BASIC ELECTRICAL AND INSTRUMENTATION ENGINEERING

About the Authors

S. Salivahanan is the Principal of SSN College of Engineering, Chennai. He obtained his B.E. degree in Electronics and Communication Engineering from PSG College of Technology, Coimbatore, M.E. degree in Communication Systems from NIT, Trichy and Ph.D. in the area of Microwave Integrated Circuits from Madurai Kamaraj University. He has four decades of teaching, research, administration and industrial experience both in India and abroad. He has taught at NIT, Trichy, A.C. College of Engineering and Technology, Karaikudi, RV College of Engineering, Bangalore, and Mepco Schlenk Engineering College, Sivakasi. He has industrial experience as Scientist/Engineer at Space Applications Centre, ISRO, Ahmedabad, Telecommunication Engineer at State Organization of Electricity, Iraq and Electronics Engineer at Electric Dar Establishment, Kingdom of Saudi Arabia.

He is the author of 40 popular books which include all-time bestsellers such as *Basic Electrical and Electronics Engineering*, *Electronic Devices and Circuits*, *Linear Integrated Circuits*, and *Digital Signal Processing*, all published by McGraw Hill Education. He has also authored the books on *Digital Circuits and Design*, *Electromagnetic Field Theory*, *Circuit Theory, Network Analysis and Synthesis* and *Control Systems Engineering*. He has published several papers at national and international levels.

Professor Salivahanan is the recipient of Bharatiya Vidya Bhavan National Award for Best Engineering College Principal for 2011 from ISTE, and IEEE Outstanding Branch Counsellor and Advisor Award in the Asia-Pacific region for 1996–97. He was the Chairman of IEEE Madras Section for two years 2008 and 2009 and Syndicate Member of Anna University.

He is a Senior Member of IEEE, Fellow of IETE, Fellow of Institution of Engineers (India), Life Member of ISTE and Life Member of Society for EMC Engineers. He is also a member of IEEE societies in Microwave Theory and Techniques, Communications, Signal Processing, and Aerospace and Electronics.

R. Rengaraj is Associate Professor in the Department of Electrical and Electronics Engineering, SSN College of Engineering, Chennai. He obtained his B.E. degree in Electrical and Electronics Engineering from Manonmaniam Sundaranar University, Tirunelveli, M.E. degree in Power Systems Engineering and Ph.D. in the area of Combined Heat and Power, both from Anna University, Chennai. He has authored a book on *Control Systems Engineering*. He has more than 13 years of teaching and research experience. He has published several research publications in refereed international journals and in the proceedings of international conferences. He has received TATA Rao Gold Medal from the Institution of Engineers (India) in 2011. He is a Life Member of ISTE and a Member of IEEE.

G.R. Venkatakrishnan is Assistant Professor in the Department of Electrical and Electronics Engineering, SSN College of Engineering, Chennai. He obtained his B.E. degree in Electrical and Electronics Engineering and M.E. degree in Control Systems from Anna University, Chennai. He has authored a book on *Control Systems Engineering* and has published many research papers in national and international journals and conferences. He is a Life Member of ISTE and a Member of IEEE.

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S. Salivahanan
Principal
SSN College of Engineering
Chennai

R. Rengaraj
Associate Professor
Department of Electrical and Electronics Engineering
SSN College of Engineering
Chennai

G.R. Venkatakrishnan
Assistant Professor
Department of Electrical and Electronics Engineering
SSN College of Engineering
Chennai

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PREFACE

Basic Electrical and Instrumentation Engineering is designed specifically to cater to the needs of second semester ECE students. The book has a perfect blend of focused content and complete coverage. Solved university question papers, which are tagged with specific topics, will be extremely helpful to students from the examination point of view. Simple, easy-to-understand and jargon-free text elucidates the fundamentals of Electrical and Instrumentation Engineering. Several solved examples, schematic diagrams and adequate questions further helps students to understand and apply the concepts.

SALIENT FEATURES

✓ Comprehensive coverage as per the latest *Basic Electrical and Instrumentation Engineering* syllabus

✓ Solutions of examination papers from 2010 to 2017 are present appropriately within the chapters
- Solved university questions as solved examples incorporated appropriately within each chapter
- Theory questions are tagged within each chapter

✓ Rich exam-oriented pedagogy
- Solved Numerical Examples within chapters: 96
- Two-Mark Questions and Answers at the end of each chapter: 150
- Unsolved Review Questions: 270

CHAPTER ORGANISATION

➤ **Chapter 1** deals with three phase power supply, three phase power measurement, transmission and distribution of electrical energy, protection of power system, tariff and power factor improvement.

➤ **Chapter 2** is devoted to introduction of ideal transformer, circuit model of transformer and determination of parameters, efficiency and auto-transformers.

➤ **Chapter 3** describes the constructional features of various motors, methods of excitation and magnetisation characteristics, starting and speed control and universal motor.

➤ **Chapter 4** focuses on the principle of operation of three-phase induction motors, equivalent circuit, single phase induction motors, types of starting and speed control methods, working principle of alternator, synchronous motors, stepper motors and brushless DC motors.

➤ **Chapter 5** concentrates on the type of electrical and electronic instruments, principles of electrical instruments, multimeters, oscilloscopes, transducers and their classifications and applications.

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S. Salivahanan
R. Rengaraj
G. R. Venkatakrishnan

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CHAPTER 1

AC Circuits and Power Systems

1.1 Introduction

Electrical power is generated, transmitted, distributed in sinusoidal form for the commercial, industrial and domestic applications. In general, two types of electrical power can be generated: single-phase power and poly-phase power. The main disadvantage of single-phase power supply is that it can carry only a reasonable amount of power but poly-phase system is normally used to generate, transmit and distribute bulk electric power. This chapter deals with the three-phase system, which is a type of poly-phase system. The generation of three-phase electric power, the relationship between voltage and current, and their power measurements are also discussed.

Further, the transmission and distribution of electric power, the necessity of protecting the power system and operation of various protective devices like circuit breaker, fuse and relay are explained. Tariff refers to the price of electrical energy that the consumer is charged for consumption. Tariff plays a major role in maintaining a healthy relation between the supplier and consumer. Hence, due consideration has to be given in fixing the tariff and the consumers must be charged with different tariffs, based on their usage. The different objectives and characteristics of tariff, factors affecting the tariff, and different types of tariff are discussed in this chapter. In an AC power system, power factor plays a major role in analysing the system performance. If the power factor is low, heavy current will flow and damage the system. The causes of low power factor, its consequences and the methods to improve the power factor are described in this chapter.

1.2 Three-Phase Supply

In an electrical power system, there are two types of systems, namely: single-phase and poly-phase systems. A single-phase system consists of two wires, where the current flows through one wire and returns through another wire, when it is energized by two terminals. Generally, in most of the households and small industries where the required capacity of a motor is not greater than 5 horsepower, single-phase systems are used. But nowadays, a three-phase system, which is a type of poly-phase system, is used to generate, transmit and distribute electrical energy.

In a three-phase system, three conductors can carry three alternating electrical quantities at the same frequency. The electrical quantities in these three conductors reach the same peak amplitude at different instances, as shown in Figure 1.1.

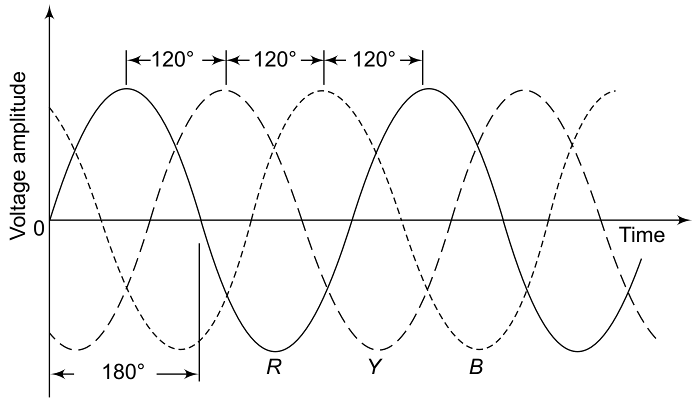


Figure 1.1 Three-phase Power System of an Electrical Quantity

If any one of the alternating quantities is taken as a reference, the other two are delayed by one-third and two-thirds of a cycle of the alternating quantity i.e., 120° apart. Therefore, three-phase power systems can be viewed as the combination of three separate single-phase systems, with 120° phase difference.

1.2.1 Advantages of a Three-phase Power System

The advantages of a three-phase system over a single-phase system are:

1. The power-to-weight ratio of a three-phase system is high when compared to a single-phase system i.e., for the same electrical power, the size of energy source required in a three-phase system is less when compared to a single-phase system.
2. For transmission of electrical power, the requirement of conductor material in a three-phase system is less, when compared to a single-phase system.
3. Instantaneous power is always constant in a three-phase system, when compared to pulsating power in a single-phase system.
4. A three-phase system will have better power factor and efficiency, when compared to a single-phase system.
5. For a given size of the system, higher output power can be obtained in a three-phase system, when compared to a single-phase system.
6. A three-phase system is efficient, reliable, economical and has better regulation when compared to a single-phase system.
7. When a fault occurs in a single line, the other two lines can be used to transmit the power to the load.
8. Motors that operate on a three-phase supply do not require any starting devices like a capacitor to run it.
9. Torque produced in the motor using a three-phase supply is uniform, when compared to pulsating torque produced by using a single-phase supply.
10. A three-phase system can be used to supply the electrical energy for a single-phase load.
11. A three-phase supply can be rectified into DC supply with very low ripple factor, when compared to a single-phase supply.
12. Parallel operation is easy in a three-phase system, when compared to a single-phase system.

1.3 Basics of a Three-Phase Power System

The colour codes of the wires used in a three-phase system vary from country to country. In India, Red (R), Yellow (Y) and Blue (B) are the colour codes used in three-phase systems. The two different configurations by which the three wires in a three-phase system are connected are: star (Y) connection and delta (Δ) connection. The different types of three-phase power systems are: (i) three-phase, three-wire system and (ii) three-phase, four-wire system. The fourth wire in a three-phase four-wire system is the neutral wire, represented in black colour. It is known that the three-phase power system can be used as source and load.

As a source, a three-phase power system can be used as either three or four-wire star connection or three-wire delta connection. Similarly, as a load, depending upon the application, the type of connection and configuration of a three-phase power system varies. The different terms used in three-phase power systems are described as follows:

- **Phase:** A branch of the circuit in a three-phase system is known as a phase.
- **Line:** The wire that connects the source and load is known as transmission line or line.
- **Neutral:** The fourth wire in the three-phase system, where all the phases in a star connection are connected together is known as neutral.
- **Phase voltage:** The voltage measured between a line and neutral or the voltage across a particular phase is called as phase voltage. It is represented as $\overline{V}_{RB} = \overline{V}_{RN} - \overline{V}_{BN}$ or simply $\overline{V}_R, \overline{V}_Y, \overline{V}_B$.
- **Line voltage or line-to-line voltage:** The voltage measured between any two lines in a three-phase power system is known as line voltage. It is represented as $\overline{V}_{RY}, \overline{V}_{YB}$ and $\overline{V}_{BR}$ and is given by $\overline{V}_{RY}, = \overline{V}_R - \overline{V}_Y, \overline{V}_{YB}, = \overline{V}_Y - \overline{V}_B$ and $\overline{V}_{BR} = \overline{V}_B - \overline{V}_R$ respectively.
- **Line currents:** The currents flowing through a particular line are called line currents, represented by $\overline{I}_R, \overline{I}_Y$ and $\overline{I}_B$.
- **Phase current:** The current flowing through a single-phase or a branch of the system is called as phase current. It is represented as $\overline{I}_{RY}, \overline{I}_{YB}$ and $\overline{I}_{BR}$ and is given by $\overline{I}_{RY} = \overline{I}_R - \overline{I}_Y, \overline{I}_{YB} = \overline{I}_Y - \overline{I}_B$ and $\overline{I}_{BR} = \overline{I}_B - \overline{I}_R$ respectively.
- **Load impedance:** For a star-connected load, the impedance between the line and neutral is called load or line impedance and for a delta-connected load, the impedance between two lines is called load or phase impedance.
- **Phase sequence:** The time order or the sequence in which the electrical quantity in the three-phase system reach their respective maximum values is known as phase sequence. If the phase sequence of a particular system is RYB, then it indicates that R phase reaches the maximum value of electrical quantity at first and then followed by Y phase and B phase.
- **Balanced condition:** The condition for having a balanced source or a balanced load is given below.
  - (i) Balanced source: A three-phase system is said to be a balanced source, if the phase voltage of each phase has the same magnitude and frequency and the phase difference between the lines is 120°.
  - (ii) Balanced load: A three-phase system is said to be a balanced load if the impedance is same for all the phases, either in star or delta connection.
- **Unbalanced condition:** If the load impedance differs in one or more phases, then the three-phase system is said to be an unbalanced load. This unbalanced condition leads to changes in line and phase currents.
- **Three-phase source**: If the three-phase system is used to generate a three-phase power supply, then it is said to be a three-phase source.
- **Three-phase load:** If the three-phase system uses the three-phase supply to perform certain functions, then it is said to be a three-phase load.

- **Power factor:** The cosine of the angle between phase voltage and the phase current is known as power factor. It can be lagging, leading or unity, depending upon the type of load connected to the system. If the phase current lags behind the phase voltage, then it is a lagging power factor load. If the phase current leads the phase voltage, then it is a leading power factor load. Similarly, if the phase current is in phase with the phase voltage, then it is a unity power factor load.
- **Phasor diagram:** The diagram that represents the line voltage, phase voltage, line current and phase current of a three-phase source or a three-phase load is known as a phasor diagram. In a star-connected three-phase system, phase voltage is taken as the reference; while, in a delta-connected three-phase system, line voltage is taken as the reference.

The schematic diagrams of a three-phase star-connected power system with three wires and four wires are shown in Figures 1.2 (a) and (b) respectively.

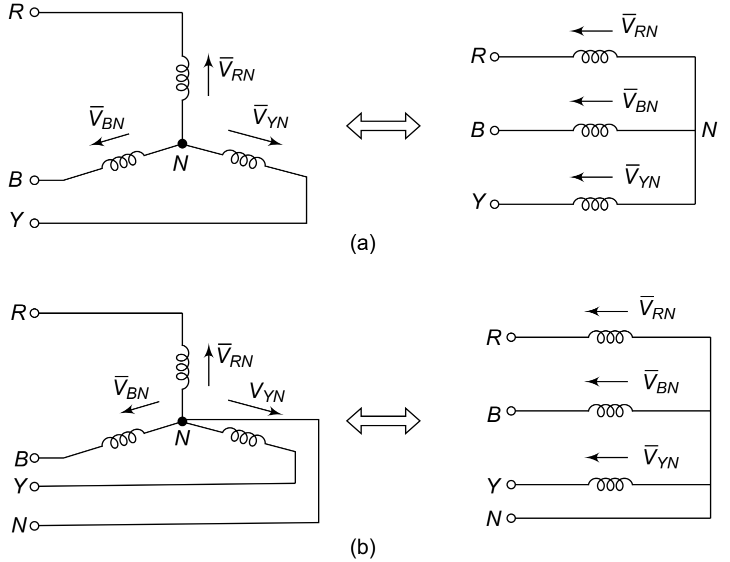


Figure 1.2 Schematic Diagram of a Star-connected Three-phase System

The schematic diagram of a three-phase delta-connected power system is shown in Figure 1.3.

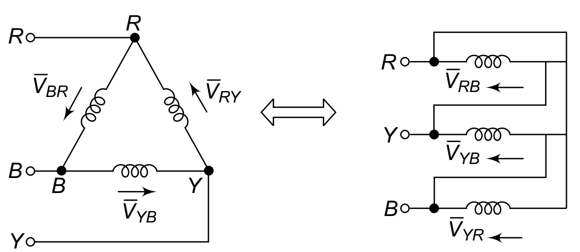


Figure 1.3 Schematic Diagram of a Delta-connected Three-phase System

The relation between the phase voltages in star-connected balanced three-phase system is given by:

$$\left.\begin{aligned} \bar{V}_{RN} &= |\bar{V}_m|\angle 0^\circ \\ \bar{V}_{YN} &= |\bar{V}_m|\angle -120^\circ \\ \bar{V}_{BN} &= |\bar{V}_m|\angle +120^\circ \text{ or } |\bar{V}_m|\angle -240^\circ \end{aligned}\right\} \tag{1.1}$$

where, V_m is the maximum value of the voltage in volts

Similarly, the relation between the line voltages in a delta-connected balanced three-phase system is given by:

$$\left.\begin{aligned} \bar{V}_{RY} &= |\bar{V}|\angle 0^\circ \\ \bar{V}_{YB} &= |\bar{V}|\angle -120^\circ \\ \bar{V}_{BR} &= |\bar{V}|\angle +120^\circ \text{ or } |\bar{V}|\angle -240^\circ \end{aligned}\right\} \tag{1.2}$$

The vector diagrams of balanced star and delta-connected power systems using the above equations are shown in Figures 1.4 (a) and (b) respectively.

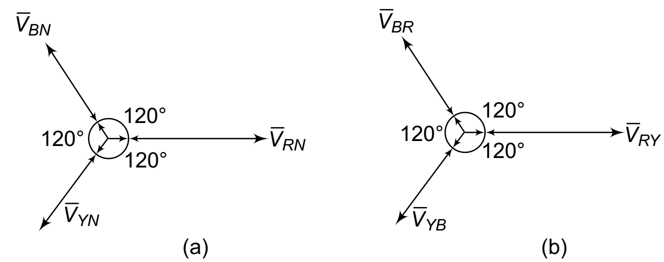


Figure 1.4 Vector Diagram of Balanced (a) Star (b) Delta-connected Systems

1.4 Generation of Three-Phase Voltages

The primary requirement in a three-phase system before analysing the balanced and unbalanced condition is the generation of three-phase voltages. A three-phase AC generator or an alternator is used to generate the three-phase voltages. The two main components of AC generator or an alternator are the field and armature, in which either field or armature is stationary. Therefore, the alternator configurations by which the three-phase voltage can be generated are: (i) stationary field with rotating armature and (ii) rotating field with stationary armature. The instantaneous phase voltages of the three-phase system, when connected in star are given by:

$$\left.\begin{aligned} \bar{V}_{RN} &= V_m \sin(\omega t) \\ \bar{V}_{YN} &= V_m \sin(\omega t - 120^\circ) \\ \bar{V}_{BN} &= V_m \sin(\omega t - 240^\circ) = V_m \sin(\omega t + 120^\circ) \end{aligned}\right\} \tag{1.3}$$

where, V_m is the maximum value of the voltage in volts.

Adding all the instantaneous voltages given in Eqn. (1.3), we get

$$\begin{aligned} \bar{V}_{RN} + \bar{V}_{YN} + \bar{V}_{BN} &= V_m \sin \omega t + V_m \sin(\omega t - 120°) + V_m \sin(\omega t + 120°) \\ &= V_m[\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 120° + \cos \omega t \sin 120°] \\ &= V_m[\sin \omega t + 2 \sin \omega t \cos 120°] = V_m\left[\sin \omega t + 2 \sin \omega t\left(\frac{-1}{2}\right)\right] = 0 \end{aligned}$$

Therefore, $\overline{V}_{RN} + \overline{V}_{YN} + \overline{V}_{BN} = 0$ (1.4)

It is clear from Eqn. (1.4) that the phasor addition of all the phase voltages at any instant in a three-phase balanced star-connected system is always zero. Similarly, if the instantaneous line voltages of the three-phase system, when connected in delta connection, are added, we get $\overline{V}_{RY} + \overline{V}_{YB} + \overline{V}_{BR} = 0$.

1.5 Analysis of The Three-Phase System

The different three-phase systems for which the relationship between phase and line voltages, phase and line currents, power, and phasor diagrams are discussed as follows:

1. Three-phase balanced star-connected source
2. Three-phase balanced delta-connected source
3. Three-phase balanced star-connected load
4. Three-phase balanced delta-connected load
5. Three-phase unbalanced delta-connected load
6. Three-phase unbalanced four-wire star-connected load
7. Three-phase unbalanced three-wire star-connected load

1.5.1 Three-Phase Balanced Star-Connected Source [AU April/May, 2015]

The circuit diagram for a three-phase balanced star-connected source with phase sequence RYB is shown in Figure 1.5.

In a balanced system, all the magnitudes of phase voltages, line voltages, phase currents and line currents are equal, which can be represented as:

$$|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}|\,;\, |\overline{V}_{RY}| = |\overline{V}_{YB}| = |\overline{V}_{BR}| = |\overline{V}_L| \quad (1.5)$$

$$|\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L|\,;\, |\overline{I}_{RY}| = |\overline{I}_{YB}| = |\overline{I}_{BR}| = |\overline{I}_{\text{ph}}| \quad (1.6)$$

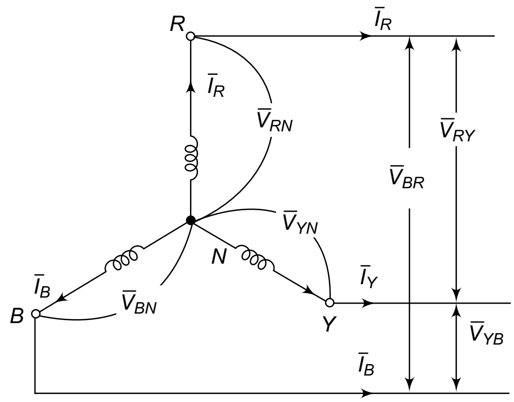


Figure 1.5 Circuit Diagram for a Three-phase Balanced Star-connected Source

Relationship Among Line and Phase Quantities

Current Relationship

Appling Kirchhoff's current law at nodes R, Y and B shown in Figure 1.5, we get:

$$\overline{I}_{RY} = \overline{I}_R;\, \overline{I}_{YB} = \overline{I}_Y\,;\, \overline{I}_{BR} = \overline{I}_B \quad (1.7)$$

From Eqns. (1.6) and (1.7), we can conclude that, in a balanced star-connected three-phase source, phase current is equal to the line current, as given by

$$\overline{I}_{\text{ph}} = \overline{I}_L \quad (1.8)$$

Voltage Relationship

It is known that, $\overline{V}_{RY} = \overline{V}_{RN} - \overline{V}_{YN}$

Using the parallelogram law of addition and the vector diagram shown in Figure 1.6, we get:

$$|\bar{V}_{RY}| = \sqrt{|\bar{V}_{RN}|^2 + |\bar{V}_{YN}|^2 + 2\,|\bar{V}_{RN}||\bar{V}_{YN}| \cos 60^\circ}$$

Using Eqn. (1.5) in the above equation and solving, we get:

$$|\bar{V}_{RY}| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \tag{1.9}$$

Similarly, we get:

$$|\bar{V}_{YB}| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \text{ and } |\bar{V}_{BR}| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \tag{1.10}$$

Therefore, using Eqns. (1.5), (1.9) and (1.10), we get the relation between the line and phase voltages, which is

$$|\bar{V}_L| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \tag{1.11}$$

Hence, it can be concluded that in a star-connected balanced three-phase source, the line voltage is $\sqrt{3}$ times the phase voltage or that the phase voltage is $\frac{1}{\sqrt{3}}$ times the line voltage. It is to be noted that the angle between the phase voltage and the line voltage is 30°.

Vector Diagram

The vector diagram for a three-phase balanced star-connected source, by considering the phase voltage as reference, is shown in Figure 1.6.

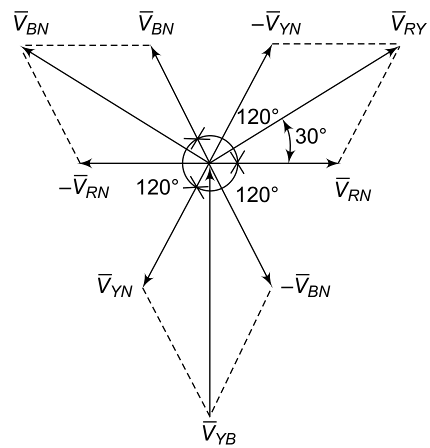


Figure 1.6 Vector Diagram for a Three-phase Balanced Star-connected Source

Power Relationship

The real power produced per phase in the system shown in Figure 1.5 is $P_{\text{ph}} = |\bar{V}_{\text{ph}}||\bar{I}_{\text{ph}}| \cos\phi$.

Therefore, the total real power produced in the system is given by

$$P = 3\,|\bar{V}_{\text{ph}}||\bar{I}_{\text{ph}}| \cos\phi \tag{1.12}$$

Using Eqns. (1.8) and (1.11), we get

$$P = 3\frac{|\bar{V}_L|}{\sqrt{3}}|\bar{I}_L| \cos\phi = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \cos\phi \qquad \text{(W)} \tag{1.13}$$

Similarly, the total reactive power, Q, and total apparent power, S, produced in the system are given by:

$$Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \sin\phi \qquad \text{(VAR)} \tag{1.14}$$

$$S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \qquad \text{(VA)} \tag{1.15}$$

1.5.2 Three-Phase Balanced Delta-Connected Source [AU April/May, 2015]

The circuit diagram for a three-phase balanced delta-connected source with phase sequence RYB is shown in Figure 1.7.

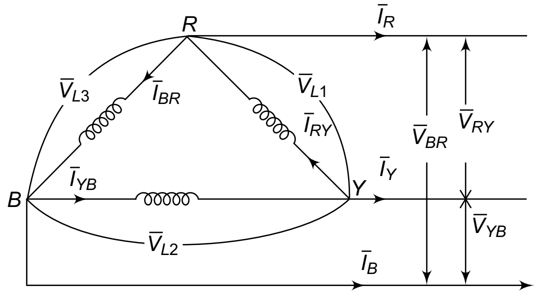


Figure 1.7 Circuit Diagram for a Three-phase Balanced Delta-connected Source

Relationship Among Line and Phase Quantities

Current Relationship

Applying Kirchhoff's current law at the node R in Figure 1.7, we have

$$\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR}$$

Using the vector diagram shown in Figure 1.8, and applying the parallelogram law of addition, we get

$$|\overline{I}_R| = \sqrt{|\overline{I}_{RY}|^2 + |\overline{I}_{BR}|^2 + 2\,|\overline{I}_{RY}||\overline{I}_{RB}| \cos 60°}$$

Solving the above equation by using Eqn. (1.6), we get

$$|\overline{I}_R| = \sqrt{3}\,|\overline{I}_{\text{ph}}| \tag{1.16}$$

Similarly, we get:

$$|\overline{I}_Y| = \sqrt{3}\,|\overline{I}_{\text{ph}}| \text{ and } |\overline{I}_B| = \sqrt{3}\,|\overline{I}_{\text{ph}}| \tag{1.17}$$

Therefore, using Eqns. (1.6), (1.16) and (1.17), we get the relation between the line and phase currents as

$$|\overline{I}_L| = \sqrt{3}\,|\overline{I}_{\text{ph}}| \tag{1.18}$$

Hence, it can be concluded that in a delta-connected balanced three-phase source, the line current is $\sqrt{3}$ times the phase current or that the phase current is $\frac{1}{\sqrt{3}}$ times the line current. It is to be noted that the phase angle between the phase current and the line current is 30°.

Voltage Relationship

Applying Kirchhoff's voltage law to the loop consisting of $\overline{V}_R$ and $\overline{V}_{RY}$ in Figure 1.7, we have:

$$\overline{V}_{RN} = \overline{V}_{RY}\,;\ \overline{V}_{YN} = \overline{V}_{YB} \text{ and } \overline{V}_{BN} = \overline{V}_{BR} \tag{1.19}$$

Using Eqns. (1.5) and (1.19), we can conclude that, in a balanced delta-connected three-phase source, phase voltage is equal to the line voltage, as given in Eqn. (1.20).

$$\overline{V}_{\text{ph}} = \overline{V}_L \tag{1.20}$$

Vector Diagram

The vector diagram for a balanced delta-connected source, by considering the phase currents as reference vector, is shown in Figure 1.8.

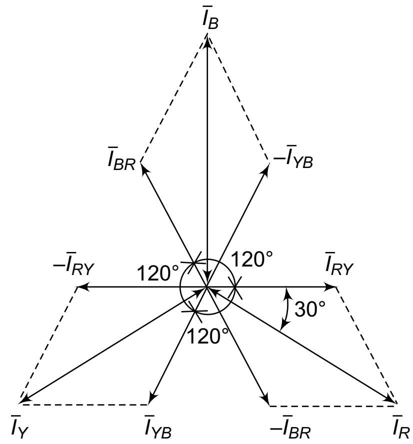


Figure 1.8 Phasor Diagram for a Balanced Delta-connected Source

Power Relationship

The real power produced per phase in the system shown in Figure 1.6 is $P_{\text{ph}} = |\bar{V}_{\text{ph}}||\bar{I}_{\text{ph}}| \cos\phi$.

Therefore, the total real power produced in the system is given by

$$P = 3|\bar{V}_{\text{ph}}||\bar{I}_{\text{ph}}| \cos\phi \tag{1.21}$$

Using Eqns. (1.18) and (1.20), we get

$$P = 3|\bar{V}_L|\frac{|\bar{I}_L|}{\sqrt{3}} \cos\phi = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \cos\phi \quad \text{(W)} \tag{1.22}$$

Similarly, the total reactive power, Q, and total apparent power, S, produced in the system are given by:

$$Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \sin\phi \quad \text{(VAR)} \tag{1.23}$$

$$S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \quad \text{(VA)} \tag{1.24}$$

1.5.3 Three-Phase Balanced Star-Connected Load [AU April/May, 2015]

The circuit diagram for a three-phase balanced star-connected load with phase sequence RYB is shown in Figure 1.9.

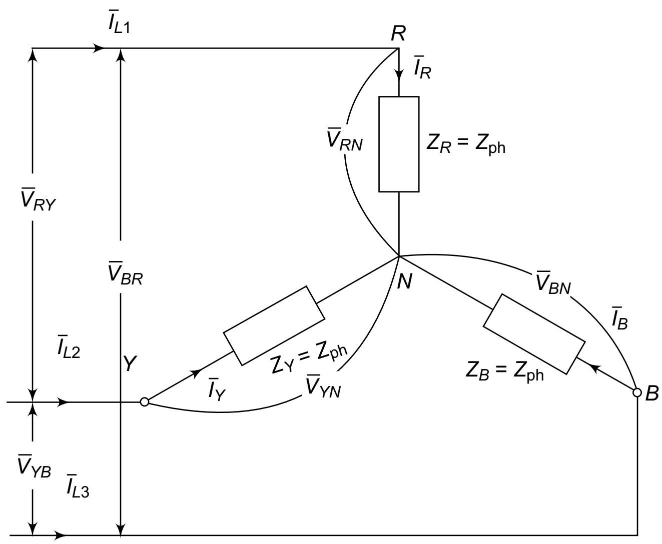


Figure 1.9 Circuit Diagram for a Three-phase Balanced Star-connected Load

Relationship Among Phase Current, Phase Voltage and Load Impedance

Let Z_R, Z_Y and Z_B be the load impedances in R, Y and B phases respectively. But in a balanced load condition, all the load impedances are equal to the load impedance per phase, Z_{ph}, represented as:

$$Z_R = Z_Y = Z_B = Z_{\text{ph}} \tag{1.25}$$

The current, voltage and power relationship between the line and phase quantities, explained in Section 1.4.1, is applicable to the balanced three-phase star-connected load, i.e.,

$$\bar{I}_{\text{ph}} = \bar{I}_L \; ; |\bar{V}_L| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \; ; P = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \cos\phi \; ; Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \sin\phi$$

and

$$S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \tag{1.26}$$

The relation between phase current, phase voltage and load impedance per phase is given by:

$$\bar{I}_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{Z_{\text{ph}}}$$

Load Impedance

If the load is lagging, leading and unity power factor in nature, then the load impedance is given by $Z_{\text{ph}} = R_{\text{ph}} + jX_{\text{ph}}$, $Z_{\text{ph}} = R_{\text{ph}} - jX_{\text{ph}}$ and $Z_{\text{ph}} = R_{\text{ph}}$ respectively.

Power Factor

The power factor of the given three-phase star-connected balanced load is $\cos\phi$.

Phasor Diagram

The phasor diagram for a three-phase balanced star-connected load with lagging and leading power factor load is shown in Figures 1.10 (a) and (b) respectively.

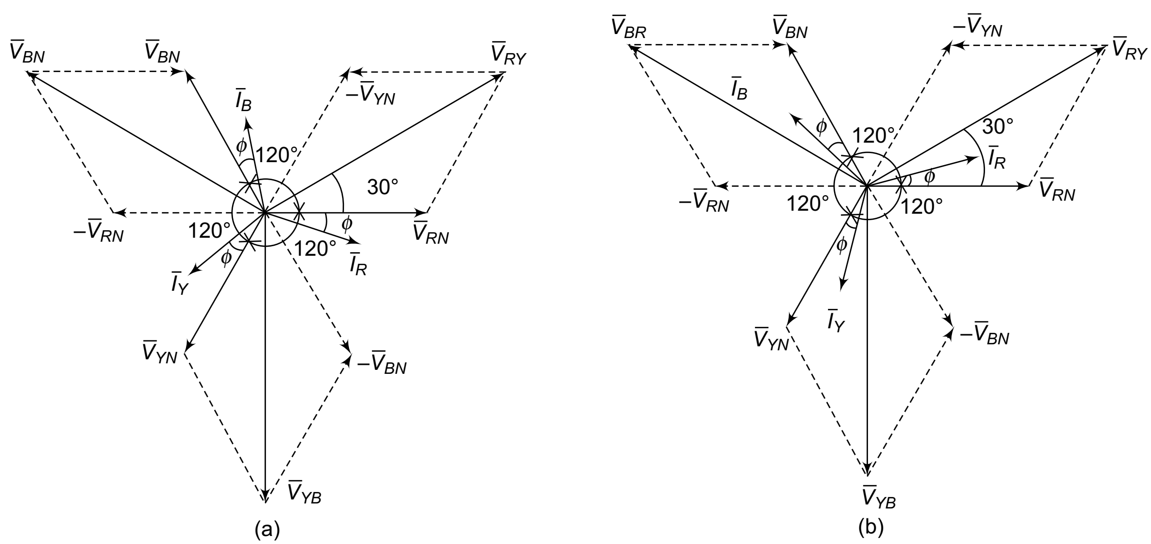


Figure 1.10 Phasor Diagram for a Three-phase Balanced Star-connected Load with (a) Lagging and (b) Leading Power Factor

1.5.4 Three-Phase Balanced Delta-Connected Load

The circuit diagram for a three-phase balanced delta-connected load with phase sequence RYB is shown in Figure 1.11.

