TURBOMACHINERY

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In memory of our teachers and mentors at
IIT Kharagpur

Late Prof. AK Mohanty

Late Prof. PK Nag

Preface

It is a proven fact that the invention of fire and then wheel changed the life of human being to a great extent. In this series, the first use of turbomachines had been the use of water wheels between third and first century B.C., for irrigation, grinding flour and the like. First real modern turbomachine as a power source did not appear until the industrial revolution in the late 1880s. Further developments in the field had tremendously contributed to the growth of civilization and well being of mankind. It was quite a challenging and thrilling task to write a textbook on this classic subject area that has diverse applications in daily life from power generation, water transportation, and use of fans to aviation.

This textbook is written to provide a single treatise on turbomachines to cater to the needs of the undergraduate and first year postgraduate students of engineering discipline. The literature on the subject is voluminous and scattered. Most of the books available on the subject are on a specific topic such as pumps, compressors, gas turbines, hydraulic turbines, etc. The ones that attempt to unify all topics require the students to acquire adequate background from several other subjects as a prerequisite. This text is written with the intention to provide handy material on the subject with useful concepts and motivate students to move to higher levels in the turbomachines field. Towards the end, care has been taken in this text to provide simple basics of subjects like thermodynamics and fluid mechanics wherever required and not depend too much on a prior knowledge.

This book of ten chapters has two objectives. The first is to provide the fundamental treatment to a general turbomachine applying basic principles of fluid dynamics and thermodynamics of flow through passages and over surfaces with one-dimensional treatment using control volume approach. The second objective is to apply these principles to the specific machines of either constant or variable density and to find major performance parameters and characteristics. Attempts have been made to obtain a balance between understanding of fundamentals and acquiring knowledge of the practical aspects for each of the machines. However, in order to achieve the balance, focus has not deviated from fundamental understanding and developing logical reasoning in readers. In the words of Leonardo da Vinci, "*He who loves practice, without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast*."

Content presentation supports outcome based learning and module-based approach. *Chapter 1* on fundamentals along with any of the remaining chapters constitutes a separate module. Main emphasis in *Chapter 2* is on the model testing of turbomachines based on affinity laws of dimensional analysis. For the readers, the module containing *Chapters 1 and 2* is a necessity before proceeding to any of the subsequent chapters. *Chapters 3 to 6* are for incompressible flow turbomachines. Contents on cavitation are presented separately in *Chapter 5,* considering its practical importance. *Chapters 7 to 9* are for compressible flow machines. *Chapter 10* on **Fluid Systems** is included to meet the course requirements of some of the universities.

Underlying principles, performance parameters and characteristics are the common features of all the machines presented from *Chapters 3 to 10*. Solved examples are given to develop the understanding of the students using analytical means and/or basic engineering practices as they progress through each section of a chapter. A Unique feature of this text is the brainstorming multiple choice questions for the preparation of competitive examinations like GATE, ESE, PSUs etc.

Additionally, the book is accompanied with supplementary learning material, accessible on McGraw Hill Education Online Learning Centre through the following link:

http://www.mhhe.com/dubey/turbomachinery

It contains the following learning resources:

For Students

- Chapter Summary Flow Charts
- Test bank (contains questions from University papers as well)

For Instructors:

- Solutions Manual
- Lecture PPTs

We would welcome and appreciate criticism and suggestions by readers for further improvement of the book, which will be gratefully acknowledged.

Maneesh Dubey

BVSSS Prasad

Archana Nema

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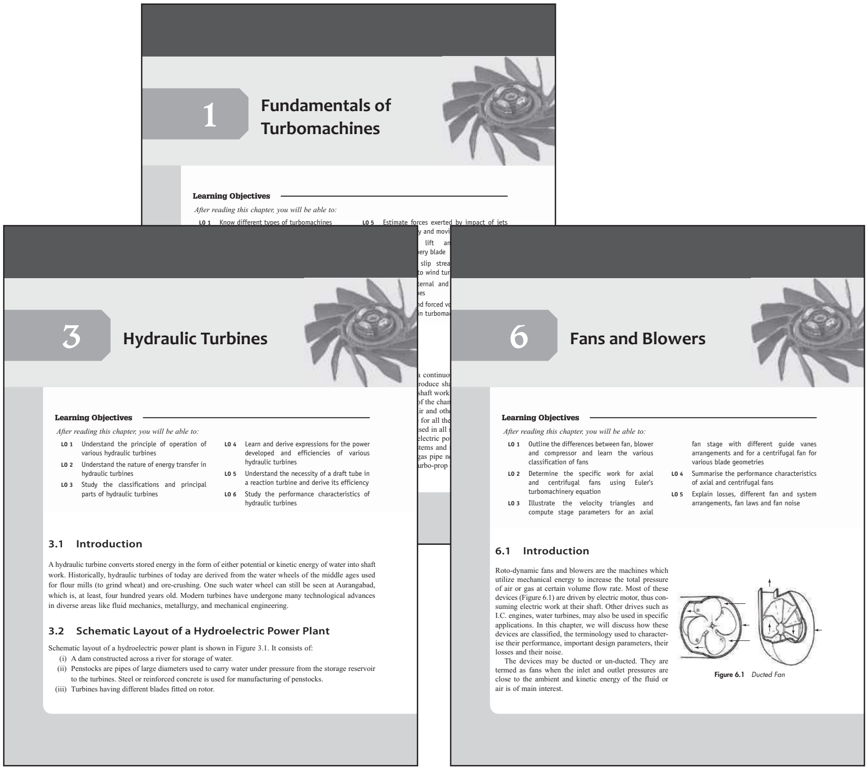
FEATURES OF


1 Fundamentals of Turbomachines

Learning Objectives

3 Hydraulic Turbines

Learning Objectives

After reading this chapter, you will be able to:

- LO 1 Understand the principle of operation of various hydraulic turbines
- LO 2 Understand the nature of energy transfer in hydraulic turbines
- LO 3 Study the classifications and principal parts of hydraulic turbines
- LO 4 Learn and derive expressions for the power developed and efficiencies of various hydraulic turbines
- LO 5 Understand the necessity of a draft tube in a reaction turbine and derive its efficiency
- LO 6 Study the performance characteristics of hydraulic turbines

3.1 Introduction

A hydraulic turbine converts stored energy in the form of either potential or kinetic energy of water into shaft work. Historically, hydraulic turbines of today are derived from the water wheels of the middle ages used for flour mills (to grind wheat) and ore-crushing. One such water wheel can still be seen at Aurangabad, which is, at least, four hundred years old. Modern turbines have undergone many technological advances in diverse areas like fluid mechanics, metallurgy, and mechanical engineering.

3.2 Schematic Layout of a Hydroelectric Power Plant

Schematic layout of a hydroelectric power plant is shown in Figure 3.1. It consists of:

(i) A dam constructed across a river for storage of water.
(ii) Penstocks are pipes of large diameters used to carry water under pressure from the storage reservoir to the turbines. Steel or reinforced concrete is used for manufacturing of penstocks.
(iii) Turbines having different blades fitted on rotor.

6 Fans and Blowers

Learning Objectives

After reading this chapter, you will be able to:

- LO 1 Outline the differences between fan, blower and compressor and learn the various classification of fans
- LO 2 Determine the specific work for axial and centrifugal fans using Euler's turbomachinery equation
- LO 3 Illustrate the velocity triangles and compute stage parameters for an axial fan stage with different guide vanes arrangements and for a centrifugal fan for various blade geometries
- LO 4 Summarise the performance characteristics of axial and centrifugal fans
- LO 5 Explain losses, different fan and system arrangements, fan laws and fan noise

6.1 Introduction

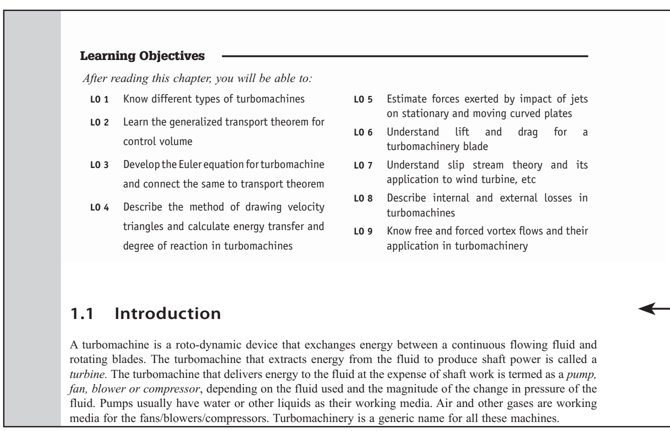
Roto-dynamic fans and blowers are the machines which utilize mechanical energy to increase the total pressure of air or gas at certain volume flow rate. Most of these devices (Figure 6.1) are driven by electric motor, thus consuming electric work at their shaft. Other drives such as I.C. engines, water turbines, may also be used in specific applications. In this chapter, we will discuss how these devices are classified, the terminology used to characterise their performance, important design parameters, their losses and their noise.

The devices may be ducted or un-ducted. They are termed as fans when the inlet and outlet pressures are close to the ambient and kinetic energy of the fluid or air is of main interest.

Figure 6.1 Ducted Fan

1. Module-based approach

Chapters are written to form modules when clubbed with the first chapter. For example, Chapters 1 & 3 form a module on Hydraulic Turbines; similarly, Chapters 1 & 6 form a module on Fans & Blowers. Hence, it offers utility to all including students, teachers and professionals!


Learning Objectives

After reading this chapter, you will be able to:

- LO 1 Know different types of turbomachines
- LO 2 Learn the generalized transport theorem for control volume
- LO 3 Develop the Euler equation for turbomachine and connect the same to transport theorem
- LO 4 Describe the method of drawing velocity triangles and calculate energy transfer and degree of reaction in turbomachines
- LO 5 Estimate forces exerted by impact of jets on stationary and moving curved plates
- LO 6 Understand lift and drag for a turbomachinery blade
- LO 7 Understand slip stream theory and its application to wind turbine, etc
- LO 8 Describe internal and external losses in turbomachines
- LO 9 Know free and forced vortex flows and their application in turbomachinery

1.1 Introduction

A turbomachine is a roto-dynamic device that exchanges energy between a continuous flowing fluid and rotating blades. The turbomachine that extracts energy from the fluid to produce shaft power is called a *turbine.* The turbomachine that delivers energy to the fluid at the expense of shaft work is termed as a *pump, fan, blower or compressor*, depending on the fluid used and the magnitude of the change in pressure of the fluid. Pumps usually have water or other liquids as their working media. Air and other gases are working media for the fans/blowers/compressors. Turbomachinery is a generic name for all these machines.

2. Outcome-based Learning

All chapters begin with Learning Objectives based on Bloom's Taxonomy, highlighting the learning outcome of the content covered.

THE BOOK

10 Fluid Systems

Learning Objectives

After reading this chapter, you will be able t

- L01 Know the significance, classifications applications of fluid systems
- L02 Explain the role of turbomachinery in systems
- L03 Outline the differences between pos displacement machines and turbomachi and comparative study between them some common basis
- L04 Explain the basic principle, work performance parameters of hydros fluid systems

10.1 Introduction

Power transmission takes place mainly in three sion takes place through shafts, gears, chains, b transformers etc. are used in electrical power tr a specific space are used for transmission of po compressible fluids is called a *fluid power tran* components in which either force or power are and then transmit when required or increase the and then transmit as and when required. A prim increases the pressure of the fluid. The pressuri actuator to perform the assigned task. A fluid sy (air/gas) or hydraulic (liquid). Extensive applic tary operations, health and recreation to name a

5.2 Cavitation in Turbines

Cavitation starts when the pressure drops too low a value. Therefore, occurrence of cavitation is more susceptible at points where the velocity or the elevation is high, particularly at those points where high velocity and high elevation are combined.

5.2.1 Cavitation in Pelton Turbine

Cavitation is not a matter of serious concern in Pelton turbines since it does not impose limits to operation. We know that Pelton turbines work under atmospheric pressure. The pressures in the system are either atmospheric as in the case of the wheel or above atmospheric as in closed conduits like the nozzle and manifold. Hence, cavitation due to negative ambient pressure does not arise. However, the velocities involved are very high and possibilities of local vacuum pressures and consequently, cavitation is possible even in Pelton turbines.

5.2.2 Cavitation in Reaction Turbines

As already discussed, the outlet of the runner, i.e. entry of the draft tube, is usually more susceptible to cavitation. Application of Bernoulli's Eq. for the flow between inlet of the draft tube and the fina pressure is atmospher the velocity at the ou small, results in,

where p_2 and C_2 repr at the outlet of the ru is the height of the t denoted by h_f. Equati ner, C_2, should be as p_2, therefore, the cha

9.13 Design of Multistage Turbines

The design of a steam turbine involves a judicious combination of theory with the results of experience, governed to a great extent by cost. The following outlined method of design is only illustrative of the theories discussed before. The following are specified to the designer: initial steam conditions, exhaust pressure, and the capacity in MW or kW. The turbine requires many stages with increase in diameter from the inlet to the exit end. All wheels turn at the same speed (rpm), but C_b, C_1, k_b, α, β, γ, leakage efficiency, disc friction and windage loss may all vary from stage to stage. The condition line, which is the logical starting point, can only be approximated until all the stage efficiencies are known.

The calculation for the casing arrangement of a multistage impulse turbine is made according to the Section 9.10. The first stage is most often a two-row Curtis stage. In order to increase the height of the nozzles, the stage is usually given a partial admission. In large condensing steam turbines, where the specific volume at the end of expansion in the turbine becomes very large, long blades of special design are selected. The design of multistage turbines is usually started with initial design considerations of first, second and last stages, whi

(c) Height of Installation

It is always advisable to install the turbines as high above the tailrace as possible. This saves cost of excavation for the draft tube. Care should be taken to ensure that cavitation does not occur.

3. Coverage

One-stop solution to all curricula requirements – dedicated chapter on Fluid Systems, which is generally a part of 'Fluid Mechanics' titles. Also, the text covers topics with industrial applications such as Cavitation, Pumps and Turbines Designs, Installation of Turbines etc.

(b) Maximum Height of Installation

Maximum permissible draft height at the plant

$$Z_2 = (Z_2)_{max} - M \quad (3)$$
$$Z_2 = 1.82 - 0.5$$
$$Z_2 = 1.32 \text{ m} \quad (4)$$

EXAMPLE 5.2 A turbine with $\sigma_c = 0.1$ is to be installed at a location where the barometric pressure is 1 bar, the summer temperature 40°C, and the net head available is 50 m. Calculate the maximum permissible height of the turbine rotor above the tailrace.

Solution

Given: $\sigma_c = 0.1$, $p_a = 1$ bar, $T_a = 40$°C, $H = 50$ m

From steam table, at 40°C, $p_v = 0.07375$ bar. σ must at least be equal to σ_c so as to avoid cavitation. The maximum permissible height of the turbine above the tailrace, i.e. the maximum draft head for a turbine setting can be obtained by,

$$(Z_2)_{max} = p_a/\rho g - p_v/\rho g - \sigma_c H \quad (1)$$
$$(Z_2)_{max} = \frac{1\times10^5}{1000\times9.81} - \frac{0.07375\times10^5}{1000\times9.81} - 0.1\times50$$
$$(Z_2)_{max} = 4.44 \text{ m} \quad (2)$$

EXAMPLE 5.3 A Francis turbine running at 120 rpm produces 11.76 MW while operating under a head of 25 m. The atmospheric pressure is 10 m of water at the site of installation of the turbine and the vapour pressure is 0.20 m of water. Calculate the maximum height of straight draft tube for the turbine.

Solution

Given: $N = 120$ rpm, $P = 11.76$ MW = 11760 kW, $H = 25$ m, $H_a = 10$ m, $H_v = 0.20$

We know that specific speed of a turbine is given by,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \quad (1)$$
$$N_s = \frac{120\times\sqrt{11760}}{25^{5/4}} \Rightarrow N_s = 232.8 \quad (2)$$

Critical Thoma's cavitation parameter for a Francis runner is given by,

$$\sigma_c = 0.044\left(\frac{N_s}{100}\right)^2 \quad (3)$$
$$\sigma_c = 0.044\left(\frac{232.8}{100}\right)^2$$

4. Solved Examples

Ample number of examples with solutions presented as per relevant topics.

FEATURES OF

Summary

Chapter-end Summary for a quick and precise recapitulation of the topics covered

SUMMARY

✦ In a general pumping system, the head between the sump level (from where the liquid is lifted) to the tank level (to where the liquid is lifted) is known as the static head, H_{st}. Various heads and expressions denoting the heads for a general pumping system are summarized in the following table.

Variable	*Expression*
Static head, H_{st}	$h_s + h_d$: suction head + delivery head
Suction head, h_s	Head developed in the suction line, the difference in the fluid energy between the sump level and the centerline of the pump.
Delivery head, h_d	Head developed in the delivery line, the difference in the fluid energy between the tank level (to where the liquid is lifted) and the center line of the pump.
Manometric head	Total head developed by the pump, the difference in the fluid energy between the outlet and inlet of the pump. $H_m = H_2 - H_1 = \left(\frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2\right) - \left(\frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1\right)$ $H_m = H_{st} + \sum$Losses in the pumping section. This is also referred simply as 'pump head' H.
Euler head or Theoretical head, H_e	$H_e = \frac{1}{g}(C_{w2}C_{b2} - C_{w1}C_{b1})$; gH_e is specific work and $\dot{m}\, gH_e$ is the the theoretical power of the pump either for a centrifugal pump, or for an axial pump, the inlet whirl component is generally negligible. In that case, $H_e = \frac{1}{g}C_{w2}C_{b2}$

✦ Theoretical fluid power developed by pump can be divided into three components

$$\dot{m}gH_e = \dot{m}\left(\frac{C_2^2 - C_1^2}{2} + \frac{C_{r1}^2 - C_{r2}^2}{2} + \frac{C_{b2}^2 - C_{b1}^2}{2}\right)$$

where, the first term is the specific kinetic energy difference of fluid (between outlet and inlet). The second term is the specific relative energy of fluid (between outlet and inlet). The third term is the centrifugal energy of fluid since $C_b^2 = r^2\omega^2$ (between outlet and inlet).

REVIEW QUESTIONS

8.1 State the assumptions made in the analysis of ideal Joule-Brayton (JB) cycle for gas turbine.
8.2 Draw the schematic $p - v$ and $T - s$ diagrams of simple Joule-Brayton cycle of gas turbine and briefly explain its working.
8.3 Derive an expression for specific work output and efficiency of simple gas turbine cycle in terms of pressure ratio and temperature ratio.
8.4 Derive an expression for optimum pressure ratio for maximum work output from an ideal Joule-Brayton cycle in terms of ratio of maximum cycle temperature to minimum cycle temperature and ratio of specific heats.
8.5 Show that the specific work output is maximum when the pressure ratio is such that the exit temperature of compressor is equal to the exit temperature of turbine.
8.6 How the actual Joule-Brayton cycle differs from the ideal Joule-Brayton cycle of a gas turbine?
8.7 Prove that the specific work output of actual Joule-Brayton gas turbine cycle is given by,

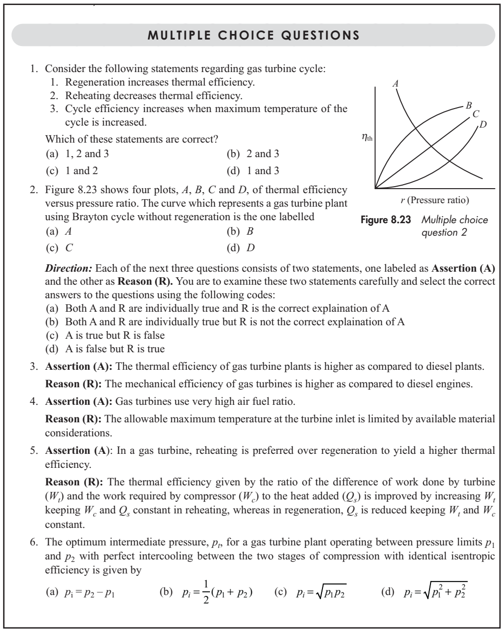
PROBLEMS

8.1 An ideal gas turbine cycle is working between the temperature limits of 350 K and 2000 K. The pressure ratio of the cycle is 1.3. The ambient pressure is 1 bar and air flow rate through the plant is 14400 m^3/min. Calculate the cycle efficiency. Take c_p = 1.005 kJ/kg – K.
[*Ans:* η = 7.23%, $\eta = f(r)$, $\eta \neq f(\theta)$]
8.2 The work ratio of an ideal Joule-Brayton cycle is 0.56 and efficiency is 35%. The temperature of the air at compressor inlet is 290 K. Determine (a) the pressure ratio, and (b) temperature drop across the turbine. [*Ans:* (a) r = 4.52, (b) $(\Delta T)_t$ = 356 K or °C]
8.3 An ideal Joule-Brayton gas turbine cycle is working between the temperature limits of 300 K and 1050 K. Determine (a) the pressure ratio of the cycle if its efficiency is equivalent to Carnot cycle efficiency, (b) optimum pressure ratio for maximum work output, (c) the cycle efficiency corresponding to maximum work, and (d) maximum specific work output.
[*Ans:* (a) $(r)_{\text{Carnot eff}}$ =80.2, (b) r_{opt} = 8.94 (c) $\eta_{\text{max work}}$ = 46.52%, (d) w = 228.64 kJ/kg]
8.4 An ideal Joule Brayton gas turbine cycle having pressure ratio of 7.5 is working between the temperature limits of 27°C and 727°C. The pressure at the inlet of compressor is 1 bar and the flow rate of air is 8.5 m^3/s. Calculate (a) the power developed, (b) cycle efficiency, and (c) the change in the work ouput and cycle efficiency in percentage, if perfect intercooling is used.
[*Ans:* (a) P = 1895.5 kW, (b) η = 43.8%, (c) Change in power = +18.6%, Change in Efficiency = –8.68%]

Problems and Review Questions

- **Problems:** Chapter-end exercise problems for practice, with answers
- **Review Questions:** Given at the end of chapter to assess clarity of concepts

THE BOOK



MULTIPLE CHOICE QUESTIONS

1. Consider the following statements regarding gas turbine cycle:
   1. Regeneration increases thermal efficiency.
   2. Reheating decreases thermal efficiency.
   3. Cycle efficiency increases when maximum temperature of the cycle is increased.

   Which of these statements are correct?
   (a) 1, 2 and 3 (b) 2 and 3
   (c) 1 and 2 (d) 1 and 3
2. Figure 8.23 shows four plots, *A*, *B*, *C* and *D*, of thermal efficiency versus pressure ratio. The curve which represents a gas turbine plant using Brayton cycle without regeneration is the one labelled
   (a) *A* (b) *B*
   (c) *C* (d) *D*

η_{th} — r (Pressure ratio) — A, B, C, D

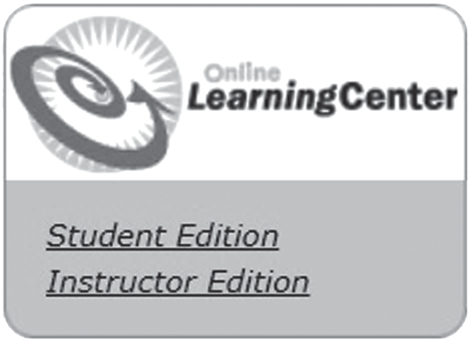
Figure 8.23 *Multiple choice question 2*

Direction: Each of the next three questions consists of two statements, one labeled as **Assertion (A)** and the other as **Reason (R).** You are to examine these two statements carefully and select the correct answers to the questions using the following codes:
(a) Both A and R are individually true and R is the correct explaination of A
(b) Both A and R are individually true but R is not the correct explaination of A
(c) A is true but R is false
(d) A is false but R is true

3. **Assertion (A):** The thermal efficiency of gas turbine plants is higher as compared to diesel plants.
   Reason (R): The mechanical efficiency of gas turbines is higher as compared to diesel engines.
4. **Assertion (A):** Gas turbines use very high air fuel ratio.
   Reason (R): The allowable maximum temperature at the turbine inlet is limited by available material considerations.
5. **Assertion (A)**: In a gas turbine, reheating is preferred over regeneration to yield a higher thermal efficiency.
   Reason (R): The thermal efficiency given by the ratio of the difference of work done by turbine (W_t) and the work required by compressor (W_c) to the heat added (Q_s) is improved by increasing W_t keeping W_c and Q_s constant in reheating, whereas in regeneration, Q_s is reduced keeping W_t and W_c constant.
6. The optimum intermediate pressure, p_i, for a gas turbine plant operating between pressure limits p_1 and p_2 with perfect intercooling between the two stages of compression with identical isentropic efficiency is given by
   (a) $p_i = p_2 - p_1$ (b) $p_i = \frac{1}{2}(p_1 + p_2)$ (c) $p_i = \sqrt{p_1 p_2}$ (d) $p_i = \sqrt{p_1^2 + p_2^2}$

Multiple Choice Questions

500+ Objective-type questions picked from previous years' GATE, IES and Public Sector Undertaking entrance examinations



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* For any queries/feedback related to OLC, please write to us at support.india@mheducation.com

Contents

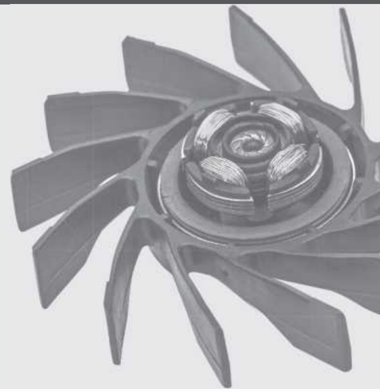
Nomenclature

Symbols			
t	Time/tip/thickness	N	Extensive property/speed
$\dot{m}$	Mass flow rate	ρ	Density/velocity ratio
V	Volume	C	Velocity/Coefficient
A	Cross sectional/flow area	η	Efficiency/Intensive property
r	Radius/pressure ratio	B	Width
F	Force or Thrust	p	Pressure/number of poles/pitch
g	Acceleration due to gravity	Z	Datum head, i.e. height from a reference
T	Temperature/Torque	R	Reaction
e	Specific energy	E	Total energy
$\dot{Q}$	Heat transfer rate	$\dot{W}$	Work transfer rate
s	Entropy	u	Specific internal energy
v	Specific volume	h	Specific enthalpy
f	Friction factor/frequency	c	Specific heat
γ	Ratio of specific heats/specific weight	M	Mach number/moment of momentum/ margin
w	Specific work	P	Power
H	Head	I	Rothalpy
α	Absolute flow angle	β	Relative flow angle
ω	Angular velocity	z	Number of blades
s	Slip factor/Thoma's cavitation parameter	R	Degree of reaction
l	Length	D	Diameter
m	Number of primary dimensions/jet ratio	μ	Viscosity
Q	Discharge or volume flow rate	a	Velocity of sound/cross sectional area of jet
R	Characteristic gas constant	k	Blade friction coefficient
φ	Flow coefficient	Ψ	Stage pressure coefficient/blade loading coefficient or temperature drop coefficient

λ	Power coefficient	θ	Temperature ratio/angle of deflection
q	Heat transfer per kg	ε	Heat exchanger effectiveness
x	Fraction of the total arc of nozzle/ dryness fraction	o	Minimum opening of flow
W	Weight/work	L	Length of stroke/length
n	Number of stages/number of strokes	S	Slip
N_{sh}	Non dimensional specific speed		
Subscripts			
0	Stagnation, no load		
1	Inlet	2	Outlet
t	Tangential/tip/turbine	h	Hub
s	Isentropic/specific/stage/static/suction/ shaft/system/slip	CV	Control volume
f	Flow/fan/frictional	B	Body
S	Surface/Supplied	i	Internal
e	Euler/external/exit	o	outer/overall
w	Whirl/water/wasted	b	Blade or vane
r	Relative/ratio/runaway	rw	Relative whirl
th	Theoretical/ideal	a	Axial/actual/atmospheric/air
P	Power	H	Head
Q	Flow or capacity or discharge	c	Critical/compressor/casing/circulation/ coupling
v	Volumetric/vapour	mano	Manometric
h	Hydraulic	m	Mechanical/model/manometric
o	Overall	tt	Total-to-total
ts	Total to static	ss	Static-to-static
p	Polytropic/pump/prototype/pressure end/constant pressure		
u	Unit	g	Gross
n	Nozzle	sn	Nozzle setting
3	Draft tube exit	fr	Friction in runner
sy	Synchronous	v	Velocity
ln	Losses in the nozzle	lb	Losses in the blades or buckets
d	Delivery/draft/drive/discharge/diffuser/ diffusion	le	Losses at exit
max	Maximum	min	Minimum
D	Diagram or blading/Drag	in	Entry/inlet

L	Lift	*q*	Change from normal discharge
l	Losses/leakage	*I*	First
II	Second	opt	Optimum
R	Rejected	*fb*	Fixed blades
mb	Moving blades	*co*	Carry over
nb	Nozzle and Blade	*tn*	Nozzle thickness
tb	Blade thickness	*T*	Torque convertor/torque
Abbreviations			
NPSHA	Net positive suction head available	*NPSHR*	Net positive suction head required
WG	Water gauge	R_e	Reynolds number
RF	Reheat factor		

1 Fundamentals of Turbomachines



Learning Objectives

After reading this chapter, you will be able to:

- **LO 1** Know different types of turbomachines
- **LO 2** Learn the generalized transport theorem for control volume
- **LO 3** Develop the Euler equation for turbomachine and connect the same to transport theorem
- **LO 4** Describe the method of drawing velocity triangles and calculate energy transfer and degree of reaction in turbomachines
- **LO 5** Estimate forces exerted by impact of jets on stationary and moving curved plates
- **LO 6** Understand lift and drag for a turbomachinery blade
- **LO 7** Understand slip stream theory and its application to wind turbine, etc
- **LO 8** Describe internal and external losses in turbomachines
- **LO 9** Know free and forced vortex flows and their application in turbomachinery

1.1 Introduction

A turbomachine is a roto-dynamic device that exchanges energy between a continuous flowing fluid and rotating blades. The turbomachine that extracts energy from the fluid to produce shaft power is called a *turbine.* The turbomachine that delivers energy to the fluid at the expense of shaft work is termed as a *pump, fan, blower or compressor*, depending on the fluid used and the magnitude of the change in pressure of the fluid. Pumps usually have water or other liquids as their working media. Air and other gases are working media for the fans/blowers/compressors. Turbomachinery is a generic name for all these machines.

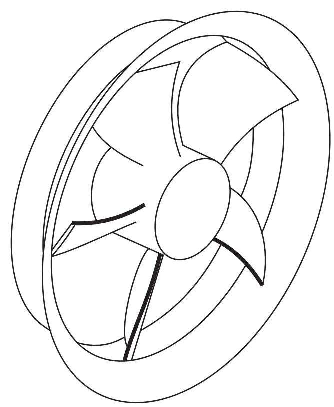
Turbomachines are essential devices in the modern world. Turbines are used in all significant electricity production plants in steam power plants, gas turbine power plants, hydro-electric power plants and wind turbines. Pumps are used to transport water in homes, municipal water systems and in several industries. Pumps and turbines are also essential in the transportation of fuel oil and gas pipe networks. Gas turbine engines are used to power all large passenger aircrafts either in the form of turbo-prop or turbo-fan engines. They also power all helicopter engines through a gearbox.

1.2 Classification of Turbomachines

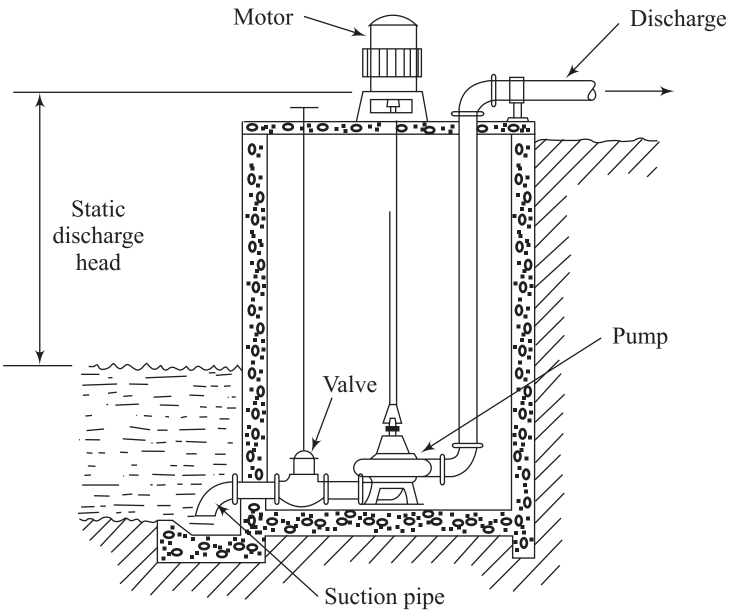
There are four major ways of classifying turbomachines.

1. **On the basis of the extent of quantity of fluid influenced by the machine:** They are grouped as: (a) *Open or unshrouded* turbomachines, and (b) *Closed or shrouded* turbomachines. Propellers, windmills [Figure 1.1(a)], and unshrouded fans, which act upon an infinite extent of fluid are called open turbomachines. Machines which are enclosed in a housing or casing [Figure 1.1(b), 1.1(c) and 1.1(e)] and operate upon only a finite quantity of fluid are called closed turbomachines.
2. **On the basis of the compressibility of the working fluid:** They are classified as (a) C*ompressible flow turbomachines*, and (b) *Incompressible flow turbomachines.* Turbo-compressors, steam and gas turbines [Figure 1.1(d)] where the working medium is a compressible fluid such as air, steam and gas flowing at high speeds, are compressible flow turbomachines. In these machines, the velocity of the fluid at times may exceed the local speed of sound. The *incompressible flow turbomachines* use either a liquid or air or gas flowing at low speeds such as in hydraulic turbines [Figure 1.1(g)], water pumps [Figure 1.1(c)], wind turbines [Figure 1.1(a)] and low speed fans [Figure 1.1(e)].
3. **On the basis of the direction of flow:** The turbomachines may be grouped as (a) axial, and (b) radial, and (c) mixed flow machines. As the fluid is flowing through the rotor of the turbomachine, the velocity of the fluid can be resolved generally into *axial*, *radial* and *tangential* components. However, if the predominant direction of the flow is parallel to the rotating axis, the device is called an axial turbomachine [Figure 1.1(d), 1.1(e)]. If the flow direction changes and has a component substantially

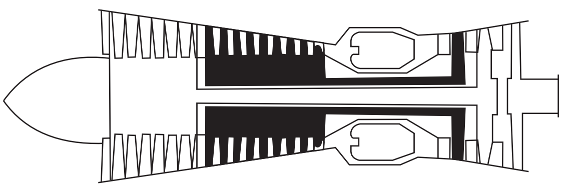
(a) *Wind Turbine: An Open, Unshrouded, Incompressible, Axial Turbomachine*



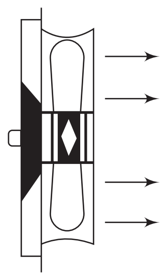
(b) *Ducted Fan: A Shrouded or Ducted, Incompressible, Axial Turbomachine*



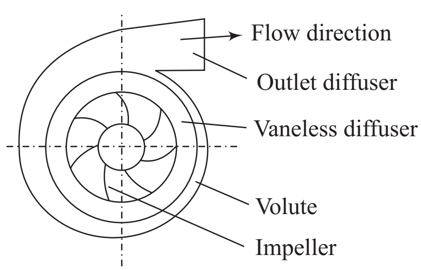
(c) *Water Pump: A Shrouded, Incompressible, Radial Turbomachine*

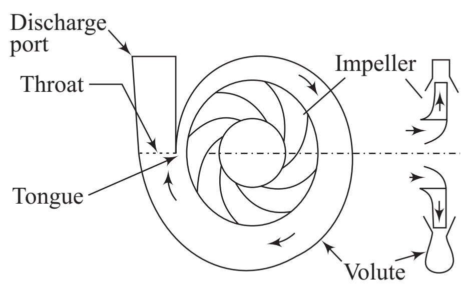


(d) *Gas Turbine Engine: Compressor and Turbine a Shrouded, Compressible, Axial Turbomachine*

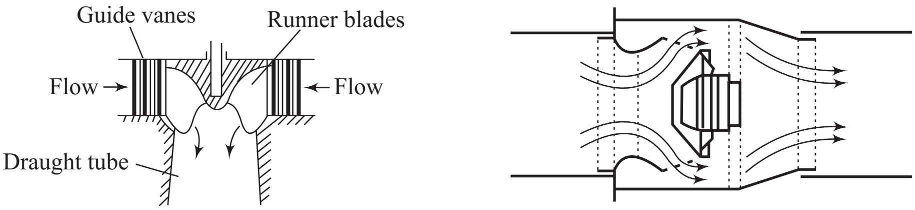


(e) *Ventilator Fan: A Shrouded, Incompressible, Axial Turbomachine*

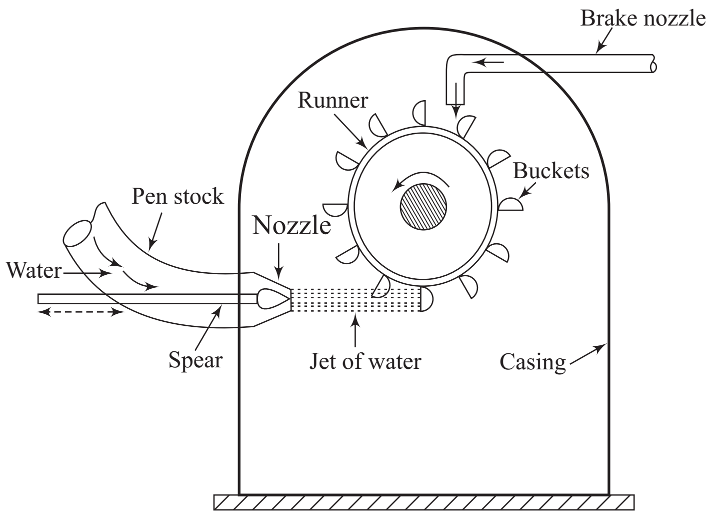




(f) *Centrifugal Compressor: A shrouded, Compressible, Centrifugal Turbomachine*



(g) *Francis Turbine: A Shrouded, Incompressible, Mixed Flow Turbomachine*



(h) *Pelton Turbine: An Incompressible, Tangential Flow Turbomachine*

Figure 1.1 *Different Turbomachines Grouped According to the Classification*

in the radial direction, the machine is known as radial or centrifugal machine [Figure 1.1(f)]. When the flow has components in radial as well as in axial direction, the machine is called mixed flow machine [Figure 1.1(g)]. Most turbomachines fall under these three categories. The rotor of a Pelton turbine [Figure 1.1(h)] which is a hydraulic turbine consists of circular disc with a number of blades (buckets) spaced around the periphery. One or more nozzles are mounted in such a way that each nozzle directs its jet along a tangent to the circle through the centres of the buckets. In this regard, it may be treated as a *tangential flow machine*. However, Pelton turbine is an *axial flow machine* as the exit flow is close to being parallel to the shaft.

4. **On the basis of direction of energy transfer:** Machines may be either power producing or power consuming turbomachines. Turbines which absorb power from the fluid and transmit to the machine shaft are power producing machines. Power consuming turbomachines transfer their shaft power to the fluid e.g. compressors, fans, or pumps absorbs power to increase fluid pressure or head.

1.3 Reynolds Transport Theorem

The Reynolds transport theorem establishes the equivalence between system approach and control volume. Consider a control volume (CV) (I + II) in a one-dimensional flow (Figure 1.2), coincident with system at $t = t_0$.

At time $t_0 + \Delta t$, we find that the control volume remains at the same position, (I + II), while the system has moved to occupy the position (II + III). During time Δt_0, the mass contained in region I has entered the control volume and the mass in region III has left the control volume.

Consider an extensive property N associated with the control volume. We can write,

$$\frac{dN_s}{dt} = \lim_{\Delta t \to 0} \frac{N_s(t_0 + \Delta t) - N_s(t_0)}{\Delta t} \tag{1.1}$$

where subscripts s denotes a system.
Further we have at $t_0 + \Delta t$

$$N_s(t_0 + \Delta t) = (N_{II} + N_{III}) \text{ at } (t_0 + \Delta t)$$

$$N_s(t_0 + \Delta t) = (N_{CV} - N_I + N_{III}) \text{ at } (t_0 + \Delta t) \tag{1.2}$$

On substituting these into Eq. (1.1) and noting that at t_0 the system and the control volume coincide, i.e. $N_s(t_0) = N_{CV}(t_0)$, we have,

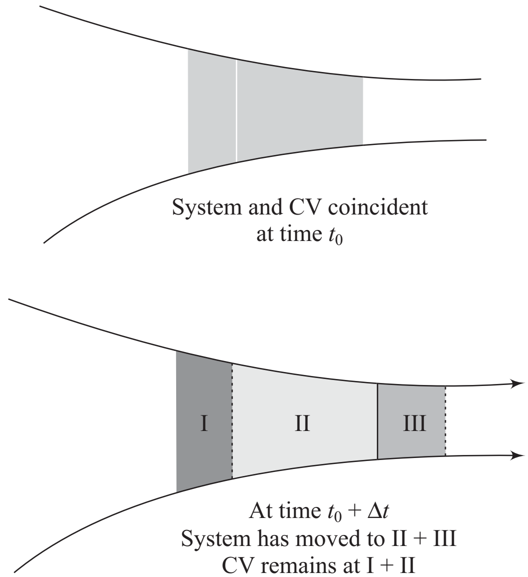


Figure 1.2 *Control Volume and System for One-Dimensional Flow*

$$\left(\frac{dN}{dt}\right)_s = \lim_{\Delta t \to 0} \frac{(N_{CV} - N_I + N_{III})(t_0 + \Delta t) - N_{CV}(t_0)}{\Delta t} \tag{1.3}$$

By readjusting the terms, we have,

$$\left(\frac{dN}{dt}\right)_s = \lim_{\Delta t \to 0} \frac{N_{CV}(t_0 + \Delta t) - N_{CV}(t_0)}{\Delta t} + \lim_{\Delta t \to 0} \frac{N_{III}(t_0 + \Delta t)}{\Delta t} - \lim_{\Delta t \to 0} \frac{N_I(t_0 + \Delta t)}{\Delta t} \tag{1.4}$$

Consider each of the three limits on the RHS of Eq. (1.4):
First limit,

$$\lim_{\Delta t \to 0} \frac{N_{CV}(t_0 + \Delta t) - N_{CV}(t_0)}{\Delta t} = \frac{\partial N_{CV}}{\partial t} = \frac{\partial}{\partial t} \int_{CV} \eta \rho dV \tag{1.5}$$

where N is an extensive property and η is the corresponding intensive property such that N = ηm, where m is the mass given by ρ times volume, i.e. $\rho \times dV$.

The second limit, which gives the rate of change of N within III could be written as,

$$\lim_{\Delta t \to 0} \frac{N_{III}(t_0 + \Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \left[\frac{\eta \rho dV}{\Delta t}\right](t_0 + \Delta t) \tag{1.6}$$

The RHS of Eq. (1.6) represents simply the rate at which N is going out of the control volume through the boundary, i.e. the control surface at right and is equal to $[\eta \rho AC]_{out}$ where A is the area of cross section of III, C is the velocity normal to the area. Therefore,

$$\lim_{\Delta t \to 0} \frac{N_{III}(t_0 + \Delta t)}{\Delta t} = [\eta \rho AC]_{out} \tag{1.7}$$

Similarly, for I, i.e. the rate at which N enters the control volume through the boundary or control surface at left, $[\eta\rho AC]_{in}$

$$\lim_{\Delta t \to 0} \frac{N_I(t_0+\Delta t)}{\Delta t} = [\eta\rho AC]_{in} \tag{1.8}$$

Substituting Eqs (1.5), (1.7) and (1.8) into Eq. (1.4), we have,

$$\left(\frac{dN}{dt}\right)_s = \frac{\partial}{\partial t}\int_{CV} \eta\rho dV + [\eta\rho AC]_{out} - [\eta\rho AC]_{in} \tag{1.9}$$

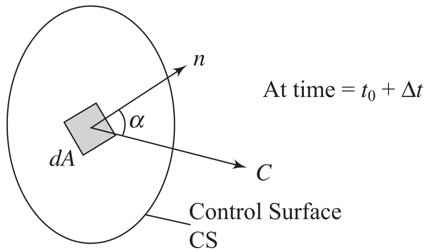


Figure 1.3 *General Control Volume and System*

Equation (1.9) is the *Reynolds transport equation* for the control volume considered.

Reynolds transport theorem states that,

Rate of change of property N within the system = (Rate of change of property N within the control volume) + (Rate of outflow of property N through the control surface) – (Rate of inflow of property through the control surface).

Equation (1.9) can be written for a generalised control volume (Figure 1.3) as,

$$\left(\frac{dN}{dt}\right)_s = \frac{\partial}{\partial t}\int_{CV} \eta\rho dV + \int_{CS} \eta\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.10}$$

The conservation equation for mass, momentum, energy, etc. can be easily written with the help of Reynolds transport theorem.

1.3.1 Conservation of Mass

Consider the extensive property as mass, i.e. $N = m$. The intensive (specific) property will then be,

$$\eta = \frac{N}{m} = \frac{m}{m} = 1 \tag{1.11}$$

Substituting N and η from Eq. (1.11) into Eq. (1.10), we have,

$$\left(\frac{dm}{dt}\right)_s = \frac{\partial}{\partial t}\int_{CV} \rho dV + \int_{CS} \rho\vec{C}\cdot\overrightarrow{dA} \tag{1.12}$$

Since m is invariant with time, the conservation of mass equation is written as,

$$\frac{\partial}{\partial t}\int_{CV} \rho dV + \int_{CS} \rho\vec{C}\cdot\overrightarrow{dA} = 0 \tag{1.13}$$

It can been easily seen that for steady flow this equation reduces to,

$$\int_{CS} \rho\vec{C}\cdot\overrightarrow{dA} = 0 \tag{1.14}$$

and for incompressible flow,

$$\int_{CS} \vec{C}\cdot\overrightarrow{dA} = 0 \tag{1.15}$$

For one-dimensional flow, as shown in Figure 1.4, we have,

$$\int_{CS} \rho \vec{C} \cdot \overrightarrow{dA} = \int_1 \rho \vec{C} \cdot \overrightarrow{dA} + \int_2 \rho \vec{C} \cdot \overrightarrow{dA} = 0 \quad (1.16)$$

A negative $\vec{C} \cdot \overrightarrow{dA}$ suggests an inflow into the control volume while a positive $\vec{C} \cdot \overrightarrow{dA}$ is an outflow from the control volume. Simplifying Eq. (1.16),

$$\rho_1 C_1 A_1 = \rho_2 C_2 A_2 \quad (1.17)$$

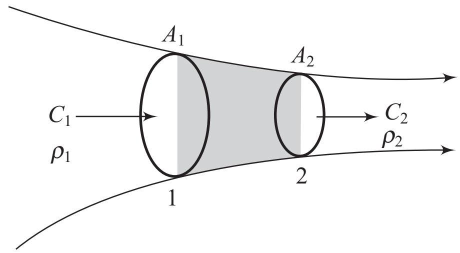


Figure 1.4 *Control Volume for One-Dimensional Steady Flow*

EXAMPLE 1.1 A nozzle is a stationary component of a certain turbomachine. Air enters the nozzle (Figure 1.5) steadily at an average velocity of 30 m/s and leaves at an average velocity of 180 m/s. If the inlet area of the nozzle is 8000 mm^2, determine (a) the mass flow rate through the nozzle, and (b) the exit area of the nozzle. Take the density of air as 2.21 kg/m^3 at the inlet, and 0.762 kg/m^3 at the exit.

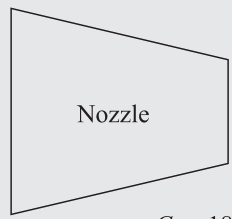


Figure 1.5 *Nozzle*

Solution

The mass flow rate of air is to be determined from the inlet conditions from Eq. (1.17) as,

$$\dot{m} = \rho_1 A_1 C_1 = 2.21 \times 0.008 \times 30 = 0.530 \text{ kg/s} \quad (1)$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

Then the exit area of the nozzle is determined to be,

$$\dot{m} = \rho_2 A_2 V_2 \quad (2)$$

$$A_2 = \dot{m}/\rho_2 V_2 = 0.530/(0.762 \times 180)$$

$$A_2 = 0.00387 \text{ m}^2 = 3870 \text{ mm}^2 \quad (3)$$

(a) Estimation of Mass Flow through a Turbomachine Rotor

Consider a typical turbomachine rotor shown in Figure 1.6. Fluid approaches the rotor from the suction and operated on by the rotor, and it is discharged into the discharge end. Consider the stream surfaces which intersect the inlet edge of the rotor at 1 and the outlet at 2. The normal fluid velocities (velocity of flow) at 1 and 2 are C_{f1} and C_{f2}, respectively. If the elemental areas of flow are da_1 and da_2, the mass flow rate at inlet and outlet are

$$\dot{m}_1 = \int \rho_1 C_{f1} da_1 \quad (1.18)$$

$$\dot{m}_2 = \int \rho_2 C_{f2} da_2 \quad (1.19)$$

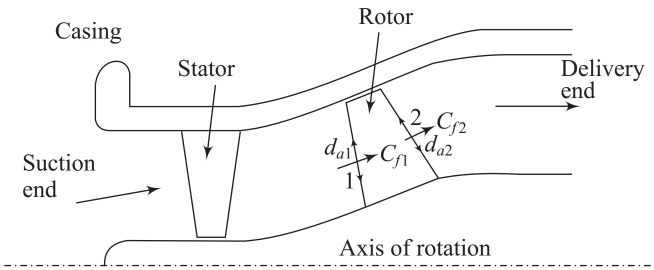


Figure 1.6 *Flow through a turbomachine*

(i) For Axial Flow Machines

Refer Figure 1.7.

$$da = \int_{r_1}^{r_2} 2\pi r dr = \pi(r_2^2 - r_1^2) \tag{1.20}$$

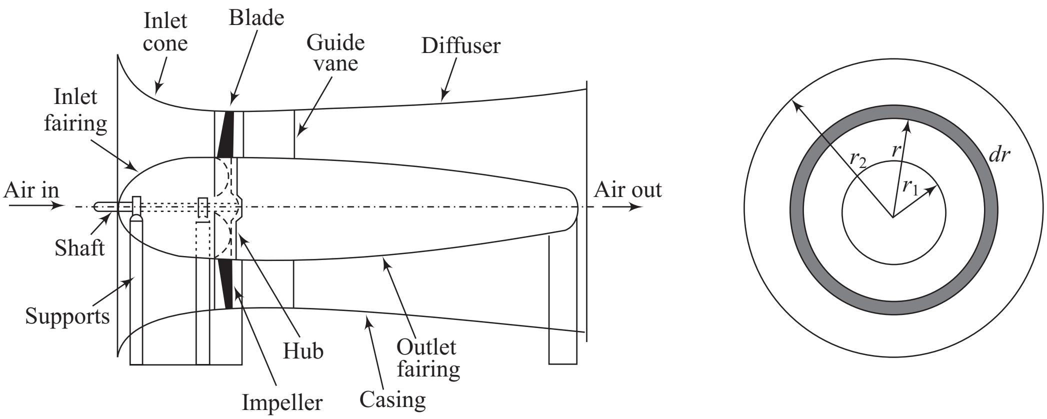


Figure 1.7 *Axial Flow Turbomachine*

(ii) For Centrifugal Flow Machines

Refer Figure 1.8.

$$da = \int_{0}^{b} 2\pi r db = 2\pi rb \tag{1.21}$$

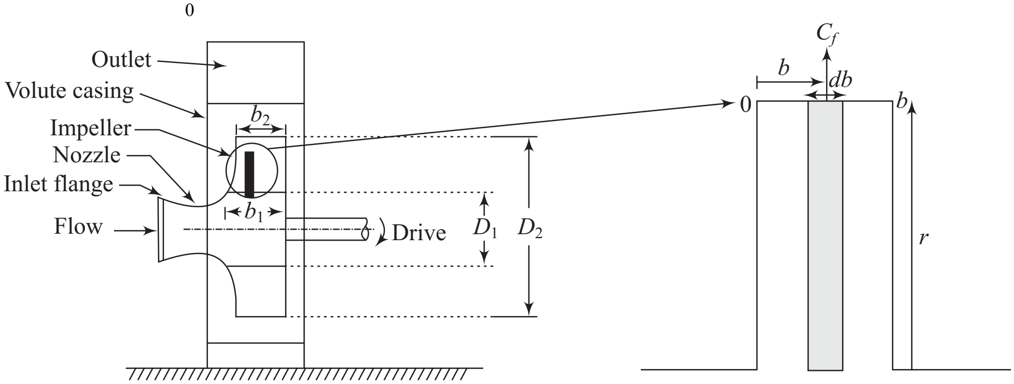


Figure 1.8 *Centrifugal Flow Turbomachine*

EXAMPLE 1.2 An axial-fan (Figure 1.9) stage has the tip diameter of 1.0 m and the hub diameter of 0.9 m. The axial velocity and density of fluid are 60 m/s and 1.2 kg/m^3, respectively. Determine the mass flow rate assuming incompressible flow.

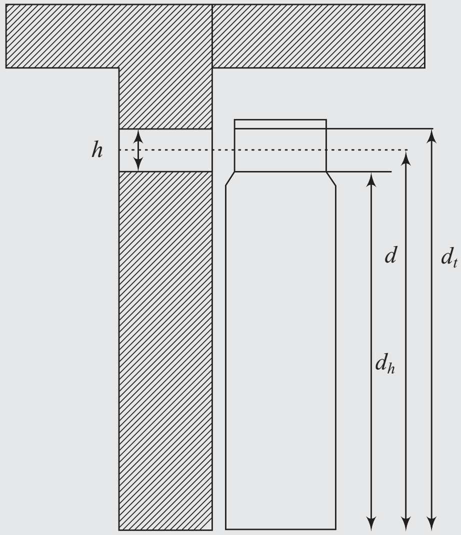


Figure 1.9 *An Axial Turbomachine*

Solution

Given: ρ = 1.2 kg/m^3, r_t = 0.5 m, r_h = 0.45 m, C_f = 60 m/s

The rate of mass flow,

$$\dot{m} = \int \rho C_f da = \rho C_f \pi (r_t^2 - r_h^2) \quad (1)$$

where,

$$a = \pi (r_t^2 - r_h^2) \quad (2)$$

$$\dot{m} = 1.2 \times 60 \times \pi (0.5^2 - 0.45^2) = 10.74 \text{ kg/s}$$

$$\dot{m} = 10.74 \text{ kg/s} \quad (3)$$

EXAMPLE 1.3 In a centrifugal pump (Figure 1.10), the impeller rotates at 600 rpm and radial velocity of 13.3 m/s at the inlet and of 4.5 m/s at the outlet. Given r_1 = 0.2 m and r_2 = 0.5 m, b_1 = 0.15 m, find the mass flow rate and exit width of the impeller assuming incompressible flow.

Solution

Since the flow of water is incompressible, take ρ = 1000 kg/m^3. Mass flow rate is,

$$\dot{m} = \int \rho C_f da = \rho Q \quad (1)$$

where, $a = 2\pi r_1 b_1$

$$Q = 2\pi \times 0.2 \times 0.15 \times 13.3 = 2.507 \text{ m}^3\text{/s}$$

$$\dot{m} = 2.507 \times 1000$$

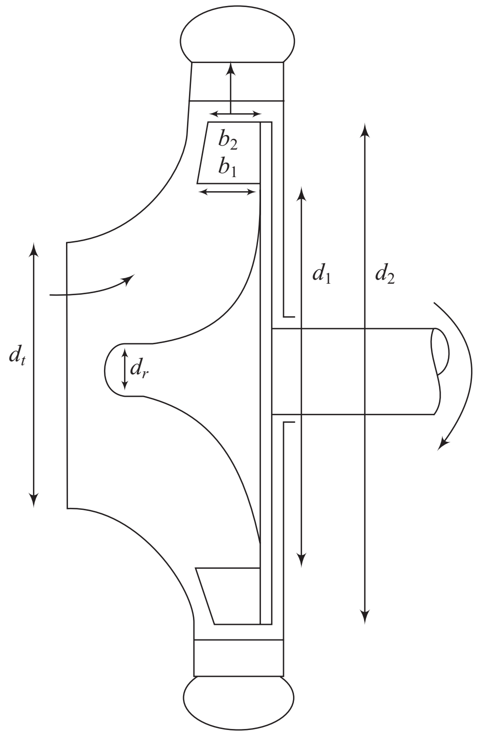


Figure 1.10 *A Radial Turbomachine*

$$\dot{m} = 2507 \text{ kg/s} \tag{2}$$

$$Q = 2\pi r_1 b_1 C_{f1} = 2\pi r_2 b_2 C_{f2} \tag{3}$$

$$Q = 2\pi \times 0.2 \times 0.15 \times 13.3 = 2\pi \times 0.5 \times b_2 \times 4.5$$

$$b_2 = 0.1773 \text{ m} \tag{4}$$

1.3.2 Momentum Equation

Using the Reynolds transport theorem [Eq. (1.10)], $N = \vec{M}$, where $\vec{M}$ is the momentum,

$$\eta = \frac{\vec{M}}{m} = \vec{C} \tag{1.22}$$

Substituting for $\vec{M}$ in Eq. (1.10), we get,

$$\frac{d\vec{M}}{dt} = \vec{F} \tag{1.23}$$

$\vec{F}$ consists of body forces $\vec{F}_B$ and surface forces, $\vec{F}_S$. Thus,

$$\frac{d\vec{M}}{dt} = \vec{F}_B + \vec{F}_S \tag{1.24}$$

Substituting for η and using Eq. (1.24) in Eq. (1.10), we have,

$$\vec{F}_B + \vec{F}_S = \frac{\partial}{\partial t}\int_{CV} \vec{C}\rho dV + \int_{CS} \vec{C}\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.25}$$

Expanding Eq. (1.25), we have,

$$\vec{F}_{Bx} + \vec{F}_{Sx} = \frac{\partial}{\partial t}\int_{CV} C_x\rho dV + \int_{CS} C_x\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.26}$$

$$\vec{F}_{By} + \vec{F}_{Sy} = \frac{\partial}{\partial t}\int_{CV} C_y\rho dV + \int_{CS} C_y\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.27}$$

$$\vec{F}_{Bz} + \vec{F}_{Sz} = \frac{\partial}{\partial t}\int_{CV} C_z\rho dV + \int_{CS} C_z\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.28}$$

Note that x momentum, that is convected in or out from the surface $\overrightarrow{dA}$ into normal direction, is given by the term $\rho C_x\vec{C}\cdot\overrightarrow{dA}$. Similar expressions in y and z directions can also be noticed.

1.3.3 Bernoulli Equation

Figure 1.11 shows a differential stream tube within a control volume. Let the velocity along the stream tube be C_s and the length of it be ds. There is no flow across the tube.

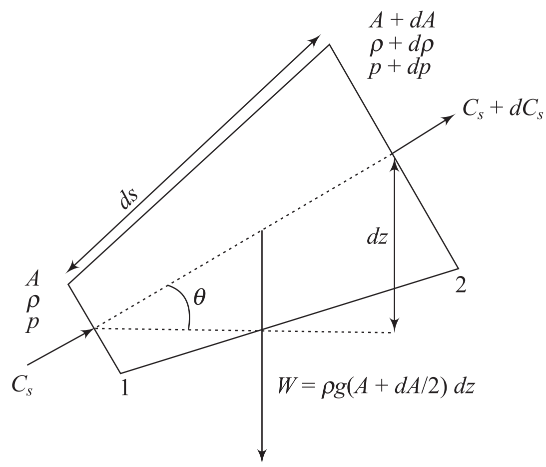


Figure 1.11 *Differential Control Volume for One-Dimensional Steady Flow*

Equation (1.13) gives,

$$\frac{\partial}{\partial t}\int_{CV}\rho dV + \int_{CS}\rho\vec{C}\cdot\overrightarrow{dA} = 0$$

The first term in the equation cancels out because of the steady flow assumption. Since all the flow takes place through 1 and 2 only, the remaining term reduces to,

$$-\rho C_S A + \rho(C_S + dC_S)(A + dA) = 0$$

Rearranging this equation, we have,

$$\dot{m} = \rho C_S A = \rho(C_S + dC_S)(A + dA) \tag{1.29}$$

where $\dot{m}$ is the mass flow rate through the control volume. From Eq. (1.25), we have the momentum equation,

$$\vec{F}_{Bs} + \vec{F}_{Ss} = \frac{\partial}{\partial t}\int_{CV} C_S\rho dV + \int_{CS} C_S\rho\vec{C}_S\cdot\overrightarrow{dA} \tag{1.30}$$

(a) Body Force

$$\vec{F}_{Bs} = -dW\sin\theta = -(m)g\sin\theta$$

$$\vec{F}_{Bs} = -(\rho dV)g\sin\theta = -\left[\rho ds\left(A + \frac{dA}{2}\right)\right]g\sin\theta$$

Noting that $ds \times \sin\theta = dz$

$$\vec{F}_{Bs} = -\rho g\left(A + \frac{dA}{2}\right)dz$$

Dropping higher order terms $dz \cdot dA$, we have,

$$\vec{F}_{Bs} = -\rho gAdz \tag{1.31}$$

(b) Surface Force

$$\vec{F}_{Ss} = pA - (p + dp)(A + dA) + \left(p + \frac{dp}{2}\right)dA$$

Rearranging and neglecting higher order terms such as $dp \cdot dA$, we have,

$$\vec{F}_{Ss} = -Adp \tag{1.32}$$

RHS of Eq. (1.30) gives,

$$\vec{F}_{Bs} + \vec{F}_{Ss} = -C_S\rho AC_S + (C_S + dC_S)\rho(A + dA)(C_S + dC_S)$$

Substituting Eq. (1.29) in above equation,

$$\vec{F}_{Bs} + \vec{F}_{Ss} = -C_S\rho AC_S + (C_S + dC_S)\rho AC_S$$

$$\vec{F}_{Bs} + \vec{F}_{Ss} = \rho AC_S dC_S \tag{1.33}$$

Substituting Eqs. (1.31) and (1.32) in LHS and RHS of Eq. (1.33),

$$\rho A C_S dC_S = -\rho g A dz - A dp$$

$$\rho A C_S dC_S + \rho g A dz + A dp = 0$$

$$C_S dC_S + g dz + dp/\rho = 0$$

$$d(C_s^2/2) + gdz + dp/\rho = 0 \tag{1.34}$$

As Eq. (1.34) is integrated for an incompressible flow, ρ = constant, we have,

$$(C^2/2) + gz + p/\rho = \text{constant} \tag{1.35}$$

1.3.4 Angular Momentum

$$T = \frac{d\tau}{dt}$$

where,

$$\tau = \int_{\text{system}} (r \times C)dm = \int_{\text{system}} (r \times C)\rho dV$$

Angular momentum about a point, say O, is given by,

$$N = \tau_0 \text{ and } \eta = \frac{d\tau_0}{dm} = r \times C \tag{1.36}$$

Substituting Eq. (1.36) in Reynolds theorem, Eq. (1.10), we get,

$$\left(\frac{d\tau_0}{dt}\right)_s = \frac{\partial}{\partial t}\int_{CV} (r \times C)\rho dV + \int_{CS} r \times C\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.37}$$

However, $\left(\frac{d\tau_0}{dt}\right)_s$ is the sum of all the moments about the point O. Therefore,

$$\Sigma(r \times C) = \frac{\partial}{\partial t}\int_{CV} (r \times C)\rho dV + \int_{CS} r \times C\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.38}$$

EXAMPLE 1.4 It is intended to move a mass M on wheels by impinging a jet of water on a vane mounted on the mass as shown in Figure 1.12. The total mass of M, including the wheel, is 350 kg, and the coefficient of rolling friction is 0.05. The vane angle at the top is horizontal, while at the bottom is 30°. The jet velocity is 15 m/s discharged from a stationary nozzle of diameter 40 mm. Estimate the starting acceleration when the water enters the vane at that top.

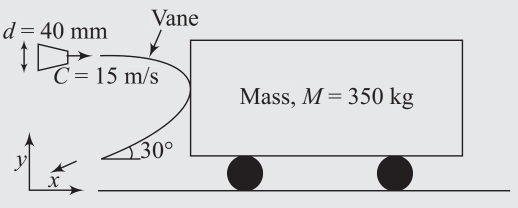


Figure 1.12 *Vane on Body of Mass, M and Wheels*

Solution

Mass flow rate of fluid,

$$\dot{m} = \rho AC = 1000 \times \frac{\pi}{4} \times 0.04^2 \times 15$$

$$\dot{m} = 18.8 \text{ kg/s} \quad (1)$$

Velocity of fluid issued from the nozzle = 15 m/s

Final velocity of fluid jet in x direction after striking the plate = component of velocity of fluid jet leaving the plate in x direction

$$C_x = -C \cos 30 = -15 \cos 30 = -13 \text{ m/s} \quad (2)$$

Hence, change in velocity of fluid jet in x direction = final velocity of fluid jet in x direction (after impingement on plate) – initial velocity of fluid jet in x direction (before impingement)

Change in velocity of fluid in x direction = –13 – 15 = –28 m/s (3)

Final velocity of fluid jet in y direction after striking the plate = component of velocity of fluid jet leaving the plate in y direction

$$C_y = -C \sin 30 = -7.5 \text{ m/s} \quad (4)$$

Hence, change in velocity of fluid jet in y direction = final velocity of fluid jet in y direction (after impingement on plate) – initial velocity of fluid jet in y direction (before impingement)

Change in velocity of fluid jet in y direction = –7.5 – 0 = –7.5 m/s (5)

$$F_x = -R_x = \text{Mass flow rate of fluid} \times \text{Change of velocity of fluid in } x \text{ direction}$$

$$F_x = -R_x = 18.8 \times (-28)$$

$$R_x = 526.4 \text{ N} \quad (6)$$

$$F_y = -R_y = \text{Mass flow rate of fluid} \times \text{Change of velocity of fluid in y direction}$$

$$F_y = -R_y = 18.8 \times (-7.5)$$

$$R_y = 141 \text{ N} \quad (7)$$

Here, the forces F_x and F_y represent the action of the water jet on the incline. R_x and R_y are the components of the force applied by the mass M to the water jet. Force F is the friction force between the incline and the ground. The application of Newton's second law to the Mass M gives,

$$\Sigma F_y = N - Mg + R_y = Ma_y = 0 \quad (8)$$

where, N is the normal reaction and acceleration in y direction, $a_y = 0$.

$$N - 350 \times 9.81 + 141 = 0$$

$$N = 3295.5 \text{ N} \quad (9)$$

$$\Sigma F_x = R_x - F = R_x - \mu N = Ma_x \quad (10)$$

$$526.4 - 0.05 \times 3292.5 = 350a_x$$

$$a_x = 1.034 \text{ m/s}^2 \quad (11)$$

1.3.5 Energy Equation

Using the Reynolds transport theorem, Eq. (1.10), here, $N = E$ and,

$$\eta = \frac{E}{m} = e \tag{1.39}$$

Substituting Eq. (1.39) in LHS of Eq. (1.10),

$$\text{LHS} = \frac{dE}{dt} = \Delta\dot{E}$$

However, according to the first law of thermodynamics for a system, the rate of change of energy of the system $\Delta\dot{E}$ is given by,

$$\Delta\dot{E} = \delta\dot{Q} - \delta\dot{W} \tag{1.40}$$

where $\dot{Q}$ is the rate at which heat is added to the system and $\dot{W}$ is the rate at which work is done by the system. Symbol δ is used in front of $\dot{Q}$ and $\dot{W}$ to indicate that they are non-properties or inexact differentials. Substituting Eq, (1.39) and Eq. (1.40) in Reynolds theorem, Eq. (1.10), we have,

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t}\int_{CV} e\rho dV + \int_{CS} e\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.41}$$

where, specific energy e is given by,

$$e = u + gz + \frac{1}{2}C^2 \tag{1.42}$$

$\dot{W}$ may have several types, for example, the flow work is given by,

$$d\dot{W}_p = pdA\cdot C$$

$$\dot{W}_p = \int_{CS} pd\,\vec{A}\cdot\vec{C} \tag{1.43}$$

Considering $\dot{W} = \dot{W}_p + \dot{W}_s +$ others (neglected)

$$\dot{W} = \int_{CS} pdA\cdot C + \dot{W}_s \tag{1.44}$$

Then Eq. (1.41) becomes,

$$\dot{Q} - \dot{W}_s - \int_{CS} pd\vec{A}\cdot\vec{C} = \frac{\partial}{\partial t}\int_{CV}\left(u + gz + \frac{1}{2}C^2\right)\rho dV + \int_{CS}\left(u + gz + \frac{1}{2}C^2\right)\rho\vec{C}\cdot\overrightarrow{dA}$$

$$\dot{Q} - \dot{W}_s = \frac{\partial}{\partial t}\int_{CV}\left(u + gz + \frac{1}{2}C^2\right)\rho dV + \int_{CS}\left(u + \frac{p}{\rho} + g\,z + \frac{1}{2}C^2\right)\rho\vec{C}\cdot\overrightarrow{dA}$$

$$\dot{Q} - \dot{W}_s = \frac{\partial}{\partial t}\int_{CV}\left(u + gz + \frac{1}{2}C^2\right)\rho dV + \int_{CS}\left(h + gz + \frac{1}{2}C^2\right)\rho\vec{C}\cdot\overrightarrow{dA} \tag{1.45}$$

where h is the specific enthalpy given by,

$$h = u + \frac{p}{\rho} = u + pv \tag{1.46}$$

Equation (1.45) is the general form of the energy equation for a control volume.

(a) One Dimensional Steady Flow

Consider the one-dimensional steady state control volume shown in Figure 1.13. Using Eq. (1.45).

$$\dot{Q} - \dot{W}_s = -(\rho CA)_1\left(h + gz + \frac{1}{2}C^2\right)_1 + (\rho CA)_2\left(h + gz + \frac{1}{2}C^2\right)_2 \tag{1.47}$$

Using continuity equation, $(\rho CA)_1 = (\rho CA)2 = \dot{m}$

$$\dot{Q} - \dot{W}_s = -\dot{m}\left(h + gz + \frac{1}{2}C^2\right)_1 + \dot{m}\left(h + gz + \frac{1}{2}C^2\right)_2$$

Let $q = \frac{\dot{Q}}{\dot{m}}$ and $w = \frac{\dot{W}}{\dot{m}}$, we then have,

$$\left(h + gz + \frac{1}{2}C^2\right)_1 = \left(h + gz + \frac{1}{2}C^2\right)_2 - q + w \tag{1.48}$$

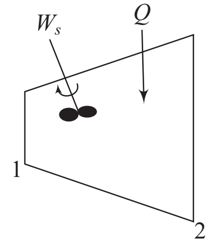


Figure 1.13 *Control Volume for One-Dimensional Steady Flow*

The term $h + gz + \frac{1}{2}C^2$ is equal to the total energy, and the term $h + \frac{1}{2}C^2$ is known as total or stagnation enthalpy denoted by h_0. Equation (1.48) may also be written for an adiabatic turbomachine control volume as,

$$w = \left(h + gz + \frac{1}{2}C^2\right)_1 - \left(h + gz + \frac{1}{2}C^2\right)_2 \tag{1.49}$$

Note that in the absence of heat and work transfer terms ($\dot{Q} = 0$, $\dot{W} = 0$) and any internal energy changes of a constant density fluid $\left(\rho = \frac{1}{V} = \text{constant}\right)$ across the inlet and outlet $(u_2 - u_1 = 0)$, Eq. (1.45) reduces to Bernoulli equation [(Eq. 1.35)].

$$(C^2/2) + gz + p/\rho = \text{constant}$$

EXAMPLE 1.5 An air compressor has the following details:

Mass flow rate $\dot{m}$ = 9.1 kg/s, $\dot{W}$ = 2.4 × 10^6 W, Inlet pressure p_1 = 96.53 kPa, Outlet pressure p_2 = 482.633 kPa, Inlet temperature T_1 = 299.8 K, Outlet temperature T_2 = 533.15 K, Inlet velocity C_1 – negligible, Outlet velocity C_2 = 152.4 m/s.

Neglect the change in potential energy of the fluid. Find how much heat is transferred from the compressor to the ambient, $\dot{Q}$?

Solution

Specific heat of air, c_p = 1005 J/kgK

Assume steady 1-D inlets and outlets (only one inlet and one outlet here)

$$\dot{Q} - \dot{W} = \dot{m}\left(h_2 + \frac{1}{2}C_2^2 + gZ_2\right) - \dot{m}\left(h_1 + \frac{1}{2}C_1^2 + gZ_1\right) \tag{1}$$

With the simplifications given, the energy equation becomes,

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1) + \dot{m}\left(\frac{1}{2}C_2^2\right) \tag{2}$$

But air is treated as an ideal gas and from thermodynamics, $h = c_pT$. Thus, $h_2 - h_1 = c_p(T_2 - T_1)$, and the heat transfer becomes,

$$\dot{Q} = \dot{W} + \dot{m}c_p(T_2 - T_1) + \dot{m}\left(\frac{1}{2}C_2^2\right)$$

$$\dot{Q} = -2.4\times10^6 + 9.1\times1005\times(533.15 - 299.8) + 9.1\times\left(\frac{1}{2}152.4^2\right) \tag{3}$$

$$\dot{Q} = -160.22\times10^3 \text{ W} = -160.22 \text{ kW} \tag{4}$$

Negative sign in heat transfer rate equation indicates that heat is transferred from the compressor to the ambient.

1.3.6 Entropy Change for Fluid Flow

Rate of degradation of energy is best measured in terms of entropy changes. Certainly the same principle must apply to the situation of fluid flow. Frequently it is desirable to evaluate the terms in energy equation and momentum equation with regard to energy degradation and hence entropy changes.

For reversible process $ds = \dfrac{\delta Q}{T}$ and for any irreversible process, $dS > \dfrac{\delta Q}{T}$. Hence, entropy change may be expressed as the sum of $\dfrac{\delta Q}{T}$ and some variable quantity depending upon the irreversibility. Thus,

$$ds = ds_e + ds_i \tag{1.50}$$

where ds_e is the entropy change resulting from external heat transfer, i.e. external entropy change, and ds_i is the internal entropy change caused by irreversibility in the process.

$$ds_e = \frac{\delta Q}{T} \tag{1.51}$$

$$ds_i \geq 0 \tag{1.52}$$

However, while checking the viability of the process, it is important to consider the interaction with surroundings as well in a non-adiabatic process. In Eq. (1.52), equality sign holds for a reversible process, whereas inequality holds for an irreversible process. We know that enthalpy change for a fluid,

$$dh = Tds_e + Tds_i + \frac{1}{\rho}dp \tag{1.53}$$

The steady flow energy equation for unit mass flow rate,

$$dh + \frac{dC^2}{2} + gdZ = \delta Q - \delta w_0$$

For no external work and substituting the value of dh in Eq. (1.53),

$$Tds_e + Tds_i + \frac{1}{\rho}dp + \frac{dC^2}{2} + gdZ = \delta Q \tag{1.54}$$

By Eq. (1.51), $Tds_e = \delta Q$, Eq. (1.54) becomes,

$$Tds_i + \frac{1}{\rho}dp + \frac{dC^2}{2} + gdZ = 0$$

Multiplying this equation by ρ and rearranging,

$$dp + \frac{\rho dC^2}{2} + g\rho dZ + \rho Tds_i = 0 \tag{1.55}$$

Equation (1.55) is called the pressure energy equation, but actually it is more generalised statement of common Bernoulli's equation. If the flow process is reversible, $ds_i = 0$. Therefore,

$$dp + \frac{\rho dC^2}{2} + g\rho dZ = 0 \tag{1.56}$$

ρTds_i is the loss of pressure resulting from irreversibility. Comparing the generalised Bernoulli's equation with the momentum equation,

$$\rho Tds_i = \frac{\rho C^2}{2}\left(f\frac{dx}{D} + dC_D\right) \tag{1.57}$$

Equation (1.57) states that pressure loss due to irreversibility is same as the pressure loss due to friction and flow interruptions.

$$\rho Tds_i = \frac{\rho C^2}{2}\left(f\frac{dx}{D} + dC_D\right) = \frac{\rho C^2}{2}dC_s \tag{1.58}$$

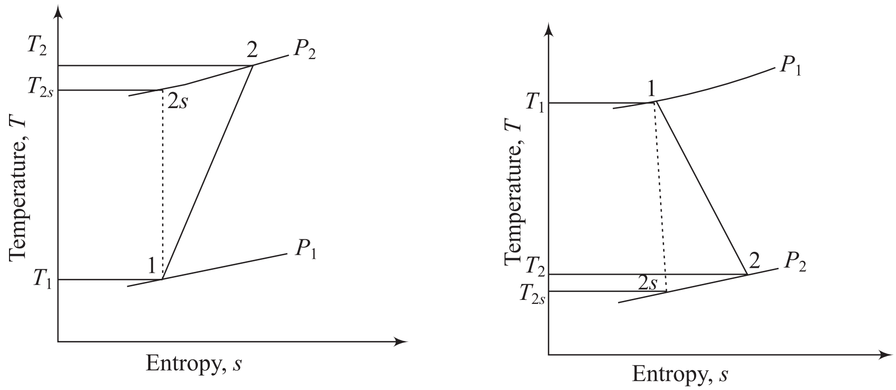
where dC_s is the internal entropy coefficient. Substituting it, momentum equation becomes,

$$dp + g\rho dZ + \frac{\rho dC^2}{2} + \frac{\rho C^2}{2}dC_s = 0 \tag{1.59}$$

In Figure 1.14 (a), the compression process through a compressor is shown with the lower pressure end and higher pressure end static conditions denoted by 1 and 2. In Figure 1.14 (b), the expansion process through a turbine is shown with lower pressure end and higher pressure end static sonditions denoted by 2 and 1. Total energy at any point in a flow field for perfect gas is given by the stagnation enthalpy, $h_0 = c_pT_0$, and the useful energy by stagnation pressure p_0. Suffix '0' denotes stagnation condition. A superscript 's' is used to indicate isentropic end condition.

$$h_0 = h + \frac{C^2}{2} \Rightarrow T_0 = T + C^2/2c_p \tag{1.60}$$

Stagnation condition is that which is obtained by isentropically bringing the velocity C to zero. On the temperature-entropy ($T - s$) diagram, stagnation point is obtained by adding at each point, a vertical height representing $C^2/2c_p$ at that point. In Figure 1.15, T_1 is the static temperature at the lower pressure ends of the compressor and T_{01} the stagnation temperature at the same location. T_2 is the static temperature at the low pressure end of the compressor and T_{02}, the stagnation temperature at the same location.



(a) Compression Process in T-s Diagram (b) Expansion Process in T-s Diagram

Figure 1.14 *Representation of Compression and Expansion Processes on T-s Diagram*

Consider that the fluid particle is brought to zero isentropically. We know that one dimensional form of energy conversion equation is,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \tag{1.61}$$

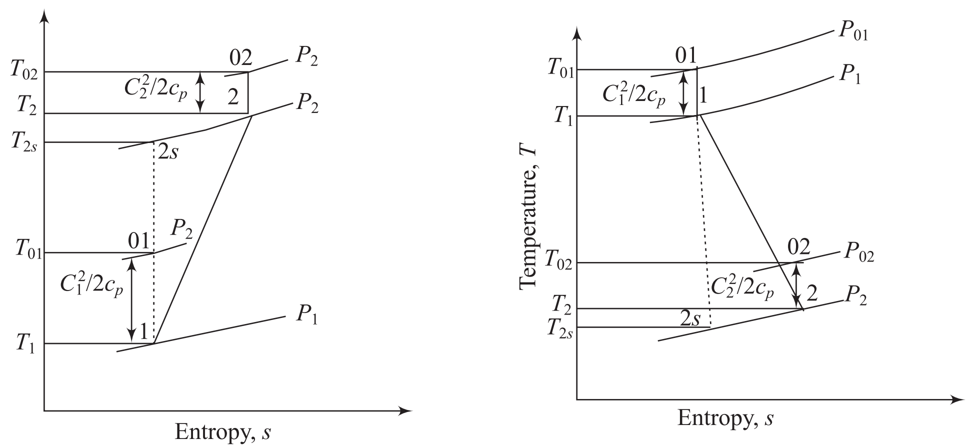


Figure 1.15 *Representation of Compression and Expansion Processes in T-s diagram*

$$c_p T_1 + \frac{C_1^2}{2} = c_p T_2 + \frac{C_2^2}{2} \tag{1.62}$$

Here, subscript 1 stands for initial state of the fluid and subscript 2 stands for final decelerated state of fluid. Since, $C_2 = 0$, lets represent $T_2 = T_{01}$ in Eq. (1.62). Then,

$$c_p T_1 + \frac{C_1^2}{2} = c_p T_{01} \tag{1.63}$$

$$c_p = \frac{\gamma R}{\gamma - 1} \tag{1.64}$$

Substituting Eq. (1.64) in Eq. (1.63), we get,

$$\frac{\gamma R}{\gamma - 1} T_1 + \frac{C_1^2}{2} = \frac{\gamma R}{\gamma - 1} T_{01} \tag{1.65}$$

Dividing the Eq. (1.65) by $\frac{\gamma R T_1}{\gamma - 1}$,

$$1 + \frac{\gamma - 1}{2} \frac{C_1^2}{\gamma R T_1} = \frac{\gamma - 1}{\gamma R T_1} \times \frac{\gamma R T_{01}}{\gamma - 1}$$

$$1 + \frac{\gamma - 1}{2} M^2 = \frac{T_{01}}{T_1} \tag{1.66}$$

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \tag{1.67}$$

Here, subscript '0' represents the stagnation condition. Without the loss of generality, subscript '1' can be deleted.

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \tag{1.68}$$

Since the process is isentropic and using isentropic relations, expression for ratio of stagnation pressure to static pressure relation can be given by,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}} \tag{1.69}$$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}} \tag{1.70}$$

The relation for p_{02} in terms of p_2 and Mach number M_2,

$$p_{02} = p_2 \left[1 + \left(\frac{\gamma - 1}{2}\right) M_2^2\right]^{\frac{\gamma}{\gamma - 1}} \tag{1.71}$$

where,

$$M_2 = C_2 / C_{2,\text{sonic}} \text{ and } C_{2,\text{sonic}} = \sqrt{\gamma R T_2} \tag{1.72}$$

EXAMPLE 1.6 A compressor delivers air at a gauge pressure of 2 bar sucking air at gauge pressure of 0.1 bar and the corresponding temperatures are 175°C and 35°C, respectively. The suction and pressure end pipe diameters are given as 300 mm and 190 mm respectively, and the mass of air delivered by the compressor is 13 kg/s. Find the specific work and the compressor coupling power.

Solution

Using *T-s* diagram (Figure 1.16) for compressor,

$$p_2 = 2 + 1 = 3 \text{ bar}$$

$$p_1 = 0.1 + 1 = 1.1 \text{ bar}$$

$$T_2 = 448 \text{ K}, T_1 = 308 \text{ K}$$

Density of air entering the suction end,

$$\rho_s = 1.1 \times 10^5/(287 \times 308) = 1.244 \text{ kg/m}^3 \quad (1)$$

Density of air leaving the pressure end,

$$\rho_p = 3 \times 10^5/(287 \times 448) = 2.333 \text{ kg/m}^3 \quad (2)$$

$$C_s = 13/[(\pi/4) \times 0.3^2 \times 1.244] = 147.8 \text{ m/s} \quad (3)$$

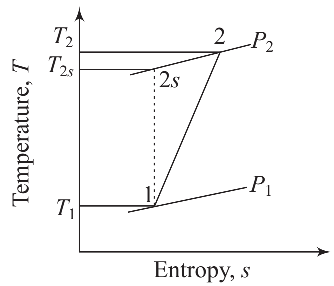


Figure 1.16 *T-s Diagram for Compression Process*

$$C_p = 13/[(\pi/4) \times 0.19^2 \times 2.333] = 196.5 \text{ m/s} \quad (4)$$

$$p_{01} = p_1 + (\rho_s C_s^2)/2 \quad (5)$$

$$p_{01} = 1.1 \times 10^5 + \frac{(1.244 \times 147.8^2)}{2} = 123587 \text{ Pa} \quad (6)$$

$$p_{02} = p_2 + (\rho_p C_p^2)/2 \quad (7)$$

$$p_{02} = 3 \times 10^5 + \frac{(2.333 \times 196.5^2)}{2} = 345041 \text{ Pa} \quad (8)$$

Since $T_0 = T + C^2/2c_p$, therefore,

$$T_{01} = 308 + 147.8^2/(2 \times 1005) = 318.9 \text{ K} \quad (9)$$

$$T_{02} = 448 + 196.5^2/(2 \times 1005) = 467.2 \text{ K} \quad (10)$$

Specific work of isentropic compression,

$$w_s = c_p(T_{02s} - T_{01}) = c_p T_{01}[(p_{02}/p_{01})^{(\gamma-1)/\gamma} - 1] \quad (11)$$

$$w_s = 1005 \times 318.9[(345041/123587)^{(1.4-1)/1.4} - 1]$$

$$w_s = 109259.56 \text{ J/kg} \quad (12)$$

Actual specific work of compression,

$$w_a = c_p(T_{02} - T_{01}) \quad (13)$$

$$w_a = 1005(467.2 - 318.9) = 149041.5 \text{ J/kg} \quad (14)$$

Coupling power,

$$P = \dot{m} w_a = 13 \times 149041.5$$

$$P = 1937539.5 \text{ W} = 1937.54 \text{ kW} \quad (15)$$

1.4 Euler Equation for a Turbomachine

It is clear from the introduction that a turbomachine has both stationary and rotating parts and the rotating parts mounted on a shaft are together called runners or rotors. The runners contain several blades on them. They are referred as impellers when used for increasing the pressure energy of the fluid (i.e. for pump/fans/compressor). The Newton's second law applied to the fluid that flows through the rotor blades under the following idealizations and assumptions is called Euler's equation for a turbomachine.

1. The flow is steady, i.e. the mass flow rate is constant across any section which implies that no storage or depletion of fluid mass in the runner,
2. The heat and work interactions between the control volume and its surroundings take place at a constant rate.
3. Velocity is uniform over any area normal to the flow. This means that the velocity at any point is representative of the total flow over a finite area. This condition also implies that there is no leakage loss and the entire fluid is undergoing the same process.
4. There are no friction and other losses in the system.
5. The fluid is assumed to have perfect guidance through the flow system. This implies that there are infinite number of thin blades on the runner.

Consider a typical turbomachine rotor shown in Figure 1.17. The moment of momentum entering the rotor at 1 and leaving at 2 are given by,

$$dM_1 = (\rho C_{f1} dA_1) C_{w1} r_1 \tag{1.73}$$

$$dM_2 = (\rho C_{f2} dA_2) C_{w2} r_2 \tag{1.74}$$

Thus, the total moments of momentum entering the plane (1) and leaving the plane (2) are, respectively,

$$M_1 = \int \rho C_{f1} C_{w1} r_1 \, dA_1 \tag{1.75}$$

$$M_2 = -\int \rho C_{f2} C_{w2} r_2 \, dA_2 \tag{1.76}$$

The fluid torque is the net effect given by,

$$T = M_1 + M_2 \tag{1.77}$$

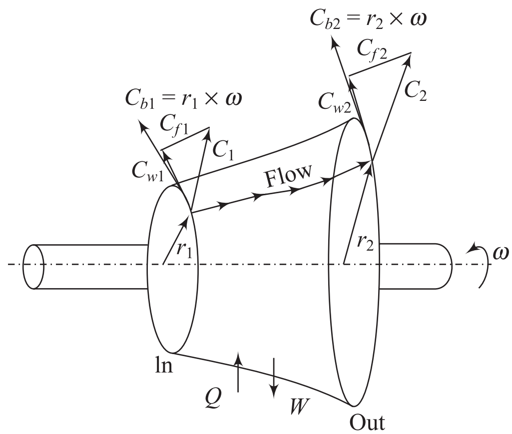


Figure 1.17 *Control Volume on Turbomachine*

$$T = \int \rho C_{f1} C_{w1} r_1 \, dA_1 - \int \rho C_{f2} C_{w2} r_2 \, dA_2 \tag{1.78}$$

It is assumed that $C_w r$ is a constant across each surface, and it is noted that $\int \rho C_f dA$ is the mass flow rate $\dot{m}$. Then Eq. (1.78) becomes,

$$T = \dot{m}(C_{w1} r_1 - C_{w2} r_2) \tag{1.79}$$

Application of Reynolds transport theorem in one-dimension also leads to the same Eq. (1.79) by taking intensive property, $\eta = r \times C$.

Energy extracted by the fluid = Power transmitted to the shaft

$$P = T \times \omega$$

$$P = \dot{m}(C_{w1} r_1 - C_{w2} r_2)\omega$$

$$\because \quad C_{b1} = \omega r_1 \text{ and } C_{b2} = \omega r_2$$

Thus, rate of work done by the rotor or runner is,

$$P = \dot{m}(C_{w1}C_{b1} - C_{w2}C_{b2}) \tag{1.80}$$

Rate of work done by the rotor per unit flow rate or specific work,

$$w = (C_{w1}C_{b1} - C_{w2}C_{b2}) \tag{1.81}$$

Equation (1.81) is known as *Euler equation for a turbomachine.* A positive value of specific work obtained from Eq. (1.81) represents that fluid does work on the rotor i.e. work output as in turbines. In such cases, Eq. (1.81) is called *Euler turbine equation*. When the value of specific work obtained is negative, it represents the work done on the fluid by the rotor i.e. work input to the machine. This machine may be a pump, a compressor, a fan or a blower. In such cases, Eq. (1.81) is known as *Euler pump equation*. Thus, for steady flow, Euler pump equation may be written neglecting the negative sign as it represents work input as,

$$w = (C_{w2}C_{b2} - C_{w1}C_{b1}) \tag{1.82}$$

For a steady turbine, Euler turbine equation is given by,

$$w = (C_{w1}C_{b1} - C_{w2}C_{b2}) \tag{1.83}$$

Referring to steady flow energy equation, for incompressible turbomachines, we get,

$$w = \Delta h_0 = \frac{(\Delta p_0)}{\rho} = gH_e = (C_{w1}C_{b1} - C_{w2}C_{b2}) \tag{1.84}$$

The hydrostatic equivalent head, H_e, is also called Euler's head. Equating Eq. (1.81) in Eq. (1.84), we get,

$$\frac{(\Delta p_0)}{\rho} = gH_e = (C_{w1}C_{b1} - C_{w2}C_{b2}) \tag{1.85}$$

$$H_e = \frac{(C_{w1}C_{b1} - C_{w2}C_{b2})}{g} \tag{1.86}$$

1.4.1 Rothalpy

In a turbomachine, the specific work done equals the change in stagnation enthalpy.

$$w = (C_{w1}C_{b1} - C_{w2}C_{b2}) = h_{01} - h_{02} \tag{1.87}$$

This relationship is true for steady, adiabatic and irreversible flow in a turbomachine.
Since $h_0 = h + C^2/2$, Eq. (1.87) can be rewritten as,

$$h_1 + C_1^2/2 - C_{w1}C_{b1} = h_2 + C_2^2/2 - C_{w2}C_{b2} \tag{1.88}$$

Now, from Eq. (1.88), a new function I is defined having the same value at exit from the impeller as at entry. This function I is called *rothalpy* and is defined as,

$$I = h + C^2/2 - C_wC_b = h_0 - C_wC_b \tag{1.89}$$

The rothalpy, (I), a short name of rotational stagnation enthalpy is a fluid mechanical property frequently used in the study of relative flows in rotating systems. The velocity of flow relative to rotor C_r is vector subtraction of tangential or peripheral velocity of blades/rotor C_b from the absolute velocity of flow C.

$$\vec{C} = \vec{C}_r + \vec{C}_b \tag{1.90}$$

Since the blade has velocity in the tangential direction, therefore the component of the relative velocity in the tangential direction is,

$$C_{rw} = C_w - C_b \tag{1.91}$$

Using Eqs (1.90) and (1.91) in Eq. (1.89), we get,

$$I = h + \frac{1}{2}(C_r^2 + C_b^2 + 2C_bC_{rw}) - C_b(C_{rw} + C_b) = h + \frac{1}{2}C_r^2 - \frac{1}{2}C_b^2$$

$$I = h + \frac{1}{2}C_r^2 - \frac{1}{2}C_b^2 = h_{0,\text{rel}} - \frac{1}{2}C_b^2 \tag{1.92}$$

where $h_{0,\text{ rel}}$ is known as *relative stagnation enthalpy*, which is given by,

$$h_{0,\text{rel}} = h + \frac{1}{2}C_r^2 \tag{1.93}$$

Equation (1.92) is the final form of Euler equation which represents that the relative stagnation enthalpy remains constant throughout the rotor if the blade speed is constant.

1.5 Velocity Triangles

The fluid moves from a stationary inlet to the rotating blade, or the fluid moves out from the rotating blades to stationary exit. The fluid velocity is termed as the absolute velocity ($\vec{C}$) if one observes the fluid from the stationary frame of reference. On the other hand, if the observer is located on the rotor, the same velocity is termed as relative velocity ($\vec{C}_r$). The relation between the absolute and relative velocity is given by the vector relation (Figure 1.18),

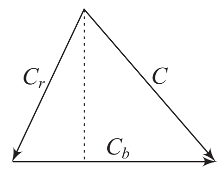


Figure 1.18 *Vector Relation of Velocity*

$$\vec{C}_r = \vec{C} - \vec{C}_b \tag{1.94}$$

where $\vec{C}_b$ is called the blade velocity. Equation (1.90) represents that these three velocities form a triangle. This means that fluid glides over the surface of the blades without separation. Hence, the relative velocity is tangential to the profile of the blade at the point under consideration. Similar velocity triangles can be drawn for a number of points on the blade profile of rotor; however, generally, velocity triangles at inlet and outlet of rotor are drawn. In Section 1.4, fluid enters the control volume with absolute velocity C_1 and leaves the control volume with absolute velocity of C_2. The above vector relation can be written at both the inlet and outlet.

These vector diagrams are called velocity triangles. A representative velocity triangle is shown in Figure 1.19.

In the velocity triangle, the following nomenclature is followed:

- C – Absolute velocity of the fluid
- C_b – Blade peripheral velocity or simply, blade velocity = $\dfrac{\pi DN}{60}$, where N is rpm of the rotor
- C_r – Relative velocity of the fluid after contact with rotor
- C_w – Whirl velocity, or tangential component, of absolute velocity C
- C_{rw} – Tangential component of relative velocity C_r

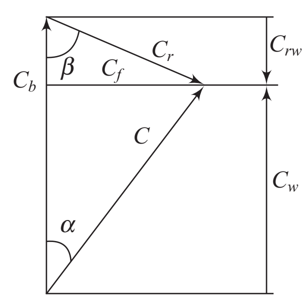


Figure 1.19 *Velocity Triangle*

C_f – Flow velocity or meridional velocity; axial component in case of axial machines, radial component in case of radial machines. This is also called meridional velocity component.

1.5.1 Flow Angles in the Velocity Triangle

Angle α between C and positive C_b direction is called *absolute flow angle*.
Angle β between C_r and negative C_b direction is called *relative flow angle*.
The above convention is not followed always. Readers should take care of the change in this convention. For instance, it is an accepted practice to measure the angles with reference to meridional velocity, C_f. In chapters 6 to 8, this notation is followed for axial flow machines.

1.5.2 Guidelines for Drawing Velocity Triangles

The following advisory is useful for better normalization and understanding of the velocity triangles:

- Each velocity has a magnitude and direction; therefore, all the velocities are vector quantities.
- The blade velocity C_b is tangent to the circular path of the blade. The blade velocity is considered positive in the direction of rotation.
- Figures 1.20 and 1.21 show the vectors drawn on the circular path of the blade for the axial flow and radial machines respectively.

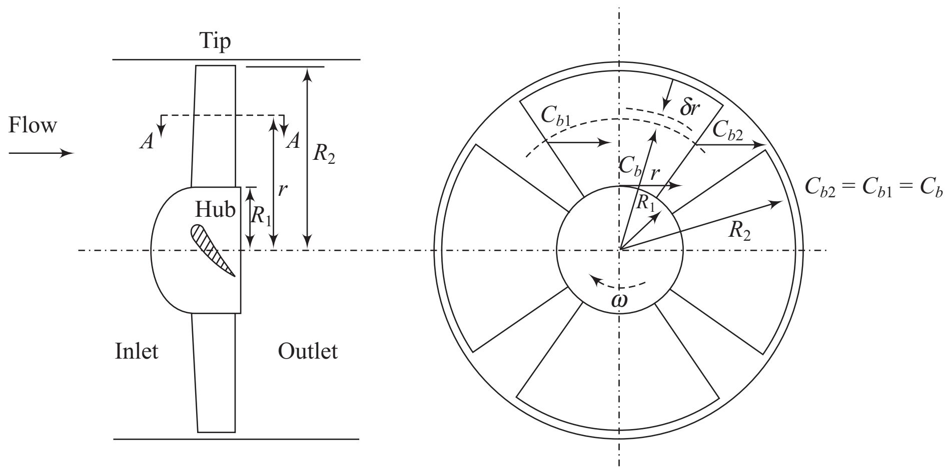


Figure 1.20 *C_b shown for Axial Flow Machine*

- For a radial flow machines, the plane of observation of velocity is perpendicular to the axis of the shaft. The blades of the rotor look as if they are encased between these circular arcs. If the rotational speed of the shaft is N rpm, the peripheral blade velocities at inlet and outlet, C_{b1} and C_{b2}, respectively, along AB and CD, as shown in Fig 1.21.

$$C_{b1} = \frac{\pi D_1 N}{60}; C_{b2} = \frac{\pi D_2 N}{60}, C_{b2} > C_{b1} \tag{1.95}$$

- For an axial flow machine, the velocity triangles are drawn on a plane parallel to the axis of the shaft and tangential to the rotor. The method of obtaining the cylindrical development surface of a plane, say A-A in Figure 1.20, is illustrated with the rotor and blades in Figure 1.22.
- The blades look like as if they are placed in a row between two imaginary parallel lines, *PQ* and *RS*. The blade velocities are along the parallel lines as the shaft rotates.
- $C_{b1} = C_{b2}$ as the tips of the blades are of the same diameter [Figures 1.20 and 1.22].

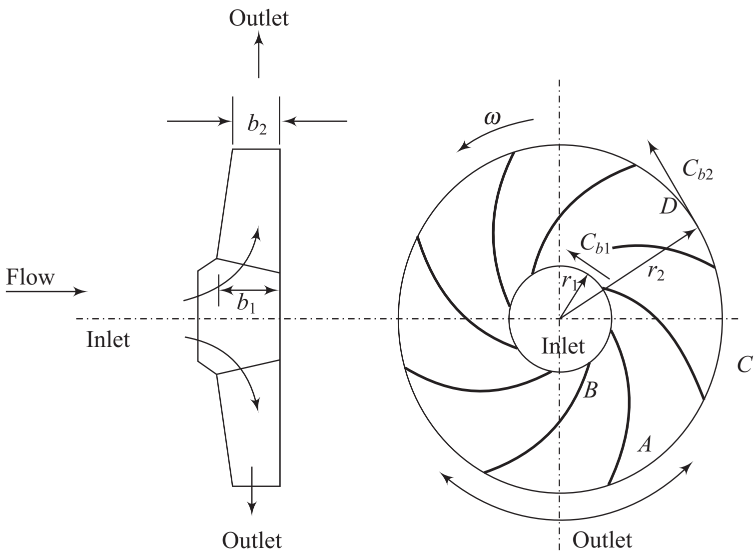


Figure 1.21 *C_b shown for Radial Flow Machine*

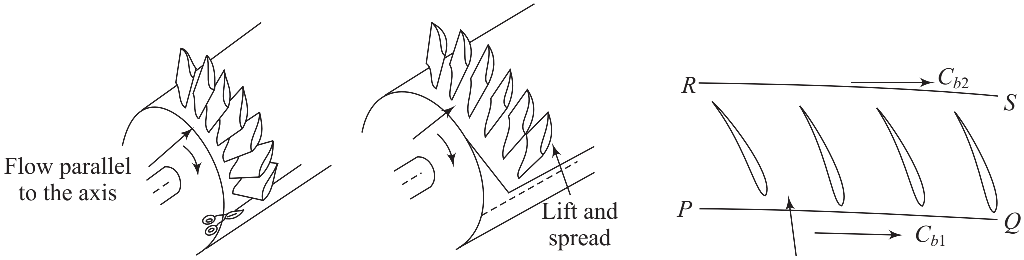


Figure 1.22 *Plane of Velocity Triangles for an Axial Flow Machine*

- The velocity of the fluid relative to the blades is tangential to the streamline of the fluid flow in the passage between two consecutive blades on the rotor. The stream line is assumed to be of the same shape as that of the blades and such flow is called vane congruent. Therefore, the blade velocity is tangential to the profile of the blade too. Figures 1.23 and 1.24 show the vane congruent flow for the radial and the axial flow rotors respectively.

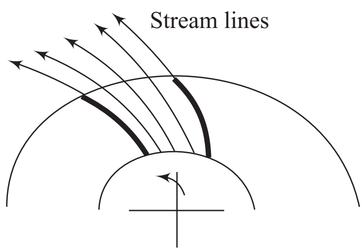


Figure 1.23 *Vane Congruent Flow for Radial Mahine*

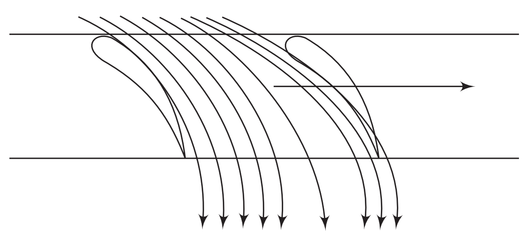


Figure 1.24 *Vane Congruent Flow in Axial Flow Machines*

- The direction of the relative velocity for different blade geometries is shown in Figure 1.25 for the radial flow machine and in Figure 1.26 for the axial flow machines. The relative velocity may be inward or outward depending on the direction of fluid flow.

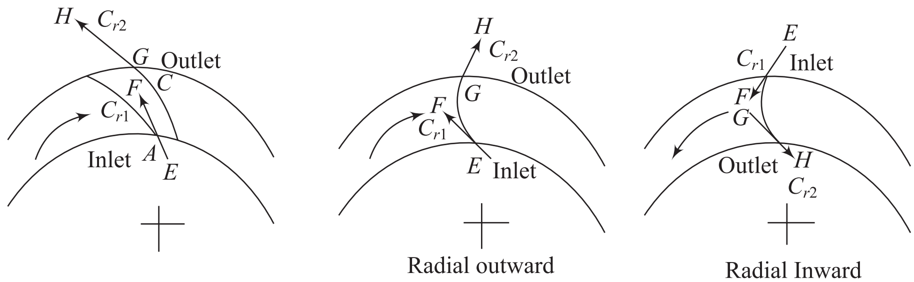


Figure 1.25 *Relative Velocities of Vectors in Radial Flow Machine*

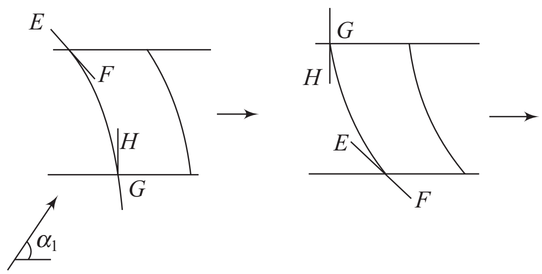


Figure 1.26 *Relative Velocity Vectors in Axial Machines*

- Tangents drawn at the tips of the vanes *A* and *C* are *EF* and *GH* as shown in Figures 1.25 and 1.26, which represent the direction of relative velocities for radial and axial flow machines, respectively. Since each blade occupies the each position during rotation, therefore, tangent drawn on either blade does not make any difference.
- The absolute velocity *C* of the fluid is the vector summation of blade velocity C_b and relative velocity C_r.
- C_{r1} and C_{r2} are radially outwards as the flow is radially outward. The sense of the same C_{r1} and C_{r2} are reversed, i.e. inward radially if the flow is inward.
- Since $\vec{C} = \vec{C}_b + \vec{C}_r$, relative velocity vector $\vec{C}_r$ starts where $\vec{C}_b$ ends. Complete the triangle to find $\vec{C}$ as shown in Figure 1.27. Some more data such as flow angles may be needed. There is absolutely nothing wrong if we write $\vec{C} = \vec{C}_r + \vec{C}_b$. Therefore, $\vec{C}_b$ starts where $\vec{C}_r$ ends. Also, the rotor may move in counter clockwise direction. Such varieties are a matter of practice.

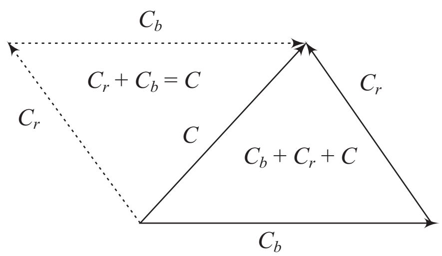


Figure 1.27 *Velocity Parallelogram*

- The absolute flow angles and relative flow angles (blade angles) are specified with respect to the blade peripheral velocity.

The following points may be noted for distinguishing between various classes of turbomachines:

- For an inward flow radial machine, $C_{b1} > C_{b2}$, whereas $C_{b2} > C_{b1}$ for an outward flow radial machine. For an axial flow machine, $C_{b1} = C_{b2}$ as tips of the blades at inlet and outlet are of the same diameter.
- Since $C_{b1} = C_{b2}$ for axial flow machines, therefore a common practice for axial machines is to draw inlet and outlet velocity triangles superimposed on a common base of blade peripheral velocity.
- $C_1 > C_2$ for power producing turbomachines such as turbines, whereas $C_2 > C_1$ for power absorbing machines such as pumps, compressors and fans.

EXAMPLE 1.7 Consider an office desk fan, Figure 1.28. It rotates at 200 rpm and has a diameter of 0.3 m. Air enters the fan at 3 m/s, parallel to the axis of rotation. Calculate the relative velocity C_r at the tip of the fan. What are the values of absolute and relative flow angles? What are the values of velocity of flow and whirl velocity?

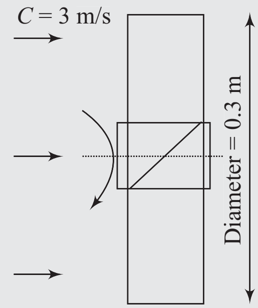


Figure 1.28 *Diagram for Desk Fan*

Solution

Given: $N = 200$ rpm, $C = 3$ m/s, $D = 0.3$ m

The blade velocity of desk fan can be obtained from the rotational speed and radius of the fan:

$$C_b = \omega r = \frac{(2\pi N)}{60} r = \frac{2\pi \times 200}{60} \times \frac{0.3}{2}$$

$$C_b = 3.14 \text{ m/s} \qquad (1)$$

The velocity triangle at outlet of the fan is shown in Figure 1.29. Since the fan is an axial fan,

$$C_f = C \sin 90 = 3 \text{ m/s} \qquad (2)$$

The magnitude of the relative velocity,

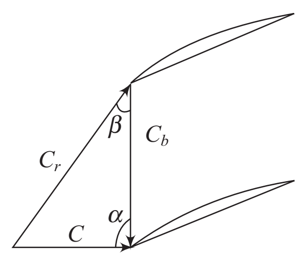


Figure 1.29 *Velocity Triangle at Outlet*

$$C_r = \sqrt{C + C_b^2} = \sqrt{3^2 + 3.14^2}$$

$$C_r = 4.34 \text{ m/s} \qquad (3)$$

$$\text{Absolute flow angle} = \alpha = 90° \qquad (4)$$

$$\text{Tangential velocity} = C_w = C \cos 90 = 0 \qquad (5)$$

$$\text{Relative flow angle} = \beta = \tan^{-1} (C_f/C_b) = \tan^{-1}(3/3.14)$$

$$\beta = 43.69° \qquad (6)$$

EXAMPLE 1.8 A pump is driven at 1470 rpm and delivers 10 litres/s with an outlet whirl component of velocity of 16.52 m/s. Pump inner diameter, $D_1 = 0.2$ m and outer diameter, $D_2 = 0.37$ m. The width of the blades at inlet and outlet remains constant at 3 mm. Sketch the inlet and outlet triangles, assuming zero inlet whirl.

Solution

Given: $N = 1470$ rpm, $Q = 10$ litres/s $= 0.01\ \text{m}^3/\text{s}$, $C_{w2} = 16.52$ m/s, $D_1 = 0.2$ m, $D_2 = 0.37$ m

$$C_{w1} = 0 \Rightarrow \alpha_1 = 0 \quad (1)$$

$$C_{b1} = \pi D_1 N/60 = \pi \times 0.2 \times 1470/60 = 15.39 \text{ m/s} \quad (2)$$

$$C_{b2} = \pi D_2 N/60 = \pi \times 0.37 \times 1470/60 = 24.48 \text{ m/s} \quad (3)$$

The radial velocities C_{f1} and C_{f2} are found using the flow rate as,

$$C_{f1} = Q/(\pi D_1 B_1) = 0.01/(\pi \times 0.2 \times 0.003) = 5.31 \text{ m/s} \quad (4)$$

$$C_{f2} = Q/(\pi D_1 B_1) = 0.01/(\pi \times 0.37 \times 0.003) = 2.87 \text{ m/s} \quad (5)$$

The shapes of the triangle at inlet of pump are shown in Figure 1.30.

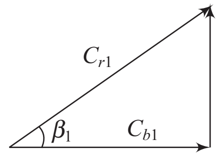


Figure 1.30 *Inlet Velocity Triangle*

$$\tan \beta_1 = \frac{C_{f1}}{C_{b1}} = \frac{5.31}{15.39}$$

$$\beta_1 = 19.04° \quad (6)$$

The shapes of the triangles at outlet of pump are shown in Figure 1.31.

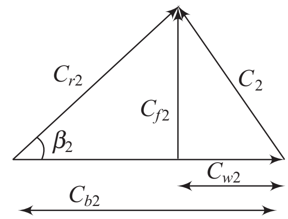


Figure 1.31 *Outlet Velocity Triangle*

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} = \frac{2.87}{24.48 - 16.52}$$

$$\beta_2 = 19.83° \quad (7)$$

1.6 Energy Transfer in Turbomachines

From velocity triangle at entry as shown in Figure 1.19,

$$C_{f1}^2 = C_1^2 - C_{w1}^2 = C_{r1}^2 - (C_{b1} - C_{w1})^2 = C_{r1}^2 - C_{b1}^2 - C_{w1}^2 + 2C_{b1}C_{w1}$$

$$C_{b1}C_{w1} = \frac{(C_1^2 + C_{b1}^2 - C_{r1}^2)}{2} \quad (1.96)$$

From velocity triangle at exit,

$$C_{f2}^2 = C_2^2 - C_{w2}^2 = C_{r2}^2 - (C_{b2} - C_{w2})^2 = C_{r2}^2 - C_{b2}^2 - C_{w2}^2 + 2C_{b2}C_{w2}$$

$$C_{b2}C_{w2} = \frac{(C_2^2 + C_{b2}^2 - C_{r2}^2)}{2} \quad (1.97)$$

Using Eq. (1.96) and Eq. (1.97) in head Eq. (1.84), we get,

$$w = gH_e = \frac{(C_1^2 + C_{b1}^2 - C_{r1}^2)}{2} - \frac{(C_2^2 + C_{b2}^2 - C_{r2}^2)}{2}$$

$$w = gH_e = \frac{(C_1^2 - C_2^2)}{2} + \frac{(C_{b1}^2 - C_{b2}^2)}{2} + \frac{(C_{r2}^2 - C_{r1}^2)}{2} \quad (1.98)$$

Equation (1.98) represents another form of Euler equation for a turbomachine. The specific work as given by Eqs (1.81), (1.83) and (1.98) is known as *Euler work.*

For steady state one dimensional isentropic flow through a turbomachine, Eq. (1.49) gives,

$$w = \frac{(C_1^2 - C_2^2)}{2} + (h_1 - h_2) + g(Z_1 - Z_2) \tag{1.99}$$

Since for isentropic incompressible flow, $dh = vdp = dp/\rho$ as ρ is constant for incompressible flow, therefore, for incompressible flow turbomachines, specific work from energy equation is,

$$w = \frac{(C_1^2 - C_2^2)}{2} + \frac{(p_1 - p_2)}{\rho} + g(Z_1 - Z_2) \tag{1.100}$$

Following components of energy transfer from Eqs. (1.98), (1.99) and (1.100) are noteworthy:

- One component is $(C_1^2 - C_2^2)/2$ which is change of the absolute kinetic energy of the fluid between the inlet and outlet of the rotor, i.e. it is the energy transfer due to interaction between the fluid and rotor which is known as the energy transfer due to *impulse effect*.
- The second term $(C_{b1}^2 - C_{b2}^2)/2$ indicates the change in fluid energy due to movement of fluid from one radius of rotation to another. Since the centrifugal force cause the rotation of the fluid element, therefore, it is the energy transfer due to the centrifugal action of the rotor. This component of energy transfer is equal to the static enthalpy change across the rotor for compressible flow turbomachines, whereas it is equal to the static pressure change of the fluid for incompressible flow turbomachines.
- The third term, $(C_{r2}^2 - C_{r1}^2)/2$, shows the energy transfer due change in fluid velocity relative to the rotor. This energy transfer is the effect of flow area on the relative velocity. A convergent passage between the blades as in turbines in the flow direction increases the relative velocity ($C_{r2} > C_{r1}$) decreasing the pressure equivalent to potential energy change. Similarly, a divergent passage between the blades as in pumps or compressors in the flow direction decreases the relative velocity ($C_{r2} < C_{r1}$) increasing the pressure equivalent to potential energy change. This component of energy transfer is equal to the potential energy change. For compressible flow machines, the contribution of energy transfer due to potential energy change is negligible.
- The second and third component of energy transfer together represents the energy transfer due to the reaction effect which is equal to the piezometric pressure change for incompressible flow machines.

EXAMPLE 1.9 Water is issuing out of a stationary nozzle of 50 mm diameter at 10 m/s velocity and atmospheric pressure glides along a curved vane moving at 3 m/s along a straight line (Figure 1.32). Estimate (a) the magnitude and direction of resultant force acting on the vane, (b) the magnitude and direction of the absolute velocity of water at outlet from the vane, (c) the work that can be done by the vane under the action of the water jet, and (d) change in kinetic energy of water in moving along the vane.

Solution

Since the vane is moving away at 3 m/s, the water with a nozzle velocity of 10 m/s, approaches the vane at $C - C_b = 7$ m/s. If the retardation of water jet due to friction on the vane surface is neglected, the relative velocity of water is unchanged from inlet to outlet. Consequently, the jet cross-sectional area is maintained all along its path. The control volume at rest is chosen to coincide with the interior of the vane and exterior of the jet.

$$\text{Nozzle area} = \frac{\pi}{4} \times 0.05^2 = 1.9635 \times 10^{-3}\ \text{m}^2 \tag{1}$$

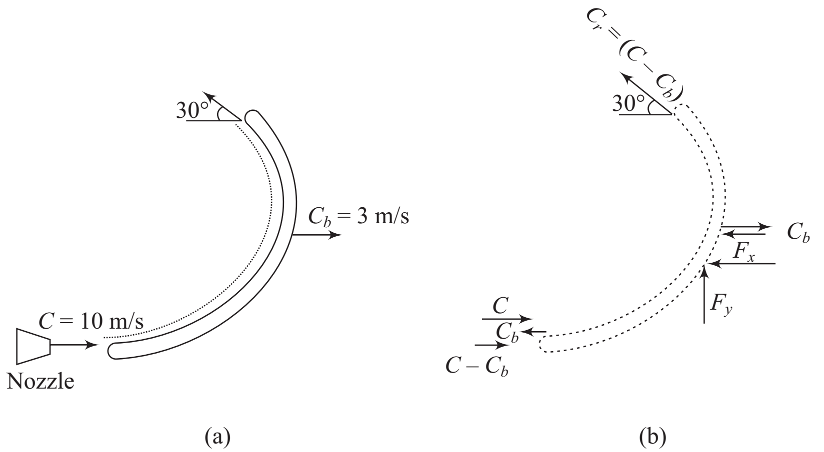


Figure 1.32 *Jet of Water on Moving Curved Plate*

Mass of water flowing along the vane i.e. relative to the control volume,

$$\dot{m} = \rho A(C - C_b) = 1000 \times 1.9635 \times 10^{-3} \times (10 - 3)$$

$$\dot{m} = 13.74 \text{ m/s} \tag{2}$$

F_x and F_y are the forces from the vane on the water jet. Since the nozzle exit is at atmospheric pressure, the net pressure effect is zero. Hence,

$$F_x = \dot{m}(C - C_b)\cos 30 - [-\dot{m}(C - C_b)] = \dot{m}(C - C_b)[1 + \cos 30] \tag{3}$$

$$F_x = 13.74 \times (10 - 3)(1 + \cos 30) = 179.47 \text{ N} \tag{4}$$

$$F_y = \dot{m}(C - C_b)\sin 30 - 0 \tag{5}$$

$$F_y = 13.74 \times (10 - 3)\sin 30 = 48.1 \text{ N} \tag{6}$$

The forces acting on the vane are due to the reaction of F_x and F_y (Figure 1.33).

$$R_x = -F_x = -179.47 \text{ N} \tag{7}$$

$$R_y = -F_y = -48.1 \text{ N} \tag{8}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{179.47^2 + 48.1^2}$$

$$R = 185.804 \text{ N} \tag{9}$$

$$\tan \varphi = \frac{R_y}{R_x} = \frac{-48.1}{-179.47}$$

$$\varphi = 15° \tag{10}$$

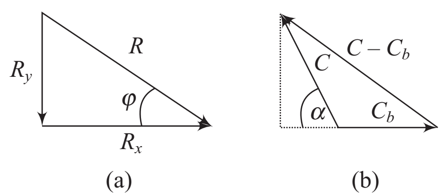


Figure 1.33 *Forces and Velocities on the Vane respectively*

The absolute velocity of water at exit from the vane is obtained by vector addition of the relative velocity and the vane velocity.

$$C_0^2 = [(C - C_b)\sin 30]^2 + [(C - C_b)\cos 30 - C_b]^2$$

$$C_0^2 = \sqrt{3.06^2 + 3.5^2}$$

$$C_0 = 4.65 \text{ m/s} \tag{11}$$

$$\tan\alpha = \frac{[(C - C_b)\sin 30]}{[(C - C_b)\cos 30 - C_b]} = \frac{[(10-3)\sin 30]}{[(10-3)\cos 30 - 3]}$$

$$\alpha = 48.82° \tag{12}$$

The vane can do work under the action of the forces on it in the direction of its motion. Thus,

$$\dot{W} = R_x C_b \tag{13}$$

$$\dot{W} = 179.47 \times 3 = 538.41 \text{ W} \tag{14}$$

The absolute velocities of water at inlet and outlet are 10 m/s and 4.65 m/s, respectively. Therefore, the change in kinetic energy is,

$$\Delta KE = \frac{1}{2}\dot{m}(C_0^2 - C_i^2) = \frac{1}{2} \times 13.74 \times (4.65^2 - 10^2)$$

$$\Delta KE = -538.45 \text{ W} \tag{15}$$

1.7 Slip

Consider infinite number of blades (ideal case) of infinitesimal thickness (here, the vane congruent flow takes place; streamlines of the flow is congruent to the vanes) on a centrifugal impeller as shown in Figure 1.34. The concave and convex sides of a blade are respectively the surfaces of positive and negative pressure, as marked in the figure. A fluid particle, besides having a primarily radial (outward) motion due to centrifugal action, has a tendency to move in the circumferential direction from the concave side. This tendency arises

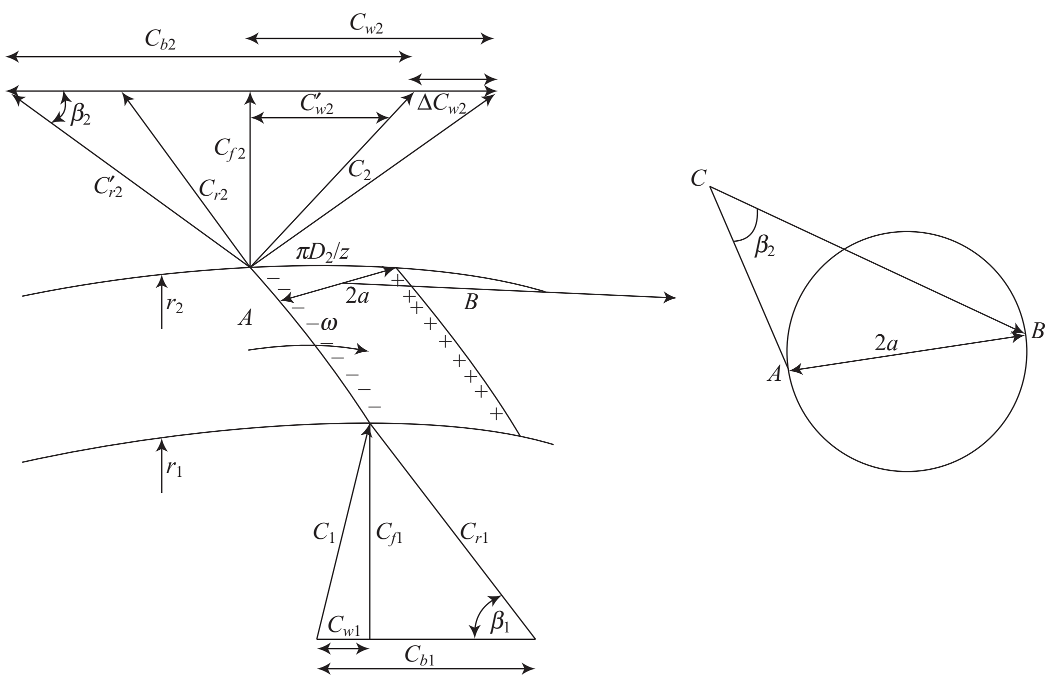


Figure 1.34 *Slip in Outlet Velocity Triangle*

due to the pressure differential between concave side of one blade to the convex side of adjacent blade. Such a cross motion is called a secondary flow since circumferential velocity is much smaller than main radial velocity.

The effect of this secondary flow is to cause a reduction of the whirl component of the outlet velocity, equal to

$$\Delta C_{w2} = C_{w2} - C'_{w2} = a\omega \tag{1.101}$$

where a is the representative radius of an imaginary circle drawn between the pressure and suction surface, as shown in Figure 1.34. Referring to Fig. 1.34,

$$BC = \frac{2a}{\sin\beta_2} \sim \text{Arc } AB = \frac{\pi D_2}{z}$$

This radius can be algebraically shown to be equal to,

$$a = \frac{\pi D_2 \sin\beta_2}{2z} \tag{1.102}$$

Here, z is the number of blades of the impeller. Substituting Equation (1.102) in Equation (1.101), we get,

$$\Delta C_{w2} = \frac{\pi C_{b2} \sin\beta_2}{z} \tag{1.103}$$

Loss of head due to secondary flow is known as slip, and is given by,

$$H_{\text{slip}} = \frac{C_{b2}\Delta C_{w2}}{g} \tag{1.104}$$

Net theoretical head by an impeller is,

$$H_{\text{th}} = H_e - H_{\text{slip}} = \frac{(C_{b2}C_{w2} - C_{b1}C_{w1})}{g} - \frac{C_{b2}\,(C_{w2} - C'_{w2})}{g} \tag{1.105}$$

$$\therefore \qquad H_{\text{th}} = \frac{(C_{b2}C'_{w2} - C_{b1}C_{w1})}{g} \tag{1.106}$$

By defining slip factor as,

$$\sigma_s = \frac{C'_{w2}}{C_{w2}} \tag{1.107}$$

$$H_{\text{th}} = \frac{\sigma_s C_{b2}C_{w2} - C_{b1}C_{w1}}{g} \tag{1.108}$$

$$C_{w2} = C_{b2} - C_{f2}\cot\beta_2 = C_{b2} - C_{f1}\cot\beta_2 \tag{1.109}$$

Substituting Eqs. (1.101) and (1.103) in Equation (1.106), we get,

$$H_{\text{th}} = \frac{\left[C_{b2}\left(C_{w2} - \dfrac{\pi C_{b2}\sin\beta_2}{z}\right) - C_{b1}C_{w1}\right]}{g} \tag{1.110}$$

For a purely radial inlet machine, prewhirl is zero, $C_{w1} = 0$

$$H_{\text{th}} = \frac{C_{b2}\left(C_{w2} - \dfrac{\pi C_{b2}\sin\beta_2}{z}\right)}{g} \tag{1.111}$$

Note that σ_s, calculated by equating Eqs. (1.108) and (1.110) is known as Stodola's slip factor.

EXAMPLE 1.10 A blower delivers 1 m^3/s of air, the impeller speed being 1450 rpm. Given $D_2 = 0.55$ m, $D_1 = 0.24$ m, $B_1 = 0.115$ m, $B_2 = 0.045$ m and the number of vanes $z = 12$. Calculate the vane angles considering outlet whirl velocity of 39.1 m/s after slip (Figure 1.35). Assume zero whirl at inlet and 28.8 m/s whirl component at outlet, density of air as 1.15 kg/m^3.

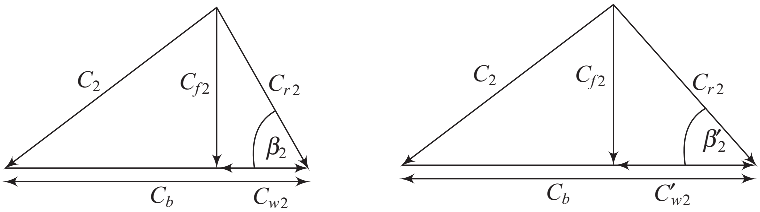


Figure 1.35 *Outlet Velocity Triangles without and with slip, respectively*

Solution

$$C_{b1} = \pi D_1 N_1/60 = \pi \times 0.24 \times 1450/60 = 18.2 \text{ m/s} \quad (1)$$

$$C_{b2} = \pi D_2 N_2/60 = \pi \times 0.55 \times 1450/60 = 41.8 \text{ m/s} \quad (2)$$

$$C_{f1} = Q/(\pi D_1 B_1) = 1.0/(\pi \times 0.24 \times 0.115) = 11.53 \text{ m/s} \quad (3)$$

$$C_{f2} = Q/(\pi D_2 B_2) = 1.0/(\pi \times 0.55 \times 0.045) = 12.86 \text{ m/s} \quad (4)$$

$$\tan \beta_1 = \frac{C_1}{C_{b1}} = \frac{C_{f1}}{C_{b1}} = \frac{11.53}{18.2}$$

$$\beta_1 = 32.35° \quad (5)$$

$$C_{w2}' = 39.1 \text{ m/s}$$

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} = \frac{12.86}{41.8 - 28.8} \quad (6)$$

$$\beta_2 = 44.69° \quad (7)$$

$$\tan \beta_2' = \frac{C_{f2}}{C_{b2} - C_{w2}'} = \frac{12.86}{41.8 - 39.1}$$

$$\beta_2' = 78.14° \quad (8)$$

1.8 Degree of Reaction

An important aspect in the design of the runners is the relative values of the impulse effect and reaction effect. Degree of reaction is a measure of the degree of energy transfer by reaction effect out of the total energy transfer in the runner or rotor. *Degree of reaction* is defined as the ratio of the energy transfer by the reaction effect i.e. by the change of static pressure to the total energy transfer in the runner.

$$\text{Degree of Reaction} = R = \frac{\text{Energy transfer due to reaction effect or pressure change}}{\text{Total energy transfer}} \quad (1.112)$$

$$R = \frac{(C_{b1}^2 - C_{b2}^2) + (C_{r2}^2 - C_{r1}^2)}{(C_1^2 - C_2^2) + (C_{b1}^2 - C_{b2}^2) + (C_{r2}^2 - C_{r1}^2)} \tag{1.113}$$

Using Eq. (1.98), Eq. (1.99) and components of energy transfer, degree of reaction for compressible flow turbomachines may also be given as,

$$\begin{aligned} R &= \frac{(h_1 - h_2) + g(Z_1 - Z_2)}{w} = \frac{(h_1 - h_2) + g(Z_1 - Z_2)}{gH_e} \\ &= \frac{(h_1 - h_2) + g(Z_1 - Z_2)}{(h_{01} - h_{02}) + g(Z_1 - Z_2)} \approx \frac{(h_1 - h_2)}{gH_e} \end{aligned} \tag{1.114}$$

Using Eq. (1.98), Eq. (1.100) and component of energy transfer, degree of reaction for incompressible flow turbomachines may also be given as,

$$\begin{aligned} R &= \frac{\left(\frac{p_1 - p_2}{\rho}\right) + g(Z_1 - Z_2)}{w} = \frac{\left(\frac{p_1 - p_2}{\rho}\right) + g(Z_1 - Z_2)}{gH_e} \\ &= \frac{1}{H_e}\left[\frac{(C_{r2}^2 - C_{r1}^2)}{2g} + \frac{(C_{b1}^2 - C_{b2}^2)}{2g}\right] = \frac{H_e - (C_1^2 - C_2^2)/2\,\text{g}}{H_e} \end{aligned} \tag{1.115}$$

Degree of reaction for a reaction turbine lies in the range of 0–1. A turbine runner with nonzero degree of reaction must necessarily be completely enclosed in a casing for the development of pressure head.

EXAMPLE 1.11 A radial turbine rotates at 150 rpm. The relative velocity at inlet C_{r1} = 1.465 m/s and equals 6 m/s at exit. The absolute velocity at inlet C_1 = 7.593 m/s and equals 6.647 m/s at exit. The physical data are: R_1 = 0.5 m, R_2 = 0.2 m. Calculate the degree of reaction.

Solution

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 15.7 \text{ rad/s} \tag{1}$$

$$C_{b1} = \omega R_1 = 15.7 \times 0.5 = 7.85 \text{ m/s} \tag{2}$$

$$C_{b2} = \omega R_2 = 15.7 \times 0.2 = 3.14 \text{ m/s} \tag{3}$$

Degree of reaction is given by,

$$R = \frac{(C_{b1}^2 - C_{b2}^2) + (C_{r2}^2 - C_{r1}^2)}{(C_{b1}^2 - C_{b2}^2) + (C_{r2}^2 - C_{r1}^2) + (C_1^2 - C_2^2)} \tag{4}$$

Substituting the values of C_b, C_r and C at inlet and outlet in above Eq. (4),

$$R = \frac{(7.85^2 - 3.14^2) + (6^2 - 1.465^2)}{(7.85^2 - 3.14^2) + (6^2 - 1.465^2) + (7.593^2 - 6.647^2)}$$

$$R = 0.864 \tag{5}$$

1.9 Impact of Jets

The jet is a stream of liquid coming out from a nozzle with a high velocity under constant pressure. When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force is exerted. Vane is a flat or curved plate fixed to the rim of the plate. The following cases of the impact of jet, i.e. the force exerted by the jet on the vane are discussed.

1.9.1 Force Exerted by Fluid on Stationary Curved Plate

The jet impinges on a curved plate at angles α_1 and α_2 at the inlet and the exit respectively. Both angles are measured with respect to x direction, as shown in Figure 1.36. Let C_1 and C_2 be the velocities of jet at the inlet and outlet, respectively. The velocities C_1 and C_2 will be same as long as there is no friction on the plate.

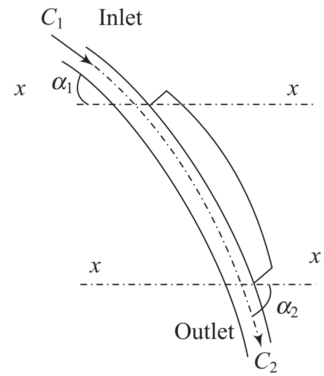


Figure 1.36 *Jet of Water on Curved Plate*

Velocity of jet at inlet in x direction = $C_1 \cos \alpha_1$

Velocity of jet at outlet in x direction = $C_2 \cos \alpha_2$

Therefore, force exerted on the jet by the plate in x direction can be determined by applying linear momentum equation.

$$F_x = \frac{m}{t} \times \text{Change of velocity in } x \text{ direction}$$

$$F_x = \rho Q(C_2 \cos \alpha_2 - C_1 \cos \alpha_1) \tag{1.116}$$

And the force exerted on the plate by the jet in x direction,

$$F_x = \rho Q(C_1 \cos \alpha_1 - C_2 \cos \alpha_2) \tag{1.117}$$

As, $$Q = aC_1 \tag{1.118}$$

Therefore, $$F_x = \rho a C_1(C_1 \cos \alpha_1 - C_2 \cos \alpha_2) \tag{1.119}$$

If the curvature of the plate at outlet is such that outlet angle $\alpha_2 > 90°$, then the second term in the bracket, i.e. $C_2 \cos \alpha_2$, will be negative. Hence, in order to get more force, the curvature of the plate at outlet should be with an obtuse angle α_2.

1.9.2 Force Exerted by Fluid on Single Moving Curved Plate

Let the angle of curvature of the plate of inlet and outlet with the reversed direction of motion of plate, i.e. $-C_b$ be β_1 and β_2 (Figure 1.37). The plate is moving with a velocity C_b in x direction. Thus, the velocity of jet relative to the motion of the plate is denoted by C_{r1}. Its direction is tangential to the point of inlet. Its magnitude is determined by the vector sum of C_{b1} and C_1.

When the jet leaves the plate, its relative velocity will remain equal to C_{r1} provided there is no decrease in velocity due to friction on the surface of flow, i.e. $C_{r1} = C_{r2}$. Now the absolute velocity of water at the outlet C_2 will be vector sum of C_{r2} and C_{b2}.

Therefore, force exerted by the jet on the plate in x direction or in the direction of motion is determined by applying linear momentum equation:

$$F_x = \frac{m}{t} \times \text{Change of velocity in } x \text{ direction}$$

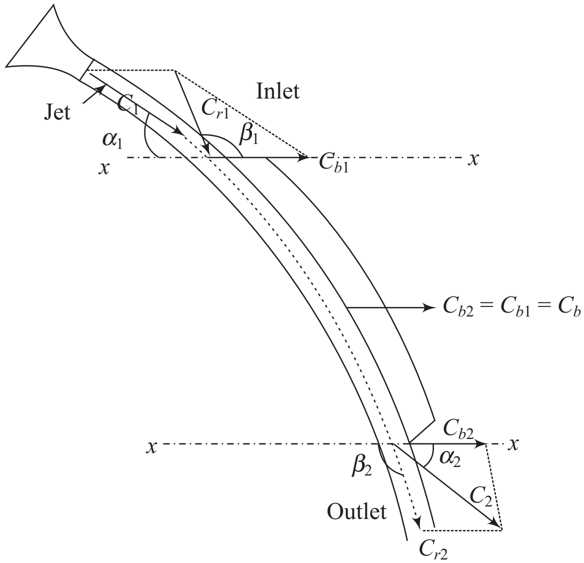


Figure 1.37 *Jet of Water on Single Moving Curved Plate*

$$F_x = \rho Q(C_1 \cos \alpha_1 - C_2 \cos \alpha_2)$$

$$Q = a(C_1 - C_{b1}) \tag{1.120}$$

$$F_x = \rho a(C_1 - C_{b1})(C_1 \cos \alpha_1 - C_2 \cos \alpha_2) \tag{1.121}$$

For $\alpha_2 > 90° \Rightarrow \cos \alpha_2 < 0$, then the second term in the bracket, $C_2 \cos \alpha_2$, will be negative. Hence, in order to get more force, the curvature of the plate should be such that α_2 is obtuse.

EXAMPLE 1.12 A jet of water (Figure 1.38) 60 mm in diameter has a velocity of 20 m/s and impinges normally on a vertical stationary plate. Find the force exerted on the plate due to the jet impingement.

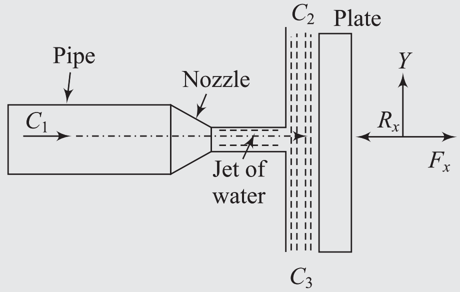


Figure 1.38 *Jet of Water Impinges on Flat Plate*

Solution

The pressure in the jet is atmospheric at the boundaries of the control volume. Since the jet impinges normally on the vertical plate and spreads radially, the force on the plate is in the x direction. By assuming the plate to be frictionless, there is no forces on the plate in the radial y direction.

Let R_x is the reaction of the plate in the x direction acting on the fluid in the control volume. The force on the plate is equal and opposite to R_x. By steady state momentum in x direction applied to the control volume,

$$\Sigma\text{Forces in } x \text{ direction} = (\text{Momentum flux})_{\text{out}} - (\text{Momentum flux})_{\text{in}}$$

$$0 - R_x = \rho Q(0 - C_1)$$

$$R_x = F_x = \rho QC_1 = \rho aC_1^2 \quad (1)$$

$$F_x = 1000 \times \frac{\pi}{4} \times 0.06^2 \times 20^2$$

$$F_x = 1130.97 \text{ N} \quad (2)$$

F_x is in positive x direction.

1.10 Aerodynamics of Turbomachinery Blading

1.10.1 Blade Element Theory

The momentum equation does not take into account number of blades, characteristics of airfoil, viz. lift, drag, zero lift angle, contour of the blades whether taper, curved, root cut-out, distribution of blade twist and effects of compressibility. Blade element theory is relatively a simple method for analyzing the performance of propellers as well as wind mills, fans, etc. The assumptions made in the blade element theory are as follows:

- The blade is composed of infinite number of small, narrow strips or sections and each section is considered as a two dimensional airfoil to produce aerodynamic forces.
- There is no interaction between the different blade sections where all the calculations are made using two dimensional aerodynamics.
- Wake expansion is not taken into account; however, the correction factors can be taken for the same.
- Yaw is not taken into account.
- Steady state conditions are considered.

In the blade element theory, each blade is considered in isolation (Figure 1.39). As the blade is made to move in the otherwise quiescent air, there is a relative motion between the air and the blade. The air flow on the upper surface experiences suction (negative) pressure and the air on the lower surface, a positive pressure.

In order to estimate the amount of power (torque × rotational speed) required to rotate the blade creating certain amount of pressure difference and flow rate, the blade element theory is applied.

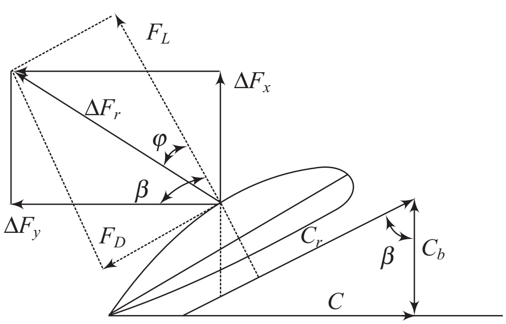


Figure 1.39 *Forces on Blade*

Here, flow has a mean velocity (C_r) and direction (β), Lift (F_L), Drag (F_D), chord length(l), dr – small radial thickness and ρ – density, z – Number of blades, Thrust - Axial forces normal to the mean velocity (ΔF_x).

Torque/radius = Tangential forces normal to the mean velocity (ΔF_y)

$$\Delta F_x = F_L \sin\beta - F_D \cos\beta \tag{1.122}$$

$$\Delta F_y = F_L \cos\beta + F_D \sin\beta \tag{1.123}$$

$$F_L = \frac{1}{2} C_l \rho C_r^2 (ldr) \tag{1.124}$$

$$F_D = \frac{1}{2} C_d \rho C_r^2 (ldr) \tag{1.125}$$

$$\tan\varphi = \frac{F_D}{F_L} = \frac{C_d}{C_l} \tag{1.126}$$

where $ldr = dA$; C_l and C_d are the coefficients of lift and drag respectively.
From Eqs (1.122) and (1.126), we get,

$$\Delta F_x = F_L(\sin\beta - \tan\varphi \cos\beta) \tag{1.127}$$

Substituting Eq. (1.124) in Eq. (1.127), we get,

$$\Delta F_x = \frac{1}{2} z C_l \rho C_r^2 (ldr)\,(\sin\beta - \tan\varphi\cos\beta) \tag{1.128}$$

Similarly, we get,

$$\Delta F_y = \frac{1}{2} z C_l \rho C_r^2 (ldr)\,(\sin\beta + \tan\varphi\cos\beta) \tag{1.129}$$

EXAMPLE 1.13 A wind turbine is designed to work at a condition with a wind speed of 10 m/s and an air density of 1.22 kg/m^3. The turbine has blades with a NACA 0012 profile and is rotating at one revolution per second. The blade chord length is 0.5 m. Ignoring the drag on the aerofoil, estimate the lift force for blade span at a radius of 6 m. Coefficient of lift, $C_l = 1.1$.

Solution

$$C_b = \omega r = 2\pi N r = 2\pi \times 1 \times 6 = 37.7 \text{ m/s} \tag{1}$$

Calculating the relative velocity,

$$C_r = \sqrt{C^2 + C_b^2} = \sqrt{10^2 + 37.7^2} = 39 \text{ m/s} \tag{2}$$

Lift is given by,

$$\Delta L = \frac{1}{2} C_l \rho C_r^2 (ldr) \tag{3}$$

Taking $dr = 1$, we get lift force as,

$$\Delta L = \frac{1}{2} \times 1.1 \times 1.22 \times 39^2 \times 0.5$$

$$\Delta L = 510.297 \text{ N} \tag{4}$$

1.10.2 Slip Stream Theory

Momentum equation and blade element theory does not account for the secondary effects caused by three dimensional flow velocities generated on the propeller by the blade tip vortex or radial components of flow velocities induced by angular acceleration due to the rotation of the propeller.

Slip stream theory is purely dynamic reaction theory for any kind of actuator which produces slipstream or jet. This theory is applied to un-shrouded turbomachines such as aircraft propellers and wind mills. This theory is very useful for optimizing the pitch of the blade setting for a given speed or in finding the optimum solidity of blade for a propeller. Assumptions of fluid stream theory are as follows:

- Fluid is incompressible and inviscid.
- The rotation of fluid within the stream tube housing the rotor of the propeller is considered.
- Stream tube is exposed infinitely far upstream from the rotor to a plane where the static pressure is constant and equal to the free stream static pressure.
- The axial component of velocity is the free stream velocity, and there is no peripheral component of velocity.
- Uniform axial velocity but angular velocity is not considered uniform.
- Pressure in the stream tube is not considered uniform in order to study the effect of rotation.

Let C_0 be the forward speed of the propeller and A be the cross-section of the disc $\left(A = \frac{\pi}{4}D^2\right)$. We know that,

$$\dot{m} = \rho A C_1 \tag{1.130}$$

Axial thrust on the disc = Change in momentum flow rate

$$F_{\text{axial}} = \text{Mass flow rate } (\dot{m}) \text{ in tube} \times \text{change in velocity}$$

$$F_{\text{axial}} = \rho A C_1 (C_2 - C_0) \tag{1.131}$$

Applying Bernoulli equation for flows in the regions upstream and downstream of the disc (Figure 1.40).

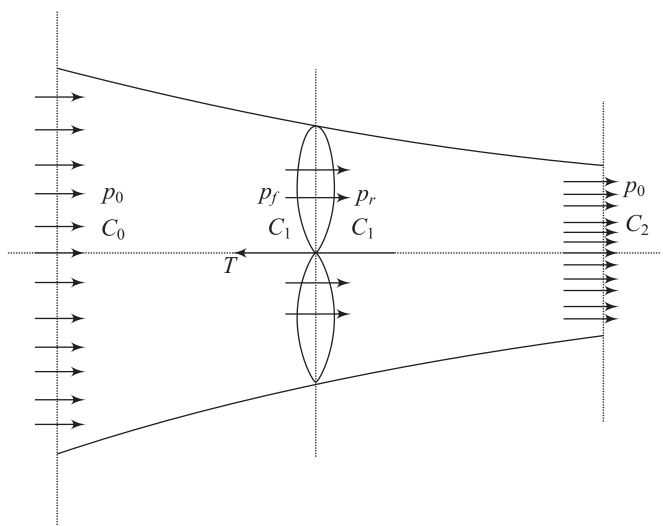


Figure 1.40 *Fluid Flow through Propeller*

$$p_0 + \frac{1}{2}\rho C_0^2 = p_f + \frac{1}{2}\rho C_1^2 \tag{1.132}$$

$$p_0 + \frac{1}{2}\rho C_2^2 = p_r + \frac{1}{2}\rho C_1^2 \tag{1.133}$$

Subtracting Eq. (1.132) from Eq. (1.133),

$$\Delta p = p_r - p_f = \frac{1}{2}\rho(C_2^2 - C_0^2) \tag{1.134}$$

Axial thrust due to the pressure difference across the disc is,

$$F_{\text{axial}} = A(p_r - p_f)$$

$$F_{\text{axial}} = \frac{1}{2}\rho A(C_2^2 - C_0^2) \tag{1.135}$$

Comparing Eqs. (1.131) and (1.135), we get,

$$C_1 = \frac{1}{2}(C_2 + C_0) \tag{1.136}$$

Applying continuity equation at the disc and the slipstream section gives,

$$C_1 \frac{\pi}{4} D^2 = C_2 \frac{\pi}{4} D_2^2$$

$$D_2^2 = \frac{C_1}{C_2} D^2 \tag{1.137}$$

Engine power (P) driving the propeller is consumed by the work of the axial thrust (F_{axial}) driving the propeller disc with velocity C_0 and partly by the increase of kinetic energy of the slip stream per unit of time.

$$P = \frac{\rho A C_1(C_2^2 - C_0^2)}{2} + F_{\text{axial}} C_0 \tag{1.138}$$

Substituting Eqs. (1.135) and (1.136) value in Eq. (1.138),

$$P = \frac{\rho A(C_2^2 - C_0^2)(C_2 + 3C_0)}{4} \tag{1.139}$$

Propulsion efficiency,

$$\eta_p = \frac{\text{Power used in propulsion}}{\text{Ideal power supplied}} = \frac{F_{\text{axial}} C_0}{P_{\text{ideal}}} \tag{1.140}$$

Ideal power given to the propeller,

$$P_{\text{ideal}} = F_{\text{axial}} C_1 = \frac{1}{2}\rho A C_1(C_2^2 - C_0^2) \tag{1.141}$$

Substituting the value of P_{ideal} from Eq. (1.141) into Eq. (1.140),

$$\eta_p = \frac{C_0}{C_1} \tag{1.142}$$

1.11 Losses in Turbomachines

The process of energy transfer in a turbomachine is always associated with the losses in the direction of flow. These losses could be divided into two groups:

1.11.1 Internal Losses

These losses occur in the inner passages of the turbomachine. These losses tend to increase the heat capacity of the flow media, thereby raising the end temperatures of the fluid discharged compared to an ideal isentropic flow.

(a) Hydraulic Losses

Losses due to friction in fluid channels, separation of the flow on the vane or shroud surfaces, diffusion, eddies and mixing of different energy levels fluids immediately at the channel exit are hydraulic losses. All are calculated from the inlet to the discharge flange of the turbomachine.

In case of a turbomachine, if w is the required specific energy to be developed and Δw_{hyd} is the equivalent of the hydraulic losses, theoretical specific work that has to be developed by the impeller blades of the pump/compressor can be represented as,

$$w_{bl} = w + w_{\text{hyd}} \tag{1.143}$$

Theoretical specific work that has to be developed by the impeller blades of the turbine can be represented as

$$w_{bl} = w - w_{\text{hyd}} \tag{1.144}$$

Hydraulic losses are made up of frictional losses, profile losses, secondary flow losses, tip clearance losses (axial machines), cavitation losses (pumps only), shock or incidence losses and annulus wall boundary layer losses (axial machines only).

(b) Leakage Losses

Leakage of the fluid media could result in a serious loss in the power output of the machine. For power absorbing machines, leakage of the energy added fluid would result in increased amount of coupling power. For power producing machines, leakage of the energy added fluid would result in loss in the power developed. Leakage occurs along the casing from the pressure side to suction side of the impeller between the casing and the impeller. It also results in loss of flow rate.

If the leakage flow could be denoted as ΔQ, then the flow through pump/compressor impeller blade channels could be given as,

$$Q' = Q + \Delta Q \tag{1.145}$$

Flow through turbine impeller blade channels could be given as,

$$Q' = Q - \Delta Q \tag{1.146}$$

The leakage volume can be estimated for a machine with known clearance size and pressure difference across the clearance.

(c) Disc Friction Losses

When disc is rotated in an enclosed chamber surrounded by fluid, a resistive torque is setup and power consumption in the case of fan or pump increases in order to overcome this resistive torque. The loss will be appreciable for axial machines with blades mounted on discs having large hub to tip ratio, D_h/D_t, and in radial machines with large D_2/D_1 diameter ratio. Disc friction is necessarily dependent on the clearances between the casing and the rotating shrouds or disc, the diameter of the impeller, roughness of the disc surfaces, viscosity and density of the fluid medium.

(d) Return Flow Losses

In case of axial flow type, particularly pumps and compressors, a return flow of the energy added takes place under off design conditions of operation. These losses are severe at discharge much less than design rates.

(e) Internal Fluid Power

Internal fluid power can be defined considering all internal losses namely leakage loss, hydraulic losses, disc friction loss and return flow loss. For pump/compressor, referring to Figure 1.41,

$$P_{\text{internal}},\ P_i = \rho g(Q + \Delta Q)(H + h_c + h_i) \tag{1.147}$$

where h_i = losses in impeller + disc friction + return flow, H = Head (manometric) and h_c = losses in casing

Internal fluid power for turbine, referring to Figure 1.42

$$P_{\text{internal}},\ P_i = \rho g(Q - \Delta Q)(H - h_c - h_i) \tag{1.148}$$

The internal specific work of any turbomachine can be expressed in the form,

$$w_{\text{internal}} = \frac{P_{\text{internal}}}{\dot{m}} \tag{1.149}$$

Referring to Figure 1.41, for pumps/compressors

Water power (WP) (ρQgH) < Impeller power (IP) {$\rho(Q + \Delta Q)\ g(H + h_c)$ or $\rho(Q + \Delta Q)\ gH_{th}$ or $\rho(Q + \Delta Q)\ gH_e$} < Internal Power or Rotor Power (RP) {$\rho(Q + \Delta Q)g(H + h_c + h_i)$ < Shaft Power (SP).

Referring to Figure 1.42, for turbines

Water power (WP) (ρQgH) > Impeller power (IP) {$\rho(Q - \Delta Q)\ g(H - h_c)$ or $\rho(Q - \Delta Q)\ gH_{th}$ or $\rho(Q - \Delta Q)\ gH_e$} > Internal Power or Rotor Power (RP) {$\rho(Q - \Delta Q)g(H - h_c - h_i)$ > Shaft Power (SP)

1.11.2 External Losses

The external losses, also known as mechanical losses, are those external to the flow medium, namely in the bearings, sealings, couplings and other auxiliary equipment that may be directly coupled to the turbomachine shaft.

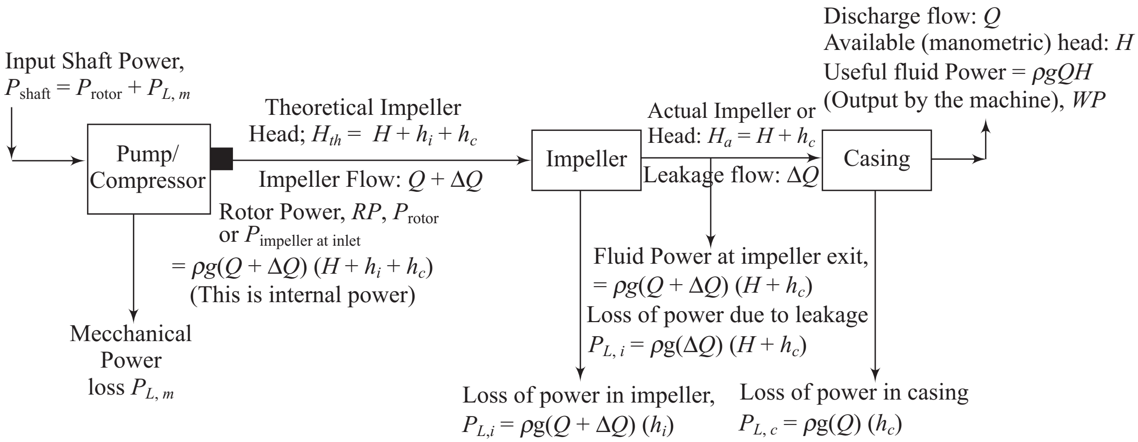
In the case of high speed compressors operated through gear, a loss in the gear will arise, which is accounted to the external losses.

Shaft Power (SP) or Coupling power $P_c = P_i + P_{L,\,m}$, for pump or compressor

Shaft Power (SP) or Coupling power $P_c = P_i - P_{L,\,m}$, for turbine

where $P_{L,\,m}$ is mechanical loss.

In Figures 1.41 and 1.42, h_i and h_c represent the respective lost heads in the impeller and casing and ΔQ represents the total leakage.



Useful Fluid Power = $P_{shaft} - \{P_{L,m} + (P_{L,i} + P_{L,l} + P_{L,c})\}$

Useful Fluid Power = $\rho gQH = P_{shaft} - \{P_{L,m} + (\rho g(Q + \Delta Q)(h_i) + \rho g(\Delta Q)(H + h_c) + \rho g(Q)(h_c))\}$

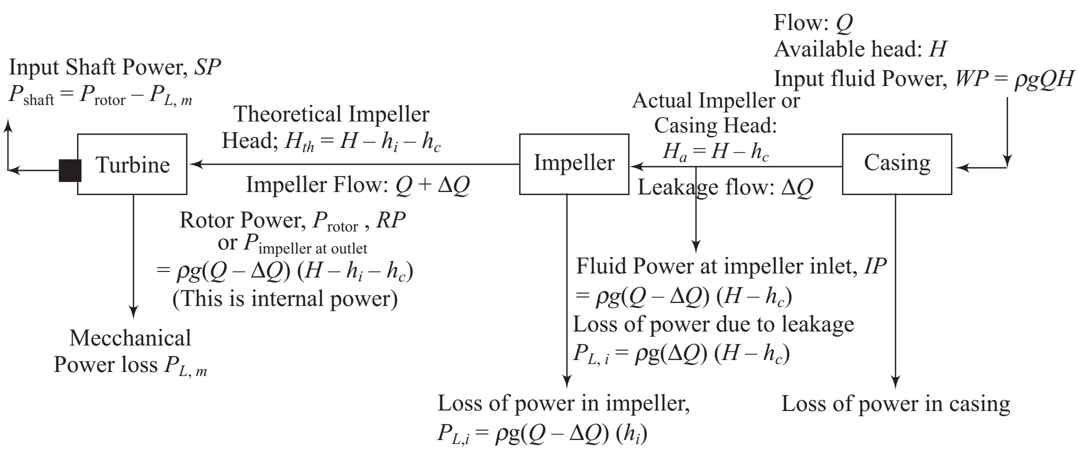
Useful Fluid Power = $\rho gQH = P_{rotor} - \{\rho g(Q + \Delta Q)(h_i) + \rho g(\Delta Q)(H + h_c) + \rho g(Q)(h_c))\}$

P_{rotor} or $P_{impeller} = \rho g(Q + \Delta_Q)(H + h_i + h_c)$

Note that Slip is neglected and $H_{euler} = H_{theoretical}$ from equation 1.99.

If the dynamic head across the pump is negligible, the available head is also called manometric head.

Figure 1.41 *Losses in Compressor/Pump*



Useful Fluid Power = $P_{shaft} - \{P_{L,m} + (P_{L,i} + P_{L,l} + P_{L,c})\}$

$P_{shaft} = \rho gQH - \{P_{L,m} + (\rho g(Q - \Delta Q)(h_i) + \rho g(\Delta Q)(H - h_c) + \rho g(Q)(h_c))\}$

$P_{rotor} = \rho gQH - \{\rho g(Q - \Delta Q)(h_i) + \rho g(\Delta Q)(H - h_c) + \rho g(Q)(h_c))\}$

P_{rotor} or $P_{impeller} = \rho g(Q - \Delta_Q)(H - h_i - h_c)$

Figure 1.42 *Losses in Turbine*

EXAMPLE 1.14 A single stage, radial flow, and double suction centrifugal pump has the following data:

Discharge = 72 litres/s, Inner diameter = 90 mm, Outer diameter = 280 mm, Revolution/minute = 1650, Head = 25 m, Width at inlet = 20 mm/side, Width at outlet = 18 mm/side, Absolute velocity angle at inlet = 90°, Leakage losses = 2 litres/s, Contraction factor due to vane thickness = 0.85, Relative velocity angle measured from tangential direction = 35° at outlet.

Determine (a) the inlet vane angle, (b) the angle at which the water leaves the wheel, and (c) the absolute velocity of water leaving impeller

Solution

Total quantity of water to be handled by the pump,

$$Q_t = Q_{\text{del}} + Q_{\text{leak}} \tag{1}$$

$$Q_t = 72 + 2 = 74 \text{ litres/s} \tag{2}$$

$$\text{Total quantity of water per side} = Q = \frac{74}{2} = 37 \text{ litres/s} \tag{3}$$

(a) Vane Angle at Impeller Inlet

$$C_{b1} = \pi D_1 N/60 = \pi \times 0.09 \times 1650/60$$

$$C_{b1} = 7.78 \text{ m/s} \tag{4}$$

$$\text{Flow area at the inlet} = \pi D_1 B_1 \times \text{Contraction factor} = \pi \times 0.09 \times 0.02 \times 0.85$$

$$\text{Flow area at the inlet} = 0.0048 \text{ m}^2 \tag{5}$$

Therefore, the velocity of flow at the inlet,

$$C_{f1} = Q/\text{Area of flow at inlet} = 0.037/0.0048$$

$$C_{f1} = 7.708 \text{ m/s} \tag{6}$$

From velocity triangle at the inlet as shown in Figure 1.43(a),

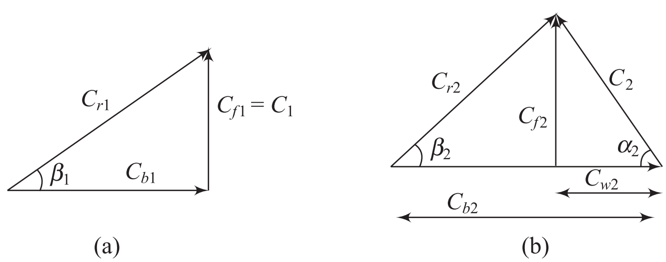


Figure 1.43 *Velocity Triangles for Example 1.16 (a) At Inlet, (b) At Outlet*

$$\tan \beta_1 = \frac{C_{f1}}{C_{b1}} = \frac{7.708}{7.78}$$

$$\beta_1 = 44.73° \tag{7}$$

(b) Angle of Water at Wheel Outlet

$$C_{b2} = \pi D_2 N/60 = \pi \times 0.28 \times 1650/60$$

$$C_{b2} = 24.2 \text{ m/s} \tag{8}$$

Flow area at the inlet = $\pi D_2 B_2 \times$ Contraction factor = $\pi \times 0.28 \times 0.018 \times 0.85$

Flow area at the outlet = 0.01346 m^2 (9)

Therefore, the velocity of flow at the outlet,

$C_{f2} = Q$/Area of flow at the inlet = 0.037/0.01346

C_{f2} = 2.749 m/s (10)

From velocity triangle at the outlet as shown in Figure 1.43(b),

$\tan \beta_2 = C_{f2}/(C_{b2} - C_{w2}) \Rightarrow \tan 35 = 2.749/(24.2 - C_{w2})$

C_{w2} = 20.27 m/s (11)

$\tan \alpha_2 = C_{f2}/C_{w2} = 2.749/20.27$

$\alpha_2 = 7.72°$ (12)

(c) Absolute Velocity of Water Leaving the Impeller

$C_2 = C_{w2}/\cos \alpha_2 = 20.27/\cos 7.72$

C_2 = 20.455 m/s (13)

1.12 Free and Forced Vortex

Vortex flow is defined as the flow of rotating mass of fluid or flow of fluid along curved path. The turbomachinery flows are akin to the vortex flows as they are also flows of rotating fluid mass.

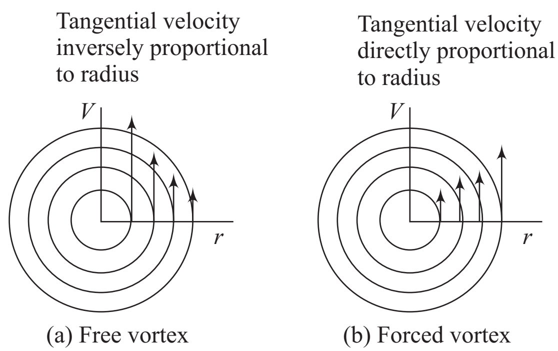


Figure 1.44 *Free and Forced Vortex*

1.12.1 Free Vortex

The fluid rotating under certain energy previously given to them is called free vortex flow [Figure 1.44(a)]. As such, the free vortex flows do not require external torque or energy. Therefore, overall energy of the flow remains constant. There is no energy interaction between an external source and the flow. There is no dissipation of mechanical energy in the flow. In this flow, fluid mass rotates due to conservation of angular momentum and velocity is inversely proportional to the radius. The free vortex flow can be represented by the equation,

$$C_w r = \text{constant} \quad (1.150)$$

At the center ($r = 0$) of rotation, velocity approaches infinity and hence, this point is singular. The free vortex flow is irrotational, and therefore, also known as the irrotational vortex. In free vortex flow, Bernoulli's equation can be applied.

In turbomachinery, flows in their casing and flows between stages are generally considered as free vortex flows. Equation (1.150) is used for the design of turbomachinery casings.

1.12.2 Forced Vortex

To maintain a forced vortex flow [Figure 1.44(b)], it requires a continuous supply of energy or external torque. All fluid particles rotate at the constant angular velocity ω as a solid body. Therefore, a flow of forced vortex is also called a solid body rotation. In forced vortex flows, therefore, the tangential velocity is directly proportional to the radius, expressed by,

$$C_w = r\omega \tag{1.151}$$

where ω = angular velocity and r = radius of fluid particle from the axis of rotation.

The surface profile of vortex flow is parabolic. In forced vortex, total energy per unit weight increases with an increase in radius. Forced vortex is not irrotational; rather, it is a rotational flow with constant vorticity 2ω.

In centrifugal flow machines, the flow inside the machine closely resembles the forced vortex motion. Equation (1.151) is used in the design of turbomachinery stages.

SUMMARY

- Turbomachines are classified as (i) incompressible and compressible flow machines, (ii) axial, radial, and mixed flow machines, (iii) power consuming (pump/fan/compressor) and power producing (turbine) machines, (iv) hydraulic/steam/gas/wind turbines, and (v) shrouded and unshrouded machines.
- Reynolds transport theorem, which links system and control volume, states:

$$\left(\frac{dN}{dt}\right)_s = \frac{\partial}{\partial t}\int_{CV} \eta\rho dV + \int_{CS} \eta\rho\vec{C}\cdot d\vec{A}$$

N is the extensive property of a system of mass, m.

η = N/m = Intensive property of the fluid flowing to control volume.

Name of the Governing Equation	*N*	η	*Equation*
Conservation of mass	m = constant	1	$\frac{\partial}{\partial t}\int_{CV}\rho dV + \int_{CS}\rho\vec{C}\cdot d\vec{A} = 0$
Conservation of linear momentum (Newton's second law)	$m\vec{C}$	$\vec{C}$	$\sum \overrightarrow{\text{Forces}} = \overrightarrow{F_B} + \overrightarrow{F_S} = \frac{\partial}{\partial t}\int_{CV}\vec{C}\rho dV + \int_{CV}\vec{C}\rho\vec{C}\cdot\overrightarrow{dA}$
Conservation of angular momentum (Euler's turbomachinery equation)	$m(\vec{r}\times\vec{C})$	$(\vec{r}\times\vec{C})$	$\sum \text{Torque} = \frac{\partial}{\partial t}\int_{CV}(r\times C)\rho dV + \int_{CS} r\times C\rho\vec{C}\cdot d\vec{A}$
Conservation of energy (first law of thermodynamics)	$\Delta E = \dot{Q} - \dot{W}_s - \dot{W}_p$	$e = \frac{E}{m}$ $e = u + gZ + C^2/2$	$\dot{Q} - \dot{W} = \frac{\partial}{\partial t}\int_{CV} e\rho dV + \int_{CS} e\rho\vec{C}\cdot d\vec{A}$ $\dot{Q} - \dot{W}_s = \frac{\partial}{\partial t}\int_{CV}\left(u + gz + \frac{1}{2}C^2\right)\rho dV + \int_{CS}\left(h + gz + \frac{1}{2}C^2\right)\rho\vec{C}\cdot d\vec{A}$

The governing equations in the table simplify for steady flow and condition such as 1-D flow and incompressible flow.

- Velocity triangles are drawn with the vector relation:

$$\vec{C} = \overrightarrow{C_b} + \overrightarrow{C_r}$$

here, $\overrightarrow{C_b}$ is the blade velocity, $\overrightarrow{C_r}$ is the relative velocity and $\vec{C}$ is the absolute velocity.

- Making the use of velocity triangle, the energy transfer in turbomachines can be grouped as in (i) centrifugal energy of the impeller, $\Delta C_b^2/2$, (ii) absolute kinetic energy change, $\Delta C^2/2$, and (iii) relative kinetic energy change $\Delta C_r^2/2$.
- The slip and slip factor are associated with the secondary flows between pressure and suction sides of a turbomachine. The slip is more prominent in radial machines. The expression for slip factor is given by,

$$\sigma_s = \frac{C'_{w2}}{C_{w2}}$$

where C'_{w2} is modified whirl component of velocity at the outlet due to the presence of secondary eddy.

- Degree of reaction indicates the ratio of energy transfer due to the reaction effect. It is defined as ratio of change in static pressure in the rotor to the total Euler head across the machine.
- Application of momentum equation is illustrated to determine the force exerted by fluid jet on a curved plate.
- The concept of lift and drag is introduced. The forces exerted on an airfoil in the x and y directions are expressed in terms of lift and drag forces.

$$\Delta F_x = F_L \sin\beta - F_D \cos\beta$$
$$\Delta F_y = F_L \cos\beta + F_D \sin\beta$$

- The actuated disc such as wind turbine rotor produces slip streams. A simple stream theory is illustrated in a flow through impeller.

$$P = \frac{\rho A(C_2^2 - C_0^2)(C_2 + 3C_0)}{4}$$

- The losses in turbomachinery are either internal or external in nature. The losses are classified and simple method of estimating the losses are illustrated.
- Flows in turbomachinery rotor are akin to forced vortex and flows in parts such as casing are related to free vortex.

REVIEW QUESTIONS

1.1 Define a turbomachine. Explain the salient features of a turbomachine.

1.2 Classify turbomachines with suitable examples.

1.3 Briefly explain the major parts of a turbomachine.

1.4 How the pumping devices for gases are classified?

1.5 Classify the turbomachines on the basis of work transfer.

1.6 Classify the turbomachine on the basis of fluid movement through the machine.

1.7 Deduce the Euler equation for a turbomachine and state its significance.

1.8 What is a vane congruent flow?

1.9 Why the airfoil shapes for the blades are used in a turbomachine?

1.10 Distinguish between impulse and reaction effect in a turbomachine.

1.11 Why, in general, radial flow turbines are inward flow machines whereas radial flow pumps or compressors are outward flow machines?

1.12 Explain the mechanism of slip as it occurs in (a) radial flow machines, and (b) axial flow machines.

1.13 Draw velocity triangles at the inlet and outlet of a turbomachine and derive an expression for the specific work.

1.14 Explain in brief the various losses that take place in turbomachines.

1.15 Derive an expression for energy transfer in terms of blade lift and drag coefficients.

1.16 Derive the Euler equation for the head extracted in a turbomachine in the terms of velocity heads of absolute and relative velocities of flow and peripheral velocity heads. Explain the significance of each of the terms.

1.17 Prove that the energy transfer as specific work is equal to the change in stagnation enthalpy of the fluid between the inlet and outlet of a turbomachine.

1.18 Derive an expression for the degree of reaction in a radial flow turbomachine and apply it on a reaction stage.

1.19 Derive the Bernoulli's equation from the general Euler equation. State the assumptions made.

1.20 Why Bernoulli's equation is not applicable to the rotor of a turbomachine?

1.21 What is the difference between free and forced vortex?

1.22 State the applications of free and forced vortex theory in turbomachines.

1.23 Deduce the one-dimensional energy equation for relative velocities for a turbomachinery.

1.24 Apply the first law of thermodynamics to power generating turbomachines.

1.25 Draw the velocity diagrams for (a) $R > 0.5$, (b) $R < 0.5$, and (c) $R = 0.5$.

1.26 Obtain the expressions for power input and theoretical efficiency for a propeller in a fluid stream.

1.27 What are the different losses that take place in a power generating turbomachine?

1.28 What are the different losses that take place in a power absorbing turbomachine?

PROBLEMS

1.1 Water is supplied to a boiler feed pump at a temperature of 90°C and 75 kN/m^2 absolute pressure. Water is delivered at the same temperature and 10 MPa absolute pressure. Under steady state condition, heat of 3.918 kJ/kg is lost by water while passing through the pump. Assume that the inlet and outlet diameters of the pump and their elevations to be the same. Determine the energy added by the pump if the discharge through it is 0.3 m^3/s.

[*Ans:* P = 4152.9 kW]

1.2 A centrifugal air compressor compresses 5.7 m^3/min of air from 85 kPa, 0.35 m^3/kg to 650 kPa, 0.1 m^3/kg. The suction and discharge line diameters are 100 mm and 62.5 mm. Determine (a) the mass flow rate of fluid, (b) specific work, and (c) the inlet and outlet velocities.

[*Ans:* (a) $\dot{m}$ = 16.3 kg/min, (b) $\dot{W}_{flow}$ = 9.57 kW, (c) C_1 = 12 m/s, C_2 = 8.83 m/s]

1.3 The gas turbine of a turbojet engine receives a steady flow of gases at a pressure of 7.2 bar, a temperature of 850°C and velocity of 160 m/s. It discharges the gases at a pressure of 1.15 bar, a temperature of 450°C and a velocity of 250 m/s. Determine the specific work output of the turbine. The process may be assumed isentropic and $c_p = 1.04$ kJ/kg – K for gases. [*Ans:* $w = 397.55$ kJ/kg]

1.4 Water flows at the rate of 25 m^3/s through a hydraulic turbine. The state at turbine inlet is 5 bar and 25°C with an elevation of 100 m above datum and a flow velocity of 1 m/s. At the turbine exit, the water is at 1.2 bar and 25.1°C with zero elevation and a flow velocity of 11 m/s. The turbine loses 5 J of heat per kg of water flowing through it. Assume specific heat of water to be 4.178 kJ/kg – K. Determine the power output of the turbine. [*Ans:* $P = 12.445$ MW]

1.5 The velocity triangles at the inlet and outlet of a turbomachine are shown in Figure 1.45. State with reasons: (a) whether the machine is radial flow type or axial flow type, (b) whether the machine is work producing type or work absorbing type, (c) specific work, (d) power if the mass flow rate is 5 kg/s, (e) degree of reaction, and (f) axial thrust.

[*Ans:* (a) Radial outward flow machine as $C_{b2} > C_{b1}$, (b) Work absorbing machine as $C_2 > C_1$, (c) $w = -100$ J/kg, (d) $P = 500$ W, (e) $R = 0.5$, (f) $F_a = 25$ N]

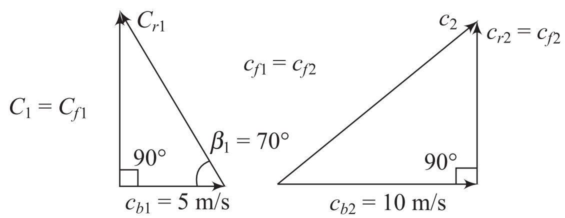


Figure 1.45 *Velocity Triangles for Problem 1.5*

1.6 The fluid enters in the rotor of a turbomachine with an absolute velocity having an axial component of 10 m/s and the tangential component in the direction of rotor motion is 15 m/s. The tangential velocity at outlet of the machine is 8 m/s. The absolute velocity of the fluid at outlet is 16 m/s in the axial direction. The peripheral velocity of the rotor at inlet is 35 m/s. Calculate (a) the energy transfer between the fluid and rotor, and (b) the type of the machine.

[*Ans:* (a) $w = 525$ J/kg, (b) Power generating]

1.7 The rotor of a radial flow machine running at 800 rpm has inlet and outlet diameter of 200 mm and 500 mm respectively. The blades on the rotor are bent backward such that the tangent at the blade outlet makes an angle of 60° with the peripheral velocity of the blade. The velocity of the flow remains constant through rotor at 10 m/s. Draw the velocity triangles at the inlet and outlet of the rotor assuming the radial entry. Calculate (a) the blade angle at inlet, (b) specific work, and (c) specific work when the blade outlet angle is 80° instead of 60°.

[*Ans:* (a) $\beta_1 = 50°$, (b) $w = -317.72$ J/kg, (c) $w = -401.7$ J/kg]

1.8 The diameters at the blade tip and hub of the rotor of an axial flow pump are 0.45 m and 0.10 m, respectively. The water enters in the pump without any whirl component and speed of the rotor is 800 rpm. The velocity of the flow may be assumed constant through the rotor as 8 m/s. Draw the velocity triangles at the inlet, separately at the tip and hub, and determine the blade angles at the inlet at both points. The blade angles at the outlet are 40° at the tip and 90° at the hub. Calculate the specific work at the tip and hub ends of the blade. If the variation of the specific work can be taken as linear between the hub and tip, find the value of specific work at 60% length of the blade. Taking this value of specific work as an average, uniform value, calculate the theoretical power required to drive the rotor.

[*Ans:* $\beta_{1t} = 23°$, $\beta_{1h} = 62.3°$, $w_t = 175.6$ J/kg, $w_h = 17.64$ J/kg, $w_{0.6l} = 112.42$ J/kg, $P = 136$ kW]

1.9 A centrifugal impeller has inner and outer diameters of 250 mm and 500 mm when rotated at a speed of 550 rpm. The inlet and outlet absolute velocities of water are 1.66 m/s and 12.4 m/s. The corresponding relative velocities are 7.42 m/s and 2.75 m/s. Calculate the degree of reaction of the impeller. [*Ans:* $R = 0.5741$]

1.10 A rectangular plate weighing 60 N is suspended vertically by a hinge on the top horizontal edge. The centre of gravity of the plate is 100 mm from the hinge. A horizontal jet of water of diameter 25 mm, whose axis is 0.15 m below the hinge, impinges normally to the plate with a velocity of 6 m/s. Calculate the horizontal force applied at the centre of gravity to maintain the plate in vertical position. Find the change in velocity of jet if the plate is deflected by 30° and the same horizontal force continues to act at the centre of gravity of the plate.[*Ans:* $R = 26.49$ N, $\Delta C = 2.48$ m/s increase]

1.11 A rocket travels at a velocity of 600 m/s discharges exhaust gases at 700 m/s velocity relative to the rocket. The mass flow rate of exhaust gases is 4.5 kg/s and the exit area is 0.07 m^2. Find the thrust on the rocket if the pressure at the exit is 9.6 kPa absolute and the ambient pressure is 8.25 kPa.

[*Ans:* $F = 3.056$ kN]

1.12 Obtain the condition for maximum power developed and maximum efficiency of a windmill and corresponding values.

$$\left[Ans: C_{\text{exit}} = \frac{C_1}{3}, P_{\max} = \frac{8}{27}\rho A C_1^3, \eta_{\max} = 59.3\%\right]$$

1.13 An airplane is moving with a velocity of 350 km/hr. At the angle of attack yielding a maximum lift-drag ratio. Calculate (a) lift and drag on the wing, and (b) percentage of the total drag due to finite span of the wing. The wing has a span of 12 m and a chord length of 2.4 m.

$$\left[Ans: \text{(a) } F_L = 66.42 \text{ kN}, F_D = 3.52 \text{ kN, (b) } \frac{C_{Di}}{C_D} \times 100 = 48.113\%\right]$$

1.14 A wing of an air-craft of airfoil section having a chord of length 2.3 m and a span of 13.8 m moves in a horizontal flight at 127 m/s. Calculate (a) Total weight the air-craft can carry, and (b) The power required for driving the airfoil. Take $C_L = 0.465$, $C_D = 0.022$ corresponding to the given angle of attack. Density of air may be assumed as 1.22 kg/m^3.

[*Ans:* (a) $W = 145.21$ kN, (b) $P = 872.51$ kW]

1.15 A free vortex is formed when a liquid is drained through a hole at the bottom of a container. A fluid element at a radial distance of 170 mm lies 120 mm below the level of water surface at the vertical boundary of the container. What will be the depth of another fluid element below that at the vertical boundary if the element is at a radial distance of 330 mm? The free water surface near the vertical boundary of the container may be assumed to be horizontal. [*Ans:* $Z_0 - Z = 31.85$ mm]

MULTIPLE CHOICE QUESTIONS

1. Which one of the following is an example of pure (100%) reaction machine?
   (a) Pelton wheel (b) Francis turbine (c) Gas turbine (d) Lawn sprinkler
2. Which of the following is not an assumption in the Bernouli's theorem?
   (a) Flow is along the stream lines (b) Flow is continuous
   (c) Flow is homogenous and compressible (d) Fluid is non viscous

3. Consider the following statements related to fluid dynamics:
    1. Momentum equation contains only vector quantities
    2. Energy equation involves scalar quantities only
    3. Irrotational flow occurs in a real fluid
    4. In a uniform flow, there is no variation of velocity at a given time with respect to distance

    Of these statements,
    (a) 1, 2 and 3 are correct (b) 2, 3 and 4 are correct
    (c) 1, 2 and 4 are correct (d) 1, 3 and 4 are correct

4. The equation $gZ + \frac{C^2}{2} + \int \frac{dp}{\rho}$ = constant along a streamline holds true for
    (a) Steady, frictionless, compressible fluid (b) Steady, uniform, incompressible fluid
    (c) Steady, frictionless. incompressible fluid (d) Unsteady incompressible fluid

5. A fluid travelling through a pipe of varying cross-section A at velocity C is observed to satisfy the Equation $A|C|$ = constant. This is most likely because the
    (a) Pressure is constant throughout the fluid (b) Density is constant throughout the fluid
    (c) Fluid is Newtonian (d) Fluid travels at constant velocity

6. The continuity equation $\frac{\partial C_x}{\partial x} + \frac{\partial C_y}{\partial y} + \frac{\partial C_z}{\partial z} = 0$
    (a) Is not valid for unsteady, incompressible fluids
    (b) Is valid for incompressible fluids whether the flow is steady or unsteady
    (c) Is valid for steady flow whether compressible or incompressible
    (d) Is valid for ideal fluid flow only

7. Bernoulli's equation is applicable between any two points
    (a) In any rotational flow of an incompressible fluid
    (b) In any type of irrotational flow of a fluid
    (c) In steady rotational flow of an incompressible fluid
    (d) In steady irrotational flow of an incompressible fluid

8. Match List 1 with List 2 and select the correct answer using the codes given below the lists:

	List 1		List 2
A.	Moment of Momentum Equation	1.	Equation to find energy loss in a pipeline having laminar flow
B.	Bernoulli's Equation	2.	Equation of motion for one dimensional steady flow of ideal and incompressible fluid
C.	Euler's Equation	3.	Equation based on conservation of momentum principle applicable to circulatory flow
D.	Hagen-Poiseullie Equation	4.	Three dimensional equation of motion based on principle of conservation of momentum for ideal and incompressible fluid flow

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	3	2	1	4
(c)	3	1	2	4
(d)	4	2	1	3

9. Consider the following conditions
   I. Fluid is ideal
   II. Flow is steady
   III. Fluid is laminar
   IV. Fluid is Newtonian and flow is turbulent
   V. Flow is along a streamline

   For $\frac{p}{\rho}+\frac{C^2}{2}+gZ=$ constant, the conditions to be satisfied are
   (a) I, II and V (b) II, III and IV (c) I, III and IV (d) II, III and V
10. A nozzle discharging water under head H has an outlet area A and discharge coefficient $C_d = 1.0$. A vertical plate is acted upon by the fluid force F_j when held across the free jet and by the fluid force F_n when held against the nozzle to stop the flow. The ratio F_j/F_n is
    (a) $\frac{1}{2}$ (b) 1 (c) $\sqrt{2}$ (d) 2
11. What does Euler's equation of turbomachines relate to?
    (a) Discharge and power (b) Discharge and velocity
    (c) Head and power (d) Head and velocity
12. Consider the following statements regarding Bernoulli's theorem for fluid flow
    1. Conservation of energy
    2. Steady flow
    3. Viscous flow
    4. Incompressible flow

    Which of these statements is/are correct?
    (a) 1, 2 and 4 only (b) 1 only (c) 2, 3 and 4 only (d) 1, 2, 3 and 4 only
13. The continuity equation in differential form is
    (a) $\frac{dA}{A}+\frac{dC}{C}+\frac{d\rho}{\rho}=$ constant (b) $\frac{A}{dA}+\frac{C}{dC}+\frac{\rho}{d\rho}=$ constant
    (c) $\frac{dA}{A}+\frac{dC}{C}+\frac{d\rho}{\rho}=0$ (d) $AdA + CdC + \rho d\rho = 0$
14. Select the correct Euler equation of a turbine relating the energy transfer per unit weight H_e.
    (a) $gH_e = C_{b1}C_{w1} - C_{b2}C_{w2}$ (b) $gH_e = C_{b1}C_{f1} - C_{b2}C_{f2}$
    (c) $gH_e = C_{b1}C_1 - C_{b2}C_2$ (d) $gH_e = C_1C_{w1} - C_2C_{w2}$
15. A turbine develops 2516 kW at 240 rpm. The torque on the shaft is
    (a) 400 kN.m (b) 3336 kN.m (c) 1000 kN.m (d)100 kN.m

16. The moment of momentum of water is reduced by 15915 N.m in a turbine rotating at 600 rpm. The power developed in kW is
(a) 1000 (b) 1500 (c) 2000 (d) 5000

17. A turbine has a runner of 4 m external diameter. The breadth at the inlet and outlet is 0.8 m and the velocity of flow is constant at 3 m/s. The discharge through the turbine is
(a) 15.08 m^3/s (b) 28.28 m^3/s (c) 30.16 m^3/s (d) 37.70 m^3/s

18. Steady flow occurs when
(a) Properties do not change with time
(b) The system is in equilibrium with its surroundings
(c) Properties change with time
(d) $\left(\dfrac{dv}{dt}\right)$ is constant

19. The first law of thermodynamics for steady flow
(a) Is an energy balance for a specified mass of fluid
(b) Accounts for all energy entering and leaving a control volume
(c) Is an expression of the conservation of linear momentum
(d) Is restricted in its application to perfect gases

20. A steam turbine receives steam steadily at 10 bar with an enthalpy of 3000 kJ/kg and discharges at 1 bar with an enthalpy of 2700 kJ/kg. The work output is 250 kJ/kg. The changes in kinetic and potential energies are negligible. The heat transfer from the turbine casing to the surroundings is equal to
(a) 0 kJ (b) 50 kJ (c) 150 kJ (d) 250 kJ

21. A small steam whistle (perfectly insulated and doing no shaft work) causes an enthalpy drop of 0.8 kJ/kg, the kinetic energy of the steam at entry is negligible, the velocity of the steam at exit is
(a) 4 m/s (b) 40 m/s (c) 80 m/s (d) 120 m/s

22. The inlet and the outlet conditions of steam for an adiabatic steam turbine are as shown in Figure 1.46.

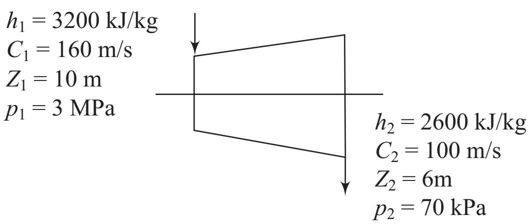


Figure 1.46 *Multiple choice question 22*

If the mass flow rate of steam through the turbine is 20 kg/s, the power output of the turbine in MW is
(a) 12.157 (b) 12.941 (c) 168.001 (d) 168.785

23. Specific enthalpy and velocity of steam at inlet and exit of a steam turbine running under steady state are as follows:

	Specific Enthaly (kJ/kg)	**Velocity (m/s)**
Inlet steam condition	3250	180
Exit steam condition	2360	5

The rate of heat loss from the turbine per kg of steam flow rate is 5 kW. Neglecting the changes in potential energy of steam, the power developed in kW by the steam turbine per kg of steam flow rate is

(a) 901.2 (b) 911.2 (c) 17072.5 (d) 17082.5

24. Hot combustion gases assumed to have properties of air at room temperature enter a gas turbine at 1 MPa and 1500 K at a rate of 0.1 kg/s and leaves at 0.2 MPa and 900 K. If heat is lost from the turbine to the surroundings at a rate of 15 kJ/s, the power output of the gas turbine is

(a) 15 kW (b) 30 kW (c) 45 kW (d) 60 kW

25. Steam expands in a turbine from 4 MPa and 500°C to 0.5 MPa and 250°C at a rate of 1350 kg/hr. Heat is lost from the turbine at a rate of 25 kJ/s during the process. The power output of the turbine is

(a) 157 kW (b) 207 kW (c) 182 kW (d) 287 kW

26. Steam is compressed by an adiabatic compressor from 0.2 MPa, 150°C to 0.8 MPa, 350°C at a rate of 1.30 kg/s. The power input to the compressor is

(a) 511 kW (b) 393 kW (c) 302 kW (d) 717 kW

27. Blade velocity is always tangential to

(a) The blade profile (b) The stream line of the fluid
(c) The rotor wheel (d) The rotor axis

28. Relative velocity is always tangential to the

(a) Stream line of fluid (b) Axis of the rotor
(c) Guide vanes (d) None of these

29. For centrifugal pump

(a) $C_{b1} = C_{b2}$ (b) $C_{b1} = 2C_{b2}$ (c) $C_{b1} < C_{b2}$ (d) $C_{b1} > C_{b2}$

30. For a radial flow machine, the fluid enters at

(a) The outer rim (b) The inner rim
(c) Perpendicular to the rim (d) Any of these

31. For an axial flow compressor, the fluid strikes

(a) On the convex side of the blade (b) On the concave side of the blade
(c) Tangential to the blade (d) Perpendicular to the blade

32. In the velocity diagram, as shown in Figure 1.47, C_b = blade velocity, C = absolute fluid velocity and C_r = relative velocity of the fluid. If the subscripts 1 and 2 refer to inlet and outlet respectively, this diagram is for

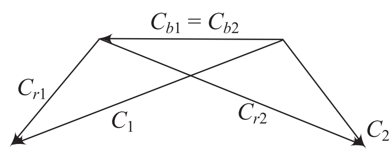


Figure 1.47 *Multiple choice question 32*

(a) An impulse turbine (b) A reaction turbine
(c) A centrifugal compressor (d) An axial flow compressor

33. The degree of reaction of a turbomachine is defined as ratio of the
    (a) Static pressure change in the rotor to that in the stator
    (b) Static pressure change in the rotor to that in stage
    (c) Static pressure change in the stator to that in rotor
    (d) Total pressure change in the rotor to that in stage
34. The degree of reaction of a turbomachine is defined as ratio of the
    (a) Energy transfer by change of pressure to total energy transfer across the rotor
    (b) Total energy transfer to static pressure drop
    (c) Change of velocity energy across the turbine to the total energy transfer
    (d) Velocity energy to pressure energy
35. Which of the following correctly completes the statement?
    The degree of reaction of an impulse turbine.
    (a) Is greater than zero but less than unity (b) Is greater than unity
    (c) Is equal to zero (d) Is equal to unity
36. A reaction turbine has degree of reaction to be 50%. If C_1 is the absolute velocity at the inlet and α_1 is the angle made by the absolute velocity to the tangent to the rotor, the blade speed is equal to
    (a) $\frac{C_1 \cos \alpha_1}{2}$ (b) $2C_1 \cos \alpha_1$ (c) $C_1 \cos^2 \alpha_1$ (d) $C_1 \cos \alpha_1$
37. If there is no change in the fluid pressure between the inlet and outlet of the rotor of an axial flow machine, then, degree of reaction R
    (a) $R \neq 0$ (b) $R = 0$ (c) $R > 0$ (d) $R < 0$
38. If the absolute velocities of the fluid at the inlet and outlet are the same in an axial flow turbomachine, the degree of reaction R
    (a) $R > 0$ (b) $R < 0$ (c) $R = 0$ (d) $R = 1$
39. In an axial flow machine with 100% reaction
    (a) $C_1 = C_2$ (b) $\beta_1 = \beta_2$ (c) $C_{r1} = C_{r2}$ (d) All of the above
40. In a radial flow turbomachine, it is not possible to have
    (a) $R = 0$ (b) $R = 1$ (c) $R > 0$ (d) None of these
41. A jet of water issues from a nozzle with a velocity of 20 m/s and it impinges normally on a flat plate moving away from it at 10 m/s. If the cross-sectional area of the jet is 0.02 m^2 and the density of water is taken as 1000 kg/m^3, then the force developed on the plate will be
    (a) 10 N (b) 100 N (c) 1000 N (d) 2000 N
42. A jet of oil with relative density 0.7 strikes normally a plate with a velocity of 10 m/s. The jet has an area of 0.03 m^2. The force exerted on the plate by the jet is
    (a) 210 N (b) 2.1 kN (c) 20.6 kN (d) 206 N
43. A water jet has an area of 0.03 m^2 and impinges normally on a plate. If a force of 1 kN is produced as a result of this impact, the velocity of the jet is
    (a) 15 m/s (b) 33.4 m/s (c) 3.4 m/s (d) 5.78 m/s
44. A jet of water with a velocity of 20 m/s impinges on a single vane moving at 5 m/s in the direction of the jet and transmits a power P_1. If the same jet drives a series of similar vanes mounted on a wheel under similar velocity conditions, the power transmitted by the jet is P_2. The value of P_1/P_2 is
    (a) 0.25 (b) 0.33 (c) 0.50 (d) 0.75

45. A two dimensional free jet of water strikes a fixed two dimensional plate at 45° to the normal of the plate. This causes the jet to split into two streams whose discharges are in the ratio
    (a) 5.83 (b) 1.00 (c) 1.41 (d) 3.00
46. A free jet of water has a velocity of 10 m/s and an area of 0.01 m^2. This jet impinges normally on a vertical plate that is moving with a velocity of 5 m/s in the direction opposite of the jet. The force on the plate due to this impingement of the jet by considering the density of water as 1000 kg/m^3 is
    (a) 2250 N (b) 1000 N (c) 500 N (d) 250 N
47. A free jet of water 0.01 m^2 area impinges normally on a stationary vertical plate at a velocity of 20 m/s. If the plate moves in the direction of the jet at a velocity of 5 m/s, what increase in the discharge at the nozzle would keep the force on the plate unaltered?
    (a) 0.005 m^3/s (b) 0.05 m^3/s (c) 0.10 m^3/s (d) 0.25 m^3/s
48. A symmetrical stationary vane experiences a force of 100 N as shown in Figure 1.48. When the mass flow rate of water over the vane is 5 kg/s with a velocity of 20 m/s without friction. The vane angle θ is

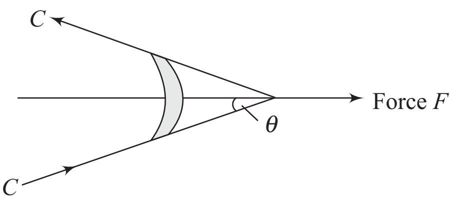


Figure 1.48 *Multiple choice question 48*

    (a) Zero (b) 30° (c) 45° (d) 60°
49. A free jet of water acts upon a water wheel which has semicircular vanes fitted on its periphery. The theoretical maximum efficiency of this wheel system is
    (a) 50% (b) 67% (c) 75% (d) 100%
50. The force exerted by a liquid jet striking normally on a flat plate is maximum when
    (a) The plate is stationary
    (b) The plate is moving away from the jet with a velocity equal to that of jet velocity
    (c) The plate is moving away from the jet with a velocity less than the jet velocity
    (d) None of these
51. A circular jet of water impinges on a vertical flat plate and bifurcates into two circular jets of half the diameter of the original. After hitting the plate
    (a) The jets move at equal velocity which is twice of the original velocity
    (b) The jets move at equal velocity which is three times of the original velocity
    (c) Data given is insufficient to calculate velocities of the two outgoing jets
    (d) The jets move at equal velocity which is equal to the original velocity
52. The drag force experienced by an object is
    (a) The component of resultant fluid dynamic force in the flow direction
    (b) The horizontal force due to pressure variation over the surface of an object
    (c) The resultant fluid dynamic force acting on the object
    (d) None of these

53. The lift force that may act on an object is
   (a) The buoyant force due to the fluid displaced by the body
   (b) The component of resultant fluid dynamic force in a direction normal to the general direction of flow
   (c) The force due to shear stress that acts on the body surface
   (d) None of these
54. The lift force on an airfoil is due to
   (a) The circulation of air around it
   (b) The pressure difference on the top and bottom surface
   (c) The formation of tip vortices
   (d) The angle of attack
55. At the stall point for the airfoil
   (a) The boundary layer separates at the leading edge
   (b) The lift is maximum and the drag is minimum
   (c) The lift is zero and the drag is maximum
   (d) The lift is maximum and the drag increases sharply beyond it
56. As the flow rate increases in a radial flow machine, the losses
   (a) Increase (b) Decrease
   (c) First remains constant and then decrease (d) First remains constant and then increase
57. The turning losses in a radial flow machine depend on
   (a) The specific work (b) The degree of reaction
   (c) The flow rate (d) None of these
58. Circulation is defined as line integral of tangential component about a
   (a) Centre (b) Closed contour in a fluid flow
   (c) Velocity profile (d) Pressure profile
59. In forced vortex flow, velocity C and radial distance r are related as
   (a) $C \propto r$ (b) $C \propto \frac{1}{r}$ (c) $C \propto \frac{1}{r^2}$ (d) $C \propto r^2$
60. The tangential velocity at a radial distance of 1 m from the axis of vortex is 5 m/s both in a free vortex and forced vortex. At a radial distance of 2 m, the ratio of velocity in forced vortex to that in free vortex will be
   (a) 2 (b) 4 (c) 6 (d) 8
61. A cylindrical vessel of radius 0.4231 m and height 1 m is open at the top. It holds water to half of its depth. Which one of the following values approximates the speed at which the cylinder is to be rotated about the vertical axis, so as to make it apex of the paraboloid just reach the centre of the bottom of the vessel?
   (a) 100 rpm (b) 150 rpm (c) 250 rpm (d) 300 rpm
62. An open circular cylinder of 1.2 m height is filled with a liquid to its top. The liquid is given a rigid body rotation about the axis of the cylinder and the pressure at the centre line at the bottom surface is found to be 0.6 m of liquid. What is the ratio of volume of liquid spilled out of the cylinder to the original volume?
   (a) $\frac{1}{4}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

63. Which of the following is correct for a forced vortex?
    (a) Turns in an opposite direction to a free vortex
    (b) Always occurs in conjunction with a free vortex
    (c) Has the tangential velocity directly proportional to the radius
    (d) Has the tangential velocity inversely proportional to the radius
64. Both free vortex and forced vortex can be expressed mathematically in terms of tangential velocity C_w at the corresponding radius r. Choose the correct combination

	Free Vortex	**Forced Vortex**
(a)	C_w/r = constant	$C_w r$ = constant
(b)	$C_w^2 r$ = constant	C_w/r = constant
(c)	$C_w r$ = constant	C_w^2/r = constant
(d)	$C_w r$ = constantt	C_w/r = constant

65. An open cylinder having 0.3 m diameter and 0.5 m height is filled with water and rotated about its axis. The amount of water spilled when the speed of rotation is 180 rpm
    (a) 12.2 l (b) 10.5 l (c) 14.4 l (d) 11.6 l
66. Which of the following is an example of free vortex flow?
    (a) A whirlpool in a river
    (b) Flow of liquid in centrifugal pump casing
    (c) Flow of liquid through a hole provided at the bottom of a container
    (d) All of these
67. In case of forced vortex, the rise of liquid level at the end is……. the fall of liquid level at the axis of rotation.
    (a) Less than (b) More than (c) Equal to (d) None of these
68. In case of closed cylindrical vessel sealed at the top and the bottom, the volume of air before rotation is……. the volume of air after rotation.
    (a) More than (b) Less than (c) Equal to (d) None of these
69. Match List 1 with List 2 and select the correct answer using the codes given below the lists:

List 1	**List 2**
A. Rotational flow	1. A fluid motion in which streamlines are concentric circles
B. Vortex flow	2. The fluid particles moving in concentric circles may not rotate about their mass centre
C. Free vortex	3. The fluid particles moving in concentric circles may rotate about their mass centre
D. Forced vortex	4. Flow near a curved solid boundary

Codes

	A	**B**	**C**	**D**
(a)	4	2	3	1
(b)	1	2	3	4
(c)	1	3	2	4
(d)	4	1	2	3

70. Match List 1 with List 2 and select the correct answer using the codes given below the lists:

List 1 Phenomenon	List 2 Condition
A. Rotational Flow	1. Velocity zero or infinite
B. Irrotational Flow	2. Proportional to velocity
C. Singularities	3. Vorticity is zero
D. Streamline Spacing	4. Vorticity exists

Codes

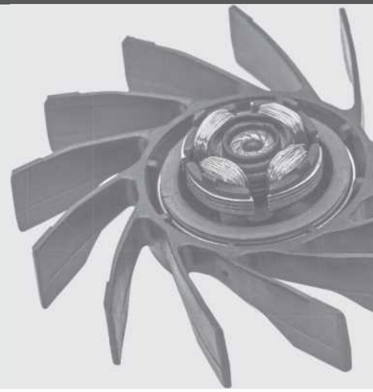
	A	B	C	D
(a)	3	4	1	2
(b)	3	4	2	1
(c)	4	3	2	1
(d)	4	3	1	2

ANSWER KEY

1. (d)	2. (c)	3. (b)	4. (a)	5. (b)	6. (b)	7. (d)	8. (d)	9. (a)	10. (b)
11. (a)	12. (a)	13. (c)	14. (a)	15. (d)	16. (a)	17. (c)	18. (a)	19. (b)	20. (b)
21. (b)	22. (a)	23. (a)	24. (c)	25. (a)	26. (a)	27. (c)	28. (a)	29. (c)	30. (d)
31. (b)	32. (b)	33. (b)	34. (a)	35. (c)	36. (d)	37. (b)	38. (d)	39. (a)	40. (a)
41. (d)	42. (b)	43. (d)	44. (d)	45. (a)	46. (a)	47. (b)	48. (d)	49. (d)	50. (a)
51. (c)	52. (a)	53. (b)	54. (a, b)	55. (d)	56. (d)	57. (c)	58. (b)	59. (a)	60. (b)
61. (a)	62. (a)	63. (c)	64. (d)	65. (c)	66. (d)	67. (c)	68. (c)	69. (a)	70. (d)

2

Dimensional Analysis and Model Testing for Turbomachines



Learning Objectives

After reading this chapter, you will be able to:

LO 1 Apply Buckingham pi theorem and express the efficiency of pump and turbine in terms of various relevant dimensionless numbers

LO 2 Outline the difference between incompressible and compressible flow turbomachinery in terms of their performance variables

LO 3 Explain the laws of affinity for turbomachines

LO 4 Understand the principle of modelling and choice of machines based on dimensionless number

LO 5 Estimate the unit quantities for turbomachinery

2.1 Introduction

The performance of turbomachines depends upon large number of variables. To describe their performance, these variables should be grouped into a set of dimensionless group or numbers. The method of forming such groups and the process of identifying important ones among them is called dimensional analysis. The dimensional analysis has two important functions:

(i) To characterise the performance of models and extend the same to prototypes

(ii) To choose machines suitable for the given characteristics specifications

Following the simple approach known from elementary thermodynamics, draw a control volume around the turbomachine, as shown in Figure 2.1. Across the control surface, a mass of fluid $\dot{m}$ enters at station 1 and leaves at station 2 steadily, the network across the control surface be $\dot{W}$. Features such as shaft speed, torque and change in fluid properties across the machine are known. All other details of the flow within the machine can be ignored.

The size of machine is characterised by the impeller diameter D, and the shape can be expressed by a number of length ratios, l_1/D , l_2/D, etc.

2.2 Buckingham's π-Theorem

The π-theorem states that, in a given problem, if the number of variables is n, the greatest number of non-dimensional groups or dimensionless numbers (also known as π-terms) is given by,

$$\pi = n - k \tag{2.1}$$

where $k \leq m$, and $m =$ number of primary dimensions.

Here, it will be assumed that $k = m$; this is true in a majority of situations.

Let there be three dependent variables, y_1, y_2, y_3, and five independent variables, x_1, x_2, x_3, x_4, x_5.

$$y_1, y_2, y_3 = f(x_1, x_2, x_3, x_4, x_5) \tag{2.2}$$

If three primary dimensions, M, L and T are involved, then theorem (2.1) gives,

$$\pi = n - k = (3 + 5) - 3 = 5$$

Thus, the number of dimensionless groups or π terms is five. This can be expressed functionally by the following relation,

$$\pi_1, \pi_2 = f(\pi_3, \pi_4, \pi_5) \tag{2.3}$$

More dimensionless groups can be formed by a combination of the π terms [as already determined in Eq. (2.3)] by the π-theorem.

The selection of π terms on the two sides in Eq. (2.3) depends on the behaviour of a given machine. The terms on the right must be the control variables whose variation will automatically vary terms on the left-hand side.

2.3 Incompressible Flow Turbomachines

Consider an incompressible flow turbomachine as a control volume of Figure 2.1. In an incompressible flow machine, fluid of density ρ flows at a volume flow rate of Q, which is determined by a valve opening. The head difference across the control volume is H, the control volume represents a turbomachine of diameter D. The machine develops a shaft power P at a speed of rotation N.

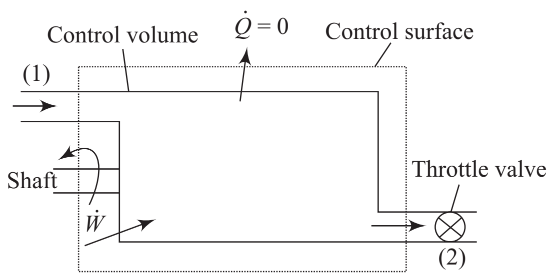


Figure 2.1 *Turbomachine as a Control Volume*

The primary turbomachinery variables for incompressible flow machines are given by Table 2.1 along with their dimensions.

TABLE 2.1 Variables of Incompressible Flow Turbomachine

Variable	*Symbol*	*Dimensions*
Specific energy or specific work	gH or w	L^2T^{-2}
Volume flow rate	Q	L^3T^{-1}
Speed	N	T^{-1}
Rotor diameter	D	L
Density	ρ	ML^{-3}
Viscosity	μ	$ML^{-1}T^{-1}$

The power output is a function of all the other variables.

$$P = f[(gH), Q, \mu, \rho, N, D] \tag{2.4}$$

$$P = \text{constant } [(gH)^a Q^b \mu^c \rho^d N^e D^f] \tag{2.5}$$

Writing the dimensions on both sides of the Eq. (2.5), we get,

$$ML^2T^{-3} = \text{constant } [L^2T^{-2})^a \times (L^3T^{-1})^b \times (ML^{-1}T^{-1})^c \times (ML^{-3})^d \times (T^{-1})^e \times L^f]$$

Equating the indices of M, L and T on the two sides, the following three equations are obtained:

$$1 = c + d \tag{2.7}$$

$$2 = 2a + 3b - c - 3d + f \tag{2.8}$$

$$-3 = -2a - b - c - e \tag{2.9}$$

Three dimensionless numbers, under the indices a, b and c, can be formed on the right-hand side. Therefore, indices d, e and f are now expressed in terms of a, b and c. Equations (2.7), (2.8) and (2.9) give,

$$d = 1 - c \tag{2.10}$$

$$e = 3 - 2a - b - c \tag{2.11}$$

$$f = 5 - 2a - 3b - 2c \tag{2.12}$$

Substitution of these values in Eq. (2.5) yields,

$$P = \text{constant } [(gH)^a \times Q^b \times \mu^c \times \rho^{1-c} \times N^{3-2a-b-c} \times D^{5-2a-3b-2c}]$$

$$P = \text{constant} \left[(gH)^a \times Q^b \times \mu^c \times \frac{\rho}{\rho^c} \times \frac{N^3}{N^{2a} \times N^b \times N^c} \times \frac{D^5}{D^{2a} \times D^{3b} \times D^{2c}} \right]$$

Rearrangement of this expression in four groups with indices 1, a, b and c gives the following relations with dimensionless numbers,

$$\frac{P}{\rho N^3 D^5} = \text{constant} \left[\left(\frac{gH}{N^2 D^2} \right)^a \times \left(\frac{Q}{ND^3} \right)^b \times \left(\frac{\mu}{\rho N D^2} \right)^c \right] \tag{2.13}$$

The third term in the brackets will be recognised as the inverse of the Reynolds number. Since the value of c is unknown, this term can be inverted, and Eq. (2.13) is expressed in a more general form as,

$$\frac{P}{\rho N^3 D^5} = f \left[\left(\frac{gH}{N^2 D^2} \right)^a \times \left(\frac{Q}{ND^3} \right)^b \times \left(\frac{\rho N D^2}{\mu} \right)^c \right] \tag{2.14}$$

Each group of variables in Eq. (2.14) is truly dimensionless and all are used in turbomachinery practise. In Eq. (2.14), the groups are known by the following names. Referring to steady flow energy equation,

$$w = \Delta h_o = \frac{\Delta p_o}{\rho} = gH$$

$$\text{Power coefficient} = C_P = \frac{P}{\rho N^3 D^5} = \frac{\rho Q g H}{\rho N^3 D^5} = \frac{\rho Q w}{\rho N^3 D^5} \tag{2.15}$$

$$\text{Flow or capacity coefficient} = \varphi = \frac{Q}{ND^3} \tag{2.16}$$

The non-dimensional group (Q/ND^3) is a volumetric flow coefficient. In compressible flow machines, an alternate to (Q/ND^3) that is frequently used is the velocity (or flow) coefficient as given below.

$$\text{Flow or capacity coefficient } \varphi = \frac{C_f \times \text{Area}}{(ND)D^2} = \frac{C_f D^2}{C_b D^2}$$

$$\text{Flow or capacity coefficient } \varphi = \frac{C_f}{C_b} \tag{2.17}$$

$$\text{Head coefficient} = \psi = \frac{gH}{N^2 D^2} = \frac{w}{N^2 D^2} = \frac{w}{(ND)^2} \tag{2.18}$$

$$\text{Energy coefficient } \psi = \frac{w}{C_b^2} \tag{2.19}$$

The term $\rho ND^2/\mu$ is equivalent to the Reynolds number, $R_e = \frac{\rho CD}{\mu}$, since $C \propto C_b$ and the peripheral velocity, C_b, is proportional to ND.

2.3.1 Efficiencies of Pump/Compressor

Refer to Figure 1.41 illustrating the energy flow in pumps/compressor. Imagine we need a discharge Q at head H at the outlet of the pump. In order to accommodate the losses, the head at inlet to the casing has to be $(H + h_c)$. That at inlet to the impeller shall be $(H + h_c + h_i)$ and the flow rate $(Q + \Delta Q)$ so as to account for the leakage. The theoretical impeller head or (Euler head, neglecting slip) has to provide for H as well as the losses i.e., $H_{th} = (H + h_c + h_i)$.

(a) Mechanical Efficiency

$$\eta_m = \frac{\text{Fluid power supplied to impeller}}{\text{Power input to shaft}} = \frac{\rho g (Q + \Delta Q) H_{th}}{P_{\text{shaft}}} \tag{2.20}$$

where P_{shaft} is power input to the pump shaft, ρ is the density of fluid, Q is the volume flow rate, ΔQ is the leakage volume, and H_{th} is the theoretical head.

(b) Impeller Efficiency

$$\eta_i = \frac{\text{Fluid power at impeller exit}}{\text{Fluid power supplied to impeller}} = \frac{\rho g(Q + \Delta Q)(H + h_c)}{\rho g(Q + \Delta Q)H_{th}} = \frac{H + h_c}{H_{th}} = \frac{H_a}{H_{th}} \tag{2.21}$$

where, Q is the volume flow rate, ΔQ is the leakage volume, $H_a = H + h_c$ is actual impeller or casing head, and H_{th} is the theoretical head.

(c) Casing or Manometric Efficiency

$$\eta_c = \eta_{\text{mano}} = \frac{\text{Fluid power at casing outlet}}{\text{Fluid power at casing inlet}} = \frac{\rho g Q H}{\rho g Q(H + h_c)} = \frac{H}{(H + h_c)} = \frac{H}{H_a} \tag{2.22}$$

(d) Volumetric Efficiency

$$\eta_v = \frac{\text{Flow rate through pump outlet (Pump discharge)}}{\text{Flow rate through impeller}} = \frac{Q}{Q + \Delta Q} \tag{2.23}$$

(e) Hydraulic Efficiency or Impeller Efficiency

$$\eta_h = \frac{\text{Actual head developed by the pump at casing inlet}}{\text{Theoretical head devloped by the impeller}} = \frac{H_a}{H_{th}} = \frac{H + h_c}{H_{th}} \tag{2.24}$$

(f) Overall Efficiency

$$\eta_o = \frac{\text{Useful fluid power developed by the pump}}{\text{Input Shaft}} = \eta_m \eta_h \eta_{\text{mano}} \eta_v = \frac{\rho g Q H}{P_{\text{shaft}}} \tag{2.25}$$

2.3.2 Efficiencies of a Turbine

Refer Fig. 1.42 illustrating energy flow in a turbine. Here Q is the flow rate at inlet to the turbine. The actual quantity available in the runner shall be ($Q - \Delta Q$) due to leakage of ΔQ. If H is the head available for a turbine,

(a) Mechanical Efficiency

$$\eta_m = \frac{\text{Output Shaft Power}}{\text{Fluid power supplied to impeller}} = \frac{P_{\text{shaft}}}{\rho g(Q - \Delta Q)H_{th}} \tag{2.26}$$

where P_{shaft} is the output Shaft Power, ρ is the density of fluid, Q is the volume flow rate, ΔQ is leakage volume, and H_{th} is theoretical head.

(b) Impeller Efficiency

$$\eta_i = \frac{\text{Fluid power at impeller exit}}{\text{Fluid power at impeller inlet}} = \frac{\rho g(Q - \Delta Q)H_{th}}{\rho g(Q - \Delta Q)(H - h_c)} = \frac{H_{th}}{H - h_c} = \frac{H_{th}}{H_a} \tag{2.27}$$

where, Q is the volume flow rate, ΔQ is leakage volume, $H_a = H - h_c$ is actual impeller or casing head, and H_{th} is theoretical head.

(c) Casing Efficiency

$$\eta_c = \frac{\text{Fluid power at casing outlet}}{\text{Fluid power at casing inlet}} = \frac{\rho g Q(H - h_c)}{\rho g Q H} = \frac{(H - h_c)}{H} \tag{2.28}$$

(d) Volumetric Efficiency

$$\eta_v = \frac{\text{Flow rate through impeller}}{\text{Flow rate through turbine inlet}} = \frac{Q - \Delta Q}{Q} \tag{2.29}$$

(e) Overall Efficiency

$$\eta_o = \frac{\text{Output Shaft Power}}{\text{Input Fluid Power}} = \eta_m \eta_i \eta_c \eta_v = \frac{P_{\text{shaft}}}{\rho Q g H} \tag{2.30}$$

EXAMPLE 2.1 A centrifugal pump delivers of 1.32 m^3/s water against a head of 100.8 m. The leakage loss is 4% of the discharge, external mechanical loss is 20 kW, manometric and hydraulic efficiencies are 85% and 80%, respectively. Determine the overall efficiency of the pump.

Solution

Given: Q = 1.32 m^3/s, H_m = 100.8 m, leakage loss = 4%, mechanical loss = 20 kW, η_m = 85%, η_h = 80%

Since leakage loss is occurring in the pump, so the quantity of fluid entering into the eye of the impeller must be greater than the discharge on the delivery side. Therefore,

$$Q_i = Q + \Delta Q = 1.32 + \frac{4}{100} \times 1.32 \Rightarrow Q_i = 1.04 \times 1.32$$

$$Q_i = 1.373 \text{ m}^3/\text{s} \tag{1}$$

We know that manometric efficiency,

$$\eta_m = \frac{H_m}{H_a}$$

$$0.85 = \frac{100.8}{H_a}$$

$$H_a = 118.59 \text{ m} \tag{2}$$

Hydraulic efficiency is defined as the actual head or power imparted by the impeller to the theoretical head (Euler head) or power, i.e. power that could be imparted in the absence of hydraulic losses. Mathematically,

$$\eta_h = \frac{H_a}{H_e} \tag{3}$$

$$0.8 = \frac{118.59}{H_e}$$

Therefore, theoretical head or Euler head,

$$H_i = H_e = 148.24 \text{ m} \tag{4}$$

Rotor power or impeller power,

$$\text{RP} = \rho g Q_i H_i \tag{5}$$

$$\text{RP} = 1000 \times 9.81 \times 1.373 \times 148.24$$

$$\text{RP} = 1996663.83 \text{ W} = 1996.67 \text{ kW} \tag{6}$$

Shaft Power or Brake Power = RP + Mechanical Losses

Shaft Power = 1996.67 + 20

Shaft Power = 2016.67 kW (7)

Mechanical efficiency,

$$\eta_m = \frac{RP}{SP} \tag{8}$$

$$\eta_m = \frac{1996.67}{2016.67}$$

$$\eta_m = 0.9901 = 99.01\% \tag{9}$$

We know that,

$$\eta_v = \frac{\text{Flow rate through pump outlet (pump discharge)}}{\text{Flow rate through impeller}} = \frac{Q}{Q + \Delta Q} \tag{10}$$

$$\eta_v = \frac{1.32}{1.373}$$

$$\eta_v = 0.9614 = 96.14\% \tag{11}$$

Overall efficiency of the pump,

$$\eta_o = \eta_h \times \eta_v \times \eta_{\text{mano}} \times \eta_m \tag{12}$$

$$\eta_o = 0.8 \times 0.9614 \times 0.85 \times 0.9901$$

$$\eta_o = 0.6473 = 64.73\% \tag{13}$$

Overall efficiency may be alternately found as follows:
We know that power imparted to the fluid or fluid power,

$$\text{FP} = \rho g Q H_m \tag{14}$$

$$\text{FP} = 1000 \times 9.81 \times 1.32 \times 100.8$$

$$\text{FP} = 1305279.36 \text{ W} = 1305.28 \text{ kW} \tag{15}$$

Overall efficiency is the ratio of fluid power to the shaft power. Therefore,

$$\eta_o = \frac{1305.28}{2016.67}$$

$$\eta_o = 0.6473 = 64.73\% \tag{16}$$

2.4 Compressible Flow Turbomachines

Dimensional analysis for compressible flow machines differs from the incompressible flow machines on account of the following factors:

1. On account of the continuously changing volume flow rate, for a given mass flow rate, the variable Q is replaced by the mass flow rate $\dot{m}$.
2. The head term, gH, is replaced by the pressure change term, Δp_o, or the pressure ratio, $p_r = \frac{p_{o1}}{p_{o2}}$.

   The change of specific enthalpy can also replace gH. This is obviously the right thing to do for blowers, compressors and turbines handling compressible fluids. It is much easier to indicate the pressure difference (often in mm W.G.) developed across a blower, or a pressure ratio developed by a compressor or compressor stage.
3. The ratio γ of specific heats also becomes one of the independent parameters.

4. In compressible flows, elasticity of the gas is an important parameter. This is taken into account by temperature, T_{o1}, or the velocity of sound at the entry, i.e.

$$a_{01} = \sqrt{\gamma R T_{01}}$$

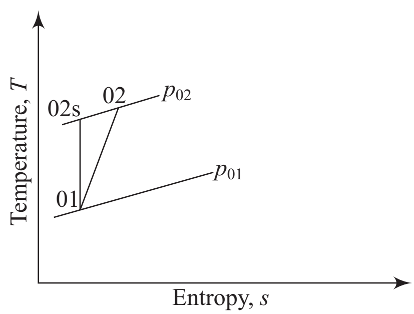
5. Since the values of the gas density and temperature vary in compressible flow machines, their values at the entry are taken.

In compressible flow machines, the variables of importance are the pressure and temperature. These variables of the gas increase in a compressor and decrease in the turbine. The primary turbomachinery variables for compressible machines are given by Table 2.2 along with their dimensions.

TABLE 2.2 Variables of Compressible Flow Turbomachines

Variable	*Symbol*	*Dimensions*
Total Pressure	p_0	$ML^{-1}T^{-2}$
Mass flow rate	$\dot{m}$	MT^{-1}
Speed	N	T^{-1}
Rotor diameter	D	L
Viscosity	μ	$ML^{-1}T^{-1}$
Density	ρ	ML^{-3}
Total temperature	T_o	θ

Refer Figure 2.2 where compression and expansion processes are represented. Considering an isentropic flow, the outlet conditions of the gas are at 02*s* whereas the actual outlet conditions are at 02. The subscript '0' refers to total conditions and 1 and 2 refers to the inlet and outlet points of the gas, respectively. '*s*' refers to constant entropy.



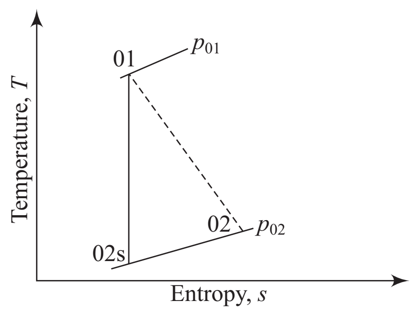


Figure 2.2 *Representation of Compression and Expansion Processes on T-s Diagram*

Now the pressure at the outlet, p_{02}, can be written as a function of the following variables to accommodate the compressibility of these types of fluids (gases).

$$p_{02} = f(D, N, \dot{m}, p_{01}, T_{01}, T_{02}, \rho_{01}, \rho_{02}, \mu) \tag{2.31}$$

Here, the pressure ratio p_{02}/p_{01} replaces the head H used in the incompressible flow turbomachine, while the mass flow rate $\dot{m}$ replaces Q. We can see that, using the equation of state, density may be written as $\rho = \dfrac{p}{RT}$. However, by examining Eq. (2.31), the density variable becomes superfluous since we already have T and p as variables. So deleting density and combining R with T, the functional relationship can be written as,

$$p_{o2} = f(p_{01}, RT_{01}, RT_{02}, \dot{m}, N, D, \mu) \quad (2.32)$$

Writing p_{02} as a product of the terms raised to powers,

$$p_{02} = \text{constant } [(p_{01})^a (RT_{01})^b (RT_{02})^c (\dot{m})^d (N)^e (D)^f (\mu)^g \quad (2.33)$$

Putting in the basic dimensions,

$$ML^{-1}T^{-2} = \text{constant } [(ML^{-1}T^{-2})^a (L^2T^{-2})^b (L^2T^{-2})^c (MT^{-1})^d (T^{-1})^e (L)^f (ML^{-1}T^{-1})^g]$$

Equating the indices,

$$M: 1 = a + d + g$$
$$L: -1 = -a + 2b + 2c + f - g$$
$$T: -2 = -2a - 2b - 2c - d - e - g$$

and solving for a, b and f in terms of d, c, e and g, we obtain,

$$a = 1 - d - g$$
$$b = \frac{d}{2} - c - \frac{e}{2} + \frac{g}{2}$$
$$f = e - 2d - g$$

Substitute for a, b and f in Eq. (2.33), then,

$$p_{02} = \text{constant}\left[(p_{01})^{1-d-g}(RT_{01})^{\frac{d}{2}-c-\frac{e}{2}+\frac{g}{2}}(RT_{02})^c(\dot{m})^d(N)^e(D)^{e-2d-g}(\mu)^g\right]$$

$$p_{02} = \text{constant} \times p_{01}\left\{\left(\frac{RT_{02}}{RT_{01}}\right)^c \left[\frac{\dot{m}(RT_{01})^{0.5}}{p_{01}D^2}\right]^d \left[\frac{ND}{(RT_{01})^{0.5}}\right]^e \left[\frac{\mu(RT_{01})^{0.5}}{p_{01}D}\right]^g\right\} \quad (2.34)$$

Now if the last term in the brackets in Eq. (2.34) is multiplied top and bottom by $(RT_{01})^{0.5}$ and noting that $\frac{p_{01}}{RT_{01}} = \rho_{01}$, then,

$$\frac{\mu RT_{01}}{p_{01}(RT_{01})^{0.5}D} = \frac{\mu}{\rho_{01}D(RT_{01})^{0.5}}$$

But the dimension of $(RT_{01})^{0.5}$ is L/T, which is a velocity. Therefore, the last term in brackets is expressible as a Reynolds number. Thus, the functional relationship may be written as,

$$\frac{p_{02}}{p_{01}} = f\left[\left(\frac{RT_{02}}{RT_{01}}\right), \frac{\dot{m}(RT_{01})^{0.5}}{p_{01}D^2}, \frac{ND}{(RT_{01})^{0.5}}, R_e\right] \quad (2.35)$$

The exact form of the function (2.35) must be obtained by experimental measurements taken from model or prototype tests. For a particular machine using a particular fluid, or for a model using the same fluid as the prototype, R is a constant and may be eliminated. The Reynolds number is in most cases so high and the flow so turbulent that changes in this parameter over the usual operating range may be neglected. However, where large changes of density take place, a significant reduction in R_e can occur, and this must then be taken into account. For a particular constant-diameter machine, the diameter D may be ignored and therefore, in view of these considerations, function (2.37) becomes,

$$\frac{p_{02}}{p_{01}} = f\left[\left(\frac{T_{02}}{T_{01}}\right), \frac{\dot{m}(T_{01})^{0.5}}{p_{01}}, \frac{N}{(T_{01})^{0.5}}\right] \tag{2.36}$$

It is usual to plot $\frac{p_{02}}{p_{01}}, \frac{T_{02}}{T_{01}}$ against the mass flow rate parameter, $\frac{\dot{m}(T_{01})^{0.5}}{p_{01}}$, for different values of the speed parameter, $\frac{N}{(T_{01})^{0.5}}$, for a particular machine. But for a family of machines, the full dimensionless group of Eq. (2.35) must be used if it is required to change the size of the machine or the gas contained. The term $\frac{ND}{(RT_{01})^{0.5}}$ can be interpreted as the Mach-number effect. This is because the impeller velocity is $C_b \propto ND$, and the acoustic velocity is $a_{01} \propto (RT_{01})^{0.5}$, while the Mach number is $M = \frac{C_b}{a_{01}}$.

2.4.1 Isentropic Efficiency

Here compressor efficiency is defined as ratio of isentropic work to actual work and turbine efficiency is defined as ratio of actual work to isentropic work; it is also called the isentropic efficiency, denoted by η_s.

It is estimated from $T - s$ and $h - s$ diagrams, in the case of compressible flow machines (high pressure ratio compressors, gas and steam turbines), referring Figure 2.3,

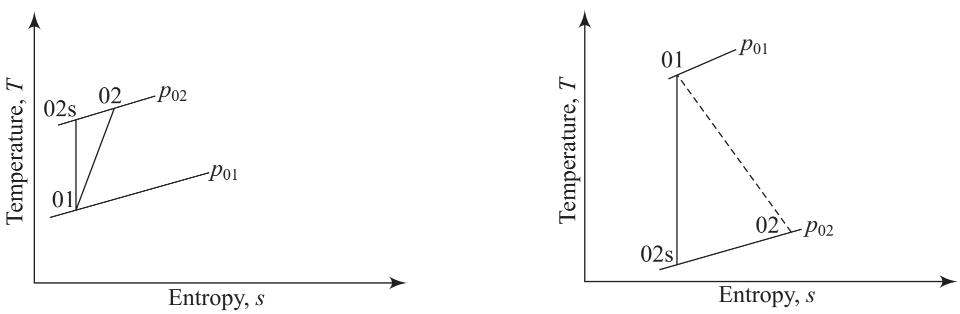


Figure 2.3 *Representation of Compression and Expansion Processes on T-s diagram*

$$\text{Compressor: } \eta_{sc} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{\Delta T_{0,s}}{\Delta T_0} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \tag{2.37}$$

$$\text{Turbine : } \eta_{st} = \frac{T_{02} - T_{01}}{T_{02s} - T_{01}} = \frac{\Delta T_0}{\Delta T_{0,s}} = \frac{h_{02} - h_{01}}{h_{02s} - h_{01}} \tag{2.38}$$

Since these expressions are based on stagnation or total conditions, the efficiencies are denoted by $(\eta_s)_{t-t}$.

In the case of turbines where the exit kinetic energy is not recoverable, a total to static efficiency is used as follows,

$$(\eta_{st})_{t-s} = \frac{T_{02} - T_{01}}{T_{2s} - T_1} = \frac{h_{02} - h_{01}}{h_{2s} - h_1} \tag{2.39}$$

For compressors, efficiency defined in terms of static to static condition is often used.

$$(\eta_{sc})_{s-s} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\Delta T_{is}}{\Delta T} = \frac{h_{2s} - h_1}{h_2 - h_1} \tag{2.40}$$

If the inlet and exit velocities are equal then,

$$(\eta_s)_{t-t} = (\eta_s)_{s-s} \tag{2.41}$$

2.4.2 Polytropic Efficiency

Pressure variation in a turbomachine may be regarded as comprising of a large number of small stages, irrespective of the actual number of stages in the machine. If each small stage has the same efficiency, then the isentropic efficiency of the whole machine will be different from the small stage efficiency. This difference is dependent upon the pressure ratio of the machine.

(a) Compressor

Consider an axial flow compressor. Referring to Figure 2.4, the pressure variation 1 – 2 is considered to be made up of several stages. Polytropic efficiency, η_p, is defined as the isentropic efficiency of an elemental stage in the process such that it is constant throughout the entire process.
For compression,

$$\eta_{p,\,c} = \frac{dT_s}{dT} = \text{constant} \tag{2.42}$$

For infinitesimal isentropic compression process,

$$\frac{dT_s}{T} = \frac{\gamma - 1}{\gamma} \frac{dp}{p} \tag{2.43}$$

Now substituting dT_s from Eq. (2.42), we have,

$$\frac{dT}{T} = \frac{\gamma - 1}{\gamma} \frac{1}{\eta_{p,c}} \frac{dp}{p} \tag{2.44}$$

Integrating Eq. (2.44),

$$\ln p^{\frac{1}{\eta_{p,c}}\left(\frac{\gamma-1}{\gamma}\right)} = \ln T + \ln \text{constant}$$

$$\frac{p^{\frac{1}{\eta_{p,c}}\left(\frac{\gamma-1}{\gamma}\right)}}{T} = \text{constant} \tag{2.45}$$

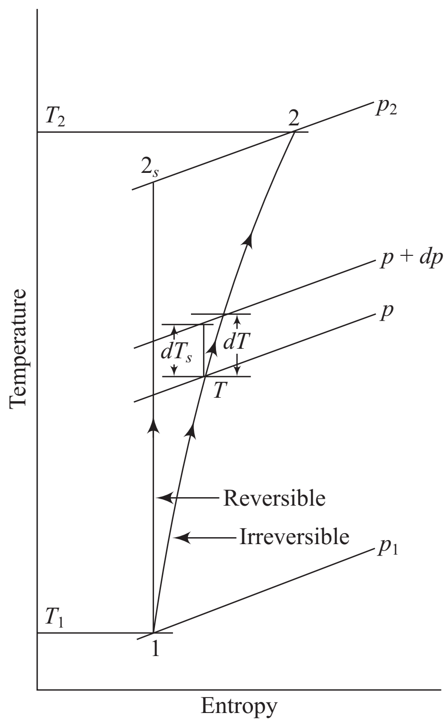


Figure 2.4 *Infinitesimal Stage in a Compression Processes*

Integrating Eq. (2.44) between the inlet 1 and outlet 2, we get,

$$\eta_{p,\,c} = \frac{\frac{\gamma-1}{\gamma}\ln(p_2/p_1)}{\ln(T_2/T_1)} = \frac{\ln(T_{2s}/T_1)}{\ln(T_2/T_1)} \tag{2.46}$$

From Eq. (2.45) between the states 1 and 2, we get,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_{p,c}}\frac{\gamma-1}{\gamma}} \tag{2.47}$$

The relation between $\eta_{p,\,c}$ and η_{sc} is given by,

$$\eta_{sc} = \frac{T_{2s}-T_1}{T_2-T_1} = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1}{\left(\frac{p_2}{p_1}\right)^{\frac{1}{\eta_{p,c}}\frac{\gamma-1}{\gamma}}-1} \tag{2.48}$$

From Eq. (2.48), if we write $\frac{1}{\eta_{p,c}}\frac{\gamma-1}{\gamma}$ as $\frac{n-1}{n}$, Eq. (2.47) is the functional relation between p and T for a polytropic process, and thus it is clear that the non-isentropic process is polytropic.

(b) Turbine

Figure 2.5 shows an infinitesimal stage for an expansion process in a turbine. For a turbine,

$$\eta_{p,\,t} = \frac{dT}{dT_s} = \text{constant} \tag{2.49}$$

For infinitesimal isentropic expansion,

$$\frac{T-dT_s}{T} = \left(\frac{p-dp}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

$$1-\frac{dT_s}{T} = \left(1-\frac{dp}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

Expanding the binomial expression on the right-hand side and ignoring terms beyond the second,

$$1-\frac{dT_s}{T} = 1-\frac{\gamma-1}{\gamma}\frac{dp}{p}$$

$$\frac{dT_s}{T} = \frac{\gamma-1}{\gamma}\frac{dp}{p} \tag{2.50}$$

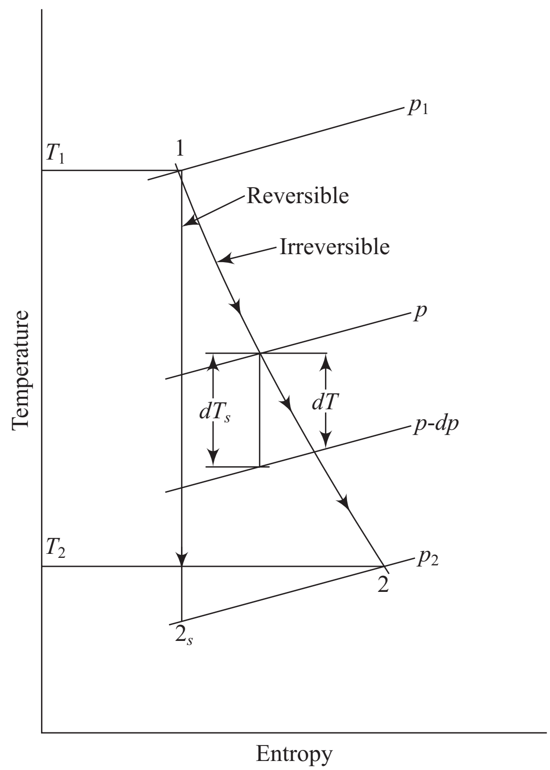


Figure 2.5 *Infinitesimal Stage in an Expansion Process*

Substituting the efficiency of such a stage in above equation,

$$\frac{dT}{T} = \frac{\gamma - 1}{\gamma} \eta_{p,\,t} \frac{dp}{p} \tag{2.51}$$

The differential Eq. (2.51) on integration results,

$$\ln T + \ln \text{constant} = \ln p^{\frac{\gamma - 1}{\gamma}\eta_{p,t}}$$

$$\frac{p^{\frac{\gamma - 1}{\gamma}\eta_{p,t}}}{T} = \text{constant} \tag{2.52}$$

Integrating Eq. (2.51) between the inlet 1 and outlet 2, we get,

$$\eta_{p,\,t} = \frac{\frac{\gamma}{\gamma - 1}\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{p_2}{p_1}\right)} = \frac{\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{T_{2s}}{T_1}\right)} \tag{2.53}$$

From Eq. (2.52), for isentropic process, we have,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}\eta_{p,t}} \tag{2.54}$$

The relation between $\eta_{p,t}$ and η_{st}, developed between the inlet 1 and outlet 2,is given by,

$$\eta_{st} = \frac{T_1 - T_2}{T_1 - T_{2s}} = \frac{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}\eta_{p,t}}}{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}} \tag{2.55}$$

where $\eta_{p,t}$ is the small stage or polytropic efficiency for the turbine. From Eq. (2.55), if we write $\frac{\gamma - 1}{\gamma}\eta_{p,t}$ as $\frac{n - 1}{n}$, then,

$$n = \frac{\gamma}{\gamma - (\gamma - 1)\eta_{p,t}} \tag{2.56}$$

If $\eta_{p,\,t} = 1 \Rightarrow n = \gamma \Rightarrow$ The actual expansion process coincides with the isentropic expansion process. Eq. (2.54) is the functional relation between p and T for a polytropic process, and thus it is clear that the non-isentropic process is polytropic.

In conclusion, if the pressure ratios for which the machines are designed are increased, isentropic efficiency of compressors tends to decrease and isentropic efficiency of turbines tends to increase.

2.5 Model Testing

Many turbomachines are so large that only a single unit might be required for model testing. Therefore, before a full-size machine is built, it is necessary to test it in model form to obtain as much information as possible about its characteristics. So that we may accurately transpose the results obtained from the model to the full-size machine, three criteria must be met. The first is that the model and the prototype must be geometrically similar; that is, the ratio of all lengths between the model and prototype must be the same. The second requirement is that of kinematic similarity, where the velocities of the fluid particles at corresponding points in the model and prototype must be related through a fixed ratio. The third requirement is that of dynamic similarity, where the forces acting at corresponding points must be in a fixed ratio between model and prototype. For a geometrically similar model, dynamic similarity implies kinematic similarity.

EXAMPLE 2.2 The specifications of centrifugal pump A are as follows:

Impeller diameter = 127 mm, Pump volume flow rate = 2.831 m^3/s, Impeller speed = 2000 rpm, Centrifugal head = 14 m

The specifications of centrifugal pump B are as follows:

Impeller diameter 102 mm, Impeller speed 2200 rpm

What is the volume flow rate of centrifugal pump B? Also determine its pump head.

Solution

Assuming that dynamic similarity exists between the first and second sized pumps, we equate the flow coefficients. Thus,

$$\frac{Q_A}{N_A D_A^3} = \frac{Q_B}{N_B D_B^3} \tag{1}$$

$$\frac{2.831}{2000 \times 127^3} = \frac{Q_B}{2200 \times 102^3}$$

$$Q_B = 1.6133 \text{ m}^3/\text{s} \tag{2}$$

Equating the head coefficients between the pumps A and B for dynamic similarity,

$$\frac{gH_A}{N_A^2 D_A^2} = \frac{gH_B}{N_B^2 D_B^2} \tag{3}$$

$$\frac{9.81 \times 14}{1400^2 \times 127^2} = \frac{9.81 \times H_B}{2200^2 \times 102^2}$$

$$H_B = 22.3 \text{ m} \tag{4}$$

EXAMPLE 2.3 Specifications for an axial flow coolant pump for one loop of a pressurised water nuclear reactor are:

Head = 85 m, Flow rate = 20000 m^3/hr, Speed 1490 rpm, Diameter = 1200 mm,

Water density 714 kg/m^3, Power 4 MW (electrical)

The manufacturer plans to build a model. Test conditions limit the available electric power to 500 kW and flow to 0.5 m^3/s of cold water. If the model and prototype efficiencies are assumed equal, find the head, speed and scale ratios of the model. Calculate the dimensionless specific speed of the prototype and confirm that it is identical with the model.

