltage controllers*

voltage controllers. As ac voltage controllers are phase controlled, thyristors or triacs are natural or line commutated and there is no requirement of commutation circuit. But the disadvantage of ac voltage controller is the introduction of harmonics in the ac supply voltage and output voltage across load.

Usually ac voltage controllers are used in lighting control system, temperature and heating control system, speed control of induction motors and on line tap changing of transformers, etc. Generally, the cycloconverters are suitable in the area of high-power low speed induction motor drives applications. This is specially used in cement mills, rolling mills. In this chapter, the operation of different types of ac controllers are explained elaborately.

9.2 INTEGRAL CYCLE CONTROL

The circuit diagram of ON-OFF or Integral Cycle ac voltage controller is depicted in Fig. 9.2. Figure 9.2(a) is the anti-parallel (back to back or inverse parallel) connection of two thyristors. When the thyristor is forward biased, thyristor T_1 is to be gated and conducts for current in positive direction. T_2 must be gated during negative half cycle and conduct. The current flows in negative direction. Gate pulse i_{g1} is applied between K_1 and G_1 for thyristor T_1. Gate pulse i_{g2} is applied for thyristor T_2 between K_2 and G_2. The ac voltage controller using triac is illustrated in Fig. 9.2(b). In this circuit, Triac is bi-directional switch and it can be conduct during positive as well as negative half cycle of supply voltage. Gate pulse of a triac is applied through i_g. Since semiconductor switches such as thyristors or triacs are switched on at the zero crossing of the input voltage and turn-OFF at zero current, this type of ac voltage controller is called zero-voltage switching ac voltage controller. This controller is also know as burst-firing or cycle selection or cycle syncopation. The supply of harmonics and radio frequency interference are very low in this controller.

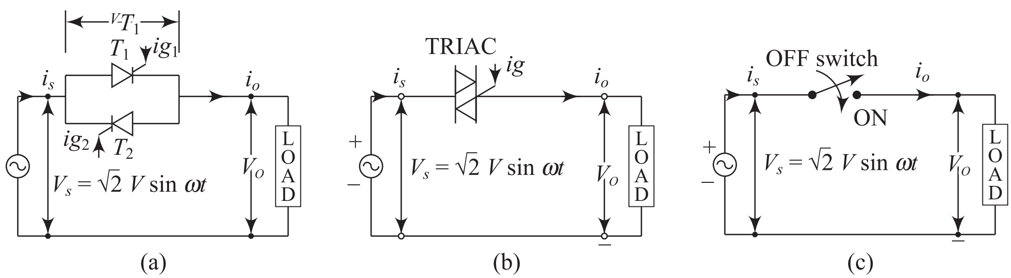


Fig. 9.2 *ON-OFF or integral cycle ac voltage Controller (a) using two thyristors, (b) using TRIAC and (c) equivalent circuit using switch*

The equivalent circuit of ac voltage controller is shown in Fig. 9.2(c). In this circuit, the switch is ON for integral number of half cycles (n) and the switch is OFF for integral number of half cycles (m). When the switch is ON, output voltage across load is the input supply voltage. If the switch is OFF, output voltage across load is 0. This method is known as *integral cycle control*, where the output voltage is function of 'n' and 'm'. In this case, switches are ON in every half-cycle. The current waveform has less distortion compared to phase control. This type of converters is used in some fields where load time constant is large. For example, temperature control system as heating time constant is large. The output voltage waveform of integral cycle control is shown in Fig. 9.3.

The RMS value of output voltage V_o is

$$V_o = \left[\frac{n}{n+m}\frac{1}{\pi}\int_0^{\pi}(\sqrt{2}V\sin\omega t)^2 d(\omega t)\right]^{1/2}$$

$$= V\left[\frac{n}{n+m}\frac{1}{\pi}\int_0^{\pi}(1-\cos 2\omega t)d(\omega t)\right]^{1/2}$$

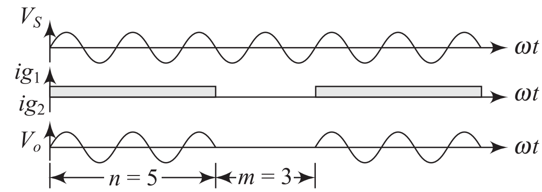


Fig. 9.3 *Output voltage waveform of integral cycle control ac voltage controller*

Then V_o can be expressed as

$$V_o = V\sqrt{\frac{n}{n+m}}$$

or $V_o = V\sqrt{K}$ where, $K = \dfrac{n}{n+m}$ is the *duty cycle*, V = RMS input voltage

In case resistive load, the RMS value of load current, I_L is

$$I_L = \frac{V_o}{R} = \frac{V\sqrt{K}}{R} \quad \text{where } R \text{ is load resistance}$$

Power output for resistive load, P_o can be expressed as

$$P_o = \frac{V_o^2}{R} = \frac{V^2K}{R} = \frac{n}{n+m}\frac{V^2}{R}$$

The Input Power $= V\cdot\dfrac{V_o}{R}$

Power factor $= \dfrac{\text{Output Power}}{\text{Input Power}} = \dfrac{V_o \times V_o/R}{V \times V_o/R} = \dfrac{V_o}{V}$

$$= \sqrt{\frac{n}{n+m}} = \sqrt{K}$$

Each thyristor conducts for 180° duration for each half cycle of 'n' integral number of half cycles, average current flows through thyristor, I_A is

$$I_A = \frac{n}{n+m}\frac{1}{2\pi}\frac{1}{R}\int_0^{\pi}\sqrt{2}V\sin\omega t d(\omega t)$$

$$= \frac{n}{n+m}\frac{1}{2\pi R}\int_0^{\pi}\sqrt{2}V\sin\omega t d(\omega t)$$

$$= \frac{n}{n+m}\frac{I_m}{\pi} = \frac{K\cdot I_m}{\pi} \qquad \text{where, } I_m = \frac{\sqrt{2}V}{R}$$

RMS current flow through thyristor, I_{RMS} is

$$I_{\text{RMS}} = \left[\frac{n}{n+m}\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\omega t\cdot d(\omega t)\right]^{1/2} = \sqrt{\frac{n}{n+m}}\frac{I_m}{2} = \frac{\sqrt{K}\cdot I_m}{2}$$

Example 9.1 In a furnace the heating resistance is 20 Ω which connected to 230 V, 50 Hz ac through a integral cycle control ac controller. Assume the switch is ON for four half cycles and OFF for two half cycles. Determine the following parameters: (a) Output voltage, (b) Power factor and (c) RMS value of load current

Solution

Given: resistance R is 20 Ω, Voltage V = 230 V, n = 4 and m = 2

The output voltage is $V_o = V\sqrt{\dfrac{n}{n+m}} = 230\sqrt{\dfrac{4}{4+2}} = 187.94 \text{ V}$

Power factor $= \sqrt{\dfrac{n}{n+m}} = \sqrt{\dfrac{4}{4+2}} = 0.818$

RMS value of load current $= I_L = \dfrac{V_o}{R} = \dfrac{187.94}{20} \text{ A} = 9.397 \text{ A}$

Example 9.2 A single-phase voltage controller is controlled by integral cycle control. Its input voltage is 230 V, 50 Hz ac. Assume it is ON for three half cycles and OFF for two half cycles and resistance R is 10 Ω. Determine the following parameters: (a) RMS output voltage, (b) Power output, (c) Power input, (d) Power factor and (e) average and RMS value of thyristor current

Solution

Given: Resistance R is 10 Ω, Voltage V = 230 V, n = 3 and m = 2

(a) The RMS output voltage is $V_o = V\sqrt{\dfrac{n}{n+m}} = 230\sqrt{\dfrac{3}{3+2}} = 178.15 \text{ V}$

(b) Power output for resistive load, P_o can be expressed as

$$P_o = \frac{V_o^2}{R} = \frac{(178.15)^2}{10} = 3173.74 \text{ Watt}$$

(c) The Input Power $= V \cdot \dfrac{V_o}{R} = 230 \times \dfrac{178.15}{10} = 4097.45 \text{ Watt}$

(d) Power factor $= \sqrt{\dfrac{n}{n+m}} = \sqrt{\dfrac{3}{3+2}} = 0.774$

(e) Average current flow through thyristor, I_A is

$$I_A = \frac{n}{n+m}\frac{I_m}{\pi} = \frac{3}{3+2}\frac{I_m}{\pi} \qquad \text{where, } I_m = \frac{\sqrt{2}V}{R} = \frac{\sqrt{2}\times 230}{10} = 32.522 \text{ A}$$

$$= \frac{3}{5} \times \frac{32.522}{\pi} \text{ A} = 6.214 \text{ A}$$

RMS current flow through thyristor, I_{RMS} is

$$I_{\text{RMS}} = \sqrt{\frac{n}{n+m}}\frac{I_m}{2} = \sqrt{\frac{3}{3+2}}\frac{32.522}{2} \text{ A} = 12.597 \text{ A}$$

Example 9.3 The resistance of 230 V, 5.5 kW furnace is 15 Ω and the furnace is controlled by integral cycle control. Determine (a) the duty ratio for power output is equal to 50% input power and power factor in this duty ratio and (b) the duty ratio for output voltage is equal to 50% rated voltage and power factor in this duty ratio.

Solution

Given: Resistance R is 15 Ω, Voltage V = 230 V

(a) The output voltage V_o is equal to

$$V_o = V\sqrt{K} \quad \text{where, } K = \frac{n}{n+m} \text{ is the } \textit{duty cycle}, V = \text{RMS input voltage}$$

Power output across load P_o can be expressed as

$$P_o = \frac{V_o^2}{R} = \frac{V^2 K}{R} = P_i K \quad \text{where, } P_i = \frac{V^2}{R} = \text{rated power input}$$

Duty ratio $K = \frac{P_o}{P_i} = \frac{0.5P_i}{P_i} = 0.5 \qquad \text{as } P_o = 0.5P_i$

Power factor $= \sqrt{K} = \sqrt{0.5} = 0.707$

(b) The output voltage V_o is equal to

$$V_o = V\sqrt{K}$$

or $V_o^2 = V^2 K$

Duty ratio $K = \frac{V_o^2}{V^2} = \frac{(0.5V)^2}{V^2} = \frac{0.25V^2}{V^2} = 0.25 \qquad \text{as } V_o = 0.5\text{ V}$

Power factor $= \sqrt{K} = \sqrt{0.25} = 0.5$

Example 9.4 A single-phase ac voltage controller is controlled by brust-firing control. It is used for heating a load of R = 5 Ω with input voltage is 230 V, 50 Hz ac. For a load power of 5 kW, find the following parameters: (a) Duty cycle (b) input power factor (c) average and RMS value of thyristor current

Solution

Given: Resistance R is 5 Ω, Voltage V = 230 V,

(a) The output voltage $V_o = V\sqrt{K}$ where, $K = \frac{n}{n+m}$ is the *duty cycle*, V = RMS input voltage

Power output across load P_o can be expressed as

$$P_o = \frac{V_o^2}{R} = \frac{V^2 K}{R}$$

Duty ratio $K = \frac{P_o \times R}{V^2} = \frac{5\times 10^3 \times 5}{230^2} = 0.4725 \qquad \text{as } P_o = 5\text{ kW}$

(b) Input power factor $= \sqrt{K} = \sqrt{0.4725} = 0.6873$

(c) Average current flow through thyristor, I_A is

$$I_A = K\frac{I_m}{\pi} \qquad \text{where, } I_m = \frac{\sqrt{2}V}{R} = \frac{\sqrt{2}\times 230}{5} = 65.053\text{ A}$$

$$= \frac{0.4725 \times 65.053}{\pi}\text{ A} = 9.78\text{ A}$$

RMS current flow through thyristor, I_{RMS} is

$$I_{\text{RMS}} = \sqrt{K}\frac{I_m}{2} = \sqrt{0.4725} \times \frac{65.053}{2}\text{ A} = 22.3582\text{ A}$$

9.3 PHASE-CONTROLLED SINGLE-PHASE AC VOLTAGE CONTROLLER

Figure 9.4(c) shows the circuit diagram using two diodes and two SCRs. This circuit consists of two switching modules. Each switching module has a SCR and an anti-parallel diode. The current flows in positive direction through T_1 and D_2, but the current flow in negative direction through T_2 and D_1. This circuit provides a common cathode connection for simplifying the gating circuit and it is very convenient from control point of view. As the current flow through two devices a thyristor and a diode, the power loss increases as forward voltage drop across two device more than Figs. 9.4(a) and (b).

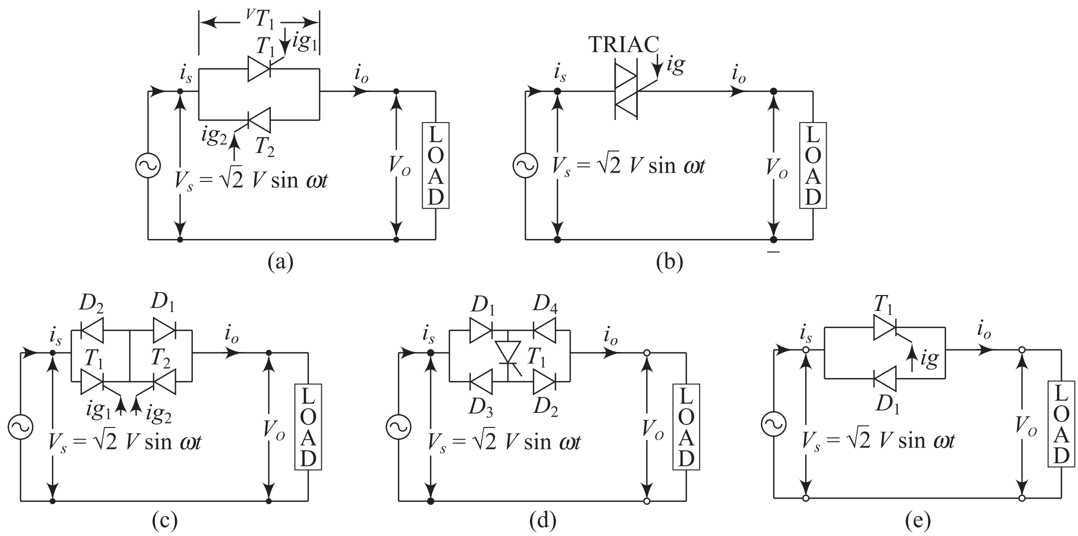


Fig. 9.4 *Single-phase ac voltage controllers (a) Full wave using two anti-parallel (back to back or inverse parallel) connection of two thyristors (b) full wave with Triac (c) full wave using two thyristors and two diodes (d) full wave using one thyristor and four diodes (e) half wave using one thyristor and one diode*

Figure 9.4(d) shows an alternative circuit for single phase controlled ac voltage controller which consists of four diodes and one thyristor. Positive current flows through diodes D_1 and D_2 and thyristor T_1. Negative current flows through D_3 and D_4 and thyristor T_1. So, the current flows through thyristor T_1 in positive as well as negative direction. At any time current flows through three devices and power loss will be more than power loss of Fig. 9.4(c).

The triac is a low power semiconductor device and it is used in voltage control circuits such as light dimmers, speed control for fan motors, etc. The advantages and disadvantages of the triac with reaspect to thyristor are given below:

Advantages

1. Triacs are triggered during both positive and negative half cycle of supply voltage by applying trigger pulse at the gate terminal.
2. A triac requires a single heat sink of slightly larger size, but anti-parallel thyristor pair needs two heat sinks of slightly smaller sizes as the clearance total space required is more for thyristors.

Disadvantages

1. Triac has $\frac{dv}{dt}$ low rating as compared to thyristor.
2. Usually Triacs are available in lower rating as compared to thyristors.
3. As a triac can be triggered in both direction (positive and negative half cycle), a trigger circuit for triac requires cautious consideration.
4. The reliability of triac is lower than that of thyristor.

9.4 SINGLE-PHASE HALF-WAVE ac VOLTAGE CONTROLLER WITH R LOAD

The single-phase half-wave ac voltage controller with R load is depicted in Fig. 9.5. This circuit consists of one thyristor T_1 in antiparallel with one diode D_1. During the positive half cycle of supply voltage, thyristor T_1 is forward biased and it is turned ON at the firing angle α and conducts up to π. In the negative half cycle of supply voltage, diode D_1 is forward biased and it conducts for π duration (from π to 2π). Therefore, the power flow through load is controlled by varying the firing angle α of T_1 in the positive half cycle of supply voltage only. Hence the control range is limited and it is applicable only for low power resistive loads such as heating and lighting. As only the positive half cycle is controlled for single-phase half-wave ac voltage controller, this circuit is also called *single-phase unidirectional voltage controller*.

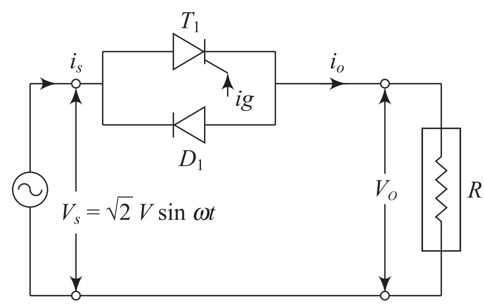


Fig. 9.5 *Single-phase half-wave ac voltage controller with R load*

When the thyristor T_1 is turned ON at the firing angle α, the voltage across load is $V_m \sin \alpha$ and current flow through load is $\frac{V_m \sin\alpha}{R}$. The thyristor is turned OFF at $\omega t = \pi$. In the negative half cycle, the diode D_1 is forward biased and conducts from $\omega t = \pi$ to 2π. Figure 9.6 shows the voltage and current waveforms of single-phase half-wave ac voltage controller with R load.

Assume the supply (input) voltage is

$V_S = \sqrt{2}V \sin \omega t$

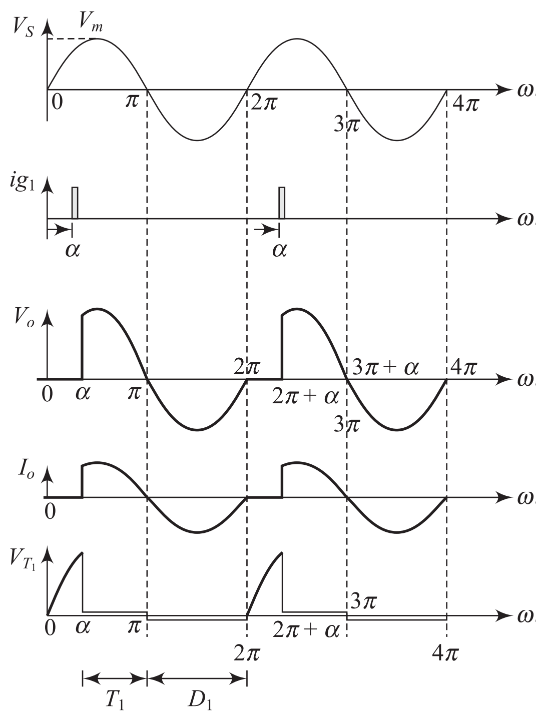


Fig. 9.6 *Voltage and current waveforms of single phase half wave ac voltage controller with R load*

RMS output voltage across load at firing angle α

$$V_o = \left[\frac{1}{2\pi}\left\{\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2\cdot d(\omega t) + \int_{\pi}^{2\pi}(\sqrt{2}V\sin\omega t)^2\cdot d(\omega t)\right\}\right]^{1/2}$$

$$= \left[\frac{V^2}{\pi}\left\{\int_{\alpha}^{\pi}(1-\cos 2\omega t)\cdot d(\omega t) + \int_{\pi}^{2\pi}(1-\cos 2\omega t)\cdot d(\omega t)\right\}\right]^{1/2}$$

$$= V\left[1 - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2} = V\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

It is clear from the above equation that RMS output voltage can be controlled from V to $\frac{V}{\sqrt{2}}$ by changing firing angle α from 0° to 180°.
The rms value of load current is

$$I_o = \frac{V_o}{R}$$

The power output at R is

$$P_{\text{out\_ac}} = V_o I_o = V_o \times \frac{V_o}{R} = \frac{V_o^2}{R} = \frac{V^2}{R}\left[1 - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right] = \frac{V^2}{R}\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]$$

ac power input is

$$P_{\text{in\_ac}} = V \times I_o = V \times \frac{V_o}{R} = \frac{V^2}{R}\left[1 - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2} = \frac{V^2}{R}\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

Input power factor is

$$pf = \frac{P_{\text{out\_ac}}}{P_{\text{in\_ac}}} = \left[1 - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2} = \left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

The average value of output voltage is

$$V_{\text{av}} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi}\sqrt{2}V\sin\omega t\cdot d(\omega t) + \int_{\pi}^{2\pi}\sqrt{2}V\sin\omega t\cdot d(\omega t)\right] = \frac{\sqrt{2}V}{2\pi}\left|-\cos\omega t\right|_{\alpha}^{2\pi} = \frac{\sqrt{2}V}{2\pi}(\cos\alpha - 1)$$

It is clear from the above equation that average output voltage can be controlled from 0 to $-\frac{\sqrt{2}V}{\pi}$ by changing firing angle α from 0° to 180°.

Example 9.5 A single-phase half-wave ac voltage controller is connected with a load of R = 10 Ω with an input voltage of 220 V, 50 Hz. When the firing angle of thyristor is 30°, find the RMS output voltage, power output at load, input power factor and average value of output voltage.

Solution

Given: V = 220 V, R = 10 Ω, $\alpha = 30° = \frac{\pi}{6}$

RMS output voltage across load at firing angle α

$$V_o = V\left[1-\frac{\alpha}{2\pi}+\frac{\sin 2\alpha}{4\pi}\right]^{1/2} = V\left[\frac{1}{2\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= 220\times\left[\frac{1}{2\pi}\left(2\pi-\frac{\pi}{6}+\frac{\sin(2\times 30)}{2}\right)\right]^{1/2} = 218.40 \text{ V}$$

The power output at R is

$$P_{\text{out\_ac}} = V_o I_o = \frac{V_o^2}{R} = \frac{218.40^2}{10} \text{ Watt} = 4769.856 \text{ Watt}$$

Input power factor is

$$pf = \frac{P_{\text{out\_ac}}}{P_{\text{in\_ac}}} = \left[1-\frac{\alpha}{2\pi}+\frac{\sin 2\alpha}{4\pi}\right]^{1/2} = \left[\frac{1}{2\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\left(2\pi-\frac{\pi}{6}+\frac{\sin(2\times 30)}{2}\right)\right]^{1/2} = 0.992$$

The average value of output voltage is

$$V_{\text{av}} = \frac{\sqrt{2}V}{2\pi}(\cos\alpha - 1) = \frac{\sqrt{2}\times 220}{2\pi}(\cos 30 - 1) = -6.63 \text{ V}$$

Example 9.6 A single-phase unidirectional ac voltage controller is connected with a load of $R = 20\ \Omega$ with an input voltage of 230 V, 50 Hz. If the firing angle of thyristor is 90°, determine (a) the RMS value of output voltage, (b) power delivered to load, (c) input power factor and average, (d) average value of thyristor current and (e) average value of diode current.

Solution

Given: $V = 230$ V, $R = 20\ \Omega$, $\alpha = 90° = \dfrac{\pi}{2}$

(a) RMS output voltage across load at firing angle α

$$V_o = V\left[1-\frac{\alpha}{2\pi}+\frac{\sin 2\alpha}{4\pi}\right]^{1/2} = V\left[\frac{1}{2\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= 230\times\left[\frac{1}{2\pi}\left(2\pi-\frac{\pi}{2}+\frac{\sin(2\times 90)}{2}\right)\right]^{1/2} = 199.185 \text{ V}$$

(b) The power delivered to load is

$$P_{\text{out\_ac}} = V_o I_o = \frac{V_o^2}{R} = \frac{199.185^2}{20} \text{ Watt} = 1983.73 \text{ Watt}$$

(c) Input power factor is

$$pf = \frac{P_{\text{out\_ac}}}{P_{\text{in\_ac}}} = \left[1-\frac{\alpha}{2\pi}+\frac{\sin 2\alpha}{4\pi}\right]^{1/2} = \left[\frac{1}{2\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\left(2\pi-\frac{\pi}{2}+\frac{\sin(2\times 90)}{2}\right)\right]^{1/2} = 0.866$$

(d) The average value of thyristor current

$$I_{T\text{av}} = \frac{1}{2\pi R}\left[\int_{\alpha}^{\pi} \sqrt{2}V \sin\omega t \cdot d(\omega t)\right] = \frac{\sqrt{2}V}{2\pi R}\left|-\cos\omega t\right|_{\alpha}^{\pi}$$

$$= \frac{\sqrt{2}V}{2\pi R}(1+\cos\alpha) = \frac{\sqrt{2}\times 230}{2\pi\times 20}(1+\cos 90) = 2.587 \text{ A}$$

(e) The average value of diode current

$$I_{D\text{av}} = \frac{1}{2\pi R}\left[\int_{\pi}^{2\pi} \sqrt{2}V \sin\omega t \cdot d(\omega t)\right] = \frac{\sqrt{2}V}{2\pi R}\left|-\cos\omega t\right|_{\pi}^{2\pi} = -\frac{\sqrt{2}V}{\pi R} = -\frac{\sqrt{2}\times 230}{\pi\times 20} = -5.174 \text{ A}$$

Example 9.7 A single-phase half-wave ac voltage controller is connected with a load of R = 5 Ω with an input voltage of 230 V, 50 Hz. If the firing angle of thyristor is 45°, determine (a) the RMS output voltage, (b) power delivered to load, (c) input power factor and (d) average value of input current voltage.

Solution

Given: V = 230 V, R = 5 Ω, $\alpha = 45° = \dfrac{\pi}{4}$

(a) RMS output voltage across load at firing angle α

$$V_o = V\left[1 - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2} = V\left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= 230\times\left[\frac{1}{2\pi}\left(2\pi - \frac{\pi}{4} + \frac{\sin(2\times 45)}{2}\right)\right]^{1/2} = 224.72 \text{ V}$$

(b) The power delivered to load R is

$$P_{\text{out\_ac}} = V_o I_o = \frac{V_o^2}{R} = \frac{224.72^2}{5} \text{ Watt} = 10099.81 \text{ Watt}$$

(c) Input power factor is

$$pf = \frac{P_{\text{out\_ac}}}{P_{\text{in\_ac}}} = \left[1 - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2} = \left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\left(2\pi - \frac{\pi}{4} + \frac{\sin(2\times 45)}{2}\right)\right]^{1/2} = 0.977$$

(d) The average value of output voltage is

$$V_{\text{av}} = \frac{\sqrt{2}V}{2\pi}(\cos\alpha - 1) = \frac{\sqrt{2}\times 230}{2\pi}(\cos 45 - 1) = -15.1679 \text{ V}$$

Average input current is equal to

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = -\frac{15.1679}{5} = -3.033 \text{ A}$$

9.5 SINGLE-PHASE FULL-WAVE ac VOLTAGE CONTROLLER WITH *R* LOAD

Figure 9.7 shows the single-phase full-wave ac voltage controller where the SCR T_1 is gated at α and SCR T_2 is also gated at $\pi + \alpha$. During the positive half cycle of supply voltage, thyristor T_1 is forward biased and it is turned ON at the firing angle α. Then T_1 starts conduction and it conducts from α to π. Just after π, thyristor T_1 is reverse biased and it is turned OFF. In the negative half cycle of supply voltage, thyristor T_2 is forward biased and it is turned ON at the firing angle $\pi + \alpha$. Consequently, T_2 conducts from $\pi + \alpha$ to 2π. Immediately after 2π, thyristor T_2 is reverse biased and it is turned OFF.

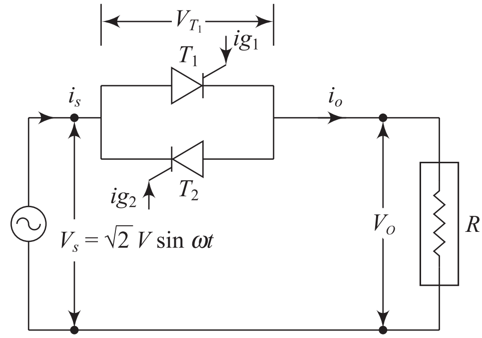


Fig. 9.7 *The single-phase full-wave ac voltage controller with R load*

The power flow is controlled through voltage control in alternative half-cycles. When T_1 is conducting, other thryristor T_2 is reverse biased by the voltage drop across the turn on SCR T_1. Similarly, if T_2 is conducting, other thryristor T_1 is reverse biased by the voltage drop across the turn on SCR T_2. Figure 9.8 shows the voltage and current waveforms for single phase ac voltage controller with resistive load. Single-phase full-wave ac voltage controllers are commonly used for the speed control of single-phase induction motor control, domestic heating and lighting loads.

Assume the supply (input) voltage is

$$V_S = \sqrt{2}V \sin \omega t$$

RMS output voltage across load at firing angle α

$$V_o = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V \sin \omega t)^2 \cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{V^2}{\pi}\int_{\alpha}^{\pi}(1 - \cos 2\omega t)\cdot d(\omega t)\right]^{1/2}$$

$$= V\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

$$= V\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

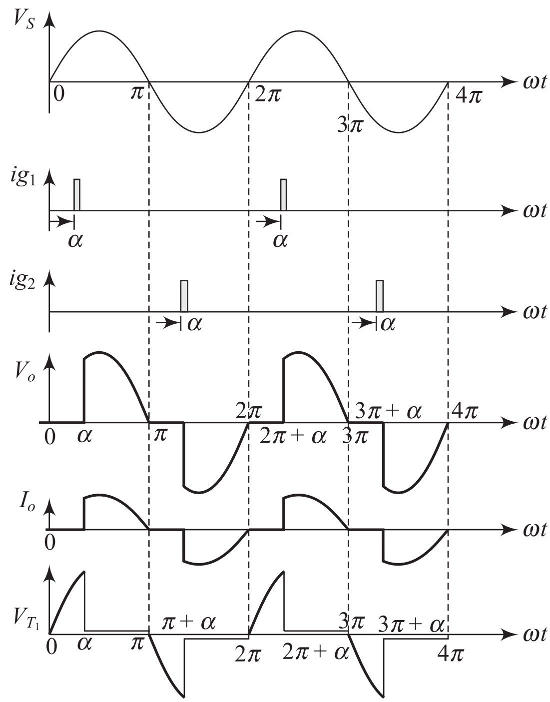


Fig. 9.8 *Voltage and current waveforms for single phase ac voltage controller with R load*

It is clear from the above equation that RMS output voltage can be controlled from V to 0 by changing firing angle α from 0° to 180°.

The rms value of current through load R is

$$I_o = \frac{V_o}{R} = \frac{V}{R}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

$$= \frac{V}{R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

The power output at load resistance R is

$$P_{\text{out\_ac}} = V_o I_o = V_o \times \frac{V_o}{R} = \frac{V_o^2}{R} = \frac{V^2}{R}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]$$
$$= \frac{V^2}{R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]$$

ac power input is equal to

$$P_{\text{in\_ac}} = V \times I_o = V \times \frac{V_o}{R} = \frac{V^2}{R}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} = \frac{V^2}{R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

Input power factor is

$$pf = \frac{P_{\text{out\_ac}}}{P_{\text{in\_ac}}} = \left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} = \left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

The average value of output voltage over any complete cycle is zero, but the half cycle average value of output voltage always exists. The average value of output voltage for half cycle duration is given by

$$V_{\text{av}} = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin(\omega t) d(\omega t) = \frac{\sqrt{2}V}{\pi}(1 + \cos\alpha)$$

When the firing angle α is changed from 0° to 180°, average output voltage can be controlled from $\frac{\sqrt{2}V}{\pi}$ to 0.

The half cycle average current is

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = \frac{\sqrt{2}V}{\pi R}(1 + \cos\alpha)$$

9.5.1 Harmonic Analysis of Output Voltage and Input Current

The output voltage can be expressed by the Fourier series:

$$v_o = \frac{1}{2}a_o + \sum_{n=1,3,5....}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

where, $$a_o = \frac{1}{\pi}\int_0^{2\pi} v_o(\omega t) d(\omega t)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v_o(\omega t) \cdot \cos n\omega t \cdot d(\omega t)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v_o(\omega t) \cdot \sin n\omega t \cdot d(\omega t)$$

$a_0/2$ is average vale of v_o. Due to resistive load, the output voltage is equal to input voltage during α to π and $\pi + \alpha$ to 2π. Due to half wave symmetry,

$$a_o = \frac{1}{\pi}\left[\int_{\alpha}^{\pi} (\sqrt{2}V \sin \omega t)^2 d(\omega t) + \int_{\pi+\alpha}^{2\pi} (\sqrt{2}V \sin \omega t)^2 d(\omega t)\right] = 0$$

$$a_n = \frac{1}{\pi}\int_{0}^{2\pi} \sqrt{2}V \sin \omega t \cdot \cos n\omega t \cdot d(\omega t)$$

$$= \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot \cos n\omega t \cdot d(\omega t) + \frac{1}{\pi}\int_{\pi+\alpha}^{2\pi} \sqrt{2}V \sin \omega t \cdot \cos n\omega t \cdot d(\omega t)$$

$$= \frac{2\sqrt{2}V}{\pi}\int_{\alpha}^{\pi} \sin \omega t \cdot \cos n\omega t \cdot d(\omega t) = \frac{\sqrt{2}V}{\pi}\int_{\alpha}^{\pi}\left[\sin(n+1)\omega t - \sin(n-1)\omega t\right]\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\left[\frac{\cos(n+1)\alpha - 1}{n+1} - \frac{\cos(n-1)\alpha - 1}{n-1}\right] \qquad \text{when } n \neq 1 \text{ and } n = 3,5,7\ldots$$

$$= 0 \qquad \text{when } n = 2,4,6\ldots$$

$$b_n = \frac{1}{\pi}\int_{0}^{2\pi} \sqrt{2}V \sin \omega t \cdot \sin n\omega t \cdot d(\omega t)$$

$$= \frac{1}{\pi}\int_{0}^{\pi} \sqrt{2}V \sin \omega t \cdot \sin n\omega t \cdot d(\omega t) + \frac{1}{\pi}\int_{\pi+\alpha}^{2\pi} \sqrt{2}V \sin \omega t \cdot \sin n\omega t \cdot d(\omega t)$$

$$= \frac{2\sqrt{2}V}{\pi}\int_{\alpha}^{\pi} \sin \omega t \cdot \sin n\omega t \cdot d(\omega t) = \frac{\sqrt{2}V}{\pi}\int_{\alpha}^{\pi}\left[\cos(n-1)\omega t - \cos(n+1)\omega t\right] d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right] \qquad \text{when } n \neq 1 \text{ and } n = 3,5,7\ldots$$

$$= 0 \qquad \text{when } n = 2,4,6\ldots$$

The RMS value of n^{th} harmonic of output voltage is

$$V_n = \left[\frac{a_n^2 + b_n^2}{2}\right]^{1/2} \quad \text{and phase angle} \quad \phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

Then output voltage can be expressed as $v_o(\omega t) = \sum_{n=1,3,5....}^{\infty} \sqrt{2}V_n \sin(n\omega t + \phi_n)$

The fundamental components of the output voltage are

$$a_1 = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cos \omega t \cdot d\omega t$$

$$= \frac{\sqrt{2}V}{\pi}\frac{\cos 2\alpha - 1}{2} = \frac{\sqrt{2}V}{2\pi}(\cos 2\alpha - 1)$$

$$b_1 = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t \cdot \sin \omega t \cdot d\omega t$$

$$= \frac{\sqrt{2}V}{\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right) = \frac{\sqrt{2}V}{2\pi}\left[2(\pi - \alpha) + \sin 2\alpha\right]$$

After combining a_1 and b_1, we can determine the fundamental component of the output voltage V_1

and $$V_1 = \left[\frac{a_1^2 + b_1^2}{2}\right]^{1/2} = \frac{V}{\pi}\left[\left(\frac{\cos 2\alpha - 1}{2}\right)^2 + \left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)^2\right]^{1/2}$$

$$\phi_1 = \tan^{-1}\left(\frac{\cos 2\alpha - 1}{2(\pi - \alpha) + \sin 2\alpha}\right)$$

When the voltage and current waveforms are sinusoidal and have same frequency, active power, reactive power and power factor can be determined by using standard equations. In case of power electronics circuits, static switches such as power diode, SCRs, Triac, transistors, MOSFETs, etc., are switched ON and OFF in some duration of a cycle. Therefore, the voltage and current waveforms are non-sinusoidal and the standard equations to determine power and power factor will not be valid. Hence harmonic analysis is needed to find out the harmonic contents of the voltage and current. For non-sinusoidal voltage and current waveforms, displacement power factor, distortion factor and total harmonic distortion (THD) are commonly used. The amplitude of harmonic voltages V_1, V_3, V_5 and V_7 in per unit with respect to firing angle α for R load is shown in Fig. 9.9.

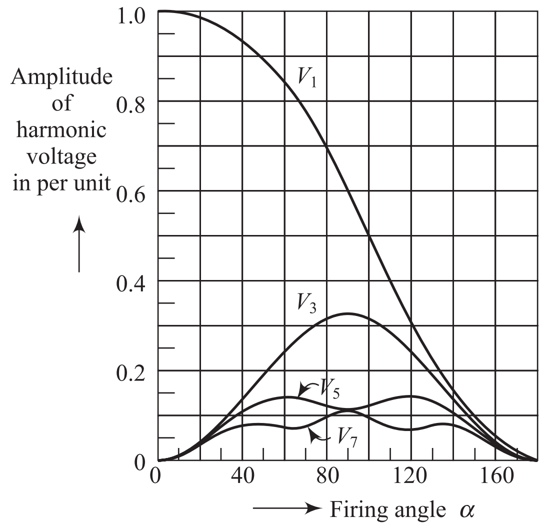


Fig. 9.9 *Amplitude of harmonic voltages V_1, V_3, V_5 and V_7 in per unit with respect to firing angle for R load*

Displacement power factor Displacement power factor is the cosine of angle ϕ_1 which is angle between input voltage and fundamental component of the current.

$$\text{DPF} = \cos\phi_1$$

Distortion factor It is the ratio of fundamental component of current, I_1 to total input current, I.

$$\text{DF} = \frac{I_1}{I}$$

Input power factor It is the ratio between average power input and the RMS value of the apparent power input. When V = input voltage, I = Input current, V_1 = fundamental component of the input voltage, I_1 = fundamental component of the input current, ϕ_1 = the phase angle between V and I_1, the input power factor is

$$\text{PF} = \frac{V_1 I_1 \cos\phi_1}{VI} = \frac{I_1}{I}\cos\phi_1 = \text{DF} \times \text{DPF} \quad \text{Assume } V = V_1$$

Total Harmonic Distortion (THD) This is the ratio of all the harmonic terms (except fundamental) to the fundamental component of current I_1 as given below.

$$\text{THD} = \frac{\sqrt{I_2^2 + I_3^2 + I_4^2 + I_5^2 \ldots}}{I_1} = \frac{\sqrt{I^2 - I_1^2}}{I_1} = \sqrt{\left(\frac{I}{I_1}\right)^2 - 1} \quad \text{as} \quad I^2 = I_1^2 + I_2^2 + I_3^2 + \cdots$$

Ripple factor It is the ratio of RMS values of all harmonic terms to the average value of current $I_{av.}$ The ripple factor can be expressed as

$$\mathrm{RF} = \frac{\sqrt{I^2 - I_1^2}}{I_{av}}$$

Power input In sinusoidal or non-sinusoidal voltage and current, power input into a system can be determined by

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) i(\omega t) d(\omega t)$$

After substituting the voltage and current values (non-sinusoidal) in the above expression, we get

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi}\left[\left\{V_{dc} + \sum_{n=1}^{\infty}\sqrt{2}V_n \sin(n\omega t + \theta_n)\right\}\left\{I_{dc} + \sum\sqrt{2}I_n \sin(n\omega t + \phi_n)\right\}d(\omega t)\right]$$

$$= V_{dc}I_{dc} + \frac{1}{2\pi}\int_0^{2\pi}\sum_{n=1}^{\infty} V_n I_n\left[\cos(\theta_n - \phi_n) - \cos(2n\omega t + \theta_n + \phi_n)\right]\cdot d(\omega t)$$

$$= V_{dc}I_{dc} + \frac{1}{2\pi}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)2\pi$$

$$= V_{dc}I_{dc} + \sum_{n=1}^{\infty} V_n I_n \cos\psi_n \qquad \text{where, } \psi_n = \theta_n - \phi_n$$

$$= V_{dc}I_{dc} + V_1 I_1 \cos\psi_1 + V_2 I_2 \cos\psi_2 + V_3 I_3 \cos\psi_3 + \dots\dots\dots + V_n I_n \cos\psi_n$$

Example 9.8 A single-phase full-wave ac voltage controller is connected with a load of R = 5 Ω with an input voltage of 230 V, 50 Hz. When the firing angle of thyristor is 60°, determine the rms output voltage, power output at load and input power factor.

Solution

Given: V = 230 V, R = 5 Ω, $\alpha = 60° = \frac{\pi}{3}$

RMS output voltage across load at firing angle α

$$V_o = \left[\frac{1}{\pi}\int_\alpha^\pi (\sqrt{2}V \sin\omega t)^2 \cdot d(\omega t)\right]^{1/2} = V\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

$$= V\left[\frac{1}{\pi}\left\{(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2} = 230 \times \left[\frac{1}{\pi}\left\{\left(\pi - \frac{\pi}{3}\right) + \frac{\sin(2\times 60)}{2}\right\}\right]^{1/2} = 206.20 \text{ V}$$

The power output at R is

$$P_{out\_ac} = V_o I_o = \frac{V_o^2}{R} = \frac{206.2^2}{5} = 8503.688 \text{ Watt}$$

ac power input is

$$P_{in\_ac} = V \times I_o = V \times \frac{V_o}{R} = 230 \times \frac{206.2}{5} = 9485.2 \text{ Watt}$$

Input power factor is

$$pf = \frac{P_{\text{ac}}}{P} = \left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} = \left[\frac{1}{\pi}\left\{(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$= \left[\frac{1}{\pi}\left\{\left(\pi - \frac{\pi}{3}\right) + \frac{\sin(2 \times 60)}{2}\right\}\right]^{1/2} = 0.8965$$

Example 9.9 A single phase full wave ac voltage controller has a load of 5 Ω and the input voltage is 230 V with 50 Hz. If the load power is 5 kW, determine (a) firing angle of thyristors, (b) input power factor and (c) rms output voltage

Solution

Given: $V = 230$ V, $R = 5\ \Omega$,

(a) At firing angle α, the rms output voltage across load is

$$V_o = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} (\sqrt{2}V \sin \omega t)^2 \cdot d(\omega t)\right]^{1/2} = V\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} = V\left[\frac{1}{\pi}\left\{(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

The power output at R is

$$P_{\text{out\_ac}} = V_o I_o$$

$$= \frac{V_o^2}{R} = \frac{V^2}{R}\left[\frac{1}{\pi}\left\{(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right\}\right] = 5000 \text{ Watt}$$

or $\dfrac{230^2}{5}\left[\dfrac{1}{\pi}\left\{(\pi - \alpha) + \dfrac{\sin 2\alpha}{2}\right\}\right] = 5000$

or $\left[1 - \dfrac{\alpha}{\pi} + \dfrac{\sin 2\alpha}{2\pi}\right] = \dfrac{5000 \times 5}{230^2} = 0.4725$

or $\dfrac{\alpha}{\pi} - \dfrac{\sin 2\alpha}{2\pi} = 1 - 0.4725 = 0.5275$

or $\alpha - \dfrac{\sin 2\alpha}{2} = 1.657$

The solution of the above equation is $\alpha = 92.5°$

Therefore, the firing angle of thyristors is $\alpha = 92.5° = 1.615$ rad

(b) Input power factor is

$$pf = \frac{P_{\text{ac}}}{P} = \left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} = \left[1 - \frac{1.615}{\pi} + \frac{\sin(2 \times 92.5)}{2\pi}\right]^{1/2} = 0.6871 \text{ lag}$$

(c) rms output voltage $V_o = \left[\dfrac{1}{\pi}\displaystyle\int_{\alpha}^{\pi} (\sqrt{2}V \sin \omega t)^2 \cdot d(\omega t)\right]^{1/2} = V\left[1 - \dfrac{\alpha}{\pi} + \dfrac{\sin 2\alpha}{2\pi}\right]^{1/2}$

$$= 230 \times \left[1 - \frac{1.615}{\pi} + \frac{\sin(2 \times 92.5)}{2\pi}\right]^{1/2} \text{V} = 158.03 \text{ V}$$

Example 9.10 A single-phase full-wave ac voltage controller is connected with a load of R =10 Ω with an input voltage of 230 V, 50 Hz. When the firing angle of thyristors is 45°, determine (a) power output at load, (b) average value of thyristor current and (c) rms value of thyristor current.

Solution

Given: $V = 230$ V, $R = 10\ \Omega$, $\alpha = 45° = \dfrac{\pi}{4}$

(a) RMS output voltage across load at firing angle α

$$V_o = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2 \cdot d(\omega t)\right]^{1/2} = V\left[1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

$$= V\left[\frac{1}{\pi}\left\{(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$= 230\times\left[\frac{1}{\pi}\left\{\left(\pi-\frac{\pi}{4}\right)+\frac{\sin(2\times 45)}{2}\right\}\right]^{1/2} = 219.301\text{ V}$$

The power output at R is

$$P_{\text{out\_ac}} = V_o I_o = \frac{V_o^2}{R} = \frac{219.301^2}{10} = 4809.29\text{ Watt}$$

(b) Average thyristor current is equal to

$$I_{\text{av\_}T} = \frac{1}{2\pi R}\int_{\alpha}^{\pi}\sqrt{2}V\sin\omega t\cdot d\omega t$$

$$= \frac{\sqrt{2}V}{2\pi R}(1+\cos\alpha) = \frac{\sqrt{2}\times 230}{2\pi\times 10}(1+\cos 45) = 8.836\text{ A}$$

(c) rms value of thyristor current is equal to

$$I_{\text{rms\_}T} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi}\left(\frac{\sqrt{2}V\sin\omega t}{R}\right)^2\cdot d\omega t\right]^{1/2} = \frac{V}{\sqrt{2}R}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{230}{\sqrt{2}\times 10}\left[\frac{1}{\pi}\left(\pi-\frac{\pi}{4}+\frac{\sin(2\times 45)}{2}\right)\right]^{1/2} = 15.506\text{ A}$$

Example 9.11 A single-phase full-wave ac voltage controller is connected with a load of $R = 5\ \Omega$ with an input voltage of 220 V, 50 Hz. Determine (a) the maximum value of average thyristor current, (b) the maximum value of rms thyristor current, (c) the minimum circuit turn-OFF time for the firing angle α, (d) the ratio of third harmonic voltage to fundamental voltage at $\alpha = 45°$, (e) the maximum value of $\dfrac{di}{dt}$ of thyristor and (f) peak inverse voltage of thyristor.

Solution

Given: $V = 220$ V, $R = 5\ \Omega$, and $\alpha = 45° = \dfrac{\pi}{4}$

(a) Average thyristor current is equal to

$$I_{\text{av\_}T} = \frac{1}{2\pi R}\int_{\alpha}^{\pi}\sqrt{2}V\sin\omega t\cdot d\omega t$$

$$= \frac{\sqrt{2}V}{2\pi R}(1+\cos\alpha)\quad \text{As } \alpha = 0, \text{average thyristor current is maximum}$$

$$= I_{\text{av\_}T(\max)} = \frac{\sqrt{2}\times 220}{2\pi\times 5}(1+\cos 0) = 19.81\text{ A}$$

(b) rms thyristor current at firing angle α

$$I_{\text{rms\_}T} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi}\left(\frac{\sqrt{2}V}{R}\sin\omega t\right)^2 \cdot d(\omega t)\right]^{1/2} = \frac{\sqrt{2}V}{2R}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

$$= \frac{V}{\sqrt{2}R}\left[\frac{1}{\pi}\left\{(\pi-\alpha) + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$ At $\alpha = 0$, rms thyristor current is maximum

$$I_{\text{rms\_}T(\max)} = \frac{V}{\sqrt{2}R}\left[\frac{1}{\pi}\left\{(\pi - 0) + \frac{\sin(2\times 0)}{2}\right\}\right]^{1/2} = \frac{V}{\sqrt{2}R} = \frac{220}{\sqrt{2}\times 5} = 31.1173\text{ A}$$

(c) The minimum circuit turn-off time for the firing angle α

$$t_{\text{turn-off}} = \frac{\pi}{\omega} = \frac{\pi}{2\pi f} = \frac{\pi}{2\pi \times 50} = 0.01\text{ sec}$$

(d) The ratio of third harmonic voltage to fundamental voltage at $\alpha = 45°$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\sqrt{2}V\sin\omega t\cdot\cos n\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\left[\frac{\cos(n+1)\alpha - 1}{n+1} - \frac{\cos(n-1)\alpha - 1}{n-1}\right]$$ when $n \neq 1$ and $n = 3, 5, 7\ldots$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\sqrt{2}V\sin\omega t\cdot\sin n\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$ when $n \neq 1$ and $n = 3, 5, 7\ldots$

For third harmonic voltage at $\alpha = 45°$

$$a_3 = \frac{\sqrt{2}V}{\pi}\left[\frac{\cos(3+1)\times 45 - 1}{3+1} - \frac{\cos(3-1)\times 45 - 1}{3-1}\right] = 0$$

$$b_3 = \frac{\sqrt{2}V}{\pi}\left[\frac{\sin(3+1)\times 45}{3+1} - \frac{\sin(3-1)\times 45}{3-1}\right] = \frac{V}{\sqrt{2}\pi}$$

$$V_3 = \left[\frac{a_3^2 + b_3^2}{2}\right]^{1/2} = \frac{b_3}{\sqrt{2}} = \frac{V}{\sqrt{2}\pi}\frac{1}{\sqrt{2}} = 0.1592\text{ V}$$

The fundamental components of the output voltage are

$$a_1 = \frac{1}{\pi}\int_{\alpha}^{\pi}\sqrt{2}V\sin\omega t\cos\omega t\cdot d\omega t$$

$$= \frac{\sqrt{2}V}{\pi}\frac{\cos 2\alpha - 1}{2} = \frac{\sqrt{2}V}{2\pi}(\cos 2\alpha - 1)$$

$$b_1 = \frac{1}{\pi}\int_{\alpha}^{\pi}\sqrt{2}V\sin\omega t\cdot\sin\omega t\cdot d\omega t$$

$$= \frac{\sqrt{2}V}{\pi}\left[\pi - \alpha + \frac{1}{2}\sin\alpha\right] = \frac{\sqrt{2}V}{2\pi}(2(\pi-\alpha) + \sin 2\alpha)$$

After combining a_1 and b_1, we can determine the fundamental component of the output voltage V_1

$$V_1 = \left[\frac{a_1^2 + b_1^2}{2}\right]^{1/2} = \frac{V}{\pi}\left[\left(\frac{\cos 2\alpha - 1}{2}\right)^2 + \left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)^2\right]^{1/2}$$

For fundamental frequency voltage at $\alpha = 45°$

$$V_{1\_at\_45°} = \frac{V}{\pi}\left[\left(\frac{\cos 2\times 45 - 1}{2}\right)^2 + \left(\pi - \frac{\pi}{4} + \frac{1}{2}\sin(2\times 45)\right)^2\right]^{1/2} = 0.428 \text{ V}$$

The ratio of third harmonic voltage to fundamental voltage at $\alpha = 45°$ is

$$\frac{V_3}{V_1} = \frac{0.1592V}{0.428V} = 0.3719$$

(e) Due to sudden rise of current from 0 to $\frac{V_m}{R}\sin\alpha$ at firing angle α, the maximum value of $\frac{di}{dt}$ of thyristor is infinite.

(f) Peak inverse voltage of thyristor $PIV = V_m = \sqrt{2}V = \sqrt{2}\times 220 = 311.08$ V

9.6 SINGLE-PHASE ac VOLTAGE CONTROLLER WITH RESISTIVE AND INDUCTIVE LOAD

The circuit diagram of a single phase ac voltage controller with resistive-inductive (R-L) load is shown in Fig. 9.10. During the interval 0 to π, thyristor T_1 is forward biased. At $\omega t = \alpha$, T_1 is triggered and $i_o = i_{T1}$ starts building up through the load. At $\omega t = \pi$, the load and source voltage are equal to zero but the current is not zero because of the presence of inductance in load circuit. Then thyristor T_1 will continue to conduct until its current falls to zero at $\omega t = \beta$. The angle β is called the *extinction angle*. Therefore, load is subjected to the source voltage from angle α to β. At $\omega t = \beta$ when i_o zero, T_1 is turned OFF as it is already reverse biased. After commutation of T_1 at β, a voltage of magnitude $-\sqrt{2}V\sin\beta$ at once appears as a reverse bias across T_1 and as a forward bias across T_2. From β to $\pi + \alpha$, no current exists in the power circuit. Thyristor T_2 is turned ON at $\omega t = \pi + \alpha$, current $i_o = i_{T2}$ starts building up in reverse direction through the load. The value of $(\pi + \alpha)$ is greater than β. At $\omega t = 2\pi$, V_s and V_o are zero but $i_{T2} = i_o$ is not zero. At $\omega t = (\pi + \alpha + \gamma)$, $i_{T2} = 0$ and T_2 turned OFF because it is already reverse biased.

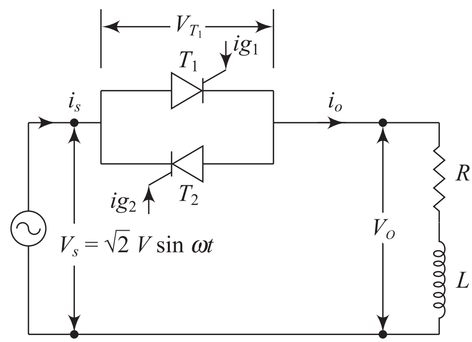


Fig. 9.10 *Single-phase ac voltage controller with resistive-inductive (RL) load*

Figure 9.11 shows the voltage and current waveforms with RL load. Due to presence of inductance in load, the load current does not reduce to zero at $\omega t = \pi$. Hence the thryristor current does not reduce to zero at $\omega t = \pi$ and the conduction has extended to the extinction angle β. Then conduction angle of each thyristor is $\gamma = \beta - \alpha$ which depends on firing angle α and the load impedance angle ϕ.

At $\omega t = (\alpha + \gamma)$, the current through thyristor T_1 becomes zero and turned OFF by the thyristor T_2 which is forward biased. As soon as the gate pulse is applied to thyristor T_2 at $\omega t = (\pi + \alpha)$, it becomes turned ON and continue to conduct up to $\omega t = (\pi + \alpha + \gamma)$.

At $\omega t = (\pi + \alpha + \gamma)$, current through thyristor T_2 is zero and T_2 is turned OFF as it is reverse bias by the voltage $V_m \sin(\pi + \alpha + \gamma)$. As T_2 is turned OFF, $V_m \sin(\pi + \alpha + \gamma)$ voltage appears as a forward bias across T_1. It is clear from Fig. 9.11 that no current exists in the power circuit during $\beta < \omega t < \pi + \alpha$ and $\pi + \alpha + \gamma < \omega t < 2\pi + \alpha$. At $(2\pi + \alpha)$, T_1 is again turned ON and current starts building up as before.

Assume input voltage $V_S = \sqrt{2}V\sin\omega t$

Now derive the expression for load current i_o and extinction angle β

For $\alpha \le \omega t \le \beta$, apply KVL on the circuit and we get

$$\sqrt{2}V\sin\omega t = Ri_o + L\frac{di_o}{dt} \qquad \alpha \le \omega t \le \beta$$

The solution of the above equation is output current i_o which can be expressed as

$$i_o = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi) + Ae^{-\frac{R}{L}t} \tag{9.1}$$

where, $Z = \sqrt{R^2 + (\omega L)^2}$ and $\phi = \tan^{-1}\frac{\omega L}{R}$

At $\omega t = \alpha$, load current $i_o = 0$.

Then $$0 = \frac{\sqrt{2}V}{Z}\sin(\alpha - \phi) + Ae^{-\frac{R}{L}\frac{\alpha}{\omega}}$$

Therefore $$A = -\frac{\sqrt{2}V}{Z}\sin(\alpha - \phi)e^{\frac{R}{L}\frac{\alpha}{\omega}}$$

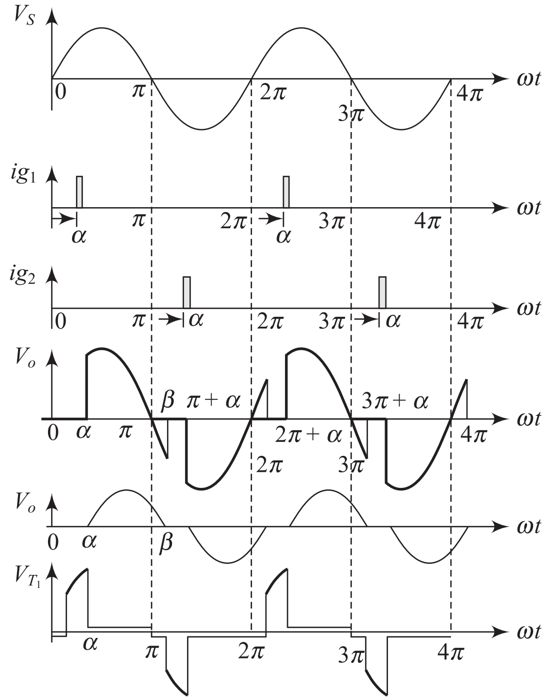


Fig. 9.11 *Voltage and current waveforms of single phase ac voltage controller with RL load*

After substituting the value of A in eq. (9.1), the output current can be expressed as

$$i_o = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{R}{\omega L}(\alpha - \omega t)}\right] \qquad \text{where, } \alpha \le \omega t \le \beta, \text{ and } \alpha \ge \phi$$

or $$i_o = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{\alpha - \omega t}{\tan\phi}}\right] \qquad \text{where, } \tan\phi = \frac{\omega L}{R}$$

At $\omega t = \beta$, load current $i_o(\beta) = 0$.

The value of β can be determined from the expression $i_o(\beta) = 0$

Therefore, $\sin(\beta - \phi) - \sin(\alpha - \phi)e^{\frac{\alpha - \beta}{\tan\phi}} = 0$

The value of extinction angle β can be computed by graph as shown in Fig. 9.12 and the variation of conduction angle γ with respect to α and ϕ is depicted in Fig. 9.13.

The RMS value of output voltage is

$$V_o = \left[\frac{1}{\pi}\int_\alpha^\beta (\sqrt{2}V\sin\omega t)^2 \cdot d(\omega t)\right]^{1/2} = \left[\frac{V^2}{\pi}\int_\alpha^\beta (1 - \cos 2\omega t)\cdot d(\omega t)\right]^{1/2}$$

$$= V\left[\frac{1}{\pi}\left(\beta - \alpha + \frac{1}{2}\sin 2\alpha - \frac{1}{2}\sin 2\beta\right)\right]^{1/2}$$

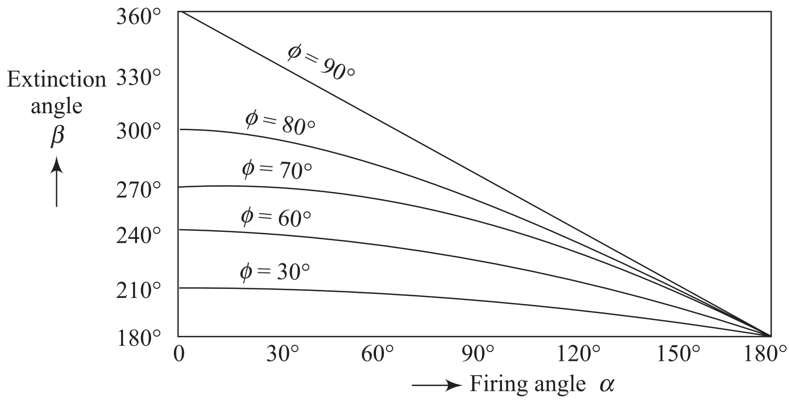


Fig. 9.12 *The variation of extinction angle β with respect to α and ϕ*

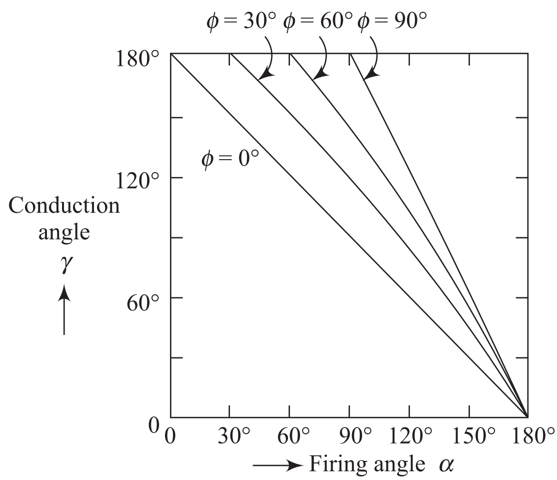


Fig. 9.13 *The variation of conduction angle β with respect to α and ϕ*

The RMS value of load current

$$I_{o\_\mathrm{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta}\left\{i_o(\omega t)\right\}^2 d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{\pi}\int_{\alpha}^{\beta}\left\{\frac{\sqrt{2}V}{Z}\left(\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{\frac{\alpha-\omega t}{\tan\phi}}\right)\right\}^2 d(\omega t)\right]^{1/2}$$

The rms output current is the combination of the rms value of both thyristor currents. If the rms value of each thyristor is I_{rms_T}, we can write

$$I_{o\_\mathrm{rms}} = \left[I^2_{\mathrm{rms}\_T} + I^2_{\mathrm{rms}\_T}\right]^{1/2} = \sqrt{2}I_{\mathrm{rms}\_T}$$

The average value of thyristor current is

$$I_{av\_T} = \frac{1}{2\pi}\int_{\alpha}^{\beta} i_o d\omega t \qquad \text{as } i_o = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{\frac{R}{\omega L}(\alpha-\omega t)}\right]$$

$$= \frac{\sqrt{2}V}{2\pi Z}\int_{\alpha}^{\beta}\left[\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{\frac{R}{\omega L}(\alpha-\omega t)}\right]d\omega t$$

9.6.1 Harmonic Analysis of Output Voltage and Input Current

The output voltage can be expressed by the Fourier series:

$$v_o = \frac{1}{2}a_o + \sum_{n}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where $$a_o = \frac{1}{\pi}\int_{0}^{2\pi} v_o(\omega t)d(\omega t)$$

$$a_n = \frac{2}{\pi}\int_{\alpha}^{\beta} v_o(\omega t)\cdot\cos n\omega t\cdot d(\omega t)$$

$$b_n = \frac{2}{\pi}\int_{\alpha}^{\beta} v_o(\omega t)\cdot\sin n\omega t\cdot d(\omega t)$$

$a_o/2$ is average vale of v_o. The output voltage is equal to input voltage during α to β and $\pi + \alpha$ to $\pi + \alpha + \gamma$. Due to half-wave symmetry, $a_o = \frac{1}{\pi}\int_{0}^{2\pi} v_o(\omega t)d(\omega t) = 0$

$$a_n = \frac{2}{\pi}\int_{\alpha}^{\beta}\sqrt{2}V\sin\omega t\cdot\cos n\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\int_{\alpha}^{\beta}\left[\sin(1-n)\omega t+\sin(1+n)\omega t\right]d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\left[\frac{\cos(1-n)\alpha-\cos(1-n)\beta}{1-n}+\frac{\cos(1+n)\alpha-\cos(1+n)\beta}{1+n}\right]$$

When, $n \neq 1$ and $n = 3,5,7\ldots$

$$b_n = \frac{2}{\pi}\int_{\alpha}^{\beta}\sqrt{2}V\sin\omega t\cdot\sin n\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\int_{\alpha}^{\beta}\left[\cos(1-n)\omega t-\cos(1+n)\omega t\right]d(\omega t)$$

$$= \frac{\sqrt{2}V}{\pi}\left[\frac{\sin(1-n)\beta-\sin(1-n)\alpha}{1-n}+\frac{\sin(1+n)\beta-\sin(1+n)\alpha}{1+n}\right]$$

When, $n \neq 1$ and $n = 3,5,7\ldots$

Fundamental components of voltage are

$$a_1 = \frac{2}{\pi}\int_{\alpha}^{\beta} \sqrt{2}V \sin\omega t \cdot \cos\omega t . d(\omega t) = \frac{\sqrt{2}V}{\pi}\int_{\alpha}^{\beta} \sin(2\omega t) d(\omega t)$$

and
$$b_1 = \frac{2}{\pi}\int_{\alpha}^{\beta} \sqrt{2}V \sin\omega t \cdot \sin\omega t \cdot d(\omega t) = \frac{\sqrt{2}V}{\pi}\int_{\alpha}^{\beta} (1-\cos 2\omega t) d(\omega t)$$

Therefore,

$$a_1 = \frac{\sqrt{2}V}{2\pi}(\cos 2\alpha - \cos 2\beta)$$

$$b_1 = \frac{\sqrt{2}V}{\pi}\left(\beta - \alpha + \frac{1}{2}\sin 2\alpha - \frac{1}{2}\sin 2\beta\right)$$

The rms value of n^{th} harmonic of output voltage is

$$V_n = \left[\frac{a_n^2 + b_n^2}{2}\right]^{1/2} \text{ and phase angle } \phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

Then output voltage can be expressed as $v_o(\omega t) = \sum_{n=1,3,5....}^{\infty} \sqrt{2}V_n \sin(n\omega t + \phi_n)$
The load current is

$$i_o(\omega t) = \sum_{n=1,3,5}^{\infty} \sqrt{2}I_n \sin(n\omega t + \phi_n - \theta_n)$$

where, $\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$,

Load impedance $Z = \sqrt{R^2 + (n\omega L)^2}\angle\theta_n$

$$\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right) \text{ and}$$

$$I_n = \frac{\left[(a_n^2 + b_n^2)/2\right]^{\frac{1}{2}}}{Z}$$

9.7 ac VOLTAGE CONTROLLERS WITH PURELY INDUCTIVE LOAD

When a purely inductive load is connected to a single-phase ac voltage controlleras depicted in Fig. 9.14, the output voltage can be controlled during ωt from $\frac{\pi}{2}$ to π only as $\phi = 90°$. Then range of firing angle is $\frac{\pi}{2} \le \alpha \le \pi$. The

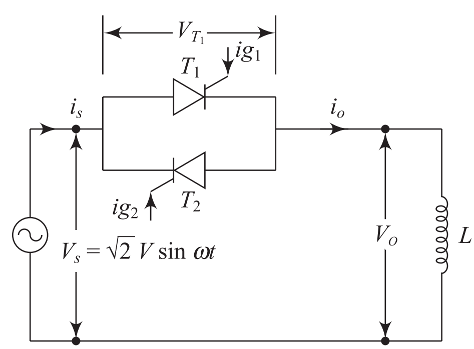


Fig. 9.14 *Single-phase ac voltage controller with purely inductive load*

waveforms of voltage and current are shown in Fig. 9.15. This circuit is also known as *thyristor controlled inductor* or *thyristor controlled reactor*. In ac power system, it is commonly called *static VAR compensation*. This unit draws lagging reactive current from utility system; hence there will be excessive voltage drops which adversely affect on stability of system.

When the load is resistive and inductive, the current flows through load is

$$i_o = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{R}{\omega L}(\alpha - \omega t)}\right]$$

$$= \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{(\alpha - \omega t)}{\tan\varphi}}\right]$$

In case of purely inductive load,

$|Z| = \omega L$, and $\phi = 90° = \frac{\pi}{2}$

Then the load current is equal to

$$i_o(\omega t) = \frac{\sqrt{2}V}{\omega L}\left[\sin\left(\omega t - \frac{\pi}{2}\right) - \sin\left(\alpha - \frac{\pi}{2}\right)e^{\frac{\alpha - \omega t}{\tan(\pi/2)}}\right]$$

$$= \frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)$$

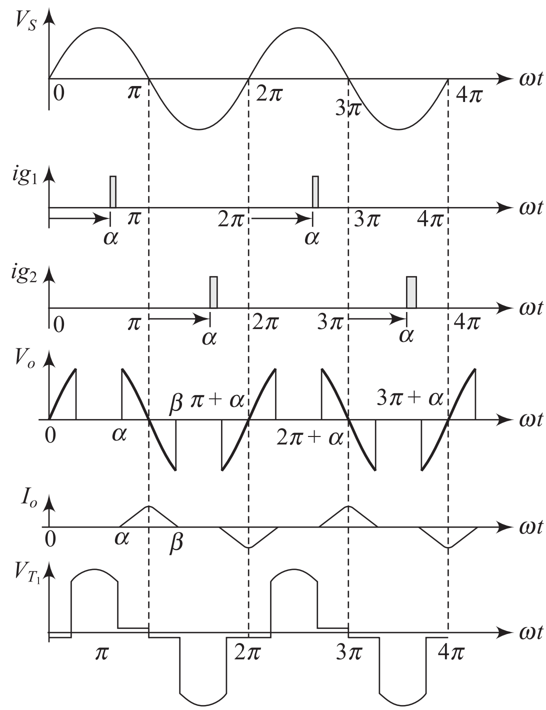


Fig. 9.15 *Voltage and current waveform of single phase ac voltage controller with purely inductive load*

Assume the load current is discontinuous and at $\omega t = \beta, i_o(\beta) = 0$.

Then, $i_o(\beta) = \cos\alpha - \cos\beta = 0$

So, $\beta = 2\pi - \alpha$

The fundamental components of load currents are

$$a_1 = \frac{2}{\pi}\int_\alpha^\beta i_o(\omega t)\cdot\cos\omega t\cdot d(\omega t) \text{ and } b_1 = \frac{2}{\pi}\int_\alpha^\beta i_o(\omega t)\cdot\sin\omega t\cdot d(\omega t)$$

After substituting the value of $i_o(\omega t)$, we obtain

$$a_1 = \frac{2}{\pi}\int_\alpha^\beta \frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)\cdot\cos\omega t\cdot d(\omega t)$$

$$= \frac{2\sqrt{2}V}{\pi\omega L}\left[\cos\alpha\,(\sin\beta - \sin\alpha) - \frac{1}{2}\left\{(\beta - \alpha) + \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right\}\right]$$

$$= \frac{\sqrt{2}V}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha] \qquad \text{when } \beta = 2\pi - \alpha$$

$$b_1 = \frac{2}{\pi}\int_\alpha^\beta \frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)\cdot\sin\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{2\pi\omega L}[-\cos 2\alpha + \cos 2\beta - 4\cos\alpha(\cos\beta - \cos\alpha)]$$

$$= 0 \qquad \text{when } \beta = 2\pi - \alpha$$

Therefore, rms value of the fundamental component of current is

$$I_1 = \left[\frac{a_1^2 + b_1^2}{2}\right]^{1/2} = \frac{V}{\pi\omega L}[2(\pi-\alpha)+\sin 2\alpha]$$

In the same way, the other harmonics components are expressed as

$$a_n = \frac{2}{\pi}\int_{\alpha}^{\beta}\frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)\cdot\cos n\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{2\pi\omega L}\left[\frac{2}{n+1}\{\sin(n+1)\alpha - \sin(n+1)\beta\} - \frac{2}{n-1}\{\sin(n-1)\alpha - \sin(n-1)\beta\} - \frac{4\cos\alpha}{n}(-\sin n\beta + \sin n\alpha)\right]$$

when $n \neq 1$ and $n = 3, 5, 7...$

$$b_n = \frac{2}{\pi}\int_{\alpha}^{\beta}\frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)\cdot\sin\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{2\pi\omega L}\left[\frac{2}{n+1}\{-\cos(n+1)\alpha + \cos(n+1)\beta\} - \frac{2}{n-1}\{-\cos(n-1)\alpha + \cos(n-1)\beta\} - \frac{4\cos\alpha}{n}(\cos n\beta - \cos n\alpha)\right]$$

when $n \neq 1$ and $n = 3, 5, 7...$

The rms value of line current and load current are as follows:

$$I = I_o = \left[\frac{1}{\pi}\int_{\alpha}^{\beta} I_m{}^2(\cos\alpha - \cos\omega t)^2 d(\omega t)\right]^{1/2}, \text{ where } I_m = \frac{\sqrt{2}V}{\omega L}$$

$$= I_m\left[\frac{1}{\pi}\int_{\alpha}^{2\pi-\alpha}(\cos^2\alpha + \cos^2\omega t - 2\cos\alpha\cos\omega t)d(\omega t)\right]^{1/2}$$

$$= I_m\left[\frac{1}{\pi}\left\{(\pi-\alpha)(1+2\cos^2\alpha) + \frac{3}{2}\sin 2\alpha\right\}\right]^{1/2}$$

Example 9.12 A single-phase ac voltage controller is fed from 220 V, 50 Hz ac supply and connected to R-L load. When $R = 5\ \Omega$ and $|\omega L| = 4\ \Omega$, determine (a) the range of firing angle, (b) maximum value of rms load current, (c) maximum power and power factor at maximum power output, (d) maximum average thyristor current, (e) maximum rms thyristor current and (f) maximum value of $\frac{di_o}{dt}$.

Solution

(a) The minimum value of firing angle $\alpha_{\min}$ = load phase angle

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{4}{5}\right) = 36.659°$$

The maximum possible value of firing angle $\alpha_{\max} = 180°$

The range of firing angle is $36.659° \leq \alpha \leq 180°$

(b) When $\alpha = \phi$, the maximum value of rms load current occurs as the ac voltage controller behaves just like load is directly connected to the ac supply.

The maximum value of rms load current is

$$I_o = \frac{V}{\sqrt{R^2 + (\omega L)^2}} = \frac{220}{\sqrt{5^2 + 4^2}} = 34.358 \text{ A}$$

(c) The maximum power output is $I_o^2 R$ = $34.358^2 \times 5$ = 1180.47 Watt

The power factor = $\dfrac{I_o^2 R}{VI_o} = \dfrac{I_o R}{V} = \dfrac{34.356 \times 5}{220} = 0.7808 \text{ (lag)}$

(d) When $\alpha = \phi$, thyristor conduction angle is $\gamma = 180° = \pi$. Then the maximum average thyristor current is equal to

$$I_{\text{av}T} = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} \frac{\sqrt{2}V}{Z} \sin(\omega t - \phi) d\omega t$$

$$= \frac{\sqrt{2}V}{\pi Z} = \frac{\sqrt{2} \times 220}{\pi\sqrt{5^2 + 4^2}} \text{A} \quad \text{as } V = 220 \text{ and } Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{5^2 + 4^2} = 15.46 \text{ A}$$

(e) The maximum rms thyristor current is equal to

$$I_{\text{rms}T} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} \left\{\frac{\sqrt{2}V}{Z} \sin(\omega t - \phi)\right\}^2 d\omega t\right]^{1/2} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} \left\{\frac{\sqrt{2}V}{Z} \sin(\omega t - \alpha)\right\}^2 d\omega t\right]^{1/2} \quad \text{as } \alpha = \phi$$

$$= \frac{\sqrt{2}V}{2Z} = \frac{\sqrt{2} \times 220}{2\sqrt{5^2 + 4^2}} \text{A} \quad \text{as } V = 220 \text{ and } Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{5^2 + 4^2} = 24.295 \text{ A}$$

(f) The value of $\dfrac{di_o}{dt}$ is equal to $\dfrac{di_o}{dt} = \dfrac{\sqrt{2}V}{Z} \omega \cos(\omega t - \phi)$ as $i_o = \dfrac{\sqrt{2}V}{Z} \sin(\omega t - \phi)$

The maximum value of $\dfrac{di_o}{dt}$ occurs at $\cos(\omega t - \phi) = 1$

Then $\left.\dfrac{di_o}{dt}\right|\max = \dfrac{\sqrt{2}V}{Z}\omega = \dfrac{\sqrt{2} \times 220}{\sqrt{5^2 + 4^2}} \times 2\pi \times 50 = 15271.16 \text{ A/sec}$

Example 9.13 A single-phase ac voltage controller is fed from 230 V, 50 Hz ac supply and connected to L load only. If $|\omega L| = 4\ \Omega$, determine (a) the control range of firing angle, (b) maximum value of rms load current, (c) maximum average thyristor current, (d) maximum rms thyristor current and (e) maximum value of $\dfrac{di_o}{dt}$.

Solution

(a) The minimum value of firing angle $\alpha_{\min}$ = Load phase angle

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{4}{0}\right) = 90°$$

The maximum possible value of firing angle $\alpha_{\max}$ = 180°

The range of firing angle is $90° \le \alpha \le 180°$

(b) When $\alpha = \phi = 90°$, the maximum value of rms load current occurs as the ac voltage controller behaves just like load is directly connected to the ac supply.

The maximum value of rms load current is

$$I_o = \frac{V}{\sqrt{(\omega L)^2}} = \frac{230}{\sqrt{4^2}} = 57.5 \text{ A}$$

(d) When $\alpha = \phi$, thyristor conduction angle is $\gamma = 180° = \pi$. Then the maximum average thyristor current is equal to

$$I_{\text{av}T} = \frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi)\,d\omega t$$

$$= \frac{\sqrt{2}V}{\pi Z} = \frac{\sqrt{2}\times 230}{\pi\sqrt{4^2}}\text{A} \qquad \text{as } V = 230 \text{ and } Z = \sqrt{(\omega L)^2} = \sqrt{4^2}$$

$$= 25.87 \text{ A}$$

(e) The maximum rms thyristor current is equal to

$$I_{\text{rms}T} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha}\left\{\frac{\sqrt{2}V}{Z}\sin(\omega t - \phi)\right\}^2 d\omega t\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha}\left\{\frac{\sqrt{2}V}{Z}\sin(\omega t - \alpha)\right\}^2 d\omega t\right]^{1/2} \quad \text{as } \alpha = \phi$$

$$= \frac{\sqrt{2}V}{2Z} = \frac{\sqrt{2}\times 230}{2\sqrt{4^2}}\text{A} \qquad \text{as } V = 230 \text{ V and } Z = \sqrt{(\omega L)^2} = \sqrt{4^2}$$

$$= 40.658 \text{ A}$$

(f) The value of $\frac{di_o}{dt}$ is equal to

$$\frac{di_o}{dt} = \frac{\sqrt{2}V}{Z}\omega\cos(\omega t - \phi) \quad \text{as} \quad i_o = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi)$$

The maximum value of $\frac{di_o}{dt}$ occurs at $\cos(\omega t - \phi) = 1$

Then $$\left.\frac{di_o}{dt}\right|\max = \frac{\sqrt{2}V}{Z}\omega = \frac{\sqrt{2}\times 230}{\sqrt{4^2}}\times 2\pi\times 50 = 25556.859 \text{ A/sec}$$

Example 9.14 Figure 9.16 shows a two stage sequence ac voltage controller. Draw the output voltage and current waveforms. Derive the rms output voltage, load current, rms current of thyristors and transformer rating.

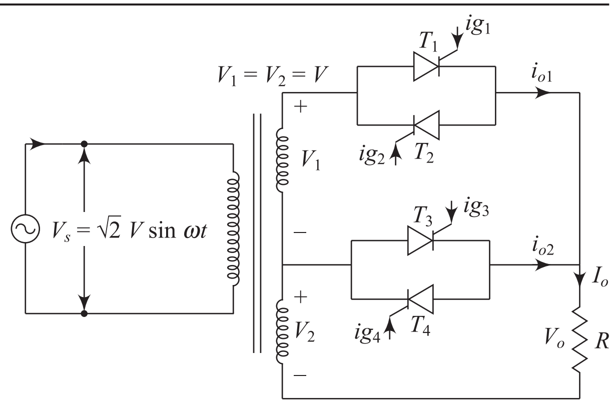


Fig. 9.16

Solution

Thyristors T_1 and T_3 are forward biased during the positive half cycle of input voltage. Assume that the firing pulses are applied to T_3 at $\omega t = 0, 2\pi, 4\pi, 6\pi\ldots$ and the firing pulses are applied to T_1 at $\omega t = \alpha, 2\pi + \alpha, 4\pi + \alpha, 6\pi + \alpha\ldots$. Then the conduction period of thyristors are given below:

For T_3:

$0 < \omega t \le \alpha, 2\pi < \omega t \le 2\pi + \alpha, 4\pi < \omega t \le 4\pi + \alpha \ldots$

For T_1: $\alpha < \omega t \le \pi, 2\pi + \alpha < \omega t \le 3\pi, 4\pi + \alpha < \omega t \le 5\pi \ldots$

Similarly thyristors T_2 and T_4 are forward biased during the negative half cycle of input voltage. Assume that the firing pulses are applied to T_4 at $\omega t = \pi, 3\pi, 5\pi, 7\pi\ldots$ and the firing pulses are applied to T_2 at $\omega t = \pi + \alpha, 3\pi + \alpha, 5\pi + \alpha, 7\pi + \alpha\ldots$. Then the conduction period of thyristors are given below:

For T_4: $\pi < \omega t \le \pi + \alpha, 3\pi < \omega t \le 3\pi + \alpha, 5\pi < \omega t \le 5\pi + \alpha \ldots$

For T_2: $\pi + \alpha < \omega t \le 2\pi, 3\pi + \alpha < \omega t \le 4\pi, 5\pi + \alpha < \omega t \le 6\pi \ldots$

The output voltage and current waveform of two stage sequence ac voltage controller are depicted in Fig. 9.17.

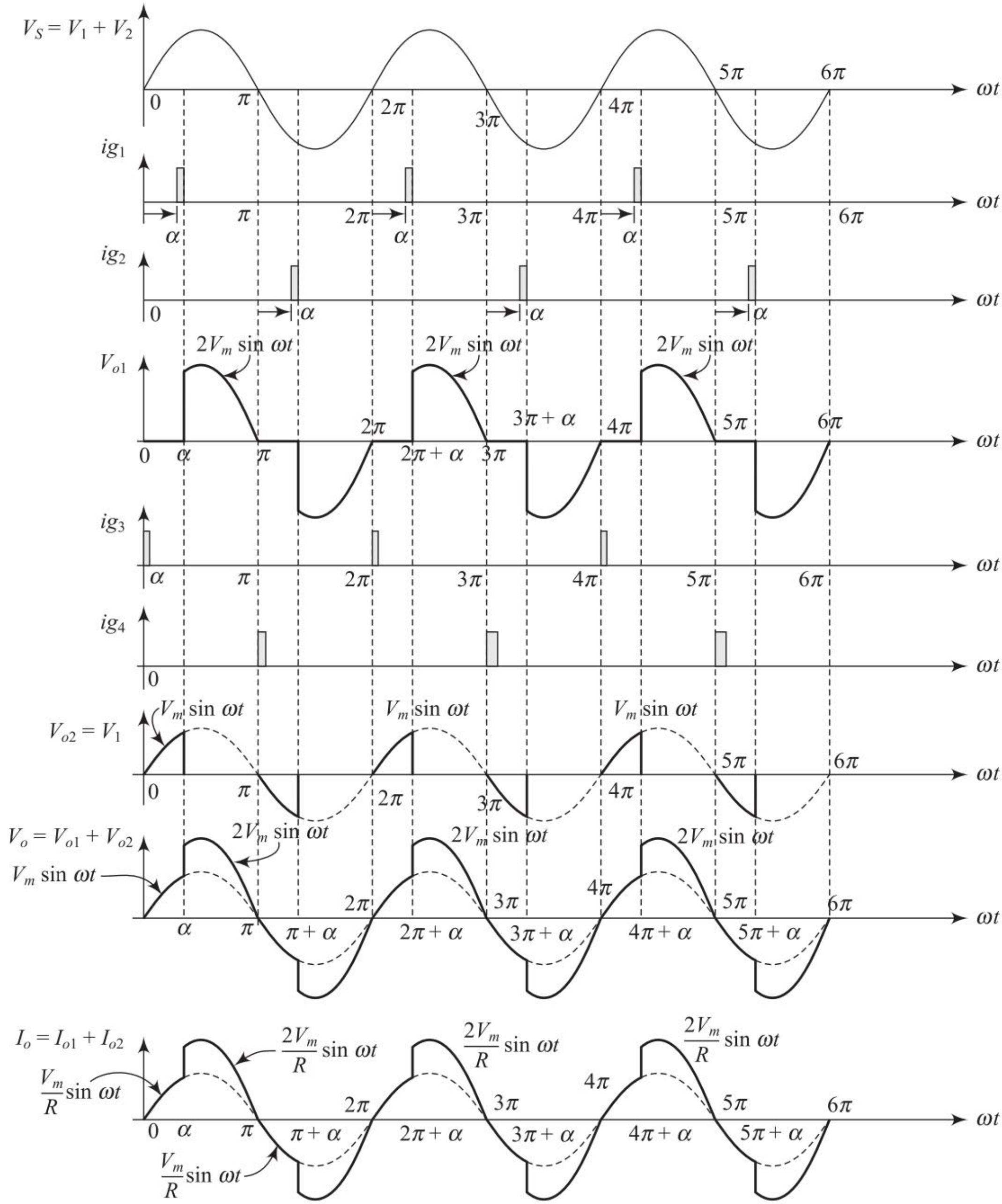


Fig. 9.17 *Output voltage and current waveforms*

Assume that $v_1 = \sqrt{2}V \sin \omega t = V_m \sin \omega t,\ v_2 = \sqrt{2}V \sin \omega t = V_m \sin \omega t$

Then $v_1 + v_2 = 2\sqrt{2}V \sin \omega t = 2V_m \sin \omega t$

The rms output voltage across load is equal to

$$V_{\text{orms}} = \left[\frac{1}{\pi}\left\{\int_0^{\alpha} v_1^2 d\omega t + \int_{\alpha}^{\pi} (v_1+v_2)^2 d\omega t\right\}\right]^{1/2} = \left[\frac{1}{\pi}\left\{\int_0^{\alpha} V_m^2 \sin^2 \omega t \cdot d\omega t + \int_{\alpha}^{\pi} (2V_m)^2 \sin^2 \omega t \cdot d\omega t\right\}\right]^{1/2}$$

$$= \left[\frac{1}{\pi}\left\{\frac{V_m^2}{2}\int_0^{\alpha}(1-\cos 2\omega t)\cdot d\omega t + \frac{4V_m^2}{2}\int_{\alpha}^{\pi}(1-\cos 2\,\omega t)\cdot d\omega t\right\}\right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi}\left\{\alpha - \frac{\sin 2\alpha}{2}\right\} + \frac{2V_m^2}{\pi}\left\{\pi - \alpha + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

The rms value of load current is

$$I_{\text{orms}} = \frac{V_{\text{orms}}}{R}$$

The rms value of current for thyristor T_3 and T_4 is

$$I_{\text{orms}-T3} = \left[\frac{1}{2\pi}\left\{\int_0^{\alpha}\left(\frac{v_1}{R}\right)^2 d\omega t\right\}\right]^{1/2} = \left[\frac{1}{2\pi R^2}\left\{\int_0^{\alpha} V_m^2 \sin^2 \omega t \cdot d\omega t\right\}\right]^{1/2}$$

$$= \left[\frac{1}{2\pi R^2}\left\{\frac{V_m^2}{2}\int_0^{\alpha}(1-\cos 2\omega t)\cdot d\omega t\right\}\right]^{1/2} = \left[\frac{V_m{}^2}{4\pi R^2}\left\{\alpha - \frac{\sin 2\alpha}{2}\right\}\right]^{1/2} = \frac{V_m}{2R}\left[\frac{1}{\pi}\left\{\alpha - \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

The rms value of current for thyristor T_1 and T_2 is

$$I_{\text{orms}-T1} = \left[\frac{1}{2\pi}\left\{\int_{\alpha}^{\pi}\left(\frac{v_1+v_2}{R}\right)^2 d\omega t\right\}\right]^{1/2} = \left[\frac{1}{2\pi R^2}\left\{\int_{\alpha}^{\pi} 4V_m^2 \sin^2 \omega t \cdot d\omega t\right\}\right]^{1/2}$$

$$= \left[\frac{1}{2\pi R^2}\left\{2V_m^2\int_{\alpha}^{\pi}(1-\cos 2\omega t)\cdot d\omega t\right\}\right]^{1/2}$$

$$= \left[\frac{V_m{}^2}{\pi R^2}\left\{\pi - \alpha + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2} = \frac{V_m}{R}\left[\frac{1}{\pi}\left\{\pi - \alpha + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

rms current rating of upper secondary of transformer is

$$I_1 = \sqrt{2} I_{\text{rms}\_T1}$$

rms current rating of lower secondary of transformer is

$$I_3 = [(\sqrt{2} I_{\text{rms}\_T1})^2 + (\sqrt{2} I_{\text{rms}\_T2})^2]^{1/2}$$

When the voltage rating of upper and lower secondary windings of transformers are equal to V, the transformer rating is

$$V(I_1 + I_2)\ \text{VA}.$$

Example 9.15 A two-stage sequence ac voltage controller is connected to a resistive load of 10 Ω. The input ac voltage is 220 V, 50 Hz and turn ratio is 1:1:1. If the firing angle of upper thyristors are 45°, determine (a) rms output voltage, (b) rms output current, (c) VA rating of transformer and (d) input power factor.

Solution

Given: $V = 220\text{ V}, V_m = \sqrt{2}V = \sqrt{2}\times 220 = 311.12\text{ V}, R = 10\,\Omega, \alpha = 45° = \dfrac{\pi}{4}$

(a) rms output voltage is

$$V_{\text{orms}} = \left[\frac{V_m^2}{\pi}\left\{\alpha - \frac{\sin 2\alpha}{2}\right\} + \frac{2V_m^2}{\pi}\left\{\pi - \alpha + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$= \left[\frac{311.12^2}{\pi}\left\{\frac{\pi}{4} - \frac{\sin(2\times 45)}{2}\right\} + \frac{2\times 311.12^2}{\pi}\left\{\pi - \frac{\pi}{4} + \frac{\sin(2\times 45)}{2}\right\}\right]^{1/2}$$

$$= 429.89\text{ V}$$

(b) rms current of T_3

$$I_{\text{rms\_}T3} = \frac{V_m}{2R}\left[\frac{1}{\pi}\left\{\alpha - \frac{\sin 2\alpha}{2}\right\}\right]^{1/2} = \frac{311.12}{2\times 10}\left[\frac{1}{\pi}\left\{\frac{\pi}{4} - \frac{\sin(2\times 45)}{2}\right\}\right]^{1/2} = 4.686\text{ A}$$

rms current of T_1

$$I_{\text{rms\_}T1} = \frac{V_m}{R}\left[\frac{1}{\pi}\left\{\pi - \alpha + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2} = \frac{311.12}{10}\left[\frac{1}{\pi}\left\{\pi - \frac{\pi}{4} + \frac{\sin(2\times 45)}{2}\right\}\right]^{1/2} = 29.66\text{ A}$$

(c) rms current rating of upper secondary of transformer is

$$I_1 = \sqrt{2}I_{\text{rms\_}T1} = \sqrt{2}\times 29.66 = 41.94\text{ A}$$

rms current rating of lower secondary of transformer is

$$I_3 = [(\sqrt{2}I_{\text{rms\_}T1})^2 + (\sqrt{2}I_{\text{rms\_}T2})^2]^{1/2}$$
$$= [(\sqrt{2}\times 29.66)^2 + (\sqrt{2}\times 4.686)^2]^{1/2} = 42.459\text{ A}$$

VA rating of the transformer is

$$V(I_1 + I_2)\text{ VA} = 220(41.94 + 42.459) = 18567.78\text{ VA}$$

(d) Output power $P_o = \dfrac{V_{\text{orms}}^2}{R} = \dfrac{429.89^2}{10} = 18480.54$ Watt

Input power factor $\cos\phi = \dfrac{P_o}{VA} = \dfrac{18480.54}{18567.78} = 0.995$ lagging

Example 9.16 A two-stage sequence ac voltage controller is connected to a resistive load of 10 Ω. The input ac voltage is 200 V, 50 Hz. Turn ratio from primary to each secondary of transformer is unity. If the firing angle of upper thyristors are 30°, determine (a) rms output voltage, (b) rms output current, (c) VA rating of transformer and (d) input power factor.

Solution

Given: $V = 200$ V, $V_m = \sqrt{2}V = \sqrt{2}\times 200 = 282.84\text{ V}, R = 10\,\Omega, \alpha = 30° = \dfrac{\pi}{6}$

(a) rms output voltage is

$$V_{\text{orms}} = \left[\frac{V_m^2}{\pi}\left\{\alpha - \frac{\sin 2\alpha}{2}\right\} + \frac{2V_m^2}{\pi}\left\{\pi - \alpha + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$= \left[\frac{282.84^2}{\pi}\left\{\frac{\pi}{6} - \frac{\sin(2\times 30)}{2}\right\} + \frac{2\times 282.84^2}{\pi}\left\{\pi - \frac{\pi}{6} + \frac{\sin(2\times 30)}{2}\right\}\right]^{1/2}$$

$$= 379.10 \text{ V}$$

(b) rms current of T_3

$$I_{\text{rms}\_T3} = \frac{V_m}{2R}\left[\frac{1}{\pi}\left\{\alpha - \frac{\sin 2\alpha}{2}\right\}\right]^{1/2} = \frac{282.84}{2\times 10}\left[\frac{1}{\pi}\left\{\frac{\pi}{6} - \frac{\sin(2\times 30)}{2}\right\}\right]^{1/2}$$

$$= 2.398 \text{ A}$$

rms current of T_1

$$I_{\text{rms}\_T1} = \frac{V_m}{R}\left[\frac{1}{\pi}\left\{\pi - \alpha + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2} = \frac{282.84}{10}\left[\frac{1}{\pi}\left\{\pi - \frac{\pi}{6} + \frac{\sin(2\times 30)}{2}\right\}\right]^{1/2}$$

$$= 27.874 \text{ A}$$

(c) rms current rating of upper secondary of transformer is

$$I_1 = \sqrt{2} I_{\text{rms}\_T1} = \sqrt{2}\times 27.874 = 39.41 \text{ A}$$

rms current rating of lower secondary of transformer is

$$I_3 = [(\sqrt{2} I_{\text{rms}\_T1})^2 + (\sqrt{2} I_{\text{rms}\_T2})^2]^{1/2}$$

$$= [(\sqrt{2}\times 27.874)^2 + (\sqrt{2}\times 2.398)^2]^{1/2} = 39.55 \text{ A}$$

VA rating of the transformer is

$$V(I_1 + I_2) \text{ VA} = 200(39.41 + 39.55) = 15792 \text{ VA}$$

(d) Output power $P_o = \dfrac{V_{\text{oms}}^2}{R} = \dfrac{379.10^2}{10} = 14371.68$ Watt t

Input power factor $\cos\phi = \dfrac{P_o}{VA} = \dfrac{14371.68}{15792} = 0.910$ lagging

9.8 THREE-PHASE ac VOLTAGE CONTROLLER

When single-phase ac voltage controllers are used in high power load applications, the distorted waveform of line current creates the unbalance in three phase line current. Usually, this unbalance line current flows through neutral which must be always avoided. Therefore, three phase ac voltage controllers should be used in high power applications in place of single phase ac voltage controllers. There are different circuit configurations of three phase Ac voltage controllers as shown in Fig. 9.18.

Figures 9.18(a) and (b) are most commonly used for Δ and Y connected loads respectively. In both the cases, only line current flows and there is no neutral current and at least two SCRs of different phases must be conduct.

In Fig. 9.18(c) and (d), three single-phase ac controllers are connected in Y and Δ respectively. Each SCR can independently control the output voltage across load. Figure 9.18(c) is only suitable for low power ac voltage controller as third harmonic current is present in the system due to a large neutral current and this third harmonic current must be restricted in line current. The analysis of single-phase

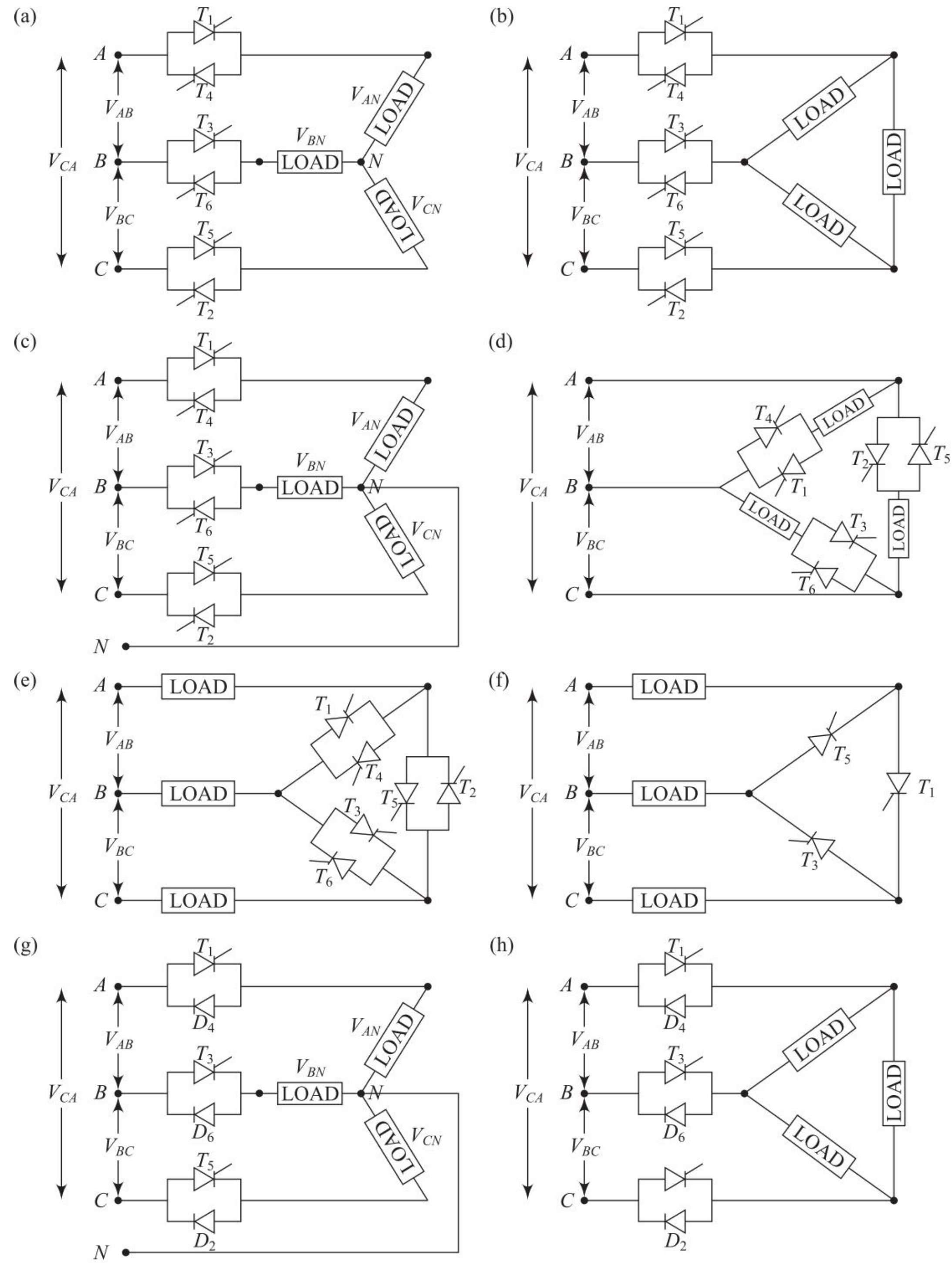


Fig. 9.18 *Different configurations of three phase ac voltage controller*

ac voltage controller can be used in these cases. In case of Fig. 9.18(b) and (d), the six terminals of loads must be available.

The other types of ac voltage controller are not commonly used. In Fig. 9.18(g) is unidirectional control (positive half cycle only) due to one SCR and one diode per phase. The output voltages waveforms are asymmetric and even harmonics are present in the line current. The presence of harmonics in line current is undesirable.

9.9 DELTA-CONNECTED THYRISTOR AND LOAD CONFIGURATION

Figure 9.19 shows a three-phase ac voltage controller, which is the delta connection of three single-phase ac voltage controllers. The conduction of each phase thyristors is independent of the other phase. The load voltages as well as load current are independent of other phases. The line current is computed as sum of two phase currents as given below.

$$i_A = i_{AB} + i_{AC}$$
$$i_B = i_{BC} + i_{BA}$$
$$i_C = i_{CA} + i_{CB}$$

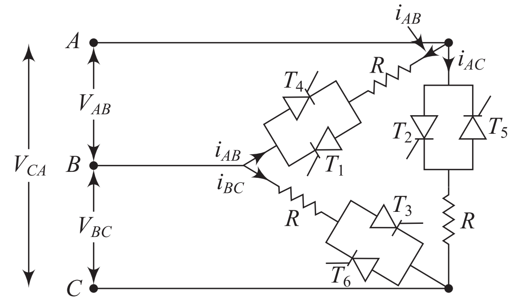


Fig. 9.19 *Delta-connected three-phase ac voltage controller*

where, line currents are i_A, i_B and i_C and phase currents are i_{AB}, i_{BC} and i_{CA}.

The waveforms for delta connected three phase ac voltage controller at firing angle $\alpha = 120°$ and $\alpha = 60°$ are depicted in Figs. 9.20 and 9.21 respectively.

Assume that the instantaneous line to line voltages are

$$v_{AB} = \sqrt{2}V_L \sin \omega t \quad \text{where, } V_L \text{ is the rms line to line voltage}$$

$$v_{BC} = \sqrt{2}V_L \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{CA} = \sqrt{2}V_L \sin\left(\omega t + \frac{2\pi}{3}\right)$$

The rms value of output voltage of phase AB is

$$V_{AB} = \left[\frac{1}{2\pi}\int_{\alpha}^{2\pi} v_{AB}^2 \, . d\omega t\right]^{1/2} = \left[\frac{2}{2\pi}\int_{\alpha}^{\pi} 2V_L^2 \sin^2 \omega t \ \ .d\omega t\right]^{1/2}$$

$$= V_L\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} = V_L\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

Similarly, the rms value of the voltage across phase BC and CA will be as follows:

$$V_{BC} = V_L\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} \text{ and}$$

$$V_{CA} = V_L\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

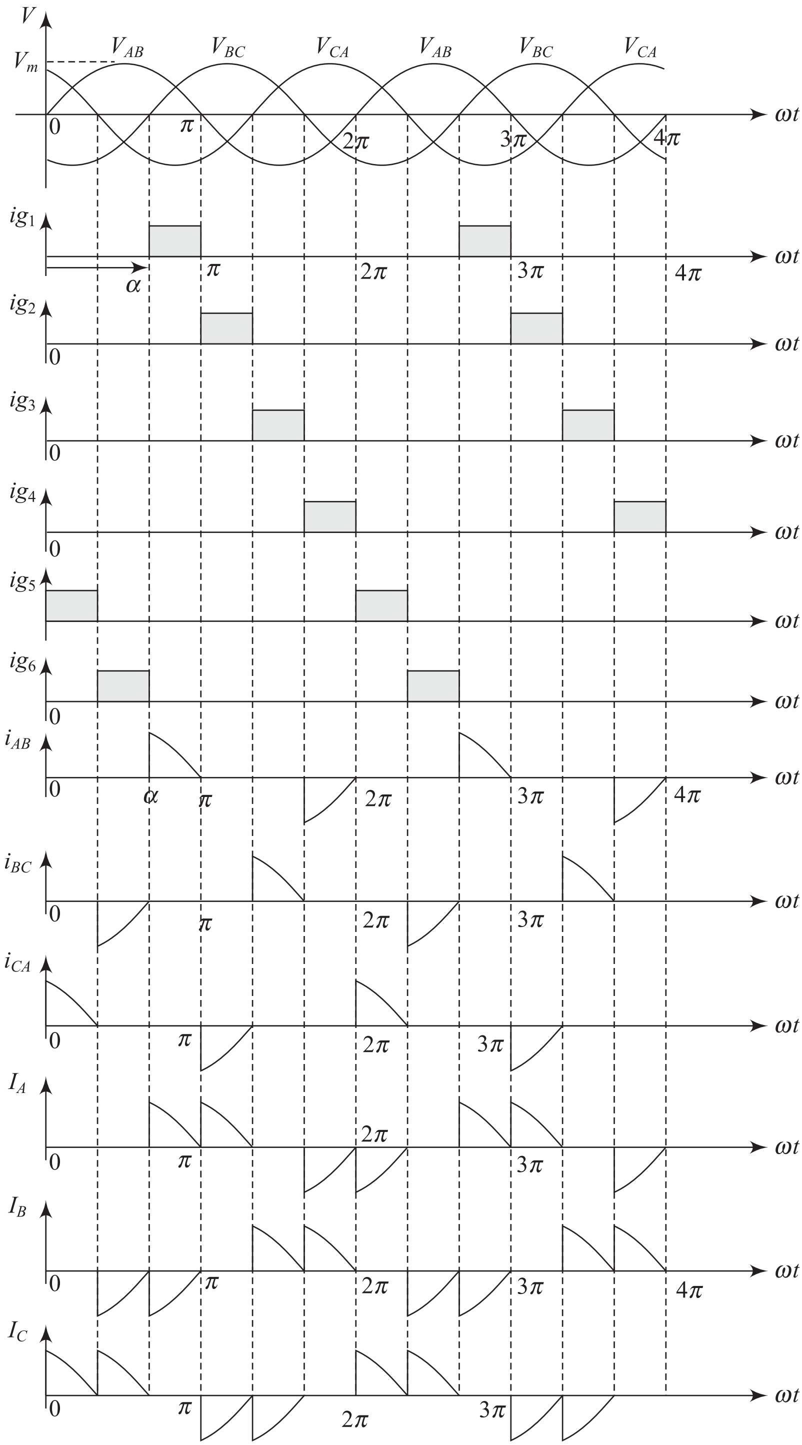


Fig. 9.20 *Waveforms of delta connected ac voltage controller at α = 120°*

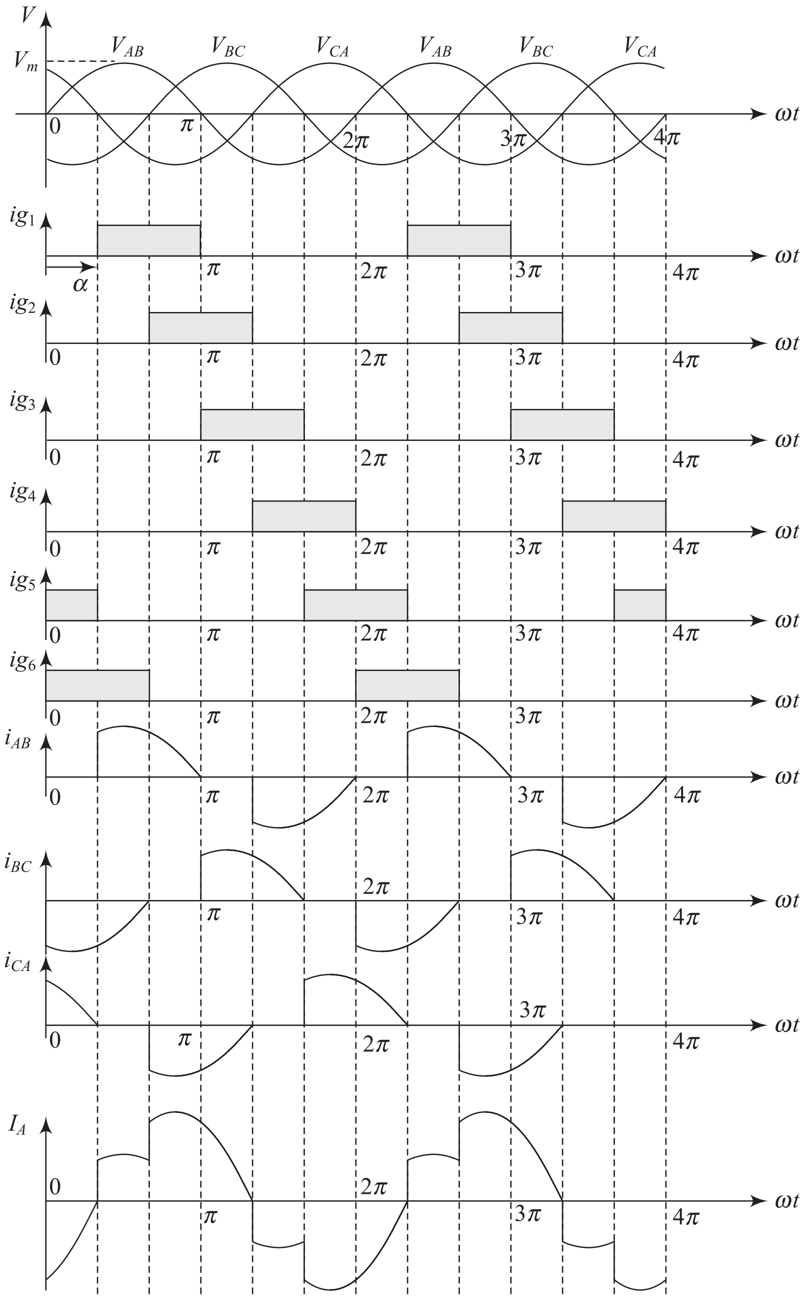


Fig. 9.21 *Waveforms of delta-connected three-phase ac voltage controller at α = 60°*

The rms value of the current is

$$I_{AB} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} i_{AB}^2(\omega t)\,d(\omega t)\right]^{1/2} \qquad \text{Assume} \quad i_{AB} = \frac{v_{AB}}{R}$$

$$= \frac{V_{AB}}{R} = \frac{V_L}{R}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

9.10 DELTA-CONNECTED *R* LOAD CONFIGURATION OF THREE PHASE ac VOLTAGE CONTROLLER

Figure 9.22 shows the three-phase ac voltage controller with delta connected resistive load. In this circuit configuration, two thyristors of two different phases must be conduct for current flows through load. When a current flows through one resistor R which is connected between A' and B', the half of this current must be flow through other two resistances as these series connected resistances are connected in parallel with the resistance between A' and B'. Waveforms of delta connected R load configuration of three phase ac voltage controller at $\alpha = 120°$ are depicted in Fig. 9.23.

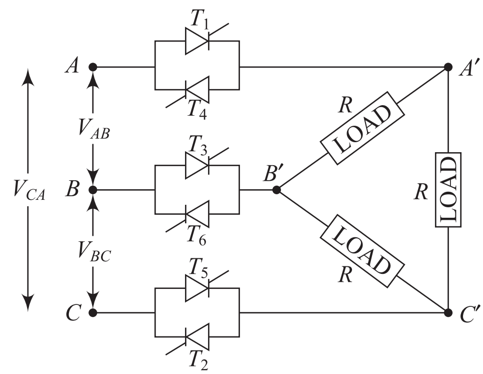


Fig. 9.22 *Three-phase ac voltage controller with delta-connected resistive load*

For the current flow through resistance connected between A' and B', thyristor pair T_1–T_4 and T_3–T_6 will conduct. When thyristors T_1, T_4, T_3 and T_6 are conducting, the output voltage can be expressed as

$$V_{A'B'} = V_L\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} \quad \text{and}$$

$$V_{A'C'} = V_{C'B'} = \frac{V_{A'B'}}{2} = \frac{V_L}{2}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

The line currents are given by

$$i_A = i_{A'B'} - i_{C'A'} = \frac{V_{A'B'} - V_{C'A'}}{R}$$

$$i_B = i_{B'C'} - i_{A'B'} = \frac{V_{B'C'} - V_{A'B'}}{R}$$

$$i_C = i_{C'A'} - i_{B'C'} = \frac{V_{C'A'} - V_{B'C'}}{R}$$

where, phase currents are $i_{A'B'}$, $i_{B'C'}$ and $i_{C'A'}$ and line currents are i_A, i_B, and i_C.

Similarly, for the current flow through resistance connected between B' and C', thyristor pair T_3–T_6 and T_2–T_5 will conduct. When thyristors T_3, T_6, T_2 and T_5 are conducting, the output voltage can be expressed as

$$V_{B'C'} = V_L\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2} \text{ and } V_{B'A'} = V_{A'C'} = \frac{V_{B'C'}}{2} = \frac{V_L}{2}\left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right]^{1/2}$$

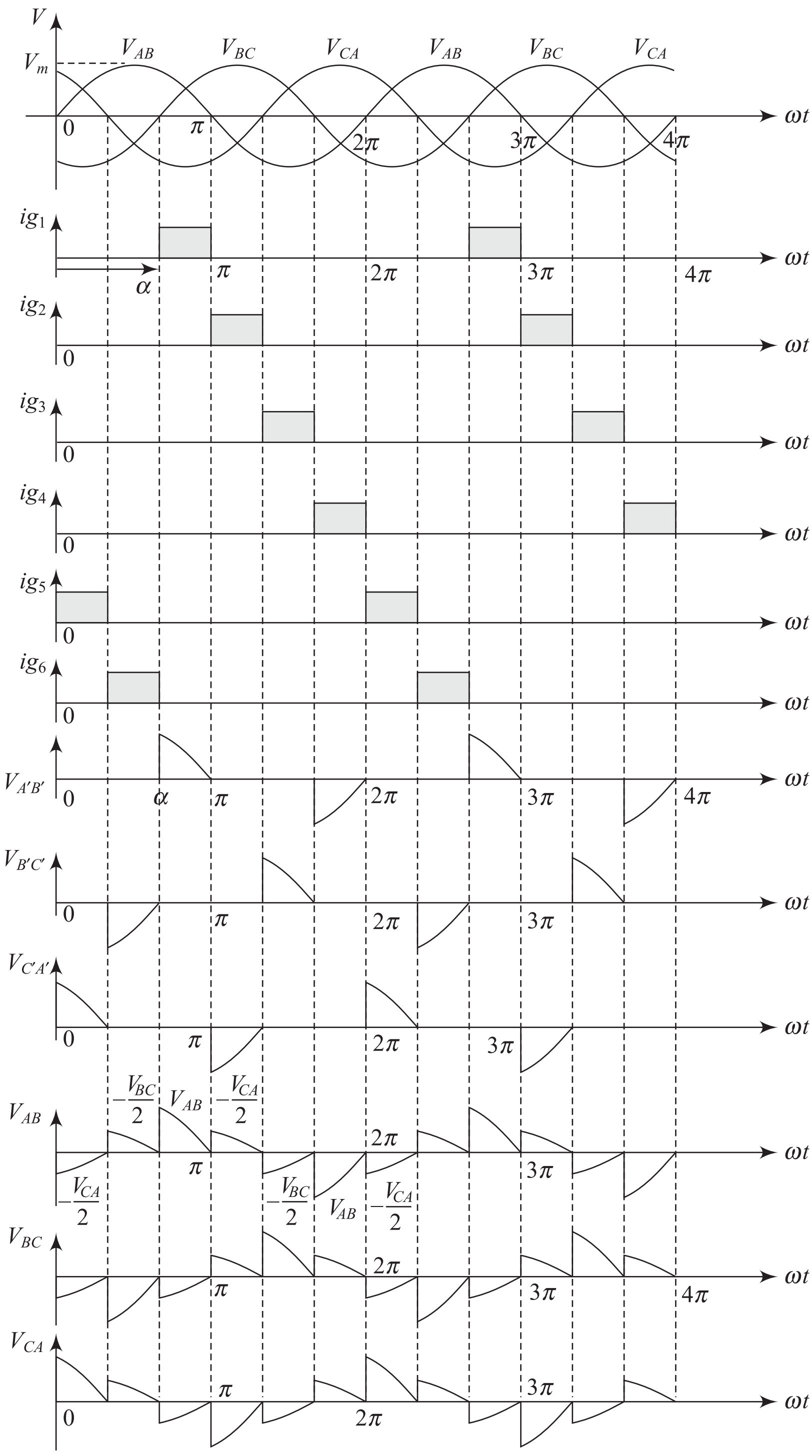


Fig. 9.23 *Waveforms of delta-connected R load configuration of three phase ac voltage controller at a = 120°*

Using the Fig. 9.23, the rms output voltage across A' and B' at firing angle $\alpha = 120°$ is computed by the following expression:

$$V_{A'B'} = \left[\frac{1}{\pi}\left\{\int_0^{\pi/3}\left(\frac{-V_{CA}}{2}\right)^2 d\omega t + \int_{\pi/3}^{2\pi/3}\left(\frac{-V_{BC}}{2}\right)^2 d\omega t + \int_{2\pi/3}^{\pi} V_{AB}^2 \cdot d\omega t\right\}\right]^{1/2}$$

where, $V_{AB} = \sqrt{2}V_L \sin \omega t$

$$V_{BC} = \sqrt{2}V_L \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$V_{CA} = \sqrt{2}V_L \sin\left(\omega t + \frac{2\pi}{3}\right)$ and V_L is the rms line to line voltage

9.11 THREE-PHASE ac VOLTAGE CONTROLLER WITH Y CONNECTED *R* LOAD

In a three-phase system, the phase voltages have same voltage with 120° phase shift. The phase voltages V_A, V_B and V_C are equal to

$$v_A = \sqrt{2}V \sin \omega t$$

$$v_B = \sqrt{2}V \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$v_C = \sqrt{2}V \sin\left(\omega t + \frac{2\pi}{3}\right)$ where V is the rms phase voltage

In a start connected system, the line voltages are $\sqrt{3}$ times of phase voltages and there 120° phase shift between two line voltages. The line voltages V_{AB}, V_{BC} and V_{CA} are equal to

$$v_{AB} = \sqrt{6}V \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{BC} = \sqrt{6}V \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{CA} = \sqrt{6}V \sin\left(\omega t - \frac{7\pi}{6}\right)$$

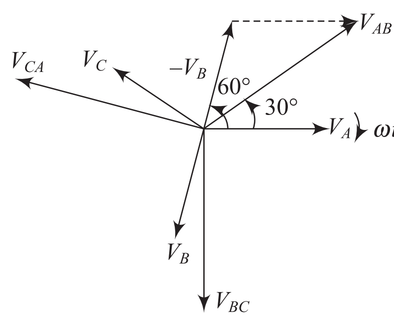


Fig. 9.24 *Phasor diagram of three phase voltages : V_A, V_B, V_C, V_{AB}, V_{BC} and V_{CA}*

The phasor diagram of a three-phase system is depicted in Fig. 9.24.

Figure 9.25 shows a three-phase ac voltage controller with Y connected R load. In this ac voltage controller, two or three thyristors of different phases must be conduct at a time. Therefore, single pulse gate signal cannot be used to turn on SCR. Actually, a train of gate trigger signal is used for the whole conduction period. In R load, thyristor conduction starts at $\omega t = 30°$ and its conduction stop at $\omega t = 180°$. Range of firing angle is 150° only

$$0° \le \alpha \le 150°$$

To get a balanced line current, the thyristors must be triggered symmetrically. Triggering sequence will be α, $\alpha + 120°$, $\alpha + 240°$ for thyristor T_1, T_3 and T_5 respectively. Thyristors T_2, T_4 and T_6 are triggered at $\alpha + 180°$, $\alpha + 300°$, $\alpha + 420°$ respectively. Depending upon the value of firing angle, the operation of converter can be divided into three different modes of operation

Mode I	$0° \le \alpha \le 60°$
Mode II	$60° \le \alpha \le 90°$
Mode III	$90° \le \alpha \le 150°$

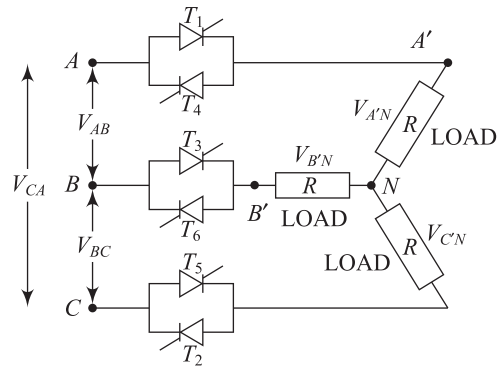


Fig. 9.25 *Three-phase ac voltage controllers with Y connected R load*

Mode I ($0° \le \alpha \le 60°$) When firing angle of thyristors $\alpha = 0°$, three thyristors of different phases always conduct and the load voltage is equal to the phase voltage. If α increases, two or three thyristors will conduct depending upon the circuit parameters. The load voltage is equal to the phase voltage or the half of the voltage $1/2V_{AB}$ or $1/2V_{AC}$. The RMS value of output voltage at firing angle α is equal to

$$V_o = \left[\frac{2}{\pi}\left\{\int_{\alpha}^{\pi/3} v_A^2 d(\omega t) + \int_{\alpha}^{\alpha+\pi/3}\left(\frac{v_{AB}}{2}\right)^2 d(\omega t) + \int_{\alpha+\pi/3}^{2\pi/3} v_A^2 d(\omega t) + \int_{2\pi/3}^{\alpha+2\pi/3}\frac{v_{AC}}{2} d(\omega t) + \int_{\alpha+2\pi/3}^{\pi} v_A^2 d(\omega t)\right\}\right]^{1/2}$$

where, $v_A = \sqrt{2}V \sin \omega t$ and

$$v_{AB} = \sqrt{6}\sin\left(\omega t + \frac{\pi}{6}\right)$$

or $$V_o = V_L\left[\frac{1}{\pi}\left(\frac{\pi}{3} - \frac{\alpha}{2} + \frac{\sin 2\alpha}{4}\right)\right]^{1/2} \quad \text{where } V_L = \sqrt{6}V$$

The gate trigger pulses for thyristors T_1, T_2, T_3, T_4, T_5 and T_6 for firing angle $\alpha = 60°$ and the output voltage waveform at $\alpha = 60°$ are shown in Fig. 9.26.

Mode II ($60° \le \alpha \le 90°$) In this mode, two thyristors of different phases always conduct and the load voltage is always equal to half of the line voltage, i.e., 1/2 V_{AB} or 1/2 V_{AC}. The rms value of output voltage of phase A is equal to

$$V_o = \left[\frac{2}{\pi}\int_{\alpha}^{\alpha+\pi/3}\left\{\frac{\sqrt{6}V}{2}\sin\left(\omega t + \frac{\pi}{6}\right)\right\}^2 d(\omega t)\right]^{1/2}$$

For simplification, if the reference of sine functions will be changed by $\frac{\pi}{6}$ and accordingly the limit of integration will also be changed by $\frac{\pi}{6}$. After simplification, we can write

$$V_o = \left[\frac{2}{\pi}\int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}}\left\{\frac{\sqrt{6}V}{2}\sin \omega t\right\}^2 d(\omega t)\right]^{1/2}$$

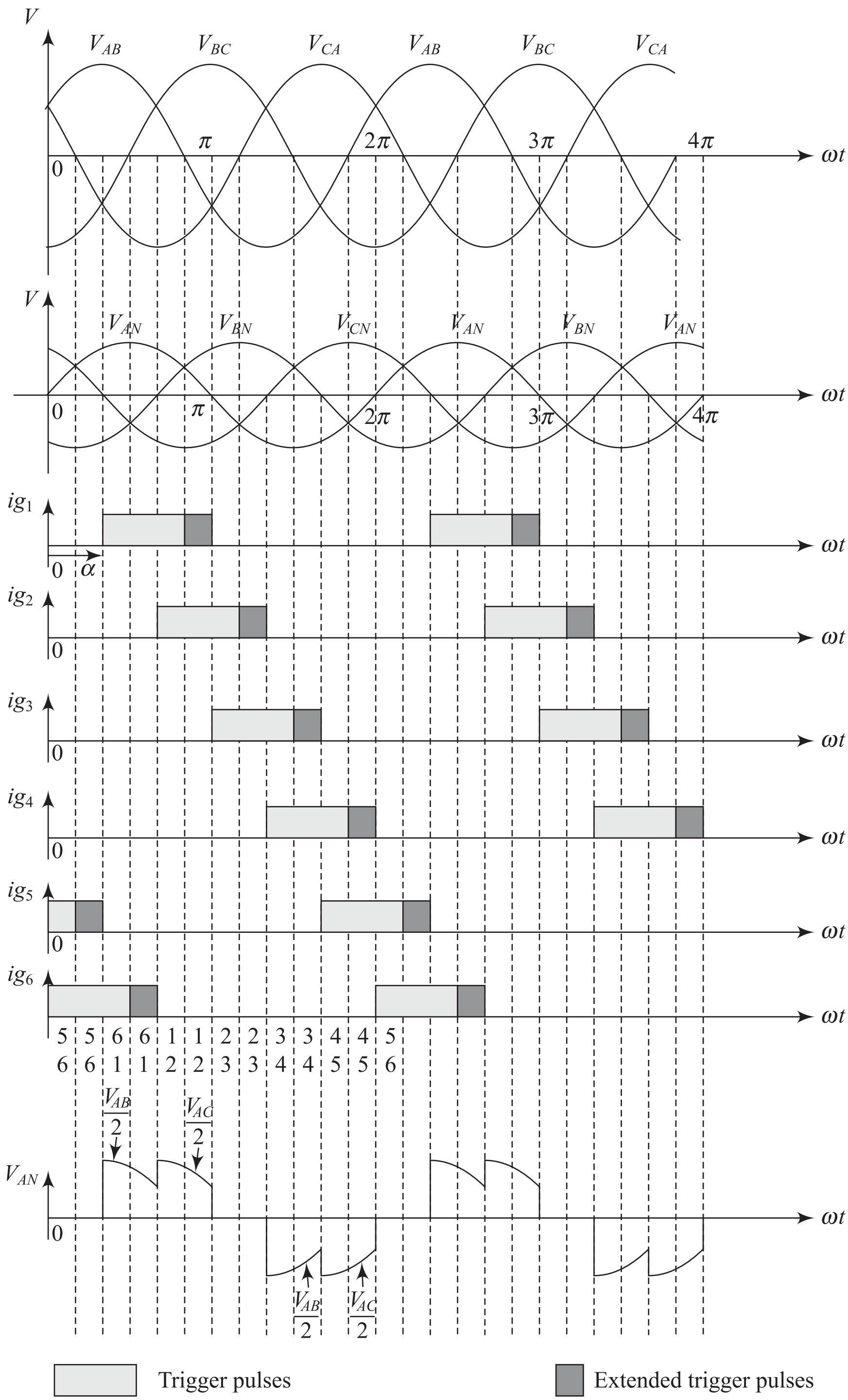


Fig. 9.26 *Waveform of three-phase ac voltage controller at* $\alpha = 60°$

or $$V_o = V_L\left[\frac{1}{2\pi}\left(\frac{\pi}{3}+\frac{3}{4}\sin 2\alpha+\frac{\sqrt{3}}{4}\cos 2\alpha\right)\right]^{1/2} \quad \text{where } V_L=\sqrt{6}\,V$$

Mode III (90° ≤ α ≤ 150°) In this case, two thyristors of different phases always conduct and sometimes none of the thyristors conduct. When firing angle α is grater than 150°, two thyristors will not conduct simultaneously. Hence the output voltage is equal to half of the line voltage or zero. Then output voltage is discontinuous and two voltage pulse will be available in each half cycle.

In this mode, the rms value of load voltage of phase A is

$$V_o=\left[\frac{2}{\pi}\int_{\alpha}^{5\pi/6}\left(\frac{\sqrt{6}V}{2}\sin\left(\omega t+\frac{\pi}{6}\right)\right)^2 d(\omega t)\right]^{1/2}$$

For simplification, if the reference of sine functions will be changed by $\frac{\pi}{6}$ and accordingly the limit of integration will also be changed by $\frac{\pi}{6}$. After simplification, we can write

$$V_o=\left[\frac{2}{\pi}\int_{\alpha+\frac{\pi}{6}}^{\pi}\left\{\frac{\sqrt{6}V}{2}\sin\omega t\right\}^2 d(\omega t)\right]^{1/2}$$

or $$V_o = V_L\left[\frac{1}{2\pi}\left(\frac{5\pi}{6}-\alpha+\frac{1}{4}\sin 2\alpha+\frac{\sqrt{3}}{4}\cos 2\alpha\right)\right]^{1/2} \quad \text{where } V_L=\sqrt{6}\,V$$

The gate trigger pulses for thyristors T_1, T_2, T_3, T_4, T_5 and T_6 for firing angle α = 120° and the output voltage waveform at α = 120° are shown in Fig. 9.27.

9.12 APPLICATIONS OF ac VOLTAGE CONTROLLER

AC voltage controllers are commonly used in electronic tap changing, ac chopper regulators and pulse width modulation control of PWM ac chopper. In this section, the operating principle of tap changing and ac chopper regulators are discussed in detail.

9.12.1 Tap Changer

The output voltages are non-sinusoidal for voltage control of ac voltage controller. These voltages become discontinuous with increasing firing angle and contain higher-order harmonics. Usually, in power system conventional tap-changer switching arrangement is used to control the output voltage across load. In conventional tap-changer, the output voltage is controlled in steps. To get smooth voltage control, now days conventional tap-changers are replaced by electronic tap changers. Figure 9.28 shows a single-phase electronic tap changer which consists of one transformer and four thyristors.

When the thyristors T_3 and T_4 are switched ON with the firing angle α = 0° and closed, the output voltage is equal to V_2. Whenever thyristors T_3 and T_4 are OFF and T_1 and T_2 are ON, the output voltage is equal to $V_1 + V_2$. The gate trigger pulses for thyristirs T_1, T_2, T_3, T_4 and the output voltage waveform are shown in Fig. 9.29. When $V = V_1 + V_2$ and $V_2 = xV$, the rms output voltage can be expressed as

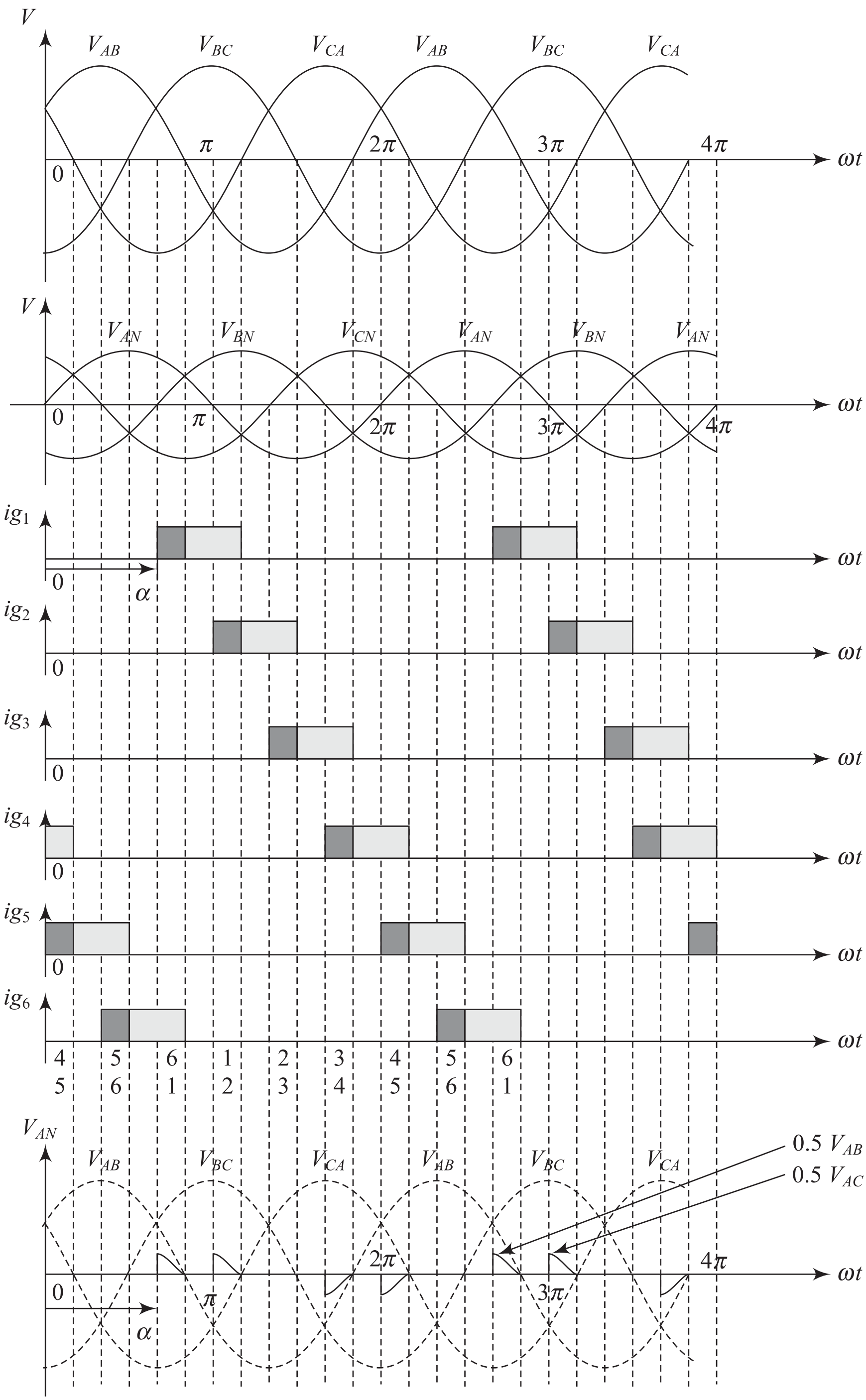


Fig. 9.27 *Waveform of three-phase ac voltage controller at α = 120°*

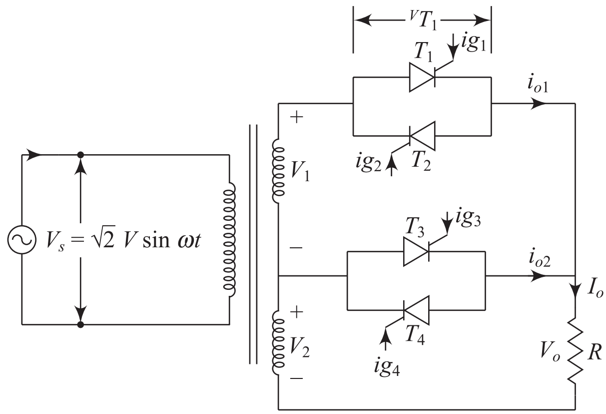


Fig. 9.28 *Single-phase electronic tap changer*

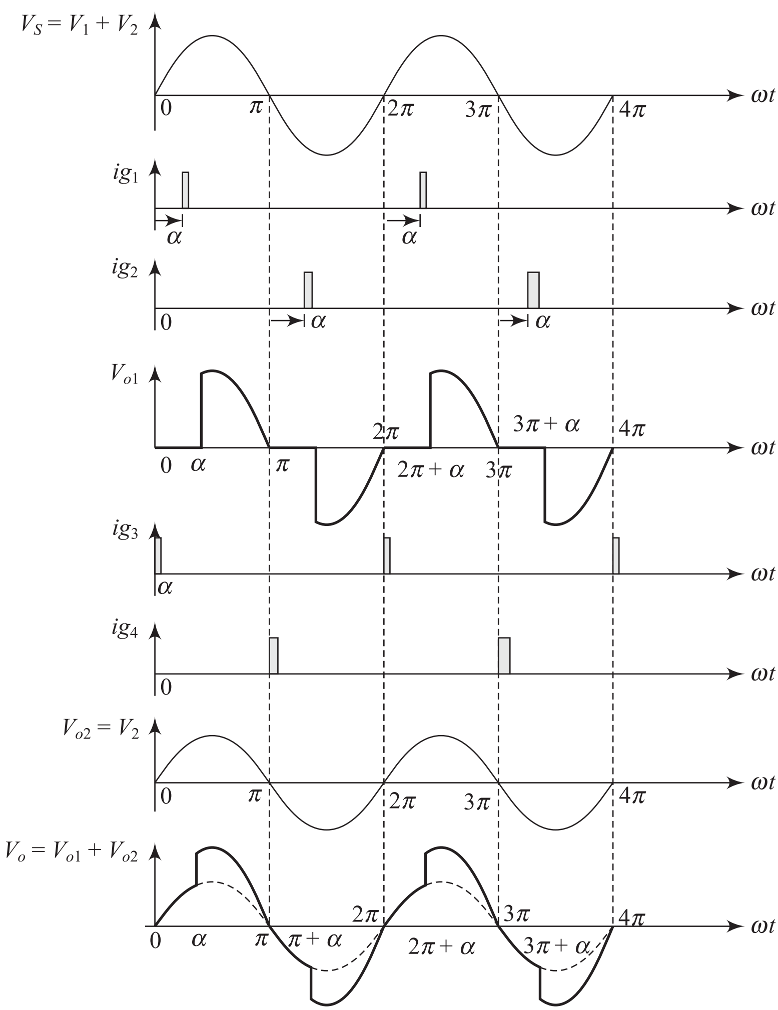


Fig. 9.29 *Gate trigger pulses for thyristirs T_1, T_2, T_3, T_4 and the output voltage waveform*

$$V_o = xV\left[1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}\right]^{1/2} = xV\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

where $V = V_1 + V_2$ and $x < 1$

The output V_o will be between xV and V when thyristor T_3 and T_4 are switched with zero firing angle and thyristor T_1 and T_2 are switched with firing angle α.

For a resistive load, the harmonics of the output voltage can be determined by Fourier series as given below.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(\omega t)\cos n\omega t\cdot d(\omega t) \qquad \text{assume} \quad v(\omega t) = V_o(\omega t)$$

$$= \frac{2}{\pi}\left[\int_0^{\alpha}\sqrt{2}V_2 \sin\omega t\cdot\cos n\omega t\cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}\,(V_1+V_2)\sin\omega t\cdot\cos n\omega t\cdot d(\omega t)\right]$$

As $V_2 = xV$ and $V_1 + V_2 = V$

$$= \frac{2}{\pi}\left[\int_0^{\alpha} x\sqrt{2}\mathrm{V}\sin\omega t\cdot\cos n\omega t\cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}\mathrm{V}\sin\omega t\cdot\cos n\omega t\cdot d(\omega t)\right]$$

$$= \frac{\sqrt{2}}{\pi}(1-x)V\left[\frac{\sin(n+1)\alpha}{n+1}-\frac{\sin(n-1)\alpha}{n-1}\right]$$

Similarly, $b_n = \frac{1}{\pi}\int_0^{2\pi} v(\omega t)\sin n\omega t\cdot d(\omega t)$ assume $v(\omega t) = V_o(\omega t)$

$$= \frac{2}{\pi}\left[\int_0^{\alpha}\sqrt{2}V_2\sin\omega t\cdot\sin n\omega t\cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}(V_1+V_2)\sin\omega t\cdot\sin n\omega t\cdot d(\omega t)\right]$$

As $V_2 = xV$ and $V_1 + V_2 = V$

$$= \frac{2}{\pi}\left[\int_0^{\alpha} x\sqrt{2}V\sin\omega t\cdot\sin n\omega t\cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}\mathrm{V}\sin\omega t\cdot\sin n\omega t\cdot d(\omega t)\right]$$

$$= \frac{\sqrt{2}}{\pi}(1-x)V\left[\frac{\cos(n+1)\alpha}{n+1}-\frac{\cos(n-1)\alpha}{n-1}\right]-\frac{\sqrt{2}V}{\pi}\left(\frac{1}{n+1}-\frac{1}{n-1}\right)$$

The values of fundamental component are

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} v(\omega t)\cos\omega t\cdot d(\omega t)$$

$$= \frac{2}{\pi}\left[\int_0^{\alpha}\sqrt{2}V_2\sin\omega t\cdot\cos\omega t\cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}(V_1+V_2)\sin\omega t\cdot\cos\omega t\cdot d(\omega t)\right]$$

After substituting $V_2 = xV$ and $V_1 + V_2 = V$ in the above equation, we get

$$= \frac{2}{\pi}\left[\int_0^{\alpha} x\sqrt{2}V\sin\omega t\cdot\cos\omega t\cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}V\sin\omega t\cdot\cos\omega t\cdot d(\omega t)\right]$$

$$= \frac{\sqrt{2}V}{2\pi}\left[(1-x)(\sin 2\alpha - 2\alpha) + 2\pi\right]$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} v(\omega t)\sin\omega t \cdot d(\omega t)$$

$$= \frac{2}{\pi}\left[\int_0^{\alpha}\sqrt{2}V_2 \sin\omega t \cdot \sin\omega t \cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}(V_1+V_2)\sin\omega t \cdot \sin\omega t \times d(\omega t)\right]$$

After substituting $V_2 = xV$ and $V_1 + V_2 = V$ in the above equation, we obtain

$$= \frac{2}{\pi}\left[\int_0^{\alpha} x\sqrt{2}V \sin\omega t \cdot \sin\omega t \cdot d(\omega t) + \int_{\alpha}^{\pi}\sqrt{2}V \sin\omega t \cdot \sin\omega t \cdot d(\omega t)\right]$$

$$= \frac{\sqrt{2}}{2\pi}(1-x)V(\cos 2\alpha - 1)$$

Therefore, the rms value of output voltage is

$$V_o = \left[\frac{1}{\pi}\left\{\int_0^{\alpha}(\sqrt{2}V_2 \sin\omega t)^2 d(\omega t) + \int_{\alpha}^{\pi}(\sqrt{2}(V_1+V_2)\sin\omega t)^2 d(\omega t)\right\}\right]^{1/2}$$

or
$$V_o = \left[\frac{1}{\pi}\left\{\int_0^{\alpha}(x\sqrt{2}V \sin\omega t)^2 d(\omega t) + \int_{\alpha}^{\pi}(\sqrt{2}V \sin\omega t)^2 d(\omega t)\right\}\right]^{1/2}$$

As $V_2 = xV$ and $V_1 + V_2 = V$

$$= V\left[\frac{1}{2\pi}\{(1-x^2)\sin 2\alpha + 2\pi\}\right]^{1/2}$$

The power factor is equal to

$$\text{power factor} = \frac{V_O}{V} = \left[\frac{1}{2\pi}\{(1-x^2)(\sin 2\alpha - 2\alpha) + 2\pi\}\right]^{1/2}$$

9.12.2 ac Chopper

AC chopper-type ac voltage controller is not commonly used as gate commutation devices such as GTOs, MOSFET, IGBT and forced commutated thyristor are required. In a thyristor-based ac chopper circuit, forced commutation technique is used. Figure 9.30 shows an ac chopper circuit which consists of four thyistors. T_1 and T_2 are main thyristors and T_{1A} and T_{2A} are auxiliary thyristors. The triggering pulses of T_1, T_2, T_{1A} and T_{2A} and the output voltage waveform of ac chopper are shown in Fig. 9.31. To generate

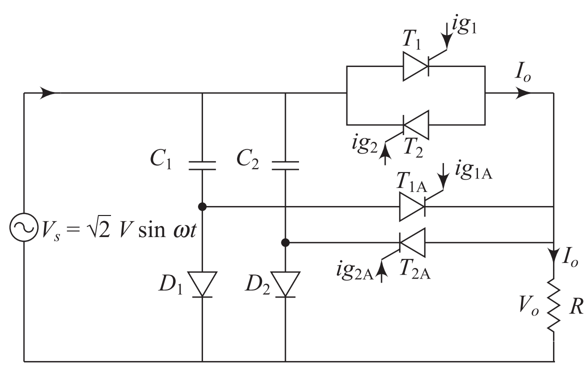


Fig. 9.30 *ac chopper using thyristors*

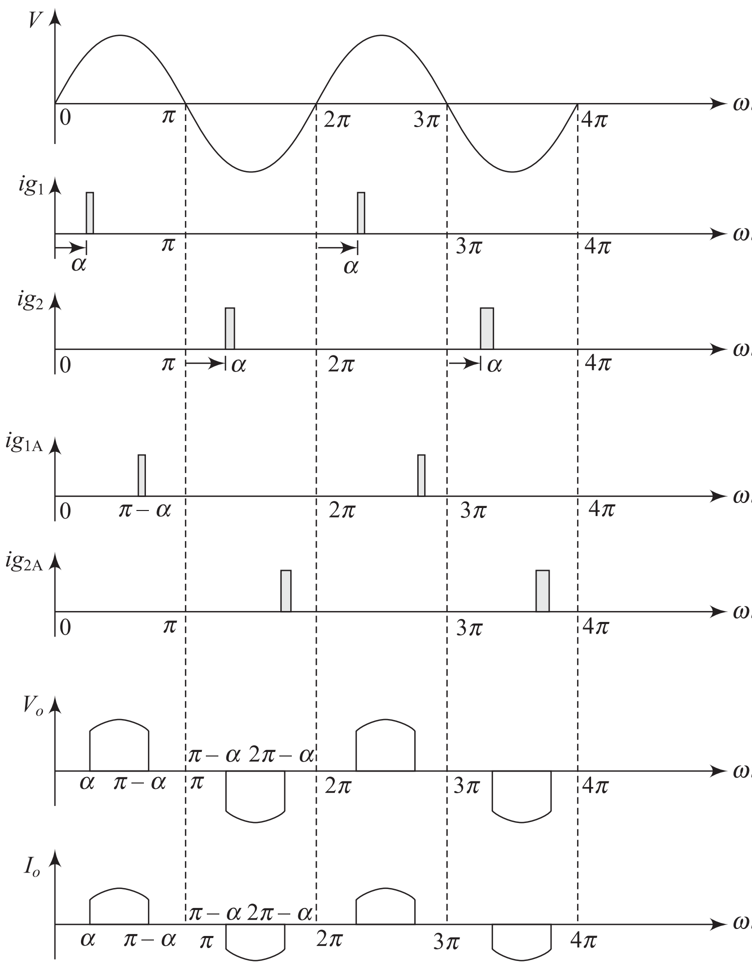


Fig. 9.31 *Gate pulses of T_1, T_2, T_{1A} and T_{2A} and output voltage waveform of ac chopper*

a symmetric voltage waveform, thyristor T_1 must be conduct from $\omega t = \alpha$ to $\omega t = \pi - \alpha$ and thyristor T_2 must be conduct from $\omega t = \pi + \alpha$ to $\omega t = 2\pi - \alpha$.

The rms value of output voltage is

$$V_o = \left[\frac{1}{\pi}\int_{\alpha}^{\pi-\alpha} (\sqrt{2}V \sin \omega t)^2 \cdot d(\omega t)\right]^{1/2} \quad \text{or} \quad V_o = V\left[\frac{1}{\pi}(\pi - 2\alpha + \sin 2\alpha)\right]^{1/2}$$

The values of the fundamental component of output voltage V_1 can be determined from the following equations:

$$a_1 = \frac{2}{\pi}\left[\int_{\alpha}^{2\pi-\alpha} \sqrt{2}V \sin \omega t \cdot \cos \omega t \cdot d(\omega t)\right]$$

$$= \frac{\sqrt{2}V}{\pi}\left[\frac{1}{2}\cos 2\omega t\right]_{\alpha}^{\pi-\alpha} = 0$$

Similarly, $$b_1 = \frac{2}{\pi}\left[\int_{\alpha}^{2\pi-\alpha} \sqrt{2}V \sin \omega t \cdot \sin \omega t \cdot d(\omega t)\right]$$

$$= \frac{\sqrt{2}V}{\pi}\left[\omega t - \frac{1}{2}\sin 2\omega t\right]_{\alpha}^{\pi-\alpha} = \frac{\sqrt{2}V}{\pi}(\pi - 2\alpha + \sin 2\alpha)$$

As $V_1 = \left[\frac{a_1^2 + b_1^2}{2}\right]^{1/2}$, we can find $V_1 = V\left[\frac{1}{\pi}(\pi - 2\alpha) + \frac{\sin 2\alpha}{\pi}\right]^{1/2}$

Power factor with resistive load is equal to

$$\text{power factor} = \frac{V_o}{V} = \left[\frac{1}{\pi}(\pi - 2\alpha) + \frac{\sin 2\alpha}{\pi}\right]^{1/2}$$

For RL load, the current can be expressed by

$$i(\omega t) = \frac{V}{Z}\left[\sin(\omega t - \varphi) - \sin(\alpha - \varphi)\, e^{\frac{\alpha - \omega t}{\tan\varphi}}\right]$$

Example 9.17 A single-phase electronic tap changer as shown in Fig. 9.28 is feeding from a 230 V, 50 Hz ac supply and is connected with a load of R = 10 Ω and the turns ratio from primary winding to each secondary winding is unity. If the firing angle of upper thyristors is 45°, determine (a) rms value of output voltage, (b) rms value of current of upper thyristors, (c) rms value of current of lower thyristors, (d) VA rating of transformer and (e) power factor.

Solution

Given: R = 10 Ω, V = 230 V and $\alpha = 45° = \frac{\pi}{4}$ radian

(a) rms value of output voltage is

$$V_o = \left[\frac{1}{\pi}\left\{\int_0^{\alpha}(\sqrt{2}V \sin \omega t)^2 \cdot d\omega t + \int_{\alpha}^{\pi} 4(\sqrt{2}V \sin \omega t)^2 \cdot d\omega t\right\}\right]^{1/2}$$

$$= \left[\frac{V^2}{\pi}\left(\alpha - \frac{\sin 2\alpha}{2}\right) + \frac{4V^2}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \left[\frac{230^2}{\pi}\left(\frac{\pi}{4} - \frac{\sin 2\times 45}{2}\right) + \frac{4\times 230^2}{\pi}\left(\pi - \frac{\pi}{4} + \frac{\sin 2\times 45}{2}\right)\right]^{1/2} = 444.0413 \text{ V}$$

(b) rms value of current of upper thyristors

$$I_{T1} = \left\{\frac{1}{2\pi R^2}\int_{\alpha}^{\pi} 4(\sqrt{2}V \sin \omega t)^2\, d\omega t\right\}^{1/2}$$

$$= \frac{\sqrt{2}V}{R}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{\sqrt{2}\times 230}{10}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{4} + \frac{\sin 2\times 45}{2}\right)\right]^{1/2} = 31.013 \text{ A}$$

(c) rms value of current of lower thyristors

$$I_{T31} = \left\{ \frac{1}{2\pi R^2} \int_0^{\alpha} (\sqrt{2}V \sin \omega t)^2 d\omega t \right\}^{1/2}$$

$$= \frac{\sqrt{2}V}{2R} \left[\frac{1}{\pi} \left(\alpha - \frac{\sin 2\alpha}{2} \right) \right]^{1/2}$$

$$= \frac{\sqrt{2} \times 230}{2 \times 10} \left[\frac{1}{\pi} \left(\frac{\pi}{4} - \frac{\sin 2 \times 45}{2} \right) \right]^{1/2} = 4.9036 \text{ A}$$

(d) The rms current rating of upper secondary winding is

$$I_1 = \sqrt{2} I_{T1} = \sqrt{2} \times 31.013 \text{ A} = 43.859 \text{ A}$$

The rms current rating of lower secondary winding is

$$I_3 = [(\sqrt{2} I_{T1})^2 + (\sqrt{2} I_{T3})^2]^{1/2} = [(\sqrt{2} \times 31.013)^2 + (\sqrt{2} \times 4.9036)^2]^{1/2}$$

$$= 44.4038 \text{ A}$$

VA rating of transformer = $V(I_1 + I_3)$ = 230(43.859 + 44.4038) VA = 20300.44 VA

(e) Power output = $\dfrac{V_o^2}{R} = \dfrac{444.0413^2}{10} = 19717.2676$ Watt

Power factor = $\dfrac{\text{Power output}}{\text{VA rating of transformer}} = \dfrac{19717.2676}{20300.44}$ = 0.9712 (lagging)

9.13 CYCLOCONVERTER

Usually ac voltage controllers are used to generate the variable amplitude of ac output voltage at the same frequency of input voltage. In ac voltage controller, the harmonic content is high especially at low output voltage range. Generally the ac variable output voltage at variable frequency can be generated by using two stage converters such as controlled rectifier (fixed ac to variable dc converter) and inverter (variable dc to variable ac at variable frequency). But cycloconverter can be used to eliminate the requirement of one or more intermediate converters. Actually *cycloconverter* is a direct frequency converter or frequency changer which converts ac power at fixed frequency to ac power at variable frequency by fixed ac to variable ac conversion. Therefore, cycloconverter is also called as one-stage frequency changer.

Frequency converter converts one frequency to another frequency or fixed frequency ac input to variable frequency output. Frequency converter is known as cycloconverter. Generally, the output frequency of cycloconverter is always less than input frequency. In cycloconverter frequency changes in steps. Usually cycloconverter is a SCR based converter with natural or line commutation.

It is used in very high power applications up to several MW. Application areas are very high power low-speed induction motor drive, low-frequency three phase/single phase induction or traction motor drives. To reduce harmonic current and to improve efficiency and power factor, cycloconverter based on forced-commutation devices are used. Sometimes, cycloconverters are used to control the output frequency. When the output voltage frequency is greater than input voltage frequency, it is called as step-up cycloconverter. In general, cycloconverter are classified as

1. Step-down cycloconverter
2. Step-up cycloconverter

1. Step-down cycloconverter In step-down cycloconverter, the output frequency f_o is less than the supply (input) frequency f_s, i.e., $f_o < f_s$. The step down cycloconverters are naturally commutated and the output frequency is limited to a value that is a fraction of input frequency. Therefore, these type of cycloconverters are commonly used in low speed ac motor drives up to 15 MW with frequencies from 0 to 20 Hz.

2. Step-up cycloconverter In step-up cycloconverter, the output frequency f_o is greater than the supply (input) frequency f_s, i.e., $f_o > f_s$. The step-up cycloconverters are forced commutated and the output frequency is limited to a value that is a multiple of input frequency. Fast switching devices and microprocessors are used to in step-up cycloconverters to implement advanced conversion strategies. The step-up cycloconverters are also known as forced commutated direct frequency changers.

Depending upon the phases, there are three types of cycloconverters such as

1. Single-phase to single-phase cycloconverters
2. Three-phase to single-phase cycloconverters
3. Three-phase to three-phase cycloconverters

The single phase to single phase cycloconverters are two types namely

1. Mid-point-type cycloconverters
2. Bridge -type cycloconverters

Applications of cycloconverters are

1. Speed control of very high power ac drives
2. Very high power low-speed induction motor drive
3. Low-frequency three phase/single phase induction or traction motor drives.
4. Static VAR compensation
5. Industrial heating

In this section, principle of operation of single phase to single-phase step-up and step-down cycloconverters, three-phase to single-phase cycloconverters and three-phase to three-phase cycloconverters are explained in detail.

9.14 SINGLE-PHASE TO SINGLE-PHASE STEP-UP CYCLOCONVERTERS

There are two types single-phase to single-phase step-up cycloconverters such as mid-point type cycloconverters and bridge-type cycloconverters. The step-up cycloconverters requires forced commutation for thyristors. In this section, the operating principle of both mid-point-type cycloconverters and bridge type cycloconverters are discussed. Assume load is purely resistive.

9.14.1 Mid-point-type Single-phase to Single-phase Step-up Cycloconverters

Figure 9.32 shows the circuit diagram of a single-phase to single-phase step-up cycloconverter mid point type with R load. This circuit consists of a single-phase transformer with the mid point of the secondary winding and four thyristors. Two of these thyristors T_1 and T_2 constitute the positive (P) group and thyristors T_3 and T_4 form the negative (N) group. The load is connected between the mid point O and the terminal C. The positive direction for output voltage V_o and output current I_o are shown in Fig. 9.32. The output voltage V_o and output current I_o are reversed in negative direction.

MODE I During the positive half cycle of supply voltage, A is positive with respect to B and thyristors T_1 and T_4 are forward biased from $\omega t = 0$ to $\omega t = \pi$. When the triggering pulse is applied to thyristor T_1 at $\omega t = 0$, it will be turned ON. Then load voltage follows the positive envelope of the supply voltage and the load current starts to flow in the positive direction through the following path:

$$A - T_1 - C - \text{Load} - O$$

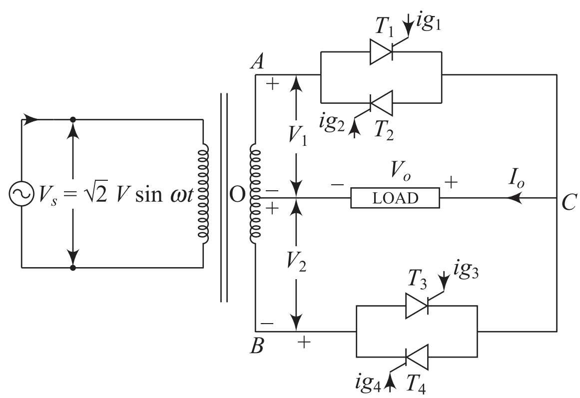


Fig. 9.32 *Single-phase step-up cycloconverter mid-point type with R load*

At the instant ωt_1, thyristor T_1 is turned OFF by forced commutation and gate signal is applied to thyristors T_4 which is already forward biased during positive half cycle of supply voltage. As soon as T_4 is turned ON, the output voltage traces the negative half cycle of the input voltage. Then load current flows in the negative direction through the following path:

$$O - \text{Load} - C - T_4 - B$$

At the time ωt_2, thyristor T_4 is turned OFF by forced commutation and gate signal is applied to thyristors T_1. Since T_1 is again turned ON, the output voltage follows the positive half cycle of the input voltage. Subsequently, the load current starts to flow in the positive direction through the following path:

$$A - T_1 - C - \text{Load} - O$$

The above process will continue for the positive half cycle of input voltage, i.e., up to $\omega t = \pi$ and the output voltage switched alternately between positive and negative envelopes at high frequency.

MODE II In the negative half cycle of supply voltage, B is positive with respect to A and thyristors T_2 and T_3 are forward biased from $\omega t = \pi$ to $\omega t = 2\pi$. When the triggering pulse is applied to thyristor T_2 at $\omega t = \pi$, it will be turned ON and the output voltage traces positive envelope of the input voltage. Then current flows in the positive direction through the following path:

$$A - T_2 - C - \text{Load} - O$$

At the instant ωt_5, thyristor T_2 is turned OFF by forced commutation and gate signal is applied to thyristors T_3. Whenever T_3 is turned ON, the output voltage traces the negative envelope of the input voltage. Then load current flows in the negative direction through the following path:

$$O - \text{Load} - C - T_3 - B$$

The above process will continue for the negative half cycle of input voltage from $\omega t = \pi$ to $\omega t = 2\pi$ and the output voltage switched alternately between positive and negative envelopes at high frequency. Therefore, output voltage frequency f_o is greater than input frequency f_s. It is clear from Fig. 9.33 that the output frequency is four times of supply frequency, i.e.,

$$f_o = 4f_s$$

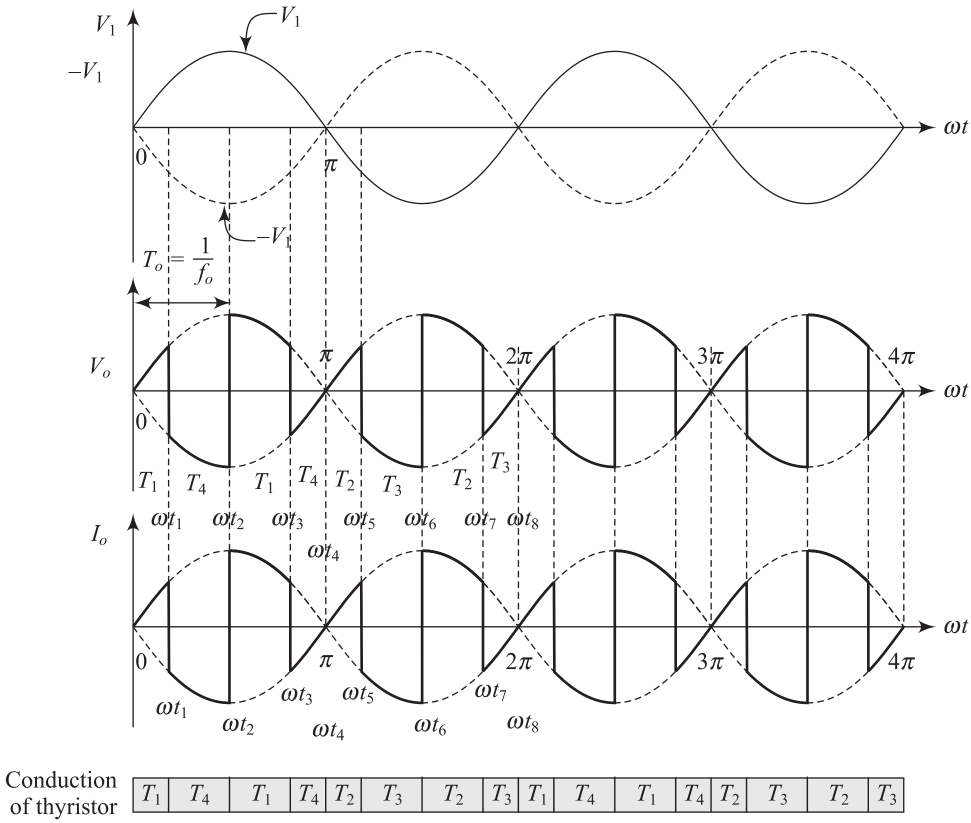


Fig. 9.33 *Waveforms of step-up cycloconverters*

9.14.2 Bridge-type Cycloconverters

Figure 9.34 shows the circuit diagram of a single-phase to single-phase step-up cycloconverter bridge type with R load. This circuit consists of eight thyristors. Four of these thyristors T_{P1}, T_{P2}, T_{P3}, and T_{P4} constitute the positive (P) group and thyristors T_{N1}, T_{N2}, T_{N3}, and T_{N4} form the negative (N) group. The load is connected between P and Q. The positive direction for output voltage V_o and output current I_o are shown in Fig. 9.34. The output voltage V_o and output current I_o are reversed in negative direction.