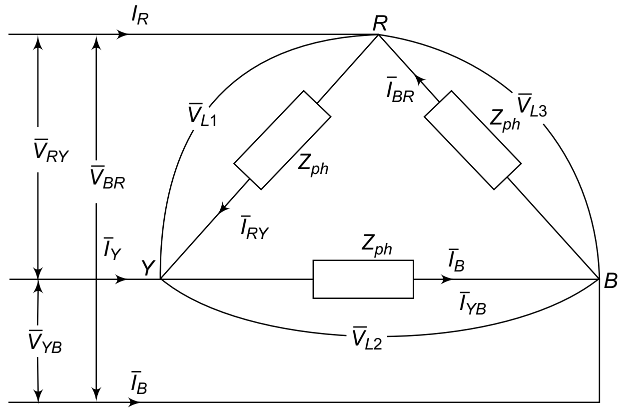


Figure 1.11 Circuit Diagram for a Three-phase Balanced Delta-connected Load

Relationship Among Phase Current, Phase Voltage and Load Impedance

The current, voltage and power relationship between the line and phase quantities, explained in Section 1.4.2, is applicable to the balanced three-phase delta-connected load, i.e.,

$$\bar{V}_{\text{ph}} = \bar{V}_L \; ; |\bar{I}_L| = \sqrt{3}\,|\bar{I}_{\text{ph}}| \; ; P = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \cos\phi \; ; Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \sin\phi$$

and

$$S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \tag{1.27}$$

The relation between phase current, phase voltage and load impedance per phase is given by:

$$\bar{I}_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{Z_{\text{ph}}}$$

Load Impedance

If the load is lagging, leading or unity power factor in nature, then the load impedance is given by $Z_{\text{ph}} = R_{\text{ph}} + jX_{\text{ph}}$; $Z_{\text{ph}} = R_{\text{ph}} - jX_{\text{ph}}$ or $Z_{\text{ph}} = R_{\text{ph}}$ respectively.

Power Factor

The power factor of the given three-phase delta-connected balanced load is cos ϕ.

Phasor Diagram

The phasor diagram for a three-phase balanced delta-connected load with lagging and leading power factor load is shown in Figures 1.12 (a) and (b) respectively.

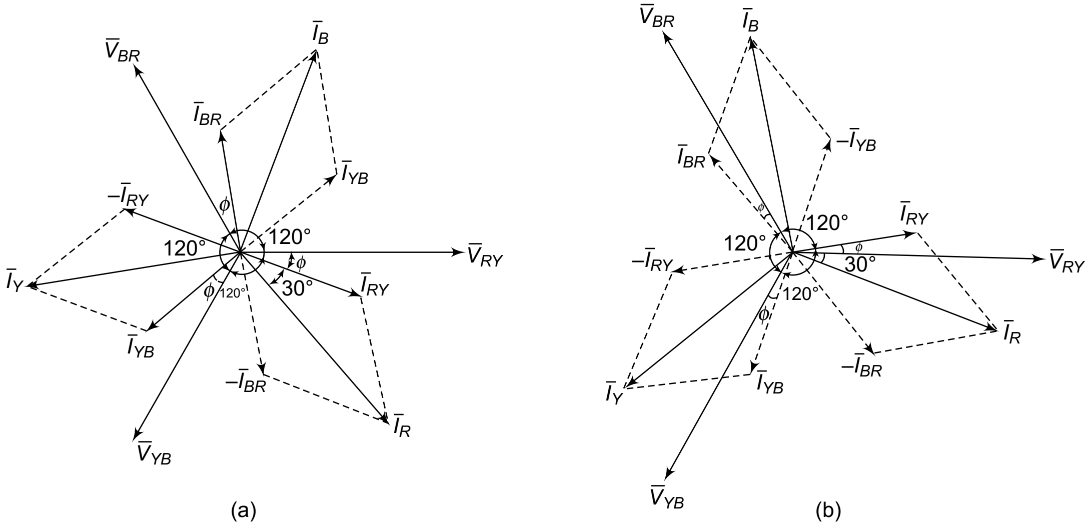


Figure 1.12 Phasor Diagram for a Three-phase Balanced Delta-connected Load with (a) Lagging and (b) Leading Power Factor

1.5.5 Three-Phase Unbalanced Delta-Connected Load **[AU May/June, 2014]**

The circuit diagram for a three-phase unbalanced delta-connected load with phase sequence RYB is shown in Figure 1.13.

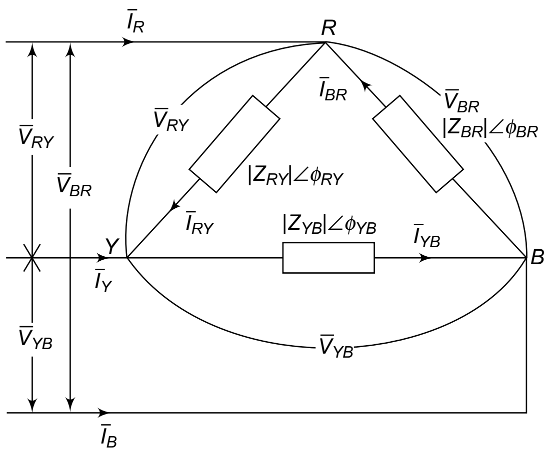


Figure 1.13 Three-phase Unbalanced Delta-connected Load

The load impedance across R-Y, Y-B and B-R terminals is given by $|Z_{RY}|\angle\phi_{RY}$, $|Z_{YB}|\angle\phi_{YB}$ and $|Z_{BR}|\angle\phi_{BR}$ respectively. It is known that in delta connection, the phase and line voltages are same. Since there is a change in load impedance, there will be changes only in the line and phase currents.

The phase and line currents in the system are given by:

$$\overline{I}_{RY} = \frac{\overline{V}_{RY}}{|Z_{RY}|\angle\varphi_{RY}} = \frac{\overline{V}_{RY}}{Z_{RY}}; \overline{I}_{YB} = \frac{\overline{V}_{YB}}{|Z_{YB}|\angle\varphi_{YB}} = \frac{\overline{V}_{YB}}{Z_{YB}}$$

and
$$\overline{I}_{BR} = \frac{\overline{V}_{BR}}{|Z_{BR}|\angle\varphi_{BR}} = \frac{\overline{V}_{BR}}{Z_{BR}} \tag{1.28}$$

$$\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR}\ ; \overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY} \text{ and } \overline{I}_B = \overline{I}_{BR} - \overline{I}_{YB} \tag{1.29}$$

Also, the total real power, reactive power and apparent power for the unbalanced delta-connected load are given in Eqns. (1.30) to (1.32) respectively.

$$P = |V_{RN}||I_{RY}|\cos\phi_{RY} + |V_{YN}||I_{YB}|\cos\phi_{YB} + |V_{BN}||I_{BR}|\cos\phi_{BR} \tag{1.30}$$

$$Q = |V_{RN}||I_{RY}|\sin\phi_{RY} + |V_{YN}||I_{YB}|\sin\phi_{YB} + |V_{BN}||I_{BR}|\sin\phi_{BR} \tag{1.31}$$

$$S = |V_{RN}||I_{RY}| + |V_{YN}||I_{YB}| + |V_{BN}||I_{BR}| \tag{1.32}$$

1.5.6 Three-Phase Unbalanced Four-Wire Star-Connected Load [AU May/June, 2014]

The circuit diagram for a three-phase four-wire unbalanced star-connected load, with phase sequence RYB, is shown in Figure 1.14.

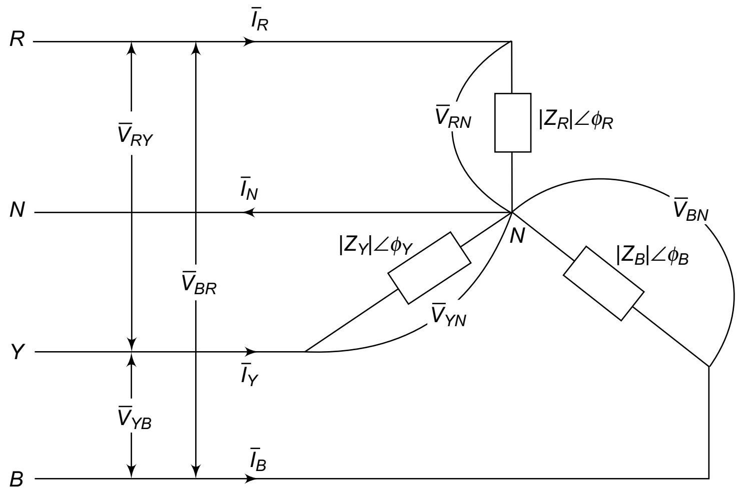


Figure 1.14 Three-phase Four-wire Unbalanced Star-connected Load

The load impedance across R-N, Y-N and B-N terminals are given by $|Z_R|\angle\phi_R$, $|Z_Y|\angle\phi_Y$ and $|Z_B|\angle\phi_B$ respectively. In star-connected load, the line and phase currents are equal as given by:

$$\overline{I}_R = \frac{\overline{V}_{RN}}{|Z_R|\angle\varphi_R} = \frac{\overline{V}_{RN}}{Z_R}; \overline{I}_Y = \frac{\overline{V}_{YN}}{|Z_Y|\angle\varphi_Y} = \frac{\overline{V}_{YN}}{Z_Y} \text{ and } \overline{I}_B = \frac{\overline{V}_{BN}}{|Z_B|\angle\varphi_B} = \frac{\overline{V}_{BN}}{Z_B} \tag{1.33}$$

The current flowing through the neutral point is obtained using Kirchhoff's current law as given by:

$$\overline{I}_N = \overline{I}_R + \overline{I}_Y + \overline{I}_B$$

The phase voltages in this system are given by:

$$|\overline{V}_{RN}| = \frac{|V_{RY}|}{\sqrt{3}} \angle\theta_R - 30^\circ \text{ or } \frac{|V_{\text{ph}}|}{\sqrt{3}} \angle\theta_R - 30^\circ \tag{1.34}$$

$$|\overline{V}_{YN}| = \frac{|V_{YB}|}{\sqrt{3}} \angle\theta_Y - 30^\circ \text{ or } \frac{|V_{\text{ph}}|}{\sqrt{3}} \angle\theta_R - 90^\circ \tag{1.35}$$

$$|\overline{V}_{BN}| = \frac{|V_{BR}|}{\sqrt{3}} \angle\theta_B - 30^\circ \text{ or } \frac{|V_{\text{ph}}|}{\sqrt{3}} \angle\theta_R - 210^\circ \tag{1.36}$$

Also, the total real power, reactive power and apparent power for the unbalanced delta-connected load are given by:

$$P = |V_{RN}||I_R|\cos\phi_{RY} + |V_{YN}||I_Y|\cos\phi_{YB} + |V_{BN}||I_B|\cos\phi_{BR} \tag{1.37}$$

$$Q = |V_{RN}||I_R|\sin\phi_{RY} + |V_{YN}||I_Y|\sin\phi_{YB} + |V_{BN}||I_B|\sin\phi_{BR} \tag{1.38}$$

$$S = |V_{RN}||I_R| + |V_{YN}||I_Y| + |V_{BN}||I_B| \tag{1.39}$$

1.5.7 Three-Phase Unbalanced Three-Wire Star-Connected Load

[AU May/June, 2014]

The circuit diagram for a three-phase three-wire unbalanced star-connected load, with phase sequence RYB, is shown in Figure 1.15.

The potential of a neutral point in this system is different from the potential of a neutral point in a balanced star-connected source. Such neutral points are called as floating neutral points, as the relation between the phase voltage, line voltage and supply voltage do not exist and the phase angle between any two phases will not be 120°. This creates difficulties in determining the line and phase voltages and currents of the load. The solution to these difficulties can be achieved by any one of the following three methods:

(a) Star-to-delta conversion
(b) Mesh analysis
(c) Millman's theorem

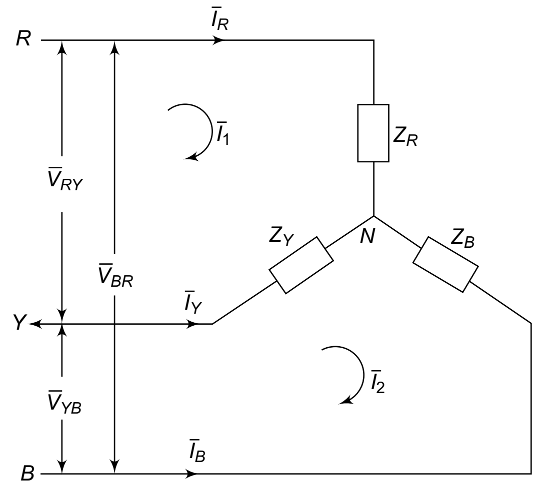


Figure 1.15 Three-phase Three-wire Unbalanced Star-connected Load

Star-to-Delta Conversion

In this method, the star-connected unbalanced load is converted into delta-connected unbalanced load, which eliminates the problem of floating neutral points. Once the star to delta conversion is performed, the system starts to act as a three-phase unbalanced delta-connected load. The line and phase voltages and currents can be obtained using the equations indicated in Section 1.4.5. The detailed description of star-to-delta conversion is discussed in Section 1.7.

Mesh Analysis

The circuit diagram of a three-phase unbalanced three-wire star-connected load, with phase sequence RYB, is shown in Figure 1.15. Let $\overline{I}_1$ and $\overline{I}_2$ be the currents flowing through loop 1 and loop 2 respectively.

Applying mesh analysis to Figure 1.15, we get
For loop 1,

$$\begin{aligned} \overline{V}_{RY} &= \overline{I}_1 Z_R + (\overline{I}_1 - \overline{I}_2) Z_Y = \overline{I}_1 Z_R + \overline{I}_1 Z_Y - \overline{I}_2 Z_Y \\ \overline{V}_{RY} &= \overline{I}_1 (Z_R + Z_Y) - \overline{I}_2 Z_Y \end{aligned} \tag{1.40}$$

For loop 2,

$$\begin{aligned} \overline{V}_{YB} &= \overline{I}_2 Z_B + (\overline{I}_2 - \overline{I}_1) Z_Y = \overline{I}_2 Z_B + \overline{I}_2 Z_Y - \overline{I}_2 Z_Y \\ \overline{V}_{YB} &= -\overline{I}_1 Z_B + \overline{I}_2 (Z_B + Z_Y) \end{aligned} \tag{1.41}$$

Upon solving Eqns. (1.40) and (1.41), the currents $\overline{I}_1$ and $\overline{I}_2$ can be determined.

From Figure 1.15, we get:

$\overline{I}_R = \overline{I}_1 ; \overline{I}_B = -\overline{I}_2$ and $\overline{I}_Y = \overline{I}_1 - \overline{I}_2$ (or) $\overline{I}_2 - \overline{I}_1$ [Depending on the magnitude of higher value]

If $\overline{I}_R, \overline{I}_Y$ and $\overline{I}_B$ are determined, then the phase and line voltages can be calculated using Eqns. (1.42) and (1.43) respectively.

$$\overline{V}_{RN} = \overline{I}_R Z_R, \overline{V}_{YN} = \overline{I}_Y Z_Y \text{ and } \overline{V}_{BN} = \overline{I}_B Z_B \tag{1.42}$$

$$\overline{V}_{RY} = \overline{V}_{RN} - \overline{V}_{YN}, \overline{V}_{YB} = \overline{V}_{YN} - \overline{V}_{BN} \text{ and } \overline{V}_{BR} = \overline{V}_{BN} - \overline{V}_{RN} \tag{1.43}$$

Millman's Theorem

According to Millman's theorem, if number of voltages sources $\overline{V}_1, \overline{V}_2, \overline{V}_3 \ldots \overline{V}_n$ with internal impedances Z_1, Z_2, …, Z_n. are in parallel, as shown in Figure 1.16 (a), then it can be replaced by an equivalent circuit consisting of a voltage source $\overline{V}_{\text{eq}}$ in series with impedance Z_{eq}, as shown in Figure 1.16(b).

Where, $$\overline{V}_{\text{eq}} = \frac{\dfrac{\overline{V}_1}{Z_1} + \dfrac{\overline{V}_2}{Z_2} + \ldots + \dfrac{\overline{V}_n}{Z_n}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \ldots + \dfrac{1}{Z_n}} = \frac{\overline{V}_1 Y_1 + \overline{V}_2 Y_2 + \ldots + \overline{V}_n Y_n}{Y_1 + Y_2 + \ldots + Y_n} \text{ and}$$

$$Z_{\text{eq}} = \frac{1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \ldots + \dfrac{1}{Z_n}} \text{ or } \frac{1}{Y_{\text{eq}}} = \frac{1}{Y_1 + Y_2 + \ldots + Y_n}$$

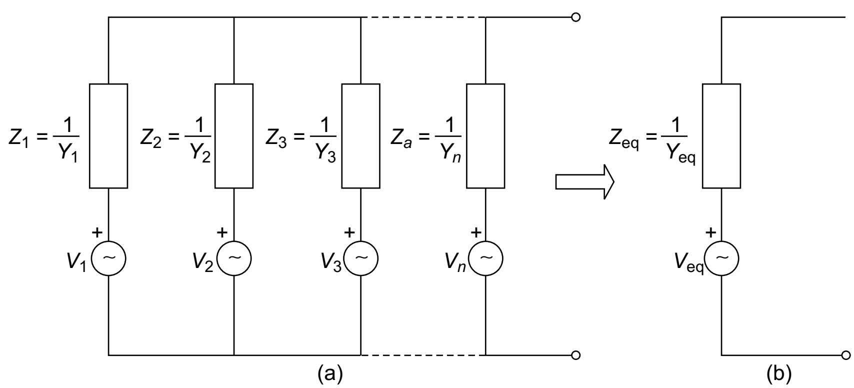


Figure 1.16 Milliman's Theorem Representation

The unbalanced three-wire star-connected load, which is supplied by a balanced star-connected source, to which Millman's Theorem is applied, is shown in Figure 1.16.

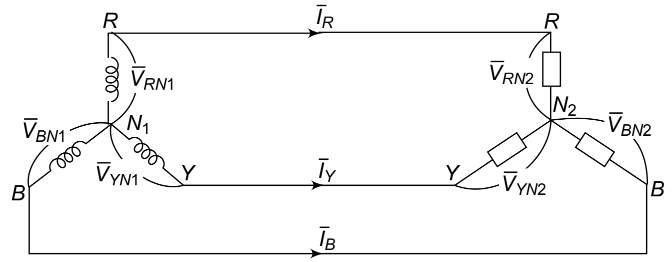


Figure 1.17 Application of Milliman's Theorem to Unbalanced Three-wire Star-connected Load

Applying Millman's theorem to Figure 1.17, we get

$$\bar{V}_{N1N2} = \frac{\bar{V}_{RN1}Y_R + \bar{V}_{YN1}Y_Y + \bar{V}_{BN1}Y_B}{Y_B + Y_Y + Y_B}$$

Where, Y_R, Y_Y and Y_B are admittances of unbalanced three-wire star load, connected such that:

$$Y_R = \frac{1}{Z_R}; Y_Y = \frac{1}{Z_Y}; Y_B = \frac{1}{Z_B}$$

Therefore, $\bar{V}_{RN1} = \bar{V}_{RN2} + \bar{V}_{N1N2}$

And $\bar{V}_{RN2} = \bar{V}_{RN1} - \bar{V}_{N1N2}$

Similarly, $\bar{V}_{YN2} = \bar{V}_{YN1} - \bar{V}_{N1N2}$ and $\bar{V}_{BN2} = \bar{V}_{BN1} - \bar{V}_{N1N2}$

The line (or phase) currents are given by:

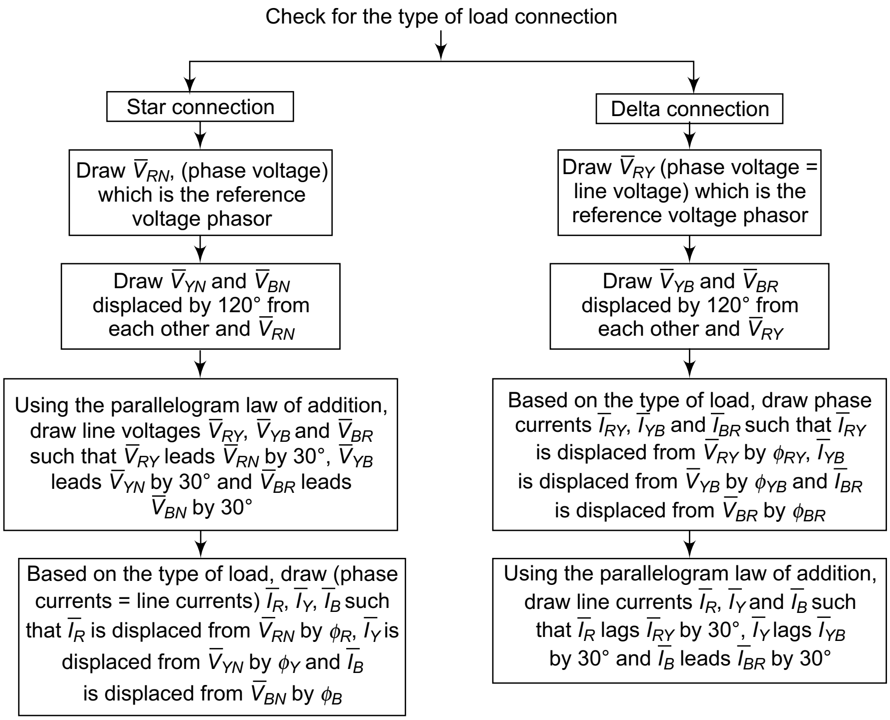
$$\bar{I}_R = \frac{\bar{V}_{RN2}}{Z_R} = \frac{\bar{V}_{RN2}}{|Z_R|\angle\theta_R}; \bar{I}_Y = \frac{\bar{V}_{YN2}}{Z_Y} = \frac{\bar{V}_{YN2}}{|Z_Y|\angle\theta_Y} \text{ and } \bar{I}_B = \frac{\bar{V}_{BN2}}{Z_B} = \frac{\bar{V}_{BN2}}{|Z_B|\angle\theta_B}$$

Different Types of Balanced Connection

The different types of connections, which can exist in three-phase systems, are:

1. Star-connected source – star-connected load
2. Star-connected source – delta-connected load
3. Delta-connected source – star-connected load
4. Delta-connected source – delta-connected load

1.6 Steps to Draw Phasor Diagram



1.7 Star–Delta Conversion **[AU May/June, 2013]**

The transformation or replacement of the three-phase star-connected load to a three-phase delta-connected load and vice versa is known star–delta conversion. This conversion is required in three-phase loads to simplify and analyse the complex circuits. As an example, if a three-phase load is connected in delta, it can be transformed into an equivalent star-connected load and after analysis, the results are converted back into their original delta equivalent. The converted equivalent circuit will have the same current and voltage levels at its network terminals, as it appears in the original circuit. The two different conversions existing in three-phase systems are: (i) star-to-delta conversion and (ii) delta-to-star conversion.

1.7.1 Star-to-Delta Conversion

In this conversion, star-connected three-phase load, shown in Figure 1.18 (a), is converted into delta-connected three-phase system, as shown in Figure 1.18 (b).

The equivalent impedances in delta-connected load are given by:

$$\left.\begin{aligned} Z_{RY} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} \\ Z_{YB} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} \\ Z_{BR} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} \end{aligned}\right\} \tag{1.44}$$

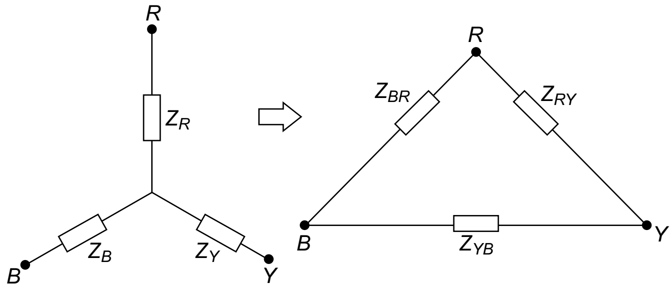


Figure 1.18 Star-to-delta Conversion

1.7.2 Delta-to-Star Conversion

In this conversion, star-connected three-phase load, shown in Figure 1.19 (a), is converted into delta-connected three-phase system, as shown in Figure 1.19 (b).

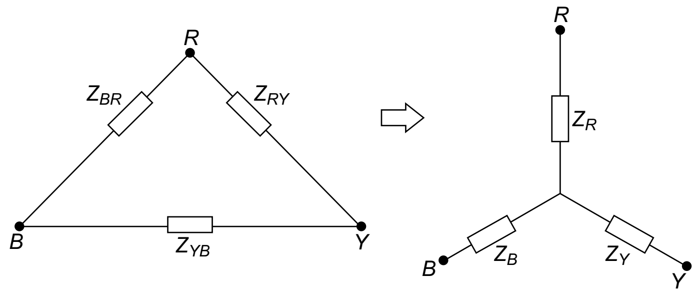


Figure 1.19 Delta-to-star Conversion

The equivalent impedances in star-connected load are given by:

$$\left.\begin{aligned} Z_R &= \frac{Z_{RY}Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}} \\ Z_Y &= \frac{Z_{YB}Z_{RY}}{Z_{RY}+Z_{YB}+Z_{BR}} \\ Z_B &= \frac{Z_{YB}+Z_{BR}}{Z_{RY}+Z_{YB}+Z_{BR}} \end{aligned}\right\} \tag{1.45}$$

Example 1.1

For the circuit shown below in Figure E1.1, calculate the line current, power and power factor. The values of R, L and C in each phase are 10 Ω, 1 H and 100 μF respectively. **[AU Nov/Dec, 2012]**

Solution

The load impedance in each phase is a parallel combination of R, L and C, as given by

$$Z_R = Z_Y = Z_B = \frac{1}{\frac{1}{R}+\frac{1}{X_L}+\frac{1}{X_C}} = 9.262 - j2.615 = 9.624\angle -15.77^\circ\ \Omega$$

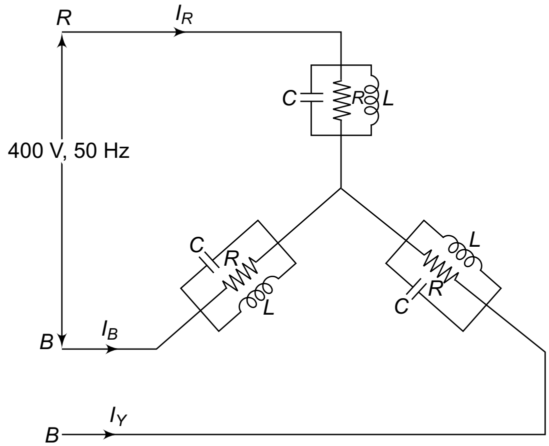


Figure E1.1

For a star connected load, $|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}| = \dfrac{|\overline{V}_L|}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94 \text{ V}$

In a balanced star-connected load, the line current or phase current is given by

$$\overline{I}_R = \overline{I}_Y = \overline{I}_B = \overline{I}_{\text{ph}} = \overline{I}_L = \frac{|\overline{V}_{\text{ph}}|}{Z_R} = \frac{230.94}{9.624\angle -15.77°} = 24\angle 15.77° \text{ A}$$

The power factor of the system is given by

$$\cos\phi = \cos(\overline{V}_{\text{ph}} \wedge \overline{I}_{\text{ph}}) = \cos(15.77°) = 0.9624$$

where $\overline{V}_{\text{ph}} \wedge \overline{I}_{\text{ph}}$ represents the phase angle between $\overline{V}_{\text{ph}}$ and $\overline{I}_{\text{ph}}$

The power consumed by the load is

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3}\ \times 400 \times 24 \times 0.9624$$
$$= 16 \text{ kW}$$

Example 1.2

A balanced star-connected load having an impedance of $15 + j20\ \Omega$ per phase is connected to three-phase, 440V, 50 Hz. Find the line current and power absorbed by the load. **[AU April/May, 2014]**

Solution

Given, load impedance, $Z_{\text{ph}} = 15 + j20\ \Omega$ and line voltage, $|\overline{V}_L| = 440$ V.

For a star-connected load, $|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}| = \dfrac{|\overline{V}_L|}{\sqrt{3}} = \dfrac{440}{\sqrt{3}} = 254.03 \text{ V}$

In a star-connected load, magnitude of line and phase currents are equal and hence,

$$\overline{I}_L = \frac{\overline{V}_{\text{ph}}}{Z_{ph}} = \frac{254.03}{15 + j20} = \frac{254.03}{25\angle 53.13} = 10.16\angle -53.13° \text{ A}$$

The power factor of the given system is cos(ϕ) = cos(53.13°) = 0.6 A
Therefore, the line currents in star-connected load are: $\overline{I}_R = 10.16\angle{-53.13°}$ A , $\overline{I}_Y = 10.16\angle{-173.13°}$ A and $\overline{I}_B = 10.16\angle{-293.13°}$ A .
The active power, reactive power and apparent power absorbed by the load are:

$$P = \sqrt{3}\,|\overline{V}_L||\overline{I}_L| \cos(\phi);\; Q = \sqrt{3}\,|\overline{V}_L||\overline{I}_L| \sin(\phi) \text{ and } S = \sqrt{3}\,|\overline{V}_L||\overline{I}_L|$$

Substituting the known values in the above equation, we get:

$$P = \sqrt{3} \times 440 \times 10.16 \times 0.6 = 4.645 \text{ kW}$$

$$Q = \sqrt{3} \times 440 \times 10.16 \times 0.8 = 6.194 \text{ kVAR}$$

and

$$S = \sqrt{3} \times 440 \times 10.16 = 7.743 \text{ kVA}$$

Example 1.3

Three equal impedances, each of 8 + j10 Ω, are connected in star. This is further connected to a 440 V, 50 Hz, three-phase supply. Calculate the active and reactive powers and line and phase currents.

Solution

Given, load impedances, Z_{ph} = 8 + j10 Ω, line voltage, $\overline{V}_L$ = 440 V

For a star-connected load, $|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}| = \dfrac{|\overline{V}_L|}{\sqrt{3}} = \dfrac{440}{\sqrt{3}} = 254.04$ V

The line or phase current in a *star-connected balanced load* is obtained as

$$\overline{I}_L \text{ or } \overline{I}_{\text{ph}} = \frac{\overline{V}_{\text{ph}}}{Z_{\text{ph}}} = 19.83\angle{-51.34°} \text{ A}$$

From the above equation, we get

$$\cos\phi = \cos(51.34°) = 0.6247$$

Therefore, $\overline{I}_R$ = 19.83∠–51.34° A; $\overline{I}_Y$ = 19.83∠–171.34° A and $\overline{I}_B$ = 19.83∠–291.34°A
The active and reactive power drawn by the star-connected load is given by

$$P = \sqrt{3} V_L I_L \cos\phi \text{ kW and } Q = \sqrt{3} V_L I_L \sin\phi \text{ kVAR}$$

Substituting the known values in the above equation, we get

$$P = 9.44 \text{ kW and } Q = 11.8 \text{ kVAR}$$

Example 1.4

A three-phase, three-wire 120 V RYB system feeds a delta-connected load, whose phase impedance is 30∠45° Ω. Find the phase and line currents in the system and draw the phasor diagram. **[AU Nov/Dec, 2012]**

Solution

Given, load impedance, Z_{ph} = 30∠45° Ω and line voltage, $|\overline{V}_L|$ = 120 V

For a delta-connected balanced load, $|\overline{V}_{\text{ph}}| = |\overline{V}_L| = 120$ V

The phase current in a delta-connected balanced load is given by

$$|\overline{I}_{RY}| = |\overline{I}_{YB}| = |\overline{I}_{BR}| = |\overline{I}_{\text{ph}}| = \frac{|\overline{V}_{\text{ph}}|}{Z_{\text{ph}}} = \frac{120}{30} = 4\text{ A}$$

The line current in a delta-connected balanced load is given by

$$|\overline{I}_{R}| = |\overline{I}_{Y}| = |\overline{I}_{B}| = |\overline{I}_{L}| = \sqrt{3}\,|\overline{I}_{\text{ph}}| = \sqrt{3}\times 4 = 6.92\text{ A}$$

For drawing the phasor diagram, the angle between $\overline{V}_{\text{ph}}(\overline{V}_L)$ and $\overline{I}_{\text{ph}}$ and the angle between $\overline{V}_{\text{ph}}(\overline{V}_L)$ and $\overline{I}_L$ are required.

Here, $$\overline{I}_{\text{ph}} = \frac{\overline{I}_{\text{ph}}}{\overline{Z}_{\text{ph}}} = \frac{120}{30\angle 45^\circ} = 4\angle -45^\circ\text{ A}$$

Therefore, the angle between $\overline{V}_{\text{ph}}(\overline{V}_L)$ and $\overline{I}_{\text{ph}}$ is 45°, for any phase and since it is negative, it indicates that the phase current lags behind the phase (line) voltage.

Also, it is known that the angle between line and phase currents is 30°. Hence, the angle between $\overline{V}_{\text{ph}}(\overline{V}_L)$ and $\overline{I}_L$ is 75° (i.e., 30° + 45°) for any phase. With the help of these details, the phasor diagram of the delta-connected load is shown in Figure E1.4.

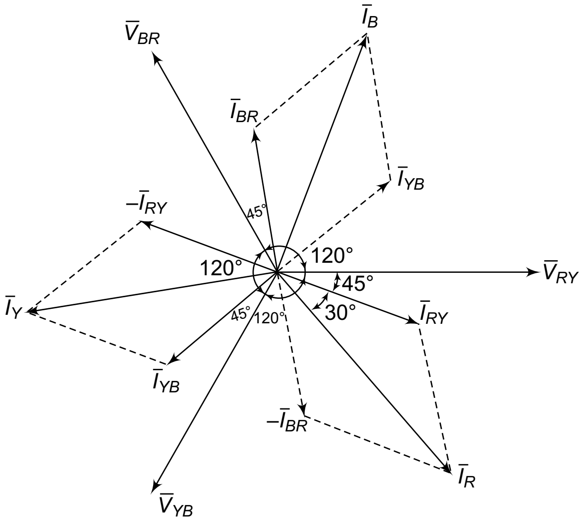


Figure E1.4 Phasor Diagram of Delta-connected Load

Example 1.5

A three-phase balanced delta-connected load of $4 + j8\ \Omega$ is connected across a 400V, three-phase balanced supply. Determine the phase currents and line currents (Phase sequence is RYB). **[AU May/Jun, 2014]**

Solution

Given, load impedance, $Z_{\text{ph}} = 4 + j8\Omega = 8.994\angle 63.43^\circ$, line voltage, $\overline{V}_L = 400$ V and RYB sequence.