Solution

Equating flow, power and head coefficients for the prototype and model, we get,

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 \tag{1}$$

$$\frac{20000}{0.5 \times 3600} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3$$

$$\left(\frac{N_1}{N_2}\right) = 11.11\left(\frac{D_2}{D_1}\right)^3 \tag{2}$$

$$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 \tag{3}$$

Substitute for $\frac{N_1}{N_2}$ from Eq. (2) in Eq. (3), then,

$$\frac{4}{0.5} = \frac{714}{1000} \times 11.11^3 \left(\frac{D_2}{D_1}\right)^9 \left(\frac{D_1}{D_2}\right)^5$$

Scale ratio of model is,

$$\frac{D_2}{D_1} = 0.30 \tag{4}$$

$$\frac{N_1}{N_2} = 11.11 \times 0.3^3 = 0.3$$

$$\frac{N_2}{N_1} = 3.33 \tag{5}$$

$$N_2 = 3.33 \times N_1 = 3.33 \times 1490$$

$$N_2 = 4961.7 \text{ rpm} \tag{6}$$

Equating the head coefficients of model and prototype, we get head ratio as,

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 \tag{7}$$

$$\frac{H_2}{H_1} = 3.3^2 \times 0.3^2$$

$$\frac{H_2}{H_1} = 0.98 \tag{8}$$

$$H_2 = 0.98 \times H_1 = 0.98 \times 85$$
$$H_2 = 83.3 \text{ m} \tag{9}$$

The dimensionless specific speed is given by,

$$N_{sh} = \frac{N\sqrt{Q}}{(gH)^{3/4}} \tag{10}$$

For the prototype,

$$N_{sh} = \frac{1490 \times \sqrt{(20000/3600)}}{(9.81 \times 85)^{3/4}} = 22.6326 \tag{11}$$

For the model,

$$N_{sh} = \frac{4961.7 \times \sqrt{0.5}}{(9.81 \times 83.3)^{3/4}} = 22.955 \tag{12}$$

Therefore, taking rounding errors into account, the dimensionless specific speeds of both model and prototype are same.

2.5.1 Laws of Affinity

Generally, the effect of Reynolds number is negligible over a limited range and as long as the operating points are lying within this range, equality of Reynold's number need not be insisted. If different operating conditions refer to the same machine using the same fluid, then D and ρ will be same.

The capacity coefficient reduces to $\frac{Q}{N}$, the energy coefficient to $\frac{w}{N^2}$ and the power coefficient to $\frac{P}{N^3}$. It must be remembered that these reduced groups are not non-dimensional. Thus, there is a simple relation between the speed of the machine 'N' and the performance variables Q, w and P, for a given machine (pump/fan) using the same fluid and working under similar conditions. These relations are known as Pump/Fan Laws or Laws of Affinity.

Referring to Eq. (2.13) or (2.14), if the performance of a pump/fan (Q_a, w_a, P_a) is known at a speed N_a, its performance at another speed N_b can be found using the laws of affinity, provided the speeds are within a range where the effect of Reynolds number on the performance is negligible. Operations under 'a' and 'b' should be under similar conditions. When the speed was changed, no other change to the system, such as valve opening, etc. should have taken place. Points 'a' and 'b' must be on the same system line or throttle line. Velocity triangles will be similar and incidence will be the same.

$$Q_b = Q_a\left(\frac{N_b}{N_a}\right),\ w_b = w_a\left(\frac{N_b}{N_a}\right)^2,\ P_b = P_a\left(\frac{N_b}{N_a}\right)^3 \tag{2.57}$$

Referring to Figure 2.6, conditions at 'a' cannot be used to find conditions at 'c', since 'a' and 'c' are on different system lines and hence, similarity conditions do not exist.

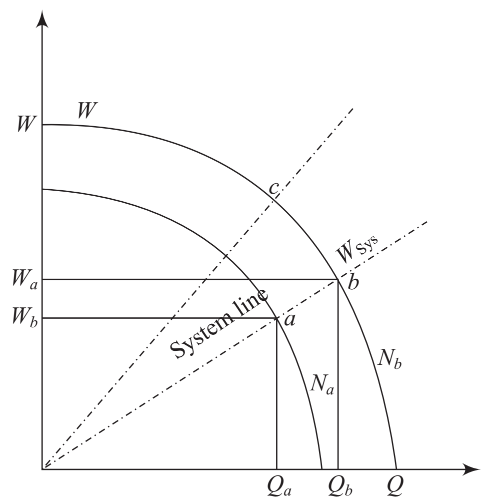


Figure 2.6 *Laws of Affinity*

EXAMPLE 2.4 The impeller diameter of centrifugal pump is 127 mm and the pump delivers 2.83 litres/s at a speed of 2000 rpm. If a 102 mm diameter impeller is fitted and the pump runs at a speed of 2200 rpm, what is the volume flow rate? Determine also the new pump head.

Solution

Assuming dynamic similarity exists between the first and second sized pumps, we equate the flow coefficients. Therefore,

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \tag{1}$$

$$\frac{2.83}{2000 \times 127^3} = \frac{Q_2}{2200 \times 102^3}$$

$$Q_2 = 1.61 \text{ litres/s} \tag{2}$$

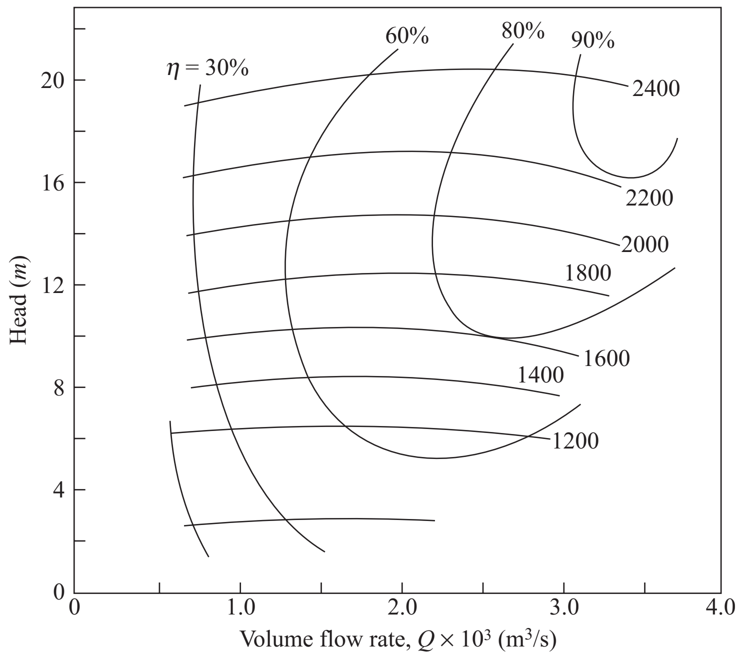


Figure 2.7 *Head vs Volume Flow Rate for Various Speeds*

From Figure 2.7, at $Q_1 = 2.83$ litres/s (2.83×10^{-3} m^3/s) and 2000 rpm, the head H_1 is 14 m. Equating head coefficients for both cases gives,

$$\frac{gH_1}{N_1^2 D_1^2} = \frac{gH_2}{N_2 D_2^2} \tag{3}$$

$$\frac{9.81 \times 14}{2000^2 \times 127^2} = \frac{9.81 \times H_2}{2200^2 \times 102^2}$$

$$H_2 = 10.9 \text{ m} \tag{4}$$

EXAMPLE 2.5 An axial flow compressor is designed to run at 4500 rpm and mass flow rate of 65 kg/s, when ambient atmospheric conditions are 101.3 kPa and 15°C. What is the correct speed at which the compressor must run and also, calculate mass flow rate when atmospheric conditions are 25°C and 60 kPa.

Solution

Using the various compressible machine dimensionless groups:

Speed parameter,

$$\frac{N_1}{\sqrt{T_{01}}} = \frac{N_2}{\sqrt{T_{02}}} \tag{1}$$

$$\frac{4500}{\sqrt{(273+15)}} = \frac{N_2}{\sqrt{273+25}}$$

$$N_2 = 4577.46 \text{ rpm} \tag{2}$$

Therefore, correct speed is 4577.46 rpm.

Now the mass flow parameter,

$$\frac{\left(\dot{m}_1\sqrt{T_{01}}\right)}{p_{01}} = \frac{\left(\dot{m}_2\sqrt{T_{02}}\right)}{p_{02}} \tag{3}$$

$$\frac{65\times\sqrt{(15+273)}}{101.3} = \frac{m_2\sqrt{(273+25)}}{60}$$

$$\dot{m}_2 = 37.848 \text{ kg/s} \tag{4}$$

In order to ensure this criteria, the values of the dimensionless groups in Eqs (2.14) and (2.35) must remain the same for both the model and the prototype.

2.5.2 Principles of Modelling

Identifying the prototype with the subscript '*p*' and the model with the subscript '*m*', we have,

Capacity coefficient,

$$\left(\frac{Q}{ND^3}\right)_p = \left(\frac{Q}{ND^3}\right)_m \tag{2.58}$$

Energy coefficient,

$$\left(\frac{w}{N^2D^2}\right)_p = \left(\frac{w}{N^2D^2}\right)_m \tag{2.59}$$

Power coefficient with same fluid used in model and prototype, i.e. $\rho_p = \rho_m$,

$$\left(\frac{P}{N^3D^5}\right)_p = \left(\frac{P}{N^3D^5}\right)_m \tag{2.60}$$

If $N_m = N_p \left(\frac{D_p}{D_m}\right)$, then these three equations reduce to,

$$Q_m = Q_p \left(\frac{D_m}{D_p}\right)^2, \; w_m = w_p, \; P_m = P_p \left(\frac{D_m}{D_p}\right)^2 \tag{2.61}$$

EXAMPLE 2.6 A one-fifth scale model of a pump was tested in a laboratory at 1000 rpm. The head developed and power input at the best efficiency point were found to be 8 m and 30 kW respectively. Determine the speed and power input to drive the pump if the prototype pump has to operate against a head of 25 m. Also find the ratio of the flow rates handled by the two pumps.

Solution

Given: Length ratio $D_r = \frac{D_m}{D_p} = \frac{1}{5}$, N_m = 1000 rpm, H_m = 8m, P_m = 30 kW, H_p = 25 m

(a) Speed of the Prototype

Since the head coefficient must besame for model and prototype, therefore,

$$\psi = \left(\frac{H}{N^2 D^2}\right)_m = \left(\frac{H}{N^2 D^2}\right)_P \tag{1}$$

$$\frac{H_m}{H_p} = \left(\frac{N_m}{N_p}\right)^2 \left(\frac{D_m}{D_p}\right)^2 \Rightarrow N_p = N_m \times \sqrt{\frac{H_P}{H_m}} \times \frac{D_m}{D_P} = 1000 \times \sqrt{\frac{25}{8}} \times \frac{1}{5}$$

$$N_p = 353.55 \text{ rpm} \approx 354 \text{ rpm} \tag{2}$$

(b) Power Input to the Pump (Prototype)

Power coefficient must be same for model and prototype, hence,

$$C_p = \left(\frac{P}{N^3 D^5}\right)_m = \left(\frac{P}{N^3 D^5}\right)_p \tag{3}$$

$$P_p = P_m \times \left(\frac{N_p}{N_m}\right)^3 \left(\frac{D_p}{D_m}\right)^5 = 30 \times \left(\frac{354}{1000}\right)^3 \times 5^5$$

$$P_p = 4158.92 \text{ kW} \tag{4}$$

(c) Flow Ratio

Discharge coefficient must be same for model and prototype.

$$\varphi = \left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p$$

Therefore, flow ratio = $Q_r = \frac{Q_m}{Q_p} = \left(\frac{N_m}{N_p}\right)\left(\frac{D_m}{D_p}\right)^3 = \frac{354}{1000}\left(\frac{1}{5}\right)^3$

$$Q_r = 2.832 \times 10^{-3} \tag{5}$$

EXAMPLE 2.7 A pump is designed for a flow rate of 0.3 m^3/s and a head of 38 m at a speed of 1500 rpm. The estimated overall efficiency is 0.8. A model of this pump is to be tested. The drive motor available has a speed of 1450 rpm and power of 4 kW. Find the maximum size of the model.

Solution

Coupling power of the pump,

$$P_p = \frac{\rho g Q H_p}{\eta_o} = \frac{1000 \times 9.81 \times 0.3 \times 38}{0.8}$$

$$P_p = 139792.5 \text{ W} = 139.7925 \text{ kW} \tag{1}$$

$$\left(\frac{P}{\rho N^3 D^5}\right)_m = \left(\frac{P}{\rho N^3 D^5}\right)_p \tag{2}$$

Since same fluid is used in model and prototype, therefore, $\rho_p = \rho_m$,

$$\frac{139.7925}{1500^3 D_p^5} = \frac{4}{1450^3 D_m^5}$$

$$\frac{D_p}{D_m} = 1.995 \tag{3}$$

The maximum model size possible is approximately half the size of the prototype.

2.6 Choice of Machines Based on Dimensionless Numbers

2.6.1 Specific Speed

Specific speed, a dimensionless parameter, may be obtained by considering two or more of the groups already found or by the formal method of combining those variables which will form a dimensionless group.

$$\text{Shape number or specific speed} = N_{sh} = \frac{(\text{Flow coefficient})^{1/2}}{(\text{Head coefficient})^{3/4}} \tag{2.62}$$

$$N_{sh} = \frac{[Q/(ND^3)]^{1/2}}{[gH/(N^2D^2)]^{3/4}}$$

$$N_{sh} = \frac{N\sqrt{Q}}{(gH)^{3/4}} \tag{2.63}$$

The expression at Eq. (2.63) is not truly dimensionless as N may be specified in rpm. Instead of N, angular velocity, ω, may also be specified in terms of rad/s. Here, the linear dimension D has been eliminated. It is important because Eq. (2.63) can be applied to geometrically similar machines of all sizes.

Or

$$\text{Shape number or specific speed} = N_{sh} = \frac{(\text{Power coefficient})^{1/2}}{(\text{Head coefficient})^{3/4}} \tag{2.64}$$

$$N_{sh} = \frac{[P/(\rho N^3 D^5)]^{1/2}}{[gH/(N^2D^2)]^{3/4}}$$

$$N_{sh} = \frac{N\sqrt{P}}{\rho^{1/2}(gH)^{5/4}} \tag{2.65}$$

In the expressions for N_{sh}, N, H, and Q are taken either at the maximum efficiency point, or, at the specification given on the nameplate of the machine.

2.6.2 Specific Diameter

We know that

$Q = C_f \times$ Flow Area $\propto C_f D^2$ and $C_b \propto ND$, then flow coefficient is

$$\frac{Q}{ND^3} = \frac{C_f}{C_b} = \varphi \tag{2.66}$$

Two dimensionless parameters shape number and specific diameter are generally used in the selection of most appropriate machine. The shape number is derived in such away that characteristic diameter (D) is eliminated.

Specific diameter is derived by eliminating the speed (N).

$$\text{Energy coefficient } \psi = \frac{2w}{C_{b2}^2} \tag{2.67}$$

A selection procedure making use of shape number can be evolved using the speed coefficient and specific diameter. A centrifugal impeller to be selected for a given volume and specific work is compared with another model impeller with identical volume and specific work, but with a volume coefficient of 1 and an energy coefficient of 1 in which $C_{f2} = C_{b2}$.The problem is to find the size D_{2m} and speed N_m of this model impeller.

The 'specific diameter' is defined in the following way:

$$\text{Specific diameter } D_s = \frac{D_2}{D_{2m}} = \frac{(\text{Energy coefficient})^{1/4}}{(\text{Volume coefficient})^{1/2}} = \frac{D(gH)^{1/4}}{Q^{1/2}} \tag{2.68}$$

2.7 Cordier Diagram

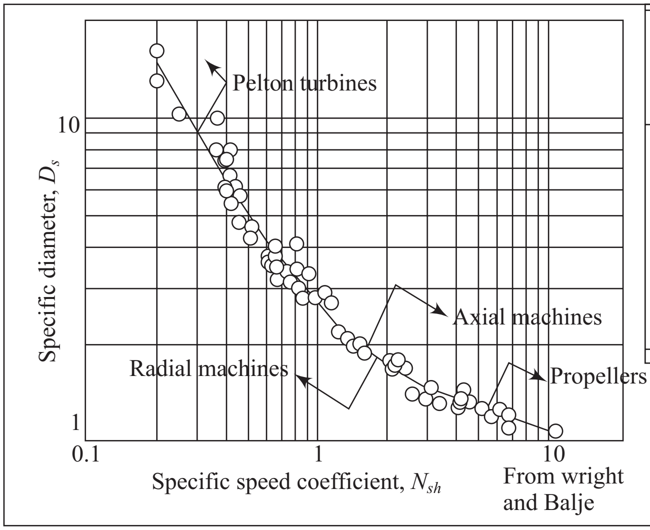
Specific coefficient parameter characterises the shape of the impeller.

$$\text{Specific shape number or speed coefficient } N_{sh} = \frac{N}{N_m} \tag{2.69}$$

$$\text{Specific shape number or speed coefficient} = \frac{(\text{Volume coefficient})^{1/2}}{(\text{Energy coefficient})^{3/4}} \tag{2.70}$$

Representation of the specific diameter on the y-axis as a function of specific shape number on the x-axis is known as the *Cordier diagram,* as shown in Figure 2.8, and is used for the design of different types of machines. Regions suitable for turbine and compressor are shown separately on the diagram. Some designers prefer to use Cordier diagram than using shape number as guidelines for the design.

Specific Speed Indicates Flowpath Shape
(Cordier Diagram)



Turbomachine	Range of specific speed coefficient or shape number N_{sh}
Pelton wheel	0.03–0.3
Francis turbine	0.3–2.0
Kaplan turbine	2.0–5.0
Centrifugal pumps	0.2–2.5
Axial-flow pumps	2.5–5.5
Centrifugal compressors	0.5–2.0
Axial-flow turbines	4.0–2.0
Axial-flow compressors	1.5–20.0

From Logan

N_{sh} is dimensionless

Figure 2.8 *Cordier Diagram*

EXAMPLE 2.8 A centrifugal pump, running at 1500 rpm and impeller diameter of 200 mm, discharges 0.12 m^3/s of water working against a head of 40 m with an efficiency of 90%. Calculate (a) the specific speed, (b) the performance of a similar pump twice its size keeping the speed constant, (c) the performance of a similar pump at twice the speed keeping diameter constant, and (d) the performance of a similar pump if the speed and size both are doubled.

Solution

Given: N_1 = 1500 rpm, D_1 = 200 mm = 0.2 m, Q_1 = 0.12 m^3/s, H_1 = 40 m, η_o = 90% = 0.9

(a) Specific Speed

Specific speed of the centrifugal pump is given by,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} \tag{1}$$

$$N_s = \frac{1500 \times \sqrt{0.12}}{40^{3/4}}$$

$$N_s = 32.67 \tag{2}$$

We know that for two geometrically similar pumps, discharge or capacity coefficient, head coefficient and power coefficient must be same. Therefore,

$$\varphi = \left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2 \Rightarrow \frac{Q_2}{Q_1} = \left(\frac{N_2}{N_1}\right)\left(\frac{D_2}{D_1}\right)^3 \tag{3}$$

$$\psi = \left(\frac{H}{N^2D^2}\right)_1 = \left(\frac{H}{N^2D^2}\right)_2 \Rightarrow \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 \tag{4}$$

$$C_P = \left(\frac{P}{\rho N^3D^5}\right)_1 = \left(\frac{P}{\rho N^3D^5}\right)_2 \Rightarrow \frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 \tag{5}$$

We know that overall efficiency of the pump is given by,

$$\eta_o = \frac{FP}{SP} = \frac{\rho gQH}{P} \tag{6}$$

$$\therefore \qquad P = \frac{\rho gQH}{\eta_o}$$

Therefore, brake power of the given pump is,

$$P_1 = \frac{1000 \times 9.81 \times 0.12 \times 40}{0.9}$$

$$P_1 = 52320 \text{ W} = 52.32 \text{ kW} \tag{7}$$

(b) Performance of the Similar Pump when the Size is Doubled

From Eqs (3), (4) and (5), we will get the discharge, head and brake power for a geometrically similar pump when the size of the pump is doubled while keeping the speed constant.

$$N_2 = N_1, D_2 = 2\,D_1$$

$$\frac{Q_2}{Q_1} = \left(\frac{D_2}{D_1}\right)^3 \Rightarrow Q_2 = Q_1\left(\frac{D_2}{D_1}\right)^3 = 0.12 \times 2^3$$

$$Q_2 = 0.96 \text{ m}^3\text{/s} \tag{8}$$

$$\frac{H_2}{H_1} = \left(\frac{D_2}{D_1}\right)^2 \Rightarrow H_2 = H_1\left(\frac{D_2}{D_1}\right)^2 = 40 \times 2^2$$

$$H_2 = 160 \text{ m} \tag{9}$$

Both the pumps are dealing the same fluid, therefore $\rho_1 = \rho_2$ and since $N_2 = N_1$, $D_2 = 2D_1$, therefore,

$$\frac{P_2}{P_1} = \left(\frac{D_2}{D_1}\right)^5 \Rightarrow P_2 = P_1\left(\frac{D_2}{D_1}\right)^5 = 52.32 \times 2^5$$

$$P_2 = 1674.24 \text{ kW} \tag{10}$$

(c) Performance of the Similar Pump when the Speed is Doubled

By Eqs (3), (4) and (5), we will get the discharge, head and brake power for a geometrically similar pump when the speed of the pump is doubled while keeping the diameter constant.

$$N_2 = 2N_1, D_2 = D_1$$

Substituting these values into Eq. (3), we get,

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1} \Rightarrow Q_2 = Q_1 \frac{N_2}{N_1} = 2 \times 0.12$$

$$Q_2 = 0.24 \text{ m}^3/\text{s} \tag{11}$$

We get,

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \Rightarrow H_2 = H_1 \left(\frac{N_2}{N_1}\right)^2 = 40 \times 2^2$$

$$H_2 = 160 \text{ m} \tag{12}$$

Both the pumps are dealing the same fluid, therefore $\rho_1 = \rho_2$ and since $N_2 = 2N_1$, $D_2 = D_1$, therefore,

$$\frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3 \Rightarrow P_2 = P_1 \left(\frac{N_2}{N_1}\right)^3 = 52.32 \times 2^3$$

$$P_2 = 418.56 \text{ kW} \tag{13}$$

(d) Performance of Geometrically Similar Pump when both the Speed and Size are Doubled

The discharge, head and brake power for a geometrically similar pump when both the speed and size of the pump are doubled,

$$N_2 = 2N_1, D_2 = 2D_1$$

Substituting these values, we get,

$$Q_2 = Q_1 \left(\frac{N_2}{N_1}\right)\left(\frac{D_2}{D_1}\right)^3 = 0.12 \times 2 \times 2^3$$

$$Q_2 = 1.92 \text{ m}^3/\text{s} \tag{14}$$

$$H_2 = H_1 \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 = 40 \times 2^2 \times 2^2$$

$$H_2 = 640 \text{ m} \tag{15}$$

Both the pumps are dealing the same fluid, therefore $\rho_1 = \rho_2$ and since $N_2 = 2N_1$, $D_2 = 2D_1$, therefore,

$$P_2 = P_1 \left(\frac{N_2}{N_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5 = 52.32 \times 2^3 \times 2^5$$

$$P_2 = 13393.92 \text{ kW} \tag{16}$$

2.8 Unit Quantities

In order to predict the performance of a turbine working under varying conditions of head, speed, power output and gate opening, the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity (1 m). The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected. The terms 'unit speed', 'unit power' and 'unit discharge' are frequently used to express the operational characteristics of hydraulic turbines. Variation of speed, power and discharge for various turbines are found out using these terms.

N, Q and P depends only on the head as,

$$N \propto H^{1/2} \tag{2.71}$$

$$Q \propto H^{1/2} \tag{2.72}$$

$$P \propto H^{3/2} \tag{2.73}$$

The constants of proportionality in Eqs (2.71), (2.72) and (2.73) are expressed in terms *unit quantities* as follows:

2.8.1 Unit Speed

The *unit speed,* N_u, is defined as the speed of a geometrically similar turbine working under a unit head (of 1m). Therefore, unit speed is given as,

$$N_u = \frac{N}{\sqrt{H}} \tag{2.74}$$

2.8.2 Unit Discharge

The *unit discharge,* Q_u, is defined as the discharge of a geometrically similar turbine working under a unit head (of 1m). Hence, unit discharge is given by,

$$Q_u = \frac{Q}{\sqrt{H}} \tag{2.75}$$

2.8.3 Unit Power

The *unit power,* N_u, is defined as the power of a geometrically similar turbine working under a unit head (of 1 m). Thus, unit power is given by,

$$P_u = \frac{P}{H^{3/2}} \tag{2.76}$$

It should be noted that the unit quantities given by Eqs (2.74), (2.75) and (2.76) are special cases of corresponding specific quantities, N_{11}, Q_{11} and P_{11},when diameter ratio is unity, i.e. when $D_m = D_p$. Therefore, for two homologous turbines 1 and 2 having the same diameter ($D_1 = D_2 = D$), the speed, discharge and power in these turbines are related as follows:

$$N_{u1} = N_{u2} = N_u \Rightarrow \left[\frac{N}{\sqrt{H}}\right]_1 = \left[\frac{N}{\sqrt{H}}\right]_2 = N_u \tag{2.77}$$

$$Q_{u1} = Q_{u2} = Q_u \Rightarrow \left[\frac{Q}{\sqrt{H}}\right]_1 = \left[\frac{Q}{\sqrt{H}}\right]_2 = Q_u \tag{2.78}$$

$$P_{u1} = P_{u2} = P_u \Rightarrow \left[\frac{P}{H^{3/2}}\right]_1 = \left[\frac{P}{H^{3/2}}\right]_2 = P_u \tag{2.79}$$

Following are the noteworthy points regarding significance of unit quantities:

- If a given turbine runs at speed N_1 and speed N_2 with corresponding discharges Q_1 and Q_2 and power P_1 and P_2; these parameters, N, Q and P,are related by the unit quantity relationships given in Eqs. (2.77), (2.78) and (2.79).
- Unit quantity relationships are particularly advantageous to compare the performance of a given turbine under varying heads.
- Unit quantities are not the basic similarity ratios but are the special cases corresponding to diameter ratio of unity, i.e. $D_1 = D_2$ in the cases under study.

EXAMPLE 2.9 The head race level in a hydel project is 200 m above the tailrace level of the plant. When the water flows at a rate of 4m^3/s through the turbine, the frictional losses in the penstock are 23 m and the head utilised in the turbine is 160 m. The leakage loss is estimated to be 0.09 m^3/s and the mechanical losses can be taken as 100 kW.

Calculate (a) the volumetric efficiency, (b) the hydraulic efficiency, (c) mechanical efficiency, and (d) overall efficiency of the system.

Solution

Given: H_g = 200 m, Q = 4m^3/s, h_f = 23 m, H_e = 160 m, ΔQ = 0.09 m^3/s, P_m = 100 kW, D_2/D_1 = 0.5

Net head on the turbine,

$$H = H_g - h_f \Rightarrow H = 200 - 23$$

$$H = 177 \text{ m} \tag{1}$$

1. **(a) Volumetric Efficiency**

$$\eta_v = \frac{Q - \Delta Q}{Q} \tag{2}$$

$$\eta_v = \frac{4 - 0.09}{4}$$

$$\eta_v = 0.9775 = 97.75\% \tag{3}$$

(b) Hydraulic Efficiency

$$\eta_h = \frac{H_e}{H} \tag{4}$$

$$\eta_h = \frac{166}{177}$$

$$\eta_h = 0.93785 = 93.785\% \tag{5}$$

(c) Mechanical Efficiency

$$P_{th} = \rho_w g(Q - Q_L)H_e \tag{6}$$

$$P_{th} = 1000 \times 9.81 \times (4 - 0.09) \times 160$$

$$P_{th} = 6137136 \text{ W} = 6137.136 \text{ kW} \tag{7}$$

Since power lost in mechanical friction is 100 W, therefore, actual shaft power produced is,

$$P = P_{th} - P_m = 6137.136 - 100$$

$$P = 6037.136 \text{ kW} \tag{8}$$

Mechanical efficiency is,

$$\eta_m = \frac{P}{P_{th}} \Rightarrow \eta_m = \frac{6037.136}{6137.136}$$

$$\eta_m = 0.983706 = 98.3706\% \tag{9}$$

(d) Overall Efficiency

$$\eta_o = \eta_v \eta_h \eta_m \tag{10}$$

$$\eta_o = 0.9775 \times 0.93785 \times 0.983706$$

$$\eta_o = 0.90181 = 90.181\% \tag{11}$$

EXAMPLE 2.10 Pump characteristics of two different sizes of same homologous series are given in the following table:

$D_1 = 0.7$ m, $N = 750$ rpm

Q	0	7	14	21	28	35	42	49	56
W	40	40.6	40.4	39.3	38	33.6	25.6	14.5	0

$D_2 = 0.51$ m, $N = 975$ rpm

Q	0	3.5	7.1	10.6	14.2	17.7	21.3	24.8	28.3
W	36	36.5	36.4	35.4	34.2	30.24	23.0	13.1	0

Plot the non dimensional characteristics.

Solution

For $D_1 = 0.7$ m, $N = 750$ rpm

φ	0	0.0272	0.054	0.082	0.109	0.136	0.163	0.190	0.218
Ψ	1.4×10^{-4}	1.47×10^{-4}	1.46×10^{-4}	1.42×10^{-4}	1.38×10^{-4}	1.22×10^{-4}	9.3×10^{-5}	5.3×10^{-5}	0

$D_2 = 0.51$ m, $N = 975$ rpm

φ	0	0.0272	0.054	0.082	0.109	0.136	0.163	0.190	0.218
Ψ	1.4×10^{-4}	1.47×10^{-4}	1.46×10^{-4}	1.42×10^{-4}	1.38×10^{-4}	1.22×10^{-4}	9.3×10^{-5}	5.3×10^{-5}	0

From the above table and Figure 2.9,we can say that,
Non dimensional characteristics of homologous series of pumps are same.

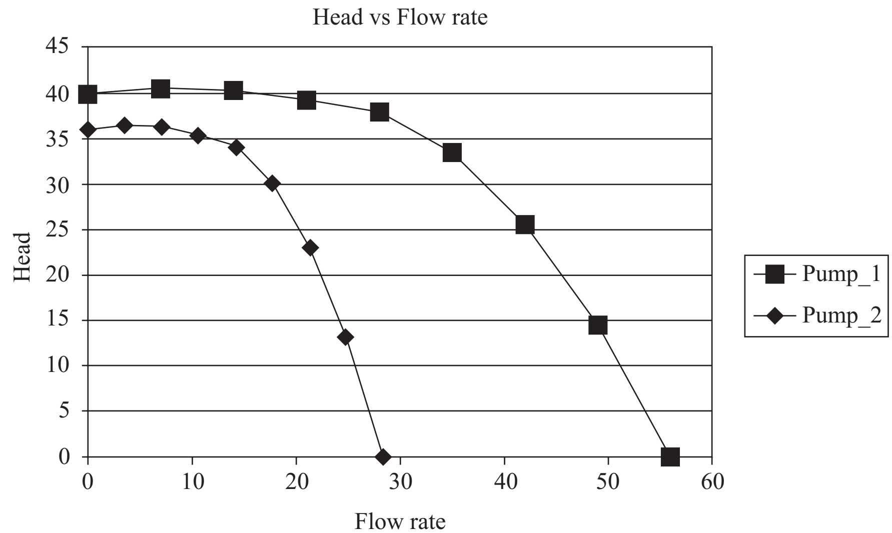


Figure 2.9 *Variation of Head with Flow Rate*

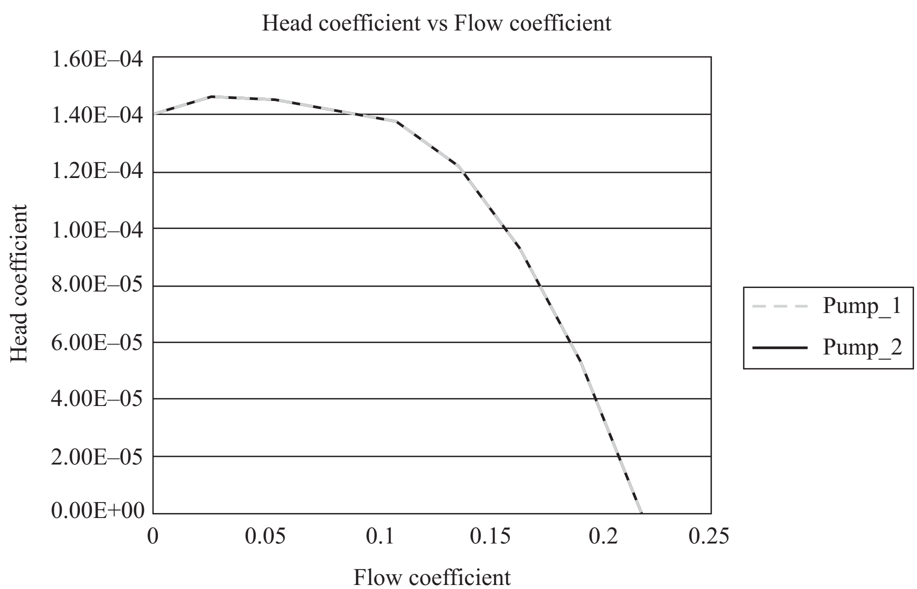


Figure 2.10 *Variation of Head Coefficient with Flow Coefficient*

SUMMARY

- The power coefficient, $\frac{P}{\rho N^3 D^5}$, of an incompressible turbomachine is calculated in terms of flow volume (capacity) coefficient, $\frac{Q}{ND^3}$, head coefficient, $\frac{gH}{N^2 D^2}$ and Reynolds number, $\frac{\rho CD}{\mu}$.
- Various efficiencies, such as mechanical efficiency, impeller efficiency, volumetric efficiency, mechanical efficiency and casing efficiency, are defined.
- The importance of $T - s$ diagram for the compression and expansion process in a compressor and turbine is illustrated.
- The pressure ratio of compressible flow machines is expressed in terms of non-dimensional mass flow parameter, speed parameter, total temperature and Reynolds number.
- The various stage efficiencies for compressor and turbine are defined.
- The specific speed and specific diameter are defined. The Cordier diagram represented as relations between specific diameter and specific speed is demonstrated for the choice of turbomachine.

REVIEW QUESTIONS

2.1 Discuss the significance of dimensional analysis in context to turbomachines.

2.2 What are the advantages of dimensional analysis for prediction of performance of turbomachines over experimental predictions?

2.3 List the primary variables with their dimensions for incompressible flow turbomachines.

2.4 List the primary variables with their dimensions for compressible flow turbomachines.

2.5 State Buckingham π theorem.

2.6 How are the repeating variables selected for dimensional analysis?

2.7 State the procedure for determining the dimensionless groups by Buckingham π theorem.

2.8 Derive an expression for incompressible flow turbomachine between power coefficient, head coefficient, flow coefficient, and Reynolds number, using Buckingham π theorem.

2.9 State the significance of power coefficient, head coefficient and flow coefficient.

2.10 State the factors on account of which dimensional analysis for compressible flow turbomachines differ from that of incompressible flow turbomachines.

2.11 Distinguish between the static and stagnation properties.

2.12 Why stagnation properties are the preferred choice over static properties for the analysis of a turbomachine.

2.13 Derive the functional relationship of stagnation pressure ratio in terms of stagnation temperature ratio, mass flow rate parameter, Mach number and Reynolds number, for a compressible flow turbomachine using Buckingham π theorem.

2.14 Using the method of dimensional analysis for a centrifugal pump, prove that,

$$\frac{Q}{ND^3} = f\left[\frac{gH/(N^2D^2)}{\mu/(\rho ND^2)}\right]$$

2.15 What is specific speed of a turbine? State its significance in the study of turbines.

2.16 Derive an expression for the specific speed and indicate the unit of various terms adopted in SI units.

2.17 How specific speed of a turbine is non-dimensionalised to obtain the shape factor?

2.18 What is meant by homologous turbines?

2.19 Why affinity laws are developed for homologous turbines?

2.20 What are the similarity laws of turbines for speed, discharge and power of two homologous turbines?

2.21 Define specific speed of a centrifugal pump. State its significance.

2.22 Derive an expression for specific speed of a centrifugal pump.

2.23 How does the specific speed of a centrifugal pump differ from that of a turbine?

2.24 Briefly describe the non-dimensional specific speed of centrifugal pumps.

2.25 Write a short note on the significance of similarity parameters in a centrifugal pump.

2.26 Describe (a) Discharge coefficient, (b) Head coefficient, and (b) Power coefficient. Establish a relation between these coefficients.

2.27 Write a brief note on the model testing of a centrifugal pump.

2.28 State the similarity rules for geometrically similar pumps of the same diameter while running at different speeds.

2.29 State the affinity laws for two geometrically similar pumps having different diameters while running at the same speed.

2.30 State the significance of affinity laws of a centrifugal pump.

2.31 What are the unit quantities? State their significance.

2.32 Explain briefly the choice based dimensionless numbers for a turbomachine.

2.33 State the significance of Cordier diagram.

2.34 Write a short note on Cordier diagram.

2.35 Define the following isentropic compression efficiencies:
(a) Total-to-total efficiency
(b) Total-to-static efficiency
(c) Static-to-static efficiency

2.36 What is the difference between four different forms of isentropic compression efficiencies? State the applicability of each type of the efficiency.

2.37 Define and differentiate between the various forms of isentropic expansion efficiencies.

2.38 What is meant by preheat effect in a multistage compression?

2.39 Prove that the overall isentropic compression efficiency is less than the individual stage isentropic compression efficiency in the case of multistage compression.

2.40 Prove that the overall isentropic expansion efficiency is more than the individual stage isentropic expansion efficiency.

2.41 Explain the polytropic efficiency in the case of compression process and derive an expression for the same.

2.42 Explain the polytropic efficiency in the case of expansion process and deduce an expression for the same.

2.43 Obtain an expression for the overall compression efficiency in terms of stage pressure ratio.

2.44 Obtain an expression for the overall expansion efficiency in terms of stage pressure ratio.

2.45 Draw the $h - s$ diagram with static and stagnation states for the compression and expansion of a gas.

PROBLEMS

2.1 Air at 100 kPa and 23°C is compressed in a stage of compressor having a static pressure ratio of 1.9.The velocities at the inlet and outlet are 15 m/s and 120 m/s, respectively. If the total-to-total efficiency is 89%, determine (a) total-to-static efficiency, (b) static-to-total efficiency, and (c) static-to-static efficiency. For air, assume γ= 1.4, c_p = 1.004 kJ/kg – K.

[*Ans:* (a) η_{t-s} = 79.52%, (b) η_{s-t} = 89.15%, (c) η_{s-s} = 79.67%]

2.2 Two fluids are to be compressed to a final total pressure of 600 kPa from an initial total pressure 100 kPa and a total temperature of 23°C. Calculate the work input if the compression is isentropic. One fluid is water with density of 1000 kg/m^3 and the other is air with c_p = 1.005 kJ/kg – K, γ= 1.4.

[*Ans:* w_{water} = 0.5 kJ/kg, w_{air} = 20 kJ/kg]

2.3 At the inlet of a gas turbine, the static pressure, temperature and velocity are 550 kPa, 677°C and 180 m/s, respectively. At the outlet of the stage, the static pressure is 250 kPa and velocity is 75 m/s. The mass flow rate of the combustion gases through the turbine is 1.5 kg/s and stage power output is 250 kW. Find the stage efficiency when, (a) the stage is first one, (b) the stage is one of the middle stages, and (c) the stage is last one. Take c_p = 1.005 kJ/kg – K, γ= 1.4.

[*Ans:* (a) η_{s1} = 80.91%, (b) η_{sM} = 80.91%, (c) η_{sL} = 79.82%]

2.4 The pressure ratio is 1.3 in each of the stages in a four stage compressor. The initial pressure and temperature at the inlet are 100 kPa and 23°C respectively. Calculate the final exit pressure and the overall isentropic efficiency if the isentropic efficiency of each stage is 0.9. Take c_p = 1.005 kJ/kg – K and γ= 1.4. What is the preheat effect in a compressor?

[*Ans:* p_5 = 285.61 kPa, η_o = 88.86%, 1.347 kJ/kg]

2.5 The pressure, temperature and velocity of air at the inlet of a single stage compressor are 100 kPa, 25°C and 10 m/s, respectively. The corresponding values at the outlet are 210 kPa, 100°C and 87 m/s. The exit of the compressor is 1 m above the inlet. Calculate (a) the actual work, (b) the isentropic work, and (c) the isentropic efficiency of the compressor. Take c_p = 1.005 kJ/kg – K and γ= 1.4.

[*Ans:* (a) w_a = 79.124 kJ/kg, (b) w_s = 70.72 kJ/kg, (c) η_c = 89.38%]

2.6 A four stage gas turbine has same stage efficiency and same pressure ratio in each of the stages. The combustion products enters in the gas turbine at a total pressure and total temperature of 600 kPa and 727°C. The stage efficiency is 0.86 and exit pressure is 100 kPa. Calculate (a) the isentropic work of all stages, (b) the reheat factor, (c) the total actual work, and (d) the overall isentropic efficiency of the turbine. Take c_p = 1.005 kJ/kg – K, γ= 1.4for the gases.

[*Ans:* (a) w_{ts} = 412.69 kJ/kg, (b) RF = 1.026, (c) w_{ts} = 354.9 kJ/kg, (d) η_o = 88.23%]

2.7 The initial and final total pressures of a fluid are 1 bar and 10 bar respectively. The initial total temperature is 10°C. What is the work of compression for isentropic steady flow with a total-to-total efficiency of 75%, if (a) the fluid is water, and (b) the fluid is air?

[*Ans:* (a) w = 1.2 kJ/kg, (b) w = 252.94 kJ/kg]

2.8 Gases from a combustion chamber enter a gas turbine at a total pressure of 7 bar and a total temperature of 827°C while the total pressure and total temperature at the turbine exit are 1.5 bar and 557°C respectively. Take molecular weight and ratio of specific heats for gases as 28.7 kg and 1.3, respectively. Compute the total-to-total efficiency and the total-to-static efficiency if the velocity at the exit of the turbine is 250 m/s. The flow through the turbine may be assumed steady isentropic flow.

[*Ans:* η_{t-t} = 82.1%, η_{t-s} = 76.3%]

2.9 The total temperature and total pressure at the inlet of a turbomachine are 30°C, and 1 bar respectively. The total specific enthalpy change in the machine is 6 kJ/kg and the isentropic total-to-total efficiency is 75%. Find (a) what general type of turbomachine would this be, (b) the total temperature at the exit, if the fluid is air, and (c) the total pressure ratio across the machine, if (i) the fluid is air, (ii) the fluid is water.

[Ans: (a) Work absorbing turbomachine, (b) T_{02} = 308.97 K, (c) $\left(\frac{p_{02}}{p_{01}}\right)_a = 1.053$, (ii) $\left(\frac{p_{02}}{p_{01}}\right)_w = 46$]

2.10 An oil of density 800 kg/m^3 flows at the rate of 0.1 m^3/s through a turbomachine which develops 100 kW power. The velocity of flow at inlet is 3 m/s while that at exit is 10 m/s, respectively. The total-to-total efficiency of the machine is 75%. Calculate (a) the change in total pressure of the oil, and (b) the change in static pressure of the oil. [*Ans:* (a) Δp_0 = –13.4 bar, (b) Δp_s = –13.8 bar]

2.11 The stagnation pressure ratio between the exit and the inlet of each stage is 0.4 in a 4 stage turbine handling air. The stage efficiencies of the first two stages are 86% whereas those of the last two stages are 84% each. Calculate the overall efficiency of the turbine. [*Ans:* η_o = 89.5%]

2.12 The pressure rise of air is 1400 mm of water gauge in a low pressure air compressor. If the initial and final states of air are p_1 = 1.0 bar, T_1 = 32°C, T_2 = 47°C. Calculate the isentropic compressor and infinitesimal stage efficiencies. [*Ans:* η_{cs} = 75.23%, η_{cp} = 75.88%]

2.13 A compressor increases the pressure of air from 1.01 bar to 3 bar. If the temperature at the inlet of compressor is 32°C. Determine the polytropic efficiency of the compressor if its isentropic efficiency is 0.75. [*Ans:* η_{cp} = 78.5%]

2.14 The isentropic efficiency of a gas turbine is 88% whereas the pressure ratio is 2.2. Determine the polytropic efficiency of the turbine if the temperature at inlet is 1227°C. Why the calculated polytropic efficiency of the turbine is less than its isentropic efficiency? [*Ans:* η_{tp} = 86.7%, Due to reheating]

2.15 The pressure ratio in each of the stages of an eight-stage air compressor is 1.3. The overall efficiency of the compressor is 80% and the mass flow rate of air through it is 45 kg/s. The state of the air at inlet of the compressor is 1 bar, 35°C. Determine (a) the state of air at compressor exit, (b) the polytropic efficiency, and (c) the efficiency of each stage.

[*Ans:* (a) p_9 = 8.16 bar, (b) η_p = 85%, (c) η_s = 84.43%]

2.16 The temperature at inlet of a three-stage gas turbine is 1227°C and temperature rise is same in each of the stages. The overall efficiency and overall pressure ratio of the turbine are 88% and 11 respectively. Find, for each stage, (a) the pressure ratio, and (b) the stage efficiency.

$$\left[Ans: \text{(a) } \frac{p_1}{p_2} = 1.93, \text{ (b) } \eta_{s1} = 84.95\%, \ \eta_{s2} = 85.2\%, \ \eta_{s3} = 85.51\% \right]$$

2.17 The mass flow rate in each of the stages in an four-stage air compressor is 45 kg/s. The pressure ratio across each stage is 1.2 with a stage efficiency of 65%. The temperature of air at the inlet is 20°C. Calculate (a) the overall efficiency, (b) the pressure ratio, and (c) the power required to drive the compressor.

$$\left[Ans: \frac{p_5}{p_1} = 2.1, \text{ (b) } \eta_o = 62.28\%, \text{ (c) } P = 5019.98 \text{ kW} \right]$$

2.18 The increase in total pressure of air is 0.2 m water gauge when it flows through a blower. The total pressure and total temperature of air at the inlet are 1.04 bar and 18°C, respectively. The total-to-total efficiency is 72%. Calculate (a) the total pressure and total temperature at the exit, (b) isentropic and actual changes in total enthalpy.

[*Ans:* (a) p_{02} = 1.0596 bar, T_{02} = 20.16°C, (b) Δh_{0s} = 1.563 kJ/kg, Δh_{0a} = 2.171 kJ/kg]

2.19 The stagnation pressure of air is decreased in the ratio of 5:1 as it flows through an air turbine. The power output of the turbine is 500 kW at the mass flow rate of 5 kg/s. The flow velocity at the exit of the turbine is 100 m/s and the total-to-total efficiency is 80%. Calculate (a) the total temperature at the inlet, (b) the actual total temperature at the exit, (c) actual static temperature at the exit, and (d) the total-to-static efficiency of the machine.

[*Ans:* (a) T_{01} = 64°C, (b) T_{02} = –35.5°C, (c) T_2 = –40.5°C, (d) η_{t-s} = 76.94%]

2.20 The pressure ratio in each of the stages in a three-stage turbine is 2 and the stage efficiency is 75%. The air initially at a temperature of 600°C flows through the machine at a rate of 25 kg/s. Calculate (a) the overall efficiency, and (b) the power developed.

[*Ans:* (a) η_{ot} = 78.76%, (b) P = 7.738 MW]

2.21 Steam enters a turbine with a static pressure, a static temperature and a flow velocity of 35 bar, 400°C and 200 m/s, respectively. At the turbine exit, the static pressure, static temperature and velocity are 1 bar, 100°C and 150 m/s, respectively. Neglecting the heat transfers during the expansion process, Calculate (a) the total-to-total efficiency, (b) total-to-static efficiency and, (c) the static-to-static efficiency. [*Ans:* (a) η_{t-t} = 0.72, (b) η_{t-s} = 0.71, (c) η_{s-s} = 0.73]

2.22 In a single-stage gas turbine, gas enters and leaves in axial direction. The nozzle efflux angle is 40°, the stagnation temperature and stagnation pressure at stage inlet are 800°C and 4 bar respectively. The exhaust static pressure is 1 bar, total-to-static efficiency is 0.85, and mean blade speed is 450 m/s. Determine (a) the work done, (b) the axial velocity which is constant through the stage, (c) the total-to-total efficiency, and (d) the degree of reaction. Assume γ= 1.33, and c_p = 1.15 kJ/kg – K.

[*Ans:* (a) w = 265 kJ/kg, (b) C_a = 485.5 m/s, (c) η_{t-t} = 85%, (d) R = 34%]

2.23 A turbine is working under a head of 20 m at a speed of 500 rpm. A 1 : 2 scale model of the turbine having the efficiency same as that of prototype is to be tested under a head of 20 m. Calculate (a) the speed of the model, (b) the scale ratios of discharge and power.

$$\left[Ans. \text{ (a) } N_m = 1000 \text{ rpm, (b) } Q_r = \frac{1}{4}, \ P_r = \frac{1}{4} \right]$$

2.24 Water flows at the rate of 12 m^3/s through a turbine which is operating under a head of 25 m at a speed of 300 rpm. Calculate the specific speed and power developed by the turbine assuming the efficiency of the turbine to be 85%. What would be the specific speed, power, discharge and speed of rotation under a head of 15 m?

[*Ans:* $N_s = 268.1$, $P_1 = 2496.5$ kW, $N_s = 268.1$, $P_2 = 1160$ kW, $Q_2 = 9.30$ m^3/s, $N_2 = 232.4$ rpm]

2.25 The diameter of the runner of a turbine is 1.5 m which develops 1500 kW power under a head of 10 m at a speed of 120 rpm. A 1:3 scale model of the turbine is tested under a head of 3.0 m. Determine the speed and power of the model. Also, calculate the discharge in the model and prototype assuming an overall efficiency of 90% for both the model and prototype.

[*Ans:* $N_m = 197.18$ rpm, $P_m = 27.39$ kW, $Q_m = 1.036$ m^3/s, $Q_p = 17.024$ m^3/s]

2.26 A turbine is working under a net head of 90 m. Water at the rate of 2.3 m^3/s flows through a 1:2 model turbine which develops 300 kW brake power under a net head of 15 m at 600 rpm. Calculate (a) the specific speed of the model, (b) overall efficiency of the model, (c) overall efficiency of the prototype, and (d) the brake power of the prototype.

[*Ans:* (a) $N_s = 352$, (b) $\eta_{om} = 88.8\%$, (c) $\eta_{op} = 90.2\%$, (d) $P_p = 17.91$ MW]

2.27 The runner diameter of a Francis turbine model is 1 m with an overall efficiency of 88.5% at its optimum performance. If the runner diameter of a homologous prototype is 2 m, calculate its maximum efficiency.

If another homologous model of the runner of diameter 0.4 m is to be tested, what maximum efficiency can be expected in this new model? [*Ans:* $\eta_{op} = 90.4\%$, $\eta_{2m} = 86.2\%$]

2.28 A Francis turbine having an overall efficiency of 85% produces 6.75 MW under a net head of 45 m at 300 rpm. This turbine has to work under homologous conditions under a net head of 60 m. What would be the speed, discharge and brake power of this turbine under the new head?

[*Ans:* $N_2 = 364.41$ rpm, $Q_2 = 20.814$ m^3/s, $P_2 = 10.392$ MW]

2.29 A turbine is required to work under a head of 35 m and at a speed of 420 rpm. A 1:4 scale model of this turbine is to be tested in a laboratory under a head of 15 m. Water at the rate of 2 m^3/s flows through the model which develops 260 kW. At what speed should the model be run? If the prototype efficiency is 3% better than that of the model, what is the power developed by the prototype turbine?

[*Ans:* $N_m = 1100$ rpm, $\eta_p = 91.2\%$, $P_p = 15.272$ MW]

2.30 The following data are obtained from a laboratory test conducted on a turbine model:

Head at the test = 8 m

Unit power = 9.2

Unit speed = 55

Unit discharge = 1.25

(a) Calculate the efficiency at this operating point. (b) If a homologous prototype turbine works under a net head of 24 m for an operating point corresponding to the above laboratory test data, find (i) the operating speed of the prototype, (ii) discharge through the prototype, and (iii) power developed by the prototype turbine.

[*Ans:* (a) $\eta_o = 75.2\%$, (b) (i) $N_p = 1082$ rpm, (ii) $Q_p = 6.123$ m^3/s, (iii) $P_p = 1082$ kW]

2.31 An axial flow compressor is designed to run at 4500 rpm and mass flow rate of 65 kg/s when ambient atmospheric conditions are 101.3 kPa and 15°C. What is the correct speed at which the compressor must run and also calculate mass flow rate when atmospheric conditions are 25°C and 65 kPa.

[*Ans:* (a) Correct speed = 4577 rpm, Mass flow obtained = 37.85 kg/s]

MULTIPLE CHOICE QUESTIONS

1. If there are m fundamental dimensions in a physical phenomenon, then the number of repeating variables for dimensional analysis by Buckingham's π theorem is
   (a) $> m$ (b) $= m$ (c) 0 (d) $\leq m$
2. The repeating variables in a dimensional analysis should
   (a) Be equal in number to that of fundamental dimensions involved in the problem variables
   (b) Include the dependent variable
   (c) Collectively contain all the fundamental dimensions
   (d) Have at least one variable containing all the fundamental dimensions
3. Which of the following rules are used in choosing the repeating variables in dimensional analysis?
   1. Repeating variables should include the dependent variables.
   2. Repeating variables should contain all primary units used in describing the variables in the problem.
   3. Repeating variables should combine among themselves to form a π.
   4. Repeating variables should not contain dependent variables.

   Of these statements:
   (a) 1 and 2 are correct (b) 2 and 3 are correct
   (c) 2 and 4 are correct (d) 3 and 4 are correct
4. With n variables and m fundamental dimensions in a system, which one of the following statements relating to the application of Buckingham's π theorem is incorrect?
   (a) With experience, π terms can be written simply by inspection of variables in a flow system.
   (b) Buckingham π-theorem is not directly applicable in compressible flow problem.
   (c) Buckingham π-theorem yields dimensionless π terms given by the difference between the number of variables and the number of fundamental dimensions.
   (d) Buckingham π-theorem reduces the number of variables by the number of fundamental dimensions involved.
5. In turbomachinery, the relevant parameters are volume flow rate, density, viscosity, bulk modulus, pressure difference, power consumption, rotational speed and a characteristic dimension. According to Buckingham π theorem, the number of independent non-dimensional groups for the case is
   (a) 3 (b) 4 (c) 5 (d) 6
6. The correct dimensionless group formed with the density ρ, speed of rotation N, diameter D, and viscosity μ is
   (a) $\dfrac{\rho ND}{\mu}$ (b) $\dfrac{\rho ND^2}{\mu}$ (c) $\dfrac{ND}{\rho\mu}$ (d) $\dfrac{ND^2}{\rho\mu}$
7. Assuming that the thrust T of a propeller depends on the diameter D, speed of advance C, angular velocity ω, dynamic viscosity μ and mass density ρ. Which of the following non-dimensional parameters can be derived by dimensional analysis?
   1. $\dfrac{T}{\rho C^2 D^2}$ 2. $\dfrac{CD}{\mu}$ 3. $\dfrac{D\omega}{C}$ 4. $\dfrac{CD\rho}{\mu}$

   Select the correct answer using the following codes.
   (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1, 2 and 4

8. A large hydraulic turbine is to generate 300 kW at 400 rpm under a head of 40 m. For initial testing, a 1:4 scale model of the turbine operates under a head of 10 m. The power generated by the model will be
(a) 2.34 kW (b) 4.68 kW (c) 9.38 kW (d) 18.75 kW
9. A hydraulic turbine develops 1000 kW of power under a head of 40 m. If the head is reduced to 20 m, the power developed is
(a) 177 kW (b) 353 kW (c) 500 kW (d) 707 kW
10. The power ratio of a pump and its 1:4 scale model, if the ratio of heads is 5:1, will be
(a) 100 (b) 3.2 (c) 179 (d) 12.8
11. The model of a propeller 3 m in diameter cruising at 10 m/s in air is tested in a wind tunnel on a 1:10 scale model. If a thrust of 50 N is measured on the model at 5 m/s wind speed, then the thrust on the prototype will be
(a) 20000 N (b) 2000 N (c) 500 N (d) 200 N
12. A turbine works at 20 m head and 500 rpm speed. Its 1:2 scale model to be tested at a head of 20 m should have a rotational speed of nearly
(a) 1000 rpm (b) 700 rpm (c) 500 rpm (d) 250 rpm
13. A Francis turbine produces 5000 kW at 280 rpm under a net head of 49 m with an overall efficiency of 80%. What would be the rpm of the same turbine under a net head of 64 m?
(a) 640 rpm (b) 480 rpm (c) 320 rpm (d) 160 rpm
14. A turbine develops 8000 kW when running at 100 rpm. The head on the turbine is 36 m. If the head is reduced to 9 m, the power developed by the turbine will be
(a) 16000 kW (b) 4000 kW (c) 1414 kW (d) 1000 kW
15. Which of the following relationships must be satisfied in the model testing of hydraulic turbines?

(a) $\dfrac{Q}{\sqrt{H}D^2}$ = constant, $\dfrac{H}{N^3 D}$ = constant (b) $\dfrac{H}{NH^3}$ = constant, $\dfrac{Q}{N^2 D^2}$ = constant

(c) $\dfrac{N\sqrt{Q}}{H^{3/2}}$ = constant, $\dfrac{N\sqrt{P}}{H^{3/4}}$ = constant (d) $\dfrac{P}{QH}$ = constant, $\dfrac{N\sqrt{P}}{H^{3/4}}$ = constant

16. The overall efficiency of a Pelton turbine is 70%. If the mechanical efficiency is 85%, what is its hydraulic efficiency?
(a) 82.4% (b) 59.5% (c) 72.3% (d) 81.3%
17. A turbine has a leakage of 0.1 m^3/s when the discharge supplied at the inlet is 2 m^3/s. If the hydraulic efficiency is 90%, overall efficiency is 75%, the mechanical efficiency of the turbine is
(a) 0.80 (b) 0.93 (c) 0.88 (d) 0.90
18. A turbine has a discharge of 3 m^3/s while operating under a head of 15 m and at a speed of 500 rpm. What would be the speed of the turbine if it has to operate under a head of 12 m?
(a) 400 rpm (b) 447 rpm (c) 559 rpm (d) 600 rpm
19. A 1:2 scale model turbine is to be tested under a head of 30 m. The speed of the prototype is 400 rpm while operating under a head of 30 m. The speed of rotation of the model would be
(a) 800 rpm (b) 566 rpm (c) 400 rpm (d) 200 rpm

20. The overall efficiency of a turbine model is found to be 88%. The diameter of the model is 400 mm. What would be the efficiency of the prototype having a diameter of 1 m?
    (a) 0.880 (b) 0.900 (c) 0.855 (d) 0.925
21. Two turbines *A* and *B* produces 400 kW and 100 kW respectively. Both the turbines are of same type having the same specific speed and are operating under the same head. What is the speed of turbine *B* if turbine *A* runs at 100 rpm?
    (a) 1250 rpm (b) 4000 rpm (c) 2000 rpm (c) 1500 rpm
22. A Francis turbine develops 500 kW power while running at a speed of 400 rpm. The unit speed of the turbine is 50 rpm. What is the effective head under which the turbine is working?
    (a) 40 m (b) 64 m (c) 62.5 m (d) 100 m
23. A hydraulic turbine develops 300 kW power under a head of 40 m while running at a speed of 400 rpm. A 1:4 scale model of the turbine is tested under a head of 10 m. The power generated by the model will be
    (a) 18.75 kW (b) 9.38 kW (c) 2.34 kW (d) 4.68 kW
24. Consider the following relations between the nozzle efficiency η_n, hydraulic efficiency η_h, wheel efficiency η_w, mechanical efficiency η_m and the overall efficiency η_o
    1. $\eta_o = \eta_n \eta_h$, 2. $\eta_h = \eta_n \eta_w$,
    3. $\eta_h = \eta_m \eta_n$, 4. $\eta_o = \eta_n \eta_w \eta_m \eta_h$

    The correct relations are
    (a) 1 and 2 only (b) 3 and 4 only (c) 1 only (d) 2, 3 and 4
25. A 1/3 scale model of a Pelton wheel is tested. The efficiency of the prototype for preliminary studies will be taken as
    (a) Same as that of the model
    (b) Greater than that of the model
    (c) Greater than that of the model if the diameter of the model runner is greater than 0.5 m
    (d) Less than that of the model

    The following data pertain to Questions 26 to 28.

    A Francis turbine under a head of 25 m produces 2000 kW at 250 rpm.
26. The power produced under a head of 1 m will be
    (a) 2 kW (b) 10 kW (c) 16 kW (d) 25 kW
27. The operating speed under a head of 1m will be
    (a) 80 rpm (b) 50 rpm (c) 24 rpm (d) 20 rpm
28. The rpm under 1 m head to produce 1 kW will be
    (a) 3.125 (b) 6.5 (c) 12 (d) 25
29. A centrifugal pump requires brake power of 18 kW while delivering 0.1 m^3/s water against a net head of 135 kPa. The specific weight of the water is 9.8 kN/m^3. What is the overall efficiency of the pump? What would be the brake power if the liquid to be pumped is a solvent instead of water having a relative density of 0.8 while keeping other factors same as before?
    (a) (75%, 14.4 kW) (b) (75%, 18 kW) (c) (80%, 22 kW) (d) 82%, 19.6 kW)
30. A centrifugal pump delivers a liquid of relative density 0.8 against a head of 12 m. If the pump is used to deliver a liquid of relative density 1.2 while the other factors remain the same, the new manometric head would be
    (a) 18 m (b) 8 m (c) 10 m (d) 12 m

31. If the speed of a centrifugal pump is doubled, its power consumption will increase
    (a) 6 times (b) 4 times (c) 2 times (d) 8 times
32. A centrifugal pump lifts water to a head of 14 m and consumes 1.73 kW of power while running at the speed of 1200 rpm. When it is operated at 2000 rpm and if the overall efficiency remains the same, its power consumption and head generated would be approximately
    (a) 4 kW and 50 m of water (b) 7 kW and 25 m of water
    (c) 6 kW and 35 m of water (d) 8 kW and 38.9 m of water
33. The following data pertain to the performance of a centrifugal pump:
    Speed = 1400 rpm, Discharge = 35 litres/s, Head = 27 m and Power = 8 kW. If the speed is increased to 1800 rpm, the power will be approximately equal to
    (a) 9.5 kW (b) 11.7 kW (c) 17 kW (d) 13.2 kW
34. A centrifugal pump delivers 120 litres/s of water when running at 1800 rpm. What would be the discharge if the speed of the pump is increased to 3000 rpm?
    (a) 9.9 m^3/s (b) 0.2 m^3/s (c) 0.5 m^3/s (d) 0.06 m^3/s
35. A centrifugal pump working against a head of 40 m delivers water at its best efficiency point at the rate of 70 litres/min while running at 600 rpm. If the speed of the pump is increased to 1200 rpm, then the head H in m and the flow Q in litres/min would be
    (a) (160, 140) (b) (40, 140) (c) (40, 560) (d) (160, 35)
36. A centrifugal pump driven by directly coupled 3 kW motor of 1450 rpm speed is proposed to be connected to another motor of 2900 rpm speed. The power of the motor should be
    (a) 6 kW (b) 12 kW (c) 18 kW (d) 24 kW
37. If the discharge in a pump is halved while keeping its speed constant, the ratio of the new head to the old head is
    (a) $\left(\frac{1}{2}\right)^{1/2}$ (b) $\left(\frac{1}{4}\right)$ (c) $\left(\frac{1}{2}\right)^{1/2}$ (d) $\left(\frac{1}{2}\right)^{1/3}$
38. A pump and its 1:4 scale model are being compared. If the ratio of the heads is 4:1, then the ratio of the power required by the prototype and model is
    (a) 100 (b) 4 (c) 128 (d) 16
39. Which one of the following equations is correct?
    (a) $h_{01} = h_1 + \frac{C_1^2}{2c_p}$ (b) $p_{01} = p_1 + \frac{C_2^2}{2c_p}$
    (c) $T_{01} = T_1 + \frac{C_1^2}{2c_p}$ (d) $T_{02} = T_2 + \frac{C_2^2}{2c_p}$
40. In a compression process
    (a) Isentropic efficiency $= \frac{\text{Actual work}}{\text{Isentropic work}}$ (b) Isentropic efficiency $= \frac{\text{Instropic work}}{\text{Actual work}}$
    (c) Isothermal efficiency $= \frac{\text{Actual work}}{\text{Isothermal work}}$ (d) Actual efficiency $= \frac{\text{Actual work}}{\text{Isentropic work}}$

41. In an expansion process

(a) Isentropic efficiency $= \dfrac{\text{Actual work}}{\text{Isentropic work}}$ (b) Isentropic efficiency $= \dfrac{\text{Isentropic work}}{\text{Actual work}}$

(c) Isothermal efficiency $= \dfrac{\text{Actual work}}{\text{Isothermal work}}$ (d) Actual efficiency $= \dfrac{\text{Actual work}}{\text{Isentropic work}}$

42. In a multistage compressor, the total-to-total efficiency is applicable
    (a) To the first stage (b) To any intermediate stage
    (c) To the last stage (d) To the intermediate stage only
43. In a multistage turbine, the total-to-static efficiency is applicable
    (a) To the first stage (b) To the last stage
    (c) To any intermediate stage (d) To the intermediate stage only
44. In all stages of multistage compressor, pressure ratios are equal and stage efficiencies are also equal. Then
    (a) Overall efficiency > Stage efficiency (b) Overall efficiency < Stage efficiency
    (c) Overall efficiency = Stage efficiency (d) Any of these
45. In all stages of multistage turbine, pressure ratios are equal and stage efficiencies are also equal. Then
    (a) Overall efficiency > Stage efficiency (b) Overall efficiency < Stage efficiency
    (c) Overall efficiency = Stage efficiency (d) Any of these
46. Which one of the following statements is incorrect?
    (a) The preheat factor is applicable to a multistage compressor
    (b) The reheat factor is applicable to a multistage turbine
    (c) In a multistage turbine, the reheat factor is greater than unity
    (d) In a multistage compressor, the preheat factor is less than unity
47. For a multistage compressor, the polytropic efficiency is
    (a) The efficiency of all stages combined together
    (b) The efficiency of one stage
    (c) Constant throughout all the stages
    (d) A direct consequence of the pressure ratio
48. Which one of the following statements is true?
    (a) In a multistage compressor, adiabatic efficiency is less than stage efficiency
    (b) In a multistage turbine, adiabatic efficiency is less than stage efficiency
    (c) Reheat factor for a multistage compressor is greater than one
    (d) Reheat factor does not affect the multistage compressor performance
49. Which one of the following expresses the isentropic efficiency, η_{sc} of the compression process in terms of enthalpy changes as shown in Figure 2.11?

    (a) $\eta_{sc} = \dfrac{\Delta h_s}{\Delta h}$

    (b) $\eta_{sc} = \dfrac{\Delta h}{\Delta h_s}$

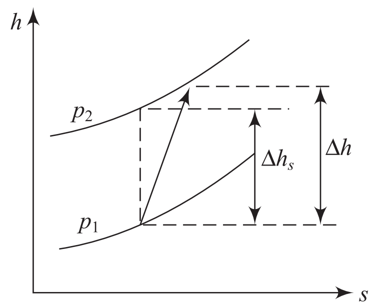


Figure 2.11 *Multiple Choice Question 49*

(c) $\eta_{sc} = \dfrac{(\Delta h - \Delta h_s)}{\Delta h}$

(d) $\eta_{sc} = \dfrac{(\Delta h - \Delta h_s)}{\Delta h_s}$

Direction: Each of the questions 50-51 consists of two statements, one labelled as **Assertion (A)** and the other as **Reason (R)**. You have to examine the two statements carefully and select the answers to these items using the following codes:

Codes

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true and R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

50. **Assertion (A):** In multistage compressors, the polytropic efficiency is always greater than the isentropic efficiency.

    Reason (R): Higher the pressure ratio, greater is the polytropic efficiency.

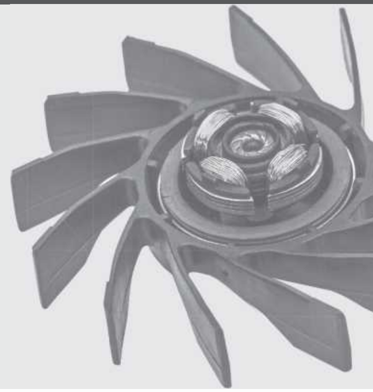
51. **Assertion (A):** The performance parameter ‘polytropic efficiency is used for axial flow gas turbines and air compressors.

    Reason (R): Polytropic efficiency is dependent on the pressure ratio.

52. The ratio of static enthalpy rise in the rotor to the static enthalpy rise in the stage of an axial flow compressor is defined as
    (a) Power input factor
    (b) Flow coefficient
    (c) Temperature coefficient
    (d) Degree of reaction

ANSWER KEY

1. (d)	2. (c)	3. (c)	4. (d)	5. (c)	6. (b)	7. (c)	8. (a)	9. (a)	10. (c)
11. (a)	12. (d)	13. (c)	14. (d)	15. (c)	16. (a)	17. (c)	18. (b)	19. (a)	20. (b)
21. (c)	22. (b)	23. (c)	24. (a)	25. (a)	26. (c)	27. (b)	28. (a)	29. (b)	30. (d)
31. (d)	32. (d)	33. (c)	34. (b)	35. (a)	36. (d)	37. (b)	38. (c)	39. (c)	40. (b)
41. (a)	42. (d)	43. (b)	44. (b)	45. (a)	46. (d)	47. (c)	48. (a)	49. (a)	50. (c)
51. (c)	52. (d)								



3 Hydraulic Turbines

Learning Objectives

After reading this chapter, you will be able to:

- **LO 1** Understand the principle of operation of various hydraulic turbines
- **LO 2** Understand the nature of energy transfer in hydraulic turbines
- **LO 3** Study the classifications and principal parts of hydraulic turbines
- **LO 4** Learn and derive expressions for the power developed and efficiencies of various hydraulic turbines
- **LO 5** Understand the necessity of a draft tube in a reaction turbine and derive its efficiency
- **LO 6** Study the performance characteristics of hydraulic turbines

3.1 Introduction

A hydraulic turbine converts stored energy in the form of either potential or kinetic energy of water into shaft work. Historically, hydraulic turbines of today are derived from the water wheels of the middle ages used for flour mills (to grind wheat) and ore-crushing. One such water wheel can still be seen at Aurangabad, which is, at least, four hundred years old. Modern turbines have undergone many technological advances in diverse areas like fluid mechanics, metallurgy, and mechanical engineering.

3.2 Schematic Layout of a Hydroelectric Power Plant

Schematic layout of a hydroelectric power plant is shown in Figure 3.1. It consists of:

(i) A dam constructed across a river for storage of water.

(ii) Penstocks are pipes of large diameters used to carry water under pressure from the storage reservoir to the turbines. Steel or reinforced concrete is used for manufacturing of penstocks.

(iii) Turbines having different blades fitted on rotor.

(iv) Tailrace is a channel to carry water away from the power plant after it has passed through the turbines. The free surface of water in the tail channel is called tailrace level.

3.2.1 Gross Head

The difference between the headrace level and tailrace level when no water is flowing is known as gross head. In other words, it is the difference between the upstream reservoir water level and the water level in the tailrace channel on the downstream. It is denoted by H_g as shown in Figure 3.1.

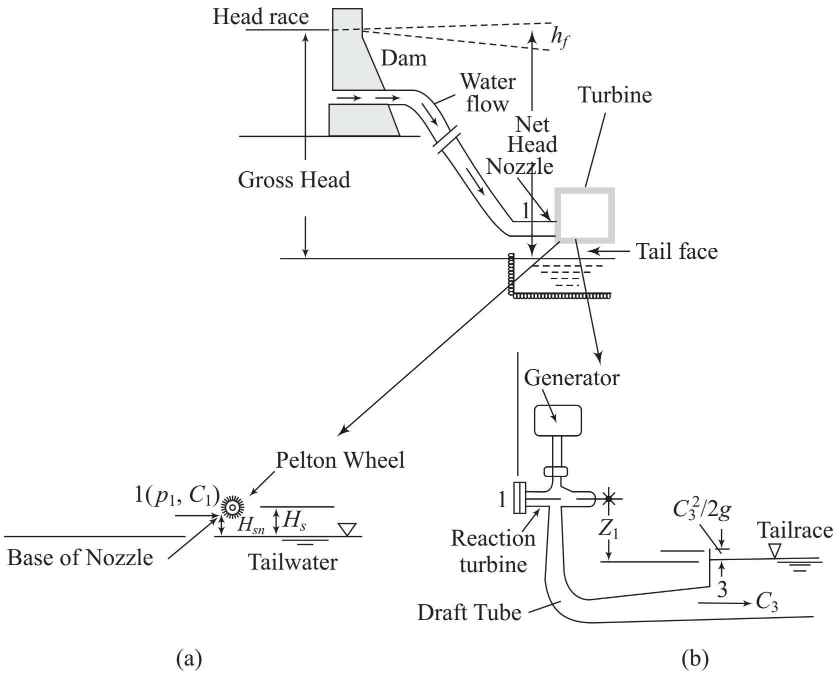


Figure 3.1 *Schematic Layout of a Hydroelectric Power Plant (a) Pelton Turbine, (b) Reaction Turbine*

3.2.2 Net Head

When water is flowing from headrace to the turbine, a loss of head due to friction between the water and penstock occurs. Though there are other losses also such as loss due to bend, pipe fittings, loss at the entrance of penstock etc., yet they are having small magnitude as compared to head loss due to friction. The head loss due to friction h_f is given by,

$$h_f = 4f\frac{L}{D^*} \times \frac{C^2}{2g} \tag{3.1}$$

where,

L = Length of penstock,
D^* = Diameter of penstock,
C = Velocity of flow in penstock.

The net head, also known as effective head, is the difference of heads between the inlet and outlet of a turbine. The net head represents the head available at the entrance of the turbine for conversion into mechanical work.

(a) Impulse Turbine

Pelton wheel, the pressure of the water does not change while flowing through the rotor of the turbine. In Pelton wheel, pressure change occurs only in the nozzle, not in the rotor. The impingement of the jet to the blades/buckets takes place at atmospheric pressure. Therefore, net head in Pelton wheel is the head available at the inlet of the turbine i.e. at the base of the nozzle as pressure everywhere is atmospheric. Refer to the layout of Pelton turbine as shown in Figure 3.1 (a).

Let H_{sn} is the height of the lowest nozzle above tailrace and H_s is turbine setting height which is the difference in elevation of the centre of the Pelton wheel and tailrace (*for* vertical axis Pelton wheel $H_{sn} = H_s$; whereas for horizontal axis Pelton wheel $H_{sn} < H_s$). Then net head on the Pelton wheel is given by,

$$H = H_g - h_f - H_{sn} \tag{3.2}$$

Net head for Pelton wheel is also given by,

$$H = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} \tag{3.3}$$

where, $p_1/\rho g$ is the pressure head at the base of the nozzle and $C_1^2/2g$ is the velocity head at the base of the nozzle.

(b) Reaction Turbine

As discussed earlier, the pressure of the water changes while water flows through the rotor of turbine. The change in water velocity and reduction in its pressure causes a reaction on the turbine blades. A reaction turbine always runs completely filled with water in an airtight casing. The tube that connects the exit of the rotor or runner of the turbine to the tailrace is known as a draft tube. The layout of a reaction turbine is shown in Figure 3.1(b). The fluid exiting from the turbine is ultimately discharged to the tailrace through a draft tube which is a divergent pipe or passage which connects the turbine exit to the tailrace. A draft tube is used to convert a large proportion of kinetic energy at the turbine exit into useful pressure energy so that velocity at its outlet is minimum.

Applying the Bernoulli's equation between the headrace and tailrace,

$$H_g = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 + h_f \tag{3.4}$$

The head at the inlet of the turbine,

$$H_1 = H_g - h_f = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 \tag{3.5}$$

The draft tube is considered as a part of the turbine. Let C_3 be the velocity at the exit of the draft tube. The velocity head $C_3^2/2g$, represents the energy head that is not utilised by the turbine in conversion of mechanical work. Therefore, the net head on the turbine is the difference of total head at the inlet of the turbine minus total head at exit of the draft tube. Hence, net head is given by,

$$H = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 - \frac{C_3^2}{2g} = H_1 - \frac{C_3^2}{2g} \tag{3.6}$$

$$H = H_g - h_f - \frac{C_3^2}{2g} \tag{3.7}$$

3.3 Classification of Hydraulic Turbines

The hydraulic turbines can be classified on the basis of (i) net head and flow rate or discharge of water available, (ii) nature of work done by the blades, (iii) direction of flow of water, and (iv) specific speed and specific diameter.

3.3.1 According to the Head and Quantity of Water Available

The difference in elevation of water surface between upstream and downstream of the turbine is the head under which turbine acts. The turbines work under a wide range of heads varying from 2 to 2000 m. A classification of turbine based on head is as shown in Table 3.1 below:

TABLE 3.1 Ranges of Operating Variables

Turbine	Net Head (H)	Discharge (Q)
Pelton Turbine	High head (300–2000 m)	Low discharge (3–4 m^3/s)
Crossflow Turbine	Medium head (100–500 m)	High discharge (6–18 m^3/s)
Francis Turbine	Low head (100–500 m)	High discharge (18–21 m^3/s)
Propeller Turbine	Low head (1.5–50 m)	High discharge (1.5–60 m^3/s)

For low heads, only Kaplan or propeller turbines are used. For medium heads, either Kaplan or Francis turbines are used. For high heads, either Francis or Pelton turbines are used. For very high heads, invariably Pelton turbines are used. Deriaz turbines are used up to a head of 300 m. Their use is, however, restricted under reversible flow conditions i.e. pumped-storage plants where the turbine also works as a pump.

Turbines can also be classified as low discharge, medium discharge and high discharge turbines, depending on the flow available. Pelton turbines are relatively low discharge turbines. Kaplan turbines are high discharge turbines, while Francis turbines occupy an intermediate position in this regard.

3.3.2 According to the Nature of Work Done by the Blades

Hydraulic turbines are classified as impulse and reaction turbines depending on the mode of energy conversion of potential energy of water in the dam into shaft work.

(a) Impulse Water Turbine

In an impulse water turbine, the pressure of water does not change while flowing through the rotor of the machine. In these turbines, all the available head of water is converted into kinetic energy in a nozzle [refer Figure 3.1 (a)] and not in the rotor. Pelton wheel belongs to this category in which the water shoots out of the nozzle in a free jet into a bucket which revolves round a shaft. During this action, the water is in contact with air all the time and the water discharged from bucket falls freely through the discharge passage into the tailrace. The free jet is at ambient pressure before and after striking the vanes.

(b) Reaction Water Turbine

In a reaction water turbine, the pressure of the water changes while it flows through the rotor of the machines. At the entrance to the runner, only a part of potential energy is converted into kinetic energy and the remaining into pressure energy. The change in velocity and reduction of pressure of water during its flow through the blade passages causes a reaction on the turbine blades. Thus, the runner converts both kinetic energy and pressure energy into mechanical energy (shaft work). In reaction turbines, the entire flow from headrace to tailrace takes place in a closed conduit system which is not open to the atmosphere at any point in its passage [refer Figure 3.1(b)]. This is because of the fact that flow passages are under pressure. Francis, Propeller, Kaplan and Deriaz turbines belong to this category.

3.3.3 According to the Direction of Flow of Water

The three orthogonal directions in the turbine flow can be referred to as radial, axial and tangential with respect to the wheel. The turbines can be classified under this category as given below:

(a) Radial Flow Turbine

If the direction of flow of water from the inlet to outlet of the turbine runner is along the radius, it is known as the *radial flow turbine*. Inward flow is the most common feature of the radial flow turbines. Early Francis turbines were of the inward radial flow type, refer Figure 3.2 (a).

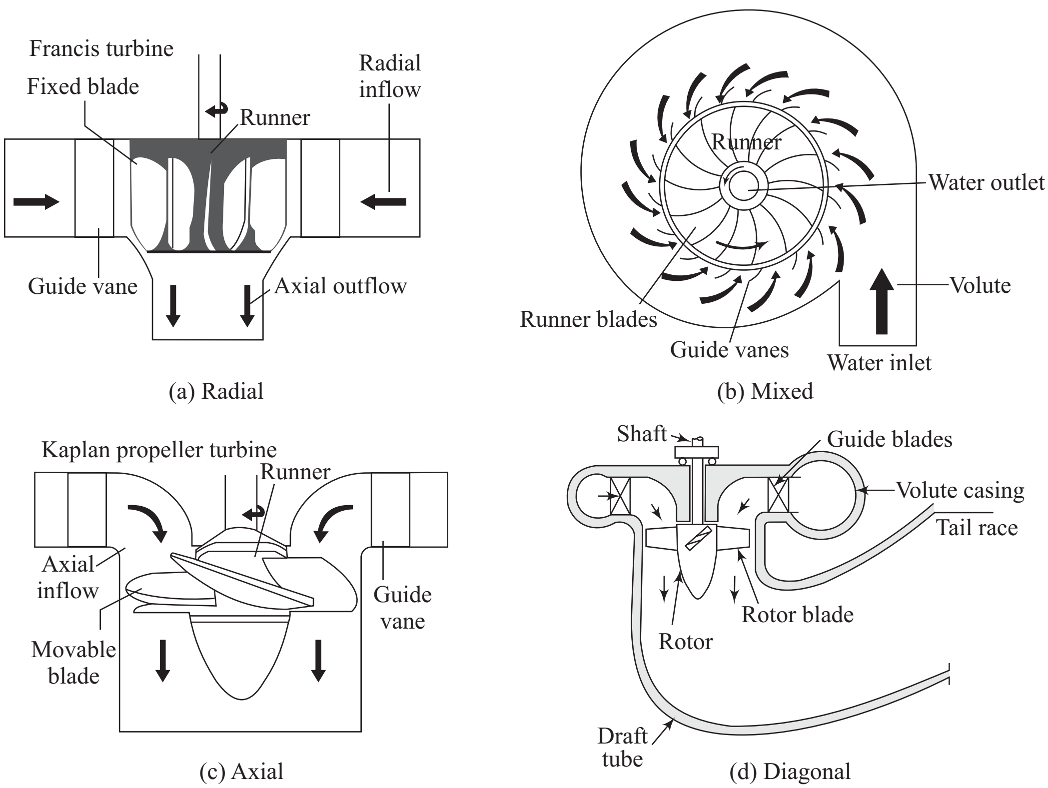


Figure 3.2 *Hydraulic Turbines according to the Direction of Flow*

(b) Mixed Flow Turbine

Sometimes, the flow direction can change between inlet and outlet of the turbine runner. It can be radial flow at the inlet and axial flow at the outlet. The turbine having a mixed flow runner is termed as a *mixed flow turbine.* Present day Francis turbine is a typical example of this category, refer Figure 3.2 (b).

(c) Axial Flow Turbines

If the flow enters and leaves the runner axially i.e. in a direction parallel to the axis of the shaft, it is known as *axial flow turbine*. The rotor blades of these turbines are essentially in the form of a propeller. Kaplan turbine, a turbine with adjustable propeller blades is an example of axial flow turbine, refer Figure 3.2 (c).

(d) Tangential Flow or Peripheral Flow Turbines

In tangential or peripheral flow turbine, the flow direction is tangential to the runner. An example of this type of turbine is Pelton wheel where the path of the jet is tangential to the path of the rotating wheel.

(e) Diagonal Flow Turbines

If the flow is neither parallel to the axis, nor perpendicular to it, but is in an angular direction with respect to the axis, it may be called *diagonal flow.* Deriaz turbine is an example of this type of turbine, refer Figure 3.2 (d). Table 3.2 summarises the flow directions of commonly used turbines.

TABLE 3.2 Flow Directions of Water Turbines

Turbine Types	*Flow Direction*
Francis turbine	Radial inward flow or mixed flow
Pelton turbine	Tangential flow
Propeller and Kaplan turbines	Axial flow
Deriaz turbine	Diagonal flow

3.3.4 According to the Specific Speed

The specific speed, N_s, is the speed of a turbine which is geometrically similar with the actual turbine that produces unit power (1 kW) when working under unit head (1 m). It is given by,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \tag{3.8}$$

where, N = normal working speed of actual turbine in rpm, P = power output of the actual turbine in kW, and H = net head in m.

Specific speed is an important parameter for the design of the turbine as it includes all the three basic parameters, viz. speed, power and head of the turbine. Although, technically, specific speed is not a dimensionless quantity, it exhibits the properties of a dimensionless number. However, by introducing ρ_w (mass density) and g (acceleration due to gravity) which are constants, one can convert it to a non-dimensional form as given by equation below,

$$N_{sh} = \frac{N\sqrt{P}}{\rho_w^{1/2}(gH)^{5/4}} \tag{3.9}$$

It should be noted that in calculating the dimensionless specific speed in revolution, N is in rev/s, power P is in Watts and head H is in m. For water $\rho_w = 1000$ kg/m^3, $g = 9.81$ m/s^2. Using Eqs. (3.9) and (3.8), we get

$$N_{sh} = 9.61 \times 10^{-4} N_s$$

Non-dimensional specific speed N_{sh} is sometimes referred as shape number since its value characterizes the shape of the machine. Two non-dimensional parameters called the non-dimensional specific speed (shape number) and specific diameter are often used to decide upon the choice of the most appropriate machine.

Referring Eqs (2.16) and (2.18) of Chapter 2, specific diameter of a turbomachine is defined as,

$$D_s = \frac{\Psi^{1/4}}{\varphi^{1/2}} = \frac{D(gH)^{1/4}}{Q^{1/2}} \tag{3.10}$$

TABLE 3.3 Turbines Classification on the Basis of Shape Number

Runner	***Specific Speed*** $N_{sh} = \frac{N\sqrt{P}}{\rho_w^{1/2}(gH)^{5/4}}$		***Specific Diameter*** $D_s = \frac{D(gH)^{1/4}}{Q^{1/2}}$
	when N is rad/s	**when N is rpm**	**when $D = 0.145$ m**
Pelton	0.05 – 0.4	0.008 – 0.06	0.62–0.86
Francis	0.4 – 2.2	0.06 – 0.35	0.14–0.26
Kaplan	1.8 – 5.0	0.286 – 0.79	0.08–0.15

The lower specific speed machines are denoted as slow runners, while high specific speed machines are known as fast runners. Table 3.3 shows the classification of turbines on the basis of the specific speed.

3.4 Euler Equation for Hydroturbines

Consider a hydro turbine as shown in Figure 3.3 and make the following assumptions.

1. The flow is steady i.e. the mass flow rate is constant across any section which implies that no storage or depletion of water mass in the runner.
2. The heat and work interactions between the runner and its surroundings takes place at a constant rate.
3. Velocity is uniform over any area normal to the flow. This means that the velocity at any point is representative of the total flow over a finite area. This condition also implies that there is no leakage loss and the entire water is undergoing the same process.
4. There are no frictional and other losses in the system.
5. The water is assumed to have perfect guidance through the flow system. This implies that there are infinite numbers of thin vanes or blades on the runner.

The angular velocity is given by,

$$\omega = \frac{2\pi N}{60} \tag{3.11}$$

Tangential or peripheral velocity of the flow at the inlet,

$$C_{b1} = \omega r_1 = \frac{\pi D_1 N}{60} \tag{3.12}$$

Tangential or peripheral velocity of the flow at the outlet,

$$C_{b2} = \omega r_2 = \frac{\pi D_2 N}{60} \tag{3.13}$$

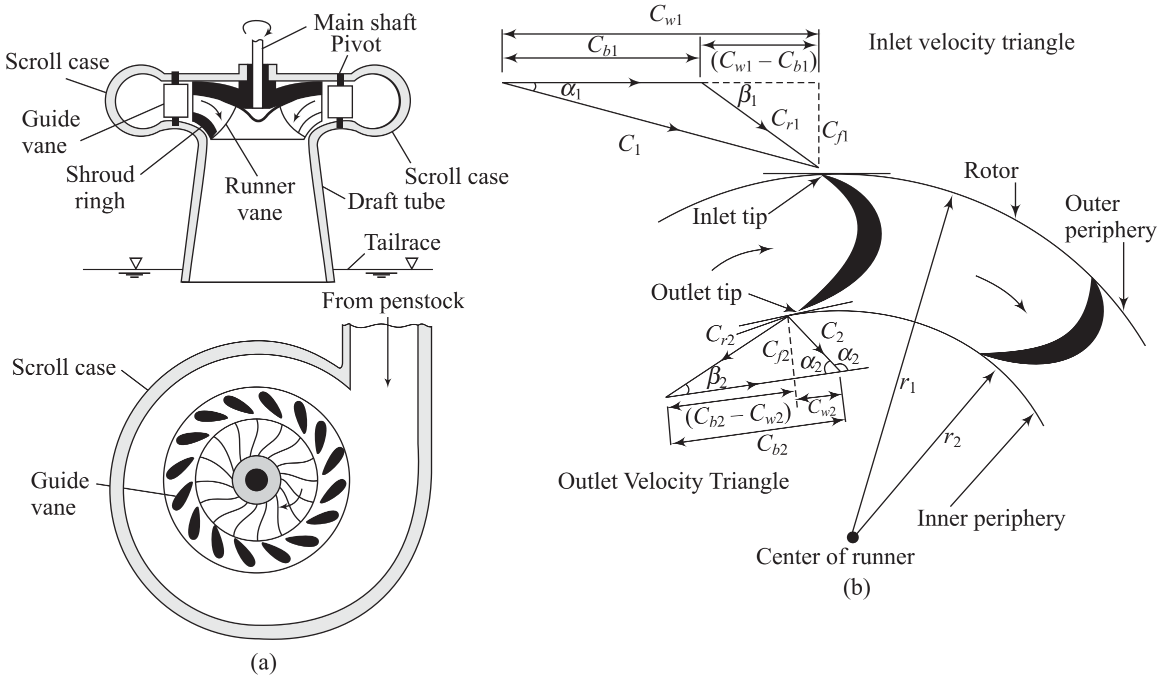


Figure 3.3 *Francis Turbine (a) Schematic Diagram, (b) Velocity Triangles at Inlet and Outlet*

There will always be a relative motion between the runner vanes and the fluid. The fluid has a component of velocity and hence a momentum in a direction tangential to the runner. The tangential velocity and hence momentum changes while flowing through the rotor. The rate at which this tangential momentum changes corresponds to a tangential force on the rotor. In a turbine, the tangential momentum of the fluid is reduced and therefore work is done by the water on the rotor. Referring to Eq. (1.80),

$$P = \dot{W} = \dot{m}\,(C_{w1}C_{b1} - C_{w2}C_{b2})$$

where, $\dot{m} = \rho Q$

$$\therefore \qquad P = \rho_w Q(C_{w1}C_{b1} - C_{w2}C_{b2}) \tag{3.14}$$

Equation (3.14) is known as *Euler equation for power in a hydroturbine*, ρ_w being density of water.

3.4.1 Euler Head or Specific Work

Energy transferred from the fluid to the runner per unit weight of the fluid or Euler head or specific work referring Eqs (1.86) and (1.98),

$$H_e = \frac{1}{g}(C_{w1}C_{b1} - C_{w2}C_{b2}) = \left[\frac{(C_1^2 - C_2^2)}{2g} + \frac{(C_{r2}^2 - C_{r1}^2)}{2g} + \frac{(C_{b1}^2 - C_{b2}^2)}{2g}\right] \tag{3.15}$$

Equation (3.15) is the *Euler equation for head extracted in a hydroturbine.* H_e is often known as the *Euler head* and represents the *ideal head* extracted by the turbine and can be transmitted to the shaft under ideal conditions. However, the actual head extracted in a turbine is smaller than the *Euler head* due to friction and other losses in the system.

3.4.2 Nature of Energy Transfer

Consider the energy equation for incompressible flow i.e. Bernoulli equation between the inlet and outlet of the turbine assuming no friction. Since an amount of energy head H_e would be extracted out of the flow between these two sections, the energy equation can be written as,

$$\frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2 + H_e \tag{3.16}$$

Substituting the value of H_e from Eq. (3.15) into Eq. (3.16),

$$\frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2 + \left[\frac{(C_1^2 - C_2^2)}{2g} + \frac{(C_{r2}^2 - C_{r1}^2)}{2g} + \frac{(C_{b1}^2 - C_{b2}^2)}{2g}\right]$$

$$\therefore \quad \left[\frac{(C_{r2}^2 - C_{r1}^2)}{2g} + \frac{(C_{b1}^2 - C_{b2}^2)}{2g}\right] = \left(\frac{p_1}{\rho g} + Z_1\right) - \left(\frac{p_2}{\rho g} + Z_2\right) \tag{3.17}$$

The RHS of Eq. (3.17) represents the change in piezometric head between inlet and outlet of the turbine. However, in a turbine $Z_1 \approx Z_2 \Rightarrow Z_1 - Z_2 \approx 0$ i.e. negligibly small. Therefore,

$$\left[\frac{(C_{r2}^2 - C_{r1}^2)}{2g} + \frac{(C_{b1}^2 - C_{b2}^2)}{2g}\right] = \left(\frac{p_1 - p_2}{\rho g}\right) \tag{3.18}$$

It is noteworthy from Eq. (3.15),

- The term $\frac{C^2}{2g}$ represents the kinetic energy per unit weight of water at a particular section i.e. velocity head. Therefore, the first term $\frac{(C_1^2 - C_2^2)}{2g}$ represents the change in kinetic energy between inlet and outlet of the turbine. This is known as *impulse effect* and represents a change in the dynamic head of the fluid while flowing through the runner.
- The second term $\frac{(C_{r2}^2 - C_{r1}^2)}{2g}$ could be interpreted as the energy change in the form of pressure head due to acceleration in a converging passage. Similarly, a diverging passage in the direction of flow causes deceleration. In an inward flow reaction turbine, the flow passage is convergent in the direction of flow i.e. from inlet to outlet, which increases the relative velocity, $C_{r2} > C_{r1}$ and hence decreases the static pressure.
- The third term represents a change in fluid energy due to movement of rotating fluid from one radius of rotation to another. It is analogous to a steady flow through a container having uniform angular velocity i.e. *forced vortex*. Since the centrifugal force field is responsible for this energy transfer, the corresponding head, $\frac{C_b^2}{2g}$, is known as *centrifugal head*. The transfer of energy due to a change in centrifugal head $\frac{(C_{b1}^2 - C_{b2}^2)}{2g}$, causes a decrease in the static pressure of the fluid as $C_{b1} > C_{b2}$, in an inward flow reaction turbine. On the other hand, in an axial flow or propeller turbine, $C_{b1} = C_{b2}$ and $\frac{(C_{b1}^2 - C_{b2}^2)}{2g} = 0$.

- The sum of the second and third terms, $\left[\frac{(C_{r2}^2 - C_{r1}^2)}{2g} + \frac{(C_{b1}^2 - C_{b2}^2)}{2g}\right]$, represents the change in the static pressure head of the fluid while flowing through the rotor as is evident from Eq. (3.18) which is called *reaction effect*.

  Note that similar discussion is presented also in Section 1.6.

3.4.3 Degree of Reaction

An important aspect in the design of the runners is the relative values of energy transfer by the impulse effect and reaction effect. Degree of reaction is a measure of the amount of energy transfer by reaction effect out of the total energy transfer in the runner or rotor. *Degree of reaction* is defined as the ratio of the energy transfer by the reaction effect i.e. by the decrease of static pressure to the total energy transfer in the runner.

$$\text{Degree of Reaction} = R = \frac{\text{Energy transfer due reaction effect or pressure drop}}{\text{Total energy transfer}}$$

Hence, by considering unit weight of water flow in the turbine,

$$R = \frac{(p_1 - p_2)/\rho g}{H_e} = \frac{1}{H_e}\left[\frac{(C_{r2}^2 - C_{r1}^2)}{2g} + \frac{(C_{b1}^2 - C_{b2}^2)}{2g}\right] = \left[\frac{H_e - (C_1^2 - C_2^2)/2g}{H_e}\right] \tag{3.19}$$

For a reaction turbine, the range of degree of reaction is 0 – 1. The runner of a turbine with non-zero degree of reaction must necessarily be completely enclosed in a casing for the development of pressure head.

(a) Impulse Turbine or Pelton Wheel

The tangential velocity of the bucket (blade or vanes speed) at the inlet and outlet of Pelton wheel is same, i.e. $C_{b1} = C_{b2}$. For an ideal Pelton wheel, there is no friction resistance i.e. $k = 1 \Rightarrow C_{r2} = C_{r1}$. Therefore, energy transfer due to pressure drop is zero which means that there is no reaction effect. Therefore, total energy transfer from fluid to rotor per kg of fluid under ideal conditions i.e. Euler Head is found from Eq. (3.15) as given below,

$$H_e = \frac{(C_1^2 - C_2^2)}{2g} \tag{3.20}$$

Hence, the degree of reaction for an ideal Pelton wheel is,

$$R = 0 \tag{3.21}$$

In the case of impulse turbine, i.e. Pelton wheel, the pressure at the inlet and outlet is same and is equal to atmospheric pressure. Therefore, the rotor or runner is not encased in a Pelton wheel.

(b) Pure Reaction turbine

If $C_1 = C_2$, total energy transfer from fluid to rotor per kg of fluid under ideal conditions, i.e. Euler head, is obtained from Eq. (3.15) and (3.18) as given below,

$$H_e = \frac{p_1 - p_2}{\rho g} \tag{3.22}$$

This holds good for a pure reaction turbine, where the wheel rotates only due to pressure drop across it, exerting a reaction by Newton's third law of motion. Hence, degree of reaction for a pure reaction turbine is unity, $R = 1$ which can be deduced by substituting the value from Eq. (3.22) into Eq. (3.19).

3.5 Efficiencies of a Turbine

The important efficiencies of a turbine are, (a) Volumetric efficiency, η_v, (b) Hydraulic efficiency, η_h, (c) Mechanical efficiency, η_m, and (d) Overall efficiency, η_o.

3.5.1 Volumetric Efficiency

The discharge of the water passing through the rotor of a turbine is slightly less than the discharge of the water supplied to it. Some discharge of the water supplied at the inlet of rotor may leave the turbine in the form of various leaks and is discharged to the tailrace directly without doing any work on the rotor of the turbine. Thus, the ratio of the discharge (volume flow rate) of the water actually doing work on the turbine rotor to the discharge of water supplied to the turbine rotor is defined as volumetric efficiency. Therefore,

$$\eta_v = \frac{\text{Discharge of water actually striking the runner}}{\text{Discharge of water supplied to the turbine}} = \frac{Q - \Delta Q}{Q} \tag{3.23}$$

where, Q is the total discharge of water supplied to the turbine rotor, and ΔQ is the leaked discharge i.e. volume of water leaked per second. Generally, leakage loss is a very small percentage of the discharge Q and is the order of 0.5%.

3.5.2 Hydraulic Efficiency

Runner is a rotating part of a turbine on which vanes or blades are fixed. The head extracted by the turbine runner or Euler head to produce mechanical power is less than the net head (H) available for the turbine rotor due to hydraulic losses. These hydraulic losses, indicated by h_i and h_c include fluid friction and form loss at the rotor including entrance and exit losses at the rotor. Thus, Euler head is,

$$H_e = H - h_c - h_i \tag{3.24}$$

Hydraulic efficiency is the ratio of head extracted by the rotor to the net head available to the rotor. The hydraulic efficiency of the turbine is given by,

$$\eta_h = \frac{\text{Head extracted by the rotor}}{\text{Net head available to the rotor}} = \frac{H - h_i - h_c}{H} = \frac{H_e}{H} \tag{3.25}$$

The power delivered to the runner by fluid i.e. power extracted by the rotor will be less than the power available at the inlet of the turbine rotor. Alternately, hydraulic efficiency may also be defined as the power given by water to the runner of a turbine to the power supplied by the water at the inlet of the turbine.

3.5.3 Impeller or Rotor Efficiency

Refer to the inlet and outlet fluid power of the impeller in Figure 1.42. Defining impeller efficiency as the ratio of the output fluid power of impeller (RP) and input fluid power of impeller (IP)

$$\eta_{imp} = \frac{\rho_w g(Q - \Delta Q)(H - h_i - h_c)}{\rho_w g(Q - \Delta Q)(H - h_c)} = \frac{RP}{IP} = \frac{H_e}{H - h_c} \tag{3.26}$$

Neglecting the casing losses, impeller efficiency will be identical with the hydraulic efficiency. On the other hand, neglecting the leakage losses and casing losses, the impeller efficiency can be written as

$$\eta_{imp} = \frac{\rho_w g(Q)(H_e)}{\rho_w g(Q)(H)} = \frac{H_e}{H} = \frac{RP}{WP} \tag{3.27}$$

In other words,

$$\eta_{imp} = \eta_h = \frac{RP}{WP} \tag{3.28}$$

where casing and leakage losses are neglected.

3.5.4 Mechanical Efficiency

The power delivered by water to the turbine rotor(runner power) is transmitted to the rotor shaft. Mechanical friction occurs between the rotor and other parts of the turbine unit such as bearings, glands, couplings etc., and also due to the presence of other mechanical losses such as windagelosses wherever applicable. Due to these mechanical losses, the power available at the shaft of the turbine (known as shaft power, SP, or brake power, BP) is lesser than the power produced by the runner of a turbine (runner power, RP).

$$\text{SP} = \text{RP} - P_{L,m} \tag{3.29}$$

The ratio of the power available at the shaft (SP) to the power delivered to the runner (RP) is defined as mechanical efficiency. Hence, mathematically,

$$\eta_m = \frac{\text{Power available at the shaft of the turbine (SP)}}{\text{Power produced by the runner (RP)}} \tag{3.30}$$

3.5.5 Overall Efficiency

It is defined as the ratio of power available at the runner shaft (SP) to the power supplied by the water at the inlet of the turbine rotor (WP). Therefore, overallefficiency,

$$\eta_o = \frac{\text{Power available at the shaft of the turbine (SP)}}{\text{Power supplied at the inlet of the turbine (WP)}} \tag{3.31}$$

$$\eta_o = \frac{\text{SP}}{\text{WP}} = \frac{\text{SP}}{\text{RP}} \times \frac{\text{RP}}{\text{WP}}$$

$$\eta_o = \frac{\text{SP}}{\text{RP}} \times \frac{\text{RP}}{\text{IP}} \times \frac{\text{IP}}{\text{WP}} \tag{3.32}$$

$$\frac{\text{IP}}{\text{WP}} = \frac{\rho_w g(Q - \Delta Q)(H - H_c)}{\rho_w gQH} = \frac{(Q - \Delta Q)}{Q} \times \frac{H - h_c}{H} = \eta_v \eta_{casing} \tag{3.33}$$

Using Eqs (3.26), (3.30) and (3.33) in Eq. (3.32), we get,

$$\eta_o = \eta_v \eta_{\text{casing}} \eta_{\text{impeller}} \eta_m \tag{3.34}$$

Generally, casing efficiency is taken to be unity i.e., $h_c = 0$ and η_{casing}, then,

$$\eta_o = \eta_v \eta_h \eta_m \tag{3.35}$$

3.6 Pelton Wheel

As discussed earlier, turbines are of two types: impulse and reaction turbines. An impulse turbine has an interaction of free jet at atmospheric pressure on a series of curved buckets on a wheel. The pressure is atmospheric throughout the runner. The Pelton turbine which was invented in 1889 is the most widely used tangential flow impulse turbine. It is extensively used for high head and low discharge installations. The water from the reservoir flows through the penstocks at the outlet of which a nozzle is fitted. The nozzle increases the kinetic energy of water flowing through the penstock. The water shoots in the form of a free jet at the outlet of the nozzle. Thus, the energy available at the inlet of the turbine is only kinetic energy. This high velocity jet strikes the buckets or vanes of the runner which revolves round a shaft. During this action, the water is in contact with air all the time and the water discharged from bucket falls freely through the discharge passage into the tailrace. The free jet is at atmospheric pressure before and after striking the vanes. Thus, the pressure at the inlet and outlet of the turbine is atmospheric. The main parts of a Pelton turbine are: (i) runner and buckets (vanes), (ii) nozzle and flow regulating arrangement (spear), (iii) casing, and (iv) braking jet.

3.6.1 Runner Assembly with Buckets

(a) Runner or Wheel

The runner consists of a large circular disc on the periphery of which a number of two-lobe ellipsoidal (double hemispherical or bowl or spoon shaped) buckets are evenly mounted as shown in Figure 3.4 (a). Two types of wheel fabrication are commonly used. In modern power equipment manufacturing plants, the preferred and common practise is to use the monocast systemm, i.e. disc and buckets are cast in a single piece. This is particularly advantageous in high capacity power units. In the other method, disc and buckets are casted separately and then the buckets are fixed to the disc through suitable nut and bolt arrangement. This method is adopted by many small and medium fabricators. The selection of material for the runners and buckets depends on the stresses, fatigue hazards and possibility of erosion due to slit content in the water and pitting caused due to cavitation. High quality alloy steel is used for small units, whereas very high head and large size plant are invariably made of stainless steel of 13% Cr and 4% Ni composition. The material used should not only be of high strength but also be adequately weldable.

The runner, nozzle and associated components of Pelton turbine should be installed well above the maximum water level of the tailrace so that the turbine remains unaffected by floods. The setting height of the lowest nozzle H_{sn}, of the Pelton wheel above the tailrace represents the non-utilization of available head, and therefore a loss. However, the setting height would normally be a small fraction of the net head as Pelton turbines are used in high head installations.

(i) Speed of Rotation or Rotational Speed

The speed of rotation in rpm is related to the tangential velocity of the wheel C_b as,

$$C_b = \frac{\pi DN}{60}$$

The tangential velocity of the wheel is proportional to the velocity of the jet. The theoretical velocity of the jet that can be generated from the net head H is known as *spouting velocity* and it is equal to $\sqrt{2gH}$. The ratio of tangential velocity of the wheel to the sprouting velocity is called speed ratio (φ).

$$\therefore \quad N = \frac{60C_b}{\pi D} = \frac{60 \times \varphi \times \sqrt{2gH}}{\pi D} \tag{3.36}$$

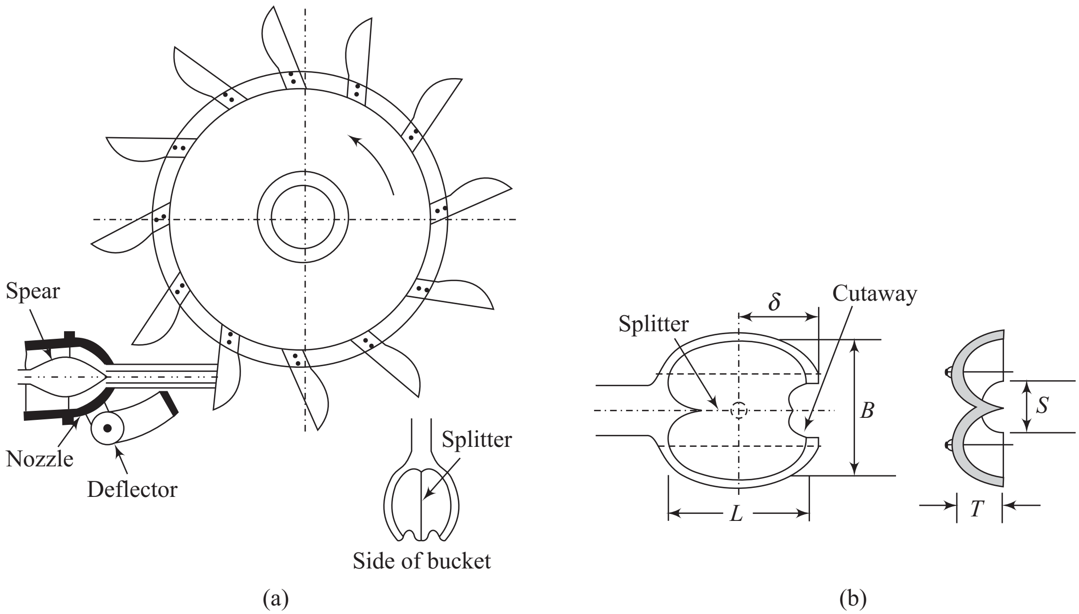


Figure 3.4 *Runner Assembly of a Pelton Wheel with Buckets*

The speed N varies inversely with wheel diameter. Pelton turbines have a speed in the wide range of 75–1000 rpm. However, the preferred range is 300–750 rpm generally. The generators are directly coupled to the turbines and the speed is adjusted to the synchronous speed. The synchronous speed of a generator is given by,

$$N_{sy} = \frac{120f}{p} \tag{3.37}$$

where, f is the frequency in Hz (In India, $f = 50$ Hz, In USA, $f = 60$ Hz); and p is the number of magnetic poles in the generator. A *speed increaser* (a geared mechanism) is used between the turbine and generator.

Generally, the highest feasible speed is preferred. The higher speed implies a higher specific speed which has the advantages as given below:

1. Smaller size of the turbine and hence lesser cost.
2. Smaller size of the generator and hence reduction in the cost of the generator.
3. The overall efficiency of a single jet Pelton turbine is relatively high in a small range of specific speed which is 13–23 with a maximum efficiency at 17. We may get the additional benefit of increased efficiency if the specific speed is obtained in this range by changing the speed.

(b) Buckets

The shape of the buckets used in present day Pelton turbines is a combination of two-lobe ellipsoidal shaped bowls or cups joined at the sides. Each bucket is divided into two symmetrical parts by a dividing wall known as ridge or splitter. The splitter is of sharp-ridge type and divides the impinging jet into two equal streams, as shown in Figure 3.4 (b). This helps to avoid the formation of any dead spots in the centre of the cups and aids lateral flow of the jet. The symmetry ensures that there is no momentum in the axial direction and hence there is no axial force on the shaft bearings.

The water jet impinges on the splitter. The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket. From the theoretical considerations, the jet must turn on itself i.e. it should have an angle of 180°. However, to avoid the interference of the oncoming jet and the return jet, the buckets are shaped to have a deflection angle in the range of 160–170° from practical point of view. The jet is deflected to this extent by each half of the bucket. The slightly oblique direction of the exiting water allows it to escape freely without hitting the back of the next bucket. The water after leaving the bucket drops freely into the tailrace. Cast iron, cast steel bronze or stainless steel are used to manufacture the bucket. The selection of buckets material depends on the available head at the inlet of the turbine runner. The surfaces of the buckets are milled, ground and polished. The shape and surface characteristics of the bucket affect the efficiency of the turbine.

A shaped notch called *cutaway* is provided symmetrically on either side of the ridge in the lower lip of the central portion. The *cutaway* helps to prevent fouling of the jet by the back of the next bucket coming into the jet. The cutaway permits smooth entrance of the jet into a bucket, and allowing gradual cutting off of water from the preceding bucket.

The main dimension of the bucket is the axial width B. The surface friction will be unnecessarily large if the axial width is too large. The exact geometrical shape of the bucket is a proprietary item of manufacturers.

(i) Diameter of the Wheel

The diameter which is used in equations and calculations of Pelton turbine is the pitch circle diameter. Pitch circle is the centerline of the circle of the buckets to which the jet axis is tangential.

3.6.2 Nozzle and Flow Regulating Arrangement

(a) Nozzle

The *nozzle* also known as *injector* forms an integral part of the Pelton turbine. The nozzle is connected at the end of *penstock*. The jet of the water necessary to drive the Pelton wheel is produced in the nozzle. The amount of water striking the vanes (buckets) of the runner is controlled by a spear needle in the nozzle, as shown in Figure 3.5.

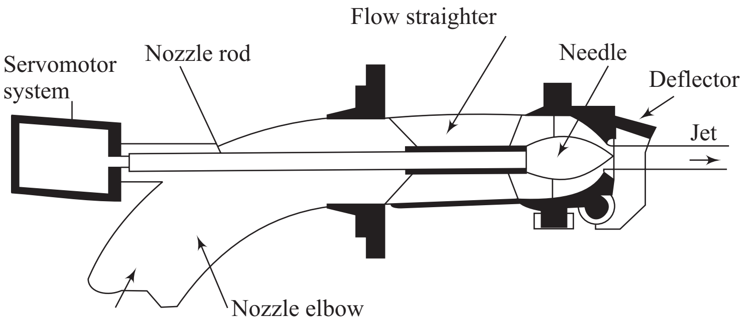


Figure 3.5 *Nozzle and Flow Regulating Assembly of a Pelton Turbine*

The spear needle is a conical valve that is operated either by hand wheel or automatically by the governor action depending upon the turbine size. The speed of the turbine increases beyond the normal speed when the load on the generator decreases. In such a case, the spear needle is pushed forward into the nozzle which reduces the quantity of water striking the runner. On the other hand, the quantity of water striking the runner increases if the spear needle is pushed back when the speed of the turbine decreases beyond the normal speed due to the increased load on the generator. It should be noted that in a nozzle, the effective head being essentially constant, the velocity of jet remains constant and the change in the area of the flow due to movement of the spear needle causes the change in the discharge.

The nozzle must be designed and manufactured to have minimum head loss. The shapes of the nozzle and spear needle are designed in such a way to achieve the perfect streamlined flow at all spear needle openings. The nozzle-spear system is made of stainless steel (13% Cr, 4% Ni alloy). It is finished to a very smooth surface in order to minimize the frictional losses. It has been found by experimental studies that the efficiency of the Pelton turbine is very sensitive to the quality of water jet that strikes on the buckets. The ideal jet of water should be solid with uniform velocity distribution and free from surface spray. The character of the jet emerging out from the nozzle depends not only on the nozzle shape but also on the flow approaching the nozzle. The flow approaching the nozzle depends on the shapes, sizes and alignment of connections of the nozzle from the penstock, and should be such so as to cause minimum of turbulence and vortices at the base of the nozzle. A *flow straighter* surrounding the needle shaft is provided to obtain good approach flow at the commencement of the nozzle contraction.

It can be observed from Figure 3.5 that the nozzle exit diameter is larger than the jet diameter due to contraction of the jet emerging from the nozzle-needle assembly.

(i) Coefficient of Velocity of the Nozzle

The jet issuing out of the nozzle has the least diameter and velocity at the *vena contracta*. The least diameter is known as *jet diameter* and the corresponding area as *jet area*. The velocity at the vena contracta is called *jet velocity*. The ratio of the jet velocity to the spouting velocity is known as *coefficient of velocity* C_v, which is given by,

$$C_v = \frac{C_1}{\sqrt{2gH}} \Rightarrow C_1 = C_v\sqrt{2gH} \tag{3.38}$$

The discharge from the nozzle = Discharge at the vena contracta = $Q = A_1C_1$

$$Q = \left(\frac{\pi}{4}d_1^2\right)C_1 = \left(\frac{\pi}{4}d_1^2\right)C_v\sqrt{2gH} \tag{3.39}$$

The value of the coefficient of velocity lies in the range of 0.98–0.99 for a well-designed nozzle.

(b) Deflectors

A *deflector,* as shown in Figure 3.5, is a device provided at the tip of the nozzle. It is used to deflect the path of a part or the full jet emerging from the nozzle in such a way that the deflected part does not strike the vanes or buckets and falls away from the runner. The other objective of the deflector is to protect the jet from the exit water spray of the runner to some extent. The deflector is needed during the governing action related to sudden decrease in load. The decrease in load would necessitate the reduction in discharge. The governor mechanism i.e. servomotor, pushes the spear-needle in the forward direction in the nozzle towards the nozzle outlet to reduce the area of the jet and thus the discharge. However, the time span within which this has to be accomplished is the concern. The quick response of the governor is desired which necessitates the spear to be pushed into the requisite position as fast as possible. But, such a rapid closure in the high velocity nozzle would result in *water hammer* problems in the penstock. A slow closure to minimize the water hammer action would result in a very slow and sluggish response of the governor when the load on the turbine is reduced. To cope with this situation of dilemma, the deflector is used. When a sudden decrease in the discharge is desired under reduced load condition, the deflector responds quickly and deflects a path of the jet away from the original path. The servomotor adjusts the spear needle in the nozzle at sufficient and appropriate speed to the correct position. While this is being done, the deflector is simultaneously withdrawn from the jet. The deflector has a designed capacity to completely deflect the jet out of the Pelton wheel and the spear needle valve follows the closure at the requisite speed. This feature is a must for quick disconnection of the jet without causing water hammer effects in the penstock.

3.6.3 Manifold, Braking Jet and Auxiliary Jet

(a) Manifold

Figure 3.6 shows schematic diagram of the manifold and jet arrangements in a 6-jet Pelton turbine. When more than one jet is used in a Pelton turbine, the flow from the penstock has to be divided into separate pipes to feed each of the nozzles. The connecting pipe assembly which is used to connect each nozzle to the main penstock is known as *manifold*. The main objective of manifold is to distribute the total flow to all the nozzles uniformly with minimum losses. The geometry of the manifold, viz. shape, size of components and alignment depends upon the number of nozzles and the flow quantity. The main components of manifold are *wye-pieces* (bifurcations), and the connecting conduits between wye-pieces. The nozzles are uniformly distributed around the wheel. The *main inlet valve (MIV)*, usually a spherical valve, is provided in the penstock at the junction with manifold. The main objectives of a good design of a manifold are:

- To get uniform distribution of discharge
- Minimization of losses
- Minimization of secondary currents and turbulence at the base of the nozzle

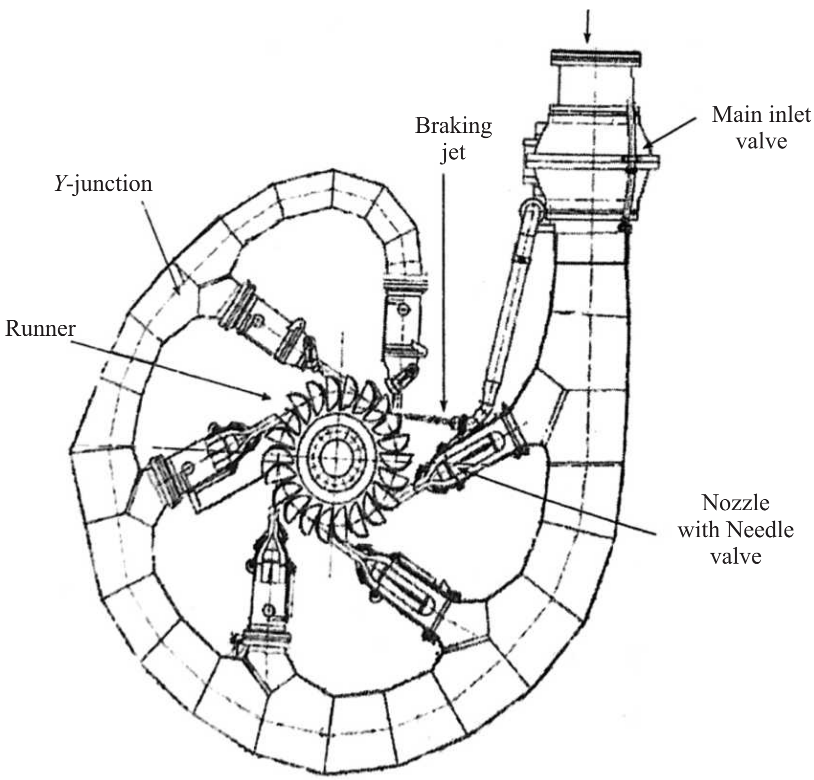


Figure 3.6 *Schematic Diagram of the Manifold and Jet Arrangements in a Multi Jet Pelton Turbine*

The preliminary design of intake manifold is done on the basis of in-house experience. The design is then fine tuned through the analysis of flow through the advanced CFD techniques and then it is finalized after performing experiments on the models. The manifold with nozzles at the end of each limb is a very costly component. It is quite obvious that in a multi jet turbine the manifold, nozzles and spear-needle valves, servomotors and governing mechanisms to operate the spear-needle valves simultaneously add up to the cost of the unit and make it very expensive.

(b) Braking Jet

The quantity of water striking the rotor decreases to zero by moving the spear in the forward direction to close the nozzle fully when a turbine is to be stopped. But the runner goes on revolving for a long period of time due to inertia and comes to rest only due to air and bearing resistance. For hastening the stoppage, a jet from a smaller nozzle is provided to act on the back of buckets. The impact of this jet on the back of the buckets causes a braking torque on the shaft by acting in a direction opposite to the direction of rotation due to power jets. The additional jet which causes braking action on the wheel to stop the turbine in a short time is called *braking jet* which can be seen in Figure 3.6. Water jet braking systems are used for emergency stoppage and fast reduction of turbine rotational speed.

Kinetic Energy of Jet

Let the nozzle produce a jet of area A_1, diameter d and velocity C_1 at the *vena contracta.* The entire head H would be converted to kinetic energy of the jet ideally if there are no losses. However, there are three types of losses, as follows:

(i) Loss in the Nozzle

If C_v is the velocity coefficient of the nozzle, then the velocity of the jet is given by,

$$C_1 = C_v\sqrt{2gH} \tag{3.40}$$

The energy loss in the nozzle, h_{ln}, is given by,

$$h_{\text{ln}} = \left(\frac{1}{C_v^2} - 1\right)\left[1 - \left(\frac{A_1}{A_b}\right)^2\right]\frac{C_1^2}{2g} \tag{3.41}$$

where, A_b = Cross sectional area at the base of the nozzle, and

A_1 = Cross sectional area of the jet

(ii) Loss of Energy in the Bucket

There is a certain amount of loss of energy during the transit of the jet from inlet to outlet of the bucket. This loss of energy in the bucket is given by,

$$h_{lb} = \frac{C_{r1}^2}{2g} - \frac{C_{r2}^2}{2g} = (1 - k^2)\frac{C_{r1}^2}{2g} \tag{3.42}$$

where C_{r1} and C_{r2} are relative velocity (i.e. relative to the bucket) of water at the entrance and exit of the bucket respectively. *Blade or bucket friction coefficient k* is used to account for the losses due to the friction. Blade friction coefficient is given by,

$$k = \frac{C_{r2}}{C_{r1}} \tag{3.43}$$

(iii) Kinetic Energy Loss at the Exit

Kinetic energy of the jet is reduced to a minimum after passing through the bucket. However, the water which leaves the bucket and discharges into the tailrace will have some residual velocity i.e. kinetic energy. This kinetic energy at the exit is not utilized in the momentum change in the bucket which would result in more work developed by the turbine. Hence, kinetic energy at the exit of the turbine is going to waste as far as turbine is concerned. This loss of energy h_{ie} is represented by,

$$h_{le} = \frac{C_2^2}{2g} \tag{3.44}$$

where C_2 is the absolute velocity of water leaving the turbine. If H_e is actual energy head transmitted to the buckets of the turbine known as Euler head, then, net head is given by,

$$H = H_e + h_{\ln} + h_{lb} + h_{ie} = H_e + h_{LKE} \tag{3.45}$$

where, h_{LKE} = Total loss of energy in the nozzle-bucket system indicated by h_i in Figure 1.42.

In addition to the above three types of kinetic energy losses, there will be other losses as given below,

(i) Mechanical losses at the bearings,

(ii) Air friction of rotating elements including wheel known as windage loss.

All the losses are generally represented under various categories of efficiencies viz. nozzle efficiency, wheel efficiency, hydraulic efficiency, and overall efficiency.

(iv) Speed Ratio

The theoretical velocity of the jet that can be generated from the net head H is known as *spouting velocity* and it is equal to $\sqrt{2gH}$. The ratio of the peripheral velocity of the buckets at the pitch circle to the spouting velocity is known as *speed ratio*. Thus,

$$\text{Speed Ratio} = \varphi = \frac{C_b}{\sqrt{2gH}} \tag{3.46}$$

The speed ratio is a coefficient whose value lies in the range of 0.43–0.47.

(v) Jet Ratio

The ratio of the pitch circle diameter of the runner to the diameter of the jet at vena contracta is known as *jet ratio*, m. Thus,

$$m = \frac{D}{d} \tag{3.47}$$

The jet ratio is a very important parameter which influences the characteristics of the Pelton turbine. For higher value of jet ratio, the cost of the wheel becomes correspondingly large and also the bearing friction and windage losses increase. On the other hand, if the jet ratio is relatively small, at some critical value of m, the bucket dimensions become unreasonable relative to the wheel diameter. However, before this point is reached; there will be increased departure from the tangential impact of the jet on the buckets. Consequently, efficiency of the Pelton wheel is reduced. Therefore, proper selection of the value of jet ratio is crucial from the economy and efficient design point of view. As indicated earlier, the specific speed range is 8–30 for the Pelton wheel and the corresponding jet ratio is 26 – 7. The plot of overall efficiency vs. specific speed for a Pelton turbine indicates that the efficiency is maximum at a specific speed of approximately 17. This is the preferred value of the specific speed. The corresponding preferred jet ratio is 12. Using two or more jets in a Pelton turbine would result in a smaller runner for a given discharge with increase in the rotational speed.

The jet ratio is a function of the specific speed. An expression which shows that specific speed depends on the jet ratio and other salient parameters of the wheel can be found in appropriate section.

(c) Relief Valve

An *auxiliary nozzle* known as *relief nozzle* has the same function as that of a deflector, viz. coping with sudden reduction of load. The auxiliary nozzle is an additional nozzle-spear valve combination connected in parallel with the main nozzle (known as *power nozzle*). The servomotors of the power nozzle also regulate the relief nozzle. The spear-needle valve is kept in a position so that when the power nozzle is open, the auxiliary or relief nozzle is closed or vice versa. When a sudden decrease in discharge is required under reduced load condition, the servomotors of the power nozzle begins the closing motion of the spear needle of the main or power nozzle, it simultaneously causes the relief nozzle to open proportionately. The relief nozzle discharges the water to tailrace without intercepting the wheel. The auxiliary nozzle also closes in slow closure mode after the complete closure of the main nozzle. It is quite clear that when the auxiliary nozzle is installed, there is no need for deflectors. Relief nozzles are seldom used presently.

3.6.4 Casing

The enclosing chamber of the turbine known as *casing* has no hydraulic function in Pelton turbine as that in reaction turbines because it works under atmospheric pressure. The purpose of the casing is to restrain the splashing of water for reduction of noise level and to discharge water to tailrace. It also acts as a safeguard against accidents. In addition, it provides a support and housing for the bearings of the turbine shaft. Arrangements for collecting splash water and provision of baffles for the reduction of windage losses is made inside the casing. It is made of cast iron or fabricated steel plates. There is considerable amount of air entrainment by high velocity jets and splashing water in the turbine system. Therefore, a regular supply of air into the casing through air vents of appropriate size is necessary. The air demand is of the order of a cubic meter per second in very large units. The Pelton turbine with casing is shown in Figure 3.7.

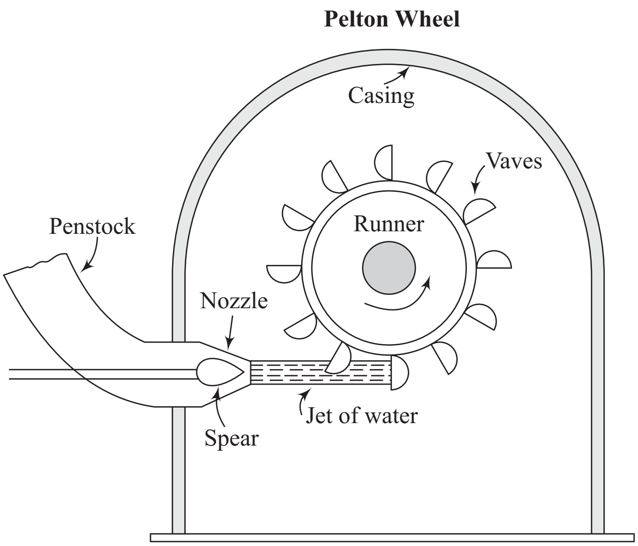


Figure 3.7 *Pelton Turbine with Casing*

3.7 Analysis of a Pelton Wheel

3.7.1 Velocity Diagrams

The shape of the buckets or vanes of a Pelton turbine is shown in Figure 3.8 (a). The jet of water shooting out from the nozzle impinges the bucket at the splitter or ridge tangentially to the pitch circle. After the impact, the jet splits into two equal streams or parts and exits at a *deflection angle* of θ with the original direction of the jet. The deflection angle of the jet at the bucket is usually obtuse. These two halves of the jet glide over the inner surfaces and come out at the outer edge. The bucket section at $Z - Z$ is shown in Figure 3.8 (a). The inlet is splitter and the outlet is outer edge of the bucket or vane. The inlet and outlet velocity triangles are drawn at the splitter and outer edge of bucket respectively as shown in Figure 3.8 (b).

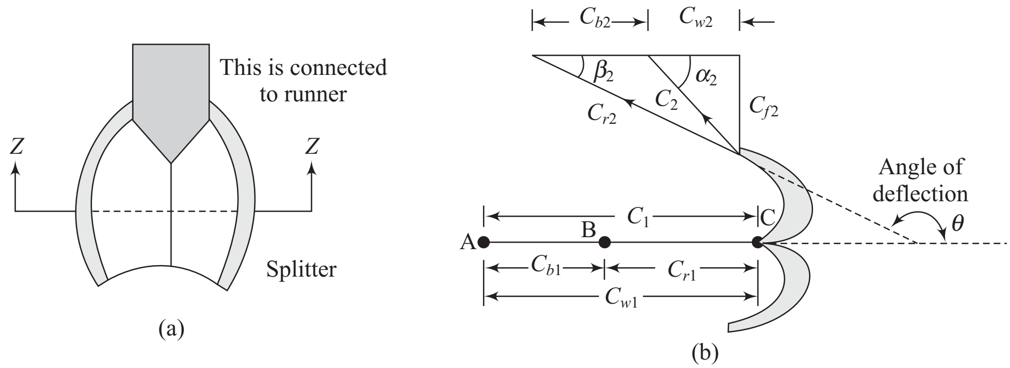


Figure 3.8 *(a) Shape of the Buckets, (b) Velocity Triangles at Inlet and Outlet of a Pelton Turbine*

The velocity of the jet shooting out from the nozzle is given by Eq. (3.38) or (3.40),

$$C_1 = C_v\sqrt{2gH}$$

The jet moves in a tangential plane before and after striking the wheel. The tangential velocity of the bucket (blade or vanes speed) at the inlet and outlet of Pelton wheel is same and is given by the equation below,

$$C_b = C_{b_1} = C_{b_2} = \omega R = \frac{\pi DN}{60} \tag{3.48}$$

where R and D are the radius and diameter of the pitch circle respectively. The angular velocity of the bucket is given by $\frac{2\pi N}{60}$, N being the rpm.

The velocity triangle at inlet degenerates into a straight line. The relative velocity and whirl velocity at inlet,

$$C_{r_1} = C_1 - C_b \tag{3.49}$$

$$C_{w_1} = C_1 \tag{3.50}$$

If k is the blade friction coefficient, then the relative velocity at exit is given by,

$$C_{r_2} = kC_{r_1} = k(C_1 - C_b) \tag{3.51}$$

Now, from the exit velocity diagram, we get

$$C_{w_2} = C_{r_2} \cos \beta_2 - C_{b_2}$$

$$C_{w_2} = k(C_1 - C_b) \cos \beta_2 - C_b \tag{3.52}$$

The blade angle at outlet is,

$$\beta_2 = 180 - \theta \tag{3.53}$$

where, θ is the deflection angle of the relative velocity of jet by buckets, also called bucket angle (≈ 165°). With the nozzle diameter d, D/d is a size parameter for the turbine. This is known as *jet ratio, m* having a value in the range of 10 to 24.

The force exerted by the jet of water in the direction of motion is obtained by Newton's second law of motion or from impulse momentum equation. Therefore, force exerted by the jet of water in the direction of motion is equal to the rate of change of momentum which is equal to the product of mass flow rate and change of velocity. The change of velocity in the direction of motion is equal to the change of velocity of whirl.

$$F_x = \text{Mass flow rate} \times \text{change of whirl velocity}$$
$$= \text{Density} \times \text{Discharge} \times \text{change of whirl velocity}$$

$$F_x = \rho_w a C_1 [C_{w_1} - (-C_{w_2})] = \rho_w a C_1 [C_{w_1} + C_{w_2}] \tag{3.54}$$

The change in whirl velocity in the direction of motion would be equal to $(C_{w_1} - C_{w_2})$ if β_2 is obtuse angle. In Eq. 3.54, a is the cross sectional area of the jet given by,

$$a = \frac{\pi}{4} d^2 \tag{3.55}$$

Energy transferred to the wheel or net work done by the jet on the runner per second i.e. power developed,

$$P = \dot{W} = \text{Force} \times \text{displacement in the direction of force} = F_x \times C_b$$

$$P = \rho_w a C_1 [C_{w_1} + C_{w_2}] C_b \tag{3.56}$$

3.7.2 Power Developed

Power given by the jet to the runner in kW is given by,

$$P = \frac{\rho_w a C_1 [C_{w_1} + C_{w_2}] C_b}{1000} \tag{3.57}$$

Putting the values of C_{w_1} and C_{w_2} from Eqs. (3.50) and (3.52) respectively into Eq. (3.56),

$$P = \rho_w a C_1 [C_1 + k(C_1 - C_b) \cos \beta_2 - C_b]\, C_b = \rho_w a C_1 (C_1 - C_b)(1 + k \cos \beta_2) C_b$$

$$P = \rho_w a C_1 (C_1 C_b - C_b^2)(1 + k \cos \beta_2) = \rho_w a\, C_1^3\, \rho (1 - \rho)(1 + k \cos \beta_2) \tag{3.58}$$

where, $\rho = \dfrac{C_b}{C_1}$ is known as the *velocity ratio*.

The noteworthy observations for a Pelton turbine from Eq. (3.58) are,

- At static condition of the runner i.e. when $C_b = 0$, Power developed $P = 0$.
- When $C_b = C_1$, power developed $P = 0$. This is the *runaway condition* of the runner that occurs when the load is completely removed from a running turbine.
- Variation of power with the bucket velocity, and hence with velocity ratio is parabolic.

From this, the head extracted or *Euler head* is obtained as,

$$H_e = \frac{P}{\rho_w g Q} = \frac{1}{g}(C_1 - C_b)(1 + k \cos \beta_2) C_b \tag{3.59}$$

Note that power P in Eq. (3.59) is rotor power, RP, if leakage ΔQ is considered.

3.7.3 Hydraulic Efficiency

The hydraulic efficiency of the nozzle-wheel system is,

$$\eta_h = \frac{H_e}{H} = \frac{(C_1 - C_b)(1 + k \cos \beta_2) C_b}{gH} \tag{3.60}$$

From Eq. (3.35), the overall efficiency of the turbine neglecting casing losses is given by

$$\eta_o = \eta_h \eta_m \eta_v = \frac{P}{\rho_w g Q H}$$

Substituting the value of hydraulic efficiency from Eq. (3.60),

$$\eta_o = \frac{(C_1 - C_b)(1 + k \cos \beta_2) C_b}{gH} \times \eta_m \eta_v = \frac{SP}{\rho_w g Q H}$$

$$\therefore \qquad SP = \eta_m \eta_v \rho_w Q (C_1 - C_b)(1 + k \cos \beta_2) C_b \tag{3.61}$$

3.7.4 Maximum Power Developed

It is clear from Eq. (3.58) or (3.61) that power developed is a function of C_b. Therefore, for maximum power developed,

$$\frac{dP}{dC_b} = 0$$

$$\frac{dP}{dC_b} = \rho_w a C_1 (1 + k \cos \beta_2)(C_1 - 2C_b) = 0$$

For a given value of C_1, k and β_2, there is a certain value of C_b for which power is maximum.

$$C_b = \frac{C_1}{2} \tag{3.62}$$

Therefore, the optimum bucket velocity for the maximum work output or power developed is equal to half of the jet velocity. The velocity ratio, ρ, is defined as ratio of the bucket velocity to the jet velocity (C_b/C_1). Hence, the optimum velocity ratio for maximum work output or power developed, $(\rho_{\text{opt}})_{\text{max work}}$, is half as can be seen from Eq. (3.62).

$$(\rho_{\text{opt}})_{\text{max work}} = \frac{1}{2} \tag{3.63}$$

Alternate method for finding the condition for maximum power developed is described below. From Eq. (3.61), power developed is,

$$SP = \eta_m \eta_v \rho_w Q (C_1 - C_b)(1 + k \cos \beta_2) C_b$$

For a given Pelton turbine having a known discharge under a net head H, η_m, η_v, ρ_w and Q are constant. Therefore, shaft power output is given by,

$$SP = K_1(C_1 - C_b)C_b = K_2 \frac{C_b}{C_1}\left(1 - \frac{C_b}{C_1}\right) = K_2 \rho(1-\rho)$$

where, K_1 and K_2 are constants given by,

$$K_1 = \eta_m \eta_v \rho_w Q(1 + k \cos \beta_2) \text{ and } K_2 = K_1 C_1^2$$

For maximum power developed,

$$\frac{dP}{d\rho} = 0 = K_2(1 - 2\rho)$$

$$\therefore \qquad (\rho_{\text{opt}})_{\text{max work}} = \frac{1}{2}$$

This is the same as given by Eq. (3.63). Substituting $C_b = \varphi\sqrt{2gH}$ and $C_1 = C_v\sqrt{2gH}$ either in Eq. (3.62) or in Eq. (3.63), the condition of maximum power results in,

$$\varphi = \frac{C_v}{2} \tag{3.64}$$

In an ideal case, $C_v = 1.0$, therefore the ideal value of speed ratio for maximum power is $\frac{1}{2}$. Substituting the result of Eq. (3.62) or (3.63) into Eq. (3.58), we get, maximum power developed in W,

$$P_{\max} = \rho_w a C_1(1 + k \cos \beta_2)\left(C_1 \times \frac{C_1}{2} - \frac{C_1^2}{4}\right)$$

$$P_{\max} = \frac{1}{4}\rho_w a(1 + k \cos \beta_2)C_1^3 \tag{3.65}$$

If there is no energy loss due to friction i.e. $k = 1$ then

$$P_{\max} = \frac{1}{4}\rho_w a(1 + \cos \beta_2)C_1^3 \tag{3.66}$$

An ideal Pelton turbine is one which has no blade friction, i.e. $k = 1$ with angle of deflection of the jet is 180°. Hence, $\beta_2 = 180 - \theta = 180 - 180 = 0$. Therefore, for an ideal Pelton Turbine,

$$P_{\max} = \frac{\rho_w a C_1^3}{2} \tag{3.67}$$

Variation of power with velocity ratio for an ideal Pelton turbine is shown in Figure 3.9. The variation is parabolic. In actual practice, angle of deflection is less than 180°. Angle of the deflection (θ) of jet is generally kept 165° so that the exiting water does not hit the following bucket. The blade angle, $\beta = 180 - \theta = 15°$. Due to inevitable resistance to flow and windage losses, the friction factor, k, lies in the range of 0.8 – 0.85. Therefore, in practice, $\rho_{\text{opt}} \approx 0.46$, instead of 0.5. Further, the power is zero at value of $C_b/C_1 \approx 0.9$

3.7.5 Maximum Hydraulic Efficiency

We know that hydraulic efficiency of the nozzle-wheel system as is given by Eq. (3.60),

$$\eta_h = \frac{H_e}{H} = \frac{(C_1 - C_b)(1 + k\cos\beta_2)C_b}{gH}$$

Since, $C_1 = C_v\sqrt{2gH} \Rightarrow gH = \frac{C_1^2}{2C_v^2}$

Substituting the value of gH in the equation of hydraulic efficiency of nozzle-wheel system, we get,

$$\eta_h = \frac{2C_v^2(C_1 - C_b)(1 + k\cos\beta_2)C_b}{C_1^2} = 2C_v^2\left(\frac{C_b}{C_1}\right)\left(1 - \frac{C_b}{C_1}\right)(1 + k\cos\beta_2) \tag{3.68}$$

velocity ratio, $\rho = \frac{C_b}{C_1}$

$$\eta_h = 2C_v^2\rho(1-\rho)(1 + k\cos\beta_2) = C\rho(1-\rho) \tag{3.69}$$

where C is a constant for a given turbine setup and is given by,

$$C = 2C_v^2(1 + k\cos\beta_2) \tag{3.70}$$

From Eq. (3.69), it is clear that hydraulic efficiency is a function of velocity ratio ρ. For maximum hydraulic efficiency,

$$\frac{d\eta_h}{d\rho} = 0$$

$$\therefore \qquad (\rho_{\text{opt}})_{\text{max hy.eff.}} = \frac{1}{2} \tag{3.71}$$

Equation (3.71) represents that for maximum hydraulic efficiency, optimum velocity ratio is $\frac{1}{2}$ i.e. $[(C_b)_{\text{opt}}]_{\text{max hy.eff.}} = \frac{C_1}{2}$. Therefore, under ideal conditions, the characteristics of the variation of hydraulic efficiency with velocity ratio are similar to that of variation of power. Noteworthy points for an ideal Pelton wheel are as follows:

- The hydraulic efficiency varies parabolically with velocity ratio and hence with tangential velocity of the blades or buckets.
- The maximum hydraulic efficiency occurs at a velocity ratio of $\frac{1}{2}$ i.e. $(C_b)_{\text{opt}} = \frac{C_1}{2}$. The maximum hydraulic efficiency and maximum power occur simultaneously at a velocity ratio of 1/2. Substituting the value of optimum velocity ratio in either Eq. (3.60) or in Eq. (3.69), maximum hydraulic efficiency is obtained as,

$$(\eta_h)_{\text{max}} = \frac{C_1^2}{4} \times \frac{(1 + k\cos\beta_2)}{gH} = \frac{C_v^2}{2}(1 + k\cos\beta_2) \tag{3.72}$$

3.7.6 Torque

Power developed as given by Eq. (3.61) is,

$$SP = \eta_m \eta_v \rho_w Q(C_1 - C_b)(1 + k \cos \beta_2)C_b$$

We know that power developed and torque is related by,

$$SP = T \times \omega$$

where, T is the torque acting on the Pelton wheel and ω is the angular velocity.

By the above two equations, torque acting on the Pelton wheel is,

$$T = \frac{SP}{\omega} = \frac{\eta_m \eta_v \rho_w Q(C_1 - C_b)(1 + k \cos \beta_2) C_b}{\omega} \tag{3.73}$$

Since angular velocity and bucket velocity are given by,

$$\omega = \frac{2\pi N}{60} \text{ rad/s}, \; C_b = \frac{\pi DN}{60} = \omega \times \frac{D}{2} \tag{3.74}$$

Substituting value of C_b from Eq. (3.74) into Eq. (3.73), we get,

$$T = \frac{SP}{\omega} = \frac{\eta_m \eta_v \rho_w QD}{2}(C_1 - C_b)(1 + k \cos \beta_2) = K_4(C_1 - C_b) \tag{3.75}$$

where, K_4 is a constant which is given by,

$$K_4 = \frac{\eta_m \eta_v \rho_w QD}{2}(1 + k \cos \beta_2) \tag{3.76}$$

It is noteworthy that in the expression of torque given by Eq. (3.75),

- Mechanical losses and volumetric losses are considered.
- The torque varies linearly with the tangential or peripheral velocity of the buckets i.e. with the speed of the runner. The torque acting on the Pelton wheel is maximum at $C_b = 0$ i.e. when the wheel is at rest.
- The torque reaches a minimum value of zero at $C_b = C_1$. As discussed in the following section, this condition of $C_b = C_1$ corresponds to the *runaway speed*.

For an ideal Pelton turbine, $k = 1$ i.e. there is no friction resistance and $\beta_2 = 180°$. For an ideal Pelton turbine, the torque varies linearly and reaches a value of zero at $C_b/C_1 = 1$ as shown in Figure 3.9. In actual turbines, because of various resistances, the torque varies slightly non linearly from a value of C_b/C_1 around 0.6 onwards and becomes zero at $C_b/C_1 \approx 0.9$.

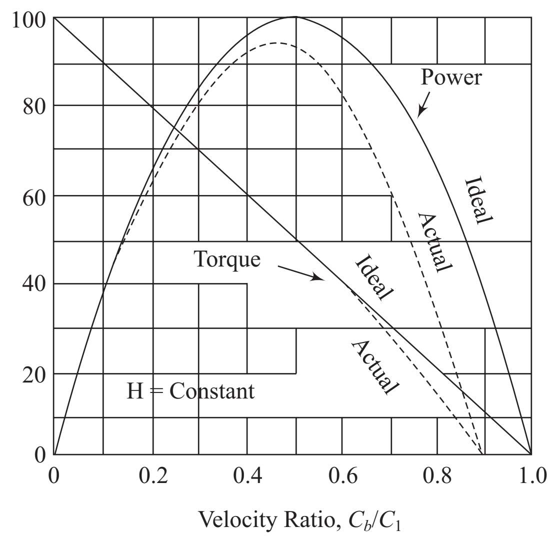


Figure 3.9 *Variation of Power and Torque with Velocity Ratio for a Pelton Turbine*

Torque acting on the Pelton wheel in terms of Euler head may be found as below,

$$T = \frac{P}{\omega} = \frac{\rho_w g Q H_e}{\omega} = \frac{\rho_w g Q H_e}{2C_b/D}$$

$$T = \rho_w g Q H_e \left(\frac{D}{2C_b}\right) \quad (3.77)$$

3.7.7 Runaway Speed

The actual power output from a Pelton turbine is given by Eq. (3.61) as,

$$P = \eta_m \eta_v \rho_w Q(C_1 - C_b)(1 + k \cos \beta_2)C_b$$

It is quite obvious that the power output $P = 0$ when,

(i) $C_b = 0$ i.e. when wheel is stationary,

(ii) $C_b = C_1$ i.e. when the Pelton wheel is running at a maximum velocity of $C_b = \frac{\pi DN}{60} = C_1$. This corresponds to the runaway speed of the turbine with the sudden removal of the entire load. Let the normal wheel or bucket velocity be C_b and let it be C_{br} under runaway conditions. Under normal operative conditions,

$$C_b = \varphi\sqrt{2gH}$$

We know that,

$$C_1 = C_v\sqrt{2gH}$$

Under runaway condition, the theoretical runaway speed is,

$$C_{br} = C_1 = C_v\sqrt{2gH} = \left(\frac{C_v}{\varphi}\right)C_b \quad (3.78)$$

Since, $C_b \propto N$, theoretical runaway speed in rpm may be found from Eq. (3.78) as,

$$N_r = \left(\frac{C_v}{\varphi}\right)N \quad (3.79)$$

where, N is the normal operative speed.

Assuming the average values of $C_v = 0.985$ and $\phi = 0.45$, the runaway speed for a Pelton turbine is $N_r \approx 2.19\ N$. However, in practice, the runaway speed is found to be approximately 1.87 times the normal speed because of windage and bearing friction that cause resistance and consequent loss of energy.

3.7.8 Diagram or Blading or Wheel Efficiency

The hydraulic efficiency was defined by considering the nozzle as an integral part of the nozzle-bucket wheel system. However, if wheel alone is considered as a system, then the energy input to the wheel is the kinetic energy of the jet and output energy is the runner power i.e. work done by the jet on the turbine rotor per second. Therefore, the wheel efficiency also known as *diagram or blading efficiency* is defined as,

$$\eta_D = \frac{\text{Output Energy}}{\text{Input Energy}} = \frac{\text{Work done by the jet on the runner per second}}{\text{Kinetic energy of the jet per second}} \quad (3.80)$$

The wheel efficiency represents the effectiveness of the wheel in converting the kinetic energy of the jet into mechanical energy of rotation of the runner. Not all of this energy of rotation of the runner is available at the shaft of the turbine because some is consumed in overcoming friction in the bearings and some in overcoming 'windage' i.e. the friction between the wheel and the atmosphere in which it rotates. In addition to these losses there is a loss in the nozzle (that is why $C_v < 1$). The overall efficiency is therefore less than the wheel efficiency. Even so, an overall efficiency of 85–90% may usually be achieved in large machines.

$$\text{Energy input to the runner/second i.e. kinetic energy of the jet/second} = \frac{1}{2}(\rho_w a C_1) \times C_1^2 \quad (3.81)$$

The power transmitted to the wheel by the jet of water known as runner power is given by Eq. (3.58) as,

$$P = \rho_w a C_1 (C_1 - C_b)(1 + k \cos \beta_2) C_b = \rho_w a C_1^3 \rho (1 - \rho)(1 + k \cos \beta_2)$$

Therefore, the blading or diagram or wheel efficiency of the Pelton turbine is,

$$\eta_D = 2(1 + k \cos \beta_2)(\rho - \rho^2) = 2\rho(1 + k \cos \beta_2)(1 - \rho) \quad (3.82)$$

It is seen from Eq. (3.82) that wheel or diagram or blading efficiency depends on the velocity ratio, blade friction coefficient and the bucket angle. It should be noted that the wheel efficiency does not include the losses in the nozzle due to which the value of wheel efficiency is larger than that of hydraulic efficiency of the same machine. From Eqs (3.69) and (3.82), the relation between wheel or diagram or blading efficiency and hydraulic efficiency can be found which is given below,

$$\eta_D = \frac{\eta_h}{C_v^2} \quad (3.83)$$

It is obvious from Eq. (3.82) that for a given value of C_1, k, β_2, there would be a certain value of ρ for which diagram or blading efficiency is maximum, which will be called optimum velocity ratio for maximum diagram efficiency. Therefore, for maximum diagram efficiency,

$$\frac{d\eta_D}{d\rho} = 0 \Rightarrow \frac{d\eta_D}{d\rho} = 2(1 + k \cos \beta_2)(1 - 2\rho) = 0$$

$$(\rho_{\text{opt}})_{\text{max wheel eff}} = \frac{1}{2} \Rightarrow C_b = \frac{1}{2} C_1 \quad (3.84)$$

Thus, from Eq. (3.84) it can be concluded that optimum velocity ratio for maximum blading or diagram or wheel efficiency is also half as was found for maximum power developed or maximum hydraulic efficiency as evident from Eqs. (3.63) and (3.71). Hence, the bucket velocity should be half of the jet velocity for maximum work output or maximum power developed as well as for maximum diagram efficiency and maximum hydraulic efficiency.

Diagram efficiency is the ratio of diagram work or work given to the runner by jet of water to the input energy i.e. kinetic energy of water, as already discussed earlier. For a given value of the jet velocity, the input energy is a constant. So, it is quite obvious that the condition for maximum diagram efficiency is same as that for maximum work output or power developed without deriving Eq. (3.84).

Similarly, substituting the result of Eq. (3.84) into Eq. (3.82), we get,

$$(\eta_D)_{\max} = 2(1 + k \cos \beta_2)\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$(\eta_D)_{\max} = \frac{(1 + k \cos \beta_2)}{2} \quad (3.85)$$

Since, diagram efficiency is the ratio of diagram work or work developed to the kinetic energy of the jet. Therefore, maximum diagram efficiency could also be found by dividing maximum power developed [Eq. (3.65)] by the kinetic energy of the jet per second [Eq. (3.81)] yielding the same result as that of Eq. (3.85).

If there is no energy loss due to friction i.e. $k = 1$,

$$(\eta_D)_{\max} = \frac{1 + \cos\beta_2}{2} \tag{3.86}$$

For an ideal Pelton turbine, $k = 1$ and angle of deflection of the jet, $\theta = 180°$, then $\beta = 180 - \theta = 0$,

$$(\eta_D)_{\max} = 1 \text{ or } 100\% \tag{3.87}$$

However, k lies between 0.8 – 0.85 and $\theta \cong 165°$, so that the exiting water does not hit the following bucket. Therefore,

$$(\eta_D)_{\max} = \frac{1 + 0.8 \times \cos 15}{2} \cong 0.886 \tag{3.88}$$

The minimum number of buckets in the wheel is approximately given by,

$$z = \frac{m}{2} + 15 \tag{3.89}$$

where m is equal to the jet ratio, D/d.

3.7.9 Nozzle Efficiency

The *nozzle efficiency* is defined as the ratio of the energy at the nozzle outlet to the energy at the base of the nozzle i.e. at the nozzle inlet. Therefore,

$$\eta_n = \frac{\text{Energy at nozzle outlet}}{\text{Energy at nozzle inlet}}$$

$$\eta_n = \frac{\left(\dfrac{C_1^2}{2g}\right)}{H} = \frac{C_1^2}{2gH} = \frac{C_v^2 \times 2gH}{2gH}$$

$$\eta_n = C_v^2 \tag{3.90}$$

From Eqs (3.83) and (3.90), it is clear that hydraulic efficiency and wheel efficiency are related as,

$$\eta_h = \eta_n \eta_D \tag{3.91}$$

3.8 Limitation of a Pelton Turbine

Efficient and reliable operations of a Pelton turbine is found under high heads and low discharge. If the turbine has to produce same power output under lower head, the discharge through the turbine has to be higher, consequently an increase in jet diameter. The increased discharge cannot be met by increasing the number of jets which are generally limited to 4 or 6 per wheel. The increase in jet diameter to obtain higher discharge results into increase in the diameter of the Pelton wheel. Therefore, the turbine becomes disproportionately large, bulky and slow running. In actual practice, reaction turbines are more suitable for lower heads and high discharge.

EXAMPLE 3.1 A Pelton wheel is operating under a gross head of 498 m in a hydraulic plant. The loss of head due to friction in the penstock is one third of the gross head. A nozzle fitted at the end of the penstock supplies water at the rate of $2 m^3/s$ to the turbine where the jet gets deflected through an angle of 170°. Determine (a) whirl velocity at inlet and outlet, (b) shaft power output of the Pelton wheel, if the mechanical efficiency is 95% , (c) hydraulic efficiency, and (d) overall efficiency by taking, Case 1

Bucket friction coefficient $k = 0.97$, speed ratio $\varphi = 0.46$, coefficient of velocity $C_v = 0.98$,

Case (2) Without any losses, i.e.,

$$\varphi = 0.5,\ k = 1,\ C_v = 1,\ H = H_g$$

Solution

Given: $H_g = 498$ m, $h_f = \frac{1}{3} H_g$, $Q = 2 m^3/s$, $\theta = 170°$

(1) With Losses

$$k = 0.97,\ \varphi = 0.46,\ C_v = 0.98,\ \eta_0 = 0.36$$

Head loss due to friction in the penstock,

$$h_f = \frac{1}{3} H_g = \frac{1}{3} \times 498 = 166 \text{ m}$$

Net head at the turbine,

$$H = H_g - h_f = 498 - 166$$

$$H = 332 \text{ m} \qquad (1)$$

Velocity of the jet is given by,

$$C_1 = C_v \sqrt{2gH} \qquad (2)$$

$$C_1 = 0.98 \times \sqrt{2 \times 9.81 \times 332}$$

$$C_1 = 79.0942 \text{ m/s} \qquad (3)$$

Peripheral velocity of the buckets /blades,

$$C_b = \varphi \sqrt{2gH} \qquad (4)$$

$$C_b = 0.46 \sqrt{2 \times 9.81 \times 332}$$

$$C_b = 37.126 \text{ m/s} \qquad (5)$$

Blade angle at the outlet of the bucket is,

$$\beta_2 = 180 - \theta = 180 - 170 = 10° \qquad (6)$$

(a) Whirl Velocity at Inlet and Outlet

Figure 3.10 shows the velocity diagrams at inlet and outlet of the turbine. From the inlet velocity triangle, velocity of whirl at inlet,

$$C_{w1} = C_1 = 79.0942 \text{ m/s} \qquad (7)$$

Relative velocity at inlet,

$$C_{r1} = C_1 - C_b = 79.0942 - 37.126$$

$$C_{r1} = 41.9682 \text{ m/s} \qquad (8)$$

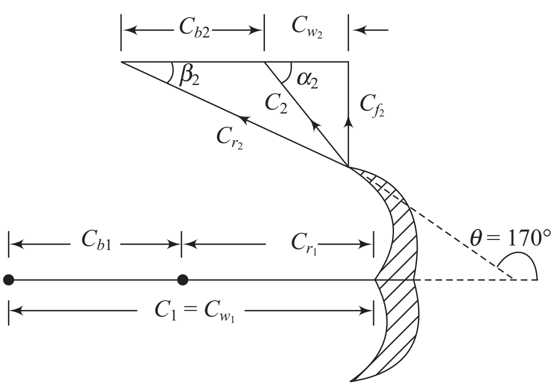


Figure 3.10 *Velocity Diagrams at Inlet and Outlet of a Pelton Turbine of Example 3.1*

$\because \qquad k = \frac{C_{r2}}{C_{r1}} = 0.97 \quad \Rightarrow \quad C_{r2} = 0.97 \times 41.9682$

$$C_{r2} = 40.71 \text{ m/s} \tag{9}$$

From the velocity triangle at outlet,

$$C_{w2} = C_{r2} \cos \beta_2 - C_b \tag{10}$$

$$C_{w2} = 40.71 \cos 10 - 37.126$$

$$C_{w2} = 2.965 \text{ m/s} \tag{11}$$

The value of C_{w2} obtained is positive, hence, the assumed velocity triangle is correct.

(b) Power Output of the Pelton Wheel

By Euler's equation, theoretical fluid power at the outlet of the runner for the assumed velocity triangle is

$$P = \rho_w Q C_b (C_{w1} + C_{w2}) \tag{12}$$

$$P = 1000 \times 2 \times 37.126 \times (79.0942 + 2.965)$$

$$P = 6093 \text{ kW} \tag{13}$$

$$\text{Shaft power} = 6093 \times 0.95 = 5788 \text{ kW} \tag{14}$$

(c) Hydraulic Efficiency

We know that, from Eq. (3.59), Euler head is given by,

$$H_e = \frac{1}{g} C_b (C_1 - C_b)(1 + k \cos \beta_2) \tag{16}$$

$$H_e = \frac{1}{9.81} \times 37.126 \times (79.0942 - 37.126) \times (1 + 0.97 \times \cos 10)$$

$$H_e = 310.55 \text{ m} \tag{17}$$

Hydraulic efficiency is,

$$\eta_h = \frac{H_e}{H}$$

$$\eta_h = \frac{310.55}{332} \tag{18}$$

$$\eta_h = 0.9354 = 93.54\% \tag{19}$$

(d) Overall Efficiency

$$\eta_o = \frac{\text{Shaft Power (SP)}}{\rho g Q H} = \frac{5788 \times 10^3}{1000 \times 9.81 \times 2 \times 332} = 88.86\% \tag{20}$$

Case (2) Without Any Losses

$$\varphi = 0.5,\ k = 1,\ C_v = 1,\ H = H_g$$

$$C_1 = C_v \sqrt{2gH} = 1 \times \sqrt{2 \times 9.81 \times 498}$$

$$C_1 = 98.8471 \text{ m/s} \tag{21}$$

$$C_b = \varphi\sqrt{2gH}$$

$$C_b = 0.5 \times \sqrt{2 \times 9.81 \times 498}$$

$$C_b = 49.42355 \text{ m/s} \tag{22}$$

(a) Velocity of Whirl at Inlet and Outlet

From the inlet velocity triangle,

$$C_{w1} = C_1 = 98.8471 \text{ m/s} \tag{23}$$

From the velocity triangle at inlet,

$$C_{r1} = C_1 - C_b = 98.8471 - 49.42355$$

$$C_{r1} = 49.42355 \text{ m/s} \tag{24}$$

Since losses due to friction in the bucket is neglected i.e. $k = \dfrac{C_{r2}}{C_{r1}} = 1$

$$\therefore \quad C_{r2} = 49.42355 \text{ m/s} \tag{25}$$

From the velocity triangle at outlet,

$$C_{w2} = C_{r2} \cos \beta_2 - C_b$$

$$C_{w2} = 49.42355 \cos 10 - 49.42355$$

$$C_{w2} = -0.8474 \text{ m/s} \tag{26}$$

As C_{w2} is negative, the assumed velocity triangle is incorrect. However, we can proceed with the assumption and formula at Eq. (12) without incurring any error, as follows.

(b) Power Output of the Pelton Wheel

By Euler's equation, for the assumed velocity triangle,

$$P = \rho_w Q C_b (C_{w1} + C_{w2})$$

$$P = 1000 \times 2 \times 49.42355 \times [98.8471 + (-0.8474)]$$

$$P = 9686986.146\ W = 9686.986 \text{ kW} \tag{27}$$

The values of P calculated by Euler's equation and from the available power differ only within 0.3 % which is due to round-off error in the calculation.

(c) Hydraulic Efficiency

$$H_e = \frac{1}{g} C_b (C_1 - C_b)(1 + k \cos \beta_2)$$

$$H_e = \frac{1}{9.81} \times 49.4235 \times (98.8471 - 49.4235) \times (1 + 1 \times \cos 10)$$

$$H_e = 494.22 \text{ m} \tag{28}$$

$$\eta_h = \frac{H_e}{H} = 99.24\% \tag{29}$$

The difference between the value of 494 m and 498 m is due to the deflection angle of blade.

(d) Overall Efficiency

$$SP = \eta_m \times RP = 0.95 \times 9686.986 = 9202.6367 \text{ kW} \tag{30}$$

$$\eta_o = \frac{SP}{\rho gQH} = \frac{9202.6367 \times 1000}{1000 \times 9.81 \times 2 \times 498} = 0.94185 = 94.185\% \tag{31}$$

Alternately, overall efficiency considering casing and volumetric efficiency to be 100%,

$$\eta_0 = \eta_h \eta_m = 0.9924 \times 0.95 = 0.942 = 94.2\%$$

EXAMPLE 3.2 In a Pelton turbine the jet is deflected by 165°. The initial jet velocity is 95 m/s and the peripheral velocity of the wheel at the pitch circle is 43 m/s. Calculate (a) the head loss due to bucket friction, (b) the magnitude and direction of the absolute velocity of the jet from the exit of the bucket, and (c) the kinetic energy head of exit stream from the buckets. Assume bucket friction coefficient $k = 0.9$.

Solution

Given: $\theta = 165°$, $C_1 = 95$ m/s, $C_b = 43$ m/s, $k = 0.9$

Figure 3.11 shows the inlet and outlet velocity triangles. Blade angle at the exit is given by,

$$\beta_2 = 180 - \theta \Rightarrow = 180 - 165 = 15°$$

Since, blade friction coefficient,

$$k = \frac{C_{r2}}{C_{r1}} = 0.9 \Rightarrow C_{r2} = 0.9C_{r1} = 0.9(C_1 - C_b) = 0.9(95 - 43)$$

$$C_{r2} = 46.8 \text{ m/s} \tag{1}$$

Let α_2 be the direction of the absolute velocity at exit of the bucket C_2 with the peripheral velocity, then velocity of flow at the outlet is given by,

$$C_{f2} = C_{r2} \sin \beta_2 \Rightarrow C_{f2} = 46.8 \times \sin 15$$

$$C_{f2} = 12.11 \text{ m/s} \tag{2}$$

Velocity of whirl at the outlet,

$$C_{w2} = C_{r2} \cos \beta_2 - C_b \Rightarrow C_{w2} = 46.8 \cos 15 - 43$$

$$C_{w2} = 2.20 \text{ m/s} \tag{3}$$

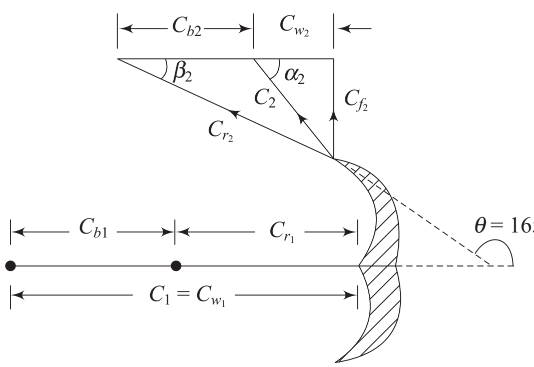


Figure 3.11 *Velocity Diagrams at Inlet and Outlet of a Pelton Turbine of Example 3.2*

(a) The Magnitude and Direction of Absolute Velocity at Exit

The direction of the absolute velocity at the exit from the buckets of the Pelton wheel is obtained by,

$$\tan \alpha_2 = \frac{C_{f2}}{C_{w2}} \Rightarrow \tan \alpha_2 = \frac{12.11}{2.20} = 5.50$$

$$\therefore \quad \alpha_2 = 79.70° \tag{4}$$

From the velocity triangle at the outlet,

$$\sin \alpha_2 = \frac{C_{f2}}{C_2} \Rightarrow C_2 = \frac{C_{f2}}{\sin \alpha_2} = \frac{12.11}{\sin 79.70}$$

$$C_2 = 12.31 \text{ m/s} \tag{5}$$

(b) Head Loss Due to Bucket Friction

Head loss or energy loss in the buckets,

$$h_{fb} = \frac{C_{r1}^2}{2g} - \frac{C_{r2}^2}{2g} \tag{6}$$

$$h_{fb} = \frac{(95-43)^2}{2\times 9.81} - \frac{46.8^2}{2\times 9.81}$$

$$h_{fb} = 26.186 \text{ m} \tag{7}$$

(c) Kinetic Energy Head of Exit Stream from the Buckets

$$\text{Kinetic energy head of exit stream from buckets} = \frac{C_2^2}{2g} \tag{8}$$

$$= \frac{12.31^2}{2\times 9.81}$$

$$\text{Kinetic energy head of exit stream from buckets} = 7.72 \text{ m} \tag{9}$$

EXAMPLE 3.3 The coefficient of velocity for a Pelton wheel nozzle is 1. The net head on the turbine is 350 m. The jet diameter and pitch circle diameter of the runner are 125 mm and 2.5 m respectively. The buckets deflect the jet through 165°and they run at 0.47 times the jet speed. The relative velocity at the outlet reduces by 7% of that at inlet due to friction in the buckets. If the mechanical efficiency is 0.9, calculate (a) hydraulic efficiency, (b) speed of the turbine, (c) the water power, (d) the shaft power, (e) power lost at the exit, (f) specific speed, and (g) shape factor.

Solution

Given: C_v = 1, H = 350 m, d = 125 mm = 0.125 m, D = 2.5 m, θ = 165°, ϕ = 0.47, C_{r2} = 0.93 C_{r1}, η_m = 0.9. The velocity of the jet is given by,

$$C_1 = C_v\sqrt{2gH} \tag{1}$$

$$C_1 = 1.0 \times \sqrt{2\times 9.81\times 350}$$

$$C_1 = 82.87 \text{ m/s} \tag{2}$$

The peripheral or tangential velocity of the buckets is,

$$C_b = \rho C_1$$

$$C_b = 0.47 \times 82.87 \tag{3}$$

$$C_b = 38.95 \text{ m/s} \tag{4}$$

(a) Hydraulic Efficiency

Since relative velocity at outlet reduces by 7% of that at inlet, therefore, $C_{r2} = 0.93\ C_{r1} \Rightarrow k = \frac{C_{r2}}{Cr_1}\ 0.93$

Theoretical or Euler head is

$$H_e = \frac{1}{g} C_b (C_1 - C_b)(1 + k\cos\beta_2) \tag{5}$$

$$H_e = \frac{1}{9.81} \times 38.95 \times (82.87 - 38.95) \times (1 + 0.93\cos 15)$$

$$H_e = 331.031 \text{ m} \tag{6}$$

Hydraulic efficiency is,

$$\eta_h = \frac{H_e}{H} \tag{7}$$

$$\eta_h = \frac{331.031}{350}$$

$$\eta_h = 0.9466 = 94.66\% \tag{8}$$

(b) Speed of the Turbine

$$C_b = \frac{\pi DN}{60} \tag{9}$$

$$38.95 = \frac{\pi \times 2.5 \times N}{60}$$

$$N = 297.56 \text{ m/s} \tag{10}$$

(c) Water Power

Discharge through the turbine is given by,

$$Q = \text{Cross sectional area of the jet} \times \text{velocity of the jet} = \frac{\pi}{4}\ d^2 C_1 \tag{11}$$

$$Q = \frac{\pi}{4} \times 0.125^2 \times 82.87$$

$$Q = 1.017 \text{ m}^3\text{/s} \tag{12}$$

Theoretical power or water power is,

$$P_{\text{ideal}} = \rho_w g Q H \tag{13}$$

$$P_{\text{ideal}} = 1000 \times 9.81 \times 1.017 \times 350$$

$$P_{\text{ideal}} = 3491869.5 \text{ W} = 3491.87 \text{ kW} \tag{14}$$

(d) Shaft Power

$$P = \eta_o \rho_w g Q H = \eta_v \eta_h \eta_m \rho_w g Q H \tag{15}$$

Assuming volumetric efficiency to be 100%, shaft power is,

$$P = 1 \times 0.9466 \times 0.9 \times 1000 \times 9.81 \times 1.017 \times 350$$

$$P = 29744863.202\ W = 2974.8633 \text{ kW} \tag{16}$$

(e) Power Lost in Discharge

The velocity diagrams at the inlet and outlet of the Pelton wheel are shown in Figure 3.12. From velocity triangle at the inlet,

$$C_{r1} = C_1 - C_b = 82.87 - 38.95$$

$$C_{r1} = 43.92 \text{ m/s} \tag{17}$$

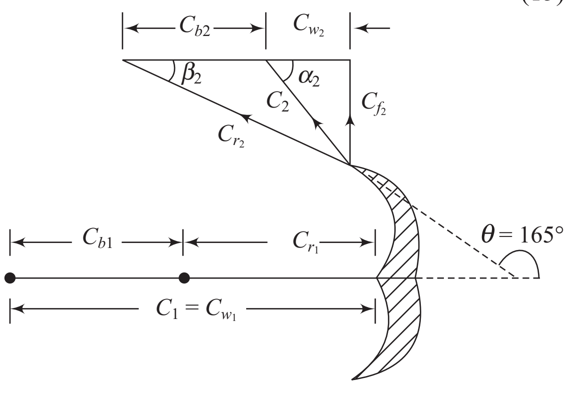


Figure 3.12 *Velocity Diagrams at Inlet and Outlet of a Pelton Turbine of Example 3.3*

Since, relative velocity at the outlet reduces by 7% of that at the inlet, therefore,

$$C_{r2} = 0.93C_{r1} = 0.93 \times 43.92$$

$$C_{r2} = 40.85 \text{ m/s} \tag{18}$$

Blade angle or bucket angle at the outlet,

$$\beta_2 = 180 - \theta = 180 - 165 = 15°$$

Velocity of flow at outlet is given by,

$$C_{f2} = C_{r2} \sin \beta_2 \Rightarrow C_{f2} = 40.85 \times \sin 15$$

$$C_{f2} = 10.573 \text{ m/s} \tag{19}$$

Velocity of whirl at the outlet,

$$C_{w2} = C_{r2} \cos \beta_2 - C_b \Rightarrow C_{w2} = 40.85 \cos 15 - 38.95$$

$$C_{w2} = 0.51 \text{ m/s} \tag{20}$$

Absolute velocity of the exit stream

$$C_2 = \sqrt{C_{f2}^2 + C_{w2}^2} \tag{21}$$

$$C_2 = \sqrt{10.573^2 + 0.51^2}$$

$$C_2 = 10.5853 \text{ m/s} \tag{22}$$

$$\text{Power lost in discharge} = P_d = \rho_w g Q \frac{C_2^2}{2g} \tag{23}$$

$$P_{L,d} = 1000 \times 9.81 \times 1.017 \times \frac{10.5853^2}{2 \times 9.81}$$

$$P_{L,d} = 56976.7 \text{ W} = 56.977 \text{ kW} \tag{24}$$

(f) Specific Speed

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \tag{25}$$

$$N_s = \frac{297.56 \times \sqrt{3491.87}}{350^{5/4}}$$

$$N_s = 11.615 \frac{\text{rev}\sqrt{\text{kW}}}{\text{min(m)}^{5/4}} \tag{26}$$

(g) Shape Factor

Specific speed in non-dimensional form i.e. shape factor in radians is given by,

$$N_{sh} = \frac{\omega\sqrt{P}}{\rho_w^{1/2}(gh)^{5/4}} = \frac{\left(\frac{2 \times \pi \times 297.56}{60}\right) \times \sqrt{3491.87 \times 10^3}}{1000^{1/2} \times (9.81 \times 350)^{5/4}}$$

$$N_{sh} = 0.070 \text{ rad} \tag{27}$$

EXAMPLE 3.4 A three jet Pelton turbine working under a net head of 400 m is required to generate 10 MW. The side clearance angle of the buckets is 15°and the relative velocity decreases by 5% while passing over the blades. Assume that overall efficiency of the wheel, η_o = 82%, coefficient of velocity, C_v = 0.97 and speed ratio, φ = 0.47. Determine (a) the diameter of the jet, (b) total discharge, (c) the force exerted by the jet on the buckets, and (iv) maximum speed of the wheel for a frequency of 50 Hz and the corresponding wheel diameter if the jet ratio is not to be less than 10.

Solution

Given: Number of jets = 3, H = 400 m, P_t = 10 MW = 10000 kW, β_2 = 15°, C_{r2} = 0.95 C_{r1}, η_o = 0.82, C_v = 0.97, φ = 0.47, frequency, f = 50 Hz, m = 10.
Velocity of the jet,

$$C_1 = C_v\sqrt{2gH} \quad (1)$$

$$C_1 = 0.97\sqrt{2\times 9.81\times 400}$$

$$C_1 = 85.93 \text{ m/s} \quad (2)$$

Peripheral or tangential velocity of the buckets,

$$C_b = \varphi\sqrt{2gH} \quad (3)$$

$$C_b = 0.47\sqrt{2\times 9.81\times 400}$$

$$C_b = 41.64 \text{ m/s} \quad (4)$$

(a) Diameter of the Jet

$\because$ Power produced by three jets, P_t = 10000 kW

$\therefore$ Power produced by one jet, $P = \dfrac{P_t}{3} = \dfrac{10000}{3}$ = 3333.33 kW (5)

We know that power produced by one jet,

$$P = \eta_o \rho_w g Q H \quad (6)$$

$$3333.33 \times 1000 = 0.82 \times 1000 \times 9.81 \times Q \times 400$$

$$Q = 1.036 \text{ m}^3\text{/s} \quad (7)$$

$$Q = \frac{\pi}{4} d^2 C_1$$

$$1.036 = \frac{\pi}{4} \times d^2 \times 85.93$$

$$d = 0.124 \text{ m} = 124 \text{ mm} \quad (8)$$

(b) Total Discharge

Total discharge = Q = Discharge of one jet × number of jets = 1.036 × 3

$$Q_t = 3.108 \text{ m}^3\text{/s} \quad (9)$$

(c) Force Exerted by a Jet

Since 5% energy is lost due to friction in the buckets, therefore, relative velocity at the outlet of the buckets is,

$$C_{r2} = 0.95C_{r1} \Rightarrow \text{Bucket friction coefficient} = k = \frac{C_{r2}}{C_{r1}} = 0.95$$

Force exerted by a single jet on the buckets in the tangential direction,

$$F_x = \rho_w Q(C_1 - C_b)(1 + k\cos\beta_2) \tag{10}$$

$$F_x = 1000 \times 1.036 \times (85.93 - 41.64) \times (1 + 0.95\cos 15)$$

$$F_x = 87989.36 \text{ N} = 87.99 \text{ kN} \tag{11}$$

Therefore, force on the buckets due to three jets,

$$F_{xt} = 87.99 \times 3 = 263.97 \text{ kN} \tag{12}$$

(d) Maximum Speed and Wheel Diameter

$$\because \quad m = \frac{D}{d} = 10 \Rightarrow D = 10\,d = 10 \times 0.124 = 1.24 \text{ m}$$

We know that,

$$C_b = \frac{\pi DN}{60} \tag{13}$$

$$41.64 = \frac{\pi \times 1.24 \times N}{60}$$

$$N = 641.34 \text{ rpm} \tag{14}$$

Synchronous speed of the generator is given by,

$$N_{sy} = \frac{120f}{p} \tag{15}$$

$$641.34 = \frac{120 \times 50}{p} \Rightarrow p = 9.3554$$

Number of poles cannot be in fraction. Therefore, number of poles is 10 by taking the next whole number. Now corresponding to 10 poles, the maximum speed of the turbine would be equal to the revised synchronous speed of the generator,

$$N = N_{sy} = \frac{120 \times f}{p} \Rightarrow N = \frac{120 \times 50}{10}$$

$$N = 600 \text{ rpm} \tag{16}$$

$$\because \quad C_b = \frac{\pi DN}{60} \Rightarrow D = \frac{60C_b}{\pi N}$$

The wheel diameter will change with the change of speed of the turbine as the peripheral velocity would remain constant. Therefore, the revised diameter of the wheel,

$$D = \frac{60 \times 41.64}{\pi \times 600}$$

$$D = 1.325 \text{ m} \tag{17}$$

New jet ratio,

$$m = \frac{D}{d} = \frac{1.325}{0.124} = 10.686 > 10 \quad (18)$$

Hence, the given condition that jet ratio is not to be less than 10 is satisfied.

EXAMPLE 3.5 A jet of water of area 8200 mm^2 strikes the buckets of a Pelton wheel which is operating under a net head of 195 m. The flow rate of water through the buckets is 0.49 m^3/s. The power available at the shaft is 800 kW and mechanical efficiency is 95%. (a) Calculate the power loss (i) in the nozzle, (ii) in the runner including the energy loss at the exit of the buckets, and (iii) in mechanical friction and windage. (b) Calculate (i) nozzle efficiency, (ii) wheel efficiency, (iii) hydraulic efficiency and (iv) overall efficiency.

Solution

Given:

A_1 = 8200 mm^2 = 0.0082 m^2, H = 195 m, Q = 0.49 m^3/s, P_{shaft} = 800 kW = 8 × 10^5 W, η_m = 95% = 0.95

Power available at the base of the nozzle i.e. water power (WP),

$$\text{WP} = \rho_w gQH \quad (1)$$

$$\text{WP} = 1000 \times 9.81 \times 0.49 \times 195$$

$$\text{WP} = 937345.5 \text{ W} = 937.35 \text{ kW} \quad (2)$$

We know that discharge is given by,

$$Q = A_1 C_1 \Rightarrow C_1 = \frac{Q}{A_1} = \frac{0.49}{0.0082}$$

$$C_1 = 59.76 \text{ m/s} \quad (3)$$

(a) Power Losses

(i) Power Loss in the Nozzle

$$\text{Kinetic or Velocity head at the nozzle exit} = \frac{C_1^2}{2g} = \frac{59.76^2}{2 \times 9.81}$$

Kinetic or Velocity head at the nozzle exit = 182.02 m (4)

$$\text{Power available at the nozzle exit i.e. jet power} = \rho_w gQ\frac{C_1^2}{2g} \quad (5)$$

$$\text{Power available at the nozzle exit} = 1000 \times 9.81 \times 0.49 \times \frac{59.76^2}{2 \times 9.81}$$

Power available at the nozzle exit i.e. jet power = 874951.94 W = 874.952 kW (6)

Power loss in the nozzle = Water Power – Power available at nozzle exit (7)

Power loss in the nozzle = 937.35 – 874.952

Power loss in the nozzle = 62.398 kW (8)

(ii) Power Loss in the Runner

We know that mechanical efficiency is given by,

$$\eta_m = \frac{\text{Power available at the shaft i.e. Shaft (SP)}}{\text{Power developed by the Runner i.e. Power (RP)}} \quad (9)$$

$$0.95 = \frac{800}{\text{RP}}$$

Power developed by the runner i.e. Runner Power (RP) = 842.10 kW (10)

Power input to the runner = Power available at the nozzle exit i.e. jet power = 874.952 kW

Power loss in the runner including energy loss at exit of the buckets = Jet Power – Runner Power

= 874.952 – 842.10 (11)

Power loss in the runner including energy loss at the exit of blades = 32.852 kW

(iii) Power Lost in Mechanical Friction and Windage

Power lost in mechanical friction and windage = Runner Power – Shaft Power (12)

Power lost in mechanical friction and windage = 842.10 – 800

Power lost in mechanical friction and windage = 42.1 kW (13)

(b) Efficiencies

(i) Nozzle Efficiency

$$\eta_n = \frac{\text{Head at the Nozzle Exit}}{\text{Head at the Nozzle Base i.e. Net Head}} = \frac{\text{Jet Power}}{\text{WP}} \quad (14)$$

$$\eta_n = \frac{182.02}{195} = \frac{874.952}{937.35}$$

$$\eta_n = 0.9343 = 93.43\% \quad (15)$$

(ii) Wheel Efficiency

$$\eta_w = \frac{\text{Runner Power}}{\text{Jet Power}} \quad (16)$$

$$\eta_w = \frac{842.10}{874.952}$$

$$\eta_w = 0.962453 = 96.2453\% \quad (17)$$

(iii) Hydraulic Efficiency

$$\eta_h = \eta_n \eta_w \quad (18)$$

$$\eta_h = 0.9343 \times 0.962453$$

$$\eta_h = 0.89922 = 89.922\% \quad (19)$$

(iv) Overall Efficiency

$$\eta_o = \eta_m \eta_h \quad (20)$$

$$\eta_o = 0.95 \times 0.89922$$

$$\eta_o = 0.85426 = 85.426\% \quad (21)$$

Overall efficiency may alternately be found from the following formula given below which will act as a check.

$$\eta_o = \frac{\text{SP}}{\text{WP}} = \frac{800}{937.35} = 0.8535 = 85.35\%$$

3.9 Reaction Turbine

The distinctive characteristic of a reaction turbine from an impulse turbine is that only a fraction of the total head available at the turbine inlet is converted into velocity head, before the runner is reached. Also, the working fluid completely fills the blade passages in the runner in reaction turbines instead of engaging one or two blades. The pressure or static head of the fluid changes gradually as it passes through the runner together with the change in its absolute kinetic energy due to impulse action between the fluid and the rotor. Therefore, the cross-sectional area of flow through the blade passages changes gradually to accommodate the variation in static pressure of the fluid. Generally, a reaction turbine is more suitable for low heads and high discharge. Reaction turbines are manufactured in a variety of configurations-radial flow, mixed flow and axial flow. Typical radial and mixed flow hydraulic turbines are called *Francis turbines*, named after James B. Francis (1815-92), an American engineer. The *Kaplan turbine*, named after an Austrian Engineer Viktor Kaplan (1876-1934), is an efficient axial flow hydraulic turbine with adjustable blades.

3.10 Radial Flow Reaction Turbines

Radial flow turbines are those turbines in which water flows in the radial direction. If the water flows radially from outwards to inwards (towards the axis of rotation) through the runner, the turbine is called inward flow reaction turbine. If the water flows from inwards to outwards, the turbine is known as outward flow reaction turbine.

In reaction turbines, the entire flow from headrace to tailrace takes place in a closed conduit system which is not open to the atmosphere at any point in its passage. At the entrance to the runner, only a part of potential energy is converted into kinetic energy and the remaining into pressure energy. Thus, the water at the inlet of the turbine possesses kinetic energy as well as pressure energy. A part of the pressure energy is converted into kinetic energy as the water flows through the runner. The runner converts both kinetic energy and pressure energy into mechanical energy. Such turbines are called reaction or pressure turbines.

3.10.1 Francis Turbine

It is an inward flow reaction turbine. Water from the casing enters the stationary guide blades or vanes which direct the water to enter the runner consisting of moving vanes. The water flows over the moving vanes in the inward radial direction and is discharged at the inner diameter of the runner. The outer diameter of the runner is the inlet and the inner diameter is the outlet.

Francis turbines are very versatile. The single unit may develop power as high as 750 MW. Initially, it was radial inward flow type flow with slow runner ($N_s \approx 60$), but now they are of the mixed flow type with radial entry and axial exit. Francis turbines are widely used for medium heads in the range of 40 m to 600 m. The discharge handled by a Francis turbine is relatively large. Present day Francis turbines have a very high overall efficiency in the range of 90–95%.

(a) Volute Casing

Casing and runner are always full of water in reaction turbines as discussed earlier. Water from the penstock enters a spiral or scroll casing which surrounds the runner, as shown in Figure 3.13 (a). The purpose of the spiral casing is to distribute water uniformly along the outer circumference of the guide blade ring (or stay vane ring if it exists), at a constant velocity. To obtain this uniform velocity, the cross-section of the spiral casing diminishes uniformly along the circumference, being maximum at the junction with the penstock and minimum at the other end. The shape of the casing for a vertical shaft turbine is such that the plan view looks like a snail shaped spiral and hence the name. This casing is also called *scroll casing*. A properly designed spiral casing produces uniform pressure all along the circumference and the flow approaches all the guide vanes at constant velocity. The radius of the spiral curve is given by,

$$r_s = r_i + \frac{\theta}{2\pi} D_p \tag{3.92}$$

where,

r_s = is the radius of the spiral casing,

r_i = is the outer radius of the stay vane ring, and

D_p = is the inside diameter of the penstock.

A spiral with $\theta = 2\pi$ is called *full spiral* and if $\theta \leq 2\pi$, it is a *half spiral* or *semi spiral*. For heads < 30 m in medium, and in large-sized units, it is economical to use a semi-spiral casing made of concrete. In a typical semi-spiral casing the approach pipe is a rectangular conduit of concrete with the aspect ratio of approximately unity. The baffle vanes are streamline shaped and are meant to divert the flow into the stay vanes. This type of semi-spiral casing is advantageously used in medium head hydroelectric plants situated next to a dam.

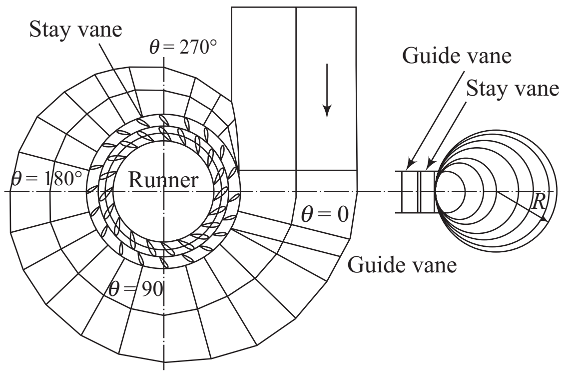


Figure 3.13 *Spiral or Scroll Casing of a Reaction Turbine*

The scroll casings are normally of welded steel-plate construction. Use of combination of cast steel and welded steel plate construction is also adopted in some instances. The scroll casing is provided with draining, air vent outlets and taps for pressure measurement. Further, an inspection port is often provided in the scroll casing. In large units, the scroll casing is completely embedded in reinforced concrete.

The inlet valve, called the *main inlet valve* (MIV) is provided at the junction of the scroll case and the penstock. MIV is used for normal operations and for isolating the unit during maintenance or for use in case of emergency. Usually, butterfly valves are used for heads ≤200 m and spherical valves for heads > 200 m.

(b) Stay Ring

The *stay ring* consists of upper and lower ring plates to which *stay vanes* are welded. Thus, the stay vanes are fixed in position and are non-adjustable. They are of streamline shape in order to minimize the losses in the flow. In addition to steering the flow to the *guide vanes*, stay vanes also support the axial forces arising mainly due to load of the generator mountings on the scroll case assembly. This aspect is particularly important in large vertical shaft installations. The number of stay vanes is the same as the number of adjustable guide vanes. The stay vanes are optional in nature and are mostly used in large installations.

(c) Adjustable Guide Blades or Wicket Gates

The *adjustable guide vanes or blades*, also known as *wicket gates*, are essential components of a Francis turbine unlike the stay vanes that are optional in nature. Adjustable guide blades are provided upstream of the runner assembly and they are usually even in numbers. Typically a set of 8 to 20 blades is used in a turbine. Each of the guide vanes is mounted on a pinion and the entire set is mounted between two rings to form a wheel known as *guide wheel*. The vanes are of streamline shape and are positioned at equal angular spacing. The entire set of blades in the guide wheel moves as one unit through external linkages connected to the regulating ring so that all the flow passages, which are formed between two adjacent blades, have the same value. The regulating ring thus controls the flow passage area to the turbine. The motion of the regulating ring, in turn, is controlled by a servomotor of the governor system through linkages. The passage between the vanes can be adjusted from a maximum value all the way down to zero opening as shown in Figure 3.14.

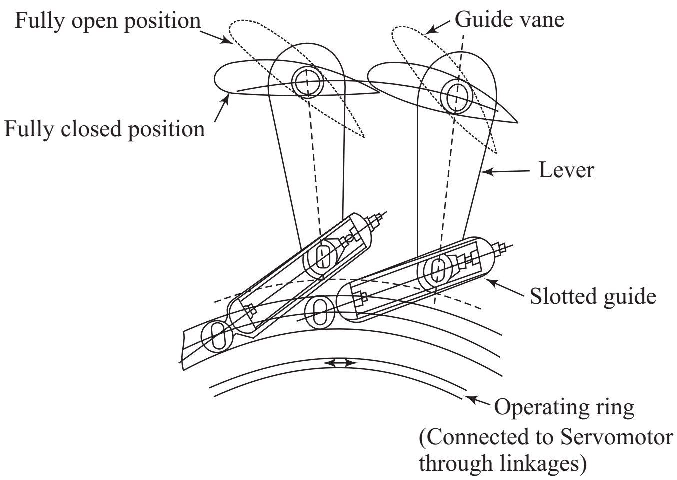


Figure 3.14 *Schematic Diagram of Adjustable Guide Blades*

The purpose of the guide blades is to direct the flow from the scroll casing into the runner with a desired level of velocity and also in a tangential direction to the tip of the runner blade. This would ensure *shock-less entry* to the runner. Therefore, guide vanes allow the water to strike the vanes fixed on the runner without shock at the inlet. Hence, the guide vanes impart a tangential velocity or angular momentum to the water before entering the runner. Further, the ability to control the width of the passage between the guide vanes helps in regulating the total discharge into the runner. This also includes complete shutting off of the flow to the runner. Thus, the guide blades form a part of the governing mechanism and are also used for starting and stopping the turbine. The guide blades are usually made of cast steel. Guide blades require regular inspection and maintenance since they are subjected to erosion due to action of sediment in the water.

(d) Runner

It is a circular wheel on which a number of blades or vanes are arranged radially round an axis of rotation. The blades are held in position by a crown (hub) plate on the top and a shroud (band) at the bottom. The crown or hub is fixed to the main shaft. The passage formed between the crown, the shroud and the adjacent blades carry the flow from the guide vanes passages. The flow in the passage is radial at the inlet but it will be in axial direction at the exit. Such a fluid passage is called *mixed flow* and the runner is called mixed flow Francis turbine runner. Pure radial flow runners are seldom used now a days. The flow from the exit of the runner enters the draft tube. The surface of the vanes is made very smooth. The radial curved vanes are so shaped that the water enters or leaves the runner without shock. The runner blades are made of stainless steel. Bronze blades are also used though not common. The runner blades are fixed in position and are usually odd in number. Typically, the numbers of blades lie in the range of 7-13. The crown and shroud are generally made of cast steel. Welded construction is adopted to join the blades, crown and shroud ring. Thus, the runner blades are fixed in position and they cannot be adjusted.

(e) Draft Tube

Water exiting the runner of a reaction turbine cannot be discharged directly to tailrace as the pressure at the exit of the runner is less than atmospheric (vacuum). After flowing past the runner, water is passed through a draft tube which is a closed flaring conduit either straight or elbow type. Thus, a draft tube is a divergent tube or pipe which is used for discharging water from the turbine exit to the tailrace. Draft tube increases the pressure and decreases the velocity of water before falling into the tailrace. Therefore, the draft tube is a conduit attachment to the exit of the turbine to achieve the following advantages:

1. To enable the turbine to be set up at a height higher than the tailrace,
2. To utilise the major part of the kinetic energy of water exiting the runner.

A draft tube is considered as an integral part of the reaction turbine.

(f) Governor

A hydro turbine is directly coupled to the electric generator on which the load changes throughout the day. Consequently, the turbine speed fluctuates. The generator is always required to run at a constant speed irrespective of the variations in the load in order to have constant frequency of electric power produced. Therefore, the speed of the turbine is required to be maintained equal to the *synchronous speed* at all loads. Changes in the load necessitate speed adjustments. The required adjustments in the speed are obtained by changing the flow rate of water over the turbine blades. The turbine governor is a system which is used for keeping the speed of the turbine to be constant at all loads by regulating the flow rate of water through it. The governor is an essential and integral component of the present day turbine unit.

3.11 Analysis of Francis Turbine

3.11.1 Hydraulic Efficiency

The *hydraulic efficiency* of the turbine is given by,

$$\eta_h = \frac{\text{Head extracted by the rotor or work head}}{\text{Net head available to the rotor}} = \frac{H_e}{H} = \frac{H - h_i}{H}$$

where, h_i represents the total head loss in the rotor. The net head of a reaction turbine can be found from Eq. (3.7). Substituting the value of H_e from Eq. (3.15), hydraulic efficiency is,

$$\eta_h = \frac{H_e}{H} = \frac{C_{w1}C_{b1} - C_{w2}C_{b2}}{gH} \tag{3.93}$$

where,

C_{w1} = Velocity of whirl or swirl at the inlet,

C_{w2} = Velocity of whirl or swirl at the outlet,

C_{b1} = Tangential velocity of the wheel at the inlet, and

C_{b2} = Tangential velocity of the wheel at the outlet.

If D_1 and D_2 are the inlet and outlet diameters of the runner, respectively, and N is the speed of the turbine in rpm, then

$$C_{b1} = \frac{\pi D_1 N}{60} \tag{3.94}$$

$$C_{b2} = \frac{\pi D_2 N}{60} \tag{3.95}$$

3.11.2 Power Developed

The velocity diagrams at inlet and outlet of the moving radial vanes are shown in Figure 3.3 (b). The energy extracted from the fluid i.e. power transferred to the runner by the fluid is given by Eq. (3.14) as follows:

$$P = \rho_w Q[C_{w1}C_{b1} - C_{w2}C_{b2}] = \rho_w a C_1[C_{w1}C_{b1} - C_{w2}C_{b2}]$$

Since C_{b1} and C_{b2} would be positive always, it is clear that power developed would be maximum when C_{w2} is negative. A negative value of C_{w2} means that whirl or swirl at the outlet of the runner is in the direction opposite to the direction of rotation of the runner (opposite to the direction of tangential velocity of the runner). This situation is called *reverse swirl*. It has been found experimentally that a small value of negative swirl may be advantageous in increasing the Euler head or power developed. Presence of large negative swirl in the flow at exit causes the efficiency to drop drastically. This is because of the fact that large reverse swirl causes a large amount of kinetic energy in exiting water which indicates reduction in the utilisation of the available head. To eliminate this problem, it has been the practice to have no reverse swirl in a turbine. Therefore, in the limit, $C_{w2} = 0$ in practice. Since, $C_{w2} = C_2 \cos \alpha_2$, therefore, C_{w2} will be zero when $\cos \alpha_2 = 0 \Rightarrow \alpha_2 = 90°$. This means that when $\alpha_2 = 90°$, the hydraulic efficiency is maximum from the practical considerations. Hence, in practice, the reaction turbine is designed in such a way so that the discharge is axial at the outlet giving $C_2 = C_{f2}$.

3.11.3 Velocity Triangles

Figure 3.15 shows the velocity triangles of a Francis turbine for which discharge is axial at the outlet, i.e. $\alpha_2 = 90° \Rightarrow C_{w2} = 0$. Hence, the energy utilised by the blades of the rotor per unit weight of water, i.e. Euler head is given by,

$$H_e = \frac{1}{g}(C_{w1}C_{b1}) \tag{3.96}$$

$$\eta_h = \frac{H_e}{H} = \frac{C_{w1}C_{b1}}{gH} \tag{3.97}$$

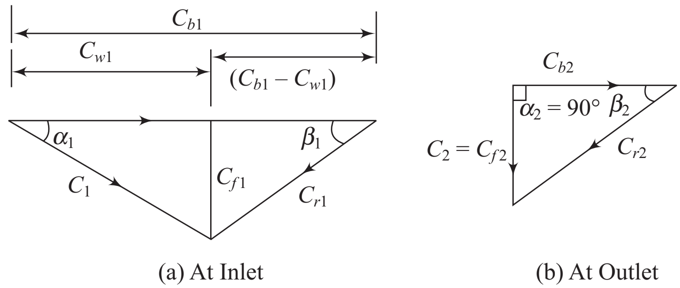


Figure 3.15 *Velocity Triangles of a Francis Turbine at Inlet and Outlet*

At the exit, the blades have to be so designed to achieve this condition of zero swirl or whirl velocity for which $\alpha_2 = 90° \therefore C_2 = C_{f2}$. It can be observed from inlet velocity triangle that $\beta_1 < 90°$. The blade angle at the inlet can also be obtuse ($\beta_1 > 90°$) or can be a right angle ($\beta_1 = 90°$). But the blade angle at the outlet $\beta_2 < 90°$ always as α_2 is always 90°.

3.11.4 Parameters Affecting Hydraulic Efficiency

Consider an ideal case of zero friction. Further, assume that the velocity of flow is constant, i.e. $C_{f2} = C_{f1}$. In a rotor, out of the total energy transferred to it by the water, only the kinetic energy of the water at the exit of the rotor is a loss to the turbine and rest of the energy is expended in doing work on the shaft.

$\therefore$ Unutilized energy i.e. loss of kinetic energy per unit weight of water $= \dfrac{C_2^2}{2g} = \dfrac{C_{f2}^2}{2g}$

Hence, when friction and other losses are neglected, and $\alpha_2 = 90°$, $C_{f2} = C_{f1}$, then net head,

$$H = H_e + \frac{C_2^2}{2g} = H_e + \frac{C_{f2}^2}{2g} = H_e + \frac{C_{f1}^2}{2g} \tag{3.98}$$

From the velocity triangle at the inlet of the turbine, Figure 3.15,

$$C_{w1} = C_{f1} \cot \alpha_1 \text{ and } C_{b1} = C_{f1} (\cot \beta_1 + \cot \alpha_1)$$

Substituting the values of C_{w1} and C_{b1} from above equations into Eq. (3.96), we get the energy utilised by the blades of the rotor per unit weight of water i.e. Euler head as,

$$H_e = \frac{1}{g}(C_{w1}C_{b1}) = \frac{1}{g}C_{f1}^2 \cot \alpha_1 (\cot \beta_1 + \cot \alpha_1) \tag{3.99}$$

Therefore, hydraulic efficiency,

$$\eta_h = \frac{H_e}{H} = \frac{gH_e}{gH_e + (C_{f1}^2/2)} = \frac{C_{f1}^2 \cot \alpha_1 (\cot \beta_1 + \cot \alpha_1)}{C_{f1}^2 \cot \alpha_1 (\cot \beta_1 + \cot \alpha_1) + (C_{f1}^2/2)}$$

$$\eta_h = \frac{2 \cot \alpha_1 (\cot \alpha_1 + \cot \beta_1)}{1 + 2 \cot \alpha_1 (\cot \alpha_1 + \cot \beta_1)} = 1 - \frac{1}{1 + 2 \cot \alpha_1 (\cot \alpha_1 + \cot \beta_1)} \tag{3.100}$$

It is quite obvious from Eq. (3.100) that when friction is neglected, the hydraulic efficiency is a function of guide blade angle and blade angle at the inlet and thus, underlines the importance of efficient design of blade geometries.

If $C_{f2} = kC_{f1}$, then hydraulic efficiency is,

$$\eta_h = 1 - \frac{2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)}{k^2 + 2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)} \tag{3.101}$$

3.11.5 Degree of Reaction

Earlier Francis turbine was of radial inward flow reaction turbine. The present day Francis turbine is a mixed flow reaction turbine in which water enters radially in the turbine and exits axially at the outlet. Therefore, $\alpha_2 = 90° \Rightarrow C_{w2} = 0$, $C_2 = C_{f1}$. Also, there is not much change in the velocity of flow. Assuming, $C_{f1} = C_{f2}$ therefore, degree of reaction for radial entry and axial discharge is obtained by Eq. (3.19) as,

$$R = \frac{H_e - \dfrac{(C_1^2 - C_2^2)}{2g}}{H_e} = 1 - \frac{(C_1^2 - C_{f2}^2)}{2gH_e} = 1 - \frac{(C_1^2 - C_{f1}^2)}{2gH_e}$$

From the velocity triangle at the inlet as shown in Figure 3.15,

$$C_1^2 - C_{f1}^2 = C_{w1}^2 = C_{f1}^2 \cot^2\alpha_1$$

Since $C_{w2} = 0$ for axial discharge, therefore the Euler head as obtained from Eq. (3.15) or (3.96) is,

$$H_e = \frac{1}{g}C_{w1}C_{b1} = \frac{1}{g}C_{f1}\cot\alpha_1 \times C_{f1}(\cot\alpha_1 + \cot\beta_1) = \frac{1}{g}C_{f1}^2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)$$

$\therefore$

$$R = 1 - \frac{C_{f1}^2\cot^2\alpha_1}{2C_{f1}^2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)}$$

$$R = 1 - \frac{\cot\alpha_1}{2(\cot\alpha_1 + \cot\beta_1)} \tag{3.102}$$

Equation (3.102) is an expression for degree of reaction of a Francis turbine in terms of geometry of blades at the inlet. It is obvious that degree of reaction is a function of α_1 and β_1 only. If $\beta_1 = 90°$ as in medium speed Francis turbine with radial entry and axial exit, $R = \frac{1}{2}$.

3.11.6 Discharge through Radial Flow Reaction Turbine

Flow ratio is defined as the ratio of the velocity of flow at the inlet to the spouting velocity given by $\sqrt{2gH}$.

$$\text{Flow ratio} = k_f = \frac{C_{f1}}{\sqrt{2gH}} \tag{3.103}$$

The discharge through a radial flow reaction turbine is given by,

$$Q = \pi D_1 B_1 C_{f1} = \pi D_2 B_2 C_{f2} \tag{3.104}$$

where,

D_1 = Diameter of runner at the inlet,
B_1 = Width of the runner at the inlet,
C_{f1} = Velocity of flow at the inlet.
D_2, B_2 and C_{f2} are the corresponding values at the outlet of the turbine.

If the thickness of the vanes is taken into consideration, then the flow area is given by $(\pi D_1 - zt)B_1$ at the inlet and by $(\pi D_2 - zt)B_2$ at the outlet of the turbine where z = is the number of the blades or vanes on the runner and t is the thickness of each blade. Therefore,

$$Q = (\pi D_1 - zt)B_1 C_{f1} = (\pi D_2 - zt)B_2 C_{f2} \tag{3.105}$$

3.11.7 Runaway Speed

Runaway speed is the speed attained by a turbine at full gate open position when generator is disconnected from the turbine and governor becomes inoperative. Runaway speed differs from manufacturer to manufacturer for a given type of turbine due to variations in design. The turbine manufacturer provides the rated runaway speed of a turbine. Therefore, all the rotating parts of a turbine must be designed to withstand the stresses due to centrifugal force occurring because of runaway speed. Generally, for a Francis turbine, the runaway speed is of the order of 175%. Table 3.4 gives the runaway speed for various variants of a Francis turbine:

TABLE 3.4 Runaway speed for various variants of a Francis turbine

S. No.	*Variant*	*Specific Speed* $\left[\frac{rpm\sqrt{kW}}{m^{5/4}}\right]$	*Runaway Speed*
1.	Slow Francis turbine	70-180	160-170% of design speed
2.	Fast Francis turbine	>200	180% of design speed

Following formulae are used to calculate runaway speed of reaction turbines which are applicable to Francis turbine and Kaplan turbine.

$$\frac{N_r}{N} = 0.63(N_s)^{1/5} \tag{3.106}$$

$$N_{r\max} = N_r\left(\frac{H_{\max}}{H_{\text{design}}}\right)^{1/2} \tag{3.107}$$

where,

N_r = Runaway speed at the head corresponding to best efficiency and full gate,
N = Rotational speed,
$N_{r\max}$ = Runaway speed at maximum head,
$H_{\max}$ = Maximum head,
H_{design} = Design head

EXAMPLE 3.6 A Francis turbine running at 200 rpm is required to produce 7.5 MW from the water flow rate of 28 m^3/s. The diameter of the runner at entrance is 2.5 m and the breadth of the blades at the inlet is 0.75 m. At what angle the wicket gates are to be set if the blade angle at the inlet is acute and the flow can be assumed to leave the runner radially?

Solution

Given:

$N = 200$ rpm, $P = 7.5$ MW $= 7500$ kW, $Q = 28\ m^3/s$, $D_1 = 2.5$ m, $B_1 = 0.75$ m, $\beta_1 < 90°$, $\alpha_2 = 90°$

The velocity diagrams at inlet and outlet are shown in Figure 3.16. The tangential or peripheral velocity at the inlet is,

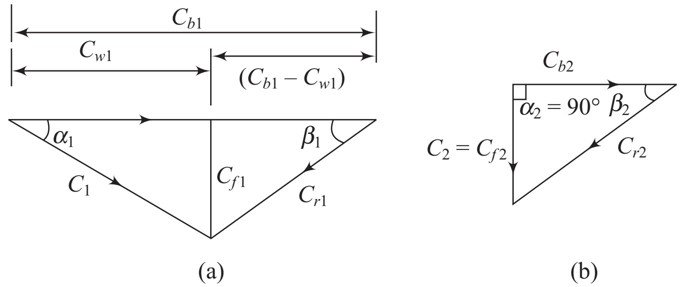


Figure 3.16 *Velocity Triangles of a Francis Turbine of Example 3.6 (a) Inlet, (b) Outlet*

$$C_{b1} = \frac{\pi D_1 N}{60} \quad (1)$$

$$C_{b1} = \frac{\pi \times 2.5 \times 200}{60}$$

$$C_{b1} = 26.18 \text{ m/s} \quad (2)$$

Discharge is given by,

$$Q = \pi D_1 B_1 C_{f1} \quad (3)$$

$$Q = \pi \times 2.5 \times 0.75 \times C_{f1} = 28$$

$$C_{f1} = 4.753 \text{ m/s} \quad (4)$$

Since the flow is radial at the outlet, $\alpha_2 = 90° \Rightarrow C_{w2} = 0$,

$$P = \rho_w Q(C_{w1}C_{b1} – C_{w2}C_{b2}) \quad (5)$$

$$7500 \times 1000 = 1000 \times 28 \times (C_{w1} \times 26.18 – 0)$$

$$C_{w1} = 10.23 \text{ m/s} \quad (6)$$

From the inlet velocity triangle,

$$\tan \alpha_1 = \frac{C_{f1}}{C_{w1}} = \frac{4.753}{10.23}$$

$$\alpha_1 = 24.92° \quad (7)$$

EXAMPLE 3.7 The peripheral and whirl velocity at the inlet of a Francis turbine are 32 m/s and 26 m/s respectively. The velocity of flow is 5 m/s and it remains constant during flow through the turbine. Assuning no velocity of whirl at exit and hydraulic efficiency of the turbine to be 90% determine (a) the blade angle at the inlet, (b) the guide vane angle at the inlet, and (c) the net head on the turbine.

Solution

Given:

$$C_{b1} = 32 \text{ m/s}, C_{w1} = 26 \text{ m/s}, C_{f1} = C_{f2} = 5 \text{ m/s}, C_{w2} = 0, \eta_h = 90\% = 0.9$$

The velocity triangle at the inlet is an acute angled triangle as $C_{w1} < C_{b1}$. The velocity triangles at the inlet and outlet are shown in the Figure 3.17.

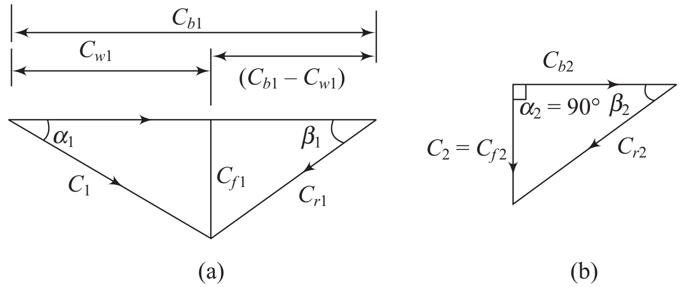


Figure 3.17 *Velocity Triangles of a Francis Turbine of Example 3.7 (a) Inlet, (b) Outlet*

(a) Inlet Blade Angle

$$\tan \beta_1 = \frac{C_{f1}}{C_{b1} - C_{w1}} = \frac{5}{32 - 26}$$

$$\beta_1 = 39.806° \tag{1}$$

(b) Inlet Guide Vane Angle

$$\tan \alpha_1 = \frac{C_{f1}}{C_{w1}} = \frac{5}{26}$$

$$\alpha_1 = 10.886° \tag{2}$$

(c) Net Head on the Turbine

$$\eta_h = \frac{C_{w1}C_{b1} - C_{w2}C_{b2}}{gH} \tag{3}$$

$$0.9 = \frac{26 \times 32}{9.81 \times H}$$

$$H = 94.235 \text{ m} \tag{4}$$

EXAMPLE 3.8 The peripheral velocity at the inlet of a Francis turbine is 16 m/s and the velocity of flow is constant at 3.5 m/s. The diameter of the runner at the entry is twice that at the exit and the discharge is radial. The degree of reaction is 0.6. Assuming no frictional losses, determine the blade angles at the entry and exit.

Solution

Given:

$$C_{b1} = 16 \text{ m/s}, C_{f1} = C_{f1} = C_{f2} = 3.5 \text{ m/s}, D_1 = 2\,D_2, \alpha_2 = 90°, R = 0.6$$

The velocity diagrams at the inlet and outlet of the turbine is shown in Figure 3.18. As no specific information about flow at inlet is given, so blade angle at the entry is assumed to be acute angle ($\beta_1 < 90°$) which is the usual practice. We know that degree of reaction is given by,

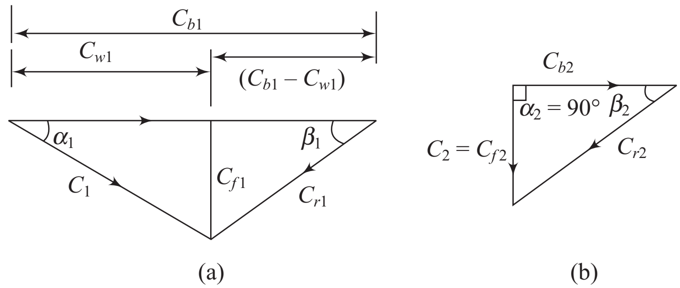


Figure 3.18 *Velocity Triangles of a Francis Turbine of Example 3.8 (a) At Inlet, (b) At Outlet*

$$R = \frac{H_e - \frac{1}{2g}(C_1^2 - C_2^2)}{H_e} \tag{1}$$

$$0.6 = \frac{H_e - \frac{1}{2g}(C_1^2 - C_2^2)}{H_e} \Rightarrow \frac{1}{2g}(C_1^2 - C_2^2) = 0.4H_e \tag{2}$$

For the radial discharge at the exit of the turbine, $\alpha_2 = 90° \Rightarrow C_{w2} = 0$

$$\because \quad H_e = \frac{1}{g}(C_{w1}C_{b1} - C_{w2}C_{b2}) \Rightarrow H_e = \frac{1}{g}C_{w1}C_{b1} \tag{3}$$

Substituting the value of H_e from Eq. (3) into Eq. (2), we get,

$$C_1^2 - C_2^2 = 0.8C_{w1}C_{b1} \tag{4}$$

From the outlet and inlet velocity triangles,

$$C_2 = C_{f2} = C_{f1}$$

$$C_1^2 = C_{f1}^2 + C_{w1}^2$$

$$\therefore \quad C_1^2 - C_2^2 = C_{f1}^2 + C_{w1}^2 - C_{f1}^2 \Rightarrow C_1^2 - C_2^2 = C_{w1}^2 \Rightarrow C_{w1}^2 = 0.8C_{w1}C_{b1}$$

$$C_{w1} = 0.8\ C_{b1} \tag{5}$$

From the inlet velocity triangle,

$$\tan\beta_1 = \frac{C_{f1}}{C_{b1} - C_{w1}} = \frac{C_{f1}}{C_{b1} - 0.8C_{b1}} = \frac{3.5}{16 - 0.8 \times 16} \quad \Rightarrow \quad \beta_1 = 47.564° \tag{6}$$

$$\because \quad C_b \propto D \Rightarrow \frac{C_{b2}}{C_{b1}} = \frac{D_2}{D_1} \Rightarrow C_{b2} = \frac{1}{2}C_{b1} = \frac{1}{2} \times 16$$

$$C_{b2} = 8 \text{ m/s} \tag{7}$$

From the outlet velocity triangle,

$$\tan\beta_2 = \frac{C_{f2}}{C_{b2}} \Rightarrow \tan\beta_2 = \frac{3.5}{8}$$

$$\beta_2 = 23.63° \tag{8}$$

EXAMPLE 3.9 The following data are for a Francis turbine which is to be designed for a power output of 6000 kW under a net head of 90 m while running at a speed of 700 rpm:

Ratio of the width of runner to external diameter of runner = 0.1

Ratio of the external diameter to the inner diameter of the runner = 2

Flow ratio = 0.28, Hydraulic Efficiency = 95%, Mechanical Efficiency = 85%,
Circumferential area occupied by the thickness of the vanes = 10%

Assuming the constant velocity of flow, calculate (a) the inlet guide vane angle, (b) the inlet runner blade angle, and (c) the runner blade angle at outlet.

Solution

Given:

$P = 6000$ kW, $H = 90$ m, $N = 700$ rpm, $B_1/D_1 = 0.1$, $D_1/D_2 = 2$, $k_f = 0.28$, $\eta_h = 0.95$, $\eta_m = 0.85$, Peripheral area occupied by the vane thickness = 10%, $C_{f1} = C_{f2}$

The velocity of flow is given by,

$$C_{f1} = C_{f2} = k_f\sqrt{2gH} \tag{1}$$

$$C_{f1} = C_{f2} = 0.28\sqrt{2 \times 9.81 \times 90}$$

$$C_{f1} = C_{f2} = 11.766 \text{ m/s} \tag{2}$$

We know that overall efficiency is given by,

$$\eta_o = \eta_m \eta_h \Rightarrow \eta_o = 0.85 \times 0.95$$

$$\eta_o = 0.8075 \tag{3}$$

$$P = \eta_o \rho_w g Q H \tag{4}$$

$$6000 \times 1000 = 0.8075 \times 1000 \times 9.81 \times Q \times 90$$

$$Q = 8.416 \text{ m}^3\text{/s} \tag{5}$$

Since the circumferential area occupied by thickness of the vanes is 10%, therefore the flow area would be $\left(1 - \frac{10}{100}\right) = 0.9$ times the circumferential area.

$\therefore$
$$Q = 0.9\,\pi D_1 B_1 C_{f1} \Rightarrow 8.416 = 0.9\pi \times D_1 \times 0.1 \times D_1 \times 11.766$$

$$D_1 = 1.60 \text{ m} \tag{6}$$

$\therefore$
$$B_1 = 0.1\,D_1 \Rightarrow B_1 = 0.16 \text{ m} \tag{7}$$

$\because$
$$\frac{D_1}{D_2} = 2$$

Therefore, diameter at the outlet of the runner,

$$D_2 = \frac{D_1}{2} = \frac{1.6}{2}$$

$$D_2 = 0.8 \text{ m} \tag{8}$$

Peripheral velocity at the inlet of turbine is given by,

$$C_{b1} = \frac{\pi D_1 N}{60} \quad (9)$$

$$C_{b1} = \frac{\pi \times 1.6 \times 700}{60}$$

$$C_{b1} = 58.64 \text{ m/s} \quad (10)$$

$$C_{b2} = \frac{\pi D_2 N}{60} \Rightarrow C_{b2} = \frac{\pi \times 0.8 \times 700}{60}$$

$$C_{b2} = 29.32 \text{ m/s} \quad (11)$$

Assuming the radial discharge at the outlet, therefore $\alpha_2 = 90° \Rightarrow C_{w2} = 0$. The hydraulic efficiency is given by,

$$\eta_h = \frac{(C_{w1}C_{b1} - C_{w2}C_{b2})}{gH} \quad (12)$$

$$0.95 = \frac{C_{w1} \times 58.64 - 0}{9.81 \times 90}$$

$$C_{w1} = 14.30 \text{ m/s} \quad (13)$$

(a) Guide Vane Angle at Inlet

From the velocity triangle at the inlet as shown in Figure 3.19,

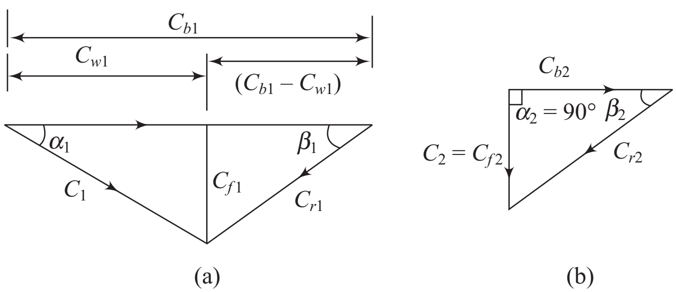


Figure 3.19 *Velocity Triangles of a Francis Turbine of Example 3.9 (a) At Inlet, (b) At Outlet*

$$\tan \alpha_1 = \frac{C_{f1}}{C_{w1}} \Rightarrow \tan \alpha_1 = \frac{11.766}{14.30}$$

$$\alpha_1 = 39.45° \quad (14)$$

(b) Blade Angle at Inlet

$\because C_{w1} < C_{b1}$, therefore angle β_1 is acute and the velocity triangle at the inlet will be as shown in Figure 3.19.

$$\tan \beta_1 = \frac{C_{f1}}{C_{b1} - C_{w1}} \Rightarrow \tan \beta_1 = \frac{11.766}{58.64 - 14.30}$$

$$\beta_1 = 14.86° \quad (15)$$

(c) Blade Angle at Outlet

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2}} = \frac{C_{f1}}{C_{b2}} = \frac{11.766}{29.32}$$

$$\beta_2 = 21.8654° \quad (16)$$

EXAMPLE 3.10 A Francis turbine is required to develop 7.5 MW while operating at a speed of 500 rpm. The runner diameter is 1.25 m and the water flows at a rate of 7.72 m^3/s. The velocity of flow is 9 m/s and the difference in piezometric heads between the entrance and exit of the runner is 60 m. Assuming the discharge to be radial at exit of the runner, calculate (a) the absolute velocity and guide vane angle at the inlet, (b) runner blade angle at the inlet, and (c) the loss of head in the runner.

Solution

Given:

$P = 7.5$ MW $= 7500$ kW $= 7.5 \times 10^6$ W, $N = 500$ rpm, $D_1 = 1.25$ m, $Q = 7.72$ m^3/s, $C_{f1} = 9$ m/s, Piezometric head difference between inlet and exit = 60 m

The peripheral velocity at inlet is,

$$C_{b1} = \frac{\pi D_1 N}{60} \quad (1)$$

$$C_{b1} = \frac{\pi \times 1.25 \times 500}{60}$$

$$C_{b1} = 32.725 \text{ m/s} \quad (2)$$

We know that power produced is given by,

$$P = \rho_w Q(C_{w1}C_{b1} - C_{w2}C_{b2}) \quad (3)$$

Since discharge is radial, $\alpha_2 = 90° \Rightarrow C_{w2} = 0$. Therefore, power produced is,

$$P = \rho_w Q C_{w1} C_{b1} \Rightarrow 7.5 \times 10^6 = 1000 \times 7.72 \times C_{w1} \times 32.725$$

$$C_{w1} = 29.687 \text{ m/s} \quad (4)$$

Velocity diagrams at the inlet and the outlet are shown in Figure 3.20. Since, $C_{w1} < C_{b1}$, therefore the angle β_1 is acute.

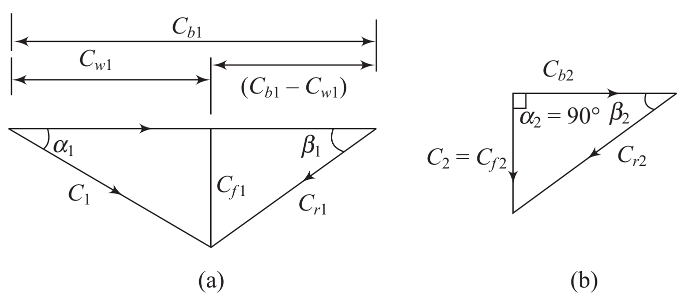


Figure 3.20 *Velocity Triangles of a Francis Turbine of Example 3.10 (a) Inlet, (b) Outlet*

(a) Absolute Velocity and Guide Vane Angle at the Inlet

From the velocity triangle at the inlet, absolute velocity at the entry of the runner is,

$$C_1 = \sqrt{C_{f1}^2 + C_{w1}^2} \Rightarrow C_1 = \sqrt{9^2 + 29.687^2}$$

$$C_1 = 31.016 \text{ m/s} \tag{5}$$

From the velocity triangle at the inlet,

$$\tan \alpha_1 = \frac{C_{f1}}{C_{w1}} \Rightarrow \tan \alpha_1 = \frac{9}{29.687}$$

$$\alpha_1 = 16.865° \tag{6}$$

(b) Blade Angle at the Inlet

$$\tan \beta_1 = \frac{C_{f1}}{C_{b1} - C_{w1}} \Rightarrow \tan \beta_1 = \frac{9}{32.725 - 29.687}$$

$$\beta_1 = 71.35° \tag{7}$$

(c) Loss of Head in the Runner

Ideal power developed by the runner due to the reaction of water flow is,

$$P = \rho_w g Q H_e \tag{8}$$

where H_e is the Euler head or theoretical head extracted by the runner.

$$7.5 \times 10^6 = 1000 \times 9.81 \times 7.72 \times H_e$$

$$H_e = 99.032 \text{ m} \tag{9}$$

Total energy head at the entrance of the runner is,

$$H_1 = \frac{p_1}{\rho_w g} + Z_1 + \frac{C_1^2}{2g}$$

Total energy head at the exit of the runner is,

$$H_2 = \frac{p_2}{\rho_w g} + Z_2 + \frac{C_2^2}{2g}$$

Applying energy equation between the entrance and exit of the runner,

$$H_1 = H_e + H_2 + h_i \tag{10}$$

where, h_i is the loss of head in the runner. Therefore,

$$\frac{p_1}{\rho_w g} + Z_1 + \frac{C_1^2}{2g} = H_e + \frac{p_2}{\rho_w g} + Z_2 + \frac{C_2^2}{2g} + h_i \tag{11}$$

$$\therefore \quad h_i = \left[\frac{p_1}{\rho_w g} + Z_1\right] - \left[\frac{p_2}{\rho_w g} + Z_2\right] + \left[\frac{C_1^2 - C_2^2}{2g}\right] - H_e$$

We know that difference in piezometric head at the inlet and outlet of the runner is given as 60 m which is,

$$\left[\frac{p_1}{\rho_w g} + Z_1\right] - \left[\frac{p_2}{\rho_w g} + Z_2\right] = 60 \text{ m}$$

$\because \quad C_2 = C_{f2} = C_{f1} = 9 \text{ m/s}$

$$\therefore \quad h_i = 60 + \frac{31.016^2 - 9^2}{2 \times 9.81} - 99.032$$

$$h_i = 5.871 \text{ m} \tag{12}$$

3.12 Axial Flow Reaction Turbines

If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine. If the head at the inlet of the turbine is the sum of pressure energy and kinetic energy, and during the flow of the water through the runner, a part of pressure energy is converted into kinetic energy, the turbine is known as a reaction turbine. The shaft of the turbine is vertical for axial flow reaction turbine. The lower end of the shaft is made larger which is known as *hub* or *boss*. The vanes are fixed on the hub and hence the hub acts as a runner for axial flow reaction turbine. Axial flow reaction turbines are of two types: (i) Propeller turbine, and (ii) Kaplan turbine.

3.12.1 Propeller Turbine

The propeller turbine is a reaction turbine used for low heads (4–80 m) and high specific speeds (300-1000). It is an axial flow device providing large flow area utilizing a large volume flow of water with low flow velocity. It consists of an axial flow runner usually with three to eight blades of airfoil shape as shown in Figure 3.21.

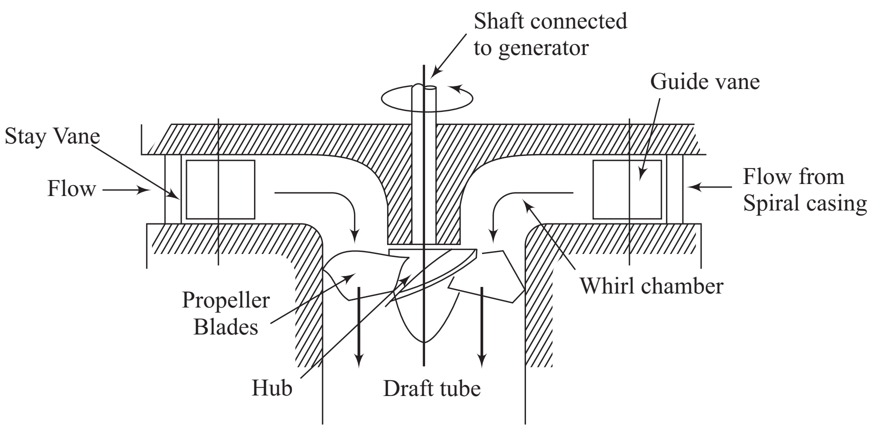


Figure 3.21 *Schematic Diagram of a Propeller Turbine*

The spiral casing and guide blades are similar to those in Francis turbine. The runner blades are fixed and non-adjustable as in Francis turbine. For a given discharge and head, propeller turbines have smaller size compared to an equivalent Francis turbine. The propeller turbines are generally mounted on a vertical axis. The variants of the propeller turbine are *tubular turbines* and *bulb turbines* which have horizontal or inclined axis and these turbines belong to the family of axial flow turbines.

3.12.2 Kaplan Turbine

A *Kaplan turbine* is a special type of propeller turbine with additional feature of mechanism to regulate the *pitch* of the blades i.e. orientation of the blades to the flow direction. A Kaplan turbine is shown in Figure 3.22 in which the runner blades are pivoted on the hub so that their inclination may be adjusted during operation in response to variations in load. The blades are adjusted automatically about pivots with the help of a governor servo-mechanism. The efficiency of a reaction turbine depends on the inlet blade angle. For maximum efficiency, it is necessary that the flow enters axially with zero incident angles. It is not possible to vary the inlet blade angle for varying demands of power i.e. load in fixed blade propeller runners. Such turbines are designed for maximum efficiency only for a particular load. At all other loads, their efficiency is less than this. In the Kaplan turbine, because of the arrangement for automatic variation of inlet blade angle with variation in load, the turbine can be run at maximum efficiency at all loads.

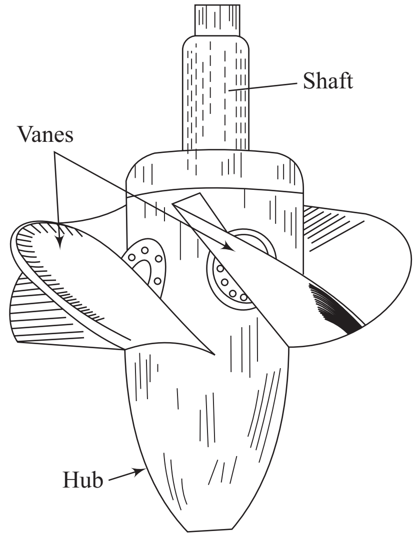


Figure 3.22 *Schematic Diagram of a Kaplan Turbine Runner*

Figure 3.23 shows a Kaplan turbine consisting of scroll casing, adjustable guide blades, whirl chamber and runner with adjustable blades. The space between the guide vanes outlet and inlet of the runner is known as *whirl chamber*. In this chamber, the flow changes from radial to axial direction. The lower portion of this chamber that is in the immediate vicinity of the blades is known as *runner chamber.* The draft tube is of inclined type. The water from the penstock enters the scroll casing and then moves to the guide vanes. From the guide vanes, the water turns through 90° and flows axially through the runner.

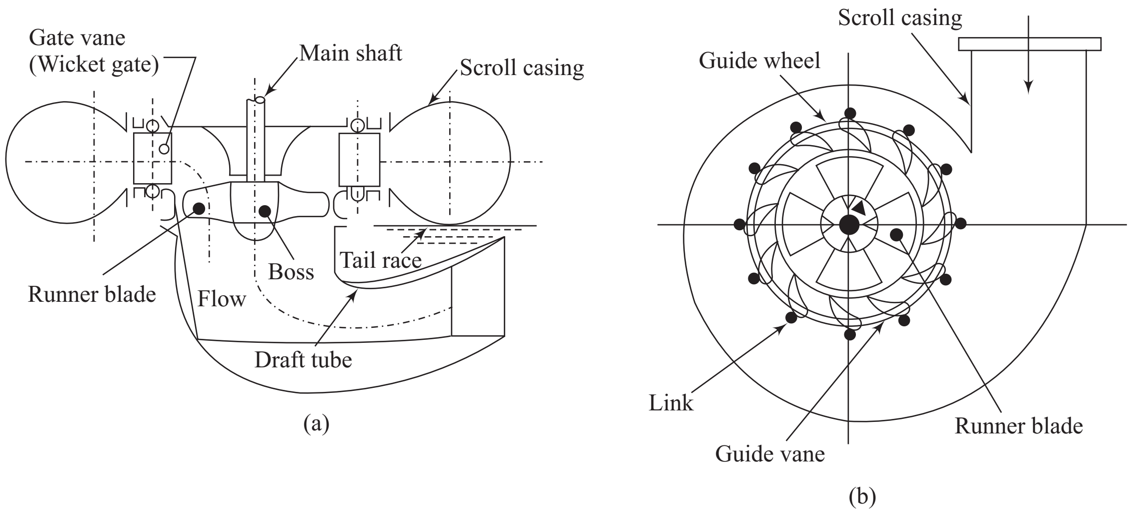


Figure 3.23 *Schematic Diagram of a Kaplan Turbine*

3.13 Analysis of Propeller and Kaplan Turbine

3.13.1 Velocity Triangles

The velocity triangles of the runner blade of a propeller turbine with fixed blades or a Kaplan turbine with variable pitch blades near the hub and near the edge of the tip are shown in Figure 3.24.

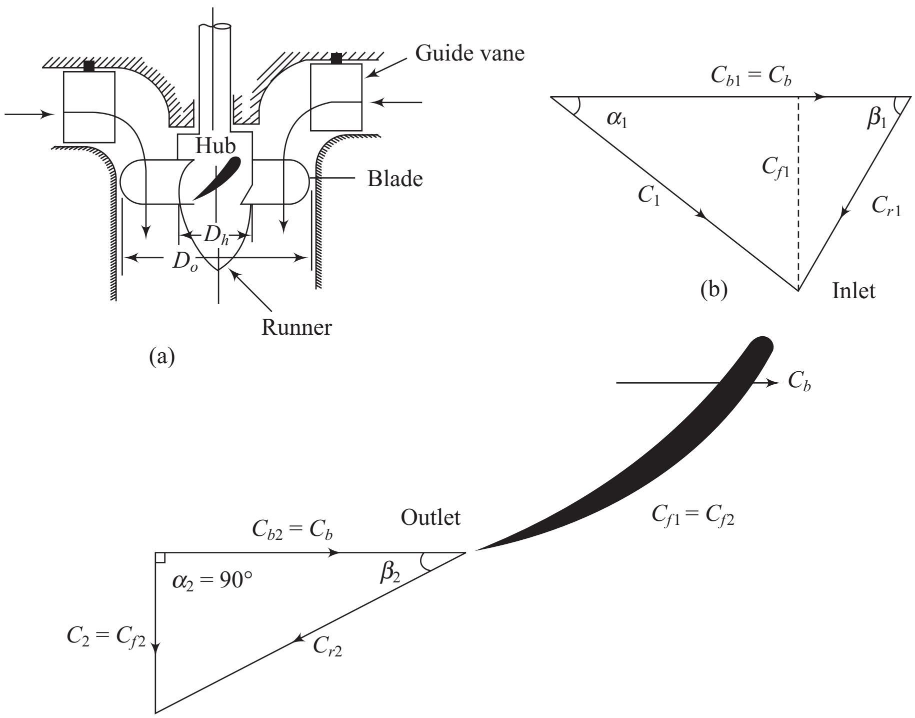


Figure 3.24 *Velocity Triangles at Inlet and Outlet of a Propeller or Kaplan Turbine*

Since the peripheral velocity varies from root to tip, so the velocity triangles are different in symmetry. The expressions for work done and efficiencies are same as those for the Francis turbine.

3.13.2 Discharge

The peripheral velocity or blade velocity and velocity of flow at the inlet and the outlet are equal. Therefore,

$$C_{b1} = C_{b2} = \frac{\pi D_o N}{60} \tag{3.108}$$

$$C_{f1} = C_{f2} = C_f \tag{3.109}$$

where, D_o is the outer diameter of runner.

Also, the area of flow at the inlet and outlet of the turbine is same and is given by,

$$A_f = \frac{\pi}{4}(D_o^2 - D_h^2)K_1 \tag{3.110}$$

where D_h is diameter of the hub or inner diameter of the runner and K_1 is the net area factor which is found after deducting the area occupied by the blades in the cross-section. It is usual to take $K_1 = 1.0$.

The discharge through the runner is given by,

$$Q = \frac{\pi}{4}(D_o^2 - D_h^2)K_1 C_f \tag{3.111}$$

The flow is assumed to leave the runner axially without any whirl component. Therefore, at the outlet, α_2 = 90° and $C_2 = C_{f2}$ as used in the case of a Francis turbine. It is also assumed that the flow entering the whirl chamber from the guide vanes creates a free vortex in the whirl and runner chambers. In a free vortex, the velocity is inversely proportional to the radius [Refer Section 1.12 (a)]. Therefore,

$$(C_{w1})_o r_o = (C_{w1})_m r_m = (C_{w1})_h r_h \tag{3.112}$$

where, $r_o = D_o/2$ is the radial distance of the tip of the blade from the axis, $r_h = D_h/2$ is the radial distance of the hub and $r_m = (D_0 + D_h)/2$ is the mean radius or the radial distance of the midpoint of the blades.

3.13.3 Euler Head or Specific Work

We know that Euler head is given by,

$$H_e = \frac{C_{w1}C_{b1} - C_{w2}C_{b2}}{g}$$

Since, $\alpha_2 = 90° \Rightarrow C_{w2} = 0$. Therefore, Euler head for axial flow turbines is given by,

$$H_e = \frac{C_{w1}C_{b1}}{g} \tag{3.113}$$

3.13.4 Hydraulic Efficiency

Further, the hydraulic efficiency is given by,

$$\eta_h = \frac{H_e}{H} = \frac{C_{w1}C_{b1}}{gH} \tag{3.114}$$

It should be noted that values of α_1, β_1 and β_2 change all along the blade length. The velocity triangle at the inlet at any radial location on the blade is an acute-angled triangle, whereas the outlet velocity triangle is a right angled triangle. Both are constructed in the same way as in Francis turbine. The nature of variation of α_1, β_1 and β_2 could be shown in a general way as in Figure 3.25.

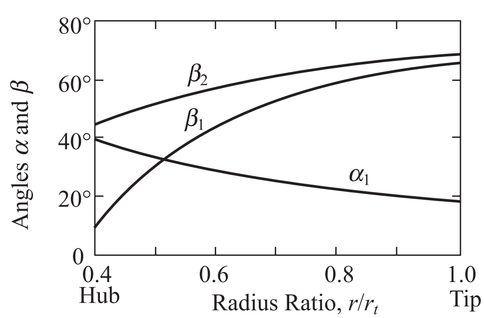


Figure 3.25 *Nature of Variation of Blade Angles in a Kaplan Turbine*

Following observations are noteworthy:

- The guide blade angle α_1 is minimum at the blade tip and increases along the radius to a maximum at the hub.
- The blade angle at the inlet, β_1, is maximum at the blade tip and decreases along the radius to a minimum at the hub.
- The blade angle at the outlet, β_2, is maximum at the blade tip and decreases along the radius to a minimum at the hub. Generally, $\beta_2 > \beta_1$ at all radial locations.