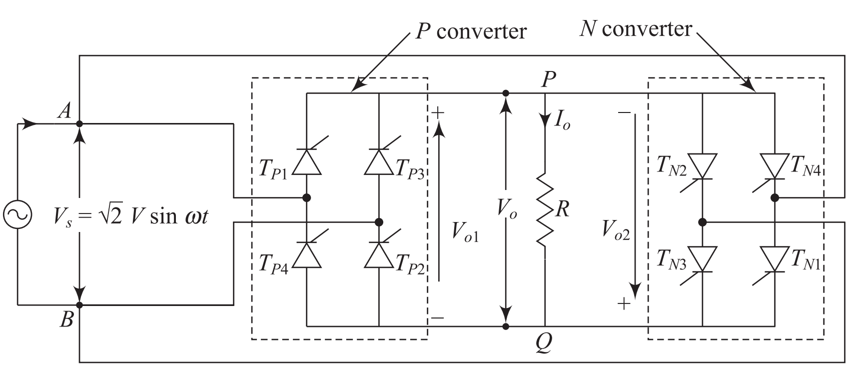


Fig. 9.34 *Single-phase to single-phase step-up cycloconverter bridge type with R load*

MODE I During the positive half cycle of supply voltage, A is positive with respect to B and thyristors T_{P1}, T_{P2}, T_{N1}, and T_{N2} are forward biased from $\omega t = 0$ to $\omega t = \pi$. When the triggering pulses are applied to thyristors T_{P1} and T_{P2} at $\omega t = 0$, these devices will be turned ON. Then load voltage follows the positive envelope of the supply voltage and the load current starts to flow in the positive direction through the following path

$$A - T_{P1} - P - \text{Load} - Q - T_{P2} - B$$

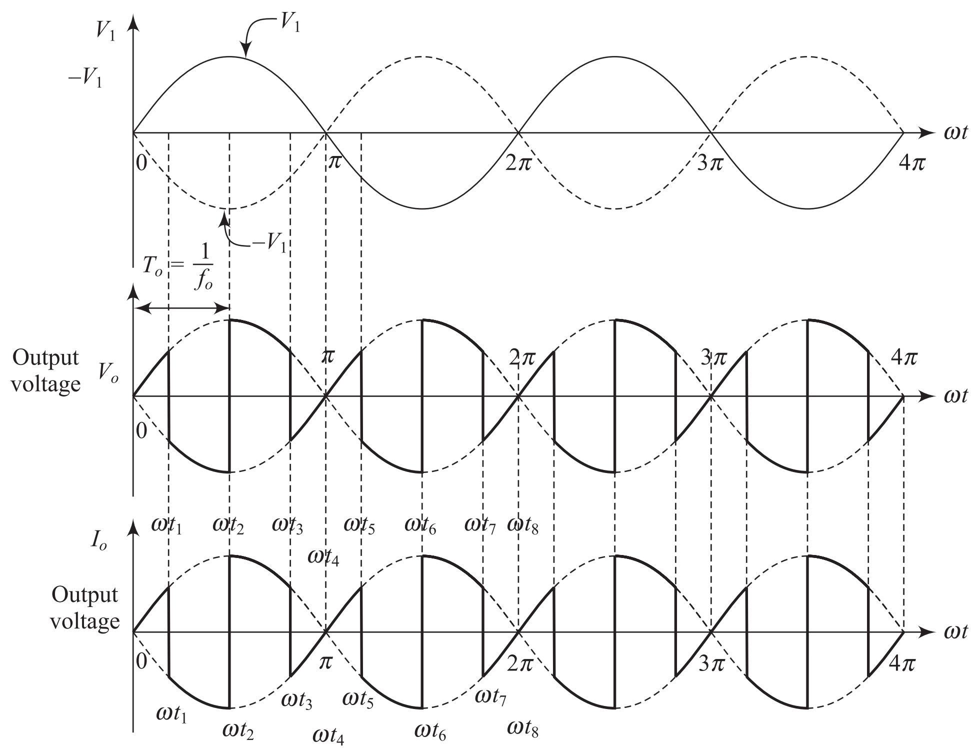
At the instant ωt_1, thyristors T_{P1} and T_{P2} are turned OFF by forced commutation and gate signals are applied to thyristors T_{N1}, and T_{N2}. As soon as T_{N1}, and T_{N2} is turned ON, the output voltage traces the negative half cycle of the input voltage. Then load current flows in the negative direction through the following path

$$A - T_{N1} - Q - \text{Load} - P - T_{N2} - B$$

At the time ωt_2, thyristors T_{N1}, and T_{N2} are turned OFF by forced commutation and gate signals are applied to thyristors T_{P1} and T_{P2}. Since T_{P1} and T_{P2} are again turned ON, the output voltage follows the positive half cycle of the input voltage. Subsequently, the load current starts to flow in the positive direction through the following path:

$$A - T_{P1} - P - \text{Load} - Q - T_{P2} - B$$

The above process will continue for the positive half cycle of input voltage, i.e., up to $\omega t = \pi$ and the output voltage switched alternately between positive and negative envelopes at high frequency.



Conduction of thyristor																
	T_{P1} T_{P2}	T_{N1} T_{N2}	T_{P1} T_{P2}	T_{N1} T_{N2}	T_{P3} T_{P4}	T_{N3} T_{N4}	T_{P3} T_{P4}	T_{N3} T_{N4}	T_{P1} T_{P2}	T_{N1} T_{N2}	T_{P1} T_{P2}	T_{N1} T_{N2}	T_{P3} T_{P4}	T_{N3} T_{N4}	T_{P3} T_{P4}	T_{N3} T_{N4}

Fig. 9.35 *Waveforms of step-up cycloconverters*

MODE II In the negative half cycle of supply voltage, B is positive with respect to A and thyristors T_{P3}, T_{P4}, T_{N3}, and T_{N4} are forward biased from $\omega t = \pi$ to $\omega t = 2\pi$. When the triggering pulses are applied to thyristors T_{P3} and T_{P4} at $\omega t = \pi$, these devices will be turned ON and the output voltage traces positive envelope of the input voltage. Then current flows in the positive direction through the following path:

$$B - T_{P3} - P - \text{Load} - Q - T_{P4} - A$$

At the instant ωt_5, thyristors T_{P3} and T_{P4} are turned OFF by forced commutation and gate signals are applied to thyristors T_{N3} and T_{N4} As T_{N3} and T_{N4} are turned ON, the output voltage traces the negative envelope of the input voltage. Then load current flows in the negative direction through the following path:

$$B - T_{N3} - Q - \text{Load} - P - T_{N4} - A$$

The above process will continue for the negative half cycle of input voltage from $\omega t = \pi$ to $\omega t = 2\pi$ and the output voltage switched alternately between positive and negative envelopes at high frequency. Therefore output voltage frequency f_o is greater than input frequency f_s. It is clear from Fig. 9.35 that the output frequency is four times of supply frequency, i.e.,

$$f_o = 4f_s$$

9.15 SINGLE-PHASE TO SINGLE-PHASE STEP-DOWN CYCLOCONVERTERS

There are two types single-phase to single-phase step-down cycloconverters such as mid-point type cycloconverters and bridge type cycloconverters. In step-down cycloconverters, thyristors are naturally commuted. In this section, both mid-point type cycloconverters and bridge-type cycloconverters are discussed.

9.15.1 Mid-point-type Cycloconverters

Figure 9.36 shows the circuit diagram of a single-phase to single-phase step-down cycloconverter mid point type with R load. The natural or line commutation is provide by the ac supply in this converter. The triggering pulses of thyristors, conduction period of thyristors and output voltage waveform for resistive load are depicted in Fig. 9.37.

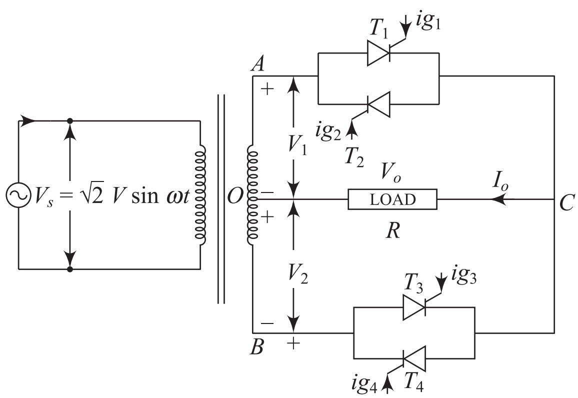


Fig. 9.36 *Single-phase to single-phase step-down cycloconverter mid point type with R load*

Mode I ($0 \le \omega t \le 2\pi$) During the positive half cycle of supply voltage, A is positive with respect to B and thyristors T_1 and T_4 are forward biased. When the triggering pulse is applied to thyristor T_1 at $\omega t = \alpha$, it will be turned ON and the current starts to flow in the positive direction through the following path:

$$A - T_1 - C - \text{Load} - O$$

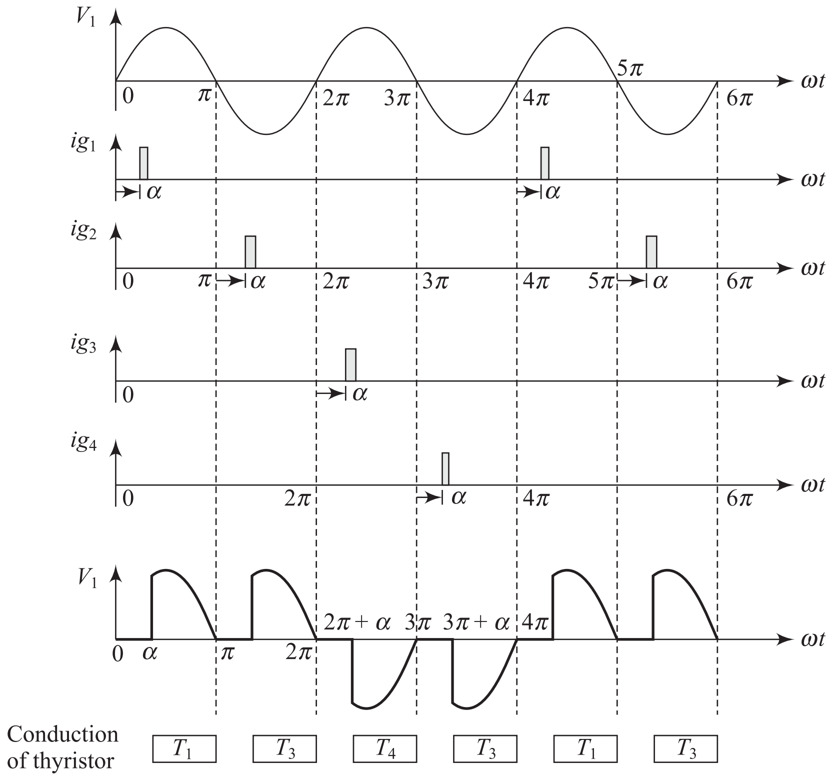


Fig. 9.37 *Gate pulses of thyristors and output voltage waveform when* $f_o = \frac{1}{2} f_s$

At $\omega t = \pi$, the input voltage becomes zero. Accordingly the load current at that instant is zero and thyristor T_1 is turned OFF.

In the negative half cycle of supply voltage, B is positive with respect to A and thyristors T_2 and T_3 are forward biased. When the triggering pulse is applied to thyristor T_2 at $\omega t = \pi + \alpha$, it will be turned on and the current starts to flow in the positive direction through the following path:

$$B - T_2 - C - \text{Load} - O$$

At $\omega t = 2\pi$, the input voltage becomes zero. Therefore, the load current at that instant is zero and thyristor T_2 is turned off.

After that the triggering pulse is applied to thyristor T_4 at $\omega t = 2\pi + \alpha$, it will be turned ON due to forward bias and the current starts to flow in the negative direction through the following path:

$$O - \text{Load} - C - T_4 - B$$

At the instant $\omega t = 3\pi$, the input voltage becomes zero. Consequently the load current is zero and thyristor T_4 is turned OFF.

Mode II ($2\pi \le \omega t \le 4\pi$) At $\omega t = 3\pi + \alpha$, the triggering pulse is applied to thyristor T_3 and it will be turned ON due to forward bias. Then the current starts to flow in the negative direction through the following path:

$$O - \text{Load} - C - T_3 - A$$

At $\omega t = 4\pi$, the input voltage becomes zero. As a result, the load current is zero and thyristor T_3 is turned OFF.

After that the above sequence of operations will be repeated to get the output voltage as shown in Fig. 9.37 when the output frequency is half of the input frequency $\left(f_o = \frac{1}{2} f_s\right)$. If the output frequency is one third of the input frequency $f_o = \frac{1}{3} f_s$, the triggering gate pulses for thyristors, conduction period of thyristors and output voltage waveform are depicted in Fig. 9.38.

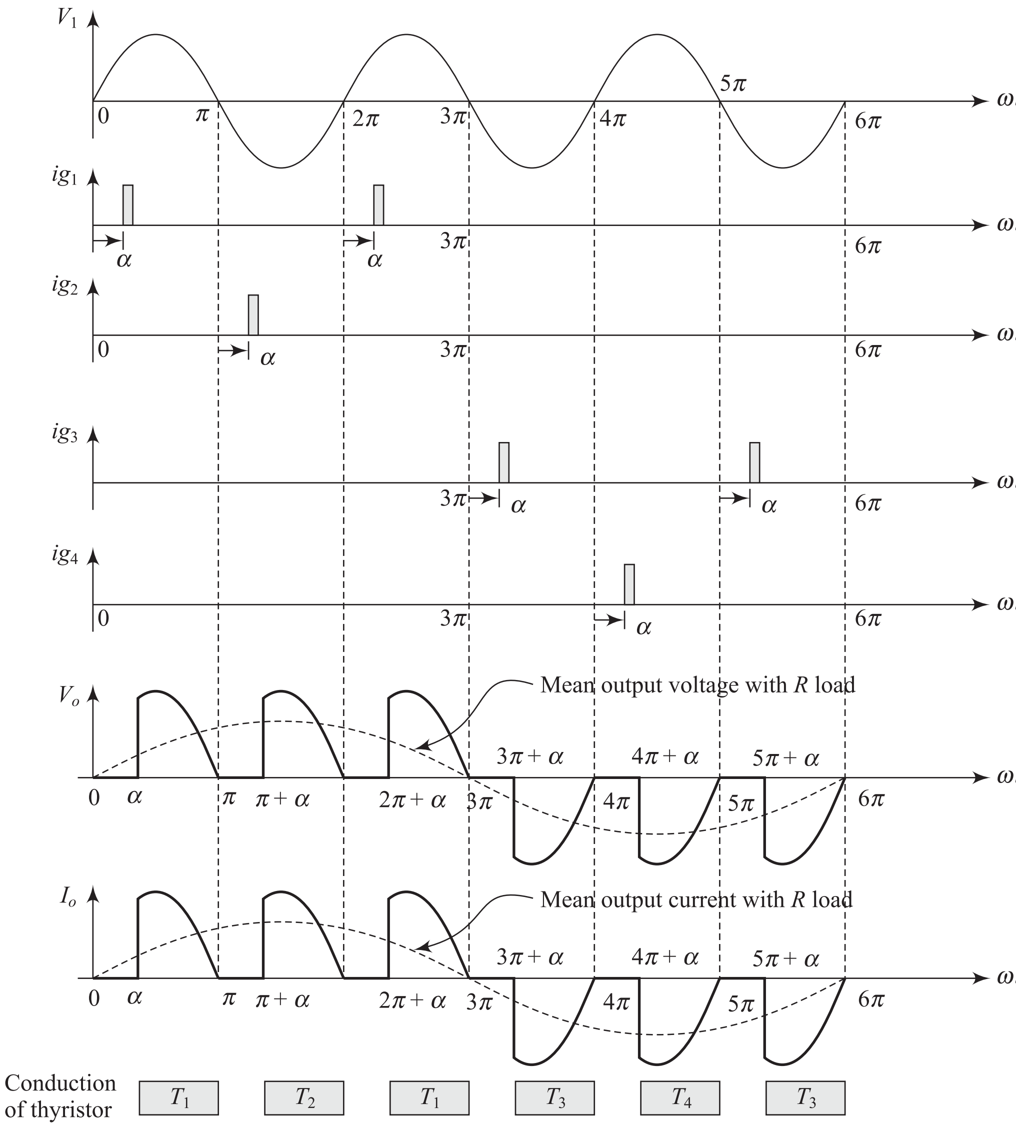


Fig. 9.38 *Gate pulses of thyristors and output voltage waveform for R load when* $f_o = \frac{1}{3} f_s$

It is clear from Fig. 9.38 that the output voltage waveform consists of three positive and three negative half cycles. Then the rms value of output voltage is equal to

$$V_o = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2\, d\omega t\right]^{1/2} \quad \text{where } V \text{ is the rms input voltage}$$

or
$$V_o = V\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

9.15.2 Mid-point-type Cycloconverters with *RL* Load

Figure 9.39 shows the circuit diagram of a single-phase to single-phase step-down cycloconverter mid point type with RL load. The natural or line commutation is provide by the ac supply in this converter. This cycloconverter operates in two modes such as continuous load current and discontinuous load current

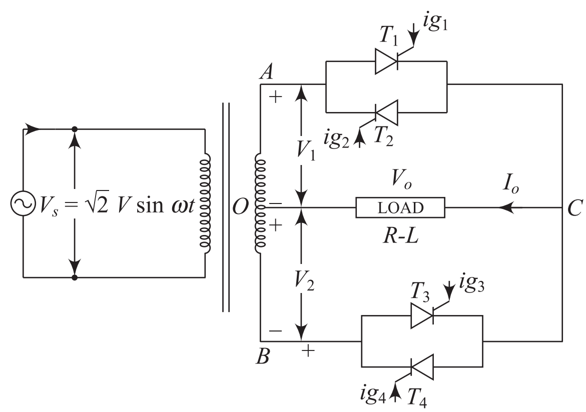


Fig. 9.39 *Single phase to single-phase step-down cycloconverter mid point type with RL load*

Discontinuous load current mode

The triggering pulses of thyristors, conduction period of thyristors and output voltage waveform for resistive-inductive (*RL*) load are depicted in Fig. 9.40.

In the positive half cycle of supply voltage, A is positive with respect to B and thyristors T_1 and T_4 are forward biased. As soon as the triggering pulse is applied to thyristor T_1 at $\omega t = \alpha$, it will be turned ON and the current starts to flow in the positive direction through the following path:

$$A - T_1 - C - \text{Load} - O$$

At $\omega t = \pi$, the input voltage becomes zero. But load current is not zero at $\omega t = \pi$ due to presence of inductive load. The conduction of thyristor can be extended to β with the conduction angle $\gamma = \beta - \alpha$. Hence the load current is zero $\omega t = \beta$ and thyristor T_1 is naturally commutated and turned OFF. During the period $\beta \le \omega t \le \pi + \alpha$, the load current is zero and it becomes discontinuous.

During the negative half cycle of supply voltage, B is positive with respect to A and thyristors T_2 and T_3 are forward biased. Whenever the triggering pulse is applied to thyristor T_2 at $\omega t = \pi + \alpha$, it will be turned ON and the current starts to flow in the positive direction through the following path:

$$B - T_2 - C - \text{Load} - O$$

At $\omega t = 2\pi$, thyristor T_2 will not be turned OFF. Due to presence of inductance, the conduction of thyristor T_2 will be extended. At $\omega t = \pi + \alpha + \gamma$, the load current becomes zero and thyristor T_2 is naturally commuted and turned OFF. The load current is zero during the period $\pi + \alpha + \gamma \le \omega t \le 2\pi + \alpha$.

At $\omega t = 2\pi + \alpha$, the triggering pulse is again applied to thyristor T_1 and it will be turned ON due to forward bias. Then the current starts to flow in the positive direction through the following path:

$$A - T_1 - C - \text{Load} - O$$

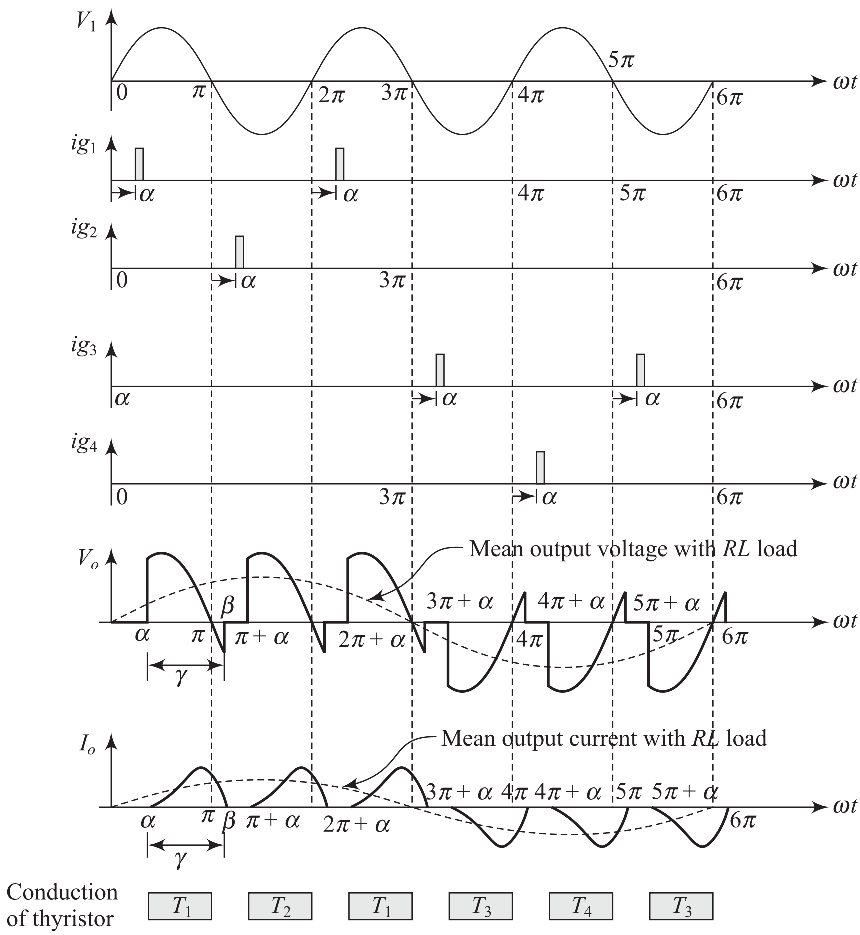


Fig. 9.40 *Gate pulses of thyristors and output voltage waveform for R-L load when $f_o = \frac{1}{3} f_s$ and discontinuous current*

The load current is zero at $\omega t = 2\pi + \alpha + \gamma$ and thyristor T_1 is turned OFF. Then load current is zero during the period $2\pi + \alpha + \gamma \le \omega t \le 3\pi + \alpha$.

After that $\omega t = 3\pi + \alpha$, the triggering pulse is applied to thyristor T_3 and it will be turned ON due to forward bias. Then the current starts to flow in the negative direction through the following path:

$$O - \text{Load} - C - T_3 - A$$

Due to the presence of inductance, thyristor T_3 will not be turned OFF at $\omega t = 4\pi$ and it will conduct up to $\omega t = 3\pi + \alpha + \gamma$. Therefore, the load current is zero at $\omega t = 3\pi + \alpha + \gamma$ and thyristor T_3 is turned OFF.

At the instant $\omega t = 4\pi + \alpha$, the triggering pulse is applied to thyristor T_4 and it will be turned ON due to forward bias and the current starts to flow in the negative direction through the following path:

$$O - \text{Load} - C - T_4 - B$$

The load current becomes zero at $\omega t = 3\pi + \alpha + \gamma$ and thyristor T_4 is turned OFF.

At $\omega t = 5\pi + \alpha$, the triggering pulse is again applied to thyristor T_3 and it will be turned ON due to forward bias. Then the current starts to flow in the negative direction through the following path:

$$O - \text{Load} - C - T_3 - A$$

The load current is zero at $\omega t = 4\pi + \alpha + \gamma$ and thyristor T_3 is turned OFF.

After that the above sequence of operations will be repeated to get the output voltage and discontinuous current as shown in Fig. 9.40 when the output frequency is one-third of the input frequency $f_o = \frac{1}{3} f_s$.

Continuous load current mode The triggering pulses of thyristors, conduction period of thyristors and output voltage waveform for resistive-inductive (*RL*) load are depicted in Fig. 9.41.

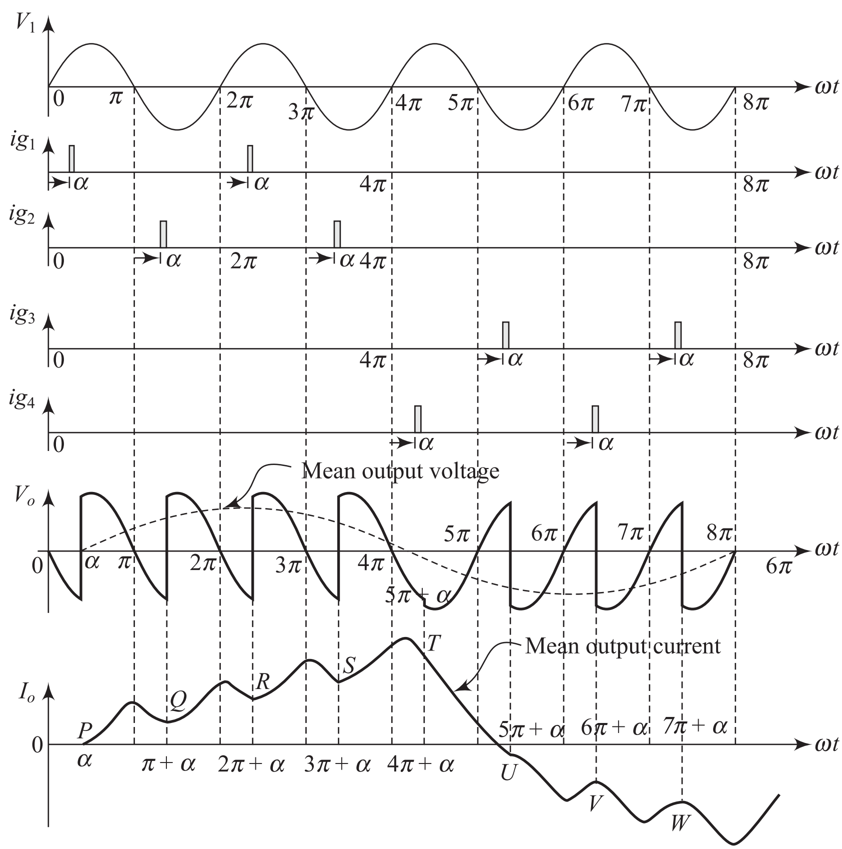


Fig. 9.41 *Gate pulses of thyristors, output voltage and output current waveforms for R-L load when $f_o = \frac{1}{4} f_s$ and continuous current*

During the positive half cycle of supply voltage, A is positive with respect to B and thyristors T_1 and T_4 are forward biased. When the triggering pulse is applied to thyristor T_1 at $\omega t = \alpha$, it will be turned ON and the current starts to flow in the positive direction through the following path:

$$A - T_1 - C - \text{Load} - O$$

At $\omega t = \pi$, the input voltage becomes zero and after $\omega t = \pi$ thyristor T_1 is not reverse biased due to inductive load. But load current is not zero at $\omega t = \pi$ due to presence of inductive load. If load is highly inductive, the conduction of thyristor can be extended upto $\omega t = \pi + \alpha$ with the conduction angle $\gamma = \pi$.

During the negative half cycle of supply voltage, B is positive with respect to A and thyristors T_2 and T_3 are forward biased. When the triggering pulse is applied to thyristor T_2 at $\omega t = \pi + \alpha$, it will be turned ON and thyristor T_1 is turned OFF due to reverse biased. Then the current starts to flow in the positive direction through the following path:

$$B - T_2 - C - \text{Load} - O$$

At $\omega t = 2\pi$, thyristor T_2 will not be turned OFF. Due to presence of inductance, the conduction of thyristor T_2 will be extended up to $\omega t = 2\pi + \alpha$ with the conduction angle $\gamma = \pi$. Since thyristor T_1 and T_2 conduct for 180° duration, the load current is continuous. Similarly, thyristors T_1, T_2, T_3 and T_4 are sequentially turned on and each thyrsistor conducts for 180° duration. The instant of gate pulse, conduction period of thyristors are given in Table 9.1.

Table 9.1 *Conduction of thyristors*

Thyrisor	Instant of gate pulses	Conduction region	Conduction angle	Path of current flow
T_1	$\omega t = \alpha$ $\omega t = 2\pi + \alpha$	$\alpha \le \omega t \le \pi + \alpha$ $2\pi + \alpha \le \omega t \le 3\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$A - T_1 - C - \text{Load} - O$
T_2	$\omega t = \pi + \alpha$ $\omega t = 3\pi + \alpha$	$\pi + \alpha \le \omega t \le 2\pi + \alpha$ $3\pi + \alpha \le \omega t \le 4\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$B - T_2 - C - \text{Load} - O$
T_3	$\omega t = 5\pi + \alpha$ $\omega t = 7\pi + \alpha$	$5\pi + \alpha \le \omega t \le 6\pi + \alpha$ $7\pi + \alpha \le \omega t \le 8\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$O - \text{Load} - C - T_3 - A$
T_4	$\omega t = 4\pi + \alpha$ $\omega t = 6\pi + \alpha$	$4\pi + \alpha \le \omega t \le 5\pi + \alpha$ $6\pi + \alpha \le \omega t \le 7\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$O - \text{Load} - C - T_4 - B$

9.15.3 Bridge-type Cycloconverters

Figure 9.42 shows the circuit diagram of a single-phase to single-phase step-down cycloconverter bridge type with RL load. The natural or line commutation is provided by the ac supply in this converter. This cycloconverter operates in two modes such as continuous load current and discontinuous load current. The simplified equivalent circuit of Fig. 9.42 is shown in Fig. 9.43.

Discontinuous load current mode The triggering pulses of thyristors, conduction period of thyristors and output voltage waveform for resistive load are depicted in Fig. 9.44.

During the positive half cycle of supply voltage, A is positive with respect to B and thyristors T_{P1}, T_{P2}, T_{N1}, and T_{N2} are forward biased. When the triggering pulse is applied to thyristors T_{P1}, and T_{P2} at $\omega t = \alpha$, these devices will be turned ON and the current starts to flow in the positive direction through the following path:

$$A - T_{P1} - P - \text{Load} - Q - T_{P2} - B$$

At $\omega t = \pi$, the input voltage becomes zero but load current is not zero due to presence of inductive load. The conduction of thyristors T_{P1}, and T_{P2} can be extended to β with the conduction angle

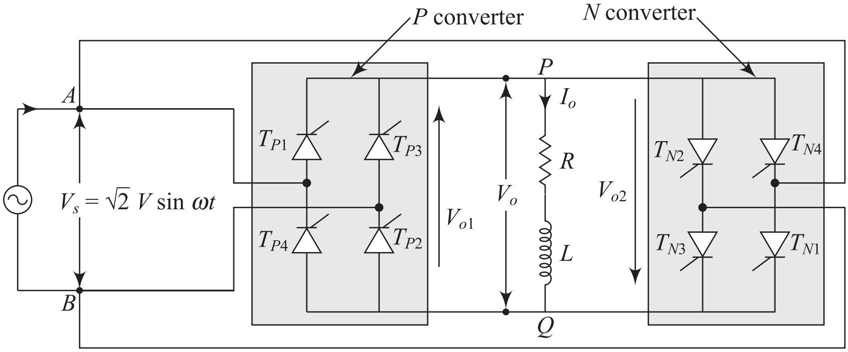


Fig. 9.42 *Single-phase to single-phase step-down cycloconverter bridge type with RL load*

$\gamma = \beta - \alpha$. Thus the load current is zero at $\omega t = \beta$ and thyristors T_{P1}, and T_{P2} are naturally commutated and turned OFF. During the period $\beta \le \omega t \le \pi + \alpha$, the load current is zero and it becomes discontinuous.

In the negative half cycle of supply voltage, B is positive with respect to A and thyristors T_{P3}, T_{P4}, T_{N3}, and T_{N4} are forward biased. Whenever the triggering pulse is applied to thyristors T_{P3} and T_{P4} at $\omega t = \pi + \alpha$, these devices will be turned ON and the current starts to flow in the positive direction through the following path:

$$B - T_{P3} - P - \text{Load} - Q - T_{P4} - A$$

At $\omega t = 2\pi$, thyristors T_{P3} and T_{P4} will not be turned OFF. Due to presence of inductance, the conduction of thyristor T_{P3} and T_{P4} will be extended. At $\omega t = \pi + \alpha + \gamma$ the load current becomes is zero and thyristor T_{P3} and T_{P4} are naturally commuted and turned OFF. The load current is zero during the period $\pi + \alpha + \gamma \le \omega t \le 2\pi + \alpha$.

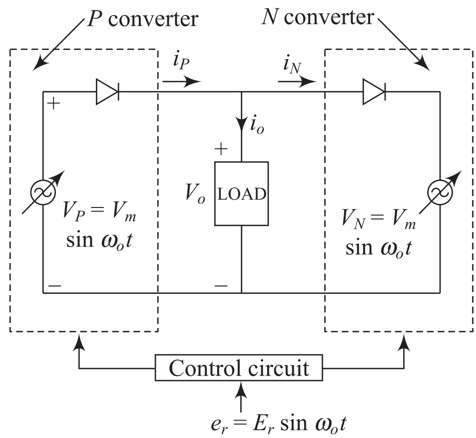


Fig. 9.43 *Simplified equivalent circuit of Fig. 9.42*

Since thyristor T_{P1}, and T_{P2} and T_{P3} and T_{P4} conduct for γ duration where $\gamma < 180°$, the load current is discontinuous. In the same way, thyristors T_{N1}, and T_{N2} and T_{N3} and T_{N4} are sequentially turned ON and each thyristor conducts for γ duration. The instant of gate pulse, conduction period of thyristors are given in Table 9.2.

Table 9.2 *Conduction of thyristors*

Thyristors	Instant of gate pulses	Conduction region	Conduction angle	Path of current flow
T_{P1}, and T_{P2}	$\omega t = \alpha$ $\omega t = 2\pi + \alpha$	$\alpha \le \omega t \le \beta$ $2\pi + \alpha \le \omega t \le 2\pi + \gamma$	$\gamma = \beta - \alpha$ and $\gamma < 180°$ γ and $\gamma < 180°$	$A - T_{P1} - P - \text{Load} - Q - T_{P2} - B$
T_{P3} and T_{P4}	$\omega t = \pi + \alpha$	$\pi + \alpha \le \omega t \le \pi + \gamma$	γ and $\gamma < 180°$	$B - T_{P3} - P - \text{Load} - Q - T_{P4} - A$
T_{N1} and T_{N2}	$\omega t = 4\pi + \alpha$	$4\pi + \alpha \le \omega t \le 4\pi + \gamma$	γ and $\gamma < 180°$	$A - T_{N1} - Q - \text{Load} - P - T_{N2} - B$
T_{N3} and T_{N4}	$\omega t = 3\pi + \alpha$ $\omega t = 5\pi + \alpha$	$3\pi + \alpha \le \omega t \le 3\pi + \gamma$ $5\pi + \alpha \le \omega t \le 5\pi + \gamma$	γ and $\gamma < 180°$ γ and $\gamma < 180°$	$B - T_{N3} - Q - \text{Load} - P - T_{N4} - A$

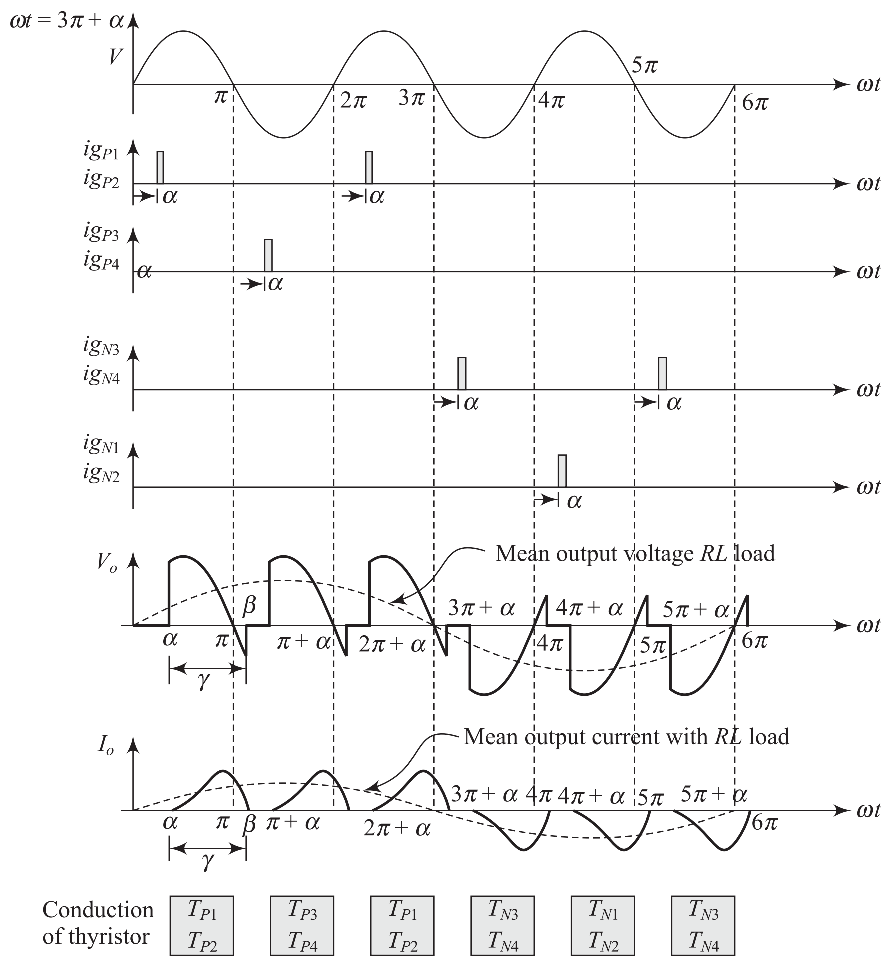


Fig. 9.44 *Gate pulses of thyristors and output voltage waveform for RL load when* $f_o = \frac{1}{3} f_s$ *and discontinuous current*

Continuous load current mode The triggering pulses of thyristors, conduction period of thyristors and output voltage waveform for resistive-inductive (RL) load are depicted in Fig. 9.45.

During the positive half cycle of supply voltage, A is positive with respect to B and thyristors T_{P1}, T_{P2}, T_{N1}, and T_{N2} are forward biased. When the triggering pulse is applied to thyristors T_{P1} and T_{P2} at $\omega t = \alpha$, these devices will be turned ON and the current starts to buildup in the positive direction through the following path

$$A - T_{P1} - P - \text{Load} - Q - T_{P2} - B$$

At $\omega t = \pi$, the input voltage becomes zero and after $\omega t = \pi$ thyristors T_{P1} and T_{P2} are not reverse biased due to inductive load. However load current is not zero at $\omega t = \pi$ due to presence of inductive load. If load is highly inductive, the conduction of thyristor can be extended up to $\omega t = \pi + \alpha$ with the conduction angle $\gamma = \pi$.

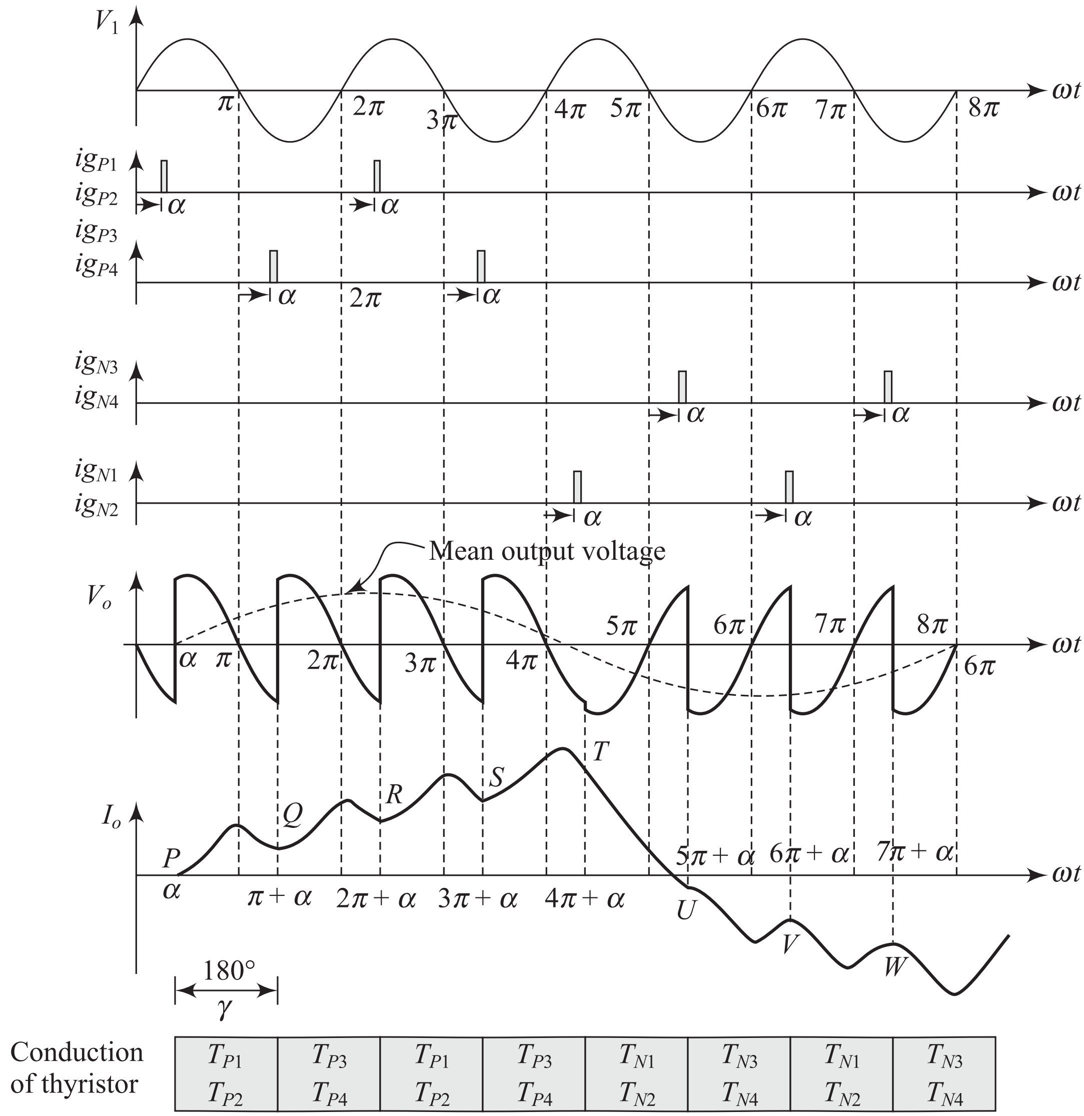


Fig. 9.45 *Gate pulses of thyristors and output voltage waveform for R-L load when $f_o = \frac{1}{4} f_s$ and continuous current*

During the negative half cycle of supply voltage, B is positive with respect to A and thyristors T_{P3}, T_{P4}, T_{N3}, and T_{N4} are forward biased. When the triggering pulse is applied to thyristor T_{P3} and T_{P4} at $\omega t = \pi + \alpha$, these devices will be turned ON and thyristors T_{P1} and T_{P2} are turned OFF due to reverse biased. Then the current starts to flow in the positive direction through the following path:

$$B - T_{P3} - P - \text{Load} - Q - T_{P4} - A$$

At $\omega t = 2\pi$, thyristors T_{P3} and T_{P4} will not be turned OFF. Due to presence of inductance, the conduction of thyristors T_{P3} and T_{P4} will be extended up to $\omega t = 2\pi + \alpha$ with the conduction angle

$\gamma = \pi$. Since thyristors T_{P1} and T_{P2} and T_{P3} and T_{P4} conducts for 180° duration, the load current is continuous. Similarly, thyristors T_{N1} and T_{N2} and T_{N3} and T_{P4} are sequentially turned ON and each thyrsistor conducts for 180° duration. The instant of gate pulse, conduction period of thyristors are given in Table 9.3.

Table 9.3 *Conduction of thyristors*

Thyristors	Instant of gate pulses	Conduction region	Conduction angle	Path of current flow
T_{P1}, and T_{P2}	$\omega t = \alpha$ $\omega t = 2\pi + \alpha$	$\alpha \le \omega t \le \pi + \alpha$ $2\pi + \alpha \le \omega t \le 3\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$A - T_{P1} - P - \text{Load} - Q - T_{P2} - B$
T_{P3} and T_{P4}	$\omega t = \pi + \alpha$ $\omega t = 3\pi + \alpha$	$\pi + \alpha \le \omega t \le 2\pi + \alpha$ $3\pi + \alpha \le \omega t \le 4\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$B - T_{P3} - P - \text{Load} - Q - T_{P4} - A$
T_{N1} and T_{N2}	$\omega t = 4\pi + \alpha$ $\omega t = 6\pi + \alpha$	$4\pi + \alpha \le \omega t \le 5\pi + \alpha$ $6\pi + \alpha \le \omega t \le 7\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$A - T_{N1} - Q - \text{Load} - P - T_{N2} - B$
T_{N3} and T_{P4}	$\omega t = 7\pi + \alpha$ $\omega t = 5\pi + \alpha$	$7\pi + \alpha \le \omega t \le 8\pi + \alpha$ $5\pi + \alpha \le \omega t \le 6\pi + \alpha$	$\gamma = \pi$ $\gamma = \pi$	$B - T_{N3} - Q - \text{Load} - P - T_{N4} - A$

Example 9.18 A single-phase bridge cycloconverter is fed from 220 V, 50 Hz ac supply and a load of 20 Ω is connected with the cycloconverter. The frequency of output voltage is half of the input frequency. If the firing angle α = 90°, determine (a) rms value of output voltage, (b) rms value of load current, (c) rms current of each thyristor and (d) input power factor

Solution

Given: V = 220 V, R = 20 ω and α = 90°

(a) rms value of output voltage

$$V_o = \left[\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V\sin\omega t)^2 d\omega t\right]^{1/2} \quad \text{where } V \text{ is the rms input voltage}$$

or $$V_o = V\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= 220 \times \left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin 2\times 90}{2}\right)\right]^{1/2} = 155.56 \text{ V}$$

(b) rms value of load current is $I_o = \dfrac{V_o}{R} = \dfrac{155.56}{20}\text{ A} = 7.778 \text{ A}$

(c) rms value of current for each converter is $I_{oP} = \dfrac{I_o}{\sqrt{2}} = \dfrac{7.778}{\sqrt{2}}\text{ A} = 5.499 \text{ A}$

rms current of each thyristor is $I_{oT} = \dfrac{I_{oP}}{\sqrt{2}} = \dfrac{7.778}{2}\text{ A} = 3.889 \text{ A}$

(d) Input VA is VI_o = 220 × 7.778 VA = 1711.16 VA

Power output is $I_o^2 R = 7.778^2 \times 20$ Watt = 1209.94 Watt

$$\text{Input power factor} = \frac{\text{power output}}{\text{input VA}} = \frac{1209.94}{1711.16} = 0.707 \text{ (lagging)}$$

9.16 THREE-PHASE TO SINGLE-PHASE CYCLOCONVERTER

Single-phase to single-phase cycloconverters are commonly used in low power industrial applications but these converters are not suitable for medium and high power applications. Therefore, three-phase to single-phase cycloconverters are developed for medium and high power applications. There are two types of three-phase to single-phase cycloconverters such as

1. Three-phase to single-phase cycloconverters using two three-phase half-wave converters
2. Three-phase to single-phase cycloconverters using two three-phase bridge converters

Figure 9.46 shows a three-phase to single-phase cycloconverter circuit using two three-phase half-wave converters. This circuit consists of two converter groups such as positive group converter and negative group converter. Each group consists of three thyristors as shown in Fig. 9.47. This circuit operates as three phase half-wave cycloconverter. During the positive half-cycle of output voltage, SCRs T_1, T_2, and T_3 will conduct and in the negative half-cycle of output voltage, SCRs T_4, T_5, and T_6 will conduct. Each SCR will conduct for 120° duration of the input voltage as SCR will conduct only when anode voltage is greater than cathode voltage or the device is forward biased. The positive half-cycle of output voltage is controlled by firing SCRs T_1, T_2, and T_3. The negative half-cycle of output voltage is controlled by firing SCRs T_4, T_5, and T_6.

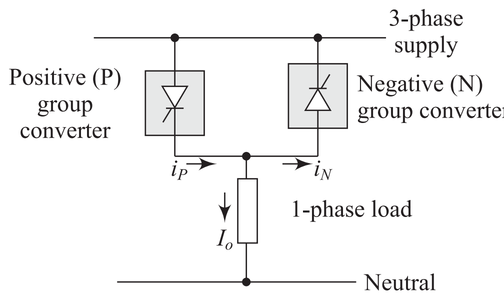


Fig. 9.46 *Three-phase to single-phase cycloconverter circuit using two three-phase half-wave converters*

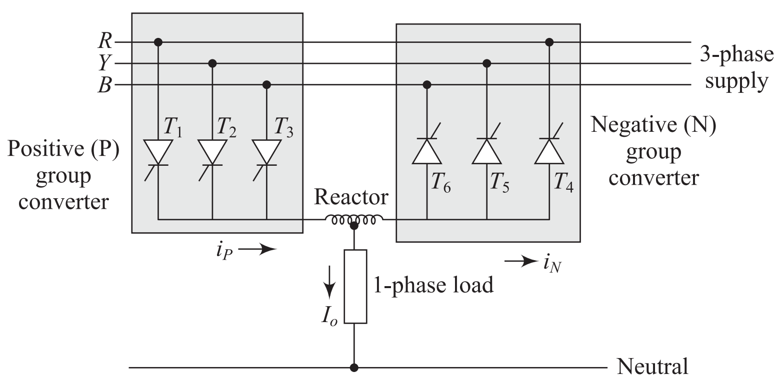


Fig. 9.47 *Three-phase to single-phase cycloconverter circuit using two three-phase half-wave converters*

Figure 9.48 shows the output voltage waveform of three-phase to single-phase cycloconverter circuit using two three-phase half-wave converters. The positive group converter generates positive output voltage and the negative group converter generates negative output voltage. For resistive load, the current waveform will be similar to output voltage waveform. Since the output voltage is not purely

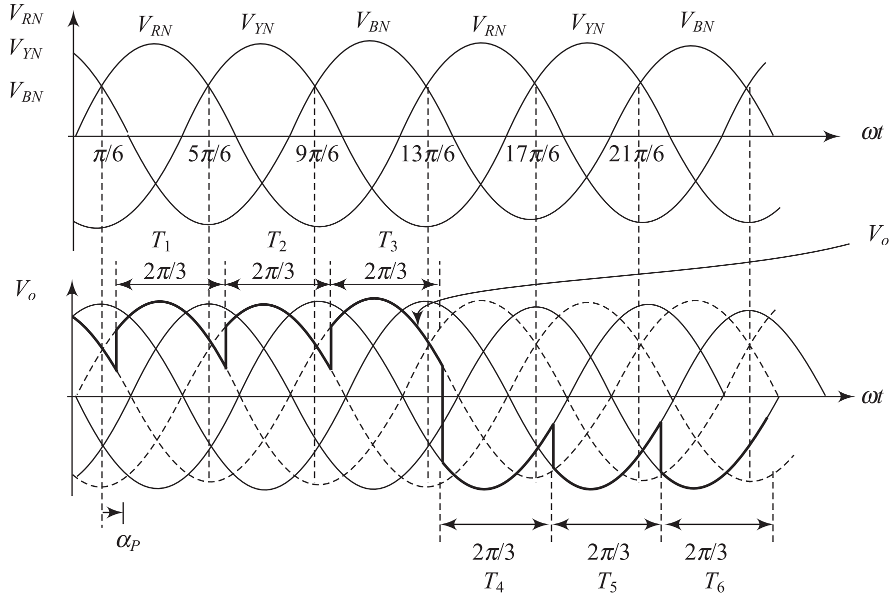


Fig. 9.48 *Output voltage waveform at firing angle α_P*

sinusoidal, the output voltage contains harmonics. Hence the power factor will be low with increasing the delay angle.

Figure 9.49 shows a three-phase to single-phase cycloconverter circuit using two three-phase bridge converters. This circuit consists of two converter groups such as positive group converter and negative group converter. Each group consists of six thyristors. This circuit operates as three-phase full-wave cycloconverter. During the positive half-cycle of output voltage, SCRs T_{P1}, T_{P2}, T_{P3}, T_{P4}, T_{P5}, and T_{P6} will conduct and in the negative half-cycle of output voltage, SCRs T_{N1}, T_{N2}, T_{N3}, T_{N4}, T_{N5}, and T_{N6} will conduct. Each SCR will conduct for 120° duration of the input voltage as SCR will conduct only when anode voltage is greater than cathode voltage or the device is forward biased. The positive half-cycle of output voltage is controlled by firing SCRs T_{P1}, T_{P2}, T_{P3}, T_{P4}, T_{P5}, and T_{P6}. The negative half-cycle of output voltage is controlled by firing T_{N1}, T_{N2}, T_{N3}, T_{N4}, T_{N5}, and T_{N6}.

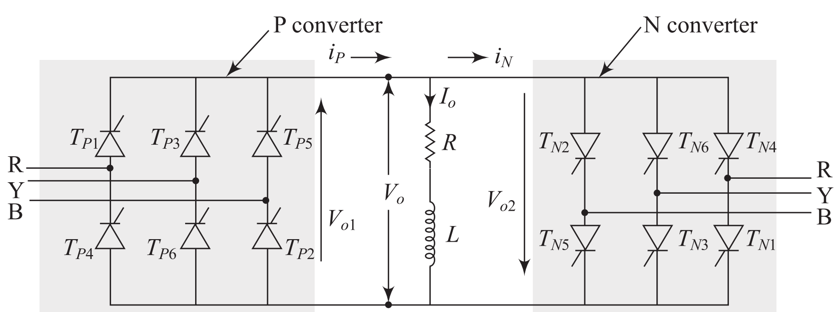


Fig. 9.49 *Three-phase to single-phase cycloconverter circuit using two three-phase bridge converters*

Figure 9.50 shows an output voltage waveform of 10 Hz. The positive converter operates for the positive half period of output voltage and it acts as a normal three-phase bridge controlled rectifier

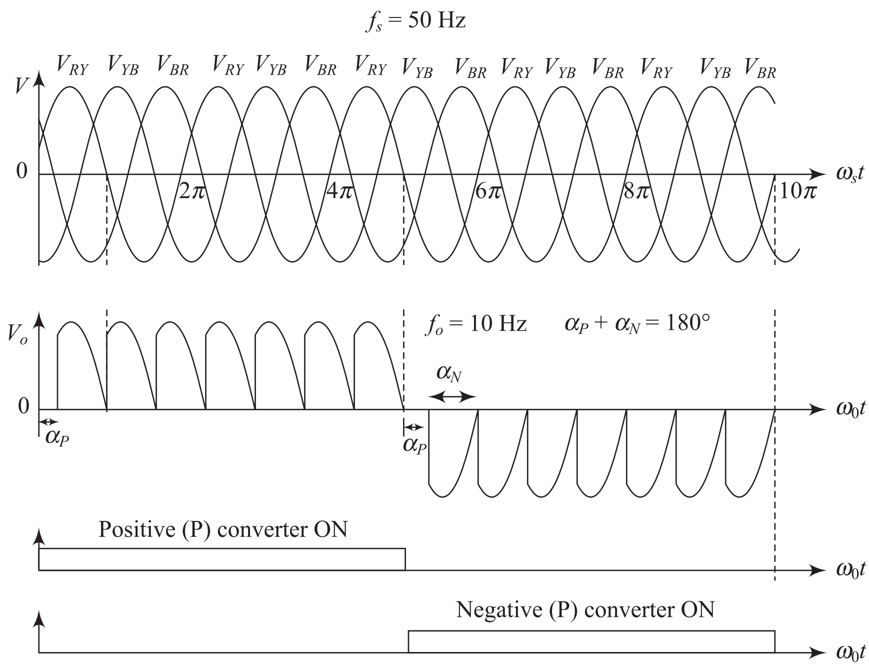


Fig. 9.50 *Output voltage waveform at firing angle α_P and α_N*

with the delay angle of α_P. The negative converter operates for the negative half period of output voltage and it also acts as a normal three-phase bridge controlled rectifier with the delay angle of α_N. The sum of α_P and α_N is 180°, i.e.,

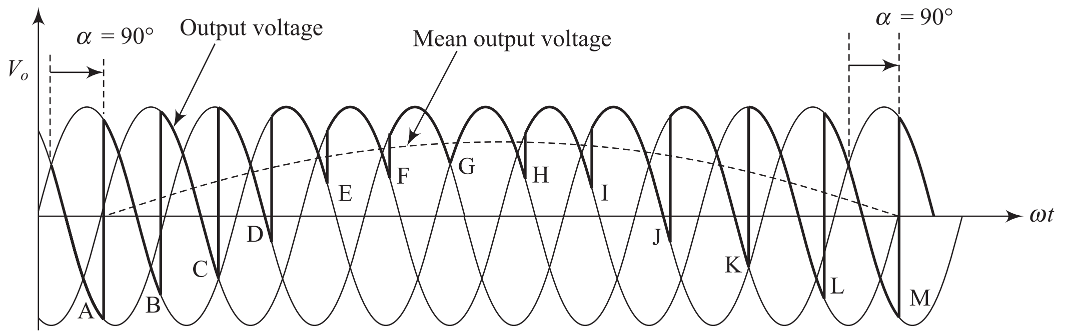
$$\alpha_P + \alpha_N = 180°$$

Since the output voltage is not purely sinusoidal, the output voltage contains harmonics. Hence the power factor will be low with increasing the delay angle. To improve the performance of three-phase to single-phase cycloconverters and to get nearly sinusoidal output voltage, the delay angle of thyristors will be varied as per requirement.

When phase angle control is applied in three-phase to single-phase cyclcoconverter, very good voltage control is possible. Thyristors can be turned ON in such delay angles, the fundamental output voltage or current for resistive load will be nearly sinusoidal. Actually, the delay angles are varied in each half cycle to get the desired sinusoidal output voltage. Hence the harmonics of the output voltage can be minimised. The delay angle of SCRs may be computed after comparing a cosine control signal at supply frequency with a sinusoidal reference voltage at output frequency.

To convert three-phase ac supply at frequency f_s into a single-phase ac supply at lower frequency ($f_o < f_s$), the firing angle of three thyristors of three-phase half wave converter can be varied progressively in steps. Therefore, a special gating circuit must be designed to introduce progressive firing angle delay. Figure 9.51 shows the output voltage waveform for a three-phase half-wave cycloconverter and mean output voltage waveforms. It is clear from Fig. 9.51 that there are eight half cycles of supply frequency in positive half period of mean output voltage. As a result, the output frequency is equal to

$$f_o = \frac{1}{8} f_s \quad \text{where } f_s \text{ is the supply frequency}$$



at A, $\alpha = 90°$ at B, $\alpha = 75°$ at C, $\alpha = 60°$ at D, $\alpha = 45°$ at E, $\alpha = 30°$

at F, $\alpha = 15°$ at G, $\alpha = 0°$ at H, $\alpha = 15°$ at I, $\alpha = 30°$

at J, $\alpha = 45°$ at K, $\alpha = 60°$ at L, $\alpha = 75°$ at M, $\alpha = 90°$

Fig. 9.51 *Output voltage and mean output voltage waveforms for a three-phase half-wave cycloconverter*

The magnitude of progressive change in firing angle is given by

$$\text{Reduction factor in frequency} \times 120°$$

where,
$$\text{Reduction factor in frequency} = \frac{\text{output frequency}}{\text{input frequency}}$$

Since the reduction factor is $\frac{f_o}{f_s} = \frac{1}{8}$, the progressive step variation in the firing angle is

$$\frac{1}{8} \times 120° = 15°$$

The firing angle at point A is 90°. Then the firing angle at point B is 90° – 15° = 75°. Similarly, the firing angle at point C is 75 – 15 = 60°, at point D is 60° – 15° = 45°, at point E is 45° – 15° = 30°, at point F is 30 – 15° = 15°, and at point G is 15° – 15° = 0°.

Hence a small delay in firing angle is introduced at B, C, D, E, F, and G. At point A, firing angle $\alpha = 90°$ and the mean output voltage is zero. At point G, the firing angle $\alpha = 0°$ and the mean output voltage is maximum. Therefore, the mean output can be expressed by

$$V_o = V_{do} \cos \alpha$$

where V_{do} is the maximum output voltage and α is delay angle.

After the point G, the firing angle is again progressively increased, firing angle at point H is 0° + 15° = 15°, at point I is 15° + 15° = 30°, at point J is 30° + 15° = 45°, at point K is 45° + 15° = 60°, at point L is 60° + 15° = 75°, and at point M is 75° + 15° = 90°. At point G, the firing angle $\alpha = 0°$ and the mean output voltage is maximum. As the firing angle is progressively increased, at point M $\alpha = 90°$ and the mean output voltage is zero. The firing angle at point A, B, C, D, E, F, G, H, I, J, K, L and M are given below

$$\begin{array}{lllll} \text{at } A, \alpha = 90° & \text{at } B, \alpha = 75° & \text{at } C, \alpha = 60° & \text{at } D, \alpha = 45° & \text{at } E, \alpha = 30° \\ \text{at } F, \alpha = 15° & \text{at } G, \alpha = 0° & \text{at } H, \alpha = 15° & \text{at } I, \alpha = 30° & \text{at } J, \alpha = 45° \\ \text{at } K, \alpha = 60° & \text{at } L, \alpha = 75° & \text{at } M, \alpha = 90° & & \end{array}$$

Figure 9.52 shows a complete of low frequency output voltage waveform which contains fundamental frequency (mean) output voltage and harmonics components and the mean output current waveform. Each half cycle of low frequency output voltage has eight half cycle of supply frequency. During the positive half cycle of low frequency output voltage, current flows through positive (P) group converter. Similarly, negative group (N) converter allow to flow of current during the negative half cycle of low frequency output voltage. This cycloconverter can able to provide current ant any power factor load. In Fig. 9.52, the mean current is lagging from mean output voltage.

In a thyristor converter circuit, current can flow in one direction only. Therefore, to flow the current in both direction during one complete cycle of load current, two three-phase half wave converters (positive group converter and negative group converter) are used as shown in Fig. 9.53. The positive group converter allows to flow current during positive half cycle of low frequency output current. Similarly, the negative group converter allows to flow current during negative half cycle of low frequency output current.

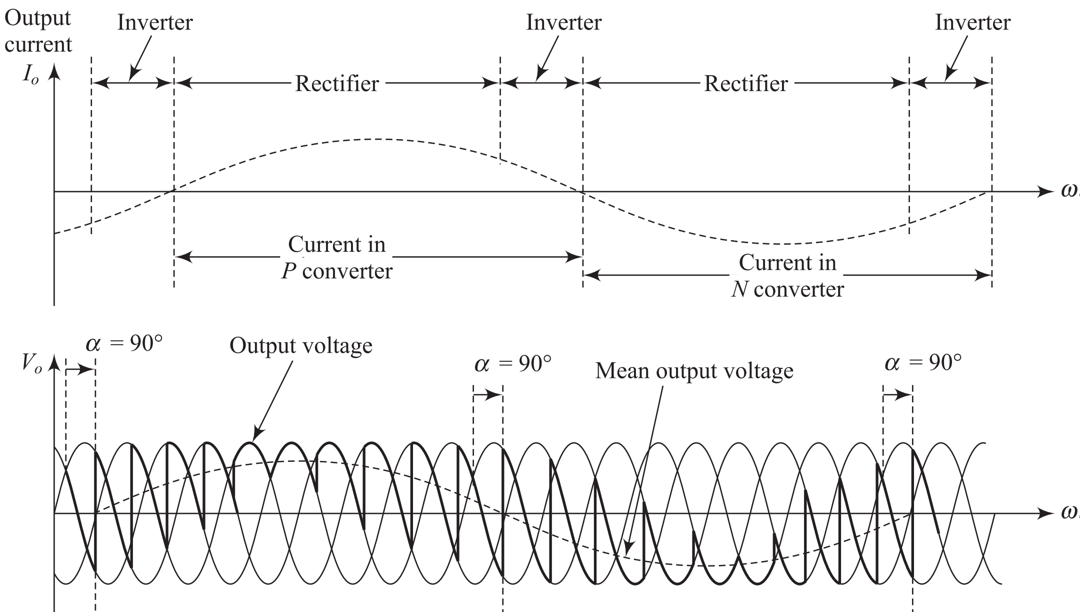


Fig. 9.52 *A complete of low frequency output voltage and current waveform for a three-phase half-wave cycloconverter*

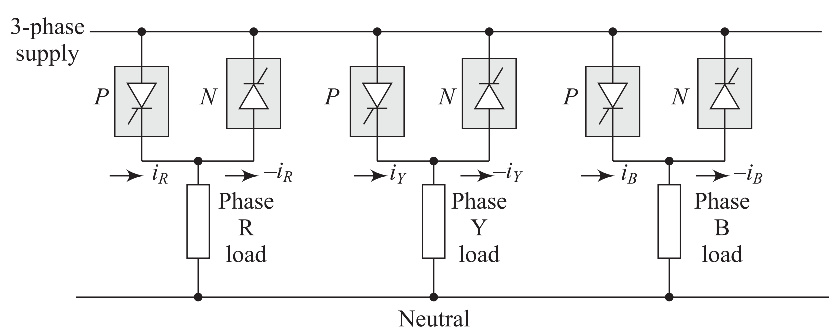


Fig. 9.53 *Three-phase to three-phase cycloconverter using three-phase half-wave converters*

It is clear from Fig. 9.52 that the mean current is negative for certain duration and it is also positive for certain duration though the mean output voltage is positive. When the output current is positive, the positive group converter works. At this condition, the positive group converter works as rectifier when the output voltage is positive. The positive group converter acts as inverter when the output voltage is negative. When the output current is negative, the negative group converter conducts. Therefore, the negative group converter works as rectifier when the output voltage is negative and acts as inverter when the output voltage is positive. Consequently, the output voltage and current have same polarity for rectifier operation and opposite polarity for inverter operation as depicted in Fig. 9.52.

In a dual converter, two phase controlled converter are connected in anti-parallel. Both the converters can be controlled simultaneously to generate the required output voltage. The output voltage of the two converters has the same average voltage, but at the same instant, output voltage is different as the output voltage waveforms are function of time. Therefore, there is a net potential difference in output voltage between two converters. Due to a net voltage, a circulating current flows in the two converters just like dual converters. The circulating current can be limited by removing the gating signal from the idle converter or by inserting an inter group reactor between positive and negative group converters as shown in Fig. 9.47. If the average values of output voltage of two converters are equal in magnitude but opposite in sign, the sum of the firing angles is 180°. When α_N and α_P are the firing angle of positive and negative group converters respectively, $\alpha_N + \alpha_P = 180°$.

9.17 THREE-PHASE TO THREE-PHASE CYCLOCONVERTER

To get three-phase low frequency output voltage, three sets of three-phase to single-phase half-wave cycloconverter circuits are inter connected as shown in Fig. 9.53. Each phase of the three-phase output must be 120° phase shifted. Since each three-phase to single-phase half-wave cycloconverter circuit requires six SCRs, a total of 18 thyristors are required to develop a three-phase to three-phase cycloconverter using three-phase half-wave converters as depicted in Fig. 9.54.

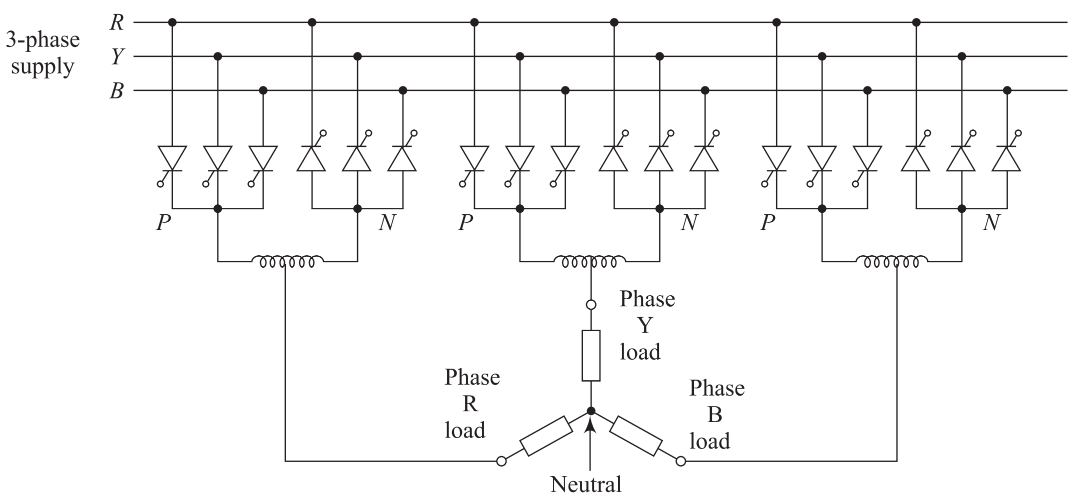


Fig. 9.54 *Three-phase to three-phase cycloconverter using three-phase half-wave converters*

Figure 9.55 shows a three-phase to three-phase cycloconverters using 36 thyristors. This circuit is also called as six-pulse three-phase to three-phase cycloconverter. In this circuit, each phase consists of a three-phase dual converter with two inter group reactors (IGR). Figure 9.55 shows a six-pulse three-phase to three-phase cycloconverter using 36 thyristors when load phases are not interconnected. In case load is connected in star or delta, each phase group must be supplied separately from three secondary windings. Figure 9.56 shows a three-phase bridge cycloconverter using 36 thyristors when load is connected in star. The amplitude of output voltage in a three-phase bridge converter

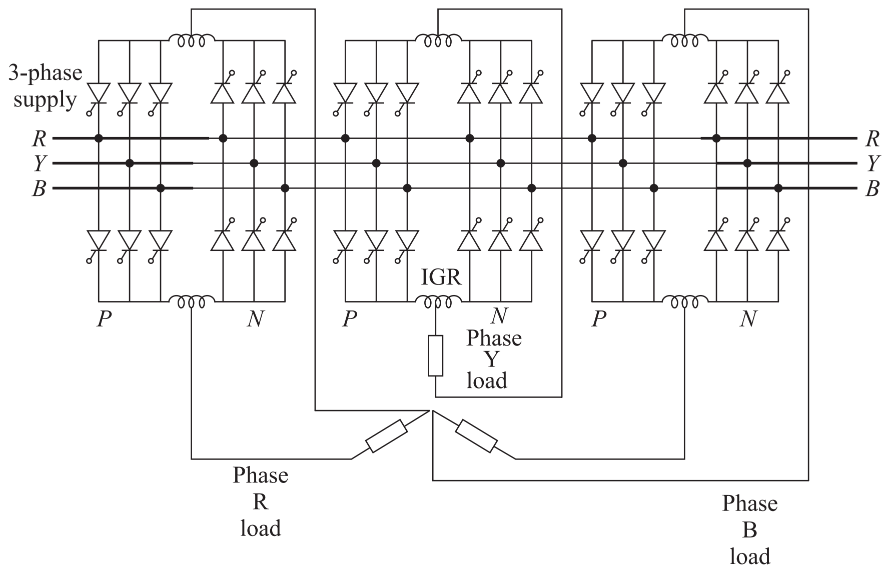


Fig. 9.55 *Six pulse three-phase to three-phase cycloconverter using 36 thyristors when load phases are not interconnected*

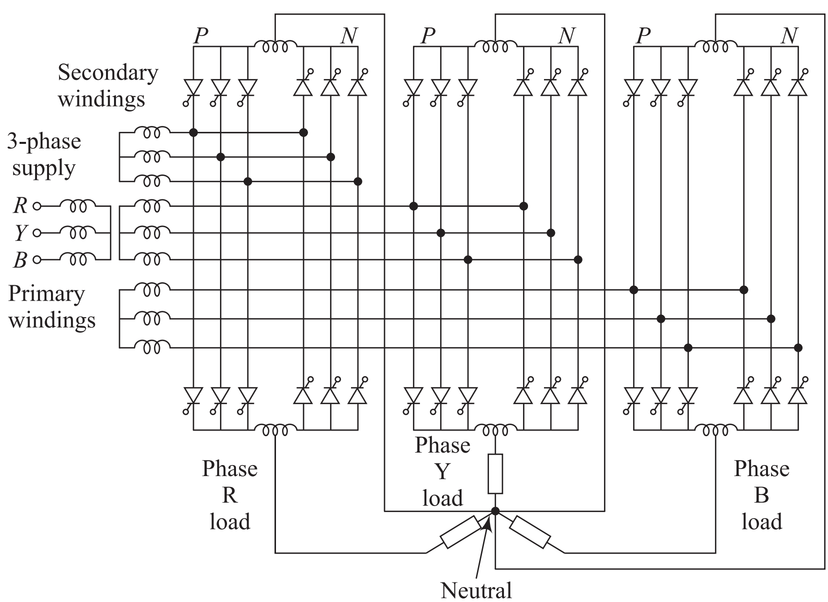


Fig. 9.56 *Six pulse three-phase to three-phase cycloconverter using 36 thyristors when load phases are not interconnected*

cycloconverter (Fig. 9.56) is double of the amplitude of output voltage in a three-phase half-wave converter cycloconverter. If same rating thyristors are used in both cyccloconverters, the total VA rating of three-phase bridge converter cycloconverter is double of that of three-phase half-wave converter cycloconverter. The three-phase bridge converter cycloconverter provides a smooth variation of output voltage, but the control circuit is very expensive due to complexity.

9.18 OUTPUT VOLTAGE OF A CYCLOCONVERTER

In a three-phase half-wave converter, each phase conducts for 120° or $\frac{2\pi}{3}$ radians of cycle of 2π radians. Similarly, in a m-phase half-wave converter, each phase conducts for $\frac{360°}{m}$ or $\frac{2\pi}{m}$ radians of cycle of 2π radians. Figure 9.57 shows the output voltage of a m-phase half-wave converter with firing angle α. Assume OO' is the time origin at which supply voltage is maximum.

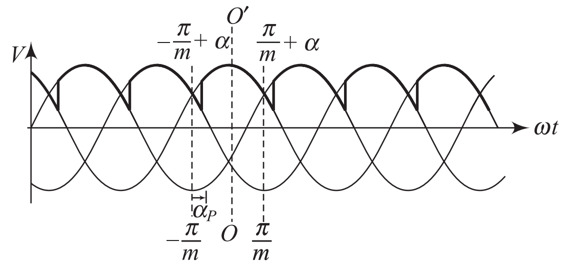


Fig. 9.57 *Output voltage of a m-phase half-wave converter with firing angle α*

The instantaneous value of phase voltage is

$$v = V_m \cos \omega t = \sqrt{2}V \cos \omega t$$

where V_m is maximum phase voltage and V is rms phase voltage.

At firing angle α = 0°, the switching device, i.e., thyristor conducts from $-\frac{\pi}{m}$ to $\frac{\pi}{m}$. Then for firing angle α, the switching device conducts from $-\frac{\pi}{m}+\alpha$ to $\frac{\pi}{m}+\alpha$. Therefore, the average value of output voltage is equal to

$$V_o = \frac{1}{2\pi/m} \int_{-\frac{\pi}{m}+\alpha}^{\frac{\pi}{m}+\alpha} V_m \cos \omega t \cdot d\omega t$$

or $$V_o = V_m \left(\frac{m}{\pi}\right) \sin\left(\frac{\pi}{m}\right) \cos \alpha$$

When firing angle α = 0°, the average value of output voltage is

$$V_{o\text{lat}\cdot\alpha=0°} = V_m \left(\frac{m}{\pi}\right) \sin\left(\frac{\pi}{m}\right) \cos 0 = V_m \left(\frac{m}{\pi}\right) \sin\left(\frac{\pi}{m}\right) \quad \text{as } \cos 0 = 1$$

$$= \sqrt{2}V \left(\frac{m}{\pi}\right) \sin\left(\frac{\pi}{m}\right)$$

The fundamental rms value per phase is equal to

$$V_{\text{orms}} = \frac{V_{O\text{at } \alpha=0}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \sqrt{2}V \left(\frac{m}{\pi}\right) \sin\left(\frac{\pi}{m}\right)$$

or, $V_{orms} = V\left(\frac{m}{\pi}\right) \sin\left(\frac{\pi}{m}\right)$ where V is the phase voltage.

Example 9.19 In a three-phase to single-phase cycloconverters, a three-pulse positive group converter and a three-pulse negative group converter are used. Each converter is fed from a delta/star transformer with turn ratio 1:1. If the input ac supply voltage is 440 V, 50 Hz, $R = 1\ \Omega$ and $|\omega L| = 2\ \Omega$, calculate (a) rms output voltage, (b) rms output current and (c) output power. Due to commutation overlap and thyristor turn-OFF timing, the firing angle in the inversion mode does not greater than 160°.

Solution

(a) Per phase input voltage of transformer is 440 V
Per phase input voltage to converter is 440 V
Voltage reduction factor is $r = \cos(180 - 160) = \cos 20°$
In a three-phase three-pulse converter, $m = 3$ and the rms value of fundamental voltage is

$$V_{orms} = rV\left(\frac{m}{\pi}\right)\sin\left(\frac{\pi}{m}\right) = \cos 20° \times 440 \times \left(\frac{3}{\pi}\right)\sin\left(\frac{\pi}{3}\right) = 342.105 \text{ V}$$

(b) Load impedance is $Z = \sqrt{R^2 + (\omega L)^2}\angle\tan^{-1}\left(\frac{\omega L}{R}\right) = \sqrt{1^2 + (2)^2}\angle\tan^{-1}\left(\frac{2}{1}\right) = 2.236\angle 63.43°$

Rms output current is

$$I_{orms} = \frac{V_{orms}}{Z} = \frac{342.105}{2.236\angle 63.43°} = 152.998\angle -63.43°$$

(c) Output power is

$$I_{orms}^2 R = 152.998^2 \times 1 = 23408.38 \text{ Watt}$$

Example 9.20 In a three-phase to single-phase cycloconverters, a three-pulse positive group converter and a three-pulse negative group converter are used. Each converter is fed from a delta/star transformer with turn ratio 2:1. If the input ac supply voltage is 400 V, 50 Hz, $R = 2.5\ \Omega$ and $|\omega L| = 2\ \Omega$, calculate (a) rms output voltage, (b) rms output current and (c) output power. Due to commutation overlap and thyristor turn-OFF timing, the firing angle in the inversion mode does not greater than 160°.

Solution

(a) Per phase input voltage of transformer is 400 V
Per phase input voltage to converter is $V_{ph} = \frac{400}{2} = 200$ V
Voltage reduction factor is $r = \cos(180 - 160) = \cos 20°$
In a three-phase three-pulse converter, $m = 3$ and the rms value of fundamental voltage is

$$V_{orms} = rV\left(\frac{m}{\pi}\right)\sin\left(\frac{\pi}{m}\right) = \cos 20° \times 200 \times \left(\frac{3}{\pi}\right)\sin\left(\frac{\pi}{3}\right) = 155.502 \text{ V}$$

(b) Load impedance is $Z = \sqrt{R^2 + (\omega L)^2}\angle\tan^{-1}\left(\frac{\omega L}{R}\right) = \sqrt{2.5^2 + (2)^2}\angle\tan^{-1}\left(\frac{2}{2.5}\right) = 3.201\angle 38.659°$

Rms output current is

$$I_{orms} = \frac{V_{orms}}{Z} = \frac{155.502}{3.201\angle 38.659°} = 48.579\angle -38.659°$$

(c) Output power is

$$I_{orms}^2 R = 48.579^2 \times 1 = 5899.79 \text{ Watt}$$

Example 9.21 A six-pulse cycloconverter is supplied from a 440 V, 50 Hz ac supply and it is delivering 50 A to a single-phase resistive load. The source inductance is 1.5 mH. Determine the output voltage at firing angle (a) $\alpha = 0°$ and (b) $\alpha = 45°$.

Solution

(a) For $\alpha = 0°$, peak value of output voltage for a three pulse converter is

$$V_o = \sqrt{2}V\left(\frac{m}{\pi}\right)\sin\left(\frac{\pi}{m}\right) = \sqrt{2}\times 440\left(\frac{3}{\pi}\right)\sin\left(\frac{\pi}{3}\right) = 514.783 \text{ V}$$

Voltage reduction due to source inductance is

$$\frac{3\omega L_s}{\pi}I_o = \frac{3\times 2\pi\times 1.5\times 10^{-3}}{\pi}\times 50 = 0.45 \text{ V}$$

The peak value of output voltage is

$$V_{o\max} = 514.783 - 0.45 = 514.333 \text{ V}$$

The rms value of output voltage is $\dfrac{V_{o\max}}{\sqrt{2}} = \dfrac{514.333}{\sqrt{2}} = 363.74 \text{ V}$

(b) For $\alpha = 45°$, peak value of output voltage for a three-pulse converter is

$$V_{o-\text{at}\_\alpha} = \sqrt{2}V\left(\frac{m}{\pi}\right)\sin\left(\frac{\pi}{m}\right)\cos\alpha = \sqrt{2}\times 440\left(\frac{3}{\pi}\right)\sin\left(\frac{\pi}{3}\right)\cos 45° = 364 \text{ V}$$

Voltage reduction due to source inductance is

$$\frac{3\omega L_s}{\pi}I_o = \frac{3\times 2\pi\times 1.5\times 10^{-3}}{\pi}\times 50 = 0.45 \text{ V}$$

The peak value of output voltage is

$$V_{o\max} = 364 - 0.45 = 363.55 \text{ V}$$

The rms value of output voltage is $\dfrac{V_{o\max}}{\sqrt{2}} = \dfrac{363.55}{\sqrt{2}} = 257.107 \text{ V}$

Summary

- AC voltage controllers control the magnitude of output voltage from a fixed voltage ac source. The voltage magnitude can be controlled by on-off (Integral Cycle) control or phase angle control.
- Integral cycle control single phase ac voltage controller, phase controlled single phase and three phase ac voltage controllers with R load and RL load are discussed elaborately.
- Application of ac voltage controller in tap changer and ac chopper are incorporated in this chapter.
- Cycloconverter is a direct frequency converter or frequency changer which converts ac power at fixed frequency to ac power at variable frequency, single phase to single phase step-up and step-down cycloconverters, three phase to single phase cycloconverters, three phase to three phase cycloconverters are explained in detail.

Multiple-Choice Questions

9.1 Phase controlled single-phase ac voltage controller converts
(a) fixed frequency ac voltage to variable frequency ac voltage
(b) fixed frequency ac voltage to fixed frequency variable ac voltage
(c) variable frequency ac voltage to fixed frequency ac voltage
(d) variable frequency ac voltage to variable frequency ac voltage

9.2 Whenaninductionmotorandaheaterare supplied from a phase controlled single-phase ac voltage controllers,
(a) only fundamental component of output voltage and current is useful in induction motor but fundamental and harmonics are useful in heater
(b) fundamental and harmonics are useful in induction motor but only fundamental component of output voltage and current is useful in heater
(c) both fundamental and harmonics are useful in induction motor and heater
(d) only harmonics are useful in induction motor and heater

9.3 In an integral cycle ac voltage controller, the switch is ON for n number of half cycles and the switch is OFF for m number of half cycles. The output voltage is equal to

(a) $V_o = V\sqrt{\dfrac{n+m}{n}}$ (b) $V_o = V\sqrt{\dfrac{n-m}{n}}$ (c) $V_o = V\sqrt{\dfrac{n}{n-m}}$ (d) $V_o = V\sqrt{\dfrac{n}{n+m}}$

9.4 A single-phase ac voltage controller is fed from a voltage source $200\sqrt{2}\ \sin 314t$ and is connected to a load resistance of R. When the firing angle of ac voltage controller is 90°, the power output to load is

(a) $\dfrac{200^2}{2R}$ (b) $\dfrac{(200\sqrt{2})^2}{R}$ (c) 200^2R (d) $\dfrac{(200\sqrt{2})^2}{R^2}$

9.5 A single-phase ac voltage controller is fed from a voltage source 200 sin 314t and is connected to a load resistance of 10 Ω. When the firing angle of ac voltage controller is 90°, the power output to load in Watt is
(a) 1000 (b) 750 (c) 500 (d) 250

In a furnace the heating resistance is 20 Ω which connected to 230 V, 50 Hz ac through a integral cycle control ac controller. Assume the switch is ON for four half cycles and OFF for two half cycles.

9.6 Output voltage is equal to
(a) 100 V (b) 125 V (c) 150 V (d) 188 V

9.7 Power factor is
(a) 0.81 leading (b) 0.81 lagging (c) 0.71 leading (d) 0.71 lagging

9.8 RMS value of load current
(a) 9.4 A (b) 8.4 A (c) 7.4 A (d) 6.4 A

9.9 A single phase voltage controller is used for controlling the power flow from 230 V, 50 Hz source into a load consisting of $R = 5\ \Omega$ and $\omega l = 3\Omega$. The load current will be maximum at the firing angle $\alpha =$

(a) 0° (b) $\tan^{-1}\left(\dfrac{3}{4}\right)$ (c) 90° (d) 180°

9.10 Average value of output voltage of half-wave ac voltage controller at firing angle α is equal to

(a) 0 (b) $\dfrac{\sqrt{2}V}{2\pi}(\cos\alpha - 1)$ (c) $\dfrac{\sqrt{2}V}{2\pi}(\cos\alpha + 1)$ (d) $\dfrac{\sqrt{2}V}{\pi}(\cos\alpha - 1)$

9.11 Average value of output voltage of full wave ac voltage controller at firing angle α is equal to

(a) 0 (b) $\dfrac{\sqrt{2}V}{2\pi}(\cos\alpha - 1)$ (c) $\dfrac{\sqrt{2}V}{2\pi}(\cos\alpha + 1)$ (d) $\dfrac{\sqrt{2}V}{\pi}(\cos\alpha - 1)$

9.12 RMS value of output voltage of half-wave ac voltage controller at firing angle α is equal to

(a) $V\left[\dfrac{1}{2\pi}\left(2\pi + \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (b) $V\left[\dfrac{1}{2\pi}\left(\pi - \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$

(c) $V\left[\dfrac{1}{\pi}\left(2\pi - \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (d) $V\left[\dfrac{1}{2\pi}\left(2\pi - \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$

9.13 Power factor of half-wave ac voltage controller at firing angle α is equal to

(a) $\left[\dfrac{1}{2\pi}\left(2\pi - \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (b) $\left[\dfrac{1}{2\pi}\left(\pi - \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$

(c) $\left[\dfrac{1}{\pi}\left(2\pi - \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (d) $\left[\dfrac{1}{2\pi}\left(2\pi + \alpha + \dfrac{\sin 2\alpha}{2}\right)\right]^{1/2}$

9.14 RMS value of output voltage of full-wave ac voltage controller at firing angle α is equal to

(a) $V\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (b) $V\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$

(c) $V\left[\frac{1}{\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (d) $V\left[\frac{1}{2\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$

9.15 Power factor of half-wave ac voltage controller at firing angle α is equal to

(a) $\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (b) $\left[\frac{1}{2\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$

(c) $\left[\frac{1}{\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$ (d) $\left[\frac{1}{2\pi}\left(2\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2}$

9.16 In a single-phase voltage controller with RL load, ac output power can be controlled when
(a) firing angle $\alpha >$ load phase angle ϕ and conduction angle $\gamma = 180°$
(b) firing angle $\alpha <$ load phase angle ϕ and conduction angle $\gamma = 180°$
(c) firing angle $\alpha >$ load phase angle ϕ and conduction angle $\alpha < \phi$
(d) firing angle $\alpha <$ load phase angle ϕ and conduction angle $\gamma < 180°$

9.17 A single-phase ac voltage controller using two thyristors in antiparallel can be operated as a half-wave controlled rectifier when
(a) load is R and pulse gating is used (b) load is R and continuous gating is used
(c) load is RL, pulse gating is used and $\alpha < \phi$ (d) load is RL and continuous gating is used

9.18 In a single-phase ac voltage controller, firing angle α, extinction angle β and conduction angle γ are related as

(a) $\gamma = \beta + \alpha$ (b) $\gamma = \beta - \alpha$ (c) $\alpha + \beta + \gamma = 0$ (d) $\beta = \gamma - \alpha$

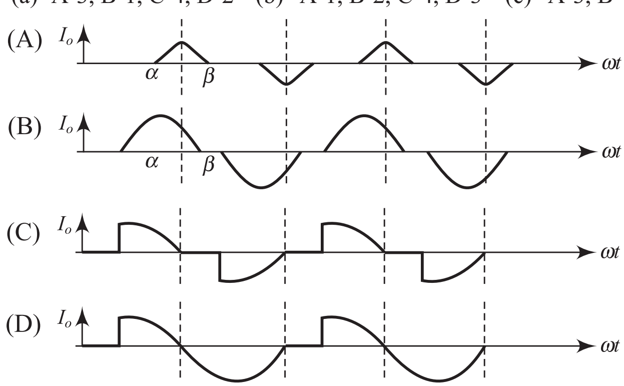
9.19 In a single-phase ac voltage controller with RL load, the ac output power can be controlled if
(a) $\alpha > \phi$ and $\gamma = 180°$ (b) $\alpha > \phi$ and $\gamma < 180°$ (c) $\alpha < \phi$ and $\gamma = 180°$ (d) $\alpha < \phi$ and $\gamma < 180°$

9.20 A single-phase ac voltage controller is fed from a 230 V, 50 Hz ac supply and is connected with a load of $R = 4\ \Omega$ and $L = 3$ mH. The control range of firing angle is equal to
(a) $13.26° < \alpha < 180°$ (b) $0° < \alpha < 180°$ (c) $13.26° < \alpha < 90°$ (b) $0° < \alpha < 90°$

9.21 In a single-phase ac voltage controller with purely inductive load, the control range of firing angle is
(a) $0° < \alpha < 180°$ (b) $90° < \alpha < 180°$ (b) $0° < \alpha < 90°$ (d) $45° < \alpha < 90°$

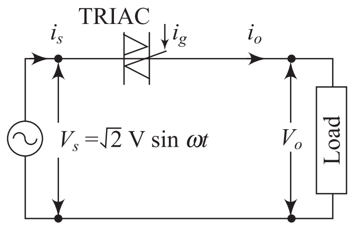
9.22 Integral cycle control ac voltage controller is suitable for
(a) loads with high time constants and limited range control (b) very fast in control action
(c) loads with very small time constants (d) R load

9.23 Match List I with List II and select the correct answer.
(a) A-3, B-1, C-4, D-2 (b) A-1, B-2, C-4, D-3 (c) A-3, B-4, C-1, D-2 (d) A-3, B-4, C-2, D-1

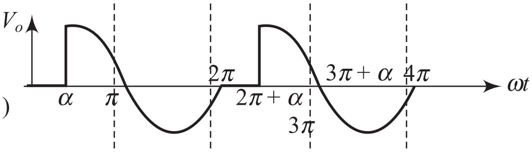


(1) Bidirectional ac voltage controller with R-L load
(2) Unidirectional ac voltage controller with R load
(3) Bidirectional ac voltage controller with purely L load
(4) Bidirectional ac voltage controller with R load

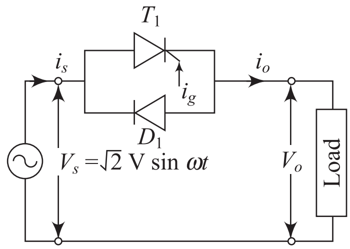
9.24 Match List I with List II and select the correct answer using the codes given below.



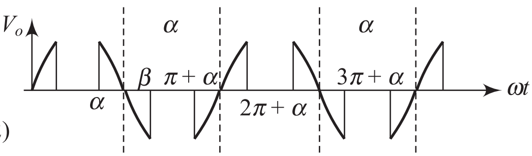
(I)



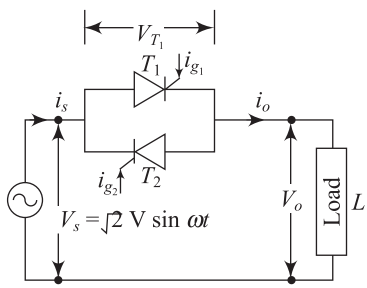
(1)



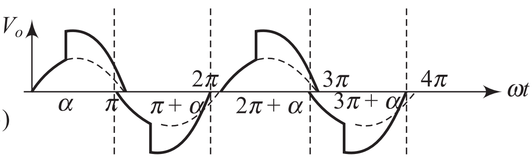
(II)



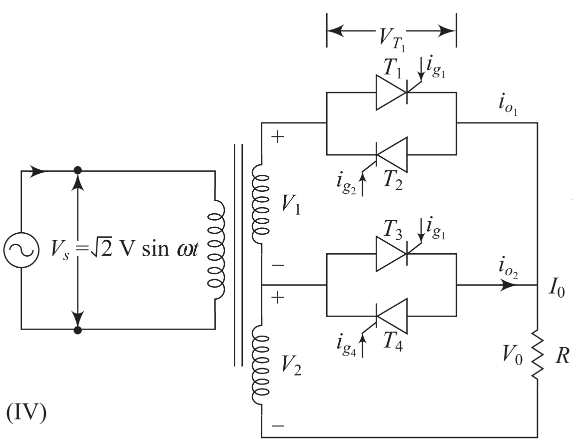
(2)



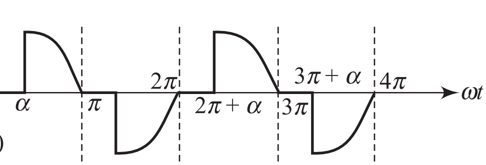
(III)



(3)



(IV)



(4)

(a) I-4, II-1, III-2, IV-3 (b) I-4, II-1, III-3, IV-2
(c) I-1, II-4, III-2, IV-3 (d) I-4, II-2, III-1, IV-3

9.25 A cycloconverter can convert power from
(a) low frequency to high frequency
(b) high frequency to low frequency
(c) supply frequency to variable frequency which is fraction of supply frequency

9.26 Cycloconverter is a frequency converter from
(a) Higher to lower frequency with two stage conversion
(b) Higher to lower frequency with one stage conversion
(c) Lower to higher frequency with two stage conversion
(d) Lower to higher frequency with one stage conversion

9.27 Which of the following statements is correct?
(a) Natural commutation is used in both step-up and step-down cycloconverters
(b) Forced commutation is used in both step-up and step-down cycloconverters
(c) Natural commutation is used in step-down cycloconvertes and Forced commutation is used in step-up cycloconverters
(d) Forced commutation is used in step-down cycloconvertes and Natural commutation is used in step-up cycloconverters

9.28 To generate approximately sinusoidal output voltage,
(a) in a single phase to single-phase cycloconverters, firing angle may be varied
(b) in a single phase to single-phase cycloconverters, firing angle may be constant
(c) in a three phase to single-phase cycloconverters, firing angle may be varied
(d) in a three phase to single phase cycloconverters, firing angle may be constant

9.29 A single phase to single-phase cycloconverter using bridge converter requires
(a) 4 SCRs (b) 8 SCRs (c) 6 SCRs (d) 18 SCRs

9.30 A three-phase to single-phase cycloconverter using three-phase bridge converter requires
(a) 4 SCRs (b) 8 SCRs (c) 6 SCRs (d) 12 SCRs

9.31 A three phase to single-phase cycloconverter using three-phase half-wave converter requires
(a) 4 SCRs (b) 8 SCRs (c) 6 SCRs (d) 12 SCRs

9.32 A three phase to three-phase cycloconverter using three-phase half-wave converter requires
(a) 18 SCRs (b) 36 SCRs (c) 6 SCRs (d) 12 SCRs

9.33 A three phase to three-phase cycloconverter using three-phase bridge-wave converter requires
(a) 18 SCRs (b) 36 SCRs (c) 6 SCRs (d) 12 SCRs

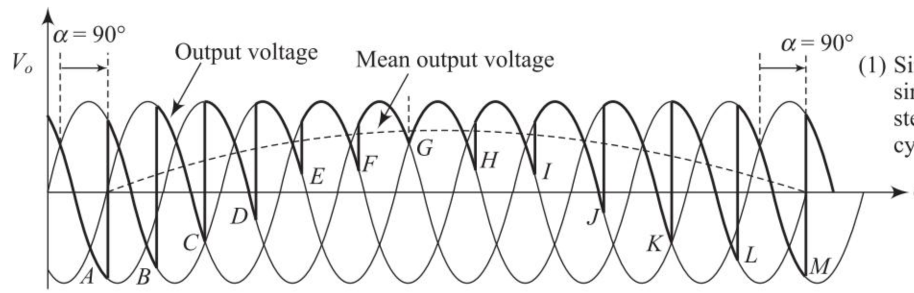
9.34 Three phase to three-phase cycloconverters using 18 thyristors and 36 thyristors have the same voltage and current ratings for thyristors. Then the ratio of VA rating of 36 thyristors to that 18 thyristors is
(a) 1 (b) 2 (c) 4 (d) 8

9.35 If α_N and α_P are the firing angle of positive and negative group converters respectively, then
(a) $\alpha_N + \alpha_P = 180°$ (b) $\alpha_N + \alpha_P > 180°$
(c) $\alpha_N + \alpha_P < 180°$ (d) $\alpha_N + \alpha_P = 360°$

9.36 Match List I with List II and select the correct answer.
(a) A-3, B-4, C-2, D-1 (b) A-1, B-2, C-4, D-3
(c) A-3, B-4, C-1, D-2 (d) A-3, B-2, C-4, D-1

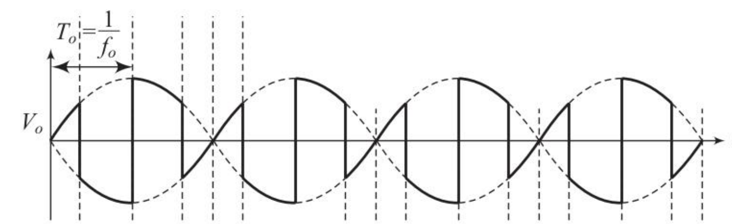
A. Controlled rectifier	1. Induction heating
B. Cycloconverter	2. Electric car
C. Chopper	3. Rolling mill drive
D. Inverter	4. Aircraft power supply

9.37 Match List I with List II and select the correct answer.
(a) A-4, B-1, C-2, D-3 (b) A-1, B-2, C-4, D-3 (c) A-3, B-4, C-1, D-2 (d) A-3, B-4, C-2, D-1



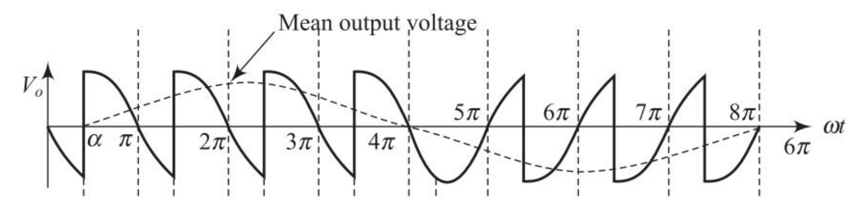
(a)

(1) Single phase to single phase step-up cycloconverter



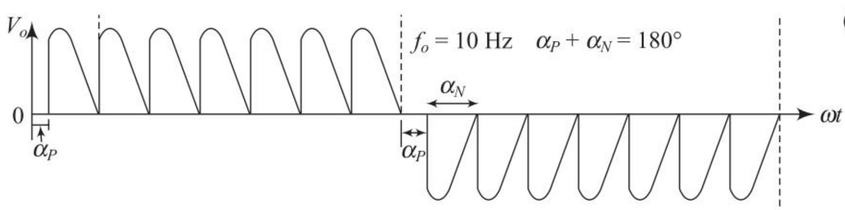
(b)

(2) Single phase to single phase step-down cycloconverter at continuous mode of operation



(c)

(3) Three phase to single phase cycloconverter using three phase bridge converter at fixed firing angle



(d)

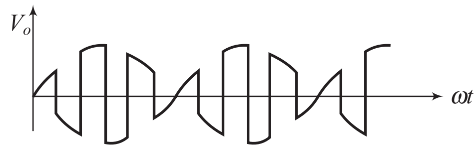
(4) Three phase to single phase cycloconverter using three phase half wave converter at variable firing angle

9.38 Match List I with List II and select the correct answer.

(a) 1-A, 2-D, 3-C, 4-B (b) 1-B, 2-C, 3-D, 4-A

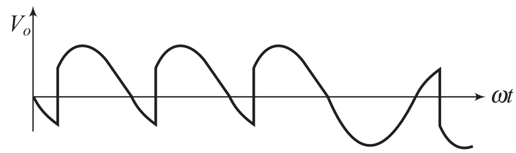
(c) 1-B, 2-A, 3-C, 4-D (d) 1-B, 2-D, 3-C, 4-A

(A)



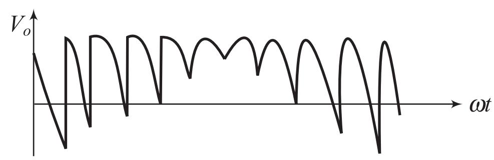
(1) Single phase to single phase cycloconverter with continuous conduction

(B)



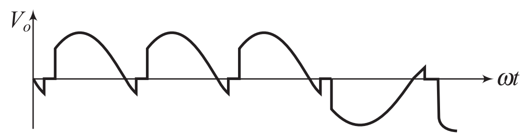
(2) Single phase to single phase cycloconverter at discontinuous mode of operation

(C)



(3) Three phase to single phase cycloconverter

(D)



(4) Step-up converter

9.39 Match List I with List II and select the correct answer.

(a) A-4, B-3, C-2, D-1 (b) A-1, B-2, C-4, D-3
(c) A-3, B-4, C-1, D-2 (d) A-3, B-2, C-4, D-1

A. Controlled rectifier	1. Solar cells
B. Cycloconverter	2. Ceiling fan drive
C. Voltage controller	3. High power ac drive
D. Inverter	4. Magnet power supply

9.40 Match List I with List II and select the correct answer.

(a) A-4, B-3, C-1, D-2 (b) A-1, B-2, C-4, D-3
(c) A-3, B-4, C-1, D-2 (d) A-3, B-2, C-4, D-1

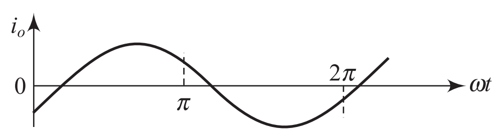
A. Controlled rectifier	1. Illumination control
B. Chopper	2. Uninterruptible power supply (UPS)
C. Voltage controller	3. Fork-lift truck
D. Inverter	4. Hydrogen production

9.41 For a single-phase ac voltage controller, match List I with List II and select the correct answer.

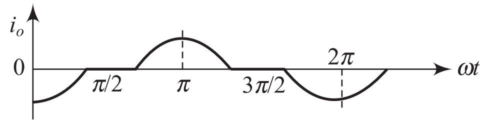
(a) 1-D, 2-C, 3-A, 4-B (b) 1-B, 2-C, 3-D, 4-A (c) 1-B, 2-A, 3-C, 4-D (d) 1-B, 2-D, 3-C, 4-A

1. *R* Load

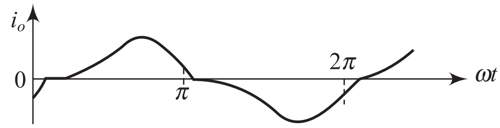
(A)



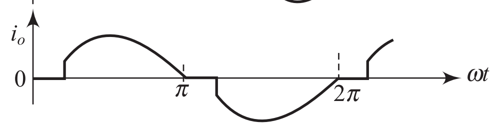
2. R-L load with $\alpha > \phi$

(B) 

3. L load with $\alpha > \phi$

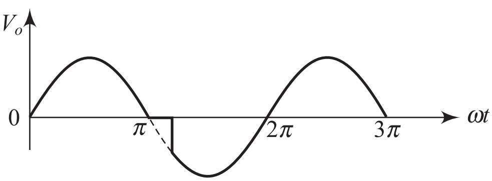
(C) 

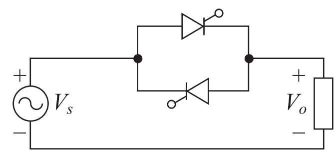
4. R-L load with $\alpha < \phi$

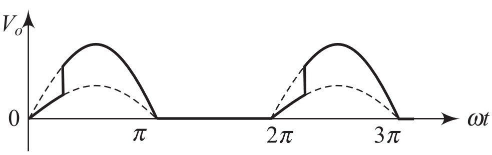
(D) 

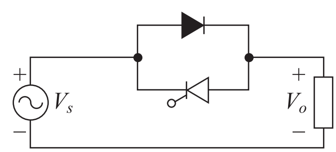
9.42 Match List I with List II and select the correct answer.

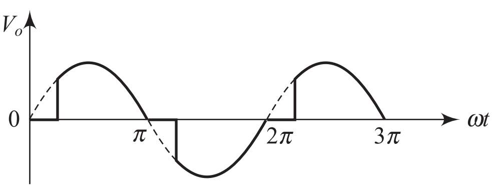
(a) 1-B, 2-D, 3-A, 4-C (b) 1-B, 2-C, 3-D, 4-A (c) 1-B, 2-A, 3-C, 4-D (d) 1-B, 2-D, 3-C, 4-A

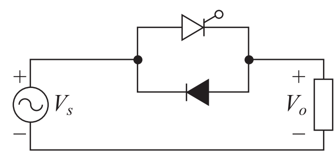
(1) 

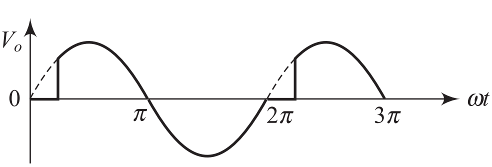
(A) 

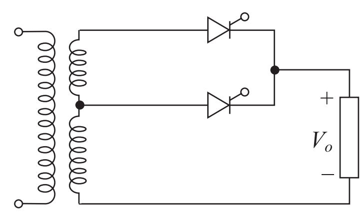
(2) 

(B) 

(3) 

(C) 

(4) 

(D) 

Fill in the Blanks

9.1 AC voltage controller using SCRs is also equivalent to an ______.

9.2 In ac voltage controllers, thyristors or triacs are ______ commutated.

9.3 The control strategies of ac voltage controllers are ______ and ______.

9.4 AC voltage controllers are used in ______.
9.5 In ac voltage controllers, the control range of firing angle is ______.
9.6 A single phase ac voltage controller with purely inductive load is commonly used as ______.
9.7 The single-phase ac voltage controller which consists of one thyristor T_1 in antiparallel with one diode D_1 is called single phase ______ .
9.8 Cycloconverter is known as ______.
9.9 In step-down cycloconverter, the output frequency f_o is ______ than the supply (input) frequency f_s.
9.10 In step-up cycloconverter, the output frequency f_o is ______ than the supply (input) frequency f_s
9.11 When α_N and α_P are the firing angle of positive and negative group converters respectively, $\alpha_N + \alpha_P =$.
9.12 The output voltage of a m-phase half wave converter with firing angle α is ______.
9.13 AC voltage controllers are used to convert ______ frequency alternating voltage directly to variable output voltage at ______ frequency.
9.14 A single-phase half-wave ac voltage controller consists of one ______ and one ______ in antiparallel.
9.15 A single-phase half-wave ac voltage controller consists of two thyristors connected in ______.
9.16 A three-phase to three-phase cycloconverter using three sets of three-phase half-wave circuits employing a total no. of ______ thyristors.
9.17 A converter circuit which convertes input power from one frequency to output power at different frequency is called ______.
9.18 A single phase to single-phase cycloconverter using bridge circuits can be developed by employing a total no of ______ thyristors.
9.19 A single phase to single-phase cycloconverter using mid-point circuits can be developed by employing a total number of ______ thyristors.
9.20 Three phase to single-phase cycloconverter using two sets of three-phase half circuits can be developed using ______ thyristors.
9.21 Three phase to three-phase using three sets of six-pulse converters can be made by using ______ thyristors.
9.22 If there is an overlap angle μ, the firing angle of inverting mode converter in cycloconverter is ______.
9.23 The fundamental rms value of output voltage of a cycloconverter is equal ______.

Review Questions

9.1 What is ac voltage controller? What are the types of ac voltage controller?
9.2 (a) What is integral cycle control? Derive the expression for rms output voltage of integral cycle ac voltage controller.
(b) What are the advantages and disadvantages of integral cycle control or ON-OFF control?
(c) Write applications of integral cycle ac voltage controller.
9.3 (a) What is phase controlled single phase ac voltage controller?
(b) What are the different circuit configurations of phase controlled single phase ac voltage controller?
(c) What are the advantages and disadvantages of phase control?
9.4 (a) Draw the single phase unidirectional ac voltage controller with R load and explain its working principle with waveforms
(b) Derive the expression for rms output voltage of unidirectional ac voltage controller.
9.5 (a) Draw the single-phase bidirectional ac voltage controller with R load and explain its working principle with waveforms.
(b) Derive the expression for rms output voltage of bidirectional ac voltage controller.
(c) What are the advantages and disadvantages of unidirectional and bidirectional ac voltage controllers? Which one of these is preferred and why?
9.6 Explain the working principle of burst firing for a single-phase ac voltage controller. Derive an expression for the rms value of output voltage.

9.7 Define power factor. Derive an expression for power factor of a bidirectional ac voltage controllers with R load.

9.8 What is extinction angle? What is the effect of load inductance on the performance of ac voltage controller?

9.9 Explain the working principle of single-phase ac voltage controller with purely inductive load.

9.10 Discuss the operation of single phase bidirectional ac voltage controller with RL load when (i) $\alpha > \phi$ and (ii) $\alpha < \phi$. Under what condition the conduction angle $\gamma = 180°$

9.11 What are the different circuit configurations of phase controlled three-phase ac voltage controller?

9.12 What are the advantages and disadvantages of delta connected converters?

9.13 Draw the circuit diagram of delta connected three-phase ac voltage controller and explain its working principle with waveforms.

9.14 Draw the circuit diagram of delta connected R load configuration of three-phase ac voltage controller and explain its working principle with waveforms.

9.15 Draw the circuit diagram of three-phase ac voltage controller with Y load and explain its working principle with waveforms.

9.16 What is the control range of the firing angle of three-phase bidirectional controllers?

9.17 What is electronic tap changer? What is ac chopper?

9.18 Discuss the operation of a two-stage sequence control of voltage controllers with R load.

9.19 What is cycloconverter? What are the types of cycloconverters?

9.20 What are the advantages and disadvantages of cycloconverters?

9.21 Explain the operating principle of single phase to single-phase step-up cycloconverter using bridge converters. Mention the conduction of various thyristors in the waveforms.

9.22 Discuss the operating principle of single phase to single-phase step-down cycloconverter using bridge converters. Mention the conduction of various thyristors in the waveforms.

9.23 What are the applications of cycloconverters?

9.24 Explain the operating principle of mid-point-type single phase to single-phase step-up cycloconverter. Mention the conduction of various thyristors in the waveforms.

9.25 Discuss the operating principle of mid-point-type single phase to single-phase step-down cycloconverter. Mention the conduction of various thyristors in the waveforms.

9.26 (a) Describe the operating principle of three phase to single-phase cycloconverter using three-phase half-wave converter. Mention the conduction of various thyristors in the waveforms.
(b) Derive the average output voltage of a m phase cycloconverter.

9.27 In a furnace the heating resistance is 10 Ω which connected to 230 V, 50 Hz ac through a integral cycle control ac controller. Assume the switch is ON for three half cycles and OFF for two half cycles. Determine the following parameters:
(a) Output voltage (b) Power factor (c) RMS value of load current

9.28 A single-phase voltage controller is controlled by integral cycle control. Its input voltage is 220 V, 50 Hz ac. Assume it is ON for five half cycles and OFF for three half cycles and resistance R is 5 Ω. Determine the following parameters:
(a) RMS output voltage (b) Power output (c) Power input
(d) Power factor (e) average and RMS value of thyristor current

9.29 The resistance of 220 V, 5.5 kW furnace is 10 Ω and the furnace is controlled by integral cycle control. Determine
(a) the duty ratio for power output is equal to 60% input power and power factor in this duty ratio
(b) the duty ratio for output voltage is equal to 60% rated voltage and power factor in this duty ratio.

9.30 A single-phase ac voltage controller is controlled by brust-firing control. It is used for heating a load of R = 10 Ω with input voltage is 220 V, 50 Hz ac. For a load power of 5 kW, find the following parameters:
(a) the duty cycle (b) input power factor (c) average and RMS value of thyristor current

9.31 A single-phase half wave ac voltage controller is connected with a load of R = 15 Ω with an input voltage of 230 V, 50 Hz. When the firing angle of thyristor is 30°, find the RMS output voltage, power output at load, input power factor and average value of output voltage.

9.32 A single phase unidirectional ac voltage controller is connected with a load of $R = 20\ \Omega$ with an input voltage of 220 V, 50 Hz. If the firing angle of thyristor is 90°, determine (a) the RMS value of output voltage, (b) power delivered to load, (c) input power factor and average, (d) average value of thyristor current and (e) average value of diode current.

9.33 A single-phase full wave ac voltage controller is connected with a load of $R = 5\ \Omega$ with an input voltage of 220 V, 50 Hz. When the firing angle of thyristor is 45°, determine the rms output voltage, power output at load and input power factor.

9.34 A single-phase full wave ac voltage controller has a load of 15 Ω and the input voltage is 230 V with 50 Hz. If the load power is 5 kW, determine (a) firing angle of thyristors, (b) input power factor and (c) rms output voltage.

9.35 A single-phase full-wave ac voltage controller is connected with a load of $R = 10\ \Omega$ with an input voltage of 220 V, 50 Hz. When the firing angle of thyristors is 30°, determine (a) power output at load, (b) average value of thyristor current and (c) rms value of thyristor current.

9.36 A single-phase ac voltage controller is fed from 230 V, 50 Hz ac supply and connected to RL load. When $R = 4\ \Omega$ and $|\omega L| = 4\ \Omega$, determine (a) the range of firing angle, (b) maximum value of rms load current, (c) maximum power and power factor at maximum power output, (d) maximum average thyristor current, (e) maximum rms thyristor current and (f) maximum value of $\frac{di_o}{dt}$

9.37 A single-phase ac voltage controller is fed from 220 V, 50 Hz ac supply and connected to L load only. If $|\omega_L| = 3\ \Omega$, determine (a) the control range of firing angle, (b) maximum value of rms load current, (c) maximum average thyristor current, (d) maximum rms thyristor current and (e) maximum value of $\frac{di_o}{dt}$.

9.38 A single-phase electronic tap changer as shown in Fig. 9.32 is feeding from a 220 V, 50 Hz ac supply and is connected with a load of $R = 20\ \Omega$ and the turns ratio from primary winding to each secondary winding is unity. If the firing angle of upper thyristors is 25°, determine (a) rms value of output voltage, (b) rms value of current of upper thyristors, (c) rms value of current of lower thyristors, (d) VA rating of transformer and (e) power factor.

9.39 A single-phase bridge cycloconverter is fed from 230 V, 50 Hz ac supply and a load of 10 Ω is connected with the cycloconverter. The frequency of output voltage is half of the input frequency. If the firing angle $\alpha = 60°$, determine (a) rms value of output voltage, (b) rms value of load current, (c) rms current of each thyristor and (d) input power factor.

9.40 A single-phase half-wave ac voltage controller is connected with a load of $R = 7.5\ \Omega$ with an input voltage of 200 V, 50 Hz. If the firing angle of thyristor is 60°, determine (a) the RMS output voltage, (b) power delivered to load, (c) input power factor and (d) average value of input current voltage.

9.41 A single-phase full-wave ac voltage controller is connected with a load of $R = 10\ \Omega$ with an input voltage of 230 V, 50 Hz. Determine (a) the maximum value of average thyristor current, (b) the maximum value of rms thyristor current, (c) the minimum circuit turn-OFF time for the firing angle α, (d) the ratio of third harmonic voltage to fundamental voltage at $\alpha = 45°$, (e) the maximum value of $\frac{di}{dt}$ of thyristor and (f) peak inverse voltage of thyristor.

9.42 Figure 9.62 shows a converter circuit. Draw the output voltage and current waveforms for the following conditions:

(a) thyristor T_1 is fired at $\omega\tau = \alpha, 2\pi + \alpha, 4\pi + \alpha, 6\pi + \alpha\ldots$

(b) thyristor T_2 is fired at $\omega t = 0, 2\pi, 4\pi, 6\pi......$

(c) thyristor T_1 is fired at $\omega\tau = \alpha, 2\pi + \alpha, 4\pi + \alpha, 6\pi + \alpha\ldots$ and thyristor T_2 is fired at $\omega t = 0, 2\pi, 4\pi, 6\pi......$

Assume the value of $R = 5\ \Omega$

9.43 A two-stage sequence ac voltage controller is connected to a resistive load of 20 Ω. The input ac voltage is 230 V, 50 Hz and turn ratio is 1:1:1. If the firing angle of upper thyristors are 60°, determine (a) rms output voltage, (b) rms output current, (c) VA rating of transformer and (d) input power factor.

9.44 A two-stage sequence ac voltage controller is connected to a resistive load of 10 Ω. The input ac voltage is 220 V, 50 Hz. Turn ratio from primary to each secondary of transformer is unity. If the firing angle of upper thyristors is 45°, determine (a) rms output voltage, (b) rms output current, (c) VA rating of transformer and (d) input power factor.

9.45 In a three phase to single-phase cycloconverters, a three-pulse positive group converter and a three-pulse negative group converter are used. Each converter is fed from a delta/star transformer with turn ratio 1:1. If the input ac supply voltage is 400 V, 50 Hz, $R = 1.5\ \Omega$ and $|\omega L| = 2\Omega$, calculate (a) rms output voltage, (b) rms output current and (c) output power.
Due to commutation overlap and thyristor turn-OFF timing, the firing angle in the inversion mode does not greater than 160°.

9.46 In a three phase to single-phase cycloconverters, a three-pulse positive group converter and a three-pulse negative group converter are used. Each converter is fed from a delta/star transformer with turn ratio 2:1. If the input ac supply voltage is 440 V, 50 Hz, $R = 3\ \Omega$ and $|\omega L| = 2\ \Omega$, calculate (a) rms output voltage, (b) rms output current and (c) output power.
Due to commutation overlap and thyristor turn-OFF timing, the firing angle in the inversion mode does not greater than 160°.

9.47 A six pulse cycloconverter is supplied from a 400 V, 50 Hz ac supply and it is delivering 50 A to a single-phase resistive load. The source inductance is 1.5 mH. Determine the output voltage at firing angle (a) $\alpha = 0°$ and (b) $\alpha = 45°$.

Answers to Multiple-Choice Questions

9.1	(b)	9.2	(a)	9.3	(d)	9.4	(a)	9.5	(a)	9.6	(d)	9.7	(b)
9.8	(a)	9.9	(a)	9.10	(b)	9.11	(a)	9.12	(d)	9.13	(a)	9.14	(a)
9.15	(a)	9.16	(c)	9.17	(c)	9.18	(b)	9.19	(b)	9.20	(a)	9.21	(b)
9.22	(a)	9.23	(d)	9.24	(a)	9.25	(c)	9.26	(b)	9.27	(c)	9.28	(c)
9.29	(b)	9.30	(d)	9.31	(c)	9.32	(a)	9.33	(b)	9.34	(b)	9.35	(a)
9.36	(a)	9.37	(a)	9.38	(d)	9.39	(a)	9.40	(a)	9.41	(a)	9.42	(a)

Answers to Fill in the Blanks

9.1 auto transformer
9.2 natural or line
9.3 ON-OFF control or Integral Cycle Control, Phase control
9.4 lighting, temperature, heating and speed control of induction motors
9.5 $\phi < \alpha < 180°$
9.6 static VAR compensation
9.7 unidirectional voltage controller
9.8 one-stage frequency changer
9.9 less
9.10 greater
9.11 180°
9.12 $V_o = V_m\left(\frac{m}{\pi}\right)\sin\left(\frac{\pi}{m}\right)\cos\alpha$
9.13 fixed, same
9.14 thyristor, diode
9.15 antiparallel
9.16 18
9.17 cycloconverter
9.18 8
9.19 4
9.20 6
9.21 36
9.22 $180° - \mu$
9.23 $V_{orms} = V\left(\frac{m}{\pi}\right)\sin\left(\frac{\pi}{m}\right)$

where V is the phase voltage

DC-TO-DC CONVERTERS (CHOPPERs)

10

10.1 INTRODUCTION

Variable voltage dc supply is widely used in electric tractions, trolley cars, golf carts, electric vehicles, switch mode power supply (SMPS), solar photovoltaic (PV) cell-based power generation etc. The variable voltage dc supply can be obtained either from fixed ac supply or from dc supply. The controlled rectifiers can be used to generate variable voltage dc supply from fixed as source. To obtain a variable dc voltage from a fixed dc supply, dc to dc converters can be used. Therefore, dc-to-dc converters can convert a fixed dc voltage into variable dc. This converter is also called a *chopper*. Generally, dc-to-dc converters are classified as

1. Step-down chopper
2. Step-up chopper

The basic operating principle of dc-to-dc converter can be represented by Fig. 10.1. Here, the input voltage is fixed dc voltage and a chopping of supply voltage is available at the output of dc to dc converter by the switching ON and OFF of any semiconductor switch. When the semiconductor switch is ON, output voltage is equal to input voltage. While the semiconductor switch is OFF, output voltage is zero. Hence, a pulse output voltage is applied to load and the average dc output voltage is controlled by controlling the ON and OFF times of switch. If the output voltage of a dc-to-dc converter is less than input voltage, this converter circuit is called as *step-down chopper*. Whenever the output voltage of a dc-to-dc converter is greater than input voltage, this converter circuit is called a *step-up chopper*.

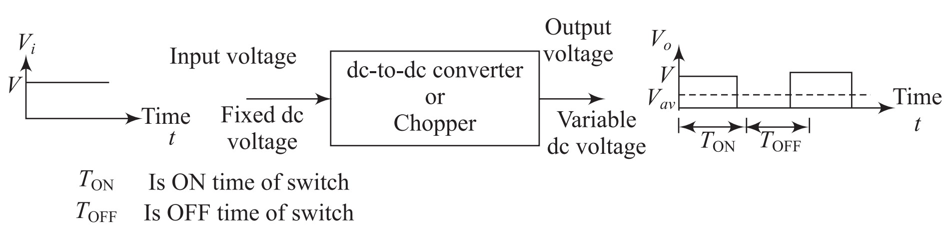


Fig. 10.1 *Block diagram of dc-to-dc converter*

The circuit configuration and operating principle of both step-down chopper and step-up chopper are explained in this chapter. The static semiconductor devices such as thyristors (SCRs), power BJTs, power MOSFETs and IGBTs are used in the chopper circuit. Usually, the dc-to-dc converters using thyristors are used for high power applications. The circuit configuration and operating principle of different choppers using thyristors (SCRs) are also discussed. The linear dc-to-dc converter using power BJ*T* are used in low power applications. Pulse width modulated dc-to-dc converters are used in medium level power applications. In this chapter, multiphase choppers and switch mode dc-to-dc converters such buck, boost, buck-boost and cuk converters are also discussed.

10.2 OPERATING PRINCIPLE OF dc-TO-dc CONVERTER

Figure 10.2 shows a circuit diagram of step-down chopper. This circuit consists of a switch (S), a diode (D), a R-L load. When the switch is ON, the output voltage is equal to input voltage. When switch is OFF, the power diode operates in the freewheeling mode and the inductor L filters the ripples in load current. This chopper circuit operates in two different modes such as

1. Mode – I during $0 \leq t \leq T_{ON}$ and
2. Mode – II during $T_{ON} \leq t \leq T$

In this circuit, input voltage is fixed and the average output voltage can be controlled by varying the ON time and OFF time of semiconductor switch. The switching devices may be thyristors (SCRs), power BJTs, power MOSFETs and IGBTs, etc.

Mode – I ($0 \leq t \leq T_{ON}$) The gating signal of semiconductor switch S is shown in Fig. 10.3. When the switch S is closed, the output load current I_o start to build up exponentially due to inductance L.

Mode – II ($T_{ON} \leq t \leq T$) During this mode, switch S is OFF, diode D is ON and provides a free wheeling path and current I_o starts to decrease.

The voltage and current waveforms V_i, V_o, I_o, and I_D are depicted in Fig. 10.3. The average output voltage is equal to

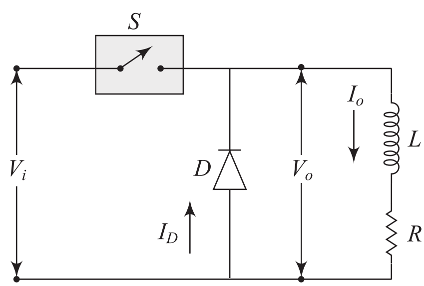


Fig. 10.2 *Step-down chopper circuit*

$$V_o = \frac{T_{ON}}{T_{ON} + T_{OFF}} V_i$$

where, T_{ON} is ON time period of the switch

T_{OFF} is OFF time period of the switch

V_i is the input voltage = V

or
$$V_o = \frac{T_{ON}}{T} V$$

$$= T_{ON} f V = DV \tag{10.1}$$

where, $T = T_{ON} + T_{OFF}$ is total time period

$f = \dfrac{1}{T}$ is frequency of switching or chopping frequency

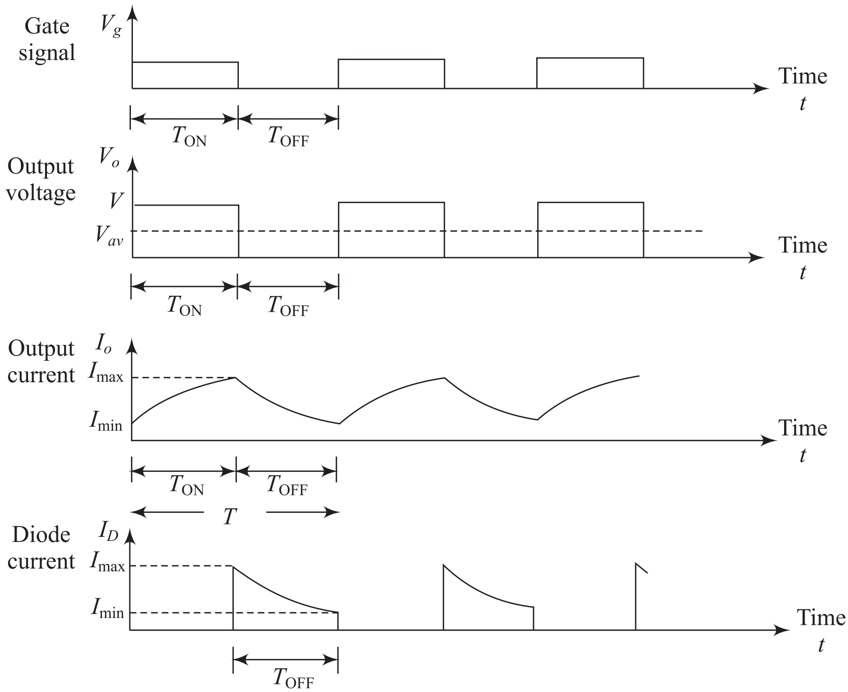


Fig. 10.3 *Voltage and current waveforms of step-down chopper circuit*

Average load current is

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = D \cdot \frac{V}{R}$$

The rms value of the output voltage is

$$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T v_o^2(t)dt\right]^{1/2} = \left[\frac{1}{T}\int_0^{T_{\text{ON}}} V^2 dt\right]^{1/2} = \sqrt{D \cdot V}$$

RMS load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{\sqrt{D}V}{R}$$

10.3 CONTROL STRATEGIES

From the Eq. (10.1), it is clear that the output voltage is directly proportional to the duty cycle. The output voltage of dc-to-dc converter can be controlled in two different ways such as

1. Time ratio control
2. Current limit control

10.3.1 Time Ratio Control

In the time ratio control method, the ON time period of the switch can be controlled keeping the total time period constant. This technique is known as *pulse width modulation* (PWM).The other method of time ratio control is that either T_{ON} or T_{OFF} is kept constant and the total time period T will be varied. This method is called as *frequency modulation*. Figure 10.4 shows the waveforms of PWM control and waveforms of frequency modulation technique is depicted in Fig. 10.5.

Pulse width modulation or constant frequency system In pulse width modulation control of chopper, the ON time of switch is varied but the total time period is constant. With the variation of ON time, the average output voltage can be varied as

$$V_{av} = \frac{T_{ON}}{T_{ON} + T_{OFF}} V = \frac{T_{ON}}{T} V \qquad \text{where, } T = T_{ON} + T_{OFF} \text{ is constant}$$

In Fig. 10.4(a), the ON time $T_{ON} = \frac{1}{2}T$ and duty ratio $D = \frac{T_{ON}}{T} = \frac{1}{2}$. Then the output voltage is equal to $V_{av} = \frac{T_{ON}}{T} V = \frac{V}{2}$.

In Fig. 10.4(b), the ON time $T_{ON} = \frac{1}{4}T$ and duty ratio $D = \frac{T_{ON}}{T} = \frac{1}{4}$. Subsequently, the output voltage is $V_{av} = \frac{T_{ON}}{T} V = \frac{V}{4}$. In this control scheme, duty cycle $D = \frac{T_{ON}}{T}$ can be varied from zero to unity. Accordingly, the output voltage can also be varied from zero to supply voltage V.

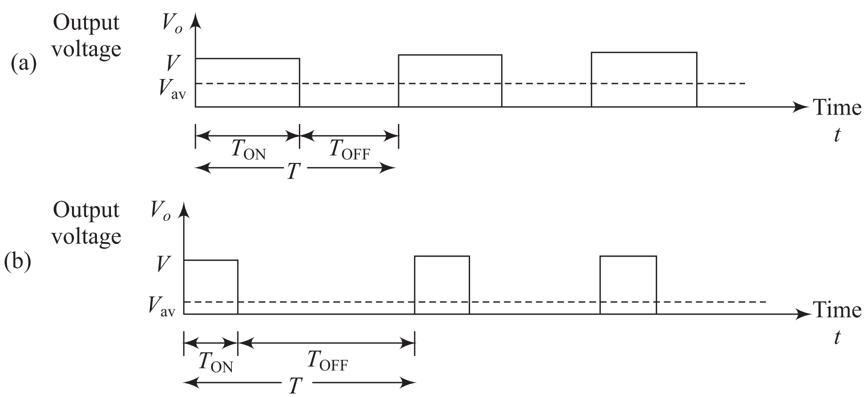


Fig. 10.4 *Pulse width modulation control of chopper*

Frequency modulation or variable frequency system In frequency modulation control of chopper, the chopping frequency f or the total time period T is varied. The frequency variation is possible in two ways such as

1. ON time T_{ON} is kept constant and OFF time T_{OFF} is variable as $f = \frac{1}{T} = \frac{1}{T_{ON} + T_{OFF}}$
2. OFF time T_{OFF} is kept constant and ON time T_{ON} is variable

When the ON time of switch is constant but the total time period is varied, the average output voltage is

$$V_{av} = \frac{T_{ON}}{T} V = T_{ON} f V \quad \text{where, } T = T_{ON} + T_{OFF} \text{ and } f \text{ are variable}$$

Figure 10.5 shows the frequency modulation of chopper with ON time constant $T_{ON.}$ In Fig. 10.5(a), the ON time $T_{ON} = \frac{1}{4}T$ and duty ratio $D = \frac{T_{ON}}{T} = \frac{1}{4}$. Then the output voltage is equal to $V_{av} = \frac{T_{ON}}{T}V = \frac{V}{4}$.

In Fig. 10.5(b), the ON time $T_{ON} = \frac{1}{2}T$ and duty ratio $D = \frac{T_{ON}}{T} = \frac{1}{2}$. Subsequently, the output voltage is $V_{av} = \frac{T_{ON}}{T}V = \frac{V}{2}$. In this control scheme, duty cycle $D = \frac{T_{ON}}{T}$ can be varied by varying total time period.

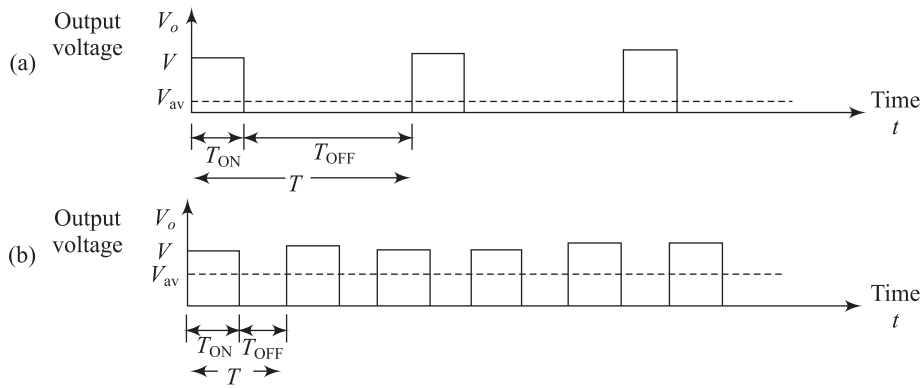


Fig. 10.5 *Frequency modulation of chopper with on-time constant T_{ON}*

Figure 10.6 shows the frequency modulation of chopper with OFF-time constant $T_{OFF.}$ When the OFF-time of switch is constant and ON-time of switch is variable, the total time period is also variable. Subsequently, the average output voltage is

$$V_{av} = \frac{T_{ON}}{T}V = T_{ON} fV \quad \text{where, } T = T_{ON} + T_{OFF} \text{ and } f \text{ are variable}$$

In Fig. 10.6(a), the on time $T_{ON} = \frac{1}{3}T$ and duty ratio $D = \frac{T_{ON}}{T} = \frac{1}{3}$. Then the output voltage is equal to $V_{av} = \frac{T_{ON}}{T}V = \frac{V}{3}$.

In Fig. 10.6(b), the ON time $T_{ON} = \frac{1}{2}T$ and duty ratio $D = \frac{T_{ON}}{T} = \frac{1}{2}$. Subsequently, the output voltage is $V_{av} = \frac{T_{ON}}{T}V = \frac{V}{2}$. Since duty cycle $D = \frac{T_{ON}}{T}$ can be varied from zero to unity, the output voltage can be varied from zero to the supply voltage V.

The frequency modulation of chopper has the following *disadvantages* over pulse width modulation of chopper:

1. Since the chopping frequency must be varied in wide range to control the output voltage in frequency modulation of chopper, the filter design is relatively difficult for wide frequency variation.

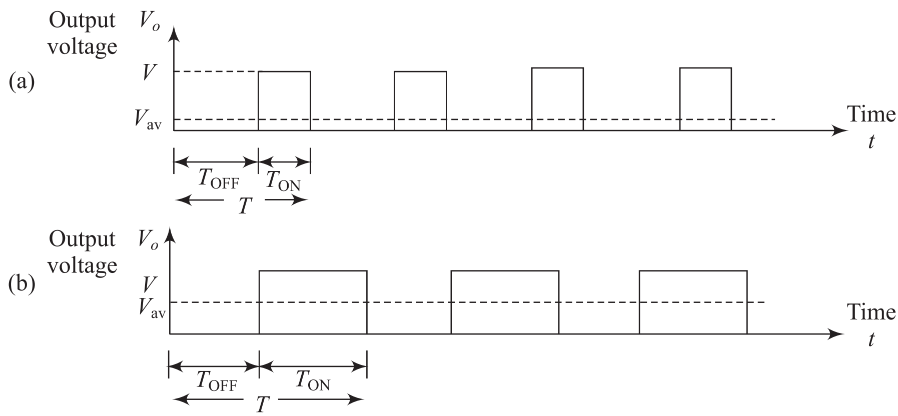


Fig. 10.6 *Frequency modulation of chopper with OFF-time constant T_{OFF}*

2. If the OFF-time of switch is large in frequency modulation of chopper, the load current will be discontinuous.
3. To control the duty ratio D, the frequency variation should be very wide. Consequently, the interference on telephone lines may be occurred.

10.3.2 Current Limit Control

In some dc-to-dc converters, it is necessary to keep the output load current constant. For this, the ON time period and OFF time period of dc chopper must be controlled in such a way that the switch is ON when I_o is less than I_{min} and the switch becomes OFF if I_o is greater than I_{max}. Hence the load current can be varied with in I_{min} and I_{max}.

Figure 10.7 shows the current limit control of chopper. The switching frequency of chopper circuit can be controlled by proper setting of maximum output current I_{max} and minimum output current I_{min}. The ripple current is equal to

$$\Delta I = I_{max} - I_{min}$$

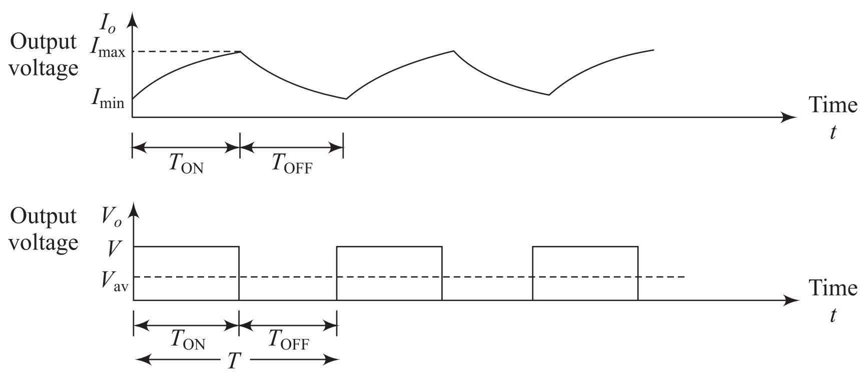


Fig. 10.7 *Current limit control of chopper*

To reduce the amplitude of ripple current, the switching frequency will be high and subsequently the switching loss will be high.

Example 10.1 A dc-to-dc converter as shown in Fig. 10.2 has a resistive load of 10 Ω and input voltage of 200 V. If the switching frequency is 1 kHz and duty cycle is 60%, determine

(a) average output voltage and current
(b) rms output voltage and current

Solution

Given: $V = 200$ V, $R = 10\ \Omega$, $f = 1$ kHz, $D = 60\% = 0.6$

(a) Average output voltage is $V_{av} = DV = 0.6 \times 200 = 120$ V
Average load current is

$$I_{av} = \frac{V_{av}}{R} = D \cdot \frac{V}{R} = 0.6 \times \frac{200}{10} = 12 \text{ A}$$

(b) rms output voltage is $V_{rms} = \sqrt{D}V = \sqrt{0.6} \times 200 = 154.9193$ V
rms load current is

$$I_{rms} = \frac{V_{rms}}{R} = \frac{154.9193}{10} = 15.491 \text{ A}$$

Example 10.2 In a step-down chopper, input voltage is 220 V and average output voltage is 140 V. If the switching frequency is 1 kHz, determine the on time and off time of switch in each cycle.

Solution

Given: $V = 220$ V, $V_{av} = 140$ V, $f = 1$ kHz
Average output voltage is

$$V_{av} = DV = \frac{T_{ON}}{T}V = T_{ON}fV$$

or, on time of switch $T_{ON} = \dfrac{V_{av}}{fV} = \dfrac{140}{1 \times 10^3 \times 220} \text{sec} = 0.6363 \text{ ms}$

OFF time of switch is $T_{OFF} = T - T_{ON} = \dfrac{1}{f} - T_{ON} = \dfrac{1}{1 \times 10^3} \text{sec} - 0.6363 \text{ ms} = 0.3637 \text{ ms}$

Example 10.3 A 100 V dc chopper operates using current limit control (CLC) strategy, the maximum value of load current is 250 A and the lower limit of current is 50 A. The ON-time and OFF-time of chopper are 20 ms and 30 ms respectively. Determine (a) the limit of current pulsation, (b) chopping frequency, (c) duty cycle and (d) output voltage.

Solution

Given: $V = 100$ V, $I_{max} = 250$ A, $I_{min} = 50$ A, $T_{ON} = 20$ ms and $T_{OFF} = 30$ ms

(a) The limit of current pulsation is $\Delta I = I_{max} - I_{min} = 250 - 50 = 200$ A

(b) Chopping frequency is $f = \dfrac{1}{T_{ON} + T_{OFF}} = \dfrac{1}{20 \text{ ms} + 30 \text{ ms}} = \dfrac{1}{50 \times 10^{-3}} = 20 \text{ Hz}$

(c) Duty cycle is $D = \dfrac{T_{ON}}{T_{ON} + T_{OFF}} = \dfrac{20 \text{ ms}}{20 \text{ ms} + 30 \text{ ms}} = 0.4$

(d) Output voltage is $V_o = DV = 0.4 \times 100 = 40$ V

Example 10.4 A chopper circuit as shown in Fig. 10.2 has input voltage of 220 V, and a resistive load of 10 Ω. When thyristor is used as a switch, the voltage across thyristor is about 2 V during on condition. If the duty cycle is 0.8, switching frequency is 1 kHz, determine (a) average output voltage (b) average output current, (c) rms output voltage, (d) rms output current, (e) average thyristor current, (f) rms thyristor current and (g) efficiency of chopper.

Solution

Given: $V = 220$ V, $R = 10\ \Omega$, $V_{S(\text{ON})} = 2$ V, $f = 1$ kHz, $D = 0.8$

(a) Average output voltage is

$$V_{\text{av}} = D(V - V_{S(\text{ON})}) = 0.8 \times (220 - 2) = 174.4 \text{ V}$$

(b) Average load current is

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = \frac{174.4}{10} = 17.44 \text{ A}$$

(c) rms output voltage is $V_{\text{rms}} = \sqrt{D}\,(V - V_{S(\text{ON})}) = \sqrt{0.8} \times (220 - 2) = 194.9851 \text{ V}$

(d) rms load current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{194.9851}{10} = 19.498 \text{ A}$$

(e) Average thyristor current is equal to average load current = 17.44 A

(f) rms thyristor current is equal to rms load current = 19.498 A

(g) Power output to load is

$$P_O = \frac{V_{\text{rms}}^2}{R} = \frac{(194.985)^2}{10} = 3801.91 \text{ Watt}$$

Power input to converter is $P_i = VI_{\text{av}} = 220 \times 17.44$ Watt $= 3836.8$ Watt

Efficiency of chopper is $\eta = \frac{P_O}{P_i} \times 100\% = \frac{3801.91}{3836.8} \times 100\% = 99.09\%$

Example 10.5 A step-down chopper has a load resistance of 20 Ω and input dc voltage is 200 V. When the chopper switch is ON, the voltage across semiconductor switch is 2 V. If the chopping frequency is 1.5 kHz and duty ratio is 40%, determine (a) average dc output voltage, (b) rms output voltage and (c) efficiency of chopper.

Solution

Given: $V = 200$ V, $R = 20\ \Omega$, $f = 1.5$ kHz, $\alpha = D = 40\% = 0.4$, $V_{S(\text{ON})} = 2$ V

(a) Average output voltage is

$$V_{\text{av}} = \alpha(V - V_{S(\text{ON})}) = D(V - V_{S(\text{ON})}) = 0.4(200 - 2) = 79.2 \text{ V}$$

(b) rms output voltage is

$$V_{\text{rms}} = \sqrt{\alpha}\,(V - V_{S(\text{ON})}) = \sqrt{D}\,(V - V_{S(\text{ON})}) = \sqrt{0.4}\,(200 - 2) = 125.22 \text{ V}$$

(c) Power output to resistive load is

$$P_o = \frac{V_{\text{rms}}^2}{R} = \frac{125.22^2}{20} = 784 \text{ Watt}$$

Average load current is $I_{\text{av}} = \frac{V_{\text{av}}}{R} = \frac{79.2}{20} = 3.96 \text{ A}$

Power input to chopper is

$$P_i = VI_{\text{av}} = 200 \times 3.96 = 792 \text{ Watt}$$

Efficiency of chopper is

$$\eta = \frac{P_o}{P_i} \times 100\% = \frac{784}{792} \times 100\% = 98.98\%$$

10.4 OPERATING PRINCIPLE OF STEP-UP CHOPPER

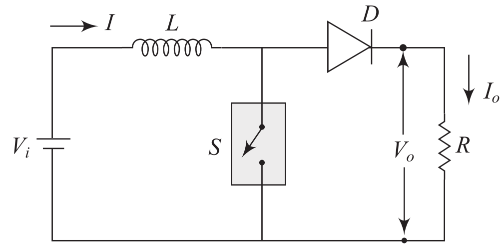


Fig. 10.8 *Step-up chopper*

Figure 10.8 shows a circuit diagram of step-up chopper which consists of a dc supply, inductance L, switch S, diode D and load. When the switch S is ON, a large inductance L is connected across the source voltage V. Then current flows through inductance and energy stored with in inductance during on-period of switch S. When the switch S is OFF, the current through inductance will not be zero instantaneously. Subsequently, this current starts to decrease and flows through diode D and load during off-period of switch S. As the current tends to decrease, the polarity of induced emf across L is reversed as shown in Figs. 10.9(a) and (b). Consequently, the output voltage across load is given by

$$V_o = V + L\frac{di}{dt} \qquad \text{and}$$

the energy stored in the inductor L is released to the load. The value of L is comparative high so that ripple current in the output should be less. Since the output voltage V_o is greater than the input voltage ($V_o > V$), this type of chopper circuit is called *step-up chopper*.

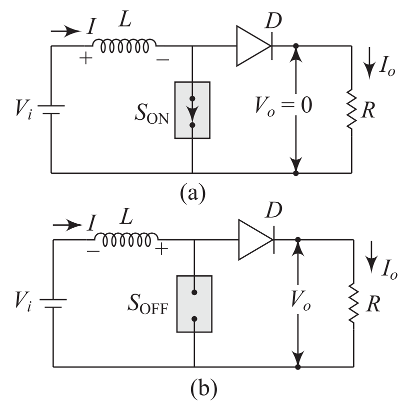


Fig. 10.9 *(a) Equivalent circuit of Fig. 10.8 when S is ON (b) Equivalent circuit of Fig. 10.8 when S is OFF*

When the switch S is ON, the current through inductance L increases from $I_{\min}$ to $I_{\max}$ and the energy input to inductor from source during T_{ON} time. If the output current varies linearly, the energy stored within the inductor is equal to

$$W_{\text{in}} = \text{Voltage across } L \times \text{Average current through } L \times T_{\text{ON}}$$

$$= V \times \left(\frac{I_{\min} + I_{\max}}{2}\right) \times T_{\text{ON}} = \left(\frac{I_{\min} + I_{\max}}{2}\right) V \cdot T_{\text{ON}}$$

When the switch S is OFF, the current though inductance L decreases from $I_{\max}$ to $I_{\min}$ and the stored energy in inductor L is released to load during T_{OFF} time. The voltage across L is $v_L = V_o - V$ as $v_L - V_o + V = 0$. Assume that the output current varies linearly; then the energy released by the inductor is equal to

$$W_{\text{out}} = \text{Voltage across } L \times \text{Average current through } L \times T_{\text{OFF}}$$

$$= (V_O - V) \times \left(\frac{I_{\min} + I_{\max}}{2}\right) \times T_{\text{OFF}} = \left(\frac{I_{\min} + I_{\max}}{2}\right)(V_O - V) \cdot T_{\text{OFF}}$$

If the system is lossless, the energy stored in the inductor W_{in} is equal to the energy released by the inductor W_{out}.

Therefore, $W_{\text{in}} = W_{\text{out}}$

or

$$\left(\frac{I_{\min} + I_{\max}}{2}\right) V \cdot T_{\text{ON}} = \left(\frac{I_{\min} + I_{\max}}{2}\right)(V_O - V) \cdot T_{\text{OFF}}$$

or $$V \cdot T_{ON} = (V_O - V) \cdot T_{OFF}$$

or $$V_O T_{OFF} = V(T_{ON} + T_{OFF}) \quad \text{as} \quad T_{ON} + T_{OFF} = T \text{ and } T_{OFF} = T - T_{ON}$$

or $$V_O = \frac{T}{T_{OFF}} V = \frac{T}{T - T_{ON}} V = \frac{1}{1 - D} V \text{ where, duty cycle } D = \frac{T_{ON}}{T}$$

If $D = 0$ and switch is always OFF, output voltage is $V_o = V$. When $D = 1$ and switch is always ON, output voltage $V_o = \propto$ as shown in Fig. 10.11. Practically duty cycle is greater than 0 and less than unity ($0 < D < 1$) and output voltage is always greater than input voltage V. Hence this circuit behaves as step-up chopper and used in regenerative breaking of dc motor. Figure 10.10 shows the voltage and current waveforms of step-up chopper.

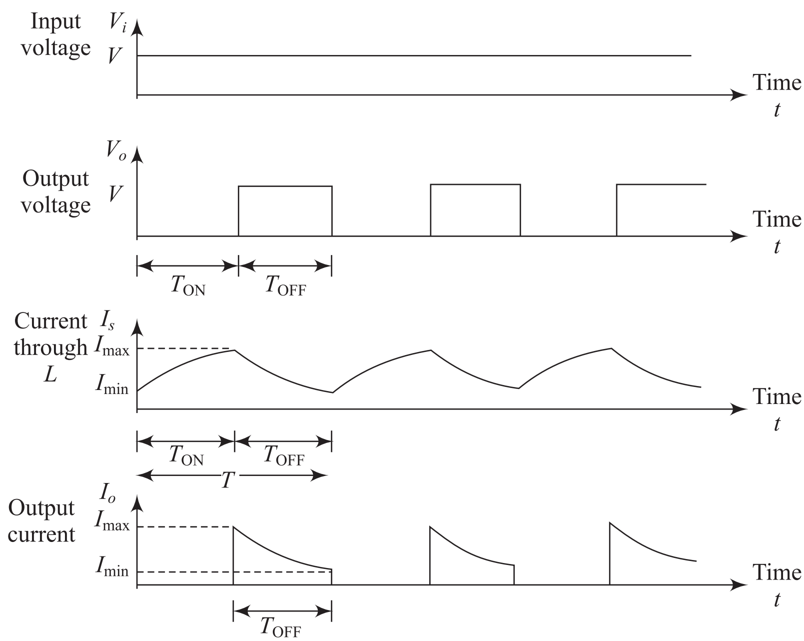


Fig. 10.10 *Voltage and current waveforms of step-up chopper*

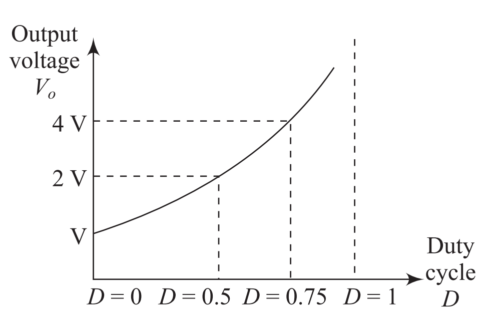


Fig. 10.11 *Variation of output voltage with Duty ratio (D)*

Example 10.6 A dc to dc step-up converter as shown in Fig. 10.8 has a resistive load of 10 Ω and input voltage of 220 V. If the switching frequency is 1 kHz and duty cycle is 50%, determine average output voltage and current.

Solution

Given: $V = 220$ V, $R = 10\ \Omega$, $f = 1$ kHz, $D = 50\% = 0.5$

Average output voltage is $V_O = \dfrac{1}{1-D}V = \dfrac{1}{1-0.5} \times 220 = 440$ V

Average load current is

$$I_O = \frac{V_O}{R} = \frac{440}{10} = 44 \text{ A}$$

Example 10.7 A step-up chopper has input voltage of 200 V and output voltage of 400 V. If the conduction time of switch is 150 μs, what is pulse width of output voltage? If the pulse width of output voltage become one fourth for constant frequency operation, compute the new average value of output voltage.

Solution

Given: $V = 220$ V, $V_O = 400$ V, $T_{\text{ON}} = 150$ μs

Average output voltage is

$$V_O = \frac{1}{1-D}V = \frac{1}{1-D} \times 200 = 400 \text{ V}$$

Therefore, $D = \dfrac{T_{\text{ON}}}{T} = 1 - \dfrac{200}{400} = 0.5$

Then $$T_{\text{ON}} = DT = 0.5T = 150\ \mu\text{s}$$

or $$T = \frac{T_{\text{ON}}}{D} = \frac{150}{0.5} = 300\ \mu\text{s}$$

The pulse width of output voltage is

$$T_{\text{OFF}} = T - T_{\text{ON}} = (300 - 150)\mu\text{s} = 150\ \mu\text{s}$$

When the pulse width of output voltage become one fourth for constant frequency operation, the OFF time is $T'_{\text{OFF}} = \dfrac{1}{4} \times T_{\text{OFF}} = \dfrac{1}{4} \times 150\ \mu\text{s} = 37.5\ \mu\text{s}$

Due to constant frequency operation, $T'_{\text{ON}} = T - T'_{\text{OFF}} = (300 - 37.7)\ \mu\text{s} = 262.5\ \mu\text{s}$

Therefore, new duty cycle $D' = \dfrac{T'_{\text{ON}}}{T} = \dfrac{262.5}{300} = 0.875$

The new average value of output voltage

$$V'_O = \frac{1}{1-D'}V = \frac{1}{1-0.875} \times 200 = 1600 \text{ V}$$

Example 10.8 A step-up chopper has input voltage of 210 V and output voltage of 550 V. (a) If the conduction time of switch is 100 μs, determine the pulse width of output voltage. (b) If the pulse width of output voltage becomes one half for constant frequency operation, find the new average value of output voltage.

Solution

Given: $V = 210$ V, $V_O = 550$ V, $T_{\text{ON}} = 100$ μs

(a) Average output voltage is

$$V_O = \frac{1}{1-D}V = \frac{1}{1-D} \times 210 = 550 \text{ V}$$

Therefore, $D = \frac{T_{\text{ON}}}{T} = 1 - \frac{210}{550} = 0.618$

Then $\quad T_{\text{ON}} = DT = 0.618T = 100\ \mu\text{s}$

or $\quad T = \frac{T_{\text{ON}}}{D} = \frac{100}{0.618} = 161.81\ \mu\text{s}$

The pulse width of output voltage is

$$T_{\text{OFF}} = T - T_{\text{ON}} = (161.81 - 100)\mu\text{s} = 61.81\ \mu\text{s}$$

(b) If the pulse width of output voltage becomes one half for constant frequency operation, the new off time is $T'_{\text{OFF}} = \frac{1}{2} \times T_{\text{OFF}} = \frac{1}{2} \times 61.81\ \mu\text{s} = 30.905\ \mu\text{s}$

As the chopper operate at constant frequency,

$$T'_{\text{ON}} = T - T'_{\text{OFF}} = (161.81 - 30.905)\mu\text{s} = 130.905\ \mu\text{s}$$

Hence, the new duty cycle is $D' = \frac{T'_{\text{ON}}}{T} = \frac{130.905}{161.81} = 0.809$

The new average value of output voltage is equal to

$$V'_O = \frac{1}{1-D'}V = \frac{1}{1-0.809} \times 210 = 1099.47 \text{ V}$$

10.5 CLASSIFICATION OF dc-TO-dc CONVERTERS OR CHOPPERS

dc-to-dc converters or choppers are classified according to input and output voltage levels, according to commutation methods, according to direction of output voltage and current.

1. According to input and output voltage levels, choppers are classified as
   (a) Step-down chopper
   (b) Step-up chopper
2. According to commutation methods, choppers are classified as
   (a) Voltage commutated chopper
   (b) Current commutated chopper
   (c) Load commutated chopper
3. Depending upon the direction of load current and voltage, dc to dc converters or choppers are classified as
   (a) Class A Chopper
   (b) Class B Chopper
   (c) Class C Chopper
   (d) Class D Chopper
   (e) Class E Chopper

10.5.1 Class A Chopper or First-Quadrant Chopper

Figure 10.12 shows a class A chopper. When the switch S is ON, output voltage V_o is equal to input voltage V and current flows through load as shown in Fig. 10.12. When the switch S is OFF, output voltage V_o is equal to zero but current flows through load in the same direction through free wheeling diode D. The average output voltage and current are always positive. Since both output voltage and current are positive, the chopper operates in the first quadrant as shown in Fig. 10.12(b). Therefore, this converter is called *first quadrant chopper*. Since the power is always flow from source to load and the output voltage is less than or equal to input voltage with $0 \le D \le 1$, this chopper is also called *step-down chopper*.

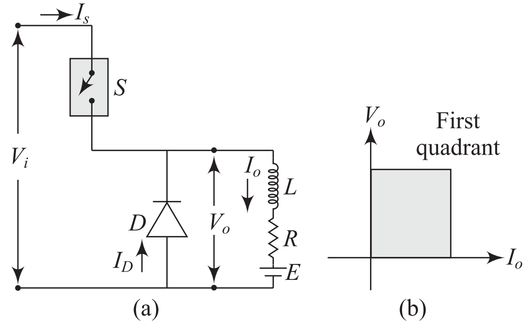


Fig. 10.12 *(a) Class A chopper (b) First quadrant operation of chopper*

10.5.2 Class B Chopper or Second-Quadrant Chopper

Figure 10.13 shows a class B chopper which consists of dc supply, diode D, switch S and a load RLE or dc motor. When the switch S is ON, output voltage V_o is equal to zero, but the current flows through inductance due to load voltage E. The energy is stored in the inductance L during T_{ON}. When the switch S is OFF, output voltage V_o is equal to $E + L\dfrac{di}{dt}$ which is greater than input voltage V. As the diode D is forward biased and starts to conduct, power will flow from load to source. Whenever switch S is ON or OFF, current flows out from load. Therefore current is negative. As output voltage is always positive and current is negative, the chopper operates in the second quadrant as shown in Fig. 10.13(b). So, this converter is known as *second quadrant chopper*. Since the output voltage is greater than input voltage, this type of chopper circuit is called *step-up chopper*.