For a delta-connected load, $\overline{V}_{RN} = \overline{V}_{YN} = \overline{V}_{BN} = \overline{V}_{\text{ph}} = \overline{V}_L = 400$ V

The phase angle between the phase voltage and phase current in a delta-connected load is $\phi = 63.43^\circ$ and the magnitude of the phase current is given by

$$\overline{I}_{\text{ph}} = \frac{\overline{V}_{\text{ph}}}{\overline{Z}_{\text{ph}}} = \frac{400}{8.944\angle 63.43^\circ} = 44.722\angle -63.43^\circ\text{ A}$$

Therefore, $\overline{I}_{RY} = 44.722\angle -63.43^\circ$; $\overline{I}_{YB} = 44.722\angle -183.43^\circ$ and $\overline{I}_{BR} = 44.722\angle -303.43^\circ$

Now, the magnitude of line current in a delta-connected load is given by

$$|\overline{I}_L| = \sqrt{3}\times|\overline{I}_{\text{ph}}| = \sqrt{3}\times 44.722 = 77.46\text{ A}$$

It is well known that the angle between phase current and line current of a phase in a delta-connected load is 30°. Therefore, $I_R = 77.46\ \angle{-93.43°}$ A, $I_Y = 77.46\ \angle{-213.43°}$ A and $I_B = 77.46\angle{-333.43°}$ A

Example 1.6

A three-phase delta-connected load has $Z_{RY} = 100 + j0\ \Omega$, $Z_{YB} = j100\ \Omega$ and $Z_{BR} = 70.7 + j70.7\ \Omega$ and is connected to a balanced three-phase 400 V supply. Determine the line currents I_R, I_Y and I_B. Assume the phase sequence as RYB.

Solution

Given, load impedances: $Z_{RY} = 100 + j0\ \Omega$, $Z_{YB} = -j100\ \Omega$ and $Z_{BR} = 70.7 + j70.7\ \Omega$. Line current, $\bar{V}_L = 400\text{ V}$ and phase sequence is RYB.

For a delta-connected load, $|\bar{V}_{\text{ph}}| = |\bar{V}_L| = 400\text{ V}$

Therefore, the phase currents in the delta-connected load are given by:

$$\bar{I}_{RY} = \frac{\bar{V}_{\text{ph}}}{Z_{RY}} = \frac{400\angle 0°}{100} = 4.4\text{ A}$$

$$\bar{I}_{YB} = \frac{\bar{V}_{\text{ph}}}{Z_{YB}} = \frac{400\angle -120°}{-j100} = 4\angle -30°\text{ A}$$

and
$$\bar{I}_{BR} = \frac{\bar{V}_{\text{ph}}}{Z_{BR}} = \frac{400\angle -240°}{70.7 + j70.7} = 4.4\angle -285°\text{ A}$$

The line currents of the system are:

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} = 4.4 - 4\angle -30° = 2.2\angle 64.92°\text{ A}$$

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} = 4\angle -30° - 4.4\angle -285° = 6.66\angle -69.59°\text{ A}$$

and
$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = 4.4\angle -285° - 4.4 = 5.357\angle 127.5°\text{ A}$$

Example 1.7

Detremine the currents for the unbalanced delta-connected load consisting of $Z_{RY} = 30 + j40\ \Omega$, $Z_{YB} = 8 - j4\ \Omega$, and $Z_{BR} = 15 + j12\ \Omega$. Assume the phase sequence to be RYB, $V = 200$ V.

Solution

Given, load impedances: $Z_{RY} = 30 + j40\ \Omega$, $Z_{YB} = 8 - j4\ \Omega$, and $Z_{BR} = 15 + j12\ \Omega$, line voltage, $\bar{V}_L = 200$ V.

For a delta-connected load, $|\bar{V}_{\text{ph}}| = |\bar{V}_L| = 200\text{ V}$

Therefore, the phase currents in the delta-connected load are given by:

$$\bar{I}_{RY} = \frac{\bar{V}_{\text{ph}}}{Z_{RY}} = \frac{200\angle 0°}{30 + j40} = 4\angle -53.13°\text{ A}$$

$$\bar{I}_{YB} = \frac{\bar{V}_{\text{ph}}}{Z_{YB}} = \frac{200\angle -120°}{8 - j4} = 22.36\angle -93.44°\text{ A}$$

and
$$\bar{I}_{BR} = \frac{\bar{V}_{\text{ph}}}{Z_{BR}} = \frac{200\angle -240°}{15 + j12} = 10.41\angle 81.34°\text{ A}$$

The line currents of the system are:

$$\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR} = 4\angle -53.13 - 10.41\angle 81.34° = 13.51\angle -86.46° \text{ A}$$

$$\overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY} = 22.36\angle -93.44° - 4\angle -53.13° = 19.48\angle -101.07° \text{ A}$$

and
$$\overline{I}_B = \overline{I}_{BR} - \overline{I}_{YB} = 10.41\angle 81.34° - 22.36\angle -93.44° = 32.74\angle 84.90° \text{ A}$$

Example 1.8

A wye load with $Z_R = 3 + j0\ \Omega$, $Z_Y = 2 + j3\ \Omega$, and $Z_B = 2 - j1\ \Omega$ is connected to a three-phase, four-wire, 100V, BYR system. Find the currents in all four lines.

Solution

Given, load impedances: $Z_R = 3 + j0\ \Omega$, $Z_Y = 2 + j3\ \Omega$, and $Z_B = 2 - j1\ \Omega$ and line voltage, $\overline{V}_L = 100$ V.

For a star-connected load, $|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}| = \dfrac{|\overline{V}_L|}{\sqrt{3}} = \dfrac{100}{\sqrt{3}} = 57.74$ V

Therefore, $\overline{V}_{BN} = 57.74\angle 0°$ V; $\overline{V}_{YN} = 57.74\angle -120°$ V and $\overline{V}_{RN} = 57.74\angle -240°$ V

The line or phase currents in a star-connected unbalanced load are:

$$\overline{I}_B = \frac{\overline{V}_{BN}}{Z_B} = \frac{57.74\angle 0°}{2 - j1} = 25.82\angle 26.56° \text{ A} = 23.09 + j11.54 \text{ A}$$

$$\overline{I}_Y = \frac{\overline{V}_{YN}}{Z_Y} = \frac{57.74\angle -120°}{2 + j3} = 16.01\angle -176.30° \text{ A} = -15.97 - j1.03 \text{ A}$$

and
$$\overline{I}_R = \frac{\overline{V}_{RN}}{Z_R} = \frac{57.74\angle -240°}{3 + j0} = 19.24\angle -240° \text{ A} = -9.62 + j16.66 \text{ A}$$

The neutral current in the system is given by

$$\overline{I}_N = \overline{I}_R + \overline{I}_Y + \overline{I}_B$$

Substituting the known values in the above equation, we get

$$\begin{aligned}\overline{I}_N &= -9.62 + j16.66 - 15.97 - j1.03 + 23.09 + j11.54 \\ &= -2.5 + j27.17 \\ &= 27.3\angle 95.28° \text{ A}\end{aligned}$$

Example 1.9

A symmetrical three-phase three-wire 440 V supplies to a star-connected load. The impedances in each branch are $Z_R = 2 + j3\ \Omega$, $Z_Y = 1 - j2\ \Omega$ and $Z_B = 3 + j4\ \Omega$. Find its equivalent delta-connected load.

[AU April/May, 2014]

Solution

The equivalent impedances for each branch in delta-connected load are:

$$\begin{aligned}Z_{RY} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} \\ &= \frac{(2 + j3)(1 - j2) + (1 - j2)(3 + j4) + (3 + j4)(3 + j3)}{3 + j4} = 3.8 - j0.4\ \Omega\end{aligned}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$= \frac{(2+j3)(1-j2)+(1-j2)(3+j4)+(3+j4)(3+j3)}{2+j3} = 5.23 - j0.85\ \Omega$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

$$= \frac{(2+j3)(1-j2)+(1-j2)(3+j4)+(3+j4)(3+j3)}{1-j2} = -3 + j8\ \Omega$$

Therefore, the equivalent load impedances of delta-connected load are: $Z_{RY} = 3.8 - j0.4\ \Omega$, $Z_{YB} = 5.23 - j0.85\ \Omega$ and $Z_{BR} = -3 + j8\ \Omega$.

Example 1.10

A symmetrical three-phase three-wire 400 V supply is connected to a delta-connected load. Impedances is each branch are $Z_{RY} = 10\ \angle 30°\ \Omega$, $Z_{YB} = 10\ \angle 45°\ \Omega$ and $Z_{BR} = 2.5\ \angle 60°\ \Omega$. Find its equivalent star-connected load. **[AU May/Jun, 2014]**

Solution

The equivalent impedances in star-connected load are given by:

$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}} = \frac{10\angle 30° \times 2.5\angle 60°}{10\angle 30° + 10\angle 45° + 2.5\angle 60°}$$

$$= 0.724 + j0.864 = 1.127\angle 50.03°\ \Omega$$

$$Z_Y = \frac{Z_{YB} Z_{RY}}{Z_{RY} + Z_{YB} + Z_{BR}} = \frac{10\angle 30° \times 10\angle 45°}{10\angle 30° + 10\angle 45° + 2.5\angle 60°}$$

$$= 3.7 + j2.59 = 4.51\angle 34.99°\ \Omega$$

$$Z_B = \frac{Z_{YB} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}} = \frac{10\angle 45° \times 2.5\angle 60°}{10\angle 30° + 10\angle 45° + 2.5\angle 60°}$$

$$= 0.48 + j1.022 = 1.129\angle 64.84°\ \Omega$$

Example 1.11

A symmetrical three-phase, 100 V, three-wire supply feeds an unbalanced star-connected load with impedances of the load as $Z_R = 5\ \angle 0°\ \Omega$, $Z_Y = 2\ \angle 90°\ \Omega$ and $Z_R = 4\ \angle{-90°}\ \Omega$. Find the line currents, voltage across the impedances and draw the phasor diagram. **[AU April/May, 2014]**

Solution

Given, load impedances: $Z_R = 5\ \angle 0°\ \Omega$, $Z_Y = 2\ \angle 90°\ \Omega$ and $Z_B = 4\ \angle{-90°}\ \Omega$

The line currents and phase voltages of unbalanced star-connected three-phase three-wire system can be determined using any one of the following methods:

(i) Star-to-delta conversion
(ii) Mesh analysis
(iii) Milliman's theorem

(i) Star-to-delta Conversion

The equivalent impedances for each branch in delta-connected load are:

$$\begin{aligned}
Z_{RY} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} \\
&= \frac{(5\angle 0°)(2\angle 90°) + (2\angle 90°)(4\angle -90°) + (4\angle -90°)(5\angle 0°)}{4\angle -90°} \\
&= \frac{12.80\angle -51.34°}{4\angle -90°} = 3.201\angle 38.65°\ \Omega \\
Z_{YB} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} \\
&= \frac{(5\angle 0°)(2\angle 90°) + (2\angle 90°)(4\angle -90°) + (4\angle -90°)(5\angle 0°)}{5\angle 0°} \\
&= \frac{12.80\angle -51.34°}{5\angle 0°} = 2.56\angle -51.34°\ \Omega \\
Z_{BR} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} \\
&= \frac{(5\angle 0°)(2\angle 90°) + (2\angle 90°)(4\angle -90°) + (4\angle -90°)(5\angle 0°)}{2\angle 90°} \\
&= \frac{12.80\angle -51.34°}{2\angle 90°} = 6.4\angle -141.34°\ \Omega
\end{aligned}$$

For a delta-connected load, $|\overline{V}_{\text{ph}}| = |\overline{V}_L| = 100\text{ V}$.

Therefore, $\overline{V}_{\text{RN}} = 100\angle 0°\text{ V}, \overline{V}_{YN} = 100\angle -120°\text{ V}$ and $\overline{V}_{BN} = 100\angle -240°\text{ V}$.

In a delta-connected load, the phase currents are obtained as:

$$\overline{I}_{RY} = \frac{\overline{V}_{RN}}{Z_{RY}} = \frac{100\angle 0°}{3.201\angle 38.65°} = 31.24\angle -38.65°\text{ A}$$

$$\overline{I}_{YB} = \frac{\overline{V}_{YN}}{Z_{YB}} = \frac{100\angle -120°}{2.56\angle -51.34°} = 39.06\angle -68.66°\text{ A}$$

and

$$\overline{I}_{BR} = \frac{\overline{V}_{BN}}{Z_{BR}} = \frac{100\angle -240°}{6.4\angle -141.34°} = 15.62\angle -98.66°\text{ A}$$

Therefore, the line currents are obtained as:

$$\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR} = (31.24\angle -38.65°) - (15.62\angle -98.66°) = 27.057\angle -8.65°\text{ A}$$

$$\overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY} = (39.06\angle -68.66°) - (31.24\angle -38.65°) = 19.705\angle -121.11°\text{ A}$$

and

$$\overline{I}_B = \overline{I}_{BR} - \overline{I}_{YB} = (15.62\angle -98.66°) - (39.06\angle -68.66°) = 26.70\angle 128.34°\text{ A}$$

(ii) Mesh Analysis

Applying mesh analysis to each loop in the system, we get:

$$100\angle 0° = \bar{I}_1(5\angle 0^{o} + 2\angle 90°) - \bar{I}_2(2\angle 90°)$$
$$= \bar{I}_1(5.38\angle 21.80°) - \bar{I}_2(2\angle 90°) \quad (1)$$

$$100\angle -120° = -\bar{I}_1(2\angle 90°) + \bar{I}_2(2\angle 90° + 4\angle -90°)$$
$$= -\bar{I}_1(2\angle 90°) + \bar{I}_2(2\angle -90°) \quad (2)$$

Subtracting Eqn. (1) from Eqn. (2), we get

$$100\angle -120° - 100\angle 0° = -\bar{I}_1(2\angle 90°) - \bar{I}_1(5.38\angle 21.80°)$$

Therefore, $\bar{I}_1 = 27.057\angle -8.65°\,\text{A}$

Substituting $\bar{I}_1 = 27.057\angle -8.65°$ in Eqn. (1), we get

$$\bar{I}_2 = 26.70\angle -51.66°\,\text{A}$$

Therefore, the line currents are given by:

$$\bar{I}_R = 27.057\angle -8.65°\,\text{A}$$
$$\bar{I}_B = -\bar{I}_B = 26.70\angle 128.34°\,\text{A}$$
$$\bar{I}_Y = 19.705\angle -121.11°\,\text{A}$$

(iii) Milliman's Theorem

Taking V_{RY} as the reference line voltage, i.e., $\bar{V}_{RY} = 100\angle 0°$ V , we get phase voltages of source as:

$$\bar{V}_{RN1} = \frac{100}{\sqrt{3}}\angle -30° = 57.73\angle -30^{o}\text{ V}; \bar{V}_{YN1} = 57.73\angle -150°\text{ V and } \bar{V}_{BN1} = 57.73\angle -270°\text{ V}$$

The load admittances of the system are:

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\angle 0°} = 0.2\angle 0°\ \mho;\ Y_Y = \frac{1}{2\angle 90°} = 0.5\angle -90°\ \mho \text{ and}$$

$$Y_B = \frac{1}{4\angle -90°} = 0.25\angle 90°\ \mho$$

We know that,

$$V_{N1N2} = \frac{\bar{V}_{RN1}\bar{Y}_R + \bar{V}_{YN1}\bar{Y}_Y + \bar{V}_{BN1}\bar{Y}_B}{Y_R + Y_Y + Y_B}$$
$$= \frac{[57.73\angle -30 \times 0.2] + [57.73\angle -150° \times 0.5\angle -90°] + [57.73\angle -270° \times 0.25\angle 90°]}{0.2 + 0.5\angle -90° + 0.25\angle 90°}$$

$$\vec{V}_{N_1N_2} = 84.15\angle -174.19°\,\text{V}$$

The voltage drops across the load are:

$$\vec{V}_{RN2} = \vec{V}_{RNI} - \vec{V}_{NIN2} = 135.26\angle -8.65°\,\text{V}$$
$$\overrightarrow{V_{YN2}} = \overrightarrow{V_{YNI}} - \overrightarrow{V_{NIN2}} = 39.39\angle -31.10°\,\text{V}$$

and $$\overrightarrow{V_{BN2}} = \overrightarrow{V_{BN1}} - \overrightarrow{V_{NIN2}} = 106.75\angle 38.35°\,\text{V}$$

The line currents are:

$$\overrightarrow{I_R} = \frac{\vec{V}_{RN2}}{Z_R} = 27.052\angle -8.65^\circ \text{ A}$$

$$\overrightarrow{I_Y} = \frac{\vec{V}_{YN2}}{Z_Y} = 19.705\angle -121.11^\circ \text{ A}$$

and $$\overrightarrow{I_B} = \frac{\vec{V}_{BN2}}{Z_B} = 26.7\angle 128.34^\circ \text{ A}$$

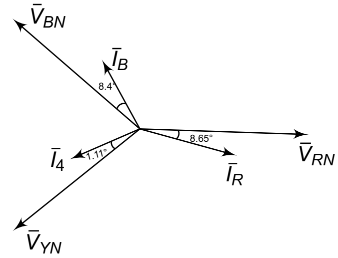


Figure E1.11

The phasor diagram for the given load is shown in Figure E1.11.

Example 1.12

An unbalanced star-connected load has balanced voltages of 100 V and RBY phase sequence. Calculate the line currents and the neutral current. Take Z_R = 15 Ω, Z_B = (10 + j5) Ω, and Z_Y = (6 – j8) Ω.

[AU April/May, 2013]

Solution

Given, three-phase four-wire unbalanced star-connected load, line voltage, $\bar{V}_L$ = 100 V, load impedances: Z_R = 15 Ω, Z_B = (10 + j5) Ω, and Z_Y = (6 – j8) Ω and an RBY phase sequence.

For a star-connected load, $|\bar{V}_{RN}| = |\bar{V}_{YN}| = |\bar{V}_{BN}| = |\bar{V}_{\text{ph}}| = \frac{|\bar{V}_L|}{\sqrt{3}} = \frac{100}{\sqrt{3}} = 57.74 \text{ V}$

In a star-connected load, the line or phase currents are given by:

$$\bar{I}_R = \frac{\bar{V}_{\text{ph}}}{Z_R} = \frac{57.74}{15} = 3.849 \text{ A}$$

$$\bar{I}_B = \frac{\bar{V}_{\text{ph}}}{Z_B} = \frac{57.74\angle -120^\circ}{10 + j5} = \frac{57.74\angle -120^\circ}{11.180\angle 26.56} = 5.164\angle -146.56^\circ = -4.309 - j2.845 \text{ A}$$

and $$\bar{I}_Y = \frac{\bar{V}_{\text{ph}}}{Z_Y} = \frac{57.74\angle -240^\circ}{6 - j8} = \frac{57.74\angle -240^\circ}{10\angle -53.13} = 5.774\angle -186.87^\circ = -5.732 + j0.6906 \text{ A}$$

The neutral current in the star-connected unbalanced load is given by

$$\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B$$

Substituting the known values in the above equation, we get:

$$\begin{aligned}\bar{I}_N &= 3.849 - 4.309 - j2.845 - j5.732 + j0.6906 \\ &= -6.192 - j2.1544 \\ &= 6.55\angle -160.81^\circ \text{ A}\end{aligned}$$

Example 1.13

A three-phase four-wire 120 V RYB system feeds an unbalanced star-connected load with Z_R = 5 ∠0° Ω, Z_Y = 10∠30° Ω, and Z_B = 20 ∠60° Ω. Obtain the four line currents. **[AU Nov/Dec, 2012]**

Solution

Given, line voltage, $|\overline{V}_L|$ = 120 V, load impedances: $Z_R = 5\ \angle 0°\ \Omega$, $Z_Y = 10\ \angle 30°\ \Omega$, and $Z_B = 20\ \angle 60°\ \Omega$

For star-connected load, $|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}| = \dfrac{|\overline{V}_L|}{\sqrt{3}} = \dfrac{120}{\sqrt{3}} = 69.3$ A

The line currents or phase currents in a star-connected load are given by:

$$\overline{I}_R = \frac{\overline{V}_{RN}}{Z_R} = \frac{69.3}{5} = 13.86\text{ A}$$

$$\overline{I}_Y = \frac{\overline{V}_{YN}}{Z_Y} = \frac{69.3\angle -120°}{10\angle 30°} = 6.93\angle -150°\text{ A}$$

and
$$\overline{I}_B = \frac{\overline{V}_{BN}}{Z_B} = \frac{69.3\angle -240°}{20\angle 60°} = 3.465\angle -300°\text{ A}$$

The current flowing through the neutral point is given by

$$\overline{I}_N = \overline{I}_R + \overline{I}_Y + \overline{I}_B$$

Substituting the line currents in the above equation, we get

$$\overline{I}_N = 13.86 + (6.93\angle -150°) + (3.465\angle -300°)$$
$$= 9.602\angle -2.77°\text{ A}$$

Example 1.14

An unbalanced star-connected load is supplied from a three-phase, 440 V symmetrical system. Determine the line currents and power input to the circuit shown in Figure E1.14 . Assume RYB sequence.

[AU April/May, 2010]

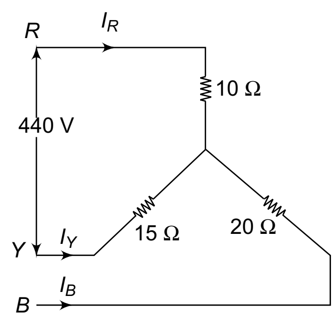


Figure E1.14

Solution

Given, line voltage, $|\overline{V}_{RY}| = 440$ V , load resistances: $R_R = 10\ \Omega$, $R_Y = 15\ \Omega$ and $R_B = 20\ \Omega$.

For a star-connected load, $|\overline{V}_{RY}| = |\overline{V}_{YB}| = |\overline{V}_{BR}| = |\overline{V}_L| = \sqrt{3}\,|\overline{V}_{\text{ph}}|$

Therefore, $|\overline{V}_{\text{ph}}| = \dfrac{440}{\sqrt{3}} = 254.03$ V

In a star-connected load, $|\overline{I}_L| = |\overline{V}_{\text{ph}}|$. Hence, the line or phase currents are obtained as:

$$\overline{I}_R = \frac{\overline{V}_{\text{ph}}}{R_R} = \frac{254.03}{10} = 25.403\text{ A}$$

$$\overline{I}_Y = \frac{\overline{V}_Y}{R_Y} = \frac{254.03\angle -120°}{15} = 16.93\angle -120°\text{ A}$$

and
$$\overline{I}_B = \frac{\overline{V}_B}{R_B} = \frac{254.03\angle -240°}{20} = 12.72\angle -240°\text{ A}$$

The total power consumed by the unbalanced star-connected load is

$$P = P_R + P_Y + P_B = |\overline{V}_{\text{ph}}|\,(|\overline{I}_R| + |\overline{I}_Y| + |\overline{I}_B|)$$

Substituting the known values in the above equation, we get

$$P = 13.98 \text{ kW}$$

Assuming the system to be lossless, the total power consumed by the load is the total power supplied to the load. Therefore, the total power supplied to the load is 13.98 kW.

Example 1.15

Determine the line currents for the unbalanced delta-connected load shown in Figure E1.15.

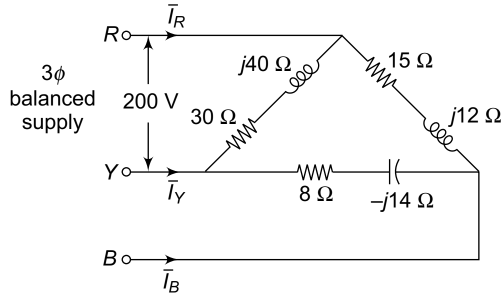


Figure E1.15

Solution

Given, line voltage, $|\bar{V}_{RY}| = 200$ V, load impedances: $Z_{\text{RY}} = 30 + j40\ \Omega = 50\angle 53.13°\ \Omega$, $Z_{\text{YB}} = 8 - j14\ \Omega = 16.124\angle{-60.255°}\ \Omega$ and $Z_{\text{BR}} = 15 - j12\ \Omega = 19.2093\angle{+\,38.66°}\ \Omega$, balanced three-phase supply and unbalanced three-phase delta-connected load.

For a delta-connected load, $|\bar{V}_{RY}| = |\bar{V}_{YB}| = |\bar{V}_{BR}| = |\bar{V}_L| = 200$ V and $|\bar{V}_{\text{ph}}| = |\bar{V}_L| = 200$ V

Taking $\bar{V}_{RY}$ as the reference voltage phasor, the phase voltages are given by:

$$\bar{V}_{RY} = 200\angle 0° \text{ V};\ \bar{V}_{YB} = 200\angle -120° \text{ V and } \bar{V}_{BR} = 200\angle -240° \text{ V}$$

Therefore, the phase currents are obtained as:

$$\bar{I}_{RY} = \frac{\bar{V}_{RY}}{Z_{RY}} = \frac{200\angle 0°}{50\angle 53.13°} = 4\angle -53.13° = 2.4 - j3.2 \text{ A}$$

$$\bar{I}_{YB} = \frac{\bar{V}_{YB}}{Z_{YB}} = \frac{200\angle -120°}{16.124\angle -60.255°} = 12.4034\angle -59.745° = 6.25 - j10.714 \text{ A}$$

and

$$\bar{I}_{BR} = \frac{\bar{V}_{BR}}{Z_{BR}} = \frac{200\angle -240°}{19.209\angle -38.66°} = 10.412\angle -278.66° = 1.56 + j10.29 \text{ A}$$

Hence, the line currents of the system are obtained as:

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} = (2.4 - j3.2) - (1.56 + j10.29)$$
$$= 0.84 - j13.49 = 13.51\angle -86.43° \text{ A}$$

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} = (6.25 - j10.714) - (2.4 - j3.2)$$
$$= 3.85 - j7.514 \text{ A} = 8.443\angle -62.87° \text{ A}$$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = (1.56 + j10.29) - (6.25 - j10.714)$$
$$= -4.69 + j21 = 21.52\angle 102.58° \text{ A}$$

Example 1.16

Three impedances, Z_1 = 17.35 + j10 Ω, Z_2 = 20 + j34.64 Ω and Z_2 = 0 – j10 Ω are delta-connected to a 400 V, three-phase system, as shown in Figure E1.16. Determine the phase currents, line currents and the total power consumed by the load.

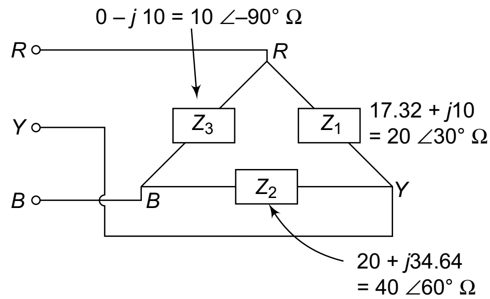


Figure E1.16

Solution

Given, line voltage, $|\overline{V}_L| = 400\ \text{V}$, load impedances: $Z_{RY} = Z_1 = 17.35 + j10\ \Omega = 20\angle 30°\ \Omega$, $Z_{YB} = Z_2 = 20 + j34.64\ \Omega = 40\angle 60°\ \Omega$ and $Z_{BR} = Z_3 = 0 - j10\ \Omega = 10\angle{-90°}\ \Omega$.

For a delta-connected load, $|\overline{V}_{\text{ph}}| = |\overline{V}_L| = 400\ \text{V}$

Taking $\overline{V}_{RY}$ as the reference voltage phasor, the phase voltages are given by:

$$\overline{V}_{RY} = 400\angle 0°\ \text{V};\ \overline{V}_{YB} = 400\angle -120°\ \text{V and } \overline{V}_{BR} = 400\angle -240°\ \text{V}$$

Therefore, the phase currents are obtained as:

$$\overline{I}_{RY} = \frac{\overline{V}_{RY}}{Z_{RY}} = \frac{400\angle 0°}{20\angle 30°} = 20\angle -30° = 17.32 - j10\ \text{A}$$

$$\overline{I}_{YB} = \frac{\overline{V}_{YB}}{Z_{YB}} = \frac{400\angle -120°}{40\angle 60°} = 10\angle -60° = 5 - j8.66\ \text{A}$$

and

$$\overline{I}_{BR} = \frac{\overline{V}_{BR}}{Z_{BR}} = \frac{200\angle -240°}{10\angle -90°} = 40\angle -150° = -34.64 + j20\ \text{A}$$

Hence, the line currents of the system are obtained as:

$$\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR} = (17.32 - j10) - (-34.64 - j20) = 51.96 + j10 = 52.913\angle 10.89°\ \text{A}$$

$$\overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY} = (5 - j8.66) - (17.32 - j10) = -12.32 + j1.34 = 12.392\angle 173.79°\ \text{A}$$

$$\overline{I}_B = \overline{I}_{BR} - \overline{I}_{YB} = (-34.64 - j20) - (5 - j8.66) = -39.64 - j11.34 = 41.2301\angle -164.03°\ \text{A}$$

The powers consumed by the load are given by:

$$P_1 = |\overline{V}_{RY}||\overline{I}_{RY}|\cos(\overline{V}_{RY} \wedge \overline{I}_{RY}) = 40 \times 20 \times \cos(30°) = 6928.2032\ \text{W}$$

$$P_2 = |\overline{V}_{YB}||\overline{I}_{YB}|\cos(\overline{V}_{YB} \wedge \overline{I}_{YB}) = 400 \times 10 \times \cos(60°) = 2000\ \text{W}$$

and

$$P_3 = |\overline{V}_{BR}||\overline{I}_{BR}|\cos(\overline{V}_{BR} \wedge \overline{I}_{BR}) = 400 \times 40 \times \cos(90°) = 0\ \text{W}$$

Therefore, the total power consumed by the load is

$$P = P_1 + P_2 + P_3 = 8.928\ \text{kW}$$

1.8 Power Measurment in a Three-Phase System

The power consumed in a three-phase system is measured using wattmeters. Generally, one wattmeter is required for measuring power in one phase and hence three wattmeters would be required to measure power in a three-phase system. But, universally it is proved that only two wattmeters are enough to measure power in three-phase systems of any type i.e., balanced or unbalanced and star or delta-connected systems. In addition, if the system is balanced, the circuit power factor can also be determined using the readings of two wattmeters. Also, in a balanced system, the total power in all the three-phases can be obtained by multiplying the power obtained in a single-phase.

1.8.1 Wattmeter

A wattmeter is an instrument that is used to measure power in watts in a single-phase or a three-phase system. The two coils that exist in a wattmeter are: fixed or current coil and moving or pressure or voltage coil. The wattmeter has four terminals: *M*, *L*, *C* and *V*; where '*M*' is the terminal to which the phase voltage of supply or mains is connected, '*L*' is the terminal to which the load is connected, '*C*' is the common terminal in the wattmeter device and '*V*' is the voltage terminal to connect the energy metre. In a wattmeter, the terminals '*M*' and '*C*' are interconnected.

Fixed or Current Coil (CC)

The coil that is connected in series with the branch or line to sense the current flowing through it is called the current coil. The characteristics of current coil are: low resistance, large cross-sectional area and less number of turns. The terminals, which are used to denote the CC, are $M - L$. In a modern wattmeter, the maximum current that is allowed to pass through CC is 20 A.

Moving or Pressure or Voltage Coil (PC)

The high-resistance coils, which are connected across or parallel to the branch or line are called moving or pressure or voltage coils. The two terminals of the pressure coil are '*C*' and '*V*', where the first one is the common terminal, which can be connected after or before the current coil and '*V*' is the specified voltage terminal with actual voltage marking. PC carries the current proportional to the voltage across the branch or line. The characteristics of PC are: large resistance, small cross-sectional area and more number of turns.

A wattmeter, which is connected to measure the power of a single-phase load, is shown in Figure 1.20. In general, the power measured by the wattmeter is a product of voltage that the pressure coil measures and the current that flows through the current coil.

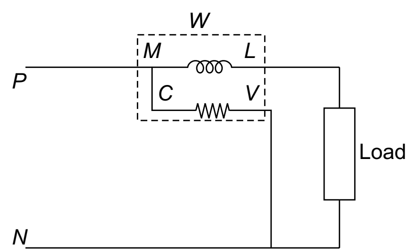


Figure 1.20 Wattmeter Connected to Measure the Power of a Single-phase Load

1.8.2 Methods of Power Measurement

The three methods that are used for the measurement of three-phase power in three-phase circuits are:

1. Three-Wattmeter Method
2. Two-Wattmeter Method
3. Single-Wattmeter Method

Blondel's Theorem

When power is supplied by an '*n*' wire AC system, then the number of wattmeters required to measure power is '$n - 1$', i.e., one less than the number of wires in the AC system, irrespective of a balanced or an unbalanced load. Therefore, for a three-phase four-wire system, the number of wattmeters required to measure the power is three and for a three-phase three-wire system, the number of wattmeters required to measure the power is two.

1.8.3 Three-Wattmeter Method

According to Blondel's theorem, the three-wattmeter method can be used only to measure the power in three-phase, four-wire star-connected balanced and unbalanced load, whose circuit diagram along with wattmeters W_1, W_2 and W_3 is shown in Figures 1.21 (a) and (b) respectively.

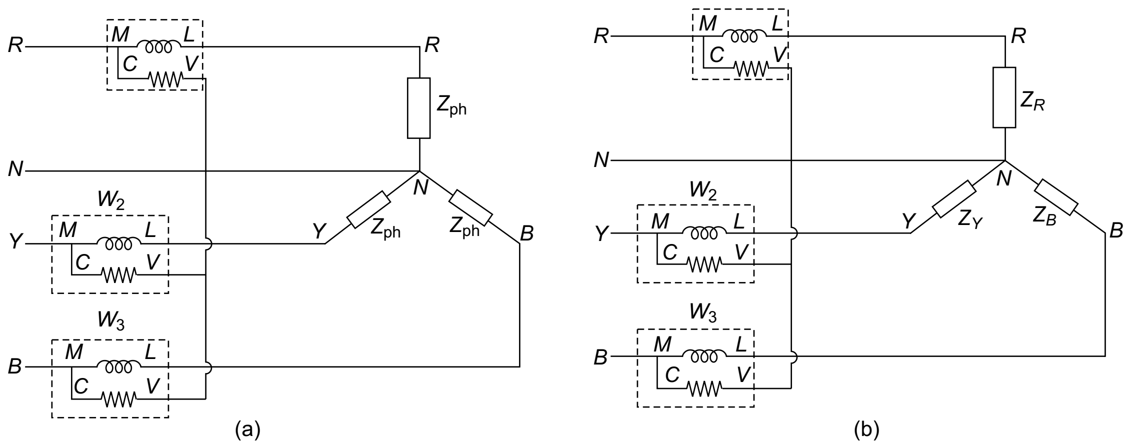


Figure 1.21 Power Measurement Using Three-Wattmeter Method

Balanced Load

Usually, the wattmeter connected across the balanced load measures the actual power measured by the load and is given by the product of root mean square (RMS) values of voltage and current. The power consumed by the three-phase load, given in Figure 1.21(a) measured by W_1, W_2 and W_3 are given by:

$$W_1 = |\overline{V}_{RN}||\overline{I}_R| \cos(\overline{V}_{RN} \wedge \overline{I}_R) \tag{1.46}$$

$$W_2 = |\overline{V}_{YN}||\overline{I}_Y| \cos(\overline{V}_{YN} \wedge \overline{I}_Y) \tag{1.47}$$

$$W_3 = |\overline{V}_{BN}||\overline{I}_B| \cos(\overline{V}_{BN} \wedge \overline{I}_B) \tag{1.48}$$

where, $\overline{V}_{RN} \wedge \overline{I}_R, \overline{V}_{YN} \wedge \overline{I}_Y$ and $\overline{V}_{BN} \wedge \overline{I}_B$ are the angles between $\overline{V}_{RN}$ and $\overline{I}_R, \overline{V}_{YN}$ and $\overline{I}_Y$ and $\overline{V}_{BN}$ and $\overline{I}_B$ respectively.