The exit guide blade angle, α_1, varies from 45° to 70°, the inlet blade angle β_1 varies from 20° to 60° and specific speed varies from 260 to 860. The hydraulic efficiency is about 91%. The degree of reaction is less than 50% and it increases gradually from the hub to the tip.

3.13.5 Runaway Speed

Equations (3.106) and (3.107) are also applicable to calculate the *runaway speed* of propeller/Kaplan turbines. The runaway speed of a propeller turbine is in the range of 2.5 to 3.0 times the design speed.

$$\frac{N_r}{N} = 0.63(N_s)^{1/5} \tag{3.115}$$

$$N_{r\max} = N_r\left(\frac{H_{\max}}{H_{\text{design}}}\right)^{1/2} \tag{3.116}$$

The Kaplan turbine however, has a theoretical runaway speed approaching infinity at the closed or flat-blade position due to its adjustable blade provision. In practice, the friction and windage of the connected generator normally will limit the runaway speed to 275% of the design speed. The specifications usually include a requirement for an adjustable stop on the rotation of blades that will limit the runaway speed to 275% to avoid excessive stresses in the generator.

EXAMPLE 3.11 The diameters of the runner and hub of a Kaplan turbine are 4.2 m and 1.2 m respectively. The turbine works under a net head of 20 m and has a speed of rotation of 150 rpm. The blade angle of the extreme edge of the runner at the inlet is 18° and flow ratio is 0.45. If the discharge at the exit of the turbine is axial, find (a) the hydraulic efficiency, (b) guide vane angle at the inlet, and (c) blade angle of the extreme edge at the outlet of the runner.

Solution

Given:

$$D_o = 4.2 \text{ m}, D_h = 1.2 \text{ m}, H = 20 \text{ m}, N = 150 \text{ rpm}, \beta_1 = 18.0°, k_f = 0.45$$

Peripheral velocities of the blades at the inlet and outlet of a Kaplan turbine are equal and are given by,

$$C_{b1} = C_{b2} = \frac{\pi D_o N}{60} \tag{1}$$

$$C_{b1} = C_{b2} = \frac{\pi \times 4.2 \times 150}{60}$$

$$C_{b1} = C_{b2} = 32.987 \text{ m/s} \tag{2}$$

The velocities of flow at the inlet and outlet of a Kaplan turbine are same and are given by,

$$C_{f1} = C_{f2} = k_f\sqrt{2gH} \tag{3}$$

$$C_{f1} = C_{f2} = 0.45\sqrt{2 \times 9.81 \times 20}$$

$$C_{f1} = C_{f2} = 8.914 \text{ m/s} \tag{4}$$

The inlet and outlet velocity triangles are shown in Figure 3.26. From the inlet velocity triangle,

$$\tan\beta_1 = \frac{C_{f1}}{C_{b1} - C_{w1}} \Rightarrow \tan 18 = \frac{8.914}{32.987 - C_{w1}}$$

$$C_{w1} = 5.55 \text{ m/s} \tag{5}$$

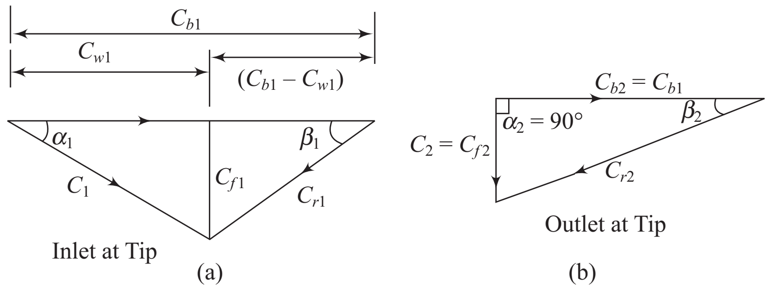


Figure 3.26 *Velocity Triangles of Example 3.11 (a) At Inlet, (b) At Outlet*

(a) Hydraulic Efficiency

Since, discharge at the exit of the turbine is axial, therefore,

$$\eta_h = \frac{C_{w1}C_{b1}}{gH} \tag{6}$$

$$\eta_h = \frac{5.55 \times 32.987}{9.81 \times 20}$$

$$\eta_h = 0.9331 = 93.31\% \tag{7}$$

(b) Guide Vane Angle at Inlet

$$\tan \alpha_1 = \frac{C_{f1}}{C_{w1}} \Rightarrow \tan \alpha_1 = \frac{8.914}{5.550}$$

$$\alpha_1 = 58.093° \tag{8}$$

(c) Blade angle of the Extreme Edge at Outlet

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2}} \Rightarrow \tan \beta_2 = \frac{8.914}{32.987}$$

$$\beta_2 = 15.122° \tag{9}$$

EXAMPLE 3.12 A Kaplan turbine is generating 12.5 MW power against a net head of 30 m. The runner tip diameter is 3.5 m and hub diameter is 1.5 m. The hydraulic efficiency is 92% and overall efficiency of the turbine is 88%. Both the inlet and outlet velocity triangles are right angled triangles at the tip. Determine (a) the speed of the turbine, (b) guide blade angle at the inlet, and (c) outlet blade angle at the tip of the blades.

Solution

Given:

$H = 30$ m, $P = 12.5$ MW $= 12500$ kW $= 12.5 \times 10^6$ W, $D_o = 3.5$ m, $D_h = 1.5$ m, $\eta_h = 0.92$, $\eta_o = 0.88$ $\beta_1 = 90°$, $\alpha_2 = 90°$

Power developed is,

$$P = \eta_o \rho_w g Q H \tag{1}$$

$$12.5 \times 10^6 = 0.88 \times 1000 \times 9.81 \times Q \times 30$$

$$Q = 48.266 \text{ m}^3/\text{s} \tag{2}$$

We know that,

$$Q = \frac{\pi}{4}(D_o^2 - D_h^2)C_{f1} \Rightarrow 48.266 = \frac{\pi}{4}(3.5^2 - 1.5^2)C_{f1}$$

$$C_{f1} = 6.145 \text{ m/s} \tag{3}$$

(a) Speed of the Turbine

Since the velocity triangles are right angled at the inlet and outlet of the turbine, as shown in Figure 3.27, i.e. $\beta_1 = 90°$, $\alpha_2 = 90° \Rightarrow C_{w1} = C_{b1}$ and $C_2 = C_{f2}$, $C_{w2} = 0$. The hydraulic efficiency for axial discharge at outlet is given by,

$$\eta_h = \frac{C_{w1}C_{b1}}{gH} \tag{4}$$

$$0.92 = \frac{C_{b1} \times C_{b1}}{9.81 \times 30} \Rightarrow C_{b1} = 16.455 \text{ m/s}$$

$$\because \quad C_{b1} = \frac{\pi D_o N}{60} \Rightarrow 16.455 = \frac{\pi \times 3.5 \times N}{60} \tag{5}$$

$$N = 89.79 \text{ rpm}$$

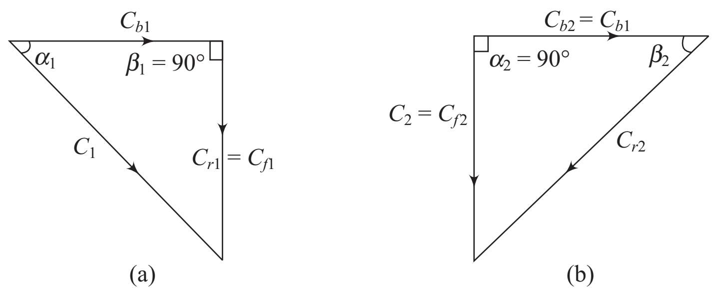


Figure 3.27 *Velocity Triangles of Example 3.12 (a) At Inlet, (b) At Outlet*

(b) Guide Blade Angle at the Inlet

$$\tan \alpha_1 = \frac{C_{f1}}{C_{b1}} = \frac{6.145}{16.455}$$

$$\alpha_1 = 20.48° \tag{6}$$

(c) Outlet Blade Angle at the Tip of the Blades

For a Kaplan turbine,

$$C_{b1} = C_{b2} \text{ and } C_{f1} = C_{f2}$$

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2}} = \frac{C_{f1}}{C_{b1}} = \tan \alpha_1$$

$$\therefore \quad \beta_2 = \alpha_1 = 20.48° \tag{7}$$

3.14 Draft Tube

The pipe of gradually increasing area i.e. diverging passage which connects the runner outlet to the tailrace is known as a draft tube. One end of the draft tube is connected to the runner exit, while the other end is submerged below the free surface of water in the tailrace. Thus, it is used for discharging water from the exit of the turbine to the tailrace. In addition to provide a diverging discharge passage for water, the draft tube is used for the following purposes also:

(i) If a reaction turbine is not fitted with a draft tube, the pressure at the outlet of the runner will be atmospheric pressure. Draft tube permits a negative head, i.e. pressure lesser than atmospheric pressure, to be established at the runner outlet. Thus, the net head on the turbine increases by using draft tube. Consequently, power developed and hydraulic efficiency increases.

(ii) The kinetic energy of water at the exit of the turbine would be a waste as an exit loss if the draft tube is not used. Draft tube provides diffuser action, i.e. deceleration of flow as it is shaped as a diverging passage. The draft tube effectively converts the kinetic energy of water at the turbine exit into useful pressure energy i.e. head energy thereby increasing the pressure difference, i.e. head on the runner of the turbine.

(iii) The draft tube permits the turbine to be installed above the tailrace to facilitate inspection and maintenance.

In Francis turbines, the kinetic energy at the outlet of the runner may amount to as much as 15% and in axial flow turbines, it amounts to 50% of the total input of energy. The recovery of kinetic energy is thus of great importance, especially in low head installations which is accomplished by using a draft tube.

Figure 3.28 shows various types of draft tubes. The conical type is used in low head turbine, i.e. low power units. The conical draft tubes and Moody spreading draft tubes are most efficient. Elbow type draft tube is more common. In the elbow type, energy is regained in the vertical portion which flattens in the elbow section to discharge water horizontally to the tailrace. Simple elbow type and elbow draft tubes with circular inlet and rectangular outlet require less space as compared to other draft tubes.

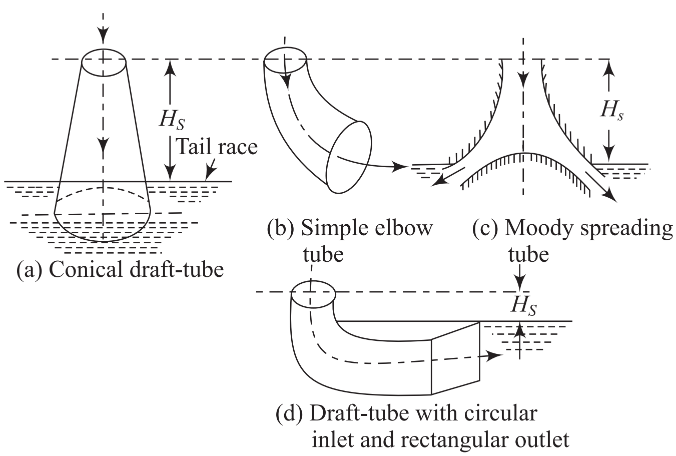


Figure 3.28 *Various Types of Draft Tubes*

3.14.1 Analysis of Draft Tube

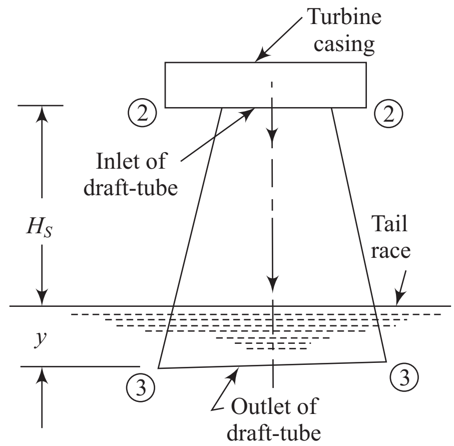


Figure 3.29 *Analysis of Flow through a Draft Tube*

Consider a reaction turbine with the flow from the runner exiting in the plane marked by section 2-2. A draft tube connects the runner and leads the outflow to the tailrace as shown in Figure 3.29. Section 3-3 marks a plane that is at a very infinitesimal distance down the end of the draft tube. The flow exits the draft tube as a free jet into the tailrace.

Let,

H_s = be the height of the draft tube above the tailrace known as *draft head* or *static head*,

y = be the depth of the draft tube submerged in water, i.e. distance of the bottom of the draft tube from the tailrace, and

h_f = be the loss of energy or pressure head due to friction in the draft tube.

Applying Bernoulli's equation to the inlet section 2-2 and outlet section 3-3 of the draft tube taking the centre line of section 3-3 as a reference or datum, we get,

$$\frac{p_2}{\rho_w g} + \frac{C_2^2}{2g} + (H_s + y) = \frac{p_3}{\rho_w g} + \frac{C_3^2}{2g} + 0 + h_f \tag{3.117}$$

We know that the pressure p at a depth d below the free surface of a static incompressible fluid is given by,

$$p = p_a + \rho g d \tag{3.118}$$

where, p_a is the atmospheric pressure which acts on the free surface.

Applying Eq. (3.118) to the outlet section 3-3 of the draft tube,

$$p_3 = p_a + \rho_w g y \Rightarrow \frac{p_3}{\rho_w g} = \frac{p_a}{\rho_w g} + y \tag{3.119}$$

Substituting the value of $p_3/\rho g$ from Eq. (3.119) into Eq. (3.117), we get,

$$\frac{p_2}{\rho_w g} + \frac{C_2^2}{2g} + (H_s + y) = \frac{p_a}{\rho_w g} + y + \frac{C_3^2}{2g} + h_f$$

$$\frac{p_2}{\rho_w g} = \frac{p_a}{\rho_w g} - \left(\frac{C_2^2 - C_3^2}{2g}\right) - (H_s - h_f) \tag{3.120}$$

$$p_2 = p_a - \rho_w \left(\frac{C_2^2 - C_3^2}{2}\right) - \rho_w g (H_s - h_f) \tag{3.121}$$

Putting the head loss,

$$h_f = k\left(\frac{C_2^2 - C_3^2}{2g}\right) \tag{3.122}$$

where, k is a coefficient which generally has a small value of the order of 0.3. Since $C_2 > C_3$ and H_s is positive, it can be found from Eq. (3.120) or (3.121) that pressure at the entry of the draft tube is lesser than atmospheric pressure i.e. the gauge pressure is negative. Thus, it is possible to keep the turbine above the tailrace by an amount H_s and this would result in negative gauge pressure at section 2-2. Further, smaller values of C_3 and k, and a larger value of H_s, would decrease the value of p_2. In other words, draft tube permits a negative head i.e. pressure lesser than atmospheric pressure to be established at the runner outlet. Thus, the net head on the turbine increases by using draft tube. Consequently, power developed and hydraulic efficiency increases.

The lowest value of the pressure at the inlet of the draft tube or runner exit p_2 that can be attained is determined by the vapour pressure of water at the prevailing temperature. When p_2 approaches the vapour pressure, *cavitation* phenomenon occur and vapor pockets may form resulting in mechanical damage, vibration and loss of efficiency. The maximum value of the draft head that can be used in a given condition depends on the cavitation susceptibility of the turbine installation. This will be discussed further at appropriate place.

Another interesting observation of Eq. (3.120) or (3.121) is that while keeping all the remaining factors to be the same, larger the friction and other losses results in higher values of draft head for a given p_v.

3.14.2 Efficiency of the Draft Tube

For the given draft tube, the kinetic energy head at the exit of the runner is $C_2^2/2g$. If there were no draft tubes, this energy head would not be recovered and hence, the kinetic energy wasted to the tailrace would have been $\rho_w gQ\frac{C_2^2}{2g} = \rho_w Q\frac{C_2^2}{2}$, where Q is the discharge through the runner. With the use of the draft tube, the energy wasted to the tailrace is $\rho_w gQ\frac{C_3^2}{2g} = \rho_w Q\frac{C_3^2}{2}$.

Energy lost due to friction and other causes in the draft tube is $\rho_w gQh_f = \rho_w Qk\left(\frac{C_2^2 - C_3^2}{2}\right)$ (3.123)

Therefore,

$$\text{The kinetic energy recovered using draft tube} = \rho_w Q\frac{C_2^2}{2} - \rho_w Q\frac{C_3^2}{2} - \rho_w k\left(\frac{C_2^2 - C_3^2}{2}\right)$$

$$\text{The kinetic energy recovered using draft tube} = \rho_w Q(1-k)\left(\frac{C_2^2 - C_3^2}{2}\right) \tag{3.124}$$

The maximum possible recovery of kinetic energy for a given C_3 is when there is no energy head loss due to friction and other causes, i.e. when $k = 0$. Hence,

$$\text{The maximum possible recovery of kinetic energy using draft tube} = \rho_w Q\left(\frac{C_2^2 - C_3^2}{2}\right) \tag{3.125}$$

The ratio of the actual recovery of kinetic energy in the draft tube to the maximum possible recovery in the given draft tube is known as the *efficiency of the draft tube,* η_d. Therefore,

$$\eta_d = 1 - \frac{h_f}{\left(\frac{C_2^2 - C_3^2}{2g}\right)} = \frac{(1-k)\left(\frac{C_2^2 - C_3^2}{2}\right)}{\left(\frac{C_2^2 - C_3^2}{2}\right)} = (1-k) \tag{3.126}$$

The draft tube efficiency gives a measure of frictional and other losses in the draft tube. Efficiency of the draft tube is also sometimes called *pressure recovery factor*.

Some authors define the efficiency of a draft tube in an alternate way. The efficiency of a draft tube is defined as the ratio of the actual conversion of kinetic head into pressure head in the draft tube to the kinetic head available at the inlet of the draft tube.

$$\text{Actual conversion of kinetic head into pressure head} = \frac{C_2^2 - C_3^2}{2g} - h_f \tag{3.127}$$

Therefore, efficiency of the draft tube is given by,

$$\eta_d = \frac{\left(\frac{C_2^2 - C_3^2}{2g}\right) - h_f}{\left(\frac{C_2^2}{2g}\right)} = 1 - \frac{\left(h_f + \frac{C_3^2}{2g}\right)}{\left(\frac{C_2^2}{2g}\right)} \tag{3.128}$$

However, the efficiency of the draft tube as given by Eq. (3.126) is preferred as it is logically sound and is used consistently throughout the book.

EXAMPLE 3.13 A conical draft tube having inlet and outlet diameters of 1 m and 1.5 m respectively, discharges water at the rate of 5 m^3/s. The turbine is set 7 m above the tailrace water. Calculate (a) the pressure head at the entrance of the draft tube, and (b) efficiency of the draft tube. Assume the loss of energy in the draft tube as 0.3 times the velocity head at the exit of the draft tube and atmospheric pressure head as 10.3 m of water.

Solution

Let subscripts 2 and 3 denote the inlet and outlet of the draft tube, respectively, as shown in Figure 3.30.
Given: $D_2 = 1.0$ m, $D_3 = 1.5$ m, $Q = 5\ m^3/s$, $H_s = 7$ m, $h_f = 0.3 \times C_3^2/2g$, $p_a/\rho_w g = 10.3$ m
Cross-sectional areas at the inlet and outlet are given by,

$$A_2 = \frac{\pi}{4} D_2^2 \Rightarrow A_2 = \frac{\pi}{4} \times 1^2 = 0.7854\ m^2$$

$$A_3 = \frac{\pi}{4} D_3^2 \Rightarrow A_3 = \frac{\pi}{4} \times 1.5^2 = 1.7671\ m^2$$

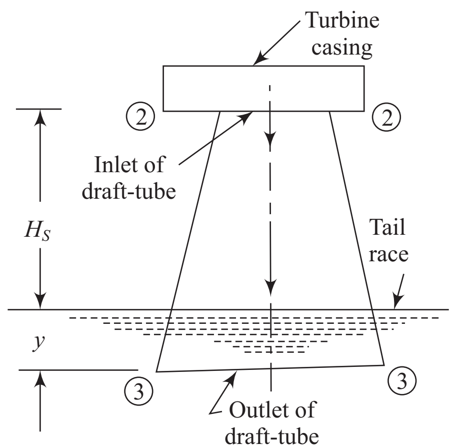


Figure 3.30 *A Draft Tube for Example 3.13*

Since, $Q = AC$, therefore velocities at entry and exit of the draft tube are,

$$C_2 = \frac{Q}{A_2} \Rightarrow C_2 = \frac{5}{0.7854} \Rightarrow C_2 = 6.366\ \text{m/s}$$

$$C_3 = \frac{Q}{A_3} \Rightarrow C_3 = \frac{5}{1.7671} \Rightarrow C_3 = 2.83\ \text{m/s}$$

Loss of head in the draft tube,

$$h_f = 0.3 \times \frac{C_3^2}{2g} \Rightarrow h_f = 0.3 \times \frac{2.83^2}{2 \times 9.81}$$

$$h_f = 0.1225\ \text{m}$$

(a) Pressure Head at the Entrance of the Draft Tube

Refer Figure 3.30. Let the depth of the draft tube immersed in water is y. Taking the bottom of the draft tube as datum and applying Bernoulli's equation between the inlet section 2-2 and outlet section 3-3,

$$\frac{p_2}{\rho_w g}+\frac{C_2^2}{2g}+Z_2=\frac{p_3}{\rho_w g}+\frac{C_3^2}{2g}+Z_3+h_f \Rightarrow \frac{p_2}{\rho_w g}+\frac{C_2^2}{2g}+Z_2=\frac{p_a}{\rho_w g}+y+\frac{C_3^2}{2g}+Z_3+h_f \quad (1)$$

$$\frac{p_2}{\rho_w g}+\frac{6.366^2}{2\times 9.81}+(7+y)=10.3+y+\frac{2.83^2}{2\times 9.81}+0+0.1225$$

$$\frac{p_2}{\rho_w g}=1.765 \text{ m (absolute)} \quad (2)$$

(b) Efficiency of the Draft Tube

$$\eta_d=1-\frac{h_f}{\left(\frac{C_2^2}{2g}-\frac{C_3^2}{2g}\right)} \quad (3)$$

$$\eta_d=1-\frac{0.1225}{\left(\frac{6.366^2}{2\times 9.81}-\frac{2.83^2}{2\times 9.81}\right)}$$

$$\eta_d = 0.9261 = 92.61\% \quad (4)$$

EXAMPLE 3.14 A Francis turbine working under a net head of 15 m develops 1 MW at an efficiency of 90%. A vertical cylindrical pipe of 2.0 m diameter is used as draft tube in this installation. If a tapering vertical draft tube having exit diameter of 2.5 m replaces the cylindrical draft tube, find (a) the increase in power developed, and (b) the increase in efficiency. Assume that the head, speed and discharge remain the same and there are no additional frictional losses in the new draft tube.

Solution

Let the subscript c is used for the cylindrical draft tube and a for the alternative conical draft tube.

Given: H = 15 m, P = 1 MW = 1000 kW, η_o = 90% = 0.9, D_c = 2.0 m, D_{a3} = 2.5 m

We know that power developed is,

$$P = \eta_o \rho_w g Q H \quad (1)$$

$$1\times 10^6 = 0.9\times 1000\times 9.81\times Q\times 15$$

$$Q = 7.551 \text{ m}^3/\text{s} \quad (2)$$

Velocity of water at exit of the cylindrical draft tube,

$$C_c=\frac{Q}{A_c}\Rightarrow C_c=\frac{Q}{\frac{\pi}{4}D_c^2}\Rightarrow C_c=\frac{7.551}{\frac{\pi}{4}\times 2^2}$$

$$C_c = 2.4036 \text{ m/s} \quad (3)$$

Velocity of water at exit in the alternate conical draft tube,

$$C_{a3} = \frac{Q}{A_{a3}} \Rightarrow C_{a3} = \frac{Q}{\frac{\pi}{4} D_{a3}^2} \Rightarrow C_{a3} = \frac{7.551}{\frac{\pi}{4} \times 2.5^2}$$

$$C_{a3} = 1.538 \text{ m/s} \tag{4}$$

$\because C_{a3} < C_c$, therefore it indicates that more kinetic energy is recovered by using alternate tapered draft tube; consequently, power developed will increase. Velocity head gained by using alternate conical draft tube,

$$\Delta H = \frac{C_c^2}{2g} - \frac{C_{a3}^2}{2g} \Rightarrow \Delta H = \frac{2.4036^2}{2 \times 9.81} - \frac{1.538^2}{2 \times 9.81}$$

$$\Delta H = 0.174 \text{ m} \tag{5}$$

(a) Increase in Power Developed

Increase in power developed, i.e. power gained is,

$$P_g = \rho_w g Q \Delta H \Rightarrow P_g = 1000 \times 9.81 \times 7.551 \times 0.174$$

$$P_g = 12889.104 \text{ W} = 12.889 \text{ kW} \tag{6}$$

(b) Increase in Efficiency

Increase in efficiency is given by,

$$\eta_g = \frac{\text{Gain in head}}{\text{Original Head}} \Rightarrow \eta_g = \frac{0.174}{15}$$

$$\eta_g = 0.116 = 1.16\% \tag{7}$$

3.15 Comparison of Turbines

The characteristics features of common types of turbine are summarised in Table 3.5.

TABLE 3.5 Comparison of Common Turbines

S. No.	*Characteristics Features*	*Pelton Turbine*	*Francis turbine*	*Kaplan/Propeller Turbine*
1.	Flow, stage and working nature	Tangential,single stage, impulse	Inward radial or mixed flow, single stage, reaction	Axial flow, single stage, reaction
2.	Maximum generation capacity	250 MW	720 MW	225 MW
3.	Number of jets/guide blades	1 to 6 jets, Maximum 2 for horizontal and 6 for vertical shaft	Adjustable guide blades	Adjustable guide blades
4.	Runner blades	Fixed	Fixed	Fixed for propeller and adjustable for Kaplan
5.	Head	100 – 1750 m	30 – 550 m	1.3 – 77.5 m
6.	Speed	75 – 1000 rpm	93 – 1000 rpm	72 – 600 rpm
7.	Hydraulic Efficiency	Single jet 85 – 90%	90 – 94%	85 – 93%

(*Contd.*)

(*Contd.*)

8.	Mechanical Efficiency	Decreases faster with time	Lesser decrease with time	Lesser decrease with time
9.	Specific Speed	6–60	50–400	280–1100
10.	Regulation Mechanism	Spear nozzle and deflector plate	Guide vanes and relief valve	Guide vanes, runner vanes and relief valve (blade stagger)-better controlled
11.	Variation in Operating Head	Less controlled	Better controlled	Better controlled
12.	Size of the Runner, Generator and Power House	Larger	Smaller	Smaller
13.	Effect of unclean Water	Less wear	High wear with loss in efficiency	High wear with loss in efficiency

3.16 Performance Characteristics of Turbines

Turbines are generally designed to work at certain values of head, discharge, power, speed, efficiency and gate opening. However, turbines are often required to work at conditions different from those for which they were designed. It is therefore necessary to determine the exact behaviour of the turbine under these varying conditions by conducting tests either on actual turbines or on their small scale models. Out of the three independent parameters-speed, head and discharge-one of the parameter is kept constant and the variation of other parameters with respect to any one of the remaining two independent variables are plotted to obtain the *characteristics curves* of the turbines. Characteristics curves of a hydraulic turbine are the curves with the help of which the exact behaviour and performance of the turbine under different working conditions can be known. The characteristics curves are plotted in terms of the unit quantities for the sake of convenience.

3.16.1 Main Characteristics Curves

Main characteristics curves, also known as *constant head curves,* are those which help in understanding the behaviour of a turbine towards selection of a turbine unit to meet specific requirements. Data sets for these curves are obtained from model testing of homologous turbines and also, sometimes from simulation studies. Tests are conducted by maintaining a constant head and constant gate opening and the speed of the turbine is varied by changing the load on the turbine. A series of values of N is thus obtained. For each value of N, the discharge Q and power P are measured. Then the overall efficiency η_o is calculated for each value of the speed N. Similar tests are performed for different gate openings. The values of Q_u, P_u, N_u and overall efficiency η_o, are computed for each gate opening and are plotted taking N_u as abscissa, as shown in Figures 3.31 to 3.34.

For Pelton wheels, discharge Q depends only on the needle valve opening (gate opening) and is independent of N_u, therefore Q_u versus N_u plots are horizontal straight lines. The velocity of the jet remains constant at a particular gate opening. The needle valve setting to any value changes only the area and not the velocity. The discharge thus remains constant for all speeds. For Francis turbines, these are dropping curves. It indicate that as the speed increases the discharge decreases since the centrifugal head developed increases with speed and retards the flow through the turbine. In a Kaplan turbine no such centrifugal head is developed to retard the flow, therefore, the discharge increases linearly with speed for all gate openings. Obviously, higher value of Q_u will be obtained for more gate opening for all types of turbines.

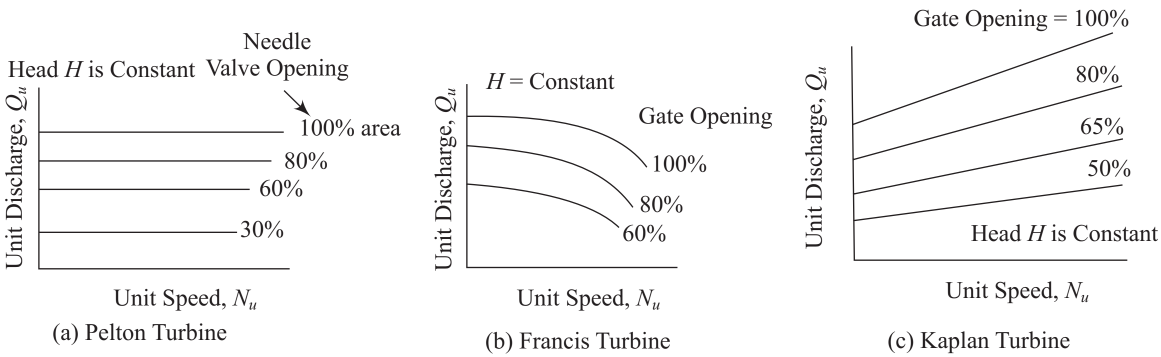


Figure 3.31 *Variation of Discharge for Various Turbines*

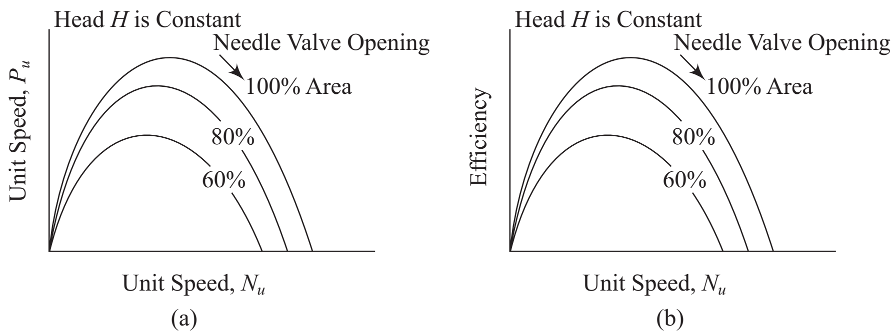


Figure 3.32 *Variation of (a) Power, and (b) Efficiency with Unit Speed in a Pelton Turbine*

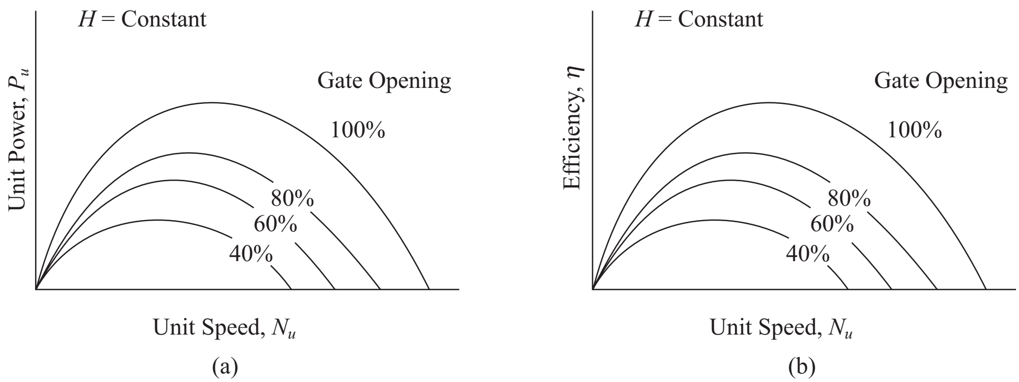


Figure 3.33 *Variation of (a) Power, and (b) Efficiency with Unit Speed in a Francis Turbine*

The variation of P_u versus N_u and η_o versus N_u are parabolic in nature for all types of turbine. It signifies that there exists a maximum value of P_u and η_o at a particular value of N_u for any gate opening. The value of N_u at which power or overall efficiency is maximum will be known as optimum value of speed for maximum power or overall efficiency. The optimum value of N_u for maximum power is different from that for maximum overall efficiency at a particular gate opening. Also, the optimum value of N_u is different for different gate opening.

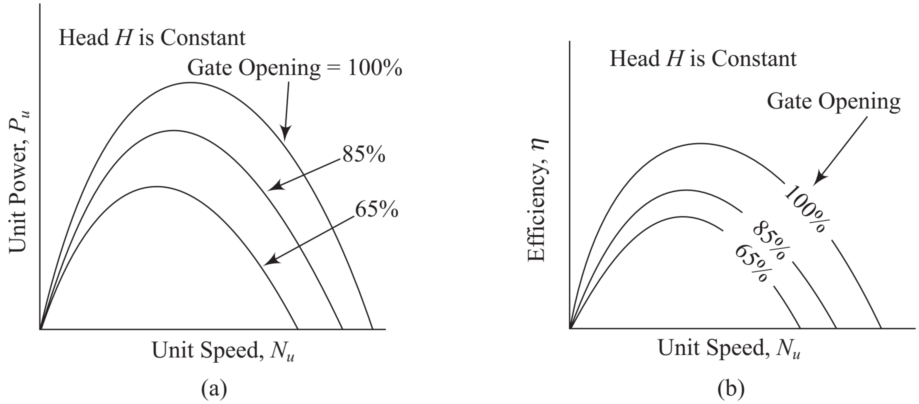


Figure 3.34 *Variation of (a) Power, and (b) Efficiency with Unit Speed in a Kaplan Turbine*

3.16.2 Constant Speed Characteristics Curves

In order to obtain constant speed characteristics curves, say efficiency versus load, the constant speed is attained by regulating the gate opening i.e. discharge as the load varies. The head may or may not remain constant. The characteristics curves of efficiency versus load for different turbines are shown in Figure 3.35 (a) and (b).

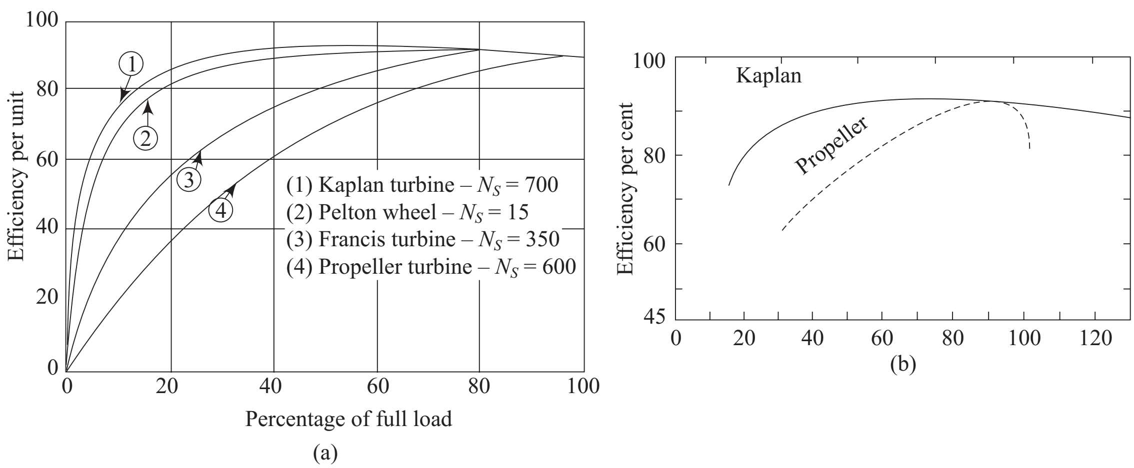


Figure 3.35 *Variation of Overall Efficiency with Load for Various Turbines*

The efficiency increases with load and attains the maximum value at full or rated load. It is observed that Kaplan and Pelton turbines maintain a high efficiency over a longer range of part load (i.e. about 40-100%) as compared with either the Francis or the fixed blade propeller turbine. Figure 3.36 shows the variation of efficiency and power with discharge where Q_0 is the maximum discharge which is required to initiate the motion of the turbine runner from the state of rest. The power increases linearly with discharge since the power ($P = g\rho_w QH$) is directly proportional to discharge if the head is constant. However, the overall efficiency increases with discharge and becomes more or less constant beyond a certain value of discharge.

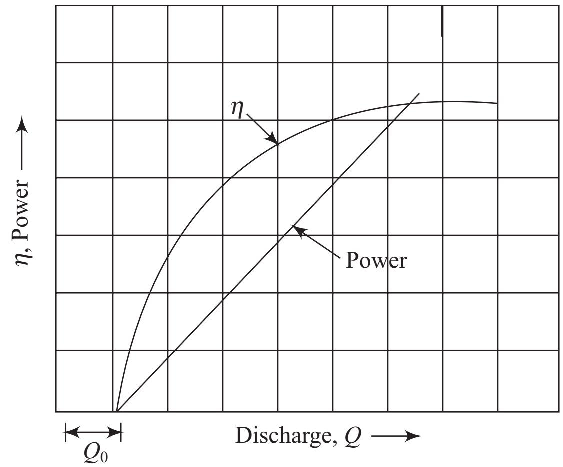


Figure 3.36 *Variation of Efficiency and Power of a Francis Turbine with Discharge*

3.16.3 Constant Efficiency Curves

Figure 3.37 shows the constant efficiency curves for all conditions of running which are also called the universal characteristic curves of the turbine. Iso-efficiency and best performance curves are plotted between N_u and Q_u for different gate openings.

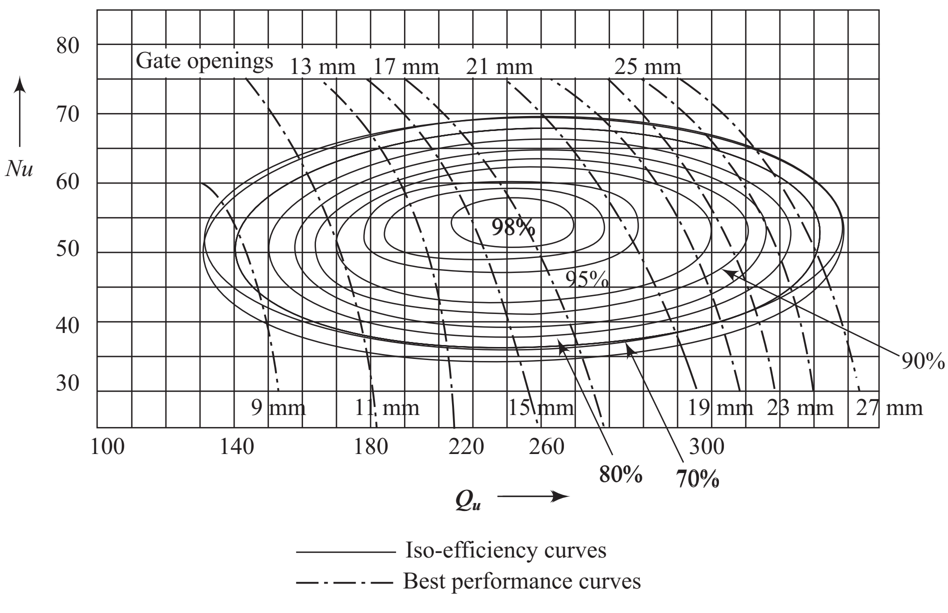


Figure 3.37 *Iso-efficiency and Best Performance Curves of a Francis Turbine*

The inner most curve represents the highest efficiency of the turbine, while outer curves represent lower efficiencies. If a vertical line is drawn at a particular value of Q_u, it will intersect an iso-efficiency (constant efficiency) curve at two points giving two values of N_u for the same efficiency while it will also touch some other efficiency curve of higher efficiency at one point. Thus, for a unit discharge or power the vertical line touches the curve of maximum efficiency at only one point. Now, if these points are joined together by a smooth curve, we obtain the best performance curve for the turbine. By drawing a horizontal line for a given N_u (at certain H and N) which cuts this best performance curve, the point of maximum efficiency is known, corresponding to which Q_u or P_u can be obtained. Hence, discharge Q and power P can be estimated at which the turbine efficiency is maximum for the given head H and speed N.

The *runaway speed* is the maximum speed of the turbine under no load and no governing action. The hydraulic design is based on the optimum speed, but it must also satisfy structurally the safe conditions at runway speed which is about 1.8 to 2.3 times the optimum speed.

3.17 Selection of Hydraulic Turbines

The selection of hydraulic turbine is based on the specific conditions under which it has to operate and attain the maximum possible efficiency. The choice depends on the available head, power to be developed and the speed at which it has to run. The following factors basically govern the selection of a suitable type of turbine.

(a) Operating Head

The present practice is to use Kaplan and propeller type of turbines for heads up to 50 m. For heads from 50 m to 400 m, Francis turbines are used. Impulse or Pelton turbines are used for heads greater than 400 m. The range of heads as mentioned is not hard and fast and may change if other conditions dominate to achieve economy.

(b) Specific Speed

It is always better to choose turbines of high specific speeds as it means compact size of turbines, generator and power house, etc. The range of specific speeds of the turbines should correspond to the synchronous speed of the generator, $N = \dfrac{120f}{p}$, where f is the frequency and p is the number of poles.

(c) Height of Installation

It is always advisable to install the turbines as high above the tailrace as possible. This saves cost of excavation for the draft tube. Care should be taken to ensure that cavitation does not occur.

(d) Performance Characteristics of Turbine

The performance characteristics of turbines as discussed in Section 3.16 should be studied carefully before selecting the type of turbine to be used. A turbine has the maximum efficiency at a certain load. If the turbine has to operate mostly at off-design conditions i.e. at part loads, only those turbines whose efficiencies do not reduce appreciably with part loads should be selected. Kaplan and Pelton turbines gives better part load performance compared to Francis and propeller turbines.

(e) Size of Turbine

Bigger size of a turbine means less number of runners for a specific power output. Therefore, the size of the turbine should be as large as possible since this results in economy of size of the power house, the number of penstocks, the generator, etc. However, the number of runners should not be less than two so that at least one unit is always available for service in the case of a plant breakdown.

Table 3.6 shows the character of acceptance of variation of head and discharges of various turbines. The turbine that has high acceptance of both the head and discharge variations is preferred.

Table 3.6 Salient Features of Various Turbines to Aid Selection

S. No.	Turbine	Specific Speed Range $\left[\frac{rpm\sqrt{kW}}{m^{5/4}}\right]$	Head Range (m)	Acceptance of Flow Variation	Acceptance of Head Variation	Part Load (%) of Designed Capacity
1.	Pelton: Single Jet	8–30	50–1500	High	Low	30–100
2.	Pelton: Multijet	8–30 (per jet)	50–1800	High	Low	10–100
3.	Francis	40–400	25–350	Medium	Low	70–100
4.	Propeller	300–900	1.5–50	Low	Low	80–100
5.	Kaplan	300–900	1.5–50	High	High	15–100

SUMMARY

- The difference between the headrace level and tailrace level when no water is flowing is known as gross head. The net head, also known as effective head, is the difference of heads between the inlet and outlet of a turbine.
- Various efficiencies of a hydraulic turbine are listed as below:

Efficiency	Formula
Volumetric efficiency	$\eta_v = \frac{Q - \Delta Q}{Q}$
Hydraulic efficiency	$\eta_h = \frac{H - h_{fr}}{H} = \frac{H_e}{H} = \frac{RP}{WP}$
Mechanical efficiency	$\eta_m = \frac{H_e - h_m}{H_e} = \frac{SP}{RP}$
Overall efficiency	$\eta_o = \eta_v \eta_h \eta_m = \frac{SP}{WP}$

- Various parameters of different hydraulic turbines are summarised in the following table:

Parameter	Pelton Turbine	Francis Turbine	Propeller/Kaplan Turbine
Net Head	$H = H_g - h_f - H_{sn}$ $= \frac{p_1}{\rho_w g} + \frac{C_1^2}{2g}$	$H = H_g - h_f - \frac{C_3^2}{2g}$ $= \frac{p_1}{\rho_w g} + \frac{C_1^2}{2g} + Z_1 - \frac{C_3^2}{2g}$	$H = H_g - h_f - \frac{C_3^2}{2g}$ $= \frac{p_1}{\rho_w g} + \frac{C_1^2}{2g} + Z_1 - \frac{C_3^2}{2g}$

Euler head	$H_e = \frac{1}{g}(C_1 - C_b)$ $(1 + k\cos\beta_2)C_b$	$H_e = \frac{1}{g}(C_{w1}C_{b1})$ $= \frac{1}{g}C_{f1}^2 \cot\alpha_1$ $(\cot\beta_1 + \cot\alpha_1)$	$H_e = \frac{1}{g}(C_{w1}C_{b1})$
Discharge	$Q = \frac{\pi}{4}d^2C_1$	$Q = (\pi D_1 - zt)\,B_1C_{f1} = (\pi D_2 - zt)B_2C_{f2}$	$Q = \frac{\pi}{4}(D_o^2 - D_h^2)K_1C_f$ Generally, net area factor $K_1 = 0$
Power developed	$P = \rho_w aC_1(C_1 - C_b)\,(1 + k\cos\beta_2)\,C_b = \rho_w aC_1^3\rho\,(1-\rho)\,(1 + k\cos\beta_2) = \eta_m\eta_v\rho_w Q(C_1 - C_b)\,(1 + k\cos\beta_2)C_b$	$P = \rho_w QC_{w1}C_{b1}$	$P = \rho_w QC_{w1}C_{b1}$
Hydraulic efficiency	$\eta_h = 2C_v^2\rho(1-\rho)\,(1 + k\cos\beta_2)$	$\eta_h = \frac{C_{w1}C_{b1}}{gH}$ $= 1 - \frac{1}{1 + 2\cot\alpha_1}$ $(\cot\beta_1 + \cot\alpha_1)$	$\eta_h = \frac{C_{w1}C_{b1}}{gH}$
Diagram or blading or wheel efficiency	$\eta_D = 2\rho(1 + k\cos\beta_2)\,(1 - p)$		
Degree of Reaction	0	$R = 1 - \frac{(C_1^2 - C_{f1}^2)}{2gH_e}$ $= 1 - \frac{\cot\alpha_1}{2(\cot\alpha_1 + \cot\beta_1)}$	$R = \frac{(C_{r2}^2 - C_{r1}^2)}{2gH_e}$
Optimum velocity ratio for maximum power maximum hydraulic and blading or wheel efficiencies	$\rho_{opt} = \frac{1}{2}$		
Maximum power	$P_{\max} = \frac{1}{4}\rho_w a$ $(1 + k\cos\beta_2)C_1^3$		
Maximum hydraulic efficiency	$(\eta_h)_{\max} = \frac{C_1^2}{4} \times \frac{(1 + k\cos\beta_2)}{gH}$ $= \frac{C_v^2}{2}(1 + k\cos\beta_2)$		
Maximum diagram or blading or wheel efficiency	$(\eta_D)_{\max} = \frac{(1 + k\cos\beta_2)}{2}$		

Runaway speed	$N_r = \left(\frac{C_v}{\varphi}\right) N$	$N_r = 0.63\, N (N_s)^{\frac{1}{5}}$ $N_{r\,max} = N_r \left(\frac{H_{\max}}{H_{\text{design}}}\right)^{1/2}$	$N_r = 0.63\, N (N_s)^{\frac{1}{5}}$ $N_{r\,max} = N_r \left(\frac{H_{\max}}{H_{\text{design}}}\right)^{1/2}$

✦ The ratio of the actual recovery of kinetic energy in the draft tube to the maximum possible recovery in the given draft tube is known as the *efficiency of the draft tube,* sometimes called *pressure recovery factor,* η_d. Therefore,

$$\eta_d = 1 - \frac{h_f}{\left(\frac{C_1^2 - C_2^2}{2g}\right)} = \frac{(1-k)\left(\frac{C_1^2 - C_2^2}{2g}\right)}{\left(\frac{C_1^2 - C_2^2}{2g}\right)} = (1-k)$$

✦ The efficiency of a draft tube is also defined as the ratio of the actual conversion of kinetic head into pressure head in the draft tube to the kinetic head available at the inlet of the draft tube.

$$\eta_d = \frac{\left(\frac{C_1^2 - C_2^2}{2g}\right)}{\left(\frac{C_1^2}{2g}\right)} = 1 - \frac{\left(h_f + \frac{C_2^2}{2g}\right)}{\left(\frac{C_1^2}{2g}\right)}$$

✦ Characteristics curves of a hydraulic turbine are the curves with the help of which the exact behavior and performance of the turbine under different working conditions can be known. The characteristics curves are plotted in terms of the unit quantities for the sake of convenience.

✦ Main characteristics curves, also known as *constant head* curves which are obtained by conducting tests on the turbine maintaining a constant head and constant gate opening and varying the speed of the turbine according to the load on the turbine.

✦ Operating characteristics curves are obtained by keeping speed to be constant by regulating the gate opening i.e. discharge as the load varies. The head may or may not remain constant.

✦ Iso-efficiency and best performance curves which are plotted between N_u and Q_u for different gate openings. Any point on this curve gives the gate opening for which maximum efficiency is the value of iso-efficiency curve at discharge and speed corresponding to the point.

✦ The *runaway speed* is the maximum speed of the turbine under no load and no governing action. The hydraulic design is based on the optimum speed, but it must also satisfy structurally the safe conditions at runway speed.

REVIEW QUESTIONS

3.1 What is the working principle of a hydraulic turbine?

3.2 Draw the general layout of a hydroelectric plant using an impulse turbine and define: (a) gross head, and (b) net head.

3.3 How the net head in a reaction turbine differs from that of a Pelton wheel?

3.4 Explain the functional differences between a Pelton or impulse turbine and a reaction turbine.

3.5 Why the theoretical head extracted by a turbine runner is less than the net head?

3.6 Define: (i) Volumetric efficiency, (ii) Hydraulic efficiency, (iii) Mechanical efficiency, and (iv) Overall efficiency of a hydraulic turbine. Establish their inter-relationship.

3.7 Differentiate the following items with respect to a turbine: (i) Power transmitted to the runner by the jet of water, (ii) Water power, and (iii) Shaft power.

3.8 How will you classify hydraulic turbines?

3.9 Differentiate between: (i) Radial and axial flow turbines, (b) Inward and outward flow reaction turbines, and (c) Kaplan and propeller turbines.

3.10 Why the runner, nozzle and associated components of a Pelton turbine should be installed well above the maximum water level of the tailrace?

3.11 Why a spear needle is used in the nozzle of a Pelton turbine?

3.12 Briefly explain the function of the following elements of a Pelton turbine: (i) Manifold, (ii) Deflector, (iii) Braking jet, and (iv) Auxiliary nozzle.

3.13 What is the function of a casing in Pelton turbine?

3.14 Why a cut-away is provided in the bucket of a Pelton turbine?

3.15 Define the following terms in a Pelton turbine: (a) Coefficient of velocity of the nozzle, (b) Speed ratio, (c) Flow ratio, and (d) Jet ratio.

3.16 Draw the velocity triangles at the inlet and outlet of the buckets of a Pelton turbine. Indicate clearly the direction of various velocities.

3.17 Derive an expression for the Euler head and hydraulic efficiency for a Pelton turbine.

3.18 Deduce an expression for the power developed by a Pelton turbine.

3.19 Prove that the optimum velocity ratio for maximum power developed by a Pelton turbine is 1/2.

3.20 Prove that the maximum hydraulic efficiency of a Pelton turbine is given by,

$$(\eta_h)_{\max} = \frac{C_1^2}{4} \times \frac{(1 + k \cos \beta_2)}{gH} = \frac{C_v^2}{2}(1 + k \cos \beta_2)$$

3.21 State the effect of speed variation on the power developed and torque in an ideal Pelton turbine.

3.22 Differentiate the following efficiencies in a Pelton turbine:

(a) Hydraulic efficiency,

(b) Nozzle efficiency and,

(c) Wheel efficiency.

Establish an inter-relationship between these above efficiencies.

3.23 What is the limitation of a Pelton turbine?

3.24 Derive the Euler equation for the head extracted in a reaction turbine in terms of absolute and relative velocities of flow and peripheral velocity heads. Explain the significance of each term in the Euler's equation.

3.25 What are the impulse effect and reaction effect in a hydraulic turbine?

3.26 Define the degree of reaction of a turbine. What is the significance of degree of reaction?

3.27 Describe in brief the components of a Francis turbine.
3.28 Why scroll casing is used in a Francis turbine?
3.29 Distinguish between (a) Semi-scroll casing and full-scroll casing, and (b) Guide vanes and stay vanes.
3.30 What is the function of stay vanes in a Francis turbine?
3.31 What is the role of wicket gates in a Francis turbine?
3.32 What is the function of shroud ring in the runner of a Francis turbine?
3.33 What is the significance of the crown on the runner of a Francis turbine?
3.34 What do you mean by reverse swirl? What are the advantages and disadvantages of having reverse swirl in a Francis turbine?
3.35 Why it is a practice not to have reverse swirl in a Francis turbine?
3.36 What is the advantage of having axial discharge at exit of a Francis turbine?
3.37 Derive an expression for the degree of reaction turbine of a Francis turbine? What would be the value of degree of reaction for a Francis turbine having radial entry and axial exit?
3.38 Compare and contrast the salient features of a common propoller and Kaplan turbine.
3.39 Why the Kaplan turbine can be run at maximum efficiency at all loads?
3.40 What is the purpose of providing whirl chamber in a Kaplan turbine?
3.41 Compare and contrast the salient features of a Francis turbine and a Kaplan turbine.
3.42 Draw a neat sketch of velocity triangles at inlet and outlet of an axial flow reaction turbine at the tip, hub and mid radius location.
3.43 What is a draft tube? Why is it used in a reaction turbine?
3.44 Describe with neat sketches different types of draft tubes.
3.45 Describe with a neat sketch the commonly used high efficiency, elbow-type draft tube for a Francis turbine.
3.46 Explain how cavitation affects the draft head of a draft tube.
3.47 What is specific speed of a turbine? State its significance in the study of hydraulic turbines.
3.48 How specific speed of a turbine is non-dimensionalised to obtain shape factor?

PROBLEMS

3.1 A Pelton wheel operating under a head of 220 m develops 3.5 MW with an overall efficiency of 85%. Calculate the jet diameter if the coefficient of velocity of the nozzle is 0.98. [*Ans:* $d = 0.194$ m]

3.2 Water at the rate of 0.8 m^3/s passes through a Pelton wheel operating under a head of 45 m. The mean peripheral speed of the bucket is 14 m/s and the bucket friction coefficient is 0.96. The jet is deflected by the buckets through an angle of 165°. Find the overall efficiency and the shaft power assuming the coefficient of velocity as 0.985 and mechanical efficiency as 0.95.
[*Ans:* $\eta_o = 88.7\%$, $P_{\text{shaft}} = 312.6$ kW]

3.3 A water jet of diameter 120 mm strikes the buckets of a Pelton wheel having a pitch circle diameter of 1.45 m. The hydraulic and mechanical efficiencies of the turbine are 0.93 and 0.92 respectively. Calculate the specific speed of the turbine assuming speed ratio as 0.46 and coefficient of velocity of the nozzle as 0.97. [*Ans:* $N_s = 17.10$]

3.4 A water jet of diameter 180 mm impinges on the buckets of a Pelton turbine having overall efficiency of 85% while operating under a net head of 500 m and at speed of 420 rpm. Find the specific speed of the turbine. Take coefficient of velocity of the jet as 0.98 and speed ratio as 0.46. [*Ans:* $N_s = 18$]

3.5 The pitch circle diameter of a Pelton wheel is 0.9 m. A jet of water of diameter 80 mm gets deflected by an angle of 170° when it impinges on the buckets. The bucket friction coefficient and coefficient of velocity of jet can be taken as 0.93 and 0.97 respectively. If the net head available on the wheel is 500 m and speed ratio is 0.45, calculate (a) the power transferred to to the wheel by the jet, (b) hydraulic efficiency, and (c) specific speed of the turbine. Take mechanical efficiency of wheel as 0.9.
[*Ans:* (a) $P_{\text{rotor}} = 2120$ kW, (b) $\eta_h = 0.897$, (c) $N_s = 17.47$]

3.6 A Pelton wheel has mean peripheral speed of buckets as 10 m/s with a jet of water discharging at a rate of 0.7 m^3/s. The bucket deflects the jet through an angle of 160° and the head on the turbine is 30 m. The coefficient of velocity of jet and bucket friction coefficient can be taken as 0.98 and 0.95, respectively. If the mechanical efficiency is 0.93, calculate the shaft power and overall efficiency of the turbine. [*Ans:* $P_{\text{shaft}} = 169.4$ kW, $\eta_o = 88.6\%$]

3.7 The following data pertains to a Pelton turbine:
The coefficient of velocity of the nozzle = 0.98, speed ratio = 0.46, bucket angle = 165°, ratio of relative velocity at the outlet to that at the inlet = 0.99. Calculate the hydraulic efficiency of the turbine. [*Ans:* $\eta_h = 0.936$]

3.8 The ratio of the wheel diameter to jet diameter is 12 in a Pelton wheel which has an overall efficiency of 90%. The coefficient of velocity of the jet is 0.95 and speed ratio is 0.45. Calculate the specific speed of the turbine in SI units and in non-dimensional form.
[*Ans:* $N_s = 17.12$, $N_{sh} = 0.01645$ revolutions]

3.9 Two jets each of 150 mm diameter impinges on a Pelton turbine. The turbine operates under a net head of 300 m and produces 6500 kW while running at 375 rpm. Calculate (a) overall efficiency, and (b) specific speed of the turbine. Take $C_v = 0.98$. [*Ans:* (a) $\eta_o = 83.3\%$, (b) $N_s = 17.12$]

3.10 Three Pelton wheels, each having two jets of the same diameter are supplied water from a single penstock of 500 m length. The total head from the reservoir to the nozzle is 250 m and water is supplied at the rate of 3 m^3/s to the powerhouse. The power transmission efficiency through the pipe is 90% and overall efficiency of the turbine is 85%. Take Darcy-Weisbach friction coefficient $f = 0.02$ and coefficient of velocity of each nozzle as 0.95. Determine (a) the shaft power, (b) diameter of the jet, and (c) the diameter of the penstock.
[*Ans:* (a) $P_{\text{shaft}} = 5617$ kW, (b) $d = 100$ mm, (c) $D_p = 0.785$ m]

3.11 A Pelton turbine of specific speed 18.5 operating under a net head of 300 m produces 6 MW power. The jet ratio is 13 and ratio of the bucket velocity to jet velocity is 0.48. The overall efficiency of the turbine is 88%. Find (a) speed, (b) diameter of the wheel, and (c) diameter of the jet.
[*Ans:* (a) $N = 298.2$ rpm, (b) $D = 2.485$ m, (c) $d = 0.191$ m]

3.12 A multijet Pelton turbine operating under a net head of 360 m produces 46 MW of power while running at a speed of 300 rpm. Find the number of jets required. [*Ans:* $n = 6$ Jets]

3.13 A Pelton wheel is installed at the foot of a dam whose reservoir level is 220 m. The head at the base of the nozzle at full nozzle opening is 200 m. The turbine is to develop a power of 3.70 MW at a speed of 200 rpm. The ratio of the bucket velocity to jet velocity is 0.46 and the coefficient of velocity of jet for the nozzle is 0.98. If the angle at the outlet of the bucket is 16°, determine (a) the wheel diameter, (b) the wheel efficiency, and (c) the hydraulic efficiency of the turbine. Neglect the friction losses in the buckets. [*Ans:* (a) $D = 2.697$ m (b) $\eta_w = 0.975$ (c) $\eta_h = 0.936$]

3.14 A Pelton wheel having mechanical efficiency of 0.96 is operating under a head of 250 m. The bucket friction coefficient is 0.95 and water jet gets deflected at an angle of 170° by the buckets. If speed ratio is 0.46, calculate (a) the hydraulic efficiency, (b) wheel efficiency, (c) nozzle efficiency, and (d) overall efficiency of the turbine. Assume coefficient of velocity of the nozzle as 0.98.

[*Ans:* (a) $\eta_h = 0.926$, (b) $\eta_w = 0.964$, (c) $\eta_n = 0.960$, (d) $\eta_o = 0.889$]

3.15 The relative velocity at the outlet of a Pelton wheel is decreased by 5% that at the inlet. The turbine working against a net head of 200 m has speed ratio of 0.45. The jet of water gets deflected at an angle of 165° by the buckets. Calculate (a) The whirl velocity at entry and exit, and (b) Absolute velocity of flow at exit of the bucket. Assume coefficient of velocity of the nozzle as 0.98.

[*Ans:* (a) $C_{w1} = 61.38$ m/s, $C_{w2} = 2.266$ m/s, (b) $C_2 = 8.47$ m/s]

3.16 Water flows at the rate of 0.2 m^3/s through a Pelton wheel which is operating against a head of 180 m to generate 300 kW power. The relative velocity of water remains unchanged till the exit while passing through the buckets. The coefficient of velocity of the nozzle is 0.985 and the angle of deflection of jet is 165°. Assuming mechanical efficiency to be 100%, calculate (a) the hydraulic efficiency, (b) the mean bucket speed, and (c) the velocity of whirl at the inlet and outlet.

[*Ans:* (a) $\eta_h = 0.851$, (b) $C_b = 19.665$ m/s, (c) $C_{w1} = 58.54$ m/s, $C_{w2} = 17.89$ m/s]

3.17 The initial velocity of the jet in a Pelton wheel is 96 m/s and the jet is deflected by 170°. The peripheral velocity of the wheel at the pitch circle is 44 m/s. Calculate (a) the kinetic energy of the jet at exit of the bucket, and (b) the magnitude and direction of absolute velocity of the jet at bucket exit. Assume the bucket friction coefficient as 1.0.

$$\left[Ans: \text{(a)}\ \frac{C_2^2}{2g} = 6.81 \text{ m, (b) } C_2 = 11.155 \text{ m/s, } \alpha_2 = 51.39°\right]$$

3.18 A water jet of diameter 120 mm impinges on the buckets of single-jet Pelton turbine operating under a net head of 150 m. The jet ratio and bucket friction coefficient are 12 and 0.96 respectively. The bucket angle and coefficient of velocity of the jet are 170° and 0.985 respectively. Calculate (a) overall efficiency, (b) starting torque, and (c) torque at normal speed. Assume mechanical efficiency as 0.96 and speed ratio as 0.45.

[*Ans:* (a) $\eta_o = 0.899$, (b) $T_0 = 45.14$ kN · m, (c) $T = 24.52$ kN·m]

3.19 A 2-jet Pelton wheel operating against a head of 60 m generates 1 MW power. The coefficient of velocity of the nozzle is 0.97 and overall efficiency of the turbine is 90%. Calculate (a) the pitch circle diameter of the wheel, (b) total discharge, and (c) specific speed of the wheel. Assume jet ratio as 12 and speed ratio as 0.46. [*Ans:* (a) $D = 2.28$ m, (b) $Q_t = 1.892$ m^3/s, (c) $N_s = 17.7$]

3.20 A Pelton wheel of diameter 2.0 m develops 5.0 MW power at a speed of 600 rpm. 5% of the energy is lost in friction while flowing through the buckets and the jet is deflected at an angle of 165° by the buckets. The mechanical efficiency of the turbine is 92% and the coefficient of velocity of the nozzle is 0.98. Compute (a) the overall efficiency, and (b) discharge through the turbine. Assume the speed ratio as 0.45. [*Ans:* (a) $\eta_o = 86.3\%$ (b) $Q = 0.596$ m^3/s]

3.21 A Pelton wheel of diameter 2.2 m is working under a head of 400 m at normal operating speed of 350 rpm. The water flows at the rate of 0.75 m^3/s and the jet deflects at an angle of 170° by the buckets. Bucket friction coefficient is 1.0 and coefficient of velocity of jet is 0.98. If the mechanical efficiency is 93%, calculate (a) hydraulic efficiency, (b) overall efficiency, (c) the power developed, (d) starting torque and torque at normal speed, and (e) theoretical runaway speed.

[*Ans:* (a) $\eta_h = 94.8\%$ (b) $\eta_o = 88.2\%$ (c) $P_{\text{shaft}} = 259$ kW
(d) $T_0 = 132$ kN·m, $T = 70.7$ kN·m (e) $N_r = 754$ rpm]

3.22 The diameter and width of the runner at the inlet of a Francis turbine are 3.0 m and 0.9 m respectively. The turbine extracts 8000 kW of power from the flow rate of 30 m^3/s while running at a speed of 200 rpm. Calculate the angle at which the wicket gates of the turbine should be set? The flow leaves the runner radially and the blade angle is obtuse. [*Ans:* $\alpha_1 = 22.58°$]

3.23 The diameters at the inlet and outlet of an inward–flow reaction turbine are 1.2 m and 0.6 m, respectively. The widths at the inlet and outlet are 0.25 m and 0.35 m respectively. The relative velocity at the entrance is 3.5 m/s and it is radial at a turbine speed of 250 rpm. Calculate (a) the magnitude and direction of absolute velocity at inlet, (b) discharge, and (c) the velocity of flow at the exit. [*Ans:* (a) $C_1 = 16.093$ m/s, $\alpha_1 = 12.56°$ (b) $Q = 3.299$ m^3/s (c) $C_{f2} = 5.0$ m/s]

3.24 The guide-blade angle and inlet blade angle in a reaction turbine are 20° and 60° respectively. The turbine working under a net head of 12 m has constant flow velocity and radial discharge at the exit. Determine the hydraulic efficiency of the turbine. [*Ans:* $\eta_h = 94.8\%$]

3.25 An inward flow reaction turbine has external diameter of 1.2 m and internal diameter of 0.6 m respectively. The width at the inlet is 0.20 m and it is 0.3 m at the outlet. The velocity of flow at the entrance of the turbine is 5 m/s. The vane thickness coefficient at both inlet and outlet is 0.95 and the speed of the turbine is 300 rpm. Assuming the discharge to be radial at outlet, calculate (a) the discharge and velocity of flow at the outlet, (b) the blade angle at the outlet.
[*Ans:* (a) $Q = 3.5814$ m^3/s, $C_{f2} = 6.67$ m/s, (b) $\beta_2 = 19.486°$]

3.26 The external and internal diameters of the runner of an inward flow reaction turbine are 1.2 m and 0.6 m respectively. The speed of rotation of the turbine is 300 rpm. The blades occupy 5% of peripheral area and the water enters radially at the rate 3 m^3/s. The width of the blades at the inlet and outlet are 0.25 m and 0.3 m respectively. Determine (a) the flow velocities at the inlet and outlet of the runner, and (b) inlet vane angle. [*Ans:* (a) $C_{f1} = 3.35$ m/s, $C_{f2} = 5.58$ m/s, (b) $\alpha_1 = 10.078°$]

3.27 Water flows at the rate of 8 m^3/s in a Francis turbine which is running at a speed of 250 rpm. The runner of the turbine has inlet diameter of 2.0 m and outlet diameter of 1.2 m whereas the breadth of the blades is constant at 0.2 m. The blades are radial at the inlet and the discharge is radial at the outlet. Calculate (a) Guide vane angle at the inlet, (b) Blade angle at the outlet.
[*Ans:* (a) $\alpha_1 = 13.67°$, (b) $\beta_2 = 34.04°$]

3.28 A Francis turbine develops 100 kW under a head of 16 m at a speed of 375 rpm. The inner and outer diameters of the runner are 0.5 m and 0.9 m respectively. The water flows at the rate of 0.75 m^3/s through the turbine and the discharge is radial at the exit with an outlet velocity of 3 m/s. Determine (a) hydraulic efficiency and overall efficiency of the turbine, and (b) the guide blade angle and blade angle at the inlet. The width of the runner is constant.
[*Ans:* (a) $\eta_h = 0.971$, $\eta_o = 0.85$, (b) $\alpha_1 = 10.935°$, $\beta_1 = 10.44°$]

3.29 The vanes of a reaction turbine are radial at the inlet and are inclined backwards to make a blade angle of 30° at the exit. The diameter of the runner at the inlet is twice that at the exit, whereas the width at the entry is one-half of that at the exit. The guide vane angle is 15° and the velocity of water leaving the guide vane is 25 m/s. Determine (a) the peripheral velocity at the entrance of the runner, and (b) the absolute velocity of water at the exit. [*Ans:* (a) $C_{b1} = 24.15$ m/s, (b) $C_2 = 6.527$ m/s]

3.30 A Francis turbine operating at a speed 300 rpm develops 4 MW power under a head of 30 m. The guide vane angle at the inlet is 30° and radial velocity of flow at the inlet is 7 m/s. The blades occupy 5% of the peripheral area. Calculate (a) diameter of the runner, and (b) width of the runner at inlet. Assume overall efficiency of the turbine as 85% and the hydraulic efficiency as 90%.
[*Ans:* (a) $D_1 = 1.39$ m, (b) $B_1 = 0.552$ m]

3.31 The diameters of the runner at the entrance and exit in a Francis turbine are 1.0 m and 0.5 m respectively. At the inlet, the guide vane angle is 15° and blade angle is 90°. The hydraulic efficiency is 95% and water leaves the blades with no whirl velocity at outlet. The radial velocity of flow is constant and the turbine operates under a net head of 30 m. Calculate (a) speed of the turbine, and (b) the blade angle at the exit. State whether the above calculated speed is synchronous with 50 Hz frequency? If it is not, what should be the speed of operation to directly couple the turbine with an alternator of 50 Hz frequency?

[*Ans:* (a) $N = 319.3$ rpm, (b) $\beta_2 = 28.18°$, Synchronous speed = 300 rpm]

3.32 The external diameter of the runner in a Francis turbine is 1.5 m whereas width of the runner is 0.22 m. The output power of the turbine is 2500 kW under an operating head of 45 m. The guide vane angle at the inlet is 20° and the blade angle at the inlet is 120°. The specific speed of the turbine is 135 and discharge at the outlet is radial. Determine (a) the hydraulic efficiency, and (b) overall efficiency.

[*Ans:* (a) $\eta_h = 89.3\%$, (b) $\eta_o = 83.3\%$]

3.33 The speed ratio in a Francis turbine is 0.80 and its flow ratio is 0.25. The width of the runner at the outer circumference is 1/4 of the external diameter. Calculate the specific speed of the turbine.

[$N_s = 179.3$]

3.34 The guide blades of an inward flow reaction turbine are at an angle of 15° with the tangent at the inlet. The turbine develops 750 kW under a net head of 100 m, while running at a speed of 750 rpm. The width of the blades at inlet is 0.1 times the inlet diameter and the blades thickness occupy 5% of the inlet area. The hydraulic efficiency of the turbine is 88% and overall efficiency is 84%. Determine (a) the diameter of the wheel at the inlet, and (b) blade angle at the inlet.

[*Ans:* (a) $D_1 = 0.519$ m, (b) $\beta_1 = 152.68°$]

3.35 The outer and inner diameters of a Francis turbine are 0.70 m and 0.35 m respectively. The turbine is working under a total head of 25 m. The blade tip is radial at inlet and the flow exits the turbine radially. The guide blade angle at the inlet is 12° and the velocity of flow is constant at 4.0 m/s. Assuming a hydraulic efficiency of 90%, calculate (a) the speed of the runner, and (b) blade angle at the exit. [*Ans:* (a) $N = 398.7$ rpm, (b) $\beta_2 = 23.03°$]

3.36 The rotor blade angle at the inlet of a Francis turbine is 61°. The turbine operates under a head of 160 m and the discharge is 110 m^3/s. The absolute velocity at outlet is radial with a magnitude of 15.5 m/s and the radial component of the velocity at the inlet is 10.3 m/s. Find (a) the power developed by the turbine, and (b) the degree of reaction. Assume 88% of the available head is converted into work.

[*Ans:* (a) $P_{shaft} = 151.63$ MW, (b) $R = 0.62$]

3.37 A Francis turbine of 76% overall efficiency generates 105 kW against a head of 12 m while operating at a speed of 150 rpm. The peripheral velocity and velocity of flow at inlet are 10 m/s are 5 m/s respectively. The hydraulic losses are 20% of the available energy. Calculate (a) the guide vane angle at the inlet, (b) blade angle at the inlet, and (c) width of the wheel at the inlet.

[*Ans:* (a) $\alpha_1 = 27.96°$, (b) $\beta_1 = 83.36°$, (c) $B_1 = 58.8$ mm]

3.38 Water flows at the rate of 10 m^3/s through a Francis turbine operating under a head of 30 m while running at a speed of 300 rpm. At the inlet tip of the runner blades, the peripheral velocity is $0.9\sqrt{2gH}$ and radial velocity of flow is $0.3\sqrt{2gH}$ where H represents the head on the turbine. The overall and hydraulic efficiencies of the turbine are 80% and 90% respectively. Determine (a) the power developed, (b) the guide blade angle at the inlet, (c) the runner blade angle at the inlet, (d) the specific speed of the turbine, and (e) diameter and width of the runner at the inlet.

[*Ans:* (a) $P_{\text{shaft}} = 2349.6$ kW, (b) $\alpha_1 = 30.98°$, (c) $\beta_1 = 36.88°$, (d) $N_s = 207$, (e) $D_1 = 1.39$ m, $B_1 = 314.6$ mm]

3.39 A Kaplan turbine with an overall efficiency of 90% is working under a net head of 5 m and develops 6 MW power. The hub diameter is 40% of the external diameter of the runner. The speed ratio of the turbine is 2.0 and flow ratio is 0.6. Determine (a) the diameter of the turbine, (b) specific speed, and (c) shape factor in revolutions as well as in radians.

[*Ans:* (a) $D_1 = 5.893$ m, (b) $N_s = 665$, (c) $N_{sh} = 0.639$ revolutions, $N_{sh} = 4.01$ radians]

3.40 A Kaplan turbine having speed ratio 1.8 and flow ratio of 0.5 develops 1 MW power while working against a head of 4.5 m. The overall efficiency is 90% and the ratio of hub diameter to blade diameter is 0.35. Find (a) the diameter of the runner, (b) speed of rotation, and (c) specific speed.

[*Ans:* (a) $D_o = 2.79$ m, (b) $N = 115.8$ rpm, (c) $N_s = 558.7$]

3.41 The water flows at the rate of 120 m^3/s through a Kaplan turbine which has external diameter of the runner as 4.5 m and hub diameter as 2.0 m. The guide vane angle is 59° and the inlet blade angle at the tip is 19°. The hydraulic and overall efficiencies of the turbine are 92% and 90% respectively. Assuming purely axial discharge at the outlet, determine (a) speed, and (b) power developed by the turbine. [*Ans:* (a) $N = 139.8$ rpm, (b) $P = 21.8$ MW]

3.42 The guide blade angle is 50° and the inlet blade angle at the tip of the blades is 14° in a propeller turbine operating at a speed of 150 rpm. The mid blade diameter of the turbine is 3.75 m and length of the blades along the radius is 1.25 m. The overall and hydraulic efficiencies of the turbine are 86% and 90% respectively. Calculate (a) the discharge through the turbine, (b) power developed, and (c) specific speed of the turbine. [*Ans:* (a) $Q = 119.25$ m^3/s, (b) $P_{\text{shaft}} = 30.335$ MW, (c) $N_s = 368.8$]

3.43 The runner of a Kaplan turbine has outer diameter of 4 m and hub diameter of 2 m. The turbine is required to develop 16 MW of power under a head of 25 m. The guide vane angle is 35° and the velocity of whirl is zero at outlet. The hydraulic efficiency is 90% and overall efficiency of the turbine is 85%. Determine (a) inlet and outlet blade angles at mid radius, and (b) specific speed of the turbine. [(a) $\beta_{1m} = 48.18°$, $\beta_{2m} = 23.30°$, (b) $N_s = 273$]

3.44 The hub diameter of a Kaplan turbine is 0.85 m and outer diameter of the blades is 2.5 m. The turbine develops 20 MW power under a head of 35 m at a rotational speed of 420 rpm. The hydraulic efficiency of the turbine is 88% and overall efficiency is 85%. Calculate (a) the discharge, (b) guide vane angle, and (c) the inlet blade angle at the tip.

[*Ans:* (a) $Q = 68.67$ m^3/s, (b) $\alpha_1 = 70.85°$, (c) $\beta_{1t} = 17.73°$]

3.45 Calculate the overall efficiency of a Kaplan turbine which develops 11.5 MW shaft power under a net head of 10 m at a speed of 96.3 rpm. The speed ratio is 1.8 m and flow ratio is 0.55. The radial length of the blades is 1.5 m. [*Ans:* $\eta_o = 92.5\%$]

3.46 A Kaplan turbine with an overall efficiency of 90% develops 15 MW power under a net head of 20 m. The diameters of the runner and hub are 4.25 m and 2.0 m respectively. The hydraulic efficiency is 93% and specific speed of the turbine is 380. The flow leaving the runner may be assumed to be completely axial. Calculate (a) inlet blade angles at the hub and tip, and (b) outlet blade angles at the hub and tip of the runner blades.

[*Ans:* (a) $\beta_{1h} = 86.57°$, $\beta_{1t} = 18.56°$, (b) $\beta_{2h} = 29.29°$, $\beta_{2t} = 14.79°$]

3.47 In a tidal power plant, a bulb turbine which is basically an axial flow turbine drives a 93% efficient generator of 5 MW at 150 rpm. The turbine is working against a head of 5.5 m with an overall efficiency of 88% and hydraulic efficiency of 94%. The tip diameter of the runner is 4.5 m and hub diameter is 2 m. Determine the vane angles of the runner at the inlet and exit at the mean diameter of the vanes with no whirl at exit. [*Ans:* $\beta_{1m} = 20.65°$, $\beta_{2m} = 19.16°$]

3.48 A propeller turbine operating at 40 rpm has a runner of diameter 4.5 m. The guide blade angle at entry is 145° and blade angle at runner outlet is 25° to the direction of blade motion. The axial flow area of water through the runner is 25 m^2. The blade angle at inlet of the turbine runner is radial. Determine (a) hydraulic efficiency, (b) discharge, (c) power generated, and (d) specific speed.
[*Ans*: (a) $\eta_h = 57.5\%$, (b) $Q = 164.75$ m^3/s, (c) $P_{\text{shaft}} = 6867$ kW, (d) $N_s = 251.62$ rpm]

3.49 A Kaplan turbine is installed in a hydroelectric power station. The turbine running at a speed of 72 rpm has runner diameter of 8.0 m and ratio of hub diameter to tip diameter of the runner as 0.4. The measured power output was 24.7 MW for a head of 10 m and discharge of 300 m^3/s during a test trial on the turbine. The installation has mechanical efficiency of 97% and generator efficiency of 96%. Determine (a) the hydraulic efficiency, and (b) the angles of the inlet and outlet velocity triangles at the tip and hub of the runner. The outlet may be assumed to be axial.
[*Ans:* (a) $\eta_h = 90.3\%$, (b) $\alpha_{1t} = 67.54°$, $\beta_{1t} = 14.63°$, $\beta_{2t} = 13.26°$,
$\alpha_{1h} = 44.06°$, $\beta_{1h} = 56.40°$, $\beta_{2h} = 30.5°$]

3.50 A propeller turbine running at 150 rpm is required to develop 22 MW of power under a head of 21 m. The external diameter of the runner is 4.5 m and hub diameter is 2.0 m. The hydraulic and overall efficiencies are 95% and 90% respectively. Assume the flow discharges through the turbine without whirl. Determine (a) runner blade angles at the outer periphery, and (b) runner blade angles at the hub. [*Ans:* (a) $\beta_{1t} = 17.36°$, $\beta_{2t} = 14.77°$, (b) $\beta_{1h} = 70.77°$, $\beta_{2h} = 30.67°$]

3.51 A water turbine has a velocity of 6 m/s at the entrance of the darft tube and a velocity of 1.2 m/s at the exit. The tailrace is 5 m below the entrance of the draft tube. Calculate the pressure head at the entrance of the draft tube if the friction losses in the draft tube are 0.1 m. [*Ans:* $p_2/\rho_w g = -6.662$ m]

3.52 A conical draft tube having diameter and pressure head at the top as 2 m and 7 m of water (vacuum) respectively. The draft tube discharges water at the outlet with a velocity of 1.2 m/s at the rate of 25 m^3/s. The losses between the inlet and outlet of the draft tube are negligible. Find the length of the draft tube immersed in water if the total length of the tube is 5 m. The atmospheric pressure head may be assumed as 10.3 m of water. [*Ans:* $y = 1.1536$ m]

3.53 The diameters at the inlet and outlet of a conical draft tube are 1.0 m and 1.5 m respectively. The total length of the draft tube is 6.0 m out of which 1.2 m is immersed in water. The draft tube discharges water at the outlet with a velocity of 2.5 m/s. The atmospheric pressure head is 10.3 m of water and loss of head due to friction in the draft tube is 0.2 times the velocity head at outlet of the tube. Find (a) pressure head at the inlet, and (b) efficiency of the draft tube.
[*Ans:* (a) $p_2/\rho_w g = 4.27$ m (abs), (b) $\eta_d = 76.3\%$]

3.54 A turbine is provided with a straight conical draft tube. The velocity at the exit of the turbine is 10 m/s and the velocity head at the exit of the draft tube is 1.0 m. The minimum pressure head in the turbine is set at 2.0 m (abs) to avoid cavitation and loss of head in the draft tube is 1.5 m. Estimate the maximum height at which turbine could be set above the tailrace. Assume atmospheric pressure as 10.3 m of water. [*Ans:* $H_s = 5.704$ m]

3.55 An elbow-type draft tube is fitted to the turbine which is set at a height of 2.0 m above the tailrace level. The draft tube has a circular section of 1.5 m^2 at the entry and a rectangular cross-section 12.5 m^2 area at the exit. The velocity at inlet of the draft tube is 12.0 m/s and frictional losses in the draft tube are 15% of the velocity head at the inlet. Calculate (a) the pressure head at the inlet of the draft tube, (b) power thrown away into the tailrace, and (c) efficiency of the draft tube.

[*Ans:* (a) $p_2/\rho_w g$ = 2.168 m (Absolute), (b) P_{LT} = 18.68 kW, (c) η_d = 84.8%]

3.56 A 80% efficient draft tube has an inlet area of 25 m^2 and an outlet area of 112.5 m^2. The inlet of the draft tube is 0.9 m above the tailrace where the velocity is 8.0 m/s. Calculate (a) the pressure head at the inlet of the draft tube, (b) the power lost in the draft tube, and (c) the power lost to the tailrace.

[*Ans:* (a) $p_2/\rho_w g$ = –3.381 m, (b) P_{Ld} = 1214 kW, (c) P_{LT} = 315 kW]

3.57 A propeller turbine operating under a net head of 25 m develops 7.0 MW of power. A vacuum gauge fitted to the exit of the turbine indicates a reading of 3.5 m pressure head. An elbow type draft tube fitted to the turbine has an inlet diameter of 3.0 m and an exit area of 38 m^2. The draft head is 2.5 m and efficiency of the draft tube is 80%. Calculate (a) the overall efficiency of the turbine, (b) the power lost in the draft tube, and (c) the power lost to the tailrace

. [*Ans:* (a) η_o = 80%, (b) P_{Ld} = 62.4 kW, (c) P_{LT} = 11.21 kW]

3.58 A turbine is working under a net head of 90 m. Water at the rate of 2.3 m^3/s flows through a 1 : 2 model turbine which develops 300 kW brake power under a net head of 15 m at 600 rpm. Calculate (a) the specific speed of the model, (b) overall efficiency of the model, (c) overall efficiency of the prototype, and (d) brake power of the prototype.

[*Ans:* (a) N_{sm} = 3.52, (b) η_{om} = 88.8%, (c) η_{op} = 90.2%, (d) $P_{shaft,\,p}$ = 17.91 MW

MULTIPLE CHOICE QUESTIONS

1. A tapered draft tube as compared to a cylindrical draft tube
   (a) Prevents hammer blow and surges
   (b) Responds better to load fluctuations
   (c) Converts more of kinetic head into pressure head
   (d) Prevents cavitation even under reduced discharges
2. A Pelton turbine is considered suitable for which of the following heads
   (a) 10–12 m (b) 20–30 m (c) 35–50 m (d) 100–250 m
3. Material for water turbine should have
   (a) High creep resistance (b) High temperature resistance
   (c) High corrosion resistance (d) Low ductility
4. A Pelton wheel is an
   (a) Outward flow reaction turbine (b) Inward flow impulse turbine
   (c) Axial flow impulse turbine (d) Mixed flow reaction turbine
5. In turbine installations, a “draft tube” converts
   (a) Kinetic head into pressure head (b) Pressure head into kinetic head
   (c) Pressure head into velocity head (d) Kinetic head into potential head
6. A bulb turbine is a
   (a) Low head turbine (b) Medium head turbine
   (c) High head turbine (d) High head, high speed turbine
7. Find the odd one out
   (a) Propeller turbine (b) Thompson’s turbine
   (c) Girard turbine (d) Four Neyron turbine

8. The specific speed of a hydraulic turbine is given by
   (a) $\frac{N\sqrt{P}}{H^{5/4}}$ (b) $\frac{P\sqrt{N}}{H^{5/4}}$
   (c) $\frac{NP}{H^{3/2}}$ (d) $\frac{\sqrt{NP}}{H^{3/2}}$
9. A turbine works at 20 m head and 500 rpm speed. Its 1:2 scale model to be tested at a head of 20m should have a rotational speed of nearly
   (a) 1000 rpm (b) 700 rpm (c) 500 rpm (d) 250 rpm
10. An impulse turbine
    (a) Always operates submerged
    (b) Makes use of a draft tube
    (c) Operates by initial conversion to kinetic energy
    (d) Converts pressure head into velocity head throughout the vanes
11. Which of the following statement is false regarding the relation of specific speed, $N_s = \frac{N\sqrt{P}}{H^{5/4}}$
    (a) The relation is valid for both impulse as well as reaction turbines
    (b) The value of specific speed depends on the system of units.
    (c) The value of specific speed will be lower than that for Kaplan turbine for any system of units.
    (d) The dimensions of specific speed are FLT^{1}
12. A nozzle kept vertical has a velocity head of 10 m at exit. The height attained by the jet is
    (a) 100 m (b) 10 m (c) 3.16 m (d) 0.316 m
13. Match List 1 with List 2 and select the correct answer using the following codes:

	List 1		List 2
A.	Draft tube	1.	Impulse turbine
B.	Surging	2.	Reciprocating pump
C.	Air vessel	3.	Reaction turbine
D.	Nozzle	4.	Centrifugal Pump

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	3	4	2	1
(c)	3	4	1	2
(d)	4	3	1	2

14. The purpose of movable wicket gates in a reaction turbine is to
    (a) Regulate discharge through the turbine
    (b) Regulate the pressure against which the turbine is working
    (c) Make the turbine casing strong
    (d) Decrease the turbine speed
15. Match List 1 and List 2 and select the correct answer using the following codes:

	List 1		List 2
A.	Propeller turbine	1.	Impulse turbine
B.	Tangential turbine	2.	Kaplan turbine
C.	Reaction is zero	3.	Gas turbine
D.	Reaction turbine	4.	Pelton turbine

Codes

	A	B	C	D
(a)	3	2	1	4
(b)	2	1	4	3
(c)	2	4	1	3
(d)	3	4	2	1

16. The available gross head of a Pelton turbine installed in a hydel power station is the vertical distance between
   (a) Forebay and tailrace
   (b) Reservoir level and turbine inlet
   (c) Forebay and turbine inlet
   (d) Reservoir level and tailrace

17. Match List I with List II and select the correct answer using the following codes:

List 1	**List 2 (Specific speeds)**
A. Kaplan turbine	1. 10–35
B. Francis turbine	2. 35–60
C. Pelton wheel with single jet	3. 60–300
D. Pelton wheel with two or more jets	4. 300–1000

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	3	4	2	1
(c)	3	4	1	2
(d)	4	3	2	1

18. Specific speed of a Kaplan turbine ranges between
   (a) 30 and 60
   (b) 60 and 300
   (c) 300 and 600
   (d) 600 and 1000

19. For a single stage impulse turbine with a rotor diameter of 2 m and a speed of 3000 rpm when the nozzle angle is 20°, the optimum velocity of steam in m/s is
   (a) 334 (b) 356 (c) 668 (d) 711

20. Match List I with List II and select the correct answer using the following codes:

List I	**List II**
A. Kaplan turbine	1. High values of speed and specific speed
B. Pelton wheel	2. Works under atmospheric pressure
C. Draft tube	3. High part load efficiency
D. Axial flow turbine	4. Pressure head recovery

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	4	3
(c)	3	2	4	1
(d)	4	3	1	2

21. In a general sense water turbines can be put under the following decreasing order of specific speeds as
   (a) Propeller turbine, Francis turbine and Pelton wheel

(b) Pelton wheel, Francis turbine and Kaplan turbine
(c) Kaplan turbine, Impulse turbine and Francis turbine
(d) Francis turbine, Kaplan turbine and Pelton wheel

22. A Francis turbine working against a net head of 49 m produces 5000 kW at 280 rpm with an overall efficiency of 80%. What would be the rpm of the same turbine under a net head of 64 m
(a) 640 rpm (b) 480 rpm (c) 320 rpm (d) 160 rpm

23. A turbine develops 8000 kW when running at 100 rpm. The head on the turbine is 36 m. If the head is decreased to 9 m, the power generated by the turbine will be
(a) 16000 kW (b) 4000 kW (c) 1414 kW (d) 1000 kW

24. In the case of Kaplan turbine the number of blades may be
(a) 100 to 125 (b) 50 to 60 (c) 20 to 25 (d) 3 to 6

25. In all reaction turbines, maximum efficiency is obtained if
(a) The guide vane angle is 90°
(b) The blade angle of the runners is 90° at the inlet
(c) The angle of the absolute velocity vector at the outlet is 90°
(d) The blade angle of the runners is 90° at the outlet

26. In a Francis turbine, the discharge leaves the runner radially at the exit. For this turbine
(a) The blade tip is radial at the exit
(b) The absolute velocity is radial at the exit
(c) The blade tip is radial at the inlet
(d) The guide vane angle is 90°

27. The specific speed for a turbine has the dimensions of
(a) T^{-1} (b) $F^{1/2}L^{-3/4/}T^{3/2}$
(c) Dimensionless (d) $F^{1/2}L^{-5/2}T^{3/2}$

28. Match List 1 and List 2 and select the correct answer using the following codes:

List 1 (Types of Turbines)	**List 2 (Characteristics of Turbines)**
A. Propeller turbine	1. Inward flow reaction turbine
B. Francis turbine	2. Tangential flow impulse turbine
C. Kaplan turbine	3. Axial flow reaction turbine with fixed blades
D. Pelton wheel	4. Axial flow reaction turbine with adjustable blades

Codes

	A	**B**	**C**	**D**
(a)	2	4	1	3
(b)	3	4	1	2
(c)	2	1	4	3
(d)	3	1	4	2

29. A single jet Pelton wheel is running at 600 rpm. The jet velocity at the exit of the nozzle is 100 m/s. If the ratio of the peripheral blade velocity to the jet velocity is 0.44, the diameter of the Pelton wheel is
(a) 0.7 m (b) 1.4 m (c) 2.1 m (d) 2.8 m

30. The maximum hydraulic efficiency of an ideal Pelton turbine in terms of blade angle at the outlet β_2, is

(a) $\frac{1+\cos\beta_2}{2}$ (b) $\frac{1-\cos\beta_2}{2}$

(c) $\frac{1-\sin\beta_2}{2}$ (d) $\frac{1+\sin\beta_2}{2}$

31. It is observed during the operation of hydraulic turbine that it has high design efficiency which remains constant over a wide range of control from the design condition. What is the type of this turbine?
(a) Pelton (b) Francis (c) Propeller (d) Kaplan

32. For the maximum efficiency of a Pelton turbine, the ratio of the blade speed to jet speed is
(a) 0.25 (b) 0.5 (c) 0.75 (d) 1

33. Which of the following options completes the correct statement?

Specific speed of a fluid machine
(a) Pertains to the speed of the machine of unit dimension
(b) Is a type number indicative of its performance
(c) Is specific to the particular machine
(d) Depends only upon the head against which the machine operates.

34. The overall efficiency of a Pelton turbine is 70%. If the mechanical efficiency is 85%, what is its hydraulic efficiency?
(a) 82.4% (b) 59.5% (c) 72.3% (d) 81.5%

35. In the statement: "In a set of reaction turbine, the requirement of water is reduced and the rpm is also reduced at a particular condition of working. The effect of each of these changes will be to *X* power delivered due to drop in head and to *Y* power delivered due to drop in rpm." *X* and *Y* stand, respectively, for
(a) Decrease and increase (b) Increase and increase
(c) Decrease and decrease (d) Increase and decrease

36. Examine the following statements given below regarding Kaplan turbine:
   1. It is a reaction turbine
   2. It is a mixed flow turbine
   3. It has adjustable blades

Which of the above statements are true?
(a) 1, 2 and 3 (b) 2 and 3 only
(c) 1 and 3 only (d) 1 and 2 only

37. Which of the following statements given below is not true in case of hydraulic turbines?
(a) Speed is proportional to 1/diameter (b) Power is proportional to speed^3
(c) Power is proportional to head$^{3/2}$ (d) Speed is proportional to head$^{1/2}$

38. Review the following statements regarding specific speed of turbomachines:
   1. It changes with shape of the runner blades and other parts of the machine
   2. Machines with higher specific speeds are restricted to low heads
   3. It is dimensionless and is independent of the type of fluid used

Which of the following statements are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

39. Which one of the following is correct?

The specific speed of a Pelton turbine if the number of jets in is n, then it is

(a) $\propto n^2$ (b) $\propto n$ (c) $\propto n^{1/2}$ (d) Independent of n

40. A turbine has a leakage of 0.1 m^3/s when the discharge supplied at the inlet is 2 m^3/s. If the hydraulic efficiency is 90%, overall efficiency is 75%, the mechanical efficiency of the turbine is

(a) 0.80 (b) 0.93 (c) 0.88 (d) 0.90

41. The head utilised in the runner of a hydraulic turbine is 72 m whereas the gross head available is 100 m. If the hydraulic efficiency of the turbine is 90%, the head lost in the pipe friction is

(a) 20 m (b) 18 m (c) 16.2 m (d) 1.8 m

42. In a two-jet Pelton turbine operating at full load, each of the nozzles issues jet of diameter 100 mm. The nozzles are so shaped that there is no further contraction of the jet. If the load is suddenly reduced to 36% of the full load, the altered jet diameter will be

(a) 18 mm (b) 30 mm (c) 36 mm (d) 60 mm

43. The length and diameter of the penstock supplying water to a turbine are 400 m and 1 m respectively. The velocity of water through the penstock is 5 m/s and Darcy-Weiesbach friction coefficient $f = 0.01$. If the gross head on the turbine is 300 m, the net head on the turbine would be approximately

(a) 200 m (b) 150 m (c) 310 m (d) 295 m

44. The net head H of a vertical axis impulse turbine is expressed as

(a) H = Gross Head – Head lost due to friction in the penstock – Height of turbine setting above trailrace
(b) H = Gross Head – Head lost due to friction in the penstock
(c) H = Gross Head – Head lost due to friction in the penstock – Head loss in the nozzle
(d) H = Gross Head + Velocity head at the nozzle-Head lost due to friction in the penstock

45. In a Pelton wheel installation, the net head H is the

(a) Difference in level between water levels at the forebay and the tailwater level,
(b) Difference in elevation between forebay water level and the nozzle outlet,
(c) Kinetic energy head of the jet issuing from the nozzle
(d) Head at the base of the nozzle

46. Consider the following water turbines:

1. Bulb 2. Kaplan 3. Pelton 4. Francis

The correct descending order of operating head while generating the same power is

(a) 2-3-1-4 (b) 4-1-3-2 (c) 3-4-2-1 (d) 1-2-4-3

47. Which of the following turbines given below retain a high efficiency during a long range of the part load operation?

1. Pelton turbine 2. Francis turbine 3. Propeller turbine 4. Kaplan turbine

Select the correct answer using the following code:

(a) 1 and 3 (B) 1 and 4 (c) 1, 2 and 4 (d) 1, 3 and 4

48. Which of the following statements is not correct with regard to selection of turbines on the basis of specific speed or head?

(a) Kaplan turbines are the best for specific speeds in the range of 10–35,
(b) Francis turbines are best for specific speed in the range of 60–300,
(c) Pelton turbines are suited for heads greater than 300 m,
(d) Francis turbines are suitable for the heads in the range of 50–150 m

49. The velocity of whirl in the inlet part of the jet impinging on a Pelton wheel is equal to
    (a) Zero (b) C_{r1} (c) C_1 (d) C_{b1}
50. Consider the following relations between the nozzle efficiency η_n, hydraulic efficiency η_h, wheel efficiency η_w, mechanical efficiency η_m and the overall efficiency η_o:
    1. $\eta_o = \eta_m \eta_h$
    2. $\eta_h = \eta_n \eta_w$
    3. $\eta_h = \eta_m \eta_n$
    4. $\eta_o = \eta_n \eta_{\mathrm{w}} \eta_m \eta_h$

    The correct relations are
    (a) 1 and 2 only (b) 3 and 4 only
    (c) 1 only (d) 2, 3 and 4
51. The degree of reaction in a Pelton turbine is
    (a) 1.0 (b) 0–0.5 (c) 0.5–1.0 (d) 0
52. Arrange the specific speed ranges of the following turbines: given below in ascending order
    1. Pelton 2. Kaplan 3. Francis
    (a) 3, 2, 1 (b) 1, 3, 2 (c) 1, 2, 3 (d) 2, 1, 3
53. A Pelton turbine having a diameter of 2.5 m is running at a speed of 300 rpm. Other data given are: coefficient of velocity = 0.975, speed ratio = 0.45 and jet ratio = 16. The theoretical runaway speed of this turbine is
    (a) 669 rpm (b) 650 rpm (c) 750 rpm (d) 4800 rpm
54. The speed ratio range for a Kaplan turbine at its best efficiency point is
    (a) 0.43–0.65 (b) 0.1–0.30 (c) 1.40–2.25 (d) 0.85–1.20
55. The head and specific speed relationship between for a Francis and Kaplan turbine is correctly represented by which one of the graphs shown in Figure 3.38?

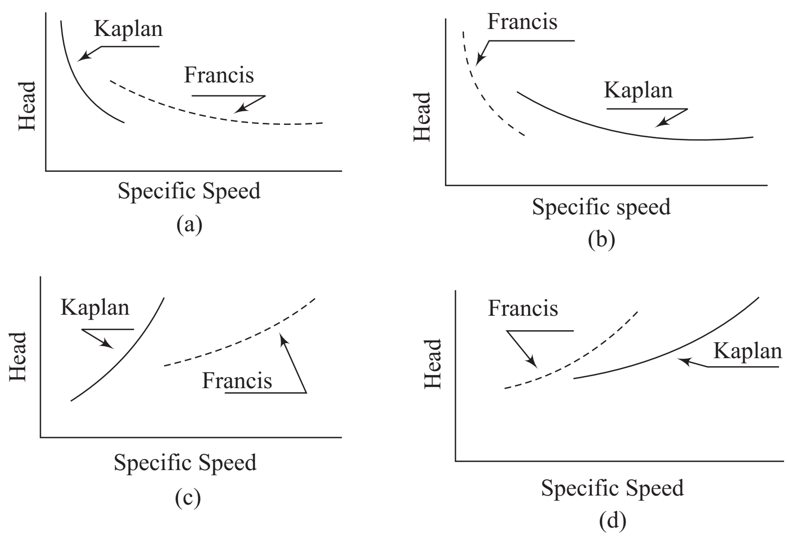


Figure 3.38 *Head vs Speed Speed for Turbines of Multiple Choice Question 55*

(a) Figure (b) (b) Figure (c) (c) Figure (a) (d) Figure (d)

56. Which of the following advantages is/are possessed by a Kaplan turbine over the Francis turbine?
    1. Relatively high part load efficiency
    2. More compact and smaller in size
    3. Low frictional losses

    The correct answer is
    (a) 2 and 3 only (b) 1 only (c) 1 and 2 only (d) 1, 2 and 3
57. The diameter of the hub is one-third of the diameter of the runner of a Kaplan turbine. The runner diameter is 3.0 m. The peripheral velocities and the velocity of flow at the inlet side of the blade at its tip are 40 m/s and 5 m/s respectively. The discharge through the runner will be
    (a) 251.3 m^3/s (b) 125.7 m^3/s (c) 31.4 m^3/s (d) 235.6 m^3/s
58. The speed ratio in a Kaplan turbine is 2.0 which is operating under a head of 35 m. If the external diameter of the runner is 2.0 m and the hub diameter is 0.6 m, the rotational speed is
    (a) 125 rpm (b) 500 rpm (c) 250 rpm (d) 150 rpm
59. The hub and runner diameter of a Kaplan turbine are 1.6 m and 4.0 m respectively. The flow velocity and whirl velocity at the inlet side of the blade at the hub are 6.0 m/s and 10.0 m/s respectively. The flow and swirl velocities at the inlet side of the tip are, respectively
    (a) 6.0 m/s and 4.0 m/s (b) 2.5 m/s and 10 m/s
    (c) 6.0 m/s and 25 m/s (d) 15 m/s and 10 m/s
60. The hub and runner diameter of a Kaplan turbine are 2.0 m and 5.0 m respectively. The peripheral velocity and whirl velocity at the inlet side of the blade tip are 40 m/s and 5.0 m/s respectively. The peripheral and swirl velocities at the mid radius of inlet section are, respectively,
    (a) 28.0 m/s and 5.0 m/s (b) 28.0 m/s and 7.1 m/s
    (c) 57.1 m/s and 3.5 m/s (d) 40.0 m/s and 7.1 m/s
61. In a Propeller turbine, the swirl velocity on the inlet side of the blades
    (a) Varies directly with the radius (b) Is constant along all the radius
    (c) Is maximum at the mid radius (d) Varies inversely with the radius
62. In a Kaplan turbine, the flow velocity
    (a) is directly proportional to the radius
    (b) is constant all along the radius
    (c) is maximum at the mid radius
    (d) is inversely proportional to the radius
63. In a Propeller turbine, the relative velocity at inlet
    (a) Is directly proportional to the radius
    (b) Is constant all along the radius
    (c) Is inversely proportional to the radius
    (d) None of these
64. The usual assumptions in one dimensional analysis of flow in a propeller turbine include which of the following statements:
    1. In frictionless, ideal flow, the relative velocity is constant all along the radius.
    2. Velocity of flow is constant all along the radius in both inlet and outlet sides of the blades.
    3. The peripheral velocity is directly proportional with the radius.
    4. The velocity of whirl is inversely proportional to the radius.

    The correct statements are
    (a) 1, 2 and 4 (b) 1, 3 and 4 (c) 2, 3 and 4 (d) Only 2

65. The angle between the absolute velocity at the inlet and the peripheral velocity at that radius
    (a) Is constant all along the radius
    (b) Is largest at the hub
    (c) Is largest at the mid radius
    (d) Is largest at the inlet
66. The ranges of speed ratio and flow ratio of Kaplan turbines are respectively
    (a) 1.30–2.25 and 0.35–0.75
    (b) 0.6–0.75 and 1.0–1.30
    (c) 0.4–0.8 and 0.1–0.4
    (d) 0.35–0.75 and 1.30–2.25
67. The absolute flow angle i.e. the angle between the peripheral velocity at any radial point and the absolute velocity of flow at that point in a Kaplan turbine
    (a) Is maximum at the mid radius
    (b) Increases from the tip of the blade towards the hub
    (c) Decreases from the tip of the blade towards the hub
    (d) Remains constant all along the radius
68. In a Kaplan turbine, the blade angle at inlet i.e. relative flow angle,
    (a) Decreases from the tip of the blade towards the hub
    (b) Increases from the tip of the blade towards the hub
    (c) Is always greater than the blade angle at outlet
    (d) Remains constant all along the radius
69. In a Kaplan turbine, the relative flow angle at the outlet i.e. the blade angle at the outlet
    (a) Increases from the tip of the blade towards the hub
    (b) Is maximum at the midradius
    (c) Decreases from the tip of the blade towards the hub
    (d) Remains constant all along the radius
70. In a multijet Pelton turbine, the flow from the penstock passes through the following element before entering the nozzles
    (a) Spiral casing
    (b) Volute chamber
    (c) Manifold
    (d) Draft tube
71. The velocity heads at the entrance and exit sections of a draft tube are 3.0 m and 0.2 m respectively. The friction and other losses in the draft tube are 0.4 m. What is the efficiency of the draft tube?
    (a) 14.4%
    (b) 92.3%
    (c) 86.7%
    (d) 85.7%
72. In axial flow turbine
    (a) Inlet is axial and outlet is radial
    (b) Inlet is axial and outlet is axial
    (c) Inlet is radial and outlet is axial
    (d) Inlet is radial and outlet is radial
73. Consider the following statements:
    1. The specific speed decreases if the degree of reaction is decreased.
    2. The degree of reaction is zero for Pelton wheel.
    3. The degree of reaction is between 0 and 1 for Francis turbine.
    4. The degree of reaction is between 0.5 and 1 for Kaplan turbine.

    Which of the above statements are correct?
    (a) 2, 3 and 4
    (b) 2 and 3
    (c) 1 and 2
    (d) 3 and 4
74. Which of the following turbines exhibits a nearly constant efficiency over 60% to 140% of design speed
    (a) Pelton wheel
    (b) Francis turbine
    (c) Deriz turbine
    (d) Kaplan turbine

75. The function of guide vanes in a reaction turbine is to
 (a) Allow the water to enter the runner without shock
 (b) Allow the water to flow through the vanes without forming eddies
 (c) Allow the required quantity of water to enter the turbine
 (d) All of these
76. The maximum number of jets generally useded in a Peltonwheel without jet interference are
 (a) 4 (b) 6 (c) 8 (d) Not defined
77. The number of buckets on the periphery of a Pelton wheel is given by
 (a) $5+\frac{m}{2}$ (b) $10+\frac{m}{2}$ (c) $15+\frac{m}{2}$ (d) $20+\frac{m}{2}$

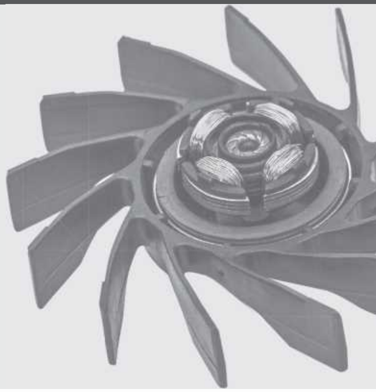
 where, m is the jet ratio given by, $m=\frac{D}{d}$
78. Hydraulic efficiency of a turbine is defined as
 (a) Power available at the rotor inlet of the turbine to the power extracted by rotor from water
 (b) Power at the rotor shaft to the power extracted by rotor
 (c) Power at the rotor shaft to the power at the rotor inlet
 (d) Power extracted by rotor from water to the power at the rotor inlet
79. Operating characteristic curves of a turbine means
 (a) Curve drawn at constant speed
 (b) Curve drawn at constant efficiency
 (c) Curve drawn at constant head
 (d) Curve drawn at constant load
80. As water flows passes through a reaction turbine rotor, pressure acting on it would vary from
 (a) Atmospheric pressure to vacuum
 (b) More than atmospheric pressure to vacuum
 (c) Less than atmospheric pressure to zero gauge pressure
 (d) Atmospheric pressure to more than atmospheric pressure

ANSWER KEY

1. (c)	2. (d)	3. (c)	4. (c)	5. (a)	6. (a)	7. (c)	8. (a)	9. (d)	10. (c)
11. (d)	12. (b)	13. (b)	14. (b)	15. (c)	16. (b)	17. (c)	18. (c, d)	19. (c)	20. (c)
21. (a)	22. (c)	23. (d)	24. (d)	25. (c)	26. (b)	27. (b)	28. (d)	29. (b)	30. (a)
31. (d)	32. (b)	33. (d)	34. (a)	35. (a)	36. (c)	37. (a)	38. (b)	39. (c)	40. (c)
41. (a)	42. (d)	43. (d)	44. (a)	45. (d)	46. (c)	47. (b)	48. (a)	49. (c)	50. (a)
51. (d)	52. (b)	53. (b)	54. (c)	55. (a)	56. (d)	57. (c)	58. (b)	59. (a)	60. (b)
61. (d)	62. (b)	63. (d)	64. (c)	65. (d)	66. (a)	67. (c)	68. (b)	69. (a)	70. (c)
71. (d)	72. (b)	73. (c)	74. (d)	75. (d)	76. (b)	77. (c)	78. (d)	79. (a)	80. (b).

4

Centrifugal and Axial Pumps



Learning Objectives

After reading this chapter, you will be able to:

- **LO 1** Explain the principle of operation and advantages of a centrifugal and axial pumps
- **LO 2** Know the classifications and principal parts of centrifugal and axial pumps
- **LO 3** Derive expressions for the work input, the various heads and efficiencies of a pump
- **LO 4** Discuss the performance characteristics of centrifugal and axial pumps
- **LO 5** Determine the minimum starting speed of a pump and loss of head due to reduced or increased flow
- **LO 6** Summarize the necessities and limitations of pumps in series and in parallel

4.1 Introduction

A pump is a device that transfers energy to the liquid. The increase in fluid energy is felt as increase in the pressure of the fluid or head of the fluid and kinetic energy of the fluid. The functions of the pump are to raise the pressure of fluid and impart desirable velocity to it so that fluid may be transported from one place to another; for example, a pump is used to raise the pressure of liquids as in municipal water works and drainage systems, agriculture and irrigation systems, tube wells, oil pumps in lubrication system, exploration of crude oil in petroleum industry etc. A pump can handle any kind of liquid, even contaminated with solid particulates to some degree. In power plants, pump is also used to handle ash slurry made with water which is highly erosive in nature. In chemical industries, it is used to handle corrosive fluids; and so on. However, its performance is affected by the increase in viscosity of the liquid and the presence of contamination in it. Large pumps are used in mines, flood water control, and for feeding water to the boilers. The size and shapes of the pumps vary widely in their vast range of usages.

In a pump, fluid flows in the direction of increasing pressure. Therefore, chances of separation and back flow are quite high, which results in decrease in efficiency and unstable operation of the pump. For this reason, the use of rotodynamic pumps is limited to those applications where moderate pressure rise (a few bar) and high volume flow rate (up to 20 m^3/s) are required.

Pumps may be broadly classified as roto-dynamic and positive displacement types. In rotodynamic pumps, energy is imparted to the fluid by means of the blades of a member on the shaft known as *impeller*. The *impeller* is mounted on a shaft driven by device such as electric motor or an IC engine. Positive displacement pumps discharge a fixed volume of liquid successfully by the action of a pumping element such as piston, vane, gear or screw in a stationary housing i.e. closed volume or cavity. Positive displacement pumps are not in the category of turbomachinery and are briefly discussed in Chapter 10.

4.2 Centrifugal Pump

This pump consists of an impeller rotating within a casing. Fluid enters axially through the *eye* of the casing. It is caught up in the impeller blades and is whirled tangentially and radially outwards until it leaves through the circumference of the impeller into the diffuser portion of the casing. The fluid gains both velocity and pressure while passing through the impeller. The doughnut-shaped diffuser, or *scroll* section of the casing decelerates the flow and further increases the pressure. Thus, the centrifugal pump, by its principle, is converse of the Francis turbine. In horizontal axis centrifugal pump, the shaft on which the impeller is mounted is disposed horizontally. Generally, pumps are provided with horizontal shafts. A low head centrifugal pump is suitable for operating under a total head of 15 m. A medium head pump is used for operating under a total head greater than 15 m but up to 40 m. If the pump works against a total head above 40 m, it is known as a high head centrifugal pump. Generally, high head pumps are multistage pumps.

4.2.1 Single Stage Centrifugal Pump

A single stage centrifugal pump has only one impeller mounted on the shaft which is used to develop pressure necessary to lift required discharge against the given head. The limiting head is about 38 m against which a single impeller may be used. It is not economical to use a single impeller for higher head because then either a high rotational speed or larger diameter impeller must be used. Consequently, high mechanical stresses and poor efficiency result in either case because of the disc friction and leakage losses. Leakage loss is the limited back flow of the fluid to the suction side due to the clearance between impeller and the surrounding casing which results in decreased efficiency of the pump. Under very high heads, the leakage loss may become considerably high due to larger difference in pressure between the discharge and suction sides of the impeller.

(a) Volute Pump

In volute type of centrifugal pumps, the impeller is surrounded by a spiral casing, the outer boundary of which is a curve called volute. The absolute velocity of the fluid leaving the impeller is reduced in the volute casing. By passing the fluid through the gradually increasing cross sectional area of the volute casing, a part of its kinetic energy gets converted into pressure energy.

(b) Volute Pump with Vortex or Whirlpool Chamber

In this type of pump, an annular space or circular chamber is provided between the impeller and volute casing. The circular chamber is known as *vortex or whirlpool chamber* and such a pump is known as *volute pump with vortex or whirlpool chamber*. The inclusion of circular chamber decreases energy loss due to the formation of eddies by a significant amount.

(c) Diffuser or Turbine Pump

The diffuser pump is a centrifugal pump having a casing with guide blades or vanned diffuser. A diffuser pump is also called turbine pump due superficial similarity with a reaction turbine. Experimental results confirm that a good design of diffuser pump has ability to convert as much as 75% of the kinetic energy of the fluid at the exit of the impeller into pressure energy. A diffuser pump is more efficient because the guide vanes provide more opportunity for gradual reduction of velocity of the fluid so that less energy is wasted in eddies.

The efficiency of a pump in any of the cases mentioned is normally lower than efficiency of a turbine. The losses of energy in the turbine and pump are of the same types. However, a pump has divergent flow passages, whereas a turbine has convergent flow passages. Therefore, there may be quick separation of flow away from the boundaries in a pump resulting in losses of energy in the form of eddies.

(d) Double Suction Centrifugal Pump

A double suction pump is shown in Figure 4.1. The fluid admits from both sides of the impeller in a double suction pump. The impeller then often resembles as two single suction impellers placed back to back. The advantage of using this arrangement is that the thrust on each side of the impeller resulting from the change in the flow direction are balanced. A double suction pump provides a large inlet area that results in high discharge of fluid.

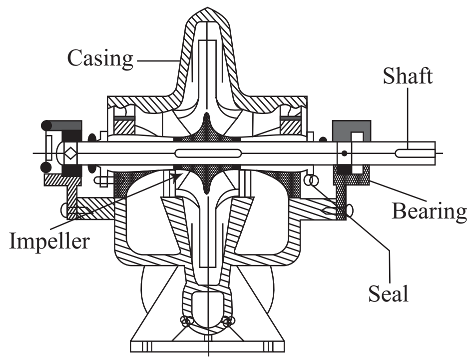


Figure 4.1 *Double Suction Centrifugal Pump*

4.2.2 Multistage Centrifugal Pump

A multistage centrifugal pump has two or more impellers connected in series, mounted on the same shaft and are enclosed in the same casing. In multistage pump, fluid discharged with increased pressure from one impeller flows to the suction of second and so on. The quantity pumped in a multistage pump is same as that passing through a single impeller, but the total head created is the sum of the pressure head developed by each impeller. Pumps using up to ten stages have been built. However, number of stages is limited to five from mechanical construction point of view. Therefore, use two such pumps in series, if required.

(a) Deep Well Pumps

Deep well pumps also known as *borewell pumps* are vertical, multistage centrifugal pumps used for pumping from deep tubewells/borewells. The volute casing of conventional centrifugal pumps is replaced by *bowl assemblies*. The bowls encase the impellers and guide vanes. All the impellers are connected to a common shaft. The whole assembly is always placed below the water surface.

(i) Submersible Pumps Submersible pump is a vertical turbine pump in which a specially designed submersible electric motor of cylindrical shape is closely coupled to the pump at the bottom as shown in Figure 4.2.

These pumps are mounted with vertical shafts as they occupy less space. The pump and the motor assembly are dipped into the well i.e. it remains submerged. A highly insulated electric cable from the motor traverses up the well and is connected to the power source at the top. The long vertical shaft and column assembly required in the vertical turbine pump is eliminated in the submersible pump. This is one of the main advantages of submersible pump that enables it to be used in very deep wells where the use of longer vertical shaft is not feasible. Further, the elimination of a long shaft reduces the bearing friction and provides an unobstructed pipe for delivery of water to the surface. In the submersible pump installations, the alignment of the well is not at all a consideration as the well need not be truly vertical. Furthermore, the need of a pump house is also eliminated. Submersible pumps are used in the domestic water supply from the small to medium sized tubewells/borewells.

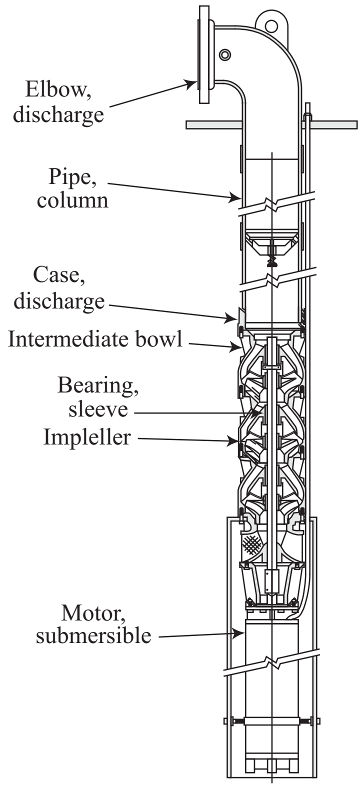


Figure 4.2 *A Submersible Pump*

4.3 Axial Flow or Propeller Pumps

In axial flow or propeller pump, the flow at outlet of the moving blades of the impeller of a pump is parallel to the shaft**.** Its working principle is just the reverse of the Kaplan or propeller turbine. It is used for larger discharge and lower head. An axial flow pump impeller is shown in Figure 4.3.

The shape of the impeller of a rotodynamic pump changes progressively with specific speed. The radial flow and mixed flow pumps are used in the specific speed ranges of $10 < N_s < 100$ and $80 < N_s < 220$, respectively.

For specific speed $N_s > 200$, the resulting impeller shape is that of a propeller pump in which flow enters and discharges axially. The propeller pump resembles the shape of a Kaplan or propeller turbine. Such low delivery head (< 12 m) and large discharge pumps with axial flow are called *axial flow or propeller pumps* for which the specific speeds lie in the range of 180–300. The suction lift has to be small for these pumps. These are suitable for irrigation, drainage, sewage etc. The axial flow pump assembly is shown in Figure 4.4. The components of the axial flow pump assembly are as follows:

- A bell shaped entry with the inlet guide vanes,
- Propeller assembly on a shaft that is connected to the prime mover,
- Exit guide vane set to remove whirl component of the flow,
- Cylindrical casing that has close clearance with the propeller. Upper part of the casing is connected to a vertical diverging section. The vertical portion of the assembly is known as *column assembly*.

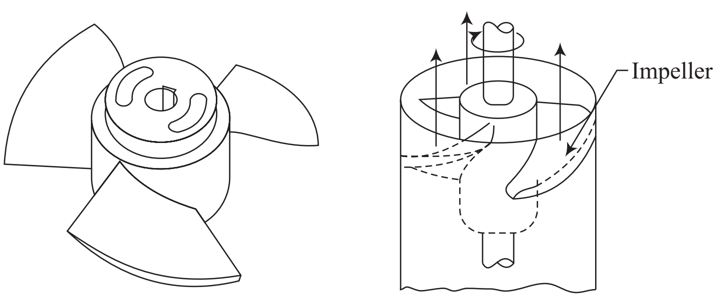


Figure 4.3: Axial Flow Pump Impeller

- The delivery pipe takes off through an easy elbow/bend from the column assembly. The drive shaft emerges out of the conduit at the bend. The discharge flange can be above or below the floor level. The intake is normally through a well-designed intake structure.

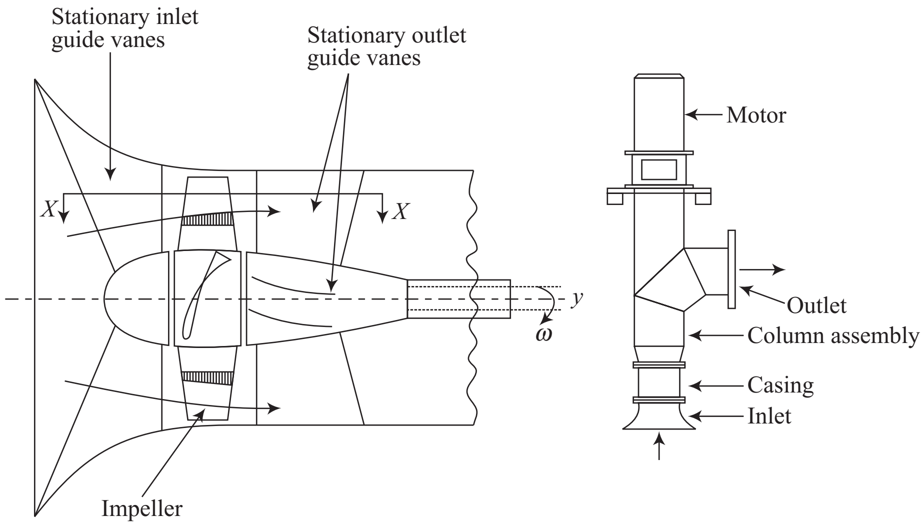


Figure 4.4 *An Axial Flow Pump Assembly*

The water from the sump well is sucked into the suction pipe in an axial direction, passed through a set of guide blades into the runner vanes (impeller) and finally discharged through another set of guide blades into the delivery pipe. Pressure is developed by flow of water over the aerofoil shaped impeller blades which looks like the propeller of a marine ship. This situation is similar to the wings of an aeroplane which produce lift. The impeller comprises a central hub on which twisted and aerofoil shaped blades are mounted, usually 3 to 8 in numbers. The impeller rotates inside a casing of sufficient length that ensures uniform incoming and outgoing flow. The clearance between the casing and rotating impeller is kept very small to reduce the leakage loss. Axial flow pumps generally have a vertical disposition of the shaft, mainly for the sake of compactness.

4.4 Mixed Flow or Half AxialPump

A mixed flow pump is the reverse of the inward flow reaction turbine (Francis turbine). The direction of the flow at the outlet of the pump is between axial and radial. Figure 4.5 shows the impeller of a mixed flow or half axial pump.

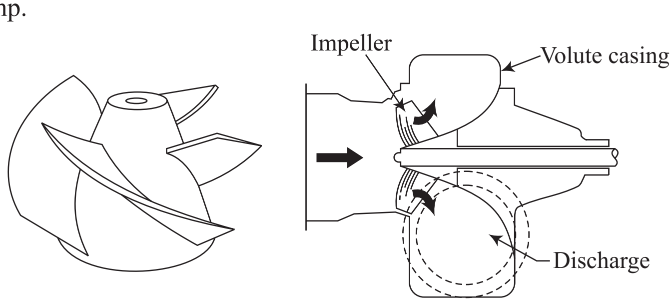


Figure 4.5 *Mixed Flower Half Axial Pump Rotor*

Common centrifugal pumps are essentially radial flow pumps in which the head is developed by the centrifugal force on the liquid. The liquid enters the impeller axially at the center and flows radially to the periphery. The flow is radial in the impeller and the exit flow is in the plane of the impeller that is normal to the axis of the shaft. The centrifugal pumps are best suited for high heads and low discharge installations. As the discharge increases and the head decreases relatively, the pure radial units become very unwieldy and slow. However, by suitably changing the configuration of the impeller geometry, relatively higher specific speeds can be obtained by the efficient sized units. In the specific speed range of 10–100, the blades are purely radial. In the specific speed range of 80–220, the shapes of the vane undergo a major change and the flow is no more purely radial. It is a mixed flow of liquid transiting from axial at entrance to exit at an angle to the radial direction. Thus, the flow through the impeller is a mixed flow i.e. the combination of axial and radial flows. The head is developed partly by the action of centrifugal force and partly by axial propulsion as a result of which the liquid entering the impeller axially is discharged in an angular direction. The liquid experiences both radial acceleration and lift force. Such pumps are called *mixed flow pumps*. Mixed flow pumps resemble the shape of Francis turbine/screw. Mixed flow pumps are also known as *Francis type pump, semi axial flow pump* and *screw pump*. These pumps are best suited for handling moderate to large quantities of liquid at moderate to low heads. Originally developed for irrigation purpose, mixed flow pumps have been improved significantly through advances in technology and materials. Typical conventional applications of mixed flow pumps include pumping of drainage and lift irrigation whereas current use includes industrial use of all types, e.g. circulating water in condenser in power plants. These pumps find applications in single stage as well in multistage configuration. There are two types of single stage mixed flow pumps:

- Volute Type: Mixed flow volute type pumps find application in condensate cooling in power plants. Sometimes volute is made of concrete for very large pumps. They are also used in water supply with reservoir intake.
- Bowl Type: Mixed flow bowl type pumps are generally in vertical or inclined settings. They are used in lift irrigation, land drainage and flood management. Also, they are used in water supply from river or reservoir intake. Bowl assembly of these pumps is submerged in the intake well as there is no priming issue.

Mixed flow pumps are available in a wide range of heads and discharges. The normal average ranges of single stage mixed flow pump variables are as follows:

- Head = 3–30 m
- Discharge = 0.10–11 m^3/s
- Size = 200–1500 mm
- Specific Speed = 80–180
- Efficiency = 80–90%

Mixed flow pumps are available in plenty with higher range than the normal average range depending on specific applications. The characteristic features of the impellers of radial and mixed flow volute type pumps are shown in Figure 4.6 (a) and (b). Figure 4.6 (c) is a schematic diagram showing the details of a vertical bowl type pump. The bulblike feature, known as *bowl*, is a characteristic feature of a vertical mixed flow pump. In a pure axial flow vertical pump, this bowl will be a cylinder.

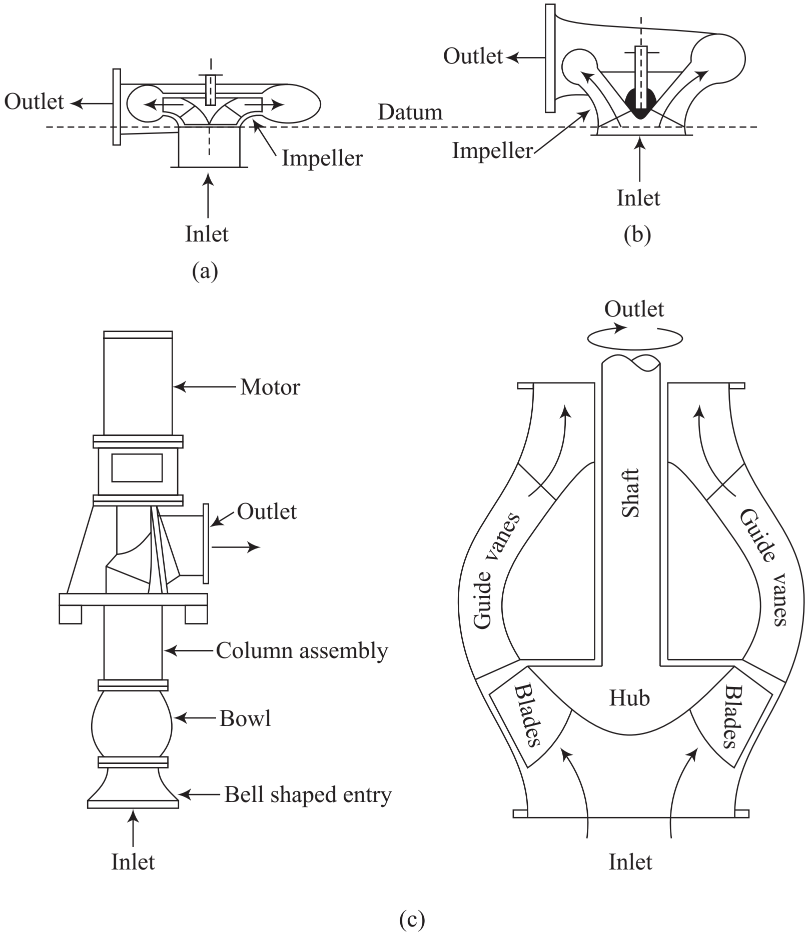


Figure 4.6 *Impeller Configuration of (a) Radial Flow Pump, (b) Mixed Flow Volute Pump, and (c) Schematic Diagram of Vertical Bowl Type Pump*

4.4.1 Vertical Turbine Pump

This is a multistage mixed flow pump. The term *turbine* refers to the fixed guide blades which replace the conventional volute chamber to act as diffuser to the flow from the impeller exit. The vertical turbine pump consists essentially of a pipe of rather limited size extending from the bottom of the well up to the top. A foot valve with a strainer is fixed at the bottom. An assembly of bowls containing impellers and fixed guide vanes is fixed above the suction pipe. All the impellers are connected in series to a common vertical shaft which extends up to the top of the well. Above the ground surface, at the top, the shaft is connected to the source of power viz. electric motor or IC engine. The diameter of the pump is limited by the diameter of the well. Figure 4.7 (a) shows a schematic diagram of a vertical turbine pump whereas Figure 4.7 (b) shows the details of the bowl assembly. This figure shows the three stage vertical turbine pump. Each stage in a pump contributes about 15 m of head to the pump. Vertical alignment of the well is very important in these pumps.

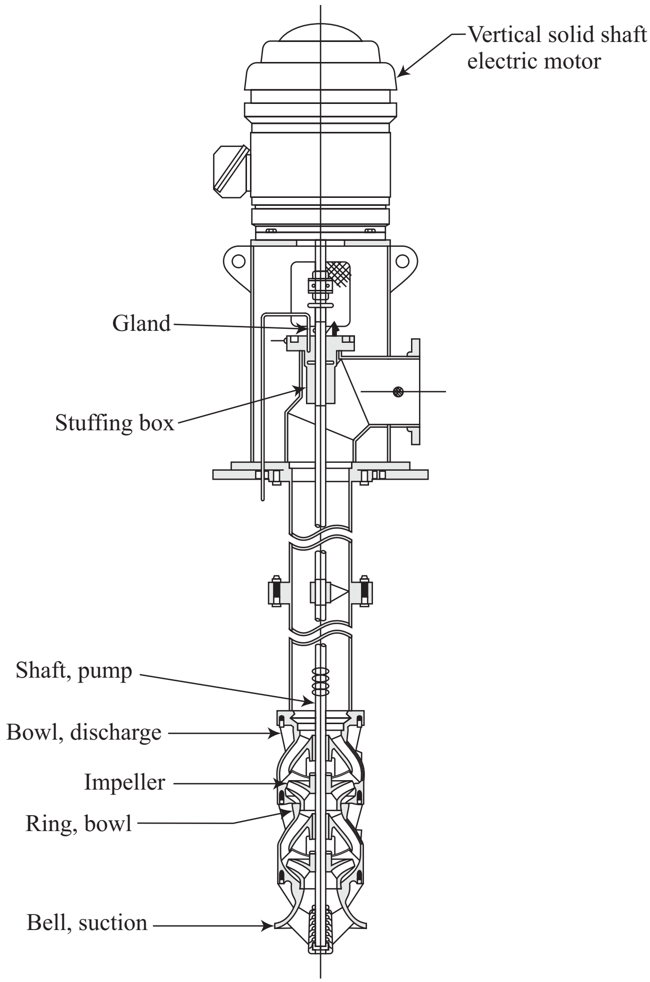


Figure 4.7 *A Vertical Turbine Pump*

The vertical turbine pump installations require a pump house at the top since the power drive is above the ground level. The pumps are robust; however, the initial cost is high. Deep well turbine pumps are widely used for pumping from large tubewells where the discharges are fairly large and heads are high. The efficiency of these pumps lies in the range of 70%.

4.5 Principal Parts of Centrifugal Pump

Figure 4.8 shows the principal parts of a centrifugal pump. They may be described as follows:

4.5.1 Impeller

The rotating part of a centrifugal pump is known as impeller. In pump terminology, impeller is a rotating assembly that consists of the shaft, the hub, the impeller blades and the shroud. The impeller is made of a

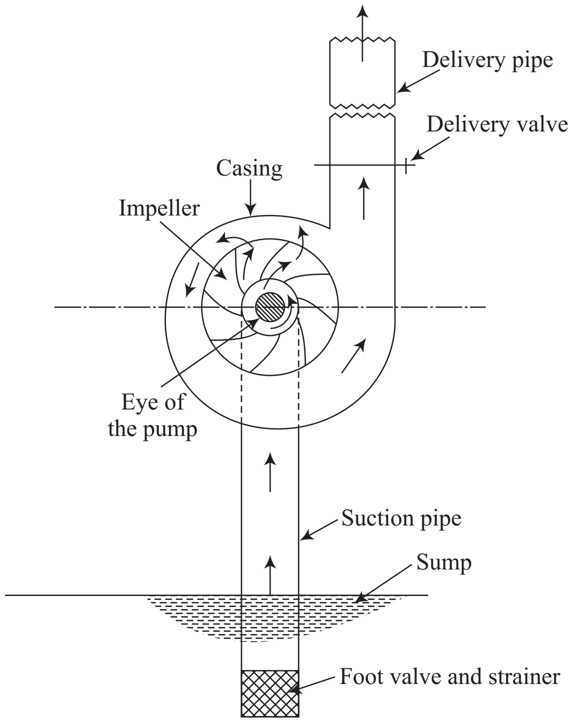


Figure 4.8 *Principal Parts of a Centrifugal Pump*

hub on which a number of vanes or blades standing vertically are casted at equiangular positions. The curvature of vanes is so designed that the fluid flows through blades without separation. The impeller blades are usually *backward curved,* but there are also *radial* or *forward curved blade designs,* which change the output pressure slightly. A shaft connects the hub to motor and passes through the pump casing which provides bearing for the shaft. A seal is provided between casing and shaft to prevent leakage of fluid.

Pressure across the passage between two consecutive vanes or blades is assumed to be the same, but due to rotation, pressure on the leading face of a blade is more than that on the trailing face (in another passage). Therefore, the fluid will have tendency to move around from leading face to trailing face in the space between casing and impeller causing losses. Therefore, the tips of the blades are sometimes covered by another flat disc or ring known as the *shroud* to prevent leaking across the blade tips from one passage to another. Sometimes the blades may be *unshrouded or open* in which they may be separated from the front casing only by a narrow clearance. It is quite obvious that pressure at the impeller exit is more than that at its entrance and that the exit and entrance are connected through space between casing and blade tip (clearance). Therefore, there are chances of back flow from exit to suction side of impeller through this casing space, i.e. clearance. Reducing clearance space can decrease the back flow.

4.5.2 Casing

The fluid enters axially into the centre of the impeller i.e. *eye* with little, if any, whirl component of the velocity. From there, it flows outward in the direction of blades, and is discharged with increased pressure and velocity into the casing due to the energy imparted by the impeller. Hence, the fluid has considerable tangential (whirl) component of velocity at the exit of the impeller which is generally much higher than that required in the delivery pipe. The kinetic energy of the fluid exiting the impeller is mostly dissipated in eddies or shock losses, consequently lowering efficiency of the pump. This necessitates arrangements to be made to decrease the velocity gradually. The conversion of greater fraction of kinetic energy at impeller outlet into useful pressure energy is obtained by the casing. Casing is an air-tight passage surrounding the impeller and is designed in such a way that the kinetic energy of water at the exit of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe. The gradual increase in the cross sectional area of the casing also provides uniform velocity of flow throughout the casing. The following types of the casings are commonly used.

(a) Volute or Spiral or Scroll Casing

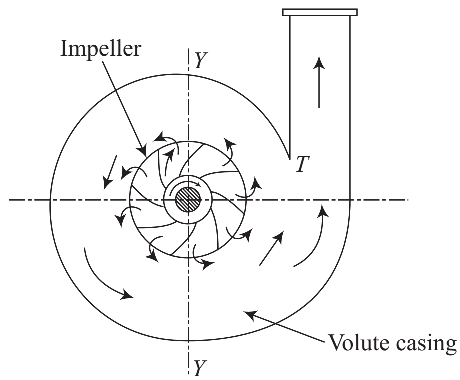


Figure 4.9 *Volute or Spiral Casing*

No external torque is exerted on the fluid particle after leaving the impeller apart from frictional effects. The angular momentum of fluid is therefore constant if friction is neglected. Thus, the fluid particles follow the path of a *free vortex.* The radial velocity at the impeller outlet remains constant round the circumference in an ideal case. The combined effect of uniform radial velocity with the free vortex ($C_w r$ = constant) provides a pattern of spiral streamlines that must be matched by the shape of the casing. This is the most important feature of the design of a pump. Therefore, the volute casing which surrounds the impeller is of spiral shape as shown in Figures 4.8 and 4.9. It is also known as the scroll casing, in which gradual increase in cross sectional area of flow around the periphery of the impeller occurs from tongue T towards the delivery pipe. The increase in area of flow decreases the velocity of flow which is quite obvious from the law of conservation of mass i.e. continuity equation. The decrease in velocity increases the pressure of the fluid flowing through the casing which is clear from the principle of conservation of energy i.e. Bernoulli's equation. The gradually increasing cross sectional area also results in maintaining a uniform velocity throughout the casing, because more and more liquid is added to the stream from the periphery of the impeller as the flow travels from the tongue T towards the discharge pipe.

The volute or spiral or scroll casing design may be such that the flow velocity is approximately equal to that of the fluid exiting the impeller. A casing design on this basis results into considerable decrease of loss of energy, but the conversion of the kinetic energy into useful pressure energy will not be possible. However, a casing design of lower casing velocity than that in the delivery pipe results into considerable energy loss due to the difference between fluid velocity in the casing and that exiting the impeller. Therefore, a compromise is often made in the design in which the casing is divergent such that gradual decrease in velocity is obtained from the velocity of fluid exiting the impeller to that in the delivery pipe.

However, in actual practice, it has been found that in volute type of casing there is only a slight increase in the efficiency of the pump because a considerable loss of energy takes place due to eddies developed in the casing.

(b) Vortex or Whirlpool Casing

If a circular chamber is introduced between the casing and the impeller as shown in Figure 4.10, the casing is known as *vortex or whirlpool casing* and such a pump is called *volute pump with vortex or whirlpool chamber*. The energy loss because of eddies formation is considerably decreased by introducing a circular chamber. The fluid from the outlet of the impeller admits with a whirling motion into the vortex chamber i.e. the fluid particles move radially away from the centre tracing a circular path during flow in this chamber. The energy of the fluid remains constant except infinitesimal loss due to friction as no work is done on it when it flows through vortex or whirlpool chamber. Therefore, torque exerted on the fluid remains the same indicating that a free vortex is formed when fluid flows through vortex chamber. There is a decrease in velocity of the fluid when it flows through the vortex chamber because the whirl velocity is inversely proportional with radial distance from the centre for a free vortex. The decrease in fluid velocity is occurred with a corresponding increase in pressure. Thus, a vortex or whirlpool chamber is used for two purposes: (i) decreasing the velocity, and (ii) increasing the pump efficiency by converting a greater portion of kinetic energy into pressure energy. The fluid exiting the whirlpool chamber flows through the surrounding volute chamber which further increases the efficiency of the pump.

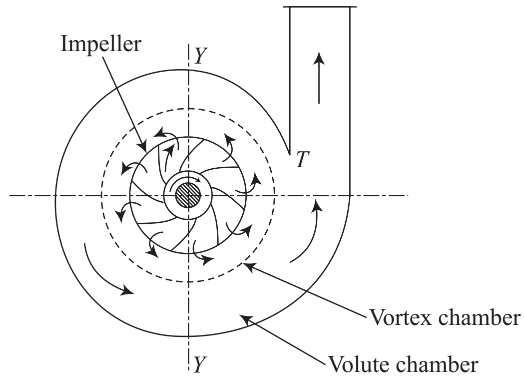


Figure 4.10 *Vortex or Whirlpool Casing*

(c) Casing with Guide Blades or Vaned Diffuser

In this type of casing, the impeller is surrounded by a series of fixed guide blades mounted on a ring which is known as diffuser as shown in Figure 4.11.

The fluid leaving the impeller first flows through a vane-less space before entering the diffuser vanes or guide vanes. The fluid exiting the impeller flows through divergent passages formed by the guide vanes (diffuser vanes) in which the flow velocity reduces with a corresponding increase in the pressure. The guide blades design is such that there is shockless entry to the guide vanes of the fluid leaving the impeller. This condition of no shock at inlet to the guide blades may be obtained by drawing the tangent at the inlet tip of the guide vanes which coincides with the direction of the absolute velocity of fluid exiting the impeller. The angle of divergence is kept 8^o-10^o to avoid separation of fluid from the walls of the diffuser passage formed by the guide blades. It should be noted that the number of diffuser vanes i.e. guide vanes should have no common factor with the number of impeller vanes to prevent resonant vibration. The number of diffuser vanes is kept less than that of moving vanes to make the flow uniform along peripheral direction. The fluid from the guide vanes then flows through the surrounding casing which may be either circular (concentric with the impeller) or of volute shape as that of the volute pump. Circular casings are used with these pumps as a normal practice.

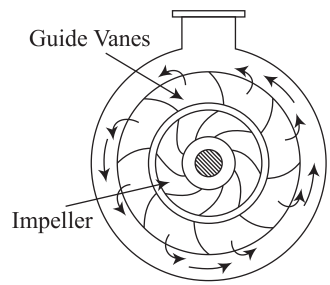


Figure 4.11 *Casing with Guide Blades or Vaned Diffuser*

The pump having a casing with guide blades or vaned diffuser is called a diffuser pump. A diffuser pump is also called turbine pump due to superficial similarity with a reaction turbine. A good design of diffuser pump converts as much as 75% of the kinetic energy of the fluid leaving the impeller into pressure energy as it has been confirmed by experimental results. However, diffuser pumps operate with the best efficiency only for a specific discharge at a particular pump speed since guide vanes are precisely installed or designed for a particular discharge only. For different values of discharge, energy loss due to shock or turbulence occurs at the inlet of the guide blades, which leads to lower efficiency. It may be possible to obtain maximum overall efficiency well above than 80% with a diffuser pump. The normal value of efficiency for a simple volute pump is from 75% to 80%, however little greater efficiencies are attainable for large machines. Moreover, turbine or diffuser pumps are costlier than simple volute pumps. Therefore, vaned diffuser ring is used only in multistage pumps.

4.6 General Pumping System

The word pumping referred to a hydraulic system commonly means to deliver liquid from a low level to a high level reservoir. A general pumping system is shown in Figure 4.12.

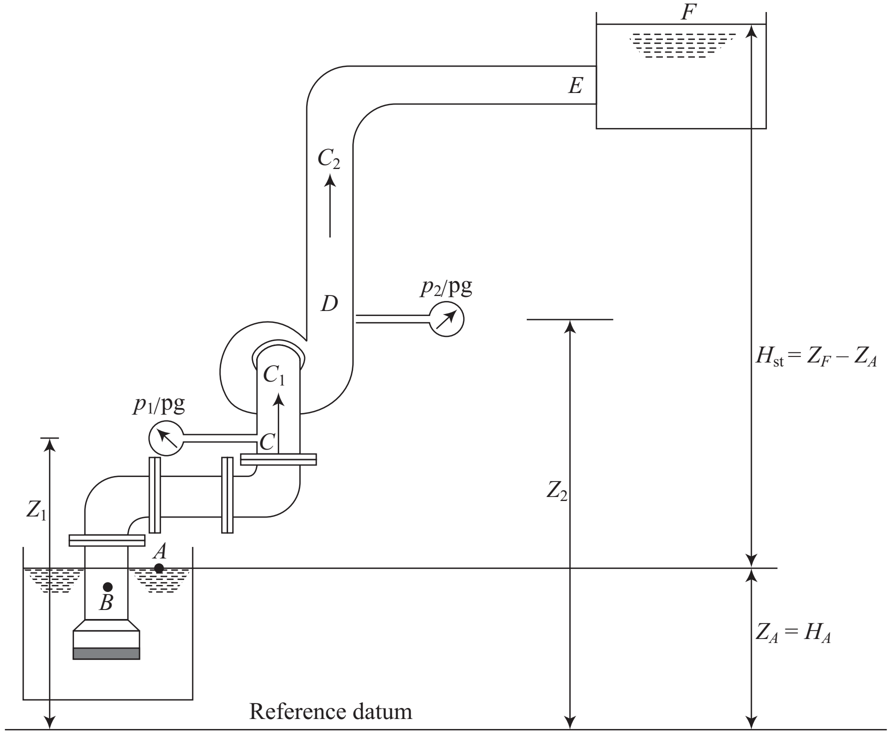


Figure 4.12 *A General Pumping System*

4.6.1 Definition of Heads

At any point in the system, the elevation or potential head is measured from a fixed reference datum line. We know from the Bernoulli's equation that the total head at any point is the sum of pressure head, velocity or kinetic head and elevation or potential head. The total head at the free surface of the lower level reservoir H_A is equal to the elevation of the free surface above the datum line as the velocity and static gauge pressure at A are zero. Similarly, the total head at the free surface of higher level reservoir or tank is $H_A + H_{st}$ and is equal to the elevation of the free surface of tank or reservoir above the reference datum.

The variation of total head as the liquid flows through the system is shown in Figure 4.13.

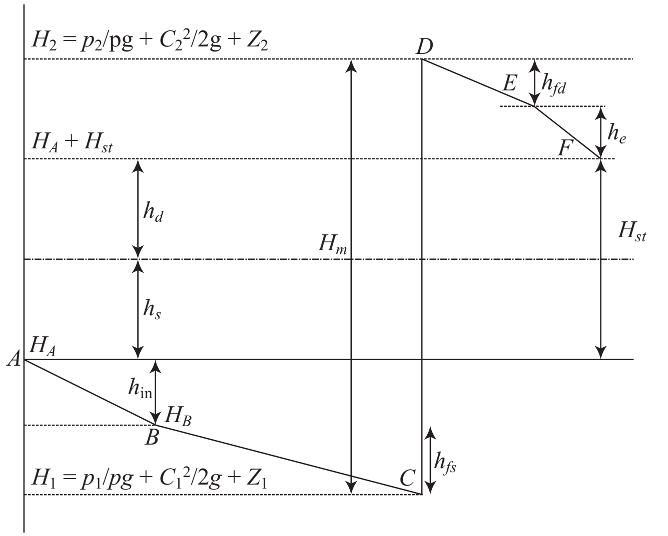


Figure 4.13 *Variation of Head in a Pumping System*

The liquid enters the inlet pipe or suction pipe causing an entry head loss h_{in} due to which the total energy line (TEL) drops to point B corresponding to a location just after the entrance to inlet or suction pipe. The total head at B is given by,

$$H_B = H_A - h_{in}$$

The total head drops further to the point C due to friction and other minor losses equivalent to h_{fs}. The fluid then enters the pump and gains energy imparted by the impeller of the pump. This increases the total head of the fluid to a point D at the exit of the pump.

During the flow of fluid from the pump outlet to the upper tank or reservoir, friction and other losses account for a total head loss of h_{fd} down to a point E. At E an exit loss $\left(h_e = \dfrac{C_2^2}{2g}\right)$ occurs when the liquid enters the upper reservoir, bringing the total head at point F to that at the free surface of the upper reservoir. In a standard pump test, the total heads are measured at the inlet and outlet flanges respectively.

$$\text{Total head at the inlet of the pump} = H_1 = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 \tag{4.1}$$

$$\text{Total head at the outlet of the pump} = H_2 = \frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2 \tag{4.2}$$

where, C_1 and C_2 are the velocities in the suction and deliver pipes respectively.

(a) Suction Head

It is the vertical distance between the centre line of the pump and the free surface of the fluid in the tank or sump from which the fluid has to be lifted. It is also known as suction lift and is denoted by h_s.

(b) Delivery Head

It is the vertical height of the liquid surface in the tank to which the liquid is delivered above the centre line of the pump. It is denoted by h_d.

(c) Static Head

The vertical distance between the liquid levels in the sump and the tank to which the liquid is delivered by the pump. Hence, the vertical distance between two levels in the reservoirs H_{st} is known as the static head or static lift. It is clear from the Figure 4.13 that the static head is equal to the sum of the suction head and delivery head.

$$H_{st} = h_s + h_d \tag{4.3}$$

(d) Euler Head or Work Head or Theoretical Head

It is the energy per unit mass of fluid imparted by the impeller of a pump. It is already defined in Eq. (1.82) as work done by the impeller on the fluid per unit mass of fluid passing through it. By considering radial entry at the inlet, the whirl component of the absolute velocity is zero ($C_{w1} = 0$) and hence, the Euler head is represented by,

$$H_e = \frac{C_{w_2} C_{b_2}}{g} \tag{4.4}$$

(e) Manometric Head

The total head that must be developed by the pump to deliver the liquid to the upper reservoir or tank is known as manometric head. From Eqs. (4.1) and (4.2), the manometric head developed by the pump,

$$H_m = H_2 - H_1$$

$$H_m = \frac{p_2 - p_1}{\rho g} + \frac{C_2^2 - C_1^2}{2g} + Z_2 - Z_1 \tag{4.5}$$

The pump is said to the working against net 'head', that head is nothing but the manometric head. The manometric head is always less than work head imparted by the rotor on the fluid $\frac{C_{w_2} C_{b_2}}{g}$ due to the energy dissipated in eddies and the friction in the impeller. If the pipes connected to inlet and outlet of the pump are of same diameter, then $C_2 = C_1$ as discharge being the same. Therefore,

$$H_m = \left[\frac{p_2}{\rho g} + Z_2\right] - \left[\frac{p_1}{\rho g} + Z_1\right] \tag{4.6}$$

Equation (4.6) represents that if the inlet and outlet pipe diameters are same, then total head developed by the pump or manometric head is equal to the increase in piezometric pressure head $\left(\frac{p}{\rho g} + Z\right)$ across the pump that could have been measured by a manometer connected between the inlet and outlet flanges of the pump. In practice, the increase in datum head $(Z_2 - Z_1)$ is neglected as it is infinitesimal compared to static pressure head $\frac{(p_2 - p_1)}{\rho g}$. Therefore, the total head developed by the pump is approximately equal to the difference of static pressure head across the pump.

Take reference datum as shown in Figure 4.12. Let p_a is atmospheric pressure which acts upon the free surfaces of sump and high level reservoir. Applying the Bernoulli's equation between A and C,

$$\frac{p_a}{\rho g} + Z_A = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 + h_{in} + h_{fs} \tag{4.7}$$

$$\frac{p_a}{\rho g} + Z_A = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_A + h_s + h_{in} + h_{fs}$$

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + h_s + h_{in} + h_{fs}$$

Applying the Bernoulli's equation between D and F,

$$\frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2 = \frac{p_a}{\rho g} + H_{st} + Z_A + h_{fd} + h_e \tag{4.8}$$

Using Eqs (4.7) and (4.8) in Eq. (4.5), manometric head can be expressed as,

$$H_m = H_{st} + h_{in} + h_{fs} + h_{fd} + h_e = h_s + h_d + \Sigma\text{losses} \tag{4.9}$$

Value of static head is substituted from Eq. (4.3) into (4.9). Equation (4.9) indicates that the total head developed by the pump is equal to the sum of suction head and delivery head plus all the losses occurring in pipe from inlet of suction pipe to the discharge point of the delivery pipe plus velocity head loss at exit of the delivery pipe.

Manomatric head is more often simply indicated by H as in Figure 1.41.

4.6.2 Suction Pipe with a Foot Valve and a Strainer

A pipe whose upper end is connected to the pump entrance (i.e. the centre of the impeller known as *eye*) and the lower end dips into the liquid in a sump is called suction pipe. A sump is a liquid storage, or a low level reservoir, from where the liquid is required to be pumped or lifted. Application of Bernoulli's equation between the free surface of the liquid in the sump and the pump centre line yields,

$$\frac{p_a}{\rho g} = \frac{p_s}{\rho g} + h_s + \frac{C_s^2}{2g} + h_{fs} \tag{4.10}$$

where,

p_a is the atmospheric pressure which acts on free surface of the liquid in the sump,
p_s is the suction pressure i.e. pressure at the inlet of pump,
h_s is the vertical distance between free surface of liquid in the sump and pump centre line (suction lift),
C_s is the velocity of liquid in the suction pipe, and
h_{fs} is the head loss due to friction in suction pipe.

From Eq. (4.10), it is clear that pressure at the pump entrance is lower than the atmospheric pressure, i.e. negative pressure or vacuum. The liquid rises in the suction pipe up to the impeller eye due to the difference between the atmospheric pressure on the free surface of the sump and the pressure at the entry of pump.

The minimum value of negative pressure at the pump entrance is the vapour pressure of the liquid at the operating temperature to avoid *cavitation*. Cavitation is a phenomenon of formation, travel and rapid collapsing of vapour bubbles in a zone where the liquid pressure drops below its vapour pressure. Due to the sudden collapsing of vapour bubbles on the metallic surface, high pressure is produced leading to pitting action on the surface. Therefore, it is quite obvious that as h_s, C_s or h_{fs} are increased, pressure at the inlet of pump may approach liquid vapour pressure p_v and *cavitation* conditions may prevail. This constraint on the inlet pressure limits the height of the suction pipe above the liquid level in the sump and requires that the pressure losses in the suction pipe be as small as possible. Therefore, suction pipe is quite often kept short and larger in diameter than the delivery pipe to decrease the velocity in it as major frictional losses are directly proportional to the square of the velocity and inversely proportional to diameter. Also, the bends in the suction pipe should be avoided as far as possible to reduce the minor losses. The velocity in the suction pipe being thus small would in turn keep the frictional losses on the lower side.

A *foot valve and strainer* is fitted at the lower end of the suction pipe. The liquid first get into the strainer which keeps away debris such as leaves, wooden, pieces etc. from the pump. Liquid then enters into the suction pipe through the foot valve. A foot valve which is a non-return or one-way valve opens only in one direction that is upward. Hence, the liquid passes through the foot valve only in upward direction and does not permit the back flow of the liquid to the sump.

4.6.3 Delivery Pipe

A pipe which is connected at its lower end to the outlet of the pump and the other end delivers the liquid to the required height is known as delivery pipe. A delivery valve is provided on the delivery pipe near the outlet of the pump. The function of the delivery valve is to control the flow from the pump into the delivery pipe.

4.6.4 Priming

Priming is the operation in which the suction pipe, casing of the pump, and the portion of the delivery pipe up to the delivery valve are completely filled with the liquid which is to be pumped before starting the pump, so that all the air (or gas or vapour) from this portion of the pump is driven out and no air pocket is left. It has been observed that even the presence of a small air pocket in any of the portion of the pump may result in no delivery of liquid from the pump.

The rotation of impeller creates a difference of pressure head between inlet and outlet of a centrifugal pump. The pressure at the inlet or eye of a pump is lower than atmospheric pressure when the pump and suction pipe is filled completely with liquid. Due to the difference in pressure head between the free surface of the sump and the eye of the pump, the atmospheric pressure causes the liquid to be rushed up from the sump into the casing. We know that the theoretical work done by the impeller per unit weight of the fluid or the Euler head generated is $\frac{C_{w2}C_{b2}}{g}$. Thus, the head created by the pump enables the delivery of liquid to the destination. If, however the pump casing is not filled with liquid at the start of the pump, the impeller rotates in air or vapour filled casing. It is clear that the Euler head generated is independent of the density of the liquid. This means that when pump is operating in the air, the head developed is column of air i.e. in of *m* of air. If the pump is primed with water, the head developed would be same column of water (*m*). Pressure developed in a centrifugal pump impeller (ρgH) is directly proportional to the density of the fluid passing through it. Since air density is very low, the pressure developed is very low as compared to that would have been generated by the same *m* of water head. Therefore, if an impeller is rotating in the presence of air, only an infinitesimal pressure would be increased resulting in no lift of liquid by the pump. Further, dry running of the pump may damage several parts of the pump. Therefore, proper priming of a centrifugal pump is absolutely necessary before starting it.

Priming requires that valves must be provided before and after the pump. A one way valve, known as foot valve is generally fitted at the lower end of the suction pipe for keeping the water in the pump. Foot valve opens to permit the liquid to enter the suction pipe when the pump is started but closes as soon as the flow stops when the pump is switched off. The closure of the foot valve does not permit the draining of the liquid into the sump. This enables the suction pipe and the pump to remain full of liquid. Thus, when the pump is restarted, it is ready for use again without any priming.

The foot valve may develop leak due to frequent closures during periodic operations over a period of time. Consequently, the suction pipe may not remain full of liquid during idle periods thus requiring maintenance or even replacement. To eliminate this, an inlet valve may be installed before the pump which should be closed before the pump is switched off so that the liquid remains inside the pump.

It should be kept in mind that the delivery valve must be closed while starting the pump and not opened until the speed has attained its normal value. The delivery valve is then opened gradually. Similarly, before stopping the pump, the delivery valve should be closed slowly with the pump running at normal speed. The closed valve does not harm the pump.

Following are some methods adopted for priming of centrifugal pumps:

- Small pumps are generally primed by pouring the liquid from an external source through an opening into the casing using a funnel. The air vent provided in the casing is opened during the filling process of liquid. When the casing is filled with liquid, the pump is started and any residual air is allowed to exit through the air vent. The air vent is closed when liquid starts escaping from the vent which shows that all the air has been removed from the suction pipe and casing of the pump.
- Larger pumps are usually primed by evacuating the casing and the suction pipe by the vacuum producing devices such as ejectors and vacuum pumps.
- Priming is not necessary when the pump is located below the sump liquid level.

In some pumps, commercially patented devices are fitted which facilitate automatic priming of pumps when pump is started and enable normal functioning of the pump in a very short time. Pumps fitted with such devices are known as self-priming pumps.

EXAMPLE 4.1 A centrifugal pump delivers water against a static head of 35 m out of which 3.5 m is the suction lift. The suction and delivery pipes are both 300 mm diameter. The frictional head loss in suction pipe is 2.3 m and that in delivery pipe is 5.7 m. The impeller is 0.4 m in diameter and 30 mm wide at the outer diameter. The speed of the pump is 1400 rpm and the blade angle at exit is 30°. The velocity of whirl at outlet is 14.39 m/s. Determine (a) discharge, (b) static pressure on the suction and delivery side of the pump, and (c) draw the energy gradient line. Assume the inlet and outlet of the pump are at the same elevation and neglect the effect of vane thickness on the area of flow.

Solution

Given: H_s = 35 m, h_s = 3.5 m, d_s = 300 mm = 0.3 m, h_{fs} = 2.3 m, h_{fd} = 5.7 m, D_2 = 0.4 m, B_2 = 30 mm = 0.03 m, N = 1400 rpm, β_2 = 30°, C_{w2} = 14.39 m/s

Blade velocity at outlet is given by,

$$C_{b2} = \frac{\pi D_2 N}{60} \qquad (1)$$

$$C_{b2} = \frac{\pi \times 0.4 \times 1400}{60}$$

$$C_{b2} = 29.32 \text{ m} \qquad (2)$$

The pumping system and velocity triangles are shown in Figures 4.14 and 4.15 respectively.

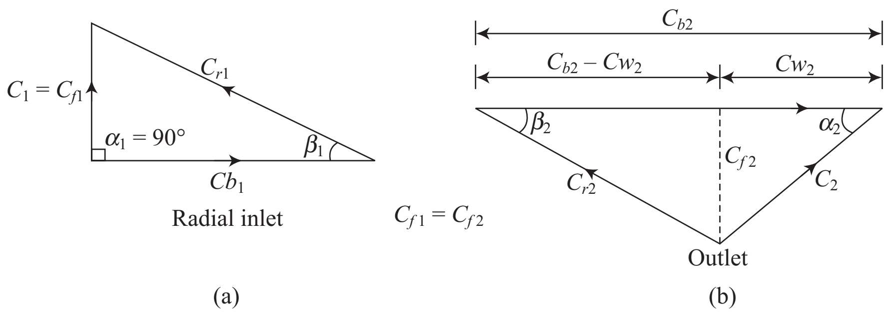


Figure 4.14 *Velocity Triangles of Example 4.1*

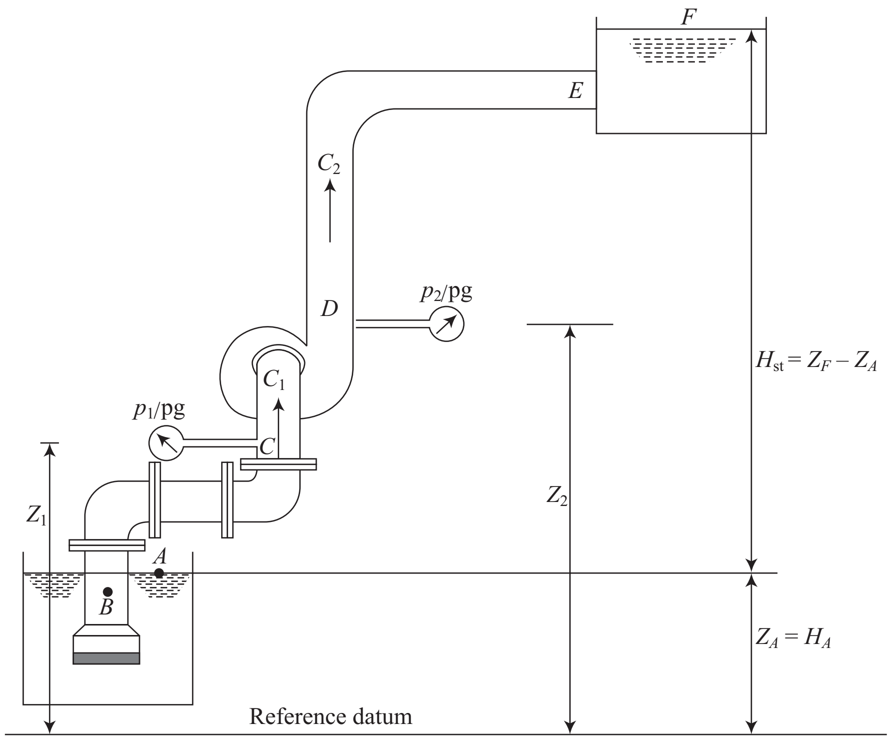


Figure 4.15 *Pumping System of Example 4.1*

From the velocity triangle at outlet,

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} \tag{3}$$

$$\tan 30 = \frac{C_{f2}}{29.32 - 14.39}$$

$$C_{f2} = 8.2 \text{ m/s} \tag{4}$$

(a) Discharge of the Pump

$$Q = \pi D_2 B_2 C_{f2} \tag{5}$$
$$Q = \pi \times 0.4 \times 0.03 \times 8.62$$
$$Q = 0.325 \text{ m}^3\text{/s} \tag{6}$$

(b) Pressure on the Suction and Delivery Side

Since the discharge through the suction and delivery pipes is same and both are of the same diameter, therefore the velocity in the suction pipe and delivery pipe would be the same. Hence,

$$C_1 = C_2 = \frac{Q}{\frac{\pi}{4} d_p^2} \tag{7}$$
$$C_1 = C_2 = \frac{0.325}{\frac{\pi}{4} \times 0.3^2}$$
$$C_1 = C_2 = 4.598 \text{ m/s} \tag{8}$$

Manometric head is given by,

$$H_m = H_{\text{st}} + h_{\text{in}} + h_{fs} + h_{fd} + h_e \tag{9}$$

Neglecting the loss of head at the suction valve and the other minor losses, then the manometric head of the pump is,

$$H_m = H_{st} + h_{fs} + h_{fd} + \frac{C_2^2}{2g} = 35 + 2.3 + 5.7 + \frac{4.598^2}{2 \times 9.81}$$
$$H_m = 44.08 \text{ m} \tag{10}$$

Applying the Bernoulli's Equation between free surface of the water in the sump (A), and entry of the pump (C),

$$\frac{p_a}{\rho g} + Z_A = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 + h_{in} + h_{fs} \tag{11}$$

Since, $Z_1 = Z_A + h_s$ and h_{in} is assumed zero, therefore,

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + h_s + h_{fs} \tag{12}$$

Assuming atmospheric pressure p_a as 1.01325 bar, therefore,

$$\frac{p_1}{\rho g} = \frac{1.01325 \times 10^5}{1000 \times 9.81} - \frac{4.598^2}{2 \times 9.81} - 3.5 - 2.3$$
$$\frac{p_1}{\rho g} = 3.4524 \text{ m} \tag{13}$$
$$p_1 = 1000 \times 9.81 \times 3.4524 = 3368.044 P_a = 33.868 \text{ kPa} \tag{14}$$

From Eqs. (13) and (14), it is obvious that $p_1 < p_{\text{atm}}$, hence, vacuum exists at the inlet of pump.

$$H_{st} = h_s + h_d \Rightarrow 35 = 3.5 + h_d \Rightarrow h_d = 31.5 \text{ m} \tag{15}$$

Applying Bernoulli's equation between the centre line of the exit of the impeller (D) and the exit of the delivery pipe (F),

$$\frac{p_2}{\rho g}+\frac{C_2^2}{2g}+Z_2=\frac{p_a}{\rho g}+H_{st}+Z_A+h_{fd}+h_e \tag{16}$$

Since, the inlet and outlet of the pump are at the same elevation as given, therefore,

$$Z_2 = Z_1 = Z_A + h_s \tag{17}$$

Using Eqs (15) and (17) and substituting $h_e = C_2^2/2g$ into Eq. (16), we get,

$$\frac{p_2}{\rho g}=\frac{p_a}{\rho g}+h_d+h_{fd} \tag{18}$$

$$\frac{p_2}{\rho g}=10.33+31.5+5.7 \Rightarrow \frac{p_2}{\rho g}=47.53\text{ m} \tag{19}$$

$$p_2 = 1000 \times 9.81 \times 47.53$$

$$p_2 = 466269.3\text{ Pa} = 466.2693\text{ kPa} \tag{20}$$

(c) Energy Gradient Line

Energy head at A,

$$H_A=\frac{p_a}{\rho g}+Z_A \Rightarrow H_A=10.33+Z_A \tag{21}$$

$$H_B = H_A - h_{\text{in}} \tag{22}$$

Since, $h_{\text{in}} = 0$, therefore,

$$H_B = H_A = 10.33 + Z_A \tag{23}$$

$$H_C = H_B - h_{fs} \Rightarrow H_C = 10.33 + Z_A - 2.3$$

$$H_C = 8.03 + Z_A$$

Alternately,

$$H_C=\frac{p_1}{\rho g}+\frac{C_1^2}{2g}+Z_1=\frac{p_1}{\rho g}+\frac{C_1^2}{2g}+Z_A+h_s=3.4524+\frac{4.598^2}{2\times 9.81}+Z_A+3.5$$

$$H_C = 8.03 + Z_A \tag{24}$$

$$H_D = H_C + H_m = 8.03 + Z_A + 44.08$$

$$H_D = 52.11 + Z_A$$

Alternately,

$$H_D=\frac{p_2}{\rho g}+\frac{C_2^2}{2g}+Z_2=\frac{p_2}{\rho g}+\frac{C_2^2}{2g}+Z_1=47.53+\frac{4.598^2}{2\times 9.81}+Z_A+3.5$$

$$H_D = 52.11 + Z_A \tag{25}$$

$$H_E = H_D - h_{fd} = 52.11 + Z_A - 5.7$$

$$H_E = 46.41 + Z_A \tag{26}$$

$$H_F = H_E - h_e = H_E - \frac{C_2^2}{2g} = 46.41 + Z_A - \frac{4.598^2}{2 \times 9.81}$$

$$H_F = 45.33 + Z_A$$

Alternately,

$$H_F = H_A + H_{st} = 10.33 + Z_A + 35$$

$$H_F = 45.33 + Z_A \tag{27}$$

Energy gradient line can be drawn using the values of H_A, H_B, H_C, H_D, H_E and H_F as given below. Energy gradient line shown below has values using sump level as the reference i.e. the value of $Z_A = 0$ is substituted in Eqs. (23), (24), (25), (26) and (27).

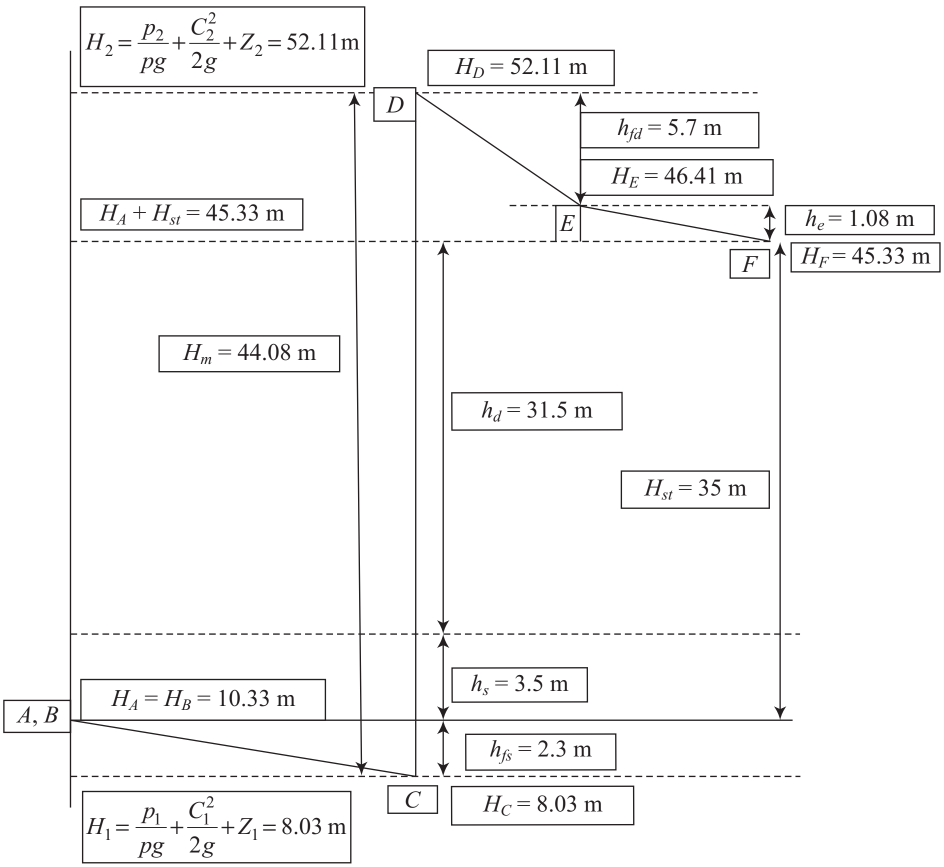


Figure 4.16 *Energy Gradient Line for Pumping System of Example 4.1*

4.7 Analysis of Centrifugal Pump

4.7.1 Discharge

Volume flow rate of the fluid or Discharge = Q × Area × Velocity of flow

Since the discharge is same at the inlet and outlet of the pump, therefore,

$$Q = \pi D_1 B_1 C_{f_1} = \pi D_2 C_{f_2} \tag{4.11}$$

where B_1 and B_2 are the width of the impeller at inlet and outlet. Mass of the water flowing through the impeller per second,

$$\dot{m} = \text{Density} \times \text{Volume Flow Rate} = \rho_w Q \tag{4.12}$$

4.7.2 Specific Work

(a) Work Imparted by the Impeller with Infinite Blades

To study an elementary theory of pump performance, following assumptions are made:

1. The flow is one dimensional steady flow i.e. no fluid mass is stored or depleted in the rotor. The mass flow rate is constant at each section.
2. The heat and work transfer between the rotor and surroundings occurs at a constant rate.
3. Flow is uniform i.e. velocity at each area normal to the flow is uniform. Hence, velocity vector at any point indicates velocity of the total flow over a finite area.
4. There are no mechanical or fluid friction and other eddy losses in the system.
5. The impeller or rotor is considered to have an infinite number of blades of infinitesional thickness having zero friction and all the flow from the inlet is assumed to be guided with uniform velocity at the exit. This assumption ensures no circulation induced cross flows in the system.

A centrifugal pump impeller with velocity triangles at entry and the exit is shown in Figure 4.17. The blades or vanes are curved at inlet and outlet. A fluid particle moves along the path shown by a broken curve in the Figure 4.17.

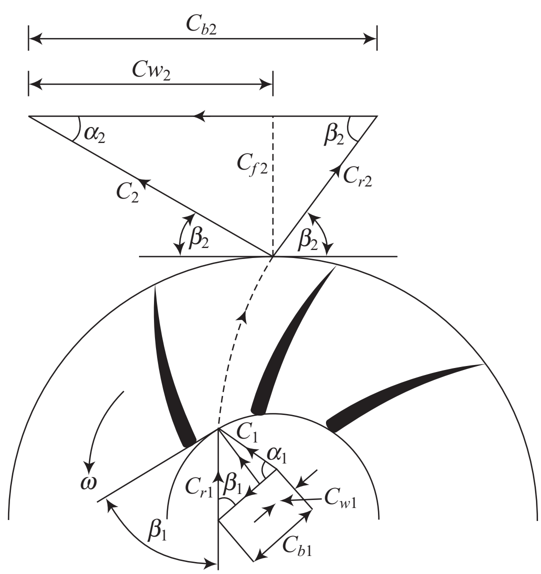


Figure 4.17 *Velocity Triangles at Inlet and Outlet of a Centrifugal Pump*

Let,

N = Speed of the impeller in rpm,

D_1 = Diameter of impeller at inlet,

C_{b_1} = Blade velocity i.e. tangential velocity of impeller at inlet,

C_1 = Absolute velocity of fluid at inlet,

C_{r_1} = Relative velocity of fluid at inlet (with respect to blade velocity),

α_1 = Angle made by the absolute velocity at inlet with the direction of motion of blades or vanes,

β_1 = Blade angle at inlet, i.e. angle made by the relative velocity at inlet with the direction of motion of the blades. D_2, C_{b_2}, C_2, C_{r2}, α_2, β_2 are corresponding values at the outlet.

A centrifugal pump is opposite of an inward flow reaction turbine. We know that the work is equal to the product of force and displacement or torque and angular displacement. Therefore, if the shaft torque (the torque exerted on the rotor shaft by motor shaft) and the rotation of the rotor are in the same direction, energy transfer is from the shaft to the rotor and from rotor to the fluid-the machine is a centrifugal pump. Conversely, if the shaft torque is opposite to the direction of rotation, the energy transfer is from the fluid to the rotor-a turbine. The value of shaft torque and hence the specific shaft work can be obtained from the Euler Eq. (1.82). Therefore,

$$\text{Specific Work} = w = (C_{w2}C_{b2} - C_{w1}C_{b1}) \tag{4.13}$$

Work done on the fluid per unit mass of the fluid is known as Euler Head H_e or theoretical head as given by Eq. (1.85) and (1.98),

$$w = gH_e = (C_{w2}C_{b2} - C_{w1}C_{b1}) = \left[\frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2}\right] \tag{4.14}$$

Equation (4.14) is an important form of Euler's equation related to a centrifugal pump because it gives the three distinct components of energy transfer by a pair of terms in the round brackets on the RHS as discussed next:

- The first term represents an increase in absolute kinetic energy or dynamic head of the fluid while passing through the impeller.
- The second term indicates an increase in static head due to retardation in fluid velocity relative to the impeller. A diverging passage in the direction of flow through the impeller reduces the relative velocity ($C_{r2} < C_{r1}$), consequently increases the static pressure which can be demonstrated by Bernoulli's equation.
- The third term represents a change in energy of the fluid due to movement of fluid from one radius of rotation to another. This is the centrifugal head imparted by impeller due to forced vortex as discussed earlier which causes an increase static pressure.

A centrifugal pump seldom has any kind of guide blades at entry. Therefore, the direction of absolute velocity at the inlet of the impeller is not directly known. It is a general assumption that the fluid enters radially into the impeller without appreciable whirl, swirl i.e. tangential component of absolute velocity. The blades are designed such that the blade inlet angle is right angle; therefore, velocity triangle at inlet is right angled one under operation at the design point. At off design conditions, the relative velocity does not remain tangential to the blade profile. Consequently, the direction of fluid is changed abruptly at the inlet of the impeller giving rise to the formation of eddies. These eddies formed cause some back flow of the fluid into the suction pipe, giving rise the fluid with some *whirl or swirl* before entering the impeller. However, considering the pump running at design conditions, i.e. the inlet whirl component, $C_{w_1} = 0$. Therefore,

$$\text{Specific Work} = w = gH_e = C_{w2}C_{b2} \tag{4.15}$$

Equation (4.15) represents the energy or work head imparted by the rotor on the fluid and H_e is usually known as the Euler head. It is quite obvious that blade velocity at the inlet does not appear in the Eq. (4.15), therefore, work done does not depend on the inlet radius.

Work done by impeller on the fluid/s = Mass of Fluid/s × Work done/kg

$$\text{Work done by the impeller on the fluid/s or Power imparted by impeller} = \rho_w Q C_{w_2} C_{b_2} \tag{4.16}$$

(b) Work Imparted by the Impeller with Finite Blades

Actually the flow is not congruent to the vanes of the pump as the real impeller has finite number of blades with finite thickness which lead to the slip as discussed in Section 1.7. Therefore, specific work, i.e. actual work head given to the fluid by the real life impeller is,

$$H'_e = \frac{C'_w C_{b_2}}{g} = \sigma_s \frac{C_{w_2} C_{b_2}}{g} \tag{4.17}$$

Therefore, the actual head imparted by impeller is less than the theoretical work head imparted by impeller, i.e. Euler head $\left(H_e = \frac{C_{w_2} C_{b_2}}{g}\right)$ as $\sigma_s < 1$.

4.7.3 Impeller Blade Angles

Blade settings are of three possible types as shown in Figure 4.18. The velocity triangles at the outlet of the impeller for all the cases are also shown in Figure 4.18 (a), (b) and (c) respectively. From velocity triangles, velocity of whirl as,

$$C_{w_2} = C_{b_2} - C_{f_2} \cot \beta_2 = C_{b_2} - (Q/A) \cot \beta_2$$

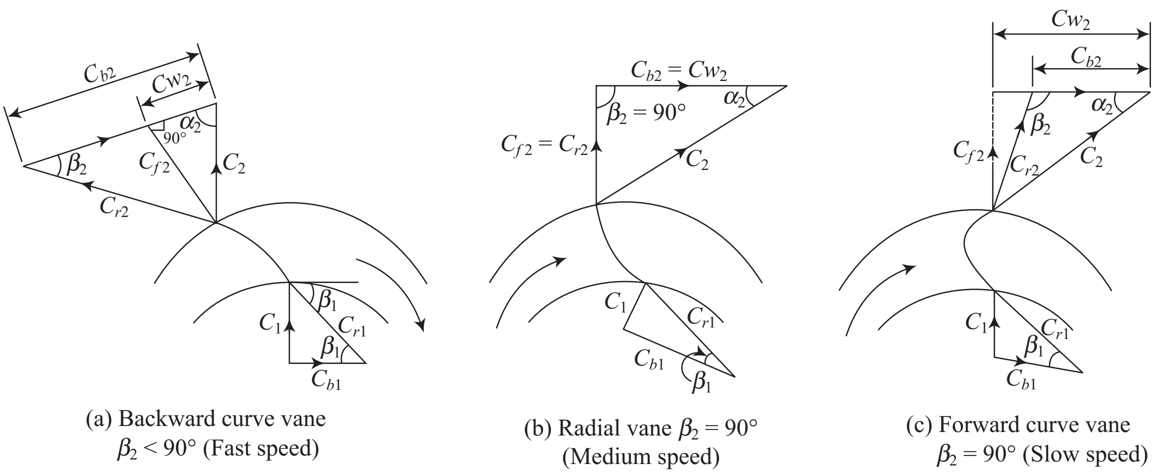


Figure 4.18 *Velocity Triangles for Different Blade Settings*

(a) Forward Facing Blades

The forward facing blades are curved in the direction of rotation, hence, $\beta_2 > 90°$. Therefore, cot β_2 is negative and $C_{w_2} > C_{b_2}$. Ideal head increases linearly with discharge for forward facing blades. The impeller having such blades is called a slow runner because of the lower value of C_{b_2} as can be seen in the Figure 4.18.

(b) Radial Blades

The radial blades in which the fluid leaves the impeller with relative velocity in a radial direction and, therefore, $\beta_2 = 90°$ and $C_{w_2} = C_{b_2}$. Therefore, Euler head is constant regardless of the discharge in radial blade pumps. Such blades are suitable for medium speed runners.

(c) Backward Facing Blades

The curvature of the blades is in a direction opposite to that of impeller rotation in backward facing blades, therefore, $\beta_2 < 90°$ and $C_{w_2} < C_{b_2}$. Ideal head decreases linearly with discharge for backward facing blades. The impeller having backward curved blades is a fast runner because of the highest C_{b_2} of the three blade geometries.

EXAMPLE 4.2 A centrifugal pump running at 1200 rpm delivers oil of specific gravity 0.85 at the rate of 80 l/s. The inner and outer diameters of the impeller are 200 mm and 400 mm respectively. The blades are curved backwards at an angle of 35°. The velocity of flow is constant at 2 m/s. Calculate (i) blade angle at inlet, and (ii) the energy given by the impeller to the fluid.

Solution

Given: N = 1200 rpm, sp. gravity = 0.85, Q = 80 l/s = 0.08 m^3/s, D_1 = 200 mm = 0.2 m, D_2 = 400 mm = 0.4 m, β_2 = 35°, $C_{f1} = C_{f2}$ = 2.0 m/s,

The inlet and outlet velocity triangles considering radial entry are shown in Figure 4.19. Let β_2 is the vane angle at the outlet.

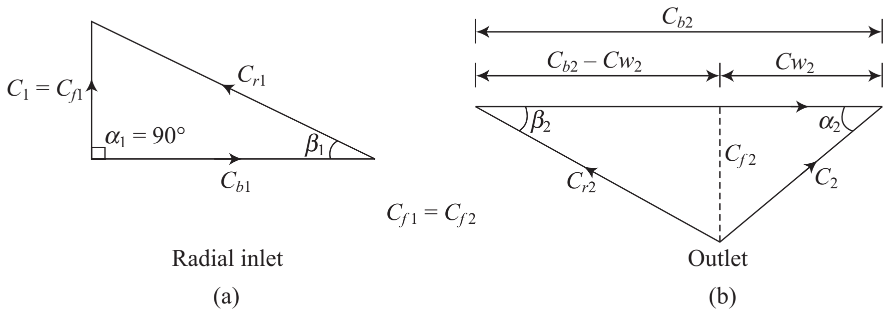


Figure 4.19 *Velocity Triangles of Example 4.2*

Tangential velocity of the impeller or blade velocity at the inlet is given by,

$$C_{b1} = \frac{\pi D_1 N}{60} \tag{1}$$

$$C_{b1} = \frac{\pi \times 0.2 \times 1200}{60}$$

$$C_{b1} = 12.57 \text{ m/s} \tag{2}$$

(a) Blade Angle at Inlet

From the inlet velocity triangle for radial entry,

$$\tan \beta_1 = \frac{C_{f1}}{C_{b1}} \tag{3}$$

$$\tan \beta_1 = \frac{2}{12.57} = 0.159 \Rightarrow \beta_1 = \tan^{-1} 0.159$$

$$\beta_1 = 9.04° \tag{4}$$

(b) Energy Imparted to the Fluid by the Impeller

Similarly, blade velocity at outlet,

$$C_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60}$$

$$C_{b2} = 25.13 \text{ m/s} \tag{5}$$

From the outlet velocity triangle,

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} \tag{6}$$

$$\tan 35 = \frac{2}{25.13 - C_{w2}} \Rightarrow C_{w2} = 25.13 - \frac{2}{\tan 35}$$

$$C_{w2} = 22.274 \text{ m/s} \tag{7}$$

Assuming no fluid frictional losses in the blade passage, no circulatory flow between the blades of the impeller due to finite number of blades and no shock losses at the entrance to the impeller; the manometric head is equal to the Euler or theoretical head. Hence,

$$H_m = H_e = \frac{C_{w2} C_{b2}}{g} \tag{8}$$

$$H_m = H_e = \frac{22.274 \times 25.13}{9.81}$$

$$H_m = H_e = 57.06 \text{ m} \tag{9}$$

We know that specific gravity,

$$s = \frac{\text{Density of the fluid}}{\text{Density of water}} = \frac{\rho}{1000} \Rightarrow 0.85 = \frac{\rho}{1000}$$

$$\rho = 850 \text{ kg/m}^3 \tag{10}$$

Energy given to the fluid by the impeller or fluid power,

$$\text{Fluid Power} = \rho g Q H_m \tag{11}$$

$$\text{Fluid Power} = 850 \times 9.81 \times 0.08 \times 57.06$$

$$\text{Fluid Power} = 38063.585 \text{ W} = 38.064 \text{ kW} \tag{12}$$

EXAMPLE 4.3 A centrifugal pump having blades which are radial at outlet discharges 100 l/s against a head of 7.5 m. The diameter of the impeller is 0.3 m and width at that diameter is 30 mm. The velocity of whirl at outlet is 9.304 m/s. Find (a) the speed of the impeller, and (b) the magnitude and direction of absolute velocity at the outlet of the impeller.

Solution

Given: $Q = 100$ l/s $= 0.1$ m^3/s, $H_m = 7.5$ m, $D_2 = 0.3$ m, $B_2 = 30$ mm $= 0.03$ m, $C_{w2} = 9.304$ m/s

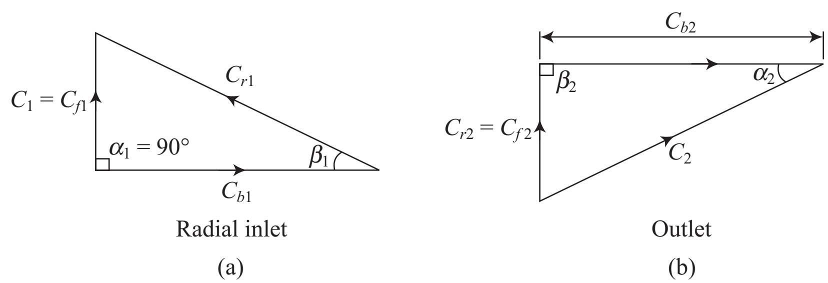


Figure 4.20 *Velocity Triangles of Example 4.3*

The velocity triangles considering the radial entry and radial discharge at the exit are shown in Figure 4.20. We know that discharge is,

$$Q = \pi D_2 B_2 C_{f2} \quad (1)$$

$$0.1 = \pi \times 0.3 \times 0.03 \times C_{f2}$$

$$C_{f2} = 2.65 \text{ m/s} \quad (2)$$

(a) Speed of the Impeller

Since vanes are radial at outlet, therefore $\beta_2 = 90° \Rightarrow C_{w2} = C_{b2}$ as can be seen from the velocity triangle. Since, tangential velocity of vanes,

$$C_{b2} = \frac{\pi D_2 N}{60} \quad (3)$$

$$9.304 = \frac{\pi \times 0.3 \times N}{60}$$

$$N = 593 \text{ rpm} \quad (4)$$

(b) Magnitude and Direction of Absolute Velocity at Exit

$$C_2 = \sqrt{C_{w2}^2 + C_{f2}^2} \quad (5)$$

$$C_2 = \sqrt{9.304^2 + 2.65^2}$$

$$C_2 = 9.674 \text{ m/s} \quad (6)$$

$$\tan \alpha_2 = \frac{C_{f2}}{C_{b2}} \quad (7)$$

$$\tan \alpha_2 = \frac{2.65}{9.304}$$

$$\alpha_2 = 15.9° \quad (8)$$

4.8 Analysis of Axial Flow or Propeller Pump

4.8.1 Velocity Triangles

The velocity diagrams for an axial flow pump are shown in Figure 4.21. The water enters the pump in an axial direction. The set of guide vanes fitted ahead of the impeller eliminate pre whirl of incoming flow. The whirl velocity is zero at inlet to impeller. The rotating impeller imparts a tangential (whirl) velocity component to the water passing over it. A pressure difference is created between the front and back of the blades due to which the fluid moves from inlet to the outlet of the pump. The stationary diffuser guide vanes are provided following the rotor to remove any whirl component from the flow and to provide the discharge in the axial direction. The guide vanes after the exit from impeller convert substantially the whirl velocity component at the outlet from the impeller into pressure. The flow velocity is constant throughout the impeller. To improve the performance of the pump at part load or overload, blades whose angle can be changed during operation are often provided in modern axial flow pumps.

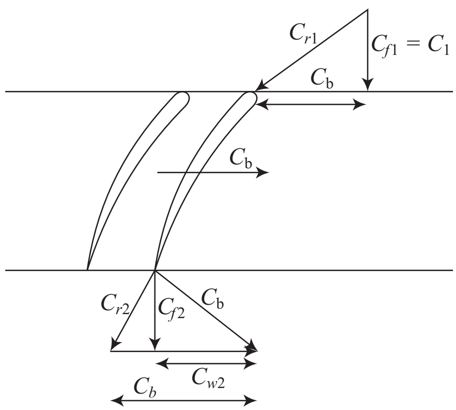


Figure 4.21 *Velocity Triangles for an Axial Flow Pump (Refer to Section X-X in Figure 4.4)*

Whereas the head developed by a centrifugal pump includes contribution of centrifugal head, the head developed by an axial flow pump is primarily due to the tangential force exerted by the rotor blades on the fluid.

4.8.2 Discharge

The impeller of an axial flow pump has specially designed blades that have the appearance of a propeller like wing of an airplane or a helicopter. No shroud is used to cover the tips of the blades. The blades are connected to drive the shaft at the hub. Generally, the blades are rigidly fixed to the hub. Axial flow pumps with adjustable pitch blades are also available as in Kaplan turbine. A preliminary analysis of axial flow pumps is same that of a Kaplan turbine as it is analogous to Kaplan turbine. Let,

D_o be the outer diameter of impeller,

D_h be the outer diameter of the hub,

The area of flow,

$$A = \frac{\pi}{4}(D_o^2 - D_h^2) \tag{4.18}$$

The guide blades at the inlet guides the flow at an angle of $\alpha_1 = 90°$ and $C_1 = C_{f1}$. The velocity of flow is taken constant all along the inlet as well as all along outlet radius.

$\therefore \quad C_{f1} = C_{f2}.$

Hence, discharge is,

$$Q = \frac{\pi}{4}(D_o^2 - D_h^2)C_f \tag{4.19}$$

4.8.3 Specific Work

(a) Specific Work with Infinite Blades

The blade or peripheral velocity at inlet and outlet are same. Therefore,

$$C_{b1} = C_{b2} = \frac{\pi D_o N}{60} \tag{4.20}$$

Since, the blade or peripheral velocity at the inlet and outlet of axial flow pump are same and the flow is assumed axially at entry of the impeller without any whirl component for maximum energy transfer. Therefore, specific work or Euler head or ideal head imparted by the impeller of an axial flow pump is obtained from Eq. 4.14 as follows:

$$w = gH_e = C_{w2}C_{b2} = \left[\frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2}\right] \tag{4.21}$$

(b) Specific Work with Finite Blades

Equation (4.21), however, is derived assuming uniform velocity over any area normal to the flow implying that velocity at any point is representative of total flow over a finite area. Also, the fluid is assumed to have perfect guidance through the flow system which indicates that there are infinite numbers of thin blades on the impeller. An axial flow pump consists of only few blades of reasonable width; a simple analysis is done by considering each blade as an isolated aerofoil. Each blade is assumed to be composed of infinite number of small, thin strips or elements and each element is considered as a two dimensional airfoil to compute aerodynamic forces.

Consider a blade element of small thickness dr at radius r from the axis. Assume momentarily that the flow passing through this element remains unaffected by the other blades or by the flow passing through other elements of the same blade, i.e. at different values of r. It means that flow is treated as two dimensional flow through each blade element. The force exerted by the fluid on this blade element depends on the magnitude and direction of the relative velocity at inlet C_{r1}. This force has two components one perpendicular and the other parallel to the direction of the velocity C_{r1} namely lift dF_L and drag dF_D respectively. The component of force acting *on the blade element* in the whirl direction is

$$dF_w = -dF_L \sin \beta_1 - dF_D \cos \beta_1$$

And the force in the downstream axial direction is,

$$dF_a = -dF_L \cos \beta_1 + dF_D \sin \beta_1$$

Expressing dF_L and dF_D in terms of lift and drag coefficients, which are discussed in aerodynamics of blading in Section 1.10, we get,

$$dF_w = \frac{1}{2}\rho C_{r1}^2 c\,dr(-C_L \sin \beta_1 - C_D \cos \beta_1) \tag{4.22}$$

$$dF_a = \frac{1}{2}\rho C_{r1}^2 c\,dr(-C_L \cos \beta_1 + C_D \sin \beta_1) \tag{4.23}$$

where, c is the chord length of the aerofoil section. The element of the blade may be regarded as being in annular control volume of radius r, thickness dr through the rotor. Applying steady flow momentum equation, as discussed in Section 1.3, if there are z number of identical blades, *the total force on the fluid* in the control volume in the axial direction, we get,

$$(p_1 - p_2)2\pi r dr + z(-dF_a) = \text{Rate of increase of axial momentum of the fluid} \quad (4.24)$$

However, if there is no radial component of velocity, continuity requires a constant density fluid to have a constant axial velocity in this annulus and therefore, rate of increase of axial momentum is zero. Non-uniformity of the axial velocity round the annulus could give a momentum correction factor slightly greater than unity downstream, but the effect of this would be negligible.

Rearranging Eq. (4.24) and putting the value of dF_a from Eq. (4.23), we obtain,

$$p_2 - p_1 = \frac{zdF_n}{2\pi r dr} = \frac{\frac{1}{2}\rho C_{r1}^2 zc}{2\pi r}(C_L \cos\beta_1 - C_D \sin\beta_1) \quad (4.25)$$

The static pressure rise $p_2 - p_1$ given by Eq. (4.25) applies in general to one value of radius r because most of the quantities on right hand side of the equation vary along the length of blade. The quantity $\frac{zc}{2\pi r}$ is equal to zc divided by circumferential distance between corresponding points on adjacent blades and it is known as the *solidity*. Solidity indicates fraction of the cross section captured by the blades. Normally, solidity changes along the blade length. Generally, solidity at hub is greater than that at the blade tips. It is to be noted that higher solidity makes the assumption less reliable that the flow passing through one blade does not affect the flow through other blades.

The force on the blade element in the whirl direction exerts a torque of an amount rdF_w on the rotor. For the elements of z blades the work done per unit time is therefore $z\omega rdF_w$, where ω is the angular velocity of the rotor. Therefore, specific work done on the fluid by rotor is,

$$w = \frac{z\omega rdF_w}{\rho C_a 2\pi r dr} \quad (4.26)$$

where, C_a is the axial component of velocity (assumed uniform) upstream of the rotor. Substituting the value of dF_w from Eq. (4.22) into Eq. (4.26), specific work done on the fluid by rotor is,

$$w = -\frac{z\omega r}{C_a 2\pi r} \times \frac{1}{2} C_{r1}^2 c(C_L \sin\beta_1 + C_D \cos\beta_1) = g\Delta H \quad (4.27)$$

where, ΔH is the increase of total head across the rotor at the radius under consideration and negative sign represents that work is done on the fluid. Hence, the negative sign may be omitted.

If there is no whirl at the inlet, $C_a = C_1 = C_{f1} = C_{b1} \tan\beta_1 = \omega r \tan\beta_1$ and $C_{r1} = C_{b1} \sec\beta_1$. Therefore, Eq. (4.27) then yields,

$$\Delta H = \frac{1}{2g}\left(\frac{zc}{2\pi r}\right)\omega^2 r^2 (C_L \sec\beta_1 + C_D \operatorname{cosec}\beta_1) \quad (4.28)$$

Equations (4.25) and (4.28) are generally applicable for a single value of r. In order to obtain pressure or head rise for the machine as a whole, integration along the blades must be performed. It is a general assumption in design that ΔH and C_a does not depend on r. However, analytical integration is not possible even then, and an empirical relation from past experience is required as a compromise.

Generally, values of lift and drag coefficients are taken from experimental results of single aerofoils past in a two dimensional flow with constant angle of attack. The magnitude and direction of relative velocity C_{r1} varies along a blade unless the blade is twisted appropriately. Also, the angle of attack is a function of radius. Practically, some flow occurs in the radial direction along the blade. Therefore, blade-element theory is not more than an approximation of the situation.

However, we can get the general outline of the pump characteristics for a fixed blade axial pump from blade element theory. For a 2D flow passing over isolated aerofoils of small camber and thickness at small angles of attack $C_L \cong 2\pi \sin \alpha$. Here, α represents the angle between direction of the upstream flow and that providing zero lift. For a particular blade element in the rotor $\alpha = \theta - \beta_1$, where θ is the fixed angle between the direction of zero lift and the direction of whirl. So substituting $C_L = 2\pi \sin (\theta - \beta_1) = 2\pi(\sin\theta \cos \beta_1 - \cos \theta \sin \beta_1)$ in Eq. (4.28), we obtain,

$$\Delta H = \frac{1}{2g}\left(\frac{zc}{2\pi r}\right)\omega^2 r^2 [2\pi(\sin\theta - \cos\theta \tan\beta_1) + C_D \operatorname{cosec}\beta_1] \tag{4.29}$$

If the discharge, and consequently the axial velocity component is raised beyond the design value keeping the blade velocity to be constant, the blade angle β_1 is increased as can be seen from the velocity triangles shown in Figure 4.21. Hence, α decreases, C_D decreases and ΔH decreases. On the other hand, a decrease of discharge results in an increase of ΔH. However, at a particular value of discharge, α will have been increased so much that causes stalling of the aerofoil, lift coefficient decreases remarkably and the increase in ΔH is shortened. Stall may not occur at all sections of the blades at the same time, but when it is sufficiently widespread, the overall performance of the machine drops.

4.8.4 Blade Angles

It is quite obvious from Eq. (4.21) that the head developed by an axial flow pump does not include the contribution of centrifugal head. The head developed by an axial flow pump is primarily due to the tangential force exerted by the rotor blades on the fluid.

$\because$ $C_{f1} = C_{f2}$, from the velocity triangle at outlet of the impeller,

$$C_{w2} = C_b - C_{f2} \cot \beta_2 = C_b - C_f \cot \beta_2 \tag{4.30}$$

From Eqs (4.21) and (4.30), the Euler head, i.e. maximum specific work imparted by the impeller to the fluid is given by,

$$gH_e = w = C_b(C_b - C_f \cot \beta_2) \tag{4.31}$$

The impeller blades of an axial flow pump are normally designed on free vortex principle i.e. $C_w r$ = constant at all radii. Further, since, $C_b = \omega r$, Eq. (4.31) indicates that the specific work or energy transfer is constant at all radii. The above equation should have the same value for all values of radius r for constant energy transfer over the entire span of the blade. Since, C_b^2 will increase with an increase in radius, therefore, an equal increase in $C_b C_f \cot \beta_2$ must take place to maintain a constant energy transfer. Since, C_f is constant, therefore cot β_2 must increase on increasing the radius. So, the blade is twisted as the radius changes.

Axial flow pumps are so designed that constant head is imparted at all radii of the propeller. The values of α_1, β_1 and β_2 change all along the blade length. Inlet and outlet blade angles β_1, β_2 are both minimum at the blade tip and increases along the radius to a maximum at the hub.

EXAMPLE 4.4 The inner and outer diameters of an axial flow pump are 0.75 m and 1.8 m repectively. Fixed stator blades lie downstream of the rotor with an inlet angle of 40° (at the mean diameter) measured from the direction of blade motion. The rotor blade outlet angle (at the mean diameter) also measured from the direction of blade motion is 30° and the rotor rotates at a speed of 250 rpm. If the whirl velocity upstream from the rotor is zero at all radii, determine

(a) the axial velocity if the flow onto the stator blade occurs at zero incidence,
(b) the rotor torque if the axial velocity is constant across the flow annulus, and
(c) the root and tip rotor blade angles for zero incidence and zero inlet whirl.

Solution

Referring to the velocity triangles in Figure 4.22, $\alpha_2 = 40°$ and $\beta_2 = 30°$ at the mean radius,

Mean diameter, $D_m = \dfrac{1.8 + 0.75}{2} = 1.275$ m

$$C_b = \frac{\pi N D}{60} \qquad (1)$$

$$C_b = \frac{\pi \times 250 \times 1.275}{60} = 16.69 \text{ m/s} \qquad (2)$$

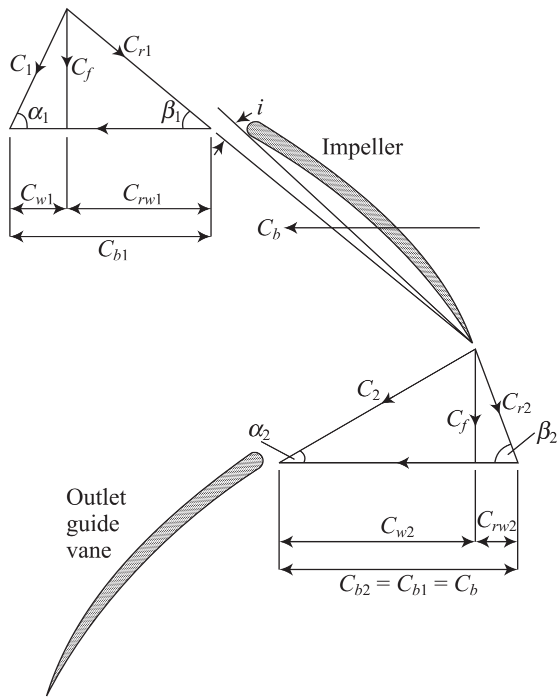


Figure 4.22 *Velocity Triangles of Example 4.4*

(a) Axial Velocity

At the mean radius,

$$C_{rw2} = \frac{C_f}{\tan 30°} \text{ and } C_{w2} = \frac{C_f}{\tan 40°}$$

$$C_{rw2} + C_{w2} = C_b = C_f\left(\frac{1}{\tan 30} + \frac{1}{\tan 40}\right) \qquad (3)$$

$$16.69 = 2.9238\, C_f$$

$$C_f = 5.71 \text{ m/s} \qquad (4)$$

(b) Rotor Torque

Flow rate through the annulus is,

$$Q = AC_f = \frac{\pi}{4}(1.8^2 - 0.75^2) \times 5.71$$

$$Q = 12 \text{ m}^3\text{/s} \qquad (5)$$

At the mean radius,

$$C_{w2} = \frac{5.71}{\tan 40°} = 6.8 \text{ m/s} \qquad (6)$$

$$H_e = \frac{C_b(C_{w2} - C_{w1})}{g} \qquad (7)$$

$$H_e = \frac{16.69 \times 6.8}{9.81}$$

$$H_e = 11.596 \text{ m} \qquad (8)$$

Power transferred,

$$P = \rho g Q H_e = 1000 \times 9.81 \times 12 \times 11.596$$

$$P = 1365081.12 \text{ W} = 1.3651 \text{ MW} \quad (9)$$

$$\text{Torque} = \frac{\text{Power}}{\text{Angular velocity}} = \frac{P}{\omega} = \frac{1365081.12}{(2\pi \times 250)/60}$$

$$\text{Torque} = 52142.26 \text{ N.m} = 52.14226 \text{ kN.m} \quad (10)$$

(c) Blade Angles
(i) At Root

$$C_{br} = \frac{\pi N D}{60}$$

$$C_{br} = \frac{\pi \times 250 \times 0.75}{60} = 9.81 \text{ m/s} \quad (11)$$

∵ $C_{w1} = 0$, therefore from velocity triangle

$$\tan \beta_{1r} = \frac{C_f}{C_{br}} = \frac{5.71}{9.8} = 5.71/9.8$$

$$B_{1r} = 30.2° \quad (12)$$

(ii) At the Tip

$$C_{bt} = \frac{\pi N D}{60} = \frac{\pi \times 250 \times 1.8}{60}$$

$$C_{bt} = 23.56 \text{ m/s} \quad (13)$$

$$\tan \beta_{1t} = \frac{C_f}{C_{bt}} = \frac{5.71}{23.56}$$

$$B_{1t} = 13.6° \quad (14)$$

4.9 Characteristics of a Pump

4.9.1 Losses in a Pump

The expression for Euler head was derived on the basis of idealized flow conditions with the assumption of radial entry of the fluid at the inlet of a pump. Significant differences in the real world pumps from the ideal conditions are: (1) The number of the blades in a pump is finite and not infinite as in an ideal pump, (2) The presence of frictional resistance to flow in the impeller passages, (3) Occurrence of non-ideal conditions at the inlet and outlet of the pump, and (4) Non-ideal diffuser performance of volute casings. Consequently, real pumps have many sources of energy losses that lead to the efficiency less than unity. These losses may be broadly classified into two categories: (a) hydraulic losses, and (b) non-hydraulic losses.

(a) Hydraulic Losses

(i) Circulation Loss or Circulatory Flow Loss

The pressure difference on the leading and trailing edge of a blade and the inertia of the fluid particles in the blade passage result in a *circulatory flow or whirl slip* as discussed in Section 1.7. The actual head imparted by impeller is less than the theoretical work head imparted by impeller. The difference $(H_e - H_e')$ is considered as the loss of head in impeller due to circulatory flow. Therefore, the circulation loss or circulatory flow loss or whirl slip is given by,

$$h_c = h_e - h_e' = \frac{C_{w_2}C_{b_2}}{g} - \sigma_s \frac{C_{w_2}C_{b_2}}{g} = (1-\sigma_s)\frac{C_{w_2}C_{b_2}}{g} = (1-\sigma_s)H_e \tag{4.32}$$

The circulation loss is practically independent of discharge. Circulation loss occurs because there is a finite numbers of a blade with finite thickness on an impeller. However, if impeller has too many blades, there will be excessive flow blockage losses and losses due to growing boundary layers which lead to non-uniform flow speeds at the outer radius of the pump, consequently lower net head efficiency of the pump. These losses are called passage losses. Therefore, some engineering optimization is necessary in order to choose both the blade shape and number of blades which is beyond the scope of this book. The literature shows that 11, 14, and 16 are common numbers of impeller blades for medium sized centrifugal pumps.

(ii) Fluid Friction Loss in Flow Passages

This loss depends on the roughness of the passage and on the area of flow. This loss is proportional to the square of the velocity.

(iii) Shock Losses at Entrance

This loss occurs due to improper entry angle of the flow with respect to the blade angle. At design conditions, this loss is practically zero and increases at reduced or increased flow from normal values.

All the above losses occur across the impeller are named hydraulic losses, denoted by h_i.

(b) Non Hydraulic Losses

(i) Leakage Loss

Leakage of fluid occurs from the seals and through the clearance space between the casing and the shroud that surrounds the impeller. Some fluid after receiving the energy by the impeller may not exit from the pump outlet. A certain amount of fluid leaks from the high pressure to the low pressure side. This back flow of fluid towards suction side carries with it energy which is subsequently wasted in eddies. A part of fluid may also leak out of the casing to waste. Therefore, such leakage loss represents energy loss and affects the efficiency of the pump.

(ii) Mechanical Losses

These are losses due to friction at bearings and packings. Mechanical losses also include the power loss due to friction between the rotating faces of the impeller or disc and the fluid.

4.9.2 Efficiencies of a Pump

In a centrifugal pump, the power is transmitted from the electric motor shaft to the pump shaft and then to the impeller. Power is imparted to the liquid by the impeller. Consider a pump unit with manometric head H_m and outlet discharge Q. Important efficiencies of a centrifugal pump are discussed below.

(a) Hydraulic Efficiency or Impeller Efficiency

Summation of losses of head due to circulation, friction in blade passages and shock loss at the eye is known as *hydraulic loss* as already discussed. Therefore, the actual work head imparted by impeller is less than that of Euler head given by Eq. below:

$$H_a = H_e - h_i \tag{4.33}$$

Hydraulic efficiency is a ratio of the actual head or power imparted to the liquid by the impeller to the ideal head (Euler head) or power i.e. which could be transmitted without considering hydraulic losses. Mathematically,

$$\eta_h = \frac{\text{Actual head imparted by the impeller}}{\text{Theoritical or Euler Head}} = \frac{H_a}{H_e} = \frac{H_e - h_i}{H_e} \tag{4.34}$$

Equations (2.21) and (2.24) may also be referred by the reader.

(b) Volumetric Efficiency

As stated earlier, some quantity of fluid leaks in the clearances between impeller and casing. These leakages include those that flow back to the suction side and those out of the casing resulting in waste. Some quantity of fluid will not be delivered at the outlet of the impeller due to leakages. Therefore, the quantity of fluid handled by the impeller is more than that it is discharged. Volumetric efficiency is used to take leakage of the fluid into account. Volumetric efficiency of a pump is defined as the ratio of the quantity of fluid discharged from the outlet of the impeller to that entering into the eye of the impeller. Let Q be the discharge of the fluid from outlet of the impeller to the delivery pipe and ΔQ be the volume of the fluid leakage per second, then the discharge entering into the eye of the impeller is $(Q + \Delta Q)$. Therefore, volumetric efficiency,

$$\eta_v = \frac{\text{Quantity of fluid discharged per second from the outlet of the impeller}}{\text{Quantity of fluid entering per second into the eye of the impeller}}$$

$$\eta_v = \frac{\rho Q}{\rho(Q + \Delta Q)}$$

$$\eta_v = \frac{Q}{Q + \Delta Q} \tag{4.35}$$

Leakage of fluid is a small percentage of discharge.

(c) Manometric Efficiency

The ratio of the manometric head to the actual work head imparted by the impeller to the fluid is known as manometric efficiency. It indicates the effectiveness of the pump in increasing the total energy of the fluid from the energy imparted to it by the impeller for this purpose. Equations (2.22) may also be referred by the reader. Mathematically,

$$\eta_{\text{mano}} = \frac{H_m}{H_a} \tag{4.36}$$

(d) Mechanical Efficiency

The power at the impeller shaft exceeds the rotor or impeller power because of friction in the bearings and other mechanical parts. Mechanical efficiency is used to take power or work lost due to friction into account. Thus, mechanical efficiency is a ratio of the power transmitted to fluid from the impeller to the power available at the pump shaft (shaft power). Therefore,

$$\eta_m = \frac{\text{Power imparted by the impeller (Rotor or Impeller Power)}}{\text{Shaft power}}$$

$$\eta_m = \frac{\rho g(Q+\Delta Q)H_e}{P} \tag{4.37}$$

(e) Overall Efficiency

It is a ratio of the power output of the pump (energy imparted to the fluid or fluid power) to the power input to the pump from the motor or prime mover (shaft power).

$$\eta_o = \frac{\text{Power gained by the fluid or Fluid Power (output)}}{\text{Shaft Power}}$$

$$\eta_o = \frac{\rho g Q H_m}{P} = \frac{\rho g Q H_m}{P} \times \frac{H_a}{H_a} \times \frac{Q+\Delta Q}{Q+\Delta Q} \times \frac{H_e}{H_e} = \frac{H_a}{H_e} \times \frac{Q}{Q+\Delta Q} \times \frac{H_m}{H_a} \times \frac{\rho g(Q+\Delta Q)H_e}{P}$$

From Eqs (4.34), (4.35), (4.36) and (4.37),

$$\eta_o = \eta_h \times \eta_v \times \eta_{\text{mano}} \times \eta_m \tag{4.38}$$

Assuming (i) shockless radial entry, (ii) uniform velocities at the inlet and outlet of an impeller, (iii) no friction in the blade passage, and (iv) no leakage loss between impeller and casing. Then the work head imparted to the fluid by the impeller is the Euler head and quantity of fluid per unit time at outlet of the impeller is same as that passing through the impeller. Consequently, $\eta_h = 1$, $\eta_v = 1$. Therefore,

Manometric efficiency,

$$\eta_{\text{mano}} = \frac{H_m}{H_e} = \frac{H_m}{C_{w_2} C_{b_2}/g}$$

$$\eta_{\text{mano}} = \frac{H_m}{H_e} = \frac{g H_m}{C_{w2} C_{b2}} \tag{4.39}$$

Mechanical efficiency,

$$\eta_m = \frac{\text{Rotoror or Impeller Power}}{\text{Shaft Power}} = \frac{\rho g Q H_e}{P} = \frac{\rho Q C_{w_2} C_{b_2}}{P} \tag{4.40}$$

Overall efficiency,

$$\eta_o = \frac{\text{Power gained by the fluid (Fluid Power)}}{\text{Shaft Power}} = \frac{\rho g Q H_m}{P} \tag{4.41}$$

$$\eta_o = \frac{\rho g Q H_m}{P} \times \frac{H_e}{H_e} = \frac{H_m}{H_e} \times \frac{\rho g Q H_e}{P} = \frac{H_m}{H_e} \times \frac{\text{Rotor Power}}{\text{Shaft Power}}$$

$$\eta_o = \eta_{\text{mano}} \times \eta_m \tag{4.42}$$

4.9.3 Theoretical Head vs Discharge Characteristics

The work done on the fluid by the impeller per unit mass with the assumption of no whirl at the eye of the impeller given by $\dfrac{C_{w2} C_{b2}}{g}$ is usually known as work head imparted by the rotor on the fluid. The head developed by the pump will be the same if the flow through the impeller is frictionless and it can be considered as the theoretical head developed (Euler head). Therefore,

$$H_e = H_{th} = \frac{C_{w_2} C_{b_2}}{g} \tag{4.43}$$

From the velocity triangle at the exit as shown in Figure 4.17,

$$C_{w_2} = C_{b_2} - C_{f_2} \cot \beta_2 = C_{b_2} - (Q/A_2) \cot \beta_2 \tag{4.44}$$

where, Q is the discharge and A_2 is flow area at the outlet of the impeller.

$$H_e = \frac{C_{b_2}}{g}[C_{b_2} - (Q/A_2) \cot \beta_2] \tag{4.45}$$

We know that blade speed in m/min is given by,

$$C_{b_2} = \pi D_2 N$$

Flow area at outlet of the impeller is,

$$A_2 = \pi D_2 B_2$$

where B_2 is the width of the blade at the outlet. Substituting the value of C_{b_2}, A_2, A_2 in Eq. (4.45), we get,

$$H_{\text{th}} = H_e = \frac{1}{g}\left[\pi^2 D_2^2 N^2 - \left(\frac{\pi D_2 N}{\pi D_2 B_2} \cot \beta_2\right) Q\right]$$

$$H_{\text{th}} = H_e = \frac{1}{g}\left[\pi D_2^2 N^2 - \frac{N}{B_2} Q \cot \beta_2\right] \tag{4.46}$$

For a pump running at a constant speed,

$$H_{th} = H_e = K_1 - K_2 Q \cot \beta_2 \tag{4.47}$$

where,

$$K_1 = \frac{\pi^2 D_2^2 N^2}{g} \text{ and } K_2 = \frac{N}{g B_2} \tag{4.48}$$

For a given impeller at constant speed, K_1 and K_2 are constants and therefore, Eq. (4.47) represents theoretical head for a given blade angle β_2. Theoretical head and discharge bears a linear relationship, i.e. theoretical head varies linearly with discharge which is plotted in Figure 4.23 as curve *I*.

Equations (4.45), (4.46) and (4.47) show that for a given blade angle, when $Q = 0$, there is a finite positive value of ideal or theoretical or Euler head H_e. This shows that even if delivery valve of a pump is completely closed or shut-off and the pump is kept running, a positive pressure head is developed by the pump. The head produced by pump in shut-off condition of delivery is known as *shut-off head*. As there is no discharge in shut-off condition of valve, the efficiency of the pump is zero and the power imparted by the pump is simply dissipated as heat at shut-off condition. Although, centrifugal pumps can be kept running for a small period when discharge valve is shut-off, any prolonged operation will lead to damage due to overheating and greater mechanical stress. From Eq. (4.45), the ideal shut-off head is,

$$H_{\text{e-shutoff}} = \frac{C_{b_2}^2}{g} \tag{4.49}$$

The actual value of shut-off head is about 60% of the ideal shut-off head due to non-recoverable losses in the impeller.

The theoretical head will be reduced by $\frac{\sigma C_{w_2} C_{b_2}}{g}$ if slip is considered. Moreover, the slip will increase with the increase in discharge. Curve II in the head-discharge plot shows the effect of slip in Figure 4.23.

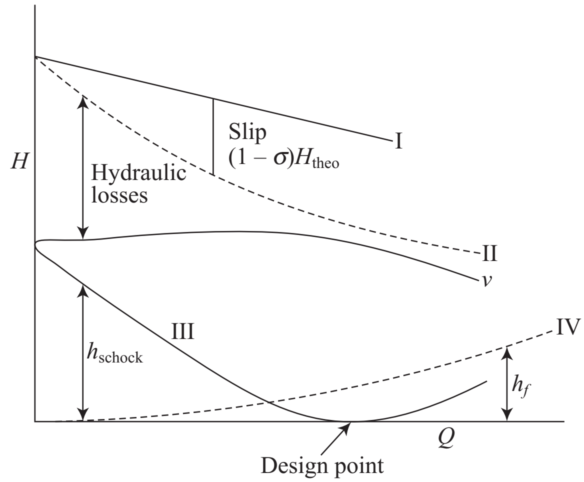


Figure 4.23 *Theoretical Head-Discharge Characteristics of a Centrifugal Pump*

The loss due to slip can occur in both, an ideal fluid and a real fluid. However, in a real fluid the shock losses at entrance and the friction losses in blade passages have also to be considered. If the pump operates at the design conditions, the shock losses are zero since the fluid moves tangentially onto the blades. At the off design conditions (either side of the design point), the shock loss increases according to the following relation:

$$h_{\text{shock}} = K_3(Q_{\text{off}} - Q)^2 \tag{4.50}$$

where, Q_{off} is the off design discharge and K_3 is a constant. Variation of head loss due to shock is shown by curve III in the Figure 4.23. The head loss due to friction is represented by curve IV and varies as the square of the discharge given by the following relation:

$$h_f = K_4 Q^2 \tag{4.51}$$

The curve V represents the actual head-discharge characteristics curve of the pump for a real fluid which is obtained by subtracting the sum of the ordinates of the curves III and IV from the ordinate of curve II at all values of abscissa.

4.9.4 Effect of Blade Outlet Angle on Head vs Discharge Characteristics

From Eq. (4.47), it is obvious that the Euler head is a linear function of $Q \cot \beta_2$ for a given impeller speed N, D_2 and B_2. Therefore, head-discharge characteristics of a centrifugal pump depends (among other parameters) on the blade outlet angle which in turn depends on the blade settings.

The variation of theoretical or ideal or Euler head with discharge for the different blade angle at the outlet of impeller is shown in Figure 4.24. It is seen that the slope of Euler head-discharge line is positive, zero and negative for forward curved, radial and backward curved blades respectively. Though, forward curved and radial blades have a very favourable head-discharge relationship, however the efficiencies are very poor due to high value of losses resulting from large swirl or turning. Additionally, larger absolute velocities at the outlet of the impeller in forward facing blades require efficient diffuser to convert the exit kinetic energy into pressure energy. Further, in forward facing blades, an instability called *surging* may occur which is an oscillatory condition in which pump hunts for proper operating point. Surging may cause only rough operation in a liquid pump, but it can be a major problem in compressor operation. Forward facing or radial vanes are seldom used in pumps due to the shortcomings discussed above. They find applications in certain special designs. Normally, backward curved blades with β_2 in the range of 20°–40° are commonly used. Backward facing blades yield the highest efficiency of the three geometries because fluid flows into and out of the blade passages with the least amount of turning. The energy transfer to the fluid and absolute velocity at pump exit is least in backward curved blades. The impeller using backward facing blades is known as a fast runner. When the impeller rotates at high speed as in rotating compressor/pump, centrifugal force acting on the vanes tries to straighten it out. For this reason radial type is mostly preferred and it is easy to manufacture. But backward curved pump is finding more and more applications with improvement in manufacturing techniques because of its higher efficiency.

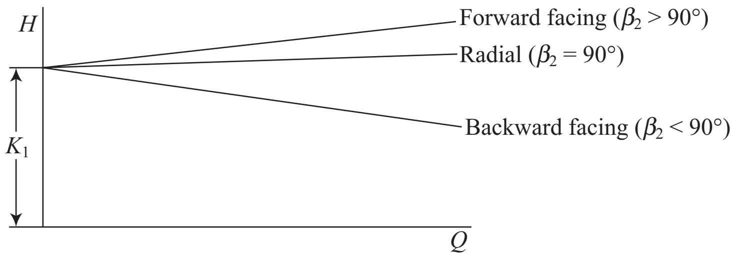


Figure 4.24 *Variation of Euler Head with Discharge for Different Blade Settings*

It is, however, of greater interest and importance to know the relationship between actual head developed by the pump and discharge. This relationship, in addition to outlet blade angle, will be affected by various losses that must be taken into account, which are proportional to the square of the discharge. The actual head-discharge characteristics after consideration of all losses, as discussed earlier, for various ranges of blade outlet angle are plotted in Figure 4.25.

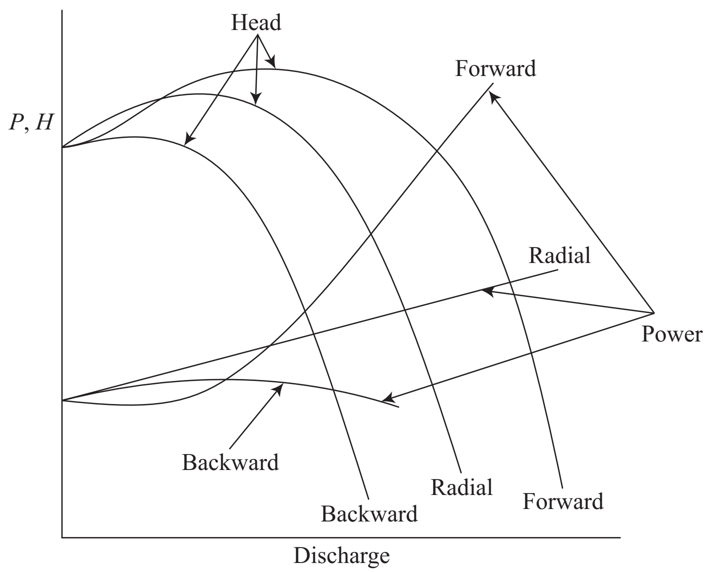


Figure 4.25 *Actual Head-Discharge Characteristics for Different Blade Settings*

The head-discharge curve is generally known as pump characteristic and is helpful in finding the suitability of a pump for given conditions of operation. For a backward curved pump ($\beta_2 < 90°$), the head-discharge curve is having steeply falling characteristics. In such a pump, the discharge would decrease as the head increases and there would be no danger of overloading the driving motor. A pump of flat characteristic ($\beta_2 = 90°$) is desirable under conditions where little variation in total head is encountered and the discharge fluctuates. A pump having rising characteristic ($\beta_2 > 90°$) is generally used when the actual lift is of small and constant amount and the friction head is large and variable with the discharge. This would be a case of a sump required to supply a long pipe system where the demand for water varies but a constant pressure is desired.

For both the radial and forward facing blades, the power increases as the discharge or flow rate is increased. In the backward curved blades, the maximum efficiency occurs in the region of maximum power. If the discharge increases beyond that at the design point, a decrease in power occurs in backward curved pumps. Therefore, the motor rated at the design point may be safely used at part load operation of the pump. This is known as self-limiting characteristic. If the pump motor is rated for maximum power in forward facing and radial blades, then it will be under-utilized for most of the time resulting in an increased cost of extra rating. If a smaller motor is employed which is rated at the design point, then if discharge increases beyond that at the design point, the motor will be overloaded. Hence, it is more difficult to decide on a choice of motor in radial and forward curved blades.

4.9.5 Loss of Head due to a Change from Normal Discharge

The efficiency of a pump will be maximum only when it is running and discharging at its design speed. This discharge at the maximum efficiency point against the designed head at design speed is known as *normal discharge or design discharge*. If the pump is running at the design speed with design discharge, the inlet velocity is radial and the relationship between the blade angle at inlet β_1 and the relative velocity is such that the relative velocity at the inlet is parallel to the tangent to the blade tip. Consider the velocity triangle Δabc as shown in 4.26 at the inlet of the pump when the flow enters tangentially at the inlet blade tip (shock-less entry). The peripheral blade velocity at the inlet shown by the line ab is C_{b_1} at the design speed and discharge. The velocity of flow at the inlet represented by ac is C_{f_1} which is equal to the absolute velocity C_1 at the inlet.

(a) Reduction from the Normal Discharge

Consider the situation when the discharge is decreased from the normal value by throttling, whereas the pump continues to run at the same speed as shown in Figure 4.26 (a).

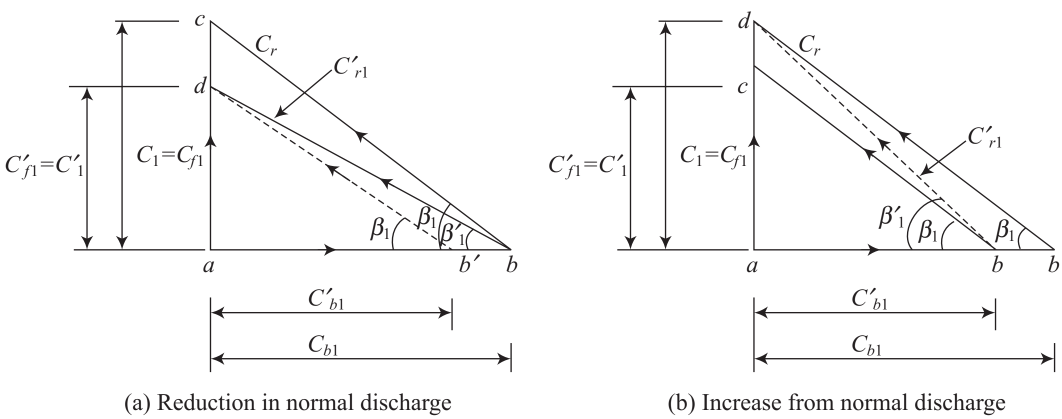


Figure 4.26 *Loss of Head due to Variation in Discharge from the Normal Value*

As the discharge is reduced, the radial velocity of flow is reduced to $C'_{f1} = C'_1$ represented by the line *ad*. Since the pump runs at the same speed, the line *ab* representing the blade peripheral velocity at the inlet remains unaltered. The new velocity triangle at inlet is Δabd and the new relative velocity is C'_{r1} represented by *bd* which will obviously not be parallel to the tangent at the blade's tip. Consequently, a shock will be experienced at entrance which indicates an energy loss. There will be an adjustment of the relative velocity such that its direction is along the blade angle β_1 and the flow velocity at inlet is $C'_{f1} = C'_1$. Hence, the relative velocity at the inlet will change to the new position indicated by the line $b'd$. This line $b'd$ which is parallel to *bc* satisfies the necessary conditions for the flow, viz, entrance angle of β_1 and the velocity of flow equal to $C'_{f1} = C'_1$. The velocity C'_{b1} represented by ab' could be considered *equivalent peripheral velocity*. Since $C'_{b1} = C'_1 \cot \beta_1$, the loss of head due to shock as a result of reduction from the normal discharge is given by,

$$h_{q_r} = \frac{(C_{b1} - C'_{b1})^2}{2g} = \frac{(C_{b1} - C'_1 \cot \beta_1)^2}{2g} = \frac{(C_{b1} - C'_{f1} \cot \beta_1)^2}{2g} \tag{4.52}$$

(b) Increase from the Normal Discharge

Consider the situation when the discharge is increased from the normal value by throttling whereas the pump continues to run at the same speed as shown in Figure 4.26 (b). The procedure for analysis if the discharge is increased beyond the normal discharge is same as that used for reduction of discharge. The loss of head due to shock as a result of increase in discharge is given by,

$$h_{qi} = \frac{(C'_{b1} - C_{b1})^2}{2g} = \frac{(C'_1 \cot \beta_1 - C_{b1})^2}{2g} = \frac{(C'_{f1} \cot \beta_1 - C_{b1})^2}{2g} \tag{4.53}$$

It is obvious from Eqs. (4.52) and (4.53) that the numerical value of the head loss would be the same whether the discharge is decreased or increased from the normal value. Thus, the general relationship for loss of head due to discharge being different from the normal discharge is given by,

$$h_q = \frac{(\text{Peripheral Blade Velocity-Equivalent Peripheral Blade Velocity})^2}{2g} = \frac{C_{b1}^2 - C_{b2}^2}{2g}$$

This energy loss due to variation in discharge from the normal discharge is called *shock loss* at entrance to the impeller.

$$\because \quad C_1 = C_{f1} \text{ and } \cot \beta_1 = \frac{C_{b1}}{C_{f1}} = \frac{C_{b1}}{(Q/A_1)}$$

$$C'_{b1} = C'_{f1} \cot \beta_1 = \frac{Q'}{A_1} \frac{A_1 C_{b1}}{Q} = \frac{Q'}{Q} C_{b1}$$

Substituting the value of C'_{b1} in the above equation of h_q, we have,

$$h_q = \frac{(C_{b1} - C'_{b1})^2}{2g} = \frac{C_{b1}^2}{2g}\left(1 - \frac{Q'}{Q}\right)^2 \tag{4.54}$$

4.9.6 Minimum Starting Speed of a Pump

When the pump is started by switching on the motor, it starts delivering fluid when the rise in pressure due to impeller action is large enough to overcome manometric head. Consider the Euler equation or ideal head developed in the form as given below that can be obtained by Eqs. (4.14) and (4.15),

$$H_e = \frac{C_{w2}C_{b2}}{g} = \left[\frac{(C_2^2 - C_1^2)}{2g} + \frac{(C_{r1}^2 - C_{r2}^2)}{2g} + \frac{(C_{b2}^2 - C_{b1}^2)}{2g}\right]$$

At the time of start of the pump, the fluid velocities are zero and the only head available is the centrifugal head $(C_{b2}^2 - C_{b1}^2)/2g$ which must overcome the manometric head. Thus, for flow to commence,

$$\frac{(C_{b2}^2 - C_{b1}^2)}{2g} \geq H_m \tag{4.55}$$

We know that manometric efficiency is,

$$\eta_{\text{mano}} = \frac{H_m}{H_e} = \frac{gH_m}{C_{w2}C_{b2}}$$

Thus, the condition governing the minimum speed of the pump so that flow can start,

$$\frac{(C_{b2}^2 - C_{b1}^2)}{2g} \geq \eta_{\text{mano}} H_e \Rightarrow \frac{(C_{b2}^2 - C_{b1}^2)}{2g} \geq \eta_{\text{mano}} \frac{C_{w2}C_{b2}}{g} \tag{4.56}$$

Let minimum speed required to start pumping action is $N_{\min}$. Then,

$$C_{b2} = \frac{\pi D_2 N_{\min}}{60} \text{ and } C_{b1} = \frac{\pi D_1 N_{\min}}{60} \tag{4.57}$$

Substituting the values of C_{b2} and C_{b1} from Eq. (4.57) into Eq. (4.55) so that flow can commence,

$$\frac{\pi^2 N_{\min}^2}{(60)^2}[D_2^2 - D_1^2] = 2gH_m$$

Therefore,

$$N_{\min} = \frac{60}{\pi\sqrt{(D_2^2 - D_1^2)}}\sqrt{2gH_m} \tag{4.58}$$

Minimum starting speed of centrifugal pump can also be found by substituting values of C_{b2} and C_{b1} from Eq. (4.57) into Eq. (4.56),

$$\frac{\pi^2 N_{\min}^2}{2g \times 60^2}[D_2^2 - D_1^2] = \eta_{\text{mano}} \frac{C_{w2}\pi D_2 N_{\min}}{60g}$$

$$N_{\min} = \frac{120\eta_{\text{mano}} C_{w2} D_2}{\pi[D_2^2 - D_1^2]} \tag{4.59}$$

Therefore, combining Eqs. (4.58) and (4.59), the minimum speed of the pump for the fluid flow to start is,

$$N_{\min} = \frac{60}{\pi\sqrt{(D_2^2 - D_1^2)}}\sqrt{2gH_m} = \frac{120\eta_{\text{mano}} C_{w2} D_2}{\pi[D_2^2 - D_1^2]} \tag{4.60}$$

4.9.7 Effect of Outlet Blade Angle on Manometric Efficiency

Consider a centrifugal pump with radial entry. At the outlet of the impeller,

$$\frac{C_{w2}C_{b2}}{g} = H_m + \frac{C_2^2}{2g}$$

$$H_m = \frac{C_{w2}C_{b2}}{g} - \frac{C_2^2}{2g} \tag{4.61}$$

From the outlet velocity triangle,

$$C_{w2} = C_{b2} - C_{f2} \cot \beta_2$$

$$C_2^2 = C_{w2}^2 + C_{f2}^2 = [C_{b2} - C_{f2} \cot \beta_2]^2 + C_{f2}^2 = C_{b2}^2 - 2C_{b2}C_{f2} \cot \beta_2 + C_{f2}^2 \cot^2 \beta_2 + C_{f2}^2$$

$$C_2^2 = C_{b2}^2 + C_{f2}^2(1 + \cot^2 \beta_2) - 2C_{b2}C_{f2} \cot \beta_2$$

$$C_2^2 = C_{b2}^2 + C_{f2}^2 \text{cosec}^2 \beta_2 - 2C_{b2}C_{f2} \cot \beta_2 \tag{4.62}$$

Therefore,

$$H_m = \frac{[C_{b2} - C_{f2} \cot \beta_2]C_{b2}}{g} - \frac{C_{b2}^2 + C_{f2}^2 \text{cosec}^2 \beta_2 - 2C_{b2}C_{f2} \cot \beta_2}{2g}$$

$$H_m = \frac{2C_{b2}^2 - 2C_{b2}C_{f2} \cot \beta_2 - C_{b2}^2 - C_{f2}^2 \text{cosec}^2 \beta_2 + 2C_{b2}C_{f2} \cot \beta_2}{2g}$$

$$H_m = \frac{C_{b2}^2 - C_{f2}^2 \text{cosec}^2 \beta_2}{2g} \tag{4.63}$$

We know that manometric efficiency,

$$\eta_{\text{mano}} = \frac{H_m}{H_e} = \frac{gH_m}{C_{w2}C_{b2}}$$

Substituting value of C_{w2} from the Eq. (4.44) and H_m from the Eq. (4.63), we get,

$$\eta_{\text{mano}} = \frac{(C_{b2}^2 - C_{f2}^2 \text{cosec}^2 \beta_2)}{2C_{b2}(C_{b2} - C_{f2} \cot \beta_2)} \tag{4.64}$$

It is clear from Eq. (4.64) that,

$$\eta_{\text{mono}} = f(C_{b2}, C_{f2}, \beta_2) \tag{4.65}$$

Case I

If $\beta_2 = 90°$, then cosec 90 = 1 and cot 90° = 0;

$$\eta_{\text{mano}} = \frac{1}{2}\left[1 - \left(\frac{C_{f2}}{C_{b2}}\right)^2\right] < 0.5 \tag{4.66}$$

Therefore, if $\beta_2 = 90°$, then manometric efficiency is always less than 50%.