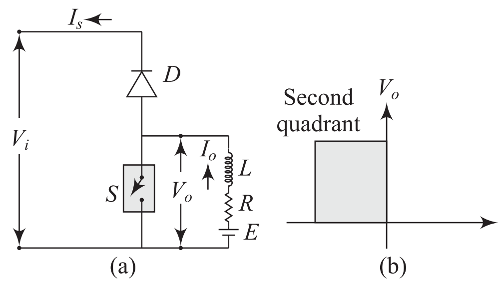


Fig. 10.13 *(a) Class B chopper (b) Second quadrant operation of chopper*

10.5.3 Class C Chopper or Two-Quadrant Type A Chopper

This type of chopper circuit is a combination of both class A and class B choppers. Figure 10.14 shows a class C chopper which consists of dc supply, two diodes D_1 and D_2, two switches S_1 and S_2 and a load RLE. Switch S_1 and diode D_1 act as class A chopper but Switch S_2 and diode D_2 act as class B chopper. The output voltage is always positive but the load current may be positive or negative. Therefore, this type of chopper circuit operates in both the first and second quadrant and is called *two quadrant* chopper.

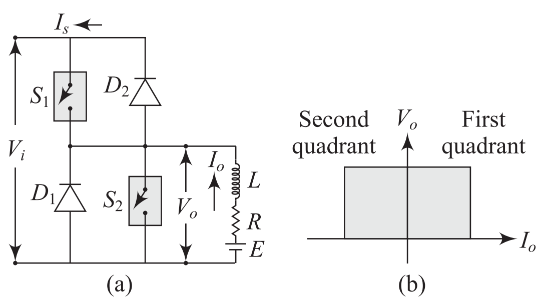


Fig. 10.14 *(a) Class C chopper (b) Two quadrant operation of chopper*

10.5.4 Class D Chopper or Two Quadrant Type B Chopper

Figure 10.15 shows a class D type chopper. When switches S_1 and S_2 are ON, the output voltage is equal to input voltage. When both switches S_1 and S_2 are OFF and diodes D_1 and D_1 are ON, the output voltage is equal to input voltage with reverse polarity. The average output voltage is positive when duty ratio is greater than 0.5 or turn-ON time of chopper is greater than turn-OFF time. But, the average output voltage is negative when duty ratio is less than 0.5 or turn-OFF time of chopper is greater than turn-ON time. The average output voltage can be expressed as

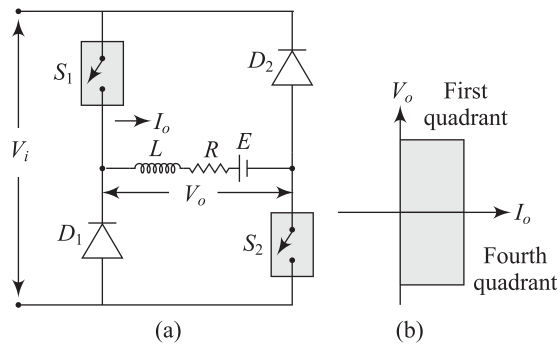


Fig. 10.15 *(a) Class D chopper and (b) Two quadrant operation of chopper*

$$V_o = \frac{T_{ON} - T_{OFF}}{T} V$$

If $T_{ON} = T_{OFF}$, duty ratio $D = \frac{1}{2} = 0.5$ and output voltage is zero.

When $T_{ON} > T_{OFF}$, duty ratio $D > \frac{1}{2} > 0.5$ and output voltage is positive.

If $T_{ON} < T_{OFF}$, duty ratio $D < \frac{1}{2} < 0.5$ and output voltage is negative.

Though the output voltage is either positive or negative depending on the value duty ratio, the load current is always positive. Therefore this type of chopper circuit is a dual converter and operates in both the first and fourth quadrant as depicted in Fig. 10.15(b).

Example 10.9 Draw the output voltage, output current, current through switches and source current waveforms for class D or two quadrant type B choppers when (i) $T_{ON} > T_{OFF}$ (ii) $T_{OFF} > T_{ON}$. Assume load is inductive.

Solution

Figure 10.16 shows the output voltage V_o, output current I_o, current through switches and source current I_S waveforms for class D or two quadrant type B choppers when $T_{ON} > T_{OFF}$.

Figure 10.17 shows the output voltage V_o, output current I_o, current through switches and source current I_S waveforms for class D or two quadrant type B choppers when $T_{OFF} > T_{ON}$.

10.5.5 Class E Chopper or Four Quadrant Chopper

Figure 10.18 shows a Class E chopper which consists of a dc supply, four switches S_1, S_2, S_3, S_4 and four diodes D_1, D_2, D_3, D_4 and *RLE* load. This chopper operates in four quadrants as explained below.

First Quadrant During first quadrant operation of chopper, switch S_1 will be operated when switch S_4 is kept ON, switch S_3 is OFF. When switch S_1 and S_4 are ON, the output voltage is equal to input voltage and load current flows from source to load. Since both the output voltage and current are positive, this chopper circuit operates in first quadrant. When the switch S_1 is OFF, current flows

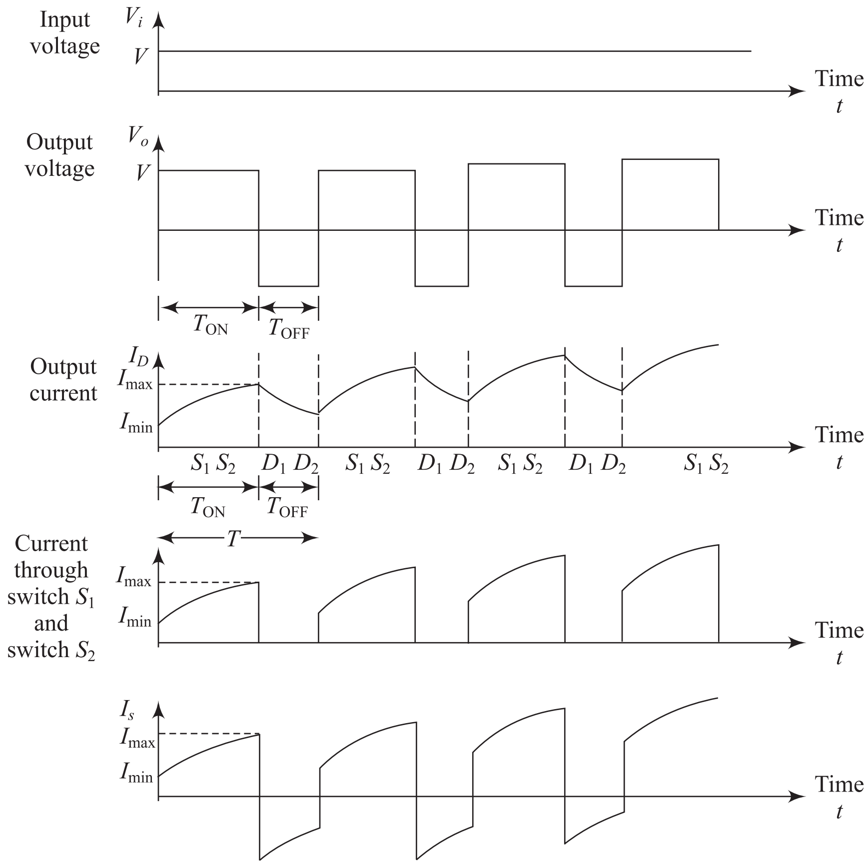


Fig. 10.16 *Voltage and current waveforms when $T_{ON} > T_{OFF}$*

through switch S_4, diode D_2 and load. Hence the type E chopper can be used as step-down chopper in the first quadrant operation.

Second quadrant In second quadrant operation of chopper, switch S_2 will be operated when switches S_1, S_3 and S_4 are kept OFF. When switch S_2 is ON, current flows through switch S_2, diode D_4, and load in reverse direction. Energy stores in the inductance during switch S_2 is ON. When switch S_2 is OFF, current flows from load to source through diodes D_1 and D_2 since $E + L\dfrac{di}{dt}$ is greater than input voltage V. As the output voltage is positive and current is negative, this chopper circuit operates in second quadrant. In the second quadrant operation, the type E chopper can be used as *step-up chopper*.

Third quadrant During third quadrant operation of chopper, switch S_3 will be operated when switch S_1 is kept OFF, switch S_2 is kept ON. The polarity of E will be reversed in this operating mode. When switch S_3 is ON, the output voltage is equal to input voltage with reverse polarity and load current is negative. Since both the output voltage and current are negative, this chopper circuit operates in third quadrant. When the switch S_3 is OFF, current flows through switch S_2, diode D_4 and load. Therefore, the type E chopper can be used as step-down chopper in the third quadrant operation.

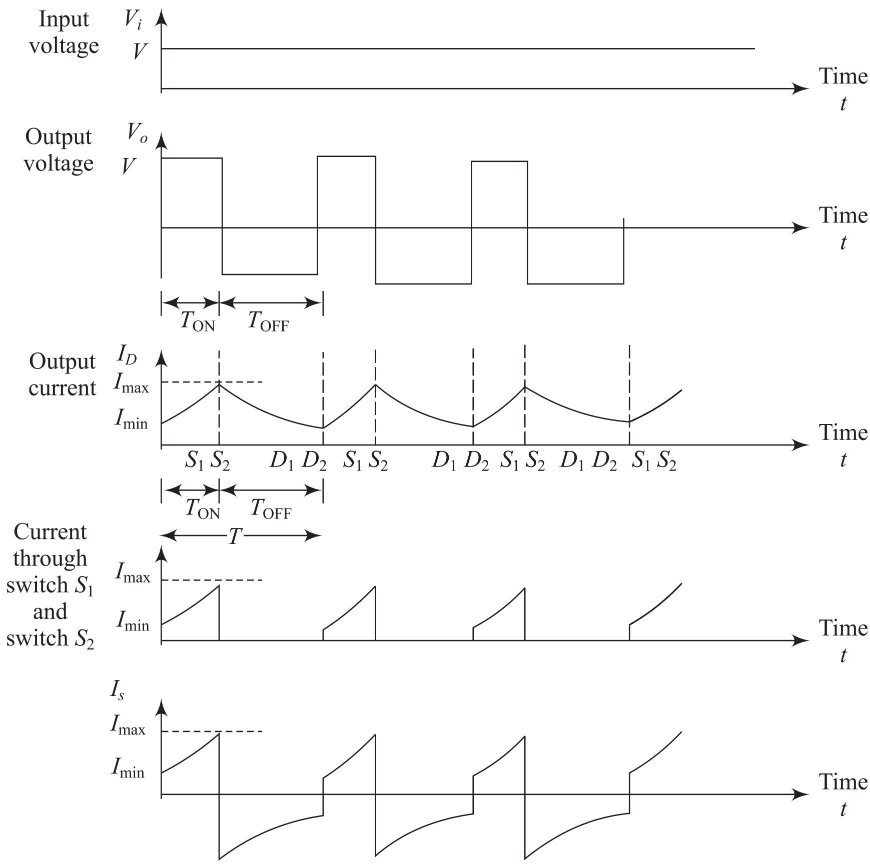
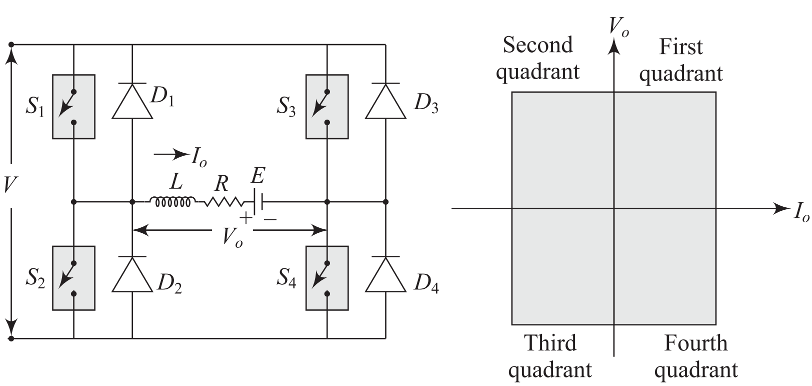


Fig. 10.17 *Voltage and current waveforms when $T_{OFF} > T_{ON}$*



(a) (b)

Fig. 10.18 *(a) Class E chopper and (b) Four quadrant operation of chopper*

Fourth quadrant In fourth quadrant operation of class –E chopper, switch S_4 will be operated when other switches S_1, S_2 and S_3 are kept OFF. The polarity of E is depicted in Fig. 10.18. When switch S_4 is ON, current flows through switch S_4, diode D_2, and load. Energy stores in the inductance during switch S_4 is ON. When switch S_4 is OFF, current flows from load to source through diodes D_2 and D_3. Since the output voltage is negative but current is positive, this chopper circuit operates in fourth quadrant. In the fourth quadrant operation, the type E chopper can be used as step-up chopper.

The conduction of semiconductor switching devices in the four quadrants is depicted in Fig. 10.19.

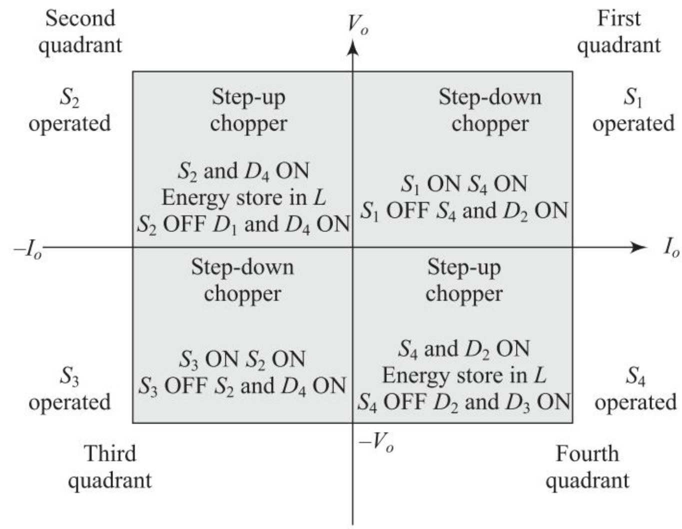


Fig. 10.19 *Operation of semiconductor switching devices in the four quadrants*

10.6 STEADY-STATE ANALYSIS OF STEP-DOWN CHOPPER

In a step-down or buck converter, the output voltage can be controlled from zero to maximum input voltage. This converter is also known as step-down converter. Figure 10.20 shows the block diagram of step-down converter. This circuit consists of a dc supply, a switch S, a diode D, inductor L and load.

When the power semiconductor switch is ON, input dc supply voltage is applied to the load. During this period, the inductor L stores energy. When the switch is OFF, free wheeling diode conducts and stored energy of inductance is transferred through free wheeling diode.

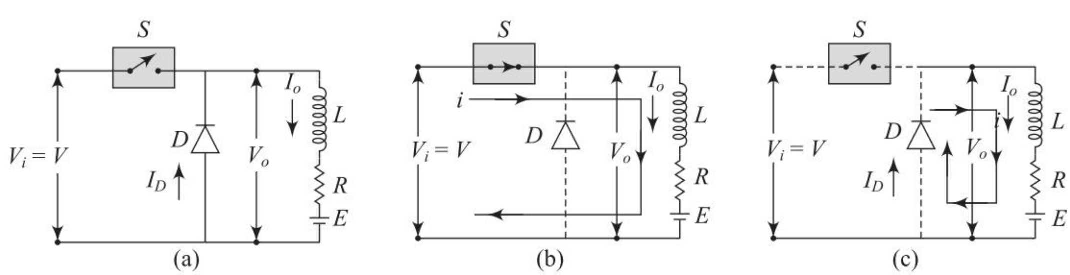


Fig. 10.20 *(a) Step-down chopper with RLE load (b) Equivalent circuit when S is ON (c) Equivalent circuit when S is OFF*

This circuit operates in two different modes such as

1. Continuous mode of operation
2. Discontinuous mode of operation

Continuous mode of operation In this mode of operation, the switch S will be turned ON before the inductor current becomes zero. Figure 10.21 shows the voltage and current waveforms.

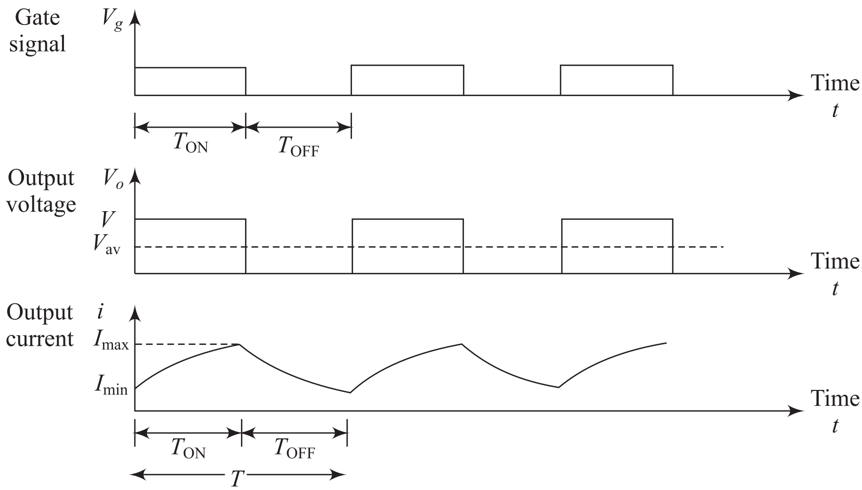


Fig. 10.21 *Voltage and current waveforms with continuous load current*

Discontinuous mode of operation The switch S will be turned ON after the inductor current becomes zero. The voltage and current waveforms are shown in Fig. 10.22.

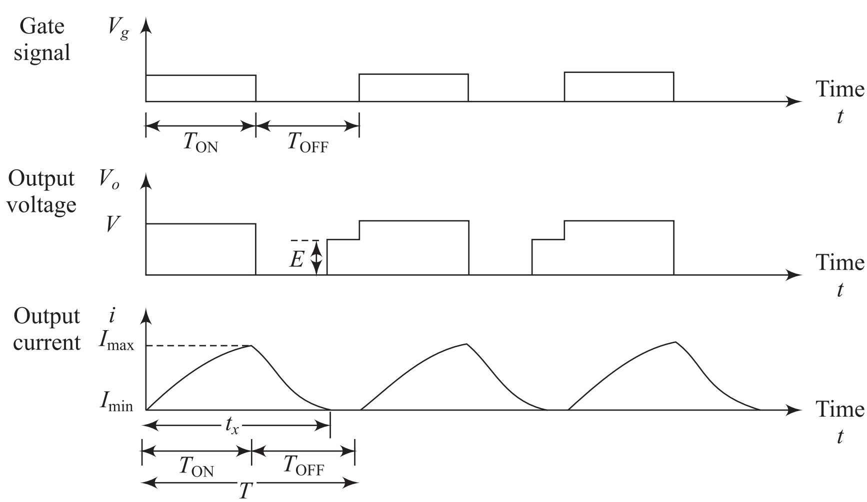


Fig. 10.22 *Voltage and current waveforms with discontinuous load current*

The switching frequency of the chopper will be about natural frequency ω_o of the LC filter, the output voltage may be higher than input voltage which will discussed in resonant converter.

Assumptions For analysis of this circuit, the following assumptions have been made:

1. The semiconductor switching device is ideal and switching loss is zero
2. The output voltage is constant as capacitance value is large
3. Rate of rise and fall of current through inductor is linear.

10.6.1 Continuous Mode of Operation

The average output voltage is

$$V_{\text{av}} = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{T}\left[\int_0^{T_{\text{ON}}} Vdt + \int_{T_{\text{ON}}}^T 0 \cdot dt\right]$$

$$= \frac{T_{\text{ON}}}{T} V = DV \quad \text{where, duty ratio } D = \frac{T_{\text{ON}}}{T}$$

Average load current is

$$I_{\text{av}} = \frac{V_{\text{av}}}{R} = D \cdot \frac{V}{R}$$

The rms value of the output voltage is

$$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T v_o^2(t)dt\right]^{1/2} = \left[\frac{1}{T}\int_0^{T_{\text{ON}}} V^2 dt\right]^{1/2} = \sqrt{D} \cdot V$$

The ripple factor is $RF = \dfrac{\sqrt{V_{\text{rms}}^2 - V_{\text{av}}^2}}{V_{\text{av}}}$

$$= \frac{\sqrt{DV^2 - D^2V^2}}{DV} = \frac{\sqrt{D - D^2}}{D}$$

The power input to the chopper is equal to

$$P_i = \frac{1}{T}\int_0^{T_{\text{ON}}} \frac{V^2}{R} dt = \frac{T_{\text{ON}}}{T}\frac{V^2}{R} = D\frac{V^2}{R}$$

The effective input resistance is

$$R_i = \frac{V}{I_{\text{av}}} = \frac{V}{D\dfrac{V}{R}} = \frac{R}{D}$$

Therefore, the effective input resistance of converter is variable as it is function of duty ratio D. The variation of input resistance with respect to duty ratio is illustrated in Fig. 10.23. When the switch is ON, the differential equation is

$$V = Ri + L\frac{di}{dt} = E$$

for $0 \le t \le T_{\text{ON}}$

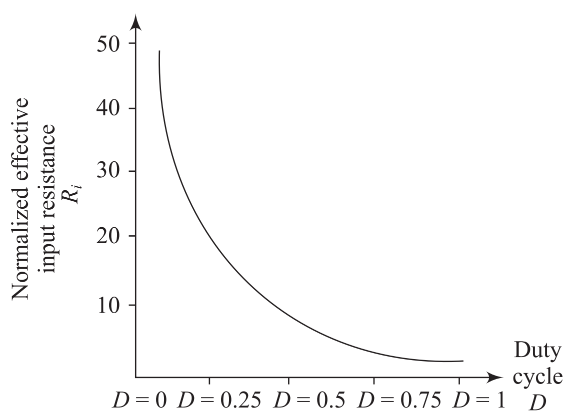


Fig. 10.23 *Variation of effective input resistance with duty ratio D*

At $t = 0$, the initial value of current is $I_{\min}$

After solving the differential equation, the load current can be expressed as

$$i(t) = I_{\min} e^{-\frac{R}{L}t} + \frac{V - E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad \text{for } 0 \le t \le T_{\text{ON}}$$

At $t = T_{\text{ON}}$, the value of load current is $I_{\max}$

$$I_{\max} = I_{\min} e^{-\frac{R}{L}T_{\text{ON}}} + \frac{V - E}{R}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right) \tag{10.2}$$

When the switch is OFF, the differential equation is

$$0 = Ri + L\frac{di}{dt} = E \;\; \text{for } T_{\text{ON}} \le t \le T$$

At $t = T_{\text{ON}}$, the initial value of current is $I_{\max}$

After solving the above differential equation, the load current can be expressed as

$$i(t') = I_{\max} e^{-\frac{R}{L}t'} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}t'}\right) \text{ where } t' = t - T_{\text{ON}} \text{ for } T_{\text{ON}} \le t \le T$$

At $t = T_{\text{ON}}$, $t' = t - T_{\text{ON}} = T_{\text{ON}} - T_{\text{ON}} = 0$,

At $t = T$, $t' = T - T_{\text{ON}} = T_{\text{ON}} - T_{\text{OFF}} - T_{\text{ON}} = T_{\text{OFF}}$, and the value of current is $I_{\min}$

Therefore $$I_{\min} = I_{\max} e^{-\frac{R}{L}T_{\text{OFF}}} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}T_{\text{OFF}}}\right) \tag{10.3}$$

Since $\frac{L}{R} = \tau$ is time constant, Eqs. (10.2) and (10.3) can be written as

$$\begin{aligned} I_{\max} &= I_{\min} e^{-\frac{T_{\text{ON}}}{\tau}} + \frac{V - E}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) \\ &= I_{\min} e^{-\frac{T_{\text{ON}}}{\tau}} + \frac{V}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) - \frac{E}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) \end{aligned} \tag{10.4}$$

$$\begin{aligned} I_{\min} &= I_{\max} e^{-\frac{T_{\text{OFF}}}{\tau}} - \frac{E}{R}\left(1 - e^{-\frac{T_{\text{OFF}}}{\tau}}\right) \\ &= I_{\max} e^{-\frac{T - T_{\text{ON}}}{\tau}} - \frac{E}{R}\left(1 - e^{-\frac{T - T_{\text{ON}}}{\tau}}\right) \end{aligned} \tag{10.5}$$

After substituting the value of $I_{\min}$ in Eq. (10.4), we obtain

$$\begin{aligned} I_{\max} &= I_{\min} e^{-\frac{T_{\text{ON}}}{\tau}} + \frac{V}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) - \frac{E}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) \\ &= I_{\max} e^{-\frac{T - T_{\text{ON}}}{\tau}} e^{-\frac{T_{\text{ON}}}{\tau}} - \frac{E}{R}\left(1 - e^{-\frac{T - T_{\text{ON}}}{\tau}}\right) e^{-\frac{T_{\text{ON}}}{\tau}} + \frac{V}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) - \frac{E}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) \end{aligned}$$

or $$I_{\max} - I_{\max} e^{-\frac{T}{\tau}} = -\frac{E}{R}\left(1 - e^{-\frac{T - T_{\text{ON}}}{\tau}}\right) e^{-\frac{T_{\text{ON}}}{\tau}} + \frac{V}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) - \frac{E}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right)$$

or $$I_{\max} - I_{\max} e^{-\frac{T}{\tau}} = -\frac{E}{R} e^{-\frac{T_{\text{ON}}}{\tau}} + \frac{E}{R} e^{-\frac{T}{\tau}} + \frac{V}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) - \frac{E}{R} + \frac{E}{R} e^{-\frac{T_{\text{ON}}}{\tau}}$$

or $$I_{\max}\left(1 - e^{-\frac{T}{\tau}}\right) = \frac{V}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) - \frac{E}{R} + \frac{E}{R} e^{-\frac{T}{\tau}}$$

or $$I_{\max}\left(1 - e^{-\frac{T}{\tau}}\right) = \frac{V}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) - \frac{E}{R}\left(1 - e^{-\frac{T}{\tau}}\right)$$

or $$I_{max} = \frac{V}{R}\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)} - \frac{E}{R}$$

After substituting the value of I_{max} in Eq. (10.5), we get

$$I_{min} = I_{max}e^{-\frac{T-T_{ON}}{\tau}} - \frac{E}{R}\left(1-e^{-\frac{T-T_{ON}}{\tau}}\right)$$

$$= \left(\frac{V}{R}\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)} - \frac{E}{R}\right)e^{-\frac{T-T_{ON}}{\tau}} - \frac{E}{R}\left(1-e^{-\frac{T-T_{ON}}{\tau}}\right)$$

$$= \frac{V}{R}\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)}\frac{e^{\frac{T_{ON}}{\tau}}}{e^{\frac{T}{\tau}}} - \frac{E}{R}e^{-\frac{T-T_{ON}}{\tau}} - \frac{E}{R} + \frac{E}{R}e^{-\frac{T-T_{ON}}{\tau}}$$

$$= \frac{V}{R}\frac{\left(e^{\frac{T_{ON}}{\tau}}-1\right)}{\left(e^{\frac{T}{\tau}}-1\right)} - \frac{E}{R}$$

When the switch S is continuously ON, the minimum current I_{min} is equal to I_{max}. The value of current is

$$I_{min} = I_{max} = \frac{V-E}{R}$$

Amplitude of ripple current at steady state At steady state condition, the amplitude of ripple current is

$$\Delta = I_{max} - I_{min} \tag{10.6}$$

After substituting the value of I_{max} and I_{min} in Eq. (10.6), we get

$$\Delta I = \frac{V}{R}\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)} - \frac{E}{R} - \frac{V}{R}\frac{\left(e^{\frac{T_{ON}}{\tau}}-1\right)}{\left(e^{\frac{T}{\tau}}-1\right)} + \frac{E}{R}$$

or $$\Delta I = \frac{V}{R}\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)} - \frac{V}{R}\frac{\left(e^{\frac{T_{ON}}{\tau}}-1\right)}{\left(e^{\frac{T}{\tau}}-1\right)} = \frac{V}{R}\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)} - \frac{V}{R}\frac{e^{\frac{T_{ON}}{\tau}}\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{e^{\frac{T}{\tau}}\left(1-e^{-\frac{T}{\tau}}\right)}$$

$$= \frac{V}{R}\left[\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)} - \frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)e^{\frac{T_{ON}-T}{\tau}}}{\left(1-e^{-\frac{T}{\tau}}\right)}\right] = \frac{V}{R}\left[\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)-\left(1-e^{-\frac{T_{ON}}{\tau}}\right)e^{\frac{T_{ON}-T}{\tau}}}{\left(1-e^{-\frac{T}{\tau}}\right)}\right]$$

$$= \frac{V}{R}\left[\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)\left(1-e^{\frac{T_{ON}-T}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)}\right] \tag{10.7}$$

It is clear from Eq. (10.7), the amplitude ripple current is independent of E.
Since $T_{ON} = DT$ and $T - T_{ON} = (1-D)T$, the ripple current is equal to

$$\Delta I = I_{max} - I_{min} = \frac{V}{R}\left[\frac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)\left(1-e^{\frac{T_{ON}-T}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)}\right] = \frac{V}{R}\left[\frac{\left(1-e^{-\frac{DT}{\tau}}\right)\left(1-e^{-\frac{(1-D)T}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)}\right]$$

The per unit ripple current is

$$\frac{\Delta I}{V/R}=\left[\frac{\left(1-e^{-\frac{DT}{\tau}}\right)\left(1-e^{-\frac{(1-DT)T}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)}\right]=\left[\frac{\left(1-e^{-\frac{DT}{\tau}}-e^{-\frac{(1-D)T}{\tau}}+e^{-\frac{T}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)}\right] \tag{10.8}$$

The ripple current is maximum, if the differentiation of Eq. (10.8) with respect to D is equal to zero.

Therefore, $$\frac{d\left(\frac{\Delta I}{V/R}\right)}{dD}=0=\left[\frac{\left(0-e^{-\frac{DT}{\tau}}\times\left(-\frac{T}{\tau}\right)-e^{-\frac{(1-D)T}{\tau}}\times\left(\frac{T}{\tau}\right)+0\right)}{\left(1-e^{-\frac{T}{\tau}}\right)}\right]$$

or $$\frac{T}{\tau}e^{-\frac{DT}{\tau}}-\frac{T}{\tau}e^{-\frac{(1-D)T}{\tau}}=0$$

Therefore, $e^{-\frac{DT}{\tau}}-e^{-\frac{(1-D)T}{\tau}}=0$ as $\frac{T}{\tau}\neq 0$

Then $\frac{DT}{\tau}=\frac{(1-D)T}{\tau}$ or, $D = 1 - D$ or, $D=\frac{1}{2}=0.5$

Hence the ripple in load current is maximum when duty cycle $D = 5$ as shown in Fig. 10.24.
The value of maximum ripple current at $D = 0.5$ is equal to

$$\Delta I_{\max}=\frac{V}{R}\left[\frac{\left(1-e^{-\frac{0.5T}{\tau}}\right)\left(1-e^{-\frac{0.5T}{\tau}}\right)}{\left(1-e^{-\frac{T}{t}}\right)}\right]=\frac{V}{R}\left[\frac{\left(1-e^{-\frac{0.5T}{\tau}}\right)\left(1-e^{-\frac{0.5T}{\tau}}\right)}{\left(1+e^{-\frac{0.5T}{\tau}}\right)\left(1-e^{-\frac{0.5T}{\tau}}\right)}\right]$$

$$=\frac{V}{R}\left[\frac{\left(1-e^{-\frac{0.5T}{\tau}}\right)}{\left(1+e^{-\frac{0.5T}{\tau}}\right)}\right]=\frac{V}{R}\tanh\frac{T}{4\tau}$$

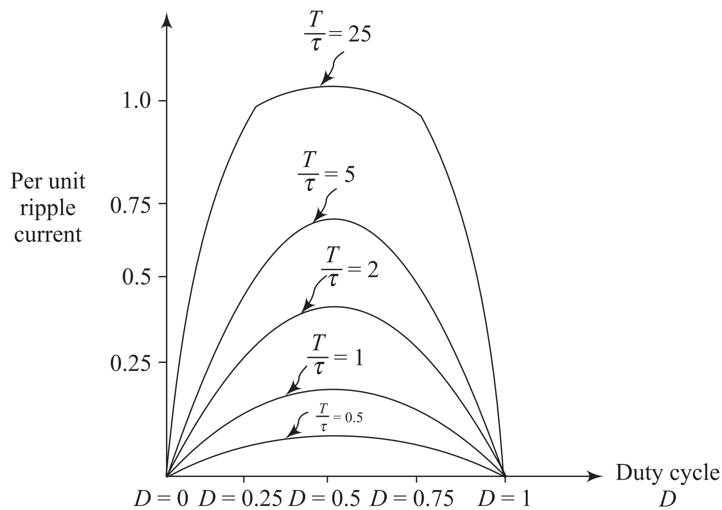


Fig. 10.24 *Per unit ripple current as a function of $\frac{T}{\tau}$ and D*

As $T=\frac{1}{f}$ and $\tau=\frac{L}{R}, \frac{T}{4\tau}=\frac{R}{4fL}$

Hence, $\Delta I_{max} = \frac{V}{R}\tanh\frac{T}{4\tau} = \frac{V}{R}\tanh\frac{R}{4fL}$

Since $4fL >> R$, $\tanh\frac{R}{4fL} \cong \frac{R}{4fL}$

Therefore, the maximum value of ripple current is

$$\Delta I_{max} = \frac{V}{R}\tanh\frac{R}{4fL} = \frac{V}{R}\times\frac{R}{4fL} = \frac{V}{4fL}$$

It is clear from above expression that the maximum value of ripple current is inversely proportional to the chopping frequency f and inductance L.

Limit of continuous conduction In step-down chopper, the ON-time T_{ON} is reduced and OFF-time T_{OFF} increases for constant chopping period T, the amplitude of load current reduces. If the ON-time is low and OFF-time is large, load current becomes zero for certain duration. Then the chopper circuit operates in discontinuous mode. The limit of continuous conduction is reached when $I_{min} = 0$. The value of duty cycle for continuous conduction is obtained from

$$I_{min} = \frac{V}{R}\frac{\left(e^{\frac{T_{ON}}{\tau}} - 1\right)}{\left(e^{\frac{T}{\tau}} - 1\right)} - \frac{E}{R} = 0$$

or
$$\frac{\left(e^{\frac{T_{ON}}{\tau}} - 1\right)}{\left(e^{\frac{T}{\tau}} - 1\right)} = \frac{E}{V} = m$$

or
$$e^{\frac{T_{ON}}{\tau}} - 1 = m\left(e^{\frac{T}{\tau}} - 1\right)$$

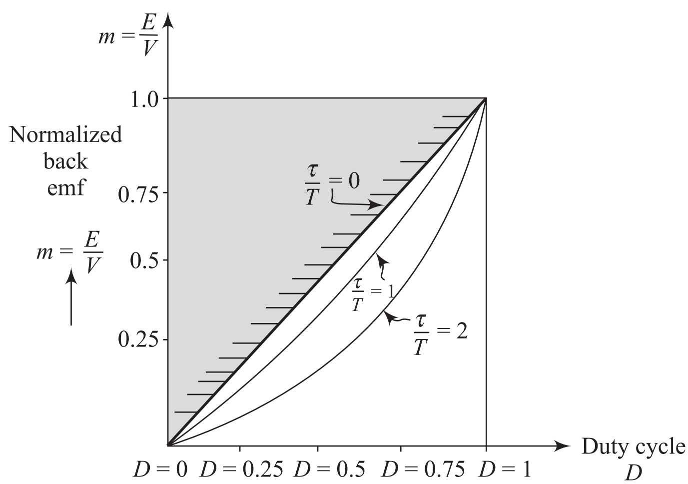


Fig. 10.25 *Limit of continuous conduction*

or
$$e^{\frac{T_{ON}}{\tau}} = 1 + m\left(e^{\frac{T}{\tau}} - 1\right)$$

or
$$T_{ON} = \tau \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right]$$

Then duty cycle $D' = \frac{T_{ON}}{T} = \frac{\tau}{T} \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right]$

If $\frac{\tau}{T} = 1$ and $m = 0.5$, $D' = \frac{\tau}{T} \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right] = 1 \cdot \ln[1 + 0.5(e^1 - 1)] = 0.6201$

If $\frac{\tau}{T} = 2$ and $m = 0.4$, $D' = \frac{\tau}{T} \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right] = 2 \cdot \ln[1 + 0.4(e^{1.5} - 1)] = 0.4614$

Fourier analysis of output voltage The output voltage is periodic in nature and it is independent of load circuit parameters. By using Fourier series, the output voltage can be expressed as

$$v_o = V_O + \sum_{n=1}^{\infty} v_n$$

where, V_O is the average output voltage and $V_O = \frac{T_{ON}}{T} V = DV$

And v_n is the value of n^{th} harmonic voltage and it can be expressed by

$$v_n = \frac{2V}{n\pi} \sin n\pi D \cdot \sin(n\omega t + \theta_n)$$

$$\text{where, } \theta_n = \tan^{-1} \frac{\sin 2\pi nD}{1 - \cos 2\pi nD} = \tan^{-1}\left(\frac{\cos \pi nD}{\sin \pi nD}\right)$$

The amplitude of harmonics voltages depends on the order of harmonics n and duty cycle D. The maximum value of n^{th} order harmonic voltage is equal to

$$v_n = \frac{2V}{n\pi} \text{ assuming } \sin n\pi D = 1$$

The rms value of harmonic voltage is

$$\frac{2V}{n\pi\sqrt{2}}$$

If Z_n is the load impedance at harmonic frequency nf and it's value is $Z_n = \sqrt{R^2 + (n\omega L)^2}$.
The harmonic current in the load is

$$i_n = \frac{v_n}{Z_n} = \frac{v_n}{\sqrt{R^2 + (n\omega L)^2}}$$

For negligible load resistance $R = 0$, the current $i_n = \frac{v_n}{n\omega L} = \frac{2V}{n^2 \pi\omega L}$ as $v_n = \frac{2V}{n\pi}$

Therefore, $i_n \propto \frac{V}{n^2}$

10.6.2 Discontinuous Mode of Operation

When the OFF time of switch is large, the load current will be discontinuous as it becomes zero at $t = t_x$. The time t_x is called *extinction time* which is measured from $t = 0$ as shown in Fig. 10.22. The value of t_x can be computed as given below.

When the switch is ON, the differential equation is

$$V = Ri + L\frac{di}{dt} + E \quad \text{for } 0 \le t \le T_{ON}$$

At $t = T_{ON}$, the initial value of current is I_{min}.

After solving the differential equation, the load current can be expressed as

$$i(t) = I_{\min} e^{-\frac{R}{L}t} + \frac{V-E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \text{ for } 0 \le t \le T_{\text{ON}}$$

Due to discontinuous conduction, the minimum value of current $I_{\min} = 0$. At $t = T_{\text{ON}}$, the value of load current is $I_{\max}$

$$I_{\max} = \frac{V-E}{R}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right) \tag{10.9}$$

When the switch is OFF, the differential equation is

$$0 = Ri + L\frac{di}{dt} + E \qquad \text{for } T_{\text{ON}} \le t \le T$$

At $t = T_{\text{ON}}$, the initial value of current is $I_{\max}$

After solving the above differential equation, the load current can be expressed as

$$i(t') = I_{\max} e^{-\frac{R}{L}t'} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}t'}\right) \text{ where, } t' = t - T_{\text{ON}} \text{ and } T_{\text{ON}} \le t \le T$$

At $t = t_x$, $t' = t_x - T_{\text{ON}}$ and $i(t') = 0$

Therefore, $i(t') = 0 = I_{\max} e^{-\frac{R}{L}(t_x - T_{\text{ON}})} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}(t_x - T_{\text{ON}})}\right)$

After substituting the value of $I_{\max} = \frac{V-E}{R}\left(1 - e^{-\frac{R}{L}T_{ON}}\right)$ in the above equation we obtain

$$0 = \frac{V-E}{R}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right) e^{-\frac{R}{L}(t_x - T_{\text{ON}})} - \frac{E}{R}\left(1 - e^{-\frac{R}{L}(t_x - T_{\text{ON}})}\right)$$

or $$E\left(1 - e^{-\frac{R}{L}(t_x - T_{\text{ON}})}\right) e^{\frac{R}{L}(t_x - T_{\text{ON}})} = (V-E)\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)$$

or $$e^{\frac{R}{L}(t_x - T_{\text{ON}})} - 1 = \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)$$

or $$e^{\frac{R}{L}(t_x - T_{\text{ON}})} = 1 + \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)$$

After taking log on both sides, we get

or $$\frac{R}{L}(t_x - T_{\text{ON}}) = \ln\left[1 + \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)\right]$$

Therefore, $$t_x = T_{\text{ON}} + \tau_a \ln\left[1 + \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)\right] \quad \text{as } \tau_a = \frac{L}{R}$$

The average value of output voltage during discontinuous load current as shown in Fig. 10.22 is equal to

$$\begin{aligned} V_{\text{av}} &= \frac{1}{T}\int_0^T v_o(t)dt \\ &= \frac{1}{T}\left[\int_0^{T_{\text{ON}}} Vdt + \int_{T_{\text{ON}}}^{t_x} 0 \cdot dt + \int_{t_x}^{T} Edt\right] \\ &= \frac{T_{\text{ON}}}{T}V + \frac{T - t_x}{T}E \end{aligned}$$

or $$V_{av} = DV + \left(1 - \frac{t_x}{T}\right)E \text{ where, duty ratio } D = \frac{T_{ON}}{T}$$

Example 10.10 A dc-to-dc step-down converter circuit with R load has input voltage of 200 V, and a resistive load of 5 Ω. If the duty cycle is 40%, switching frequency is 1 kHz, determine (a) average output voltage, (b) rms output voltage, and (c) ripple factor

Solution

Given: $V = 200$ V, $R = 5\ \Omega$, $f = 1$ kHz, $D = 40\% = 0.4$

(a) Average output voltage is

$$V_{av} = DV = 0.4 \times 200 = 80 \text{ V}$$

(b) RMS output voltage is $V_{rms} = \sqrt{D}V = \sqrt{0.4} \times 200 = 126.491$ V

(c) The ripple factor is

$$RF = \frac{\sqrt{V_{rms}^2 - V_{av}^2}}{V_{av}} = \frac{\sqrt{D - D^2}}{D} = \frac{\sqrt{0.4 - 0.4^2}}{0.4} = 1.2247$$

Example 10.11 Prove that the critical inductance of the filter in a step-down chopper circuit is $L = \dfrac{V_o^2(V - V_o)}{2fVP_o}$ where, V_o is output voltage, V is input voltage, P_o is output power and f is chopping frequency.

Solution

For the critical inductance L of the filter, load current starts from 0 when the semiconductor switch is ON for T_{ON} duration and it reaches at a maximum value I_{max}. When the switch is OFF for T_{OFF} duration, the load current falls from I_{max} to 0 at $t = T$.

As the load current varies from 0 to I_{max} during T_{ON} and current falls from I_{max} to zero during T_{OFF} and the rate of increase or decrease is linear, the average value of output load current is equal to

$$I_o(T_{ON} + T_{OFF}) = I_oT = \frac{1}{2}I_{max}T_{ON} + \frac{1}{2}I_{max}T_{OFF} = \frac{1}{2}I_{max}(T_{ON} + T_{OFF}) = \frac{1}{2}I_{max}T$$

Therefore, $I_oT = \frac{1}{2}I_{max}T$ or, $I_{max} = 2I_o$

Since filter inductor L is connected in series with load, the input voltage is equal to

$$V = V_o + L\frac{di}{dt}$$

or $$V = V_o + L\frac{I_{max}}{T_{ON}}$$

or $$V = V_o + L\frac{2I_o}{T_{ON}} \text{ as } I_{max} = 2I_o$$

Therefore, $$L = \frac{V - V_o}{2I_o}T_{ON} \tag{i}$$

Since the average output voltage is $V_o = VfT_{ON}$ and output power is equal to $P_o = V_oI_o$,

$$T_{ON} = \frac{V_o}{Vf} \quad \text{and} \quad I_o = \frac{P_o}{V_o}$$

After substituting the value of T_{ON} and I_o in the above equation, we obtain

$$L = \frac{V_o^2(V - V_o)}{2fVP_o}$$

Example 10.12 Prove that the critical inductance in the load circuit of a step-down chopper is directly proportional to $x(1 - x)$ where x is the duty cycle.

Solution

The critical inductance of a step-down chopper is

$$L = \frac{V - V_o}{2I_o} T_{ON} \tag{ii}$$

We know that $T_{ON} = DT$ and $V_o = DV$

After substituting the value of T_{ON} and V_o in Eq. (ii), we obtain

$$L = \frac{V - V_o}{2I_o} T_{ON} = \frac{V - DV}{2I_o} \times DT = \frac{V(1 - D)DT}{2I_o}$$

$$= \frac{V(1 - x)xT}{2I_o} = \frac{VT}{2I_o}(1 - x)x \quad \text{as } D = x \text{ is the duty cycle}$$

or $\quad L \propto (1 - x)x$

Therefore, the critical inductance is directly proportional to $x(1 - x)$ as $\dfrac{VT}{2I_o}$ is constant.

Example 10.13 A dc-to-dc converter is connected to a 150 V dc source with an inductive load $R = 10\ \Omega$ and $L = 10$ mH. A free wheeling diode is also connected across load. Assume the load current varies from 10 A to 15 A. Find the time ratio of dc to dc converter.

Solution

Given: $V = 150$ V, $I_{max} = 15$ A, $I_{min} = 10$ A, $R = 10\ \Omega$

Average value of load current is $I_O = \dfrac{I_{max} + I_{min}}{2} = \dfrac{15 + 10}{2} = 12.5$ A

The maximum value of load current $= \dfrac{V}{R} = \dfrac{150}{10} = 15$ A

Average value of output voltage is

$$V_{av} = \frac{V \times I_O}{\text{Maximum value of load current}} = \frac{150 \times 12.5}{15} = 125 \text{ V}$$

We know that $V_{av} = \dfrac{T_{ON}}{T} V$

Then $\quad \dfrac{T_{ON}}{T} = \dfrac{V_{av}}{V} = \dfrac{125}{150} = 0.8333$

$$\frac{T_{ON}}{T} = \frac{T_{ON}}{T_{ON} + T_{OFF}} = \frac{y}{y + 1} = 0.8333 \quad \text{as the time ratio of dc to dc converter is } y = \frac{T_{ON}}{T_{OFF}}$$

or $\quad y = \dfrac{0.8333}{1 - 0.8333} = 4.998$

Example 10.14 A step-down chopper with *RLE* load as shown in Fig. 10.20 has continuous constant load current. Determine the maximum value of average thyristor current rating. Assume that switch S is thyristor.

Solution

Average output voltage is $V_{av} = DV$

As the load current is continuous and constant, the value of load current is

$$I_O = \frac{V_{av} - E}{R}$$

The current flow through thyristor is shown in Fig. 10.26. The average value of thyristor current is equal to

$$I_T = \frac{T_{ON}}{T} I_O = D\frac{V_{av} - E}{R} = D \cdot \frac{DV - E}{R} = \frac{D^2V - DE}{R} \tag{iii}$$

For maximum value of $I_T, \frac{dI_T}{dD} = 0$

Therefore, $\frac{dI_T}{dD} = \frac{2DV - E}{R} = 0$

Then duty ratio is $D = \frac{E}{2V}$

After substituting the value of D in Eq. (iii), we get

$$I_T = \frac{D^2V - DE}{R} = \frac{1}{R}\left[\left(\frac{E}{2V}\right)^2 V - \left(\frac{E}{2V}\right)E\right] = \frac{E^2}{4VR}$$

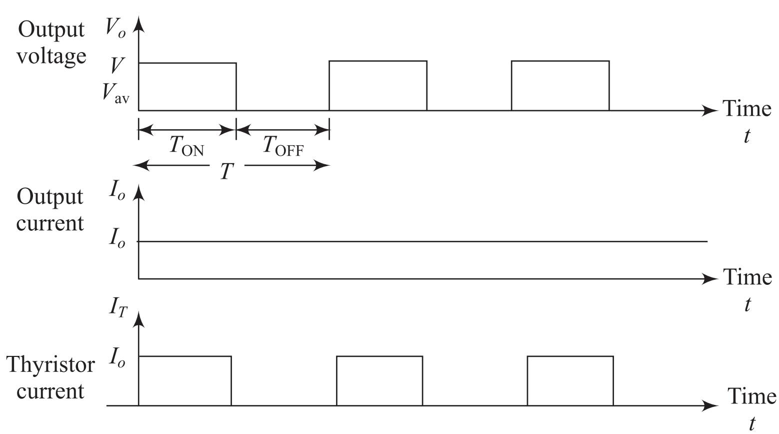


Fig. 10.26 *Waveforms of V_o, I_o and I_T*

Example 10.15 A step-down chopper with *RLE* load as shown in Fig. 10.20 has input voltage of 200 V, R = 2 ohms, L = 10 mH, E = 50 V, T_{ON} = 500 μs, T = 1000 μs.

(a) Determine whether load current is continuous or discontinuous,
(b) Find the value of average load current,
(c) Calculate the maximum and minimum value of load current,
(d) Draw the load current, free wheeling diode current and current through switch and
(e) Ripple current

Solution

Given: $V = 200$ V, $R = 2$ ohms, $L = 10$ mH, $E = 50$ V, $T_{ON} = 500$ µs, $T = 1000$ µs.

$$\tau_a = \frac{L}{R} = \frac{10\times 10^{-3}}{2} = 5 \text{ ms} \quad \text{and} \quad m = \frac{E}{V} = \frac{50}{200} = 0.25 \text{, duty cycle } D = \frac{T_{\text{ON}}}{T} = \frac{500\ \mu\text{s}}{1000\ \mu\text{s}} = 0.5$$

$$\frac{T_{\text{ON}}}{\tau} = \frac{500\ \mu\text{s}}{5\text{ ms}} = \frac{500\times 10^{-6}}{5\times 10^{-3}} = 0.1 \text{ and } \frac{T}{\tau} = \frac{1000\ \mu\text{s}}{5\text{ ms}} = \frac{1000\times 10^{-6}}{5\times 10^{-3}} = 0.2$$

(a) Duty cycle $D' = \frac{\tau}{T}\cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right] = \frac{5\times 10^{-3}}{1000\times 10^{-6}}\cdot \ln\left[1 + 0.25\left(e^{\frac{1000\times 10^{-6}}{5\times 10^{-3}}} - 1\right)\right] = 0.2693$

Actual duty cycle $D = 0.5$ is greater than $D' = 0.2693$, load current is continuous.

(b) The value of average load current is

$$I_o = \frac{V_{\text{av}} - E}{R} = \frac{DV - E}{R} = \frac{0.5\times 200 - 50}{2} = 25 \text{ A}$$

(c) The maximum value of load current is

$$I_{\max} = \frac{V}{R}\frac{\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right)}{\left(1 - e^{-\frac{T}{\tau}}\right)} - \frac{E}{R} = \frac{200}{2}\frac{(1 - e^{-0.1})}{(1 - e^{-0.2})} - \frac{50}{2} = 27.5096 \text{ A}$$

The minimum value of load current is

$$I_{\min} = \frac{V}{R}\frac{\left(e^{\frac{T_{\text{ON}}}{\tau}} - 1\right)}{\left(e^{\frac{T}{\tau}} - 1\right)} - \frac{E}{R} = \frac{200}{2}\frac{(e^{0.1} - 1)}{(e^{0.2} - 1)} - \frac{50}{2} = 22.4998 \text{ A}$$

(d) The load current, free wheeling diode current and current through switch are depicted in Fig. 10.27

(e) The ripple current is equal to

$$\Delta I = I_{\max} - I_{\min} = 27.5096 - 22.4998 = 5.0098 \text{ A}$$

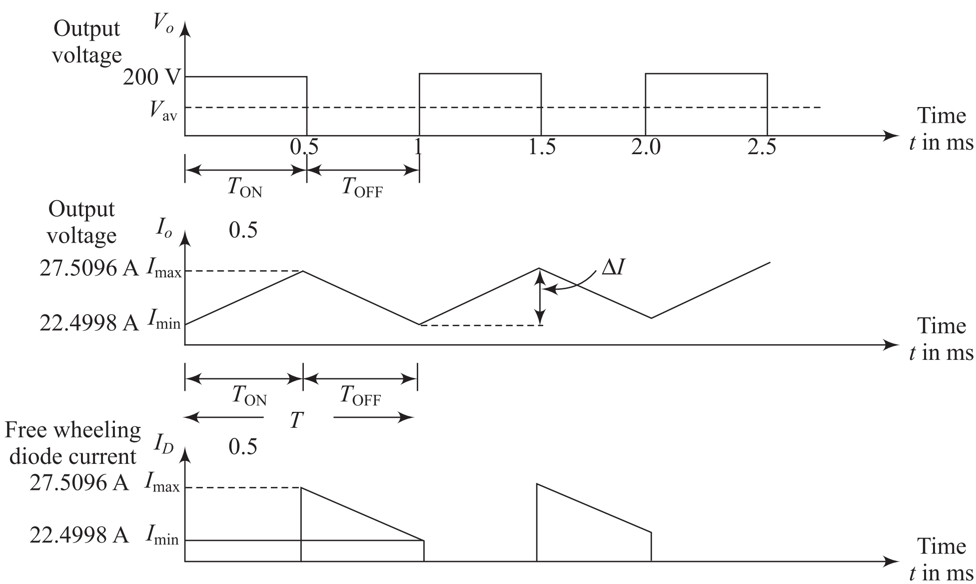


Fig. 10.27

Example 10.16 Figure 10.28 shows a type A chopper with LE load. This chopper circuit has input voltage of 400 V, $L = 0.05$ H and constant E. When the duty cycle is 0.4, determine the chopping frequency to limit the load ripple current to 15 A.

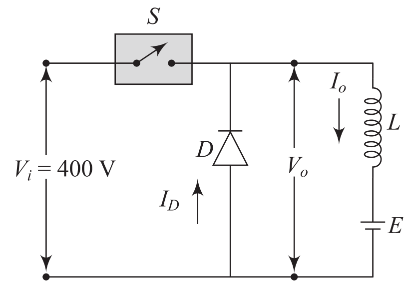


Fig. 10.28

Solution

Given: $V = 400$ V, $L = 0.05$ H, $I_{max} = 15$ A, and $D = 0.4$

Figure 10.29 shows the output voltage and current waveform.

The average output voltage is

$$V_{av} = DV = 0.4 \times 400 = 160 \text{ V}$$

Since average value of voltage drop across L is zero,

$$E = V_{av} = DV = 160 \text{ V}$$

When the switch is ON, the voltage across inductance L is $(V - E)$

The volt time area applied to inductance L during T_{ON} is equal to

$$(V - E)T_{ON} = (400 - 160)T_{ON} \tag{i}$$

During $0 \le t \le T_{ON}$, the current through inductance L increases from I_{min} to I_{max} and the volt-time area across L is equal to

$$\int_0^{T_{ON}} v_L dt = \int_0^{T_{ON}} L\frac{di}{dt}dt = \int_{I_{min}}^{I_{max}} Ldi = L(I_{max} - I_{min}) = L \times \Delta I \quad \text{(ii) as } (I_{max} - I_{min}) = \Delta I$$

The value of Eq. (i) should be equal to the value of Eq. (ii)

Therefore, $(400 - 160)T_{ON} = L\Delta I$

or
$$T_{ON} = \frac{L\Delta I}{400 - 160} = \frac{0.05 \times 15}{240}\text{sec} = 3.125 \text{ ms}$$

Duty ratio $D = \dfrac{T_{ON}}{T}$ and $T = \dfrac{T_{ON}}{D}$

The frequency of chopper is

$$f = \frac{1}{T} = \frac{D}{T_{ON}} = \frac{0.4}{3.125 \times 10^{-3}} = 128 \text{ Hz}$$

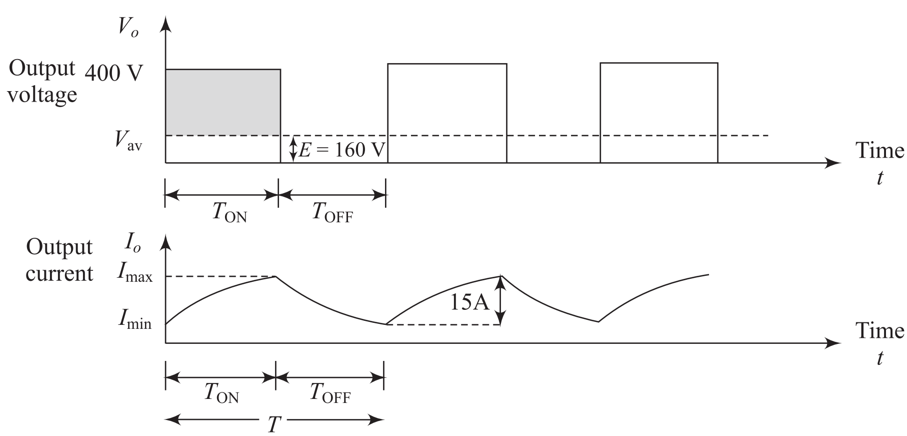


Fig. 10.29

Example 10.17 A type A chopper feed to an *RLE* load. Prove that the maximum value of rms current rating of freewheeling diode is $0.385\frac{V}{R}\left(1-\frac{E}{V}\right)^{3/2}$ when load current is ripple free.

Solution

If the duty cycle of chopper is D, the output voltage is $V_o = V_{av} = DV = aV$ as $D = \alpha$.

The load current is $I_o = \frac{V_o - E}{R} = \frac{\alpha V - E}{R}$

When the chopper switch is in OFF state, the load current flows through freewheeling diode. The rms value of freewheeling diode current is equal to

$$I_{D_F} = \sqrt{\frac{T_{\text{OFF}}}{T}} I_o = \left(1 - \frac{T_{\text{ON}}}{T}\right)^{1/2} I_o = (1-\alpha)^{1/2}\frac{\alpha V - E}{R} \qquad \text{(i)}$$

$$= \frac{V}{R}\alpha(1-\alpha)^{1/2} - \frac{E}{R}(1-\alpha)^{1/2}$$

$$= \frac{V}{R}(\alpha^2-\alpha^3)^{1/2} - \frac{E}{R}(1-\alpha)^{1/2}$$

The rms value of freewheeling diode current is maximum when $\frac{dI_{D_F}}{d\alpha} = 0$

Therefore, $\frac{dI_{D_F}}{d\alpha} = 0 = \frac{V}{R}\frac{1}{2}\frac{2\alpha - 3\alpha^2}{\sqrt{\alpha^2-\alpha^3}} + \frac{E}{R}\frac{1}{2}\frac{1}{\sqrt{1-\alpha}}$

or $$V\frac{2\alpha - 3\alpha^2}{\sqrt{\alpha^2-\alpha^3}} = -E\frac{1}{\sqrt{1-\alpha}}$$

or $$3\alpha - 2 = \frac{E}{V}$$

or $$\alpha = \frac{1}{3}\left[2 + \frac{E}{V}\right]$$

After substituting the value of α in Eq. (i), we obtain

$$I_{D_F} = (1-\alpha)^{1/2}\frac{\alpha V - E}{R} = \left[1 - \frac{1}{3}\left(2+\frac{E}{V}\right)\right]^{1/2}\frac{\frac{1}{3}\left(2+\frac{E}{V}\right)V - E}{R}$$

$$= \frac{1}{R}\left[\frac{1}{3} - \frac{1}{3}\frac{E}{V}\right]^{1/2}\frac{2V-2E}{3}$$

$$= \frac{1}{R}\frac{1}{\sqrt{3}}\left[1-\frac{E}{V}\right]^{1/2}\frac{2V}{3}\left[1-\frac{E}{V}\right] = \frac{V}{R}\frac{2}{3\sqrt{3}}\left[1-\frac{E}{V}\right]^{1/2}\left[1-\frac{E}{V}\right]$$

$$= \frac{V}{R}\frac{2}{3\sqrt{3}}\left[1-\frac{E}{V}\right]^{3/2} = 0.385\frac{V}{R}\left(1-\frac{E}{V}\right)^{3/2}$$

Example 10.18 A step-up chopper is used to provide 500 V from a 200 V dc input voltage. If the blocking period of thyristor is 90 μs, what is the conduction period of thyristor?

Solution

The output voltage is $V_{av} = \frac{T_{\text{ON}} + T_{\text{OFF}}}{T_{\text{OFF}}}V$

or $$500 = \frac{T_{\mathrm{ON}} + 90}{90} 200$$

Therefore, $$T_{\mathrm{ON}} = \frac{500}{200} \times 90 - 90 = 135\ \mu\mathrm{s}$$

Example 10.19 A type A copper operates at 1 kHz from 200 V dc supply and it is connected to a RL load. The load time constant is 5 ms and the load resistance is 15 Ω. Determine the average load current and the amplitude of current ripple for mean value of output voltage of 60 V. Find the maximum value and minimum value of load current.

Solution

The load time constant is $\frac{L}{R} = 5\ \mathrm{ms}$ and load resistance R = 15 Ω

Therefore, $L = 5 \times 10^{-3} \times R = 5 \times 10^{-3} \times 15 = 75\ \mathrm{mH}$

The time period $T = \frac{1}{f} = \frac{1}{1 \times 10^3} = 1\ \mathrm{ms}$

The average output voltage $V_{\mathrm{av}} = DV$

or $$D = \frac{V_{\mathrm{av}}}{V} = \frac{60}{200} = 0.3$$

The ON period of switch is $T_{\mathrm{ON}} = DT = 0.3 \times 1$ ms = 0.3 ms and the OFF period of switch is

$$T_{\mathrm{OFF}} = T - T_{\mathrm{ON}} = 1\ \mathrm{ms} - 0.3\ \mathrm{ms} = 0.7\ \mathrm{ms}$$

During on period of switch, current increases from $I_{\min}$ to $I_{\max}$. We assume that the rate of increment is linear.

Therefore, $V - V_o = L\frac{I_{\max} - I_{\min}}{T_{\mathrm{ON}}}$

or $$I_{\max} - I_{\min} = \frac{(V - V_o)T_{\mathrm{ON}}}{L} = \frac{(V - DV)T_{\mathrm{ON}}}{L} \quad \text{as } V_{\mathrm{av}} = V_o = DV$$

or $$I_{\max} - I_{\min} = \frac{V}{L}(1 - D)T_{\mathrm{ON}} = \frac{V}{L}\left(1 - \frac{T_{\mathrm{ON}}}{T}\right)T_{\mathrm{ON}}$$

During the OFF period of switch, the amplitude of ripple current is

$$\Delta I = I_{\max} - I_{\min} = \frac{V_o}{L}T_{\mathrm{OFF}} = \frac{60}{75 \times 10^{-3}} \times 0.7 \times 10^{-3} = 0.56\ \mathrm{A}$$

as $V_o = DV = 0.3 \times 200 = 60$ V

The average load current is $I_o = \frac{I_{\max} + I_{\min}}{2} = \frac{V_o}{R} = \frac{60}{15}\mathrm{A} = 4\ \mathrm{A}$

The maximum value of load current is $I_{\max} = I_o + \frac{1}{2}\Delta I = 4 + \frac{1}{2} \times 0.56 = 4.28\ \mathrm{A}$

The minimum value of load current is $I_{\min} = I_o - \frac{1}{2}\Delta I = 4 - \frac{1}{2} \times 0.56 = 3.72\ \mathrm{A}$

Example 10.20 The speed of separately excited dc motor is controlled by a class A chopper and the motor operates at below rated speed. The armature resistance is R_a = 0.25 Ω and inductance is L_a = 25 mH. The dc input voltage is V = 200 V, the motor constant is k = 0.1 V/rpm. When motor operates at constant load torque with an average load current of 40 A, determine (a) the range of speed control and (b) the range of duty cycle.

Solution

Given: $R_a = 0.25\ \Omega$, $L_a = 215$ mH, $V = 200$ V, $k = 0.1$ V/rpm, $I_o = 40$ A

(a) The minimum speed of motor is zero. The back emf at zero speed $E_b = 0$.

We know that $V_o = V_{av} = DV = E_b + I_a R_a$

As $E_b = 0,\ D = \dfrac{I_a R_a}{V} = \dfrac{40 \times 0.25}{200} = 0.05$

The maximum duty cycle is $D = 1$

Then $\quad DV = E_b + I_a R_a$ and $E_b = DV - I_a R_a = 200 - 40 \times 0.25 = 190$ V

As the motor constant is $k = 0.1$ V/rpm, the operating speed of motor is

$$N = \frac{E_b}{k} = \frac{190}{0.1} = 1900 \text{ rpm}$$

The range of speed control is $0 \le N \le 1900$ rpm

(b) The range of duty cycle is $0.05 \le D \le 1$.

Example 10.21 In a battery operated chopper fed dc drive, the maximum possible value of accelerating current is 425 A, the lower limit of current pulsation is 180 A. The ON period of switch is 14 ms and OFF period of switch is 11 ms and the time constant is 63.5 ms. Find (a) the higher limit of current pulsation, (b) chopping frequency and (c) duty cycle. Assume battery voltage $V = 200$ V and $R = 0.1\ \Omega$.

Solution

Given: $I_{max} = 425$ A, lower limit of current pulsation is $\Delta I = 180$ A, $T_{ON} = 14$ ms, $T_{OFF} = 11$ ms, $\tau_a = \tau = 63.5$ ms

(a) The minimum value of current is $I_{min} = I_{max} - \Delta I = 425 - 180 = 245$ A

The maximum current is equal to

$$I_{max} = I_{min} e^{-\frac{T_{ON}}{\tau}} + \frac{V-E}{R}\left(1 - e^{-\frac{T_{ON}}{\tau}}\right)$$

or
$$425 = 245^{-\frac{14}{63.5}} + \frac{V-E}{R}\left(1 - e^{-\frac{14}{63.5}}\right)$$

or
$$\frac{V-E}{R} = [(425 - 245 \times 0.80214)]/(1 - 0.80214) = 1154.72$$

As $V = 200$ V and $R = 0.1\ \Omega$, the back emf is $E = 200 - 1154.72 \times 0.1 = 84.528$

$T_{ON} = 14$ ms for the higher current pulsation. Here the duty ratio is

$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}} = \frac{14}{14 + 11} = 0.56$$

The current pulsation will be highest at $D = 0.5$, therefore $T_{ON} = 14$ ms and $T_{OFF} = 14$ ms.

We know that $I'_{min} = I_{max} e^{-\frac{T_{OFF}}{\tau}} - \dfrac{E}{R}\left(1 - e^{-\frac{T_{OFF}}{\tau}}\right)$

or
$$I'_{min} = 425 e^{-\frac{14}{63.5}} - \frac{84.528}{0.1}\left(1 - e^{-\frac{14}{63.5}}\right) = 173.66 \text{ A}$$

The higher limit of current pulsation is $\Delta I_{max} = I_{max} - I'_{min} = 425 - 173.66 = 251.34$ A

(b) Chopping frequency $f = \dfrac{1}{T} = \dfrac{1}{T_{ON} + T_{OFF}} = \dfrac{1}{14 \text{ ms} + 14 \text{ ms}} = 35.714$ Hz

(c) Duty cycle is $D = \dfrac{T_{ON}}{T_{ON} + T_{ON}} = \dfrac{14}{14 + 14} = 0.5$

Example 10.22 A chopper controlled separately excited dc motor is operated by a dc battery. When dc motor operates at full rating, $V = 72$ V, $I = 200$ A and $N = 2500$ rpm. The current pulsation is maintained in between 180 A and 230 A. Determine (a) chopping frequency and (b) duty cycle when armature resistance $R_a = 0.045\ \Omega$, armature inductance $L_a = 7$ mH, battery resistance $R_b = 0.055\ \Omega$ and speed = 100 rpm.

Solution

Given: $V = 72$ V, $I = 200$ A, $N = 2500$ rpm, $R_a = 0.045\ \Omega$, $L_a = 7$ mH, $R_b = 0.065\ \Omega$

For a separately excited dc motor $V_o = E_b + I_aR_a$

As back emf $E_b = kN$, $V_a = kN + I_aR_a$ or $72 = k \times 2500 + 200 \times 0.045$

Then $k = \dfrac{72 - 200 \times 0.045}{2500} = 0.0252$ V/rpm

When the motor operate at 1000 rpm, the back emf is equal to

$$E_b = kN = 0.0252 \times 1000 = 25.2 \text{ V}$$

Time constant $\tau = \dfrac{L_a}{R_a + R_b} = \dfrac{7 \text{ mH}}{(0.045 + 0.055)\Omega} = 0.070$ s

The maximum current is equal to

$$I_{\max} = I_{\min} e^{-\frac{T_{\text{ON}}}{\tau}} + \frac{V - E_b}{R}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right) \text{ where, } R = R_a + R_b = 0.045 + 0.055 = 0.1\ \Omega$$

As $I_{\max} = 230$ A, $I_{\min} = 180$ A, $230 = 180e^{-\frac{T_{\text{ON}}}{\tau}} + \dfrac{72 - 25.2}{0.1}\left(1 - e^{-\frac{T_{\text{ON}}}{\tau}}\right)$

or $$230 = 180x + 468(1 - x) \quad \text{as } x = e^{-\frac{T_{\text{ON}}}{\tau}}$$

or $$x = \frac{468 - 230}{468 - 180} = 0.826$$

Therefore, $x = e^{-\frac{T_{\text{ON}}}{\tau}} = 0.826 = e^{-\frac{T_{\text{ON}}}{0.07}}$

The ON time of chopper is $T_{\text{ON}} = 0.01338$ s

The minimum current $I_{\min} = I_{\max} e^{-\frac{T_{\text{OFF}}}{\tau}} - \dfrac{E}{R}\left(1 - e^{-\frac{T_{\text{OFF}}}{\tau}}\right)$

or $$180 = 230e^{-\frac{T_{\text{OFF}}}{\tau}} - \frac{25.2}{0.1}\left(1 - e^{-\frac{T_{\text{OFF}}}{\tau}}\right)$$

or $$180 = 230y - 252(1 - y) \quad \text{as } y = e^{-\frac{T_{\text{OFF}}}{\tau}}$$

Therefore, $y = \dfrac{180 + 252}{230 + 252} = 0.8962$

Therefore, $y = e^{-\frac{T_{\text{OFF}}}{\tau}} = 0.8962 = e^{-\frac{T_{\text{OFF}}}{0.07}}$

The OFF time of chopper is $T_{\text{OFF}} = 0.00767$ s

(a) The chopping frequency is $f = \dfrac{1}{T_{\text{ON}} + T_{\text{OFF}}} = \dfrac{1}{0.01338 + 0.00767} = 47.50$ Hz

(b) Duty ratio $D = \dfrac{T_{\text{ON}}}{T_{\text{ON}} + T_{\text{OFF}}} = \dfrac{0.01338}{0.02105} = 0.6356$

Example 10.23 Figure 10.30 shows a chopper circuit where the semiconductor switch is ideal, V is the input voltage and V_o is the load emf. Prove that this circuit can be operating either as a step-down or a step-up chopper.

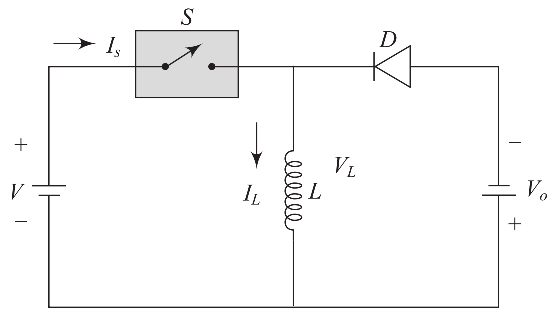


Fig. 10.30

Solution

When switch S is closed, V is applied to inductance L and current increases from I_{min} to I_{max} during T_{ON} time. The voltage across inductance L is

$$V_L = L\frac{di}{dt}$$

or $$V_L = L\frac{I_{max} - I_{min}}{T_{ON}}$$

If $I_{max} - I_{min} = \Delta I$, $V_L = L\dfrac{\Delta I}{T_{ON}}$ or, $\Delta I \cdot L = V_L T_{ON}$ (i)

When switch S is opened, current flows through L, V_o and diode D and the decreases from I_{max} to I_{min} during T_{OFF} time.

Then $$V_o = L\frac{I_{max} - I_{min}}{T_{OFF}} = L\frac{\Delta I}{T_{OFF}}$$

or $$\Delta I \cdot L = V_o T_{OFF} \quad \text{(ii)}$$

As per Eqs. (i) and (ii), we obtain

$$\Delta I \cdot L = V_L T_{ON} = V_o T_{OFF}$$

or $$V_o = V_L \cdot \frac{T_{ON}}{T_{OFF}} = V_L \cdot \frac{T_{ON}}{T - T_{ON}} = V_L \cdot \frac{T_{ON}/T}{1 - (T_{ON}/T)} = V_L \frac{D}{1-D} \quad \text{as } D = \frac{T_{ON}}{T}$$

As $V_o = V_L \dfrac{D}{1-D}$, this circuit operates as step-down chopper when $D < 0.5$. If $D > 0.5$, this circuit operates as step-up chopper.

Example 10.24 The switching frequency of chopper as shown in Fig. 10.31(a) is 1 kHz. This chopper circuit is operated at the boundary of continuous and discontinuous conduction. If the current waveform follows the Fig. 10.31(b), determine the on time of chopper and the value of peak current.

Solution

Given: $V = 200$ V, $V_o = 500$ V, $L = 150$ mH

When switch S is closed, V is applied to inductance L and current increases from 0 to I_{max} during T_{ON} time. The voltage across inductance L is

$$V_L = L\frac{di}{dt}$$

or $$V_L = L\frac{I_{max}}{T_{ON}}$$

or $$I_{max} \cdot L = V_L T_{ON} \quad \text{(i)}$$

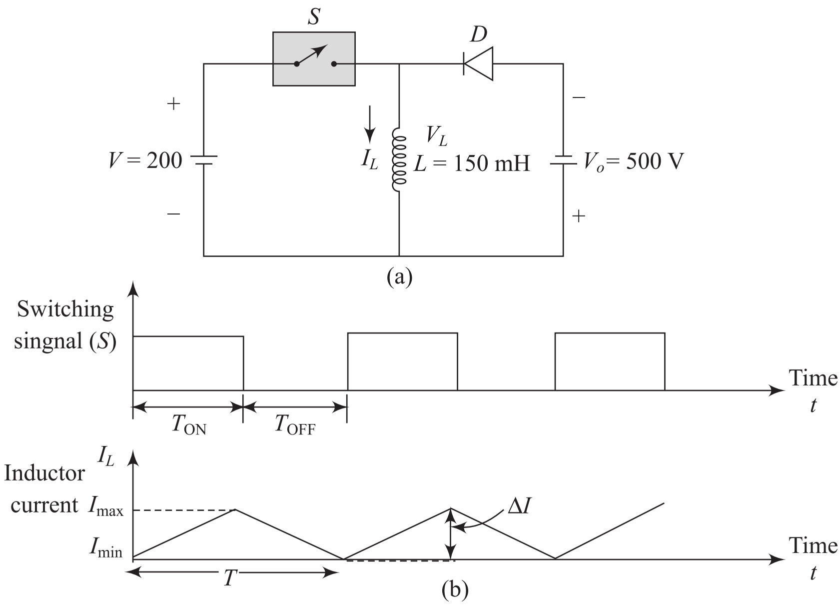


Fig. 10.31 *(a) Chopper circuit and (b) Switching signal and inductor current*

When switch S is opened, current flows through L, V_o and diode D and the decreases from I_{max} to 0 during T_{OFF} time.

Then $V_o = L\dfrac{I_{max}}{T_{OFF}}$

or $$I_{max}L = V_oT_{OFF} \tag{ii}$$

As per Eqs. (i) and (ii), we obtain

$$V_LT_{ON} = V_oT_{OFF}$$

or $$V_o = V_L \cdot \frac{T_{ON}}{T_{OFF}} = V_L \cdot \frac{T_{ON}}{T - T_{ON}} = V_L \cdot \frac{T_{ON}/T}{1-(T_{ON}/T)} = V_L\frac{D}{1-D} \quad \text{as } D = \frac{T_{ON}}{T}$$

or $$V_o = V_L\frac{D}{1-D}$$

or $$500 = 200\frac{D}{1-D} \quad \text{as } V_L = 200 \text{ V}, \quad V_o = 500 \text{ V}$$

The duty ratio $D = 0.7142$

If $f = 1$ kHz, $T = 1$ ms, $T_{ON} = DT = 0.7142 \times 1 = 0.7142$ ms

The maximum value of current is $I_{max} = \dfrac{V_LT_{ON}}{L} = \dfrac{200 \times 0.7142 \times 10^{-3}}{150 \times 10^{-3}}\text{A} = 0.9522 \text{ A}$

10.7 NON-ISOLATED dc-TO-dc CONVERTERS (CHOPPERS)

In an non-isolated dc to dc converters or choppers, the load impedance is directly connected with the input dc voltage. As the load is not electrically isolated from input dc supply, these converters are simple in construction and commonly used in different industrial applications as variable voltage dc supply. These converters are also called switch mode dc supply or switch mode dc to dc converters. There are different circuit configurations available for dc-to-dc converter without isolation, but the following four circuit configurations are most commonly used:

1. Buck converter
2. Boost converter
3. Buck-Boost converters
4. Cuk converters

10.7.1 Buck Converter

Figure 10.32 shows the circuit topology of buck converter which consists of a switch S, a diode D_F, inductor L, capacitor C and load. In this converter, the output voltage can be controlled from zero to the maximum input dc voltage V by varying the duty cycle of switch S. Therefore, this circuit is also called *step-down chopper*.

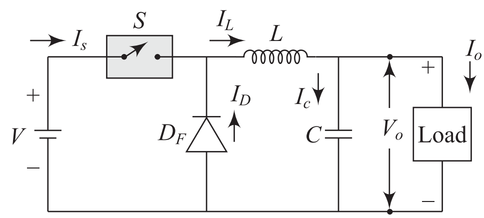


Fig. 10.32 *The circuit diagram of buck converter*

High frequency semiconductor switches such as MOSFET and IGBT are generally used in buck converter. When the switch S is ON, energy is stored with in the inductor L. When the switch S is OFF, the stored energy of inductor is transferred to capacitor C through freewheeling diode D_F. This circuit operates in two different operating modes such as

1. Continuous conduction mode
2. Discontinuous conduction mode

In continuous conduction mode of operation, the switch S must be turned on before the inductor current i_L reaches zero. But in case of discontinuous conduction mode of operation, the switch S must be turned ON after the inductor current i_L becomes zero. In this section, both the continuous conduction mode and discontinuous conduction mode of buck converters are explained in detail.

Assumptions The following assumptions are required to simplify the circuit analysis:

1. All semiconductor switches are ideal and there is no switching loss with in devices
2. The current through inductor L increases and decreases linearly. Therefore, the rate of change of current $\frac{di}{dt}$ is constant and the voltage across inductor $V_L = L\frac{di}{dt}$ is constant
3. Since the value of capacitor is large, the output voltage is constant.

Continuous conduction mode Figure 10.33 shows the voltage and current waveforms of buck converter during continuous conduction mode of operation. When the switch S is ON, the voltage across free wheeling diode is V and the output voltage is V_o. Then the voltage across inductor is

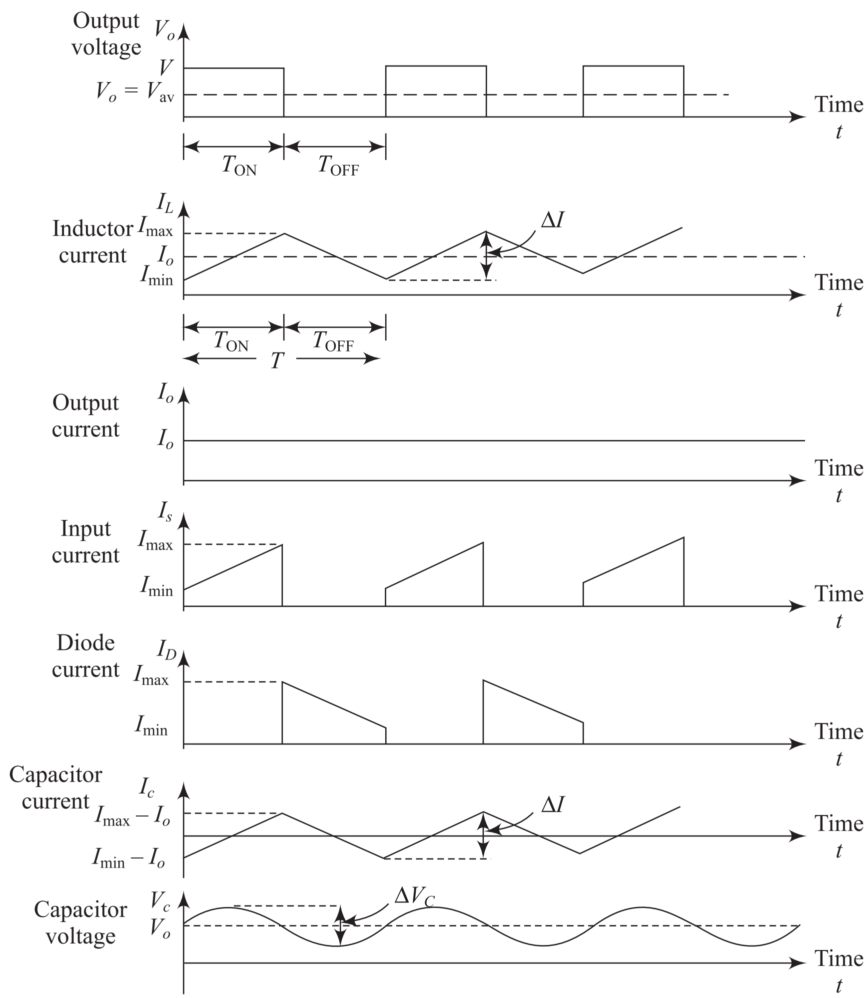


Fig. 10.33 *Voltage and current waveforms of buck converter in continuous conduction mode*

$$V_L = L\frac{di_L}{dt} = V - V_o \quad \text{where, } i_L \text{ current flows through inductor } L$$

During the ON-period T_{ON}, the inductor current i_L increases from I_{min} to I_{max} linearly. Therefore, the changes in inductor current is given by

$$\Delta I = I_{max} - I_{min} = \frac{di_L}{dt} \times T_{ON} = \frac{V - V_o}{L} T_{ON} \quad \text{where } \frac{di_L}{dt} = \frac{V - V_o}{L}$$

$$\therefore \qquad T_{ON} = \frac{L\Delta I}{V - V_o}$$

When the switch S is OFF, the free wheeling diode conducts and the voltage across D_F is zero and the negative output voltage appears across inductor. Therefore, the voltage across inductor is equal to

$$V_L = L\frac{di_L}{dt} = -V_o \qquad \text{where, } i_L \text{ current flows through inductor } L$$

During the OFF-period T_{OFF}, the inductor current i_L decreases from $I_{\max}$ to $I_{\min}$ linearly. Consequently, the changes in inductor current is expressed by

$$-\Delta I = I_{\min} - I_{\max} = \frac{di_L}{dt} \times T_{\text{OFF}} = \frac{-V_o}{L} T_{\text{OFF}} \quad \text{where, } \frac{di_L}{dt} = \frac{-V_o}{L} \text{ during } T_{\text{OFF}}$$

$\therefore$
$$T_{\text{OFF}} = \frac{L\Delta I}{V_o}$$

The switching frequency $f = \dfrac{1}{T} = \dfrac{1}{T_{\text{ON}} + T_{\text{OFF}}}$

After substituting the value of T_{ON} and T_{OFF} in the above equation, we obtain

$$f = \frac{1}{T_{\text{ON}} + T_{\text{OFF}}} = \frac{1}{\dfrac{L\Delta I}{V - V_o} + \dfrac{L\Delta I}{V_o}} = \frac{V_o(V - V_o)}{VL\Delta I}$$

The amplitude of ripple current is

$$\Delta I = \frac{V_o(V - V_o)}{VfL}$$

Since the duty cycle varies in between zero and unity and the output voltage is $V_o = DV$, the amplitude of ripple current is equal to

$$\Delta I = \frac{V_o(V - V_o)}{VfL} = \frac{D(1 - D)V}{fL}$$

If the average value of load current is I_o, the maximum and minimum value of inductor current are given by

$$I_{\max} = I_o + \frac{1}{2}\Delta I \text{ and } I_{\min} = I_o - \frac{1}{2}\Delta I$$

Applying the Kirchhoff's current law, we obtain the inductor current

$$I_L = I_C + I_o$$

Assume that the load ripple current ΔI_o is very small and it is neglected. Then $\Delta I_L = \Delta I_C$

The average capacitor current, which flows for $\dfrac{T_{\text{ON}}}{2} + \dfrac{T_{\text{OFF}}}{2} = \dfrac{T}{2}$ duration is

$$I_C = \frac{\Delta I}{4}$$

The voltage across capacitor is

$$V_C = \frac{1}{C}\int I_C dt + V_C(t = 0)$$

The peak-to-peak ripple voltage across capacitor is

$$\Delta V_C = V_C - V_C(t = 0) = \frac{1}{C}\int_0^{T/2} \frac{\Delta I}{4} dt = \frac{\Delta I T}{8C} = \frac{\Delta I}{8fC} \tag{10.10}$$

After substituting the value of ΔI in Eq. (10.10), we obtain

$$\Delta V_C = \frac{\Delta I}{8fC} = \frac{V_o(V - V_o)}{8LCf^2 V} \qquad \text{as } \Delta I = \frac{V_o(V - V_o)}{VfL}$$

or
$$\Delta V_C = \frac{\Delta I}{8fC} = \frac{D(1 - D)V}{8LCf^2} \qquad \text{as } \Delta I = \frac{D(1 - D)V}{fL}$$

The condition for continuous inductor current and capacitor voltage, the inductor ripple current ΔI will be two times of average inductor current I_o and it is represented by $\Delta I = 2I_o$

$$\Delta I = \frac{D(1-D)V}{fL} = 2I_o = 2\frac{DV}{R} \qquad \text{as } I_o = \frac{DV}{R}$$

Then the critical inductance value is equal to

$$L = \frac{(1-D)R}{2f}$$

When V_C is the average capacitor voltage, the capacitor ripple voltage is $\Delta V_C = 2V_o$

As $$\Delta V_C = \frac{\Delta I}{8fC} = \frac{D(1-D)V}{8LCf^2}, \qquad \frac{D(1-D)V}{8LCf^2} = 2V_o = 2DV$$

Then the critical capacitance value is equal to

$$C = \frac{1-D}{16Lf^2}$$

Discontinuous conduction mode In discontinuous conduction mode of operation, initially inductor current i_L is zero. When the switch S is ON, the voltage across inductor is $V_L = L\frac{di_L}{dt} = V - V_o$

In the T_{ON} period, the inductor current i_L increases from 0 to $I_{\max}$ linearly and the changes in inductor current is given by

$$\Delta I = I_{\max} = \frac{di_L}{dt} \times T_{\text{ON}} = \frac{V - V_o}{L} T_{\text{ON}} \qquad \therefore T_{\text{ON}} = \frac{L\Delta I}{V - V_o}$$

The average load current is

$$I_o = \frac{\Delta I}{2} = \frac{V - V_o}{2L} T_{\text{ON}} \qquad \text{as } \Delta I = \frac{V - V_o}{L} T_{\text{ON}}$$

or $$I_o = \frac{V - V_o}{2L} T_{\text{ON}} = \frac{V - V_o}{2L} \frac{T_{\text{ON}}}{T} T$$

$$= \frac{V - V_o}{2L} DT = \frac{(V - V_o)D}{2fL} \qquad \text{as } D = \frac{T_{\text{ON}}}{T} \text{ and } T = \frac{1}{f}$$

Due to discontinuous i_L current, a dead time t_d is shown in Fig. 10.34. During dead time t_d, the inductor current is zero. Then fall time of inductor current is equal to

$$T - T_{\text{ON}} - t_d$$

When the switch S is OFF, the voltage across inductor is $V_L = L\frac{di_L}{dt} = -V_o$

In the time period $T - T_{\text{ON}} - t_d$, the inductor current i_L decreases from $I_{\max}$ to zero linearly and the changes in inductor current is equal to

$$-\Delta I = 0 - I_{\max} = \frac{di_L}{dt} \times (T - T_{\text{ON}} - t_d) = \frac{-V_o}{L}(T - T_{\text{ON}} - t_d)$$

or $$\Delta I = \frac{V_o}{L}(T - T_{\text{ON}} - t_d)$$

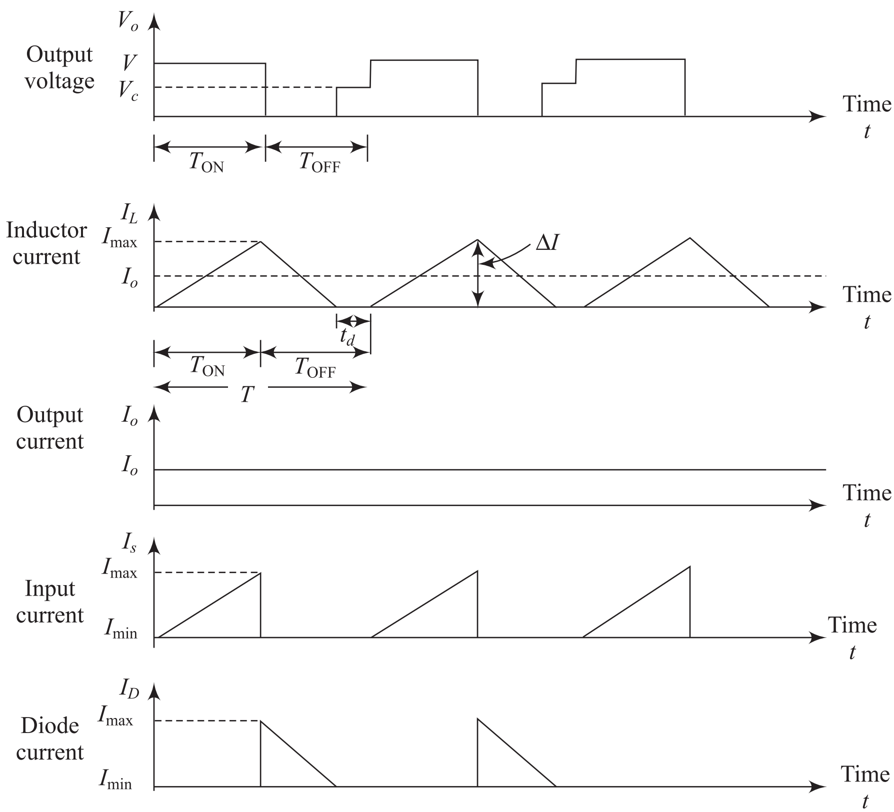


Fig. 10.34 *Voltage and current waveforms of buck converter in discontinuous conduction mode*

The ratio of output voltage to input voltage is equal to

$$\frac{V_o}{V_i} = \frac{D}{1 - \dfrac{t_d}{T}}$$

If the load current is continuous, $t_d = 0$ and the average value of load current is equal to inductor current ($I_o = I_L$).

Example 10.25 A buck converter has input voltage of 220 V and it operates at 500 Hz. The average load current is 50 A. The load resistance is 2 Ω. Determine the value of inductance to limit the maximum peak to peak ripple current through inductor to 10%. Find the value of inductance for maximum ripple current.

Solution

Given: $V = 200$ V, $f = 500$ Hz, $I_o = 50$ A, $R = 2\ \Omega$, $\Delta I = 10\%$ of I_o

The output voltage $V_o = I_oR = 50 \times 2 = 100$ V

The duty ratio is $D = \dfrac{V_o}{V} = \dfrac{100}{220} = 0.4545$

The amplitude of ripple current through inductor is $\Delta I = 10\%$ of $I_o = 0.1 \times 50 = 5$ A

We know that $\Delta I = \dfrac{V_o(V - V_o)}{VfL} = \dfrac{D(1 - D)V}{fL}$

or $$L = \frac{D(1 - D)V}{f\Delta I} = \frac{0.4545(1 - 0.4545)}{500 \times 5} = 0.0991 \text{ mH}$$

For maximum ripple current, duty ratio of buck converter is $D = 0.5$.
Then the value of inductance is

$$L = \frac{D(1 - D)V}{f\Delta I} = \frac{0.5(1 - 0.5)}{500 \times 5} = 0.1 \text{ mH}$$

Example 10.26 A buck converter has input voltage of 15 V and the required average output voltage is 6 V at $R = 400\ \Omega$ and the peak to peak output ripple voltage is 20 mV. If it operates at 20 kHz and peak to peak ripple current of inductor is 0.75 A, determine (a) duty cycle ratio, (b) filter inductance L, (c) filter capacitance C and (d) critical values of L and C.

Solution

Given: $V = 15$ V, $V_o = 6$ V, $R = 400\ \Omega$, $\Delta V_C = 20$ mV, $f = 20$ kHz, $\Delta I = 0.75$ A

(a) As $V_o = DV$, duty cycle ratio is $D = \dfrac{V_o}{V} = \dfrac{6}{15} = 0.4$

(b) $L = \dfrac{D(1 - D)V}{f\Delta I} = \dfrac{0.4(1 - 0.4) \times 15}{20 \times 10^3 \times 0.75} = 0.24 \text{ mH}$

(c) As $\Delta V_C = \dfrac{\Delta I}{8fC}$, $C = \dfrac{\Delta I}{8f\Delta V_C} = \dfrac{0.75}{8 \times 20 \times 10^3 \times 20 \times 10^{-3}} = 234.375\ \mu\text{F}$

(d) The critical inductance value is equal to

$$L = \frac{(1 - D)R}{2f} = \frac{(1 - 0.4) \times 400}{2 \times 20 \times 10^3} = 6 \text{ mH}$$

The critical capacitance value is equal to

$$C = \frac{1 - D}{16Lf^2} = \frac{1 - 0.4}{16 \times 6 \times 10^{-3} \times (20 \times 10^3)^2} = 0.0156\ \mu\text{F}$$

10.7.2 BOOST Converter

The circuit topology of boost converter is depicted in Fig. 10.35. This circuit consists of a inductor L, switch S, diode D, capacitor C and load. In this converter, the output voltage can be above the input dc voltage V by varying the duty cycle of switch S. So, this circuit is also called *step-up chopper*.

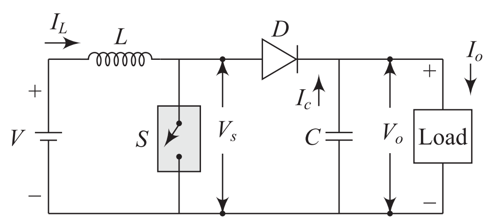


Fig. 10.35 *Circuit diagram of boost converter*

When the switch S is ON, energy is stored with in the inductor L and inductor current i_L from certain minimum value to maximum value. The voltage across inductor L is equal to input voltage. Since diode D is OFF, the output voltage V_o is the voltage across capacitor. As the capacitor value is large, the load current is constant. Figure 10.36 shows the equivalent circuit diagram of Fig. 10.35 during T_{ON} period.

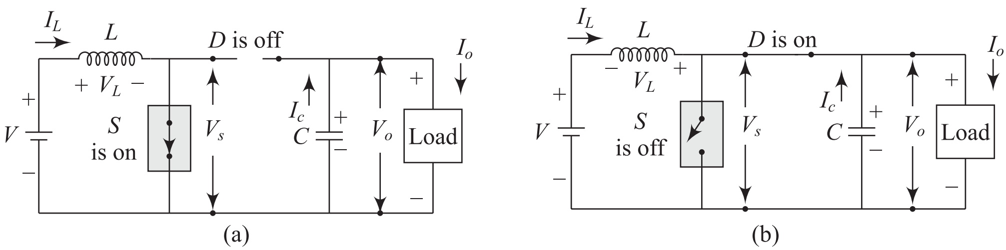


Fig. 10.36 *(a) Equivalent circuit of Fig. 10.35 when switch S is ON and (b) Equivalent circuit of Fig. 10.35 when switch S is OFF*

When the switch S is turned OFF at $t = T_{ON}$, a negative voltage $V_L = L\dfrac{di_L}{dt}$ is developed across inductor L. Consequently, the voltage across switch will be $V + V_L$ which is greater than input voltage V. Then the stored energy of inductor is transferred to capacitor C through diode D and the inductor current decreases linearly. During this period, not only the stored energy of inductor flows but also the energy flows from dc source to load. The equivalent circuit diagram of Fig. 10.35(b) during T_{OFF} period is illustrated in Fig. 10.36(b).
The boost circuit operates in two different operating modes such as

1. continuous conduction mode
2. discontinuous conduction mode

In continuous conduction mode of operation, the switch S must be turned ON before the inductor current i_L reaches zero. On the other hand, the switch S must be turned ON after the inductor current i_L becomes zero in discontinuous conduction mode of operation. In this section, both the continuous conduction mode and discontinuous conduction mode of boost converters are discussed elaborately.

Continuous conduction mode Figure 10.37 shows the voltage and current waveforms of boost converter during continuous conduction mode of operation. When the switch S is ON, the voltage across the switch S is zero and the voltage across inductor is

$$V_L = L\frac{di_L}{dt} = V \quad \text{where, } i_L \text{ current flows through inductor } L$$

Then the output voltage is V_o which the voltage is across capacitor C.
The polarities of inductor voltage V_L and output voltage V_o are depicted in Fig. 10.36.
During the ON-period T_{ON}, the inductor current i_L increases from I_{min} to I_{max} linearly. Hence, the changes in inductor current is given by

$$\Delta I = I_{max} - I_{min} = \frac{di_L}{dt} \times T_{ON} = \frac{V}{L} T_{ON} \tag{10.11}$$

where, $\dfrac{di_L}{dt} = \dfrac{V}{L}$

$\therefore \quad T_{ON} = \dfrac{L\Delta I}{V}$

When the switch S is OFF, the diode conducts and the $(V_o - V)$ voltage appears across inductor L. In the OFF-period T_{OFF}, the inductor current i_L decreases from I_{max} to I_{min} linearly. Accordingly, the changes in inductor current is expressed by

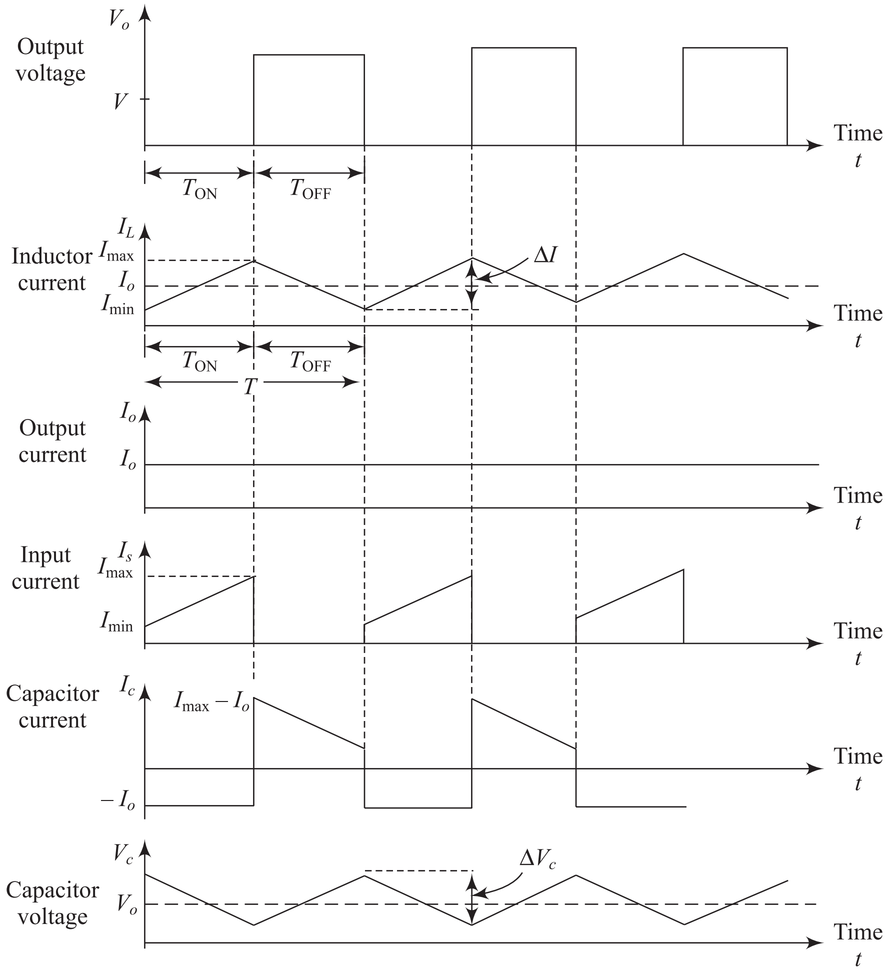


Fig. 10.37 *Voltage and current waveforms of a boost converter in continuous mode*

$$-\Delta I = I_{min} - I_{max} = \frac{di_L}{dt} \times T_{OFF} = \frac{-(V_o - V)}{L} T_{OFF} \quad \text{where, } \frac{di_L}{dt} = \frac{-(V_o - V)}{L} \text{during } T_{OFF}$$

or $$\Delta I = \frac{(V_o - V)}{L} T_{OFF} \tag{10.12}$$

$\therefore$ $$T_{OFF} = \frac{L\Delta I}{V_o - V}$$

From Eqs. (10.11) and (10.12), we get

$$\Delta I = \frac{V}{L} T_{ON} = \frac{(V_o - V)}{L} T_{OFF}$$

or $$V \cdot T_{ON} = (V_o - V) T_{OFF}$$

or $$V_o = \frac{V \cdot T_{\text{ON}} + V \cdot T_{\text{OFF}}}{T_{\text{OFF}}} = \frac{T_{\text{ON}} + T_{\text{OFF}}}{T_{\text{OFF}}} V = \frac{T}{T - T_{\text{ON}}} V$$

or $$V_o = \frac{V}{1 - D} \tag{10.13}$$

It is clear from Eq. (10.13) that if duty cycle D varies from zero to unity, the output voltage will be equal or greater than input voltage ($V_o \geq V$)

Since all semiconductor switches are ideal and lossless, the power input to converter P_i is equal to the power output from converter P_o.

Therefore, $P_i = P_o$

or $$VI = V_o I_o = \frac{V}{1 - D} I_o \quad \text{or, } I = \frac{I_o}{1 - D}$$

The switching frequency $f = \frac{1}{T} = \frac{1}{T_{\text{ON}} + T_{\text{OFF}}}$

After substituting the value of T_{ON} and T_{OFF} in the above equation, we obtain

$$f = \frac{1}{T_{\text{ON}} + T_{\text{OFF}}} = \frac{1}{\frac{L\Delta I}{V} + \frac{L\Delta I}{V_o - V}} = \frac{V(V_o - V)}{V_o L \Delta I}$$

Then amplitude of ripple current is

$$\Delta I = \frac{V(V_o - V)}{V_o f L}$$

or $$\Delta I = \frac{V}{fL}\left(1 - \frac{V}{V_o}\right)$$

As $V_o = \frac{V}{1 - D}$, $\quad \Delta I = \frac{V}{fL}[1 - (1 - D)] = \frac{VD}{fL}$

When the chopper switch S is ON, the capacitor supplies the load current for T_{ON} duration. The average capacitor current during T_{ON} duration is $I_C = I_o$ and the peak-to-peak ripple voltage across capacitor is

$$\Delta V_C = V_C - V_C(t = 0) = \frac{1}{C} \int_0^{T_{\text{ON}}} I_o dt = \frac{I_o T_{\text{ON}}}{C} \tag{10.14}$$

When know that $V_o = \frac{V}{1 - D}$. Then $D = \frac{V_o - V}{V_o} = \frac{T_{\text{ON}}}{T}$

or $$T_{\text{ON}} = \frac{V_o - V}{V_o} \cdot T = \frac{V_o - V}{f V_o}$$

After substituting the value of T_{ON} in Eq. (10.14), we obtain

$$\Delta V_C = \frac{I_o T_{\text{ON}}}{C} = \frac{I_o}{C} \frac{V_o - V}{f V_o}$$

or $$\Delta V_C = \frac{I_o}{C} \frac{V_o - V}{f V_o} = \frac{I_o D}{fC} \quad \text{as} \quad V_o = \frac{V}{1 - D} \quad \text{and} \quad D = \frac{V_o - V}{V_o}$$

The condition for continuous inductor current and capacitor voltage, the inductor ripple current ΔI will be two times of average inductor current I_o and it is represented by $\Delta I = 2I_o$

$$\Delta I = \frac{VD}{fL} = 2I_o = 2\frac{V}{(1-D)R} \qquad \text{as} \quad I_o = \frac{V_o}{R} = \frac{V}{(1-D)R}$$

or
$$\frac{VD}{fL} = 2\frac{V}{(1-D)R}$$

Then the critical inductance value is equal to

$$L = \frac{D(1-D)R}{2f}$$

When V_C is the average capacitor voltage, the capacitor ripple voltage is $\Delta V_C = V_o$

As
$$\Delta V_C = \frac{I_o}{C}\frac{V_o - V}{fV_o} = \frac{I_o D}{fC}, \quad \frac{I_o D}{fC} = 2V_o = 2I_o R \quad \text{as} \quad V_o = I_o R$$

Then the critical capacitance value is equal to

$$C = \frac{D}{2fR} \quad \text{as} \quad \frac{I_o D}{fC} = 2I_o R$$

Discontinuous conduction mode In discontinuous conduction mode of operation, inductor current i_L is discontinuous as shown in Fig. 10.38. During dead time t_d, the inductor current is zero. Therefore, the fall time of inductor current is equal to

$$T_{\text{OFF}} - t_d = T - T_{\text{ON}} - t_d$$

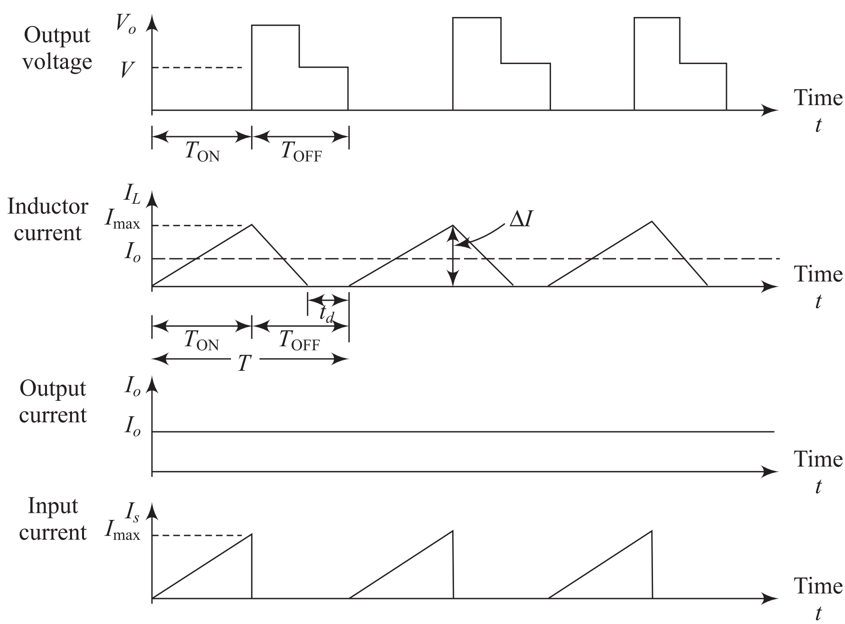


Fig. 10.38 *Voltage and current waveforms of a boost converter in discontinuous mode*

and ripple current $\Delta I = \dfrac{V}{L}T_{\text{ON}} = \dfrac{(V_o - V)}{L}(T_{\text{OFF}} - t_d)$

or $$VT_{\text{ON}} = (V_o - V)(T_{\text{OFF}} - t_d)$$

or $$\frac{V_o}{V} = \frac{1 - \dfrac{t_d}{T}}{1 - D - \dfrac{t_d}{T}}$$

Example 10.27 A boost converter has input voltage of 12 V and it operates at 25 kHz. When the duty cycle is 0.25, L = 150 μH, C = 147 μH and average load current is 1.5 A, determine the average output voltage, peak to peak ripple current through inductor.

Solution

Given: V = 12 V, f = 2 kHz, D = 0.25, L = 150 μH, C = 147 μF

We know that $V_o = \dfrac{V}{1-D} = \dfrac{12}{1-0.25} = 16 \text{ V}$

The amplitude of ripple current through inductor is

$$\Delta I = \frac{V}{fL}[1-(1-D)] = \frac{VD}{fL} = \frac{12 \times 0.25}{25 \times 10^3 \times 150 \times 10^{-6}} = 0.8 \text{ A}$$

Example 10.28 A boost converter has input voltage of 5 V and it operates at 20 kHz. When the average output voltage V_o = 10 V, the average load current I_o = 0.8 A, L = 100 μH, and C = 147 μF, determine (a) duty cycle, (b) ripple current of inductor ΔI, (c) the maximum current flows through inductor I_{max}, (d) ripple voltage across capacitor and (e) critical values of L and C.

Solution

Given: V = 5 V, f = 20 kHz, V_o = 10 V, L = 100 μH, C = 147 μF

(a) Output voltage $V_o = \dfrac{V}{1-D}$. Then $D = \dfrac{V_o - V}{V_o} = \dfrac{10-5}{10} = 0.5$

(b) The amplitude of ripple current through inductor is

$$\Delta I = \frac{V}{fL}[1-(1-D)] = \frac{VD}{fL} = \frac{5 \times 0.5}{20 \times 10^3 \times 100 \times 10^{-6}} = 1.25 \text{ A}$$

(c) Inductor current $I = \dfrac{I_o}{1-D} = \dfrac{0.8}{1-0.5} = 1.6 \text{ A}$

The maximum current flows through inductor $I_{\text{max}} = I + \dfrac{\Delta I}{2} = 1.6 + \dfrac{1.25}{2} \text{ A} = 2.225 \text{ A}$

(d) Ripple voltage across capacitor $\Delta V_C = \dfrac{I_o}{C}\dfrac{V_o - V}{fV_o} = \dfrac{I_o D}{fC} = \dfrac{0.8 \times 0.5}{20 \times 10^3 \times 147 \times 10^{-6}} = 136.05 \text{ mV}$

(e) $R = \dfrac{V_o}{I_o} = \dfrac{10}{0.8} = 12.5\ \Omega$

The critical inductance value is equal to

$$L = \frac{D(1-D)R}{2f} = \frac{0.5(1-0.5) \times 12.5}{2 \times 20 \times 10^3} = 78.125 \text{ μH}$$

The critical capacitance value is equal to

$$C = \frac{D}{2fR} = \frac{0.5}{2 \times 20 \times 10^3 \times 12.5} = 1\,\mu\text{F}$$

10.7.3 Buck-Boost Converter

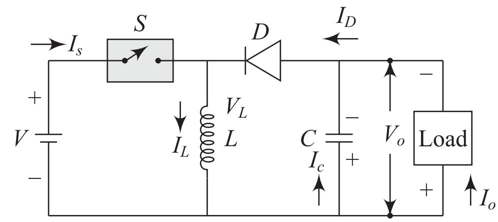


Fig. 10.39 *Circuit diagram of buck-boost converter*

Figure 10.39 shows the circuit topology of buck-boost dc-to-dc converter which consists of a switch S, inductor L, diode D, capacitor C and load. In this converter, the output voltage can be controlled by varying the duty cycle of switch S. When duty cycle is less than 50%, the output voltage will be less than input voltage and this converter acts as buck converter. When duty cycle is greater than 50%, the output voltage will be more than input voltage and this converter works as boost converter. Therefore, this circuit is also called *buck-boost-down converter*.

When the switch S is ON, energy is stored with in the inductor L. When the switch S is turned OFF at $t = T_{ON}$, the inductor current decreases and negative voltage is generated across inductor L.

The stored energy of inductor is transferred to capacitor C and load through diode D. This circuit operates in two different operating modes such as

1. continuous conduction mode
2. discontinuous conduction mode

In continuous conduction mode of operation, the switch S must be turned ON before the inductor current i_L becomes zero. However, in case of discontinuous conduction mode of operation, the switch S must be turned ON after the inductor current i_L reaches zero. In this section, both the continuous conduction mode and discontinuous conduction mode of buck converters are explained in detail.

Continuous conduction mode The voltage and current waveforms of buck-boost converter during continuous conduction mode of operation is shown in Fig. 10.40. If the switch S is ON, the voltage across inductor is

$$V_L = L\frac{di_L}{dt} = V \text{ where, } i_L \text{ current flows through inductor } L$$

During the ON-period T_{ON}, the inductor current i_L increases from I_{min} to I_{max} linearly. As a result, the changes in inductor current is equal to

$$\Delta I = I_{max} - I_{min} = \frac{di_L}{dt} \times T_{ON} = \frac{V}{L} T_{ON} \tag{10.15}$$

where, $\dfrac{di_L}{dt} = \dfrac{V}{L}$

$\therefore$ $$T_{ON} = \frac{L\Delta I}{V}$$

When the switch S is OFF, the voltage across inductor is negative and its value is $-V_o$. Therefore, the voltage across inductor is equal to

$$V_L = L\frac{di_L}{dt} = -V_o \text{ where, } i_L \text{ current flows through inductor } L$$

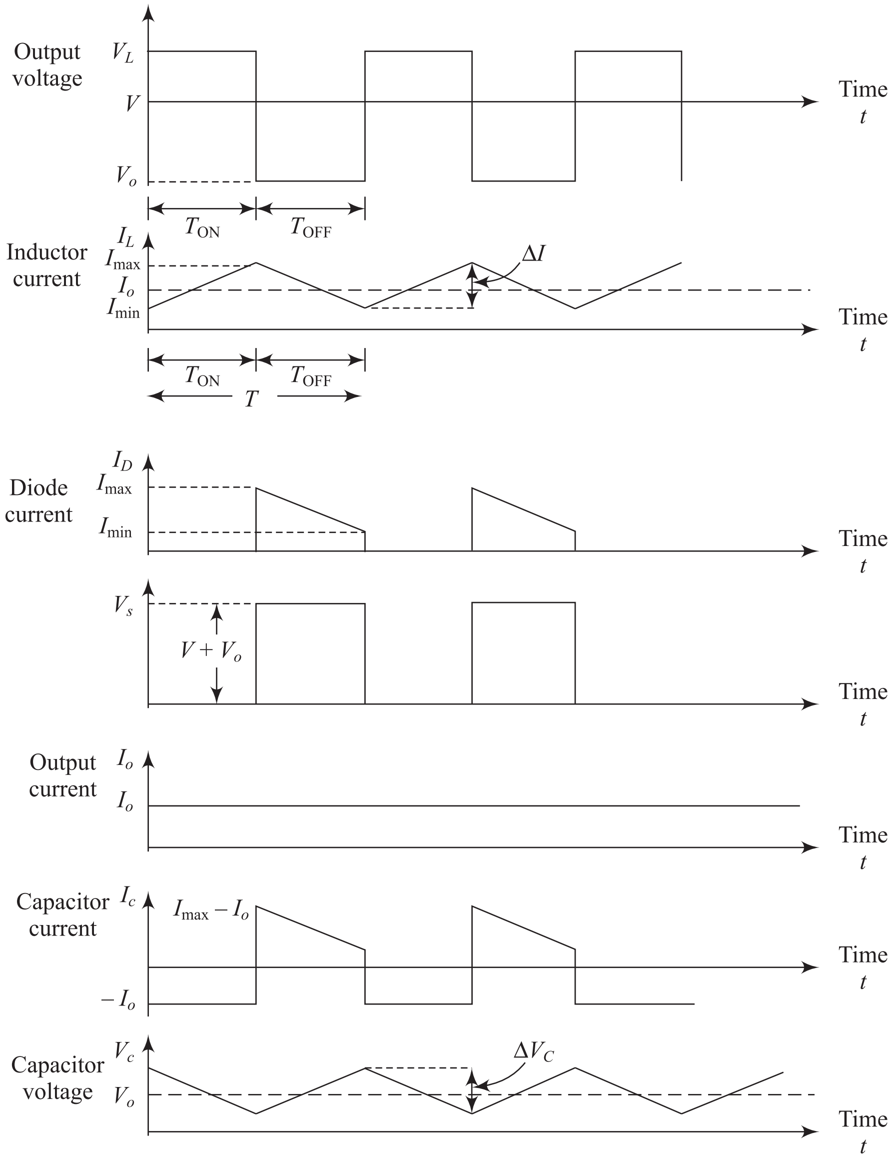


Fig. 10.40 *Voltage and current waveforms of a buck-boost converter in continuous mode*

In the OFF-period T_{OFF}, the inductor current i_L decreases from I_{max} to I_{min} linearly. So, the changes in inductor current is expressed by

$$\Delta I = \frac{di_L}{dt} \times T_{OFF} = \frac{-V_o}{L} T_{OFF} \tag{10.16}$$

where, $\dfrac{di_L}{dt} = \dfrac{-V_o}{L}$ during T_{OFF}

$$\therefore \qquad T_{\text{OFF}} = -\frac{L\Delta I}{V_o}$$

Using equation (10.15) and (10.16), we get

$$\Delta I = \frac{V}{L}T_{\text{ON}} = \frac{-V_o}{L}T_{\text{OFF}}$$

or

$$V_o = -\frac{T_{\text{ON}}}{T_{\text{OFF}}}V = -\frac{T_{\text{ON}}/T}{T_{\text{OFF}}/T}V = -\frac{D}{1-D}V \tag{10.17}$$

as $D = \frac{T_{\text{ON}}}{T}$ and $\frac{T_{\text{OFF}}}{T} = 1 - D$

It is clear from Eq. (10.17) that this circuit acts as buck converter when duty cycle $D < 0.5$ and output voltage is less than input voltage ($V_o < V$). This circuit can also operate as boost converter when duty cycle $D > 0.5$ and output voltage is greater than input voltage ($V_o > V$).

Assume that all semiconductor switching devices are ideal and lossless. Then the power input to converter P_i is equal to the power output from converter P_o.

Therefore, $P_i = P_o$

or

$$VI = V_o I_o = -\frac{D}{1-D}VI_o \text{ or, } I = -\frac{D}{1-D}I_o$$

The switching frequency $f = \frac{1}{T} = \frac{1}{T_{\text{ON}} + T_{\text{OFF}}}$

After substituting the value of T_{ON} and T_{OFF} in the above equation, we obtain

$$f = \frac{1}{T_{\text{ON}} + T_{\text{OFF}}} = \frac{1}{\frac{L\Delta I}{V} - \frac{L\Delta I}{V_o}} = \frac{V_o V}{L\Delta I(V_o - V)}$$

The amplitude of ripple current through inductor is

$$\Delta I = \frac{V_o V}{fL(V_o - V)} = \frac{VD}{fL} \qquad \text{as } \frac{V_o}{V_o - V} = D$$

When the chopper switch S is ON, the capacitor supplies the load current for T_{ON} duration. The average discharging current of capacitor during T_{ON} duration is $I_C = I_o$ and the peak-to-peak ripple voltage across capacitor is

$$\Delta V_C = V_C - V_C(t=0) = \frac{1}{C}\int_0^{T_{\text{ON}}} I_o dt = \frac{I_o T_{\text{ON}}}{C} \tag{10.18}$$

Since $V_o = -\frac{D}{1-D}V, D = \frac{V_o}{V_o - V} = \frac{T_{\text{ON}}}{T}$

or

$$T_{\text{ON}} = \frac{V_o}{V_o - V}T = \frac{V_o}{f(V_o - V)}$$

After substituting the value of T_{ON} in Eq. (10.18), we obtain

$$\Delta V_C = \frac{I_o T_{\text{ON}}}{C} = \frac{I_o}{C}\frac{V_o}{f(V_o - V)} = \frac{I_o D}{fC} \qquad \text{since} \quad D = \frac{V_o}{V_o - V}$$

The condition for continuous inductor current and capacitor voltage, the inductor ripple current ΔI will be two times of average inductor current I_o and it is represented by $\Delta I = 2I_o$

$$\Delta I = \frac{VD}{fL} = 2I_o = 2\frac{DV}{(1-D)R} \qquad \text{as} \quad I_o = \frac{V_o}{R}$$

or $$\frac{VD}{fL} = 2\frac{DV}{(1-D)R}$$

Then the critical inductance value is equal to

$$L = \frac{(1-D)R}{2f}$$

When V_C is the average capacitor voltage, the capacitor ripple voltage is $\Delta V_C = 2V_o$

As $$\Delta V_C = \frac{I_o T_{\text{ON}}}{C} = \frac{I_o}{C}\frac{V_o}{f(V_o - V)} = \frac{I_o D}{fC}, \qquad \frac{I_o D}{fC} = 2V_o = 2I_o R$$

Therefore, the critical capacitance value is equal to

$$C = \frac{D}{2fR}$$

Discontinuous conduction mode During discontinuous conduction mode of operation, inductor current i_L is discontinuous as shown in Fig. 10.41. The inductor current reaches zero at $t = T - t_d$. During dead time t_d, the inductor current is zero. Therefore, the fall time of inductor current is equal to

$$T_{\text{OFF}} - t_d = T - T_{\text{ON}} - t_d$$

and ripple current $$\Delta I = \frac{V}{L}T_{\text{ON}} = -\frac{V_o}{L}(T_{\text{OFF}} - t_d)$$

or $$VT_{\text{ON}} = -V_o(T_{\text{OFF}} - t_d)$$

or $$\frac{V_o}{V} = -\frac{T_{\text{ON}}}{T_{\text{OFF}} - t_d} = -\frac{D}{1 - D - \frac{t_d}{T}}$$

Example 10.29 A buck-boost converter has input voltage of 24 V and it operates at 30 kHz. When the duty cycle is 0.4, L = 500 μH, C = 147 μF and average load current is 1 A, determine the average output voltage, peak to peak ripple current through inductor.

Solution

Given: V = 24 V, f = 30 kHz, D = 0.4, L = 500 μH, C = 147 μF

We know that $$V_o = -\frac{D}{1-D}V = -\frac{0.4}{1-0.4} \times 24 = -16 \text{ V}$$

The amplitude of ripple current through inductor is

$$\Delta I = \frac{V_o V}{fL(V_o - V)} = \frac{VD}{fL} = \frac{24 \times 0.4}{30 \times 10^3 \times 500 \times 10^{-6}} = 0.64 \text{ A}$$

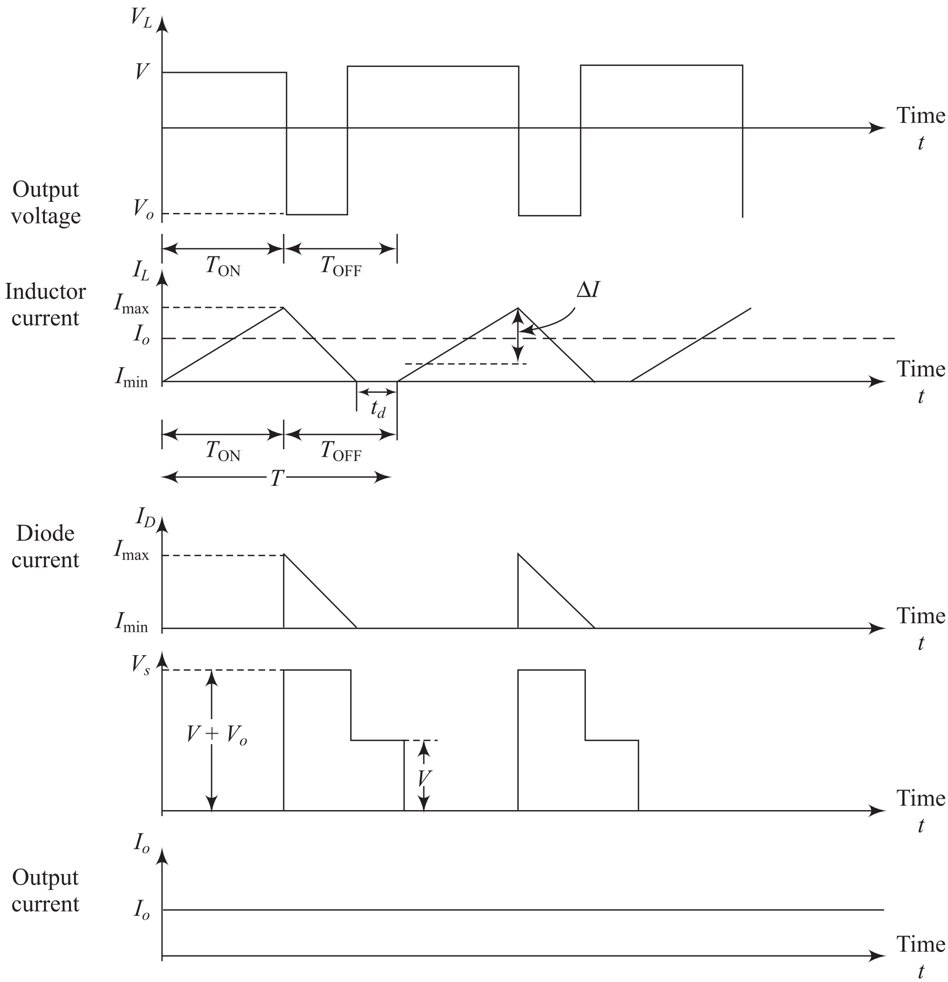


Fig. 10.41 *Voltage and current waveforms of a buck-boost converter in discontinuous mode*

Example 10.30 A buck-boost converter has input voltage of 15 V. The duty cycle is 0.25 and it operates at 20 kHz. When $L = 250\ \mu$H, $C = 220\ \mu$F and average load current is 1.5 A, determine (a) average output voltage, (b) peak-to-peak output voltage ripple, (c) peak-to-peak ripple current through inductor, (d) maximum current flows through switch and (e) the critical values of L and C.

Solution

Given: $V = 15$ V, $f = 20$ kHz, $D = 0.25$, $L = 250\ \mu$H, $C = 220\ \mu$F

(a) We know that $V_o = -\dfrac{D}{1-D}V$

Therefore $V_o = -\dfrac{0.25}{1-0.25}\times 15 = -5$ V

(b) The peak-to-peak output voltage ripple $\Delta V_C = \dfrac{I_o D}{fC} = \dfrac{1.5\times 0.25}{20\times 10^3 \times 220\times 10^{-6}} = 85.227$ mV

(c) Peak-to-peak ripple current through inductor $\Delta I = \frac{VD}{fL} = \frac{15 \times 0.25}{20 \times 10^3 \times 250 \times 10^{-6}} = 0.75$ A

(d) Average current $I = \frac{D}{1-D} I_o = \frac{0.25}{1-0.25} \times 1.5 = 0.5$ A

Maximum current flows through switch is $I_{\max} = \frac{I}{D} + \frac{\Delta I}{2} = \frac{0.5}{0.25} + \frac{0.75}{2} = 2.375$ A

(e) $R = \frac{-V_o}{I_o} = \frac{5}{1.5} = 3.333\ \Omega$

(f) The critical inductance value is equal to

$$L = \frac{(1-D)R}{2f} \frac{(1-0.25) \times 3.333}{2 \times 20 \times 10^3} = 62.493\ \mu\text{H}$$

The critical capacitance value is equal to

$$C = \frac{D}{2fR} = \frac{0.25}{2 \times 20 \times 10^3 \times 3.333} = 1.875\ \mu\text{F}$$

10.7.4 CUK Converters

In CUK converter, the energy can be transferred from source to load during ON-time T_{ON} and OFF-time T_{OFF} of switching. Figure 10.42 shows a CUK converter which consists of two inductors L_1 and L_2, two capacitors C_1 and C_2, a switch S, diode D and load. In this converter, the output voltage may be more or less than the input dc voltage V by varying the duty cycle of switch S. Therefore, this converter circuit operates in both buck and boost converter. In this converter, capacitors are used as energy storing device.

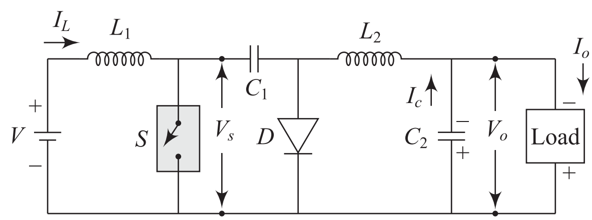


Fig. 10.42 *Circuit diagram of CUK converter*

Figure 10.43(a) shows the equivalent circuit diagram of Fig. 10.42 during T_{ON} period. When the switch S is ON, energy is stored with in the inductor L_1 and inductor current i_{L1} from certain minimum value $I_{L1\min}$ to maximum value $I_{L1\max}$. The voltage across inductor L_1 is equal to input voltage V. The inductor voltage is given by

$$V_{L1} = L_1 \frac{di_{L1}}{dt} = V \quad \text{where, } i_{L1} \text{ current flows through inductor } L_1$$

The output voltage is V_o which is the voltage across capacitor C_2. The polarities of inductor voltage V_L and output voltage V_o are depicted in Fig. 10.43. Figure 10.44 shows voltage and current waveforms of a CUK converter. During the ON-period T_{ON}, the inductor current i_{L1} increases from $I_{L1\min}$ to $I_{L1\max}$ linearly. Consequently, the changes in inductor current is given by

$$\Delta I_{L1} = I_{L1\max} - I_{L1\min} = \frac{di_L}{dt} \times T_{ON} = \frac{V}{L_1} T_{ON} \tag{10.19}$$

where, $\frac{di_{L1}}{dt} = \frac{V}{L_1}$

$$\therefore \quad T_{ON} = \frac{L_1 \Delta I_{L1}}{V}$$

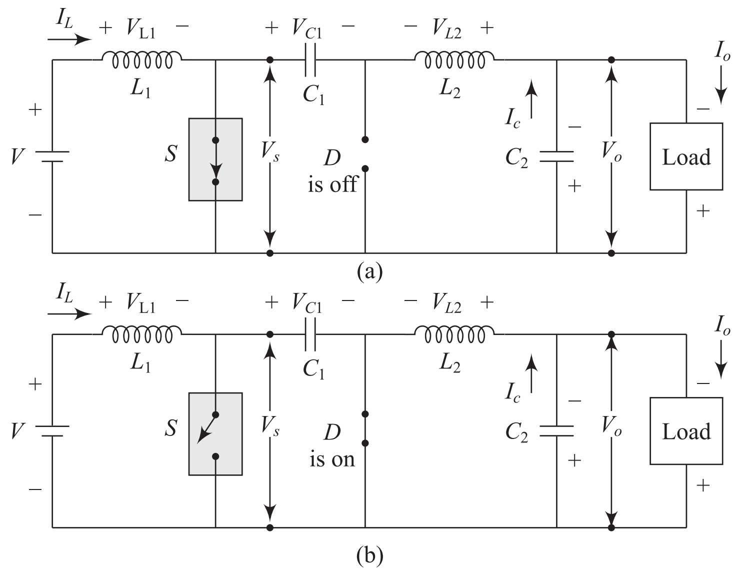


Fig. 10.43 *(a) Equivalent circuit of Fig. 10.42 when switch S is ON and (b) Equivalent circuit of Fig. 10.42 when switch S is OFF*

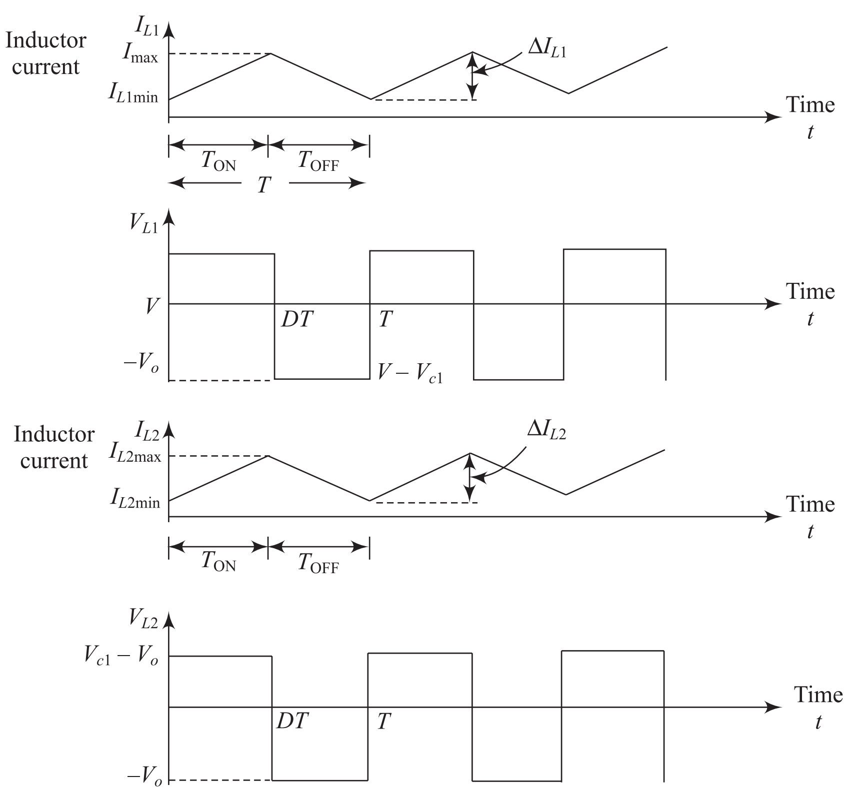


Fig. 10.44 *Voltage and current waveforms of a CUK converter*

When the switch S is OFF, the inductor current i_{L1} decreases from $I_{L1\max}$ to $I_{L1\min}$. The voltage across inductor is

$$V_{L1} = -(V_{C1} - V) = L_1 \frac{-\Delta I_{L1}}{T_{\text{OFF}}}$$

$$\therefore \qquad T_{\text{OFF}} = \frac{L_1 \Delta I_{L1}}{V_{C1} - V}$$

Therefore, $\Delta I_{L1} = \dfrac{V}{L_1} T_{\text{ON}} = \dfrac{V_{C1} - V}{L_1} T_{\text{OFF}}$

or $$V \cdot T_{\text{ON}} = (V_{C1} - V) T_{\text{OFF}}$$

or $$V_{C1} = \frac{V}{1 - D}$$

In the same way, the inductor current i_{L2} increases from $I_{L2\min}$ to $I_{L2\max}$ linearly during the ON-period T_{ON} of switch. The voltage across inductor is $V_{L2} = V_{C1} - V_o$. So the changes in inductor current is given by

$$\Delta I_{L2} = I_{L2\max} - I_{L2\min} = \frac{di_{L2}}{dt} \times T_{\text{ON}} = \frac{V_{C1} - V_o}{L_2} T_{\text{ON}} \tag{10.20}$$

where, $\dfrac{di_{L2}}{dt} = \dfrac{V_{C1} - V_o}{L_2}$

$$\therefore \qquad T_{\text{ON}} = \frac{L_2 \Delta I_{L2}}{V_{C1} - V_o}$$

When the switch S is OFF, the inductor current i_{L1} falls from $I_{L2\max}$ to $I_{L2\min}$. The voltage across inductor is

$$V_{L2} = -V_o = L_2 \frac{-\Delta I_{L2}}{T_{\text{OFF}}}$$

$$\therefore \qquad T_{\text{OFF}} = \frac{L_2 \Delta I_{L2}}{V_o}$$

$$\Delta I_{L2} = \frac{V_{C1} - V_o}{L_2} T_{\text{ON}} = \frac{V_o}{L_2} T_{\text{OFF}}$$

or $$(V_{C1} - V_o) \cdot T_{\text{ON}} = V_o T_{\text{OFF}}$$

or $$V_{C1} = \frac{V_o}{D}$$

As $V_{C1} = \dfrac{V}{1 - D}$ and $V_{C1} = \dfrac{V_o}{D}$, the output voltage is equal to

$$V_o = \frac{D}{1 - D} V \tag{10.21}$$

It is clear from Eq. (10.21) that if duty cycle D is greater than 50%, this converter acts as boost converter. When duty cycle D is less than 50%, this converter acts as buck converter. Since all

semiconductor switches are ideal and lossless, the power input to converter P_i is equal to the power output from converter P_o.
Therefore, $P_i = P_o$

or $$VI = V_o I_o = \frac{D}{1-D} V I_o \quad \text{or, } I = \frac{D}{1-D} I_o$$

The switching frequency $f = \frac{1}{T} = \frac{1}{T_{\text{ON}} + T_{\text{OFF}}}$

After substituting the value of T_{ON} and T_{OFF} in the above equation, we obtain

$$f = \frac{1}{T_{\text{ON}} + T_{\text{OFF}}} = \frac{1}{\frac{L_2 \Delta I_{L2}}{V_{C1} - V_o} + \frac{L_2 \Delta I_{L2}}{V_o}} \quad \text{as} \quad T_{\text{ON}} = \frac{L_2 \Delta I_{L2}}{V_{C1} - V_o} \quad \text{and} \quad T_{\text{OFF}} = \frac{L_2 \Delta I_{L2}}{V_o}$$

Then amplitude of ripple current is

$$\Delta I_{L2} = \frac{V_o (V_{C1} + V_o)}{V_{C1} f L_2}$$

After substituting the value of V_{C1} and V_o, we obtain

$$\Delta I_{L2} = \frac{DV}{fL_2}$$

Example 10.31 Assume that a CUK converter operates at 30 kHz to get an output voltage 150 V when the dc input voltage is about 24 V. Find the duty ratio and the voltage across switch during OFF-period.

Solution

Given: $f = 30$ kHz, $V_o = 150$ V, $V = 24$ V

We know that $V_o = \frac{D}{1-D} V \quad \text{or } 150 = \frac{D}{1-D} 24$

or $$D = \frac{V_o}{V_o + V} = \frac{150}{150 + 24} = 0.862$$

The voltage across switch during OFF-period is

$$V_{C1} = \frac{V}{1-D} = \frac{24}{1 - 0.862} = 173.91 \text{ V}$$

10.8 ISOLATED dc-TO-dc CONVERTERS (CHOPPERS)

In several industrial applications, dc voltage of different voltage levels with respect to ground such as +5 V, –5 V, +6 V, –6 V, +9 V, –9 V, +10 V, –10 V, +12 V, –12 V are required. These different voltage levels should be available from independent of dc supply. The electrical isolation between the dc source and load is used to provide protection. Transformers are used for electrical isolation. If normal silicon steel core transformers are used, there will be high hysteresis loss due to high switching frequency of dc-to-dc converters and the wide hysteresis-loop. Therefore, ferrite-core transformer is used for electrical isolation to reduce hysteresis loss with narrow hysteresis-loop. Figure 10.45 shows a circuit diagram for isolated dc-to-dc converter. This circuit generates two voltages V_{o1} and V_{o2}. The

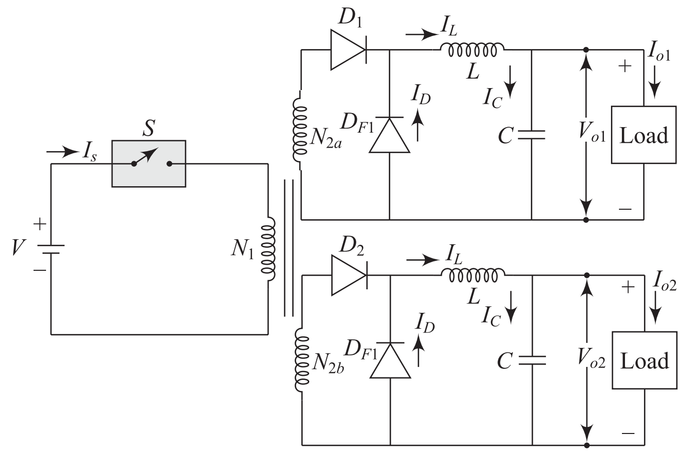


Fig. 10.45 *Circuit diagram for isolated dc-to-dc converter*

amplitude of output voltage depends upon the turn ratio of isolating transformers. These converters can be act as step-up, step-down, buck, boost, buck-boost converters. This type of isolated dc to dc converter is known as *switch mode power supply (SMPS)*. The advantages and disadvantages of SMPS are given below:

Advantages of SMPS

1. Due to high frequency operation, SMPS has smaller size, less weight and high efficiency compared to conventional linear dc power supply.
2. The output voltage of SMPS is less sensitive with respect to input voltage variation.

Disadvantages of SMPS

1. SMPS generates both the electromagnetic and radio frequency interference due to high switching frequency.
2. SMPS has high ripple in output voltage and its regulation is bad.
3. To reduce radio frequency noise, filter circuit should be used on both input and output of SMPS

The most commonly used isolated dc to dc converters are:

1. Fly-back converter
2. Forward converter
3. Push-pull converter
4. Half bridge converter
5. Full-bridge converter

In this section, the operating principle of fly-back converters, forward converters, push-pull converters, half bridge and full-bridge converters are explained in detail.

10.8.1 Fly-back Converter

Figure 10.46 shows a circuit topology of an isolated fly-back converter which consists of a dc supply, switch *S*, transformer, diode *D*, capacitor *C* and a load. When the switch *S* is ON, the input dc voltage *V* is directly applied to the primary winding of transformer. Subsequently a voltage is induced in the

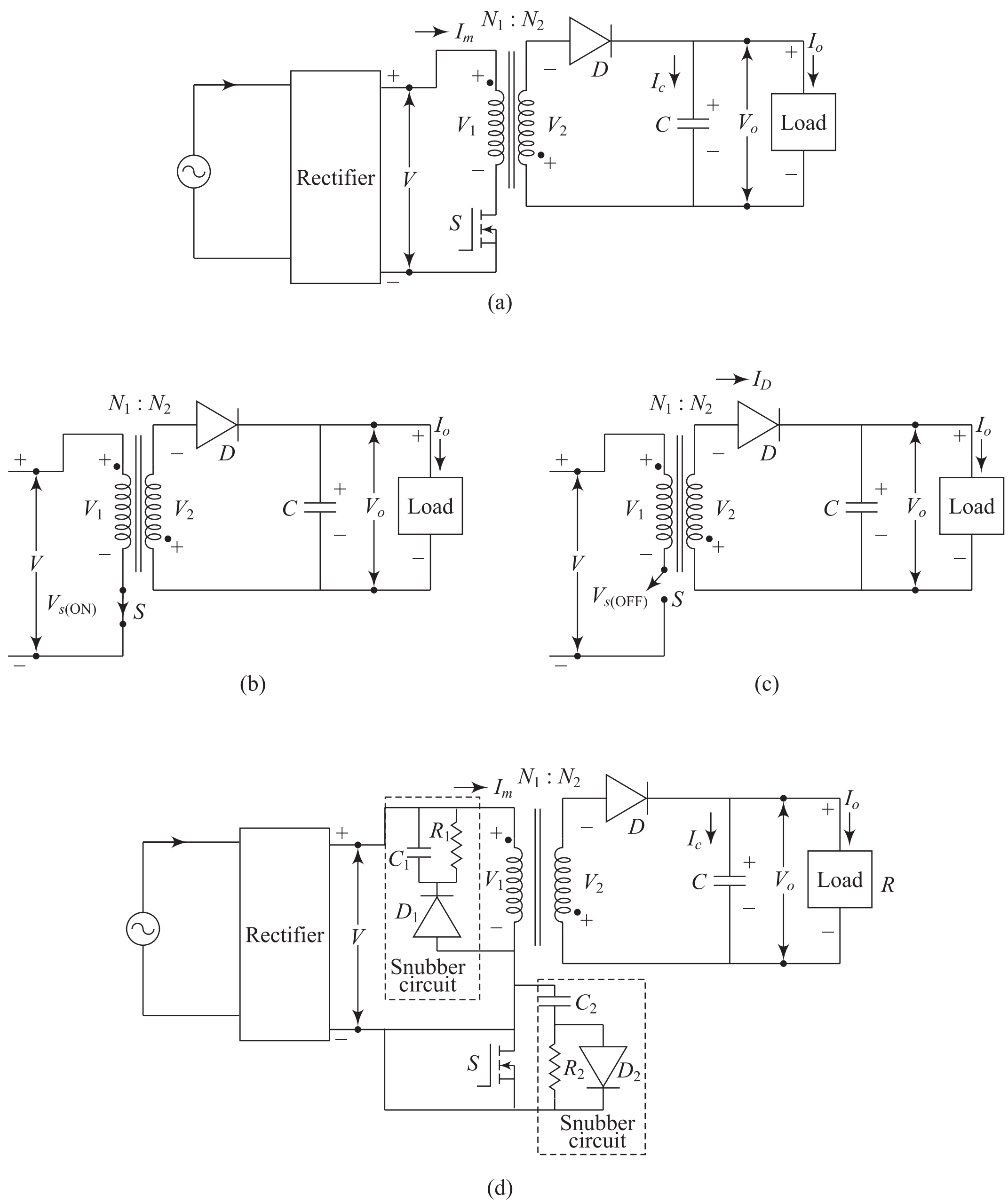


Fig. 10.46 *(a) Circuit topology of an isolated fly-back converter, (b) Equivalent circuit of Fig. 10.46(a) when S is ON, (c) Equivalent circuit of Fig. 10.46(a) when S is OFF and (d) actual circuit of an isolated fly-back converter*

secondary winding but the polarity of induced voltage in secondary winding is opposite of primary winding voltage. Due to reverse polarity, diode D is reverse biased and current will not flow through secondary winding. Hence the primary winding of transformer stores the magnetic energy.

When the switch S is OFF, a negative voltage is induced in the secondary winding as current decreases. Then diode D is forward biased and conducts. The stored energy is transferred to load. Fig. 10.47 shows the voltage and current waveforms of an isolated fly back converter.

When the switch S is ON, the voltage across the primary winding of transformer is

$$V_1 = V - V_{S(\mathrm{ON})}$$

where, V is the dc supply voltage, $V_{S(\mathrm{ON})}$ is the voltage across switch S when it is ON.
Then the voltage induced in the secondary winding of transformer is

$$V_2 = \frac{N_2}{N_1}V_1 = \frac{N_2}{N_1}[V - V_{S(\mathrm{ON})}]$$

where, N_1 and N_2 are number of turns of primary and secondary winding respectively.
When the switch S is OFF, the voltage across the secondary winding of transformer is

$$V_2 = V_D + V_C$$

Where, V_D is the voltage across diode D, V_C is the voltage across capacitor C and $V_C = V_o$
Therefore, $V_2 = V_D + V_o$ and output voltage is equal to $V_o = V_2 - V_D$

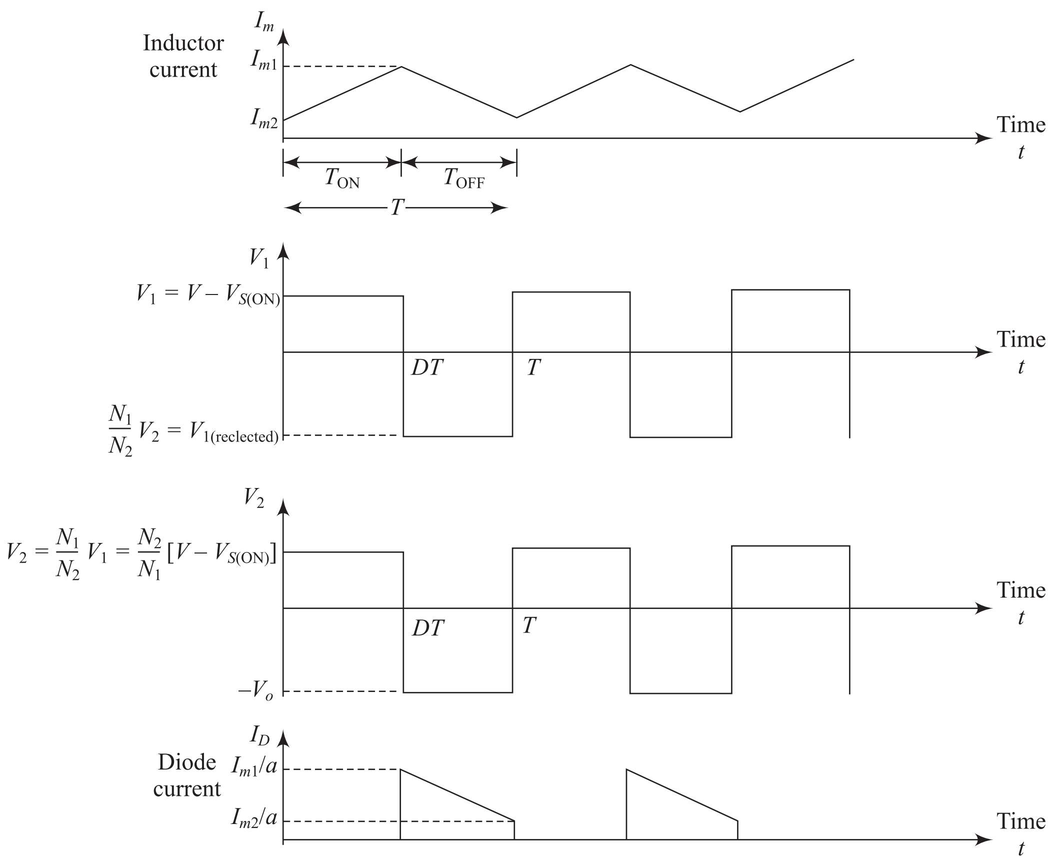


Fig. 10.47 *Voltage and current waveforms of an isolated fly back converter*

The reflected voltage induced in the primary winding is

$$V_{1(\text{reflected})} = \frac{N_1}{N_2} V_2 \text{ and}$$

$$V_2 = \frac{N_2}{N_1} V_{1(\text{reflected})}$$

Figure 10.48 shows the voltage waveform across the primary winding of transformer. The volt time area during T_{ON} is equal to the volt time area during T_{OFF}

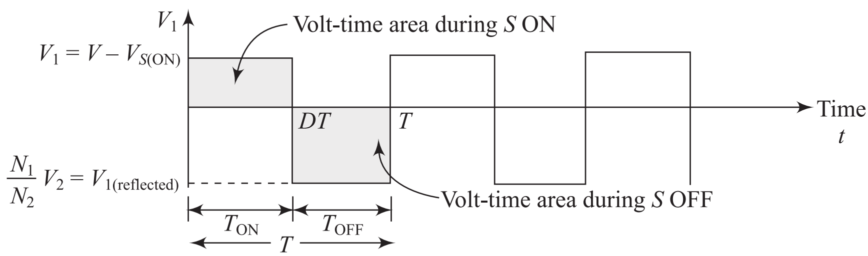


Fig. 10.48 *The voltage waveform across the primary winding of transformer*

Therefore, $[V - V_{S(ON)}]T_{ON} = V_{1(\text{reflected})} T_{OFF}$

or $$V_{1(\text{reflected})} = \frac{T_{ON}}{T_{OFF}} [V - V_{S(ON)}]$$

or $$V_{1(\text{reflected})} = \frac{D}{1-D} [V - V_{S(ON)}]$$

The output voltage is equal to

$$V_o = V_2 - V_D \tag{10.22}$$

After substituting the value of V_2 in Eq. (10.22), we obtain

$$V_o = \frac{N_2}{N_1} V_{1(\text{reflected})} - V_D \qquad \text{as } V_2 = \frac{N_2}{N_1} V_{1(\text{reflected})}$$

or $$V_o = \frac{N_2}{N_1} \frac{D}{1-D} [V - V_{S(ON)}] - V_D$$

The maximum voltage across switch is

$$V_{S(ON)\max} = V + V_{1(\text{reflected})} = V + \frac{N_1}{N_2} V_o$$

The energy stored in the primary winding of transformer is

$$W_P = \frac{1}{2} L_P I_1^2 \qquad \text{where, } I_1 = I_{m1}$$

L_P is the inductance of primary winding and I_1 is the rms value of current of primary winding
Average power output to load is

$$P_O = \frac{1}{2} f L_P I_1^2$$

When the switch is ON, the current through the primary winding of transformer increases linearly. Then the voltage across of primary winding is

$$V = L_P \frac{I_{m1}}{T_{ON}} = L_P I_{m1} \frac{f}{D} \quad \text{as} \quad T_{ON} = \frac{D}{f}$$

or
$$L_P I_{m1} = \frac{DV}{f} \text{ and } I_{m1} = \frac{2P_O}{VD}$$

As per energy conservation, input power is equal to output power

$$P_i = V_i I_i = P_o = V_o I_o$$

or
$$V_i I_i = V_o I_o$$

or
$$V I_i = V_o I_o \text{ as } V_i = V$$

The average input current is

$$I_i = \frac{V_o I_o}{V} = \frac{P_O}{V} = \frac{DVI_{m1}}{2V} = \frac{DI_{m1}}{2} \quad \text{as } P_O = \frac{DVI_{m1}}{2} \text{ and } I_i = I_{m(av)}$$

The rms value of primary winding current is

$$I_{1(rms)} = I_{m1}\sqrt{\frac{D}{3}}$$

The ripple voltage is inversely proportional to the time constant RC and the per unit ripple voltage is

$$\frac{\Delta V_o}{V_o} = \frac{T_{ON}}{RC}$$

10.8.2 Forward Converters

Figure 10.49 shows the circuit topology of a forward converter which consist of an isolating transformer with three windings such as primary winding, secondary winding and feed back winding, switch S, three diodes D_1, D_2 and D_F, inductor L, capacitor C and load. The energy can be transferred from dc supply to load when the switch is ON. Therefore, this converter circuit is called *forward converters*.

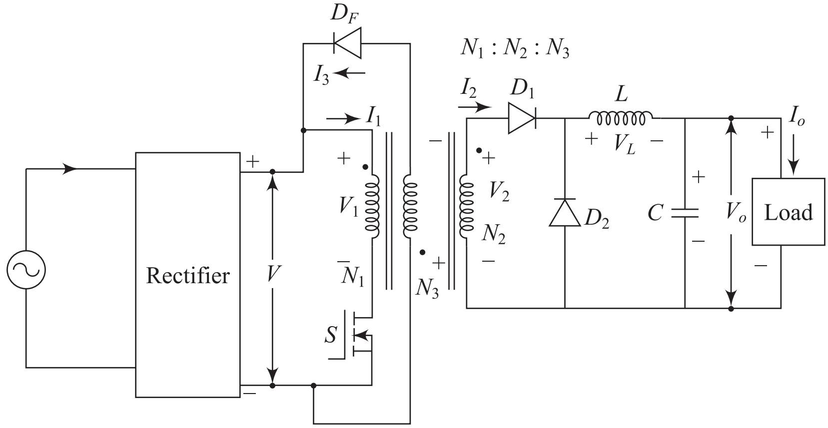


Fig. 10.49 *Circuit topology of an isolated forward converter*

When the switch S is ON, the input dc voltage V is directly applied to the primary winding of transformer and a voltage is induced in the secondary winding. The polarity of induced voltage in secondary winding is same as primary winding voltage as depicted in Fig. 10.49. Subsequently, diode D_1 is forward biased and conducts. The energy from dc supply is transferred to load through diode D_1 and inductor L stores the energy. During T_{ON} the energy is also stored with in the magnetic field of transformer. As diode D_F is reverse biased, it will be turned OFF.

When the switch S is turned OFF, the current through primary winding starts to fall and a negative voltage is induced across primary winding. The induced voltage in the feedback winding is reversed. As the voltage across the feed back winding is greater than the input voltage, diode D_F is forward biased and conducts. The energy stored in the common magnetic field of transformer is feedback to the supply. As diode D_1 becomes reverse biased, it will be turned OFF. During T_{OFF} diode D_2 conducts and the energy stored in inductor L is transferred to load. When the voltage across feedback winding becomes supply voltage V, the diode D_F stops conduction and all energy stored in the common magnetic field of transformer is feedback to the supply.

Figure 10.50 shows the voltage waveforms of an isolated forward converter.

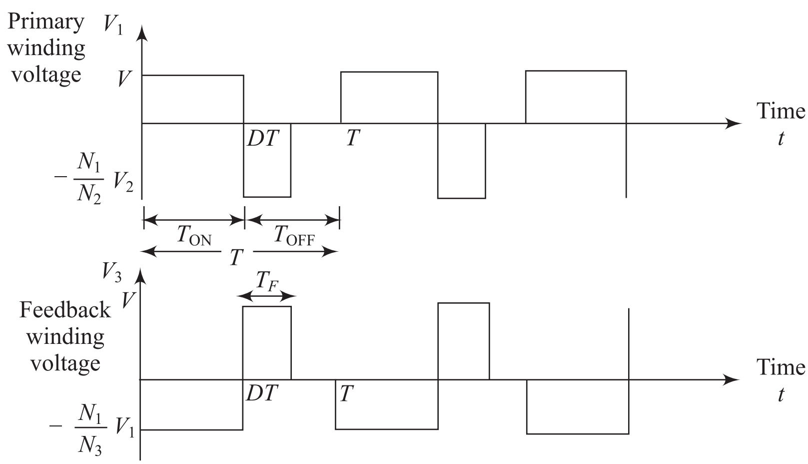


Fig. 10.50 *Voltage waveforms of primary winding and feedback winding*

When the switch S is ON, the induced voltage across the secondary winding of transformer is

$$V_2 = \frac{N_2}{N_1}V_1 = \frac{N_2}{N_1}V \qquad \text{as } V_1 = V$$

where, N_1 and N_2 are the number of turns of primary and secondary winding of transformer. As diode D_1 conducts, the voltage across inductance L is

$$V_L = V_2 - V_o = \frac{N_2}{N_1}V - V_o \qquad \text{during } 0 \le t \le T_{ON}$$

When the switch S is OFF, the diode D_2 conducts and the inductor voltage

$$V_L = -V_o \qquad \text{during } T_{ON} \le t \le T$$

We know that the energy stored in an inductor over a cycle is zero.

Therefore, $\int_0^T V_L \, dt = 0$

or $$\int_0^{T_{ON}} V_L dt = \int_{T_{ON}}^{T} V_L dt$$

or $$\left[\frac{N_2}{N_1}V - V_o\right]T_{ON} = -V_o(T - T_{ON})$$

The output voltage of forward converter is expressed by

$$V_o = \frac{N_2}{N_1}DV$$

The open circuit voltage across the switch is

$$V_{S(OC)} = \left(1 + \frac{N_1}{N_3}\right)V$$

where, N_3 is the number of turns of feedback winding of transformer.

When $\frac{N_1}{N_3} = 1,$ the maximum voltage across switch is $2V$.

10.8.3 Push-Pull Converters

Figure 10.51 shows the circuit topology of push-pull converter which consists of a transformer with mid points on both primary and secondary windings, two switches S_1 and S_2, two diodes D_1, D_2, inductor L, capacitor C, load and a rectifier circuit. LC circuit is used as output filter.
When the switch S_1 is ON, the input voltage V is applied to lower half of the primary winding of transformer. Then $V_1 = V$.

There will be an induced voltage across both lower half and upper half of secondary winding of transformer. The induced voltage is equal to

$$V_2 = \frac{N_2}{N_1}V_1 = \frac{N_2}{N_1}V \text{ as } V_1 = V$$

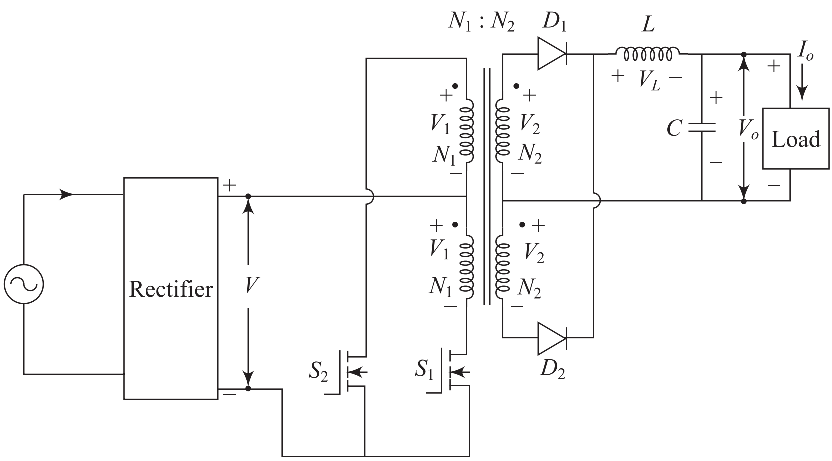


Fig. 10.51 *Circuit topology of push-pull converter*

The diode D_1 is forward biased due to upper half of secondary winding of transformer and conducts. Then the output voltage is given by

$$V_o = V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1} V$$

When the switch S_2 is turned ON, the input voltage $-V$ is applied to upper half of the primary winding of transformer. Then $V_1 = -V$.