From the phasor diagram shown in Figures 1.21 (a) and (b), the angles $\overline{V}_{RN} \wedge \overline{I}_R, \overline{V}_{YN} \wedge \overline{I}_Y$ and $\overline{V}_{BN} \wedge \overline{I}_B$ will be ϕ and $-\phi$ for lagging and leading power factors. But it is known that $\cos(-\phi) = \cos(\phi)$. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 + W_3$$

Substituting Eqns. (1.46) to (1.48) in the above equation, we get

$$W = |\overline{V}_{RN}||\overline{I}_R|\cos\phi + |\overline{V}_{YN}||\overline{I}_Y|\cos\phi + |\overline{V}_{BN}||\overline{I}_B|\cos\phi \tag{1.49}$$

But in a balanced star-connected load, we have

$$|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\mathrm{ph}}| = \frac{|\overline{V}_L|}{\sqrt{3}} \text{ and } |\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L| \tag{1.50}$$

Substituting Eqn. (1.50) in Eqn. (1.49) and solving, we get the total average power measured by the three wattmeters connected across a star-connected balanced load as

$$W = \sqrt{3}\,|\overline{V}_L||\overline{I}_L|\cos\phi \tag{1.51}$$

Unbalanced Load

Usually, the wattmeter connected across the unbalanced load measures only the instantaneous power rather than the actual power. The instantaneous powers measured by W_1, W_2 and W_3 in Figure 1.21(b) are given by

$$W_1 = \frac{1}{T}\int \overline{v}_{RN} \times \overline{i}_R \, dt, W_2 = \frac{1}{T}\int \overline{v}_{YN} \times \overline{i}_Y \, dt \text{ and } W_3 = \frac{1}{T}\int \overline{v}_{BN} \times \overline{i}_B \, dt \tag{1.52}$$

where, $\overline{v}_{RN}, \overline{v}_{YN}, \overline{v}_{BN}$ are the instantaneous phase voltages, $\overline{i}_R, \overline{i}_Y, \overline{i}_B$ are the instantaneous line or phase currents and T is the time period of the voltage or current.

Therefore, the total power measured by the wattmeters is given by

$$W = W_1 + W_2 + W_3$$

Substituting Eqn. (1.52) in the above equation, we get the total average power measured by the three wattmeters connected across star-connected unbalanced load as

$$W = \frac{1}{T}\int ((\overline{v}_{RN} \times \overline{i}_R) + (\overline{v}_{YN} \times \overline{i}_Y) + (\overline{v}_{BN} \times \overline{i}_B))\, dt \tag{1.53}$$

1.8.4 Two-Wattmeter Method **[AU May/June, 2013]**

Two-wattmeter method is used to measure the total power in a three-phase, three-wire star or delta-connected balanced or unbalanced load. In this method, the current coils of the wattmeter are connected with any two lines (i.e., R and Y) and the pressure coil of the wattmeter is connected between the above two lines and the third line (i.e., B). The different phase systems for which the two-wattmeter method can be used to measure the power consumed by the load are:

1. Star-connected balanced load
2. Star-connected unbalanced load
3. Delta-connected balanced load
4. Delta-connected unbalanced load

Star-connected Balanced Load

The circuit diagram for two-wattmeter method applied to a three-phase balanced star-connected load is shown in Figure 1.22.

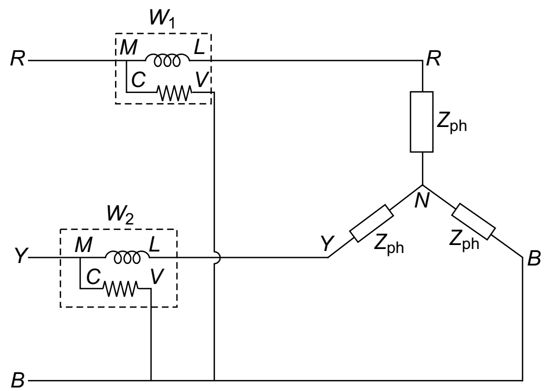


Figure 1.22 Two-Wattmeter Method for Three-phase Balanced Star-connected Load

The readings of wattmeter shown in Figure 1.22 for a balanced star-connected load is given below:

$$W_1 = |\bar{I}_R||\bar{V}_{RB}|\cos(\bar{I}_R \wedge \bar{V}_{RB}) \text{ and } W_2 = |\bar{I}_Y||\bar{V}_{YB}|\cos(\bar{I}_Y \wedge \bar{V}_{YB})$$

It is known that, $\bar{V}_{RB} = \bar{V}_{RN} - \bar{V}_{BN}$ and $\bar{V}_{YB} = \bar{V}_{YN} - \bar{V}_{BN}$. The phasor diagram for two-wattmeter method, applied to a three-phase balanced star-connected load with lagging power factor (inductive load), is shown in Figure 1.23(a).

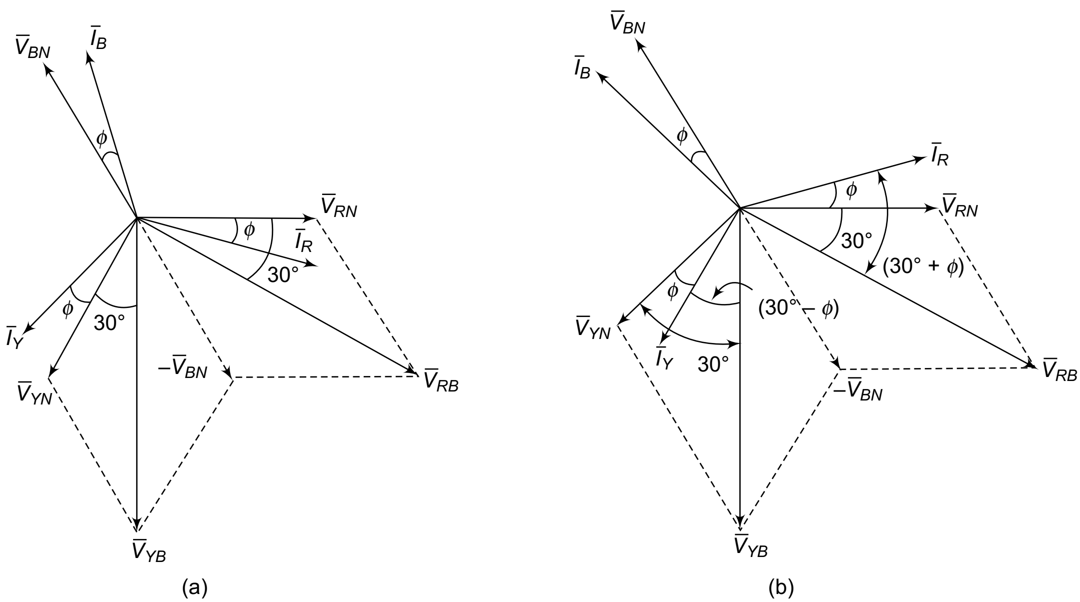


Figure 1.23 Phasor Diagram for Two-Wattmeter Method with (a) Lagging and (b) Leading Power Factor

From the phasor diagram shown in Figure 1.23 (a), the angles $\bar{V}_{RB} \wedge \bar{I}_R$ and $\bar{V}_{YB} \wedge \bar{I}_Y$ are (30° – ϕ) and (30° + ϕ) respectively. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 = |\bar{V}_{RB}||\bar{I}_R|\cos(30° - \phi) + |\bar{V}_{YB}||\bar{I}_Y|\cos(30° + \phi) \tag{1.54}$$

But, in a balanced star-connected load, we have

$$|\overline{V}_{RB}| = |\overline{V}_{YB}| = |\overline{V}_{BR}| = |\overline{V}_L| \text{ and } |\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L| \tag{1.55}$$

Substituting Eqn. (1.55) in (1.54) and solving, we get

$$W = |\overline{V}_L||\overline{I}_L|[\cos(30° - \phi) + \cos(30° + \phi)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin B \sin A$ in the above equation and solving, we get the total average power consumed by the load as

$$W = \sqrt{3}\,|\overline{V}_L||\overline{I}_L|\cos\phi \tag{1.56}$$

Similarly, it is possible to obtain the total active power consumed by the load for a leading power factor. The phasor diagram for two-wattmeter method applied to a three-phase balanced star-connected load with leading power factor (capacitive load) is shown in Figure 1.23 (b).

From the phasor diagram shown in Figures 1.23 (b), it can be seen that the angles $\overline{V}_{RB} \wedge \overline{I}_R$ and $\overline{V}_{YB} \wedge \overline{I}_Y$ are $(30° + \phi)$ and $(30° - \phi)$ respectively. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 = |\overline{V}_{RB}||\overline{I}_R|\cos(30° + \phi) + |\overline{V}_{YB}||\overline{I}_Y|\cos(30° - \phi) \tag{1.57}$$

But, in a balanced star-connected load, we have

$$|\overline{V}_{RB}| = |\overline{V}_{YB}| = |\overline{V}_{BR}| = |\overline{V}_L| \text{ and } |\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L| \tag{1.58}$$

Substituting Eqn. (1.58) in (1.57) and solving, we get

$$W = |\overline{V}_L||\overline{I}_L|[\cos(30° + \phi) + \cos(30° - \phi)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin B \sin A$ in the above equation and solving, we get the total average power consumed by the load as

$$W = \sqrt{3}\,|\overline{V}_L||\overline{I}_L|\cos\phi \tag{1.59}$$

It is noted from Eqns. (1.56) and (1.59) that, the total active power consumed by the load with lagging or leading power factor is same.

Star-connected Unbalanced Load

The circuit diagram for two-wattmeter method applied to a three-phase unbalanced star-connected load is shown in Figure 1.24.

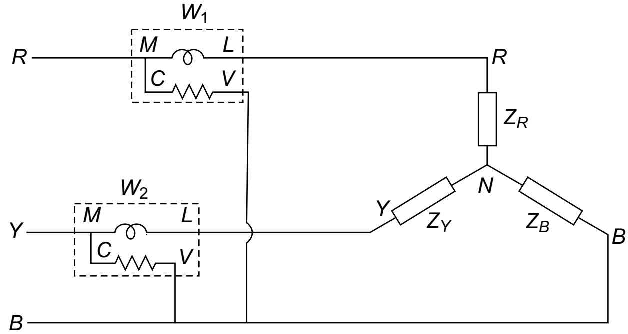


Figure 1.24 Two-Wattmeter Method for Three-phase Unbalanced Star-connected Load

It is known that for an unbalanced load, wattmeter reads the instantaneous power. Therefore, the readings of wattmeter shown in Figure 1.24 are given by

$$W_1 = \overline{v}_{RB} \times \overline{i}_R \text{ and } W_2 = \overline{v}_{YB} \times \overline{i}_Y \tag{1.60}$$

But, we know that,

$$\overline{v}_{RB} = \overline{v}_{RN} - \overline{v}_{BN} \text{ and } \overline{v}_{YB} = \overline{v}_{YN} - \overline{v}_{BN} \tag{1.61}$$

Substituting Eqn. (1.61) in Eqn. (1.60), we get

$$W_1 = (\overline{v}_{RN} - \overline{v}_{BN}) \times \overline{i}_R \text{ and } W_2 = (\overline{v}_{YN} - \overline{v}_{BN}) \times \overline{i}_Y$$

Therefore, the total average power consumed by the load is given by:

$$\begin{aligned} W &= W_1 + W_2 = (\overline{v}_{RN} - \overline{v}_{BN}) \times \overline{i}_R + (\overline{v}_{YN} - \overline{v}_{BN}) \times \overline{i}_Y \\ &= (\overline{v}_{RN}\overline{i}_R - \overline{v}_{BN}\overline{i}_R) + (\overline{v}_{YN}\overline{i}_Y - \overline{v}_{BN}\overline{i}_Y) \\ &= \overline{v}_{RN}\overline{i}_R + \overline{v}_{YN}\overline{i}_Y - \overline{v}_{BN}(\overline{i}_R + \overline{i}_Y) \end{aligned} \tag{1.62}$$

Applying KCL to the neutral point N, we get

$$\overline{i}_R + \overline{i}_Y + \overline{i}_B = 0$$

Therefore, $\overline{i}_B = -(\overline{i}_R + \overline{i}_Y)$ (1.63)

Substituting Eqn. (1.63) in Eqn. (1.62), we get

$$W = \overline{v}_{RN}\overline{i}_R + \overline{v}_{YN}\overline{i}_Y + \overline{v}_{BN}\overline{i}_B$$

Therefore, the total average power consumed by the load is

$$W = P_R + P_Y + P_B$$

Where, P_R, P_Y and P_B are instantaneous values of powers consumed by each phase of the load at the instant considered, regardless of the power factor. Hence, at any instant, addition of two wattmeter readings gives the instantaneous total average power consumed by a three-phase load.

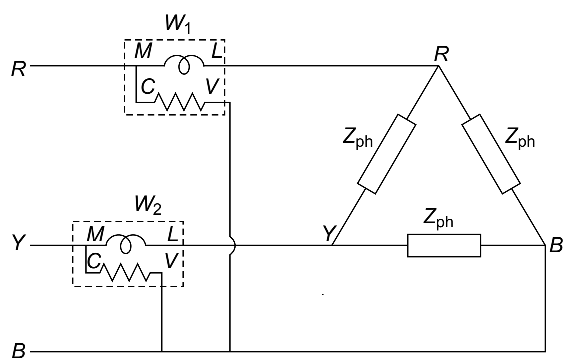


Figure 1.25 Two-Wattmeter Method for Three-phase Balanced Delta-connected Load

Delta-connected Balanced Load

The circuit diagram for two-wattmeter method applied to a three-phase balanced delta-connected load is shown in Figure 1.25

The readings of wattmeter, shown in Figure 1.25 are given by:

$$W_1 = |\overline{V}_{RB}||\overline{I}_R|\cos(\overline{V}_{RB} \wedge \overline{I}_R) \text{ and } W_2 = |\overline{V}_{YB}||\overline{I}_Y|\cos(\overline{V}_{YB} \wedge \overline{I}_Y)$$

It is known that, $\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR}$ and $\overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY}$. The phasor diagram for two-wattmeter method, applied to a three-phase balanced delta-connected load with lagging power factor (inductive load) is shown in Figure 1.26 (a).

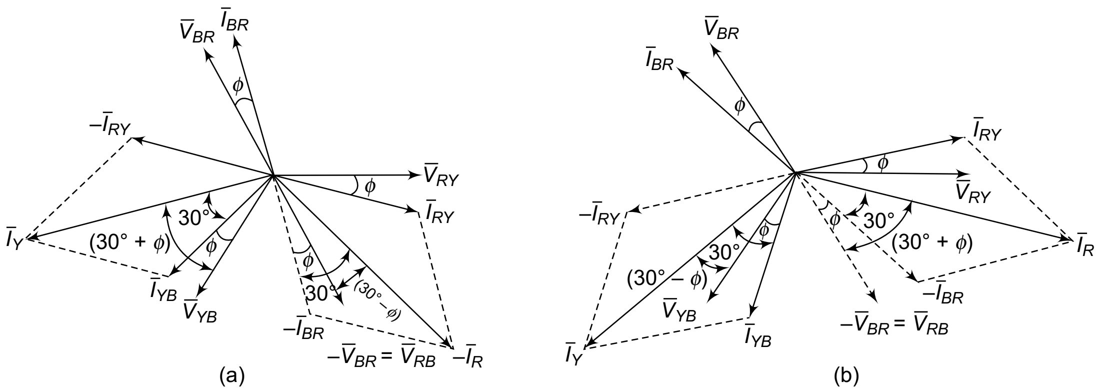


Figure 1.26 Phasor Diagram for Three-phase Balanced Delta-connected Load with (a) Lagging and (b) Leading Power Factor

From the phasor diagram shown in Figure 1.26 (a), it can be observed that the angles $\overline{V}_{RB} \wedge \overline{I}_R$ and $\overline{V}_{YB} \wedge \overline{I}_Y$ are (30° – ϕ) and (30° + ϕ) respectively. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 = |\overline{V}_{RB}||\overline{I}_R|\cos(30° - \phi) + |\overline{V}_{YB}||\overline{I}_Y|\cos(30° + \phi) \tag{1.64}$$

But, in a balanced star-connected load, we have

$$|\overline{V}_{RB}| = |\overline{V}_{YB}| = |\overline{V}_{BR}| = |\overline{V}_L| \text{ and } |\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L| \tag{1.65}$$

Substituting Eqn. (1.65) in (1.64) and solving, we get

$$W = |\overline{V}_L||\overline{I}_L|[\cos(30° - \phi) + \cos(30° + \phi)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin B \sin A$ in the above equation and solving, we get the total average power consumed by the load as

$$W = \sqrt{3}|\overline{V}_L||\overline{I}_L|\cos\phi \tag{1.66}$$

Similarly, it is possible to obtain the total active power consumed by the load for a leading power factor. The phasor diagram for two-wattmeter method applied to a three-phase balanced delta-connected load with leading power factor (capacitive load) is shown in Figure 1.26 (b).

From the phasor diagram shown in Figure 1.26 (b), it can be noted that the angles $\overline{V}_{RB} \wedge \overline{I}_R$ and $\overline{V}_{YB} \wedge \overline{I}_Y$ are (30° + ϕ) and (30° – ϕ) respectively. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 = |\overline{V}_{RB}||\overline{I}_R|\cos(30° + \phi) + |\overline{V}_{YB}||\overline{I}_Y|\cos(30° - \phi) \tag{1.67}$$

But, in a balanced star-connected load, we have

$$|\overline{V}_{RB}| = |\overline{V}_{YB}| = |\overline{V}_{BR}| = |\overline{V}_L| \text{ and } |\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L| \tag{1.68}$$

Substituting Eqn. (1.68) in Eqn. (1.67) and solving, we get

$$W = |\overline{V}_L||\overline{I}_L|[\cos(30° + \phi) + \cos(30° - \phi)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin B \sin A$ in the above equation and solving, we get the total average power consumed by the load as

$$W = \sqrt{3}\,|\overline{V}_L||\overline{I}_L|\cos\phi \tag{1.69}$$

It is noted from Eqns. (1.66) and (1.69), that the total active power consumed by the balanced delta-connected load with lagging or leading power factor is same.

Delta-connected Unbalanced Load

The circuit diagram for two-wattmeter method, applied to a three-phase unbalanced delta-connected load is shown in Figure 1.27.

It is known that, for an unbalanced load, wattmeter reads the instantaneous power in the load. Therefore, the readings of wattmeter shown in Figure 1.27 are given by:

$$W_1 = \bar{v}_{RB} \times \bar{i}_R \text{ and } W_2 = \bar{v}_{YB} \times \bar{i}_Y \quad (1.70)$$

But, we know that,

$$\bar{i}_R = \bar{i}_{RY} - \bar{i}_{BR} \text{ and } \bar{i}_Y = \bar{i}_{YB} - \bar{i}_{RY} \quad (1.71)$$

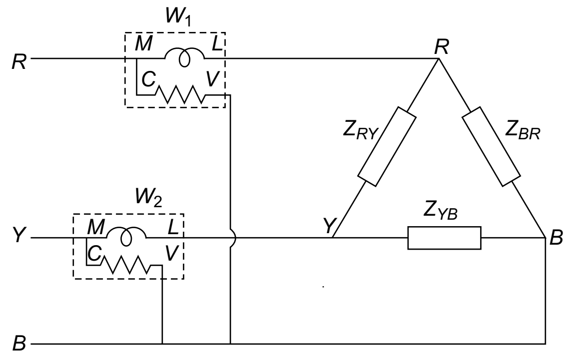


Figure 1.27 Two-Wattmeter Method for Three-phase Unbalanced Delta-connected Load

Substituting Eqn. (1.71) in Eqn. (1.70), we get

$$W_1 = (\bar{i}_{RY} - \bar{i}_{BR}) \times \bar{v}_{RB} \text{ and } W_2 = (\bar{i}_{YB} - \bar{i}_{RY}) \times \bar{v}_{YB}$$

Therefore, the total average power consumed by the load is given by

$$\begin{aligned} W &= W_1 + W_2 = (\bar{i}_{RY} - \bar{i}_{BR}) \times \bar{v}_{RB} + (\bar{i}_{YB} - \bar{i}_{RY}) \times \bar{v}_{YB} \\ &= \bar{i}_{RY}\bar{v}_{RB} - \bar{i}_{BR}\bar{v}_{RB} + \bar{i}_{YB}\bar{v}_{YB} - \bar{i}_{RY}\bar{v}_{YB} \end{aligned}$$

Substituting $\bar{v}_{RB} = -\bar{v}_{BR}$ in the above equation, we get

$$W = \bar{v}_{BR}\bar{i}_{BR} + \bar{v}_{YB}\bar{i}_{YB} - (\bar{v}_{BR} + \bar{v}_{YB})\bar{i}_{RY} \quad (1.72)$$

Applying KVL to the circuit shown in Figure 1.27, we get

$$\bar{v}_{RY} + \bar{v}_{YB} + \bar{v}_{BR} = 0$$

i.e.,

$$\bar{v}_{RY} = -(\bar{v}_{YB} + \bar{v}_{BR})$$

Substituting the above equation in Eqn. (1.72), we get

$$W = \bar{v}_{RY}\bar{i}_{RY} + \bar{v}_{YB}\bar{i}_{YB} + \bar{v}_{BR}\bar{i}_{BR}$$

Therefore, the total average power consumed by the load is

$$W = P_R + P_Y + P_B$$

Where, P_R, P_Y and P_B are instantaneous values of powers consumed by each phase of the load at the instant considered, regardless of the power factor. Hence, at any instant, adding the two wattmeter readings gives the instantaneous total average power consumed by a three-phase load.

1.8.5 Power Factor Calculation by Two-Wattmeter Method [AU May/June, 2016]

In case of a balanced star or a delta-connected load, the power factor can be calculated from readings of W_1 and W_2.

The wattmeter readings in two-wattmeter method for lagging power factor loads are:

$$W_1 = |\bar{V}_L||\bar{I}_L|\cos(30° - \phi) \text{ and } W_2 = |\bar{V}_L||\bar{I}_L|\cos(30° + \phi)$$

Let, $$W_1 + W_2 = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi \tag{1.73}$$

$$W_1 - W_2 = |\bar{V}_L||\bar{I}_L|[\cos(30° - \phi) - \cos(30° + \phi)]$$
$$= |\bar{V}_L||\bar{I}_L|[\cos 30° \cos\phi + \sin 30° \sin\phi - \cos 30° \cos\varphi + \sin 30° \sin\varphi]$$
$$= |\bar{V}_L||\bar{I}_L|[2\sin 30° \sin\phi] = |\bar{V}_L||\bar{I}_L|\left[2 \times \frac{1}{2}\sin\phi\right]$$

Therefore, $$W_1 - W_2 = |\bar{V}_L||\bar{I}_L|\sin(\phi) \tag{1.74}$$

Taking ratio for the Eqn. (1.74) and Eqn. (1.73), we get

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{|\bar{V}_L||\bar{I}_L|\sin\phi}{\sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

On taking the inverse of tan function on both sides, we get

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]$$

Therefore, power factor, $\cos\phi = \cos\left\{\tan^{-1}\left[\dfrac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$

For leading power factor, we get negative value for tan ϕ. But, cosine of negative angle is always positive. Hence, cos ϕ is always positive but its nature must be determined by observing the sign of tan ϕ.

1.8.6 Effect of Power Factor on Wattmeter Readings

For a lagging power factor load, we have the wattmeter readings as:

$$W_1 = |\bar{V}_L||\bar{I}_L|\cos(30° - \phi) \text{ and } W_2 = |\bar{V}_L||\bar{I}_L|\cos(30° + \phi)$$

The effect of power factor on wattmeter readings, for different values of ϕ, is analysed as follows:
Case (i): Here, cos $\phi = 0$, Therefore, $\phi = 90°$
Therefore,

$$W_1 = |\bar{V}_L||\bar{I}_L|\cos(30° - 90°) = +\frac{1}{2}|\bar{V}_L||\bar{I}_L|$$

$$W_2 = |\bar{V}_L||\bar{I}_L|\cos(30° - 90°) = +\frac{1}{2}|\bar{V}_L||\bar{I}_L|$$

i.e., $W_1 + W_2 = 0$ (or) $W_2 = -W_1$ (or) $|W_1| = |W_2|$

It is noted that as a wattmeter has a positive scale, it cannot show negative readings. Negative reading is indicted when the pointer tries to deflect in negative direction i.e., to the left of zero. In such a case, reading can be converted to positive by interchanging either pressure coil connections i.e., ($C \leftrightarrow V$) or by interchanging current coil connections ($M \leftrightarrow L$). By interchanging connections of both the coils, there will not be any effect on wattmeter reading.

Such a reading obtained by interchanging connections of either of the two coils will be positive but it must be taken as negative for calculation purposes.

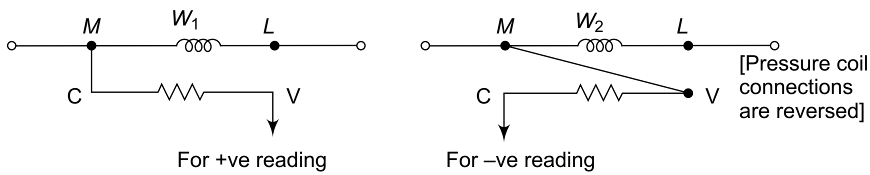


Figure 1.28 Positive and Negative Reading on Wattmeter

In this case, the reading on W_2 must be taken by reversing the connections as shown in Figure 1.28 and must be taken as negative for calculation purpose.

Case (ii): Here, $\cos\phi = 0.5$, Therefore, $\phi = 60°$

Therefore, $$W_1 = |\overline{V}_L||\overline{I}_L|\cos(30° - 60°) = |\overline{V}_L||\overline{I}_L|\cos 30° = \text{positive}$$
$$W_2 = |\overline{V}_L||\overline{I}_L|\cos(30° + 60°) = 0$$

Therefore, $W_1 + W_2 = W_1 = \text{total power}$

One wattmeter shows zero reading for $\cos\phi = 0.5$. For all power factors between 0 and 0.5, W_2 shows negative and W_1 shows positive readings, for a lagging power factor.

Case (iii): Here, $\cos\phi = 1$, Therefore, $\phi = 0°$

$$W_1 = |\overline{V}_L||\overline{I}_L|\cos(30° + 0) = |\overline{V}_L||\overline{I}_L|\cos 30° = \text{positive}$$
$$W_2 = |\overline{V}_L||\overline{I}_L|\cos(30° - 0) = |\overline{V}_L||\overline{I}_L|\cos 30° = \text{positive}$$

Therefore, both W_1 and W_2 are equal and positive. For all power factors between 0.5 and 1, both wattmeters give positive reading.

The results can be summarized as given in Table 1.1

Table 1.1 *Variations of W_1 and W_2 for Different Power Factors*

Range of p.f.	Range of 'ϕ'	W_1 sign	W_2 sign	Remark
$\cos\phi = 0$	$\phi = 90°$	Positive	Negative	$\|W_1\| = \|W_2\|$
$0 < \cos\phi < 0.5$	$90° < \phi < 60°$	Positive	Negative	—
$\cos\phi = 0.5$	$\phi = 60°$	Positive	0	—
$0.5 < \cos\phi < 1$	$60° < \phi < 0°$	Positive	Positive	—
$\cos\phi = 1$	$\phi = 0°$	Positive	Positive	$W_1 = W_2$

It is noted that Table 1.1 is applicable for lagging power factor loads, but the same Table is applicable for leading power factor loads by interchanging the columns of W_1 and W_2. The effect of power factor on wattmeter readings i.e., curve of power factor against K, is shown in Figure 1.29, where K is given by:

$$K = \frac{\text{Smaller reading}}{\text{Larger reading}}$$

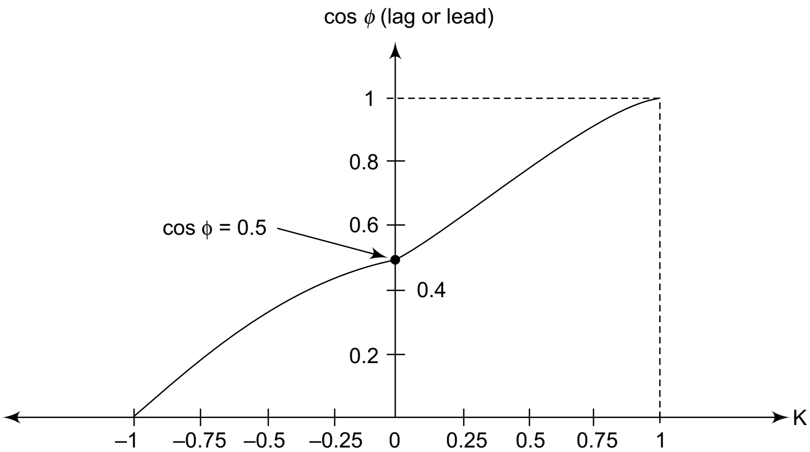


Figure 1.29 Effect of Power Factor on Wattmeter Readings

1.8.7 Reactive Volt-Amperes by Two-Wattmeter Method

The reactive Volt-Amperes can be determined as follows:

It is known that, $W_1 - W_2 = |\bar{V}_L||\bar{I}_L| \sin \phi$

The total reactive Volt-Amperes for a three-phase circuit is given by,

$$Q = \sqrt{3}\, |\bar{V}_L||\bar{I}_L| \sin \phi = \sqrt{3}\, (W_1 - W_2) \text{ VAR}$$

Thus, reactive Volt-Amperes of a three-phase circuit can be obtained by multiplying $\sqrt{3}$ with the difference of the two-wattmeter readings.

Therefore, apparent power, $S = \sqrt{3}\, |\bar{V}_L||\bar{I}_L|$ VA

Active power, $P = \sqrt{3}\, |\bar{V}_L||\bar{I}_L| \cos \phi = W_1 + W_2 \quad$ W

Reactive power, $Q = \sqrt{3}\, |\bar{V}_L||\bar{I}_L| \sin \phi = \sqrt{3}\, (W_1 - W_2)$ VAR

Example 1.17

Two wattmeters are connected to measure the power in a three-phase, three-wire balanced load. Determine the total power and power factor if the two wattmeters read: (i) 1000 W each, both positive (ii) 1000 W each, of opposite sign. **[AU April/May, 2010]**

Solution

Case (i) When $W_1 = W_2 = 1000$ W

The total power consumed by the load is given by

$$W = W_1 + W_2 = 2000 \text{ W}$$

The power factor of the given load is

$$\cos \phi = \cos \left\{ \tan^{-1} \left[\frac{\sqrt{3}\, (W_1 - W_2)}{(W_1 + W_2)} \right] \right\}$$

Substituting the known values, we get

$$\cos\phi = 1$$

Therefore, the system is a unity power factor system.

Case (ii) When $W_1 = 1000$ W and $W_2 = -1000$ W

The total power consumed by the load is given by

$$W = W_1 + W_2 = 0 \text{ W}$$

The power factor of the given load is

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

Substituting the known values, we get

$$\cos\phi = 0$$

Therefore, the system is a zero power factor system.

Example 1.18

The two-wattmeters method produces wattmeter readings: $W_1 = 1560$ W and $W_2 = 2100$ W, when connected to a delta-connected load. If line voltage is 220 V, calculate (i) the per-phase average power (ii) the per-phase reactive power (iii) the power factor and (iv) the phase impedance. **[AU May/June, 2003]**

Solution

Given, $W_1 = 1560$ W and $W_2 = 2100$ W

(i) The total active power is given by

$$P = W_1 + W_2 = 3660 \text{ W}$$

Therefore, active power per phase is $\dfrac{3660}{3} = 1220$ W

(ii) The total reactive power is given by

$$Q = \sqrt{3}\,(W_1 \sim W_2) = \sqrt{3}\,(2100 - 1560)$$
$$= 935.31 \text{ VAR}$$

Therefore, reactive power per phase is $\dfrac{935.31}{3} = 311.77$ VAR

(iii) The power factor of the system is given by

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

Substituting the known values, we get

$$\cos\phi = 0.97 \text{ and } \phi = 14.06°$$

(iv) Line voltage, $V_L = 220$ V

The total active power is given by

$$P = \sqrt{3} V_L I_L \cos\phi$$

Substituting the known values in the above equation, we get

$$\bar{I}_L = \frac{3660}{\sqrt{3} \times 220 \times 0.97} = 9.9024 \text{ A}$$

The phase current in the delta-connected load is obtained as

$$\bar{I}_{\text{ph}} = \frac{\bar{I}_L}{\sqrt{3}} = 5.7173 \text{ A}$$

But, the phase current in the delta-connected load is given by

$$\bar{I}_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{Z_{\text{ph}}}$$

Substituting the known values in the above equation, we get

$$Z_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{\bar{I}_{\text{ph}}} = \frac{220}{5.7173} = 38.47\ \Omega$$

Impedance angle will be $30° + \phi = 44.06°$

Therefore, the phase impedance existing in the delta-connected load is $38.47\angle 44.06°\ \Omega$

Example 1.19

Two wattmeters, connected to measure the input to a balanced, three-phase circuit, indicate 2000 W and 500 W respectively. Find the power factor of the circuit: (i) when both readings are positive and (ii) when the latter is obtained after reversing the connections to the current coil of one instrument.

Solution

Case (i) W_1 = 2000 W and W_2 = 500 W

The power factor of the system is given by

$$\cos\phi = \cos\left[\tan^{-1}\left\{\frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}\right\}\right]$$

Substituting the known values and solving, we get

$$\cos\phi = 0.69$$

Case (ii) W_1 = 2000 W and W_2 = –500 W

The power factor of the system is given by

$$\cos\phi = \cos\left[\tan^{-1}\left\{\frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}\right\}\right]$$

Substituting the known values and solving, we get

$$\cos\phi = 0.33$$

Example 1.20

The power input to a 2000 V, 50 Hz three-phase motor, running on full load at an efficiency of 90% is measured by two wattmeters, which indicate 300 kW and 100 kW respectively. Calculate (i) input power (ii) power factor (iii) line current and (iv) HP output.