Case II

If $\beta_2 \to 0$ then, cosec $\beta_2 \to \infty$, cot $\beta_2 \to \infty$, $\Rightarrow$ When $\beta_2 \to 0$, then cosec $\beta_2 \approx$ cot β_2

$$\eta_{\text{mano}} = \frac{C_{b2}^2 - C_{f2}^2 \cot^2 \beta_2}{2C_{b2}(C_{b2} - C_{f2} \cot \beta_2)}$$

$$\therefore \qquad \eta_{\text{mano}} = \left(\frac{C_{b2} + C_{f2} \cot \beta_2}{2C_{b2}}\right)$$

When $\beta_2 \to 0$, $C_{w2} \to 0 \Rightarrow [(C_{b2} - C_{w2}) = C_{f2} \cot \beta_2] \to C_{b2}$ and $\left(\frac{C_{b2} + C_{f2} \cot \beta_2}{2C_{b2}}\right) \to 1$

Therefore,

$$\eta_{\text{mano}} \to 1 \tag{4.67}$$

Hence, if the outlet blade angle approaches zero then manometric efficiency approaches the highest possible value of 100%. Therefore, it is clear that smaller the value of outlet blade angle β_2, higher would be the value of manometric efficiency. Infinitesimal values of blade angles mean very long blades which consequently results in excessive resistance to flow inside the impeller. Therefore, the usual practice is not to have outlet blade angle less than 20° and the value of outlet blade angle is kept in the neighbourhood of 20° to obtain high manometric efficiency.

4.10 Performance Characteristics of Actual Centrifugal Pumps

Centrifugal pump performance characteristics curves are graphical representation of the results obtained through a number of experiments or tests performed on it. We know that the head, discharge, power, efficiency and speed are prime variables on which the performance of a pump depends. Some of the variables, out of these, are kept constant and the interdependency of the others is represented in the form of charts. Performance characteristics curves are necessary to forecast the operation and performance of a pump working under different head, flow rate and speed. Performance characteristics of centrifugal pumps are usually studied under following four categories: (1) Main characteristics curves, (2) Iso-efficiency (Hill/Muschel) curves, (3) Operating characteristics curves, and (4) Constant-head or constant-discharge curves.

4.10.1 Main Characteristics Curves

Main characteristics curves are plotted from the laboratory test data of a particular type of pump having a fixed geometry with head, power and efficiency as ordinates and discharge as a common abscissa for various pump speeds. Main characteristics curves of a pump in academic circles including research publications, characteristics curves are usually plotted for a fixed impeller diameter with the speed N as the third variable. The practice adopted by pump manufacturers' literature is to make characteristics curves for a fixed nominal speed with impeller diameter as the third variable. Figure 4.27 shows the main characteristics of a pump.

The head here refers to manometric head developed by the pump. The maximum volume flow rate through a pump occurs when net head on the pump is zero. The maximum discharge is called the pump's free delivery. The condition of free delivery is obtained when there is no restriction to the flow at the pump inlet or outlet, i.e. when there is no load on the pump. It is obvious from actual head-discharge curves shown

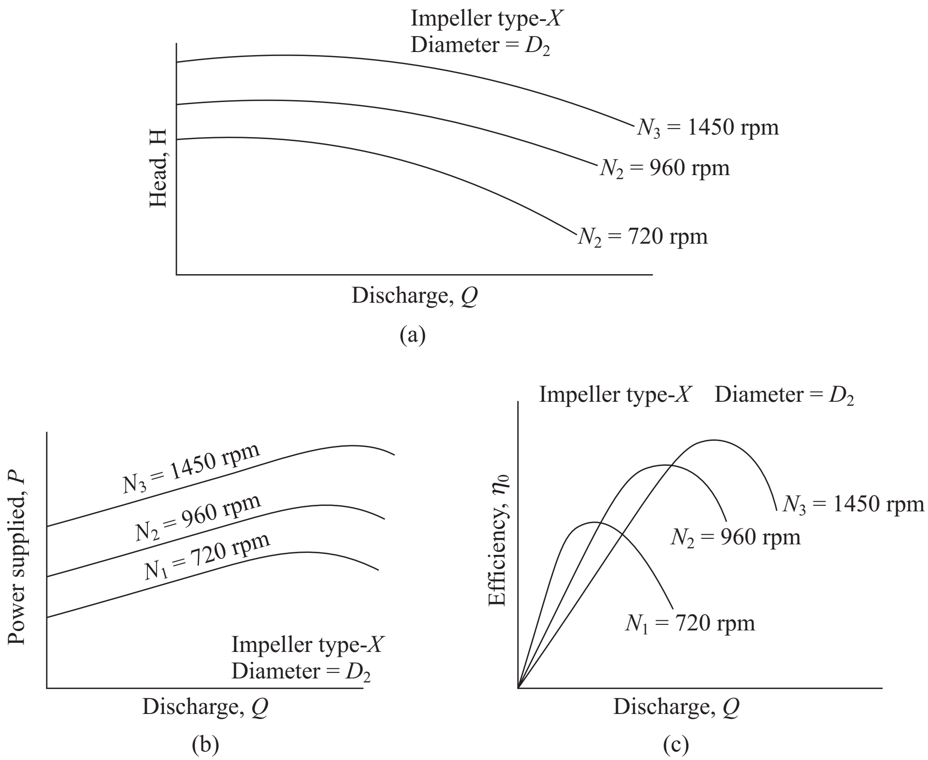


Figure 4.27 *Main Characteristics of a Centrifugal Pump*

in Figure 4.27 (a) that for a particular speed, the variation of actual head with discharge is non-linear and it is approximately parabolic. Actual head decreases with increase in discharge at a given speed and head developed increases with increase in speed at a fixed flow rate or discharge. There is a finite intercept of head on the y axis for each speed representing a non-zero shut-off head. The shut-off head increases with speed. The actual shut-off head at a given speed is 60% of the theoretical shut-off head at the same speed which is given by Eq. (4.49). The theoretical head-discharge relationship for a given speed is linear as indicated by Eq. (4.47); however, the actual head discharge curve is of parabolic type as hydraulic losses are proportional to the square of the discharge. The variation of power required with the discharge is shown in Figure 4.27 (b) for various pump speeds. The power increases non-linearly with discharge at a particular speed of the pump. It is also evident that power required for pumping a given discharge will be more at higher speeds of the pump.

Figure 4.27 (c) represents the variation of the overall efficiency of a centrifugal pump with discharge for various speeds of the pump. For a constant speed of the pump, each $\eta_0 - Q$ curve starts from zero, reaches a maximum at a particular discharge and then decreases with further increase in the value of discharge. At the *free delivery* condition, the efficiency is zero because the net head is zero though discharge being maximum at this point. At the other extreme i.e. at the *shut-off condition*, the efficiency is again zero because the pump is doing no useful work ($Q = 0$). The point at which the efficiency reaches the maximum is called *Best Efficiency Point* (BEP). The peak efficiency values increases with increase in the speed of the pump.

4.10.2 Iso-Efficiency Curves

Pump manufacturers offer several choices of impeller diameter in single casing. There are several advantages of this: (i) manufacturing costs are reduced, (ii) capacity of the pump is increased by simply replacing impeller, (iii) installation mountings are standardized, and (iv) equipment for a different application may be reused. When plotting the characteristics curves for such a 'family' of pumps, manufacturers do not plot the separate curves of H, η_o and P for various impeller diameters in the form drawn in Figure 4.27. Instead, manufacturers prefer to combine the performance curves for an entire family of pumps of different diameters into a single graph as shown in Figure 4.28 (a). They draw the $H - Q$ curve for each impeller diameter in the same way as in Figure 4.27, but create *contour lines* of constant efficiency by drawing smooth curves through the points that have the same value of η_o for the different impeller diameters. Contour lines of constant power are also drawn on the same graph in similar fashion as shown in Figure 4.28 (b). Thus, iso-efficiency curves also known as *Hill Charts/Muschel Curves* represent the entire characteristics of the pump in one plot which help in locating best regions of operation in a given problem. It should be noted that identical pump casing is used for the different impeller diameters for plotting these iso-efficiency curves. It is clear from Figure 4.28 (b) that pump manufacturers do not always plot performance characteristics curves to the free delivery. This is because of the fact that pumps are not operated for free delivery due to the low values of net head and efficiency. If the higher values of discharge and head are required, then the customer should step up to the next casing of larger size, or consider using additional pumps in series or parallel. Each such horizontal line indicating a constant efficiency gives two values of discharge for each speed due to nature of the efficiency-discharge curve. Similar intercepts with different horizontal lines provide a set of points $f(Q, N, \eta_o)$. These points are then mapped to Figure 4.27 (a) to get $f(Q, N, \eta_o, H)$. Points having the same efficiency are joined (with interpolation wherever necessary) to obtain iso-efficiency curves in the appropriate $H - Q$ reference axes. Figure 4.28 (b) is one such schematic chart for three impellers of diameters D_1, D_2 and D_3 all running at nominal speed N_1. The figure also presents iso-efficiency curves as well as curves corresponding to power P_1, P_2 and P_3. It is clear from the performance characteristics plot of Figure 4.28 (b) that the larger the impeller for a given pump casing, higher is the maximum achievable efficiency. Then why should anyone buy the smaller impeller pump? It is noteworthy in answering this question that the customer's application requires a certain combination of discharge and head. If the

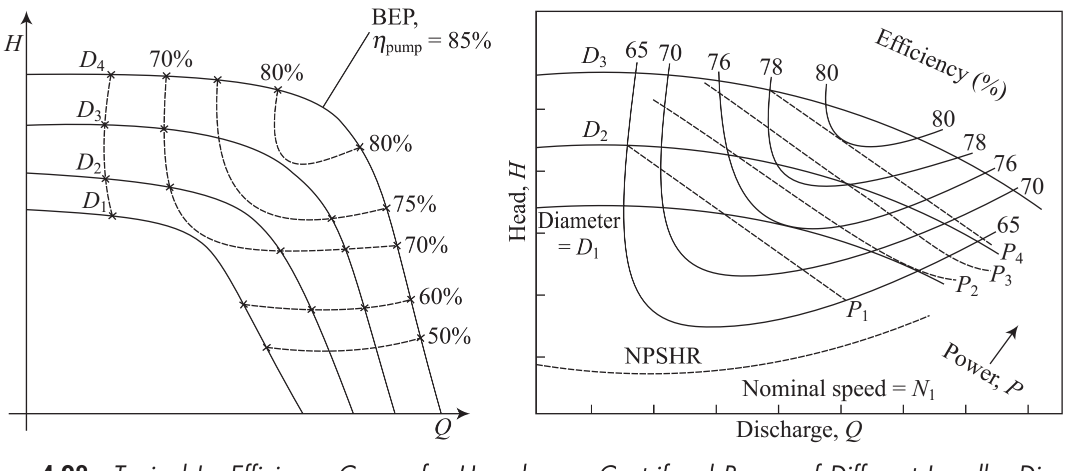


Figure 4.28 *Typical Iso-Efficiency Curves for Homologous Centrifugal Pumps of Different Impeller Diameters but Same Casing*

requirements match with a particular impeller diameter, it may be more cost effective to sacrifice pump efficiency in order to satisfy these requirements. It is to be noteworthy that an extra curve labelled NPSHR is also shown. This curve is important for the suction side conditions of the pump that must also be carefully considered during selection and installation of a pump. It is clear that as the efficiency increases, width of the curve decreases and ultimately it shrinks to a point at some efficiency which corresponds to the point of maximum efficiency for that pump. Also for each speed, there is only one point of maximum efficiency. Thus, the line joining these points is the line at which the pump is to be operated for that efficiency.

4.10.3 Operating Characteristics Curves

Operating characteristics of a pump are the characteristics of the given type of pump of fixed geometry at constant speed. Operating characteristics of a pump are extracted from the main characteristics curves. The variation of head, power and efficiency with respect to discharge gives the operating characteristics of a pump at a pre-fixed value of speed or design speed. The normal operating condition of a pump corresponds to the best efficiency point (BEP).

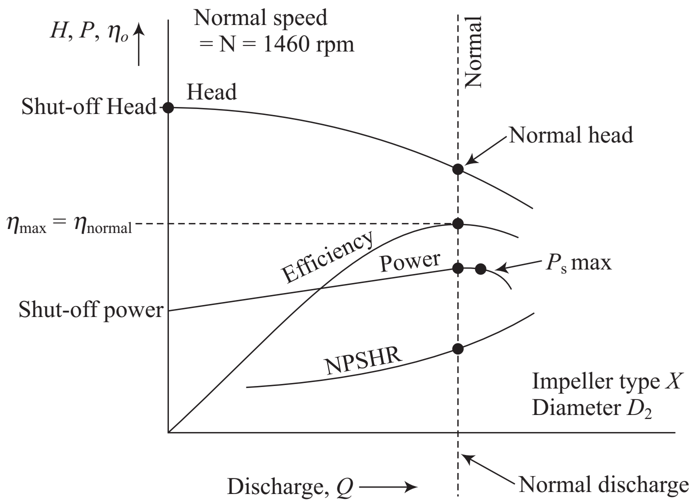


Figure 4.29 *Operating Characteristics of a Centrifugal Pumps*

Figure 4.29 shows the operating characteristics curves of a pump. Since the efficiency curve is quite sharp near the operating point (BEP), it is desirable to run the pump at the design flow rate. The head corresponding to maximum efficiency is known as *normal or design head*. The efficiency is a function of the flow rate or discharge. It attains a maximum value at a specific value of flow rate, generally known as the *normal or design flow rate or discharge or capacity* of the pump. Similarly, values of power and speed corresponding to maximum efficiency point (BEP) are known as normal or *design power* and *normal or design speed*. The NPSHR being an essential parameter is also represented as a function of discharge. The following points are noteworthy:

- The shut-off power is smaller than that consumed at normal or design conditions.
- The difference between maximum power and normal or design operating power is small. Thus, there is no danger of overloading the motor of the pump at any point.

EXAMPLE 4.5 A centrifugal pump is delivering 480 l/s of water while running at 600 rpm. The head developed by the pump is 9.6 m and manometric efficiency is 85%. The water enters the impeller without any whirl and the velocity of flow is 3.0 m/s. The loss of head in the pump due to fluid friction may be assumed to be $0.02C_2^2$ where the velocity with which water leaves the impeller is C_2. Find (a) diameter of the impeller, (b) blade angle at outlet, (c) flow area at outlet.

Solution

Given: Q = 480 l/s = 0.48 m^3/s, N = 600 rpm, H_m = 9.6 m, η_{mano} = 0.85, C_{f2} = 3.0 m/s, $h_f = 0.02C_2^2$

(a) Diameter of the Impeller

We know that manometric efficiency is,

$$\eta_{\text{mano}} = \frac{gH_m}{C_{w2}C_{b2}} \tag{1}$$

$$0.85 = \frac{g \times 9.6}{C_{w2}C_{b2}}$$

$$\frac{C_{w2}C_{b2}}{g} = \frac{9.6}{0.85}$$

$$\frac{C_{w2}C_{b2}}{g} = 11.294 \tag{2}$$

We know that if there is no whirl at the inlet i.e. radial entry, no slip or circulation (slip), no friction in the blades passages and no shock losses in the impeller; then the head supplied to the fluid by the impeller is the theoretical or Euler or working head. However, in actual cases, hydraulic losses (slip, shock losses and friction in blade passages) occur. A fraction of the velocity head at exit of the impeller cannot be converted into pressure head due to formation of eddies in the volute casing. Therefore, the head imparted by the impeller will be less than the Euler head. Considering the head loss due to fluid friction, the head developed will be given by the following relation:

$$H_m = \frac{C_{w2}C_{b2}}{g} - h_f = \frac{C_{w2}C_{b2}}{g} - 0.02C_2^2 \tag{3}$$

Substituting the value of $\dfrac{C_{w2}C_{b2}}{g}$ from Eq. (2) into Eq. (3),

$$9.6 = 11.294 - 0.02C_2^2$$

$$C_2 = 9.20 \text{ m/s} \tag{4}$$

From the velocity triangle at outlet as shown in Figure 4.30,

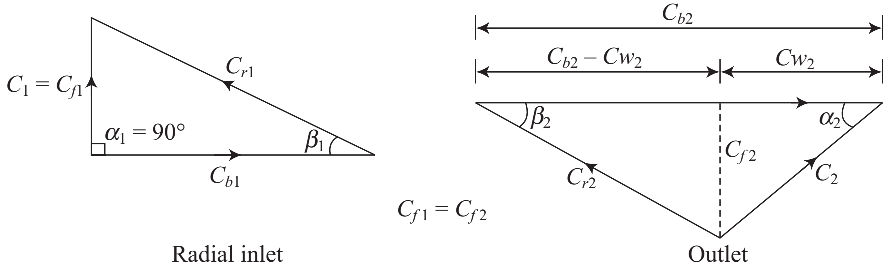


Figure 4.30 *Velocity Triangles of Example 4.5*

$$\sin \alpha_2 = \frac{C_{f2}}{C_2} \tag{5}$$

$$\sin \alpha_2 = \frac{3.0}{9.20}$$

$$\alpha_2 = 19.03° \tag{6}$$

Velocity of whirl at outlet,

$$C_{w2} = C_2 \cos \alpha_2 \tag{7}$$

$$C_{w2} = 9.2 \cos 19.03$$

$$C_{w2} = 8.70 \text{ m/s} \tag{8}$$

Substituting value of C_{w2} from Eq. (8) into Eq. (2), we get,

$$C_{b2} = 12.74 \text{ m/s} \tag{9}$$

Tangential velocity of vanes is given by,

$$C_{b2} = \frac{\pi D_2 N}{60} \tag{10}$$

$$12.74 = \frac{\pi \times D_2 \times 600}{60}$$

Diameter of impeller,

$$D_2 = 0.406 \text{ m} = 406 \text{ mm} \tag{11}$$

(b) Blade Angle at Outlet
From the velocity triangle at outlet,

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} \tag{12}$$

$$\tan \beta_2 = \frac{3.0}{12.74 - 8.70}$$

$$\beta_2 = 36.6° \tag{13}$$

(c) Flow Area at Outlet

$$\text{Discharge} = \text{Flow Area} \times \text{Velocity} \Rightarrow Q = A_{f2} \times C_{f2} \tag{14}$$

$$0.48 = A_{f2} \times 3.0$$

$$A_{f2} = 0.16 \text{ m}^2 \tag{15}$$

EXAMPLE 4.6 The inlet and outlet diameters of a centrifugal pump running at 1000 rpm are 0.20 m and 0.5 m respectively. It delivers 0.27 m^3/s of water against a head of 14 m. Determine (a) the loss of head due to shock when the discharge is reduced by 25% by closing the delivery valve while keeping the pump speed to be constant, and (b) minimum starting speed of the pump.

Solution

Given: $N = 1000$ rpm, $D_1 = 0.20$ m, $D_2 = 0.5$ m, $Q = 0.27\ m^3/s$, $H_m = 14$ m

The peripheral blade velocity at inlet,

$$C_{b1} = \frac{\pi D_1 N}{60} \tag{1}$$

$$C_{b1} = \frac{\pi \times 0.20 \times 1000}{60}$$

$$C_{b1} = 10.472 \text{ m/s} \tag{2}$$

(a) Loss of Head due to Reduced Discharge

Since the discharge is reduced by 25% from the normal discharge, therefore the value of reduced discharge is,

$$Q' = \left(1 - \frac{25}{100}\right)Q \tag{3}$$

$$Q' = 0.75 \times 0.27$$

$$Q' = 0.2025 \text{ m}^3\text{/s} \tag{4}$$

We know that the loss of head due to shock when discharge is reduced,

$$h_{qr} = \frac{C_{b1}^2}{2g}\left(1 - \frac{Q'}{Q}\right)^2 \tag{5}$$

$$h_{qr} = \frac{10.472^2}{2 \times 9.81}\left(1 - \frac{0.2025}{0.27}\right)^2$$

$$h_{qr} = 0.349 \text{ m} \tag{6}$$

(b) Minimum Starting Speed

We know that minimum starting speed of the centrifugal pump is given by,

$$N_{\min} = \frac{60}{\pi\sqrt{D_2^2 - D_1^2}} \times \sqrt{2gH_m} \tag{7}$$

$$N_{\min} = \frac{60}{\pi\sqrt{0.5^2 - 0.20^2}} \times \sqrt{2g \times 14}$$

$$N_{\min} = 685 \text{ rpm} \tag{8}$$

EXAMPLE 4.7 A centrifugal pump running at 1400 rpm has an impeller of diameter 0.3 m with backward curved blades. The blade angle at the outlet is 30°. Determine the theoretical head and velocity at exit (a) when the pump is running as it is designed, (b) when the pump is running in reverse direction at the same speed. The velocity of flow at the outlet is 15% of the peripheral velocity of the blades in both the cases.

Solution

Given: $N = 1400$ rpm, $D_2 = 0.3$ m, $\beta_2 = 30°$, $C_{f2} = 15\%$ of C_{b2}

(a) Pump Running as Designed

$$C_{b2} = \frac{\pi D_2 N}{60} \tag{1}$$

$$C_{b2} = \frac{\pi \times 0.3 \times 1400}{60}$$

$$C_{b2} = 21.99 \text{ m/s} \tag{2}$$

Velocity of flow at the outlet,

$$C_{f2} = 0.15 C_{b2} = 0.15 \times 21.99$$

$$C_{f2} = 3.2985 \text{ m/s} \tag{3}$$

Velocity triangles for the Case (a) when the pump has backward curved blades are shown in Figure 4.31. From the velocity triangle at the outlet,

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} \tag{4}$$

$$\tan 30 = \frac{3.2985}{21.99 - C_{w2}}$$

$$C_{w2} = 16.28 \text{ m/s} \tag{5}$$

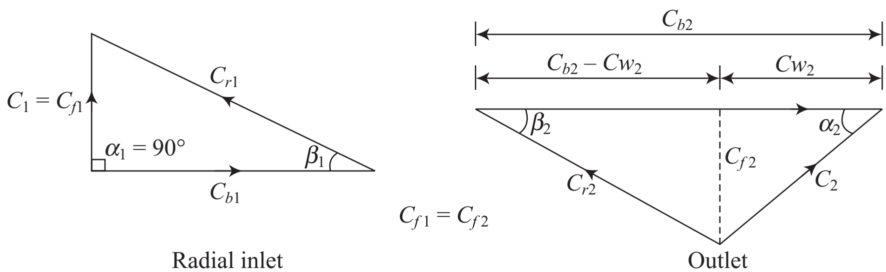


Figure 4.31 *Velocity Triangles of Example 4.7 when the Pump has Backward Curved Vanes*

Theoretical head developed by the pump or Euler head,

$$H_e = \frac{C_{w2} C_{b2}}{g} \tag{6}$$

$$H_e = \frac{16.28 \times 21.99}{9.81}$$

$$H_e = 36.493 \text{ m} \tag{7}$$

Referring again the velocity triangle at outlet,

$$C_2 = \sqrt{C_{f2}^2 + C_{w2}^2} \tag{8}$$

$$C_2 = \sqrt{3.2985^2 + 16.28^2}$$

$$C_2 = 16.611 \text{ m/s} \tag{9}$$

(b) When the Direction of Rotation is Reversed

When the direction of rotation of the pump is reversed, the blades will act as the forward curved blades with blade angle at outlet, $\beta_2 = 180 - \beta_2' = 180 - 30 = 150°$. The outlet velocity triangle is as shown in Figure 4.32.

The peripheral velocity of blades will remain the same as diameter and speed of rotation are same. Similarly, the velocity of flow at outlet is same because it is 15% of the peripheral velocity in both the cases. From the velocity triangle at outlet,

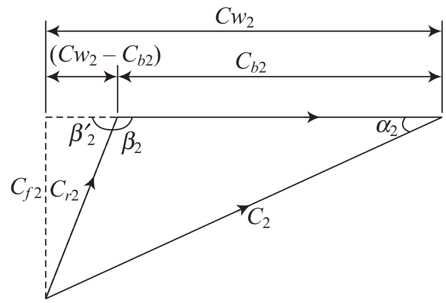


Figure 4.32 *Velocity Triangles of Example 4.7 at Outlet when Running in Reverse Direction*

$$\tan\beta_2' = \frac{C_{f2}}{C_{w2} - C_{b2}} \Rightarrow \tan 30 = \frac{3.2985}{C_{w2} - 21.99}$$

$$C_{w2} = 27.70 \text{ m/s} \quad (10)$$

The theoretical head imparted by the pump or Euler head,

$$H_e = \frac{C_{w2}C_{b2}}{g} \Rightarrow H_e = \frac{27.70 \times 21.99}{9.81}$$

$$H_e = 62.1 \text{ m} \quad (11)$$

Absolute velocity at exit,

$$C_2 = \sqrt{C_{f2}^2 + C_{w2}^2 = 3.2985^2 + 27.70^2}$$

$$C_2 = 27.90 \text{ m/s} \quad (12)$$

EXAMPLE 4.8 A centrifugal pump has backward curved vanes with outlet blade angle of 30°. The diameter and speed of the impeller is 0.3 m and 1200 rpm respectively. The velocity of flow remains constant at the inlet and outlet and it is 2 m/s. Calculate (a) Kinetic head flow at the exit of the impeller, and (b) the theoretical rise in pressure head across the impeller.

If the volute casing can convert 55% of the kinetic energy exiting the impeller into pressure energy, determine (c) theoretical total pressure head difference between the outlet and inlet of the pump.

Solution

Given: $\beta_2 = 30°$, $D_2 = 0.3$ m, $N = 1200$ rpm, $C_{f1} = C_{f2} = 2$ m/s

The peripheral velocity at outlet,

$$C_{b2} = \frac{\pi D_2 N}{60} \quad (1)$$

$$C_{b2} = \frac{\pi \times 0.3 \times 1200}{60}$$

$$C_{b2} = 18.85 \text{ m/s} \quad (2)$$

From the velocity triangles as shown in Figure 4.33,

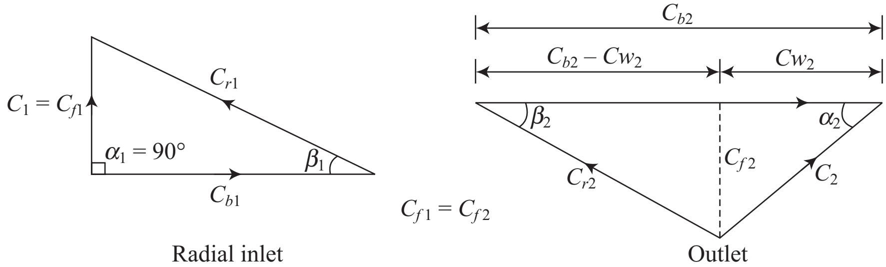


Figure 4.33 *Velocity Triangles of Example 4.8*

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} \quad (3)$$

$$\tan 30 = \frac{2.0}{18.85 - C_{w2}}$$

$$C_{w2} = 15.39 \text{ m/s} \quad (4)$$

(a) Kinetic Head Flow at Exit

From the velocity triangle at exit,

$$C_2 = \sqrt{C_{w2}^2 + C_{f2}^2} \quad (5)$$

$$C_2 = \sqrt{15.39^2 + 2^2}$$

$$C_2 = 15.52 \text{ m/s} \quad (6)$$

Kinetic head at the exit i.e. the kinetic energy per unit weight of liquid at the exit is,

$$\text{Kinetic head at exit} = \frac{C_2^2}{2g} = \frac{15.52^2}{2 \times 9.81}$$

$$\text{Kinetic head at exit} = 12.28 \text{ m} \quad (7)$$

(b) Theoretical Rise in Pressure Head

Kinetic energy per unit weight of liquid at the entry is,

$$\text{Kinetic head at entry} = \frac{C_1^2}{2g} = \frac{C_{f1}^2}{2g} = \frac{2^2}{2 \times 9.81}$$

$$\text{Kinetic head at entry} = \frac{C_1^2}{2g} = 0.204 \text{ m} \quad (8)$$

Theoretical head imparted by the pump or the Euler head,

$$H_e = \frac{C_{w2} C_{b2}}{g} \quad (9)$$

$$H_e = \frac{15.39 \times 18.85}{9.81}$$

$$H_e = 29.57 \text{ m} \quad (10)$$

Applying Bernoulli's equation between impeller inlet and outlet considering no frictional losses, we get,

$$\frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1 + \frac{C_{w2}C_{b2}}{g} = \frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2$$

Assuming no change in datum head i.e. $Z_1 = Z_2$,

$$\frac{p_1}{\rho g} + \frac{C_1^2}{2g} + \frac{C_{w2}C_{b2}}{g} = \frac{p_2}{\rho g} + \frac{C_2^2}{2g}$$

Therefore, theoretical or ideal difference in pressure head across the impeller,

$$\Delta H_{pi} = \left(\frac{p_2}{\rho g} - \frac{p_1}{\rho g}\right) = \left(\frac{C_1^2}{2g} - \frac{C_2^2}{2g}\right) + \frac{C_{w2}C_{b2}}{g} = \left(\frac{C_1^2}{2g} - \frac{C_2^2}{2g}\right) + H_e \quad (11)$$

Substituting the values from Eqs (6), (9) and (10),

$$\Delta H_p = 0.204 - 12.28 + 29.57$$

$$\Delta H_p = 17.494 \text{ m} \quad (12)$$

(c) When there is Recovery of Pressure Head in Casing

Since the volute casing converts 55% of the kinetic energy exiting the impeller into pressure energy, therefore the absolute velocity at the exit will be (1 – 55/100) = 0.45 times the exit velocity of the case (a). Hence, in this case, recovery of the pressure head in the casing by Eqs. (8) and (9),

$$\frac{C_2^2}{2g} = 0.45 \times 12.28 = 5.526 \text{ m} \quad (13)$$

Therefore, by substituting the values from Eqs. (8), (10) and (13), we get,

$$\Delta H_{pi} = 0.204 - 5.526 + 29.57$$

$$\Delta H_{pi} = 24.248 \text{ m} \quad (14)$$

EXAMPLE 4.9 A centrifugal pump running at 1200 rpm has impeller diameter of 0.4 m. It has 12 backward curved blades at 28° each of 5 mm thickness. The width of the flow passages at the outlet is 15 mm. The pump is delivering 54 l/s of water. The pressure gauges are fitted close to the pump at the suction and discharge pipes at 3 m above the water level of supply reservoir. The pressure readings are 5 m water (vacuum) and 18 m water (gauge) on the suction and delivery sides respectively. If 45% of velocity is recovered as static pressure head in the volute casing, find (a) Euler head, (b) manometric efficiency, (c) losses in the impeller, (d) motor power to run the pump if mechanical efficiency is 92%. Assume no whirl at entrance and no circulatory flow or slip in the impeller.

Solution

Given: N = 1200 rpm, D_2 = 0.4 m, z = 12, β_2 = 28°, t = 5 mm, B_2 = 15 mm, Q = 54 l/s = 0.054 m^3/s,

$Z_1 = Z_2 = 3$ m, $\frac{p_1}{\rho_w g}$ = 5 m water (vacuum), $\frac{p_2}{\rho_w g}$ = 18 m water (gauge), $\eta_m = 0.92$

Pressure recovery in casing = 45% of velocity head at exit.

Tangential blade velocity at the outlet,

$$C_{b2} = \frac{\pi D_2 N}{60} \quad (1)$$

$$C_{b2} = \frac{\pi \times 0.4 \times 1200}{60}$$

$$C_{b2} = 20.944 \text{ m/s} \quad (2)$$

Area of the flow at the exit,

$$A_{f2} = (\pi D_2 - zt)B_2 \quad (3)$$

$$A_{f2} = (\pi \times 0.4 - 12 \times 0.005) \times 0.015$$

$$A_{f2} = 0.018 \text{ m}^2 \quad (4)$$

We know that discharge is the product of flow area and velocity of flow. Therefore,

$$Q = A_{f2} \times C_{f2} \quad (5)$$

$$0.054 = 0.018 \times C_{f2}$$

$$C_{f2} = 3 \text{ m/s} \quad (6)$$

From the velocity triangle at outlet of the impeller as shown in Figure 4.34,

$$\tan \beta_2 = \frac{C_{f2}}{C_{b2} - C_{w2}} \quad (7)$$

$$\tan 28 = \frac{3}{20.944 - C_{w2}}$$

$$C_{w2} = 15.30 \text{ m} \quad (8)$$

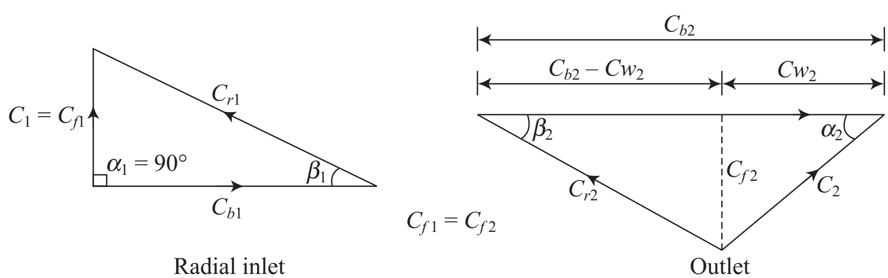


Figure 4.34 *Velocity Triangles of Example 4.9*

(a) Theoretical Head

Theoretical head developed by the pump or Euler head is,

$$H_e = \frac{C_{w2} C_{b2}}{g} \quad (9)$$

$$H_e = \frac{15.30 \times 20.944}{9.81}$$

$$H_e = 32.67 \text{ m} \quad (10)$$

(b) Manometric Efficiency

Actual increase in pressure head across the pump is equal to the difference of readings of pressure gauges on the delivery and suction sides. Therefore, actual rise in pressure head or actual manometric head is,

$H_{ma} = \Delta H_{pa}$ = Pressure head on the delivery side – Pressure head on the suction side

$$H_{ma} = 18 - (-5)$$

$$H_{ma} = \Delta H_{pa} = 23 \text{ m} \tag{11}$$

Actual manometric efficiency,

$$(\eta_{mano})_a = \frac{H_{ma}}{H_e} \tag{12}$$

$$(\eta_{mano})_a = \frac{23}{32.67}$$

$$(\eta_{mano})_a = 0.7040 = 70.40\% \tag{13}$$

(c) Losses in the Impeller

From the velocity triangle at the outlet,

$$C_2 = \sqrt{C_{w2}^2 + C_{f2}^2} \tag{14}$$

$$C_2 = \sqrt{15.30^2 + 3^2}$$

$$C_2 = 15.6 \text{ m/s} \tag{15}$$

Velocity head or kinetic head at the exit of the impeller is given by,

$$\frac{C_2^2}{2g} = \frac{15.6^2}{2 \times 9.81}$$

$$\frac{C_2^2}{2g} = 12.404 \text{ m} \tag{16}$$

$$\text{Kinetic head recovered in casing as pressure head} = \frac{45}{100} \times \frac{C_2^2}{2g} = 0.45 \times 12.404$$

$$\text{Kinetic head recovered in casing} = 5.582 \text{ m} \tag{17}$$

Change of kinetic energy in the impeller,

$$\Delta KE = \left(\frac{C_2^2}{2g} - \frac{C_1^2}{2g}\right) = 12.404 - \frac{C_1^2}{2g} \tag{18}$$

Unrecovered kinetic head,

$$\Delta KE_{unrec} = \Delta KE - \text{Kinetic head recovered} = 12.404 - \frac{C_1^2}{2g} - 5.582$$

$$\Delta KE_{unrec} = 6.822 - \frac{C_1^2}{2g} \tag{19}$$

Ideal head imparted by the impeller,

$$H_e = \Delta H_{pa} + \Delta KE_{unrec} + \text{Losses in the impeller } (h_i)$$

$$32.67 = 23 + 6.822 - \frac{C_1^2}{2g} + h_i$$

$$\text{Total losses in the impeller} = H_{iL} = h_i - \frac{C_1^2}{2g} = 32.67 - 23 - 6.822$$

$$\text{Total losses in the impeller} = H_{iL} = 2.848 \text{ m} \quad (20)$$

(d) Power of the Motor

Power imparted to the fluid or fluid power (FP) is,

$$\text{FP} = \rho g Q H_m = 1000 \times 9.81 \times 0.054 \times 23$$

$$\text{FP} = 12184.02 \text{ W} = 12.184 \text{ kW} \quad (21)$$

In the absence of hydraulic and leakage losses, overall efficiency is represented as,

$$\eta_o = \eta_{mano} \times \eta_m \quad (22)$$

$$\eta_o = 0.704 \times 0.92$$

$$\eta_o = 0.648 \quad (23)$$

$$\eta_o = \frac{\text{FP}}{\text{SP or BP}} \quad (24)$$

$$0.648 = \frac{12.184}{\text{SP}}$$

$$\text{SP or BP} = 18.80 \text{ kW} \quad (25)$$

Power required by the motor is the brake power.

4.11 Performance Characteristics of Axial Pump

The variation of head, brake power and efficiency with discharge in non-dimensional form for an axial flow pump is shown in Figure 4.35. The abscissa is the percentage of best efficiency discharge and ordinate is the percentage of best efficiency value. All the three curves show a common value of (100,100) at the design point. As can be seen from the Figure 4.35, the head decreases steeply with increase in the discharge. The shut-off head is 250%, i.e. 2.5 times the design head.

The head drops by about 50% of the design head if there is only 20% increase in normal or design discharge. Axial flow pumps with fixed blades are suspected to have flow instability at around 50% of normal flow.

The efficiency reduces on either side of BEP and limits the useful application range. The operation range is limited by the head and efficiency curves to 60% – 120% of the normal value. Axial flow pumps with variable pitch which are known as *Kaplan pumps* exhibit fairly good efficiency over a wide range of discharge. Brake power increases steeply with decrease in flow and reaches approximately 180% of normal value at the shut-off. The low flow operation of the pump should be eliminated if the motor of the pump is not designed to take this variation in discharge, otherwise overloading of the motor may take place. In other words, axial flow pumps are not suitable where the load is subjected to wide range of fluctuation. It is clear that the shut-off conditions of axial flow pumps are completely different from the radial flow pumps. Axial flow pumps are started with control valves open due to high head and power requirement at shut-off.

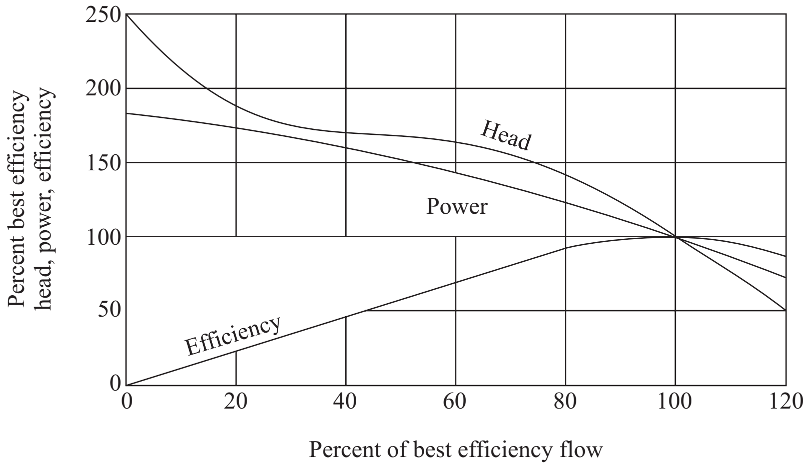


Figure 4.35 *Performance Characteristics of an Axial Flow Pump*

EXAMPLE 4.10 The design speed of an axial flow pump is 1450 rpm. The external and hub diameters of the pump impeller are 0.4 m and 0.2 m respectively. The blade angle at entry is 15° and that at outlet is 30° respectively. Assume that the flow is axial at entery (with no whirl), draw velocity triangles at the inlet and outlet. Calculate knowing that flow approaches the impeller axially,

(a) The actual head imparted and discharge of the pump, and (b) power input of the pump.

The guide vanes at pump inlet (upstream of the rotor) are used to control discharge. The vanes cause some prewhirl of the flow at inlet, determine (c) the new head imparted by the pump if the guide blades reduce discharge by 20%. If all of the efficiencies remain constant except hydraulic efficiency which is decreased to 80%, what will be the percentage decrease in the power consumption of pump?

Assume if needed $\eta_v = 98\%$, $\eta_h = 83\%$, $\eta_m = 96\%$

Solution

Given: $N = 1450$ rpm, $D_o = 0.4$ m, $D_h = 0.2$ m, $\beta_1 = 15°$, $\beta_2 = 30°$, $C_{w1} = 0$, $\eta_v = 98\%$, $\eta_h = 83\%$, $\eta_m = 96\%$

(a) Actual Head Developed and Discharge

$$\pi r_m^2 = \pi(r_o^2 - r_h^2)$$

$$r_m = \sqrt{0.2^2 - 0.1^2}$$

$$r_m = 0.1732 \text{ m} \tag{1}$$

Tangential or peripheral velocity of the blades at mean radius,

$$C_b = \omega r_m = \frac{2\pi N}{60} r_m \tag{2}$$

$$C_b = \frac{2\pi \times 1450}{60} \times 0.1732$$

$$C_b = 26.3 \text{ m/s} \tag{3}$$

The area of flow is,

$$A = \frac{\pi}{4}(D_o^2 - D_h^2) \tag{4}$$

$$A = \frac{\pi}{4} \times (0.4^2 - 0.2^2)$$

$$A = 0.09425 \text{ m}^2 \tag{5}$$

The velocity triangles at inlet and outlet are shown in Figure 4.36.

$$C_{f1} = C_{b1} \tan \beta_1 \Rightarrow C_{f1} = 26.3 \tan 15$$

$$\therefore \quad C_{f1} = C_{f2} = 7.047 \text{ m/s} \tag{6}$$

From the velocity triangle at outlet,

$$C_{w2} = C_b - C_f \cot \beta_2 \tag{7}$$

$$C_{w2} = 26.3 - 7.047 \cot 30$$

$$C_{w2} = 14.094 \text{ m/s} \tag{8}$$

Since the flow is axial at the inlet, therefore, $C_{w1} = 0$, Euler head is given by,

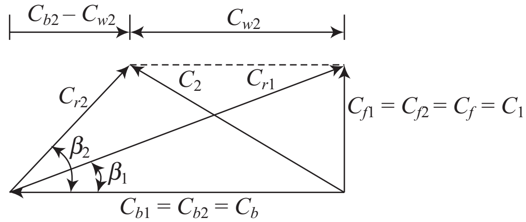


Figure 4.36 *Inlet and Exit Velocity Diagrams with No Whirl at Inlet for Example 4.10*

$$H_e = \frac{1}{g} C_{w2} C_{b2} \tag{9}$$

$$H_e = \frac{1}{9.81} \times 14.094 \times 26.3$$

$$H_e = 37.785 \text{ m} \tag{10}$$

Actual head developed,

$$H_a = \eta_h H_e \tag{11}$$

$$H_a = 0.83 \times 37.785$$

$$H_a = 31.36 \text{ m} \tag{12}$$

Actual discharge of the pump is,

$$Q = \eta_v A C_f \tag{13}$$

$$Q = 0.98 \times 0.09425 \times 7.047$$

$$Q = 0.651 \text{ m}^3/\text{s} \tag{14}$$

(b) Brake Power

Overall efficiency of the pump is expressed as,

$$\eta_o = \eta_v \eta_h \eta_m \tag{15}$$

$$\eta_o = 0.98 \times 0.83 \times 0.96$$

$$\eta_o = 0.78 \tag{16}$$

$$\eta_o = \frac{\text{FP}}{\text{BP}} = \frac{\rho g Q H_a}{\text{BP}} \tag{17}$$

$$0.78 = \frac{1000 \times 9.81 \times 0.651 \times 31.36}{\text{BP}}$$

$$\text{BP} = 256762.41 \text{ W} = 256.7624 \text{ kW} \tag{18}$$

(c) Head and Power Input with Pre-rotation at Inlet

The velocity diagram with pre-rotation at the inlet is shown in Figure 4.37.

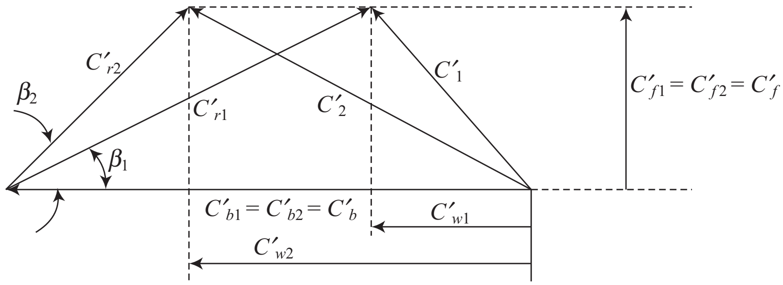


Figure 4.37 *Inlet and Exit Velocity Diagrams with Pre-rotation at Inlet using Guide Vanes*

Since the discharge is decreased by 20% due to the guide vanes rotation, therefore new discharge,

$$Q' = 0.8Q \Rightarrow Q' = 0.8 \times 0.651$$

$$Q' = 0.521 \text{ m}^3\text{/s} \tag{19}$$

$$Q' = \eta_v A C'_f \Rightarrow 0.521 = 0.98 \times 0.09425 \times C'_f$$

$$C'_f = 5.641 \text{ m/s} \tag{20}$$

$$C'_{w2} = C_b - C'_f \cot \beta_2 \Rightarrow C'_{w2} = 26.3 - 5.641 \cot 30$$

$$C'_{w2} = 16.53 \text{ m/s} \tag{21}$$

$$C'_{w1} = C_b - C'_f \cot \beta_1 \Rightarrow C'_{w1} = 26.3 - 5.641 \cot 15$$

$$C'_{w1} = 5.2475 \text{ m/s} \tag{22}$$

Euler head is given by,

$$H'_e = \frac{1}{g}(C'_{w2} - C'_{w1})C_b \Rightarrow H'_e = \frac{1}{9.81} \times (16.53 - 5.2475) \times 26.3$$

$$H'_e = 30.25 \text{ m} \tag{23}$$

The new actual head developed by the pump is,

$$H'_a = \eta_h H'_e \Rightarrow H'_a = 0.80 \times 30.25$$

$$H'_a = 24.2 \text{ m} \tag{24}$$

$$\eta'_o = \eta_v \eta'_h \eta_m \Rightarrow \eta'_o = 0.98 \times 0.80 \times 0.96$$

$$\eta'_o = 0.753 \tag{25}$$

$$BP' = \frac{1000 \times 9.81 \times 0.521 \times 24.2}{0.753}$$

$$BP' = 164258.22 \text{ W} = 164.26 \text{ kW} \tag{26}$$

$$\% \text{ Reduction in Power Input} = \frac{\text{BP-BP}'}{BP} \times 100 = \frac{256.7624 - 164.26}{256.7624} \times 100$$

$$\% \text{ Reduction in Power Input} = 36.03\% \tag{27}$$

4.12 Performance Characteristics of Mixed Flow Pump

The performance characteristic of a mixed flow vertical pump is shown in Figure 4.38. From the head-discharge curve, it is obvious that the head falls much steeply with the discharge as compared to the similar plot of a centrifugal pump. The shut-off head is about 18% of the normal head. The operating range is limited 60 – 120% of the normal value.

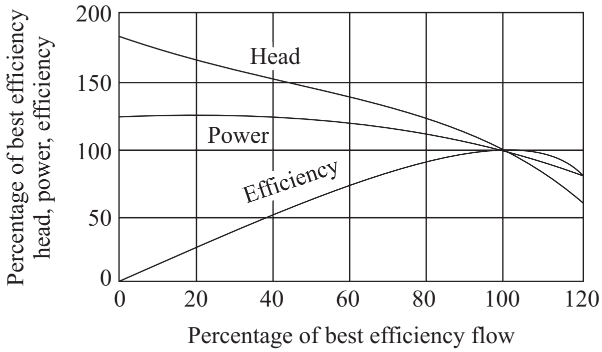


Figure 4.38 *Performance Characteristics of Mixed Flow Vertical Pump*

TABLE 4.1 Performance Comparison of Centrifugal and Axial Pumps

S. No.	*Parameter/Characteristic*	*Centrifugal Pumps*	*Axial Pumps*
1	Specific speed	100-110	180-300
2.	Mechanical losses in the pump	Significant	Negligible
3.	Intake design	Moderately important	Very important
4.	NPSHR	Low	High
5.	Air handling	Poor	Good
6.	When pump is reversed	Flow direction is same as before	Flow reverses
7.	$H - Q$ curve	Flat to moderately steep	Steep
8.	Ratio of safe operating head to normal head	1.1–1.6	2.0–3.0
9.	Ratio of safe operating power to normal power	< 1.0	1.5–2.5

4.13 Net Positive Suction Head (NPSH)

In a case of a pump, net positive suction head is the total suction head (including both static and dynamic heads) available at entry of the pump higher than the vapour pressure head. It can be considered as an amount of margin for a pump against vapourization of the liquid and therefore susceptibility of the pump for cavitation, which is discussed in detail in Chapter 5.

$$\text{NPSHA} = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} - \frac{\rho_v}{\rho g} \tag{4.68}$$

Net positive suction head required (NPSHR), a pump characteristic, is ascertained by the pump manufacturer through the experimental results.

EXAMPLE 4.11 Figure 4.39 shows a centrifugal pump used to pump gasoline (γ = 8.5 kN/m^3, p_v = 60 kPa) from reservoir *A* to another reservoir *B* not shown in the figure at flow rate of 0.046 m^3/s. It is required to determine available net positive suction head. The minor losses may be neglected.

Take p_a = 101 kPa, D_s = 0.15 m, L_s = 50 m, f = 0.018.

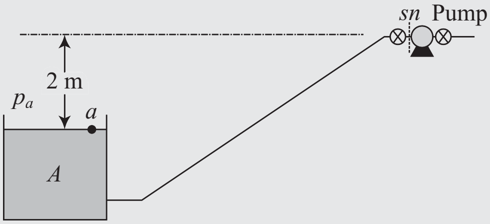


Figure 4.39 *Schematic Diagram of the Pumping System*

Solution

Given: γ = 8.5 kN/m^3, p_v = 60 kPa, Q = 0.046 m^3/s, p_a = 101 kPa, D_s = 0.15 m, L_s = 50 m, f = 0.018

Velocity of flow in the suction pipe,

$$C_s = \frac{Q}{\frac{\pi}{4}D_s^2} \Rightarrow C_s = \frac{0.046}{\frac{\pi}{4} \times 0.15^2} \tag{1}$$

$$C_s = 2.6031 \text{ m/s} \tag{2}$$

Using Darcy's formula,

$$h_{fs} = f\frac{L_s}{D_s}\frac{C_s^2}{2g} + \text{Minor Losses} \tag{3}$$

$$h_{fs} = 0.018 \times \frac{50}{0.15} \times \frac{2.6031^2}{2 \times 9.81} + 0$$

$$h_{fs} = 2.072 \text{ m} \tag{4}$$

Applying the energy equation between points (a) and (sn) shown in Figure 4.39,

$$\frac{p_a}{\gamma} + \frac{C_a^2}{2g} + Z_a = \frac{p_{sn}}{\gamma} + \frac{C_{sn}^2}{2g} + Z_{sn} + h_{fs} \tag{5}$$

$\because$ $C_a = 0$, $\frac{p_{sn}}{\gamma} = h_{sn}$ $\therefore$ Above equation can be simplified to,

$$h_{sn} + \frac{C_{sn}^2}{2g} = \frac{p_a}{\gamma} + (Z_a - Z_{sn}) - h_{fs} \tag{6}$$

We know that,

$$\text{NPSHA} = \frac{p_1}{\gamma} + \frac{C_1^2}{2g} - \frac{p_v}{\gamma} \tag{7}$$

$$\therefore \qquad \text{NPSHA} = \frac{p_a}{\gamma} + (Z_a - Z_{sn}) - h_{fs} - \frac{p_v}{\gamma}$$

$$\text{NPSHA} = \frac{101}{8.5} + (-2) - 2.072 - \frac{60}{8.5}$$

$$\text{NPSHA} = 0.7515 \text{ m} \tag{8}$$

4.14 Specific Speed

Specific speed of a pump is defined as the speed of a geometrically similar pump operating at maximum efficiency under a unit head and delivering unit discharge. Specific speed is a characteristic parameter to compare the performance of different pumps.

The discharge Q for a pump is given by,

$$Q = \text{Area} \times \text{Velocity of Flow}$$

$$Q = \pi DBC_f \Rightarrow Q \propto DBC_f$$

$$\because \qquad B \propto D$$

$$Q \propto D^2 C_f \tag{4.69}$$

We know that,

$$C_b = \frac{\pi DN}{60} \Rightarrow C_b \propto DN$$

$$\because \qquad C_b \propto C_f \propto \sqrt{H_m}$$

$$\therefore \qquad DN \propto \sqrt{H_m} \Rightarrow D \propto \frac{\sqrt{H_m}}{N} \tag{4.70}$$

Substituting the value of D from Eq. (4.70) into Eq. (4.69), we get,

$$Q \propto \frac{H_m}{N^2} C_f \propto \frac{H_m}{N^2} \times \sqrt{H_m} \Rightarrow Q \propto \frac{H_m^{3/2}}{N^2}$$

$$Q = K \frac{H_m^{3/2}}{N^2} \tag{4.71}$$

If $H_m = 1$ m, $Q = 1$ m^3/s, then $N = N_s$, therefore Eq. (4.71) results,

$$1 = \frac{K}{N_s^2} \Rightarrow K = N_s^2 \tag{4.72}$$

Substituting value of K from Eq. (4.72) into Eq. (4.71),

$$Q = N_s^2 \frac{H_m^{3/2}}{N^2}$$

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}} \tag{4.73}$$

The specific speed, as given by Eq. (4.73), is not a dimensionless quantity and hence depends upon the units of N, Q and H_m. In SI units, speed N is in rpm, head H_m is in m and discharge Q is in m^3/s. It is quite obvious from the Eq. (4.73) that specific speed is a size independent parameter because it does not involve the size of the impeller. Therefore, a single value of specific speed represents a family of homologous pumps of a given shape of impeller. The value of specific speed changes with the shape of the impeller. Specific speed is the most important single parameter used for selecting the type and number of pumps for a specific job. It is also used for comparing the characteristics e.g. best efficiency of different types of rotodynamic pumps on a common base.

Figure 4.40 shows the efficiency versus specific speed for different three main types of dynamic pumps. It shows the approximate range of optimum efficiency of centrifugal pumps as a function of specific speed. As the specific speed increases, transition occurs from the radial flow to axial flow which requires the corresponding changes in the shapes of the impeller, also shown schematically in the figure. The radial flow pumps have smaller specific speed and the axial flow pumps have the highest range of specific speed. Specific speeds of mixed flow pumps lie in between the radial and axial flow pumps. The efficiency curves of the three types of pumps have overlap regions as indicated in the figure.

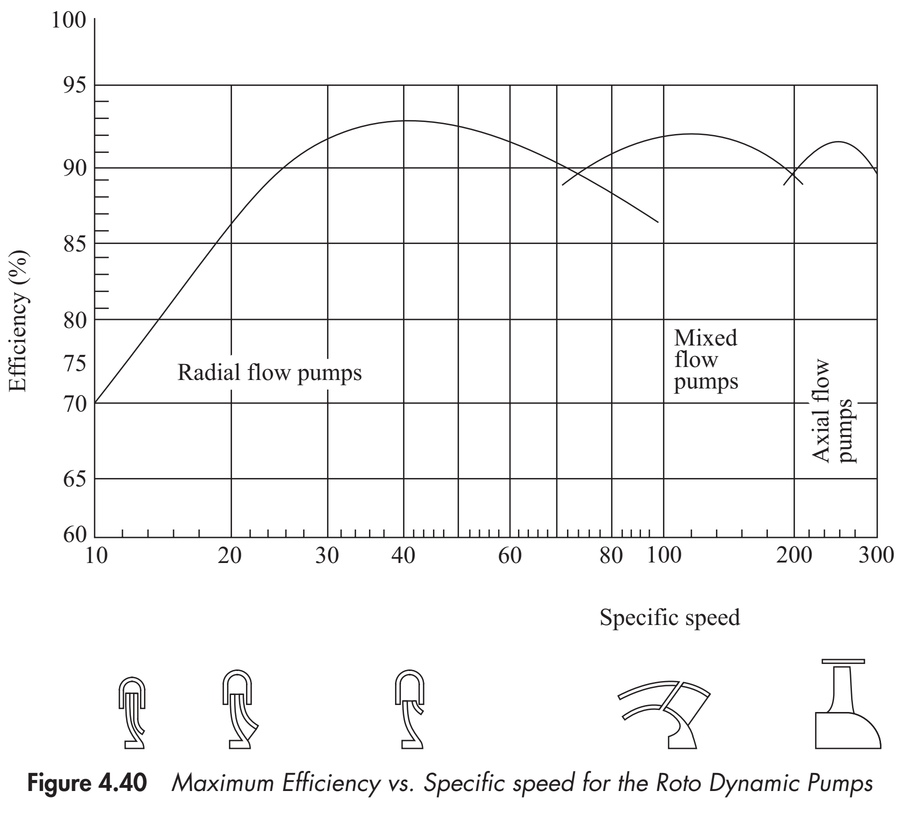


Figure 4.40 *Maximum Efficiency vs. Specific speed for the Roto Dynamic Pumps*

High discharge and low head results in high value of specific speeds. Axial flow pumps are best suited in such cases. For pumps with low discharge and high head, the specific speed is low and radial flow centrifugal pumps are the best choice. In the overlap regions either of the pumps may be used. It is clear from the shapes of the impellers shown in Figure 4.40 that low specific speeds pumps have outlet diameters much higher than the inlet diameters and relatively narrow flow passages. Similarly, high specific speed pumps have outlet diameters which are same as the inlet diameters and large open flow passages.

Figure 4.41 shows typical best efficiency curves of normal commercial pumps versus specific speed for different values of discharge or capacity. The optimum efficiency increases with increase in the capacity at any given speed. However, the rate of increase in efficiency becomes smaller as the discharge increases. For capacity greater than 600 l/s, the increase in efficiency with discharge is marginal and further, only one curve represents all values of capacity beyond 600 l/s. It may be noted from the diagram that the optimum efficiency drops off rapidly for low discharges and specific speed below 15. The following common conventions are used for computing specific speed:

- For double suction pumps, the specific speed is calculated on the basis of one half of the total discharge of the pump as the double suction pump is treated as two pumps connected back to back.
- If all the impellers are of same size, a multistage centrifugal pump of *m* stages is considered as *m* number of single stage pumps connected in series. Therefore, head per stage (total head divided by number of stages) is used for calculating the specific speed.

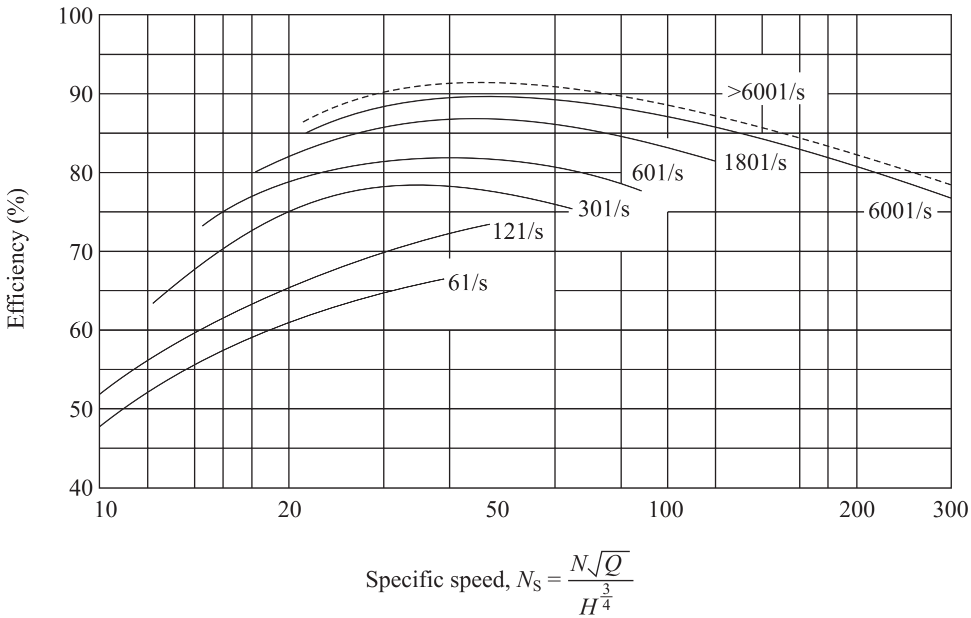


Figure 4.41 *Variation of Best Efficiency with Specific Speed for Various Discharges*

Table 4.2 shows the values of specific speeds for various centrifugal pumps.

TABLE 4.2 Ranges of Specific Speeds for Various Types of Centrifugal Pumps

Type of the Pump	*Subclass*	*Specific Speed* $\frac{rpm\sqrt{\frac{m^3}{s}}}{m^{3/4}}$	*Shape Number revolutions*
Radial Flow Pumps	Slow speed	10–30	0.03–0.09
	Medium speed	30–50	0.09–0.15
	High speed	50–100	0.15–0.30
Mixed flow pumps	—	75–220	0.225–0.66
Axial flow pumps	—	180–450	0.54–1.35

Non-dimensional specific speed, known as *shape factor or shape number,* for pumps is obtained by raising the discharge or flow coefficient (φ) to 1/2 and dividing this by head coefficient (ψ) raised to exponent 3/4,

$$N_{sh} = \frac{\varphi^{1/2}}{\psi^{3/4}} = \frac{N\sqrt{Q}}{(gH)^{3/4}} \tag{4.74}$$

In calculating the shape factor, speed N should be in rps, discharge Q in m^3/s and head H in m.

EXAMPLE 4.12 It is required to pump 1076 litres/s of water against a head of 31 m. Calculate the number of pumps required if the specific speed of each pump is 35 and running at 1200 rpm. The dynamic head in the system can be neglected.

Solution

Given: Q_t = 1076 litres/s = 1.076 m^3/s, H_m = 31 m, N_s = 35 rpm, N = 1200 rpm

Let Q is the discharge of each pump. We know that specific speed of the pump is expressed as,

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}} \tag{1}$$

$$35 = \frac{1200\sqrt{Q}}{31^{3/4}}$$

$$Q = 0.1468 \text{ m}^3/\text{s} \tag{2}$$

Since the total discharge is more than the discharge of water by one pump, therefore the pumps have to be connected in parallel to meet the required job.

$$\text{Number of the pumps required} = \frac{\text{Total Discharge}}{\text{Discharge of one pump}} \tag{3}$$

$$\text{Numbers of the pumps required} = \frac{1.076}{0.1468} = 7.33$$

Since the number of the pumps cannot be in fraction practically, therefore,

$$\text{Number of the pumps required} = 8 \tag{4}$$

EXAMPLE 4.13 A single stage centrifugal pump running at 1400 rpm delivers 350 litres/s of water against a head of 34 m. The diameter of the impeller is 0.4 m. Find the number of stages and diameter of each impeller of a similar multistage pump running at 1200 rpm to deliver 600 litres/s of water against a head of 194 m.

Solution

Given: N_1 = 1400 rpm, Q_1 = 350 litres/s = 0.35 m^3/s, H_{m1} = 34 m, D_1 = 0.4 m, N_2 = 1200 rpm, Q_2 = 600 litres/s, H_{2t} = 194 m.

Subscript 1 is used here for single stage pump and subscript 2 for its geometrically similar multistage pumps arrangement. For geometrically similar pumps, specific speed must be the same. Therefore, the specific speed of single pump and geometrically similar multistage pumps arrangement should be the same. Let H_{m2} is the head developed by multistage pumps arrangement. Therefore,

$$N_s = \frac{N_1\sqrt{Q_1}}{H_{m1}^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_{m2}^{3/4}} \tag{1}$$

$$\therefore \quad \frac{1400 \times \sqrt{0.35}}{34^{3/4}} = \frac{1200 \times \sqrt{0.6}}{H_{m2}^{3/4}} \Rightarrow H_{m2} = 34 \times \left[\frac{1200}{1400} \times \sqrt{\frac{0.6}{0.35}}\right]^{4/3}$$

$$H_{m2} = 39.653 \text{ m} \tag{2}$$

Since the total head against which the multistage pumps arrangement has to work 194 m is more than the head developed by considering this multistage arrangement as a single entity 39.653 m, therefore each pump should be connected in series to meet the requirement. Therefore,

$$\text{Number of stages} = \frac{H_{2t}}{H_{m2}} = \frac{194}{39.653}$$

$$\text{Number of stages} = 5 \tag{3}$$

We know from the affinity laws that for the geometrically similar pumps, power, head and discharge coefficients must be the same. Therefore,

$$\varphi = \left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2 \tag{4}$$

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \Rightarrow \frac{D_2}{D_1} = \left[\frac{Q_2}{Q_1} \times \frac{N_1}{N_2}\right]^{1/3} = \left[\frac{0.6}{0.35} \times \frac{1400}{1200}\right]^{1/3} = 1.26$$

$$D_2 = 1.26\, D_1 = 1.26 \times 0.4$$

$$D_2 = 0.504 \text{ m} = 504 \text{ mm} \tag{5}$$

EXAMPLE 4.14 The specific speed of an axial pump is 1150 and flow velocity is 2.5 m/s. The external and internal diameters of the impeller are 0.9 m and 0.45 m, respectively. Determine appropriate speed of rotation of the pump to develop a head of 5.5 m. Also, find blade angle at the inlet of the pump.

Solution

Given: N_s = 1150, C_f = 2.5 m/s, D_o = 0.9 m, D_h = 0.45 m, H = 5.5 m

Discharge is given by,

$$Q = \frac{\pi}{4}(D_o^2 - D_h^2)C_f \tag{1}$$

$$Q = \frac{\pi}{4}(0.9^2 - 0.45^2) \times 2.5$$

$$Q = 1.193 \text{ m}^3/\text{s} = 1193 \text{ litres/s} \tag{2}$$

Specific speed is,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} \tag{3}$$

$$1150 = \frac{N\sqrt{1.193}}{5.5^{3/4}}$$

$$N = 3781.374 \text{ rpm} \tag{4}$$

$$C_{b1} = \frac{\pi D_h N}{60} \Rightarrow C_{b1} = \frac{\pi \times 0.45 \times 3781.374}{60}$$

$$C_{b1} = 89.1 \text{ m/s} \tag{5}$$

From the velocity triangle at inlet as shown in Figure 4.42,

$$\tan\beta_1 = \frac{C_{f1}}{C_{b1}} \Rightarrow \tan\beta_1 = \frac{2.5}{89.1}$$

$$\beta_1 = 1.607° \tag{6}$$

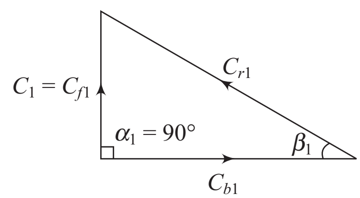


Figure 4.42 *Velocity Triangle at Inlet of Example 4.14*

4.15 Pump and System

4.15.1 Matching of System Characteristics

The performance characteristics of a pump consist of the $H - Q$ curve, $P - Q$ curve, $\eta_o - Q$ curve, and NPSHR – Q curve and are provided by the manufacturer to the customer. A general pumping system is shown in Figure 4.12 which involves piping with necessary fittings, valves and some measuring instruments besides pump. The complete set of elements consisting of supply and receiving reservoirs, liquid, piping and other apparatuses is called a *pumping system*. The cumulative resistance offered to the pumping by the system is known as *system head requirement* and forms a system characteristic. In a pump installation, the ideal objective is to match the pump characteristics to the system characteristics and achieve the best efficiency in the process. In other words, the exact operating point of a pump in practice is determined from the matching of pump characteristics with the head loss flow characteristics of the external system, i.e. pipe network, valves etc. to which the pump is connected.

Let us consider the pump and the piping system as shown in Figure 4.12. The head losses are proportional to the square of flow velocities in a highly turbulent flow. The head losses in both the suction and delivery sides consist of major losses due to friction between fluid and pipe wall, and minor losses due to valves, bends, etc. Therefore, the losses in suction and delivery sides can be written as,

$$h_1 = f\frac{l_1}{d_1} \times \frac{C_1^2}{2g} + K_1\frac{C_1^2}{2g} \tag{4.75}$$

$$h_2 = f\frac{l_2}{d_2} \times \frac{C_2^2}{2g} + K_2\frac{C_2^2}{2g} \tag{4.76}$$

where,

f = Darcy's Friction Factor

h_1 = Loss of head in suction side

l_1 = Length of suction pipe,

D_1 = Diameter of suction pipe,

C_1 = Average flow velocity in suction pipe,

K_1 = Loss coefficient on the suction side

h_2, l_2, D_2, C_2 are the corresponding values of delivery pipe.

The first terms in Eqs. (4.75) and (4.76) represent the head loss due to friction between fluid and pipe wall while the second terms represent the sum of all the minor losses which include losses due to valves and pipe bends, entry and exit losses etc.

Therefore, the total head the pump has to develop in order to supply the fluid from the lower (supply) reservoir to upper or receiving reservoir is,

$$H = H_{st} + h_1 + h_2 \tag{4.77}$$

We know that the discharge or flow rate is directly proportional to the flow velocity. Therefore, flow resistance in the form of losses is proportional to the square of the discharge as follows:

$$h_1 + h_2 = \text{System Resistance} = KQ^2 \tag{4.78}$$

where K is a constant which includes the lengths and diameters of the pipes and various loss coefficients. Flow resistance in the form of losses given by Eq. (4.78), known as *system resistance* represents the loss of head at a particular discharge through the system. If any of the parameters in the system such as adjustment of valve opening, or insertion of a new bend etc., then K will change. Therefore, total head H can be considered as the total opposing head of the pumping system which must be overcome by the pump for the fluid to be pumped from the supply reservoir to the upper reservoir. From Eqs. (4.77) and (4.78),

$$H = H_{st} + KQ^2 \tag{4.79}$$

System characteristics Eq. (4.79) when plotted on the $H – Q$ plane gives the system characteristics curve, represented in Figure 4.43 by uv. Consider the situation when a pump installed in a pumping system starts. The discharge from the pump starts with zero value and corresponding shut-off head, and then it gradually increases. The corresponding head generated by the pump varies along the pump curve mn. The starting point is m and the pumping state moves along the pump curve from m towards n. Simultaneously, the system starts responding from the point u corresponding to zero discharge on the system characteristics curve. Corresponding to every pumping state on the pump curve mn, there will be a corresponding point on the system characteristics curve uv representing system resistance. The point of intersection of the system characteristics curve and pump characteristics curve on the

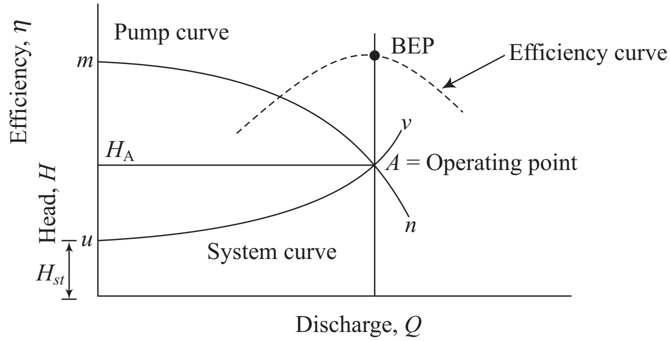


Figure 4.43 *Operating Point of a Centrifugal Pump*

$H - Q$ plane is known as the *operating point or duty point*. At the operating point A, the requirements of the system are perfectly matched by pump. The operating or duty point may or may not lie at the design point of the pump which corresponds to maximum efficiency. Variations in the values of different parameters over time, operational patterns and errors in estimation of various losses may lead deviation of operating point from design point. In such a situation, one must be satisfied if the operating point lies close or in the neighborhood of the design point, i.e. BEP. Coincidence of the operating point with the design point (BEP) is a matter of chance which is an ideal situation.

(a) Effect of Speed Variation

Head-Discharge characteristic of a given pump is always plotted at a constant speed. If the pump characteristic at one speed is known, it is possible to predict the characteristic at other speed by using affinity laws or similarity laws. Let A, B and C are three points on the characteristic curve shown in Figure 4.44 at speed N_1. For points A, B and C, the corresponding heads and flow rates at a new speed N_2 can be found by using affinity laws or similarity laws for a given pump diameter as follows:

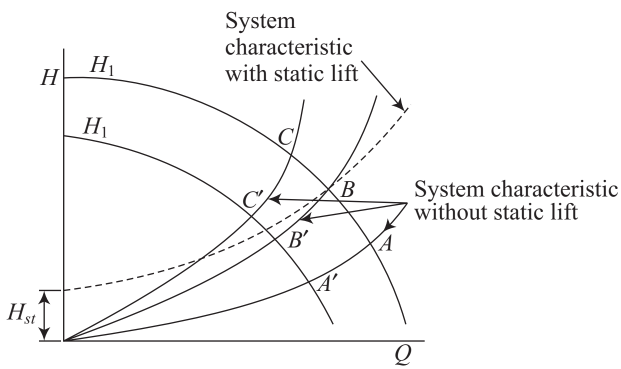


Figure 4.44 *Effect of Speed Variation on Operating Point of a Centrifugal Pump*

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1}$$

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \Rightarrow \frac{H_2}{H_1} = \left(\frac{Q_2}{Q_1}\right)^2$$

Therefore,

$$H \propto Q^2 \tag{4.80}$$

Equation (4.80) represents that all corresponding points on $H - Q$ characteristic curves lie on a parabola through origin at different speeds. If the static head or lift H_{st} is zero, then the curve for system characteristic and the locus of similar operating points will be the same parabola passing through the origin. In other words, in case of zero static lift, for an operating point at speed N_1, it is only necessary to apply the affinity laws directly to find the corresponding operating point at the new speed because it will lie in the same system characteristic curve itself.

(b) Effect of Variation in Pump Diameter

The effect of variation in pump diameter may also be examined by applying similarity laws. For a constant speed,

$$\frac{Q_2}{Q_1} = \frac{D_2^3}{D_1^3}$$

$$\frac{H_2}{H_1} = \frac{D_2^2}{D_1^2} \Rightarrow \frac{H_2}{H_1} = \left(\frac{Q_2}{Q_1}\right)^{2/3}$$

$$\therefore \quad H \propto Q^{2/3} \tag{4.81}$$

4.15.2 Pumps in Series

When the head or discharge of a single pump is not sufficient for an application, pumps are arranged in series or parallel to meet the desired requirement. When it is required to pump a liquid to a head larger than the designed head of an available pump, the pumping could be achieved by connecting two or more pumps in series. It is to be noted that combining two dissimilar pumps in series may lead to problems, especially if one pump is larger than other, because the discharge through each pump must be the same but the increase in overall head is equal to the sum of head rise of both the pumps.

Hence, when pumps are connected in series, the combined net head is equal to the sum of the net heads of each pump at a particular discharge or volume flow rate. Consider three pumps are connected in series. It is quite obvious that dissimilar pumps have widely different performance characteristic curves as shown in Figure 4.45.

The *shut-off head* of the three pumps combined in series is equal to the sum of the shut-off head of pump 1, 2 and 3. The total net head developed by the three pumps in series arrangement is equal to $H_1 + H_2 + H_3$. The free deliveries of the pumps are shown by the dashed lines. Beyond the free delivery of pump 1, it must be shut-off or closed and bypassed. Otherwise it would be running beyond its maximum discharge point and the pump or its motor may get damaged. Furthermore, the net head developed by this pump would be negative contributing to a net head loss in the system. With the pump 1 shut-off and bypassed, the combined net head becomes $H_2 + H_3$. Similarly, pump 2 should be shut-off and bypassed beyond the free delivery of pump 2. Consequently, the net head generated is then equal to H_3 alone, as indicated to the right of the second vertical dashed line. In this case, the combined free delivery is the same as that of pump 3 alone assuming that the other two pumps are bypassed. If the three pumps were identical, it would not be necessary to shut-off any of the pumps since the free delivery of each pump would occur at the same discharge.

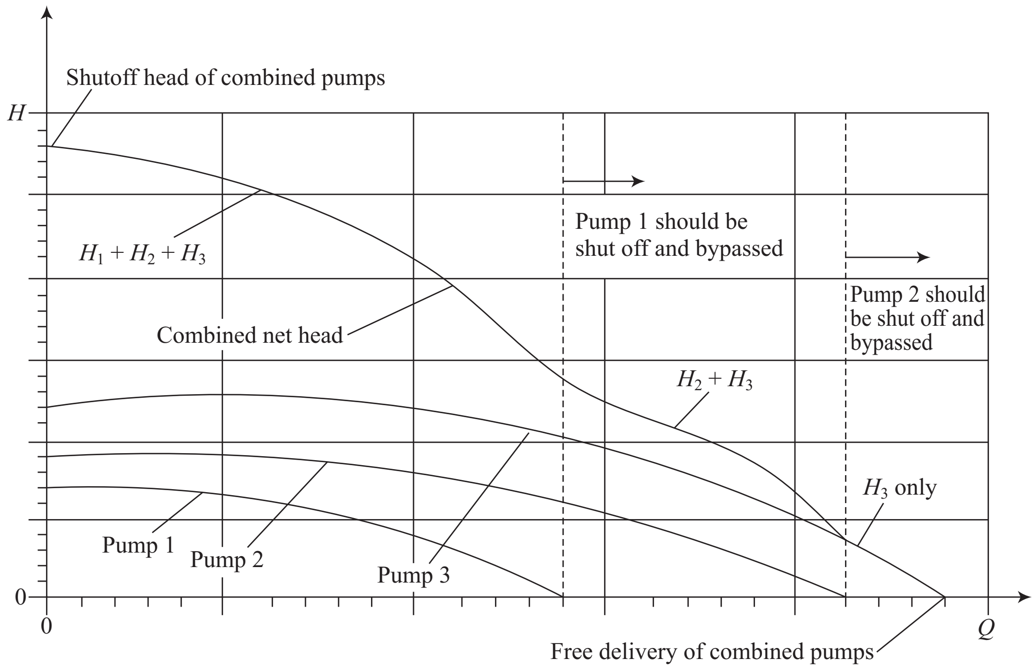


Figure 4.45 *Three Dissimilar Pumps in Series*

Now consider two similar pumps connected in series. The $H-Q$ characteristic of the individual pumps will be the same as represented by the curve *mn* in Figure 4.46. We know that under no friction, i.e. ideal conditions in the system when the two similar pumps are in series arrangement, the discharge passing through both the pumps will be the same and the head developed will be double of that of a single pump. If the effect of system friction is taken into consideration, the actual result will be different from that in the ideal conditions. The combined $H-Q$ characteristic curve for the cases of similar pumps connected in series is known as the *series duplex curve* shown by the curve *rs* in Figure 4.46. In order to obtain series duplex curve, the ordinates of the single pump characteristic curve *mn* are multiplied by 2, representing number of pumps in series. Then the new coordinates are plotted on the same abscissa and ordinate using the same scale as the single pump characteristic curve *mn*. The system characteristic curve *uv* is plotted using Eq. (4.79).

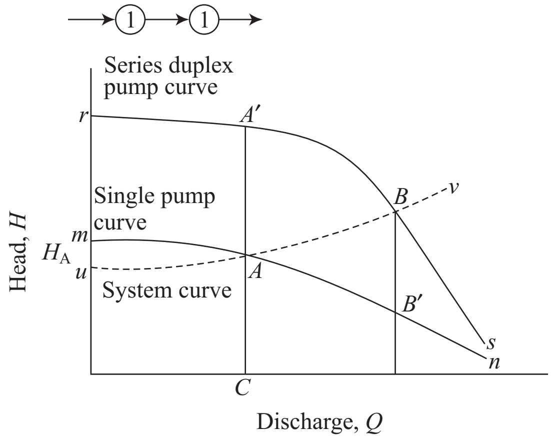


Figure 4.46 *Similar Pumps in Series*

The operating point of single pump is *A* where system characteristic curve intersects the pump characteristic curve. The system characteristic curve *uv* intersects the series duplex curve *rs* at *B* which is the operating point of two similar pumps in series arrangement. The following points are noteworthy:

- Let there be no friction in the system and *A* be the operating point or design point of single pump. Then, the operating point of two identical pumps in series is *A′* which is obtained as the point of intersection of a vertical line from *A* and the series duplex curve *rs*.
- If friction is taken into account, then the actual point of operation is the point of intersection of system characteristic curve and series duplex curve which is *B*. It is obvious that two similar pumps in series at actual operating point, work under a system resistance consisting of dynamic head loss, would develop a net head that is less than twice the single pump head, i.e. $H_B < H'_A$ and the discharge of the series duplex pump would be larger than the discharge of single pump i.e. $Q_B > Q_A$. This change in the coordinates of the actual operating point *B* is due to the effect of dynamic head loss.
- If a vertical line is drawn from operating point *B* of series duplex pump to intersect the single pump characteristic curve at *B′*, then the coordinates of *B′* are $\left[(Q'_B = Q_B), \left(H'_B = \dfrac{H_B}{2}\right)\right]$ by the nature of the curves *mn* and *rs*. Point *B′* represents the contribution of discharge and head of individual pumps 1 and 2. At the operating point of similar pumps in series arrangement, the total head developed (H_B) is smaller than $2H_A$ and the extra discharge passed by each pump to the system is $(Q_B - Q_A)$. It is quite interesting to note that in series arrangement, the discharge passing through each of the pumps is more than what would have passed through each of them if they were running singly. Similarly, the head produced by each of the pumps in series is less than what each of them would have produced individually.
- If the BEP of the pump were made to coincide with the operating point *A* of the single pump operation, then the operating point *B* of the series duplex pumps will be to the right of the *BEP* (Design Point) and thus will have less than the optimal efficiency.
- The operation of pumps in series is very rare. Multistage pumps (in series) are preferred when very high head and low discharge are required. For high heads and small discharges, *booster pumps* are used which is a special case of pumps in series. A booster pump is installed at some distance from the primary pump mainly for purposes of increasing the pressure in the downstream such as in areas of sub-main in water distribution system.

4.15.3 Pumps in Parallel

When the flow rate or discharge of a single pump is not sufficient for an application, pumps are connected in parallel. Each of the pumps sucks liquid from a common sump and discharges it to a common pipe to which delivery pipes of each pump is connected. The pumps connected in parallel need not be similar. Combining dissimilar pumps in parallel may create problems because head rise must be the same but the flow rate or discharge is the sum of the discharge of three individual pumps. It is quite obvious that dissimilar pumps will have different performance characteristic curve. If the pumps size is not matched properly while combining them in parallel, the smaller pump may not be able to develop the large head imposed on it resulting in possibility of back flow in its piping branch. This would reduce the overall head rise and the power supplied to the smaller pump would be merely a waste.

Consider three dissimilar pumps connected in parallel as shown in Figure 4.47. The characteristic curves of individual pumps and combined performance characteristic curve of three pumps arranged in parallel are shown in Figure 4.47. The free delivery of three pumps arranged in parallel is equal to the sum of the free delivery of each individual pump. For low values of net head, the discharge or capacity of three pumps arranged in parallel is equal to $Q_1 + Q_2 + Q_3$. For the head greater than the shut-off head of pump 1, then the pump 1 should be shut-off and its piping branch should be blocked with a valve. Otherwise it would be running beyond its extreme designed operating point and the pump or its motor would be damaged. Furthermore, the discharge through the pump 1 would be negative contributing to a net loss in the system. With the pump 1 shut-off and blocked in parallel arrangement, the combined capacity becomes $Q_2 + Q_3$ as shown by the first horizontal dashed line in Figure 4.47. Similarly, for the head greater than the shut-off head of pump 2, then the pump 2 should also be shut-off and blocked. The combined capacity of the parallel arrangement is then equal to Q_3 only as shown by second horizontal dashed line in Figure 4.47. In this case, the combined shut-off head is the same as that of pump 3 alone assuming that the other two pumps are shut-off and their piping branches are blocked. Therefore, the identical or similar pumps are connected in parallel.

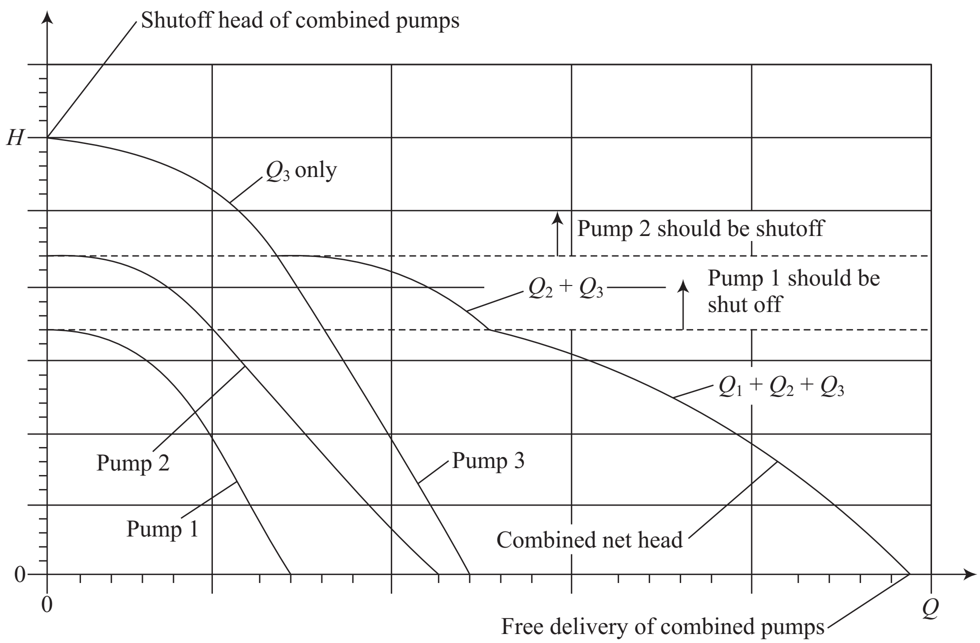


Figure 4.47 *Performance Characteristics of Three Dissimilar Pumps in Parallel*

Consider two identical pumps connected in parallel. In the ideal case of no friction in the system, the head would remain same and the discharge of the two pumps arranged in parallel will be double of the single pump. Figure 4.48 shows the combined $H - Q$ characteristic for the similar pumps connected in parallel. The $H - Q$ characteristic curve of either of the pumps would be the same (since pumps are identical) which is shown by curve *mn* in the figure. In the ideal case of frictionless system, the two pumps arranged in parallel gives the output discharge equal to double of discharge of a single pump. The characteristic curve for two identical pumps in parallel arrangement, shown by curve *mr* is obtained by adding the flow rates at the same head. The curve *mr* is also known as the *parallel duplex pump curve*. The system characteristic curve representing the variation of total frictional head against discharge is represented by curve *uv*.

The point of intersection of system characteristic curve with the single pump characteristic curve is the operating point *A* when either of the pumps 1 or 2 is working. The coordinates of operating point of a single pump are (Q_A, H_A). The operating point of two similar pumps arranged in parallel (Point *B*) is the intersection point of the system characteristic curve *uv* with the parallel duplex pump characteristic curve *mr*. The coordinates of point *B* are (Q_B, H_B). Following are the noteworthy points:

- Let, the operating point of a single pump is *A*. In case of no friction in the system, the operating point of two similar pumps in parallel arrangement is point *A′* having the coordinates ($2Q_A$, H_A) which is point of intersection of a horizontal line from *A* (because head being the same) with the parallel duplex pump characteristic curve *mr*.

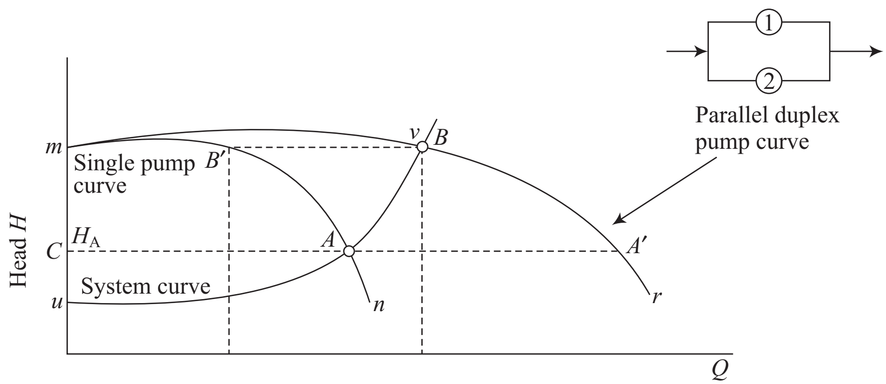


Figure 4.48 *Similar Pumps in Parallel*

- The actual point of operation of parallel arrangement of two similar pumps is the point *B*. It is clear that $Q_B < Q'_A$ and $H_B > H_A$ or H'_A. This change in the coordinates of the actual operating point with the ideal operating point of two similar pumps in parallel arrangement is due to the effect of dynamic head loss of the system. Therefore, two similar pumps arranged in parallel when working against a system resistance head would deliver a net discharge which is less than that of twice of a single pump and also the net head developed would be larger than the head generated by a single pump operating in the same system.
- Let a horizontal line drawn from point *B* intersect the single pump characteristic curve at point *B′*. Then by the nature of the characteristic curves *mn* and *mr*, the coordinates of point *B′* are $\left[\left(Q'_B = \frac{Q_B}{2}\right), (H'_B = H_B)\right]$. Point *B′* represents the contribution of head and discharge of each of the pumps 1 and 2. Therefore, it can be concluded that each of two similar pumps arranged in parallel would deliver discharge which is less than that each of them would have delivered if they were running individually. Also the net head developed by each of the pumps in parallel is greater than what each of them would have generated if they were running singly.

EXAMPLE 4.15 Two identical pumps having the characteristics as shown in Table 4.3 are to be set in a pumping station to discharge sewage to a settling tank through a 200 mm PVC pipeline 2.5 km long. The static lift is 15 m. Permitting for minor head losses of 10.0 $C^2/2g$ and taking an effective roughness of 0.15 mm, calculate the discharge and power consumption if the pumps were to be connected in parallel.

TABLE 4.3 Characteristics of Pump of Example 4.15

Discharge (*litres/s*)	0	10	20	30	40
Total Head (*m*)	30	27.5	23.5	17	7.5
Overall Efficiency (%)		44	58	50	18

Solution

Calculated system heads H are tabulated in Table 4.4 for discrete discharges Q

$$H = H_{st} + h_f + h_m$$

TABLE 4.4 System Heads for Discrete Discharges

Q (litres/s)	10	20	30	40
H (m)	16.53	20.8	23.37	36.48

The predicted head versus discharge curve for dual pump operation in parallel mode is obtained by doubling the discharge over the range of heads (since the pumps are identical in this case). The system and efficiency curves are added as shown in Figure 4.49.

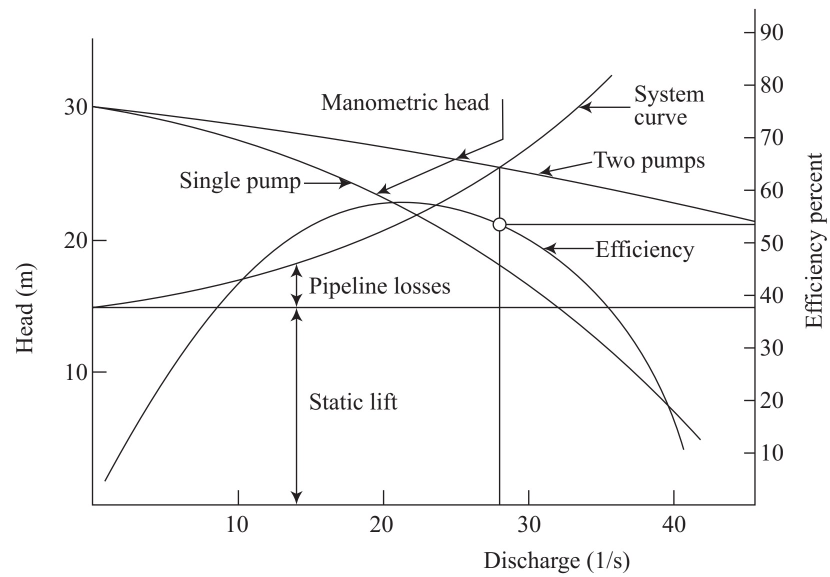


Figure 4.49 *Pump and System Characteristics of Example 4.15 for Parallel Operation*

From the intersection of the characteristic and system curves the following results are obtained:

Single pump operation: $Q = 22.5$ l/s, $H_m = 24$ m, $\eta = 0.58$; $H_m = 24$ m; $\eta = 0.58$

Power consumption = 9.13 kW

Parallel operation: $Q = 28.5$ l/s, $H_m = 26$ m, $\eta = 0.51$

(Corresponding with 14.25 l/s per pump)

Power input = 14.11 kW

SUMMARY

✦ In a general pumping system, the head between the sump level (from where the liquid is lifted) to the tank level (to where the liquid is lifted) is known as the static head, H_{st}. Various heads and expressions denoting the heads for a general pumping system are summarized in the following table.

Variable	*Expression*
Static head, H_{st}	$h_s + h_d$: suction head + delivery head
Suction head, h_s	Head developed in the suction line, the difference in the fluid energy between the sump level and the centerline of the pump.
Delivery head, h_d	Head developed in the delivery line, the difference in the fluid energy between the tank level (to where the liquid is lifted) and the center line of the pump.
Manometric head	Total head developed by the pump, the difference in the fluid energy between the outlet and inlet of the pump. $H_m = H_2 - H_1 = \left(\frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2\right) - \left(\frac{p_1}{\rho g} + \frac{C_1^2}{2g} + Z_1\right)$ $H_m = H_{st} + \sum \text{Losses in the pumping section}$ This is also referred simply as 'pump head' H.
Euler head or Theoretical head, H_e	$H_e = \frac{1}{g}(C_{w2}C_{b2} - C_{w1}C_{b1})$; gH_e is specific work and $\dot{m}\, gH_e$ is the the theoretical power of the pump either for a centrifugal pump, or for an axial pump, the inlet whirl component is generally negligible. In that case, $H_e = \frac{1}{g}C_{w2}C_{b2}$

✦ Theoretical fluid power developed by pump can be divided into three components

$$\dot{m}gH_e = \dot{m}\left(\frac{C_2^2 - C_1^2}{2} + \frac{C_{r1}^2 - C_{r2}^2}{2} + \frac{C_{b2}^2 - C_{b1}^2}{2}\right)$$

where, the first term is the specific kinetic energy difference of fluid (between outlet and inlet). The second term is the specific relative energy of fluid (between outlet and inlet). The third term is the centrifugal energy of fluid since $C_b^2 = r^2\omega^2$ (between outlet and inlet).

✦ Slip is an effect of non–congruent of the fluid flow with the blades of the pump. The outlet whirl component of the fluid C_{w2} gets modified by the slip factor. The modified outlet whirl velocity, $C_{w2} = \sigma_s C_{w2}$ where Σ_s is slip factor and is a function of β_2 and no of blades.

✦ Euler or Theoretical head, considering slip, if $C_{w1} = 0$,

$$H_e = \frac{\sigma_s C_{w2} C_{b2}}{g}$$

✦ The actual work head imparted by impeller is given by,

$$H_a = H_e - h_i$$

✦ The various efficiency expressions for hydraulic pumps are summarized in the following table.

Efficiency	***Expressions***
Hydraulic efficiency or impeller efficiency	$\eta_h = \frac{\text{Actual head developed by the pump at casing inlet}}{\text{Theoretical head devloped by the impeller}}$ $\eta_h = \frac{H_a}{H_{\text{th}}} = \frac{H_e - h_i}{H_e} = \frac{H + h_c}{H_{\text{th}}}$
Manometric (or casing) efficiency	$\eta_c = \eta_{\text{mano}} = \frac{\text{Fluid power at casing outlet}}{\text{Fluid power at casing inlet}}$ $\eta_c = \eta_{\text{mano}} = \frac{\rho g Q H}{\rho g Q (H + h_c)} = \frac{H}{(H + h_c)} = \frac{H}{H_a}$
Volumetric efficiency	$\eta_v = \frac{\text{Flow rate through pump outlet (Pump discharge)}}{\text{Flow rate through impeller}} = \frac{Q}{Q + \Delta Q}$
Mechanical efficiency	$\eta_m = \frac{\text{Fluid power supplied to impeller}}{\text{Power input to shaft}} = \frac{\rho g (Q + \Delta Q) H_{\text{th}}}{P_{\text{shaft}}}$
Overall efficiency	$\eta_0 = \frac{\text{Fluid power at casing outlet}}{\text{Power input to shaft}} = \frac{\rho g Q H}{P_{\text{shaft}}}$ $\eta_o = \eta_m \eta_h \eta_{\text{mano}} \eta_v$

✦ The expression for the minimum starting speed of a centrifugal pump is given by:

$$N_{\min} = \frac{60}{\pi\sqrt{(D_2^2 - D_1^2)}}\sqrt{2gH_m} = \frac{120\eta_{\text{mano}} C_{w2} D_2}{\pi[D_2^2 - D_1^2]}$$

✦ The blades which are curved in the direction of rotation are called forward facing blades. For forward facing blades, $\beta_2 > 90° \Rightarrow C_{w2} > C_{b2}$.

✦ The radial blades in which the fluid leaves the impeller with relative velocity in a radial direction and, therefore, $\beta_2 = 90°$, $C_{w2} = C_{b2}$.. Euler head is constant regardless of the discharge in radial blade pumps. Such blades are suitable for medium speed runners.

✦ In the backward facing blades, the blades curvature is in a direction opposite to that of impeller rotation, therefore, $\beta_2 < 90°$ and $C_{w2} < C_{b2}$. Ideal head decreases linearly with discharge for backward facing blades. The impeller having backward curved blades is a fast runner because of the highest C_{b2}.

- Specific speed of a centrifugal pump is given by,

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

- The radial flow pumps have smaller specific speed and the axial flow pumps have the highest range of specific speed. Specific speeds of mixed flow pumps lie in between the radial and axial flow pumps. Low specific speeds pumps have outlet diameters much higher than the inlet diameters and relatively narrow flow passages. Similarly, high specific speed pumps have outlet diameters which are same as the inlet diameters and large open flow passages.
- Main characteristics curves of centrifugal pumps are plotted from the laboratory test data of a particular type of pump having a fixed geometry with head, power and efficiency as ordinates and discharge as a common abscissa for various pump speeds and usually for a fixed impeller diameter.
- Operating characteristic curves represent the variation of head, power and efficiency with respect to discharge of a pump at a pre–fixed value of speed or design speed. The normal operating condition of a pump corresponds to the Best Efficiency Point (BEP).
- The point of intersection of the system characteristics curve and pump characteristics curve on the H–Q plane is known as the *operating point or duty point*. At the operating point, the requirements of the system are perfectly matched by pump. The operating or duty point may or may not lie at the design point of the pump which corresponds to maximum efficiency.
- When it is required to pump a liquid to a head larger than the designed head of an available pump, the pumping could be achieved by connecting two or more pumps in series.
- When the flow rate or discharge of a single pump is not sufficient for an application, pumps are connected in parallel. Each of the pumps sucks liquid from a common sump and discharges it to a common pipe to which delivery pipes of each pump is connected.
- The propeller pump resembles the shape of a Kaplan or propeller turbine. Such low delivery head (< 12 m) and large discharge pumps with axial flow are called *axial flow or propeller pumps* for which the specific speeds lie in the range of 180 – 300 $\frac{\text{rpm}\sqrt{\frac{m^3}{s}}}{m^{3/4}}$.

REVIEW QUESTIONS

4.1 List and define the main categories of roto-dynamic pumps.

4.2 What is the difference between radial flow type, mixed flow type and axial flow type pumps?

4.3 How the centrifugal pumps are classified? Briefly describe each of them.

4.4 What is the difference between single suction and double suction centrifugal pumps?

4.5 What are the advantages of centrifugal pumps over reciprocating pumps?

4.6 Why the speed of operation of a centrifugal pump is higher compared to reciprocating pumps?

4.7 Explain the working principle of a single stage centrifugal pump?

4.8 Why a turbine pump has efficiency lower than that of a turbine though the losses of energy in the turbine and pump are of the same kind?

4.9 Explain the principal parts of a centrifugal pump with suitable diagrams.

4.10 What is the *shroud*? Why it is used?

4.11 Why the clearance between the casing and blade tip should be small?

4.12 What is the function of the casing used in a centrifugal pump?

4.13 The volute casing which surrounds the impeller is of spiral shape. Why?

4.14 Why a vortex or whirlpool chamber is introduced between the casing and impeller?

4.15 Differentiate between the volute casing and vortex casing for the centrifugal pump?

4.16 What is the function of providing guide vanes or diffuser vanes in the casing?

4.17 Describe the various types of casings used in a centrifugal pump with neat sketches.

4.18 Why a diffuser pump will operate with the maximum efficiency only for specific value discharge at a particular impeller speed?

4.19 Why the suction pipe is quite often kept short in length and larger in diameter than delivery pipe?

4.20 A foot valve fitted in the suction pipe is one way valve. Why?

4.21 State the assumptions made while calculating the work head or Euler head imparted by an impeller to the fluid.

4.22 Derive the Euler equation related to a centrifugal pump? Discuss the three distinct components of energy transfer in it?

4.23 Define (a) Suction head, (b) Delivery head, (c) Static head, (d) Euler head or theoretical head, and (d) Manometric head.

4.24 Distinguish between Euler head and manometric head?

4.25 Describe the slip or circulatory flow in the inter-vane passage in a centrifugal pump?

4.26 Why too few blades on the impeller of a centrifugal pump lead to excessive circulatory flow or slip?

4.27 What are the disadvantages of too many blades on the impeller of a centrifugal pump?

4.28 Explain various energy losses which occur in a centrifugal pump.

4.29 Explain (a) hydraulic losses, (b) leakage losses, and (c) mechanical losses occurring in a centrifugal pump.

4.30 Define (a) hydraulic efficiency, (b) volumetric efficiency, (c) manometric efficiency, (d) mechanical efficiency, and (e) overall efficiency of a centrifugal pump?

4.31 Establish the relation between the different types of efficiencies referred to a centrifugal pump?

4.32 What is the difference between fluid power and brake power of a centrifugal pump? Define efficiency of the pump in terms of these quantities.

4.33 Discuss the effect of blade exit angle on the performance of a centrifugal pump in relation to the head developed, power required and manometric efficiency of a centrifugal pump?

4.34 Which blade profile is preferred out of the backward curved, radial and forward curved blades and why?

4.35 Why forward facing or radial facing blades are seldom used in pumps?

4.36 Why the centrifugal pump impeller vanes are usually backward curved?

4.37 Why the backward curved blades yield the highest efficiency of the three blade geometries for the pumps?

4.38 Define specific speed of a centrifugal pump? State significance of it.

4.39 Derive an expression for specific speed of a centrifugal pump.
4.40 How does the specific speed of a centrifugal pump differ from that of a turbine?
4.41 Briefly describe the non-dimensional specific speed of centrifugal pumps.
4.42 Derive an expression for minimum starting speed of a centrifugal pump.
4.43 Obtain an expression for loss of head due to a change from normal discharge.
4.44 Prove that theoretical head-discharge curve of a centrifugal pump is linear.
4.45 Why the actual head-discharge curve is non-linear while theoretical head-discharge curve is linear?
4.46 Define (a) free delivery, (b) shut-off head, and (c) best efficiency point of a centrifugal pump.
4.47 Explain the head-discharge characteristics of a centrifugal pump.
4.48 What is meant by performance characteristics of an actual centrifugal pump? State its significance also.
4.49 Draw and discuss the main and operating characteristics of a centrifugal pump.
4.50 What do you understand by iso-efficiency curves? Explain their importance.
4.51 Compare the operating characteristics of radial flow, axial flow and mixed flow pumps.
4.52 Explain the variation of maximum efficiency of a centrifugal pump as a function of specific speed and maximum discharge with a neat sketch.
4.53 Draw a graph representing the variation of maximum efficiency of large pumps with specific speed. Mark clearly the operation zones of radial flow, mixed flow and axial flow pumps.
4.54 Why two or more dissimilar pumps are not connected in series or in parallel usually?
4.55 Why centrifugal pumps are sometimes connected in series and sometimes in parallel?
4.56 Describe a multistage centrifugal pump.
4.57 How the operating point of a pump-pipeline is determined when (a) a single pump is working, (b) series duplex pumps are working, and (c) parallel duplex pumps are working?
4.58 What is priming? Describe the need of priming in a centrifugal pump. How is it achieved?
4.59 If a centrifugal pump does not deliver any water when started, what may be the probable causes and how they can be remedied?
4.60 Describe briefly the methods adopted for priming of centrifugal pumps.
4.61 Write a short note on axial flow pumps.
4.62 Draw and discuss the performance characteristic curves of an axial flow pump.

PROBLEMS

4.1 Following are the values of some parameters which refer to a centrifugal pump installation:

Suction lift = 3.0 m	Overall efficiency of the pump = 0.70
Head loss in suction pipe 1.0 m	Relative density of liquid to be pumped = 0.89
Delivery head = 20 m	Speed of rotation 1200 rpm
Head loss in delivery pipe = 10.0 m	Efficiency of the motor = 90%
Discharge = 40 litres/s	

Calculate the brake power and torque applied to the pump shaft.

[*Ans:* $P = 18.85$ kW, $T = 0.15$ kN · m]

4.2 A centrifugal pump operating at 1200 rpm supplies water at the rate of 90 litres/s against a head of 10 m. The diameter of the impeller is 250 mm with the width of 60 mm at outlet. The manometric efficiency is 0.85. Calculate the blade angle at outlet of the impeller. [*Ans:* $\beta_2 = 12.87°$]

4.3 A centrifugal pump running at 1200 rpm has external and internal diameter of the impeller of 400 mm and 200 mm respectively. The blades of the impeller are curved backward at an angle of 28° at the exit. Calculate (a) velocity and direction of flow at the exit, (b) head developed by the pump, and (c) blade angle at inlet. Assume manometric efficiency of pump as 80%.

[*Ans:* (a) $C_2 = 21.462$ m/s, $\alpha_2 = 5.35°$, (b) $H_m = 43.79$ m, (c) $\beta_1 = 9.04°$]

4.4 A centrifugal pump working against a head of 9 m is delivering water at the rate of 108 litres/s while running at 600 rpm. The pump has forward curved blades which make an angle of 150° at exit with the direction of motion. The flow velocity through the impeller is constant at 2 m/s and the manometric efficiency is 75%. Find (a) the diameter, and (b) the width of the impeller.

[*Ans:* (a) $D_2 = 0.295$ m, (b) $B_2 = 5.83$ mm]

4.5 A centrifugal pump is developing a total head of 39 m. The manometric efficiency is 90% and the flow velocity through the impeller remains unchanged at 2 m/s. The blade angle, diameter and breadth of the impeller at the outlet are 30°, 0.5 m and 100 mm respectively. Calculate (a) the speed of the impeller, (b) specific speed, and (c) shape factor of the pump.

[*Ans:* (a) $N = 856.46$ rpm, (b) $N_s = 30.76$, (c) $N_{sh} = 0.0925$]

4.6 A centrifugal pump working against a head of 11 m delivers 33 litres/s of water while running at 1000 rpm. The manometric and overall efficiency of the pump are 85% and 70% respectively. The impeller blades are curved backwards at an angle of 30° at the exit and the flow velocity is constant at 1.8 m/s. Taking the inner diameter to be half of the external diameter of the impeller, calculate (a) the blade angle at inlet, and (b) the brake power. [*Ans:* (a) $\beta_1 = 15.56°$, (b) $BP = 5.0872$ kW]

4.7 A centrifugal pump delivers 8640 litres/min of water against a head of 10 m while running at 800 rpm. The diameter and width of the impeller at outlet are 300 mm and 60 mm respectively. If the blades are curved backwards at an angle of 40° with the tangent to the periphery of the impeller at exit, determine the manometric efficiency and specific speed of the pump.

[*Ans:* $\eta_{\text{mano}} = 81.9\%$, $N_s = 53.985$]

4.8 A centrifugal pump has an impeller of diameter 250 mm whose width at the exit is 50 mm. The flow velocity through the impeller having radial vanes is constant at 3.0 m/s. The speed of the pump is 1200 rpm and manometric efficiency is 85%. Find (a) the head developed by the pump, and (b) discharge. [*Ans:* (a) $H_m = 21.38$ m, (b) $Q = 117.81$ litres/s]

4.9 A centrifugal pump is working against a head of 51 m while running at 1000 rpm. The vanes on the impeller are bent backwards with an exit angle of 30° and the velocity of flow across the impeller is constant at 2.4 m/s. The diameter and width of the impeller at outlet are 0.48 m and 60 mm respectively. The external diameter of the impeller is three times the internal diameter. Calculate (a) the blade angle at inlet, (b) Euler head (c) Manometric efficiency.

[*Ans:* (a) $\beta_1 = 15.986°$, (b) $H_e = 53.74$ m, (c) $\eta_{\text{mano}} = 94.90\%$]

4.10 A centrifugal pump is developing a net head of 25 m. Find the minimum speed of the pump to start if the impeller has external and internal diameter as 0.5 m and 0.25 m, respectively.

[*Ans:* $N_{\text{min}} = 976.84$ rpm]

4.11 A centrifugal pump is to run at 1400 rpm against a head of 44 m. If the manometric efficiency is 0.8 and inner diameter is one third of the outer diameter, what is the minimum impeller diameter that will just start delivery of water? [*Ans:* $D_{2\,min}$ = 0.425 m]

4.12 A centrifugal pump working against a head of 6.5 m delivers 102 litres/s of water. The speed of the pump is 600 rpm. The diameters of the impeller at the inlet and outlet are 0.2 m and 0.4 m respectively. Determine the head loss due to shock when the discharge of the pump is reduced by 35% from the normal value through throttling while keeping the speed of the pump to be the same.

[*Ans:* h_q = 0.2465 m]

4.13 A centrifugal pump running at 1450 rpm delivers 193 litres/s of water against a head of 32 m. The diameter and width at the exit of the impeller are 0.3 m and 60 mm respectively. The manometric efficiency is 0.8 and inner diameter of the impeller is 0.15 m. Determine (a) the blade angle at outlet, and (b) the blade angle at the eye of impeller. [*Ans:* (a) β_2 = 31.56°, (b) β_1 = 16.683°]

4.14 The external and internal diameters of the impeller of a centrifugal pump are 0.3 m and 0.15 m respectively. The rotational speed of the pump is 1200 rpm and the vanes make an angle of 28° with the peripheral velocity direction at the outlet. The velocity of the flow through the impeller is constant at 1.7 m/s. Assuming the manometric efficiency of 82%, find (a) the vane angle at the inlet, (b) absolute velocity and direction at outlet, and (c) manometric head of the pump.

[*Ans:* (a) β_1 = 10.23°, (b) C_2 = 15.745 m/s, α_2 = 6.2°, (c) H_m = 24.66 m]

4.15 A centrifugal pump is running at 1000 rpm delivers water against a net head of 7 m. The blade angle at outlet is 28° with the direction of peripheral velocity. The diameter and width at the outlet of impeller are 0.25 m and 50 mm respectively. Calculate (a) discharge, and (b) power required for running the pump. Assume manometric and overall efficiency as 90% and 75% respectively.

[*Ans:* (a) Q = 0.1516 m^3/s, (b) P = 13.88 kW]

4.16 A centrifugal pump with an impeller of outer diameter of 0.5 m and inner diameter 0.25 m delivers 135 litres/s of water while running at a speed of 900 rpm. The width of the impeller is 70 mm at inlet and 50 mm at the outlet. The vanes are curved backwards with an angle of 26° with the tangential velocity direction at outlet. The manomeric and mechanical efficiencies are 0.8 and 0.9 respectively. Determine (a) the net head produced by the pump, (b) brake power of the pump, and (c) specific speed and shape factor of the pump.

[*Ans:* (a) H_m = 38.5 m, (b) P = 70.816 kW, (c) N_s = 21.395, N_{sh} = 0.06433 rev.]

4.17 The outer diameter of a centrifugal pump impeller is 0.4 m whereas inner diameter is 0.15 m. The impeller blades make an angle of 40° and 35° with the direction of peripheral velocity at outlet and inlet, respectively. The speed of the pump is 1000 rpm and velocity of flow through the impeller remains constant. If the manometric efficiency of the pump is 82%, estimate the head developed by the pump. [*Ans:* H_m = 25.2 m]

4.18 A single stage centrifugal pump delivers 0.28 m^3/s against a head 26 m while running at 1400 rpm. It is required to raise 0.374 m^3/s of water over a height of 92 m at another site by a similar pump operating at the same speed but in multistage. Determine the number of stages required.

[*Ans:* 3 Stages]

4.19 A four-stage centrifugal pump is running at 1000 rpm delivering 20 litres/s of water under a total head of 32 m. The width and diameter of each impeller at outlet is 40 mm and 0.2 m respectively. The blades of each impeller are bent backwards with an angle of 30° at outlet. Calculate the manometric efficiency. [η_{mano} = 82.42%]

4.20 A centrifugal pump delivers 130 litres/s of water while running at 1000 rpm. The diameter and width of the impeller at the exit is 400 mm and 80 mm, respectively. The blades on the impeller are bent backward at an angle of 30° at the exit. Determine (a) the theoretical rise in pressure head across the impeller and the velocity head at the exit of the impeller, (b) if a volute casing converts 50% of the velocity head at exit of the impeller into the pressure head; determine the actual total pressure head across the pump. $\left[Ans: \text{(a)} \left(\frac{\Delta p_i}{\rho g}\right)_{\text{th}} = 22.10 \text{ m}, \frac{C_2^2}{2g} = 17.92 \text{ m, (b)} \left(\frac{\Delta p_i}{\rho g}\right)_{\text{actual}} = 31.06 \text{ m}\right]$

4.21 A centrifugal pump having manometric efficiency of 80%. The blades on the impeller of the pump are backward facing with an angle of 28° at the outlet and the velocity of flow through the impeller remains constant at 1.7 m/s. The tangential velocity of the blades at the exit is 22 m/s. Calculate (a) the difference in pressure head across the pump considering no loss in the impeller, (b) the percentage of kinetic head at the exit recovered as pressure head in the volute casing.

$$\left[Ans: \text{(a)} \left(\frac{\Delta p_i}{\rho g}\right)_{\text{th}} = 24.145 \text{ m, (b)} (\Delta KE)_{\text{recov}} = 62.37\%\right]$$

4.22 A diffuser type centrifugal pump with an impeller of diameter 0.5 m develops a head of 24 m. The flow enters radially in the eye of the impeller and the blade angle at the exit is 28°. The velocity of flow across the impeller is constant at 2.15 m/s. The manometric efficiency of the pump is 90% and the loss in the impeller is 5% of the velocity head at the exit. Determine (a) the speed of the impeller, (b) the percentage of velocity head at the exit recovered by the diffuser, and (c) the direction of the diffuser blades. [*Ans:* (a) $N = 700$ rpm, (b) $(\Delta KE)_{\text{recov}} = 77.644\%$, (c) $\alpha_2 = 8.564°$]

4.23 A centrifugal pump is working under a head of 17 m delivers 417 litres/s of water while running at 1000 rpm. The overall efficiency of the pump is 75%. If the speed of the pump is increased to 1500 rpm, find (a) the discharge, (b) the head, and (c) the brake power of the pump?

[*Ans:* (a) $Q_2 = 625.5$ litres/s, (b) $H_2 = 32.25$ m, (c) $P_2 = 312.9435$ kW]

4.24 Find the number of stages required in a multistage pump to deliver 3750 litres/min against a total head of 290 m. The speed of operation and specific speed of the pump is 1500 rpm and 25 respectively. [*Ans:* $n = 8$ stages]

4.25 A centrifugal pump running at 1400 rpm is required to draw 235 litres/s of water from a deep well against a head of 200 m. Find the number of stages required if the specific speed of the pump is limited to 20. [*Ans:* $n = 2$ stages]

MULTIPLE CHOICE QUESTIONS

1. A centrifugal pump has which of the following advantages?
   1. Low initial cost
   2. Compact, occupies small floor space
   3. Handles highly viscous fluids easily

   Choose the right answer using the codes given below:

   (a) 1, 2, and 3 (b) 1 and 2 only
   (c) 1 and 3 only (d) 2 and 3 only

2. Which pump would you select for pumping sewage?
   (a) Reciprocating pump
   (b) Open impeller centrifugal pump
   (c) Multistage centrifugal pump
   (d) Screw pump
3. The correct order of components through which the fluid flows in a centrifugal pump is
   (a) Impeller, suction pipe, foot valve and strainer, delivery pipe
   (b) Impeller, suction pipe, delivery pipe, foot valve and strainer
   (c) Foot valve and strainer, suction pipe, strainer pipe, impeller and delivery pipe
   (d) Suction pipe, delivery pipe, impeller, foot valve and strainer
4. Match the List 1 and List 2 and choose the right answer using the codes given

List 1	List 2
1. Closed impeller pump	A. Sandy water
2. Semi open impeller pump	B. Acids
3. Open impeller	C. Sewage water

Codes

	1	**2**	**3**
(a)	A	C	C
(b)	C	A	B
(c)	A	B	C
(d)	B	C	A

5. The functions of a volute casing in a centrifugal pump are
   1. To collect water from the periphery of the impeller and to discharge it to the delivery pipe at constant speed
   2. To increase the pump discharge
   3. To increase the pump efficiency
   4. To decrease the head loss due to change in velocity

   Select the correct answer using the following codes,
   (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) 1 and 2
6. A turbine pump is primarily a centrifugal pump provided additionally with
   (a) Backward facing vanes
   (b) Casing with vane diffuser
   (c) Guide blades at entry
   (d) Adjustable blades
7. Match List 1 with List 2 and select the answers using the codes given,

List 1 (Type of Pumps)	List 2 (Features)
A. Centrifugal Pumps	1. Air Vessel
B. Gear Pump	2. Draft tube
C. Reciprocating Pump	3. Guide Vanes
D. Turbine Pump	4. Rotary pump
	5. Rotor having blades

Codes

	A	**B**	**C**	**D**
(a)	4	2	5	3
(b)	5	4	1	2
(c)	4	2	3	1
(d)	5	4	1	3

8. In a centrifugal pump the inlet angle will be designed to have
   (a) Relative velocity vector in the radial direction
   (b) Absolute velocity vector in the radial direction
   (c) Velocity of flow to be zero
   (d) Peripheral velocity to be zero

9. The following statements are about a centrifugal pump:
   1. A foot valve and strainer is fitted in the suction pipe
   2. The manometric head is the head developed by the pump
   3. The delivery pipe is fitted with a foot valve and a strainer
   4. The suction pipe has generally a larger diameter compared to discharge pipe

   Of these statements
   (a) 1 and 2 are correct   (b) 1 and 4 are correct
   (c) 2 and 4 are correct   (d) 1, 2, 3 and 4 are correct

10. The manometric efficiency η_{mano} in a centrifugal pump with radial entry of liquid is given by,
   (a) $\eta_{\text{mano}} = \dfrac{C_{b2}C_{w2}}{2gH_m}$ (b) $\eta_{\text{mano}} = \dfrac{C_{b2}C_{w2}}{gH_m}$
   (c) $\eta_{\text{mano}} = \dfrac{2gH_m}{C_{b2}C_{w2}}$ (d) $\eta_{\text{mano}} = \dfrac{gH_m}{C_{b2}C_{w2}}$

11. A centrifugal pump running at 1200 rpm has impeller diameter of 0.5 m. The absolute velocity at the exit is 9 m/s and makes an angle of 40° with the direction of whirling. If the manometric efficiency is 83%, the actual head produced is approximately
   (a) 18.33 m   (b) 15 m   (c) 10 m   (d) 30 m

12. A centrifugal pump is used to deliver water at the rate of 0.1 m^3/s to a height of 11 m. The frictional and other losses are 5 m. If the pump efficiency is 0.8 m, the power necessary to run the pump should be
   (a) 13 kW   (b) 26.9 kW   (c) 19.6 kW   (d) 17.8 kW

13. A centrifugal pump requires brake power of 18 kW while delivering 0.1 m^3/s water against a net head of 135 kPa. The specific weight of the water is 9.8 kN/m^3. What is the overall efficiency of the pump? What would be the brake power if the liquid to be pumped is a solvent instead of water having a relative density of 0.8 while keeping other factors same as before?
   (a) (75%, 14.4 kW)   (b) (75%, 18 kW)
   (c) (80%, 22 kW)   (d) (82%, 19.6 kW)

14. A centrifugal pump delivers a liquid of relative density 0.8 against a head of 12 m. If the pump is used to deliver a liquid of relative density 1.2 while the other factors remain the same, the new manometric head would be
   (a) 18 m   (b) 8 m   (c) 10 m   (d) 12 m

15. Use of large diameter pipes in a pump results in the reduction of
   (a) Static head   (b) Frictional head
   (c) Both static head and frictional head   (d) Manometric head

16. If the diameter of the impeller is reduced, it will result in
   (a) Discharge decreases and pressure increases
   (b) Both the discharge and pressure decrease
   (c) Both the flow and pressure increase
   (d) Discharge increases and the pressure decreases

17. In a centrifugal pump when delivery valve is fully closed, the pressure of fluid inside the pump will
(a) Become zero
(b) Reduce
(c) Increase
(d) Remain unaltered

18. The theoretical shut-off head of a centrifugal pump is given by
(a) $\frac{C_{b2}^2}{g}$
(b) $\frac{C_{b1}^2}{g}$
(c) $\frac{1}{2g}(C_{b2}^2 - C_{b1}^2)$
(d) $\frac{1}{g}(C_{b2}^2 - C_{b1}^2)$

19. A centrifugal pump running at 1200 rpm has external and internal diameters of 0.4 m and 0.2 m respectively. If the delivery valve is completely shut-off, the theoretical head produced is approximately
(a) 53 m (b) 64.4 m (c) 16.3 m (d) 105 m

20. A centrifugal pump is started keeping its delivery valve
(a) Fully open
(b) Fully shut
(c) Partially open
(d) 50% open

21. Throttling the delivery valve of a centrifugal pump results in
(a) Decreased head
(b) Increased head
(c) Increased power and head
(d) Increased power

22. A centrifugal pump delivers 0.4 m^3/s while running at a speed of 750 rpm. The outer and inner diameters of the impeller are 0.6 m and 0.3 m respectively. The discharge is reduced by 40% by throttling while keeping the speed to be the same. The loss of head due to shock is
(a) 2.55 m (b) 1.13 m (c) 4.52 m (d) 1.81 m

23. The centrifugal pump will start delivering water if the pressure rise in the impeller is
(a) More than or equal to the manometric head
(b) Less than manometric head
(c) Less than suction head
(d) More than or equal to suction head

24. A centrifugal pump with an impeller having inlet diameter of 0.25 m and outlet diameter of 0.5 m. The pump is working against a head of 25 m. The minimum starting speed in rpm of the pump is
(a) 675 rpm (b) 740 rpm (c) 977 rpm (d) 1440 rpm

25. The blades of a centrifugal pump are normally
(a) Radial
(b) Forward curved
(c) Twisted
(d) Backward curved

26. A centrifugal pump provides maximum efficiency if its blades are
(a) Forward facing
(b) Backward facing
(c) Straight
(d) Shaped like a wave

27. The blades of a slow centrifugal pump impeller are
(a) Bent forward
(b) Radial
(c) Bent backward
(d) Propeller type

28. For attaining a non-overloading characteristic in centrifugal pumps
(a) Backward curved blades are preferred over forward curved blades
(b) Forward curved blades are preferred over backward curved blades
(c) Forward curved blades are preferred over radial blades at exit
(d) Blades radial at exit are preferred over backward curved blades

29. Manometric efficiency of a centrifugal pump
    (a) Decreases with decrease in the blade angle at the exit
    (b) Increases with the decrease in the blade angle at the exit
    (c) Is independent of the blade angle at the exit
    (d) Is maximum at the value of 90° for the blade angle at the exit
30. The specific speed of a hydraulic pump is the speed of a geometrically similar pump working against a unit head and
    (a) Delivering unit quantity of water (b) Consuming unit power
    (c) Consuming unit velocity of flow (d) Having unit radial velocity
31. In terms of speed of rotation of the impeller (N), discharge (N) and the manometric head (H_m), the specific speed for a pump is
    (a) $\dfrac{N\sqrt{Q}}{H_m^{5/4}}$ (b) $\dfrac{N\sqrt{Q}}{H_m^{3/4}}$ (c) $\dfrac{N\sqrt{Q}}{H_m^{1/4}}$ (d) $\dfrac{N\sqrt{Q}}{H_m^{5/4}}$
32. The dimension of the specific speed of centrifugal pump is
    (a) $LT^{3/2}$ (b) $LT^{-3/4}T^{3/2}$ (c) $L^{3/4}T^{-3/2}$ (d) $M^{1/2}L^{1/4}T^{-5/2}$
33. Examine the following statements related to specific speed of turbomachines:
    1. Variation of specific speed occurs with shape of the runner and other components of the machine
    2. Higher specific speed machines are restricted to low heads
    3. Specific speed is dimensionless and is independent of variation of type of fluid used

    Which of these statements are correct?
    (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3
34. A centrifugal pump is delivering 135 litres/s of water at a head of 45 m when running at 1800 rpm. The specific speed of the pump is
    (a) 17 (b) 90 (c) 38 (d) 128
35. If N_{s1} is the specific speed of a centrifugal pump having large discharge at a low head and N_{s2} is the specific speed of a centrifugal pump having relatively low discharge at a relatively high head, which of the following is true?
    (a) $N_{s1} > N_{s2}$
    (b) $N_{s1} = N_{s2}$
    (c) $N_{s1} < N_{s2}$
    (d) All the other three are possible under different conditions
36. The specific speeds of radial vane pump, mixed flow pump and axial flow pump are N_{s1}, N_{s2} and N_{s3}, respectively. Then
    (a) $N_{s1} > N_{s2} > N_{s3}$ (b) $N_{s3} > N_{s2} > N_{s1}$
    (c) $N_{s2} > N_{s3} > N_{s1}$ (d) $N_{s3} > N_{s1} > N_{s2}$
37. If the speed of a centrifugal pump is doubled, its power consumption will increase
    (a) 6 times (b) 4 times (c) 2 times (d) 8 times
38. A centrifugal pump lifts water to a head of 14 m and consumes 1.73 kW of power while running at the speed of 1200 rpm. When it is operated at 2000 rpm and if the overall efficiency remains the same, its power input and head developed would be approximately
    (a) 4 kW and 50 m of water (b) 7 kW and 25 m of water
    (c) 6 kW and 35 m of water (d) 8 kW and 38.8 m of water

39. The following data pertains to the performance of a centrifugal pump

Speed = 1400 rpm, Discharge = 35 litres/s, Head = 27 m and power = 8 kW. If the speed is raised to 1800 rpm, the power input will be approximately equal to

(a) 9.5 kW (b) 11.7 kW (c) 17 kW (d) 13.2 kW

40. A centrifugal pump delivers 120 litres/s of water when running at 1800 rpm. What would be the discharge if the speed of the pump is raised to 3000 rpm?

(a) 0.9 m^3/s (b) 0.2 m^3/s (c) 0.5 m^3/s (d) 0.06 m^3/s

41. A centrifugal pump working against a head of 40 m delivers water at its best efficiency point at the rate of 70 litres/min while running at 600 rpm. If the speed of the pump is increased to 1200 rpm, then the head H in m and the discharge Q in l/min would be

(a) (160, 140) (b) (40, 140) (c) (40, 560) (d) (160, 35)

42. A centrifugal pump run by directly coupled 3 kW motor of speed 1450 rpm is proposed to be connected to another motor of 2900 rpm speed. The power of the motor should be

(a) 6 kW (b) 12 kW (c) 18 kW (d) 24 kW

43. If the discharge in a pump is halved while keeping its speed unchanged, the ratio of the new head to the old head is

(a) $\left(\frac{1}{2}\right)^{1/2}$ (b) $\left(\frac{1}{4}\right)$ (c) $\left(\frac{1}{2}\right)^{3/2}$ (d) $\left(\frac{1}{2}\right)^{1/3}$

44. A pump and its 1:4 scale model are being compared. If the ratio of the heads is 4:1, then the ratio of the power required by the prototype and model is

(a) 100 (b) 4 (c) 128 (d) 16

45. Susceptibility of cavitation are high if the

(a) Local pressure becomes very high
(b) Local temperature becomes low
(c) Thoma cavitation parameter exceeds a certain limit
(d) Local pressure falls below the vapour pressure

46. The operating point of a pump is the point of intersection of

(a) The system characteristic curve and the pump curve
(b) The pump curve and the theoretical power curve
(c) The system characteristic curve and the efficiency curve
(d) The pump characteristic curve and the Y-axis

47. The following statements are related to a centrifugal pump,

1. The efficiency is zero at the shut-off head of the pump
2. The discharge at the free delivery point of the pump is greater than that at its BEP.
3. The efficiency of the pump is zero at its free delivery point
4. At the BEP, the net head has its maximum value

Of these statements

(a) 1, 3 and 4 are correct (b) 1, 2 and 3 are correct
(c) 1, 2 and 4 are correct (d) 1, 2, 3 and 4 are correct

48. Centrifugal pumps operating in series will result in

(a) Higher discharge (b) Reduced power consumption
(c) Low speed operation (d) Higher head

49. The comparison between pumps operating in series and in parallel is
    (a) Pumps operating in series boost the discharge, whereas the pumps operating in parallel boost the head
    (b) Pumps operating in parallel boost the discharge, whereas the pumps in series boost the head
    (c) In both cases there would be a boost in discharge only
    (d) In both cases there would be a boost in head only
50. If two pumps identical in all respects and each capable of delivering a discharge Q against a head H are connected in series, the resulting discharge is
    (a) $2Q$ against head of $2H$
    (b) $2Q$ against head of H
    (c) Q against head of $2H$
    (d) $2Q$ against head of $\sqrt{2}H$
51. If two pumps identical in all respects and each capable of delivering a discharge Q against head H are connected in parallel, the resulting discharge is
    (a) $2Q$ against head of $2H$
    (b) $2Q$ against head of H
    (c) Q against head of $2H$
    (d) $\sqrt{Q}$ against head of $\sqrt{2}H$

Refer Figure 4.50 for Q. No. 52 and 53.

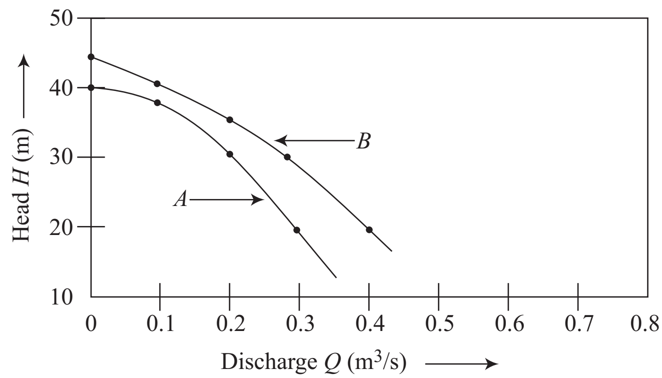


Figure 4.50 *Multiple choice questions, 52-53*

52. Two centrifugal pumps A and B are available for use in a pipe flow system. Their characteristics are shown in Fig. 4.50

    When operating in parallel, the discharge for a head of 20 m will be nearly
    (a) 0.35 m^3/s (b) 0.5 m^3/s
    (c) 0.6 m^3/s (d) 0.7 m^3/s
53. For the series operation of the two pumps shown in the above figure, for a discharge of 0.2 m^3/s, the head will be nearly
    (a) 90 m (b) 65 m
    (c) 45 m (d) 30 m

54. Referring Figure 4.51, the head and discharge variation for a centrifugal pump is represented by

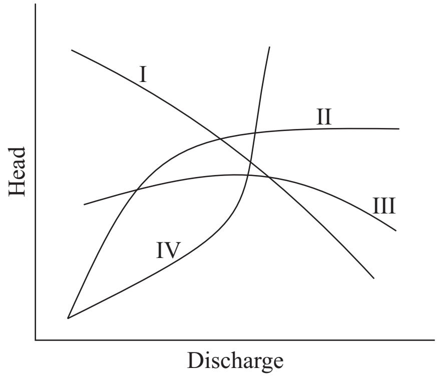


Figure 4.51 *Multiple choice question, 54*

(a) Curve I (b) Curve II
(c) Curve III (d) Curve IV

55. Which curve represents the condition for backward curved vanes in a centrifugal pump referring Figure 4.52?

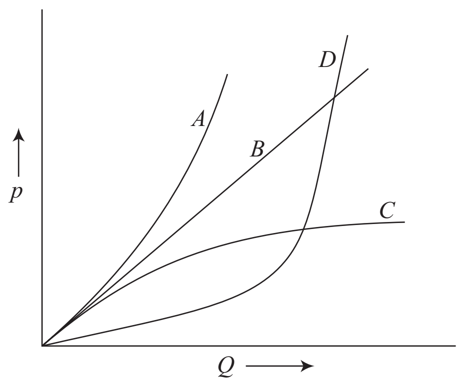


Figure 4.52 *Multiple choice question 55*

(a) Curve A (b) Curve B
(c) Curve C (d) Curve D

For Q. No. 56 and 57, refer Table 4.5 representing the characteristics of a centrifugal pump running at 1500 rpm.

TABLE 4.5 Multiple Choice Questions 56-57

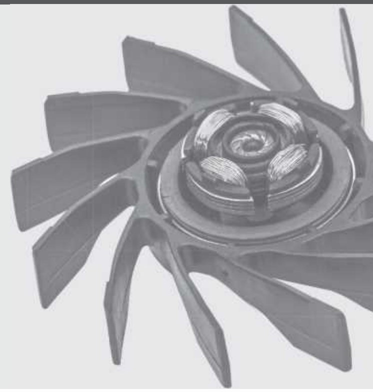
Q(litres/s)	10	15.5	21.5	30.3	36.0	42.5	48.0
H(m)	35	34.5	33.3	30.0	26.8	22.8	18.5
η(%)	60	67.5	72.0	73.5	71.0	66.0	60.0

56. The specific speed corresponding to the best efficiency is nearly
    (a) 20 (b) 30 (c) 40 (d) 60
57. The shut-off head will be nearly
    (a) 0 (b) 18.5 m (c) 25.5 m (d) 35.0 m
58. Priming is necessary in
    (a) Centrifugal pumps to lift water from a greater depth
    (b) Centrifugal pumps to remove air in the suction pipe and casing
    (c) Hydraulic turbine to remove air in the turbine casing
    (d) Hydraulic turbine to increase the speed of turbine and to generate more power.
59. Typical range of specific speeds for axial flow pumps is
    (a) 380 – 950 (b) 80 – 200 (c) 10 – 100 (d) 200 – 300

ANSWER KEY

1. (a)	2. (b)	3. (c)	4. (d)	5. (c)	6. (b)	7. (d)	8. (b)	9. (b)	10. (d)
11. (a)	12. (c)	13. (b)	14. (d)	15. (b)	16. (b)	17. (d)	18. (a)	19. (b)	20. (b)
21. (c)	22. (b)	23. (a)	24. (c)	25. (d)	26. (b)	27. (a)	28. (a)	29. (b)	30. (a)
31. (b)	32. (c)	33. (b)	34. (c)	35. (a)	36. (b)	37. (d)	38. (d)	39. (c)	40. (b)
41. (a)	42. (d)	43. (b)	44. (c)	45. (d)	46. (a)	47. (b)	48. (d)	49. (b)	50. (c)
51. (b)	52. (d)	53. (b)	54. (a)	55. (c)	56. (a)	57. (d)	58. (b)	59. (d).	

5 Cavitation in Turbines and Pumps



Learning Objectives

After reading this chapter, you will be able to:

LO 1 Explain the inception of cavitation, probable damages and methods to avoid it

LO 2 Describe the concepts of net positive suction head, Thoma's cavitation parameter, their significance and relation between them

LO 3 Define the suction specific speed and its significance

LO 4 Understand the locations those are susceptible to cavitation

5.1 Inception of Cavitation

In the flow of liquid/water in a conduit, local pressure can attain vacuum values depending upon the velocity, datum head and nature of flow boundary. The pressure even at a particular cross section varies widely due to non-uniform flow in machines. Therefore, there may be regions on the low pressure side of the rotor in which the pressure drops to values greatly below atmospheric pressure. If the pressure at any point in a liquid drops to the vapour pressure at the prevailing temperature, the liquid there boils and small bubbles of vapour are formed in large numbers. When these bubbles are carried along by the flow to higher pressure zones of the flow, they get compressed, decrease in size and after a period of time may instantly collapse as the vapour condenses to liquid again. The collapse of vapour bubbles is a phenomenon of implosion (internal bursting). A pocket or cavity results due to instantaneously collapsing of vapour bubbles. Consequently, the surrounding liquid rushes in to fill the pocket or cavity produced by the collapsing of bubbles. The liquid rushing from all directions hits violently at the centre of the cavity, thus increasing the pressure to a very high value (up to 1000 MPa). Any neighboring solid surface is also subjected to these extreme pressures; because, even though the pockets or voids are not formed actually at the solid surface, the pressures are propagated from the cavities by the pressure waves like that occur in water hammer. The process of formation, travel and sudden collapse of vapour bubbles in negative pressure zones of a liquid flow is known as *cavitation*. Cavitation causes severe damages to the surfaces.

The liquid such as water has air dissolved in it. The air is released if the pressure of the liquid decreases resulting in *air cavitation*. It is found that air cavitation causes lesser damages to the surfaces than vapour cavitation.

Cavitation is not always undesirable as it may be a blessing in super cavitation such as in very fast super cavitating torpedoes and some propellers. *Super cavitation* is the use of cavitation sufficient to produce an enough large bubble of gas inside a liquid to enclose an object travelling through the liquid. Super cavitation greatly decreases the drag on the object due to skin friction, consequently, ensuring the achievement of very high speed.

5.2 Cavitation in Turbines

Cavitation starts when the pressure drops too low a value. Therefore, occurrence of cavitation is more susceptible at points where the velocity or the elevation is high, particularly at those points where high velocity and high elevation are combined.

5.2.1 Cavitation in Pelton Turbine

Cavitation is not a matter of serious concern in Pelton turbines since it does not impose limits to operation. We know that Pelton turbines work under atmospheric pressure. The pressures in the system are either atmospheric as in the case of the wheel or above atmospheric as in closed conduits like the nozzle and manifold. Hence, cavitation due to negative ambient pressure does not arise. However, the velocities involved are very high and possibilities of local vacuum pressures and consequently, cavitation is possible even in Pelton turbines.

5.2.2 Cavitation in Reaction Turbines

As already discussed, the outlet of the runner, i.e. entry of the draft tube, is usually more susceptible to cavitation. Application of Bernoulli's Eq. for the flow between inlet of the draft tube and the final discharge into the tailrace (where the pressure is atmospheric), as shown in Figure 5.1, assuming the velocity at the outlet of the draft tube to be negligibly small, results in,

$$\frac{p_2}{\rho g} + \frac{C_2^2}{2g} + Z_2 = \frac{p_a}{\rho g} + h_f \qquad (5.1)$$

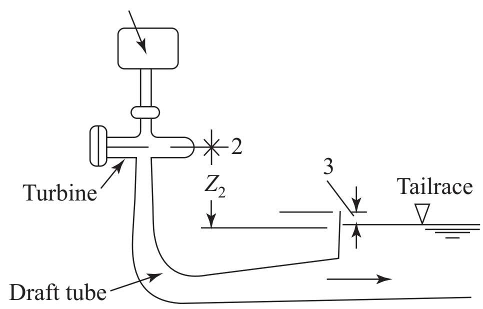


Figure 5.1 *Application of Bernoulli's Eq. between Turbine Exit and Tailrace*

where p_2 and C_2 represents the static pressure and velocity at the outlet of the runner or inlet of the draft tube and Z_2 is the height of the turbine runner outlet above the tailrace. Head loss due to friction in the draft tube is denoted by h_f. Equation (5.1) incidentally shows a further reason why the velocity at the outlet of the runner, C_2, should be as small as possible. It is obvious that larger the value of C_2, the smaller is the value of p_2, therefore, the chances of cavitation susceptibility is more.

An important parameter in the context of cavitation is the available suction head defined as the total head (static and dynamic head) at exit of the turbine above the vapour pressure known as NPSH (net positive suction head), which is given by,

$$\text{NPSH} = \frac{p_2}{\rho g} + \frac{C_2^2}{2g} - \frac{p_v}{\rho g} \qquad (5.2)$$

From Eq. (5.1) and Eq. (5.2),

$$\text{NPSH} = \left(\frac{p_a}{\rho g} + h_f - Z_2\right) - \left(\frac{p_v}{\rho g}\right)$$

$$\text{NPSH} = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - (Z_2 - h_f) = H_a - H_v - (Z_2 - h_f) \tag{5.3}$$

Rearranging Eq. (5.1) gives,

$$\frac{C_2^2}{2g} - h_f = \frac{p_a}{\rho g} - \frac{p_2}{\rho g} - Z_2 \tag{5.4}$$

For a particular design of machine operated under its design conditions, the left hand side of Eq. (5.4) may be considered as a particular proportion, say σ_c, of the net head H across the turbine. Then,

$$\sigma_c = \frac{p_a/\rho g - p_2/\rho g - Z_2}{H} \tag{5.5}$$

For cavitation not to occur, $p_2 > p_v$ i.e. $\sigma > \sigma_c$, where,

$$\sigma = \frac{\text{NPSH}}{H} = \frac{p_a/\rho g - p_v/\rho g - (Z_2 - h_f)}{H} = \frac{H_a - H_v - (Z_2 - h_f)}{H} \tag{5.6}$$

where, σ is the Thoma's cavitation parameter and σ_c is the critical cavitation parameter. From Eq. (5.1), it is clear that $(p_a/\rho g - Z_2)$ or $(H_a - Z_2)$ is a measure of the pressure at the discharge end of the turbine. Therefore, σ is a measure of how much the pressure at the exit of the turbine differs from the vapour pressure. Further, Eq. (5.6) indicates that σ is decreased if either Z_2 or H is increased. The value of σ may be calculated to determine whether cavitation is likely to occur in a particular installation. For cavitation not to occur, $p_2 > p_v \Rightarrow \sigma > \sigma_c$.

In practice, the value of σ_c is used to find the maximum height of the installation of turbine runner above the tailrace to avoid cavitation. The critical Thoma's cavitation parameter σ_c increases with an increase in the specific speed of the turbine. Therefore, higher specific speed turbines must be installed closer to the tailrace. The following correlations are used for finding the value of σ_c for different turbines:

- **For Francis Turbines**

$$\sigma_c = 0.044 \times \left(\frac{N_s}{100}\right)^2 \tag{5.7}$$

- **For Propeller Turbines**

$$\sigma_c = 0.3 + 0.0032\left(\frac{N_s}{100}\right)^{2.73} \tag{5.8}$$

For Kaplan turbines, σ_c is taken 10% higher than that of a similar propeller turbine.

(a) Efficiency versus Thoma's Cavitation Parameter Plot

The Thoma's cavitation parameter, σ, is a function of the net head, H, for a given speed and discharge of the turbine. When a turbine is tested at constant speed, fixed discharge by increasing the values of net head, it amounts to testing the turbine at lowering the values of σ. Figure 5.2 shows the result of such a turbine test. The general effect of cavitation parameter on the turbine efficiency is represented by the Figure 5.2.

Let us study the effect on the efficiency of the turbine when σ is reduced from a large initial value, say σ_a, towards zero i.e. following the curve from right to left. The curve is generally a constant efficiency curve with a rapid decrease in the value of efficiency below a certain critical value of σ. The rapid fall in the efficiency is due to inception of cavitation. Before the start of rapid fall, there can be a momentary rise followed by rapid fall. However, the curve suddenly breaks off representing a plunging efficiency for values of σ below the onset point of cavitation. The lowest value of σ at which incipient cavitation manifests at the turbine under study is identified by the criteria of 1% drop in efficiency. This point marked $\sigma_{1\%}$ in Figure 5.2 represents the lowest value of σ that can be permitted in the turbine without serious occurrence of cavitation in the turbine. This point is considered as a critical point and the value of σ at this point is called *critical cavitation parameter or critical Thoma's coefficient* and is designated by σ_c. The parameter σ_c is a characteristic of the turbine under study. Thus, every turbine will have a unique σ_c. The operation zone of the turbine can be defined as follows:

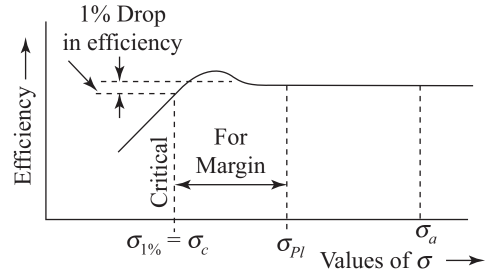


Figure 5.2 *Efficiency versus Cavitation Parameter*

1. If $\sigma \geq \sigma_c$, there will be no danger of cavitation, and
2. If $\sigma < \sigma_c$, the turbine installation will suffer cavitation problem.

In practice, using σ_c, the maximum elevation of the turbine, i.e. the maximum draft head for a turbine setting can be obtained from Eq. (5.5) as,

$$(Z_2)_{\max} = p_a/\rho g - p_v/\rho g - \sigma_c H = H_a - H_v - \sigma_c H \tag{5.9}$$

Equation (5.9) shows that greater the net head, H, on which a given turbine operates, the lower it must be placed relative to tailrace level. It should be noted that the value of draft head given by Eq. (5.9) is the maximum value of Z_2 that could be adopted at the site and it indicates the incipient cavitation conditions at that setting. It is a usual practice to allow a margin (M) of up to 0.5 m. Therefore, the actual height of the turbine above the tail race, i.e. draft head, will be less than or equal to ($Z_{2-\max} - M$). The factor of safety is also incorporated sometimes into the value of σ_c by considering the operational σ_c to be a certain percentage higher than σ_c obtained from 1% criteria. Such operational σ_c is known as *plant sigma*, σ_{pl}.

(b) Variation of Critical Thoma's Coefficient with Specific Speed

The critical cavitation parameter varies with the specific speed of the turbine. The normal variation of σ_c with specific speed is summarized in Table 5.1.

TABLE 5.1 Cavitation Factor for Different Specific Speeds and Turbines

Francis Turbine		*Kaplan turbine*	
Specific Speed (N_s) $\frac{\text{rpm}/\sqrt{\text{kw}}}{\text{m}^{5/4}}$	**Cavitation Factor (σ_c)**	**Specific Speed (N_s) $\frac{\text{rpm}/\sqrt{\text{kw}}}{\text{m}^{5/4}}$**	**Cavitation Factor (σ_c)**
50	0.04	300-450	0.35-0.4
100	0.05	450-550	0.4-0.45
150	0.07	550-600	0.46-0.6
200	0.1	650-700	0.85
250	0.14	700-800	1.05
300	0.2		
350	0.27		

These are the typical values and they can be obtained by simple linear interpolation for intermediate values of specific speed. Many empirical relations are available for the estimation of σ_c for turbines. These values of σ_c, at best, are suited for preliminary studies. For final-stage designs and installations, more reliable and definitive values are needed. For each turbine, σ_c is obtained by model tests and turbine manufacturer furnishes its value.

Figure 5.3 shows the variation in σ_c with shape number. Figure 5.3 and Table 5.1 show that turbines of higher specific speeds have higher values of σ_c. Therefore, turbines of higher specific speeds must be installed much lower as compared to turbines of lower specific speeds. For a high net head, it may be required to install the turbine below the tailrace, thus greatly increasing the difficulties of construction and maintenance. This factor limits the use of propeller turbines to low heads for which, they are best suited in other ways. Study of Eq. (5.9) reveals that at some high specific speed turbine settings, i.e. at low head installations, the conditions may necessitate negative settings of the turbine. In such cases, the turbines will have to be below the tailrace. The correct estimation of tailrace is very important to realize, because the draft head is provided relative to the tailrace. It should also be noted that Figure 5.3 provides useful general guidelines only, not more than this. In practice, the occurrence of cavitation depends very much on the design parameters.

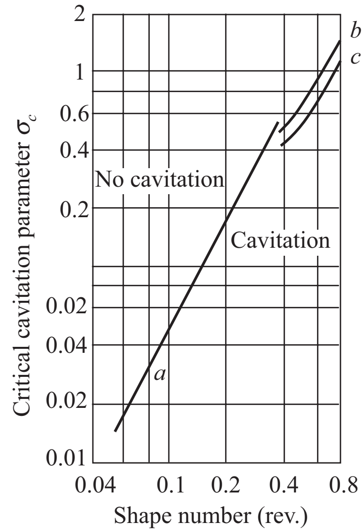


Figure 5.3 *Variation of Critical Cavitation Parameter with Shape Number*

Cavitation is a phenomenon which is not limited to turbines only. Wherever it occurs, it is an additional factor to be thought of carefully for dynamic similarity between one situation and another. Similarity of cavitation requires the *cavitation number* $(p - p_v)/\frac{1}{2}\rho C^2$, to be the same at corresponding points. However, similarity of cavitation is difficult to achieve as suggested by experiments.

(c) Suction Specific Speed

Suction specific speed provides another useful basis for similarity with respect to cavitation in the turbines in addition to Thoma's criterion. The *suction specific speed* is the speed of a geometrically similar turbine which develops 1 kW power under the available net positive suction head (NPSH) of 1 m. According to this definition, the formula for suction specific speed is obtained by substituting the net head on the turbine in the expression of specific speed by NPSH. Therefore,

$$(N_s)_{\text{suction}} = \frac{N\sqrt{P}}{(\text{NPSH})^{5/4}} \tag{5.10}$$

The similarity of cavitation can be established if the suction specific speed for the model and prototype turbines has the same value. The NPSH available is given by Eq. (5.3). Substituting the value of NPSH available from Eq. (5.6) into Eq. (5.10), we get,

$$(N_s)_{\text{suction}} = \frac{N\sqrt{P}}{(\sigma H)^{5/4}}$$

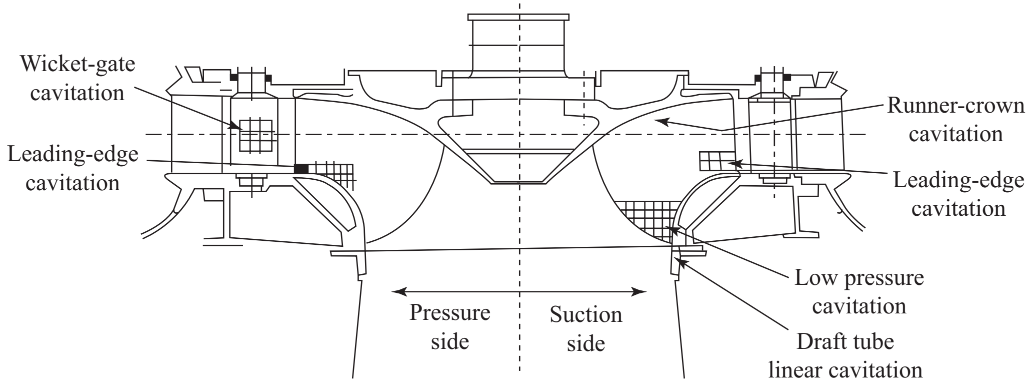
$$\sigma = \left[\frac{N_s}{(N_s)_{\text{suction}}}\right]^{4/5} \tag{5.11}$$

Equation (5.11) relates Thoma's cavitation parameter with suction specific speed. Both these parameters σ and $(N_s)_{suction}$, are used to establish the similarity of cavitation between model and prototype turbines. However, the concept of suction specific speed is generally used in the case of pumps.

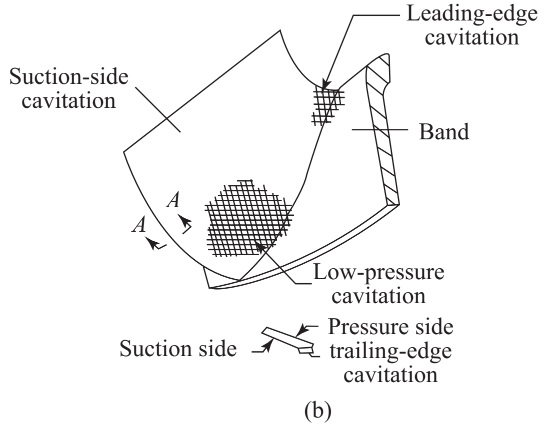
5.2.3 Locations Susceptible to Cavitation

In a Pelton turbine, some of the locations where cavitation phenomenon and damage to the surface occur are: (a) the tip of the needle, (b) inner surface of the nozzle, (c) seal rings in the nozzle, and (d) buckets. In the needle, streak like corrugations appear which affect the performance of the needle. In the buckets, the erosion may result in faulty splitter geometry and hence lowers the efficiency of the wheel. Mostly, imperfections in the geometrical boundaries are responsible for the cavitation in these locations.

The presence of silt in the water is sometimes responsible to cavitation inception. The silt particles erode the surfaces in the high velocity zone due to abrasion. Consequently, these surfaces become rough or uneven losing their smooth finish. The uneven surfaces may initiate cavitation and consequent damage to the surface in the neighborhood of cavitation source.



(a)



(b)

Figure 5.4 *Susceptible Locations of Cavitation in Francis Turbine*

For each rotating element in a reaction turbine, there will be a low pressure side called suction side and the other high pressure side called pressure side. Cavitation can occur on both suction and pressure side in a reaction turbine.

The major locations that are susceptible to cavitation damage in a Francis turbine are listed below and shown in Figure 5.4:

(i) Runner
- Leading edge of blade near band on suction side
- Leading edge of blade near band on pressure side
- Crown and at air vents in crown

(ii) Discharge rings (Opposite runner band)
(iii) Draft tube liner (under bottom of band; below band)
(iv) Wicket gates (on the side of wicket gates)

A Kaplan turbine is highly susceptible to cavitation because of its high specific speed and low head. The critical Thoma's cavitation parameter is high, being of the order of 1.0 for a specific speed of about 700.

Therefore, safeguard against cavitation is an important basis in the design and installation of Kaplan turbine. Possible locations of cavitation in a Kaplan turbine are listed and shown in Figure 5.5:

(i) Suction side of blade from centerline to trailing edge
(ii) Leading edge of blade on suction side
(iii) Leading edge of blade on pressure side
(iv) Trailing edge of blade on pressure side
(v) Blade periphery on suction side
(vi) Hub
(vii) Discharge ring
(viii) Draft tube liner

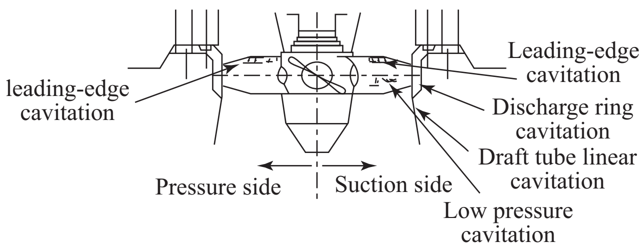


Figure 5.5 *Susceptible Locations of Cavitation in Kaplan Turbine*

EXAMPLE 5.1 A Francis turbine operating under a net head of 20 m is tentatively proposed to be setup with a draft head of 2 m at a certain location. The local atmospheric and vapour pressure heads are 10 m and 0.18 m respectively. The critical Thoma's cavitation parameter for the given turbine can be taken as 0.4. Find (a) whether there is cavitation susceptibility, (b) maximum height of installation of the turbine above the tailrace if a safety margin of 0.5 m for the draft height is mandatory?

Solution

Given: $H = 20$ m, $Z_2 = 2$ m, $H_a = 10$ m, $H_v = 0.18$ m, $\sigma_c = 0.4$, $M = 0.5$ m

Maximum draft head for a turbine setting is given by,

$$(Z_2)_{\max} = H_a - H_v - \sigma_c H \quad (1)$$

$$(Z_2)_{max} = 10 - 0.18 - 0.4 \times 20$$

$$(Z_2)_{max} = 1.82 \text{ m} \quad (2)$$

(a) Cavitation Suspectibility

Proposed turbine setting $Z_2 = 2.0$ m. Since, $Z_2 > (Z_2)_{max}$, therefore, proposed turbine is susceptible to cavitation.

(b) Maximum Height of Installation

Maximum permissible draft height at the plant

$$Z_2 = (Z_2)_{max} - M \quad (3)$$

$$Z_2 = 1.82 - 0.5$$

$$Z_2 = 1.32 \text{ m} \quad (4)$$

EXAMPLE 5.2 A turbine with $\sigma_c = 0.1$ is to be installed at a location where the barometric pressure is 1 bar, the summer temperature 40°C, and the net head available is 50 m. Calculate the maximum permissible height of the turbine rotor above the tailrace.

Solution

Given: $\sigma_c = 0.1$, $p_a = 1$ bar, $T_a = 40$°C, $H = 50$ m

From steam table, at 40°C, $p_v = 0.07375$ bar. σ must at least be equal to σ_c so as to avoid cavitation. The maximum permissible height of the turbine above the tailrace, i.e. the maximum draft head for a turbine setting can be obtained by,

$$(Z_2)_{max} = p_a/\rho g - p_v/\rho g - \sigma_c H \quad (1)$$

$$(Z_2)_{max} = \frac{1 \times 10^5}{1000 \times 9.81} - \frac{0.07375 \times 10^5}{1000 \times 9.81} - 0.1 \times 50$$

$$(Z_2)_{max} = 4.44 \text{ m} \quad (2)$$

EXAMPLE 5.3 A Francis turbine running at 120 rpm produces 11.76 MW while operating under a head of 25 m. The atmospheric pressure is 10 m of water at the site of installation of the turbine and the vapour pressure is 0.20 m of water. Calculate the maximum height of straight draft tube for the turbine.

Solution

Given: $N = 120$ rpm, $P = 11.76$ MW $= 11760$ kW, $H = 25$ m, $H_a = 10$ m, $H_v = 0.20$

We know that specific speed of a turbine is given by,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \quad (1)$$

$$N_s = \frac{120 \times \sqrt{11760}}{25^{5/4}} \Rightarrow N_s = 232.8 \quad (2)$$

Critical Thoma's cavitation parameter for a Francis runner is given by,

$$\sigma_c = 0.044\left(\frac{N_s}{100}\right)^2 \quad (3)$$

$$\sigma_c = 0.044\left(\frac{232.8}{100}\right)^2$$

$$\sigma_c = 0.238 \quad (4)$$

Maximum draft head for a turbine setting is given by,

$$(Z_2)_{\max} = H_a - H_v - \sigma_c H \quad (5)$$

$$(Z_2)_{\max} = 10 - 0.20 - 0.238 \times 25$$

$$(Z_2)_{\max} = 3.85 \text{ m} \quad (6)$$

EXAMPLE 5.4 A Kaplan turbine of specific speed 600 is required to generate 60 MW at 50 Hz in hydroelectric power station. The overall efficiency of the turbine is 90% when the available head is 20 m. Find the maximum safe elevation of the runner to avoid cavitation taking vapour pressure as 0.26 m of water and the critical value of Thoma's cavitation factor as 0.7. Tests on a 1:8 scale model of this turbine are conducted in a laboratory where the available head is 10 m. What will be the power developed and elevation of the model turbine above sump in the laboratory?

Solution

Given: N_s = 600, P_p = 60 MW = 60000 W, f = 50 Hz, η_o = 90% = 0.9, H_p = 20 m, H_v = 0.26 m, σ_c = 0.7, $\frac{D_m}{D_p} = \frac{1}{8}$, H_m = 10 m

(a) Maximum Elevation of the Runner to avoid Cavitation

Assuming standard atmospheric pressure head, H_a = 10.3 m

$$(Z_2)_{\max} = H_a - H_v - \sigma_c H \quad (1)$$

$$(Z_2)_{\max} = 10.3 - 0.26 - 0.7 \times 20$$

$$(Z_2)_{\max} = -3.96 \text{ m} \quad (2)$$

The negative sign shows that the turbine is to be installed below the tailrace.

(b) Elevation of Model to avoid Cavitation

For cavitation similarity between model and prototype, $(\sigma_c)_m = (\sigma_c)_p = 0.7$

$$[(Z_2)_{\max}]_{\text{model}} = H_a - H_v - (\sigma_c)_m H_m \Rightarrow [(Z_2)_{\max}]_{\text{model}} = 10.3 - 0.26 - 0.7 \times 10$$

$$[(Z_2)_{\max}]_{\text{model}} = 3.04 \text{ m} \quad (3)$$

EXAMPLE 5.5 The model of a hydraulic turbine tested in the laboratory at a head of 20 m just began to cavitate when the suction head was raised to 7 m. Its optimum hydraulic efficiency was 82%. The prototype turbine is required to work at a head of 50 m. Determine the safe working setting for the turbine above the tailrace. Atmospheric conditions in the laboratory and site are given below:

	Atmospheric Pressure	*Vapour Pressure Corresponding to Water Temperature*
At site	1×10^5 N/m^2	0.035×10^5 N/m^2
In laboratory	1.012×10^5 N/m^2	0.033×10^5 N/m^2

Solution

Given: H_m = 20 m, $(Z_2)_{max,\, m}$ = 7 m, η_o = 82% = 0.82, H_p = 50 m, $p_{a,\, p} = 1 \times 10^5$ N/m^2, $p_{v,\, p} = 0.035 \times 10^5$ N/m^2 $p_{a,\, m} = 1.012 \times 10^5$ N/m^2, $p_{v,\, m} = 0.033 \times 10^5$ N/m^2

For similarity between model and prototype,

$$(\sigma_c)_m = (\sigma_c)_p \Rightarrow \left[\frac{p_a/\rho g - p_v/\rho g - Z_2}{H}\right]_m = \left[\frac{p_a/\rho g - p_v/\rho g - Z_2}{H}\right]_p \quad (1)$$

$$\left[\frac{1.012\times10^5/1000\times9.81 - 0.033\times10^5/1000\times9.81 - 7}{20}\right]$$

$$= \left[\frac{1\times10^5/1000\times9.81 - 0.035\times10^5/1000\times9.81 - Z_{2,p}}{50}\right]$$

$$Z_{2,\, p} = 2.387 \text{ m} \quad (2)$$

The maximum safe setting of the turbine above the tailrace is 2.387 m.

EXAMPLE 5.6 Calculate the value of Thoma's cavitation parameter for a Kaplan turbine having non-dimensional specific speed of 19.36 and non-dimensional suction specific speed of 17.6.

Solution

Given: $N_{sh} = 19.36$, $(N_{sh})_{\text{suction}} = 17.6$

The non dimensional specific speed of a turbine is given by,

$$N_{sh} = 9.61 \times 10^{-4} N_s \quad (1)$$

$$\therefore \quad 19.36 = 9.61 \times 10^{-4} \times N_s \Rightarrow N_s = 20145.68158 \quad (2)$$

From Eq. (1), suction specific speed of a turbine is,

$$(N_s)_{\text{suction}} = \frac{(N_{sh})_{\text{suction}}}{9.61\times10^{-4}} \Rightarrow (N_s)_{\text{suction}} = \frac{17.6}{9.61\times10^{-4}}$$

$$(N_s)_{\text{suction}} = 1.831426 \times 10^4 \quad (3)$$

We know that,

$$\sigma = \left[\frac{N_s}{(N_s)_{\text{suction}}}\right]$$

$$\sigma = \frac{20145.682}{1.83\times10^4} \quad (4)$$

$$\sigma = 1.1 \quad (5)$$

5.3 Cavitation in Pumps

Refer to general pumping system as shown in Figure 4.12. In a centrifugal pump, the suction side of the pump or inlet of the impeller and blade passages is most susceptible to cavitation. Low pressure regions of blade tips and junction of the blades with the shroud at the inlet are the common spots where the effects of cavitation can be seen. In a vanned diffuser, the inlet region of the vanes is also susceptible.

5.3.1 Thoma's Cavitation Factor

Refer to general pumping system shown in Figure 4.12. The pressure is minimum at the inlet of the pump impeller. Applying the Bernoulli's equation between the surface of liquid in the sump and inlet of the impeller,

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + h_s + \frac{C_1^2}{2g} + h_{fs} \tag{5.12}$$

where,

- p_a Pressure at the free surface in the sump
- p_1 Pressure at the pump inlet,
- C_1 Fluid velocity in the suction pipe at the pump inlet,
- h_s Suction head i.e. Vertical distance between free surface of liquid level in the sump and pump centre line (suction lift),
- h_{fs} Friction head loss in suction pipe,

The term h_{fs} must therefore include the loss of head resulting by liquid passing through strainers and non-return valves fitted to the suction pipes in addition to losses occurred by 'ordinary' friction and by bends in pipe.

For a pump running under given conditions, $C_1^2/2g$ may be considered as a certain proportion of the head developed by the pumps, say $\sigma_c H_m$, then rearranging Eq. (5.12),

$$\sigma_c = \frac{p_a/\rho g - p_1/\rho g - h_s - h_{fs}}{H_m} \tag{5.13}$$

For cavitation not to occur, $p_1 > p_v$, the vapour pressure of the liquid, i.e. $\sigma > \sigma_c$, where,

$$\sigma = \frac{p_a/\rho g - p_v/\rho g - h_s - h_{fs}}{H_m} \tag{5.14}$$

σ is known as the Thoma cavitation parameter and σ_c is critical cavitation parameter (critical Thoma cavitation parameter). Thoma cavitation coefficient is a function of manometric head which varies with discharge for a given speed of the pump. When a pump is tested at a constant speed with fixed discharge and at lower inlet heads, it amounts to testing the pump at lower values of Thoma cavitation coefficient. Figure 5.6 shows the efficiency versus σ plot from the results of such a pump test.

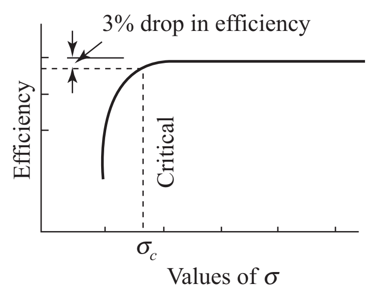


Figure 5.6 *Efficiency vs Cavitation Factor Plot*

Consider the curve from right to left to study the variation of efficiency with decreasing value of σ. With decrease in σ, efficiency is approximately constant or gradually decreases until a certain value σ_c (critical cavitation coefficient) is reached. For $\sigma < \sigma_c$, the efficiency suddenly drops due to cavitation. The minimum value of σ at which incipient cavitation may be easily seen, is fixed by the criteria of 3% drop in efficiency. Hence, this point represents the least value of σ, known as critical cavitation coefficient or critical Thoma coefficient, which can be permitted in the pump without serious cavitation effects. σ_c is a characteristic of a particular pump. Thus, every pump will have a different σ_c. It is noteworthy that, (i) If $\sigma \geq \sigma_c$, there will be no cavitation, and (ii) If $\sigma < \sigma_c$, the pump will suffer cavitation problem. In order that σ should be as large as possible, suction lift h_s must be as small as possible. It may necessitate placing the pump below the liquid level of the sump in some installations, i.e. with negative value of h_s to eliminate cavitation.

Conditions in axial flow pumps are even more favourable for cavitation as compared to those in centrifugal pumps. Equation (5.14) is not applicable for axial flow pumps and more complicated analysis is required as liquid does not enter at a single radius in axial flow machines. The critical cavitation parameter being unique to a pump depends upon its specific speed. An empirical relation between σ_c and pump specific speed for a single suction pump suggested by Shepherd, is as follows:

$$\sigma_c = \frac{N_s^{4/3}}{825} \tag{5.15}$$

Equation (5.15) is valid for specific speed in the range of 10-240 and thus, it covers radial, mixed flow and axial flow pumps. The value of critical Thoma cavitation coefficient for a pump is generally obtained by model tests and is specified by pump manufacturer, either directly as σ_c or as *NPSHR*. If it is not specified, then Eq. (5.15) is used to compute the value of σ_c. The change in atmospheric pressure with altitude should be considered in the major pump installations in mountainous areas. The vapour pressure, p_v, is a function of temperature of the liquid. It increases with increase in the temperature of the liquid.

5.3.2 Net Positive Suction Head (NPSH)

(a) Net Positive Suction Head available (NPSHA)

Net positive suction head in case of a pump is defined as the available suction head (including both static and dynamic heads, i.e. stagnation head) at inlet of the pump above the vapour pressure head. It can be regarded as measure of margin for a pump against vapourization of the liquid and hence, susceptibility of the pump to cavitation.

$$\text{NPSHA} = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} - \frac{p_v}{\rho g} \tag{5.16}$$

Application of Bernoulli's equation between the free surface of the liquid level in the sump and the pump centre line yields,

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + h_s + h_{fs} \Rightarrow \frac{p_1}{\rho g} + \frac{C_1^2}{2g} = \frac{p_a}{\rho g} - h_s - h_{fs} \tag{5.17}$$

Therefore, by Eq. (5.12) or (5.17) and (5.16),

$$\text{NPSHA} = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{fs} = H_a - H_v - h_s - h_{fs} \tag{5.18}$$

NPSHA is a function of pump setting height and pump operating characteristics. From Eq. (5.14) and (5.18), Thoma cavitation coefficient,

$$\sigma = \frac{\text{NPSHA}}{H_m} = \frac{p_a/\rho g - p_v/\rho g - h_s - h_{fs}}{H_m} = \frac{H_a - H_v - h_s - h_{fs}}{H_m} \tag{5.19}$$

(b) Net Positive Suction Head Required (NPSHR)

The least value of σ at which incipient cavitation may be easily seen, is fixed by the criteria of 3% drop in efficiency or head at constant discharge. Net positive suction head corresponding to critical cavitation coefficient is called NPSHR or $NPSH_{min}$. NPSHR is defined as the suction head (including both static and dynamic heads) required at inlet of the pump above the vapour pressure head to obtain satisfactory head (i.e. not greater than 3% decrease in efficiency or head at constant discharge) to prevent excessive cavitation. Therefore,

$$\sigma_c = \frac{\text{NPSHR}}{H_m} \Rightarrow \text{NPSHR} = \sigma_c H_m \tag{5.20}$$

NPSHR, a pump characteristic, is determined by the pump manufacturer through the experiments and the results are represented as the variation of NPSHR vs head in $H - Q$ pump performance chart. It should be noted that NPSHR is specified by the manufacturer, whereas NPSHA is dependent on pump setting and pump operating characteristics.

(c) Margin

The setting of the pump relative to liquid level in the sump should be such as to obtain NPSHR of the pump with adequate factor of safety through the provision of a margin. The difference between NPSHA and NPSHR is known as the margin M which is given by,

$$M = \text{NPSHA} - \text{NPSHR} \tag{5.21}$$

The provision of margin is to account for unforeseen conditions always. Margin is kept about 10% of NPSHR or 0.5 – 1.0 m depending up on the type of liquid and type of the pump. It is noteworthy that:

(i) If NPSHA > NPSHR, the pump setting is OK,
(ii) If NPSHA < NPSHR, the pump setting is not OK and cavitation will be a problem,
(iii) Always provide adequate margin as a factor of safety.

The objective of the pump setting is to have maximum NPSHA at a given location by providing larger margin to minimize the possibilities of cavitation. To maximize NPSHA,

- Decrease the suction lift,
- Reduce the head loss due to friction by (i) reducing suction pipe length, (ii) increasing the suction pipe diameter, and (iii) minimizing the number of bends, tees and valves in the suction pipe.
- Increasing the ambient pressure if possible,
- Lowering the operating temperature of liquid wherever applicable thereby reducing vapour pressure.

(d) NPSH Plots

Manufacturers test their pumps for cavitation in a test facility by varying discharge and inlet pressure in a controlled manner. Specifically, at a given discharge and liquid temperature, the pressure at inlet is slowly decreased until cavitation occurs somewhere inside the pump. The value of NPSHA is calculated using Eq. (5.16) and is noted at this operating condition. This process is repeated at several other values of discharge and the manufacturer then publishes NPSHR. The measured NPSHR varies with discharge, and therefore NPSHR is often plotted on the same pump performance curve as net head as shown in Figure 5.7.

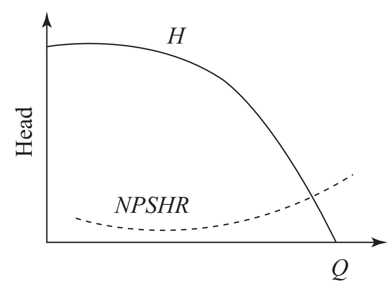


Figure 5.7 *Head and NPSHR vs Discharge Plot for a Pump*

When NPSHR is expressed in terms of head of the liquid being pumped, the NPSHR is independent of the type of liquid. However, if NPSHR is expressed for a particular liquid in pressure units such as P_a, one must be careful to convert this pressure to equivalent column height of the actual liquid being pumped. NPSHR increases with discharge, although for some pumps it decreases with discharge first at low flow rates where the pump is not operating very efficiently, as shown in Figure 5.7.

In order to ensure that pump should not cavitate, NPSHA > NPSHR. It should be noted that NPSHA varies not only with discharge, but also with temperature of the liquid since vapour pressure is a function of temperature. NPSHA also depends on the type of liquid being pumped, since there is a unique p_v versus T curve for each liquid. Since, head losses through the piping system upstream of the inlet increases with discharge, the pump inlet stagnation pressure head decreases with discharge. Therefore, the value of NPSHA decrease with discharge. The maximum discharge that can be delivered by the pump without cavitation is the discharge at the intersection point of NPSHA and NPSHR, as shown in Figure 5.8.

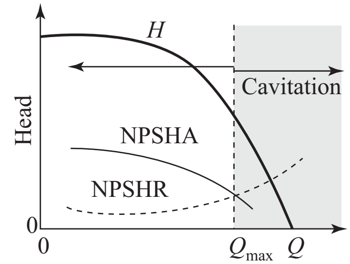


Figure 5.8 *Plot to represent Maximum Discharge of a Pump without Cavitation*

(e) Effect of Cavitation on Pump Performance

Figure 5.9 shows the variation of head and discharge of a pump subjected to cavitation. The ordinate is the head and efficiency, both as percentage of design value. The abscissa is the non-dimensional discharge, ratio of discharge to the design discharge (Q/Q_d). The full curves are the operating characteristics without cavitation. The dotted lines show the performance of the pump when cavitation is induced in the pump. The mismatch of efficiency and large loss of head when pump is cavitating is clearly shown. It is seen that at the inception of cavitation, there is a sudden drop in the head and in efficiency. This decrease in the operating head at cavitation is made use of defining the inception of cavitation. A criterion commonly used is to identify the point at which there is 3% reduction in the manometric head at the point corresponding to the incipient cavitation.

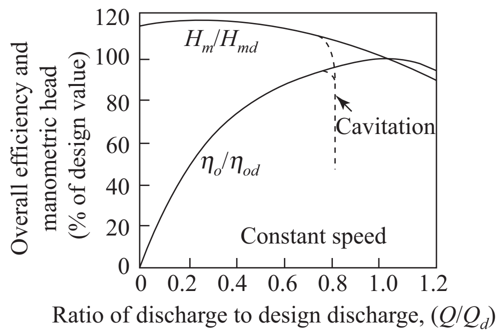


Figure 5.9 *Effect of Cavitation on Performance of a Pump*

5.3.3 Suction Specific Speed

In addition to Thoma's cavitation parameter, suction specific speed is another cavitation parameter which is commonly used in the case of pumps to indicate whether the cavitation will occur. The *suction specific speed* may be defined as the speed of a geometrically similar pump which would deliver unit discharge when the available net positive suction head is 1 m. According to this definition, suction specific speed may be expressed by substituting the manometric head H_m in the expression of specific speed by NPSHA. Therefore,

$$(N_s)_{\text{suction}} = \frac{N\sqrt{Q}}{\text{NPSHA}^{3/4}} \tag{5.22}$$

Substituting the value of NPSHA from Eq. (5.13) into Eq. (5.16), we get,

$$\sigma = \left[\frac{N_s}{(N_s)_{\text{suction}}}\right]^{4/3} \tag{5.23}$$

For cavitation free flow in most of the centrifugal and propeller pumps, limiting value of the suction specific speed have been found in the range of 4700 – 6700 when it is expressed in [rpm/$(l/s)^{1/2}/m^{3/4}$].

By having the same value of Thoma's cavitation parameter, σ, or the suction specific speed $(N_s)_{suction}$, for the model and prototype, the similarity in respect of cavitation is achieved. However, for the pumps, suction specific speed is a more pertinent cavitation parameter for achieving the similarity between the model and prototype in respect of cavitation. This is so because although σ and $(N_s)_{suction}$ depend on the inlet flow conditions, σ also depends on the specific speed N_s of the pump. A constant value of σ can be achieved only if in addition to inlet flow conditions, N_s is constant and only then it will ensure similar cavitation conditions. On the other hand, $(N_s)_{suction}$ is independent of the specific speed of the pump. Therefore, a constant value of $(N_s)_{suction}$ alone will ensure similarity of cavitation in the model and prototype. For example, if specific speed of the pump is changed by changing the external diameter of the impeller keeping the inlet flow conditions unchanged, it can be assumed that cavitation conditions remain the same. The suction specific speed $(N_s)_{suction}$ will not change since the inlet flow conditions remain the same, however, σ will change since its value depends on the head H and hence on the external diameter of the impeller. As such, even if the value of σ is not the same, as long as $(N_s)_{suction}$ is same for the model and prototype, similarity of cavitation can be ensured.

5.3.4 Locations Susceptible to Cavitation

The possible locations for cavitation in a centrifugal pump are listed as follows:

(i) The suction end of the pump,
(ii) Blade passages,
(iii) The low pressure regions of vane tips,
(iv) The junction area of the vanes with shroud at inlet,
(iv) In a vanned diffuser, the inlet region of vanes

In an axial flow pump, cavitation generally starts on the back of blade tips since the pressure is minimum there. However, separation of the flow from a blade may give rise to cavitation at other radii. The minimum pressure in recently developed 'super-cavitating' machines generally has lesser values. Therefore, cavitation takes place not in the form of small bubbles which collapse violently on the blade surfaces, but in the form of large bubbles which are carried away from the surfaces. A super-cavitating machine for a particular discharge may be made smaller without a limiting condition on the speed which cavitation imposes normally. Also, limiting conditions on the installing of the machine are less critical because a lower minimum pressure is permissible. However, a super-cavitating machine has a lower efficiency than the conventional one. Therefore, these machines are preferred in specialized applications, where the advantages override the decrease of efficiency.

5.4 Limitations of Affinity Laws

Any effects of fluid viscosity or retention period effects of vapor bubbles are not considered in affinity or model testing laws. Any cavitation parameter should include a Reynolds number term to consider the viscosity effects. The problem involves further complications in pumps than in turbines because pumps handle a wide variety of fluids having very dissimilar characteristics, ranging from Newtonian to time-dependent viscoelastic fluids. Therefore, the physical properties of the fluid may have an intense consequence on cavitation. Material of construction and the surface finish are common parameters in pumps and turbines.

The two fluid characteristics that are not considered generally in the analysis of cavitation are:

(i) Viscosity

(ii) Compressibility

Fluids with viscosities significantly greater than those of water must have Reynolds number dependent cavitation phenomenon. Besides this, fluids with high viscosity do not have a fixed single-vapor pressure. The rapid collapse of a vapor bubble is also a function of the retention time of a bubble in a low-pressure zone. The retention time of vapour bubble will increase with the size of machine. The fluid velocities will increase linearly, and if bubble growth is proportional to time, then similarity will be achieved. However, this is not true due to the effect of surface tension. Therefore, fluid viscosity and surface tension both have an effect on bubble growth.

Compressibility is another parameter that was not considered. Compressibility seems unimportant as pumps deal with essentially incompressible fluids. This is true only in bubble collapse zones of a pump, not true in other zones of a pump. The study of compressibility leads to the conclusion that pressures exerted by rapid collapse of bubbles increase according to the first power of velocities for fluids of constant acoustic velocity. Application of affinity laws reveal that rapid collapse of bubbles should increase according to the square of fluid velocities.

5.5 Methods to Avoid Cavitation

The following methods are used to avoid cavitation in hydraulic turbines.

(a) Installation of Turbine below Tailrace

The propeller turbine runners which find applications in low head and high specific speed installations are more susceptible to cavitation as the value of $(C_1^2 - C_2^2)/2g$ would be considerably large due to high velocity at the outlet of the turbine. In order to keep the value of pressure at the exit of the runner (p_2) within the permissible limit of cavitation, the value of Z_2 is found negative representing that the turbine runner be placed below the tailrace. For such installations, the turbines always remain under water. The difficulty in inspection and maintenance of the turbine are chief drawbacks of this method.

(b) Cavitation Free Design

The problem of cavitation is avoided by designing the proper shape of the blade, blade angle and thickness of the blade of the runner of a turbine.

(c) Use of Material

The effect of cavitation can be decreased by choosing a more resistant material. Cast steel, stainless steel and alloy steel for runner are in order of increasing resistance to cavitation. Welded parts have been found more resistant to cavitation than casted one.

(d) Polishing

Cavitation effect is minimized by using polished surface. Coatings of stainless steel to the runners of cast steel are used to reduce cavitation.

(e) Selection of Specific Speed

It is possible to avoid cavitation by selecting a runner of appropriate specific speed.

5.6 Damage due to Cavitation

The alternate formation and instantaneously collapse of vapour bubbles in cavitation phenomenon may be repeated with a frequency of many thousand times a second. The extremely high pressures, even though exerted for only infinitesimal time (microsecond) over a very small area can cause intense damage to the surface. The material finally fails by fatigue, supported perhaps by corrosion; therefore, resulting in badly scored and pitted surface. Parts of the surface may even be ripped away completely.

In addition to material damage, the cavitation phenomenon is also accompanied by a characteristic noise and vibration of the enclosing boundary. Sound such as gravel were passing through the machine can be heard when cavitation occurs in a turbine or pump. The milky appearance of water at the exit of the draft tube is another sign of cavitation in a reaction turbine. Cavitation not only causes great and irreparable damage, the larger vapour cavities may so interrupt the flow that the efficiency of a machine is lowered. The force exerted by water on the turbine blades decreases as the surface of the turbine blades become rough due to pitting action. Consequently, the work done by water on the runner or runner power decreases which results in decrease of efficiency. Everything possible should therefore be done to eliminate cavitation in a turbine, i.e. the pressure of the liquid/water at every point must be ensured greater than the vapour pressure of water at the given temperature.

The main effects of cavitation are:

1. Pitting/torn away damage in the wall region of the component undergoing cavitation.
2. Occurrence of noise and vibration of the component. It may sound as though gravel were passing through the machine. Low frequency noise up to about 10 kHz causes no or low damage, whereas high frequency noise in the range of 10-200 kHz causes significant damage to the parts.
3. Change in the performance of the system; for instance reduction of lift, increase in the drag, drop in the turbo-machine efficiency,
4. Significant reduction in the life of the machinery/equipment due to mechanical damage.
5. Torque fluctuation and vibration of casing and bearing house.

Thus, Cavitation is undesirable in the operation of hydraulic machines and must be avoided in the design and installation of hydro machinery and devices. However, if cavitation is detected in an existing unit, the remedial measures are site specific.

EXAMPLE 5.7 A centrifugal pump delivers water at the rate of 300 litres/s from a sump whose water level is 2 m below the eye. The NPHSR as provided by the manufacturer is 7.6 m. The atmospheric pressure at the site is 1 bar absolute and the vapour pressure at the relevant temperature is 2.4 kPa absolute. The total head loss in the suction pipe is found to be 0.97 m. A margin of 1.98 m is essential. Determine the NPHSA and state the suitability of the installation against the cavitation.

Solution

Given: Q = 300 litres/s = 0.3 m^3/s, h_s = 2 m, NPSHR = 7.6 m, p_a = 1 bar = 10^5 Pa, p_v = 2.4 kPa = 2400 Pa, h_{fs} = 0.97 m, M = 1.98 m

We know that NPSHA is given by,

$$\text{NPSHA} = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{fs}$$

$$\text{NPSHA} = \frac{10^5}{1000 \times 9.81} - \frac{2400}{1000 \times 9.81} - 2 - 0.97 \tag{1}$$

$$\text{NPSHA} = 6.98 \text{ m} \tag{2}$$

Since the NPSHA < NPSHR, the pump will have cavitation problem.

EXAMPLE 5.8 A single stage centrifugal pump has a specific speed of 32 pumps water against a head of 38 m. Find the height of the pump above the sump level so that the NPSHA has a margin of 2 m. The atmospheric pressure and vapour pressure head can be taken as 10.3 m and 0.3 m respectively. The head loss in the suction pipe can be taken as 12% of the suction lift.

Solution

Given: $N_s = 32$, $H_m = 38$ m, $M = 2$ m, $H_a = 10.3$ m, $H_v = 0.3$ m, $h_{fs} = 0.12h_s$

We know that critical cavitation parameter or critical Thoma's coefficient is given by,

$$\sigma_c = \frac{N_s^{4/3}}{825}$$

$$\sigma_c = \frac{32^{4/3}}{825} \quad (1)$$

$$\sigma_c = 0.123 \quad (2)$$

NPSHR is given by,

$$\text{NPSHR} = \sigma_c H_m \quad (3)$$

$$\text{NPSHR} = 0.123 \times 38$$

$$\text{NPSHR} = 4.674 \text{ m} \quad (4)$$

We know that NPSHA is given by,

$$\text{NPSHA} = \text{NPSHR} + M = H_a - H_v - h_s - h_{fs} \quad (5)$$

$$4.674 + 2 = 10.3 - 0.3 - h_s - 0.12h_s$$

$$h_s = 3.78 \text{ m} \quad (6)$$

EXAMPLE 5.9 A centrifugal pump delivers water at the rate of 0.12 m^3/s against a head of 28 m. The critical cavitation parameter for the pump is found to be 0.12. The pump is to be installed at a location where the reading of the barometer is 95 kPa absolute and the vapour pressure is 3.2 kPa absolute. The head loss due to friction in intake pipe is 0.33 m. Calculate, (a) NPSHR, and (b) The maximum permissible elevation above the free surface of the sump at which pump could be installed?

The margin of the pump is 2 m.

Solution

Given: $Q = 0.12$ m^3/s, $H_m = 28$ m, $\sigma_c = 0.12$, $p_a = 95$ kPa = 95000 Pa, $p_v = 3.2$ kPa = 3200 Pa, $h_{fs} = 0.33$ m, $M = 2$ m

(a) NPSHR

The critical Thoma's cavitation parameter is given by,

$$\sigma_c = \frac{\text{NPSHR}}{H_m} \Rightarrow \text{NPSHR} = \sigma_c H_m \quad (1)$$

$$\text{NPSHR} = 0.12 \times 28$$

$$\text{NPSHR} = 3.36 \text{ m} \quad (2)$$

(b) Maximum Possible Suction Lift

We know that,

$$\text{NPSHA} = \text{NPSHR} + M = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{fs}$$

$$3.36 + 2 = \frac{95000}{1000 \times 9.81} - \frac{3200}{1000 \times 9.81} - h_s - 0.33 \quad (3)$$

$$h_s = 3.67 \text{ m} \quad (4)$$

EXAMPLE 5.10 During a laboratory test it is found that appreciable cavitation begins in a pump when the stagnation head at entry is decreased to 3.40 m. The pump delivers 46 litres/s of water against a total head of 37 m across the pump. Barometer reading is 750 mm of Hg and vapour pressure of water is 1.8 kPa. What is the value of cavitation parameter? The pump has to be installed at a different site where the barometric pressure is 620 mm of Hg and temperature is decreased such that corresponding vapour pressure of water is 830 Pa. Calculate the value of the cavitation coefficient if discharge and total head developed by the pump remains the same. Is it essential to decrease the height of the pump at the new site, and if so by how much?

Solution

Given: $\frac{p_1}{\rho g} + \frac{C_1^2}{2g} = 3.40$ m, $Q = 46$ litres/s $= 0.046$ m^3/s, $H_m = 37$ m, $p_a = 750$ mm Hg, $p_v = 1.8$ kPa, $p_a' = 620$ mm Hg, $p_v' = 830$ Pa

where subscripts 1 refers to the state at entry of the pump and superscript ′ refers to the new location.

Since the cavitation just starts when $p_1 = p_v = 1.8$ kPa $= 1800$ Pa

$$\therefore \quad \frac{C_1^2}{2g} = 3.40 - \frac{1800}{1000 \times 9.81}$$

$$\frac{C_1^2}{2g} = 3.22 \text{ m} \quad (1)$$

For a given pump running under given conditions, $C_1^2/2g$ may be considered as a certain proportion of the head developed by the pumps, say $\sigma_c H_m$, where σ_c is the critical Thoma's cavitation parameter. Hence,

$$\sigma_c H_m = \frac{C_1^2}{2g} \Rightarrow \sigma_c \times 37 = 3.22$$

$$\sigma_c = 0.087 \quad (2)$$

This dimensionless parameter will remain same for both the locations as H_m and $C_1^2/2g$ are constant.

$$\because \quad 760 \text{ mm Hg} = 101.325 \text{ kPa}$$

$$\therefore \quad 750 \text{ mm Hg} = \frac{101.325}{760} \times 750 = 99.992 \text{ kPa}$$

Hence, atmospheric pressure, $p_a = 99.992$ kPa $= 99992$ Pa

Applying Bernouli's equation between the free surface of the sump and entry of the pump,

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{C_1^2}{2g} + h_s + h_{fs} \Rightarrow (h_s + h_{fs}) = \frac{p_a}{\rho g} - \frac{p_1}{\rho g} - \frac{C_1^2}{2g}$$

$\because$ $\quad p_1 = p_v$

$$(h_s + h_{fs}) = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - \frac{C_1^2}{2g} \tag{3}$$

$$(h_s + h_{fs}) = \frac{99992}{1000 \times 9.81} - \frac{1.8 \times 1000}{1000 \times 9.81} - 3.22$$

$$(h_s + h_{fs}) = 6.8 \text{ m} \tag{4}$$

For the new location,

$$p'_a = \frac{620}{760} \times 101.325 = 82.66 \text{ kPa} = 82660 \text{ Pa}$$

Hence atmospheric pressure at the new location, p'_a = 82660 Pa

Applying Bernouli's equation between the free surface of the sump and entry of the pump at the new site, we get,

$$(h'_s + h'_{fs}) = \frac{p'_a}{\rho g} - \frac{p'_v}{\rho g} - \frac{C_1^2}{2g} \Rightarrow (h'_s + h'_{fs}) = \frac{82660}{1000 \times 9.81} - \frac{830}{1000 \times 9.81} - 3.22$$

$$(h'_s + h'_{fs}) = 5.12 \text{ m} \tag{5}$$

Total head loss in the suction pipe at both the site are same as discharge is same because $h_f \propto Q^2$.

$\therefore$ $$h_{fs} = h'_{fs} \tag{6}$$

From Eqs (4) and (5), it is clear that at the new site pump has to be lowered by,

$$(h_s - h'_s) = 6.8 - 5.12$$

$$(h_s - h'_s) = 1.68 \text{ m} \tag{7}$$

EXAMPLE 5.11 A centrifugal pump running at 1000 rpm delivers water at the rate of 160 litres/s under a head of 30 m. At the site of installation of the pump, atmospheric pressure is 1 bar (abs) and vapour pressure of water is 3 kPa (abs). The head loss in the suction pipe is equivalent to 0.2 m of water. Calculate (a) Minimum NPSH, and (b) Maximum allowable height of the pump from the free surface of water in the sump.

Solution

Given: N = 1000 rpm, Q = 160 litres/s = 0.16 m^3/s, H_m = 30 m, p_a = 1 bar, p_v = 3 kPa, h_{fs} = 0.2 m

(a) Minimum NPSH or NPSHR

We know that specific speed of the pump is given by,

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}} \quad (1)$$

$$N_s = \frac{1000 \times \sqrt{0.16}}{30^{3/4}}$$

$$N_s = 31.205 \quad (2)$$

Critical Thoma's Cavitation parameter for specific speed in the range of 10–240 is given by,

$$\sigma_c = \frac{N_s^{4/3}}{825} \quad (3)$$

$$\sigma_c = \frac{31.205^{4/3}}{825}$$

$$\sigma_c = 0.1191 \quad (4)$$

Minimum net positive suction head corresponds to critical cavitation coefficient is called NPSHR or NPSH_{min}. It is given by,

$$\text{NPSHR} = \sigma_c H_m \quad (5)$$

$$\text{NPSHR} = 0.1191 \times 30$$

$$\text{NPSHR} = 3.573 \quad (6)$$

(b) Maximum Height of the Pump from Sump Level

Assuming a margin of 0.5 m, NSPHA = NSPHR + M = 3.573 + 0.5 = 4.073 m

$$\text{NPSHA} = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{fs} \quad (7)$$

$$4.073 = \frac{1 \times 10^5}{1000 \times 9.81} - \frac{3000}{1000 \times 9.81} - h_s - 0.2$$

$$h_s = 5.615 \text{ m} \quad (8)$$

EXAMPLE 5.12 A single stage centrifugal pump running at 500 rpm delivers 300 m^3/min of water against a head of 120 m. The diameter of the pump impeller is 2 m and it has a positive suction lift of 3 m including velocity head and friction head. Laboratory tests are to be conducted on a model with 0.45 m impeller diameter and on reduced head of 95 m. Calculate the speed, discharge and suction lift for the laboratory tests. Assume vapour head H_v = 0.34 m and atmospheric head H_a = 10.15 m of water.

Solution

N_p = 500 rpm, Q_p = 300 m^3/min, H_p = 120 m, D_p = 2 m, $h_s + h_{fs}$ = 3 m, D_m = 0.45 m, H_m = 95 m, H_v = 0.34, H_a = 10.15 m

(a) Speed of the Model

Head coefficient for model and prototype should be the same.

$$\psi = \left(\frac{H}{N^2D^2}\right)_m = \left(\frac{H}{N^2D^2}\right)_p \tag{1}$$

$$\frac{95}{N_m^2 \times 0.45^2} = \frac{120}{500^2 \times 2^2}$$

$$N_m = 1977 \text{ rpm} \tag{2}$$

(b) Discharge of the Model

Discharge coefficient for model and prototype should be the same.

$$\varphi = \left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p \tag{3}$$

$$\frac{Q_m}{1977 \times 0.45^3} = \left(\frac{300}{500 \times 2^3}\right)$$

$$Q_m = 13.51 \text{ m}^3\text{/min} \tag{4}$$

(c) Suction Lift of the Model

$$\sigma = \left(\frac{H_a - H_v - h_s - h_{fs}}{H}\right)_m = \left(\frac{H_a - H_v - h_s - h_{fs}}{H}\right)_p \tag{5}$$

$$\frac{10.15 - 0.34 - 3}{120} = \frac{10.15 - 0.34 - (h_s + h_{fs})_m}{95}$$

$$(h_s + h_{fs})_m = 4.418 \text{ m} \tag{6}$$

SUMMARY

- Cavitation is defined as the phenomenon of formation, travel and sudden collapsing of vapour bubbles in a region when the pressure of the liquid drops below its vapour pressure.
- The available suction head inclusive of both static and dynamic heads at exit from the turbine known as NPSH (Net positive suction head), is given by,

$$\text{NPSH} = \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - (Z_2 - h_f) = H_a - H_v - (Z_2 - h_f)$$

- Thoma's cavitation coefficient is used to find the susceptibility of turbines to cavitation which is as follows:

$$\sigma = \frac{p_a/\rho g - p_v/\rho g - Z_2}{H} = \frac{H_a - H_v - Z_2}{H} = \frac{\text{NPSH}}{H}$$

- The maximum elevation of the turbine above the tailrace i.e. the maximum draft head for a turbine setting is,

$$(Z_2)_{max} = p_a/\rho g - p_v/\rho g - \sigma_c H = H_a - H_v - \sigma_c H$$

- The *suction specific speed* is the speed of a geometrically similar turbine which develops 1 kW power under the available net positive suction head (NPSH) of 1 m.

$$(N_s)_{suction} = \frac{N\sqrt{P}}{(\text{NPSH})^{5/4}}$$

- Thoma's cavitation coefficient is used to find the susceptibility of pumps to cavitation which is given below:

$$\sigma = \frac{\text{NPSHA}}{H_m} = \frac{p_a/\rho g - p_v/\rho g - h_s - h_{fs}}{H_m} = \frac{H_a - H_v - h_s - h_{fs}}{H_m}$$

- The critical Thoma's cavitation coefficient is given by,

$$\sigma_c = \frac{\text{NPSHR}}{H_m} = \frac{p_a/\rho g - p_1/\rho g - h_s - h_{fs}}{H_m}$$

- For cavitation not to occur, $p_1 > p_v$, the vapour pressure of the liquid, i.e. $\sigma > \sigma_c$. NPSHR is specified by the manufacturer whereas NPSHA is dependent on pump setting and pump operating characteristics.
- The maximum discharge that can be delivered by the pump without cavitation is the discharge at the intersection point of NPSHA and NPSHR.
- The *suction specific speed* may be defined as the speed of a geometrically similar pump which would deliver unit discharge when the available net positive suction head is 1 m.

$$(N_s)_{suction} = \frac{N\sqrt{Q}}{\text{NPSHA}^{3/4}}$$

$$\sigma = \left[\frac{N_s}{(N_s)_{suction}}\right]^{4/3}$$

REVIEW QUESTIONS

5.1 Define cavitation. What are the effects of cavitation on the performance of a hydraulic turbine?

5.2 Write a short note on cavitation with special reference to a Francis turbine.

5.3 Discuss how cavitation affects the draft head of a draft tube.

5.4 What factors affect the occurrence of cavitation in turbines? If cavitation was to occur, at what locations the cavitation damage is likely to take place?

5.5 Write a brief note on the cavitation problem in axial flow turbines.

5.6 Derive an expression for Thoma's cavitation coefficient and state its significance.

5.7 State the relation between Thoma's cavitation coefficient and critical Thoma's cavitation coefficient for cavitation not to occur in hydraulic turbines.

5.8 Describe briefly cavitation problem in a Pelton turbine.

5.9 Explain how the height of a turbine above the tailrace is limited by cavitation.

5.10 Define suction specific speed of a turbine.

5.11 Deduce a relation between suction specific speed, specific speed and cavitation parameter for a turbine.

5.12 How the possibility of cavitation to occur in a centrifugal pump is determined?

5.13 What are the effects of cavitation on the performance of a centrifugal pump?

5.14 State the relation between Thoma's cavitation coefficient and critical Thoma's cavitation coefficient for cavitation not to occur in a centrifugal pump.

5.15 What are the limitations for installing a centrifugal pump above the liquid surface in the pump?

5.16 Why the suction lift of a pump cannot exceed a certain limit?

5.17 What is NPSHR? How it is related to the cavitation coefficient of the pump?

5.18 Define NPSHR and NPSHA. Explain how these two quantities are used to ensure that cavitation does not occur in a pump.

5.19 Define margin of a centrifugal pump and state its significance?

5.20 Why suction specific speed is a more pertinent cavitation parameter for achieving the cavitation similarity between the model and prototype?

5.21 Define suction specific speed of a pump and state its significance.

5.22 Express Thoma's cavitation parameter in terms of specific speed and suction specific speed for a pump.

5.23 What are the limitations of similarity laws in the study of cavitation?

5.24 What are the various methods to avoid cavitation?

PROBLEMS

5.1 A turbine is provided with a straight conical draft tube. The velocity at the exit of the turbine is 10 m/s and the velocity head at the exit of the draft tube is 1.0 m. The minimum pressure head in the turbine is set at 2.0 m (abs) to avoid cavitation and loss of head in the draft tube is 1.5 m. Estimate the maximum height at which turbine could be set above the tailrace. Assume atmospheric pressure as 10.3 m of water. [*Ans:* $(Z_2)_{max} = 5.704$ m]

5.2 A Francis turbine of design power 2.5 MW under a head of 35 m at a speed of 200 rpm is selected for installation at a site. The atmospheric pressure head at the site is 10.0 m and vapour pressure head is 0.24 m. The empirical relation between critical cavitation parameter σ_e and specific speed of the turbine is given by: $\sigma_c = 0.0317\left(\dfrac{N_s}{100}\right)^2$.

State whether the proposed turbine setting at a draft height of 7.0 m at the site is safe from cavitation? A safety margin of 0.5 m is mandatory in the draft height.

[*Ans:* Z_2 = 7.0 m < $(Z_2)_{max} - M$, Hence safe]

5.3 A turbine whose runner has an exit velocity of 10 m/s is to be fitted with a straight conical draft tube. The loss of head due to friction and other causes in the draft tube and exit velocity head at the tailrace is limited to 2 m. What maximum height of setting would you recommend for the turbine if it is to be free from cavitation? The minimum pressure at the outlet of the turbine runner should be 19.26 kN/m^2. Take atmospheric pressure as 98.1 kN/m^2. [*Ans:* $(Z_2)_{max}$ = 4.91 m]

5.4 A turbine with critical Thoma's cavitation factor of 0.12 is to be installed at a location where the barometric pressure is 1 bar, summer temperature is 40°C, and the net head available is 45 m. Find the maximum permissible height of the turbine rotor above the tailrace. At 40°C, the vapour pressure for water is 0.0737 bar. [*Ans:* $(Z_2)_{max}$ = 4.04 m]

5.5 A Kaplan turbine running against a net head of 20 m has a design power of 40 MW at a speed of 100 rpm. The atmospheric pressure head at the site is 10 m of water and the relevant vapour pressure head is 0.33 m of water. The variation of critical Thoma's cavitation parameter with specific speed for the given turbine is as under:

Specific Speed, (N_s)	400	600	800
Critical Thoma Cavitation Parameter	0.43	0.73	1.50

Calculate the maximum draft head for installation of the turbine at the site if the safety margin is 0.30 m. [*Ans:* $(Z_2)_{max}$ = 0.81 m]

5.6 A Kaplan turbine is having runner diameter of 1.25 m discharges 12 m^3/s. The area at the exit of draft tube is 6 times of that at inlet. The pressure head at entry to the draft tube must not be more than 1.5 m below atmospheric pressure. Estimate the maximum height of installation of the runner relative to the tailrace. The efficiency of the draft tube may be assumed as 70%. [*Ans:* $(Z_2)_{max}$ = 3.317 m]

5.7 The critical cavitation parameter for a turbine of non-dimensional specific speed of 3.5 was found to be 0.9. The runner is located at a depth of 1.5 m relative to tailrace. Calculate the maximum permissible velocity in the draft tube where the working temperature is 45°C and the atmospheric pressure is 10.3 m of water. [*Ans:* $(C_2)_{max}$ = 14.572 m/s]

5.8 A centrifugal pump discharging water has a suction head of 6.2 m. Calculate the NPSHA if the atmospheric pressure and vapour pressure is equivalent to head of 10.2 m and 0.3 m respectively. The friction loss head in the suction pipe is 0.45 m. State whether the installation is safe against cavitation if the NPSHR is 2.9 m and a margin of 1.1 m is mandatory. [*Ans:* NPSHA = 3.25 m, Not safe]

5.9 The NPSHR for a centrifugal pump as provided by manufacturer is 6.2 m. The pump is to be installed to lift water at the rate of 0.3 m^3/s when the water level in the supply reservoir is 1.6 m below the axis of the pump. The atmospheric pressure is at the site is 97 kPa and vapour pressure is 2.35 kPa absolute. The friction head loss in the suction pipe is 1.6 m. Examine whether the proposed pump installation is safe against cavitation if the margin of 1.0 m is mandatory. [*Ans:* Not safe]

5.10 A pump is to be installed at an elevation of 1.5 m below the water level in the supply reservoir. The loss of head due to friction in the suction pipe is 0.7 m. The atmospheric and vapour pressure heads are 10.3 and 0.4 m (absolute) respectively. Calculate the margin if the NPSHR of this pump is 5.5 m. [*Ans:* $M = 2.2$ m]

5.11 Tests on a model pump indicate a critical cavitation parameter to be 0.10. A homologous unit is to be installed at a certain location where atmospheric pressure is 90 kPa, vapour pressure is 3.5 kPa absolute, and is to pump water against a head of 25 m. What is the maximum permissible suction lift? [*Ans:* $(h_s)_{max} = 5.82$ m if $M = 0.5$ m]

MULTIPLE CHOICE QUESTIONS

1. Chances of occurrence of cavitation are high if the
   (a) Local pressure becomes very high
   (b) Local temperature becomes low
   (c) Thoma cavitation parameter exceeds a certain limit
   (d) Local pressure falls below the vapour pressure
2. Cavitation in a hydraulic turbine is most likely to occur at the turbine
   (a) Entry (b) Exit (c) Stator exit (d) Rotor exit
3. In an installation of a centrifugal pump, if the temperature of the liquid in the sump is raised by 20° C, then the
   (a) NPSHR is reduced (b) NPSHR is increased
   (c) NPSHR is increased (d) NPSHR is reduced
4. Consider the following statements:

   Cavitation in hydraulic machines occurs at the
   1. Outlet of the pump
   2. Inlet of the pump
   3. Outlet of a turbine

   Of these statements
   (a) 2 and 3 are correct (b) 1 and 3 are correct
   (c) 1, 2 and 3 are correct (d) 1 and 2 are correct
5. Cavitation in a centrifugal pump will occur if
   (a) It operates above the minimum net positive suction head
   (b) It operates below the minimum net positive suction head
   (c) The pressure at the pump entry is more than atmospheric pressure
   (d) The pressure at the pump entry is equal to the atmospheric pressure
6. The following statement pertains to a centrifugal pump:

   If the pump NPSH requirements are not satisfied, then
   1. Efficiency will be lowered
   2. Cavitation will occur
   3. Sufficient head will not be developed to lift the water
   4. Very low discharge will be delivered

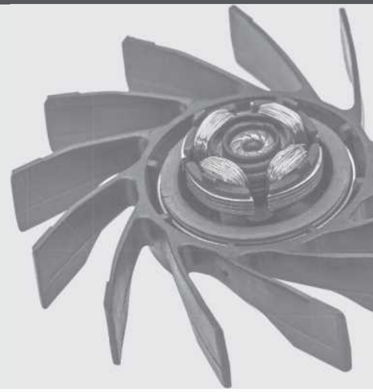
Of these statements

(a) 2 and 3 are correct
(b) 1, 2 and 4 are correct
(c) 1, 3 and 4 are correct
(d) All the 4 statements are correct

7. A centrifugal pump is located at a height of 5.1 m above the free surface of water in the sump. Loss of head due to friction in the suction pipe is 0.7 m. The atmospheric pressure head is 10.3 m of water and the vapour pressure is 0.3 m absolute. The NPSH of the pump is
(a) 3.8 m
(b) 4.0 m
(c) 4.2 m
(d) 4.5 m

ANSWER KEY

1. (d) 2. (d) 3. (d) 4. (a) 5. (b) 6. (a) 7. (c)



6 Fans and Blowers

Learning Objectives

After reading this chapter, you will be able to:

- **LO 1** Outline the differences between fan, blower and compressor and learn the various classification of fans
- **LO 2** Determine the specific work for axial and centrifugal fans using Euler's turbomachinery equation
- **LO 3** Illustrate the velocity triangles and compute stage parameters for an axial fan stage with different guide vanes arrangements and for a centrifugal fan for various blade geometries
- **LO 4** Summarise the performance characteristics of axial and centrifugal fans
- **LO 5** Explain losses, different fan and system arrangements, fan laws and fan noise

6.1 Introduction

Roto-dynamic fans and blowers are the machines which utilize mechanical energy to increase the total pressure of air or gas at certain volume flow rate. Most of these devices (Figure 6.1) are driven by electric motor, thus consuming electric work at their shaft. Other drives such as I.C. engines, water turbines, may also be used in specific applications. In this chapter, we will discuss how these devices are classified, the terminology used to characterise their performance, important design parameters, their losses and their noise.

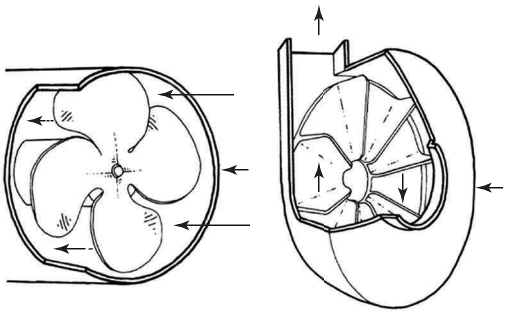


Figure 6.1 *Ducted Fan*

The devices may be ducted or un-ducted. They are termed as fans when the inlet and outlet pressures are close to the ambient and kinetic energy of the fluid or air is of main interest.

6.2 Terminology of a Fan

$$\text{Fan Total Power} = \gamma_w Q h_{tw} \tag{6.1}$$

$$\text{Fan Static Power} = \gamma_w Q h_{sw} \tag{6.2}$$

where Q is the capacity in m^3/s, γ_w is the specific weight of water (gauge fluid) in N/m^3 or kN/m^3, h_{tw} is the total head in m of water gauge (WG) in m, and h_{sw} is the static head in m of water gauge (WG) in m.

Static power is that part of the total air power which is used to produce the change in static head.

$$\text{Fan total efficiency} = \frac{\text{Fan Power (Total)}}{\text{Fan Input Power}} \times 100\% \tag{6.3}$$

$$\text{Fan static efficiency} = \frac{\text{Fan Power (Static)}}{\text{Fan Input Power}} \times 100\% \tag{6.4}$$

6.3 Difference between a Fan, Blower and a Compressor

Air or gas is sucked into a fan or blower or a compressor as the impeller is made to rotate by the drive. As the fluid flows out of the impeller, energy is imparted to the fluid through the impeller blades and the energy of the fluid increases.

Referring to steady flow energy Eq. (1.49), the mechanical energy at the impeller shaft of fan/blower/compressor is used to increase the following forms of fluid energy.

- Kinetic energy
- Internal energy
- Static pressure
- Potential energy

The roto-dynamic machine is called a fan when the primary concern is to increase the kinetic energy of the fluid and all other forms of energy are small or negligible. For example, in a domestic ceiling fan, the comfort by the air velocity is of primary interest. The machine is termed as blower if the rise in fluid energy in both kinetic and static pressure forms are important. An example is a blower supplying air to an air conditioning duct, providing a rise in pressure to overcome various flow resistances and also to provide necessary velocity to the air flow. More often, the velocity of flow is small enough to consider the flow in the fans and blower to be incompressible. The density of fluid is taken as a constant value.

In a compressor, in addition to increase in also kinetic energy and pressure, increase of internal energy is also significant. Thus, change in enthalpy (internal energy + flow work) is of interest for the compressor. The pressure rise is quite high in compressor; the change in pressure is usually expressed by pressure ratio. The density variations of fluid flow are significant; the flow is considered as a compressible flow in the compressors.

EXAMPLE 6.1 Three roto-dynamic ducted devices have the following inlet and outlet conditions. Identify them as fan, blower and compressor. The ambient conditions are 300 K and 1 bar. Take $R = 288$ J/kgK. Calculate the pressure ratio for the compressor.

(a) Static pressure at the inlet is –10 Pa, static pressure at the outlet is 15 Pa and velocity at outlet is 5 m/s.

(b) Static pressure at the inlet is –30 Pa, static pressure at the outlet is 10 kPa and velocity at outlet is 20 m/s.
(c) Static pressure at the inlet is –100 Pa, static pressure at the outlet is 1 MPa and velocity at outlet is 30 m/s.

Solution

The density of ambient air,

$$\rho = \frac{p}{RT} = \frac{10^5}{288 \times 300} = 1.157 \text{ kg/m}^3 \qquad (1)$$

(a) The device is a fan because the magnitude of dynamic head $\rho C^2/2 = (1.157 \times 5^2)/2 = 14.46$ Pa. It is of same order of magnitude as static pressure at the outlet ($p_{\text{outlet}} = 15$ Pa). Note that the velocity is low (5 m/s); static pressure rise is low, equal to 25 Pa.
(b) Dynamic head is $\rho C^2/2 = (1.157 \times 20^2)/2 = 231.4$ Pa and static pressure rise is $(\Delta p)_s = 10000 - (-30) = 10030$ Pa. Therefore, the device is a blower because the static pressure rise is more significant than the increase in kinetic energy, although the velocity is higher than in the part (a) of the example.
(c) Dynamic head is $\rho C^2/2 = (1.157 \times 30^2)/2 = 520.65$ Pa and static pressure rise is $(\Delta p)_s = 1000000 - (-100) = 1000100$ Pa. Therefore, the device is a compressor because both dynamic head and pressure rise are much higher than the previous example, the blower. The inlet absolute pressure is $p_1 + p_{\text{atm}} = 10^5 - 100 = 99900$ Pa. Static pressure ratio of the compressor is $p_{2s}/p_{1s} = 10^6/99900 = 10.01$
Pressure ratio for the compressor = Outlet absolute pressure/Inlet absolute pressure
Pressure ratio for the compressor = $(10^6 + 10^5)/(-100 + 10^5) = 11.01$

6.4 General Layout of a Fan and System

A fan system consists of a fan and a duct system connected near the inlet and the outlet. The duct system can have various components installed within the air pathway such as air control dampers, heat exchangers, filters on the inlets or outlets, diffusers and noise attenuators, as seen in Figure 6.2. The fan drives the air

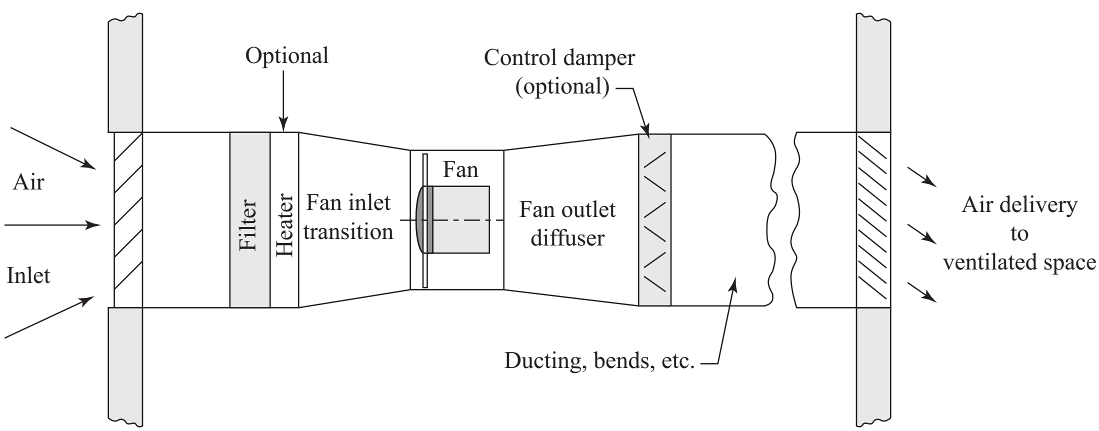


Figure 6.2 *Fan and System*

stream to overcome resistance caused by the ductwork and other components. The ducting directs the air flow to the required locations and also acts as a support and housing for the other components used in the particular process.

Fan Total Pressure = System (total) Pressure Loss,
Fan total pressure is the increase in total pressure (measured by facing Pitot tubes) across the fan.

Fan Static Pressure + Fan Velocity Pressure = System Pressure Loss,
Fan static pressure is the difference between the fan total pressure and fan velocity pressure. Fan velocity pressure is the average velocity pressure at the fan outlet only.

Fan Static Pressure = System Pressure Loss –Fan Velocity Pressure,

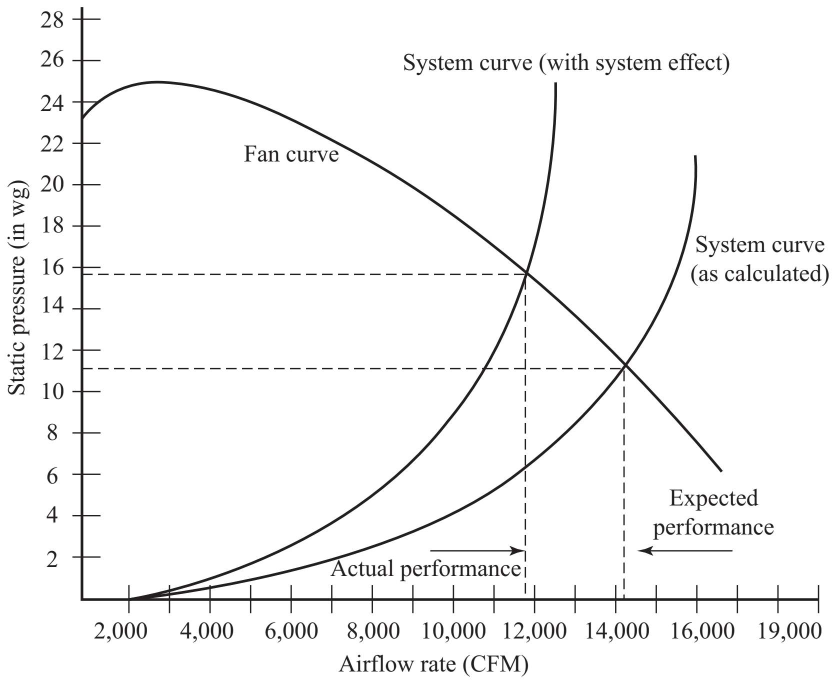


Figure 6.3 *Machine and System Characteristics*

The assumption is often made that the fan velocity pressure is very nearly equal to the system discharge velocity pressure. In calculating system pressure loss, discharge velocity pressure is ignored and the resulting total pressure taken as being the required fan static pressure.

The fan and system static pressure curves are shown in Figure 6.3.

6.5 Classification of Fans

1. On the Basis of Application

(a) Domestic fans which are un-ducted and pressure difference across the fan is negligible. [Figure 6.4(a) and (b)]
(b) Exhaust fans [Figure 6.4(c)]

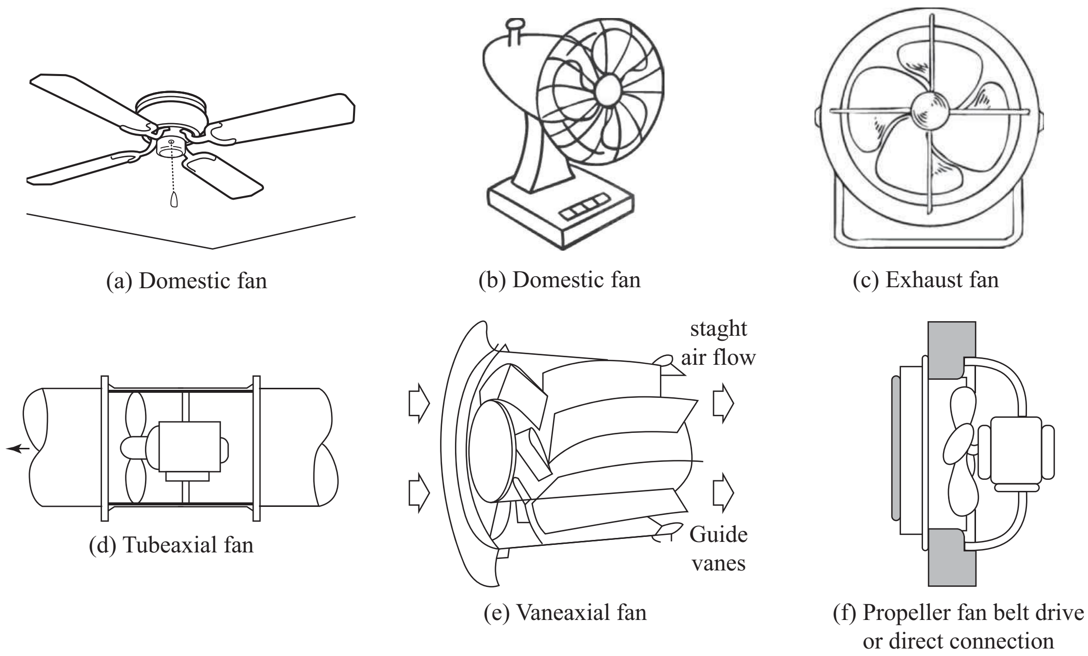


Figure 6.4 *Applications of Fans*

(c) Industrial fans, e.g. cooling tower fans-induced draft fans and forced draft in power plants, etc.
(d) Cooling fans, e.g. automotive radiators [Figure 6.4(d) and (e)]
(e) Propeller fans, e.g. turbofan in an aircraft engine [Figure 6.4(f) and 6.5]

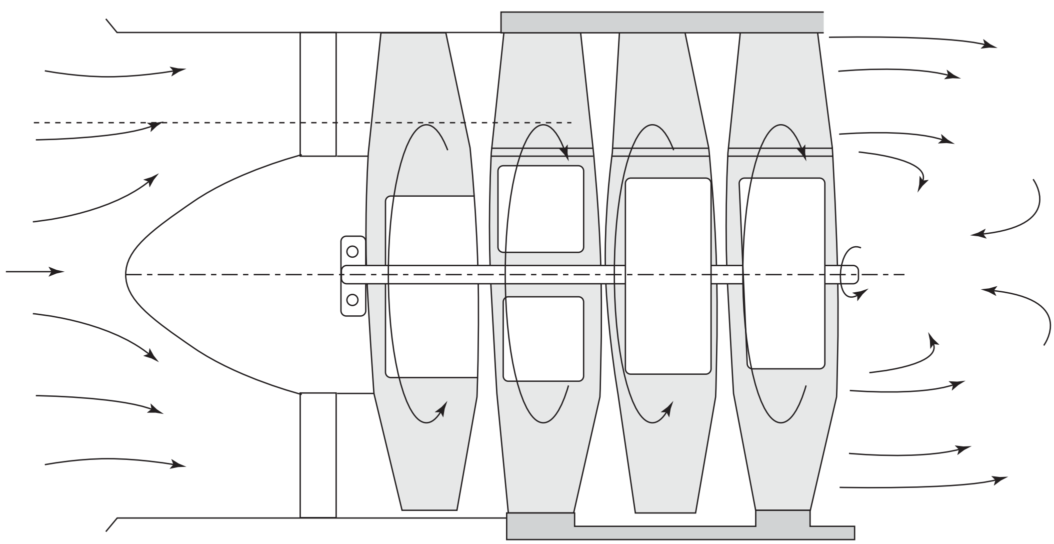


Figure 6.5 *Multistage Fan*

2. On the Basis of Predominant Direction of Fluid Motion through the Impeller

(a) Axial flow fans [Figure 6.6(a)]
(b) Radial flow or centrifugal fans [Figure 6.6(b)]
(c) Mixed flow fans

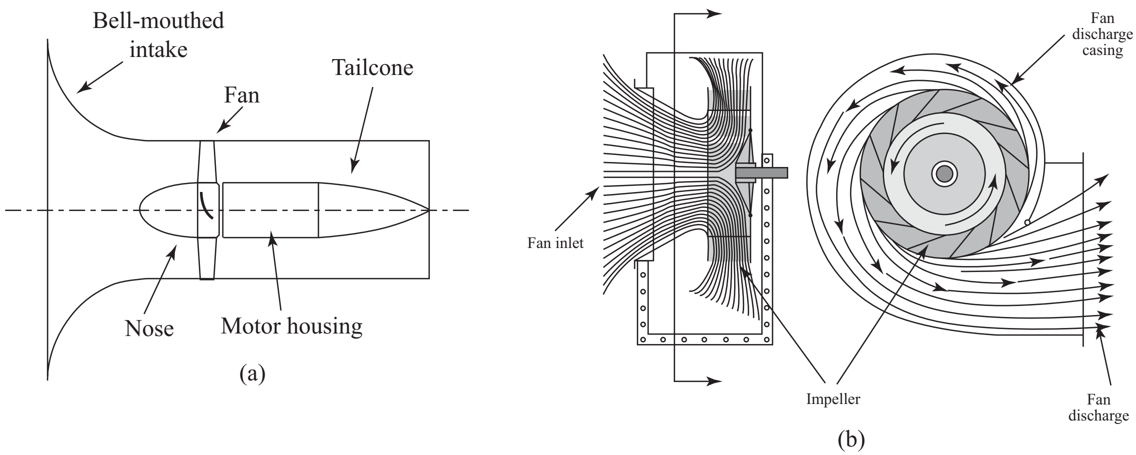


Figure 6.6 *(a) Axial Fan, and (b) Centrifugal Fan*

3. On the Basis of Outlet Blade Angle (Figure 6.7)

(a) **Forward Curved Blade**: Forward-curved blades are curved in the direction of the fan wheel's rotation. These are especially sensitive to particulates. Forward-curved blades provide a low noise level and relatively small air flow with a high increase in static pressure.

(a) Paddle blade (radial blade) (b) Forward curved (multi-vane) (c) Backward curved

Figure 6.7 *Types of Centrifugal Fans*

(b) **Backward Curved Blade:** Backward-curved blades are curved against the direction of the fan wheel's rotation. Smaller blowers may have backward-inclined blades, which are straight, not curved. Larger backward inclined/curved blowers have blades whose backward curvatures mimic that of an aerofoil cross section, but both designs provide good operating efficiency with relatively economical construction techniques. These types of blowers are designed to handle gas streams with low to moderate particulate loadings. They can be easily fitted with wear protection but certain blade curvatures can be prone to solids build-up. Backward curved wheels are often heavier than corresponding forward curved equivalents as they run at higher speeds and require stronger construction.

Backward curved fans can have a high range of specific speeds but are most often used for medium specific speed applications—high pressure, medium flow applications. Backward curved fans are much more energy efficient than radial blade fans, hence, for high power applications, may be a suitable alternative to the lower cost radial bladed fan.

(c) **Aerofoil Backward Curved Blade**

(d) **Radial Tipped Blade:** Radial blowers have wheels whose blades extend straight out from the centre of the hub. Radial bladed wheels are often used on particulate-laden gas streams because they are the least sensitive to solid build-up on the blades, but they are often characterized by greater noise output. High speeds, low volumes and high pressures are common with radial blowers, and are often used in vacuum cleaners, pneumatic material conveying systems, and similar processes.

4. On the Basis of Degree of Reaction (R)

(a) Low degree of reaction fan (R < 1)

(b) High degree of reaction fan (R > 1)

5. On the Basis of Specific Speed (Figure 6.8)

(a) Low specific speed fans—which produce high pressures at low flow volume

(b) High specific speed fans—which produce lower pressures at high flow volume

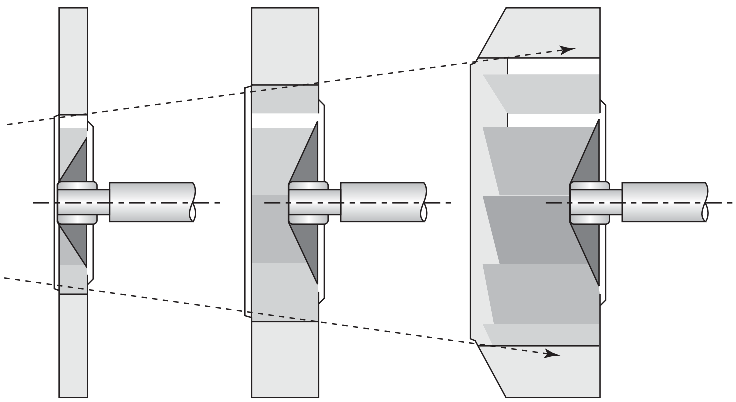


Figure 6.8 *Fans Based on Specific Speed*

6.6 Specific Work from Euler's Turbomachinery Equation

Refer Euler's Eq. (1.82) to the power absorbing machine,

$$w = C_{b2}C_{w2} - C_{b1}C_{w1}$$

In Figure 6.10 (a), for axial flow machines at the midsection, blade velocities at the inlet and the outlet at midsection are equal, i.e. $C_{b1} = C_{b2} = C_b$. However, for centrifugal fan, blade velocity at the outlet (i.e. at the outer radius of impeller) is larger than the velocity at the inlet (i.e. at the inner radius of impeller), i.e. $C_{b2} > C_{b1}$.

Therefore, for centrifugal fan, specific work is equal to,

$$w = C_{b2}C_{w2} - C_{b1}C_{w1} \tag{6.5}$$

For axial fan,

$$w = C_b(C_{w2} - C_{w1}) \quad (6.6)$$

From the blade velocity triangles (Figure 1.19) at the inlet and the outlet, the specific work for a centrifugal fan is given by,

$$w = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \quad (6.7)$$

Since $C_{b1} = C_{b2} = C_b$ for an axial fan, therefore, specific work is,

$$w = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \quad (6.8)$$

Applying steady flow energy Eq. (1.49) to a fan which is an incompressible flow machine, we have,

$$w = \frac{(p_2 - p_1)}{\rho} + \frac{(C_2^2 - C_1^2)}{2} \quad (6.9)$$

Comparing Eqs. (6.7) and (6.9), we get,

$$\frac{(p_2 - p_1)}{\rho} = \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \quad (6.10)$$

Equation (6.10) represents that the static pressure rise across the fan comprises two parts. First is the centrifugal part $(C_{b2}^2 - C_{b1}^2)/2$ and the second is the relative kinetic energy part, $(C_{r1}^2 - C_{r2}^2)/2$.

Recall that in the case of centrifugal machines, the pressure rise across the impeller is given only by first term, i.e. $\omega(r_2^2 - r_1^2)/2$, neglecting the second term as $C_{r1} \approx C_{r2}$. The pressure rise for axial flow machines is given by $(C_{r1}^2 - C_{r2}^2)/2$.

EXAMPLE 6.2 A fan discharges air of density 1.15 kg/m^3 at the rate of 1 m^3/s. The suction end pipe diameter is 250 mm and the pressure end pipe diameter is 300 mm. The pressures measured at the suction and pressure ends of the blower are –30 mm *WC* and 300 mm *WC* respectively. Calculate the mass of the air discharge in kg/s and the specific work in kJ/kg. Neglect the height difference between the suction and pressure end centre lines. Also, neglect all the losses.

Solution

Suction end area of cross section,

$$A_s = \frac{\pi}{4} \times 0.25^2 = 0.0491 \text{ m}^2 \quad (1)$$

Pressure end area of cross section,

$$A_p = \frac{\pi}{4} \times 0.3^2 = 0.071 \text{ m}^2 \quad (2)$$

$$p_{\text{net}} = p_p - p_s = 300 - (-30) = 330 \text{ mm of } WC$$

$$p_{net} = \frac{(330 \times 1000 \times 10^{-3})}{1.15} = 286.96 \text{ m of air column}$$

$$p_{net} = 286.96 \times 9.81 \times 1.15 = 3237.34 \text{ Pa} \quad (4)$$

Suction end velocity,

$$C_s = Q/A_s = 1/0.0491 = 20.37 \text{ m/s} \quad (5)$$

Pressure end velocity,

$$C_p = Q/A_p = 1/0.071 = 14.08 \text{ m/s} \quad (6)$$

From the steady flow energy equation by taking the fluid as control volume,

$$\dot{W} = \dot{Q} + \dot{m}\left[(h_s - h_p) + \frac{(C_s^2 - C_p^2)}{2} + g(Z_s - Z_p)\right] \quad (7)$$

Since fan is an incompressible flow machine, i.e. $\rho_1 = \rho_2 = \rho$ treating the flow to be adiabatic, i.e. $\dot{Q}$ = 0 and neglecting internal energy change, i.e. $(u_s - u_p) = 0$, and potential energy change $g(Z_s - Z_p) = 0$, Eq. (7) results,

$$w = -\left[\frac{(p_p - p_s)}{\rho} + \frac{(C_p^2 - C_s^2)}{2}\right] \quad (8)$$

$$w = -\left[\frac{3237.34}{1.15} + \frac{(14.08^2 - 20.37^2)}{2}\right]$$

$$w = -2706.733 \text{ J/kg} \quad (9)$$

Specific work is negative, because work is done on the fluid.
Mass of air discharged,

$$\dot{m}_a = \rho Q = 1.15 \times 1$$

$$\dot{m}_a = 1.15 \text{ kg/s} \quad (10)$$

EXAMPLE 6.3 A centrifugal fan has the following data:

Inner diameter of the impeller = 80 mm
Outer diameter of the impeller = 200 mm
Speed = 1450 rpm
The relative and absolute velocities, respectively, are
At entry = 20 m/s, 21 m/s
At exit = 17 m/s, 25 m/s
Flow rate = 0.5 kg/s
Motor efficiency = 78%

Determine (a) the stage pressure rise, (b) degree of reaction, and (c) power required to drive the fan. Take air density as 1.25 kg/m^3

Solution

$$C_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.08 \times 1450}{60} = 6.074 \text{ m/s}$$

$$C_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.20 \times 1450}{60} = 15.184 \text{ m/s}$$

$$\frac{(C_{b2}^2 - C_{b1}^2)}{2} = \frac{(15.184^2 - 6.074^2)}{2} = 96.83 \text{ J/kg} \quad (1)$$

$$\frac{(C_{r1}^2 - C_{r2}^2)}{2} = \frac{(20^2 - 17^2)}{2} = 55.5 \text{ J/kg} \quad (2)$$

$$\frac{(C_2^2 - C_1^2)}{2} = \frac{(25^2 - 21^2)}{2} = 92 \text{ J/kg} \quad (3)$$

The static pressure rise in the rotor is,

$$(\Delta p)_{\text{rotor}} = (p_2 - p_1) = \rho\left[\frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2}\right] \quad (4)$$

$$(\Delta p)_{\text{rotor}} = 1.25(96.83 + 55.5) = 190.4125 \text{ Pa (N/m}^2) \quad (5)$$

(a) Stage Pressure Rise

Referring Eqs (1.84) and (1.98), the total pressure rise across the stage is,

$$(\Delta p_0)_{\text{stage}} = \rho w = \rho\left[\frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2}\right] \quad (6)$$

$$(\Delta p_0)_{\text{stage}} = 1.25(92 + 96.83 + 55.5)$$

$$(\Delta p_0)_{\text{stage}} = 305.4125 \text{ Pa (N/m}^2) = 305.4125/9.81 \text{ mm WG}$$

$$(\Delta p_0)_{\text{stage}} = 305.4125 \text{ Pa (N/m}^2) = 31.133 \text{ mm WG} \quad (7)$$

(b) Degree of reaction

$$R = \frac{(\Delta p)_{\text{rotor}}}{(\Delta p)_{\text{stage}}} \quad (8)$$

$$R = \frac{190.4125}{305.4125} = 0.6233 \quad (9)$$

(c) Power Required to Drive the Fan

Specific stage work is,

$$w = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \quad (10)$$

$$w = 92 + 96.83 + 55.5 = 244.33 \text{ J/kg} \quad (11)$$

$$\eta_m = \frac{\text{Stage Power}}{\text{Motor Power}} = \frac{\dot{m}w_{\text{stage}}}{P} \quad (12)$$

$$0.78 = \frac{0.5 \times 244.33}{P}$$

$$P = 156.622 \text{ W} \quad (13)$$

6.7 Axial Fans

The predominant flow in the axial fans is in the direction parallel to the axis while flowing through the impeller. This can be viewed by the streamlines drawn in the meridional view as shown in Figure 6.9. In addition to the axial, there may be another component of velocity. For example, a significant vortex action may be imparted to the air due to tangential component. The characteristics of axial fans depend largely upon aerodynamic design, number of impeller blades and angle of attack. They are compact and can be easily combined to series and parallel configurations. The tip speed of axial fans is high and hence they tend to be noisy. For high flow resistance, they also tend to stall.