Accordingly, there will be an induced voltage across both lower half and upper half of secondary winding of transformer. The induced voltage in lower half of secondary winding of transformer is equal to

$$V_2 = \frac{N_2}{N_1} V_1 = -\frac{N_2}{N_1} V \quad \text{as } V_1 = -V$$

The diode D_2 is forward biased due to voltage of lower half of secondary winding of transformer and conducts. Then the output voltage is given by

$$V_o = \frac{N_2}{N_1} V$$

Since the voltage of primary winding swings from $+V$ to $-V$, each switch must be able to withstand at $2V$. Therefore, this converter is only suitable for low voltage applications.

10.8.4 Half-Bridge Converter

Figure 10.52 shows the circuit configuration of half-bridge converter which consists of a rectifier, two capacitors C_1 and C_2, two switches S_1 and S_2, a transformer with mid-point on the secondary winding, two diodes D_1 and D_2, inductor L, capacitor C and load. LC circuit is used as output filter.

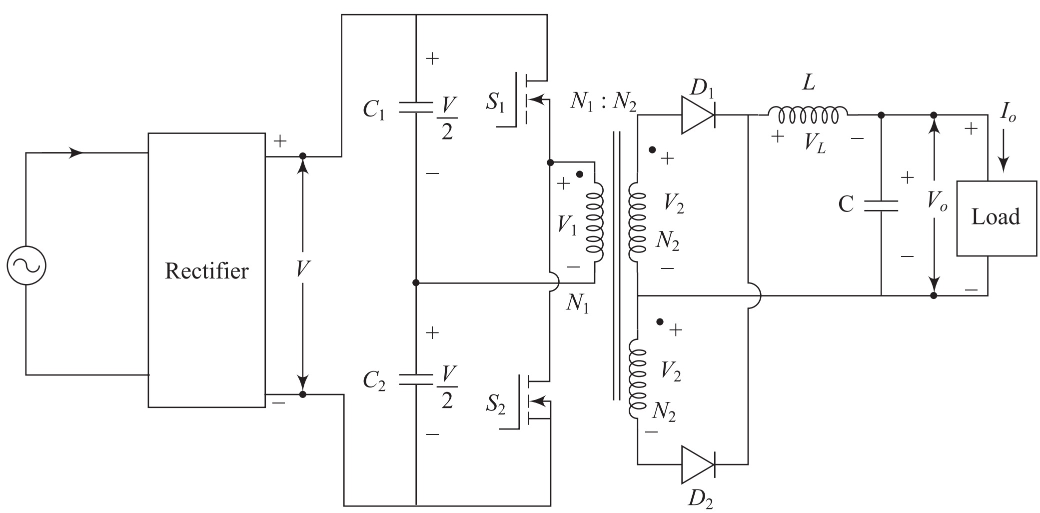


Fig. 10.52 *Half bridge dc-to-dc converter*

The capacitors C_1 and C_2 have equal value so that the voltage across each capacitor is equal to $\frac{V}{2}$. When the switch S_1 is ON, the voltage across capacitor C_1 is applied to the primary winding of transformer. Then $V_1 = \frac{V}{2}$.

There will be an induced voltage across both lower half and upper half of secondary winding of transformer. The induced voltage in each half of secondary winding is given by

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1} \frac{V}{2} \qquad \text{as} \quad V_1 = \frac{V}{2}$$

Since the diode D_1 is forward biased due to voltage of upper half of secondary winding of transformer and conducts, the output voltage is equal to

$$V_o = V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1} \frac{V}{2}$$

When the switch S_2 is turned ON, a reverse voltage of $-\frac{V}{2}$ is applied to the primary winding of transformer. Then $V_1 = -\frac{V}{2}$.

Consequently, there will be an induced voltage across both lower half and upper half of secondary winding of transformer. The induced voltage in lower half of secondary winding of transformer is equal to

$$V_2 = \frac{N_2}{N_1} V_1 = -\frac{N_2}{N_1} \frac{V}{2} \qquad \text{as} \quad V_1 = -\frac{V}{2}$$

Since the diode D_2 is forward biased due to voltage of lower half of secondary winding of transformer and conducts, the output voltage is equal to

$$V_o = \frac{N_2}{N_1} \frac{V}{2}$$

As the voltage of primary winding swings from $+\frac{V}{2}$ to $-\frac{V}{2}$, the each switch must be able to withstand at V. Therefore, the half bridge converter is preferred over push-pull converter for high voltage applications.

10.8.5 Full-Bridge Converter

The circuit configuration of full-bridge converter is shown in Fig. 10.53. This circuit consists of a rectifier, four switches S_1, S_2, S_3 and S_4, a transformer with mid-point on the secondary winding, two diodes D_1 and D_2, inductor L, capacitor C and load. The LC circuit is used as output filter.

When the switch S_1 and S_2 are turned ON and switches S_3 and S_4 are in OFF, the voltage V is applied to the primary winding of transformer. Then $V_1 = V$.

Subsequently, there will be an induced voltage across both lower half and upper half of secondary winding of transformer. The induced voltage in each half of secondary winding is expressed by

$$V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1} V \qquad \text{as } V_1 = V$$

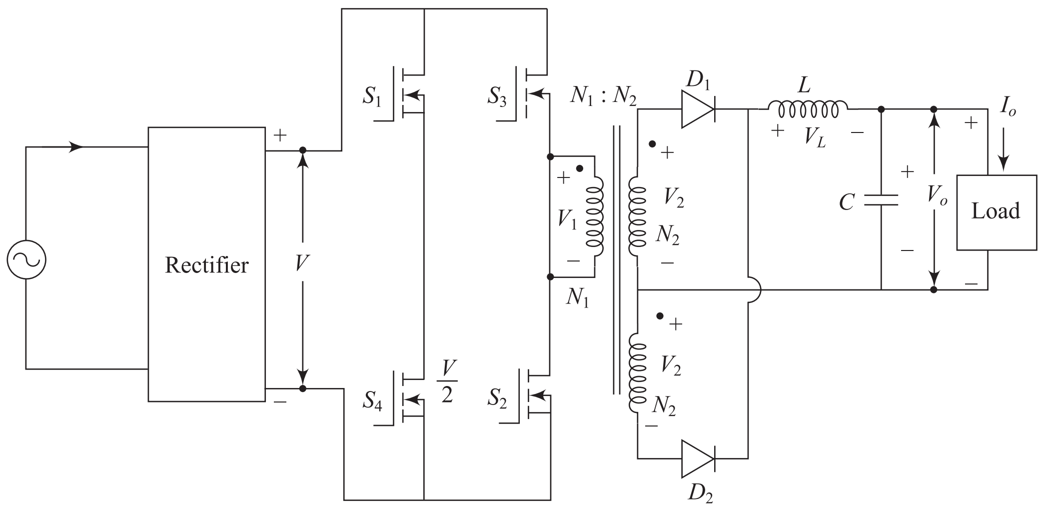


Fig. 10.53 *Full-bridge dc-to-dc converter*

As the diode D_1 is forward biased due to voltage of upper half of secondary winding of transformer and conducts, the output voltage is equal to

$$V_o = V_2 = \frac{N_2}{N_1} V_1 = \frac{N_2}{N_1} V$$

When the switch S_3 and S_4 are turned ON and switches S_1 and S_2 are in OFF, a reverse voltage of $-V$ is applied to the primary winding of transformer. Then $V_1 = -V$.

Accordingly, there will be an induced voltage across both lower half and upper half of secondary winding of transformer. The induced voltage in lower half of secondary winding of transformer is equal to

$$V_2 = \frac{N_2}{N_1} V_1 = -\frac{N_2}{N_1} V \qquad \text{as } V_1 = -V$$

Since the diode D_2 is forward biased due to voltage of lower half of secondary winding of transformer and conducts, the output voltage is equal to

$$V_o = \frac{N_2}{N_1} V$$

As the voltage of primary winding swings from $+V$ to $-V$, the each switch must be able to withstand at V. As the full-bridge converter operates with minimum voltage and current stress on switches, these converters are preferred over half bridge for high power applications.

10.9 VOLTAGE OR IMPULSE COMMUTATED CHOPPER

In voltage commutation, a conducting thyristor is switched OFF or commutated when a large reverse voltage is applied across the thyristor for a certain period of time. The time duration should be more than the thyristor recovery time t_q. Usually the reverse voltage is applied through a previously charged capacitor. When the reverse voltage is suddenly applied across the conducting thyristor, the anode current becomes zero and thyristor will be turned OFF. This chopper is also known *as impulse commutated chopper* or *parallel capacitor turn-OFF chopper*. Generally these chopper circuits are used in high power circuits. This circuit is also called *oscillation chopper*.

Figure 10.54 shows the circuit topology of voltage commutated chopper which consists of two thyristors such as main thyristor T_1 and auxiliary thyristor T_A, free wheeling diode D_F, *RLE* load and a commutation circuit. The commutation is formed by capacitor C, diode D, inductor L and auxiliary thyristor T_A. The resistance R_C is used to charge the capacitor. For analysis the operation of the chopper, following assumptions are required:

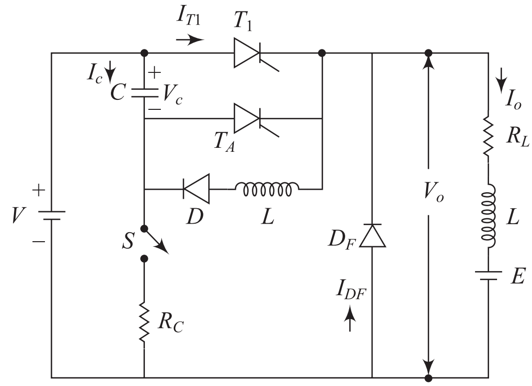


Fig. 10.54 *Circuit topology of voltage commutated chopper*

1. All semiconductor switching devices are ideal. The voltage drop across each device is zero and the loss is zero.
2. The load current is constant during the entire commutation period.
3. The inductor L should operate in linear region and there is no saturation.

Before starting the operation of chopper, the switch S will be ON and the capacitor C will be charged to input voltage V with polarity as shown in Fig. 10.54. The chopper circuit should operate in the following modes as given below.

Mode I ($0 < t < t_1$) At $t = 0$, the main thyristor T_1 is turned ON. Then load current I_o flows from dc source to load through thyristor T_1. The diode D is forward biased and conducts. Then current i_C flows through the following path:

$$T_1 - L - D - C$$

Subsequently, the capacitor C discharges through the LC resonating circuit. The current through the thyristor T_1 is

$$i_{T1} = i_o + i_C$$

The equivalent circuit of mode-I during $0 < t < t_1$ is illustrated in Fig. 10.55(a). At $t = t_1$ capacitor C is recharged to $-V$ with reverse polarity as depicted in Fig. 10.56 and capacitor current becomes zero. Since current can flow in the forward direction only though diode D, diode will be OFF at $t = t_1$.

Mode II ($t_1 < t < t_2$) Just after $t = t_1$, the main thyristor continue its conduction and the load current I_o remains constant and flows from dc source to load through thyristor T_1. The equivalent circuit of mode-II during $t_1 < t < t_2$ is illustrated in Fig. 10.55(b).

Mode III ($t_2 < t < t_3$) At $t = t_2$, a triggering pulse is applied to auxiliary thyristor T_A and it is turned on to commutated the main thyristor T_1. As soon as T_A is turned ON, the capacitor voltage is applied across T_1 and the load voltage is suddenly increased from V to $V + V_C$ or $2V$ as $V_C = V$. After that the output voltage decreases linearly as capacitor C discharges. The load current I_o flows through capacitor C and thyristor T_A. Then current i_C flows through the following path:

$$C - T_A - \text{LOAD}$$

The capacitor recharges to $+V$. At $t = t_3$, the output voltage is equal to zero as shown in Fig. 10.56. The equivalent circuit of mode-III during $t_2 < t < t_3$ is illustrated in Fig. 10.55(c).

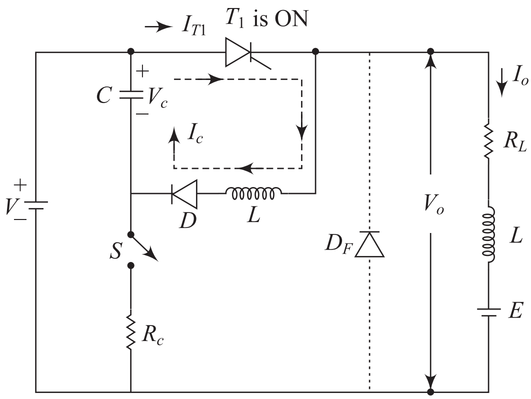


Fig. 10.55(a) *Equivalent circuit in Mode I during $0 < t < t_1$*

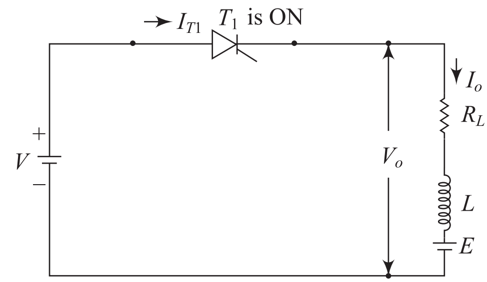


Fig. 10.55(b) *Equivalent circuit in Mode II during $t_1 < t < t_2$*

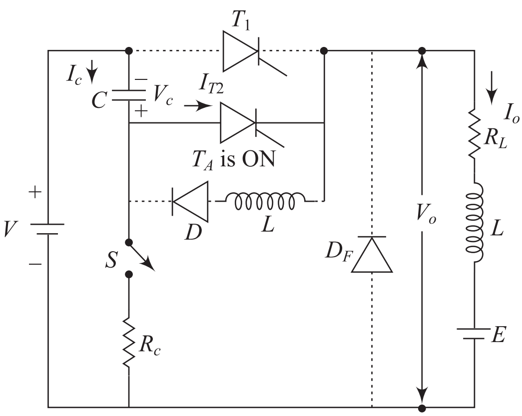


Fig. 10.55(c) *Equivalent circuit in Mode III during $t_2 < t < t_3$*

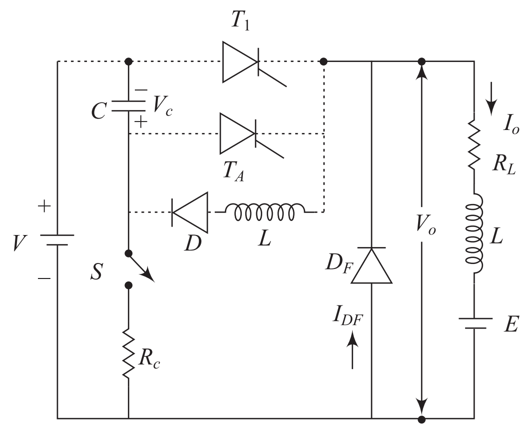


Fig. 10.55(d) *Equivalent circuit in Mode IV during $t_3 < t < t_4$*

Mode IV ($t_3 < t < t_4$): When the capacitor C is slightly over charged than V, the current through the capacitor becomes zero. Due to load commutation, thyristor T_A will be turned OFF. Now all semiconductor switches are in OFF condition and the load current I_o flows through free wheeling diode D_F as shown in Fig. 10.56. The equivalent circuit of mode-IV during $t_3 < t < t_4$ is illustrated in Fig. 10.55(d).

After that the switching cycle will be repeated from Modes I to IV. Figure 10.56 shows the voltage and current waveforms of voltage commutated chopper.

10.9.1 Design of Commutating Capacitance *C* and Commutating Inductance *L*

In the mode I, the capacitor current i_C flows though inductance L and diode D. The KVL equation for the loop $T_1 - L - D - C$ is

$$L\frac{di_C}{dt} + \frac{1}{C}\int i_C dt = 0$$

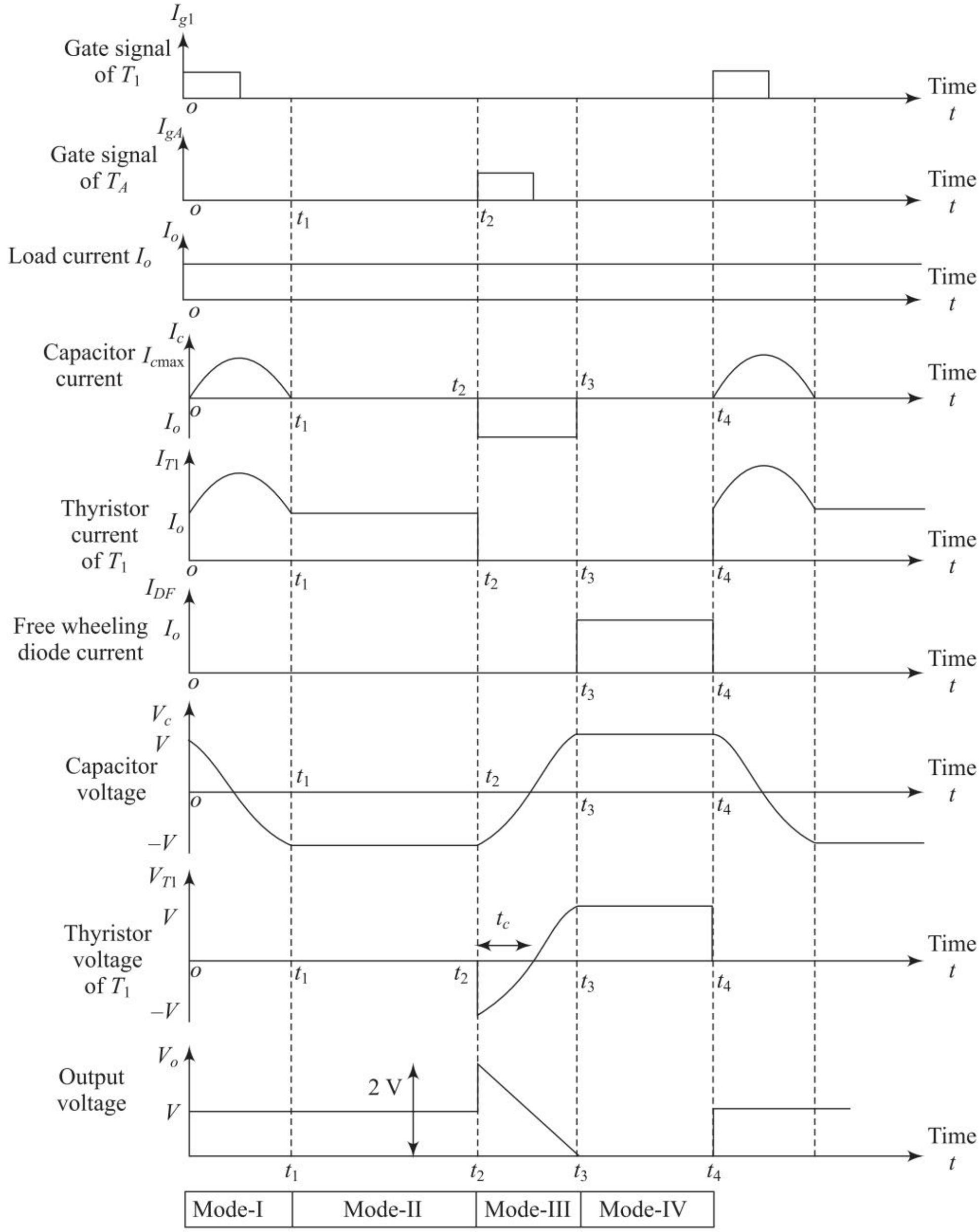


Fig. 10.56 *Voltage and current waveforms of voltage commutated chopper*

At $t = 0$, $i_C = 0$ and $V_C = V$. Then the capacitor current can be expressed as

$$i_C = \frac{V}{\sqrt{L/C}} \sin \omega_O t \quad \text{and the capacitor voltage is equal to}$$

$$V_C = V \cos \omega_o t \qquad \text{for} \qquad 0 \le \omega_o t \le \pi$$

where, $\omega_O = \frac{1}{\sqrt{LC}}$, and the maximum capacitor current is $i_{C\max} = V\sqrt{\frac{C}{L}}$

At $t = t_1$, current $i_C = 0$ and $\omega_o t_1 = \pi$

Therefore, $t_1 = \frac{\pi}{\omega_o} = \pi\sqrt{LC}$

For proper commutation, $i_{C\max}$ should be less than or equal to the load current I_o

Therefore, $V\sqrt{\frac{C}{L}} \le I_o$ as $i_{C\max} \le I_o$

or $\left(\frac{V}{I_o}\right)^2 \le \frac{L}{C}$ or $\left(\frac{V}{I_o}\right)^2 C \le L$

For constant load during $t_2 < t < t_3$, the value of capacitor should be such that

$$I_o = \frac{dQ}{dt} = C\frac{V}{t_C}$$

where, t_C is the turn-OFF time of commutation circuit. The value of t_C must be greater than thyristor turn-OFF time t_q and the value of C is given by

$$C = \frac{I_o}{V}t_C = \frac{I_o}{V}(t_q + \Delta t) \qquad \text{as } t_C = t_q + \Delta t$$

It is clear from Fig. 10.57, the load voltage is V at $t = 0$ and after $t = 0$ voltage increases from V to $2V$. At $t = t_2$, load voltage is $2V$. Subsequently, the load voltage decreases and reaches to 0 V at $t = t_3$. Therefore, average load or output voltage is equal to

$$V_o = \frac{Vt_2 + \frac{1}{2}2V(t_3 - t_2)\cdot}{T} = \frac{V}{T}(t_2 + (t_3 - t_2)) = \frac{V}{T}[T_{\text{ON}} + (t_3 - t_2)] \tag{10.23}$$

In the time interval, $(t_3 - t_2)$, the voltage across capacitor changes from $-V$ to $+V$ and the total change in voltage across capacitor is $2V$.

Therefore, $I_o = C\frac{2V}{t_3 - t_2}$ or, $t_3 - t_2 = \frac{2CV}{I_o}$

After substituting the value of $t_3 - t_2 = \frac{2CV}{I_o}$ in Eq. (10.23), we obtain

$$V_o = \frac{V}{T}[T_{\text{ON}} + (t_3 - t_2)] = \frac{V}{T}\left[T_{\text{ON}} + \frac{2CV}{I_o}\right] = V\cdot\frac{T'_{\text{ON}}}{T}$$

as $T'_{\text{ON}} = \left[T_{\text{ON}} + \frac{2CV}{I_o}\right]$ is known as an effective ON period.

The minimum ON period of chopper is equal to $t_1 = \frac{\pi}{\omega_o} = \pi\sqrt{LC}$

The minimum duty cycle is $\alpha_{\min} = \frac{t_1}{T} = \pi f\sqrt{LC}$

The minimum load or output voltage is $V_{o\min} = \alpha_{\min}V + \frac{1}{2}\frac{2V \times 2t_c}{T} = \alpha_{\min}V + 2fVt_c$ (10.24)

After substituting the value of $\alpha_{\min}$ in Eq. (10.24), we get

$$V_{o l\min} = \alpha_{\min} V + 2fVt_c = \pi f\sqrt{LC}V + 2fVt_c = V(\pi f\sqrt{LC} + 2ft_c)$$

The maximum ON period of chopper is equal to $\alpha_{\max} = \dfrac{T - 2t_c}{T} = (1 - 2ft_c))$

The maximum load or output voltage is

$$V_{o l\min} = \alpha_{\max} V + \frac{1}{2}\frac{2V \times 2t_c}{T} = \alpha_{\max} V + 2fVt_c \tag{10.25}$$

After substituting the value of $\alpha_{\max}$ in equation (10.25), we obtain

$$V_{o l\max} = \alpha_{\max} V + 2fVt_c = (1 - 2ft_c)V + 2fVt_c = V$$

Example 10.32 A battery operated electric car is operated by voltage commuted chopper. The battery voltage is 20 V, starting current is 5 A and thyristor turn-OFF time is 20 μs. Determine the commutating inductor L and commutating capacitor C.

Solution

Given: $V = 20$ V, $I_o = 5$ A, $t_q = 20$ μs

The turn-OFF time of circuit is $t_c = t_q + \Delta t = (20 + 20)$ μs = 40 μs

The value of C is $C = \dfrac{I_o}{V} t_C = \dfrac{I_o}{V}(t_q + \Delta t) = \dfrac{5}{20}(20 + 20)\mu\text{F} = 10\,\mu\text{F}$

The value of inductance L is $L \geq \left(\dfrac{V}{I_o}\right)^2 C = \left(\dfrac{20}{5}\right)^2 \times 10 \times 10^{-6} = 160\,\mu\text{H}$

Example 10.33 A voltage commutated chopper has the following parameters:

$V = 200$ V, RLE load parameters ($R = 0.5\ \Omega$ $L = 4$ mH, $E = 50$ V)

The commutation circuit parameters are: $L = 25$ μH, $C = 50$ μF, $T_{\text{ON}} = 500$ μs, $T = 1000$ μs.

If the load current is 60 A, determine (a) effective ON time of chopper, (b) the peak current flows through T_1 and T_A, (c) turn-OFF time of T_1 and T_A and (d) total commutation interval.

Solution

Given: $V = 200$ V, $R = 0.5\ \Omega$, $L = 4$ mH, $E = 50$ V, $L = 25$ μH, $C = 50$ μF, $T_{\text{ON}} = 500$ μs, $T = 1000$ μs, $I_o = 60$ A

(a) The effective ON period of chopper is $T'_{\text{ON}} = \left[T_{\text{ON}} + \dfrac{2CV}{I_o}\right] = 500\,\mu\text{s} + \dfrac{2 \times 50 \times 200}{60}\mu\text{s} = 833.33\,\mu\text{s}$

(b) The peak current through T_1 is $I_{T1\max} = I_o + V\sqrt{\dfrac{C}{L}} = 60 + 200\sqrt{\dfrac{50}{25}} = 342.842$ A

(c) The turn-OFF time of T_1 is $t_c = \dfrac{CV}{I_o} = \dfrac{50 \times 10^{-6} \times 200}{60} s = 166.667\,\mu\text{s}$

The turn-OFF time of T_A is $t_{c1} = \dfrac{\pi}{2}\sqrt{LC} = \dfrac{\pi}{2}\sqrt{25 \times 10^{-6} \times 50 \times 10^{-6}} = 55.558\,\mu\text{s}$

(d) Total commutation interval is $2t_c = 2 \times 166.667 = 333.334\,\mu\text{s}$

Example 10.34 A voltage commutated (impulse commutated) chopper is connected to an inductive load with a load current of 100 A. If input voltage $V = 200$ V, chopping frequency is 250 Hz, turn-OFF time of thyristor is 20 μs, peak current through main thyristor is 1.5 times of constant load current, determine (a) the value of commutating components L and C and (b) minimum and maximum output voltage. Assume safety factor is 2.

Solution

Given: $I_o = 100$ A, $V = 200$ V, $f = 250$ Hz, $t_q = 20$ μs

(a) As the safety factor is 2, the turn-OFF time of commutation circuit for main thyristor is

$$t_c = 2t_q = 2 \times 20\ \mu s = 40\ \mu s$$

The value of C is $C = \frac{I_o}{V} t_C = \frac{100}{200} \times 40\ \mu F = 20\ \mu F$

The value of inductance L is $L \geq \left(\frac{V}{I_o}\right)^2 C = \left(\frac{200}{100}\right)^2 \times 20 \times 10^{-6} = 80\ \mu H$

(b) The minimum value of duty cycle is $\alpha_{\min} = \pi f \sqrt{LC} = \pi \times 250 \sqrt{80 \times 10^{-6} \times 20 \times 10^{-6}} = 0.0314$

The minimum output voltage is $V_{o|\min} = \alpha_{\min} V + 2fVt_c = 0.0314 \times 200 + 2 \times 250 \times 200 \times 40 \times 10^{-6} = 10.28$ V

The maximum output voltage is $V_{o\max} = V = 200$ V

10.10 LOAD COMMUTATED CHOPPER

Figure 10.57 shows a load commutated chopper circuit. This circuit consists of four thyristors T_1, T_2, T_3, T_4, a commutating capacitor C, free wheeling diode D_F and a load. In this circuit, thyristor T_1 and T_2 work at a time where as thyristors T_3 and T_4 also work simultaneously. The load current I_o is shared by either T_1 and T_2 or T_3 and T_4 alternatively. When T_1 and T_2 are conducting, thyristors T_1 and T_2 are behave as main thyristors and T_3, T_4 and C are act as commutation components. Similarly, if T_3 and T_4 are conduction, T_1, T_2 and C act as commutation components. When T_1, T_2, T_3 and T_4 are not conducting, the load current flows through the freewheeling diode.

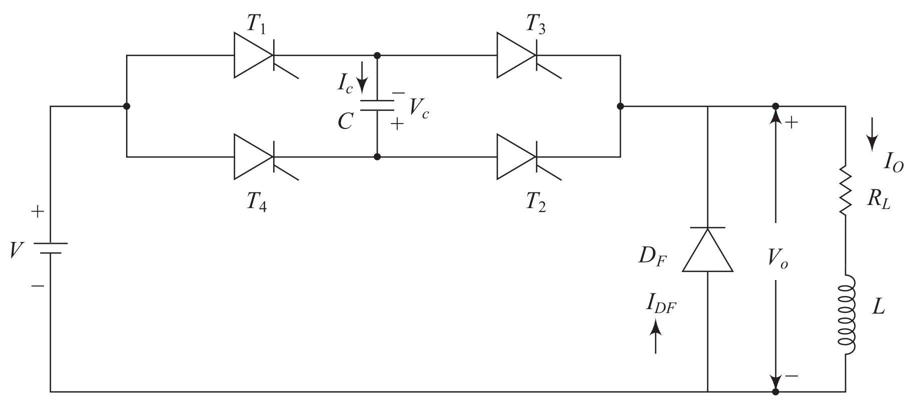


Fig. 10.57 *Load Commuted Chopper Circuit*

For load commutated chopper circuit analysis, we assume that capacitor C is initially charged to V with upper plate negative and lower plate positive as depicted in Fig. 10.57. This circuit operates in three different operating modes which as explained below:

Mode-I When the capacitor C is charged with lower plate positive and upper plate negative, the load commutated chopper circuit is ready to start operation. At $t = 0$, firing pulses are applied to T_1 and T_2 and subsequently thyristors T_1 and T_2 will be turned on. Fig. 10.58(a) shows the equivalent circuit for

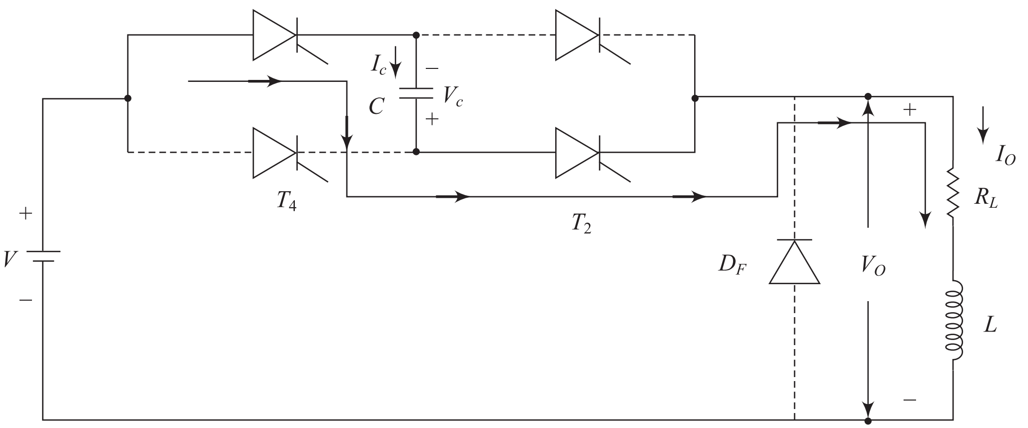


Fig. 10.58 *(a) Equivalent circuit during mode I, $0 \leq t < t_1$*

mode I. The output voltage across load is $V_o = V + VC = V + V = 2V$ as $V_C = V$. Then load current flows from dc supply to load through T_1, C and T_2. At $t = 0$, $V_{T3} = V_{T4} = -V$. Thyristors T_3 and T_4 are reverse biased and they are operating in off state. At $t = 0$, capacitor C is charged to +V and load current flows through capacitor, subsequently capacitor C is charged linearly by constant load current I_o. At $t = t_1$, the capacitor voltage is $V_C = -V$ and the load voltage falls from 2 V to 0 as $V_o = V - V_C = 0$.

Mode II At $t = t_1$, capacitor C is slightly overcharged by ΔV, the upper plate of capacitor is positively charged and lower plate is negatively charged. Since freewheeling diode is forward biased and load current is transferred from T_1 and T_2 to D_F, and hence load current flows through freewheeling diode D_F. Until trigger pulses are applied to T_3 and T_4, $V_{T3} = V_{T4} = V$, $V_{T1} = V_{T2} = -\Delta V$ as capacitor C is overcharged by a small voltage ΔV. In the time interval $t_2 - t_1$, $V_C = -V$, $V_o = 0$, $i_c = 0$, $i_{fd} = I_o$, $i_{T1} = i_{T2} = i_{T3} = i_{T4} = 0$. Figure 10.58(b) shows the equivalent circuit during mode II.

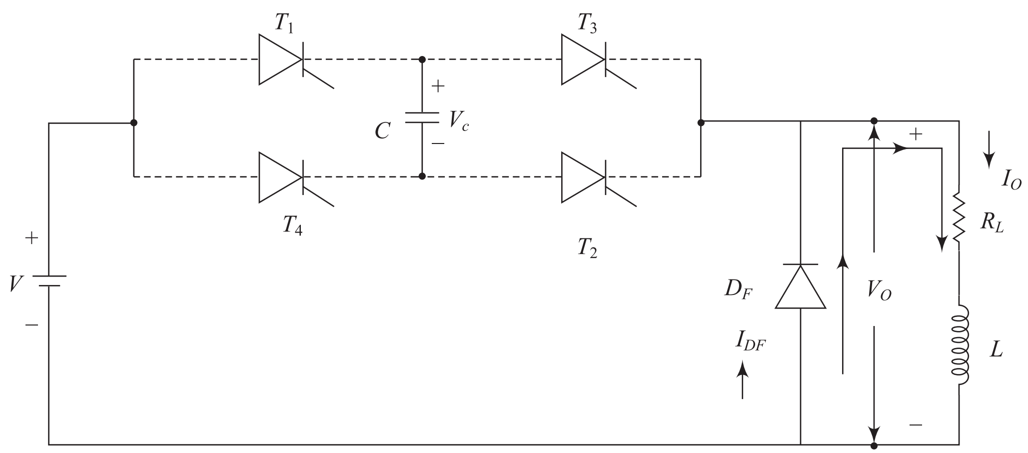


Fig. 10.58 *(b) Equivalent circuit during mode II, $t_1 \leq t < t_2$*

Mode III When triggering pulses are applied to T_3 and T_4 at $t = t_2$, T_3 and T_4 are conducting, the output voltage is $V_o = V + V_C = 2V$ as $V_C = V$. Thyristors T_1 and T_2 are reverse biased by V_C and these devices will be turned off at $t = t_2$. The load current flows through T_4, C, T_3 and the capacitor C is charged linearly from $-V$ to $+V$. The output voltage is 2 V at t_2 and it falls from 2 V to zero at t_3. At $t = t_3$, capacitor C is small overcharged, free wheeling diode gets forward biased and load current passes through freewheeling diode and load. Figure 10.59(c) shows the equivalent circuit during mode III. The voltage and current waveforms of load commuted chopper is depicted in Fig. 10.59.

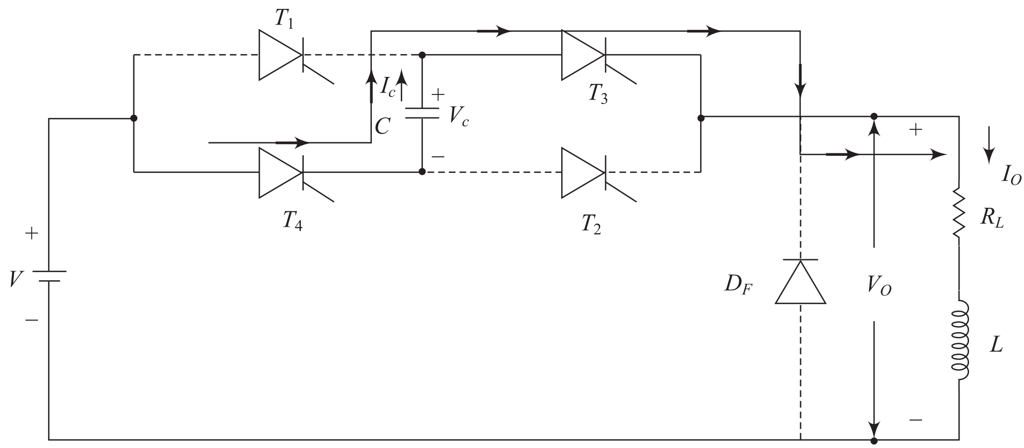


Fig. 10.58 *(c) Equivalent circuit during mode III, $t_2 \le t < t_3$*

10.10.1 Design of Commutation Capacitance C

Assume load current I_o is constant, capacitor voltage V_C changes from $-V$ to V in time interval T_{ON}, i.e., $0 \le t < t_1$.

The change in voltage is 2 V in the time interval T_{ON} and the current I_o is expressed by

$$I_o = C\frac{2\text{ V}}{T_{ON}}$$

Therefore, $C = \dfrac{I_o T_{ON}}{2\text{ V}}$

The output voltage is $V_o = \dfrac{1}{2}\cdot 2\text{ V}\cdot T_{ON}\cdot\dfrac{1}{T} = VT_{ON}f$

Since $T_{ON} = \dfrac{2VC}{I_o}$, the average output voltage is equal to

$$V_o = VT_{ON}f = V\cdot\frac{2VC}{I_o}f = \frac{2V^2Cf}{I_o}$$

The minimum chopping period is $T_{minimum} = T_{ON} = T_{min}$.
Then maximum chopping frequency is

$$f_{max} = \frac{1}{T_{min}} = \frac{1}{T_{ON}}$$

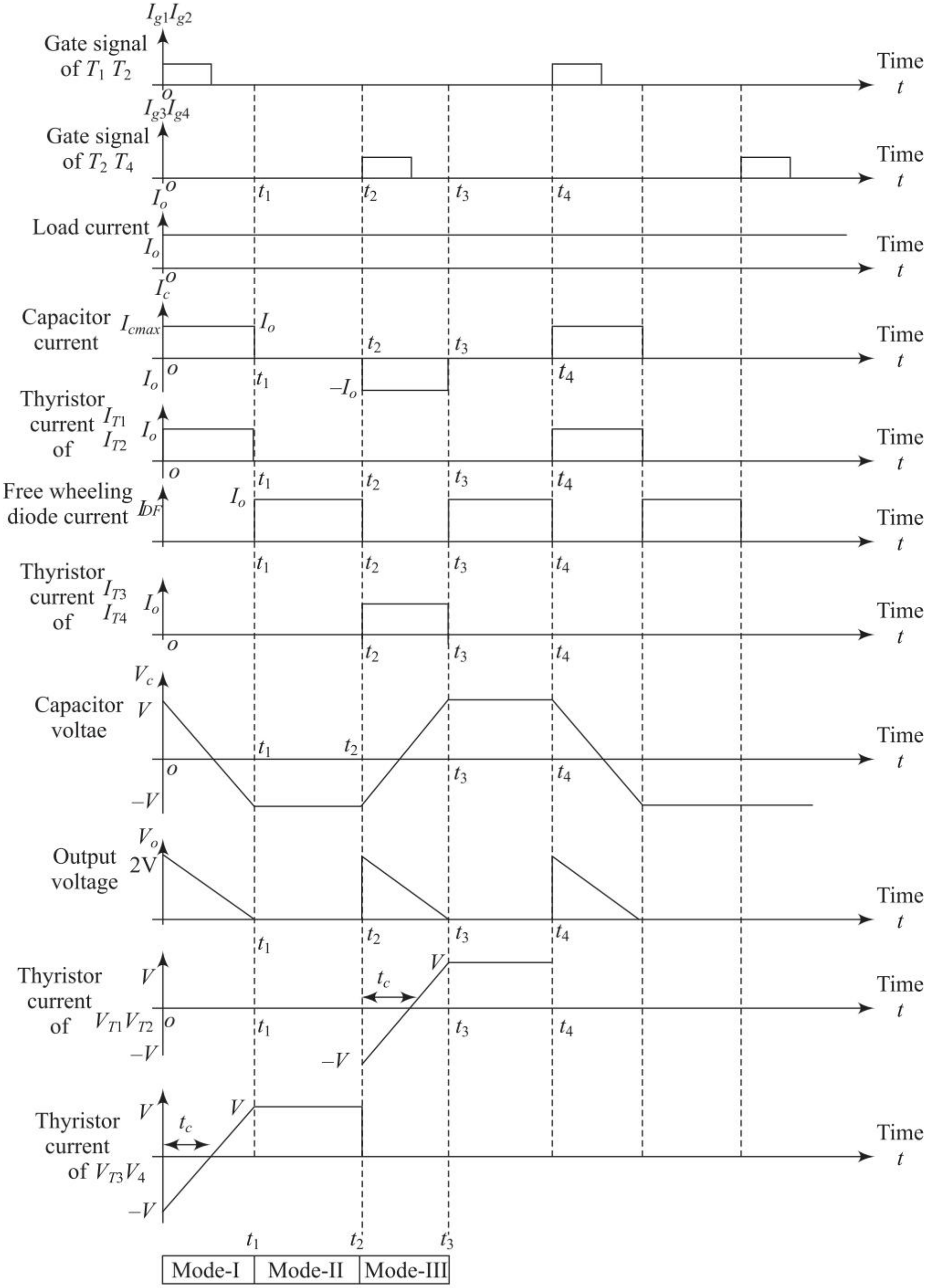


Fig. 10.59 *Voltage and current waveforms of load commuted chopper*

The value of capacitor in terms of maximum chopping frequency is given by

$$C = \frac{I_o}{2V}\frac{1}{f_{\max}}$$

It is clear from Fig. 10.61 that the circuit turn–off time for each thyristor is

$$t_c = \frac{1}{2}T_{\text{ON}} = \frac{1}{2}C \cdot \frac{2V}{I_o} = \frac{CV}{I_o}$$

Then total commutation interval is

$$T_{\text{ON}} = \frac{2CV}{I_o}$$

The load commutated chopper has the following advantages and disadvantages.

Advantages

(i) This circuit has the capability of commutation at any amount of load current.
(ii) Commutating inductor is used in this circuit. Hence the size of circuit will not be bulky.
(iii) Since the circuit operates at high frequency in order of kHz, the size of filter is less.

Disadvantages

(i) The peak output voltage is equal to the two times input voltage.
(ii) In high frequency applications, the switching frequency is high and switching losses are more. Hence, in high power applications, the efficiency is low.
(iii) Freewheeling diode should have high voltage handling capability i.e. 2 V, twice the supply voltage.
(iv) The commutation capacitor C should carry full load current at a frequency of half the chopping frequency.
(v) Always one pair of thyristors will be conducting where as other pair of thyristor will not be conducting.

Example 10.35 A load commutated chopper fed from 230 V DC supply has a constant load current of 30 A for a duty cycle of 0.4 and a chopping frequency of 2 kHz, determine (a) the value of commutating capacitance, (b) average output voltage, (c) circuit turn off time for thyristors and (d) total commutation interval.

Solution

The value of capacitor in terms of maximum chopping frequency is

$$C = \frac{I_o}{2V}\frac{1}{f_{\max}} = \frac{30}{2 \times 230 \times 2 \times 10^3} = 32.60\ \mu\text{F}$$

(b) The average output voltage is

$$V_o = VT_{\text{ON}}f = V\frac{T_{\text{ON}}}{T} = 230 \times 0.4 = 92\ \text{V}$$

(c) The circuit turn-off time for each thyristor is

$$t_c = \frac{1}{2}T_{\text{ON}} = \frac{1}{2}DT = \frac{1}{2} \times 0.4 \times \frac{1}{2 \times 10^3} = 100\ \mu\text{s}$$

(d) The total commutation interval is

$$T_{\text{ON}} = 200\ \mu\text{s}$$

10.11 CURRENT COMMUTED CHOPPER

Figure 10.60 shows a current commuted chopper circuit where T_1 is the main thyristor, T_A is the auxiliary thyristor, capacitor C, inductor L, diodes D_1 and D_2, freewheeling diode D_F and the charging resistance R_C.

For current commuted chopper circuit analysis, we assume that

(i) The load current I_o is constant,
(ii) All semiconductor switches T_1, T_A, D_1, D_2, and D_F are ideal and
(iii) The charging resistance R_C is very large so that it can be considered as open circuit during commutation interval.

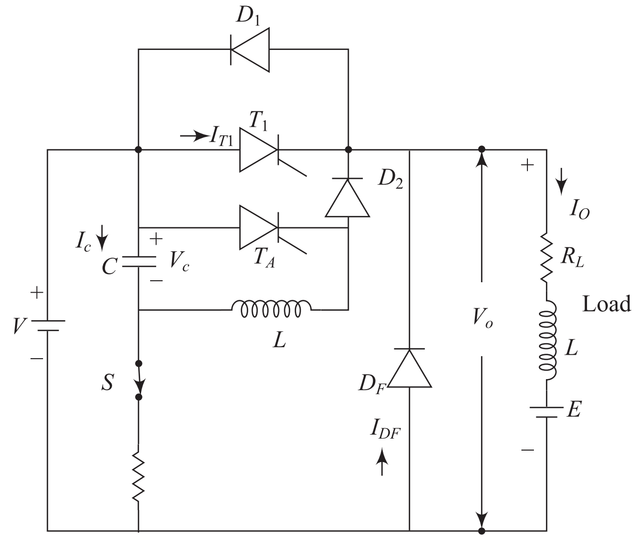


Fig. 10.60 *Current commuted chopper circuit*

Just like voltage commutation chopper, the energy for current commutation comes from the stored energy in the capacitor. Initially, capacitor C is charged to a voltage V so that energy for commutation process is readily available. The capacitor C is charged to a voltage V through voltage source V, capacitor C and the charging resistor R_C. The upper plate of capacitor is positively charged and lower plate is negatively charged. When the main thyristor T_1 is fired at $t = 0$ so that the output voltage $V_o = V$ and current flows through load is $i_o = I$ up to $t = t_1$. Whenever the auxiliary thyristor T_A is turned on, the commutation process of main thyristor T_1 starts. The complete commutation process of T_1 is divided into following modes as given below:

Mode I Assume thyristor T_1 is conducting from $t = 0$ and at $t = t_1$ triggering pulses are applied to thyristor T_A to turn-off thyristor T_1. Subsequently, an oscillating current $i_c = \frac{V}{\omega_o L}\sin\omega_o t$ flows through C, T_A and L. During the time interval $t_1 \le t \le t_2$, capacitor current i_c and capacitor voltage v_c vary sinusoidally as depicted in Fig. 10.60. When capacitor voltage $V_C = 0$, the current i_c is maximum negative. At $t = t_2$, i_c starts to flow in reverse direction, the auxiliary thyristor T_A becomes naturally commutated. At $t = t_2$, the voltage across capacitor is $V_c = -V$ with lower plate positive and upper plate negative. Since T_1 is unaffected, load current I_o and output voltage V_o will be constant. Figure 10.61(a) shows the equivalent circuit of current commuted chopper.

Mode II At $t = t_2$ thyristor T_A is turned off, the oscillating current i_c flows through C, L, D_2 and T_1 as depicted in Fig. 10.61(b). Just after $t = t_2$, current i_c flows through T_1 and not through D_1 as D_1 is reverse biased by the small voltage drop across T_1. In thyristor T_1, current i_c flows in opposite to the load current I_o. Therefore, the current flows through thyristor T_1 is $i_{T1} = I_o - i_c$. At $t = t_3$, i_c becomes equal to I_o. Consequently, $i_{T1} = I_o - i_c = I_o - I_o = 0$ and the main thyristor T_1 will be turned off at $t = t_3$. As the oscillating current flows through T_1 turns it off, it is called current commutated chopper. During the time interval $t_2 \le t \le t_3$, the output voltage across load is V.

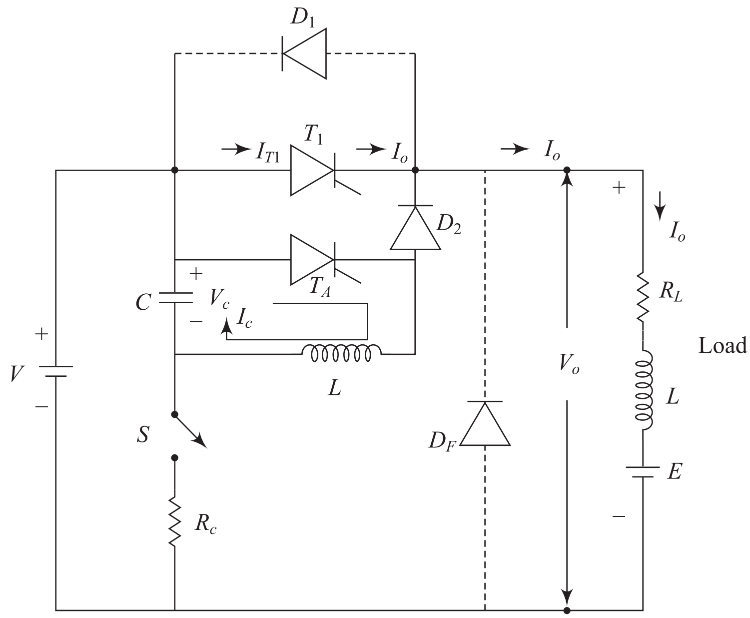


Fig. 10.61 *(a) The equivalent circuit of current commuted chopper $t_1 < t < t_2$*

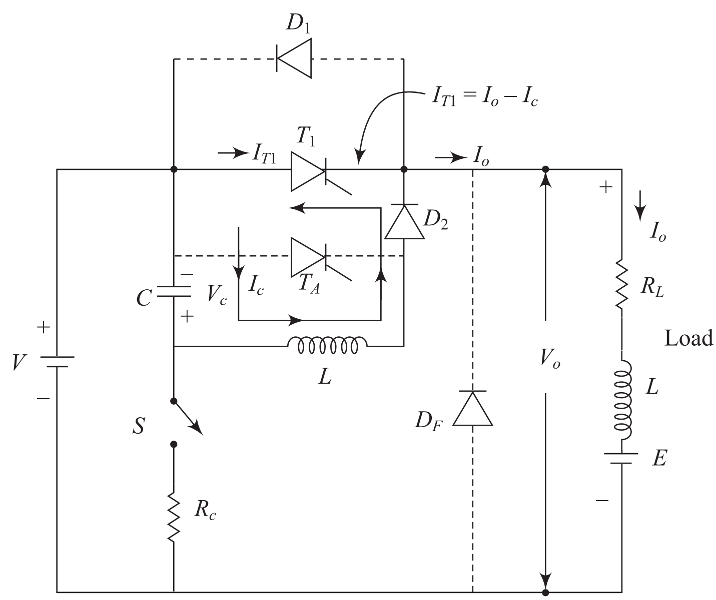


Fig. 10.61 *(b) The equivalent circuit of current commuted chopper $t_2 < t < t_3$*

Mode III At $t = t_3$, thyristor T_1 is turned off, and the current i_c becomes more than load current I_o. Just after $t = t_3$, i_c provides the load current I_o and the excess current $i_c - I_o$ flows through diode D_1 as it is conducting. Hence, $i_{D_1} = i_c - I_o$. Due to voltage drop across D_1, thyristor T_1 is reverse biased for duration $t_c = t_4 - t_3$. At $t = t_4$, the capacitor voltage V_C is greater than V and the freewheeling becomes conducting, otherwise mode –IV operation starts. During mode III, capacitor current reaches peak value $I_{c\max} = \dfrac{V}{\omega_o L}$ when $V_C = 0$. After this instant, capacitor voltage reverses. At $t = t_4$, the upper plate of capacitor is positive and lower plate of capacitor becomes negative. Figure 10.61(c) shows the equivalent circuit of current commuted chopper in this operating mode.

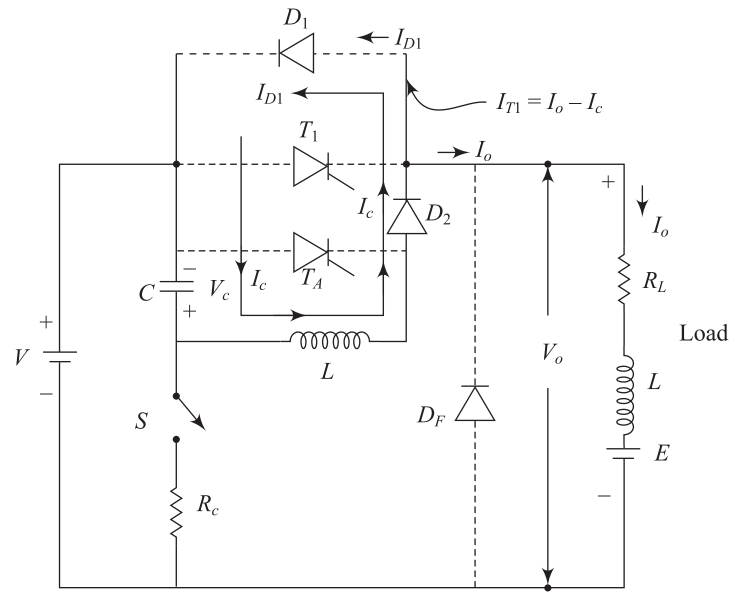


Fig. 10.61 *(c) The equivalent circuit of current commuted chopper $t_3 < t < t_4$*

Mode IV At $t = t_4$, the current i_c is equal to I_o. Consequently, $i_{D_1} = i_c - I_o = 0$ and diode D_1 is turned off. Just after $t = t_4$, a constant load current flows through C, L, D_2 and load. Subsequently, the capacitor C is charged linearly to dc input voltage V at $t = t_5$. During the time interval $t_4 < t < t_5$, $i_c = I_o$. Since D_1 is turned off at $t = t_4$, $V_{T_1} = V_{T_A} = V_C$. The output voltage is $V_o = V - V_C$. At $t = t_5$, $V_C = V$ and the output voltage is equal to $V_o = V - V_C = 0$. During the time interval $t_4 < t < t_5$, capacitor voltage increases linearly. Subsequently, the load voltage V_o decreases to zero during this interval. Figure 10.61(d) shows the equivalent circuit of current commuted chopper in this operating mode.

Mode V At $t = t_5$, the capacitor C is overcharged which is slightly more than input voltage V. Consequently, the free wheeling diode becomes forward biased and starts to conduct. Then load current I_o flows through freewheeling diode D_F. The load voltage at $t = t_5$ is equal to zero. Since current i_c is not zero, the capacitor C is still connected to load through C, L, D_2 and load. As C is overcharged when the stored energy of L is transferred to C. At $t = t_6$, $i_c = 0$ and capacitor voltage V_c is more than V. During the time interval $t_5 < t < t_6$, load current is equal to $I_o = I_C + I_{D_F}$. At $t = t_6$, $I_C = 0$ and $I_o = I_{D_F}$. The commutation process will be completed at $t = t_6$. Figure 10.61(e) shows the equivalent circuit of current commuted chopper in this operating mode. Current and Voltage waveforms of current commuted chopper are depicted in Fig. 10.62. The *total turn-off time* or commutation interval $t_6 - t_1$ can be computed by the following method:

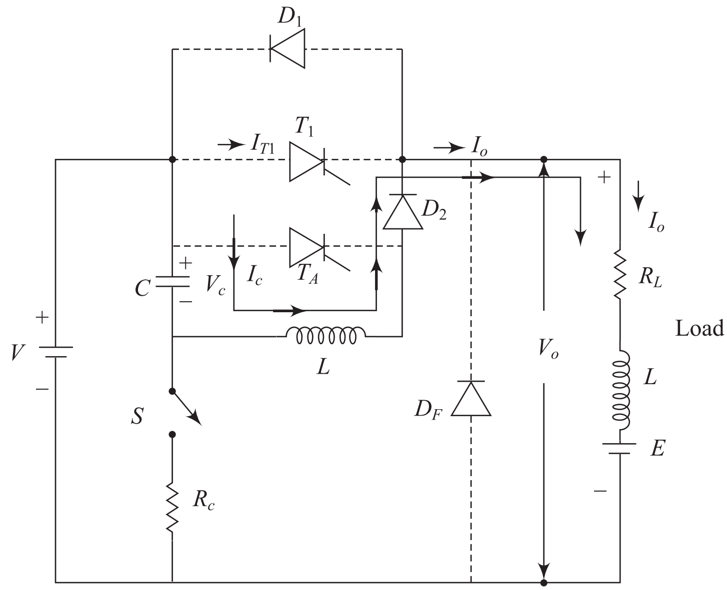


Fig. 10.61 *(d) The equivalent circuit of current commuted chopper $t_4 < t < t_3$*

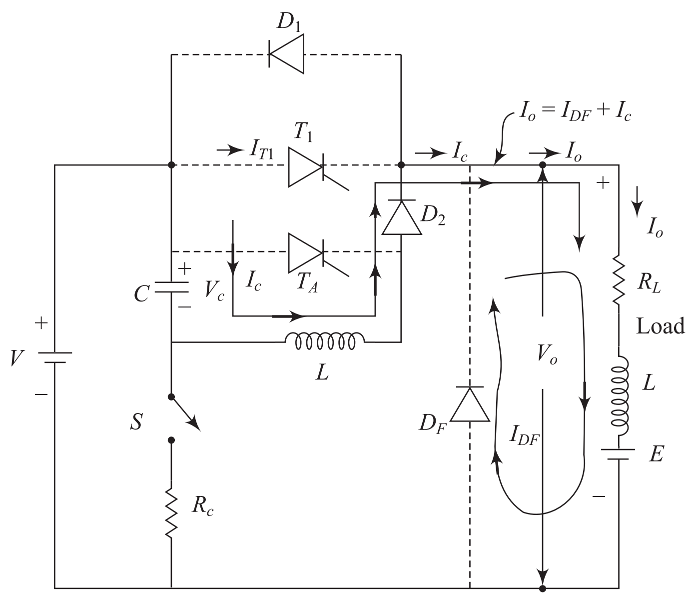


Fig. 10.61 *(e) The equivalent circuit of current commuted chopper $t_5 < t < t < t_6$*

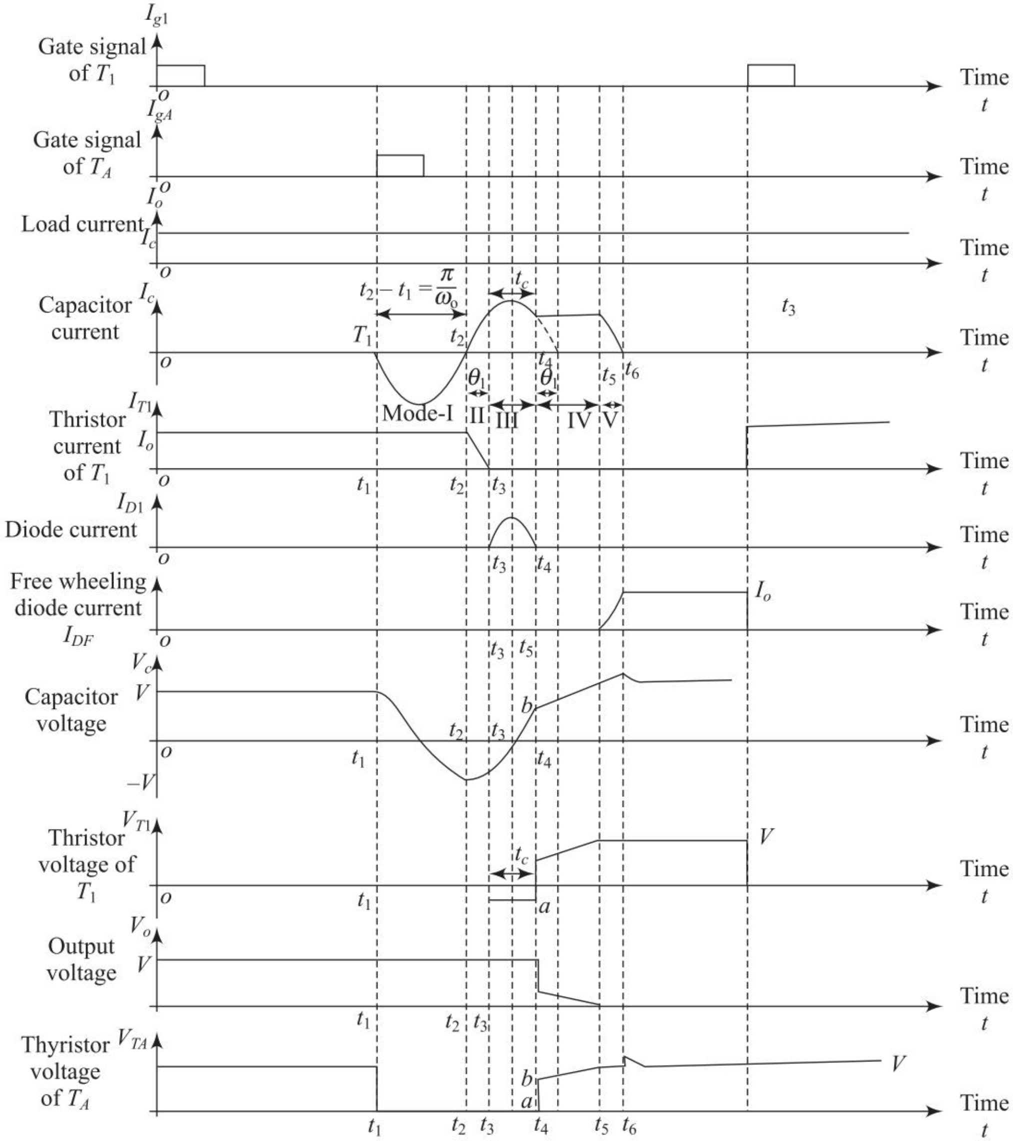


Fig. 10.62 *Current and Voltage waveforms of current commuted chopper*

10.11.1 Total Turn-off Time or Total Commutation Interval

The total turn-off time or total commutation interval of current commuted chopper is $t_6 - t_1$ and it is expressed by the following components:

$$(t_6 - t_1) = (t_6 - t_5) + (t_5 - t_4) + (t_4 - t_2) + (t_2 - t_1)$$

Time interval ($t_2 - t_1$) In the time interval $(t_2 - t_1)$, current waveform i_c completes one negative half cycle of π radians at the oscillating frequency ω_o.

$$\text{time period of half cycle of oscillating current} = (t_2 - t_1) = \frac{\pi}{\omega_o} = \pi\sqrt{LC}$$

Time interval ($t_4 - t_2$) At $t = t_2$, $i_c = 0$ and at $t = t_4$, $i_c = I_o$ after passing through its peak. During the time interval $(t_4 - t_2)$, current i_c cover $\pi - \pi_1$ radian. The value of $(t_4 - t_2)$ is expressed by

$$(t_4 - t_2) = \frac{\pi - \theta_1}{\omega_o} = (\pi - \theta_1)\sqrt{LC}$$

Time interval ($t_5 - t_4$) At $t = t_4$, the capacitor voltage is $V_C = V \sin(90 - \theta_1)$. *The voltage across C at* $t = t_5$ *is* $V_C = V$

The increase in voltage across capacitor during *time interval* $(t_5 - t_4)$ is $V - V\sin(90 - \theta_1)$

Since $i = C\dfrac{dv}{dt}$ *and* I_o *is constant,*

$$I_o = C\frac{V - V\sin(90 - \theta_1)}{t_5 - t_4}$$

or
$$t_5 - t_4 = CV\frac{1 - \sin(90 - \theta_1)}{I_o} = CV\frac{1 - \cos\theta_1}{I_o}$$

Time interval ($t_6 - t_5$) During *time interval* $(t_6 - t_5)$, the current $i_c = I_o \cos \omega_o t$. It is clear from Fig. 10.62 that $(t_6 - t_5)$ is equal to $\dfrac{\pi}{2}$ radian of a sine wave,

$$(t_6 - t_5) = \frac{\pi}{2\omega_o} = \frac{\pi}{2}\sqrt{LC}$$

$$\begin{aligned}
(t_6 - t_1) &= (t_6 - t_5) + (t_5 - t_4) + (t_4 - t_2) + (t_2 - t_1) \\
&= \frac{\pi}{2}\sqrt{LC} + CV\frac{1 - \cos\theta_1}{I_o} + (\pi - \theta_1)\sqrt{LC} + \pi\sqrt{LC} \\
&= \left(\frac{5\pi}{2} - \theta_1\right)\sqrt{LC} + CV\frac{1 - \cos\theta_1}{I_o} \\
&= \left(\frac{5\pi}{2} - \theta_1\right)\sqrt{LC} + 2CV\frac{\sin^2\dfrac{\theta_1}{2}}{I_o}
\end{aligned}$$

The turn off time of main thyristor is

$$t_4 - t_3 = t_c = (\pi - 2\theta_1)\sqrt{LC}$$

The turn off time of auxiliary thyristor is

$$t_4 - t_2 = t_{c1} = (\pi - \theta_1)\sqrt{LC}$$

It is clear from Fig. 10.62 that the capacitor voltage is maximum at $t = t_6$. The maximum capacitor voltage can be determined from $V_{C\,max}$ = Voltage at $t = t_5$ + Increase in voltage due to the energy transferred from L to C during time interval $(t_6 - t_5)$.

At $t = t_5$, the energy stored in L is $\frac{1}{2}LI_o^2$ and $t = t_6$, the total energy stored in L is transferred to C. Therefore, voltage across capacitor will be increased due to the transfer of energy from L to C and the increase in voltage is equal to V_C. As per energy conservation, $\frac{1}{2}CV_C^2 = \frac{1}{2}LI_o^2$

Therefore, $V_C = I_o\sqrt{\frac{L}{C}}$

The maximum value of capacitor voltage is $V_{C\max} = V + V_C = V + I_o\sqrt{\frac{L}{C}}$

10.11.2 Design Parameters of Current Commuted Chopper

The value of commutating inductance L and commutating capacitance C are determined from the following equations:

The peak commutating current $I_{C\max}$ must be more than the maximum possible load I_o. The oscillating current in the commutation circuit is equal to

$$I_c = \frac{V}{\omega_o L}\sin\omega_o t = V\sqrt{\frac{C}{L}}\sin\omega_o t = I_{c\max}\sin\omega_o t$$

As per design requirement, $I_{c\max} = V\sqrt{\frac{C}{L}} > I_o$

Therefore, the peak commutating current $I_{C\max}$ is certain multiple of load current I_o.

We can write, $I_{c\max} = V\sqrt{\frac{C}{L}} = xI_o$

or, $$x = \frac{I_{c\max}}{I_o} \quad \text{where } 1.4 < x < 3$$

Assume that the turn-off time of main thyristor is t_q. The circuit turn-off time t_c must be greater than the turn-off time of main thyristor and $t_c = t_q + \Delta t$.

It clear from Fig. 10.62 that $t_c = t_4 - t_3$

or $$\omega_o t_c = \pi - 2\theta_1$$

The load current $I_o = I_{c\max}\sin\theta_1$

Therefore, $\theta_1 = \sin^{-1}\left(\frac{I_o}{I_{c\max}}\right) = \sin^{-1}\left(\frac{1}{x}\right)$

The circuit turn-off time of main thyristor is

$$t_c = \frac{1}{\omega_o}(\pi - 2\theta_1) = \frac{1}{\omega_o}\left(\pi - 2\sin^{-1}\left(\frac{I_o}{I_{c\max}}\right)\right)$$

After substituting the value of ω_o in the above equation, we get

$$t_c = \sqrt{LC}\left(\pi - 2\sin^{-1}\left(\frac{I_o}{I_{c\max}}\right)\right) \qquad \text{as } \omega_o = \frac{1}{\sqrt{LC}}$$

Therefore, $\sqrt{C} = t_c \Big/ \left[\sqrt{L}\left(\pi - 2\sin^{-1}\left(\frac{I_o}{I_{c\max}}\right)\right)\right]$

After substituting the value of $\sqrt{C}$ in equation $V\sqrt{\frac{C}{L}} = xI_o$, we obtain

$$\frac{V}{L}\cdot\frac{t_c}{\pi - 2\sin^{-1}(1/x)} = xI_o$$

or

$$L = \frac{V}{xI_o}\cdot\frac{t_c}{\pi - 2\sin^{-1}(1/x)}$$

The value of $\sqrt{L}$ is

$$\sqrt{L} = t_c \Big/ \left[\sqrt{C}\left(\pi - 2\sin^{-1}\left(\frac{I_o}{I_{c\max}}\right)\right)\right]$$

After substituting the value of $\sqrt{L}$ in equation $V\sqrt{\frac{C}{L}} = xI_o$, we obtain

$$\frac{V}{t_c}(\pi - 2\sin^{-1}(1/x))C = xI_o$$

The value of C is $C = \dfrac{xI_o t_c}{V[\pi - 2\sin^{-1}(1/x)]}$

The value of $I_{c\max}$ and $V_{c\max}$ are the peak ratings of thyristors T_1 and T_A. The value of charging resistance R_c is such that the periodic time $T \geq 3CR_C$.

Example 10.36 In a current commuted chopper, the peak commutating current is twice the maximum possible load current. The input voltage is 220 V dc and the turn-off time of main thyristor is 20 μs. If the maximum load current is 150 A, determine (a) the values of commutating components L and C, (b) maximum capacitor voltage and (c) the peak commutating current.

Solution

Given: $V = 220$ V, $t_q = 20$ μs, $I_o = 150$ A, $x = 2$

(a) $t_c = (20 + 20)$ μs $= 40$ μs

The value of L is $L = \dfrac{V}{xI_o}\cdot\dfrac{t_c}{\pi - 2\sin^{-1}(1/x)} = \dfrac{220}{2\times 150}\cdot\dfrac{40\times 10^{-6}}{\pi - 2\sin^{-1}(1/2)} = 11.20\ \mu\text{H}$

The value of C is $C = \dfrac{xI_o t_c}{V[\pi - 2\sin^{-1}(1/x)]} = \dfrac{2\times 150\times 40\times 10^{-6}}{200[\pi - 2\sin^{-1}(1/2)]} = 22.90\ \mu\text{F}$

(b) The maximum value of capacitor voltage is

$$V_{C\max} = V + V_C = V + I_o\sqrt{\frac{L}{C}} = 220 + 150\times\sqrt{\frac{11.20}{22.90}} = 324.90\ \text{V}$$

(c) The peak commutating current is

$$I_{C\max} = xI_o = 2\times 50 = 300\ \text{A}$$

Example 10.37 In a current commuted chopper, the input voltage is 220 V dc. The commutating components of chopper are L = 25 μH and C = 40 μF. If the maximum load current is 200 A, determine (a) Turn-off time of main thyristor, (b) Turn-off time of auxiliary thyristor, (c) Total commutation interval.

Solution

Given: $V = 220$ V, $L = 25$ μH, $C = 40$ μF, $I_o = 200$ A

(a) The peak commutating current is $I_{C\max} = V\sqrt{\dfrac{C}{L}} = 220\sqrt{\dfrac{40}{25}} = 278.28$ A

Then $x = \dfrac{I_{C\max}}{I_o} = \dfrac{278.28}{200} = 1.391$

Turn-off time of main thyristor $t_c = \sqrt{LC}\left(\pi - 2\sin^{-1}\left(\dfrac{I_o}{I_{c\max}}\right)\right)$

$$= \sqrt{25\times10^{-6}\times40\times10^{-6}}\left(\pi - 2\sin^{-1}\left(\frac{200}{278.28}\right)\right) = 48.638\ \mu\text{s}$$

(b) $\theta_1 = \sin^{-1}\left(\dfrac{I_o}{I_{c\max}}\right) = \sin^{-1}\left(\dfrac{1}{x}\right) = \sin^{-1}\left(\dfrac{1}{1.391}\right) = 45.96°\ °$

Turn-off time auxiliary thyristor is

$$t_4 - t_2 = t_{c1} = (\pi - \theta_1)\sqrt{LC} = \sqrt{LC}\left(\pi - \sin^{-1}\left(\frac{I_o}{I_{c\max}}\right)\right)$$

$$= \sqrt{25\times10^{-6}\times40\times10^{-6}}\left(\pi - \sin^{-1}\left(\frac{200}{278.28}\right)\right) = 74.011\ \mu\text{s}$$

(c) Total commutation interval is

$$t_6 - t_1 = \left(\frac{5\pi}{2} - \theta_1\right)\sqrt{LC} + 2CV\frac{\sin^2\dfrac{\theta_1}{2}}{I_o}$$

$$= \left(\frac{5\pi}{2} - 45.96°\right)\sqrt{25\times10^{-6}\times40\times10^{-6}} + 2\times40\times10^{-6}\times220\frac{\sin^2\dfrac{45.96°}{2}}{200}$$

$$= 223.085\ \mu\text{s} + 13.412\ \mu\text{s} = 236.497\ \mu\text{s}$$

10.12 JONES CHOPPER

Figure 10.63 shows the Jones chopper circuit which consists of main thyristor T_1, auxiliary thyristor T_A. The commutation circuit for main thyristor is capacitor C, diode D_1, thyristor T_A and autotransformer L. The advantage of using auto transformer is that it can eliminate the commutation failure as the energy stored in inductor L-L_2 slightly enhances the capacitor voltage to a value greater than V and the definite commutation process occur as inductance L_1 and L_2 are closely coupled. In this chopper circuit, class D commutation technique is used. This chopper operates in different modes as given below:

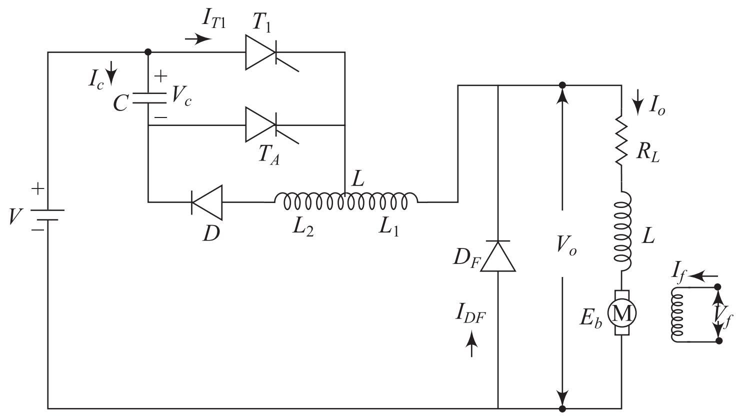


Fig. 10.63 *Jones chopper circuit*

Mode I We assume that initially the capacitor C is charged to a voltage V and the polarity of capacitor is depicted in Fig. 10.64. When the trigger pulse is applied to main thyristor T_1 at $t = t_1$, thyristor T_1 will be turned on, the current follows through the path C-T_1-L-L_2-D_1-C as depicted in Fig. 10.65. Therefore, the capacitor C will be charged to the opposite polarity with upper plate negatively charged and lower plate positively charged. Diode D_1 is used to prevent further oscillation of L_2-C circuit. The capacitor C should retain its charge till the thyristor T_2 gets trigger pulse. If the thyristor T_1 is ON for long time duration, the dc motor operates at steady state speed. The speed of dc motor can be determined by the dc input voltage and characteristics of dc motor.

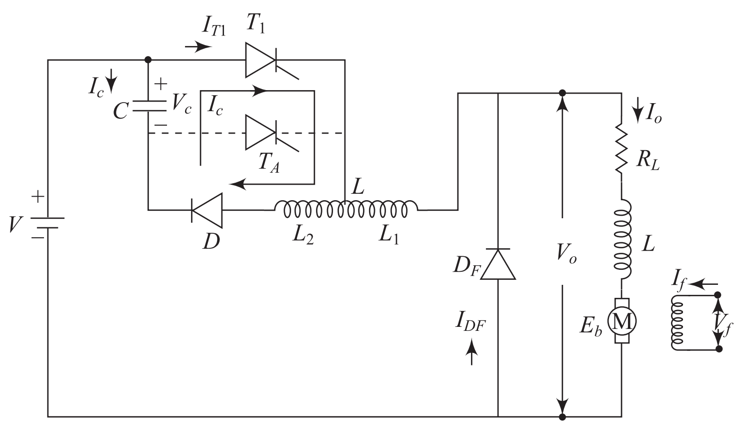


Fig. 10.64 *Jones chopper circuit in mode I*

Mode II At $t = t_3$, the auxiliary thyristor T_A is turned on and current starts to flow through T_A as depicted in Fig. 10.65. The path of current flow is C-T_A-T_1-C.

As the capacitor discharges, thristor T_1 will be reverse biased and it will be turned off. Whenever the capacitor C is recharged, the thyristor T_A gets turned off as the current flows below holding current. When both thyristor T_1 and T_A are not conducting, the free wheeling diode starts to conduct and load current flows through the path L-R-M and D_F. The electromagnetic torque developed with in motor is directly proportional to load current. At $t = t_3$, the voltage at lower plate of capacitor C is greater than V. The time interval $t_4 - t_3$ is called as circuit turn-off time. Figure 10.66 shows the voltage and current waveforms of a Jones Chopper.

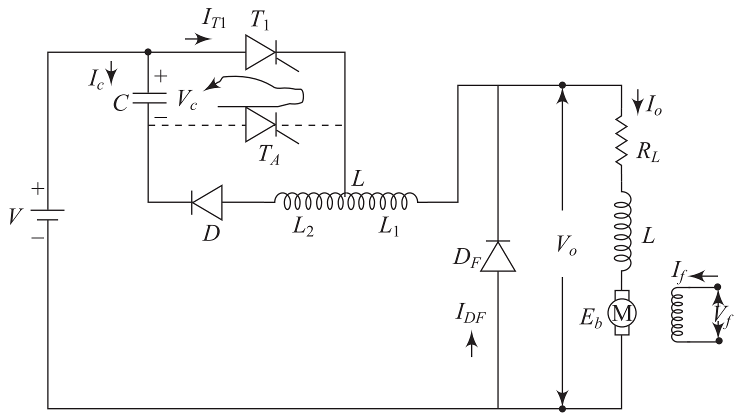


Fig. 10.65 *Jones chopper circuit in mode II*

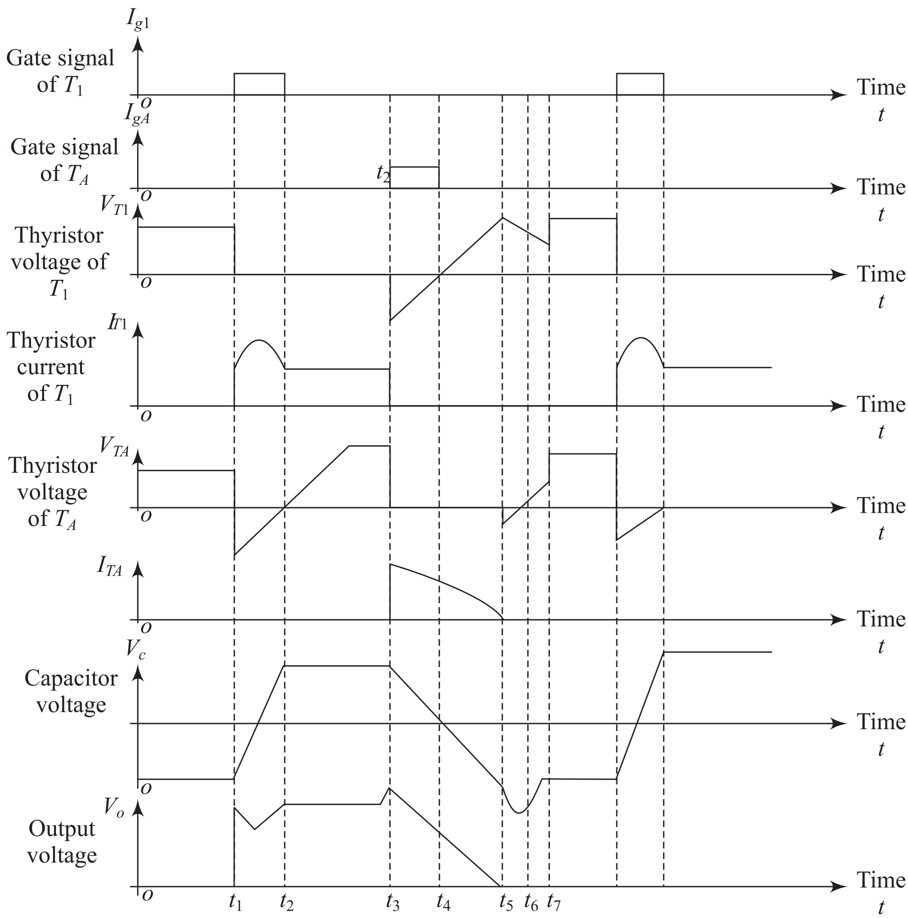


Fig. 10.66 *Voltage and current waveforms of a Jones Chopper*

10.12.1 Design Parameters of Jones Chopper

During the turn-off time of thyristor T_1, the energy stored in inductance L_1 is transferred to capacitor C.

Therefore, $\frac{1}{2}L_1 I_O^2 = \frac{1}{2}CV_C^2$

or $$\frac{V_C^2}{I_O^2} = \frac{L_1}{C}$$

or $$V_C = I_o\sqrt{\frac{L_1}{C}} \tag{10.26a}$$

During the turn-off of the thyristor T_1, the capacitor voltage will be changed from V_C to 0.
The turn-off time of thyristor is

$$t_q = \frac{V_C C}{I_o}$$

After substituting the value of V_C in the above equation, we obtain

$$t_q = \frac{I_o\sqrt{\frac{L_1}{C}}C}{I_o} = \sqrt{L_1 C}$$

After dividing the equation (1) by V, we get

$$\frac{V_C}{V} = \frac{I_o}{V}\sqrt{\frac{L_1}{C}} \tag{10.26b}$$

If we assume that $g = \frac{V_C}{V}$ and $R_m = \frac{V}{I_o}$, Eq. (10.26) can be expressed as

$$g = \frac{1}{R_m}\sqrt{\frac{L_1}{C}}$$

The voltage across thyristors T_1 and T_2 is $V_C = gV$
As the value of g increases, the voltage rating of thyristor increases.

Example 10.38 The speed of a separately excited dc is controlled by Jones Chopper. When the input voltage is 100 V dc, turn-off time of thyristor is 20 μs and the current flows through main thyristor is 80 A and conductance is 4 mho, determine the value of commutating capacitor C and transformer inductances L_1 and L_2.

Solution

Given: $V = 100$ V, $t_q = 20$ μs, $I_o = 80$ A, $g = 4$ mho

$$R_m = \frac{V}{I_o} = \frac{100}{80} = 1.25\,\Omega$$

We know that $g = \frac{1}{R_m}\sqrt{\frac{L_1}{C}}$

Therefore, $4 = \frac{1}{1.25}\sqrt{\frac{L_1}{C}}$ or, $\sqrt{\frac{L_1}{C}} = 4 \times 1.25 = 5$ (1)

$$t_q = \sqrt{L_1 C} = 20\,\mu\text{s} \quad (2)$$

From Eqs. (1) and (2), we can write

$$\sqrt{\frac{L_1}{C}} \times \sqrt{L_1 C} = 5 \times 20 \times 10^{-6}$$

or $$L_1 = 100 \times 10^{-6} = 100\,\mu\text{H}$$

As $\sqrt{\frac{L_1}{C}} = 5,\ C = \frac{L_1}{25} = \frac{100 \times 10^{-6}}{25} = 4\,\mu\text{F}$

Assume $L_1 = L_2 = 100\ \mu\text{H}$

10.13 MORGAN CHOPPER

Figure 10.67 shows the Morgan Chopper which consists of a thyristor, diode *D*, capacitor *C*, saturate reactor SR. The advantage of this circuit is that the cost is very low due to presence of only one thyristor. The commutating elements of the circuit are capacitor *C*, saturate reactor (SR) and diode (D). The characteristics of saturate reactor are different from air core inductor. An air core can take any amount of flux and the air core inductor never saturates whereas the inductance offered by the air core inductor is very large and saturate reactor can be saturated for low value of exciting current. This chopper operates in three different modes as given below:

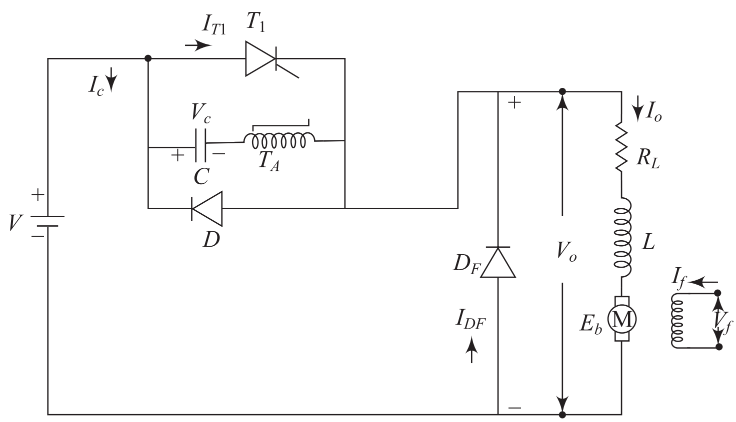


Fig. 10.67 *Morgan Chopper*

Mode 0 When the thyristor T_1 operates in off state, the capacitor *C* will be charged to the supply voltage *V*. The charging path of capacitor will be *V-C-SR-L-M-V* as depicted in Fig. 10.68. The inductance offered by the saturable reactor is very low. When the capacitor is charged to *V*, the charging will be stopped and the staurable reactor is placed in positive saturation condition.

Mode I When a triggering pulse is applied to thyristor T_1 at $t = t_1$, *T*1 will be turned on. Since T_1 is ON, the voltage across capacitor is applied to the satiable reactor and the core flux direction within *SR* is changed from positive saturation to negative saturation. When the *SR* changes from positive saturation to negative saturation completely, the capacitor *C* discharges through the path *C-T*1*-SR-C*.

Then the LC circuit forms a resonating circuit with a discharging time of $T_1\sqrt{L_sC}$ where L_s is the post saturation inductance. As the discharging time is very small, the capacitor voltage will be reversed very quickly. Subsequently, the capacitor voltage $-V$ is applied on the saturable reactor in the reverse direction. Then the core is driven from negative saturation to positive saturation. After certain time interval, the core flux reaches the positive saturation and the capacitor will discharge the charge in opposite direction to thyristor T_1. Consequently, thyristor T_1 will be turned off.

Mode II When the thyristor T_1 is turned-off, the free wheeling diode D_F will be forward biased due to the stored energy in the inductor. Subsequently, the load current flows through D_F and the load voltage is zero. The time required to saturate the core depends on the volt-time integral. The conduction of thyristor is fixed and it is function of commutating inductance L and capacitance C. The average output voltage can be controlled by changing the operating frequency. The on time of thyristor T_1 is determined by the time required for the reactor to move from positive saturation to the negative saturation and again back to positive saturation. The voltage and current waveforms in a Morgan chopper is depicted in Fig. 10.68.

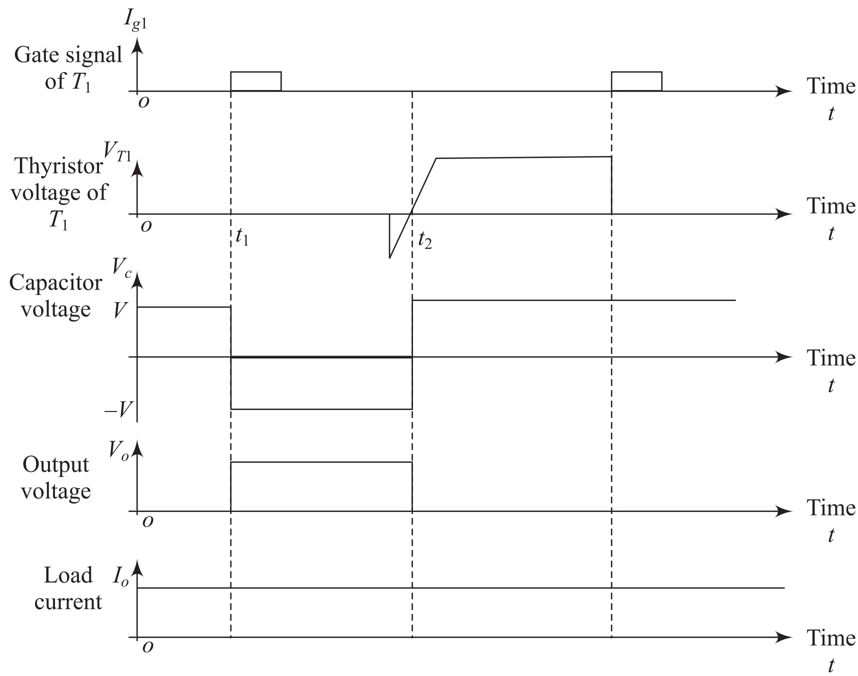


Fig. 10.68 *Voltage and current waveforms of a Morgan chopper*

10.14 MULTIPHASE CHOPPERS

In a multiphase chopper circuit, several choppers are connected in parallel. All the choppers should be identical and operating with the same frequency and duty cycle. But the turn-on period of switches will be phase displaced or shifted with respect to each other. Figure 10.69(a) shows a two-phase or biphase chopper where two choppers are connected in parallel. Similarly, Fig. 10.69(b) shows a three-phase or triphase chopper which consists of three parallel connected choppers. Figure 10.69(c) shows a m-phase

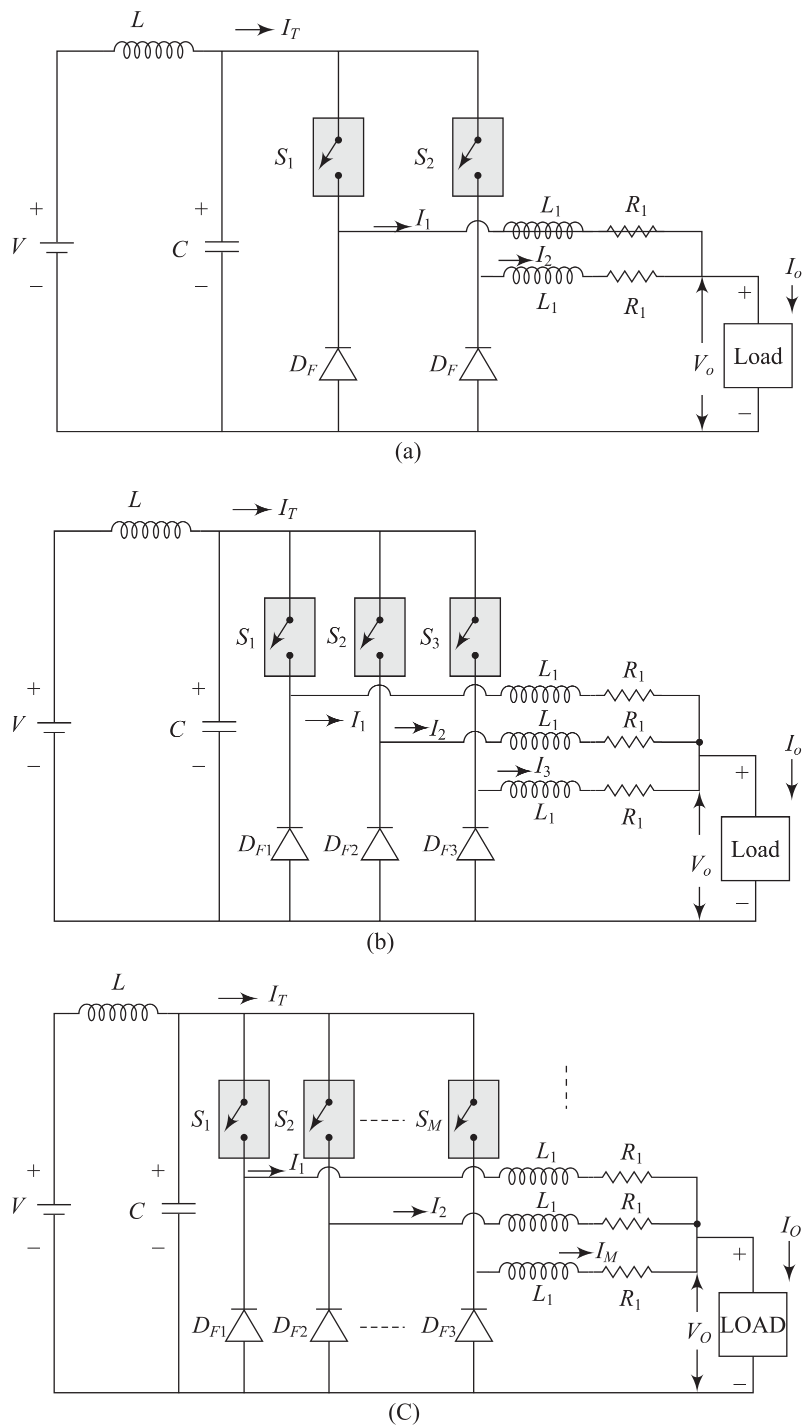


Fig. 10.69 *(a) A two-phase or biphase chopper, (b) A three-phase or triphase chopper, (c) A m-phase chopper*

chopper where m choppers are connected in parallel. Each chopper circuit consists of an input filter with L and C components. In the same way, each chopper should have a separate but identical output smoothing inductance L_1. The resistance R_1 is the resistance of the inductive coil. The load will be either resistive R, or inductive R-L, or RLE, or a dc motor.
A multiphase chopper can be operated in two different modes such as

- In phase operation mode
- Phase shifted operation mode

In the phase operation mode, all parallel connected choppers are turned ON and turned OFF simultaneously. In case of phase shifted operation mode choppers are not turned ON and turned OFF simultaneously but the chopping frequency is same.

If the load current is ripple free due to very high inductive load and its value is represented by I_o, the current waveform of a two phase chopper when it operates in phase operation mode is depicted in Fig. 10.70. I_1 current flows through chopper-1 and I_2 current flows through chopper-2. The total input current can be obtained after addition of I_1 and I_2. Therefore, $I_T = I_1 + I_2 = I_o$. When a multiphase chopper operates in phase operation mode, it behaves just like a single phase chopper but the current rating of the switch which is used in multiphase phase chopper will be reduced with respect to single phase chopper. When a multiphase chopper operates in phase operation mode, it behaves just like a single phase chopper but the current rating of the switch which is used in multiphase phase chopper will be reduced with respect to single phase chopper.

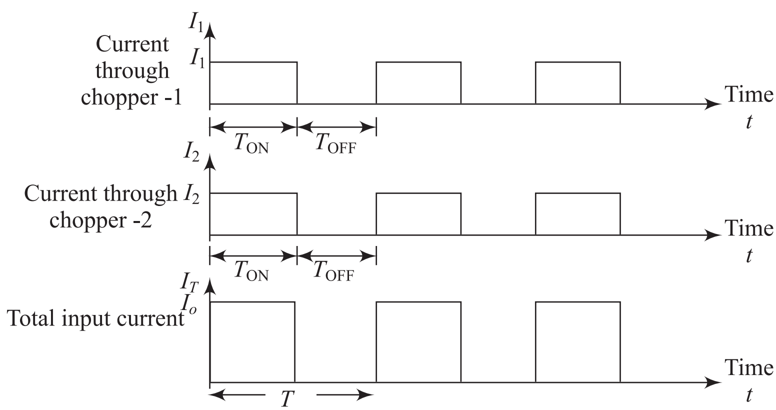


Fig. 10.70 *Current waveforms of two-phase chopper in phase operation mode*

Figure 10.71 shows the current waveform of a two phase chopper when it operates in phase shifted operation mode. Assume that I_1 current flows through chopper -1 and I_2 current flows through chopper -2. The total input current is the addition of I_1 and I_2. Therefore, $I_T = I_1 + I_2$.

In Fig. 10.71, the frequency of harmonics in the input current is equal to the switching frequency of chopper. However, in case of phase shifted operation mode, the frequency of harmonics in the input current is equal to twice the switching frequency of chopper as depicted in Fig. 10.72. Since the frequency of harmonics in the input current is two times of chopping frequency, the size of the filter will be reduced in phase shifted multiphase chopper. For this reason, phase shifted multiphase chopper is commonly used in industrial applications.

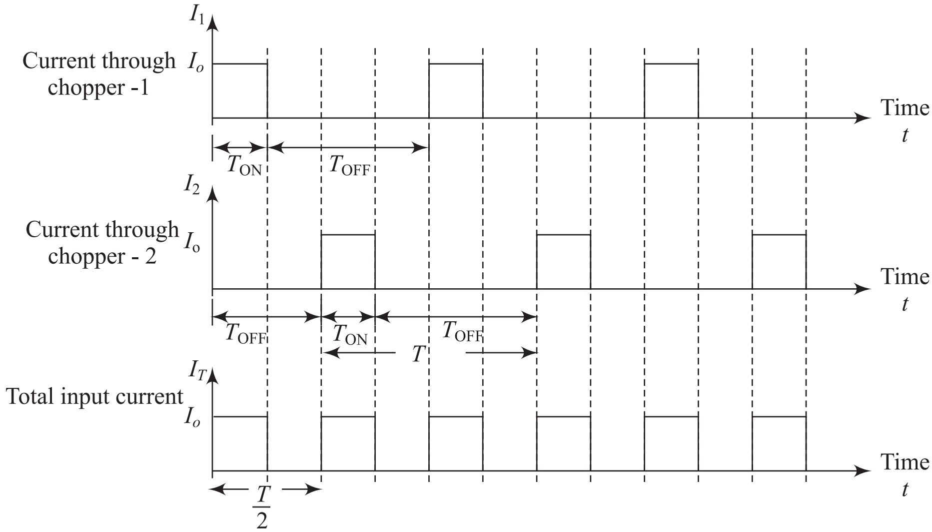


Fig. 10.71 *Current waveforms of two-phase chopper in phase shifted operation mode*

If the load is less inductive, the ripple must be exist in the output current. When a multiphase chopper operates in phase operation mode, the ripple in output current will be high as shown in Fig. 10.72(a). However, the ripple in output current will be less when a multiphase chopper operates in phase shifted operation mode as depicted in Fig. 10.72(b).

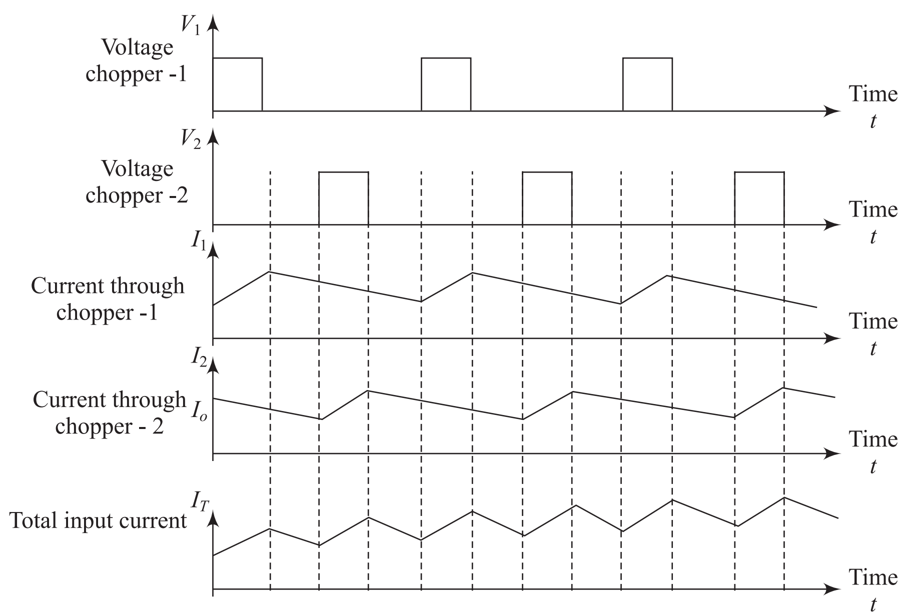


Fig. 10.72 *(a) Voltage and current waveforms of a two-phase chopper*

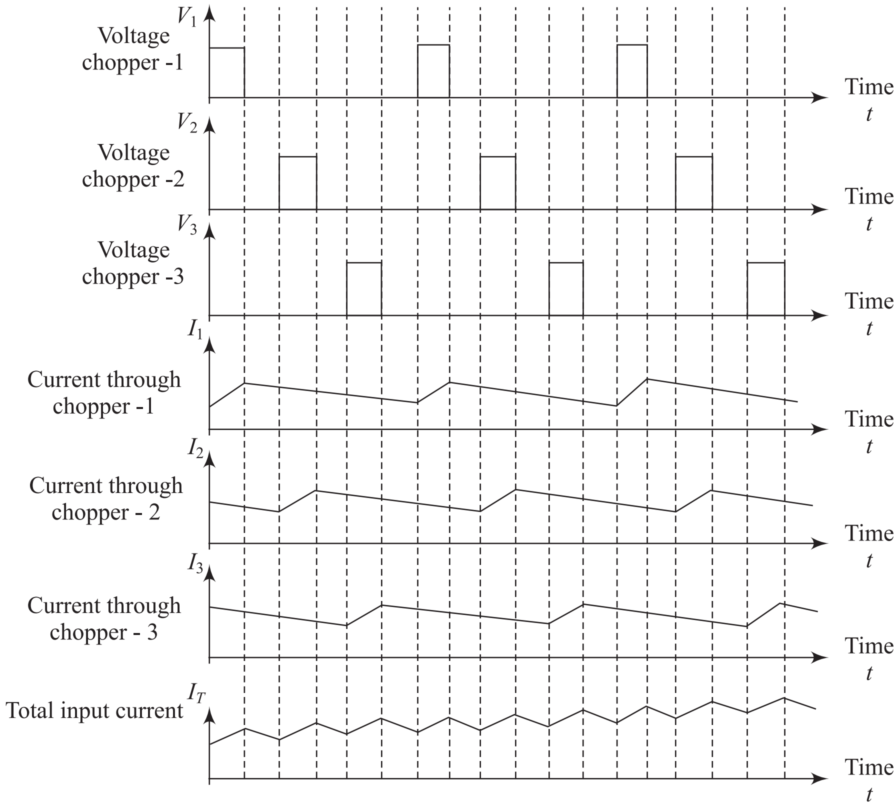


Fig. 10.72 *(b) Voltage and current waveforms of a three-phase chopper*

Multiphase choppers are commonly used in such industrial applications where large load current is required. The advantages of multiphase choppers with respect to single phase choppers are

- Amplitude of ripple in input current is reduced significantly
- Ripple frequency of input current is increased.
- Size of filer will be reduced

The disadvantages of multiphase choppers with respect to single phase choppers are

- External inductors are used in each phase
- Extra commutation circuits are required
- The control circuit of chopper will be complex

Summary

- A dc to dc converter (chopper) is a switch-mode converter which is used to get an adjustable dc output voltage from a fixed dc power supply.
- DC to DC converters are classified as step-down and step-up chopper. The operating principle and control strategies of step-down and step-up choppers are discussed elaborately.

- Non-isolated dc to dc converters such as Buck, Boost, Buck-Boost and Cuk converters, voltage or impulse commutated chopper are incorporated in this chapter.
- Isolated dc to dc converters such as Fly-back converter, Forward converter, Push-Pull converters, half-bridge and full-bridge converters are explained in detail.

Multiple-Choice Questions

10.1 If a chopper circuit has input voltage of V, T_{ON} is on time of switch and f is the chopping frequency, then the output voltage of chopper is equal to
(a) $T_{ON}fV$ (b) $T_{OFF}fV$ (c) $T_{ON}f/V$ (d) fV/T_{ON}

10.2 The output voltage of dc to dc converter (chopper) can be controlled by
(a) amplitude modulation (b) frequency modulation
(c) pulse width modulation (d) pulse width modulation and frequency modulation

10.3 In pulse width modulation control of a chopper, the following statements are correct:
(i) On time T_{ON} is constant and f is varied (ii) On time T_{ON} is varied and f is constant
(iii) Off time T_{OFF} is constant and f is varied (iv) Off time T_{OFF} is varied and f is constant
(a) (i) and (ii) (b) (ii) and (iii) (c) (iii) and (i) (d) (ii) and (iv)

10.4 In a dc-to-dc converter, the input voltage and output voltage waveforms are ________ and ____________ respectively.
(a) continuous, discontinuous (b) continuous, continuous
(c) discontinuous, discontinuous (d) discontinuous, continuous,

10.5 In frequency modulation control of a chopper, the following statements are correct:
(i) On time T_{ON} is constant and f is varied
(ii) On time T_{ON} is varied and T_{OFF} is constant
(iii) Off time T_{OFF} is constant and f is varied
(iv) Off time T_{OFF} is varied and f is constant
(a) (i), (ii) and (iii) (b) (ii) and (iii) (c) (iii) and (i) (d) (ii) and (iv)

10.6 When a stepdown copper operates in the continuous conduction mode with a duty cycle D, the ratio of average output voltage to input voltage $\frac{V_{av}}{V}$ is
(a) $1 - D$ (b) $\frac{1}{1-D}$ (c) $\frac{D}{1-D}$ (d) D

10.7 If a step-down copper operates in the continuous conduction mode with a duty cycle D, the rms value of the output voltage is
(a) $\sqrt{D \cdot V}$ (b) DV (c) $\sqrt{(1-D) \cdot V}$ (d) $(1 - D)V$

10.8 When a step-up copper operates in the continuous conduction mode with a duty cycle D, the ratio of average output voltage to input voltage $\frac{V_{av}}{V}$ is
(a) $1 - D$ (b) $\frac{1}{1-D}$ (c) $\frac{D}{1-D}$ (d) D

10.9 If a step-down copper operates in the continuous conduction mode with a duty cycle D, the ripple factor is
(a) $\frac{\sqrt{D-D^2}}{D}$ (b) $\frac{\sqrt{D^2-D}}{D}$ (c) $\frac{D}{\sqrt{D^2-D}}$ (d) $\frac{D}{\sqrt{D-D^2}}$

10.10 The effective input resistance of a step-down chopper is
(a) $R_i = \frac{R}{D}$ (b) $R_i = DR$ (c) $R_i = \frac{R}{1-D}$ (d) $R_i = (1 - D)R$

10.11 If a step-down copper operates in the continuous conduction mode, the ripple in load current is maximum when duty cycle ___________.

(a) $D = 0.25$ (b) $D = 0.5$ (c) $D = 0.75$ (d) $D = 1$

10.12 A step-down chopper *RLE* load is operated in the continuous conduction mode with duty ratio D, the maximum value of ripple current is

(a) $\Delta I_{\max} = \frac{V}{4fL}$ (b) $\Delta I_{\max} = \frac{4fL}{V}$ (c) $\Delta I_{\max} = \frac{V}{2fL}$ (d) $\Delta I_{\max} = \frac{2fL}{V}$

10.13 A step-down chopper *RLE* load is operated in the continuous conduction mode, the limit of continuous conduction is reached when

(a) $I_{\min} \neq 0, D > \frac{\tau}{T} \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right]$ (b) $I_{\min} \neq 0, D = \frac{\tau}{T} \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right]$

(c) $I_{\min} = 0, D \geq \frac{\tau}{T} \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} - 1\right)\right]$ (d) $I_{\min} = 0, D \leq \frac{\tau}{T} \cdot \ln\left[1 + m\left(e^{\frac{T}{\tau}} + 1\right)\right]$

10.14 A step-down chopper *RLE* load is operated in the discontinuous conduction mode with duty ratio D, the extinction time is __________ .

(a) $t_x = T_{\text{ON}} + \tau_a \ln\left[1 + \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)\right]$ (b) $t_x = T_{\text{OFF}} + \tau_a \ln\left[1 + \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)\right]$

(c) $0\, t_x = T_{\text{ON}} - \tau_a \ln\left[1 + \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)\right]$ (d) $t_x = T_{\text{OFF}} - \tau_a \ln\left[1 + \frac{(V-E)}{E}\left(1 - e^{-\frac{R}{L}T_{\text{ON}}}\right)\right]$

10.15 A step-down chopper *RLE* load is operated in the discontinuous conduction mode with duty ratio D. If t_x is the extinction time, the average output voltage is

(a) $V_{\text{av}} = DV + \left(1 - \frac{t_x}{T}\right)E$ (b) $V_{\text{av}} = DV + \left(1 + \frac{t_x}{T}\right)E$

(c) $V_{\text{av}} = DV - \left(1 - \frac{t_x}{T}\right)E$ (d) $V_{\text{av}} = DV - \left(1 + \frac{t_x}{T}\right)E$

10.16 In a type A chopper, V is the input voltage, R is the load resistance and D is the duty ratio. The average value of output voltage and load current are _____ and _____ respectively.

(a) $DV, \frac{DV}{R}$ (b) $(1-D)V, \frac{(1-D)V}{R}$ (c) $\sqrt{D}V, \frac{\sqrt{D}V}{R}$ (d) $\frac{V}{1-D}, \frac{V}{(1-D)R}$

10.17 In a type A chopper, V, R, I_o and D are the input voltage, load resistance, load current and duty ratio respectively. The average and rms value of freewheeling diode currents are

(a) $DI_o, \sqrt{D}I_o$ (b) $(1-D)I_o, \sqrt{(1-D)}I_o$ (c) $\frac{I_o}{1-D}, \frac{I_o}{\sqrt{1-D}}$ (d) $\frac{I_o}{D}, \frac{I_o}{\sqrt{D}}$

10.18 In step-up chopper, input voltage is 200 V and output voltage is 400 V. If the conduction time of semiconductor switch is 150 μs, the non-conduction time of semiconductor switch is _______.

(a) 150 μs (b) 200 μs (c) 250 μs (d) 300 μs

10.19 A buck converter has input voltage of V, output voltage of V_o, inductance of L and it operates at frequency f, then the amplitude of ripple current is equal to

(a) $\Delta I = \frac{V_o(V - V_o)}{VfL}$ (b) $\Delta I = \frac{V_o(V + V_o)}{VfL}$ (c) $\Delta I = \frac{V_o(V - V_o)}{fL}$ (d) $\Delta I = \frac{V_o(V - V_o)}{VL}$

10.20 If buck converter has input voltage of V, duty ratio D, inductance L and it operates at frequency f, the amplitude of ripple current is equal to

(a) $\Delta I = \dfrac{D(1-D)V}{fL}$ (b) $\Delta I = \dfrac{D(1+D)V}{fL}$ (c) $\Delta I = \dfrac{D(1-D)}{fL}$ (d) $\Delta I = \dfrac{D(1-D)V}{L}$

10.21 The output voltage of buck-boost converter is equal to

(a) $V_o = -\dfrac{D}{1-D}V$ (b) $V_o = \dfrac{D}{1-D}V$ (c) $V_o = DV$ (d) $V_o = \sqrt{D}V$

10.22 In a dc chopper using thyristor, the best performance can be obtained from ________
(a) line commutation (b) voltage commutation (c) current commutation (d) load commutation

10.23 In a _______ chopper output voltage and current are positive.
(a) Type A (b) Type B (c) Type C (d) Type D

10.24 In a _______ chopper output voltage is positive and current is negative
(a) Class A (b) Class B (c) Class C (d) Class D

10.25 In which chopper, the output current remains positive but output voltage is either positive or negative
(a) Class A (b) Class B (c) Class C (d) Class D

10.26 If the duty cycle of a chopper is exactly 50%, the pulse is considered to be a
(a) square wave (b) sine wave (c) low duty cycle (d) high duty cycle

10.27 In a load commutated chopper, during conduction of T_1 and T_3, the load voltage V_o varies from
(a) V to 0 (b) V to –V (c) 2 V to 0 (d) 2 V to V

10.28 In a load commutated chopper, the free wheeling diode D_F conducts only when ________ are not conducting.
(a) T_1, T_2, T_3, T_4 and C (b) T_1, T_3, and C (c) T_2, T_4 and C (d) T_1, T_2, T_4 and C

10.29 In a Jones chopper, the turn-off time of thyristor is ________.

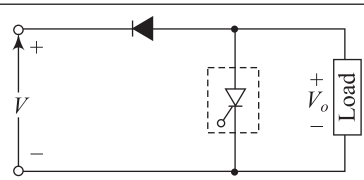
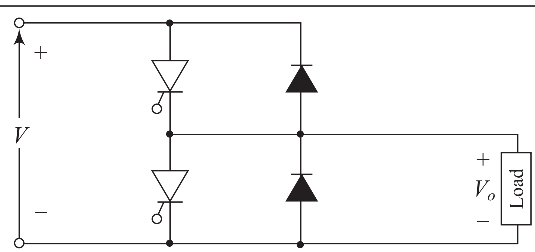
(a) $t_q = \dfrac{V_C C}{I_o}$ (b) $t_q = \dfrac{V_C I_o}{C}$ (c) $t_q = \dfrac{I_o C}{V_C}$ (d) $t_q = \sqrt{L_1 C}$

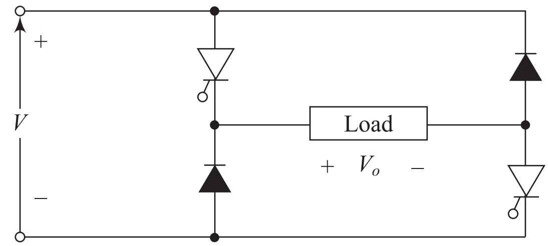
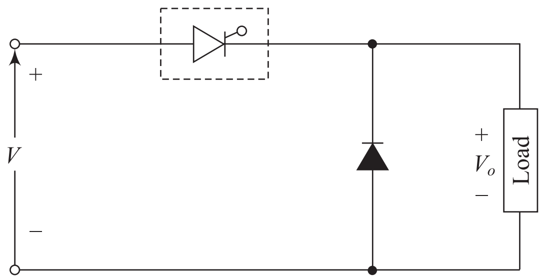
10.30 In a Jones chopper,

(a) $g = \dfrac{1}{R_m}\sqrt{\dfrac{L_1}{C}}$ (b) $g = \dfrac{1}{R_m}\sqrt{\dfrac{C}{L_1}}$ (c) $g = R_m\sqrt{\dfrac{L_1}{C}}$ (a) $g = \dfrac{1}{R_m}\sqrt{L_1 C}$

10.31 In a thyristor-based dc chopper circuit, which type of commutation results the best performance?
(a) Current commutation (b) Load commutation
(c) Voltage commutation (d) All of these

10.32 Match the List I and List II and find the correct matching.

List-I	List-II
1. Type –A chopper	A. 
2. Type –B chopper	B. 

3. Type –C chopper	C. 
4. Type –D chopper	D. 

(a) 1-D, 2-A, 3-C, 4-B (b) 1-A, 2-B, 3-C, 4-D
(c) 1-A, 2-D, 3-C, 4-B (d) 1-D, 2-A, 3-B, 4-C

10.33 The circuit as shown in Fig. 10.73 is initially in steady state with thyristor T OFF. When T is turned ON, the peak current flows through thyristor is equal to

(a) 42 A
(b) 40 A
(c) 22 A
(d) 2 A

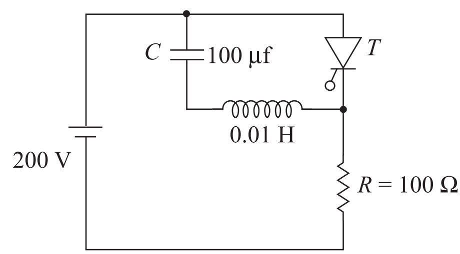
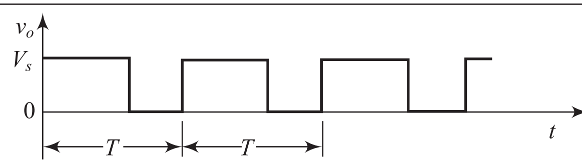
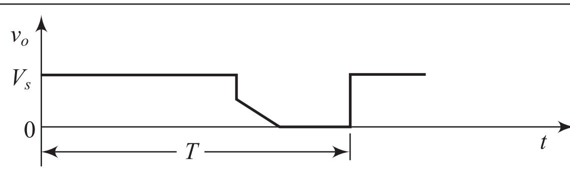
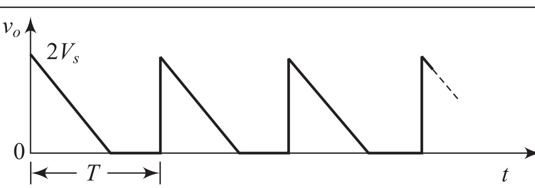
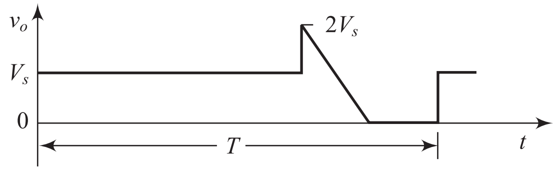


Fig. 10.73

10.34 Match the List I and List II and find the correct matching.

List-I	List -II
1. Voltage Commuted Chopper	A. 
2. Current Commuted Chopper	B. 
3. Load Commuted Chopper	C. 

4. Ideal DC Chopper	D. 

(a) 1-D, 2-B, 3-A, 4-C
(b) 1-A, 2-B, 3-C, 4-D
(c) 1-D, 2-B, 3-C, 4-A
(a) 1-B, 2-D, 3-C, 4-A

10.35 The maximum current flows through main thyristor as shown in Fig. 10.74 is ______.
(a) 70.7 A (b) 141.4 A (c) 170.7 A (d) 200 A

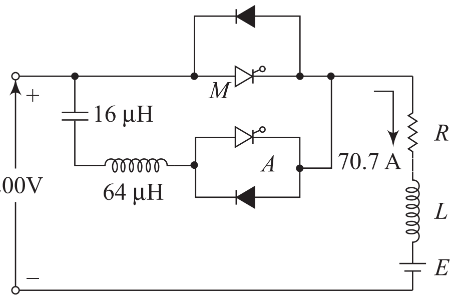


Fig. 10.74

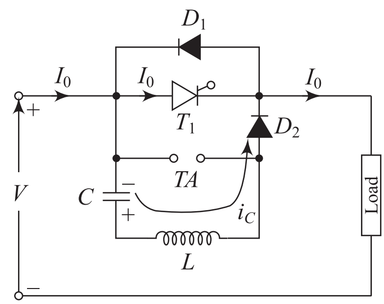


Fig. 10.75

10.36 Figure 10.75 shows a current commutated chopper. When T_1 is conducting, load current is I_o. If T_A is turned ON with the capacitor polarity as shown in Fig. 10.75, capacitor current flows through ______.
(a) Thyristor T_1 as it is already conducting
(b) Diode D_1 as it provides an easy path
(c) Thyristor T_1 as diode D_1 is reverse biased by voltage drop across T_1
(d) Diode D_1 as thyristor T_1 is unidirectional device.

Fill in the Blanks

10.1 A ______ is converter which converts fixed dc voltage to variable dc voltage.
10.2 The output voltage of a dc to dc converter is less than input voltage, this converter circuit is called_________.
10.3 When the output voltage of a dc to dc converter is greater than input voltage, the converter circuit is known as________
10.4 Switching devices such as ____, ______, _____, and _____ are used in chopper circuit.
10.5 The output voltage of dc to dc converter (chopper) can be controlled by ____ and _____.
10.6 In a chopper with current limit control, the switch is ON when I_o is less than _________ and the switch becomes OFF if I_o is greater than ___________.
10.7 The relationship between input and output voltage of a stepdown (buck) converter is _______.
10.8 The relationship between input and output voltage of a step-up (boost) converter is _______.
10.9 The relationship between input and output voltage of a buck-boost converter is _____.
10.10 The relationship between input and output voltage of a cuk converter is _____.
10.11 In case of a step-down chopper, the per unit ripple is maximum when duty ratio is ______
10.12 In a pulse width modulated method of controlling the average output voltage in a chopper, the on-time T_{ON} is ______ and the chopping frequency is _______.

10.13 Isolated dc to dc converters such as buck, boost, buck-boost converters are known as _______________.
10.14 A voltage commutated chopper consists of ______ and _____ diode.
10.15 When a dc to dc step-down converter operates in discontinuous mode and t_x extinction time, the average output voltage is equal to___________.
10.16 In a step-up chopper, duty cycle is greater than ________ and less than______.
10.17 In a step-up chopper, output voltage is always _______ than input voltage and duty cycle is______.
10.18 In a step-down chopper, output voltage is always _______ than input voltage with duty cycle ______.
10.19 A class______ chopper is a four quadrant chopper.
10.20 A class______ chopper is a two quadrant chopper.
10.21 The first quadrant chopper is a class_______ chopper.
10.22 The second quadrant chopper is a class ________chopper.
10.23 The average output voltage in terms of T_{ON}, f and V is __________.
10.24 In a step-down chopper with *RLE* load, the maximum value of ripple is ___________ proportional to the chopping frequency f and inductance L.

Review Questions

10.1 Define chopper. What are the types of chopper? What are the applications of chopper circuit?
10.2 Explain the operating principle of dc chopper with a suitable diagram. Draw the voltage and current waveforms of chopper. Derive expressions for average output voltage and rms output voltage.
10.3 (a) What are the different control strategies?
(b) Discuss the time ratio control in a dc chopper.
(c) What is frequency modulation? What is pulse width modulation?
(a) Write the disadvantages of frequency modulation of chopper over pulse width modulation?
10.4 State the current limit control? Explain the difference between current limit control and time ratio control. Why current limit control is preffered over any other control strategies?
10.5 Draw the circuit diagram of step-up chopper and describe its operating principle. Draw the voltage and current waveforms of step-up chopper. Derive expressions for average output voltage and rms output voltage. What are the applications of step-up chopper?
10.6 Describe the classifications of chopper or dc-to-dc converter.
10.7 Write short notes on the following:
(a) Class A Chopper or First Quadrant Chopper (b) Class B Chopper or Second Quadrant Chopper
(c) Class C Chopper or Two Quadrant Type A Chopper
(d) Class D Chopper or Two Quadrant Type B Chopper
(e) Class E Chopper or Four Quadrant Chopper
10.8 In a class A chopper with a *RLE* load, derive the expressions for the following parameters when it operates in continuous mode:
(a) Average output voltage, (b) Average load current, (c) rms value of the output voltage,
(d) rms value of the output current, (e) Ripple factor, (f) Effective input resistance,
(g) I_{max}, (h) I_{min}, (i) Amplitude of ripple current and (j) Limit of continuous conduction.
10.9 Prove that in a class A chopper with a *RLE* load, the ripple in load current is maximum when duty cycle $D = 0.5$.
10.10 Prove that in a class A chopper with a *RLE* load, the maximum value of ripple current is $\Delta I_{max} = \dfrac{V}{4fL}$
10.11 If a class A chopper with a *RLE* load operates in discontinuous mode, derive the expressions for the following parameters:
(a) Extinction time t_x, (b) Average output voltage and (c) I_{max}
10.12 Describe the continuous and discontinuous modes operation of a type A chopper. What is the limit of continuous conduction?

10.13 What is multiphase chopper? What are the different operation modes of multiphase chopper?

10.14 Why phase shifted operation of multiphase chopper is preferred over any other operating modes?

10.15 Draw a biphase and a triphase chopper and explain their operating principle with waveforms.

10.16 Describe the voltage commutated chopper with waveforms and equivalent circuits. Write the applications of voltage commutated chopper.

10.17 What is the effect of chopping frequency on load ripple current?

10.18 What are the performance parameters of a converter?

10.19 What is switch mode power supply? What is the advantage of switch mode power supply? What are the disadvantages of switch mode power supply?

10.20 Write the advantages and disadvantages of the following converters:
(a) Buck converter (b) Boost converter
(c) Buck-Boost converter and (d) CUK converter

10.21 Write the short notes on the following:
(a) Fly-back converter, (b) Forward converter, (c) Push-pull converter,
(d) Half-bridge converter and (e) Full-bridge converter

10.22 List the various types of SMPS. Describe flyback SMPS with equivalent circuits and waveforms. Derive expression for output voltage.

10.23 A dc-to-dc converter as shown in Fig. 10.2 has a resistive load of 20 Ω and input voltage of 200 V. If the switching frequency is 2 kHz and duty cycle is 40%, determine
(a) average output voltage and current
(b) rms output voltage and current

10.24 In a step-down chopper, input voltage is 220 V and average output voltage is 150 V. If the switching frequency is 1 kHz, determine the ON time and OFF time of switch in each cycle.

10.25 A chopper circuit as shown in Fig. 10.2 has input voltage of 200 V, and a resistive load of 10 10 Ω. When thyristor is used as a switch, the voltage across thyristor is about 2 V during on condition. If the duty cycle is 0.75, switching frequency is 1 kHz, determine
(a) average output voltage, (b) average output current, (c) rms output voltage,
(d) rms output current, (e) average thyristor current, (f) rms thyristor current and
(g) efficiency of chopper.

10.26 A dc to dc step-up converter as shown in Fig. 10.8 has a resistive load of 5 Ω and input voltage of 100 V. If the switching frequency is 1 kHz and duty cycle is 40%, determine average output voltage and current.

10.27 A step-up chopper has input voltage of 220 V and output voltage of 450 V. If the conduction time of switch is 130 μs, what is pulse width of output voltage? If the pulse width of output voltage become one fourth for constant frequency operation, compute the new average value of output voltage.

10.28 Draw the output voltage, output current, current through switches and source current waveforms for class D or two quadrant type B choppers when $T_{ON} < T_{OFF}$. Assume load is inductive

10.29 A dc-to-dc stepdown converter circuit with R load has input voltage of 200 V, and a resistive load of 10 Ω. If the duty cycle is 70%, switching frequency is 1.5 kHz, determine (a) average output voltage, (b) rms output voltage, (c) ripple factor and (d) effective input resistance.

10.30 A dc-to-dc converter is connected to a 120 V dc source with an inductive load $R = 8$ Ω and $L = 10$ mH. A free wheeling diode is also connected across load. Assume the load current varies from 12 A to 16 A. Determine the time ratio of dc-to-dc converter.

10.31 A stepdown chopper with *RLE* load has continuous constant load current. Determine the maximum value of average thyristor current rating. Assume that switch S is thyristor.

10.32 A stepdown chopper with *RLE* load has input voltage of 220 V, $R = 2$ ohms, $L = 12$ mH, $E = 55$ V, $T_{ON} =$ 500 μs, $T = 1000$ μs.
(a) Determine whether load current is continuous or discontinuous.
(b) Find the value of average load current.
(c) Calculate the maximum and minimum value of load current.
(d) Draw the load current, free wheeling diode current and current through switch.
(e) Ripple current.

10.33 Prove the following equations for a stepdown chopper with *RLE* load with continuous load current:

(i) $I_{max} = \dfrac{V}{R}\dfrac{\left(1-e^{-\frac{T_{ON}}{\tau}}\right)}{\left(1-e^{-\frac{T}{\tau}}\right)} - \dfrac{E}{R}$ (ii) $I_{min} = \dfrac{V}{R}\dfrac{\left(e^{\frac{T_{ON}}{\tau}}-1\right)}{\left(e^{\frac{T}{\tau}}-1\right)} - \dfrac{E}{R}$

10.34 A stepdown chopper with *RLE* load has input voltage of 220 V, R = 2 ohms, L = 12 mH, E = 55 V, T_{ON} = 500 µs, T = 1000 µs.
(a) Determine the first, second and third harmonics of load current.
(b) Compute the average value of supply current.
(c) Find power input, power absorbed by load emf and power loss in resistance.
(d) Calculate the rms value of load current.

10.35 A type-A chopper with *RLE* load has input voltage of 210 V, R = 1 ohms, L = 4.5 mH, E = 62 V, T_{ON} = 400 µs, T = 1600 µs. (a) Find whether load current is continuous or discontinuous. (b) Determine the value of average output voltage and average load current. (c) Calculate the maximum and minimum value of load current. (d) Draw the load current, free wheeling diode current and current through switch.

10.36 Figure 10.76 shows a type-A chopper with LE load. This chopper circuit has input voltage of 400 V, L = 0.1 H and constant E. When the duty cycle is 0.35, determine the chopping frequency to limit the load ripple current to 10 A.

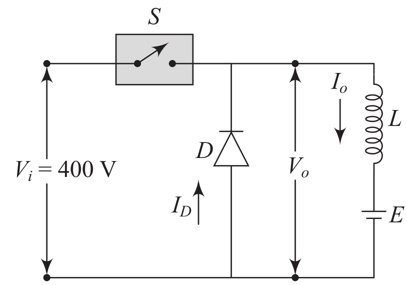


Fig. 10.76

10.37 A buck converter has input voltage of 210 V and it operates at 600 Hz. The average load current is 450 A. The load resistance is 2 Ω. Determine the value of inductance to limit the maximum peak to peak ripple current through inductor to 12%. Find the value of inductance for maximum ripple current.

10.38 A boost converter has input voltage of 15 V and it operates at 30 kHz. When the duty cycle is 0.3, L = 150 µH, C = 147 µF and average load current is 1.5 A, determine the average output voltage, peak-to-peak ripple current through inductor.

10.39 A buck-boost converter has input voltage of 24 V and it operates at 25 kHz. When the duty cycle is 0.35, L = 500 µH, C = 147 µF and average load current is 1 A, determine the average output voltage, peak-to-peak ripple current through inductor.

10.40 Assume that a CUK converter operates at 30 kHz to get an output voltage 160 V when the dc input voltage is about 24 V. Find the duty ratio and the voltage across switch during OFF period.

10.41 A stepdown chopper has a load resistance of 20 Ω and input dc voltage is 220 V. When the chopper switch is ON, the voltage across semiconductor switch is 2.2 V. If the chopping frequency is 1.5 kHz and duty ratio is 40%, determine (a) average dc output voltage, (b) rms output voltage and (c) efficiency of chopper.

10.42 A step-up chopper has input voltage of 200 V and output voltage of 500 V. (a) If the conduction time of switch is 110 µs, determine the pulse width of output voltage. (b) If the pulse width of output voltage becomes one half for constant frequency operation, find the new average value of output voltage.

10.43 A stepup chopper is used to provide 550 V from a 200 V dc input voltage. If the blocking period of thyristor is 80 µs, what is the conduction period of thyristor?

10.44 Show that the critical inductance of the filter in a step-down chopper circuit is

$$L = \frac{V_o^2(V - V_o)}{2fVP_o}$$

where V_o is output voltage, V is input voltage, Po is output power and f is chopping frequency.

10.45 Show that the critical inductance in the load circuit of a stepdown chopper is directly proportional to $x(1-x)$ where x is the duty cycle.

10.46 A 120 V dc chopper operates using current limit control (CLC) strategy, the maximum value of load current is 200 A and the lower limit of current is 50 A. The ON time and OFF time of chopper are 20 ms and 30 ms respectively. Determine (a) the limit of current pulsation, (b) chopping frequency, (c) duty cycle and (d) output voltage.

10.47 A type A copper operates at 1 kHz from 220 V dc supply and it is connected to a RL load. The load time constant is 5 ms and the load resistance is 12 Ω. Determine the average load current and the amplitude of current ripple for mean value of output voltage of 60 V. Find the maximum value and minimum value of load current.

10.48 A type A chopper feed to an *RLE* load. Prove that the maximum value of rms current rating of freewheeling diode is $0.385\frac{V}{R}\left(1-\frac{E}{V}\right)^{3/2}$ when load current is ripple free.

10.49 The speed of separately excited dc motor is controlled by a class A chopper and the motor operates at below rated speed. The armature resistance is $R_a = 0.3$ Ω and inductance is $L_a = 20$ mH. The dc input voltage is $V = 200$ V, the motor constant is $k = 0.1$ V/rpm. When motor operates at constant load torque with an average load current of 40 A, determine (a) the range of speed control and (b) the range of duty cycle.

10.50 In a battery operated chopper fed dc drive, the maximum possible value of accelerating current is 425 A, the lower limit of current pulsation is 200 A. The ON period of switch is 14 ms and OFF period of switch is 11 ms and the time constant is 63.5 ms. Find (a) the higher limit of current pulsation, (b) chopping frequency and (c) duty cycle. Assume battery voltage $V = 220$ V and $R = 0.1$ Ω.

10.51 A chopper controlled separately excited dc motor is operated by a dc battery. When dc motor operates at full rating, $V = 80$ V, $I = 200$ A and $N = 2500$ rpm. The current pulsation is maintained in between 180 A and 230 A. Determine (a) chopping frequency and (b) duty cycle when armature resistance $R_a = 0.045$ Ω, armature inductance $L_a = 7$ mH, battery resistance $R_b = 0.055$ Ω and speed = 100 rpm.

10.52 The switching frequency of chopper as shown in Fig. 10.77(a) is 1.5 kHz. This chopper circuit is operated at the boundary of continuous and discontinuous conduction. If the current waveform follows the Fig. 10.77(b), determine the on time of chopper and the value of peak current.

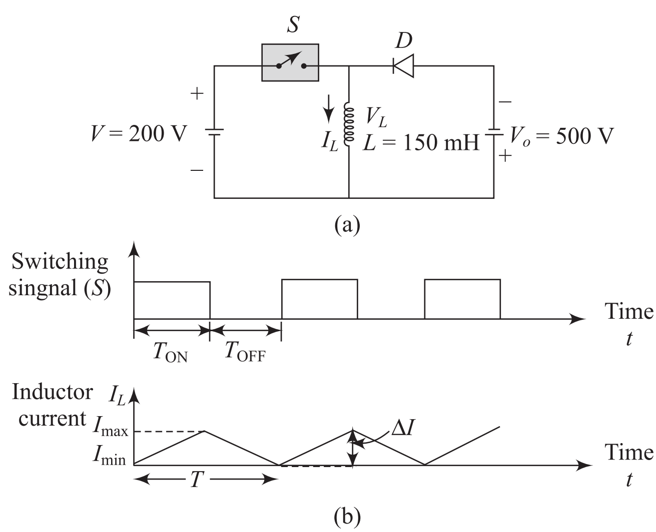


Fig. 10.77 *(a) Chopper circuit and (b) Switching signal and inductor current*

10.53 A buck converter has input voltage of 12 V and the required average output voltage is 6 V at $R = 400$ Ω and the peak to peak output ripple voltage is 20 mV. If it operates at 20 kHz and peak to peak ripple current of inductor is 0.8 A, determine (a) duty cycle ratio, (b) filter inductance L, (c) filter capacitance C and (d) critical values of L and C.

10.54 A boost converter has input voltage of 5 V and it operates at 25 kHz. When the average output voltage V_o = 12 V, the average load current I_o = 0.8 A, L = 100 μH, and C = 147 μF, determine (a) duty cycle, (b) ripple current of inductor ΔI, (c) the maximum current flows through inductor I_{max}, (d) ripple voltage across capacitor and (e) critical values of L and C.

10.55 A buck-boost converter has input voltage of 15 V. The duty cycle is 0.3 and it operates at 30 kHz. When L = 300 μH, C = 220 μF and average load current is 1.5 A, determine (a) average output voltage, (b) peak-to-peak output voltage ripple, (c) peak-to-peak ripple current through inductor, (d) maximum current flows through switch and (e) the critical values of L and C.

10.56 A battery operated electric car is operated by voltage commuted chopper. The battery voltage is 22 V, starting current is 5 A and thyristor turn-OFF time is 15 μs. Determine the commutating inductor L and commutating capacitor C.

10.57 A voltage commutated chopper has the following parameters:
V = 220 V, *RLE* load parameters (R = 0.25 Ω L = 4mH, E = 50 V)
The commutation circuit parameters are: L = 30 μH, C = 50 μF, T_{ON} = 500 μs, T = 1000 μs.
If the load current is 60 A, determine (a) effective ON time of chopper, (b) peak current flows through T_1 and $T_{A,}$ (c) turn-OFF time of T_1 and T_A and (d) total commutation interval.

Answers to Multiple-Choice Questions

10.1 (a)	10.2 (d)	10.3 (d)	10.4 (a)	10.5 (a)	10.6 (d)	10.7 (a)
10.8 (b)	10.9 (b)	10.10 (a)	10.11 (b)	10.12 (a)	10.13 (c)	10.14 (a)
10.15 (a)	10.16 (a)	10.17 (b)	10.18 (a)	10.19 (a)	10.20 (a)	10.21 (a)
10.22 (b)	10.23 (a)	10.24 (b)	10.25 (d)	10.26 (a)	10.27 (c)	10.28 (a)
10.29 (d)	10.30 (a)	10.31 (c)	10.32 (a)	10.33 (c)	10.34 (c)	10.35 (c)
10.36 (c)						

Answers to Fill in the Blanks

10.1 chopper
10.2 step-down chopper
10.3 step-up chopper
10.4 thyristors (SCRs), power BJTs, power MOSFETs and IGBTs
10.5 Time ratio control, Current limit control
10.6 I_{min}, I_{max}
10.7 $V_o = DV$
10.8 $\left(V_o = \frac{1}{1-D}V\right)$
10.9 $\left(V_o = -\frac{D}{1-D}V\right)$
10.10 $\left(V_o = \frac{D}{1-D}V\right)$
10.11 D = 0.5)
10.12 varied, constant
10.13 switch mode power supply (SMPS)
10.14 main thyristor T_1 , auxiliary thyristor $T_{A,}$ free wheeling
10.15 $\left(V_{av} = DV + \left(1 - \frac{t_x}{T}\right)E\right)$
10.16 0, unity
10.17 greater, $0 < D < 1$
10.18 less, $0 \le D \le 1$
10.19 E
10.20 C
10.21 A
10.22 B
10.23 $V_o = T_{ON} fV$
10.24 inversely

INVERTERS

11

11.1 INTRODUCTION

An inverter is a converter circuit which is used to convert dc power into ac power at desired output voltage and frequency. The block diagram representation of inverter is shown in Fig. 11.1. In an inverter, input is dc voltage, but output is ac at desired voltage with required frequency. The output voltage may be fixed or variable. Similarly, the frequency of output voltage is also fixed or variable. Usually the output voltage can be controlled by pulse width modulation (PWM) technique, gain control of inverter (ac output voltage/dc input voltage) and controlling modulation index.

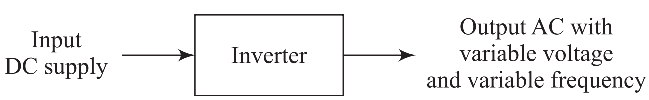


Fig. 11.1 *Block diagram representation of inverter*

Inverters are most commonly used in the following applications:

- Variable speed induction motor drives
- Adjustable speed ac drives
- Induction heating
- Uninterruptible power supply (UPS)
- Standby power supply
- HVDC power transmission
- Variable voltage and variable frequency power supply
- Battery operated vehicle drives

The phase controlled or line commutated full converters can operate inverter mode at the line frequency only. These converters are called *line commutated inverters*. But the line commutated inverters require an ac supply which is used for commutation of thyristors, at the output. Therefore, line commutated inverters can not work as isolated ac voltage source with variable voltage and variable frequency. When a thyristor-based inverter is used to provide an isolated ac voltage source, the forced commutation technique must be used to turn OFF SCRs. Consequently, thyristor-based inverters are costly and bulky. But these inverters are suitable for high power applications. In low power and moderate power applications, the gate commutation switching devices such as GTOs, BJTs, MOSFETs and IGBTs are used in inverter.

In this chapter, the classification of inverters, principle of operation of single-phase and three-phase inverters with *R* load and *RL* load, three-phase bridge inverters, performance parameters of inverters, different methods of voltage control and harmonic reduction of inverters are discussed in detail. The operating principles of resonant converters such as series, parallel, series-parallel, quasi-resonant, zero current and zero-voltage converters are also incorporated in this chapter.

11.2 CLASSIFICATION OF INVERTERS

Inverters can be classified depending upon the following factors:

1. Input source
2. Commutation
3. Circuit configuration
4. Wave shape of output voltage

11.2.1 Input Source

Based on the nature of input source, inverters are classified as current source inverter (CSI) and voltage source inverter (VSI).

Current Source Inverter (CSI) In this type of inverter, a current source with high internal impedance is used as input of inverter. Figure 11.2 shows the block diagram representation of current source inverter. In CSI, the supply current does not change very rapidly, but the load current can be controlled by varying dc input voltage of CSI. This inverter is commonly used in very high power applications such as induction motor drives.

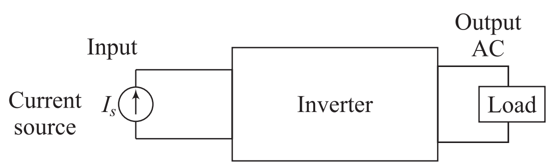


Fig. 11.2 *Block diagram representation of current source inverter (CSI)*

Voltage Source Inverter (VSI) In voltage source inverter (VSI), a dc voltage source with very small internal impedance is used as input of inverter. Figure 11.3 shows the block diagram representation of voltage source inverter. The dc side terminal voltage is constant, but the ac side output voltage may be constant or variable irrespective of load current. The VSI can be classified as half-bridge VSI and full bridge VSI.

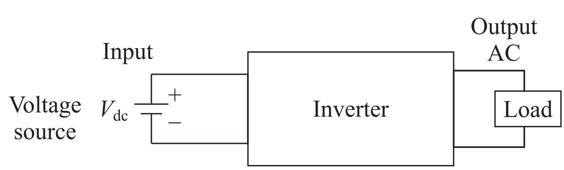


Fig. 11.3 *Block diagram representation of voltage source inverter (CSI)*

11.2.2 Commutation

According to commutation method, inverters may be classified as line commutated inverters and forced commutated inverters.

Line commutated inverters Single-phase or three-phase fully controlled converter acts as an inverter when the firing angle α is greater than $90°\left(\frac{\pi}{2}\text{ radian}\right)$. This is a voltage source inverter and the used switching devices such as thyristors are naturally commutated.

Forced commutated inverters In these type of inverters, additional circuits are required for commutation of thyristors. Depending upon the commutation technique, these inverters are classified as an auxiliary commutated inverters and complementary commutated inverters.

Auxiliary commutated inverters In case of auxiliary commutated inverters, an auxiliary thyristor must be used to turn OFF a conducting thyristor. For example, modified McMurray inverters or McMurray inverters are auxiliary commutated inverters.

Complementary commutated inverters In complementary commutated inverters, tightly couple inductors can be used to turn OFF the other thyristor of a pair. For example, McMurray Bedford inverters are complementary commutated inverters.

11.2.3 Circuit Configuration

According to circuit topology or connection of semiconductor switches, inverters can be classified as series inverters, parallel inverters, half bridge inverters and full bridge inverters.

Series inverters In series inverters, inductor L and capacitor C are connected in series with the load. In this inverter L and C are used as commutating elements and the performance of inverter depends on the value of L and C.

Parallel inverters In case of parallel inverters, commutating elements are connected in parallel with the conducting thyristor.

Half-bridge inverters and full bridge inverters In half-bridge inverters, only one leg of bridge exists. In case of full bridge inverters, either two legs or three legs are existing for single-phase or three-phase inverters respectively.

11.2.4 Wave Shape of Output Voltage

In an ideal inverter, output voltage must be purely sinusoidal. But due to switching of semiconductor devices as per requirement of inverter operation, output voltage is non-sinusoidal and it contains harmonics. Depending upon the output voltage waveform this inverter can also be classified as square wave inverters and pulse width modulation inverters.

Square wave inverters A square wave inverter generates a square wave ac output voltage of constant amplitude. The amplitude of the output voltage of inverter can be controlled by varying the input dc voltage.

Pulse width modulation inverter In pulse width modulation inverters, the output voltage contains one or more pulses in each half cycle. By varying the width of these pulses, the amplitude of output voltage can be controlled though the input dc voltage is constant. There are different control techniques of pulse width modulation which are explained in Section 11.7.

11.3 PERFORMANCE PARAMETERS OF INVERTERS

In an ideal inverter, the output voltage must be purely sinusoidal. However, the output voltage of practical inverters is non-sinusoidal and it contains fundamental components as well as harmonic components. Usually the performance of an inverter is measured by the following performance parameters:

Harmonic factor of n^{th} harmonic (HF_n) The Harmonic factor is a measure of the individual harmonic contribution in the output voltage of an inverter. This is defined by the ratio of the rms voltage of a particular harmonic component to the rms voltage of the fundamental component. It is represented by

$$HF_n = \frac{V_n}{V_1}$$

where, V_n is rms value of the n^{th} harmonic component and
V_1 is rms value of the fundamental component

Total Harmonic Distortion (THD) The total harmonic distortion is a measure of closeness in a shape between the output voltage waveform and its fundamental component. This is defined by the ratio of the rms value of the total harmonic component of the output voltage to the rms value of the fundamental component. It is represented by

$$THD = \frac{\left[\sum_{n=2,3,4,5...}^{\infty} V_n^2\right]^{1/2}}{V_1} \frac{[V_{\text{rms}}^2 - V_1^2]^{1/2}}{V_1}$$

where, V_{rms} is the rms value of output voltage.

Distortion Factor (DF) The distortion factor is used to measure the amount of harmonics that remains in the output voltage waveform, after the waveform has been subjected to second order attenuation (divided by n^2). The distortion factor is represented by

$$DF = \frac{\left[\sum_{n=2,3,4,5...}^{\infty} \left(\frac{V_n}{n^2}\right)^2\right]^{1/2}}{V_1}$$

Lowest Order Harmonics (LOH) It is the lowest frequency harmonic with a magnitude greater than or equal to three percent of the magnitude of the fundamental component of the output voltage. For higher the frequency of LOH, the distortion will be lower in the current waveform.

11.4 SINGLE-PHASE HALF-BRIDGE VOLTAGE SOURCE INVERTER

Figure 11.4 shows the circuit configuration of a single-phase half-bridge voltage source inverter (VSI). Here switches S_1 and S_2 are power semiconductor switches such as IGBT, BJT, MOSFET, etc. When the switch is closed, current flows through devices. While switches are opened, current flows through diode. In this section, the operating principle of half bridge VSI with *R* and *RL* load is explained in detail.

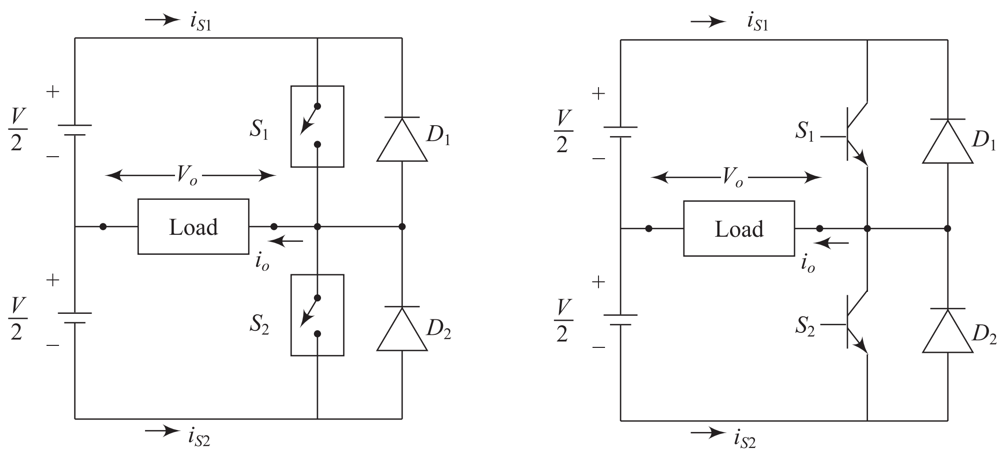


Fig. 11.4 *Single-phase half-bridge inverter*

11.4.1 Single-Phase Half-Bridge Voltage Source Inverter with *R* load

The operation of a single-phase half-bridge inverter with R load can be divided into two different modes such as

1. Mode I $\left(0 \le t \le \frac{T}{2}\right)$: Switch S_1 conducts and S_2, D_1, and D_2 are in OFF
2. Mode II $\left(\frac{T}{2} \le t \le T\right)$: Switch S_2 conducts and S_1, D_1, and D_2 are in OFF

1. Mode I $\left(0 \le t \le \frac{T}{2}\right)$ When the switch S_1 is closed for $\frac{T}{2}$, i.e., half of the time period, $\frac{V}{2}$ is applied across load and current flows through load is $\frac{V}{2R}$. The switching diagram of mode I is shown in Fig. 11.5(a) and the path of current flow during $0 \le t \le \frac{T}{2}$ is also shown in Fig. 11.5(a).

2. Mode II $\left(\frac{T}{2} \le t \le T\right)$ At $t = \frac{T}{2}$, switch S_1 is opened and switch S_2 is closed for $\frac{T}{2}$ duration. Then again $-\frac{V}{2}$ is applied across load and $-\frac{V}{2R}$ current will flow through load. The switching diagram of mode II is given in Fig. 11.5(b) and the path of current flow during $\frac{T}{2} \le t \le T$ is also depicted in Fig. 11.5(b).

The gating signals of S_1 and S_2, output voltage and current waveforms are shown in Fig. 11.6. Here the output voltage waveform is square wave. The current waveform is also similar to output voltage waveform. During R load, diodes D_1 and D_2 are not conducting. The frequency of output voltage is $f = \frac{1}{T}$. The output frequency can be controlled by varying the ON time and OFF time of switches. The operation of half-bridge inverter is represented by Table 11.1.

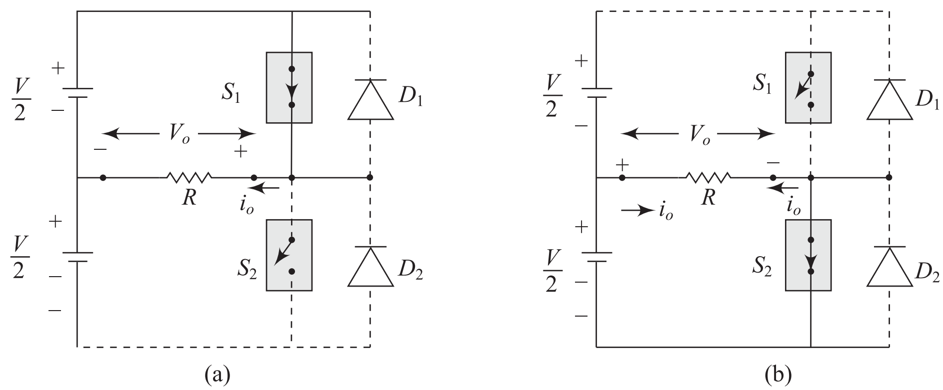


Fig. 11.5 *(a) Switching diagram of Fig. 11.4 in Mode I (b) Switching diagram of Fig. 11.4 in Mode II*

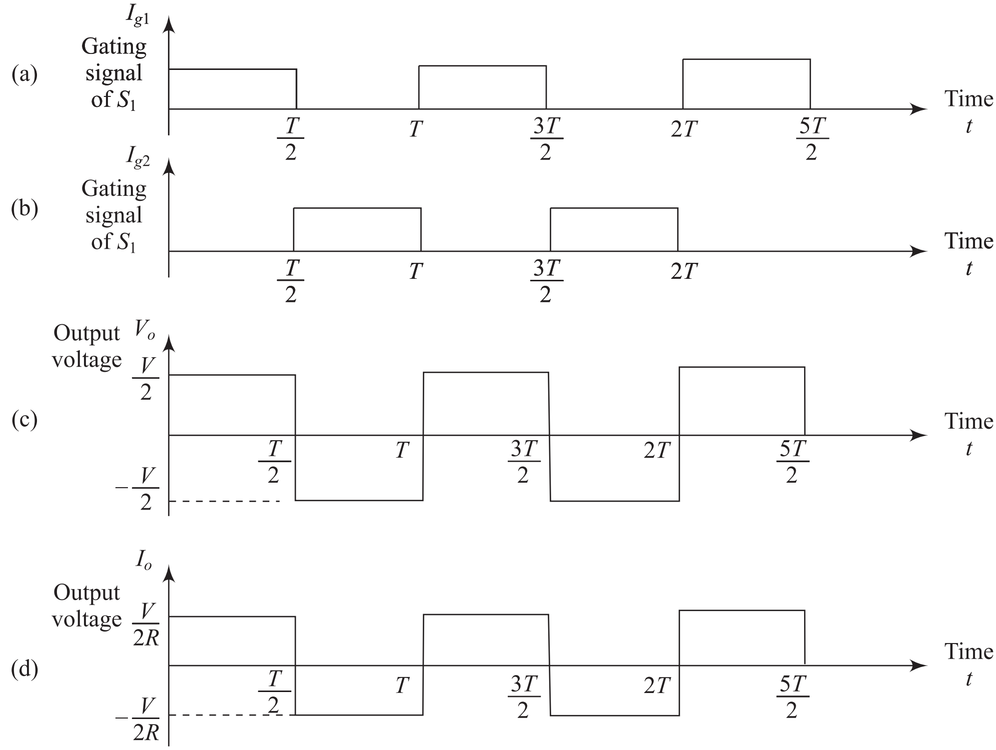


Fig. 11.6 *(a) i_{g1}, gating signals of switch S_1 (b) i_{g2}, gating signals of switch S_2 (c) Output voltage V_o (d) Output current i_o*

Table 11.1 *Operation of half-bridge inverter.*

Mode	Time	Load Voltage	Load Current	Conducting Device
I	$0 < t \le \frac{T}{2}$	$\frac{V}{2}$	$\frac{V}{2R}$	S_1
II	$\frac{T}{2} \le t \le T$	$-\frac{V}{2}$	$-\frac{V}{2R}$	S_2

Figure 11.6(c) shows the output voltage waveform. The average value of output voltage is

$$V_{o(\mathrm{av})} = \frac{1}{T}\int_0^T v_o(t)\cdot dt = 0$$

The rms value of output voltage is

$$V_{o(\mathrm{rms})} = \left[\frac{1}{T/2}\int_0^{\frac{T}{2}} v_o^2(t)\cdot dt\right]^{1/2} = \left[\frac{1}{T/2}\int_0^{\frac{T}{2}} \left(\frac{V}{2}\right)^2 \cdot dt\right]^{1/2} = \frac{V}{2}$$

Hence, the rms value of square wave is equal to the peak value.
The output voltage V_o can be expressed using Fourier series as

$$v_o(t) = \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

where,
$$a_n = \frac{1}{\pi}\int_0^{2\pi} v_o(t)\cos n\omega t\cdot d\omega t \quad \text{and}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v_o(t)\sin n\omega t\cdot d\omega t$$

Due to half-wave symmetry, only b_n components are present.

Therefore,
$$b_n = \frac{2}{\pi}\int_0^{\pi} v_o(t)\sin n\omega t\cdot d\omega t$$

or
$$b_n = \frac{2}{\pi}\int_0^{\pi} \frac{V}{2}\sin n\omega t\cdot d\omega t \quad \text{as } v_o(t) = \frac{V}{2}$$

$$= \frac{2V}{n\pi} \quad \text{where, } n = 1, 3, 5\ldots$$

The output voltage can be expressed as

$$v_o(t) = \sum_{n=1,2,3\ldots}^{\infty} \frac{2V}{n\pi}\sin n\omega t$$

The rms value of the n^{th} component is

$$V_n = \frac{1}{\sqrt{2}}\frac{2V}{n\pi} \text{ where, } n = 1, 3, 5$$

The rms value of fundamental component is V_1 = 0.45 V

Example 11.1 A single-phase half-bridge inverter feeds a resistive load of 5 Ω. When the voltage $\frac{V}{2}$ is 120 V, determine

(a) rms value of the fundamental component of output voltage
(b) the output power
(c) the average and peak current of transistors which are used in inverter
(d) the peak inverse voltage (PIV) of transistors
(e) the lowest order harmonics and the corresponding harmonic factor
(f) third harmonic distortion factor

Solution

Given: $\frac{V}{2} = 120$ V and $R = 5\ \Omega$

Therefore, $V = 2 \times 120\text{ V} = 240\text{ V}$

The output voltage of single phase half bridge inverter is expressed as

$$v_o(t) = \sum_{n=1,2,3...}^{\infty} \frac{2V}{n\pi} \sin n\omega t$$

(a) The rms value of fundamental component of output voltage is

$$V_1 = \frac{1}{\sqrt{2}}\frac{2V}{\pi} = \frac{1}{\sqrt{2}}\frac{2 \times 240\text{ V}}{\pi} V = 107.994\text{ V}$$

(b) The rms value of output voltage is $V_{o(\text{rms})} = \frac{V}{2} = 120\text{ V}$

The output power is equal to $P_o = \frac{V_{o(\text{rms})}^2}{R} = \frac{120^2}{5}$ Watt = 2880 Watt

(c) The current and voltage waveform of inverter is depicted in Fig. 11.7.

The average current of each transistor = $0.5 I_{s(\text{peak})} = 0.5 \times \frac{V}{2R} = 0.5 \times \frac{240}{2 \times 5} = 12\text{ A}$

The peak current of each transistor is $I_{s(\text{peak})} = \frac{V}{2R} = \frac{240}{2 \times 5} = 24\text{ A}$

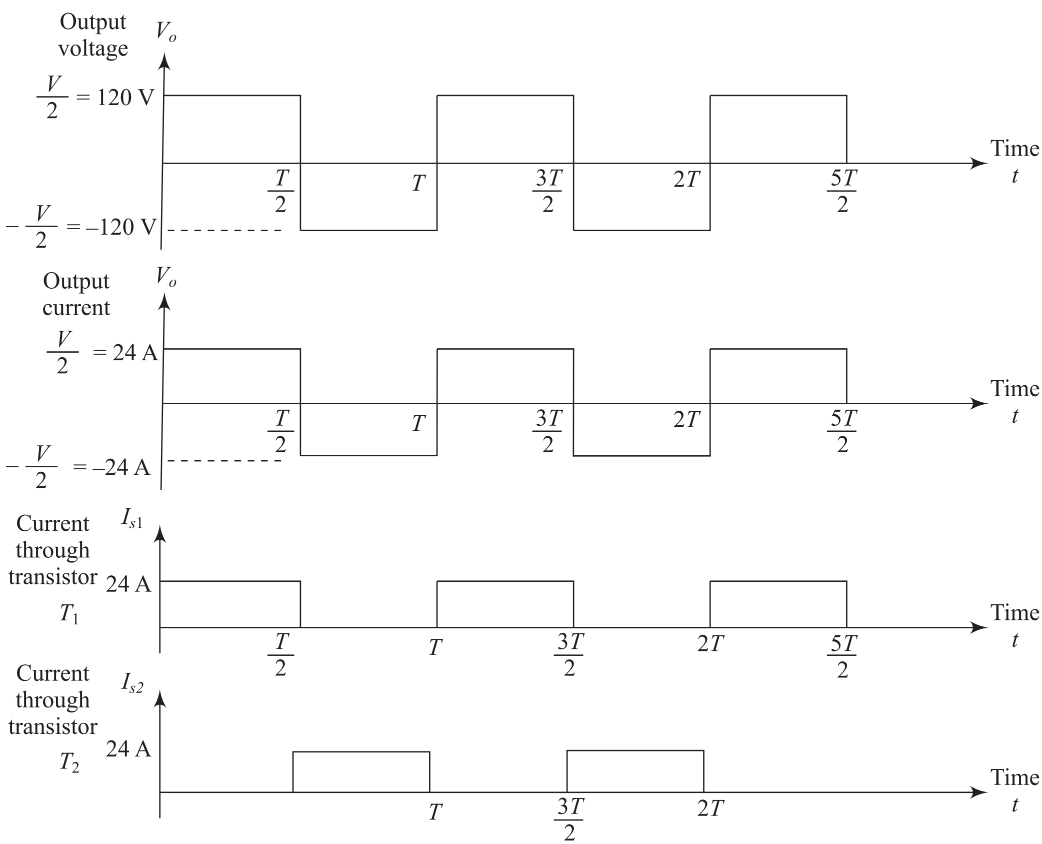


Fig. 11.7 *Voltage and current waveforms of single-phase half-wave inverter*

(d) The peak inverse voltage (PIV) of each transistor is equal to $2 \times \dfrac{V}{2} = V = 240$ V

(e) The lowest order harmonics is $V_{3(\mathrm{rms})} = \dfrac{2V}{3\pi} \times \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}V}{3\pi} = \dfrac{\sqrt{2} \times 240}{3\pi} = 35.998$ V

The corresponding harmonic factor is $HF_3 = \dfrac{V_{3(\mathrm{rms})}}{V_{1(\mathrm{rms})}} = \dfrac{35.998}{107.994} = 0.3333$

(f) Third harmonic distortion factor

$$THD = \frac{[V_{\mathrm{rms}}^2 - V_1^2]^{1/2}}{V_1} = \frac{[120^2 - 107.994^2]^{1/2}}{107.994} = 0.4844$$

where, V_{rms} is the rms value of output voltage.