Solution

Given, W_1 = 300 kW, W_2 = 100 kW, efficiency = 90%, line voltage, V_L = 2000 V

(i) The input power is given by $P_i = W_1 + W_2 = 300 = 400$ kW

(ii) The power factor of the system is given by

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(300-100)}{(400)}\right]\right\} = 0.76$$

(iii) Line current is obtained by

$$I_L = \frac{P_i}{\sqrt{3}V_L\cos\theta} = \frac{400\times10^3}{\sqrt{3}\times2000\times0.76} = 151.93\text{ A}$$

(iv) Output power is obtained by

$$P_0 = P_i \times \eta\; 400 \times 0.9 = 360\text{ kW}$$

We know that 1 kW = 1.34 HP. Therefore, the output power in HP is given by

$$P_0 = 360 \times 1.34\text{ HP} = 482.4\text{ HP}$$

1.9 Introduction to Electric Power System **[AU May/June, 2016]**

A basic necessity in the economic development of a country is the amount of energy generated, transmitted and distributed. The amount of energy consumed by a person is defined as the per capita consumption of energy. There is a close relationship between the per capita consumption and a country's development. If the per capita consumption of energy is high, then the standard of living of the people is high. In nature, though energy exists in different forms, electrical energy is the most important form of energy, which has high impact on the modern society and has become a part of our daily life. In this section, the generation, transmission and distribution of electrical energy, which together forms the electric supply system or the electric power system is discussed. The transmission and distribution systems are further classified as primary and secondary transmissions, and primary and secondary distributions respectively. A typical single-line diagram of an AC power system is shown in Figure 1.30.

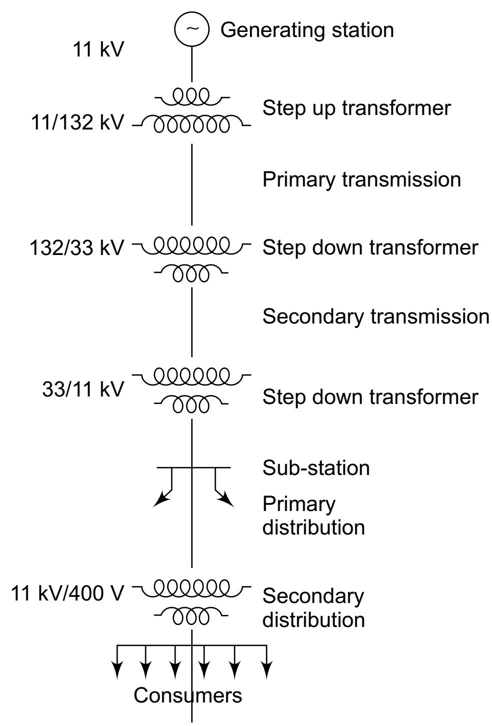


Figure 1.30 Single-line Diagram of an AC Power System

1.9.1 Advantages of Electrical Energy

The electrical energy is superior to all other forms of energy and has the following advantages:

1. Convenient form, as it can be converted to any other form of energy.
2. Easier control.

3. It has greater flexibility, as it can be transmitted from one place to another.
4. Economically available to domestic, commercial and industrial consumers.
5. Does not create pollution by generating smoke, fumes and so on.
6. Electrical energy can be transmitted effectively.

1.9.2 Generation of Electrical Energy

In general, generation of electrical energy is a process that involves the conversion of available energy in nature to electrical energy. As electrical energy can be generated from the resources available, the power stations that produce electrical energy are located mostly in remote places.

The general schematic diagram to represent the generation of electrical energy is shown in Figure 1.31, which comprises of a prime mover and an alternator. The prime mover is driven by the energy, which is obtained from various energy sources and the output of the prime mover is converted into electrical energy using an alternator. A generating station or a power plant is defined as the place where a bulk amount of electrical energy is generated.

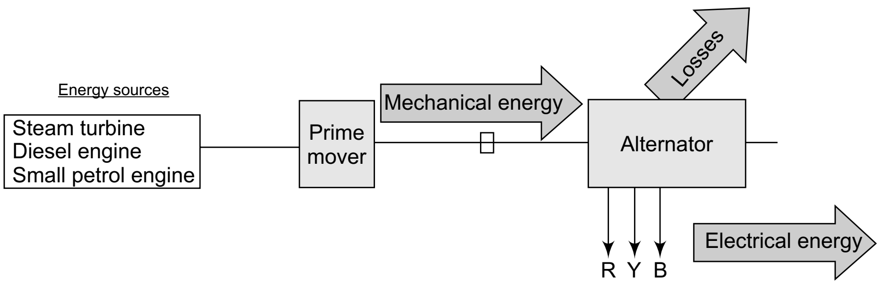


Figure 1.31 Schematic Diagram for the Generation of Electrical Energy

The different sources of energy from which the electrical energy can be produced or generated are: sun, wind, water, fuel, nuclear energy and so on, and their corresponding generating stations are solar power station, wind power station, hydroelectric power station, diesel power station, nuclear power station and so on respectively. The unit used to represent the electrical energy is watt-sec or joule, which is defined as: one watt-second energy transferred between two points when a potential difference of one volt exists between them and one ampere current passes between them for one second i.e., electrical energy in watt-sec = voltage in volt × current in ampere × time in second. But, the power stations or generating stations are represented based on the amount of kV they generate. Generally, the electrical energy is generated at 11 kV or 6.6 kV and it can be stepped up to 110 kV or 230 kV or 400 kV or still higher with the help of power transformers.

1.9.3 Transmission of Electrical Energy **[AU May/June, 2016]**

Electrical energy generated at power stations can be transmitted to different load centres, using overhead lines or underground cables. The two ways in which the electrical energy is transmitted are: primary transmission and secondary transmission. Generally, the electrical energy is transmitted at higher voltages, using overhead lines in primary transmission and using underground cables in secondary transmission.

The advantages of transmitting electrical energy at higher voltages are:
1. Less volume of conductor material is required.
2. The current flowing through the line is reduced, which in turn reduces line losses and helps in increasing the efficiency of transmission.

3. Voltage regulation of the transmission system gets improved.

In primary transmission, the electrical energy is transmitted to main-load centres, where the voltage level is 1500 kV or 700 kV or 400 kV or 230 kV. In secondary transmission, the electrical energy is transmitted to sub-load centres, where the voltage level is 33 kV or 66 kV or 110 kV. The electrical energy can also be transmitted using DC transmission. The comparison between AC and DC transmissions is listed in Table 1.2

Table 1.2 *Comparison Between AC and DC Transmissions*

AC Transmission	DC Transmission
Requires three conductors for transmission and hence requires more copper material.	Requires two conductors for transmission and hence requires less copper material.
Inductance, capacitance, phase displacement and surge problems exist.	No such problems exist.
Voltage drop in the transmission line is more due to the presence of inductance.	Since no inductance exists, voltage drop is less.
Poor voltage regulation	Better voltage regulation
Skin effect exists	Absence of skin effect
Requires more insulation, as the potential stress on the conductors is more.	Potential stress on the conductor is less and hence requires less insulation.
Corona loss and interference with communication circuits are more.	Less corona loss and reduced interference with communication circuits.
Dielectric loss exists in the system.	Free from dielectric loss in the system.
Stability problem and synchronizing difficulties exist in the system.	DC transmission system is free from these problems.
AC voltage can be stepped up for transmission of power.	DC voltage cannot be stepped up for transmission.
Electrical energy can be generated at a high AC voltage.	Commutation problem exists in generating electrical energy at a high DC voltage.
Easy and cheaper to maintain an AC system.	Difficult and costlier to maintain a DC system.
Difficult to construct	Easier to construct

1.9.4 Distribution of Electrical Energy

The electrical system, which distributes electrical energy to the consumers, is known as a distribution system. The electrical system between the sub-stations and service mains is known as a distribution system. There are two types of distribution systems: primary and secondary distributions. The power, which is received at 33 kV, 66 kV or 110 kV, is stepped down to 11 kV, where the power is distributed along main roads and is called the primary distribution system. The power is distributed through distribution lines called distributors, which run along the streets of consumers. The distribution lines, which feed the power from primary distribution to the distributor, are called feeders. The power is then distributed to the consumers from the distributors using service lines. Therefore, feeders, along with service lines, make up the secondary distribution system.

1.9.5 Essential Components of Transmission and Distribution Systems

Both transmission and distribution of electrical energy from generating stations to consumers can be done using overhead lines or underground cables.

Components of Overhead Lines

The essential components of overhead lines, which can be used for transmitting or distributing electrical energy are:

- Conductors, which are used to carry electrical power from the sending-end to the receiving-end of a particular station.
- Support, which is provided using poles or towers, helps in keeping the conductors at a suitable height from the ground.
- Insulators, which are used to insulate the conductors from the ground.
- Cross-arms are used to provide support to the insulators.
- Miscellaneous items like lightning arrestors, phase plates and so on.

Components of Underground Cables **[AU May/June, 2016]**

The essential components of underground cables are shown in Figure 1.32, which can be used for transmitting or distributing electrical energy. The various components of the cables are explained as follows:

- Cores or conductors, which are made up of stranded tinned copper or aluminium to provide flexibility to the cable.
- Insulation is a layer, which is placed above the conductor, with a suitable thickness that decides the maximum voltage that can be transmitted through the cable.
- Metallic sheath is provided in the cable to protect the cable from moisture, gases or other liquids, which could damage the cable.
- Bedding, which is made up of fibrous material like jute or hessian tape, is provided over the metallic sheath to prevent it from corrosion and other mechanical injuries.
- Armouring, which is made up of galvanized steel wire or steel tape, to protect the cable while it is being laid or handled.
- Serving is used for protecting the armouring from atmospheric conditions and is made up of a fibrous material called jute.

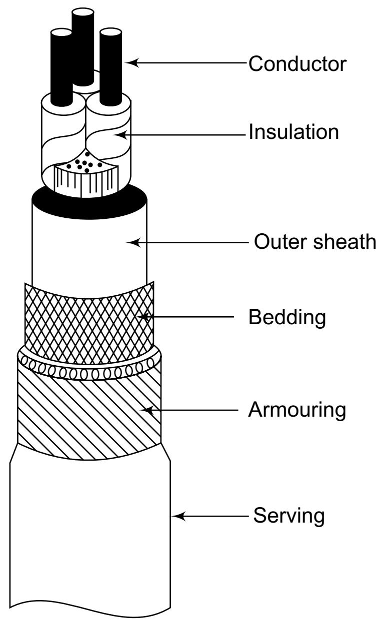


Figure 1.32 Schematic Diagram of an Underground Cable

1.9.6 Comparison Between Overhead Lines and Underground Cables

Even though the electrical energy can be transmitted or distributed using overhead lines and underground cables, the selection of one over the other is based on the following reasons listed in Table 1.3.

Table 1.3 *Comparison Between Overhead Lines and Underground Cables*

Parameter	Overhead Lines	Underground Cables
Public safety	More dangerous to the public	Less hazardous to the public
Initial cost	Less costly	It is 10 times the cost of an overhead line
Flexibility	More flexible and can be modified easily	Less flexible

(*Contd.*)

Faults	Chance of fault occurrence is more	Less chance of fault occurrence
Appearance	Bad, since the wiring is visible	Good, since the wiring is not visible
Fault location and repair	Easier to locate the fault and repair it	Difficult to locate the fault and repair it
Useful life	Less	More
Maintenance cost	High	Very low
Interference with communication circuits	Faces problems due to interference	No interference with the communication circuits
Lightning	Liable to hazards from lightning	Not liable to hazards from lightning
Submarine crossings	Cannot be used	Can be used
Transmission	Used for long-distance transmission	Used for short-distance transmission

1.10 Introduction to Power System Protection

In general, the operating voltage of an electrical power system ranges from 415 V to 400 kV or even higher. The different equipment used in the electrical power system are: machines, transformers, transmission lines, insulators, bus bars, cables and so on, which may be placed in open or closed conditions. Due to various reasons, all the equipment may undergo abnormal or faulty conditions in their life span. Some examples of faulty conditions are: insulation failure in the cable due to lightning surges, increase in the voltage (i.e., overvoltage) of the alternator due to sudden loss of load, undesirable heating in the transformer due to open-circuit condition, deterioration of insulation winding in transformer due to winding short-circuits and so on. Hence, it becomes necessary to protect equipment from various faulty conditions like over-voltage, lightning surges, insulation failure, resonance, improper earthing, short-circuit or open-circuit conditions, and balanced and unbalanced faults, which ensures the continuous working of the equipment. Also, it is important to safeguard the human personnel who get exposed to power system equipment under faulty or abnormal conditions. In general, the protection scheme is classified as: (i) primary protection and (ii) back-up protection. The primary protection is designed in such a way that the components of the power system are protected. The back-up protection is the secondary defence mechanism, which comes into operation when the primary protection scheme fails.

The different protective equipment or devices that are used in the power system, to safeguard different equipment, are: circuit breakers, fuses and protective relays. During normal operation of the power system, these protective devices allow to switch on or switch off the electrical equipment. On the other hand, if a fault occurs in any part of the power system, the protective device detects these faults and disconnects the unhealthy portion of the system, thereby protecting the power system from damage, to ensure continuity of the supply.

1.10.1 Essential Features of Protective Devices [AU May/June, 2014]

The essential features of protective devices are:

- **Complete reliability:** It is the most important feature, which all the protective devices should have, in the power system. If a fault occurs in any part of the power system, then the protective device should operate in such a way that the faulty section gets isolated from the other part of the power system.
- **Absolutely certain discrimination:** Clear and accurate discrimination between the faulty section and healthy section is required to isolate the faulty section. This feature will ensure the continuity of supply.

- **Quick operation:** The time taken by the protective device to isolate the faulty section must be minimum so that other parts of the system do not get damaged.
- **Provision for manual control:** Even if the protective device can be made automatic, there should be a provision for manual control to carry out the necessary operations when the automatic control fails.

1.11 Circuit Breaker [AU April/May 2015]

A switching device, which can be used to make or break a circuit manually, automatically or with the help of remote control, under different conditions i.e., normal and under faulty conditions, is known as a circuit breaker. Special attention must be given while designing a circuit breaker, to safely interrupt the arc produced during its operation, as modern power systems deal with very high currents.

1.11.1 Working of a Circuit Breaker

The fixed and moving contacts, which are called electrodes, exist in a normal circuit breaker. The medium in which these contacts are placed could either be oil or air. When the power system is operating normally, these contacts will remain closed and will not open automatically until a faulty condition occurs in the system. Whenever a fault occurs in the system, these contacts can be opened either manually or automatically or by using a remote control. During a faulty condition, due to energization of trip coils of the circuit breaker, the moving contact is pulled apart, which opens the circuit and an arc is formed between these contacts. The schematic diagram of the circuit breaker under normal and fault conditions is shown in Figure 1.33.

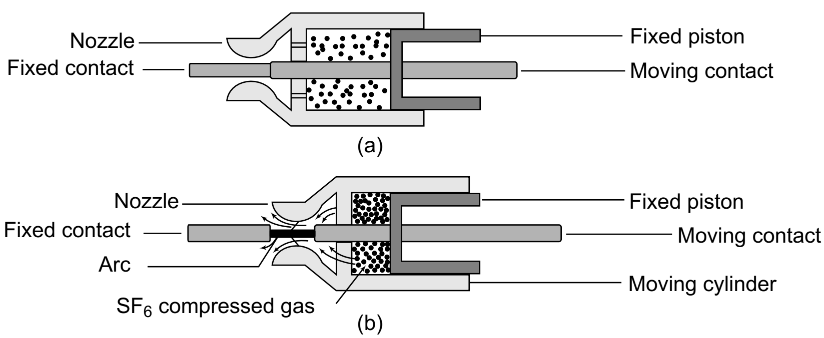


Figure 1.33 Circuit Breaker in (a) Normal and (b) Faulty Conditions

An arc develops between the fixed and moving contacts, when a fault occurs in the power system. The faulty current in the power system will continue to flow until this arc is extinguished or stopped. Therefore, the formation of the arc not only delays the interruption of faulty current, but also generates huge amounts of heat, which might cause damage to the power system or the circuit breaker itself. Hence, it is necessary to extinguish the arc developed in a short interval of time, so that the magnitude of heat generated will not exceed a maximum value.

1.11.2 Arc Phenomenon [AU May/June, 2014]

When a faulty condition occurs before the fixed and moving contacts are separated, a huge amount of current starts flowing in the power system. During the separation of contacts, due to the rapid decrease in the contact area and the large amount of fault current, the current density increases, which in turn increases the temperature. The increase in temperature in the medium either ionizes the air or vaporizes and ionizes the oil, which then act as conductors and help in striking an arc between the contacts. The developed arc is maintained, as there exists only a small potential difference between the contacts. Since the arc developed a low-resistance path, the fault current existing in the power system will remain uninterrupted till the persistence of arc between the contacts.

The period till which the arc exists between the contacts is known as arcing period. In arcing period, the arc resistance plays a vital role and is inversely proportional to the magnitude of current flowing through the contacts. The factors that affect the arc resistance are:

- **Degree of ionization:** An inverse relation exists between the arc resistance and the number of ionized particles in the medium.
- **Length of the arc:** When the length of the arc or distance between the contacts increases, the arc resistance increases.
- **Cross section of arc:** An inverse relation exists between the arc cross-section and the arc resistance.

It is very important to extinguish or quench the arc as early as possible, before it could cause serious damage to the system. As the arc developed forms a conductive path for electricity, the fault current flowing through the circuit breaker will not be interrupted till the arc is extinguished. The most important designing criteria of a circuit breaker is providing proper technique for quenching the arc, to ensure a quick and safe fault-current interruption.

1.11.3 Principles of Arc Extinction [AU Nov/Dec, 2012]

The principle factors responsible for arc maintenance between the contacts are:

- **Potential difference between the contacts:** In general, the potential difference between two points is inversely proportional to the distance between the points. Hence, one way of quenching the arc is to separate the contacts such that the potential difference between the contacts will be inadequate to maintain the arc. But, in practice, this method is not possible in a high-voltage system, where the required distance between the contacts is very high to quench the arc.
- **Ionized particles in the medium between the contacts:** The particles in the medium, which are ionized between the contacts, help to maintain the arc. Therefore, by either cooling the arc or by removing the ionized particles existing between the contacts, the arc can be extinguished.

1.11.4 Methods of Arc Extinction [AU Nov/Dec, 2012]

The two different methods by which the arc can be extinguished are:

- High-resistance method and
- Low-resistance method or current zero method

High-Resistance Method

In this method, arc resistance is allowed to increase with time to a particular value, which will be insufficient to maintain the arc and consequently interrupts the fault current or extinguishes the arc. The main disadvantage of this method is the large amount of energy that is dissipated in the arc, as it is directly proportional to the arc resistance. Hence, this method is applicable only to DC and low-capacity circuit breakers. The different ways by which the arc resistance can be increased are:

- **Increasing the length of the arc:** When the gap between the contacts is increased to increase the length of the arc, the arc resistance increases, as there exists a direct relation between the arc resistance and the length of the arc.

- **Cooling the arc:** Efficient cooling of the medium, directed along the arc using gas blast, helps in deionization of the particles in the medium, which increases the arc resistance.
- **Reducing cross-sectional area of the arc:** Allowing the arc to pass through a small opening or by having a smaller contact area will reduce the cross-sectional area of the arc, which in turn increases the arc resistance, as the cross-sectional area is inversely proportional to the arc resistance.
- **Arc splitting:** Splitting of the arc into more number of smaller arcs, using conducting plates between the two contacts, will experience the length-decreasing effect and cooling effect, which helps in increasing the arc resistance.

Low-resistance or Current-Zero Method

The extinction of an arc in modern high power AC circuits is carried out using this method. In this method, the resistance of the arc is kept as low as possible till the fault current becomes zero. The arc is naturally extinguished and helps in preventing the re-strike of the arc, even when there is a rise in potential difference across the contacts. In an AC system, whenever the current drops to zero at every half-cycle, there will be a brief moment where the arc gets extinguished. Here, the dielectric strength of the ions and the electrons existing in the medium between the contacts is small, which can breakdown easily due to the rising contact voltage called as re-striking voltage. Due to such a breakdown, the arc developed between the contacts exists for another half cycle. If the dielectric strength of the medium is made to increase rapidly than the voltage across the contacts, the arc fails to re-strike near the current zero and hence the fault current gets interrupted. The different ways by which the dielectric strength can be increased rapidly are:

- Recombination of ionized particles with neutral molecules in the medium between the contacts.
- Swiping the ionized particles with the unionized particles.

Therefore, the rapid de-ionization of the particles of the medium as soon the current becomes zero is the most important problem in an AC arc interruption. The de-ionization of the particles in the medium can be achieved by using any one of the following methods:

- **Lengthening of the gap:** Rapid opening of the contacts increases the dielectric strength of the medium, as it depends on the length of the gap between the contacts and dielectric strength of the medium.
- **High pressure:** Increasing the pressure in the surrounding area of the arc increases the density of the discharging particles, which in turn increases the speed of de-ionization and helps in increasing the dielectric strength of the medium.
- **Cooling:** If the ionized particles are allowed to cool, there will be a natural combination of ionized particles and thereby, the dielectric strength of the medium increases.
- **Blast effect:** Complete removal of the ionized particles using gas blast or forced oil along the discharge and replacing it with unionized particles increases the dielectric strength of the medium.

1.11.5 Terms Associated with Circuit Breaker **[AU Nov/Dec, 2014]**

The terms which are associated with circuit breaker are: (i) arc voltage (ii) re-striking voltage and (iii) recovery voltage.

- **Arc voltage:** It is the voltage that is obtained across the contacts of the circuit breaker during the arcing period.
- **Re-striking voltage:** It is the voltage that appears across the contacts at the instant of arc extinction.
- **Recovery voltage:** It is the normal frequency RMS voltage that appears across the contacts of circuit breaker after the final arc extinction and it is approximately equal to the system voltage.

1.11.6 Classification of Circuit Breakers **[AU Nov/Dec, 2014]**

The different categories, based on which the circuit breaker is classified, are: (i) medium in which the circuit-breaker operates (ii) actuating signal in which it works (iii) construction type (iv) voltage level and so on.

Category 1: Working Medium

Based on the working medium, the circuit breaker is further classified as:
- Oil circuit breaker
- Air Blast circuit breaker
- SF6 circuit breaker
- Vacuum circuit breaker

Category 2: Actuating Signal

Based on the actuating signal, which is required to actuate the circuit breaker, it is further classified as:
- Spring-operated circuit breaker
- Pneumatic circuit breaker
- Hydraulic circuit breaker

Category 3: Voltage Level

Based on the voltage level in which the circuit breakers operate, it is further classified as:
- Low-voltage circuit breakers (< 1 kV)
- Medium-voltage circuit breakers (1-72 kV)
- High-voltage circuit breakers (> 72 kV)

Category 4: Location of Circuit Breaker

Based on the location of circuit breaker, it is further classified into:
- Outdoor circuit breaker
- Indoor circuit breaker

1.12 Fuse

A fuse is a short piece of wire or a thin strip of metal, which is inserted in series to the circuit. When the fault current flows through the fuse for a sufficient time, it melts the fuse, thus isolating the circuit. Under normal operation, the fuse is kept at a temperature below the melting point of the material used, which helps in carrying the normal current without any rise in temperature. But when fault occurs in the power system due to a short circuit or when an overload current, which is greater than the normal current, flows through the fuse, this fault current will increase the temperature above the melting point of the material used for the fuse. Hence, the material melts or blows, thereby isolating the healthy part and protects the circuit. The magnitude of excessive current flowing in the circuit is an important factor in deciding the time taken for melting or blowing out the fuse. Greater the fault current, lesser the time required to melt or blow out the fuse. The inverse time-current characteristics of a fuse are shown in Figure 1.34.

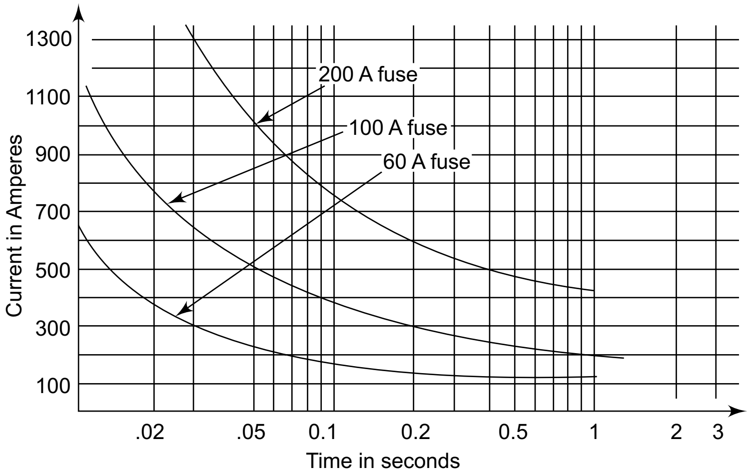


Figure 1.34 Inverse Time–Current Characteristics of a Fuse

1.12.1 Advantages and Disadvantages

The advantages and disadvantages of fuse are given as follows:

Advantages

1. Cheapest form of protection device.
2. Requires no maintenance.
3. Operation of fuse is completely automatic.
4. Easily breaks a large amount of fault current.
5. Pollution-free protection device i.e., does not create any smoke or noise.
6. Suitable for over-current conditions due to its inverse current–time characteristics.
7. Requires less time for isolating the faulty part of the circuit.

Disadvantages

1. Rewiring or replacing a fuse takes a considerable time.
2. Discrimination between fuses connected in series is not possible.
3. Correlation of the characteristics of fuse with the protected device is not always possible.

1.12.2 Desirable Characteristics of Fuse Element Materials

The desirable characteristics of the material used for the fuse to perform satisfactorily are:

- Low melting point, e.g., tin, lead
- High conductivity, e.g., silver, copper
- Least reactive to oxidation, e.g., silver
- Affordable, e.g., lead, tin, copper

It can be noted that no element possesses all the desirable characteristics of a fuse and hence a compromise is to be made in selecting a material for the fuse. Lead, tin, copper, zinc and silver are the most commonly used fuse materials. Tin, or an alloy of lead and tin (0.37 and 0.63 respectively) is used as a fuse element material, where the rating of current is up to 10 A. For larger currents, copper or silver is used as fuse element material. Usually, the copper is tinned to prevent oxidation effect. Zinc, in strip form, is used where a considerable time-delay is required. In day-to-day activities, silver is used as a fuse element due to the following characteristics:

- Does not get affected or deteriorate when used in dry air.
- As the expansion coefficient of silver is very small, it can carry the rated current continuously for a long time.
- Conductivity is very high.
- Instantaneous transition to vapour state from melting state, when compared to other materials, is possible due to low specific heat.
- Faster operation is possible at higher currents.
- Quick interruption of fault current is possible as the element vaporises at a temperature much lower than the temperature required to ionize it.

1.12.3 Important Terms **[AU Nov/Dec, 2014]**

Following are the terms required in fuse analysis:

- **Current rating of fuse element:** It is the amount of current that the fuse element can carry under normal operation, without overheating or melting. It depends on temperature rise in the fuse holder, the fuse material and the surroundings of the fuse.
- **Fusing current:** It is the minimum current at which the fuse element melts or blows away and isolates the healthy portion of the power system. It is higher than the current rating of the fuse element.
- **Fusing factor:** It is the ratio of the fusing current to the current rating of fuse element and its value is always greater than 1.
- **Prospective current:** It is the RMS value of the fault current, which is obtained by replacing the fuse with a conductor of negligible resistance.
- **Cut-off current:** It is the maximum value of fault current obtained before the fuse element melts.
- **Pre-arcing time:** The time taken to cut off the fault current from its commencement is known as pre-arcing time.
- **Arcing time:** The time taken to extinguish the arc after the pre-arcing time is known as arcing time.
- **Total operating time:** It is the summation of pre-arcing and arcing time.
- **Breaking capacity:** The RMS value of the maximum prospective current, which a fuse can deal at rated voltage, is known as breaking capacity.

1.12.4 Classification of Fuses

The general classification of fuses is given as follows:

1. Low-voltage fuses
   - Semi-enclosed re-wireable fuse
   - High rupturing capacity (HRC) cartridge fuse with and without tripping device
2. High-voltage fuses
   - Cartridge type
   - Liquid type
   - Metal-clad fuses

1.12.5 Comparison Between Circuit Breaker and Fuse [AU May/June, 2014]

Specification	Circuit Breaker	Fuse
Function	Performs interruption function	Performs detection and interruption functions
Operation	Requires more equipment for automatic operation.	Requires less equipment for full automatic operation
Breaking capacity	Very large	Small
Operating time	Large (0–1 or 0–2 sec)	Very small (0.002 sec)
Replacement	It need not be replaced after its operation.	Requires continuous replacement after every operation.

1.13 Protective Relays [AU April/May, 2015]

A device that detects the fault in the system and initiates the circuit breaker operation and helps in isolating the faulty element from the healthy portion of the system is known as a protective relay. Electrical quantities whose values get changed during normal and abnormal conditions are: voltage, current, frequency and phase angle. Protective relay constantly measures these electrical quantities. If any one of the electrical quantities changes, the protective relay detects the abnormal condition and operates in such a way that the circuit breaker isolates the faulty portion from the healthy portion. A typical relay circuit is shown in Figure 1.35.

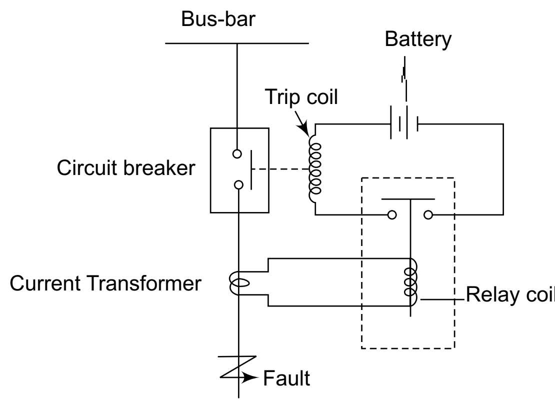


Figure 1.35 Simple Protective Relay Circuit With Circuit Breaker

The three different parts of a simple relay circuit are: (i) primary winding of the current transformer (CT), which is placed in series with the transmission line to be protected, (ii) secondary winding of CT and operating coil of relay and (iii) tripping circuit, which has an AC or DC source, trip coil of the circuit breaker and relay contacts. When a fault occurs at point F, the current through the transmission line increases and it starts flowing through the relay coil. Then, the increased current causes the relay contact to close the tripping circuit of the circuit breaker, which makes the circuit breaker to open, thereby isolating the faulty section from rest of the system.

1.13.1 Fundamental Requirements of Protective Relay [AU Nov/Dec, 2014]

The fundamental requirements of a protective relay to detect the fault and trigger the circuit breaker are given below:

- **Selectivity:** It is the ability of the protective relay to select the exact location of the faulty system and disconnect it without affecting the other parts of the system.
- **Speed:** The disconnection of the faulty section using protective relay should be as fast as possible. Otherwise, (i) the electrical equipment may get damaged (ii) the system voltage may get reduced and (iii) one type of fault may develop into other types of faults.

- **Sensitivity:** The minimum value of the actuating quantity, which is required to operate the protective relay system.
- **Reliability:** It is the ability of the protective relay system to operate under predetermined conditions.
- **Simplicity:** Maintenance of the protective relay system should be simple.
- **Economy:** Economically, a particular type of protective relay system should be selected.

1.13.2 Operating Principle of Protective Relays

The most commonly used protective relay in the power system is an electro-mechanical type relay. The operating principles on which these protective relays work are: electromagnetic attraction and electromagnetic induction.

Electromagnetic Attraction Relays

The basic concept of armature attraction by poles of an electromagnet or a plunger attraction using solenoid is implemented in electromagnetic attraction relays. These relays are actuated either by DC or AC sources.

Electromagnetic Induction Relays

In the electromagnetic induction relays, the initial force is developed on the moving element due to the interaction of electromagnetic fluxes with eddy current. The electromagnetic induction relays are widely used in applications involving AC quantities and not preferred with DC quantities.

1.13.3 Relay Timing

The operation time is an important characteristic of the relay. Operation time is the time taken between the instant at which the actuating signal is energized and the instant at which the relay contacts are closed. Based on the relay timing or the operation time of the relay, protective relays are classified as:

- **Instantaneous relay:** The contact in the relay circuit gets closed immediately when the electrical quantity in the relay coil exceeds the maximum limit, without any intentional time delay.
- **Inverse-time relay:** The operating time of the inverse-time relay is inversely proportional to the magnitude of the electrical quantity.
- **Definite time-lag relay:** There exists a definite time lag between the instant at which the electrical quantity exceeds the maximum value and the instant at which the relay contacts are closed. This type of time setting is independent of the magnitude of the electrical quantity flowing through the relay coil.

1.13.4 Important Terms **[AU Nov/Dec, 2014]**

- **Pick-up current:** The minimum value of current required to flow through the relay coil to make the relay operational is known as pick-up current. When the current flowing through the relay coil is less than this value, the relay does not operate. The magnitude of pick-up current is the product of the rated secondary current of CT and the current setting.
- **Current setting:** The current setting is the setting of pick-up current to any required value and is achieved using tappings in the operating coil of the relay.
- **Plug-setting multiplier (PSM):** It is given by the ratio of the magnitude of fault current in the relay coil to the pick-up current value.
- **Time-setting multiplier:** The adjustment provided in the relay to control or adjust the operation time of the relay is known as time-setting multiplier.

The operating time vs. P.S.M of a protective relay is shown in Figure 1.36. The operating time of the relay is calculated if all the above terms are known.

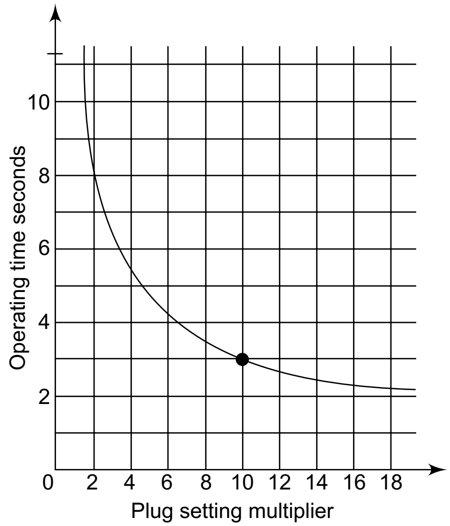


Figure 1.36 Operating Time vs. Plug-Setting Multiplier

1.13.5 Functional Relays

The most important functional relays are:

- **Induction type over-current relay (non-directional):** It works on the induction principle and operates the circuit breaker when the current in the circuit exceeds a predetermined value.
- **Induction type directional power relay:** It works on the induction principle and operates when the power in the circuit flows in a particular direction.
- **Distance or impedance relay:** Its operation is based on the ratio of applied voltage to the current flowing in the circuit. Two types of distance or impedance relays are: (a) definite distance-type impedance relay and (b) time-distance impedance relay.
- **Differential relay:** When the phasor difference of two or more similar electrical quantities exceeds a maximum value, the relay becomes operational. Two types of differential relays are: (a) current differential relay and (b) voltage-balance differential relay.
- **Transley scheme:** It is a balanced voltage scheme with the addition of a directional feature.