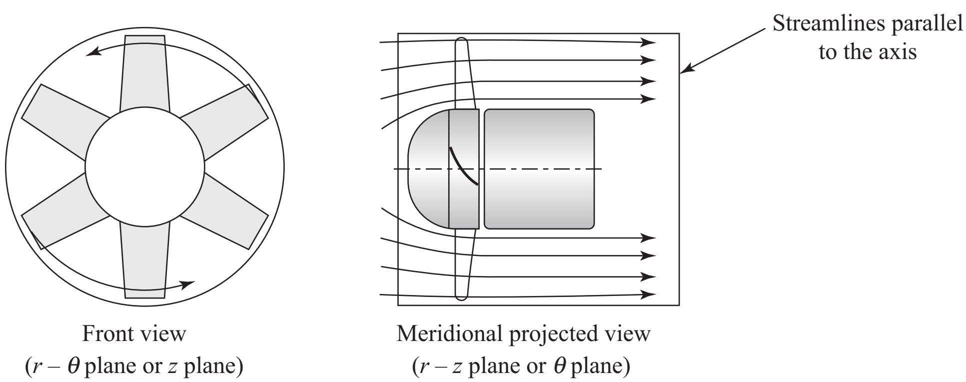
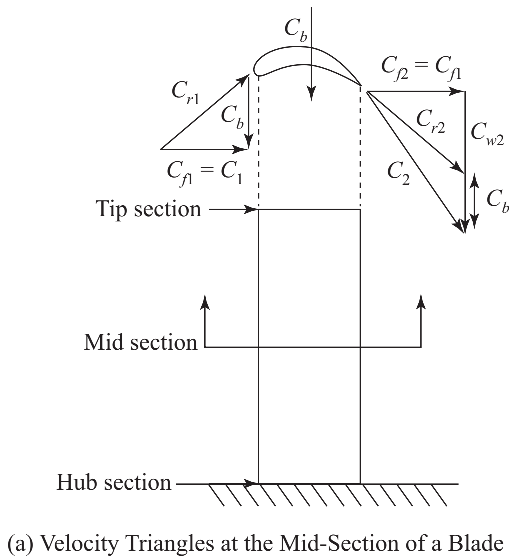


Figure 6.9 *Front and Meridional Views of an Axial Fan*

6.7.1 Velocity Triangles

Referring to Section 1.5, where the construction of velocity triangles is explained, for an axial fan, velocity at the inlet is assumed to be axial. In other words, the meridional component or the velocity of flow is parallel to the axis. In general, the value of axial velocity is assumed constant even if the fan has several stages. At any given section between the hub and tip, the blade velocity is constant at the inlet and the outlet. Considering these aspects, the inlet and outlet velocity triangles of axial fan at the midsection of rotor blades are shown in Figure 6.10 (a). The superposed velocity triangles with C_b as common base are shown in Figure 6.10 (b).



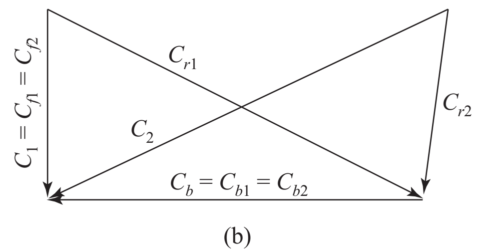


Figure 6.10 *Velocity Triangles of an Axial Fan (a) At the Mid-Section of a Blade, (b) Superimposed with C_b as common base*

6.8 Axial Fan Stage Parameters

An axial fan in its simplest form has an impeller with or without a casing or duct as noticed in Figures 6.1 and 6.2. However, fans can be of multi stage as in Figure 6.5. A fan stage consists of not only impeller (rotor) but also a stationary row of vanes for guiding the flow and for energy conversion of the fluid.

As the fan impeller is made to rotate by the drive, air is sucked into it. This inlet section is called suction end. As the fluid flows out of the impeller, pressure of the fluid is increased. This outlet section is called pressure end. In order to guide the flow to the desired axial direction, a set of stator vanes are placed in the downstream of the rotor. This rotor and stator is together called stage of fan. Figure 6.11 shows the cascade view of the stage.

6.8.1 Stage with Upstream Guide Vanes

(a) Velocity Triangles

Velocity triangles at the suction and pressure ends of the impeller are shown in Figure 6.12. We assume that the flow approaches and leaves the blades with uniform velocities and the flow is congruent to the blades. The following vector relation can be written as,

$$C = C_b + C_r$$

where C is the absolute velocity (velocity observed with references to stationary frame), C_r is the relative velocity (velocity observed with references to rotating frame) and C_b is the linear speed ($C_b = \omega r$). Denoting the inlet, outlet of rotor with suffixes 1 and 2 and the inlet of stator is suffixed with 0. C_0, C_{f0} are the inlet absolute velocity and axial component of absolute velocity at the upstream guide vanes respectively. C_{r1}, α_1, β_1, C_1, C_{f1} and C_{w1} are the inlet relative velocity, inlet flow angle, inlet blade angle, inlet absolute

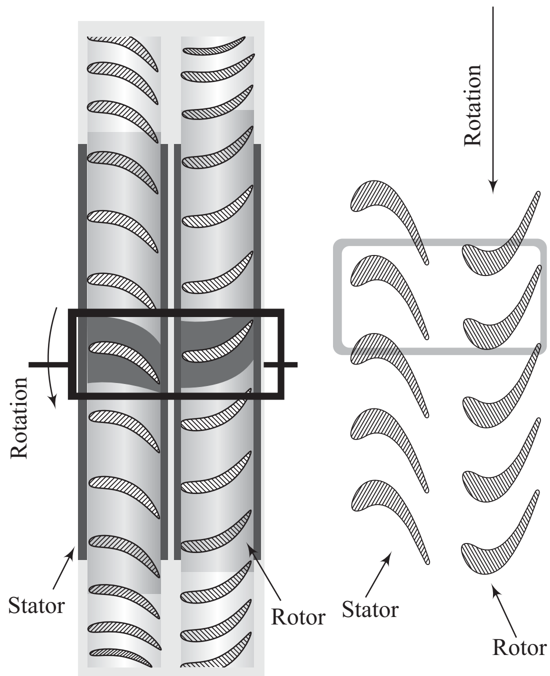


Figure 6.11 *Cascade View of an Axial Stage*

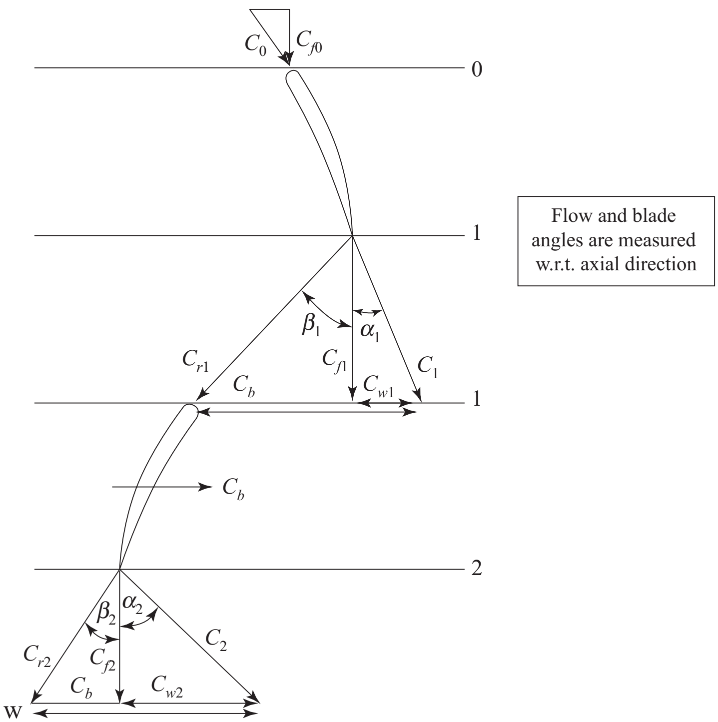


Figure 6.12 *Velocity Triangles of Fan with Guide Vanes*

velocity, axial component of inlet absolute velocity and inlet tangential component of absolute velocity at the rotor, respectively. C_{r2}, α_2, β_2, C_2, C_{f2} and C_{w2} are the outlet relative velocity, outlet flow angle, outlet blade angle, outlet absolute velocity, outlet axial component of absolute velocity and outlet tangential component of absolute velocity at the rotor, respectively.

We assume that the flow approaches the rotor smoothly and hence, the relative velocity vector is tangential at the blade leading edges as shown in inlet of cascade. We also assume that the flow leaves the rotor smoothly and hence, the relative velocity vector is tangential at the blade trailing edges as shown in outlet of cascade.

(b) Mass Flow Rate

Mass flow through the axial fan can be estimated by knowing the meridional component of velocity, i.e., axial velocity as follows:

$$\dot{m} = \rho \frac{\pi}{4} D_1^2 C_{f1} = \rho \frac{\pi}{4} D_2^2 C_{f2} \tag{6.11}$$

(c) Stage Work

Velocity triangles for axial fan stage with guide vanes are shown in Figure 6.12. Since for axial fans, $C_{b1} = C_{b2} = C_b$, therefore, from Euler's Eq. (1.82), specific stage work is given by,

$$w = C_b(C_{w2} - C_{w1}) \tag{6.12}$$

In an ideal case with perfect deflection and adiabatic flow, the entire input to the rotor must appear as the stagnation enthalpy rise in air or gas,

$$(\Delta h_0)_{\text{stage}} = C_b(C_{w2} - C_{w1}) \tag{6.13}$$

From velocity triangles in Figure 6.12,

$$C_{w2} = C_b - C_{f2} \tan \beta_2 \tag{6.14}$$

Dividing above Eq. (6.14) by C_b, we get,

$$\frac{C_{w2}}{C_b} = 1 - \frac{C_{f2} \tan \beta_2}{C_b} \tag{6.15}$$

We know that the flow coefficient is,

$$\varphi = \frac{C_{f2}}{C_b} \tag{6.16}$$

Using Eq. (6.16) in Eq. (6.15),

$$\frac{C_{w2}}{C_b} = 1 - \varphi \tan \beta_2 \tag{6.17}$$

From velocity triangles in Figure 6.12,

$$C_{w1} = C_b - C_{f1} \tan \beta_1 \tag{6.18}$$

$$\frac{C_{w1}}{C_b} = 1 - \frac{C_{f1} \tan \beta_1}{C_b} \tag{6.19}$$

We know that the flow coefficient is,

$$\varphi = \frac{C_{f1}}{C_b} \tag{6.20}$$

Using Eq. (6.20) in Eq. (6.19),

$$\frac{C_{w1}}{C_b} = 1 - \varphi \tan \beta_1 \tag{6.21}$$

Rearranging Eq. (6.21),

$$C_{w1} = C_b(1 - \varphi \tan \beta_1) \tag{6.22}$$

Using Eqs. (6.17) and (6.22) in Eqs. (6.12) and (6.13), stage work is given by,

$$w = (\Delta h_0)_{\text{stage}} = C_b(C_{w2} - C_{w1}) = C_b^2 \varphi(\tan \beta_1 - \tan \beta_2) \tag{6.23}$$

(d) Stage Pressure Rise

For isentropic flow and incompressible flow,

As $$w = (\Delta h_0)_{\text{stage}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho} \tag{6.24}$$

Therefore, stagnation pressure rise in the stage is given by,

$$(\Delta p_0)_{\text{stage}} = \rho w_{\text{stage}} = \rho(\Delta h_0)_{\text{stage}} = \rho C_b(C_{w2} - C_{w1}) \tag{6.25}$$

$$(\Delta p_0)_{\text{stage}} = \rho C_b^2 \varphi(\tan \beta_1 - \tan \beta_2) \tag{6.26}$$

$$(\Delta p)_{\text{rotor}} = \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} \tag{6.27}$$

$$(\Delta p)_{\text{stator}} = \frac{\rho(C_2^2 - C_1^2)}{2} \tag{6.28}$$

(e) Stage Pressure Coefficient

Stage pressure coefficient for the rotor is given by,

$$\Psi = \frac{(\Delta p_0)_{\text{stage}}}{0.5 \rho C_b^2} \tag{6.29}$$

Using Eq. (6.26) in Eq. (6.29), we get,

$$\Psi = 2\varphi(\tan \beta_1 - \tan \beta_2) \tag{6.30}$$

(f) Degree of Reaction

It is defined as the ratio of static pressure rise in the rotor and stagnation rise over the stage.

$$R = \frac{(\Delta p)_{\text{rotor}}}{(\Delta p_0)_{\text{stage}}} \tag{6.31}$$

We assume constant axial velocity and from velocity triangles in Figure. 6.12,

$$C_{r1}^2 - C_{r2}^2 = [C_{f1}^2 + (C_b - C_{w1})^2 - C_{f2}^2 - (C_b - C_{w2})^2]$$

$$C_{r1}^2 - C_{r2}^2 = C_{f1}^2 + C_b^2 + C_{w1}^2 - 2C_bC_{w1} - C_b^2 - C_{f2}^2 - C_{w2}^2 + 2C_{w2}C_b$$

$$C_{r1}^2 - C_{r2}^2 = C_{w1}^2 - C_{w2}^2 - 2C_b(C_{w1} - C_{w2}) \tag{6.32}$$

Using Eq. (6.32) in Eq. (6.27), we get,

$$(\Delta p)_{\text{rotor}} = \rho \frac{[C_{w1}^2 - C_{w2}^2 - 2C_b(C_{w1} - C_{w2})]}{2} \tag{6.33}$$

Substituting Eqs. (6.25) and (6.33) in Eq. (6.31),

$$R = \frac{[C_{w1}^2 - C_{w2}^2 - 2C_b(C_{w1} - C_{w2})]}{[2C_b(C_{w2} - C_{w1})]} \tag{6.34}$$

(g) Fan Efficiencies

Isentropic work input is always less than actual work input. The actual air power is lower than ideal air power. The actual air power $\rho Q(\Delta h_0)_{\text{stage}}$ and the ideal air power (Euler's work) is given by, $\rho QC_b(C_{w2} - C_{w1})$,

$$\rho Q(\Delta h_0)_{\text{stage}} < \rho QC_b(C_{w2} - C_{w1})$$

$$\frac{(\Delta p_0)_{\text{stage}}}{\rho} < C_b(C_{w2} - C_{w1})$$

Fan efficiency is the ratio of isentropic work to the actual work input or it is the ratio of the actual air power to ideal air power.

$$\eta_{f\,\text{total}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho C_b(C_{w2} - C_{w1})} \tag{6.35}$$

Actual power input to the stage is,

$$P = \dot{m}C_b(C_{w2} - C_{w1}) = Q\frac{[\rho C_b^2 \varphi(\tan \beta_1 - \tan \beta_2)]}{\eta_{f\,\text{total}}} \tag{6.36}$$

where $\dot{m}$ and Q are the air mass flow rate and volume flow rate, respectively.

(i) Overall Efficiency

Taking mechanical and electrical efficiency of the drive as η_d, the overall efficiency is,

$$\eta_0 = \eta_d \eta_{f\,\text{total}} \tag{6.37}$$

(ii) Volumetric Efficiency

From Figure 6.13, the volumetric efficiency is defined as the ratio of flow rates delivered and entering the impeller of the fan.

$$\eta_v = \frac{Q}{Q'} = \frac{Q}{Q + \Delta Q} \tag{6.38}$$

where Q is the air flow rate delivered to the fan, Q' is the air flow rate entering the impeller and ΔQ is the leakage due to $\Delta p = p_e - p_i$, here i is the inlet to fan and e is the exit to fan.

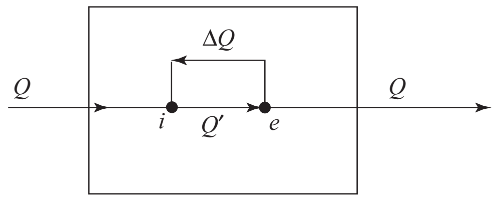


Figure 6.13 *Representation of Volumetric Efficiency*

6.8.2 Stage without Guide Vanes

(a) Velocity Triangles

The general velocity triangles at the inlet (Figure 6.14) and outlet (Figure 6.15), for the stage without guide vanes, are drawn in Figure 6.16. The inlet and outlet are denoted with suffixes 1 and 2. C_{r1}, β_1, C_1 and C_{f1} are the inlet relative velocity, inlet blade angle, inlet absolute velocity and axial component of inlet absolute velocity, respectively. Similarly, C_{r2}, α_2, β_2, C_2 C_{f2} and C_{w2} are the outlet relative velocity, outlet flow angle, outlet blade angle, outlet absolute velocity, outlet axial component of absolute velocity and outlet tangential component of absolute velocity, respectively.

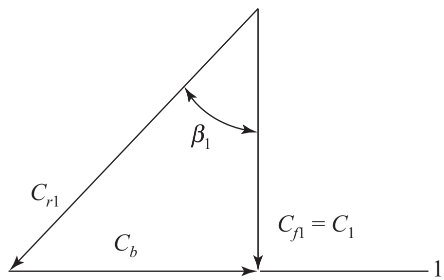


Figure 6.14 *Inlet Velocity Triangle of a Fan without Guide Vane*

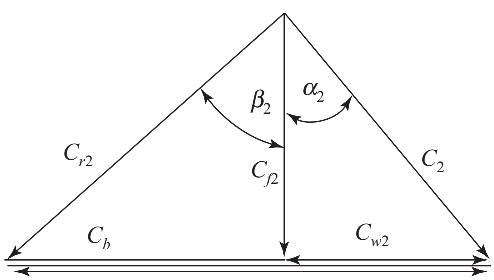


Figure 6.15 *Outlet Velocity Triangle of a Fan without Guide Vane*

(b) Stage Work

Velocity triangles for axial fan stage without guide vanes are shown in Figure 6.16.

From Euler's Eq. (1.82), stage work is given by,

$$w = C_b(C_{w2} - C_{w1}) \tag{6.39}$$

In an ideal case with perfect deflection and adiabatic flow, the entire input to the rotor must appear as the stagnation enthalpy rise in air or gas,

$$(\Delta h_0)\text{stage} = w = C_b(C_{w2} - C_{w1}) = \frac{(\Delta p_0)_{\text{stage}}}{\rho} \tag{6.40}$$

For no guide vanes, if $C_{w1} = 0$,

$$w = C_b C_{w2} \tag{6.41}$$

From velocity triangle as shown in Figure 6.16,

$$C_{w2} = C_b - C_f \tan \beta_2 \tag{6.42}$$

Dividing Eq. (6.42) by C_b, we get,

$$\frac{C_{w2}}{C_b} = 1 - \frac{C_f \tan \beta_2}{C_b} \tag{6.43}$$

We know that the flow coefficient is,

$$\varphi = \frac{C_f}{C_b} \tag{6.44}$$

Using Eq. (6.44) in Eq. (6.43),

$$C_{w2} = C_b(1 - \varphi \tan \beta_2 \tag{6.45}$$

Substituting Eq. (6.45) in Eq. (6.39) or Eq. (6.41), we get,

$$w = C_b^2(1 - \varphi \tan \beta_2) \tag{6.46}$$

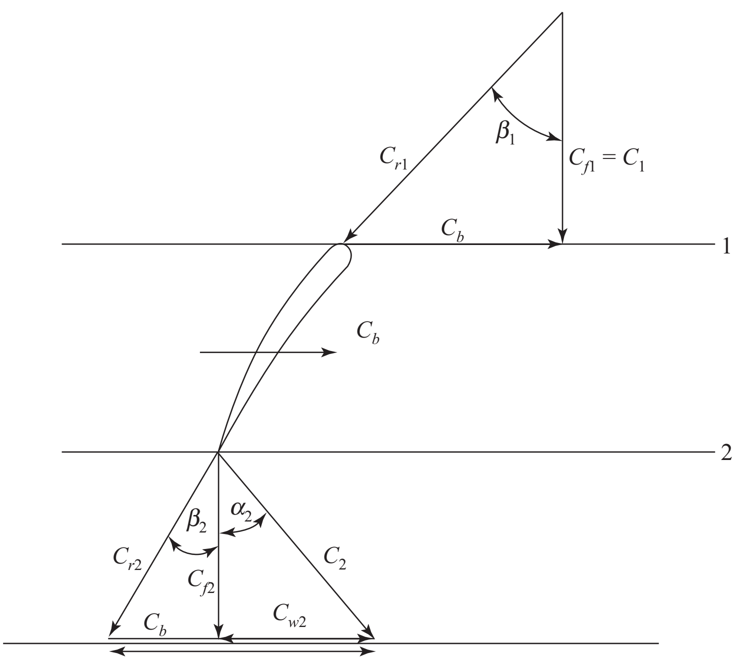


Figure 6.16 *Velocity Triangles of a Fan without Guide Vanes*

(c) Stage Pressure Rise

For isentropic and incompressible flow,

$$(\Delta h_0)_{\text{stage}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho} \tag{6.47}$$

Total change in pressure in the rotor and fixed blade rings,

$$(\Delta p_0)_{\text{stage}} = \rho C_b (C_{w2} - C_{w1}) \tag{6.48}$$

Here, only rotor is considered for the stage. Therefore, the static pressure rise of stage is same as that of the rotor.

$$(\Delta p)_{\text{stage}} = (\Delta p)_{\text{rotor}} = \frac{\rho (C_{r1}^2 - C_{r2}^2)}{2} \tag{6.49}$$

We assume constant axial velocity and from velocity triangles in Figure. 6.16,

$$\begin{aligned} C_{r1}^2 - C_{r2}^2 &= C_{f1}^2 + C_b^2 - [C_{f2}^2 + (C_b - C_{w2})^2] \\ C_{r1}^2 - C_{r2}^2 &= C_{f1}^2 + C_b^2 - C_{f2}^2 - C_b^2 - C_{w2}^2 + 2C_{w2}C_b \\ C_{r1}^2 - C_{r2}^2 &= 2C_{w2}C_b - C_{w2}^2 \end{aligned} \tag{6.50}$$

Using Eq. (6.50) in Eq. (6.49), we get,

$$(\Delta p)_{\text{stage}} = (\Delta p)_{\text{rotor}} = \rho C_b C_{w2} - \rho \frac{C_{w2}^2}{2}$$

Rearranging the above equation,

$$(\Delta p)_{\text{stage}} = \rho C_b^2 \left[\frac{C_{w2}}{C_b} - \frac{1}{C_b^2} \frac{(C_{w2})^2}{2} \right] \tag{6.51}$$

Using Eq. (6.45) in Eq. (6.51), static pressure rise in the rotor or stage is,

$$(\Delta p)_{\text{stage}} = (\Delta p)_{\text{rotor}} = \rho C_b^2 \left[1 - \varphi \tan \beta_2 - \frac{(1 - \varphi \tan \beta_2)^2}{2} \right] \tag{6.52}$$

Rearranging Eq. (6.52), we get,

$$(\Delta p)_{\text{stage}} = (\Delta p)_{\text{rotor}} = \rho \frac{C_b^2}{2} (1 - \varphi^2 \tan^2 \beta_2) \tag{6.53}$$

(d) Stage Pressure Coefficient

Stage pressure coefficient for the rotor is given by,

$$\Psi = \frac{(\Delta p_0)_{\text{stage}}}{0.5 \rho C_b^2} \tag{6.54}$$

Using Eq. (6.53) in Eq. (6.54),

$$\psi_{stage} = \psi_{rotor} = (1 - \varphi^2 \tan^2 \beta_2) \tag{6.55}$$

(e) Stage Reaction

It is defined as the ratio of static pressure rise in the rotor and stagnation rise over the stage.

$$R = \frac{(\Delta p)_{rotor}}{(\Delta p_0)_{stage}}$$

$$(\Delta p)_{rotor} = \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} + \frac{\rho(C_{b2}^2 - C_{b1}^2)}{2} \tag{6.56}$$

For axial machines, $C_{b1} = C_{b2} = C_b$. Therefore,

$$(\Delta p)_{rotor} = \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} = \rho\frac{C_b^2}{2}(1 - \varphi^2 \tan^2 \beta_2) \tag{6.57}$$

$$(\Delta p_0)_{stage} = \rho C_b C_{w2} \tag{6.58}$$

Using Eqs (6.58) and (6.57) in Eq. (6.56), we get,

$$R = \frac{C_b^2(1 - \varphi^2 \tan^2 \beta_2)}{2C_b C_{w2}} \tag{6.59}$$

Substituting the value of C_{w2} from Eq. (6.45) in Eq. (6.59), we get,

$$R = \frac{C_b^2(1 - \varphi^2 \tan^2 \beta_2)}{2C_b^2(1 - \varphi \tan \beta_2)}$$

Rearranging this equation,

$$R = \frac{(1 + \varphi \tan \beta_2)}{2} \tag{6.60}$$

(f) Fan Efficiencies

Isentropic work input is always less than actual work input. The actual air power is lower than ideal air power. The actual air power is $\rho Q(\Delta h_0)_{stage}$ and the ideal air power (Euler's work) is given by $\rho Q C_b C_{w2}$,

$$\rho Q(\Delta h_0)_{stage} < \rho Q C_b C_{w2}$$

$$\frac{(\Delta p_0)_{stage}}{\rho} < C_b C_{w2}$$

Fan efficiency is the ratio of actual air power to ideal air power or it is the ratio of isentropic to actual work input.

$$\eta_{f\,\text{total}} = \frac{(\Delta p_0)_{\text{stage}}}{(\rho C_b C_{w2})} \tag{6.61}$$

Actual power input to the stage is,

$$P = \dot{m} C_b (C_{w2}) = \frac{Q(\Delta p_0)_{\text{stage}}}{\eta_{f\,\text{total}}}$$

$$P = Q \frac{\rho C_b^2 (1 - \varphi \tan \beta_2)}{\eta_{f\,\text{total}}} \tag{6.62}$$

where $\dot{m}$ and Q are the air mass flow rate and volume flow rate, respectively. Taking mechanical and electrical efficiency of the drive as η_d, the overall efficiency is,

$$\eta_0 = \eta_d \eta_{f\,\text{total}} \tag{6.63}$$

EXAMPLE 6.4 An axial fan stage consists of only a rotor that requires 1 kW of power and builds a total pressure difference of 60 mm of water gauge W.G. The fan rotates at a speed of 1400 rpm. The hub and tip diameter of the rotor blade are 250 mm and 500 mm, respectively. If the flow coefficient is 0.25, determine (a) rotor blade angle at entry, (b) rotor blade angle at exit, (c) flow rate, (d) stage pressure rise, (e) overall efficiency, (f) degree of reaction, and (g) specific speed.

Inlet flow conditions are $p_1 = 1.02$ bar and $T_1 = 316$ K.

Solution

$$D = \frac{(D_t + D_h)}{2} = \frac{(0.5 + 0.25)}{2} = 0.375 \text{ m}$$

$$C_b = \frac{\pi D N}{60} = \frac{\pi \times 0.375 \times 1400}{60} = 27.49 \text{ m/s} \tag{1}$$

Here, $C_{f1} = C_{f2} = C_f$

$$C_f = \varphi C_b = 0.25 \times 27.49 = 6.8725 \text{ m/s} \tag{2}$$

$$\rho = \frac{p}{RT} = \frac{1.02 \times 10^5}{(287 \times 316)} = 1.125 \text{ kg/m}^3 \tag{3}$$

(a) Rotor Blade Angle at Entry

Referring to Figure 6.16,

$$\tan \beta_1 = \frac{C_b}{C_f} = \frac{27.49}{6.8725}$$

$$\beta_1 = 75.96° \tag{4}$$

(b) Rotor blade Angle at Exit

$$w = C_b C_{w2} = \frac{(\Delta p_0)_{\text{stage}}}{\rho} = \frac{60 \times 9.81}{1.125} = 523.2 \text{ J/kg}$$

$$C_{w2} = 19.032 \text{ m/s} \tag{5}$$

$$\tan \beta_2 = \frac{(C_b - C_{w2})}{C_f} = \frac{27.49 - 19.032}{6.8725}$$

$$\beta_2 = 50.905° \tag{6}$$

(c) Flow Rate

$$A = \frac{\pi}{4}(D_t^2 - D_h^2) = \frac{\pi}{4}(0.5^2 - 0.25^2) = 0.147 \text{ m}^2$$

$$Q = AC_f = 0.147 \times 6.8725 = 1.010 \text{ m}^3/\text{s} \tag{7}$$

(d) Stage Pressure Rise

The static pressure rise in the stage is,

$$(\Delta p)_{\text{stage}} = \frac{\rho C_b^2}{2}(1 - \varphi^2 \tan^2 \beta_2)$$

$$(\Delta p)_{\text{stage}} = \frac{1.125 \times 27.49^2}{2} \times (1 - 0.25^2 \times \tan^2 50.905) \tag{8}$$

$$(\Delta p)_{\text{stage}} = 384.84 \text{ N/m}^2 = 384.84/9.81 \text{ mm WG} = 39.23 \text{ mm WG} \tag{9}$$

Since, here only rotor is considered for the stage, therefore, the static pressure rise of stage is same as that of the rotor.

$$(\Delta p)_{\text{stage}} = (\Delta p)_{\text{rotor}} = 39.23 \text{ mm WG} \tag{10}$$

(e) Overall Efficiency

$$(\Delta p_0)_{\text{stage}} = 60 \text{ mm WG}$$

The ideal power required to drive the fan is Euler's work given as,

$$P_{\text{ideal}} = \frac{\dot{m}(\Delta p_0)_{\text{stage}}}{\rho} = Q(\Delta p_0)_{\text{stage}} = 1.010 \times 60 \times 9.81 = 594.486 \text{ W} = 0.5945 \text{ kW} \tag{11}$$

The overall efficiency of the fan is,

$$\eta_0 = \frac{\text{Ideal Power}}{\text{Actual Power}} = \frac{0.594}{1}$$

$$\eta_0 = 0.594 = 59.4\% \tag{12}$$

(f) Degree of Reaction

$$R = \frac{(\Delta p)_{\text{rotor}}}{(\Delta p_0)_{\text{stage}}} = \frac{39.23}{60}$$

$$R = 0.654 \tag{13}$$

(g) Specific Speed

$$gH = 9.81 \times (60/1000) \times (1000/1.125) = 523.2 \text{ m}^2/\text{s}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1400}{60} = 146.61 \text{ rad/s}$$

The dimensionless specific speed is,

$$N_{sh} = \frac{\omega\sqrt{Q}}{(gH)^{3/4}} \tag{14}$$

$$N_{sh} = \frac{146.61\sqrt{1.010}}{523.2^{3/4}}$$

$$N_{sh} = 1.347 \text{ rad} \tag{15}$$

6.8.3 Stage with Downstream Guide Vanes

(a) Velocity Triangles

Velocity triangles at the suction and pressure ends of the impeller for axial fan stage with downstream guide vanes are shown in Figure 6.17. C_{r1}, β_1, C_1 and C_{f1} are the inlet relative velocity, inlet blade angle, inlet absolute velocity and axial component of inlet absolute velocity, respectively. C_{r2}, α_2, β_2, C_2, C_{f2} and C_{w2} are the outlet relative velocity, outlet flow angle, outlet blade angle, outlet absolute velocity, axial component of outlet absolute velocity and tangential component of outlet absolute velocity, respectively. C_3, C_{f3} are the outlet absolute velocity and axial component of absolute velocity at the downstream guide vanes, respectively.

(b) Stage Work

From Euler's Eq. (1.82), stage work is given by,

$$w = C_b(C_{w2} - C_{w1})$$

In an ideal case with perfect deflection and adiabatic flow, the entire input to the rotor must appear as the stagnation enthalpy rise in air or gas,

$$(\Delta h_0)_{\text{stage}} = w = C_b(C_{w2} - C_{w1})$$

For no guide vanes, if $C_{w1} = 0$,

$$w = C_b C_{w2} \tag{6.64}$$

From the velocity triangle as shown in Figure 6.17,

$$C_{w2} = C_b - C_f \tan \beta_2 \tag{6.65}$$

Dividing Eq. (6.65) by C_b, we get,

$$\frac{C_{w2}}{C_b} = 1 - \frac{C_f \tan \beta_2}{C_b} \tag{6.66}$$

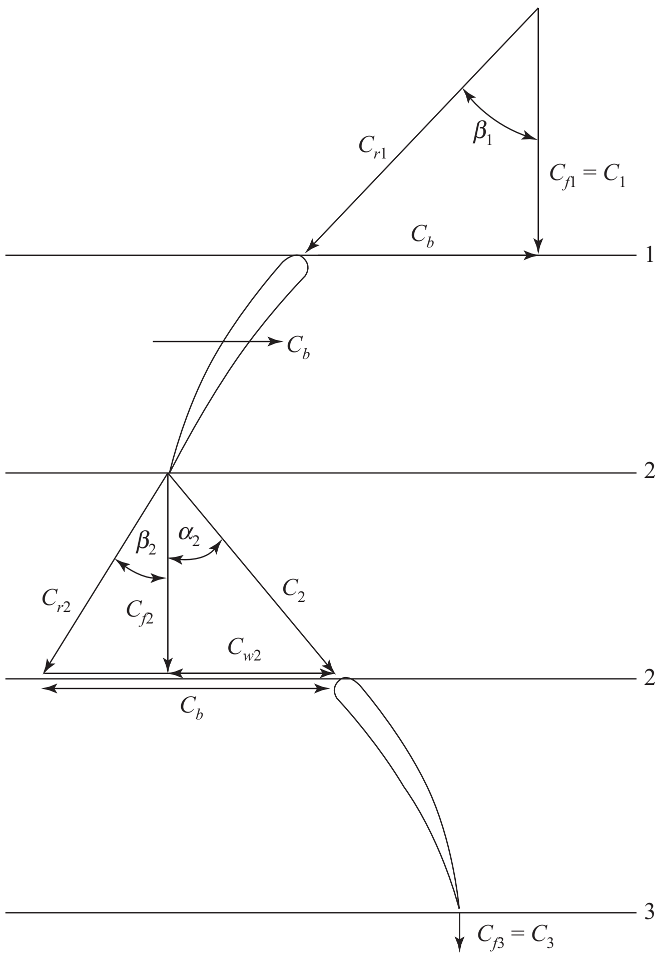


Figure 6.17 *Velocity Triangles of a Fan with Downstream Guide Vanes*

We know that the flow coefficient is,

$$\varphi = \frac{C_f}{C_b} \tag{6.67}$$

Using Eq. (6.67) in Eq. (6.66),

$$C_{w2} = C_b(1 - \varphi \tan \beta_2) \tag{6.68}$$

Substituting Eq. (6.68) in Eq. (6.64), we get,

$$w = (\Delta h_0)_{\text{stage}} = C_b^2(1 - \varphi \tan \beta_2) \tag{6.69}$$

(c) Stage Pressure Rise

For isentropic flow and incompressible flow,

$$(\Delta h_0)_{\text{stage}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho} \tag{6.70}$$

The stage pressure rise is,

$$(\Delta p_0)_{\text{stage}} = \rho(\Delta h_0)_{\text{stage}} = \rho C_b C_{w2} = \rho C_b^2 (1 - \varphi \tan \beta_2) \tag{6.71}$$

The pressure rise of stage in the rotor is given by,

$$(\Delta p)_{\text{rotor}} = \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} \tag{6.72}$$

We assume constant axial velocity and from velocity triangles in Figure 6.17,

$$C_{r1}^2 - C_{r2}^2 = C_{f1}^2 + C_b^2 - [C_{f2}^2 + (C_b - C_{w2})^2]$$

$$C_{r1}^2 - C_{r2}^2 = C_{f1}^2 + C_b^2 - C_{f2}^2 - C_b^2 - C_{w2}^2 + 2C_{w2}C_b$$

$$C_{r1}^2 - C_{r2}^2 = 2C_{w2}C_b - C_{w2}^2 \tag{6.73}$$

Using Eq. (6.73) in Eq. (6.72), we get,

$$(\Delta p)_{\text{rotor}} = \rho C_b C_{w2} - \rho \frac{C_{w2}^2}{2}$$

Rearranging this equation,

$$(\Delta p)_{\text{rotor}} = \rho C_b^2 \left[\frac{C_{w2}}{C_b} - \frac{1}{2}\left(\frac{C_{w2}}{C_{b2}}\right)^2 \right] \tag{6.74}$$

Using Eq. (6.68) in Eq. (6.74), static pressure rise in the rotor is,

$$(\Delta p)_{\text{rotor}} = \rho C_b^2 \left[1 - \varphi \tan \beta_2 - \frac{(1 - \varphi \tan \beta_2)^2}{2} \right]$$

Rearranging this equation gives,

$$(\Delta p)_{\text{rotor}} = \frac{\rho C_b^2}{2} (1 - \varphi^2 \tan^2 \beta_2) \tag{6.75}$$

(d) Stage Pressure Coefficient

Stage pressure coefficient for the rotor is given by,

$$\Psi = \frac{(\Delta p_0)_{\text{stage}}}{0.5 \rho C_b^2} \tag{6.76}$$

Using Eq. (6.71) in Eq. (6.76),

$$\Psi = 2(1 - \varphi \tan \beta_2) \tag{6.77}$$

(e) Stage Reaction

It is defined as the ratio of static pressure rise in the rotor and stagnation rise over the stage.

$$R = \frac{(\Delta p)_{\text{rotor}}}{(\Delta p_0)_{\text{stage}}}$$

$$(\Delta p)_{\text{rotor}} = \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} + \frac{\rho(C_{b2}^2 - C_{b1}^2)}{2} \tag{6.78}$$

For axial machines, $C_{b1} = C_{b2} = C_b$. Therefore,

$$(\Delta p)_{\text{rotor}} = \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} = \frac{\rho C_b^2}{2}(1 - \varphi^2 \tan^2 \beta_2)$$

$$(\Delta p_0)_{\text{stage}} = \rho C_b C_{w2}$$

Using these equations in Eq. (6.78), we get,

$$R = \frac{C_b^2(1 - \varphi^2 \tan \beta_2^2)}{2C_b C_{w2}} \tag{6.79}$$

Substituting the value of C_{w2} from Eq. (6.68) in Eq. (6.79), we get,

$$R = \frac{[C_b^2(1 - \varphi^2 \tan \beta_2^2)]}{2C_b^2(1 - \varphi \tan \beta_2)}$$

Rearranging this equation,

$$R = \frac{(1 + \varphi \tan \beta_2)}{2} \tag{6.80}$$

(f) Fan Efficiencies

$$\rho Q(\Delta h_0)_{\text{stage}} < \rho Q C_b C w_2$$

$$\frac{(\Delta p_0)_{\text{stage}}}{\rho} < C_b C_{w2}$$

Fan efficiency is the ratio of actual air power to ideal air power or it is the ratio of isentropic to actual work input.

$$\eta_{f\,\text{total}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho C_b C_{w2}} \tag{6.81}$$

Actual power input to the stage is,

$$P = \dot{m}C_b(C_{w2}) = \frac{Q(\Delta p_0)_{\text{stage}}}{\eta_{f\,\text{total}}} \tag{6.82}$$

where $\dot{m}$ and Q are the air mass flow rate and volume flow rate, respectively. Taking mechanical and electrical efficiency of the drive as η_d, the overall efficiency is,

$$\eta_0 = \eta_d \eta_{f\,\text{total}} \tag{6.83}$$

6.8.4 Counter Rotating Fan Stage

(a) Velocity Triangles

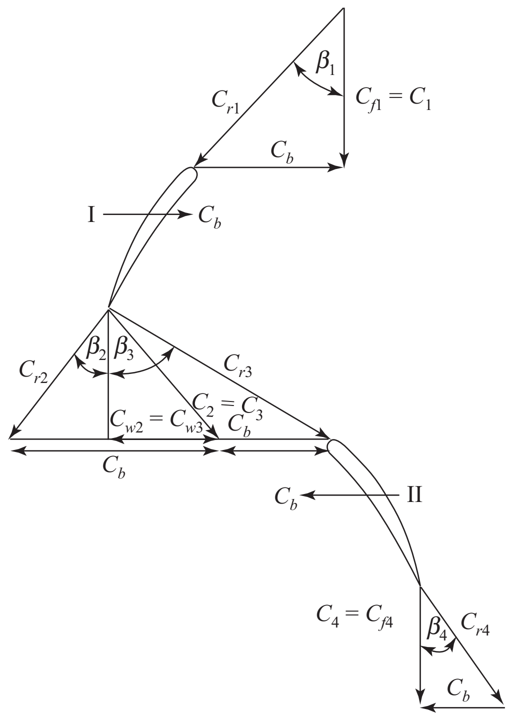


Figure 6.18 *Velocity Triangles of Counter Rotating Fan*

Velocity triangles for counter rotating fan stage are shown in Figure 6.18. In this stage, two axial fans are rotating in opposite directions. Two rotors are assumed to have same peripheral speeds. Axial velocity is constant throughout the stage. Assume that $C_{w1} = 0$, $C_3 = C_2$, $C_{w3} = C_{w2}$, $C_{w4} = 0$.

(b) Specific Work

Work done by the first rotor is given by,

$$w_{\text{I}} = C_b(C_{w2} - C_{w1}) = C_b C_{w2} \tag{6.84}$$

Work done by the second rotor is given by,

$$w_{\text{II}} = C_b[0 - (-C_{w3})]$$

$$W_{\text{II}} = C_b C_{w3} = C_b C_{w2} \tag{6.85}$$

Equations (6.84) and (6.85) show that work done and pressure rise are the same in the two rotors. Total stage work,

$$w = w_{\mathrm{I}} + w_{\mathrm{II}} = 2C_b C_{w2}$$

From velocity triangles shown in Figure 6.18,

$$C_{w2} = C_b(1 - \varphi \tan \beta_2)$$

$$w = 2C_b C_{w2} = 2C_b^2(1 - \varphi \tan \beta_2) \tag{6.86}$$

(c) Stage Pressure Rise

Stage pressure rise is given by,

$$(\Delta p)_{\text{stage}} = \rho(\Delta h_0)_{\text{stage}}$$

$$(\Delta p)_{\text{stage}} = 2\rho C_b^2(1 - \varphi \tan \beta_2) \tag{6.87}$$

Static pressure rise in the first rotor is given by,

$$(\Delta p)_{\mathrm{I}} = \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2}$$

From velocity triangle (Figure. 6.18) at the entry and exit, we get,

$$(\Delta p)_{\mathrm{I}} = \rho C_b C_{w2} - \frac{\rho C_{w2}^2}{2} \tag{6.88}$$

Static pressure rise in the second rotor is,

$$(\Delta p)_{\mathrm{II}} = \frac{\rho(C_{r3}^2 - C_{r4}^2)}{2}$$

From velocity triangles at the entry and exit, we get,

$$(\Delta p)_{\mathrm{II}} = \rho C_b C_{w2} + \frac{\rho C_{w2}^2}{2} \tag{6.89}$$

Adding Eqs (6.88) and (6.89), total static pressure rise in the stage is,

$$(\Delta p_0)_{\text{stage}} = (\Delta p)_{\text{stage}} = (\Delta p)_{\mathrm{I}} + (\Delta p)_{\mathrm{II}}$$

$$(\Delta p_0)_{\text{stage}} = (\Delta p)_{\text{stage}} = 2\rho C_b C_{w2} \tag{6.90}$$

(d) Stage Pressure Coefficient

From pressure coefficient definition,

$$\Psi = \frac{w}{0.5C_b^2} \tag{6.91}$$

Using Eq. (6.86) in Eq. (6.91), we get,

$$\psi = 4(1 - \varphi \tan \beta_2) \tag{6.92}$$

(e) Fan Efficiencies

$$\rho Q(\Delta h_0)_{\text{stage}} < \rho Q \times 2C_b C_{w2}$$

$$\frac{(\Delta p_0)_{\text{stage}}}{\rho} < 2C_b C_{w2}$$

Fan efficiency is the ratio of actual air power to the ideal air power or it is the ratio of isentropic to actual work input.

$$\eta_{f\,\text{total}} = \frac{(\Delta p_0)_{\text{stage}}}{2\rho C_b C_{w2}} \tag{6.93}$$

Actual power input to the stage is,

$$P = 2\dot{m} C_b C_{w2} = \frac{Q(\Delta p_0)_{\text{stage}}}{\eta_{f\,\text{total}}} \tag{6.94}$$

where $\dot{m}$ and Q are the air mass flow rate and volume flow rate, respectively.

EXAMPLE 6.5 A single-stage axial flow blower with no inlet guide vanes runs at 3600 rpm. The rotor tip and hub diameter are 200 mm and 125 mm, respectively. The mass flow rate of air is 0.5 kg/s. The turning angle of the rotor is 20° towards the axial direction during air flow over the blade. The blade angle at inlet is 55°. If the atmospheric temperature and pressure are 1 atm and 25°C, respectively, and assuming constant axial velocity through the machine, find (a) the total pressure of the air at the exit of the rotor, the rotor total to total efficiency being 90% and the total pressure drop across the intake is 2.5 mm of water, (b) the static pressure rise across the rotor, (c) the static pressure rise across the stator, if the stator efficiency is 75%, (d) the change in total pressure across the stator, (e) the overall total-to-total efficiency, and (f) the degree of reaction for the stage.

Solution

Given: N = 3600 rpm, D_t = 0.2 m, D_h = 0.125 m, p_1 = 1 atm = 1.013 bar, T_1 = 298 K, $\dot{m}$ = 0.5 kg/s, $\beta_1 - \beta_2 = 20°$, $\beta_1 = 55°$, $\eta_{t-t} = 0.9$, $\eta_{\text{stator}} = 0.75$

(a) Total Pressure of Air at Rotor Exit

The pressure changes involved are small so that the flow may be treated as incompressible. At the inlet, the density of air is,

$$\rho_0 = \frac{p_0}{RT_0} = \frac{1.013 \times 10^5}{287 \times 298} = 1.184 \text{ kg/m}^3 \tag{1}$$

Area of flow,

$$A = \frac{\pi}{4}(D_t^2 - D_h^2) = \frac{\pi}{4}(0.2^2 - 0.125^2) = 0.01914 \text{ m}^2 \tag{2}$$

The axial velocity is constant throughout the machine, therefore, $C_{f1} = C_{f2} = C_f$.

$$C_f = \frac{\dot{m}}{\rho A} = \frac{0.5}{1.184 \times 0.01914} = 22.1 \text{ m/s} \tag{3}$$

Mean rotor blade velocity,

$$C_b = \frac{\pi(D_h + D_t)N}{2 \times 60} = \frac{\pi(0.125 + 0.2) \times 3600}{120} = 30.6 \text{ m/s} \tag{4}$$

Since the flow is axial, $C_{w1} = 0$ and from velocity triangle at the outlet, refer Figure 6.17,

$$C_{w2} = C_b - C_f \tan \beta_2$$

$$C_{w2} = 30.6 - 22.1 \times \tan(\beta_1 - 20) = 30.6 - 22.1 \tan 35$$

$$C_{w2} = 15.13 \text{ m/s} \tag{5}$$

The actual total enthalpy rise across the rotor is,

$$w = (\Delta h_0)_{\text{rotor}} = C_b(C_{w2} - C_{w1}) \tag{6}$$

$$(\Delta h_0)_{\text{rotor}} = 30.6 \times (15.13 - 0) = 462.98 \text{ J/kg} \tag{7}$$

The isentropic total enthalpy rise across the rotor is,

$$(\Delta h_{0s})_{\text{rotor}} = \eta_{t-t} \times (\Delta h_0)_{\text{rotor}} \tag{8}$$

$$(\Delta h_{0s})_{\text{rotor}} = 0.9 \times 462.98 = 416.7 \text{ J/kg}$$

The total pressure rise across the rotor is,

$$(\Delta p_0)_{\text{rotor}} = \rho \times (\Delta h_{0s})_{\text{rotor}} \tag{9}$$

$$(\Delta p_0)_{\text{rotor}} = 1.184 \times 416.7$$

$$(\Delta p_0)_{\text{rotor}} = 493.37 \text{ N/m}^2 = 50.293 \text{ mm of } H_2O \tag{10}$$

$$\text{Stagnation pressure at the rotor exit} = (\Delta p_0)_{\text{rotor}} - \text{Pressure drop at intake} \tag{11}$$

$$\text{Stagnation pressure at the rotor exit} = 50.293 - 2.5 = 47.793 \text{ mm of } H_2O \tag{12}$$

(b) Static Pressure Rise across the Rotor

$$C_1 = C_f = 22.1 \text{ m/s} \tag{13}$$

$$C_2 = \sqrt{C_f^2 + C_{w2}^2} = \sqrt{22.1^2 + 15.13^2} = 26.78 \text{ m/s} \tag{14}$$

$$(\Delta p)_{\text{rotor}} = (\Delta p_0)_{\text{rotor}} - \frac{\rho(C_2^2 - C_1^2)}{2} \tag{15}$$

$$(\Delta p)_{\text{rotor}} = 493.37 - \frac{1.184(26.78^2 - 22.1^2)}{2}$$

$$(\Delta p)_{\text{rotor}} = 357.94 \text{ N/m}^2 = 36.5 \text{ mm of water} \tag{16}$$

(c) Static Pressure Rise across Stator

The actual static enthalpy change across the stator is,

$$(\Delta h)_{\text{stator}} = \frac{C_2^2 - C_1^2}{2} = \frac{26.78^2 - 22.1^2}{2} = 114.38 \text{ J/kg} \tag{17}$$

The theoretical static enthalpy change across the stator is,

$$(\Delta h_s)_{\text{stator}} = \eta_{\text{stator}} \times \Delta h_{\text{stator}} = 0.75 \times 114.38 = 85.79 \text{ J/kg} \tag{18}$$

The static pressure rise across the stator,

$$(\Delta p)_{\text{stator}} = \rho \times (\Delta h_s)_{\text{stator}} \tag{19}$$

$$(\Delta p)_{\text{stator}} = 1.184 \times 85.79 = 101.58 \text{ N/m}^2 = 10.4 \text{ mm } H_2O \tag{20}$$

(d) Total Pressure Change across Stator

The change in total pressure across the stator is,

$$(\Delta p_0)_{\text{stator}} = (\Delta p)_{\text{stator}} + \frac{\rho(C_1^2 - C_2^2)}{2} = 101.58 + 1.184 \times (-114.38) = -33.85 \text{ N/m}^2 \tag{21}$$

i.e. the total pressure across the stator drops by amount 33.85 N/m^2 or 3.5 mm of water.

(e) Overall Total-to-Total Efficiency

Total pressure at stator exit:

$$p_{03} = \text{Total pressure at rotor exit} - (\Delta p_0)_{\text{stator}} \tag{22}$$

$$p_{03} = 47.8 - 3.5 = 44.293 \text{ mm water}$$

Theoretical total enthalpy change across the stage,

$$(\Delta h_{0s})_{\text{stage}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho} = \frac{\rho_w g h_w}{\rho} \tag{23}$$

$$(\Delta h_{0s})_{\text{stage}} = \frac{1000 \times 9.81 \times \left(\dfrac{44.293}{1000}\right)}{1.184} = 366.99 \text{ J/kg} \tag{24}$$

$$\eta_{t-t} = \frac{(\Delta h_{0s})_{\text{stage}}}{(\Delta h_0)_{\text{stage}}} = \frac{(\Delta h_{0s})_{\text{stage}}}{(\Delta h_0)_{\text{rotor}}} \tag{25}$$

$$\eta_{t-t} = \frac{366.99}{462.98} = 79.27\% \tag{26}$$

(f) Degree of Reaction

$$R = \frac{(\Delta p)_{\text{rotor}}}{(\Delta p)_{\text{stage}}} \tag{27}$$

$$R = \frac{357.94}{(357.94 + 101.58)} = 0.779 \tag{28}$$

6.9 Centrifugal Fan

It is a device which handles air and gas, and raises its pressure mainly by centrifugal action similar to axial flow machines. The centrifugal fan also consists of an inlet duct, casing, rotor, diffuser space, and inter connecting duct (for multistage machine).

6.9.1 Velocity Triangles

Flow enters axially through the eye of the impeller of the fan as shown in Figure 6.19. Flow then take place radially outward through the impeller, guided by the vanes. Assuming uniform flow throughout the periph-

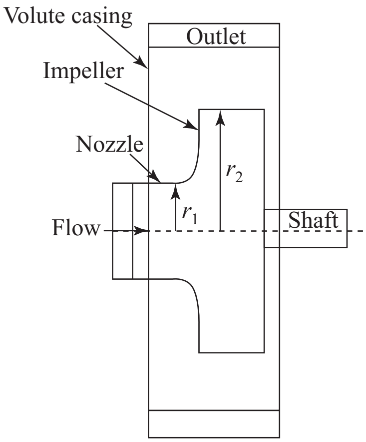


Figure 6.19 *Centrifugal Fan*

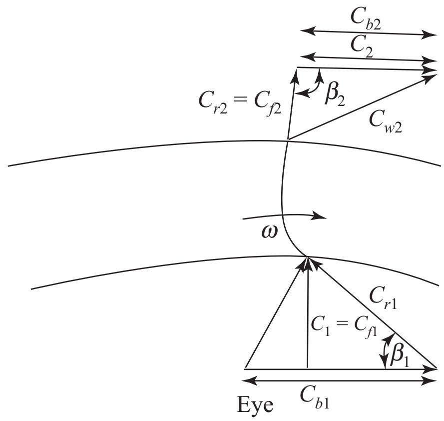


Figure 6.20 *Velocity Triangles of Backward Curved Centrifugal Fan*

ery at the inlet as well as outlet of the impeller, velocity triangles are drawn in Figure 6.20. As velocity is radial at the inlet, the meridional or radial component of velocity is also the velocity of flow, i.e. $C_{f1} = C_1$. The triangle is a right angle triangle with tangential component being equal to zero ($C_{w1} = 0$). At the outlet, the relative velocity depends upon the blade angle. In the figure, the impeller is shown with radial lines. At the outlet, therefore, the outlet velocity triangle is also right angle triangle; the outlet relative velocity being parallel to the blades, as shown in Figure 6.20.

If the blades are changed, the velocity triangles will change. Velocity triangles for different geometries are as shown in Figure 6.20 to 6.22. Figure 6.20 shows backward curved blade, whereas Figure 6.21 shows the forward curved blade and Figure 6.22 for radial tipped blades. The students may redraw the velocity triangles by changing the direction of rotation for practice.

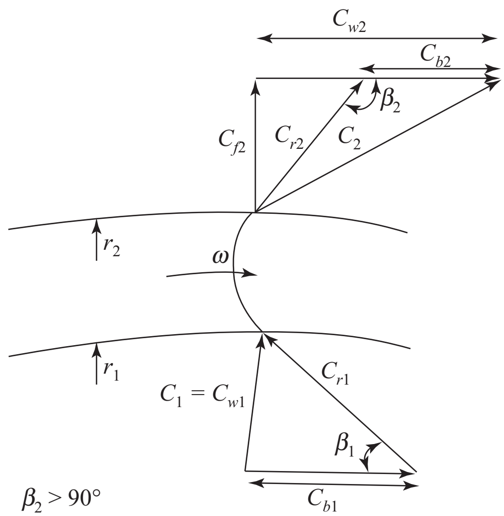


Figure 6.21 *Velocity Triangles for Forward Curved Centrifugal Fan*

6.9.2 Stage Parameters

(a) Mass Flow Rate

Mass flow rate through the centrifugal fan can be estimated as follows by knowing the radial component of velocity,

$$\dot{m} = \rho \pi D_1 B_1 C_{f1} = \rho \pi D_2 B_2 C_{f2} \tag{6.95}$$

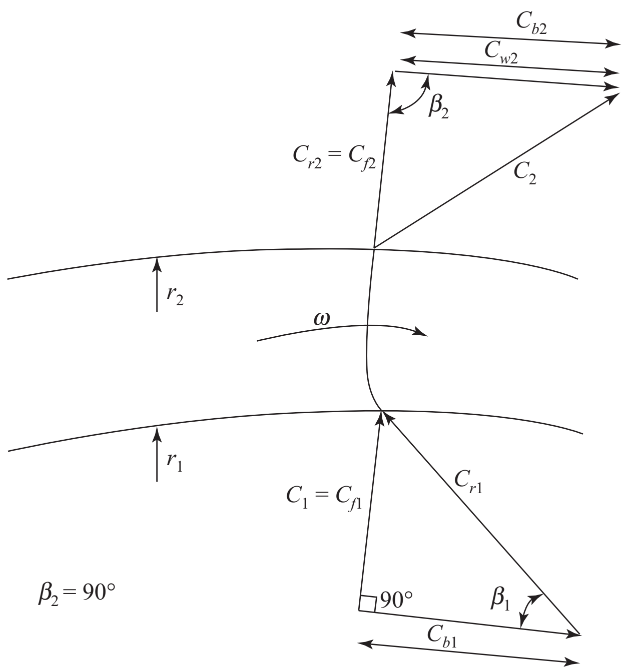


Figure 6.22 *Velocity Triangles of Radial Tipped Centrifugal Fan*

(b) Specific Work

Considering one blade element as in Figure 6.23 and the velocity triangles at its inlet and outlet are drawn with the following nomenclature.

C_1, C_2 – Absolute velocity at entry and exit

C_{r1}, C_{r2} – Relative velocity at entry and exit

β_1, β_2 – Relative velocity angle at entry and exit

C_{w1}, C_{w2} – Tangential velocity at entry and exit

C_{f1}, C_{f2} – Axial velocity at entry and exit

If air enters radially in the impeller, $C_{w1} = 0$. The expression for specific work for general turbomachines is given by Euler's Eq. (1.82). Therefore specific work done by the impeller,

$$w = C_{w2}C_{b2} \tag{6.96}$$

We know that,

$$(\Delta p_0)_{\text{stage}} = \rho C_{w2}C_{b2} \tag{6.97}$$

From outlet velocity triangle, we get generalised form for C_{w2} as,

$$C_{w2} = C_{b2} - C_{f2} \cot \beta_2 \tag{6.98}$$

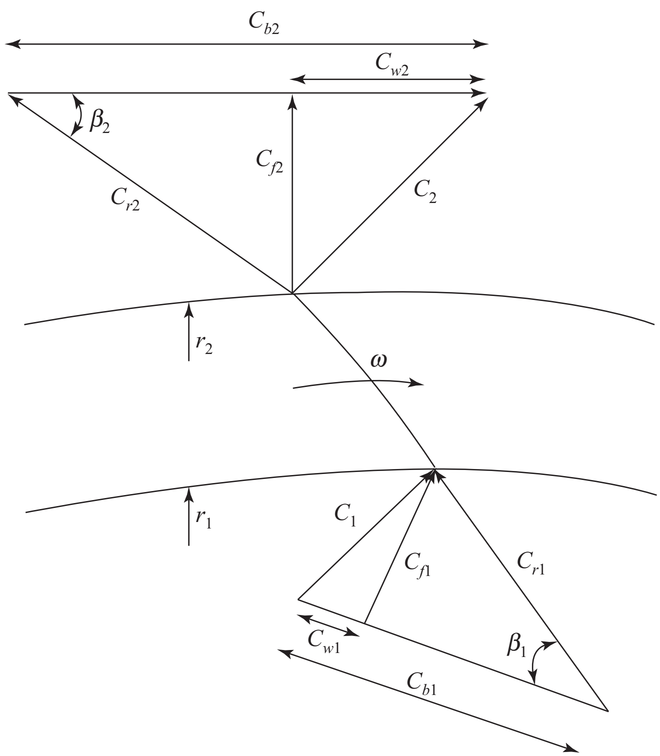


Figure 6.23 *Velocity Triangles of a Backward Curve Centrifugal Fan*

Substituting Eq. (6.98) in Eq. (6.97),

$$(\Delta p_0)_{\text{stage}} = \rho C_{b2}(C_{b2} - C_{f2} \cot \beta_2) \tag{6.99}$$

Volume flow rate through the impeller is,

$$Q = \pi D_1 B_1 C_{f1} = \pi D_2 B_2 C_{f2} \tag{6.100}$$

where B_1 and B_2 are the axially blade width at diameter D_1 and D_2 respectively.

Substituting C_{f2} from Eq. (6.100) in Eq. (6.99), we get,

$$(\Delta p_0)_{\text{stage}} = \rho C_{b2}[C_{b2} - (Q/\pi D_2 B_2)\cot \beta_2] \tag{6.101}$$

Blade coefficient,

$$\Psi = \frac{(\Delta p_0)_{\text{stage}}}{0.5 \rho C_{b2}^2} \tag{6.102}$$

Flow coefficient,

$$\varphi = \frac{C_{f1}}{C_{b1}} = \frac{C_{f2}}{C_{b2}} = \frac{Q}{\pi D_2 B_2 C_{b2}} \tag{6.103}$$

Power coefficient,

$$\lambda = \psi\varphi \tag{6.104}$$

Substituting Eqs (6.102), (6.103) and (6.104) in Eq. (6.101), we get,

$$\psi = \frac{(\Delta p_0)_{\text{stage}}}{\left(\frac{\rho C_{b2}^2}{2}\right)} = 2\left[1 - \left(\frac{Q}{\pi D_2 B_2 C_{b2}}\right)\cot\beta_2\right] \tag{6.105}$$

Rearranging Eq. (6.105), we get,

$$\psi = 2(1 - \varphi \cot \beta_2) \tag{6.106}$$

$$\lambda = 2\varphi(1 - \varphi \cot \beta_2) \tag{6.107}$$

(i) For Backward Curved Blade, $\beta_2 < 90°$

$$C_{w2} = C_{b2} - C_{f2} \cot \beta_2 \tag{6.108}$$

From Eqs. (6.108), (6.97), (6.102), (6.103) and (6.104), we get,

$$\psi = 2(1 - \varphi \cot \beta_2) \tag{6.109}$$

$$\lambda = 2\varphi(1 - \varphi \cot \beta_2) \tag{6.110}$$

(ii) For Radial Blade, $\beta_2 = 90°$

$$C_{w2} = C_{b2} \tag{6.111}$$

$$(\Delta p_0)_{\text{stage}} = \rho C_{b2}^2 \tag{6.112}$$

Substituting Eq. (6.112) in Eq. (6.102), we have,

$$\psi = 2 \tag{6.113}$$

Thus, using Eq. (6.113) in Eq. (6.104),

$$\lambda = 2\varphi \tag{6.114}$$

(iii) For Forward Curved Blade, $\beta_2 = 90°$

$$C_{w2} = C_{b2} + C_{f2} \cot(180 - \beta_2) \tag{6.115}$$

Using Eqs. (6.115), (6.97), (6.102), (6.103) and (6.104), we get,

$$\psi = 2(1 - \varphi \cot \beta_2) \tag{6.116}$$

$$\lambda = 2\varphi(1 - \varphi \cot \beta_2) \tag{6.117}$$

(c) Stage Pressure Rise

For isentropic incompressible flow,

$$Tds = dh - vdp \Rightarrow 0 = dh - \frac{dp}{\rho}$$

$$w = (\Delta h_0)_{\text{stage}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho} \tag{6.118}$$

Using Eqs. (6.96) and (6.98) in Eq. (6.118), we get,

$$(\Delta p_0)_{\text{stage}} = \rho C_{w2} C_{b2} = \rho C_{b2}(C_{b2} - C_{f2} \cot \beta_2) \tag{6.119}$$

Rearranging Eq. (6.119) and using the flow coefficient definition (φ) in Eq. (6.103), we get,

$$(\Delta p_0)_{\text{stage}} = \rho C_{b2}^2 (1 - \varphi \cot \beta_2) \tag{6.120}$$

Static pressure rise through the impeller is due to the change in the centrifugal energy and the diffusion of the relative flow. Therefore,

$$(\Delta p)_{\text{impeller}} = \frac{\rho(C_{b2}^2 - C_{b1}^2)}{2} + \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} \tag{6.121}$$

Stagnation pressure rise through the stage can be obtained from Euler's Eqs (1.84) and (1.98),

$$(\Delta p_0)_{\text{stage}} = \frac{\rho(C_2^2 - C_1^2)}{2} + \frac{\rho(C_{b2}^2 - C_{b1}^2)}{2} + \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2} \tag{6.122}$$

Substituting Eq. (6.121) in Eq. (6.122), we get,

$$(\Delta p_0)_{\text{stage}} = (\Delta p)_{\text{impeller}} + \frac{\rho(C_2^2 - C_1^2)}{2} \tag{6.123}$$

(d) Stage Pressure Coefficient

Blade coefficient for the stage,

$$\Psi = \frac{(\Delta p_0)_{\text{stage}}}{0.5 \rho C_{b2}^2}$$

Substituting $(\Delta p_0)_{\text{stage}}$ value from Eq. (6.97), we get,

$$\Psi = \frac{\rho C_{w2} C_{b2}}{0.5 \rho C_{b2}^2}$$

$$\Psi = \frac{2 C_{w2}}{C_{b2}} \tag{6.124}$$

An impeller pressure coefficient,

$$\Psi_{\text{impeller}} = \frac{(\Delta p)_{\text{impeller}}}{0.5 \rho C_{b2}^2} \tag{6.125}$$

(e) Stage Reaction (R)

Degree of reaction of the stage is given by,

$$R = \frac{(\Delta p)_{\text{impeller}}}{(\Delta p_0)_{\text{stage}}} \tag{6.126}$$

From entry velocity triangles,

$$C_{r1}^2 - C_{b1}^2 = C_1^2$$

$$(\Delta p)_{\text{impeller}} = \frac{\rho(C_{b2}^2 - C_{b1}^2)}{2} + \frac{\rho(C_{r1}^2 - C_{r2}^2)}{2}$$

$$(\Delta p)_{\text{impeller}} = \frac{\rho(C_1^2 + C_{b2}^2 - C_{r2}^2)}{2}$$

Since $C_1 = C_{f1} = C_{f2}$, therefore,

$$(\Delta p)_{\text{impeller}} = \frac{\rho(C_{b2}^2 - C_{r2}^2 + C_{f2}^2)}{2} \tag{6.127}$$

From exit velocity triangles,

$$C_{r2}^2 - C_{f2}^2 = (C_{b2} - C_{w2})^2$$

$$C_{b2}^2 + C_{f2}^2 - C_{r2}^2 = 2C_{b2}C_{w2} - C_{w2}^2 \tag{6.128}$$

Substituting Eq. (6.128) in Eq. (6.127),

$$(\Delta p)_{\text{impeller}} = \frac{\rho(2C_{b2}C_{w2} - C_{w2}^2)}{2} \tag{6.129}$$

Substituting Eqs. (6.129) and (6.97) in Eq. (6.126),

$$R = \frac{\rho(2C_{b2}C_{w2} - C_{w2}^2)}{2\rho C_{w2}C_{b2}}$$

Rearranging this equation,

$$R = 1 - \frac{C_{w2}}{2C_{b2}} \tag{6.130}$$

(i) For Backward Curved Vanes ($\beta_2 < 90°$)

$$\frac{C_{w2}}{C_{b2}} < 1\text{, therefore } R < 1$$

(ii) For Radial Vanes ($\beta_2 = 90°$)

$$\frac{C_{w2}}{C_{b2}} = 1\text{, therefore } R = \frac{1}{2}$$

(iii) For Forward Curved Vanes ($\beta_2 > 90°$)

$$\frac{C_{w2}}{C_{b2}} > 1, \text{ therefore } R < \frac{1}{2}$$

(f) Stage Efficiency

Fan stage efficiency is defined as the ratio of isentropic work to actual work. Isentropic work can be given by Eq. (6.118), whereas actual work can be given by Eq. (6.96),

Therefore, efficiency is,

$$\eta_{st} = \frac{(\Delta p_0)_{stage}}{\rho C_{b2} C_{w2}} \tag{6.131}$$

EXAMPLE 6.6 A centrifugal blower with a radial impeller produces a pressure equivalent to 1 m column of water. The pressure and temperature at its entry are 0.98 bar and 310 K, respectively. The electric motor driving the blower runs at 3000 rpm. The efficiencies of the fan and drive are 82% and 88%, respectively. The radial velocity remains constant and has a value of $0.2C_{b2}$. The velocity at the inlet eye is $0.4C_{b2}$. If the blower handles 200 m^3/min of air at the entry conditions, determine (a) power required by the electric motor, (b) impeller diameter, (c) inner diameter of the blade ring, (d) air angle at entry, (e) impeller widths at entry and exit, (f) number of impeller blades, and (g) the specific speed.

Solution

(a) Power Required by Electric Motor

$$Q = \frac{200}{60} = 3.334 \text{ m}^3/\text{s} \tag{1}$$

Ideal power,

$$P_i = Q\Delta p_0 = Q\rho_w g h_w \tag{2}$$

$$P_i = 3.334 \times 1000 \times 9.81 \times 1 = 32706.54 \text{ W} = 32.706 \text{ kW} \tag{3}$$

Actual power,

$$P = \frac{P_i}{\eta_f \eta_d} = \frac{32.706}{0.82 \times 0.88}$$

$$P = 45.32 \text{ kW} \tag{4}$$

(b) Impeller Diameter

$$\rho = \frac{p}{RT} = \frac{0.98 \times 10^5}{(287 \times 310)} = 1.10 \text{ kg/m}^3$$

For radial impeller,

$$\eta_{fan} = \frac{(\Delta p_0)_{stage}}{\rho C_{b2}^2} = \frac{\rho_w g h_w}{\rho C_{b2}^2} \tag{5}$$

$$0.82 = \frac{1000 \times 9.81 \times 1}{1.1 \times C_{b2}^2}$$

$$C_{b2} = 104.287 \text{ m/s} \tag{6}$$

$$C_{b2} = \frac{(\pi D_2 N)}{60} = \frac{\pi D_2 \times 3000}{60} = 104.287$$

$$D_2 = 0.664 \text{ m} = 664 \text{ mm} \tag{7}$$

(c) Inner Diameter of Blade Ring

$$C_{f1} = C_{f2} = 0.2C_{b2} = 0.2 \times 104.287$$

$$C_{f1} = C_{f2} = 20.857 \text{ m/s} \tag{8}$$

$$C_1 = 0.4C_{b2} = 0.4 \times 104.287$$

$$C_1 = 41.715 \text{ m/s} \tag{9}$$

$$Q = \frac{\pi}{4} D_1^2 C_1 \tag{10}$$

$$3.334 = \frac{\pi}{4} D_1^2 \times 41.715$$

$$D_1 = 0.319 \text{ mm} = 319 \text{ mm} \tag{11}$$

(d) Air Angle at Entry

$$C_{b1} = \frac{D_1}{D_2} C_{b2} = \frac{319}{664} \times 104.287$$

$$C_{b1} = 50.10 \text{ m/s} \tag{12}$$

$$\tan \beta_1 = \frac{C_{f1}}{C_{b1}} = \frac{20.857}{50.1}$$

$$\beta_1 = 22.6° \tag{13}$$

(e) Impeller Widths at Entry and Exit

$$Q = \pi D_1 B_1 C_{f1} \tag{14}$$

$$B_1 = \frac{Q}{\pi D_1 C_{f1}} = \frac{3.334}{\pi \times 0.319 \times 20.857}$$

$$B_1 = 0.1595 \text{ m} = 159.5 \text{ mm} \tag{15}$$

$$B_2 = \frac{D_1}{D_2} B_1 = \frac{0.319}{0.664} \times 0.1595$$

$$B_2 = 0.07663 \text{ m} = 76.63 \text{ mm} \tag{16}$$

(f) Number of Impeller Blades

Number of blades can be given by correlation as follows:

$$z = \frac{(8.5 \sin \beta_2)}{\left(1 - \frac{D_1}{D_2}\right)} \tag{17}$$

$$z = \frac{8.5 \sin 90}{\left(1 - \frac{0.319}{0.664}\right)}$$

$$z = 16.35 \approx 17 \tag{18}$$

Therefore, the number of blades can be taken as 17.

(g) Specific Speed
The head produced by the blower for radial blades is,

$$gH = w = C_{b2}^2 = 104.287^2 = 10875.7783 \text{ m}^2/\text{s}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60} = 314.16 \text{ rad/s}$$

The dimensionless specific speed is,

$$N_{sh} = \frac{\omega\sqrt{Q}}{(gH)^{3/4}} \tag{19}$$

$$N_{sh} = \frac{314.16\sqrt{3.334}}{10875.7783^{3/4}} = 0.538$$

$$N_{sh} = 0.5386 \text{ rad} \tag{20}$$

6.10 Slip Factor

Referring to Section 1.7 and Figure 1.34, change in whirl component of outlet velocity due to slip is

$$\Delta C_{w2} = \frac{\pi C_{b2} \sin \beta_2}{z} \tag{6.132}$$

Where z is the number of blades on the impeller. Loss of head due to slip is

$$H_{slip} = \frac{C_{b2} \Delta C_{w2}}{g} \tag{6.133}$$

Net theoretical head by an impeller is

$$H_{th} = \frac{\sigma_s C_{b2} C_{w2} - C_{b1} C_{w1}}{g} \tag{6.134}$$

$$H_{th} = \frac{\left[c_{b2}\left(C_{w2} - \frac{\pi C_{b2} \sin \beta_2}{z}\right) - C_{b1} C_{w1}\right]}{g} \tag{6.135}$$

For a purely radial inlet machine, prewhirl is zero, $C_{w1} = 0$

$$H_{th} = \frac{C_{b2}\left(C_{w2} - \dfrac{\pi C_{b2} \sin \beta_2}{z}\right)}{g} \tag{6.136}$$

Note that σ_s is the slip factor, given by Eq. 1.107.

6.11 Losses in Fans

6.11.1 Impeller Entry Losses

It occurs due to the flow at eye and its turning from axial to radial direction.

$$\Delta p_i = \frac{k_i(\rho C_0^2)}{2} \tag{6.137}$$

where k_i is the loss factor in order of 0.5–0.8 and C_0 is the air velocity in the impeller eye. Impeller blade losses due to friction and separation on account of change of incidence can also be included under this loss. Larger flow rates lead to negative incidence and less flow rates leads to positive incidence.

6.11.2 Leakage Loss

Clearance between rotating periphery of impeller and casing at entry in the main flow field leads to leakage of some air and disturbance in the main flow field. Leakage also occurs through the clearance between the fan shaft and the casing.

$$\Delta Q = C_d \pi d_1 \delta \sqrt{(2\Delta p_s/\rho)} \tag{6.138}$$

where ΔQ is the leakage volume, C_d is discharge coefficient and Δp_s is the static pressure difference, across the clearance between impeller and casing (δ).

6.11.3 Impeller Losses

These losses arise from passage friction and separation. They depend on the relative velocity, rate of diffusion and blade geometry.

$$\Delta p_{ii} = \frac{k_{ii}\rho(C_{r1}^2 - C_{r2}^2)}{2} \tag{6.139}$$

where k_{ii} is in order of 0.2–0.3 for sheet metal blades and less for blades of aerofoil section.

6.11.4 Diffuser and Volute Losses

Losses in the diffuser also occur due to friction and separation. At off design conditions, there are losses due to incidence. Flow from the impeller expands to larger cross-sectional area in the volute. This leads due to eddy formation.

$$\Delta p_{iii} = \frac{k_{iii}\rho(C_2^2 - C_3^2)}{2} \tag{6.140}$$

where k_{iii} is 0.4 at maximum efficiency and C_3 is the average velocity at the fan outlet. Further losses occur due to the volute passage friction and flow separation.

6.11.5 Disc Friction Losses

This is due to fluid drag on the back surface of the impeller disc.

$$dT = f \cdot 2\pi r dr \cdot \frac{1}{2}\rho C_b^2 \times r \tag{6.141}$$

where f is friction factor of the order of 0.005 and Cb = ωr is the peripheral velocity of the element. On integration,

$$T = \frac{f\rho\omega^2 r^5}{5} \tag{6.142}$$

If the back plate is close to the casing, spaced by the distance 's', viscous forces predominate and,

$$dT = \mu\frac{dC}{dy} \times 2\pi r dr \times rN\frac{\mu\omega r}{s}2\pi r^2 dr \tag{6.143}$$

On integration,

$$T = \pi\mu\omega\frac{r^4}{2s} \tag{6.144}$$

6.12 Performance Characteristics of Centrifugal Fan

6.12.1 Theoretical Characteristics

Theoretical characteristics of centrifugal fan are obtained based upon blade angle from Eq. (6.106). They are shown in Figure 6.24. Theoretically, the pressure remains constant at all flows for radial blade (β_2 = 90°). The pressure falls with increasing flow for backward blade (β_2 < 90°) whereas the pressure rises linearly with increasing flow for forward blade (β_2 > 90°).

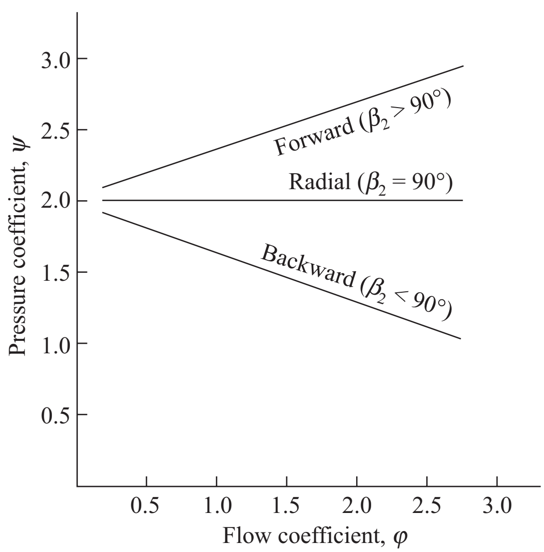


Figure 6.24 *Theoretical Fan Characteristics*

6.12.2 Actual Characteristics

The actual characteristics, as shown in Figure 6.25, are obtained by deducting the stage losses from the theoretical head or pressure coefficient (Figure 6.25).

(a) Forward Curved Centrifugal Fan

Figure 6.26 shows the performance characteristics of a forward curved centrifugal fan. The forward curve fan is used to deliver high air volumes against static pressures up to 152.4 mm water gauge. Any selection to the left of the 40% free delivery point (peak) will result in an unstable pulsating airflow that will lead to impeller structural damage. Even though good peak efficiencies are on either side of the peak, selections should be limited to 45% or greater of free delivery.

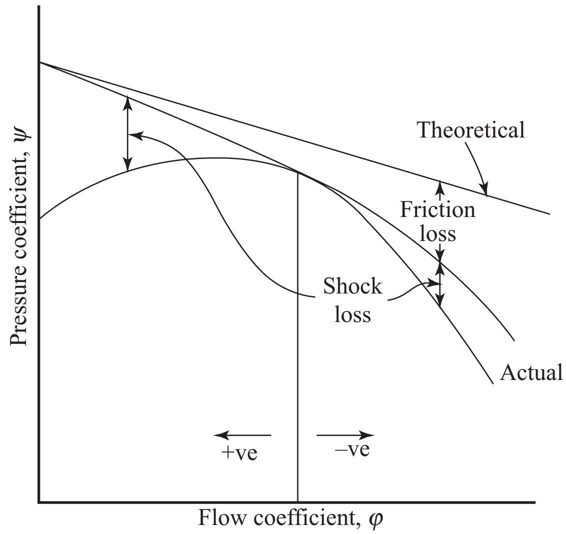


Figure 6.25 *Actual Fan Characteristics*

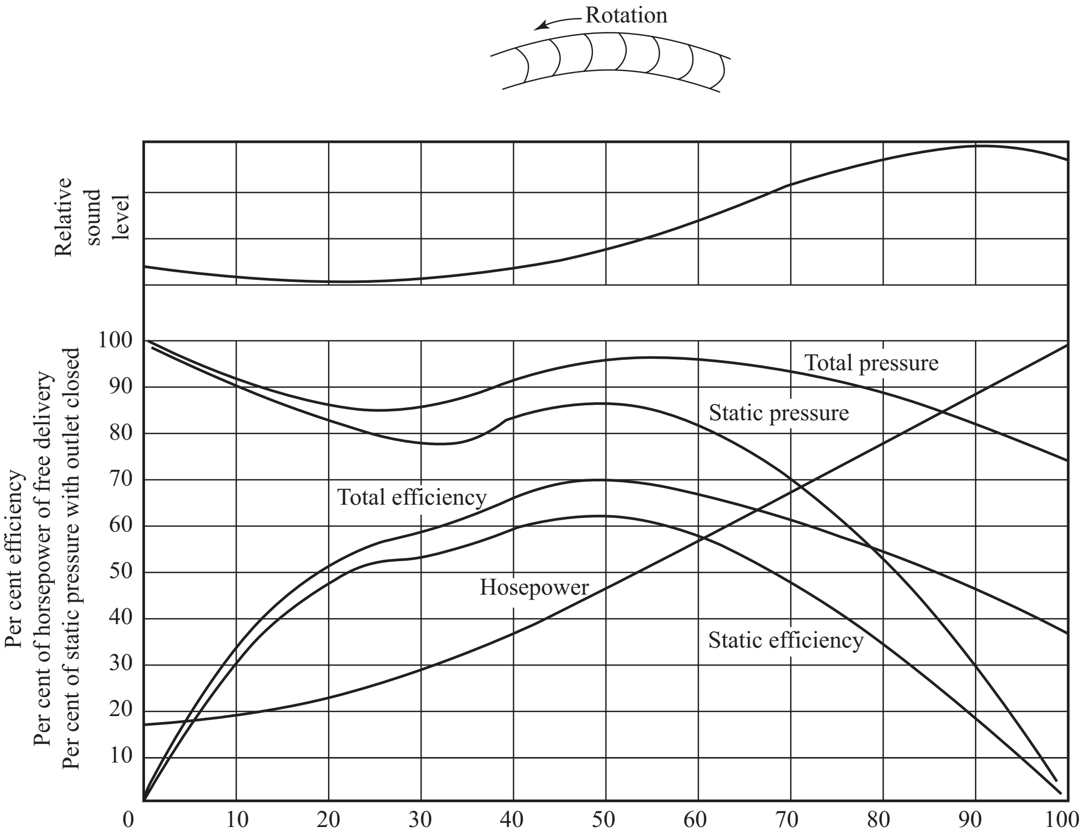


Figure 6.26 *Performance Characteristics of Forward Curved Centrifugal Fan*

The advantage of the forward curve fan is its low speed and quiet operation. The light construction results in a low cost fan and its relatively high airflow results in a small fan requiring minimum space, making it ideal for the residential and commercial heating and cooling market. Disadvantages are its high power requirements at or near free delivery.

(b) Backward Curved Centrifugal Fan

Figure 6.27 shows the performance characteristics of a backward curved centrifugal fan. Backward inclined fans are used to deliver medium to high airflow at static pressures up to 508 mm water gauge. The normal selection range for quiet, efficient performance is from 40% to 85% of free delivery. The power increases to a maximum as airflow increases, and then drops off again toward free delivery. This means that a motor selected to accommodate the peak power will not overload, despite variations in the system resistance or airflow, as long as the fan speed remains constant.

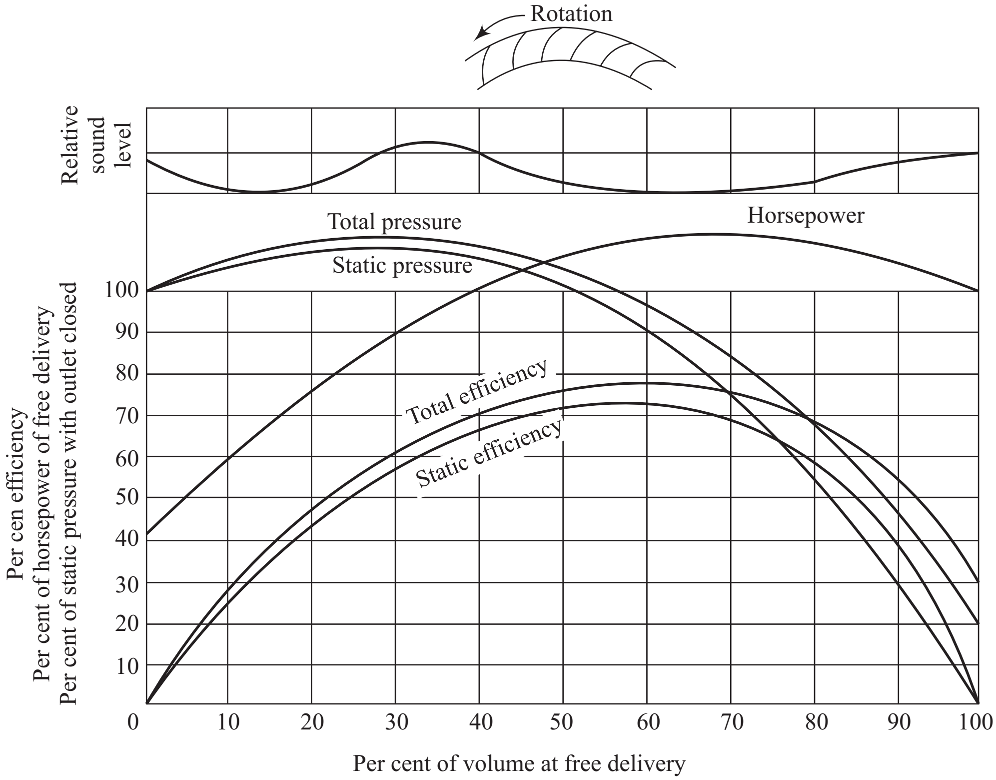


Figure 6.27 *Performance Characteristics of Backward Curved Centrifugal Fan*

The air leaving the backward inclined impeller has less of its total energy in the form of velocity pressure than does the air leaving a forward curve impeller. Because more of its energy is in the form of static pressure, a backward inclined impeller loses less energy in the process of converting from velocity pressure to static pressure in the housing.

(c) Radial Tip Centrifugal Fan

Figure 6.28 shows the performance characteristics of a radial tipped centrifugal fan. These fans are generally selected to operate from 35% to 80% of free delivery. They are ideal for high static pressure applications. Fan efficiencies are lower than both the forward curved and backward inclined type, but this is generally offset by their ability to adapt to harsh environments. These are medium speed fans and are used to deliver low air volumes at medium to high pressure. The impeller design and high operating velocities exhibit the same rising power characteristic as the forward curved fans.

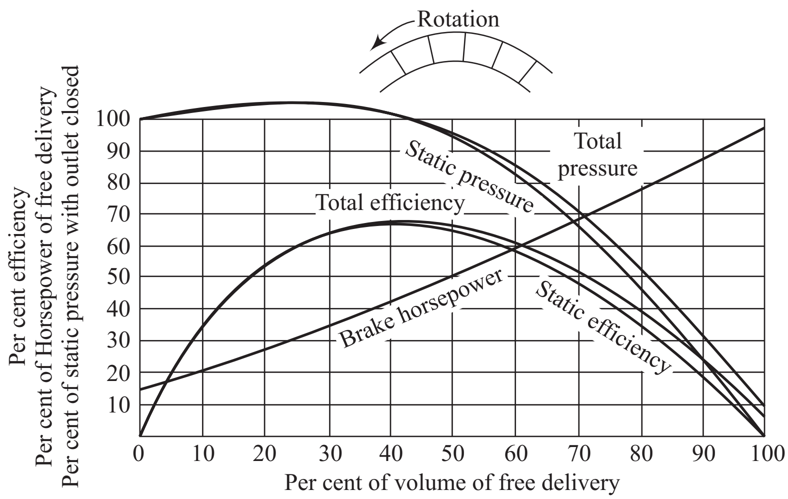


Figure 6.28 *Performance Characteristics of Radial Tipped Centrifugal Fan*

6.13 Performance Characteristics of Axial Fan

Figure 6.29 shows characteristics of an axial fan. Axial flow fans are high specific speed machines with high efficiencies. The efficiency and delivery pressure fall when the fan operates at high flow rates, i.e. on the right of the point *s* in the stable range. The flow through the fan can be varied either through a valve control or by regulating guide vanes upstream or downstream of the fan. When the flow rate through the fan is reduced below the value corresponding to the peak (point *s*) of the performance curve, the flow becomes unstable .

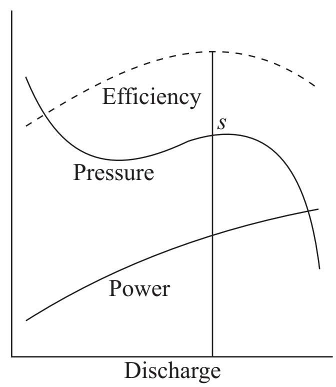


Figure 6.29 *Characteristics of An Axial Fan*

6.14 Fan and System

Combining fans in series or parallel can achieve the desired airflow without greatly increasing the system package size or fan diameter.

6.14.1 Fans in Parallel

Parallel operation is defined as having two or more fans blowing together side by side. The performance of two fans in parallel will result in doubling the volume flow, but only at free delivery. As Figure 6.30 shows, when a system curve is overlaid on the parallel performance curves, the higher the system resistance, the less increase in flow results with parallel fan operation. Thus, this type of application should only be used when the fans can operate in a low resistance almost in a free delivery condition. Series operation can be defined as using multiple fans in a push-pull arrangement.

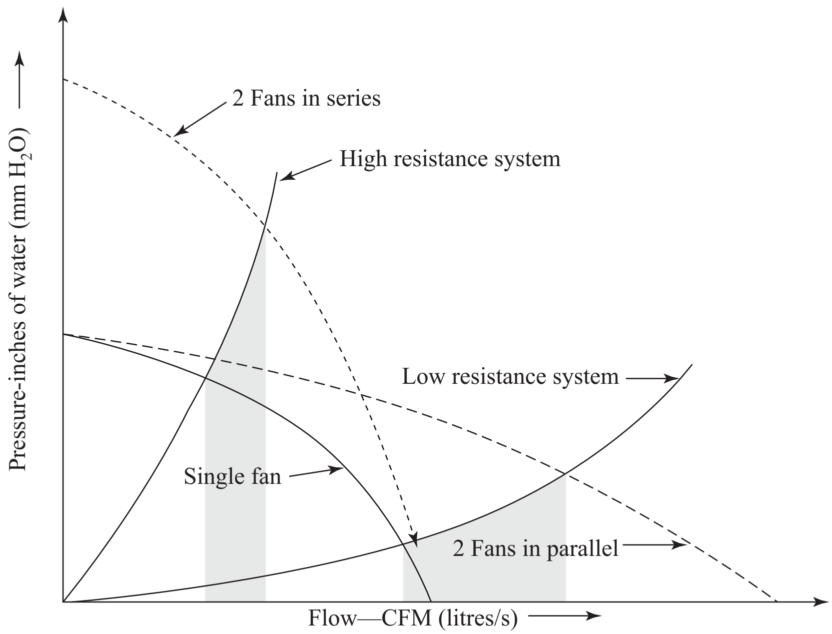


Figure 6.30 *Characteristics of Fan in Series and Parallel*

6.14.2 Fan in Series

By staging two fans in series, the static pressure capability at a given airflow can be increased, but again, not to double at every flow point, as Figure 6.30 shows. In series operation, the best results are achieved in systems with high resistances. In both series and parallel operation, particularly with multiple fans, certain areas of the combined performance curve will be unstable and should be avoided. This instability is unpredictable and is a function of the fan and motor construction and the operating point.

6.15 Fan Laws

Generally, fan performance is described as the characteristic curves of pressure, efficiency and shaft power versus airflow for particular values of fan speed, air density and fan dimensions. It is advantageous to find the operating characteristic of the fan at other speeds and air densities by using affinity laws. Affinity laws are also useful to predict the performance of larger fans from results obtained by conducting tests on geometrically smaller models. Euler's equation and stage parameters (discussed earlier) can be used to find useful relations known as the fan laws.

6.15.1 Fan Pressure Law

From Euler's equation, the pressure rise by a fan without any whirl at inlet is,

$$(\Delta p) = \rho C_b C_{w2}$$

However, both the peripheral speed, C_b, and the rotational (whirl) component of outlet velocity, C_{w2}, vary with the speed of rotation N and the impeller diameter D. Hence,

$$(\Delta p) \propto (\mathrm{ND})(\mathrm{ND})$$

$$\frac{(\Delta p)}{N^2 D^2} = \text{constant} \tag{6.145}$$

The fan pressure law applies for both static and stagnation pressure.

6.15.2 Fan Air Flow Law

For a fan, the discharge is,

$$Q = \pi DBC_f$$

However, for a fan,

$$B \propto D \text{ and } C_f \propto C \propto C_b \propto ND\text{, therefore,}$$

$$Q \propto ND^3$$

$$\frac{Q}{ND^3} = \text{constant} \tag{6.146}$$

From Euler's equation, it is clear that fan pressure varies directly with air density. However, we normally accept volume flow rate, i.e. discharge Q, rather than mass flow rate as the basis of flow measurement in fans. In other words, if the density changes, we still compare operating points at corresponding values of discharge.

6.15.3 Fan Air Power Law

For incompressible flow, the mechanical power transmitted from the impeller to the air is given as,

$$P = (\Delta p_0)_{\text{stage}}\, Q$$

$$P \propto N^3 D^5$$

$$\frac{P}{N^3 D^5} = \text{constant} \tag{6.147}$$

6.15.4 Application of Fan Laws to Geometrically Similar Fans

In the practical utilization or design of fans, we are generally interested in varying only one of the independent variables (speed, air density, impeller diameter) at a particular time while keeping the other two constant. The fan laws are then be summarized as shown in Table 6.1.

TABLE 6.1 Application of Fan Laws with Variation of One Independent Variable

Variable Speed (N)	*Variable Diameter (D)*	Variable Density (ρ)
$\Delta p \propto N^2$	$\Delta p \propto D^2$	$\Delta p \propto \rho$
$Q \propto N$	$Q \propto D^3$	Q = constant
$P \propto N^3$	$P \propto D^5$	$P \propto \rho$

These laws can be applied to compare the performance of a given fan at changed speeds or air densities, or to compare the performance of different sized fans provided that those two fans are geometrically similar.

If the two sets of operating conditions or the two geometrically similar fans are identified by subscripts 1 and 2, then the fan laws may be written, more generally, as the following equations.

$$\frac{(\Delta p)_1}{(\Delta p)_2} = \frac{N_1^2 D_1^2 \rho_1}{N_2^2 D_2^2 \rho_2} \tag{6.148}$$

$$\frac{Q_1}{Q_2} = \frac{N_1 D_1^3}{N_2 D_2^3} \tag{6.149}$$

$$\frac{P_1}{P_2} = \frac{N_1^3 D_1^5 \rho_1}{N_2^3 D_2^5 \rho_2} \tag{6.150}$$

6.16 Fan Noise

Fan is considered as a source of noise, as it creates disturbances in an elastic medium. Noise is a random phenomenon and certainly the majority of the sound emitted by a fan is random both with respect to frequency and time. The noise propagated from the fan inlet or outlet is predominantly the noise of aerodynamic origin which may have a number of components, such as:

(a) Thickness noise due to the passage through the air of blades of finite thickness;
(b) Noise due to forces exerted by the fan blades on the air;
(c) Rotation noise due to the passage of blades past any fixed point resulting in sound at the blade passage frequency (number of blades × speed of rotation) and harmonic multiples of this frequency;
(d) Vortex shedding noise due to flow separation from solid/air boundaries in decelerating flow;
(e) Air turbulence noise due to shear forces in fluid regions remote from boundaries; and
(f) Interference noise due to contact by turbulent wakes on obstructions and guide vanes, the interference by rotating wakes from blades on closely spaced guide vanes also causing discrete frequency components.

The most prominent source of sound in ventilating fans is that due to interaction of fluid forces with solid boundaries mentioned in (d) above. Sound power due to this source can be expressed by,

$$W \propto \rho_0^{(5/2)} N^6 D^5 f(Re), \tag{6.151}$$

where D is the impeller diameter. D is typical dimension for a range of homologous fans. Impeller peripheral velocity, $C_b = \pi DN$ and speed of sound in air, $C = (\gamma p_0/\rho_0)^{(1/2)}$.

Using the fans laws, i.e. volume flow, $Q \propto ND^3$ and fan pressure, $p \propto p_0 N^2 D^2$,

For average ambient air conditions,

$$W \propto p^{(5/2)} Qf(R_e) \tag{6.152}$$

or since air power and fan power p, Q and ignoring $f(R_e)$ over a moderate range of variation,

$$W \propto (\text{fan power}) \times (\text{fan pressure})^{(3/2)} \tag{6.153}$$

Sound power level in dB can be calculated using the equation: 10 log $(W/10^{-12})$.

EXAMPLE 6.7 A fan having a duty of 9 m^3/s at a pressure of 900 Pa has a sound power level of 100 dB.

Assuming that sound power varies as tip speed[5], find the specific sound power level for the homologous series to which fan belongs.

Solution

The term specific sound power level may be applied to a fan in the same sense as specific speed. Therefore, from Eqs. (6.150) to (6.152), it can be seen if sound power varies as tip speed[6], sound power W varies $p^{5/2}$ $Qf(Re)$. By the same arguments, if W ∝ (fan power) × (fan pressure)$^{(3/2)}$, then $p \propto C_b^2$ and $Q \propto N$, $W \propto p^2Q$. Over a moderate range of volume and pressure, then $f(Re)$ term may be neglected. Taking specific sound power W_s to be the sound power of a fan in the same homologous series which gives a duty of 0.0005 m^3/s at a pressure of 249 Pa.

$$\frac{W_s}{W} = \frac{31}{p^2Q} \Rightarrow W_s = \frac{31W}{p^2Q} = \frac{W}{0.03p^2Q}$$

Specific sound power level = 10 log $(W_s/10^{-12})$ = 10 log $(W/10^{-12})$-10 log $(0.03p^2Q)$ dB.

$$L_s = L_w + 15 - 20 \log p - 10 \log Q \text{ dB}$$

and for the series of fans given,

$$L_s = 100 + 15 - 20 \log 900 - 10 \log 9$$

$$L_s = 46.4 \text{ dB}$$

SUMMARY

- ✦ Application of Euler's turbomachinery equation for axial and centrifugal fans are expressed as, $w = C_b(C_{w2} - C_{w1})$ and $w = C_{b2}C_{w2} - C_{b1w1}$, respectively.
- ✦ Different stage parameters: mass flow rate, stage work, stage pressure rise, stage pressure coefficient, degree of reaction and stage efficiency for both axial and centrifugal fans are summarized in the following table.

Stage Parameters	*Axial Compressor*	*Centrifugal Compressor*
Mass flow rate	$\dot{m} = \rho\frac{\pi}{4}D_1^2C_{f1} = \rho\frac{\pi}{4}D_2^2C_{f2}$	$\dot{m} = \rho\pi D_1B_1C_{f1} = \rho\pi D_2B_2C_{f2}$
Stage work	● With upstream guide blades $w = C_b^2\varphi\,(\tan\beta_1 - \tan\beta_2)$ ● Without guide blades $w = C_b^2\,(1 - \varphi\tan\beta_2)$	$w = C_{w2}C_{b2}$

	• With downstream guide blades $w = C_b^2(1-\varphi \tan\beta_2)$ • Counter rotating stage $w = 2C_b^2(1-\varphi \tan\beta_2)$	
Stage pressure rise	• With upstream guide blades $(\Delta p_0)_{stage} = \rho C_b^2 \varphi(\tan\beta_1 - \tan\beta_2)$ • Without guide blades $(\Delta p)_{stage} = \rho \dfrac{C_b^2}{2}(1-\varphi^2\tan^2\beta_2)$ • With downstream guide blades $(\Delta p_0)_{stage} = \rho C_b^2(1-\varphi\tan\beta_2)$ • Counter rotating stage $(\Delta p)_{stage} = 2\rho C_b C_{w2}$	$(\Delta p_0)_{stage} = \rho C_{b2}[C_{b2} - (Q/\pi D_2 B_2)\cot\beta_2]$ $(\Delta p_0)_{stage} = \rho C_{b2}^2(1-\varphi\cot\beta_2)$
Stage pressure coefficient	• With upstream guide blades $\psi = 2\varphi(\tan\beta_1 - \tan\beta_2)$ • Without guide blades $\psi = (1-\varphi^2\tan^2\beta_2)$ • With downstream guide blades $\psi = 2(1-\varphi\tan\beta_2)$ • Counter rotating stage $\psi = 4(1-\varphi\tan\beta_2)$	• Backward and forward curved blades $\psi = 2(1-\varphi\cot\beta_2)$ • Radial blades $\psi = 2$
Degree of reaction	• With upstream guide blades $R = \dfrac{[C_{w1}^2 - C_{w2}^2 - 2C_b(C_{w1}-C_{w2})]}{[2C_b(C_{w2}-C_{w1})]}$ • Without guide blades $R = \dfrac{(1+\varphi\tan\beta_2)}{2}$ • With downstream guide blades $R = \dfrac{(1+\varphi\tan\beta_2)}{2}$	$R = 1 - \dfrac{C_{w2}}{2C_{b2}}$ • For backward curved vanes ($\beta_2 < 90°$) $\dfrac{C_{w2}}{C_{b2}} < 1$, therefore $R < 1$ • For radial vanes ($\beta_2 = 90°$) $\dfrac{C_{w2}}{C_{b2}} = 1$, therefore $R = \dfrac{1}{2}$ • For forward curved vanes ($\beta_2 > 90°$) $\dfrac{C_{w2}}{C_{b2}} > 1$, therefore $R < \dfrac{1}{2}$
Fan efficiency	• With upstream guide blades $\eta_{f\,total} = \dfrac{(\Delta p_0)_{stage}}{\rho C_b(C_{w2}-C_{w1})}$ • Without guide blades $\eta_{f\,total} = \dfrac{(\Delta p_0)_{stage}}{(\rho C_b C_{w2})}$	$\eta_{st} = \dfrac{(\Delta p_0)_{stage}}{\rho C_{b2} C_{w2}}$

	• With downstream guide blades $\eta_{f\text{ total}} = \frac{(\Delta p_0)_{\text{stage}}}{\rho C_b C_{w2}}$ • Counter rotating stage $\eta_{f\text{ total}} = \frac{(\Delta p_0)_{\text{stage}}}{2\rho C_b C_{w2}}$	

✦ The performance characteristics of axial and centrifugal fans are described.

✦ The slip, losses, affinity laws and fan noise are described.

REVIEW QUESTIONS

6.1 What is meant by a fan?

6.2 List the types of fans, their characteristics and typical applications.

6.3 What is the fan total power and fan static power?

6.4 What is the fan total efficiency and fan static efficiency?

6.5 State the specific differences between fan, blower and compressor.

6.6 What is the fan total pressure, fan static pressure and fan velocity pressure?

6.7 What are the various classifications of a fan?

6.8 Differentiate between centrifugal and axial fans.

6.9 Prove that static pressure rise across the fan is the summation of the centrifugal part, $(C_{b2}^2 - C_{b1}^2)/2$ and the relative kinetic energy part, $(C_{r1}^2 - C_{r2}^2)/2$.

6.10 Prove that specific work for a fan is given by,

$$w = \frac{p_2 - p_1}{\rho} + \frac{C_2^2 - C_1^2}{2}$$

6.11 Compare the forward curved, backward curved and radially tipped centrifugal fans.

6.12 Draw the velocity triangles for an axial fan stage with upstream guide vanes.

6.13 Prove that for an axial fan stage with upstream guide blades,

$$w = (\Delta h_0)_{\text{stage}} = C_b(C_{w2} - C_{w1}) = C_b^2\varphi\,(\tan\beta_1 - \tan\beta_2)$$

where φ is the flow coefficient and β_1 and β_2 are the blade angles at inlet and outlet of the impeller.

6.14 Prove that stage pressure rise for an axial fan stage with upstream guide vanes is,

$$(\Delta p_0)_{\text{stage}} = \rho C_b^2\varphi\,(\tan\beta_1 - \tan\beta_2)$$

6.15 Derive an expression for stage pressure coefficient for an axial fan stage with upstream guide vanes.

6.16 Prove that degree of reaction for an axial fan stage with upstream guide vanes is,

$$R = \frac{[C_{w1}^2 - C_{w2}^2 - 2C_b(C_{w1} - C_{w2})]}{[2C_b(C_{w2} - C_{w1})]}$$

6.17 Derive an expression for fan total efficiency of an axial fan stage with upstream guide vanes.

6.18 Prove that specific work in an axial fan stage without guide vanes is,

$$w = (\Delta h_0)_{\text{stage}} = C_b^2\,(1 - \varphi \tan \beta_2)$$

6.19 Show that the static pressure rise in the stage and impeller of an axial fan stage without guide vanes is equal and is given by,

$$(\Delta p)_{\text{stage}} = (\Delta p)_{\text{rotor}} = \rho \frac{C_b^2}{2}(1 - \varphi^2 \tan^2 \beta_2)$$

6.20 Prove that degree of reaction for an axial fan stage without guide vanes is,

$$R = \frac{(1 + \varphi \tan \beta_2)}{2}$$

6.21 Derive an expression for fan total efficiency of an axial fan stage without any guide vanes.

6.22 Draw the velocity triangles for an axial fan stage with downstream guide vanes.

6.23 What is the effect of providing downstream guide vanes on the stage parameters of an axial fan as compared to stage without guide vanes?

6.24 Show that the stage pressure coefficient for an axial fan stage with downstream guide vanes is, $\psi = 2(1 - \varphi \tan \beta_2)$

6.25 Draw the velocity triangles for counter rotating fans stage.

6.26 Prove that pressure rise in the counter rotating axial fan stage is, $(\Delta p)_{\text{stage}} = 2\rho C_b C_{w2}$.

6.27 Show that for a counter rotating axial fan stage, stage pressure coefficient is, $\psi = 4(1 - \varphi \tan \beta_2)$.

6.28 Derive an expression for the power required for a counter rotating axial fan stage.

6.29 Draw the velocity triangles for a centrifugal fan stage.

6.30 Prove that power coefficient for a centrifugal pump stage is,

$$\lambda = 2\varphi(1 - \varphi \cot \beta_2)$$

6.31 Prove that the power coefficient is two times the flow coefficient for a centrifugal fan having radial vanes.

6.32 Prove that degree of reaction for a centrifugal fan is,

$$R = 1 - \frac{C_{w2}}{2C_{b2}}$$

6.33 Show that degree of reaction for a backward curved centrifugal fan is less than 1.

6.34 Show that degree of reaction for a centrifugal fan having radial blades is 1/2.

6.35 Show that degree of reaction for a forward curved centrifugal fan is less than 1/2.

6.36 Prove that theoretical head of a fan having slip is,

$$H_{\text{th}} = \frac{C_{b2}\left(C_{w2} - \dfrac{\pi C_{b2} \sin \beta_2}{z}\right)}{g}$$

6.37 What are the various losses that occur in a fan stage?

6.38 State the affinity laws governing the performance of a fan.

6.39 Draw and discuss the performance characteristics of a centrifugal fan.

6.40 Draw and discuss the performance characteristics of an axial fan.

6.41 Explain with the help of fan and system curves that why fans are connected in series?

PROBLEMS

6.1 The air at 100 kPa and 27°C enters the impeller of a centrifugal blower without any whirl component. The speed of the blower is 6000 rpm and external diameter of the impeller is 0.6 m. Total-to-total efficiency is 70% and degree of reaction is 0.6. The radial component of velocity is 95 m/s which remains constant throughout. Determine (a) exit blade angle, (b) pressure at outlet, and (c) power input. [*Ans:* (a) β_2 = 68.35°C, (b) p_2 = 125.1 kPa, (c) w = 28.426 kJ/kg]

6.2 Air at a flow rate of 3 kg/s enters in a centrifugal fan at 100 kPa, 20°C from still atmosphere. The fan running at 580 rpm raises the static pressure by 2 kPa. The power input is 44 kW. If the speed is changed to 490 rpm and inlet conditions to 98 kPa, 200°C find (a) static pressure at exit, (b) the mass flow rate, and (c) input power. [*Ans:* (a) p_2 = 98.867 kPa, (b) $\dot{m}$ = 1.539 kg/s, (c) P = 16.11 kW]

6.3 An overhead fan with a vertical shaft is fitted to extract air through a small cooling tower from its top. The speed of the fan is 500 rpm. The tip and hub diameters of the fan blades are 2 m and 0.5 m, respectively. The blade loading is uniform through its length, i.e. specific work is same at any section of the blade. The exit blade angle at tip is 12° and the flow velocity remains constant throughout the rotor at 10.5 m/s. The state of the air at inlet is 1 bar, 15°C, total-to-total efficiency is 85% and mechanical efficiency is 90%. Determine (a) inlet blade angle of the tip, (b) inlet and outlet blade angles at the hub, (c) total pressure rise, (d) mass flow rate, and (e) the Input power.

[*Ans:* (a) β_{1t} = 11.34°, (b) β_{1h} = 38.73°, β_{2h} = 83.2°,
(c) $(\Delta p_0)_{\text{stage}}$ = 0.0022 bar = 22 mm of water, (d) $\dot{m}$ = 37.41 kg/s, (e) P = 6.443 kW]

6.4 Characteristic curves are available for a fan running at 850 rpm and passing air of inlet density 1.2 kg/m^3. Readings from the curves indicate that at an airflow of 150 m^3/s, the fan pressure is 2.2 kPa and the shaft power is 440 kW. Assuming that the efficiency remains unchanged, calculate the corresponding points if the fan is running at 1100 rpm in air of density 1.1 kg/m^3.

[*Ans:* p_{02} = 3.377 kPa, Q_2 = 194.1 m^3/s, P_2 = 874 kW]

6.5 Air flow rate at the inlet of a fan is 30 m^3/s. The fan develops a pressure of 2.5 kPa. The barometric pressure at the fan inlet is 97 kPa. The motor consumes an electrical power of 1100 kW. If the combined motor/transmission efficiency is 95%, determine the isentropic efficiency of the impeller and, also, of the total unit. [*Ans:* $(\eta_s)_{\text{rotor}}$ = 71.2%, $(\eta_s)_o$ = 67.3%]

6.6 A centrifugal fan is used for air circulation in a water cooling tower. The fan running at 1200 rpm delivers 16 m^3/s of air against a stagnation pressure difference of 2.8 kPa. This operating condition represents the design point at which there are no shock losses and no whirl at inlet. The fan sucks air of density 1.2 kg/m^3 from the atmosphere. The air enters the impeller radially (with no pre-whirl) at the design point. The inlet and exit diameters of the impeller are 0.9 m and 1.35 m respectively. The velocity of flow is constant through the impeller at 20 m/s. Take η_v = 96%, η_{vane} = 88%, η_h = 85%,

η_m = 95%. Determine (a) vane angles at inlet and at exit, (b) impeller widths at inlet and exit, and (c) fan power consumption.

[*Ans:* (a) β_1 = 19.5°, β_2 = 22.6°, (b) B_1 = 295 mm, B_2 = 197 mm, (c) P = 57.8 kW]

6.7 A centrifugal fan equipped with inlet guide vanes is running at a speed of 750 rpm. The fan is designed to deliver air of density 1.2 kg/m^3 at a rate of 4.25 m^3/s. The flow has no pre-rotation at inlet. The diameter of impeller at inlet and outlet is 0.525 m and 0.75 m, respectively. The width of the blades at inlet is 172 mm while that at outlet is 100 mm. The blade angle at exit is 70°. The pressure recovery in the volute casing is 40% of the actual velocity head at the impeller exit and the leakage is negligibly small. The blade, hydraulic and mechanical efficiencies are 88%, 85% and 96%, respectively. Determine (a) the actual velocity and pressure at the fan discharge section, and (b) fan brake power. [*Ans:* (a) C_2 = 21 m/s, P_2 = 340 kPa (gauge), (b) P = 3.15 kW]

6.8 Assume that the fan given in Example 6.7 is now equipped with upstream guide vanes for flow rate control. The flow rate is required to be reduced to 3.2 m^3/s by using upstream guide vanes to impose pre-whirl. What should be the flow angle at the inlet? In this case, the hydraulic efficiency is reduced to 80% due to the increase in shock losses. What will be the new brake power?

[*Ans:* α_1 = 66°, P = 2.18 kW]

6.9 The tip and hub diameters of the rotor of an axial fan stage are 0.6 m and 0.3 m respectively. The speed of the fan is 960 rpm and the blade angle at exit is 10°. The flow conditions at the inlet of the fan are 1.02 bar and 43°C. The flow coefficient is 0.245 and power required by the fan is 1 kW. Find (a) blade angle at inlet, (b) discharge, (c) stage pressure rise, (d) overall efficiency, (e) degree of reaction, and (f) specific speed.

[*Ans:* (a) Q = 1.175 m^3/s, (b) β_1 = 76.23°, (c) $(\Delta p_0)_{stage}$ = 56.14 mm of water, (d) η_o = 64.7%, (e) R = 0.52, (f) N_{sh} = 1.047]

6.10 Calculate all the parameters if the fan of Problem 6.9 is provided with downstream guide vanes. What is the guide blade angle at inlet?

[*Ans:* (a) Q = 1.175 m^3/s, (b) β_1 = 76.23°, (c) $(\Delta p_0)_{stage}$ = 56.14 mm of water, (d) η_o = 63.7%, (e) R = 0.52, (f) N_{sh} = 1.047, α_3 = 75.63°]

6.11 The upstream guide blades are provided in the fan of Problem 6.9 for negative swirl. The blade angle at inlet is 86°. Calculate (a) the static pressure rise in the rotor and the stage pressure rise, (b) stage pressure coefficient, (c) degree of reaction, (d) exit air angle of the rotor blades and upstream guide blades, and (e) power input if the overall efficiency of the drive is 64.7%.

[*Ans:* (a) $(\Delta p)_{rotor}$ = $(\Delta p_0)_{stage}$ = 146.86 mm of water, (b) ψ = 5.006, (c) R = 2.25, (d) β_2 = 76.23°, α_1 = 84.4°, (e) P = 2.629 kW)]

6.12 The impeller and upstream guide vanes of Example 6.9 are symmetrical and degree of reaction is 0.5. The blade angles at inlet and outlet are 30° and 10° respectively. Determine (a) the stage pressure rise, (b) pressure coefficient, and (c) power required if the fan efficiency is 80% and drive efficiency is 88%. [*Ans:* (a) $(p_0)_{stage}$ = 4.61 mm of water, (b) ψ = 0.196, (c) P = 0.075 kW]

6.13 The diameter of an open air propeller fan is 0.5 m. The velocities on the upstream and downstream of the fan are 5 m/s and 25 m/s, respectively. The overall efficiency of the fan is 40% and ambient conditions are 1.02 bar and 37°C. Determine (a) mass flow rate, (b) total pressure developed by the fan, and (c) power input. [*Ans:* (a) $\dot{m}$ = 3.37 kg/s, (b) $(\Delta p_0)_{stage}$ = 35.04 of water, (c) P = 2.52 kW]

MULTIPLE CHOICE QUESTIONS

1. The parameter used by ASME to define fans, blowers and compressors is
   (a) Fan Ratio (b) Specific Ratio (c) Blade Ratio (d) Twist Factor
2. Which of the following axial fan types is most efficient?
   (a) Propeller (b) Tube axial (c) Vane axial (d) Radial
3. Which of the following is not a centrifugal fan type?
   (a) Vane axial (b) Radial
   (c) Airfoil type, backward curved (d) Forward curved
4. Match the List 1 and List 2 for centrifugal fan types and select the correct answer using the given codes.

List 1	**List 2**
1. Backward curved	A. High pressure, medium flow
2. Forward curved	B. Medium pressure, high flow
3. Radial	C. High pressure, high flow

Codes

	1	**2**	**3**
(a)	A	C	B
(b)	C	B	A
(c)	A	B	C
(d)	B	C	A

5. The relation between discharge and speed of a fan is
   (a) $\frac{Q_1}{Q_2} = \frac{N_1}{N_2}$ (b) $\frac{Q_1}{Q_2} = \frac{N_1^2}{N_3^2}$ (c) $\frac{Q_1}{Q_2} = \frac{N_1^3}{N_2^3}$ (d) None of these
6. The choice of a fan depends on
   (a) Flow (b) Static pressure (c) Both (a) and (b) (d) None of these
7. The efficiency of backward curved fans compared to forward curved fans is
   (a) Higher (b) Lower
   (c) Nothing can be said (d) Same
8. For high pressure applications, the more suitable fan is
   (a) Propeller type fans (b) Tube axial fans
   (c) Backward curved centrifugal fans (d) None of these
9. Axial fans are best suited for
   (a) Large flow, low head (b) Low flow, high head
   (c) High head, large flow (d) Low flow, low head
10. The efficiency of forward curved fans as compared to backward curved fans is
    (a) Lower (b) Higher (c) Same (d) None of these
11. Vane axial fans have the efficiency in the range of
    (a) 78–85% (b) 60–70% (c) 90–95% (d) 50–60%

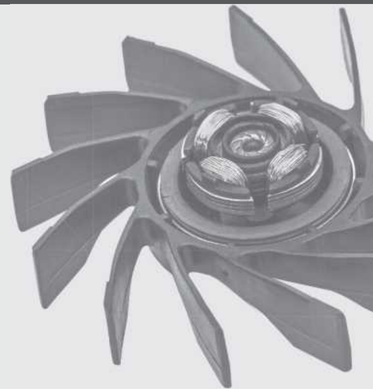
12. The efficiency of the backward curved fans lie in the range of
    (a) 65–70% (b) 75–85% (c) 90–95% (d) 50–60%
13. The pressure to be considered for calculating power required for centrifugal fans is
    (a) Discharge static pressure (b) Static plus dynamic pressure
    (c) Total static pressure (d) Static plus ambient pressure
14. The efficiency of aerofoil fan handling clean air lie in the range of
    (a) 40–50% (b) 80–90% (c) 60–70% (d) 70–80%
15. The clearance required for efficient operation of impeller of 1 meter plus diameter in radial type fans is
    (a) 5–10 mm (b) 1–2 mm (c) 20–30 mm (d) 0.5–1.5 mm
16. Which type of control gives maximum benefits for fans from maximum saving in energy point of view?
    (a) Discharge damper control (b) Inlet guide vane control
    (c) Variable pitch control (d) Speed control
17. Axial flow fans are equipped with
    (a) Fixed blades (b) Curved blades (c) Flat blades (d) Variable pitch blades
18. The ratio of maximum to minimum flow rate is called
    (a) Turn-up ratio (b) Turn-down ratio (c) Up-down ratio (a) None of these

ANSWER KEY

1. (b)	2. (c)	3. (a)	4. (b)	5. (a)	6. (c)	7. (a)	8. (c)	9. (a)	10. (a)
11. (a)	12. (b)	13. (c)	14. (b)	15. (c)	16. (d)	17. (d)	18. (b)		

7

Axial and Centrifugal Compressors



Learning Objectives

After reading this chapter, you will be able to:

- **LO 1** Explain the applications of Euler's turbomachinery equation for centrifugal and axial compressors
- **LO 2** Demonstrate the specific work on h-s (Mollier) diagram for centrifugal and axial compressors
- **LO 3** Illustrate the velocity triangles for a stage for centrifugal and axial compressors
- **LO 4** Evaluate different stage parameters, both for centrifugal and axial compressors
- **LO 5** Understand and compute the losses and efficiency of axial and centrifugal compressors
- **LO 6** Describe the performance characteristics of compressors including choking, surging and stalling phenomenon

7.1 Introduction

Roto-dynamic compressors are functionally similar to fans and blowers. However, the static pressure rise in compressors is much larger and is generally a few bars. In contrast with the fans, the velocity (Mach number) in the compressor is much larger and the flow should be considered as compressible. The density of fluid varies from the inlet to the outlet of the compressor. The compressor could be centrifugal or axial. In this chapter, both these types are discussed.

7.2 Specific Work

Referring to Euler's turbomachinery Eq. (1.82), expression for the specific work of a centrifugal compressor is,

$$w = C_{b2}C_{w2} - C_{b1}C_{w1} \tag{7.1}$$

and for an axial compressor (as $C_{b1} = C_{b2} = C_b$), the expression for specific work is given by,

$$w = C_b(C_{w2} - C_{w1}) \tag{7.2}$$

Considering the velocity triangle relations [refer Figure 1.19 and Eq. (1.98)], Eq. (7.2) will be modified as,

$$w = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \tag{7.3}$$

for a centrifugal compressor, and

$$w = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \tag{7.4}$$

for an axial compressor.

Applying the steady flow energy equation for a turbomachine control volume [Eq. (1.49)] to a compressor, the specific work done on the control volume is also given by,

$$w = (h_2 - h_1) + \frac{(C_2^2 - C_1^2)}{2} = (h_{02} - h_{01}) \tag{7.5}$$

By combining Eq. (7.5) with Eq. (7.3), the stagnation or total enthalpy difference is the specific work. Thus,

$$w = (h_{02} - h_{01}) = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \tag{7.6}$$

By combining Eq. (7.6) with Eq. (7.5), the static enthalpy difference is expressed as,

$$(h_2 - h_1) = \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2} \tag{7.7}$$

7.2.1 Isentropic Pressure and Temperature Ratio

The isentropic compression process is represented on $T - s$ diagram for a perfect gas ($h = c_pT$), as shown in Figure 7.1.

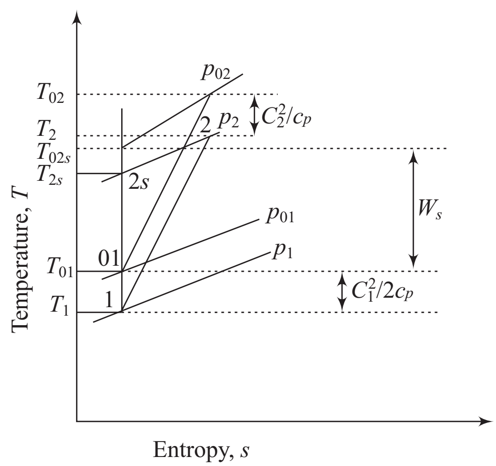


Figure 7.1 *Representation of Compression Process on T-s diagram*

If the compression process is isentropic, then the isentropic work is given by,

$$w_s = (h_{02s} - h_{01}) = c_p(T_{02s} - T_{01}) \tag{7.8}$$

$$T_{02s} - T_{01} = \frac{(C_2^2 - C_1^2)}{2c_p} + \frac{(C_{b2}^2 - C_{b1}^2)}{2c_p} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_p} \tag{7.9}$$

$$\frac{(T_{02s} - T_{01})}{T_{01}} = \frac{(C_2^2 - C_1^2)}{2c_pT_{01}} + \frac{(C_{b2}^2 - C_{b1}^2)}{2c_pT_{01}} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_pT_{01}}$$

$$\frac{T_{02s}}{T_{01}} - 1 = \frac{(C_2^2 - C_1^2)}{2c_pT_{01}} + \frac{(C_{b2}^2 - C_{b1}^2)}{2c_pT_{01}} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_pT_{01}}$$

Thus, the isentropic total temperature ratio is given by,

$$\frac{T_{02s}}{T_{01}} = 1 + \frac{(C_2^2 - C_1^2)}{2c_pT_{01}} + \frac{(C_{b2}^2 - C_{b1}^2)}{2c_pT_{01}} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_pT_{01}} \tag{7.10}$$

Referring to Figure 7.1, the relation for isentropic process (01-02s) is,

$$\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \tag{7.11}$$

The total pressure ratio is therefore given by,

$$\frac{p_{02}}{p_{01}} = \left[1 + \frac{(C_2^2 - C_1^2)}{2c_pT_{01}} + \frac{(C_{b2}^2 - C_{b1}^2)}{2c_pT_{01}} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_pT_{01}}\right]^{\frac{\gamma}{\gamma-1}} \tag{7.12}$$

Further, from Eq. (7.9), we have,

$$T_{2s} - T_1 = \frac{(C_{b2}^2 - C_{b1}^2)}{2c_p} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_p} \tag{7.13}$$

Hence, the isentropic static temperature ratio, T_{2s}/T_1 is given by,

$$\frac{T_{2s}}{T_1} = 1 + \frac{(C_{b2}^2 - C_{b1}^2)}{2c_pT_1} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_pT_1} \tag{7.14}$$

Static pressure ratio for the process (1 – 2*s*) is,

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{p_2}{p_1} = \left[1 + \frac{(C_{b2}^2 - C_{b1}^2)}{2c_pT_1} + \frac{(C_{r1}^2 - C_{r2}^2)}{2c_pT_1}\right]^{\frac{\gamma}{\gamma-1}} \tag{7.15}$$

The $T - s$ diagram in Figure 7.1 and the relations in Eqs (7.9) to (7.15) are valid for both axial and centrifugal compressors. In the following sections, we will first discuss about the axial compressor and then the centrifugal compressor. Notice that the $T - s$ diagram and the expressions (7.9) to (7.15) are not presented for the fans as the flow through the fans is considered as incompressible.

7.3 Axial Compressor

7.3.1 Velocity Triangles

In a multistage axial flow machine, a rotor and a stator together are called a stage. Figure 7.2 shows an axial compressor stage.

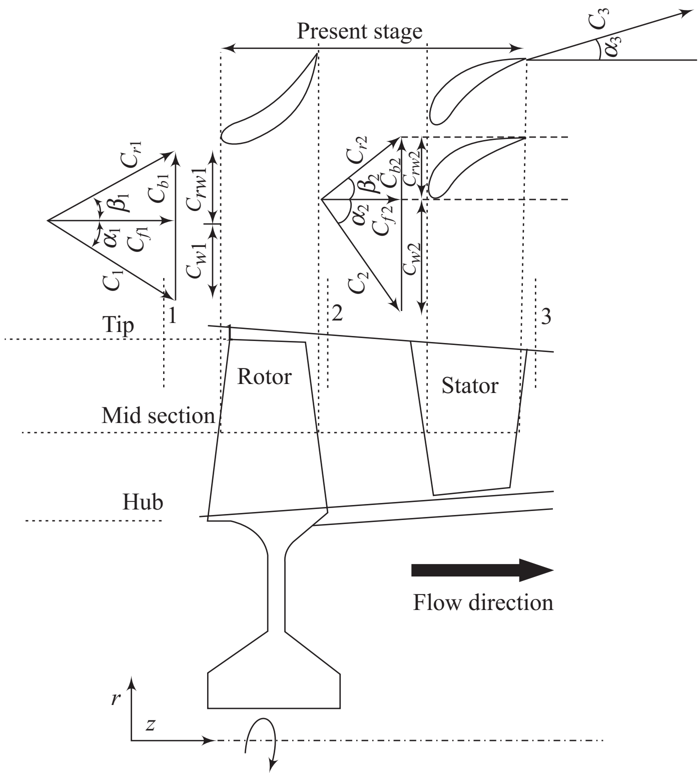


Figure 7.2 *Axial Compressor Stage and its Velocity Triangles*

The absolute velocity entering the rotor from stator of the previous stage is C_1. The absolute velocity leaving the stator of the present stage is C_3. If $C_1 = C_3$ for a stage, then such a stage is called a normal stage or repeating stage. For the positive static pressure rise, $p_2 > p_1$ or $\frac{p_2}{p_1} > 1$; for an axial compressor, $C_{b1} = C_{b2} = C_b$. Equation (7.15) yields that C_{r1} should be greater than C_{r2}.

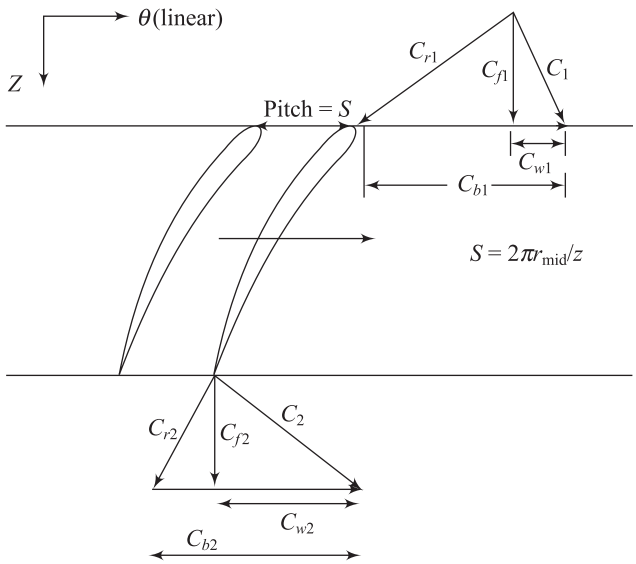


Figure 7.3 *Rotor Cascade and Velocity Triangles*

The velocity triangles at the inlet and outlet of the mid-section (at $r = r_{mid}$) of the rotor are represented by a linear cascade as shown in Figure 7.3. The repeating stage, shown in Figure 7.4, is drawn by adding a stator cascade to the rotor cascade of Figure 7.3. As a special case, the velocity triangles for the repeating stage with the flow entering and exit axially are shown in Figure 7.5. For such a repeating stage, following conditions hold:

$$C_1 = C_{f1},\ C_{b1} = C_{b2} = C_b,\ C_1 = C_3$$

7.3.2 Stage Parameters

(a) Mass Flow Rate

Mass flow through the axial compressor can be estimated by knowing the meridional component of velocity, i.e. axial velocity, as follows:

$$\dot{m} = \rho_1 \pi \frac{D_1^2}{4} C_{f1} = \rho_2 \pi \frac{D_2^2}{4} C_{f2} \tag{7.16}$$

where density, $\rho = \dfrac{p_{01}}{RT_{01}}$ for a perfect gas such as air.

(b) Stage Work

Assuming $C_{f1} = C_{f2} = C_f$, two basic equations follow immediately from the geometry of the velocity triangles from Figure 7.4. These are,

$$\frac{C_b}{C_f} = \tan\alpha_1 + \tan\beta_1 \tag{7.17}$$

$$\frac{C_b}{C_f} = \tan\alpha_2 + \tan\beta_2 \tag{7.18}$$

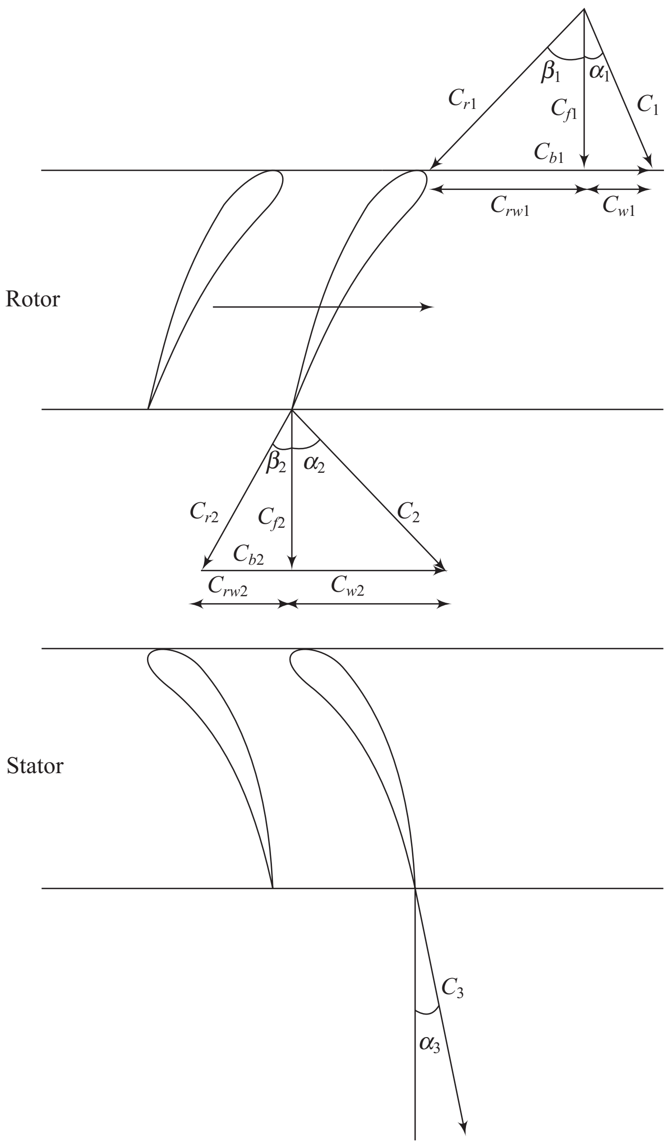


Figure 7.4 *Repeating Stage*

Substituting values of C_{w1} and C_{w2} from velocity triangles of Figure 7.4 in Eq. (7.2), we have,

$$w = C_b C_f (\tan \alpha_2 - \tan \alpha_1) \tag{7.19}$$

Using Eqs (7.17) and (7.18) in Eq. (7.19), specific work in terms of the rotor blade air angles (β_1, β_2) is given by,

$$w = C_b C_f (\tan \beta_1 - \tan \beta_2) \tag{7.20}$$

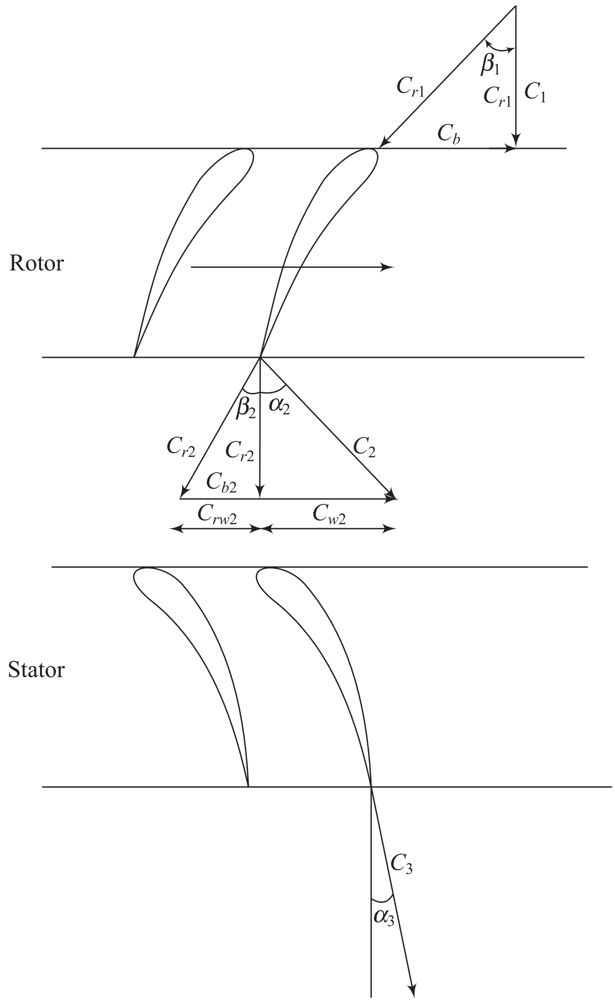


Figure 7.5 *Repeating Stage with Zero Inlet Whirl*

From Figure 7.4, stagnation temperature rise in the stage ($\Delta T_{0\ \text{stage}}$) is given by,

$$\Delta T_{0\ \text{stage}} = T_{03} - T_{01}$$

$T_{03} = T_{02}$, because there is no work input in the stator. Using these equations and Eq. (7.20), stagnation temperature in the stage is,

$$\Delta T_{0\text{stage}} = T_{02} - T_{01} = \frac{w}{c_p} = \frac{C_b C_f}{c_p}(\tan \beta_1 - \tan \beta_2) \tag{7.21}$$

Here, ($\tan \beta_1 - \tan \beta_2$) is called the blade deflection. The higher this quantity, higher is the blade turning and the flow turning in the blade passages.

(c) Degree of Reaction

Degree of reaction provides a measure of the extent to which the rotor contributes to the overall static pressure rise in the stage (rotor and stator).

$$R = \frac{\text{Static enthalpy rise in the rotor}}{\text{Static enthalpy rise in the stage}} = \frac{c_p \Delta T_{\text{rotor}}}{c_p \Delta T_{\text{stage}}} = \frac{c_p \Delta T_{\text{rotor}}}{c_p (\Delta T_{\text{rotor}} + \Delta T_{\text{stator}})} \tag{7.22}$$

Here, $\Delta T_{\text{rotor}} = (T_2 - T_1)$ and $\Delta T_{\text{stator}} = (T_3 - T_2)$ denote the respective values of static temperature rise in the rotor and stator. For a repeating stage, $C_1 = C_3$; hence,

$\Delta T_{\text{repeating stage}} = \Delta T_{0\ \text{stage}}$. Therefore, from Eqs. (7.19) and (7.20), we have,

$$w = c_p \Delta T_{0\ \text{stage}} = c_p \Delta T_{\text{repeating stage}} = c_p(\Delta T_{\text{rotor}} + \Delta T_{\text{stator}}) = C_b C_f(\tan \beta_1 - \tan \beta_2)$$

$$= C_b C_f(\tan \alpha_2 - \tan \alpha_1) \tag{7.23}$$

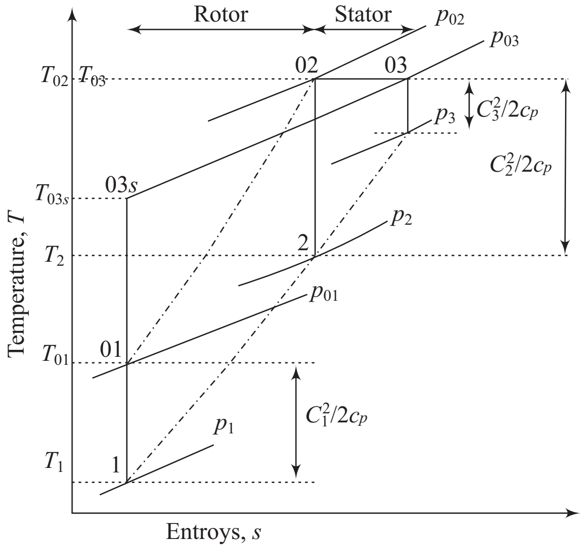


Figure 7.6 *Representation of Compression Process in a Compressor Stage on T-s Diagram*

Figure 7.6 represents a compression process in a compressor stage on $T - s$ diagram. Work input to the stage takes place in the rotor, the steady flow energy Eq. (1.49) yields

$$w = c_p \Delta T_{\text{rotor}} + \frac{1}{2}(C_2^2 - C_1^2) \tag{7.24}$$

From Eqs (7.23) and (7.24),

$$c_p(\Delta T_{\text{rotor}}) = C_b C_f(\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2}(C_2^2 - C_1^2) \tag{7.25}$$

From velocity triangles in Figure7.4,

$$C_1 = C_f \sec \alpha_1,\ C_2 = C_f \sec \alpha_2 \tag{7.26}$$

Using Eq. 7.26 in Eq. 7.25

$$c_p(\Delta T_{\text{rotor}}) = C_b C_f(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}C_f^2(\tan^2\alpha_2 - \tan^2\alpha_1) \tag{7.27}$$

Using Eqs (7.27) and (7.23) in Eq. (7.22), we have,

$$R = \frac{C_b C_f(\tan\alpha_2 - \tan\alpha_1) - \frac{1}{2}C_f^2(\tan^2\alpha_2 - \tan^2\alpha_1)}{C_b C_f(\tan\alpha_2 - \tan\alpha_1)} \tag{7.28}$$

$$R = 1 - \frac{C_f}{2C_b}(\tan\alpha_2 + \tan\alpha_1) \tag{7.29}$$

By adding Eqs (7.17) and (7.18),

$$\frac{2C_b}{C_f} = \tan\alpha_1 + \tan\beta_1 + \tan\alpha_2 + \tan\beta_2 \tag{7.30}$$

From Eqs (7.29) and (7.30),

$$R = 1 - \frac{C_f}{2C_b}\left[\frac{2C_b}{C_f} - (\tan\beta_1 + \tan\beta_2)\right] \tag{7.31}$$

$$R = \frac{C_f}{2C_b}(\tan\beta_1 + \tan\beta_2) \tag{7.32}$$

(d) Stage Pressure Ratio

The stage pressure ratio is given by,

$$r_0 = \frac{p_{03}}{p_{01}} = \left(\frac{T_{03s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \tag{7.33}$$

We know that the isentropic efficiency of the stage is denoted by η_{stage}, where

$$\eta_{\text{stage}} = \frac{T_{03s} - T_{01}}{T_{03} - T_{01}} = \frac{T_{03s} - T_{01}}{\Delta T_{0\,\text{stage}}} \tag{7.34}$$

From Eqs (7.33) and (7.34), stage pressure ratio can be rewritten as,

$$r_0 = \left[1 + \frac{\eta_{\text{stage}}\,\Delta T_{0\,\text{stage}}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \tag{7.35}$$

If the basic purpose is to increase pressure ratio (r_0), temperature rise in a stage ($\Delta T_{0\ \text{stage}}$) should be increased. In other words, the higher the temperature rise in a stage, the lower is the number of stages for a given overall pressure ratio. From Eq. (7.21), it can be inferred that to obtain high temperature rise in a stage, the designer must combine:

(i) High blade speed (C_b) – the limitation being the blade stresses.
(ii) High axial velocity (C_f) – the limitation being the mass flow rate and passage area.
(iii) High fluid deflection ($\beta_1 - \beta_2$) in the rotor blades – the limitation being the adverse pressure gradient.

(e) Stage Pressure Coefficient

Stage Pressure or loading or work coefficient is also given by,

$$\psi = \frac{w}{0.5C_b^2} = \frac{2w}{C_b^2} \tag{7.36}$$

$$\psi = \frac{2c_p \Delta T_{0\text{stage}}}{C_b^2} = \frac{2C_f(\tan\beta_1 - \tan\beta_2)}{C_b} \tag{7.37}$$

(f) Efficiencies

The ideal work, i.e. isentropic works in the stage is,

$$w_s = (h_{03s} - h_{01}) = c_p(T_{03s} - T_{01}) = \left[c_p T_{01}\left(r_0^{\frac{\gamma-1}{\gamma}} - 1\right)\right] \tag{7.38}$$

The actual work in the stage is,

$$w_a = (h_{03} - h_{01}) = (h_{02} - h_{01}) = c_p(T_{03} - T_{01}) \tag{7.39}$$

The total-to-total efficiency of the stage is defined by,

$$\eta_{tt} = \frac{\text{Ideal stage work between total conditions at entry and exit}}{\text{Actual stage work}} \tag{7.40}$$

$$\eta_{tt} = \frac{w_s}{w_a} = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}} = \frac{T_{03s} - T_{01}}{T_{03} - T_{01}} \tag{7.41}$$

Substituting Eqs (7.20) and (7.38) in Eq. (7.41), we get,

$$\eta_{tt} = \frac{\left[c_p T_{01}\left(r_0^{\frac{\gamma-1}{\gamma}} - 1\right)\right]}{C_b C_f(\tan\beta_1 - \tan\beta_2)} \tag{7.42}$$

The static-to-static efficiency of the stage is defined by,

$$\eta_{ss} = \frac{(h_{3s} - h_1)}{(h_3 - h_1)} = \frac{(T_{3s} - T_1)}{(T_3 - T_1)} \tag{7.43}$$

The total-to-total stage efficiency is given by,

$$\eta_{tt} = \frac{(h_{03s} - h_{01})}{(h_{03} - h_{01})} = \frac{(T_{03s} - T_{01})}{(T_{03} - T_{01})} = \frac{[(h_{03} - h_{01}) - (h_{03} - h_{03s})]}{(h_{03} - h_{01})} \tag{7.44}$$

$$\eta_{tt} = 1 - \frac{(h_{03} - h_{03s})}{(h_{03} - h_{01})} \tag{7.45}$$

EXAMPLE 7.1 The following data refers to an axial flow compressor:
β_1 = 60°, turning angle = 30° and ΔC_w = 100 m/s, degree of reaction 50%, speed 36000 rpm, mean diameter = 140 mm, inlet pressure = 2 bar and inlet temperature = 57°C. Find α_1, the pressure rise, the amount of air handled and power, if the blade height is 20 mm.

Solution

Given:

β_1 = 60°, $\beta_1 - \beta_2$ = 30°, ΔC_w = 100 m/s, R = 0.5, N = 36000 rpm, D = 0.14 m, p_1 = 2 bar, T_1 = 330 K, B = 0.02 m

(a) Air flow Angle (α_1)

$$\beta_1 - \beta_2 = 30° \Rightarrow 60 - \beta_2 = 30$$

$$\beta_2 = 30°$$

For degree of reaction, R = 0.5,

$$\alpha_1 = \beta_2 = 30° \tag{1}$$

(b) The Pressure Rise

Blade mean speed

$$C_b = \frac{\pi DN}{60} = \frac{\pi \times 0.14 \times 36000}{60} = 263.89 \text{ m/s} \tag{2}$$

Stagnation temperature rise ($T_{02} - T_{01}$) is equal to static temperature rise ($T_2 - T_1$) assuming C_1, C_2 to be same.

$$w = C_b(C_{w2} - C_{w1}) = c_p(T_{02} - T_{01}) = c_p(T_2 - T_1) \tag{3}$$

$$T_2 = \frac{C_b(C_{w2} - C_{w1})}{c_p} + T_1 \tag{4}$$

$$T_2 = \frac{263.89 \times 100}{1005} + 330 = 356.26 \text{ K} \tag{5}$$

Assuming that there are no losses in the compressor, the pressure ratio can be determined from,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_2}{2} = \left(\frac{356.26}{330}\right)^{\frac{1.4}{0.4}} \tag{6}$$

$$p_2 = 2.62 \text{ bar} \tag{7}$$

Pressure rise

$$\Delta p = p_2 - p_1 = 2.62 - 2$$

$$\Delta p = 0.62 \text{ bar} \tag{8}$$

(c) The Amount of Air Handled

$$R = \frac{C_f}{2C_b}(\tan \beta_1 + \tan \beta_2) \Rightarrow C_f = \frac{2C_b R}{\tan \beta_1 + \tan \beta_2} \tag{9}$$

$$C_f = \frac{2 \times 263.89 \times 0.5}{\tan 60 + \tan 30} = 114.27 \text{ m/s} \tag{10}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{2 \times 10^5}{287 \times 330} = 2.11 \text{ kg/m}^3 \tag{11}$$

$$\dot{m} = \rho_1 A_1 C_f = \rho_1 \pi D B C_f \tag{12}$$

$$\dot{m} = 2.11 \times \pi \times 0.14 \times 0.02 \times 114.27$$

$$\dot{m} = 2.121 \text{ kg/s} \tag{13}$$

(d) Power Input

$$P = \dot{m}w = \dot{m}c_p(T_{02} - T_{01}) = \dot{m}c_p(T_2 - T_1) \tag{14}$$

$$P = 2.121 \times 1.005 \times (356.26 - 330)$$

$$P = 55.976 \text{ kW} \tag{15}$$

EXAMPLE 7.2 An eight stage axial flow compressor provides an overall pressure ratio of 6 : 1 with an overall isentropic efficiency of 90%, when the temperature of air at inlet is 20°C. The work is divided equally between the stages. A 50% reaction is used with a mean blade speed 188 m/s and a constant axial velocity 100 m/s through the compressor. Estimate the blade angles and power required. Assume air to be a perfect gas.

Solution

Given: $n = 8$, $\dfrac{p_{0,n+1}}{p_{01}} = 6$, $\eta_s = 0.9$, $T_{01} = 293$ K, $R = 0.5$, $C_b = 188$ m/s, $C_f = 100$ m/s

For 50% reaction turbine, the blades are symmetrical and so the velocity diagrams are identical. Therefore,

$$\alpha_1 = \beta_2, \ \alpha_2 = \beta_1 \tag{1}$$

If the compression process was isentropic then the temperature of air leaving the compressor stage would be,

$$T_{0,(n+1)s} = T_{01}\left(\frac{p_{0,n+1}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 293(6)^{\frac{0.4}{1.4}} = 488.9 \text{ K} \tag{2}$$

The overall isentropic efficiency is given by,

$$\eta_s = \frac{T_{0,(n+1)s} - T_{01}}{T_{0,n+1} - T_{01}} \tag{3}$$

$$T_{0,n+1} = T_{01} + \frac{T_{0,(n+1)s} - T_{01}}{\eta_s} = 293 + \frac{488.9 - 293}{0.9}$$

$$T_{0,\,n+1} = 510.67 \text{ K} \tag{4}$$

Specific work for n stages,

$$w = nC_bC_f(\tan\alpha_2 - \tan\alpha_1) = c_p(T_{0,\,n+1} - T_{01}) \tag{5}$$

$$8 \times 188 \times 100(\tan\alpha_2 - \tan\alpha_1) = 1.005 \times 10^3 \times (510.67 - 293)$$

$$\tan\alpha_2 - \tan\alpha_1 = 1.45 \tag{6}$$

For 50% degree of reaction,

$$\alpha_2 = \beta_1 \tag{7}$$

Since $\alpha_2 = \beta_1$, then

$$\tan\beta_1 - \tan\alpha_1 = 1.45 \tag{8}$$

From inlet velocity configuration,

$$C_b = C_{w1} + C_{rw1} = C_f(\tan\alpha_1 + \tan\beta_1)$$

$$(\tan\alpha_1 + \tan\beta_1) = \frac{C_b}{C_f} = \frac{188}{100} = 1.88$$

$$(\tan\alpha_1 + \tan\beta_1) = 1.88 \tag{9}$$

From Eqs (8) and (9),

$$\tan\beta_1 = \frac{1.45 + 1.88}{2} = 1.665$$

$$\beta_1 = \alpha_2 = 59° \tag{10}$$

Substituting the value of $\tan\beta_1 = 1.665$ in Eq. (8), we get,

$$\tan\alpha_1 = \tan\beta_1 - 1.45 = 1.665 - 1.45 = 0.215$$

$$\alpha_1 = \beta_2 = 12.134 \tag{11}$$

Power required for air flow rate of 1 kg/s,

$$P = \dot{m}c_p(T_{0,\,n+1} - T_{01}) = 1 \times 1.005 \times (510.67 - 293)$$

$$P = 218.76 \text{ kW} \tag{12}$$

7.4 Centrifugal Compressor

7.4.1 Velocity Triangles

Meridonial view of a centrifugal compressor is shown in Fig. 7.7. Air enters into the impeller at 1 and leaves at 2. Assuming the following conditions, the velocity triangle can be drawn as shown in Figure 7.8.

$$C_{f1} = C_1,\ C_{w1} = 0,\ C_{b2} > C_{b1} \tag{7.46}$$

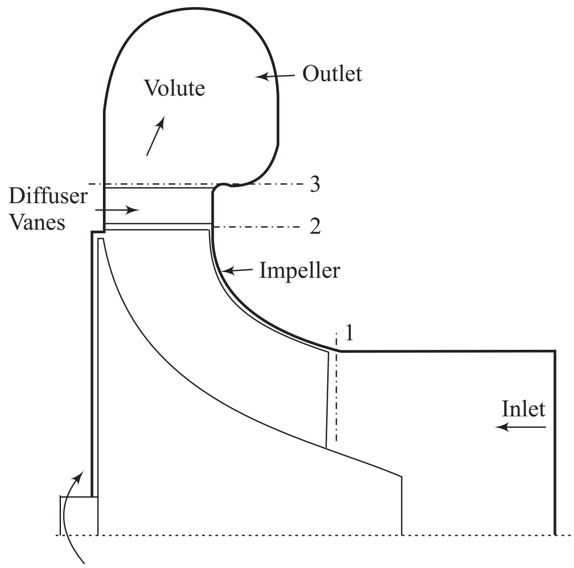


Figure 7.7 *Meridional View of Centrifugal Compressor*

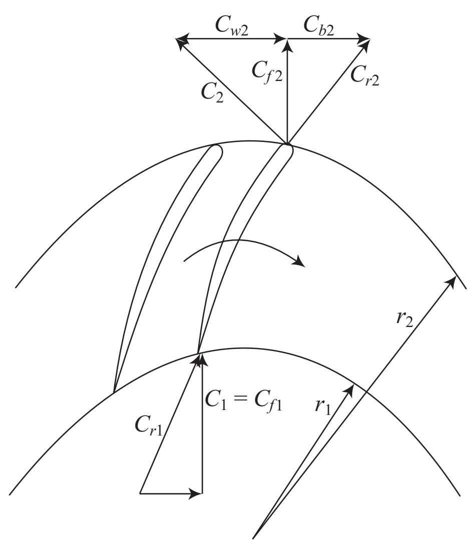


Figure 7.8 *Velocity Triangle for Centrifugal Compressor*

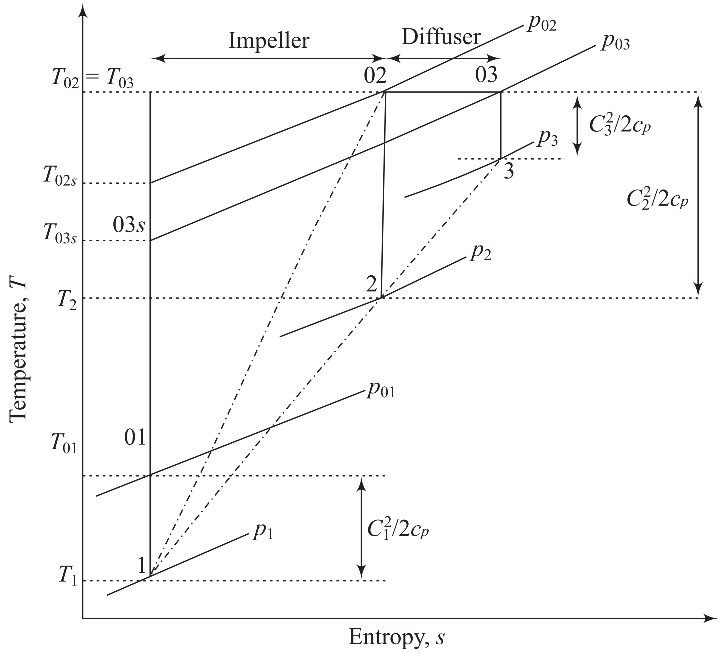


Figure 7.9 *Compression Process in a Centrifugal Compression Stage on T – s Diagram.*

7.4.2 Stage Parameters

(a) Mass Flow Rate

Mass flow rate through the centrifugal compressor can be estimated as follows, by knowing the radial component of velocity:

$$\dot{m} = \rho_1 \pi D_1 B_1 C_{f1} = \rho_2 \pi D_2 B_2 C_{f2} \tag{7.47}$$

Mass flow rate through impeller inlet annulus area is given by,

$$\dot{m} = \rho_1 \pi (r_t^2 - r_h^2) C_{f1} \tag{7.48}$$

Where r_h and r_t denote hub and tip radii at the eye, respectively, and density,

$$\rho_1 = \frac{p_1}{RT_1} \tag{7.49}$$

(b) Stage Work as a Function of Flow Coefficient and Blade Angle

(i) Backward Curved Blade Compressor

For backward curved blades, $\beta_2 < 90°$.
From velocity triangle in Figure 7.10,

$$C_{f1} = C_1$$

$$C_{w1} = 0 \tag{7.50}$$

$$C_{f2} = C_2 \sin \alpha_2 = C_{r2} \sin \beta_2 \tag{7.51}$$

$$C_{w2} = C_2 \cos \alpha_2 = C_{f2} \cot \alpha_2 = C_{b2} - C_{f2} \cot \beta_2 \tag{7.52}$$

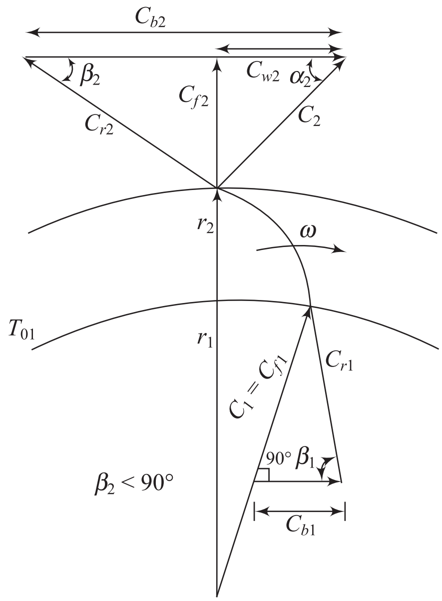


Figure 7.10 *Velocity Triangle for Backward Curved Blade*

Substituting Eqs. (7.50) and (7.52) in Eq. (7.1),

$$w = C_{b2}(C_{b2} - C_{f2} \cot \beta_2) \tag{7.53}$$

We know that flow coefficient at the impeller exit is defined as,

$$\varphi_2 = \frac{C_{f2}}{C_{b2}} \tag{7.54}$$

From Eq. (7.54) in Eq. (7.53),

$$w = C_{b2}^2(1 - \varphi_2 \cot \beta_2) \tag{7.55}$$

(ii) Forward Curved Blade Compressor

For forward curved blades, $\beta_2 > 90°$.

From velocity triangle as shown in Figure 7.11,

$$C_{w1} = 0 \tag{7.56}$$

$$C_{f2} = C_2 \sin \alpha_2 = C_{r2} \sin (180 - \beta_2) = C_{r2} \sin \beta_2 \tag{7.57}$$

$$C_{w2} = C_2 \cos \alpha_2 = C_{f2} \cot \alpha_2 = C_{b2} - C_{f2} \cot \beta_2 \tag{7.58}$$

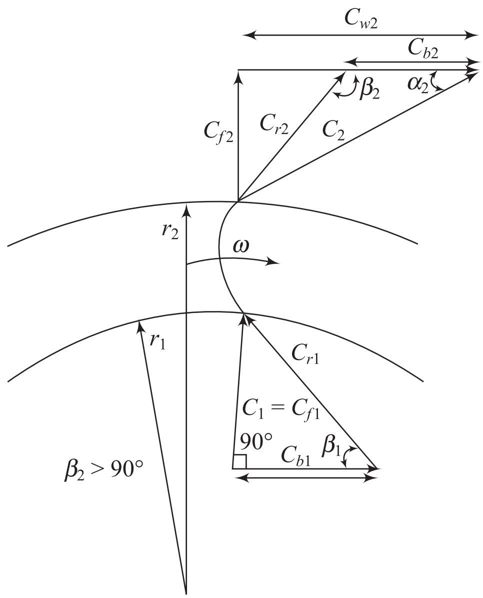


Figure 7.11 *Velocity Triangle for Forward Curved Blade*

Substituting Eq. (7.54) and (7.58) in Eq. (7.1),

$$w = C_{b2}^2(1 - \varphi_2 \cot \beta_2) \tag{7.59}$$

(iii) Radial Tipped Blade Compressor

For radial tipped blades, $\beta_2 = 90°$.

From velocity triangle as shown in Figure 7.12,

$$C_{w1} = 0 \tag{7.60}$$

$$C_{f2} = C_{r2} = C_2 \sin \alpha_2 \tag{7.61}$$

$$C_{w2} = C_{b2} = C_2 \cos \alpha_2 = C_{f2} \cot \alpha_2 \tag{7.62}$$

Substituting Eqs. (7.60) and (7.62) in Eq. (7.1),

$$w = C_{b2}^2 \tag{7.63}$$

(c) Degree of Reaction

Degree of reaction can be defined as,

$$R = \frac{\text{Change in static enthalpy in the impeller}}{\text{Change in stagnation enthalpy in the stage}} \tag{7.64}$$

$$R = \frac{h_2 - h_1}{h_{03} - h_{01}} = \frac{h_2 - h_1}{h_{02} - h_{01}} \tag{7.65}$$

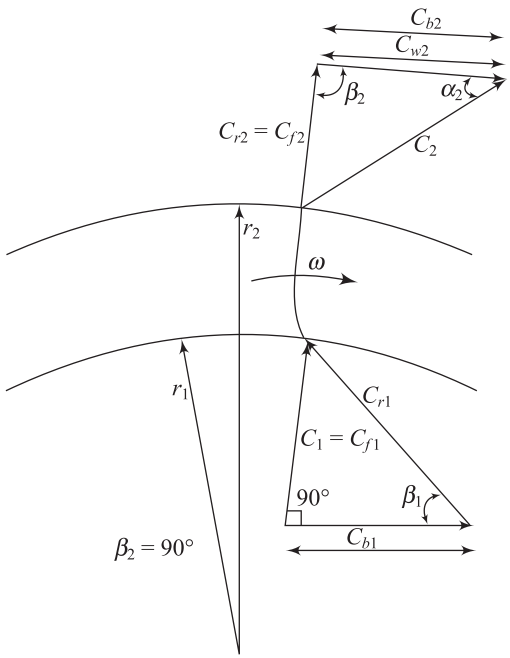


Figure 7.12 *Velocity Triangle for Radial Tipped Blade*

From Eqs. (7.6) and (7.7), we have,

$$w = (h_{02} - h_{01}) = \frac{(C_2^2 - C_1^2)}{2} + \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2}$$

$$(h_2 - h_1) = \frac{(C_{b2}^2 - C_{b1}^2)}{2} + \frac{(C_{r1}^2 - C_{r2}^2)}{2}$$

Also, for zero whirl at the entry of the impeller,

$$w = C_{b2}C_{w2}$$

Substituting this equation in Eq. (7.65),

$$R = \frac{(C_{b2}^2 - C_{b1}^2) + (C_{r1}^2 - C_{r2}^2)}{2C_{b2}C_{w2}} = \frac{(C_{b2}^2 - C_{r2}^2) + (C_{r1}^2 - C_{b1}^2)}{2C_{b2}C_{w2}} \tag{7.66}$$

For constant radial velocity component,

$$C_1 = C_{f1} = C_{f2}$$

$$C_{r1}^2 - C_{b1}^2 = C_{f1}^2 = C_{f2}^2$$

(i) Backward Curved Blade Compressor

From the velocity triangles at the entry and exit of the impeller as shown in Figure 7.10,

$$C_{r2}^2 = C_{f2}^2 + (C_{b2} - C_{w2})^2 = C_{f2}^2 + C_{b2}^2 - 2C_{b2}C_{w2} + C_{w2}^2$$

$$C_{b2}^2 - C_{r2}^2 = 2C_{b2}C_{w2} - C_{w2}^2 - C_{f2}^2 \tag{7.67}$$

$$C_{r1}^2 = C_{b1}^2 + C_{f1}^2 \tag{7.68}$$

Using Eqs. (7.67) and (7.68) in Eq. (7.66), we have,

$$R = 1 - \frac{1}{2}\left(\frac{C_{w2}}{C_{b2}}\right) \tag{7.69}$$

Substituting Eqs. (7.52) and (7.54) in Eq. (7.69),

$$R = \frac{1}{2} + \frac{1}{2}(\varphi_2 \cot \beta_2) \tag{7.70}$$

(ii) Forward Curved Blade Compressor

From the velocity triangles at the entry and exit of the impeller as shown in Figure 7.11,

$$C_{r2}^2 = C_{f2}^2 + (C_{b2} - C_{w2})^2 = C_{f2}^2 + C_{b2}^2 - 2C_{b2}C_{w2} + C_{w2}^2$$

$$C_{b2}^2 - C_{r2}^2 = 2C_{b2}C_{w2} - C_{w2}^2 - C_{f2}^2 \tag{7.71}$$

$$C_{r1}^2 = C_{b1}^2 + C_{f1}^2 \tag{7.72}$$

Using Eqs. (7.71) and (7.72) in Eq. (7.66), we have,

$$R = 1 - \frac{1}{2}\left(\frac{C_{w2}}{C_{b2}}\right) \tag{7.73}$$

Substituting the Eqs (7.54), (7.58) in Eq. (7.73),

$$R = \frac{1}{2} + \frac{1}{2}(\varphi_2 \cot \beta_2) \tag{7.74}$$

(iii) Radial Tipped Blade Compressor

From the velocity triangles at the entry and exit of the impeller as shown in Figure 7.12,

$$C_2^2 = C_{r2}^2 + C_{b2}^2 \Rightarrow C_{r2}^2 = C_2^2 - C_{b2}^2 = C_{f2}^2 \tag{7.75}$$

$$C_{r1}^2 = C_{b1}^2 + C_{f1}^2 \tag{7.76}$$

Using Eqs (7.75) and (7.76) in Eq. (7.66), we have,

$$R = \frac{C_{b2}^2 + C_{f1}^2 - C_{f2}^2}{2C_{b2}C_{w2}} \tag{7.77}$$

For constant radial velocity component, from Eq. (7.77), we get

$$R = \frac{[2C_{b2}C_{w2} - (C_2^2 - C_1^2)]}{2C_{b2}C_{w2}}$$

$$R = \frac{1}{2}\left(\frac{C_{w2}}{C_{b2}}\right) \tag{7.78}$$

Substituting Eq. (7.62) in Eq. (7.78),

$$R = \frac{1}{2} \tag{7.79}$$

(d) Stage Pressure Ratio

From Figure 7.9 and Eq. (7.8), we have,

$$w_s = c_p(T_{03s} - T_{01}) = c_p T_{01}\left(\frac{T_{03s}}{T_{01}} - 1\right)$$

$$w_s = c_p T_{01}\left[r_0^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{7.80}$$

where stagnation pressure ratio, $r_0 = \frac{p_{03s}}{p_{01}} = \frac{p_{03}}{p_{01}}$.

(i) Forward and Backward Curved Blade Compressor
Using Eqs. (7.55) and (7.59) in Eq. (7.80),

$$w = c_p T_{01}\left[r_0^{\frac{\gamma-1}{\gamma}} - 1\right] = C_{b2}^2(1 - \varphi_2 \cot \beta_2) \tag{7.81}$$

$$r_0 = \frac{p_{03}}{p_{01}} = \left[1 + \frac{C_{b2}^2}{c_p T_{01}}(1 - \varphi_2 \cot \beta_2)\right]^{\frac{\gamma}{\gamma-1}} \tag{7.82}$$

(ii) Radial Tipped Blade Compressor
Using Eq. (7.63) in Eq. (7.80), we have,

$$w = c_p T_{01}\left[r_0^{\frac{\gamma-1}{\gamma}} - 1\right] = C_{b2}^2$$

$$r_0 = \frac{p_{03}}{p_{01}} = \left(1 + \frac{C_{b2}^2}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \tag{7.83}$$

(e) Stage Pressure Coefficient

Stage pressure or loading or work coefficient is given by,

$$\psi = \frac{2w}{C_{b2}^2} \tag{7.84}$$

(i) Forward and Backward Curved Blade Compressor
Using Eqs. (7.55) and (7.59) in Eq. (7.84),

$$\psi = 2(1 - \varphi_2 \cot \beta_2) \tag{7.85}$$

(ii) Radial Tipped Blade Compressor

Using Eq. (7.63) in Eq. (7.84),

$$\psi = 2 \tag{7.86}$$

(f) Efficiency

The actual work in the stage is,

$$w_a = h_{03} - h_{01} = c_p(T_{02} - T_{01}) = C_{b2}^2(1 - \varphi_2 \cot \beta_2) \tag{7.87}$$

As $T_{03} = T_{02}$

The ideal work in the stage is isentropic work given as,

$$w_s = h_{03s} - h_{01} = c_p(T_{03s} - T_{01}) \tag{7.88}$$

$$w_s = c_p T_{01}(T_{03s}/T_{01} - 1)$$

$$w_s = c_p T_{01}\left[r_0^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{7.89}$$

The total-to-total efficiency of the stage can be defined by,

$$\eta_{tt} = \frac{w_s}{w_a} = \frac{(h_{03s} - h_{01})}{(h_{02} - h_{01})} \tag{7.90}$$

$$\eta_{tt} = \frac{c_p T_{01}\left[r_0^{\frac{\gamma-1}{\gamma}} - 1\right]}{C_b^2(1 - \varphi_2 \cot \beta_2)} \tag{7.91}$$

EXAMPLE 7.3 A single sided centrifugal compressor draws air from the atmosphere having ambient conditions of 100 kPa and 15°C. The hub diameter is 0.13 m and the eye tip diameter is 0.3 m. If the mass flow into the eye of the compressor is 8 kg/s and the speed is 16200 rpm, calculate the blade inlet angle at the root and tip of the eye. Assume zero whirl at inlet and no losses in the intake duct.

Solution

Let the hub and tip radii at the eye are denoted by r_h and r_t, respectively. Then the flow area of the impeller inlet annulus is,

$$A = \pi(r_t^2 - r_h^2) = \pi(0.15^2 - 0.065^2) = 0.0574 \text{ m}^2 \tag{1}$$

Assume a value for density ρ_1 as equal to ρ_{01}, which is based on stagnation conditions.

$$\rho_1 = \rho_{01} = \frac{p_{01}}{RT_{01}} = \frac{10^5}{287 \times 288} = 1.21 \text{ kg/m}^3 \tag{2}$$

Then from the continuity equation,

$$C_f = \frac{\dot{m}}{\rho_1 A_1} = \frac{8}{1.21 \times 0.0574} = 115.18 \text{ m/s} \tag{3}$$

Since, there is no inlet whirl component, the absolute inlet velocity $C_1 = C_f$, and the temperature equivalent of this velocity is,

$$\frac{C_1^2}{2c_p} = \frac{115.18^2}{2 \times 1005} = 6.6\ \text{K} \tag{4}$$

Therefore,

$$T_1 = T_{01} - \frac{C_1^2}{2c_p} = 288 - 6.6 = 281.4\ \text{K} \tag{5}$$

Assuming isentropic flow at the inlet,

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \tag{6}$$

$$p_1 = 10^5 \left(\frac{281.4}{288}\right)^{\frac{1.4}{1.4-1}} = 92206.3\ \text{Pa} = 92.2\ \text{kPa} \tag{7}$$

Then new,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{9.22 \times 10^4}{287 \times 281.4} = 1.14\ \text{kg/m}^3 \tag{8}$$

And new,

$$C_f = \frac{\dot{m}}{\rho_1 A_1} = \frac{8}{1.14 \times 0.0574} = 122.25\ \text{m/s} \tag{9}$$

Repeat,

$$\frac{C_1^2}{2c_p} = \frac{122.25^2}{2 \times 1005} = 7.43\ \text{K} \tag{10}$$

Therefore,

$$T_1 = T_{01} - \frac{C_1^2}{2c_p} = 288 - 7.43 = 280.57\ \text{K} \tag{11}$$

$$\frac{p_1}{p_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \tag{12}$$

$$p_1 = 10^5 \left(\frac{280.57}{288}\right)^{\frac{1.4}{1.4-1}} = 91257.93\ \text{Pa} = 91.258\ \text{kPa} \tag{13}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{91.258 \times 10^3}{287 \times 280.57} = 1.13\ \text{kg/m}^3 \tag{14}$$

Further iterations are unnecessary and the value $\rho_1 = 1.13\ \text{kg/m}^3$ may be taken as the inlet density and $C_1 = C_f = 122.25$ m/s as the inlet velocity.

At the hub,

$$C_{b1} = \frac{\pi D_h N}{60} = \frac{\pi \times 16200 \times 0.13}{60} = 110.3 \text{ m/s} \tag{15}$$

$$\tan \beta_{1h} = \frac{C_{b1}}{C_f}$$

$$\tan \beta_{1h} = \frac{110.3}{122.25} \tag{16}$$

$$\tan\beta_{1b} = 42.06° \tag{17}$$

At the tip,

$$C_{b2} = \frac{\pi D_t N}{60} = \frac{\pi \times 16200 \times 0.3}{60} = 254.47 \text{ m/s} \tag{18}$$

Blade angle

$$\tan \beta_{1t} = \frac{C_{b2}}{C_f} \tag{19}$$

$$\tan \beta_{1t} = \frac{254.47}{122.25}$$

$$\tan \beta_{1t} = 64.34° \tag{20}$$

EXAMPLE 7.4 20 m^3/s of air at 1 bar and 15°C is to be compressed in a centrifugal compressor through a pressure ratio 1.5:1. The compression follows the law $pV^{1.5}$ = constant. The velocity of flow at the inlet and outlet remains constant and equal to 60 m/s. If the inlet and outlet impeller diameters are, respectively, 0.6 m and 1.2 m, and speed of rotation is 5000 rpm, find (a) the blade angles at the inlet and outlet of the impeller and the angle at which the air enters the impeller casing, and (b) breadth of impeller blade at the inlet and outlet. It may be assumed that no diffuser is fitted and the whole pressure increase occurs in the impeller and that the blades have a negligible thickness. Assume pre-whirl at inlet to be zero.

Solution

Given:

$Q_1 = 20$ m^3/s, $p_1 = 1$ bar, $T_1 = 15°\text{C} = 288$ K, $\frac{p_2}{p_1} = 1.5$, $C_{f1} = C_{f2} = C_1 = 60$ m/s, $D_1 = 0.6$ m, $D_2 = 1.2$ m, $N = 5000$ rpm

(a) Blade Angles and Flow Angle

Peripheral velocity of impeller at the inlet,

$$C_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.6 \times 5000}{60} = 157.08 \text{ m/s} \tag{1}$$

From the inlet velocity vector diagram,

$$\tan \beta_1 = \frac{C_1}{C_{b1}} = \frac{60}{157.08}$$

$$\beta_1 = 20.9° \tag{2}$$

Peripheral velocity of impeller top at the outlet,

$$C_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 5000}{60} = 314.16 \text{ m/s} \tag{3}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = 288(1.5)^{0.5/1.5} = 329.68 \text{ K} \tag{4}$$

$$w = C_{b2}C_{w2} = c_p(T_2 - T_1) \tag{5}$$

$$C_{w2} = \frac{1005(329.68 - 288)}{314.16}$$

$$C_{w2} = 133.33 \text{ m/s} \tag{6}$$

Now,

$$C_{rw2} = C_{b2} - C_{w2} = 314.16 - 133.33 = 180.83 \text{ m/s}$$

From the outlet velocity vector diagram,

$$\tan \beta_2 = \frac{C_{f2}}{C_{rw2}} \tag{7}$$

$$\beta_2 = \tan^{-1}(60/180.83)$$

$$\beta_2 = 18.36° \tag{8}$$

$$\tan \alpha_2 = \frac{C_{f2}}{C_{w2}} \tag{9}$$

$$\alpha_2 = \tan^{-1}(60/133.83)$$

$$\alpha_2 = 24.2° \tag{10}$$

(b) Breadth of Impeller Blades at Inlet and Outlet

If Q_1 is the flow rate (discharge), in m^3/s, at the inlet, then,

$$Q_1 = \pi D_1 B_1 C_{f1} \tag{11}$$

$$B_1 = \frac{20}{\pi \times 0.6 \times 60}$$

$$B_1 = 0.177 \text{ m} \tag{12}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{13}$$

$$V_2 = \frac{p_1 V_1}{T_1} \times \frac{T_2}{p_2} = \frac{1 \times 10^5 \times 20}{288} \times \frac{329.68}{1.5 \times 10^5}$$

$$V_2 = 15.26 \text{ m}^3/\text{s} \tag{14}$$

Then,

$$Q_2 = V_2 = \pi D_2 B_2 C_{f2} \tag{15}$$

$$B_2 = \frac{15.26}{\pi \times 1.2 \times 60}$$

$$B_2 = 0.0675 \text{ m} \tag{16}$$

7.5 Losses and Efficiencies

In the above descriptions of stage parameters, isentropic efficiency in various forms is introduced. The isentropic efficiency (Eqs. 7.40 and 7.43) is also called internal efficiency. This efficiency considers the following losses:

(i) Internal losses consists of aerodynamic (sometimes called hydraulic) losses and leakage losses. The internal losses are further divided into several losses such as frictional losses, profile losses, a secondary flow losses, clearance losses, cavitation losses, shock or incidence losses and annulus wall boundary losses. In addition there are disc friction and return flow losses.

(ii) Mechanical losses are those external to the flow medium namely in bearing, sealing, coupling and other auxiliary equipment that may be directly, or through a gear, coupled to the compressor shaft.

Referring to $T - s$ diagram, the isentropic specific work is denoted by,

$$w_s = h_{03s} - h_{01} = c_p (T_{03s} - T_{01}) = c_p T_{01}\left(\frac{T_{03s}}{T_{01}} - 1\right)$$

$$w_s = c_p T_{01}\left[r_0^{\frac{\gamma-1}{\gamma}} - 1\right]$$

where $r_0 = \dfrac{p_{03}}{p_{01}}$, is the pressure ratio of the compressor.

If the internal losses are added to the isentropic work, internal power results from internal work as,

$$\text{Internal power} = w_s + w_{\text{internal losses}} \tag{7.92}$$

$$h_{03} - h_{01} = (h_{03s} - h_{01}) + (h_{03} - h_{03s}) \tag{7.93}$$

Thus, the internal losses are obtained from above equations as,

$$w_{\text{internal losses}} = (h_{03} - h_{03s}) \tag{7.94}$$

where h_{03} is the actual state and h_{03s} is the isentropic state. From the ideal condition of infinite number of blades and infinitesimal thickness and vane congruent fluid flow, the specific work is given by gH_e. In the previous sections w_e is simply indicated as w only.

As the number of blades is finite with finite thickness, slip occurs. Because of slip, the work absorbing capacity of the impeller reduces. After subtracting slip power, which is not a loss, the theoretical specific work of the impeller [Eq. (1.105)] is,

$$gH_{\text{th}} = gH_e - gH_{\text{slip}} \tag{7.95}$$

Using the definition of slip, σ_s, Eq. (1.08) is restated as

$$gH_{\text{th}} = \sigma_s C_{b2} C_{w2} - C_{b1} C_{w1} \tag{7.96}$$

As the velocity triangles are drawn for vane-congruent and ideal conditions, the velocity components, C_{w2} and C_{w1}, calculated from those triangles, will give the value of w_e or gH_e. Using Stodola's approximation, as in Eq. (1.110),

$$w_s = gH_{\text{th}} = \left[C_{b2}\left(C_{w2} - \frac{\pi C_{b2}\sin\beta_2}{z} \right) - C_{b1}C_{w1} \right] \tag{7.97}$$

If prewhirl is zero, $C_{w1} = 0$

$$w_s = gH_{\text{th}} = C_{b2}\left(C_{w2} - \frac{\pi C_{b2}\sin\beta_2}{z} \right) \tag{7.98}$$

A popular formula for σ_s is given by,

$$\sigma_{\text{Stantiz}} = 1 - \frac{0.63\pi}{z} \tag{7.99}$$

for centrifugal compressor, where z is the number of vanes.

7.5.1 Power Input Factor

We have included the internal losses in the definition of isentropic efficiency. However, the disc friction and return flow were not included. To account for these losses, power input factor (ψ_w), is introduced.

Internal power considering the disc friction and return flow losses is equal to,

$$\text{Internal power} = h_{03} - h_{01} = \psi_w \sigma_s w_e \tag{7.100}$$

Or,

$$T_{03} - T_{01} = \frac{\psi_w \sigma_s w_e}{c_p} \tag{7.101}$$

Note that the power input factor ψ_w is more than unity, representing an increase in compressor work input, which is required to overcome the disc friction losses.

$$T_{03} - T_{01} = \frac{T_{03s} - T_{01}}{\eta_{tt}} = \frac{T_{01}\left(\frac{T_{03s}}{T_{01}} - 1 \right)}{\eta_{tt}} = \frac{T_{01}\left[\left(\frac{p_{03}}{p_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\eta_{tt}}$$

From the above equation, the pressure ratio can be estimated by

$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_{tt}(T_{03} - T_{01})}{T_{01}} \right]^{\frac{\gamma}{\gamma-1}} \tag{7.102}$$

For examples, pressure ratio for centrifugal compressor for radial tipped blades is obtained from the Eq. (7.102) as,

$$\frac{p_{03}}{p_{01}} = \left(1 + \frac{\eta_{tt}\Psi_w \sigma_s C_{b2}^2}{T_{01}c_p} \right)^{\frac{\gamma}{\gamma-1}} \tag{7.103}$$

EXAMPLE 7.5 In a radial tipped centrifugal compressor with inlet guide vanes, air leaving the guide vanes has a velocity of 91.5 m/s at 70° to the tangential direction. Determine the inlet relative Mach number, assuming frictionless flow through the guide vanes and impeller total head isentropic efficiency. The other operating conditions are: Impeller diameter at inlet = 457 mm, Impeller diameter at exit = 762 mm, Radial component of velocity at impeller exit = 53.4 m/s, Slip factor = 0.9, Impeller speed = 11000 rpm, and Static pressure at impeller exit = 223 kPa (*abs*).
Take T_{01} = 288 K and p_{01} = 1.013 bar.

Solution

$$C_1 = 91.5 \text{ m/s}$$

$$C_{w1} = C_1 \cos \alpha_1 = 91.5 \cos 70$$

$$C_{w1} = 31.295 \text{ m/s} \quad (1)$$

$$C_{f1} = C_1 \sin \alpha_1 = 91.5 \sin 70$$

$$C_{f1} = 85.98 \text{ m/s} \quad (2)$$

$$C_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.457 \times 11000}{60} = 263.21 \text{ m/s} \quad (3)$$

$$C_{rw1} = C_{b1} - C_{w1} \quad (4)$$

$$C_{rw1} = 263.21 - 31.295$$

$$C_{rw1} = 231.915 \text{ m/s} \quad (5)$$

$$C_{r1}^2 = C_{rw1}^2 + C_{f1}^2 = 231.915^2 + 85.98^2$$

$$C_{r1} = 247.34 \text{ m/s} \quad (6)$$

$$T_1 = T_{01} - \frac{C_1^2}{2c_p} = 288 - \frac{91.5^2}{2 \times 1005}$$

$$T_1 = 283.835 \text{ K} \quad (7)$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 283.835}$$

$$a_1 = 337.7 \text{ m/s} \quad (8)$$

Inlet relative Mach number

$$M_{1,r} = \frac{C_{r1}}{a_1} = \frac{247.34}{337.7}$$

$$M_{1,r} = 0.732 \quad (9)$$

$$C_{b2} = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.762 \times 11000}{60} = 438.88 \text{ m/s} \quad (10)$$

In the radial tipped centrifugal compressor with slip and pre-whirl, the specific work can be written as

$$w = c_p(T_{02} - T_{01}) = \sigma_s C_{b2}^2 - C_{b1} C_{w1} \quad (11)$$

$$c_p(T_{02} - T_{01}) = 0.9(438.88)^2 - (263.21 \times 31.295)$$

$$T_{02} - T_{01} = 164.295$$

$$T_{02} = 164.295 + 288 = 452.295 \text{ K} \quad (12)$$

$$C_{f2} = 53.4 \text{ m/s} \quad (13)$$

$$C'_{w2} = \sigma_s C_{w2} = \sigma_s \times C_{b2} = 0.9 \times 438.88 = 394.992 \text{ m/s} \quad (14)$$

$$C_2^2 = C'^2_{w2} + C_{f2}^2 = 394.992^2 + 53.4^2$$

$$C_2 = 398.585 \text{ m/s} \tag{15}$$

$$T_2 = T_{02} - \frac{C_2^2}{2c_p} = 452.295 - \frac{398.585^2}{2 \times 1005}$$

$$T_2 = 373.255 \text{ K} \tag{16}$$

$$\frac{p_{02}}{p_2} = \left(\frac{T_{02}}{T_2}\right)^{\frac{r}{r-1}} = \left(\frac{452.295}{373.255}\right)^{\frac{1.4}{0.4}}$$

$$p_{02} = 1.95856 \times 223 = 436.76 \text{ kPa} = 4.37 \text{ bar} \tag{17}$$

Total head isentropic efficiency

$$\eta_{tt} = \frac{T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{02} - T_{01}} \tag{18}$$

$$\eta_{tt} = \frac{288\left[\left(\frac{4.37}{1.013}\right)^{\frac{0.4}{1.4}} - 1\right]}{164.295}$$

$$\eta_c = 0.9088 = 90.88\% \tag{19}$$

7.6 Performance Characteristics

7.6.1 Centrifugal Compressor

The characteristics of compressible flow machines are usually described in terms of the groups of variables derived in Eq. (2.36). The characteristics are given as a series of curves of p_{03}/p_{01}, plotted against the mass flow parameter, $mT_{01}^{0.5}/p_{01}$, for fixed speed intervals of $N/T_{01}^{0.5}$. Idealized fixed-speed characteristics are shown in Figure 7.13.

Consider a centrifugal compressor which delivers compressed air. Let the air flow through a flow control valve situated after the diffuser. If the valve is fully closed, a certain pressure ratio from the inlet across the diffuser outlet will be developed, as shown with point *A* in Figure 7.13. This pressure ratio is developed only because the vanes churn the air about the impeller. As the flow control valve is opened and air flow begins, the diffuser contributes to the pressure ratio and maximum pressure ratio is reached at point *B*. The compressor efficiency at this pressure ratio may be just below its maximum value. A further increase in mass flow brings conditions to point *C*, where the pressure may have dropped but the efficiency is now the maximum. This is generally the design mass flow rate and pressure ratio. A further increase in mass flow sees the slope of the curve increasing until it is almost vertical at point *D*, where the pressure rise is zero. Theoretically, point *D* would have reached when all the input power is absorbed in overcoming internal friction. The curve *ABCD* is called the characteristics curve, drawn by joining a series of operating points by varying the resistance across the throttle.

(a) Surging

Surge is a condition which occurs beyond the stable limit of operation. In Figure 7.13, let the surge line pass through the point B. It means stable operation occurs right of B, say E, at which the slope of the characteristics is negative. If the operating point is left of B, say F, the slope of the characteristics is positive and hence unstable.

Let us first consider stable operation at E. If a small reduction of flow momentarily occurs due to some reason, the compressor will produce a greater pressure rise, because of which the throttle resistance falls. Consequently, the flow rate will increase and the original operating point, E, is resorted. Similar argument can be used even if a small increase of flow takes place momentarily. The stable operation ensures restoration of original operating point despite small perturbations in flow rate.

If the operating point is at F, a small reduction in mass flow causes a greater reduction in compressor pressure ratio than the corresponding pressure ratio across the throttle. As a consequence of this increased throttle resistance, the flow will further decrease leading to unstable operation.

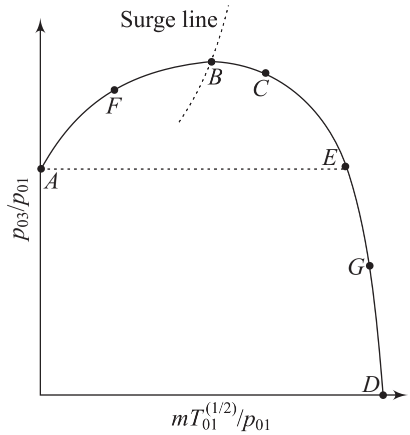


Figure 7.13 *General (Ideal) Characteristics of a Centrifugal Compressor*

Thus, if the operating point is close to the surge line, even a small reduction in mass flow may shift operation to the unstable region. In this unstable region (region of positive slope of the characteristic), the delivery pressure continues to decrease causing drop in mass flow and so on until point A (where mass flow is less) or even beyond (negative mass flow through compressor) is reached. When the back pressure reduces itself sufficiently due to reduced flow rate, the positive flow becomes established. The compressor then picks up until the same restricted mass flow is reached, when the pressure reduction again takes place. The pressure therefore surges back and forth. This sequence of events repeats cyclically at certain frequencies. If this unstable operation is severe enough, it could lead to not only noise and vibration, but also failure of parts of the compressor.

It may be noted that because of the reduction of mass flow, the axial velocity, C_f, into the eye is reduced and therefore, the relative flow angle onto the blade β_1 is increased. The air flow onto the blade will no longer be tangential. Surging tends to originate in diffuser passages where frictional effects of the fluid next to the vane surfaces retard the flow. Indeed the flow may well be in reverse from one blade passage to the next. The oscillations of mass flow are the sources for noise and vibration. Commonly, there are a small number of predominant frequencies. The lowest frequencies are usually associated with a Helmholtz-type of resonance of the flow through the compressor, with the inlet or outlet flow volumes. The likelihood of surging can be reduced by making the number of diffuser vanes an odd number multiple of the impeller vanes. In this way, a pair of diffuser vanes will be supplied with air from an odd number of vanes and pressure fluctuations are more likely to be evened out around the circumference than if exact multiples of diffuser vanes are employed. In actual practice, most of the curves between point A and B are not realized.

(b) Rotating Stall

Rotating stall is a separate phenomenon which may lead to surging. However, it can exist on its own in stable operating conditions. This phenomenon also causes poor performance.

Figure 7.14 illustrates the air flow directions in a number of blade passages.

If the air angle of incidence onto blade A is excessive, perhaps due to a partial blockage or uneven flow in the diffuser, the blade may stall. Because of the partial mass flow decrease in the blade passage, the deflected air causes the angle of incidence below blade A to increase. The angle of incidence above blade A will tend to decrease considering the direction of rotation, shown in Figure 7.14, blade B will be the next to stall while blade A will be un-stalled. The process is repeated about the periphery of the disc. Prolonged cyclic loading and un-loading of the rotor blades may lead to fatigue failure or even immediate catastrophic failure. The stall propagates in the opposite direction to blade motion at a frequency related to shaft speed.

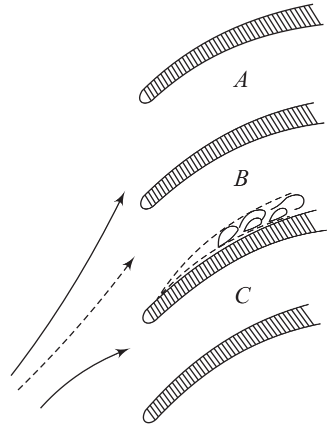


Figure 7.14 *Representation of a Rotating Stall*

(c) Choking

If the mass flow is increased to the right of point C on the negative slope of the characteristic (Figure 7.13), a point G is reached where no further increase in mass flow is possible, no matter how wide open the flow control valve is. This indicates that at some point within the compressor, sonic conditions have been reached, causing the limiting maximum mass flow rate to be set as in the case of compressible flow through a converging-diverging nozzle. Indeed, should this condition arise, shock waves may well be formed within certain passages.

Choking may take place at the inlet, within the impeller, or in the diffuser section. It will occur in the inlet if stationary guide vanes are fitted, the maximum mass flow being governed by the following equation for isentropic flow at the throat of the nozzle.

$$\frac{\dot{m}}{A} = \left[\gamma p_0 \rho_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]^{\frac{1}{2}} \tag{7.104}$$

where stagnation conditions (p_0, ρ_0) at the inlet are known and A is the flow area. In stationary passages, the velocity that is choked is the absolute velocity. In the rotating impeller, it is the relative velocity, C_r, that is the choked velocity. Now,

$$h_{01} = h_1 + \frac{C_1^2}{2} \tag{7.105}$$

$$h_{01} = h_1 + \frac{C_{r1}^2 - C_{b1}^2}{2} \tag{7.106}$$

Choking occurs when the relative velocity equals the acoustic velocity, $C_{r1,\,\text{sonic}}$. Eq. (7.106) becomes,

$$T_{01} = T_1 + \frac{\gamma R T_1 - C_{b1}^2}{2c_p} \tag{7.107}$$

$$\frac{T_1}{T_{01}} = \frac{2\left(1+\dfrac{C_{b1}^2}{2c_pT_{01}}\right)}{\gamma+1} \tag{7.108}$$

For isentropic flow,

$$\frac{\rho_1}{\rho_{01}} = \left(\frac{T_1}{T_{01}}\right)^{\frac{1}{\gamma-1}} \tag{7.109}$$

and therefore, $\dot{m} = \rho a A$

$$\frac{\dot{m}}{A} = \rho_{01}a_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Since $a \propto T^{0.5}$, substituting from Eq. (7.104) and rearranging gives,

$$\frac{\dot{m}}{A} = \rho_{01}a_{01}\left[\frac{2\left(1+\dfrac{C_{b1}^2}{2c_pT_{01}}\right)}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{\dot{m}}{A} = \left\{\gamma p_{01}p_{01}\left[\frac{2\left(1+\dfrac{C_{b1}^2}{2c_pT_{01}}\right)}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\right\}^{\frac{1}{2}} \tag{7.110}$$

Equation (7.110) is simply a modified version of Eq. (7.104) and shows that the choking mass flow rate increases with impeller speed. In the diffuser passages, Eq. (7.110) also applies with the subscripts changed to the impeller outlet conditions

$$\frac{\dot{m}}{A} = \left\{\gamma p_{02}p_{02}\left[\frac{2\left(1+\dfrac{C_{b1}^2}{2c_pT_{01}}\right)}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}\right\}^{\frac{1}{2}} \tag{7.111}$$

The areas in Eqs (7.104), (7.110) and (7.111) refer to the flow areas at the respective locations.

(d) Actual Characteristic Curves

Figure 7.15 shows the actual characteristics curve, drawn for overall pressure ratio and efficiency plotted against $mT_{01}^{0.5}/p_{01}$ at fixed speed intervals of $N/T_{01}^{0.5}$. It is usual to transfer constant efficiency points onto the corresponding speed curves of the pressure ratio characteristics and then join those points together to form constant-efficiency contours.

It is evident that at all speeds in the range of mass flow over which the centrifugal compressor will operate before surging or choking occurs is quite wide. However, this width (margin) decreases as the speed increases. The onset of surge occurs at increasingly high mass flows as the speed increases; refer to the locus of limit of stability of the surge line in Figure 7.15.

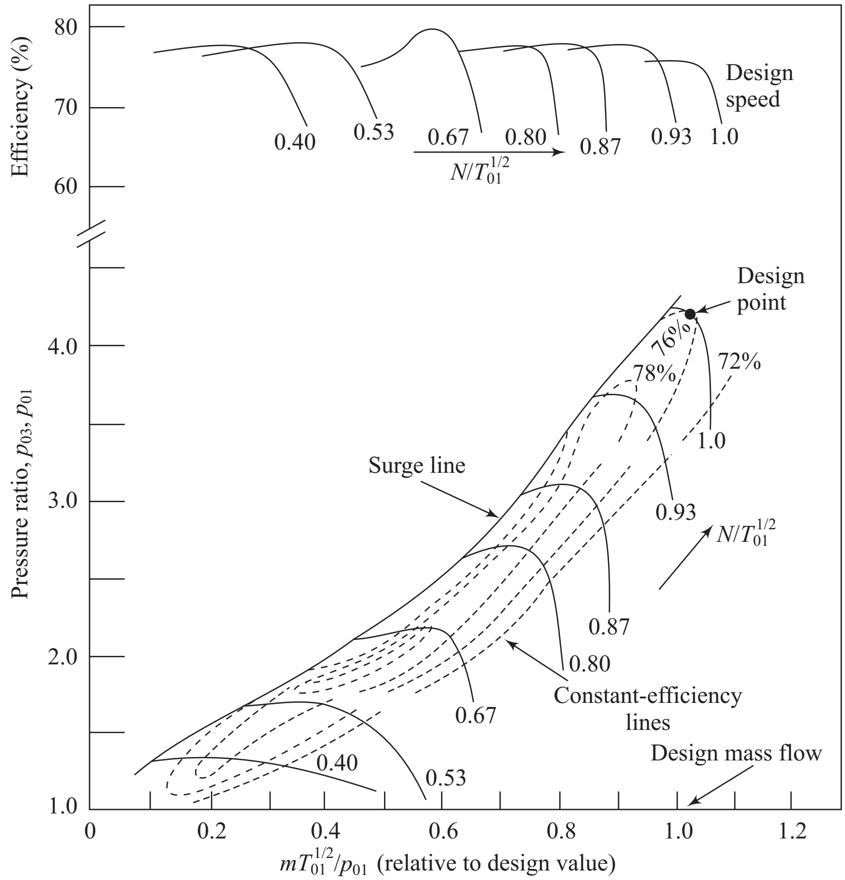


Figure 7.15 *Typical (Actual) Characteristics of a Centrifugal Compressor*

7.6.2 Axial Compressor

The performance characteristics (Figure 7.16) of axial compressor are similar to the centrifugal compressor but with distinct differences. The following dimensionless groups are used to represent the performance characteristics.

Figure 7.16 shows typical ideal and actual characteristics lines. Difference between the ideal and actual curve arises due to stage loss. Stages losses in compressor are mainly due to blade friction, flow separation, unsteady flow and vane-blade spacing.

Let pressure is p, flow rate is Q, non dimensional flow rate is $mT_{01}^{0.5}/p_{01}$, flow coefficient is φ, and stage loading coefficient is $\psi = C/C_b^2$. The performance of a compressor is defined according to its design. Axial compressors, particularly near design conditions, are, on the whole, amenable to analytical treatment. Substituting Eq. (7.19) in Eq. (7.36), we have,

$$\psi = 2\varphi(\tan \alpha_2 - \tan \alpha_1) \tag{7.112}$$

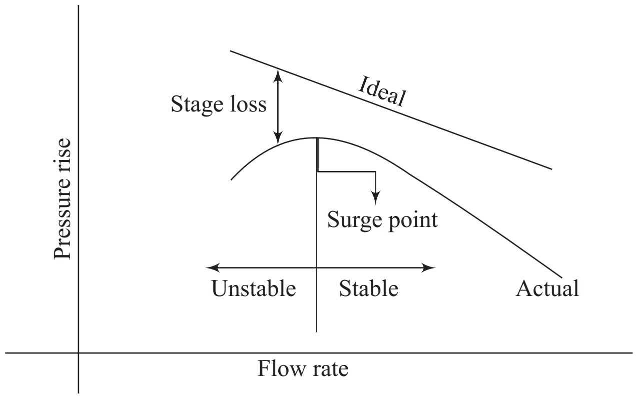


Figure 7.16 *General Representation of Ideal and Actual Characteristics of an Axial Compressor*

Rearranging Eq. (7.18) and using the definition of flow coefficient φ,

$$\frac{C_b}{C_f} = \frac{1}{\varphi} = \tan\alpha_2 + \tan\beta_2$$

$$\tan\alpha_2 = \left[\frac{1}{\varphi} - \tan\beta_2\right] \quad (7.113)$$

From Eqs (7.112) and (7.113),

$$\psi = 2[1 - \phi(\tan\beta_2 + \tan\alpha_1)] \quad (7.114)$$

The value of ($\tan\beta_2 + \tan\alpha_1$) does not change for a wide range of operating point till stalling. Also, $\alpha_1 = \alpha_3$ because of minor change in air angle at the rotor and stator, where α_3 is diffuser blade angle.

(a) Off-Design Operation

In actual practice, the operating point of the compressor deviates from the design point which is known as off-design operation.

Let $J = (\tan\beta_2 + \tan\alpha_3) = \text{constant}$

Representing design values with superscript '′'.

$$\psi' = 1 - J\varphi'$$

$$J = \frac{1-\psi'}{\varphi'} \quad (7.115)$$

For off design operations from Eq. (7.115),

$$\psi = 1 - J\varphi$$

$$\psi = 1 - \frac{\varphi(1-\psi')}{\varphi'} \quad (7.116)$$

Note that for positive values of J, slope of the ψ versus φ curve is negative and for negative values of J, slope of the curve is positive; refer Figure 7.17.

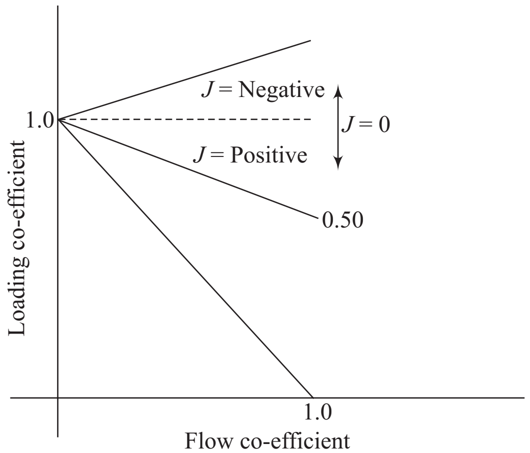


Figure 7.17 *Ideal Off-Design Performance*

(b) Surging

In the plot of characteristics, the line separating the graph between two regions—unstable and stable—is known as the surge line (Figure 7.18). This line is formed by joining the surge points at different rotational speeds. As discussed earlier for centrifugal compressor, the unstable flow in axial compressors due to complete breakdown of the steady conditions through flow is termed as surging.

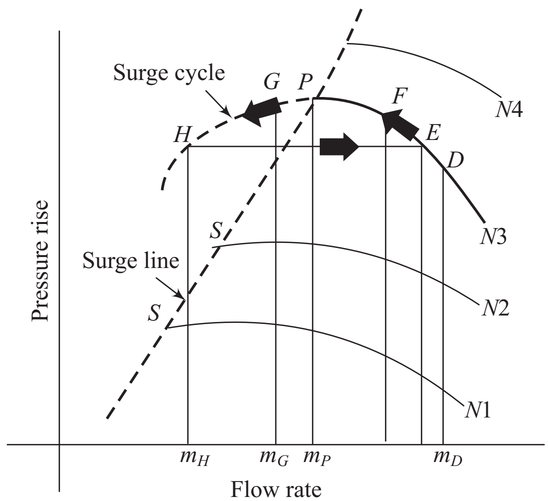


Figure 7.18 *Representation of Axial Flow Characteristics*

Suppose the initial operating point, $D(\dot{m},\, p_D)$, at some speed N_3 rpm. On decreasing the flow rate by partial closing of the valve, at the same rpm, along the characteristic curve, the pressure in the outlet pipe system increases. This will be taken care by increase in input pressure at the compressor. Further increase in pressure till point P (surge point) will increase the compressor pressure. Further moving towards left, keeping rpm constant, pressure in the outlet pipe will increase but compressor pressure will decrease leading to back air-flow towards the compressor. Due to this back flow, pressure in the outlet pipe will decrease because this unequal pressure condition cannot stay for a long period of time. Though valve position is set for lower flow rate, say point G, but compressor will work according to normal stable operation point, say E. The path $E - F - P - G - H$ will be followed leading to breakdown of flow, hence pressure in the compressor falls further to point (p_H). This increase and decrease of pressure in the outlet pipe will occur repeatedly in the outlet pipe and compressor following the cycle $E - F - P - G - H - E$. This cycle is known as the surge cycle.

This surge cycle phenomenon will cause vibrations in the whole machine and may lead to mechanical failure. That is why left portion of the curve from the surge point is called unstable region and may cause damage to the machine. So the recommended operation range is on the right side of the surge line.

(c) Stalling

Stalling is an important phenomenon that affects the performance of the compressor. An analysis is made of rotating stall in compressors of many stages, finding conditions under which a flow distortion can occur which is steady in a travelling reference frame, even though upstream total and downstream static pressure are constant. In an axial compressor, the phenomenon of separation of air flow at the aero-foil blades is called stalling, as shown in Figure 7.19, which leads to reduced compression and drop in engine power.

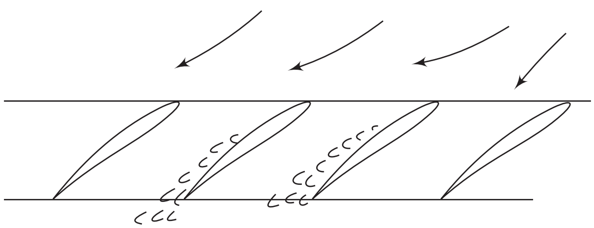


Figure 7.19 *Rotating Stall in an Axial Compressor*

Flow separation occurring on the suction side of the blade is called positive stalling. The flow separation that occurs on the pressure side of the blade is called negative stalling.

Negative stall is negligible compared to the positive stall because flow separation is least likely to occur on the pressure side of the blade.

In a multistage compressor, at the high pressure stages, axial velocity is very small. Stalling value decreases with a small deviation from the design point causing stall near the hub and tip regions whose size increases with decreasing flow rates. They grow larger at very low flow rate and affect the entire blade height. Delivery pressure significantly drops with large stalling which can lead to flow reversal. The stage efficiency drops with higher losses.

(i) Rotating Stalling

Non-uniformity of air flow in the rotor blades may disturb local air flow in the compressor without upsetting it. The compressor continues to work normally but with reduced compression. Thus, rotating stall decreases the effectiveness of the compressor.

The rotor blades of axial compressors may stall in separate patches. These stall patches travel around the compressor annulus. Consider a portion of a blade, as illustrated in Figure 7.19, to be affected by a stall patch. This patch must cause a partial obstruction to the flow which is deflected on both sides of it. Thus, the incidence of the flow on to the blades on the right of the stall cell is reduced, but the incidence to the left is increased. As these blades are already close to stalling, the net effect is for the stall patch to move to the left; the motion is then self-sustaining.

The stall is generally noticed in a compressor if the flow coefficient is reduced lower than the designed value.

(ii) Effects

- This reduces efficiency of the compressor.
- Forced vibrations in the blades may occur due to passage through stall compartment.
- These forced vibrations may match with the natural frequency of the blades causing resonance and hence, failure of the blade.

(d) Choking

This phenomenon in an axial compressor is similar to that of centrifugal compressor. In contrast to the stalling phenomenon which occurs at low flow coefficient, increase of flow coefficient leads to choking of the flow passages.

(e) Actual Characteristic Curve

Actual flow characteristics of axial flow compressor are shown in Figure 7.20. Actual characteristics of axial flow compressor describe variation of stagnation pressure ratio and efficiency with mass flow rate parameter. *Choking* is obtained at higher speeds of rotation. The *surge line* limits the characteristics with the low mass flow rate parameter on the left side of it. But it is found that surge usually only occurs at higher speeds of rotation. The characteristics curve therefore has a sharp maximum and this maximum is the starting point of a possible surge cycle, independently from the capacity of the downstream reservoir and compressor casing. At lower speeds, the maxima are smooth usually and surge can only start at an operating point with lower mass flow rate than at the maxima. The limiting phenomenon is then rotating stall if it becomes so intense that there is a risk for damage. Conventionally, start of rotating stall is considered as a limiting instability phenomenon. Also, the limiting instability line in the performance chart is called the surge line, although in practice, surge only occurs for the higher speeds.

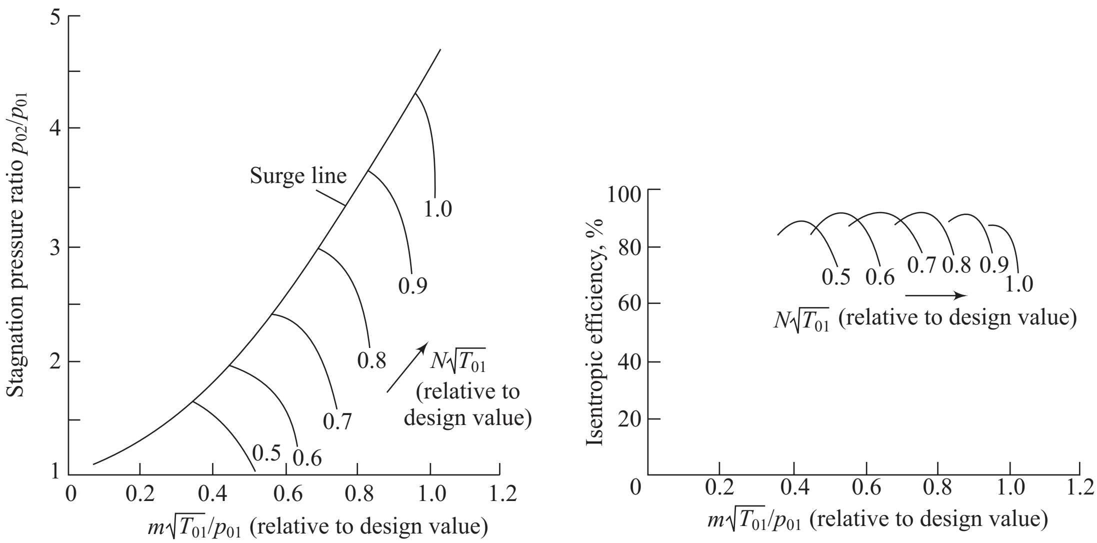


Figure 7.20 *Actual Characteristics of an Axial Compressor*

SUMMARY

- Application of Euler's turbomachinery equation and centrifugal and axial compressors are expressed as, $C_{b2}C_{w2} - C_{b1}C_{w1}$ and $C_b(C_{w2} - C_{w1})$, respectively.
- The specific works on $h - s$ (Mollier) diagram for centrifugal and axial compressor are given by, $w_s = c_p(T_{02s} - T_{01})$
- Different stage parameters: mass flow rate, stage work, degree of reaction, stage pressure ratio, stage pressure coefficient and stage efficiency for both centrifugal and axial compressor, are summarized in the following table.

Stage Parameters	*Axial Compressor*	*Centrifugal Compressor*
Mass flow rate	$\dot{m} = \rho\pi\frac{D_1^2}{4}C_{f1} = \rho\pi\frac{D_2^2}{4}C_{f2}$	$\dot{m} = \rho\pi D_1 B_1 Cf_1 = \rho\pi D_2 B_2 C_{f2}$
Stage work	$w = C_b C_f(\tan\alpha_2 - \tan\alpha_1)$ $w = C_b C_f(\tan\beta_1 - \tan\beta_2)$	• Backward curved blade $w = C_{b2}^2(1 - \varphi_2\cot\beta_2)$ • Forward curved blade $w = C_{b2}^2(1 - \varphi_2\cot\beta_2)$ • Radial tipped blade $w = C_{b2}^2$
Degree of reaction	$R = \frac{C_f}{2C_b}(\tan\beta_1 + \tan\beta_2)$	• Backward curved blade $R = \frac{1}{2} + \frac{1}{2}(\varphi_2\cot\beta_2)$ • Forward curved blade $R = \frac{1}{2} + \frac{1}{2}(\varphi_2\cot\beta_2)$ • Radial tipped blade $R = \frac{1}{2}$
Stage pressure ratio	$r_0 = \frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_{\text{stage}}\Delta T_{0\text{stage}}}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$	• Backward curved blade $r_0 = \frac{p_{03}}{p_{01}} = \left[1 + \frac{C_{b2}^2}{c_p T_{01}}(1 - \varphi_2\cot\beta_2)\right]^{\frac{\gamma}{\gamma-1}}$ • Forward curved blade $r_0 = \frac{p_{03}}{p_{01}} = \left[1 + \frac{C_{b2}^2}{c_p T_{01}}(1 - \varphi_2\cot\beta_2)\right]^{\frac{\gamma}{\gamma-1}}$ • Radial tipped blade $r_0 = \frac{p_{03}}{p_{01}} = \left(1 + \frac{C_{b2}^2}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$

Stage pressure coefficient	$\psi = \dfrac{2c_p \Delta T_{0\text{stage}}}{C_b^2} = \dfrac{2C_f(\tan\beta_1 - \tan\beta_2)}{C_b}$	• Backward curved blade $\psi = 2(1 - \varphi_2 \cot\beta_2)$ • Forward curved blade $\psi = 2(1 - \varphi_2 \cot\beta_2)$ • Radial tipped blade $\psi = 2$
Stage efficiency	$\eta_{tt} = \dfrac{\left[c_p T_{01} r_0^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]}{C_b C_f(\tan\beta_1 - \tan\beta_2)}$	$\eta_{tt} = \dfrac{c_p T_{01}\left[r_0^{\frac{\gamma-1}{\gamma}} - 1\right]}{C_b^2(1 - \varphi_2 \cot\beta_2)}$

✦ The expressions for the losses and the efficiency of axial and centrifugal compressors are described.

✦ The characteristics of compressors including choking, surging and stalling phenomenon, are also described.

REVIEW QUESTIONS

7.1 Using the Euler's turbomachinery equation and steady flow energy equation, prove that specific work for a compressor is given by,

$$w = h_{02} - h_{01} = \frac{C_2^2 - C_1^2}{2} + \frac{C_{b2}^2 - C_{b1}^2}{2} + \frac{C_{r1}^2 - C_{r2}^2}{2}$$

7.2 Applying steady flow energy equation to a compressor, prove that specific work is given by,

$$w = h_{02} - h_{01} = h_2 - h_1 + \frac{C_2^2 - C_1^2}{2}$$

7.3 Prove that static enthalpy rise in a compressor is,

$$h_2 - h_1 = \frac{C_{b2}^2 - C_{b1}^2}{2} + \frac{C_{r1}^2 - C_{r2}^2}{2}$$

7.4 Draw the isentropic compression process on $T - s$ diagram and prove that total pressure ratio is,

$$\frac{p_{02}}{p_{01}} = \left[1 + \frac{C_{b2}^2 - C_{b1}^2}{2c_p T_{01}} + \frac{C_{r1}^2 - C_{r2}^2}{2c_p T_{01}} + \frac{C_2^2 - C_1^2}{2c_p T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

7.5 Draw the isentropic compression process on $T - s$ diagram and obtain an expression for static temperature ratio.

7.6 Draw the schematic diagram and velocity triangles for an axial compressor stage at mean radius.

7.7 Draw the velocity triangles for rotor cascade of an axial flow compressor.

7.8 What is the normal stage or repeating stage of a compressor?

7.9 What is the difference between the rotor cascade and repeating stage of a compressor?

7.10 Draw the velocity triangles for repeating stage of an axial compressor with axial flow at entry and axial exit.

7.11 Why relative velocity at inlet is greater than that at outlet for an axial compressor?

7.12 Prove that the specific work for an axial compressor stage is,

$$w = C_b C_f(\tan a_2 - \tan a_1) = C_b C_f(\tan \beta_1 - \tan \beta_2) = c_p \Delta T_{\text{rotor}} + \frac{1}{2}(C_2^2 - C_1^2)$$

7.13 Show that the stagnation temperature rise in an axial compressor stage is expressed by,

$$(\Delta T_0)_{\text{stage}} = \frac{C_b C_f}{c_p}(\tan \beta_1 - \tan \beta_2)$$

What is the significance of blade deflection in axial compressor?

7.14 What factors limit the high temperature rise in an axial compressor stage?

7.15 Derive an expression for stage pressure rise of an axial compressor in terms of stage efficiency and stage temperature rise.

7.16 What parameters limit the pressure rise in an axial compressor stage?

7.17 Prove that a lesser number of stages are required for axial compressor for a given overall pressure ratio if the temperature rise in a stage is higher.

7.18 Obtain an expression for stage pressure or loading or work coefficient in an axial compressor stage.

7.19 Prove that total-to-total efficiency of an axial compressor stage is expressed as,

$$\eta_{tt} = \frac{\left[c_p T_{01}\left(r_0^{\frac{\gamma-1}{\gamma}}\right)\right]}{C_b C_f(\tan \beta_1 - \tan \beta_2)}$$

7.20 Obtain an expression for static-to-static efficiency of an axial compressor.

7.21 Derive an expression for static-to-total efficiency of an axial compressor.

7.22 Draw the meridional view and velocity triangles for a centrifugal compressor.

7.23 Draw the $T-s$ diagram for a compression process in a centrifugal compressor stage.

7.24 Obtain a relationship in terms of flow coefficient and blade angle at exit for specific work of centrifugal compressor having (a) backward curved blades, and (b) forward curved blades.

7.25 Show that the specific work for a centrifugal compressor having radial tipped blades is equal to the square of the blade peripheral velocity at the outlet.

7.26 Derive expressions in terms of flow coefficient and blade angle at exit for degree of reaction of a centrifugal compressor having (a) backward curved blades, and (b) forward curved blades.

7.27 Prove that the degree of reaction for a centrifugal compressor having radial tipped blades is 1/2.

7.28 Prove that stagnation pressure ratio of a centrifugal compressor having either backward curved or forward curved blades is given by,

$$r_0 = \frac{p_{03}}{p_{01}} = \left[1 + \frac{C_{b2}^2}{c_p T_{01}}(1 - \varphi_2 \cot \beta_2)\right]^{\frac{\gamma}{\gamma-1}}$$

7.29 Prove that the stagnation pressure ratio of a centrifugal compressor having radially tipped blades is,

$$r_0 = \frac{p_{03}}{p_{01}} = \left(1 + \frac{C_{b2}^2}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma - 1}}$$

7.30 What parameters limit the pressure rise in a centrifugal compressor stage?

7.31 Why the pressure rise per stage in a centrifugal compressor is more than that in an axial compressor?

7.32 Show that the stage pressure coefficient for forward curved and backward curved centrifugal compressor is expressed as,

$$\psi = 2(1 - \varphi_2 \cot \beta_2)$$

7.33 Show that the stage pressure coefficient for radial tipped blade centrifugal compressor is 2.

7.34 Derive an expression for total-to-total efficiency of a centrifugal compressor in terms of stagnation pressure ratio, flow coefficient at exit and blade angle at exit.

7.35 Why radial tipped blades are most widely used in centrifugal compressor stages?

7.36 Describe in brief the various losses which occur in a compressor.

7.37 What is the effect of finite number of blades with finite thickness on the specific work of a compressor?

7.38 What is the significance of power input factor in a compressor?

7.39 Prove that the actual stagnation temperature rise in a compressor is,

$$T_{03} - T_{01} = \frac{\psi \sigma C_{b2}^2}{c_p}$$

where ψ is the power input factor and σ is the slip factor.

7.40 Draw a graph between stagnation pressure ratio versus mass flow parameter for fixed speed intervals and discuss the ideal general characteristics of a centrifugal compressor.

7.41 What is surging? How does it affect the stable operation of a centrifugal compressor?

7.42 What are the effects of surging on the performance of a centrifugal compressor?

7.43 How surging can be minimized in a centrifugal compressor?

7.44 What is rotating stall in centrifugal compressor and why does it occur?

7.45 How does the angle of incidence effect the stalling of blades in a centrifugal compressor?

7.46 What are the effects of rotating stall in a centrifugal compressor?

7.47 What is meant by choking in a compressor?

7.48 State the locations where the choking may take place.

7.49 When the choking at the inlet of a centrifugal compressor takes place? State the governing equation for choking at the inlet.

7.50 State the condition and governing equation for choking in the rotating impeller.

7.51 Prove that the mass flow rate at choking condition increases with the impeller speed.

7.52 State the governing equation for choking mass flow rate in diffuser passages.

7.53 Draw and discuss the actual characteristics of a centrifugal compressor.

7.54 Draw and discuss the ideal and actual performance curves for an axial compressor.

7.55 Deduce the expressions of loading coefficient in terms of flow coefficient, air angle at inlet and blade angle at outlet for the on design and off design operation of an axial compressor.

7.56 Draw and discuss the loading coefficient versus flow coefficient characteristics at off design conditions for an axial compressor.

7.57 Discuss in brief the surging and its effects in an axial flow compressor.

7.58 What is stalling in an axial compressor stage? How is it developed?

7.59 What is positive stalling and negative stalling?

7.60 State the conditions which lead to stalling and the locations susceptible to stalling in multistage axial compressor.

7.61 What is meant by rotating stall? Explain in brief the development of large and small stall patches in an axial compressor stage.

7.62 What are the effects of stalling in an axial compressor?

7.63 Draw and discuss the actual characteristics curves of an axial compressor

PROBLEMS

7.1 A compressor having stagnation pressure ratio of 3 sucks the air with a velocity of 70 m/s from atmosphere at 1 bar, 17°C. The mass flow rate through the compressor is 30 kg/min. Calculate (a) the stagnation pressure and stagnation temperature at outlet, (b) specific work of the compressor, and (c) power of the driving motor, if mechanical efficiency is 95% and isentropic efficiency of the compressor is 72%.

[*Ans:* (a) p_{02} = 3.09 bar, T_{02} = 170.15°C, (b) w = 153.763 kJ/kg, (c) P = 80.93 kW]

7.2 A compressor draws air from atmosphere at 1 bar, 20°C. At exit, the stagnation pressure and temperature are 3.5 bar, 150°C, whereas static pressure is 3 bar. Calculate (a) total-to-total efficiency, (b) polytropic efficiency, and (c) air velocity at the exit.

[*Ans:* (a) η_{t-t} = 97.11%, (b) η_p = 97.48%, (c) C_2 = 191.5 m/s]

7.3 Air enters a three stage axial flow compressor at 1 bar, 23°C. The specific work input per stage is 25 kJ/kg. If the stage efficiency is 86%, calculate (a) the static temperature at the exit, (b) static pressure ratio, and (c) compressor efficiency. [*Ans:* (a) T_4 = 101.64°C, (b) $\dfrac{p_4}{p_1}$ = 1.9617, (c) η_c = 85.1%]

7.4 A multistage axial flow compressor absorbs 4.5 MW while delivering 20 kg/s of air from inlet stagnation conditions of 1 bar, 15°C. The polytropic efficiency of the compressor is 90% and the stage stagnation pressure ratio is constant. The temperature rise in the first stage may be taken as 20°C. Calculate (a) pressure at the compressor outlet, (b) the number of stages, and (c) the overall isentropic efficiency of the compressor. [*Ans:* (a) $(p_0)_{n+1}$ = 6.12 bar, (b) n = 9, (c) η_{t-t} = 87.5%]

7.5 An eight stage axial flow compressor provides an overall pressure ratio of 4 : 1 with an overall isentropic efficiency of 82%, when the temperature of air at inlet is 20°C. The work is divided equally between the stages. 50% reaction is used with a mean blade speed 180 m/s and a constant axial velocity 90 m/s through the compressor. Estimate the power required and blade angles. Assume air to be a perfect gas. Calculate (a) specific work required to drive the compressor, and (b) the blade angles. [*Ans:* (a) w = 174.57 kJ/kg, (b) $\beta_1 = \alpha_2$ = 59.16°, $\beta_2 = \alpha_1$ = 18°]

7.6 An axial compressor of mean diameter 600 mm is delivering air at the rate of 57 kg/s while running at 15000 rpm. The initial temperature of the air is 35°C, actual temperature rise is 30°C and pressure ratio of the compressor is 1.3. Determine (a) power required to drive the compressor if mechanical efficiency is 86%, (b) stage efficiency, and (c) degree of reaction if the temperature at the exit of rotor is 55°C. [*Ans:* (a) $P = 1998.31$ kW, (b) $\eta_{stage} = 79.92\%$, (c) $R = 0.67$]

7.7 The initial stagnation state is 1 bar, 20°C and the final stagnation state is 5 bar, 270°C in a multistage axial compressor. Calculate (a) overall isentropic efficiency, (b) polytropic efficiency of the compressor, and (c) number of stages required if the actual total temperature rise per stage does not exceed 16°C and assume that the polytropic efficiency is the stage efficiency.
[*Ans:* (a) $\eta_{t-t} = 76.4\%$, (b) $\eta_{poly} = 74.53\%$, (c) $n = 12$]

7.8 The overall pressure ratio is 6.3 in a sixteen stage axial compressor having stage efficiency of 89.5%. The state of the air at the inlet is 1 bar, 15°C. Calculate (a) the polytropic efficiency, and (b) overall efficiency. [*Ans:* (a) $\eta_p = 89.67\%$, (b) $\eta_0 = 86.75\%$, (c) $PF = 0.9693$]

7.9 The state of air at entry of a 50% reaction axial compressor is 1 bar, 37°C. The flow coefficient is 0.6 and total-to-total efficiency is 82%. The stagnation pressure ratio is 1.7, coefficient of pressure is 0.6 and work done factor is 0.85. Determine (a) the mean blade speed, (b) blade angles at inlet and outlet, and (c) power input for a mass flow rate of 10 kg/s.
[*Ans:* (a) $C_b = 439.3$ m/s, (b) $\beta_1 = 41.57°$, $\beta_2 = 61.65°$, (c) $P = 579$ kW]

7.10 An axial flow compressor running at 12000 rpm sucks air at the rate of 20 kg/s from atmosphere which is at a state 1 bar, 15°C. The tip diameter of the impeller is 0.6 m, blade angle at inlet is 40° and blade angle at outlet of the impeller is 70°. The entry of the air in the impeller is axial without any whirl component. The axial flow components of the air velocity remain constant through the impeller and stator blades. The diffuser blades downstream of the rotor make the flow again axial for the next stage. The total-to-total efficiency for the rotor is 85%, mechanical efficiency of the drive is 98%, work done factor is 0.88 and diffusion efficiency is 80%. Calculate (a) static pressure ratio across the rotor, (b) static pressure across the stage, (c) degree of reaction, and (d) power input.

[*Ans:* (a) $\frac{p_2}{p_1} = 2.518$, (b) $\frac{p_3}{p_1} = 3.4721$, (c) $R = 0.6527$, (d) $P = 2015$ kW]

7.11 The stagnation pressure and temperature at the inlet of an axial compressor are1 bar, 320 K. The stagnation pressure ratio is 4:1 and overall stagnation isentropic (total-to-total) efficiency is 86%. The mean peripheral speed of the blades is 190 m/s and the degree of reaction is 0.5 at the mean radius. The relative air angles at inlet and outlet are 30° and 10° respectively. The work done factor is 0.88. Calculate (a) stagnation polytropic efficiency, (b) number of stages, (c) inlet pressure and temperature, and (d) blade height in the first stage if the hub to tip ratio is 0.4 and mass flow rate is 20 kg/s. [*Ans:* (a) $\eta_{poly} = 88.4\%$, (b) $n = 11$, (c) $p_1 = 0.68664$ bar, $T_1 = 14.39°C$ (d) $h_b = 114$ mm]

7.12 Stagnation pressure and temperature at the inlet of a 50% reaction axial compressor stage are 1 bar and 20°C, respectively. The mean diameter of the blade ring is 0.35 m and speed of the compressor is 18000 rpm. Air angle at rotor and stator exit is 60° and height of the blade at entry is 50 mm. The flow coefficient is 0.5 and work done factor is 0.88. Total-to-total efficiency of the compressor is 85% and mechanical efficiency is 0.96. Calculate (a) air angle at the rotor and stator entry, (b) mass flow rate of air, (c) power required, (d) loading coefficient, (e) stagnation pressure ratio of the stage, and (f) Mach number at the entry of the rotor.
[*Ans:* (a) $\beta_1 = \alpha_2 = 35.1°$, (b) $\dot{m} = 9.199$ kg/s, (c) $P = 372.376$, (d) $\psi = 0.372$, (e) $r_0 = 1.472$, $M_{1r} = 0.4622$]

7.13 The mean blade speed is 250 m/s in the second stage of an axial compressor. The axial velocity is 150 m/s. The blades are symmetrical and the blade deflection is 0.6. The isentropic efficiency of the stage is 85% and stagnation pressure ratio of the stage is 4. Air at the rate of 1000 kg/min enters the stage with a stagnation temperature of 300 K. If the mechanical efficiency is 99%, calculate (a) the number of stages, and (b) power required to compress.

Assume that the kinetic energy of the air entering the successive stage is negligible. Also assume the absolute and relative air angles with respect to axial direction.

[*Ans:* (a) $n = 7$, (b) $P = 2651.51$ kW]

7.14 The mean diameter of the blade ring of an axial compressor is 0.6 m and its speed of rotation is 15000 rpm. The velocity of whirl at inlet is 85 m/s and axial velocity is 225 m/s which remains constant throughout. The state of air at inlet is 1 bar, 300 K and stage efficiency is 89%. The specific work and power required by the compressor are 45 kJ/kg and 425 kW. Calculate (a) pressure ratio, (b) fluid deflection angle, (c) degree of reaction, (d) flow rate of air, and (e) shaft power if mechanical efficiency of the drive is 95%.

[*Ans:* $\dfrac{p_2}{p_1} = 1.547$, (b) $\beta_2 - \beta_1 = 7.565°$, (c) $R = 0.719$, (d) $\dot{m} = 9.444$ kg/s, (e) $BP = 447.37$ kW]

7.15 The total head pressure and temperature at inlet of a ten stage axial compressor is 1 bar and 300 K. The overall isentropic efficiency is 85%. All the stages are designed for 50% reaction. The work is divided equally in all the stages. The blade speed at the mean height is 210 m/s and the axial velocity is 175 m/s at a particular stage. The absolute air angle at entry of the rotor is 75° and the work done factor is 0.92. Calculate (a) relative air angle at the inlet, (b) stagnation pressure ratio of the stage, (c) polytropic efficiency, (d) static temperature of the air at entry of the rotor, (e) static temperature of the air leaving the first stage, and (f) rotor inlet relative Mach number.

[*Ans:* (a) $\beta_1 = 47.01°$, (b) $\left(\dfrac{p_{11}}{p_1}\right)_0 = 5.565$, (c) $\eta_p = 89.14\%$, (d) $T_1 = 283.7$ K,
(e) $T_2 = 314.19$ K, (f) $M_{1r} = 0.71$]

7.16 The speed of an axial compressor designed on the free vortex principle is 5000 rpm. The hub to tip ratio is 0.6 and the specific work is 25 kJ/kg. The work done factor is 0.94 and total-to-total efficiency of the stage is 89%. The absolute velocity of air at inlet is 150 m/s and the ambient conditions are 1 bar and 300 K. Inlet air angle at blade tip is 25°. Calculate (a) the tip radius and corresponding rotor air angle at exit if the Mach number relative to tip is limited to 0.92, (b) mass flow rate, (c) stagnation pressure ratio, (d) power required, and (e) rotor air angles at the root section.

[*Ans:* $r_{tip} = 0.731$ m, $(\beta_2)_{tip} = 31.7°$, (b) $\dot{m} = 150.7$ kg/s, (c) $\dfrac{p_{02}}{p_{01}} = 1.283$,
(d) $P = 3768.16$ kW, (b) $\beta_{1r} = 39.76°$, $\beta_{2r} = 71.9°$]

7.17 Calculate the stage parameters of compressor of Example 7.16 if the absolute velocity at inlet is axial. Ignore the given value of inlet air angle at blade tip and also, compute it.

[*Ans:* (a) $\beta_{1t} = 28.595°$, (b) $r_{tip} = 0.526$ m, $(\beta_2)_{tip} = 40.03°$, (c) $\dot{m} = 88.33$ kg/s,
(d) $\dfrac{p_{02}}{p_{01}} = 1.283$, (e) $P = 2208.6$ kW, (f) $\beta_{1r} = 42.25°$, $\beta_{2r} = 8.45°$]

7.18 The specific work input and work done factor of a free vortex design axial compressor are 25 kJ/kg and 0.95, respectively. The blade velocities at the root, mean radius and tip are 160 m/s, 215 m/s and

270 m/s respectively. The axial velocity is 155 m/s which remains constant throughout from root to tip. The degree of reaction at the mean radius is 50%. Calculate (a) inlet and exit relative air angles at root, mean radius and tip of the blades, and (b) reaction at the root and tip.

[*Ans:* (a) $\beta_{1m} = 42.576°$, $\beta_{2m} = 73.4°$, $\beta_{1t} = 33.62°$, $\beta_{2t} = 48.79°$, $\beta_{1r} = 57.595°$, $\beta_{2r} = -66.72°$, (b) $R_t = 0.6831$, $R_r = 0.00979$]

7.19 The air flows at the rate of 25 kg/s in the first stage of an axial flow compressor of 50% reaction. The specific work input is 0.25 kJ/kg and speed of the compressor is 10200 rpm. The absolute velocity at inlet makes an angle of 75° with the tangent. The mean blade speed is 200 m/s and the work done factor is 0.95. The stagnation and static temperatures at inlet are 300 K and 285 K respectively. The total-to-total efficiency is 89% and stagnation pressure at inlet is 1 bar. Calculate (a) the mean radius, (b) relative and absolute air angles at the mean radius, (c) blade height, and (d) overall pressure ratio.

[*Ans:* (a) $r_m = 0.1872$ m, (b) $\beta_{1m} = \alpha_{2m} = 45.32°$, $\beta_{2m} = \alpha_{1m} = 78.49°$, (c) $B = 0.124$ m, (d) $r_0 = 1.284$]

7.20 A centrifugal compressor running at 12000 rpm delivers 600 m^3/min of free air. The diameter ratio of the impeller is 2 and pressure ratio is 4 with an isentropic efficiency of 84%. The air is sucked into the compressor from the atmosphere at 1 bar, 23°C. The blades are radial at the outlet of the impeller and the flow velocity of 60 m/s may be assumed constant throughout. The entry of the air is radial at inlet. Calculate (a) power input, (b) impeller diameters at the inlet and outlet, and (c) blade angle at inlet. [*Ans:* (a) $P = 2023.856$ kW, (b) $D_1 = 0.3322$ m, $D_2 = 0.6644$ m, (c) $\beta_1 = 16.04°$]

7.21 The inlet and outlet diameters of the impeller of a centrifugal compressor are 0.3 m and 0.6 m respectively. The compressor sucks air from the atmosphere at 1 bar, 23° without any whirl component. The speed of the compressor is 10000 rpm and velocity of flow is constant throughout at 120 m/s. The blade angle at outlet is 75° and blade width at inlet is 60 mm. Calculate (a) specific work, (b) stagnation pressure at exit, (c) mass flow rate, (d) power to drive compressor if overall efficiency is 70%. [*Ans:* (a) $w = 88.596$ kJ/kg, (b) $p_{02} = 246.6$ kPa, (c) $\dot{m} = 7.88$ kg/s, (d) $P = 997.34$ kW]

7.22 A centrifugal compressor running at 15000 rpm sucks air from atmosphere at 1 bar, 300 K and develops a stagnation pressure ratio of 4. The entry of air is radial at inlet of the compressor. The blades are radial at outlet and flow component of velocity at exit is 135 m/s. The diameters of the impeller at inlet and outlet are 0.25 m and 0.58 m respectively. The total-to-total efficiency of the compressor is 78%. Draw the velocity triangles and calculate (a) blade angle at inlet, (b) slip, and (c) slip factor. [*Ans:* (a) $\beta_1 = 34.51°$, (b) slip = 43.547 m/s, (c) $\sigma_s = 0.9044$]

7.23 A centrifugal compressor running at 15000 rpm delivers 18.2 kg/s of air with a stagnation pressure ratio of 4. The stagnation temperature at inlet is 288 K, slip factor 0.9, power input factor 1.04 and total isentropic efficiency 80%. Calculate the external diameter of the impeller and power input.

[*Ans:* $D_2 = 0.5518$ m, $P = 3200.13$ kW]

7.24 The mass flow rate of air through a centrifugal compressor used as a supercharger for aero-engines is 180 kg/min. The suction pressure and temperature are 1 bar and 280 K. The suction velocity is 90 m/s. The conditions of the air at the end of isentropic compression are 1.5 bar, 335 K and 230 m/s. Calculate (a) isentropic efficiency of the compressor, (b) power required to drive the compressor, and (c) overall efficiency of the unit.

Assume that the kinetic energy of the air gained in the impeller is entirely converted into pressure energy in the diffuser. [*Ans:* (a) $\eta_s = 73.34\%$, (b) $P = 233.025$ kW, (c) $\eta_o = 71.52\%$]

7.25 A centrifugal compressor running at 20000 rpm has diameter of the impeller at exit as 0.60 m. The total-to-total efficiency is 80% and the slip factor is 0.8. The absolute velocities at inlet and outlet of the compressor may be assumed constant. Stagnation temperature at inlet is 17°C. Calculate (a) the temperature rise of the air passing through the compressor, and (b) the stage pressure ratio.

[*Ans:* (a) $(T_{03s} - T_{01}) = 251.41$ K or °C, (b) $r_0 = 8.89$]

7.26 The static conditions at the inlet of a centrifugal compressor 1 bar, 20°C. The velocity of air at the inlet is 60 m/s. The free air delivered by a compressor is 20 kg/min. The isentropic efficiency of the compressor is 70% and total head (stagnation) pressure ratio is 3. Determine (a) the total head (stagnation) temperature at exit, and (b) power required to drive the compressor if the mechanical efficiency of the drive is 95%. [*Ans:* (a) $T_{03} = 450.09$ K, (b) $P = 54.76$ kW]

7.27 The suction conditions are 1 bar, 288 K in a two stage centrifugal compressor which delivers 500 m^3/min of free air. The pressure ratio and total-to-total efficiency of each stage are 1.25 and 80%, respectively. Find the isentropic efficiency (total-to-total) for the entire compression process.

[*Ans:* $\eta_{tt} = 79.38\%$]

7.28 A centrifugal compressor running at 10000 rpm having inlet static conditions of air 1 bar, 30°C delivers free air at the rate of 1.5 m^3/s. The static-to-static efficiency is 90% and static pressure ratio is 5. The velocity of flow is constant throughout at 50 m/s. The slip factor is 0.92 and the blades are radial at outlet. Assuming power factor of 1.11, calculate (a) static temperature of air at the outlet, (b) power required, (c) diameter of the impeller, (d) blade angle at inlet, and (e) diffuser inlet angle.

[*Ans:* (a) $T_2 = 499.55$ K, (b) $P = 339.05$ kW, (c) $D_2 = 1.1$ m, (d) $\beta_1 = 10.94°$, (e) $\alpha_2 = 5.39°$]

7.29 The static temperature at inlet of a centrifugal compressor is 27°C and tip speed is 550 m/s. The power factor is 1.05 and slip factor is 0.85. The static-to-static efficiency of the compressor is 82%. Calculate the static pressure ratio of the compressor.

$$\left[Ans: \frac{p_2}{p_1} = 6.87 \right]$$

7.30 The static pressure and temperature of air at the entry of a centrifugal compressor are 1 bar and 283 K. The power input to the compressor is 450 kW and total pressure at outlet is 5 bar. The rotational speed of the compressor is 20000 rpm and the velocity of air at the inlet is 150 m/s. The total-to-total efficiency is 80% and hub diameter is 120 mm. If the slip factor is 0.9, calculate (a) the change in total temperature, (b) diameter of the impeller at outlet and inlet, (c) mass flow rate of air.

[*Ans:* (a) $T_{02} - T_{01} = 185.13$ K or °C, (b) $D_2 = 0.434$ m, $D_1 = 0.12046$ m, (c) $\dot{m} = 2.42$ kg/s]

7.31 The blade speed in a centrifugal compressor is 550 m/s with no pre whirl. The total-to-total efficiency of the compressor is 85% and slip factor is 0.95. The ambient temperature is 15°C and mass flow rate of the compressor is 25 kg/s. Calculate for standard sea level (a) the pressure ratio, (b) specific work input, and (c) power required. [*Ans:* (a) $r_0 = 1.19$, (b) $w = 28.737$ kJ/kg, (c) $P = 718.45$ kW]

7.32 A centrifugal compressor delivers 30 kg/s of air while running at a speed of 15000 rpm. The exit relative velocity at exit radius of 0.35 m is 100 m/s at an angle of 75°. The entry of the air is axial in the compressor and stagnation state at inlet is 1 bar, 23°C. Calculate (a) torque, (b) power input to the compressor, (c) specific work, (d) ideal head developed, and (e) stagnation pressure at the exit.