Example 11.2 A single-phase half-bridge inverter has a resistive load of 10 Ω and the center tap dc input voltage is 100 V. Determine (a) rms value of output voltage, (b) rms value of fundamental component of output voltage, (c) first three harmonics of the output voltage waveform, (d) fundamental power consumption in load and (e) rms power consumed by load.

Solution

Given: $\dfrac{V}{2} = 100$ V and $R = 10\ \Omega$

Therefore, $V = 2 \times 100 = 200$ V

(a) The rms value of output voltage is $V_{o(\mathrm{rms})} = \dfrac{V}{2} = 100$ V

(b) The rms value of fundamental component of output voltage

$$V_1 = \frac{1}{\sqrt{2}} \frac{2V}{\pi} = \frac{1}{\sqrt{2}} \frac{2 \times 200}{\pi} = 90.07 \text{ V}$$

(c) Since $V_n = \dfrac{V_1}{n}$, first three harmonics of the output voltage waveform is

rms value of third harmonic voltage is $V_3 = \dfrac{V_1}{3} = \dfrac{90.07}{3} = 30.023$ V

rms value of fifth harmonic voltage is $V_5 = \dfrac{V_1}{5} = \dfrac{90.07}{5} = 18.014$ V

rms value of seventh harmonic voltage is $V_7 = \dfrac{V_1}{7} = \dfrac{90.07}{7} = 12.867$ V

(d) Fundamental power consumption in load is $P_1 = \dfrac{V_1^2}{R} = \dfrac{90.07^2}{10} = 811.26$ Watt

(e) rms power consumed by load $P_{\mathrm{rms}} = \dfrac{V_{o(\mathrm{rms})}^2}{R} = \dfrac{100^2}{10} = 1000$ Watt

11.4.2 Single-Phase Half-Bridge Voltage Source Inverter with *RL* Load

For *RL* load, the output voltage wave form is similar to that with *R* load but the load current waveform is different from the load current with resistive load. The output voltage and current waveforms are illustrated in Fig. 11.8. The operation of a single-phase half-wave inverter with *RL* load can be divided into four different modes such as

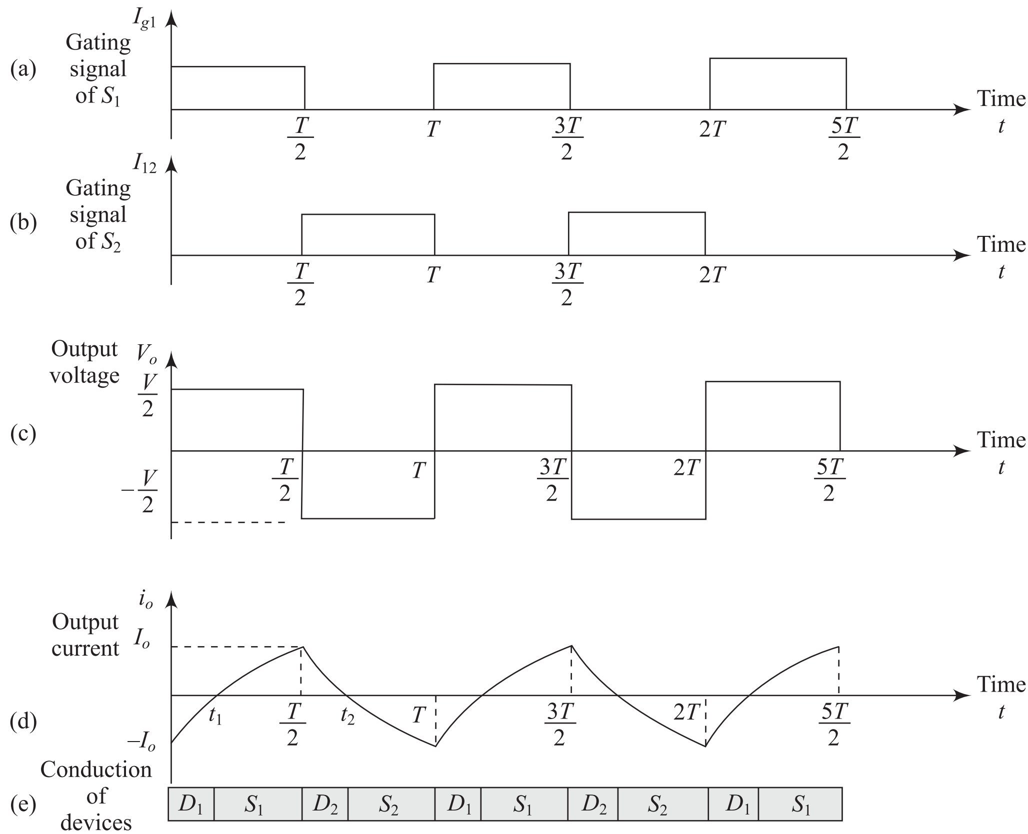


Fig. 11.8 *(a) i_{g1} gating signals of switch S_1, (b) i_{g2} gating signals of switch S_2, (c) Output voltage V_o, (d) Output current i_o and (e) Conduction of devices*

1. **Mode I** $(0 \le t \le t_1)$: Diode D_1 conducts
2. **Mode II** $\left(t_1 \le t \le \frac{T}{2}\right)$**:** Switch S_1 conducts
3. **Mode III** $\left(\frac{T}{2} \le t \le t_2\right)$**:** Diode D_2 conducts
4. **Mode IV** $(t_2 \le t \le T)$**:** Switch S_2 conducts

1. Mode I (0 ≤ *t* ≤ t_1) At $t = 0$, the gating signal is removed from switch S_2 and it becomes OFF. At this instant the load current is i_o which is equal to its negative peak value ($-I_o$). Due to inductive load, the load current can not be reversed instantly and then diode D_1 starts to conduct at $t = 0$. Subsequently, the output voltage across load is $\frac{V}{2}$ and the load current i_o increases form its negative peak value ($-I_o$) as the current cannot reverse instantaneously due to inductive load. Then the load current flows through diode D_1. In the time interval $0 \le t \le t_1$, the voltage across load is positive, but load current is negative. Hence the energy stored in inductance L during previous cycle must be fed back to dc supply through D_1 and the load current decreases slowly. At $t = t_1$, the load current becomes zero. The switching diagram of mode I is given in Fig. 11.9(a) and the path of current flow during $0 \le t \le t_1$ is also shown in Fig. 11.9(a).

2. Mode II $\left(t_1 \le t \le \frac{T}{2}\right)$ At the instant $t = t_1$, diode D_1 becomes OFF but switch S_1 is ON. The current starts to flow in positive direction and it reaches its maximum positive peak value I_o at $t = \frac{T}{2}$. During the time interval $t_1 \le t \le \frac{T}{2}$, both the output voltage as well as current is positive and energy stored in inductance L. The switching diagram of mode II is shown in Fig. 11.9(b) and the path of current flow during $t_1 \le t \le \frac{T}{2}$ is shown in Fig. 11.9(b).

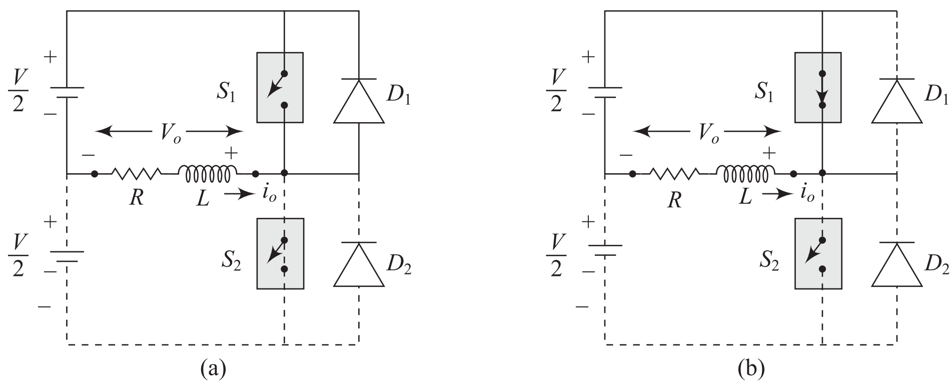


Fig. 11.9 *(a) Switching diagram of half bridge inverter with R-L Load in mode I (b) Switching diagram of half bridge inverter with R-L Load in mode II*

3. Mode III $\left(\frac{T}{2} \le t \le t_2\right)$ At $t = \frac{T}{2}$, switch S_1 becomes OFF. At this instant, the load current i_o is equal to its positive peak value (I_o). Due to inductive load, the load current can not be reversed instantly and then diode D_2 starts to conduct at $t = \frac{T}{2}$. After that the output voltage across load is $-\frac{V}{2}$ and the load current i_o decreases form its positive peak value I_o as the current cannot reverse instantaneously due to inductive load. This current flows through diode D_2. During the time interval $\frac{T}{2} \le t \le t_2$, the output voltage is negative but the load current is positive. It decreases slowly and reaches zero at $t = t_2$. The energy stored in inductance will be released and fed back to dc supply during this period. Figure 11.10(a) shows the load current flow path through diode D_2 and load and the switching diagram of mode III.

4. Mode IV ($t_2 \le t \le T$) At instant $t = t_2$, diode D_2 becomes OFF and switch S_2 is ON. The load current starts to flow in negative direction and it reaches maximum negative $-I_o$ at $t = T$. The switching diagram of mode IV is shown in Fig. 11.10(b) and the current flow path through S_2 and load is also depicted in Fig. 11.10(b). The operation of half-bridge inverter is represented by Table 11.2.

Table 11.2

Mode	Time	Load Voltage	Load Current	Conducting Device
I	$0 \le t \le t_1$	$\frac{V}{2}$	Negative	D_1
II	$t_1 \le t \le \frac{T}{2}$	$\frac{V}{2}$	Positive	S_1
III	$\frac{T}{2} \le t \le t_2$	$-\frac{V}{2}$	Positive	D_2
IV	$t_2 \le t \le T$	$-\frac{V}{2}$	Negative	S_2

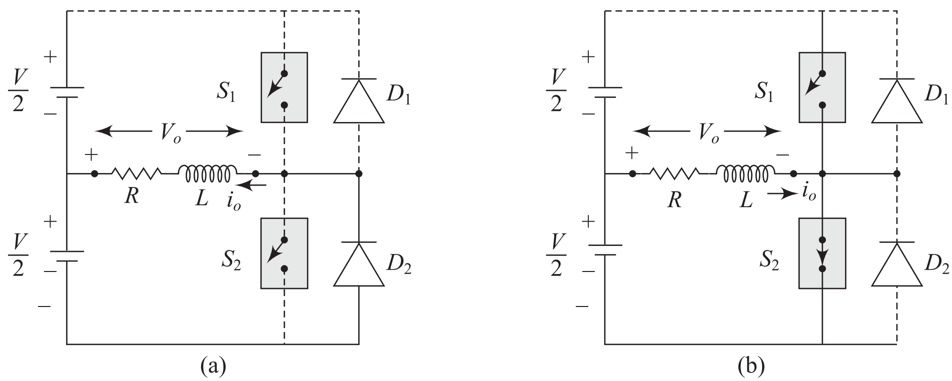


Fig. 11.10 *(a) Switching diagram of half bridge inverter with RL Load in mode III (b) Switching diagram of half bridge inverter with RL Load in mode IV*

The output voltage can be expressed as

$$\frac{V}{2} = i_o(t)R + L\frac{di_o(t)}{dt} \quad \text{for } 0 < t \le \frac{T}{2}$$

Assume that the initial condition is $i_o(t=0) = -I_o$. Then the output current can be expressed as

$$i_o(t) = \frac{V}{2R}\left(1 - e^{-\frac{t}{\tau}}\right) - I_o e^{-\frac{t}{\tau}} \tag{11.1}$$

where, $\tau = \frac{L}{R}$

Assume at $t = \frac{T}{2}, i_o\left(t = \frac{T}{2}\right) = I_o$

Then $$i_o\left(\frac{T}{2}\right) = \frac{V}{2R}\left(1 - e^{-\frac{T}{2\tau}}\right) - I_o e^{-\frac{T}{2\tau}} = I_o$$

or $$\frac{V}{2R}\left(1 - e^{-\frac{T}{2\tau}}\right) = I_o\left(1 + e^{-\frac{T}{2\tau}}\right)$$

or $$I_o = \frac{V}{2R}\frac{\left(1-e^{-\frac{T}{2\tau}}\right)}{\left(1+e^{-\frac{T}{2\tau}}\right)}$$

After substituting the value of I_o in Eq. (11.1), we get

$$i_o(t) = \frac{V}{2R}\left(1-e^{-\frac{t}{\tau}}\right) - \frac{V}{2R}\frac{\left(1-e^{-\frac{T}{2\tau}}\right)}{\left(1+e^{-\frac{T}{2\tau}}\right)}e^{-\frac{t}{\tau}}$$

or $$i_o(t) = \frac{V}{2R}\left(1-\frac{2}{1+e^{-\frac{T}{2\tau}}}e^{-\frac{t}{\tau}}\right)$$

During $\frac{T}{2} \le t \le T$, the voltage equation can be written as

$$-\frac{V}{2} = i_o(t')R + L\frac{di_o(t')}{dt} \quad \text{where, } t' = t - \frac{T}{2}$$

Assume $i_o\left(t=\frac{T}{2}\right) = I_o$ and $i_o(t=T) = -I_o$

The output current can be expressed as

$$i_o(t) = -\frac{V}{2R}\left(1-\frac{2}{1+e^{-\frac{T}{2\tau}}}e^{-\frac{(t-T/2)}{\tau}}\right) \qquad \text{for} \quad \frac{T}{2} \le t \le T$$

The rms output voltage is

$$V_{\text{rms}} = \left[\frac{1}{T/2}\int_0^{T/2}\left(\frac{V}{2}\right)^2 dt\right]^{1/2} = \frac{V}{2}$$

Due to half-wave symmetry, only b_n components are present

$$b_n = \frac{2}{\pi}\int_0^{\pi}\frac{V}{2}\sin n\omega t \cdot d\omega t$$

$$= \frac{2V}{n\pi} \quad \text{where, } n = 1, 3, 5...$$

The rms value of fundamental component is

$$V_{1\text{rms}} = \frac{2V}{\sqrt{2}\pi} = 0.45 \text{ V}$$

The instantaneous load current can be expressed as

$$i_o(t) = \sum_{n=1,3,5...}^{\infty}\frac{2V}{n\pi\sqrt{R^2+(n\omega L)^2}}\sin(n\omega t - \phi_n) \text{ and } \phi_n = \tan^{-1}\frac{n\omega L}{R}$$

where, the impedance offered by the load to the n^{th} harmonic is $Z_n = \sqrt{R^2+(n\omega L)^2}$

The fundamental load current is

$$i_{o1} = \frac{2V}{\pi\sqrt{R^2+(\omega L)^2}}\sin(\omega t - \phi_1) \quad \text{and} \quad \phi_1 = \tan^{-1}\frac{\omega L}{R}$$

Example 11.3 A single-phase half-bridge inverter feeds an *RL* load with $R = 10\ \Omega$ and $L = 0.1$ H. When the voltage $\frac{V}{2}$ is 120 V and the frequency of output voltage is 50 Hz, find

(a) The output current for the first two half cycles of output voltage.
(b) Third harmonic distortion factor (THD) of load current.

Solution

Given: $\frac{V}{2} = 120$ V, $R = 10\ \Omega$ and $L = 0.1$ H.

Therefore, $V = 2 \times 120 \text{ V} = 240 \text{ V}$

(a) For the first or positive half cycles of output voltage $\left(0 < t \le \frac{T}{2}\right)$, the output voltage can be expressed as

$$\frac{V}{2} = i_o(t)R + L\frac{di_o(t)}{dt}$$

As $i_o(t=0) = -I_o,\ i_o(t) = \frac{V}{2R}\left(1 - e^{-\frac{t}{\tau}}\right) - I_o e^{-\frac{t}{\tau}}$ where, $\tau = \frac{L}{R} = \frac{0.1}{10} = 0.01$

At $t = \frac{T}{2}, i_o\left(t = \frac{T}{2}\right) = I_o = \frac{V}{2R}\frac{\left(1 - e^{-\frac{T}{2\tau}}\right)}{\left(1 + e^{-\frac{T}{2\tau}}\right)}$ where, $T = \frac{1}{f} = \frac{1}{50} = 0.02$ s and $\frac{T}{2} = 0.01$ s

Then $i_o(t) = \frac{V}{2R}\left(1 - \frac{2}{1 + e^{-\frac{T}{2\tau}}}e^{-\frac{t}{\tau}}\right) = \frac{240}{2 \times 10}\left(1 - \frac{2}{1 + e^{-\frac{0.2}{2 \times 0.01}}}e^{-\frac{t}{0.01}}\right) = 12(1 - 1.9999e^{-100t})$

For the second or negative half cycles of output voltage $\left(\frac{T}{2} \le t \le T\right)$, the output current can be expressed as

$$i_o(t) = -\frac{V}{2R}\left(1 - \frac{2}{1 + e^{-\frac{T}{2\tau}}}e^{-\frac{(t - T/2)}{\tau}}\right) = -\frac{240}{2 \times 10}\left(1 - \frac{2}{1 + e^{-\frac{0.2}{2 \times 0.01}}}e^{-\frac{(t - T/2)}{0.01}}\right)$$

$$= -12(1 - 1.9999e^{-100(t - 0.01)})$$

(b) The instantaneous load current can be expressed as

$$i_o(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V}{n\pi\sqrt{R^2 + (n\omega L)^2}}\sin(n\omega t - \phi_n) \text{ and } \phi_n = \tan^{-1}\frac{n\omega L}{R}$$

The rms value of fundamental current

$$I_1 = \frac{2V}{\pi\sqrt{R^2 + (2\pi fL)^2}} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}V}{\pi\sqrt{R^2 + (2\pi fL)^2}}$$

$$= \frac{\sqrt{2} \times 240}{\pi\sqrt{10^2 + (2\pi \times 50 \times 0.1)^2}} = 1.6372 \text{ A}$$

Similarly, rms value of third and fifth harmonics current are

$$I_3 = \frac{\sqrt{2}V}{3\pi\sqrt{R^2 + (2\pi \times 3fL)^2}} = \frac{\sqrt{2} \times 240}{3\pi\sqrt{10^2 + (2\pi \times 3 \times 50 \times 0.1)^2}} = 0.001349 \text{ A}$$

$$I_5 = \frac{\sqrt{2}V}{5\pi\sqrt{R^2 + (2\pi \times 5fL)^2}} = \frac{\sqrt{2} \times 240}{5\pi\sqrt{10^2 + (2\pi \times 5 \times 50 \times 0.1)^2}} = 2.91554 \times 10^{-4} \text{ A}$$

THD of load current is

$$= \frac{\sqrt{I_3^2 + I_5^2 + I_7^2 + I_9^2 + \cdots}}{I_1} = \frac{\sqrt{0.001349^2 + (2.91554 \times 10^{-4})^2 + \cdots}}{1.6372}$$

$$= 0.001178$$

11.5 SINGLE-PHASE FULL BRIDGE INVERTER

Figures 11.11 and 11.12 show single-phase full bridge inverters which consists of two pairs of controlled switches (S_1, S_2, and S_3, S_4) and two pairs of diodes (D_1, D_2 and D_3, D_4). Among these devices, only one pair of devices conducts simultaneously. When switches S_1 and S_2 are ON, the output voltage across load is $+V$. Similarly, when switches S_3 and S_4 are ON, the output voltage across load is $-V$. Diodes D_1, D_2, D_3, and D_4 are used as feedback diodes. The load of full bridge inverter will be either resistive or inductive. The sequence of switching of switches and diodes for R load and RL load are given below.

For R Load

$$S_1\,S_2\,/\,S_3\,S_4\,/\,S_1\,S_2\,/\,S_3\,S_4\,/...$$

For RL Load

$$D_1\,D_2\,/\,S_1\,S_2\,/\,D_3\,D_4\,/\,S_3\,S_4\,/\,D_1\,D_2\,/\,S_1\,S_2\,/\,D_3\,D_4\,/\,S_3\,S_4\,/$$

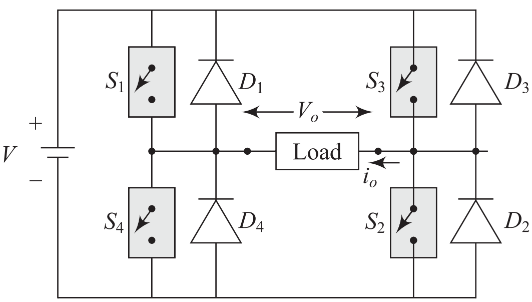


Fig. 11.11 *Single-phase full bridge inverter*

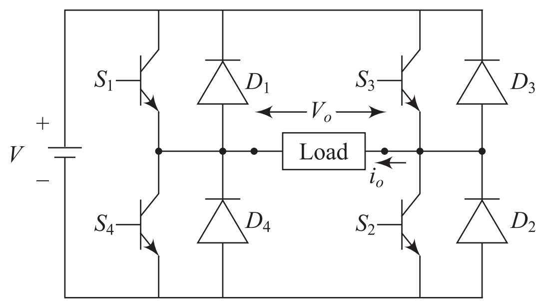


Fig. 11.12 *Single-phase full bridge inverter using BJT*

11.5.1 Single-Phase Full Bridge Inverter with R Load

Similar to single-phase half bridge inverter, the operation of a single-phase full bridge inverter with R load can be divided into two different modes such as

1. Mode I $\left(0 \le t \le \frac{T}{2}\right)$: Switches S_1 and S_2 conduct
2. Mode II $\left(\frac{T}{2} \le t \le T\right)$: Switches S_3 and S_4 conduct

1. Mode I $\left(0 \le t \le \frac{T}{2}\right)$ In this mode, switches S_1 and S_2 are closed for $\frac{T}{2}$, i.e., half of the time period, and the voltage V is applied across load. Then current flows through load is $\frac{V}{R}$. The switching diagram of mode I is illustrated in Fig. 11.13(a) and the path of current flow during $0 \le t \le \frac{T}{2}$ is also given in Fig. 11.13(a).

2. Mode II $\left(\frac{T}{2} \le t \le T\right)$ At $t = \frac{T}{2}$, switches S_1 and S_2 are turned OFF and switches S_3 and S_4 are turned ON for $\frac{T}{2}$ duration. Then again $-V$ is applied across load and $-\frac{V}{R}$ current will flow through load. The switching diagram of mode II is shown in Fig. 11.13(b) and the path of current flow during $\frac{T}{2} \le t \le T$ is also depicted in Fig. 11.13(b). At $t = T$, again S_1 and S_2 are turned ON and S_3 and S_4 turn OFF and cyclically switches are ON and OFF repeatedly.

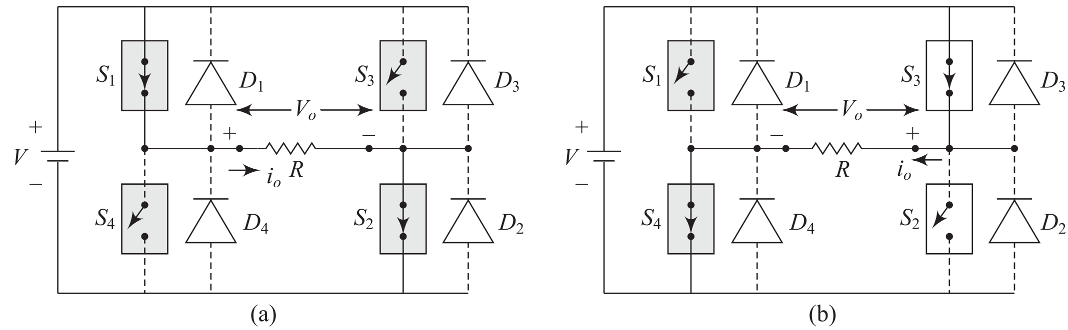


Fig. 11.13 *(a) Switching diagram of Fig. 11.11 in mode I and (b) Switching diagram of Fig. 11.11 in mode II*

The gating signals of switches S_1, S_2, S_3 and S_4, output voltage and current waveforms are shown in Fig. 11.14. At this time, the output voltage waveform is a square wave. The current waveform is also similar to output voltage waveform. During R load, diodes D_1, D_2, D_3 and D_4 are not conducting. The frequency of output voltage is $f = \frac{1}{T}$. The output frequency can be controlled by varying the ON time and OFF time of switches. The operation of full-bridge inverter with R load is represented by Table 11.3.

Table 11.3

Mode	Time	Load Voltage	Load Current	Conducting Device
I	$0 < t \le \frac{T}{2}$	V	$\frac{V}{R}$	S_1 and S_2
II	$\frac{T}{2} \le t \le T$	$-V$	$-\frac{V}{R}$	S_3 and S_4

Figure 11.14(c) shows the output voltage waveform. The average value of output voltage is

$$V_{o(\text{av})} = \frac{1}{T}\int_0^T v_o(t) \cdot dt = 0$$

The rms value of output voltage is

$$V_{o(\text{rms})} = \left[\frac{1}{T/2}\int_0^{\frac{T}{2}} v_o^2(t) \cdot dt\right]^{1/2} = \left[\frac{1}{T/2}\int_0^{\frac{T}{2}} V^2 \cdot dt\right]^{1/2} = V$$

Therefore, the rms value of square wave is equal to the peak value.

The output voltage V_o can be expressed using Fourier series as

$$v_o(t) = \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

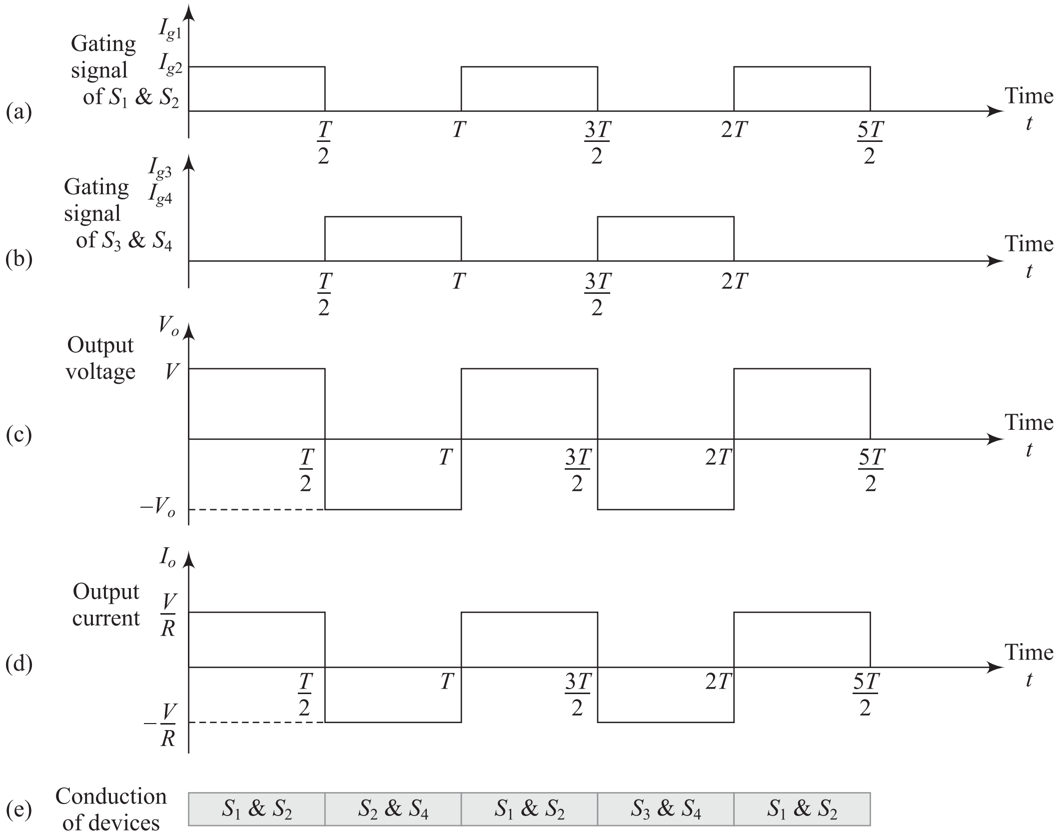


Fig. 11.14 *(a) i_{g1}, i_{g2} gating signals of switches S_1 and S_2, (b) i_{g3}, i_{g4} gating signals of switches S_3 and S_4 (c) Output voltage V_o, (d) Output current i_o (e) Conduction of devices*

where $$a_n = \frac{1}{\pi}\int_0^{2\pi} v_o(t)\cos n\omega t \cdot d\omega t \text{ and } b_n = \frac{1}{\pi}\int_0^{2\pi} v_o(t)\sin n\omega t \cdot d\omega t$$

Due to half-wave symmetry, only b_n components are present.

Therefore, $$b_n = \frac{2}{\pi}\int_0^{\pi} v_o(t)\sin n\omega t \cdot d\omega t$$

or $$b_n = \frac{2}{\pi}\int_0^{\pi} V \sin n\omega t \cdot d\omega t \qquad \text{as } v_o(t) = V$$

$$= \frac{4V}{n\pi} \qquad \text{where } n = 1, 3, 5\ldots$$

The output voltage can be expressed as

$$v_o(t) = \sum_{n=1,2,3\ldots}^{\infty} \frac{4V}{n\pi}\sin n\omega t$$

The rms value of the n^{th} component is

$$V_n = \frac{1}{\sqrt{2}}\frac{4V}{n\pi} \quad \text{where, } n = 1, 3, 5...$$

The rms value of fundamental component is

$$V_{1\text{rms}} = \frac{4V}{\sqrt{2}\pi} = 0.9 \text{ V}$$

Example 11.4 A single-phase full bridge inverter feeds a resistive load of 10 Ω. When the dc voltage source is 200 V, determine

(a) rms value of the fundamental component of output voltage
(b) the output power
(c) the average and peak current of transistors which are used in inverter
(d) the peak inverse voltage (PIV) of transistors
(e) the lowest order harmonics and the corresponding harmonic factor
(f) third harmonic distortion factor

Solution

Given: $V = 200$ V and $R = 10$ Ω

The output voltage of single-phase full bridge inverter is expressed as

$$v_o(t) = \sum_{n=1,2,3...}^{\infty} \frac{4V}{n\pi}\sin n\omega t$$

(a) The rms value of fundamental component of output voltage is

$$V_1 = \frac{1}{\sqrt{2}}\frac{4V}{\pi} = \frac{1}{\sqrt{2}}\frac{4\times 200 \text{ V}}{\pi}V = 179.9908 \text{ V}$$

(b) The rms value of output voltage is $V_{o(\text{rms})} = V = 200$ V

The output power is equal to $P_o = \frac{V_{o(\text{rms})}^2}{R} = \frac{200^2}{5}$ Watt $= 8000$ Watt

(c) The current and voltage waveform of inverter is depicted in Fig. 11.15.

The average current of each transistor $= 0.5I_{s(\text{peak})} = 0.5\times\frac{V}{R} = 0.5\times\frac{200}{10} = 10$ A

The peak current of each transistor is $I_{s(\text{peak})} = \frac{V}{R} = \frac{200}{10} = 20$ A

(d) The peak inverse voltage (PIV) of each transistor is equal to $V = 200$ V

(e) The lowest order harmonics is $V_{3(\text{rms})} = \frac{4V}{3\pi}\times\frac{1}{\sqrt{2}} = \frac{4\times 200}{3\pi}\times\frac{1}{\sqrt{2}} = 59.9969$ V

The corresponding harmonic factor is $HF_3 = \frac{V_{3(\text{rms})}}{V_{1(\text{rms})}} = \frac{59.9969}{179.9908} = 0.3333$

(f) Third harmonic distortion factor

$$THD = \frac{[V_{\text{rms}}^2 - V_1^2]^{1/2}}{V_1} = \frac{[200^2 - 179.9908^2]^{1/2}}{179.9908} = 0.48445$$

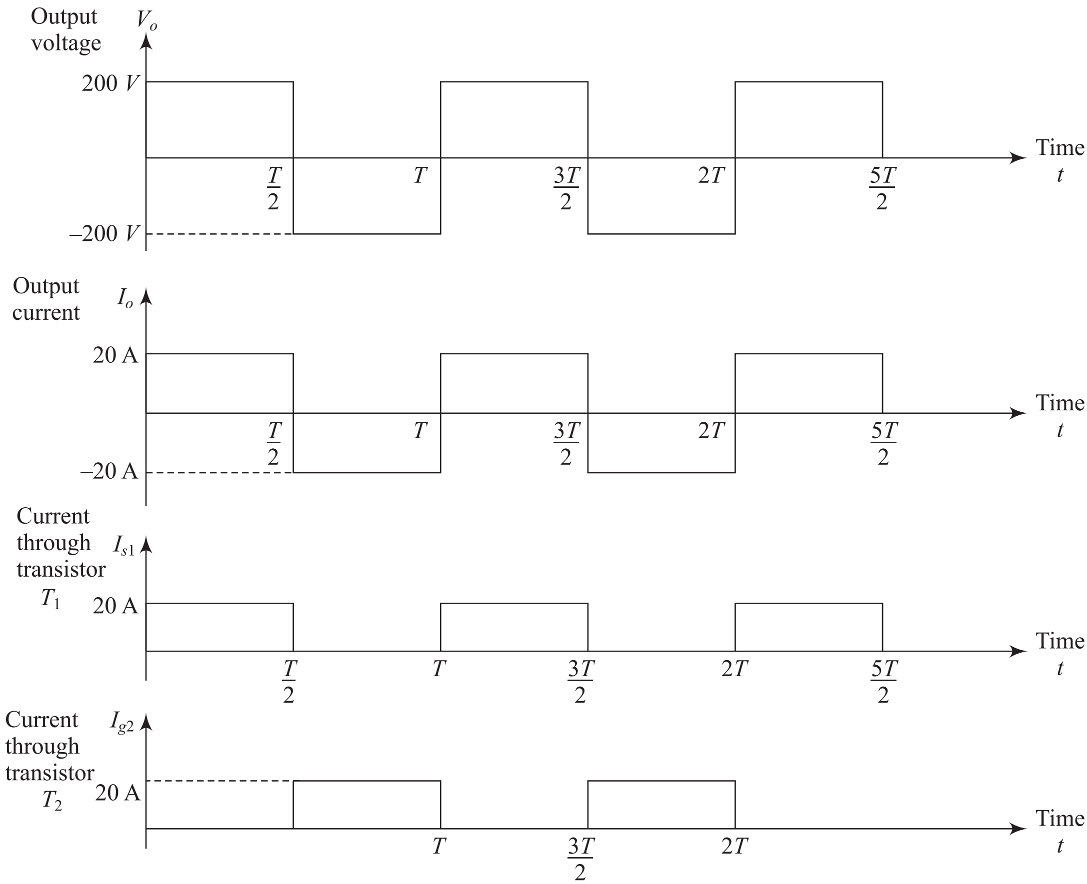


Fig. 11.15 *Voltage and current waveforms of single-phase half-wave inverter*

11.5.2 Single-Phase Full Bridge Inverter with *RL* Load

The output voltage and current waveforms of full bridge inverter with *RL* load are given in Fig. 11.16. Similar to half-bridge inverter with *RL* load, the full bridge inverter with *RL* load operates in four different modes such as

1. Mode I $(0 \le t \le t_1)$: Diodes D_1 and D_2 conduct
2. Mode II $\left(t_1 \le t \le \frac{T}{2}\right)$: Switches S_1 and S_2 conduct
3. Mode III $\left(\frac{T}{2} \le t \le t_2\right)$: Diode D_3 and D_4 conduct
4. Mode IV $(t_2 \le t \le T)$: Switches S_3 and S_4 conduct

1. Mode I (0 ≤ *t* ≤ *t*) At $t = 0$, the gating signals are removed from switch S_3 and S_4 and these switches will be turned OFF. At this instant, the load current is i_o and is equal to its negative peak value $(-I_o)$. Since the load is inductive, the load current cannot be reversed instantly and then diodes

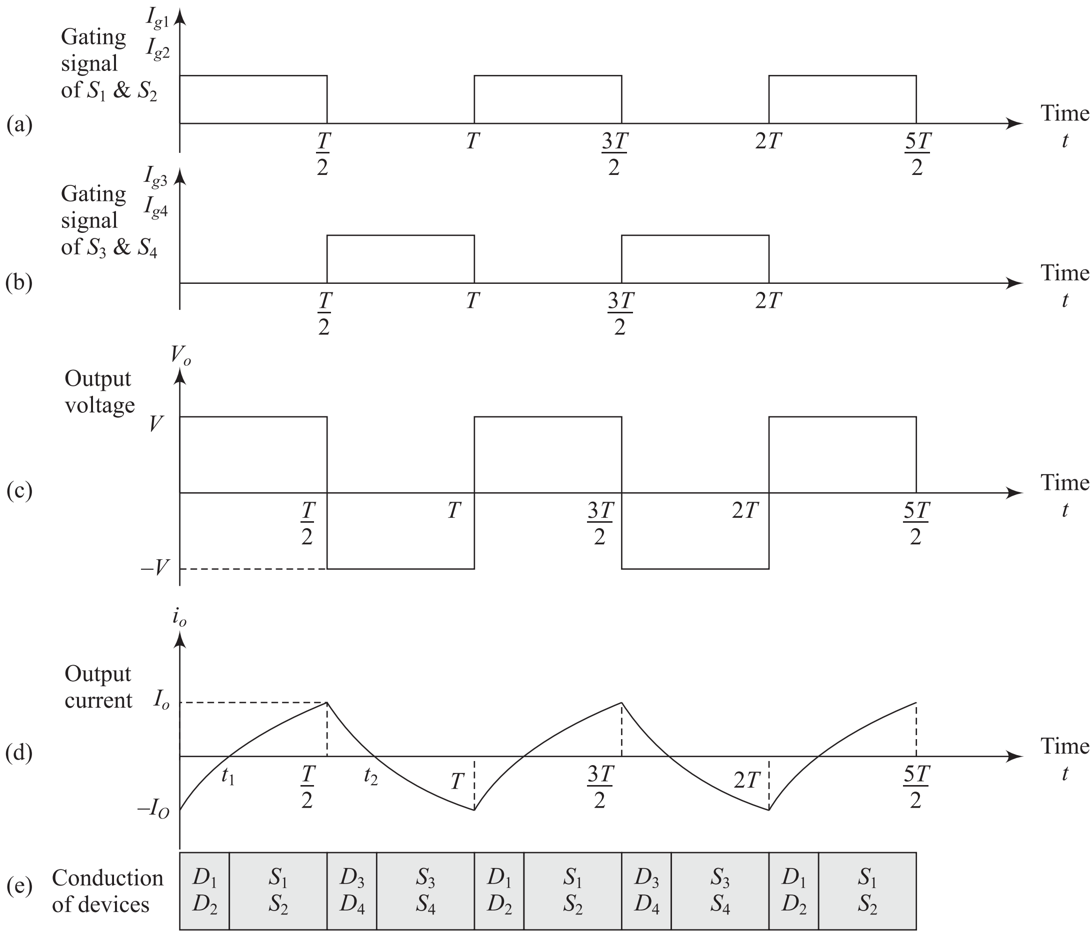


Fig. 11.16 *(a) i_{g1}, i_{g2} gating signals of switches S_1 and S_2, (b) i_{g3}, i_{g4} gating signals of switches S_3 and S_4, (c) Output voltage V_o, (d) Output current i_o and (e) Conduction of devices*

D_1 and D_2 starts to conduct at $t = 0$. Subsequently, the output voltage across load is $+V$ and the load current i_o increases from its negative peak value $(-I_o)$ as the current cannot reverse instantaneously due to inductive load. Then the load current flows through diode D_1 and D_2. During the time interval $0 \le t \le t_1$, the voltage across load is positive, but load current is negative. Therefore, the energy stored in inductance L during previous cycle must be fed back to dc supply through feedback diodes D_1 and D_2 and the load current decreases slowly. At $t = t_1$, the load current becomes zero. The switching diagram of mode I is shown in Fig. 11.17(a) and the path of current flow during $0 \le t \le t_1$ is also depicted in Fig. 11.17(a).

2. Mode II $\left(t_1 \le t \le \frac{T}{2}\right)$ At the instant $t = t_1$, feedback diodes D_1 and D_2 are turned OFF but switches S_1 and S_2 are turned ON. Then load current starts to flow in positive direction through switches S_1 and S_2 and it reaches its maximum positive peak value I_o at $t = \frac{T}{2}$. In the time interval $t_1 \le t \le \frac{T}{2}$, both the output voltage as well as current is positive and energy stored in inductance L. The switching diagram of mode II is shown in Fig. 11.17(b) and the path of current flow during $t_1 \le t \le \frac{T}{2}$ is also shown in Fig. 11.17(b).

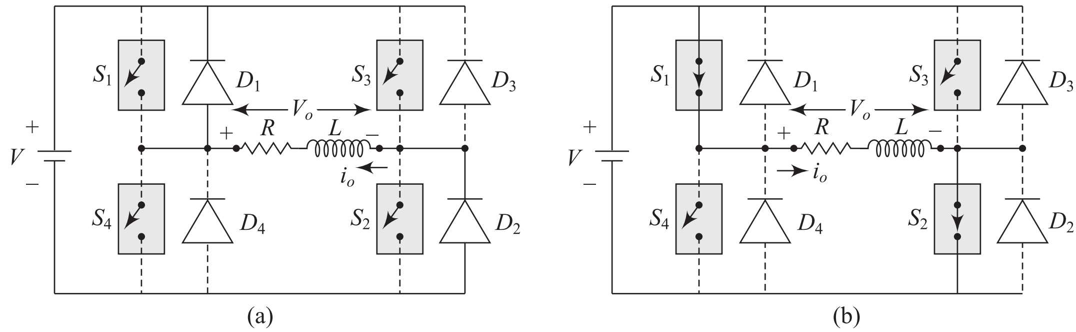


Fig. 11.17 *(a) Switching diagram of Fig. 11.11 in mode I, (b) Switching diagram of Fig. 11.11 in mode II*

3. Mode III $\left(\frac{T}{2} \le t \le t_2\right)$ At $t = \frac{T}{2}$, switches S_1 and S_2 are turned OFF. At this instant, the load current is i_o is equal to its positive peak value (I_o). As the load is inductive, the load current cannot be reversed instantly and then diodes D_3 and D_4 start to conduct at $t = \frac{T}{2}$. Then the output voltage across load is $-V$ and the load current i_o decreases form its positive peak value I_o as the current cannot reverse instantaneously due to inductive load. During the time interval $\frac{T}{2} \le t \le t_2$, the output voltage is negative but the load current is positive. The load current decreases gradually and reaches zero at $t = t_2$. Hence, the energy stored in inductance will be released and fed back to dc source during this period. Fig. 11.18(a) shows the load current flow path through diodes D_3 and D_4 and load and the switching diagram of mode III.

4. Mode IV ($t_2 \le t \le T$) At instant $t = t_2$, diodes D_3 and D_4 are turned OFF and switches S_3 and S_4 are turned ON. Then load current starts to flow in negative direction through switches S_3 and S_4 and it reaches maximum negative $-I_o$ at $t = T$. The switching diagram of mode IV is shown in Fig. 11.18(b) and the current flow path through S_3, S_4 and load is also depicted in Fig. 11.18(b). The operation of full-bridge inverter is represented by Table 11.4.

Table 11.4 *Operation of full bridge inverter*

Mode	Time	Load Voltage	Load Current	Conducting Device
I	$0 \le t \le t_1$	V	Negative	D_1 and D_2
II	$t_1 \le t \le \frac{T}{2}$	V	Positive	S_1 and S_2
III	$\frac{T}{2} \le t \le t_2$	$-V$	Positive	D_3 and D_4
IV	$t_2 \le t \le T$	$-V$	Negative	S_3 and S_4

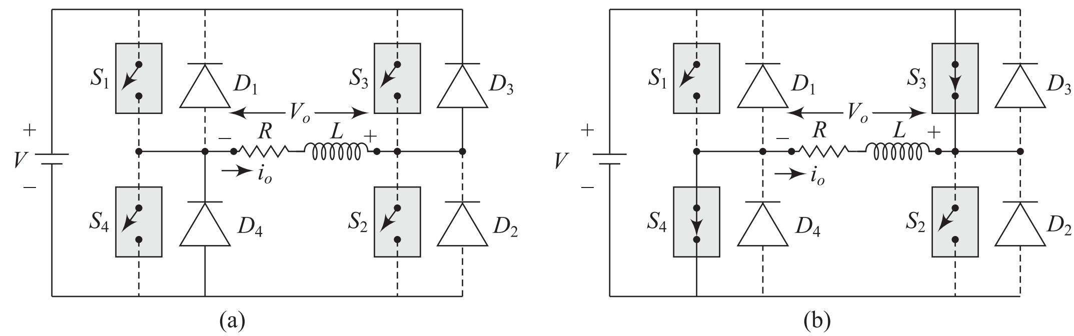


Fig. 11.18 *(a) Switching diagram of Fig. 11.11 in mode III, (b) Switching diagram of Fig. 11.11 in mode IV*

The rms value of output voltage is

$$V_{o(\text{rms})} = \left[\frac{1}{T/2}\int_0^{\frac{T}{2}} v_o^2(t)\cdot dt\right]^{1/2} = \left[\frac{1}{T/2}\int_0^{\frac{T}{2}} V^2\cdot dt\right]^{1/2} = V$$

Hence, the rms value of square wave is equal to the peak value.

The load current can be expressed as

$$i_o(t) = \frac{V}{R}\left(1 - \frac{2}{1+e^{-\frac{T}{2\tau}}}e^{-\frac{t}{\tau}}\right) \qquad \text{for } 0 \le t \le \frac{T}{2}$$

and

$$i_o(t) = -\frac{V}{R}\left(1 - \frac{2}{1+e^{-\frac{T}{2\tau}}}e^{-\frac{(t-T/2)}{\tau}}\right) \qquad \text{for } \frac{T}{2} \le t \le T$$

where, $\tau = \dfrac{L}{R}$

The output voltage V_o can be expressed using Fourier series as

$$v_o(t) = \sum_{n=1,2,3\ldots}^{\infty} \frac{4V}{n\pi}\sin n\omega t$$

The rms value of the n^{th} component is

$$V_n = \frac{1}{\sqrt{2}}\frac{4V}{n\pi} \quad \text{where, } n = 1, 3, 5\ldots$$

The rms value of fundamental component is

$$V_{1\text{rms}} = \frac{4V}{\sqrt{2}\pi} = 0.9\text{ V}$$

The instantaneous load current i_o can be expressed as

$$i_o(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V}{n\pi\sqrt{R^2+(n\omega L)^2}}\sin(n\omega t - \phi_n) \text{ and } \phi_n = \tan^{-1}\frac{n\omega L}{R}$$

where, $Z_n = \sqrt{R^2+(n\omega L)^2}$ is the impedance offered by the load to the nth harmonic.

The fundamental load current is

$$i_{o1} = \frac{4V}{\pi\sqrt{R^2+(\omega L)^2}}\sin(\omega t - \phi_1) \text{ and } \phi_1 = \tan^{-1}\frac{\omega L}{R}$$

Example 11.5 A full bridge inverter has a dc source voltage of 240 V. The inverter supplies a *RLC* load with $R = 10\ \Omega$, $L = 0.1$ H and $C = 4.7\ \mu$F. The operating frequency of inverter is 500 Hz. Determine (a) rms load current at fundamental frequency, (b) the rms value of load current, (c) power output, (d) average supply current ad (e) THD in load current.

Solution

Given: $V = 240$ V, $R = 10\ \Omega$, $L = 0.1$ H, $C = 4.7\ \mu$F and $f = 500$ Hz

The inductive reactance at fundamental frequency is

$$X_L = 2\pi fL = 2\pi \times 500 \times 0.1\ \Omega = 314.2857\ \Omega$$

The capacitive reactance at fundamental frequency is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 500 \times 4.7 \times 10^{-6}}\ \Omega = 67.6982\ \Omega$$

The impedance offered to the nth harmonic component is

$$Z_n = \sqrt{R^2 + \left(nX_L - \frac{X_C}{n}\right)^2}$$

Therefore, $$Z_1 = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (314.2857 - 67.698)^2} = 246.79\ \Omega$$

$$Z_3 = \sqrt{R^2 + \left(3X_L - \frac{X_C}{3}\right)^2} = \sqrt{10^2 + \left(3 \times 314.2857 - \frac{67.698}{3}\right)^2} = 920.3254\ \Omega$$

$$Z_5 = \sqrt{R^2 + \left(5X_L - \frac{X_C}{5}\right)^2} = \sqrt{10^2 + \left(5 \times 314.2857 - \frac{67.698}{5}\right)^2} = 1557.92\ \Omega$$

$$Z_7 = \sqrt{R^2 + \left(7X_L - \frac{X_C}{7}\right)^2} = \sqrt{10^2 + \left(7 \times 314.2857 - \frac{67.698}{7}\right)^2} = 2190.3516\ \Omega$$

$$Z_9 = \sqrt{R^2 + \left(9X_L - \frac{X_C}{9}\right)^2} = \sqrt{10^2 + \left(9 \times 314.2857 - \frac{67.698}{9}\right)^2} = 2821.067\ \Omega$$

The rms value of nth harmonic component of output voltage is

$$V_n = \frac{0.9V}{n} = \frac{0.9 \times 240}{n} = \frac{216}{n}$$

The rms value of of nth harmonic component of output current is

$$I_n = \frac{V_n}{Z_n} = \frac{216}{nZ_n}$$

(a) The rms value of load current at fundamental frequency is

$$I_1 = \frac{V_1}{Z_1} = \frac{216}{Z_1} = \frac{216}{246.79} A = 0.875\text{ A} \quad \text{as } V_1 = 216\text{ V}$$

Similarly, $I_3 = \dfrac{216}{3Z_3} = \dfrac{216}{3 \times 920.3254} = 0.078233$ A, $I_5 = \dfrac{216}{5Z_5} = \dfrac{216}{5 \times 1557.92} = 0.027729$ A,

$$I_7 = \frac{216}{7Z_7} = \frac{216}{7 \times 2190.3516} = 0.014087\text{ A}, I_9 = \frac{216}{9Z_9} = \frac{216}{9 \times 2821.067} = 0.00850\text{ A}$$

(b) The rms value of load current is equal to

$$I = \sqrt{I_1^2 + I_3^2 + I_5^2 + I_7^2 + I_9^2 + \cdots}$$
$$= \sqrt{0.875^2 + 0.0782^2 + 0.0277^2 + 0.0140^2 + 0.0085^2 + \cdots} = 0.8790\text{ A}$$

(c) The power output is

$$P_o = I^2 R = 0.879^2 \times 10 = 7.72 \text{ Watt}$$

(d) The average supply current is

$$I_{\text{av}} = \frac{P_o}{V} = \frac{7.72}{240} = 0.032 \text{ A}$$

(e) *THD* in load current is

$$THD = \frac{\sqrt{I^2 - I_1^2}}{I_1} = \frac{\sqrt{0.879^2 - 0.875^2}}{0.875} = 0.095$$

Example 11.6 A single-phase full bridge inverter has a resistive load of 10 Ω and it is operated from a 120 V dc input voltage. Determine (a) rms value of output voltage, (b) rms value of fundamental component of output voltage, (c) first three harmonics of the output voltage waveform, (d) fundamental power consumption in load, (e) rms power consumed by load and (f) transistor rating.

Solution

Given: V = 120 V and R = 10 Ω

(a) The rms value of output voltage is $V_{o(\text{rms})} = V = 120 \text{ V}$

(b) The rms value of fundamental component of output voltage

$$V_1 = \frac{1}{\sqrt{2}} \frac{4V}{\pi} = \frac{1}{\sqrt{2}} \frac{4 \times 120}{\pi} = 108.09 \text{ V}$$

(c) As $V_n = \dfrac{V_1}{n}$, first three harmonics of the output voltage waveform is

rms value of third harmonic voltage is $V_3 = \dfrac{V_1}{3} = \dfrac{108.09}{3} = 36.03 \text{ V}$

rms value of fifth harmonic voltage is $V_5 = \dfrac{V_1}{5} = \dfrac{108.09}{5} = 21.618 \text{ V}$

rms value of seventh harmonic voltage is $V_7 = \dfrac{V_1}{7} = \dfrac{108.09}{7} = 15.441 \text{ V}$

(d) Fundamental power consumption in load is $P_1 = \dfrac{V_1^2}{R} = \dfrac{108.09^2}{10} = 1168.34 \text{ Watt}$

(e) rms power consumed by load $P_{\text{rms}} = \dfrac{V_{o(\text{rms})}^2}{R} = \dfrac{120^2}{10} = 1440 \text{ Watt}$

(f) Transistor rating: Voltage rating $V_{\text{CE}} \geq V \geq 120 \text{ V}$ and

Current rating $I_{T\,\text{peak}} \geq \dfrac{V}{R} \geq 12 \text{ A}$ and $I_{T\text{rms}} \geq \dfrac{V}{\sqrt{2}R} \geq \dfrac{12}{\sqrt{2}} = 8.485 \text{ A}$

Example 11.7 A single-phase full bridge inverter is connected to a *RL* load where R = 10 Ω and L = 0.2 H. If the inverter is supplied from 220 V dc source and its output voltage frequency is 50 Hz, determine the expressions for steady state current for the first two half cycles.

Solution

Given: V = 220 V, f = 50 Hz, R = 10 Ω and L = 0.2 H

During the first half cycle, $0 \leq t \leq \dfrac{T}{2}$, the KVL equation for *RL* load is

$$V = Ri_o + L\frac{di_o}{dt}$$

Assuming $i_o(t=0)=0$, the solution of above equation is

$$i_o(t)=\frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)=\frac{220}{10}\left(1-e^{-\frac{10}{0.2}t}\right)=22(1-e^{-50t})$$

At $t=\frac{T}{2}$, the value of current is $i_o\left(t=\frac{T}{2}\right)=\frac{V}{R}\left(1-e^{-\frac{RT}{2L}}\right)$

During the second half cycle, $\frac{T}{2}\le t\le T$ or $0\le t'\le\frac{T}{2}$ where $t'=t-\frac{T}{2}$ the KVL equation for RL load is

$$-V=Ri_o+L\frac{di_o}{dt'}$$

Assuming initial condition $i_o\left(t=\frac{T}{2}\right)=\frac{V}{R}\left(1-e^{-\frac{RT}{2L}}\right)$, the solution of above equation is

$$i_o(t')=-\frac{V}{R}+\frac{V}{R}\left(2-e^{-\frac{RT}{2L}}\right)e^{-\frac{R}{L}t'}\qquad\text{where, } 0\le t'\le\frac{T}{2}$$

$$=-\frac{220}{10}+\frac{220}{10}\left(2-e^{-\frac{10\times20\times10^{-3}}{2\times0.2}}\right)e^{-\frac{10}{0.2}t'}=-22+22(2-e^{-0.5})e^{-50t'}$$

At $$t=\frac{T}{2}, i_o\left(t=\frac{T}{2}\right)=I_o=\frac{V}{R}\frac{\left(1-e^{-\frac{RT}{2L}}\right)}{\left(1+e^{-\frac{RT}{2L}}\right)}=\frac{220}{10}\frac{\left(1-e^{-\frac{10\times20\times10^{-3}}{2\times0.2}}\right)}{\left(1+e^{-\frac{10\times20\times10^{-3}}{2\times0.2}}\right)}=22\frac{(1-e^{-0.5})}{(1+e^{-0.5})}=5.385\text{ A}$$

Steady state current for the first half cycle $\left(0\le t\le\frac{T}{2}\right)$ is

$$i_o(t)=22(1-e^{-50t})-5.385e^{-50t}$$

Steady state current for the second half cycle $\left(0\le t'\le\frac{T}{2}\right)$ is

$$i_o(t')=-22+22(2-e^{-0.5})e^{-50t'}+5.385e^{-50t'}\quad\text{where, } t'=t-\frac{T}{2}$$

Example 11.8 A single-phase transistorised full bridge inverter has a resistive load of 5 Ω and it is operated from a 96 V dc input voltage. Determine (a) total harmonic distortion, (b) distortion factor, (c) harmonic factor and distortion factor at lowest order harmonic, (d) transistor ratings.

Solution

Given: V = 96 V and R = 5 Ω

(a) The rms value of output voltage is $V_{o(\text{rms})}=V=96$ V
The rms value of fundamental component of output voltage

$$V_{1\text{rms}}=0.9\text{ V}=0.9\times96=86.4\text{ V}$$

rms harmonic voltage is $V_n=\sqrt{V_{o(\text{rms})}^2-V_{1\text{rms}}^2}=\sqrt{96^2-86.4^2}=41.845$ V

Total harmonic distortion is TDH = $\frac{V_n}{V_{1\text{rms}}}=\frac{41.845}{86.4}V=0.4843$

(b) Distortion factor is $DF=\frac{\left[\sum_{n=3,5,7...}^{\infty}\left(\frac{V_n}{n^2}\right)^2\right]^{1/2}}{V_1}$

As $V_n=\frac{V_1}{n}, V_3=\frac{V_1}{3}=\frac{86.4}{3}=28.8\text{ V}, V_5=\frac{V_1}{5}=\frac{86.4}{5}=17.28\text{ V}$ and $V_7=\frac{V_1}{7}=\frac{86.4}{7}=12.34$ V

Therefore $\sum_{n=3,5,7}\left(\frac{V_n}{n^2}\right)^2=\left(\frac{V_3}{3^2}\right)^2+\left(\frac{V_5}{5^2}\right)^2+\left(\frac{V_7}{7^2}\right)^2=\left(\frac{28.8}{3^2}\right)^2+\left(\frac{17.28}{5^2}\right)^2+\left(\frac{12.34}{7^2}\right)^2=10.7811$

Then $DF=\frac{\left[\sum_{n=3,5,7...}^{\infty}\left(\frac{V_n}{n^2}\right)^2\right]^{1/2}}{V_1}=\frac{\sqrt{10.7811}}{86.4}=0.03800=3.8\%$

(c) Harmonic factor and distortion factor at lowest order harmonic

rms value of third harmonic voltage is $V_3=\frac{V_1}{3}=\frac{86.4}{3}=28.8$ V

Harmonic factor $HF_3=\frac{V_{3\text{rms}}}{V_{1\text{rms}}}=\frac{28.8}{86.4}=0.3333=33.33\%$

Distortion factor $DF_3=\frac{\frac{V_{3\text{rms}}}{3^2}}{V_{1\text{rms}}}=\frac{\frac{28.8}{3^2}}{86.4}=0.03703=3.703\%$

(f) *Transistor rating:* Voltage rating $V_{\text{CE}}\geq V\geq 96$ V and

Current rating $I_{T\text{peak}}\geq\frac{V}{R}\geq 19.2$ A and $I_{T\text{rms}}\geq\frac{V}{\sqrt{2}R}\geq\frac{19.2}{\sqrt{2}}=13.57$ A

Average transistor current is $I_{T\text{av}}=\frac{I_{T\text{peak}}}{2}=\frac{13.57}{2}=6.785$ A

Example 11.9 A single-phase full bridge inverter is connected to a RC load where R = 10 Ω and C = 50 μF. If the inverter is supplied from 220 V dc source and its output voltage frequency is 50 Hz, determine the expressions for steady state current for the first two half cycles.

Solution

Given: V = 220 V, f = 50 Hz, R = 10 Ω and C = 50 μF.

During the first half cycle, $0\leq t\leq\frac{T}{2}$, the KVL equation for RC load is

$$V=Ri_o+\frac{1}{C}\int i_o\cdot dt$$

As $i_o=\frac{dq}{dt}$, the above equation can be written as

$$V=R\frac{dq}{dt}+\frac{q}{C}$$

Solution of the above equation is $q(t)=CV(1-e^{-\frac{t}{RC}})$ and $V_C(t)=\frac{q(t)}{C}=V(1-e^{-\frac{t}{RC}})$

Then $$i_o(t)=C\frac{dV_C(t)}{dt}=\frac{V}{R}e^{-\frac{t}{RC}}$$

Therefore, $V_C(t)=V(1-e^{-\frac{t}{RC}})=220(1-e^{-\frac{t}{10\times 50\times 10^{-6}}})=220(1-e^{-2000t})$ and

$$i_o(t)=\frac{V}{R}e^{-\frac{t}{RC}}=\frac{220}{10}e^{-\frac{t}{10\times 50\times 10^{-6}}}=22e^{-2000t}$$

At $t=\frac{T}{2}$, the value of current is $V_C\left(t=\frac{T}{2}\right)=V\left(1-e^{-\frac{T}{2RC}}\right)$

During the second half cycle, $\frac{T}{2}\leq t\leq T$ or $0\leq t'\leq\frac{T}{2}$ where $t'=t-\frac{T}{2}$ the KVL equation for *RL* load is

$$-V=Ri_o+\frac{1}{C}\int i_o dt'$$

As $i_o = \dfrac{dq}{dt}$, the above equation can be written as $-V = R\dfrac{dq}{dt'} + \dfrac{q}{C}$

Assuming initial condition $V_C\left(t = \dfrac{T}{2}\right) = V\left(1 - e^{-\frac{T}{2RC}}\right)$, the solution of above equation is

$$q(t') = -CV\left(1 - e^{-\frac{t'}{RC}}\right) + CV_C\left(\frac{T}{2}\right)e^{-\frac{t'}{RC}} \qquad \text{where, } 0 \le t' \le \frac{T}{2}$$

$$V_C(t') = \frac{q(t')}{C} = -V\left(1 - e^{-\frac{t'}{RC}}\right) + V_C\left(\frac{T}{2}\right)e^{-\frac{t'}{RC}} = -V + V\left(2 - e^{-\frac{T}{2RC}}\right)e^{-\frac{t'}{RC}}$$

$$i_o(t') = C\frac{dV_C(t')}{dt} = -\frac{V}{R}\left(2 - e^{-\frac{T}{2RC}}\right)e^{-\frac{t'}{RC}}$$

Therefore,
$$V_C(t') = -V + V\left(2 - e^{-\frac{T}{2RC}}\right)e^{-\frac{t'}{RC}} = -220 + 220\left(2 - e^{-\frac{20\times10^{-3}}{2\times10\times50\times10^{-6}}}\right)e^{-\frac{t'}{10\times50\times10^{-6}}}$$
$$= -220 + 220(2 - e^{-20})e^{-2000t'}$$

$$i_o(t') = -\frac{V}{R}\left(2 - e^{-\frac{T}{2RC}}\right)e^{-\frac{t'}{RC}} = -\frac{220}{10}\left(2 - e^{-\frac{20\times10^{-3}}{2\times10\times50\times10^{-6}}}\right)e^{-\frac{t'}{10\times50\times10^{-6}}} = -22(2 - e^{-20})e^{-2000t'}$$

At
$$V_C\left(t = \frac{T}{2}\right) = V_o = V\frac{1 - e^{-\frac{T}{2RC}}}{1 + e^{-\frac{T}{2RC}}} = 220\frac{1 - e^{-\frac{20\times10^{-3}}{2\times10\times50\times10^{-6}}}}{1 + e^{-\frac{20\times10^{-3}}{2\times10\times50\times10^{-6}}}} = 220\frac{1 - e^{-20}}{1 + e^{-20}} \approx 220\text{ V}$$

Steady state voltage for the first half cycle is

$$V_C(t) = 220(1 - e^{-2000t}) - 220e^{-2000t} \text{ and}$$

Steady state voltage for the second half cycle is

$$V_C(t') = -220 + 220(2 - e^{-20})e^{-2000t'} + 220e^{-2000t'}$$

Example 11.10 A single-phase bridge inverter delivers power to *RLC* series load with $R = 5\ \Omega$ and $\omega L = 10\ \Omega$. The time period of output voltage is 0.2 ms. What is the value of *C* to obtain load commutation? Assume thyristor turn-OFF time is 16 μs and circuit turn-off time is $1.5t_q$. Assume that load current contains fundamental component only.

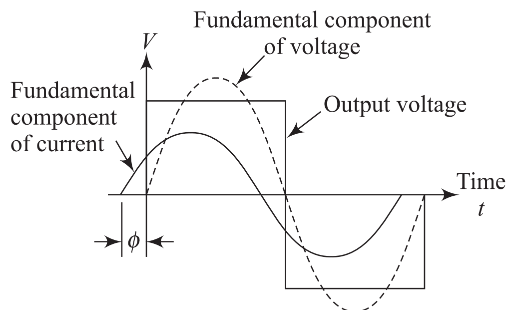


Fig.11.19 *Waveforms of fundamental component voltage and fundamental component current*

Solution

The value of *C* should be such that the circuit operates in underdamped condition with *RLC* load. Figure 11.19 shows the waveforms of fundamental component voltage and fundamental component current. The current leads voltage by an angle ϕ.

Then
$$\tan\phi = \frac{X_C - X_L}{R}$$

Since current leads voltage by an angle ϕ, $X_C > X_L$

$\dfrac{\phi}{\omega}$ must be equal to the circuit turn-OFF time $1.5t_q = 1.5 \times 16 \times 10^{-6} = 24\ \mu\text{s} = t_c$

Frequency
$$f = \frac{1}{T} = \frac{1}{0.2\times10^{-3}} = 5\text{ kHz}$$

$$\phi = \omega t_c = 2\pi f t_c = 2\pi \times 5 \times 10^3 \times 24 \times 10^{-6} = 0.7536 \text{ rad} = 43.16°$$

$$\tan\phi = \frac{X_C - X_L}{R} \qquad \text{or,} \quad \tan 43.16 = \frac{X_C - 10}{5}$$

or $$0.9367 = \frac{X_C - 10}{5} \qquad \text{or,} \quad X_C = 10 + 5 \times 0.9367 = 14.6835$$

or $$\frac{1}{2\pi \times 50 \times C} = 14.6835 \qquad \text{or,} \quad C = \frac{1}{2\pi \times 50 \times 14.6835} = 216.89\ \mu\text{F}$$

11.6 THREE-PHASE INVERTER

The output of a single-phase inverter is a non-sinusoidal waveform and it consists of harmonics component. This is suitable only for low power industrial applications. When the more number of switching elements are used in an inverter, the output waveform will be non-sinusoidal, but its harmonics content will be reduced. Hence the output is nearer to sinusoidal. Therefore, three-phase inverters are used for high power industrial applications.

A three-phase inverter can be formed after combining three half-bridge single phase inverters in parallel. It consists of six switching devices (SCRs, power BJTs, MOSFETs, IGBTs, etc.) and six diodes. The circuit configuration of a three-phase inverter is shown in Fig. 11.20(a) and (b). The load may be connected either in delta or star. The delta connected load and star connected load are depicted in Fig. 11.21(a) and (b) respectively.

The circuit arrangement of inverter as shown in Fig. 11.20 may be operated with star connected or delta connected load and the switching devices can be triggered in two different modes of operation such as

1. 180° conduction mode
2. 120° conduction mode

In this section, the operating principle of above two modes of an inverter are discussed elaborately.

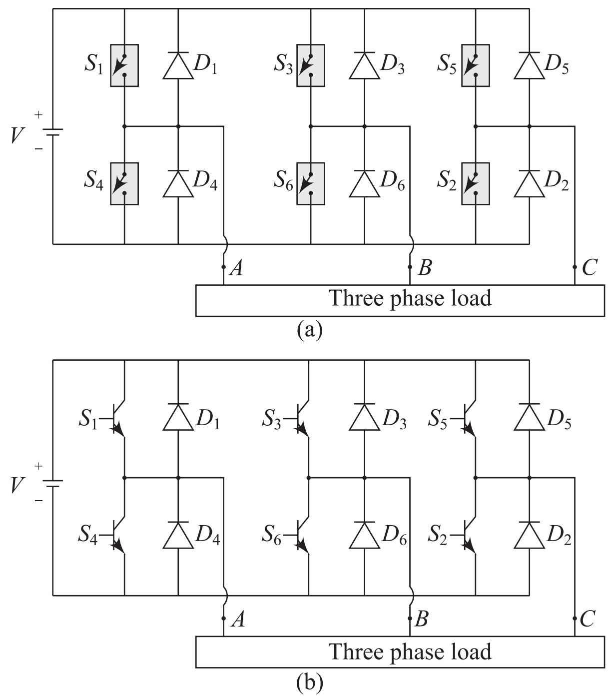


Fig. 11.20 *(a) Three-phase inverter using switches and diodes and (b) Three-phase inverter using transistors and diodes*

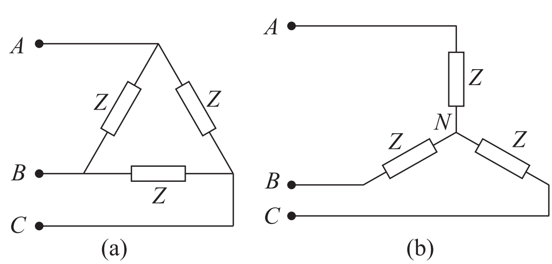


Fig. 11.21 *(a) Delta connected load and (b) Star connected load*

11.6.1 180° Conduction Mode

In this mode, each switch conducts for 180° duration in each cycle of the output voltage. Each leg of three-phase inverter consists of two switches, one is a part of positive group switches and other is a part of negative group switches. When a positive group switch of a leg conducts for 180° duration, its corresponding negative group switch of same leg conducts for next 180° duration as one complete cycle is equal to 360° duration. For example, if the switch S_1 of leg 1 is ON for 180° duration during $0° \le \omega t \le 180°$, then the switch S_4 of leg 1 is ON for 180° duration during $180° \le \omega t \le 360°$. The sequence of switching of semiconductor switches S_1, S_2, S_3, S_4, S_5, S_6 are depicted in Fig. 11.22. In this switching scheme, three switches from three different legs are conducted at a time. Hence, two switches from the same leg are not switched on simultaneously. The one complete cycle of switching can be operated into six modes and each mode operates only for 60° duration. The different operating modes are given in Table 11.5. The switching sequence is

$$\boxed{S_1, S_6, S_5 \,/\, S_1, S_6, S_2 \,/\, S_1, S_3, S_2 \,/\, S_4, S_3, S_2 \,/\, S_4, S_3, S_5 \,/\, S_4, S_6, S_5}$$

Fig. 11.22 *Sequence of switching of semiconductor switches*

Table 11.5 *Operating modes of inverter*

Mode	Operating duration	Conduction of switches
Mode-I	$0 \le \omega t \le \frac{\pi}{3}$	S_1, S_6, S_5
Mode-II	$\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$	S_1, S_6, S_2
Mode-III	$\frac{2\pi}{3} \le \omega t \le \pi$	S_1, S_3, S_2
Mode-IV	$\pi \le \omega t \le \frac{4\pi}{3}$	S_4, S_3, S_2
Mode-V	$\frac{4\pi}{3} \le \omega t \le \frac{5\pi}{3}$	S_4, S_3, S_5
Mode-VI	$\frac{5\pi}{3} \le \omega t \le 2\pi$	S_4, S_6, S_5

It is clear from Table 11.5 that during each interval, only three switches conducts and two from the positive group and other one from negative group or vice versa. Assume that the load is connected in star and the phase voltages are V_{AN}, V_{BN} and V_{CN}. The line to line voltages are V_{AB}, V_{BC} and V_{CA}. The equivalent circuits of inverter for six different modes are given below:

1. Mode I $\left(0 \le \omega t \le \frac{\pi}{3}\right)$ During mode I, switches S_1, S_6, and S_5 conduct for $0 \le \omega t \le \frac{\pi}{3}$ and the equivalent circuit is shown in Fig. 11.23.

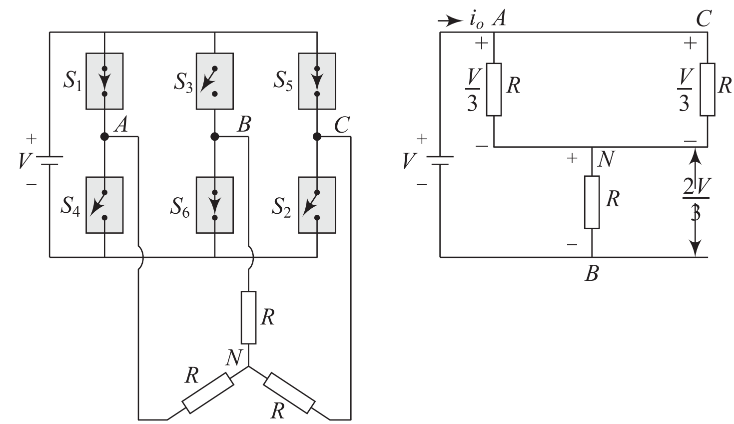


Fig. 11.23 *Equivalent circuit of Fig. 11.20 during $0 \le \omega t \le \frac{\pi}{3}$ when S_1, S_6 and S_5 are closed*

The current i_o is given by

$$i_o = \frac{V}{R \| R + R} = \frac{V}{\frac{R}{2} + R} = \frac{2}{3}\frac{V}{R} \quad \text{as } R \| R = \frac{R}{2}$$

The lines to neutral voltages are

$$V_{AN} = V_{CN} = \frac{i_o}{2} R = \frac{V}{3} \qquad \text{as } i_o = \frac{2}{3}\frac{V}{R}$$

$$V_{NB} = i_o R = \frac{2}{3} V \quad \text{then } V_{BN} = -V_{NB} = -\frac{2}{3} V$$

2. Mode II $\left(\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}\right)$ In mode II, switches S_1, S_6, and S_2 conduct for $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$ and the equivalent circuit is shown in Fig. 11.24.

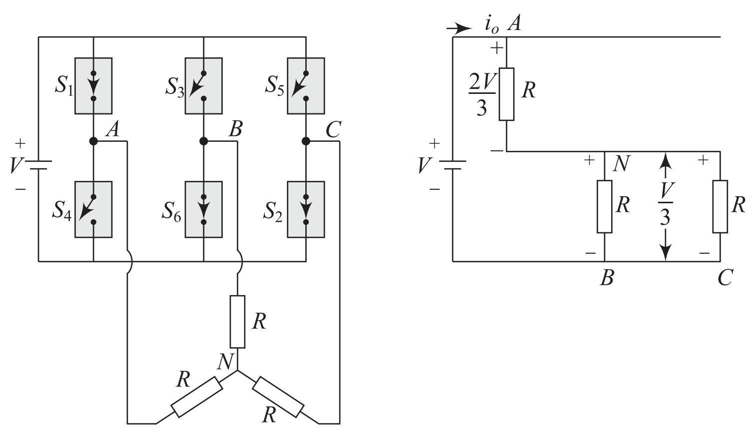


Fig. 11.24 *Equivalent circuit of Fig. 11.20 during $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$ when S_1, S_6, and S_2 are closed*

The current i_o is given by

$$i_o = \frac{V}{R + R\|R} = \frac{V}{R + \frac{R}{2}} = \frac{2}{3}\frac{V}{R} \qquad \text{as } R\|R = \frac{R}{2}$$

The lines to neutral voltages are

$$V_{AN} = i_o R = \frac{2}{3}V \qquad \text{as } i_o = \frac{2}{3}\frac{V}{R}$$

$$V_{NB} = V_{NC} = \frac{i_o}{2}R = \frac{V}{3} \quad \text{then } V_{BN} = -V_{NB} = -\frac{V}{3} \text{ and } V_{CN} = -V_{NC} = -\frac{V}{3}$$

3. Mode III $\left(\frac{2\pi}{3} \le \omega t \le \pi\right)$ During mode III, switches S_1, S_3, and S_2 conduct for $\frac{2\pi}{3} \le \omega t \le \pi$ and the equivalent circuit is shown in Fig. 11.25.

The current i_o is given by

$$i_o = \frac{V}{R\|R + R} = \frac{V}{\frac{R}{2} + R} = \frac{2}{3}\frac{V}{R} \qquad \text{as } R\|R = \frac{R}{2}$$

The lines to neutral voltages are

$$V_{AN} = V_{BN} = \frac{i_o}{2}R = \frac{V}{3} \qquad \text{as } i_o = \frac{2}{3}\frac{V}{R}$$

$$V_{NC} = i_o R = \frac{2}{3}V \quad \text{then } V_{CN} = -V_{NC} = -\frac{2}{3}V$$

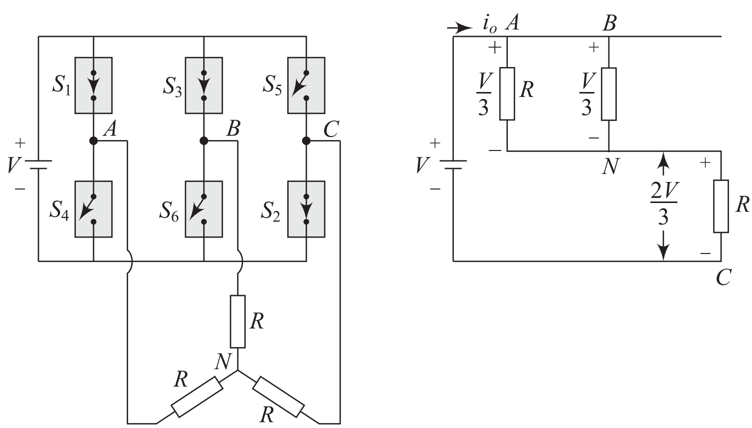


Fig. 11.25 *Equivalent circuit of Fig. 11.20 during $\frac{2\pi}{3} \le \omega t \le \pi$ when S_1, S_3 and S_2 are closed*

Similarly, during mode IV $\left(\pi \le \omega t \le \frac{4\pi}{3}\right)$, the line to neutral voltages are

$$V_{AN} = V_{CN} = -\frac{V}{3} \text{ and } V_{BN} = -\frac{2V}{3}$$

In mode V $\left(\frac{4\pi}{3} \le \omega t \le \frac{5\pi}{3}\right)$, the line to neutral voltages are

$$V_{AN} = -\frac{2V}{3} \text{ and } V_{BN} = V_{CN} = \frac{V}{3}$$

During mode VI $\left(\frac{5\pi}{3} \le \omega t \le 2\pi\right)$, the line to neutral voltages are

$$V_{AN} = V_{BN} = -\frac{V}{3} \text{ and } V_{CN} = \frac{2V}{3}$$

When the lines to neutral voltages are known for a complete cycle, the line-to-line voltages are computed by using following expressions:

$$V_{AB} = V_{AN} - V_{BN}$$
$$V_{BC} = V_{BN} - V_{CN}$$
$$V_{CA} = V_{CN} - V_{AN}$$

The output phase voltages V_{AN}, V_{BN}, V_{CN} and line voltages V_{AB}, V_{BC}, V_{CA} are depicted in Fig. 11.26. The instantaneous line voltages can be expressed in Fourier series as they are periodic non-sinusoidal waveform as given below.

$$v_{AB} = \sum_{n=1,3,5...}^{\infty} \frac{4V}{n\pi}\cos\frac{n\pi}{6}\sin\left(n\omega t + \frac{n\pi}{6}\right)$$

$$v_{BC} = \sum_{n=1,3,5...}^{\infty} \frac{4V}{n\pi}\cos\frac{n\pi}{6}\sin\left(n\omega t - \frac{n\pi}{2}\right)$$

$$v_{CA} = \sum_{n=1,3,5...}^{\infty} \frac{4V}{n\pi}\cos\frac{n\pi}{6}\sin\left(n\omega t + \frac{5n\pi}{6}\right)$$

If the triple harmonics (n = 3, 9, 15, …) are zero, $\cos\frac{n\pi}{6} = 0$.

The rms line to line voltage is expressed by

$$V_{L(\text{rms})} = \left[\frac{1}{\pi}\int_0^{\frac{2\pi}{3}} V^2 d\omega t\right]^{1/2} = \left[\frac{1}{\pi}V^2\left(\frac{2\pi}{3} - 0\right)\right]^{1/2}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}V = 0.8165\text{ V}$$

The rms value of nth component of the line voltage is given by

$$V_{Ln(\text{rms})} = \frac{4V}{\sqrt{2}n\pi}\cos\frac{n\pi}{6}$$

The rms value of fundamental component of the line voltage is

$$V_{L1(\text{rms})} = \frac{4V}{\sqrt{2}\pi}\cos\frac{\pi}{6} = 0.7797\text{ V}$$

The rms value of phase voltages is

$$V_{\text{ph(rms)}} = \frac{V_{L(\text{rms})}}{\sqrt{3}} = \frac{\sqrt{2}}{3}V = 0.4714\text{ V}$$

The power output of inverter with resistive load is equal to

$$P = 3\frac{V_{\text{ph(rms)}}^2}{R} = \frac{2}{3}\frac{V^2}{R}$$

The rms value of switch current is given by

$$I_{\text{Switch(rms)}} = \frac{I_{\text{ph(rms)}}}{\sqrt{2}} = \frac{V_{\text{ph(rms)}}}{\sqrt{2}R} = \frac{V}{3R}$$

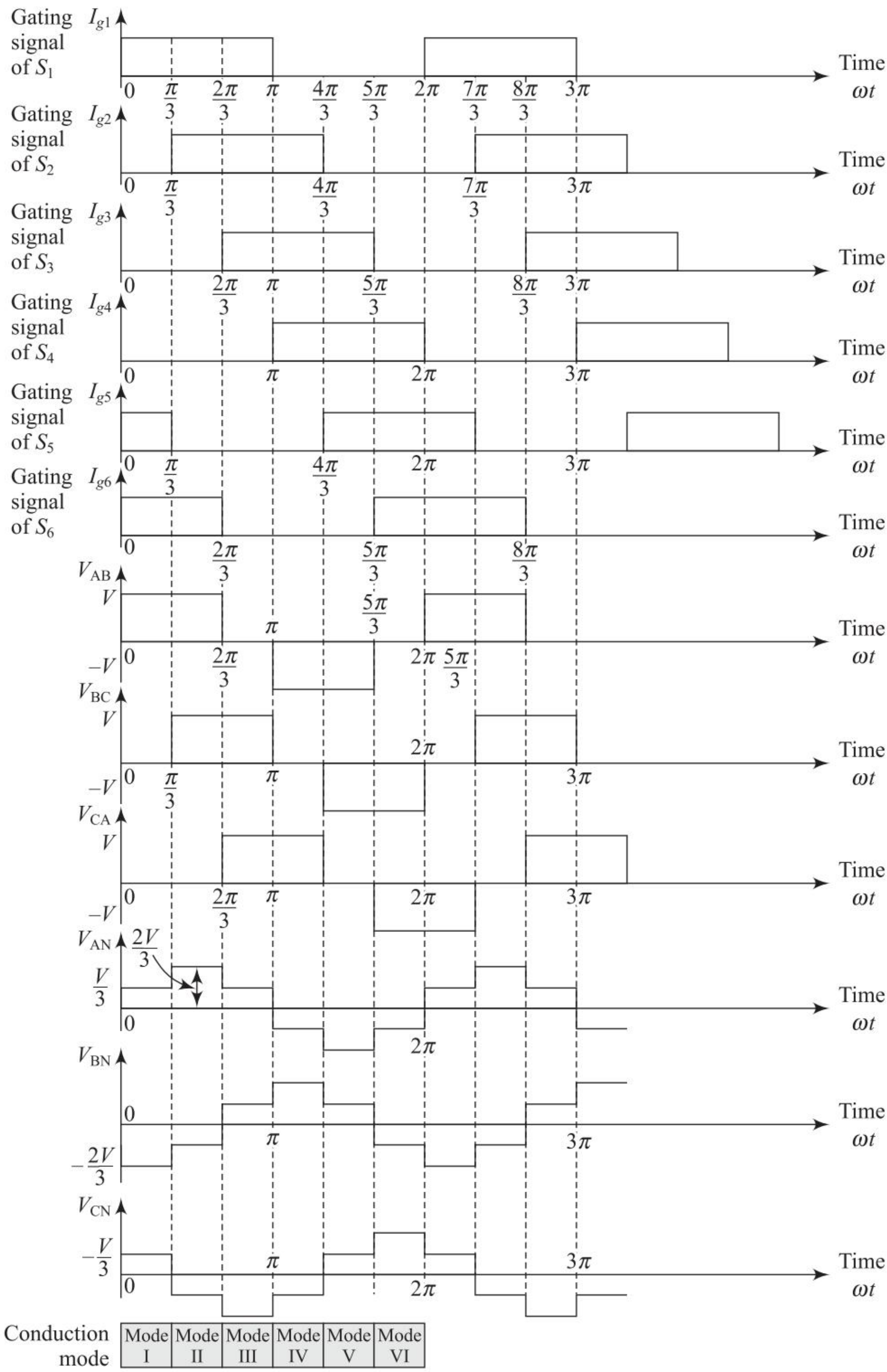


Fig. 11.26 *Gating signals of S_1, S_2, S_3, S_4, S_5, S_6, line voltages V_{AB}, V_{BC}, V_{CA} and phase voltages V_{AN}, V_{BN}, V_{CN}*

Example 11.11 A three-phase bridge inverter is fed from 400 V dc supply. If the semiconductor switches (transistors) which are used in inverter conducts for 180° duration and the inverter is supplying a star connected resistive load of 10 Ω, determine
(a) rms value of per phase voltage and line voltage
(b) rms value of load current
(c) rms value of current flows through transistors
(d) Power delivered to load
(e) Average source current

Solution

Given: $V = 400$ V, $R = 10\ \Omega$
(a) The rms value of per phase load voltage is

$$V_{\text{ph(rms)}} = \frac{V_{L(\text{rms})}}{\sqrt{3}} = \frac{\sqrt{2}}{3}V = 0.4714\text{ V} = 0.4714 \times 400\text{ V} = 188.56\text{ V}$$

The rms line to line voltage is

$$V_{L(\text{rms})} = \frac{\sqrt{2}}{\sqrt{3}}V = 0.8165\text{ V} = 0.8165 \times 400\text{ V} = 326.6\text{ V}$$

(b) rms value of load current per phase

$$I_{\text{ph}} = \frac{V_{\text{ph(rms)}}}{R} = \frac{188.56}{10} = 18.856\text{ A}$$

(c) rms value of current flows through transistors

$$I_{\text{rms(transistor)}} = \frac{V}{3R} = \frac{400}{3 \times 10} = 13.3333\text{ A}$$

(d) Power delivered to load

$$P_O = 3\frac{V^2_{\text{ph(rms)}}}{R} = 3 \times \frac{188.56^2}{10}\text{Watt} = 10.6664\text{ kW}$$

(e) Average source current is

$$I_{\text{av}} = \frac{P_O}{V} = \frac{10666.4}{400} = 26.666\text{ A}$$

Example 11.12 A three-phase bridge inverter operates in 180° conduction mode and it is operated by a 120 V dc battery and it is connected with a *R* load of 5 Ω. Determine (a) rms line to line output voltage, (b) fundamental output voltage and (c) distortion factor and harmonic factor of output voltage waveform.

Solution

Given: $V = 120$ V, conduction of transistor is 180° and $R = 5\ \Omega$
(a) rms line-to-line output voltage

$$V_{L(\text{rms})} = \frac{\sqrt{2}}{\sqrt{3}}V = 0.8165\text{ V} = 0.8165 \times 120 = 97.98\text{ V}$$

(b) Fundamental output voltage

$$V_{L1(\text{rms})} = \frac{4V}{\sqrt{2}\pi}\cos\frac{\pi}{6} = 0.7797\text{ V} = 0.7797 \times 120 = 93.564\text{ V}$$

(c) Distortion factor is

$$DF = \frac{V_{L1(\text{rms})}}{V_{L(\text{rms})}} = \frac{93.564}{97.98} = 0.9545$$

Harmonic factor of output voltage waveform

$$HF = \sqrt{\frac{1}{DF^2} - 1} = \sqrt{\frac{1}{0.9545^2} - 1} = 0.31256$$

Example 11.13 A three-phase bridge inverter operates is operated by 400 V dc input voltage and it operates in 180° conduction mode. Find (a) rms value line to line output voltage, (b) rms value line to phase output voltage and (c) fundamental output voltage in terms of line voltage and phase voltage.

Solution

Given: $V = 400$ V, and conduction of transistor is 180°

(a) rms line-to-line output voltage is

$$V_{L(\text{rms})} = \frac{\sqrt{2}}{\sqrt{3}} V = 0.8165\text{ V} = 0.8165 \times 400 = 326.6\text{ V}$$

(b) rms value line-to-phase output voltage is

$$V_{\text{Phase(rms)}} = \frac{V_{L(\text{rms})}}{\sqrt{3}} = \frac{0.8165V}{\sqrt{3}} = \frac{0.8165 \times 400}{\sqrt{3}} = 188.568\text{ V}$$

(c) Fundamental output voltage in terms of line voltage

$$V_{L1(\text{rms})} = \frac{4V}{\sqrt{2}\pi} \cos\frac{\pi}{6} = 0.7797\text{ V} = 0.7797 \times 400 = 311.88\text{ V}$$

Fundamental output voltage in terms of phase voltage

$$V_{\text{Phase1(rms)}} = \frac{V_{L1(\text{rms})}}{\sqrt{3}} = \frac{311.88}{\sqrt{3}} = 180.069\text{ V}$$

Example 11.14 A three-phase bridge inverter operates is operated by 240 V dc supply and it operates in 180° conduction mode. When the inverter is connected to a star connected R load with $R = 5\ \Omega$ and $L = mH$ and inverter output frequency $f_o = 50$ Hz, compute (a) instantaneous line-to-line voltage and line current in Fourier series, (b) rms value line-to-line output voltage, (c) rms value line to phase output voltage and (d) fundamental output voltage in terms of line voltage and phase voltage.

Solution

Given: $V = 240$ V, conduction of transistor is 180°, $R = 5\ \Omega$, $L = 10$ mH and $f_o = 50$ Hz

(a) Instantaneous line-to-line voltage in Fourier series

$$v_{AB} = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t + \frac{n\pi}{6}\right)$$

where, the triple harmonics (n = 3, 9, 15…) are zero, as $\cos\frac{n\pi}{6} = 0$.

$$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ and } \frac{\pi}{6} = 30°$$

$$v_{AB1} = \frac{4V}{\pi}\cos\frac{\pi}{6}\sin\left(\omega t + \frac{\pi}{6}\right) = \frac{4 \times 240}{\pi}\cos\frac{\pi}{6}\sin(314t + 30°) = 264.772\sin(314t + 30°)$$

$$v_{AB5} = \frac{4V}{5\pi}\cos\frac{5\pi}{6}\sin 5\left(\omega t + \frac{\pi}{6}\right) = \frac{4 \times 240}{5\pi}\cos\frac{5\pi}{6}\sin 5(314t + 30°)$$
$$= -52.954\sin 5(314t + 30°)$$

$$v_{AB7} = \frac{4V}{7\pi}\cos\frac{7\pi}{6}\sin 7\left(\omega t + \frac{\pi}{6}\right) = \frac{4 \times 240}{7\pi}\cos\frac{7\pi}{6}\sin 7(314t + 30°) = -37.82\sin 7(314t + 30°)$$

$$v_{AB11} = \frac{4V}{11\pi}\cos\frac{11\pi}{6}\sin 11\left(\omega t + \frac{\pi}{6}\right)$$
$$= \frac{4 \times 240}{11\pi}\cos\frac{11\pi}{6}\sin 11(314t + 30°) = 24.07\sin 11(314t + 30°)$$

The instantaneous line to line voltage in Fourier series

$$v_{AB} = 264.772\sin(314t+30°) - 52.954\sin 5(314t+30°) - 37.82\sin 7(314t+30°)\cdots$$
$$\cdots + 24.07\sin 11(314t+30°) + \cdots$$

$$Z_L = \sqrt{R^2 + (n\omega L)^2}\angle\tan^{-1}\left(\frac{n\omega L}{R}\right)$$

$$Z_{L1} = \sqrt{5^2 + (314\times 10\times 10^{-3})^2}\angle\tan^{-1}\left(\frac{314\times 10\times 10^{-3}}{5}\right) = 5.90\angle 32.21°$$

$$Z_{L5} = \sqrt{5^2 + (5\times 314\times 10\times 10^{-3})^2}\angle\tan^{-1}\left(\frac{5\times 314\times 10\times 10^{-3}}{5}\right) = 16.476\angle 72.33°$$

$$Z_{L7} = \sqrt{5^2 + (7\times 314\times 10\times 10^{-3})^2}\angle\tan^{-1}\left(\frac{7\times 314\times 10\times 10^{-3}}{5}\right) = 22.54\angle 77.18°$$

$$Z_{L11} = \sqrt{5^2 + (11\times 314\times 10\times 10^{-3})^2}\angle\tan^{-1}\left(\frac{11\times 314\times 10\times 10^{-3}}{5}\right) = 34.9\angle 81.76°$$

The instantaneous line to line currents in Fourier series

$$i_{AB} = 44.87\sin(314t - 2.21°) - 3.214\sin(5\times 314t + 77.67°) - 1.6779\sin(7\times 314t + 132.82°)\cdots$$
$$\cdots + 0.689\sin(11\times 314t + 248.24°) + \cdots$$

(b) rms line-to-line output voltage

$$V_{L(\text{rms})} = \frac{\sqrt{2}}{\sqrt{3}}V = 0.8165\text{ V} = 0.8165\times 240 = 195.96\text{ V}$$

(c) rms value line-to-phase output voltage is

$$V_{\text{Phase(rms)}} = \frac{V_{L(\text{rms})}}{\sqrt{3}} = \frac{0.8165V}{\sqrt{3}} = \frac{0.8165\times 240}{\sqrt{3}} = 113.14\text{ V}$$

(d) Fundamental output voltage in terms of line voltage

$$V_{L1(\text{rms})} = \frac{4V}{\sqrt{2}\pi}\cos\frac{\pi}{6} = 0.7797\text{ V} = 0.7797\times 240 = 187.128\text{ V}$$

Fundamental output voltage in terms of phase voltage

$$V_{\text{Phase1(rms)}} = \frac{V_{L1(\text{rms})}}{\sqrt{3}} = \frac{187.128}{\sqrt{3}} = 108.04\text{ V}$$

11.6.2 120° Conduction Mode

$S_1, S_6 / S_1, S_2 / S_3, S_2 / S_3, S_4 / S_4, S_5 / S_5, S_6$

Fig. 11.27 *Sequence of switching of semi-conductor switches*

In this mode, each switch conducts for 120° duration in each cycle of the output voltage. Each leg of three phase inverter consists of two switches, one is a part of positive group switches and other is a part of negative group switches. The gating signals of switches are given for every 60°. The sequence of switching of semiconductor switches S_1, S_2, S_3, S_4, S_5, S_6 are depicted in Fig. 11.27. In this switching scheme, only two switches conduct at any instant of time, one from the positive group of switches and the other from the negative group switches. The one complete cycle of switching can be operated into six modes and each mode operates only for 60° duration. The different operating modes are given in Table 11.6. The switching sequence is shown in Fig 11.27.

Table 11.6 *Operating modes of inverter*

Mode	Operating duration	Conduction of switches
Mode-I	$0 \le \omega t \le \dfrac{\pi}{3}$	S_1, S_6
Mode-II	$\dfrac{\pi}{3} \le \omega t \le \dfrac{2\pi}{3}$	S_1, S_2
Mode-III	$\dfrac{2\pi}{3} \le \omega t \le \pi$	S_3, S_2
Mode-IV	$\pi \le \omega t \le \dfrac{4\pi}{3}$	S_3, S_4
Mode-V	$\dfrac{4\pi}{3} \le \omega t \le \dfrac{5\pi}{3}$	S_4, S_5
Mode-VI	$\dfrac{5\pi}{3} \le \omega t \le 2\pi$	S_5, S_6

It is clear from Table 11.6 that during each interval, only two switches conducts and one from the positive group and other one from negative group. Assume that the load is connected in star and the phase voltages are V_{AN}, V_{BN} and V_{CN}. The line-to-line voltages are V_{AB}, V_{BC} and V_{CA}. The equivalent circuits of inverter for six different modes are given below.

1. Mode I $\left(0 \le \omega t \le \dfrac{\pi}{3}\right)$ During mode I, switches S_1 and S_6 conduct for $0 \le \omega t \le \dfrac{\pi}{3}$ and the equivalent circuit is shown in Fig. 11.28.

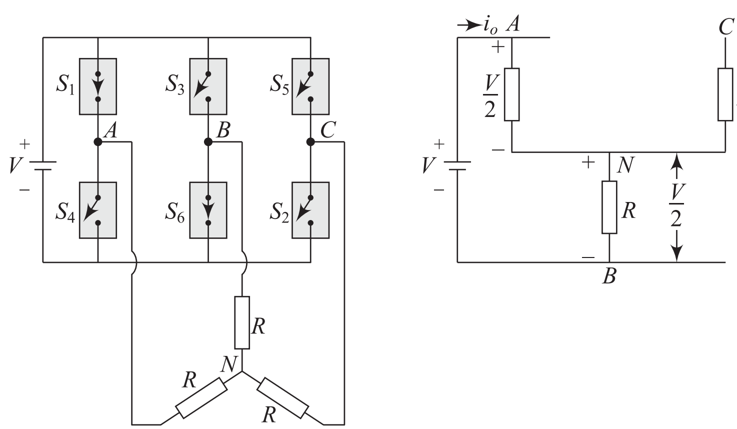


Fig. 11.28 *Equivalent circuit of Fig. 11.20 during* $0 \le \omega t \le \dfrac{\pi}{3}$ *when* S_1 *and* S_6 *are closed*

The current i_o is given by

$$i_o = \frac{V}{R+R} = \frac{V}{2R}$$

The lines to neutral voltages are

$$V_{AN} = i_o R = \frac{V}{2} \qquad \text{as } i_o = \frac{V}{2R}$$

$$V_{NB} = i_o R = \frac{V}{2} \text{ then } V_{BN} = -V_{NB} = -\frac{V}{2}, V_{CN} = 0$$

2. Mode II $\left(\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}\right)$ In mode II, switches S_1 and S_2 conduct for $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$ and the equivalent circuit is shown in Fig. 11.29.

The current i_o is given by

$$i_o = \frac{V}{R+R} = \frac{V}{2R} \qquad \text{as } R \| R = \frac{R}{2}$$

The lines to neutral voltages are

$$V_{AN} = i_o R = \frac{V}{2} \qquad \text{as } i_o = \frac{2}{3}\frac{V}{R}, \text{and } V_{BN} = 0$$

$$V_{NC} = i_o R = \frac{V}{2} \text{ then } V_{CN} = -V_{NC} = -\frac{V}{2}$$

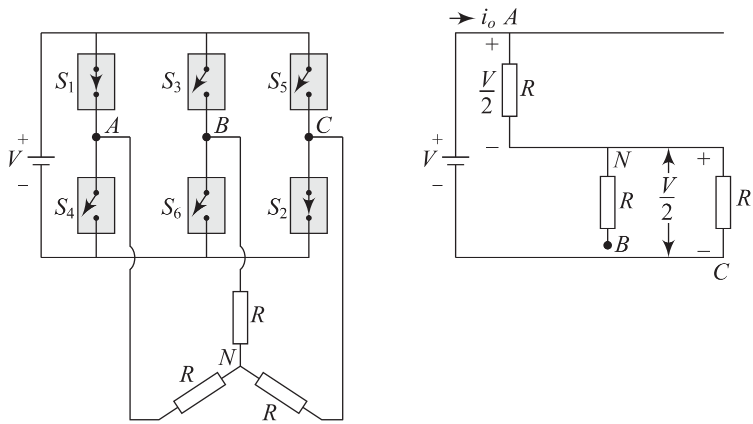


Fig. 11.29 *Equivalent circuit of Fig. 11.20 during $\frac{\pi}{3} \le \omega t \le \frac{2\pi}{3}$ when S_1 and S_2 are closed*

3. Mode III $\left(\frac{2\pi}{3} \le \omega t \le \pi\right)$ During mode III, switches S_3 and S_2 conduct for $\frac{2\pi}{3} \le \omega t \le \pi$ and the equivalent circuit is shown in Fig. 11.30.

The current i_o is given by

$$i_o = \frac{V}{R+R} = \frac{V}{2R}$$

The lines to neutral voltages are

$$V_{AN} = 0, \quad V_{BN} = V_{NC} = i_o R = \frac{V}{2}$$

Then $$V_{CN} = -V_{NC} = -\frac{V}{2}$$

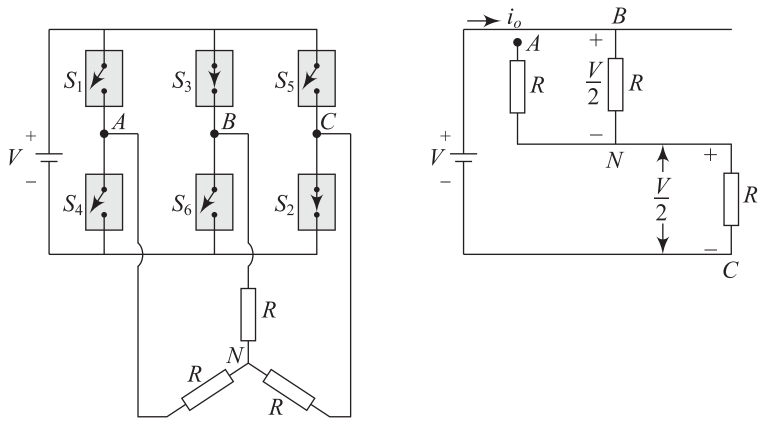


Fig. 11.30 *Equivalent circuit of Fig. 11.20 during* $\frac{2\pi}{3} \le \omega t \le \pi$ *when* S_3 *and* S_2 *are closed*

Similarly, during mode IV $\left(\pi \le \omega t \le \frac{4\pi}{3}\right)$, S_3 and S_4 are closed and the line to neutral voltages are

$$V_{AN} = -\frac{V}{2}, V_{BN} = \frac{V}{2} \text{ and } V_{CN} = 0$$

In mode V $\left(\frac{4\pi}{3} \le \omega t \le \frac{5\pi}{3}\right)$, S_5 and S_4 are closed and the line to neutral voltages are

$$V_{AN} = -\frac{V}{2}, V_{BN} = 0 \text{ and } V_{CN} = \frac{V}{2}$$

During mode VI $\left(\frac{5\pi}{3} \le \omega t \le 2\pi\right)$, S_5 and S_6 are closed and the line to neutral voltages are

$$V_{AN} = 0, V_{BN} = -\frac{V}{2} \text{ and } V_{CN} = \frac{V}{2}$$

As the lines to neutral voltages are known for a complete cycle, the line to line voltages are computed by using following expressions:

$$V_{AB} = V_{AN} - V_{BN}, V_{BC} = V_{BN} - V_{CN} \text{ and } V_{CA} = V_{CN} - V_{AN}$$

The output phase voltages V_{AN}, V_{BN}, V_{CN} and line voltage V_{AB}, V_{BC}, V_{CA} are shown in Fig. 11.31. The instantaneous phase voltages can be expressed in Fourier series since these voltages are periodic non-sinusoidal waveform as given below.

$$v_{AN} = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t + \frac{n\pi}{6}\right)$$

$$v_{BN} = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t - \frac{n\pi}{2}\right)$$

$$v_{CN} = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t + \frac{5n\pi}{6}\right)$$

The Fourier series expression of line voltage waveform V_{AB} is given by

$$v_{AB} = \sum_{\substack{n=6k\pm 1 \\ k=0,1,2,3\ldots}}^{\infty} \frac{3V}{n\pi} \sin\left(n\omega t + \frac{n\pi}{3}\right)$$

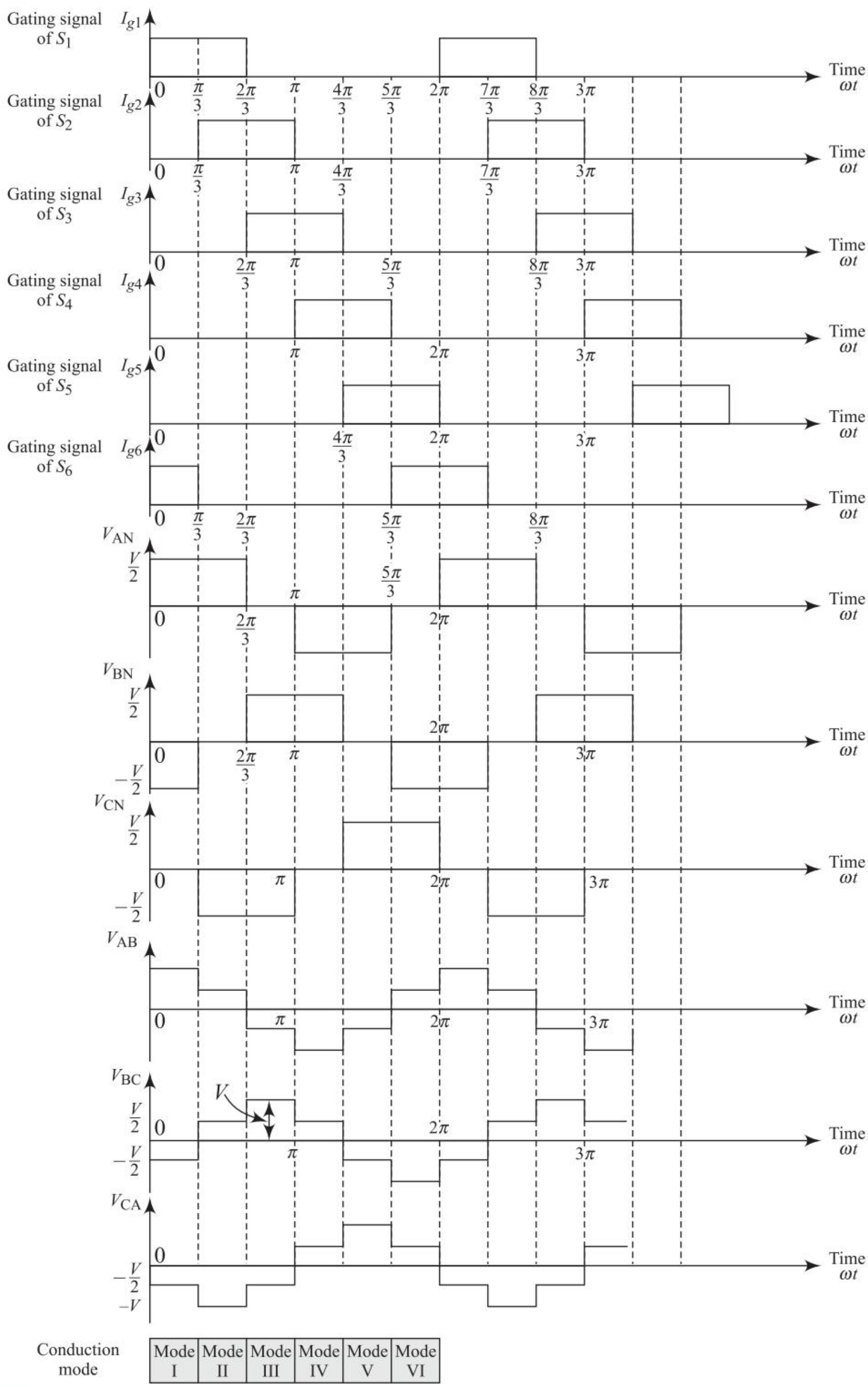


Fig. 11.31 *Gating signals of S_1, S_2, S_3, S_4, S_5, S_6, phase voltages V_{AN}, V_{BN}, V_{CN} and line voltages V_{AB}, V_{BC}, V_{CA}*

Similarly, line voltages V_{BC} and V_{CA} are represented by

$$v_{BC} = \sum_{\substack{n=6k\pm1\\k=0,1,2,3\ldots}}^{\infty} \frac{3V}{n\pi}\sin n\left(\omega t + \frac{\pi}{3} - \frac{2\pi}{3}\right) = \sum_{\substack{n=6k\pm1\\k=0,1,2,3\ldots}}^{\infty} \frac{3V}{n\pi}\sin n\left(\omega t - \frac{\pi}{3}\right)$$

$$v_{CA} = \sum_{\substack{n=6k\pm1\\k=0,1,2,3\ldots}}^{\infty} \frac{3V}{n\pi}\sin n\left(\omega t - \frac{\pi}{3} - \frac{2\pi}{3}\right) = \sum_{\substack{n=6k\pm1\\k=0,1,2,3\ldots}}^{\infty} \frac{3V}{n\pi}\sin n(\omega t - \pi)$$

The rms value of phase voltage is expressed by

$$V_{\text{phase(rms)}} = \left[\frac{1}{\pi}\int_0^{\frac{2\pi}{3}}\left(\frac{V}{2}\right)^2 d\omega t\right]^{1/2} = \left[\frac{1}{\pi}\left(\frac{V}{2}\right)^2\left(\frac{2\pi}{3} - 0\right)\right]^{1/2}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}\frac{V}{2} = \frac{V}{\sqrt{6}} = 0.4082\text{ V}$$

The rms value of line voltages is

$$V_{L(\text{rms})} = \sqrt{3}V_{\text{ph(rms)}} = \sqrt{3}\times\frac{V}{\sqrt{6}} = \frac{V}{\sqrt{2}} = 0.707\text{ V}$$

The rms value of nth component of the line voltage is given by

$$V_{Ln(\text{rms})} = \frac{3V}{\sqrt{2}n\pi}$$

The rms value of fundamental component of the line voltage is

$$V_{L1(\text{rms})} = \frac{3V}{\sqrt{2}\pi} = 0.6752\text{ V}$$

The power output of inverter is

$$P_o = 3\frac{V^2_{\text{ph(rms)}}}{R} = \frac{V^2}{2R} \qquad \text{as} \quad V_{\text{phase(rms)}} = \frac{V}{\sqrt{6}}$$

The rms current flows through switch (transistor) is

$$I_{\text{switch(rms)}} = \frac{I_{\text{ph(rms)}}}{\sqrt{2}} = \frac{V}{2\sqrt{3}R} \qquad \text{as} \quad I_{\text{phase(rms)}} = \frac{V_{\text{ph(rms)}}}{R}$$

Example 11.15 A three-phase bridge inverter is fed from 600 V dc supply. If the semiconductor switches (transistors) which are used in inverter conducts for 120° duration and the inverter is supplying a star connected resistive load of 10 Ω, determine (a) rms value of per phase voltage and line voltage, (b) rms value of load current, (c) rms value of current flows through transistors, (d) power delivered to load, (e) average source current.

Solution

Given: $V = 600$ V, $R = 10\ \Omega$

(a) The rms value of phase voltage is expressed by

$$V_{\text{ph(rms)}} = \frac{V}{\sqrt{6}} = 0.4082\times 600 = 244.92\text{ V}$$

The rms value of line voltages is

$$V_{L(\text{rms})} = \sqrt{3}V_{\text{ph(rms)}} = \sqrt{3}\times 244.92 = 424.21\text{ V}$$

(b) rms value of load current per phase

$$I_{ph} = \frac{V_{ph(rms)}}{R} = \frac{244.92}{10} = 24.492 \text{ A}$$

(c) rms value of current flows through transistors

$$I_{rms(transistor)} = \frac{V}{2\sqrt{3}R} = \frac{600}{2\sqrt{3} \times 10} = 17.320 \text{ A}$$

(d) Power delivered to load

$$P_O = 3I_{ph(rms)}^2 R = 3 \times 24.492^2 \times 10 \text{ Watt} = 17.9957 \text{ kW}$$

(e) Average source current is

$$I_{av} = \frac{P_O}{V} = \frac{17995.7}{600} = 29.992 \text{ A}$$

Example 11.16 A three-phase bridge inverter operates is operated by dc supply and it operates in 120° conduction mode, determine (a) input dc voltage for fundamental line voltage of 420 V, (b) rms value line-to-line output voltage, (c) rms value line-to-phase output voltage and (d) transistor voltage rating.

Solution

Given: Conduction of transistor is 120°, $V_{L1(rms)}$ = 420 V

(a) The rms value of fundamental component of the line voltage is

$$V_{L1(rms)} = \frac{3V}{\sqrt{2}\pi} = 0.6752 \text{ V} = 420$$

Therefore, dc supply is $V = \frac{420}{0.6752} V = 622 \text{ V}$

(b) rms value line-to-line output voltage is

$$V_{L(rms)} = \sqrt{3}V_{ph(rms)} = \sqrt{3} \times \frac{V}{\sqrt{6}} = \frac{V}{\sqrt{2}} = 0.707 \text{ V} = 0.707 \times 622 \text{ V} = 439.754 \text{ V}$$

(c) rms value line-to-phase output voltage

$$V_{phase(rms)} = \frac{\sqrt{2}}{\sqrt{3}}\frac{V}{2} = \frac{V}{\sqrt{6}} = 0.4082 \text{ V} = 0.4082 \times 622 = 253.90 \text{ V}$$

(d) Transistor voltage rating $V_{CEO} \geq 1.5V \geq 1.5 \times 622 = 933 \text{ V}$

11.7 PULSE WIDTH MODULATED INVERTERS

Usually the output voltage of single-phase inverter is a square wave. The major drawbacks of square wave inverters are:

1. Output voltage of inverter is constant and it is equal to supply voltage V. This output voltage can not be controlled.
2. Output voltage consists of third harmonic and other harmonics as shown in Fig. 11.32. The rms value of nth harmonic component is

$$V_n = \frac{4V}{n\pi\sqrt{2}} \quad \text{for } n = 1, 3, 5, 7...$$

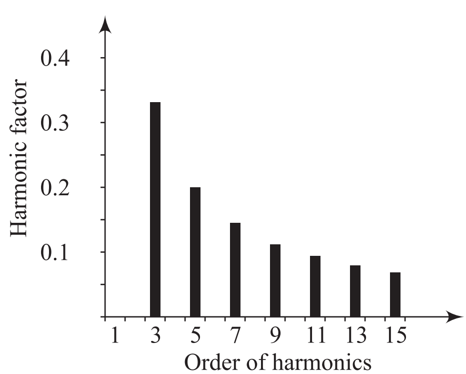


Fig. 11.32 *Harmonic spectrum of square wave inverter*

When $n = 1$, the rms value of fundamental component is $V_1 = \frac{4V}{\pi\sqrt{2}} = 0.9$ V . Then the ratio $\frac{V_1}{V} = 0.9$. The harmonic factor is equal to

$$HF_n = \frac{V_n}{V_1} = \frac{1}{n} \quad \text{for } n = 3, 5, 7, 9...$$

After substituting the values of n, we get

$$HF_3 = \frac{V_3}{V_1} = \frac{1}{3} = 0.333, \quad HF_5 = \frac{V_5}{V_1} = \frac{1}{5} = 0.2, \quad HF_7 = \frac{V_7}{V_1} = \frac{1}{7} = 0.143,$$

$$HF_9 = \frac{V_9}{V_1} = \frac{1}{9} = 0.111, \quad HF_{11} = \frac{V_{11}}{V_1} = \frac{1}{11} = 0.091, \quad HF_{13} = \frac{V_{13}}{V_1} = \frac{1}{13} = 0.0769$$

and $$HF_{15} = \frac{V_{15}}{V_1} = \frac{1}{15} = 0.0666 .$$

To control the output voltage, inverter must be fed from an ac-to-dc converter or dc-to-dc converter. However, to control output voltage as well as to reduce harmonics in the inverter output voltage, the pulse width modulation (PWM) technique should be used in inverter. In PWM control, the output pulse duration is modulated or varied to control the output voltage. There are different methods of modulations, but the most commonly used modulation techniques in inverter are

1. Single-pulse width modulation (SPWM)
2. Multi-pulse width modulation (MPWM)
3. Sinusoidal Pulse Width Modulation (SinPWM)
4. Modified Sinusoidal Pulse Width Modulation

In this section, the above modulation techniques are discussed elaborately.

11.7.1 Single-Pulse Width Modulation (SPWM)

In a single-pulse width modulation inverter, there is only one pulse on each half cycle of output voltage. The width of the pulse should be varied to control the inverter output voltage. Figure 11.33 shows the gating signals and the output voltage waveform of a single-phase full-bridge inverter.

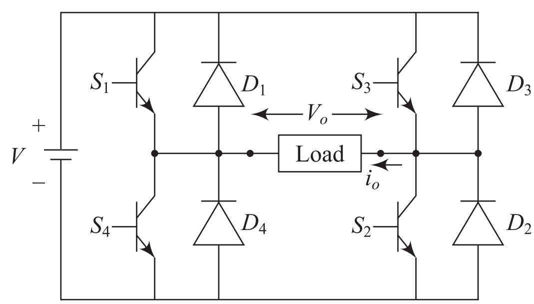


Fig. 11.33 *Single-phase full-bridge inverter*

The gating signals of switches (BJTs, MOSFETs, IGBT, Thyristors and GTOs, etc.) are determined by comparing a reference signal v_r and a carrier signal v_c. The maximum amplitude of reference and carrier signals are A_r and A_c respectively. The frequency of reference and carrier signal is same as fundamental frequency of output voltage. The output voltage can be controlled by controlling the width of pulse d which depends upon the amplitude modulation index (M). The amplitude modulation index can be determined from the expression

$$M = \frac{A_r}{A_c}$$

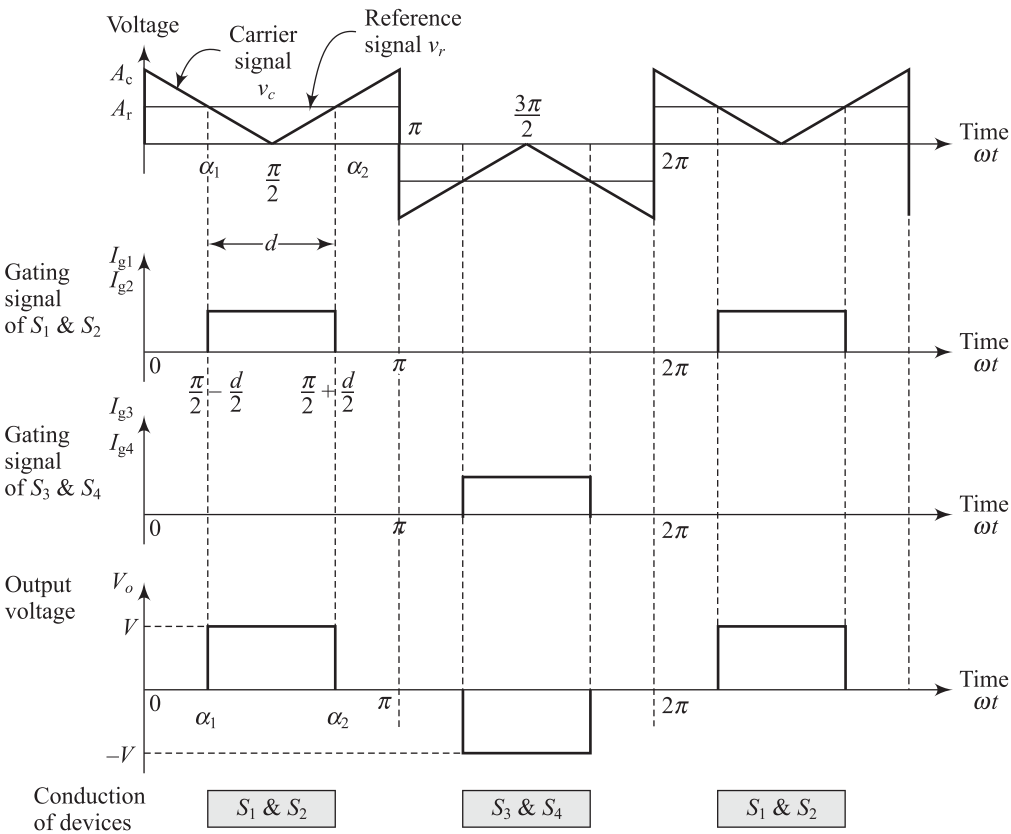


Fig. 11.34 *(a) Carrier signal and reference signal, (b) Gating signals of S_1 and S_2, (c) Gating signals of S_3 and S_4 (d) Output voltage of single phase full-bridge inverter with one pulse per half cycle*

The angle and time of intersections for reference and carrier signals are

$$\alpha_1 = \omega t_1 \quad \text{or,} \quad t_1 = \frac{\alpha_1}{\omega} = (1-M)\frac{T}{4} \text{ and}$$

$$\alpha_2 = \omega t_2 \quad \text{or,} \quad t_2 = \frac{\alpha_2}{\omega} = (1+M)\frac{T}{4}$$

where, T is the time period of signals

The pulse width in angle is $d = \alpha_2 - \alpha_1$

The pulse width in time is $t_2 - t_1 = M\frac{T}{2}$

The rms value of output voltage is

$$V_{o(\text{rms})} = \left[\frac{1}{\pi}\int_{\frac{(\pi-d)}{2}}^{\frac{(\pi+d)}{2}} V^2 dt\right]^{1/2} = V\sqrt{\frac{d}{\pi}}$$

When the amplitude of reference signal A_r is varied from 0 to A_c, the pulse width d can be changed from 0° to 180° or π. Accordingly the rms value of output voltage $V_{o(\text{rms})}$ can be varied from 0 to V. The output voltage waveform is depicted in Fig. 11.35. The shape of output voltage is called quasi-square wave which can be expressed in Fourier series.

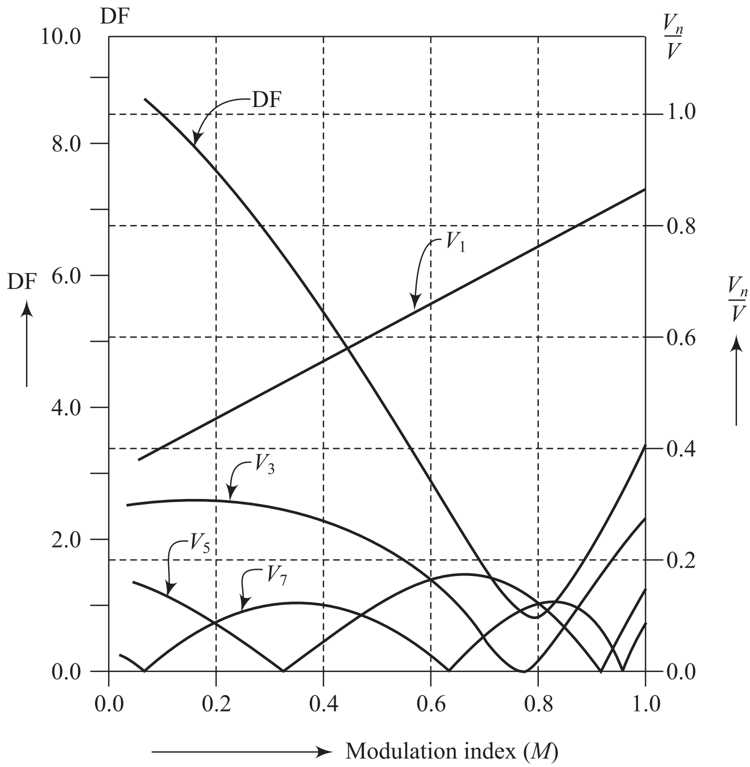


Fig. 11.35 *Harmonic profile of single pulse width modulation inverter*

Due to half-wave symmetry of output voltage, the even harmonics are absent and only odd b_n components are present. The values of b_n are given by

$$b_n = \frac{2}{\pi}\int_{(\pi-d)/2}^{(\pi+d)/2} V \sin n\omega t \cdot d\omega t$$

$$= \frac{4V}{n\pi}\sin\frac{n\pi}{2}\sin\frac{nd}{2} \quad \text{where, } n = 1, 3, 5...$$

or

$$b_n = \frac{4V}{n\pi}\sin\frac{nd}{2} \quad \text{where, } n = 1, 5, 9...$$

and

$$b_n = -\frac{4V}{n\pi}\sin\frac{nd}{2} \quad \text{where, } n = 3, 7, 11...$$

The output voltage can be expressed as

$$v_o = \sum_{n=1,3,5....}^{\infty} \frac{4V}{n\pi}\sin\frac{n\pi}{2}\sin\frac{nd}{2}\sin n\omega t \quad \text{where, } n = 1, 3, 5...$$

or

$$v_o = \frac{4V}{\pi}\left[\sin\frac{d}{2}\sin\omega t - \frac{1}{3}\sin\frac{3d}{2}\sin 3\omega t + \frac{1}{5}\sin\frac{5d}{2}\sin 5\omega t - \frac{1}{7}\sin\frac{7d}{2}\sin 7\omega t + \cdots\right]$$

To eliminate nth harmonic, $\sin\frac{nd}{2} = 0$

or $$\frac{nd}{2} = k\pi \quad \text{where, } k \text{ is an integer}$$

Therefore, the width of pulse is $d = \frac{2k\pi}{n} = \frac{360k}{n}$ in degree. To eliminate third harmonic, the required pulse width is equal to $d = \frac{360k}{n} = \frac{360}{3} = 120°$ assuming $k = 1$ and $n = 3$. The variations of fundamental component and other harmonic components and the rms value of output voltage is depicted in Fig. 11.35.

The disadvantages of single pulse modulation are as follows:

1. Harmonic content is high.
2. The maximum rms value of fundamental component is only about 90.09% of dc input voltage V.
3. Third harmonic dominate.

Example 11.17 A single-phase PWM inverter is fed from a 220 V dc supply and it is connected to a *RL* load with $R = 10\ \Omega$ and $L = 10$ mH. Determine the total harmonic distortion in the load current. Assume width of each pulse is $\frac{\pi}{2}$ and the output frequency is 50 Hz.

Solution

Given: $V = 220$ V, $R = 10\ \Omega$, $L = 10$ mH, $d = \frac{\pi}{2} = 90°$

The rms value of nth harmonic component of the output voltage is

$$V_n = \frac{4V}{\sqrt{2}n\pi}\sin\frac{nd}{2}$$

The impedance offered to the nth harmonic current is

$$Z_n = \sqrt{R^2 + (n\omega L)^2}$$

Therefore, $$V_1 = \frac{4V}{\sqrt{2}\pi}\sin\frac{d}{2} = \frac{4\times 220}{\sqrt{2}\pi}\sin\frac{90}{2} = 139.9999 \text{ V} \quad \text{and}$$

$$Z_1 = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi\times 50\times 10\times 10^{-3})^2} = 10.482\ \Omega$$

Similarly, $$V_3 = \frac{4V}{3\sqrt{2}\pi}\sin\frac{3d}{2} = \frac{4\times 220}{3\sqrt{2}\pi}\sin\frac{3\times 90}{2} = 46.6666 \text{ V} \quad \text{and}$$

$$Z_3 = \sqrt{R^2 + (3\omega L)^2} = \sqrt{10^2 + (2\pi\times 3\times 50\times 10\times 10^{-3})^2} = 13.744\ \Omega$$

$$V_5 = \frac{4V}{5\sqrt{2}\pi}\sin\frac{5d}{2} = \frac{4\times 220}{5\sqrt{2}\pi}\sin\frac{5\times 90}{2} = 27.9999 \text{ V} \quad \text{and}$$

$$Z_5 = \sqrt{R^2 + (5\omega L)^2} = \sqrt{10^2 + (2\pi\times 5\times 50\times 10\times 10^{-3})^2} = 18.626\ \Omega$$

$$V_7 = \frac{4V}{7\sqrt{2}\pi}\sin\frac{7d}{2} = \frac{4\times 220}{7\sqrt{2}\pi}\sin\frac{7\times 90}{2} = 19.9999 \text{ V} \quad \text{and}$$

$$Z_7 = \sqrt{R^2 + (7\omega L)^2} = \sqrt{10^2 + (2\pi\times 7\times 50\times 10\times 10^{-3})^2} = 24.165\ \Omega$$

$$V_9 = \frac{4V}{9\sqrt{2}\pi}\sin\frac{9d}{2} = \frac{4\times 220}{9\sqrt{2}\pi}\sin\frac{9\times 90}{2} = 15.5555 \text{ V} \quad \text{and}$$

$$Z_9 = \sqrt{R^2 + (9\omega L)^2} = \sqrt{10^2 + (2\pi\times 9\times 50\times 10\times 10^{-3})^2} = 30.00\ \Omega$$

The harmonics currents are

$$I_1 = \frac{V_1}{Z_1} = \frac{139.9999}{10.482} A = 13.356 \text{ A}, \quad I_3 = \frac{V_3}{Z_3} = \frac{46.6666}{13.7444} A = 3.3954 \text{ A}$$

$$I_5 = \frac{V_5}{Z_5} = \frac{27.9999}{18.626} A = 1.503 \text{ A}, \quad I_7 = \frac{V_7}{Z_7} = \frac{19.9999}{24.165} A = 0.8276 \text{ A}$$

$$I_9 = \frac{V_9}{Z_9} = \frac{15.5555}{30.0} A = 0.5185 \text{ A}$$

The total harmonic distortion in the load current is

$$THD = \frac{\sqrt{I_3^2 + I_5^2 + I_7^2 + I_9^2 + \cdots}}{I_1}$$

$$= \frac{\sqrt{3.3954^2 + 1.503^2 + 0.8276^2 + 0.5185^2 + \cdots}}{13.356} = 0.2874$$

11.7.2 Multi-Pulse Width Modulation (MPWM)

To reduce the harmonic content in output voltage, more than one pulse should be present on each half cycle of output voltage. The number of output pulses depends on the frequency of carrier signal f_c and reference signal f_r. The number pulses (p) per half cycle of output voltage can be determined from

$$p = \frac{f_c}{2f_r} = \frac{m_f}{2}$$

where, $m_f = \frac{f_c}{f_r}$ is the frequency modulation ratio.

Hence, the frequency modulation index m_f controls the output voltage. This type of modulation is also called *uniform pulse-width modulation* (UPWM).

Figures 11.36 and 11.37 show the gating signals and the output voltage waveform of a single phase full-bridge inverter with single pulse width modulation and multi-pulse width modulation respectively. The rms output voltage is

$$V_{\text{rms}} = \left[\frac{2p}{2\pi} \int_{\left(\frac{\pi}{p}-d\right)/2}^{\left(\frac{\pi}{p}+d\right)/2} V^2 d\omega t \right]^{1/2} = V\sqrt{\frac{pd}{\pi}}$$

where, p is number of pulses per half cycle and d is the duration of each pulse.

With the variation of modulation index M from 0 to 1, the pulse width varies from 0 to $\frac{\pi}{p}$ or $\frac{180°}{p}$ and the rms output voltage varies from 0 to V.

Due to half-wave symmetry of output voltage, the even harmonics are absent and only odd b_n components are present. The values of b_n are given by

$$b_n = \frac{2}{\pi} \int_0^{\pi} v_o \sin n\omega t \cdot d\omega t$$

or $$b_n = p \times \frac{2}{\pi} \int_{\left(\gamma - \frac{d}{2}\right)}^{\left(\gamma + \frac{d}{2}\right)} V \sin n\omega t \cdot d\omega t = \frac{2pV}{n\pi} |\cos n\omega t|_{\gamma - \frac{d}{2}}^{\gamma + \frac{d}{2}}$$

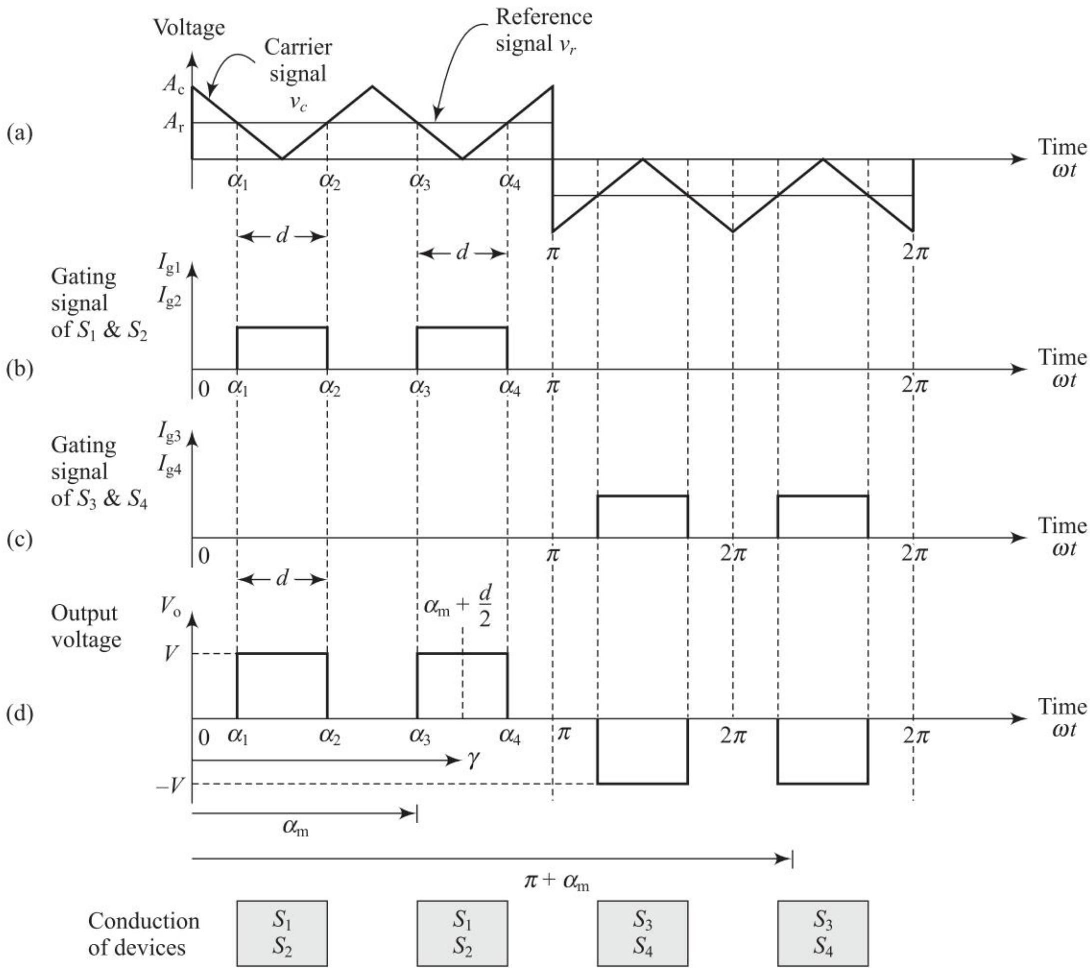


Fig. 11.36 *(a) Carrier signal and reference signal, (b) Gating signals of $S_1$and S_2, (c) Gating signals of S_3 and S_4, (d) Output voltage of single phase full-bridge inverter with two pulse per half cycle*

where, p is number of pulse per half cycle and $\gamma = \alpha_m + \dfrac{d}{2}$

Then $$b_n = \frac{4pV}{n\pi} \sin n\gamma \sin \frac{nd}{2} \quad \text{where, } n = 1, 3, 5 \ldots.$$

In case of $p = 2, b_n = \dfrac{8V}{n\pi} \sin n\gamma \sin \dfrac{nd}{2}$ where, $n = 1, 3, 5 \ldots.$

Then output voltage can be expressed by Fourier series as given below.

$$v_o = \sum_{n=1,3,5\ldots}^{\infty} \frac{8V}{n\pi} \sin n\gamma \sin \frac{nd}{2} \sin n\omega t \quad \text{where, } n = 1, 3, 5\ldots$$

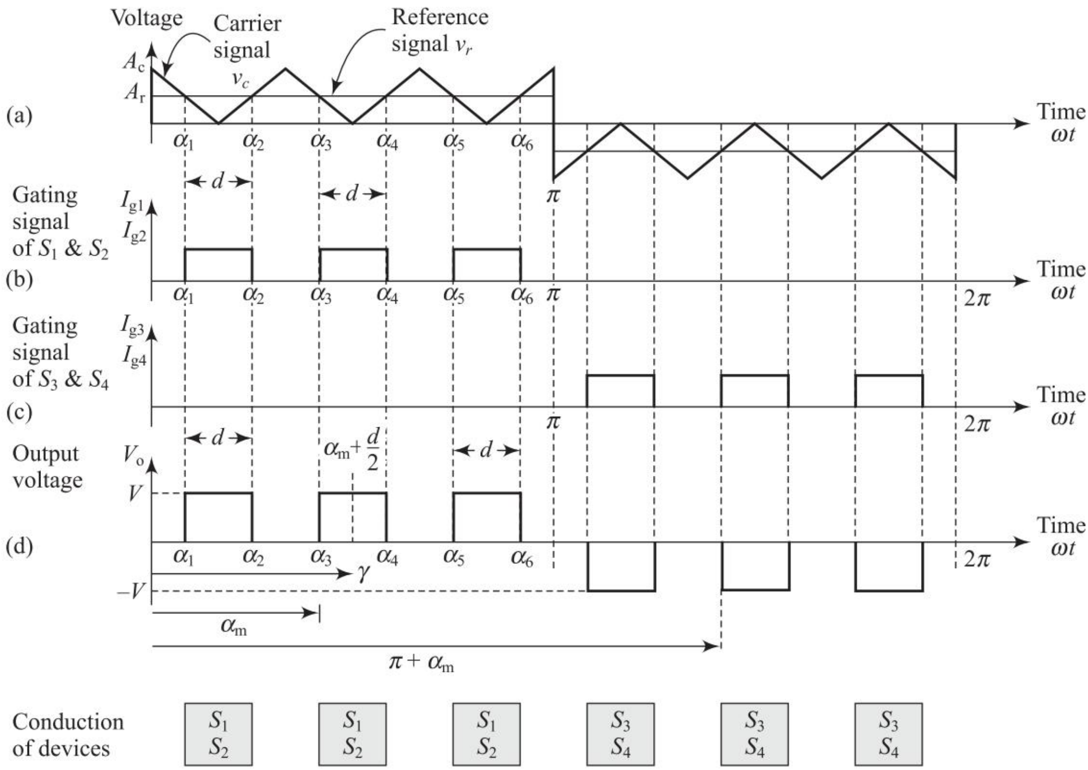


Fig. 11.37 *(a) Carrier signal and reference signal, (b) Gating signals of S_1 and S_2, (c) Gating signals of S_3 and S_4, (d) Output voltage of single phase full-bridge inverter with two pulse per half cycle*

or
$$v_o = \frac{8V}{\pi}\left[\sin\gamma\,\sin\frac{d}{2}\sin\omega t + \frac{1}{3}\sin 3\gamma\,\sin\frac{3d}{2}\sin 3\omega t + \frac{1}{5}\sin 5\gamma\,\sin\frac{5d}{2}\sin 5\omega t + \frac{1}{7}\sin 7\gamma\,\sin\frac{7d}{2}\sin 7\omega t + \cdots\right]$$

when $p = 2$

The amplitude of the nth harmonic voltage is equal to

$$V_n = \frac{8V}{n\pi}\sin n\gamma\,\sin\frac{nd}{2} \quad \text{when } p = 2 \text{ and } n = 1, 3, 5 \ldots$$

It is clear from above equation that the amplitude of the nth harmonic voltage depends on γ and d. To eliminate nth harmonic, $\sin\gamma = 0$ or, $n\gamma = \pi$

$$\text{and } \sin\frac{nd}{2} = 0 \text{ or, } \frac{nd}{2} = k\pi \quad \text{where, } k \text{ is integer}$$

Therefore, $\gamma = \frac{\pi}{n} = \frac{180°}{n}$ and the width of pulse is $d = \frac{2k\pi}{n} = \frac{360k}{n}$ in degree.

The variations of fundamental component and other harmonic components and the rms value of output voltage is depicted in Fig. 11.38.

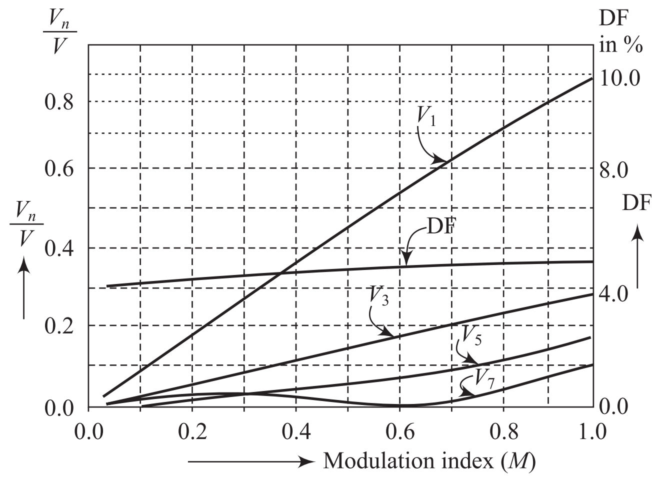


Fig. 11.38 *Harmonic profile of multi pulse modulation with p = 5*

The angle and time of intersections for reference and carrier signals are

$$\alpha_m = \omega t_m \quad \text{or,} \quad t_m = \frac{\alpha_m}{\omega} = (m - M)\frac{T}{4p} \quad \text{for } m = 1, 3, 5, \ldots 2p \text{ and}$$

$$\alpha_m = \omega t_m \quad \text{or,} \quad t_m = \frac{\alpha_m}{\omega} = (m - 1 + M)\frac{T}{4p} \quad \text{for } m = 2, 4, 6, \ldots 2p$$

where, T is the time period of signals.
The pulse width in angle is $d = \alpha_{m+1} - \alpha_m$

The pulse width in time is $t_{m+1} - t_m = M\frac{T}{2p}$

Example 11.18 The output voltage of multi-pulse modulation inverter is shown in Fig. 11.39. Determine (a) the rms value of output voltage, (b) the rms value of fundamental component of output voltage, (c) the total harmonic distortion.

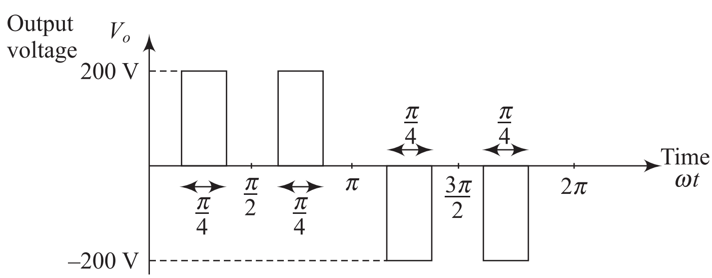


Fig. 11.39 *Output voltage of multi-pulse modulation inverter*

Solution

(a) The rms value of output voltage is equal to

$$V_o = \left(\frac{200^2 \times \frac{\pi}{4}}{\frac{\pi}{2}}\right)^{1/2} = 200\sqrt{\frac{1}{2}} = 141.42 \text{ V}$$

Due to half-wave symmetry, only odd harmonics are present in the Fourier series.

$$b_n = \frac{4}{\pi}\int_0^{\pi/2} V_o(t)\sin n\omega t \cdot d\omega t = \frac{4}{\pi}\int_{\pi/8}^{3\pi/8} 200 \sin n\omega t \cdot d\omega t \quad \text{where, } n = 1, 3, 5$$

Therefore, $b_1 = \dfrac{4}{\pi}\displaystyle\int_{\pi/8}^{3\pi/8} 200 \sin \omega t \cdot d\omega t = \dfrac{800}{\pi}[-\cos \omega t]_{22.5^\circ}^{67.5^\circ}$

$$= \frac{800}{\pi}[-\cos 67.5 + \cos 22.5] = 137.75$$

(b) The rms value of fundamental component of output voltage is

$$V_1 = \frac{b_1}{\sqrt{2}} = \frac{137.75}{\sqrt{2}} = 97.41 \text{ V}$$

(c) The total harmonic distortion is equal to

$$THD = \frac{\sqrt{V_o^2 - V_1^2}}{V_1} = \frac{\sqrt{141.42^2 - 97.41^2}}{97.41} = 1.052$$

Example 11.19 A single-phase transistorized half bridge bipolar PWM inverter is operated from a center tap 96 V dc input voltage. The fundamental output frequency is 50 Hz and the carrier frequency is 1 kHz and modulation index is 0.75. Determine (a) carrier ratio m_f, (b) number of pulses per cycle, (c) fundamental output voltage, (d) distortion factor of output voltage waveform and (e) harmonic factor of output voltage waveform.

Solution

Given: $\dfrac{V}{2} = 96$ V , f_r = 50 Hz, f_c = 1 kHz and m = 0.75

(a) Carrier ratio m_f is $m_f = \dfrac{f_c}{f_r} = \dfrac{1000}{50} = 20$

(b) Number of pulses per cycle $p = \dfrac{m_f}{2} = \dfrac{20}{2} = 10$

(c) Fundamental output voltage $V_{o1(\text{rms})} = \dfrac{1}{\sqrt{2}} m \dfrac{V}{2} = \dfrac{1}{\sqrt{2}} \times 0.75 \times 96 = 50.919$ V

(d) Distortion factor of output voltage waveform $DF = \dfrac{V_{o1(\text{rms})}}{V_{o(\text{rms})}} = \dfrac{50.919}{96} = 0.5304$

(e) Harmonic factor of output voltage waveform $HF = \sqrt{\dfrac{1}{DF^2} - 1} = \sqrt{\dfrac{1}{0.5304^2} - 1} = 1.598$

Example 11.20 A single-phase transistorized full bridge bipolar PWM inverter is operated from a 120 V dc battery and it is connected with a *RL* load. If the modulation index is 0.9, determine (a) rms output voltage, (b) fundamental output voltage, (c) distortion factor of output voltage waveform, (d) harmonic factor of output voltage waveform and (e) gain of inverter.

Solution

Given: V = 120 V and m = 0.9

(a) rms output voltage $V_{\text{orms}} = V = 120$ V

(b) Fundamental output voltage $V_{o1(\text{rms})} = \dfrac{1}{\sqrt{2}} mV = \dfrac{1}{\sqrt{2}} \times 0.9 \times 120 = 76.379$ V

(c) Distortion factor of output voltage waveform $DF = \dfrac{V_{o1(\text{rms})}}{V_{o(\text{rms})}} = \dfrac{76.379}{120} = 0.6364$

(d) Harmonic factor of output voltage waveform $HF = \sqrt{\dfrac{1}{DF^2} - 1} = \sqrt{\dfrac{1}{0.6364^2} - 1} = 1.212$

(e) Gain of inverter is $G = 0.707m = 0.707 \times 0.9 = 0.6363$

11.7.3 Sinusoidal Pulse Width Modulation

In sinusoidal pulse width modulation, several pulses are used in each half cycle of output voltage but the width of the pulses is not same as in the case of multiple pulse modulations (uniform pulse width modulation). But in both sinusoidal pulse width modulation and multiple pulse modulations, the width of each pulse varies with the amplitude of sine wave reference voltage. In case of sinusoidal pulse width modulation, the width of the pulse at the centre of the half cycle is maximum and decreases on either side. The generation of gating signal of switches by comparing a sinusoidal reference signal v_r with a triangular carrier signal v_c is depicted in Fig. 11.40. A comparator is used to compare a sinusoidal reference signal v_r with respect to a triangular carrier signal v_c. When the amplitude of sinusoidal reference signal is greater than the amplitude of triangular carrier signal, comparator output is high. If the amplitude of sinusoidal reference signal is less than the amplitude of triangular carrier signal, comparator output is low.

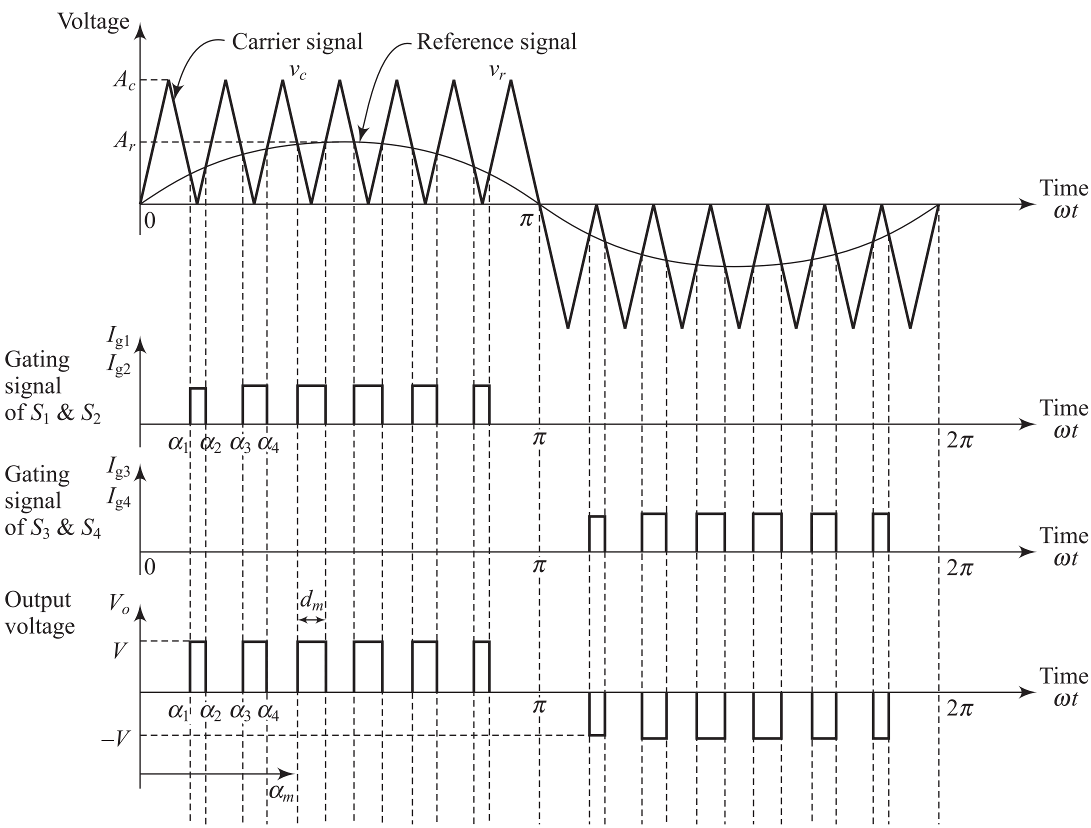


Fig. 11.40 *Sinusoidal pulse width modulation*

If the frequency of carrier signal is f_c and the frequency of sinusoidal reference signal is f_r, there are $m = \frac{f_c}{2f_r}$ carrier pulses per half cycle. The number of pulses in each half cycle is equal to $\left(\frac{f_c}{2f_r} - 1\right) = (m-1)$.

The modulation index is $M = \frac{A_r}{A_c}$ and it controls the rms output voltage and the harmonic content of the output voltage waveform.

When d_m is the width of the mth pulse, the rms value of output voltage is

$$V_o = V\left[\sum_{m=1}^{2p} \frac{d_m}{\pi}\right]^{1/2}$$

From the harmonic analysis of output voltage of sinusoidal pulse width modulation inverter, the following features will be observed:

1. In linear modulation (modulation index is less than 1), the largest amplitudes in the output voltage are associated with harmonics of order
   $\frac{f_c}{f_r} \pm 1$ or, $2m \pm 1$ where, m in the number of carrier pulses per half cycle.
2. By increasing the number of pulses per half cycle, the order of dominant harmonic frequency can be increases so that the higher order harmonic frequency signals can be filtered easily and size of filter is minimized. If $m = 5$, the 9th and 11th order harmonics will be significant in the output voltage and these signal can be filtered easily. But with the increase of m, the switching frequency of semiconductor devices will be increased. Therefore the switching loss will be more and inverter efficiency will be reduced. During design of inverter, there should be a compromise state between the filter requirement and inverter efficiency.

In case of over modulation (modulation index is greater than 1), the lower order harmonics will be present in output signal and pulse width no longer a sine function of the angular position of pulse.

The output voltage waveform of sinusoidal pulse width modulation inverter can be expressed by Fourier series as given below.

$$v_o(t) = b_n \sin n\omega t \quad \text{for } n = 1, 3, 5, 7\ldots$$

where, $$b_n = \sum_{m=1}^{2p} \frac{4V}{n\pi} \sin\frac{nd_m}{4}\left\{\sin n\left(\alpha_m + \frac{3d_m}{4}\right) - \sin n\left(\pi + \alpha_m + \frac{d_m}{4}\right)\right\} \quad \text{for } n = 1, 3, 5, 7\ldots$$

The angle $\alpha_m = \omega t_m$ and time $t_m = \frac{\alpha_m}{\omega} = t_x + m\frac{T}{4(p+1)}$

where, t_x is computed from the following equations:

$$1 - \frac{4t(p+1)}{T} = M \sin\left\{\omega\left(t_x + m\frac{T}{4(p+1)}\right)\right\} \quad \text{for } m = 1, 3, 5, 7 \ldots 2p$$

and $$\frac{4t(p+1)}{T} = M \sin\left\{\omega\left(t_x + m\frac{T}{4(p+1)}\right)\right\} \quad \text{for } m = 2, 4, 6, \ldots 2p$$

The width of mth pulse in angle is $d_m = \alpha_{m+1} - \alpha_m$

The pulse width in time is $t_{m+1} - t_m = \frac{\alpha_{m+1}}{\omega} - \frac{\alpha_m}{\omega}$

The variations of fundamental component and other harmonic components and the rms value of output voltage is depicted in Fig. 11.41. The variation of $\frac{V_1}{V}$ with respect to M is illustrated in Fig. 11.42.

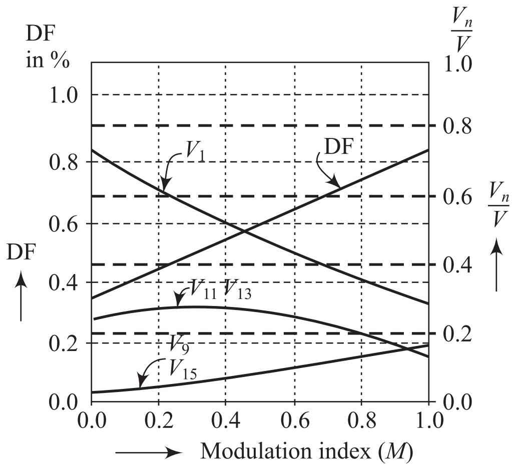


Fig. 11.41 *Harmonics of sinusoidal pulse-width modulation with p = 5*

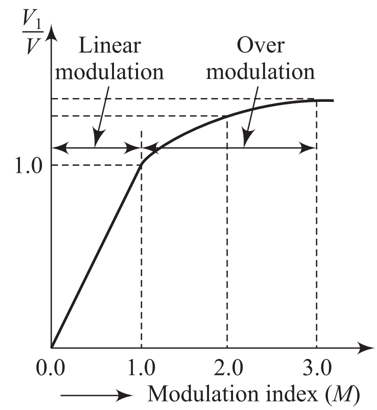


Fig.11.42 *The variation of* $\frac{V_1}{V}$ *with respect to M*

11.7.4 Modified Sinusoidal Pulse Width Modulation

It is clear from output voltage of Fig. 11.43 that the widths of the pulses can not change significantly with the variation of modulation index at the middle of half cycle due to the characteristics of the sine wave reference signal. When the carrier signal is applied during the fast and last $\frac{\pi}{3}(60°)$ interval of each half cycle $\left(0 \le \omega t \le \frac{\pi}{3} \text{ and } \frac{2\pi}{3} \le \omega t \le \pi\right)$, the width of the pulses will be changed significantly. This type of modulation is known as modified sinusoidal pulse width modulation.

Due to change in output voltage waveform as shown in Fig. 11.43, the fundamental component is increased and its harmonics characteristics are improved. As the number of switching of power devices are reduced, the switching losses are also reduced.

The angle $\alpha_m = \omega t_m$ and time $t_m = \frac{\alpha_m}{\omega} = t_x + m\frac{T}{12(p+1)}$ for $m = 1, 3, 5, 7 \ldots p$

where, t_x is computed from the following equations:

$$1 - \frac{12t(p+1)}{T} = M \sin\left\{\omega\left(t_x + m\frac{T}{12(p+1)}\right)\right\} \quad \text{for } m = 1, 3, 5, 7 \ldots p$$

and
$$\frac{12t(p+1)}{T} = M \sin\left\{\omega\left(t_x + m\frac{T}{12(p+1)}\right)\right\} \quad \text{for } m = 2, 4, 6, \ldots 2p$$

The width of mth pulse in angle is $d_m = \alpha_{m+1} - \alpha_m$

The pulse width in time is $t_{m+1} - t_m = \frac{\alpha_{m+1}}{\omega} - \frac{\alpha_m}{\omega}$

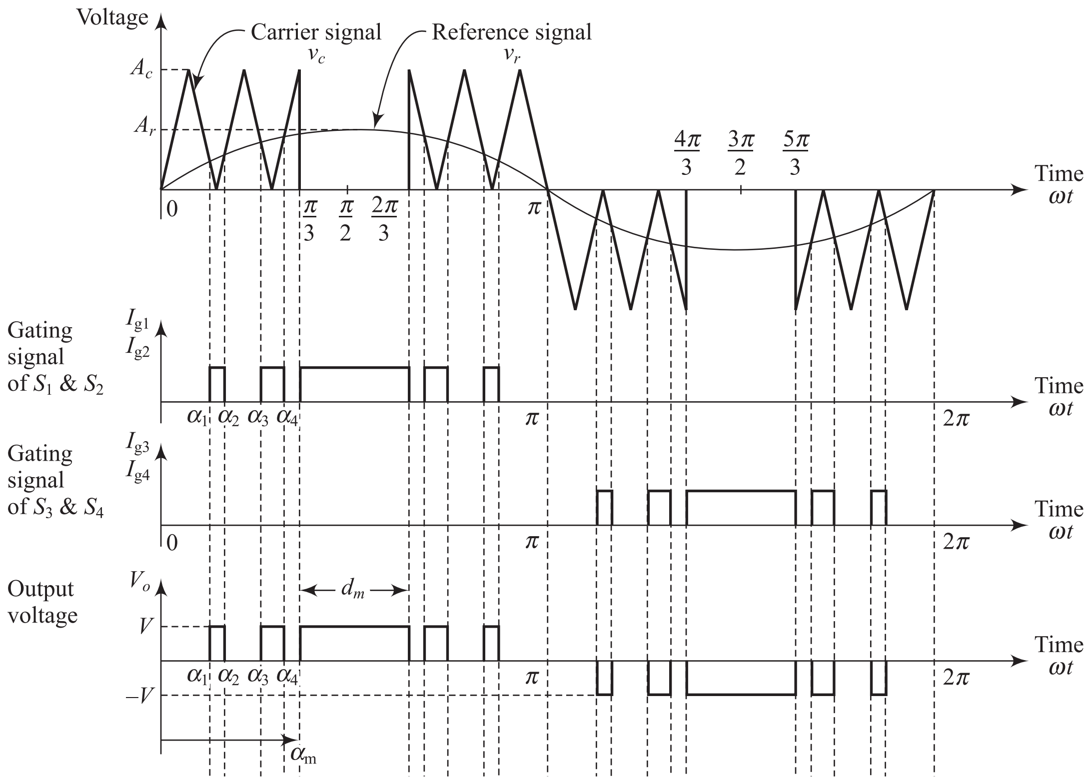


Fig. 11.43 *Modified sinusoidal pulse width modulation inverter*

The variations of fundamental component and other harmonic components and the rms value of output voltage is depicted in Fig. 11.44.

11.8 RESONANT CONVERTERS

When pulse width modulation (PWM) techniques are used for dc-to-dc and dc-to-ac converters, the semiconductor switches interrupts the entire load current during turn-ON and turn-OFF process. Therefore, these semiconductor switches are subjected to high switching stresses in the switch-mode operation and the switching power loss is considerably high. The other problem of PWM techniques is that this switching operation generates electromagnetic interference (EMI) due to large $\frac{di}{dt}$ and $\frac{dv}{dt}$. To maintain low switching loss, fast switching devices should be used in switch mode converters.