1.14 Introduction to Tariff

A large number of consumers can be influenced to use electrical energy produced by numerous power stations, if it is sold at a reasonable price or rate. The reasonable price or rate at which the produced electrical energy is supplied to the consumer is defined as tariff. In other words, the price charged by the supplier to supply electrical energy to various types of consumers in known as tariff. Based on tariff, there will be a healthy competition between the companies that supply electrical energy. Each supply company will fix the tariff for the produced electric energy in such a way that it earns profit on its capital investment, in addition to the total cost spent in producing electrical energy. The tariff at which the electrical energy is charged is not uniform for all the consumers (domestic, commercial, and industrial), as it depends on the magnitude of consumption and its load condition. Therefore, in fixing the tariff, due consideration has to be given to the type of consumer, which leads to more complications.

1.14.1 Objectives of Tariff

Like in the case of other commodities, electricity tariff covers the production and supply cost of electrical energy with a reasonable profit. Therefore, certain objectives and requirements are to be satisfied before fixing a tariff for the electrical energy. They are:

- Cost of producing electrical energy at a place should be recovered.
- Cost of the capital investment for generation, transmission and distribution of electrical energy should be recovered.

- Cost of the operation, raw materials, maintenance and losses of electrical energy must be recovered.
- Cost of miscellaneous services like metering, billing, collection and so on, should be recovered.
- Tariff should ensure the satisfactory net return or profit on the capital investment.
- Tariff should be uniform for large number of population.
- Tariff should be simple and easily understandable to the consumers.
- Proper advantage must be given to the consumers using electrical energy during off-peak hours.
- Proper charges must be provided to the consumers demanding more electrical energy during peak hours.
- Penalty must be imposed on the consumers for low power factors.

1.14.2 Factors Affecting the Tariff

The factors for fixing the tariff for electrical energy are:

Nature of Load

The three different types of loads are: domestic, commercial and industrial. The tariff allocated for electrical energy varies for these different types of loads. The industrial load consumes more energy for a longer time when compared to domestic and commercial loads. Hence, the tariff must be decided based on the nature of load and it should not be uniform for all loads.

Maximum Demand

The maximum of all the demands that a particular station supplies in a given period is called as maximum demand. It depends on the maximum installed-capacity of the station and the maximum kWh generated from the station. The tariff allocated for the electrical energy generated from a particular generation station is directly proportional to the maximum demand it supplies. Hence, increase in maximum demand increases the installed capacity of the generating station, which increases the capital investment cost and as a result, the tariff increases.

Load Requirement Time

In general, the time of consumption of electrical energy is classified as peak and off-peak times. The time at which the maximum demand is consumed is called peak time. If the consumer demands the power in peak time, tariff will be higher and if the consumer demands the same power in off-peak time, the tariff will be less.

Load Power Factor

The power factor of the load is inversely proportional to the tariff of the electrical energy consumed. If the power factor is low, additional devices are required to correct the power factor and hence the tariff will be high.

1.14.3 Characteristics of Tariff

The desirable characteristics of a tariff are given as follows:

Proper Return

Proper return from each consumer must be ensured by the tariff i.e., the total cost obtained from the consumers must match the production and supply cost of electrical energy incurred, along with a reasonable profit to the generation station. This will ensure continuous and reliable service from the generation company to the consumers.

Fairness

Fairness of tariff to different consumers depends on: (i) the amount of energy consumed and (ii) the deviation in the load pattern. Tariff should be low for consumers who consume more amount of energy when compared to those consuming small amounts of energy. Similarly, the consumer whose load pattern does not deviate much should be charged low when compared to other consumers.

Simplicity

Tariff calculation of the electrical energy consumed should be easily understandable by any consumer. It helps in maintaining a smooth relation between the consumer and the supplier. Otherwise, it will make the consumers distrust the supply companies.

Reasonable Profit

The profit that the supplier receives through the tariff charged should be reasonable. To maintain a good relation between the supplier and the consumer, the profit should be restricted to 8 per cent per annum.

Attractive

Tariff charged for the electrical energy should attract more number of consumers, encouraging them to utilize the services provided by the company. It can be achieved by fixing the tariff such that the consumer can easily afford it.

1.14.4 Types of Tariff

The different forms by which the tariff for electrical energy consumed is determined are:

- Simple tariff
- Flat-rate tariff
- Block-rate tariff
- Two-part tariff
- Maximum-demand tariff
- Power-factor tariff
- Three-part tariff

They are explained in detail as follows.

Simple Tariff

When a fixed rate is applied to each unit of consumed energy, it is known as simple tariff or uniform tariff. In this type, the rate per unit of energy consumed is constant and it does not depend on the quantity of energy consumed by the consumer. The electrical energy consumed by the consumer is recorded using an energy metre. This is the simplest of all tariffs and is represented graphically as shown in Figure 1.37.

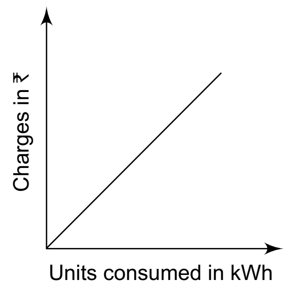


Figure 1.37 Simple Tariff

Advantages

1. Simple method
2. Easy to understand and to apply
3. Consumers pay according to their usage of electrical energy.

Disadvantages

1. There is no discrimination between different types of consumers.
2. Higher cost per unit of energy consumed
3. As it does not provide any incentives, this type of tariff does not encourage the use of more electrical energy.
4. Even though there exists a connection, the supplier cannot charge any amount when the consumer does not consume any energy.

Flat-rate Tariff

In flat-rate tariff, different types of consumers are charged at different fixed costs, per unit of electrical energy consumed. Here, the consumers are grouped into different categories and each category is charged a fixed rate, similar to the simple tariff method. The fixed rate for each category of consumers is decided based on their loads and the power factor of the loads. The flat-rate tariff for different categories is represented in Figure 1.38.

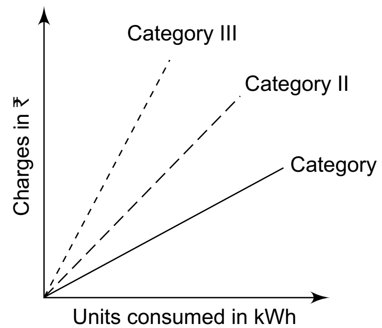


Figure 1.38 Flat-rate Tariff

Advantages

1. It is fair to the consumers.
2. Calculation is simple.

Disadvantages

1. Though the consumers are categorized, incentives are not provided to them.
2. Since separate metres are required for different loads like lighting load, power load and so on, it makes the whole arrangement complex and expensive.
3. Each category of consumer is charged a fixed rate, irrespective of the amount of energy consumed by the consumer.

Block-rate Tariff

In this type of tariff, the energy consumed by the consumers is divided into different blocks and then each block will be charged a particular fixed rate. It is to be noted that if a block of energy is charged at a specified rate, then the succeeding blocks will be charged at a progressively reduced rate i.e., the rate per unit in the first block is the highest and it is progressively reduced for the succeeding blocks of energy. Graphically, it is represented as shown in Figure 1.39.

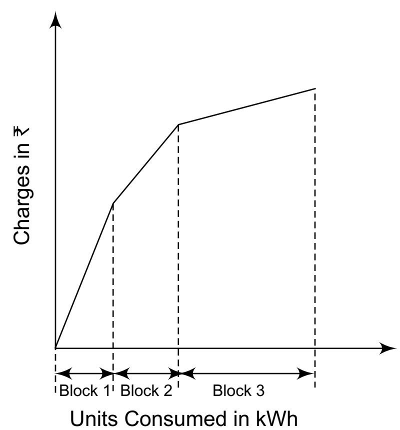


Figure 1.39 Block-rate Tariff

Advantages

1. Requires only one energy-metre.
2. Incentives are provided to the consumers in terms of reduced tariff, which attract the consumers to use more energy.
3. Increased load factor of the system.
4. Reduced cost of electrical energy generation.

Disadvantage

Even though there exists a connection, the supplier cannot charge any amount when the consumer does not consume any energy.

Two-part Tariff

In this type of tariff, the total cost that the consumer is charged by the supplier consists of two components: fixed charges and running charges. It depends on maximum demand and number of units consumed by the consumer respectively. Mathematically, it is expressed as

$$\text{Total cost} = [a \times MD + b \times UC] \tag{1.75}$$

where, MD is the maximum demand of the consumer in kW, UC is the number of units consumed by the consumer in kWh, a is the charge per kW, which when multiplied by MD gives the fixed charge and b is the charge per kWh, which when multiplied by UC gives the total running charges.

The capital cost of investment, taxes and some part of operating cost, which is independent of total energy generated, will be covered by the fixed charges. The running charge covers the operating cost, which varies with respect to the variation in energy generated or supplied.

Advantages

1. Consumers can easily understand this type of tariff.
2. As the fixed charge is independent of the units consumed by the consumer, the supplier will receive the fixed charges, even when the consumer does not consume any energy.

Disadvantages

1. Irrespective of the usage of electrical energy, the consumer has to pay the fixed charges.
2. Error exists in assessing the maximum demand of the consumer.

Maximum-demand Tariff

The only difference between the two-part tariff and maximum-demand tariff is that the maximum demand of a particular consumer is determined by installing a maximum-demand metre in the premises of the consumer. This type of tariff eliminates the error that exists in assessing the maximum demand.

It is not suitable for small residential consumers, as a separate maximum-demand metre is required.

Power-factor Tariff **[AU May/June, 2014]**

In this type of tariff, the power factor of the consumers' load is taken into consideration. The power factor plays an important role in an AC system. High power factor helps in an optimal operation of the system. Low power factor of the system causes more line losses and imbalance to the system and hence, the consumer will have to be penalized. The power-factor tariff is further classified into:

- **kVA maximum-demand tariff:** It is also known as a modified form of two-part tariff, where the fixed charges are calculated based on the maximum demand in kVA instead of kW, since the power factor is inversely proportional to kVA demand. Therefore, a consumer having low power factor has to pay more fixed charges. This type of tariff encourages the consumers to operate their load at an improved power factor.
- **Sliding-scale tariff:** It is also known as average power-factor tariff, where a power factor of 0.8 lagging is considered as the reference, which helps in penalizing the consumer whose power factor is less than the reference. On the other hand, if the power factor of the consumer is greater than the reference power factor, an incentive in the form of a discount will be provided to the consumer.

- **kW and kVAR tariff:** In this type of tariff, both active power (kW) and reactive power (kVAR) consumptions are measured and charged separately. As there is an inverse relation between kVAR and power factor, low power factor consumes more reactive power and as a result, the consumer will be charged heavily.

Three-part Tariff

In this type of tariff, the total cost that the consumer is charged by the supplier is split into three parts: fixed charge, semi-fixed charge and running charge, and is expressed mathematically as

$$\text{Total cost} = [c + a \times MD + b \times UC] \tag{1.76}$$

where, c is the fixed charge for every billing period, which includes the capital investment cost of secondary distribution and labour cost of collecting revenues.

Applications of different types of tariff are listed in Table 1.4.

Table 1.4 *Applications of Different Types of Tariff*

Tariff Type	Application
Simple tariff	Tube wells used for irrigation purposes.
Flat-rate tariff	Domestic consumers
Block-rate tariff	Major residential and small commercial consumers
Two-part tariff	Industrial consumers whose maximum demand is significant.
Maximum demand tariff	Large industrial consumers
Three-part tariff	Big consumers

Example 1.21

The monthly electric consumption of a residential home is 123 units. Block-rate tariff is used to determine the monthly bill, and the rate for different blocks are: (i) INR 2.74 per unit for the first 15 units; (ii) INR 2.70 per unit for the next 25 units; and (iii) INR 2.32 per unit for the remaining units. Also, a constant charge of INR 7 is charged per month. Determine the monthly bill.

Solution

Given,
The monthly electric consumption of a residential home = 123 units
Using block-rate tariff and given data,
The monthly bill = $(15 \times 2.74) + (25 \times 2.70) + (83 \times 2.32)$ = INR 301.16
Considering the constant charge of INR 7, the net monthly bill for the given residential load = 301.16 + 7 = INR 308.16

Example 1.22

An industrial consumer has a single-phase 230 V supply. The monthly energy consumption is 2020 kWh. A maximum demand indicator installed at the consumer premises indicates the total energy consumption per month for maximum demand is 552 kWh and is charged at INR 3.50 per kWh. The remaining units are charged at INR 1.80 per kWh. Determine the monthly bill and the average tariff for the consumer.

Solution

Maximum demand energy consumption = 552 kWh
Charge per kWh of maximum demand = INR 3.50
Total energy consumption of the consumer = 2020 kWh
Therefore, the monthly bill for the consumer = (552 × 3.5) + ((2020 – 552) × 1.8)
= INR 4574.40

Average tariff for the consumer = $\frac{4574.4}{2020}$ = INR 2.2645 per kWh

Example 1.23

The total energy consumption of a consumer per year is 5,25,600 kWh. The maximum demand of the consumer is 80 kW. Determine the yearly monthly bill if the tariff used is INR 6,000 + INR 700 per kW of maximum demand per year + INR 1.80 per kWh.

Solution

The total yearly bill for the consumer = 6,000 + (700 × 80) + (1.8 × 5,25,600)
= INR 10,08,080

Example 1.24

An electrical supply company offers two tariffs: (i) (30 + (0.03 per kWh)) and (ii) 0.06 per kWh for the first 400 units and 0.05 per kWh for the remaining units. If the total bill obtained using the two tariffs is same, determine the energy consumption per month for the consumer.

Solution

Let x be the total energy in kWh consumed by the consumer per month.
Then, the monthly bill using first tariff = $30 + (0.03 \times x)$ and
the monthly bill using second tariff = $(0.06 \times 400) + (0.05 \times (x - 400))$
Since the monthly bill obtained using two tariffs are equal, we get

$$30 + (0.03 \times x) = (0.06 \times 400) + (0.05 \times (x - 400))$$

Solving the above equation, we get $x = 1300$
Hence, the total monthly energy consumption of the consumer is 1300 kWh.

1.15 POWER FACTOR **[AU Nov/Dec, 2012]**

The ratio of the active power (P), measured in kW, and the apparent power (S), measured in kVA, is defined as the power factor. The real power assumed in an AC circuit is the active power and the power produced in the AC circuit when the voltage and current are not in phase is the apparent power. Also, the power factor is used to represent the fraction of total energy that is used for doing useful work and the fraction of energy is stored in the form of magnetic energy in the inductor and capacitor of the circuit, and its value lies between 0 and 1. The most economical value of power factor lies between 0.9 and 0.95. The power triangle of an AC circuit with ϕ as the phase-angle difference between voltage and current is shown in Figure 1.40

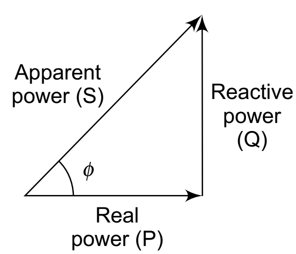


Figure 1.40 Power Triangle

The power factor of a circuit can be lagging, leading or unity. If the voltage in the circuit leads the current flowing in the circuit, then the power factor is said to be leading. If the current flowing in the circuit leads the voltage in the circuit, then the power factor is said to be lagging. If the current and voltage of the circuit are in phase, then the power factor is unity. The pictorial representation of the lagging, leading and unity power factors, based on the phase angle between the voltage and current, are shown in Figures 1.41. (a) to (c) respectively. The power factor does not depend on the magnitude of the voltage or current.

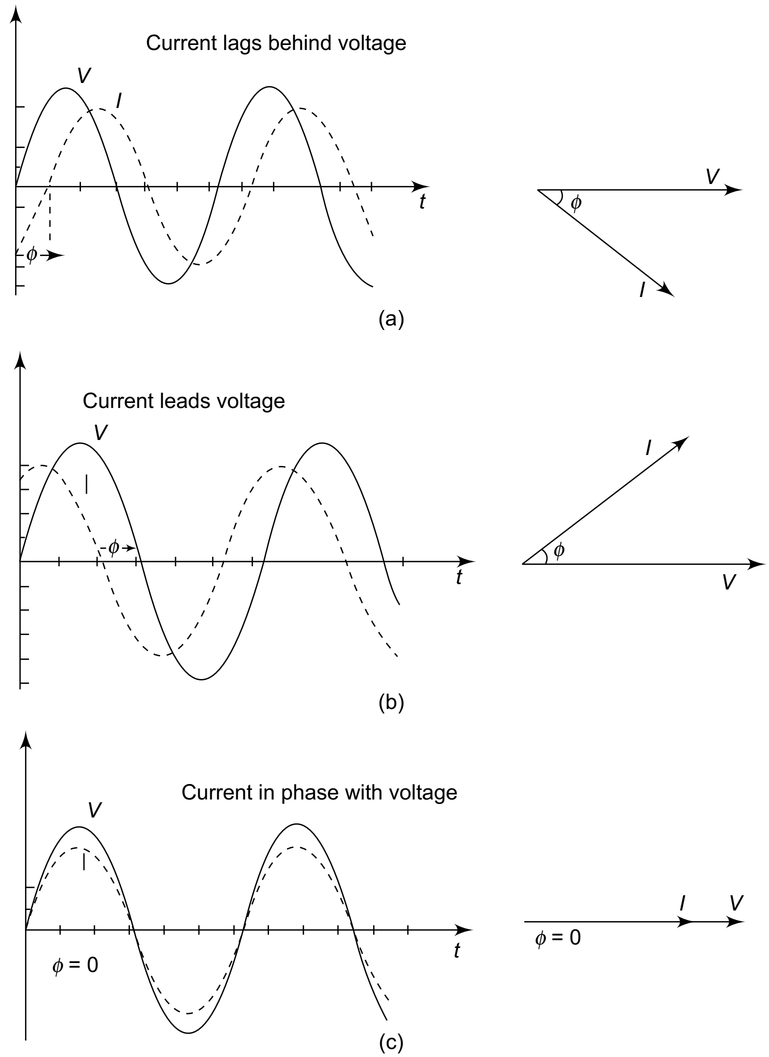


Figure 1.41 Pictorial Representation of Lagging, Leading and Unity Power Factor

In general, for an AC circuit, the power factor is given by the cosine of the angle between the voltage and current i.e., $\cos\phi$. Therefore, using the power triangle, we get

$$\cos\phi = \frac{P}{S} = \frac{P}{VI}$$

The current in the AC circuit is given by

$$I = \frac{P}{V\cos\phi}$$

Therefore, the current flowing through an AC circuit is inversely proportional to the power factor. Hence, the current consumed by the load is more at a low power factor than it does at a high power factor. Therefore, it becomes necessary to maintain the power factor as high as possible. If the power factor of the system becomes low, it has to be compensated using any device.

1.15.1 Causes of Low Power Factor

The different causes of low power factor are given below:

1. Single-phase and three-phase induction motors, which are normally used in AC circuits, where the current lags the voltage by 90°, as it is purely inductive in nature.
2. Other inductive equipment, which cause low power factors are: transformers, generators, arc lamps, electric furnaces and so on.
3. Load variation in the power system i.e., if the power system is loaded lightly, the power factor becomes low, as the current drawn by the equipment increases due to increase in voltage.
4. Existence of harmonic current reduces the power factor.
5. Imbalance in the power system due to improper wiring or electrical accident.

1.15.2 Disadvantages of Low Power Factor

The disadvantages of having low power factor are:

1. Current drawn by the circuit will be high.
2. Copper loss in the equipment will be high and hence, the efficiency of the equipment decreases.
3. Equipment gets overheated due to the copper loss, which in turn increases the stress on the insulation of the cable and makes it weak.
4. As the current drawn by the circuit increases, the size of the conductor has to be increased to carry such current, which in turn increases the cost.
5. Increases the kVA rating of the machine, which increases the size of the equipment, thereby increasing the cost.
6. Voltage drop in the equipment will be increased, thereby affecting the voltage regulation of the equipment.
7. Leads to decrease in the active power, which results in uneconomic operation of the equipment.

1.16 Power Factor Improvement or Correction [AU Nov/Dec, 2013]

If the power factor of the system is low, improvement or correction of power factor is necessary. Adjusting the power factor of the system closer to unity using some equipment is known as power-factor improvement or correction. Power-factor improvement or correction reduces the apparent power consumed by the load and hence, the current drawn by the load will decrease. In other words, power-factor correction or improvement is the injection of reactive power into the circuit to neutralize the effect of lagging current. Static capacitors or synchronous condensers or phase advancers are used to improve or correct the power factor of the system.

1.16.1 Using Static Capacitors

Static capacitors, whose ratings vary from 15 kVAR to 10000 kVAR, are used as devices to improve or correct power factor of the system. In a three-phase system, the capacitors arranged in star or delta connection are used to improve the power factor, as shown in Figure 1.42.

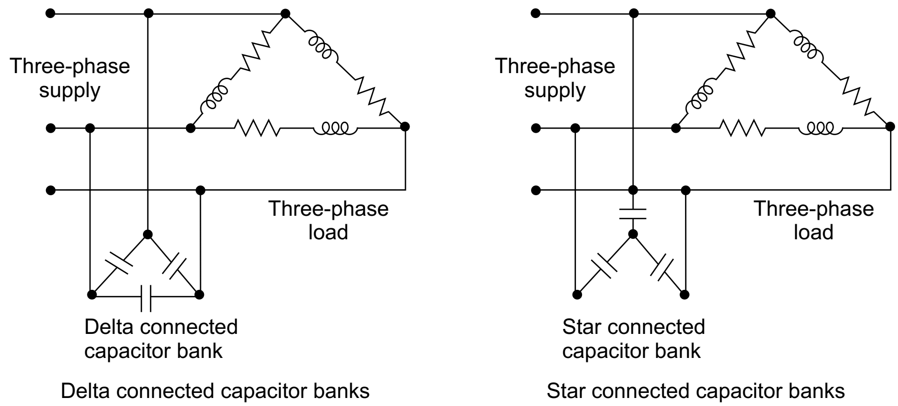


Figure 1.42 Static Capacitor in Delta and Star Connections

Advantages of Static Capacitors

1. Simplest method of power factor improvement or correction.
2. Maintenance required in the system is less.
3. Easier installation and less weight.

Disadvantages of Static Capacitors

1. Life span of the capacitor is short i.e., 8–10 years.
2. Gets easily damaged due to over voltages.
3. Once the capacitor is damaged, it is difficult to replace.
4. Production of switching surges and harmonics happen due to constant switching.

Applications of Static Capacitors

Capacitors in the range of a few hundred kVAR are used in industrial distribution circuits; capacitors of 500–3000 kVAR rating are used in small distribution substations and those with larger ratings are used in large substations.

1.16.2 Using Synchronous Condensers

The working of a synchronous motor can be varied, based on the excitation given to its windings. The excitation to the windings is classified as over-excitation, under-excitation and normal-excitation. A synchronous motor in over-excitation and no load condition draws leading current and starts to act like a capacitor and is known as a synchronous condenser. Varying the field excitation can control the amount of kVAR supplied to the load using synchronous condensers.

Advantages of Synchronous Condensers

1. Lifespan of the equipment is longer i.e., almost 25 years.
2. Power factor control is flexible and reliable.
3. Harmonics have no effect on synchronous condensers, as there is no switching mechanism.

Disadvantages of Synchronous Condensers

1. Losses in the synchronous condensers are high.
2. They are expensive and have a high maintenance cost.
3. The noise generated by them pollutes the environment.
4. Auxiliary device is required, as a synchronous condenser is not self-starting.
5. Use of synchronous condenser is uneconomical for equipment below 500 kVA.

Applications of Synchronous Condensers

Synchronous condensers are used in large factories, industries and large substations to improve the power factor and voltage regulation.

1.16.3 Comparison Between Static Capacitors and Synchronous Condensers

The comparison between static capacitors and synchronous condensers is listed in Table 1.5.

Table 1.5 *Synchronous Condensers vs. Static Capacitors*

Synchronous Condensers	Static Capacitors
Better control of power factor is possible.	Better control of power factor is not possible.
Overloading for a small time-interval is possible.	Overloading is not possible.
Greater kVAR is supplied when there is a voltage drop.	Less kVAR is supplied when there is a voltage drop.
Stability of the system is improved due to existence of inertia.	Stability of the system remains unchanged.
Power loss in the system is high.	Power loss in the system is low.
Installation can be at only one point.	Can be installed at any point in the system.
Rating cannot be changed.	Rating can be changed.
Difficulties exist in the installation.	Installation is easier.
Less failure rate of the system.	High failure rate of the system.
Suitable and economical, where requirement of kVAR is more than 10,000.	Suitable and economical, where requirement of kVAR is small.

1.16.4 Using Phase Advancers

In an induction motor, the power factor is low, as the stator winding draws lagging current from the supply. Phase advancer, which can be used only in induction motors to improve the power factor, is a simple AC exciter. It is mounted on the shaft of the induction motor and is connected to the rotor circuit. Phase advancer supplies exciting ampere-turns to the rotor circuit of induction motor at slip frequency, which improves the power factor of the induction motor.

Two-Mark Questions and Answers

1. **What is the phase sequence of a three-phase system?** **[AU April/May, 2013]**

   The order or the sequence in which the electrical quantity in the three-phase system reaches its respective maximum value is known as phase sequence. If the phase sequence of a particular system is RYB, then it indicates that R phase reaches the maximum value of electrical quantity at first, followed by Y phase and B phase.

2. **Write the relation between the line and phase value of voltage and current in a balanced delta-connected system.** **[AU April/May, 2014]**

   The relation between the line and phase currents is $|\bar{I}_L| = \sqrt{3}\,|\bar{I}_{\text{ph}}|$

   The relation between the line and phase voltages is $\bar{V}_{\text{ph}} = \bar{V}_L$

3. **When the load is balanced, what is the amount of current in the neutral wire for a three-phase four-wire system?** **[AU Nov/Dec, 2014]**

   The current flowing through the neutral point is given by $\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B$

4. **Write the expression for the three-phase total power.** **[AU Nov/Dec, 2014]**

   The total average power consumed by the load in the balanced system is $W = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi$.

5. **Write the equation for the phasor difference between the potentials of the delta-connected network.** **[AU Nov/Dec, 2014]**

   In a delta-connected network, the magnitudes of both the phase and line voltages are same. If line voltage $\bar{V}_{RY}$ is taken as the reference, we have other line voltages as

   $$\bar{V}_{YB} = |\bar{V}_{YB}|\angle -120^\circ \text{ and } \bar{V}_{BR} = |\bar{V}_{BR}|\angle -240^\circ$$

   The phase voltages are given by:

   $$\bar{V}_{RN} = |\bar{V}_{RN}|\angle 0^\circ, \bar{V}_{YN} = |\bar{V}_{YN}|\angle -120^\circ \text{ and } \bar{V}_{BN} = |\bar{V}_{BN}|\angle -240^\circ$$

6. **What are the advantages of a three-phase system?**

   [AU April/May, 2015; Nov/Dec, 2015; April/May 2016]

   Refer Section 1.2.1 for the list of advantages of a three-phase system.

7. **Define line voltage and line current.** **[AU April/May, 2015]**

   Line voltage or line-to-line voltage is the voltage, which is measured between any two lines in a three-phase power system. They are denoted by $\bar{V}_{RY}, \bar{V}_{YB}$ and $\bar{V}_{BR}$ and are given by $\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$, $\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$ and $\bar{V}_{BR} = \bar{V}_B - \bar{V}_R$ respectively.

   Line currents are the currents flowing through a particular line and is represented by $\bar{I}_R, \bar{I}_Y$ and $\bar{I}_B$.

8. **When is a three-phase supply system called a balanced supply system?** **[AU Nov/Dec, 2015]**

   A three-phase system is said to be a balanced source, if the phase voltage of each phase has the same magnitude and frequency, and the phase difference between the lines is 120°.

9. Give a delta circuit having resistors and write the required expressions to transform the circuit to a star circuit. **[AU Nov/Dec, 2012]**

If the resistance in a delta connection are R_{RY}, R_{YB} and R_{BR}, then the corresponding resistance in a star connection are:

$$R_R = \frac{R_{RY} R_{BR}}{R_{RY} + R_{YB} + R_{BR}}$$

$$R_Y = \frac{R_{YB} R_{RY}}{R_{RY} + R_{YB} + R_{BR}}$$

$$R_B = \frac{R_{YB} R_{BR}}{R_{RY} + R_{YB} + R_{BR}}$$

10. A three-phase 400 V is given to balanced star-connected load of impedance 8 + *j*6 Ω. Calculate line current. **[AU April/May, 2016]**

Given, $V_L = 400$ V, $Z_{\text{ph}} = 8 + j6\ \Omega$

$$V_{\text{ph}} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 254.03 \text{ V}$$

Therefore, $\bar{I}_L = \dfrac{\bar{V}_{\text{ph}}}{Z_{\text{ph}}} = \dfrac{254.03}{8 + j6} = \dfrac{254.03}{10\ \angle 36.86} = 25.40\ \angle -36.86^{\circ}$

11. What are the requirements of protection? **[AU April/May, 2010]**

Refer to Section 1.10: Introduction to power system protection

12. What is the importance of arc resistance? On which factor does it depend? **[AU April/May, 2010]**

In arcing period, the arc resistance plays a vital role and is inversely proportional to the magnitude of current flowing through the contacts. The factors that affect the arc resistance are:

- ***Degree of ionization:*** An inverse relation exists between the arc resistance and the number of ionized particles in the medium.
- ***Length of the arc:*** When the length of the arc or distance between the contacts increases, the arc resistance increases.
- ***Cross-section of the arc:*** An inverse relation exists between the arc cross-section and the arc resistance.

13. Distinguish between recovery voltage and re-striking voltage. **[AU Nov/Dec, 2011]**

The recovery voltage is the normal frequency RMS voltage, which appears across the contacts of a circuit breaker after the final arc extinction and it is approximately equal to the system voltage, whereas the re-striking voltage is the transient voltage, which is obtained across the contacts when the current becomes zero during the arcing period.

14. What are the basic requirements of a circuit breaker? **[AU Nov/Dec, 2011]**

A switching device, which can be used to make or open a circuit either manually or automatically or by using remote control under different conditions like normal and under fault conditions, is known as a circuit breaker. Special attention must be given while designing a circuit breaker, as the modern power systems deal with very high currents, to safely interrupt the arc produced during its operation.

15. Write the effects of arc resistance. **[AU Nov/Dec, 2012]**

It is very important to extinguish the arc as early as possible before it causes serious damage to the system. As the arc developed forms a conductive path for electricity, the fault current flowing through the circuit breaker will not be interrupted till the arc is extinguished. The most important designing criteria of a circuit breaker is to provide proper technique in quenching the arc for a quick and safe fault current interruption.

16. What is meant by relay operating time? **[AU Nov/Dec, 2012]**

The relay operation time is the time taken between the instant at which the actuating signal is energized and the instant at which the relay contacts are closed.

17. List out the various methods of arc interruption. **[AU Nov/Dec, 2012]**

The two different methods by which the arc can be extinguished are:

- High-resistance method
- Low-resistance method or zero-current method

18. How do you classify the circuit breaker? **[AU Nov/Dec, 2012]**

The circuit breaker is classified as:

- Medium, in which the circuit breaker operates
- Actuating signal, in which it works
- Construction type
- Voltage level

19. Write the operational difference between a fuse and a circuit breaker. **[AU Nov/Dec, 2012]**

Refer to Section 1.13.5 for the operational difference between a fuse and a circuit breaker.

20. What are the functions of protective relays? **[AU April/May, 2015]**

Refer to Section 1.14.

21. Give the two methods of arc interruptions. **[AU April/May 2015]**

The two different methods by which the arc can be extinguished are:

- High-resistance method and
- Low-resistance method or zero-current method

22. Differentiate between a fuse and a protective relay. **[(AU April/May, 2010])**

Refer to Sections 1.13 and 1.14.

23. Define the term "maximum demand". **[AU Nov/Dec, 2012]**

The maximum of all the demands that a particular station supplies in a given period is called as the maximum demand. It depends on the maximum installed capacity of the station and the maximum kWh of energy generated by the station.

24. Write the effect of power factor in energy consumption billing. **[AU April/May, 2014]**

The power factor of the load is inversely proportional to the tariff of the electrical energy consumed. If the power factor is low, additional devices are required to correct the power factor and hence the tariff will be high.

Review Questions

1. Prove that for balanced supply and unbalanced load of a three-phase system, two wattmeters are sufficient to measure power.
2. Derive the formula for total power consumed in an unbalanced star-connected load.
3. Explain a method to determine power factor in a three-phase system.
4. What are the advantages of three-phase systems?
5. Prove that the total instantaneous power in a balanced three-phase system is constant and is equal to the average power whether the load is star or delta-connected.
6. Discuss in detail, the three-phase three-wire circuits with star-connected balanced loads.
7. Explain in detail, the phasor diagram of the voltages and currents of three-phase unbalanced circuits.
8. When is a three-phase supply system called a balanced supply system?
9. Show that three-phase power can be measured by two wattmetres. Draw the phasor diagrams. Derive an expression for power factor in terms of wattmetre readings.
10. With a neat circuit diagram and a phasor diagram, explain the three-phase power measured by two-wattmetre method and also derive the expression for power factor.
11. What is tariff? What are its objectives?
12. Name the different types of tariffs.
13. What is two-part tariff?
14. What are the causes of low-power factor?
15. What are the effects of low-power factor?
16. What are the advantages of power factor improvement?
17. Compare shunt capacitors and synchronous condensers.
18. Discuss the reasons for the following statements:
    (a) Industrial consumers usually install capacitors for power factor correction at their premises.
    (b) Two-part tariff is used for industrial consumers and not for residential consumers.
    (c) Residential consumers are generally charged on block metre rate.
    (d) Flat demand rate is very rarely used.
    (e) It is generally not economical to improve the power factor of an installation to unity.
    (f) Installation of capacitors improves voltage regulation.
    (g) Low-power factor means higher energy losses.
19. Discuss the different types of tariffs used for charging the consumers of electric energy.
20. What objectives should a utility keep in mind while deciding the tariffs for consumers?
21. What are the special features of two-part tariff? For which category of consumers is it used?
22. How do demand factor, load factor and diversity factor, in a power system, affect the fix action of tariffs?
23. Discuss the features that distinguish an electric utility from other forms of business.
24. What are the various costs that have to be taken into account for fixing tariff?
25. Discuss the importance of encouraging customers to use electricity during off-peak hours?
26. Give a simple example to illustrate why a part of electric supply cost must be based on maximum demand.
27. Why is it not economical for customers to raise power factor to unity? How can the most economical power factor be determined for a consumer installation?
28. Compare the advantages and disadvantages of using a synchronous condenser and a capacitor for power-factor improvement.