[*Ans:* (a) $T = 5.501$ kN.m, (b) $P = 8640.8$ kW, (c) $w = 288.1428$ kW, (d) $H_e = 29372.35$ m, (e) $p_{02} = 10.44$ bar]

7.33 A centrifugal compressor delivers 15 kg/min of air while running at 15000 rpm. The root and tip diameters of the eye are 0.18 m and 0.31 m respectively. The ambient conditions are 1 bar, 288 K. The pre-whirl is at an angle of 20° and the inlet flow velocity is constant at 150 m/s. Calculate (a) the vane angles at root and tip of the eye, (b) static pressure, temperature and density at inlet, and (c) relative Mach number at the tip of the eye.

[*Ans:* (a) $\beta_{1r} = 59.95°$, $\beta_{1t} = 38.5°$, (b) $p_1 = 85.42$ kN/m^2, $T_1 = 275.3$ K, $\rho_1 = 1.033$ kg/m^3, (c) $M_{1r-t} = 0.7252$]

7.34 A 600 kW motor is used to drive a centrifugal compressor running at 20000 rpm. The outer diameter of the impeller is 0.5 m and the blade angle at exit is 26° measured from the radial direction. The velocity of flow at outlet is 120 m/s. The mechanical efficiency is 95% and there is no slip. The velocity at inlet is 100 m/s without any whirl. The stagnation pressure ratio of the stage is 6. The flow at inlet is assumed incompressible and the ambient conditions are 1.01325 bar and 288 K. Calculate (a) the mass flow rate of air, (b) eye tip and hub diameter if the radius ratio is 0.3 for the eye , and (c) total-to-total efficiency. [*Ans:* (a) $\dot{m} = 2.313$ kg/s, (b) $D_h = 48.79$ mm, $D_t = D_1 = 162.5$ mm, (c) $\eta_{t-t} = 78.5\%$]

7.35 A centrifugal compressor sucks air from atmosphere at state 1.01325 bar, 288 K. There is no whirl at inlet and blades are radial. The tip speed of the impeller is 375 m/s, radial velocity at exit is 30 m/s and flow area at the exit of the impeller is 0.095 m^2. The slip factor is 0.9 and total-to-total efficiency is 90%. Calculate (a) the Mach number at the impeller tip, and (b) mass flow rate.

[*Ans:* (a) $M_2 = 0.895$, (b) $\dot{m} = 5.36$ kg/s]

7.36 A centrifugal compressor running at 45000 rpm has 10 radial blades with impeller tip diameter of 170 mm. There is no whirl at inlet and the slip factor is 0.9. The mass flow rate is 30 kg/min. The mean diameter of the eye is 65 mm while the annulus height at the eye is 25 mm. The static pressure and temperature at the inlet of impeller is 95 kPa and 22°C, respectively. Calculate (a) theoretical power transferred to the air, (b) blade angle at the mean diameter of impeller inlet, (c) stagnation temperature at impeller outlet, and (d) stagnation pressure at impeller exit if the total-to-total efficiency is 90%.

[*Ans:* (a) $P = 72.2$ kW, (b) $\beta_1 = 29.7°$, (c) $T_{02} = 442.47$ K, (d) $p_{02} = 3.4974$ bar]

7.37 A centrifugal compressor having stage pressure ratio of 4 delivers 600 kg/min of air while running at 12000 rpm. The slip and power factors are 0.91 and 1.06, respectively. The isentropic efficiency of the compressor is 92%. There are sonic conditions at the exit of the impeller to have shockless entry. The ambient conditions are 1.01325 bar and 288 K. The entry of the air in the compressor is without any whirl and overall isentropic efficiency is 83%. Calculate (a) the outer diameter of the impeller, and (b) axial depth of the impeller. [*Ans:* (a) $D_2 = 0.667$ m, (b) $B_2 = 0.025$ m]

7.38 A single-sided centrifugal compressor having total pressure ratio of 5 delivers 600 kg/min of air while running at 15000 rpm. The air enters axially with a velocity of 140 m/s from atmosphere at 1 bar, 300 K. The slip factor is 0.92 and isentropic efficiency is 81%. Calculate (a) the stagnation temperature rise, (b) tip speed of the impeller, (c) tip diameter, (d) annulus area of the eye, and (e) theoretical power required to drive the compressor.

[*Ans:* (a) $(T_{02} - T_{01}) = 216.23$°C or K, (b) $C_{b2} = 486$ m/s, (c) $D_2 = 0.619$ m, (d) $A_1 = 0.0615$ m^2, (e) $P = 2173.1$ kW]

7.39 Air enters axially in a centrifugal compressor fitted with radial blades from a practically quiescent atmosphere of 1 bar, 288 K. The velocity at the outlet of the diffuser is negligible. The speed of the compressor is 20000 rpm and tip diameter of the impeller is 0.45. Assuming no losses, calculate the static exit temperature and pressure. [*Ans:* $T_3 = 508.99$ K, $p_3 = 7.338$ bar]

7.40 A centrifugal supercharger (centrifugal compressor) is connected to the manifold of the combustion chamber of an engine. It develops a total pressure of 2 bar while the ambient conditions are 0.65 bar, –8°C. The mass flow rate is 60 kg/min at a speed of 20000 rpm. The velocity at inlet of the impeller is 110 m/s. The isentropic efficiency of the machine is 81%. Assuming no pre-whirl and radial discharge, calculate (a) the impeller diameter, (b) annulus area at the entry of impeller, and (c) theoretical power required to run the compressor.

[*Ans:* (a) $D_3 = 0.337$ m, (b) $A_2 = 0.00732$ m^2, (c) $P = 124.52$ kW]

7.41 A centrifugal compressor having radial blades is running at 15000 rpm. The intake of air is axial at stagnation state of 1 bar, 32°C. The flow velocity is 150 m/s at rotor exit diameter of 0.6 m and total-to-total efficiency is 81%. If the slip is 50 m/s, calculate (a) the slip coefficient, and (b) pressure ratio of the stage. [*Ans:* (a) $\sigma_s = 0.894$, (b) $r_0 = 4.376$]

MULTIPLE CHOICE QUESTIONS

1. The inlet and exit velocity diagrams of a turbomachine rotor are shown in Figure 7.21 (a) and (b). The turbomachine is

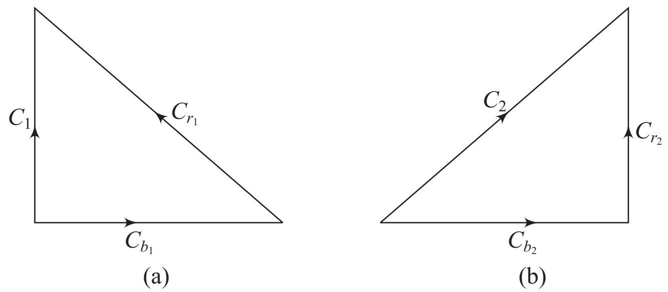


Figure 7.21 *Figure of Multiple Choice Question 1*

(a) An axial compressor with radial blades
(b) A radial compressor with radial blades
(c) A radial compressor with curved blades
(d) An axial compressor with forward curved blades

2. In a centrifugal compressor, assuming the same overall dimensions, blade inlet angle and rotational speeds, which of the following bladings will be given the maximum pressure rise?
(a) Forward curved blades
(b) Backward curved blades
(c) Radial blades
(d) All three types of bladings have the same pressure rise

3. In a centrifugal compressor, the highest Mach number leading to shockwave in the fluid flow occurs at
(a) Diffuser inlet radius (b) Diffuser outlet radius
(c) Impeller inlet radius (d) Impeller outer radius

4. **Assertion (A):** The specific work input for an axial flow compressor is lower than that of the centrifugal compressor for the same pressure ratio.

Reason (R): The isentropic efficiency of axial flow compressor is much higher than that of a centrifugal compressor.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

5. The curve shown in Figure 7.22 represents the variation of theoretical pressure ratio with mass flow rate of a compressor running at constant speed. The permissible operating range of the compressor is represented by the part of the curve from
(a) A to B (b) B to C
(c) B to D (d) D to E

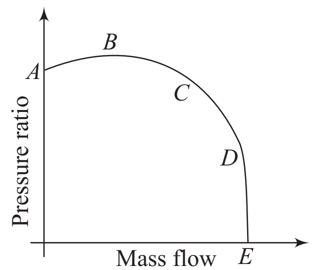


Figure 7.22 *Figure of Multiple Choice Question 5*

6. For a multistage compressor, the polytropic efficiency is
(a) The efficiency of all stages combined together
(b) The efficiency of one stage
(c) Constant throughout for all the stages
(d) A direct consequence of the pressure ratio

7. Which of the following is the effect of blade shape on the performance of centrifugal compressor?
(a) Backward curved blades have poor efficiency
(b) Forward curved blades have higher efficiency
(c) Backward curved blades lead to stable performance
(d) Forward curved blades produce lower pressure ratio

8. Surging basically implies
(a) Unsteady, periodic and reversed flow
(b) Forward motion of air at a speed above sonic velocity
(c) The surging action due to the blast of air produced in a compressor
(d) Forward movement of aircraft

9. Degree of reaction in an axial compressor is defined as the ratio of static enthalpy rise in the
(a) Rotor to static enthalpy rise in the stator (b) Stator to static enthalpy rise in the rotor
(c) Rotor to static enthalpy rise in the stage (d) Stator to static enthalpy rise in the stage

10. The usual assumption in elementary compressor cascade theory is that
(a) Axial velocity through the cascade changes
(b) The pressure rise across the cascade is given by equation of state
(c) Axial velocity through the cascade does not change
(d) With no change in axial velocity between inlet and outlet, the velocity diagram is formed

11. Phenomenon of choking in compressor means
(a) No flow of air
(b) Fixed mass flow rate regardless of pressure ratio
(c) Reducing mass flow rate with increase in pressure ratio
(d) Increased inclination of the chord with air stream

12. Centrifugal compressors are suitable for large discharge and wider range of mass flow but a relatively low discharge pressure of the order of 10 bar, because of
(a) Low pressure ratio (b) Limitation of size of receiver
(c) Large speeds (d) High compression index

13. The degree of reaction of a centrifugal compressor is equal to
    (a) $1-\frac{C_{w2}}{2C_{b2}}$ (b) $1-\frac{C_{b2}}{2C_{w2}}$ (c) $1-\frac{2C_{w2}}{C_{b2}}$ (d) $1-\frac{C_{w2}}{C_{b2}}$
14. The value of polytropic index of compression in the centrifugal air compressor design is generally taken as
    (a) 1.2 (b) 1.3 (c) 1.4 (d) 1.5
15. The turbomachine used to circulate refrigerant in large refrigeration plant is
    (a) A centrifugal compressor (b) A radial turbine
    (c) An axial compressor (d) An axial turbine
16. In an axial flow compressor stage, air enters and leaves the stage axially. If the whirl component of air leaving the rotor is half of the mean peripheral velocity of the rotor blades, then degree of reaction will be
    (a) 1 (b) 0.75 (c) 0.5 (d) 0.25
17. If an axial flow compressor is designed for a constant velocity through all stages, then the area of annulus of the succeeding stages will
    (a) Remain the same (b) Progressively decrease
    (c) Progressively increase (d) Depend upon the number of stages
18. What will be the shape of the velocity triangle at the exit of a radial bladed centrifugal impeller taking into account slip?
    (a) Right angled (b) Isosceles
    (c) All angles less than 90° (d) One angle greater than 90°
19. Which one of the following statements is true?
    (a) In a multistage compressor, adiabatic efficiency is less than stage efficiency
    (b) In a multistage turbine, adiabatic efficiency is less than stage efficiency
    (c) Reheat factor for a multistage compressor is greater than one
    (d) Reheat factor does not affect the multistage compressor performance
20. At the eye tip of a centrifugal impeller, blade velocity is 200 m/s, while the uniform axial velocity at inlet is 150 m/s. If the sonic velocity is 300 m/s, then the inlet Mach number of the flow will be
    (a) 0.5 (b) 0.66 (c) 0.83 (d) 0.87
21. In a centrifugal compressor terminology, vaneless space refers to the space between
    (a) The inlet and blade inlet edge (b) Blades in the impeller
    (c) Diffuser exit and volute casing (d) Impeller tip and diffuser inlet edge
22. If the static temperature rise in the rotor and stator are ΔT_A and ΔT_B, respectively, the degree of reaction in axial flow compressor is given by
    (a) $\frac{\Delta T_A}{\Delta T_B}$ (b) $\frac{\Delta T_A}{\Delta T_A + \Delta T_B}$ (c) $\frac{\Delta T_B}{\Delta T_A + \Delta T_B}$ (d) $\frac{\Delta T_B}{\Delta T_A}$
23. The capacity of an air compressor is specified as 3 m^3/min. It means that the compressor is capable of
    (a) Supplying 3 m^3 of compressed air per minute (b) Compressing 3 m^3 of free air per minute
    (c) Supplying 3 m^3 of compressed air at NTP (d) Compressing 3 m^3 of standard air per minute
24. Which one of the following pairs of features and compressors type is not correctly matched?
    (a) Vane compressor: Intake and delivery ports compressor is attained by back flow and internal compression cylindrical rotor set to eccentric casing

(b) Reciprocating compressor: Intermittent discharge requires receiver, produces high pressure, slow speed and lubrication problems
(c) Centrifugal compressor: Continuous flow, radial flow, handles large volume, much higher speed and fitted into design of aero-engines
(d) Axial flow compressor: Successive pressure drops through contracting passages, blades are formed from a number of circular arcs, axial flow

25. Consider the following statements.

In centrifugal compressors, there is a tendency of increasing surge when
1. The number of diffuser vanes is less than the number of impeller vanes
2. The number of diffuser vanes is greater than the number of impeller vanes
3. The number of diffuser vanes is equal to the number of impeller vanes
4. Mass flow is greatly in excess of that corresponding to the design mass flow

Which of these statements is/are correct?
(a) 1 and 4 (b) 2 alone (c) 3 and 4 (d) 2 and 4

26. In an axial flow compressor design, velocity diagrams are constructed from the experimental data of aerofoil cascades. Which one of the following statements in this regard is correct?
(a) Incidence angle of the approaching air is measured from the trailing edge of the blade
(a) δ is the deviation angle between the angle of incidence and tangent to the camber line
(c) The deflection angle ε of the gas stream while passing through the cascade is given by $\varepsilon = \alpha_1 - \alpha_2$
(d) ε is the sum of the angle of incidence and camber less any deviation angle i.e. $\varepsilon = i + \theta - \delta$

27. The flow in the vaneless space between the impeller exit and diffuser inlet of a centrifugal compressor can be assumed as
(a) Free vortex (b) Forced vortex
(c) Solid body rotation (d) Logarithmic spiral

28. Which portion of the centrifugal compressor characteristics shown in the Figure 7.23 is difficult to obtain experimentally?

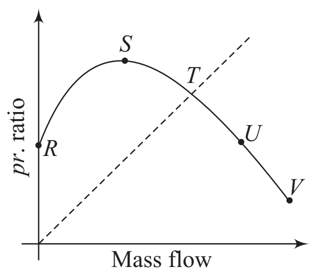


Figure 7.23 *Figure of Multiple Choice Question 28*

(a) *RS* (b) *ST* (c) *TU* (d) *UV*

29. Consider the following statements regarding the axial flow in an air compressor.
1. Surging is a local phenomenon while stalling affects the entire compressor
2. Stalling is a local phenomenon while surging affects the entire compressor
3. The pressure ratio of an axial compressor stage is smaller than that of a centrifugal compressor stage

Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

30. The pressure rise in the impeller of centrifugal compressor is achieved by
    (a) The decrease in volume and diffusion action
    (b) The centrifugal action and decrease in volume
    (c) The centrifugal and diffusion action
    (d) The centrifugal and push-pull action
31. Centrifugal compressors are more suitable as compared to axial compressor for
    (a) High head, low flow rate
    (b) Low head, low flow rate
    (c) Low head, high flow rate
    (d) High head, high flow rate
32. Stalling of blades in an axial flow compressor is the phenomenon of
    (a) Air stream blocking the passage
    (b) Motion of air at sonic velocity
    (c) Unsteady, periodic and reversed flow
    (d) Air stream not able to follow the blade contour
33. The inlet and exit velocity diagrams of a turbomachine rotor are shown in Figure 7.24. The turbomachine is
    (a) An axial compressor with backward curved blades
    (b) A radial compressor with backward curved blades
    (c) A radial compressor with forward curved blades
    (d) An axial compressor with forward curved blades

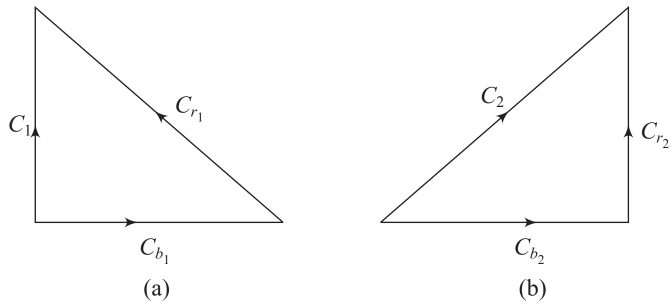


Figure 7.24 *Figure of Multiple Choice Question 33*

34. In an axial flow compressor:

    α_1 = Exit angle from stator, β_1 = Inlet angle to rotor, α_2 = Inlet angle to stator, β_2 = Outlet angle from rotor. The condition to have 50% degree of reaction is
    (a) $\alpha_1 = \beta_2$
    (b) $\alpha_2 = \beta_1$
    (c) $\alpha_1 = \beta_2$ and $\beta_1 = \alpha_2$
    (d) $\alpha_1 = \alpha_2, \beta_1 = \beta_2$
35. Consider the following statements with reference to supercharging of IC Engine.
    1. Reciprocating compressor are invariably used for high degree of supercharging
    2. Rotary compressor like roots blower are quite suitable for low degree of supercharging
    3. Axial flow compressors are most commonly employed for supercharging. Diesel engine used in heavy duty transport vehicles
    4. Centrifugal compressors are used for turbocharging

    Which of these statements are correct?
    (a) 1 and 2
    (b) 2 and 3
    (c) 1 and 4
    (d) 2 and 4

36. Which one of the following expresses the isentropic efficiency η of the compression process in terms of enthalpy changes as indicated in Figure 7.25?

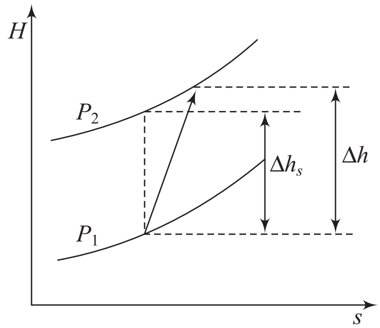


Figure 7.25 *Figure of Multiple Choice Question 36*

(a) $\eta = \frac{\Delta h_s}{\Delta h}$ (b) $\eta = \frac{\Delta h}{\Delta h_s}$ (c) $\eta = \frac{\Delta h - \Delta h_s}{\Delta h}$ (d) $\eta = \frac{\Delta h - \Delta h_s}{\Delta h_s}$

37. While flowing through the rotor blades in an axial flow air compressor, the relative velocity of air
(a) Continuously decreases (b) Continuously increases
(c) First increases and then decreases (d) First decreases and then increases

38. For centrifugal compressors, which one of the following is the correct relationship between pressure coefficient ψ, slip factor σ_s, work input factor ψ_w and isentropic efficiency η_s?

(a) $\psi = \frac{\sigma_s \psi_w}{\eta_s}$ (b) $\psi = \frac{\psi_w}{\sigma_s \eta_s}$ (c) $\psi = \sigma_s \psi_w \eta_s$ (d) $\psi = \frac{\sigma_s \eta_s}{\psi_w}$

39. **Assertion (A):** In multistage compressors, the polytropic efficiency is always greater than the isentropic efficiency.

Reason (R): Higher the pressure ratio, the greater is the polytropic efficiency.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

40. In a radial blade centrifugal compressor, the velocity of blade tip is 400 m/s and slip factor is 0.9. Assuming the absolute velocity at inlet to be axial, the specific work of compressor is
(a) 36 kJ (b) 72 kJ (c) 144 kJ (d) 360 kJ

41. Consider the following statements.

For a large aviation gas turbine, an axial flow compressor is usually preferred over centrifugal compressor because
1. The maximum efficiency is higher
2. The frontal area is lower
3. The pressure rise per stage is more
4. The cost is lower

Which of these statements is/are correct?
(a) 1 and 4 (b) 1 and 2 (c) 1, 2 and 3 (d) 2, 3 and 4

42. For an axial flow compressor, when the degree of reaction is 50%, it implies that
    (a) Work done in compression will be the least
    (b) 50% stages of the compressor will be ineffective
    (c) Pressure after compression will be optimum
    (d) The compressor will have symmetrical blades
43. How can the pressure ratio be increased in a centrifugal compressor
    (a) Only by increasing the tip speed
    (b) Only by decreasing the inlet temperature
    (c) By both (a) and (b)
    (d) Only by increasing the inlet temperature
44. What does application of centrifugal air compressor lead to?
    (a) Large frontal area of aircraft
    (b) Higher flow rate through the engine
    (c) Higher aircraft speed
    (d) Lower frontal area of the aircraft
45. The power required to drive a turbo compressor for a given pressure ratio decreases when
    (a) Air is heated at entry
    (b) Air is cooled at entry
    (c) Air is cooled at exit
    (d) Air is heated at exit
46. Which one of the following can be the cause/causes of an air cooled compressor getting overheated during operation?
    1. Insufficient lubricating oil
    2. Broken valve strip
    3. Clogged intake filter

    Which of these statements is/are correct?
    (a) Only 3 (b) 1 and 2 (c) 2 and 3 (d) 1, 2 and 3
47. Match the List 1 and List 2 and select the correct answer using the codes given below the lists.

List 1	**List 2**
1. Fan	A. 1.1
2. Blower	B. 2.5
3. Centrifugal air compressor	C. 4
4. Axial flow air compressor	D. 10

Codes

	1	**2**	**3**	**4**
(a)	B	A	C	D
(b)	A	B	C	D
(c)	A	B	D	C
(d)	B	A	D	C

48. Consider the following statements regarding axial flow compressors.
    1. An axial flow air compressor is often described as a reversed reaction turbine
    2. With 50% degree of reaction, the velocity diagrams are symmetrical

    Which of these statements is/are correct?
    (a) Only 1 (b) Only 2 (c) 1 and 2 (d) Neither 1 nor 2
49. Stalling phenomenon in an axial compressor stage is caused due to which one of the following?
    (a) Higher mass flow rate than the designed value
    (b) Lower mass flow rate than the designed value
    (c) Higher mass flow rate or non-uniformity in the blade profile
    (d) Lower mass flow rate or non-uniformity in the blade profile

50. In a multistage axial flow compressor with equal temperature rise in all stages, the pressure ratio in subsequent stages
 (a) Remains constant (b) Increases gradually (c) Decreases (d) Increases rapidly

51. Which one of the following types of impeller vanes are commonly used in centrifugal type compressors?
 (a) Forward curved (b) Radial (c) Backward curved (d) Tangential

52. A centrifugal compressor is suitable for which of the following?
 (a) High pressure ratio, low mass flow (b) Low pressure ratio, low mass flow
 (c) High pressure ratio, high mass flow (d) Low pressure ratio, high mass flow

53. The suction pressure is 1 bar and delivery pressure is 125 bar. What is the ideal intermediate pressure at the end of first stage for a 3 – stage air compressor?
 (a) 25 bar (b) 5 bar (c) 10 bar (d) 20 bar

54. Consider the following statements regarding axial flow compressors.
 1. Like centrifugal compressor, axial flow compressors are limited by surge at low mass flow rates
 2. Axial flow compressors experience choking at low flow rates
 3. The design point is close to surge point
 4. As mass flow decreases the compressor blades stall causing flow separation

 Which of these statements are correct?
 (a) 1 and 2 only (b) 1, 2 and 3 (c) 1, 3 and 4 (d) 3 and 4 only

55. In the graph shown in Figure 7.26, for an axial compressor, surging is likely to occur in which one of the following zones?

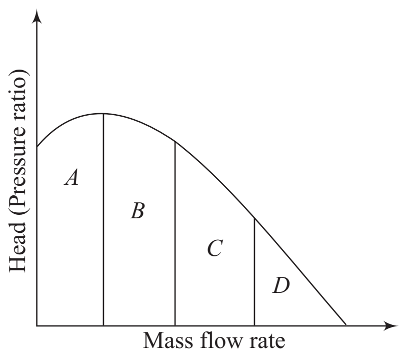


Figure 7.26 *Figure of Multiple Choice Question 55*

 (a) A (b) B (c) C (d) D

56. Which one of the following diagrams, shown in Figure 7.27, correctly depicts the radial distribution of axial velocity over the blades in the last stage of multistage axial flow compressor?

Figure 7.27 *Figure of Multiple Choice Question 56*

57. Consider the following statements referring to axial flow compressors as compared to centrifugal compressors.
    1. Can be designed for higher pressure ratios than centrifugal compressors
    2. Have higher maximum efficiency than centrifugal compressors but in a narrow speed range
    3. More suitable for aviation gas turbines due to lower frontal area
    4. Lighter in mass than centrifugal compressors

    Which of these statements are correct?
    (a) 1 and 3 only (b) 2 and 3 only (c) 1, 2 and 3 only (d) 1, 2, 3 and 4
58. **Assertion (A):** The performance parameter 'polytropic efficiency' is used for axial flow gas turbines and air compressors.

    Reason (R): Polytropic efficiency is dependent on the pressure ratio.
    (a) Both A and R are true and R is the correct explanation of A
    (b) Both A and R are individually true but R is not the correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true
59. **Assertion (A):** In centrifugal compressors, sometimes guide vanes are provided at inlet.

    Reason (R): The guide vanes provide pre-whirl which helps in restricting the Mach number at inlet to an acceptable value below supersonic.
    (a) Both A and R are true and R is the correct explanation of A
    (b) Both A and R are individually true but R is not the correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true
60. Losses in a centrifugal compressor are due to
    (a) Impeller channel losses only (b) Inlet losses
    (c) Both (a) and (b) (d) Neither (a) nor (b)
61. Compressors used in gas turbines are
    (a) Reciprocating type (b) Centrifugal type (c) Axial flow type (d) All of these
62. In centrifugal compressor, the work input is equal to the sum of
    (a) Pressure head, relative head and dynamic head
    (b) Dynamic head, centrifugal head and relative head
    (c) Pressure head, centrifugal head and dynamic head
    (d) Pressure head, centrifugal head and relative head
63. For a centrifugal compressor with radial vanes, slip factor is the ratio of
    (a) Isentropic work to Euler work
    (b) Whirl velocity to the blade velocity at the impeller exit

(c) Stagnation pressure to static pressure
(d) Isentropic temperature rise to actual temperature rise

64. The specific speed of a centrifugal compressor is generally
(a) Less than that of reciprocating compressor
(b) Independent of compressor type but depends only on the size of compressor
(c) Higher than that of axial compressor
(d) More than specific speed of reciprocating but less than axial compressor

65. In a centrifugal compressor, an increase in speed at a given pressure ratio causes
(a) Increase in flow and increase in efficiency (b) Increase in flow and decrease in efficiency
(c) Decrease in flow and increase in efficiency (d) Decrease in flow and decrease in efficiency

66. **Statement I:** The term 'surge' indicates the phenomenon of instability which takes place at low flow values and which involves an entire system including not only of the centrifugal compressor but also the group of components traversed by the fluid upstream and downstream of it.

Statement II: Choking is defined as separation of fluid from rotor blades of centrifugal compressor
(a) Both statements are true and Statement II is the correct explanation of Statement I
(b) Both statements are individually true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

67. Consider the following statements referring to axial flow compressors as compared to centrifugal compressors.
1. Stalling is the separation of flow from the blade surface
2. Surging leads to physical damage due to impact loads and high frequency vibrations
3. Mass flow rate is minimum if choking occurs

Which of these statements are correct?
(a) 1, 2 and 3 only (b) 1 and 3 only (a) 1 and 2 only (d) 2 and 3

68. Which of the following statements are correct?
1. Velocity compounded impulse turbine gives less speed and less efficiency
2. For an ideal centrifugal compressor, the pressure developed depends on the impeller velocity and diameter
3. While flowing through the rotor blades in a gas turbine, the relative velocity of gas continuously decreases
4. While flowing through the rotor blades in an axial flow compressor, the relative velocity of air continuously decreases

Which of these statements are correct?
(a) 1 and 3 (a) 2 and 3 (c) 1 and 4 (d) 2 and 4

69. What is the power required to drive a centrifugal air compressor when impeller diameter is 0.45 m and speed is 7200 rpm?
(a) 28.78 kJ/kg (b) 30.78 kJ/kg (c) 27.78 kJ/kg (d) 26.78 kJ/kg

70. What is the correct sequence in increasing order of air handling/compressing machines based on the pressure ratio?
(a) Air blower, axial flow fan, centrifugal compressor and reciprocating compressor
(b) Air axial flow fan, centrifugal compressor, air blower and reciprocating compressor
(c) Air blower, centrifugal compressor, axial flow fan and reciprocating compressor
(d) Air axial flow fan, air blower, centrifugal compressor and reciprocating compressor

71. The head developed is maximum keeping other parameters, viz. rotor diameter, speed, width, inlet angle, etc. constant for a centrifugal compressor with
    (a) Rotor with backward curved blades
    (b) Rotor with forward curved blades
    (c) Rotor with radial blades
    (d) All of these

72. **Assertion (A):** Axial flow compressors need many stages to develop high pressure ratios.

    Reason (R): The amount of turning of air flow in blade row is limited by the occurrence of separation, a phenomenon caused by adverse pressure gradient.
    (a) Both A and R are true and R is the correct explanation of A
    (b) Both A and R are individually true but R is not the correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true

73. Match the List 1 and List 2 and select the correct answer using the codes given below the lists.

List 1	List 2
A. Slip	1. Reduction of whirl velocity
B. Stall	2. Fixed mass flow rate regardless of pressure ratio
C. Choking	3. Flow separation
	4. Flow area separation

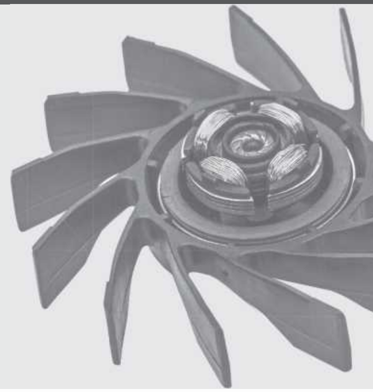
Codes

	A	B	C
(a)	4	3	2
(b)	1	3	2
(c)	4	1	3
(d)	2	3	4

ANSWER KEY

1. (a)	2. (a)	3. (c)	4. (a)	5. (c)	6. (c)	7. (c)	8. (a)	9. (c)	10. (b)
11. (b)	12. (a)	13. (a)	14. (c)	15. (a)	16. (b)	17. (b)	18. (c)	19. (a)	20. (c)
21. (d)	22. (b)	23. (b)	24. (d)	25. (b)	26. (c)	27. (d)	28. (b)	29. (c)	30. (c)
31. (c)	32. (d)	33. (c)	34. (c)	35. (d)	36. (a)	37. (a)	38. (c)	39. (c)	40. (c)
41. (b)	42. (d)	43. (c)	44. (a)	45. (b)	46. (d)	47. (b)	48. (c)	49. (d)	50. (b)
51. (c)	52. (d)	53. (b)	54. (a)	55. (a)	56. (c)	57. (c)	58. (c)	59. (a)	60. (c)
61. (c)	62. (b)	63. (b)	64. (d)	65. (b)	66. (c)	67. (c)	68. (c)	69. (a)	70. (d)
71. (b)	72. (a)	73. (b)							

8 Gas Turbine



Learning Objectives

After reading this chapter, you will be able to:

- **LO 1** Describe the different thermodynamic cycles of gas turbine which are used for power generation and aircraft propulsion
- **LO 2** Define and obtain expression for different stage parameters for an axial and radial turbine
- **LO 3** Understand the performance characteristics of an axial and radial turbines.

8.1 Introduction

Gas turbine is a popular turbomachine for aircraft propulsion and for peak load power stations because of its compact nature, low weight to power ratio and quick start.

Before discussing gas turbine as turbomachine, some of the shaft power cycles and a propulsion cycle are briefly discussed.

8.1.1 Simple Joule-Brayton Cycle

The Joule-Brayton cycle consists of two isentropic processes and two constant pressure processes. The schematic diagram of a simple gas turbine cycle is shown in Figure 8.1(a). Air or gas is taken into the compressor at pressure p_1 and temperature T_1, and is isentropically compressed to a state of (p_2, T_2). Heat is added in the combustion chamber at constant pressure to raise the temperature of gas to T_3. Gas entering the turbine at pressure p_3 and temperature T_3 is expanded isentropically to reach a state of p_4, T_4 ($p_4 = p_1$).

The ideal cycle takes air as the standard medium. Hence, this cycle is called air standard cycle for gas turbine. This thermodynamic cycle is shown in Figure 8.1(b).

(a) Net Work in Terms of Pressure Ratio and Temperature Ratio

Heat supplied per kg (q_S) = $c_p (T_3 - T_2)$ (8.1)

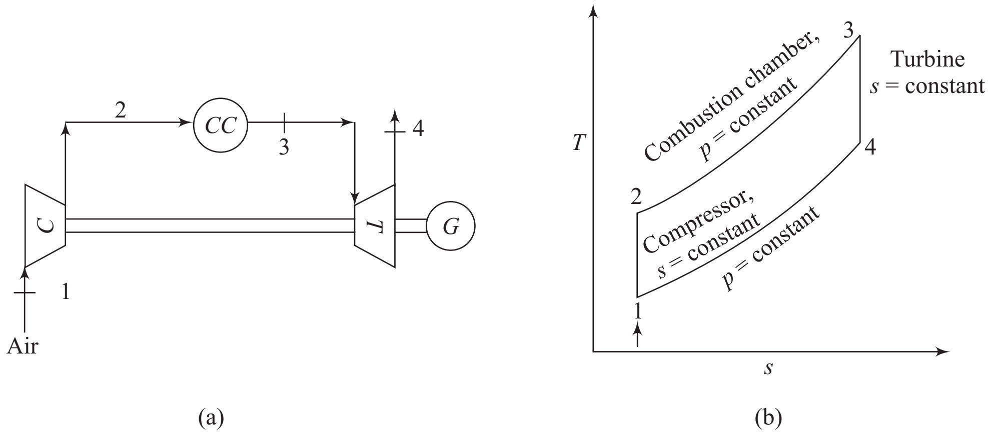


Figure 8.1 *Simple Joule-Brayton Cycle (a) Schematic Diagram (b) T-s Diagram*

Heat rejected per kg (q_R) = c_p (T_4 – T_1) (8.2)

Work required by compressor per kg (w_c) = c_p (T_2 – T_1) (8.3)

Work done by turbine per kg (w_t) = c_p (T_3 – T_4) (8.4)

Using Eqs (8.3) and (8.4), we have,

Specific net work output (w) = $w_t - w_c = c_p (T_3 - T_4) - c_p(T_2 - T_1)$ (8.5)

We define pressure ratio (r) = $p_2/p_1 = p_3/p_4$ and temperature ratio $\theta = T_3/T_1$, maximum to minimum cycle temperature ratio. (8.6)

Since the process 1-2 and 3-4 are isentropic,

$$r^{(\gamma-1)/\gamma} = T_2/T_1 = T_3/T_4 = x \tag{8.7}$$

Equation (8.5) can be rewritten using Eqs. (8.6) and (8.7) as,

$$w = c_pT_3 (1 - T_4/T_3) - c_pT_1 (T_2/T_1 - 1) = c_pT_3 (1 - 1/x) - c_pT_1(x - 1)$$

$$w = c_p\theta T_1 (1 - 1/x) - c_pT_1(x - 1)$$

$$w = c_pT_1[\theta(1 - 1/x) - (x - 1)]$$

$$w = c_pT_1[\theta(1 - 1/r^{(\gamma-1)/\gamma}) - (r^{(\gamma-1)/\gamma} - 1)] \tag{8.8}$$

(b) Cycle Efficiency

We know that the cycle efficiency is defined as,

$$\eta = (q_S - q_R)/q_S \tag{8.9}$$

Using Eqs. (8.1) and (8.2) in Eq. (8.9),

$$\eta = [c_p (T_3 - T_2) - c_p (T_4 - T_1)]/c_p(T_3 - T_2) = 1 - [c_p(T_4 - T_1)/c_p(T_3 - T_2)] \tag{8.10}$$

Using Eq. (8.7), Eq. (8.10) can be written as,

$$\eta = 1 - [(T_4 - T_1)/(T_3 - T_2)] = 1 - [(T_4 - T_1)/x(T_4 - T_1)] = 1 - 1/x$$

$$\eta = 1 - \frac{1}{r^{(\gamma-1)/\gamma}} \tag{8.11}$$

(c) Optimum Pressure Ratio for Maximum Work Output

The optimum pressure ratio for maximum work output can be obtained from the condition,

$$\frac{dw}{dx} = 0 \tag{8.12}$$

Using Eq. (8.12) in Eq. (8.8), we get,

$$0 = \frac{d[c_p T_1[\theta(1 - 1/x) - (x - 1)]]}{dx}$$

$$0 = c_p T_1 (\theta/x^2 - 1)$$

$$1 = \theta/x^2$$

$$x^2 = \theta \tag{8.13}$$

Using Eq. (8.7) in Eq. (8.13), we have,

$$r^{2\left(\frac{\gamma-1}{\gamma}\right)} = T_3/T_1 = \theta$$

$$(r_{\text{opt}})_{\text{max work}} = \theta^{\frac{\gamma}{2(\gamma-1)}} \tag{8.14}$$

8.1.2 Actual Joule-Brayton Cycle

The ideal Joule-Brayton cycle does not take into account of the component efficiency for compressor and turbine. The modified cycle taken in account of component efficiencies is called actual cycle, shown in Figure 8.2.

The expressions for net work done, cycle efficiency, and optimum pressure ratio for actual cycle, are derived as follows.

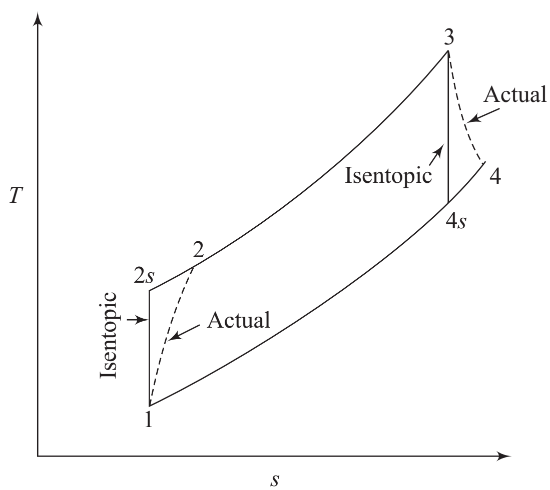


Figure 8.2 *Representation of Actual Joule-Brayton Cycle on T-s Plane*

(a) Net Work Done in Terms of Pressure Ratio and Temperature Ratio

We know that the isentropic efficiencies of a compressor and turbine are defined as,

$$\eta_c = (T_{2s} - T_1)/(T_2 - T_1) \tag{8.15}$$

$$\eta_t = (T_3 - T_4)/(T_3 - T_{4s}) \tag{8.16}$$

We also know that,

$$x = T_{2s}/T_1 = T_3/T_{4s} = r^{(\gamma-1)/\gamma} \tag{8.17}$$

$$\theta = T_3/T_1 \tag{8.18}$$

Work required by compressor per kg,

$$w_c = c_p(T_2 - T_1) = c_p(T_{2s} - T_1)/\eta_c = c_p T_1 (x - 1)/\eta_c \tag{8.19}$$

Work done by turbine per kg,

$$w_t = c_p(T_3 - T_4) = \eta_t c_p(T_3 - T_{4s}) = \eta_t c_p T_3 (1 - T_{4s}/T_3) \quad (8.20)$$

$$w_t = \eta_t c_p T_1 \theta(1 - 1/x) \quad (8.21)$$

Using Eqs. (8.18), (8.19), (8.20) and (8.21), we have specific work output as,

$$w = w_t - w_c = \eta_t c_p T_1 \theta(1 - 1/x) - c_p T_1 (x - 1)/\eta_c = c_p T_1 [\theta\eta_t\eta_c(1 - 1/x) - (x - 1)]/\eta_c$$

$$w = c_p T_1 [\theta\eta_t\eta_c (1 - 1/r^{(\gamma-1)/\gamma}) - (r^{(\gamma-1)/\gamma} - 1)]/\eta_c \quad (8.22)$$

(b) Cycle Efficiency

Heat supplied per kg = $(q_S) = c_p(T_3 - T_2) = c_p T_1 [T_3/T_1 - (1 + (x - 1) \times (1/\eta_c))]$

$$q_s = c_p T_1 [(\theta - 1)\eta_c - (x - 1)]/\eta_c \quad (8.23)$$

Using Eqs (8.19), (8.21) and (8.23) in efficiency formula, we have,

$$\eta_{\text{actual}} = (w_t - w_c)/q_s = \{c_p T_1 [\theta\eta_t\eta_c(1 - 1/x) - (x - 1)]/\eta_c\}/\{c_p T_1 [(\theta - 1)\eta_c - (x - 1)]/\eta_c\}$$

$$\eta_{\text{actual}} = \theta\eta_t\eta_c(1 - 1/x) - (x - 1)/[(\theta - 1)\eta_c - (x - 1)] \quad (8.24)$$

(c) Optimum Pressure Ratio for Maximum Work Output

For maximum work output, $\dfrac{dw}{dx} = 0$ (8.25)

Using Eq. (8.22) in Eq. (8.25), we have,

$$0 = \frac{d}{dx}\{c_p T_1(\theta\eta_t\eta_c(1 - 1/x) - (x - 1]/\eta_c\}$$

$$0 = (\theta\eta_t\eta_c/x^2) - 1$$

$$x^2 = \theta\eta_t\eta_c \quad (8.26)$$

$$x = \sqrt{\alpha}\text{ ; where } \alpha = \theta\eta_t\eta_c \quad (8.27)$$

Using Eq. (8.17) in Eq. (8.27), we have,

$$(r_{\text{opt}})_{\text{max work}} = \alpha^{\frac{\gamma}{2(\gamma-1)}} = [\theta\eta_t\eta_c]^{\frac{\gamma}{2(\gamma-1)}} \quad (8.28)$$

(d) Optimum Pressure Ratio for Maximum Efficiency

Using Eq. (8.27) in Eq. (8.24), efficiency can be rewritten as,

$$\eta = [\alpha(1 - 1/x) - (x - 1)]/[(\theta - 1)\eta_c - (x - 1)] \quad (8.29)$$

For maximum efficiency, $\dfrac{d\eta}{dx} = 0$ (8.30)

Using Eq. (8.30) in Eq. (8.29), we have,

$$\frac{d}{dx}\{[\alpha(1 - 1/x) - (x - 1)]/[(\theta - 1)\eta_c - (x - 1)]\} = 0$$

$$\left[\alpha\left(1-\frac{1}{x}\right)-(x-1)\right]\times(-1)-[(\theta-1)\eta_c-(x-1)]\times\left(\frac{\alpha}{x^2}-1\right)=0$$

$$-\alpha+\frac{\alpha}{x}+x-1-\frac{\theta\eta_c\alpha}{x^2}+\frac{\alpha\eta_c}{x^2}+\frac{\alpha}{x}-\frac{\alpha}{x^2}+\theta\eta_c-\eta_c-x+1=0$$

$$\frac{\alpha}{x^2}(\eta_c-\theta\eta_c-1)+\frac{2\alpha}{x}+(\theta\eta_c-\eta_c-\alpha)=0$$

$$x^2(\alpha+\eta_c-\theta\eta_c)-2\alpha x+\alpha(1+\theta\eta_c-\eta_c)=0 \tag{8.31}$$

Let,

$$\beta = 1+\theta\eta_c-\eta_c \tag{8.32}$$

$$x^2(1+\alpha-\beta)-2\alpha x+\alpha\beta=0$$

Solving the above quadratic equation, we get,

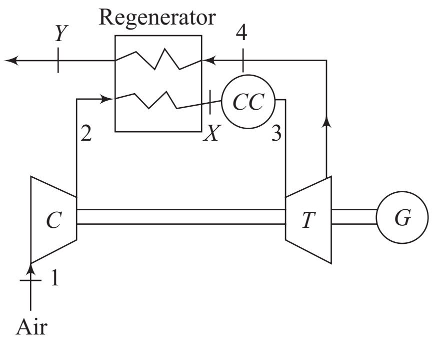
$$x=\frac{2\alpha\pm\sqrt{4\alpha^2-4(1+\alpha-\beta)\alpha\beta}}{2(1+\alpha-\beta)} \tag{8.33}$$

Substituting the value of x from Eq. (8.17) into Eq. (8.33), we get,

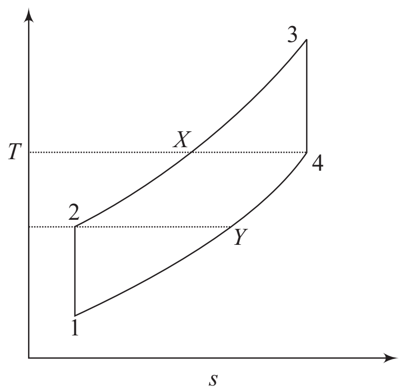
$$r^{\gamma-1/\gamma}=\frac{2\alpha\pm\sqrt{4\alpha^2-4(1+\alpha-\beta)\alpha\beta}}{2(1+\alpha-\beta)} \tag{8.34}$$

8.1.3 Ideal Joule-Brayton Cycle with Heat Exchanger

In the ideal Joule-Brayton cycle, the outlet temperature (T_4) is quite high and hence, enthalpy (h_4) of the gas that is leaving the turbine is also high. In order to recover some of the exhaust heat, a regenerative heat exchanger is used to increase the enthalpy of incoming air from the compressor. Figure 8.3 (a) shows exchanges between exhaust gas and compressed air. The exhaust temperature decreases to T_Y from T_4, whereas the compressed air temperature increases to T_X from T_2 [Figure 8.3 (b)], before entering the combustion chamber.



(a) Schematic Diagram



(b) T-s Diagram

Figure 8.3 *Simple Open Gas Turbine Cycle with Regeneration*

(a) Cycle Efficiency with Perfect Regeneration

We can define heat exchanger effectiveness as,

ε = Actual heat transfer/Maximum heat that can be transferred

$$\varepsilon = (T_X - T_2)/(T_4 - T_2) \text{ or } (T_4 - T_Y)/(T_4 - T_2) \quad (8.35)$$

For perfect regeneration $\varepsilon = 1 \Rightarrow T_X = T_4$ and $T_2 = T_Y$

Maximum possible heat transfer from heat exchanger per kg,

$$q_T = c_p(T_4 - T_2) \quad (8.36)$$

We know that,

$$r^{(\gamma-1/\gamma)} = T_2/T_1 = T_3/T_4 = x \quad (8.37)$$

$$\theta = T_3/T_1 \quad (8.38)$$

$$\text{Heat supplied per kg } (q_S) = c_p(T_3 - T_X) = c_p(T_3 - T_4) \quad (8.39)$$

$$\text{Heat rejected per kg } (q_R) = c_p(T_Y - T_1) = c_p(T_2 - T_1) \quad (8.40)$$

We know that efficiency, $\eta = (q_S - q_r)/q_S = 1 - (q_R/q_S)$ (8.41)

Using Eqs. (8.39) and (8.40) in Eq. (8.41), efficiency can rewritten as,

$$\eta = 1 - (T_2 - T_1)/(T_3 - T_4) \quad (8.42)$$

$$\eta = 1 - \{[(T_2/T_1)T_1 - T_1]/[(T_3/T_4)T_4 - T_4]\} = 1 - [(T_3/T_4)/(T_3/T_1) = 1 - (x/\theta)$$

Using Eqs. (8.37) and (8.38), we have,

$$\eta = 1 - [(r^{(\gamma-1)/\gamma}/(T_3/T_1)] \quad (8.43)$$

(b) Net Work Done in Terms of Pressure Ratio and Temperature Ratio

Specific work output (w) = $w_t - w_c = c_p(T_3 - T_4) - c_p(T_2 - T_1)$ (8.44)

$$w = c_p\, T_3\, (1 - T_4/T_3) - c_pT_1(T_2/T_1 - 1) = c_p\, T_3\, (1 - 1/x) - c_pT_1(x - 1)$$

Using Eqs. (8.37), (8.38) in Eq. (8.44), we get,

$$w = c_p\, \theta T_1(1 - 1/x) - c_pT_1\, (x - 1)$$

$$w = c_pT_1\, [\theta(1 - 1/x) - (x - 1)] \quad (8.45)$$

$$w = c_pT_1\, [\theta(1 - 1/\, r^{(\gamma-1)/\gamma}) - (r^{(\gamma-1)/\gamma} - 1)] \quad (8.46)$$

(c) Optimum Pressure Ratio for Maximum Specific Work Output with Perfect Regeneration

For maximum work output, $\dfrac{dw}{dx} = 0$ (8.47)

Substituting Eq. (8.46) in Eq. (8.47), we have,

$$0 = \frac{d}{dx}\{c_pT_1[\theta(1 - 1/x) - (x - 1)]\}$$

$$0 = c_pT_1\, (\theta/x^2 - 1)$$

$$1 = \theta/x^2$$

$$x^2 = \theta \quad (8.48)$$

$$r^{2\left(\frac{\gamma-1}{\gamma}\right)} = T_3/T_1 \tag{8.49}$$

Using Eqs (8.37) and (8.38) in Eq. (8.49),

$$(r_{\text{opt}})_{\text{max work}} = \theta^{\frac{\gamma}{2(\gamma-1)}} \tag{8.50}$$

(d) Efficiency for Ideal Cycle with Heat Exchanger in Terms of Heat Exchanger Effectiveness

Here, heat exchanger effectiveness, $\varepsilon \neq 1$

We know that, heat supplied and heat rejected per kg is,

$$q_s = c_p(T_3 - T_X) = c_p\left\{(T_3 - T_4) + \frac{[(T_4 - T_2) - (T_X - T_2)]}{(T_4 - T_2)} \times (T_4 - T_2)\right\} \tag{8.51}$$

Using Eq. (8.35) in Eq. (8.51),

$q_s = c_p\,[(T_3 - T_4) + (1 - \varepsilon)\,(T_4 - T_2)]$

Using $\theta = T_3/T_1$ and Eq. (8.37),

$$q_s = c_pT_1\,[\theta(1 - 1/x) + (1 - \varepsilon)(\theta/x - x)] \tag{8.52}$$

$$q_r = c_p\,(T_Y - T_1)$$

Using Eq. (8.35) in the above equation,

$$q_r = c_p\,(T_Y - T_1) = c_p\,[(T_4 - T_2) - \varepsilon(T_4 - T_2) + (T_2 - T_1)]$$

Using $\theta = T_3/T_1$ and Eq. (8.37),

$$q_r = c_pT_1\,[(x - 1) + (1 - \varepsilon)\,(\theta/x - x)] \tag{8.53}$$

We know that ideal efficiency is given by,

$$\eta = (q_s - q_r)/q_s = 1 - q_r/q_s \tag{8.54}$$

Using Eqs. (8.52) and (8.53) in Eq. (8.54), we have,

$$\eta = 1 - \frac{[(x-1) + (1-\varepsilon)\,(\theta/x - x)]}{[\theta(1 - 1/x) + (1-\varepsilon)(\theta/x - x)]} \tag{8.55}$$

8.1.4 Actual Joule-Brayton Cycle with Heat Exchanger

Work done by compressor per kg,

$$w_c = c_p(T_2 - T_1) = c_p(T_{2s} - T_1)/\eta_c = c_pT_1\,(x - 1)/\eta_c \tag{8.56}$$

Work done by turbine per kg,

$$w_t = c_p(T_3 - T_4) = \eta_t\,c_p(T_3 - T_{4s}) = \eta_t\,c_pT_3\,(1 - T_{4s}/T_3)$$

$$w_t = \eta_t\,c_pT_1\theta(1 - 1/x) \tag{8.57}$$

Using Eqs (8.56) and (8.57), we get net work output,

$$w = w_t - w_c = \eta_t\,c_pT_1\theta(1 - 1/x) - c_pT_1\,(x - 1)/\eta_c$$

$$w = c_pT_1\,[\theta\eta_t\eta_c\,(1 - 1/x) - (x - 1)]/\eta_c \tag{8.58}$$

Heat supplied per kg,

$$q_S = c_p[(T_3 - T_4) + (1 - \varepsilon)(T_4 - T_2)] \tag{8.59}$$

Using Eq. (8.58) and (8.59), we have,

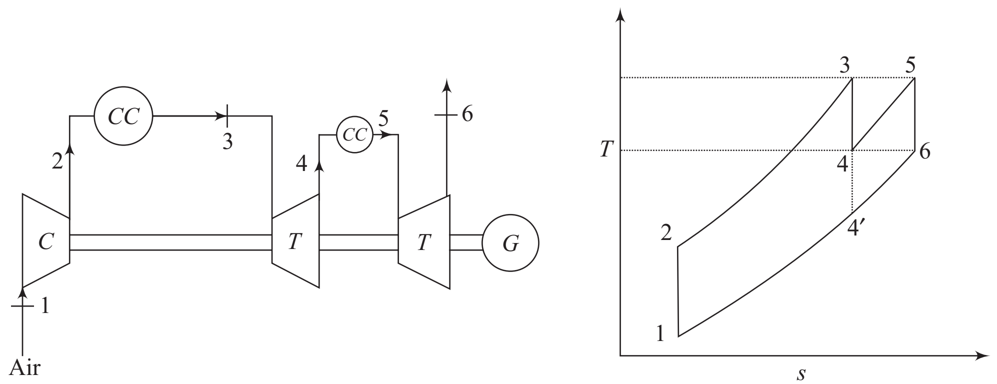
$$\eta = w/q_S$$

$$\eta = \{c_pT_1\,[\theta\eta_t\eta_c(1 - 1/x) - (x - 1)]/\eta_c\}/\{c_p[(T_3 - T_4) + (1 - \varepsilon)(T_4 - T_2)]\} \quad (8.60)$$

$$\eta = T_1\,[\alpha(1 - 1/x) - (x - 1)]/[T_1\ \alpha(1 - 1/x) + \eta_c(1 - \varepsilon)(T_4 - T_2)] \quad (8.61)$$

8.1.5 Ideal Joule-Brayton Cycle with Reheating

The schematic and T-s diagrams of a simple open gas turbine with reheating between two stages of expansion are shown in Figure 8.4.



(a) Schematic Diagram (b) *T-s* Diagram

Figure 8.4 *Simple Open Gas Turbine with Reheationg*

Let,

$$r^{(\gamma-1)/\gamma} = T_2/T_1 = x;\ r = p_2/p_1 \quad (8.62)$$

$$r_3^{(\gamma-1)/\gamma} = T_3/T_4 = x_3;\ r_3 = p_3/p_4 \quad (8.63)$$

$$r_4^{(\gamma-1)/\gamma} = T_5/T_6 = x_4;\ r_4 = p_5/p_6 \quad (8.64)$$

$$\theta = T_3/T_1 \quad (8.65)$$

$$T_5 = T_3, p_3 = p_2, p_4 = p_5, p_6 = p_1 \quad (8.66)$$

$$p_2/p_1 = p_3/p_6 = p_3/p_4 \times p_5/p_6$$

$$r = r_3r_4 \quad (8.67)$$

From Eqs (8.62), (8.63) and (8.64),

$$x = x_3x_4 \quad (8.68)$$

Specific work output,

$$w = w_t - w_c = c_p(T_3 - T_4) + c_p(T_5 - T_6) - c_p(T_2 - T_1)$$

$$w = c_pT_1[(T_3/T_1 - T_4/T_1) + (T_5/T_1 - T_6/T_1) - (T_2/T_1 - 1)] \quad (8.69)$$

Using Eqs (8.62), (8.63), (8.64), (8.65), (8.66) and (8.67), we have,

$$w = c_pT_1[2\theta - (\theta/x_3) - (\theta/x_4) - x + 1] \quad (8.70)$$

Heat supplied per kg,

$$q_S = c_p(T_3 - T_2) + c_p(T_5 - T_4) = c_pT_1(2\theta - x - \theta/x_3) \quad (8.71)$$

For maximum work output, $$\frac{dw}{dx_3} = 0 \quad (8.72)$$

Substituting Eq. (8.70) in Eq. (8.72) and then by using Eqs (8.68) and (8.67), we get,

$$x_3 = x_4 = \sqrt{x} \quad (8.73)$$

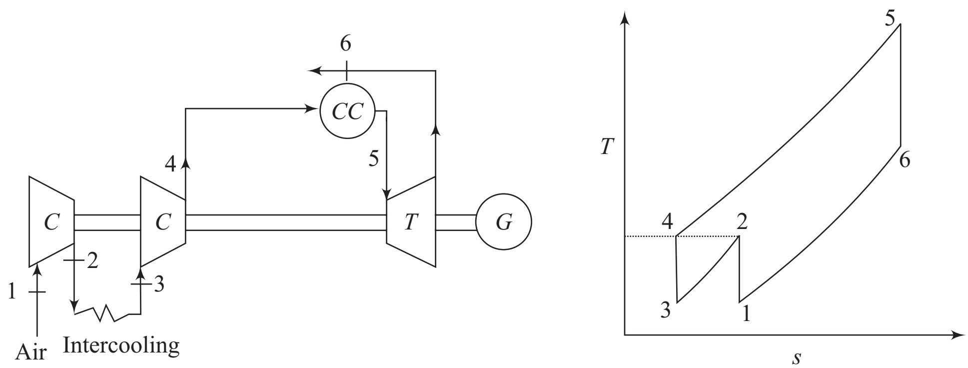
$$r_3 = r_4 = \sqrt{r} \quad (8.74)$$

Using Eqs (8.70) and (8.73), maximum efficiency can be written as,

$$\eta_{max} = w_{max}/q_s = [2\theta(1 - 1/\sqrt{x}) - (x - 1)/]/[2\theta - x - (\theta/\sqrt{x})] \quad (8.75)$$

8.1.6 Ideal Joule-Brayton Cycle with Intercooling

The simple open gas turbine with intercooling between two stages of compression is represented schematically and on T-s diagram as shown in Figure 8.5.



(a) Schematic diagram (b) *T*-s diagram

Figure 8.5 *Simple Open Gas Turbine with Intercooling*

Here, $$r^{(\gamma-1)/\gamma} = T_5/T_6 = x;\ r = p_5/p_6 \quad (8.76)$$

$$r_1^{(\gamma-1)/\gamma} = T_2/T_1 = x_1;\ r_1 = p_2/p_1 \quad (8.77)$$

$$r_2^{(\gamma-1)/\gamma} = T_4/T_3 = x_2;\ r_2 = p_4/p_3 \quad (8.78)$$

$$\theta = T_5/T_1 \quad (8.79)$$

For perfect intercooling between two stages of compression,

$$T_3 = T_1,\ T_4 = T_2 \quad (8.80)$$

Specific work output,

$$w = w_t - w_c = c_p(T_5 - T_6) - [c_p(T_2 - T_1) + c_p(T_4 - T_3)] \quad (8.81)$$

$$w = c_pT_1\{(T_5/T_1 - T_6/T_1) - [(T_2/T_1 - 1) + (T_4/T_1 - T_3/T_1)]\} \quad (8.82)$$

Heat supplied per kg,

$$q_s = c_p (T_5 - T_4) \tag{8.83}$$

For maximum work output, $\dfrac{dw}{dx_1} = 0$ (8.84)

Using Eqs (8.76), (8.77), (8.78), (8.79) and (8.80) in Eqs (8.82) and (8.84), we have,

$$x_1 = x_2 = \sqrt{x} \tag{8.85}$$

$$w_{max} = [\theta - \theta/x - 2(\sqrt{x} - 1) \tag{8.86}$$

Using Eqs (8.83) and (8.86), maximum efficiency can be written as,

$$\eta_{max} = w_{max}/q_S = c_p T_1[\theta - (\theta/x) - 2(\sqrt{x} - 1)]/c_p(T_5 - T_4)$$

$$\eta_{max} = c_p T_1[\theta - (\theta/x) - 2(\sqrt{x} - 1)]/c_p T_1[(T_5/T_1) - (T_4/T_1)]$$

$$\eta_{max} = \left[\theta - \left(\frac{\theta}{x}\right) - 2(\sqrt{x} - 1)\right]/[\theta - \sqrt{x}] \tag{8.87}$$

EXAMPLE 8.1 In an intercooled Joule Brayton cycle, the maximum and minimum temperatures are 1200 K and 300 K, respectively. The compression is carried out in two intercooled stages of equal pressure ratio. The working fluid is brought to the minimum temperature of the cycle after the first stage of compression with intercooling. The entire expansion is carried out in one turbine stage. The isentropic efficiency of both compressor stages is 0.85. The isentropic efficiency of turbine is 0.9. Find the overall pressure ratio that would give the maximum net work per kg of working fluid. Take $\gamma = 1.4$.

Solution

Given: $T_5 = 1200$, $T_1 = 300$ K, $p_2/p_1 = p_4/p_3$, $T_3 = 300$ K, $\eta_c = 0.85$, $\eta_t = 0.9$

The schematic and T-s diagrams of Joule-Brayton gas turbine cycle with intercooling between two stages of compression is shown in Figure 8.6.

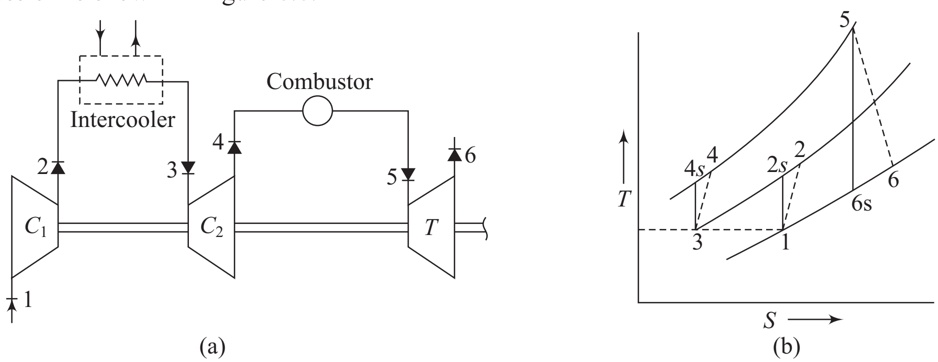


Figure 8.6 *Joule-Brayton Cycle Gas Turbine Power Plant with Intercooling*

As $p_2/p_1 = p_4/p_3$, so $p_4/p_3 = p_2/p_1 = \sqrt{p_4/p_1} = \sqrt{r}$ (1)

Process 1-2: $T_2 = T_1(p_2/p_1)^{(\gamma-1)/\gamma} = T_1(r)^{(\gamma-1)/2\gamma}$ (2)

Process 3-4: $T_{4s} = T_3(p_4/p_3)^{(\gamma-1)/\gamma} = T_3(r)^{(\gamma-1)/2\gamma}$ (3)

Process 5-6: $T_5 = T_{6s}(p_4/p_1)^{(\gamma-1)/\gamma} = T_{6s}(r)^{(\gamma-1)/2\gamma}$ (4)

$$T_{6s} = T_5/(r)^{(\gamma-1)/\gamma} \quad (5)$$

Specific compressor work, $w_c = w_{c1} + w_{c2} = c_p(T_2 - T_1) + c_p(T_4 - T_3)$

$$w_c = c_p(T_2 - T_1) + c_p(T_4 - T_1) \quad (6)$$

Using isentropic efficiency of compressor,

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{T_{4s} - T_3}{T_4 - T_3} = \frac{T_{4s} - T_1}{T_4 - T_1} \quad (7)$$

$$w_c = c_p\frac{(T_{2s} - T_1)}{\eta_c} + c_p\frac{(T_{4s} - T_1)}{\eta_c} \quad (8)$$

$$w_c = \frac{c_p}{\eta_c}[(r^{(\gamma-1)/2\gamma}T_1 - T_1) + (r^{(\gamma-1)/2\gamma}T_1 - T_1)]$$

$$w_c = \frac{2c_pT_1}{\eta_c}[r^{(\gamma-1)/2\gamma} - 1] \quad (9)$$

Isentropic efficiency of turbine,

$$\eta_t = \frac{T_5 - T_6}{T_5 - T_{6s}} \quad (10)$$

$$w_t = c_p(T_5 - T_6) = c_p\eta_t(T_5 - T_{6s}) = c_pT_5\eta_t[1 - 1/r^{(\gamma-1)/\gamma}] \quad (11)$$

$$w = w_t - w_c = c_pT_5\eta_t[1 - 1/r^{(\gamma-1)/\gamma}] - \frac{2c_pT_1}{\eta_c}[r^{(\gamma-1)/2\gamma} - 1] \quad (12)$$

For maximum specific work output, i.e. net work done per kg of the working fluid

$$\frac{dw}{dr} = 0 \quad (13)$$

Differentiating specific work with respect to r,

$$c_pT_5\eta_t\left(\frac{\gamma-1}{\gamma}\right)r^{(1-2\gamma)/\gamma} - \frac{2c_pT_1}{\eta_c}\left(\frac{\gamma-1}{2\gamma}\right)r^{(-1-\gamma)/2\gamma} = 0$$

$$r_{\text{opt}} = [T_1/(T_5\eta_t\eta_c)]^{\frac{2\gamma}{3(1-\gamma)}} \quad (14)$$

Substituting the known values,

$$r_{\text{opt}} = [300/(1200 \times 0.85 \times 0.9)]^{\frac{2\times14}{3(1-1.4)}}$$

$$r_{\text{opt}} = 13.6 \quad (15)$$

EXAMPLE 8.2 In an intercooled Jouble Brayton cycle power plant, 30 kg/s of air is supplied at 1 bar, 27°C. Compression of air is achieved to 10 bar in two stages with perfect intercooling in between at optimum pressure. The maximum temperature in cycle is 1000 K. First stage expansion of gas in the turbine occurs up to 3 bar. Gas is subsequently reheated up to 995 K before being sent to second stage. Fuel used for heating in combustion chamber has a calorific value of 42,000 kJ/kg. Considering a constant c_p of 1.0032 kJ/kg.K throughout the cycle, determine the net output, thermal efficiency and the air fuel ratio. The isentropic efficiency values of compression and expansion are 85% and 90%, respectively.

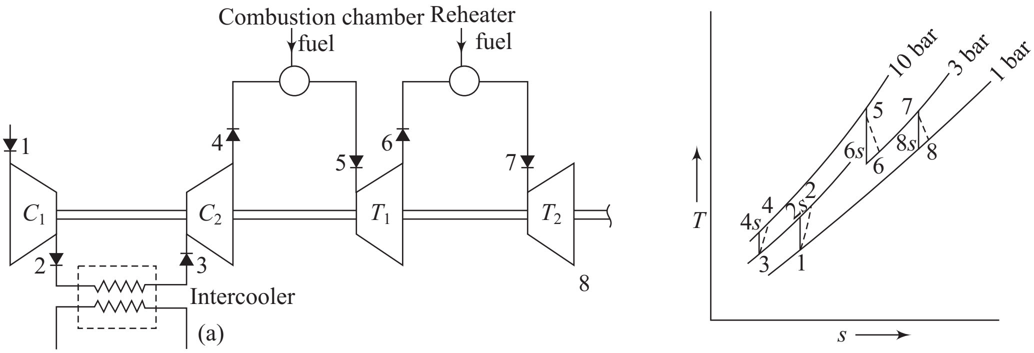


Figure 8.7 *Gas turbine Plant with Reheating and Intercooling*

Solution

For perfect intercooling, the pressure ratio of each compressor stage = $\sqrt{10}$ = 3.16

For process 1-2, $T_{2s} = T_1 \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300(3.16)^{\frac{1.4-1}{1.4}} = 416.76\text{ K}$ (1)

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{416.76 - 300}{T_2 - 300}$$

$T_2 = 437.36$ K (2)

For perfect intercooling, $T_3 = T_1 = 300$ K,

For process 3-4, $T_{4s} = T_3 \left(\dfrac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = 300(3.16)^{\frac{1.4-1}{1.4}} = 416.76\text{ K}$ (3)

Again due to compression efficiency,

$$\eta_{\text{isen},c} = \frac{T_{4s} - T_1}{T_4 - T_1} = \frac{416.76 - 300}{T_2 - 300}$$

$T_4 = 437.36$ K (4)

Total compressor work, $w_c = 2c_p(437.36 - 300) = 275.59$ kJ/kg (5)

$T_5 = 1000$ K

For expansion process 5-6',

$$T_{6s} = T_5 \left(\frac{p_6}{p_5}\right)^{\frac{\gamma-1}{\gamma}} = 1000\left(\frac{3}{10}\right)^{\frac{1.4-1}{1.4}} = 708.93\text{ K} \quad (6)$$

Considering expansion efficiency, $0.9 = \dfrac{T_5 - T_6}{T_5 - T_{6s}} = \dfrac{1000 - T_6}{1000 - 708.93}$

$T_6 = 738.04$ K and $T_7 = 995$ K (7)

For expansion process 7-8*s*,

$$T_{8s} = T_7\left(\frac{p_8}{p_7}\right)^{\frac{\gamma-1}{\gamma}} = 995\left(\frac{1}{3}\right)^{\frac{1.4-1}{1.4}} = 726.95\text{ K} \quad (8)$$

Considering expansion efficiency, $0.9 = \dfrac{T_7 - T_8}{T_7 - T_{8s}}$

$$T_8 = 753.75\text{ K} \quad (9)$$

Expansion work output per kg air = $c_p(T_5 - T_6) + c_p(T_7 - T_8)$

$$w_t = 1.00032\ (1000 - 738.04) + 1.0032\ (995 - 753.75)$$

$$w_t = 504.821\text{ kJ/kg} \quad (10)$$

$$q_s = c_p(T_5 - T_4) + c_p(T_7 - T_6)$$

$$q_s = 1.0032\{(1000 - 437.36) + (995 - 738.04)\} = 822.22\text{ kJ/kg} \quad (11)$$

Fuel required per kg of air, m_f = 822.22/42000 = 0.01958 kg of fuel/kg of air

Air fuel ratio = 1/ 0.01958 = 51.07 (12)

Net output = $w_t - w_c$ = 504.821 – 275.59 = 229.231 kJ/kg (13)

Output for air flowing at 30 kg/s = 229.231 × 30 = 6876.93 W = 6.877 kW (14)

Thermal efficiency = $(w_t - w_c)/q_s$ = 229.231/822.22 = 27.88 % (15)

EXAMPLE 8.3 In a regenerative-reheat gas turbine cycle, air enters at 1 bar, 300 K. The compressor has intercooling between the two stages of compression. Air leaves the first stage of compression at 290 K at 4 bar pressure after intercooler. The air is subsequently compressed up to 8 bar. Compressed air leaving second stage compressor is passed through a regenerator of effectiveness 0.80. The combustion chamber yields maximum temperature of 1300 K at inlet to turbine having expansion up to 4 bar and then reheated up to 1300 K. The turbine has expanded gases further down to 1 bar. Exhaust from the turbine is passed through the regenerator and then discharged out of the cycle. The heating value of the fuel is 42000 kJ/kg. Find the fuel-air ratio in each combustion chamber, total turbine work and thermal efficiency. Consider compression and expansion to be isentropic and air as working fluid throughout the cycle.

Solution

Given: T_1 = 300 K, p_1 = 1 bar, $p_2 = p_3$ = 4 bar, T_3 = 290 K, T_6 = 1300 K

$$p_6 = p_4 = 8\text{ bar},\ T_8 = 1300\text{ K},\ p_8 = 4\text{ bar}$$

For process 1-2, $T_2 = T_1(p_2/p_1)^{(\gamma-1)/\gamma} = 300 \times (4/1)^{\frac{1.4-1}{1.4}} = 445.79\text{ K}$ (1)

For process 3-4, $T_4 = T_3(p_4/p_3)^{(\gamma-1)/\gamma} = 290 \times (8/4)^{\frac{1.4-1}{0.4}} = 353.51\text{ K}$ (2)

For process 6-7, $T_7 = T_6(p_7/p_8)^{(\gamma-1)/\gamma} = 1300 \times (4/8)^{\frac{1.4-1}{1.4}} = 1066.43\text{ K}$ (3)

For process 8-9, $T_9 = T_8(p_9/p_8)^{(\gamma-1)/\gamma} = 1300(1/4)^{\frac{1.4-1}{1.4}} = 874.83\text{ K}$ (4)

In regenerator, effectiveness = 0.80 $= \dfrac{T_5 - T_4}{T_9 - T_4} = \dfrac{T_5 - 353.51}{874.83 - 353.51}$

$$T_5 = 770.56\text{ K} \quad (5)$$

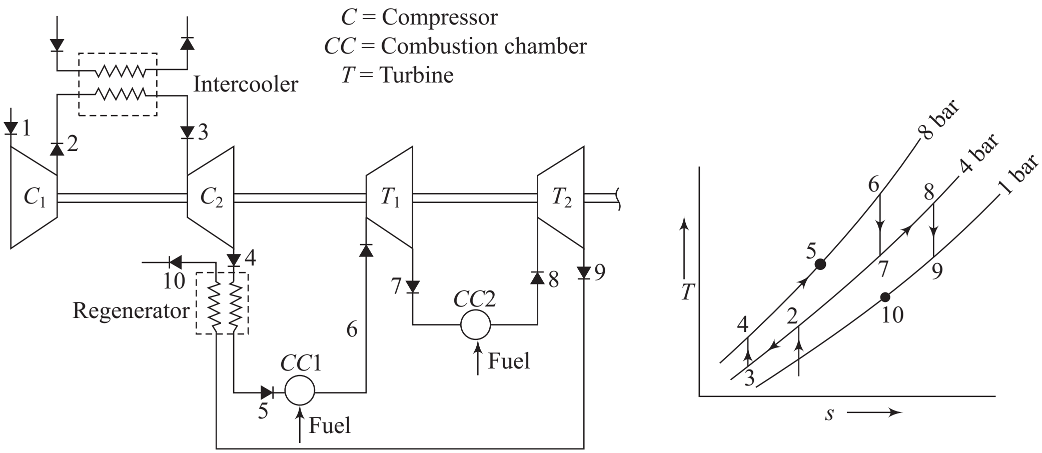


Figure 8.8 *Brayton Cycle Gas Turbine Power Plant with Regeneration, Reheat and Intercooling*

Assume $c_p = 1.005$ kJ/kg-K.

Compressor work per kg of air, $w_c = c_p(T_2 - T_1) + c_p(T_4 - T_3) = 210.3465$ kJ/kg (6)

Turbine work per kg of air, $w_t = c_p(T_6 - T_7) + c_p(T_8 - T_9) = 662.034$ kJ/kg (7)

Heat added per kg of air, $q_s = c_p(T_6 - T_5) + c_p(T_8 - T_7) = 766.825$ kJ/kg (8)

Net work, $w = w_t - w_c = 451.688$ kJ/kg (9)

Cycle thermal efficiency, $\eta = w/q_s = 451.688/766.825 = 58.9$ % (10)

Fuel added per kg in combustion chamber, 1 F:A of 1 = $c_p(T_6 - T_5)/42000$ = 0.0126 7 kg of fuel/kg of air (11)

Fuel added per kg in combustion chamber 2, F:A of 1 = $c_p(T_8 - T_7)/42000$
= 0.0056 kg of fuel/kg of air (12)

8.2 Aircraft Propulsion Cycles

8.2.1 Simple Turbojet Engine and its Ideal Cycle

The nomenclature to be adopted is shown in Figure 8.9, which illustrates a simple turbojet engine and the ideal cycle upon which it operates. The turbine produces just sufficient work to drive the compressor, and the remaining part of the expansion is carried out in the propelling nozzle.

Because of the significant effect of forward speed, the intake must be considered as a separate component. If η_d is the efficiency of the diffuser, then the total pressure at the end of diffusion process is given by,

$$\frac{p_{01}}{p_a} = \left(1 + \frac{\eta_d \times (\gamma - 1)}{2} \times M^2\right)^{\frac{\gamma}{(\gamma-1)}} \tag{8.85}$$

If η_c is the efficiency of the compressor then,

$$(h_{02} - h_{01}) = \frac{1}{\eta_c} \times (h_{02s} - h_{01})$$

$$(h_{02} - h_{01}) = \frac{c_p}{\eta_c} \times (T_{02s} - T_{01}) \tag{8.86}$$

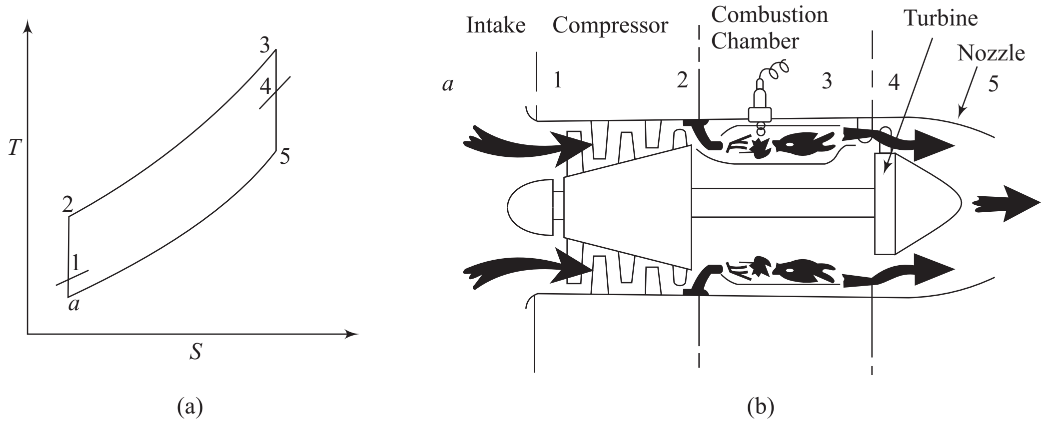


Figure 8.9 *Simple Turbo Jet Engine and its Ideal Cycle*

$$(h_{02} - h_{01}) = \frac{c_p \times T_{01}}{\eta_c} \times \left[\left(\frac{T_{02s}}{T_{01}}\right) - 1\right] \tag{8.87}$$

$$(h_{02} - h_{01}) = \frac{c_p \times T_{01}}{\eta_c} \times \left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \tag{8.88}$$

The heat input per kg of air in the combustion chamber is given by,

$$q = h_{03} - h_{02} \tag{8.89}$$

We assume that there is no pressure loss in the combustion chamber so that the pressure remains constant and full pressure is available for expansion in the turbine and nozzle.

We have assumed that the turbine and compressor works are same.

$$h_{02} - h_{01} = h_{03} - h_{04} \tag{8.90}$$

If η_t is the efficiency of the turbine then,

$$h_{03} - h_{04} = \eta_t \times c_p \times T_{03}\left[1 - \frac{p_{04}}{p_{03}}\right]^{\frac{\gamma-1}{\gamma}} \tag{8.91}$$

If η_{nozzle} is the efficiency of the nozzle then,

$$\eta_{\text{nozzle}} = \frac{h_{04} - h_5}{h_{04} - h_{5s}} \tag{8.92}$$

Here, h_5 is used instead of total enthalpy h_{05} because the exhaust nozzle efficiency is an indication of the percentage of total energy converted into velocity energy. Therefore,

$$(h_{04} - h_5) = c_p \times T_{04} \times \eta_{\text{nozzle}} \times \left[1 - \left(\frac{p_5}{p_{04}}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{8.93}$$

8.2.2 Actual Turbojet Cycle

Figure 8.10 shows the real turbojet cycle on the T-s diagram. The major differences between simple cycle (Figure 8.9) and turbojet cycle (Figure 8.10) are as follows:

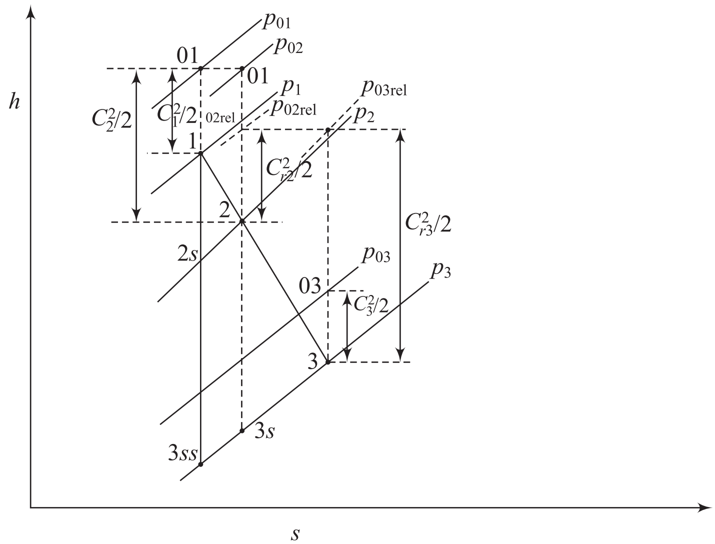


Figure 8.10 *Actual Turbojet Cycle*

- In the actual cycle, the intake, the compressor, the turbine and the propelling nozzle will have individual component efficiency unlike the simple cycle.
- The combustion pressure loss is expressed as the pressure loss Δp_b is non zero and is expressed as certain percentage of compressor delivery pressure.
- The specified efficiency is used in the following fashion:
  (a) Intake efficiency (η_i) is used in estimating compressor inlet pressure.

$$p_{01} = p_a[1 + \eta_i \frac{C_a^2}{2c_pT_a}]^{\frac{\gamma}{\gamma-1}}$$

  (b) Compressor efficiency (η_c) is used to compute the total exit temperature of the compressor.

$$T_{02} = T_{01} + \frac{T_{01}}{\eta_c}\left[\left(\frac{p_{02}}{p_{o1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

  (c) The turbine inlet pressure is calculated using the combustion pressure loss (Δp_b) as,

$$p_{03} = p_{02}\left(1 - \frac{\Delta p_b}{p_{02}}\right)$$

  (d) The turbine exit pressure can be calculated using the expression of turbine efficiency (η_t) as,

$$p_{04} = p_{03}\left[1 - \frac{1}{\eta_t}\left(1 - \frac{T_{04}}{T_{03}}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

  (e) The critical pressure (p_c) in the nozzle is estimated using the nozzle efficiency (η_n) as,

$$p_c = p_{04}\left[1 - \frac{1}{\eta_n}\left(1 - \frac{T_c}{T_{04}}\right)\right]^{\frac{\gamma}{\gamma-1}}$$

8.3 Velocity Triangles and Temperature-Entropy Diagrams

8.3.1 Velocity Triangles

(a) Axial Turbine

An axial turbine stage along with its velocity triangles at entry and exit are shown in Figure 8.11. The change in mean diameter between the entry and exit is negligible since the stage is axial. Therefore, the peripheral velocity remains constant.

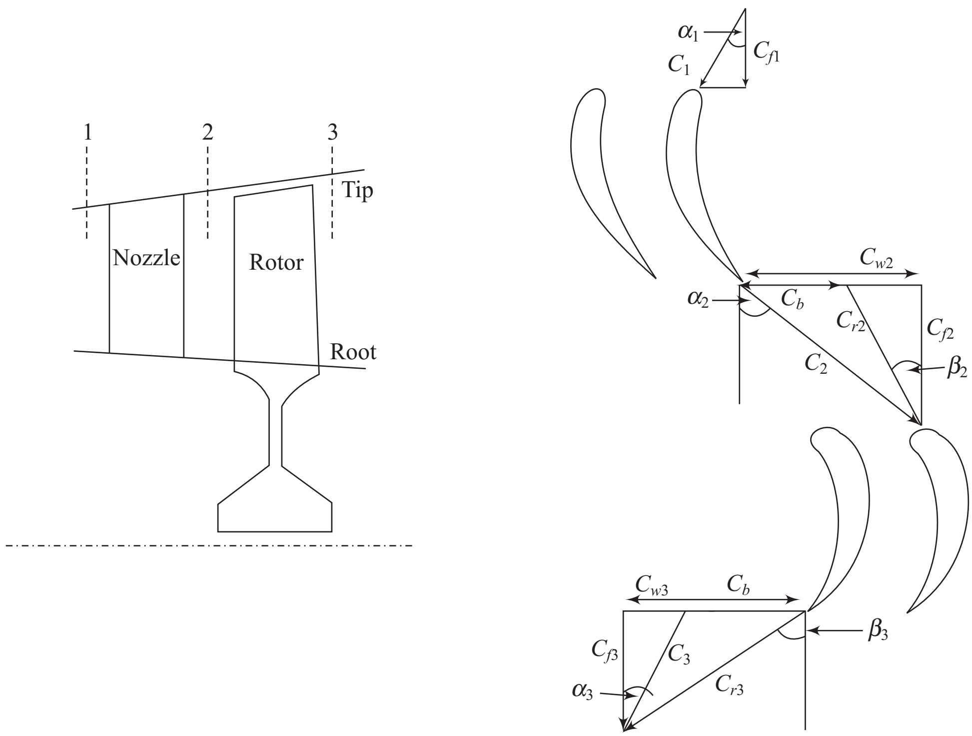


Figure 8.11 *Axial Turbine and its Velocity Triangle*

(b) Radial Turbine

Figure 8.12 shows a radial turbine stage along with various components and velocity triangles. An inward flow volute or scroll casing distributes gas properly over the entire periphery of the nozzle ring or rotor blades when the high pressure gas enters the turbine through a duct or pipe. The nozzle ring is not used in some of the applications. In such cases, the flow receives some degree of acceleration accompanied by a pressure drop in the volute casing.

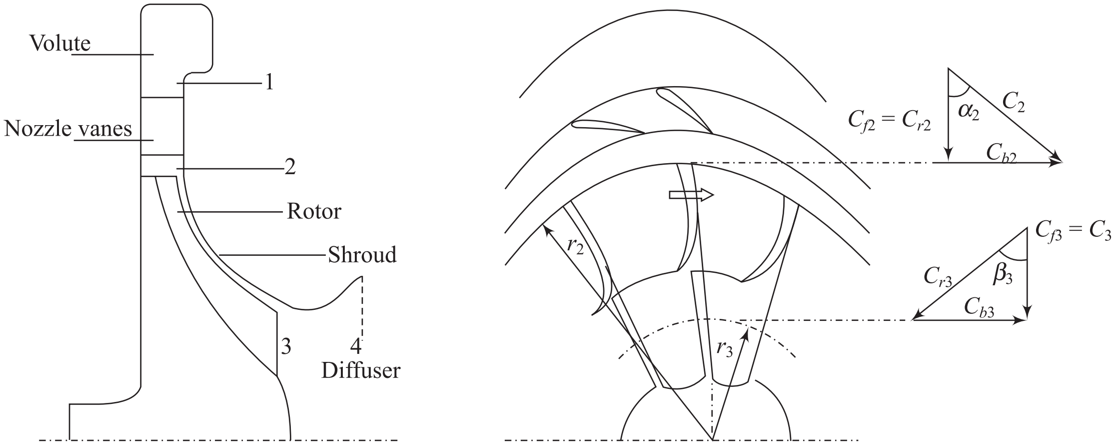


Figure 8.12 *Radial Turbine and Its Velocity Triangle*

8.3.2 Enthalpy-Entropy Diagram

(a) Axial Turbine

The enthalpy (temperature) entropy diagram for flow through an axial turbine stage with some degree of reaction is shown in Figure 8.13. Static and stagnation values of the pressure and temperatures are indicated at various points. The isentropic expansion of the gas through fixed and moving rows of the blades is represented by the process 1-2s and 2s-3s, respectively. The actual expansion process is represented by the curve 1-2-3, and the actual states of the gas at the exits of the fixed and moving blades are represented by points 2 and 3, respectively. Since there is only energy transformation and no energy transfer (work) between the states 1 and 2, therefore, the stagnation enthalpy remains constant.

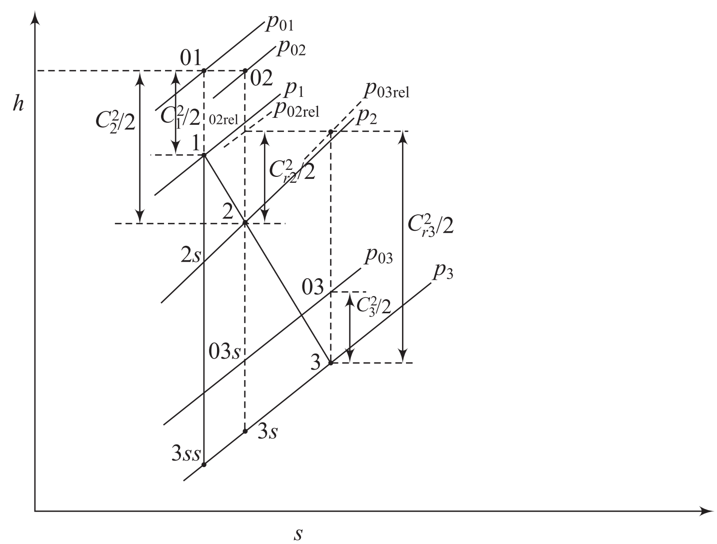


Figure 8.13 *Representation of Expansion in an Axial Turbine on T-s Diagram*

(b) Radial Turbine

Figure 8.14 shows the various flow processes on temperature (enthalpy) diagram of a radial gas turbine. The gas expands isentropically in the nozzles (process 1-2s) from pressure p_1 to p_2 with an increase in velocity from C_1 to C_2. Since this is an energy transformation process, the stagnation enthalpy remains constant but the stagnation pressure decreases due to losses. The expansion in the rotor 2-3 represents energy transfer accompanied by an energy transformation process.

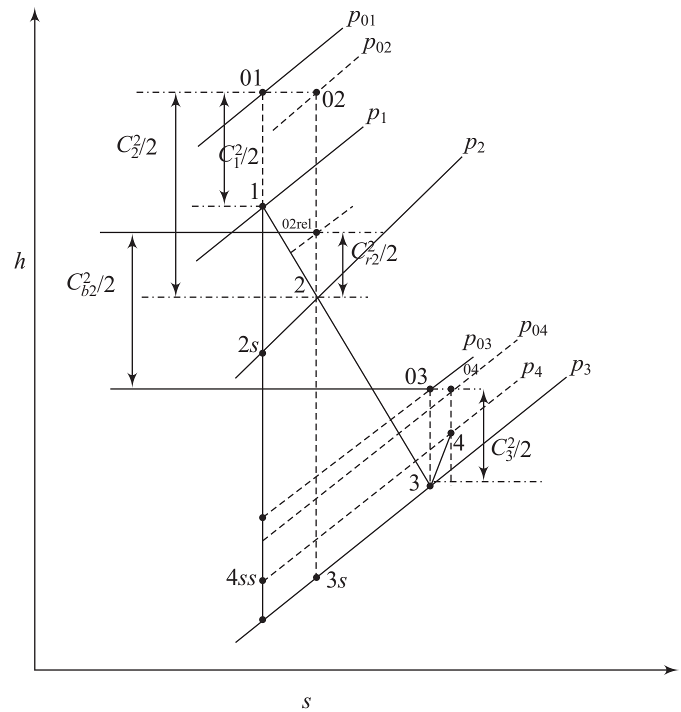


Figure 8.14 *Representation of Expansion in Radial Turbine on T-s Diagram*

8.4 Stage Parameters of Axial Turbine

Here, $C_1 = C_3$ (for repeating stage); $C_{f3} = C_{f2} = C_f$ (8.97)

8.4.1 Stage Work

From the axial turbine velocity triangle, as shown in Figure 8.11, we get,

$$C_b/C_f = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3 \quad (8.98)$$

Stage work output per unit mass flow is

$$w = C_b[C_{w2} - (-C_{w3})] = C_b(C_{w2} + C_{w3}) \quad (8.99)$$

From the velocity triangle, we get the value of $C_{w2} + C_{w3}$

$$w = C_b C_f (\tan \alpha_2 + \tan \alpha_3) \tag{8.100}$$

Substituting the value of Eq. (8.98) in Eq. (8.100),

$$w = C_b\, C_f (\tan \beta_2 + \tan \beta_3) \tag{8.101}$$

From the steady flow energy equation, we have,

$$w = c_p \Delta T_{0,\text{ stage}}$$

where $\Delta T_{0,\text{ stage}}$ is the stagnation temperature drop in the stage, and hence,

$$c_p \Delta T_{0,\text{ stage}} = C_b\, C_f (\tan \beta_2 + \tan \beta_3) \tag{8.102}$$

8.4.2 Stage Pressure Ratio

The stage pressure ratio of the stage, p_{o3}/p_{01}, can be found from

$$\Delta T_{0,\text{stage}} = T_{01} - T_{03} = \eta_{\text{stage}} T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{p_{03}}{p_{o1}} = \left[1 - \frac{\Delta T_{0,\text{stage}}}{\eta_{\text{stage}} T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \tag{8.103}$$

where η_{stage} is the isentropic efficiency or stage efficiency described in the following paragraph.

8.4.3 Stage Efficiency

Isentropic efficiency or stage efficiency is the ratio of actual stage work to the ideal stage work between total conditions at entry and exit, which is given by,

$$\eta_{\text{stage}} = \eta_{t-t} = (T_{01} - T_{03})/(T_{01} - T_{03s}) = (h_{01} - h_{03}) / (h_{01} - h_{03s}) \tag{8.104}$$

This isentropic efficiency, based upon stagnation temperature when applied to a stage, is often called the total-to-total efficiency.

The kinetic energy at the exit of a turbine stage is treated as a loss in the definition of the total-to-static efficiency.

$$\eta_{t-s} = (h_{01} - h_{03}) / (h_{01} - h_{3s}) = (h_{01} - h_{03})/(h_{01} - h_{03s} + 1/2 \times C_{3s}^2)$$

where $C_{3s} = C_3$

$$\eta_{t-s} = (h_{01} - h_{03})/(h_{01} - h_{03s} + 1/2 \times C_3^2) \tag{8.105}$$

8.4.4 Blade Loading Coefficient or Temperature Drop Coefficient

$$\Psi = w/0.5\ C_b^2 = (2c_p\, \Delta T_{0,\text{ stage}})/ C_b^2 \tag{8.106}$$

$$\Psi = 2C_f (\tan\beta_2 + \tan \beta_3)/C_b \tag{8.107}$$

Using flow coefficient definition,

$$\varphi = C_f/C_b \tag{8.108}$$

Substituting Eq. (8.108) in Eq. (8.107), we get,

$$\Psi = 2\varphi\,(\tan\beta_2 + \tan\beta_3) \tag{8.109}$$

8.4.5 Degree of Reaction

Degree of reaction expresses the fraction of stage expansion which occurs in the rotor, which is also defined in terms of static temperature or (enthalpy) drop in the rotor.

$$R = \frac{c_p(T_2 - T_3)}{c_p(T_1 - T_3)} = \frac{c_p(T_2 - T_3)}{c_p(T_{01} - T_{03})} \tag{8.110}$$

$$c_p\,(T_{01} - T_{03}) = C_b C_f\,(\tan\beta_2 + \tan\beta_3) \tag{8.111}$$

Relative to the rotor blades, the flow does no work and the steady flow energy equation yields,

$$C_p(T_2 - T_3) = \frac{1}{2}(C_{r3}^2 - C_{r2}^2) \tag{8.112}$$

Using the velocity triangle, we get,

$$(C_{r3}^2 - C_{r2}^2) = C_b^2(\sec^2\beta_3 - \sec^2\beta_2) \tag{8.113}$$

$$(C_{r3}^2 - C_{r2}^2) = C_b^2(\tan^2\beta_3 - \tan^2\beta_2) \tag{8.114}$$

Using Eqs. (8.111), (8.112) and (8.114) in Eq. (8.110), we get,

$$R = \frac{C_f}{2C_b}(\tan\beta_3 - \tan\beta_2) = \frac{1}{2}\varphi(\tan\beta_3 - \tan\beta_2) \tag{8.115}$$

Using the relation of Eq. (8.98) in these equations, we get,

$$\tan\beta_3 = \tan\alpha_3 + \frac{1}{\varphi} \tag{8.116}$$

$$\tan\beta_2 = \tan\alpha_2 - \frac{1}{\varphi} \tag{8.117}$$

Using the value of $\tan\beta_3$ from Eq. (8.116) in Eq. (8.115), we get,

$$R = \frac{1}{2} + \frac{1}{2}\varphi[\tan\alpha_3 - \tan\beta_2] \tag{8.118}$$

Using the value of $\tan\beta_2$ from Eq. (8.117) in Eq. (8.115), we get,

$$R = \frac{1}{2} + \frac{1}{2}\varphi[\tan\beta_3 - \tan\alpha_2] \tag{8.119}$$

Using the value of $\tan\beta_3$, $\tan\beta_2$ from Eqs. (8.116) and (8.117) in Eq. (8.115),

$$R = 1 + \frac{1}{2}\varphi[\tan\alpha_3 - \tan\alpha_2] \tag{8.120}$$

Note that if $\alpha_3 = \beta_2$ or $\alpha_2 = \beta_3$, the degree of reaction R is 50% and if $\alpha_2 = \alpha_3$, the degree of reaction is 100%.

The gas angles can now be expressed in terms of Ψ, R and φ as follows. Adding and subtracting Eq. (8.109) and Eq. (8.115), we get,

$$\tan \beta_3 = \frac{1}{2\varphi}\left(\frac{1}{2}\Psi + 2R\right) \quad (8.121)$$

$$\tan \beta_2 = \frac{1}{2\varphi}\left(\frac{1}{2}\Psi - 2R\right) \quad (8.122)$$

8.5 Performance Characteristics of Axial Turbines

8.5.1 Effect of Blade Loading Coefficients and Flow Coefficients

Considering 50% reaction design, where the expansion is reasonably evenly divided between the stator and rotor rows, substituting $R = 0.5$ in Eq. (8.115) and comparing with Eq. (8.98), we get,

$$\beta_3 = \alpha_2 \text{ and } \alpha_3 = \beta_2 \quad (8.123)$$

and the velocity triangle diagram becomes symmetrical.

As $C_1 = C_3$, both in magnitude and direction for repeating stage, we have $\alpha_1 = \alpha_3 = \beta_2$ and also, the stator and rotor blades have the same inlet and outlet angles. Equations (8.121), (8.122), and (8.123), for $R = 0.5$ give all the gas angles in terms of Ψ and φ.

Note that low Ψ means more stages for a given overall turbine output, whereas low φ means a larger turbine annulus area for a given mass flow.

In industrial gas turbines, when size and weight are of little consequence and low specific fuel consumption is vital, it is sensible to design with a low Ψ and a low φ.

For aero engines, it is desired to keep the weight and frontal area minimum and this requires using higher values of Ψ and φ.

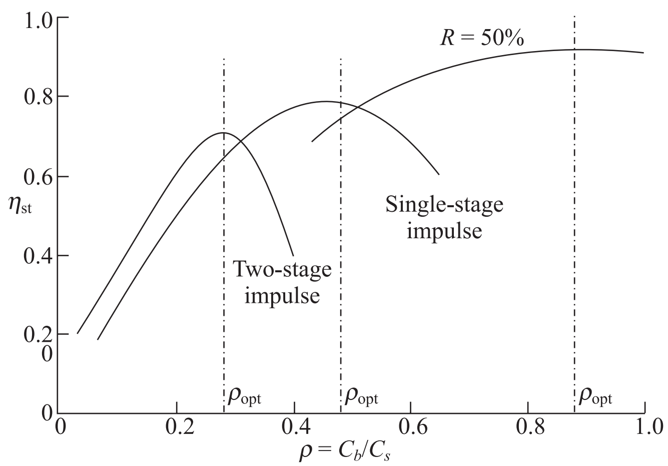


Figure 8.15 *Variation of Stage Efficiency with Blade-to-Gas Speed Ratio*

Stage work is governed by the blade-to-gas speed ratio parameter. Blade-to-gas speed ratio is the ratio of blade velocity and the isentropic gas velocity that would be obtained in its isentropic expansion through stage pressure ratio.

$$\rho_s = \frac{C_b}{C_s} \tag{8.124}$$

From Figure 8.13

$$h_{01} - h_{03s} = \frac{1}{2}C_s^2 = C_p T_{01}\left\{1 - \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right\} \tag{8.125}$$

Using Eqs (8.102), (8.108), (8.109) and (8.125) in Eq. (8.104),

$$\boxed{\eta_{\text{stage}} = \eta_{t-t} = \frac{C_b C_f \Psi}{C_s^2 \varphi} = \rho^2 \Psi} \tag{8.126}$$

Efficiency [Eq. (8.126)] of the turbine stages can also be plotted against this ratio. Such plots for some impulse and reaction stages are shown in Figure 8.15. The figure also shows the optimum values of the velocity ratio and the range of off-design for various types of stages.

EXAMPLE 8.4 The values of stagnation and static pressures of a zero-reaction gas turbine stage at the mean blade height are as follows:

Stagnation pressure: At nozzle entry = 414 kPa, and at nozzle exit = 400 kPa

Static pressure: At nozzle exit = 207 kPa, and at rotor exit = 200 kPa

The mean blade speed is 291 m/s. The inlet stagnation temperature is 1100 K. The flow angle at nozzle exit is 70° measured from the axial direction. Consider that the magnitude and direction of the flow velocities at entry and exit of the stage are same. Find the total-to-total efficiency of the stage. Assume a perfect gas with c_p =1.148 kJ/(kg°C) and γ= 1.333.

Solution

The total-to-total efficiency of a turbine stage is defined, in the usual notation, as,

$$\eta_{t-t} = \frac{h_{01} - h_{03}}{h_{01} - h_{03ss}} \tag{1}$$

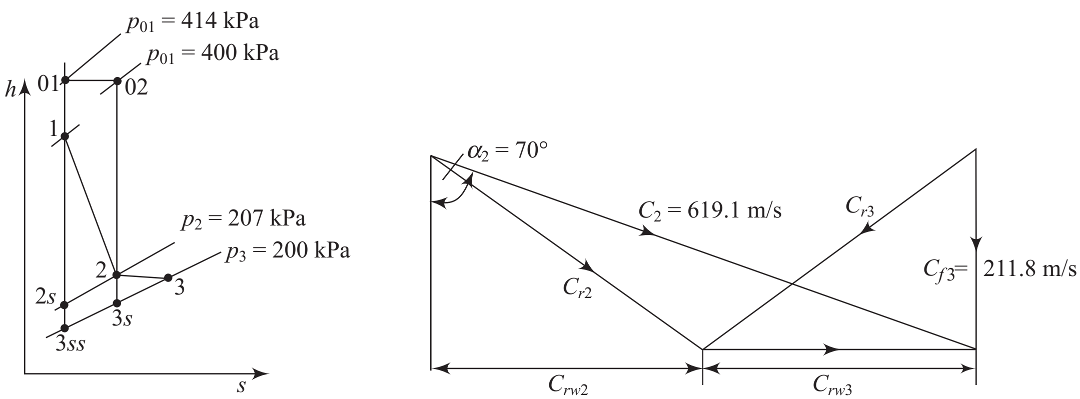


Figure 8.16 *T-s Diagram and Velocity Triangle of Zero-Reaction Gas Turbine Stage*

With $C_1 = C_3$, this can be rewritten as,

$$\eta_{t-t} = \frac{1}{\left[1 + \frac{(h_3 - h_{3ss})}{(h_1 - h_3)}\right]} \tag{2}$$

$$\eta_{t-t} = \frac{1}{\left[1 + \frac{(T_3 - T_{3ss})}{(T_1 - T_3)}\right]} \tag{3}$$

with the perfect gas assumption. In order to determine the efficiency of the stage, the velocity diagram must first be solved.

The stage reaction is defined as $R = (h_2 - h_3)/(h_1 - h_3)$, so that zero reaction means h_2 equals h_3. The relative stagnation enthalpy, $h_{0,\mathrm{rel}} = h + \frac{1}{2}C_r^2$, is constant in the rotor, then

$$h_2 + \frac{1}{2}C_{r2}^2 = h_3 + \frac{1}{2}C_{r3}^2 \tag{4}$$

and, therefore $C_{r2} = C_{r3}$. The velocity at nozzle exit, C_2, must be determined to complete the velocity diagram.

$$T_2 = T_{02}\left(\frac{p_2}{p_{02}}\right)^{(\gamma-1)/\gamma} \tag{5}$$

$$T_2 = 1100(207/400)^{0.2498} = 933.1 \text{ K} \tag{6}$$

$$C_2^2 = 2c_p(T_{02} - T_2) = 2 \times 1148(1100 - 933.1) \tag{7}$$

$$C_2 = 619.1 \text{ m/s}$$

Referring to the velocity diagram,

$$C_{f2} = C_2 \cos \alpha_2 = 619.1 \cos 70° = 211.8 \text{ m/s} \tag{8}$$

$$C_{w2} = C_2 \sin \alpha_2 = 619.1 \sin 70° = 581.8 \text{ m/s} \tag{9}$$

$$C_{rw2} = C_{w2} - C_b = 581.8 - 291 = 290.8 \text{ m/s} \tag{10}$$

An important point to note is that $C_{rw3} = C_{rw2}$ as $C_{f3} = C_{f2}$ and $C_{rw2} = C_b$. Thus,

$$C_{w3} = C_{rw3} - C_{b2} = C_{rw2} - C_b = 290.8 - 291 = -0.2 \text{ m/s} \tag{11}$$

i.e. the flow leaving the stage is very nearly axial in direction with a small angle of swirl, $\alpha_3 = \tan^{-1}(-0.2/211.8) = -0.05°$. Effectively, $C_3 = C_1 = C_{f3} = 211.8$ m/s.

Thus, with $T_3 = T_2 = 933.1$ K and $T_{01} = T_{01} = 1100$ K,

$$T_1 - T_3 = T_{01} - \frac{C_1^2}{2c_p} - T_3 \tag{12}$$

$$T_1 - T_3 = 1100 - 211.8^2/(2 \times 1148) - 933.1 = 147.4°\text{C} \tag{13}$$

Using the isentropic relation between temperature and pressure,

$$T_{3ss} = T_{01}\left(\frac{p_3}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

$$T_{3ss} = 1100(200/414)^{0.2498} = 917.1 \text{ K} \quad (14)$$

Therefore, $T_3 - T_{3ss} = 933.3 - 917.1 = 16°C$ (17)

$$\eta_{t-t} = \frac{1}{\left[1 + \frac{16}{147.4}\right]} = 90.2\% \quad (18)$$

EXAMPLE 8.5 An axial flow gas turbine stage develops 3.36 MW for a mass flow rate of 27.2 kg/s. The stagnation pressure and stagnation temperature at stage entry are 772 kPa and 1000 K. The axial velocity is constant throughout the stage. The gases enter and leave the stage without any swirl. At nozzle exit, the static pressure is 482 kPa and the flow direction is at 18° to the plane of the wheel. Find

(i) The axial velocity and degree of reaction for the stage when the entropy increase in the nozzles is 12.9 J/(kg K). Assume that the specific heat at constant pressure of the gas is 1.148 kJ/(kg K) and the gas constant is 0.287 kJ/(kg K).

(ii) The total-to-total efficiency of the stage given that the increase in entropy of the gas across the rotor is 2.7 J/(kg K).

Solution

Referring to the Mollier diagram, the nozzle exit velocity, C_2, is solved by determining T_{2s} from the isentropic temperature-pressure relationship and then estimating the temperature difference $T_2 - T_{2s}$ from the entropy increase across the nozzle. Thus,

$$T_{2s} = T_{01}\left(\frac{p_2}{p_{01}}\right)^{(\gamma-1)/\gamma} = 1000(482/772)^{0.25} = 888.9 \text{ K} \quad (1)$$

Using the relation $Tds = dh - dp/\rho$, at constant pressure, $T\Delta S \cong \Delta h$,

$$h_2 - h_{2s} \cong T_{2s}(s_2 - s_{2s}) = 888.9 \times 12.9 = 11.47\frac{\text{kJ}}{\text{kg}} \quad (2)$$

$$T_2 - T_{2s} = \frac{(h_2 - h_{2s})}{c_p} = \frac{11.47}{1.148} = 10°C \quad (3)$$

Therefore, $T_2 = 10 + 888.9 = 898.9$ K (4)

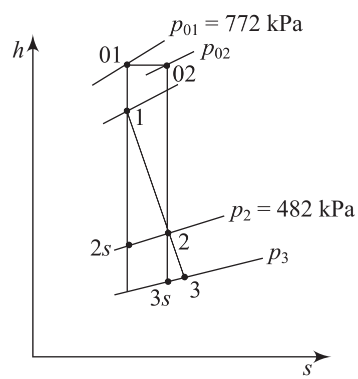


Figure 8.17 *T-s Diagram of an Axial Gas Turbine Stage*

$$C_2^2 = 2c_p(T_{01} - T_2) = 2 \times 1148(1100 - 898.8) = 23.21 \times 10^4$$

$$C_2 = 481.6 \text{ m/s} \quad (5)$$

The axial velocity is easily obtained,

$$C_f = C_2 \cos\alpha_2 = 481.6 \cos(90 - 18) = 148.9 \text{ m/s} \quad (6)$$

The stage reaction is defined as,

$$R = 1 - \frac{C_f}{2C_b}(\tan\alpha_2 - \tan\alpha_3) \quad (7)$$

which, with $\alpha_3 = 0$, becomes

$$R = 1 - (C_f/2C_b)\tan\alpha_2 \tag{8}$$

The blade speed C_b can be found from the equation for specific work, viz.,

$$w = C_b C_{w2} = C_b C_f \tan\alpha_2 \tag{9}$$

$$C_b = \frac{w}{C_f \tan\alpha_2} = \left(\frac{\dot{W}/\dot{m}}{C_f \tan\alpha_2}\right) \tag{10}$$

$$= 3.36 \times 10^6/(27.2 \times 148.9 \times \tan 72°) = 269.5 \text{ m/s} \tag{15}$$

$$R = 1 - 148.9 \times \tan 72°/(2 \times 269.5) = 1 - 0.850 = 0.150 \tag{16}$$

The total-to-total efficiency of the stage is

$$\eta_{t-t} = \frac{1}{\left[1 + \dfrac{(h_2 - h_{2s}) + (h_3 - h_{3s})}{(h_{01} - h_{03})}\right]} \tag{17}$$

$$\eta_{t-t} = \frac{1}{\left[1 + \dfrac{c_p(T_2 - T_{2s}) + c_p(T_3 - T_{3s})}{w}\right]} \tag{18}$$

The temperature difference $T_3 - T_{3s} = T_3(s_3 - s_{3s})/c_p$ and requires the evaluation of T_3. Now,

$$w = h_{01} - h_{03} = h_{01} - h_3 - \frac{1}{2}C_f^2, \text{ (as } C_1 = C_2 = C_f) \tag{19}$$

$$T_3 = T_{01} - \left(\Delta W + \frac{1}{2}C_f^2\right)\Big/c_p = 1000 - (123.5 \times 10^3 + 1/2(148.9^2)/1148 = 1000 - 117.2 = 882.8 \text{ K} \tag{20}$$

$$T_3 - T_{3s} = 882.8 \times 2.7/1148 = 2.08°\text{C} \tag{21}$$

Therefore,

$$\eta_{t-t} = \frac{1}{\left[1 + \dfrac{1.148(10 + 2.08)}{123.5}\right]} = 89.9\% \tag{22}$$

8.6 Stage Parameters of Radial Turbine

The relative velocity at rotor tip is radial and the absolute velocity at exit is axial.

$$C_{w3} = 0;\ C_{w2} = C_{b2};\ \boldsymbol{C_{f3} = C_3;\ C_{r3} = C_{r2}} \tag{8.127}$$

8.6.1 Stage Work

The specific work output is given by,

$$w = c_p \Delta T_{0,\text{ stage}} = c_p(T_{01} - T_{03}) = C_{w2}C_{b2} \tag{8.128}$$

As $C_{w2} = C_{b2,}$ thus,

$$\boxed{w = C_{b2}^2} \tag{8.129}$$

8.6.2 Stage Pressure Ratio

In the ideal isentropic turbine with perfect diffuser, the specific work output would be

$$w_s = c_p (T_{01} - T_{4s}) = (C_0^2)/2 \tag{8.130}$$

where the velocity equivalent of the enthalpy drop, C_0.

In terms of turbine pressure ratio,

$$\frac{C_0^2}{2} = c_p T_{01} \left\{1 - \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right\} \Rightarrow \frac{p_{03}}{p_{o1}} = \left[1 - \frac{C_0^2}{2c_p T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \tag{8.131}$$

8.6.3 Stage Efficiency

The overall isentropic efficiency of the turbine and diffuser may be expressed by,

$$\eta_0 = (T_{01} - T_{03}) / (T_{01} - T_{4s}) \tag{8.132}$$

because $(T_{01} - T_{4s})$ is the temperature equivalent of the maximum work that could be produced by an isentropic expansion from the inlet state (p_{01}, T_{01}) to $p_{a.}$

Considering the turbine alone, above efficiency is expressed by,

$$\eta_{t-t} = (T_{01} - T_{03}) / (T_{01} - T_{03s}) \tag{8.133}$$

where η_{t-t} is referred as the total-to-total efficiency.

8.6.4 Blade Loading Coefficient or Temperature Drop Coefficient

$$\Psi = (w)/0.5\ C_{b2}^2 = (2c_p \Delta T_{0,\text{ stage}})/ C_{b2}^2 \tag{8.134}$$

$$\Psi = 2 \tag{8.135}$$

8.6.5 Degree of Reaction

R = Static enthalpy drop in the rotor/Stagnation enthalpy drop in the stage

$$R = \frac{(h_2 - h_3)}{(h_{01} - h_{03})} = \frac{(T_2 - T_3)}{(T_{01} - T_{03})} \tag{8.136}$$

$$h_2 - h_3 = (C_{b2}^2 - C_{b3}^2)/2 + (Cr_3^2 - Cr_2^2)/2 \tag{8.137}$$

$$(h_{02} - h_{03}) = (C_{b2}^2 - C_{b3}^2)/2 + (C_2^2 - C_3^2)/2 + (Cr_3^2 - Cr_2^2)/2$$

$$(h_{02} - h_{03}) = w = C_{b2} C_{w2}, \tag{8.138}$$

for zero swirl at the entry of the impeller.

Substituting Eqs. (8.137) and (8.138) in Eq. (8.136),

$$R = \frac{(C_{r3}^2 - C_{b3}^2) + (C_{b2}^2 - C_{r2}^2)}{2C_{b2}C_{w2}} \tag{8.139}$$

From the velocity triangles at the entry and exit of the impeller,

$$C_{f2}^2 = C_{r2}^2 = C_2^2 - C_{b2}^2 \tag{8.140}$$

$$C_{r3}^2 = C_{f3}^2 + C_{b3}^2 = C_3^2 + C_{b3}^2 \tag{8.141}$$

Using Eqs. (8.140) and (8.141) in Eq. (8.139), we have,

$$R = \frac{[C_3^2 + (2C_{b2}^2 - C_2^2)]}{2C_{b2}C_{w2}} \tag{8.142}$$

Using Eqs. (8.129) and (8.138) in Eq. (8.142), we get,

$$R = \frac{[2C_{b2}C_{w2} - (C_2^2 - C_3^2)]}{2C_{b2}C_{w2}} \tag{8.143}$$

If $C_{r2} = C_{f3} = C_3$, $(C_2^2 - C_3^2) = C_{w2}^2$

$$\therefore \quad R = 1 - \frac{1}{2}\left(\frac{C_{w2}}{C_{b2}}\right) \tag{8.144}$$

As $C_{w2} = C_{b2}$ from Eq. 8.127,

Therefore, $R = 1/2$.

Assuming conditions at the inlet other than the mentioned condition and same conditions at the outlet, and using general velocity triangle at inlet and outlet, we find C_{w2} in terms of air angle (β_2) at the inlet.

$$C_{w2} = C_{b2} + C_{f2} \cot \beta_2 \tag{8.145}$$

Substituting Eq. (8.145) in (8.144), we get,

$$R = \frac{1}{2}(1 - \varphi_2 \cot \beta_2) \tag{8.146}$$

where $\varphi_2 = C_{f2}/C_{b2}$

Variation of degree of reaction using Eq. (8.146) with flow coefficient for various values of the air angles at the rotor is shown in Figure 8.18. The degree of reaction at a given flow coefficients increases with the air angle at the rotor entry; it decreases with the increase in flow coefficient for $\beta_2 < 90°$ and increase with the flow coefficient for $\beta_2 > 90°$. The degree of reaction of the fifty percent reaction stage remains constant at all values of the flow coefficient.

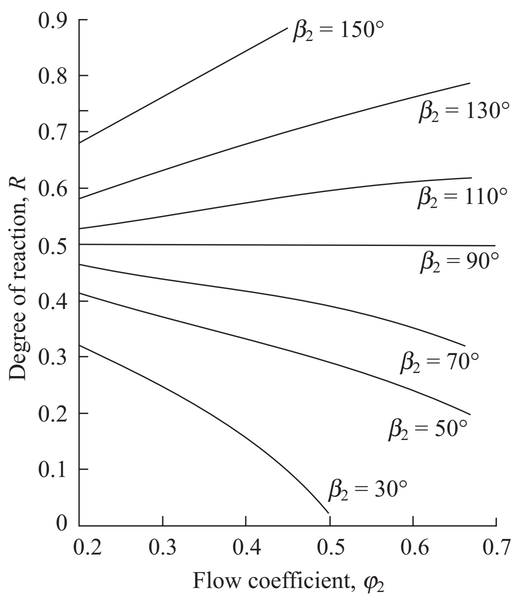


Figure 8.18 *Variation of Degree of Reaction (R) with Flow Coefficient (φ_2) for Different Inlet Air Angle (β_2)*

8.7 Performance Characteristics of Radial Turbine

Plots of the stage loading coefficient with flow coefficient for various values of β_2 are shown in Figure 8.19. The loading coefficient at a given flow coefficient decreases with the increases in the air angle, β_2; it increase with the flow coefficient for $\beta_2 < 90°$ and decrease with the increase in flow coefficient for $\beta_2 > 90°$.

The performance characteristics of turbines are often expressed in terms of plots between the stage efficiency and the blade-to-gas speed ratio. The blade-to-gas speed ratio can be expressed in terms of the isentropic stage terminal velocity C_0.

For an ideal or isentropic radial turbine stage with $C_{w3} = 0$ and complete recovery of the kinetic energy at the exit,

$$h_{01} - h_{03s} = \frac{1}{2}C_0^2 = C_{b2}C_{w2}$$

Using Eqs (8.145) and (8.146), we get,

$$\rho = \frac{C_{b2}}{C_0} = [2(1+\varphi_2 \cot \beta_2)]^{-1/2} \tag{8.147}$$

for $\beta_2 = 90°$

$$\rho = \frac{C_{b2}}{C_0} = 0.707$$

Using Eqs (8.128), (8.145) in Eq. (8.133), we get,

$$\eta_{t-t} = \frac{C_{b2}(C_{b2} + C_{f2} \cot \beta_2)}{h_{01} - h_{03s}}$$

The ideal work between total conditions at the entry and exit of the stage is,

$$w_s = h_{01} - h_{03s} = c_p T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Using above relation in η_{t-t},

$$\eta_{t-t} = \frac{C_{b2}(C_{b2} + C_{f2} \cot \beta_2)}{c_p T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]} = \frac{C_b^2(1+\varphi_2 \cot \beta_2)}{c_p T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

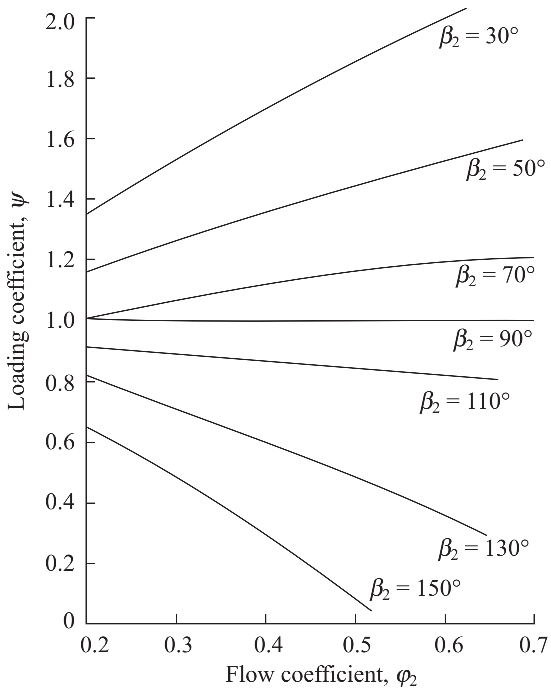


Figure 8.19 *Variation of Loading Coefficient with Flow Coefficient (φ_2) for Different Inlet Air Angle (β_2)*

Here, $$\Psi = \frac{w_{sp}}{0.5C_b^2} = \frac{2C_{w2}C_{b2}}{C_{b2}^2} = 2(1+\varphi_2 \cot \beta_2)$$

Using Ψ, we get,

$$\eta_{t-t} = \frac{\Psi C_b^2}{2c_p T_{01}\left[1-\left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]} = \frac{\Psi C_b^2}{2c_p T_{01}[1-r_0^{\frac{\gamma-1}{\gamma}}]} \tag{8.148}$$

where is r_0 is stagnation pressure ratio.

Similarly, for total-to-static efficiency is given by,

$$\eta_{t-s} = \frac{\Psi C_b^2}{2c_p T_{01}\left[1-\left(\frac{p_3}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]} = \frac{\Psi C_b^2}{2c_p T_{01}[1-r^{\frac{\gamma-1}{\gamma}}]} \tag{8.149}$$

where r is static pressure ratio.

Using blade-to-speed ratio value, Eqs (8.131 and 8.147) in Eq. (8.148), we get,

$$\eta_{\text{stage}} \text{ or } \eta_{t-t} = \rho^2(1+\varphi_2 \cot\beta_2) = \rho^2\Psi \tag{8.150}$$

For an isentropic stage, η_{stage} or $\eta_{t-t} = 1$. (8.151)

Figure 8.20 depicts various curves of η_{stage} or η_{t-t} versus blade-to gas speed ratio (ρ) for various values of the nozzle exit air angle.

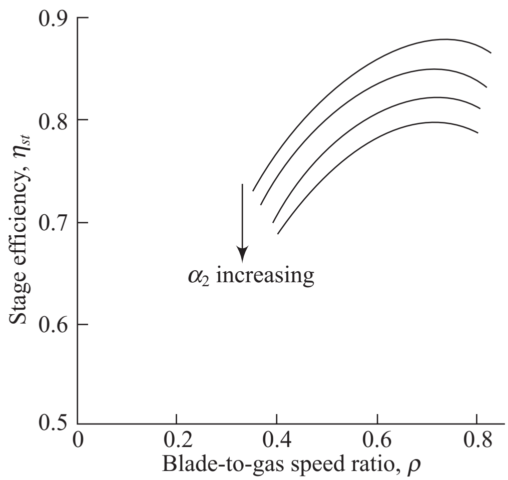


Figure 8.20 *Variation of stage efficiency, η_{stage} with blade-to gas speed ratio (ρ) for nozzle exit air angle*

8.8 Mach Number Limitations

The flow in a turbine chokes when the Mach number reaches sonic value; this can occur at the nozzle throat or anywhere in the rotor flow passage up to its exit. Besides this, the flow can reach supersonic velocities due to local acceleration in which the deceleration, if any, to subsonic flow will result in shock waves.

EXAMPLE 8.6 A small inward flow radial gas turbine operates at its design point. Its total-to-total efficiency is 0.90. The stagnation pressure and temperature of the gas at turbine entry are 400 kPa and 1140 K. The flow leaves the turbine at a pressure of 100 kPa and has negligible final velocity. The flow is just choked at the nozzle exit. Determine the impeller peripheral speed and the flow outlet angle from the nozzles.

For the gas, assume $\gamma = 1.333$ and $R = 287$ J/(kg°C).

Solution

Figure 8.21 shows a meridional section of a 90º inward flow radial turbine and diffuser together with the design point velocity triangles at rotor inlet and rotor outlet. At this condition, the relative velocity C_{r2} at the rotor inlet is in the radial direction and at the rotor outlet, the absolute velocity C_3 is in the axial direction (i.e. $C_{w2} = C_{b2}$ and $C_{w3} = 0$). Thus, the specific work equation is,

$$\Delta W = h_{01} - h_{03} = C_{b2}C_{w2} - C_{b3}C_{w3} = C_{b2}^2$$

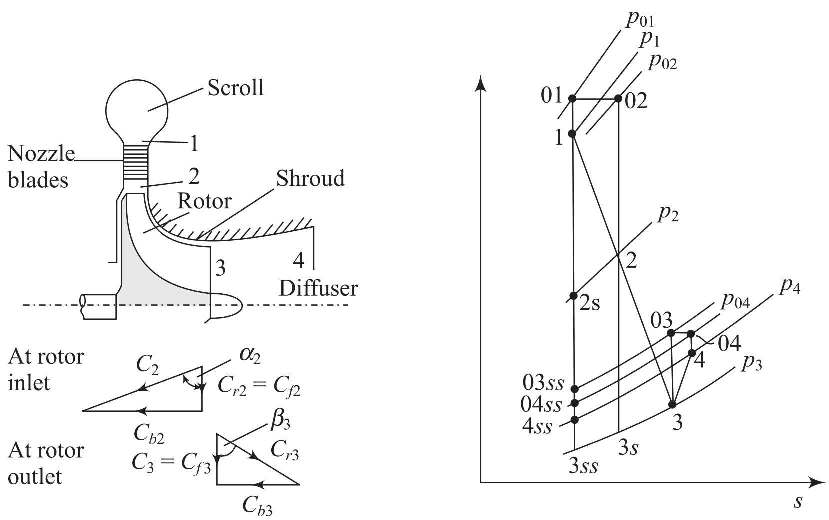


Figure 8.21 *Velocity Triangle and T-s Diagram of Inward Flow Radial Gas Turbine*

Referring to the simplified Mollier diagram, the total-to-total efficiency of the combined turbine stage and diffuser is,

$$\eta_{t-t} = \frac{h_{01} - h_{04}}{h_{01} - h_{04ss}} = \frac{h_{01} - h_{03}}{h_{01} - h_{04ss}} = C_{b2}^2/[c_p T_{01}(1 - T_{04ss}/T_{01}]$$

After transposing and substituting for the isentropic temperature ratio in terms of the pressure ratio,

$$C_{b2}^2 = \eta_{t-t} c_p T_{01}\left[1 - \left(\frac{p_{04}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right] = 0.9 \times 1149 \times 1140\left[1 - \left(\frac{100}{400}\right)^{0.249}\right]$$

$$= 1.179 \times 10^6 (1 - 0.7073) = 0.3451 \times 10^6$$

Hence, the blade tip speed is,

$$C_{b2} = 587.4 \text{ m/s}$$

At nozzle exit, the absolute flow Mach number is,

$$M_2 = \frac{C_2}{a_2} = \frac{C_{b2} \operatorname{cosec} \alpha_2}{a_2} \tag{1}$$

In Eq. (1), the values of both a_2 and α_2 are unknown and another condition must be used to solve the flow angle α_2. Across the nozzle, the stagnation enthalpy remains constant, i.e.

$$h_{01} = h_{02}$$

$$c_p T_{01} = c_p T_2 + \frac{1}{2} C_2^2 = c_p T_2 + \frac{1}{2} C_{b2}^2 \operatorname{cosec}^2 \alpha_2$$

After rearranging,

$$\frac{T_2}{T_{01}} = 1 - \frac{\frac{1}{2} C_{b2}^2 \operatorname{cosec}^2 \alpha_2}{c_p T_{01}} = 1 - \frac{1}{2}(\gamma - 1)\left(\frac{C_{b2}}{a_{01}}\right)^2 \operatorname{cosec}^2 \alpha_2 \tag{2}$$

where $C_p T_{01} = \gamma R T_{01}/(\gamma - 1) = a_{01}^2/(\gamma - 1)$.

Using Eq. (1),

$$\frac{T_2}{T_{01}} = (a_2/a_{01})^2 = \frac{C_{b2}^2 \operatorname{cosec}^2 \alpha_2}{(M_2 a_{o1})^2} \tag{3}$$

Combining Eqs (1), (2) and (3) and rearranging,
Substituting the values,

$$a_{01} = (\gamma R T_{01})^{1/2} = (1.33 \times 287 \times 1140)^{1/2} = 660.4 \text{ m/s} \tag{4}$$

$$\sin \alpha_2 = (C_{b2}/a_{01})\left[\frac{1}{2}(\gamma - 1) + 1/M_2^2\right]^{1/2} \tag{5}$$

$\gamma = 1.333$ (given) and

$M_2 = 1$ for choked condition

Therefore, $\sin \alpha_2 = (587.4/660.4)(1 + 0.1665)^{1/2} = 0 - 0.9607$

Hence, the nozzle flow outlet angle is $\alpha_2 = 73.88°$ (6)

8.9 Application of Specific Speed

The concept of specific speed was applied almost exclusively to incompressible flow machines as an important parameter in the selection of the optimum type and size of unit. The volume flow rate through hydraulic machines remains constant. But in radial flow gas turbine, volume flow rate changes significantly, and this change must be taken into account.

Using dimensionless form of specific speed,

$$N_{sh} = N(Q_3)^{1/2}/(\Delta h_{0,s})^{3/4} \tag{8.152}$$

where N is in rev/s, Q_3 in in m^3/s and isentropic total-to-total enthalpy drop (from turbine inlet to outlet) is in J/kg. For the radial inflow radial turbine,

$$C_{b2} = \pi N D_2 \text{ and } \Delta h_{0s} = \frac{1}{2}C_0^2$$

$$N_{sh} = \left\{ Q_3^{1/2} / \left[\left(\frac{1}{2}\right) C_0^2 \right] \right\}^{3/4} (C_{b2}/\pi D_2)(C_{b2}/\pi N D_2)^{1/2}$$

$$N_{sh} = (\sqrt{2}/\pi)^{3/2} (C_{b2}/C_o)^{3/2} (Q_3/ND_2^3)^{1/2} \tag{8.153}$$

For inward flow radial turbine, $\dfrac{C_{b2}}{C_0} = 0.707$, substituting this value in Eq. (8.153), we get,

$$N_{sh} = 0.18(Q_3/ND_2^3)^{1/2} \text{ rev} = 0.18\,\varphi^{1/2} \text{ rev.} \tag{8.154}$$

Equation 8.154 shows that specific speed is directly proportional to the square root of the volumetric flow coefficient. Assuming a uniform axial velocity at rotor exit C_3, so that $Q_3 = A_3C_3$, rotor disc area $A_d = \pi D_2^2/4$, then

$$N = \frac{C_{b2}}{\pi D_2} = \frac{C_0\sqrt{2}}{2\pi D_2} \tag{8.155}$$

$$(Q_3/ND_2^3) = \frac{A_3 C_3 2\pi D_2}{\sqrt{2} C_o D_2^2} = \left(\frac{A_3}{A_d}\right)\left(\frac{C_3}{C_0}\right)\left(\frac{\pi^2}{2\sqrt{2}}\right) \tag{8.156}$$

Using Eq. (8.156) in Eq. (8.155), we have,

$$N_{sh} = 0.336(C_3/C_0)^{1/2}(A_3/A_d)^{1/2} \text{ rev} \tag{8.157}$$

$$N_{sh} = 2.11(C_3/C_0)^{1/2}(A_3/A_d)^{1/2} \text{ rad} \tag{8.158}$$

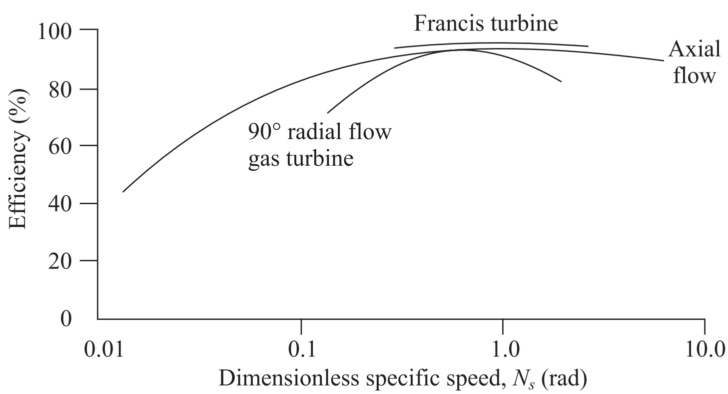


Figure 8.22 *Variation of Efficiency with Specific Speed for Different Turbines*

Suggested values for C_3/C_0 and A_3/A_d are as follows:

$0{:}04 < C_3/C_0 < 0.3$

$0.1 < A_3/A_d < 0.5$

Then, $0.3 < Ns < 1.1.$

Thus, the N_{sh} range is very small and Figure 8.22 shows the variation of efficiency with N_{sh}, where it is seen to match the axial flow gas turbine over the limited range of N_{sh}.

EXAMPLE 8.7 The following particulars relate to a small inward flow radial gas turbine:

Rotor inlet tip diameter = 92 mm
Rotor outlet tip diameter = 64 mm
Rotor outlet hub diameter = 26 mm
Ratio $C_3/C_0 = 0.447$
Ratio C_{b2}/C_0 (ideal) = 0.707
Blade rotational speed = 30500 rpm
Density at impeller exit = 1.75 kg/m^3

Determine:

(a) The dimensionless specific speed of the turbine.
(b) The volume flow rate at impeller outlet.
(c) The power developed by the turbine.

Solution

(a) Dimensionless Specific Speed

Now,

$$A_3 = \frac{\pi(D_{3t}^2 - D_{3h}^2)}{4} = \frac{\pi(0.064^2 - 0.026^2)}{4} = 2.73 \times 10^{-3}\ \text{m}^2 \quad (1)$$

$$A_d = \frac{\pi(D_2^2)}{4} = \frac{\pi(0.092^2)}{4} = 6.65 \times 10^{-3}\ \text{m}^2 \quad (2)$$

$$N_{sh} = 0.336(C_3/C_0)^{1/2}(A_3/A_d)^{1/2}\ \text{rev} \quad (3)$$

Dimensionless specific speed,

$$N_{sh} = 0.336(0.447 \times 2.73/6.65)^{1/2} = 0.144\ \text{rev} = 0.904\ \text{rad} \quad (4)$$

(b) The Volume Flow Rate at Outlet

$$N_{sh} = 0.18(Q_3/ND_2^3)^{1/2} \quad (5)$$

$$0.144 = 0.18\left(Q_3 \times \frac{60}{30{,}500} \times 0.092^3\right)^{1/2}$$

$$Q_3 = 0.253\ \text{m}^3/\text{s} \quad (6)$$

(c) The Power Developed by the Turbine

$$P_t = W_t = \dot{m}C_{b3}^2 = \rho_3 Q_3 C_{b3}^2 = 1.75 \times 0.253 \times (\pi D_2 N/60)^2$$

$$P_t = W_t = 1.75 \times 0.253 \times (\pi \times 0.092 \times 30{,}500/60)^2 = 9.565\ \text{kW} \quad (7)$$

SUMMARY

- The following expressions and T-s diagram for different thermodynamic cycles are presented:

  Simple Joule-Brayton cycle, actual Joule-Brayton cycle, heat exchanger cycle, cycle with intercooling and combination cycles, and aircraft propulsion cycle

- Expressions for different stage parameters for an axial and radial turbine are summarized in the following table:

Stage Parameters	***Axial Turbine***	***Radial Turbine***
Stage work	$w = C_b C_f(\tan\alpha_2 + \tan\alpha_3)$ $w = C_b C_f(\tan\beta_2 + \tan\beta_3)$	$w = C_{b2}^2$
Stage pressure ratio	$\dfrac{p_{03}}{p_{01}} = \left[1 - \dfrac{\Delta T_0, \text{stage}}{\eta_{\text{stage}} T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$	$\dfrac{p_{03}}{p_{01}} = \left[1 - \dfrac{C_0^2}{2c_p T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$ C_0 = velocity equivalent of the enthalpy drop
Stage efficiency	$\eta_{\text{stage}} = (T_{01} - T_{03})/(T_{01} - T_{03s})$	$\eta_{t\text{-}t} = (T_{01} - T_{03})/(T_{01} - T_{03s})$
Blade loading coefficient	$\Psi = 2\varphi(\tan\beta_2 + \tan\beta_3)$	$\Psi = 2$
Degree of reaction	$R = \left(\dfrac{1}{2}\right) \times \varphi \times (\tan\beta_3 - \tan\beta_2)$	$R = 1/2$

- Performance characteristics of an axial and radial turbine are described.

REVIEW QUESTIONS

8.1 State the assumptions made in the analysis of ideal Joule-Brayton (JB) cycle for gas turbine.

8.2 Draw the schematic $p - v$ and $T - s$ diagrams of simple Joule-Brayton cycle of gas turbine and briefly explain its working.

8.3 Derive an expression for specific work output and efficiency of simple gas turbine cycle in terms of pressure ratio and temperature ratio.

8.4 Derive an expression for optimum pressure ratio for maximum work output from an ideal Joule-Brayton cycle in terms of ratio of maximum cycle temperature to minimum cycle temperature and ratio of specific heats.

8.5 Show that the specific work output is maximum when the pressure ratio is such that the exit temperature of compressor is equal to the exit temperature of turbine.

8.6 How the actual Joule-Brayton cycle differs from the ideal Joule-Brayton cycle of a gas turbine?

8.7 Prove that the specific work output of actual Joule-Brayton gas turbine cycle is given by,

$$w = \frac{c_p T_1 \{[\eta_t \eta_c (T_3/T_1)(1 - 1/r^{(r-1)/\gamma}) - (r^{((r-1)/\gamma)})]\}}{\eta_c}$$

where T_1 and T_3 are the minimum and maximum cycle temperatures, η_t is the isentropic turbine efficiency, η_c is the isentropic compressor efficiency and r is the pressure ratio.

8.8 Derive an expression for efficiency of the actual Joule-Brayton gas turbine cycle in terms of isentropic turbine efficiency, isentropic compressor efficiency, ratio of maximum cycle temperature to minimum cycle temperature and pressure ratio.

8.9 Prove that the optimum pressure ratio for specific work output of an actual Joule-Brayton cycle is given by,

$$r_{\text{opt}} = [\eta_t \eta_c (T_3/T_1)]^{\gamma/2(\gamma-1)}$$

8.10 Derive an expression for optimum pressure ratio for efficiency of an actual gas turbine cycle in terms of isentropic efficiencies of turbine and compressor, maximum and minimum cycle temperatures.

8.11 Why a regenerative heat exchanger is incorporated between the exhaust gas and compressed air?

8.12 Draw the schematic and $T-s$ diagrams of an ideal gas turbine cycle with regenerative heat exchanger.

8.13 Prove that the efficiency of an ideal Joule-Brayton cycle with a heat exchanger of unity effectiveness is, $\eta = 1 - (T_1/T_3)r^{(\gamma-1)/\gamma}$

8.14 Derive an expression for specific work output of an ideal Joule-Brayton cycle with a heat exchanger of effectiveness 1 and find the optimum pressure ratio.

8.15 Derive an expression for efficiency of an ideal Joule-Brayton cycle with a heat exchanger of effectiveness less than unity.

8.16 Derive an expression for efficiency of an actual Joule-Brayton cycle with a heat exchanger in terms of isentropic efficiencies of turbine and compressor, ratio of maximum to minimum temperature, pressure ratio and effectiveness of heat exchanger.

8.17 Why expansion is carried out in multistages in a gas turbine with reheat between two stages?

8.18 Obtain the condition for maximum specific work output of ideal Joule-Brayton cycle with reheating.

8.19 Derive an expression for maximum specific work output and maximum efficiency of ideal Joule-Brayton cycle with reheating.

8.20 Draw the schematic and $T-s$ diagrams for an ideal gas turbine cycle with intercooling between the two stages of compression and obtain an expression for the maximum efficiency.

8.21 Explain the working of an ideal turbojet engine cycle and also derive an expression for the thrust developed. State the assumptions made in the analysis.

8.22 Show the real turbojet cycle on the $T-s$ diagram. What are the major differences between the ideal and real turbojet engine cycle?

8.23 Draw the velocity triangles and $T-s$ diagram for an axial gas turbine.

8.24 Draw the velocity triangles and $T-s$ diagram for a radial gas turbine.

8.25 Prove that the specific stage work for an axial gas turbine is,

$$w = c_p(\Delta T_0)_{\text{stage}} = C_b C_f(\tan \beta_2 + \tan \beta_3)$$

where β_2 and β_3 are the blade angles at the rotor inlet and outlet, respectively.

8.26 Prove that the stage pressure ratio of an axial gas turbine stage is expressed by,

$$\frac{p_{03}}{p_{01}} = \left[1 - \frac{(\Delta T_0)_{\text{stage}}}{\eta_{\text{stage}} T_{01}}\right]^{\frac{\gamma}{\gamma-1}}$$

8.27 What is the total-to-total efficiency of an axial gas turbine stage?

8.28 Define the total-to-static efficiency of an axial gas turbine stage. What is its significance?

8.29 Prove that the blade loading or temperature drop coefficient is given by,

$$\psi = 2\varphi(\tan \beta_2 + \tan \beta_3)$$

where φ, β_2 and β_3 are the flow coefficient, blade angle at the rotor inlet, and blade angle at the outlet, respectively.

8.30 Prove that the degree of reaction for an axial gas turbine is expressed by,

$$R = \frac{1}{2}\varphi(\tan \beta_3 - \tan \beta_2)$$

8.31 Express the relative and absolute gas angles at the inlet and outlet of an axial gas turbine stage in terms of flow coefficient, blade loading or temperature drop coefficient and degree of reaction.

8.32 Draw and discuss the temperature drop coefficient versus flow coefficient of a 50% reaction stage of an axial gas turbine for various nozzle outlet angles, α_2, stage outlet swirl angles, α_3, on blade coefficient basis and for various stage efficiencies.

8.33 Prove that the stage efficiency of an axial turbine in terms of ratio of blade velocity to the isentropic gas velocity, ρ, and blade loading or temperature drop coefficient, ψ, is given by,

$$\eta_{\text{stage}} = \eta_{t-t} = \rho^2 \Psi$$

8.34 Draw and discuss the variation of stage efficiency versus blade to speed ratio for single stage impulse, double stage impulse and 50% reaction stage of an axial gas turbine.

8.35 Prove that the specific work for a radial gas turbine stage, if the relative velocity at rotor tip is radial and absolute velocity at exit is axial, is given by,

$$w = C_{b2}^2$$

8.36 Obtain an expression for stage pressure ratio of a radial gas turbine stage for an ideal isentropic turbine with perfect diffuser.

8.37 What is the total-to-total efficiency of a radial gas turbine considering turbine and diffuser as the system? What would be total-to-total efficiency if turbine alone is considered as a system?

8.38 Show that the blade loading or temperature drop coefficient for a radial gas turbine stage is two times the blade peripheral velocity.

8.39 Prove that the degree of reaction in a radial gas turbine is given by,

$$R = 1 - \frac{1}{2}\left(\frac{C_{w2}}{C_{b2}}\right) = \frac{1}{2}(1 - \varphi_2 \cot \beta_2)$$

8.40 Prove that the degree of reaction for a radial gas turbine stage is 1/2 if the relative velocity at the rotor tip is radial and absolute velocity at the exit is axial.

8.41 Draw and discuss the variation of degree of reaction characteristics with flow coefficient for different air angles at the inlet of a radial gas turbine.

8.42 Prove that the total-to-total efficiency of a radial gas turbine is given by,

$$\eta_{t-t} = \frac{\psi C_b^2}{2c_p T_{01}\left[1 - (r_0)^{\frac{\gamma-1}{\gamma}}\right]} = \rho^2 \psi$$

where Ψ is the loading or temperature drop coefficient and r_0 is the stagnation pressure ratio of the stage, $\rho = \dfrac{C_{b2}}{C_0}$, and C_0 is the velocity equivalent of the enthalpy drop in the stage.

8.43 Prove that the total-to-static efficiency of a radial gas turbine stage is given by,

$$\eta_{t-t} = \frac{\psi C_b^2}{2c_p T_{01}\left[1-(r)^{\frac{\gamma-1}{\gamma}}\right]}$$

where r is the static pressure ratio.

8.44 Draw and discuss the variation of loading coefficient characteristics with flow coefficient for different inlet air angles in a radial gas turbine.

8.45 Draw and discuss the variation of stage efficiency characteristics with ratio of blade-to-gas at the exit for different nozzle exit air angles in a radial gas turbine.

8.46 What are the Mach number limitations in a gas turbine?

8.47 Derive an expression for non dimensional specific speed for radial gas turbines.

8.48 Draw and discuss the efficiency versus dimensionless specific speed characteristics for different gas turbines.

PROBLEMS

8.1 An ideal gas turbine cycle is working between the temperature limits of 350 K and 2000 K. The pressure ratio of the cycle is 1.3. The ambient pressure is 1 bar and air flow rate through the plant is 14400 m^3/min. Calculate the cycle efficiency. Take c_p = 1.005 kJ/kg – K.

[*Ans:* η = 7.23%, $\eta = f(r)$, $\eta \neq f(\theta)$]

8.2 The work ratio of an ideal Joule-Brayton cycle is 0.56 and efficiency is 35%. The temperature of the air at compressor inlet is 290 K. Determine (a) the pressure ratio, and (b) temperature drop across the turbine. [*Ans:* (a) r = 4.52, (b) $(\Delta T)_t$ = 356 K or °C]

8.3 An ideal Joule-Brayton gas turbine cycle is working between the temperature limits of 300 K and 1050 K. Determine (a) the pressure ratio of the cycle if its efficiency is equivalent to Carnot cycle efficiency, (b) optimum pressure ratio for maximum work output, (c) the cycle efficiency corresponding to maximum work, and (d) maximum specific work output.

[*Ans:* (a) $(r)_{\text{Carnot eff}}$ =80.2, (b) r_{opt} = 8.94 (c) $\eta_{\text{max work}}$ = 46.52%, (d) w = 228.64 kJ/kg]

8.4 An ideal Joule Brayton gas turbine cycle having pressure ratio of 7.5 is working between the temperature limits of 27°C and 727°C. The pressure at the inlet of compressor is 1 bar and the flow rate of air is 8.5 m^3/s. Calculate (a) the power developed, (b) cycle efficiency, and (c) the change in the work ouput and cycle efficiency in percentage, if perfect intercooling is used.

[*Ans:* (a) P = 1895.5 kW, (b) η = 43.8%, (c) Change in power = +18.6%,
Change in Efficiency = –8.68%]

8.5 The mass flow rate through an ideal gas turbine cycle is 12.8 kg/s and the pressure ratio is 9. The temperature limits of the cycle is 290 K and 1200 K. Calculate (a) power output, and cycle efficiency (b) the percentage change in power output and cycle efficiency if a perfect reheater is used, and (c) the heat added in the reheater per minute.

[*Ans:* (a) $P = 3943.2$ kW, $\eta = 47\%$ (b) Increase in $P = 28.75\%$, Decrease in $\eta = 66.596\%$ (c) $Q_{rh} = 248832$ kJ/min]

8.6 An ideal gas turbine power plant of pressure ratio 8 is designed to produce 5 MW power. The state of the air at inlet is 1 bar, 27° and maximum temperature of the cycle is 727°C. Calculate (a) mass flow rate of the air and cycle efficiency, (b) percentage reduction in mass flow rate and efficiency if the cycle is modified by incorporating heat exchanger and reheater.

[*Ans:* (a) $m = 24.424$ kg/s, $\eta = 44.8\%$, (b) Decrease in $m = 24.39\%$, $\eta_{\text{modified}} = 47.5\%$]

8.7 An ideal gas turbine cycle of pressure ratio 6.5 develops 3 MW power. The maximum temperature in the cycle does not exceed 700°C and the inlet conditions are 1 bar, 10°C. Determine (a) mass flow rate of air and cycle efficiency, and (b) power generated, percentage increase in power generated and percentage increase in cycle efficiency for the same mass flow rate if the cycle is modified by perfect intercooling, regeneration and reheating.

[*Ans:* (a) $m = 14.732$ kg/s, $\eta = 41.2\%$, (b) $P = 4190$ kW, Increase in Power = 39.67%, Increase in $\eta = 50.485\%$]

8.8 An ideal Joule-Brayton working between 1 bar, 300 K and 5 bar, 1250 K has two stages of compression with perfect intercooling and two stages of expansion. The output of the first stage expansion is being used to drive the two compressors where the intermediate pressure is optimized for the compressor. The exhaust of the first stage turbine is reheated to 1250 K and expanded in the second stage. Calculate (a) the specific work output, (b) cycle efficiency without and with perfect regeneration, and (c) percentage increase in the cycle efficiency due to regeneration.

[*Ans:* (a) $w = 350.8$ kJ/kg, (b) $\eta = 33.96\%$, $\eta_{\text{modified}} = 69.2\%$, (c) Increase in cycle efficiency = 50.9%]

8.9 A Joule-Brayton cycle is working between temperature range of 30°C and 800°C. The isentropic efficiencies of the compressor and turbine are 80% and 90%, respectively. Assuming air is used as working medium, find the pressure ratio for (a) maximum specific work output, and (b) maximum efficiency. [*Ans:* (a) $(r)_{\text{opt-sp work}} = 5.15$, $(r)_{\text{opt-max eff}} = 25.5$]

8.10 The air is taken in at a pressure of 1 bar, 303 K in a gas turbine of pressure ratio 6. The temperature of the gases after the combustion of fuel of calorific value 43100 kJ/kg in the combustion chamber is 700°C. The flow rate of air is 80 kg/min and the effect of fuel in the mass flow rate is negligible. The isentropic efficiencies of the compressor and turbine are 87% and 85%, respectively. Determine (a) air-fuel ratio, (b) the final temperature of the exhaust gases, (c) power output, and (c) cycle efficiency. Take $c_p = 1.005$ kJ/kg – K, $\gamma = 1.4$ for air and $c_p = 1.147$/kg – K, $\gamma = 1.33$ for gases.

[*Ans:* (a) $A : F = 86 : 1$, (b) $T_4 = 676.2$ K (c) $P = 142$ kW, (d) $\eta = 21.25\%$]

8.11 The air at initial temperature of 280 K is compressed from 1 bar to 4 bar with isentropic compression efficiency of 82% in a gas turbine power plant. The maximum temperature of the cycle is 800°C. The calorific value of the fuel used is 43100 kJ/kg and heat losses are 15% of the heat librerated by combustion. The isentropic efficiency of the turbine is 85%. Determine (a) specific work output, (b) work ratio, (c) cycle efficiency, (d) air-fuel ratio, and (e) specific fuel consumption.

Assume for air c_p = 1.005 kJ/kg-K, γ= 1.4, For gases c_p = 1.147 kJ/kg-K, γ= 1.33

[*Ans:* (a) w = 137.7 kJ/kg, (b) Work ratio = 0.4522, (c) η = 16.3%, (d) $A : F$ = 49.94 : 1 (e) $\dot{m}_{sp}$ = 0.524 kg/kW-hr]

8.12 A gas turbine working against a pressure ratio of 11.3137 produces zero net work output when heat added is 476.354 kJ/kg. The total temperature at inlet of compressor is 27°C and total isentropic efficiency of the turbine is 71%. Calculate the maximum to minimum temperature ratio of the cycle and the total isentropic efficiency of the compressor. Assume c_p = 1.005 kJ/kg-K, γ= 1.4, for the whole cycle. $\left[Ans: \frac{T_{03}}{T_{01}} = 4, \eta_c = 70.4\%\right]$

8.13 The work ratio of a gas turbine plant is 0.0544 and the heat supplied in the combustion chamber is 476.354 kJ/kg. The stagnation temperature of air at the inlet is 300 K. The isentropic efficiency of the compressor and turbine are 70.42% and 70%, respectively. Determine the ratio of maximum to minium total temperature and the suitable pressure ratio. [*Ans:* T_{03}/T_{01} = 2.934, r = 3.144]

8.14 The work ratio is 0.3 of a practical gas turbine cycle having isentropic efficiencies of compressor and turbine as η_c and η_t, respectively. The pressure ratio and temperature ratio are 12 and 4, respectively. Calculate (a) the minimum temperature ratio required to drive the compressor, and (b) the efficiency of the compressor under ideal conditions.

[*Ans:* (a) Minimum Temperature Ratio = 2.8 (b) η_{ideal} = 50.8%]

8.15 The polytropic efficiency of the compressor in a gas turbine plant is 85%. The ideal exit temperature of the compressor is twice that of inlet. Calculate (a) the isentropic efficiency of the compressor, if the polytropic efficiency of the turbine is same as that of compressor, and (b) isentropic efficiency of the turbine assuming there is no pressure loss. Take γ = 1.33 for gases

[*Ans:* (a) η_c = 79.4%, (b) η_t = 88.5%]

8.16 The isentropic efficiency of a compressor is 85% in a gas turbine plant having pressure ratio of 4. Calculate the corresponding polytropic efficiency of the compressor. [*Ans:* η_{cp} = 87.7%]

8.17 The stagnation pressure, static pressure and temperature at inlet of a compressor in a gas turbine plant are 1 bar, 0.93 bar and 10°C, respectively. The total head pressure and temperature at the exit of compressor are 6 bar and 230°C, respectively. The total temperature at the exit of the turbine is 733 K and turbine power output is 5.1 MW. Calculate (a) the total-to-total efficiency of the compressor, (b) air rate if area at inlet of compressor is 0.10 m^2, and (c) the temperature of the gases at inlet of the turbine.

It may be assumed that there are no losses at compressor entry and the velocity distribution is uniform. The increase of mass flow due to fuel addition is negligible. Take c_p = 1.005 kJ/kg-K, γ= 1.4 for air and c_p = 1.147 kJ/kg-K, γ= 1.33 for gases.

[Ans: (a) $(\eta_c)_{t-t}$ = 90.35%, (b) $\dot{m}_a$ = 12.51 kg/s, (c) T_{03} = 1088.43 K]

8.18 A gas turbine cycle equipped with a regenerator of effeciency 0.75 has pressure ratio of 7. The air enters the compressor at 1 bar, 54°C with mass flow rate of 20 kg/s and maximum cycle temperature of the cycle is 1000 K. The isentropic efficiencies of the compressor and turbine are 82% and 85%, respectively. The combustion efficiency is 97% and calorific value of the fuel used is 43100 kJ/kg. The mechanical efficiency is 99% and the regenerator gas side pressure loss is 0.1 bar. Calculate

(a) the power output, specific fuel consumption and cycle efficiency assuming that the pressure losses in the air side of the regenerator and combustion chamber are accounted for in the compressor efficiency, and (b) compare the results with those determined for the same cycle without regenerator and without pressure losses. The effect of fuel mass addition in the heat balance is to be neglected but to be considered in turbine calculations.

[Ans: (a) $P = 1222.96$ kW. $\dot{m}_{sp} = 0.53$ kg/kW-hr, $\eta = 15.8\%$,
(b) $P = 1522.37$ kW, $\dot{m}_{sp} = 0.49$ kg/kW-hr, $\eta = 17.06\%$]

8.19 A gas turbine plant is employing a separate power turbine, regenerator and intercooler between two stages of compression. The pressure ratio and isentropic efficiency of compression of each stage is 3 and 80%, respectively. The ambient pressure and temperature are 1 bar and 54°C. The temperature after intercooler is 24°C and maximum temperature at turbine inlet is 1000 K. The isentropic efficiency of compressor turbine as well as power turbine is 88%. The transmission efficiency of turbine to compressor is 98%. The regenerator effectiveness is 0.8 and gas side pressure loss in the regenerator is 0.1 bar. The mass flow rate of air is 15 kg/s and calorific value of the fuel is 43100 kJ/kg. Assume that the pressure loss in the air side of the renerator and combustion chamber are accounted for in the compressor efficiency. Calculate (a) the power output, (b) the specific fuel consumption, and (c) the overall thermal efficiency.

Take $c_p = 1.005$ kJ/kg-K, $\gamma = 1.4$ for air and $c_p = 1.147$ kJ/kg-K, $\gamma = 1.33$ for gases

[*Ans:* (a) $P = 1811.69$ kW, (b) $\dot{m}_{sp} = 0.32$ kg/kW-hr, (c) $\eta = 26.1\%$]

8.20 An open cycle gas turbine plant of generation capacity 1850 kW is equipped with an intercooler, a reheater and a renererator. The high pressure turbine drives the low pressure compressor on one shaft. On another shaft, the low pressure turbine drives the high pressure compressor and the load. The ambient state is 1 bar, 300 K and the pressure ratio in each stage of compression is 2.5. The isentropic efficiencies of the turbine and compressor are same as 80%. The pressure drop in each heater and each side of regenerator is 3% and effectiveness of the regenerator is 75%. Assume that intercooling is done to ambient temperature and reheating to maximum cycle temperature. Maximum cycle temperature is 993 K, the combustion efficiency is 98% and the calorific value of the fuel used is 43100 kJ/kg. Calculate (a) the specific work of each turbine and compressor, (b) the air rate and thermal efficiency, and (c) fuel consumption per hour at rated load and specific fuel consumption. Mass of the fuel and mechanical efficiencies may be neglected

[Ans: (a) $(w_c)_{HP} = (w_c)_{LP} = 112.86$ kJ/kg, $(w_t)_{LP} = (w_c)_{HP} = 231$ kJ/kg,
(b) $\dot{m}_a = 15.7$ kg/s, $\eta = 26.1\%$, (c) $\dot{m}_f = 605.65$ kg/hr, $\dot{m}_{sp} = 0.327$ kg/kW-hr]

8.21 A gas turbine consists of two compresors and two turbines mounted on the same shaft. The pressure and temperature at the inlet of first stage compressor are 2 bar and 25°C. The maximum cycle pressure and temperature are limited to 8 bar and 850°C. A perfect intercooler is used between the two compressors. A reheater is used between the two turbines in which the gases are reheated to 850°C before entering into the low pressure turbine. The isentropic efficiencies of the turbine and compressors are same as 83%. Calculate (a) the cycle efficiency without regenerator, (b) cycle efficiency with regenerator of effectiveness 0.65, and (c) air rate if power developed is 310 kW.

Assume $\gamma = 1.4$ and $c_p = 1$ kJ/kg-K [*Ans:* (a) $\eta = 20.2\%$, (b) $\eta = 34.3\%$, (c) $\dot{m}_a = 1.66$ kg/s]

8.22 The compressor of a gas turbine plant is driven by the high pressure turbine of a two stage turbine and compresses 5 kg/s of air from 1 bar to 5 bar with an isentropic efficiency of 85%. The high pressure turbine stage has 675°C temperature at inlet with an isentropic efficiency of 87%. The isentropic efficiency of the low pressure turbine which is independent mechanically is 82%. The expansion pressure ratios of the two turbines are not equal. The low pressure turbine exhaust passes through a heat exchanger which transfers 70% of the available heat in cooling the exhaust gases to raise the temperature of delivery of the compressor. The pressure and temperature at the inlet of compressor are 1 bar and 288 K. Calculate (a) the total intermediate temperature and pressure between the two stages of expansion, (b) the power output of the low pressure stage, and (c) the overall efficiency. Assuming that working fluid is air throughout of constant specific heats and there are no pressure losses. [*Ans:* (a) T_{04} = 749.94 K, p_{04} = 1.91 bar, (b) $(P_t)_{LP}$ = 522.10 kW, (c) η = 29.69%]

8.23 The air enters at 1 bar, 288 K in a propulsion cycle. The pressure at the exit of the compressor is 5 bar and the maximum temperature is 900°C. The air expands in the turbine to such a pressure that the turbine work is just equal the compressor work. On leaving the turbine, the air expands isentropically in a nozzle to 1 bar. Calculate the velocity of air leaving the nozzle. [*Ans:* C_5 = 728.4 m/s]

8.24 A turbojet engine consists of a single stage compressor, single stage turbine and a nozzle. The pressure and temperature at the inlet of compressor are 0.8 bar and 280 K. The discharge pressure of the compressor is 4 bar and maximum temperature of the cycle is 550°C. The static back pressure of the nozzle is 0.6 bar and aircraft speed is 720 km/hr. The isentropic efficiencies of the compressor and turbine are 80% and 85%, respectively. The isentropic nozzle efficiency is 90%. The combustion efficiency is 98% and mechanical efficiency is 95%. The calorific value of the fuel used is 42000 kJ/kg. Determine (a) the air fuel ratio, (b) the minimum power required to drive the compressor, and (c) the thrust developed.

Take c_p = 1 kJ/kg-K, γ= 1.4 for air and c_p = 1.2 kJ/kg-K, γ= 1.35 for gases.

[*Ans:* (a) $A : F$ = 100.5 : 1, (b) P_c = 4100 kW, (c) F = 4.828 kN]

8.25 The maximum temperature at the inlet of turbine is 1100 K in a simple turbo-jet engine having pressure ratio of 4 and mass flow rate of 22.7 kg/s at design conditions. The state of the air at inlet of the compressor is 1.013 bar, 15°C. The lower calorific value of the fuel used is 43125 kJ/kg-K and pressure loss in the combustion chamber is 0.21 bar. The isentropic efficiencies of the compressor and turbine are 85% and 90%, respectively. The propelling nozzle efficiency is 95% and transmission efficiency is 99%. Calculate the design thrust and specific fuel consumption when the unit is stationary at sea level. Take c_p = 1.0035 kJ/kg-K, γ= 1.4 for air and c_p = 1.147 kJ/kg-K, γ= 1.33 for gases.

[*Ans:* F = 13.086 kN/s, $\dot{m}_{sp}$ = 0.1097 kg/N-hr]

8.26 The gas enters the turbine at 4.5 bar and 800°C and exhaust at 1.75 bar in a turbojet engine cycle. The turbine absorbs 75% of the available enthalpy drop. The exhaust gas from the turbine expands in a nozzle to 1.03 bar. There is no heat loss and conversion of the kinetic energy is 100% of the available isentropic enthalpy drop. The velocity of the gas entering the turbine and nozzle may be assumed negligible. Calculate (a) the temperature of the gas entering the nozzle, and (b) the velocity of the gas at the exit of the nozzle. Assume c_p = 1.05 kJ/kg-K and γ= 1.38.

[*Ans:* (a) T_{04} = 888.92 K, (b) C_5 = 503.2 m/s]

8.27 A turbojet aircraft is flying at an altitude of 6.1 km with a speed of 805 km/hr where the ambient conditions are 0.458 bar and 248 K. The pressure ratio and isentropic efficiency of the compressor are 4 and 85%, respectively. The lower calorific value of the fuel used is 43000 kJ/kg and pressure loss in the combustion chamber is 0.21 bar. The temperature at the turbine inlet is 827°C, intake duct efficiency is 95% and isentropic efficiency of the turbine is 90%. The mechanical transmission efficiency is 99%, nozzle efficiency is 95% and exit area of the convergent nozzle used is 0.0935 m^2. Take c_p = 1.005 kJ/kg-K, γ = 1.4 for air and c_p = 1.147 kJ/kg-K, γ = 1.33 for gases. Determine (a) the thrust, and (b) specific fuel consumption. [*Ans:* (a) F = 6745.17 N, (b) $\dot{m}$ = 0.140 kg/N-hr]

8.28 The hub and tip diameters of a rotor of an axial gas turbine stage running at 6000 rpm are 0.45 m and 0.75 m, respectively. The air angle at nozzle exit is 75°. The air angle at rotor entry and rotor exit are 45° and 76°, respectively. Assume that the radial equilibrium and free vortex flow exist in the stage. Determine at the hub, mean and tip sections (a) the relative and absolute air angles, (b) degree of reaction, (c) blade to gas speed ratio, (d) specific work, and (e) the loading coefficient.

[*Ans:* At Mean Section: (a) Air angles-already given, (b) R_m = 0.551, (c) ρ_m = 0.707, w = 65.157 kJ/kg, ψ_m = 1.833, At Hub: (a) α_{2h} = 78.63°, β_{2h} = 71.13°, β_{3h} = 75.08°, (b) R_h = 0.202, (c) ρ_h = 0.404, (d) w = 65.16 kJ/kg, ψ_h = 3.260, At Tip: (a) α_{2t} = 71.48°, β_{2t} = –23.36°, β_{3t} = 77.3°, (b) R_t = 0.7126, (c) ρ_t = 1.085, (d) w = 65.17 kJ/kg, (e) ψ_t = 1.174]

8.29 A 50% reaction stage of a gas turbine is running at 12000 rpm. The pressure and temperature at inlet are 10 bar and 1500 K. Mass flow rate of the gas is 70 kg/s. Stage pressure ratio is 2 and stage efficiency is 87%. The fixed and moving blade exit air angles are same at 60°. Assuming optimum blade to gas speed ratio, determine (a) flow coefficienct, (b) mean diameter of the stage, (c) power developed, (d) pressure ratio across the stator and rotor blade rings, (e) hub-to-tip ratio the rotor, and (f) degree of reaction at the hub and tip.

[*Ans:* (a) φ = 0.5773, (b) D = 0.6693 m, (c) P = 16.508 MW,
(d) r_{stator} = 1.387, r_{rotor} = 1.442, (e) $\dfrac{r_h}{r_t}$ = 0.768, (f) R_h = 0.336, R_t = 0.6078]

8.30 Combustion gases flow through an axial turbine stage with designed flow coefficient of 0.8 and the blade-loading coefficient of 1.7. The stage is normal with flow into the stator at angle of –21.2°. The gases leave the stator with an absolute velocity of 463 m/s. The stagnation temperature at inlet is 1200 K, and the total-to-total efficiency is 89%. Take γ = 1.33, c_p = 1.148 kJ/kg-K. Calculate (a) the amount of turning by the stator and the rotor, (b) the specific stage work and the drop in the stagnation temperature, and (c) the static pressure ratio across the stage.

[*Ans:* (a) $\alpha_2 - \alpha_3$ = 81.27°, $\beta_2 - \beta_3$ = 84.57°,
(b) w = 141.7 kJ/kg, $(\Delta T_0)_{\text{stage}}$ = 123.4 K or °C, (c) r = 1.652]

8.31 Combustion gases flow through a 50% reaction stage of an axial turbine that has a total-to-total efficiency of 91% and design flow coefficient of 0.8. The flow into the stator is at an angle of –14°, and the axial velocity is constant across the stage at 240 m/s. The stagnation temperature at inlet is 1200 K. Take γ = 1.33 and R = 0.287 kJ/kg-K. Determine (a) the flow angles for a normal stage, (b) the specific work, and (c) the total-to-static efficiency.

[*Ans:* (a) $\beta_2 = -\alpha_3$ = 14°, $\beta_3 = -\alpha_2$ = –56°, (b) w = 125.9 kJ/kg, (c) η_{t-s} = 74.5%]

8.32 A normal axial gas turbine stage of zero reaction has an axial entry. The nozzle turns the flow by 64°. Determine (a) the flow coefficient, (b) the velocity at exit of the nozzles if the axial velocity is 240 m/s, and (c) the specific stage work and the drop in the stagnation temperature. Take c_p = 1.148 kJ/kg-K [*Ans:* (a) φ = 0.9755, (b) C_2 = 547.5 m/s, (c) w = 121.1 kJ/kg, $(\Delta T_0)_{\text{stage}}$ = 105.5 K or °C]

8.33 The gases leave the stator of an axial turbine stage at an angle of 60° with a velocity of 350 m/s. The blade-loading oefficient is 1.8 and flow coefficient is 0.7. If the axial velocity is reduced by 25 m/s from its design condition, find the percentage reduction in the reaction. [*Ans:* ΔR = 3.9%]

8.34 An inward flow radial gas turbine having radial blades at inlet is running at 62000 rpm. The diameter at inlet is 126 mm. The gases enter the blades radially with a mass flow rate of 18.6 kg/min and leave without any whirl. The stagnation supply temperature is 1150 K and the pressure ratio of the turbine is $p_{01}/p_3 = 2$. The ratio of specific heats is 1.35 and the gas constant is 0.287 kJ/kg-K. Find (a) C_{b2}/C_0, where C_0 is velocity equivalent of enthalpy drop or the spouting velocity, (b) the total-to-static efficiency, and (c) the power output of the turbine.

[*Ans:* (a) $C_{b2}/C_0 = 0.632$, (b) $\eta_{t-s} = 79.9\%$, (c) $P = 51.87$ kW]

8.35 Combustion gases at stagnation pressure 390 kPa and stagnation temperature of 1150 K flow through an inward radial flow turbine to an exit pressure $p_3 = 100$ kPa. The total-to-static efficiency is 80%, and the flow leaving the stator is choked with a Mach number of unity. The relative velocity at the inlet of the rotor is radial and the velocity of whirl at exit is zero. Calculate (a) the specific work developed by the turbine, and (b) the angle of the absolute velocity at the inlet of the rotor.

Take $\gamma = 1.35$, $c_p = 1.107$ kJ/kg-K, R = 0.287 kJ/kg-K.

[*Ans:* (a) $w = 302.80$ kJ/kg, (b) $\alpha_2 = 63.3°$]

8.36 The combustion gases leave the stator of an inward flow gas turbine at an angle of 67° with a flow rate of 0.34 kg/s. The rotor blades of radius 58 mm at the inlet are radial. The shroud radius of the blade at the outlet is 45.6 mm, the hub-to-shroud radius ratio is 0.35. The relative gas angle at exit is –38° with axial direction. The relative velocity at the inlet of the rotor is radial. The power developed by the gas turbine unit is 58.2 kW while running at 64000 rpm. The stagnation temperature and pressure at the entry of the stator are 1100 K and 2.5 bar. The static enthalpy loss coefficient for the flow across the stator is 0.08. The static pressure at outlet is 1 bar. Find (a) the Mach number at the inlet of the rotor, (b) the total-to-static efficiency, and (c) the static enthalpy loss coefficient of the rotor. Take $c_p = 1.33$, $\gamma = 1.148$ kJ/kg-K. [*Ans:* (a) $M_2 = 0.675$, (b) $\eta_{t-s} = 66.2\%$, (c) $\zeta_r = 0.6399$]

8.37 The total-to-static pressure ratio of a ninety degree inward flow radial gas turbine is 3.5 while running at 16000 rpm and exit pressure is 1 bar. The stagnation temperature at inlet is 923 K and blade to isentropic speed ratio is 0.66. The diameter ratio of the rotor is 0.45 and width of the rotor at inlet is 50 mm. The air angle at exit of the nozzle is 20° and nozzle efficiency is 95%. The velocity of flow (meridional velocity) is constant throughout and the exit is axial. Assume the properties of the gas as those of air. Determine (a) the rotor diameter, (b) the blade air angle at exit, (c) the mass flow rate, (d) the diameters of the hub and tip at rotor exit, (e) the power output, and (f) the total-to-static efficiency of the stage.

[*Ans:* (a) $D_2 = 0.589$ m, (b) $\beta_3 = 38.96°$, (c) $\dot{m} = 14.21$ kg/s,
(d) $D_{3h} = 83.6$ mm, $D_{3t} = 446.4$ mm, (e) $P = 3.458$ MW, (f) $\eta_{t-s} = 87.11\%$]

8.38 An inward flow radial gas turbine with cantilever blades develops 100 kW while running at 12000 rpm. The flow coefficient is 0.4 and total-to-total efficiency is 90%. The mass flow rate of air is 60 kg/min at inlet temperature of 400 K. Determine (a) the diameters of the rotor, (b) air angles at the inlet and outlet, (c) the nozzle exit air angle, and (d) the stagnation pressure ratio across the stage.

Assume diameter of the rotor at exit is 80% of that at the inlet, no whirl at outlet and constant flow velocity throughout.

[*Ans:* (a) $D_2 = 0.3559$ m, $D_3 = 0.2847$ m, (b) $\beta_2 = 21.80°$, $\beta_3 = 26.56°$,
(c) $\alpha_2 = 11.31°$, (d) $r_0 = 3.10$]

MULTIPLE CHOICE QUESTIONS

1. Consider the following statements regarding gas turbine cycle:
   1. Regeneration increases thermal efficiency.
   2. Reheating decreases thermal efficiency.
   3. Cycle efficiency increases when maximum temperature of the cycle is increased.

   Which of these statements are correct?

   (a) 1, 2 and 3 (b) 2 and 3
   (c) 1 and 2 (d) 1 and 3

2. Figure 8.23 shows four plots, *A*, *B*, *C* and *D*, of thermal efficiency versus pressure ratio. The curve which represents a gas turbine plant using Brayton cycle without regeneration is the one labelled

   (a) *A* (b) *B*
   (c) *C* (d) *D*

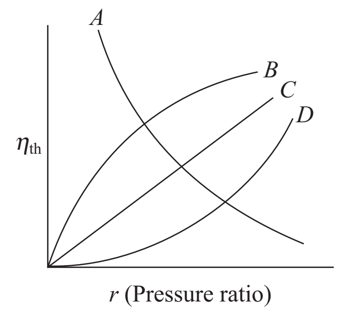


Figure 8.23 *Multiple choice question 2*

Direction: Each of the next three questions consists of two statements, one labeled as **Assertion (A)** and the other as **Reason (R).** You are to examine these two statements carefully and select the correct answers to the questions using the following codes:

(a) Both A and R are individually true and R is the correct explaination of A
(b) Both A and R are individually true but R is not the correct explaination of A
(c) A is true but R is false
(d) A is false but R is true

3. **Assertion (A):** The thermal efficiency of gas turbine plants is higher as compared to diesel plants.

   Reason (R): The mechanical efficiency of gas turbines is higher as compared to diesel engines.

4. **Assertion (A):** Gas turbines use very high air fuel ratio.

   Reason (R): The allowable maximum temperature at the turbine inlet is limited by available material considerations.

5. **Assertion (A)**: In a gas turbine, reheating is preferred over regeneration to yield a higher thermal efficiency.

   Reason (R): The thermal efficiency given by the ratio of the difference of work done by turbine (W_t) and the work required by compressor (W_c) to the heat added (Q_s) is improved by increasing W_t keeping W_c and Q_s constant in reheating, whereas in regeneration, Q_s is reduced keeping W_t and W_c constant.

6. The optimum intermediate pressure, p_t, for a gas turbine plant operating between pressure limits p_1 and p_2 with perfect intercooling between the two stages of compression with identical isentropic efficiency is given by

   (a) $p_i = p_2 - p_1$ (b) $p_i = \frac{1}{2}(p_1 + p_2)$ (c) $p_i = \sqrt{p_1 p_2}$ (d) $p_i = \sqrt{p_1^2 + p_2^2}$

7. In a gas turbine cycle, the turbine output is 600 kJ/kg, the compressor work is 400 kJ/kg and the heat supplied is 1000 kJ/kg. The thermal efficiency of this cycle is
   (a) 80% (b) 60% (c) 40% (d) 20%
8. In a single stage open cycle gas turbine, the mass flow through the turbine is higher than the mass flow through the compressor, because
   (a) The specific volume of air increases by the use of intercooler
   (b) The temperature of air increases in the reheater
   (c) The combustion of fuel takes place in the combustion chamber
   (d) The specific heats at constant pressure for incoming air and exhaust gases are different
9. Figure 8.24 shows the effect of substitution of an isothermal compression process for the isentropic compression process in the gas turbine cycle. The shaded area 1 – 5 – 2 – 1 on the $p - v$ diagram represents
   (a) Reduction in the compression work
   (b) Reduction in the specific volume
   (c) Increment in the compression work
   (d) Increment in the specific volume

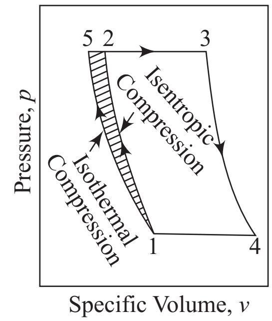


Figure 8.24 *Multiple choice question 9*

10. A gas turbine develops 120 kJ of work while the compressor absorbed 60 kJ of work and the heat supplied is 200 kJ. If a regenerator recovers 40% of the heat in exhaust gases, then the increase in overall thermal efficiency would be
    (a) 10.2% (b) 8.6% (c) 6.9% (d) 5.7%
11. Which one of the thermodynamic cycles shown in Figure 8.25 represents that of a Brayton cycle?

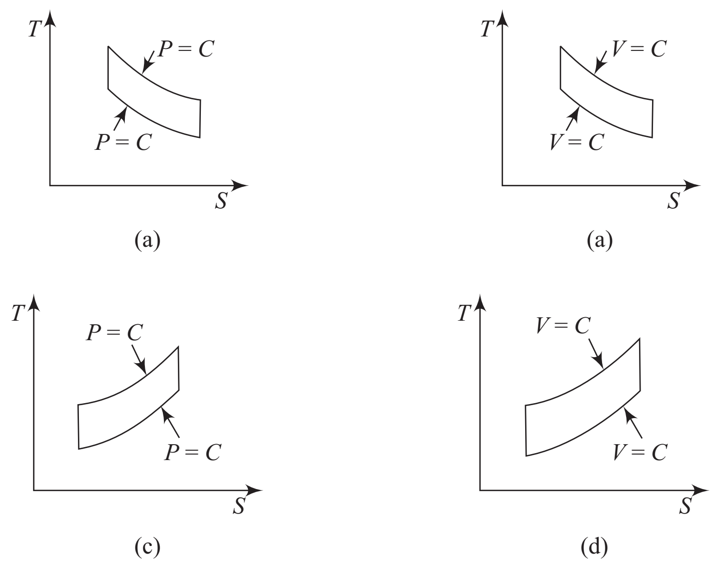


Figure 8.25 *Multiple choice question 11*

Each of the next five questions consists of two statements, one is labeled as A**ssertion (A)** and the other **as Reason (R).** You are to examine these two statements carefully and select the answers to these questions using the following codes:

(a) Both A and R are individually true and R is the correct explaination of A
(b) Both A and R are individually true but R is not the correct explaination of A
(c) A is true but R is false
(d) A is false but R is true

12. **Assertion (A):** The thermal efficiency of a gas turbine plant is low as compared to a reciprocating IC engine.

    Reason (R): In a gas turbine plant, the maximum pressure and temperature are low as compared to those of reciprocating IC engines**.**

13. **Assertion (A):** In gas turbines, regenerative heating always improves the efficiency unlike that in the case of reheating.

    Reason (R): Regenerative heating is isentropic.

14. **Assertion (A):** The thermal efficiency of Brayton cycle with regeneration decreases as the pressure ratio of the compressor increases.

    Reason (R): As the pressure ratio of compressor increases, the range of temperature in the regenerator decreases and the amount of heat recovered reduces.

15. **Assertion (A):** The thermal efficiency of Brayton cycle would not necessarily increase with reheat.

    Reason (R): Constant pressure lines on the $T - s$ diagram slightly diverge with increase in entropy.

16. **Assertion (A):** The air-fuel ratio employed in a gas turbine is approximately 60 : 1.

    Reason (R): A lean mixture of 60 : 1 in a gas turbine is mainly used for complete combustion.

17. A gas turbine works on which one of the following cycles?
    (a) Brayton (b) Rankine (c) Stirling (d) Otto

18. Reheating in a gas turbine
    (a) Increases the compressor work (b) Decreases the compressor work
    (c) Increase the turbine work (d) Decreases the turbine work

19. Consider the following statements regarding a closed gas turbine cycle:
    1. The cycle can employ mono-atomic gas like helium instead of air to increase the cycle efficiency if other conditions are same.
    2. The efficiency of heat exchanger increases with the use of helium.
    3. The turbine blades suffer higher corrosion damages.
    4. Higher output can be obtained for the same size.

    Which of these statements are correct?
    (a) 1, 2 and 3 (b) 1, 2 and 4 (c) 2, 3 and 4 (d) 1, 3 and 4

20. The efficiency of a simple gas turbine can be improved by using a regenerator because the
    (a) The work of compression is reduced (b) The heat required to be supplied is reduced
    (c) The work output of the turbine is increased (d) The heat rejected is increased

21. The thermal efficiency of a gas turbine cycle with regeneration in terms of maximum temperature T_3, minimum temperature T_1, pressure ratio r and $\gamma = c_p/c_v$ is given by

(a) $\eta = 1 - \frac{T_1}{T_3} r^{\gamma/(\gamma-1)}$ (b) $\eta = 1 - \frac{T_3}{T_1} r^{\gamma/(\gamma-1)}$ (c) $\eta = 1 - \frac{T_3}{T_1} r^{(\gamma-1)/\gamma}$ (d) $\eta = 1 - \frac{T_1}{T_3} r^{(\gamma-1)/\gamma}$

22. Brayton cycle with infinite intercooling and reheating stages would approximate a
(a) Stirling cycle (b) Ericsson cycle (c) Otto cycle (d) Atkinson cycle

23. Consider the following statements regarding modifications in a gas turbine power plant working on a simple Brayton cycle:
1. Incorporation of regeneration process increases specific work output as well as thermal efficiency.
2. Incorporation of regeneration process increases thermal efficiency but specific work output remains unchanged.
3. Incorporation of intercooling process in a multistage compression increases specific work output but the heat input also increases.
4. Incorporation of intercooling process in a multi-stage compression system increases specific work output, the heat addition remains unchanged.

Which of these statements are correct?
(a) 1 and 3 (b) 1 and 4 (c) 2 and 3 (d) 2 and 4

24. **Assertion (A):** In a constant pressure type gas turbine, large quantity of air is used in excess of its combustion requirements.

Reason (R): Excess air is used to compensate for inevitable air loss due to leakages in the system.
(a) Both A and R are true and R is the correct explaination of A
(a) Both A and R are individually true but R is not the correct explaination of A
(a) A is true but R is false
(a) A is false but R is true

25. Consider the following statements:
1. The speed of rotation of the moving elements of gas turbines is much higher than those of steam turbines.
2. Gas turbine plants are heavier and larger in size than steam turbine plants.
3. Gas turbines require cooling water for its operations.
4. Almost any kind of fuel can be used with gas turbines.

Which of these statements are correct?
(a) 1 and 2 (b) 1 and 3 (c) 1 and 4 (d) 3 and 4

26. Which of the following increase the work ratio in a simple gas turbine plant?
1. Heat exchanger
2. Intercooling
3. Reheating

Select the current answer using the following codes:
(a) 1 and 2 (b) 2 and 3 (c) 1 and 3 (d) 1, 2 and 3

27. In a Brayton cycle, the value of optimum pressure ratio for maximum net work done between temperatures T_1 and T_3, where T_3 is the maximum temperature and T_1 is the minimum temperature, is

(a) $(r_{\text{opt}})_{\text{max work}} = \left(\dfrac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$ (b) $(r_{\text{opt}})_{\text{max work}} = \left(\dfrac{T_3}{T_1}\right)^{\frac{\gamma-1}{2\gamma}}$

(c) $(r_{\text{opt}})_{\text{max work}} = \left(\dfrac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$ (d) $(r_{\text{opt}})_{\text{max work}} = \left(\dfrac{T_3}{T_1}\right)^{\frac{2(\gamma-1)}{\gamma}}$

28. Use of maximum pressure ratio corresponding to maximum to minimum cycle temperature in case of Joule cycle gives which one of the following?
(a) Maximum efficiency but very low specific work output
(b) Maximum efficiency and very high specific work output
(c) Minimum efficiency and very low specific work output
(d) Minimum efficiency and very high specific work output

29. An open cycle constant pressure gas turbine uses a fuel of calorific value 40000 kJ/kg with air fuel ratio of 80 : 1 and develops a net output of 80 kJ/kg of air. The thermal efficiency of the cycle is
(a) 61% (b) 16% (c) 18% (d) None of these

30. For the same net power output, if the speed is constant
(a) The turbine used in gas turbine power plants is larger than that used in steam power plants
(b) The turbine used in gas turbine power plants is smaller than that used in steam power plants
(c) The same turbine can be used for both plants
(d) None of these

31. Consider the following statements regarding gas turbine cycle:
1. Supersonic flow leads to decrease in efficiency.
2. Supersonic flow leads to decrease in flow rate.

Which of these statements is/are correct?
(a) 1 only (b) 2 only (c) 1 and 2 (d) Neither 1 nor 2

32. In a gas turbine cycle with regeneration
(a) Pressure ratio increases (b) Work output decreases
(c) Thermal efficiency increases (d) Heat input increases

33. Consider the following statements regarding gas turbines:
1. The degree of reaction of a reaction turbine is the ratio of energy transfer in fixed blade to the overall energy transfer across a stage.
2. The overall pressure drop in a turbine is the product of pressure drop per stage and number of stages.
3. Gas turbine cycle (Brayton cycle) is not as efficient as Rankine cycle for steam.

Which of these statements is/are correct?
(a) 1 only (b) 2 only (c) 2 and 3 (d) 3 only

34. In a simple single stage gas turbine plant, if T_1 is the minimum temperature and T_3 is the maximum temperature, then the work ratio in terms of pressure ratio r is

(a) $1-\dfrac{T_3}{T_1}(r)^{\frac{\gamma-1}{\gamma}}$ (b) $1-\dfrac{T_1}{T_3}(r)^{\frac{\gamma-1}{\gamma}}$ (c) $1-\dfrac{T_1}{T_3}(r)^{\frac{\gamma}{\gamma-1}}$ (d) $1-\dfrac{T_1}{T_3}(r)^{\frac{1}{\gamma}}$

35. Increasing the number of reheating stages in a gas turbine to infinity makes the expansion tending
    (a) Reversible adiabatic (b) Isothermal
    (c) Isobaric (d) Adiabatic
36. The $T-s$ diagram for a gas turbine plant, as shown in Figure 8.26, is drawn for the case where
    (a) Compression is done in two stages incorporating an intercooler between the two
    (b) Expansion of gases is done in two stages followed by regeneration
    (c) Expansion is done in two stages with a reheater between the two
    (d) Expansion is done in two stages with a reheater between the two followed by regeneration

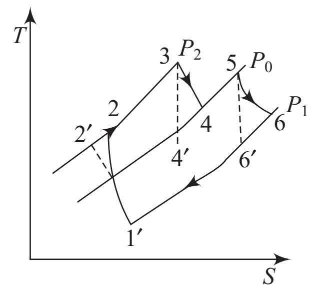


Figure 8.26 *Multiple choice question 36*

37. In which modification of simple gas turbine cycle is the work ratio increased?
    1. Regenerative gas turbine cycle
    2. Gas turbine cycle with reheating
    (a) Both 1 and 2 (b) 1 only (c) 1 only (d) Neither 1 nor 2
38. Reheat between multistage expansion in Joule cycle increases
    1. Overall work output
    2. The work ratio
    3. The thermal efficiency

    Which of these are correct?
    (a) 1, 2 and 3 (b) 1 and 2 only (c) 2 and 3 only (d) 1 and 3 only
39. The gas in a cooling chamber of a closed cycle gas turbine is cooled at
    (a) Constant volume (b) Constant temperature
    (c) Constant pressure (d) None of these
40. In a gas turbine plant, regeneration is done to
    (a) Increase compression work (b) Decrease turbine work
    (c) Limit the maximum temperature (d) Improve the plant efficiency
41. Consider a gas turbine supplied with gas at 5 bar and 1000 K to expand adiabatically to 1 bar. The mean specific heat at constant pressure is 1.0425 kJ/kg-K and at constant volume is 0.7662 kJ/kg-K. Calculate the power developed in kW per kg of gas per second and exhaust gas temperature.
    (a) 462 kW/kg/s and 647 K (b) 362 kW/kg/s and 653 K
    (c) 462 kW/kg/s and 653 K (d) 362 kW/kg/s and 647 K
42. The thermal efficiency of a simple open gas turbine plant is improved by regeneration as this
    (a) Decreases the temperature of the gases at the turbine inlet
    (b) Decrease the quantity of heat supplied in combustion chamber
    (c) Increases the turbine output
    (d) Lowers the work input to compressor
43. In a two stage gas turbine plant with interccoling and reheating
    (a) Both work ratio and thermal efficiency increases
    (b) Work ratio increases but thermal efficiency decreases
    (c) Thermal efficiency increases but work ratio decreases
    (d) Both work ratio and thermal efficiency decreases

44. The maximum net specific work obtainable in an ideal Brayton cycle for T_{max} = 900 K and T_{min} = 400 K is given by
(a) $100c_p$ (b) $500c_p$ (c) $700c_p$ (b) $800c_p$

45. In a gas turbine, hot combustion products with specific heats, c_p = 0.98 kJ/kg-K and c_v = 0.7538 kJ/kg-K, enter the turbine at 20 bar, 1500 K and exit at 1 bar. The isentropic efficiency of the turbine is 94%. The work developed by the turbine per kg of gas flow is
(a) 689.64 kJ/kg (b) 794.66 kJ/kg (c) 1009.72 kJ/kg (d) 1313.00 kJ/kg

46. An ideal Brayton cycle operating between the pressure limits of 1 bar and 6 bar has minimum and maximum temperatures of 300 K and 1500 K. The ratio of specific heats of the working fluid is 4. The approximate final temperature, in Kelvin, at the end of the compression and expansion processes are, respectively
(a) 500 and 900 (b) 900 and 500 (c) 500 and 5400 (d) 900 and 900

47. The thermal efficiency of an air standard Brayton cycle in terms of pressure ratio r and $\gamma = \dfrac{C_p}{C_v}$

(a) $\eta = 1 - \dfrac{1}{r^{\gamma-1}}$ (b) $\eta = 1 - \dfrac{1}{r^{\gamma}}$ (c) $\eta = 1 - \dfrac{1}{r^{1/\gamma}}$ (d) $\eta = 1 - \dfrac{1}{r^{(\gamma-1)/\gamma}}$

Common Data Questions 48 and 49

In a simple Brayton cycle, the pressure ratio is 8 and the temperatures at the entrance of the compressor and turbine are 300 K and 1400 K, respectively. Both the compressor and gas turbine have isentropic efficiencies equal to 80%. For the gas, assume a constant value of specific heat at constant pressure equal to 1 kJ/kg K and ratio of specific heats as 1.4. Neglect the changes in kinetic and potential energies.

48. The power required by the compressor in kW for 1 kg/s of gas flow rate is
(a) 194.7 (b) 243.4 (c) 304.3 (d) 378.5

49. The thermal efficiency of the cycle in % is
(a) 24.8 (b) 38.6 (c) 44.8 (d) 53.1

50. Match the features given in List 1 and List 2 for a Brayton cycle and select the correct answer using the following codes:

List 1	List 2
A. Compression	1. Isothermal
B. Rejection	2. Isobaric
C. Expansion	3. Isentropic
D. Heat addition	4. Isoenthalpic

Codes

	A	B	C	D
(a)	1	2	3	4
(a)	2	3	4	1
(c)	4	3	2	1
(d)	3	2	3	2

51. Consider the following statements:

When air is to be compressed to reasonably high pressure, it is usually carried out by a multistage compressor with an intercooler between the stages because

1. Work supplied is saved

2. Weight of compressor is reduced
3. More uniform torque is obtained leading to reduction in the size of the flywheel
4. Volumetric efficiency is increased

Which of these statements is/are correct?
(a) 1 only (b) 2 and 4 (c) 1, 2 and 3 (d) 1, 2, 3 and 4

52. Consider the following statements:
    1. Intercooling is effective only at lower pressure ratios and high turbine inlet temperatures.
    2. There is a very little gain in thermal efficiency when intercooling is used without the benefit of regeneration.
    3. With the high values of γ and c_p of the working fluid, the net power output of Brayton cycle will increase.

    Which of these statements is/are correct?
    (a) 1, 2 and 3 (b) 1 and 2 (c) 1 and 3 (d) 2 and 3

53. **Assertion (A):** A gas turbine plant is very sensitive to turbine and compressor inefficiencies.

    Reason (R): In a gas turbine plant, a large portion of the turbine work is consumed by the compressor.
    (a) Both A and R are true and R is the correct explaination of A
    (b) Both A and R are individually true but R is not the correct explaination of A
    (c) A is true but R is false
    (d) A is false but R is true

54. Figure 8.27 shows an open cycle gas turbine employing reheat on $T-s$ diagram. Assuming that the specific heats are same for both air and gas, and neglecting the increase in mass flow due to addition of fuel, the efficiency is

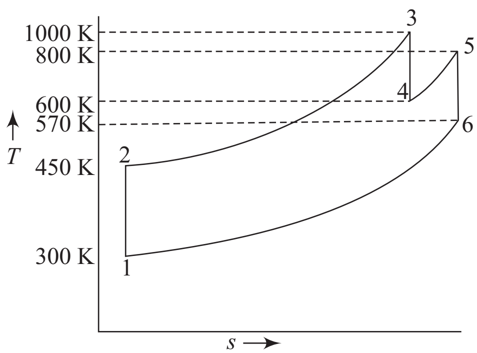


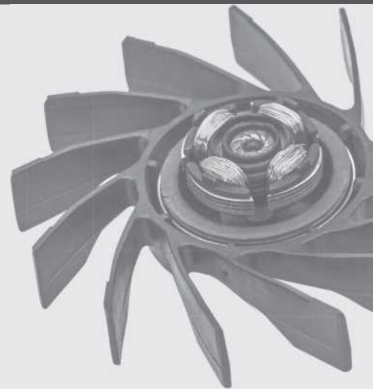
Figure 8.27 *Multiple choice question 54*

(a) 33.3% (b) 64% (c) 72.7% (d) 84%

ANSWER KEY

1. (d)	2. (b)	3. (b)	4. (b)	5. (d)	6. (c)	7. (d)	8. (c)	9. (a)	10. (a)
11. (c)	12. (a)	13. (c)	14. (a)	15. (b)	16. (c)	17. (a)	18. (c)	19. (b)	20. (b)
21. (d)	22. (b)	23. (c)	24. (c)	25. (c)	26. (b)	27. (c)	28. (a)	29. (b)	30. (a)
31. (c)	32. (c)	33. (b)	34. (b)	35. (b)	36. (c)	37. (c)	38. (b)	39. (c)	40. (d)
41. (b)	42. (b)	43. (b)	44. (a)	45. (a)	46. (a)	47. (d)	48. (b)	49. (c)	50. (d)
51. (d)	52. (d)	53. (a)	54. (b)						

9 Steam Turbines



Learning Objectives

After reading this chapter, you will be able to:

- **LO1** Explain the various terms associated with steam turbines and learn their classification
- **LO2** Explain the necessity and the various methods of compounding of steam turbines
- **LO3** Draw the velocity diagrams and analyze the performance of simple impulse, Rateau, Curtis and reaction turbines with the effects of various parameters on the performance
- **LO4** Compare the performance of simple impulse, Rateau, Curtis and reaction turbines
- **LO5** Determine the nozzle and blade heights, number of parallel exhausts in different stages and casing arrangements of steam turbines
- **LO6** Analyze the reheat factor in multistage steam turbines, factors affecting reheat and effects of such reheat
- **LO7** Illustrate the design of multistage steam turbine

9.1 Introduction

A steam turbine is a prime mover or power producing turbomachine in which energy of high pressure, high temperature steam supplied from a steam generator is converted continuously into shaft work and the lower pressure, lower temperature steam at its outlet is discharged.

Just as gas turbine is a prime mover in the power plants on Joule-Brayton cycle and its variants (Chapter 8), steam turbine is a prime mover in the plants working on Rankine cycle and its variants. Rankine cycles and their analyses however, are not presented in this text. Most of the analysis methods used for gas turbines (Chapter 8) are also applicable for steam turbines, the major difference being the treatment of the participating medium.

A steam turbine is basically an assemblage of nozzle and ring of moving blades mounted on a rotor shaft. The motion of the turbine shaft is determined solely by the dynamic action between steam and rotor. The high pressure and high temperature steam first expands in a nozzle and comes out as a high velocity jet because of the static pressure drop in the nozzle. This high velocity steam jet exiting the nozzle impinges on the blades mounted on a rotor. Consequently, the steam flow direction is changed from inlet to exit of the rotor. The change of momentum due to the steam flow direction change results in dynamic force which acts as a driving thrust for shaft rotation. Thus, the rate of change of momentum of a high velocity steam jet striking on a curved blade which is free to rotate determines the *motive power* of a steam turbine. It is noteworthy that any impact of the jet does not cause motive force on the blade which is designed in such a way that steam jet glides over the blade without any tendency to hit it. The conversion of energy of the steam in the blades to the driving thrust takes place by impulse, reaction or impulse reaction principle.

The first steam turbine was built by the Greek inventor, Hero of Alexandria, in 120 BC, which worked on a pure reaction principle and no power was produced by it. In fact, it was considered as an amazing toy, as shown in Figure 9.1 (a). It consisted of a simple hollow spherical vessel mounted in between two pivots. The vessel was equipped with four tangential converging tubes (nozzles). The steam was supplied to the vessel from a cauldron or boiler. The steam was expanded through the converging tubes to atmospheric pressure, consequently causing a reaction force on the ball and making it to rotate between the pivots.

Giovanni Branca made the first impulse turbine after many centuries later in 1629. In 1878, Carl Gustav Patrik de Laval, a Swedish engineer, made a simple impulse turbine using supersonic nozzle for a maximum speed of 10,000 rpm for separating cream from milk. De Laval further in 1897 developed a velocity compounded impulse turbine (a two moving blades row turbine with a fixed blade row between them), fed by a single set of high velocity nozzles. French engineer, Auguste Rateau, developed the pressure compounded impulse turbine in 1900 by conducting experiments on De Laval turbine in 1894. A velocity compounded turbine, similar to a two-stage de Laval turbine, was patented in 1896 by Charles G. Curtis in the USA. In 1884, in England, Charles A Parsons developed a multistage axial flow reaction turbine with blades of brass on a steel rotor. Berger Ljungstorm of Sweden, in 1912, developed a double flow radial reaction turbine.

Steam undergoes a continuous steady flow process in the turbine and velocity of steam is very high. Therefore, steam turbine handles a large mass of steam and produces a large power. Steam turbines are used in thermal power plants to drive the electric generators to produce electricity. Steam turbines are not only used in coal based and nuclear power plants to drive electric generators in order to generate electricity, but they are also used (a) to propel large ships, ocean liners, submarines and so on, and (b) to drive power absorbing machines like large compressors, blowers, fans and pumps.

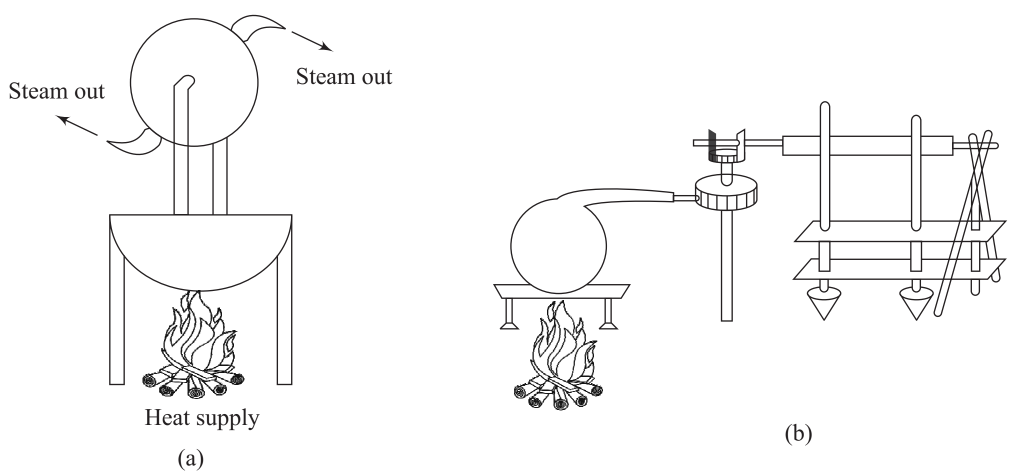


Figure 9.1 *(a) Hero's Reaction Turbine, (b) Branca's Impulse Turbine*

9.2 Classification of Steam Turbines

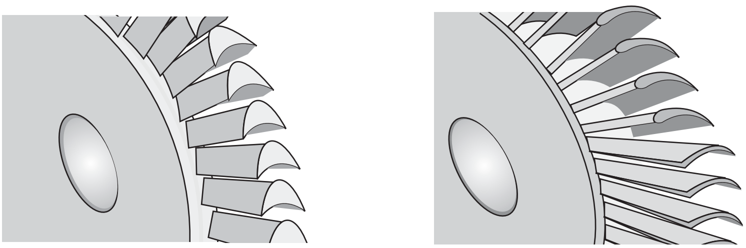
The objective of the turbine technology is to convert maximum amount of working fluid energy into useful mechanical work with maximum efficiency by means of a plant having maximum reliability, minimum cost, minimum supervision, and minimum starting time. Steam turbines are grouped into several classes on various bases as under:

9.2.1 On the Basis of Type of Blading and Mode of Steam Action

On the basis of blade types and the mode of action between steam and rotor wheel, the turbines may be of two types: (a) Impulse turbines, and (b) Reaction turbines.

(a) Impulse Turbine

The blades of impulse turbines are symmetrical, as shown in Figure 9.2 (a). Consequently, the cross-sectional area (flow area of steam) from blade inlet to exit is constant. Therefore, static pressure decreases mainly in stationary nozzle and there is little or no decrease in static pressure through rotor blade passage in an impulse turbine. Hence, driving thrust in case of impulse turbine is obtained due to change of momentum because of change in the direction of steam velocity while flowing across the blade from inlet to outlet.



(a) (b)

Figure 9.2 *(a) Impulse Turbine Blades, (b) Reaction Turbine Blades*

(b) Reaction Turbine

The blade profile of reaction turbine is of aerofoil section, as shown in Figure 9.2 (b), resulting in varying cross-sectional area (flow area for steam) from blade inlet to outlet. Thus, the blades of reaction turbine provide a suitable passage for steam to expand like that in the nozzle. Therefore, static pressure decreases in stationary nozzle as well as in blade passage in the reaction turbine. The reduction of static pressure in the blade passages of rotor further converts thermal energy into kinetic energy, consequently, generation of resultant reactive force. Driving thrust in reaction turbine consists of reactive force and force corresponding to change of momentum because of the change in direction of steam velocity.

9.2.2 On the basis of Direction of Steam Flow

On the basis of direction of steam flow, turbines may be classified into (a) axial flow turbine, (b) radial flow turbine, and (c) tangential flow turbine.

(a) Axial Flow Turbine

In axial flow turbines, the steam flows over the blades along the axis of the turbine shaft. These turbines are most appropriate for big turbo-generators and are commonly used in modern steam power plants.

(b) Radial Flow Turbine

In this turbine, steam is admitted in the central part near shaft and it flows in radial outwards direction through the successive moving blades mounted concentrically. Radial flow turbine contains two shafts end to end and each one drives a separate generator. A rotor is fixed to each shaft on which rings of 50% reaction radial flow blades are mounted. The two rings of concentric blades revolve opposite to each other, as shown in Figure 9.3. In this way, a relative speed of two times the running speed is achieved and each blade row is made to work. The final stages may be designed of axial flow type in order to get larger flow area. Radial flow turbines can be warmed and started quickly. Therefore, they are used as standby turbines during peak load hours. Smaller unit sizes are most successful in radial flow turbines. The challenging difficulties in design have restricted the evolution of larger units of radial flow turbines. These are also called Ljungstrom turbines, after the name of Swedish inventor.

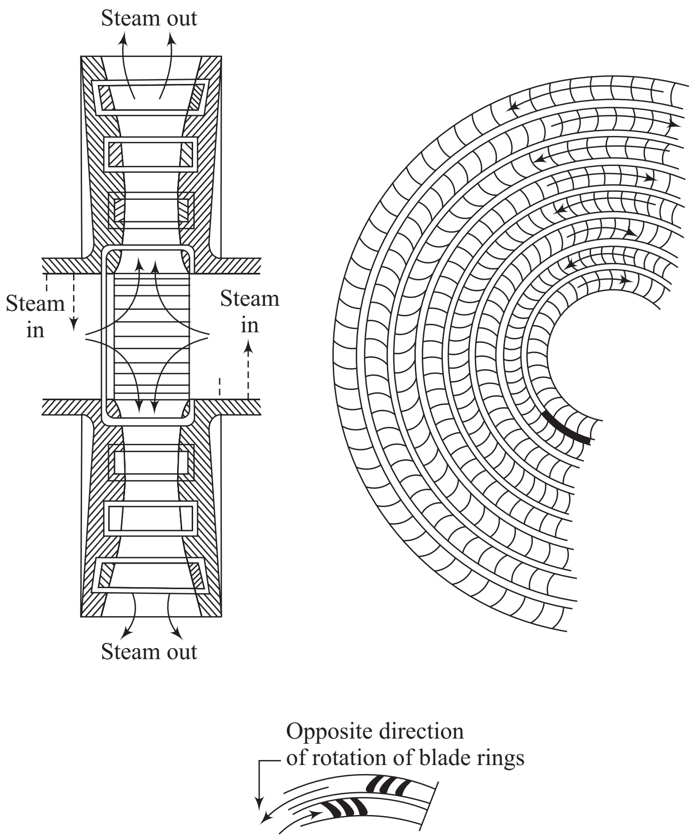


Figure 9.3 *Radial Flow Steam Turbine (Ljungstorm Turbine)*

(c) Tangential Flow Turbine

In tangential flow turbines, the flow of steam is in the tangential direction. It consists of circular disc with a number of blades (buckets) spaced around the periphery. One or more nozzles are mounted in such a way that each nozzle directs its jet along a tangent to the circle through the centres of the buckets. The steam reverses back and re-enters other bucket round the periphery. This is repeated several times as steam follows the helical path. These turbines are very robust but have poorer efficiency. They find applications for driving the auxiliaries of power plant.

As the radial flow turbines are in rare use, this chapter deals only with axial turbines.

9.2.3 On the Basis of Application

On the basis of application, turbines can be condensing or non-condensing, depending on whether the back pressure is below or equal to atmospheric pressure.

For small units without reheat, the steam turbine may consist of a single turbine when the steam expanding through the turbine exhausts to a condenser or a process line. Figure 9.4 (a) and (b) shows straight flow condensing turbines. For large unit without reheat, the steam after expansion in an initial section may be exhausted to another turbine, as shown in Figure 9.4(c). The first turbine is known as the high-pressure (HP) turbine, whereas the second one is the low-pressure (LP) turbine. The steam from outlet of low pressure turbine may then be discharged to a condenser or to a process.

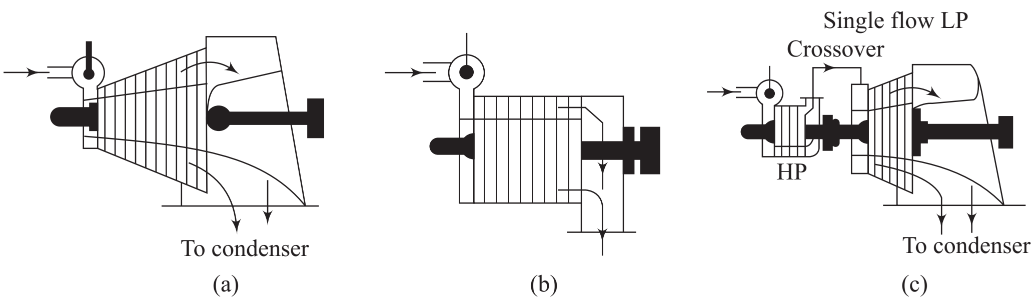


Figure 9.4 *(a) Straight Flow Condensing Turbine, (b) Straight Flow Non-Condensing Turbine, (c) Two Section Condensing Turbine*

(a) Condensing Turbine

In the condensing steam turbines, steam exiting from the turbine admits into condenser, as shown in Figure 9.5 (a). The steam after expansion is discharged into the condenser where vacuum can be maintained. Therefore, by using condenser, the pressure at the end of expansion can be reduced much lesser than atmospheric pressure. The recirculation of condensate as feed water for the boiler is possible in these steam turbines. All steam power plants use condensing turbines.

(b) Non-Condensing Turbine

Non-condensing steam turbine, as shown in Figure 9.5 (b), is the one in which steam exiting from the turbine is discharged to atmosphere instead of condenser as in a condensing turbine.

(c) Back Pressure Turbine

In this type of turbine, steam exits the turbine at a pressure much higher than the atmospheric pressure. The exhaust steam from the turbine can be used for some other purposes, viz. process heating or running smaller condensing turbines.

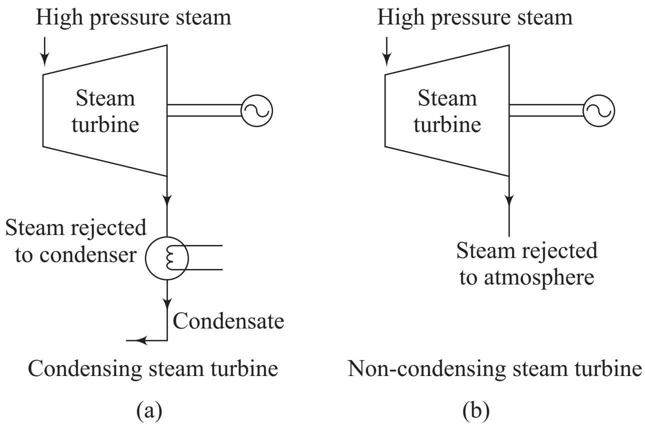


Figure 9.5 *Condensing and Non-Condensing Steam Turbine*

(d) Pass-out or Extraction Turbine

In pass out turbines, definite amount of steam is continuously withdrawn from the turbine for the purpose of process heating and remaining steam quantity is permitted to expand in the lower pressure sections of turbine. However, process heating applications require that turbines must supply steam at a constant pressure. Regulating valve is used to regulate the pressure because the pressure varies with load. This is known as a controlled or automatic extraction turbine, as shown in Figure 9.6 (a). Therefore, pressure regulating valve and control gear is essential to maintain the turbine speeds and steam pressure to be constant regardless of changes of power and heating loads.

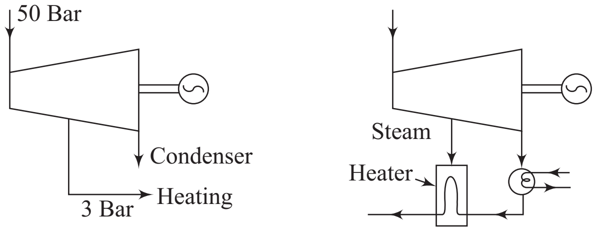


Figure 9.6 *(a) Pass-out or Extraction Turbine, (b) Regenerative Turbine*

(e) Regenerative Turbine

Regenerative turbine contains several extraction points through which small quantities of steam are continuously withdrawn for heating the boiler feed water in feed water heaters for the purpose of increasing plant thermal efficiency. Pressure and quantity of steam extracted for feed water heating varies as a function of load. This variation is permissible; therefore, no effort is made to control the pressure. This is known as simple or uncontrolled or non-automatic extraction, as shown in Figure 9.6 (b). Presently, reheating and regenerative arrangements are provided in all steam power plants.

9.2.4 On the Basis of Steam Supply Conditions

On the basis of steam supply conditions, turbines are classified as (a) single pressure turbine, (b) mixed or dual pressure turbine, and (c) reheat turbine.

(a) Single Pressure Turbine

If there is a single source of steam supply for the turbine, as shown in Figure 9.7(a), then it is called *single pressure turbine*.

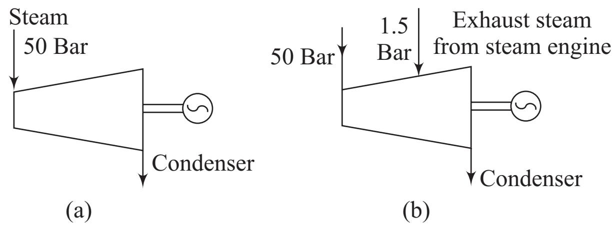


Figure 9.7 *(a) Single Pressure Turbine, (b) Mixed Pressure Turbine*

(b) Mixed or Dual Pressure Turbine

If a turbine uses two sources of steam at different pressures, it is called *mixed or dual pressure turbine,* as shown in Figure 9.7 (b). The dual pressure turbine finds application in nuclear power plants where it uses both the sources continuously. It is also used in industrial plants, e.g. rolling mill, colliery, etc. where there are two supplies of steam and use of one supply is more economical than the other; for example, the exhaust steam from an engine is the economical steam which may be utilised in the low pressure stages of a steam turbine.

(c) Reheat Turbine

Steam after expansion in high pressure (HP) turbine is passed back to the boiler for reheating in a single reheat cycle. The reheated steam from the boiler further expands in the intermediate-pressure (IP) or reheat turbine and exhausts into a crossover line that supplies steam to the double-flow low pressure (LP) turbine. After further expansion, the exiting steam from the LP turbine is exhausted to a condenser, as shown in Figure 9.8 (a).

Figure 9.8 (b) shows double-reheat cycle where the steam is reheated twice, having four turbine sections, viz. HP, IP, reheat and double-flow LP. Flow classification of triple-flow, four-flow, six-flow and eight-flow is also used. A triple-flow classification represents the use of one double-flow LP turbine and a single-flow LP turbine, as shown in Figure 9.8 (c). Four flow classification represents the use of two double-flow LP turbines, as shown in Figure 9.8 (d).

9.2.5 On the Basis of Number of Stages

Turbines are classified as single stage turbine and multistage turbine on the basis of number of stages.

(a) Single Stage Turbine

When the expansion of steam occurs in a single stage, then it is known as *single stage turbine*. One shaft is used in a single stage turbine.

(b) Multistage Turbine

The number of stages required is very high in larger power output turbine necessitating extra bearings to support the shaft. Multistage turbines are used in such situations. If the steam expands in two stages, it is known as *two stage or double stage turbine* and if it expands in three stages, it is known as a *three stage or triple stage turbine*.

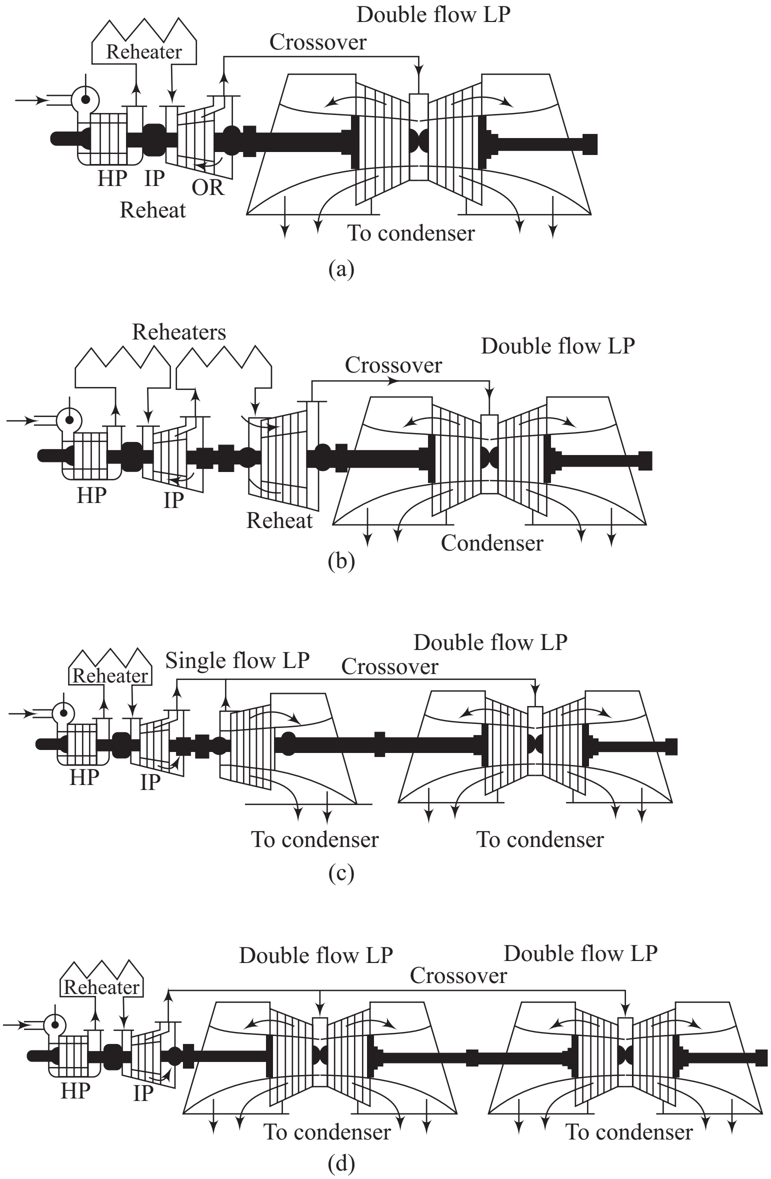


Figure 9.8 *(a) Single Reheat Cycle, (b) Double Reheat Cycle, (c) Triple Flow Low Pressure Turbine Cycle, (d) Four Flow Low Pressure Turbine Cycle*

9.2.6 On the Basis of Number of Cylinders

If all the stages are enclosed in one cylinder, it is known as *single cylinder turbine*. If all the stages are housed in more than one cylinder, say two or three, it is called a multi cylinder steam turbine.

9.2.7 On the Basis of Cylinder Flow Arrangement

(a) Single Flow

If the steam after entering at one end passes through the blading in a direction almost parallel to the shaft axis and comes out at the other end, it is called *single flow turbine,* as shown in Figure 9.4 (a), (b) and (c). High pressure cylinder and smaller units generally use single flow.

(b) Double Flow

If the steam after entering at the centre is divided into two parts moving axially away from the other across different set of bladings on the same shaft, it is known as *double flow turbine,* as shown in Figure 9.8 (a) and (b). Double flow is generally used in low pressure cylinder. Complete balancing against end thrust is obtained in a double flow turbine. This turbine also provides larger flow area through two sets of blades. The blade height is also decreased since mass flow rate is halved in double flow as compared with single flow under the same conditions. Triple flow and four flow low pressure turbines are shown in Figure 9.8 (c) and 9.8 (d), respectively.

(c) Reversed Flow

Reversed flow arrangement, as shown in Figure 9.9, is occasionally used in high pressure cylinder where higher temperature steam is used on the larger sets to minimize differential expansion, i.e. unequal expansion of rotor and casing.

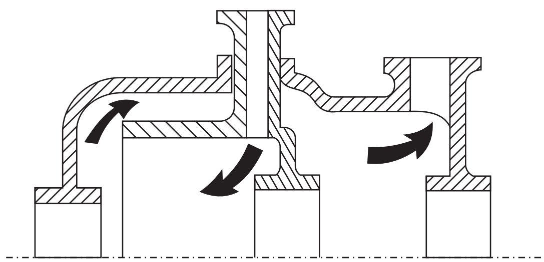


Figure 9.9 *Reversed Flow Turbine*

9.2.8 On the Basis of Number of Shafts

The overall steam turbine generator layout of a power plant is classified as *tandem-compound* or *cross-compound* depending on the disposition of shaft.

(a) Tandem Compound

The tandem-compound unit, as shown in Figure 9.10, has all turbines and the generator in-line, connected to the same shaft and therefore operates at constant speed.

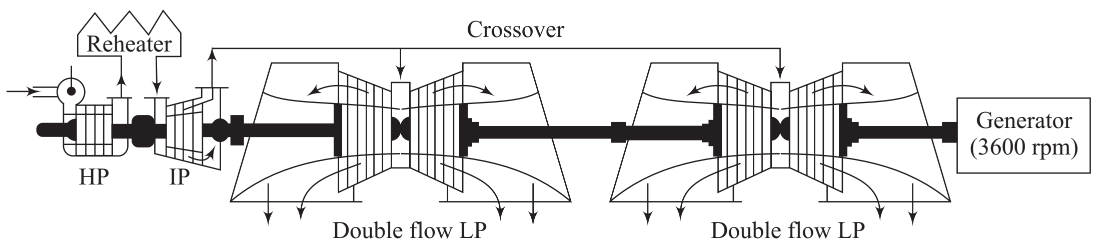


Figure 9.10 *Tandem Compound Turbine Unit*

(b) Cross Compound

The cross-compound arrangement, as shown in Figure 9.11, usually comprises of HP and IP turbines running at 3600 rpm [for frequency of 60 Hz and a bipolar generator, rpm = $\frac{(120 \times 60)}{2}$ = 3600 rpm as in the

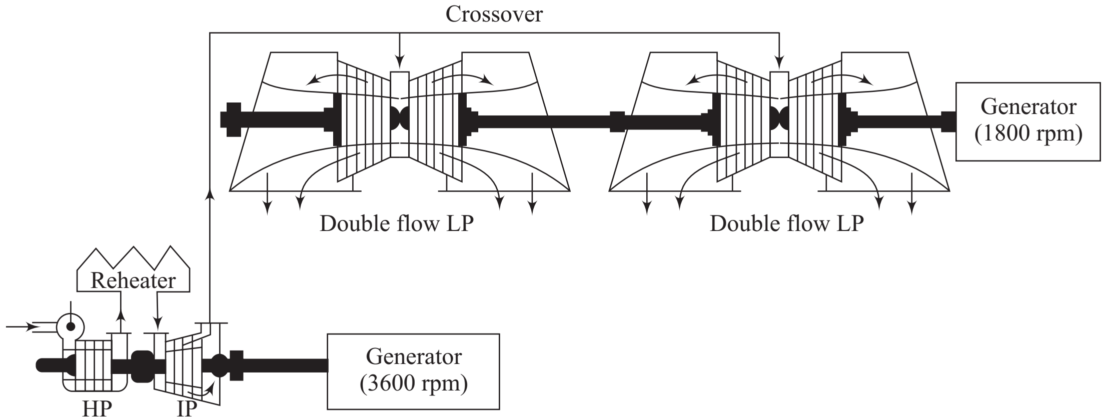


Figure 9.11 *Cross Compound Turbine Unit*

USA, while in India with 50 Hz frequency, it is 3000 rpm] that drive a generator. The steam exiting from the IP turbine passes through an LP turbine that operates at 1800 rpm (1500 rpm for 50 Hz and four poles) driving a separate generator. The LP turbine running at lower speed permits the use of longer blades in the last-stage of turbine with expansion to greater moisture contents, i.e. lesser dryness. Consequently, exhaust losses are reduced which result in higher efficiencies of the turbine.

Large cross-compound turbines with both shaft dispositions running at 3600 rpm have been developed in 1300 MW range. The two-shaft layout was used to reduce shaft length and any single generator size.

Steam turbines used in power plants are thus usually labelled by shaft disposition, number of LP turbines, steam flow path and the last stage blade length of the LP turbine. A turbine labelled as TC4F30, for example, indicates a tandem-compound (TC) unit having two double-flow (4*F*) LP turbines with 30 inch (0.762 m) blade length of last stage. A *CC*2*F*23 represents a cross-compound (CC) unit having one double-flow (2*F*) LP turbine with very long (more than 0.5 m) blades of last stage.

9.2.9 On the Basis of Speed of Rotation

Turbines are classified as (a) constant speed turbines, and (b) variable speed turbines on the basis of speed of rotation.

(a) Constant Speed Turbines

Specifications of rotational speed are most important in turbines that are directly related to electric generators as these must be alternating current units except the smallest sizes ones. Therefore, turbines must operate at speeds corresponding to the standard number of cycles per second as given by the following expression:

$$N = \frac{120 \times \text{Number of cycles per second}}{\text{Number of Poles}} = \frac{120f}{p} \tag{9.1}$$

The minimum number of poles in a generator is two and the corresponding maximum possible speed for 60 cycles/s (Hz) is 3600 rpm; for 50 cycles/s (Hz), the speed would be 300, 1500 and 750 rpm for 2, 4 and 8 poles generators, respectively.

(b) Variable Speed Turbines

Any speed ratio between the driver (turbine) and the driven machine may be obtained practically in variable speed turbines as they have gear mechanism. Therefore, the turbine may be designed for its own best efficiency point. These turbines are used for driving ships, compressors, blowers and generators of variable frequency.

9.2.10 On the Basis of Pressure of Steam

Based on the steam pressure at the inlet, turbine may be classified as: (a) low pressure steam turbine, (b) medium pressure steam turbine, (c) high pressure turbine, and (d) super pressure turbine.

The inlet pressure is less than 20 *bar* in low pressure steam turbines, whereas inlet pressure is in the range of 20–40 *bar* in medium pressure turbines. High pressure steam turbines have steam inlet pressure in the range of 40–70 *bar*.

9.3 Compounding of Steam Turbines

Figure 9.12 shows the schematic arrangement of a single stage impulse turbine. A single stage consists of a set of nozzles attached in the casing accompanied with a set of moving blades attached to the rotor which is mounted on a shaft. The change of pressure and velocity along the turbine axis is plotted in Figure 9.12. In this impulse stage, whole pressure drop of steam from boiler pressure to condenser pressure takes place in the nozzles and theoretically there is no drop in pressure, while steam passes across the passage between two blades.

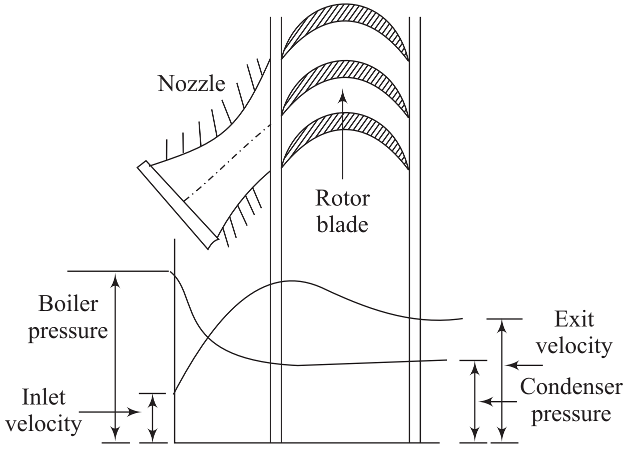


Figure 9.12 *Schematic Diagram of a Single Stage Impulse Turbine*

If the whole expansion of steam from boiler pressure to condenser pressure (vacuum) is permitted in a single set of nozzles, there would be a large enthalpy drop resulting in very high velocity of steam exiting from the nozzle (C_1) (refer Figure 9.12). Since $\frac{C_b}{C_1} = \cos\frac{\alpha}{2}$, the peripheral velocity of blades (C_b) also becomes very large. As $C_b = \frac{\pi DN}{60}$, therefore, if value of D is chosen from economy point of view, N becomes exceedingly high. Hence, single-stage impulse turbines, known as De Laval turbines, essentially have very high speed of rotation (N) of the order of 30000 rpm.

Such high speeds cannot be used effectively. It results in large amount of losses due to friction. High speed of rotation results in very high centrifugal stresses. Alternatively, if N is fixed, the rotor diameter becomes exceedingly high. Furthermore, a single stage results in the sufficiently high steam velocity at the exit, consequently, more loss of kinetic energy with steam leaving the stage. These difficulties are overcome by compounding or multistaging of turbines, where steam is allowed to expand in a number of stages instead of only in a single stage. The speed of turbine is decreased, whereas obtaining the same enthalpy drop of steam.

Compounding is a thermodynamic method for decreasing the turbine speed without using a gear box. Fundamentally, compounding of steam turbines is of two types and third type is the combination of the two:

1. Pressure compounding or Rateau staging
2. Velocity compounding or Curtis staging
3. Pressure and velocity compounding staging

9.3.1 Pressure Compounding or Rateau Staging

In pressure compounding or Rateau staging, entire stage pressure drop takes place in parts instead of in a single stroke in nozzles as in simple impulse turbine. Reasonable steam velocities as a consequence of division of pressure drop results in permissible turbine speed. The pressure compounding consists of a row of stationary nozzles between the two moving blade rows. Hence, pressure compounding of impulse turbine comprises a series of simple impulse stages or De Laval turbine stages, as shown in Figure 9.13. Exhaust of each row of moving blades is supplied to stationary nozzle row of the following stage. In pressure compounded turbine or Rateau staging, part expansion of high pressure steam in the first stationary nozzles row causes decrease in pressure and increase in steam velocity. Thereafter, steam enters moving blade row where no drop in pressure results ideally because of smooth blades of symmetrical profile but velocity decreases. Steam exiting the row of moving blades then enters row of stationary nozzles, where further part expansion takes place and steam exiting nozzles row enters the succeeding row of blades. The variation of pressure and velocity in a pressure compounded impulse turbine stage is shown in Figure 9.13.

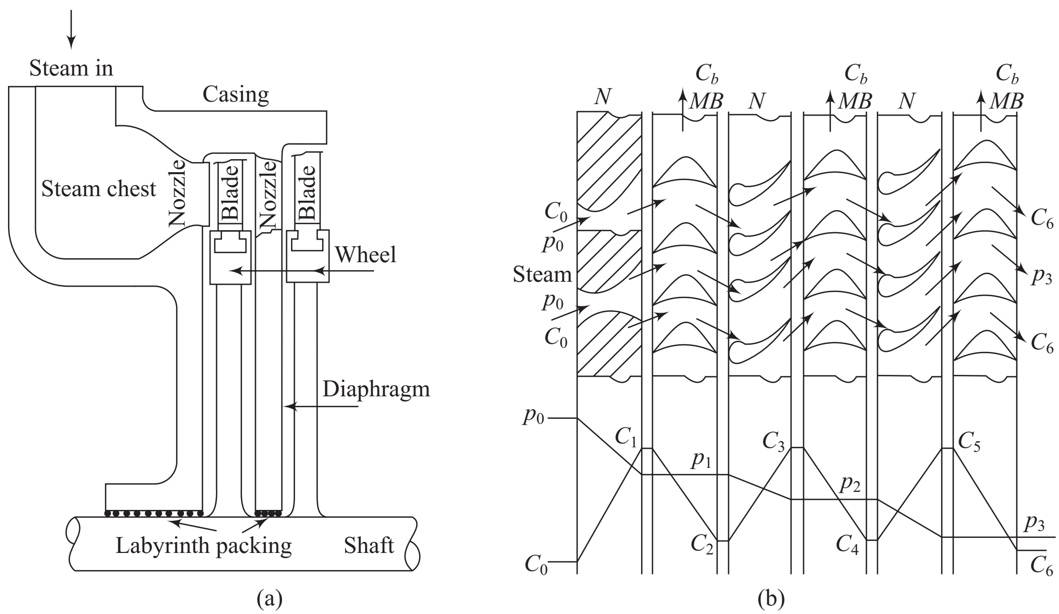


Figure 9.13 *Pressure Compounded (or Rateau) Impulse Staging (a) Schematic Diagram of Pressure Compounded Impulse Turbine, (b) Three Pressure or Rateau Stages*

9.3.2 Velocity Compounding or Curtis Staging

Velocity compounded impulse turbine is also known as 'Curtis' turbine. Staging of velocity is exercised in velocity compounded impulse turbine in order to make practical and effective use of the high velocity steam jet with permissible speed of rotation. In this turbine, soaking up of kinetic energy is divided into two or more rows of moving blades with guide blades in between them rather than in a single row of moving blades. Schematic diagram of Curtis stage, i.e. velocity compounded impulse turbine stage is shown in Figure 9.14 along with variations of pressure and velocity in the stage.

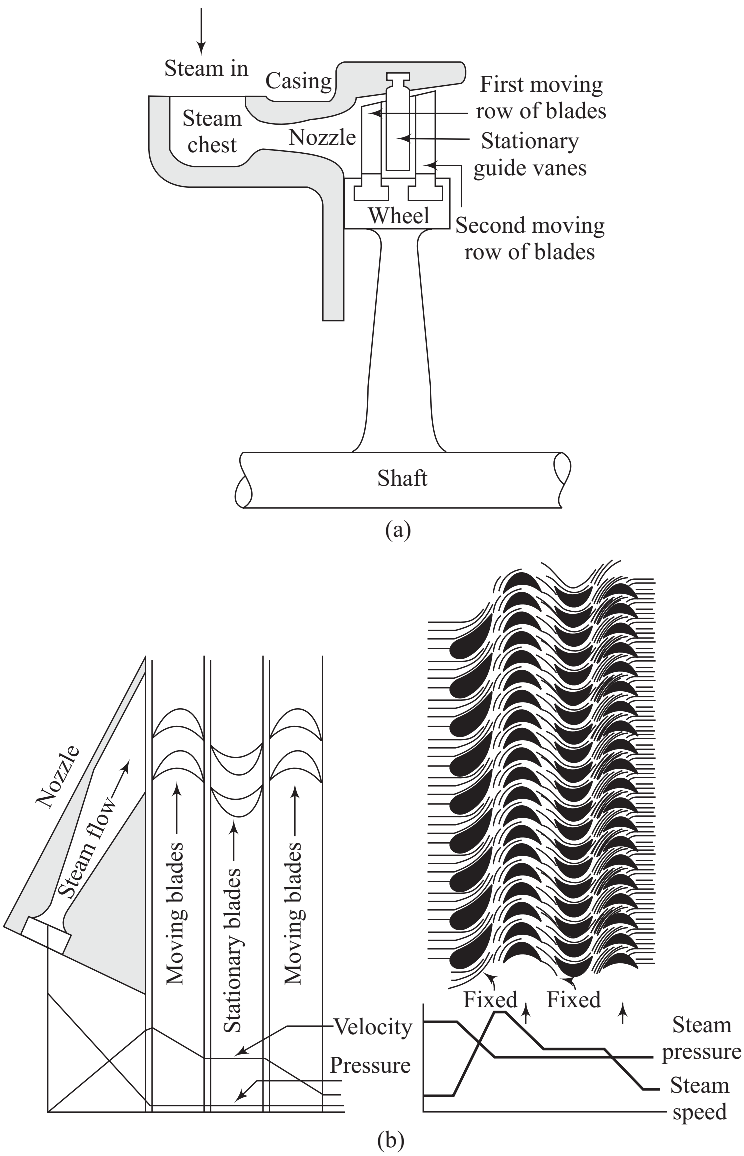


Figure 9.14 *A Two-Row Curtis (Velocity Compounded) Stage (a) Nozzles and Blades Arrangement, (b) Pressure and Velocity Variation*

The high pressure, high temperature steam from boiler expands from p_0 to p_1 with velocity increasing from C_0 to C_1 in the first row (ring) of nozzles. The pressure p_1 remains fundamentally constant thereafter. The kinetic energy of steam jet $\left(\frac{1}{2}\dot{m}C_1^2\right)$ exiting the nozzle is partially converted into shaft work in the first moving blades row resulting in decrease of steam velocity decreasing from C_1 to C_2. Thereafter, steam enters the fixed blades which act as guide blades. The stationary guide blades deflect exiting steam jets from first row of moving blades smoothly to the succeeding moving blades rows where partial conversion of remaining kinetic energy $\left(\frac{1}{2}\dot{m}C_2^2\right)$ into shaft work takes place. A two-row Curtis or velocity compounded turbine has two rings (sets) of moving blades with one ring of guide blades between them. The entire pressure reduction takes place in nozzle only, whereas the velocity decrease takes place in two segments (stages) in two moving blade rows, respectively. No decrease in steam velocity occurs across the stationary guide blades row since smooth and symmetrical impulse turbine blades are used in stationary guide blades row.

A three row Curtis stage consists of a two-row stage succeeded by a second ring of guide blades and then a third ring of moving blades. Hence, conversion of fluid energy to rotor shaft occurs in three stages. High kinetic energy steam leaving the nozzles is partially converted into shaft work in the first ring of moving blades. First set of stationary guide blades deflect exiting steam of first ring of moving blades smoothly to the second moving blades row where further partial conversion of remaining kinetic energy into shaft work takes place. Again, steam leaving the second row of moving blades gets deflected by the second ring of stationary guide blades smoothly to the third ring of moving blades where steam expands further to produce shaft work.