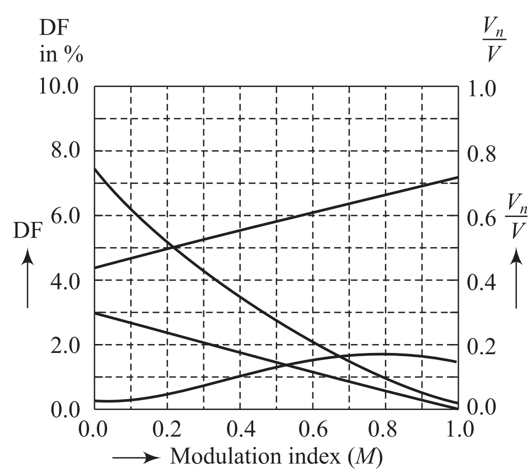


Fig. 11.44 *Harmonics of modified sinusoidal pulse-width modulation with p = 5*

If the switching frequency is increased in order to reduce the converter size and weight, the power density should be increased. The switching stress, switching losses and electromagnetic interference (EMI) are also increased linearly with switching frequency. If a semiconductor switch operates with snubber circuit in a dc to dc converter, trajectories of voltage across device (v_d), current through device (i_d) and power loss (p_d) are depicted in Fig. 11.45. It is clear from Fig. 11.45 that the switching losses are significant through the snubber circuit is present across the device.

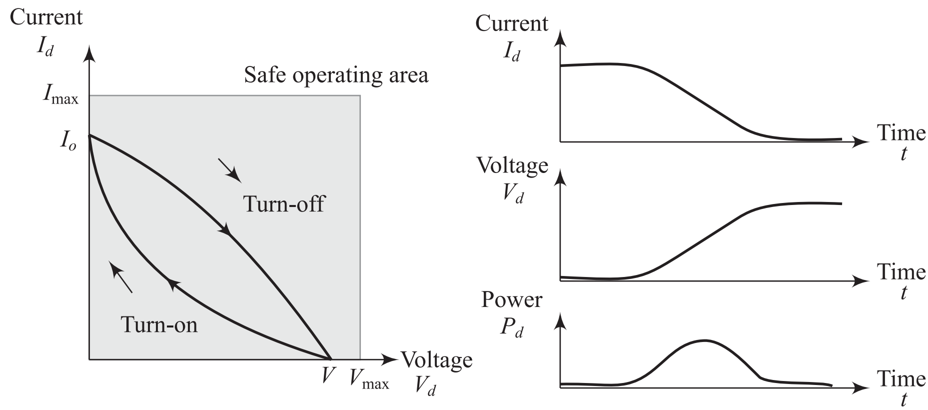


Fig. 11.45 *Trajectories of voltage across device (v_d), current through device (i_d) and power loss (p_d) with snubber circuit*

Consequently, to implement high switching frequency in converter circuits, all shortcomings can be minimized if each semiconductor switch of converter circuit changes its state either from OFF to ON or ON to OFF when voltage across device becomes zero or current through device is zero. This type of switching is called zero voltage switching (ZVS) or zero-current switching (ZCS). Figure 11.46 shows trajectories of voltage across device (v_d), current though device (i_d) and power loss (p_d) with ZVS or ZCS. By using the concept of ZVS or ZCS, the switching losses, stress on semiconductor devices, generation of EMI will be reduced significantly. The converter topologies using the ZVS or ZCS strategies are called resonant converters. Most of the resonant converters operate satisfactory using LC resonance circuit.

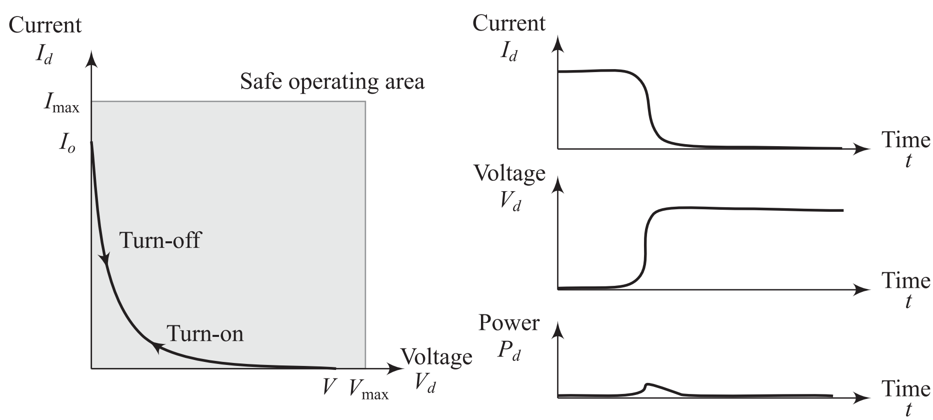


Fig. 11.46 *Trajectories of voltage across device (v_d), current though device (i_d) and power loss (p_d) with ZVS or ZCS*

In a typical resonant converter, the basic operation of switches is to generate square wave ac waveforms from dc supply and the resonating components (inductor *L* and capacitor *C*) are used to filter the unwanted harmonic components from the square wave. The resonating LC circuit must be proper tuned at the switching frequency of converter. There are different circuit topologies of resonant converters. Figure 11.47(a), (b) and (c) show the block diagrams of resonant converters.

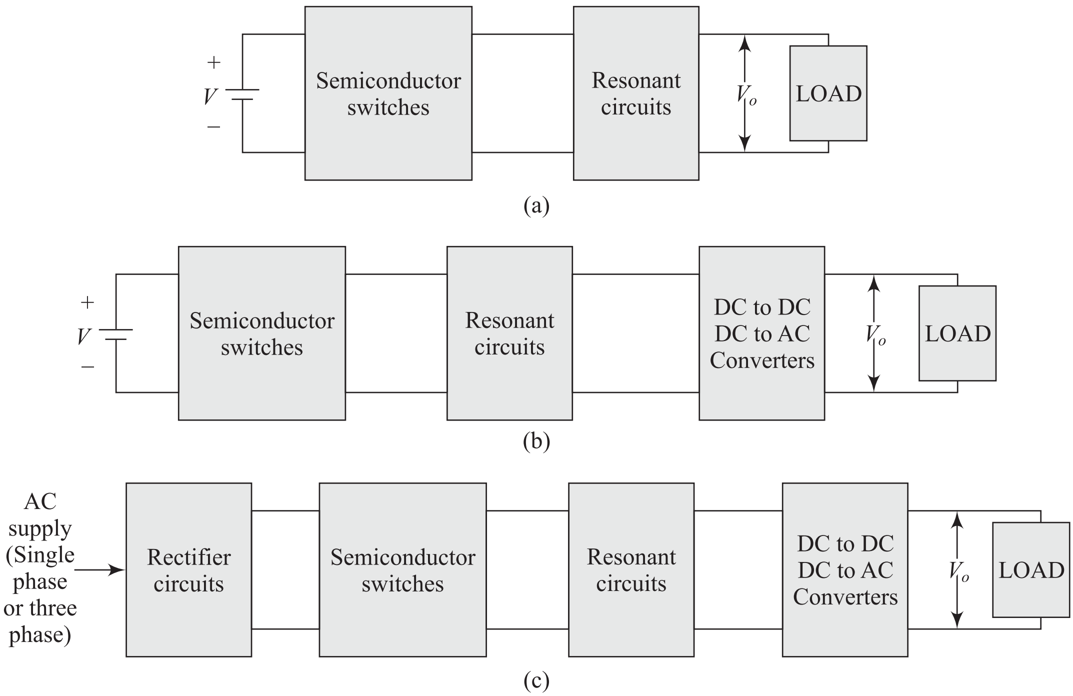


Fig. 11.47 *Block diagram of resonant converters*

The LC circuit does not act as a simple low-pass filter, but it operates in ideally undamped or highly underdamped oscillation of current and/or voltage. For ideally underdamped condition, $R = 0$ and the output waveform is sinusoidal. For highly underdamped condition, $R \neq 0$ and the current through *R* and the voltage across *R* are approximately sinusoidal. Usually, LC circuits are two types such series resonant and parallel resonant. When a series LC resonant circuit is used in converter, it is called *series-resonant converters*. If a parallel LC resonant circuit is used in converter, it is called *parallel-resonant converters*. In this section, both the series-resonant converters and parallel -resonant converters are discussed elaborately.

11.8.1 Undamped Series Resonant Circuits

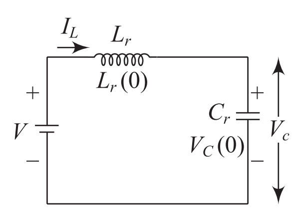


Fig. 11.48 *An undamped series resonant circuit*

A undamped series resonant circuit is shown in Fig. 11.48 where *V* is the input voltage at time t_o. The initial current of inductor is $I_L(0)$ and the initial voltage of capacitor is $V_C(0)$.

When a current i_L flow through the circuit, the KVL equation of the circuit is

$$V = L_r \frac{di_L}{dt} + v_C \text{ and } i_L = C_r \frac{dv_C}{dt}$$

The solution of above equations with time is

$$i_L(t) = I_L(0)\cos\omega_0(t-t_0) + \frac{V - V_C(0)}{Z_O}\sin\omega_0(t-t_0) \qquad \text{for } t > t_o$$

where, $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{L_r C_r}}$ = resonance frequency

and characteristic impedance $Z_0 = \sqrt{\frac{L_r}{C_r}}$ in Ω

The voltage across capacitor is

$$v_C(t) = V - \{V - V_C(0)\}\cos\omega_0(t-t_0) + Z_0 I_L(0)\sin\omega_0(t-t_0) \quad \text{for } t > t_o$$

The plot of $i_L(t)$ and $v_C(t)$ are shown in Fig. 11.49.

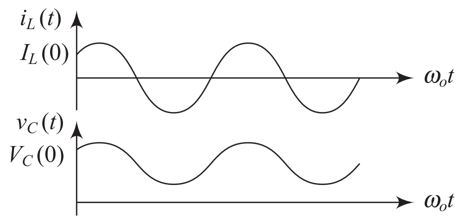


Fig. 11.49 *$i_L(t)$ and $v_C(t)$ waveforms of a undamped series resonant circuit*

11.8.2 Highly Underdamped Series Resonant Circuits

Figure 11.50 shows a *RLC* series resonant circuit where *R* is load resistance and L_r and C_r are resonating components (inductance and capacitance).

The KVL equation of the circuit is

$$V = Ri + L_r\frac{di}{dt} + \frac{1}{C_r}\int i\,dt$$

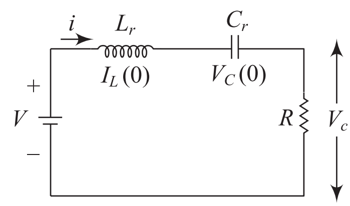


Fig. 11.50 *RLC series resonant circuit*

Assume $V_C(0) = 0$ and $I_L(0) = 0$

The solution of the above equation is

$$i(t) = \frac{V}{\omega_O L_r}e^{-\xi\omega_O t}\sin\omega_r t$$

or
$$i(t) = \frac{V}{\omega_O L_r}e^{-\alpha t}\sin\omega_r t$$

where, ω_o = undamped resonance frequency = $\frac{1}{\sqrt{L_r C_r}}$

damped frequency is $\omega_r = \sqrt{\frac{1}{L_r C_r} - \frac{R^2}{4L_r^2}}$

and
$$\xi\omega_o = \alpha = \frac{R}{2L_r} \quad \text{or,} \quad \xi = \frac{\alpha}{\omega_o} = \frac{R}{2\omega_o L_r} = \frac{R}{2}\sqrt{\frac{C_r}{L_r}}$$

The voltage across capacitor is

$$v_C(t) = V - (V_o - V)e^{-\alpha t}\left(\frac{\alpha}{\omega_r}\sin\omega_r t + \cos\omega_r t\right)$$

or
$$v_C(t) = V - (V_o - V)e^{-\alpha t}\left[\left\{1 + \left(\frac{\alpha}{\omega_r}\right)^2\right\}^{1/2}\cos(\omega_r t + \theta)\right]$$

where, $\theta = \tan^{-1}\left(\frac{\alpha}{\omega_r}\right), V_o =$ output voltage

The quality factor Q is equal to

$$Q = \frac{\omega_o L_r}{R} = \frac{1}{\omega_o C_r R} = \frac{Z_o}{R} \text{ where, } Z_o = \text{characteristic impedance} = \omega_o L_r = \frac{1}{\omega_o C_r}$$

The impedance of RLC circuit is

$$Z(s) = R + sL + \frac{1}{sC}$$

The admittance of RLC circuit is

$$Y(s) = \frac{1}{Z(s)} = \left[R + sL + \frac{1}{sC}\right]^{-1}$$

$$= \frac{1}{R}\frac{2\alpha s}{s^2 + 2\alpha s + \omega_O^2}$$

where, roots are $s_1 = -\alpha + j\omega_r$ and $s_2 = -\alpha - j\omega_r$ and $\omega_r = \sqrt{\omega_O^2 - \alpha^2} = \omega_O\sqrt{1-\xi^2}$

At resonance frequency, $s = j\omega_O$ and $Y(s) = \dfrac{1}{R}$

The impedance of the circuit is a function of frequency with Q. When R is constant, at resonance frequency $\omega = \omega_o$, the impedance becomes pure resistance R. At frequency $\omega_r > \omega_o$ and $\omega_r < \omega_o$, Z increases with ω as shown in Fig. 11.51.

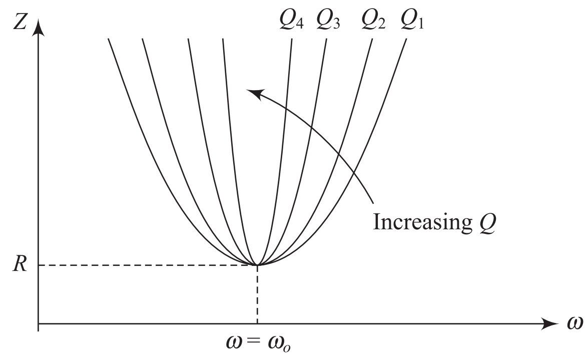


Fig. 11.51 *Impedance Z varies with ω*

The voltage gain of RLC series circuit is equal to

$$G(s) = \frac{V_o(s)}{V(s)} = \frac{R}{R + sL + \dfrac{1}{sC}}$$

or $$G(j\omega) = \frac{V_o(j\omega)}{V(j\omega)} = \frac{R}{R + j\omega L - j\dfrac{1}{\omega C}} = \frac{1}{1 + jQ\left[\dfrac{\omega}{\omega_O} - \dfrac{\omega_O}{\omega}\right]}$$

or $$|G(j\omega)| = \left[\frac{1}{1 + Q^2\left(u - \dfrac{1}{u}\right)}\right]^{1/2} \text{ where, } u = \frac{\omega}{\omega_O}$$

The variation of gain of an RLC series circuit is depicted in Fig. 11.52.

Figure 11.53 shows a RLC series resonant circuit with ac input voltage. When a square wave voltage signal is applied to Fig. 11.53 as input signal, the output voltage and current wave forms are depicted in Fig. 11.54.

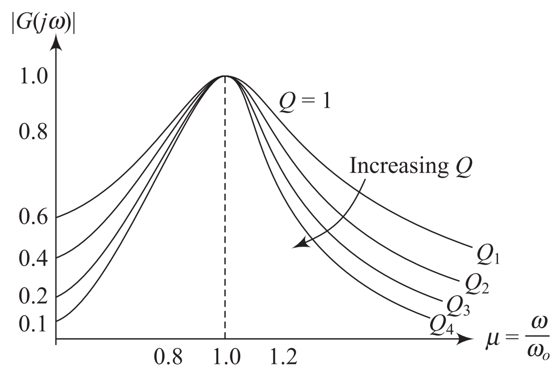


Fig. 11.52 *Gain variation of RLC series circuit*

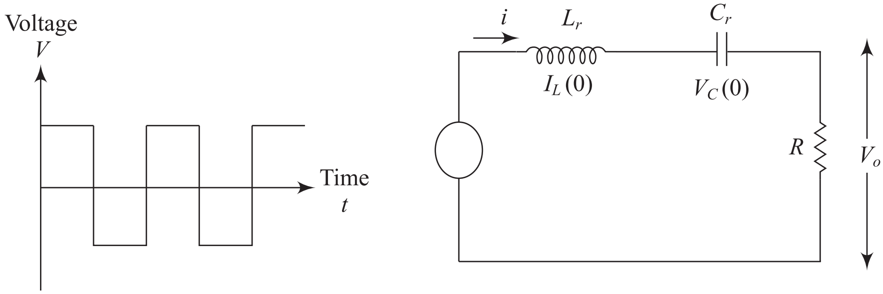


Fig. 11.53 *RLC series resonant circuit with square wave input voltage signal*

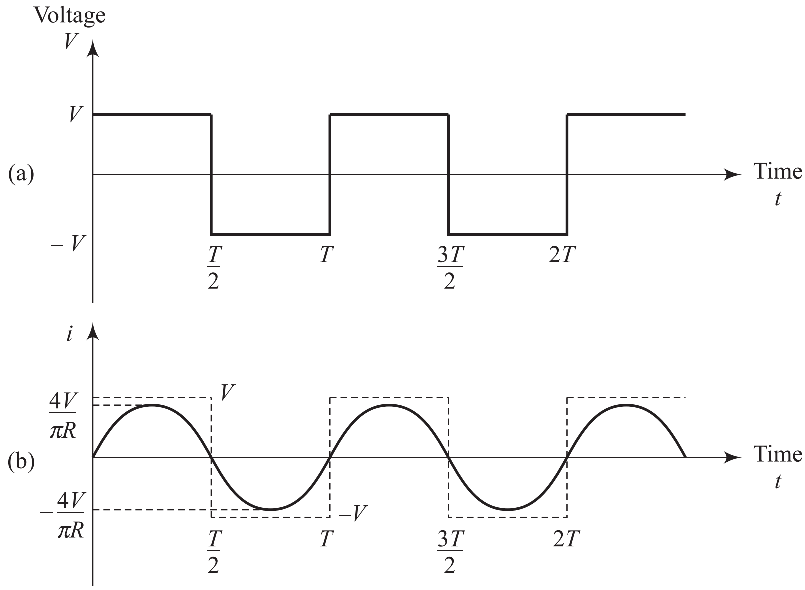


Fig. 11.54 *(a) Square wave input voltage and (b) Sinusoidal current signal*

11.8.3 Parallel Resonant Circuit

An undamped parallel resonant circuit is depicted in Fig. 11.55. This circuit is supplied by a dc current I. $i_L(0)$ is the initial current and $V_C(0)$ is the capacitor voltage.

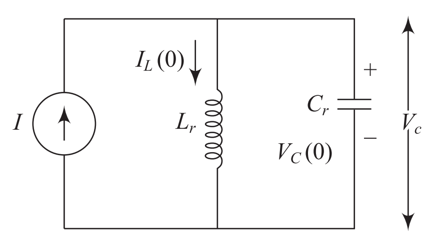


Fig. 11.55 *An undamped parallel resonant circuit*

The KCL equation of the circuit is

$$I = i_L + C_r \frac{dV_C}{dt} \text{ and}$$

$$L\frac{di_L}{dt} = V_C$$

The solution of the above equations are

$$i_L(t) = I + [I_L(0) - I]\cos\omega_0(t - t_0) + \frac{V_C(0)}{Z_O}\sin\omega_o(t - t_0) \quad \text{for } t > t_o$$

and the voltage across capacitor is

$$V_C(t) = Z_o[I - I_L(0)]\sin \omega_0(t - t_0) + V_C(0)\cos \omega_o(t - t_0)$$

where, resonant frequency $\omega_0 = \dfrac{1}{\sqrt{L_r C_r}}$ and the characteristic impedance $Z_o = \sqrt{\dfrac{L_r}{C_r}}$

11.8.4 Highly Underdamped Parallel Resonant Circuits

The *RLC* parallel resonant circuit is shown in Fig. 11.56 which is supplied by current *I*.

The quality factor $Q = \omega_o R C_r = \dfrac{R}{\omega_o L_r} = \dfrac{R}{Z_o}$

The amplitude of impedance Z is a function of frequency with Q as a parameter with R is constant:

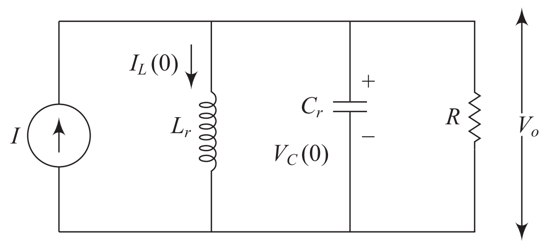


Fig. 11.56 *Parallel resonant circuit*

$$Z(s) = \frac{R \cdot \dfrac{s}{Q\omega_o}}{1 + \dfrac{s^2}{\omega_o^2} + \dfrac{s}{Q\omega_o}}$$

The gain of *RLC* parallel circuit is

$$G(j\omega) = \frac{\dfrac{R/j\omega C_r}{R + 1/j\omega C_r}}{j\omega L_r + \dfrac{R/j\omega C_r}{R + 1/j\omega C_r}} = \frac{1}{1 - \omega^2 L_r C_r + \dfrac{j\omega L_r}{R}} = \frac{1}{1 - \left(\dfrac{\omega}{\omega_0}\right)^2 + j\dfrac{\omega/\omega_o}{Q}}$$

or

$$|G(j\omega)| = \frac{1}{\sqrt{(1 - u^2)^2 + \left(\dfrac{u}{Q}\right)^2}}$$

$$|G(j\omega)|_{\max} = Q \quad \text{at } u = 1 \quad \text{or } \omega = \omega_O$$

The variation of impedance Z with respect to ω and the current and output voltage waveforms of a parallel resonant circuit are depicted in Fig. 11.57.

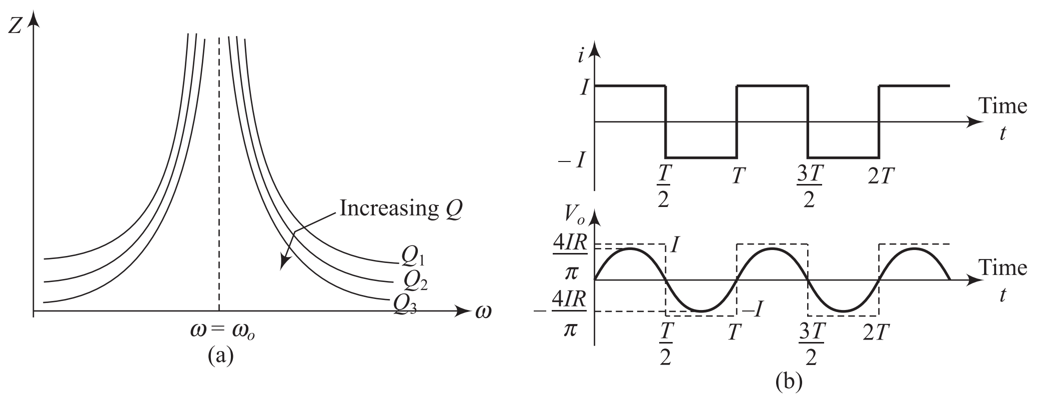


Fig. 11.57 *(a) Impedance Z varies with ω and (b) Current and output voltage waveforms*

11.8.5 Parallel Loaded Resonant LC Converters

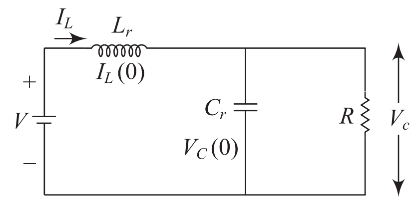


Fig. 11.58 Parallel loaded resonant converter

Figure 11.58 shows a parallel loaded resonant converter. The operating principle of parallel load resonant LC converter is similar to the series loaded resonant converters. The voltage across capacitor appears across the load.

The voltage gain of this circuit is

$$G(j\omega) = \frac{1}{1 - \omega^2 L_r C_r + j\dfrac{\omega L_r}{R}} \quad (11.2)$$

Resonant frequency is equal to $\omega_o = \dfrac{1}{\sqrt{L_r C_r}}$

and the quality factor $Q = \dfrac{R}{\omega_o L_r}$

After substituting the value of ω_o and Q in Eq. (11.2), we get

$$G(j\omega) = \frac{1}{\left[1 - \left(\dfrac{\omega}{\omega_o}\right)^2\right] + j\dfrac{\omega}{\omega_o}Q}$$

or $$|G(j\omega)| = \frac{1}{\left[(1-\mu^2)^2 + \left(\dfrac{\mu}{Q}\right)^2\right]^{1/2}} \quad \text{as } \mu = \frac{\omega}{\omega_o}$$

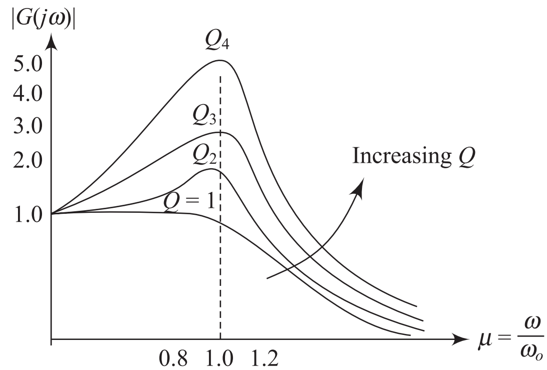


Fig. 11.59 Frequency response of $|G(j\omega)|$ for parallel loaded resonant converter

The plot of voltage gain $|G(j\omega)|$ with respect to μ is shown in Fig. 11.59 where $\mu = \dfrac{\omega}{\omega_o}$

11.9 CLASSIFICATION OF RESONANT CONVERTERS

There are different circuit topologies of resonant converters. The most commonly used resonant converters are given below.

1. Self-commuted or load commutated resonant converters
2. Zero voltage switching (ZVS)/Zero current switching (ZCS) resonant converters
3. DC link resonant converters
4. AC link resonant converters

In this section, the operating principle of self-commuted or load commutated resonant converters is discussed.

11.9.1 Self-commuted or Load Commutated Resonant Converters

In these converters, both series and parallel LC resonating circuits are used. The switching frequency ω_s is controlled about resonance frequency ω_o to control the input power. The series and parallel load resonating converters can also be classified as

1. Series/parallel resonant converters using unidirectional switches
2. Series/parallel resonant converters using bidirectional switches

11.9.2 Series Resonant Converters Using Unidirectional Switches

The load resistance R with resonating LC components develops a series RLC underdamped circuit as shown in Fig. 11.60. The series resonant circuit operates in three operating modes such as mode I, mode II and mode III. The triggering pulse of thyristors, voltage across capacitors and current waveforms at three different modes are depicted in Fig. 11.61.

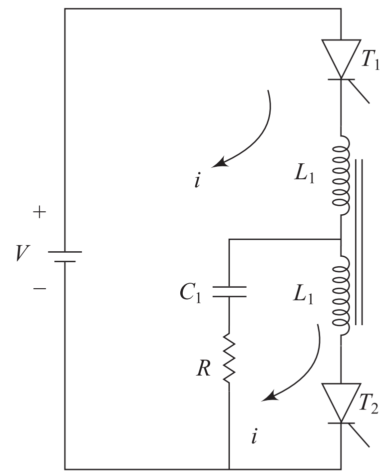


Fig. 11.60 *Series resonant inverter with $L_1 = L_2 = L$ and $C_1 = C$*

1. Mode I ($0 \le t \le t_1$) When the thyristor T_1 is fired and turned ON, a resonant current flows though T_1, L, C and R. The KVL equation of the circuit is

$$V = L\frac{di}{dt} + Ri + \frac{1}{C}\int i dt$$

or $$0 = L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i(t)}{C} \text{ as } V \text{ is constant and } \frac{dV}{dt} = 0$$

or $$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i(t)}{LC} = 0$$

The solution of the above equation is

$$i(t) = c_1 e^{(-a+b)t} + c_2 e^{(-a-b)t}$$

where $a = \dfrac{R}{2L}, b = \sqrt{\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}}$, c_1 and c_2 are constants

The resonant frequency is

$$\omega_r = \beta = jb = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Where, damping ratio $\xi = \dfrac{R}{2}\sqrt{\dfrac{C}{L}}$ and angular underdamped natural frequency $\omega_o = \dfrac{1}{\sqrt{LC}}$

Angular damped resonating frequency is

$$\omega_r = \beta = \omega_o\sqrt{1-\xi^2}, \text{ and } \alpha = a = \frac{R}{2L} = \frac{R}{2}\sqrt{\frac{C}{L}}\frac{1}{\sqrt{LC}} = \xi\omega_o$$

When $$i(0) = 0 \text{ and } V_C(0) = V, i(t) = \frac{V + V_C(0)}{\omega_r L} e^{-at} \sin \omega_r t \quad \text{for } 0 \le \omega_r t \le \pi$$

Resonant frequency $\omega_r = \beta = jb = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2}$

At the instant $t = t_m$, the current $i(t)$ is maximum and $\dfrac{di}{dt} = 0$

Therefore, $$\omega_r e^{-at_m} \cos \omega_r t_m - \alpha e^{-at_m} \sin \omega_r t_m = 0$$

or $$t_m = \frac{1}{\omega_r}\tan^{-1}\frac{\omega_r}{\alpha}$$

The voltage across capacitor is

$$V_C(t) = \frac{1}{C}\int_0^t i(t)dt - V_C(0) = -(V + V_C(0))e^{-at}\left(\frac{\alpha \sin \omega_r t + \omega_r \cos \omega_r t}{\omega_r}\right) + V$$

$$i(t = t_1) = 0, t_1 = \frac{\pi}{\omega_r}, V_C(t_1) = (V + V_C(0))e^{-\frac{\alpha\pi}{\omega_r}} + V$$

2. Mode II ($t_1 \le t \le t_2$**)** In this operating mode, both the thyristors T_1 and T_2 are OFF. The current flow through RLC circuit is $i(t) = 0$, and voltage across capacitor is $V_C = V_C(t_1) = V_C(t_2)$.

3. Mode III ($t_2 \le t \le t_3$**)** T_2 is ON and a reverse resonant current flows through the load. The KVL equation is

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i dt + V_C(t_2) = 0$$

Assume $i(t_2) = 0, V_C(t=0) = -V_C(t_2) = -V_C(t_1) = -V_C$

The solution of the equation is

$$i(t) = \frac{V_C(t_1)}{\omega_r L} e^{-\alpha t} \sin \omega_r t \qquad \text{for } t_2 \le t \le t_3$$

The capacitor voltage is

$$V_C(t) = \frac{1}{C}\int_0^t i(t)dt - V_C(t_1) \qquad \text{assuming } t_2 = 0 \text{ and } t = t_2 + t = t$$

$$= -V_C(t_1) \cdot e^{-\alpha t} \frac{\alpha \sin \omega_r t + \omega_r \cos \omega_r t}{\omega_r}$$

In this mode, current i flows during $t_2 \le t \le t_3$, and $i(t) = 0$ at $t = t_2$ and $t = t_3$.

Therefore, $$t_3 - t_2 = \frac{\pi}{\omega_r} = t$$

At steady state condition, $V_C(t_3) = V_C(t_1) e^{-\frac{\alpha\pi}{\omega_r}}$

For proper operation of series inverter, current $i(t = t_1)$ must be zero and thyristor T_1 must be turned OFF before applying the trigger pulse to thyristor T_2. If both thyristors T_1 and T_2 are turned ON at a time, the dc supply will be short circuit through thyristors. Therefore, a blanking period is required for thyristors to avoid short circuit. This blanking period ($t_1 \le t \le t_2$) is called *dead zone*. The dead zone must be greater than the turn-OFF time of thyristors, t_q.

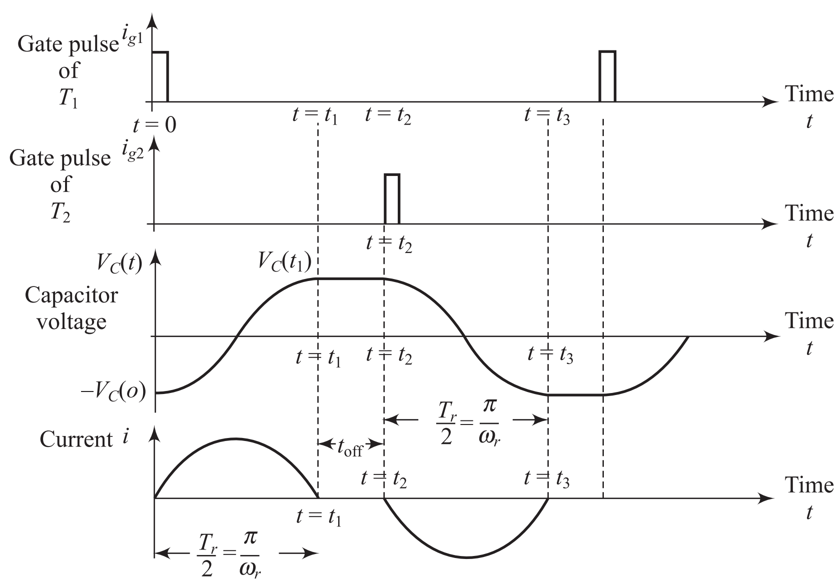


Fig. 11.61 *Triggering pulse of thyristors, voltage across capacitors and current waveforms of series resonant inverter*

The dead zone is equal to $t_{\text{off}} = \dfrac{\pi}{\omega_o} - \dfrac{\pi}{\omega_r}$ and $t_{\text{off}} > t_q$

where, t_q is turn-OFF time and ω_O is the frequency of output voltage.

The maximum possible output frequency is $f_{\max} = \dfrac{1}{T_r + 2t_q} = \dfrac{1}{2\left(t_q + \dfrac{\pi}{\omega_r}\right)}$ where, $T_r = \dfrac{2\pi}{\omega_r}$.

Example 11.21 A series resonant *RLLC* inverter using thyristors has the following parameters: $R = 1\,\Omega, L_r = 0.1\,\text{mH}, C_r = 10\,\mu\text{F}$ and $t_q = 10\,\mu\text{s}$

Determine the maximum switching frequency for non-overlap operation of the series resonant inverter.

Solution

Given: $R = 1\,\Omega, L_r = 0.1\,\text{mH}, C_r = 10\,\mu\text{F}$ and $t_q = 10\,\mu\text{s}$

The resonant frequency is

$$\omega_r = \sqrt{\frac{1}{L_r C_r} - \frac{R^2}{4L_r^2}} = \sqrt{\frac{1}{0.1\times 10^{-3}\times 10\times 10^{-6}} - \frac{1^2}{4\times(0.1\times 10^{-3})^2}} = 31.23k - \text{rad/s}$$

As $\omega_r = 2\pi f_r,\ f_r = \dfrac{\omega_r}{2\pi} = \dfrac{31.23\times 10^3}{2\pi}\,\text{Hz} = 4.968\text{ kHz}$ and $T_r = \dfrac{2\pi}{\omega_r} = \dfrac{2\pi}{31.23\times 10^3} = 0.2012\text{ ms}$

The maximum switching frequency is

$$f_{\max} = \frac{1}{T_r + 2t_q} = \frac{1}{0.2012\times 10^{-3} + 2\times 10\times 10^{-6}}\,\text{Hz} = 4.5207\text{ kHz}$$

Example 11.22 A single-phase series resonant *RLC* inverter delivers power to load with $R = 2\ \Omega$ and $X_L = 10\ \Omega$. If the time period is 0.1 ms, find the value of C so that load commutation of thyristor is possible. Assume thyristor turn-OFF time is 15 µs.

Solution

Given: $R = 2\ \Omega$, $X_L = 10\ \Omega$, $t_q = 15\ \mu\text{s}$

The time period is equal to

$$\frac{T_r}{2} + t_q = 0.1\text{ ms} = 100\ \mu\text{s}$$

or $$T_r = 200\ \mu\text{s} - 2t_q = 200\ \mu\text{s} - 2\times 15\ \mu\text{s} = 170\ \mu\text{s}$$

We know that $T_r = \dfrac{2\pi}{\omega_r}$. Therefore, $\omega_r = \dfrac{2\pi}{T_r} = \dfrac{2\pi}{170\times 10^{-6}} = 36.974k - \text{rad/s}$

As $$X_L = \omega_r L = 10,\ L = \frac{X_L}{\omega_r} = \frac{10}{36.974\times 10^3} = 0.2704\text{ mH}$$

The resonant frequency is

$$\omega_r = \sqrt{\frac{1}{L_r C_r} - \frac{R^2}{4L_r^2}}$$

or $$\omega_r^2 = \frac{1}{L_r C_r} - \frac{R^2}{4L_r^2}$$

or
$$C_r = \frac{1}{L_r\left(\omega_r^2 + \frac{R^2}{4L^2}\right)}$$
$$= \frac{1}{0.2704\times10^{-3}\left(36974^2 + \frac{2^2}{4(0.2704\times10^{-3})^2}\right)} = 2.678\ \mu\text{F}$$

Example 11.23 Determine the value of inductance of a series resonant *RLC* inverter when it operates at frequency 10 kHz and its capacitor value is 1 μF. Assume inverter operates at angular undamped natural frequency ω_o.

Solution

Given: f_r = 10 kHz, C = 1 μF

Angular undamped natural frequency $\omega_o = 2\pi f = \frac{1}{\sqrt{LC}}$

or
$$4\pi^2 f^2 = \frac{1}{LC}$$

Therefore,
$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(10\times10^3)^2\times1\times10^{-6}} = 0.2535\ \text{mH}$$

Example 11.24 A series resonant *RLC* inverter using thyristors has the following parameters:

$R = 100\ \Omega$, $L_r = 6.0$ mH, $C_r = 1.2$ μF and $t_{\text{off}} = 0.20$ ms

(a) Find the value of output frequency.
(b) If the load resistance varies from 40 Ω to 140 Ω, determine the range of output frequency for non-overlap operation of the series resonant inverter.

Solution

Given: $R = 100\ \Omega$, $L_r = 6.0$ mH, $C_r = 1.2$ μF and $t_{\text{off}} = 0.20$ ms

(a) The resonant frequency is
$$\omega_r = \sqrt{\frac{1}{L_rC_r} - \frac{R^2}{4L_r^2}} = \sqrt{\frac{1}{6.0\times10^{-3}\times1.2\times10^{-6}} - \frac{100^2}{4\times(6.0\times10^{-3})^2}} = 8.3333\ \text{k-rad/s}$$

As
$$\omega_r = 2\pi f_r,\ f_r = \frac{\omega_r}{2\pi} = \frac{8.3333\times10^3}{2\pi}\ \text{Hz} = 1.3257\ \text{kHz}$$

and
$$T_r = \frac{2\pi}{\omega_r} = \frac{2\pi}{8.3333\times10^3} = 0.7542\ \text{ms}$$

The output frequency is
$$f = \frac{1}{T_r + 2t_q} = \frac{1}{0.7542\times10^{-3} + 2\times0.2\times10^{-3}}\ \text{Hz} = 866.40\ \text{kHz}$$

(b) If R = 40 Ω,
$$\omega_r = \sqrt{\frac{1}{L_rC_r} - \frac{R^2}{4L_r^2}} = \sqrt{\frac{1}{6.0\times10^{-3}\times1.2\times10^{-6}} - \frac{40^2}{4\times(6.0\times10^{-3})^2}} = 11.3038\ \text{k-rad/s}$$
$$T_r = \frac{2\pi}{\omega_r} = \frac{2\pi}{11.3038\times10^3} = 0.556\ \text{ms}$$

The output frequency is

$$f = \frac{1}{T_r + 2t_q} = \frac{1}{0.556 \times 10^{-3} + 2 \times 0.2 \times 10^{-3}} \text{Hz} = 1046 \text{ Hz}$$

If R = 140 Ω,

$$\omega_r = \sqrt{\frac{1}{L_r C_r} - \frac{R^2}{4L_r^2}} = \sqrt{\frac{1}{6.0 \times 10^{-3} \times 1.2 \times 10^{-6}} - \frac{140^2}{4 \times (6.0 \times 10^{-3})^2}} = 1.6666 \text{ k-rad/s}$$

$$T_r = \frac{2\pi}{\omega_r} = \frac{2\pi}{1.6666 \times 10^3} = 3.771 \text{ ms}$$

The output frequency is

$$f = \frac{1}{T_r + 2t_q} = \frac{1}{3.771 \times 10^{-3} + 2 \times 0.2 \times 10^{-3}} \text{Hz} = 239.75 \text{ Hz}$$

Therefore, the range of output frequency is $239.75 \text{ Hz} < f < 1046 \text{ Hz}$

11.10 VOLTAGE CONTROL OF INVERTERS

Depending on the nature of load, variable ac voltage or variable voltage with variable frequency are required at the input terminals of load. When the output voltage of inverter is applied to load, the output voltage of inverter can be controlled to get desired output from the system. There are different methods to control the output voltage of inverter. The most commonly used control methods are:

1. External control of dc input voltage of inverter
2. External control of ac output voltage of inverter
3. Internal control of inverter

11.10.1 External Control of dc Input Voltage of Inverter

By controlling the dc input voltage of inverter, the output voltage inverter can be controlled. The different schemes of external control of dc input voltage of inverter is depicted in Fig. 11.62(a), (b), (c) and (d). In Fig. 11.62(a), the fixed dc voltage is applied to a dc-to-dc converter or chopper to obtain variable dc voltage. When the variable dc voltage is applied to an inverter through filter, the controllable ac output voltage can be obtained from inverter.

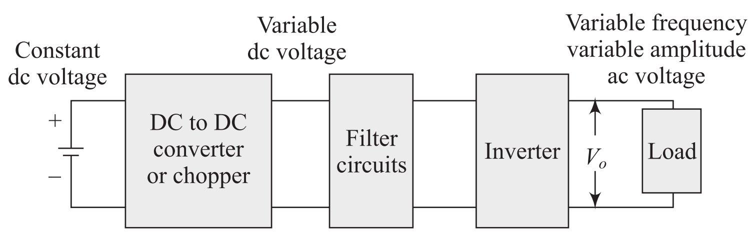


Fig. 11.62 *(a) Input voltage control of using dc-to-dc converter*

In Fig. 11.62(b), the ac input voltage is initially converted into a variable ac voltage by using ac voltage controller and subsequently it converts into dc using an uncontrolled rectifier. In this scheme, the variable voltage and variable frequency ac output is obtained just after three conversion stages. Consequently, the efficiency of system is poor and the input power factor is poor at low voltages.

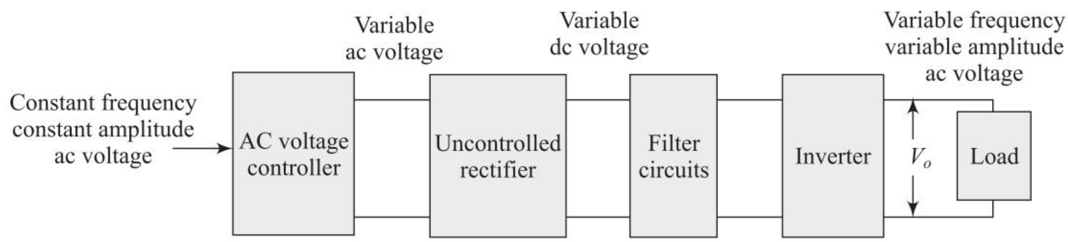


Fig. 11.62 *(b) Input voltage control of Inverter using ac voltage controller and uncontrolled rectifier*

In Fig. 11.62(c), the variable dc voltage is obtained from a controlled rectifier and then this variable dc voltage is applied to inverter to get variable ac output voltage from inverter. In this scheme, two conversion stages are required and the efficiency of the system is better than previous scheme. For low output voltage, the input power factor is low. Since the output voltage contains low frequency harmonics, the size of filter is bulky and the system response will be sluggish.

Similarly in Fig. 11.62(d), ac voltage is applied to uncontrolled rectifier to get fixed dc voltage which is applied to chopper to obtain variable dc voltage. As the chopper operates at very high frequency, the constant dc voltage is converted into a variable dc voltage at high frequency. Due to very high frequency, the size of filter is reduced significantly. The fundamental power factor remains unity for all operating conditions but the system loss increases due to an extra converter.

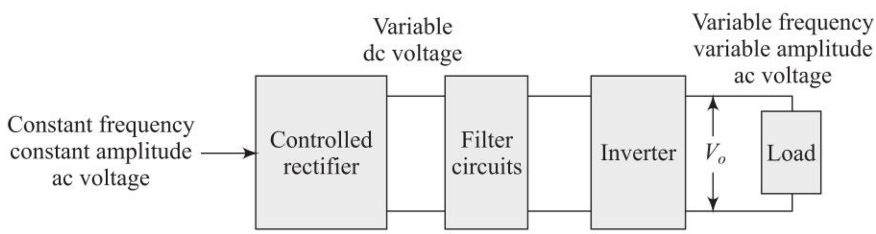


Fig. 11.62 *(c) Input voltage control of inverter using controlled rectifier*

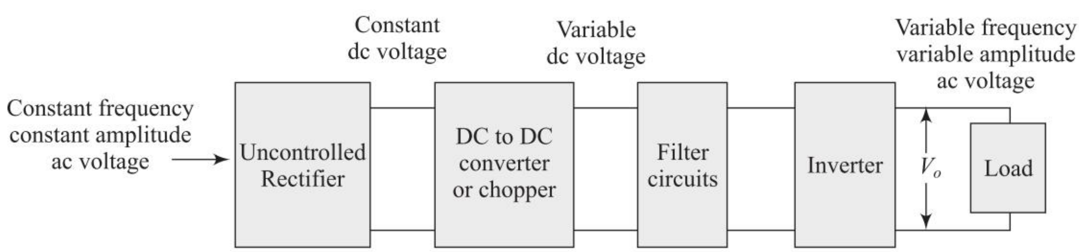


Fig. 11.62 *(d) Input voltage control of Inverter using uncontrolled rectifier and dc to dc converter*

11.10.2 External Control of ac Output Voltage of Inverter

The external control of ac output voltage of inverter can be possible by the following methods:

1. AC voltage controller
2. Series-connected inverters

1. AC voltage controller Figure 11.63 shows the external control of ac output voltage of inverter using ac voltage controller. In this method, an ac voltage controller is incorporated in between the output voltage of inverter and the load terminals. When the firing angle of thyristors of ac voltage controller is varied, the variable ac voltage will be applied to load terminals. Since the harmonics content of output voltage of ac voltage controller is high, this method of voltage control is not widely used and it is suitable for low power applications only.

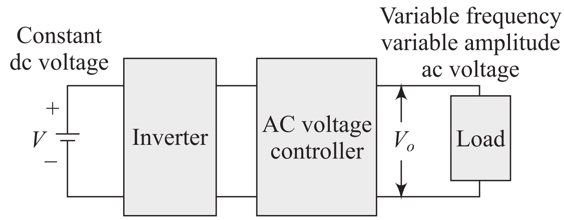


Fig. 11.63 *External control of ac output voltage of inverter using ac voltage controller*

2. Series-connected inverters In series connected inverters method of voltage control, two or more inverters are connected in series. Figure 11.64 shows that the two square wave inverters are connected in series to get a variable ac output voltage. The output voltage of inverters I and II are applied to the primary windings of two transformers but the secondary windings of transformers are connected in series. The output voltage of transformers V_{o1} and V_{o2} have same magnitude but they have a phase difference ϕ. The phasor sum of two fundamental voltages V_{o1} and V_{o2} provides the resultant fundamental voltage

$$V_o = \sqrt{V_{01}^2 + V_{02}^2 + 2 \cdot V_{01} \cdot V_{02} \cos \phi}$$

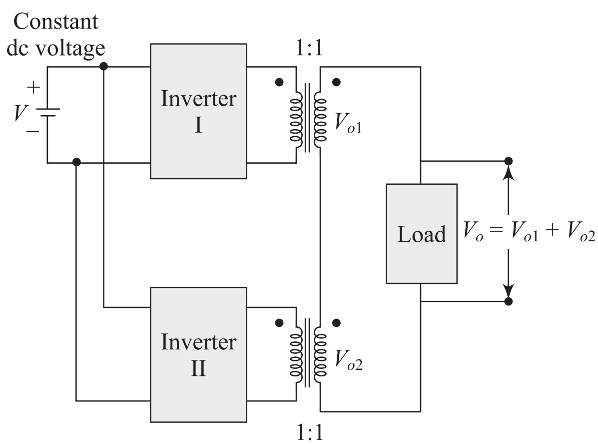


Fig. 11.64 *Series connection of two inverters and its output voltage $V_o = V_{o1} + V_{o2}$*

Figure 11.65 shows the output voltage waveform of series connected inverters. Since the frequency of output voltages V_{o1} and V_{o2} is same, if the phase difference ϕ is zero, the output voltage is $V_o = V_{o1} + V_{o2}$. When the phase difference ϕ is π, the output voltage is $V_o = V_{o1} - V_{o2} = 0$. As the phase difference can be varied by changing the firing angle of two inverters, the output voltage can be controlled. The voltage control using series connected inverters is also called *multiple converter control*. Since the harmonics content in the output voltage is large, this method of voltage control is used for low output voltage levels, i.e., 25% to 30% of the rated voltage.

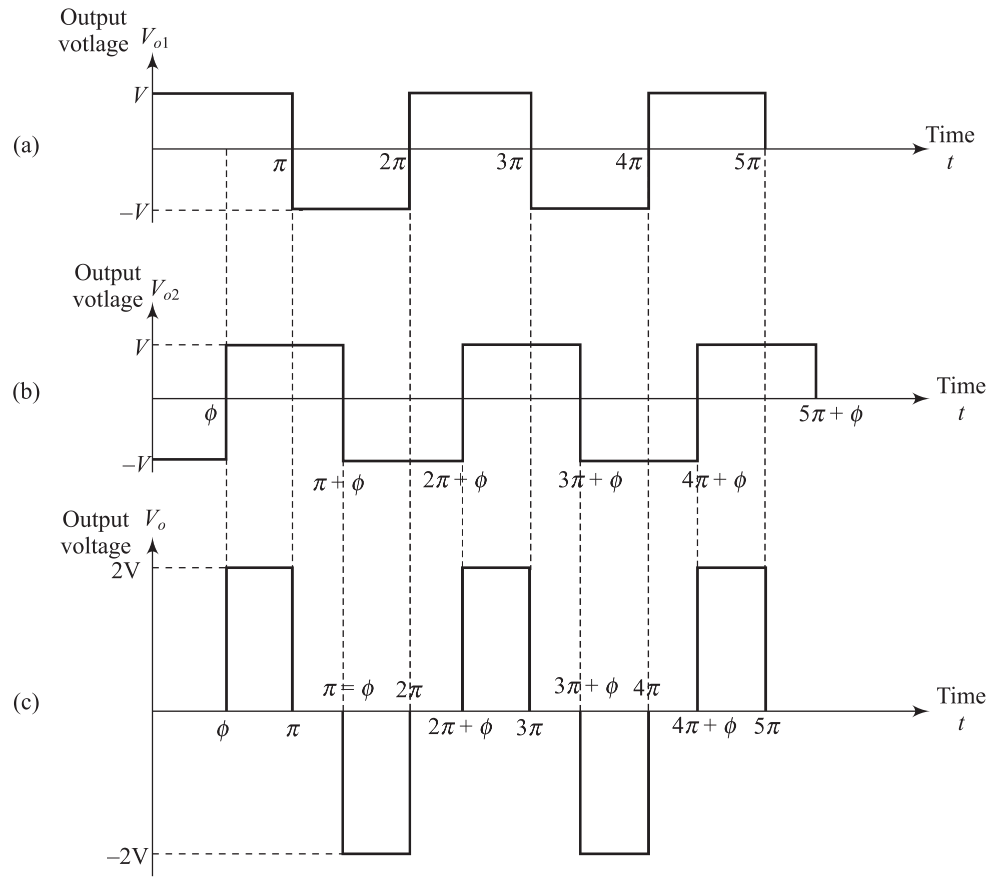


Fig. 11.65 *Output voltage waveforms of series connected inverters (a) V_{o1} (b) V_{o2} (c) V_o*

11.10.3 Internal Control of Inverter

In this method, the voltage control of inverter is possible within the inverter. The most efficient method of internal control of inverter is pulse-width modulation control.

Pulse-width modulation control In pulse-width modulation control, a fixed dc voltage is applied to the inverter and a variable ac output voltage can be obtained by controlling the width of output pulses. In this method, the variable ac output voltage will be available with out any additional components and the lower order harmonics can be eliminated. Since the higher order harmonics can filter very easily, the filtering circuit requirement should be minimized. Therefore, pulse-width modulation (PWM) inverters are very popular in industrial applications.

11.11 HARMONIC REDUCTION IN OUTPUT VOLTAGE OF INVERTER

In most of the industrial applications, the fundamental component of output voltage of inverter is only useful. But due to presence of harmonics in inverter output voltages, the system performance becomes sluggish. Therefore, the harmonics content must be limited with in 5% of its fundamental component of output voltage. If the harmonic content is more than 5%, filter circuit should be designed as per requirement and must be inserted in between inverter and load. When the inverter contains high frequency harmonics, the size will be reduced and cost of filter will be less. If low frequency harmonics are present within the output voltage of inverter, to eliminate or attenuate low frequency harmonics, filter size will be bulky and subsequently the cost of filter will be high. Therefore, lower order harmonics must be eliminated from the inverter output voltage by applying the following techniques:

1. Harmonic Reduction Using PWM Technique
2. Harmonic Reduction by Series Connected Inverters
3. Harmonic Reduction by Stepped Wave Inverters

11.11.1 Harmonic Reduction Using PWM Technique

The lower order harmonics in output voltage of inverter can be eliminated when there are several pulses per half cycle in output voltage waveform. Figure 11.66 shows an output voltage waveform of a single phase inverter. It is clear from Fig. 11.66 that ten commutation of thyristors per cycle are required and the amplitude of voltage varies in between $+V$ and $-V$. The output voltage waveform is symmetrical at $\frac{\pi}{2}$ and π.

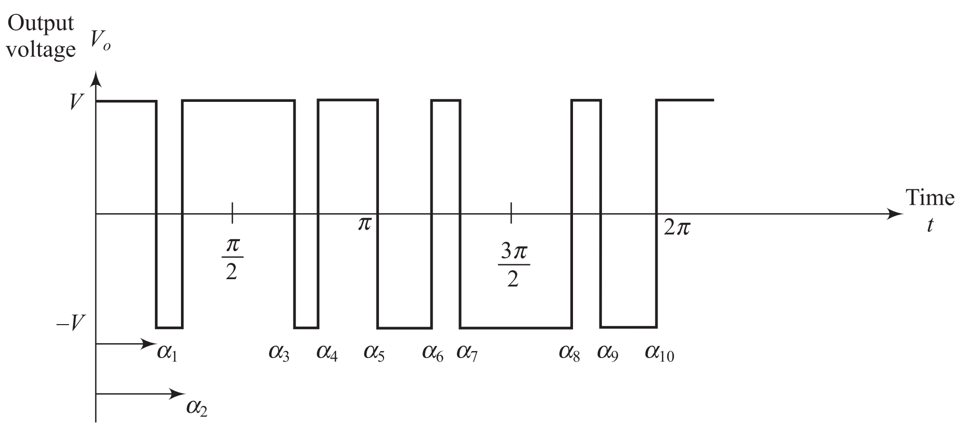


Fig. 11.66 *Output voltage waveform of a single phase inverter using PWM technique*

After Fourier analysis of output voltage waveform, we can represent the output voltage by the Fourier series

$$v_o = \sum_{n=1,3,5,7...}^{\infty} b_n \sin n\omega t$$

where $$b_n = \frac{4V}{\pi}\left[\int_0^{\alpha_1} \sin n\omega t \cdot d\omega t - \int_{\alpha_1}^{\alpha_2} \sin n\omega t \cdot d\omega t + \int_{\alpha_2}^{\pi/2} \sin n\omega t \cdot d\omega t\right]$$

or $$b_n = \frac{4V}{n\pi}[1 - 2\cos n\alpha_1 + 2\cos n\alpha_2]$$

To eliminate the third and fifth harmonics from the output voltage, $b_3 = 0$ and $b_5 = 0$.

Therefore, $$b_3 = \frac{4V}{3\pi}[1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2] = 0 \text{ and}$$

$$b_5 = \frac{4V}{5\pi}[1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2] = 0$$

As $V \neq 0$, $[1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2] = 0$ and

$$[1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2] = 0 \quad \text{where, } 0 < \alpha_1 < \frac{\pi}{2} \text{ and } \alpha_1 < \alpha_2 < \frac{\pi}{2}$$

After solving the above equations, we find α_1 = 23.62° (approx) and α_2 = 33.30° (approx).

When α_1 = 23.62° and α_2 = 33.30°, the amplitude of 7th, 9th, 11th, and 13th harmonics voltages are as follows:

$$b_7 = \frac{4V}{7\pi}[1 - 2\cos 7\alpha_1 + 2\cos 7\alpha_2]$$

$$= \frac{4V}{7\pi}[1 - 2\cos(7 \times 23.62) + 2\cos(7 \times 33.30)] = 0.3154 \text{ V}$$

$$b_9 = \frac{4V}{9\pi}[1 - 2\cos 9\alpha_1 + 2\cos 9\alpha_2]$$

$$= \frac{4V}{9\pi}[1 - 2\cos(9 \times 23.62) + 2\cos(9 \times 33.30)] = 0.5198 \text{ V}$$

$$b_{11} = \frac{4V}{11\pi}[1 - 2\cos 11\alpha_1 + 2\cos 11\alpha_2]$$

$$= \frac{4V}{11\pi}[1 - 2\cos(11 \times 23.62) + 2\cos(11 \times 33.30)] = 0.3866 \text{ V}$$

$$b_{13} = \frac{4V}{13\pi}[1 - 2\cos 13\alpha_1 + 2\cos 13\alpha_2]$$

$$= \frac{4V}{13\pi}[1 - 2\cos(13 \times 23.62) + 2\cos(13 \times 33.30)] = 0.0374 \text{ V}$$

The amplitude of fundamental component is

$$b_1 = \frac{4V}{\pi}[1 - 2\cos\alpha_1 + 2\cos\alpha_2] = \frac{4V}{\pi}[1 - 2\cos(23.62) + 2\cos(33.30)] = 1.068 \text{ V}$$

11.11.2 Harmonic Reduction by Series Connected Inverters

Figure 11.67(a) shows the series connection of two inverters and the output voltage waveform is depicted in Fig. 11.67(b). By using Fourier analysis, the output voltage V_{o1} can be expressed as

$$V_{o1} = \frac{4V}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \frac{1}{7}\sin 7\omega t + \frac{1}{9}\sin 9\omega t + \cdots\right]$$

Since the output voltage V_{o2} is phase shifted by $\phi = \frac{\pi}{3}$, V_{o2} can be expressed as

$$V_{o2} = \frac{4V}{\pi}\left[\sin\left(\omega t - \frac{\pi}{3}\right) + \frac{1}{3}\sin 3\left(\omega t - \frac{\pi}{3}\right) + \frac{1}{5}\sin 5\left(\omega t - \frac{\pi}{3}\right)\right.$$
$$\left. + \frac{1}{7}\sin 7\left(\omega t - \frac{\pi}{3}\right) + \frac{1}{9}\sin 9\left(\omega t - \frac{\pi}{3}\right) + \cdots\right]$$

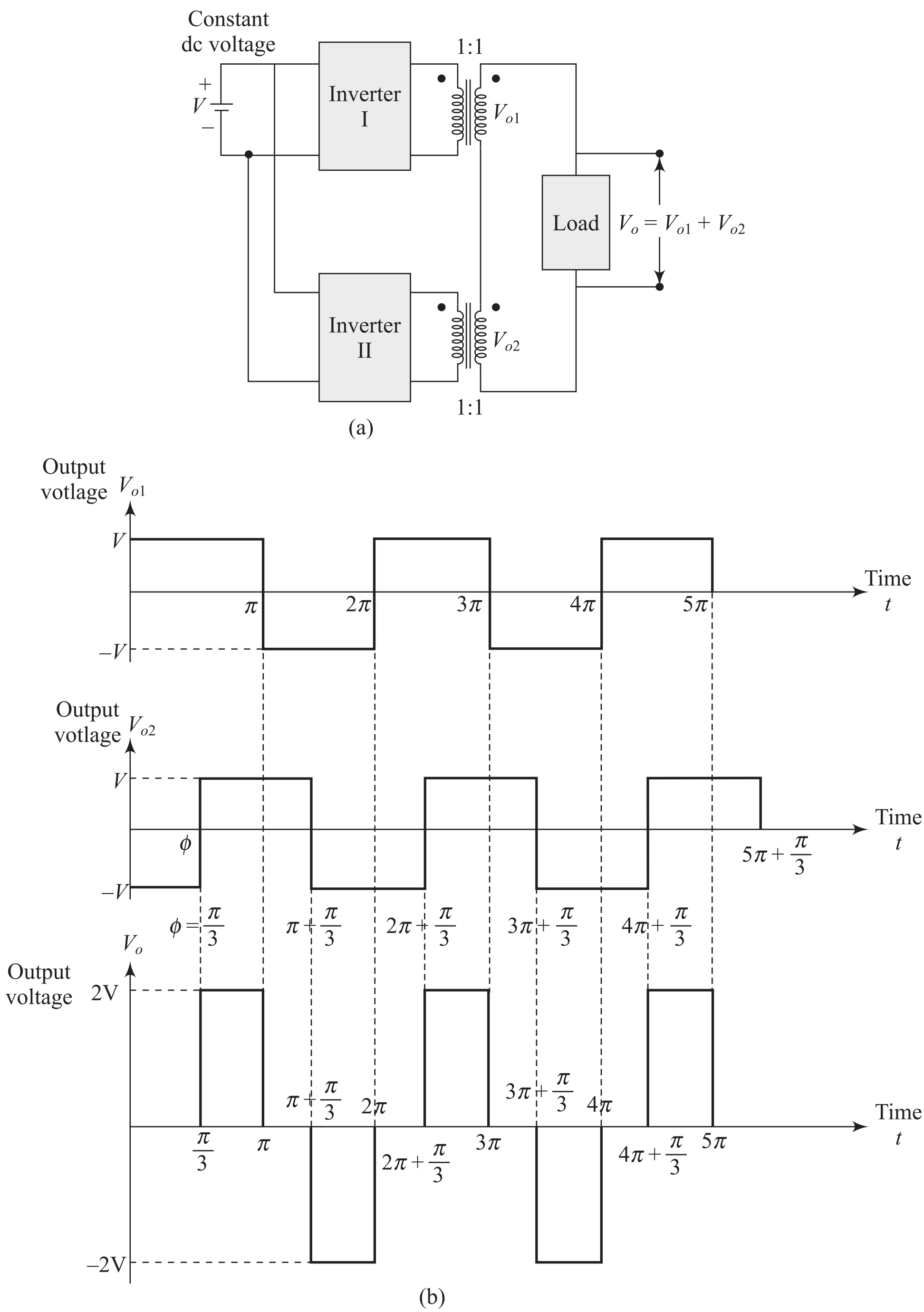


Fig. 11.67 *(a) Harmonic reduction using series connection of inverters and (b) Output voltage waveforms to eliminate third harmonics*

The output voltage is equal to

$$V_o = V_{o1} + V_{o2}$$
$$= \frac{4\sqrt{3}V}{\pi}\left[\sin\left(\omega t - \frac{\pi}{6}\right) + \frac{1}{5}\sin\left(5\omega t - \frac{\pi}{6}\right) + \frac{1}{7}\sin\left(7\omega t - \frac{\pi}{6}\right) + \cdots\right]$$

The summation of the first, third, fifth and seventh harmonics components of voltage are depicted in Fig. 11.68. The fundamental component of output voltage is $\sqrt{3}$ times of V_{o1} and it lags by 30° from V_{o1}. In Fig. 11.68(b) the third harmonic voltage of V_{o2} lags by 180° from the third harmonic voltage of V_{o1}. Hence, the sum of third harmonic voltages is equal to zero. It is clear from Fig. 11.68 (c) that the fifth harmonic component of output voltage is $\sqrt{3}$ times of $V_{o1(5)}$ and it leads by 30° from $V_{o1(5)}$. Similarly the seventh harmonic component of output voltage is $\sqrt{3}$ times of $V_{o1(7)}$ and it lags by 30° from $V_{o1(7)}$. It is clear from above discussion that when $\phi = \frac{\pi}{3}$, the third harmonic component can be eliminated from output voltage.

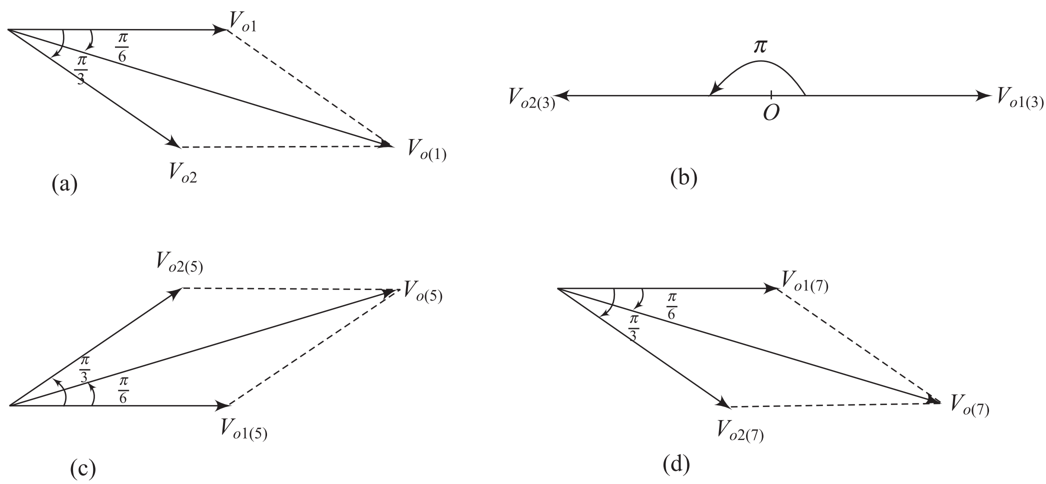


Fig. 11.68 *Addition of fundamental, third, fifth and seventh harmonics voltages*

11.11.3 Harmonic Reduction by Stepped Wave Inverters

Figure 11.69 shows the series connection of two inverters using transformers with different turn's ratio. Assume the turn ratio of transformer-I is 1: 3 and this transformer is connected to Inverter –I. The turn ratio of transformer II is 1: 1 and it is connected to inverter II. The output voltages of inverters I and II are depicted in Fig. 11.69. Due to different turn ratio of transformer, the output

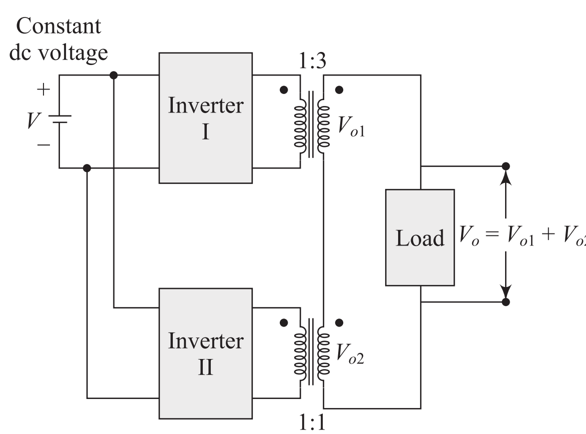


Fig. 11.69 *Harmonic reduction by step wave inverters*

voltage at the secondary winding of transformers will be different. After addition of output voltage of transformers I and II, we get stepped wave output voltage as depicted in Fig. 11.70. After Fourier analysis of Fig. 11.70, we find that the amplitude of harmonics voltages depends on the values of d_1, d_2, d_3, d_4 and V. If we choose the width of d_1, d_2, d_3 and d_4 properly, third, fifth and seventh harmonics can be eliminated or attenuated significantly from output voltage. Therefore, when pulses of different widths and amplitudes are superimposed, a stepped wave output voltage can be obtained with reduced harmonic content.

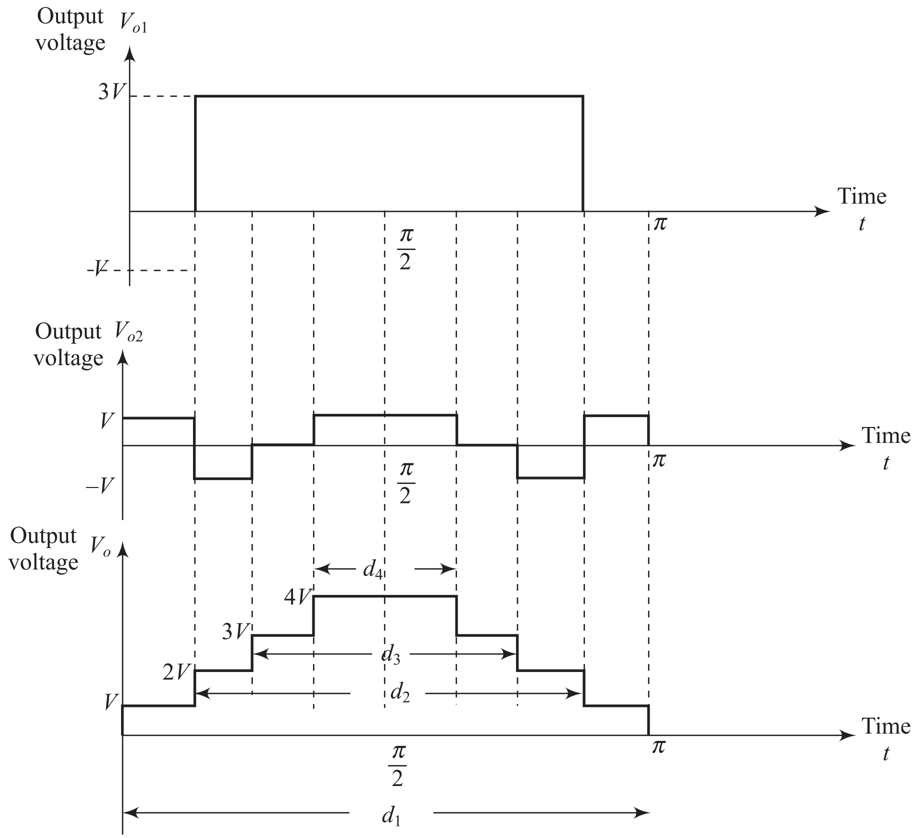


Fig. 11.70 *The output voltage waveforms of stepped wave inverter*

11.12 McMURRAY HALF-BRIDGE INVERTER (AUXILIARY IMPULSE COMMUTED INVERTER)

Figure 11.71 shows the circuit diagram of a McMurray half-bridge Inverter which consists of two main thyristors T_1 and T_2, two main diodes D_1 and D_2 and two auxiliary thyristors T_{A1} and T_{A2}. In this circuit, the centre tapped dc supply is used as depicted in Fig. 11.71. As auxiliary thyristors T_{A1} and T_{A2} are used for commutation process of main thyristor T_1 and T_2 respectively, this circuit is also known as *auxiliary thyristor commuted inverter* or *auxiliary impulse commuted inverter*. The commutating elements of this inverter circuit are an inductance-capacitance (LC) circuit and two auxiliary thyristors T_{A1} and

T_{A2}. The main thyristors T_1 and T_2 are gated to conduct current to the load during alternate half cycles. When the load is inductive (R-L), the feedback diodes D_1 and D_2 conduct during part of each half cycle to return power from the load to the dc supply. The commutation of main thyristors T_1 and T_2 is done by auxiliary thyristors T_{A1} and T_{A2} in conjunction with the capacitor C and the inductor L. To commutate thyristor T_1, auxiliary thyristors T_{A1} is fired. For commutation of thyristor T_2, auxiliary thyristors T_{A2} is fired.

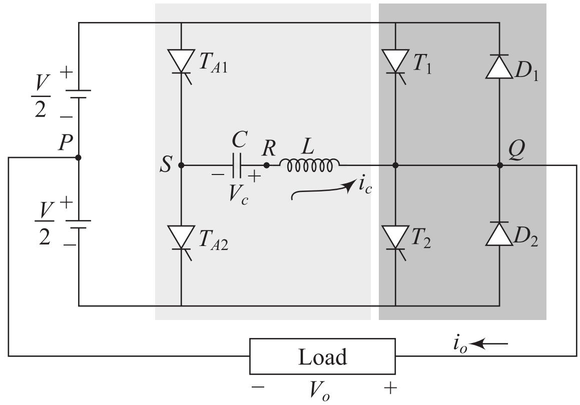


Fig. 11.71 *McMurray half-bridge inverter*

11.12.1 Operating Principle

Assume that thyristor T_1 is conducting and the load current I_o flows through T_1 and load. Initially the capacitor C is charged to a voltage which more than V. The polarity of capacitor voltage is shown in Fig. 11.71. Assume that all other devices are in OFF-state.

To turn OFF (commutate) thyristor T_1, auxiliary thyristor T_{A1} is turned ON by applying trigger pulse. As soon as T_{A1} starts to conduct, the capacitor C starts to discharge. The equivalent circuit of the inverter at this instant is depicted in Fig. 11.72(a) The path of current flow is C^+ - L - T_1 - T_{A1} - C^-. The discharge current pulse through T_{A1}, C and L builds up to exceed the load current I_o assumed to flow from Q to P through load at this instant.

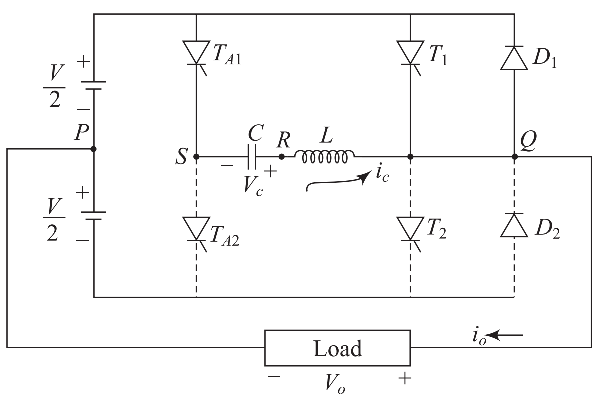


Fig. 11.72 *(a) The equivalent circuit of the inverter during first part of commutating interval*

When the capacitors discharge current I_C is equal to the load current I_o, the net current flow through thyristor T_1 becomes zero. If the commutating impulse current I_C is greater than the load current I_o, the net current flow through thyristor T_1 is zero, I_o current flows through load and the excess current flows through feedback diode D_1. Due to the forward threshold voltage of D_1, thyristor T_1 gets turned OFF. The capacitor continues to discharge through diode D_1 and load current flows continuously. After certain time interval, the discharging capacitor C again starts to recharge in the reverse polarity and it behaves oscillatory in nature due to inductance L.

The capacitor current reaches peak value whenever the voltage across capacitor becomes zero. After reaching the peak value, the capacitor current I_C starts to decay and a charge of reversed polarity builds up on capacitor C.

At the instant I_C is equal to I_o after the peak value attainment, thyristor T_2 must be triggered. To control the capacitor voltage, the delay of triggering thyristor T_2 with respect to T_{A1} is maintained. Whenever thyristor T_2 is triggered and turned ON, the discharge current flows through it instead of flowing through diode D_1. Therefore the load current I_o should be maintained by load inductance and charging of capacitor C. A much smaller pulse of current i_C will flows from dc supply through the path T_{A1}- C- L and T_2 to make up the losses incurred during first pulse and complete the charge of the capacitor to the initial value V but polarity of capacitor will be reversed. When the second pulse is applied to T_{A1}, it is reverse biased and ceases to conduct. Then the capacitor C is ready to commutate thyristor T_2 at the end of its conducting half cycle. Figure 11.72(b) shows the equivalent circuit of the inverter during second part of commutating interval.

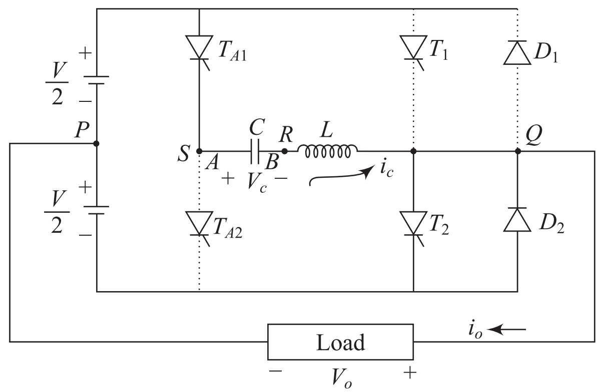


Fig. 11.72 *(b) The equivalent circuit of the inverter during second part of commutating interval*

After completion of charging of capacitor C (reverse polarity), plate A is positive and plate B is negative, the net current flows through the thyristor T_2 reduces to a zero value and the excess current ($I_C - I_o$) flows through diode D_2. For the forward threshold voltage of D_2, thyristor T_2 will be turned OFF. Since the load inductance L feeds the energy back to lower half of the source. Due to inductance, current flow through diode D_2 cannot be increased instantaneously from zero to a full load current. As load current Io flows, current though T_{A1} reduces to zero and the thyristor T_{A1} is turned OFF.

The diode D_2 continues to conduct until the load current I_o becomes zero and thyristor T_2 can be conducted if the gate signal is present and reverse current flows. When the auxiliary thyristor T_{A2} is turned ON, thyristor T_2 will be turned OFF and the cycle will be repeated. The voltage and current waveforms of McMurray half-bridge inverter are depicted in Fig. 11.73.

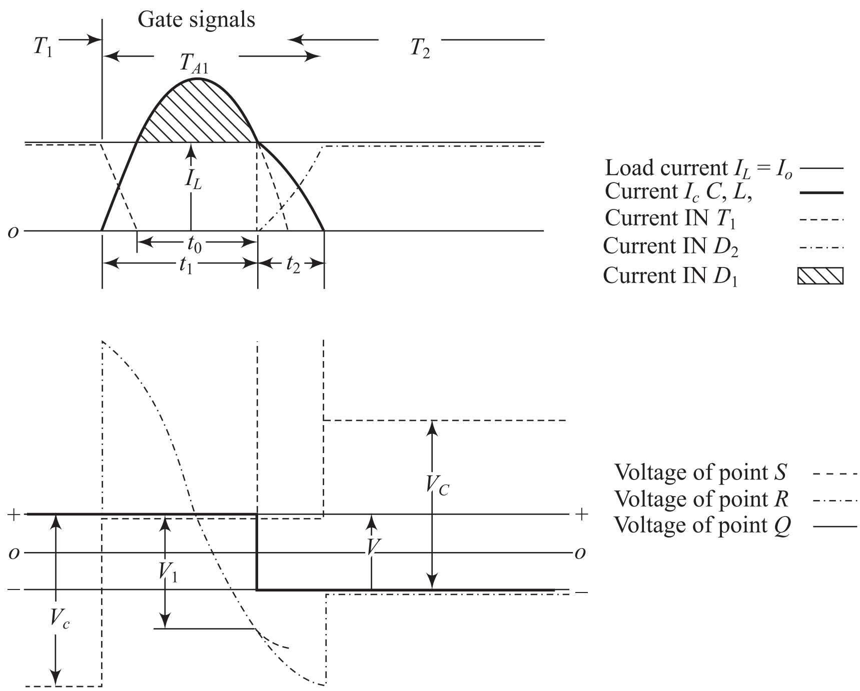


Fig. 11.73 *Voltage and current waveforms of McMurray half-bridge inverter*

11.13 MODIFIED McMURRAY HALF-BRIDGE INVERTER

Figure 11.74 shows a modified McMurray half-bridge inverter. This inverter is a current commuted voltage source inverter. This inverter circuit consists of two main thyristors T_1 and T_2, two main diodes D_1 and D_2, two auxiliary thyristors T_{A1} and T_{A2}, two auxiliary diodes D_{A1} and D_{A2}, damping resistor R_d, inductor L and capacitor C. The three wire dc supply is used and ac load is connected between points P and Q. The commutation circuit is formed by two auxiliary thyristors T_{A1} and T_{A2}, two auxiliary diodes D_{A1} and D_{A2}, damping resistor R_d, inductor L and capacitor C. The capacitor C is used to provide the required energy for commutating main thyristors. The inductance L is used to limit $\frac{di}{dt}$ to a safe value in main and auxiliary thyristors. Since auxiliary thyristors are used for commutation of main thyristors, this inverter is called as *auxiliary commuted inverter*.

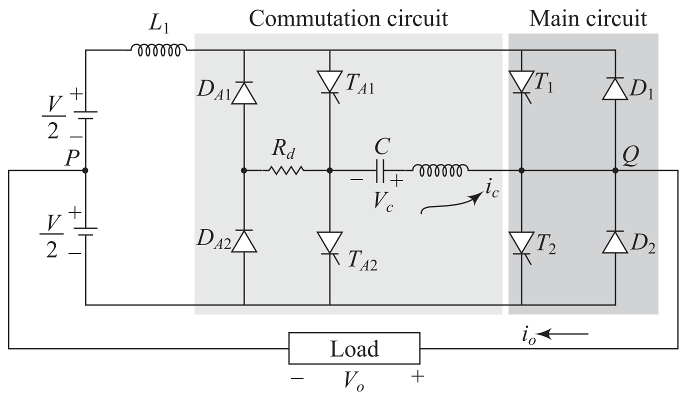


Fig. 11.74 *Modified McMurray half-bridge inverter*

This circuit operates at lagging power factor load. For circuit analysis, the following assumptions are required:

1. During the commutation interval, the load current remains constant.
2. All semiconductor switches (thyristors and diodes) are ideal
3. Inductor L and capacitor C are ideal and they have no resistance.

Figure 11.75 shows the voltage and current waveforms of a modified McMurray half-bridge inverter. The complete operation is subdivided into seven different modes which are explained below.

Mode I Assume thyristor T_1 operates in conducting state and a constant load current I_o flows through T_1, i.e., $I_{T1} = I_o$. The capacitor C is charged to a voltage V with plate A positive and plate B is negative due to the commutation of previously conducting thyristor T_2. Figure 11.76(a) shows the equivalent circuit during this mode. When thyrsitor T_1 is conducting, the commutation circuit operates in passive mode.

Mode II Whenever auxiliary thyristors T_{A1} is triggered at $t = 0$ to turn-OFF main thyristor T_1, the capacitor current i_C starts to build up through resonant circuit whose path is

$$L^+ \; T_1 - T_{A1} - C - L^-.$$

The voltage drop across the main thyristor T_1 can reverse biased diode D_1. Consequently, thyristor T_1 carries only the capacitor current i_C and current does not flow through diode D_1. As the load current

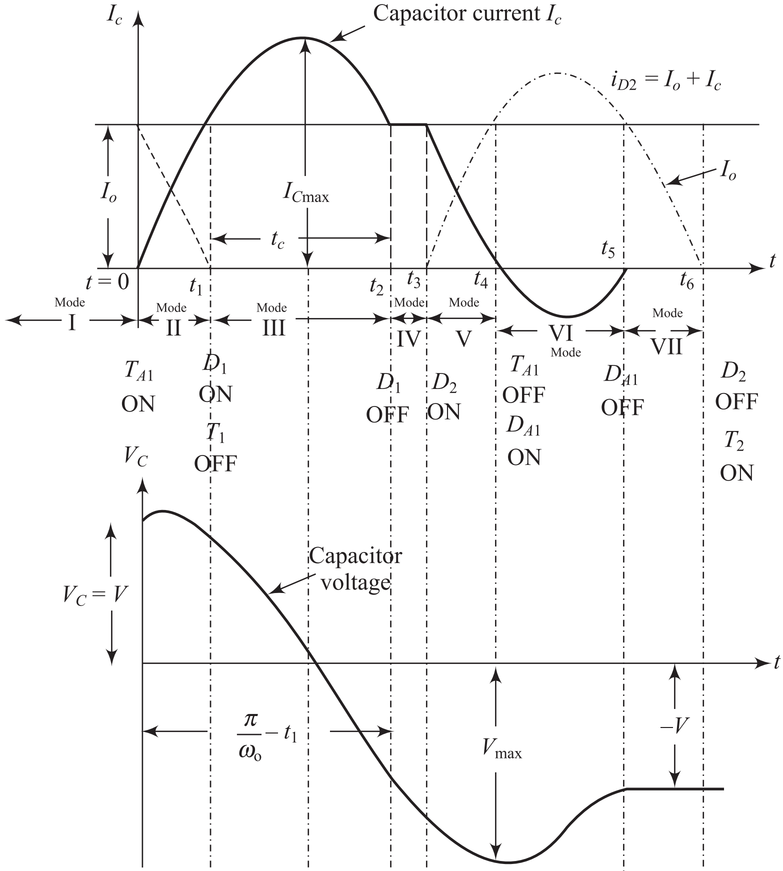


Fig. 11.75 *Voltage and current waveforms of a modified McMurray half-bridge inverter*

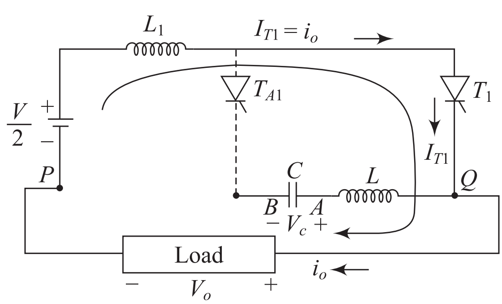


Fig. 11.76 *(a) The equivalent circuit during mode-I (t < 0)*

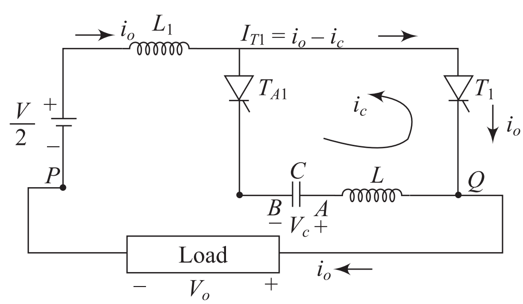


Fig. 11.76 *(b) The equivalent circuit during mode-II ($0 < t < t_1$) and $i_C < i_o$*

I_o is constant, an increase in current i_C, there is a decrease in thyristor current as $I_{T1} = I_o - i_C$. At $t = t_1$, current i_C reaches I_o and $I_{T1} = I_o - i_C = 0$. Therefore, thyristor T_1 will be turned OFF at $t = t_1$. Figure 11.76(b) shows the equivalent circuit during this mode.

Mode III After $t = t_1$, the discharging current i_C exceeds the load current I_o and the excessive current flows through diode D_1. Hence the diode D_1 is called as energy recovery diode. The voltage drop across D_1 can reverse biases T_1 to bring it to forward blocking mode. At $t = t_2$, the capacitor current i_C reaches the peak or maximum value $I_{C\,max}$ whenever the capacitor voltage V_C becomes zero. After $I_{C\,max}$, the capacitor current starts to decrease and the capacitor begins to charge in reverse direction. Figure 11.76(c) shows the equivalent circuit during this mode; diode current $i_{D1} = i_C - I_o$.

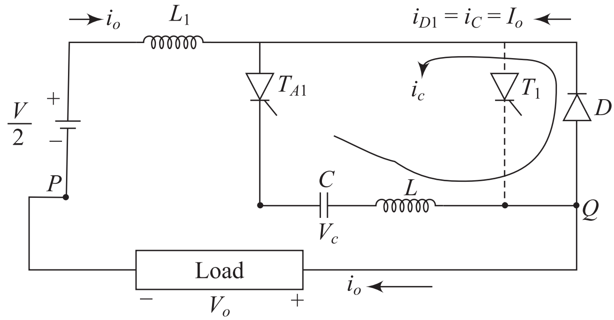


Fig. 11.76 *(c) The equivalent circuit during mode-III ($t_1 < t < t_2$) and $i_C > i_o$*

Mode IV At $t = t_2$ the capacitor current i_C becomes I_o. Then diode D_1 stops conduction. The constant load current I_o flows through the path $\frac{V^+}{2} - T_{A1} - C - L - \text{Load} - \frac{V^-}{2}$ as the auxiliary thyristor T_{A1} is triggered. Subsequently, the load current charges capacitor C linearly with reverse polarity. At $t = t_3$ voltage V_C is greater than V. Figure 11.76(d) shows the equivalent circuit during this mode.

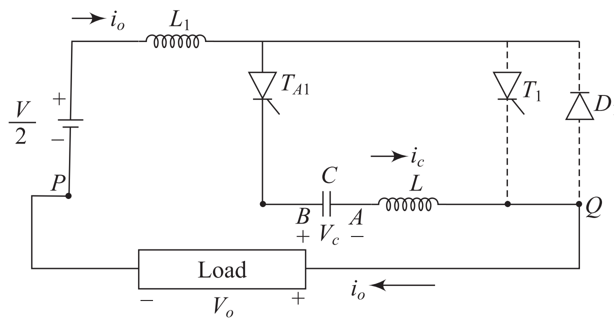


Fig. 11.76 *(d) The equivalent circuit during mode-IV ($t_2 < t < t_3$) and $i_C = i_o$*

Mode V At $t = t_3$, capacitor voltage V_C is slightly greater than V. It is clear from Fig. 11.76 that diode D_2 gets forward biased. As diode D_2 starts to conduct, a current starts to flow through diode D_2. The load current flows through the path

$$\frac{V^+}{2} - T_{A1} - C - L - \text{Load} - \frac{V^-}{2}$$

The path of current flows through diode D_2 is

$$\frac{V^-}{2} - D_2 - \text{Load} - \frac{V^+}{2}$$

Just after $t = t_3$, the capacitor current i_C begins to decrease whereas current through the diode D_2 starts building up so that the sum of i_C and i_{D2} is equal to load current I_o, i.e., $I_o = i_C + i_{D2}$.

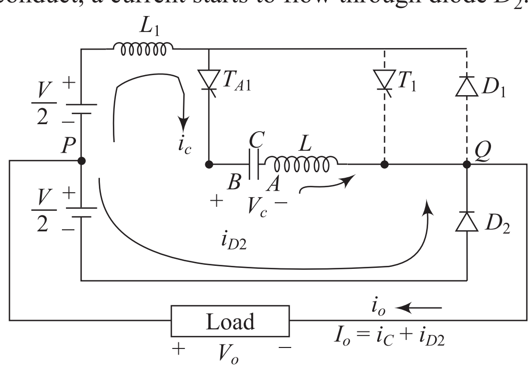


Fig. 11.76 *(e) The equivalent circuit during mode-V ($t_3 < t < t_4$) and $I_o = i_C + i_{D2}$*

The input dc voltage V is impressed across the resonant circuit through the diode D_2. The energy stored in the inductance L will be transferred to C. Consequently, capacitor C is overcharged to a peak value V_{max} at $t = t_4$. Thyristor T_2 does not turned ON as the reverse bias voltage applied to it by the forward threshold voltage of diode D_2. Figure 11.76(e) shows the equivalent circuit during this mode.

Mode VI At $t = t_4$, current through diode D_2 reaches to load current I_o and the capacitor discharge current falls to zero. Auxiliary thyristor T_{A1} gets turned OFF at the instant $t = t_4$ as the discharging current tends to reverse.

As the capacitor voltage is greater than the input dc voltage V, capacitor C gets discharged through the path

$$C^+ - R_d - D_{A1} - V - D_2 - L - C^-$$

After $t = t_4$, the current direction gets reversed to that of the previous current direction. Since the load current I_o is constant, by applying KCL at point Q we obtain $i_{D2} = i_C + I_o$. In this mode of operation, $i_{D2} > I_o$

After that the voltage across the capacitor gradually decreases to V as the circuit is usually critically damped. At $t = t_5$, current i_C is equal to zero and $V_C = -V$, $i_{D2} = I_o$. The voltage drop across R_d and D_{A1} are applied a reverse bias across T_{A1}. Consequently, the commutation of thyristor T_{A1} occurs. Figure 11.76(f) shows the equivalent circuit during this mode.

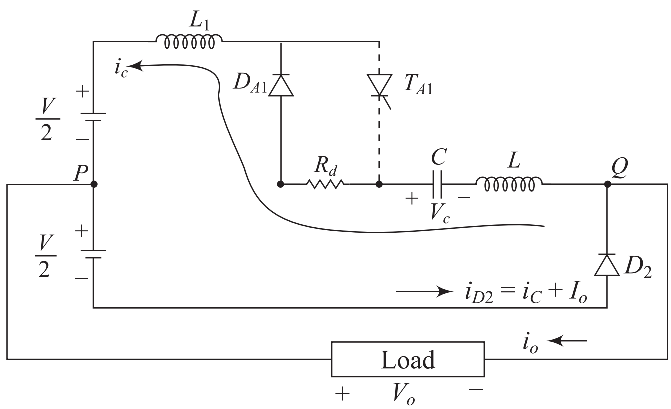


Fig. 11.76 *(f) The equivalent circuit during mode-VI ($t_4 < t < t_5$) and $i_{D2} > I_o$*

Mode VII As the load current decreases, load current $I_o = i_{D2}$ becomes zero at time $t = t_6$. The main thyristor T_2 is triggered between the time interval $t_4 - t_3$, i.e., $\pi\sqrt{LC}$ seconds time delay after the auxiliary thyristor T_{A1} is triggered to turned ON. But the auxiliary thyristor T_{A1} will not be turned ON at this moment as the reverse bias voltage applied to it by the voltage drop across diode D_2. At $t = t_6$, current $i_{D2} = 0$ and thyristor T_2 is not reverse biased. Consequently, the input dc voltage applied a forward bias across thyristor T_2 and trigger pulse is already applied to it to turn ON. The load is subjected to negative voltage through D_2 at $t = t_6$ at the instant of its conduction.

Just after $t = t_6$, capacitor C is charged to voltage $-V$ and it is ready for next commutation process. The commutation process from thyristor T_2 to D_1 is identical to the commutation process from T_1 to D_2. Figure 11.76(g) shows the equivalent circuit during mode VII.

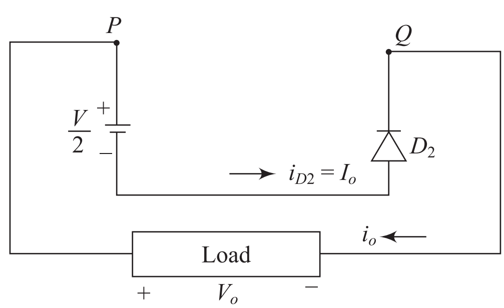


Fig. 11.76(g) *The equivalent circuit during mode-VII ($t_5 < t < t_6$) and $i_C = 0$, $i_{D2} = I_o$*

11.13.1 Design of Commutation Circuit Components of Modified McMurray Half-bridge Inverter

During modes II and III of modified McMurray half-bridge inverter, the circuit parameters inductance and capacitance only play an important role. The commutation or capacitor current i_C during these two modes is equal to

$$i_C = V\sqrt{\frac{C}{L}}\sin\omega_o t = I_{C\max}\sin\omega_o t \tag{11.3}$$

where, $$I_{C\max} = V\sqrt{\frac{C}{L}} \text{ and } \omega_o = \frac{1}{\sqrt{LC}}$$

At $t = t_1$, $$i_C = I_o = I_{C\max}\sin\omega_o t_1$$

or $$t_1 = \frac{1}{\omega_o}\sin^{-1}\left(\frac{I_o}{I_{C\max}}\right)$$

It is clear from Fig. 11.77 that

$$\omega_o t_2 = \pi - \omega_o t_1 = \pi - \sin^{-1}\left(\frac{I_o}{I_{C\max}}\right)$$

or $$t_2 = \frac{1}{\omega_o}(\pi - \omega_o t_1) = \frac{1}{\omega_o}\left[\pi - \sin^{-1}\left(\frac{I_o}{I_{C\max}}\right)\right]$$

The circuit turn-OFF time of the main thyristor T_1 is equal to

$$t_c = t_2 - t_1 = \frac{1}{\omega_o}\left[\pi - 2\sin^{-1}\left(\frac{I_o}{I_{C\max}}\right)\right] \tag{11.4}$$

The time t_c must be greater than the thyristor turn-OFF time t_q. Practically, this condition must be satisfied by several different combinations of L and C. The current commutation pulse i_C provides the required turn-OFF time with the minimum amount of capacitor energy $\frac{1}{2}CV^2$.

From Eq. (11.4), we obtain $\frac{1}{2}\omega_o t_c = \frac{\pi}{2} - \sin^{-1}\left(\frac{I_o}{I_{C\max}}\right)$

or $$\frac{I_o}{I_{C\max}} = \sin\left(\frac{\pi}{2} - \frac{\omega_o t_c}{2}\right) = \cos\frac{\omega_o t_c}{2}$$

If $$\frac{I_{C\max}}{I_o} = x, \quad \cos\frac{\omega_o t_c}{2} = \frac{1}{x}$$

or $$\frac{t_c}{\sqrt{LC}} = 2\cos^{-1}\left(\frac{1}{x}\right) = g(x) \quad \text{as } \omega_o = \frac{1}{\sqrt{LC}}$$

The commutating capacitor must provide the commutating energy

$$W = \frac{1}{2}CV^2 = \frac{1}{2}LI_o^2 \tag{11.5}$$

After substituting the value of $V = I_{C\max}\sqrt{\frac{L}{C}}$ in Eq. (11.5), we obtain

$$W = \frac{1}{2}CV^2 = \frac{1}{2}CV \cdot I_{C\max}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{LC} \cdot VI_{C\max}$$

Since $\sqrt{LC} = \dfrac{t_c}{2\cos^{-1}\left(\dfrac{1}{x}\right)}$, the above equation can be written as

$$W = \frac{1}{2}\sqrt{LC} \cdot VI_{C\max} = \frac{1}{2}\frac{t_c}{2\cos^{-1}\left(\dfrac{1}{x}\right)}VI_{C\max}$$

or $$W = \frac{t_c \cdot V \cdot x \cdot I_o}{4\cos^{-1}\left(\dfrac{1}{x}\right)} \quad \text{as} \quad I_{C\max} = xI_o \qquad (11.6)$$

The product of t_cVI_o has the dimensions of energy. In normalised form, Eq. (11.6) can be expressed by

$$\frac{W}{t_cVI_o} = \frac{x}{4\cos^{-1}\left(\dfrac{1}{x}\right)} = h(x)$$

Figure 11.77 shows the plotting of $h(x)$ with respect to x. It is clear from Fig. 11.77 that the normalized commutation energy $h(x)$ has a minimum value of 0.446 when $x = 1.5$

Subsequently, $$g(x) = 2\cos^{-1}\left(\frac{1}{1.5}\right) = 1.682$$

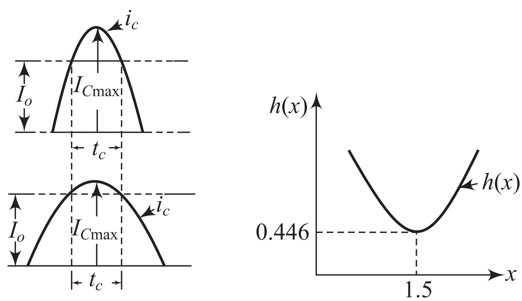


Fig. 11.77 *The plotting of h(x) with respect to x*

The design of commutation circuit is carried out on the basis of worst operating conditions which consists of minimum supply voltage $V_{\min}$ and maximum load current $I_{o\max}$.

Then we can write, $$V_{\min}\sqrt{\frac{C}{L}} = I_{C\max} = xI_{o\max} = 1.5I_{o\max}$$

or $$\sqrt{\frac{C}{L}} = \frac{1.5I_{o\max}}{V_{\min}} \qquad (11.7a)$$

We know that $$\sqrt{LC} = \frac{t_c}{g(x)} = \frac{t_c}{1.682} \qquad (11.7b)$$

After multiplication of Eqs. (11.7a) and (11.7b), we obtain

$$C = \frac{1.5I_{o\max} \cdot t_c}{1.682V_{\min}} = 0.892\frac{t_cI_{o\max}}{V_{\min}}$$

Equation (11.7a) can be expressed as

$$\sqrt{\frac{L}{C}} = \frac{V_{\min}}{1.5I_{o\max}} \qquad (11.7c)$$

After multiplication of Eqs. (11.7b) and (11.7c), we get

$$L = \frac{t_cV_{\min}}{1.682 \times 1.5I_{o\max}} = 0.3964\frac{t_cV_{\min}}{I_{o\max}}$$

For critical damping of the resonant circuit which consists of R_d, L and C, the following condition must be satisfied:

$$\sqrt{\frac{1}{LC}-\left(\frac{R_d}{2L}\right)}=0$$

Form the above equation, we can find the value of resistance which provides critical damping and R_d is equal to

$$R_d = 2\sqrt{\frac{L}{C}}$$

11.14 McMURRAY FULL-BRIDGE INVERTER

Figure 11.78 shows the modified McMurray full-bridge inverter circuit which consists of four main thyristors T_1, T_2, T_3 and T_4, four main diodes D_1, D_2, D_3 and D_4, four auxiliary thyristors T_{A1}, T_{A2}, T_{A3} and T_{A4} and inductance L and capacitance C. The commutation circuit consists of four auxiliary thyristors T_{A1}, T_{A2}, T_{A3} and T_{A4} and inductance L and capacitance C. This circuit operates in four different modes as explained below.

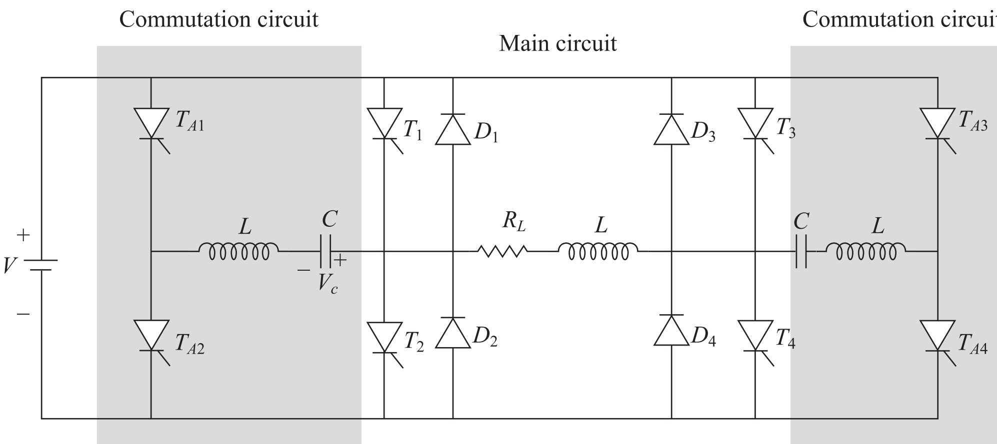


Fig. 11.78 *Modified McMurray full-bridge inverter circuit*

Mode I Assume that initially triggering pulses are applied to thyristors T_1 and T_4 and these thyristors will be turned ON and conducting. Then a load current I_o flows through load and thyristors T_1 and T_4. The capacitor C gets charged to the supply voltage V approximately and the polarity of capacitor voltage is depicted in Fig. 11.79. The load current flows through the following path:

$$V^+ - T_1 - \text{LOAD} - T_4 - V^-$$

Mode II In order to turn-OFF the main thyristors T_1 and T_4, auxiliary thyristors T_{A1} and T_{A4} are triggered. Then discharging current of capacitor follows through the following paths:

$$C^+ - T_1 - T_{A1} - L - C^- \text{ (circuit} - 1\text{) and } C^+ - T_1 - T_{A4} - L - C^- \text{ (circuit} - 2\text{)}$$

Whenever the capacitor discharge current i_C is greater than the load current I_o, i.e., $i_C > I_o$, then the excess current flows through the diodes D_1 and D_4. Since the forward threshold voltage of diodes D_1 and D_4 are applied across thyristors T_1 and T_4, these thyristors will be turned OFF or force commutated.

After that thyristors T_1 and T_4 are turned OFF, the load current does not become zero due to the presence of inductive load and commutation inductance. The energy trapped in the inductance L is to be transferred to the capacitance C to maintain the load current in the same direction.

The capacitor current i_C should reaches the maximum value $i_{C\,max}$ when the capacitor voltage V_C becomes zero. After that the capacitor current i_C starts to decrease and it is charged with a reverse polarity.

Mode III When the capacitor current i_C is less than load current I_o, i.e., $i_C < I_o$, diodes D_1 and D_4 stop conduction and therefore the current flow through diodes become zero. Hence, thyristors T_1 and T_4 returns to the forward blocking mode. At this instant, the load current does not become zero due to presence of commutating inductance L and load. Then the energy trapped in the inductance L will be transferred to the capacitance C to maintain the load current in the same direction. Subsequently the load current I_o flows through the following path:

$$L^+ - C - \text{LOAD} - D_3 - T_{A1} - L^- (\text{circuit}-1) \text{ and } L^+ - T_{A4} - D_2 - \text{LOAD} - C - L^- (\text{circuit}-2)$$

The load current flows through the diodes D_3 and D_2 and the energy back to supply. Then the load current flows through the following path:

$$L^+ - R_L - D_3 - V - D_2 - L_L^-$$

Mode IV In this mode, thyristors T_2 and T_3 are fired and the load current flows through the path:

$$V^+ - T_3 - \text{LOAD} - T_2 - V^-$$

In this mode of operation, the load current direction is reversed to that of the previous mode load current direction. After that thyristors T_2 and T_3 will be forced commuted or turned OFF by the turning ON of the auxiliary thyristors T_{A3} and T_{A2} respectively. Then the above processes will be repeated cyclically.

11.15 MODIFIED McMURRAY FULL-BRIDGE INVERTER

Figure 11.79 shows the modified McMurray full-bridge inverter circuit which consists of four main thyristors T_1, T_2, T_3, and T_4, four feedback diodes D_1, D_2, D_3, and D_4, four auxiliary thyristors T_{A1}, T_{A2}, T_{A3}, and T_{A4}, four auxiliary diodes D_{A1}, D_{A2}, D_{A3}, and $D_{A4.}$ It is clear from Fig. 11.80 that the resistor R_d and the auxiliary diodes D_{A1}, D_{A2}, D_{A3} and D_{A4} are incorporated into the basic McMurray full-bridge inverter. The operating principle of this inverter is explained briefly below.

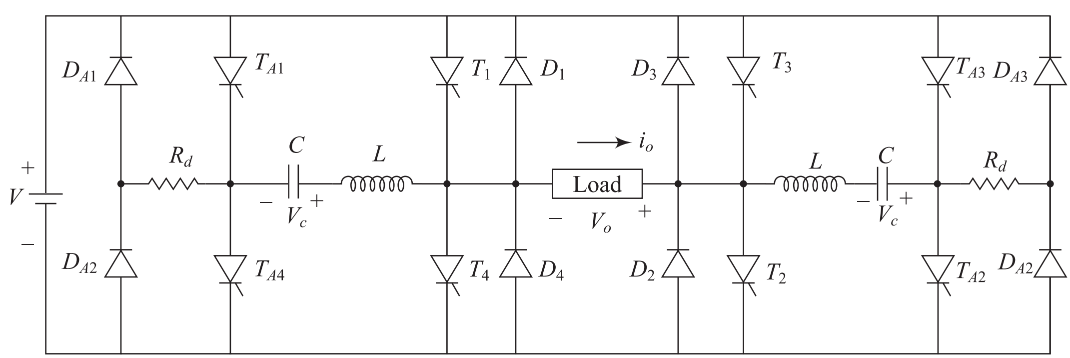


Fig. 11.79 *Modified McMurray full-bridge inverter*

Mode I In this mode of operation, thyristors T_1 and T_2 are triggered simultaneously and these devices operate in the conducting state. Then load current flows through the following path:

$$V^+ - T_1 - \text{LOAD} - T_2 - V^-$$

Mode II During this mode, the auxiliary thyristors T_{A1}, and T_{A2} are triggered simultaneously to turn off the main thyristors T_1 and T_2.

Mode III In this operating mode, the commutating current I_c in the both circuits flows beyond the load current I_o. Consequently, thyristors T_1 and T_2 are turned OFF.
In this way, this inverter circuit operates just like McMurray half-bridge inverter.

Example 11.25 For a modified McMurray half-bridge inverter, determine the value of commutating components L and C for the following parameters:

(a) maximum load current is 100 A, (b) thyristor turn-OFF time t_q = 30 μs and (c) $V_{\min}$ = 200 V

Solution

Given: $I_{o\max}$ = 100 A, t_q = 30 μs and $V_{\min}$ = 200 V
The circuit turn-OFF time is always greater than turn-OFF time of thyristors. Assume $t_c = 1.5t_q = 1.5 \times 30\ \mu\text{s} = 45\ \mu\text{s}$

The value of C is $$C = \frac{1.5I_{o\max} \cdot t_c}{1.682V_{\min}} = 0.892\frac{t_c I_{o\max}}{V_{\min}}$$

$$= 0.892\frac{45 \times 10^{-6} \times 100}{200} = 20.07\ \mu\text{F}$$

The value of L is $$L = \frac{t_c V_{\min}}{1.682 \times 1.5I_{o\max}} = 0.3964\frac{t_c V_{\min}}{I_{o\max}}$$

$$= 0.3964\frac{45 \times 10^{-6} \times 200}{100} = 35.676\ \mu\text{H}$$

Example 11.26 A single-phase modified McMurray full-bridge inverter is fed by 250 V. The dc voltage varies ±5%. The current during commutation vary from 25 A to 120 A. Determine (a) the value of commutating components L and C if thyristor turn-OFF time t_q = 40 μs, (b) the value of R_d. Assume that the factor of safety is equal to 2.

Solution

Given: $I_{o\max}$ = 120 A, t_q = 40 μs and $V_{\min} = 250 - \frac{250 \times 5}{100}\text{V} = 237.5\ \text{V}$

(a) The circuit turn-OFF time is always greater than turn-OFF time of thyristors. Assume $t_c = 2t_q = 2 \times 40\ \mu\text{s} = 80\ \mu\text{s}$ as the factor of safety is equal to 2.

The value of C is $$C = \frac{1.5I_{o\max} \cdot t_c}{1.682V_{\min}} = 0.892\frac{t_c I_{o\max}}{V_{\min}}$$

$$= 0.892\frac{80 \times 10^{-6} \times 120}{237.5} = 36.05\ \mu\text{F}$$

The value of L is $$L = \frac{t_c V_{\min}}{1.682 \times 1.5I_{o\max}} = 0.3964\frac{t_c V_{\min}}{I_{o\max}}$$

$$= 0.3964\frac{80 \times 10^{-6} \times 237.5}{120} = 62.76\ \mu\text{H}$$

(b) The value of resistance is

$$R_d = 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{62.76}{36.05}} = 2.638\ \Omega$$

11.16 McMURRAY BEDFORD HALF-BRIDGE INVERTER

Figure 11.80 shows the McMurray Bedford half-bridge inverter circuit which consists of two main thyristors T_1 and T_2, two feedback diodes D_1 and D_2, two inductors L_1 and L_2, two capacitors C_1 and C_2. The commutation circuit consists of two capacitors C_1 and C_2 and two inductors L_1 and L_2 which are magnetically coupled. Actually inductors L_1 and L_2 form one inductor with a center tap so that $L_1 = L_2 = L$. The value of inductance of the centre tap inductor is about 50 μH. Actually this inductor is wound on a core with proper air gap so that it does not provide any saturation. The value of capacitors C_1 and C_2 are same, i.e., $C_1 = C_2 = C$. This inverter circuit operates as a *voltage commuted voltage source inverter*.

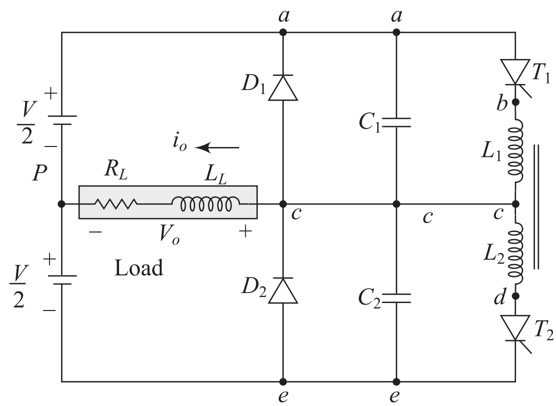


Fig. 11.80 *Single-phase McMurray Bedford half-bridge inverter circuit*

This inverter circuit has less number of thyristors and diodes as compared to modified McMurray half-bridge inverter, but the number of inductors and capacitors are large. In one branch of this inverter circuit, two tightly coupled inductors are connected in series with two thyristors. When one thyristor is turned ON, the other conducting thyristor will be turned OFF. This commutation circuit is also called *complementary commutation*. Since complementary commutation is used in this inverter circuit, this circuit is called *complementary commutated inverter*. This inverter circuit operates in *six* different modes which are explained below.

Mode I ($t \le 0$) During this mode of operation, thyristor T_1 conducts and load current I_o flows from the upper dc source to load. Since the load current is constant, the voltage drop across the commutating inductance L_1 is negligible as $L_1 \frac{di}{dt} = 0$. As the voltage drop across L_1 and T_1 is equal to zero, the voltage drop across C_1 is zero. The voltage across C_2 is V as the point a is connected to point c through T_1 and L_1 and the lower plate of capacitor C_2 is connected to point e. Figure 11.81(a) shows the equivalent circuit of McMurray Bedford half-bridge inverter for this mode. The voltage at nodes b, c and d with respect to e is V.

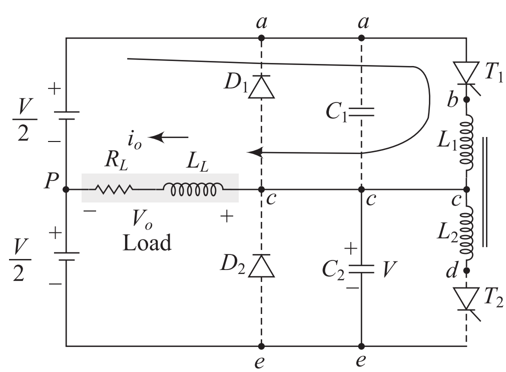


Fig. 11.81 *(a) Equivalent circuit of McMurray Bedford half-bridge inverter Mode I ($t \le 0$)*

Mode II At $t = 0^+$, thyristor T_2 is triggered to initiate the commutation of T_1. When the thyristor T_2 is turned ON, the point d will be connected to the point e, i.e., the negative supply voltage terminal. The voltage across capacitors C_1 and C_2 cannot be changed instantaneously and the voltage V will be applied across L_2. Since inductors L_1 and L_2 are magnetically coupled, an equal voltage is induced across inductance L_1 with terminal b positive. The voltage across thyristor T_1 is determined by the KVL equation in the loop a-b-c-d-e. Therefore,

$$V_{T1} - \frac{V}{2} - \frac{V}{2} + V + V = 0$$

So that $\quad V_{T1} = -V$

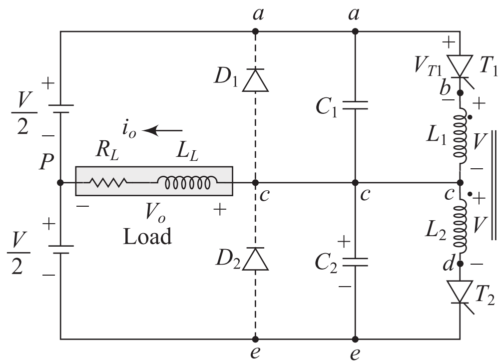


Fig. 11.81 *(b) Equivalent circuit of McMurray Bedford half-bridge inverter Mode II (t = 0+)*

Figure 11.81(b) shows the equivalent circuit of McMurray Bedford half-bridge inverter for this mode. It is clear from Fig. 11.81(b) that point b is positive with respect to a by the voltage V. Hence thyristor T_1 must be subjected to a reverse voltage of $-V$ and it will be turned off at $t > 0^+$ as shown in Fig. 11.81(c). The load current I_o which flows through thyristor T_1 and inductor L_1 is transferred to L_2 and T_2 to maintain constant mmf in the centre tapped inductor and maintain constant flux linkage. The current directions i_{C1} and i_{C2} are shown in Fig. 11.81(c). The KVL equation for the loop C_1, C_2 and V is expressed by

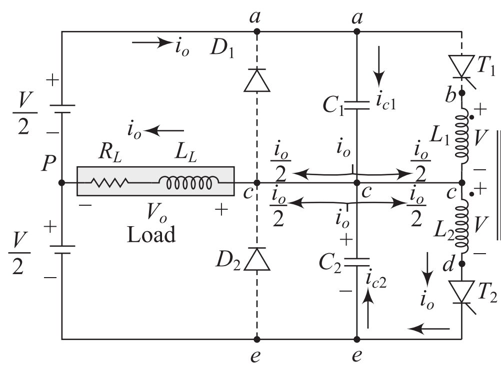


Fig. 11.81 *(c) Equivalent circuit of McMurray Bedford half-bridge inverter Mode II (t > 0+)*

$$-V(\text{voltage across } C_2) + V(\text{input voltage}) - \frac{1}{C_1}\int i_{C1} \cdot dt + \frac{1}{C_2}\int i_{C2} \cdot dt = 0$$

or $\quad i_{C1} = i_{C2}$

Therefore, $\quad C_1 = C_2$

The KCL equation at node c is expressed by

$$i_{C1} + i_{C2} = I_o + I_o$$

or $\quad i_{C1} = i_{C2} = I_o$

At $t = 0+$, the current I_o flows through capacitor C_1 and C_2. The half of i_{C1} flows the load and the half of i_{C1} flows through L_2. Similarly, half of i_{C2} flows through the load and the half of i_{C2} flows through L_2. The capacitor C_1 is getting charged from zero voltage and the capacitor C_2 is getting discharged from V at the same rate. Since the capacitor C_2 is connected across L_2, an oscillating current flows through C_2, L_2 and T_2. At one fourth of a cycle, the oscillating current increases from initial value I_o to a maximum value of $I_{\max}$ in L_2 and T_2 and the voltage across C_2, i.e., V_{C2} reduces to zero. At the instant of one fourth of a cycle, thyristor T_2 is triggered, the KCL at node c is expressed by

$$i_{C1} + i_{C2} = I_o + I_m$$

I_o is the load current and I_m is current flows through L_2 and T_2.

As $i_{C1} = i_{C2}, i_{C1} = i_{C2} = \frac{I_o + I_m}{2}$ Fig. 11.81(d) shows the equivalent circuit of McMurray Bedford half-bridge inverter for this mode. The variations of i_{C1}, i_{C2}, i_{T2} and I_o from $t = 0$ to $t = t_1$ are depicted in Fig. 11.82. The voltage of node *c* drops to zero in one-fourth of a cycle of $V \cos \omega_o t$. In the same way, the voltage of node *b* *decreases* from 2 V to zero at $t = t_1$, i.e., one fourth of a cycle.

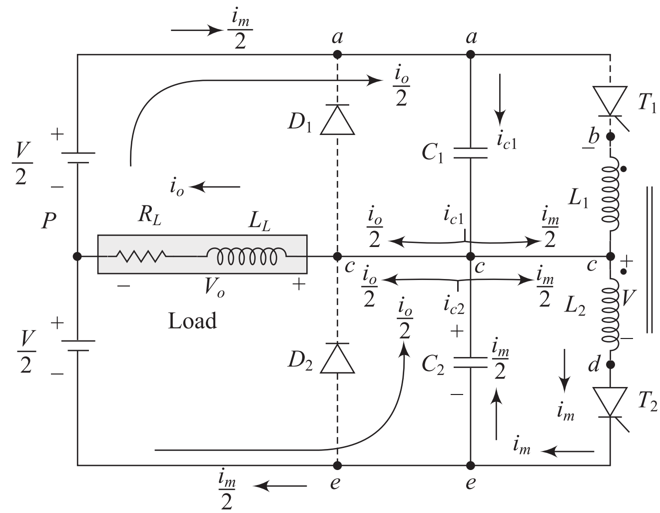


Fig. 11.81 *(d) Equivalent circuit of McMurray Bedford half-bridge inverter Mode II ($t < t_1$)*

As thyristor T_1 is reversed when voltage of node *b* falls to *V*, the commutation time for T_1 is t_c as shown in Fig. 11.82. When the circuit consists of C_2, L_2 and T_2, the ringing frequency is equal to

$$\omega_o = \frac{1}{\sqrt{LC}}$$

The periodic time is equal to $T_o = \frac{1}{f_o} = \frac{2\pi}{\omega_o} = 2\pi\sqrt{LC}$

At one-fourth of a cycle $t_1 = \frac{T_o}{4} = \frac{1}{2}\pi\sqrt{LC}$

where, *L* is the inductance of L_2 and *C* is the capacitance of C_2.
The circuit turn-OFF time t_c is less than one quarter of a cycle.

Therefore, $\quad t_c < t_1$

or $\quad t_c < \frac{1}{2}\pi\sqrt{LC} \qquad$ as $t_1 = \frac{1}{2}\pi\sqrt{LC}$

Mode III At $t = t_1$, the capacitor C_1 is charged to input dc voltage *V* and there is no current flow through C_1, i.e., $i_{C1} = 0$. Just after one-fourth of a cycle form the instant $t = 0$ to at $t = t_1$, $V_{C2} = 0$ and $\frac{1}{2}(I_o + I_m)$ flows through capacitor C_2 and it is charged with the bottom plate positive. Consequently, diode D_2 will be forward biased at $t = t_1$ and the current $(I_o + I_m)$ is transferred to diode D_2. Subsequently, $i_{C1} = i_{C2} = 0$ after $t = t_1$ but $i_{D2} = (I_o + I_m)$. Figure 11.81(e) shows the equivalent circuit of McMurray Bedford half-bridge inverter for this mode.

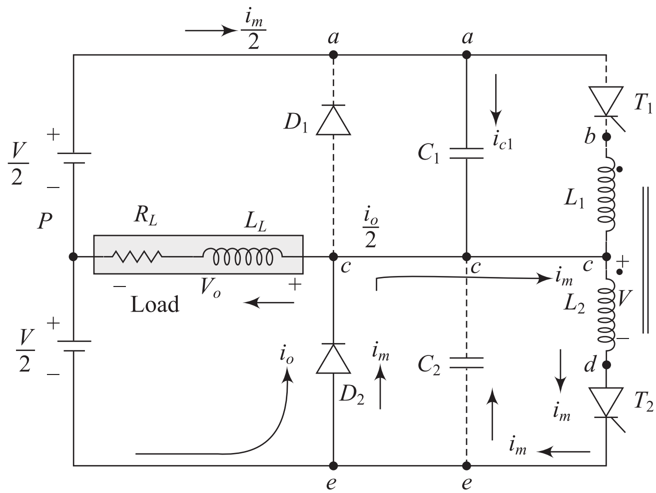


Fig. 11.81 *(e) Equivalent circuit of McMurray Bedford half-bridge inverter Mode III ($t = t_1$)*

The stored energy in inductor L_2 at $t = t_1$ is dissipated in the circuit which is formed by L_2, T_2 and D_2. At $t = t_2$, the total energy stored in L_2 is completely dissipated. As a result, current i_{T2} decays to zero and thyristor T_2 will be turned OFF at $t = t_2$. Generally, a small resistance is connected in series with the diode to dissipate the stored energy of L_2 firstly. Since thyristor current i_{T2} decays from I_m at $t = t_1$ to zero at $t = t_2$, the diode current i_{D2} decays from $i_{D2} = (I_o + I_m)$ at $t = t_1$ to $i_{D2} = i_o$ at $t = t_2$. Actually the load current decreases from I_o at $t = t_1$ to $i_{D2} = i_o$ at $t = t_2$.

Mode IV Figure 11.81(f) shows the equivalent circuit of McMurray Bedford half-bridge inverter for this mode. The current i_{T2} through L_2 and T_2 has reduced to zero but a load current $i_o = i_{D2}$ flows through the diode D_2 during the time interval $t_3 - t_2$.

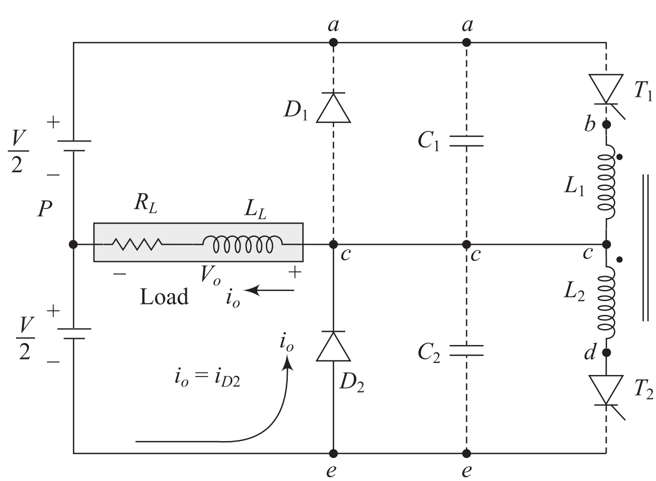


Fig. 11.81 *(f) Equivalent circuit of McMurray Bedford half-bridge inverter Mode IV ($t > t_2$)*

Mode V In this mode, the load current flows through diode D_2 ($i_o = i_{D2}$) decays to zero at $t = t_3$ and then its value is reversed. Then diode D_2 is blocked. As the voltage drop across D_2 does not exist, thyristor T_2 is reversed biased. Consequently, thyristor T_2 is turned ON during the time interval $t_3 - t_2$ to carry the load current in reverse direction. The capacitor C_1 is charged to the dc input voltage V and it is ready for commutation the main thyristor T_2.

The value of commutating circuit parameters L and C for minimum trapped energy is equal to

$$L = 2.35 \frac{Vt_q}{I_{o\max}} \text{ and } C = 2.35 \frac{I_{o\max} t_q}{V}$$

where, t_q is the thyristor turn-OFF time and $I_{o\,\max}$ is the maximum load current.

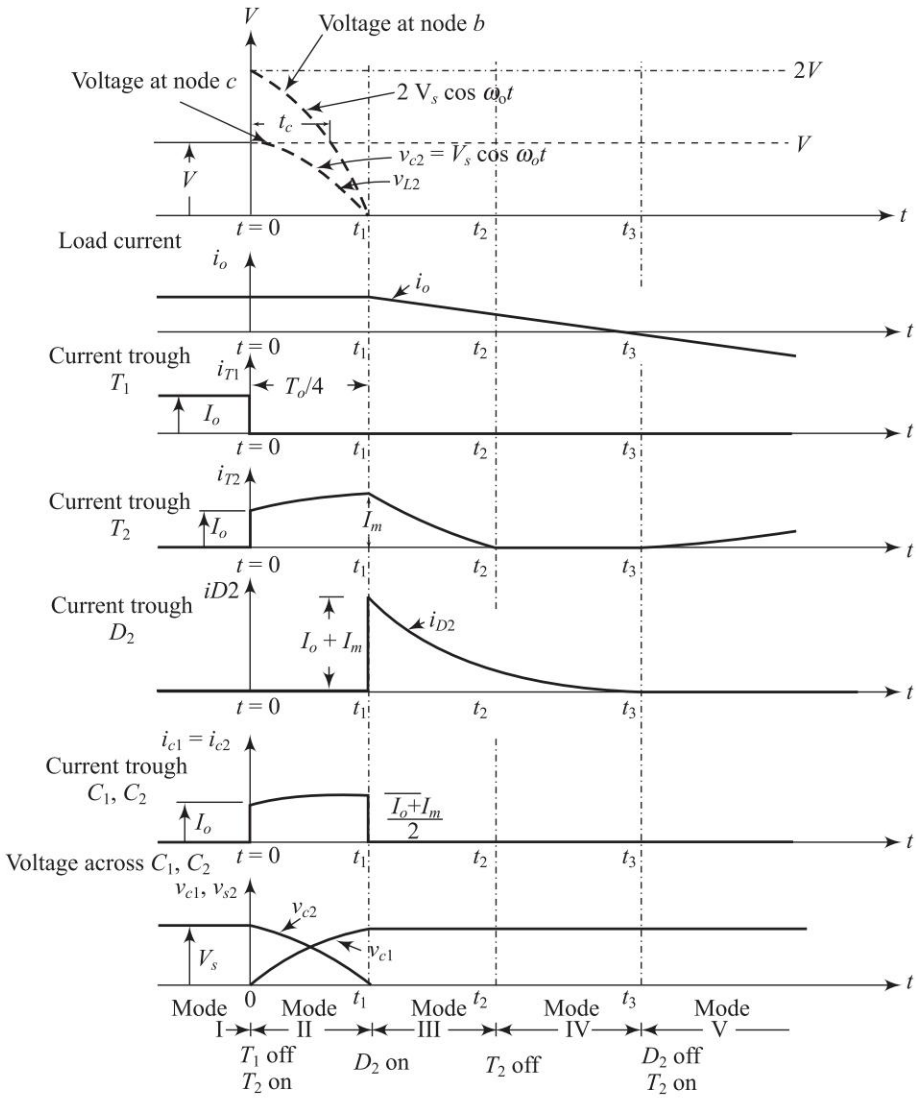


Fig. 11.82 *Voltage and current waveforms of Mc Murray Bedford inverter*

11.17 McMURRAY BEDFORD FULL-BRIDGE INVERTER

Figure 11.83 shows a single-phase McMurray Bedford full-bridge inverter circuit which can be made by connecting two half-bridge inverters. This circuit consists of four main thyristors T_1, T_2, T_3 and T_4, four feedback diode D_1, D_2, D_3 and D_4, four inductors L_1, L_2, L_3 and L_4, and four capacitors C_1, C_2, C_3 and C_4. The working principal of this inverter is similar to that half-bridge inverter. During mode-I, thyristors T_1 and T_2 are conducting and load current flows through V -T_1-L_1-LOAD-L_2 and T_2. The voltage across C_1 and C_2 is zero but capacitor voltage across C_3 and C_4 are V. For commutation of thyristors T_1 and T_2, thyristors T_3 and T_4 are triggered. Consequently, thyristor T_1 and T_2 will be reverse biased by voltage -V and thyristors T_1 and T_2 are turned OFF. After that, the cycle will be repeated.

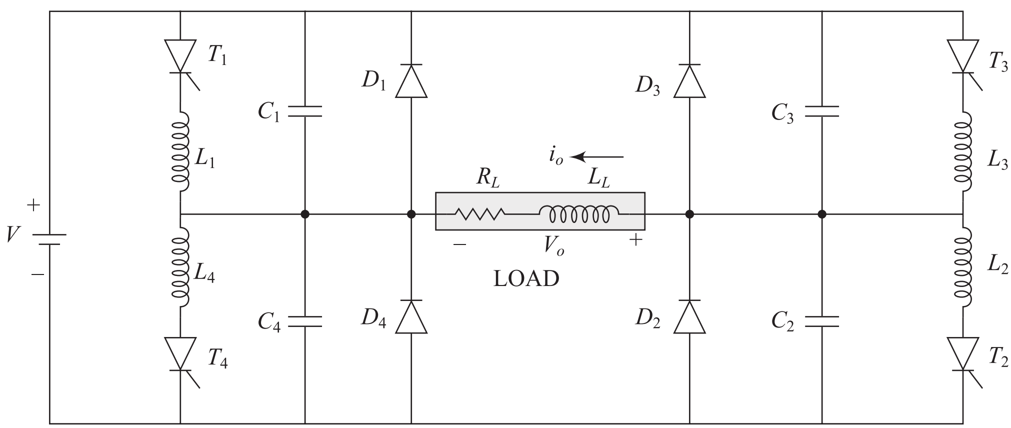


Fig. 11.83 *McMurray Bedford full-bridge inverter*

11.18 CURRENT SOURCE INVERTER

In a current source inverter (CSI), the current which is supplied from current source is maintained constant but is adjustable. The magnitude of current is independent of load impedance, but the amplitude of output voltage and its nature of waveform depends on the load impedance in CSI. The dc input current to CSI is obtained from a fixed ac voltage source though a bridge rectifier and a chopper or a controlled rectifier. Usually, the current input to CSI is almost ripple-free as L filter is used before CSI. A current source inverter converts the dc input current to ac current at inverters output terminals. The output frequency of CSI depends upon the rate of firing or triggering pulses of thyristors. The amplitude of ac current can be varied by changing the dc input current.

Since CSI is a constant current system, it is used typically to supply high power factor loads where impedance will be remain constant or decreases at harmonic frequencies in order to prevent problems either on switching or with harmonics voltage. An VSI requires feedback diode whereas a CSI does not require any feedback diode. The commutation circuit of CSI is very simple as it contains only capacitors. Since semiconductor switches which are used in CSI must be withstand at reverse voltage, power transistors, power MOSFETs and GTOs can not be used in CSI. Generally current source inverters are used in the following fields:

1. Synchronous motor starting
2. Induction heating

3. Lagging VAR compensation
4. Speed control of ac motors (induction motors)

In this section, the operating principle of single phase current source inverter with ideal switches and a single phase capacitor commutated current source inverter with R load are discussed elaborately.

11.18.1 Single-phase Current Source Inverter

Figure 11.84 shows a single-phase current source inverter. For circuit analysis, we assume that all switching devices are ideal. Thyristor is an ideal switch and has zero commutation time. Figure 11.85 shows the current and voltage waveforms of a single-phase current source inverter. In this inverter circuit, the current source consists of a dc voltage source V with a large inductance L in series with it. The high impedance reactor is connected in series with voltage source to maintain constant current at the input terminals of current source inverter (CSI) and a constant dc current I is maintained at the input terminals of CSI.

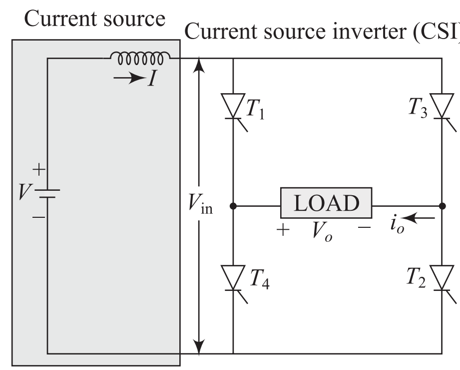


Fig. 11.84 *Single-phase current source inverter*

When thyristors T_1 and T_2 are turned ON, the load current I flows through the load and is positive. When thyristors T_3 and T_4 are turned ON, the load current I flows through load and is negative. The output current I_o is a square wave with amplitude of I. The frequency of current I_o can be controlled by controlling the switching frequency of triggering signals of thyristor pairs T_1 and T_2 and T_3 and T_4.

We assume that load consists of a capacitor C. The load current is equal to

$$i_o = C\frac{dV_o}{dt}$$

Since i_o is constant, the slope $\frac{dV_o}{dt}$ must be constant over each half cycle. The slope $\frac{dV_o}{dt}$ is positive during the time interval $0 < t < \frac{T}{2}$ and the slope $\frac{dV_o}{dt}$ is negative during the time interval $\frac{T}{2} < t < T$. The input voltage to the CSI is $V_{in} = V_o$ when thyristors T_1 and T_2 conduct. The input voltage to the CSI is $V_{in} = -V_o$ when thyristors T_3 and T_4 conduct. The frequency of output voltage is equal to the frequency of output current but the frequency of input voltage is two times of frequency of output voltage as depicted in Fig. 11.85.

The amplitude of dc current I is constant and it is always unidirectional, if the average value of input voltage is positive, power flows from current source to load. When average value of input voltage is negative, the power flows from load to source and regenerative action takes place. In ideal case load current waveform is a square wave but practically the load current waveform is not a square wave as the rise and fall of current cannot be instantaneous. Since each switching device (thyristor) has finite commutation time, all practical inverter has finite rise time of current and fall time of current.

Current source inverters can be developed by using forced commutation or load commutation. Forced commutation is required for lagging power factor load where as load commutation is possible for leading power factor load.

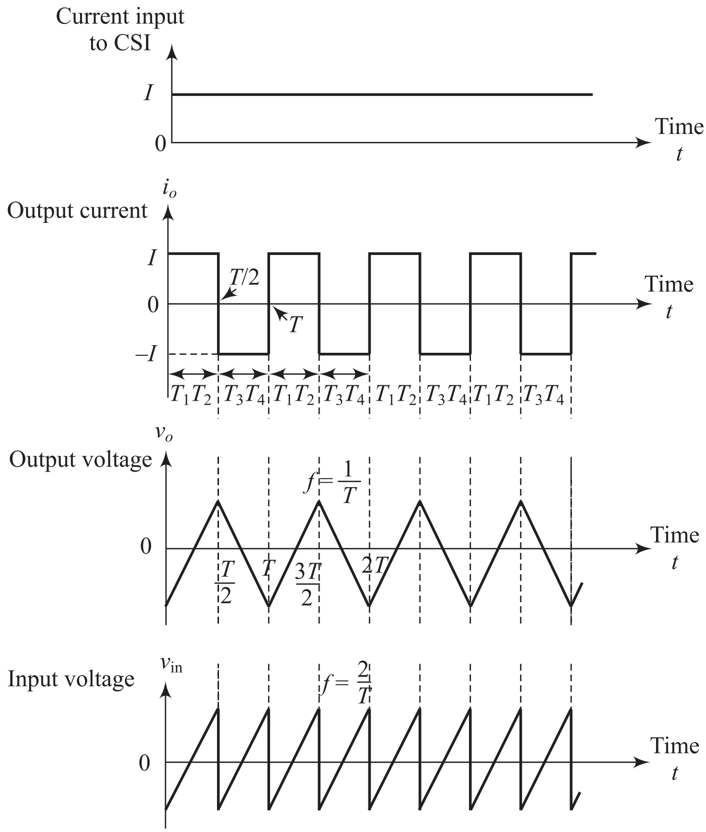


Fig. 11.85 *Voltage and current waveforms of a CSI with ideal switches*

11.18.2 Single-phase Capacitor Commutated CSI with *R* load

Current source inverters are load or force commutated. Load commutation is only possible when the load power factor is leading. Force commutation is possible for lagging load. Figure 11.86 shows

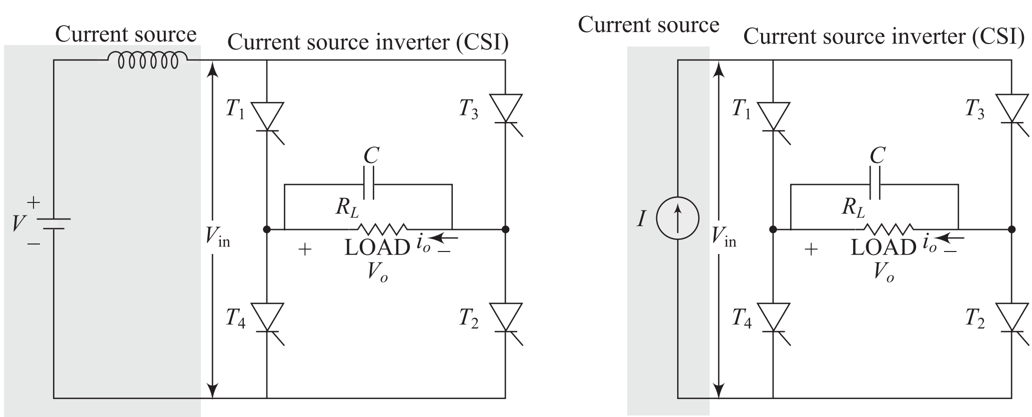


Fig. 11.86 *Single-phase capacitor commutated CSI with R load*

a single-phase capacitor commutated CSI with R load. A dc current source is used to provide constant current I. A capacitor C is connected in parallel with the load and is used for storing the charge for commutation of thyristors. Thyristor pair T_1 and T_2 are turned ON by applying the trigger pulse i_{g1} and i_{g2} respectively. Similarly, Thyristor pair T_3 and T_4 are turned ON by applying the trigger pulse i_{g3} and i_{g4} respectively. The load current waveform is depicted in Fig. 11.87.

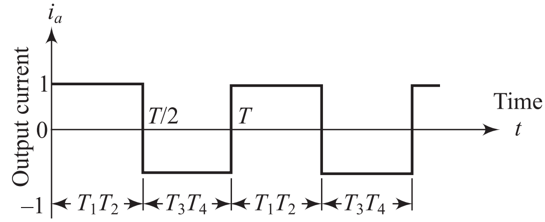


Fig. 11.87 *Load current waveform*

Circuit analysis Figure 11.88(a) shows the equivalent circuit of CSI during $0 < t < \frac{T}{2}$. Assume that initially the capacitor is charged to a voltage $V_C = -V_1$ with capacitor right plate positive and left plate negative. When thyristors T_1 and T_2 are gated at $t = 0$, the capacitor voltage V_C reverse biased the conducting thyristors T_3 and T_4, then these thyristors are commutated immediately. Subsequently, the source current I flows through T_1, parallel combination of R and C and T_2. In the time interval 0 to $\frac{T}{2}, i_{T1} = i_{T2} = I$, the output current $i_o = I$. The capacitor voltage V_C changes from $-V_1$ to $+V_1$ through the charging of capacitor C by current i_C. The output voltage is $V_o = V_C$. The amplitude of current waveform is $i_o = \frac{V_o}{R} = \frac{V_C}{R}$. The current and voltage waveforms of single phase CSI with R load is depicted in Fig. 11.89.

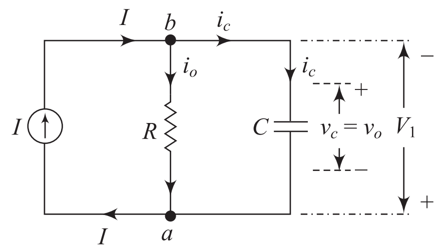


Fig. 11.88 *(a) Equivalent circuit of CSI during $0<t<\frac{T}{2}$*

When triggering pulses are applied to thyristors T_3 and T_4 at $t = \frac{T}{2}, V_C = V_1$ which reverse biased thyristors T_1 and T_2. Therefore, these thyristors are turned OFF immediately. Subsequently, the source current flows through T_3, parallel combination of R, C and T_4. During the time interval $t = \frac{T}{2}$ to T, $i_{T3} = I_{T4} = I$ and output load current is $i_o = -I$. Figure 11.88(b) shows the equivalent circuit of CSI during $\frac{T}{2} < t < T$.

At steady state operation of current source inverter, the voltage and current waveforms are depicted in Fig. 11.89. At $t = 0$ the capacitor is charged to voltage $-V_1$, the output voltage is equal to $V_o = V_C = -V_1$ and load current $i_o = -\frac{V_1}{R} = -I_1$. During $t = 0$ to $t = \frac{T}{2}$, capacitor charges from $-V_1$ to $+V_1$. At $t = \frac{T}{2}, i_o = \frac{V_o}{R} = \frac{V_C}{R} = \frac{V_1}{R} = I_1$ current. The input voltage $V_{in} = V_o$ during $t = 0$ to $t = \frac{T}{2}$, but the input voltage $V_{in} = -V_o$ during $t = \frac{T}{2}$ to $t = T$.

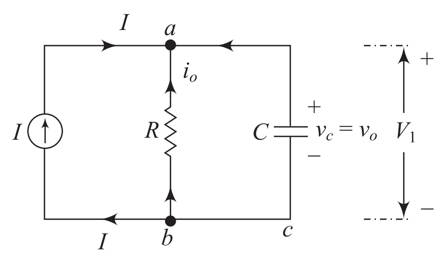


Fig. 11.88 *(b) Equivalent circuit of CSI during $\frac{T}{2}<t<T$*

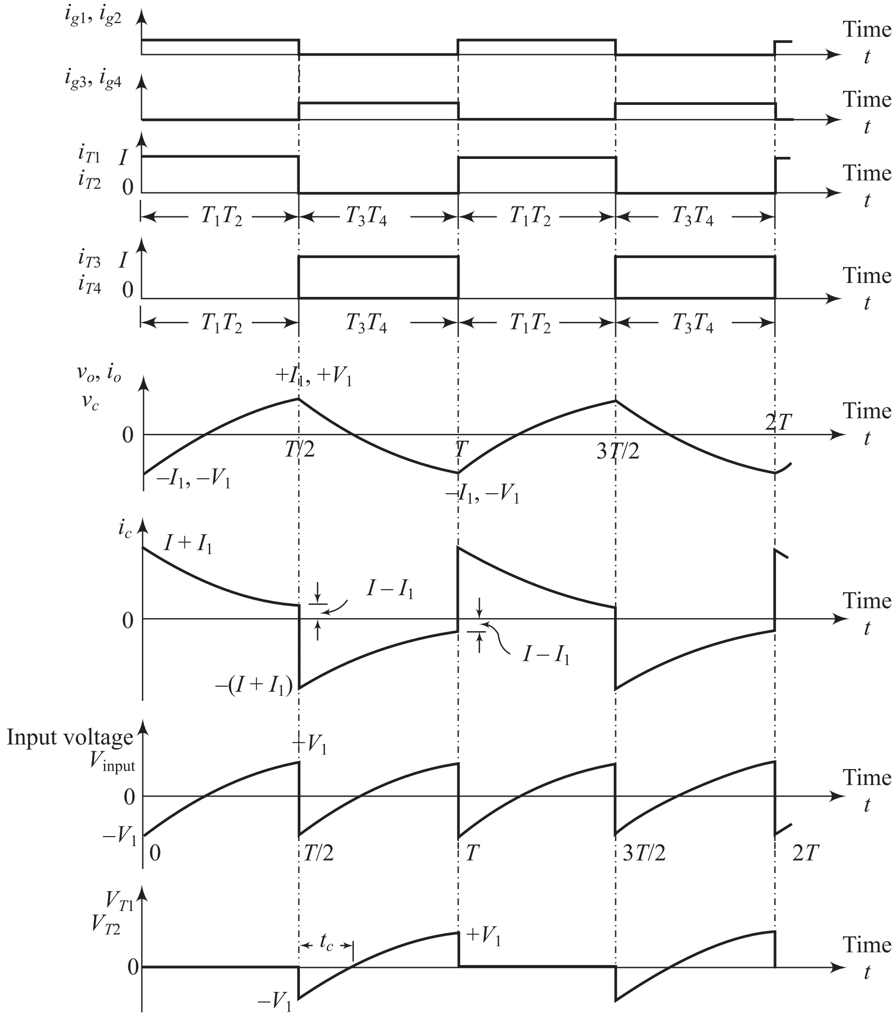


Fig. 11.89 *Current and voltage waveforms of single phase CSI with R load*

It is clear from Fig. 11.88(a) that whenever thyristors T_1 and T_2 are conducting during $t = 0$ to $t = \dfrac{T}{2}$, current i_C, i_o are out from node a and current I is in the node b. The KCL equation at node a and node b are expressed as

$$I = i_C + i_o \qquad \text{or,} \quad i_C = I - i_o$$

At time $t = 0$, current $i_o = -I_1$. Consequently, $i_C = I + I_1$ as $i_o = -I_1$. At $t = \dfrac{T}{2}$, $i_o = I_1$. Then $i_C = I - I_1$ as $i_o = I_1$

After $t = \dfrac{T}{2}$, thyristors T_1 and T_2 are turned OFF and thyristors T_3 and T_4 are conducting, but currents i_C and i_o flow in the same direction. Figure 11.88(b) shows the equivalent circuit of CSI during $\dfrac{T}{2} < t < T$. The KCL at node a is equal to

$$i_o + i_C + I = 0$$

or $$i_C = -I - i_o$$

At $t = \frac{T}{2}, i_o = I_1$. Therefore, $i_C = -I - i_o = -I - I_1 = -(I + I_1)$. At $t = T, i_o = -I_1$. Then $i_C = -I - i_o = -I + I_1 = -(I - I_1)$.

The voltage across thyristor T_1 is zero when T_1 and T_2 are ON. When T_3 and T_4 are turned ON, $V_{T1} = V_{T2} = -V_C = -V_o = V_{\text{in}}$ during $\frac{T}{2} < t < T$.

During the time interval $0 < t \le \frac{T}{2}$, the equivalent circuit of the CSI is depicted in Fig. 11.88(a). The capacitor is initially charged to a voltage $-V_1$. The KVL equation of the closed path is expressed by

$$Ri_o - \frac{1}{C}\int (I - i_o)dt + V_1 = 0 \tag{11.8}$$

After differentiating Eq. (11.8), we get

$$R\frac{di_o}{dt} + \frac{i_o}{C} = \frac{I}{C}$$

or $$\left(Rp + \frac{1}{C}\right)I_o = \frac{I}{C}$$

The complementary function of the solution of above equation is obtained from force-free equation is $\left(Rp + \frac{1}{C}\right)I_o = 0$ or, $p = -\frac{1}{RC}$

Therefore, $$I_{Cp} = Ae^{-\frac{t}{RC}}$$

For particular integral, $p = 0$. Then, $\frac{i_o}{C} = \frac{I}{C}$

Therefore, $i_o = I$

The complete solution of load current is

$i_o = PI + CF$ where, PI = particular integral and CF = complementary function

or $$i_o = I + Ae^{-\frac{t}{RC}} \tag{11.9}$$

The load current at $t = 0$, $i_o = -I_1$ at steady state condition

Therefore, $-I_1 = I + A$ or, $A = -(I + I_1)$

After substituting the value of A in Eq. (11.9), we obtain

$$i_o = I - (I + I_1)e^{-\frac{t}{RC}}$$

or $$i_o = I\left(1 - e^{-\frac{t}{RC}}\right) - I_1 e^{-\frac{t}{RC}} \quad \text{for } 0 < t \le \frac{T}{2} \tag{11.10}$$

At $t = \frac{T}{2}$, current i_o is equal to I_1. Then Eq. (11.10) can be written as

$$I_1 = I\left(1 - e^{-\frac{T}{2RC}}\right) - I_1 e^{-\frac{T}{2RC}}$$

Therefore, $$I_1 = I\left(\frac{1-e^{-\frac{T}{2RC}}}{1+e^{-\frac{T}{2RC}}}\right)$$

If $$T >> RC, \frac{T}{2RC} >> 1 \text{ and } e^{-\frac{T}{2RC}} = 0, \text{ then } I_1 = I$$

After substituting the value of I_1 in Eq. (11.10), we obtain

$$i_o = I\left(1-e^{-\frac{t}{RC}}\right) - I\left(\frac{1-e^{-\frac{T}{2RC}}}{1+e^{-\frac{T}{2RC}}}\right)e^{-\frac{t}{RC}}$$

or $$i_o = I\left(1 - 2\frac{e^{-\frac{t}{RC}}}{1+e^{-\frac{T}{2RC}}}\right)$$

The output voltage V_o or voltage across capacitor V_C is expressed by

$$V_o = V_C = i_o R = IR\left(1 - 2\frac{e^{-\frac{t}{RC}}}{1+e^{-\frac{T}{2RC}}}\right)$$

If t_c is the turn-OFF time of each thyristor, at $t = t_c, V_o = V_C = i_o R = 0$.

Therefore, $$V_o = V_C = i_o R = IR\left(1 - 2\frac{e^{-\frac{t_c}{RC}}}{1+e^{-\frac{T}{2RC}}}\right) = 0$$

or $$e^{-\frac{t_c}{RC}} = \frac{1}{2}\left(1+e^{-\frac{T}{2RC}}\right)$$

or $$t_c = RC\ \ln\left(\frac{2}{1+e^{-\frac{T}{2RC}}}\right) \qquad (11.11)$$

The average value of input voltage is

$$V_{\text{input}} = \frac{2}{T}\int_0^{\frac{T}{2}} i_o R \cdot dt$$

or $$V_{\text{input}} = \frac{2}{T} IR \int_0^{\frac{T}{2}}\left(1 - 2\frac{e^{-\frac{t}{RC}}}{1+e^{-\frac{T}{2RC}}}\right)dt$$

or $$V_{\text{input}} = IR\left[1 - \frac{4RC}{T}\left(\frac{1-e^{-\frac{T}{2RC}}}{1+e^{-\frac{T}{2RC}}}\right)\right]$$

If $V_{\text{input}} \cdot I$ is positive, power delivered to load.

It is observed from Eq. (11.11) that if the inverter frequency is increased, the turn-OFF time t_c which is provided by the circuit decreases. The circuit commutating time t_{off} must be more than the thyristor turn-OFF time t_q for proper operation. Always there is an upper limit to the inverter frequency beyond which thyristors fails to commutate.

If the inverter frequency is low and time period is large, the plot of load current and output voltage with respect to time is shown in Fig. 11.90. Since the waveforms are approximately a square wave, it can be inferred that for low inverter frequencies, inverter has square-wave output for load current I_o or output voltage V_o.

If the inverter frequency is high and time period T is small, waveform of V_o or I_o is approximately sine wave. Therefore, at higher inverter frequency, CSI has sinusoidal wave shape for output load current or load voltage.

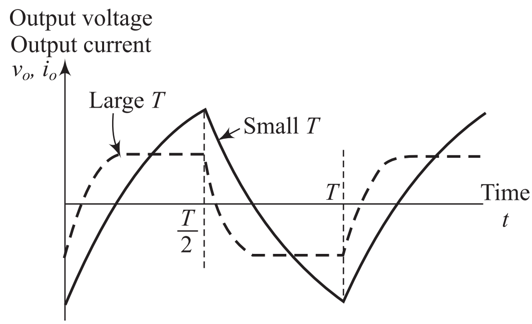


Fig. 11.90 *Output voltage and output current waveforms of a single-phase CSI*

Square-wave current To get square wave output of the load current, $\frac{T}{2RC} > 5.0$

If t_q is the thyristor turn-OFF time, $t_q = t_{\text{off}} = RC \ln \frac{2}{1+e^{-5}} = RC \ln(2) = 0.69RC$

or $$C = \frac{t_q}{0.69R} \tag{11.12}$$

As $$\frac{T}{2RC} = 5, \quad T = 10RC$$

The maximum inverter frequency is expressed by

$$f_{\max} = \frac{1}{T} = \frac{1}{10RC} \tag{11.13}$$

After substituting the value of C in Eq. (11.13), we obtain

$$f_{\max} = \frac{1}{10R} \times \frac{0.69R}{t_q} = \frac{0.069}{t_q}$$

Sine-wave current For sinusoidal output load current, the capacitive reactance X_C at three times the minimum frequency $f_{\min}$ must be less than $\frac{R}{2}$

At $3f_{\min}$, $$X_C < \frac{R}{2}$$

or $$\frac{1}{3 \times 2\pi f_{\min} C} \le \frac{R}{2}$$

or $$C \ge \frac{0.106}{Rf_{\min}}$$

Assuming that the inverter operates at higher frequencies higher $f_{\min}$ in order to obtain the sinusoidal waveform.

Example 11.27 A single-phase capacitor commutated inverter is operated at 50 Hz output frequency with a load resistance of 10 Ω. If the thyristor turn-OFF time is 40 μs, compute (a) the value of commutating capacitor C for proper thyristor commutation, (b) load current, (c) frequency for reliable commutation and (d) value of resistance R.

Solution

Given: $f = 50$ Hz, $R = 10\ \Omega$, $t_q = 40\ \mu$s

(a) If we assume factor of safety is 5, $t_{\text{off}} = 5t_q = 5 \times 40 = 200\ \mu\text{s}$. As f = 50 Hz, T = 20 ms

As $\quad t_{\text{off}} = RC \ln \dfrac{2}{1+e^{-5}} = RC \ln(2) = 0.69RC, \quad t_{\text{off}} = 0.69RC$

Therefore, $\quad 200 \times 10^{-6} = 0.69 \times 10 \times C$

The value of C is C = 28.98 μF

(b) We know that $I_1 = I\left(\dfrac{1-e^{-\frac{T}{2RC}}}{1+e^{-\frac{T}{2RC}}}\right)$ and $e^{-\frac{T}{RC}} = e^{-\frac{20\times10^{-3}}{2\times10\times28.98\times10^{-6}}} = e^{-34.50}$

Then load current $I_1 = I\left(\dfrac{1-e^{-34.50}}{1+e^{-34.50}}\right) \approx I$

(c) For reliable operation is $t_{\text{off}} \geq 1.5t_q \geq 1.5 \times 40 = 60\ \mu\text{s}$

We know that $t_c = RC \ln\left(\dfrac{2}{1+e^{-\frac{T}{2RC}}}\right)$

or $\quad 60 \times 10^{-6} = 10 \times 28.98 \times 10^{-6} \ln\left(\dfrac{2}{1+e^{-\frac{T}{2\times10\times28.98\times10^{-6}}}}\right)$

or $\quad \ln\left(\dfrac{2}{1+e^{-\frac{T}{2\times10\times28.98\times10^{-6}}}}\right) = \dfrac{60}{289.8} = 0.207$

Then $\quad T$ = 271.472 μs

The frequency for reliable commutation is $f = \dfrac{1}{T} = \dfrac{1}{271.472 \times 10^{-6}} = 3683.62\ \text{Hz}$

(c) For successful commutation $t_{\text{off}} \geq 1.5t_q \geq 1.5 \times 40 = 60\ \mu\text{s}$

As $\quad t_{\text{off}} = 0.69RC, 60 \times 10^{-6} = 0.69 \times R \times 28.98 \times 10^{-6}$

Value of resistance is R = 3 Ω

11.19 COMPARISON BETWEEN VOLTAGE SOURCE INVERTER AND CURRENT SOURCE INVERTER

The comparison between voltage source inverter and current source inverter is given in Table 11.7.

Table 11.7 *Comparison between voltage source inverter and current source inverter*

Voltage Source Inverter (VSI)	Current Source Inverter (CSI)
Polarity of dc input voltage is unidirectional.	Polarity of dc input current is unidirectional
Polarity of dc current changes with dc power flow.	Polarity of dc voltage changes with dc power flow.
In VSI, input voltage is maintained constant, but the current may not be constant.	In CSI, input current is maintained constant, but the voltage may not be constant.
DC large capacitance maintains dc voltage constant.	DC large smoothing reactor maintains dc current constant.

Contd.

In VSI, the misfiring of switching devices creates short circuit across source. This is severe problem.	In CSI, input current is maintained constant and the short circuit across the source due to misfiring of switching devices is not a severe problem.
In VSI, amplitude of output voltage does not depends on the load, but amplitude of output current depends on the load.	In CSI, amplitude of output current does not depends on the load, but amplitude of output voltage depends on the load.
The commutation circuits of thyristors in VSI are comparatively complex with respect to CSI.	The commutation circuits of thyristors in CSI are comparatively simpler than that in VSI.
In VSI, freewheeling diodes are required to handle reactive power load.	CSI can be able to handle reactive or regenerative load without freewheeling diodes. Therefore, freewheeling diodes are not required.
In VSI, the fault current contributed by the converter to a dc line fault can not be limited by control action since diodes in the converter will feed into the fault.	In CSI, the fault current contributed by the converter to a dc line fault can be limited by control action and minimized by the large dc smoothing reactor.

Summary

- An inverter is a converter circuit which is used to convert dc power into ac power at desired output voltage and frequency.
- Inverters can provide single-phase and three-phase ac voltages from a fixed or variable dc voltage.
- The classification of inverters is explained in this chapter. The circuit configuration and operating principle of single-phase and three-phase inverters with *R* and *RL* load are discussed elaborately.
- The pulse width modulation inverters are also incorporated in this chapter. Resonant converters and Mc Murray Full-bridge inverters are explained in detail.