CHAPTER 2

Transformers

2.1 Introduction

The AC system is generally used instead of a DC system for the generation, transmission and distribution of electrical power because the AC voltage can be increased or decreased according to the requirement. Single-phase transformer is one such device used in power systems. A transformer is a static device used for coupling two or more electric circuits. It works on the principle of mutual induction and transfers the electric energy from one circuit to another when there is no electrical connection between the two circuits. When compared to the input voltage, the output voltage of transformer can be increased or decreased with a proportional decrease or increase in the current ratings. The size of the single-phase transformer can vary from very small to large size. But irrespective of its shape or size, a transformer is used only to transfer electric energy from one voltage level to another with same frequency. The analysis of a single-phase transformer, including its working principle and different tests are explained in this chapter. Also, the construction and operation of a three-phase transformer and an auto-transformer are discussed.

2.2 Single-Phase Transformer

A transformer is a stationary apparatus by which electric power in one circuit is transformed to another circuit with the same frequency. It transforms electrical power without any direct electrical connection between input and output, but with the help of mutual induction between the two windings. The working principle and construction of single-phase transformer is explained below.

2.2.1 Working Principle **[AU April/May, 2014]**

The working principle of a two-winding transformer can be understood with the help of the diagram shown in Figure 2.1. Based on the principle of Faraday's law of electromagnetic induction and mutual induction, the transformer transfers electric power from one circuit to another. Faraday's law of electromagnetic induction states that whenever a current-carrying conductor cuts the magnetic flux, an emf gets induced in the conductor. When another coil is brought near to the former coil, another emf gets induced in the latter

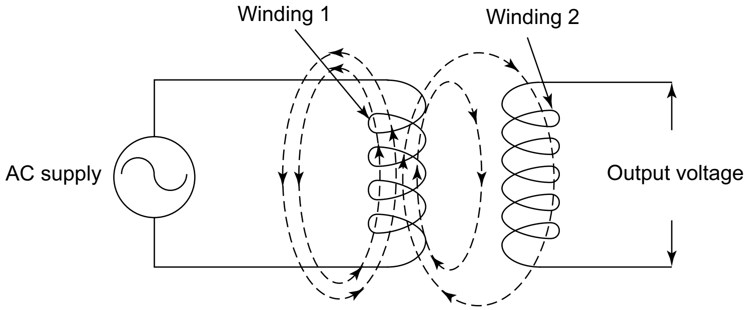


Figure 2.1 Two-winding Transformer

coil due to mutual induction. This induced emf in the stationary conductor is called statically induced emf. Therefore, when the latter coil forms a closed loop, the induced emf produces a current that flows through the loop and load.

2.2.2 Construction

The essential components of the transformer are:

1. Magnetic core
2. Two windings, namely primary and secondary windings
3. A time varying magnetic flux

The constructional diagram of a single-phase transformer is shown in Figure 2.2.

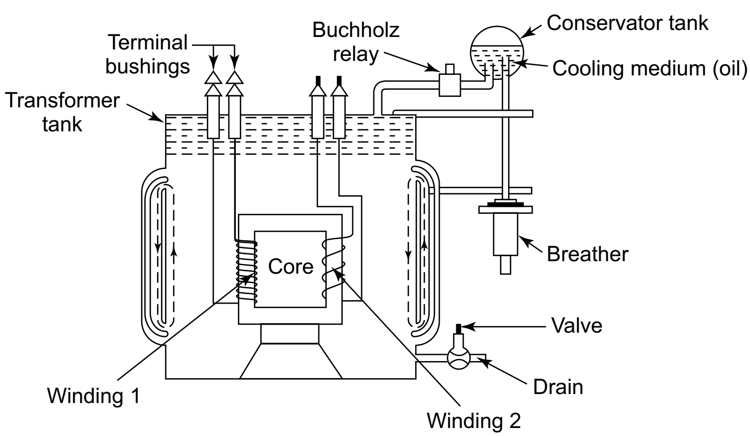


Figure 2.2 Constructional Diagram of a Single-phase Transformer

The different components of a single-phase transformer are:

1. Core
2. Limb
3. Yoke

4. Windings
5. Conservator
6. Cooling medium
7. Breather
8. Explosion vent
9. Buchholz relay

These components can be grouped as magnetic circuit, electrical circuit, dielectric circuit, tanks and accessories.

Magnetic Circuit

The core, yoke and limb of the transformer form the magnetic circuit. The core of the transformer is made of high-grade silicon steel or sheet steel lamination, which has low hysteresis loss and provides a continuous magnetic path, when it is assembled. The vertical position of the core on which the coil is wound is called limb, while the horizontal position of the core is known as yoke. The main functions of the magnetic circuit are:

1. Provides low reluctance path for carrying the flux.
2. Carries the windings required for electric power transfer.

Electrical Circuit

In the transformer, there are two windings, namely primary and secondary windings that form the electrical circuit. The winding connected to the AC source is called the *primary winding,* normally indicated using '1'. Similarly, the winding connected to the load is called the *secondary winding,* normally indicated using '2'. These windings are made of copper and its cross-section can be either rectangular or circular, depending on the voltage level. The rectangular cross-section is used for both low and high-voltage windings in large transformers, whereas, the circular cross-section is used for high-voltage windings in small transformers.

According to the core construction and the manner in which these windings are wound, transformers are classified as core-type and shell-type transformers. In the core-type transformer, the windings surround a considerable part of the core, whereas in the shell-type transformer, the core surrounds a considerable portion of the windings.

Core-type Transformer **[AU April/May, 2015]**

In core-type transformer, rectangular frame laminations are formed to build the core of the transformer. The laminations are pressed or punched out from larger steel sheets and arranged into thin steel strips to assemble the letters "E", "I", "L" and "U" as shown Figure 2.3.

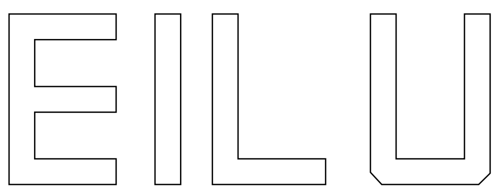


Figure 2.3 Thin Steel-strip Laminations

These laminations, when joined together, form the required core-type. The core-type transformer can be obtained using the laminations "I", "L" and "U" as "L-L" laminations or "U-I" laminations shown in Figure 2.4(a). But "L-L" laminations are mostly preferred when compared to "U-I" laminations. High reluctance at the joints is avoided by pressing against each other. The continuous joint is eliminated using the alternate layers. Interleaving the windings of transformer, as shown in Figure 2.4(b), reduces leakage fluxes. For simplicity, the primary and secondary windings are located on the separate limbs of the core.

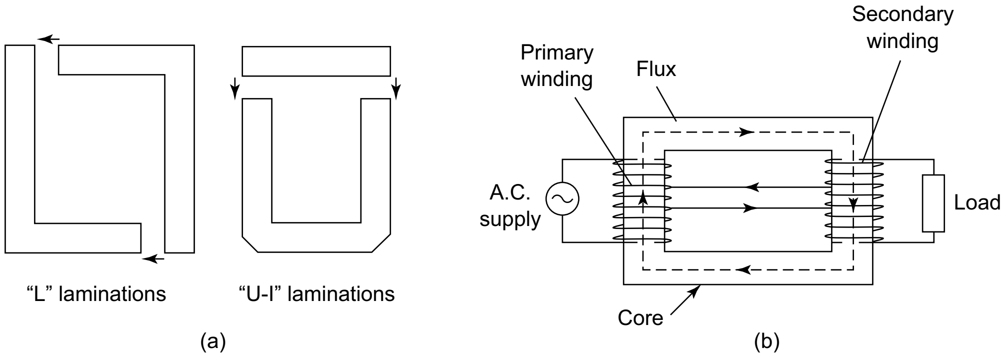


Figure 2.4 Core-type Transformer (a) Laminations Used (b) With Windings

Shell-type Transformer

The shell-type transformer can be obtained using the laminations "E", and "I" as "E-I" laminations or "E-E" laminations shown in Figure 2.5(a). But "E-I" laminations are mostly preferred when compared to "E-E" laminations. The techniques used in core-type transformer are also used in shell-type transformer to reduce the leakage fluxes and continuous joints. This type of transformer has three limbs or legs in which the width of the central limb is twice the width of the outer limbs. The centre limb carries the whole flux generated due to input voltage and the side limb carries half of the flux. The two windings of the transformer are wound on the centre limb are shown in Figure 2.5(b).

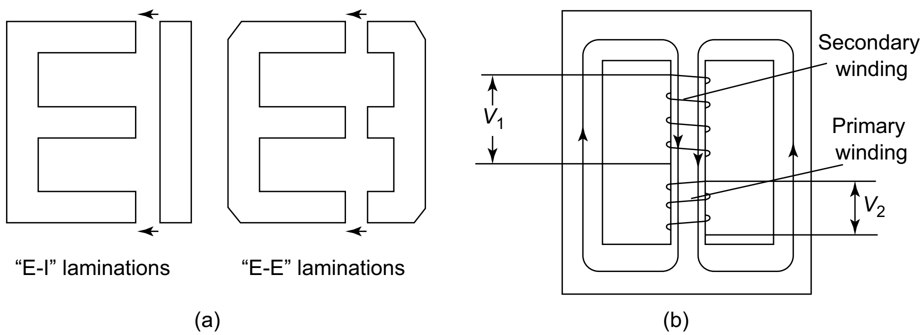


Figure 2.5 Shell-type Transformer (a) Laminations Used (b) With Windings

The comparison between core and shell-type transformers is given in Table 2.1.

Table 2.1 *Core-type vs. Shell-type Transformer* **[AU April/May, 2015]**

Core-type Transformer	Shell-type Transformer
The windings surround the core and are placed on the side limbs.	The core surrounds a considerable portion of the windings and is placed on the central limb
Lamination is cut in the form of L, I or U strips.	Lamination is cut in the form of long strips that are arranged in the shape of E and I.
Requires more copper material	Requires less copper material

(*Contd.*)

Two limbs exist	Three limbs exist
Requires more insulation	Requires less insulation
Equal distribution of flux on the limbs	Unequal distribution of flux in the limbs
Two magnetic circuits are formed by the windings.	One magnetic circuit is formed by the windings.
More loss exists in terms of leakage flux	Less loss exists in the system
Easy to maintain	Difficult to maintain
High output can be achieved	Less output can be achieved
Natural cooling is possible	Cooling mechanism exists

Dielectric Circuit

In a transformer, to insulate the conducting parts from each other, insulations are used. These insulations comprise of the dielectric circuit and are used in various places to reduce eddy current losses. A light coating of varnish or any oxide is used to insulate the lamination whose thickness varies from 0.35 mm to 0.5 mm for a normal AC operation.

Tanks and Accessories

The essential protective devices attached to the transformer that increase the life span of a transformer are as follows:

Conservator

A cylindrical tank that is placed at the top or roof of the transformer main tank is called a conservator. It is provided with a large cover for proper maintenance and cleaning of the transformer. In addition to acting as the transformer-cooling medium, it acts like a reservoir. An adequate space is provided in the conservator since the volume of the cooling medium might increase due to rise in transformer temperature, when it is fully loaded.

Cooling Medium

When the transformer is loaded, some losses occur within. These losses appear in the form of heat, which increases the transformer temperature. Hence, a proper provision should be made in the transformer to dissipate this heat and to maintain the transformer temperature within its limits. Therefore, a cooling medium in the form of air or oil is required to remove the heat generated during loading.

Breather

The heart of the transformer is the breather, which is similar to the human heart. The breather transports fresh air in and out of the transformer. This component is required to maintain the cooling-medium level in the conservator. In addition, the breather is provided with silica gel to eliminate moisture content in the cooling medium and to maintain the quality of cooling medium.

Explosion Vent

A thin aluminium pipe that is placed at the ends of the transformer to prevent it from damage is called an explosion vent. It helps in maintaining the pressure inside the transformer, which drastically increases when there is an increase in temperature of the transformer.

Buchholz Relay

A gas-actuated relay placed in the large-size transformer to protect it from internal fault is called Buchholz relay. It is used in the transformer with a rating greater than 500 kVA. Its working principle is that, when an internal fault takes place, evaporation of oil in the form of gas occurs due to increase in temperature. The evaporated gas activates the Buchholz relay and alarms the personnel, which helps in disconnecting the transformer from the supply.

2.2.3 Working of a Transformer

The schematic diagram to explain the working of a transformer is shown in Figure 2.6. It consists of two inductive coils, which are electrically separated but are wound on a high-permeability, laminated-steel magnetic core to maximise the coupling. These coils have high mutual inductance and are used to transfer electric energy from one voltage level to another. One of the two coils connected to a source of alternating voltage, V_1, with frequency, f, is called primary winding. The second coil connected to a load is called the secondary winding. The load draws out the electrical energy transformed to this winding. The primary winding has N_1 turns while the secondary winding has N_2 turns. When an alternating voltage excites the primary winding, it circulates an alternating current. This current produces an alternating flux, ϕ, which completes its path through the common magnetic core. Thus, an alternating flux links with the secondary winding and a mutually induced emf develops in the secondary winding. If the load is connected to the secondary winding, this emf drives a current through it.

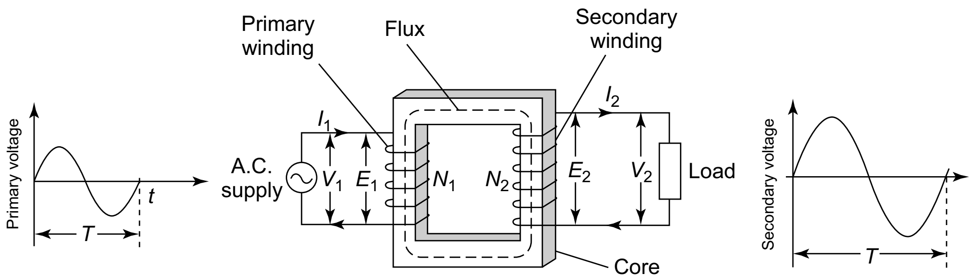


Figure 2.6 Working of a Transformer

Here, the rms values of the induced voltages in the primary and the secondary windings are E_1 and E_2 volts respectively. These voltages will have sinusoidal waveform with same frequency as that of the applied voltage V_1. The currents which flow in the closed primary and the secondary circuits are I_1 and I_2 respectively. Hence, the electrical energy is transferred from the primary circuit to the secondary circuit without changing the frequency of the input voltage.

Turns ratio: It is the ratio of the number of turns in primary to the number of turns in secondary. It is given by

$$\text{Turns ratio } = \frac{N_1}{N_2}$$

If $N_2 > N_1$, the transformer is called a step-up transformer and if $N_2 < N_1$, the transformer is called a step-down transformer.

2.3 EMF Equation of the Transformer

[AU April/May, 2016]

Consider that N_1 and N_2 are the number of turns in the primary and secondary windings respectively, ϕ is the alternating magnetic flux generated in the primary winding with its maximum value as ϕ_m in Weber and f is the frequency of the supply voltage in Hertz. When a supply voltage V_1, with frequency f, is applied to the primary winding, an alternating magnetic flux ϕ with same frequency is generated, as shown in Figure 2.7.

Using Faraday's law, the average emf induced per turn in the primary winding, e_1, is proportional to the average rate of change of flux.

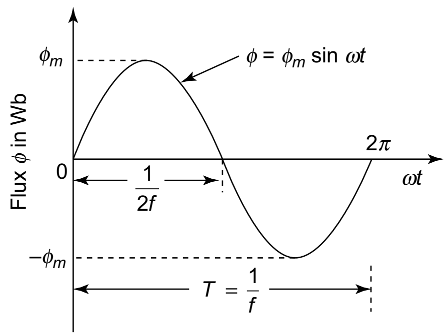


Figure 2.7 Alternating Flux Generated at the Primary Winding

i.e.,
$$e_1 = \frac{d\phi}{dt} = \frac{\phi_m - 0}{\frac{1}{4f} - 0} = 4f\phi_m \text{ Wb/sec}$$

Here, the rate of change of flux per turn is the induced emf in volts. Therefore, the average emf/turn = $4f\phi_m$ V.

If the flux ϕ variation is sinusoidal, then rms value of the induced emf is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{\text{rms value}}{\text{Average value}}$$

For a sinusoidal waveform, the form factor is 1.11.
Therefore, the rms value of the induced emf per turn, in the primary winding is

$$e_{1r} = 1.11 \times 4f\phi_m = 4.44f\phi_m \text{ V}$$

Hence, for N_1, the rms value of the induced emf in the primary winding is given by

$$E_1 = 4.44f\phi_m N_1 \tag{2.1}$$

Similarly, the rms value of the induced emf in the secondary winding is given by

$$E_2 = 4.44f\phi_m N_2 \tag{2.2}$$

In an ideal transformer on no load, the relation between the input and output voltages and the induced emfs in the windings are given by:

$$V_1 = E_1 \text{ and } V_2 = E_2$$

where, V_2 is the terminal voltage.

Using the above equations, we get the transformation ratio as

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

Therefore, the transformation ratio (K) is defined as the ratio of the secondary induced voltage (E_2) to the primary induced voltage (E_1).

Also, the input and output powers of the transformer in lossless condition is given by V_1I_1 and V_2I_2 respectively. Assuming both the powers are equal, we get

$$V_1I_1 = V_2I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Here, the currents are in the inverse ratio of the (voltage) transformation ratio.

2.4 Reasons for not Using DC Supply as a Source **[AU Nov/Dec, 2013]**

The following are the reasons for not connecting a transformer to a DC source or supply:

1. A constant flux is produced in the primary winding when a DC source is applied to the primary winding of the transformer. Due to this constant flux, no emf is induced in the secondary winding. Hence, no current will be delivered to the load connected to the secondary winding.
2. When AC source is used, due to the alternating flux produced in the primary winding, a self-induced emf is generated in it, which opposes the applied voltage and limits the primary current. But, when a DC source is used, there will be no induced emf and hence heavy current flows in the primary winding, which results in the failure of the transformer.

2.5 Types of Transformers

1. **Based on transformation ratio, K, or number of turns in the windings**
   - ***Step-up transformer:*** It transfers a high current, low AC voltage into a low current, high AC voltage. Here, $N_2 > N_1$ and $V_2 > V_1$. Therefore, the transformation ratio is greater than 1, i.e., $K > 1$.
   - ***Step-down transformer:*** It is the opposite of a step-up transformer i.e., it transfers a low current, high AC voltage into a high current, low AC voltage. Here, $N_2 < N_1$ and $V_2 < V_1$. Therefore, the transformation ratio is less than 1, i.e., $K < 1$.
2. **Based on the service it provides**
   - ***Power transformer*:** The power transformer, whose voltage ratings are 400 kV, 200 kV, 100 kV, 66 kV and 33 kV, with a rating greater than 200 MVA, is used in transmission networks for transmitting higher voltages. These large size transformers with 100% efficiency are placed nearer to generating and transmission stations.
   - ***Distribution transformer:*** The distribution transformer, whose voltage ratings are 11 kV, 6.6 kV, 3.3 kV, 400 V and 230 V, with a rating less than 200 MVA, is used in distribution networks for distributing voltages to the consumers. This distribution transformer can be further classified as pole mount, pad mount or underground transformer, liquid immersed or dry-type transformer and single or three-phase supply, based on the mounting location, the insulation type and the nature of supply respectively.
   - ***Instrument transformer:***
     - Current transformer
     - Potential transformer
     - Auto-transformer

3. **On the basis of the supply**
   - Single-phase transformer
   - Three-phase transformer
4. **On the basis of cooling medium**
   - Air-cooled transformer
   - Oil-cooled transformer

2.6 Ideal Transformer **[AU April/May, 2016]**

In an ideal transformer, there is no dissipation loss, as the primary and secondary windings have zero resistance, and the core has infinite permeability. The coils should be tightly coupled so that there is no leakage flux and the coefficient of coupling should be equal to one i.e., $k = 1$. As compared to the load connected, the inductive reactances of primary and secondary windings should be extremely large. Self-inductance of primary or secondary winding is proportional to the square of the number of turns of the coil.

An ideal transformer has the following properties:

- ***Absence of winding resistance:*** The primary and secondary windings are purely inductive, without any resistance, and are wound on the core in the ideal transformer. Since it has no winding resistance, the copper loss in an ideal transformer is zero.
- ***Very high or infinite permeability of the core:*** The permeability of the core material used in an ideal transformer is very high or infinite i.e., a very less magnetising current is required to magnetise the core and produce the required flux since an inverse relation exists between the permeability of the core and the magnetising current required to produce the flux.
- ***No leakage flux:*** In an ideal transformer, 100% of the total flux produced in the primary winding links the secondary winding. Hence, there is no leakage flux in such an ideal transformer.
- ***100% efficiency:*** In an ideal transformer, as there is no loss, the output power developed is exactly equal to the input power supplied to the transformer. Therefore, the ideal transformer is free from losses and has 100% efficiency.

2.7 Accounting for Finite Permeability and Core Loss

2.7.1 Ideal Transformer on No load **[AU Nov/Dec, 2015]**

The ideal transformer with no load is shown in Figure 2.8. Here, V_1 is the voltage supplied to the primary winding, with N_1 turns. Since the windings used in an ideal transformer are free from losses and the core has finite permeability, the current drawn from the source is used to produce the magnetic flux by magnetising the iron core. Hence, the current drawn from the primary winding is called magnetising current and is denoted by I_m. As the winding is purely inductive, the magnetising current I_m lags the supply voltage by 90°. Due to I_m, an alternating flux ϕ in phase with I_m is produced. This

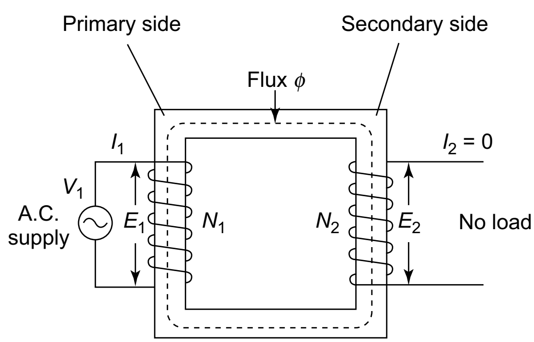


Figure 2.8 Ideal Transformer on No Load

alternating flux ϕ induces an emf in the primary winding, E_1, which opposes the supply voltage i.e., the phase angle between E_1 and V_1 is 180°.

In an ideal transformer, there is no leakage flux. Hence, all the flux produced in the primary winding ϕ links the secondary winding. Since an alternating flux ϕ links the secondary winding, an emf, E_2, is induced in the secondary winding. Here, E_2 is in phase with E_1 and in anti-phase with V_1. Though E_1 and E_2 are in phase with each other, the magnitude of induced emf in secondary winding, E_2, depends on the number of turns, N_2, in the secondary winding. Since the secondary circuit is not closed, the secondary current, I_2 = 0. The phasor diagram for the ideal transformer on no load with $N_2 > N_1$ is shown in Figure 2.9.

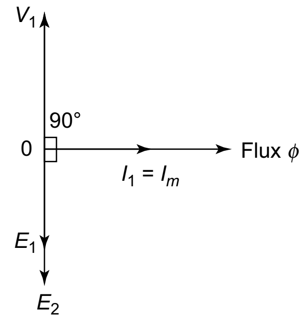


Figure 2.9 Phasor Diagram of an Ideal Transformer on No Load

Though the supply voltage is given to the primary winding, the input power supplied is $V_1I_1 \cos(V_1 \wedge I_1) = V_1I_1 \cos 90° = 0$. The output power developed in the ideal transformer is zero. As the efficiency in the ideal transformer is 100%, the input power supplied in the transformer is equal to the output power developed in it and the input power is also zero. The no-load power factor of ideal transformer is zero lagging.

2.7.2 Practical Transformer on No-load **[AU Nov/Dec, 2016]**

In practice, there exists no ideal transformer i.e., any transformer has losses in it. The hysteresis and eddy current losses exist in the transformer core due to the alternating flux produced by the AC source. These losses can be reduced by using: (i) high-grade materials like silicon steel and (ii) laminations or stacks of thin laminations for building the core. No winding in the transformer is purely inductive and it has small resistance, which contributes to copper loss when the current starts flowing through the winding. The primary current drawn from the source that contributes to both core loss and copper loss is called no-load current, which is denoted as I_0. Therefore, the two components of no-load current are:

- ***Wattless component:*** The magnetizing current, I_m, which is a purely reactive component of I_o that lags V_1 by 90°, is required to magnetise the core and to produce the magnetic flux.
- ***Wattful component:*** The power component, I_c, supplies the total losses of the transformer under no-load condition. It is the active or core loss component of I_o, which is in phase with V_1.

Hence, the total no-load current of a transformer, I_o is the vector addition of I_m and I_c and is given by

$$\overline{I}_o = \overline{I}_m + \overline{I}_c$$

Also, the phase angle between I_o and V_1 is ϕ_0 and the no-load power factor of the transformer is cos ϕ_0. The phasor diagram of practical transformer on no-load is shown in Figure 2.10.

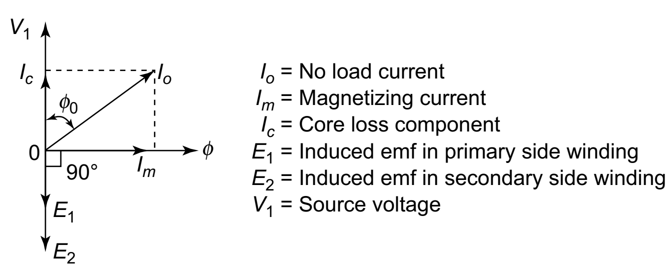


Figure 2.10 Phasor Diagram of Practical Transformer on No Load

Using Figure 2.10, the two components of I_o are given by

$$I_c = I_o \cos \phi_0 \text{ and } I_m = I_o \sin \phi_0$$

The magnitude of no-load current is given by

$$I_o = \sqrt{I_m^2 + I_c^2}$$

The input power on no-load, W_o, supplied to the primary winding of the transformer is given by

$$W_o = V_1 I_o \cos \phi_o = V_1 I_c$$

Since I_c is very small, approximately 0.04 times the full-load rated current, it contributes more to core loss or iron loss when compared to primary copper loss. Hence, the input power, W_o, supplied is the iron loss of the transformer, P_i, which is constant for different load conditions.

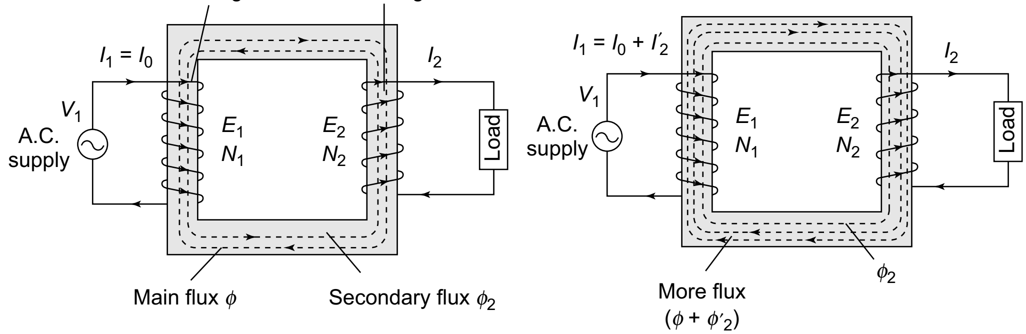
i.e., $W_o = V_1 I_o \cos \phi_o = P_i$ = iron loss

Since the secondary circuit is not closed, the secondary current, $I_2 = 0$ and hence, the output power developed in the transformer is zero.

2.8 Transformer on Load **[AU Nov/Dec, 2016]**

The practical transformer connected with load is shown in Figure 2.11. When the primary winding of the transformer is connected to the AC source, V_1 an emf E_2 is induced in the secondary winding due to transformer action. Since the load is connected to the secondary winding, the current I_2 flows through the winding and an output voltage V_2 is obtained across the load. The phase difference between the output voltage V_2 and I_2 depends on the load i.e., if load is inductive, I_2 lags V_2 by ϕ_2, if load is capacitive, I_2 leads V_2 by ϕ_2 and if load is resistive, I_2 is in phase with V_2.

The current I_2 flowing through secondary winding produces its own magnetic flux ϕ_2, which opposes the magnetic flux ϕ i.e., the direction of ϕ_2 is opposite to ϕ, as shown in Figure 2.11(a). Since ϕ_2 opposes ϕ, there is a momentary reduction in the magnetic flux ϕ, which further decreases the induced emf E_1. This decrease in E_1 increases the difference between V_1 and E_1 that causes the primary winding to draw extra current I_2', thereby producing an additional flux ϕ_2' opposite to ϕ_2, as shown in Figure 2.11(b). This extra current drawn by the primary winding is called load component of I_1. Since this current I_2' neutralizes ϕ_2, the flux is maintained constant in the transformer and hence, it is also known as constant flux machine.



(a) ϕ_2 opposes ϕ (b) Primary drawn more current

Figure 2.11 Transformer on Load

The ampere-turns, which produce the flux ϕ_2 is $N_2 I_2$ and the ampere-turns, which produce the flux, ϕ_2' is $N_1 I_2'$. As these ampere-turns get balanced, the flux in the transformer is maintained constant for any load condition.

i.e., $$N_1 I_2' = N_2 I_2$$

Therefore, $$I_2' = \frac{N_2}{N_1} I_2 = K I_2$$

where, K is the transformation ratio.

2.8.1 Components of Primary Current, I_1

When the transformer is connected to load, the primary current I_1 has two components:

1. No-load current, I_o which has two components I_m and I_c. The phase difference between V_1 and I_o is ϕ_o which is the no-load power factor.
2. Load component I_2', which neutralizes the effect of I_2. The phase difference between I_2 and I_2' is 180°.

Therefore, in vector form, the primary current is given by

$$\overline{I}_1 = \overline{I}_o + \overline{I}_2'$$

The phasor diagram of inductive, resistive and capacitive load when it is connected to transformer is shown in Figures 2.12 (a) to (c) respectively.

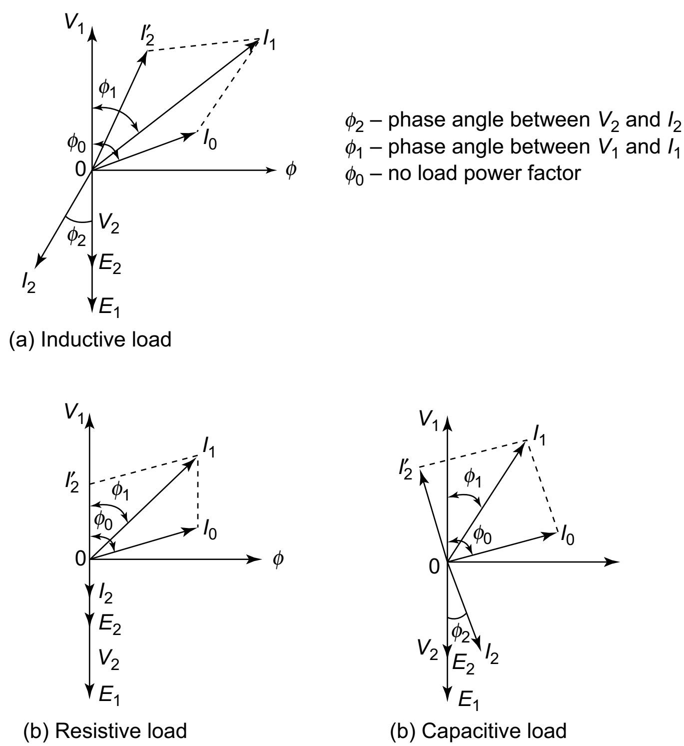


Figure 2.12 Phasor Diagram of Transformer on Load

Example 2.1

The required no-load ratio in a single-phase, 50 Hz, core type transformer is 6600/250 V. Find the number of turns in each winding, if the flux is 0.06 Wb. **[AU April/May, 2016]**

Solution

Given, $f = 50$ Hz and $\phi_m = 0.06$ Wb

At no load, $E_1 = V_1$ 6600 V and $E_2 = V_2 = 250$ V

The induced emf in the primary winding is given by

$$E_1 = 4.44\phi_m f N_1$$

Substituting the given values, we get

$$6600 = 4.44 \times 0.06 \times 50N_1$$

Therefore, the number of turns in primary winding, $N_1 = 495.495 = 495$

The transformation ratio is given by

$$K = \frac{E_2}{E_1} = \frac{250}{6600} = 0.03787$$

The number of turns in secondary winding is given by

$$N_2 = KN_1 \left(\text{since } K = \frac{N_2}{N_1} \right)$$

Therefore, $N_2 = 0.03787 \times 496 = 18.78 \approx 19$

Example 2.2

A single-phase transformer is rated at 240/120 V, 50 Hz. Find voltage and frequency of secondary at no-load: (i) if primary voltage is 120 V, 25 Hz and (ii) if primary voltage is 240 V DC. **[AU Nov/Dec, 2014]**

Solution

Given, $V_1 = 240$ V, $V_2 = 120$ V and $f = 50$ Hz

The transformation ratio is given by

$$K = \frac{V_2}{V_1} = \frac{120}{240} = \frac{1}{2}$$

(i) If $V_1 = 120$ V and $f = 25$ Hz, then

$$\frac{1}{2} = \frac{V_2}{120} \text{ since } \left(K = \frac{V_2}{V_1} \right)$$

Therefore, the voltage of secondary at no-load, $V_2 = 60$ V

Due to transformer action, the frequency of secondary at no-load is also 25 Hz.

(ii) When a DC source is applied to the primary winding, a constant flux will be produced in the winding. Due to this constant flux, no emf is induced in the secondary winding. Hence, no current will be delivered to the load connected to the secondary winding. Therefore, $V_2 = 0$.