The advantages of velocity compounded impulse turbine are that lesser number of stages are required relative to pressure compounding at lower cost. It also needs lesser space and is comparatively more reliable and easy to start. In multistage velocity compounded impulse turbine, large pressure drop occurs in the first stage and pressure drop goes on decreasing in the succeeding turbine stages, i.e. they are subjected to lower pressure range, therefore, lesser number of stages are required. The whole pressure drop takes place only in nozzles in velocity compounded impulse turbine. Therefore, rest of turbine and its casing is not required to be manufactured very strong. However, more frictional losses due to high initial velocity and 'non-optimum value of ratio of blade peripheral velocity to steam velocity for all blade rows' results in low efficiency of velocity compounded impulse turbine which goes on decreasing with increase in number of stages.

9.3.3 Pressure-Velocity Compounded Impulse Staging

Pressure-velocity compounded impulse turbine is a combination of the earlier discussed two types of compounding. In this type of turbine, steam from boiler is supplied to the stationary nozzle row succeeded by moving blade row which is followed by fixed blade row that is succeeded by moving blade row. Steam exiting moving blade row passes through the fixed nozzle row succeeded by moving blade, fixed blade and moving blade rows, respectively. Schematic arrangement of pressure-velocity compounded impulse turbine stage is illustrated in Figure 9.15, which also depicts the variation of pressure and velocity at different sections.

In this type of staging, both pressure decrease and velocity decrease are divided across various sections, as illustrated in Figure 9.15. Thus, in pressure-velocity compounded impulse staging, one or more 'Curtis stage' (velocity compounded) succeeded by 'Rateau stage' (pressure compound) are allocated. Curtis stages decrease pressure to a reasonable level with high fraction of work done per stage and thereafter the highly efficient 'Rateau stages' absorb the remaining energy available. By using this combined arrangement, it is possible to decrease the overall length of turbine, consequently, saving in initial cost which balances the effect of lower efficiency.

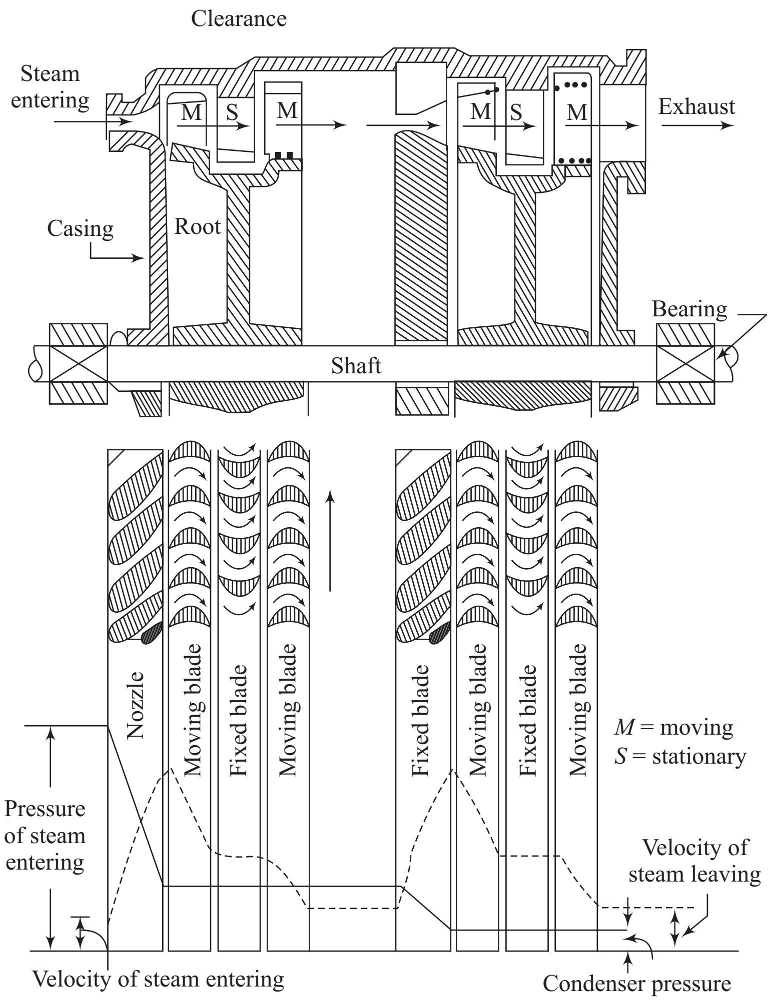


Figure 9.15 *Pressure-Velocity Compounded Impulse Turbine*

9.4 Analysis of Simple Impulse Turbine Staging

In this impulse stage, whole pressure drops of steam take place in the nozzles and ideally no pressure drop takes place while steam passes across the passage between the two blades. In Figure 9.16 (a), steam from the boiler enters the nozzle through piping at pressure p_0 with velocity C_0, and exits the nozzle at predefined angle in order to flow smoothly over the moving blades. The velocity of the steam increases to C_1 as it undergoes expansion up to pressure p_1 when it passes through the nozzle. High velocity steam jets emerging out from the nozzles strike the blades with velocity C_1, get deflected at an angle, and exit at a lower velocity C_2; consequently, a torque is exerted on the blades. The pressure of steam p_1 remains fundamentally constant during its flow through the blade passages because of the constant cross-sectional area.

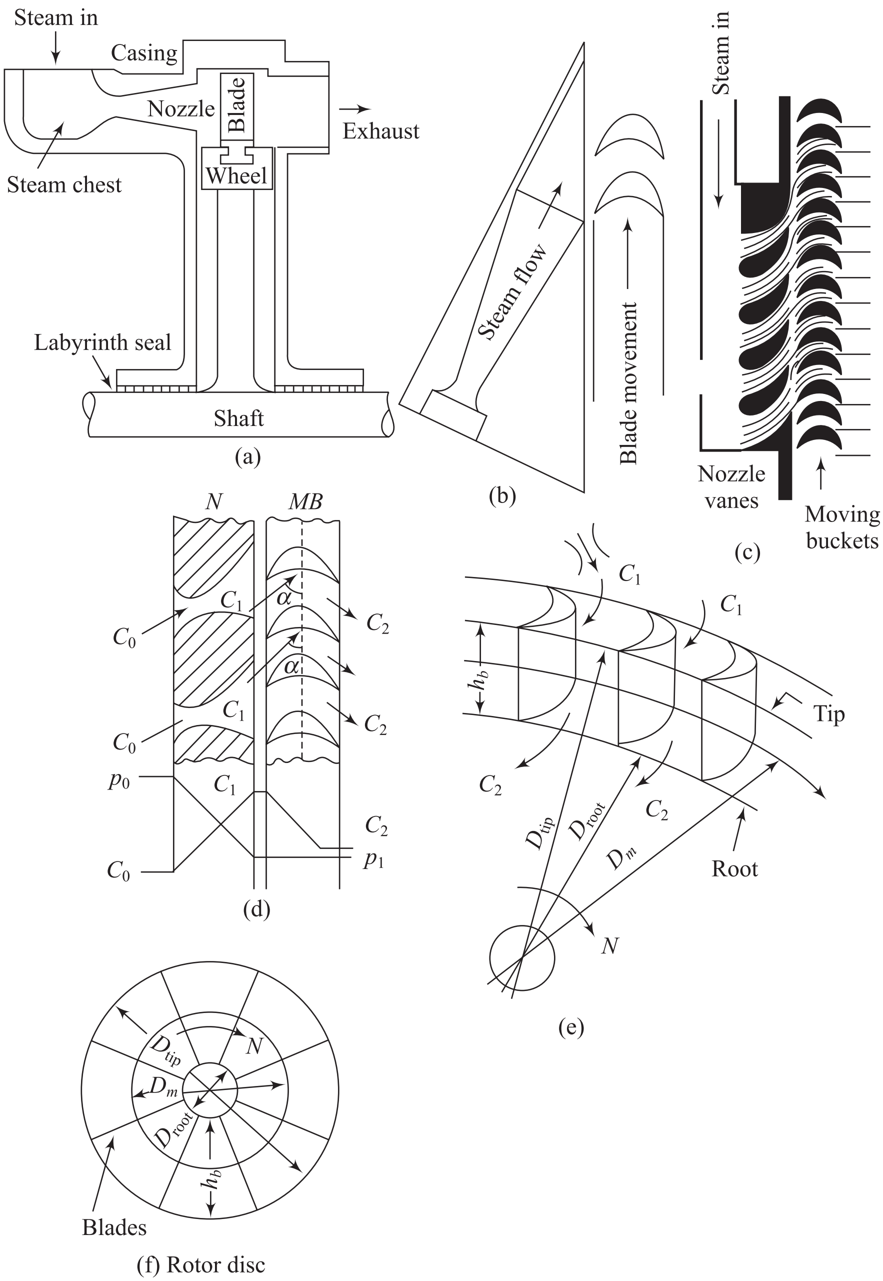


Figure 9.16 *An Impulse Steam Turbine Stage*

From the conservation law of momentum, difference of momentums of steam jets at the inlet and outlet of blades both resolved in the whirling direction of the rotor is equal to the angular momentum absorbed by the rotor in developing shaft work.

In Figures 9.16 (b) to 9.16 (d), it is observed that the wheel rotates only due to the impulse action of the jets (i.e. the difference of momentums due to deflection of the jets by the blades). The blades of such a rotor or wheel are known as impulse blades. Simple impulse turbine finds applications where low power output at very high speed is needed or only a small drop in pressure is available. These turbines are not suitable for applications which require conversion of large thermal energy into work.

The mean peripheral velocity of the blades, also called the mean blade velocity, C_b, is given by,

$$C_b = \frac{\pi D_m N}{60} \tag{9.2}$$

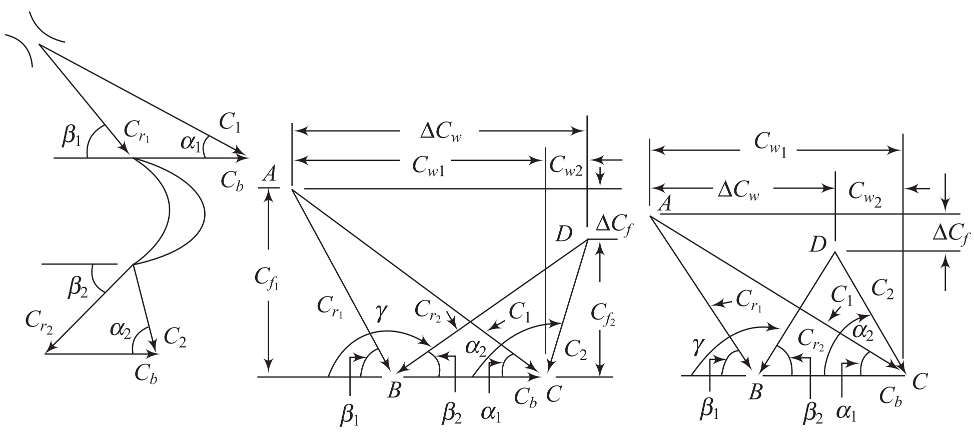
where D_m is the mean diameter of the wheel and N is its rpm. The area of flow or blade annulus [Figure 9.16 (e)], A_b, is given by,

$$A_b = \frac{\pi}{4}(D_t^2 - D_h^2) = \pi\left(\frac{D_t + D_h}{2}\right)\left(\frac{D_t - D_h}{2}\right) = \pi D_m h_b \tag{9.3}$$

where D_1 is the tip diameter, D_2 is the hub diameter, and h_b is the height of the blades.

9.4.1 Velocity Diagrams

Figure 9.17 shows velocity triangles at the entrance and exit of moving blade. Velocity triangles at the inlet and outlet, as shown in Figure 9.17 (a), have been superposed on a common C_b in Figure 9.17 (b) to obtain combined velocity diagram for the stage. If all the angles are measured clockwise, then γ is the exit blade angle ($\gamma = 180 - \beta_2$) and α_2 is the angle made by absolute velocity of steam leaving the blades with the plane of rotation of the wheel.



(a) (b) $\alpha_2 > 90°$ (c) $\alpha_2 < 90°$

Figure 9.17 *Velocity Triangles for Simple Impulse Turbine*

The steam exiting the nozzle glides smoothly into the row of moving blades. Steam exits the nozzle at an angle α_1 with absolute velocity C_1. This steam jet will be supplied to moving blade with velocity C_1 at an angle α_1. Because of the whirling motion of moving blade, the stream of steam in fact glides over the moving blade with velocity C_{r1} at inlet blade angle β_1. The velocity C_{r1} is actually the velocity of steam at the inlet relative to moving blade which is known as relative velocity of steam at the inlet. The relative velocity of steam at the inlet of moving blade C_{r1} is the vector summation of two velocity vectors C_1 and

C_{b1}. Steam exits the moving blade with velocity C_{r2} which is the relative velocity of steam at the outlet of moving blade. Therefore, relative velocity, in fact, is the velocity with which steam passes through the moving blade. The relative velocities at the blade outlet and inlet should be same for a perfectly smooth and frictionless blade. The reason for this is that no expansion of steam takes place in moving blade due to symmetrical blade profiles and constant area of flow passage between two consecutive moving blades from inlet to outlet. In fact, the relative velocity at the exit will be smaller than that at the inlet, i.e. $C_{r2} < C_{r1}$, due to friction over the blade. This decrease in relative velocity is accounted by a parameter known as blade friction factor, $k = \frac{C_{r2}}{C_{r1}}$.

It is quite clear looking at the inlet section 1 that to obtain the maximum change in momentum, steam should impinge horizontally on the moving blade, i.e. $\alpha_1 = 0$, and should also exit horizontally from the blade horizontally, i.e. $\alpha_2 = 0$, resulting in the semi-circular shape of moving blade. Semi-circular moving blades are not feasible since a moving blades row has a number of blades one after the another and each blade has to receive steam from number of nozzles one after the another. That is why nozzles are set at some angle to the blade, say α_1 in this case.

Since the steam leaving the nozzle at an angle α_1 with velocity C_1 passes over the blade, it has two components of velocity. One is tangential component which is parallel to the direction of rotation and other one is axial component which is perpendicular to the direction of rotation of blades. Tangential component of velocity known as whirl velocity is accountable for producing thrust due to change in momentum. Axial component of velocity known as flow velocity is accountable for keeping flow of steam through the moving blade row. Discharge through the moving blade row is the product of velocity flow and effective flow passage area. Hence, magnitude of the flow velocity decides the size of rotor for given discharge of the steam.

Both the whirl and flow velocities are the two mutually perpendicular components of absolute velocity which depend mostly on the nozzle angle, i.e. the angle of absolute velocity α_1. The nozzle angle α_1 should be decided on the basis of requirements of thrust and flow rate through the blade row. The velocity of whirl, $C_1 \cos \alpha_1$, decreases, whereas the velocity of flow, $C_1 \sin \alpha_1$, increases with increase in nozzle angle α_1. Thus, the maximization of one causes minimization of the other, therefore, a compromise should be made while choosing the angle of absolute velocity at the inlet or nozzle angle α_1. Let the absolute velocity at the outlet is C_2 at an angle α_2 (which will be inlet angle for succeeding nozzles if more than one similar stage are used). Hence, there will be whirl velocity $C_2 \cos \alpha_2$ and flow velocity $C_2 \sin \alpha_2$ components at the exit of blade also. In a simple stage of impulse turbine, the velocity of whirl component at the outlet is a type of loss at outlet. Therefore, this component should be minimum in order to minimize the loss at exit. For minimizing the whirl velocity at the outlet, i.e. $C_2 \cos \alpha_2$, should be minimum, i.e. zero, which gives $\alpha_2 = 90°$ for minimum loss at the exit. This is a particular case in which discharge from the turbine is axial, i.e. $\alpha_2 = 90°$, and such turbines are also known as axial discharge turbines.

Therefore, it is quite clear that the moving blade merely deflects the steam in an impulse turbine stage. The change in direction of steam from inlet to outlet causes momentum change, consequently, thrust is produced. No expansion of steam takes place in moving blades due to symmetrical blade profiles and constant area of flow passage between two consecutive moving blades from inlet to outlet. The expansion of steam takes place only in the nozzle. Also, pressure of steam from inlet to outlet of moving blades remains constant under ideal conditions as there is no expansion across moving blade. For symmetrical blading, the blade angles at the inlet and outlet are same, i.e. $\beta_1 = \beta_2$.

Steam coming out from the nozzles at absolute velocity C_1 strikes the blades with relative velocity C_{r1}, while the blades rotate with mean peripheral velocity C_b. Steam leaves with relative velocity C_{r2} while its absolute velocity is C_2. As shown in Figure 9.17 (a), α_1 is the nozzle angle subtended by the nozzle axis with the direction of rotation of the wheel, β_1 is the inlet blade angle and β_2 is the exit blade angle.

9.4.2 Stage Parameters

(a) Tangential Thrust or Driving Thrust

It is the difference of whirl or tangential components or cosine components of the velocities of steam which drives the wheel and produces the torque. It is called the change in the velocity of whirl, ΔC_w, which is given by,

$$\Delta C_w = \text{velocity of whirl at inlet - velocity of whirl at outlet}$$

$$\Delta C_w = C_{w1} - C_{w2} = C_1 \cos \alpha_1 - C_2 \cos \alpha_2 \tag{9.4}$$

If $\alpha_2 > 90°$, C_{w1} and C_{w2} become additive in estimating ΔC_w [Figure 9.17 (b)].
If $\alpha_2 < 90°$, C_{w2} is to be subtracted from C_{w1} to determine ΔC_w [Figure 9.17 (c)].
From the inlet velocity triangle ABC [Figure 9.17 (b)],

$$C_1 \cos \alpha_1 - C_b = C_{r1} \cos \beta_1$$

$$C_1 \sin \alpha_1 = C_{r1} \sin \beta_1$$

Therefore, on division,

$$\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - C_b} \tag{9.5}$$

From the exit velocity triangle DBC [Figure 9.17 (b)],

$$C_2 \cos (180 - \alpha_2) + C_b = C_{r2} \cos (180 - \gamma)$$

$$C_b - C_2 \cos \alpha_2 = C_{r2} \cos \gamma \tag{9.6}$$

The same result can also be obtained from Figure 9.17 (c):

$$C_b - C_2 \cos \alpha_2 = C_{r2} \cos (180 - \gamma) = -C_{r2} \cos \gamma$$

$$C_2 \cos \alpha_2 = C_b + C_{r2} \cos \gamma \tag{9.7}$$

The ratio of relative velocity at the exit to that at the inlet is called 'the blade friction factor', k,

$$k = \frac{C_{r2}}{C_{r1}} \tag{9.8}$$

$$\text{Energy loss due to friction in the blades} = \frac{C_{r1}^2 - C_{r2}^2}{2} \tag{9.9}$$

Now, from Eqs (9.4) and (9.7),

$$\Delta C_w = C_1 \cos \alpha_1 - C_2 \cos \alpha_2 = C_1 \cos \alpha_1 - C_b - kC_{r1} \cos \gamma$$

$$\Delta C_w = C_1 \cos \alpha_1 - C_b - k\frac{C_1 \cos \alpha_1 - C_b}{\cos \beta_1} \cos \gamma$$

$$\Delta C_w = (C_1 \cos \alpha_1 - C_b)\left[1 - k\frac{\cos \gamma}{\cos \beta_1}\right] \tag{9.10}$$

Since $\gamma = 180 - \beta_2$; $\cos \gamma = -\cos \beta_2$ [refer Figure 9.17(b)],
Therefore,

$$\Delta C_w = (C_1 \cos \alpha_1 - C_b)\left(1 + k\frac{\cos \beta_2}{\cos \beta_1}\right)$$

Also,

$$\Delta C_w = C_{r1} \cos \beta_1 + C_{r2} \cos \beta_2 = C_{r1} \cos \beta_1 + kC_{r1} \cos \beta_2$$

$$\Delta C_w = C_{r_1} \cos \beta_1 \left(1 + k\frac{\cos \beta_2}{\cos \beta_1}\right) = (C_1 \cos\alpha_1 - C_b)\left(1 + k\frac{\cos\beta_2}{\cos\beta_1}\right) \quad (9.11)$$

Blades are said to be symmetrical if blade angles are equal, i.e. $\beta_1 = \beta_2$. Impulse turbines mostly have symmetrical blades. Therefore, by Eq. (9.11),

$$\Delta C_w = C_{r1} \cos \beta_1(1 + k) = (C_1 \cos \alpha_1 - C_b)(1 + k) \quad (9.12)$$

The tangential thrust exerted by the jets on the blades, F_t is

$$F_t = m\Delta C_w \quad (9.13)$$

where $\dot{m}$ is the steam flow rate.

(b) Axial Thrust

The velocity component in the axial direction is known as velocity of flow. The axial thrust, i.e. the thrust produced by the difference in the axial components of the velocities (along the axis of the shaft) or flow velocities,

$$F_a = m\Delta C_f \quad (9.14)$$

$$\Delta C_f = C_{r1} \sin \beta_1 - C_{r2} \sin \beta_2 \quad (9.15)$$

The bearing should be designed for taking the axial thrust load.

(c) Blading Work or Diagram Work

The rate at which work is done by the jets on the steam turbine blades is called the *blading work or diagram work*, W_D. Diagram work is the work available at the rotor of the turbine. Diagram work is obtained by Euler's equation as follows:

$\dot{W}_D = F_t \times$ Displacement per unit time in tangential direction $= \dot{m}(C_{w1}C_{b1} - C_{w2}C_{b2}) = m(C_{w1} - C_{w2})C_b$

$\because C_{b1} = C_{b2} = C_b$ and $C_{w1} - C_{w2} = \Delta C_w$, Displacement/unit time in tangential direction $= C_b$

$$\dot{W}_D = F_t C_b = \dot{m}\Delta C_w C_b \quad (9.16)$$

Alternate Method of Finding Diagram Work

Diagram work can also be found by applying steady flow energy equation between the inlet 1 and outlet 2 of the blades. Assuming no potential energy change from inlet to exit across the moving blades and no heat transfer across the stage, consequently, steady flow energy equation results,

$$\dot{m}\left(h_1 + \frac{C_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{C_2^2}{2}\right) + \dot{W}$$

$$\dot{W} = \dot{m}\left[(h_1 - h_2) + \frac{C_1^2 - C_2^2}{2}\right] = \dot{m}(h_{01} - h_{02}) \quad (9.17)$$

Equation (9.17) is same as Eq. (1.99) by neglecting change in potential energy.

The change in static enthalpy in case of impulse turbine from section 1 to section 2 is the change in kinetic energy associated with relative velocity from section 1 to 2.

$$h_1 - h_2 = \frac{C_{r2}^2 - C_{r1}^2}{2} \quad (9.18)$$

$$\dot{W} = \dot{m}\left[\frac{C_{r2}^2 - C_{r1}^2}{2} + \frac{C_1^2 - C_2^2}{2}\right] \quad (9.19)$$

For perfectly smooth blades, there will be no friction, hence, $C_{r1} = C_{r2} \Rightarrow h_1 - h_2 = 0$. Therefore, for impulse stage with smooth blades,

$$\dot{W} = \dot{m}\left(\frac{C_1^2 - C_2^2}{2}\right) \quad (9.20)$$

From velocity triangles,

$$C_1^2 = C_{r1}^2 + C_b^2 + 2C_{r1}C_b\cos\beta_1$$

$$C_2^2 = C_{r2}^2 + C_b^2 - 2C_{r2}C_b\cos\beta_2 \quad (9.21)$$

$$C_1^2 - C_2^2 = C_{r1}^2 - C_{r2}^2 + 2(C_{r1}\cos\beta_1 + C_{r2}\cos\beta_2)C_b \quad (9.22)$$

From velocity tringles,

$$C_{r1}\cos\beta_1 + C_{r2}\cos\beta_2 = C_1\cos\alpha_1 + C_2\cos\alpha_2 = \Delta C_w$$

$$C_1^2 - C_2^2 = C_{r1}^2 - C_{r2}^2 + 2\Delta C_w \cdot C_b \Rightarrow (C_1^2 - C_2^2) + (C_{r2}^2 - C_{r1}^2) = 2\Delta C_w \cdot C_b \quad (9.23)$$

Substituting value of $(C_1^2 - C_2^2)$ from Eq. (9.23) into Eq. (9.20), we get,

$$\dot{W} = \dot{m}\Delta C_w \cdot C_b$$

Eq. (9.16) can also be obtained from Eq. (1.86) by taking $C_{b1} = C_{b2}$ for the axial maxhine and $\frac{\dot{W}}{\dot{m}} = w = gH_e = gH_{\text{th}}$

(d) Diagram Efficiency or Blade Efficiency or Wheel Efficiency

If the rotor alone is considered as the system, the energy input to the blades is the kinetic energy of jets issuing out from nozzles. It is the fraction of energy input to the blades, i.e. kinetic energy of jets of steam issuing from nozzles which is converted to blade work.

Rate of energy input to the blades = Rate of kinetic energy at the inlet of blades = $\frac{1}{2}\dot{m}C_1^2$ (9.24)

Therefore, the blading or diagram or wheel efficiency,

$$\eta_D = \frac{\text{Rate of work done on the blades}}{\text{Rate of energy input on the blades}} = \frac{\dot{m}\Delta C_w C_b}{\frac{1}{2}\dot{m}C_1^2}$$

$$\eta_D = \frac{2\Delta C_w C_b}{C_1^2} \quad (9.25)$$

The velocity C_1 can be obtained by applying steady flow energy equation across the nozzle.

(e) Optimum Velocity Ratio and Maximum Diagram Efficiency

Substituting ΔC_w from Eq. (9.12) in Eq. (9.25), the diagram efficiency is given by,

$$\eta_D = \frac{2(C_1\cos\alpha_1 - C_b)(1+k)C_b}{C_1^2} = \frac{2C_b^2\left(\frac{C_1\cos\alpha_1}{C_b} - 1\right)(1+k)}{C_1^2} \quad (9.26)$$

Defining the *velocity ratio* ρ as $\frac{C_b}{C_1}$, the ratio of mean blade velocity C_b to the jet velocity C_1,

$$\eta_D = 2\rho^2\left(\frac{\cos\alpha_1}{\rho} - 1\right)(1+k) = 2(\rho\cos\alpha_1 - \rho^2)(1+k) \tag{9.27}$$

There is a certain value of ρ which makes η_D the maximum. Differentiating η_D with respect to ρ and making it equal to zero,

$$\frac{d\eta_D}{d_\rho} = 2(\cos\alpha_1 - 2\rho)(1+k) = 0$$

Therefore, the optimum velocity ratio for impulse blading is,

$$\rho_{\text{opt}} = \frac{\cos\alpha_1}{2} \tag{9.28}$$

Substituting this optimum value of ρ in Eq. (9.27), the maximum blading or diagram efficiency is,

$$(\eta_D)_{\max} = 2\left[\frac{\cos^2\alpha_1}{2} - \frac{\cos^2\alpha_1}{4}\right](1+k)$$

$$(\eta_D)_{\max} = \frac{1+k}{2}\cos^2\alpha_1 \tag{9.29}$$

For perfectly smooth blades, $k = 1$. If the energy loss due to friction in blades is small, $k \approx 1$, then

$$(\eta_D)_{\max} = \cos^2\alpha_1 \tag{9.30}$$

where α_1 is the nozzle angle. The lower is the nozzle angle, higher is the blading efficiency. However, too low a nozzle angle may cause energy loss at blade inlet. Therefore, the nozzle angle has to be maintained within a certain range, which varies from 16° to 22°.

(f) Nozzle Efficiency

Nozzle efficiency is the ratio of the kinetic energy available at the exit of the nozzle to the enthalpy change occurring across the nozzle, i.e. between entrance and exit (sections 0 and 1).

$$\eta_n = \frac{\text{Kinetic energy available at the outlet of the nozzle}}{\text{Enthlpy change across the nozzle}} = \frac{\dot{m}C_1^2/2}{\dot{m}(h_0 - h_1)}$$

$$\eta_n = \frac{C_1^2}{2(h_0 - h_1)} \tag{9.31}$$

(g) Stage Efficiency or Gross Efficiency

Stage efficiency is defined as the ratio of rate of work done on the blades (or specific work) and the rate of energy input to the stage (or specific energy input to the stage). Energy supplied to the stage is the enthalpy change between section 0 and 1, i.e. the inlet of the nozzle to the exit of the nozzle. Therefore, stage efficiency is the ratio of the stage output to input energy for the stage.

$$\text{Rate of the energy supplied to the stage} = \dot{m}(h_0 - h_1) \tag{9.32}$$

$$\eta_{\text{stage}} = \frac{\text{Rate of the work done on the blades}}{\text{Rate of energy supplied to the stage}} = \frac{\dot{m}\Delta C_w \cdot C_b}{\dot{m}(h_0 - h_1)}$$

$$\eta_{\text{stage}} = \frac{\Delta C_w \cdot C_b}{(h_0 - h_1)} \tag{9.33}$$

By Eqs (9.25), (9.31) and (9.33),

$$\eta_{\text{stage}} = \eta_n \eta_D \tag{9.34}$$

The stage efficiency can also be defined using total properties as in Eqs (8.104) and (8.105). This practice is however, less used in steam turbines.

(h) Overall Efficiency

The overall stage efficiency is the ratio of rate of work imparted to turbine shaft (or specific turbine shaft work) to the rate of energy input to the stage (or specific energy input to the stage).

$$\eta_o = \frac{\text{Rate of work delivered at the turbine shaft}}{\text{Rate of energy input to the stage}} = \frac{P_{\text{shaft}}}{\dot{m}(h_0 - h_1)} \tag{9.35}$$

$$\eta_o = \frac{P_{\text{shaft}}}{P_D} \times \frac{P_D}{\dot{m}C_1^2/2} \times \frac{\dot{m}C_1^2/2}{\dot{m}(h_0 - h_1)} = \eta_m \eta_D \eta_n$$

$$\eta_o = \eta_n \eta_D \eta_m \tag{9.36}$$

The student may easily draw the similarity between Eqs (9.35) and (2.30).

9.4.3 Graphical Method

The diagram or blading work or diagram or blading efficiency under a certain operating condition can also be estimated graphically by drawing the inlet and exit velocity triangles to scale. Velocity at exit from the nozzle may be found if the state of steam at the inlet to the nozzles, the exit pressure and nozzle efficiency are known. Steady flow energy equation is,

$$\dot{m}\left(h_1 + \frac{C_1^2}{2} + gZ_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{C_2^2}{2} + gZ_2\right) + \dot{W}$$

Let us consider that the state of steam at the inlet to the nozzle and the exit pressure is known. Also, considering the flow of steam through the nozzle to be isentropic, $\dot{Q} = 0$, and since there is no work interaction involved, $\dot{W} = 0$, and neglecting the potential energy change, the steady flow energy equation for the nozzle reduces to

$$h_0 + \frac{C_0^2}{2} = h_1 + \frac{C_1^2}{2} \tag{9.37}$$

If the velocity of approach, i.e. the inlet velocity, C_0, is small, the velocity at the exit from the nozzle is given by,

$$C_1 = [2(h_0 - h_1) \times 10^3]^{1/2} = 44.72(h_0 - h_1)^{1/2} \text{ m/s} \tag{9.38}$$

where, $(h_0 - h_1)$ is the enthalpy drop across the nozzle in kJ/kg. Due to the friction between the fluid and walls of the nozzle, the expansion process is irreversible; although, still approximately adiabatic, i.e. the flow through the nozzle is not isentropic. Nozzle efficiency η_n is used to take friction into account, which is defined as the ratio of the actual enthalpy drop to the isentropic enthalpy drop. Therefore, the velocity at the exit from the nozzle is given by,

$$C_1 = 44.72[(h_0 - h_1)\eta_n]^{1/2} \tag{9.39}$$

Knowing D and N, C_b is found out from

$$C_b = \frac{\pi DN}{60} \tag{9.40}$$

With α_1 and k being given and with the blades being symmetrical ($\beta_1 = \beta_2$), a scale for the diagram is chosen, say, 1 cm = 50 m/s A horizontal line AB is drawn, the length of which is proportional to C_b, as shown in Figure 9.18.

A straight line is drawn from B making an angle α_1 and a length BC proportional to C_1 be cut off. CA is joined. The angle $\angle CAD$ is measured. This is the inlet blade angle β_1. CA is measured, which represents C_{r1}. Again, a straight line AE is drawn from A making an angle ($\beta_1 = \beta_2$) and a length AE is cut off proportional to C_{r2}(= kC_{r1}). EB is joined. The velocity diagram is thus completed. From this diagram, ΔC_w, C_1 and C_2 are measured. Then the desired particulars,

$$F_t = \dot{m}\,\Delta C_w,\ F_a = \dot{m}\,(C_{f1} - C_{f2}),\ P_D = \dot{W}_D = F_t \cdot C_b = \dot{m}\,\Delta C_w C_b,\ \eta_D = \frac{2\Delta C_w \cdot C_b}{C_1^2} \text{ are estimated.}$$

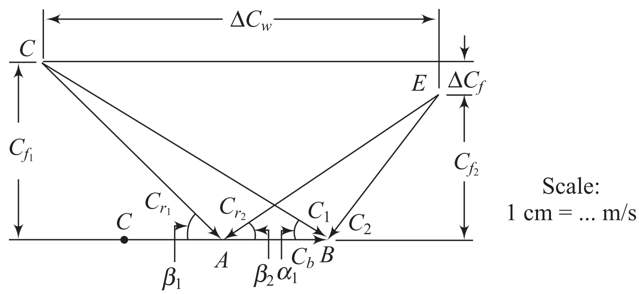


Figure 9.18 *Drawing of Velocity Diagram on Scale*

9.4.4 Alternative Way of Drawing Velocity Diagrams

The velocity diagrams with symmetrical impulse blading can also be drawn in a different way, as shown in Figure 9.19. The inlet velocity triangle ABC is drawn first as before. A length CD equal to (kC_{r1}) is cut off from C to represent C_{r2}, since $\beta_1 = \beta_2$. ED is drawn from D, the length being proportional to $C_b \cdot CE$ is joined. The absolute exit velocity C_2 is the vector sum of C_b and C_{r2}, as shown. Then, C_{w1}, C_{w2}, C_{f1}, and C_{f2} are measured from the diagram to estimate the different output parameters.

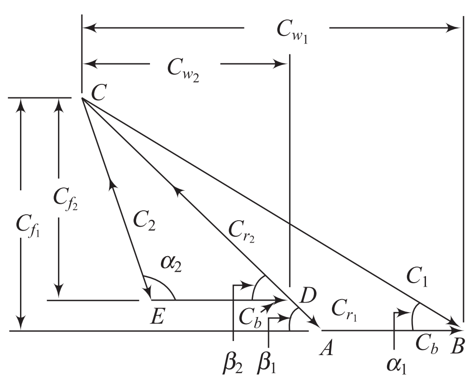


Figure 9.19 *Velocity Diagram for Impulse Turbine with Symmetrical Blades*

If the blade friction factor k is unity, i.e. there is no energy loss due to friction as steam flows through the blades, and the discharge is axial, i.e. parallel to the axis of the shaft, for which $\alpha_2 = 90°$, the resultant velocity diagram becomes as shown in Figure 9.20. The horizontal line $AB(=2C_b)$ is drawn such that $AC = CB = C_b$. From A, a perpendicular is drawn to AB and from B, a straight line is drawn making an angle α_1, which cuts the perpendicular at D, then ΔDCA is the exit velocity triangle.

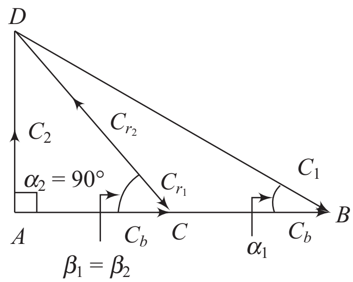


Figure 9.20 *Velocity Diagram for Frictionless Symmetrical Blades with Axial Discharge*

EXAMPLE 9.1 The steam leaves the nozzle at an angle of 18° and enters in a simple impulse turbine with a velocity of 1,200 m/s. The blades of the turbine are symmetrical and the mean peripheral velocity of the blades is 500 m/s. If the steam is to enter the blades without shock, (a) What will be the blade angles? (b) Neglecting the friction effects on the blades, calculate the tangential force on the blades and diagram power if the mass flow rate is 0.78 kg/s. Also determine the axial thrust and diagram efficiency. (c) If the relative velocity at the exit is reduced by friction to 85% of that at the inlet, calculate the axial thrust, diagram power, and diagram efficiency.

Solution

Given: $\alpha_1 = 18°$, $C_1 = 1{,}200$ m/s, $\beta_1 = \beta_2$, $C_b = 500$ m/s, $m = 0.78$ kg/s

The velocity triangles are shown in Figure 9.21.

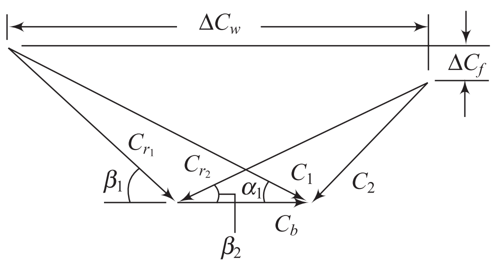


Figure 9.21 *Velocity Diagram of Example 9.1*

(a) Blade Angles

From the velocity triangle at the inlet,

$$C_{f1} = C_{r1} \sin \beta_1 = C_1 \sin \alpha_1 \tag{1}$$

$$C_{r1} \cos \beta_1 + C_b = C_1 \cos \alpha_1 \Rightarrow C_{r1} \cos \beta_1 = C_1 \cos \alpha_1 - C_b \tag{2}$$

Dividing Eq. (1) by (2), we get,

$$\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - C_b} \Rightarrow \tan \beta_1 = \frac{1200 \times \sin 18}{1200 \times \cos 18 - 500}$$

$$\therefore \quad \beta_1 = \beta_2 = 30.039° \tag{3}$$

(b) Neglecting Blade Friction

Since the friction on the blades is neglected, therefore, blade friction coefficient is unity, i.e. $k = 1 \Rightarrow C_{r2} = C_{r1}$. Substituting the values in Eq. (1), we get,

$$C_{r1} \sin 30.039 = 1200 \times \sin 18$$

$$\therefore \quad C_{r1} = C_{r2} = 740.768 \text{ m/s} \tag{4}$$

$$C_{w1} = C_1 \cos \alpha_1 \Rightarrow C_{w1} = 1200 \times \cos 18$$

$$C_{w1} = 1141.268 \text{ m/s} \tag{5}$$

$$C_{w2} = C_{r2} \cos \beta_2 - C_b \Rightarrow C_{w2} = 740.768 \times \cos 30.039 - 500$$

$$C_{w2} = 141.272 \text{ m/s} \tag{6}$$

Tangential force on the blades, i.e. whirling force,

$$F_t = \dot{m}\,(C_{w1} - C_{w2}) \tag{7}$$

$$F_t = 0.78 \times [1141.268 - (-141.272)]$$

C_{w2} is taken negative as it is found to be in opposite direction to that of C_{w1}.

$$F_t = 1000.38 \text{ N} \tag{8}$$

Diagram power is given by,

$$P_D = \dot{m}\,(C_{w1} - C_{w2})\, C_b = F_t C_b \tag{9}$$

$$P_D = 0.78 \times [1141.268 - (-141.272)] \times 500$$

$$P_D = 500190.6 \text{ W} = 500.1906 \text{ kW} \tag{10}$$

The energy input to the blades is the kinetic energy of jets issuing out from the nozzles.

$$\text{Energy input to the blades/s} = \text{KE of jets issuning out from nozzles/s} = \frac{1}{2}\dot{m}C_1^2 \tag{11}$$

where $\dot{m}$ is the mass flow rate in kg/s.

$$\text{Energy input to the blades/s} = \frac{1}{2} \times 0.78 \times 1200^2$$

$$\text{Energy input to the blades/s} = 561600 \text{ W} = 561.6 \text{ kW} \tag{12}$$

$$\eta_D = \frac{\text{Diagram work/s, i.e. Diagram Power}}{\text{Energy input to the blades}/s} \tag{13}$$

$$\eta_D = \frac{\text{Diagram work/s, i.e. Diagram Power}}{\text{Energy input to the blades/s}} = \frac{500.1906}{561.6}$$

$$\eta_D = 0.890653 = 89.0653\% \tag{14}$$

$$F_a = \dot{m}\Delta C_f = \dot{m}(C_{r1} \sin \beta_1 - C_{r2} \sin \beta_2) \tag{15}$$

Since, blades are equiangular, i.e. $\beta_1 = \beta_2$, and the friction on the blades is neglected, therefore, blade friction coefficient is zero, i.e. $k = 1 \Rightarrow C_{r2} = C_{r1} \Rightarrow C_{f1} = C_{f2}$. Hence,

$$F_a = 0 \tag{16}$$

(c) Considering Friction

$$C_{r2} = 0.85\ C_{r1} \Rightarrow C_{r2} = 0.85 \times 740.768$$

$$C_{r2} = 629.653 \text{ m/s} \tag{17}$$

$$F_a = \dot{m}\Delta C_a = \dot{m}(C_{r1} \sin\beta_1 - C_{r2} \sin\beta_2)$$

$$F_a = 0.78 \times (740.768 \sin 30.039 - 629.653 \sin 30.039)$$

$$F_a = 43.386 \text{ N} \tag{18}$$

$$C_{w2} = C_{r2} \cos\beta_2 - C_b \Rightarrow C_{w2} = 629.653 \times \cos 30.039 - 500$$

$$C_{w2} = 45.0811 \text{ m/s} \tag{19}$$

Diagram power is given by,

$$P_D = \dot{m}\,(C_{w1} - C_{w2})\,C_b = F_t C_b = 0.78 \times [1141.268 - (-45.0811)] \times 500$$

$$P_D = 462676.149 \text{ W} = 462.676 \text{ kW} \tag{20}$$

The energy input to the blades is the kinetic energy of jets issuing out from the nozzles.

$$\text{Energy input to the blades/s} = \text{KE of jets issuing out from nozzles/s} = \frac{1}{2}\dot{m}C_1^2$$

where $\dot{m}$ is the mass flow rate in kg/s. Energy input to the blades will remain the same as that of Case (b) as $\dot{m}$ and C_1 are same.

$$\text{Enegy input to the blades/s} = 561600 \text{ W} = 561.6 \text{ kW}$$

$$\eta_D = \frac{\text{Diagram work/s i.e. Diagram Power}}{\text{Energy input/to the blades/s}}$$

$$\eta_D = \frac{\text{Diagram work/s i.e. Diagram Power}}{\text{Energy input/to the blades/s}} = \frac{462.676}{561.6}$$

$$\eta_D = 0.82385 = 82.385\% \tag{21}$$

EXAMPLE 9.2 The nozzles of an impulse turbine stage receive steam at 16 bar, 350°C and discharge is at 12 bar. The nozzle angle is 18° and nozzle efficiency is 97%. The turbine runs at the speed which is required for maximum work. The inlet angle of the blades is designed such that steam enters without shock. The blade angle at the exit is 5° less than that at the inlet. The blade friction coefficient is 0.88 and flow rate of the steam is 1340 kg/hr. Calculate (a) axial thrust, (b) diagram power, and (c) diagram efficiency.

Solution

$p_0 = 16$ bar, $t_0 = 350$°C, $p_1 = 12$ bar, $\alpha_1 = 18°$, $\eta_n = 97\% = 0.97$, $\beta_2 = \beta_1 - 5$, $k = 0.88$, $\dot{m} = 1340$ kg/hr. The states of the steam at nozzle entry and exit can be located by the steam table or Mollier chart as shown in Figure 9.22 (a). From the properties of steam,

$$h_0 = 3145.4 \text{ kJ/kg},\ s_0 = 7.0694 \text{ kJ/kg-K},\ (s_g)_{12\text{ bar}} = 6.5233 \text{ kJ/kg-K}$$

$$\because \quad s_{1s} = s_0 = 7.0694 \text{ kJ/kg-K} > (s_g)_{12\text{ bar}}$$

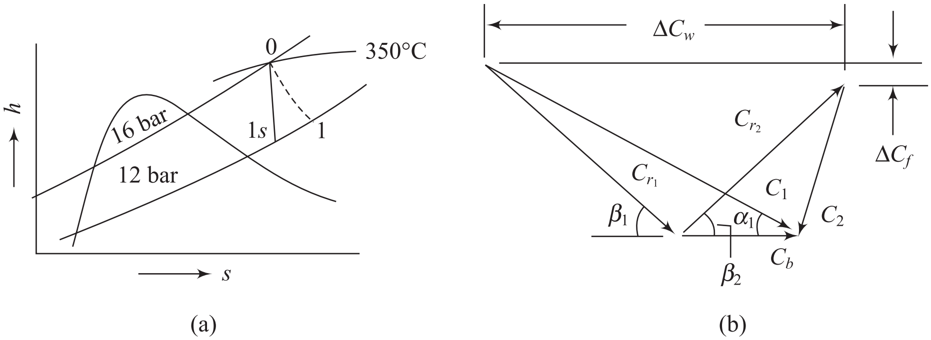


Figure 9.22 *(a) State of Steam at Nozzle Entry and Exit, (b) Velocity Diagram of Example 9.2*

Therefore, state 1*s* is in the superheated region.

t_{1s} = 310.45°C, h_{1s} = 3068.33 kJ/kg.. The velocity of the jet of steam at nozzle outlet,

$$C_1 = \sqrt{2\eta_n (h_0 - h_{1s}) + C_0^2} \tag{1}$$

$$C_1 = \sqrt{2 \times 0.97 \times 1000 \times (3145.4 - 3068.33) + 0} \Rightarrow C_1 = 386.673 \text{ m/s} \tag{2}$$

Optimum velocity ratio for maximum efficiency of impulse blading,

$$\rho_{\text{opt}} = \frac{\cos \alpha_1}{2} \Rightarrow \frac{C_b}{C_1} = \frac{\cos \alpha_1}{2} \tag{3}$$

$$\frac{C_b}{386.673} = \frac{\cos 18}{2}$$

$$C_b = 183.874 \text{ m/s} \tag{4}$$

The velocity triangles are shown in Figure 9.22 (b).

$$\tan \beta_1 = \frac{C_{f1}}{C_{w1} - C_b} \Rightarrow \tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - C_b} \tag{5}$$

$$\tan \beta_1 = \frac{386.673 \sin 18}{386.673 \cos 18 - 183.874}$$

$$\beta_1 = 33.017° \tag{6}$$

$$\beta_2 = \beta_1 - 5 = 33.017 - 5$$

$$\beta_2 = 28.017 \tag{7}$$

$$C_{f1} = C_{r1} \sin \beta_1 = C_1 \sin \alpha_1 \Rightarrow C_{r1} = \frac{C_1 \sin \alpha_1}{\sin \beta_1} \tag{8}$$

$$C_{r1} = \frac{386.673 \sin 18}{\sin 33.017}$$

$$C_{r1} = 219.29 \text{ m/s} \tag{9}$$

$\because \qquad k = \dfrac{C_{r2}}{C_{r1}} \Rightarrow 0.88 = \dfrac{C_{r2}}{219.29}$

$C_{r2} = 192.985$ m/s (10)

(a) Axial Thrust

$$F_a = \dot{m}\Delta C_f = \dot{m}(C_{r1} \sin \beta_1 - C_{r2} \sin \beta_2) \quad (11)$$

$$F_a = \frac{1340}{3600} \times (219.29 \times \sin 33.017 - 192.975 \times \sin 28.017$$

$F_a = 10.7355$ N (12)

(b) Diagram Power

$\Delta C_w = C_{r1} \cos \beta_1 + C_{r2} \cos \beta_2 \Rightarrow \Delta C_w = 219.29 \cos 33.017 + 192.975 \cos 28.017$

$\Delta C_w = 354.23655$ m/s (13)

Diagram power,

$$P_D = \dot{m}(C_{w1} - C_{w2})C_b = \dot{m}(\Delta C_w)C_b \quad (14)$$

$$P_D = \frac{1340}{3600} \times (354.23655) \times 183.874$$

$P_D = 24244.654$ W = 24.2465 kW (15)

(c) Diagram Efficiency

Energy input to the blades/s = KE of jets issuing out from nozzles/s = $\frac{1}{2}\dot{m}C_1^2$ (16)

where $\dot{m}$ is the mass flow rate in kg/s.

$$\text{Energy input to the blades/s} = \frac{1}{2} \times \frac{1340}{3600} \times 386.673^2$$

Energy input to the blades/s = 27826.59055 W = 27.8266 kW (17)

$$\eta_D = \frac{\text{Diagram work/}s\text{ i.e. Diagram Power}}{\text{Energy input /to the blades/}s} \quad (18)$$

$$\eta_D = \frac{24.2465}{27.8266}$$

$\eta_D = 0.87134 = 87.134\%$ (19)

9.5 Analysis of Pressure Compounding or Rateau Staging

The pressure drops occurs only in the nozzles in pressure compounded or Rateau staging of impulse turbines. There is no pressure drop (theoretically) while steam flows through the blades. The kinetic energy of steam increases in the nozzles at the expense of the pressure drop and it is absorbed (partially) by the blades in each stage, in producing torque or power. The total enthalpy drop is divided equally among the stages in pressure compounded steam turbine, as shown in Figure 9.23.

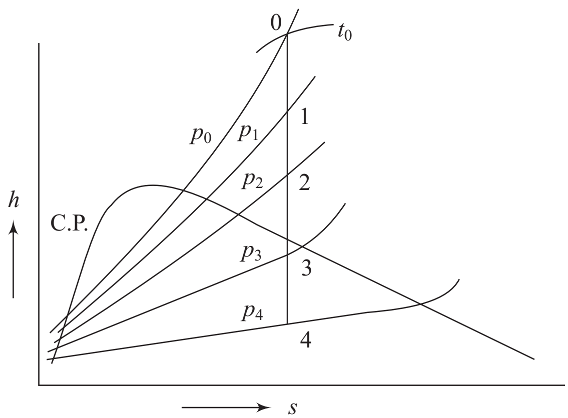


Figure 9.23 *Enthalpy Drop per stage in a 4-Stage Pressure Compounded Turbine*

The total isentropic enthalpy drop of steam ($h_0 - h_4$) is divided equally among the four stages of the turbine. In Mollier diagram, the enthalpy drop ($h_0 - h_4$) is measured, then the enthalpy drop, $h_0 - h_1 = h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = (h_0 - h_4/4)$ is computed and inserted on the isentropic line. The inter stage pressures noted from the diagram are p_1 , p_2 and p_3, i.e. the pressure after first stage is p_1, the pressure after second stage is p_2 and so on.

The velocity of steam at the exit from the first row of nozzles is

$$C_1 = 44.72(h_0 - h_1)^{½} = 44.72[(h_0 - h_4)/4]^{½}$$

$$C_1 = \frac{1}{2}(44.72)[(h_0 - h_4)]^{1/2} \tag{9.41}$$

If the kinetic energy of steam at the inlet to each subsequent row of nozzles is small and neglected, then the velocity of steam leaving the nozzles in each row will be the same as given by Eq. (9.41). For a single-stage turbine, the jet velocity would have been

$$C_1 = 44.72[h_0 - h_4]^{½} \tag{9.42}$$

Therefore, for a 4-stage turbine, the velocity of steam leaving the nozzles in each stage is half of that for a single-stage turbine. For a 9-stage turbine, it will be one third.

For each impulse stage operating at its maximum blading efficiency, the blade velocity is given by,

$$C_b = \frac{\cos\alpha_1}{2}C_1 \tag{9.43}$$

Therefore, along with C_1, C_b also gets halved for a 4-stage turbine, and for a given D, N gets halved or for a given N, D gets halved. With more stages, N or D would further decrease.

If there are n stages in series in the turbine, the isentropic enthalpy drop per stage would be,

$$(\Delta h_s)_{\text{stage}} = \frac{(\Delta h_s)_{\text{total}}}{n} \tag{9.44}$$

From Eq. (9.43), and for an impulse stage,

$$C_1 = \frac{2}{\cos\alpha_1}C_b = 44.72[(\Delta h_s)_{\text{stage}}]^{1/2}$$

$$(\Delta h_s)_{\text{stage}} = \left[\frac{2C_b}{44.72 \cos \alpha_1}\right]^2 = 4\left[\frac{C_b}{44.72 \cos \alpha_1}\right]^2 \tag{9.45}$$

From Eqs. (9.44) and (9.45), the number of impulse stages required for a certain enthalpy drop $(\Delta h_s)_{\text{total}}$ can thus be estimated under ideal condition, i.e.,

$$n = \frac{(\Delta h_s)_{\text{total}}}{(\Delta h_s)_{\text{stage}}} \tag{9.46}$$

EXAMPLE 9.3 An impulse steam turbine has a number of pressure stages, each having a row of nozzles and a single ring of moving blades. The velocity of steam leaving the nozzles is 350 m/s. The nozzle angle in the first stage is 20° and blade angle at the outlet is 30° to the plane of rotation. The mean peripheral speed of the blades is 140 m/s. Determine the specific work done in the stage and stage efficiency if the blade friction coefficient is 0.85 and nozzle efficiency is 90%.

Solution

Given: $C_1 = 350$ m/s, $\alpha_1 = 20°$, $\beta_2 = 30°$, $C_b = 140$ m/s, $k = 0.85$, $\eta_n = 0.9$.

(a) Stage Specific Work and Stage Efficiency

The velocity triangles for the turbine are shown in Figure 9.24.

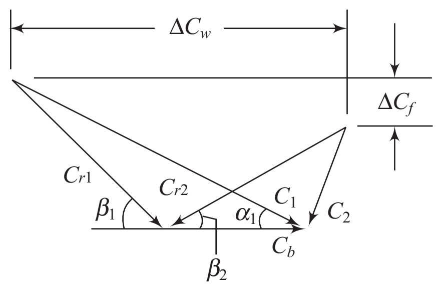


Figure 9.24 *Velocity Diagram of Example 9.3*

$$\tan \beta_1 = \frac{C_{f1}}{C_{w1} - C_b} = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - C_b} \tag{1}$$

$$\tan \beta_1 = \frac{350 \sin 20}{350 \cos 20 - 140}$$

$$\beta_1 = 32.364° \tag{2}$$

$$C_{f1} = C_{r1} \sin \beta_1 = C_1 \sin \alpha_1 \Rightarrow C_{r1} = \frac{C_1 \sin \alpha_1}{\sin \beta_1} \tag{3}$$

$$C_{r1} = \frac{350 \times \sin 20}{\sin 32.364}$$

$$C_{r1} = 223.628 \text{ m/s} \tag{4}$$

$$k = \frac{C_{r2}}{C_{r1}} \Rightarrow 0.85 = \frac{C_{r2}}{223.628}$$

$$C_{r2} = 190.084 \text{ m/s} \tag{5}$$

$$\Delta C_w = C_{r1} \cos \beta_1 + C_{r2} \cos \beta_2 \tag{6}$$

$$\Delta C_w = 223.628 \cos 32.364 + 190.084 \cos 30$$

$$\Delta C_w = 353.5082 \text{ m/s} \tag{7}$$

Specific diagram work,

$$w_D = (C_{w1} - C_{w2})\, C_b = \Delta C_w C_b \tag{8}$$

$$W_D = 353.5082 \times 140$$

$$w_D = 49491.148 \text{ J/kg} = 49.49115 \text{ kJ/kg} \tag{9}$$

$$\text{Specific input energy to the blade} = \frac{1}{2}C_1^2 \tag{10}$$

Diagram efficiency,

$$\eta_D = \frac{w_D}{\frac{1}{2}C_1^2} \Rightarrow \eta_D = \frac{49491.148}{\frac{1}{2}\times 350^2}$$

$$\eta_D = 0.80802 = 80.802\% \tag{11}$$

Stage efficiency is given by,

$$\eta_s = \eta_n \eta_D \tag{12}$$

$$\eta_s = 0.9 \times 0.80802$$

$$\eta_s = 0.7272 = 72.72\% \tag{13}$$

9.6 Analysis of Velocity Compounding or Curtis Staging

9.6.1 Velocity Diagrams

As discussed earlier, in velocity compounding, there is only one row of nozzles and two or more rows of moving blades. There is a row of fixed guide blades in between the moving blade rows. The enthalpy or pressure drop takes place only in the nozzles in the first stage and it is converted into kinetic energy.

The velocity diagrams for the first and second row of moving blades are shown, respectively, in Figure 9.25 (a) and (b).

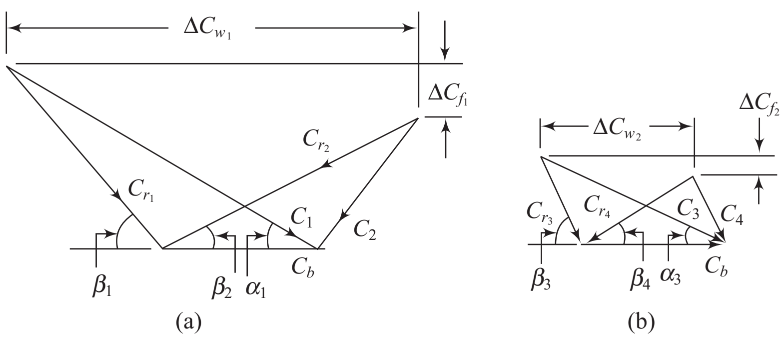


Figure 9.25 *Velocity Diagrams for a 2—Row Curtis Stage (a) First Row of Moving Blades, (b) Second Row of Moving Blades*

Expansion of high pressure steam takes place in the nozzle first and the steam exiting the nozzle passes through the first row of moving blades. Exiting steam of first row of moving blades then passes through a row of fixed blades. Fixed blades attached to the casing just act like guide blades. Fixed blades guide the steam to enter smoothly into the following (second) row of moving blades. Therefore, nozzle, first row of moving blades, fixed blades row and second moving blades row, together constitute a stage of 2-row velocity compounded impulse turbine. Assuming that outlet of nozzle at section 1-1, first moving blade row outlet at section 2-2, fixed blade row outlet at section 3-3 and second row of moving blades outlet at section 4-4.

Absolute and relative velocities and various angles at sections 1-1, 2-2, 3-3 and 4-4 are illustrated in velocity triangles of Figure 9.25.

Steam admits into first moving blades row at section 1-1 with absolute velocity C_1 and exits it with velocity C_2 at an angle α_2. Steam flows through stationary blades row and exits it with absolute velocity C_3 at an angle α_3. Finally, steam exits the last row of moving blades with absolute velocity C_4 at an angle α_4. For symmetrical blades $\beta_1 = \beta_2$ and $\beta_3 = \beta_4$, the velocity triangles can also be drawn as shown in Figure 9.26.

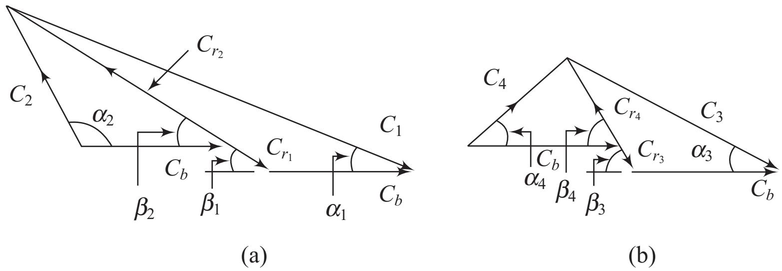


Figure 9.26 *Velocity Diagrams for a 2—Row Curtis Stage with Symmetrical Blades (a) First Row of Moving Blades, (b) Second Row of Moving Blades*

9.6.2 Stage Parameters

The blade friction factor k may be assumed the same for both moving and guide blades. Hence,

$$\frac{C_{r2}}{C_{r1}} = \frac{C_3}{C_2} = \frac{C_{r4}}{C_{r3}} = k \tag{9.47}$$

α_1 = Exit angle of the guide blades

β_1, β_2 = Inlet and exit angles of the first row of moving blades

β_3, β_4 = Inlet and exit angles of the second row of moving blades

ΔC_{w1}, ΔC_{w2} = Changes in the velocity of whirl in the first and second rows of moving blades

ΔC_{f1}, ΔC_{f2} = Changes in the axial components of velocity in the first and second rows of moving blades

The tangential thrust,

$$F_t = \dot{m}\sum \Delta C_w = \dot{m}(\Delta C_{w1} + \Delta C_{w2}) \tag{9.48}$$

The axial thrust,

$$F_a = \dot{m}\sum \Delta C_f = \dot{m}(\Delta C_{f1} + \Delta C_{f2}) \tag{9.49}$$

Blading or diagram work,

$$\dot{W}_D = F_t C_b = \dot{m}(\Delta C_{w1} + \Delta C_{w2}) C_b \tag{9.50}$$

Blading or diagram efficiency,

$$\eta_D = \frac{2(\Sigma \Delta C_w) C_b}{C_1^2} = \frac{2(\Delta C_{w1} + \Delta C_{w2}) C_b}{C_1^2} \tag{9.51}$$

These parameters can be estimated either graphically by drawing the velocity diagrams to scale or trigonometrically.

9.6.3 Effectiveness of Moving Rows

Consider a two-row Curtis stage with frictionless symmetrical blading for simplicity. Therefore, $\beta_1 = \beta_2$, $\beta_3 = \beta_4$, and the blade friction factor $k = 1$ Also, let steam leaves the stage axially, therefore, $\alpha_4 = 90°$. The velocity diagrams of the first and second rows of moving blades can be combined into one diagram as shown in Figure 9.27.

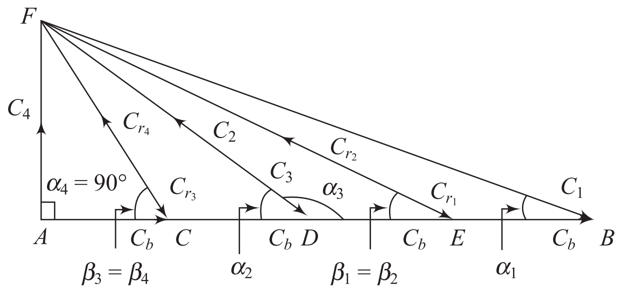


Figure 9.27 *Velocity Diagram of a 2-Row Curtis Stage with Frictionless Symmetrical Blading and Axial Discharge*

The horizontal line $AB = 4C_b$ is drawn such that $AC = CD = DE = EB = C_b$. At A, a perpendicular is drawn, and from B, a line is drawn making an angle α_1 with AB, which cuts the perpendicular at point F. FC, FD and FE are joined. The directions of velocities are shown in the diagram.

Since, $\alpha_2 > 90°, \Delta C_{w1} = C_1 \cos \alpha_1 + C_2 \cos \alpha_2 = 4C_b + 2C_b$

$$\Delta C_{w1} = 6C_b \tag{9.52}$$

Now, $\Delta C_{w2} = C_3 \cos \alpha_3 - C_4 \cos \alpha_4$

$$\Delta C_{w2} = 2C_b \tag{9.53}$$

Work done by the steam jets on the first row of moving blades,

$$W_{D1} = \dot{m}\Delta C_{w1}C_b = \dot{m} \times 6C_b \times C_b$$

$$\dot{W}_{D1} = 6\dot{m}C_b^2 \tag{9.54}$$

Work done by the steam jets on the second row of moving blades,

$$\dot{W}_{D2} = \dot{m}\Delta C_{W2}C_b = \dot{m} \times 2C_b \times C_b$$

$$\dot{W}_{D2} = 2\dot{m}C_b^2 \tag{9.55}$$

Thus,

$$W_{D1} : W_{D2} = 6 : 2 = 3 : 1 \tag{9.56}$$

Equation (9.56) represents that three-fourth of the total work is done by the steam jets on the first row of moving blades and one-fourth of the total work is done on the second row of moving blades.

In a 3-row Curtis stage, it can similarly be shown that,

$$W_{D1} : W_{D2} : W_{D3} = 5 : 3 : 1 \tag{9.57}$$

Equation (9.57) represents that only one-ninth of the total work in the stage is done in the third row of moving blades. Therefore, in a Curtis stage, the effectiveness of succeeding (last) rows decreases with the number of rows of moving blades. In a conventional Curtis stage design, only two rows of moving blades are used. It is not justifiable to use more than two rows.

9.6.4 Optimum Velocity Ratio

Let us again consider a velocity compounded two-row Curtis turbine having frictionless symmetrical blading but with discharge not being axial. The velocity diagram is shown in Figure 9.28.

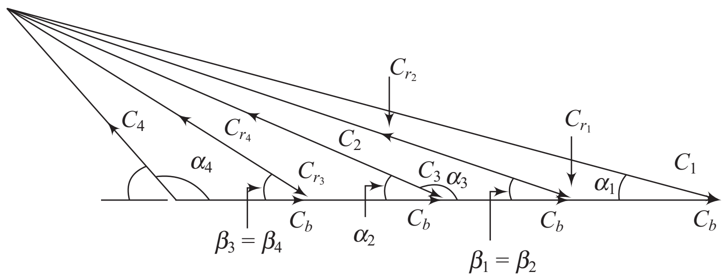


Figure 9.28 *Velocity Diagram for a 2—Row Curtis Stage with Frictionless Symmetrical Blading and Non-Axial Discharge*

$$\Delta C_{w1} = C_1 \cos \alpha_1 + C_2 \cos \alpha_2 = C_1 \cos \alpha_1 + C_1 \cos \alpha_1 - 2C_b$$

$$\Delta C_{w1} = 2(C_1 \cos \alpha_1 - C_b) \tag{9.58}$$

$$\Delta C_{w2} = C_3 \cos \alpha_3 + C_4 \cos \alpha_4 = C_1 \cos \alpha_1 - 2C_b + C_1 \cos \alpha_1 - 4C_b$$

$$\Delta C_{w2} = 2(C_1 \cos \alpha_1 - 3C_b) \tag{9.59}$$

$$\Sigma\Delta C_w = \Delta C_{w1} + \Delta C_{w2} = 4(C_1 \cos \alpha_1 - 2C_b) \tag{9.60}$$

The rate of energy transfer from fluid to rotor,

$$\dot{W}_D = \dot{m}(\Sigma\Delta C_w)C_b \tag{9.61}$$

And the diagram efficiency (fraction of fluid energy converted to work that is available at the rotor),

$$\eta_D = \frac{2\Sigma\Delta C_w \cdot C_b}{C_1^2} = \frac{2 \times 4(C_1 \cos\alpha_1 - 2C_b)C_b}{C_1^2} = \frac{8C_b^2\left(\dfrac{C_1 \cos\alpha_1}{C_b} - 2\right)}{C_1^2}$$

$$\eta_D = 8\rho^2\left(\frac{\cos\alpha_1}{\rho} - 2\right) = 8(\rho \cos\alpha_1 - 2\rho^2) \tag{9.62}$$

For maximum diagram efficiency,

$$\frac{d\eta_D}{d\rho} = 0$$

$$\frac{d\eta_D}{d\rho} = 8(\cos\alpha_1 - 4\rho) = 0$$

$$\rho_{\text{opt}} = \frac{\cos\alpha_1}{4} \tag{9.63}$$

For a three-row Curtis stage, it can similarly be shown that,

$$\rho_{opt} = \frac{\cos\alpha_1}{6} \tag{9.64}$$

Therefore, for a Curtis stage having n-rows of moving blades,

$$\rho_{opt} = \frac{\cos\alpha_1}{2n} \tag{9.65}$$

If $n = 1$, i.e. for a simple impulse stage, $\rho_{opt} = \frac{\cos\alpha_1}{2}$, as obtained earlier.
For a two-row Curtis stage, $n = 2$,

$$\rho_{opt} = \frac{C_b}{C_1} = \frac{\cos\alpha_1}{4}$$

Hence, $C_b = \frac{1}{2}\left(\frac{\cos\alpha_1}{2} C_1\right)$

Therefore, in a 2-row Curtis stage, the blade velocity is half of the value for a single-stage turbine. For certain D, N gets halved. Substituting in Eq. (9.62),

$$(\eta_D)_{max} = 8\left(\frac{\cos^2\alpha_1}{4} - \frac{\cos^2\alpha_1}{8}\right)$$

$$(\eta_D)_{max} = \cos^2\alpha_1 \tag{9.66}$$

Equations (9.63) and (9.66) are valid for the conditions as specified before, i.e. there is no friction and blades are symmetrical, and the discharge is non-axial. The same efficiency, i.e. $\cos^2\alpha_1$, was also obtained for symmetrical and frictionless simple impulse blading [Eq. (9.30)]. However, had friction been taken into account, the blading efficiency of the Curtis stage would have been considerably lower than the efficiency of the simple impulse blading.

$$C_1 = \frac{4C_b}{\cos\alpha_1} = 44.72[(\Delta h_s)_{Curtis}]^{1/2}$$

Hence, isentropic enthalpy drop in the 2-row Curtis stage,

$$(\Delta h_s)_{Curtis} = 16\left[\frac{C_b}{44.72\cos\alpha_1}\right]^2 \tag{9.67}$$

EXAMPLE 9.4 The following data pertains to a two-row velocity compounded impulse turbine:
Steam velocity at nozzle exit = 610 m/s, Nozzle angle = 18°, Mean blade speed 130 m/s,
Exit angles from: First row moving blades = 20°, fixed guide blades = 22°, Second row moving blades = 33°, Steam flow rate = 6 kg/s, Blade friction coefficient = 0.9.
Determine (a) whirling thrust, (b) axial thrust, (c) power developed, and (d) diagram efficiency.

Solution

Given: C_1 = 610 m/s, α_1 = 18°, C_b = 130 m/s, β_2 = 20°, α_3 = 22°, β_4 = 33°, $\dot{m}$ = 6 kg/s, k = 0.9

The velocity triangles are shown in Figure 9.29.

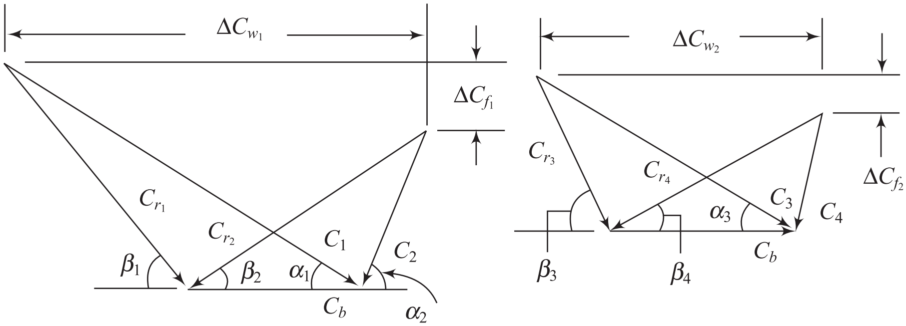


Figure 9.29 *Velocity Diagrams of Example 9.4*

$$\tan\beta_1 = \frac{C_{f1}}{C_{w1} - C_b} = \frac{C_1 \sin\alpha_1}{C_1\cos\alpha_1 - C_b} \tag{1}$$

$$\tan\beta_1 = \frac{610\sin 18}{610\cos 18 - 130}$$

$$\beta_1 = 22.722° \tag{2}$$

$$C_{f1} = C_{r1}\sin\beta_1 = C_1\sin\alpha_1 \Rightarrow C_{r1} = \frac{C_1\sin\alpha_1}{\sin\beta_1} \tag{3}$$

$$C_{r1} = \frac{610\sin 18}{\sin 22.722}$$

$$C_{r1} = 488.014 \text{ m/s} \tag{4}$$

$$k = \frac{C_{r2}}{C_{r1}} \Rightarrow C_{r2} = kC_{r1} \tag{5}$$

$$C_{r2} = 0.9 \times 488.014$$

$$C_{r2} = 439.2126 \text{ m/s} \tag{6}$$

$$\tan\alpha_2 = \frac{C_{r2}\sin\beta_2}{C_{r2}\cos\beta_2 - C_b} \Rightarrow \tan\alpha_2 = \frac{439.2126 \times \sin 20}{439.2126\cos 20 - 130}$$

$$\alpha_2 = 27.983° \tag{7}$$

$$\sin\alpha_2 = \frac{C_{f2}}{C_2} = \frac{C_{r2}\sin\beta_2}{C_2} \Rightarrow C_2 = \frac{C_{r2}\sin\beta_2}{\sin\alpha_2}$$

$$C_2 = \frac{439.2126 \times \sin 20}{\sin 27.983}$$

$$C_2 = 320.1545 \text{ m/s} \tag{8}$$

$$k = \frac{C_3}{C_2} \Rightarrow C_3 = kC_2 \Rightarrow C_3 = 0.9 \times 320.1545$$

$$C_3 = 288.139 \text{ m/s} \tag{9}$$

$$\Delta C_{w1} = C_1 \cos \alpha_1 + C_2 \cos \alpha_2 = C_{r1} \cos \beta_1 + C_{r2} \cos \beta_2 \tag{10}$$

$$\Delta C_{w1} = 610 \cos 18 + 320.1545 \cos 27.983$$

$$\Delta C_{w1} = 862.869 \text{ m/s} \tag{11}$$

Change in axial components of velocities or flow velocities in the first stage,

$$\Delta C_{f1} = C_1 \sin \alpha_1 - C_2 \sin \alpha_2 \Rightarrow \Delta C_{f1} = 610 \sin 18 - 320.1545 \sin 27.983$$

$$\Delta C_{f1} = 38.281 \text{ m/s} \tag{12}$$

$$\tan \beta_3 = \frac{C_3 \sin \alpha_3}{C_3 \cos \alpha_3 - C_b} \Rightarrow \tan \beta_3 = \frac{288.139 \sin 22}{288.139 \cos 22 - 130}$$

$$\beta_3 = 38.20° \tag{13}$$

$$C_{f3} = C_{r3} \sin \beta_3 = C_3 \sin \alpha_3 \Rightarrow C_{r3} = \frac{C_3 \sin \alpha_3}{\sin \beta_3} \tag{14}$$

$$C_{r3} = \frac{288.139 \sin 22}{\sin 38.20}$$

$$C_{r3} = 174.543 \text{ m/s} \tag{15}$$

$$k = \frac{C_{r4}}{C_{r3}} \Rightarrow C_{r4} = kC_{r3} = 0.9 \times 174.543$$

$$C_{r4} = 157.089 \text{ m/s} \tag{16}$$

$$\Delta C_{w2} = C_{r3} \cos \beta_3 + C_{r4} \cos \beta_4 \tag{17}$$

$$\Delta C_{w2} = 174.543 \cos 38.20 + 157.089 \cos 33$$

$$\Delta C_{w2} = 268.912 \text{ m/s} \tag{18}$$

$$\Delta C_{f2} = C_3 \sin \alpha_3 - C_{r4} \sin \beta_4 \Rightarrow \Delta C_{f2} = 288.139 \sin 22 - 157.089 \sin 33$$

$$\Delta C_{f2} = 22.382 \text{ m/s} \tag{19}$$

(a) Whirling Thrust

$$F_t = \dot{m}\Sigma\Delta C_w = \dot{m}(\Delta C_{w1} + \Delta C_{w2}) \tag{20}$$

$$F_t = 6 \times (862.869 + 268.912)$$

$$F_t = 6790.686 \text{ N} = 6.791 \text{ kN} \tag{21}$$

(b) Axial Thrust

$$F_a = \dot{m}\Sigma\Delta C_f = m(\Delta C_{f1} + \Delta C_{f2}) \tag{22}$$

$$F_a = 6 \times (38.281 + 22.382)$$

$$F_a = 363.978 \text{ N} = 0.36398 \text{ kN} \tag{23}$$

(c) Power Developed

$$P_D = \dot{m}\,(\Delta C_{w1} + \Delta C_{w2})\, C_b = F_t C_b \quad (24)$$

$$P_D = 6.791 \times 130$$

$$P_D = 882.83 \text{ kW} \quad (25)$$

(d) Diagram Efficiency

$$\eta_D = \frac{2\Delta C_w C_b}{C_1^2} = \frac{2(\Delta C_{w1} + \Delta C_{w2})C_b}{C_1^2} \quad (26)$$

$$\eta_D = \frac{2 \times (862.869 + 268.912) \times 130}{610^2}$$

$$\eta_D = 0.79082 = 79.082\% \quad (27)$$

9.7 Reaction Turbines

In reaction turbines, pressure drop occurs both in the nozzles or the fixed row of blades, as well as in the moving row of blades as shown in Figure 9.30, since the moving blade passage are also of converging type, i.e. of the nozzle shape. The expansion of steam while flowing through the blades causes an increase in kinetic energy, consequently, a reactive force in the opposite direction is produced by Newton's third law of motion.

Blades rotate due to both the impulse effect of the jets (due to change in their momentum) and the reaction force of the exiting jets exerted on the blades in the opposite direction. Such turbines are called impulse-reaction turbines, or to distinguish them from impulse turbines, simply reaction turbines.

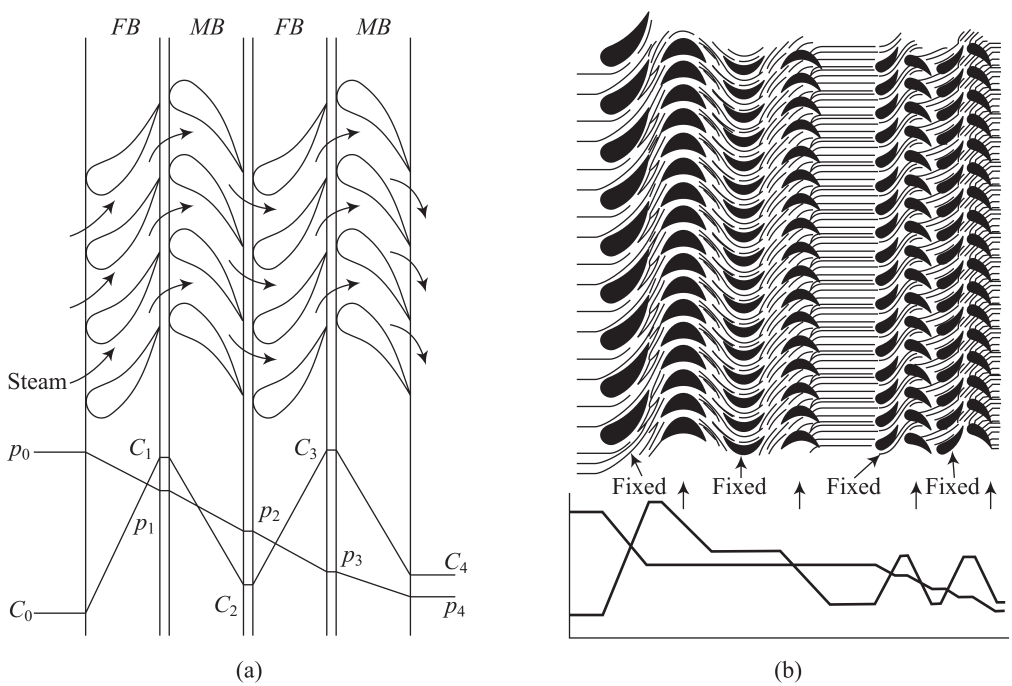


Figure 9.30 *(a) A 2—Stage Reaction Turbine, (b) A 2—Row Curtis Stage Followed by 2—Reaction Stages*

9.7.1 Velocity Diagrams

The velocity diagrams for the moving blades of a 50% reaction turbine are shown in Figure 9.31. In a simple impulse turbine, the value of C_{r2} (= kC_{r1}) would be given by BE, but in the reaction turbine this velocity is increased to BC by further expansion of the steam in the nozzle-shape blade channels.

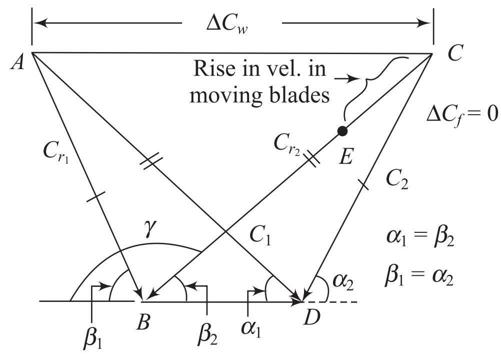


Figure 9.31 *Velocity Diagram for a 50% Reaction Turbine*

9.7.2 Stage Parameters

(a) Degree of Reaction

The degree of reaction (R) of these turbines is defined as,

$$R = \frac{\Delta h_{mb}}{\Delta h_{fb} + \Delta h_{mb}} \tag{9.68}$$

Where the subscripts *mb* and *fb* represent moving blades and fixed blades, respectively.

- If $\Delta h_{mb} = 0$, $R = 0$, which is the case of pure impulse turbines where there is no enthalpy drop of steam in the moving blades, and all the enthalpy drop of the stage takes place only in nozzles.
- If $\Delta h_{fb} = 0$, $R = 1$, which is the case of a pure reaction (R = 100%) turbine, e.g. Hero's turbine.
- If equal enthalpy drops occur in the fixed and moving blades, i.e. if $\Delta h_{fb} = \Delta h_{mb} = (\Delta h_{\text{stage}})/2$, $R = 1/2$ or 50%. Sometimes, 50% reaction turbines are also called *Parsons Turbines.*

(b) Specific Diagram Work

For manufacturing advantage, both fixed blades and moving blades are made similar in shape so that they can be extruded from the same set of dies.

Since, $\Delta h_{fb} = \Delta h_{mb}$, $C_1 = C_{r2}$.

Again, for similar geometry, $\alpha_1 = \beta_2 = 180° - \gamma$.

The triangles ABD and DBC are similar, therefore, $C_{r1} = C_2$ and $\beta_1 = \alpha_2$.

Since $\beta_1 \neq \beta_2$, the blades are unsymmetrical. Again, since $\Delta C_f = 0$, there is no axial thrust exerted on the blades due to change in axial velocity in a 50% reaction turbine. However, there will be considerable axial thrust produced due to the pressure difference across the blades in each rotor disc since there is pressure drop of steam across the moving blades.

$$\Delta C_w = C_1 \cos \alpha_1 + C_2 \cos \alpha_2 = C_1 \cos \alpha_1 + C_{r2} \cos \beta_2 - C_b$$

$$\Delta C_w = 2C_1 \cos \alpha_1 - C_b$$

Also, $$\Delta C_w = C_{r1} \cos \beta_1 + C_{r2} \cos \beta_2 = C_1 \cos \alpha_1 - C_b + C_1 \cos \alpha_1$$

$$\Delta C_w = 2C_1 \cos \alpha_1 - C_b \tag{9.69}$$

The diagram work per kg of steam, i.e. specific diagram work,

$$w_D = \Delta C_w C_b = (2C_1 \cos \alpha_1 - C_b)C_b \tag{9.70}$$

(c) Diagram Efficiency

$$\text{Energy input/kg of steam} = \frac{C_1^2}{2} + \frac{C_{r2}^2 - C_{r1}^2}{2} = \frac{C_1^2}{2} + \frac{C_1^2}{2} - \frac{C_{r1}^2}{2}$$

$$\text{Energy input/kg of steam} = C_1^2 - \frac{C_{r1}^2}{2}$$

Now, $\quad C_{r1}^2 = C_1^2 + C_b^2 - 2C_1C_b \cos\alpha_1$

$\therefore \quad$ $$\text{Energy input/kg of steam} = C_1^2 - \frac{C_1^2 + C_b^2 - 2C_1C_b\cos\alpha_1}{2}$$

$$\text{Energy input/kg of steam} = \frac{C_1^2 - C_b^2 + 2C_1C_b\cos\alpha_1}{2} \qquad (9.71)$$

Diagram efficiency of the blades,

$$\eta_D = \eta_b = \frac{2(2C_1\cos\alpha_1 - C_b)C_b}{C_1^2 - C_b^2 + 2C_1C_b\cos\alpha_1} = \frac{2C_b^2\left[2\dfrac{C_1\cos\alpha_1}{C_b} - 1\right]}{C_1^2[1 - (C_b^2/C_1^2) + 2(C_b/C_1)\cos\alpha_1]}$$

Putting $C_b/C_1 = \rho$, the velocity ratio,

$$\eta_D = \frac{2\rho^2\left[2\dfrac{\cos\alpha_1}{\rho} - 1\right]}{1 - \rho^2 + 2\rho\cos\alpha_1}$$

$$\eta_D = \frac{2(2\rho\cos\alpha_1 - \rho^2)}{1 - \rho^2 + 2\rho\cos\alpha_1} \qquad (9.72)$$

(d) Optimum Velocity Ratio, Maximum Specific Diagram Work and Maximum Diagram Efficiency

There is a particular value of ρ for which η_D is a maximum. Differentiating η_D with respect to ρ and equating it to zero,

$$\frac{d\eta_D}{d\rho} = \frac{(1-\rho^2+2\rho\cos\alpha_1)2(2\cos\alpha_1 - 2\rho) - 2\rho(2\cos\alpha_1 - \rho)(-2\rho + 2\cos\alpha_1)}{(1-\rho^2+2\rho\cos\alpha_1)^2} = 0$$

$$4(1 - \rho^2 + 2\rho\cos\alpha_1)(\cos\alpha_1 - \rho) - 4\rho(2\cos\alpha_1 - \rho)(\cos\alpha_1 - \rho) = 0$$

$$4(\cos\alpha_1 - \rho)(1 - \rho^2 + 2\rho\cos\alpha_1 - 2\rho\cos\alpha_1 + \rho^2) = 0$$

$\therefore \quad$ $$\rho_{opt} = \cos\alpha_1 \qquad (9.73)$$

or $\quad C_b = C_1\cos\alpha_1$

Substituting the value of ρ_{opt} in Eq. (9.72),

$$(\eta_D)_{max} = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1} \qquad (9.74)$$

From Eqs. (9.70) and (9.73), the specific blading work corresponding to maximum blading efficiency,

$$(w_D)_{max} = (2C_b - C_b)C_b$$

$$(w_D)_{max} = C_b^2 \quad (9.75)$$

The velocity diagrams for a 50% reaction turbine operating with maximum blading efficiency are shown in Figure 9.32.

$$\Delta C_w = C_1 \cos \alpha = C_b$$

$$\therefore \quad (w_D)_{max} = \Delta C_w C_b = C_b^2 \quad (9.76)$$

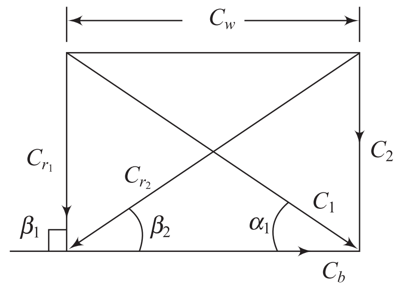


Figure 9.32 *Velocity Diagram for a 50% Reaction Turbine with Maximum Diagram Efficiency*

(e) Specific Enthalpy Drop in Fixed Blades, Moving Blades and in the Stage

It is no longer convenient to talk of the nozzles and blades, since in the reaction turbine both fixed and moving blades act like nozzles. It is usual to refer to the two sets of blades as the stator or fixed blades and the rotor or moving blades.

Since the isentropic enthalpy drop is equally distributed between the fixed blades and moving blades in a 50% reaction turbine stage as shown in Figure 9.33.

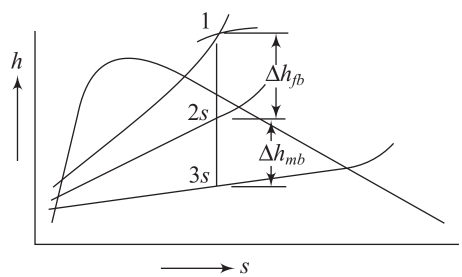


Figure 9.33 *Enthalpy Drop in a 50% Reaction Turbine*

$$(\Delta h_s)_{fb} = h_1 - h_{2s} = \frac{C_1^2 - C_2^2}{2} \quad (9.77)$$

$$(\Delta h_s)_{mb} = h_{2s} - h_{3s} = \frac{C_{r2}^2 - C_{r1}^2}{2} = \frac{C_1^2 - C_2^2}{2} \quad (9.78)$$

$$\therefore \quad (\Delta h_s)_{fb} = (\Delta h_s)_{mb} \quad (9.79)$$

$$(\Delta h_s)_{stage} = h_1 - h_{3s} = 2(h_{2s} - h_{3s}) = 2(\Delta h_s)_{mb} \quad (9.80)$$

$$\therefore \quad R = \frac{h_{2s} - h_{3s}}{h_1 - h_{3s}} = \frac{(\Delta h_s)_{mb}}{2(\Delta h_s)_{mb}} = \frac{1}{2} \quad (9.81)$$

9.7.3 Carry-Over Efficiency

For symmetrical staging, i.e. 50% reaction staging, the isentropic enthalpy drop across the stage is evenly distributed between the stationary and moving rows of blades, as shown in Figure 9.33, so that,

$$(\Delta h_s)_{fb} = (\Delta h_s)_{mb} = \frac{(\Delta h_s)_{stage}}{2} = (\Delta h_s)_{row}$$

Assuming the same nozzle efficiency for both fixed and moving rows, the kinetic energy of fluid gained per row, fixed as well as moving, is

$$\text{Kinetic energy of fluid gained per row} = \eta_n \cdot \frac{(\Delta h_s)_{stage}}{2} \quad (9.82)$$

The expansion of steam occurs in fixed passages in addition to that in the nozzles of the first ring. At the entry of the fixed blade passages, the inlet velocity is the exit velocity of the preceding rotor blades (moving blades row) which cannot be neglected. However, the exit velocity of the preceding rotor blades C_2 is somewhat reduced when approaching the fixed blades of succeeding stage for further expansion due to the gap between the moving and fixed blade rings and due to the possible eddies, discontinuities and a sort of 'fluid shear' between the blade rings. This reduction is accounted by employing the *carry-over efficiency,* η_{CO}. The value of the carry-over efficiency depends on the gap between the respective rings, the speed of rotation, the diameter, the eddies, etc. and is of the order of 0.96–0.97 Hence, the kinetic energy exiting one stage that is available to the following stage is given by $\eta_{co}C_2^2/2$, where η_{co} is the carry-over efficiency.

The energy balance for the fixed row of blades gives,

$$(C_1^2/2) + h_1 = \eta_{co}\frac{C_2^2}{2} + h_2$$

$$\frac{C_1^2 - \eta_{co}C_2^2}{2} = h_2 - h_1 = \eta_n \frac{(\Delta h_s)_{\text{stage}}}{2}$$

$$\therefore \qquad (\Delta h_s)_{\text{stage}} = \frac{C_1^2 - \eta_{co}C_2^2}{\eta_n} \tag{9.83}$$

This is the energy available for conversion in one stage. The energy output or diagram work per kg of steam is given by Eq. (9.70). Therefore, the combined nozzle and blade efficiency, η_{nb}, is given by,

$$\eta_{nb} = \frac{(2C_1\cos\alpha_1 - C_b)C_b}{(C_1^2 - \eta_{co}C_2^2)/\eta_n}$$

$$\eta_{nb} = \eta_n \frac{C_b^2}{C_1^2}\frac{\left(\dfrac{2C_1\cos\alpha_1}{C_b} - 1\right)}{1 - \eta_{co}\left(\dfrac{C_2^2}{C_1^2}\right)} \tag{9.84}$$

Again, $C_2^2 = C_{r_2}^2 + C_b^2 - 2C_{r_2}C_b\cos\alpha_1$

$$\therefore \qquad \frac{C_2^2}{C_1^2} = 1 + \rho^2 - 2\rho\cos\alpha_1, \text{ where } \rho = C_b/C_1$$

Substituting in Eq. (9.84),

$$\eta_{nb} = \eta_n \frac{\rho^2\left(\dfrac{2\cos\alpha_1}{\rho} - 1\right)}{1 - \eta_{co}(1 + \rho^2 - 2\rho\cos\alpha_1)}$$

$$\eta_{nb} = \eta_n \frac{2\rho\cos\alpha_1 - \rho^2}{1 - \eta_{co}(1 + \rho^2 - 2\rho\cos\alpha_1)} \tag{9.85}$$

When $\eta_{co} = 1$, $\eta_{nb} = \eta_n$, i.e. losses are confined to the nozzle friction only.
When $\eta_{co} = 0$, there is no carry-over of kinetic energy to the next stage (velocity of approach is zero), and

$$\eta_{nb} = \eta_n(2\rho\cos\alpha_1 - \rho^2) \tag{9.86}$$

which is the efficiency of a single row of blades.
If $\eta_n = 1$,

$$\eta_{nb} = \eta_b = \eta_D = 2\rho \cos \alpha_1 - \rho^2 \quad (9.87)$$

For maximum blading efficiency,

$$\frac{d\eta_D}{d\rho} = 2 \cos \alpha_1 - 2\rho = 0$$

$$\rho_{opt} = \cos \alpha_1$$

which is same as Eq. (9.73). Substituting in Eq. (9.87),

$$(\eta_b)_{max} = \cos^2 \alpha_1 \quad (9.88)$$

which is somewhat less than that is obtained from Eq. (9.74). A comparison of Eqs. (9.30), (9.66) and (9.88) reveals that the optimum efficiencies for simple impulse, Curtis and reaction blading are all equal. However, when friction is taken into account, the reaction stage is found to be the most efficient, followed by Rateau and Curtis staging in that order. The friction losses are less significant in the reaction stage since the flow velocities are lower.

EXAMPLE 9.5 The steam leaving the moving blade row is at 1.5 bar, 0.88 dry at a certain point in a Parson's turbine running at 3000 rpm. The axial velocity of flow at the inlet of the moving blade row is 0.65 times the mean blade velocity, whereas that at the exit from the row is 0.7 times the mean blade velocity. The outlet angles of both fixed and moving blades are 22° measured from the plane of rotation. The ratio of the height of the moving blades at the outlet to the mean diameter is 0.1. If the mass flow rate of steam is 6.5 kg/s, determine (a) the height of the moving blades at the outlet, and (b) power developed in the blade row.

Solution

Given: p_2 = 1.5 bar, x_2 = 0.88, N = 3000 rpm, $C_{f1} = 0.65C_b$, $C_{f2} = 0.7C_b$, $\alpha_1 = \beta_2 = 22°$, h_{b2}/D = 0.1, $\dot{m}$ = 6.5 kg/s
For a Parson's turbine,

$$\alpha_1 = \beta_2 = 22°,\ C_{r2} = C_1,\ C_{r1} = C_2 \quad (1)$$

(a) Blade Height at Outlet

From steam table: At a pressure of 1.5 bar, v_{f2} = 0.001053 m³/kg, v_{g1} = 1.1593 m³/kg

$$\therefore \quad v_2 = (1 - x_2)v_{f2} + x_2 v_{g2} \Rightarrow v_2 = (1 - 0.88) \times 0.001053 + 0.88 \times 1.1593$$

$$v_2 = 1.0203 \text{ m}^3/\text{kg} \quad (2)$$

$$C_b = \frac{\pi DN}{60} \Rightarrow C_b = \frac{\pi D \times 3000}{60} = \frac{\pi \times 10h_{b2} \times 3000}{60}$$

$$C_b = 157.08D = 1570.8h_{b2} \quad (3)$$

$$\dot{m} = \frac{\pi D h_{b2} C_{f2} k_{tb}}{v_2} = \frac{\pi D h_{b2} \times 0.7C_b}{v_2} k_{tb} \Rightarrow 6.5 = \frac{\pi \times 10h_{b2} \times h_{b2} \times 0.7 \times 1570.8h_{b2}}{1.0203} \times 1$$

$$h_{b2} = 0.0577 \text{ m} = 57.7 \text{ mm} \quad (4)$$

(b) Power Developed

$$C_b = 1570.8 h_{b2} = 1570.8 \times 0.0577$$

$$C_b = 90.635 \text{ m/s} \quad (5)$$

$$C_{f1} = 0.65 C_b = C_1 \sin \alpha_1 \Rightarrow 0.65 \times 90.635 = C_1 \sin 22$$

$$C_1 = 157.266 \text{ m/s} \quad (6)$$

The velocity diagram is shown in Figure 9.34.

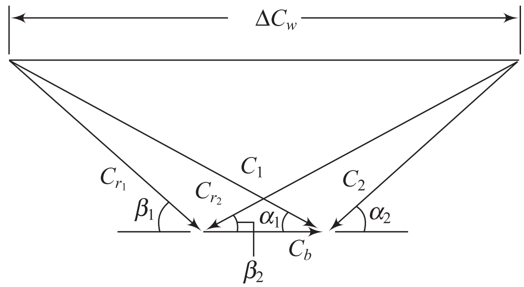


Figure 9.34 *Velocity Diagram of Example 9.5*

$$\Delta C_w = C_{w1} + C_{w2} = C_1 \cos \alpha_1 + C_{r2} \cos \beta_2 - C_b = 2C_1 \cos \alpha_1 - C_b \quad (7)$$

$$\Delta C_w = 2 \times 157.266 \times \cos 22 - 90.635$$

$$\Delta C_w = 200.944 \text{ m/s} \quad (8)$$

$$P_D = \dot{m} \Delta C_w C_b \quad (9)$$

$$P_D = 6.5 \times 200.994 \times 90.635$$

$$P_D = 118411.1 \text{ W} = 118.411 \text{ kW} \quad (10)$$

EXAMPLE 9.6 In a multistage Parson's reaction turbine, at one of the stages, the rotor diameter is 1.25 m and speed ratio is 0.72. The speed of the rotor is 3000 rpm. Determine (a) the inlet blade angle if the blade angle at outlet is 22°, (b) diagram efficiency, and (c) percentage increase in diagram efficiency and rotor speed if the turbine is designed to run at the best theoretical speed.

Solution

Given: $D = 1.25$ m, $\rho = 0.72$, N = 3000 rpm, $\beta_2 = 22°$

$$C_{b1} = C_{b2} = C_b = \frac{\pi DN}{60} \quad (1)$$

$$C_{b1} = C_{b2} = C_b = \frac{\pi \times 1.25 \times 3000}{60}$$

$$C_{b1} = C_{b2} = C_b = 196.35 \text{ m/s} \quad (2)$$

$\because$ $$\rho = \frac{C_b}{C_1} \Rightarrow C_1 = \frac{C_b}{\rho} \quad (3)$$

$$C_1 = \frac{196.35}{0.72}$$

$$C_1 = 272.708 \text{ m/s} \quad (4)$$

The velocity triangles at the inlet and outlet for the Parson's turbine are shown in Figure 9.35.

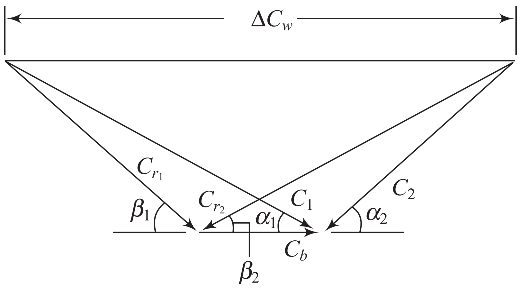


Figure 9.35 *Velocity Diagram of Example 9.6*

(a) Blade Angle at Inlet

For Parson's turbine $\alpha_1 = \beta_2 = 22°$. From the velocity triangle at the inlet,

$$C_{w1} = C_1 \cos \alpha_1 \Rightarrow C_{w1} = 272.708 \times \cos 22$$

$$C_{w1} = 252.85 \text{ m/s} \quad (5)$$

$$C_{f1} = C_1 \sin \alpha_1 \Rightarrow C_{f1} = 272.708 \times \sin 22$$

$$C_{f1} = 102.158 \text{ m/s} \quad (6)$$

$$\tan \beta_1 = \frac{C_{f1}}{C_{w1} - C_{b_1}} \Rightarrow \tan \beta_1 = \frac{102.158}{252.85 - 196.35}$$

$$\beta_1 = 61.055° \quad (7)$$

(b) Diagram Efficiency

For the Parson's reaction turbine, $\alpha_1 = \beta_2$, $\alpha_2 = \beta_1$, $C_{r2} = C_1 = 272.708$ m/s, $C_{r1} = C_2$.
From the outlet velocity triangle,

$$C_{b2} + C_{w2} = C_{r2} \cos \beta_2 \Rightarrow 196.35 + C_{w2} = 272.08 \times \cos 22$$

$$C_{w2} = 56.50 \text{ m/s} \quad (8)$$

By Euler's turbine equation, specific diagram or blade work,

$$w = (C_{w1}C_{b1} - C_{w2}C_{b2}) = C_b(C_{w1} - C_{w2}) \quad (9)$$

$$w = 196.35 \times [252.85 - (-56.50)]$$

Negative sign is taken with C_{w2} as its direction is opposite to that of C_{w1}.

$$w = 60740.87 \text{ J/kg} = 60.41 \text{ kJ/kg} \quad (10)$$

From the inlet velocity triangle,

$$\frac{C_{w1} - C_{b1}}{C_{r1}} = \cos\beta_1 \Rightarrow \frac{252.85 - 196.35}{C_{r1}} = \cos 61.055$$

$$C_{r1} = 116.743 \text{ m/s} \tag{11}$$

Total energy supplied to the stage per kg of steam or specific input energy is, $(\Delta h)_s$ = Specific kinetic energy supplied to the fixed blades + Specific kinetic energy supplied to the moving blades

$$(\Delta h)_s = \frac{C_1^2}{2} + \frac{(C_{r2}^2 - C_{r1}^2)}{2} \tag{12}$$

$$(\Delta h)_s = \frac{272.708^2}{2} + \frac{(272.708^2 - 116.743^2)}{2}$$

$$(\Delta h)_s = 67555.189 \text{ J/kg} = 67.5552 \text{ kJ/kg} \tag{13}$$

$$\eta_D = \frac{w}{(\Delta h)_s} \tag{14}$$

$$\eta_D = \frac{60.41}{67.5552}$$

$$\eta_D = 0.894232 = 89.4232\% \tag{15}$$

(c) Best Efficiency Point

For maximum efficiency of Parson turbine,

$$\rho_{\text{opt}} = \frac{C_b}{C_1} = \cos\alpha_1 \tag{16}$$

$$\frac{C_{b1}}{272.708} = \cos 22$$

$$(C_{b1})_{\text{BEP}} = 252.85 \text{ m/s} \tag{17}$$

$$(C_{b1})_{\text{BEP}} = \frac{\pi D N_{\text{BEP}}}{60} \Rightarrow 252.85 = \frac{\pi \times 1.25 \times N_{\text{BEP}}}{60}$$

$$N_{\text{BEP}} = 3863.26 \text{ rpm} \tag{18}$$

$$\% \text{ Increase in speed} = \frac{N_{\text{BEP}} - N}{N} \times 100 = \frac{3863.26 - 3000}{3000} \times 100$$

$$\% \text{ Increase in speed} = 28.7753\% \tag{19}$$

Maximum blading or diagram efficiency of Parson turbine is,

$$(\eta_D)_{\max} = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1} \tag{20}$$

$$(\eta_D)_{\max} = \frac{2\cos^2 22}{1 + \cos^2 22}$$

$$(\eta_D)_{\max} = 0.92454 = 92.54\% \tag{21}$$

$$\% \text{ Increase in diagram or blading efficiency} = \frac{(\eta_D)_{max} - \eta_D}{\eta_D} \times 100 = \frac{0.9245 - 0.894232}{0.894232} \times 100$$

% Increase in blading or diagram efficiency = 3.385% (22)

9.8 Comparison of Enthalpy Drops in Various Stages

Assuming the blades are operating at maximum efficiency, for a symmetrical (50%) reaction stage,

$$\rho_{opt} = \cos \alpha_1 = C_b/C_1$$

Therefore, velocity of steam exiting the nozzles or fixed blades is,

$$C_1 = C_b/\cos \alpha_1 = 44.72\left[\frac{(\Delta h_s)_{stage}}{2}\right]^{½}$$

$$(\Delta h_s)_{stage} = 2\left[\frac{C_b}{44.72\cos\alpha_1}\right]^2 \tag{9.89}$$

This is the isentropic enthalpy drop in a 50% reaction stage. If we compare this with the isentropic enthalpy drop for a simple impulse stage, Eq. (9.45) and that for a 2-row Curtis stage, Eq. (9.67),

$$(\Delta h_s)_{impulse stage} = 4\left[\frac{C_b}{44.72 \cos \alpha_1}\right]^2$$

and $$(\Delta h_s)_{2\text{-row Curtis stage}} = 16\left[\frac{C_b}{44.72\cos\alpha_1}\right]^2$$

We find for the same values of C_b and α_1,

$$(\Delta h_s)_{50\% \text{ reaction stage}} : (\Delta h_s)_{impulse\ stage} : (\Delta h_s)_{2\text{-row Curtis stage}} = 1 : 2 : 8 \tag{9.90}$$

Therefore, a 2-row Curtis stage is ideally equivalent to four simple impulse stages and eight 50% reaction stages.

Since the number of stages required for a certain total isentropic enthalpy drop is,

$$n = \frac{(\Delta h_s)_{total}}{(\Delta h_s)_{stage}}$$

The lower is the enthalpy drop per stage, the higher is the number of stages required for the same output. For example let C_b = 320 m/s and α_1 = 16°,

$$(\Delta h_s)_{impulse\ stage} = 221.64 \text{ kJ/kg}$$

$$(\Delta h_s)_{50\% \text{ reaction stage}} = 110.82 \text{ kJ/kg}$$

$$(\Delta h_s)_{2\text{-row Curtis stage}} = 886.56 \text{ kJ/kg}$$

Thus, for the same output of a turbine, the number of reaction (50%) stages required will be about twice the number of impulse or pressure (Rateau) stages and about eight times the number of 2-row Curtis stages. So, reaction turbines are costlier than impulse turbines for the same output.

Again, for the same mean blade speed C_b,

$$C_1 = C_b/\cos\alpha_1 \text{ for a 50\% reaction stage}$$

$$C_1 = 2C_b/\cos\alpha_1 \text{ for a simple impulse stage}$$

$$C_1 = 4C_b/\cos\alpha_1 \text{ for a 2-row Curtis stage}$$

The energy loss due to friction is proportional to the square of the velocity of a fluid (steam). Since the fluid velocity is the highest for a 2-row Curtis stage and lowest in the 50% reaction stage, so the energy loss due to friction in the reaction stage is the least, while that in the Curtis stage is the highest. The energy loss in the impulse stage will be in between these two values. Therefore, the efficiency of the reaction stage will be the highest and that of the Curtis stage will be the lowest, while that of impulse stage will lie in between.

$$\eta_{\text{50\% reaction stage}} > \eta_{\text{simple impulse stage}} > \eta_{\text{2-row Curtis stage}} \tag{9.91}$$

Figure 9.36 gives a comparison of diagram efficiencies with respect to the velocity ratio for various turbine stages. Reaction turbines are more efficient but are more costly. In modern turbines, a 2-row Curtis stage is generally used as the first stage followed by a series of reaction stages. Sometimes impulse stages are also used after the Curtis stage. The use of the initial Curtis stage decreases the rotor length, i.e. number of stages required since a 2-row Curtis stage can replace about eight 50% reaction stages and hence, the cost of the rotor significantly, at the cost of some loss in efficiency. It is often called the control stage, where steam at the highest pressure and temperature is immediately expanded down into a region of more moderate pressures and temperatures, such that leakage between stages is reduced, expansion problems are simplified and the overall length of the rotor is reduced. Thus, it is a good compromise to use one 2-row Curtis stage initially, sacrificing some loss in efficiency. A large drop in enthalpy occurs in this Curtis stage. The remaining enthalpy drop occurs in the subsequent stages, the number of which can be estimated from

$$n = \frac{(\Delta h_s)_{\text{total}} - (\Delta h_s)_{\text{2-row Curtis stage}}}{(\Delta h_s)_{\text{stage}}} \tag{9.92}$$

where Δh_s represents the isentropic enthalpy drop.

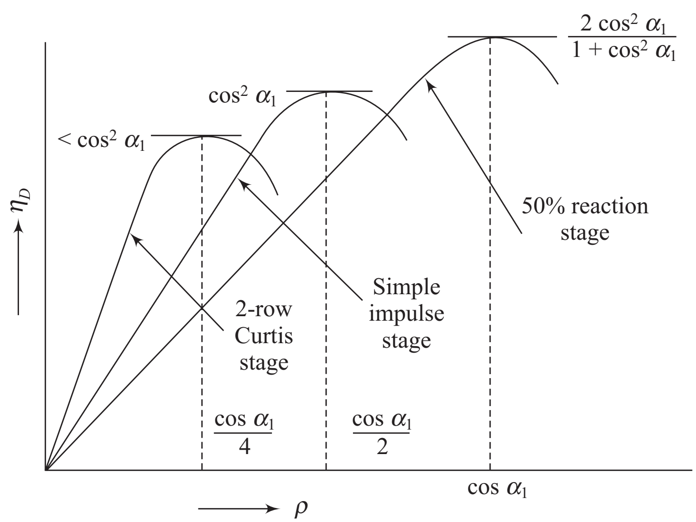


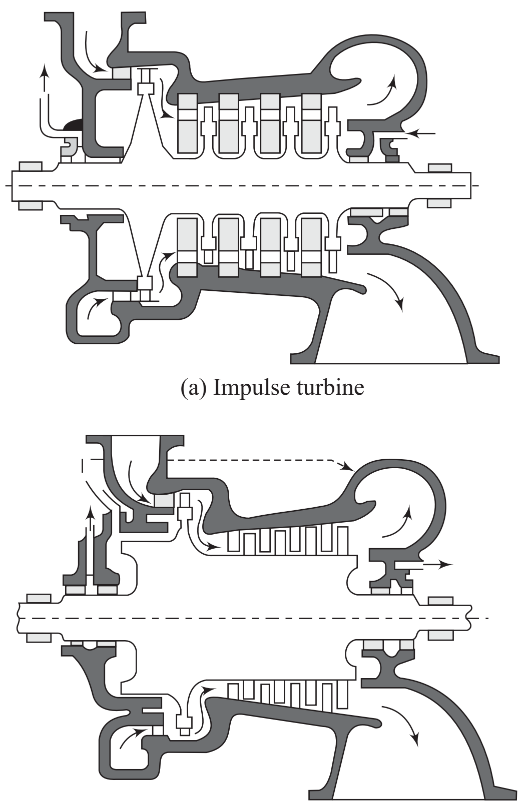
Figure 9.36 *Variation of Diagram Efficiency with Speed Ratio for Various Turbine Stages*

The arrangement of stages for a 15-stage turbine is as follows:

$$\frac{\text{2-row Curtis stage}}{\text{N MB GB MB}}\ \frac{1}{\text{FB MB}}\ \frac{2}{\text{FB MB}}\ \frac{3}{\text{FB MB}}\ \frac{4}{\text{FB MB}}\cdots\frac{13}{\text{FB MB}}\ \frac{14}{\text{FB MB}},$$

where N = nozzle, MB = moving blades, GB = guide blades, and FB = fixed blades.

These stages are distributed in a number of cylinders or casing, as discussed later. Simple layouts of an impulse and reaction turbine are shown in Figure 9.37. Figure 9.38 shows typical single cylinder designs of impulse and reaction turbines. In practice, the numbers of impulse and reaction turbines in operation are approximately equal, the choice being dependent on the preference of the turbine manufacturer. In some designs, the stage may have some degree of reaction, 10% or less, to produce a small pressure drop across the bucket. This small drop can be very useful in keeping the buckets running full of steam, with the shaft thrust positive in one direction.



(a) Impulse turbine

(b) Reaction turbine

Figure 9.38 *Some Typical Designs of Impulse and Reaction Turbines*

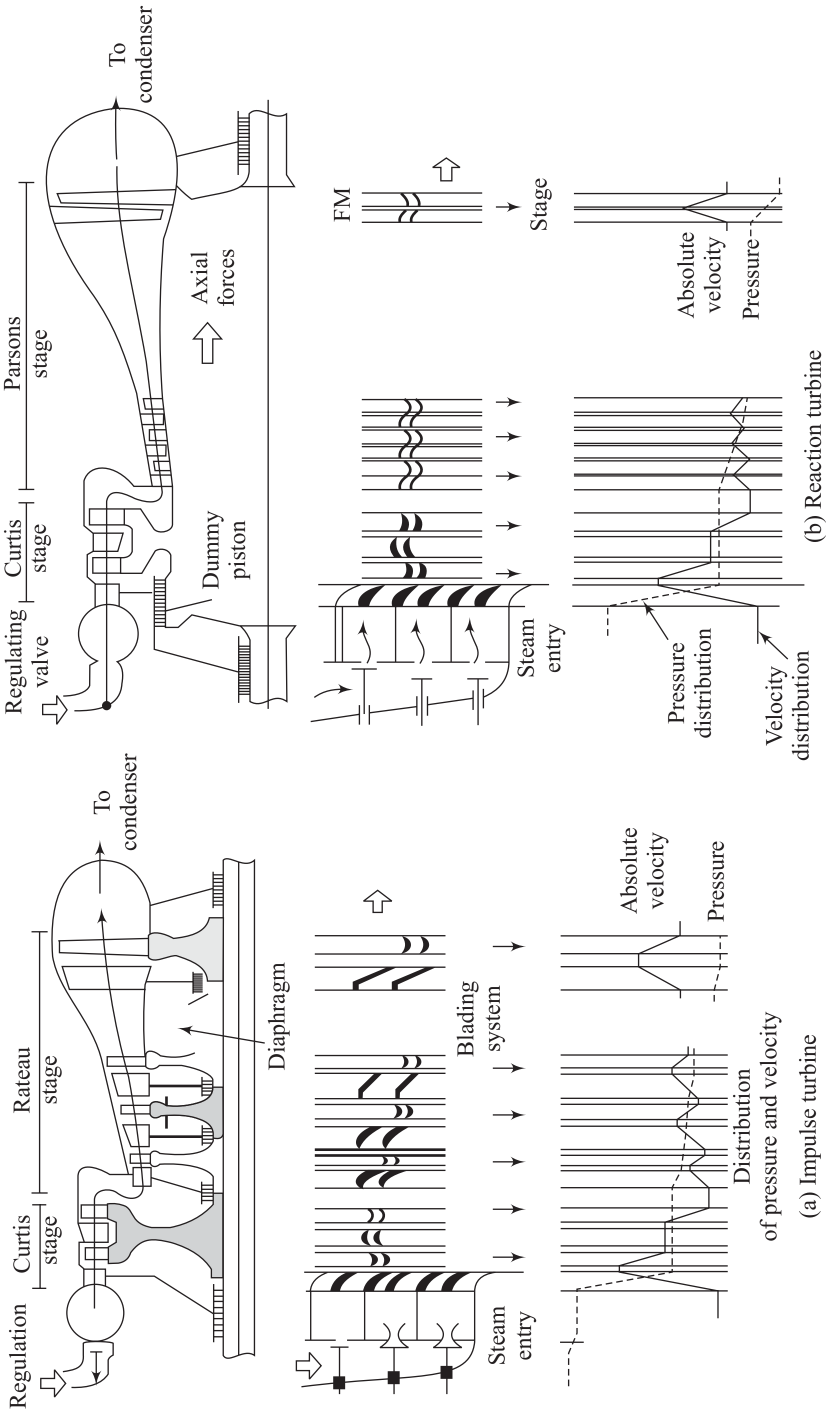


Figure 9.37 *Layouts of Impulse and Reaction Turbines*

9.9 Variation of Blade Velocity along Blade Height

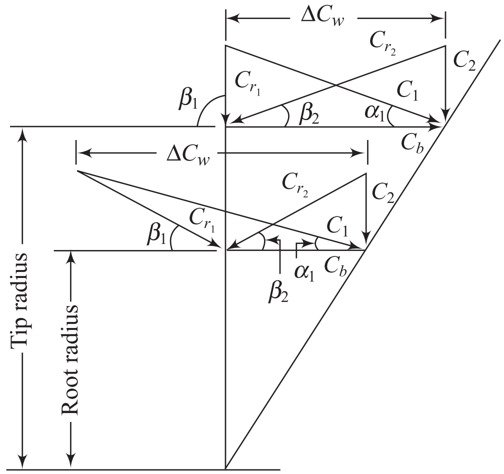


Figure 9.39 *Variation of Blade Angles with Diameter*

In the low pressure region, the blade heights are quite large. The velocity at the blade root $\left[\frac{(\pi D_h N)}{60}\right]$ will be much smaller than that at the mid-point, and still smaller than that at the tip $\left[\frac{(\pi D_t N)}{60}\right]$. Hence, for a good efficiency, the blade angles should vary with the diameter, as shown in Figure 9.39. For this reason, *twisted (or warped) blades are used in the later stages of the turbine.* The vertical line in Figure 9.39 represents the radius of the wheel. The vertical distance between the tip and root radii is the blade height. Steam enters and leaves the blades with velocities C_1 and C_2, respectively. Since, C_b is proportional to the radius, angle β_1 will vary accordingly. It is seen that the vector diagram at the root looks like one for an impulse blade, while the diagram at the tip looks like one for a reaction blade. The blade work $C_b \Delta C_w$ is constant.

EXAMPLE 9.7 Steam expands isentropically from 35 bar, 600°C to 0.075 bar in a turbine running at 3500 rpm. The nozzle angle is 17°. Determine the mean diameter of the runner assuming ideal conditions if the turbine were of (a) single impulse stage, (b) single 50% reaction stage, (c) four pressure or Rateau stages, (d) one two-row Curtis stage, and (e) four 50% reaction stages.

Solution

Given: p_1 = 35 bar, t_1 = 600°C, p_2 = 0.075 bar, N = 3500 rpm, α_1 = 17°

From steam table, properties of superheated steam at 35 bar, 600°C,

$$h_1 = 3678.4 \text{ kJ/kg}, \; s_1 = 7.4339 \text{ kJ/kg-K}$$

$$s_{2s} = s_1 = 7.4339 \text{ kJ/kg-K which is less than } (s_g)_{0.075\text{ bar}} = 8.2515 \text{ kJ/kg-K.}$$

Hence, point 2s is in wet region. At condenser pressure of 0.075 bar, from properties of saturated steam,

$$s_g = 8.2515 \text{ kJ/kg-K}, \; s_f = 0.5764 \text{ kJ/kg-K}, \; s_{fg} = 7.6750 \text{ kJ/kg-K}, \; h_f = 168.79 \text{ kJ/kg}, \; h_{fg} = 2406 \text{ kJ/kg}$$

$$s_{2s} = 7.4339 = 0.5764 + x_{2s} \times 7.6750$$

$$x_{2s} = 0.8935 \quad (1)$$

$$h_{2s} = 168.79 + 0.8935 \times 2406$$

$$h_{2s} = 2318.551 \text{ kJ/kg} \quad (2)$$

$$(\Delta h_s)_{\text{total}} = h_1 - h_{2s} \Rightarrow (\Delta h_s)_{\text{total}} = 3678.4 - 2318.551$$

$$(\Delta h_s)_{\text{total}} = 1359.849 \text{ kJ/kg} \quad (3)$$

(a) Single Impulse Stage

$$(\Delta h_s)_{\text{stage}} = (\Delta h_s)_{\text{total}} = 1359.849 \text{ kJ/kg} \quad (4)$$

$$C_1 = 44.72(\Delta h_s)^{1/2}_{\text{stage}} \tag{5}$$

$$C_1 = 44.72(1359.849)^{1/2}$$

$$C_1 = 1649.1 \text{ m/s} \tag{6}$$

For ideal condition of maximum efficiency for single stage impulse turbine,

$$\frac{C_b}{C_1} = \frac{\cos \alpha_1}{2} \tag{7}$$

$$\frac{C_b}{1649.1} = \frac{\cos 17}{2}$$

$$C_b = 788.521 \text{ m/s} \tag{8}$$

$$C_b = \frac{\pi DN}{60} \Rightarrow 788.521 = \frac{\pi \times D \times 3500}{60}$$

$$D = 4.303 \text{ m} \tag{9}$$

(b) Single 50% Reaction Stage

In a 50% reaction turbine,

$$(\Delta h_s)_{\text{stage}} = (\Delta h_s)_{\text{total}} = 1359.849 \text{ kJ/kg} \tag{10}$$

$$C_1 = 44.72\left[\frac{(\Delta h_s)_{\text{stage}}}{2}\right]^{1/2} \tag{11}$$

$$C_1 = 44.72\left(\frac{1359.849}{2}\right)^{1/2}$$

$$C_1 = 1166.1 \text{ m/s} \tag{12}$$

For ideal condition of maximum efficiency for single stage 50% reaction turbine,

$$\frac{C_b}{C_1} = \cos \alpha_1 \tag{13}$$

$$\frac{C_b}{1166.1} = \cos 17$$

$$C_b = 1115.147 \text{ m/s} \tag{14}$$

$$C_b = \frac{\pi DN}{60} \Rightarrow 1115.147 = \frac{\pi \times D \times 3500}{60}$$

$$D = 6.085 \text{ m} \tag{15}$$

(c) Four Pressure or Rateau Stages

$$(\Delta h_s)_{\text{stage}} = \frac{(\Delta h_s)_{\text{total}}}{4} = \frac{1359.849}{4} \tag{16}$$

$$(\Delta h_s)_{\text{stage}} = 339.96225 \text{ kJ/kg} \tag{17}$$

$$C_1 = 44.72(\Delta h_s)^{1/2}_{\text{stage}} \tag{18}$$

$$C_1 = 44.72(339.96225)^{1/2}$$

$$C_1 = 824.55 \text{ m/s} \tag{19}$$

For ideal condition of maximum efficiency for Rateau stages,

$$\frac{C_b}{C_1} = \frac{\cos \alpha_1}{2} \tag{20}$$

$$\frac{C_b}{824.55} = \frac{\cos 17}{2}$$

$$C_b = 394.26 \text{ m/s} \tag{21}$$

$$C_b = \frac{\pi DN}{60} \Rightarrow 394.26 = \frac{\pi \times D \times 3500}{60}$$

$$D = 2.1514 \text{ m} \tag{22}$$

(d) Two-Row Curtis Stage

$$(\Delta h_s)_{\text{stage}} = (\Delta h_s)_{\text{total}} = 1359.849 \text{ kJ/kg} \tag{23}$$

$$C_1 = 44.72(\Delta h_s)^{1/2}_{\text{stage}} \tag{24}$$

$$C_1 = 44.72(1359.849)^{1/2}$$

$$C_1 = 1649.1 \text{ m/s} \tag{25}$$

For a two-row Curtis stage,

$$\frac{C_b}{C_1} = \frac{\cos \alpha_1}{4} \tag{26}$$

$$\frac{C_b}{1649.1} = \frac{\cos 17}{4}$$

$$C_b = 394.26 \text{ m/s} \tag{27}$$

$$C_b = \frac{\pi DN}{60} \Rightarrow 394.26 = \frac{\pi \times D \times 3500}{60}$$

$$D = 2.1514 \text{ m} \tag{28}$$

(e) Four 50% Reaction Stages

Isentropic enthalpy drop per reaction stage,

$$(\Delta h_s)_{\text{stage}} = \frac{(\Delta h_s)_{\text{total}}}{4} \Rightarrow (\Delta h_s) = \frac{1359.849}{4} = 339.96225 \text{ kJ/kg} \tag{29}$$

$$C_1 = 44.72\left[\frac{(\Delta h_s)_{\text{stage}}}{2}\right]^{1/2} \tag{30}$$

$$C_1 = 44.72\left(\frac{339.96225}{2}\right)^{1/2}$$

$$C_1 = 583.045 \text{ m/s} \tag{31}$$

For ideal condition of maximum efficiency for 50% reaction turbine,

$$\frac{C_b}{C_1} = \cos\alpha_1 \tag{32}$$

$$\frac{C_b}{583.045} = \cos 17$$

$$C_b = 557.57 \text{ m/s} \tag{33}$$

$$C_b = \frac{\pi DN}{60} \Rightarrow 557.57 = \frac{\pi \times D \times 3500}{60}$$

$$D = 3.0425 \text{ m} \tag{34}$$

9.10 Nozzle and Blade Heights

Nozzle or blade height depends on the total annular area required to pass the desired quantity of fluid. From Figure 9.40, it can be seen that the available flow area at the exit of one nozzle passage is approximately,

$$A = oh_n$$

where o is the width of the flow passage at the exit at mean nozzle height, and h_n is the nozzle height.

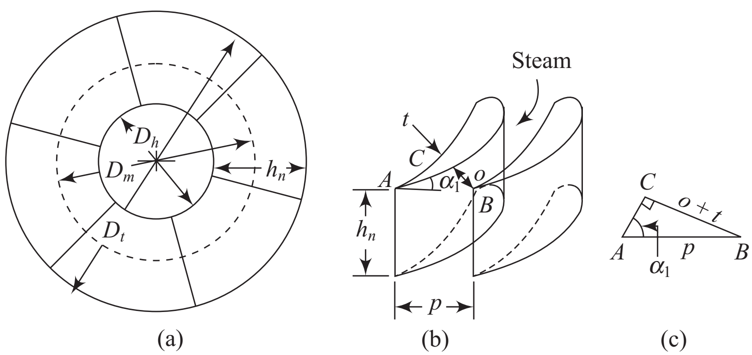


Figure 9.40 *Flow through a Nozzle Passage*

From Figure 9.40 (c),

$$p \sin\alpha_1 = o + t$$

where p is the pitch of nozzles at mean nozzle height, o is the minimum opening for flow, and t is the nozzle wall thickness as shown.

$$\therefore \quad A = (p \sin\alpha_1 - t)h_n$$

Assuming full peripheral admission, i.e. the nozzle diaphragm is completely occupied by nozzles, the number of nozzles, z, is given by

$$z = (\pi D)/p$$

Total nozzle area is,

$$A_n = p \sin \alpha_1 \left(1 - \frac{t}{p \sin \alpha_t}\right) h_n \frac{\pi D}{P}$$

$$A_n = pDh_n k_{tn} \sin \alpha_1 \tag{9.93}$$

where k_{tn} is the nozzle thickness factor.

Similarly, it can be obtained that the area available for flow at the entrance to the blades is,

$$A_b = \pi D h_b k_{tb} \sin \beta_1 \tag{9.94}$$

where k_{tb} is the edge thickness factor for blades.

From the continuity equation, it follows that,

$$\dot{m} = \frac{A_n C_1}{v_1} = \frac{A_b C_{r1}}{v_1}$$

$$(\pi D h_n k_{tn} \sin \alpha_1) C_1 = (\pi D h_b k_{tb} \sin \beta_1) C_{r1} \tag{9.95}$$

For all practical purposes, it can be assumed that $k_{tn} = k_{tb}$, so that,

$$h_b/h_n = (C_1 \sin \alpha_1)/(C_{r1} \sin \beta_1) = 1 \tag{9.96}$$

Equation (9.96) indicates that the blade height at entrance is equal to the nozzle height at the exit, but it is customary to increase the blade entrance height slightly. This is done in order to avoid spilling of the fluid issuing from the nozzles. This increase in the blade entrance height is called 'overlap' and is given equally at the root and tip of the blade. Thus, h_b exceeds h_n by 1.6 mm in high pressure stages and by about 20 mm in low pressure stages of large turbines, i.e. the overlap varies from 1.6 mm to 20 mm.

9.10.1 First Stage Nozzles and Blades

From the continuity equation, the nozzle area required for a given flow rate $\dot{m}$ is given by,

$$A_n = \frac{\dot{m} v_1}{C_1}$$

At high pressure of steam, v_1 is small and $C_1 = \frac{C_b}{\cos \alpha_1}$ for 50% reaction blading, and $C_1 = \frac{2C_b}{\cos \alpha_1}$ for impulse blading if the blades are assumed to operate at the maximum efficiency. Thus, at the initial stage of turbine, the flow area (A_n) required is small. Now,

$$A_n = \pi D h_n \sin \alpha_1 \cdot k_{tn}$$

where D is obtained from the given value of $C_b = (\pi DN)/60$. Since A_n is small for given values of α_1 and k_{tn}, the estimated value of h_n is often found to be too small to manufacture. If the nozzles are conical, the diameter of each nozzle is also found to be very small. It has been experimentally found that small diameter nozzles are less efficient than nozzles of large diameter.

Furthermore, the blades following the nozzles are thus very short, which is difficult to fabricate. Such short blades are also less efficient. It is advisable to use a minimum height of 20 mm for the initial blades. Therefore, the nozzle height in the first stage cannot be much less than 20 mm, or

$$(h_b)_{\min} \cong (h_n^*)_{\min} \cong 20 \text{ mm}$$

The required nozzle height h_n for a given mass flow is much less than this. Thus,

$$A_n = x\pi D(h_n)_{\min} \sin \alpha_1 \cdot k_{tn} \tag{9.97}$$

where x is the fraction of the total arc of nozzles which is available for steam flow. This is called *partial admission of steam*. Some nozzles in the arc are blocked by steel plates, as shown in Figure 9.41.

Thus, by increasing the estimated h_n to its minimum practicable value (20 mm) and simultaneously blocking a part of the periphery of the nozzle diaphragm, the required flow area (A_n) for a certain mass flow of steam at the given condition is obtained. For full admission, the entire arc or periphery of the nozzle diaphragm (πD) is open for steam flow. For partial admission, the arc or periphery open for steam flow is ($x\pi D$), where x is less than unity. With partial admission, a given blade passage will not receive flow from the nozzles at all times. Thus, exposed to alternate flows of high velocity steam, blades are subjected to vibrations which may prove dangerous in long blades.

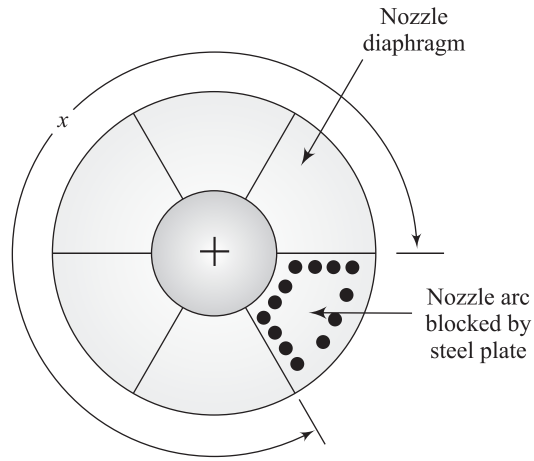


Figure 9.41 *Partial Admission of Steam in the Initial Stage*

9.10.2 Last Stage Blade Height

The last stage is very important in steam turbine design. As the pressure of the steam decreases during expansion, the specific volume increases. The volume flow of steam increases which requires an increased flow area. Consequently, h_b and D increase with decrease of pressure. The pressure is least in the last stage, therefore, h_b and D should have maximum values.

Blades are held at one end with the rotor while the other end is free; so they act as cantilevers with distributed load of steam on them. They are subjected to bending stresses. Blades are also subjected to centrifugal stresses since they are rotating at a high rpm. Both the bending and centrifugal stresses increase as the blade height increases. Due to these stresses, both blade height and blade diameter get restricted. The maximum blade velocity is also limited depending on the material of the blades, which is about 350 – 400 m/s.

$$(C_b)_{\max} = 350 \text{ m/s} = \frac{\pi (D)_{\max} N}{60}$$

The rpm of the rotor is decided from the generator side. For a 2-pole, 50 Hz alternator,

$$N = \frac{120 f}{p} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

$$\therefore \quad (D)_{\max} = \frac{350 \times 60}{p \times 3000} = 2.23 \text{ m}$$

For straight blades, the maximum blade height is about 20% of the mean blade ring diameter, or $(h_b/D)_{\max} = 0.2$.

Where it is necessary to exceed this ratio because of flow requirements, the blades may be tapered or twisted, as shown in Figure 9.42,

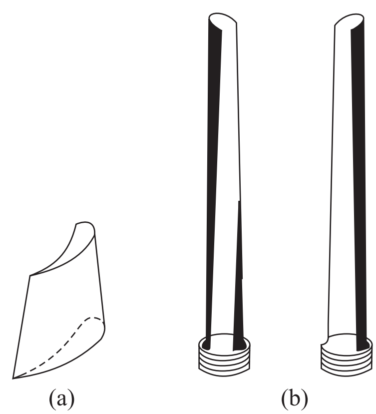


Figure 9.42 *(a) Tapered Blade, (b) Twisted Blades*

resulting in reduced bending and centrifugal stresses. With these modifications, the blade height may approach about 30% of the mean blade ring diameter. Therefore, for twisted or tapered blades,

$$(h_b/D)_{max} = 0.3$$

$$\therefore \quad (h_b)_{max} = 2.23 \times 0.3 = 0.67 \text{ m}$$

The flow area (A_b) could also be increased by increasing the blade angles, which would, in turn, increase the nozzle angle α_1. This would, however, decrease the blade efficiency. By reducing the rpm, both h_b and D can be increased. But, this will also increase the weight and bulk of the rotor along with the cost. So, this is not justified.

When the blade height becomes a significant part of the total stage diameter, the ratio of steam to blade speed changes over the height of the blade. Figure 9.43 shows the velocity diagrams at the root and tip of the twisted blades that receives a steam jet moving in a vortex flow ($C_1r_1 = C_2r_2$). The blade root has been designed for impulse flow, when the tip is exposed to a pure reaction force.

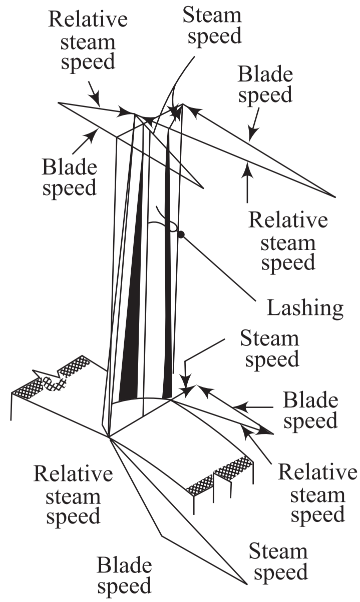


Figure 9.43 *Velocity Diagram at the Root and Tip of Twisted Blades*

9.10.3 Parallel Exhausts: Number of Last Stages

The maximum flow area is provided in the last stage where the volume flow rate is maximum.

$$(A_b)_{max} = \pi(Dh_b)_{max} \sin \beta_1 k_{tb}$$

The maximum volume flow rate or discharge that the last stage can accommodate,

$$\text{Discharge} = \pi(Dh_b)_{max} \sin \beta_1 k_{tb} C_{r1}$$

$$\because \quad C_{r1} \sin \beta_1 = C_1 \sin \alpha_1$$

Therefore,

$$\text{Discharge} = \pi(Dh_b)_{max}\, k_{tb}\, C_1 \sin \alpha_1 \tag{9.98}$$

If impulse blading is used,

$$\frac{C_b}{C_1} = \frac{\cos \alpha_1}{2}$$

The maximum blade velocity is limited depending on the material of the blades, which is about 350-400 m/s. Assuming C_b = 350 m/s and $C_1 = 700/\cos \alpha_1$,
From Eq. (9.98), for $\alpha_1 = 20°$ and k_{tb} = 0.9 (assumed),

$$\text{Maximum volume flow rate} = \pi \times 2.23 \times 0.67 \times \frac{700}{\cos 20°} \sin 20° \times 0.9 = 1075 \text{ m}^3\text{/s}$$

The maximum mass flow rate of steam that the last stage can accommodate,

$$(\dot{m})_{max} = \frac{1075}{v_2} \text{kg/s}$$

For the turbine exhaust condition at 0.075 bar, 0.88 quality,

$$v_2 = 0.001 + 0.88 \times 19.24 = 16.93 \text{ m}^3/\text{kg}$$

$$(\dot{m})_{\max} = \frac{1075}{16.93} = 63.5 \text{ kg/s}$$

The number of parallel exhausts or last stages required for a given steam flow rate of m is,

$$n = \frac{\dot{m}}{(\dot{m})_{\max}} = \frac{\dot{m}}{63.5} \tag{9.99}$$

9.10.4 Casing Arrangement

If the number of parallel exhausts is estimated to be 4, i.e. $n = 4$, the casing arrangement of the turbine may be as shown in Figure 9.44. Steam first expands in the HP (high pressure) turbine (1 – 2), the exhaust from which is taken back to the steam generator for reheating (2 – 3). The reheated steam (3) then expands in the IP (intermediate pressure) turbine (3 – 4). The steam leaving the IP turbine (4) is split into four equal streams expanding in two DFLP (double flow low pressure) turbines (4 – 5). The four parallel exhaust streams from the LP turbines (5) then enter the condenser. A double flow low pressure turbine (DFLPT) not only provides two parallel exhausts, but also helps the turbine to balance the axial thrust. Equal and opposite axial thrusts operating in the two similar turbines of the DFLPT neutralize each other.

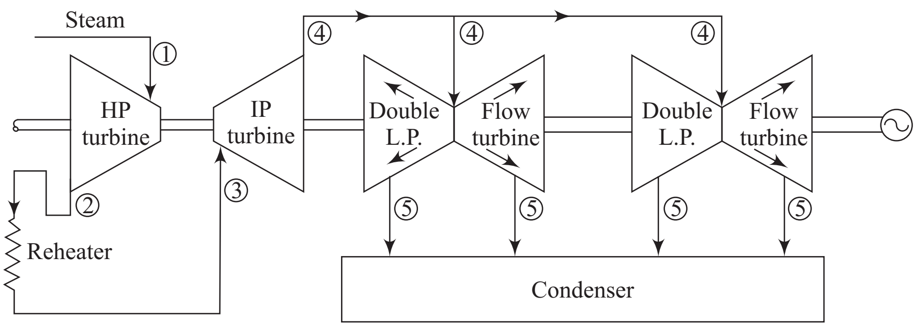


Figure 9.44 *Casing Arrangement of a Steam Turbine for Four Parallel Exhausts*

The HP and IP turbines are installed in the way as shown in order to reduce the axial thrust. Steam at states 1 and 3 is more or less at the same temperature. This will cause no thermal stresses at the HP and IP turbine inlets. If the IP turbine is oriented in the opposite way, there will be a thermal gradient along the shaft, which would cause considerable thermal stresses. The HP turbine outer casing is often made of double shell construction, with intermediate pressure steam filling the annular space so that the pressure difference across the casing wall is reduced.

To reduce the temperature gradient around and periphery and hence, the thermal stress at turbine entrance, steam is admitted at two or three feed points in both HP and IP turbines, instead of having single entry.

If $n = 3$, one DFLP turbine and one single flow LP turbine will be used to accommodate the required flow rate of steam, as shown in Figure 9.45.

Figures 9.44 and 9.45 are called *tandem compounded* steam turbines as all turbine cylinders mounted on the same shaft. If the numbers of cylinders are large, or the cylinders are of heavy weights, then the cylinders may be mounted on two shafts, each coupled to a separate electric generator as shown in Figure 9.46. Such an arrangement of turbine cylinders on two shafts is called a *cross-compounded* steam turbine.

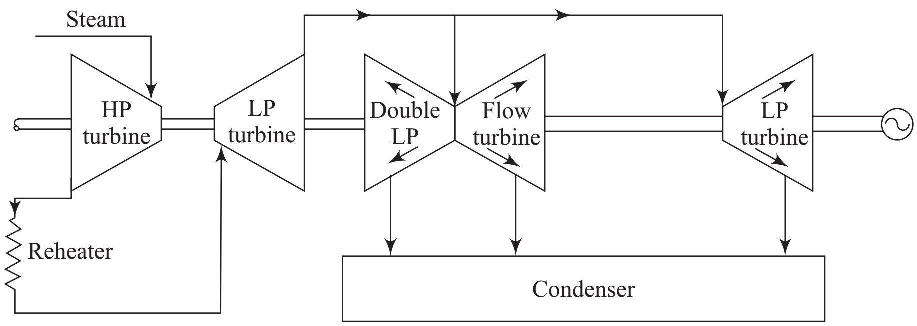


Figure 9.45 *Casing Arrangement of a Steam Turbine for Three Parallel Exhausts*

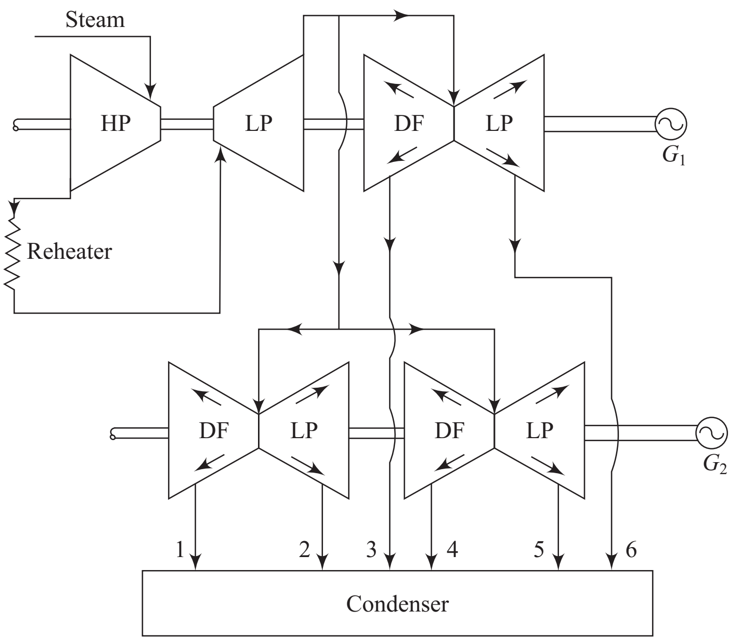


Figure 9.46 *Casing Arrangement of a Two Shaft Cross Compounded Steam Turbine with Six Parallel Exhausts*

EXAMPLE 9.8 The first stage of a steam turbine is a 2-row velocity compounded impulse runner. The mean peripheral velocity of the blades is 125 m/s and the velocity of steam at the inlet is 625 m/s. The nozzles are set at an angle of 18° and angles at outlet of the first rows of moving and fixed blades, and second row of moving blades are 19°, 22° and 36°, respectively. The height of the nozzle is 20 mm and mass flow rate of steam is 5.5 kg/s. Calculate the length of the nozzle arc neglecting the nozzle wall thickness. The specific volume of the steam at nozzles exit is 0.38 m^3/kg. Assume that all the blades have a pitch of 25 mm and a thickness of 0.4 mm at the tip exit. Calculate the height of the blades at the exit from each row. Take blade friction coefficient as 0.88 for all the blade rows.

Solution

Given: C_b = 125 m/s, C_1 = 625 m/s, α_1 = 18°, β_2 = 19°, α_3 = 22°, β_4 = 36°, h_n = 20 mm = 0.02, $\dot{m}$ = 5.5 kg/s, v_1 = 0.38 m^3/kg p = 25 mm = 0.025 m, t_2 = 0.4 mm, k = 0.88

(a) Length of the Nozzle Arc

$$\dot{m} = \frac{x\pi D h_n C_1 \sin\alpha_1 k_{tn}}{v_1} \tag{1}$$

$$5.5 = \frac{x\pi D \times 0.02 \times 625 \sin 18 \times 1}{0.38}$$

Length of the nozzle arc = $x\pi D$ = 0.541 m (2)

(b) Blade Height at Exit from Each Row

From the Figure 9.47,

$$o + t = p \sin\beta_2 \Rightarrow p \sin\beta_2 - t = o = \text{opening for steam flow}$$

$$A_b = o \times h_b \times z$$

where z is the number of such openings = $\dfrac{x\pi D}{p}$

$$A_b = \frac{x\pi D}{p} h_b (p \sin\beta_2 - t)$$

$$\dot{m} = \frac{x\pi D}{p} h_b (p \sin\beta_2 - t) C_{r2} \times \frac{1}{v_1} \tag{3}$$

$$\tan\beta_1 = \frac{C_1 \sin\alpha_1}{C_1 \cos\alpha_1 - C_b} \Rightarrow \beta_1 = \frac{625 \sin 18}{625 \cos 18 - 125}$$

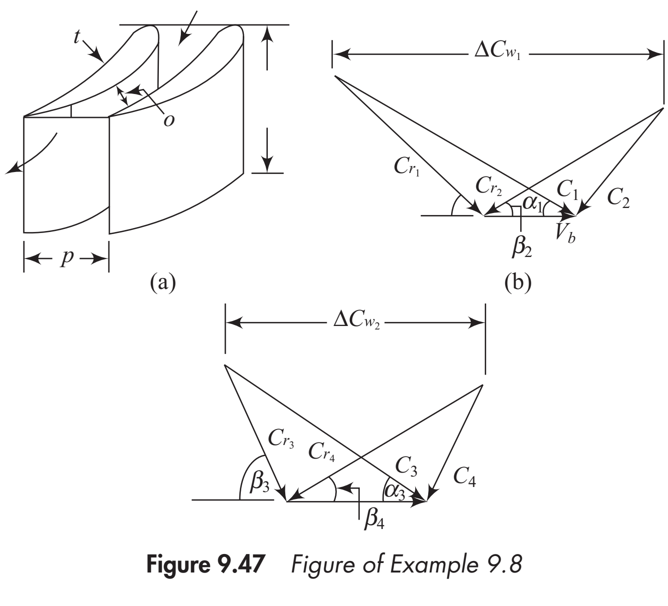


Figure 9.47 *Figure of Example 9.8*

$$\beta_1 = 22.364° \quad (4)$$

$$C_{f1} = C_{r1} \sin \beta_1 = C_1 \sin \alpha_1 \Rightarrow C_{r1} \sin 22.364 = 625 \sin 18$$

$$C_{r1} = 507.598 \text{ m/s} \quad (5)$$

$$C_{r2} = kC_{r1} \Rightarrow C_{r2} = 0.88 \times 507.598$$

$$C_{r2} = 446.686 \text{ m/s} \quad (6)$$

$$C_2^2 = C_{r2}^2 + C_b^2 - 2C_{r2}C_b \cos \beta_2 \Rightarrow C_2^2 = 446.686^2 + 125^2 - 2 \times 446.686 \times 125 \times \cos 19$$

$$C_2 = 331/\text{s} \quad (7)$$

$$C_3 = kC_2 \Rightarrow C_3 = 0.88 \times 331$$

$$C_3 = 291.28 \text{ m/s} \quad (8)$$

$$\tan \beta_3 = \frac{C_3 \sin \alpha_3}{C_3 \cos \alpha_3 - C_b} \Rightarrow \tan \beta_3 = \frac{291.28 \times \sin 22}{291.28 \cos 22 - 125}$$

$$\beta_3 = 36.95° \quad (9)$$

$$C_{f3} = C_{r3} \sin \beta_3 = C_3 \sin \alpha_3 \Rightarrow C_{r3} \sin 36.95 = 291.28 \sin 22$$

$$C_{r3} = 181.521 \text{ m/s} \quad (10)$$

$$C_{r4} = kC_{r3} \Rightarrow C_{r4} = 0.88 \times 181.521$$

$$C_{r4} = 159.74 \text{ m/s} \quad (11)$$

Substituting values in Eq. (3), we get,

I. Row of Moving Blades

$$5.5 = \frac{0.541}{0.025} h_{b1} (0.025 \sin 19 - 0.0004) \times 446.686 \times \frac{1}{0.38}$$

$$h_{b1} = 0.02794 \text{ m} = 27.94 \text{ mm} \quad (12)$$

Fixed Row of Guide Blades

$$5.5 = \frac{0.541}{0.025} h_n (0.025 \sin 22 - 0.0004) \times 291.28 \times \frac{1}{0.38}$$

$$h_n = 0.037 \text{ m} = 37 \text{ mm} \quad (13)$$

II. Row of Moving Blades

$$5.5 = \frac{0.541}{0.025} h_{b2} (0.025 \sin 36 - 0.0004) \times 159.74 \times \frac{1}{0.38}$$

$$h_b = 0.0423 \text{ m} = 42.3 \text{ mm} \quad (14)$$

9.11 Losses in Steam Turbines

A number of losses occur in a steam turbine which is a power producing turbomachine running with higher speed. These losses take place during the expansion of steam within a turbine. Sum of all these losses is a considerable quantity. Therefore, proper attention should be paid to the losses during selection of a turbine. These losses can be broadly classified broadly into two categories: (i) Internal losses, related with the steam flow, and (ii) External losses, that takes place external to the turbine casing.

9.11.1 Internal Losses

(a) Losses in Regulating Valves

Throttling of the steam occurs while passing through the main valve and the regulating valves before entering the turbine, consequently, pressure losses. However, the reduced enthalpy drop in the turbine results in lesser specific output. Hence, throttling which is an irreversible process results in loss of some available energy of steam. The pressure drop varies from 3 – 5% of the inlet steam pressure p_0.

(b) Nozzle Friction Losses

A nozzle is designed to obtain an increase in velocity from entry to exit considering isentropic expansion. The actual velocity of steam at the nozzle exit may not be equal to the designed velocity value corresponding to isentropic enthalpy drop. This change in actual working conditions of nozzle occurs due to non-isentropic expansion. The deviation from isentropic to non-isentropic expansion is taken in account by nozzle efficiency. Nozzle efficiency takes into account the effect of friction. However, in addition to friction, losses also occur due to boundary layer growth and eddies formation in the wake which depends on the height and length of flow passage. Losses in a turbulent boundary layer are greater than those in a laminar boundary layer. Pressure or enthalpy drop per stage in a reaction turbine is less resulting in lower velocity; therefore, laminar flow condition exists over a greater length of flow passage. Hence, loss due to friction in a reaction stage is lesser than the impulse stage. However, since more number of reaction stages is needed, so the total surface area exposed to flow is larger, consequently friction loss is increased. Therefore, losses in nozzle loss depends on its size, surface roughness, nozzle length, roundness of entry, angle of divergence, gap between nozzles, moisture and trailing edge.

(c) Blade Friction Losses

Various factors that cause losses in moving blades are discussed as follows:

(i) **Impingement Losses:** Steam from the exit of nozzles touches the blades leading edges. Energy may be lost because of flow separation if the entry is not sufficiently smooth and eddies are formed. These losses, known as impingement losses, are lesser for laminar flow relative to the turbulent flow.

(ii) **Frictional Losses:** These losses depend on the roughness of blade surface and occur in blade passages. Friction of the steam with blades causes frictional losses that are considered by the blade friction factor. Blade friction factor mainly depends on the Reynolds number; however, it is defined earlier as the ratio of relative velocity of steam exiting blades to the relative velocity of steam entering blades. Friction loss is also known as 'passage loss'.

(iii) **Boundary Layer separation:** Sharp deflection of fluid in the blade passage results in losses due to separation of boundary layer. Deflection of fluid exerts a centrifugal force that causes compression in the vicinity of concave surface and the rarefaction in the vicinity of convex surface of blade, therefore, resulting in boundary layer separation.

(iv) **Wake Losses:** Turbulence at the exit of previous nozzles row because of finite thickness of nozzle exit edge results in loss of energy. Mixing of steam jet exiting nozzles and entering moving blade occurs. Formation of eddies and turbulence takes place due to this flow transition from nozzle passage to the blade passage. This turbulence is normally in the form of trailing vortices which keep on disappearing at high velocities. These results in the decease of kinetic energy imparted to blades and are known as *wake losses*. Because of thickness of trailing edge, wake losses are observed at the trailing edge of fixed blades also.

(v) **Carry-over Losses:** When steam passes from one stage to the following stage, energy loss due to gap between fixed and moving blades rows is known as carry over losses. The kinetic energy exiting a stage is $\frac{C_2^2}{2}$ and available to the next is given by $\eta_{co}\left(\frac{C_2^2}{2}\right)$, where η_{co} is the carry-over efficiency. The carry-over loss is least if the gap between consecutive rows is little.

The various losses just discussed are taken into account by assuming the profile loss coefficient k_p, incidence loss coefficient k_i, and carry-over loss coefficient k_c. Losses due to turbulence, friction, fluid deflection in blade passage, curvature of blade and the actual outlet flow angle being different from the blade exit angle are taken into account by profile loss coefficient. Incidence loss coefficient considers turbulence losses caused by incidence angle. The carry over loss coefficient takes into account loss of kinetic energy during transition of flow between the rows. Therefore, the actual relative velocity exiting the blade is expressed by,

$$C_{r2} = k_p k_i k_c C_{r1} \tag{9.100}$$

(d) Disc Friction Losses

This type of energy loss takes place when an object, say disc, is rotated in air or any other fluid. The disc would cut the surrounding fluid and impart motion to it. There is always a relative motion between the disc surface and surrounding air or fluid. Surrounding fluid always exerts a resistance to motion of moving disc because of this relative motion. This may result in the energy loss due to friction that may be sensed by the enthalpy increase of surrounding fluid. Steam turbines rotor is also completely surrounded by the steam that offers resistance to the rotor motion. The energy loss of rotor may transfer into the surrounding steam. This loss of energy is called *disc friction losses*. The disc friction loss may result in heating of steam surrounding the rotor, i.e. a portion of rotor kinetic energy is transferred to steam causing heating of steam. Disc friction loss is considerable in impulse stages relative to reaction stages, where it is infinitesimal and can be neglected.

(e) Windage/Partial Admission Loss

Windage loss takes place when the blades of rotor come in contact with stationary fluid (steam). Turbines in which admission is partial, viz. an impulse stage, steam churns in the region of turbine that does not have active steam. Energy transfer from blade to steam occurs when moving blades come in contact with inactive steam. This energy loss of energy from rotor blades to steam is called *windage loss*. The full admission turbines have negligible region inside turbine for inactive steam. Therefore, windage loss is approximately negligible. Turbine should be filled with moving team (active steam) in order to minimize windage. Windage losses are infinitesimal in case of low pressure stages. Since reaction turbines are full admission turbines, therefore, windage losses are negligible.

(f) Gland Leakage Losses

Steam can leak between stages and along the shaft at entrance and leaving points of the casing. In both impulse and reaction stages, leakage from diaphragm occurs through the radial gap (clearance) between the fixed nozzle diaphragm and the shaft or drum. In reaction stages, leakage from tip takes place through the clearance between the outer periphery of the moving blades and the casing since pressure difference exist across the blades. Radial gap between the shaft and casing at both high and low pressure ends of turbines causes leakage loss known as shaft leakage. Steam leaks out to the atmosphere at the high pressure side, whereas air from atmosphere leaks into the shell at the low pressure side since the pressure being less than atmospheric. Leakage represents loss of energy since the leaked steam does not impart work to the blades. Leakages from diaphragm and tip both can be minimized by decreasing the radial clearances, but rubbing or metal-to-metal contact must be avoided. The clearance may be as low as 0.5 mm. However, both static and dynamic balancing of the rotor properly is essential to avoid any such rubbing. Use of seals or packings is essential to further decrease the leakage loss. Labyrinths, carbon rings, water or steam seals, or gland leak-off seals may be used. Labyrinths with carbon rings and gland leak-off seals may be used to prevent shaft leakage. Labyrinths seals comprise of a series of thin strips attached to the casing which maintains the least possible clearance with the shaft, as shown in Figure 9.48. The small constrictions throttle the steam to lower pressures several times until only an infinitesimal quantity leaks out. Carbon ring seals consist of a ring of carbon divided into parts. The rings are fitted snugly to the shaft by springs in order to stop leakage. These seals may be used in large turbines along with labyrinth seals in series.

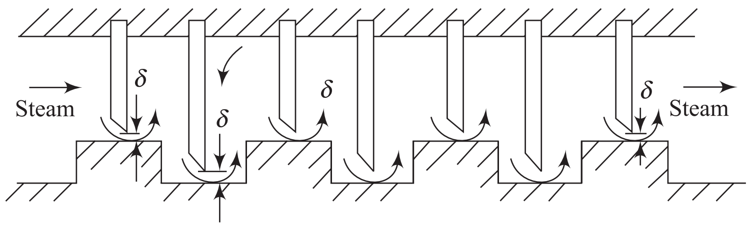


Figure 9.48 *Stepped Labyrinth Seals*

(g) Bearing Losses

Bearings are crucially important elements as they support turbine shaft of high rotational speed. Generally, a loss to the extent of 1% of turbine output takes place in bearings. However, bearing loss depends upon bearing load, viscosity of oil, shaft speed, surface area of bearing and thickness of oil film.

(h) Residual Velocity Loss

Steam exiting the turbine at last stage has a definite velocity which indicates a quantity of kinetic energy that cannot be imparted to the turbine shaft and hence a waste.

There are additional energy losses also because of the moisture of steam. Some kinetic energy of steam is consumed for dragging the water particles along with steam. Erosion and also corrosion occurs if the dryness fraction of steam is less than 0.88. Energy losses occur due to friction in the exhaust hood of the turbine as the exiting velocity of steam from the last stage of turbine is fairly large (100-120 m/s). Area is gradually increased like that of a diffuser in exhaust hoods of the condenser. Therefore, velocity of steam is further decreased with a corresponding increase in pressure when steam enters the condenser. The use of such hoods permits the exiting pressure of turbine to be slightly lesser than that needed by the condenser (depending on temperature and flow of cooling water, and air extraction from its shell). Therefore, turbine work is increased.

(i) Radiation Losses

Radiation losses also take place in steam turbines though these are very small compared with other losses. Therefore, radiation losses may be neglected. The high temperature steam in steam turbines is restricted to small portion of casing. Therefore, radiation losses are infinitesimal. Pipings, casing of turbine, etc. to carry hot steam should be insulated well to prevent radiation losses.

9.11.2 External Losses

Some energy losses occur in the bearings and governing systems that could be decreased by the better lubrication systems. Oil pumps also consume some energy. As the turbines are insulated well, the heat loss from surface due to convection and radiation is small. Modern well-designed and very efficient large electric generators are cooled by hydrogen in which the energy losses are within 2 – 3%.

9.12 Reheat Factor and Condition Line

The expansion process in a steam turbine must be isentropic under ideal condition. But actual expansion in the steam turbine is not isentropic process as various losses such as friction, leakage, entry and exit losses, etc. occur. Consequently, available energy converts lesser into the shaft work due to these losses. Therefore, steam at the exit of turbine will have higher enthalpy as compared with that under isentropic expansion. The ideal and actual expansion processes of steam in a single stage turbine are illustrated in Figure 9.49 (a). Steam enters into the turbine at state 1 and exits at 2*s* in ideal isentropic expansion, whereas at state 2 in actual expansion; therefore, $h_2 > h_{2s}$. Comparing isentropic expansion 1 – 2*s* with actual expansion 1 – 2 represents that total available energy $(h_1 - h_{2s})$ is not completely converted into shaft work as various losses occur in the turbine. A fraction of the available energy expressed by $[(h_1 - h_{2s}) - (h_1 - h_2) = (h_2 - h_{2s})]$ lasts with expanding steam itself. On Mollier chart ($h - s$ diagram), the shift of state at the end of expansion from 2*s* to 2 because of non-isentropic effects is termed as reheating of steam at constant pressure. The reheating from 2*s* to 2 is assumed to be at constant pressure for the sake of simplification, whereas actually it may not be so. Non-isentropic expansion increases the quality of steam after expansion, i.e. temperature at exit $T_2 > T_{2s}$, if the state of steam at turbine exit lies in superheat region or dryness fraction at exit $x_2 > x_{2s}$, if the state of steam at turbine exit lies in wet region. Reheating of steam leads to entropy increase, i.e. $s_2 > s_{2s}$. The ideal and actual expansions of steam in a single stage turbine are shown on $T - s$ diagram in Figure 9.49 (b). The amount of heat losses or energy dissipation due to the friction, etc. are shown by $T - s$ diagram and subsequent recovery of energy losses by expansion in steam turbine due to reheating by the area 2*s* – 2 – *B* – *A*.

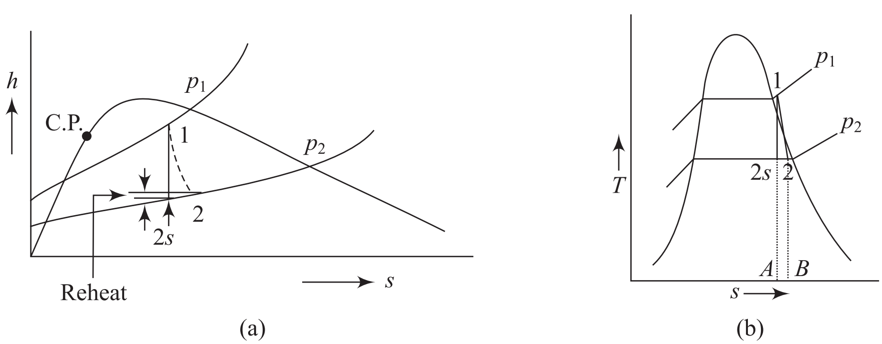


Figure 9.49 *Expansion in a Single Stage with Reheat on (a) h – s Diagram, (b) T – s Diagram*

Expansion in multistage shown on $h - s$ and $T - s$ diagrams in Figure 9.50 (a) and (b) indicate the expansion in four stages beginning from state 1. The ideal expansion in the first stage is shown by the line $1 - 2s$, whereas actual expansion is $1 - 2$ Expansion in second stage starts from state 2. Isentropic expansion in second stage is $2 - 3s$, whereas non-isentropic expansion is $2 - 3$. Actual state at the end of second stage expansion is 3 from which the succeeding stage expansion starts. The theoretical and actual states are 4*s* and 4, respectively, at the end of third stage expansion. The theoretical and actual states are 5*s* and 5, respectively, at the end of last stage expansion.

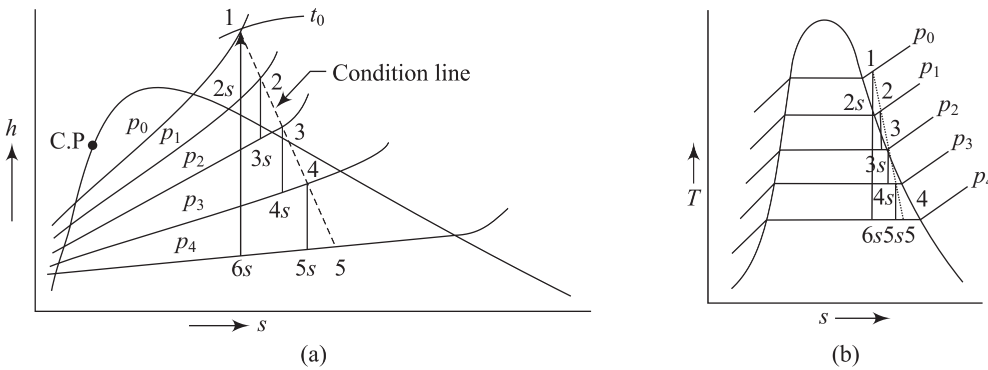


Figure 9.50 *Expansion in Multistage and Condition Line on (a) h – s Diagram, (b) T – s Diagram*

The sum of the isentropic enthalpy drops in stages, i.e. $[(h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) + (h_4 - h_{5s})]$ represents the sum of available energies in individual stages which is known as *cumulative enthalpy or heat drop*. The isentropic enthalpy drop for the whole turbine, $(h_1 - h_{6s})$, known as *Rankine enthalpy or heat drop,* is the available energy to produce work if expansion is carried out in a single stage.

Following conclusions are drawn in multistage expansion taking into account the effects of reheat:

- Reheat takes place with increase in entropy.
- The reheat in a given stage indicates higher availability of energy for the next stage except for the last where reheat is a loss because of the absence of further stages.
- The constant pressure lines diverge from one another thereby increasing the enthalpy drop for the same pressure drop.
- Because of (ii) and (iii), the cumulative enthalpy drop is greater than the Rankine enthalpy drop.
- The locus of actual states 1, 2, 3, 4 and 5 is known as *condition line* which gives the actual state of steam exiting the turbine stages.

Reheat factor is the ratio of cumulative enthalpy drop to the Rankine enthalpy drop. Thus,

$$RF = \frac{\text{Cumulative enthalpy drop}}{\text{Rankine enthalpy drop}} = \frac{(h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) + (h_4 - h_{5s})}{(h_1 - h_{6s})} \quad (9.101)$$

The value of reheat factor is always greater than unity since cumulative enthalpy drop is always greater than Rankine enthalpy drop, as already discussed. The usual values of reheat factor are of the order of 1.04 – 1.08 signifying 4 – 8% increase in work output due to multistaging. The reheat factor depends on the initial state of the steam, the pressure ratio of expansion of each stage, the stage efficiency of each stage and the number of stages. The value of reheat factor will increase if number of stages for a given pressure range is increased. For constant pressure range and number of stages, value of reheat factor increases with decrease in stage efficiency. The value of reheat factor must be unity under ideal conditions from designer's perspective. As value of reheat factor 1 cannot be obtained practically, therefore, attempts must be made to decrease the value of reheat factor to approximately unity. Unit value of reheat factor represents that actual expansion is identical to ideal expansion and cumulative enthalpy drop equals to Rankine enthalpy drop. The reheat factor is applicable to pressure compounded steam turbines with two or more stages of pressure drops. Although, compounding is done to reduce the velocity of steam in the turbine passages and speed of the turbine, the reheat effect is an unintentional advantage of multistaging.

The ratio of the actual enthalpy drop to the isentropic enthalpy drop is known as *stage efficiency*. Assuming the stage efficiencies of all the stages to be the same, then,

$$\eta_{\text{stage}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{h_4 - h_5}{h_4 - h_{5s}} \tag{9.102}$$

$$\therefore \quad h_1 - h_{2s} = \frac{h_1 - h_2}{\eta_{\text{stage}}},\; h_2 - h_{3s} = \frac{h_2 - h_3}{\eta_{\text{stage}}},\; h_3 - h_{4s} = \frac{h_3 - h_4}{\eta_{\text{stage}}},\; h_4 - h_{5s} = \frac{h_4 - h_5}{\eta_{\text{stage}}} \tag{9.103}$$

Substituting values from Eq. (9.103) into Eq. (9.102), we get,

$$RF = \frac{\dfrac{h_1 - h_2}{\eta_{\text{stage}}} + \dfrac{h_2 - h_3}{\eta_{\text{stgae}}} + \dfrac{h_3 - h_4}{\eta_{\text{stgae}}} + \dfrac{h_4 - h_5}{\eta_{\text{stage}}}}{h_1 - h_{6s}} = \frac{1}{\eta_{\text{stage}}} \times \frac{h_1 - h_5}{h_1 - h_{6s}} \tag{9.104}$$

Internal efficiency of the turbine is defined as the ratio of sum of actual stage outputs to the available energy in turbine. In other words, internal efficiency may also be expressed by the ratio of internal work output of turbine to the Rankine enthalpy drop. Therefore,

$$\eta_{\text{internal}} = \frac{h_1 - h_5}{h_1 - h_{6s}} \tag{9.105}$$

By Eqs (9.104) and (9.105),

$$\eta_{\text{int ernal}} = R_F \times \eta_{\text{stage}} \tag{9.106}$$

Since, $RF > 1$ always, therefore,

$$\eta_{\text{int ernal}} > \eta_{\text{stage}} \tag{9.107}$$

In other words, due to reheat, the sum of $(h_2 - h_{2s}) + (h_3 - h_{3s}) + (h_4 - h_{4s}) + (h_5 - h_{5s})$ is greater than $(h_5 - h_{6s})$.

EXAMPLE 9.9 Steam at 35 bar, 450°C enters in a 5-stage steam turbine and leaves at 0.07 bar, 0.89 dry. Determine (a) steam state at entry of each stage, (b) stage efficiency or efficiency ratio of each stage, (c) reheat factor, and (b) overall turbine efficiency. Assume that all the stages develop equal work.

Solution

Given: p_1 = 35 bar, t_1 = 450°C, n = 5, p_6 = 0.07 bar, x_6 = 0.89

The states of the steam at the turbine inlet and outlet are located on the Mollier chart as shown in Figure 9.51. From Mollier chart,

$$\Delta h_a = h_1 - h_6 = 1028 \text{ kJ/kg}$$

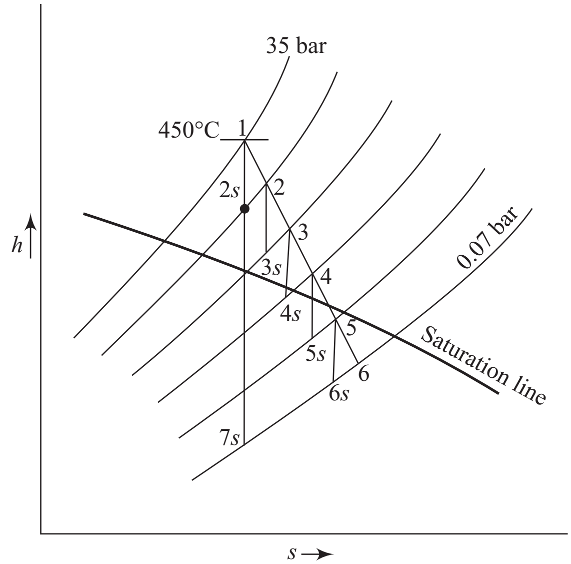


Figure 9.51 *h—s* Diagram of Example 9.9

Since all the stages develop equal work, therefore,

$$\text{Actual enthalpy drop in each stage} = \frac{h_1 - h_6}{5} = \frac{1028}{5}$$

Actual enthalpy drop in each stage = 205.6 kJ/kg

(a) Steam State at Entry of Each Stage

Drawing the actual enthalpy drop line 1 – 6 on the Mollier chart, steam state at the entry of each stage can be located as in the Table 9.2.

TABLE 9.2 States of Steam at Entry of Each Stage for Problem 9.9

Stage	*State of Steam at Inlet of Stage*	
	Pressure in bar	*Temperature/Dryness Fraction*
1	35	450°C
2	14.5	342°C
3	5.5	235°C
4	1.7	125°C
5	0.35	0.95

(b) Stage Efficiency

From Mollier chart,

$$h_1 - h_{2s} = 264 \text{ kJ/kg}, \; h_2 - h_{3s} = 246 \text{ kJ/kg},$$

$$h_3 - h_{4s} = 240 \text{ kJ/kg}, \; h_4 - h_{5s} = 240 \text{ kJ/kg}, \; h_5 - h_{6s} = 235 \text{ kJ/kg} \qquad (1)$$

∵ Actual enthalpy drop in each stage = 205.6 kJ/kg, therefore,

∵ $$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = h_4 - h_5 = h_5 - h_6 = 205.6 \text{ kJ/kg}$$

∵ $$\eta_{\text{stage}} = \frac{\text{Actual enthalpy drop in the stage}}{\text{Isentropic enthalpy drop in the stage}} = \frac{\Delta h_a}{\Delta h_s} \Rightarrow \Delta h_s = \frac{\Delta h_a}{\eta_{\text{stage}}} \tag{2}$$

$$(\eta_{\text{stage}})_{\text{I}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{205.6}{264}$$

$$(\eta_{\text{stage}})_{\text{I}} = 0.77888 = 77.888\% \tag{3}$$

$$(\eta_{\text{stage}})_{\text{II}} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{205.6}{246}$$

$$(\eta_{\text{stage}})_{\text{II}} = 0.8357724 = 83.577\% \tag{4}$$

$$(\eta_{\text{stage}})_{\text{III}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{205.6}{240}$$

$$(\eta_{\text{stage}})_{\text{III}} = 0.856667 = 85.667\% \tag{5}$$

$$(\eta_{\text{stage}})_{\text{IV}} = \frac{h_4 - h_5}{h_4 - h_{5s}} = \frac{205.6}{240}$$

$$(\eta_{\text{stage}})_{\text{IV}} = 0.856667 = 85.667\% \tag{6}$$

$$(\eta_{\text{stage}})_{\text{V}} = \frac{h_5 - h_6}{h_5 - h_{6s}} = \frac{205.6}{235}$$

$$(\eta_{\text{stage}})_{\text{V}} = 0.874894 = 87.4894\% \tag{7}$$

(c) Reheat Factor

From Mollier chart,

$$h_1 - h_{7s} = 1162 \text{ kJ/kg} \tag{8}$$

$$RF = \frac{\text{Cumulative enthalpy drop}}{\text{Rankine enthalpy drop}} = \frac{\sum_{n=1}^{n} \Delta h_i}{h_1 - h_{7s}} = \frac{h_1 - h_{2s} + h_2 - h_{3s} + h_3 - h_{4s} + h_4 - h_{5s} + h_5 - h_{6s}}{h_1 - h_{7s}} \tag{9}$$

$$RF = \frac{264 + 246 + 240 + 240 + 235}{1162}$$

$$RF = 1.05422 \tag{10}$$

(d) Overall Turbine Efficiency

$$\eta_{\text{internal}} = \frac{\text{Cumulative actual enthalpy drop}}{\text{Rankine anthalpy drop}} = \frac{\sum_{n=1}^{n} \Delta h_a}{h_1 - h_{7s}} = \frac{h_1 - h_6}{h_1 - h_{7s}} \tag{11}$$

$$\eta_{\text{internal}} = \frac{1028}{1162}$$

$$\eta_{\text{internal}} = 0.8846816 = 88.4682\% \tag{12}$$

EXAMPLE 9.10 Steam is supplied to an impulse steam turbine at 40 bar, 400°C and the condenser pressure is 0.075 bar. The first stage of the turbine is velocity compounded with a row of fixed guide blades in between the two rows of moving blades. The isentropic enthalpy drop for this stage is 1/4 of that for the whole turbine unit. The nozzle angle is 18° and nozzle efficiency is 90%. The mean peripheral velocity of both the moving blades rows is 0.2 times the velocity of steam leaving the nozzle. The exit blade angles for both the fixed and moving blades are 30° and the blade friction coefficient for all the blades is 0.87. If the internal efficiency of the turbine is 78%, calculate (a) efficiency of the first stage, and (b) percentage of the total power developed by the turbine in this stage.

Solution

Given: p_1 = 40 bar, t_1 = 400°C, p_2 = 0.075 bar, $(\Delta h_s)_{\text{stage}} = \frac{1}{4}(\Delta h_s)_{\text{total}}$, $\alpha_1 = 18°$, $\eta_n = 90\% = 0.9$, $C_b = 0.2C_1$, β_2, β_4 = 30°, $\alpha_3 = 30°$, $k = 0.87$, $\eta_i = 78\% = 0.78$

Figure 9.52 illustrates the states of steam at the turbine inlet and outlet on the *h-s* diagram.

From steam table, properties of superheated steam,

$$h_1 = 3213.6 \text{ kJ/kg}, s_1 = 6.7690 \text{ kJ/kg-K}$$

$s_{2s} = s_1$ = 6.7690 kJ/kg-K which is less than $(s_g)_{0.075 \text{ bar}}$ = 8.2515 kJ/kg-K. Hence, point 2*s* is in wet region. At condenser pressure of 0.075 bar, from properties of saturated steam,

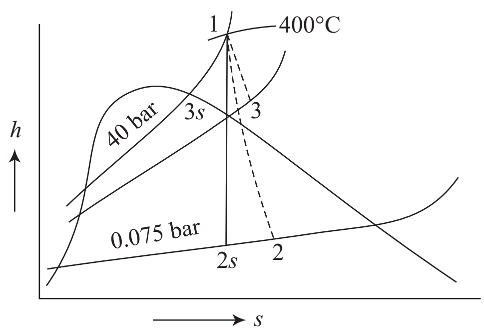


Figure 9.52 *States of Steam at Turbine Inlet and Outlet on h-s Diagram.*

$$s_g = 8.2515 \text{ kJ/kg – K}, s_f = 0.5764 \text{ kJ/kg-K}, s_{fg} = 7.6750 \text{ kJ/kg-K},$$
$$h_f = 168.79 \text{ kJ/kg}, h_{fg} = 2406 \text{ kJ/kg}$$

$$s_{2s} = 6.7690 = 0.5764 + x_{2s} \times 7.6750$$

$$x_{2s} = 0.807 \tag{1}$$

$$h_{2s} = 168.79 + 0.807 \times 2406$$

$$h_{2s} = 2110.432 \text{ kJ/kg} \tag{2}$$

$$(\Delta h_s)_{\text{total}} = h_1 - h_{2s} \Rightarrow (\Delta h_s)_{\text{total}} = 3213.6 - 2110.432$$

$$(\Delta h_s)_{\text{total}} = 1103.168 \text{ kJ/kg} \tag{3}$$

Isentropic specific enthalpy drop in the two-row velocity compounded or Curtis stage is,

$$h_1 - h_{3s} = \frac{1}{4}(\Delta h_s)_{\text{total}} = \frac{1}{4} \times 1103.168$$

$$h_1 - h_{3s} = 275.792 \text{ kJ/kg} \quad (4)$$

Velocity of steam at the exit of the nozzle,

$$C_1 = \sqrt{2\eta_n(\Delta h_s) + C_0^2} = \sqrt{2\eta_n(h_1 - h_{3s}) + C_0^2} \quad (5)$$

$$C_2 = \sqrt{2 \times 1000 \times 0.9 \times 275.792}$$

$$C_1 = 704.575 \text{ m/s} \quad (6)$$

$$C_b = 0.2C_1 \Rightarrow C_b = 0.2 \times 704.575$$

$$C_b = 140.915 \text{ m/s} \quad (7)$$

Velocity triangles are shown in Figure 9.53.

$$\tan \beta_1 = \frac{C_{f1}}{C_{w1} - C_b} = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - C_b} \Rightarrow \tan \beta_1 = \frac{704.575 \sin 18}{704.575 \cos 18 - 140.915}$$

$$\beta_1 = 22.364° \quad (8)$$

$$C_{f1} = C_{r1} \sin \beta_1 = C_1 \sin \alpha_1 \Rightarrow C_{r1} = \frac{C_1 \sin \alpha_1}{\sin \beta_1} = \frac{704.575 \sin 18}{\sin 22.364}$$

$$C_{r1} = 572.225 \text{ m/s} \quad (9)$$

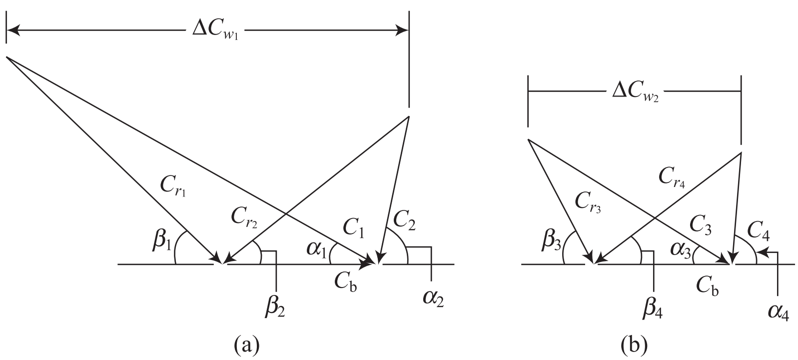


Figure 9.53 *Velocity Diagram of Example 9.10*

$$k = \frac{C_{r2}}{C_{r1}} \Rightarrow C_{r2} = kC_{r1} \Rightarrow C_{r2} = 0.87 \times 572.225$$

$$C_{r2} = 497.836 \text{ m/s} \quad (10)$$

$$\Delta C_{w1} = C_{r1} \cos \beta_1 + C_{r2} \cos \beta_2 = 572.225 \cos 22.364 + 497.836 \cos 30$$

$$\Delta C_{wt} = 960.324 \text{ m/s} \quad (11)$$

$$C_2^2 = (C_{r2} \sin \beta_2)^2 + (C_{r2} \cos \beta_2 - C_b)^2$$

$$\Rightarrow \quad C_2^2 = (497.836 \sin 30)^2 + (497.836 \cos 30 - 140.915)^2$$

$$C_2 = 382.348 \text{ m/s} \tag{12}$$

$$C_3 = kC_2 \Rightarrow C_3 = 0.87 \times 382.348$$

$$C_3 = 332.643 \text{ m/s} \tag{13}$$

$$\tan \beta_3 = \frac{C_{f3}}{C_{w3} - C_b} = \frac{C_3 \sin \alpha_3}{C_3 \cos \alpha_3 - C_b} \Rightarrow \tan \beta_3 = \frac{332.643 \sin 30}{332.643 \cos 30 - 140.915}$$

$$\beta_3 = 48.5° \tag{14}$$

$$C_{f3} = C_{r3} \sin \beta_3 = C_3 \sin \alpha_3 \Rightarrow C_{r3} \sin 48.5 = 332.643 \sin 30$$

$$C_{r3} = 222.0712 \text{ m/s} \tag{15}$$

$$C_{r4} = kC_{r3} \Rightarrow C_{r4} = 0.87 \times 222.0712$$

$$C_{r4} = 193.202 \text{ m/s} \tag{16}$$

$$\Delta C_{w2} = C_{r3} \cos \beta_3 + C_{r4} \cos \beta_4 \Rightarrow \Delta C_{w2} = 222.0712 \cos 48.5 + 193.202 \cos 30$$

$$\Delta C_{w2} = 314.467 \text{ m/s} \tag{17}$$

(a) Stage Efficiency

$$\eta_D = \frac{2(\Sigma \Delta C_w) C_b}{C_1^2} = \frac{2(\Delta C_{w1} + \Delta C_{w2}) C_b}{C_1^2} \tag{18}$$

$$\eta_D = \frac{2(960.324 + 314.467)140.915}{704.575^2}$$

$$\eta_D = 0.723722 = 72.3722\% \tag{19}$$

$$\eta_s = \eta_n \eta_D \tag{20}$$

$$\eta_s = 0.9 \times 0.723722$$

$$\eta_s = 0.65135 = 65.135\% \tag{21}$$

(b) Stage Power

$$(\Delta h)_{\text{total}} = \eta_i (h_1 - h_{2s}) \tag{22}$$

$$(\Delta h)_{\text{total}} = 0.78(1103.168)$$

$$(\Delta h)_{\text{total}} = 860.47 \text{ kJ/kg} \tag{23}$$

$$(\Delta h)_{\text{Curtis}} = \eta_s (h_1 - h_{3s}) \tag{24}$$

$$(\Delta h)_{\text{Curtis}} = 0.65135(275.792)$$

$$(\Delta h)_{\text{Curtis}} = 179.637 \text{ kJ/kg} \tag{25}$$

$$\% \text{ Power developed in Curtis stage} = \frac{(\Delta h)_{\text{Curtis}}}{(\Delta h)_{\text{total}}} \times 100 = \frac{179.637}{860.471} \times 100$$

$$\% \text{ Power developed in Curtis stage} = 20.877\% \quad (26)$$

EXAMPLE 9.11 For the turbine of Example 9.3, calculate the number of stages required assuming that the stage efficiency and work done are the same for all the stages. Steam is supplied to the first stage at 20 bar, 300°C and the condenser pressure is 0.065 bar Take reheat factor as 1.05.

Solution

For the turbine of Example 9.3, stage efficiency is found to be 72.72%.

Internal efficiency of the turbine is,

$$\eta_{\text{internal}} = h_s \times RF \quad (1)$$

$$\eta_{\text{internal}} = 0.7272 \times 1.05$$

$$\eta_{\text{internal}} = 0.7636 \quad (2)$$

The states of the steam at inlet and outlet are shown in Figure 9.54. From steam table, properties of superheated steam at 20 bar, 300°C,

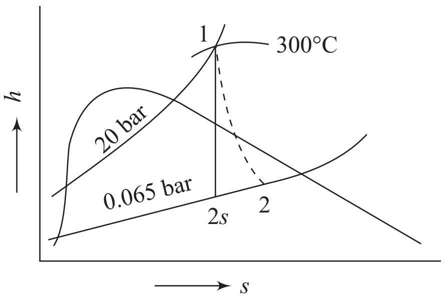


Figure 9.54 *States of Steam at Turbine Inlet and Outlet on Mollier Chart*

$$h_1 = 3023.5 \text{ kJ/kg}, \ s_1 = 6.7664 \text{ kJ/kg-K}$$

At condenser pressure of 0.065 bar, from properties of saturated steam,

$$s_g = 8.30894 \text{ kJ/kg-K}, \ s_f = 0.5364 \text{ kJ/kg-K}, \ s_{fg} = 7.77248 \text{ kJ/kg-K}, \ h_f = 156.402 \text{ kJ/kg},$$

$h_{fg} = 2413.08$ kJ/kg $\because$ $s_{2s} = s_1 = 6.7664$ kJ/kg-K $< (s_g)_{0.065 \text{ bar}} \Rightarrow$ Steam will be wet at '2*s*'.

$$s_{2s} = 6.7664 = 0.5364 + x_{2s} \times 7.77248$$

$$x_{2s} = 0.80155 \quad (3)$$

$$h_{2s} = 156.402 + 0.80155 \times 2413.08$$

$$h_{2s} = 2090.6063 \text{ kJ/kg} \quad (4)$$

Specific enthalpy drop in all the stages,

$$(\Delta h)_{\text{total}} = h_1 - h_2 = \eta_{it}(h_1 - h_{2s}) \quad (5)$$

$$(\Delta h)_{\text{total}} = 0.7636 \times (3023.5 - 2090.6063)$$

$$(\Delta h)_{\text{total}} = 712.358 \text{ kJ/kg} \quad (6)$$

$$\text{Number of stages} = n = \frac{(\Delta h)_{\text{total}}}{(\Delta h)_{\text{stage}}} = \frac{(\Delta h)_{\text{total}}}{w_D} \quad (7)$$

$$n = \frac{712.358}{49.49115}$$

$$n = 14.394 \approx 15 \text{ Stages} \quad (8)$$

9.13 Design of Multistage Turbines

The design of a steam turbine involves a judicious combination of theory with the results of experience, governed to a great extent by cost. The following outlined method of design is only illustrative of the theories discussed before. The following are specified to the designer: initial steam conditions, exhaust pressure, and the capacity in MW or kW. The turbine requires many stages with increase in diameter from the inlet to the exit end. All wheels turn at the same speed (rpm), but C_b, C_1, k_{tb}, α, β, γ, leakage efficiency, disc friction and windage loss may all vary from stage to stage. The condition line, which is the logical starting point, can only be approximated until all the stage efficiencies are known.

The calculation for the casing arrangement of a multistage impulse turbine is made according to the Section 9.10. The first stage is most often a two-row Curtis stage. In order to increase the height of the nozzles, the stage is usually given a partial admission. In large condensing steam turbines, where the specific volume at the end of expansion in the turbine becomes very large, long blades of special design are selected. The design of multistage turbines is usually started with initial design considerations of first, second and last stages, while the intermediate stages are designed later.

9.13.1 First Stage

The first stage blade velocities range from 120 m/s to 150 m/s. For a bipolar 50 Hz alternator, N = 3000 rpm.

$$\therefore \qquad C_b = \frac{\pi DN}{60} = 120 \text{ m/s (say)}$$

The mean diameter *D* can be estimated. The optimum velocity ratio for a 2-row Curtis stage,

$$\rho_{\text{opt}} = \cos\frac{\alpha_1}{4} = \frac{C_b}{C_1}$$

For the actual stage, the velocity ratio will be less, say, 0.95 of this value.
Therefore, for steam leaving the nozzles, C_1 can be computed from

$$C_1 = \left(\frac{4C_b}{\cos\alpha_1}\right) \times 0.95$$

Again, $$C_1 = 44.72[\eta_n(\Delta h_s)_{\text{Curtis}}]^{1/2}.$$

From which Δh_s (=enthalpy drop in the Curtis stage) can be estimated, assuming a suitable value of nozzle efficiency.

The total isentropic enthalpy drop is first noted from the Mollier Chart with the given steam condition at the inlet and exit of the turbine. The enthalpy drop that would take place in the subsequent stages would become,

$$(\Delta h_s)_{\text{impulse}} = (\Delta h_s)_{\text{total}} - (\Delta h_s)_{\text{Curtis}}$$

Assuming nozzle angle α_1, the blade exit angles β_2 and β_4, and the blade friction factor *k* for all the blades, the velocity diagrams of the Curtis stage can be drawn, from which $\Sigma\Delta C_w$, W_D and η_D can be determined.

Now, $$\eta_{\text{stage}} = \eta_n\eta_D(1 - \eta_{df})$$

where η_{df} is the coefficient that accounts for the disc friction, windage and other losses (≈3%).

Assuming a mechanical efficiency of, say, 97%,

$$\dot{m} = \frac{MW \times 10^3}{\eta_{\text{stage}} \times \eta_{\text{mech}} \times (\Delta h_s)_{\text{total}}} \text{kg/s}$$

$$\dot{m} = \frac{A_1 C_1}{v_1}$$

from which A_1, the nozzle exit area, can be estimated.

Now, $$A_1 = \pi D h_n k_{tn} C_1 \sin \alpha_1$$

from which h_n can be determined. If this is found to be very small, the minimum practicable height with partial admission is to be adopted.

(a) First Row of Moving Blades

$$h_{b1} = h_n + 1.60 \text{ mm}$$

$$h_{b2} = h_n \times \left(\frac{C_{f1}}{C_{f2}} \right)$$

(b) Guide Blades

$$h_{b3} = h_{b2} + 1.60 \text{ mm}$$

$$h_{b4} = h_n \times \frac{C_{f1}}{C_{f3}}$$

(c) Second Row of Moving Blades

$$h_{b3} = h_{b4} + 1.60 \text{ mm}$$

$$h_{b6} = h_n \times \frac{C_{f1}}{C_{f4}}$$

The ratio of the axial components $\frac{C_{f1}}{C_{f2}}$ and so on can be determined from the velocity diagrams. A constant blade diameter, D, has been assumed for the Curtis stage.

9.13.2 Second Stage

For impulse stages, the optimum velocity ratio

$$\rho_{\text{opt}} = \cos \frac{\alpha_1}{2} = \frac{C_b}{C_1}$$

where α_1 is the nozzle angle. For the actual stage, a value somewhat less than this optimum value may be assumed. If the average blade velocity is assumed, then the average nozzle exit velocity can be estimated.

$$C_1 = 44.72[\eta_n (\Delta h_s)_{\text{stage}}]^{1/2}$$

Assuming suitable value of η_n, the average enthalpy drop per stage can be computed.

$$\text{Number of Stages Required} = \frac{(\Delta h_s)_{\text{total}}}{(\Delta h_s)_{\text{stage}}}$$

The absolute velocity of steam exiting the second row of moving blades (C_4) of the Curtis stage is known. From the energy balance across the nozzles of the second stage (impulse), the absolute velocity of steam (C_1) entering the blades can be estimated. By assuming suitable values of relevant parameters, the velocity diagrams can be plotted, from which η_D can be estimated, and

$$\eta_{\text{stage}} = \eta_n \times \eta_D \times (1 - \eta_{df})$$

can similarly be determined.
In the stage, $\Delta h_{\text{act}} = \eta_{\text{stage}} \times (\Delta h_s)_{\text{stage}}$

$$\dot{m} = \frac{\pi D h_b \times k_{\text{th}} C_1 \sin \alpha_1}{v_1}$$

Assuming a suitable value of (h_b/D), h_b and D can be estimated. The nozzle height will then be, $h_n = (h_b - 1.60)$ mm

9.13.3 Last Stage

Assuming the maximum blade velocity consistent with the blade material (350 – 400 m/s), the last stage diameter is estimated. From the stress consideration, the maximum height to diameter ratio for twisted or tapered blades can be assumed to be 0.3. If the blades are assumed to operate close to the maximum efficiency, the jet velocity of steam and hence, Δh_s can be determined. The velocity diagrams can be drawn from which η_D can be estimated.

$$\dot{m} = \frac{\pi D h_b \times k_{\text{th}} C_1 \sin \alpha_1}{v_1}$$

From which both h_b and D are estimated.
The procedure has also been illustrated while discussing the casing arrangements earlier.

9.13.4 Intermediate Stages

The isentropic enthalpy drops that would take place in HP, IP and LP cylinders are known. Since the enthalpy drop per stage (Δh_s) is also known, the number of stages in each cylinder can be determined. The blade and nozzle dimensions can similarly be estimated.

By knowing the stage efficiencies of all the stages, the condition line can be drawn on the Mollier diagram from which the final condition (x, v) of steam can be noted. The turbine internal efficiency is then determined, and the steam flow rate $\dot{m}$ is calculated. If this $\dot{m}$ does not tally with the value used earlier, calculations have to be repeated.

SUMMARY

- ✦ Various classifications and the terms associated with steam turbines are presented.
- ✦ Necessity and various methods of compounding of steam turbines have been discussed.

✦ Velocity diagrams for different steam turbines have been drawn. Stage parameters for a simple impulse and 2-row Curtis stages are listed in the following table:

Stage Parameters	*Simple Stage*	*Velocity Compounded or 2- Row Curtis Stage*
Whirling Thrust	$F_t = \dot{m}\Delta C_w$	$F_t = \dot{m}\Sigma\Delta C_w = \dot{m}(\Delta C_{w1} + \Delta C_{w2})$
Axial Thrust	$F_a = \dot{m}\Delta C_f$	$F_a = \dot{m}\Sigma\Delta C_f = \dot{m}(\Delta C_{f1} + \Delta C_{f2})$
Blading Work or Diagram Work	$\dot{W}_D = F_t C_b = \dot{m}\Delta C_w C_b$	$\dot{W}_D = F_t C_b = \dot{m}(\Delta C_{w1} + \Delta C_{w2})C_b$
Diagram or Blade or Wheel Efficiency	$\eta_D = \dfrac{2(C_1 \cos\alpha_1 - C_b)(1+k)C_b}{C_1^2}$	$\eta_D = \dfrac{2(\Sigma\Delta C_w)C_b}{C_1^2} = \dfrac{2(\Delta C_{w1} + \Delta C_{w2})C_b}{C_1^2}$
Optimum Velocity Ratio	$\rho_{opt} = \dfrac{\cos\alpha_1}{2}$	$\rho_{opt} = \dfrac{\cos\alpha_1}{4}$
Maximum Diagram or Blade Efficiency	$(\eta_D)_{max} = \cos^2\alpha_1$	$(\eta_D)_{max} = \cos^2\alpha_1$

✦ The total enthalpy drop is divided equally among the stages in pressure compounded or Rateau stage.

✦ Three-fourth of the total work is done by the steam jets on the first row of moving blades and one-fourth of the total work is done on the second row of moving blades in a 2-row Curtis stage.

✦ Only one-ninth of the total work in the stage is done in the third row of moving blades.

✦ In a Curtis stage, the effectiveness of last row decreases with the number of rows of moving blades. In a conventional Curtis stage design, only two rows of the moving blades are used.

✦ Stage parameters for a reaction turbine are given in the following table:

Stage Parameters	*Reaction Turbine*
Diagram or Blade Work	$w_D = \Delta C_w C_b = (2C_1 \cos\alpha_1 - C_b)C_b$
Diagram or Blade efficiency	$\eta_D = \dfrac{2(2\rho\cos\alpha_1 - \rho^2)}{1 - \rho^2 + 2\rho\cos\alpha_1}$
Optimum Velocity Ratio	$\rho_{opt} = \cos\alpha_1$
Maximum Diagram or Blade Work	$(w_D)_{max} = C_b^2$
Maximum Diagram or Blade Efficiency	$(\eta_D)_{max} = \dfrac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1}$

✦ The exit velocity of the preceding rotor blades is somewhat reduced when approaching the fixed blades of succeeding stage for further expansion due to the gap between the moving and fixed blade rings and due to the possible eddies, discontinuities and a sort of 'fluid shear' between the blade rings. This reduction is accounted by employing the *carry-over efficiency,* η_{co}.

✦ The combined nozzle and blade efficiency η_{nb} is given by,

$$\eta_{nb} = \eta_n \frac{2\rho\cos\alpha_1 - \rho^2}{1 - \eta_{co}(1 + \rho^2 - 2\rho\cos\alpha_1)}$$

✦ The relation between isentropic enthalpy drop in various steam turbine stages is,

$$(\Delta h_s)_{50\%\text{ reaction stage}} : (\Delta h_s)_{\text{impulse stage}} : (\Delta h_s)_{\text{2-rwo Curtis stage}} = 1 : 2 : 8$$

✦ The efficiency of the reaction stage will be the highest and that of the Curtis stage will be the lowest, while that of impulse stage will lie in between.

$$\eta_{50\%\text{ reaction stage}} > \eta_{\text{simple impulse stage}} > \eta_{\text{2-row Curtis stage}}$$

✦ For a good efficiency, the blade angles should vary with the diameter, therefore, twisted (or warped) blades are used in the later stages of the turbine.

✦ The nozzle and blade heights, number of parallel exhausts in different stages and casing arrangements of steam turbines have been found.

✦ Various losses occurring in a steam turbine have been discussed.

✦ Reheat factor is the ratio of cumulative enthalpy drop to the Rankine enthalpy drop. The usual values of reheat factor are of the order of 1.04 – 1.08 signifying 4 – 8% increase in work output due to multistaging.

✦ Relation between internal efficiency, reheat factor and stage efficiency is,

$$\eta_{\text{int ernal}} = RF \times \eta_{\text{stage}}$$

✦ Design of the first stage, second stage, last stage and intermediate stages of a multistage steam turbine has been discussed.

REVIEW QUESTIONS

9.1 How does energy conversion occur in a steam turbine?

9.2 Classify steam turbines.

9.3 Differentiate between impulse and reaction turbine.

9.4 Explain the working of a De Laval steam turbine with a neat sketch.

9.5 Define (a) Diagram power, (b) Diagram efficiency, and (c) Stage efficiency.

9.6 Why are steam turbines compounded?

9.7 Explain the velocity compounding in steam turbines with a neat sketch.

9.8 Explain the pressure compounding in steam turbines with a neat sketch.

9.9 Distinguish between pressure compounding and velocity compounding of a steam turbine.

9.10 What is pressure-velocity compounding of a steam turbine? What are its advantages?

9.11 Why velocity compounded impulse stages are generally placed at the high pressure inlet side of a steam turbine?

9.12 Draw the velocity triangles for an impulse turbine and derive the expression for (a) Tangential thrust, (b) Axial thrust, (c) Diagram power, and (d) Diagram efficiency.

9.13 What is the significance of (a) Whirl velocity component, and (b) Axial or flow velocity component in a steam turbine?

9.14 For a steam turbine prove that, $\eta_{\text{overall}} = \eta_n \, \eta_{\text{stage}} \, \eta_m$.

9.15 What is the optimum velocity ratio for an impulse turbine? Find the corresponding diagram power and diagram efficiency.

9.16 Explain how the blade efficiency varies with the velocity ratio in an impulse turbine.

9.17 How the number of stages in a turbine is decided?

9.18 Draw the velocity triangles for a two row Curtis stage and derive expressions for diagram power and diagram efficiency.

9.19 What is the optimum velocity ratio for a 2-row Curtis turbine? Find the corresponding diagram power and diagram efficiency.

9.20 Explain how the blade efficiency varies with the velocity ratio for a two stage velocity compounded impulse turbine.

9.21 Prove that three-fourth of the total work is done by the steam jets on the first row of moving blades and one-fourth of the total work is done on the second row of moving blades in a 2-row Curtis stage.

9.22 Prove that the work done in the first, second and third row in a 3-moving blade row Curtis turbine is in the ratio of 5:3:1.

9.23 Why the effectiveness of a Curtis turbine decreases as the number of rows of moving blades increases?

9.24 Why a 2-row Curtis turbine is most often used as the first stage in large steam turbines?

9.25 Explain the working of a single stage reaction turbine with a neat sketch. Why pure reaction turbine is not possible practically?

9.26 Why the reaction turbine blades are unsymmetrical?

9.27 Define degree of reaction. What is a 50% reaction turbine?

9.28 Draw the velocity triangles for a Parson's turbine and derive the expressions for diagram work and diagram efficiency.

9.29 Prove that for a Parson's turbine $\beta_1 = \alpha_2$, $\beta_2 = \alpha_1$, where α_1 is the nozzle angle, β_1, β_2 are the blade angles at the inlet and outlet of the turbine and α_2 is the angle of the absolute velocity at the outlet with the tangent.

9.30 Prove that the fixed blades and moving blades of a 50% reaction turbine are identical in shape.

9.31 Prove that for a 50% reaction turbine, $C_1 = C_{r2}$ and $C_{r2} = C_{r1}$, where C_1, C_2 are absolute velocities at the inlet and outlet, respectively, and C_{r1}, C_{r2} are relative velocities at the inlet and outlet, respectively.

9.32 What is the optimum velocity ratio for a 50% reaction turbine? Find the corresponding diagram power and diagram efficiency.

9.33 Prove for 50% reaction turbine, the specific work, w, is C_b^2, where C_b is the mean blade velocity.

9.34 Prove that $\eta_{50\%\text{ reaction}} > \eta_{\text{impulse}} > \eta_{\text{2-row Curtis}}$

9.35 Prove that $(\Delta h_s)_{50\%\text{ reaction stage}} : (\Delta h_s)_{\text{impulse stage}} : (\Delta h_s)_{\text{2-row Curtis stage}} = 1:2:8$.

9.36 What is meant by carryover efficiency? What is its significance?

9.37 Explain how the mass flow rate can be obtained from given blade height, mean diameter and steam condition at that stage?

9.38 Why twisted (or warped) blades are used in the later stages of the turbine?

9.39 How nozzle and blade heights are fixed in a steam turbine?

9.40 What is meant by *overlap*? Why overlap is provided at the root and tip of the blades?

9.41 What is meant by partial admission of steam?

9.42 How the blade dimensions of the last stage of a steam turbine are decided?

9.43 What do you understand by parallel exhausts in a steam turbine? Why are they needed?

9.44 What are the various losses in a steam turbine?

9.45 Explain (a) Disc friction loss, and (b) Windage loss.

9.46 Why there is some energy loss due to wetness of the steam?

9.47 What is leakage loss and how is it minimised?

9.48 What are the labyrinth seals? Where are they used?

9.49 What is reheat factor? Discuss its significance with the help of $T-s$ and $h-s$ diagram.

9.50 Prove that reheat factor of a steam turbine is greater than unity.

9.51 Prove that $\eta_{\text{stage}} = \eta_n \eta_D (1 - \eta_{df})$ where η_n is the nozzle efficiency, η_D is the diagram efficiency and η_{df} is the coefficient that accounts for the disc friction, windage and other losses.

9.52 Write a short note on the performance characteristics of steam turbine.

9.53 Prove that internal efficiency of a multistage turbine is always greater than stage efficiency.

PROBLEMS

9.1 An impulse turbine stage has equiangular blades in which the friction effects may be neglected. The flow rate of steam is 0.75 kg/s and mean blade velocity is 200 m/s. If the turbine operates close to maximum blading efficiency, find (a) the discharge angle at which steam leaves the blades, and (b) diagram power. [*Ans:* $\alpha_2 = 90°$, $P_D = 60$ kW]

9.2 The mean diameter of rotor of a single stage impulse steam turbine is 1 m while running at 3000 rpm. Steam exits the nozzle with a velocity of 300 m/s at an angle of 20°. The axial thrust on blades is 98 N. Calculate the power generated. Equiangular blades are used and 19% of kinetic energy is lost in friction during flow of steam across the blades. [*Ans:* $P_D = 1.136$ kW]

9.3 The nozzle angle is 20° with the plane of blades rotation in an impulse steam turbine and steam leaves the nozzle at 375 m/s. The mass flow rate of steam is 10 kg/s and blade velocity coefficient is 0.85. The mean peripheral velocity of the blades is 165 m/s. If there is no axial thrust in the stage, calculate (a) inlet and outlet angles for blades, and (b) the power developed.

[*Ans:* (a) $\beta_1 = 34.4°$, $\beta_2 = 41.6°$, (b) $P_D = 532$ kW]

9.4 The velocity of steam at the exit of nozzle is 900 m/s in a symmetrical blading impulse turbine. The nozzles are set at angle of 20° with the tangent. The blade velocity is 300 m/s and blade friction coefficient is 0.7. If the steam flow rate is 1 kg/s, calculate (a) blade angle at the inlet, (b) whirling thrust on the wheel, (c) axial thrust, (d) diagram power, and (e) blading efficiency.

[*Ans:* (a) $\beta_1 = 29.24°$, (b) $F_t = 927.7$ N, (c) $F_a = 92.3$ N, (d) $P_D = 278.3$ kW, (e) $\eta_D = 68.7\%$]

9.5 The mean diameter of the blade ring of an impulse turbine running at 3000 rpm is 0.8 m. The blade angles both at the inlet and outlet are 30°. Mass flow rate of the steam is 1 kg/s and blade friction coefficient is 0.85. If the discharge is axial at outlet, calculate (a) nozzle angle, (b) absolute velocity of steam at the outlet of nozzle, (c) specific enthalpy drop in the stage, (d) driving force on the wheel, (e) axial thrust, (f) diagram power, and (g) diagram efficiency.

[*Ans:* (a) $\alpha_1 = 17.33°$, (b) $C_1 = 72.56$ m/s, (c) $\Delta h = 41.06$ kJ/kg,
(d) $F_t = 273.5$ N, (e) $F_a = 12.8$ N, (f) $P_D = 34.37$ kW, (g) $\eta_D = 83.7\%$]

9.6 The steam leaves the nozzle at 400 m/s in a single row impulse turbine which has blade speed of 175 m/s. Turbine develops 180 kW power. Steam flow rate is 163.2 kg/min and steam leaves the turbine axially. The blade velocity coefficient is 0.9. Find (a) nozzle angle, (b) inlet and outlet blade angles, (c) axial thrust, (d) energy loss at the exit, (e) energy loss in blades, and (f) diagram efficiency.

[*Ans:* (a) $\alpha_1 = 19°$, (b) $\beta_1 = 32.6°$, $\beta_2 = 36.3°$, (c) $F_a = 4.35$ N, (d) $(KE_l)_{exit} = 8.27$ kJ/kg, (e) $(KE_l)_b = 5.53$ kJ/kg, (f) $\eta_D = 41.36\%$]

9.7 Steam at the rate of 10 kg/s enters in the rotor of mean diameter of 1.05 m running at 3000 rpm in an impulse stage. Nozzles are set at an angle of 18° and the ratio of velocity of blades to velocity of steam is 0.42. The ratio of relative velocity at the outlet to that at the inlet is 0.84. The blade angle at the outlet is 3° lesser than that at the inlet. Determine (a) axial thrust on the blades, (b) whirling thrust on blades, (c) resultant thrust on blades, (d) power output, and (e) Diagram efficiency.

[*Ans:* (a) $F_a = 250$ N, (b) $F_t = 3900$ N, (c) $F = 3908$ N, (d) $P_D = 641.5$ kW (e) $\eta_D = 70\%$]

9.8 The velocity of steam is 900 m/s at the exit of nozzles which are set at an angle of 20° in a single stage impulse turbine. The mean diameter of the blade ring is 0.5 m and speed of rotation is 10000 rpm. The loss of energy in the equiangular blades is 15%. The steam flow rate is 750 kg/hr. Determine (a) blade inlet angle for shockless entry, (b) diagram power, (c) diagram efficiency, (d) axial thrust, and (e) power lost in the blades due to friction.

[*Ans:* (a) $\beta_1 = \beta_2 = 28°$, (b) $P_D = 56.8$ kW, (c) $\eta_D = 70\%$, (d) $F_a =$ (e) $P_{lb} = 11.98$ kW]

9.9 The mean diameter of rotor of an impulse turbine is 250 mm and the blades are equiangular. The velocity of steam at the inlet of turbine is 930 m/s at an angle of 20° with the tangent. The power generated is 10 kW and the blade friction coefficient is 0.85. Determine (a) best angles of the blades, (b) turbine speed, (c) steam flow rate, and (d) the blade efficiency.

[*Ans:* (a) $\beta_1 = \beta_2 = 36.1$, (b) $N = 33300$ rpm, (c) $\dot{m} = 102.1$ kg/hr, (d) $\eta_D = 81.6\%$]

9.10 Dry saturated steam at 8 bar enters a single stage simple impulse turbine and leaves at 0.2 bar. The nozzles of 90% efficiency are set at an angle of 15°. The internal and mechanical efficiencies of the turbine are 75% and 92%, respectively. If the efficiency of generator is 90%, determine (a) mean blade speed for maximum diagram efficiency, and (b) mass flow rate of steam for generating 200 kW power.

[*Ans:* (a) $C_b = 490$ m/s, (b) $\dot{m} = 0.564$ kg/s]

9.11 The nozzle angle is 20° and blade friction coefficient is 0.83 in a stage of an impulse turbine with single row of equiangular moving blades. Determine (a) maximum blade efficiency and corresponding velocity ratio.

If the blade efficiency is 90% to that of maximum, find (b) the values of the velocity ratios, and (c) blade angles.

[*Ans:* (a) $(\eta_b)_{max} = 80.7\%$, $\rho_{opt} = 0.47$, (b) $\rho = 0.62$, $\rho = 0.32$, (c) $\beta_1 = 36°$, $\beta_2 = 48.6°$]

9.12 The isentropic specific enthalpy drop in a stage of velocity compounded steam turbine with two rows of moving blades is 320 kJ/kg. The nozzle angle is 16°, blade velocity is 150 m/s and velocity ratio is 0.95. The blade friction factor for all rows of the blades is 0.9. The mass flow rate of steam is 20 kg/s and all the blades are symmetrical. Determine (a) blade angles, (b) power output, (c) stage efficiency, and (b) kinetic energy of the steam leaving the stage.

[*Ans:* (a) $\beta_1 = 19.8°$, $\beta_3 = 26.8°$, $\alpha_2 = 42.4°$, (b) $P_D = 4264$ kW, (c) $\eta_s = 66.7\%$, (d) $(P_l)_{exit} = 234$ kW]

9.13 The nozzles are inclined at an angle of 20° to the plane of rotation in an impulse turbine. The blade angles at the inlet and outlet are same. The mean diameter of the blades is 500 mm and blade friction coefficient is 0.8. The velocity of steam at the inlet of turbine is 750 m/s and power developed is 20 kW. Determine (a) optimum blade angles, (b) mass flow rate of steam, and (c) diagram efficiency.

[*Ans:* (a) $\beta_1 = \beta_2 = 36.042°$, (b) $\dot{m} = 323$ kg/hr, (c) $\eta_D = 79.5\%$]

9.14 The nozzles of a single stage impulse turbine delivers working fluid at an angle of 25° with the plane of rotation of the blades. The fluid leaves the blades at a trailing angle of 60° to the plane of rotation with an absolute velocity of 290 m/s. The blades are equiangular and there is 5% decrease in the axial velocity while passing over the blades.

(a) Determine (i) blade angles, (ii) specific blade work, (iii) blade velocity coefficient, and (iv) diagram efficiency.

(b) If the fluid is steam and state of the steam at the nozzle inlet is 1.5 bar, 150°C determine the pressure at the moving blades and express as a percentage of the isentropic enthalpy drop (i) the nozzle loss, (ii) loss in blade passage, (iii) disc friction loss, (iv) residual velocity loss, and (v) net work available.

The efficiency of the nozzle is 87% and disc friction loss is 3% of the work done on the blades. Show the losses for the stage on $h - s$ diagram.

9.15 A stage of an axial flow velocity compounded steam turbine has two rows of moving blades separated by a row of fixed blades. The blade angles at the entry and exit of both the moving blade rows are 30°. The blade speed of each moving blade row is 130 m/s and blade friction factor is 0.85 for both fixed and moving blades. The discharge of steam from the second row of moving blades is axial. Determine (a) absolute velocity of steam at the inlet, (b) specific work done, and (c) blade efficiency.

[*Ans:* (a) $C_1 = 650$ m/s, (b) $w = 149.5$ kJ/kg, (c) $\eta_D = 70.7\%$]

9.16 The first stage of a combination turbine is a two-row velocity compounded impulse turbine in which the steam flows at the rate of 2.6 kg/s. The nozzle angle is 16°, velocity at the exit of nozzle is 630 m/s and mean blade speed is 125 m/s. The relative velocity at the outlet is 84% of that at the inlet for all the blade rows. The blade angles are 18°, 22° and 36° at the outlet of first moving blades row, fixed guide blades row and second moving blades row, respectively. Determine (a) the change in the velocity of whirl, (b) whirling thrust on the blades, (c) axial thrust on the blades, (d) power developed, and (e) blading efficiency.

[*Ans:* (a) $\Delta C_w = 1127.5$ m/s, (b) $F_t = 2.93$ kN, (c) $F_a = 188.5$ N, (d) $P_D = 366.4$ kW, (e) $\eta_D = 71\%$]

9.17 The nozzles are set at an angle of 16° with the plane of rotation of the blades and the velocity at the exit of the nozzle is 800 m/s in a stage of a velocity compounded steam turbine. The blade angles are 20°, 25° and 30° at the exit of first moving blades row, fixed guide blades and second moving blades row, respectively. The mean speed of the blades is 150 m/s and steam flow rate is 5 kg/s. The blade friction coefficient for first moving blades row, fixed blades row and second moving blades row are 0.8, 0.85 and 0.85, respectively. Calculate (a) diagram efficiency, (b) energy carried by steam at the exit, (c) axial thrust on each blade row, and (d) power produced.

[*Ans:* (a) $\eta_D = 66.2\%$, (b) $(KE_l)_{exit} = 3.79$ kJ/kg, (c) $(F_a)_{\text{I moving}} = 20.9$ N,
$(F_a)_{fixed} = 20.9$ J/kg, $(F_a)_{\text{II moving}} = 27$ N, (d) $P_D = 1059$ kW]

9.18 The velocity of steam leaving the nozzle in a two moving blade rows velocity compounded impulse turbine is 500 m/s. Angle at exit of the second moving blade is 30° from the plane of rotation of blades. Blade velocity coefficient for each blade row is 0.9. The discharge from the stage is axial and there is no axial thrust on any of the moving blade row. The axial velocity in first moving blade row is twice that of the second moving blade row. Neglect the increase in volume of steam during expansion. Determine (a) blade and nozzle angles, (b) the blade speed, (c) specific work, (d) ratio of the height required for moving blade rows and fixed blade row with respect to the nozzle height.

[*Ans:* (a) $\beta_3 = 26.764°$ $\alpha_3 = 15.05°$, $\alpha_2 = 27.885°$, $\beta_2 = 19.944°$, $\beta_1 = 17.88°$, $\alpha_1 = 14.152°$, (b) C_b = 105.8583 m/s, (c) $w = 99.826$ kJ/kg, (d) $\left[\frac{h_{bm}}{h_n}\right]_{\text{I}} = 1, \left[\frac{h_{bf}}{h_n}\right] = 1, \left[\frac{h_{bm}}{h_n}\right]_{\text{II}} = 2$

9.19 Steam at 3 bar, 0.94 dry leaves the fixed blades of a reaction turbine stage with a velocity of 143 m/s. Ratio of axial flow velocity to blade velocity is 0.7 at the inlet and 0.75 at the outlet of the moving blades. Fixed and moving blade exit angles are same. The height of the blades is 18 mm and steam flow rate is 2.5 kg/s. If the mean blade velocity is 70 m/s, determine the degree of reaction.

[*Ans:* $R_d = 0.582$]

9.20 The mean diameter and height of the blades are 0.5 m and 30 mm, respectively in a particular stage of a 50% reaction turbine. The blade angles at the inlet and outlet are 60° and 160° respectively. The speed of turbine is 3000 rpm and the density of steam is 2.7 kg/m^3. Calculate (a) the mass flow rate of steam, (b) power developed, and (c) stage efficiency.

[*Ans:* (a) $\dot{m} = 4.61$ kg/s, (b) $P_D = 43.57$ kW, (c) $\eta_{\text{stage}} = 91.5\%$]

9.21 The absolute velocity of steam at the inlet to moving blades of a 50% reaction stage of a turbine is 240 m/s. The blade velocity is 210 m/s and nozzle angle is 20°. The steam flow rate is 1 kg/s. Determine (a) the height of the blades at the inlet and outlet, (b) specific enthalpy drop in the moving blades and in the stage, (c) diagram power for a mass flow rate of 1 kg/s, and (d) diagram efficiency.

9.22 Steam at 60 bar, 600°C is supplied to a 50% reaction turbine and leaves at 0.07 bar. The diagram power developed is 25 MW. The reheat factor is 1.04 and the stage efficiency is 80%. Calculate the mass flow rate of steam. [*Ans:* $\dot{m} = 21$ kg/s]

9.23 The mean blade speed is 220 m/s and velocity ratio of 0.7 in a reaction turbine. Find the inlet blade angle and specific work output if exit angle is 20°. Also, calculate the percentage increase in diagram efficiency if turbine is running at the optimum velocity ratio.

[*Ans:* (a) $\beta_1 = 55°$, $w = 81.5$ kJ/kg, % increase in $\eta_D = 93.8\%$]

9.24 A 14-stage 50% reaction turbine has same enthalpy drop and isentropic efficiency in each stage. The steam enters at 20 bar, 400°C and leaves at 0.2 bar as dry saturated steam. All the blades have an outlet blade angle of 22° and the average value of the blade velocity ratio is 0.82. The speed of the turbine is 2400 rpm and steam flow rate is 34000 kg/hr. If the reheat factor is 1.05, calculate (a) stage efficiency, (b) blading power, (c) mean diameter of the drum, (d) blade height of the last row of moving blades, and (e) pressure at inlet of last stage.

[*Ans:* (a) $\eta_s = 67.7\%$, (b) $P_D = 6025$ MW, (c) $D = 1.34$ m, (d) $h_{b14} = 0.175$ m, (e) $p_{14} = 0.32$ bar]

9.25 Dry saturated steam at 2.7 bar enters a stage of 50% reaction turbine with a velocity of 90 m/s. The mean height of the blades is 40 mm and the moving blade exit angle is 20°. The axial velocity of flow is 0.75 times the mean blade velocity. Mass flow rate of steam is 9000 kg/hr and the effect of blade tip thickness on the annular area can be neglected. Calculate (a) speed of the rotor, (b) diagram power, (c) diagram efficiency, and (d) specific enthalpy drop in the stage.

[*Ans:* (a) $N = 1824$ rpm, (b) $P_D = 13.14$ kW, (c) $\eta_D = 78.7\%$, (d) $\Delta h = 5.26$ kJ/kg]

9.26 Steam is supplied at 140 bar, 560°C and exits at 0.075 bar in a steam turbine. The maximum blade velocity is 320 m/s and nozzle efficiency in all the stages is 90%. Nozzle angle is 15° for impulse stages and 25° for reaction stages. All the stages operate close to the best efficiency point. Calculate the number of stages required for each of the following arrangements: (a) all simple stages, (b) all 50% reaction stages, (c) a 2-row Curtis stage followed by simple impulse stages, and (d) a 2 row Curtis stage followed by 50% reaction stages.

9.27 Steam at 20 bar, 350°C enters in a 5-stage steam turbine and leaves at 2 bar. The stage efficiency is 75% in all stages. Determine (a) reheat factor, and (b) inter-stage pressures, and (c) turbine internal efficiency. Assume that enthalpy drop is same in all stages.

9.28 Steam at 35 bar, 450°C enters in a 5-stage steam turbine and leaves at 0.07 bar, 0.89 dry. Determine (a) state of steam at the entry of each stage, (b) stage efficiency or efficiency ratio of each stage, (c) reheat factor, and (b) overall turbine efficiency. Assume that all stages develop equal work.

[*Ans:* (a) $p_2 = 14.5$ bar, $t_2 = 342$°C, $p_3 = 5.5$ bar, $t_3 = 235$°C, $p_4 = 1.7$ bar, $t_4 = 125$°C, (b) $\eta_{s1} = 77.88\%$, $\eta_{s2} = 83.58\%$, $\eta_{s3} = 85.67\%$, $\eta_{s4} = 85.67\%$, $\eta_{s5} = 87.49\%$, (c) $RF = 1.05$, (d) $\eta_o = 88.47\%$]

9.29 Steam at 23 bar, 345°C enters in a 4-stage pressure compounded impulse turbine and leaves at 0.07 bar. The internal efficiency of the turbine is 72%. Determine (a) inter-stage pressures, (b) stage efficiency or efficiency ratio for each stage, and (c) reheat factor. Assume that all stages develop equal work and condition line straight.

[*Ans:* (a) $p_2 = 7.25$ bar, $p_3 = 1.9$ bar, $p_4 = 0.39$ bar, (b) $\eta_{s1} = 61.68\%$, $\eta_{s2} = 68.14\%$, $\eta_{s3} = 70.41\%$, $\eta_{s4} = 72.84\%$, (c) $RF = 1.0574$]

9.30 State of steam at the inlet is 35 bar, 420°C in a 4-stage velocity compounded impulse turbine. The pressure at the exit is 0.07 bar. The average efficiency is 70% in each stage. Determine (a) the pressure and quality of steam at the entry of each stage, (b) turbine internal efficiency, and (c) reheat factor.

[*Ans:* (a) $p_2 = 11.3$ bar, $t_2 = 304$°C, $p_3 = 2.9$ bar, $t_3 = 189$°C, $p_4 = 0.55$ bar, $x_4 = 0.99$, (b) 74.6%, (c) $RF = 1.07$]

9.31 Steam at 30 bar, 350°C enters a three stage turbine and exits first, second and third stages at 7 bar, 1 bar and 0.1 bar respectively. The stage efficiency is 70% which is same in all the stages. Determine (a) final steam quality, (b) reheat factor, and (c) overall thermal efficiency.

[*Ans:* (a) $x_4 = 0.94$, (b) $RF = 1.05$, (c) $\eta_o = 24.5\%$]

9.32 Steam is supplied at 20 bar, 350°C to a 4-stage turbine and exits at 0.05 bar. The overall efficiency of the turbine is 80% and work from all the stages is equal. Assuming condition line to be straight, find (a) inter-stage pressure, (b) stage efficiency of all the stages, and (c) reheat factor.

[*Ans:* (a) $p_2 = 9$ bar, $p_3 = 2.4$ bar, $p_4 = 0.5$ bar, (b) $\eta_{s1} = 72.5\%$, $\eta_{s2} = 76\%$, $\eta_{s3} = 78.4\%$, $\eta_{s4} = 82.5\%$, (c) $RF = 1.04$]

MULTIPLE CHOICE QUESTIONS

1. Consider the following statements with reference to steam turbines.
   1. A single unit can be designed for output of 1000 MW or more
   2. Much higher speed may be possible compared with a reciprocating engine
   3. They are more compact as compared to a gas turbine power plant
   4. The maintenance and running costs may increase with years of service

   Which of these statements are correct?

   (a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 2 and 4 (d) 1, 3 and 4

2. The enthalpy drop of steam in impulse turbine occurs in
   (a) Nozzles (b) Rotor blades (c) Stator blades (d) Exhaust pipe
3. Expansion line *EFG* of a 2-stage steam turbine on $h - s$ plane is shown in Figure 9.55. The reheat factor for this turbine is

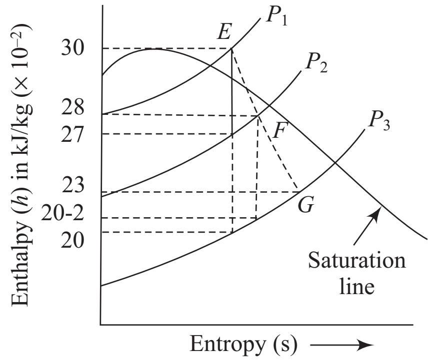
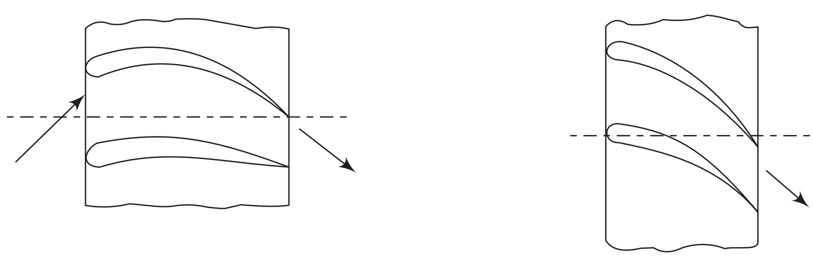
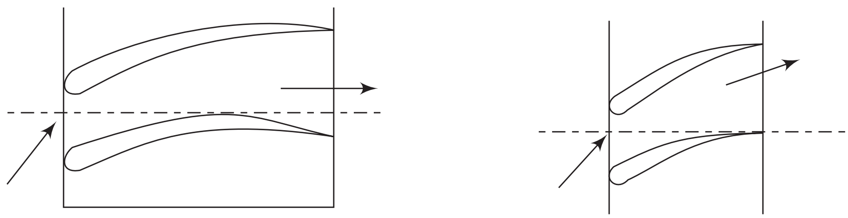


Figure 9.55 *Multiple Choice Question 3*

   (a) 1.08 (b) 0.7 (c) 0.648 (d) 1.43
4. Ratio of enthalpy drop in moving blades to the total enthalpy drop in the stage is known as
   (a) Reheat factor (b) Blade efficiency
   (c) Degree of reaction (d) Internal efficiency
5. What is the cause of reheat factor in a steam turbine?
   (a) Reheating (b) Superheating
   (c) Supersaturation (d) Blade friction
6. An impulse turbine is designed for free vortex flow. The tangential velocity of steam at the root radius of 250 mm is 430 m/s and the blade height is 100 mm, then the tangential velocity of the steam at the tip will be
   (a) 602 m/s (b) 504 m/s (c) 409 m/s (d) 307 m/s
7. If in a steam turbine stage, heat drop in moving blade ring is 40 kJ/kg and that in the fixed blade ring is 60 kJ/kg, then the degree of reaction is
   (a) 0.2 (b) 0.4 (c) 0.6 (d) 0.7
8. The blade passage for the nozzle blade row of the first stage of an impulse turbine as shown in Figure 9.56 is best represented as



(a) (b)



(c) (d)

Figure 9.56 *Multiple Choice Question 8*

9. Review the following statements related to an impulse turbine.
   1. Relative velocities at the entry and outlet of the rotor blades are the same
   2. Absolute velocities at the entrance and outlet of the rotor blades are the same
   3. Static pressure across the rotor blade passage is constant
   4. Total pressure across the rotor blade passage is constant

   Which of these statements are correct?
   (a) 1 and 4 (b) 2 and 3 (c) 1 and 3 (d) 2 and 4
10. The absolute and relative velocities of steam at the inlet of the rotor of a reaction turbine are 236 m/s and 132 m/s, respectively. Steam exits the rotor with a relative velocity of 232 m/s and absolute velocity of 126 m/s. The specific power output is
   (a) 38.1 kW (b) 40.1 kW (c) 43.8 kW (d) 47.4 kW
11. The power output of a reaction turbine stage is 280 kW for steam flow rate of 1 kg/s. If the nozzle efficiency is 0.92 and rotor efficiency is 0.90, the isentropic static enthalpy drop will be
   (a) 352 kW (b) 347 kW (c) 338 kW (d) 332 kW
12. If α is the nozzle angle, maximum blade efficiency of a Parson's turbine is given by

   (a) $\dfrac{\cos^2\alpha_1}{1+2\cos^2\alpha_1}$ (b) $\dfrac{\cos\alpha_1}{1+\cos^2\alpha_1}$ (c) $\dfrac{2\sin^2\alpha_1}{1+\sin^2\alpha_1}$ (d) $\dfrac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$
13. Consider the following characteristics:
   1. High steam and blade velocities
   2. Low steam and blade velocities
   3. Low speeds of rotation
   4. High carry-over loss

   A simple impulse turbine possesses which of characteristics mentioned above?
   (a) 1 and 2 (b) 2 and 3 (c) 1 and 4 (d) 3 and 4
14. The nozzle angle at inlet is 30° in a simple impulse turbine. For maximum diagram efficiency, the blade speed ratio is
   (a) 0.433 (b) 0.25 (c) 0.5 (d) 0.75
15. In a two row Curtis turbine stage having symmetrical blades
   (a) Work done by both rows of moving blades are same
   (b) Work done by the first row of moving blades is twice that of second row of moving blades
   (c) Work done by the first row of moving blades is thrice that of second row of moving blades
   (d) Work done by the first row of moving blades is four times that of second row of moving blades

16. In a reaction turbine, decreasing the inlet pressure results in
    (a) Increase of blade heights since the specific volume of steam reduces
    (b) Increase in blade heights since the specific volume of steam increases
    (c) Decrease in blade heights since specific volume of steam increases
    (d) Decrease in blade heights since specific volume of steam reduces
17. Match **List-I (Feature)** with **List-II (Turbine/Staging)** and select the correct answer using the codes given.

List-I	**List-II**
A. Single stage impulse turbine	1. Parsons turbine
B. Pressure compounding	2. De Laval turbine
C. Velocity compounding	3. Rateau staging
D. Reaction turbine	4. Curtis Staging

Codes

	A	**B**	**C**	**D**
(a)	4	1	2	3
(b)	2	3	4	1
(c)	4	3	2	1
(d)	2	1	4	3

18. Velocity diagram for a stage of impulse turbine is shown in Figure 9.57. What are the driving and axial thrusts, respectively, for steam flow rate of 1 kg/s?
    (a) 450 N, 8 N (b) 560 N, 8 N (c) 680 N, 4 N (d) 910 N, 4 N

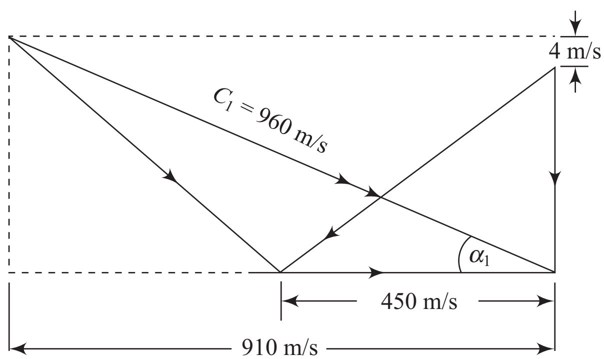


Figure 9.57 *Multiple Choice Question 18*

19. **Assertion (A):** Velocity compounding is used for high pressure end of a multistage turbine.
    Reason (R): Shock wave losses are reduced in high pressure end.
    (a) Both A and R are true and R is a correct explanation of A
    (b) Both A and R are true but R is not a correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true
20. Which one of the following is true?
    De Laval turbine is a
    (a) Simple reaction turbine (b) Simple impulse turbine
    (c) Velocity compounded impulse turbine (d) Pressure compounded impulse turbine

21. For a steam jet angle of 30° (from the tangential direction) at the entrance, what is the blade efficiency (approximate) of a Parson's turbine?
(a) 0.85 (b) 0.49 (c) 0.4 (d) 0.3

22. Despite the other factors, poorer part-load performance of De Laval turbine is due to the
(a) Formation of shock waves in the nozzle
(b) Formation of expansion waves at the nozzle
(c) Turbulent mixing at the nozzle exit
(d) Increases profile losses in the rotor

23. Match List-I (various velocities in the velocity diagram of a two-stage impulse turbine) with List-II (blade angles) and select the correct answer using the codes given.

List-I	List-II
A. Relative velocity of steam at the inlet tip of blade	1. Nozzle angle
B. Absolute velocity of steam at the inlet tip of blade	2. Moving blade leading edge angle
C. Relative velocity of steam at the outlet tip of blade	3. Moving blade trailing edge angle
D. Absolute velocity of steam at the outlet tip of blade	4. Fixed blade leading edge angle

Codes

	A	B	C	D
(a)	1	2	4	3
(b)	2	1	4	3
(c)	2	1	3	4
(d)	1	2	3	4

24. The variation of the absolute velocity of steam along with blade length as shown in the Figure 9.58 is for
(a) Curtis turbine (b) De Laval turbine (c) Radial turbine (d) Parson's turbine

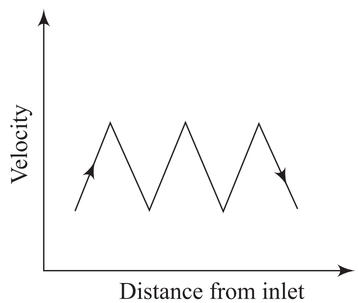


Figure 9.58 *Multiple Choice Question 24*

25. The correct ascending order of efficiency at the design points is
(a) Rateau, De Laval, Parson's, Curtis
(b) Curtis, De Laval, Rateau, Parson's
(c) De Laval, Curtis, Rateau, Parson's
(d) Parson's, Curtis, Rateau, De Laval

26. Which one of the following relations between fixed and moving blade angles given below is true for a Parson's turbine?
(a) $\alpha_1 = \alpha_2$ (b) $\alpha_1 = \beta_2$ (c) $\alpha_2 = \beta_2$ (d) $\beta_1 = \beta_2$

27. The following data pertains to an axial flow turbine stage:

Relative velocity of steam at entry of the rotor 79.0 m/s, Relative velocity of steam leaving the rotor 152 m/s. Degree of reaction is approximately
(a) 0.9 (b) 0.8 (c) 0.7 (d) 0.6

28. The flow in clearance between the blade tips and casing of a steam turbine is
    (a) Greater in the reaction turbine than in the impulse type
    (b) Greater in the impulse turbine than in the reaction turbine
    (c) Independent of the type of turbine
    (d) Independent of the size of the turbine
29. Rotational speeds of steam turbine can be reduced to practical values by which of the following method (s)?
    1. By using heavy flywheel
    2. By using a quick response governor
    3. By compounding
    4. By reducing fuel feed to the furnace

    (a) 3 alone (b) 1, 2, 3 and 4 (c) 1, 2 and 4 (d) 2 and 3
30. **Assertion (A):** For power plant applications, a large number of stages in reaction turbines are common in practice.

    Reason (R): A pressure drop occurs in the moving blades in a reaction turbine unlike that in impulse turbine where pressure does not change across the moving blade.
    (a) Both A and R are individually true and R is the correct explanation of A
    (b) Both A and R are individually true and R is not the correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true
31. The net result of pressure-velocity compounding of steam turbine is
    (a) Lesser number of stages (b) Large turbine for a given pressure drop
    (c) Shorter turbine for a given pressure drop (d) Lower friction loss
32. Given C_b = Blade speed, C_1 = Absolute velocity of steam at the inlet of the blade, α_1 = Nozzle angle. The efficiency of an impulse turbine is maximum when
    (a) $C_b = 0.5C_1 \cos \alpha_1$ (b) $C_b = C_1 \cos \alpha_1$ (c) $C_b = 0.5C_1^2 \cos \alpha_1$ (d) $C_b = C_1^2 \cos \alpha_1$
33. An impulse turbine generates 50 kW power if the mean blade speed is 400 m/s. The rate of change of momentum tangential to the rotor is
    (a) 200 N (b) 175 N (c) 150 N (d) 125 N
34. At a certain section of a reaction turbine, the blade diameter is 1.8 m. The flow velocity of steam is 49 m/s and the volume flow rate of steam is 5.4 m^3/s. The blade height at this section will be approximately
    (a) 40 mm (b) 20 mm (c) 10 mm (d) 5 mm
35. Examine the following statements.

    If steam is reheated during the expansion through the turbine stages
    1. Blades erosion will reduce
    2. The overall pressure ratio will increase
    3. Specific enthalpy drop will increase

    Which of these statements are correct?
    (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

Direction: Each of the questions 36 to 42 consists of two statements, one marked as '**Assertion (A)**' and the other as **'Reason (R)'**. You have to examine these two statements carefully and choose the answers to these statements using the codes given.

Codes:
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true and R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

36. **Assertion (A):** The work done in Parson's reaction turbine is double of the work done during the expansion in the moving blades.

    Reason (R): The steam expands in the moving as well as in the fixed blades in a reaction turbine and in the Parson's turbine; the fixed and moving blades are similar.

37. **Assertion (A):** Reaction turbines are not built on pure reaction principle.

    Reason (R): Pure reaction is difficult to realize in practice.

38. **Assertion (A):** Work output per stage of an impulse turbine is double that of a 50% reaction stage at the same speed.

    Reason (R): Maximum speed ratio is limited for any class of turbine.

39. **Assertion (A):** Parson turbine has a degree of reaction equal to 50%.

    Reason (R): It is a reaction turbine with symmetrical fixed and moving blades.

40. **Assertion (A):** Modern turbines have velocity compounding at the initial stages and pressure compounding in following stages.

    Reason (R): Excessive tip leakage occurs in the high pressure region of reaction blading.

41. **Assertion (A):** Reaction blading is commonly used in intermediate and low pressure parts of steam turbines.

    Reason (R): Reaction blading gives higher efficiency than impulse blading.

42. **Assertion (A):** In conventional impulse steam turbine designs, only two rows of moving blades are used in a Curtis stage.

    Reason (R): As the number of rows of moving blades in a Curtis stage increases, the effectiveness of the later rows decreases.

43. If D is the diameter of the turbine rotor and C_b is the peripheral blade velocity, then the disc friction loss is proportional to

    (a) $(DC_b)^3$ (b) $D^2C_b^3$ (c) $D^3C_b^2$ (d) DC_b^4

44. The steam turbines are compounded to

    (a) Increase efficiency (b) Decrease turbine speed
    (c) Increase blade speed ratio (d) Decrease axial thrust

45. Match **List-I** (different turbine stages) with **List-II** (turbines) and choose the correct answer using the codes given.