Multiple-Choice Questions

11.1 In a voltage source inverter (VSI),
(a) the output voltage waveform V_o does not depend on load impedance Z_L but load current waveform I_o depends on load impedance Z_L
(b) the output voltage waveform V_o depends on load impedance Z_L but load current waveform I_o does not depend on load impedance Z_L
(c) both V_o and I_o waveforms depend on load impedance Z_L
(d) both V_o and I_o waveforms do not depend on load impedance Z_L

11.2 In a current source inverter(CSI),
(a) the output voltage waveform V_o does not depend on load impedance Z_L but load current waveform I_o depends on load impedance Z_L
(b) the output voltage waveform V_o depends on load impedance Z_L but load current waveform I_o does not depend on load impedance Z_L
(c) both V_o and I_o waveforms depend on load impedance Z_L
(d) both V_o and I_o waveforms do not depend on load impedance Z_L

11.3 If the amplitude of output voltage of a single-phase half-bridge inverter is $\frac{V}{2}$ and the output power is P_O, then the output voltage and output power of a single-phase half-bridge inverter are ______ and ______ respectively
(a) $\frac{V}{2}, P_O$ (b) $\frac{V}{2}, 4P_O$ (c) V, P_O (d) $V, 4P_O$

11.4 A single-phase full-bridge voltage source inverter operates with R load, the nature of output current is ______

(a) square wave (b) sine wave (c) triangular wave (d) pulse wave

11.5 A single-phase full-bridge inverter can operate in load commutation mode when the load consists of ______

(a) RLC under damped (b) RLC critically damped
(c) RLC over damped (d) RL or RC load

11.6 A single-phase full-bridge voltage source inverter operates with RL load, the nature of output current is ______

(a) square wave (b) sine wave (c) triangular wave (d) pulse wave

11.7 A single phase full-bridge inverter delivers power to a series connected RLC load with $R = 2\ \Omega$, $j\omega L = j8\Omega$. The load commutation is feasible in inverter operation when $\left|\dfrac{1}{j\omega C}\right|$ is equal to ______

(a) zero Ω (b) $6\ \Omega$ (c) $8\ \Omega$ (d) $10\ \Omega$

11.8 When a three-phase voltage source inverter as shown in Fig. 11.91 operates in 180° conduction mode, the sequence of conduction of switches in the first two steps, beginning with the initiation of switch S_1, are ______ and ______ respectively.

(a) S_1, S_6, S_5 and S_1, S_6, S_2
(b) S_1, S_6, S_2 and S_1, S_6, S_5
(c) S_1, S_6, S_5 and S_1, S_3, S_2
(d) S_1, S_3, S_2 and S_1, S_6, S_2

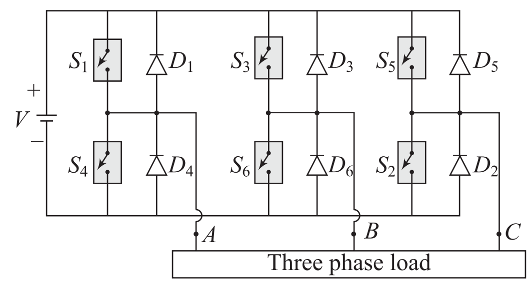


Fig. 11.91

11.9 When a three-phase voltage source inverter as shown in Fig. 11.92 operates in 120° conduction mode, the sequence of conduction of switches in the first two steps, beginning with the initiation of switch S_1, are ______ and ______ respectively.

(a) S_1, S_6 and S_1, S_2 (b) S_1, S_2 and S_1, S_6
(c) S_1, S_6 and S_3, S_2 (d) S_1, S_2 and S_1, S_4

11.10 In single-phase half-bridge inverter, the rms value of fundamental component of output voltage is ______.

(a) $V_{1\text{rms}} = 0.45\ V$ (b) $V_{1\text{rms}} = 0.75\ V$ (c) $V_{1\text{rms}} = 0.9\ V$ (d) $V_{1\text{rms}} = V$

11.11 In single-phase full-bridge inverter, the rms value of fundamental component of output voltage is ______.

(a) $V_{1\text{rms}} = 0.45\ V$ (b) $V_{1\text{rms}} = 0.75\ V$ (c) $V_{1\text{rms}} = 0.9\ V$ (d) $V_{1\text{rms}} = V$

11.12 When a three-phase voltage source inverter operates in 180° conduction mode, the rms line to line voltage is ______.

(a) $V_{L(\text{rms})} = \dfrac{\sqrt{2}}{\sqrt{3}}V$ (b) $V_{L(\text{rms})} = \dfrac{\sqrt{2}}{3}V$ (c) $V_{L(\text{rms})} = \dfrac{1}{\sqrt{3}}V$ (d) $V_{L(\text{rms})} = V$

11.13 When a three-phase voltage source inverter operates in 120° conduction mode, the rms line to line voltage is ______.

(a) $V_{L(\text{rms})} = \dfrac{\sqrt{2}}{\sqrt{3}}V$ (b) $V_{L(\text{rms})} = \dfrac{1}{\sqrt{2}}V$ (c) $V_{L(\text{rms})} = \dfrac{\sqrt{3}}{\sqrt{2}}V$ (d) $V_{L(\text{rms})} = V$

11.14 In a single-pulse width modulation inverter, if the input voltage is V and the pulse width is d, the rms value of output voltage is equal to ______.

(a) $V_{o(\text{rms})} = V\sqrt{\dfrac{d}{\pi}}$ (b) $V_{o(\text{rms})} = V\sqrt{\dfrac{\pi}{d}}$ (c) $V_{o(\text{rms})} = \dfrac{V}{2}\sqrt{\dfrac{d}{\pi}}$ (d) $V_{o(\text{rms})} = \dfrac{V}{2}\sqrt{\dfrac{\pi}{d}}$

11.15 In a single-pulse width modulation inverter, if the input voltage is 220 V and the pulse width is 120°, the rms value of output voltage is equal to ______.
(a) 210.6 V (b) 200.42 V (c) 190.5 V (d) 179.62 V

11.16 In a single-pulse width modulation inverter, if the third harmonic is eliminated, the width of pulse is equal to ______.
(a) 150° (b) 120° (c) 60° (d) 30°

11.17 In a single-pulse width modulation inverter, if the fifth harmonic is eliminated, the width of pulse is equal to ______.
(a) 108° (b) 72° (c) 60° (d) 36°

11.18 In a single-pulse width modulation inverter, if the third harmonic is eliminated, the amplitude of rms value of fundamental component output voltage and the width of pulse are ______ and ______ respectively.
(a) $\frac{\sqrt{6}V}{\pi}, 120°$ (b) $\frac{2\sqrt{3}V}{\pi}, 120°$ (c) $\frac{\sqrt{6}V}{\pi}, 60°$ (d) $\frac{2\sqrt{3}V}{\pi}, 60°$

11.19 In a multi-pulse width modulation (uniform PWM) inverter, if p is number of pulses per half cycle and d is the duration of each pulse, then the rms output voltage is equal to ______.
(a) $V_{\text{rms}} = V\sqrt{\frac{pd}{\pi}}$ (b) $V_{\text{rms}} = V\sqrt{\frac{\pi}{pd}}$ (c) $V_{\text{rms}} = pV\sqrt{\frac{d}{\pi}}$ (d) $V_{\text{rms}} = pV\sqrt{\frac{\pi}{d}}$

11.20 In an uniform pulse width modulation inverter, the amplitude of reference square wave is 1 V and the amplitude of triangular carrier wave is 2 V, If five pulse per half cycle is generated, the pulse width will be ______
(a) 12° (b) 18° (c) 24° (d) 36°

11.21 In a multi-pulse width modulation inverter, the amplitude and frequency of reference square wave and the triangular carrier wave are 4 V, 5 kHz and 1 V, 1 kHz respectively. The number of pulse per half cycle and the pulse width will be ______.
(a) 3, 45° (b) 3, 60° (c) 5, 45° (d) 5, 60°

11.22 In a sinusoidal pulse width modulation inverter, the amplitude and frequency of sinusoidal reference wave and the triangular carrier wave are 1 V, 50 Hz and 5 V, 1 kHz respectively. If the zeros of the triangular carrier wave and sinusoidal reference wave coincide, the modulation index and the order of significant harmonics are ______.
(a) 0.4, 9 and 11 (b) 0.2, 9 and 11 (c) 0.4, 17 and 19 (d) 0.2, 17 and 19

11.23 A single-phase CSI is connected to a capacitor load. For constant current source, the voltage across capacitor is equal to ______.
(a) square wave (b) sine wave (c) triangular wave (d) pulse wave

11.24 In a current source inverter, when the frequency of output voltage is f, the frequency of voltage input to CSI is ______.
(a) f (b) $2f$ (c) $3f$ (d) $4f$

11.25 In a sinusoidal pulse width modulation inverter, there are m cycles of the triangular carrier wave in the half cycle of sinusoidal reference wave. If the zero of the triangular carrier wave coincides with zero of the sinusoidal reference wave, the number of pulses generated in each half cycle are ______.
(a) m (b) $m - 1$ (c) $2m$ (d) $m + 1$

11.26 The simplest method of eliminating third harmonic from the output voltage of a single-phase full-bridge inverter is ______.
(a) single-pulse width modulation (b) uniform pulse width modulation
(c) multi-pulse width modulation (d) stepped wave inverter

11.27 A series capacitor commuted inverter operate satisfactory, when ______.
(a) $\frac{1}{LC} < \left(\frac{R}{2L}\right)^2$ (b) $\frac{1}{LC} = \left(\frac{R}{2L}\right)^2$ (c) $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2$ (d) None of these

11.28 The pulse width modulation technique is used in single-phase inverter for which of the following reasons:
(a) Reduce lower order harmonics and increase higher order harmonics.
(b) Increase lower order harmonics and reduce higher order harmonics.
(c) Reduce the total harmonic distortion.
(d) None of the above

11.29 The output voltage waveform of a three-phase square wave inverter consists of
(a) only odd harmonics (b) only even harmonics
(c) both odd and even harmonics

11.30 In a series resonant inverter,
(a) The load current is a square waveform
(b) The output voltage waveform depends on the damping factor of load impedance
(c) The trigger frequency is higher than the damped resonant frequency
(d) None of the above

11.31 In a single-phase bridge inverter, the maximum value of fundamental component of load current is equal to I. If the load is purely resistive, the maximum value of nth harmonic component of load current is equal to
(a) I (b) $\frac{I}{n}$ (c) $\frac{I}{n^2}$ (d) $\frac{I}{n\sqrt{n}}$

11.32 In a single-phase bridge inverter, the maximum value of fundamental component of load current is equal to I. If the load is highly inductive, the maximum value of nth harmonic component of load current is equal to
(a) I (b) $\frac{I}{n}$ (c) $\frac{I}{n^2}$ (d) $\frac{I}{n\sqrt{n}}$

11.33 In a single-phase bridge inverter, the maximum value of fundamental component of load current is equal to I. If the load is highly capacitive, the maximum value of nth harmonic component of load current is equal to
(a) I (b) $\frac{I}{n}$
(c) $\frac{I}{n^2}$ (d) $\frac{I}{n\sqrt{n}}$

11.34 The rms value of output voltage waveform as depicted in Fig. 11.92 is
(a) 100 V (b) $\frac{100}{\pi}V$
(c) 200 V (d) $\frac{200}{\pi}V$

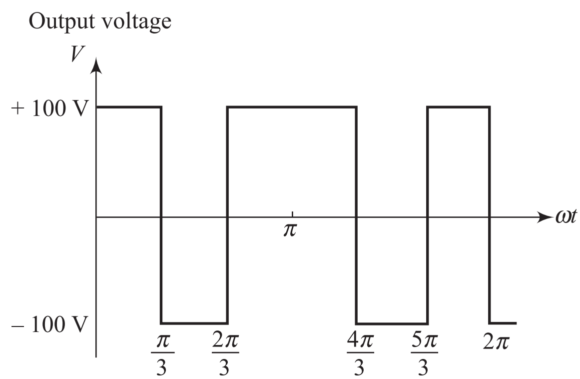


Fig. 11.92

11.35 The output voltage of a three-phase square wave inverter does not contain third harmonics for
(i) line voltages in 180° conduction mode
(ii) phase voltages in 180° conduction mode
(iii) line voltages in 120° conduction mode
(iv) phase voltages in 120° conduction mode
(a) i, ii, iii (b) i, iv (c) ii, iii, iv (d) i, ii, iii, iv

11.36 If input dc voltage is constant, the output voltage of a single-phase bridge inverter can be controlled by
(a) pulse width modulation (b) changing the switching frequency
(c) pulse amplitude modulation (d) All of these

11.37 The output voltage waveforms of a voltage source inverter V_{o1} and V_{o2} are depicted in Fig. 11.93. The rms output voltage V_{12} between terminal 1 and 2 is

(a) $V\sqrt{\dfrac{\phi}{2\pi}}$ (b) $V\sqrt{\dfrac{\phi}{\pi}}$ (c) $\dfrac{V}{\pi}$ (d) $\dfrac{V\phi}{\pi\sqrt{2}}$

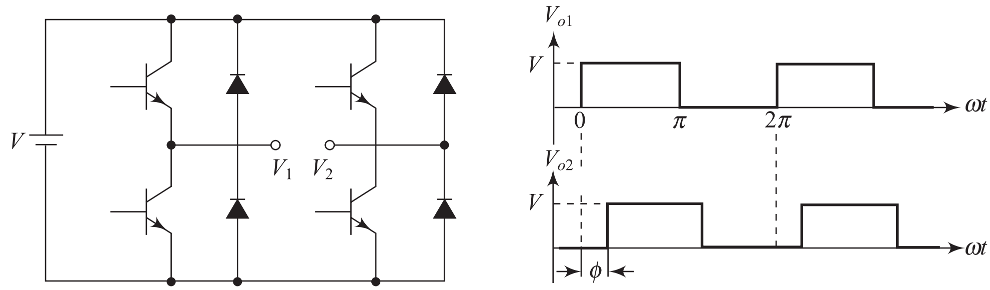


Fig. 11.93

11.38 A three-phase voltage source inverter is connected to purely inductive three phase load. From Fourier analysis of output voltage waveform, it is found that the amplitude of nth order harmonic voltage is α_n times of fundamental component where $\alpha_n < 1$. Then the amplitude of nth order harmonic current is

(a) α_n times of fundamental component current
(b) $n \cdot \alpha_n$ times of fundamental component current
(c) $\dfrac{\alpha_n}{n}$ times of fundamental component current
(d) zero

11.39 Match List I and List II and find the correct matching.

List I	List II
1. Voltage source inverter	A. DC to AC converter (inverter) with inductive load
2. Current source inverter	B. Voltage spites in the output voltage
3. Freewheeling diode	C. Phase controlled converter with inductive load
4. Feedback diode	D. Peaks in the inverter current

(a) 1-D, 2-A, 3-C, 4-B (b) 1-D, 2-B, 3-C, 4-A
(c) 1-A, 2-D, 3-C, 4-B (d) 1-D, 2-A, 3-B, 4-C

11.40 Figure 11.94 shows a periodic output voltage waveform of VSI. If the conduction angle is $\alpha = 120°$, the rms value of fundamental component of the output voltage is

(a) 100 V (b) 110 V
(c) 90 V (d) 78 V

11.41 A periodic output voltage waveform of VSI is shown in Fig. 11.94. What will be the conduction angle α for which the output voltage will be free from 5th harmonic

(a) $\alpha = 180°$ (b) $\alpha = 150°$
(c) $\alpha = 120°$ (d) $\alpha = 72°$

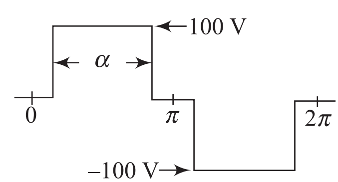


Fig. 11.94

11.42 McMurray commutation is superior compared to parallel capacitor commutation in respect of

(a) over voltage spike at the output (b) number of components used
(c) reduction in current through thyristor (d) trigger circuit

11.43 A single-phase bridge inverter can be implemented using thyristors without forced commutation circuit when the load is
(a) series combination of *R*, *L* and *C* with resonant frequency of the circuit is greater than the inverter switching frequency
(b) series combination of *R*, *L* and *C* with resonant frequency of the circuit is less than the inverter switching frequency
(c) series combination of *R* and *L*
(d) series combination of *R* and *C*

11.44 Match the List I and List II and find the correct matching.

List I	List I
1. Voltage source inverter	A. Voltage depend on the values of *R* and *L* of load
2. Current source inverter	B. Voltage constant and independent of *R* and *L* of load
3. Phase controlled rectifier with *RL* load	C. Depends on firing angle
4. Single-pulse converter with *RL* load	D. Depends on firing angle and also impedance angle of load

(a) 1-D, 2-A, 3-C, 4-B (b) 1-B, 2-A, 3-C, 4-D
(c) 1-A, 2-D, 3-C, 4-B (d) 1-D, 2-A, 3-B, 4-C

11.45 In McMurray commutation inverter circuit, the circuit turn-OFF time is
(a) independent of load current and depend on operating frequency
(b) depend on load current and independent of operating frequency
(c) depend on load current and also load power factor
(d) independent of load current and depend on recovery period

11.46 The operating frequency of a self-oscillating inverter using a saturable core depends on
(a) battery voltage and saturation flux density
(b) battery voltage
(c) load power factor
(d) battery voltage, saturation flux density and number of turns on primary winding

Fill in the Blanks

11.1 An ______ is a converter circuit which is used to convert dc power into ac power at desired output voltage and frequency.
11.2 The gate commutation switching devices such as ______ are used in inverter.
11.3 The ______ is commonly used in very high power applications such as induction motor drives.
11.4 ______ inverters are complementary commutated inverters.
11.5 ______ is defined by the ratio of the rms value of the total harmonic component of the output voltage to the rms value of the fundamental component.
11.6 ______ is defined by the ratio of the rms voltage of a particular harmonic component to the rms voltage of the fundamental component.
11.7 When a single phase full-bridge inverter operates with dc source voltage *V*, the output voltage can be expressed as ______
11.8 A three-phase inverter can be formed after combining ______ single-phase inverters in parallel.
11.9 A three-phase inverter can be operated in ______ conduction modes.
11.10 The output voltage of single-pulse width modulated inverter can be controlled by controlling the width of pulse *d* which depends upon ______.

11.11 To eliminate third harmonic from the output voltage of single-pulse width modulated inverter, the required pulse width is equal to ______.

11.12 In modified sinusoidal pulse width modulation, the carrier signal is applied during the fast and last ______ interval of each half cycle.

11.13 In ______ each semiconductor switch of converter circuit changes its state either from OFF to ON or ON to OFF when voltage across device becomes zero.

11.14 In ______ each semiconductor switch of converter circuit changes its state either from OFF to ON or ON to OFF when current through device is zero.

11.15 Resonant converters operates using ______ switching technique.

11.16 For proper operation of modified McMurray half-bridge inverter, the value commutating capacitor is ______.

11.17 For proper operation of modified McMurray half-bridge Inverter, the value commutating inductor is ______.

11.18 The value of resistance which provides critical damping in modified McMurray half-bridge inverter is ______.

11.19 In McMurray Bedford half-bridge inverter circuit, the circuit turn-OFF time t_c is ______ than one quarter of a cycle.

11.20 For proper operation of Modified McMurray Bedford half-bridge inverter, the value commutating capacitor is ______.

11.21 For proper operation of Modified McMurray Bedford Half-bridge Inverter, the value commutating inductor is ______.

11.22 In a three-phase bridge inverter, the Fourier series expansion of line to neutral voltage is expressed by ______.

11.23 If two inverters are connected in series, the phasor sum of two fundamental voltages V_{o1} and V_{o2} is ______.

11.24 In ______, amplitude of output voltage does not depend on the load, but amplitude of output current depends on the load.

11.25 In ______, amplitude of output current does not depend on the load, but amplitude of output voltage depends on the load.

Review Questions

11.1 Define inverter. What are the types of inverter? What are the applications of inverter circuit? What are the switching devices used in inverter circuit?

11.2 Explain the operating principle of inverter with a suitable diagram. Draw the voltage and current waveforms of inverter. Derive the expression for rms output voltage.

11.3 Discuss the classification of inverters. What are the performance parameters of inverters?

11.4 What do you mean by VSI and CSI?

11.5 (a) Draw the circuit diagram of a single phase half-bridge voltage source inverter and explain its operating principle with R load and (b) Derive the expression for
(i) average value of output voltage, (ii) rms value of output voltage and (iii) rms value of fundamental component.

11.6 What are the drawbacks of single-phase half-bridge voltage source inverter? Explain how these drawbacks can be overcome.

11.7 (a) Draw the circuit diagram of a single-phase half-bridge voltage source inverter with RL load and explain its operating principle and (b) Derive the expression for (i) rms value of output voltage, (ii) rms value of fundamental component and (iii) output load current.

11.8 Prove that 'the output power of a single-phase full-bridge inverter is four times of the output power of a single-phase half-bridge inverter'.

11.9 Why diodes are connected in anti-parallel with semiconductor switches in inverter circuit?

11.10 (a) Draw the circuit diagram of a single phase full-bridge voltage source inverter and explain its operating principle with R load and (b) Derive the expression for (i) Average value of output voltage, (ii) rms value of output voltage and (iii) rms value of fundamental component

11.11 (a) Draw the circuit diagram of a single-phase full-bridge voltage source inverter with RL load and explain its operating principle and (b) Derive the expression for (i) rms value of output voltage, (ii) rms value of fundamental component and (iii) output load current.

11.12 Write Fourier series expression for output voltages of single-phase half-bridge and full-bridge inverters.

11.13 What is a three-phase inverter? What are the applications of three-phase inverter? What are the switching devices used in three-phase inverter circuit?

11.14 Discuss the operating principle of a three-phase bridge inverter with a suitable diagram when each semiconductor switch conducts for 180°.

11.15 (a) Draw the phase voltage and line voltage waveforms of a three-phase bridge inverter with star connected R load when each semiconductor switch conducts for 180° and (b) Derive the expression for (i) rms value of line to line voltage, (ii) rms value of phase voltage, (iii) rms value of fundamental component of line voltage, (iv) power delivered to load, (v) rms value of load current, (vi) rms value of current flows through semiconductor switch and (vii) average source current.

11.16 Explain the operating principle of a three-phase bridge inverter with a suitable diagram when each semiconductor switch conducts for 120°.

11.17 (a) Draw the phase voltage and line voltage waveforms of a three-phase bridge inverter with star connected R load when each semiconductor switch conducts for 120° and (b) Derive the expression for (i) rms value of phase voltage, (ii) rms value of line voltage, (iii) rms value of fundamental component of line voltage, (iv) power delivered to load, (v) rms value of load current, (vi) rms value of current flows through semiconductor switch and (vii) average source current.

11.18 What is pulse width modulated inverters? What are the different PWM techniques used in inverter? Explain any one PWM technique with suitable diagram.

11.19 What is the need for controlling the output voltage of an inverter? What are different techniques used to control the output voltage of an inverter? Differentiate between different techniques.

11.20 Write short notes on the following:
(a) Single pulse width modulation (SPWM)
(b) Multi pulse width modulation (MPWM)
(c) Sinusoidal Pulse Width Modulation (SinPWM)
(d) Modified Sinusoidal Pulse Width Modulation

11.21 Prove that the output voltage of single pulse modulation inverter can be expressed as

$$v_o = \frac{4V}{\pi}\left[\sin\frac{d}{2}\sin\omega t - \frac{1}{3}\sin\frac{3d}{2}\sin 3\omega t + \frac{1}{5}\sin\frac{5d}{2}\sin 5\omega t - \frac{1}{7}\sin\frac{7d}{2}\sin 7\omega t + \cdots\right]$$

where d is the pulse width.

11.22 Describe sinusoidal pulse width modulation inverter with proper diagram. Discuss the condition under which the number of pulses generated per half cycle is $\left(\frac{f_c}{2f_r} - 1\right) = (m-1)$ where f_c is the frequency of carrier signal and f_r is the frequency of sinusoidal reference signal.

11.23 What is the need for harmonic reduction in output voltage of an inverter? What are the different methods used for harmonic reduction in output voltage of inverter?

11.24 Write short notes on the following:
(a) Harmonic Reduction Using PWM Technique
(b) Harmonic Reduction by Series Connected Inverters
(c) Harmonic Reduction by Stepped Wave Inverters

11.25 Compare the advantages and disadvantages of half-bridge and full-bridge inverters.

11.26 What are the advantages of eliminating lower-order harmonics from the output voltage of an inverter?

11.27 What are the advantages and disadvantages of a current source inverter?

11.28 Compare 180° and 120° conduction mode of three phase bridge inverter.

11.29 Draw the block diagram of resonant converter and explain its operating principle. What are advantages of resonant converter over other converters?

11.30 State the advantages of ZVS and ZCS.

11.31 Draw the circuit diagram of series resonant converter and explain its operation with voltage and current waveforms.

11.32 What are type's resonant converters? Discuss the operation of self-commutating or load resonating converters with circuit diagram and voltage and current waveforms.

11.33 Draw the circuit diagram of parallel loaded resonant converter and explain its operation with voltage and current waveforms.

11.34 A single-phase half-bridge inverter feeds a resistive load of 15 Ω. When the voltage $\frac{V}{2}$ is 200 V, determine

(a) rms value of the fundamental component of output voltage
(b) the output power
(c) the average and peak current of transistors which are used in inverter
(d) the peak inverse voltage (PIV) of transistors
(e) the lowest order harmonics and the corresponding harmonic factor
(f) third harmonic distortion factor

11.35 A single-phase half-bridge inverter feeds an *RL* load with $R = 10\ \Omega$ and $L = 0.01$ H. When the voltage $\frac{V}{2}$ is 100 V and the frequency of output voltage is 50 Hz, find

(a) The output current for the first two half cycles of output voltage
(b) Third harmonic distortion factor (THD) of load current

11.36 A single-phase full-bridge inverter feeds a resistive load of 5 Ω. When the dc voltage source is 120 V, determine

(a) rms value of the fundamental component of output voltage
(b) the output power
(c) the average and peak current of transistors which are used in inverter
(d) the peak inverse voltage (PIV) of transistors
(e) the lowest order harmonics and the corresponding harmonic factor
(f) third harmonic distortion factor

11.37 A full-bridge inverter has a dc source voltage of 240 V. The inverter supplies a *RLC* load with $R = 5\ \Omega$, $L = 0.1$ H and $C = 4.7\ \mu$F. The operating frequency of inverter is 600 Hz. Determine (a) rms load current at fundamental frequency, (b) the rms value of load current, (c) power output, (d) average supply current and (e) THD in load current.

11.38 A three-phase bridge inverter is fed from 200 V dc supply. If the semiconductor switches (transistors) which are used in inverter conducts for 180° duration and the inverter is supplying a star connected resistive load of 20 Ω, determine

(a) rms value of per phase voltage and line voltage
(b) rms value of load current
(c) rms value of current flows through transistors
(d) power delivered to load
(e) average source current

11.39 A three-phase bridge inverter is fed from 400 V dc supply. If the semiconductor switches (transistors) which are used in inverter conducts for 120° duration and the inverter is supplying a star connected resistive load of 5 Ω, determine

(a) rms value of per phase voltage and line voltage
(b) rms value of load current
(c) rms value of current flows through transistors
(d) power delivered to load
(e) average source current

11.40 A single phase PWM inverter is fed from a 200 V dc supply and it is connected to a RL load with $R = 5\ \Omega$ and $L = 5$ mH. Determine the total harmonic distortion in the load current. Assume width of each pulse is $\frac{\pi}{2}$ and the output frequency is 50 Hz.

11.41 The output voltage of multipluse modulation inverter is shown in Fig. 11.95. Determine
(a) the rms value of output voltage
(b) the rms value of fundamental component of output voltage
(c) the total harmonic distortion

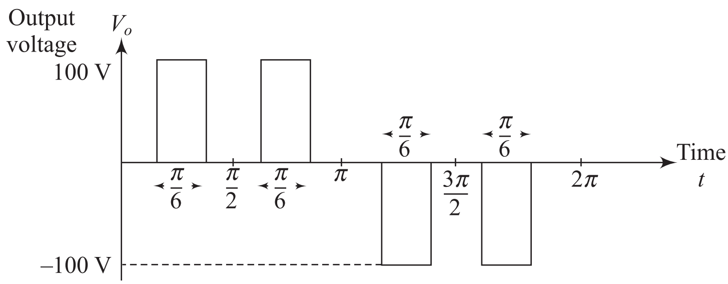


Fig. 11.95

11.42 A series resonant RLC inverter using thyristors has the following parameters:

$$R = 1\ \Omega,\ L_r = 0.2\text{ mH},\ C_r = 10\ \mu\text{F and } t_q = 15\ \mu\text{s}$$

Find the maximum switching frequency for non-overlap operation of the series resonant inverter.

11.43 A single-phase series resonant RLC inverter delivers power to load with $R = 2.5\ \Omega$ and $X_L = 15\ \Omega$. If the time period is 0.1 ms, determine the value of C so that load commutation of thyristor is possible. Assume thyristor turn-OFF time is 12 μs.

11.44 Find the value of inductance of a series resonant RLC inverter when it operates at frequency 15 kHz and its capacitor value is 1 μF. Assume inverter operates at angular undamped natural frequency ω_o

11.45 A series resonant RLC inverter using thyristors has the following parameters:

$$R = 90\Omega,\ L_r = 6.0\text{ mH},\ C_r = 1.2\ \mu\text{F and } t_{\text{off}} = 0.25\text{ ms}$$

(a) Determine the value of output frequency.
(b) If the load resistance varies from 50 Ω to 150 Ω, find the range of output frequency for non-overlap operation of the series resonant inverter.

11.46 A single-phase half-bridge inverter has a resistive load of 5 Ω and the centre tap dc input voltage is 96 V. Determine (a) rms value of output voltage, (b) rms value of fundamental component of output voltage, (c) first three harmonics of the output voltage waveform, (d) fundamental power consumption in load and (e) rms power consumed by load.

11.47 A single-phase full-bridge inverter has a resistive load of 12 Ω and it is operated from a 120 V dc input voltage. Determine (a) rms value of output voltage, (b) rms value of fundamental component of output voltage, (c) first three harmonics of the output voltage waveform, (d) fundamental power consumption in load, (e) rms power consumed by load and (f) transistor rating.

11.48 A single-phase transistorized full-bridge inverter has a resistive load of 5 Ω and it is operated from a 48 V dc input voltage. Determine (a) total harmonic distortion, (b) distortion factor, (c) harmonic factor and distortion factor at lowest order harmonic and (d) transistor ratings.

11.49 A single-phase transistorized half-bridge bipolar PWM inverter is operated from a center tap 96 V dc input voltage. The fundamental output frequency is 50 Hz and the carrier frequency is 1.5 kHz and modulation index is 0.8. Determine (a) carrier ratio m_f, (b) number of pulses per cycle, (c) fundamental output voltage, (d) distortion factor of output voltage waveform and (e) harmonic factor of output voltage waveform.

11.50 A single-phase transistorized full-bridge bipolar PWM inverter is operated from a 120 V dc battery and it is connected with a *RL* load. If the modulation index is 0.8, determine (a) rms output voltage, (b) fundamental output voltage, (c) distortion factor of output voltage waveform, (d) harmonic factor of output voltage waveform and (e) gain of inverter.

Answers to Multiple-Choice Questions

11.1 (a)	11.2 (b)	11.3 (d)	11.4 (a)	11.5 (a)	11.6 (c)	11.7 (d)
11.8 (a)	11.9 (a)	11.10 (a)	11.11 (c)	11.12 (a)	11.13 (b)	11.14 (a)
11.15 (d)	11.16 (b)	11.17 (b)	11.18 (a)	11.19 (a)	11.20 (b)	11.21 (a)
11.22 (d)	11.23 (c)	11.24 (b)	11.25 (b)	11.26 (a)	11.27 (c)	11.28 (a)
11.29 (a)	11.30 (b)	11.31 (b)	11.32 (c)	11.33 (a)	11.34 (a)	11.35 (d)
11.36 (a)	11.37 (b)	11.38 (c)	11.39 (b)	11.40 (d)	11.41 (d)	11.42 (a)
11.43 (a)	11.44 (b)	11.45 (b)	11.46 (c)			

Answers to Fill in the Blanks

11.1 inverter

11.2 SCRs, GTOs, BJTs, MOSFETs and IGBTs

11.3 CSI

11.4 McMurray Bedford

11.5 THD

11.6 HF_n

11.7 $v_o(t) = \sum_{n=1,2,3...}^{\infty} \frac{4V}{n\pi} \sin n\omega t$

11.8 three half-bridge

11.9 180° and 120°

11.10 amplitude modulation index

11.11 $d = 120°$

11.12 $\frac{\pi}{3}$ or 60°

11.13 *ZVS*

11.14 *ZCS*

11.15 *ZVS or ZCS*

11.16 $C = 0.892 \frac{t_c I_{o\max}}{V_{\min}}$

11.17 $L = 0.3964 \frac{t_c V_{\min}}{I_{o\max}}$

11.18 $R_d = 2\sqrt{\frac{L}{C}}$

11.19 less

11.20 $C = 2.35 \frac{I_{o\max} t_q}{V}$

11.21 $L = 2.35 \frac{V t_q}{I_{o\max}}$

11.22 $V_{an} = \sum_{n=6k\pm1}^{\infty} \frac{2V}{n\pi} \sin n\omega t$ where $k = 0, 1, 2, 3...$

11.23 $V_o = \sqrt{V_{01}^2 + V_{02}^2 + 2 \cdot V_{01} \cdot V_{02} \cos\phi}$

11.24 VSI

11.25 CSI

APPLICATIONS OF POWER ELECTRONICS IN ELECTRICAL DRIVES, POWER FACTOR IMPROVEMENT, UPS

12

12.1 INTRODUCTION

Electrical drives are available in very wide power range from a few watts to several MW. These drives are commonly used in many industrial applications starting from very precise high performance position control systems in robotics to variable speed drives in pumps for flow control. In all electrical drives, the position or speed is controlled by a power electronics converter. Basically, power electronics converter is used for interfacing between input power supply and electric motor. Figure 12.1 shows a block diagram of a electric drive system which consists of electric motor, power electronics converters and energy transmitting shafts to transfer mechanical energy from motor to load.

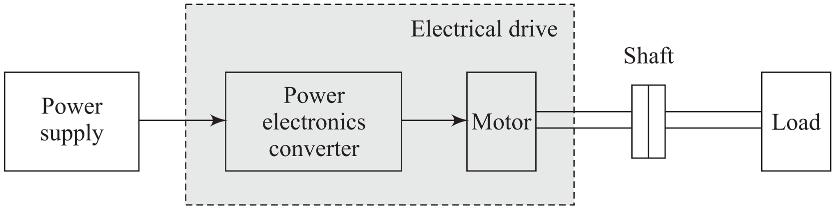


Fig. 12.1 *An open loop electrical drive*

Presently, all modern electrical drive systems operate with closed loop system as depicted in Fig. 12.2. The output of a *power electronics converter* may be a variable dc or a variable ac with variable voltage and frequency. The output of a power converter depends upon the requirement of load. When the load is a three-phase induction motor, the converter output will be adjustable ac voltage and frequency. If the load is dc motor, the converter output will be adjustable dc voltage.

The *feedback signals* are the measured parameters of the load, i.e., speed and position. These signals are used as input signals of controller. The command signals are also applied to the controller. Then the feedback signals are compared with the reference or

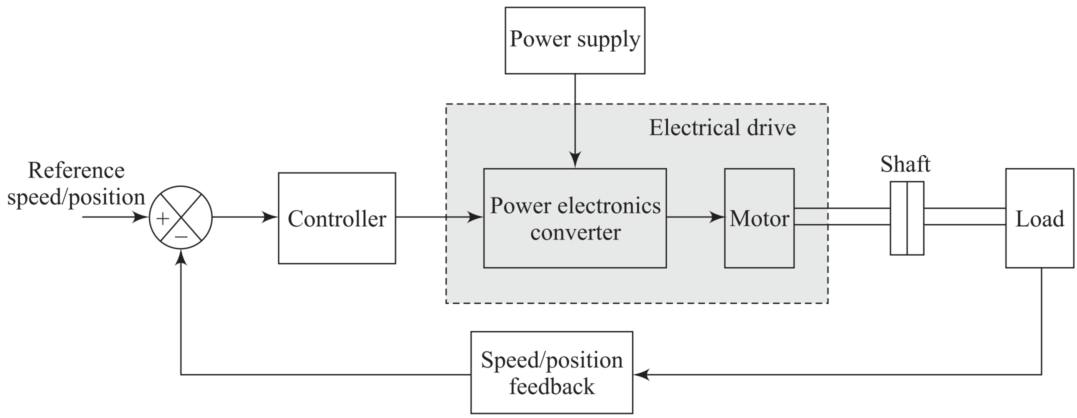


Fig. 12.2 *A closed loop electrical drive*

command signals and accordingly the control signals are generated by the controller to turn ON the semiconductor switches of power converter. Consequently, we can get the required output at the load.

The *control circuit* or *controller* is the heart of power electronics converters. The control circuit generates triggering pulses to control thyristors of power electronics converters. The control circuit is a low power circuit which is built using analog circuits, digital circuits, microprocessors or microcontrollers or personnel computer.

Usually, there are three types of electrical drives such as

- DC motor drives
- Induction motor drives
- Synchronous motor drives

In this chapter, power electronics-based dc motor drives, induction motor drives and synchronous motor drives are discussed elaborately.

12.2 DC MOTOR DRIVES

DC motors are extensively used in variable or adjustable speed and position control drives. They provide high starting torque and the speed control of dc motor is possible over a wide range. Usually, separately excited dc motors and dc series motors are used in variable speed drives but dc series motors are extensively employed for traction applications. The speed of dc motor can be controlled by armature voltage control and field-flux control. In armature voltage control, the motor speed can be controlled below base speed or rated speed but using field-flux control, the motor speed can be controlled above base speed or rated speed.

Controlled rectifiers provide a variable dc output voltage from a fixed ac input voltage by changing the firing angle of thyristors. DC-to-dc converters or choppers can provide a variable dc output voltage from a fixed dc voltage. Since controlled rectifiers and choppers can provide a continuously variable dc voltage, the applications of controlled rectifiers and choppers in speed control of dc motors is a revolution in modern industrial variable speed drives from a few watts to several MW. Figure 12.3 shows a controlled rectifier fed dc drives and the block diagram of chopper fed dc drives is depicted in Fig. 12.4. Depending upon the type of power source, dc drives can be classified as

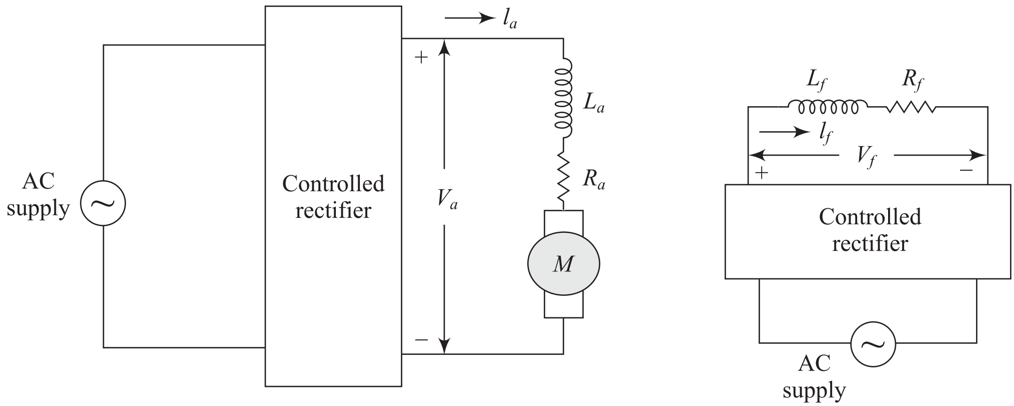


Fig. 12.3 *Controlled rectifier fed dc drive*

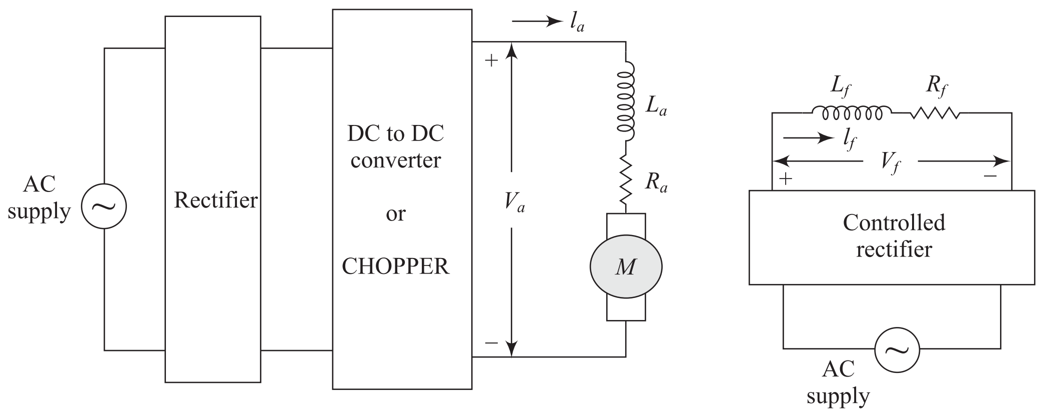


Fig. 12.4 *Chopper fed dc drive*

1. Single-phase controlled rectifier fed dc drive
2. Three-phase controlled rectifier fed dc drive
3. Chopper fed dc drive

In this chapter, initially the basic characteristics of dc motors are discussed and subsequently the speed control strategies of dc motors are explained.

12.2.1 Separately Excited DC Motor

The equivalent circuit of separately excited dc motor which is couple to a load is shown in Fig. 12.5. In Fig. 12.5,

V_a = Armature voltage,	I_a = Steady state armature current,
i_a = Instantaneous armature current,	E_b = Back emf,
V_f = Field voltage	I_f = Steady state field current,
i_f = Instantaneous field current,	L_a = Inductance of armature winding,
R_a = Resistance of armature winding,	L_f = Inductance of field winding,
R_f = Resistance of field winding,	T_e = Electromagnetic torque,
T_L = Load torque,	ω = Speed and B = Damping constant

The field current i_f is independent of the armature current i_a of a separately excited dc motor. Therefore, the change in the armature current has no effect on the field current. The value of field current is less than the armature current. The KVL equation in the field circuit is

$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

The KVL equation in the armature circuit is

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E_b$$

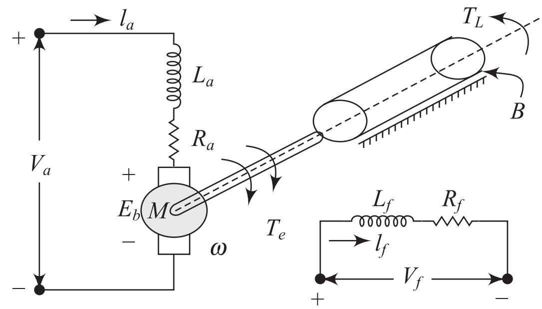


Fig. 12.5 *Equivalent circuit of a separately excited dc motor*

The back emf of motor is $E_b = k_b \omega i_f$ where, k_b is the back emf constant

The electromagnetic torque developed by the motor is

$$T_e = k_t i_f i_a \quad \text{where, } k_t \text{ is the torque constant}$$

The relation between the electromagnetic torque and load torque is

$$T_e = J\frac{d\omega}{dt} + B\omega + T_L$$

At steady state condition, the time derivative in the above equations becomes zero and the above equations can be written as

$$V_f = R_f I_f$$

$$V_a = R_a I_a + E_b = R_a I_a + k_b \omega I_f \qquad \text{as } E_b = k_b \omega I_f$$

$$T_e = B\omega + T_L$$

Electromagnetic power developed is

$$P_d = T_e \omega$$

Then $$\omega = \frac{V_a - I_a r_a}{k_b I_f}$$

or $$\omega \propto \frac{V_a - I_a r_a}{\phi} \text{ where, } \phi \propto I_f$$

It is clear from above equation that the speed of a separately excited dc motor can be controlled by

1. armature voltage control
2. field current or field flux control

The speed at rated armature voltage is called *rated speed* or *base speed*. The speed less than base speed is controlled by armature control, armature and field current are constant to maintain the torque demand but armature voltage is varied.

In the field control, speed is greater than the base speed is possible where armature current maintained at its rated value and the field current varied to control the speed of motor. Power developed by the motor is $P_d = T_e\omega$ which is remaining constant. The characteristics of a separately excited dc motor are depicted in Fig. 12.6.

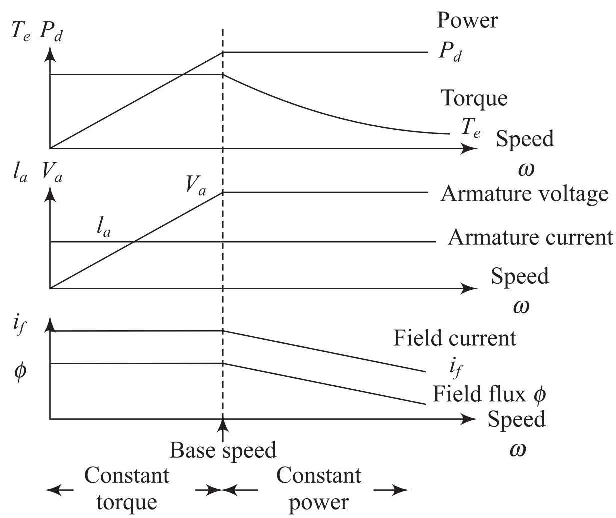


Fig. 12.6 *Characteristics of a separately excited dc motor*

12.2.2 DC Series Motor

Figure 12.7 shows the equivalent circuit of a dc series motor where field winding is connected in series with armature. The back emf E_b, armature voltage V_a, electromagnetic torque T_e of dc series motor can be expressed by

$$E_b = k_b \omega I_f = k_b \omega I_a \quad \text{as} \quad I_a = I_f$$

$$V_a = R_f i_f + L_f \frac{di_f}{dt} + R_a i_a + L_a \frac{di_a}{dt} + E_b$$

$$T_e = k_t i_f i_a$$

$$T_e = J \frac{d\omega}{dt} + B\omega + T_L$$

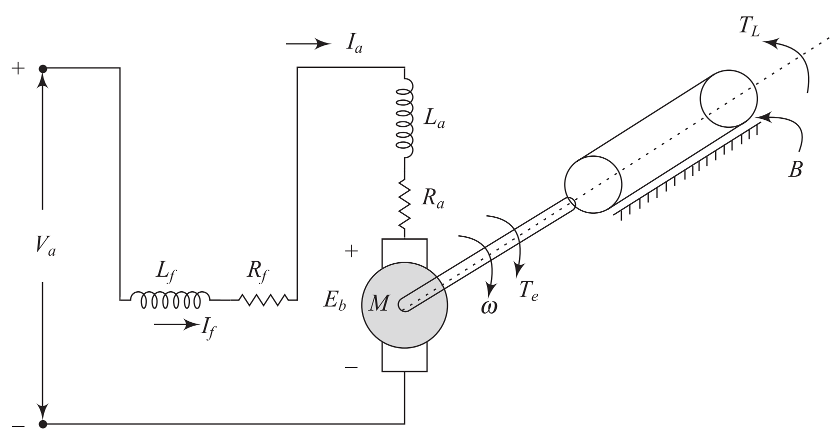


Fig. 12.7 *Equivalent circuit of a dc series motor*

At steady state condition, the time derivative in the above equations becomes zero and the above equations can be written as

$$E_b = k_b \omega I_f = k_b \omega I_a$$

$$V_a = (R_f + R_a)I_a + E_b = (R_f + R_a)I_a + k_b \omega I_a$$

$$T_e = B\omega + T_L$$

The speed of the motor is equal to

$$\omega = \frac{V_a - (R_f + R_a)I_a}{k_b I_a}$$

The speed can be controlled by armature voltage and armature current.

At starting dc series motor provides very high torque during starting. Therefore, dc series motors are commonly used in electric traction applications.

To control speed below base speed, armature voltage is varied when armature current kept constant. Therefore, power P_d linearly varies with armature voltage V_a as $P_d = V_a I_a = T_e \omega$ and the electromagnetic torque $T_e = k_t I_a^2$ remains constant. For speed control above base speed, the flux ϕ must be decreased by using a diverter and I_a is constant. Due to decrease of flux ϕ, electromagnetic torque decreases as $T_e = k_t \phi I_a$. Subsequently, power P_d remains constant. The characteristics of dc series motor are shown in Fig. 12.8.

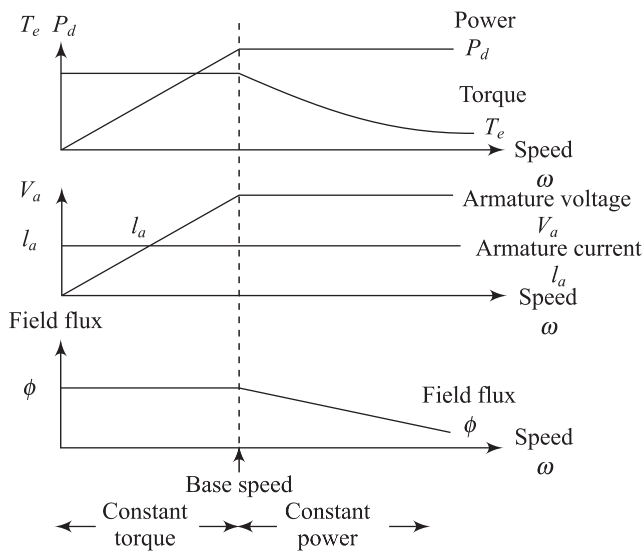


Fig. 12.8 *Characteristics of a dc series motor*

12.3 SINGLE-PHASE DC DRIVE

The simplest representation of a single phase dc drive is shown in Fig. 12.9 where armature and field circuits of dc motor are connected to the output of controlled rectifier. By varying the firing angle of thyristors of the controlled rectifier, the voltage applied to armature and field windings of dc motor can be controlled. This drive can be operating either in continuous or discontinuous operating mode.

At low value of firing angle, armature current is continuous. If the firing angle is increased, armature current becomes discontinuous. Due to discontinuous armature current, the losses in motor increased significantly with poor speed regulation. The smoothing inductor L is connected in series with the armature circuit to reduce the armature ripple current. The armature current will be continuous at low value of motor speed. Based on the power electronics converter circuit, single-phase dc drives are classified as the following:

1. Single-phase half wave converter fed dc drives

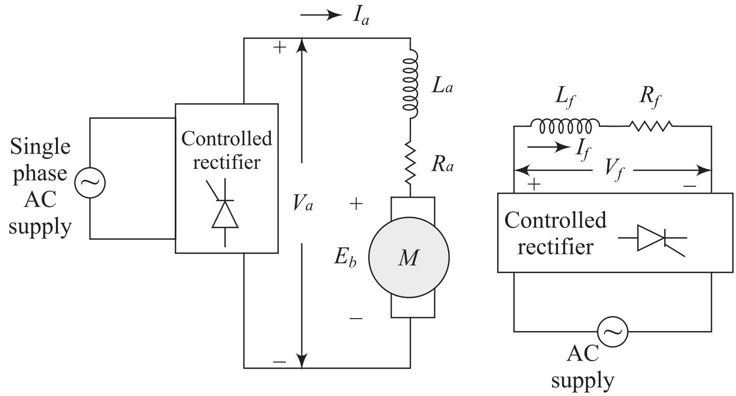


Fig. 12.9 *Representation of a single-phase dc drive*

2. Single-phase semi converter fed dc drives
3. Single-phase full converter fed dc drives
4. Single-phase dual converter fed dc drives

12.3.1 Single-phase Half-wave Converter Fed DC Drives

Figure 12.10 shows a single-phase half-wave converter fed separately excited dc drives. The motor field winding is supplied by a single-phase semi-converter and the amplitude of field current through field winding is ripple free. If a half-wave converter is applied in the field circuit, the field current contains high ripple and subsequently the magnetic losses of the dc motor will be increased significantly.

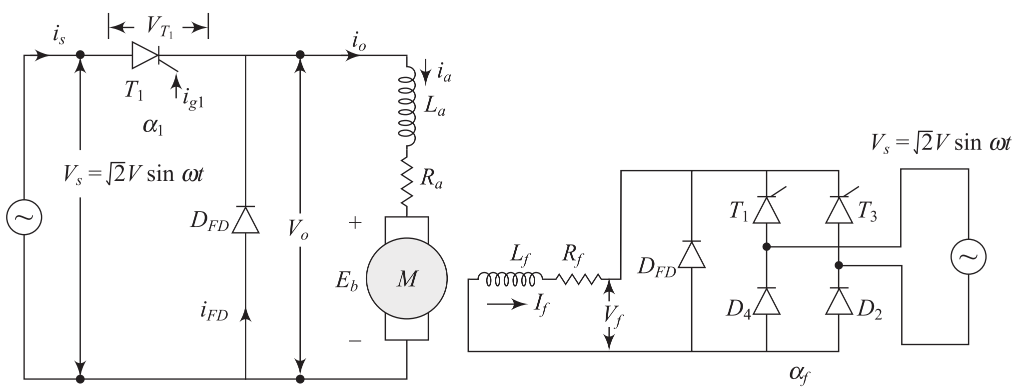


Fig. 12.10 *Single-phase half-wave converter fed DC drive*

Usually, the armature current is discontinuous unless a very large inductance L is connected in series with armature. A freewheeling diode must be connected across armature and the drive operates in one quadrant as shown Fig. 12.11. Assume that armature current is ripple free. Then the waveforms of input ac voltage V_s, armature terminal voltage V_o, source current i_S, thyristor current i_{T1} and current through freewheeling diode i_{FD} for this drive are depicted in Fig. 12.11. This drive is suitable up to $\frac{1}{2}$ kW dc power drive and can operate in the first quadrant only as depicted in Fig. 12.11.

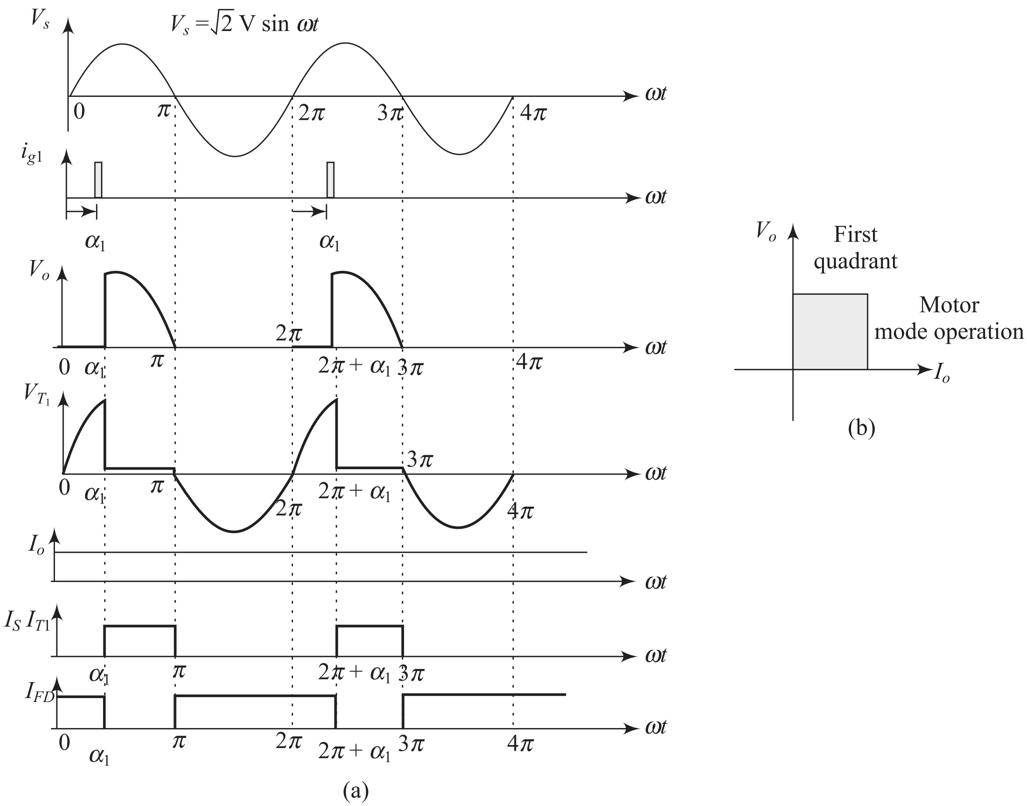


Fig. 12.11 *(a) Waveforms of single-phase half-wave converter fed DC drive (b) First quadrant diagram*

If the firing angle of thyristor T_1 is α_1, the average output voltage of a single-phase half-wave converter is

$$V_o = \frac{\sqrt{2}V}{2\pi}(1 + \cos\alpha_1) \qquad \text{for} \quad 0 \le \alpha_1 \le \pi$$

or

$$V_o = \frac{V_m}{2\pi}(1 + \cos\alpha_1)$$

as $V_m = \sqrt{2}V$ is the maximum value of supply voltage and V is the rms value of supply voltage.

When the firing angle of thyristors of single-phase semi-converter in field circuit is α_f, the average output voltage of single-phase semi-converter across the field winding is

$$V_f = \frac{\sqrt{2}V}{\pi}(1 + \cos\alpha_f) \qquad \text{for} \quad 0 \le \alpha_f \le \pi$$

or

$$V_f = \frac{V_m}{\pi}(1 + \cos\alpha_f) \qquad \text{as } V_m = \sqrt{2}V$$

It is clear from Fig. 12.11 that the rms value of source current I_S or thyristor current I_T is

$$I_S = I_T = \left[\frac{1}{2\pi}\int_{\alpha_1}^{\pi} I_o^2 d\omega t\right]^{1/2} = \sqrt{I_o^2\left(\frac{\pi - \alpha_1}{2\pi}\right)}$$

or $$I_S = I_T = I_o\sqrt{\left(\frac{\pi - \alpha_1}{2\pi}\right)}$$

rms value of armature current is $I_{a(\text{rms})} = I_o$

rms value of free wheeling diode current is

$$I_{FD} = \sqrt{I_o^2\left(\frac{\pi + \alpha_1}{2\pi}\right)} = I_o\sqrt{\left(\frac{\pi + \alpha_1}{2\pi}\right)}$$

Power input = rms value of input supply (source) voltage × rms value of source current

$$= VI_S = VI_o\sqrt{\frac{\pi - \alpha_1}{2\pi}}$$

Power deliver to motor is equal to

$$= E_b I_a + I_a^2 R_a = V_o I_o$$

Power factor of input supply

$$= \frac{\text{Power delivered to motor}}{\text{Power input}} = \frac{E_b I_a + I_a^2 R_a}{V \cdot I_{S(\text{rms})}} = \frac{V_o I_o}{VI_S} \quad \text{as } I_S = I_{S(\text{rms})}$$

12.3.2 Single-phase Semiconverter Fed DC Drives

Figure 12.12 shows a single-phase semiconverter fed separately excited dc drive. Both the armature and field windings of dc motor are fed by the single-phase semiconverters. This drive operates in one-quadrant and it can be used up to 15 kW dc drives. Assume that the armature and field currents are ripple free. The voltage and current waveforms for a highly inductive load are shown in Fig. 12.13.

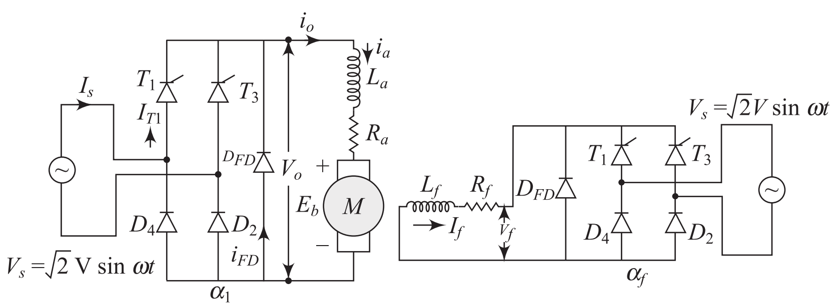


Fig. 12.12 *Single-phase semiconverter fed DC drive*

As single-phase semiconveter is connected in the armature circuits and the firing angle of thyristor is α, the average armature output voltage is

$$V_o = \frac{\sqrt{2}V}{\pi}(1 + \cos\alpha_1) \qquad \text{for} \qquad 0 \le \alpha_1 \le \pi$$

or $$V_o = \frac{V_m}{\pi}(1 + \cos\alpha_1) \text{ as } V_m = \sqrt{2}V \text{ and } V \text{ is the rms value of supply voltage}$$

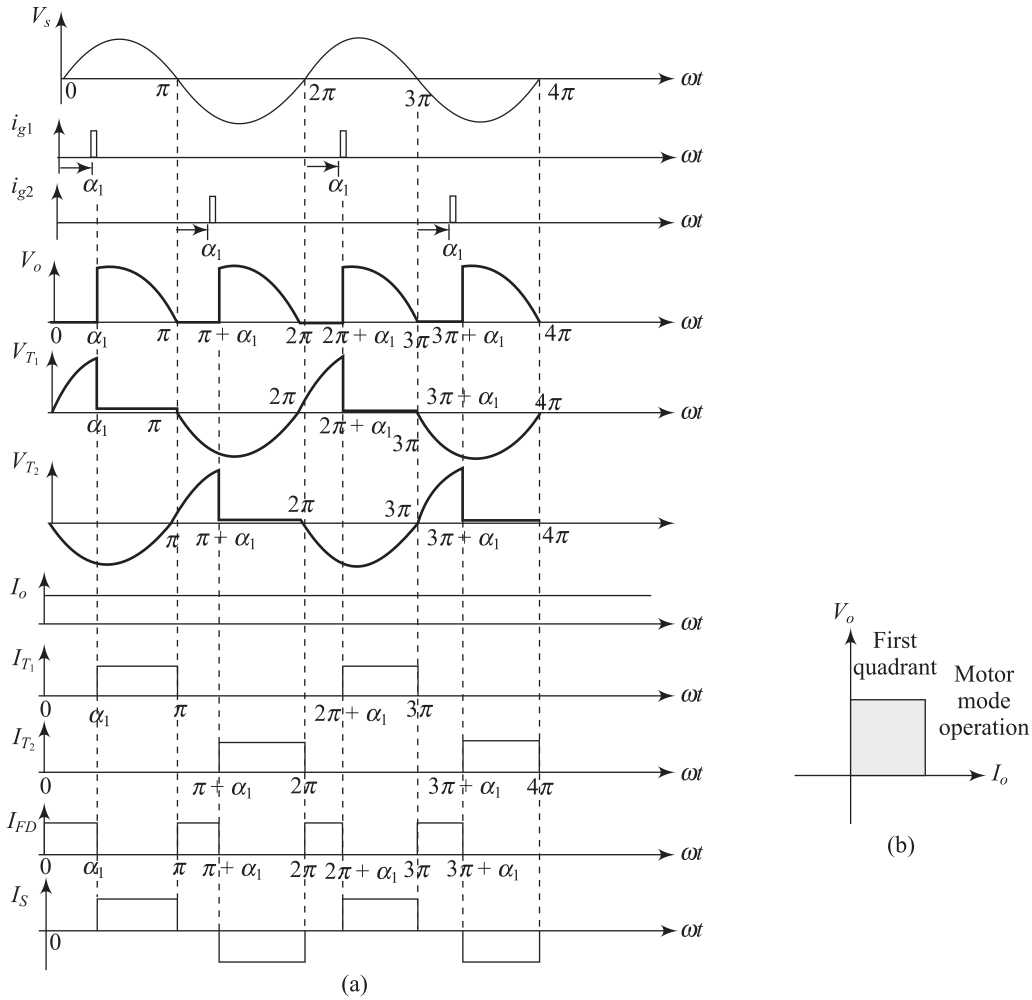


Fig. 12.13 *(a) Waveforms of single-phase half-wave converter fed DC drive (b) First quadrant diagram*

Since the field winding is fed by a semiconverter and the firing angle of thyristor is α_f, the average voltage across the field winding is

$$V_f = \frac{\sqrt{2}V}{\pi}(1 + \cos\alpha_f) \quad \text{for} \quad 0 \le \alpha_f \le \pi$$

or $$V_f = \frac{V_m}{\pi}(1 + \cos\alpha_f) \text{ as } V_m = \sqrt{2}V \text{ and } V \text{ is the rms value of supply voltage}$$

The rms value of supply current is

$$I_S = I_T = \left[\frac{2}{2\pi}\int_{\alpha_1}^{\pi} I_o^2 d\omega t\right]^{1/2} = \sqrt{I_o^2\left(\frac{\pi - \alpha_1}{\pi}\right)}$$

or $$I_S = I_T = I_o\sqrt{\left(\frac{\pi - \alpha_1}{\pi}\right)}$$

As armature current is constant, rms value of armature current is

$$I_{a(\text{rms})} = I_o$$

rms current of thyristor is

$$I_{T(\text{rms})} = \sqrt{I_o^2\left(\frac{\pi - \alpha_1}{2\pi}\right)} = I_o\sqrt{\left(\frac{\pi - \alpha_1}{2\pi}\right)}$$

rms value of free wheeling diode current is

$$I_{FD} = \sqrt{I_o^2\left(\frac{\alpha_1}{\pi}\right)} = I_o\sqrt{\frac{\alpha_1}{\pi}}$$

Power factor of input supply

$$= \frac{\text{Power delivered to motor}}{\text{Power input}} = \frac{E_b I_a + I_a^2 R_a}{V \cdot I_{S(\text{rms})}} = \frac{V_o I_o}{V I_{S(\text{rms})}}$$

Example 12.1 The speed of a 15 HP, 220 V, 1000 rpm dc series motor is controlled by a single-phase half-controlled bridge (semiconverter) which is supplied from 230 V, 50 Hz ac supply. The total armature and field winding resistance is $R_a + R_f = 0.5\ \Omega$. When the motor current is continuous and ripple free, and the motor operates at 1000 rpm with firing angle of $\alpha = 25°$, determine the (a) motor current and (b) motor torque.
Assume motor constant $k_b = 0.03$ V/A rad/s, $k_t = 0.03$ N-m/Amp^2 and the current waveforms are ripple free.

Solution

Given: 220 V, $\alpha_1 = 25°$, $N = 1000$ rpm, $R_a + R_f = 0.5\ \Omega$, $k_b = 0.03$ V/A rad/s, $k_t = 0.03$ N-m/Amp^2

(a) The output voltages of single-phase half-controlled bridge is

$$V_o = \frac{\sqrt{2}V}{\pi}(1 + \cos\alpha_1) \text{ and } V_o = E_b + I_a(R_a + R_f)$$

The electromagnetic torque of motor is

$$T_e = k_t I_f I_a = k_t I_a^2 \quad \text{as } I_f = I_a$$

The back emf of motor is $E_b = k_b \omega I_f = k_b \omega I_a$

Therefore, $V_o = E_b + I_a(R_a + R_f) = k_b\omega I_a + I_a(R_a + R_f) \quad \text{as } E_b = k_b\omega I_a$

or $$V_o = \frac{\sqrt{2}V}{\pi}(1 + \cos\alpha_1) = k_b\omega I_a + I_a(R_a + R_f)$$

Speed $$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1000}{60} = 104.7619 \text{ rad/s}$$

At $\alpha_1 = 25°, \dfrac{\sqrt{2} \times 220}{\pi}(1 + \cos 25) = 0.03 \times 104.7619 \times I_a + I_a \times 0.5$

Armature current is $I_a = 51.80$

(b) Motor torque is $T_e = k_t I_a^2 = 0.03 \times 51.80^2 \text{ Nm} = 80.49 \text{ N-m}$

12.3.3 Single-Phase Full Converter Fed DC Drives

Figure 12.14 shows a single-phase full converter fed dc drives where full converter-1 is connected to armature circuit of dc motor and the full converter-2 is applied to field winding of dc motor. This is a two quadrant drive as shown in Fig. 12.15 and its applications limited up to 15 kW.

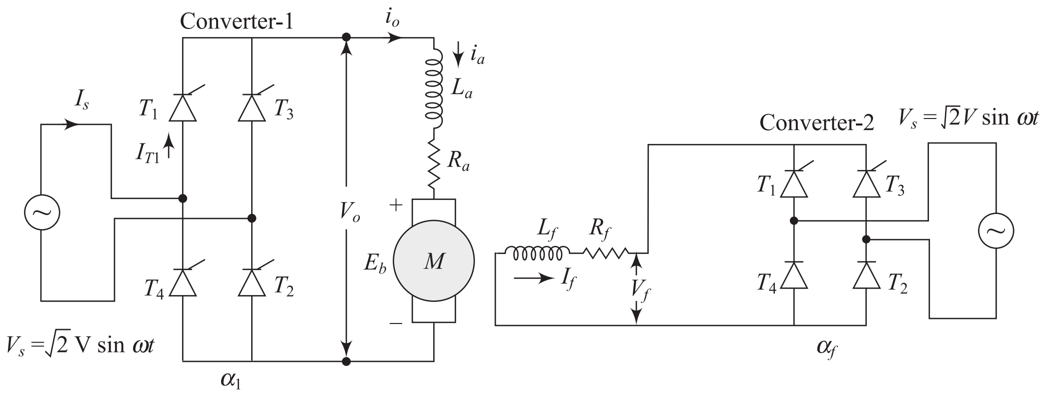


Fig. 12.14 *Single-phase full converter fed dc drive*

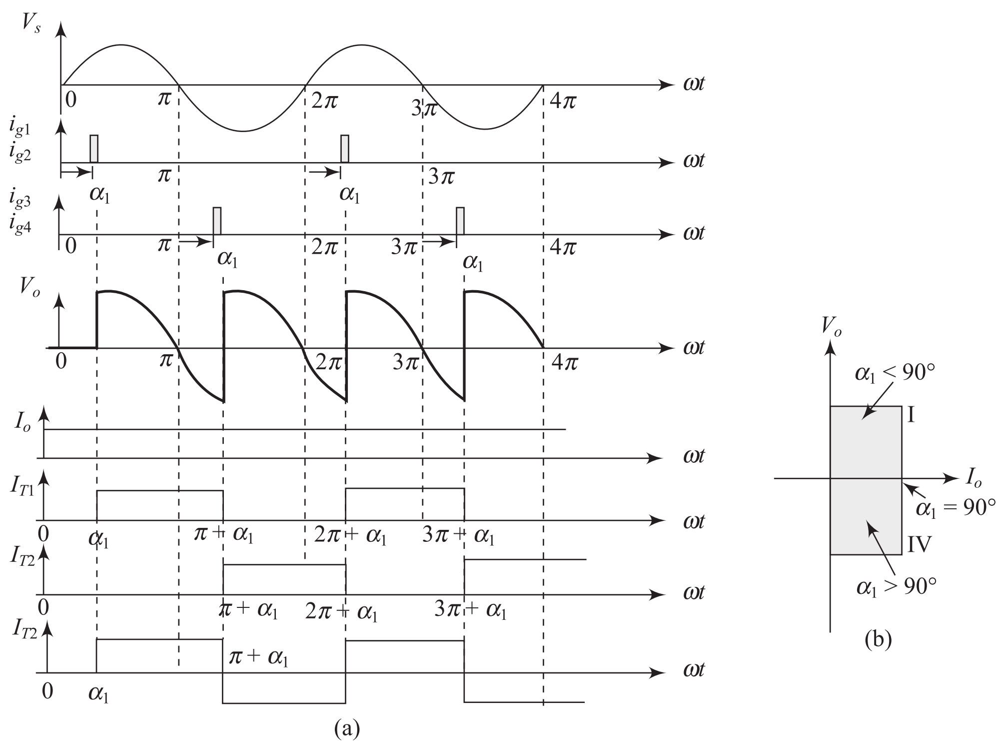


Fig. 12.15 *(a) Waveforms of single-phase half-wave converter fed dc drive and (b) First and fourth quadrant diagram*

The armature converter-1 provides the output voltage $+V_o$ to $-V_o$ by varying the firing angle of converter from 0° to 180°. Hence this drive operates in the first and fourth quadrant. For regenerative breaking of dc motor, the power must flow in reverse direction, i.e., from motor to ac supply. This is possible if the back of motor is reversed by reversing the field excitation (field current or field flux). Subsequently, the $E_b I_a$ is negative. Since thyristors are unidirectional device, current can flow in one direction only. Therefore, for regenerative braking, the polarity of back emf must be reversed. This is feasible if firing angle of the field converter-2 is greater than 90°. The polarity of applied voltage will be reversed (-negative), and the field winding is energized by negative voltage through a full converter. The average output voltage of armature converter-1 is

$$V_o = V_a = \frac{2\sqrt{2}V}{\pi}\cos\alpha_1 \quad \text{for} \quad 0 \le \alpha_1 \le \pi$$

or $$V_o = V_a = \frac{2V_m}{\pi}\cos\alpha_1 \quad \text{as} \quad V_m = \sqrt{2}V \text{ and } V \text{ is the rms value of supply voltage}$$

The average output voltage of field converter-2 is

$$V_f = \frac{2\sqrt{2}V}{\pi}\cos\alpha_f \quad \text{for} \quad 0 \le \alpha_f \le \pi$$

or $$V_f = \frac{2V_m}{\pi}\cos\alpha$$

Since armature current of separately excited dc motor is constant, the rms value of armature current is

$$I_{a(\text{rms})} = I_o$$

The rms value of source current is

$$I_S = I_{S(\text{rms})} = \left(I_o^2 \frac{\pi}{\pi}\right)^{1/2} = I_o$$

The rms value of thyristor current is

$$I_{T1(\text{rms})} = \left(I_o^2 \frac{\pi}{2\pi}\right)^{1/2} = \frac{I_o}{\sqrt{2}}$$

Supply power factor is

$$\frac{V_t I_a}{V_s I_s} = \frac{\frac{2\sqrt{2}V}{\pi}\cos\alpha \, . \, I_o}{VI_o} = \frac{2\sqrt{2}}{\pi}\cos\alpha$$

It is clear from above equation that the input power factor depends on the firing angle of converter.

Example 12.2 A separately excited dc motor is controlled by a single-phase full converter which is supplied from 440 V, 50 Hz ac supply. If the field circuit is fed through a single-phase full-converter with 0° firing angle, the delay angle of armature converter is 30° and load current is 20 A, determine (a) voltage across field winding, (b) the field current, (c) electromagnetic torque and load torque, (d) voltage across armature winding, (e) back emf, (f) motor speed and (g) input power factor. Assume armature resistance R_a = 0.5 Ω, field resistance R_f = 140 Ω, motor constant k_b = 0.7 V/A rad/s, k_t = 0.03 N-m/Amp^2 and the current waveforms are ripple free.

Solution

Given: 240 V, α_f = 0°, α_1 = 30°, R_a = 0.5 Ω, R_a = 0.5 Ω, R_f = 140 Ω, k_b = 0.7 V/A rad/s, k_t = 0.7 N-m/Amp^2

(a) Voltage across the field winding is

$$V_f = \frac{2\sqrt{2}V}{\pi}\cos\alpha_f = \frac{2\sqrt{2}\times 440}{\pi}\cos 0 = 395.97 \text{ V}$$

(b) The field current is

$$I_f = \frac{V_f}{R_f} = \frac{395.97}{140} = 2.828 \text{ A}$$

(c) Motor constant $k_t = 0.7$ V/A rad/s
At steady state operation, the motor electromagnetic torque = Load torque

or $\quad T_e = k_t I_f I_a = T_L$

or $\quad T_e = k_t I_f I_a = 0.7 \times 2.828 \times 20 = 39.592$ N-m

(d) The average output voltage of a single-phase full converter which is connected to armature circuit

$$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha_1 = \frac{2\sqrt{2}\times 440}{\pi}\cos 30° = 342.928 \text{ V}$$

(e) The back emf is

$$E_b = V_o - I_a R_a = 342.928 - 20 \times 0.5 = 332.928 \text{ V}$$
$$E_b = k_b \omega I_f = 0.7 \times \omega \times 2.828 \text{ V} = 342.928$$

(f) Speed $\omega = \dfrac{E_b}{k_b I_f} = \dfrac{342.928}{0.7 \times 2.828} = 173.23$ rad/s

(g) The total power input from the supply is

$$P_i = V_o I_a + V_f I_f = 342.928 \times 20 + 395.97 \times 2.828 = 7078.36 \text{ Watt}$$

The rms value of supply current is

$$I_S = \sqrt{I_a^2 + I_f^2} = \sqrt{20^2 + 2.828^2} = 20.198$$

Input power factor is

$$\frac{P_i}{VI_S} = \frac{7078.36}{440 \times 20.198} = 0.796 \text{ (lagging)}$$

Example 12.3 In Example 12.2, the polarity of induced emf is reversed by reversing the field excitation to its maximum value. Compute (a) firing angle of the field converter, (b) delay angle of armature converter at 1000 rpm to maintain armature current at 40 A and (c) the power fed back to the supply due to regenerative breaking of the motor.

Solution

(a) As the field excitation is reversed, the voltage across the field winding is

$$V_f = \frac{2\sqrt{2}V}{\pi}\cos\alpha_f = \frac{2\sqrt{2}\times 440}{\pi}\cos\alpha_f = -395.97 \text{ V}$$

or $\quad \alpha_f = 180°$

Therefore, firing angle of the field converter is $\alpha_f = 180°$

(b) The field current is

$$I_f = \frac{V_f}{R_f} = -\frac{395.97}{140} = -2.828 \text{ A}$$

$$E_b = k_b \omega I_f = -0.7 \times \frac{2\pi \times 1000}{60} \times 2.828 = -207.386 \text{ V}$$

where, k_b = 0.7 V/A, N = 1000 rpm and $\omega = \dfrac{2\pi N}{60}$

The polarity of induced emf is reversed by reversing the field excitation. Then the average output voltage across armature terminals is

$$V_o = V_t = -E_b + I_a R_a$$

or $$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha_1 = -E_b + I_a R_a$$

or $$\frac{2\sqrt{2}\times 440}{\pi}\cos\alpha_1 = -207.386 + 40\times 0.5 = -145.17 \text{ V}$$

or $$\alpha_1 = \cos^{-1}\left(\frac{-145.17\times\pi}{2\sqrt{2}\times 440}\right) = 111.506°$$

(c) The power fed back to the supply due to regenerative breaking of the motor is equal to

$$P = V_o I_a = 145.17\times 40 = 5806.8 \text{ Watt}$$

Example 12.4 A 230 V, 1500 rpm, 15 A separately excited dc motor is controlled by a single-phase full bridge controlled rectifier which is supplied from 220 V, 50 Hz ac supply. Assume that armature resistance R_a = 1 Ω, field resistance R_f = 120 Ω load current is continuous and ripple free. Determine (a) motor speed at the firing angle α = 45° and load torque of 10 N-m and (b) torque developed at the firing angle α = 30° and speed 1000 rpm.

Solution

Given: $V = 220$ V, $\alpha_1 = 45°$, $R_a = 1\ \Omega$, $R_f = 120\ \Omega$

When the separately excited dc motor operates at rated operating condition,

$$V_t = E_b + I_a R_a = k_b I_f \omega + I_a R_a$$

Rated speed $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi\times 1500}{60} = 157.142$ rad/s, $V_t = 230$ V, $I_a = 15$ A

Therefore, $230 = k_b I_f \times 157.142 + 15\times 1$

Then $k_b I_f = (230 - 15)/157.142 = 1.3681$ V-s/rad or N-m/A

(a) At steady-state operation, the motor electromagnetic torque = Load torque

$$T_e = k_t I_f I_a = T_L$$

When torque is 10 N-m, $T_e = k_t I_f I_a = 1.3681\times I_a = 10$ N-m   As $k_t I_f$ = 1.3681 N-m/A

Armature current $I_a = \dfrac{10}{1.3681}$ A = 7.309 A

At firing angle α_1 = 45°, the output voltage of converter-1 is

$$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha_1 = E_b + I_a R_a = k_b I_f \omega + I_a R_a$$

or $$\frac{2\sqrt{2}\times 220}{\pi}\cos 45 = 1.3681\,\omega + 7.309\times 1\ 1$$

Speed $\omega = (139.99 - 7.309)/1.3681$ rad/s = 96.988 rad/sec

(b) At the firing angle $\alpha = 30°$, speed is equal to 1000 rpm,
The output voltage of converter-1 is

$$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha_1 = E_b + I_a R_a = k_b I_f \omega + I_a R_a$$

or
$$\frac{2\sqrt{2}\times 220}{\pi}\cos 30 = 1.3681\times\frac{2\pi\times 1000}{60} + I_a \times 1$$

Armature current $I_a = 28.14$ A
The electromagnetic torque developed in motor at the firing angle $\alpha = 30°$ and speed 1200 rpm is equal to

$$T_e = k_t I_f I_a = 1.3681\times 28.14 = 38.49 \text{ N-m}$$

12.3.4 Single-Phase Dual Converter Fed DC Drives

Figure 12.16 shows a single-phase dual converter fed separately excited dc drive where a single-phase dual converter is feeding the armature circuit. In a dual converter, two full converters, i.e., converter-1 and converter-2 are connected in anti parallel. When the converter-1 operates, a positive armature voltage V_a is applied to armature terminals of motor, but the converter-2 operates to provide a negative armature voltage ($-V_a$). Consequently, the converter-1 operates in first and fourth quadrants and the converter-2 operates in second and third quadrants. Hence, this drive operates in four quadrants and this drive also operates in four operating modes such as

1. Forward motoring – First quadrant
2. Forward regenerative braking – Fourth quadrant
3. Reverse motoring – Third quadrant
4. Reverse regenerative braking – Second quadrant

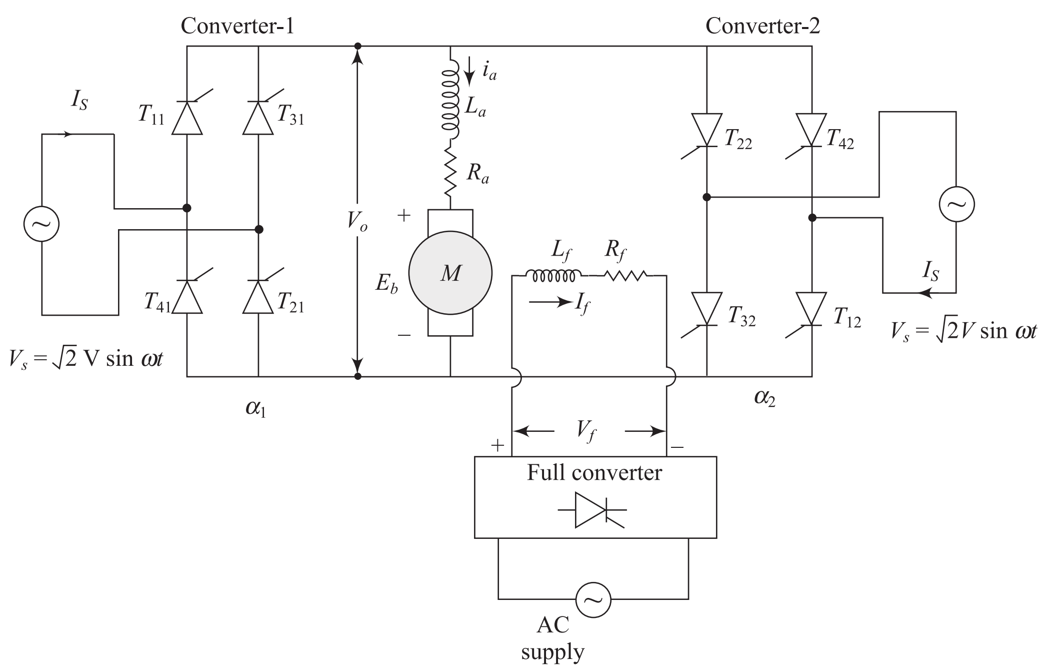


Fig. 12.16 *Single-phase dual converter fed DC drive*

When the converter-1 operates with the firing angle α_1, the average output voltage of armature converter-1 is

$$V_o = V_a = \frac{2\sqrt{2}V}{\pi}\cos\alpha_1 \quad \text{for } 0 \le \alpha_1 \le \pi$$

or
$$V_o = V_a = \frac{2V_m}{\pi}\cos\alpha_1 \text{ as } V_m = \sqrt{2}V \text{ and } V \text{ is the rms value of supply voltage}$$

If the converter-2 operates with the firing angle α_2, the average output voltage of armature converter-2 is

$$V_o = V_a = \frac{2\sqrt{2}V}{\pi}\cos\alpha_2 \quad \text{for } 0 \le \alpha_2 \le \pi$$

or
$$V_o = V_a = \frac{2V_m}{\pi}\cos\alpha_2$$

where, $\alpha_1 + \alpha_2 = \pi$ and $\alpha_2 = \pi - \alpha_1$

The average output voltage applied to field winding is

$$V_f = \frac{2\sqrt{2}V}{\pi}\cos\alpha_f = \frac{2V_m}{\pi}\cos\alpha_f \quad \text{for } 0 \le \alpha_f \le \pi$$

When the converter-1 operates with $\alpha_1 < 90°$, motor operates in forward motoring mode, i.e., first quadrant operation. If the converter-1 operates with $\alpha_1 > 90°$, motor operates in forward regenerative braking mode, i.e., fourth quadrant operation.

While converter-2 operates with $\alpha_2 < 90°$, motor operates in reverse regenerative braking mode, i.e., second quadrant operation. If the converter-2 operates with $\alpha_2 > 90°$, motor operates in reverse motoring mode, i.e., third quadrant operation. Four quadrant operation of single-phase dual converter fed dc drive is given in Fig. 12.17.

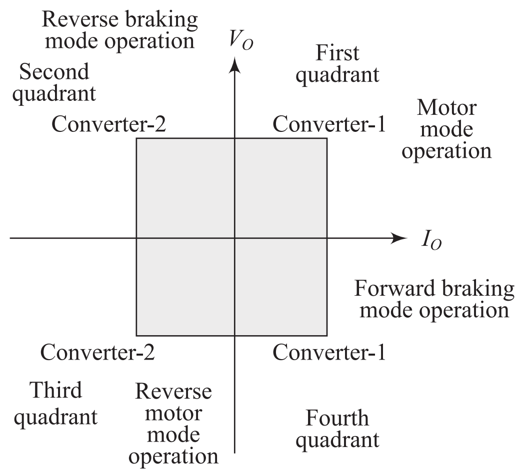


Fig. 12.17 *Four quadrant diagram*

12.4 THREE-PHASE DC DRIVES

Single-phase dc drives have limited applications up to 15 kW. For high power applications, single-phase dc drives are not suitable. Therefore, three-phase dc drives are used for high power applications up to some megawatt power. When a three-phase controlled rectifier feeds power to the armature circuit, the motor operates at below base speed. While a three-phase controlled rectifier is connected in the field winding, its speed will be controlled above base speed.

The ripple frequency of the armature voltage using three phase controlled rectifier fed drives is higher than that of single-phase drives. As a result armature current ripple is reduced. Consequently, less inductance is required in the armature circuit to reduce ripple in armature current. Since the armature current is mostly continuous, the performance of three-phase dc drive is better than that of single-phase drives. The three-phase dc drives are classified as

1. Three-phase half-wave converter fed dc drives
2. Three-phase semi-converter fed dc drives

3. Three-phase full converter fed dc drives
4. Three-phase dual converter fed dc drives

In this section, the above three-phase drives are discussed elaborately. Assume that the armature current is ripple free.

12.4.1 Three-Phase Half-wave Converter Fed DC Drives

Figure 12.18 shows a three-phase half-wave converter fed dc drive where armature circuit is fed from a three-phase half-wave converter and the field circuit is also fed from a three-phase semi converter. This drive operates in first quadrant only. This drive is suitable for applications up to 40 kW power level. This drive can be operating in two quadrants when the field circuit is drive by a single-phase full converter or a three-phase full converter.

When a three-phase half-wave converter is connected to the armature circuit, the average armature voltage at firing angle α is equal to

$$V_o = V_a = \frac{3V_{m(\text{line})}}{2\pi}\cos\alpha_1 \quad 0 \le \alpha_1 \le \pi$$

where, $V_{m(\text{line})}$ is the maximum line voltage.

If V_m is the maximum phase voltage of a star connected three ac supply, the output voltage of three-phase half-wave converter is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha_1 \quad \text{as } V_{m(\text{line})} = \sqrt{3}V_m$$

In the field circuit dc voltage is supplied by a three-phase semiconverter and the average output voltage across the field winding is equal to

$$V_f = \frac{3V_{m(\text{line})}}{2\pi}(1+\cos\alpha_f) \quad \text{for } 0 \le \alpha_f \le \pi$$

or

$$V_f = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha_f) \quad \text{as } V_{m(\text{line})} = \sqrt{3}V_m$$

Since three-phase half-wave converter introduces dc components in the ac supply, this drive is not generally used in industrial applications.

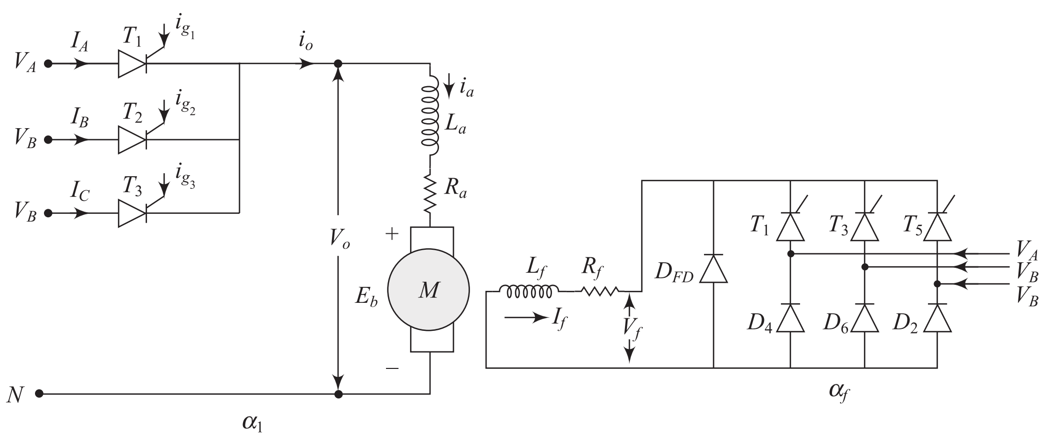


Fig. 12.18 *Three-phase half-wave converter fed dc drive*

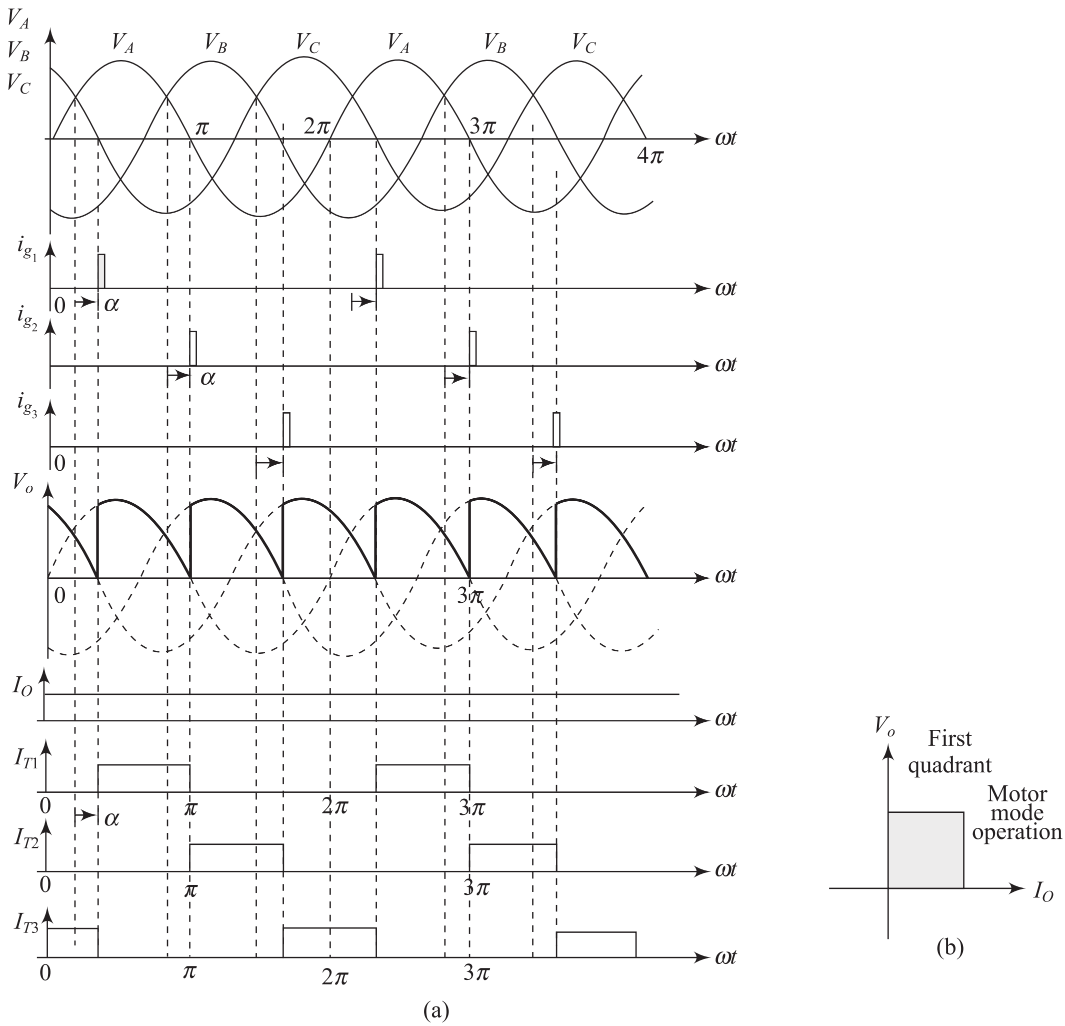


Fig. 12.19 *(a) Waveforms of three phase half-wave converter fed dc drive and (b) First quadrant diagram*

rms value of armature current is

$$I_{a(\text{rms})} = I_o$$

rms value of phase current is

$$I_{S(\text{rms})} = \left(I_o^2 \frac{2\pi/3}{2\pi} \right)^{1/2} = \frac{I_o}{\sqrt{3}}$$

Average thyristor current is

$$I_{T1(\text{av})} = \left(I_o \frac{2\pi/3}{2\pi} \right) = \frac{I_o}{3}$$

The rms value of thyristor current is equal to the rms value of source current

$$I_{T1(\text{rms})} = I_{S(\text{rms})} = \left(I_o^2 \frac{2\pi/3}{2\pi} \right)^{1/2} = \frac{I_o}{\sqrt{3}}$$

Example 12.5 The speed of a 10 HP, 300 V, 1500 rpm separately excited dc motor is controlled by a three-phase half-converter from a Y connected, 220 V, 50 Hz ac supply. The field current is controlled by a three-phase semiconverter and is set to its maximum value. Assume armature resistance $R_a = 0.1\ \Omega$, field resistance $R_f = 210\ \Omega$, and motor constant is 0.6 V/A-rad/s. Compute

(a) the firing angle of armature converter at rated power and rated speed,
(b) the no-load speed if the firing angle is same as (a) and no-load current is 12% of rated current and
(c) the speed regulation.

Solution

Given: $R_a = 0.1\ \Omega$, $R_f = 210\ \Omega$, $k_b = 1.4$ V/A-rad/s, N = 1500 rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1500}{60} = 157.1428 \text{ rad/s}$$

Rated armature current $I_a = \dfrac{P_{\text{rated}}}{V_{\text{rated}}} = \dfrac{10 \times 746}{300} = 29.84$ A

Phase voltage $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{220}{\sqrt{3}}\text{V} = 127.017$ V

Maximum phase voltage $V_m = \sqrt{2}V_p = \sqrt{2} \times 127.017 = 179.629$ V

The field current will be maximum when field voltage will be maximum. Therefore, $\alpha_f = 0$.

Voltage across field winding $V_f = \dfrac{3\sqrt{3}V_m}{2\pi}(1 + \cos\alpha_f) = \dfrac{3\sqrt{3}V_m}{\pi} = \dfrac{3\sqrt{3} \times 179.629}{\pi} = 296.98$ V

The field current $I_f = \dfrac{V_f}{R_f} = \dfrac{296.98}{210}\text{A} = 1.414$ A

(a) At rated speed, the generated back emf is

$$E_b = k_b I_f \omega = 0.6 \times 1.414 \times 157.1428 \text{ V} = 133.31 \text{ V}$$

Voltage across armature is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha_1 = E_b + I_a R_a$$

or
$$\frac{3\sqrt{3} \times 179.629}{2\pi}\cos\alpha_1 = 133.31 + 29.84 \times 0.1 = 136.294 \text{ V}$$

$$\alpha_1 = \cos^{-1}\left(\frac{136.294 \times 2\pi}{3\sqrt{3} \times 179.629}\right) = 23.38°$$

The firing angle of armature converter at rated power and rated speed is $\alpha_1 = 23.38°$

(b) No load armature current is 12% of rated current. Then $I_a = 0.12 \times 29.84 = 3.58$ A
The generated back emf at no-load is

$$E_b = V_o - I_a R_a = 136.294 - 3.58 \times 0.1 = 135.936 \text{ V}$$

$$E_b = k_b I_f \omega = 0.6 \times 1.414 \times \omega = 135.936 \text{ V}$$

or
$$\omega = \frac{135.936}{0.6 \times 1.414} = 160.22 \text{ rad/s}$$

$$N_{\text{no-load}} = \frac{60\omega}{2\pi} = \frac{60 \times 160.22}{2\pi} = 1529.37 \text{ rpm}$$

(c) Speed regulation is

$$\frac{N_{\text{no-load}} - N_{\text{full-load}}}{N_{\text{full-load}}} = \frac{1529.37 - 1500}{1500} = 0.01958 = 1.958\%$$

12.4.2 Three-phase Semiconverter Fed dc Drives

Figure 12.20 shows the circuit diagram of three-phase semiconverter fed separately excited dc motor. The field winding of the motor is also connected to three-phase semiconverter. This drive operates in the first quadrant and its application is limited to 115 kW power ratings.

Assume that the armature current is ripple free and continuous. Figure 12.21 shows the voltage and current waveforms of a semiconverter fed drive at firing angle $\alpha = 30°$.

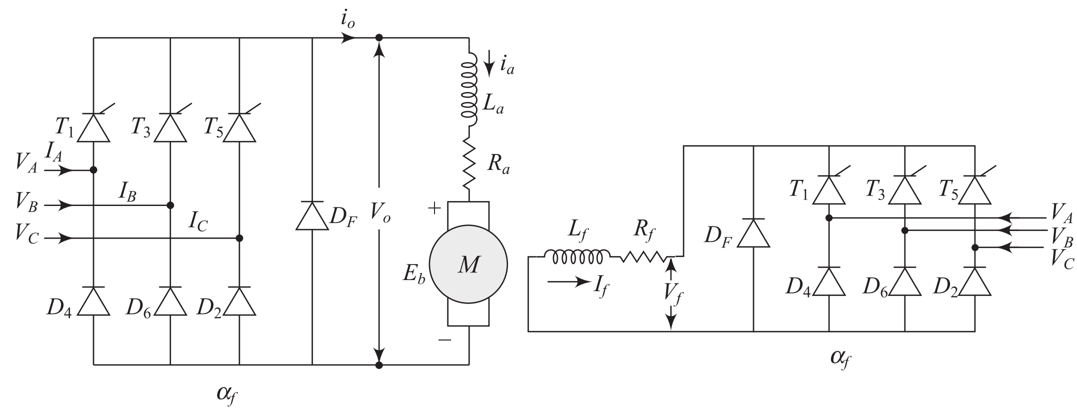


Fig. 12.20 *Three-phase semi-converter fed dc drive*

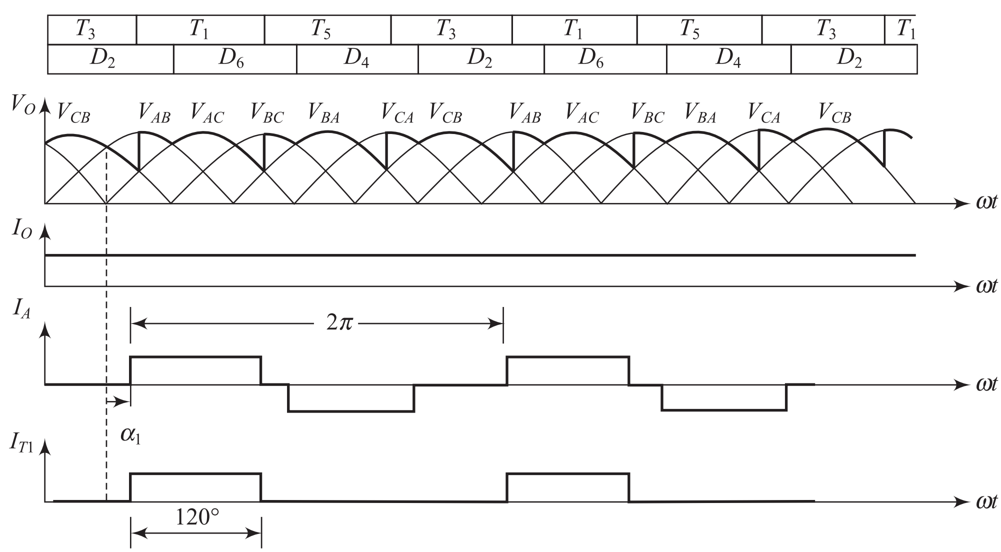


Fig. 12.21 *Voltage and current waveforms of a three-phase semi converter fed DC drive at α_1 = 30°*

If the firing angle $\alpha \leq 60°$, each thyristor conducts for 120° duration. When α is greater than 60° and less than 180°, each thyristor conducts for 180° – α duration. Free wheeling diodes conduct while $\alpha > 60°$ and it conducts for α – 60° duration. While the armature current is continuous, the sum of conduction angle of thyristor and conduction angle of freewheeling diode is 120°.

As a three-phase semiconverter is connected across the armature and its firing angle is α_1, average armature voltage is

$$V_o = \frac{3V_{m(\text{line})}}{2\pi}(1+\cos\alpha_1) \quad \text{for} \quad 0 \leq \alpha_1 \leq \pi$$

or
$$V_o = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha_1) \quad \text{as} \quad V_{m(\text{line})} = \sqrt{3}V_m$$

Since a three-phase semiconverter is connected across the field winding, the voltage V_f is equal to

$$V_f = \frac{3V_{m(\text{line})}}{2\pi}(1+\cos\alpha_f) \quad \text{for} \quad 0 \leq \alpha_f \leq \pi$$

$$V_f = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha_f)$$

Since armature current is ripple free, rms value of armature current is

$$I_a = I_o$$

When $\alpha \leq 60°$, the rms value of line current I_A is

$$I_{S(\text{rms})} = \left(I_o^2\,\frac{2\pi/3}{\pi}\right)^{1/2} = I_o\,\frac{\sqrt{2}}{\sqrt{3}}$$

The rms value of thyristor current I_{T1} is

$$I_{T1(\text{rms})} = \left(I_o^2\,\frac{2\pi/3}{2\pi}\right)^{1/2} = \frac{I_o}{\sqrt{3}}$$

When $\frac{\pi}{3} < \alpha < \pi$, the rms value of supply line current I_A is

$$I_{S(\text{rms})} = \left(I_o^2\,\frac{\pi-\alpha}{\pi}\right)^{1/2} = I_o\left(\frac{\pi-\alpha}{\pi}\right)^{1/2}$$

The rms value of thyristor current I_{T1} is

$$I_{T1(\text{rms})} = \left(I_o^2\,\frac{\pi-\alpha}{2\pi}\right)^{1/2} = I_o\left(\frac{\pi-\alpha}{2\pi}\right)^{1/2}$$

If $\alpha < \frac{\pi}{3}$, the average value of thyristor current is $\frac{I_o}{3}$ and for $\frac{\pi}{3} < \alpha < \pi$, the average value of average thyristor current is

$$I_{T1(\text{av})} = \left(\frac{\pi-\alpha}{2\pi}\right)I_o$$

Average value of free wheeling diode is

$$I_{FD(\text{av})} = I_o\left(\frac{\alpha-\pi/3}{2\pi/3}\right) = I_o\,\frac{\alpha-60}{120} \qquad \text{for } \frac{\pi}{3} < \alpha < \pi$$

rms value of free wheeling diode is

$$I_{FD(\text{rms})} = \left(I_o^2 \frac{\alpha - \pi/3}{2\pi/3}\right)^{1/2} = I_O\left(\frac{\alpha - \pi/3}{2\pi/3}\right)^{1/2} \qquad \text{for } \frac{\pi}{3} < \alpha < \pi$$

12.4.3 Three-phase Full Converter Fed dc Drives

Figure 12.22 shows a three-phase full converter fed dc drive where a three-phase full converter is connected in the armature circuit and another three-phase full converter is applied to the field winding. This drives operates in two quadrants and it is used up to 1500 kW ratings.

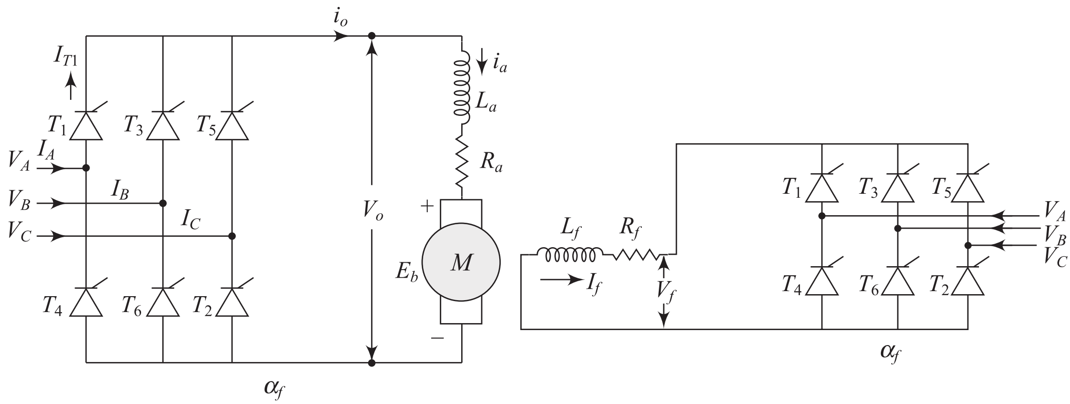


Fig. 12.22 *Three -phase full converter fed dc drive*

The average output voltage across armature of dc motor is

$$V_o = \frac{3V_{m(\text{line})}}{\pi}\cos\alpha_1 \qquad \text{for } 0 \le \alpha_1 \le \pi$$

or

$$V_o = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha_1 \qquad \text{where, } V_{m(\text{line})} = \sqrt{3}V_m$$

The average output voltage is applied to field winding is

$$V_f = \frac{3V_{m(\text{line})}}{\pi}\cos\alpha_f \qquad \text{for } 0 \le \alpha_f \le \pi$$

or

$$V_f = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha_f$$

For regenerative braking of dc motor, the polarity of induced emf E_b must be reversed. The polarity of back emf can be reversed by reversing the field excitation with increasing the firing angle α_f greater than 90°(α_f > 90°). During regenerative braking of dc motor, power flow in reverse direction due to reverse polarity of back emf.

The voltage and current waveforms of a three-phase full converter fed dc drive are depicted in Fig. 12.23 at α = 30° assuming the armature current is continuous.

The rms value of armature current is

$$I_{a(\text{rms})} = I_o$$

rms value of supply current I_A is

$$I_{S(\text{rms})} = \left(I_o^2 \frac{2\pi/3}{\pi} \right)^{1/2} = I_o \frac{\sqrt{2}}{\sqrt{3}}$$

The rms value of thyristor current I_{T1} is

$$I_{T1(\text{rms})} = \left(I_o^2 \frac{2\pi/3}{2\pi} \right)^{1/2} = \frac{I_O}{\sqrt{3}}$$

Average value of thyristor current is equal to

$$I_{T1(\text{av})} = I_o \frac{2\pi}{3} \frac{1}{2\pi} = \frac{I_o}{3}$$

Figure 12.23(c) shows the A phase current i_A. Similarly, B phase current and C phase current are phase shifted by 120°.

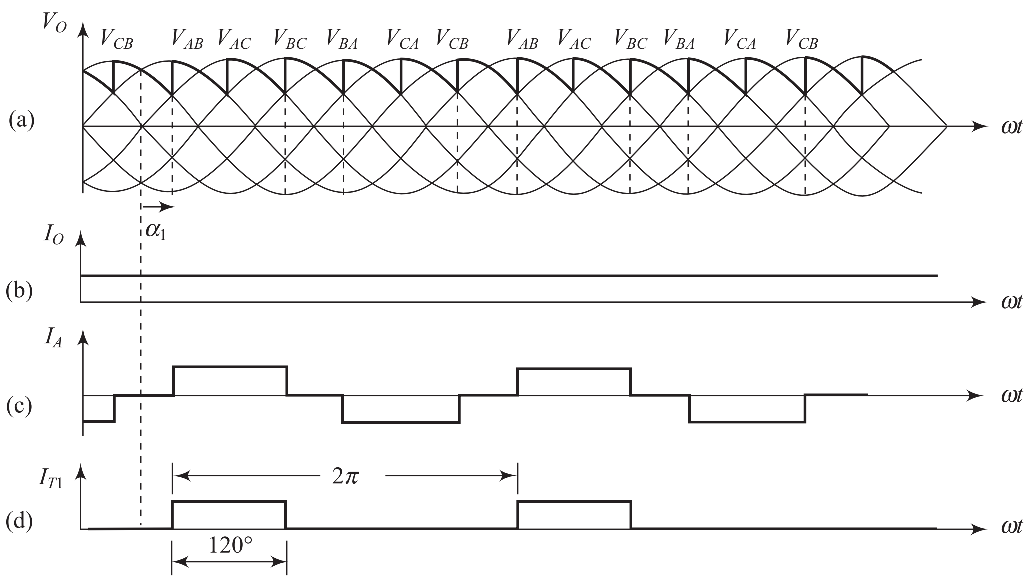


Fig. 12.23 *Voltage and current waveforms of a three-phase full converter fed dc drive at α_1 = 30°*

12.4.4 Three-Phase Dual Converter Fed dc Drives

Figure 12.24 shows a three-phase dual converter fed dc drive. In this drive, two three-phase full-wave converters are connected in anti-parallel and applied across the armature of dc motor. Converter-1 is used to operate motor in first and fourth quadrant but converter-2 is used to operate motor in second and third quadrant. This converter operates in four quadrants and it has limited applications up to 2 MW power rating. Just like single-phase dc drive, a full-wave converter or a semi-converter can be used as field converter. For regenerative braking purpose, the polarity of generated emf must be reversed when the field circuit is energized from single-phase or three-phase full converter.

When the converter-1 operates with a delay angle of α_1, the average output voltage is

$$V_o = \frac{3V_{m(\text{line})}}{\pi} \cos \alpha_1 \qquad \text{for } 0 \le \alpha_1 \le \pi$$

or $$V_o = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_1 \qquad \text{when } V_{m(\text{line})} = \sqrt{3}V_m$$

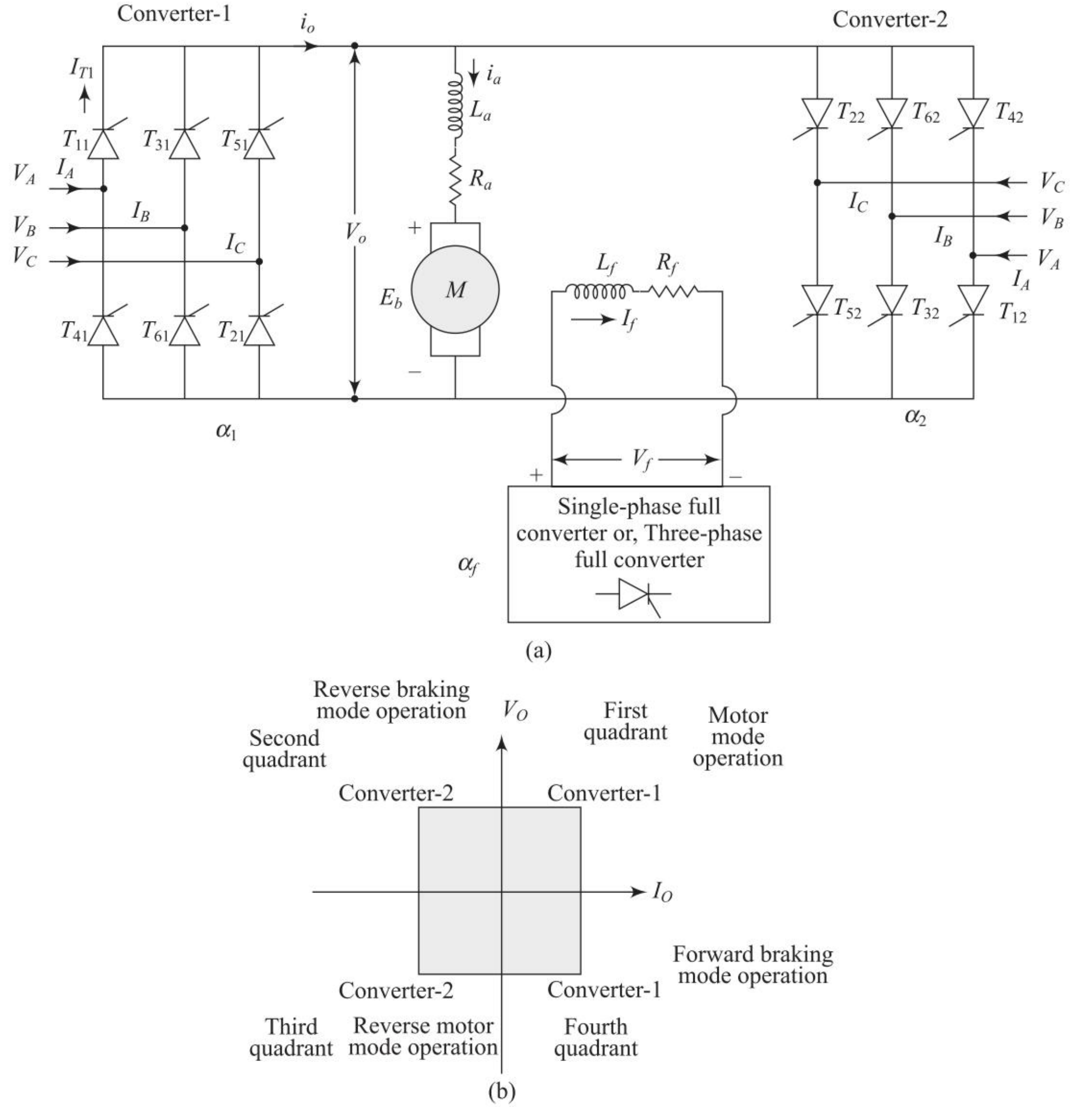


Fig. 12.24 *(a) Three-phase dual converter fed DC drive and (b) Four quadrant operation of dual converter fed dc drive*

If the firing angle of converter-2 is α_2, the average output voltage is

$$V_o = \frac{3V_{m(\text{line})}}{\pi} \cos \alpha_2 \qquad \text{for } 0 \le \alpha_2 \le \pi$$

or $$V_o = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_2 \qquad \text{where, } \alpha_1 + \alpha_2 = \pi \text{ and } \alpha_2 = \pi - \alpha_1$$

When a three-phase full converter is applied across field winding, the average field voltage is

$$V_f = \frac{3V_{m(\text{line})}}{\pi}\cos\alpha_f = \frac{3\sqrt{3}V_m}{2\pi}\cos\alpha_f \quad \text{for } 0 \le \alpha_f \le \pi$$

Example 12.6 A separately excited dc motor can be controlled by a three-phase semiconverter which is supplied by Y connected 208 V, 50 Hz ac supply. The dc motor has the following parameters:
Armature resistance R_a = 0.5 Ω, armature inductance L_a = 5 mH, machine constant k = 1.2 V/rad/s or N-m/A.
Determine the speed of motor at 20 N-m load with firing angle α = 30°

Solution

Given: $R_a = 0.5\ \Omega, L_a = 5\text{ mH}, T_L = 20\text{ N-m}, \alpha = 30°, \quad k = 1.2\text{ V/rad/s or N-m/A}.$

The motor torque $T_e = T_L = kI_a$

$\therefore$ $$I_a = \frac{T_L}{k} = \frac{20}{1.2}\text{A} = 16.666\text{ A}$$

The phase voltage $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{208}{\sqrt{3}} = 120.08\text{ V}$

The maximum phase voltage $V_m = \sqrt{2}V_p = \sqrt{2}\times 120.08\text{ V} = 169.818\text{ V}$

The voltage across armature is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha_1) = E_b + I_aR_a = k\omega + I_aR_a$$

or $$\frac{3\sqrt{3}\times 169.818}{2\pi}(1+\cos 30) = 1.2\omega + 16.666\times 0.5$$

or ω = 211.35 rad/s

Motor speed is $N = \dfrac{60\omega}{2\pi} = \dfrac{60\times 211.35}{2\times\pi} = 2017.43\text{ rpm}$

Example 12.7 A 15 HP, 300 V, 1000 rpm separately excited dc motor can be controlled by a three-phase semiconverter which is supplied by Y connected 220 V, 50 Hz ac supply. The dc motor armature resistance R_a = 1.0 Ω and armature current is continuous and ripple free. If the motor operates at 900 rpm at firing angle α = 45°, determine the rms value of source and thyristor currents, average value of thyristor current and input power factor.

Solution

Given: $R_a = 1.0\ \Omega, \alpha = 45°, N = 900\text{ rpm}$

At rated operating condition, $I_{\text{rated}} = \dfrac{P_{\text{rated}}}{V_{\text{rated}}} = \dfrac{15\times 746}{300}\text{A} = 37.3\text{ A}$

Voltage across armature $V_o = E_b + I_aR_a = k\omega + I_aR_a$

or $$300 = k\times\frac{2\pi\times 1000}{60} + 37.3\times 1.0 \quad \text{At rated speed } N = 1000\text{ rpm}$$

or $$k = \frac{(300 - 37.3) \times 60}{2\pi \times 1000} \text{ V-s/rad} = 2.507 \text{ V-s/rad}$$

The phase voltage $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{220}{\sqrt{3}} = 127.01$ V

The maximum phase voltage $V_m = \sqrt{2}V_p = \sqrt{2} \times 127.01 \text{ V} = 179.619$ V

The voltage across armature is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos\alpha_1) = E_b + I_a R_a = k\omega + I_a R_a$$

or $$\frac{3\sqrt{3} \times 179.619}{2\pi}(1 + \cos 45) = 2.507 \times \frac{2\pi \times 900}{60} + I_a \times 0.5$$

or $$253.47 = 236.37 + 0.5I_a$$

$$I_a = \frac{253.47 - 236.37}{1.0} = 17.1 \text{ A}$$

rms value of source current

$$I_{S(\text{rms})} = I_o\frac{\sqrt{2}}{\sqrt{3}} = 17.1\frac{\sqrt{2}}{\sqrt{3}} = 13.962 \text{ A}$$

rms value of thyristor current I_{T1} is

$$I_{T1(\text{rms})} = \frac{I_O}{\sqrt{3}} = \frac{17.1}{\sqrt{3}} = 9.872 \text{ A}$$

Average value of thyristor current is

$$I_{T1(\text{av})} = \frac{I_O}{3} = \frac{17.1}{3} = 5.7 \text{ A}$$

Input power factor is

$$\frac{V_o I_a}{\sqrt{3}V_L I_{S(\text{rms})}} = \frac{253.47 \times 17.1}{\sqrt{3} \times 220 \times 13.962} = 0.814 \text{ (lagging)}$$

Example 12.8 A dc motor is driven by a three-phase full converter as shown in Fig. 12.25 and 100 A dc current flow through armature with negligible ripple.

(a) Draw the ac line current i_A for one cycle at $\alpha_1 = 30°$.

(b) Determine the third and fifth harmonic components of line current as a percentage of the fundamental current.

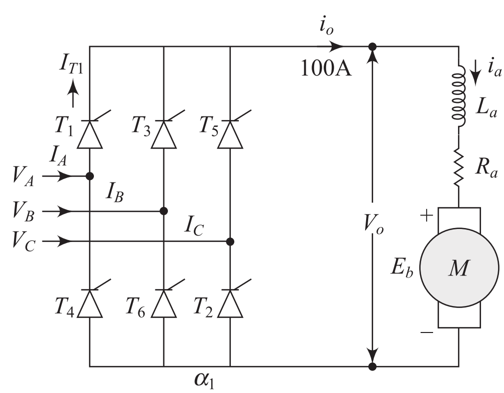


Fig. 12.25

Solution

The ac line current i_A at firing angle $\alpha_1 = 30°$ is depicted in Fig. 12.26. The amplitude of armature current $I_a = 100$ A The waveform i_A can be expressed by Fourier series as given below.

$$i_{An} = \sum_{n=1,3,5....}^{\infty} \frac{4I_a}{n\pi}\cos\frac{n\pi}{6}\sin(n\omega t - n\alpha)$$

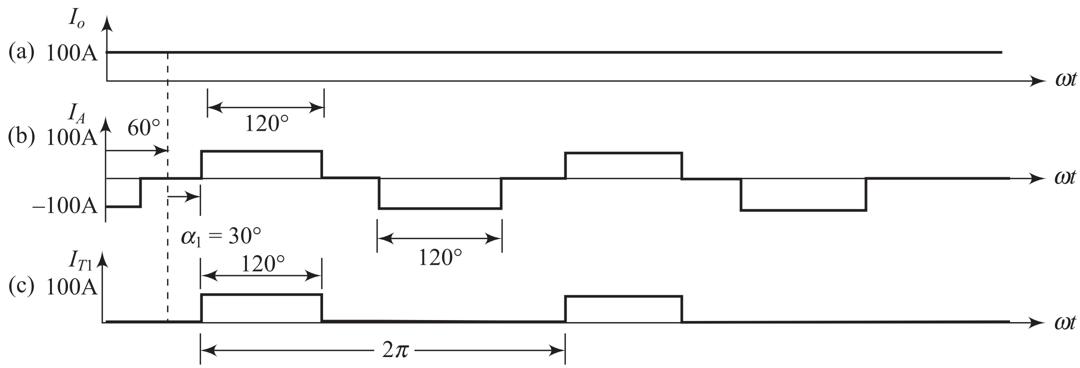


Fig. 12.26 *(a) load current i_o (b) ac line current i_A at firing angle α = 30° (c) current through T_1*

rms value of nth harmonic current is

$$I_{An} = \frac{4I_a}{\sqrt{2}n\pi}\cos\frac{n\pi}{6}$$

rms value of fundamental component of current is

$$I_{A1} = \frac{4I_a}{\sqrt{2}\pi}\cos\frac{\pi}{6} = \frac{4\times 100}{\sqrt{2}\pi}\cos\frac{\pi}{6} = 77.983 \text{ A}$$

rms value of third harmonic component of current is

$$I_{A3} = \frac{4I_a}{\sqrt{2}.3\pi}\cos\frac{3\pi}{6} = 0 \text{ A}$$

rms value of fifth harmonic component of current is

$$I_{A5} = \frac{4I_a}{\sqrt{2}.5.\pi}\cos\frac{5\pi}{6} = \frac{4\times 100}{\sqrt{2}..5\pi}\cos\frac{5\pi}{6} = -15.5966 \text{ A}$$

The third harmonic components of line current as a percentage of the fundamental current is

$$\frac{I_{A3}}{I_{A1}} = 0 = 0\%$$

The fifth harmonic components of line current as a percentage of the fundamental current is

$$\frac{I_{A5}}{I_{A1}} = \frac{-15.5966}{77.983} = -0.2 = -20\%$$

Example 12.9 In speed control of dc motor, the load torque is 40 N-m. Assume that under steady state condition the motor operates at speed 500 rpm and $t = 0$. At $t \geq 0^+$, the electromagnetic torque is suddenly increased to 100 N-m. The inertia of dc motor is 0.01 N-m.sec^2/rad.

(a) Write differential equation which governs the speed of motor for $t \geq 0$.
(b) Compute the time taken to reach the speed 1000 rpm from 500 rpm.

Assume the friction torque is negligible.

Solution

(a) At steady state condition, the electromagnetic torque is equal to load torque.
Then, at $t = 0$, $T_e = T_L$

Under dynamic condition, the electromagnetic torque T_e is equal to the sum inertia torque $J\frac{d\omega}{dt}$, friction torque $D\omega$ and load torque T_L

Therefore, $T_e = J\frac{d\omega}{dt} + D\omega + T_L$

(b) As the friction torque is negligible, $D\omega = 0$

$\therefore \qquad T_e = J\frac{d\omega}{dt} + T_L$

or $\qquad 100 = 0.01\frac{d\omega}{dt} + 40 \quad$ as $T_e = 100$ N-m, $J = 0.01$ N-m sec^2/rad, $T_L = 40$ N-m

or $\qquad \frac{d\omega}{dt} = \frac{100 - 40}{0.01} = 6000$

or $\qquad dt = \frac{d\omega}{6000}$

After integrating, we get $t = \frac{\omega}{6000} + C$

At $t = 0$, $\omega = \frac{2\pi \times 500}{60}$ rad/s

Therefore, $0 = \frac{\omega}{6000} + C$ or, $C = -\frac{\omega}{6000} = -\frac{1}{6000} \times \frac{2\pi \times 500}{60} = -\frac{\pi}{360}$

Then, $t = \frac{\omega}{6000} - \frac{\pi}{36}$

The time taken to reach the speed 1000 rpm from 500 rpm

$$t = \frac{\omega}{6000} - \frac{\pi}{36} = \frac{1}{6000} \times \frac{2\pi \times 1000}{60} - \frac{\pi}{360} = \frac{\pi}{360} \text{ sec}$$

Example 12.10 The speed of a 15 HP, 400 V, 1000 rpm separately excited dc motor is controlled by a three-phase full converter from a Y connected, 250 V, 50 Hz ac supply. Assume armature resistance $R_a = 0.5\ \Omega$, field resistance $R_f = 150\ \Omega$, and motor constant is 0.8 V/A-rad/s.

(a) The field current is controlled by a three-phase full converter and is set to its maximum value. If the motor develop 100 N-m torque at 1000 rpms, determine the firing angle of armature converter.

(b) If the field current is set to its maximum value, firing angle of armature converter is 30° and developed torque is 100 N-m, find the operating speed of motor.

Solution

Given: $R_a = 0.5\ \Omega$, $R_f = 150\ \Omega$, $k_b = 0.8$ V/A-rad/s, $N = 1000$ rpm

(a) Speed $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1000}{60} = 104.76$ rad/s

Phase voltage $V_p = \frac{V_L}{\sqrt{3}} = \frac{250}{\sqrt{3}}\text{V} = 144.337$ V

Maximum phase voltage $V_m = \sqrt{2}V_p = \sqrt{2} \times 144.337 = 204.123$ V

The field current will be maximum when field voltage will be maximum. Therefore, $\alpha_f = 0$.

Voltage across field winding $V_f = \frac{3\sqrt{3}V_m}{\pi}\cos\alpha_f = \frac{3\sqrt{3}V_m}{\pi} = \frac{3\sqrt{3} \times 204.123}{\pi} = 337.48$ V

The field current $I_f = \frac{V_f}{R_f} = \frac{337.48}{150} \text{A} = 2.249 \text{ A}$

The toque developed $T_e = kI_f I_a$

Armature current is $I_a = \frac{T_e}{kI_f} = \frac{100}{0.8 \times 2.249} \text{A} = 55.58 \text{ A}$

At rated speed, the generated back emf is

$$E_b = k_b I_f \omega = 0.8 \times 2.249 \times 104.76 \text{ V} = 188.48 \text{ V}$$

Voltage across armature is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_1 = E_b + I_a R_a$$

or
$$\frac{3\sqrt{3} \times 204.123}{\pi} \cos \alpha_1 = 188.48 + 55.58 \times 0.5 = 216.27 \text{ V}$$

$$\alpha_1 = \cos^{-1}\left(\frac{216.27 \times \pi}{3\sqrt{3} \times 204.123}\right) = 50.145°$$

The firing angle of armature converter is α_1 = 50.145°

(b) at α_1 = 30°, Voltage across armature is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_1 = \frac{3\sqrt{3} \times 204.123}{\pi} \cos 30 = 292.26 \text{ V}$$

We know that $V_o = E_b + I_a R_a$

Therefore, $E_b = V_o - I_a R_a = 292.26 - 55.58 \times 0.5 = 264.47 \text{ V}$

The back emf $E_b = k_b I_f \omega = 0.8 \times 2.249 \times \omega = 264.47 \text{ V}$

Speed $\omega = \frac{264.47}{0.8 \times 2.249} = 146.993$ rad/s = 1403.11 rpm

Example 12.11 A 220 V, 1450 rpm, 20 A separately excited dc motor is supplied from a three-phase full converter. When the full converter is connected to 440 V, 50 ac supply through a delta-star transformer, (a) determine the turn ratio of transformer and (b) find the value of firing angle of armature converter when (i) motor rotates at 1200 rpm with rated torque and (ii) motor operates at -1000 rpm with half rated torque.

Assume armature resistance is 0.5 Ω and motor terminal voltage is rated voltage when converter firing angle is zero.

Solution

(a) At α_1 = 0°, Voltage across armature is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_1 = \frac{3\sqrt{3} \times V_m}{\pi} \cos 0 = 220 \text{ V}$$

or
$$V_m = \frac{220\pi}{3\sqrt{3}} = 133.065 \text{ V}$$

Phase voltage (secondary) $V_p = \frac{V_m}{\sqrt{2}} = \frac{133.065}{\sqrt{2}} = 94.09 \text{ V}$

Phase voltage (primary) = 440 V

$$\text{Turn ratio} = \frac{\text{Phase Voltage (Primary)}}{\text{Phase Voltage (Secondary)}} = \frac{440}{94.09} = 4.67$$

(b) (i) Motor rotates at 1200 rpm with rated torque
At 1450 rpm and I_a = 20 A, the induced back emf

$$E_b = V_o - I_a R_a = 220 - 20 \times 0.5 = 210 \text{ V}$$

At 1200 rpm, the induced back emf is $210 \times \frac{1200}{1450} \text{V} = 173.79 \text{ V}$

The voltage across armature at rated torque is

$$V_o = E_b + I_a R_a = 173.79 + 20 \times 0.5 = 183.79 \text{ V}$$

Therefore, $V_o = V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_1 = 183.79 \text{ V}$

or $\frac{3\sqrt{3} \times 133.065}{\pi} \cos \alpha_1 = 183.79 \text{ V}$

$$\alpha_1 = \cos^{-1}\left(\frac{183.79\pi}{3\sqrt{3} \times 133.065}\right) = 33.34°$$

(ii) Motor operates at -1000 rpm with half rated torque

Armature current $I_a = \frac{1}{2} \times \text{rated current} = \frac{1}{2} \times 20 = 10 \text{ A}$

At 1200 rpm, the induced back emf is $-210 \times \frac{1000}{1450} \text{V} = -144.8275 \text{ V}$

The voltage across armature at rated torque is

$$V_o = V_a = \frac{3\sqrt{3}V_m}{\pi} \cos \alpha_1 = E_b + I_a R_a = -144.8275 + 10 \times 0.5 = -139.8275 \text{ V}$$

Therefore, $\frac{3\sqrt{3} \times 133.065}{\pi} \cos \alpha_1 = -139.8275 \text{ V}$

$$\alpha_1 = \cos^{-1}\left(\frac{-139.8275\pi}{3\sqrt{3} \times 133.065}\right) = 129.46°$$

12.5 dc-TO-dc CONVERTER (CHOPPER) FED dc DRIVES

Variable dc voltage can be obtained from dc-to-dc converters or choppers. Therefore, choppers can be used for speed control of dc motor. If the field winding of dc motor is fed from a constant dc source and a chopper is connected across armature circuit, the voltage across armature can be controlled by varying the duty cycle of chopper. Due to armature voltage variation, the speed of dc motor can be controlled below base speed. For speed control of dc motor above base speed, another chopper may be connected across field winding. The dc-to-dc converter or chopper fed dc drives are widely used in electric traction applications.

The chopper fed dc drive is not only used in speed control but also it can be used for dynamic and regenerative braking of dc motors. A chopper fed dc drive operates in four quadrants by controlling both armature voltage and field current. When the field flux is in the same direction, but the polarity of armature voltage can be reversed. The motor will operate in reverse direction, it is called reverse motoring.

During regenerative braking, motor voltage must be greater than input voltage. If the input voltage is non-receptive during regenerative braking, motor voltage will be less than input voltage and subsequently regenerative may not be possible. In that case, rheostatic braking is possible in chopper fed dc drives.

Chopper fed dc drives are most commonly used in battery operated electric vehicles. The different operating modes of chopper fed drives are:

1. Power control or motoring control
2. Rheostatic or dynamic braking control
3. Regenerative braking control
4. Combination of regenerative and rheostatic braking control

12.5.1 Operating Principle of Power Control

Figure 12.27 shows the arrangement of chopper fed separately excited dc motor. In this dc-to-dc converter, the semiconductor switch such as transistor, IGBT, MOSFET or force commutated thyristor may be used. This drive operates as first quadrant drive as shown in Fig. 12.28(b). The waveforms of input voltage V_i, armature voltage V_o, load current i_o, input current i_S and free wheeling diode current i_{FD} are depicted in Fig. 12.28(a). Assume that load is highly inductive and the load current is ripple free. The average voltage across armature of dc motor is

$$V_o = \frac{T_{\text{ON}}}{T} V = DV = f T_{\text{ON}} V$$

where D = Duty cycle $= \dfrac{T_{\text{ON}}}{T}$ and f = Chopping frequency $= \dfrac{1}{T}$

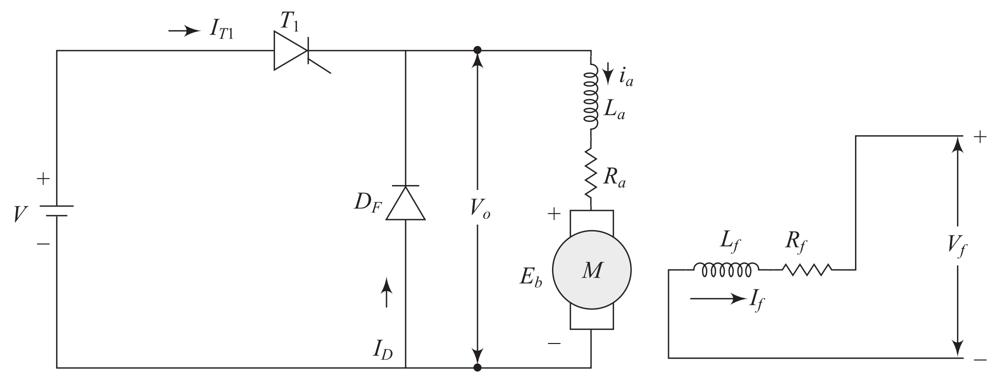


Fig. 12.27 *Chopper fed dc drive*

The power supplied to the motor = Average armature voltage × Average armature current

$$= V_o I_a = DVI_a$$

Average value of input current is equal to

$$I_i = \frac{T_{\text{ON}}}{T} I_a = DI_a$$

The equivalent input resistance of chopper drive seeing from input is

$$R_{\text{eq}} = \frac{V_i}{I_i} = \frac{V}{DI}$$

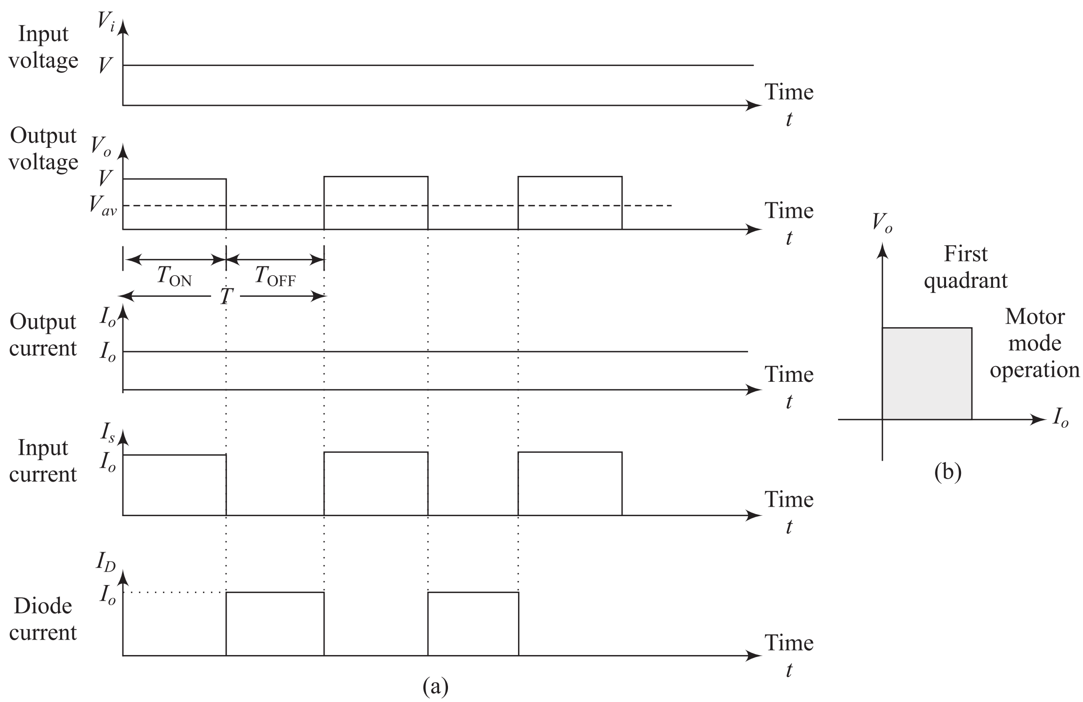


Fig. 12.28 *(a) Voltage and current waveforms chopper fed dc drive (b) Quadrant of chopper*

By varying duty cycle D, R_{eq} can be varied.
Applying KVL in the armature circuit, we get

$$V_o = DV = E_b + I_a R_a = k_b \omega + I_a R_a$$

or
$$\omega = \frac{DV - I_a R_a}{k_b}$$

It is clear from above equation that by varying the duty cycle D of chopper, the voltage across armature can be controlled and subsequently the speed of dc motor will be controlled.

If the load is not highly inductive, the armature current will not be ripple free. When the switch is ON, input voltage is applied across armature, the armature current will increase from I_{min} to I_{max}. If the switch is OFF, the free wheeling diode conducts and the voltage across armature becomes zero and armature current falls from I_{max} to I_{min}. The waveforms of armature voltage, armature current, source current and current through freewheeling diode are shown in Fig. 12.29.

The instantaneous armature current when the switch is ON is expressed by

$$I_a(t) = \frac{V - E_b}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{min} e^{-\frac{t}{\tau}} \qquad \text{for } 0 < t \le T_{ON}$$

where, $\tau = \dfrac{L}{R}$

When the switch is OFF, the instantaneous armature current is

$$I_a(t) = -\frac{E_b}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{max} e^{-\frac{t}{\tau}} \qquad \text{for } T_{ON} \le t \le T$$

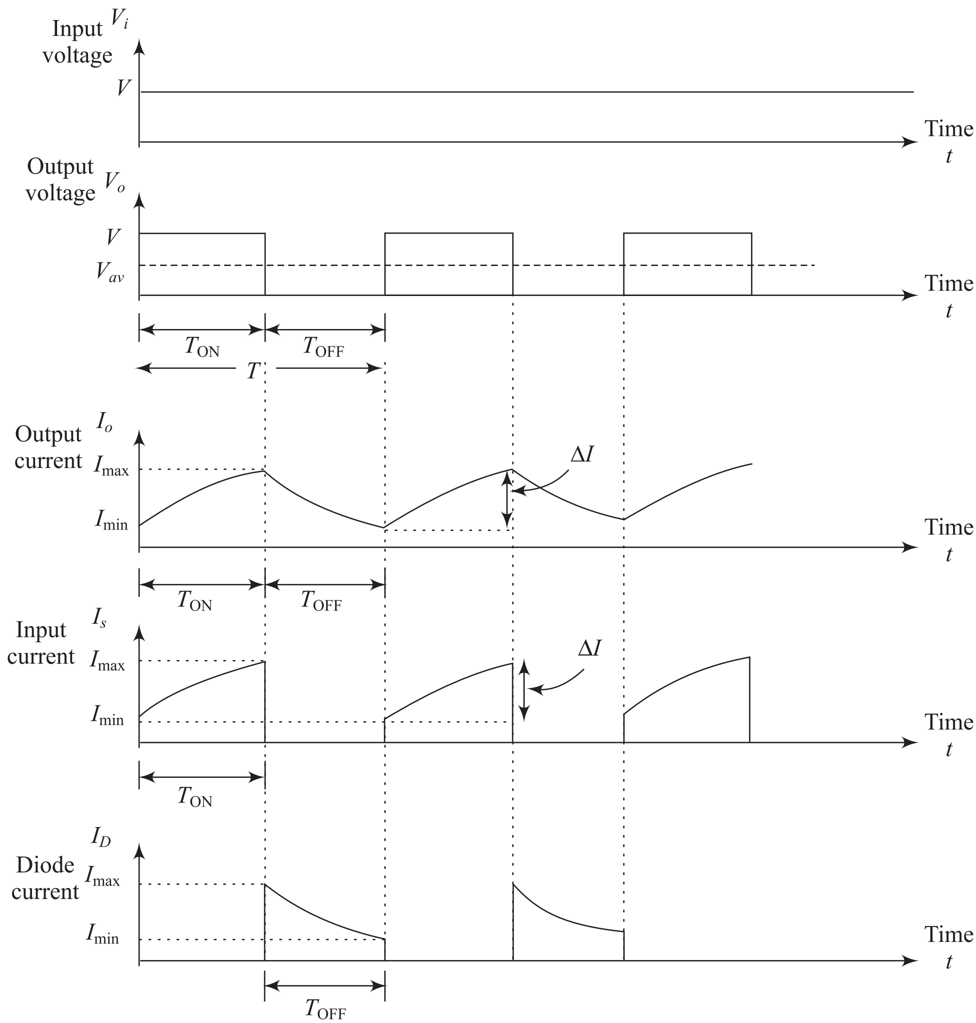


Fig. 12.29 *Voltage and current waveforms chopper fed dc drive*

The maximum peak-to-peak ripple current is

$$\Delta I_{max} = \frac{V}{R}\tanh\frac{R}{4fL}$$

where, R is the total resistance of armature circuit

L is the total resistance of armature circuit

If motor armature and field windings are connected in series of a series motor,

$$R = R_a(\text{armature resistance}) + R_f(\text{field resistance}) + \text{ any series resistance and}$$

$$L = L_a(\text{armature inductance}) + L_f(\text{field.inductance}) + \text{ any series inductance.}$$

At steady state operating condition, voltage across motor terminals is equal to

$$V_o = DV = E_b + I_a R$$

Example 12.12 The speed of separately excited dc motor is controlled by a chopper. The input voltage of chopper is 220 V, armature resistance R_a = 0.25 Ω and motor constant K = 0.08 V/rpm. When the motor drives a constant load, average armature current of 25 A flows through armature winding. Determine (a) the range of speed control and (b) the range of duty cycle D. Assume motor current is ripple free.

Solution

Given: $V = 220$ V, $R_a = 0.25\ \Omega$, $k = 0.08$, $I_a = 25$ A

When the duty cycle of chopper is D, the voltage across armature is

$$V_o = DV = E_b + I_aR_a = k_b\omega + I_aR_a$$

Since the motor drives a constant load torque, motor torque is constant and armature current is constant at 25 A. The minimum possible speed is zero, i.e., N = 0 rpm

Therefore, $DV = k_b\omega + I_aR_a$

or $$D \times 220 = 0.08 \times 0 + 25 \times 0.25 \quad \text{as} \quad \omega = \frac{2\pi N}{60} = 0$$

∴ $$D = \frac{25 \times 0.25}{220} = 0.0284$$

The spped of dc motor will be maximum at D = 1

$$DV = k_b\omega + I_aR_a$$

or $$\omega = \frac{DV - I_aR_a}{k_b} = \frac{1 \times 220 - 25 \times 0.25}{0.08} \text{ rad/s} = 2671.87 \text{ rpm}$$

The range of speed control is $0 \le N \le 2671.87$ and the range of duty cycle $0.284 \le D \le 1$

Example 12.13 A dc series motor is drived by a chopper. The motor has the following parameters:

$$R_a = 0.1\ \Omega,\ R_f = 0.2\ \Omega,\ K = 0.004 \text{ N-m/Amp}^2$$

If the dc supply voltage of chopper is 400 V, duty cycle of chopper is 50% and the average armature current is 100 A, determine (a) input power from supply, (b) motor speed and (c) motor torque.
Assume current is ripple free.

Solution

Given: V = 400 V, D = 50% = 0.5 $I_a = 100$ A, $R_a = 0.1\ \Omega$, $R_f = 0.2\ \Omega$, $K = 0.004$ N-m/Amp^2

(a) Input power from supply $V_oI_a = DVI_a = 0.5 \times 400 \times 100 = 20$ kW

(b) Voltage across motor terminals is

$$V_o = DV = E_b + I_aR \quad \text{where,} \quad R = R_a + R_f = (0.1 + 0.2)\ \Omega = 0.3\ \Omega$$

or $$E_b = DV - I_aR = 0.5 \times 400 - 100 \times 0.3 = 170 \text{ V}$$

Since $E_b = k_bI_a\omega$, $$\omega = \frac{E_b}{k_bI_a} = \frac{170}{0.004 \times 100} = 425 \text{ rad/s}$$

(c) Motor torque $T_e = kI_a^2 = 0.004 \times 100^2 \text{ N-m} = 40$ N-m

Example 12.14 A chopper circuit is used to control a separately excited dc motor. The chopper is ON for 10 ms and OFF for 20 ms. When the chopper is supplied by 200 V dc supply, determine the average load current when the motor operates in 1200 rpm and machine constant k = 0.5 V/rad per sec. The armature resistance is 1 Ω. Assume armature current is continuous and ripple free.

Solution

Given: $V = 200$ V, $T_{ON} = 10$ ms, $T_{OFF} = 20$ ms, $N = 1200$ rpm, $k = 0.5$ V/rad per sec, $R_a = 1\ \Omega$

Duty cycle $D = \dfrac{T_{ON}}{T_{ON} + T_{OFF}} = \dfrac{10}{10+20} = \dfrac{1}{3}$

Voltage across armature terminals is

$$V_o = DV = E_b = I_a R = k\omega + I_a R_a$$

or
$$\frac{1}{3} \times 200 = 0.5 \times \frac{2\pi \times 1200}{60} + I_a \times 1 \quad \text{as } \omega = \frac{2\pi N}{60}$$

The average load current $I_a = \dfrac{200}{3} - 0.5 \times \dfrac{2\pi \times 1200}{60} = 3.80$ A

Example 12.15 The speed of a separately excited dc motor is controlled by a chopper circuit. Assume that the chopper is supplied by 250 V dc supply and it operates at 200 Hz. The motor has the following parameters:

$$R_a = 0.25\ \Omega,\ L_a = 25 \text{ mH},\ k = 1.5 \text{ V/rad per sec or N-m/A}$$

When the motor operates at 1200 rpm with load torque 30 N-m, determine (a) the minimum and maximum values of armature current, (b) ripple current and (c) derive the expression of armature current during ON and OFF period of chopper.

Solution

Torque developed in the motor $T_e = kI_a$

At 30 N-m load torque, the average armature current $I_a = \dfrac{T_e}{k} = \dfrac{30}{1.5} = 20$ A

The induced back emf $E_b = k\omega = k\dfrac{2\pi N}{60} = 1.5 \times \dfrac{2\pi \times 1200}{60} = 188.57$ V

Voltage across armature terminals

$$V_o = DV = E_b + I_a R_a = 188.57 + 20 \times 0.25 = 193.57 \text{ V}$$

or
$$D = \frac{193.57}{250} = 0.774$$

Time period $T = \dfrac{1}{f} = \dfrac{1}{200} = 5$ ms,

$$T_{ON} = DT = 0.774 \times 5 \text{ ms} = 3.87 \text{ ms} \quad \text{and} \quad T_{OFF} = T - T_{ON} = 5 \text{ ms} - 3.87 \text{ ms} = 1.13 \text{ ms}$$

$$\tau = \frac{L_a}{R_a} = \frac{25 \times 10^{-3}}{0.25} = 100 \times 10^{-3}, \quad \frac{1}{\tau} = \frac{1}{100 \times 10^{-3}} = 10$$

The instantaneous armature current when the switch is ON is expressed by

$$I_a(t) = \frac{V - E_b}{R_a}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{min} e^{-\frac{t}{\tau}} \quad \text{for } 0 < t \le T_{ON}$$

$$= \frac{250 - 188.57}{0.25}(1 - e^{-10t}) + I_{min} e^{-10t} = 245.72(1 - e^{-10t}) + I_{min} e^{-10t}$$

At $t = 3.87$ ms, $I_a(t) = I_{max}$.

Therefore, $I_{max} = 245.72(1 - e^{-10 \times 3.87 \times 10^{-3}}) + I_{min} e^{-10 \times 3.87 \times 10^{-3}}$

or
$$I_{max} = 9.314 + 0.962 I_{min} \tag{i}$$

When the switch is OFF, the instantaneous armature current is

$$I_a(t) = -\frac{E_b}{R}\left(1 - e^{-\frac{t}{\tau}}\right) + I_{\max}e^{-\frac{t}{\tau}} \quad \text{for } T_{ON} \le t \le T$$

$$= -\frac{188.57}{0.25}(1 - e^{-10t}) + I_{\max}e^{-10t} = -754.28(1 - e^{-10t}) + I_{\max}e^{-10t}$$

At t = 1.13 ms, $I_a(t) = I_{\min}$.

Therefore, $I_{\min} = -754.28(1 - e^{-10\times1.13\times10^{-3}}) + I_{\max}e^{-10\times1.13\times10^{-3t}}$

$$I_{\min} = -8.428 + 0.988I_{\max} \tag{ii}$$

After substituting the value of $I_{\min}$ in Eq. (i), we get

$$I_{\max} = 9.314 + 0.962 \times (-8.428 + 0.988I_{\max})$$

or $$I_{\max} = 1.206 + 0.95I_{\max}$$

$\therefore$ $$I_{\max} = 24.12 \text{ A}$$

Then $$I_{\min} = -8.428 + 0.988I_{\max} = -8.428 + 0.988 \times 24.12 = 15.40 \text{ A}$$

(b) Ripple current is $\Delta I = I_{\max} - I_{\min} = 24.12 - 15.40 = 8.72$ A

The instantaneous armature current when the switch is ON is expressed by

$$I_a(t) = 245.72(1 - e^{-10t}) + 15.4e^{-10t} \quad \text{for } 0 < t \le T_{\text{ON}}$$

The instantaneous armature current when the switch is OFF is expressed by

$$I_a(t) = -754.28(1 - e^{-10t}) + 24.12e^{-10t} \quad \text{for } T_{\text{ON}} \le t \le T$$

12.5.2 Principle of Regenerative Braking of Separately Excited dc Motor

During regenerative braking, the dc motor acts as a generator and the kinetic energy of the motor and load are returned to the dc supply. In the motor mode operation, the armature current is positive and its value is $I_a = \frac{V_o - E_b}{R_a}$.

When the armature of a separately excited motor rotate at speed ω, the kinetic energy is stored in the inertia of motor and load. In the transport system, the energy is stored in vehicles, train, rotating shaft. The regenerative braking of separately excited dc motor can be done by circuit as shown in Fig. 12.30. When the switch (thyristor T_1) is ON, the armature current increases due to short circuit of armature terminals of dc motor. When the switch (thyristor T_1) is OFF and the diode conducts, the energy stored in the armature inductance would be returned back to the supply and the drive operates in second quadrant. The waveforms of input voltage, armature voltage, input current, armature current are depicted in Fig. 12.31. Assume that the armature current is continuous and ripple free.

The average voltage across armature terminals (chopper) is

$$V_O = \frac{T_{OFF}}{T_{ON} + T_{OFF}}V = \frac{T_{OFF}}{T}V = (1 - D)V \quad \text{where, } D = \text{duty cycle}$$

Power generated by the dc motor is

$$P_g = V_o I_a = (1 - D)VI_a$$

When the motor operates as generator, its generated voltage is equal to

$$E_g = k_b \omega I_f = V_o + I_a R = (1 - D)V + I_a R_a$$

where, k_b is machine's constant, ω is machine speed in rads per second.

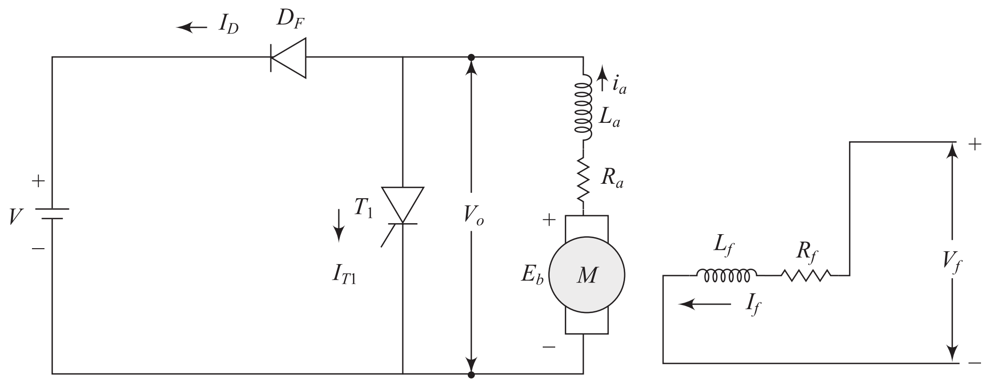


Fig. 12.30 *Regenerative braking of separately excited dc motor*

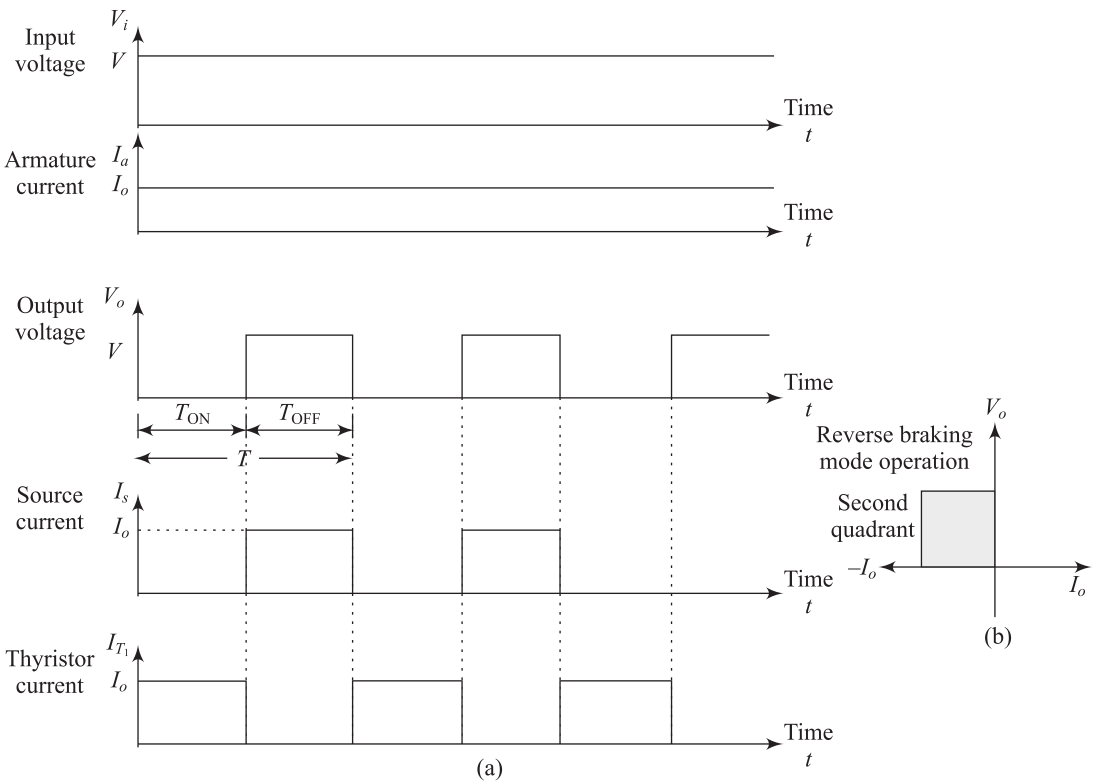


Fig. 12.31 *(a) Voltage and current waveforms of regenerative braking of dc motor and (b) Second quadrant operation of dc motor*

The condition of regenerative braking is $E_g > V_o + I_a R_a$

or $$E_g > (1-D)V + I_a R_a \quad \text{as} \quad V_o = (1-D)V$$

If the minimum braking speed is $\omega_{\min}$, the motor speed during regenerative braking is

$$\omega = \frac{(1-D)V + I_a R_a}{k_b I_f} \quad \text{where, } \omega > \omega_{\min}$$

The maximum braking speed of a dc series motor can be found from

$$k_b \omega_{\max} I_f - I_a R = V$$

or $$\omega_{\max} = \frac{V + I_a R}{kI_f} \quad \text{where, } R = R_a + R_f$$

When the switch is ON, $V_t = 0 = E_g - I_a R_a - L_a \dfrac{di_a}{dt}$

or $$E_g - I_a R_a = L_a \frac{di_a}{dt}$$

As $\dfrac{di_a}{dt}$ is positive and current rises in inductance, energy is stored in inductance L_a

Since $\dfrac{di_a}{dt} \geq 0, \quad E_g - I_a R_a \geq 0$

When the switch is OFF, $E_g - I_a R_a - L_a \dfrac{di_a}{dt} = V$

or $$E_g - I_a R_a - V = L_a \frac{di_a}{dt}$$

Therefore, $V - (E_g - I_a R_a) = -L_a \dfrac{di_a}{dt}$,

For regenerative braking, $E_g - I_a R_a > V$

Therefore, $V - (E_g - I_a R_a)$ must be negative, current decreases and $\dfrac{di_a}{dt}$ becomes negative.

∴ $$V - (E_g - I_a R_a) \leq 0$$

or $$-(E_g - I_a R_a) \leq -V$$

or $$(E_g - I_a R_a) \geq V$$

At minimum braking speed

$$(E_g - I_a R_a) = 0 \qquad \text{or} \qquad k_b \omega I_f = I_a R_a \quad \text{as} \quad E_g = k_b \omega I_f$$

The maximum braking speed $\omega_{\min} = \dfrac{I_a R_a}{k_b I_f}$

During maximum braking speed,

$$(E_g - I_a R_a) = V$$

The maximum braking speed $\omega_{\max} = \dfrac{V + I_a R_a}{k_b I_f}$

For regenerative braking, the motor speed must be greater than $\omega_{\min}$ and less than $\omega_{\max}$ and it is expressed by

$$\omega_{\min} < \omega < \omega_{\max}$$

or $$\frac{I_a R_a}{kI_f} < \omega < \frac{V + I_a R_a}{kI_f}$$

The regenerative braking of chopper fed separately excited dc drive provides stable operation, but dc series motor operates with an unstable operating characteristic. Therefore, regenerative braking of dc series motor is difficult one.

Example 12.16 Regenerative braking is applied to a separately excited dc motor using a dc chopper. The chopper circuit is supplied from 250 V dc supply. The average armature current during regenerative braking is kept constant 200 A with negligible ripple. The machine has the following parameters:

$$R_a = 0.25\ \Omega,\ k = 1.2 \text{ V/rad per sec}$$

If the duty cycle of chopper is 50%, determine (a) power feedback to dc supply, (b) minimum and maximum permissible braking speeds and the speed range of regenerative braking and (c) speed during regenerative braking.

Solution

Average armature voltage is $V_o = (1 - D)V = (1 - 0.5) \times 250 = 125$ V

(a) Power feedback to dc supply is $V_o I_a = 125 \times 200 = 25$ kW

(b) Minimum braking speed $\omega_{\min} = \dfrac{I_a R_a}{k} = \dfrac{200 \times 0.25}{1.2} = 41.666$ rad/s

Maximum braking speed $\omega_{\max} = \dfrac{V + I_a R_a}{k} = \dfrac{250 + 200 \times 0.25}{1.2} = 250$ rad/s

The speed range of regenerative braking $41.66 \le \omega \le 250$ rad/s

(c) During regenerative braking, the generated voltage is equal to

$$E_b = V_o + I_a R_a = 125 + 200 \times 0.25 = 175 \text{ V}$$

We know that $E_b = k\omega$, then $\omega = \dfrac{E_b}{k} = \dfrac{175}{1.2} = 145.833$ rad/s

12.5.3 Principle of Rheostatic Braking of dc Drive

In rheostatic braking of chopper fed dc drive, the stored energy must be dissipated in the rheostat. Therefore, the system will be heated. The rheostatic braking is also known as dynamic braking. Figure 12.32 shows the arrangement of rheostatic braking of a separately excited dc motor. This drive operates in second quadrant. The waveforms of voltage and current for rheostatic braking of a separately excited dc motor are shown in Fig. 12.33. Assume that the armature current is continuous and ripple free.