List-I	List-II
A. 50% reaction stage	1. Rateau
B. Two-stage velocity compounded turbine	2. Parson
C. Single stage impulse	3. Curtis
D. Two-stage pressure compounded	4. De-Laval
	5. Hero

Codes

	A	B	C	D
(a)	5	1	2	3
(b)	5	3	2	1
(c)	2	3	4	1
(d)	3	1	4	2

46. Partial admission turbine refers to the situation where the
   (a) Steam is admitted partially into the blades through nozzles
   (b) Nozzles occupy the complete circumference leading into the blade annulus
   (c) Nozzles do not occupy the complete circumference leading into the blade annulus
   (d) Steam is admitted partially into the blades directly
47. Consider the following statements regarding a 100% reaction turbine.
   1. Change in absolute velocity of the steam across the moving blades is zero.
   2. Change in absolute velocity of the steam across the moving blades is negative.
   3. Enthalpy drop in the fixed blades is zero.

   Which of these statements is/are correct?
   (a) 1 only (b) 2 only (c) 2 and 3 (d) 1 and 3
48. Which one of the following pairs is not correctly matched?
   (a) Internal efficiency: Product of stage efficiency and reheat factor of steam turbine
   (b) Stage efficiency: Ratio of adiabatic enthalpy drop to the isentropic enthalpy drop of a turbine
   (c) Dryness fraction: Decreases due to reheating of steam within a stage
   (d) Steam condensation: Enhance blade erosion during expansion through the turbine
49. Velocity triangle for a reaction turbine stage is shown in Figure 9.59, in which $AB = C_1$ = Absolute velocity at the rotor blade inlet; $CB = C_{r1}$ = Relative velocity at the rotor blade inlet; $CE = C_{r2}$ = Relative velocity at the rotor blade exit and $CD = CB$. The ratio of reaction force to impulse force is

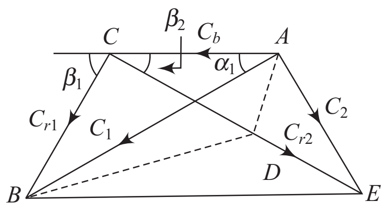


Figure 9.59 *Multiple Choice Question 49*

   (a) *CE*/*CB* (b) *CD*/*CE* (c) *DE*/*BD* (d) *AE*/*AB*
50. Which one of the following statements is correct?
   (a) Reheat factor is zero if efficiency of the turbine is close to unity
   (b) Lower the efficiency, higher will be the reheat factor
   (c) Reheat factor is independent of steam conditions at the turbine inlet
   (d) Availability of reheat is higher at the low pressure end
51. Figure 9.60 shows the variation of certain parameter in case of simple impulse turbine. The curve *A* – *B* – *C* represents the variation of
   (a) Pressure in nozzle and blades (b) Velocity in nozzles and blades
   (c) Temperature in nozzle and blades (d) Enthalpy in nozzle and blades

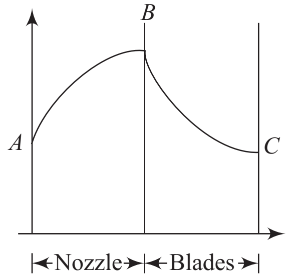


Figure 9.60 *Multiple Choice Question 51*

52. Steam admits a De Laval turbine with a velocity of 30 m/s and exits with a velocity of 10 m/s. The specific work done is
 (a) 400 N.m (b) 600 N.m (c) 800 N.m (d) 1200 N.m
53. In a 50% reaction stage, absolute velocity angle at entrance is 45°, mean peripheral speed is 75 m/s and the absolute velocity at the exit is axial. The stage specific work is
 (a) 2500 m^2/s^2 (b) 3270 m^2/s^2 (c) 4375 m^2/s^2 (d) 5625 m^2/s^2
54. Match **List-I** (blades) with **List-II** (features) and select the correct answer using the codes given.

List-I	**List-II**
A. Ceramic blades	1. High creep strength
B. Steam turbine blades	2. Forged and machined
C. Alloy steel blades	3. Precision cast
D. Compressor blades	4. Thick at mid chord
	5. Thin trailing edge

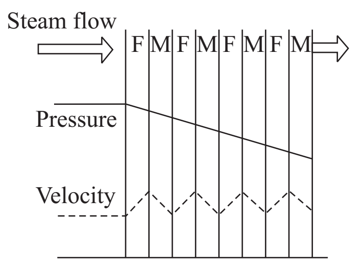
Codes

	A	**B**	**C**	**D**
(a)	2	1	5	4
(b)	3	4	5	1
(c)	2	4	3	5
(d)	3	2	1	5

55. Which one of the following is the characteristic of pressure-compounding (Rateau staging)?
 (a) Low efficiency at low speeds of rotation
 (b) High efficiency with low steam velocities
 (c) High efficiency with high steam velocities
 (d) Low efficiency with high speeds of rotation
56. In Parson's reaction turbines, the velocity triangles at the entry and exit are
 (a) Asymmetrical (b) Isosceles (c) Right-angled (d) Congruent
57. What is the value of the reheat factor in multistage turbines?
 (a) 1.03–1.04 (b) 1.10–1.20 (c) 0.9–1.00 (d) 1.20–1.25
58. A 4-row velocity compounded steam turbine develops total 6400 kW power. What is the power developed by the last row?
 (a) 200 kW (b) 400 kW (c) 800 kW (d) 1600 kW
59. What is the ratio of the isentropic work to Euler's work known as?
 (a) Pressure coefficient (b) Slip factor
 (c) Work factor (d) Degree of reaction

60. Review the following statements regarding an impulse turbine.
    1. Blade passages are of constant cross-section
    2. Partial admission of steam is permissible
    3. Axial thrust is only due to the change in flow velocity of steam at the inlet and outlet of moving blades

    Which of these statements are correct?
    (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3
61. Blade erosion in steam turbine takes place due to
    (a) High temperature steam (b) Droplets in steam
    (c) High rotational speed (d) High flow rate
62. For maximum blade efficiency (utilization factor), the work done (J/kg) in a single stage 50% reaction turbine is
    (a) $2C_b^2$ (b) $1/C_b^2$ (c) C_b^3 (d) C_b^2
63. The pressure and velocity diagram as shown in Figure 9.61 for a steam turbine refers to which of the following?



(Where : M-moving blade, F-fixed blade)

Figure 9.61 *Multiple Choice Question 64*

    (a) Impulse turbine-Velocity compounded
    (b) Impulse turbine-Pressure compounded
    (c) Impulse turbine-Pressure and velocity compounded
    (d) Reaction turbine stages
64. Examine the following statements regarding velocity compounded impulse turbine having two rows of moving blades.
    1. The work done in each row of moving blades is equal
    2. The efficiency of a velocity compounded stage is less than that of a pressure compounded stage
    3. The velocity compounded stage is often used in the first stage of a multistage impulse turbine

    Which of these statements are correct?
    (a) 1 and 3 (b) 2 only (c) 2 and 3 (d) 1, 2 and 3
65. **Assertion (A):** Single stage impulse steam turbines are not used in practice.

    Reason (R): Single stage impulse turbines have very low rpm.
    (a) Both A and R are true and R is a correct explanation of A
    (b) Both A and R are true but R is not a correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true

66. **Assertion (A):** In the steam turbines, supersaturated flow means that vapour does not condense immediately as it crosses the dry saturated line.

    Reason (R): The mass flow with super-saturation flow is greater than the mass flow with isentropic flow.
    (a) Both A and R are true and R is a correct explanation of A
    (b) Both A and R are true but R is not a correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true

67. **Assertion (A):** The pressure compounded impulse steam turbine is the most efficient type of impulse turbine.

    Reason (R): Ratio of blade velocity to steam velocity remains constant.
    (a) Both A and R are true and R is a correct explanation of A
    (b) Both A and R are true but R is not a correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true

68. To improve the quality of steam at the turbine exit, which of the following will be used?
    1. Reheat cycle
    2. Increase the maximum pressure when maximum and minimum temperatures are fixed
    3. Use superheated steam, instead of saturated steam when the maximum and minimum pressures are fixed

    (a) 1, 2 and 3 (b) 1 and 2 (c) 1 and 3 (d) 2 and 3

69. **Assertion (A):** Impulse staging is commonly employed in high pressure side and reaction staging in intermediate low pressure side of the steam turbine.

    Reason (R): The tip leakage across the moving blades is less in impulse staging as the pressure drop is small and there can be large pressure drop across the fixed blades and nozzles.
    (a) Both A and R are true and R is a correct explanation of A
    (b) Both A and R are true but R is not a correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true

70. In which of the following steam turbines, steam is taken from various points along the turbine, solely for the feed-water heating?
    (a) Extraction turbine (b) Bleeder turbine
    (c) Regenerative turbine (d) Reheat turbine

71. In a half-degree reaction Parson's turbine, operating at the design conditions, the enthalpy drop of steam in one stage of the turbine occurs
    (a) Entirely in the fixed blades
    (b) Entirely in the moving blades
    (c) Half in the fixed blades and half in the moving blades
    (d) None of the above

72. Examine the statements given below regarding steam turbines:
    1. The maximum blade efficiency of a single stage impulse turbine will be $\cos^2\alpha_1$, where α_1 is the nozzle angle.
    2. For a reaction steam turbine with identical stator and rotor blades, the blade velocity for maximum blade efficiency is equal to the inlet steam velocity.
    3. Velocity compounded impulse steam turbine gives less speed and less efficiency.

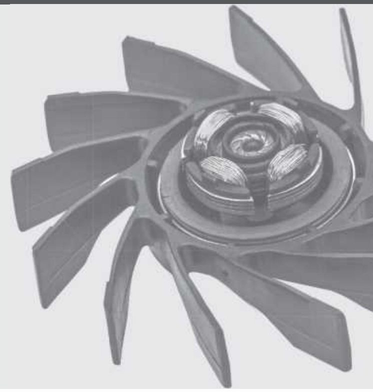
Which of these statements are correct?
(a) 1 and 2 (b) 1 and 3 (c) 2 and 3 (d) 1, 2 and 3

73. The optimum ratios of blade speed to tangential component of jet speed for the De Laval and Parson's turbine are
(a) 1 for both
(b) 1/2 for De Laval turbine and 1 for Parson's turbine
(c) 1 for De Laval turbine and 1/2 for Parson's turbine
(d) 1/2 for both

ANSWER KEY

1. (c)	2. (a)	3. (a)	4. (c)	5. (d)	6. (d)	7. (b)	8. (a)	9. (c)	10. (a)
11. (c)	12. (d)	13. (c)	14. (a)	15. (c)	16. (b)	17. (b)	18. (d)	19. (a)	20. (b)
21. (a)	22. (d)	23. (c)	24. (d)	25. (b)	26. (b)	27. (d)	28. (a)	29. (a)	30. (b)
31. (b)	32. (a)	33. (d)	34. (b)	35. (d)	36. (a)	37. (b)	38. (b)	39. (a)	40. (b)
41. (b)	42. (a)	43. (c)	44. (b)	45. (c)	46. (a)	47. (d)	48. (c)	49. (c)	50. (b)
51. (b)	52. (a)	53. (d)	54. (c)	55. (c)	56. (d)	57. (a)	58. (b)	59. (c)	60. (c)
61. (b)	62. (d)	63. (d)	64. (c)	65. (c)	66. (b)	67. (a)	68. (c)	69. (a)	70. (c)
71. (c)	72. (b)	73. (b)							

10 Fluid Systems



Learning Objectives

After reading this chapter, you will be able to:

LO1 Know the significance, classifications and applications of fluid systems

LO2 Explain the role of turbomachinery in fluid systems

LO3 Outline the differences between positive displacement machines and turbomachinery and comparative study between them on some common basis

LO4 Explain the basic principle, working, performance parameters of hydrostatic fluid systems

LO5 Describe the basic principle of hydrodynamic transmission systems

LO6 Discuss the working, performance parameters and characteristics of fluid coupling and torque converter

LO7 Summarize the need and various methods of governing of hydraulic and steam turbines

10.1 Introduction

Power transmission takes place mainly in three ways, viz. mechanical, electrical and fluid. When transmission takes place through shafts, gears, chains, belts or pulleys, it is mechanical power transmission. Cables, transformers etc. are used in electrical power transmission systems. Fluids i.e. liquids or gases restricted in a specific space are used for transmission of power. This system of power transfer using incompressible or compressible fluids is called a *fluid power transmission system*. A fluid system is a network of connected components in which either force or power are transmitted using the fluid. Fluid systems store fluid energy and then transmit when required or increase the energy of the fluid mostly the pressure energy several times and then transmit as and when required. A prime mover is required in a fluid system to run a pump which increases the pressure of the fluid. The pressurized fluid then passes through pipes and hoses to operate an actuator to perform the assigned task. A fluid system by virtue of the kind of fluid used is either pneumatic (air/gas) or hydraulic (liquid). Extensive applications of fluid systems are found in many industries, military operations, health and recreation to name a few. For example, pneumatic drill used by dentists and the spring which holds the seat in the revolving chair is operated pneumatically. Hydraulic systems are used in agricultural farms to operate equipment either for lifting or lowering down or for better control and torque adjustments such as in harvesters and tractors. In the construction industry, backhoes, cranes and vibrators make use of hydraulic systems. In manufacturing industries such as in fabrication of automobile body panels hydraulic and pneumatic systems are used to make the equipment operate as needed. Defense sector uses fluid systems to assist to move personnel, supplies, and equipment to support their operations. Several other applications range from air braking system, hydraulic braking system, and power-steering of land vehicles to the precise positioning of rocket launchers for air defense. Open as well as underground mining operations involve use of fluid systems. Equipments like large shovels and other material handling systems are examples.

In the recreation sector, roller coasters get their launch at the beginning with the help of hydraulic and pneumatic systems that pumps the liquid into rows of accumulators for storing energy as compressed gas. At the starting, the energy is suddenly released into a hydraulic motor. The output shaft of hydraulic motor drives a drum of cable which rapidly brings the roller coaster from rest to very high velocities.

10.2 Advantages and Disadvantages of Fluid Systems

Fluid power systems have several advantages and disadvantages when compared with mechanical and electrical power transfer systems. Following are the advantages and disadvantages of fluid systems relative to the other modes of power transmission.

10.2.1 Advantages

1. Ease in the force and torque multiplication and control.
2. Variable speed control for both linear and rotary motions to a great extent.
3. In case of system overloading, the actuator stops working without damaging the other elements.
4. Precision control of the speed of machine or its parts.
5. Reverse action and instant stopping of the linear and rotary actuators minimizing shocks to the entire system.
5. Suitable for a wide range of machine sizes and designs.
6. Easily adapt to other control systems viz. mechanical, electrical and electronic.
7. Readily provide lubrication of the components.

10.2.2 Disadvantages

1. High safety requirement as compressed fluids are used.
2. High noise level of pneumatic systems if air is exhausted directly to atmosphere.
3. Higher component wear due to susceptibility to polluted environments without effective filtration.
4. Fluid leakage and spills may create a safety hazard in the working environment, for example, may result in fire in hydraulic systems that use combustible oil.
5. Strict environmental norms to be followed for handling and disposal of oils.
6. Cost of compressing and conditioning air in pneumatic systems is very high.
7. The control of actuator speed in pneumatic systems may have less accuracy due to compressibility of air.

10.3 Basic Components of a Fluid System

10.3.1 Pneumatic System

Figure 10.1 shows a schematic diagram of a pneumatic system with main components.

Brief description of various components of a pneumatic system is as given below-

- The pneumatic actuator transforms the fluid energy into mechanical energy to perform useful task.
- A compressor to increase the pressure of the air sucked from the atmosphere.
- A reservoir to store compressed air known as storage reservoir.
- Valves to regulate the direction, quantity and pressure of compressed air.
- Electric motor to run the compressor.

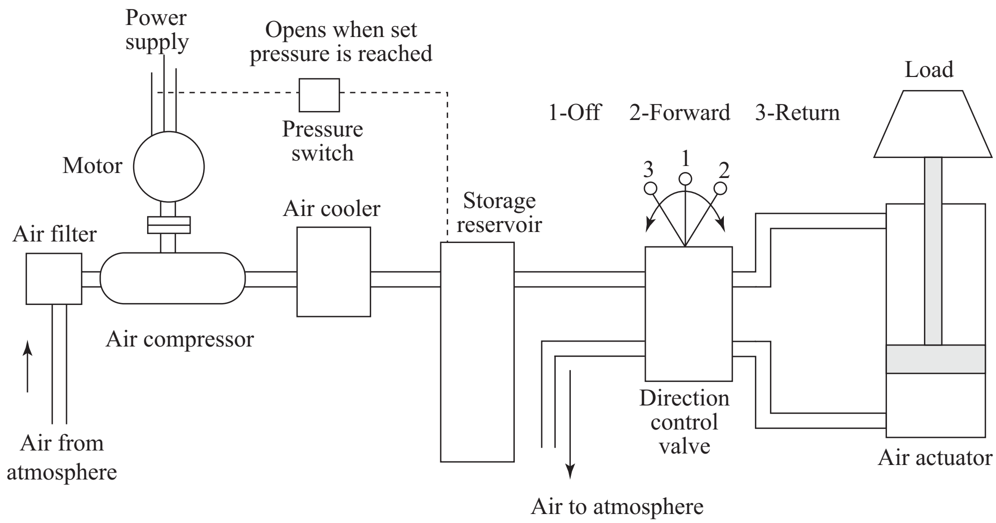


Figure 10.1 *Basic Components of a Pneumatic System*

The compressor sucks the air from atmosphere through an air filter and compresses it to the required pressure. An air cooler is used for cooling the compressed air with some prior treatment to remove moisture. The treated compressed air is then stored in the storage tank to maintain the pressure. A pressure switch is provided in the storage reservoir for starting and stopping the electric motor if the pressure drops up to a certain value.

10.3.2 Hydraulic System

Hydraulic systems are networks using pressurized liquid as a fluid for transmitting force or power from a generating source to the point of application to perform assigned task. Figure 10.2 shows the schematic network of a hydraulic system with the basic components.

- Actuator is the device to convert fluid energy into mechanical work to accomplish the useful task. The actuator may be either linear (e.g. hydraulic cylinder) or rotary (e.g. hydraulic motor) to obtain linear or rotary motion respectively.
- A hydraulic pump for the circulation of the oil from the oil tank to rest of the network by conversion of mechanical energy into pressure energy.
- Valves to regulate the direction, quantity and pressure of oil flowing in the network.

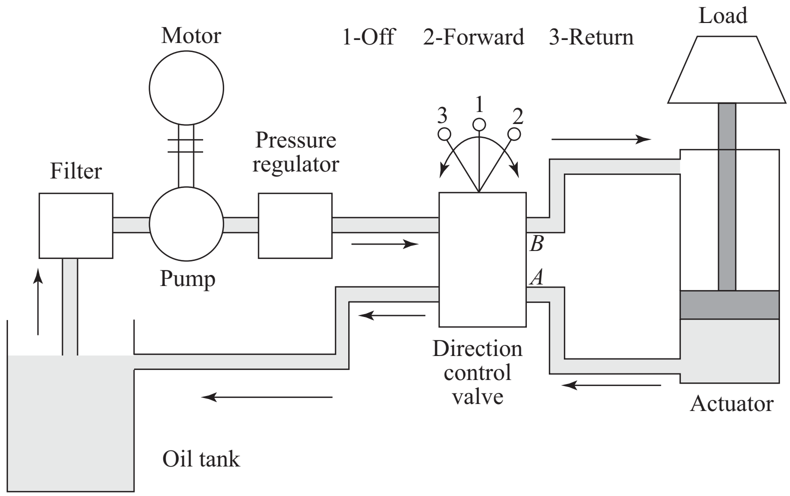


Figure 10.2 *Basic Components of a Hydraulic System*

- External power supply (motor) to run the pump.
- A tank or reservoir to store the liquid usually oil.
- Piping system for transportation of oil from one place to another.
- Filters to remove dirt, dust and other impurities to keep the clean and efficient system as well as to avoid damage to the actuator and valves.
- Pressure regulator to maintain the designed pressure in the system

The pump sucks the oil from the oil tank through a filter when the electric motor is switched on. The pressurized oil through the outlet of the pump passes through the regulating valve and the load is lifted as oil does work on the actuator. Oil from the other side of the actuator returns back to the tank through return piping line. Reciprocating motion of the piston in the cylinder is regulated by a three position control valve as given below.

- When the valve is positioned to right i.e. 2, the pressurized oil passes through the pipe line connected to port *A* lifting the load.
- When the position of the valve is changed to left i.e. 3, the oil passes through the piping connected to port *B* lowering the load.
- When the valve is at center position i. e. 1, it locks the fluid into the cylinder, thereby holding it in its position and all the oil from the delivery side of the pump returns back to the tank through pressure relief valve.

10.4 Comparison of Hydraulic and Pneumatic Systems

Hydraulic and pneumatic systems are similar on the basis of energy transfer by means of fluid pressure and flow; however, they differ in how and where they are applied. These differences are briefly discussed as under:

10.4.1 Characteristics of the Fluid

Compressibility of the fluid is the characteristic attribute that is responsible for the difference between hydraulic and pneumatic systems. A gas is highly compressible as compared to the liquid that has infinitesimal compressibility. This characteristic makes a hydraulic system suitable for more accurate, easily controlled movement of cylinders and motors than pneumatic systems. High compressibility of the gases produces a more 'cushiony' operation in pneumatic systems which are not suitable where movements of high accuracy are required.

10.4.2 Operating Pressure

The operating pressure of hydraulic systems is much higher as compared to pneumatic systems. Pressures higher than 70 MPa are used in special applications. On the contrary, pneumatic systems generally operate between 550–700 kPa. Normally extremely high pressure pneumatic systems are not used in practice.

10.4.3 Actuator Speed

Pneumatic systems normally find applications where high speed movement is required. Rotational speeds of more than 20,000 rpm are possible. Cylinder operation is responded quickly by the pneumatic systems. Pneumatic systems find application where lighter loads and lower accuracy is required. Hydraulic actuators are used where large forces are required. The fluid used in hydraulic actuator is highly incompressible so that pressure applied can be transmitted instantaneously to the member attached to it.

10.4.4 Component Weight

Hydraulic systems operate at higher pressures, requiring the use of robust construction and huge designs to withstand the high pressure. Pneumatic systems can be manufactured using lighter materials and small designs which minimizes the material requirement as they have to withstand much lower pressures. Hydraulic systems handles heavier weights, therefore, they require both higher system operating pressure and physical strength of machine parts. Pneumatic systems are used where ease of handling and lower weight are critical for effective operation of the tool or system.

10.4.5 Cost

In general, the operational cost of pneumatic systems is more compared to hydraulic systems. This high cost results due to the compression, conditioning, and distribution of air. Careful maintenance of pneumatic systems to eliminate leakage can greatly reduce operating cost.

In this chapter we will focus on hydraulic systems as their analysis is simple as compared to pneumatic system.

10.5 Role of Turbomachinery in Fluid Systems

Turbomachines are used to transmit power or torque from one shaft to another shaft through a fluid without any mechanical or electrical means. Turbomachinery plays a vital role in fluid-dynamic power transmission which consists of a centrifugal pump or centrifugal compressor connected in series. The fluid exiting the pump or compressor drives the turbine. The turbine exhaust is returned back to the inlet of the pump. The turbine and pump are housed in a single casing to minimize the energy losses. The pump element is known as *primary* or *transmitter* which receives energy from an internal combustion engine or electric motor. The turbine or fluid motor which receives power is known as *secondary* or *receiver*.

Unlike electric motors, air or hydraulic motors can produce a large amount of torque while operating at low speeds. Some hydraulic and pneumatic motors can even maintain torque at a very slow speed without overheating. Fluid systems can deliver constant torque or force regardless of speed changes. Fluid systems can transmit power more economically over greater distances than mechanical types. However, fluid systems are restricted to shorter distances compared to electrical systems.

10.6 Positive Displacement Machines vs. Turbomachinery

All fluid machines may be either of positive displacement type or rotodynamic type. In positive displacements, machine's fluid is directed into a closed volume. Energy transfer to the fluid is accomplished by the movement of the boundary of the closed volume, causing the volume to expand or contract, thereby sucking fluid in or squeezing fluid out respectively. *Heart* is a good example of a positive displacement pump which has one way valves that open to admit blood in-as heart chambers expand, and other one way valves which open as blood is pushed out of those chambers when they contract. *A positive displacement turbine* may be considered as a positive displacement pump running in reverse. The fluid turns a shaft or displaces a reciprocating rod as the fluid is pushed into a closed volume. The closed volume of fluid is then pushed out as more fluid admits into the device. Thus, the energy is extracted from the flowing fluid and is converted into mechanical energy. However, positive displacement turbines are generally not used for power generation, but rather for flow rate or flow volume measurement. For example, water meter which is used in houses. In a water meter, water forces itself into a closed chamber of expanding volume connected to an output shaft which rotates as the water enters the chamber. The boundary of the volume then collapses, turning the output shaft some more and letting the water continue on its way to sink, shower, etc. The water meter records each 360° rotation of the output shaft and the meter is precisely calibrated to the known volume of the fluid in the chamber. Considering the limitation of the positive displacement turbines, only positive displacement pumps are discussed briefly.

Positive displacement pumps transfer a captive volume of liquid successfully through the action of a device such as piston, vane, gear or screw in a closed chamber or cavity. Broadly, there are two types of positive displacement pumps: reciprocating and rotary displacement pumps.

A reciprocating pump consists primarily of a piston or a plunger reciprocating inside a cylinder, thus performing suction and delivery stroke. It is a positive displacement pump which means that it is a displacement pump which creates lift and pressure by displacing liquid with a piston or plunger. The chamber or cylinder is alternately filled or emptied by forcing and drawing the liquid by mechanical motion.

The rotary positive displacement pumps consist essentially of a stationary housing in which a power driven unit carrying one or more pumping elements, viz. vane, gear, lobe etc. is made to rotate, as shown in Figure 10.3. Due to rotation, the suction and delivery ports open and close at appropriate times. The requisite pressure is built up either by pure rotation or by combined rotation and oscillation of the pumping elements. Though the pumping elements rotate in positive displacement pumps, yet its action is not dynamic and it merely serves to displace the liquid.

Positive displacement pumps supply a fixed volume of fluid in a cycle. The fluid quantity discharged per revolution is constant in these pumps. Positive displacement pumps provide fluid flow proportional to their displacement and speed of the rotor and they are used in most of the industrial fluid power applications. The fluid flow at the output is constant and does not depend on the pressure of the system i.e. load. One of the important advantages of these pumps is that the areas of high-pressure and low-pressure (i.e. output and input area) are separate from each other and hence the fluid cannot leak back due to higher pressure at the outlets. These significant characteristics make the positive displacement pump most suitable and universally accepted for hydraulic systems.

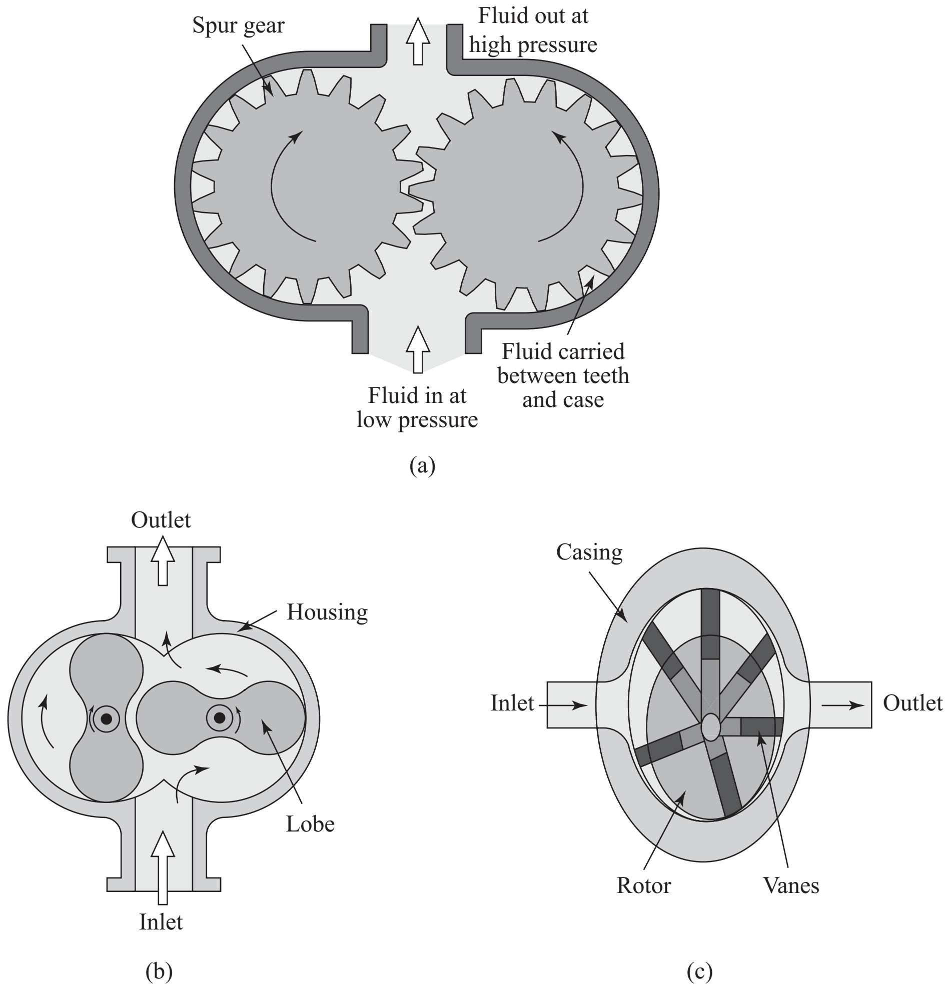


Figure 10.3 *Rotary Positive Displacement Pumps (a) Gear Pump, (b) Lobe Pump, and (c) Vane Pump*

In a turbomachine or rotodynamic machine, both thermodynamic and dynamic interaction between the flowing fluid and the rotating element (rotor or runner or impeller) takes place and involves energy transfer with change in both pressure and momentum. Turbomachines are distinguished from positive displacement machines in requiring that there exists a relative motion between the flowing fluid and rotating element (vanes) to transfer the energy and also aids the fluid to flow in a particular direction. Comparison between positive displacement machines and turbomachines is as follows:

10.6.1 Action

A positive displacement machine creates thermodynamic and mechanical action between near static fluid and relatively slow moving surface and involves in volume change and displacement of fluid as in IC engines. A turbomachine creates thermodynamic and dynamic action between flowing fluid and rotating element involving energy transfer with pressure and momentum changes. If discharge from a positive-displacement pump is closed, the pressure inside the pump increases and either the pump stops or some part of the casing bursts; if the discharge valve of a rotodynamic pump is closed, however, the rotating impeller merely churns the fluid round, and the energy consumed is converted to heat.

10.6.2 Operation

The positive displacement machine commonly involves reciprocating motion and unsteady flow of fluids like in reciprocating IC engines or slow rotating fluids like in gear pumps. A turbomachine involves steady flow of fluid with pure rotary motion of mechanical elements. Only unsteadiness will be there during starting, stopping and changes in loads on the machine.

10.6.3 Mechanical Features

A positive displacement machine commonly works at low speeds and involves complex mechanical design. It may have valves and normally will have heavy foundation. A turbomachine works at high speeds, simpler in design, light in weight, have less vibration problems and require light foundation.

10.6.4 Efficiency of Energy Conversion

A positive displacement machine gives higher efficiency due to energy transfer near static conditions either in compression or expansion processes. A turbomachine gives less efficiency in energy transfer. The energy transfer due to dynamic action will be less during compression process of fluid like in pumps and compressors and will be slightly more during expansion processes like in turbines but still lower than reciprocating machines.

10.6.5 Volumetric Efficiency

The volumetric efficiency of a positive displacement machine is low due to closing and opening of the valves during continuous operation. In turbomachines, since there are no valves under steady flow conditions, the volumetric efficiency is very high. A turbomachine has high fluid handling capacity. Turbomachines also have the fluid phase changes during the cavitation phenomenon in hydraulic pumps and turbines and surge and stall in compressors, blowers and fans if the machines are operated at off-design condition leading to associated vibrations and stoppage of flow and damage to blades. Positive displacement machines have no such problems.

10.7 Hydrostatic Systems

Hydrostatics uses the principles of equilibrium to deal with mechanics of stationary fluids. Hydrostatic system is that in which the primary function of the fluid is the transmission of force and power by the fluid static pressure. The hydrostatic system develops high pressure which is used to drive a linear or rotational actuator through a transmission line and a control element. The motivating force in such systems is a change in pressure, whereas the velocity of the fluid usually remains constant. It consists of two basic elements, a pumping unit to convert mechanical work into fluid energy and a hydraulic motor of reciprocating or rotary type to convert fluid energy into mechanical work. A lead which forms a circuit connects the two main components. The pumping unit transmits fluid pressure and is therefore known as *transmitter*. The hydraulic motor which receives force and power by means of fluid pressure is known as *receiver*. For example, a pumping unit is operating a hydraulic press. In this case the pumping unit is a transmitter and hydraulic press is a receiver. The work done by the pump is utilised for displacement of oil against a force which arises from resistance to motion of the plunger in the hydraulic press. Hydraulic accumulator and hydraulic intensifier are examples of hydrostatic systems.

10.7.1 Basic Principles

The basic principles of hydrostatic systems are very simple. An incompressible fluid is to be used and hence oil is the most common working medium in these systems. The system consists of a closed unit and the liquid is confined in a set of interconnected containers and connecting tubes. The four basic principles applicable to these systems are:

(i) Force applied at one point is transmitted to another point.

(ii) The applied force is multiplied or divided depending on the geometry of the system.

These two principles can be represented by the following example. Consider two interconnected piston-cylinder units, as shown in Figure 10.4 (a).

The cylinders 1 and 2 are interconnected and contain an incompressible fluid, i.e. oil. The force F_1 applied on the piston at 1 transforms into pressure $p = F_1/A_1$, where A_1 is the cross sectional area of the cylinder 1. By Pascal's law, the pressure is same at all points in the fluid and hence, it will also be acting on the bottom of the piston/ram of the cylinder 2, i.e. $p_1 = p_2 = p$. The force applied at the bottom of the piston of larger cylinder 2 having an area A_2 is then,

$$F_2 = pA_2 = F_1 \frac{A_2}{A_1} \tag{10.1}$$

If the volume of the oil is held constant, the displacement of the larger piston will be proportionately smaller as compared to the smaller piston.

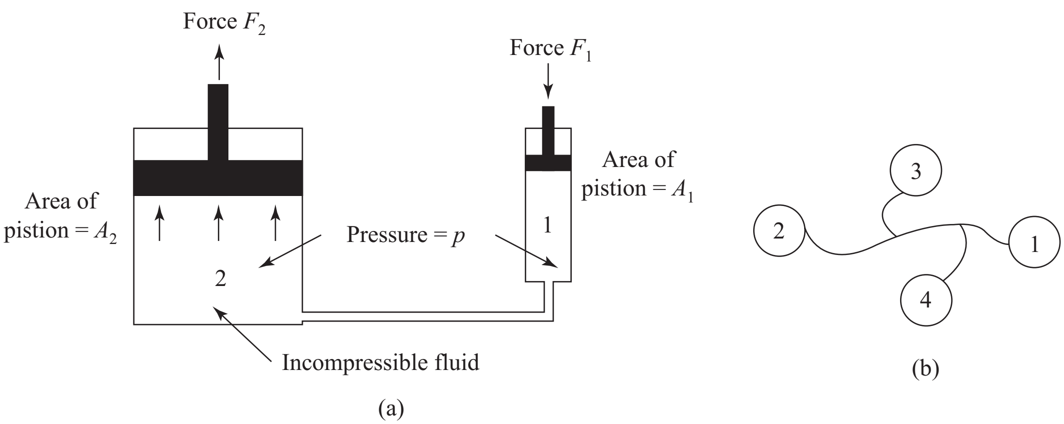


Figure 10.4 *Basic Principles of Hydrostatic Systems*

Equation (10.1) represents that the force F_2 is related to force F_1 through a multiplication factor A_2/A_1. Thus, if the diameter ratio is 2, force applied at 1 is multiplied by a factor of 4 at the bottom of the cylinder 2.

(iii) The connecting pipe can be of any shape, size, length and planar layout.

(iv) The pipe can have any branching and force will be transmitted at all branches with multiplying factors depending on the individual geometries.

For example, consider two additional cylinders 3 and 4 with pistons, are connected to the pipe connecting 1 and 2, as shown in Figure 10.4 (b). For a given force F_1 applied to the piston of the cylinder 1, the pistons in all the other three cylinders, 2, 3 and 4, will experience forces F_2, F_3 and F_4, respectively. The forces F_3 and F_4 will have multiplication factors depending on the areas of the piston 3 and 4, respectively. It should be noted that F_2 remains unchanged due to the introduction of additional branches in the tube connecting 1 and 2. This results in an important concept of *master cylinder* which drives more than one cylinder, e.g. automobile braking system.

10.7.2 Hydraulic Press

The *hydraulic press* is a device which is used for providing compressive or lifting force by the application of a much smaller force. Equation (10.1) represents the relationship between the applied force and output force. Based on the fundamental principles of hydrostatic systems as already discussed, working devices have been developed and hydraulic press have been used as a standard device for obtaining large compressive forces. A schematic diagram of a hydraulic press is shown in Figure 10.5 (a). It consists of a ram (sometimes also called *plunger*) sliding in fixed cylinder to which high pressure liquid is forced by a pump. A movable platform is attached to the ram. The ram is activated by pump providing the hydraulic pressure that causes the displacement of a movable platform. The materials placed between stationary platform and movable platform undergo high compressive forces. Common industrial applications of hydraulic presses include compression forming, blanking, forging and punching. This type of press is called *direct-type hydraulic press* as the movable platform acts directly against the object to be pressed. The stationary platform takes the reaction load and transmits it to the foundation. Similar to the principle of a hydraulic press, the arrangement of a ram working in a cylinder under hydraulic pressure created by a pump finds many industrial applications such as *hydraulic jack* and *hydraulic lift*. The press acts like a simple hydraulic jack or a simple hydraulic lift if the stationary platform of the press shown in Figure 10.5 (a) is omitted.

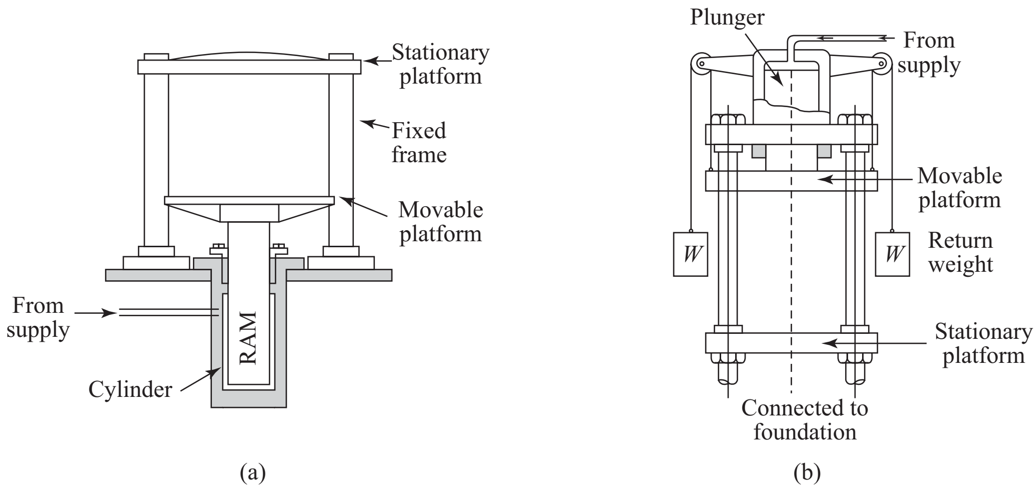


Figure 10.5 *Schematic Diagram of (a) Direct-type Hydraulic Press, (b) Inverted-type Hydraulic Press*

Figure 10.5 (b) is a schematic diagram of an *inverted type of hydraulic press*. It consists of a fixed cylinder in which a ram is sliding. A movable platform is attached to the lower end of the ram. As the ram moves up and down, the movable platform attached to the ram also moves up and down between the two fixed plates. When any liquid under higher pressure created by a pump is supplied into the cylinder, moves vertically downwards and exerts a force equal to the product of pressure and area of the ram on the object placed between lower fixed plate and the movable platform. Thus, the object gets pressed. The reaction force is directly transmitted to the foundation. After the pressing job is over, the liquid from the cylinder is taken out in order to bring the ram and movable plate to the initial position. The return weights assist in the quick return of the movable plate to original position.

10.7.3 Hydraulic Accumulator

A hydraulic accumulator is a device like that of a storage battery used for temporary storing the energy of a liquid in the form of pressure energy when not required by the system. The stored energy may be used for any sudden or intermittent requirement. Generally, pumps in a hydraulic system work continuously. For example, a crane or lift requires large amount of energy in the form of high pressure liquid which is required during upward motion of the load only. No energy is required during the downward motion. However, the pump supplies the liquid under pressure continuously at uniform rate. During the idle stroke of the machine when there is no load, the liquid under pressure can be stored in the accumulator and it can be supplied as and when required.

(a) Simple Accumulator

Figure 10.6 shows a simple or common type of accumulator which consists of a fixed vertical cylinder containing a sliding ram. One side of the cylinder is connected to the pump (input) and the other side to the machine (output). The ram is at the lowermost position at the starting of the cycle. When the liquid is not required by the machine, the high pressure liquid delivered by the pump flows into the cylinder (accumulator) and causes the ram to move up. A resistance load on the movement of the ram is provided either through a dead weight or in the form of compression spring. This arrangement keeps the liquid in the accumulator under desired high pressure. When the ram is at the uppermost position, the cylinder is full of liquid and the accumulator has stored the maximum amount of pressure energy. When the machine requires a large amount of energy at increased load, this pressure liquid from the accumulator is released through the outlet to the chosen hydraulic machine or device. The release of the pressure liquid causes the ram of the accumulator to move downwards towards its original position. The commonly used accumulators can be the following types:

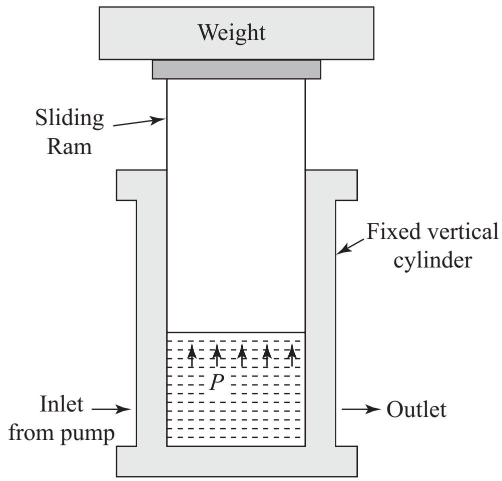


Figure 10.6 *Simple Hydraulic Accumulator*

(i) Raised weight type
(ii) Spring type
(iii) Metal bellows type
(iv) Compressed gas type

Figure 10.6 is of the raised weight accumulator. Let D be the diameter of the ram, L is the stroke of the ram or length of the ram movement or lift of the ram, p is the pressure of the liquid in the ram supplied by the pump, and W is the weight placed on the ram including the weight of the ram. Then,

$$W = Ap = \frac{\pi}{4} D^2 p \tag{10.2}$$

The work done in lifting the ram or the energy stored in the ram $= WL = \frac{\pi}{4} D^2 pL$ (10.3)

The energy stored in the accumulator is also known as the *capacity* of the accumulator given by,

$$\text{Capacity of the accumulator} = WL = \frac{\pi}{4} D^2 pL = p \times \text{Volume of the accumulator} \quad (10.4)$$

The power supplied by the accumulator when the ram falls uniformly through a distance L in time t is,

$$P = \frac{WL}{t} = p \times Q \quad (10.5)$$

where,

$$Q = \text{Discharge through accumulator} = \frac{\text{Volume of the accumulator}}{t} \quad (10.6)$$

(b) Differential Accumulator

A *differential accumulator* is a variant of a simple or common accumulator. It differs from the common accumulator that comparatively high pressure can be obtained by a relatively small load on the ram. Figure 10.7 shows a differential accumulator. It consists of a fixed vertical ram or fixed cylinder having a central vertical hole throughout its length and is surrounded by a closely fitted brass bush to make the lower portion larger than its upper portion. The fixed vertical cylinder or fixed ram of small diameter is surrounded by an inverted sliding cylinder having circular collars outwards at the base on which weights are placed. The liquid from the pump is supplied to the fixed cylinder at its bottom. The liquid enters the sliding cylinder through the central hole. Liquid under pressure is admitted through the fixed ram and fills the annular space between the fixed ram and the cylinder available above the top of the bush. The liquid can enter and exit through the valve controlled openings at the bottom of fixed ram.

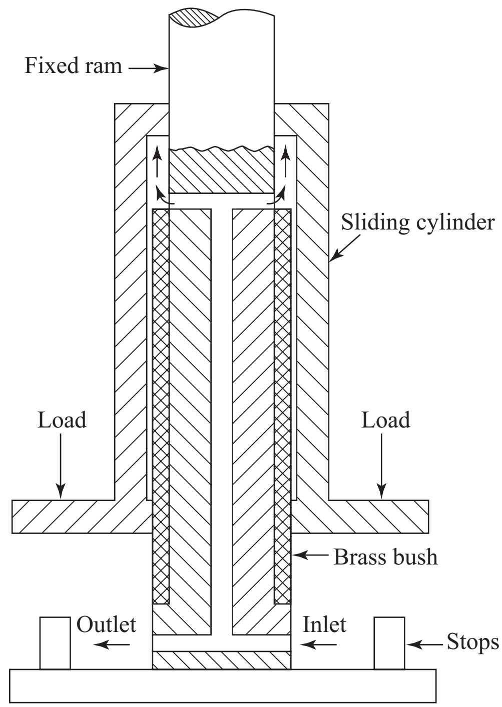


Figure 10.7 *Differential Accumulator*

The liquid from the pump enters from the bottom of the fixed ram and lifts the inverted cylinder upwards during loading. The accumulator is fully charged or loaded when the inverted cylinder is at the end of its stroke. The liquid will be under pressure due to the load W. During the unloading of the accumulator i.e. release of the energy, the liquid passes out of the outlet at the bottom of the fixed cylinder and the inverted sliding cylinder moves downwards.

Let D is the external diameter of the bush and d is the external diameter of the fixed ram, then the annular area is,

$$A = \frac{\pi}{4}(D^2 - d^2) \quad (10.7)$$

If W is the load carried by the sliding or inverted cylinder including its weight, then pressure of the liquid in the accumulator is,

$$p = \frac{W}{A} = \frac{4W}{\pi(D^2 - d^2)} \tag{10.8}$$

It is obvious from Eq. (10.8) that for a given load, pressure can be increased by making the cross sectional area of the bush (annular space) small.

Capacity of the accumulator is given by,

$$C = WL = pAL = p \times \text{Volume of the accumulator} \tag{10.9}$$

The major applications of accumulator in hydraulic systems are as follows:

- Stores pressures and hydraulic fluid
- As a backup power source
- As a leakage compensator
- As a shock absorber of the system
- Maintains pressure and reduces pump size
- Reduces pump pulsations

10.7.4 Hydraulic Intensifier

A *hydraulic intensifier* is a device which converts the low pressure from a cylinder into high pressure in a smaller cylinder. Intensifiers, also known as *boosters,* consist essentially of two different sized cylinders with pistons connected by a common piston rod.

Intensifiers operate on the ratio-of-areas principle in interconnected cylinders. There are two types of hydraulic intensifiers described as follows:

(a) Simple Hydraulic Intensifier

A simple hydraulic intensifier is shown in Figure 10.8. A common piston rod connects the pistons of two cylinders of different diameters. The force exerted by a lower pressure fluid on the larger piston is transmitted mechanically by the piston rod to the smaller piston. The smaller piston develops a higher pressure on the fluid in its bore. The pressure ratio is inversely proportional to the area ratio.

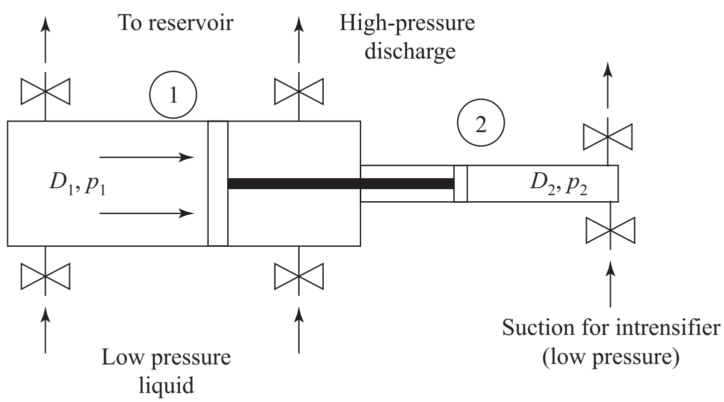


Figure 10.8 *Simple Hydraulic Intensifier*

Assume that there is no acceleration head. The force exerted by the fluid on the pistons of two cylinders must be the same. Therefore,

$$\frac{\pi}{4}D_1^2 p_1 = \frac{\pi}{4}D_2^2 p_2$$

$$p_2 = p_1\left(\frac{D_1}{D_2}\right)^2 \tag{10.10}$$

Since $D_1 > D_2 \Rightarrow p_2 > p_1$. Equation (10.10) represents that the input pressure is increased by a factor which is equal to the square of the ratio of the inlet to outlet cylinder diameters. The pressure ratio p_2/p_1 is known as *intensification ratio*.

(b) Co-Axial Hydraulic Intensifier

A single acting co-axial type intensifier is shown in Figure 10.9. It consists of a fixed ram surrounded by a sliding cylinder or movable ram. Initially the low pressure liquid from the main supply enters through the fixed ram and enters into the sliding cylinder or movable ram. The sliding cylinder is surrounded by a fixed inverted cylinder which contains the low pressure liquid from the main supply. The weight of this liquid pushes the sliding cylinder to move downwards till it reaches its full stroke length and thereby the pressure of the liquid in the sliding cylinder is increased. The high pressure liquid is supplied to the machine through the fixed ram. The action is similar to that of a syringe. The ratio of the area of the outer fixed cylinder (inverted cylinder) to the area of the fixed ram is the intensification ratio of this device.

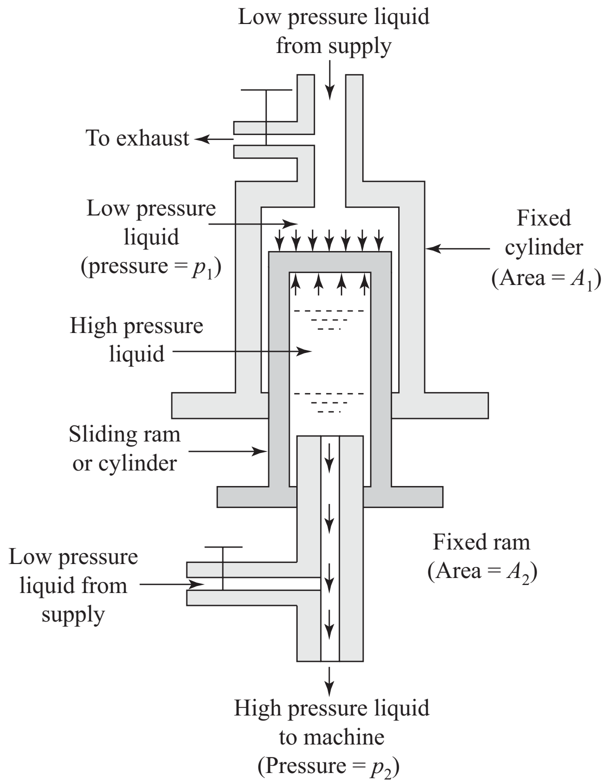


Figure 10.9 *Coaxial Hydraulic Intensifier*

Let p_1 is the pressure of liquid from the main supply (low pressure supply), A_1 is the external area of the sliding cylinder or inside cross sectional area of the fixed cylinder of diameter D_1, A_2 is the inside cross sectional area of the fixed ram of inner diameter D_2. Therefore, discharge pressure at the outlet if considering no frictional loss in the movement of the ram is,

$$p_2 = p_1\left(\frac{A_1}{A_2}\right) = p_1\left(\frac{D_1}{D_2}\right)^2 \tag{10.11}$$

If, however, there is frictional loss amounting to $\varepsilon\%$ at each of the ram, considering equilibrium of the moving ram at any position,

$$p_1 A_1\left(1-\frac{\varepsilon}{100}\right)=\frac{p_2 A_2}{\left(1-\frac{\varepsilon}{100}\right)}$$

$$p_2 = p_1\left(\frac{A_1}{A_2}\right)\left(1-\frac{\varepsilon}{100}\right)^2 \tag{10.12}$$

(c) Location of Intensifier

In a hydraulic system operating a machine, viz. hydraulic press, crane, winch, etc.,intensifiers are located after the accumulator and before the machine. A layout of a hydraulic device is shown in Figure 10.10. The hydraulically driven machine is supplied with high pressure liquid from a supply reservoir through the action of a pump. The pump is driven either by an electric motor or an internal combustion engine. Normally the pumps run continuously. The working of the accumulator and intensifier depends on the load on the machine.

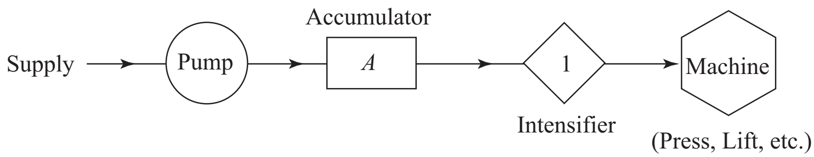


Figure 10.10 *Layout of a Hydraulic Device*

10.7.5 Hydraulic Ram

A *hydraulic ram* is a type of pump which raises water without any external power for its operation. When large quantity of water is available at a small height, a small quantity of water can be lifted to a considerable height with the help of a hydraulic ram. It works on the principle of water hammer. The main components of a hydraulic ram are shown in the Figure 10.11. The valve chamber has two valves-delivery valve V_1 and waste valve V_2. The delivery valve opens to an air vessel whereas waste valve opens into a waste water channel. When the inlet valve fitted to the supply tank is opened, water from the supply tank starts flowing into the valve chamber. This increases the level of water in the valve chamber that exerts a dynamic thrust on the waste valve due to which the waste valve, V_2, starts moving upward. A stage is reached when the waste valve gets suddenly closed due to the build-up of dynamic pressure within the valve chamber at the seat of the valve, V_2. The instantaneous closure of the waste valve brings the water in the supply line suddenly to rest which results in a further increase in pressure in the valve chamber. This high pressure force opens the delivery valve, V_1. The water from the valve chamber enters into the air vessel and compresses the air in it. This compressed air exerts force on the water in the air vessel and small quantity of water is raised to a greater height through the delivery pipe into delivery tank. The flow of water through the delivery valve continues until the pressure in the valve chamber is reduced, delivery valve closes and the waste valve opens. This allows the flow of water from the supply tank to the valve chamber and the cycle is repeated.

Let Q_s be the discharge of water supplied to the ram, Q_d be the discharge of water into the delivery tank, Q_w be the discharge wasted by the ram, H_1 is the height of the water surface in the supply tank above the valve chamber, H_2 is the height of the delivery tank above the valve chamber, h_{fs} be the frictional head loss in the supply pipe and h_{fd} be the frictional head loss in the delivery pipe.

$$Q_s = Q_d + Q_w \tag{10.13}$$

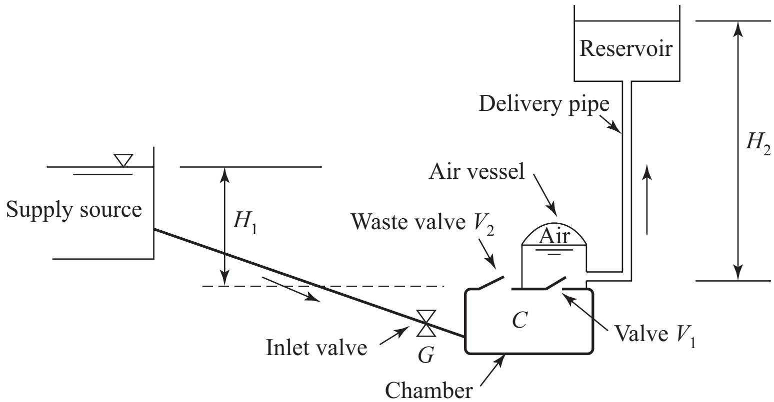


Figure 10.11 *A Hydraulic Ram*

(a) D' Aubuisson Efficiency

Power supplied to the ram, i.e. input energy rate,

$$P_i = \rho_w g Q_s (H_1 - h_{fs}) = \rho_w g (Q_d + Q_w)(H_1 - h_{fs}) \tag{10.14}$$

Power delivered by the ram, i.e. output energy rate,

$$P_o = \rho_w g Q_d (H_2 + h_{fd}) \tag{10.15}$$

The efficiency of the hydraulic ram is,

$$\eta = \frac{P_o}{P_i} \Rightarrow \eta = \frac{Q_d (H_2 + h_{fd})}{Q_s (H_1 - h_{fs})} \tag{10.16}$$

If friction losses are neglected,

$$\eta = \frac{Q_d H_2}{Q_s H_1} \tag{10.17}$$

(b) Rankine Efficiency

According to Rankine, the water surface in the supply tank is taken as the datum. The water is initially at a height of H_1 from the ram and hence, the water is raised to a height of $(H_2 - H_1)$, not H_2. Assume that the friction losses are small, hence neglected.

Power supplied to the ram, i.e. input energy rate,

$$P_i = \rho_w g (Q_s - Q_d) H_1 \tag{10.18}$$

Power delivered by the ram, i.e. output energy rate,

$$P_o = \rho_w g Q_d (H_2 - H_1) \tag{10.19}$$

The efficiency of the hydraulic ram is,

$$\eta = \frac{P_o}{P_i} \Rightarrow \eta = \frac{Q_d (H_2 - H_1)}{(Q_s - Q_d) H_1} \tag{10.20}$$

The value of the Rankine's efficiency is always less than the D' Aubuisson's efficiency.

10.7.6 Hydraulic Lift

The *hydraulic lift* is a device used for carrying the persons and goods from one floor to another in a multi-storied building. Commonly used hydraulic lifts are of two types described as follows:

(a) Direct Acting Type

It consists of a vertical ram sliding in a cylinder as shown in Figure 10.12. At the top of the ram, a cage or platform is fitted on which persons stand or goods may be placed. The liquid under pressure is admitted into the fixed cylinder at its bottom. This liquid exerts force on the sliding ram which moves vertically up. Thus the cage is raised to the required height. By removing the liquid from the fixed cylinder, the cage can be moved in downward direction.

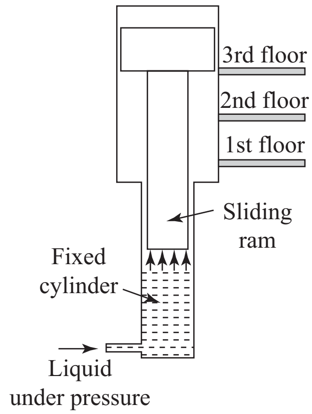


Figure 10.12 *Direct Acting Hydraulic Lift*

(b) Suspended Hydraulic Lift

A modified form of direct acting hydraulic lift is a suspended hydraulic lift which is shown in Figure 10.13. It consists of a cage on which persons may stand or goods may be placed. A jigger consisting of a fixed cylinder, a sliding ram and a set of two pulleys is provided at the foot of the hole of the cage. One of the pulley blocks is movable while the other is fixed. The end of the sliding ram is connected to the movable pulley block. A wire rope with one of its ends fixed to a movable pulley attached to the ram of the jigger is passed over all the pulleys and is then taken over the guide pulleys as shown in the figure. The cage is suspended from the other end of the rope.

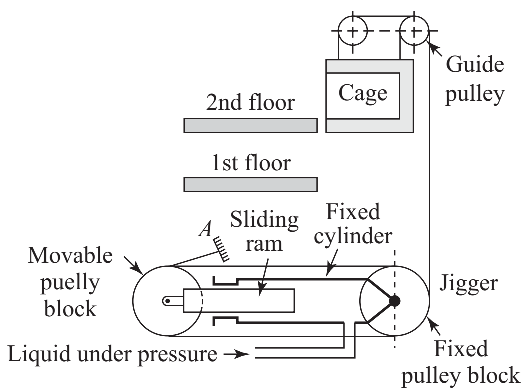


Figure 10.13 *Suspended Hydraulic Lift*

The raising or lowering of the cage of the lift is done by the jigger. The water under pressure is admitted into the fixed cylinder of the jigger. Due to the high water pressure, the sliding ram is forced to move towards left. The movable pulley block also moves towards left as one end of the sliding ram is connected to movable pulley block. This in turn increases the distance between the two sets of pulley blocks. This will result in winding the wire passing over the guide pulleys and the cage is lifted. For lowering the cage, the water is taken out from the cylinder and the ram moves towards the right. The movable pulley block attached to the sliding ram also moves towards right. This in turn decreases the distance between the two sets of pulley blocks and the length of the rope is increased.

10.7.7 Hydraulic Crane

A *hydraulic crane* is a device widely used for lifting or transferring heavy loads in workshops, docks and warehouses. It consists of a mast, jib, tie, guide pulleys and jigger as shown in Figure 10.14. The mast has its own pedestal for the support. The jib can be raised or lowered in order to decrease or increase the radius of rotation of the crane. The mast along with the jib can revolve about its vertical axis. The jigger consists of a movable ram sliding in a fixed cylinder. A movable pulley block is connected to the top of the sliding ram, whereas a fixed pulley block is connected to the lower end of the fixed cylinder. The pulley block attached to the ram moves up and down while the pulley block attached to the fixed cylinder does not move.

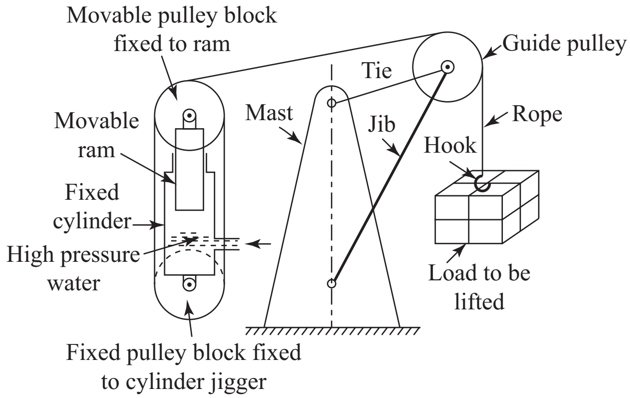


Figure 10.14 *A Hydraulic Crane*

A wire rope with one of its end fixed to a movable pulley attached to the sliding ram of the jigger is taken round over all the pulleys of two sets of pulleys and is then finally taken over the guide pulley attached to the jib. The other end of the rope is provided with a hook for suspending the load.

The water under pressure is admitted into the cylinder of the jigger when a load is to be lifted. The high pressure water forces the sliding ram to move vertically up and the movable pulley block attached to the ram also move upwards. Consequently, the distance between the two pulley blocks increases and hence the wire passing over the guide pulleys is pulled by the jigger. This lifts the load attached to the hook to any place within the crane's working area.

EXAMPLE 10.1 A hydraulic press has a ram of 300 mm and plunger of 120 mm diameter. The frictional resistance of ram operating the loading platform is 30 kN whereas a compressive force of 495 kN is required at a job. Calculate the force to be applied on the plunger if the frictional resistance at the plunger assembly can be taken as 1.5% of the external load acting on it.

Solution

Given: $D = 300$ mm = 0.3 m, $d = 120$ mm = 0.12 m, $(F_f)_{ram} = 30$ kN, $F_{job} = 495$ kN, $(F_f)_{plunger} = 1.5\%$ of external load on plunger

Total force to be provided at the base of the ram by the fluid pressure,

$$F_2 = F_{job} + (F_f)_{ram} = 495 + 30 = 525 \text{ kN} \tag{1}$$

Fluid pressure at the base of the ram,

$$p_2 = \frac{F_2}{A_2} \Rightarrow p_2 = \frac{F_2}{\frac{\pi}{4}D^2} \tag{2}$$

$$p_2 = \frac{525}{\frac{\pi}{4} \times 0.3^2}$$

$$p_2 = 7427.231 \text{ kN/m}^2 \tag{3}$$

By Pascal's Law, the pressure is same at all points in the fluid, therefore, pressure acting on the plunger $p_1 = p_2 = 7427.231$ kN/m^2.
Force required on the plunger without frictional resistance,

$$F_p = p_1 A_1 = \frac{\pi}{4} d_p^2 p_1 \tag{4}$$

$$F_p = \frac{\pi}{4} \times 0.12^2 \times 7427.231$$

$$F_p = 84 \text{ kN} \tag{5}$$

Frictional resistance at the plunger assembly,

$$(F_f)_{\text{plunger}} = \frac{1.5}{100} F_p \Rightarrow (F_f)_{\text{plunger}} = 0.015 \times 84$$

$$(F_f)_{\text{plunger}} = 1.26 \text{ kN} \tag{6}$$

Total force required at the plunger,

$$F_1 = F_p + (F_f)_{\text{plunger}} \Rightarrow F_1 = 84 + 1.26$$

$$F_1 = 85.26 \text{ kN} \tag{7}$$

EXAMPLE 10.2 The weight raised by a hydraulic press is 3.0 kN and the distance moved by the weight is 2 m in 35 min. The ram of the press has 200 mm diameter. The plunger has 50 mm diameter with stroke length of 200 mm. Calculate (a) force applied on the plunger, (b) number of strokes performed by the plunger, and (c) power required to drive the plunger.

Solution

$F_2 = 3.0$ kN = 3000 N, $y = 2.0$ m, $t = 35$ min = 2100 s, $D = 200$ mm = 0.2 m, $d = 50$ mm = 0.05 m, L_p = 200 mm = 0.2 Area of plunger,

$$A_1 = \frac{\pi}{4} d^2 \Rightarrow A_1 = \frac{\pi}{4} \times 0.05^2$$

$$A_1 = 0.0019635 \text{ m}^2 \tag{1}$$

Area of the ram,

$$A_2 = \frac{\pi}{4} D^2 \Rightarrow A_2 = \frac{\pi}{4} \times 0.2^2$$

$$A_2 = 0.031416 \text{ m}^2 \tag{2}$$

(a) Force Applied on the Plunger

By Pascal's law, the pressure is same at all points in the fluid and hence pressure acting on the plunger and bottom of the piston/ram are the same, i.e. $p_1 = p_2$. Therefore, force applied on the plunger,

$$F_1 = F_2 \left(\frac{A_1}{A_2} \right) = F_2 \left(\frac{d}{D} \right)^2 \tag{3}$$

$$F_1 = 3 \times \left(\frac{0.05}{0.2} \right)^2$$

$$F_1 = 0.1875 \text{ kN} = 187.5 \text{ N} \tag{4}$$

(b) Number of Strokes of Plunger

The volume of the liquid displaced by the ram and plunger movements is same in both the cylinders. The plunger has to travel a great distance to move the ram by a short distance. If L_p is the stroke of the plunger and n is the number of strokes of the plunger to move the ram through a distance y, then,

$$nA_1L_p = A_2y \tag{5}$$

$$n \times 0.0019635 \times 0.2 = 0.031416 \times 2$$

$$n = 160 \tag{6}$$

(c) Power Required to Drive the Plunger

Work done by the press is,

$$W = \text{Force (Weight) Lifted} \times \text{Displacement} \tag{7}$$

$$W = 3 \times 2 \Rightarrow W = 6 \text{ kJ} = 6000 \text{ J} \tag{8}$$

Power required to drive the plunger is equal to the work done by the press per second.

$$P = \frac{6000}{35 \times 60}$$

$$P = 2.857 \text{ W} \tag{9}$$

EXAMPLE 10.3 The total weight on the ram including its self-weight in a hydraulic accumulator is 620 kN. The diameter of the sliding ram is 750 mm. The frictional resistance against the movement of the ram is 5% of the total weight. Determine the pressure of the water when (a) the ram is moving up with a uniform velocity, and (b) ram is moving down with a uniform velocity.

Solution

Given: W = 620 kN, D = 750 mm = 0.75 m, Frictional resistance = 5% of W

Frictional resistance,

$$F_f = 5\% \text{ of } W = \frac{5}{100}W \Rightarrow F_f = 0.05 \times 620$$

$$F_f = 31 \text{ kN} \tag{1}$$

Area of the ram,

$$A = \frac{\pi}{4}D^2 \Rightarrow A = \frac{\pi}{4} \times 0.75^2$$

$$A = 0.44179 \text{ m}^2 \tag{2}$$

(a) Ram Moving Up

When the ram is moving upwards, both the weight and frictional resistance are acting in the downward direction. Therefore, total force on the ram when it is moving up,

$$F_1 = W + F_f \Rightarrow F_1 = 620 + 31$$

$$F_1 = 651 \text{ kN} \tag{3}$$

Therefore, pressure acting on the water when the ram is moving up,

$$p = \frac{F_1}{A} \Rightarrow p = \frac{651}{0.44179}$$

Pressure of water,

$$p = 1473.551 \text{ kN/m}^2 \quad (4)$$

(b) Ram Moving Down

When the ram is moving downwards, the frictional resistance is acting in the upward direction. Therefore, total force on the ram when it is moving upwards,

$$F_2 = W - F_f \Rightarrow 620 - 31$$

$$F_2 = 589 \text{ kN} \quad (5)$$

Therefore, pressure acting on the water when the ram is moving down,

$$p = \frac{F_2}{A} \Rightarrow p = \frac{589}{0.44179}$$

Pressure of water,

$$p = 1333.22 \text{ kN/m}^2 \quad (6)$$

EXAMPLE 10.4 An accumulator of a hydraulic system has a sliding ram of 300 mm diameter and 6 m stroke length. The ram is loaded with 500 kN which includes its self-weight also. The packing friction amounts to 5% of the load on the accumulator. If the ram falls steadily through the full stroke in 2 min and at the same time pumps 10 litres/s to the machinery, what power is delivered to the machinery?

Solution

Given: D = 300 mm = 0.3 m, L= 6 m, W = 500 kN, Packing friction = 5% of W, t = 2 min = 120 s, Q= 10 litres/s = 0.01 m^3/s

Energy is delivered to the machine by the accumulator as well as by the pump.

Effective load on the ram when the ram is descending = Total load – Packing friction

$$W = 500 - 0.05 \times 500$$

$$W = 475 \text{ kN} \quad (1)$$

Power supplied by the accumulator,

$$P_{acc} = \frac{WL}{t} \quad (2)$$

$$P_{acc} = \frac{475 \times 6}{120}$$

$$P_{acc} = 23.75 \text{ kW} \quad (3)$$

Pressure of the water in the accumulator when ram is descending,

$$p = \frac{W}{\frac{\pi}{4} D^2} \Rightarrow p = \frac{475}{\frac{\pi}{4} \times 0.3^2}$$

$$p = 6719.8754 \text{ kN/m}^2 \quad (4)$$

Head of the water developed by the pump,

$$H = \frac{p}{\rho g} \Rightarrow H = \frac{6719.8754 \times 1000}{1000 \times 9.81}$$

$$H = 685.0026 \text{ m} \tag{5}$$

Power supplied by the pump,

$$P_p = \rho g Q H \Rightarrow P_p = 1000 \times 9.81 \times 0.01 \times 685.0026$$

$$P_p = 67198.76 \text{ W} = 67.199 \text{ kW} \tag{6}$$

Total power supplied to the hydraulic system,

$$P = P_{\text{acc}} + P_p \tag{7}$$

$$P = 23.75 + 67.199$$

$$P = 90.949 \text{ kW} \tag{8}$$

EXAMPLE 10.5 It is required to transmit 40 kW of power from a hydraulic accumulator through a pipe of 100 mm diameter and 1.5 km long. The friction in the pipeline is 3% of the total power being transmitted. Find out the diameter of the ram if friction factor $f = 0.01$ for the pipeline and the ram is loaded with a weight of 1500 kN.

Solution

Given: P = 40 kW, D_p = 100 mm = 0.1, L= 1.5 km = 1500 m, Friction loss in the pipe = 3% of P, f = 0.01, W = 1500 kN

$$\text{Power lost due to friction} = \frac{3}{100} \times 40 = 1.2 \text{ kW} \tag{1}$$

The loss of head due to friction in the pipeline,

$$h_f = 4f \frac{L}{D} \frac{C^2}{2g}$$

$$h_f = 4 \times 0.01 \times \frac{1500}{0.1} \times \frac{C^2}{2 \times 9.81}$$

$$h_f = 30.58\, C^2 \tag{2}$$

$$\text{Power lost due to friction} = \rho g Q h_f = \rho g \frac{\pi}{4} D_p^2 C \times h_f$$

$$1.2 \times 1000 = 1000 \times 9.81 \times \frac{\pi}{4} \times 0.1^2 \times C \times 30.58 C^2$$

$$C = 0.7986 \text{ m/s} \tag{3}$$

Discharge through the pipe line and accumulator are the same.

$$Q = \frac{\pi}{4} \times D_p^2 \times C \Rightarrow Q = \frac{\pi}{4} \times 0.1^2 \times 0.7986$$

$$Q = 0.0062722 \text{ m}^3/\text{s} \tag{4}$$

Power developed by accumulator,

$$P = \rho g Q H \tag{5}$$

$$40 \times 1000 = 1000 \times 9.81 \times 0.0062722\ \text{H}$$

$$H = 650.086\ \text{m} \tag{6}$$

Pressure of the water in the accumulator,

$$p = \rho g H \Rightarrow p = 1000 \times 9.81 \times 650.086$$

$$p = 6.377 \times 10^6\ \text{N/m}^2 = 6377\ \text{kN/m}^2 \tag{7}$$

For the ram,

$$p = \frac{W}{A} = \frac{W}{\frac{\pi}{4}D^2} \tag{8}$$

$$6377 = \frac{4 \times 1500}{\pi \times D^2}$$

$$D = 0.54726\ \text{m} \tag{9}$$

EXAMPLE 10.6 A hydraulic intensifier has a fixed ram of 100 mm diameter and a sliding ram of 0.5 m diameter. The supply pressure is 2.5 bar and the loss due to friction at each of the packing of the intensifier is 2.5% of the total force on each of the packings. Calculate the pressure at the outlet of the intensifier.

Solution

Given: D_2 = 100 mm = 0.1 m, D_1 = 0.5 m, p_1 = 2.5 bar = 250 kPa, ε = 2.5% = 0.025

Considering equilibrium of the moving ram at any position,

$$p_1 A_1 \left(1 - \frac{\varepsilon}{100}\right) = \frac{p_2 A_2}{\left(1 - \frac{\varepsilon}{100}\right)}$$

$$p_2 = p_1 \left(\frac{A_1}{A_2}\right)\left(1 - \frac{\varepsilon}{100}\right)^2 = p_1 \left(\frac{D_1}{D_2}\right)^2 \left(1 - \frac{\varepsilon}{100}\right)^2 \tag{1}$$

$$p_2 = 250 \times \left(\frac{0.5}{0.1}\right)^2 \times \left(1 - \frac{2.5}{100}\right)^2$$

$$p_2 = 5941.41\ \text{kPa} = 5.94141\ \text{MPa} \tag{2}$$

EXAMPLE 10.7 The liquid at a pressure of 50 bar is supplied in a hydraulic intensifier and delivered to a machine at a pressure of 200 bar. The stroke of the intensifier is 1.2 m and capacity is 25 litres. Determine the inside diameters of the high pressure fixed ram and sliding ram.

Solution

$p_1 = 50$ bar $= 5000$ kPa, $p_2 = 200$ bar $= 20000$ kPa, $L_1 = 1.2$ m, Capacity $= 25$ litres

Capacity of intensifier = Displacement volume

= Area of the fixed ram × Stroke length (1)

$0.025 = A_1 \times 1.2$

$A_1 = 0.02083 \text{ m}^2$ (2)

Area of the low pressure fixed ram/cylinder,

$$A_1 = \frac{\pi}{4} d_1^2 \Rightarrow 0.02083 = \frac{\pi}{4} d_1^2$$

$d_1 = 0.16285$ m (3)

Considering the equilibrium of the sliding cylinder,

$p_1 A_1 = p_2 A_2$ (4)

Therefore, cross sectional area of high pressure sliding cylinder,

$$A_2 = \frac{\pi}{4} d_2^2 = A_1 \left(\frac{p_1}{p_2} \right) = 0.02083 \times \left(\frac{50}{200} \right)$$

$d_2 = 0.08143 \text{ m} = 81.43 \text{ mm}$ (5)

EXAMPLE 10.8 A hydraulic ram lifts 5 litres/*s* of water to a tank 20 m above the ram through a 85 m long pipe of 75 mm diameter. The supply tank is 4 m above the ram and it is estimated that water wasted at the ram is 75 litres/*s*. Calculate the efficiency of the ram assuming that frictional loss in the supply pipe is negligible. Take Darcy's friction coefficient, $f = 0.015$ for the delivery pipe.

Solution

Given: $Q_d = 5$ litres/*s* $= 0.005 \text{ m}^3/\text{s}$, $H_2 = 20$ m, $L_d = 85$ m, $D_d = 75$ mm $= 0.075$ m, $H_1 = 4$m, $Q_w = 75$ litres/*s* $= 0.075 \text{ m}^3/\text{s}$, $f = 0.015$

Area of the delivery pipe,

$$A_d = \frac{\pi}{4} D_d^2 \Rightarrow A_d = \frac{\pi}{4} \times 0.075^2$$

$A_d = 0.004418 \text{ m}^2$ (1)

Velocity in the delivery pipe,

$$C_d = \frac{Q_d}{A_d} \Rightarrow C_d = \frac{0.005}{0.004418}$$

$C_d = 1.1317$ m/s (2)

$$h_{fd} = 4f \frac{L_d}{D_d} \frac{C_d^2}{2g} \Rightarrow h_{fd} = 4 \times 0.015 \times \frac{85}{0.075} \times \frac{1.1317^2}{2 \times 9.81}$$

$h_{fd} = 4.4389$ m (3)

Discharge supplied to the ram,

$$Q_s = Q_d + Q_w \tag{4}$$

$$Q_s = 0.005 + 0.075 \Rightarrow Q_s = 0.08 \text{ m}^3/\text{s} \tag{5}$$

Efficiency of the ram,

$$\eta = \frac{Q_d(H_2 + h_{fd})}{Q_s(H_1 - h_{fs})} \tag{6}$$

Since, frictional loss in the supply pipe is negligible, therefore, $h_{fs} = 0$

$$\eta = \frac{0.005 \times (20 + 4.4389)}{0.08 \times (4 - 0)}$$

$$\eta = 0.38186 = 38.186\% \tag{7}$$

EXAMPLE 10.9 A hydraulic ram receives water from a source under a head of 7 m and delivers 9 litres/s to a reservoir 18 m above the ram. The delivery pipe is 100 m long of diameter 100 mm whereas supply pipe is 12 m long of diameter 200 mm. The ratio of the water lifted to water wasted by the ram is 1 : 10. Calculate the efficiency of the ram (a) Assuming a friction factor $f = 0.01$ for both the pipes, and (b) If the friction in the pipes is neglected.

Solution

Given: H_1 = 7m, Q_d = 9 litres/s = 0.009 m^3/s, H_2 = 18m, L_d = 100 m, D_d = 100 mm = 0.1 m, L_s = 12 m, D_s = 200 mm = 0.2 m, $\frac{Q_d}{Q_w} = \frac{1}{10}$, $f = 0.01$

Area of the delivery pipe,

$$A_d = \frac{\pi}{4} D_d^2 \Rightarrow A_d = \frac{\pi}{4} \times 0.1^2$$

$$A_d = 0.007854 \text{ m}^2 \tag{1}$$

$$C_d = \frac{Q_d}{A_d} \Rightarrow C_d = \frac{0.009}{0.007854}$$

$$C_d = 1.1459 \text{ m/s} \tag{2}$$

$$h_{fd} = 4f \frac{L_d}{D_d} \frac{C_d^2}{2g} \Rightarrow h_{fd} = 4 \times 0.01 \times \frac{100}{0.1} \times \frac{1.1459^2}{2 \times 9.81}$$

$$h_{fd} = 2.677 \text{ m} \tag{3}$$

$$\because \quad \frac{Q_d}{Q_w} = \frac{1}{10} \Rightarrow Q_w = 10Q_d \Rightarrow Q_w = 10 \times 0.009$$

$$Q_w = 0.09 \text{ m}^3/\text{s} \tag{4}$$

$$Q_s = Q_d + Q_w \Rightarrow Q_s = 0.009 + 0.09$$

$$Q_s = 0.099 \text{ m}^3/\text{s} \tag{5}$$

$$A_s = \frac{\pi}{4}D_s^2 \Rightarrow A_s = \frac{\pi}{4} \times 0.2^2$$

$$A_s = 0.031416 \text{ m}^2 \tag{6}$$

$$C_s = \frac{Q_s}{A_s} \Rightarrow C_s = \frac{0.099}{0.031416}$$

$$C_s = 3.1513 \text{ m/s} \tag{7}$$

$$h_{fs} = 4f\frac{L_s}{D_s}\frac{C_s^2}{2g} \Rightarrow h_{fs} = 4 \times 0.01 \times \frac{12}{0.2} \times \frac{3.1513^2}{2 \times 9.81}$$

$$h_{fs} = 1.12476 \text{ m} \tag{8}$$

(a) Efficiency Considering Friction

$$\eta = \frac{Q_d(H_2 + h_{fd})}{Q_s(H_1 - h_{fs})} \tag{9}$$

$$\eta = \frac{0.009 \times (18 + 2.677)}{0.099 \times (7 - 1.12476)}$$

$$\eta = 0.351935 = 35.1935\% \tag{10}$$

(b) Efficiency Neglecting Friction

$$\eta = \frac{Q_d H_2}{Q_s H_1} \tag{11}$$

$$\eta = \frac{0.009 \times 18}{0.099 \times 7}$$

$$\eta = 0.23377 = 23.377\% \tag{12}$$

EXAMPLE 10.10 A hydraulic lift is required to lift a load of 12 kN through a height of 18 m once in every 2 min. If the speed of the lift is 0.75 m/s, determine (a) power required to drive the lift, (b) working period of the lift, and (c) idle period of the lift.

Solution

Given: W = 12 kN = 12000 N, H = 18 m, t = 2 min = 120 s, C_l = 0.75 m/s

(a) Power Required to Drive the Lift

Work done in lifting the load,

$$\text{Work done} = WH \Rightarrow \text{Work done} = 12 \times 18$$

$$\text{Work done} = 216 \text{ kN} \cdot \text{m} \tag{1}$$

Power required to drive the lift,

$$P = \frac{\text{Work done}}{\text{Time}} = \frac{216}{120}$$

$$P = 1.8 \text{ kW} \tag{2}$$

(b) Working Period of the Lift

$$\text{Working period of the lift} = \frac{H}{C_l} = \frac{18}{0.75}$$

$$\text{Working period of the lift} = 24 \text{ s} \tag{3}$$

(c) Idle Period of the Lift

Idle period of lift = Total Time – Working period of lift = 120 – 24

$$\text{Idle period of lift} = 96 \text{ s} \tag{4}$$

EXAMPLE 10.11 A hydraulic lift is required to raise a load of 200 kN through a height of 15 m once in every 2 min. The speed of the lift is 0.75 m/s. Lift is operated from an accumulator which is being continuously charged by a pump. The pressure of the water is 3000 kPa. The efficiency of the lift is 80% and that of the pump is 85%. Determine the power required to run the pump and minimum capacity of the accumulator. Neglect all other losses in the system.

Solution

Given: $W = 200 \text{ kN} = 2 \times 10^5 \text{ N}$, $H = 15$ m, $t = 2 \text{ min} = 120$ s, $C_l = 0.75$ m/s, $p = 3000 \text{ kPa} = 3 \times 10^6$ *Pa*, $\eta_l = 80\% = 0.8$, $\eta_p = 85\% = 0.85$

Work done by water (which is supplied from accumulator and pump) in raising the lift per second, i.e. useful power,

$$P_{th} = WC_l = 2 \times 10^5 \times 0.75$$

$$P_{th} = 150000 \text{ W} = 150 \text{ kW} \tag{1}$$

Actual power supplied to the lift,

$$P = \frac{P_{th}}{\eta_l} \Rightarrow P = \frac{150}{0.8}$$

$$P = 187.5 \text{ kW} \tag{2}$$

$$\text{Time taken to lift the weight to 15 m height} = \frac{H}{C_l} = \frac{15}{0.75} = 20\ s \tag{3}$$

$$\text{Energy spent when the lift raises the weight by 15 m} = 187.5 \times 20 = 3750 \text{ kJ} \tag{4}$$

The pump supplies this energy in every 120 s.

$$\text{Continuous power transferred by the pump to the fluid} = \frac{3750}{120} = 31.25 \text{ kW}$$

$$P_p = \text{Power required to run the pump} = \frac{31.25}{0.85} = 36.765 \text{ kW} \tag{5}$$

$$\text{Idle period of lift} = 120 - 20 = 100 \text{ s} \tag{6}$$

Thus, during idle period of lift, the energy will be stored in the accumulator and during working period of the lift of 20s, the energy will be supplied by the accumulator to the lift.

$$\text{Energy supplied to recharge the accumulator} = \text{Output of the pump} \times \text{Idle period}$$

$$E_s = \text{Energy supplied to recharge the accumulator} = 31.25 \times (120 - 20) = 3125 \text{ kJ}$$

If C is the capacity of the accumulator and p is its pressure, then,

$$pC = E_s \Rightarrow C = \frac{E_s}{p} \Rightarrow C = \frac{3125}{3000}$$

$$C = 1.0417 \text{ m}^3 \tag{7}$$

Check for Energy Balance

Energy supplied by the accumulator = 3125 kJ
Energy supplied by the pump during operation of the lift = 31.25 × 20 = 625 kJ
Total energy supplied = 3125 + 625 = 3750 kJ = energy spent in raising the weight

EXAMPLE 10.12 Water is supplied to a hydraulic crane at a pressure of 650 kPa for lifting a load through a height of 10 m. The diameter of the ram is 180 mm and velocity ratio is 6. The efficiency of the crane is 65%. Determine (a) the weight lifted by the crane, and (b) the quantity of water needed to lift the load.

Solution

Given: $p = 650$ kPa $= 6.5 \times 10^5$ Pa, $H = 10$m, $D = 180$ mm $= 0.18$ m, Velocity ratio = 6, $\eta = 65\% = 0.65$
Area of the fixed ram is,

$$A = \frac{\pi}{4}D^2 \Rightarrow A = \frac{\pi}{4} \times 0.18^2$$

$$A = 0.02545 \text{ m}^2 \tag{1}$$

$$\text{Pressure force on the ram} = pA = 6.5 \times 10^5 \times 0.02545$$

$$\text{Pressure force on the ram} = 16.5425 \text{ kN} \tag{2}$$

$$\text{Velocity ratio} = \frac{\text{Distance moved by the load (weight)}}{\text{Distance moved by the force}} = 6 \tag{3}$$

(a) Weight Lifted by Crane

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Weight} \times \text{Distance moved by the weight}}{\text{Pressure Force} \times \text{Distance moved by the force}} \tag{4}$$

$$= \frac{\text{Weight}}{\text{Pressure Force}} \times \text{Velocity ratio}$$

$$0.65 = \frac{\text{Weight}}{16.5425} \times 6$$

$$W = 1.792 \text{ kN} \tag{5}$$

(b) Quantity of Water Required to Lift Load

Distance moved by the pressure force is the stroke of the ram in lifting the load.

$$\text{Velocity ratio} = \frac{\text{Distance moved by the weight}}{\text{Distance moved by the force}} \quad (6)$$

$$= \frac{\text{Distance moved by the weight}}{\text{Stroke of the ram}}$$

$$6 = \frac{10}{L} \Rightarrow L = 1.67 \text{ m} \quad (7)$$

Therefore, quantity of water (volume) required to lift the weight,

V = Area of the ram × Stroke of the ram = 0.02545 × 1.67

$V = 0.0425 \text{ m}^3 = 42.5$ litres (8)

EXAMPLE 10.13 A hydraulic crane is required to lift a weight of 15 kN through a height of 10 m with a speed of 15 m/min in every two minutes. The efficiency of the crane is 65% and water is supplied to the crane at a pressure of 5000 kPa. The crane is fed from an accumulator to which water is supplied by a pump. Find (a) the capacity of cylinder of the jigger, (b) capacity of the accumulator, and (c) minimum power input for the pump.

Solution

Given: W = 15 kN, H = 10m, speed of weight lifting = 15 m/min, t = 2 min, η = 65% = 0.65, p = 5000 kPa

(a) Capacity of Cylinder of Jigger

Output of crane = Weighted lifted × Height = 15 × 10 = 150 kJ (1)

Input of crane = Work done by water on ram

= Force on the ram × Distance travelled by ram

Input of crane = $pAL = p$ × Volume of the cylinder (2)

Input of crane = 5000 × Volume of the cylinder (3)

Efficiency of the crane,

$$\eta = \frac{\text{Output of crane}}{\text{Input of crane}} \Rightarrow 0.65 = \frac{150}{5000 \times V}$$

Capacity of the Jigger Cylinder = $V = 0.04615 \text{ m}^3$ = 46.15 litres (4)

(b) Capacity of the Accumulator

Input of the crane = 5000 × 0.04615

Input of the crane = 230.5 kN.m (5)

This input is given to the crane once in every two minutes.

$$\text{Input to the crane/min} = \frac{230.5}{2} = 115.25 \text{ kJ/min} \quad (6)$$

The weight 15 kN is raised to a height of 10 m with a speed of 15m/min

$$\text{Time required to lift the weight through a height of 10 m} = \frac{H}{\text{Speed}} = \frac{10}{15} = \frac{2}{3}\text{ min} \tag{7}$$

Work done by the pump during raising of weight = Input to the crane/min × Time to lift

$$\text{Work done by the pump during lifting} = 115.25 \times \frac{2}{3} = 76.833\text{ kJ} \tag{8}$$

Energy supplied by the accumulator = Total input energy to the crane
– Work done by pump during lifting

$$\text{Energy supplied by the accumulator} = 230.5 - 76.833 = 153.667\text{ kJ} \tag{9}$$

Energy supplied by the accumulator = Force on the ram of accumulator × Lift of ram

$$\begin{aligned}\text{Energy supplied by the accumulator} &= p \times A \times H \\ &= p \times \text{Capacity (Volume) of the accumulator}\end{aligned} \tag{10}$$

153.667 = 5000 × Capacity of the accumulator

$$\text{Capacity of the accumulator} = 0.030733\text{ m}^3 = 30.733\text{ litres} \tag{11}$$

(c) Minimum Power Input for the Pump

$$\text{Minimum power input for the pump} = \frac{\text{Work input to the crane/min}}{60} = \frac{115.25}{60}$$

$$\text{Minimum power input for the pump} = 1.921\text{ kW} \tag{12}$$

10.8 Hydrodynamic Systems

Hydrodynamics deals with the mechanics of moving fluid and hydrodynamic systems use fluid motion to transmit power. The objective of a hydrodynamic system is to transmit power and the required effect is obtained primarily by virtue of changes in the kinetic energy of the flow of the working media and the changes of pressure are avoided as far as possible. A hydrodynamic transmitter consists essentially of a centrifugal pump impeller mounted on the driving shaft and an oil turbine or runner mounted on driven shaft. Power is transmitted from the driving to driven shaft through circulation of oil between the impeller and runner. A rotodynamic, i.e. non-positive displacement pump is used in hydrodynamic systems. The relative spatial position of the prime mover (e.g., turbine) is fixed. Hydraulic coupling and torque converter are the examples of hydrodynamic systems.

10.8.1 Analysis of Hydrodynamic Transmission

The schematic diagram of a hydrodynamic transmission system is shown in Figure 10.15.

The power input shaft (driver) is shaft S_1 on which an impeller P is mounted. The oil enters the impeller at A and leaves at B. The oil then enters the blades of casing C. These stator blades guide the oil at the designated angle to the inlet of the turbine blades T at D. The energy of the oil is absorbed by the blades of the turbine and then the oil leaves the turbine runner at E. The oil is now ready to enter the inlet of the

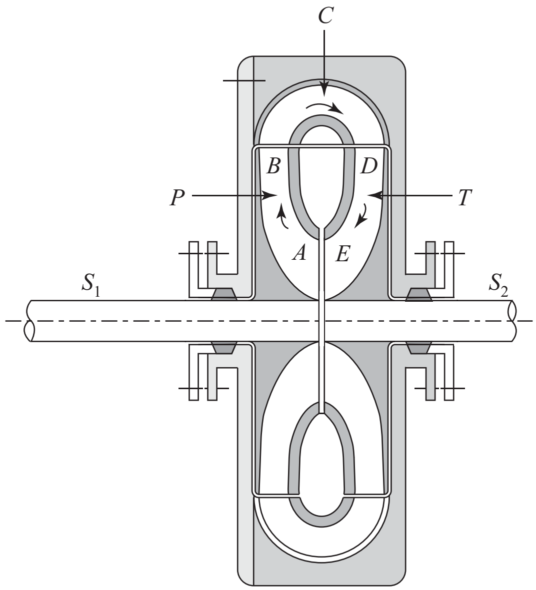


Figure 10.15 *Schematic Diagram of a Hydrodynamic Transmission System*

pump and the process is repeated continuously. For clearer and better understanding of the device, impeller P is shaped like that of a centrifugal pump and the turbine runner, like that of a Francis turbine shown in Figure 10.15. Different orientations of the stator blades can vary the speed of the driven shaft resulting in different values of the output torque. Alternately, for equal torques, the stator blades may be completely absent and the blades of the impeller and runner can be shaped such that to make the flow of the fluid from the impeller directly to the inlet of the turbine blades.

Assume that the speed of the input shaft is N_1 and that of output shaft is N_2. The speed ratio N_r is given by,

$$N_r = \frac{N_2}{N_1} \tag{10.21}$$

If the torques on the input and output shaft be T_1 and T_2 respectively then the torque ratio, T_r is,

$$T_r = -\frac{T_2}{T_1} \tag{10.22}$$

The input torque is negative and therefore the torque ratio becomes positive. The efficiency of the hydrodynamic transmission is the ratio of the output power and input power. Therefore,

$$\eta = \frac{P_2}{P_1} = \frac{2\pi N_2 T_2}{2\pi N_1 T_1} \Rightarrow \eta = N_r T_r \tag{10.23}$$

Since $\eta < 1$ always, the following conclusions can be drawn from Eq. (10.23):

(i) If $T_r = 1$, then, $N_r < 1$.

(ii) If $N_r = 1$ then, $T_r < 1$.

These conclusions are drawn on the basis of assumption that there are losses, and therefore, $\eta < 1$, with a consequent reduction of either the speed or the torque on the output shaft. The losses can be visualised as the power losses or energy losses. A general energy balance can be written as,

Energy input – Energy loss in dissipation = Energy output

or in the rate form as,

Power input – Power losses = Power output

$-2\pi N_1 T_1$ – Power losses $= 2\pi N_2 T_2$

The power losses in the above equation can be written in the same form as other terms, therefore,

$$-2\pi N_1 T_1 - 2\pi N T_l = 2\pi N_2 T_2$$

$$-N_1 T_1 - N T_l = N_2 T_2 \tag{10.24}$$

where, T_l is the loss of torque. The loss of torque may be associated with a speed that may be either the input speed or output speed. Assume that N in Eq. (10.24) is N_1, i.e. input speed. Therefore,

$$-N_1 T_1 - N_1 T_l = N_2 T_2 \tag{10.25}$$

Substituting value of $N_2 T_2$ from Eq. (10.25) into Eq. (10.23), we get,

$$\eta = \frac{N_2 T_2}{-N_1 T_1} = \frac{-N_1 T_1 - N_1 T_l}{-N_1 T_1}$$

$$\eta = \frac{T_1 + T_l}{T_1} \tag{10.26}$$

If N is N_2 in Eq.(10.24), similarly it can be shown that,

$$\eta = \frac{T_2}{T_l + T_2} \tag{10.27}$$

In Eqs. (10.24), (10.26) and (10.27), the loss of torque is due to dissipation of energy in the fluid. The increase or decrease of torque can also be designed to get a torque multiplier or torque divider as required. This is achieved by introducing stationary stator (reaction member) blades with different outlet angles. Hence, Eqs. (10.24), (10.26) and (10.27) require some revisions. In such a case, the casing or stator also participates in the torque transmission process. Therefore, in addition to the input and output torques, now there is another torque T_{reaction} of the reaction member (stator). A balance of torques gives the following equation,

$$T_1 + T_2 + T_{\text{reaction}} = 0$$

$$-\frac{T_2}{T_1} = 1 + \frac{T_{\text{reaction}}}{T_1} \Rightarrow T_r = 1 + \frac{T_{\text{reaction}}}{T_1} \tag{10.28}$$

Following are the noteworthy points of Eq. (10.28),

- $T_r = 1 \Rightarrow T_{\text{reaction}} = 0$, the stator blades do not participate in the transmission process. This is a simple coupling.
- $T_r > 1 \Rightarrow \frac{T_{\text{reaction}}}{T_1} > 0$. This is the case of torque multiplier. Since T_1 is negative, T_{reaction} must also be negative to get $T_r > 1$. This implies that the stator blades should exert a torque in the same direction as that of the input torque (torque on the driver shaft or torque on the impeller shaft).

- $T_r < 1 \Rightarrow \frac{T_{\text{reaction}}}{T_1} < 0$. This is the case of torque divider. Since T_1 is negative, T_{reaction} must also be positive to get $T_r < 1$. This implies that the stator blades should exert a torque in the opposite direction as that of the input torque (torque on the driver shaft or torque on the impeller shaft), i.e. in the same direction as that a turbine or driven shaft.

With the above background, the fluid coupling and torque converter are studied in the following sub sections.

10.8.2 Fluid Coupling

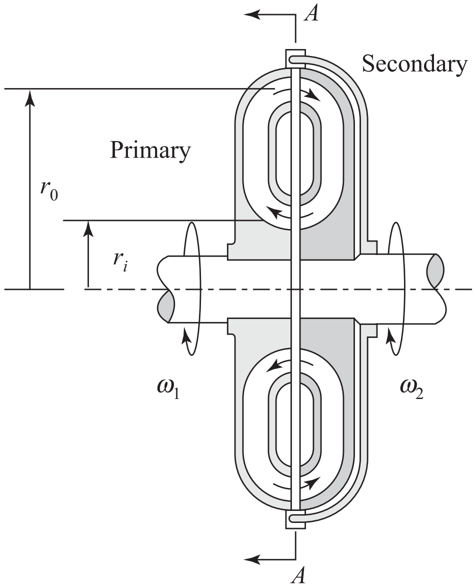
Fluid coupling is a device which is used to transmit power with the same torque on the driving and driven shaft with fluid as working medium. The purpose of fluid coupling is to act as flexible power transmitting coupling. The essential features of the coupling are illustrated in Figure 10.16.

In a fluid coupling, a pump impeller is driven by an input shaft. The impeller of the pump is closely coupled to a turbine runner which transmits the torque to an output shaft coaxial with the input shaft. The fluid used is hydraulic oil. A cooling system is generally provided with the device to dissipate the heat generated. In a typical fluid coupling used, for example, in a propulsion system of ship, the pump and turbine are mounted back to back with little separation between the leading and trailing edges of the two impellers.

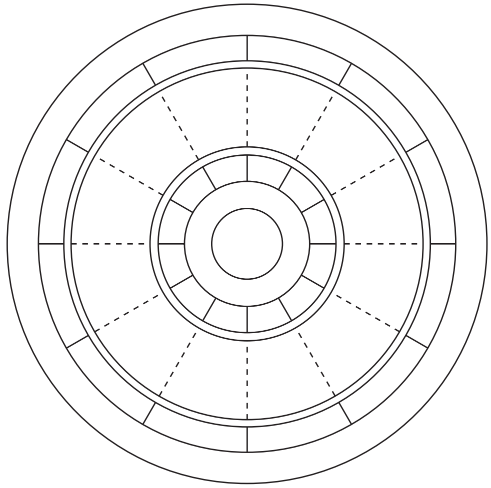
The primary and secondary runners are the only elements involved. The primary runner is an impeller of a centrifugal pump which is driven by an electric motor or an IC engine and secondary runner is the radial reaction turbine runner. There is no mechanical connection between the driving and driven shaft. Generally, fluid used is an oil of low viscosity e.g. ordinary mineral oil which flows directly from one runner (primary) to the other (secondary) without passing through any intervening stationary passages. The casing is generally fastened to one of the runners and rotates with it, otherwise the two runners are otherwise similar. Straight radial blades are used with each runner which has a similar appearance of half a grapefruit with the pulp removed from the segments. If the driver shaft is allowed to rotate, the oil will pass through the impeller blades (primary runner or driver), spun round and will flow radially outwards with higher energy by centrifugal force. However, since the oil is being carried round with the rotating impeller (driver), it is thrown into the driven member. The fluid (oil) will thus strike the turbine runner blades (secondary runner or driven) at an angle while flowing radially inwards, transfer power or torque to the turbine runner (driven) due to transfer of kinetic energy. Flow occurs in this direction because the speed of the primary is greater than that of the secondary. The head produced in the primary is thus greater than the centrifugal head resisting flow through the secondary. If the driver and driven rotate at the same speed, the heads would balance each other and the circulation of oil cannot take place, and no torque would be transmitted.

The inlet angles of the runners are set such that the flow from the impeller enters without any shock. The oil in the turbine moves radially inwards, i.e. towards the shaft of the runner, from where it flows to the pump to complete a closed circuit. Since the discharge and the change in velocity vector are numerically same in both the pump impeller and turbine runner, the torque of the driver (input) and driven (output) shafts of a fluid coupling are same at all speeds if the friction and other losses are neglected.

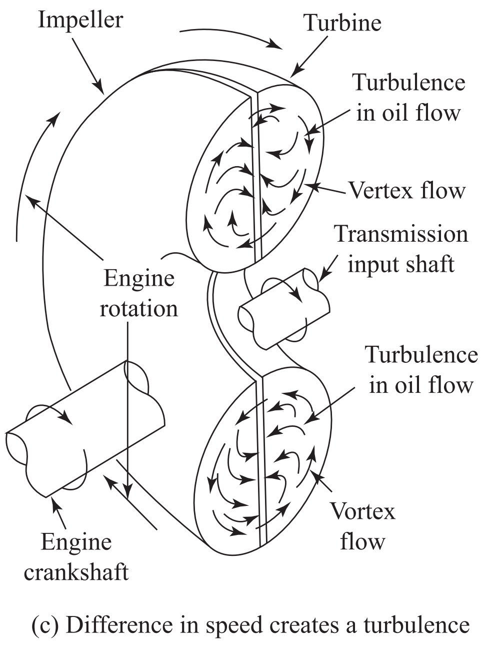
As already discussed, there would be no flow of oil if the speed of both the pump impeller and turbine runner are same. However, due to fluid friction and turbulence effects, the angular velocity of the driven shaft (output) ω_2 will be a little less than that of the driving shaft (input), ω_1. The ratio, $S = \left(\frac{\omega_1 - \omega_2}{\omega_1}\right)$, is known as *slip*. The slip is generally very small being about 2–3% at peak speed. The decrease of the speed of the



(a) Assembly



(b) Section *A-A*



(c) Difference in speed creates a turbulence

Figure 10.16 *Fluid Coupling*

driven shaft is necessary to maintain continuous flow of oil from the impeller to the runner. Consequently, the power loss is very small and the torque ratio is very close to 1. The condition at which the speeds of the driving and driven shafts are the same is known as *stall*. At the stall value, the efficiency drops down to zero very rapidly.

(a) Velocity Triangles

The interacting faces of the two elements pump impeller and turbine runner of a fluid coupling guide the oil flow in the axial direction. The flow from the pump to the turbine is at a larger radius. The back flow from the turbine to the pump is at a smaller radius. The velocity triangles can be drawn corresponding to these two elements in order to determine the transmission of the torques or power. The velocity triangles are drawn on a plane parallel to the axis as the flow is parallel to the axis. The velocity triangles are drawn with reference to the respective machines. It is quite possible that the outlet of the pump impeller and the inlet of the turbine runner may be in the radial direction or in the axial direction. But these possibilities do not alter the analysis of the fluid coupling as the torque or power transmission depend on the whirl components of the fluid velocities. The additional subscripts p and t represent the pump and turbine respectively in spite of the symbols and subscripts used throughout the text of this book. It is to be kept in mind that just as there are different designs of the pumps or turbines with varying parameters, D_1, D_2, β_1, β_2, α_1, α_2, etc., there are various designs of fluid coupling. The power input to the pump must be higher than the power output of the turbine unit.

The general velocity triangles for a fluid coupling are shown in Figure 10.17. It is to be noted that:

(i) The speed of the turbine shaft is less than that of the pump shaft. Hence, $C_{b1t} < C_{b2p}$, at the same radius, i.e. the mean radius at the exit of the pump or the inlet of the turbine, $r_{2p} = r_{1t}$.
(ii) Both C_{b1t} and C_{b2p} are at a larger radius compared to the velocities at the other ends, i.e. at a smaller radius, $r_{1p} = r_{2t}$.
(iii) The flow velocities are the same, i.e. $C_{f1t} = C_{f2p}$.
(iv) Similarly, $C_{b2t} < C_{b1p}$ at the same radius.
(v) Both C_{b2t} and C_{b1p} are at a smaller radius.
(vi) The flow velocities are the same, i.e. $C_{f1p} = C_{f2t}$.

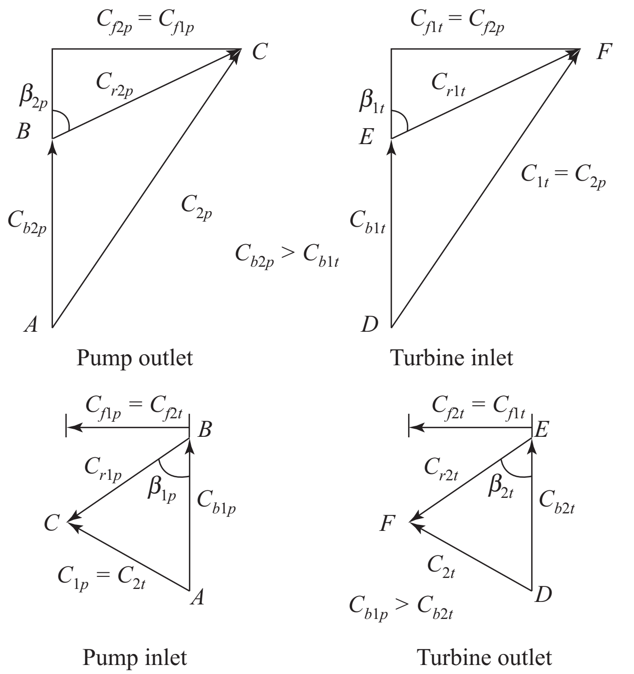


Figure 10.17 *Velocity Triangles for Fluid Coupling*

(b) Analysis

Let T_1 be the input torque or pump torque and ω_1 the angular velocity of the input shaft (driver), then the power input to the coupling is,

$$P_1 = T_1\omega_1 \tag{10.29}$$

Similarly, the output power or turbine power is,

$$P_2 = T_2\omega_2 \tag{10.30}$$

where T_2 is the output torque or turbine torque and ω_2 is the angular velocity of the output shaft (driven).

The efficiency of the coupling is the ratio of the power output to the power input which is given by,

$$\eta_c = \frac{P_2}{P_1} = \frac{T_2\omega_2}{T_1\omega_1} \tag{10.31}$$

If the frictional resistances at the surfaces of the impeller and rotor are neglected as being infinitesimal, the torques of the input shaft (driver) and the output shaft are equal and hence the efficiency in this case would be the ratio of angular velocities, $\frac{\omega_2}{\omega_1}$.

Since the slip is $S = \left(\frac{\omega_1 - \omega_2}{\omega_1}\right)$, then the efficiency is related to slip as,

$$\eta_c = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = 1 - S \tag{10.32}$$

The efficiency is less than 100% because of two reason. First is that the energy dissipation occurs by friction because the fluid moves relative to the solid surfaces. Secondly, additional losses occur as the fluid from one runner strikes the blades of the other runner moving at slightly different velocity. Although such losses could be reduced by rounding the inlet edges of the blades, such a refinement may not justify the additional expense.

The Euler equations derived for pump and turbine rotors apply to the elements of the coupling. Specific work of the turbine is,

$$w_t = (C_{w1t}C_{b1t} - C_{w2t}C_{b2t})$$

If zero whirl slip is assumed and since radial blades are used in both the pump impeller and turbine runner, then the whirl velocity of the fluid at inlet of the turbine runner (driven or secondary) is equal with the blade velocity of the pump impeller (primary or driver) at that radius, i.e. $C_{w1t} = C_{b2p} = \omega_1 r_2$. Moreover, C_{w2} for the turbine is equal to the blade velocity at outlet of the turbine, i.e. $C_{w2t} = C_{b2t} = \omega_2 r_1$. Therefore,

$$w_t = (\omega_1\omega_2 r_2^2 - \omega_2^2 r_1^2) \tag{10.33}$$

Since $w_t = T_2\omega_2$, therefore, torque on the turbine shaft or driven shaft is,

$$T_2 = (\omega_1 r_2^2 - \omega_2 r_1^2) = (C_{b2p} r_2 - C_{b2t} r_1) \tag{10.34}$$

Similarly, for pump impeller or primary, specific work,

$$w_p = (C_{w2p}C_{b2p} - C_{w1p}C_{b1p})$$

Since the blades on pump impeller are radial, therefore, $\beta_{2p} = 90° \Rightarrow C_{w2p} = C_{b2p} = \omega_1 r_2$ and $C_{w1p} = C_{b2t} = \omega_2 r_1$. Therefore, specific pump work,

$$w_p = (\omega_1^2 r_2^2 - \omega_1\omega_2 r_1^2) \tag{10.35}$$

Since $w_p = T_1\omega_1$, therefore, torque on the pump shaft or driver shaft is,

$$T_1 = (\omega_1 r_2^2 - \omega_2 r_1^2) = (C_{b2p} r_2 - C_{b2t} r_1) \tag{10.36}$$

The difference in the specific turbine work (output) and specific pump work, as represented by Eqs. (10.33) and (10.35), is the energy dissipated per kg mass of fluid and, as the flow is highly turbulent, is proportional to Q^2, where Q represents the discharge round the circuit.

In practice, however, there is a significant variation of radius, and therefore of blade velocity, across the inlet and outlet sections of both runners. Moreover, the rate at which fluid passes from one runner to the other varies with the radius in a manner not readily determined. Consequently, Eqs. (10.33) and (10.35) can be regarded as no better than first approximations to the truth even if mean values are used for r_2 and r_1 .

(c) Characteristics

A graph is plotted as shown in Figure 10.18 using Eq. (10.31) or (10.32) to study the variation of efficiency with the angular velocity ratio, $\dfrac{\omega_2}{\omega_1}$, which is same as the speed ratio $\dfrac{N_2}{N_1}$. The efficiency is zero when the speed ratio is zero and increases linearly up to a value of $\dfrac{\omega_2}{\omega_1} = \dfrac{N_2}{N_1} \approx 0.95$ and then the efficiency drops to zero value at a speed ratio of 1. Hence, at the stall value, the efficiency drops down to zero very rapidly.

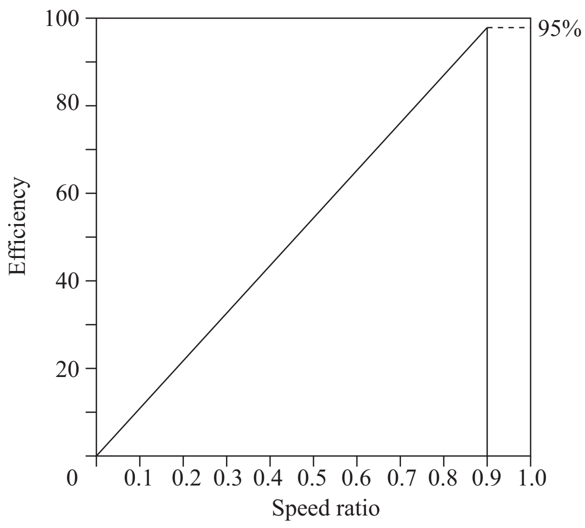


Figure 10.18 *Variation of Efficiency of Hydraulic Coupling with Speed ratio*

The torque coefficient, C_T, and power coefficient, C_P, of a hydraulic coupling can be found by dimensional analysis as follows:

$$C_T = f(S) = \frac{T}{\rho N_1^2 D^5} \tag{10.37}$$

$$C_P = f(S) = \frac{P}{\rho N_1^3 D^5} \tag{10.38}$$

where D is the diameter of the impeller. These two non-dimensional coefficients, C_T and C_P, enable the scaling up of parameters for homologous fluid couplings. Following are the noteworthy points that can be drawn from Eqs. (10.37) and (10.38):

(i) Torque is directly proportional to the square of the speed of rotation of the impeller.
(ii) Power is directly proportional to the cubic power of the speed of rotation of the impeller.
(iii) Both power and torque are directly proportional to the density of the liquid used in the fluid coupling.
(iv) Fluid couplings are bi-directional and can be used in either direction of rotation.

Low viscosity fluids are generally preferred in the fluid coupling. Physical properties of the oil like density and viscosity decides the operation characteristics of the coupling. For example, increasing the density of the oil increases the torque that can be transmitted at a given speed. SAE 10, SAE 10w, oils are used in fluid coupling.

Figure 10.19 shows the general characteristics of a coupling. When secondary shaft is locked, the torque transmitted is known as *stall torque*. The maximum speed at which the primary can rotate under conditions of stall is given by the intersection of the curves of stall torque and input torque. For motorcar engines, this speed is of the order of 100 rad/s (16 rev/s).

Fluid couplings are very widely used in diesel locomotives, automobiles, aviation and marine machinery and agriculture machinery. A fluid coupling used in a motorcar transmission is normally incorporated in the engine flywheel. It is then loosely known as a fluid flywheel.

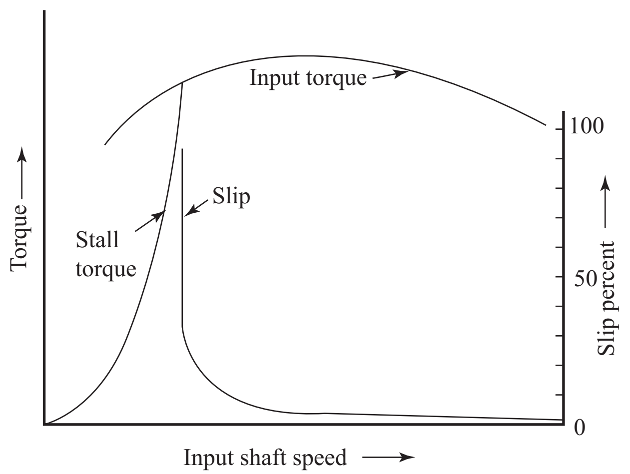


Figure 10.19 *Torque and Slip versus Input Shaft Speed for a Hydraulic Coupling*

(d) Advantages

- It provides acceleration pedal control to regulate automatic disengagement of drive to gearbox at a particular speed.
- Engine vibrations are not transmitted to wheels. Similarly, there is no effect on the engine from shock loads of transmission side.
- No adjustments are to be made as there is no wear on moving parts.
- When gear engages, no jerks are transmitted. It damps all shocks and strains incident with connecting a revolving engine to transmission.

- Vehicle can be stopped in gear and move off by pressing acceleration only.
- Engines and wheels are not connected directly. Therefore, in case of engine overloading, it will not stop. But, it results in slip within coupling.
- Slip in the coupling does not cause damage to working components as in the case of friction clutches.

EXAMPLE 10.14 The torque on the input shaft of a fluid coupling is 20 N·m at a speed of 1500 rpm Calculatethe output torque and efficiency of the coupling if the slip is 4%.

Solution

Given: T_1 = 20 N·m, N_1 = 1500 rpm, S = 4% = 0.04

1. Efficiency of the Coupling

Efficiency of the coupling,

$$\eta_c = 1 - S \tag{1}$$

$$\eta_c = 1 - 0.04$$

$$\eta_c = 0.96 = 96\% \tag{2}$$

2. Output Torque

We know that,

$$S = \frac{\omega_1 - \omega_2}{\omega_1} = \frac{N_1 - N_2}{N_1} \tag{3}$$

∴ $$N_2 = (1 - S)\, N_1 = (1 - 0.04) \times 1500$$

$$N_2 = 1440 \text{ rpm} \tag{4}$$

$$\omega_1 = \frac{2\pi N_1}{60} \Rightarrow \omega_1 = \frac{2\pi \times 1500}{60}$$

$$\omega_1 = 157.08 \text{ rad/s} \tag{5}$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 1440}{60}$$

$$\omega_2 = 150.8 \text{ rad/s} \tag{6}$$

Efficiency of the coupling is,

$$\eta_c = \frac{P_2}{P_1} = \frac{T_2 \omega_2}{T_1 \omega_1} \tag{7}$$

$$0.96 = \frac{T_2 \times 150.8}{20 \times 157.08}$$

$$T_2 = 20 \text{ N.m} \tag{8}$$

EXAMPLE 10.15 The mean diameter at inlet and outlet of the pump in a fluid coupling are 0.18 m and 0.25 m. The oil of specific gravity 0.8 enters the impeller without any whirl and the areas of flow remain constant throughout. The blade angle at the inlet is 45° and blades are at 90° to the blade velocity at the outlet. The width of the blades at outlet is 15 mm. The speed of the driver is 1440 rpm whereas that of driven is 1350 rpm. Calculate (a) mass flow rate of the oil, (b) torque transmitted, (c) power lost to cooling system, (d) efficiency of the coupling, (e) angles of the turbine blades at the inlet and outlet, and (f) absolute velocity of the fluid from the pump to the turbine and its direction.

Solution

Given: D_1 = 0.18 m, D_2 = 0.25 m, s = 0.8, C_{w1p} = 0, β_{1p} = 45°, β_{2p} = 90°, B_{2p} = 15 mm = 0.015 m, N_1 = 1440 rpm, N_2 = 1350 rpm

Area of flow at pump outlet,

$$A_{2p} = \pi D_{2p} B_{2p} \Rightarrow A_{2p} = \pi \times 0.25 \times 0.015$$

$$A_{2p} = 0.01178\ \text{m}^2$$

Since flow area remains constant throughout, therefore,

$$\therefore \quad A_{1p} = A_{2p} = 0.01178\ \text{m}^2 \qquad (1)$$

Peripheral velocity of the pump blades at inlet,

$$C_{b1p} = \frac{\pi D_1 N_1}{60} \qquad (2)$$

$$C_{b1p} = \frac{\pi \times 0.18 \times 1440}{60}$$

$$C_{b1p} = 13.57\ \text{m/s} \qquad (3)$$

Peripheral velocity of the pump blades at outlet,

$$C_{b2p} = \frac{\pi D_2 N_1}{60} \Rightarrow C_{b2p} = \frac{\pi \times 0.25 \times 1440}{60}$$

$$C_{b2p} = 18.85\ \text{m/s} \qquad (4)$$

The velocity triangles of the pump and turbine elements are shown in Figure 10.20. The velocity of the fluid at pump inlet,

$$C_{1p} = C_{f1p} = C_{b1p} \tan \beta_{1p} \qquad (5)$$

$$C_{1p} = C_{f1p} = 13.57 \tan 45$$

$$C_{1p} = C_{f1p} = 13.57\ \text{m/s} \qquad (6)$$

(a) Mass Flow Rate

$$\dot{m} = \rho A_{1p} C_{f1p} = s \rho_w A_{1p} C_{f1p} \qquad (7)$$

$$\dot{m} = 0.8 \times 1000 \times 0.01178 \times 13.57$$

$$\dot{m} = 127.88\ \text{kg/s} \qquad (8)$$

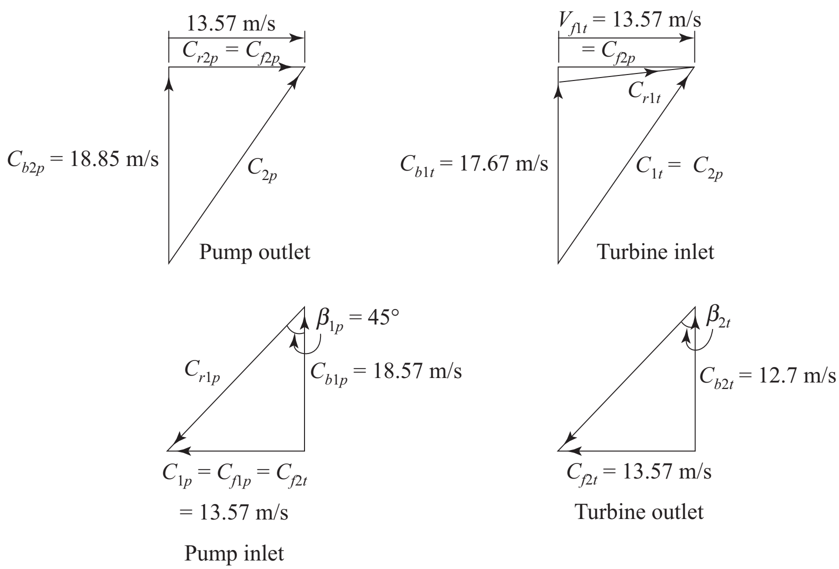


Figure 10.20 *Velocity Triangles for Example 10.15*

(b) Torque Transmitted

Tangential or whirling force at the pump impeller is,

$$F_{wp} = \dot{m}(C_{w1p} - C_{w2p}) \tag{9}$$

Since, blades are at 90° to the blade velocity at the outlet, $C_{w2p} = C_{b2p}$ = 18.85 m/s and $C_{w1p} = 0$ as given,

$$F_{wp} = 127.88 \times (0 - 18.85)$$

$$F_{wp} = -2410.6 \text{ N} \tag{10}$$

Input torque to the pump,

$$T_1 = F_{wp} r_2 = F_{wp} \frac{D_2}{2} = 2410.6 \times \frac{0.25}{2}$$

$$T_1 = 301.326 \text{ N.m} \tag{11}$$

Since the torques on the driving and driven shaft is equal in a fluid coupling, therefore torque transmitted to the turbine rotor is,

$$T_2 = T_1 = 301.326 \text{ N.m} \tag{12}$$

(c) Power Lost to Cooling System

Power input is,

$$P_1 = T_1 \omega_1 = \frac{2\pi N_1}{60} T_1 \tag{13}$$

$$P_1 = \frac{2\pi \times 1440}{60} \times 301.326$$

$$P_1 = 45438.89 \text{ W} \tag{14}$$

Power output, i.e. at turbine,

$$P_2 = T_2\omega_2 = \frac{2\pi N_2}{60}T_2 \tag{15}$$

$$P_2 = \frac{2\pi \times 1350}{60} \times 301.326$$

$$P_2 = 42598.96 \text{ W} \tag{16}$$

The loss in the power is (45438.89 – 42598.96) = 2839.93 W which is lost to the oil in the coupling, consequently oil gets heated up. This heat is required to be dissipated to some cooling system.

(d) Efficiency of the Coupling

$$\eta_c = \frac{P_2}{P_1} \tag{17}$$

$$\eta_c = \frac{42598.96}{45438.89}$$

$$\eta_c = 0.9375 = 93.75\% \tag{18}$$

(e) Turbine Blade Angles at Inlet and Outlet

Blade velocities at turbine inlet and outlet are,

$$C_{b1t} = \frac{\pi D_2 N_2}{60} \Rightarrow C_{b1t} = \frac{\pi \times 0.25 \times 1350}{60}$$

$$C_{b1t} = 17.67 \text{ m/s} \tag{19}$$

$$C_{b2t} = \frac{\pi D_1 N_2}{60} \Rightarrow C_{b2t} = \frac{\pi \times 0.18 \times 1350}{60}$$

$$C_{b2t} = 12.72 \text{ m/s} \tag{20}$$

Output torque, i.e. torque on turbine runner,

$$T_2 = \dot{m}(C_{w1t}r_2 - C_{w2t}r_1) \tag{21}$$

Since the oil enters in the pump impeller without whirl, i.e. no whirl at turbine outlet, hence, $C_{w2t} = 0$.

$\therefore$
$$C_{w1t} = \frac{T_2}{\dot{m}r_2} \Rightarrow C_{w1t} = \frac{301.326}{127.88 \times 0.25/2}$$

$$C_{w1t} = 18.85 \text{ m/s} \tag{22}$$

Since flow area remains constant throughout,

$\therefore$
$$C_{f1p} = C_{f2p} = C_{f1t} = C_{f2t} = 13.57 \text{ m/s}$$

From the velocity triangles of turbine at inlet and outlet as shown in Figure 10.20,

$$\tan\beta_{1t} = \frac{C_{f1t}}{C_{w1t} - C_{b1t}} \tag{23}$$

$$\tan\beta_{1t} = \frac{13.57}{18.85 - 17.67}$$

$$\beta_{1t} = 85.03° \tag{24}$$

$$\tan \beta_{2t} = \frac{C_{f2t}}{C_{b2t}} \tag{25}$$

$$\tan \beta_{2t} = \frac{13.57}{12.7}$$

$$\beta_{2t} = 46.9° \tag{26}$$

(f) Absolute Velocity of Fluid from Pump to Turbine and its Direction

$$C_{2p} = C_{1t} = \sqrt{C_{f2p}^2 + C_{b2p}^2} \tag{27}$$

$$C_{2p} = C_{1t} = \sqrt{13.57^2 + 18.85^2}$$

$$C_{2p} = C_{1t} = 23.23 \text{ m/s} \tag{28}$$

Fluid angle at the outlet of the pump and inlet of turbine will be equal, i.e. $\alpha_{2p} = \alpha_{1t}$. From velocity triangle at pump outlet,

$$\tan \alpha_{2p} = \frac{C_{f2p}}{C_{b2p}} \Rightarrow \tan \alpha_{2p} = \frac{13.57}{18.85}$$

$$\therefore \quad \alpha_{2p} = \alpha_{1t} = 35.75° \tag{29}$$

10.8.3 Torque Converter

A torque converter is a modified form of fluid coupling. The torque converter, similar to a fluid coupling, acts like a mechanical clutch allowing the load to be disengaged from the source of the power. Unlike a fluid coupling, however, torque ratio gets varied automatically in a continuous variable manner in a torque converter when there is a significant difference between the speeds of rotation of driver and driven, thus providing the equivalent of a reduction gear.

A torque converter as shown in Figure 10.21 consists of three main components: (i) A primary runner i.e. a pump impeller connected mechanically to the driving shaft, (b) A secondary runner i.e. a turbine runner connected to the driven shaft, and (c) A stator, usually known as a reaction member, positioned in the middle of the flow from the pump impeller and turbine runner. Thus, the basic difference between a fluid coupling and a torque converter is that the later has a stator i.e. a set of stationary blades in addition to the primary and secondary runners.

A centrifugal pump is used in a torque converter. As it rotates, fluid is thrown outwards, similar to the spin cycle of washing machine which throws water and clothes to the outside of the wash tub. As fluid is pushed outwards, a vacuum is created that draws more fluid in at the centre. The fluid then impinges on the blades of a turbine runner mounted on the driven shaft. The blades of the turbine are curved so that the fluid, which enters the turbine from the outside, has to change direction before it exits the centre of the turbine. Moreover, the blades also accelerate the flow. As the turbine causes the fluid to change direction, the fluid causes the turbine to spin. The fluid exits the turbine at the centre, moving in a different direction than when it entered.

The fluid exits the turbine in a direction opposite to the direction in which the pump (driver) is turning. If the fluid were allowed to hit the pump, it would slow down the driver or primary runner (pump impeller), thus wasting power. This is why a torque converter has a stator. The stator is positioned in the very centre of the pump impeller and turbine runner. Its function is to redirect the fluid returning from the turbine so that

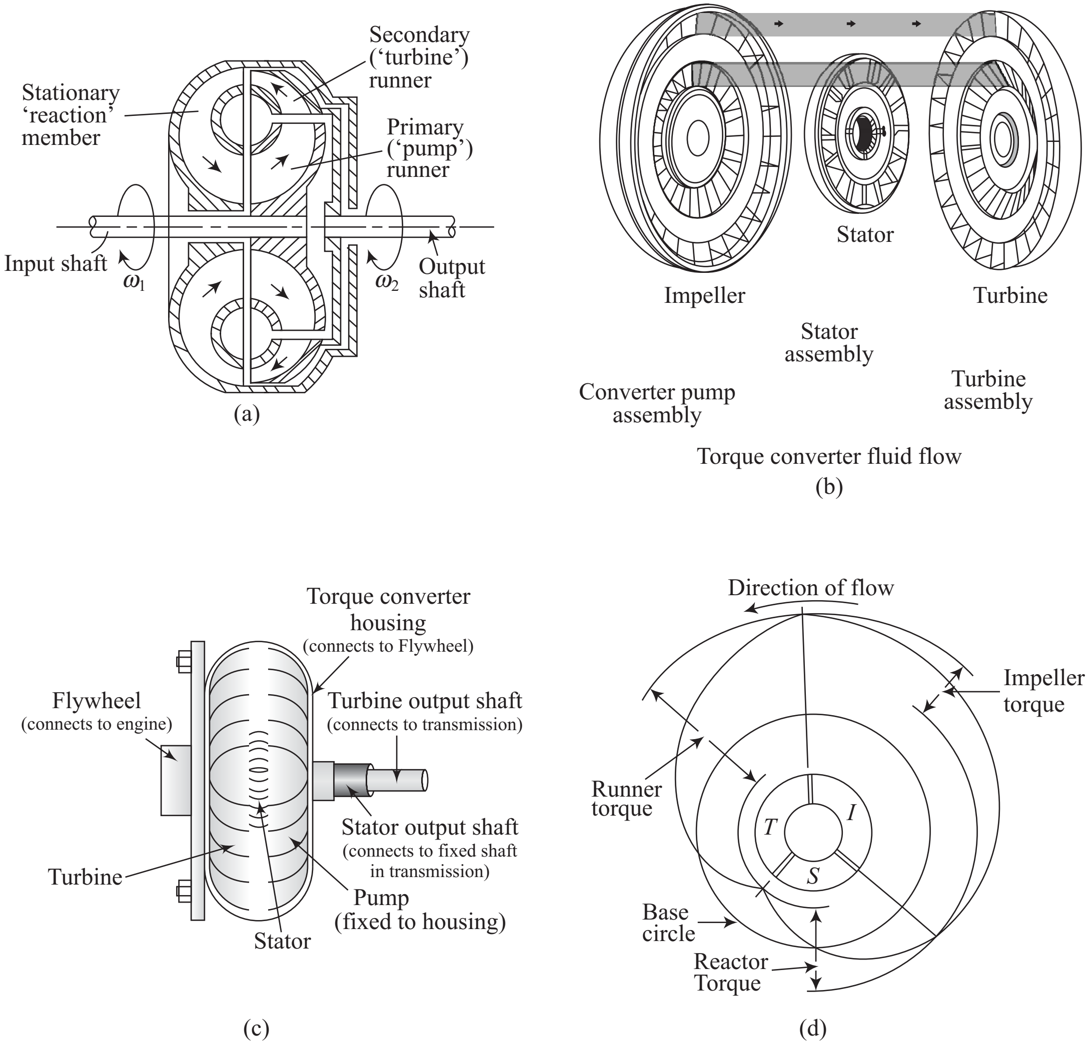


Figure 10.21 *(a) Fluid or Hydraulic Torque Convertor, (b) Direction of Fluid Flow, (c) Blades of Components, and (d) Polar Diagram*

it aids the rotation of the pump instead of impeding it. This will result in the recovery of the energy in the returning fluid and addition to the energy being supplied by the pump itself. This action causes a substantial increase in the mass of the fluid being directed to the turbine, producing an increase in the output torque.

A torque is exerted on the stator blades as stationary blades are designed to change the angular momentum of the fluid passing through them. Therefore, in order to prevent rotation of stator blades, an opposing torque must be applied from the casing/housing. Because of this additional torque on the assembly as a whole, the torque on the turbine runner no longer equals that on the impeller of the pump. The torque on the secondary may be as much as five times the input torque at the primary if stator (reaction) blades are designed appropriately. Since, however, the stator (reaction) blades remains stationary i.e. the do not move, work done on them is zero. Therefore, the power output of the turbine runner is equal to the pump input power minus the power lost in turbulence.

For a greater torque on the driven shaft, the change in angular momentum in the turbine runner should be greater than that in the pump. The stator (reaction) blades are so shaped as to increase the angular momentum of the fluid which is further increased in course of flow through the pump impeller. Thus, the stator contributes to an additional torque over that of the driving shaft (pump shaft). The amplification of torque depends on the design of the stator blades and the speed ratio.

Unlike the straight radial blades used in a fluid coupling, the turbine and stator of a torque converter use angled and curved blades. The shape of the blades is important as even minor variations can cause significant variations in the performance.

Many arrangements of the three elements of a torque converter are possible; one for a single-stage converter is shown in Figure 10.21. For some purposes, particularly where a large torque multiplication is required, more complicated arrangements are used having two or even three turbine stages.

(a) Velocity Diagrams

Velocity diagrams of the torque converter are shown in Figure 10.22. The orientation of the stator blades is different for torque multiplication and torque reduction. The same Figure 10.22 is used corresponding to the two situations for the sake of comparison and to show the difference.

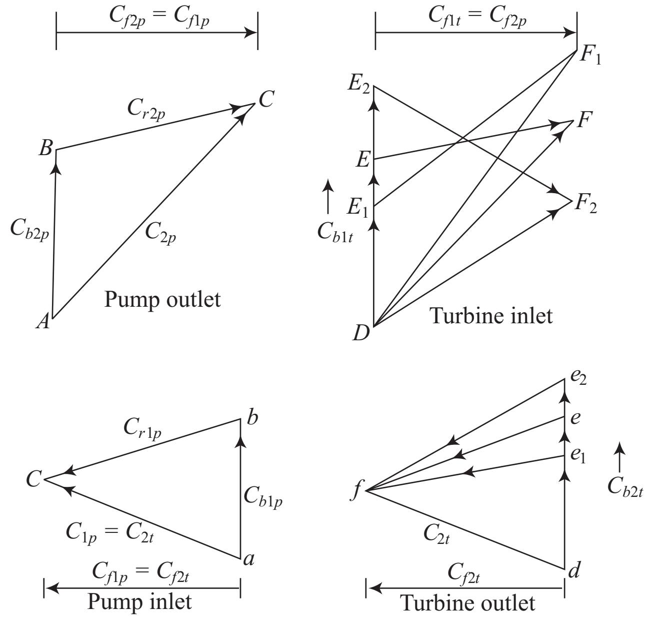


Figure 10.22 *Velocity Triangles of a Torque Converter*

Suppose that the load on the turbine is more. Consequently, the required torque on turbine is more. We know that if the required torque is to be increased, the speed gets reduced. The stator blades are oriented such that the velocity of the fluid is now DF_1 and the blade velocity is DE_1. The flow area and mass flow rate in such a case would be more. The stator blades have an effect of adding the reaction torque and the net torque on the turbine is also increased.

If the load on the turbine is decreased, consequently torque on the turbine shaft is decreased. In such a case, the fluid velocity is DF_2 and the blade speed is increased to DE_2. The stator exerts a torque in the opposite direction to that of the impeller. Hence, the net torque on the turbine is decreased.

(b) Analysis

The relationships between torques of the input (pump) and output (turbine) shafts can be given as,

$$T_2 = T_1 + T_{reaction} \tag{10.39}$$

where T_1 is the torque of the input or driver (pump) shaft, T_2 is the torque of the output or driven (turbine) shaft, and $T_{reaction}$ is the torque of the stator (reaction member).

$$\text{Power Input} = P_1 = T_1\omega_1 \tag{10.40}$$

$$\text{Power output} = P_2 = T_2\omega_2 \tag{10.41}$$

Efficiency of torque converter,

$$\eta_T = \frac{P_2}{P_1} = \frac{T_2\omega_2}{T_1\omega_1} \tag{10.42}$$

Depending on the design of stator blades, $T_{reaction}$ may be positive which results $T_2 > T_1$; in this case, it is a *torque multiplier*. If $T_{reaction}$ is negative, $T_2 < T_1$, the converter will be a *torque divider* in this case. Generally, torque converters are employed for torque multiplication.

(c) Characteristics

Figure 10.23 shows schematically the variation of torque ratio and efficiency with the speed ratio in a torque converter.

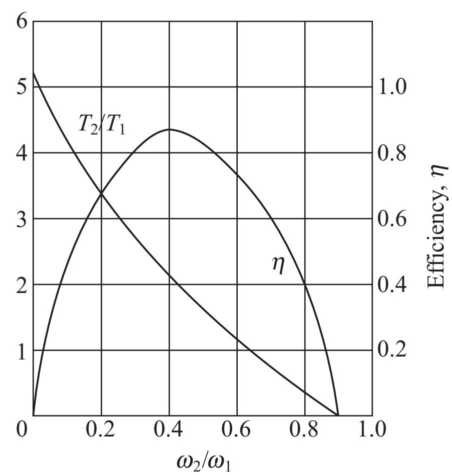


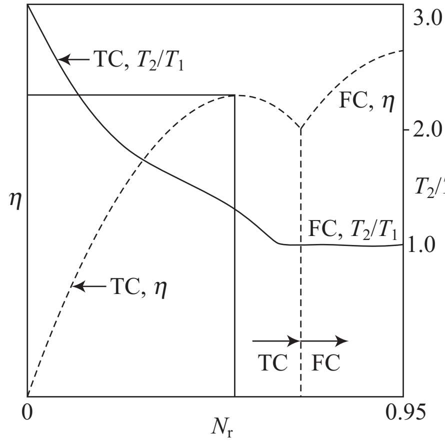
Figure 10.23 *Characteristics of a Torque Converter*

Figure 10.23 represents that the efficiency of torque converter is maximum at a speed ratio of approximately 0.5 and efficiency drops at higher speed ratios. The efficiency is zero at speed ratio of zero and once again it is zero at a speed ratio of approximately 0.9. The torque converter has a lesser maximum efficiency as compared to a fluid coupling due to more complex flow conditions. If the speed ratio is changed from that which gives maximum efficiency, there is a corresponding change of the velocity triangles for each transition from one element to another, and much energy is lost in turbulence when the directions of the relative velocities of the fluid are not tangential to the inlet edges of the blades. The torque ratio has highest value at zero speed ratio and drops continuously in non linear way to have a zero value at a speed ratio of approximately 0.9. Figure shows that, if a torque multiplier is designed to obtain a large increase of torque, the efficiency has a maximum value at a speed ratio much lesser than unity and drops off steeply as the speed ratio tends to unity. In many applications, this characteristic is a serious disadvantage. There is one way of coping with this difficulty. $\frac{T_2}{T_1}$ drops below unity at the higher speed ratios, that is, the reaction torque becomes negative. If a ratchet device is fitted whereby the reaction member may rotate in a forward direction only, the reaction blades automatically begin to turn as the reaction torque changes sign. The entire device then behaves like that of a simple coupling; the efficiency is equal to the speed ratio, and the torque ratio remains at unity. Thus, a combination of torque converter and coupling is obtained in which each is used in its best operating range. As discussed, torque converters have better efficiencies in the lower speed regions.

Hence, it is a practice to retain the device as a torque converter in the initial working conditions of higher efficiency, higher torque and then shift the conditions of working as a fluid coupling to take the advantage of higher efficiencies of the fluid coupling at higher loads. Such combined characteristics are shown in Figure 10.24.

An alternative solution is to substitute a direct drive when the speed ratio reaches a certain value; this is achieved by a clutch that is required to slip until the speeds of the primary and secondary rotors are equalized. The slip represents the inefficiency of the system and hence the energy lost as heat in both the fluid coupling and torque converter. Dissipation of heat generated in these units is an important factor in the construction of these devices. External cooling system may be required for large units.

Torque converters are extensively used in (a) automatic transmission systems of automobiles such as cars, buses and light trucks, (b) marine propulsion systems, and (c) industrial power transmission.



TC: Torque Converter, FC: Fluid Coupling

Figure 10.24 *Operation of Hydrodynamic Transmission System First as Torque Converter and then as Fluid Coupling*

10.8.4 Comparison of Torque Converter

(a) With Fluid Coupling

- Fluid coupling acts as an automatic clutch without torque multiplication. Torque converter is essentially an automatic clutch and torque multiplying device.
- Fluid coupling has two principal components – impeller and runner. Torque converter has three components: impeller, runner, reactor (or) stator.
- The vanes of fluid coupling are straight and radial shaped. The vanes of torque converter are curved shape.
- Torque converter can be converted into fluid coupling while transmitting torque ratio of 1:1.

(b) With Gear Box

- Both are torque multiplying devices.
- In torque converter, torque ratio varies automatically in a continuously variable manner. Gear box has definite speed ratio and changes are in steps.
- In torque converter during torque ratio variation, there is no interruption of power from the engine to road wheels. In gear box during gear shifting engine power is cut-off.
- Torque converter is smooth, vibration less and silent in operation. Gear box has vibrations, jerks and noises.
- Separate gear drive is necessary for reversing in torque converter. Gear box is self-contained with reversing mechanism.
- Efficiency of torque converter is maximum for a part of its speed range. Gear box efficiency is constant throughout.

EXAMPLE 10.16 The speed of rotation of the input shaft in a torque converter is 2400 rpm while that at output shaft is 900 rpm. The input and output torques are 50 N·m and 110 N·m, respectively. Find (a) power of the input and output shafts, and (b) efficiency of the torque converter.

Solution

Given: N_1 = 2400 rpm, N_2 = 900 rpm, T_1 = 50 N·m, T_2 = 110 N·m

$$\omega_1 = \frac{2\pi N_1}{60} \Rightarrow \omega_1 = \frac{2 \times \pi \times 2400}{60}$$

$$\omega_1 = 251.33 \text{ rad/s} \tag{1}$$

$$\omega_2 = \frac{2\pi N_2}{60} \Rightarrow \omega_2 = \frac{2 \times \pi \times 900}{60}$$

$$\omega_2 = 94.25 \text{ rad/s} \tag{2}$$

(a) Power at the Input and Output Shafts

$$P_1 = T_1\omega_1 \Rightarrow P_1 = 50 \times 251.33$$

$$P_1 = 12566.5 \text{ W} = 12.5665 \text{ kW} \tag{3}$$

$$P_2 = T_2\omega_2 \Rightarrow P_2 = 110 \times 94.25$$

$$P_2 = 10367.5 \text{ W} = 10.3675 \text{ kW} \tag{4}$$

(b) Efficiency of Torque Converter

$$\eta_T = \frac{P_2}{P_1} \tag{5}$$

$$\eta_T = \frac{10.3675}{12.5665}$$

$$\eta_T = 0.8250 = 82.50\% \tag{6}$$

EXAMPLE 10.17 A torque converter has an input torque of 220 N·m whereas the stator exerts a torque of 80 N·m in the same direction as that of input torque. The speeds of the input and output shafts are 2750 rpm and 1750 rpm respectively. Calculate the output torque and efficiency of torque converter.

Solution

Given: T_1 = 220 N.m, $T_{reaction}$ = 80 N.m, N_1 = 2750 rpm, N_2 = 1750 rpm

1. Output Torque

Since the stator torque is in the same direction of the input torque, therefore, torque converter will act as torque multiplier. Hence, output torque is,

$$T_2 = T_1 + T_{reaction} \tag{1}$$

$$T_2 = 220 + 80$$

$$T_2 = 300 \text{ N.m} \tag{2}$$

2. Efficiency of Torque Converter

$$\omega_1 = \frac{2\pi N_1}{60} \Rightarrow \omega_1 = \frac{2 \times \pi \times 2750}{60}$$

$$\omega_1 = 287.98 \text{ rad/s} \tag{3}$$

$$\omega_2 = \frac{2\pi N_2}{60} \Rightarrow \omega_2 = \frac{2\pi \times 1750}{60}$$

$$\omega_2 = 183.26 \text{ rad/s} \tag{4}$$

$$\eta_T = \frac{T_2\omega_2}{T_1\omega_1} \tag{5}$$

$$\eta_T = \frac{300 \times 183.26}{220 \times 287.98}$$

$$\eta_T = 0.8678 = 86.78\% \tag{6}$$

EXAMPLE 10.18 The fluid coupling of Example 10.15 is proposed to be converted into a torque multiplier so that the torque is increased by 25% of the input torque by introducing stator blades between impeller of the pump and turbine runner. The diameters at the pump outlet and turbine inlet remains the same as 0.25 m. The diameter at the pump inlet and turbine outlet also remains the same as 0.18 m. The stator blades are radial at the entry and exit. The efficiency may be assumed as 90%. Calculate (a) speed of the output shaft, (b) stator blade angles at inlet and outlet, (c) ratio of the outlet to inlet velocity of oil in the stator blades, and (d) turbine blade angles at inlet and outlet.

Solution

The data of the pump is same as that in Example 10.15 except that outlet is now in radially outward direction instead of axial direction. Hence, the following results of Example 10.15 hold good:

$$T_1 = 301.326 \text{ N.m}, \dot{m} = 127.88 \text{ kg/s}, P_1 = 45438.89 \text{ W}, C_{2p} = 23.23 \text{ m/s}, \alpha_{2p} = 35.75° \tag{1}$$

(a) Speed of Output Shaft

Since the torque is increased by 25% of the input torque by introducing stator blades, therefore, torque at the turbine shaft,

$$\therefore \quad T_2 = 1.25\, T_1 \Rightarrow T_2 = 1.25 \times 301.326$$

$$T_2 = 376.66 \text{ N.m} \tag{2}$$

$$P_2 = \eta_T P_1 \Rightarrow P_2 = 0.9 \times 45438.89$$

$$P_2 = 40895 \text{ W} \tag{3}$$

$$P_2 = T_2\omega_2 = \frac{2\pi N_2}{60} T_2 \tag{4}$$

$$40895 = \frac{2\pi \times N_2}{60} \times 376.66$$

$$N_2 = 1036.79 \text{ rpm} \tag{5}$$

(b) Stator Blade Angles at Inlet and Outlet

Stator blade inlet angle,

$$\alpha_{1s} = \alpha_{2p} = 35.75° \tag{6}$$

$$C_{b1t} = \frac{\pi D_2 N_2}{60} \Rightarrow C_{b1t} = \frac{\pi \times 0.25 \times 1036.79}{60}$$

$$C_{b1t} = 13.57 \text{ m/s} \tag{7}$$

$C_{w2t} = 0$, since the oil enters in the pump impeller without whirl.

Power output at the turbine shaft,

$$P_2 = \dot{m}(C_{w1t}C_{b1t} - C_{w2t}C_{b2t}) \tag{8}$$

$$40895 = 127.88 \times (C_{w1t} \times 13.57 - 0)$$

$$C_{w1t} = 23.57 \text{ m/s} \tag{9}$$

The velocity triangles at turbine inlet and outlet are shown in Figure 10.25.

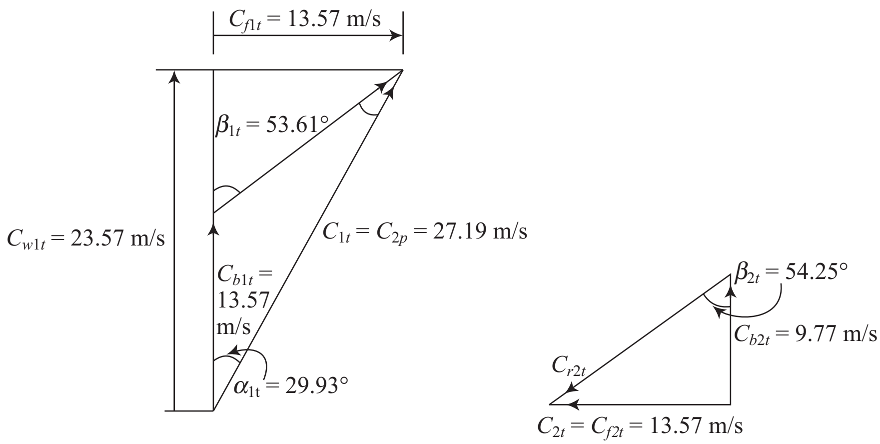


Figure 10.25 *Velocity Triangles of Turbine for Example 10.18*

From velocity triangle at inlet,

$$C_{1t} = \sqrt{C_{f1t}^2 + C_{w1t}^2} \Rightarrow C_{1t} = \sqrt{13.57^2 + 23.57^2}$$

$$C_{1t} = 27.197 \text{ m/s} \tag{10}$$

$$\tan \alpha_{1t} = \frac{C_{f1t}}{C_{w1t}} \Rightarrow \tan \alpha_{1t} = \frac{13.57}{23.57}$$

$$\alpha_{1t} = 29.93° \tag{11}$$

Stator blade outlet angle,

$$\alpha_{2s} = \alpha_{1t} = 29.93° \tag{12}$$

(c) Ratio of the Outlet to Inlet Velocity in the Stator Blades

$$\frac{C_{2s}}{C_{1s}} = \frac{C_{1t}}{C_{2p}} \tag{13}$$

$$\frac{C_{2s}}{C_{1s}} = \frac{27.197}{23.23}$$

$$\frac{C_{2s}}{C_{1s}} = 1.171 \tag{14}$$

(d) Turbine Blade Angles at Inlet and Outlet

$$\tan \beta_{1t} = \frac{C_{f1t}}{C_{w1t} - C_{b1t}} \tag{15}$$

$$\tan \beta_{1t} = \frac{13.57}{23.57 - 13.57}$$

$$\beta_{1t} = 53.61° \tag{16}$$

$$C_{b2t} = \frac{\pi D_1 N_2}{60} \Rightarrow C_{b2t} = \frac{\pi \times 0.18 \times 1036.79}{60}$$

$$C_{b2t} = 9.77 \text{ m/s} \tag{17}$$

$$\tan \beta_{2t} = \frac{C_{f2t}}{C_{b2t}} \tag{18}$$

$$\tan \beta_{2t} = \frac{13.57}{9.77}$$

$$\beta_{2t} = 54.25° \tag{19}$$

EXAMPLE 10.19 The pump impeller running at 1500 rpm has the mean diameter as 0.3 m at the inlet and outer diameter of 0.4 m. The oil enters in the pump impeller without any whirl. The inlet is axial and outlet is radial in the pump. The width of the blades at the inlet is 20 mm. Blade angle at the inlet and outlet of the pump are 50° and 80° respectively. The pitch circle or mean diameter of the turbine is 0.3 m, whereas the diameter at inlet is 0.4 m. The inlet of the fluid is radially inward whereas fluid leaves axially at outlet. The blade angle at the inlet of turbine is 75°. The fixed blades of the stator guide the oil of specific gravity 0.82 from the pump outlet to the turbine inlet. The velocity of oil in the stator blades is increased by 10% by designing the areas. Assume the velocity of flow constant throughout. Calculate (a) mass flow rate, (b) stator blade angle at inlet, (c) stator blade angle at outlet, (d) turbine blade outlet angle, (e) input torque and input power, (f) output torque and output power, (g) efficiency, and (h) torque ratio.

Solution

Given: N_1 = 1500 rpm, D_1 = 0.3 m, D_2 = 0.4 m, C_{w1p} = 0, α_{1p} = 90°, B_{1p} = 0.02 m, β_{1p} = 50°, β_{2p} = 80°, β_{2t} = 90°, β_{1t} = 75°, s = 0.82, C_{2s} = 1.1 C_{1s}

(a) Mass Flow Rate

$$C_{b1p} = \frac{\pi D_1 N_1}{60} \Rightarrow C_{b1p} = \frac{\pi \times 0.3 \times 1500}{60}$$

$$C_{b1p} = 23.562 \text{ m/s} \quad (1)$$

The velocity diagram at inlet of pump is shown in Figure 10.26. From velocity triangle at pump inlet,

$$C_{f1p} = C_{b1p} \tan \beta_{1p} \Rightarrow C_{f1p} = 23.562 \tan 50$$

$$C_{f1p} = 28.08 \text{ m/s} \quad (2)$$

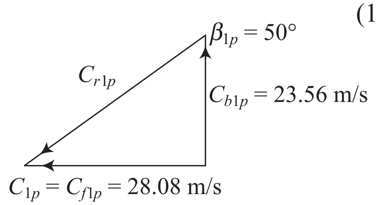


Figure 10.26 *Velocity Triangle at Pump Inlet for Example 10.19*

Flow area at inlet of the pump,

$$A_{1p} = \pi D_1 B_{1p} \Rightarrow A_{1p} = \pi \times 0.3 \times 0.2$$

$$A_{1p} = 0.01885 \text{ m}^2 \quad (3)$$

$$\dot{m} = \rho A_{1p} C_{f1p} \quad (4)$$

$$\dot{m} = 0.82 \times 1000 \times 0.01885 \times 28.08$$

$$\dot{m} = 434.033 \text{ kg/s} \quad (5)$$

(b) Stator Blade Angle at Inlet

Since velocity of flow is constant throughout, therefore,

$$C_{f2p} = C_{f1p} = 28.08 \text{ m/s} \quad (6)$$

$$C_{b2p} = \frac{\pi D_2 N_1}{60} \Rightarrow C_{b2p} = \frac{\pi \times 0.4 \times 1500}{60}$$

$$C_{b2p} = 31.416 \text{ m/s} \quad (7)$$

From velocity triangle at pump outlet as shown in Figure 10.27,

$$C_{w2p} = C_{b2p} - \frac{C_{f2p}}{\tan \beta_{2p}} \Rightarrow C_{w2p} = 31.416 - \frac{28.08}{\tan 80}$$

$$C_{w2p} = 26.465 \text{ m/s} \quad (8)$$

$$\tan \alpha_{2p} = \frac{C_{f2p}}{C_{w2p}} \Rightarrow \tan \alpha_{2p} = \frac{28.08}{26.465}$$

$$\alpha_{2p} = 46.7° \quad (9)$$

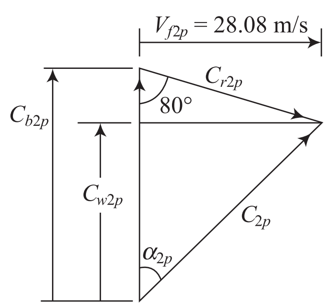


Figure 10.27 *Velocity Triangle at Pump Outlet for Example 10.19*

Stator blade angle at the inlet,

$$\beta_{1s} = \alpha_{2p} = 46.7° \quad (10)$$

(c) Stator Blade Angle at Outlet

$$C_{2p} = \sqrt{C_{f2p}^2 + C_{w2p}^2} \Rightarrow C_{2p} = \sqrt{28.08^2 + 26.465^2}$$

$$C_{2p} = 38.59 \text{ m/s} \quad (11)$$

Velocity of oil at the inlet of stator,

$$C_{1s} = C_{2p} = 38.59 \text{ m/s} \quad (12)$$

$$C_{2s} = 1.1\ C_{1s} \Rightarrow C_{2s} = 1.1 \times 38.59$$

$$C_{2s} = 42.45 \text{ m/s} \quad (13)$$

Fluid velocity at turbine inlet,

$$C_{1t} = C_{2s} = 42.45 \text{ m/s} \quad (14)$$

Since velocity of flow is constant throughout, therefore,

$$C_{f1t} = C_{f2p} = C_{f1p} = 28.08 \text{ m/s}$$

From the velocity triangle at turbine inlet as shown in Figure 10.28,

$$\sin \alpha_{1t} = \frac{C_{f1t}}{C_{1t}} \Rightarrow \sin \alpha_{1t} = \frac{28.08}{42.45}$$

$$\alpha_{1t} = 41.41°$$

Therefore, stator blade angle at outlet,

$$\beta_{2s} = \alpha_{1t} = 41.41° \quad (15)$$

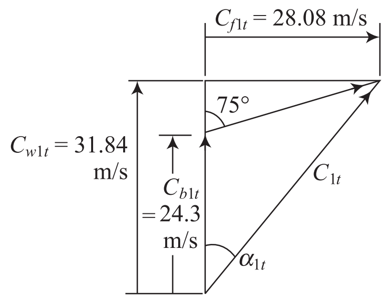


Figure 10.28 *Velocity Triangle at Turbine Inlet for Example 10.19*

(d) Turbine Blade Outlet Angle

$$C_{w1t} = C_{1t} \cos \alpha_{1t} = C_{w1t} = 42.45 \cos 41.41$$

$$C_{w1t} = 31.84 \text{ m/s} \quad (16)$$

$$C_{b1t} = C_{w1t} - \frac{C_{f1t}}{\tan \beta_{1t}} \Rightarrow C_{b1t} = 31.84 - \frac{28.08}{\tan 75}$$

$$C_{b1t} = 24.316 \text{ m/s} \quad (17)$$

$\because$ $$C_{b1t} = \frac{\pi D_2 N_2}{60} \Rightarrow N_2 = \frac{60 C_{b1t}}{\pi D_2} = \frac{60 \times 24.316}{\pi \times 0.4}$$

$$N_2 = 1161 \text{ rpm} \quad (18)$$

$$C_{b2t} = \frac{\pi D_1 N_2}{60} \Rightarrow C_{b2} = \frac{\pi \times 0.3 \times 1161}{60}$$

$$C_{b2t} = 18.237 \text{ m/s} \quad (19)$$

Since velocity of flow is constant throughout, therefore,

$$C_{f2t} = C_{f1t} = C_{f2p} = C_{f1p} = 28.08 \text{ m/s}$$

Now it is possible to draw velocity triangle at turbine outlet as shown in Figure 10.29.

$$\tan \beta_{2t} = \frac{C_{f2t}}{C_{b2}} \Rightarrow \tan \beta_{2t} = \frac{28.08}{18.237}$$

$$\beta_{2t} = 57° \quad (20)$$

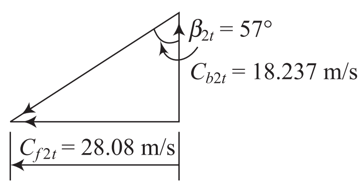


Figure 10.29 *Velocity Triangle at Turbine Outlet for Example 10.19*

(e) Input Torque and Input Power

Given $C_{w1p} = 0$

$$T_1 = \dot{m}(C_{w2p}r_{2p} - C_{w1p}r_{1p}) \Rightarrow T_1 = 434.033(26.465 \times 0.2 - 0)$$

$$T_1 = 2297.337 \text{ N.m} \tag{21}$$

Input power P_1 is given by,

$$P_1 = T_1\omega_1 = \frac{2\pi N_1}{60}T_1 \tag{22}$$

$$P_1 = \frac{2 \times \pi \times 1500}{60} \times 2297.337$$

$$P_1 = 3608648 \text{ W} = 360.865 \text{ kW} \tag{23}$$

(f) Output Torque and Output Power

Since the fluid leaves axially at outlet of the turbine, therefore, $C_{w2} = 0$. Torque at turbine, i.e. output torque is,

$$T_2 = \dot{m}(C_{w1t}r_{1t} - C_{w2}r_{2t}) \tag{24}$$

$$T_2 = 434.033 \times (31.84 \times 0.2 - 0)$$

$$T_2 = 2763.922 \text{ N.m} \tag{25}$$

$$P_2 = T_2\omega_2 = \frac{2\pi N_2}{60}T_2 \tag{26}$$

$$P_2 = \frac{2 \times \pi \times 1161}{60} \times 2763.922$$

$$P_2 = 336036.63 \text{ W} = 336.037 \text{ kW} \tag{27}$$

(g) Efficiency of Torque Converter

$$\eta_T = \frac{P_2}{P_1} \tag{28}$$

$$\eta_T = \frac{336.037}{360.865}$$

$$\eta_T = 0.9312 = 93.12\% \tag{29}$$

(h) Torque Ratio

$$T_r = \frac{T_2}{T_1} \Rightarrow T_r = \frac{2763.922}{2297.337}$$

$$T_r = 1.2031 \tag{30}$$

Equation (30) represents that this torque converter acts as a torque multiplier.

10.9 Control Systems

Often the fluid flow in a turbomachine may not meet design requirements or in the conditions of fluctuating load (demand), the variable fluid low is desired. The control of fluid flow in a turbomachine can be obtained in a number of ways as given below:

- Valve or damper control, i.e., change the system curve
- Speed control, i.e., change the machine curve
- Speed and damper/valve control, i.e., manage both curves

The system curve is shifted to the left or right by means of a variable resistance by the valve or damper control. This results in a decrease or an increase in flow while the prime mover moves at constant speed.

The prime mover curve is shifted up or down, consequently, increasing or decreasing the fluid flow in the system with the same system curve.

Finally, the combined speed and valve/damper control provide more flexibility in the operation of a system. This combined mode of operation is the most efficient from a cost point of view. We will focus our attention to control the speed of the hydraulic and steam turbines under varying load condition in this section.

10.9.1 A Generalised Governing System

The turbine governor is a system that regulates the quantity of water flowing through runner. The governor receives information on the existing rotational speed of the turbine and adjusts the inflow of water through the turbine to maintain the speed equal to the synchronous speed.

A generalised governing system consists essentially of: (a) control system, and (b) mechanical or hydraulic actuating system. Figure 10.30 shows a schematic diagram of a basic control system. The control system may be: (a) mechanical, and (b) electronic: analogue or digital. The actuator can be hydraulically controlled or mechanical motors or load actuator. While each of these has relative advantages, hydraulically controlled actuators are most commonly used. Pure mechanical controllers are also known as first generation (1G) controllers. In these controllers, belt drive control devices were directly coupled to the prime mover, i.e. turbine. Flyball type pendulum was the standard speed-sensing device.

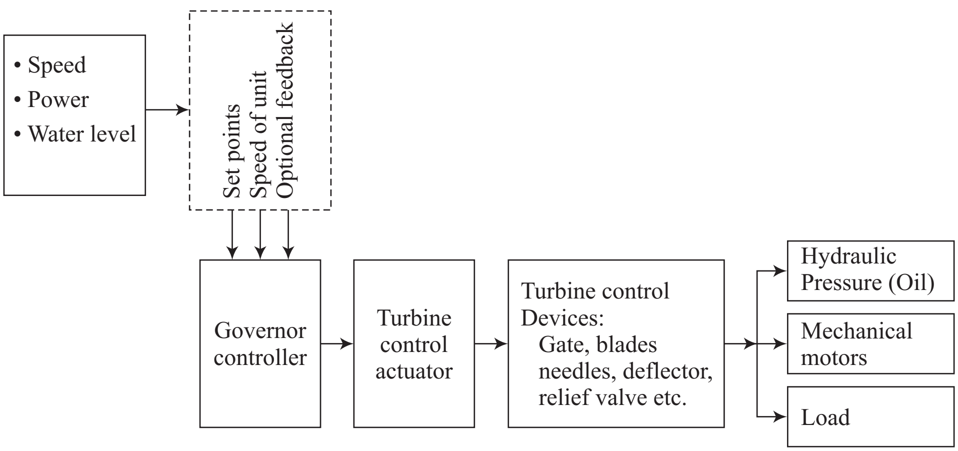


Figure 10.30 *Schematic Diagram of a Basic Control System*

In the second generation (2G) mechanical controllers, speed sensing was achieved by pendulum motors and permanent magnet generators. Mechanical dashpots, springs, and link mechanisms were used in achieving the desired settings. This type of controller belongs to the *PI* category which will be discussed in the next sub-section.

The third generation (3G) controller is called *electro-hydraulic controller*. In this controller, mechanical components are reduced in comparison to 1G and 2G controllers. Electrical or electronic devices were used for sensing the speed. Analogue circuitry was used to develop set points signal that would help in positioning of the control actuators of hydraulic units. This type of controller belongs to the *PID* category which will be discussed in the next sub-section.

The fourth generation (4G) controllers which are currently used are also known as *digital controllers*. Digital control hardware running on application software achieves the required control function. Signals of speed, power, water level, etc. are acquired through electronic devices and transmitted in digital form. Hydraulic devices controlled by digital signals perform the turbine control. The 4G controllers are very flexible, highly reliable, have quick response time and are adoptable to remote control. Self-diagnostic features and ability to change the control functions through software are some of the additional advantages. 1G and 2G controllers are obsolete.

10.9.2 Theory of Controllers

Basically, the controller uses an algorithm that provides the control signal in a feedback loop. For every deviation from a set point value, a correction is sent by a controller. There are three functions involved in calculating the correction.

(a) Proportional Function (P)

It deals with the present values, multiplying the current error by a set value *P* and subtracting the resultant value from the input of the process. The chief disadvantage of proportional controller is that it will overreact and cause oscillation in the system.

(b) Integral Stage (I)

It handles past values, integrating the errors over a period of time. This is then multiplied by a constant and subtracted from the input of the process. This helps to reduce the oscillations of a proportional controller. In addition, the integral stage ensures that the stable error is reduced to zero. A control system which uses the *P* and *I* stages only is said to belong to *PI* category.

(c) Derivative Term (D)

The *PI* controller reacts slowly to changes in the control variable. To overcome this difficulty, the derivative term is incorporated to predict the future performance of the system. The derivative of error over time is the parameter which is multiplied by a constant and subtracted from the input of the process. This enables the controllers to respond to a change in the system much faster than a *PI* system. Most of the present day controller algorithms use the principle of *PID* functions and such controllers are called *Proportional Integrated Derivative (PID) controllers*.

Though most of the present day turbine governors are of the digital type, a brief description of second generation 2*G* mechanical governor is given in the following sub-sections as it helps a lot to intuitively understand the salient working of various components of the governor.

10.9.3 Governing of Hydraulic Turbines

Hydraulic turbines are directly coupled to the electric generators. The frequency of power generation by a generator of constant number of pair of poles under all varying conditions should be fairly constant within a small permissible deviation of ±2%. Otherwise the electrical appliances will burn. This is only possible when the speed of the generator under all varying load conditions should be constant. Therefore, the generators are always required to run at a constant speed irrespective of the variations in the load. This constant speed (rpm) of the generator is given by,

$$N = \frac{120f}{p} \tag{10.43}$$

where f is the frequency for power generated in cycles per second and p is the number of poles for the generator. The speed of the generator can be maintained at a constant level only if the speed of the turbine runner is constant as given by Eq. (10.43). It is known as the synchronous speed of the turbine runner for which it is designed. If the load on the generator goes on varying with the variation of consumer's demand and if the input for the turbine remains the same, then the speed of the runner tends to increase if the load goes down or tends to decrease if the load on the generator goes up. Therefore, the speed of the generator and hence, the frequency will vary accordingly, which is not desired. Therefore, the speed of the runner is always required to be maintained at a constant level at all loads. It is done automatically by a governor which regulates the quantity of water flowing through runner in proportion to the load so that the power output from the turbine is equal to the load and by this way the speed is maintained constant. Therefore, the governing of a turbine is defined as the operation by which the speed of the turbine (and hence of generator) is maintained constant under varying condition of load.

(a) Pelton Wheel

In a Pelton turbine, water flow to the runner is regulated by the combined action of the spear and the deflector plate. There is a centrifugal governor, as in the case of a steam turbine, where its sensitivity to load variation is augmented by an oil-operated servo-mechanism as shown in Figure 10.31. When the load on the generator drops, the speed of turbine runner increases beyond the rated or synchronous speed due to the accelerating torque caused by the difference between turbine output and generator load. The centrifugal governor which is connected to the turbine main shaft will be rotating at an increased speed. The flyballs of the centrifugal governor fly outward due to more centrifugal force because of higher speed. Due to the outward/upward movement of the flyballs, the sleeve will also move upward. A horizontal lever, supported on the fulcrum, connects the sleeve and piston rod of the control valve. As the sleeve moves up, the lever turns about the fulcrum and the portion of the lever to the right of the fulcrum moves down pushing the piston rod of the control valve downwards. With the downward motion of the piston rod, valve V_1 closes and valve V_2 opens, as shown in Figure 10.31. A gear pump pumps oil from the oil sump to the control or relay valve. Oil flows through the valve V_2 and exerts force on the face L of the piston of relay cylinder or servomotor. The piston rod along with spear moves to the right, thus decreasing the flow area and hence, the rate of water flow (or discharge of water) to the turbine which consequently reduces the speed of the turbine. The speed of the turbine falls till it becomes normal when the flyballs, sleeve, lever, etc. also come to its normal position. The reverse happens when the load on the generator increases, speed decreases, flyballs, fly inward with less centrifugal force due to decreased speed, the sleeve moves down, the piston rod of control valve goes up, valve V_1 opens and valve V_2 closes, the oil under pressure flows through valve V_1 and exerts a force on the face M of the piston of relay cylinder or servomotor. The piston rod and the spear move to the left as a result of which more water flows to the turbine to take up more load and the speed becomes normal, i.e. attains its rated value.

It is noteworthy that the spear or needle valve is used normally for small load fluctuations. When there is a sudden fall of load, the spear has to move rapidly to close the nozzle. This rapid closing may cause water hammer problem. It is serious in large capacity plants with long penstocks. To avoid the water hammer effects during a sudden fall of load, a *deflector* is introduced in the system which is not shown in Figure 10.31. The function of the deflector is to deflect some water from the jet advancing to the turbine runner when the load on the turbine suddenly decreases. The quantity of water flowing through the nozzle remains the same, but a certain part of water coming out from the nozzle is deflected and is not allowed to

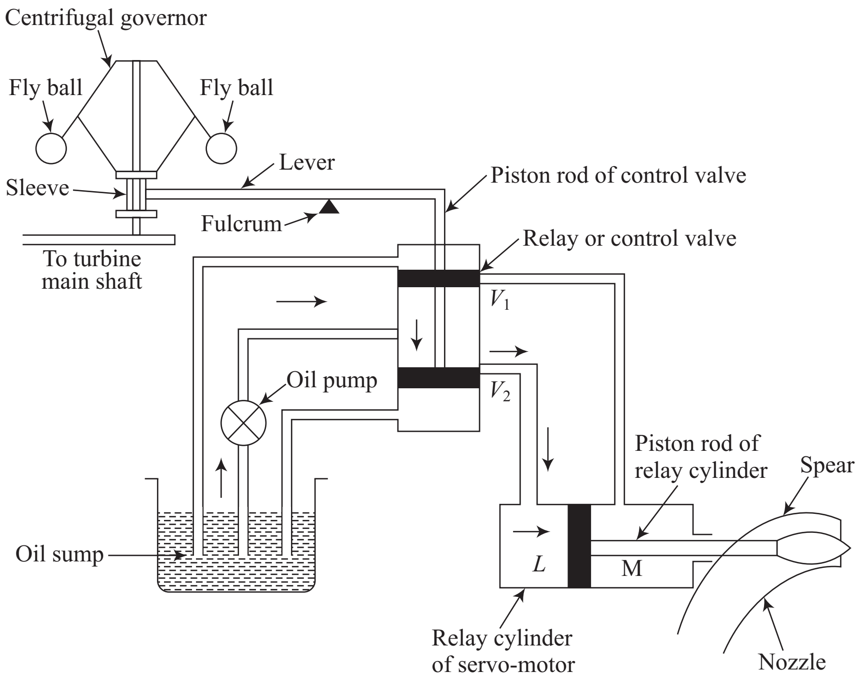


Figure 10.31 *Governing of a Pelton Turbine*

strike the buckets. The deflected water goes waste into the tailrace. Thus, in the case of sudden rapid drop in load, the speed is maintained constant by the joint action of spear and deflector plate.

(b) Reaction Turbines

The governing of Francis turbine is similar to that of Pelton wheel except that motion of piston in servomotor is used to partially close or open the guide vanes through which the water is supplied to the turbine instead of spear in the nozzle of the Pelton turbine. The governing system is shown in Figure 10.32.

The position of the control valve and the servomotor correspond to the design load on the turbine and operate in the same way as in the case of Pelton wheel. When the load on the generator is less than the design load, the centrifugal governor senses the speed and valve V_1 closes while V_2 of relay valve opens which allows the high pressure oil to enter the left portion of servomotor. This pushes the piston forward and partially closes the passage of guide blades through rack and pinion arrangement via linkages. Reverse action takes place when the load on the generator is more than the designed load.

A compensating device is, however, added to prevent the governor from overshooting. The compensating device consists of a bell crank lever with a pivot. When the servomotor piston moves to the right, the bell crank lever *EFG* is rotated downward about fulcrum *F* and the arm *G* is lowered. This pulls down the pivot *A*, which in turn, lowers the fulcrum *B*. Thus, the relay port *b* is partially or fully closed, restricting the piston motion of the servomotor to the right. In this way, it prevents the governor from overshooting.

The governor is always operated with a pressure relief valve (not shown in figure) in order to protect the system from water hammer effects. A sudden closure of wicket gates or guide vanes due to sudden drop in load will open the relief valve due to sudden increase in pressure and protect the conduit from inertia effects of speeding water. The relief valve consists of a spear and is held by fluid (oil) pressure to close the bypass

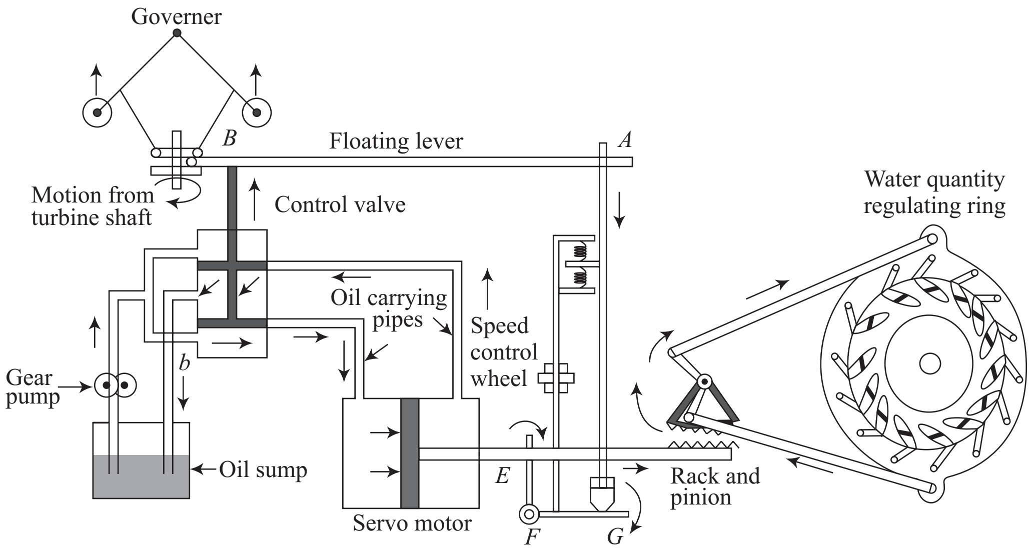


Figure 10.32 *Governing of a Francis Turbine*

of water from the spiral casing to the tailrace at design load. When the load decreases suddenly, a bell crank lever opens the pilot valve of the pressure chamber so that the pressure on the spear is reduced, thereby permitting the spear to be lifted up and allows a portion of water to flow directly from the spiral casing to the tailrace through the bypass without striking the runner blades. Thus, both the deflector of Pelton wheel and the relief valve of Francis or Kaplan turbine perform the same function of protecting the system from water hammer effects when the load suddenly decreases.

In the case of Kaplan turbine, the runner blades are also adjustable in addition to guide vanes. Hence, the governor is required to operate both sets of blades simultaneously. The runner blades are also operated by a separate servomotor and a control valve which are interconnected with those of the guide blades to ensure that for a given guide vane opening, there is a definite runner vane inclination.

10.9.4 Governing of Steam Turbines

The function of a governor is to maintain the speed of the shaft to be constant as the load varies. The simplest type of governor is the centrifugal flyball type as shown in Figure 10.33. The power available at the shaft is equal to $2\pi NT/60$, where N is the speed in rpm and T is the torque. As load (or torque) decreases, speed increases. Consequently, with the increase of centrifugal force, the fly-balls fly apart and raise the sleeve which is operated through a lever and a fulcrum actuates the main valve to close and reduce the mass flow of steam admitted to the turbine.

An oil operated servo mechanism in addition may be used to enhance the sensitivity of the governor as shown in Figure 10.34. The governor force is amplified to move a light and almost frictionless pilot valve which controls the flow of high pressure oil to a piston. The piston powered by the oil can thus operate the governor valve as desired. The steady-state speed regulation R_s is given by,

$$R_s = \frac{N_0 - N}{N_r} \times 100 \tag{10.44}$$

where N_0 is the speed at no load, N is the speed at rated load, and N_r is the rated speed.

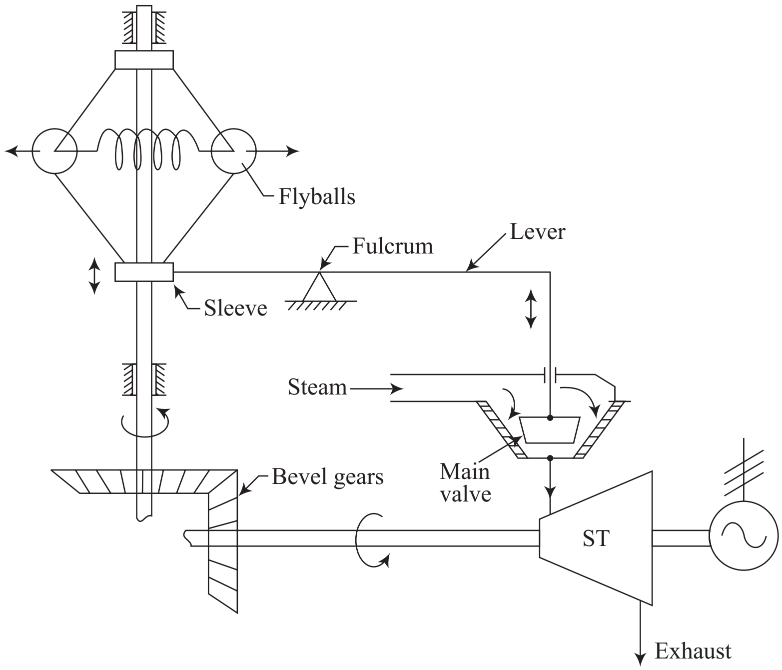


Figure 10.33 *Centrifugal Flyball Governor*

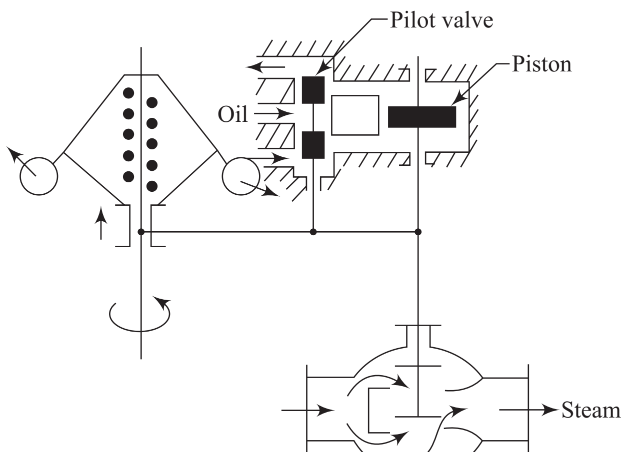


Figure 10.34 *Centrifugal Governor with Servo Mechanism*

Usually, the speed change from full load to no load is limited to 3-4% of the speed at rated load. If the rated speed is 3000 rpm, the speed at full load would be 3060 rpm and that at no load would be 2940 rpm. Therefore, in this case,

$$R_s = \frac{3060 - 2940}{3000} \times 100 = 4\%$$

(a) Throttle Governing

In throttle governing, the mass flow of steam through the turbine is regulated to control the output of turbine as per load or demand by throttling the steam.

The steam passing through the turbine is regulated by opening and closing of valve in throttle governing. The throttling process which is an isenthalpic i.e. constant enthalpy process occurs across the valve if the valve is closed. Throttling is accompanied with reduction in pressure, an increase in entropy and corresponding decrease in availability of energy per unit mass flow of steam. Also, as a result of throttling, the state of steam at inlet of turbine stage gets changed and the modified expansion line for each load is obtained. It may be found that the pressure drop occurs even in the full open position of governor valve. Therefore, it can be said that throttling is evident at all loads on turbine. Throttle governing on $h - s$ diagram as shown in Figure 10.35 (a) shows that the steam is available at state 1 at p_0 pressure in the main steam line. As the load decreases, shaft speed increases, then the throttle valve is partially closed causing throttling, to admit less steam to the turbine and to produce less power according to the demand. Due to restriction of passage in the valve, steam is throttled, say, from p_0 to $p_{throttle}$ as shown by 1–3. The specific ideal output of turbine thus reduces from $(h_1 - h_{2s})$ to $(h_3 - h_{4s})$. With further closure of the valve, $p_{throttle}$ will still be less to produce a still lower output.

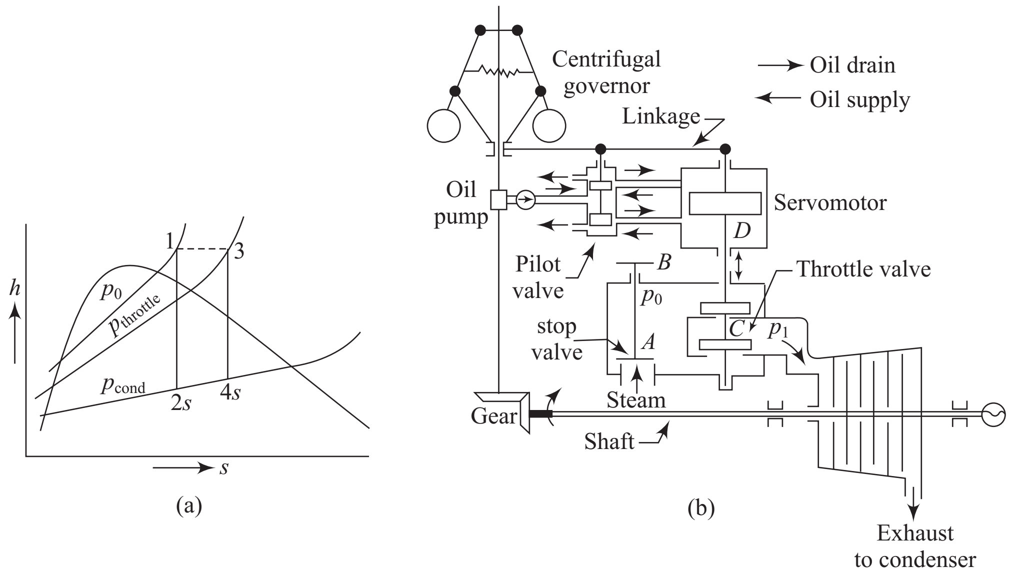


Figure 10.35 *(a) h – s Diagram for Throttling, (b) Schematic Diagram of Throttle Governing*

Schematic diagram of simple throttle governing in steam turbines is shown in Figure 10.35 (b). The change in speed of shaft is sensed by a centrifugal governor. The relay system consists of a pilot valve and servomotor. Either upward or downward displacement of servomotor piston decides the opening of throttle valve C. The high pressure oil entering from pilot valve to upper or lower half of servomotor piston D actuates the servomotor piston. Under normal operating condition, the servomotor piston is at the middle position and the inlet and exit ports remain in closed position by the pilot valves. As the load is decreased, then oil enters the upper half of servomotor, causing lowering down the servomotor piston and the throttle valve starts closing resulting in reduction of mass flow rate of steam and consequently the output till the speed is maintained to normal running speed. Simultaneously, the oil from lower half of servomotor gets drained out through pilot valve port. In the case of increased load on the turbine, the high pressure oil enters lower half of servomotor, then servomotor piston gets lifted up causing lift of throttle valve.

The steam consumption plotted against the turbine load shows a linear relationship, which is called *Willian's Line* as shown in Figure 10.36, and given by

$$\dot{m} = a + bL \tag{10.45}$$

where, a is the no load steam consumption, kg/s (intercept on y-axis of Figure 10.36), b is the steam rate (or specific steam consumption), kg/kW·s; (slope of the Willan's line) and L is the load, kW.

Following are the disadvantages of the throttle governing,

- The initial superheat at inlet increases due to throttling and the greatest variations in steam velocity occur in the later stages.
- In the later stages, the wetness of steam gets decreased due to throttling. Therefore, reduction in stage efficiency occurs at part load operation of turbine due to this decreased wetness.

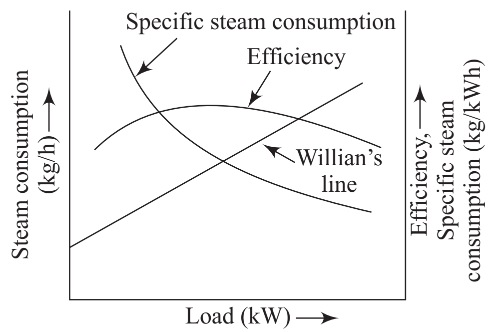


Figure 10.36 *Characteristics of Throttle Governing*

(b) Nozzle Governing

If throttle governing is done at low loads, the turbine efficiency is considerably reduced. The nozzle control may then be a better method of governing. The nozzles are made up in sets, each set being controlled by a separate valve as shown in Figure 10.37. The steam inlet to the rotor blades is through the sets of nozzles that are distributed over the annular space. When the turbine is running at full load, all the nozzle sets are open and the torque exerted on the rotor is uniform over the entire periphery.

When the load on the generator decreases, if all the nozzle sets are continued to remain open, the work produced by the turbine exceeds the demand of the energy (by the generator). Consequently, the speed of the turbine and generator tend to increase. The flyballs of the governor connected to the main shaft move outwards due to increased centrifugal force because of the higher speed. The upward movement of the sleeve of the governor is converted to affect the closure of some sets of the nozzles. Generally, the closure is by sets of segmental plates so that sets of the opposite nozzles are closed. Thus, the mass flow rate of steam is regulated as per load on the generator. The inlet (boiler) pressure and outlet (condenser) pressure are not changed. Also, the specific enthalpy drop is not changed in nozzle governing.

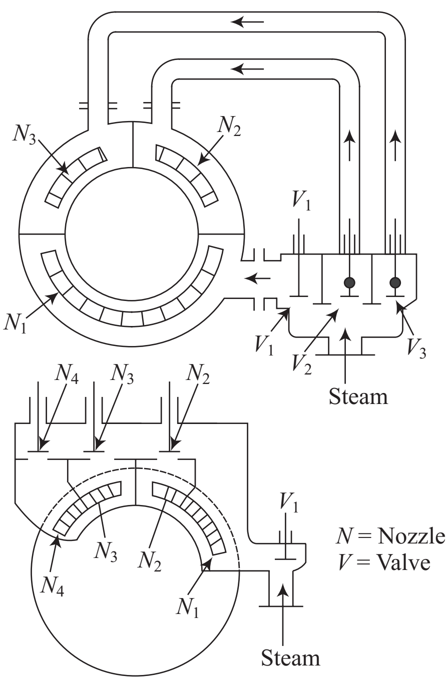


Figure 10.37 *Schematic Diagram of Nozzle Governing*

(c) By-Pass Governing

Schematic diagram of bypass governing is shown in Figure 10.38. Steam from main line enters the main valve which is controlled by speed governor. Steam from main valve enters the nozzle box or steam chest. Within the economical loads, the turbine is governed by the speed governor through nozzle or throttle governing. But for all loads greater than economical load, a bypass valve is opened. Bypass valve is provided on the nozzle box. Bypass valve is connected to a passage which delivers steam being bypassed to the later stages of turbine when turbine is overloaded. Bypass valve is actuated when load varies, thus allowing only part of steam entering main valve to contribute in power output. By pass valve is controlled by speed governor for all loads within its range. There may be more than one by pass valves in this kind of governing depending upon turbine and its application.

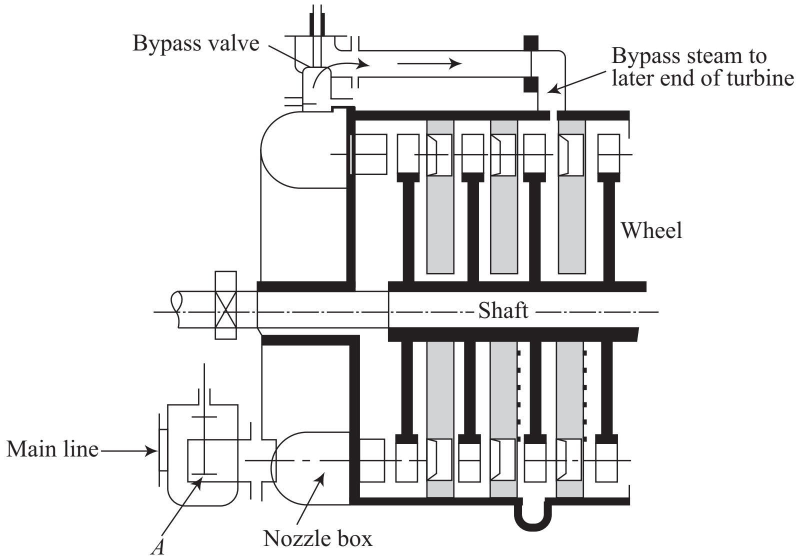


Figure 10.38 *By-Pass Governing*

(d) Combined Governing

Sometimes any one of throttle, nozzle control and by pass governing is not sufficient to meet the governing requirements. In such situations, the combination of two governing systems is used. These popular combinations are throttle and nozzle control combined governing and throttle and by pass combined governing.

(e) Emergency Governor

Every turbine is provided with some form of an emergency governor which trips the turbine (closes the stop valve and stops the steam supply) when,

(a) Shaft exceeds 110% of its rated speed, i.e. 3300 rpm
(b) Lubrication system fails
(c) Balancing (static as well as dynamic) of turbine is not proper
(d) Condenser becomes hot (due to inadequate cooling water circulation) or vacuum is less.

One common type of over speed trip employs a pin or weight on the turbine shaft. Centrifugal force acting on the pin is opposed by a spring until about 10% overspeed is reached, whereupon, the centrifugal force overcomes the spring force and the pin flies out and strikes a trigger which, in turn, releases a spring to close the stop valve immediately.

10.9.5 Desirable Qualities of a Governor

The desirable qualities of a good governor for a turbine are as follows:

(a) Sensitivity

It is a measure of the smallest change in a parameter that can be detected and corrected.

(b) Response Time

This is a conflicting requirement. A very quick response and changing the flow into the turbine may set up *water hammer* problems in the penstock. On the other hand, a very slow response may endanger the performance and operation of the electrical system. Provision of a bypass valve in reaction turbines and deflector for the jet in a Pelton turbine enables the quick responses without undue water hammer problems.

(c) Stability

The governor system must be stable and return to stable equilibrium after each episode of deviation from normal conditions. Further, quick convergence to set design point values is desired; i.e. no hunting, no undue oscillations etc. A typical stability value of a good governor is that the speed fluctuations should not exceed ±0.15% of rated speed.

(d) Reliability

The governor system should be reliable for operation of the turbine and for safety devices which are incorporated to take account of unforeseen events. Safety shutoff overrides should be available for emergencies. Adequate redundancy of essential components is usually provided to take care of this aspect.

It is seen that these requirements could be adequately found in an efficient manner in the present day digital governing systems

SUMMARY

- Fluid systems use the prime mover to drive a *pump* that pressurizes a fluid, which is then transferred through pipes and hoses to an actuator.
- Applications, significance and classification of fluid systems have been discussed. Pneumatic systems use gas, generally air, while hydraulic systems use liquids, usually oil. Other fluids are often used in special applications.
- Turbomachinery is extensively used in fluid-dynamic power transmission which consists of a centrifugal pump or centrifugal compressor connected in series. The fluid discharged from the pump or compressor drives the turbine. The discharge from the turbine is returned back to the inlet of the pump.
- In rotary positive displacement pumps, pressure is built up either by pure rotation or by combined rotation and oscillation of the pumping elements such as vane, gear, lobe etc.
- The basic principle of hydrostatic fluid systems is Pascal's law, or the principle of transmission of fluid pressure,

$$F_2 = pA_2 = F_1 \frac{A_2}{A_1}$$

Force F_2 is related to force F_1 through a multiplication factor A_2/A_1. The *hydraulic press* is a device which is used for providing compressive or lifting force by the application of a much smaller force.

- In a *direct-type hydraulic press*, the materials placed between stationary platform and movable platform undergo high compressive forces. The movable platform acts directly against the object to be pressed.
- A hydraulic accumulator is a device similar to a storage battery used for temporary storing the energy of a liquid in the form of pressure energy when not required by the system.
- The energy stored or capacity of simple accumulator is given by,

$$\text{Capacity of the accumulator} = W_L = \frac{\pi}{4} D^2 pL = p \times \text{Volume of accumulator}$$

- The power supplied by the simple accumulator when the ram falls uniformly through a distance L in time t is,

$$P = \frac{WL}{t} = p \times Q$$

- If W is the load carried by the sliding or inverted cylinder including its weight, then pressure of the liquid in the differential accumulator is,

$$p = \frac{W}{A} = \frac{4W}{\pi(D^2 - d^2)}$$

- Capacity of the differential accumulator is given by,

$$C = WL = pAL = p \times \text{Volume of the accumulator}$$

- A *hydraulic intensifier* also known as *booster* is a device which converts the low pressure from a cylinder into high pressure in a smaller cylinder.
- The discharge pressure in a hydraulic intensifier considering no frictional losses is,

$$p_2 = p_1 \left(\frac{D_1}{D_2} \right)^2$$

- Since $D_1 > D_2 \Rightarrow p_2 > p_1$. Hence, the input pressure is increased by a factor which is equal to the square of the ratio of inlet to outlet cylinder diameters. The pressure ratio p_2/p_1 is known as *intensification ratio*.
- A *hydraulic ram* working on the principle of water hammer is a type of pump which raises a small quantity of water to greater height without any external power when large quantity of water is available at a small height.
- D' Aubuisson efficiency of the Hydraulic Ram is,

$$\eta = \frac{P_o}{P_i} \Rightarrow \eta = \frac{Q_d (H_2 + h_{fd})}{Q_s (H_1 - h_{fs})}$$

- The Rankine efficiency of the Hydraulic Ram is,

$$\eta = \frac{P_o}{P_i} \Rightarrow \eta = \frac{Q_d(H_2 - H_1)}{(Q_s - Q_d)H_1}$$

The value of the Rankine's efficiency is always less than the D' Aubuisson's efficiency.

- Hydrodynamic transmission systems use fluid motion to transmit power. The required effect is obtained primarily by virtue of change in kinetic energy of the flow of working medium and the changes of pressure are avoided as far as possible. Examples are fluid coupling and torque converter.
- The efficiency of the hydrodynamic transmission, if losses are neglected, is,

$$\eta = \frac{P_2}{P_1} = \frac{2\pi N_2 T_2}{2\pi N_1 T_1} \Rightarrow \eta = N_r T_r$$

Since $\eta > 1$

If $T_r = 1$, then $N_r < 1$.

If $N_r = 1$ then $T_r < 1$

- A general energy balance considering losses is,

$$-N_1 T_1 - NT_l = N_2 T_2$$

where T_l is the loss of torque. The loss of torque may be associated with a speed that may be either the input speed or output speed.

- The increase or decrease of torque is achieved by introducing stationary stator (reaction member) blades with different outlet angles. In such a case, the casing or stator also participates in the torque transmission process. Torque balance equation is,

$$-\frac{T_2}{T_1} = 1 + \frac{T_{\text{reaction}}}{T_1} \Rightarrow T_r = 1 + \frac{T_{\text{reaction}}}{T_1}$$

- Noteworthy points from Torque Balance Equation

$T_r = 1 \Rightarrow T_{\text{reaction}} = 0$, the stator blades do not participate in the transmission process. This is a simple coupling.

$T_r > 1 \Rightarrow \frac{T_{\text{reaction}}}{T_1} > 0$. This is the case of torque multiplier. Since T_1 is negative, T_{reaction} must also be negative to get $T_r > 1$. This implies that the stator blades should exert a torque in the same direction as that of the input torque (torque on the driver shaft or torque on the impeller shaft).

$T_r < 1 \Rightarrow \frac{T_{\text{reaction}}}{T_1} < 0$. This is the case of torque divider. Since T_1 is negative, T_{reaction} must also be positive to get $T_r < 1$. This implies that the stator blades should exert a torque in the opposite direction as that of the input torque (torque on the driver shaft or torque on the impeller shaft) i.e. in the same direction as that a turbine or driven shaft.

✦ Differences between Fluid Coupling and Torque Converter

Characteristics	***Fluid Coupling***	***Torque Converter***
Output Torque	$T_2 = T_1$	$T_2 = T_1 + T_{\text{reaction}}$ Torque Multiplier if if T_{reaction} is positive, $T_2 > T_1$ Torque Divider if T_{reaction} is negative, $T_2 < T_1$
Working	No Torque multiplication	Torque multiplication takes place
Components	2 principal components-Impeller and runner	3 principal components-Impeller, runner and Stator(reactor)
Vane Shape	Vanes are straight and radial	Vanes are curved
Efficiency	$\eta_c = \frac{P_2}{P_1} = \frac{T_2\omega_2}{T_1\omega_1}$	$\eta_T = \frac{P_2}{P_1} = \frac{T_2\omega_2}{T_1\omega_1}$
Efficiency Variation	Efficiency is zero when speed ratio is zero and increases linearly up to a value of 0.95 and then the efficiency drops to zero at a speed ratio of 1.	The efficiency is maximum at a speed ratio of approximately 0.5 and efficiency drops at higher speed ratios. The efficiency is zero at speed ratio of zero and once again it is zero at a speed ratio of 0.9. The maximum efficiency of a torque converter is less than that of a fluid coupling because of more complicated flow conditions.
Slip	$S = \frac{\omega_1 - \omega_2}{\omega_1} = \frac{N_1 - N_2}{N_1}$	

✦ A combination of torque converter and fluid coupling is used to obtain efficiency in best operating range. Torque converters have better efficiencies in the lower speed regions. Hence, it is a practice to retain the device as a torque converter in the initial working conditions of higher efficiency, higher torque and then shift the conditions of working as a fluid coupling to take the advantage of higher efficiencies of the fluid coupling at higher loads.

✦ Hydraulically controlled actuators are generally used for controlling the governing of turbines.

✦ Governing of a turbine is defined as an operation through which speed of a turbine is kept constant under varying conditions of load. A comparison of governing mechanism in Hydraulic Turbines (Pelton, Francis and Kaplan turbine) is

	Pelton	***Francis***	***Kaplan***
Mechanism	Flow is regulated by the combined action of the spear and the deflector plate.	Similar to Pelton turbine except motion of piston in servomotor is used to partially close or open the guide vanes through which water is supplied to the turbine instead of spear in the nozzle of Pelton turbine.	The governor is required to operate adjustable moving blades and guide blades both simultaneously. Runner blades are also operated by a separate servomotor and a control valve interconnected with those of guide blades.

✦ Governing of Steam Turbines

	Throttle governing	*Nozzle governing*	*By Pass governing*
Mechanism	Throttling of steam to a suitable pressure and regulation of mass flow rate.	Set of Nozzles are controlled by a separate valve. The steam inlet to the rotor blades is through the sets of nozzles to regulate mass flow rate of steam.	Steam is by passed to the later stages of turbine when turbine is overloaded.

✦ The steam consumption plotted against the turbine load shows a linear relationship, which is called *Willian*'s *Line* and given by

$$\dot{m} = a + bL$$

REVIEW QUESTIONS

10.1 What is meant by a fluid system?

10.2 What is the significance of a fluid system?

10.3 What is the basic principle of operation of a fluid system?

10.4 What are the basic functions of a fluid system?

10.5 What is the difference between the mechanical, electrical and fluid transmission systems for transmitting power?

10.6 What are the two broad classifications of a fluid system? Differentiate between them in brief.

10.7 What are the advantages and disadvantages of fluid systems?

10.8 Draw a neat sketch of a general pneumatic system and explain the function of each element of the system.

10.9 Draw a neat sketch of a general hydraulic system and explain the function of each element of the system.

10.10 Compare the pneumatic systems with hydraulic systems.

10.11 What is the role of turbomachinery in fluid systems?

10.12 What is a positive displacement machine? What is the basic difference between positive displacement machines and turbomachines?

10.13 Compare the positive displacement machines with turbomachinery.

10.14 Why positive displacement turbine is used in flow rate or flow volume measurement?

10.15 What is a hydrostatic system? Distinguish between hydrostatic and hydrodynamic systems with suitable examples.

10.16 Explain the basic principle of operation of a hydraulic press?

10.17 Explain briefly (a) Direct type hydraulic press, and (b) Inverted type hydraulic press.

10.18 Explain the working principle of (a) Ordinary hydraulic accumulator, and (b) Differential type hydraulic accumulator.

10.19 Explain the working of a co-axial type intensifier with suitable diagram.

10.20 What is the significance of hydraulic intensifier? Differentiate between the simple and co-axial intensifiers.

10.21 Differentiate between the hydraulic accumulator and hydraulic intensifier.

10.22 Explain the working of a hydraulic ram.

10.23 Obtain an expression for the efficiencies of a hydraulic ram.

10.24 Describe briefly the working of (a) Direct acting hydraulic lift, and (b) Suspended type hydraulic lift.

10.25 What is the difference between the direct acting hydraulic lift and suspended type hydraulic lift?

10.26 Explain the working of a hydraulic crane with a suitable sketch.

10.27 What is a hydrodynamic power transmission system?

10.28 What is the basic principle of operation of a hydrodynamic power transmission system?

10.29 Explain the working of a power transmitting hydrodynamic system.

10.30 Explain how the stator blades give rise to increase or decrease the torque at the output shaft in a hydrodynamic power transmission system.

10.31 Explain the working of a fluid coupling. What are its main applications?

10.32 Draw the operating characteristics of a fluid coupling and explain.

10.33 Explain the working of a torque converter. What are its main applications?

10.34 What is the difference between a fluid coupling and a torque converter?

10.35 Draw and discuss the operating characteristics of a torque converter.

10.36 Explain how the characteristics of a fluid coupling and torque converter can be integrated for higher efficiencies.

10.37 What is the governing of a turbine?

10.38 Why governing of a turbine is necessary?

10.39 Why an oil operated servo mechanism is used in a governor?

10.40 Describe with the help of a neat sketch, the governing of a Pelton turbine.

10.41 Describe with the help of a neat sketch, the governing of hydraulic reaction turbines.

10.42 How the governing of a Francis turbine differs from that of a Pelton wheel?

10.43 How the governing of a Kaplan turbine differs from that of a Francis turbine?

10.44 What are the desirable qualities of a governor?

10.45 What is Willian's line? State its significance.

10.46 Explain the nozzle governing of steam turbine?

10.47 Explain the throttle governing in steam turbine?

10.48 Explain bypass governing of a steam turbine?

10.49 Differentiate among various methods of governing of steam turbine.

10.50 Why an emergency governor is provided in a turbine unit? When does it become operative?

PROBLEMS

10.1 The plunger of a hydraulic press has 25 mm diameter, 0.25 m stroke and makes 30 strokes/min. The diameter of the ram is 0.25 m. If a weight of 40 kN is to be raised by the press, determine (a) force on the plunger, and (b) rate of rise of the weight. [*Ans:* (a) F_1 = 400 N, (b) C_{ram} = 0.075 m/min]

10.2 A hydraulic press has to lift a weight of 60 kN through a height of 1.2 m in 1.5 min. The diameters of the ram and plunger are 0.25 m and 25 mm respectively. The stroke of the plunger is 0.25 m. Calculate (a) force required to raise the weight, (b) input power of the plunger, (c) volume of the oil to be pumped, and (d) number of strokes of the plunger per minute.

[*Ans:* (a) F_p = 600 N, (b) P = 800 W, (c) V = 58.9 litres, (d) N = 320 strokes/min]

10.3 The diameters of the ram and plunger of a hydraulic press are 150 mm and 20 mm respectively. A load of 40 kN is required to be raised through 1.5 m. The stroke of the plunger is 0.375 m. Calculate (a) number of strokes of the plunger to lift the load, and (b) power of the motor required to drive the plunger if the time taken to raise the weight is 2 min and overall efficiency of the motor is 70%.

[*Ans:* (a) N = 225 strokes, (b) P_{motor} = 714 W]

10.4 A hydraulic accumulator with 0.305 m diameter ram and 1.2 m stroke length is loaded with a weight of 800 kN. The friction may be taken as 3% of the total load on the ram. Find (a) power supplied by the accumulator when the ram is moving down through the full stroke length in 100 s, and (b) power supplied by the pump if the pump is supplying 6 litres/s of liquid to the machine while the accumulator is moving down. [*Ans: (a)* P_1 = 9.312 kW, (b) P_2 = 63.73 kW]

10.5 The diameter of the sliding ram of a hydraulic accumulator is 0.5 m. The total weight on the sliding ram including its self-weight is 800 kN. The frictional resistance against the movement of the ram is 5% of the total weight. Calculate the pressure of water when the ram is (a) moving up with a uniform velocity, and (b) moving down with a uniform velocity. [*Ans:* (a) p = 4.278 kPa, (b) p = 3.871 kPa]

10.6 An accumulator with 0.25 m diameter ram and 0.3 m stroke length is loaded with a weight of 500 kN. The friction may be taken as 5% of the load on the accumulator. The ram is moving down through the full stroke length with a uniform velocity in 100 s. A pump connected to the accumulator system supplies 300 litres/min of oil to the machine when the ram is moving down. Calculate the total power supplied to the machine. [*Ans:* P_t = 62.61 kW]

10.7 A hydraulic intensifier has a fixed ram of 100 mm diameter and a sliding ram of 0.4 m diameter. The pressure on the high pressure side is 30 MPa and the loss due to friction at each of the packing of the intensifier is 4% of the total force on each of the packings. Calculate the pressure of the liquid on the low pressure side. [*Ans:* p_1 = 2034 kPa]

10.8 The pressure of water supplied to an intensifier is 200 kPa whereas the pressure of water leaving the intensifier is 1000 kPa. The external diameter of the sliding cylinder is 0.2 m. Find the diameter of the fixed ram of the intensifier. [*Ans:* D_2 = 89.4 mm]

10.9 A hydraulic ram delivers 120 litres/min of water to a delivery tank which is 15 m above the ram. The amount of water wasted is 15 litres/s. The ram is working against a head of 4 m. The loss of head in the delivery pipe is 1.5 m, whereas the loss in the supply pipe is negligible. Find the efficiency of the ram. [*Ans:* η = 48.5%]

10.10 A hydraulic ram receives water at the rate of 100 litres/s from a source against a head of 5 m and delivers to a tank 20 m above the ram at the rate 10 litres/s. The diameter of the delivery pipe is 0.1 m and length 50 m. The length and diameter of the supply pipe are 15 m and 0.2 m respectively. Calculate the efficiency of the ram assuming $f = 0.02$ for both the supply and delivery pipes. [*Ans:* $\eta = 49.3\%$]

10.11 A hydraulic ram raises water to a height of 20 m at the rate of 5 litres/*s*. The water wasted at the ram is 70 litres/*s*. The source is 4 m above the ram. Assume 0.4 m loss of head in the supply pipe and 0.6 m in the delivery pipe. Calculate the efficiency of the ram. [*Ans:* $\eta = 38.1\%$]

10.12 A hydraulic ram receives water against a head of 3 m and delivers to an effective head of 21 m above the ram. The ratio of water delivered to water wasted is 1 : 15. Calculate the efficiency of the ram neglecting frictional losses in the pipe. [*Ans:* $\eta = 43.8\%$]

10.13 A hydraulic lift is required to lift a load of 98.1 kN to a height of 12 m once in every 100 *s*. The speed of the lift is 0.6 m/s. Determine (a) power required to drive the lift, (b) working period of the lift, and (c) idle period of the lift. [*Ans:* (a) $P = 11.772$ kW, (b) Working period = 20 s, (c) Idle period = 80 s]

10.14 A hydraulic lift is required to raise 100 kN through a height of 28 m once in every 3 min. The speed of the lift is 0.75 m/s. Water from accumulator and pump is supplied to the lift at a pressure of 3 MPa during working stroke of the lift. The efficiency of the pump is 75% and that of the lift is 80%. Calculate (a) power required to drive the pump, and (b) minimum capacity of the accumulator. Neglect all other losses in the system. [*Ans:* (a) $P_p = 25.92$ kW, (b) $C = 0.925$ m^3]

10.15 Water is supplied to a hydraulic crane at a pressure of 700 kPa for raising a weight through a height of 10 m. The diameter of the ram is 150 mm and velocity ratio is 6. The efficiency of the crane is 60%. Determine (a) weight lifted by the crane, and (b) quantity of water required to lift the load.

[*Ans:* (a) $W = 1.2369$ kN, (b) $V = 29.45$ litres]

10.16 The jigger of a hydraulic crane has velocity ratio of 8 : 1 and a ram of diameter 0.25 m. Water is supplied to the jigger at a pressure of 11000 kPa. The efficiency of the crane is 60% when the load is lifted to a height of 10 m. Calculate (a) load lifted by the crane, and (b) quantity of water supplied.

[*Ans:* (a) $W = 40.5$ kN, (b) $V = 6.136$ litres]

10.17 The input shaft of a fluid coupling driven by a motor is running at 600 rpm whereas the output shaft at 510 rpm. The power output of the motor is 48 kW. Calculate (a) input torque, (b) output power, and (c) efficiency of the coupling. [*Ans:* (a) $T_1 = 764$ N.m, (b) $P_2 = 40.8$ kW, (c) $\eta_c = 85\%$]

10.18 The pumping element of a fluid coupling has a mean inlet diameter as 75 mm whereas mean outlet diameter of 0.11 m. The blades of the pump impeller are bent backwards having inlet and outlet blade angles 60° and 75° respectively. The blade width at inlet is 10 mm. The speed of the driver shaft is 1000 rpm while that of driven shaft is 900 rpm respectively. The velocity of flow remains constant at inlet and outlet. The velocity of the fluid at the entry of the pump is at 75° to the plane of rotation of the impeller. Draw the velocity triangles at the inlet and outlet of pump impeller and turbine runner and calculate (a) blade angles at inlet and outlet of the turbine runner, (b) mass flow rate, (c) torque transmitted, (d) power at the input shaft, (e) power at the output shaft, and (f) efficiency of the coupling. Assume specific gravity of the oil used in the coupling as 0.85.

[*Ans:* (a) $\beta_{1t} = 81.85°$, $\beta_{2t} = 63.81°$, (b) $\dot{m} = 9.3$ kg/s, (c) $T_2 = T_1 = 0.9378$ N.m,
(d) $P_1 = 98.2$ W, (e) $P_2 = 88.38$ W, (f) $\eta_c = 90\%$]

10.19 The pump impeller of a torque converter having blade width at inlet as 20 mm is running at a speed of 3000 rpm. The blades of the pumping element are bent backwards with inlet blade angle as 40°, whereas outlet blade angle as 50°. The mean radius at inlet and outlet are 0.12 m and 0.16 m, respectively. The velocity of flow remains constant throughout the system as 25 m/s. The fixed blades of the stator change the flow path of the fluid by an angle of 15° to accelerate the fluid. The efficiency of the converter is 85% and the specific gravity of the oil used in the coupling is 0.85. Draw the velocity triangles at inlet and outlet of the pump impeller and turbine runner. Calculate (a) mass flow rate of the fluid, (b) torque at the input shaft, (c) input power, (d) output power, (e) speed of the turbine runner, (f) output torque, and (g) ratio of output torque to input torque.

[*Ans:* (a) $\dot{m}$ = 320.45 kg/s, (b) T_1 = 1198 N.m,, (c) P_1 = 376.36 kW, (d) P_2 = 319.9 kW, (e) N_2 = 1703 rpm, (f) T_2 = 1793.8 N · m, (g) T_2/T_1 = 1.497]

10.20 The pump element of a torque converter running at 3000 rpm has backward curved blades with inlet blade angle of 40° and outlet blade angle of 65°. The mean radius at inlet is 0.1 m while that at outlet is 0.15 m and the width of the blades at inlet is 20 mm. The velocity of flow remains constant throughout the system at 20 m/s. The converter having efficiency of 85% uses oil of specific gravity 0.85. The fixed blades of the stator change the flow path of the oil by an angle of 10° so as to retard the flow. Draw the velocity triangles at inlet and outlet of the pump impeller and turbine runner. Determine (a) mass flow rate, (b) input torque, (c) input power, (d) output power, (e) speed of the turbine runner, (f) torque at the turbine runner, and (g) torque ratio.

[*Ans:* (a) $\dot{m}$ = 213.63 kg/s, (b) T_1 = 1049.32 N.m, (c) P_1 = 329.654 kW, (d) P_2 = 280.2 kW, (e) N_2 = 4044 rpm, (f) T_2 = 661.6 N · m, (g) T_2/T_1 = 0.63]

MULTIPLE CHOICE QUESTIONS

1. **Assertion (A):** No solid connection exists between the driving shaft and driven shaft.

   Reason (R): Energy transfer is by the change in moment of momentum.
   (a) Both A and R are true and R is a correct explanation of A
   (b) Both A and R are true but R is not a correct explanation of A
   (c) A is true but R is false
   (d) A is false but R is true
2. Fluid machines use the principle of either (i) supplying energy to the fluid, or (ii) extracting energy from the fluid. Some fluid machines are combination of both (i) and (ii). They are classified as
   (a) Compressors
   (b) Hydraulic turbines
   (c) Torque converters
   (d) Wind mills
3. In a hydrodynamic transmission machine, the entry and exit of fluid in the pump are at
   (a) Equal radii
   (b) Smaller radius to larger radius
   (c) Larger radius to smaller radius
   (d) None of these

4. In a hydrodynamic transmission machine, the entry and exit of fluid in the turbine are at
   (a) Equal radii
   (b) Smaller radius to larger radius
   (c) Larger radius to smaller radius
   (d) None of these
5. Which one of the following combinations represents the power transmission systems?
   (a) Pump, hydraulic accumulator, hydraulic intensifier and hydraulic coupling
   (b) Pump, turbine, hydraulic accumulator and hydraulic coupling
   (c) Turbine, accumulator, intensifier and hydraulic coupling
   (d) Accumulator, intensifier, hydraulic coupling and torque converter
6. The most suitable condition for fluid coupling is
   (a) $N_r > 1$ (b) $T_r > 1$ (c) $T_r = 1$ (d) $N_r = 1$
7. Consider the following statements regarding the fluid coupling
   1. Efficiency increases with increase in speed ratio
   2. Neglecting friction the output torque is equal to input torque
   3. At the same input speed, higher slip requires higher input torque

   Which of these statements are correct?
   (a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3
8. A hydraulic coupling transmits 1 kW of power at an input speed of 200 rpm with a slip of 2%. If the input speed is changed to 400 rpm, the power transmitted with the same slip is
   (a) 2 kW (b) 1/2 kW (c) 4 kW (d) 8 kW
9. In a hydraulic coupling, what is the ratio of speed of the turbine runner to that of the pump impeller to maintain circulatory motion of oil?
   (a) < 1 (b) = 1 (c) > 1 (d) Can be any value
10. Which one of the following is correct for hydraulic coupling?
    (a) It connects two shafts rotating at about the same speed
    (b) It connects the two shafts running at different speeds
    (c) It is used to divide the torque to the driven shaft
    (d) It is used to connect the centrifugal pump and its electric motor for efficient operation
11. In a hydraulic coupling
    (a) The magnitude of input and output torques are equal
    (b) The magnitude of input torque is greater than output torque
    (c) The magnitude of input torque is less than output torque
    (d) The magnitude of input torque is negligible as compared to output torque
12. **Assertion (A):** In a fluid coupling, hydrodynamic transmission is done by pump and turbine.

    Reason (R): Fluid coupling is a type of machine in which fluid is used as a means of energy transfer.
    (a) Both A and R are true and R is a correct explanation of A
    (b) Both A and R are true but R is not a correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true

13. If ω_1 and ω_2 represent the angular velocities of driver and driven members of a fluid coupling respectively, then the slip is equal to

    (a) $1-\dfrac{\omega_1}{\omega_2}$ (b) $\dfrac{\omega_1}{\omega_2}$ (c) $\dfrac{\omega_2}{\omega_1}$ (d) $1-\dfrac{\omega_2}{\omega_1}$

14. In a fluid coupling, the torque transmitted is 500 kN.m when the speed of the driving and driven shaft is 900 rpm and 720 rpm, respectively. The efficiency of the fluid coupling will be
    (a) 20% (b) 25% (c) 80% (d) 90%
15. In a fluid coupling, as speed ratio N_r varies from zero to 0.95
    (a) The efficiency starts from zero and increases continuously
    (b) The efficiency starts from zero, increases and then decreases
    (c) The efficiency starts from 1.0 and then decreases continuously
    (d) None of these
16. The energy flow in a torque converter is
    (a) Input shaft-Fluid-Pump impeller-Turbine runner-Output shaft
    (b) Input shaft-Pump impeller-Turbine runner-Output shaft
    (c) Input shaft-Turbine runner-Pump impeller-Output shaft
    (d) Input shaft-Pump impeller-Fluid-Turbine runner-Output shaft
17. Consider the following statements regarding torque converter.
    1. It has stationary set of blades in addition to the primary and secondary rotors
    2. It can be used for multiplication of torques
    3. The maximum efficiency of converter is less than that of a fluid coupling
    4. In a converter, designed to give a large increase of torque, the efficiency at smaller speed ratio approaches unity

    Which of these statements are correct?
    (a) 1, 2, 3 and 4 (b) 1, 2 and 3 (c) 1, 2 and 4 (d) 3 and 4
18. In contrast to fluid coupling, torque converters are operated
    (a) While completely filled with liquid (b) While partially filled with liquid
    (c) Without liquid (d) While completely filled with air
19. Consider the following statements regarding torque converter.
    1. Its maximum efficiency is less than that of the fluid coupling
    2. It has two runners and a set of stationary vanes interposed between them
    3. It has only one runner and a stator
    4. The ratio of secondary to primary torque is zero for the zero value of angular velocity of secondary runner

    Which of these statements are correct?
    (a) 1 and 2 (b) 3 and 4 (c) 1 and 4 (d) 2 and 4
20. The component of the torque converter that allows torque multiplication is
    (a) Turbine (b) Impeller (c) Stator (d) Free wheel

21. Which one of the following graphs as shown in Figure 10.39 represents the characteristics of a torque converter? (Where suffix 2 stands for turbine runner and 1 stands for pump impeller)

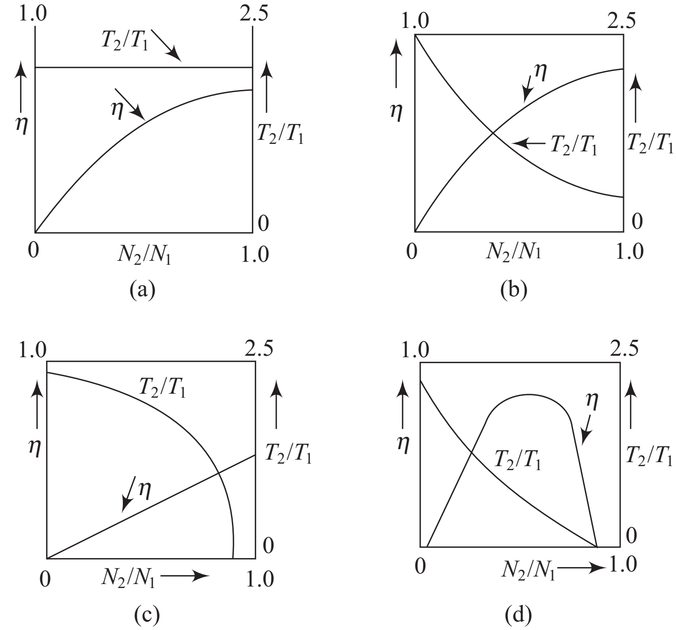


Figure 10.39 *Multiple Choice Question 21*

22. The stalling of a torque converter is due to
    (a) Input torque less than the stator blades torque
    (b) The negative input torque
    (c) Input torque more than the stator blades torque
    (d) Input torque less than the frictional torque
23. In a torque multiplier
    (a) The stator blade torque is zero
    (b) The stator blade torque is in the same direction as the input torque
    (c) The stator blade torque is in the opposite direction of the input torque
    (d) The stator blade torque is more than the input torque
24. Which one of the following statements is wrong about the transmitted torque?
    (a) Proportional to the radius of the impeller at the outlet
    (b) Proportional to the square of the radius of impeller at the outlet
    (c) Proportional to the speed of the impeller
    (d) Proportional to the mass flow rate
25. In a torque converter, as the input shaft speed varies from zero to rated value
    (a) The slip starts from zero and reaches 95%
    (b) The slip starts from 1.0 and reaches zero
    (c) The slip starts from 1.0 and reaches 5%
    (d) The slip starts from 1.0 and remains constant

26. The function of which of the following hydraulic devices is analogous to that of the flywheel of a reciprocating steam engine and an electric storage battery?
    (a) Hydraulic ram
    (b) Hydraulic accumulator
    (c) Hydraulic intensifier
    (d) Hydraulic jack
27. Which one of the following pairs is not a positive displacement pump?
    (a) Reciprocating pump
    (b) Jet pump
    (c) Sliding vane pump
    (d) Lobe pump
28. Which one of the following pairs is not correctly matched?
    (a) Centrifugal Pump: Rotating blades in the rotor create centrifugal head
    (b) Reciprocating Pump: Positive displacement pump
    (c) Turbine Pump: Centrifugal pump with guide vanes
    (d) Gear Pump: Gear teeth work like rotating blades to create centrifugal head
29. A hydraulic press has a ram of 0.2 m diameter and a plunger of 50 mm diameter. The force required at the plunger to lift a weight of 16×10^4 N shall be
    (a) 256×10^4 N
    (b) 64×10^4 N
    (c) 4×10^4 N
    (d) 1×10^4 N
30. An accumulator is a device to store
    (a) Sufficient quantity of liquid to compensate the change in discharge
    (b) Sufficient energy to drive the machine when the normal energy source does not function
    (c) Sufficient energy in case of machines which work intermittently to supplement the discharge from the normal source
    (d) Liquid which otherwise would have gone to waste
31. **Assertion (A):** A hydraulic ram is a device used to lift water from deep wells.

    Reason (R): Hydraulic ram works on the principle of water hammer.
    (a) Both A and R are true and R is a correct explanation of A
    (b) Both A and R are true but R is not a correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true
32. Consider the following statements about a hydraulic ram.
    1. Hydraulic ram does not need any external power
    2. It works on the fundamental principle of water hammer
    3. The efficiency of a hydraulic ram is only of the order of 8 to 10%
    4. It can be termed as low, intermittent discharge low head pumping installation

    Which of these statements are correct?
    (a) 2 and 3
    (b) 1, 2, 3 and 4
    (c) 1 and 2
    (d) 1 and 3
33. A hydraulic ram is a pump which works on the principle of
    (a) Water hammer
    (b) Centrifugal action
    (c) Reciprocating action
    (d) Hydraulic press
34. A simple hydraulic intensifier has a piston diameter of 40 mm and 80 mm. If the supply pressure is 750 kPa, the pressure to which the liquid is raised is
    (a) 1500 kPa
    (b) 2250 kPa
    (c) 3000 kPa
    (d) 6000 kPa

35. A hydraulic accumulator has a ram of cross-sectional area of 3×10^4 mm^2 and stroke of 2 m. It is supplied with water at 500 kPa. The capacity of the accumulator is
(a) 500 kN.m (b) 0.06 kN.m (c) 15 kN.m (d) 30 kN.m

36. A total load of 50 kN produces a pressure of 375 kPa in a hydraulic accumulator. If 5% of the load is consumed in overcoming the frictional resistance at packing, what additional load is required to produce a pressure of 750 kPa in the cylinder?
(a) 47.5 kN (b) 50 kN (c) 52.5 kN (d) 55 kN

37. Which one of the following devices is used to lift a small stream of water to a great height?
(a) Hydraulic ram (b) Hydraulic crane (c) Hydraulic lift (d) Hydraulic coupling

38. A hydraulic ram delivers 2 litres/*s* of water to a height of 28 m above the ram. The water is supplied to the ram from a height of 2.8 m at the rate of 35 litres/*s*. The efficiency of the ram is
(a) 57% (b) 55% (c) 52% (d) 44%

39. A hydraulic crane utilizes 60 litres/*s* of water at 6 MPa pressure to lift a weight of 20 kN to a height of 12 m. The efficiency of the crane is
(a) 87.7 % (b) 66.7% (c) 50.3% (d) 45.5%

40. **Assertion (A):** Throttle governing is thermodynamically more efficient than nozzle control governing for steam turbines.

Reason (R): Throttling process conserves the total enthalpy.
(a) Both A and R are true and R is a correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

41. The lines *abc* and *cd* shown in the Figure 10.40 below are known as
(a) Wilson's line (b) Willian's line (c) S.C. line (d) Throttling line

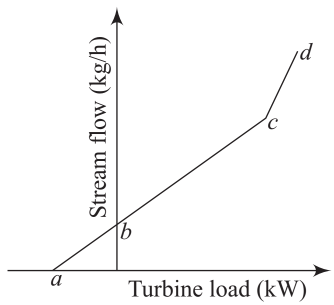


Figure 10.40 *Multiple Choice Question 41*

42. Consider the following statements regarding the nozzle governing of steam turbines.
1. Working nozzles receive steam at full pressure
2. High efficiency is maintained at all loads
3. Stage efficiency suffers due to partial admission
4. In practice, each nozzle of the first stage is governed individually

Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2 (c) 1, 3 and 4 (d) 1, 2 and 4

43. **Assertion (A):** The speed of the governed turbine is constant irrespective of the load.

Reason (R): In governing, the steam supply is regulated to maintain the speed.
(a) Both A and R are true and R is a correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

44. (A): With throttle governing of a steam turbine, the turbine power is reduced by reduction in available heat drop together with decrease in the rate of heat flow.

Reason (R): The pressure and the rate of steam flow are simultaneously decreased with the help of a throttle valve.
(a) Both A and R are true and R is a correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

45. Consider the following statements:
1. Throttle governing improves quality of steam in the last few stages.
2. Internal efficiency of steam is not seriously affected by throttle governing.
3. Throttling governing is better than nozzle governing.

Which of these statements is/are correct?
(a) 1, 2 and 3 (b) 1 and 3 (c) 2 and 3 (d) 1 and 2

46. An emergency governor of a steam turbine trips the turbine when
1. Shaft exceeds 100% of its rated speed
2. Condenser becomes hot due to inadequate cooling water circulation
3. Lubrication system fails
4. Balancing of turbine is not proper

Which of these statements is/are correct?
(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 3, 4 and 1 (d) 4, 1 and 2

47. **Assertion (A):** Throttle governing is used only in small steam turbines.

Reason (R): At part loads, the efficiency of steam turbine reduces considerably with throttle governing.
(a) Both A and R are true and R is a correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

48. **Assertion (A):** Nozzle control governing cannot be used in reaction steam turbines.

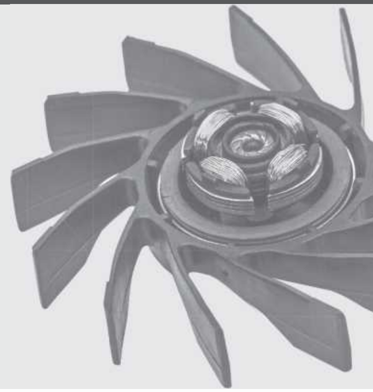
Reason (R): In reaction steam turbines, full admission of steam must take place.
(a) Both A and R are true and R is a correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

49. Throttle governing in steam turbines
(a) Leads to significant pressure loss (b) Increases the efficiency
(c) Increases heat losses (d) Decreases steam temperature

50. Willian's line represents
    (a) Total steam consumption vs. power output with throttle governing
    (b) Total steam consumption vs. power output with cut-off governing
    (c) Behaviour of supersaturated steam through nozzles
    (d) Condensation of steam while flowing through the turbine

ANSWER KEY

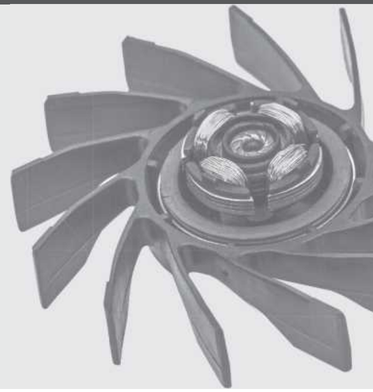
1. (a)	2. (c)	3. (b)	4. (c)	5. (d)	6. (c)	7 (a)	8. (d)	9. (a)	10. (b)
11. (a)	12. (a)	13. (d)	14. (c)	15. (a)	16. (d)	17. (a)	18. (a)	19. (a)	20. (c)
21. (d)	22. (d)	23. (b)	24. (a)	25. (c)	26. (b)	27. (b)	28. (d)	29. (d)	30. (c)
31. (d)	32. (c)	33. (a)	34. (c)	35. (d)	36. (b)	37. (a)	38. (a)	39. (b)	40. (d)
41. (b)	42. (a)	43. (a)	44. (a)	45. (d)	46. (b)	47. (d)	48. (a)	49. (a)	50. (a)



Bibliography

1. Denis G Shepard, *Principles of Turbomachinery,* Macmillan, 1956.
2. SL Dixon, C Hall, *Fluid Mechanics and Thermodynamics of Turbomachinery*, Butterworth-Heinemann, 2014.
3. William W Peng, *Fundamentals of Turbomachinery*, John Wiley and Sons, 2008.
4. Terry Wright and Philip Gerhant, *Fluid Machinery: Application, Selection and Design*, CRC Press, 2009.
5. Larry Bachus and Angel Custodio, *Know and Understand Centrifugal Pumps*, Elsevier, 2013.
6. RK Turton, *Principles of Turbomachinery*, Springer, 1995.
7. Val S Lobanoff and Robert R Ross, *Centrifugal Pumps: Design and applications,* Gulf Professional Publishing, 1992.
8. S M Yahya, *Turbines, Compressors and Fans*, Tata McGraw Hill Education Private Limited, New Delhi, 2011.
9. K Subramanya, *Hydraulic Machines*, Tata McGraw Hill Education Private Limited, New Delhi, 2013.
10. George Frederick Round, *Incompressible Flow Turbomachines: Design, Selection, Application and Theory,* Butterworth, Heinemann, 2004.
11. AT Sayers, *Hydraulic and Compressible Flow Turbomachines*, McGraw Hill Book Company, 1990.
12. Yunus A Cengel and John M Cimbala, *Fluid Mechanics: Fundamentals and Applications,* Tata McGraw Hill Publishing Company Limited, New Delhi, 2007.
13. Irving H Shames, *Mechanics of Fluids,* McGraw Hill Higher Education, 2003.
14. BS Messay, *Mechanics of Fluids,* ELBS and Van Nustrand Reinhold (UK) Co. Limited, New Delhi, 1983.
15. S K Som and G Biswas, *Introduction to Fluid Mechanics and Fluid Machines*, Tata McGraw Hill Publishing Company Limited, New Delhi, 2008.
16. AK Mohanty, *Fluid Mechanics*, PHI Learning Private Limited, New Delhi, 2012.
17. Brucer R Munson, Donald F Young, Theodore H Okiishi and Wade W Huebsch; *Fundamentals of Fluid Mechanics*, Wiley India (P) Limited, New Delhi, 2010.
18. G Gopalkrishnan and D Prithvi Raj, *A Treatise on Turbomachines*, Scitech Publications, Chennai, 2008.
19. SW Yuvan, *Foundation of Fluid Mechanics*, Prentice Hall of India Pvt. Ltd., New Delhi, 1998.
20. AurelStodola, *Steam and Gas Turbines*, McGraw Hill Book Company, New York, 1927.
21. FR Young, *Cavitation*, McGraw HillBook Company, New York, 1989.

22. Adrian Bejan, Advanced Engineering Thermodynamics, 4th Edition, John Wiley & Sons Inc., New Jersey, 2016.
23. MJ Moran and HN Shapiro et al., *Fundamentals of Engineering Thermodynamics,* John Wiley & Sons Inc., New Jersey, 2013.
24. Yunus A Cengel and MA Boles, *Thermodynamics An Engineering Approach*, 7th Edition, McGraw Hill Education (India) Private Ltd, New Delhi, 2014.
25. PK Nag, *Engineering Thermodynamics*, 6th Edition, McGraw Hill Education, New Delhi, 2018.
26. Frank M White, *Fluid Mechanics*, Tata McGraw Hill Publishing Company Limited, New Delhi, 2008.
27. V Kadambai and Manohar Prasad, *Energy Conversion Systems, Volume-III, Turbomachines,* New Age International Publishers, New Delhi, 2005.
28. Rama SR Gorla and Aijaz Khan, *Turbomachinery Design and Theory*, Marcel Dekker, Inc, New York, 2003
29. Ahmad Nourbaksh, Andre Jaumotte, Charles Hirsch and Hamideh B Parizi, *Turbopumps and Pumping Systems*, Springer, Verlag Berlin, Heidelberg, 2008.
30. Erik Dick, *Fundamentals of Turbomachines*, Springer, Verlag Berlin, Heidelberg, 2015.
31. Hassan M Badr and Wael H Ahmad, *Pumping Machinery Theory and Practice*, John Wiley and Sons Ltd., West Sussex, 2015.
32. Jack D Mattingly, *Elements of Propulsion: Gas Turbines and Rockets*, American Institute of Aeronautics and Astronautics, Virginia, 2006.
33. KM Srinivasan, *Rotodynamic Pumps (Centrifugal and Axial)*, New Age International Publishers, New Delhi, 2008.
34. Lev Nelik, Centrifugal and Rotary Pumps, CRC Press, Florida, 1999.
35. Seppo A Korpela, *Principles of Turbomachinery*, John Wiley & Sons Inc., New Jersey, 2011.
36. H Cohen, GFC Rogers and HIH Saravanamutto, *Gas Turbine Theory, 4Th Edition,* Longman Group Limited, Essex, 1996.
37. BudugurLakshminarayana, *Fluid Dynamics and Heat Transfer of Turbomachinery*, John Wiley & Sons Inc., New York, 1996.
38. Wilson DG, *The design of High Efficiency Turbomachinery and Gas Turbines*, MIT Press, Cambridge, 1984.
39. Wallis RA 1983, Axial Flow Fans and Ducts, John Wiley & Sons Inc., New Jersey, 1983.
40. Eck B, *Fans*, Pergaman Press, New York, 1973.
41. AJ Stepanoff, *Centrifugal and Axial Flow Pumps: Theory, Design and Applications*, John Wiley & Sons Inc., New York, 1948.
42. Michael Volk, *Pump Characteristics and Applications*, 3rd Edition, CRC Press, Florida, 2014.
43. Viktor Gelpke and Aaron Howell Van Cleve, *Hydraulic Turbines: Their Design and Applications,* Mc Graw Hill Book Company, 1910.
44. Robert Long Daugherty, *Hydraulic Turbines*, Mc Graw Hill Book Company, 1920.
45. Fox and Mc Donald, *Introduction to Fluid Mechanics*, 8th Edition, Jon Wiley & Sons Inc., 2011.
46. NS Govinda Rao, *Fluid Flow Machines,* Tata Mc Graw Hill, New Delhi, 1983.
47. EF Church, *Steam Turbines*, McGraw Hill, New York, 1950.
48. Z Hussain, *Steam Turbines*, Tata McGraw Hill, New Delhi, 1984.
49. WJ Kearton, *Steam Turbine Theory and Practice*, ELBS and Pitman, London, 1958.
50. JF Lee, *Theory and Design of Steam and Gas Turbines, McGraw Hill Company, New York, 1954.*
51. BU Pai, *Turbomachines*, Wiley India Pvt. Ltd. New Delhi, 2015.
52. SC Gupta, *Fluid Mechanics and Hydraulic Machines,* Pearson, New Delhi, 2012.



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