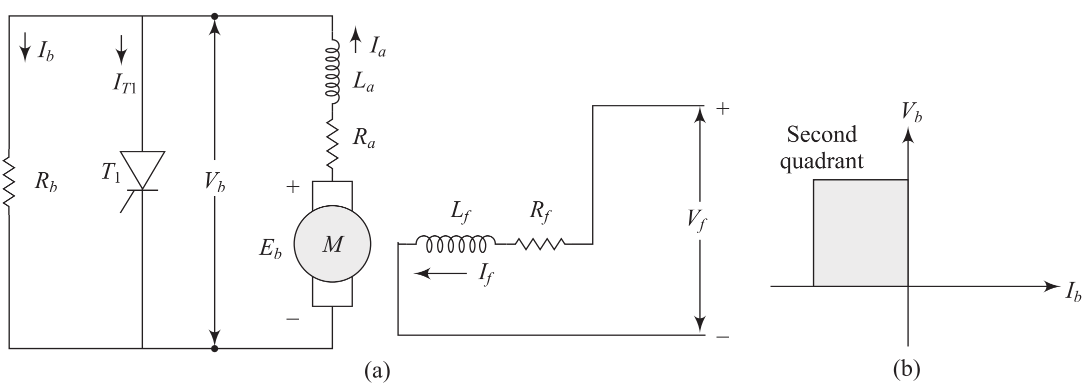


Fig. 12.32 *(a) Rheostatic braking of a separately excited dc motor (b) Quadrant*

The average current through braking resistance is

$$I_b = (1 - D)I_a$$

The average voltage across braking resistance is

$$V_b = I_b R_b = (1 - D)I_a R_b$$

The equivalent load resistance of the generator is

$$R_{\text{eq}} = \frac{V_b}{I_a} = \frac{(1-D)I_aR_b}{I_a} = (1-D)R_b$$

The power dissipation in the braking resistance R_b is

$$P_b = I_a^2 R_b(1-D) = (1-D)I_a^2 R_b$$

It is clear from above equation that by varying the duty cycle D, the braking power can be controlled.

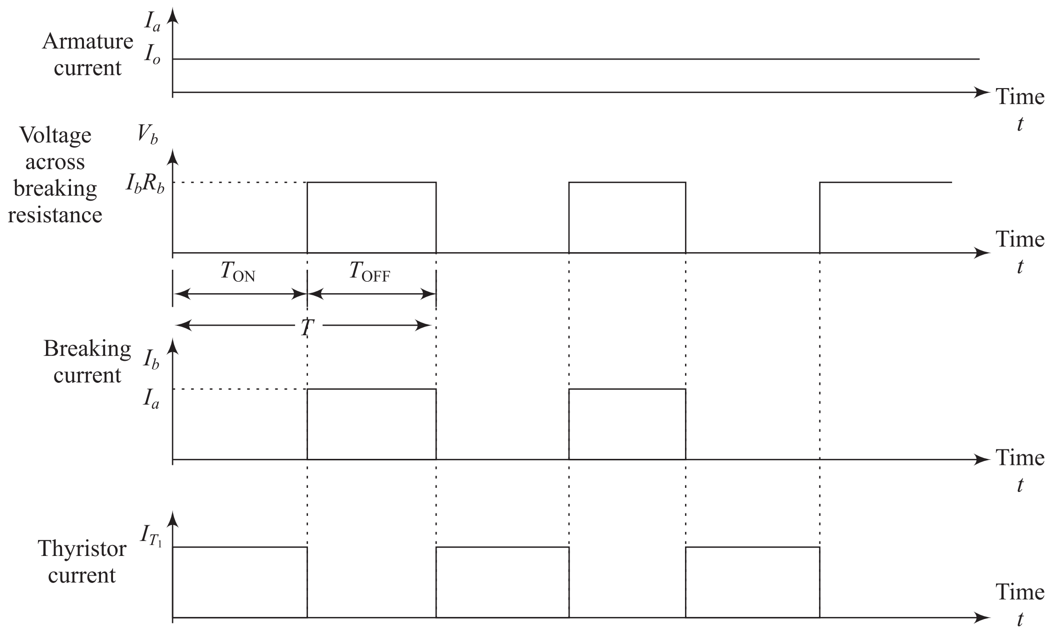


Fig. 12.33 *Waveforms of rheostatic braking of a separately excited dc motor*

Example 12.17 A dc chopper is used for rheostatic braking of a separately excited dc motor. The machine has the following parameters:

Armature resistance $R_a = 0.1\ \Omega$, braking resistance $R_b = 5\ \Omega$ and $k_b = 1.2$ V/A rad per sec

The average armature current during braking is kept constant 100 A with negligible ripple. The field current is 1.5 A.

If the duty cycle of chopper is 60%, determine (a) average current through braking resistance, (b) average voltage across dc converter, (c) equivalent load resistance of the generator, (d) power dissipated across braking resistance and (e) speed during braking.

Solution

(a) The average current through braking resistance is

$$I_b = (1-D)I_a = (1-0.6)\times 100 = 40\ \text{A}$$

(b) The average voltage across dc converter

$$V_{\text{Chopper}} = V_b = I_bR_b = (1-D)I_aR_b = (1-0.6)\times 100\times 5 = 200\ \text{V}$$

(c) The equivalent load resistance of the generator is

$$R_{\text{eq}} = \frac{V_b}{I_a} = \frac{200}{40} = 5\ \Omega$$

(d) The power dissipation in the braking resistance R_b is

$$P_b = I_a^2 R_b (1-D) = (1-D) I_a^2 R_b = (1-0.6) \times 100^2 \times 5 = 20 \text{ kW}$$

(e) The generated emf $E_b = V_b + I_a R_a = 200 + 100 \times 0.1 = 210$ V

We know that $E_b = k_b I_f \omega$, then $\omega = \dfrac{E_b}{k_b I_f} = \dfrac{210}{1.2 \times 1.5} = 116.666$ rad/s

12.5.4 Combination of Regenerative and Rheostatic Braking of dc Motor

The stored energy in the inertia of machine and load is dissipated as heat during rheostatic braking. In case of regenerative braking, the stored energy is fed back to dc supply. Therefore, regenerative braking is energy efficient braking. If the dc supply is partially receptive, the combination of regenerative and rheostatic brake control will be the most energy efficient. This method is practically suitable for electric traction. Figure 12.34 shows the circuit diagram for regenerative and rheostatic braking.

Initially regenerative braking is applied to motor and the line voltage is sensed continuously. Whenever the amplitude of voltage exceeds a preset value, i.e., about 20% the line voltage, the regenerative braking is removed and the rheostatic braking is applied to motor. Thyristor T_2 will be turned ON to divert the current to the braking resistance R_b.

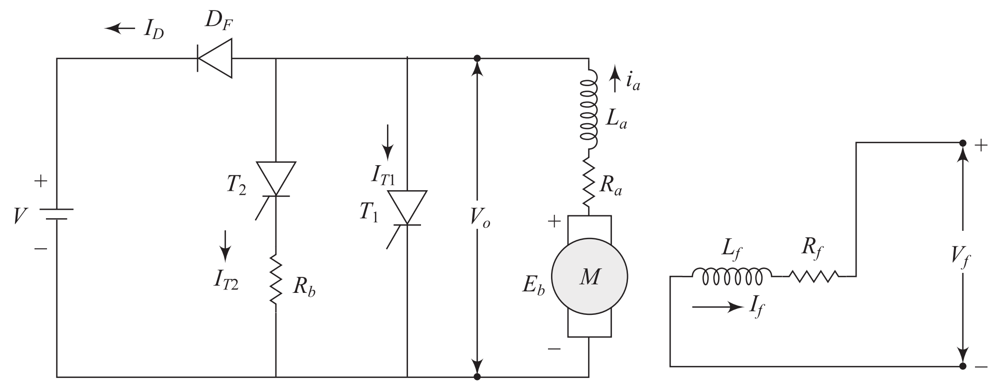


Fig. 12.34 *Combination of regenerative and rheostatic braking of dc motor*

12.6 TWO-QUADRANT CHOPPER-FED DC DRIVES

For motor mode operation, the dc-to-dc converter or chopper fed dc drives operates in the first quadrant where both the armature voltage and armature current are positive. During regenerative braking, the dc-to-dc converter fed dc drives operates in second quadrant where armature voltage is positive but the armature current is negative. Figure 12.35 shows the basic arrangement of two quadrant dc-to-dc converter fed drives where thyristors and transistors can be used in place of semiconductor switches. This drive operates on

1. motoring mode
2. regenerative braking mode

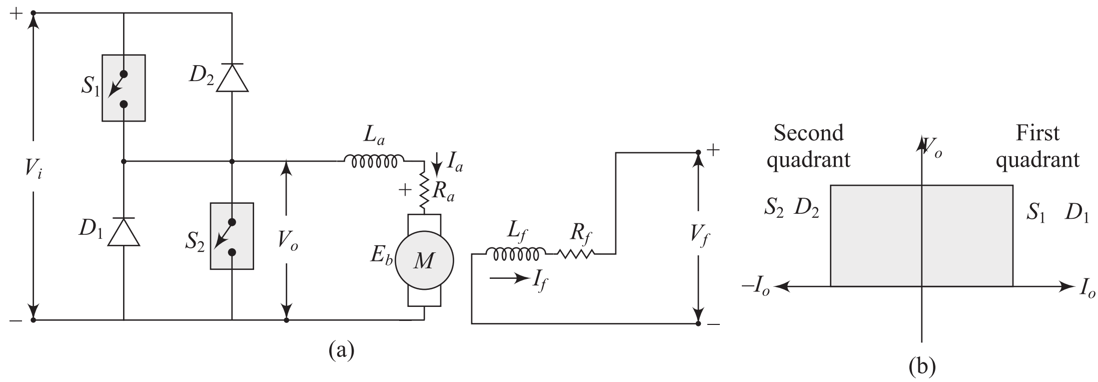


Fig. 12.35 *(a) Two quadrant chopper fed dc drive (b) Two-quadrant diagram*

12.6.1 Motoring Mode

In this operating mode, the semiconductors switch S_1 and diode D_1 operate. When switch S_1 is turned ON, the input voltage V_i is applied to the armature terminals and armature current I_a increases. When switch S_1 is turned OFF, the free wheeling diode D_1 conducts and the armature current I_a flows through diode D_1. In motor mode operation, this drive operates in first quadrant as shown in Fig. 12.35(b). The first quadrant operation of dc motor is also called as *forward motoring mode*.

12.6.2 Regenerative Braking Mode

In this operating mode, the semiconductors switch S_2 and diode D_2 operate. When switch S_2 is turned ON, the motor acts as a generator. The armature current rises and energy is stored in armature inductance L_a. When the switch S_2 is turned OFF, diode D_2 becomes turn ON, the direction of armature current I_a is reversed. Subsequently, the energy stored in inductance L_a will be returned to dc supply. During regenerative braking mode, this drive operates in second quadrant as shown in Fig. 12.35(b). The second quadrant operation of dc motor is known as *forward regenerative braking mode*.

12.7 FOUR-QUADRANT CHOPPER-FED DC DRIVES

Figure 12.36(a) shows a four quadrant chopper fed dc drive which consists of four semiconductor switches S_1, S_2, S_3, S_4 and four diodes D_1, D_2, D_3 and D_4. This dc drive operates on the following modes:

1. First quadrant (Forward motoring)
2. Second quadrant (Forward regenerative braking mode)
3. Third quadrant (Reverse motoring)
4. Fourth quadrant (Reverse regenerative braking)

In this section, all four operating modes are discussed.

1. Forward motoring During forward motoring mode operation, this drive operates in the first quadrant. In this mode, semiconductor switches S_1 and S_2 operate and switches S_3 and S_4 are OFF. When switches S_1 and S_2 are turned ON, the input voltage V_i is applied across armature terminals of dc motor and the armature current I_a increases. While switch S_1 is turned OFF and switch S_2 is ON condition, the armature current start to decrease and flows through switch S_2 and diode D_4. If both switches S_1 and S_2 are turned OFF, armature current is forced to decay through diodes D_3 and D_4.

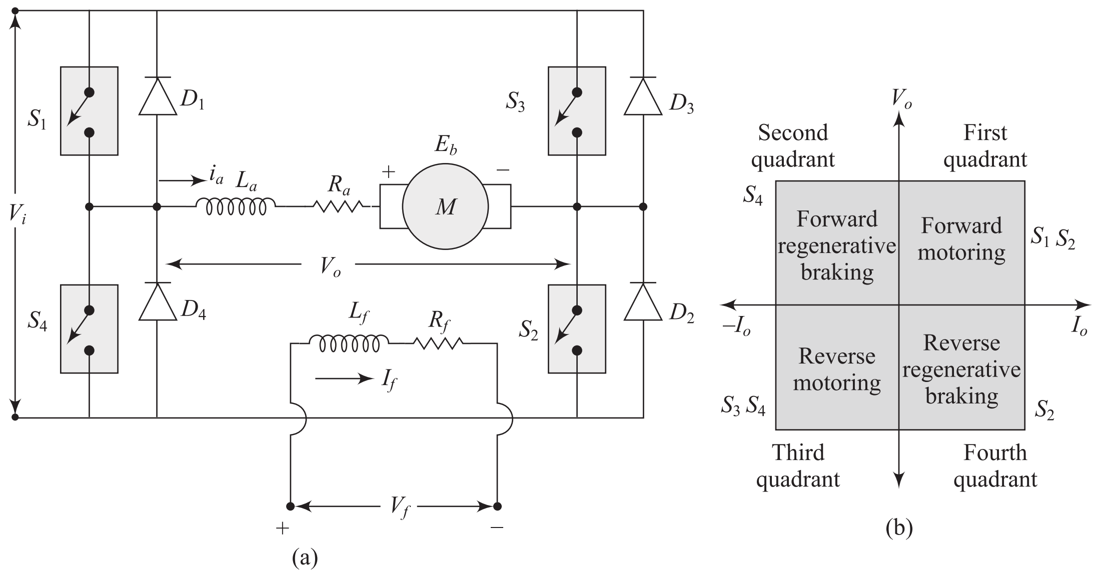


Fig. 12.36 *(a) Four quadrant chopper fed dc drive (b) Four quadrant diagram*

2. Forward regenerative braking mode In this operating mode, the motor works as a generator, the induced emf of motor is greater than input voltage. When semiconductor switches S_1, S_2 and S_3 are turned off and switch S_4 is turned ON, armature current increases and flows through switch S_4 and diode D_2. When the switch S_4 becomes OFF, the motor acts as a generator and the stored energy in inductance must be returned back to dc source through diodes D_1 and D_2. In forward regenerative braking mode, motor operates in second quadrant.

3. Reverse motoring In this operating mode, semiconductor switches S_3 and S_4 operate and switches S_1 and S_2 are turned OFF. When switches S_3 and S_4 are turned ON, the armature current increases, but it flows in reverse direction. When switch S_4 is turned ON and switch S_3 is turned OFF, the armature current starts to decay and flows through switch S_4 and diode D_2. When both switches S_3 and S_4 are turned OFF, armature current falls and flows through diodes D_1 and D_2. This operating mode is opposite to forward motoring mode.

4. Reverse regenerative braking mode When semiconductor switches S_1, S_3 and S_4 are turned OFF and switch S_2 is turned ON, armature current increases through switch S_2 and diode D_4. Then energy is stored in inductance. When switch S_2 is turned OFF, the armature current falls. As the generated voltage is greater than input voltage, the stored energy of motor must be returned to the dc supply through diodes D_3 and D_4. In reversed regenerative braking mode, motor operates in fourth quadrant.

12.8 ac DRIVES

Generally, most of the motion control systems are derived by electric motors. Drives employing electric motors are called electric drives. Most commonly used electric drives are:

1. *DC motor drives* employing shunt, series, and compound dc motors.
2. *Induction motor drives* employing squirrel cage and wound rotor (slip ring) induction motors.
3. *Synchronous motor drives* employing permanent magnet and wound field synchronous motors.

4. *Brushless dc motor drives* employing brushless dc motors.
5. *Stepper motor drives* employing stepper motors.
6. *Switch reluctance motor* drives employing Switch reluctance motor.

Based on the application of motors, electric drives are classified as dc drives and ac drives. The dc drives are already discussed in Section 12.2 to Section 12.7. AC motors contain highly coupled stator and rotor windings, non-linear and multivariable structure but separately excited DC motors has comparatively simpler decoupled structure than AC motors. The comparison between AC and DC drives are given in Table 12.1.

Table 12.1 *Comparison between AC and DC drives*

AC Drives	DC Drives
AC motors are lightly decoupled, non-liner and multi-variable structure.	DC motors are simple decoupled structure.
AC drives require complex control algorithms.	DC drive have simple control algorithms.
AC motors are light weight and about 20% to 40% light weight than same rating dc motor.	DC motors are heavy weight compared to AC motors.
The cost of AC motors less compared to DC motors for same kW rating.	The cost of dc motor is high compared to AC motor for same kW rating.
AC motor requires less maintenance compared to DC motors.	DC motor requires more maintenance compared to AC motors.
VSI, CSI, AC voltage controller, PWM inverters are used in variable speed induction motor drive.	Controlled rectifiers and choppers are used to control DC motors.
Power converters used in AC drives are relatively complex and more expensive.	Power converters used in DC drives are relatively simple and less expensive.

The AC drives have certain advantages and disadvantages over DC drives which are given below.

Advantages of AC drives compared to DC drives

1. For the same kW rating, ac motors are 20% to 40% light weight as compared to dc motors
2. AC motors are less expensive as compared to same kW rating dc motors
3. AC motors have low maintenance as compared with dc motors
4. AC motors can work in hazard areas like chemical and petrochemical etc. but dc motors are not suitable for hazard areas due to commutator sparking

Disadvantages of AC drives compared to DC drives

1. The control of ac drives require complex control algorithms such as model reference, adaptive control, sliding mode control and field-oriented control which can be performed by microprocessors and microcontrollers
2. Power electronics converters such as converters, inverters and ac voltage controllers are used to control the ac motors and these converters are expensive
3. Power electronics converters for ac drives generate harmonics in the supply system and load circuit. Therefore, ac motors will be derated.

The advantages of ac drives *outweigh* their disadvantages. AC drives are used for several industrial applications. Generally, there are two types of ac drives:

1. Induction motor drives
2. Synchronous motor drives

In this section, induction motor drives and synchronous motor drives are discussed elaborately.

12.9 INDUCTION MOTOR DRIVES

Three-phase induction motor have stator and rotor windings. The stator windings are supplied with balanced three-phase ac supply. Due to transformation action, voltage is induced in the rotor winding. Based on rotor construction, three-phase induction motors are of two types such as squirrel cage induction motors and wound rotor slip-ring induction motors. The rotor of squirrel cage induction motor is made of aluminum or copper bars which are short circuited by two end rings. In case of wound rotor induction motor, the rotor has a balanced three-phase distributed winding with same number of poles as stator winding. Actually, three-phase rotor windings are connected to three slip rings on the rotor shaft. When a balanced three-phase ac supply is connected to three-phase stator winding, the rotating magnetic field is developed and rotates at synchronous speed N_s.
The synchronous speed is equal to

$$N_s = \frac{120f}{P} \text{ in rpm} \quad \text{or} \quad n_s = \frac{2f}{P} \text{ in rps}$$

The synchronous speed can be expressed in rad/sec

$$\omega_s = \frac{4\pi f_s}{P} = \frac{2\omega_1}{P} \text{ rad/sec}$$

where $f_s = f_1 = f$ supply frequency in Hz,

$\omega_1 = 2\pi f_1$ = supply frequency in rad/s

P = Number of stator poles.

The speed of rotor of induction motor N_r is always less than synchronous speed N_s

The slip $s = \dfrac{N_s - N_r}{N_s} = \dfrac{\omega_s - \omega_m}{\omega_s}$ where, ω_m is the rotor speed or shaft speed in rad/sec.

Then rotor speed is equal to

$$N_r = N_s(1-s) \text{ in rpm} \quad \text{or,} \quad \omega_m = \omega_s(1-s) \text{ in rad/s}$$

The equivalent circuit of one phase of rotor is represented by Fig. 12.37(a) and one phase stator and rotor circuit is depicted in Fig. 12.37(b).

where, r_2 = resistance per phase of rotor winding

x_2 = reactance per phase of rotor winding at supply frequency

E_2 = induced rms phase voltage at zero speed $N_r = 0$

r_1 = resistance per phase of stator winding

x_1 = reactance per phase of stator winding at supply frequency

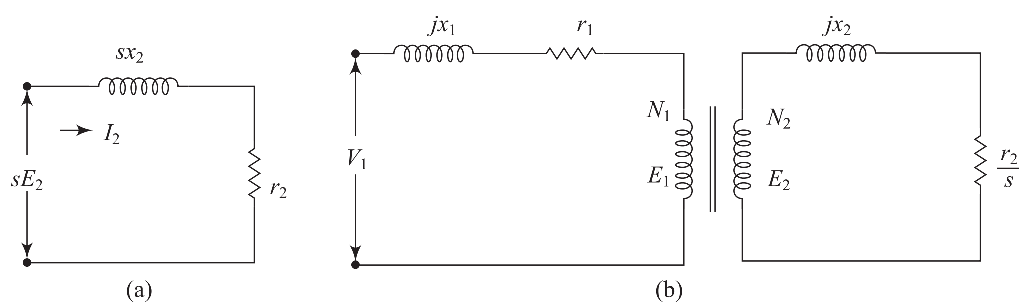


Fig. 12.37 *(a) One phase of rotor circuit and (b) One phase stator and rotor circuit*

The per phase equivalent circuit of three-phase induction motor is depicted in Fig. 12.38(a), (b) and (c). where

x_2' = reactance per phase of rotor winding referred to stator

r_2' = resistance per phase of rotor winding referred to stator

R_m = magnetizing resistance, X_m is the magnetizing reactance

I_1 = stator current, I_2' is the rotor current referred to stator, V_1 is input voltage per phase

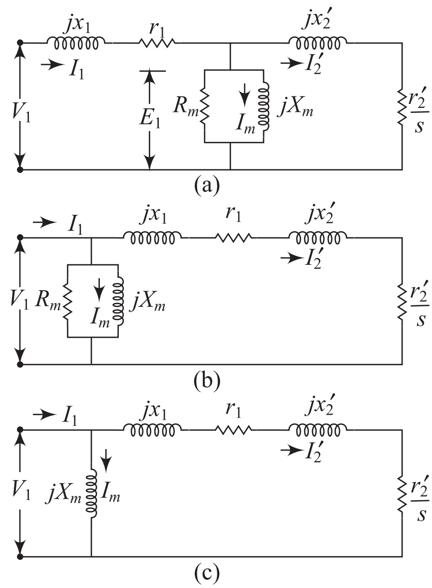


Fig. 12.38 *(a) Equivalent circuit of induction motor (b) Alternative circuit of Fig. 12.37(a) (c) Simplified equivalent circuit of induction motor*

Rotor current referred to stator is $I_2' = \dfrac{V_1}{\left(r_1 + \dfrac{r_2'}{s}\right) + j(x_1 + x_2')}$

The air gap power, i.e., power transferred from stator to rotor is

$$P_g = 3I_2'^2 \frac{r_2'}{s}$$

Rotor copper loss (ohmic loss) is $P_{Cu} = 3I_2'^2 r_2'$

Power developed in rotor is

$$P_d = P_g - P_{Cu}$$

$$= 3I_2'^2 \frac{r_2'}{s} - 3I_2'^2 r_2' = 3I_2'^2 r_2' \left(\frac{1-s}{s}\right) = P_g(1-s)$$

Torque developed by motor

$$T_e = \frac{P_d}{\omega_r} = 3I_2'^2 r_2' \left(\frac{1-s}{s}\right) \cdot \frac{1}{\omega_s(1-s)} \qquad \text{as } \omega_r = \omega_s(1-s)$$

$$= \frac{1}{\omega_s} \cdot 3I_2'^2 \frac{r_2'}{s} = \frac{P_g}{\omega_s} \tag{12.1}$$

After substituting the value of I_2' in Eq. (12.1), we obtain

$$T_e = \frac{3}{\omega_s} \frac{V_1^2}{\left(r_1 + \dfrac{r_2'}{s}\right)^2 + (x_1 + x_2')^2} \cdot \frac{r_2'}{s}$$

The electromagnetic torque is function of slip s. The developed torque will be maximum when $\dfrac{dT_e}{ds} = 0$. The slip at which maximum torque occurs is equal to

$$s_m = \frac{r_2'}{\sqrt{r_1^2 + (x_1 + x_2')^2}}$$

The maximum electromagnetic torque is equal to

$$T_{e\max} = \frac{3}{2\omega_s} \frac{V_1^2}{r_1 + \sqrt{r_1^2 + (x_1 + x_2')^2}}$$

The maximum torque is also known as pull out torque or breakdown torque which is independent of rotor resistance r_2', but slip at maximum torque s_m is directly proportional to rotor resistance r_2'.

Neglecting the value of resistance r_1, $T_e = \dfrac{3}{\omega_s} \dfrac{V_1^2}{\left(\dfrac{r_2'}{s}\right)^2 + (x_1 + x_2')^2} \cdot \dfrac{r_2'}{s}$ (12.2)

and the slip at maximum torque is $s_m = \dfrac{r_2'}{x_1 + x_2'}$

After substituting the value of s_m in Eq. (12.2), the maximum torque is

$$T_{e\max} = \frac{3}{2\omega_s} \frac{V_1^2}{x_1 + x_2'} \tag{12.3}$$

From Eqs. (12.2) and (12.3), we obtain

$$\frac{T_e}{T_{e\max}} = \frac{2}{\dfrac{s}{s_m} + \dfrac{s_m}{s}} = \frac{2ss_m}{s_m^2 + s^2}$$

If $s << s_m$ and $\dfrac{s}{s_m} << \dfrac{s_m}{s}$, $\quad \dfrac{T_e}{T_{e\max}} = \dfrac{2s}{s_m} = \dfrac{2(\omega_s - \omega_m)}{s_m \omega_s}$ (12.4)

or $\quad \omega_s - \omega_m = \dfrac{\omega_s s_m T_e}{2T_{e\max}}$

or motor shaft speed $\omega_m = \omega_s - \dfrac{\omega_s s_m T_e}{2T_{e\max}} \omega_m = \omega_s \left(1 - \dfrac{s_m T_e}{2T_{e\max}}\right)$ (12.5)

When the slip is small, the motor torque is directly proportional to slip as per Eq. (12.4). The motor speed decreases with increasing load torque as shown in Eq. (12.5).

After substituting the value of s_m in Eq. (12.5), we get

$$\omega_r = \omega_s \left(1 - \frac{r_2}{x_1 + x_2} \frac{T_e}{T_{e\max}}\right) \tag{12.6}$$

It is clear from Eq. (12.6) that the speed drop of induction motor from no-load to full load depends on rotor resistance.

The input power to motor is

$$P_i = 3V_1 I_1 \cos\phi_1 \text{ where, } \phi_1 \text{ is the angle between } V_1 \text{ and } I_1$$
$$= P_g + \text{stator core loss} + \text{stator copper loss}$$
$$= P_g + P_{\text{core}} + P_{\text{Cu}}$$

Shaft power output is $P_O = P_d - \text{fixed loss} = P_d - P_{\text{no-load}}$

Fixed loss is equal to friction and windage loss, $P_{\text{no-load}}$

Motor efficiency is $\eta = \dfrac{P_O}{P_i} = \dfrac{P_d - P_{\text{no-load}}}{P_g + P_{\text{core}} + P_{\text{Cu}}}$

If $P_g >> (P_{\text{core}} + P_{\text{Cu}})$ and $P_d >> P_{\text{no-load}}$,

$$\text{efficiency } \eta \approx \frac{P_d}{P_g} = \frac{P_g(1-s)}{P_g} = 1 - s \qquad \text{as } P_d = P_g(1-s)$$

Torque output at shaft is $T_{sh} = \dfrac{P_{sh}}{\omega_m} = \dfrac{P_{sh}}{\omega_s(1-s)}$ where, shaft output $P_{sh} = P_o$ output power

12.9.1 Torque-Speed Characteristics of Induction Motor

The torque developed in machine is function of slip s or speed. Figure 12.39 shows the torque-slip (speed) characteristics of induction motor.

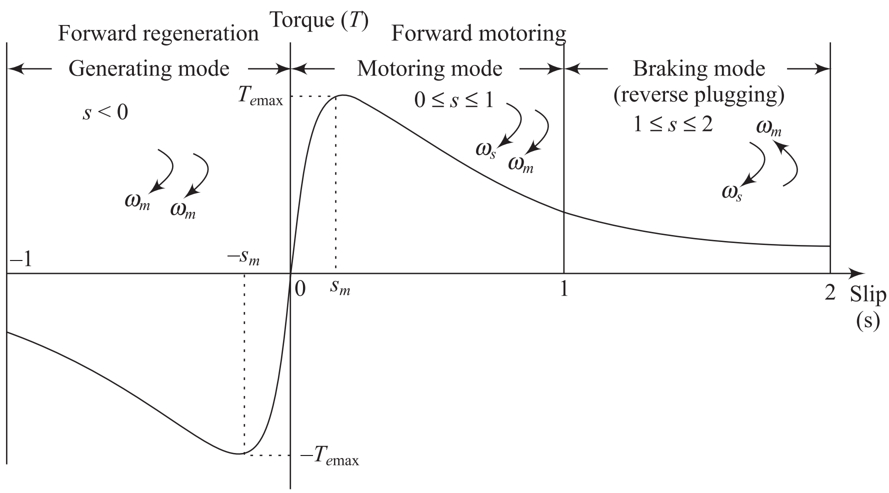


Fig. 12.39 *Torque-slip (speed) characteristics of induction motor*

The torque-speed characteristics of induction motor are subdivided into three operating regions such as

1. Forward motoring, $0 \le s \le 1$
2. Regeneration, $s < 0$
3. Reverse plugging, $1 \le s \le 2$

1. Forward motoring ($0 \le s \le 1$) During forward motoring operation, the induction motor rotates in the same direction as the electromagnetic field. When slip increases, the torque increases while air gap flux remains constant. At $s = s_m$, torque is equal to maximum torque. When slip $s > s_m$, torque decreases with the reduction of the air gap flux.

2. Regeneration ($s < 0$) During regeneration operation, the motor speed ω_m is greater than the synchronous speed ω_s. As ω_m and ω_s rotate in the same direction, slip s is negative. Then $\dfrac{r_2'}{s}$ is negative. Therefore, power is fed back to the ac supply and induction motor acts as an induction generator. The torque-speed characteristic of induction motor during regeneration is similar to that of forward motoring but torque becomes negative.

3. Reverse plugging ($1 \le s \le 2$) In reverse plugging, the speed of induction motor is opposite to the direction of the electromagnetic field and slip is greater than unity. Plugging operation is possible when the sequence of ac supply is reversed and the direction of filed is reversed. The developed torque opposes the motion and it acts as braking torque. As slip $s > 1$, the motor current will be high and the developed torque will be low. During plugging, the energy will be dissipated with in the motor. Subsequently induction motor will be heated excessively. Therefore, this type of breaking is not commonly used in induction motor.

Example 12.18 A 3-ϕ 50 Hz, 8 pole, 400 V, 720 rpm Y connected induction motor has the following parameters:

$r_1 = 0.25\ \Omega$, $x_1 = 0.75\ \Omega$, $r_2' = 0.2\ \Omega$, $x_2' = 0.75\ \Omega$, and $X_m = 25\ \Omega$

When the motor is driving a load torque $T_L = 0.015\omega_m^2$, determine slip, full load rotor current, power factor at full load and load torque.

Solution

The synchronous speed $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{8} = 750$ rpm

The full load slip $s = \dfrac{N_s - N_r}{N_r} = \dfrac{750 - 720}{750} = 0.04$

The full load rotor current is

$$I_2' = \frac{V_1}{\sqrt{\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2}} = \frac{\frac{400}{\sqrt{3}}}{\sqrt{\left(0.25 + \frac{0.2}{0.04}\right)^2 + (0.75 + 0.75)^2}} = 42.296 \text{ A}$$

$$\phi_2 = \cos^{-1}\frac{r_1 + \frac{r_2'}{s}}{\sqrt{\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2}} = \cos^{-1}\frac{0.25 + \frac{0.2}{0.04}}{\sqrt{\left(0.25 + \frac{0.2}{0.04}\right)^2 + (0.75 + 0.75)^2}} = \cos^{-1} 0.961$$

$$= 15.94^\circ \text{ (lagging)}$$

Then $I_2' = 42.296\angle{-15.94^\circ}$

The magnetizing current $I_m = \dfrac{V_1}{jX_m} = \dfrac{\frac{400}{\sqrt{3}}}{j25} = 9.2376\angle{-90^\circ}$

The full load stator current is

$$\begin{aligned} I_s &= I_2' + I_m = 42.296\angle{-15.94^\circ} + 9.2376\angle{-90^\circ} \\ &= 40.669 - j11.615 - j9.2376 = 40.669 - j20.8526 \\ &= 45.703\angle{-27.146^\circ} \end{aligned}$$

Power factor is $\cos\phi = \cos(-27.146) = 0.8898$ (lagging)

Synchronous speed $\omega_s = \dfrac{4\pi f}{P} = \dfrac{4\pi \times 50}{8} = 78.5714$ rad/s

Motor speed in rad/s $\omega_m = \omega_s(1 - s) = 78.5714(1 - 0.04)$ rad/s = 75.4285 rad/s

Load torque $T_L = 0.015\omega_m^2 = 0.015 \times 75.4285^2$ N-m $= 85.341 \cdot$ N-m

12.10 SPEED CONTROL OF THREE-PHASE INDUCTION MOTOR

Three-phase induction motors are most commonly used in adjustable speed drives as these motors are highly suitable to fulfil the demand of loads. In different industrial applications, adjustable speeds are required for efficient operations. Presently power electronics converters are employed for speed control of three-phase induction motors. The speed and torque of three-phase induction motors can be controlled by the following methods:

1. Stator voltage control
2. Stator frequency control
3. Stator voltage and frequency control
4. Stator current control
5. Rotor resistance control
6. Slip-power recovery control
7. Rotor voltage control

The first four methods are applicable for both squirrel cage induction motor (IM) and wound rotor induction motor, but other three methods are only feasible in wound rotor or slip ring induction motors only. In this section, only stator voltage control, stator frequency control, $\frac{V}{f}$ control, stator current control, rotor resistance control and slip power recovery control are discussed.

12.10.1 Stator Voltage Control of Induction Motor

The electromagnetic torque of induction motor is directly proportional to the square of the stator input voltage and it is given by

$$T_e = \frac{3}{\omega_s} \frac{V_1^2}{\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2} \cdot \frac{r_2'}{s}$$

where, V_1 is input voltage, r_1 and x_1 are stator resistance and reactance, r_2' and x_2' are rotor resistance and reactance, s is slip and ω_s is synchronous speed.
When the stator input voltage is reduced by a fraction, the electromagnetic torque is expressed by

$$T_e = \frac{3}{\omega_s} \frac{(xV_1)^2}{\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2} \cdot \frac{r_2'}{s} \qquad \text{where, } 0 \le x \le 1$$

The stator voltage control is possible by using a three-phase ac voltage controller; variable voltage fed variable dc link inverters and pulse width modulation inverters. Figure 12.40 shows a three-phase ac voltage controller fed induction motor drive. By varying the firing angle of thyristors, the rms voltage across the stator of induction motor can be controlled. Subsequently, motor electromagnetic torque will be changed. Hence the speed of the induction motor will be controlled.

It is clear from Fig. 12.41 that the torque-speed characteristic and load characteristic intersect at point A, and B. At point A, $x = 1$ and input voltage is rated, motor speed is O-a, electromagnetic torque T_e is equal load torque, i.e., $T_e = T_L = T_{A-a}$. When x is reduced to 0.75, the operating point becomes B, the motor speed is reduced to O-b , the electromagnetic torque and load torque are equal to T_{B-b}.

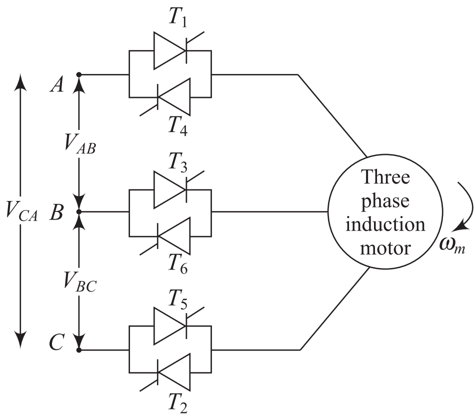


Fig. 12.40 *Three-phase ac voltage controller fed induction motor drive*

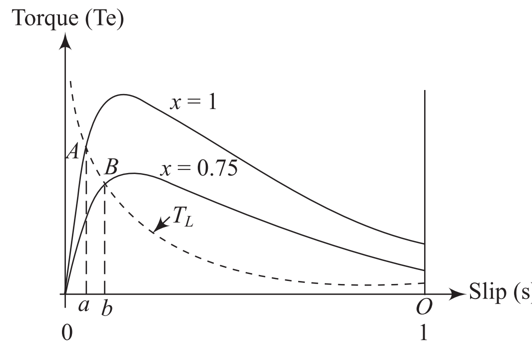


Fig. 12.41 *Torque speed characteristics of induction motor at x = 1 and x = 0.75*

The stator voltage control method is only suitable for induction motors having large value of s_m and the wide control range. The speed control using ac voltage controller is very simple but the harmonic contents are high and input power factor of ac supply is low. Therefore, this drive is only suitable for low power applications such as electric fans, blowers, pumps where the starting torque requirement is low. The drive is not suitable for a constant torque drive and for low slip motors, the speed control range will be narrow.

In case of blower type load, the load torque is directly proportional to the speed square and it is given by

$T_L = k\omega_m^2$ where, T_L is load torque and ω_m is machine speed

The mechanical power developed in the motor is

$$P_m = T_L\omega_m = P_g(1-s)$$

Since $P_g = 3I_2'^2 \dfrac{r_2'}{s}$, $\quad T_L\omega_m = P_g(1-s) = 3I_2'^2 \dfrac{r_2'}{s}(1-s)$

Therefore, $I_2'^2 = \dfrac{T_L\omega_m}{3r_2'}\dfrac{s}{1-s}$ or $I_2' = \sqrt{\dfrac{T_L\omega_m}{3r_2'}\dfrac{s}{1-s}}$ (12.7)

We know that $\omega_m = \omega_s(1-s)$ and $T_L = k\omega_m^2$. After substituting the value of T_L in Eq. (12.7), we obtain

$$I_2' = \sqrt{\frac{k\omega_m^2\omega_m}{3r_2'}\frac{s}{1-s}} = \omega_m\sqrt{\frac{k\omega_m}{3r_2'}\frac{s}{1-s}}$$

or $$I_2' = \omega_s(1-s)\sqrt{\frac{k\omega_s(1-s)}{3r_2'}\frac{s}{1-s}} = \omega_s(1-s)\sqrt{\frac{k\cdot s\cdot\omega_s}{3r_2'}} = \sqrt{s}(1-s)\sqrt{\frac{k\cdot\omega_s^2}{3r_2'}}$$

Therefore, $I_2' = \sqrt{s}(1-s)\sqrt{\dfrac{k\cdot\omega_s^2}{3r_2'}}$

The rotor current I_2' is function of slip and I_2' will be maximum when $\dfrac{dI_2'}{ds} = 0$.

Then, $\dfrac{dI_2'}{ds} = \dfrac{1}{2}\dfrac{1}{\sqrt{s}}(1-s)\sqrt{\dfrac{k\cdot\omega_s^2}{3r_2'}} + \sqrt{s}(-1)\sqrt{\dfrac{k\cdot\omega_s^2}{3r_2'}} = 0$

or $$\frac{1-s}{2} = s \text{ or, } s = \frac{1}{3}$$

Therefore, the maximum current at $s = \dfrac{1}{3}$ is equal to

$$I_2'|\max = \sqrt{s}(1-s)\sqrt{\frac{k\cdot\omega_s^2}{3r_2'}} = \sqrt{\frac{1}{3}}\left(1-\frac{1}{3}\right)\sqrt{\frac{k\cdot\omega_s^2}{3r_2'}} = \frac{1}{\sqrt{3}}\frac{2}{3}\frac{1}{\sqrt{3}}\sqrt{\frac{k\cdot\omega_s^2}{r_2'}}$$

$$= \frac{2}{9}\sqrt{\frac{k\cdot\omega_s^2}{r_2'}}$$

Example 12.19 A three-phase induction motor drives a blower where load torque is directly proportional to speed squared. If the motor operates at 1450 rpm, determine the maximum current in terms of rated current.

Solution

Synchronous speed $N_s = 1500$ rpm and rotor speed $N_r = 1450$ rpm

Slip $s = \dfrac{N_s - N_r}{N_s} = \dfrac{1500 - 1450}{1500} = 0.0333$

$$I_2'|\max = \frac{2}{9}\sqrt{\frac{k\cdot\omega_s^2}{r_2'}} \text{ and } I_2' = \sqrt{s}(1-s)\sqrt{\frac{k\cdot\omega_s^2}{3r_2'}}$$

The ratio $$\frac{I_2'|_{\max}}{I_2'} = \frac{\frac{2}{9}\sqrt{\frac{k\omega_s^2}{r_2'}}}{\sqrt{s}(1-s)\sqrt{\frac{k\omega_s^2}{3r_2'}}} = \frac{2}{9}\frac{\sqrt{3}}{\sqrt{s}(1-s)} = \frac{2}{9}\frac{\sqrt{3}}{\sqrt{0.0333}(1-0.0333)} = 2.1819$$

12.10.2 Stator Frequency Control of Induction Motor

When the frequency of supply voltage has been changed, the synchronous speed of induction motor will be changed. Hence the torque-speed characteristics will be modified and the speed of motor can be controlled. The voltage induced in stator winding is directly proportional to the product of supply frequency f_1 and air-gap flux ϕ. The per phase voltage induced in stator winding is expressed by

$$V_1 = \sqrt{2}\pi f_1 N_1 \phi k w_1$$

Due to reduction in supply frequency without change in amplitude of supply voltage, air-gap flux increases. Actually induction motors are designed to operate at the knee point of the magnetization characteristic so that the magnetic material is fully utilized. Since flux increases, the induction motor will operate in saturation region. Therefore, the magnetization current will be increased, the line current and voltage become distorted, the core loss and stator copper loss increase and the motor parameters would not be valid to determine the torque-speed characteristics. Accordingly, the motor efficiency is reduced. At low frequency, the reactance of motor decreases and the motor current will be high. For this reason, the speed control of induction motor with constant voltage and reduced supply frequency is not commonly used in industry.

When the frequency is increased above rated frequency, air-gap flux decreases and subsequently electromagnetic torque will be decreased. The performance of induction motor can be determined by using the equivalent circuit as shown in Fig. 12.38(c).

The rotor current is

$$I_2' = \frac{V_1}{\sqrt{\left(\frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2}} \quad \text{neglecting } r_1$$

At supply frequency f_1, synchronous speed $\omega_s = \frac{4\pi f_1}{P} = \frac{2\omega_1}{P}$ rad/s as $\omega_1 = 2\pi f_1$

The electromagnetic torque is $T_e = \frac{3}{\omega_s} I_2^2 \cdot \frac{r_2'}{s}$ (12.8)

After substituting the value of I_2' in Eq. (12.8), we obtain $T_e = \frac{3}{\omega_s} \frac{V_1^2}{\left(\frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2} \cdot \frac{r_2'}{s}$ (12.9)

If the rotor frequency is $f_r = f_2$, the relation between supply frequency $f_s = f_1$ and rotor frequency is $f_r = sf_s$ or $f_2 = sf_1$

$$\therefore \qquad s = \frac{f_r}{f_s} = \frac{f_2}{f_1} = \frac{2\pi f_2}{2\pi f_1} = \frac{\omega_2}{\omega_1}$$

After substituting the value of $\omega_s = \frac{2\omega_1}{P}$, $x_1 = \omega_1 l_1$, $x_2' = \omega_2 l_2'$, $s = \frac{\omega_2}{\omega_1}$ in Eq. (12.9), we get

$$T_e = \frac{3P}{2\omega_1} \frac{V_1^2}{\left(\frac{r_2'\omega_1}{\omega_2}\right)^2 + \omega_1^2 (l_1 + l_2')^2} \cdot \frac{r_2'\omega_1}{\omega_2}$$

or

$$T_e = \frac{3P}{2\omega_1^2} \frac{V_1^2}{r_2'^2 + \omega_2^2 (l_1 + l_2')^2} \cdot \omega_2 r_2'$$

The slip at which the electromagnetic torque occurs is $s_{\max} = \frac{r_2'}{x_1 + x_2'}$

The rotor frequency at which the electromagnetic torque is maximum in rad/s.

$$\omega_2 = s_{\max}\omega_1 = \frac{\omega_1 r_2'}{\omega_1 (l_1 + l_2')} = \frac{r_2'}{(l_1 + l_2')}$$

It is clear from above equation that ω_2 does not depend on the supply frequency ω_1. After substituting $r_2' = \omega_2 (l_1 + l_2')$, the maximum torque $T_{e,\max}$ is equal to

$$T_{e\max} = \frac{3P}{2\omega_1^2} \frac{V_1^2}{r_2'^2 + \omega_2^2 (l_1 + l_2')^2} \cdot \omega_2 r_2'$$

$$= \frac{3P}{2\omega_1^2} \frac{V_1^2}{\omega_2^2 (l_1 + l_2')^2 + \omega_2^2 (l_1 + l_2')^2} \cdot \omega_2^2 (l_1 + l_2')$$

$$= \frac{3P}{2\omega_1^2} \frac{V_1^2}{(l_1 + l_2')^2 + (l_1 + l_2')^2} \cdot (l_1 + l_2')$$

$$= \frac{3P}{4\omega_1^2} \frac{V_1^2}{(l_1 + l_2')}$$

Hence, the $T_{e,\max}$ is inversely proportional to supply frequency squared, i.e., $T_{e\max} \propto \dfrac{1}{\omega_1^2}$.

Since supply voltage V_1 is constant, the value of $\dfrac{3P}{4}\dfrac{V_1^2}{(l_1 + l_2')}$ is constant. Therefore, $T_{e\max}\omega_1^2$ is also constant. If the operating frequency ω_1 increases and $T_{e\max}\omega_1^2$ remains constant, the maximum torque at increased frequency ω_1 will be reduced. This behavior is similar to the working of dc series motors. With constant voltage and increased frequency operation, air gap flux is reduced. Therefore, during stator frequency control method, induction motor should be work in field weakening mode. Figure 12.42 shows torque speed characteristics of a three-phase induction motor with stator frequency control with constant supply voltage.

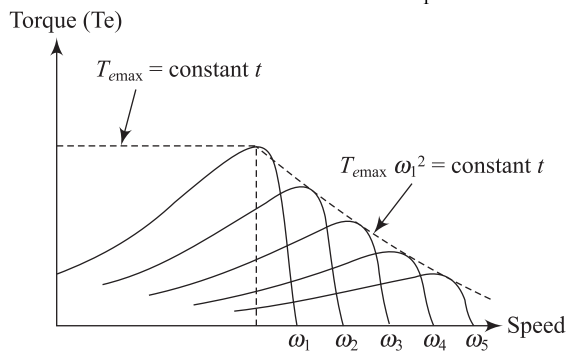


Fig. 12.42 *Torque speed characteristics of induction motor at stator frequency control*

Example 12.20 A three-phase 50 Hz, four pole, 440 V, 1420 rpm, 20 kW, Y connected induction motor has the following parameters:

$$r_2' = 0.30\ \Omega \text{ and } x_2' = 1.1\ \Omega.$$

The value of magnetizing reactance X_m and stator leakage impedance and rotational losses are neglected. When the motor is operated by three phase 440 V, 100 Hz supply, determine (a) the motor speed at rated load, (b) slip at maximum torque, and (c) the maximum torque.

Solution

At three-phase 440 V, 50 Hz supply

$$\omega_m = \frac{2\pi N_r}{60} = \frac{2\pi \times 1420}{60} = 148.76 \text{ rad/s}$$

The full load torque $T_{e\text{-full-load}} = \dfrac{P}{\omega_m} = \dfrac{20 \times 1000}{148.76} = 134.44$ N-m

(a) When three-phase 440 V, 100 Hz supply is applied to induction motor,

the reactance will be $x_2' = 1.1 \times \dfrac{100}{50}\Omega = 2.2\ \Omega$ and resistance $r_2' = 0.30\ \Omega$

Synchronous speed $\omega_s = \dfrac{4\pi f}{P} = \dfrac{4\pi \times 100}{4} = 314.2857$ rad/s

The full load torque $T_e = \dfrac{3}{\omega_s}\dfrac{V_1^2}{\left(r_1 + \dfrac{r_2'}{s}\right)^2 + (x_1 + x_2')^2}\cdot\dfrac{r_2'}{s}$

or $$T_e = \frac{3}{\omega_s}\frac{V_1^2}{\left(\dfrac{r_2'}{s}\right)^2 + (x_2')^2}\cdot\frac{r_2'}{s} \quad \text{As } r_1 \text{ and } x_1 \text{ are neglected}$$

$$= \frac{3}{314.2857}\frac{\left(\dfrac{440}{\sqrt{3}}\right)^2}{\left(\dfrac{0.3}{s}\right)^2 + (1.1)^2}\cdot\frac{0.3}{s} = 134.44$$

or $$\frac{184.8s}{0.3^2 + 1.1^2 s^2} = 134.44 \quad \text{or, } 162.67s^2 - 184.8s + 12.09 = 0$$

Therefore, $s = \dfrac{-(-184.8) \pm \sqrt{(-184.8)^2 - 4 \times 162.67 \times 12.09}}{2 \times 162.67}$

Slip s = 0.0696

The motor speed at rated load is $\omega_m = \omega_s(1 - s) = 314.2857(1 - 0.0696) = 292.41$ rad/s

(b) Slip at maximum torque $s_m = \dfrac{r_2'}{x_2'} = \dfrac{0.3}{2.2} = 0.1363$

(c) The maximum torque

$$T_{e\max} = \frac{3}{2\omega_s}\frac{V_1^2}{x_1 + x_2'} \cdot = \frac{3}{2\omega_s}\frac{V_1^2}{x_2'} \cdot \text{ as } x_1 \text{ is neglected}$$

$$= \frac{3}{2 \times 314.2857}\frac{\left(\frac{440}{\sqrt{3}}\right)^2}{2.2} \cdot = 140 \text{ N-m}$$

Example 12.21 A three-phase 50 Hz, four pole, 440 V, 10 kW delta connected induction motor has the following parameters:

$$r_1 = 0.5\ \Omega,\ x_1 = 1.2\ \Omega,\ r_2' = 0.30\ \Omega, \text{ and } x_2' = 1.2\ \Omega,$$

The value of magnetizing reactance X_m is neglected.

(a) If the motor is started with DOL starting, determine starting current, power factor at starting, maximum torque.

(b) When the motor is operated by 220 V, 25 Hz supply and is started with DOL starting, determine starting current, power factor at starting, maximum torque.

Solution

(a) The starting current is

$$I_s = \frac{V_1}{\sqrt{\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2}} = \frac{V_1}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} \quad \text{as } s = 1$$

$$= \frac{440}{\sqrt{(0.5 + 0.3)^2 + (1.2 + 1.2)^2}} = 173.926 \text{ A}$$

The power factor

$$\cos\phi = \frac{r_1 + r_2'}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} = \frac{0.5 + 0.3}{\sqrt{(0.5 + 0.3)^2 + (1.2 + 1.2)^2}} = 0.3162 \text{ (lagging)}$$

Slip at maximum torque $s_m = \dfrac{r_2'}{\sqrt{r_1^2 + (x_1 + x_2')^2}} = \dfrac{0.3}{\sqrt{0.5^2 + (1.2 + 1.2)^2}} = 0.1223$

Synchronous speed $\omega_s = \dfrac{4\pi f}{P} = \dfrac{4\pi \times 50}{4} = 157.1428$ rad/s

The maximum electromagnetic torque is equal to

$$T_{e\max} = \frac{3}{2\omega_s}\frac{V_1^2}{r_1 + \sqrt{r_1^2 + (x_1 + x_2')^2}} = \frac{3}{2 \times 157.1428}\frac{440^2}{0.5 + \sqrt{0.5^2 + (1.2 + 1.2)^2}} = 626.1225 \text{ N-m}$$

(b) At 25 Hz, the stator and rotor reactance will be

$$x_1 = 1.2 \times \frac{25}{50}\Omega = 0.6\ \Omega \text{ and } x_2' = 1.2 \times \frac{25}{50}\Omega = 0.6\ \Omega$$

The starting current is

$$I_s = \frac{V_1}{\sqrt{\left(r_1 + \frac{r_2'}{s}\right)^2 + (x_1 + x_2')^2}} = \frac{V_1}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} \quad \text{as } s = 1$$

$$= \frac{220}{\sqrt{(0.5 + 0.3)^2 + (0.6 + 0.6)^2}} = 152.565 \text{ A}$$

The power factor

$$\cos\phi = \frac{r_1 + r_2'}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} = \frac{0.5 + 0.3}{\sqrt{(0.5 + 0.3)^2 + (0.6 + 0.6)^2}} = 0.5547 \text{ (lagging)}$$

Slip at maximum torque $s_m = \dfrac{r_2'}{\sqrt{r_1^2 + (x_1 + x_2')^2}} = \dfrac{0.3}{\sqrt{0.5^2 + (0.6 + 0.6)^2}} = 0.2453$

Synchronous speed $\omega_s = \dfrac{4\pi f}{P} = \dfrac{4\pi \times 25}{4} = 78.574$ rad/s

The maximum electromagnetic torque is equal to

$$T_{e\max} = \frac{3}{2\omega_s} \frac{V_1^2}{r_1 + \sqrt{r_1^2 + (x_1 + x_2')^2}} = \frac{3}{2 \times 78.574} \frac{220^2}{0.5 + \sqrt{0.5^2 + (0.6 + 0.6)^2}} = 536.412 \text{ N-m}$$

12.10.3 Stator Voltage and Frequency Control of Induction Motor

In a three-phase induction motor, the stator voltage per phase is

$$V_1 = \sqrt{2}\pi f_1 N_1 \phi k w_1$$

Then, $$\frac{V_1}{f_1} = \sqrt{2}\pi N_1 \phi k w_1$$

It is clear from above equation that if the ratio of supply voltage to supply frequency $\left(\dfrac{V}{f}\right)$ is remaining constant, subsequently the air gap flux ϕ remains constant. In stator voltage and frequency $\left(\dfrac{V}{f}\right)$ control, frequency and voltage are varied in such a way that $\dfrac{V}{f}$ ratio is maintained constant at its rated value. The starting torque of induction motor is

$$T_e|_{\text{starting}} = \frac{3}{\omega_s} \frac{V_1^2}{(r_1 + r_2')^2 + (x_1 + x_2')^2} \cdot r_2' \qquad \text{as slip } s = 1$$

Since $(x_1 + x_2) >> (r_1 + r_2)$ and $\omega_s = \dfrac{4\pi f_1}{P} = \dfrac{2\omega_1}{P}$ as $\omega_1 = 2\pi f_1$

Therefore, $T_e|_{\text{starting}} = \dfrac{3}{\omega_s} \dfrac{V_1^2}{(x_1 + x_2')^2} \cdot r_2' = \dfrac{3P}{2\omega_1} \dfrac{V_1^2}{(x_1 + x_2')^2} \cdot r_2'$ as $\omega_s = \dfrac{2\omega_1}{P}$

$$= \frac{3P}{2\omega_1} \frac{V_1^2}{\omega_1^2 (l_1 + l_2')^2} \cdot r_2' \qquad \text{as } x_1 = \omega_1 l_1 \text{ and } x_2 = \omega_1 l_2'$$

$$= \frac{3P}{2\omega_1} \left(\frac{V_1 \cdot}{\omega_1}\right)^2 \frac{r_2'}{(l_1 + l_2')^2}$$

When $\frac{V_1}{f_1}$ or $\frac{V_1}{\omega_1}$ the air-gap flux ϕ is kept constant, the starting torque is inversely proportional to supply frequency ω_1.

The maximum electromagnetic torque is

$$T_{e\max} = \frac{3}{\omega_s} \cdot \frac{V_1^2}{2(x_1 + x_2')}$$

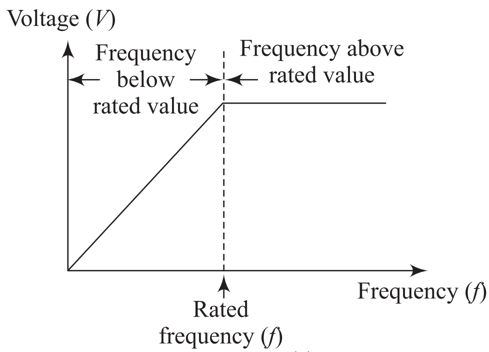
or
$$T_{e\max} = \frac{3P}{2\omega_1} \cdot \frac{V_1^2}{2\omega_1(l_1 + l_2')} = \frac{3P}{2}\left(\frac{V_1}{\omega_1}\right)^2 \frac{1}{l_1 + l_2'}$$

It is clear from the above expression that since air gap flux is kept constant; the maximum torque will be constant.

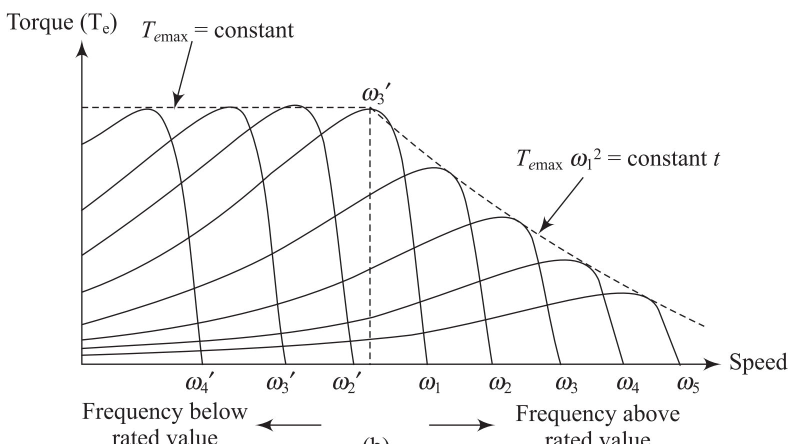
When the stator resistance r_1 is neglected, the slip at maximum torque is

$$s_m = \frac{r_2'}{x_1 + x_2'} = \frac{r_2'}{\omega_1(l_1 + l_2')}$$

If the supply frequency ω_1 in rad/s is reduced, the slip at maximum torque increases.



(a)



(b)

Fig. 12.43 *(a) Voltage variation with frequency and (b) Torque-speed characteristics of induction motor*

The variation of voltage with frequency is shown in Fig. 12.43(a). The applied voltage will be constant above the base speed. Below the base speed, $\frac{V}{f}$ ratio is maintained constant but at low frequencies, $\frac{V}{f}$ ratio is increased to maintain the maximum torque constant. The torque-speed characteristics for motoring operations with $\frac{V}{f}$ control are depicted in Fig. 12.43(b). The speed control and braking operation of induction motor is feasible from about zero speed to above synchronous speed. The variable voltage and variable frequency control induction motor drives using voltage source inverters are used in low and medium power drives.

12.10.4 Stator Current Control of Induction Motor

The torque developed in induction motor and speed control of three phase induction motor can be controlled by stator current control. The behavior of an induction motor using stator current control is different from the stator voltage control with the help of inverters.

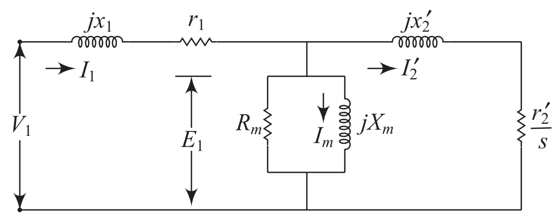


Fig. 12.44 *Equivalent circuit of single-phase induction motor*

Figure 12.44 shows the equivalent circuit of induction motor where current I_1 is fed into the stator windings of a three-phase induction motor, the rotor current is I_2, stator impedance is $r_1 + jx_1$.

The rotor current is

$$I_2' = I_1 \frac{jX_m}{\frac{r_2'}{s} + j(x_2' + X_m)} \qquad \text{neglecting } R_m$$

The electromagnetic torque developed T_e is

$$T_e = \frac{3}{\omega_s} I_2'^2 \frac{r_2'}{s}$$

After substituting the value of I_2' in the above equation, we obtain

$$T_e = \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{\left(\frac{r_2'}{s}\right)^2 + (x_2' + X_m)^2} \cdot \frac{r_2'}{s}$$

Applying the condition for maximum power transfer in the equivalent circuit, we find

$$\frac{r_2'}{s_m} = x_2' + X_m \quad \text{when } r_1 + jX_1 = 0 \text{ and current } I_1 \text{ is constant}$$

or
$$s_m = \frac{r_2'}{x_2' + X_m}$$

The maximum torque $T_{e\max}$ is

$$T_{e\max} = \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{\left(\frac{r_2'}{s_m}\right)^2 + (x_2' + X_m)^2} \cdot \frac{r_2'}{s_m} = \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{\left(\frac{r_2'}{r_2'}(x_2' + X_m)\right)^2 + (x_2' + X_m)^2} \cdot \frac{r_2'}{r_2'}(x_2' + X_m)$$

$$= \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{2(x_2' + X_m)^2} \cdot (x_2' + X_m) = \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{2(x_2' + X_m)}.$$

As $\omega_s = \frac{2\omega_1}{P}, X_m = \omega_1 L_m, x_2' = \omega_1 l_2',$

$$T_{e\max} = \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{2(x_2' + X_m)} \cdot = \frac{3P}{2\omega_1} \frac{\omega_1^2 (I_1 L_m)^2}{2\omega_1 (l_2' + L_m)} \cdot = \frac{3P}{4} \frac{(L_m)^2}{(l_2' + L_m)} \cdot I_1^2$$

It is clear from the above equation that the maximum torque is proportional to stator current squared, independent of supply frequency f_1 and also independent of rotor resistance.
The starting torque of induction motor is

$$T_{e|\text{starting}} = \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{\left(\frac{r_2'}{s}\right)^2 + (x_2' + X_m)^2} \cdot \frac{r_2'}{s} \qquad \text{where, } s = 1$$

$$= \frac{3}{\omega_s} \frac{(I_1 X_m)^2}{(r_2')^2 + (x_2' + X_m)^2} \cdot r_2' = \frac{3}{\omega_s} \frac{(X_m)^2}{(r_2')^2 + (x_2' + X_m)^2} \cdot r_2' I_1^2$$

The torque-speed characteristics of induction motor at different stator current is depicted in Fig. 12.45.

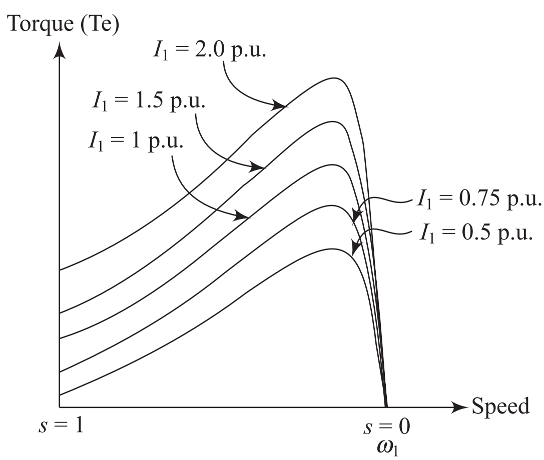


Fig. 12.45 *Torque-speed characteristics of induction motor*

Example 12.22 A three-phase 440 V, four pole, 50 Hz delta connected induction motor has the following parameters:

$$r_1 = 0 \cdot \Omega, x_1 = 1.1\ \Omega, r_2' = 0.30\ \Omega, x_2' = 1.1\ \Omega \text{ and } X_m = 40\ \Omega$$

When the induction motor is fed from (a) 440 V, 50 Hz voltage source inverter and (b) 30 A, 50 Hz constant current source inverter, determine the slip for maximum torque, maximum torque and starting torque for cases (a) and (b).

Solution

(a) When the induction motor is fed from 440 V, 50 Hz voltage source inverter

The slip at maximum torque is $s_m = \dfrac{r_2'}{x_1 + x_2'} = \dfrac{0.3}{1.1 + 1.1} = 0.1363$

Synchronous speed $\omega_s = \dfrac{4\pi f}{P} = \dfrac{4\pi \times 50}{4} = 157.1428$ rad/s

The maximum torque is $T_{e\max} = \dfrac{3}{2\omega_s}\dfrac{V_1^2}{x_1 + x_2'} \cdot = \dfrac{3}{2 \times 157.1428}\dfrac{440^2}{1.1 + 1.1} \cdot = 840$ N-m

When resistance $r_1 = 0, T_e = \dfrac{3}{\omega_s}\dfrac{V_1^2}{\left(\dfrac{r_2'}{s}\right)^2 + (x_1 + x_2')^2} \cdot \dfrac{r_2'}{s}$

At starting s = 1, starting torque will be

$$T_{e\,\text{starting}} = \frac{3}{\omega_s}\frac{V_1^2}{\left(\dfrac{r_2'}{1}\right)^2 + (x_1 + x_2')^2} \cdot \frac{r_2'}{1} = \frac{3}{157.1428}\frac{440^2}{(0.3)^2 + (1.1 + 1.1)^2} \cdot 0.3 = 224.90 \text{ N-m}$$

(b) When the induction motor is fed from 30 A, 50 Hz constant current source inverter

The slip at maximum torque is $s_m = \dfrac{r_2'}{x_2' + X_m} = \dfrac{0.3}{1.1 + 40} = 0.00729$

The maximum torque is $T_{e\max} = \dfrac{3}{\omega_s}\dfrac{(I_1 X_m)^2}{2(x_2' + X_m)} \cdot = \dfrac{3}{157.1428}\dfrac{\left(\dfrac{30}{\sqrt{3}} \times 40\right)^2}{2(1.1 + 40)} \cdot = 111.479$ N-m

The starting torque of induction motor is

$$T_{e|\text{starting}} = \frac{3}{\omega_s}\frac{(I_1 X_m)^2}{\left(\dfrac{r_2'}{s}\right)^2 + (x_2' + X_m)^2} \cdot \frac{r_2'}{s} \qquad \text{where, } s = 1$$

$$= \frac{3}{\omega_s}\frac{(I_1 X_m)^2}{(r_2')^2 + (x_2' + X_m)^2} \cdot r_2' = \frac{3}{157.1428}\frac{\left(\dfrac{30}{\sqrt{3}} \times 40\right)^2}{(0.3)^2 + (1.1 + 40)^2} \cdot 0.3 = 5.424 \text{ N-m}$$

12.10.5 Static Rotor Resistance Control of Induction Motor

In a wound rotor or slip-ring induction motor, a three-phase variable resistance $R_{r\,\text{external}}$ can be inserted in the rotor circuit as depicted in Fig. 12.46. By changing the rotor resistance $R_{r\,\text{external}}$, the motor electromagnetic torque can be controlled. The starting torque and starting current can also be varied by controlling the rotor resistance $R_{r\,\text{external}}$. The torque-speed characteristics of induction motor and the effect of rotor resistance on stator current are shown in Fig. 12.46(a) and (b) respectively. The disadvantages of speed control using rotor resistance control are as follows:

1. Efficiency is low at low speeds.
2. Speed changes very widely with the change of load.
3. Voltages and currents become unbalance if rotor circuit resistances are not equal.

The above speed control method is commonly used in overhead cranes and load equalization.

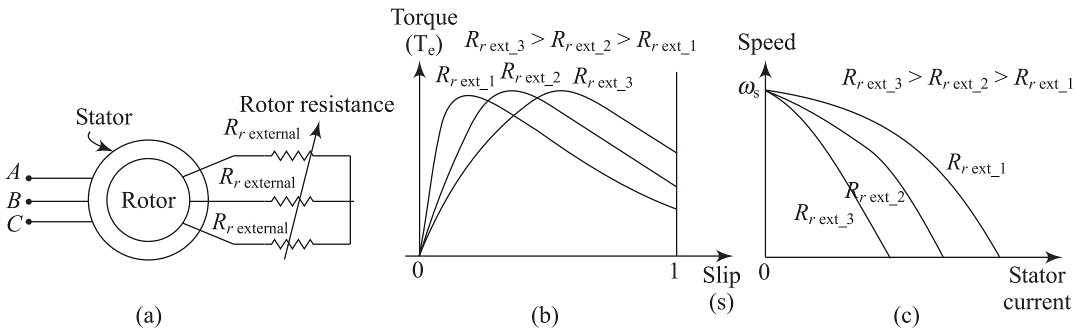


Fig. 12.46 *(a) External rotor resistance control of induction motor, (b) Effect of external rotor resistance in torque-slip characteristics and (c) Effect of external rotor resistance in stator current and speed*

The three-phase resistor can be replaced by a three-phase rectifier, a chopper and one resistor as shown in Fig. 12.47. The inductor L_d is used to smooth the current I_d. The effective rotor circuit resistance may be varied by changing the duty cycle of chopper and subsequently the speed control of slip ring induction motor is possible. Rectifier circuit is used to convert the slip frequency input power to dc power at output terminals A-B.

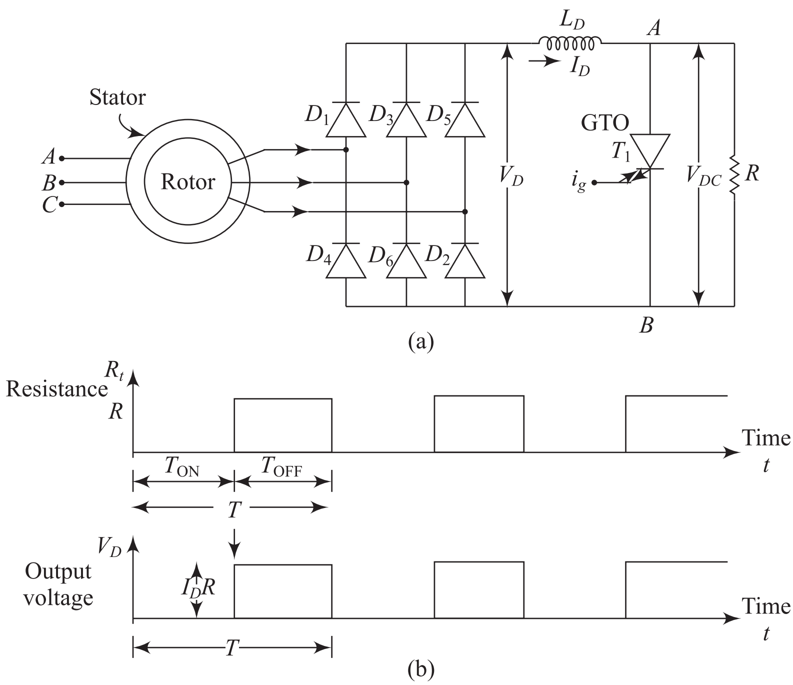


Fig. 12.47 *(a) Static rotor resistance control using rectifier and chopper and (b) Waveforms of output voltage V_d and external resistance*

When the chopper is ON, the resistance R is short circuited and $V_{dc} = 0 = V_d$. When chopper is OFF, $V_{dc} = V_d$ neglecting the effect of L_d

If the chopper is ON for T_{ON} time and is OFF for T_{OFF} time. Then effective external resistance R_{A-B} is expressed by

$$R_{A-B} = R\frac{T_{OFF}}{T} = R\frac{T - T_{ON}}{T} = R(1-\delta) \text{ where, } \delta = \frac{T_{ON}}{T} \text{ is duty cycle of chopper}$$

The equivalent circuit of three-phase induction motor, rectifier and chopper circuit are depicted in Fig. 12.48(a). When stator impedance $r_1 + jx_1$ and rotor impedance referred to stator $r_2' + jx_2'$ are neglected as compared to inductor L_d, the equivalent circuit can be represented in simplified form as depicted in Fig. 12.48(b).

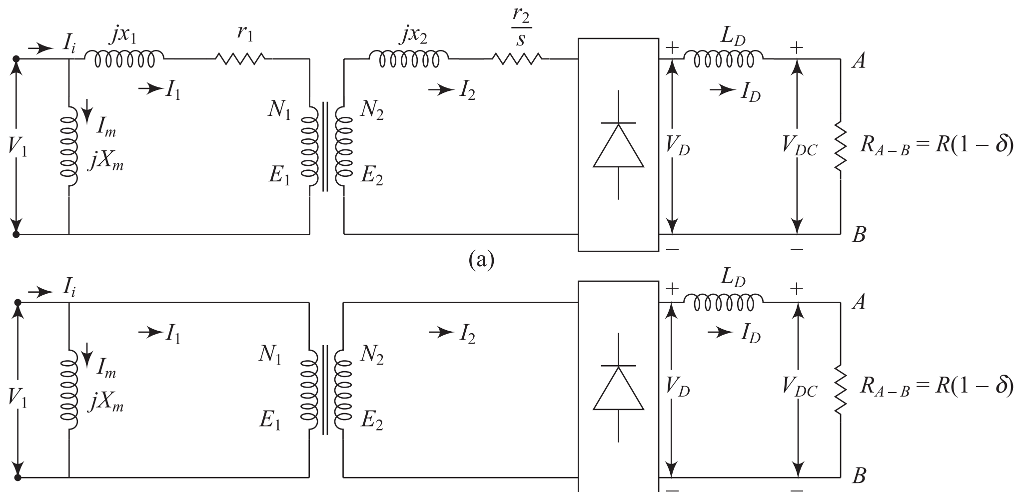


Fig. 12.48 *(a) Equivalent circuit of Fig. 12.47(a), and (b) Approximate equivalent circuit of Fig. 12.48(a)*

Stator voltage V_1 referred to rotor circuit is E_2

$$\frac{V_1}{E_2} = \frac{N_1}{N_2} \quad \text{or,} \quad E_2 = \frac{N_2}{N_1}V_1 = aV_1$$

where, $a = \dfrac{N_2}{N_1} = \dfrac{\text{rotor effective turns per phase}}{\text{stator effective turns per phase}}$ = per phase turns ratio from rotor to stator

E_2 is rotor induced emf per phase at stand-still

V_1 is stator voltage per phase

Voltage $E_2 = saV_1$

The output voltage of a three-phase bridge appears is V_d and it is expressed by

$$V_d = \frac{3V_{m\,\text{line}}}{\pi} = \frac{3\sqrt{3}V_{m\,\text{phase}}}{\pi} = \frac{3\sqrt{3}}{\pi}\sqrt{2}saV_1 = \frac{3\sqrt{6}}{\pi}saV_1 = 2.338saV_1$$

where, $V_{m\text{line}}$ is maximum line voltage and $V_{m\text{phase}}$ is maximum phase voltage

$V_{m\text{phase}}$ is the maximum value of phase voltage $= \sqrt{2}saV_1$

Total slip power is $3sP_g$. If there is no loss in rectifier, dc output power is equal to V_dI_d. Actually dc power is equal to slip power.

Therefore, $3sP_g = V_d I_d$

$$\therefore \qquad P_g = \frac{V_d I_d}{3s}$$

The per phase mechanical power developed is $P_m = P_g(1-s)$

Therefore, $P_m = P_g(1-s) = \dfrac{V_d I_d}{3s}(1-s) = \dfrac{V_d I_d}{3}\dfrac{1-s}{s}$

The per phase mechanical power is $P_m = T_e\omega_m = T_e\omega_s(1-s)$

Therefore, $\dfrac{V_d I_d}{3}\dfrac{1-s}{s} = T_e\omega_s(1-s)$

or
$$I_d = \frac{3sT_e\omega_s}{V_d} \tag{12.10}$$

After substituting the value of V_d in Eq. (12.10), we get

$$I_d = \frac{3sT_e\omega_s}{2.338saV_1} = 1.2831\frac{T_e\omega_s}{aV_1} \qquad \text{as } V_d = 2.338saV_1$$

Total load torque $T_L = 3T_e \quad \therefore T_e = \dfrac{T_L}{3}$

The current I_d in terms of load torque T_L is

$$I_d = \frac{3sT_e\omega_s}{2.338saV_1} = \frac{3s\omega_s}{2.338saV_1}\frac{T_L}{3} = \frac{T_L\omega_s}{2.338aV_1}$$

The current through inductor I_d is independent of motor speed.

The dc voltage at the rectifier output is $V_d = I_d R(1-\delta)$

Then, $V_d = 2.338saV_1 = I_d R(1-\delta)$

or Slip $s = \dfrac{I_d R(1-\delta)}{2.338aV_1}$

Motor speed $\omega_m = \omega_s(1-s)$

After substituting the value of slip in $\omega_m = \omega_s(1-s)$, we find

$$\omega_m = \omega_s(1-s) = \omega_s\left(1 - \frac{I_d R(1-\delta)}{2.338aV_1}\right)$$

or
$$N_r = N_s\left(1 - \frac{I_d R(1-\delta)}{2.338aV_1}\right) \qquad \text{as } \omega_m = \frac{2\pi N_r}{60} \text{ and } \omega_s = \frac{2\pi N_s}{60}$$

After substituting the value of I_d in the above equation, we obtain

$$\omega_m = \omega_s\left(1 - \frac{I_d R(1-\delta)}{2.338aV_1}\right) = \omega_s\left(1 - \frac{R(1-\delta)}{2.338aV_1}\frac{T_L\omega_s}{2.338aV_1}\right) \quad \text{as } I_d = \frac{T_L\omega_s}{2.338aV_1}$$

$$= \omega_s\left(1 - \frac{T_L\omega_s R(1-\delta)}{(2.338aV_1)^2}\right)$$

It is clear from above equation that speed of induction motor decreases with increasing load torque T_L when duty cycle is constant.

In three-phase rectifier, each diode conducts for 120°. The waveform of per phase rotor current I_r is depicted in Fig. 12.49. Assume the output current I_d is ripple free. Then, the rms value of rotor current is

$$I_{r(\text{rms})} = \sqrt{I_d^2 \frac{2\pi}{3}\frac{1}{\pi}} = \frac{\sqrt{2}}{\sqrt{3}} I_d$$

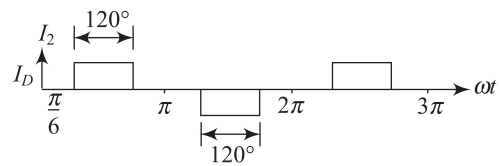


Fig. 12.49 *Per phase rotor current waveform as $I_r = I_2$*

The rotor current referred to stator

$$I_1 = \frac{N_2}{N_1} I_2 = aI_2 = \frac{\sqrt{2}}{\sqrt{3}} aI_d \qquad \text{as } I_2 = I_{r(\text{rms})}$$

After Fourier series analysis of I_2 current waveform, we find

$$b_1 = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_d \sin \omega t \cdot d\omega t = \frac{2}{\pi} I_d \,|-\cos \omega t|_{\pi/6}^{5\pi/6} = \frac{2\sqrt{3}}{\pi} I_d$$

The rms value of fundamental component of rotor current is

$$I_{2(\text{rms})} = \frac{b_1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{2\sqrt{3}}{\pi} I_d = \frac{\sqrt{6}}{\pi} I_d$$

The rms value of fundamental component of rotor current referred to stator is

$$I_{1(\text{rms})} = \frac{N_2}{N_1} I_{2(\text{rms})} = aI_{2(\text{rms})} = \frac{\sqrt{6}}{\pi} aI_d$$

Example 12.23 A three phase, 440 V, 50 Hz, 1450 star connected wound rotor induction motor has the following parameters:

$$r_1 = 0.2\ \Omega,\ x_1 = 0.45\ \Omega,\ r_2 = 0.15\ \Omega,\ x_2' = 0.45\ \Omega \text{ and negligible } X_m$$

The speed of motor can be controlled by a GTO based dc-to-dc converter (chopper) where inductor L_d is very large and $R = 5\ \Omega$. When the motor operates at 1000 rpm and inductor current is 100 A, determine (a) duty cycle of chopper, (b) efficiency at 25 kW power output and (c) rms value of fundamental component of rotor current referred to stator.

Assume per phase rotor to stator turn ratio is 0.75 and no-load loss is negligible.

Solution

(a) Per phase stator voltage $V_1 = \dfrac{440}{\sqrt{3}} = 254.034$ V

We know $\omega_m = \omega_s(1-s) = \omega_s\left(1 - \dfrac{I_d R(1-\delta)}{2.338aV_1}\right)$

Therefore, $N_r = N_s\left(1 - \dfrac{I_d R(1-\delta)}{2.338aV_1}\right)$ or, $1000 = 1500\left(1 - \dfrac{100\times 5(1-\delta)}{2.338\times 0.75\times 254.034}\right)$

Duty cycle of chopper $\delta = 0.7277$

(b) Power loss in chopper resistance is $I_d^2 R(1-\delta = 100^2 \times 5\times(1-0.7277) = 13615$ Watt

Rotor current is $I_2 = \dfrac{\sqrt{2}}{\sqrt{3}} I_d = \dfrac{\sqrt{2}}{\sqrt{3}}\times 100 = 81.65$ A

Rotor copper loss $= 3I_2^2 r_2 = 3\times 81.65^2 \times 0.15 = 3000.025$ Watt

Stator current is $I_1 = aI_2 = 0.75 \times 81.65 = 61.2375$ A

Stator copper loss = $3I_1^2 r_1 = 3 \times 61.2375^2 \times 0.2 = 2250.018$ Watt

Power output P_{output} = 25 kW

Input power $P_{\text{input}} = P_{\text{output}}$ + stator copper loss + rotor copper loss + Loss in chopper

$$= 25 \times 10^3 + 2250.018 + 3000.025 + 13615 = 43865.043 \text{ Watt}$$

Efficiency $\eta = \dfrac{P_{\text{output}}}{P_{\text{input}}} \times 100 = \dfrac{25 \times 10^3}{43865.043} \times 100 = 56.99\%$

(c) rms value of fundamental component of rotor current referred to stator is

$$I_{1(\text{rms})} = \frac{\sqrt{6}}{\pi} aI_d = \frac{\sqrt{6}}{\pi} \times 0.75 \times 100 \text{ A} = 58.45 \text{ A}$$

12.11 SYNCHRONOUS MOTOR DRIVES

Synchronous motors have two windings such as a three-phase armature winding on the stator and a *field winding* on the rotor. The winding on the stator is also known as *armature winding*. The three-phase winding on stator of synchronous motor is similar to the three-phase winding on the stator of three phase induction motor. When the field winding is excited by dc and dc current flows through field winding to develop magnetic field. There are two mmf in synchronous motor such as field mmf and armature mmf. Whenever the field winding is excited with dc, a mmf is produced and it is called *field mmf*. Whenever three-phase stator winding is excited by three-phase ac voltage, a three-phase balanced current flows through armature and is called *armature current*. Due to armature current, a rotating armature mmf is generated. Then two mmfs are combined together to create a resultant mmf. Subsequently, the field mmf interacts with the resultant mmf to generate electromagnetic torque. Though armature of synchronous motor is similar to the stator of three-phase induction motor, there is no induction in the rotor. A synchronous motor always operates at synchronous speed (constant speed) with zero slip. The power factor of synchronous motors can be controlled by varying its field current. There are different types synchronous motors such as

1. Cylindrical rotor synchronous motors
2. Salient pole synchronous motors
3. Reluctance synchronous motors
4. Permanent magnet synchronous motors

The speed control of synchronous motors can be possible using both inverter and cycloconverters. In this section, cylindrical rotor motors, salient pole motors, reluctance motors, permanent magnet motors and speed control of synchronous motors are explained in detail.

12.11.1 Cylindrical Rotor Synchronous Motors

In cylindrical rotor synchronous motors, the field winding is wound on the cylindrical rotor. These motors have uniform air gap and the reactances are independent of the rotor position. The per phase equivalent circuit for a cylindrical rotor synchronous motor is depicted in Fig. 12.50 where,

$$V_f = \sqrt{2}\pi f \phi_f N_1 k_w, \text{ the excitation voltage per phase,} \tag{12.11}$$

r_a = armature resistance per phase,
X_s = synchronous reactance per phase,
V_t = armature terminal voltage per phase,
$Z_s = r_a + jX_s$ = synchronous impedance per phase, and
I_a = armature current per phase.

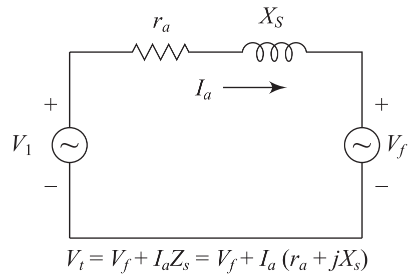


Fig. 12.50 *Equivalent circuit for a cylindrical rotor synchronous motor*

It is clear from the Fig. 12.50 that

$$V_t = V_f + I_a Z_s = V_f + I_a (r_a + jX_s)$$

Then excitation voltage per phase

$$V_f = V_t - I_a (r_a + jX_s)$$

Therefore, current

$$I_a = \frac{V_t - V_f}{Z_s} = \frac{V_t}{Z_s} - \frac{V_f}{Z_s}$$

If $\vec{V}_t = V_t \angle 0, \vec{V}_f = V_f \angle -\delta$, and current I_a is θ angle lagging from voltage V_t, Fig. 12.51 shows the phasor diagram of synchronous motor at lagging load.

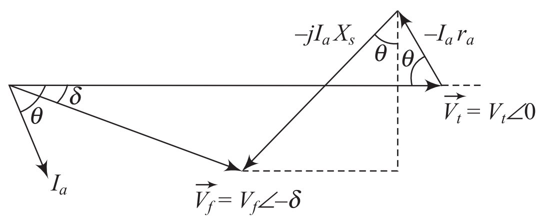


Fig. 12.51 *Phasor diagram of synchronous motor at lagging load*

From phasor diagram, we can write

$$\begin{aligned} V_f &= V_t \angle 0 - I_a (r_a + jX_s) \\ &= V_t - (I_a \cos\theta - jI_a \sin\theta)(r_a + jX_s) \qquad \text{as } \vec{I}_a = (I_a \cos\theta - jI_a \sin\theta) \\ &= V_t - I_a r_a \cos\theta - I_a X_s \sin\theta - j(I_a X_s \cos\theta - I_a r_a \sin\theta) = V_f \cos\delta - jV_f \sin\delta = V\angle -\delta \end{aligned}$$

where, $V_f \cos\delta = V_t - I_a r_a \cos\theta - I_a X_s \sin\theta$ and $V_f \sin\delta = (I_a X_s \cos\theta - I_a r_a \sin\theta)$

Then $\tan\delta = \dfrac{V \sin\delta}{V \cos\delta} = \dfrac{(I_a X_s \cos\theta - I_a r_a \sin\theta)}{V_t - I_a r_a \cos\theta - I_a X_s \sin\theta}$ and $\qquad \delta = \tan^{-1}\left[\dfrac{(I_a X_s \cos\theta - I_a r_a \sin\theta)}{V_t - I_a r_a \cos\theta - I_a X_s \sin\theta}\right]$

$$\text{Current } I_a = \frac{V_t}{Z_s} - \frac{V_f}{Z_s} = \frac{V_t}{r_a + jX_s} - \frac{V_f \cos\delta - jV_f \sin\delta}{r_a + jX_s}$$

$$= \frac{V_t (r_a - jX_s)}{r_a^2 + X_s^2} - \frac{(V_f \cos\delta - jV_f \sin\delta)(r_a - jX_s)}{r_a^2 + X_s^2} \tag{12.12}$$

Then $I_a \cos\theta = \dfrac{V_t r_a}{r_a^2 + X_s^2} - \dfrac{V_f r_a \cos\delta - V_f X_s \sin\delta}{r_a^2 + X_s^2}$

The power input to the machine is

$$P_{\text{input}} = 3V_t I_a \cos\theta = 3V_t \left[\frac{V_t r_a}{r_a^2 + X_s^2} - \frac{V_f r_a \cos\delta - V_f X_s \sin\delta}{r_a^2 + X_s^2}\right]$$

$$= \frac{3V_t^2 r_a}{r_a^2 + X_s^2} - \frac{3V_t V_f r_a \cos\delta - 3V_t V_f X_s \sin\delta}{r_a^2 + X_s^2}$$

Stator copper loss of synchronous motor is $P_{\text{stator\_cu\_loss}} = 3I_a^2 r_a$

Air gap power is $P_g = P_{\text{input}} - P_{\text{stator\_cu\_loss}}$

The developed power is $P_d = P_g$

If the synchronous speed of synchronous motor is ω_s, the torque developed is T_d.

Therefore, $T_d \omega_s = P_d$ or, $T_d = \dfrac{P_d}{\omega_s}$

If armature resistance (stator resistance) r_a is very small, $P_{\text{stator\_cu\_loss}} = 3I_a^2 r_a = 0$

Then $P_{\text{input}} = \dfrac{3V_t^2 r_a}{r_a^2 + X_s^2} - \dfrac{3V_t V_f r_a \cos\delta - 3V_t V_f X_s \sin\delta}{r_a^2 + X_s^2} \approx \dfrac{3V_t V_f X_s \sin\delta}{X_s^2} = \dfrac{3V_t V_f \sin\delta}{X_s}$

and $\delta = \tan^{-1}\left[\dfrac{(I_a X_s \cos\theta - I_a r_a \sin\theta)}{V_t - I_a r_a \cos\theta - I_a X_s \sin\theta}\right] \approx \tan^{-1}\left[\dfrac{I_a X_s \cos\theta}{V_t - I_a X_s \sin\theta}\right]$

If armature resistance is neglected,

$$P_{\text{input}} = P_{\text{output}} = \frac{3V_t V_f}{X_s}\sin\delta$$

When per phase developed torque is T_e, $3T_e = \dfrac{P_{\text{output}}}{\omega_s}$

$$\therefore \quad T_e = \frac{P_{\text{output}}}{3\omega_s} = \frac{V_t V_f}{\omega_s X_s}\sin\delta \quad \text{where, } \omega_s = \frac{4\pi f}{P} = \text{synchronous speed in rad/s} \quad (12.13)$$

δ = load or power angle.

The torque T_e vs load angle δ characteristics for cylindrical rotor synchronous motor is depicted in Fig. 12.52(a). During motor mode operation, δ is negative and torque or power becomes positive. For generator mode operation, δ is positive and torque or power becomes negative.

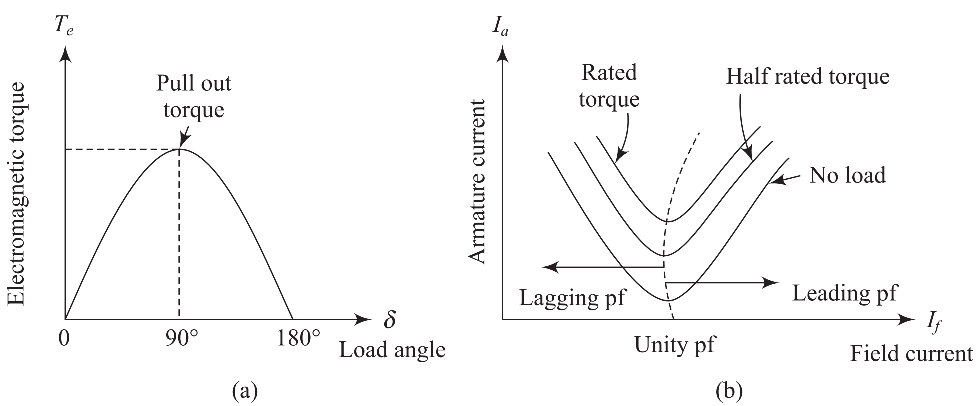


Fig. 12.52 *(a) Torque-angle characteristics and (b) V-curve of synchronous motor*

If the fixed voltage at constant frequency is applied to synchronous motor, torque depends on the load angle δ and the excitation voltage V_f. When V_f and δ are constant, torque depends on the voltage to frequency $\dfrac{V}{f}$ ratio. If $\dfrac{V}{f}$ ratio is constant, speed control at constant torque is possible. When V_t, V_f

and δ are remain constant, torque decreases with increase in speed and the synchronous motor operates in the field weakening mode.

At $\delta = 90°$, the torque T_e is maximum and its value is $T_{e\max} = \dfrac{V_t V_f}{\omega_s X_s}$. The maximum developed torque is also known as *pull-out torque*. For stable operation, synchronous motor should operate in the positive slope of $T_e - \delta$ curve and the range of torque angle is $-90° \le \delta \le 90°$

The power factor of a synchronous motor depends on the field current I_f. The typical variation of armature current I_a of a synchronous motor with respect to field current I_f at different loads is depicted in Fig. 12.52(b). Since the shape of curves is similar to the letter "V", these curves are called *V curves* of a synchronous motor. For the same armature current, the power factor may be lagging or leading depending upon the field excitation. The unity power factor curve is represented by dotted line when the synchronous motor is normally excited. During under excitation, i.e., below normal excitation, synchronous motors operate at lagging power factor. When the field current is increased beyond the normal excitation, i.e., over excitation, synchronous motors operate at leading power factor.

If the power angle $\delta = 90°$, the pull-out power is $P_{\max} = \dfrac{V_t V_f}{X_s}$

The pull-out torque is $T_{e\max} = \dfrac{1}{\omega_s}\dfrac{V_t V_f}{X_s}$

When the field excitation is constant, the excitation emf V_f is directly proportional to supply frequency and the synchronous reactance X_s is directly proportional to frequency. Therefore, $\dfrac{V_t}{X_s}$ is independent of frequency variation. When the change of supply voltage V_t is directly proportional to frequency and $\dfrac{V_t}{f}$ or $\dfrac{V_t}{\omega s}$ is constant, the pull-out torque $T_{e\,\max}$ is constant. Subsequently, the pull out power is $P_{\max} = T_{e\max}\,\omega_s$ increases linearly.

At ω_s = 1.0 per unit, synchronous motor operates at rated voltage and rated frequency. When $\omega_s > 1.0$ per unit and synchronous motor operates at supersynchronous speed, rated voltage must be applied and is kept constant, after that inverter frequency can be increased to operate at higher speeds.

For constant field excitation V_f is directly proportional to supply frequency and increase in frequency keeps $\dfrac{V_f}{X_s}$ constant. Since input voltage remain constant above base speed, the pull-out power remain constant but the pull-out torque decreases with increase in speed as $T_{e\max} = \dfrac{P_{\max}}{\omega_s}$. Fig. 12.53 shows variation of $T_{e\,\max}$, $P_{\max}$, V_t and I_a with respect to frequency for synchronous motor.

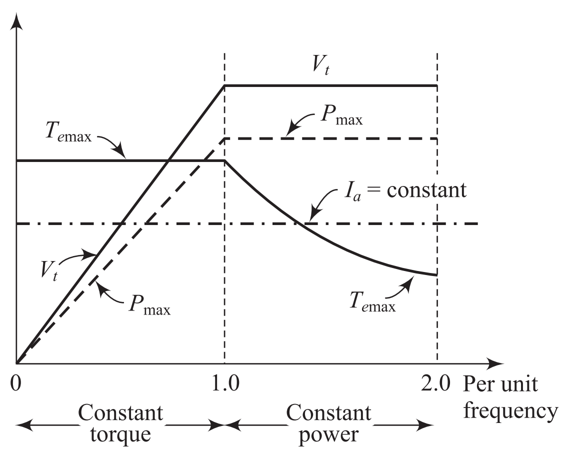


Fig. 12.53 *Variation of $T_{e\,max}$, P_{max}, V_t and I_a with respect to frequency for synchronous motor*

12.11.2 Salient Pole Synchronous Motors

The armature winding of salient pole synchronous motor is similar to that of cylindrical rotor synchronous motors. The armature winding is placed in the stator and the field winding is placed on the rotor of motor. The field winding is a concentrated winding on the salient poles. Due to saliency, the air-gap is not uniform and the flux depends on the position of the rotor. To analyze a salient pole synchronous motor, armature current and reactances are resolved into two components namely direct axis (d-axis) and quadrature axis (q-axis) components. Armature current I_a is represented by direct axis current I_d and quadrature axis current I_q. Similarly, reactance is represented by d-axis synchronous reactance X_d and q axis synchronous reactance X_q.

The phasor diagram of a salient-pole synchronous motor with negligible armature resistance r_a is depicted in Fig. 12.54.

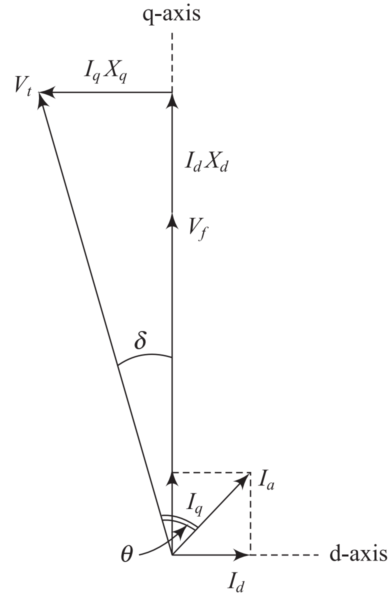


Fig. 12.54 *Phasor diagram of salient pole synchronous motors*

From Fig. 12.54, we can write $V_t = V_f + jI_d X_d + jI_q X_q$

Therefore, $V_f = V_t - jI_d X_d - jI_q X_q$

Since the angle between V_f and I_a is $\theta - \delta$,

$$I_d = I_a \sin(\theta - \delta) \text{ and } I_q = I_a \cos(\theta - \delta)$$

Then, $\quad V_t \sin\delta = I_q X_q = X_q I_a \cos(\theta - \delta)$

$$= X_q I_a \cos\theta \cos\delta + X_q I_a \sin\theta \sin\delta$$

or $$\sin\delta (V_t - I_a X_q \sin\theta) = I_a X_q \cos\theta \cos\delta$$

or $$\tan\delta = \frac{I_a X_q \cos\theta}{V_t - I_a X_q \sin\theta} \qquad \therefore \quad \delta = \tan^{-1}\left(\frac{I_a X_q \cos\theta}{V_t - I_a X_q \sin\theta}\right)$$

The terminal voltage is resolved into d-axis and q-axis components:

$$V_{td} = -V_t \sin\delta \text{ and } V_{tq} = V_t \cos\delta$$

Input power is $$P = 3(V_{td} I_d + V_{tq} I_q) = -3V_t I_d \sin\delta + 3V_t I_q \cos\delta \tag{12.14}$$

$$= 3V_t I_q \cos\delta - 3V_t I_d \sin\delta$$

Since $V_t \sin\delta = I_q X_q$, $\quad I_q = \dfrac{V_t}{X_q} \sin\delta$

and $V_t \cos\delta = V_f + I_d X_d$

or $$I_d X_d = V_t \cos\delta - V_f \qquad \therefore \quad I_d = \frac{V_t \cos\delta - V_f}{X_d}$$

As $$I_q X_q = V_t \sin\delta, \qquad I_q = \frac{V_t \sin\delta}{X_q}$$

After substituting the value of I_d and I_q in equation (12.14), we obtain

$$P = \frac{V_f V_t}{X_d} \sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta \tag{12.15}$$

It is clear from Eq. (12.15) that the power has two components. The first component is $\frac{V_f V_t}{X_d}\sin\delta$ which is same as cylindrical rotor motor and it is called the *electromagnetic power*. The other part is $\frac{1}{2}V_t^2\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta$ which is called the *reluctance power*. The reluctance power component is developed due to the reluctance variation along *d*-axis and *q*-axis. The torque developed in synchronous motor is equal to

$$T_e = \frac{P}{\omega_s} = \frac{1}{\omega_s}\frac{V_f V_t}{X_d}\sin\delta + \frac{V_t^2}{2\omega_s}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta$$

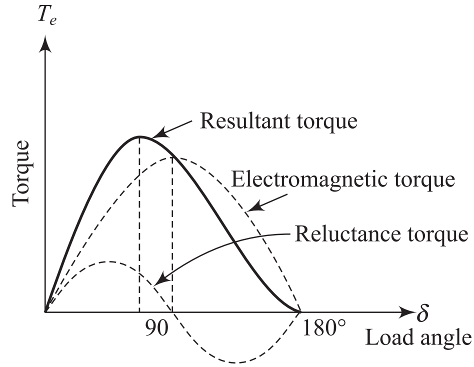


Fig. 12.55 *Torque versus load angle characteristics of salient pole synchronous motors*

First component is electromagnetic torque and the other component is reluctance torque. Figure 12.55 shows the torque T_e vs load angle δ characteristics. The torque is maximum at a load angle less than 90°.

12.11.3 Reluctance Synchronous Motors

The reluctance motor is similar to the salient-pole motors but there is no field winding on the rotor. When three-phase armature winding of a salient pole synchronous motor is connected to ac voltage source, a rotating magnetic field is developed in the air gap. This rotating flux induces a field in the rotor and it has a tendency to align with the armature field. Consequently, a reluctance torque is developed at synchronous speed.

Whenever a salient-pole synchronous motor is connected to ac voltage source, it runs at synchronous speed. If field current of salient-pole synchronous motor is switched off, it runs at synchronous speed continuously and operates as a reluctance motor. Therefore, a reluctance motor is similar to a salient pole motor with no field winding on the rotor. Reluctance motors are used for low power drives where constant speed operation is required. These motors are also used in some applications where a number of motors must be rotate in synchronism.

As there is no field current ($I_f = 0$), $V_f = 0$. Figure 12.56 shows the phasor diagram for synchronous reluctance motor. Per phase input power is

$$P = V_t\cos\delta\cdot I_q - V_t\sin\delta\cdot I_d \qquad (12.16)$$

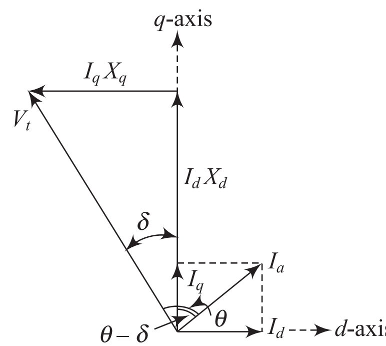


Fig. 12.56 *Phasor diagram for synchronous reluctance motor*

Direct axis and quadrature axis currents are $I_q = \frac{V_t\sin\delta}{X_q}$ and $I_d = \frac{V_t\cos\delta}{X_d}$

After substituting the values of I_q and I_d in Eq. (12.16), we obtain

$$P = V_t\cos\delta\cdot I_q - V_t\sin\delta\cdot I_d = V_t\cos\delta\cdot\frac{V_t\sin\delta}{X_q} - V_t\sin\delta\cdot\frac{V_t\cos\delta}{X_d}$$

$$= \frac{V_t^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta$$

From Fig. 12.56, we can write

$$V_t \sin\delta = I_q X_q = X_q I_a \cos(\theta - \delta) \quad \text{as} \quad I_q = I_a \cos(\theta - \delta)$$

or

$$\tan\delta = \frac{I_a X_q \cos\theta}{V_t - I_a X_q \sin\theta}$$

Therefore, $\delta = \tan^{-1} \dfrac{I_a X_q \cos\theta}{V_t - I_a X_q \sin\theta}$

Three-phase input power is

$$P_{3\text{-phase}} = \frac{3V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

Then reluctance torque

$$T_e = \frac{P_{3\text{-phase}}}{\omega_s} = \frac{3V_t^2}{2\omega_s}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

At $\delta = 45°$, the pull-out torque is equal to

$$T_p = T_{e\max} = \frac{3V_t^2}{2\omega_s}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \quad \text{as} \quad \sin 2\delta = 1$$

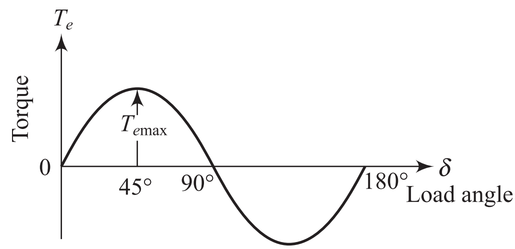


Fig. 12.57 *Variation of reluctance torque with load angle for synchronous reluctance motor*

Figure 12.57 shows the variation of reluctance torque with load angle.

12.11.4 Permanent Magnet Synchronous Motors

Permanent magnet synchronous motors are similar to salient pole synchronous motors without the field winding on the rotor. In permanent magnet synchronous motor, the required field flux is developed by permanent magnets which are mounted on the rotor. In these motors, the excitation voltage V_f can not be varied. For the same frame size, permanent magnet synchronous motor has higher pull-out torque and more efficiency compared to salient pole synchronous motor. All the equations for evaluating the performance of a salient pole synchronous motor are applicable to permanent magnet synchronous motor with constant excitation voltage V_f. Due to absence of field winding, dc supply to field winding and two slip rings leads, the motor losses will be reduced and the complexity of motor construction is also reduced.

These motors are known as *brushless motors* and most commonly used in robots and machine tools. Permanent magnet synchronous motors can be operated by rectangular current source or sinusoidal current source. A rectangular current fed synchronous motor used in low power drives when the concentrated windings on the stator are excited by square or trapezoidal voltage. A sinusoidal current fed synchronous motor has distributed windings on the stator and usually this motor is used in high power drives.

12.11.5 Closed Loop $\frac{V}{f}$ Control of Synchronous Motors

Synchronous motors can be operated as constant torque drive and constant power drive. In the constant torque region, the $\frac{V}{f}$ is maintained constant and in the constant power region, torque decrease with increasing frequency. The torque-speed characteristics of synchronous motor at different frequencies are shown in Fig. 12.58.

The speed of synchronous motors can be controlled by changing voltage, frequency and current. Figure 12.59 shows the block diagram of closed loop $\frac{V}{f}$ control of synchronous motors. The speed error $\omega_{\text{error}} = (\omega_r - \omega_{\text{ref}})$ is generated by an error detector. After that the speed error generates the frequency and voltage command for pulse width modulation inverter. To get proper voltage, firing angle of controlled rectifier is adjusted and frequency is also adjusted by controlling PWM voltage source inverter. As the speed of synchronous motor depends on the supply frequency only, these drives are commonly used in paper mills, spinning mills, textile mills and machine tools.

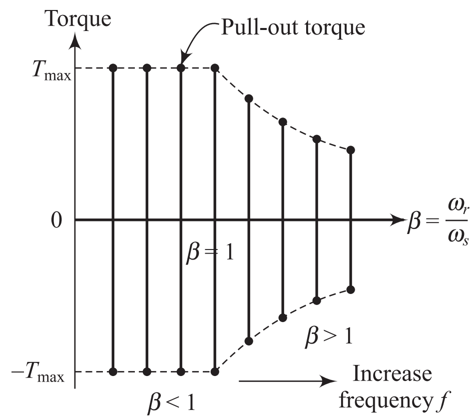


Fig. 12.58 *Frequency control of synchronous motor*

Example 12.24 A four pole, three-phase, 420 V, 50 Hz star-connected reluctance type synchronous motor has negligible armature resistance, $X_d = 6\ \Omega$ and $X_q = 4\ \Omega$. If the load torque of synchronous motor is about 40 N-m, determine (a) load angle, (b) line current and (c) input power factor.

Solution

The synchronous speed $\omega_s = \frac{4\pi f}{P} = \frac{4\pi \times 50}{4} = 157$ rad/sec

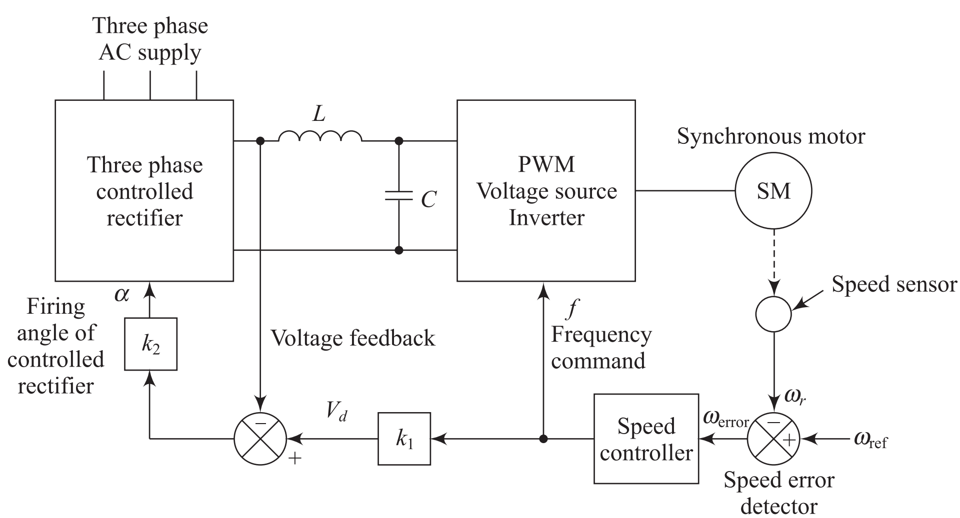


Fig. 12.59 *Closed loop $\frac{V}{f}$ control of synchronous motors*

Three-phase power $P_{3\text{-phase}} = T_e\omega_s = \frac{3V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$

(a) As $T_e = 40$ N-m, $V_t = \dfrac{420}{\sqrt{3}}V$, $X_d = 6\ \Omega$ and $X_q = 4\ \Omega$,

$$40 \times 157 = \frac{3 \times \left(\dfrac{420}{\sqrt{3}}\right)^2}{2}\left(\frac{1}{4} - \frac{1}{6}\right)\sin 2\delta \quad \text{or, } \sin 2\partial = 0.854$$

Therefore, $\delta = \dfrac{\sin^{-1}(0.854)}{2} = \dfrac{58.69°}{2} = 29.34°$

(b) Phase voltage $V_t = \dfrac{420}{\sqrt{3}}V = 242.494$ V

Direct axis-current $I_d = \dfrac{V_t \cos\delta}{X_d} = \dfrac{242.494 \times \cos 29.34}{6} = 35.23$ A

Quadrature axis-current $I_q = \dfrac{V_t \sin\delta}{X_q} = \dfrac{242.494 \times \sin 29.34}{4} = 29.704$ A

Armature current $I_a = [I_d^2 + I_q^2]^{1/2} = \sqrt{35.23^2 + 29.704^2} = 46.08$ A

(b) We know that $\sqrt{3}V_t I_a \cos\phi = T_e \omega_s$

Power factor is $\cos\phi = \dfrac{T_e \omega_s}{\sqrt{3}V_t I_a} = \dfrac{40 \times 157}{420 \times 46.08} = 0.3244$ (lagging)

12.12 POWER FACTOR IMPROVEMENT

In electrical industry, the most commonly used load are induction motors, induction furnace, arc furnace, fluorescent tube and electric fans which operate at low power factor. Due to low factor, these loads draw a large amount of reactive power. Consequently, the consumer terminal voltage is reduced significantly and the performances of utility devices are also reduced. To improve system performance, the power factor must be improved. There are different methods of power factor improvement such as using

1. capacitor banks
2. synchronous condensers
3. static VAR compensations

When a bank of capacitors is connected across the load, the overall reactive power drawn from the system will be decreased as capacitor banks provide leading reactive power. As a result, the system power factor can be improved. As the system load is dynamic in nature, the reactive power demand is also dynamic. If a bank of capacitors is used for power factor improvement, sometimes system operate at leading power factor and sometimes in lagging power factor and unity power operation will be very rare. To operate the system at unity power factor continuously, dynamic VAR compensation or dynamic power factor control must be carried out through dynamic switching in or out of capacitors. The dynamic switching is feasible by using thyristors which are controlled by applying trigger pulses. Thyristor controlled reactors (TCRs) and Static VAR compensators are most commonly used power factor improvement techniques through which we can (i) regulate the reactive power flow, (ii) control power factor and (iii) maintain better voltage profile as per consumer requirement. In this section, Thyristor controlled reactors (TCRs) and Static VAr compensators are explained elaborately.

12.12.1 Thyristor Controlled Reactors (TCRs)

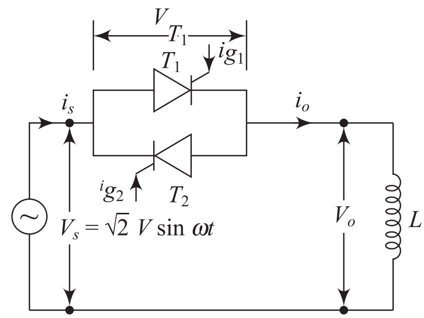


Fig. 12.60 *Thyristor controlled reactor*

Figure 12.60 shows a circuit diagram of thyristor controlled reactors where a linear inductor L is connected to ac source through anti-parallel connection of two thyristors. This circuit is also known as *thyristor controlled inductor*. In ac power system, it is commonly called as *static VAR compensation*. This is similar to the circuit where a purely inductive load is connected to a single-phase ac voltage controller. The output voltage can be controlled during ωt from $\frac{\pi}{2}$ to π only as $\phi = 90°$. Then range of firing angle is $\frac{\pi}{2} \le \alpha \le \pi$. The waveforms of voltage and current at different firing angles are shown in Fig. 12.61. This circuit draws lagging reactive current from utility system; hence there will be excessive voltage drops which adversely affect on stability of system.

During the positive half cycle of supply voltage, thyristor T_1 will conduct whenever the trigger pulse is applied to T_1. Similarly, during the negative half cycle of supply voltage, thyristor T_2 will conduct whenever the trigger pulse is applied to T_2. If the firing angle $\alpha \le 90°$, the source current i_s is continuous in nature and it lags 90° from input voltage. In this case the fundamental component i_{s1} is same as i_s and its value is maximum. Subsequently, the reactance offered by the reactor is minimum i.e., $X_L = \frac{V}{I_s} = \frac{V}{I_{s1}}$

where, V is rms value of source voltage and $I_s = I_{s1}$ is rms value of source current.

When the firing angle $\alpha > 90°$, the source current is discontinuous and it flows from α to β.

The load current is equal to

$$i_o(\omega t) = \frac{\sqrt{2}V}{\omega L}\left[\sin\left(\omega t - \frac{\pi}{2}\right) - \sin\left(\alpha - \frac{\pi}{2}\right)e^{\frac{\alpha - \omega t}{\tan(\pi/2)}}\right]$$

$$= \frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)$$

Assume the load current is discontinuous and at $\omega t = \beta, i_o(\beta) = 0$

Then, $i_o(\beta) = \cos\alpha - \cos\beta = 0$

So, $\beta = 2\pi - \alpha$

The fundamental components of load currents are

$$a_1 = \frac{2}{\pi}\int_\alpha^\beta i_o(\omega t)\cdot\cos\omega t\cdot d(\omega t) \text{ and } b_1 = \frac{2}{\pi}\int_\alpha^\beta i_o(\omega t)\cdot\sin\omega t\cdot d(\omega t)$$

After substituting the value of $i_o(\omega t)$, we obtain

$$a_1 = \frac{2}{\pi}\int_\alpha^\beta \frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)\cdot\cos\omega t\cdot d(\omega t)$$

$$= \frac{2\sqrt{2}V}{\pi\omega L}\left[\cos\alpha(\sin\beta - \sin\alpha) - \frac{1}{2}\left\{(\beta-\alpha) + \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right\}\right]$$

$$= \frac{\sqrt{2}V}{\pi\omega L}[2(\pi-\alpha) + \sin 2\alpha] \qquad \text{when } \beta = 2\pi - \alpha$$

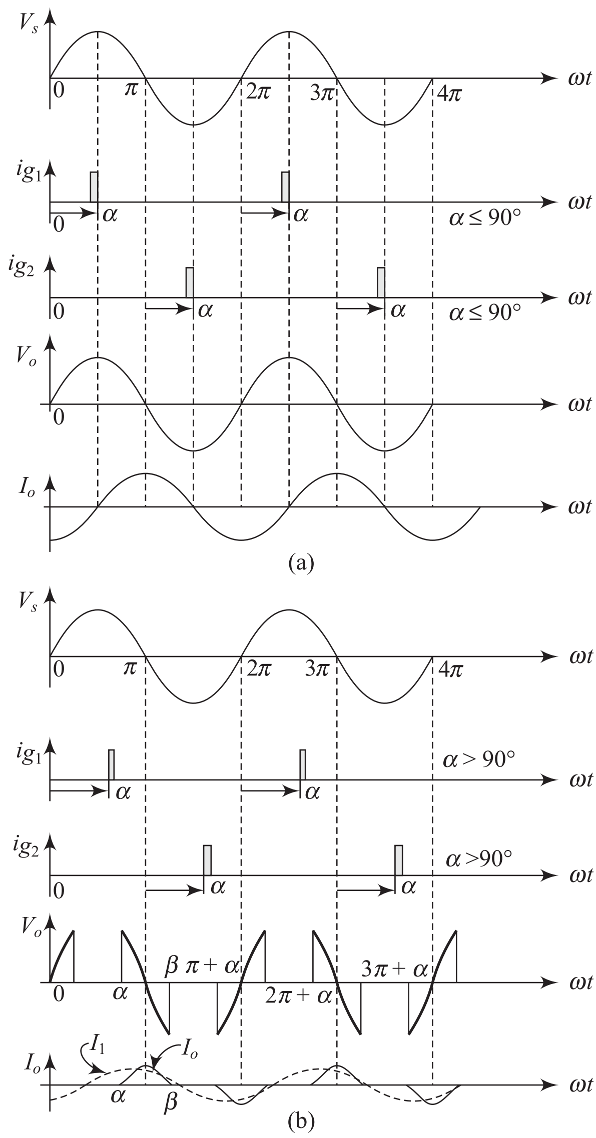


Fig. 12.61 *Voltage and current waveforms of thyristor controlled reactor (a) $\alpha \le 90°$ and (b) $\alpha > 90°$*

$$b_1 = \frac{2}{\pi}\int_{\alpha}^{\beta}\frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)\cdot\sin\omega t\cdot d(\omega t)$$

$$= \frac{\sqrt{2}V}{2\pi\omega L}[-\cos 2\alpha + \cos 2\beta - 4\cos\alpha(\cos\beta - \cos\alpha)]$$

$$= 0 \qquad \text{when } \beta = 2\pi - \alpha$$

Therefore, rms value of the fundamental component of current is

$$I_1 = \left[\frac{a_1^2 + b_1^2}{2}\right]^{1/2} = \frac{V}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha] \quad \text{for } \alpha > 90°$$

The reactive power drawn during $0 \le \alpha \le 90°$ is equal to

$$Q = VI_{s1} = VI_s = V\frac{V}{\omega L} = \frac{V^2}{\omega L}$$

During $90° \le \alpha \le 180°$. The reactive power drawn is equal to

$$Q = VI_1 = V\frac{V}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha] = \frac{V^2}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha]$$

It is clear from above equation that at $\alpha = 90°$, the reactive power drawn is maximum, i.e., $Q = \dfrac{V^2}{\omega L}$ and at $\alpha = 180°$, the reactive power drawn is minimum, i.e., zero.

12.12.2 Static VAR Compensator (SVC)

Figure 12.62 shows a circuit diagram for Static VAR Compensator (SVC) where a thyristor controlled reactor (TCR) is connected in parallel with a fixed capacitor C. It is clear from Fig. 12.62 that an SVC is a combination of TCR and a fixed capacitor and a load is connected in parallel with static VAR Compensator. The Static VAR Compensator is also known as thyristor controlled compensator (TCC) or static VAR compensating system (SVS).

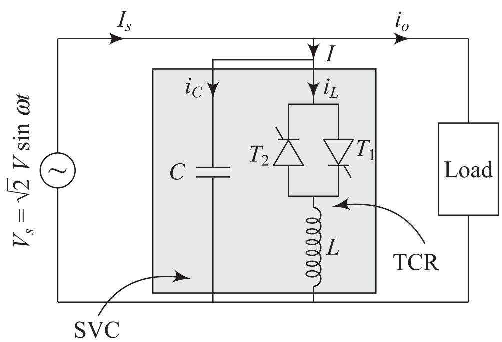


Fig. 12.62 *Static VAR Compensator (SVC)*

As fixed capacitor is connected to ac supply, the constant leading power ωCV^2 is supplied to power system. When thyristors T_1 and T_2 are fired at $\alpha =$ 90°, the value of reactance will be minimum and TCR provides maximum lagging reactive power i.e. $\dfrac{V^2}{\omega L}$. The values of C and L are selected in such a way that $\dfrac{V^2}{\omega L}$ is more than ωCV^2. Therefore, at $\alpha = 90°$ the SVC draws lagging reactive power from the source and accordingly the power factor of the combination of load and SVC is lagging in nature.

At $\alpha = 90°$ (Fig. 12.63(a)), the current flows through inductor is I_L and the value of I_L is maximum. The current flows through capacitor is I_C which is less than I_L and I_o is the load current. The phasor diagram of I_C, I_L, I_o and I_S is depicted in Fig. 12.63. As the drawing of lagging – reactive power from source is increased by $V(I_L - I_C)$, the reactive power of supply system has been increased from $VI_o \sin \theta_o$ to $VI_s \sin \theta_s$.

When firing angle $\alpha_1 > 90°$ (Fig. 12.63(b)), the inductive reactance becomes more and as a result the fundamental current of I_L will be reduced. The resultant of I_C and I_L will be modified from OA to OB. Then power factor has been improved from θ_o to θ_1. In this case, the total reactive power drawn from the ac system has been reduced from $VI_o \sin \theta_o$ to $VI_1 \sin \theta_1$.

If firing angle is further increased, at $\alpha = \alpha_2$ ($\alpha_2 > \alpha_1$) (Fig. 12.63(c)) the inductive reactance be more than the inductance at $\alpha = \alpha_1$. As the value of inductive current decreases, the fundamental

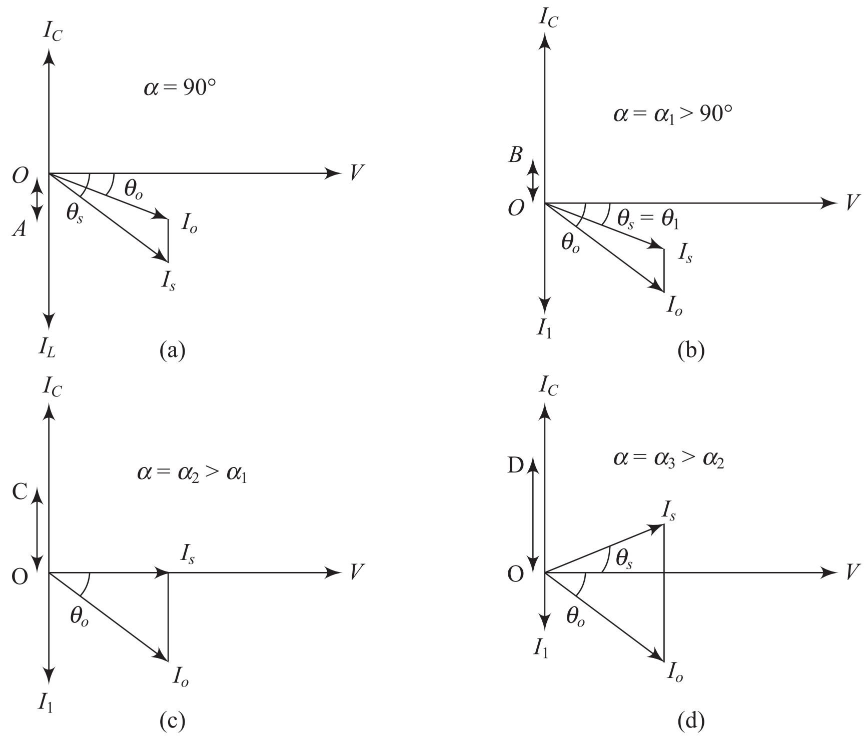


Fig. 12.63 *Phasor diagram of I_C, I_L, I_o and I_S (a) at α = 90°, (b) at $\alpha = \alpha_1 > 90°$, (c) at $\alpha = \alpha_2 > \alpha_1$ and (d) at $\alpha = \alpha_3 > \alpha_2$*

component I_1 is further reduced. If we assume that at $\alpha = \alpha_2$, $(I_C - I_1) = I_o \sin\theta_o$ and the power factor of system becomes unity.

When the firing angle is further increased from $\alpha = \alpha_2$ to $\alpha = \alpha_3$ $(\alpha_3 > \alpha_2)$ (Fig. 12.63(d)) the inductive reactance be more than the inductance at $\alpha = \alpha_2$. As the inductive current decreases, the fundamental component I_1 is further reduced. If we assume that at $\alpha = \alpha_3$, $(I_C - I_1) > I_o \sin\theta_o$ and the system power factor can be changed from unity power factor to leading power factor.

At any firing angle of TCR, the fundamental component of SVC is expressed by

$$I = \frac{V}{X_C} - I_1 = \frac{V}{X_C} - \frac{V}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha] \qquad \text{for } \alpha > 90°$$

$$= \omega CV - \frac{V}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha] = V\left\{\omega C - \frac{1}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha]\right\}$$

When the current of SVC is positive, the SVC provides reactive power to load to improve power factor as well as voltage profile. If the current of SVC is negative, the SVC absorbs reactive power from source and weakens the system power factor as well as voltage profile. Hence by controlling the firing angle of TCR from α = 90° to α = 180°, the reactive power can be continuously regulated and system power factor can also be improved.

12.13 UNINTERRUPTIBLE POWER SUPPLY (UPS)

There are several critical applications where a sudden power failure causes a great deal of public inconvenience and there will be huge economic losses. Examples of some critical load are process control plants, a large computer network systems, safety monitoring system of a plant, communication systems, and hospital intensive care units etc. An uninterruptible power supply (UPS) system is required to provide power without interruption in all critical load. UPS is also known *as standby power supply*. Usually, the two types of UPS configurations are commonly used such as

(a) Load normally connected to ac supply
(b) Load normally connected to inverter

12.13.1 Load Connected to AC Supply

Figure 12.64 shows that load is normally connected to ac supply and the rectifier maintains the full charge of the battery. Whenever the supply fails, the load is switched to the output of the inverter that takes over the main supply. In this configuration, the circuit must be break momentarily and the transfer by solid state switches which takes about 4 to 5 ms. Inverter operates only during the time when the power supply fails.

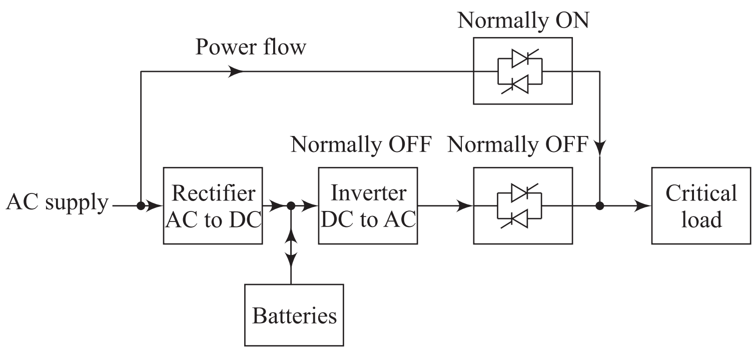


Fig. 12.64 *Load is normally connected to ac supply (short-break UPS)*

12.13.2 Load Connected to Inverter

Figure 12.65 shows that the inverter operates continuously and its output is connected to load. Therefore, there is no need for breaking the power supply. The rectifier unit supplies power to the inverter and

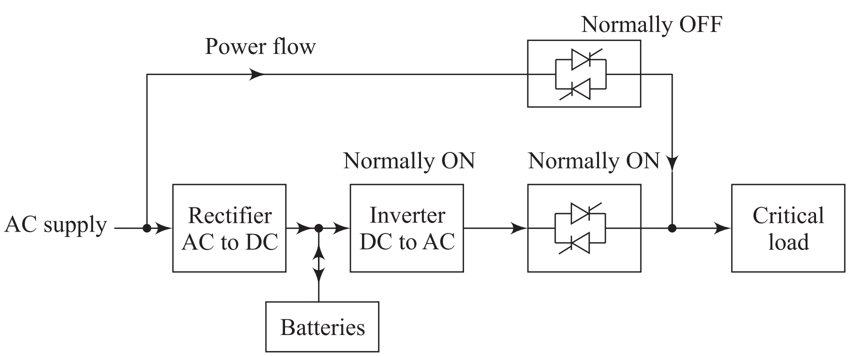


Fig. 12.65 *Load normally connected to inverter (no-break UPS)*

maintains the charge on the standby battery. In this configuration, inverter can be used to provide power supply, to protect the load from the transients in the main supply and to maintain the desired frequency at load. Whenever inverter fails to give supply, the load is switched to ac power supply. The advantages of no-break UPS are

1. Load will be protected from transients in the ac supply.
2. Inverter is used to provide power supply to load.
3. Inverter output frequency must be maintained at the desired value.

Nickel-cadmium and lead-acid-type batteries are used in the UPS system. Nickel-cadmium batteries have the following advantages over lead-acid type batteries:

1. The electrolyte of Nickel-cadmium (NC) batteries does not emit any explosive gas during charging.
2. The electrolyte of NC batteries is non-corrosive.
3. NC batteries can not be damaged by overcharging or discharging and these batteries have longer life.

But the cost of NC batteries is two or three times that of lead-acid batteries. The time period during which a bank of NC batteries can deliver power to load depends on battery size and nature of load.

Summary

- In dc drives, the voltage across armature and field windings can be controlled by dc to dc converters and ac to dc converters. DC to DC converter fed drives are mostly used in traction applications where as ac to dc converter fed drives are used as variable speed drives.
- In this chapter, single phase and three phase controlled rectifier fed dc drives and chopper fed dc drives are explained in detail.
- AC drives have advantages over dc drives. Torque speed characteristics and different speed control scheme of ac drives such as induction motor drives and synchronous motor drives are discussed elaborately.
- Power factor improvement using TCRs and SVC and uninterruptible power supply (UPS) are also incorporated in this chapter.

Multiple-Choice Questions

12.1 In a separately excited dc motor, constant horsepower control of speed is achieved by ______.
(a) armature voltage (b) armature current (c) field voltage (d) field current

12.2 In a separately excited dc motor, constant torque control of speed is achieved by ___________.
(a) armature voltage (b) armature current (c) field voltage (d) field current

12.3 In armature speed control method of dc motor, when the applied armature voltage has been decreased by a large amount, the motor __________.
(a) speed increases (b) speed decreases
(c) works as generator as back emf exceeds the applied voltage (d) gets overloaded

12.4 If a separately excited dc motor is to be operated in the first quadrant only, the ________ converter is used.
(a) Single-phase half wave (b) Single-phase full-wave wave
(c) Single-phase dual (d) Four-quadrant chopper

12.5 When a separately excited dc motor is to be operated in the two quadrants, the __________ converter is used.
(a) Single-phase half wave (b) Single-phase semi
(c) Single-phase full-wave wave (d) Single-phase dual

12.6 If a separately excited dc motor is to be operated in the four quadrant, the __________converter is used
(a) Single-phase half wave (b) Single-phase full-wave wave
(c) Single-phase dual (d) Four-quadrant chopper

12.7 When a separately excited dc motor is to be controlled from a 3-phase source for operation in first quadrant only, which converter will be used?
(a) three-phase half-wave converter (b) three-phase full converter
(c) three-phase semi converter (d) three-phase dual converter

12.8 A single-phase half-wave converter with freewheeling diode fed separately excited dc drive operates at 1000 RPM at firing angle α 45°. If single-phase half-wave converter is replaced by single phase semi-converter, the motor rotates at
(a) 2000 RPM (b) 1500 RPM (c) 1000 RPM (d) 500 RPM

12.9 When a separately excited dc motor is fed from a single-phase full converter with firing angle α, it runs at a speed of 'N' RPM. If the motor is fed from single-phase semi-converter with same firing angle of full converter, the motor speed is found to be 2N rpm. The value of firing angle of converters is _____ degree.
(a) 68.528° (b) 69.528° (c) 70.528° (d) 71.528°

12.10 During the discontinuous conduction mode of a dc drive with armature control using a single phase converter, the reduction in speed ω_m for an increase in armature current i_a or load torque T_L will be ________ that for the continuous conduction mode.
(a) more than (b) equal (c) less than (d) zero

12.11 During the regenerative braking operation of a chopper based dc drive, the energy transfer takes place ____
(a) from the load at high voltage to the source at low voltage
(b) from the source at high voltage to the load at low voltage
(c) from the source at low voltage to the load at high voltage
(d) from the load at low voltage to the source voltage at high voltage

12.12 In a dc drive supplied by a single phase ac source and a controlled rectifier, discontinuous conduction can be avoided by _____________
(a) decreasing the frequency of the ac supply
(b) adding high inductor in series with the armature of the dc motor
(c) adding a capacitor in series with the armature of the dc motor
(d) increasing the frequency of the ac supply

12.13 A three-phase full-wave controlled rectifier is connected to a separately excited dc motor and the machine has the following data:
$T_e = 150$ N-m, $\omega = 75$ rad/s and $I_a = 50$. What will be the back emf of the motor?
(a) 200 V (b) 225 V (c) 250 V (d) 275 V

12.14 If a separately excited dc motor is supplied from a single-phase full converter with firing angle of $\alpha = 60°$, it operates at a speed of 800 rpm. When the same motor is connected to 1-phase semi-converter with the same firing angle of $\alpha = 60°$, the motor runs at speed _____ rpm
(a) 800 (b) 1000 (c) 1200 (d) 1400

12.15 A three-phase full wave controlled rectifier is connected to a separately excited dc motor and the machine has the following data
$E_b = 150$ V, $\omega = 75$ rad/s and $I_a = 50$. What will be the electromagnetic torque of the motor?
(a) 50 N-m (b) 75 N-m (c) 100 N-m (d) 125 N-m

12.16 A single phase half wave converter with freewheeling diode drives a separately excited dc motor at 500 rpm with the firing angle $\alpha = 30°$. What will be the motor speed if single phase half wave converter is replaced by a full converter with $\alpha = 30°$, the motor speed would be
(a) 750 rpm (b) 828 rpm (c) 928 rpm (d) 1000 rpm

12.17 When a single-phase full converter with firing angle α drives a separately excited dc motor, the dc machine runs at a speed of 1200 rpm and load current is continuous. If one of the four thyristors gets open-circuited, the motor speed will be ________.
(a) 1200 rpm (b) 1000 rpm (c) 800 rpm (d) 600 rpm

12.18 A single-phase semi-converter drives a separately excited dc motor and the armature current is 25 A which is ripple free. If the firing angle of converter is $\alpha = 45°$, the rms values of freewheeling diode current and thyristor current would be ______ A, _______ A respectively.
(a) 12.5 A, 15.3 A (b) 15.3 A, 12.5 A (c) 12.5 A, 15 A (d) 12 A, 15.3 A

12.19 DC drives that use series motors are more advantages than drives that employ separately excited motors because they have ________
(a) high starting torque (b) frequent starting
(c) frequent torque overloads (a) low starting torque

12.20 A single-phase half-controlled rectifier drives a separately excited dc motor and the dc motor has back emf constant of 0.75 V/rpm. The armature current is 10 A which is ripple free and the armature resistance is 0.2 Ω. If the converter fed from 220 V, 50 Hz single-phase ac supply with a firing angle of $\alpha = 60°$, the speed of the motor will be
(a) 393 rpm (b) 390 rpm (c) 339 rpm (d) 330 rpm

12.21 A three-phase semi-converter feeds the armature of a separately excited dc motor and developing a non-zero torque in motor. Under steady-state operation, the motor armature current is found to drop to zero at certain instances of time. At these instances, the input voltage is
(a) equal to the instantaneous value of the ac phase voltage
(b) equal to the instantaneous value of the motor back emf
(c) zero
(d) greater than the instantaneous value of the motor back emf

12.22 Which of the following statements are correct for speed control of a squirrel cage induction motor using three phase bridge inverter?
(a) If frequency deceases, starting torque decreases with constant supply voltage
(b) If frequency deceases, starting torque increases with constant supply voltage
(c) If frequency increases, starting torque increases with constant supply voltage
(d) If frequency increases, starting torque remains constant with constant supply voltage

12.23 A delta connected induction motor is fed by a three-phase ac-to-dc inverter and operated in constant $\frac{V}{f}$ control mode during starting with a __________.
(a) star-delta starter (b) DOL starter (c) auto-transformer starter

12.24 The slip-power control scheme of a three-phase induction motor can able to provide the speed control range of ________.
(a) 0 to N_s (b) 0 to $2N_s$ (c) N_s to $-N_s$ (d) 0 to $-N_s$

12.25 A three-phase induction motor is used for speed control applications. It is drive from an inverter with a constant $\frac{V}{f}$ control. The motor name plate details are as follows:

$V = 415$ V, Phase = 3, $f = 50$ Hz, $N = 2845$ RPM

If the induction motor run with the inverter output frequency set at 45 Hz and with half the rated slip, the running speed of the motor is equal to______________
(a) 2010 rpm (b) 2003 rpm (c) 1998 rpm (d) 1885 rpm

12.26 The most accurate and adaptable method for improving reactive power compensation is
(a) satiable reactor with controlled reactor (b) saturable reactor with capacitor bank
(c) switched capacitors (d) fixed capacitor with thyristor controlled rector

12.27 Static VAR compensator is usually designed to operate at
(a) slightly lagging power factor (b) slightly leading power factor
(c) unity power factor (d) zero power factor lagging

12.28 Reactive power provided by a thyristor controlled reactor when $\alpha \le 90°$ is ______.
(a) $\frac{V^2}{\omega L}$ (b) $\frac{V}{\omega L}$ (c) ωLV^2 (d) ωLV

12.29 If the firing angle of a thyristor controlled reactor $\alpha > 90°$, the fundamental current through inductor is equal to

(a) $\frac{V}{\pi\omega L}[2(\pi - \alpha) + \sin\alpha]$ (b) $\frac{V}{\pi\omega L}[2(\pi + \alpha) + \sin 2\alpha]$

(c) $\frac{V}{\pi\omega L}[(\pi - \alpha) + \sin 2\alpha]$ (d) $\frac{V}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha]$

12.30 During $\frac{\pi}{2} < \alpha < \pi$, the current provided by SVC is equal to

(a) $V\left\{\omega C - \frac{1}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha]\right\}$ (b) $V\left\{\omega C - \frac{1}{\pi\omega L}[2(\pi + \alpha) + \sin 2\alpha]\right\}$

(c) $V\left\{\omega C - \frac{1}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha]\right\}$ (d) $V\left\{\omega C + \frac{1}{\pi\omega L}[2(\pi - \alpha) + \sin 2\alpha]\right\}$

12.31 Which of the following statements is correct for SVC?
(i) Reduce the source current (ii) reduce load reactive power
(iii) improves voltage profile (iv) improves supply power factor
(a) (i), (iii), (iv) (b) (ii) (c) (i), (ii) (d) (iii), (iv)

12.32 Which of the following statements of thyristor controlled reactor is correct?
(i) Maximum reactive power at firing angle $\alpha = 90°$
(ii) Maximum reactive power at firing angle $\alpha = 180°$
(iii) No reactive power at $\alpha = 0°$
(iv) Maximum reactive power at firing angle $\alpha = 30°$
(a) (i), (iv) (b) (ii) (c) (i), (ii) (d) (iii), (iv)

12.33 The effective inductance offered by a thyristor controlled inductor (TCR) is 50 mH. If firing angle is varied, the effective inductance will be changed.
(i) 50 mH for firing angle $\alpha = 60°$
(ii) 50 mH for firing angle $\alpha = 90°$
(iii) Less than 50 mH for firing angle $\alpha = 120°$
(iv) More than 20 mH for firing angle $\alpha = 120°$
(a) (i), (ii), (iv) (b) (ii) (c) (i), (ii) (d) (iii), (iv)

Fill in the Blanks

12.1 DC drives are widely used in ______________.
12.2 DC series motors are preferred to separately excited dc motor due to ________.
12.3 The variable dc voltage is provided to dc motor by using ________.
12.4 The normal drop in speed of dc motor from no load to full load is about __________.
12.5 The most commonly used speed control methods of dc are _______.
12.6 The armature control is convenient for ________ control.
12.7 When the armature voltage is kept constant at the base value and field current is varied for speed control, this method is known as ___________.
12.8 Single-phase converter fed drive can be able to implement constant ____ and constant _____ control operation.
12.9 The regenerative braking of a separately excited dc motor is possible with a _______ quadrant chopper.
12.10 The speed control of dc motor in smooth four-quadrant operation is possible using a _______ converter.
12.11 For the same HP rating, induction motors are ___________ lighter than dc motors.
12.12 If an induction motor has *P* number poles and operates at frequency *f*, the synchronous speed of induction motor is __________.

12.13 In a torque-speed characteristics of an induction motor, the complete range of slip is _______.
12.14 In a torque-speed characteristics of an induction motor, the range of slip for forward motoring is __________, for plugging is _______ and for regeneration is _______.
12.15 The most commonly used speed control methods of induction motor are _______ and ______.
12.16 AC voltage controller fed induction motor drive is suitable for _______ power applications.

Review Questions

12.1 How power electronics can be used in electric drives?
12.2 What are the different types of dc drives based on the input supply?
12.3 What is the purpose of a converter in dc drives?
12.4 Define the base speed of a dc motor.
12.5 What are the different parameters to be varied for speed control of a separately excited dc motor?
12.6 Why dc series motors are used in traction system?
12.7 Define speed regulation of a dc drive.
12.8 Draw the equivalent circuit of a separately excited dc motor and derive the mathematical model of machine. Sketch the characteristics of motor in constant torque and power region.
12.9 Draw the equivalent circuit of a dc series motor and derive the mathematical model of machine. Sketch the characteristics of motor in constant torque and power region.
12.10 Explain the working principle of single phase full converter fed dc drive with waveforms and appropriate equations.
12.11 Explain the working principle of single phase semi-converter fed dc drive with waveforms and appropriate equations.
12.12 What are the advantages and disadvantages of single-phase full converter fed dc motor drives?
12.13 What are the advantages and disadvantages of single phase semi converter fed dc motor drives?
12.14 Explain the working principle of three-phase half converter fed dc drive with waveforms and appropriate equations.
12.15 Explain the working principle of three-phase semi-converter fed dc drive with waveforms and appropriate equations.
12.16 What are the advantages and disadvantages of three-phase full converter fed dc motor drives?
12.17 What are the advantages and disadvantages of three-phase semi converter fed dc motor drives?
12.18 What are the advantages and disadvantages of three-phase dual converter fed dc motor drives?
12.19 What is first quadrant dc drive? What is two quadrant dc drive? What is four quadrant dc drive?
12.20 Discuss the working principle of chopper fed dc drive with waveforms and appropriate equations.
12.21 What is the working principle of regenerative braking of dc-dc converter fed dc motor drives?
12.22 Write short notes on the followings:
(a) Rheostatic braking (b) two quadrant chopper fed dc drive (c) four quadrant chopper fed dc drive
12.23 What is ac drive? What are the advantages and disadvantages of ac drive over dc drive?
12.24 Write the comparison between ac drive and dc drive.
12.25 What are the different types of induction motor?
12.26 Define synchronous speed, slip, slip frequency of induction motor.
12.27 Draw the torque-speed characteristics of induction motor. What are the different methods for speed control of induction motor? What are the advantages of $\frac{V}{f}$ control?
12.28 What is a static Kamer drive? What is a static Scherbius drive?
12.29 What is a field-weakening mode of induction motor? What is the effect of frequency control of induction motor?
12.30 What are the different types of synchronous motor?
12.31 Define torque angle of synchronous motor. What are the differences between salient-pole motors and reluctance motors?

12.32 What are the differences between salient-pole motors and permanent-magnet motors?
12.33 What is pull-out torque of synchronous motor?
12.34 What are the torque-speed characteristics of synchronous motor?
12.35 What are the V-curves of synchronous motor?
12.36 What is thyristor controlled reactor? Explain how the inductance of thyristor controlled reactor can be varied with firing angle α.
12.37 What is SVC? Describe how SVC regulates the reactive power flow and improves system power factor.
12.38 What is UPS? What are the types of UPS? Draw a block diagram of UPS system and explain its working principle. Give a list of industrial applications of UPS.
12.39 Assume that a separately excited dc motor is controlled by a single phase half wave controlled rectifier which is supplied from 220 V, 50 Hz ac supply. When the field circuit is fed through a single-phase semi-converter with 30° firing angle and the motor operates at 900 rpm with 10 N-m load, determine the following:
(a) firing angle of thyristor of a single phase half wave controlled rectifier
(b) rms value of source current, thyristor current and free wheeling diode current
(c) input power factor
Assume, armature resistance $R_a = 1\ \Omega$, field resistance $R_f = 180\ \Omega$ motor constant = 0.6 V/A rad/s and the current waveforms are ripple free.
12.40 A separately excited dc motor is controlled by a single phase semi-converter which is supplied from 230 V, 50 Hz ac supply. When the field circuit is fed through a single phase semi-converter with 10° firing angle and the motor operates at 1200 rpm with 22 N-m load, determine the following:
(a) Voltage across field winding
(b) Field current
(c) Armature current
(d) Delay angle of thyristor of a single-phase semi-converter
(e) rms value of source current, thyristor current and free wheeling diode current
(f) Input power factor
Assume, armature resistance $R_a = 0.5\ \Omega$, field resistance $R_f = 120\ \Omega$ motor constant = 0.7 V/A rad/s and the current waveforms are ripple free.
12.41 The speed of a 18 HP, 220 V, 1000 rpm dc series motor is controlled by a single-phase half-controlled bridge (semi-converter) which is supplied from 230 V, 50 Hz ac supply. The total armature and field winding resistance is $R_a + R_f = 0.55\ \Omega$. When the motor current is continuous and ripple free, and the motor operates at 1000 rpm with firing angle of $\alpha 1 = 30°$, determine the following:
(a) Motor current, (b) Motor torque
Assume, motor constant $k_b = 0.03$ V/A rad/s, $k_t = 0.03$ N-m/Amp^2 and the current waveforms are ripple free.
12.42 A separately excited dc motor is controlled by a single-phase full converter which is supplied from 400 V, 50 Hz ac supply. If the field circuit is fed through a single-phase full-converter with 0° firing angle, the delay angle of armature converter is 30° and load current is 25 A, determine the following:
(a) Voltage across field winding (b) Field current
(c) Electromagnetic torque and load torque (d) Voltage across armature winding
(e) Back emf (f) Motor speed
(g) Input power factor
Assume, armature resistance $R_a = 0.5\ \Omega$, field resistance $R_f = 140\ \Omega$, motor constant $k_b = 0.7$ V/A rad/s, $k_t = 0.03$ N-m/Amp^2 and the current waveforms are ripple free.
12.43 In Example 12.42, the polarity of induced emf is reversed by reversing the field excitation to its maximum value. Compute (a) firing angle of the field converter, (b) delay angle of armature converter at 1000 rpm to maintain armature current at 45 A and (c) the power fed back to the supply due to regenerative breaking of the motor.
12.44 A 220 V, 1500 rpm, 16 A separately excited dc motor is controlled by a single-phase full bridge controlled rectifier which is supplied from 220 V, 50 Hz ac supply. Assume that armature resistance $R_a = 1\ \Omega$, field resistance $R_f = 120\ \Omega$ load current is continuous and ripple free.
Determine (a) motor speed at the firing angle $\alpha = 45°$ and load torque of 12 N-m (b) torque developed at the firing angle $\alpha = 30°$ and speed 1000 rpm

12.45 A 210 V, 1000 rpm, 50 A separately excited dc motor is controlled by a single-phase full bridge controlled rectifier which is supplied from 230 V, 50 Hz ac supply. Assume that armature resistance $R_a = 0.2\ \Omega$ and load current is continuous and ripple free.

Find (a) the firing angle of armature converter for rated motor torque at speed 640 rpm, (b) the firing angle of armature converter for rated motor torque at speed - 600 rpm and (c) motor speed at $\alpha = 145°$ and half rated torque.

12.46 The speed of a 12 HP, 300 V, 1500 rpm separately excited dc motor is controlled by a three-phase half converter from a Y connected, 210 V, 50 Hz ac supply. The field current is controlled by a three-phase semi-converter and is set to its maximum value. Assume armature resistance $R_a = 0.1\ \Omega$, field resistance $R_f = 210\ \Omega$, and motor constant is 0.6 V/A-rad/s. Compute (a) the firing angle of armature converter at rated power and rated speed, (b) the no-load speed if the firing angle is same as (a) and no-load current is 12% of rated current and (c) the speed regulation.

12.47 A separately excited dc motor can be controlled by a three-phase semi-converter which is supplied by Y connected 220 V, 50 Hz ac supply. The dc motor has the following parameters:

Armature resistance $R_a = 0.5\ \Omega$, armature inductance $L_a = 5$ mH, machine constant 1.2 V/rad/s or N-m/A. Determine the speed of motor at 20 N-m load with firing angle $\alpha = 30°$.

12.48 A 12 HP, 300 V, 1000 rpm separately excited dc motor can be controlled by a three-phase semi-converter which is supplied by Y connected 220 V, 50 Hz ac supply. The dc motor armature resistance $R_a = 1.0\ \Omega$ and armature current is continuous and ripple free. If the motor operates at 900 rpm at firing angle $\alpha = 45°$, determine the rms value source and thyristor currents, average value of thyristor current and input power factor.

12.49 A dc motor is driven by a three phase full converter as shown in Fig. 12.66 and 100 A dc current flow through armature with negligible ripple.

(a) Draw the ac line current i_A for one cycle at $\alpha_1 = 45°$.

(b) Determine the third and fifth harmonic components of line current as a percentage of the fundamental current.

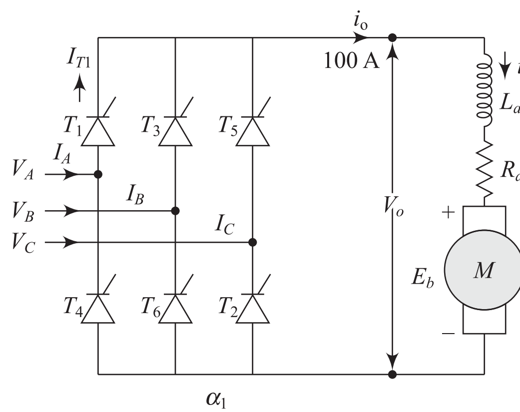


Fig. 12.66

12.50 In speed control of dc motor, the load torque is 42 N-m. Assume that under steady state condition the motor operates at speed 500 rpm and $t = 0$. At $t \geq 0^+$, the electromagnetic torque is suddenly increased to 100 N-m. The inertia of dc motor is 0.01 N-m.sec^2/rad.

(a) Write differential equation which govern the speed of motor for $t \geq 0$

(b) Compute the time taken to reach the speed 1000 rpm from 500 rpm

Assume the friction torque is negligible.

12.51 The speed of a 14 HP, 400 V, 1000 rpm separately excited dc motor is controlled by a three-phase full converter from a Y connected, 250 V, 50 Hz ac supply. Assume armature resistance $R_a = 0.5\ \Omega$, field resistance $R_f = 150\ \Omega$, and motor constant is 0.8 V/A-rad/s.

(a) The field current is controlled by a three-phase full converter and is set to its maximum value. If the motor develop 100 N-m torque at 1000 rpms, determine the firing angle of armature converter

(b) If the field current is set to its maximum value, firing angle of armature converter is 30° and developed torque is 100 N-m, find the operating speed of motor

12.52 220 V, 1445 rpm, 20 A separately excited dc motor is supplied from a three-phase full converter. When the full converter is connected to 440 V, 50 ac supply through a delta-star transformer, (a) determine the turn ratio of transformer and (b) find the value of firing angle of armature converter when (i) motor rotates at 1200 rpm with rated torque and (ii) motor operates at -1000 rpm with half rated torque.

Assume armature resistance is 0.5 Ω and motor terminal voltage is rated voltage when converter firing angle is zero.

12.53 The speed of separately excited dc motor is controlled by a chopper. The input voltage of chopper is 220 V, armature resistance $R_a = 0.3\ \Omega$ and motor constant $K = 0.08$ V/rpm. When the motor drives a constant load, average armature current of 25 A flows through armature winding. Determine (a) the range of speed control and (b) the range of duty cycle D. Assume motor current is ripple free.

12.54 A dc series motor is driven by a chopper. The motor has the following parameters:

$$R_a = 0.15\ \Omega,\ R_f = 0.25\ \Omega,\ K = 0.004\ \text{N-m/Amp}^2$$

If the dc supply voltage of chopper is 400 V, duty cycle of chopper is 50% and the average armature current is 100 A, determine (a) input power from supply, (b) motor speed and (c) motor torque. Assume current is ripple free.

12.55 A chopper circuit is used to control a separately excited dc motor. The chopper is ON for 10 ms and OFF for 18 ms. When the chopper is supplied by 200 V dc supply, determine the average load current when the motor operates in 1200 rpm and machine constant $k = 0.5$ V/rad per sec. The armature resistance is 1 Ω. Assume armature current is continuous and ripple free.

12.56 The speed of a separately excited dc motor is controlled by a chopper circuit. Assume that the chopper is supplied by 250 V dc supply and it operates at 200 Hz. The motor has the following parameters:

$$R_a = 0.2\ \Omega,\ L_a = 25\ \text{mH},\ k = 1.5\ \text{V/rad per sec or N-m/A}$$

When the motor operates at 1200 rpm with load torque 30 N-m, determine
(a) The minimum and maximum values of armature current, (b) Ripple current and (c) Derive the expression of armature current during on and off period of chopper.

12.57 The speed of a separately excited dc motor is controlled by a chopper circuit. Assume that the chopper is supplied by 220 V dc supply and it operates at 100 Hz. The motor has the following parameters:

$$R_a = 0\ \Omega,\ L_a = 30\ \text{mH},\ k = 1.2\ \text{V/rad per sec or N-m/A}$$

When the motor operates at 1000 rpm with load torque 24 N-m, determine (a) the minimum and maximum values of armature current, (b) ripple current, (c) derive the expression of armature current during on and off period of chopper.

12.58 Regenerative braking is applied to a separately excited dc motor using a dc chopper. The chopper circuit is supplied from 250 V dc supply. The average armature current during regenerative braking is kept constant 200 A with negligible ripple. The machine has the following parameters:

$$R_a = 0.2\ \Omega,\ k = 1.2\ \text{V/rad per sec}$$

If the duty cycle of chopper is 50%, determine (a) power feedback to dc supply, (b) minimum and maximum permissible braking speeds and the speed range of regenerative braking and (c) speed during regenerative braking.

12.59 A dc chopper is used for rheostatic braking of a separately excited dc motor. The machine has the following parameters:
Armature resistance $R_a = 0.15\ \Omega$, braking resistance $R_b = 5\ \Omega$ and $k_b = 1.2$ V/A rad per sec.
The average armature current during braking is kept constant at 100 A with negligible ripple. The field current is 1.5 A.
If the duty cycle of chopper is 60%, determine (a) average current through braking resistance, (b) average voltage across dc converter, (c) equivalent load resistance of the generator, (d) power dissipated across braking resistance and (e) speed during braking.

12.60 A separately excited dc motor is operated by a single-phase semi-converter which is fed from 220 V, 50 Hz ac supply. The motor parameters are:

$$r_a = 1.75\ \Omega,\ L_a = 0.05\ \text{H},\ k_m = 1\ \text{V/rad/sec and}\ k_T = 1\ \text{Nm/A}$$

If the firing angle of semi-converter is 60°, the motor runs at 1400 rpm.
(a) Find the expression of armature current. (b) Draw waveforms of output voltage and armature current.
(c) Determine the average motor torque.

12.61 A three-phase 400 V, 50 Hz, four pole, star connected induction motor has the following parameters

$$r_1 = 0.2\ \Omega,\ x_1 = 0.4\ \Omega,\ r_2' = 0.2\ \Omega,\ x_2' = 0.4\ \Omega\ \text{and}\ X_m = 20\ \Omega$$

If the no load loss of induction motor is 50 W and motor rotates ate 1440 rpm, determine the following: (a) Synchronous speed in rpm and rad/s, (b) slip, (c) input current, (d) input power factor, (e) input power, (f) air gap power and (g) motor efficiency.

12.62 A three-ϕ 50 Hz, eight pole, 420 V, 725 rpm Y connected induction motor has the following parameters:

$$r_1 = 0.25\ \Omega,\ x_1 = 0.75\ \Omega,\ r_2' = 0.2\ \Omega,\ x_2' = 0.75\ \Omega, \text{ and } X_m = 25\ \Omega$$

When the motor is driving a load torque $T_L = 0.015\omega_m^2$, determine slip, full load rotor current, power factor at full load and load torque.

12.63 A three-phase induction motor drives a blower where load torque is directly proportional to speed squared. If the motor operates at 1460 rpm, determine the maximum current in terms of rated current.

12.64 A three-phase 50 Hz, four pole, 42 V, 1425 rpm, 20 kW, Y connected induction motor has the following parameters:

$$r_2' = 0.30\ \Omega \text{ and } x_2' = 1.1\ \Omega.$$

The value of magnetizing reactance X_m and stator leakage impedance and rotational losses are neglected. When the motor is operated by three phase 440 V, 100 Hz supply, determine (a) the motor speed at rated load, (b) slip at maximum torque and (c) the maximum torque.

12.65 A three-phase 50 Hz, four pole, 400, 10 kW delta connected induction motor has the following parameters:

$$r_1 = 0.5\ \Omega,\ x_1 = 1.2\ \Omega,\ r_2' = 0.30\ \Omega, \text{ and } x_2' = 1.2\ \Omega,$$

The value of magnetizing reactance X_m is neglected.

(a) If the motor is started with DOL starting, determine starting current, power factor at starting, maximum torque.

(b) When the motor is operated by 220 V, 25 Hz supply and is started with DOL starting, determine starting current, power factor at starting, maximum torque.

12.66 A three-phase induction motor which is derived by an inverter has the following parameters:

$$r_1 = 0.45\ \Omega,\ x_1 = 1.2\ \Omega,\ r_2' = 0.30\ \Omega,\ x_2' = 1.2\ \Omega \text{ and } X_m = 30\ \Omega$$

(a) Find the value of stator current input and full load torque at slip 0.035 if the inverter output voltage is 440 V, 50 Hz

(b) When the inverter output voltage is changed to 400 V, 45 Hz, determine the value of stator current input and full load torque at the instant of voltage change.

Answers to Multiple-Choice Questions

12.1	(d)	12.2	(a)	12.3	(c)	12.4	(a)	12.5	(c)	12.6	(c) & (d)	12.7	(a)
12.8	(a)	12.9	(c)	12.10	(a)	12.11	(a)	12.12	(b)	12.13	(b)	12.14	(c)
12.15	(c)	12.16	(c)	12.17	(d)	12.18	(a)	12.19	(a)	12.20	(a)	12.21	(b)
12.22	(b)	12.23	(b)	12.24	(b)	12.25	(b)	12.26	(d)	12.27	(a)	12.28	(a)
12.29	(d)	12.30	(a)	12.31	(a)	12.32	(a)	12.33	(a)				

Answers to Fill in the Blanks

12.1 Electric traction, machine tools

12.2 High starting torque

12.3 Controlled rectifier and chopper

12.4 5%

12.5 Armature control and field control

12.6 Constant torque

12.7 field weakening

12.8 torque, power

12.9 second

12.10 dual

12.11 20%-40%

12.12 $N_s = \dfrac{120f}{P}$

12.13 $-1 \le s \le 2$

12.14 $0 \le s \le 1,\ s > 1,\ s < 0$

12.15 stator voltage control, $\dfrac{V}{f}$ control

12.16 low