Example 2.3

The no-load current of a transformer is 15 A at a power factor of 0.2, when connected to a 460 V, 50 Hz supply. If the primary winding has 550 turns, calculate: (i) the magnetising component of no-load current, (ii) the iron loss and (iii) the maximum value of the flux in the core. **[AU Nov/Dec, 2015]**

Solution

Given, I_o = 15 A, cos ϕ_o = 0.2, $V_o = V_1$ = 460 V, f = 50 Hz and N_1 = 550

(i) The magnetising component of no-load current is given by

$$I_m = I_o \sin \phi_o$$
$$= 15 \times \sin(\cos^{-1}(0.2)) = 14.6969 \text{ A}$$

(ii) The iron loss of the transformer is given by

$$P_i = V_o I_o \cos \phi_o$$
$$P_i = 460 \times 15 \times 0.2 = 1380 \text{ W}$$

(iii) At no load, $E_1 = V_o = V_1 = 4.44 f \phi_m N_1$
Substituting the given values, we get

$$460 = 4.44 \times 50 \times \phi_m \times 550$$

Therefore, the maximum value of the flux in the core is ϕ_m = 3.76 mWb

Example 2.4

A 400/200 V transformer takes 1 A, at a power factor of 0.4, on no load. If the secondary supplies a load current of 50 A at 0.8 lagging power factor, calculate the primary current. **[AU April/May, 2012]**

Solution

Given, I_o = 1 A, E_1 = 400 V, E_2 = 200 V, cos ϕ_o = 0.4, I_2 = 50 A and cos ϕ_2 = 0.8
The transformation ratio is given by

$$K = \frac{E_2}{E_1} = \frac{200}{400} = 0.5$$

The magnitude of extra current to be consumed by the primary winding is

$$I_2' = KI_2 = 0.5 \times 50 = 25 \text{ A}$$

The angle of this extra current is decided based on load power-factor, cos ϕ_2. Therefore, $\phi_2 = \cos^{-1}(0.8) = 36.86°$.

This additional extra current, I_2', is in anti-phase with I_2 that lags E_2 by 36.86°. The phasor diagram of such transformer is shown in Figure E2.4(a), by considering flux ϕ as the reference.

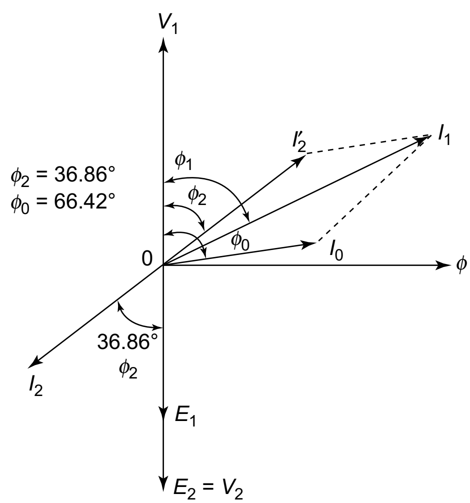


Figure E2.4(a)

As there is no winding resistance or leakage reactance in both the windings, the vectors $\overline{V}_1$ and $-\overline{E}_1$ are in phase. Hence, $\overline{V}_1$ is drawn opposite to $\overline{E}_1$.

Therefore, the phase angle between $\overline{V}_1$ and $\overline{I}_o$ is

$$\phi_o = \cos^{-1}(0.4) = 66.42°.$$

The total primary current in the primary winding is given by

$$\overline{I}_1 = \overline{I}_2' + \overline{I}_o$$

Since both currents $\overline{I}_2'$ and $\overline{I}_o$ are in vector form, it is resolved into their respective components, as shown in Figures E2.4 (b) and (c) respectively.

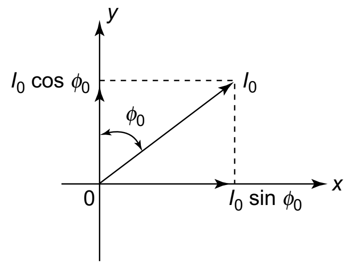


Figure E2.4(b)

Using Figure E2.4 (b), the two components of $\overline{I}_o$ are:

$$(I_o)_x = I_o \sin \phi_o = 1 \times \sin 66.42° = 0.9165 \text{ A}$$
$$(I_o)_y = I_o \cos \phi_o = 1 \times \cos 66.42° = 0.4 \text{ A}$$

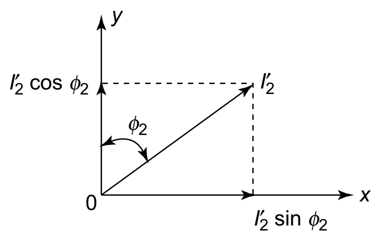


Figure E2.4(c)

Similarly, using Figure E2.4(c), the two components of $\overline{I}_2'$ are:

$$(I_2')_x = I_2' \sin \phi_2 = 25 \times \sin 36.86° = 15 \text{ A}$$
$$(I_2')_y = I_2' \cos \phi_2 = 25 \times \cos 36.86° = 20 \text{ A}$$

Therefore, the no-load current and the additional component of primary current are given by:

$$\overline{I}_o = 0.4 + j0.9165 \text{ A}$$

and $$\overline{I}_2' = 20 + j15 \text{ A}$$

Hence, the total primary current of the transformer is given by:

$$\overline{I}_1 = \overline{I}_o + \overline{I}_2'$$
$$= 0.4 + j0.9165 + 20 + j15 = 20.4 + j15.9165 \text{ A}$$

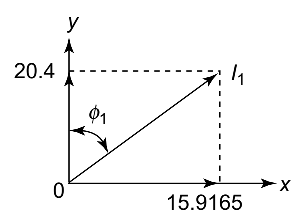


Figure E2.4(d)

Thus, the horizontal and vertical components of $\overline{I}_1$ are shown in Figure E2.4(d), and are given by:

$$(I_1)_x = 15.9165 \text{ A and } (I_1)_y = 20.4 \text{ A}$$

Therefore, the magnitude of the primary current is given by

$$|\overline{I}_1| = \sqrt{(I_1)_x^2 + (I_1)_y^2} = \sqrt{15.9165^2 + 20.4^2} = 25.874 \text{ A}$$

Also, the primary power factor angle is given by

$$\phi_1 = \tan^{-1}\frac{(I_1)_x}{(I_1)_y} = \tan^{-1}\left(\frac{15.9165}{20.4}\right) = 37.96°$$

Therefore, the primary power factor is given by cos ϕ_1 = cos 37.96° = 0.788. Since the primary current always lags the supply voltage, V_1, the primary power factor is lagging power factor.

Example 2.5

A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at the power factor of 0.8 lagging, the primary current is 25 A at the power factor of 0.707 lagging. Determine the no-load current of the transformer and its phase with respect to the voltage. **[AU Nov/Dec, 2014]**

Solution

Given, $N_1 = 800$, $N_2 = 200$, $I_2 = 80$ A, $\cos\phi_2 = 0.8$ lagging, $I_1 = 25$ A and $\cos\phi_1 = 0.707$ lagging.

The transformation ratio, $K = \dfrac{N_2}{N_1} = \dfrac{200}{800} = \dfrac{1}{4}$

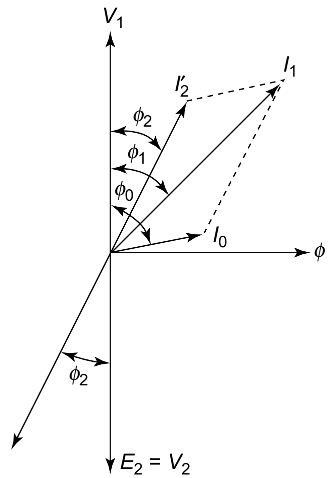


Figure E2.5

The input primary angle and the load angle is given by:

$$\phi_1 = \cos^{-1} 0.707 = 45°$$

and $$\phi_2 = \cos^{-1} 0.8 = 36.8698°$$

The extra current that is drawn by the primary winding is

$$I_2' = KI_2 = \frac{1}{4} \times 80 = 20 \text{ A}$$

Therefore, the primary current in vector form is given by

$$\overline{I}_1 = \overline{I}_2' + \overline{I}_o \qquad (1)$$

Assuming there is no winding resistances or leakage reactance, the phasor diagram of the transformer with load is shown in Figure E2.5.

Both $\overline{I}_1$ and $\overline{I}_2'$ can be resolved into horizontal and vertical components, based on their respective angles ϕ_1 and ϕ_2.

Therefore,

$$(I_1)_x = I_1 \sin\phi_1 = 25 \times \sin 45° = 25 \times \frac{1}{\sqrt{2}} = 17.675 \text{ A}$$

$$(I_1)_y = I_1 \cos\phi_1 = 25 \times \cos 45° = 25 \times \frac{1}{\sqrt{2}} = 17.675 \text{ A}$$

$$(I_2')_x = I_2' \sin\phi_2 = 20 \times \sin 36.86° = 20 \times 0.6 = 12 \text{ A}$$

$$(I_2')_y = I_2' \cos\phi_2 = 20 \times \cos 36.86° = 20 \times 0.8 = 16 \text{ A}$$

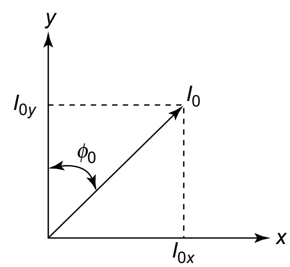


Figure E2.5(b)

Using Eqn. (1), the horizontal and vertical components of no-load current are shown in Figure E2.5(b), and are given by

$$(I_o)_x = (I_1)_y - (I_2')_x = 17.675 - 12 = 5.675 \text{ A}$$

$$(I_o)_y = (I_1)_y - (I_2')_y = 17.675 - 16 = 1.675 \text{ A}$$

Therefore, the magnitude of no-load current is given by

$$|I_o| = \sqrt{(I_o)_x^2 + (I_o)_y^2} = \sqrt{5.675^2 + 1.675^2} = 5.917 \text{ A}$$

Also, the no-load angle is given by

$$\phi_0 = \tan^{-1}\frac{(I_o)_x}{(I_o)_y} = \tan^{-1}\frac{5.675}{1.675} = 73.55°$$

Hence, the no-load current in the transformer in vector form is given by

$$\overline{I}_o = 5.917\angle 73.55° \text{ A}$$

Therefore, the no-load power factor is given by

$$\cos\phi_o = \cos 73.55° = 0.283 \text{ (lagging)}$$

Example 2.6

A single-phase transformer with a ratio of 6.6 kV/415 V takes a no-load current of 0.75 A at the power factor of 0.22. If the secondary supplies a current of 120 A at the power factor of 0.8, calculate the total current taken by the primary. **[AU April /May, 2016]**

Solution

Given, I_o = 0.75 A, E_1 = 6.6 kV, E_2 = 415 V, cos ϕ_o = 0.22, I_2 = 120 A and cos ϕ_2 = 0.8

The transformation ratio is given by

$$K = \frac{E_2}{E_1} = \frac{415}{6.6\times 10^3} = 0.06287$$

The magnitude of extra current to be consumed by the primary winding is

$$I_2' = KI_2 = 0.06287\times 120 = 7.5454 \text{ A}$$

The angle of this extra current is decided based on load power factor, cos ϕ_2. Therefore, $\phi_2 = \cos^{-1}(0.8) = 36.86°$.

This additional extra current, I_2' is in anti-phase with I_2, which lags E_2 by 36.86°. The phasor diagram of such transformer is shown in Figure E2.6 (a), by considering flux ϕ as the reference.

As there is no winding resistance or leakage reactance in both the windings, the vectors $\overline{V}_1$ and $-\overline{E}_1$ are in phase. Hence, $\overline{V}_1$ is drawn opposite to $\overline{E}_1$.

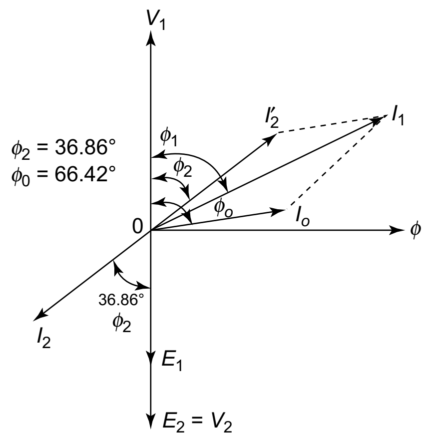


Figure E2.6(a)

Therefore, the phase angle between $\overline{V}_1$ and $\overline{I}_o$ is $\phi_o = \cos^{-1}(0.22)$ = 77.2909°.

The total primary current in the primary winding is given by

$$\overline{I}_1 = \overline{I}_2' + \overline{I}_o$$

Since both currents $\overline{I}_2'$ and $\overline{I}_o$ are in vector form, it is resolved into their respective components, as shown in Figures E2.6 (b) and (c) respectively.

Using Figure E2.6 (b), the two components of $\overline{I}_o$ are:

$$(I_o)_x = I_o \sin\phi_o = 0.75\times\sin 77.2909° = 0.7316 \text{ A}$$
$$(I_o)_y = I_o \cos\phi_o = 0.75\times\cos 77.2909° = 0.165 \text{ A}$$

Similarly, using Figure E2. (c), the two components of $\overline{I_2'}$ are:

$$(I_2')_x = I_2' \sin \phi_2 = 7.5454 \times \sin 36.86° = 4.5262 \text{ A}$$
$$(I_2')_y = I_2' \cos \phi_2 = 7.5454 \times \cos 36.86° = 6.0371 \text{ A}$$

Therefore, the no-load current and the additional component of primary current is given by:

$$\overline{I_o} = 0.1645 + j0.7316 \text{ A}$$

and $$\overline{I_2'} = 6.0371 + j5.2578 \text{ A}$$

Hence, the total primary current of the transformer is given by

$$\begin{aligned}\overline{I_1} &= \overline{I_o} + \overline{I_2'}\\ &= 0.165 + j0.7316 + 6.0371 + j4.5262\\ &= 6.2021 + j5.2578 \text{ A}\end{aligned}$$

Thus, the horizontal and vertical components of $\overline{I_1}$ are shown in Figure E2.6 (d), and are given by:

$$(I_1)_x = 5.2578 \text{ A} \quad \text{and} \quad (I_1)_y = 6.2021 \text{ A}$$

Therefore, the magnitude of the primary current is given by

$$\begin{aligned}|\overline{I_1}| &= \sqrt{5.2578^2 + 6.2021^2}\\ &= \sqrt{27.644 + 38.466} = 8.130 \text{ A}\end{aligned}$$

Also, the primary power factor angle is given by

$$\phi_1 = \tan^{-1}\left(\frac{5.2578}{6.2021}\right) = 40.289°$$

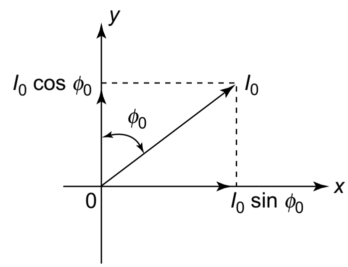


Figure E2.6(b)

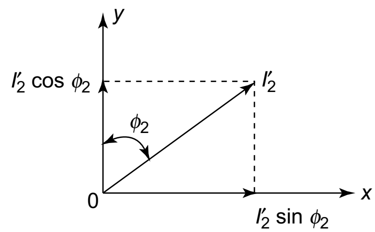


Figure E2.6(c)

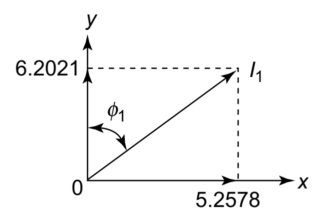


Figure E2.6(d)

Therefore, the primary power factor is given by cos ϕ_1 = cos 40.289° = 0.7628. Since the primary current always lags the supply voltage, V_1, the primary power factor is lagging power factor.

2.9 Effect of Winding Resistances

In a practical transformer, some resistances exist in both primary and secondary windings, which cause the copper loss and voltage drop in the transformer. Let R_1 and R_2 be the primary and secondary winding resistances in ohms. When an AC supply is connected to the primary winding, a current I_1 flows through it, causing a voltage drop I_1R_1 across the winding. Due to this voltage drop, the induced emf in the primary winding, E_1 in vector form, is given by

$$\overline{E_1} = \overline{V_1} - \overline{I_1R_1} \tag{2.3}$$

Similarly, when the load is connected to the transformer, a current I_2 flowing through the secondary winding causes a voltage drop I_2R_2 in the load. Therefore, the terminal or load voltage, $\overline{V}_2$ appears across the load in vector form is given by

$$\overline{V}_2 = \overline{E}_2 - \overline{I_2R_2} \tag{2.4}$$

Since this drop is purely resistive, it is always in phase with its respective currents I_1 and I_2.

2.9.1 Equivalent Winding Resistance

In order to make the calculations simple, the two winding resistances R_1 and R_2 can be transferred to either primary or secondary side, which will not affect the transformer performance. The methodology of this transfer is given below:

The total copper loss due to the resistances R_1 and R_2 is

$$P_c = I_1^2R_1 + I_2^2R_2 \tag{2.5}$$

When the resistance R_2 is transferred to the primary side, we get

$$P_c = I_1^2\left(R_1 + \frac{I_2^2}{I_1^2}R_2\right)$$

Since $\dfrac{I_1}{I_2} = K$, we get

$$P_c = I_1^2\left(R_1 + \frac{R_2}{K^2}\right) = I_1^2(R_1 + R_2') = I_1^2R_{1e} \tag{2.6}$$

where, $R_2' = \dfrac{R_2}{K^2}$ is the secondary winding resistance and R_{1e} is the total winding resistance when referred to primary side is given by

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

It is noted that, the copper loss, P_c given by Eqn. (2.5) and by Eqn. (2.6) is same.

Similarly, when the winding resistance R_1 is transferred to the secondary winding, we get

$$P_c = I_2^2\left(\frac{I_1^2}{I_2^2}R_1 + R_2\right)$$

$$I_2^2(K^2R_1 + R_2) = I_2^2(R_1' + R_2) = I_2^2R_{2e} \tag{2.7}$$

where, $R_1' = K^2R_1$ is the primary winding resistance and R_{2e} is the total winding resistance when referred to secondary side and is given by

$$R_{2e} = R_1' + R_2 = K^2R_1 + R_2$$

It is noted that, the copper loss, P_c generated by Eqn. (2.5) and by Eqn. (2.7) is same.

The concepts of these winding resistances are shown in Figures 2.13(a), (b) and (c).

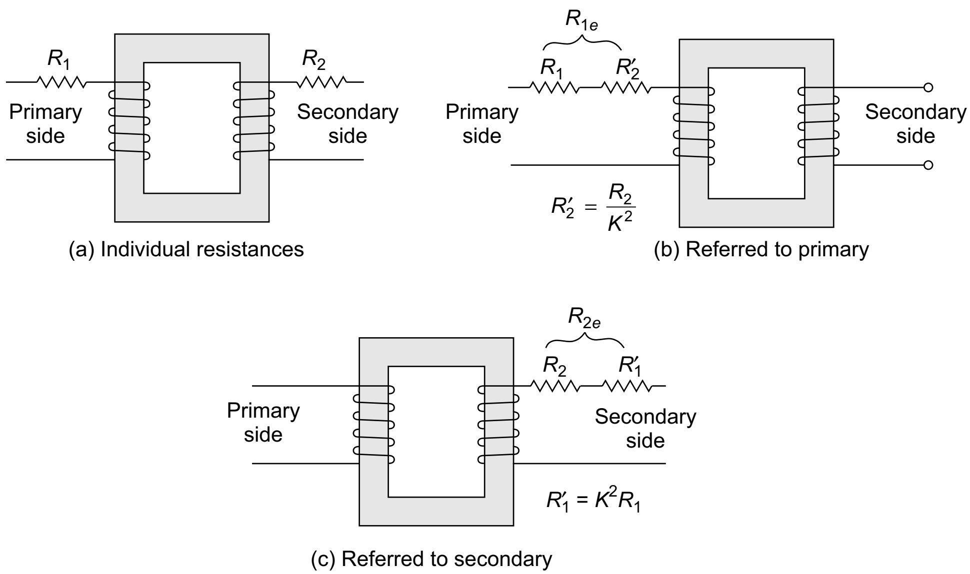


Figure 2.13 Winding Resistances

Note:

(i) For calculation purpose, when winding resistances are transferred to primary side, the secondary winding resistance becomes zero. Hence, the total copper loss occurs due to R_{1e}.

(ii) Similarly, for calculation purpose, when winding resistances are transferred to secondary side, the primary winding resistance becomes zero. Hence, the total copper loss occurs due to R_{2e}.

(iii) $R_1' = K^2 R_1$ and $R_2' = \dfrac{R_2}{K^2}$ where $K = \dfrac{N_2}{N_1} = \dfrac{I_1}{I_2}$. This result can be cross-checked, as follows: When only low-voltage side is considered for calculation, the high-voltage winding resistance gets decreased, as it is transferred to low-voltage side. Also, when only high-voltage side is considered for calculation, the low-voltage winding resistance gets increased, as it is transferred to low-voltage side.

2.10 Effect of Leakage Reactance

It is known that the total magnetic flux generated in the primary winding links the secondary winding. But in a practical transformer, some of the magnetic flux, ϕ_1 links only with the primary winding. Similarly, some of the magnetic flux, ϕ_2 produced in the secondary winding links only with it. These fluxes, which link only with primary and secondary windings, are called leakage flux as represented in Figure 2.14.

These leakage fluxes generate self-induced emfs e_{L1} and e_{L2} in the primary and secondary windings

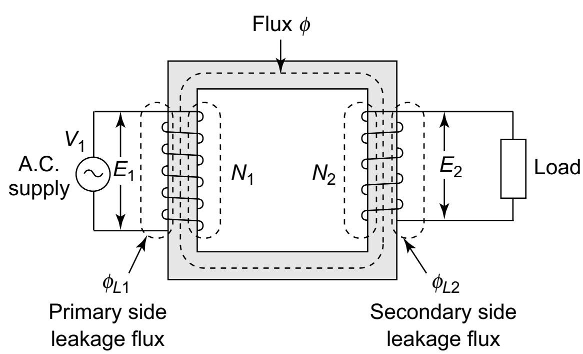


Figure 2.14 Leakage Fluxes in Transformer

respectively. Therefore, the supply voltage V_1 and the induced emf E_2 have to overcome these self-induced emfs to generate E_1 and V_2 respectively. These self-induced emfs are considered as the voltage drop across fictitious reactance called leakage reactance of the winding. The leakage reactances in the primary and secondary windings are represented as X_1 and X_2 respectively, such that the drops I_1X_1 and I_2X_2 equals the self-induced emfs e_{L1} and e_{L2} respectively.

2.10.1 Equivalent Leakage Reactance

Similar to the winding resistances of the transformer, the leakage reactances can also be transferred to either primary or secondary side. The total winding resistance referred to primary and secondary represented by R_{1e} and R_{2e} respectively holds true for leakage reactances. Therefore, the total leakage reactance of the transformer when referred to primary side, X_{1e} is given by

$$X_{1e} = X_1 + X_2'$$

where $X_2' = \dfrac{X_2}{K^2}$ is the secondary leakage reactance when referred to primary side.

Similarly, the total leakage reactances of the transformer when referred to secondary side, X_{2e} is given by,

$$X_{2e} = X_1' + X_2$$

where $X_1' = K^2 X_1$ is the primary leakage reactance when referred to secondary side.

The concepts of these leakage reactances are shown in Figures 2.15(a), (b) and (c).

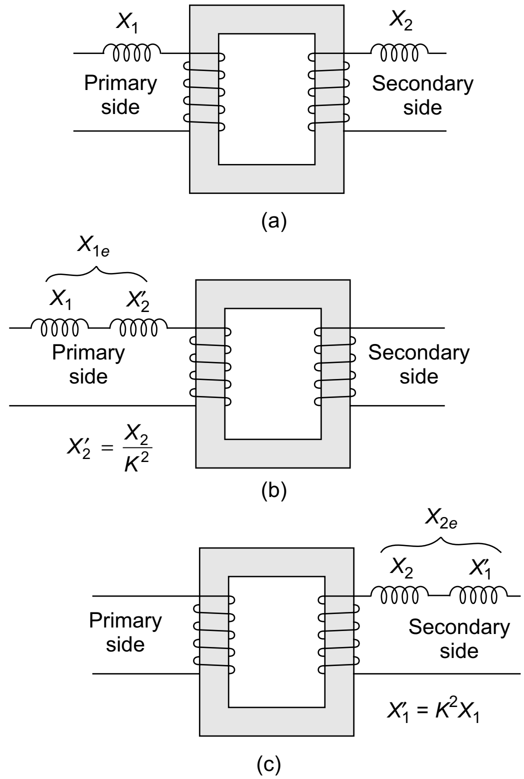


Figure 2.15 Leakage reactances (a) in both windings (b) referred to primary side (c) referred to secondary side

2.11 Winding Impedance

The primary and secondary windings of the transformer have their own winding resistances and leakage reactances. Therefore, the total winding impedances in the primary and secondary windings are given by $Z_1 = R_1 + jX_1\ \Omega$ and $Z_2 = R_2 + jX_2\ \Omega$ respectively, as shown in Figure 2.16.

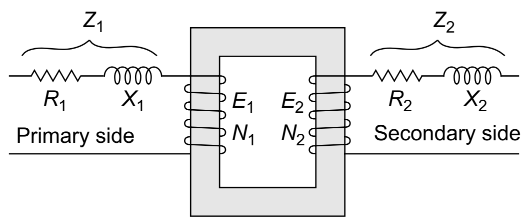


Figure 2.16 Winding Impedance of a Transformer

2.11.1 Equivalent Winding Impedances

Similar to winding resistances and leakage reactances, the winding impedance can also be transferred to either primary or secondary side.

The total winding impedance when referred to primary side is given by

$$Z_{1e} = Z_1 + Z_2'$$

where, $Z_2' = \dfrac{Z_2}{K^2}$ is the secondary impedance when referred to primary side.

Similarly, the total winding impedance when referred to secondary side is given by

$$Z_{2e} = Z_1' + Z_2$$

where, $Z_1' = K^2 Z_1$ is the primary impedance when referred to secondary side.

Also, the total winding impedances, when referred to primary side and secondary side, are given by

$$Z_{1e} = R_{1e} + jX_{1e}$$

and
$$Z_{2e} = R_{2e} + jX_{2e}$$

The concept of equivalent winding impedance when referred to primary and secondary side is shown in Figure 2.17(a) and Figure 2.17(b) respectively.

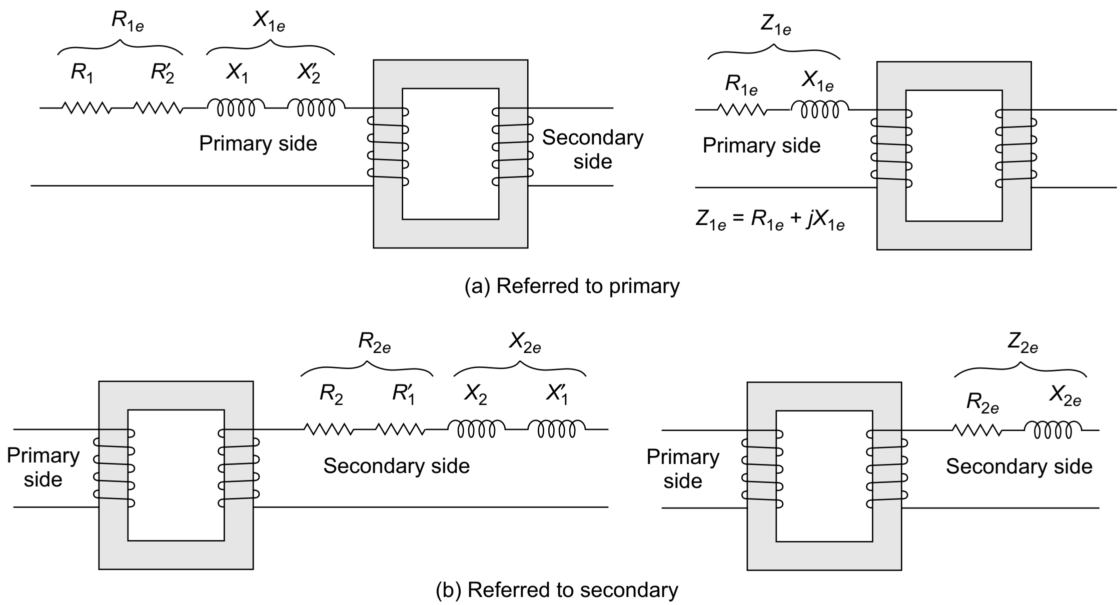


Figure 2.17 Equivalent Winding Impedances

Example 2.7

A 2000/200 V transformer has primary resistance and reactance of 2 Ω and 4 Ω respectively. The corresponding secondary values are 0.025 Ω and 0.04 Ω. Determine (i) the equivalent resistance and reactance of primary referred to secondary (ii) total resistance and reactance referred to secondary (iii) equivalent resistance and reactance of secondary referred to primary and (iv) total resistance and reactance referred to primary.

[AU Nov/Dec, 2015]

Solution

Given, $R_1 = 2\ \Omega$, $X_1 = 4\ \Omega$, $R_2 = 0.025\ \Omega$, $X_2 = 0.04\ \Omega$, $V_1 = 2000$ V and $V_2 = 200$ V

The transformation ratio is given by

$$K = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$$

(i) The equivalent resistance and reactance of primary when referred to secondary are

$$R_1' = K^2 R_1 = (0.1)^2 \times 2 = 0.02\ \Omega$$

and $$X_1' = K^2 X_1 = (0.1)^2 \times 4 = 0.04\ \Omega$$

(ii) The total resistance and reactance when referred to secondary are

$$R_{2e} = R_2 + R_1' = 0.025 + 0.020 = 0.045\ \Omega$$

and $$X_{2e} = X_2 + X_1' = 0.04 + 0.04 = 0.08\ \Omega$$

(iii) The equivalent resistance and reactance of secondary when referred to primary are

$$R_2' = \frac{R_2}{K^2} = \frac{0.025}{(0.1)^2} = 2.5\ \Omega$$

and $$X_2' = \frac{X_2}{K^2} = \frac{0.04}{(0.1)^2} = 4\ \Omega$$

(iv) The total resistance and reactance when referred to primary are

$$R_{1e} = R_1 + R_2' = 2 + 2.5 = 4.5\ \Omega$$

and $$X_{1e} = X_1 + X_2' = 4 + 4 = 8\ \Omega$$

2.12 Transformer on Load with Winding Impedances

The circuit diagram of a transformer on load with winding impedances is shown in Figure 2.18.

Here, the supply voltage V_1 in vector form is given by

$$\begin{aligned} \overline{V_1} &= -\overline{E_1} + \overline{I_1 R_1} + j\overline{I_1 X_1} \\ &= -\overline{E_1} + \overline{I_1 Z_1} \end{aligned} \tag{2.8}$$

where, $\overline{E_1}$ is the induced emf in the primary winding, which opposes $\overline{V_1}$. Here, $\overline{I_1 R_1}$ is the primary winding resistance drop, which is in phase with $\overline{I_1}$, and $\overline{I_1 X_1}$ is the primary leakage reactance drop, which leads $\overline{I_1}$ by 90°.

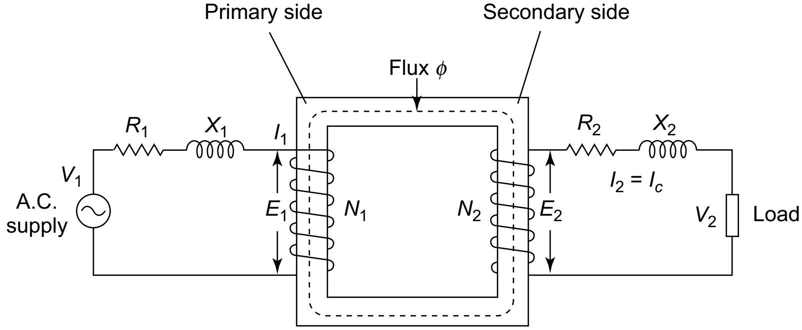


Figure 2.18 Transformer on Load with Winding Impedances

Similarly, the induced emf in secondary winding, E_2 in vector form is given by:

$$\begin{aligned}\overline{E}_2 &= \overline{V}_2 + \overline{I_2R_2} + j\overline{I_2X_2}\\ &= \overline{V}_2 + \overline{I_2Z_2}\end{aligned} \tag{2.9}$$

where, $\overline{V}_2$ is the load or terminal voltage obtained across the load. Here, $\overline{I_2R_2}$ is the secondary winding resistance drop, which is in phase with $\overline{I}_2$ and $\overline{I_2X_2}$ is the leakage reactance drop, which leads $\overline{I}_2$ by 90°.

The other equations to be considered in drawing the phasor diagram of different loads are:

No-load primary current, $\overline{I}_o = \overline{I}_c + \overline{I}_m$ (2.10)

Primary current, $\overline{I}_1 = \overline{I}_o + \overline{I}_2'$ (2.11)

and load component of $\overline{I}_1$, $\overline{I}_2' = -K\overline{I}_2$ (2.12)

The phase angle between different currents and voltages to be considered in drawing a phasor diagram of different loads are:

ϕ_0 is the phase angle between $\overline{V}_1$ and $\overline{I}_o$

ϕ_1 is the phase angle between $\overline{V}_1$ and $\overline{I}_1$

and ϕ_2 is the phase angle between $\overline{V}_2$ and $\overline{I}_2$

2.12.1 Steps to Draw a Phasor Diagram

The steps to be followed in drawing the phasor diagram of a transformer on load condition are:

(i) Consider the magnetic flux ϕ as the reference.

(ii) Draw the induced emf $\overline{E}_1$ such that it lags ϕ by 90°.

(iii) Draw $\overline{E}_2$ in phase with $\overline{E}_1$. The magnitude of $\overline{E}_2$ depends on the type of transformer. If it is a step-up transformer, $|\overline{E}_2| > |\overline{E}_1|$ and if it is a step-down transformer, $|\overline{E}_1| > |\overline{E}_2|$.

(iv) Draw $\overline{V}_2$ such that it lags $\overline{E}_2$ by a very small angle.

(v) Depending on the type of load connected to the transformer, $\overline{I}_2$ is drawn i.e., (i) if lagging power-factor load is connected, $\overline{I}_2$ lags $\overline{V}_2$ by ϕ_2, (ii) if leading power-factor load is connected, $\overline{I}_2$ leads $\overline{V}_2$ by ϕ_2 and (iii) if unity power-factor load is connected, $\overline{I}_2$ is in phase with $\overline{V}_2$.

(vi) Using Eqn. (2.9), obtain $\overline{E}_2$ such that $\overline{I_2R_2}$ is in phase with $\overline{V}_2$ and $\overline{I_2X_2}$ leads $\overline{I}_2$ by 90°.

(vii) Draw $\overline{I}_2'$ exactly opposite to $\overline{I}_2$ such that $\overline{I}_2' = K\overline{I}_2$.

(viii) Using Eqn. (2.10), obtain $\overline{I}_o$.

(ix) The primary current $\overline{I}_1$ can be obtained by using Eqn. (2.11).

(x) Draw $-\overline{E}_1$ exactly opposite to $\overline{E}_1$.

(xi) Using Eqn. (2.8), obtain $\overline{V}_1$ such that $\overline{I_1R_1}$ is in phase with $-\overline{E}_1$ and $\overline{I_1X_1}$ leads $\overline{I}_1$ by 90°. The phase angles, ϕ_o, ϕ_1 and ϕ_2 can be marked.

Using the above steps, the phasor diagram of the transformer with lagging, leading and unity power-factor loads are drawn, as shown in Figures 2.19(a) to (c) respectively.

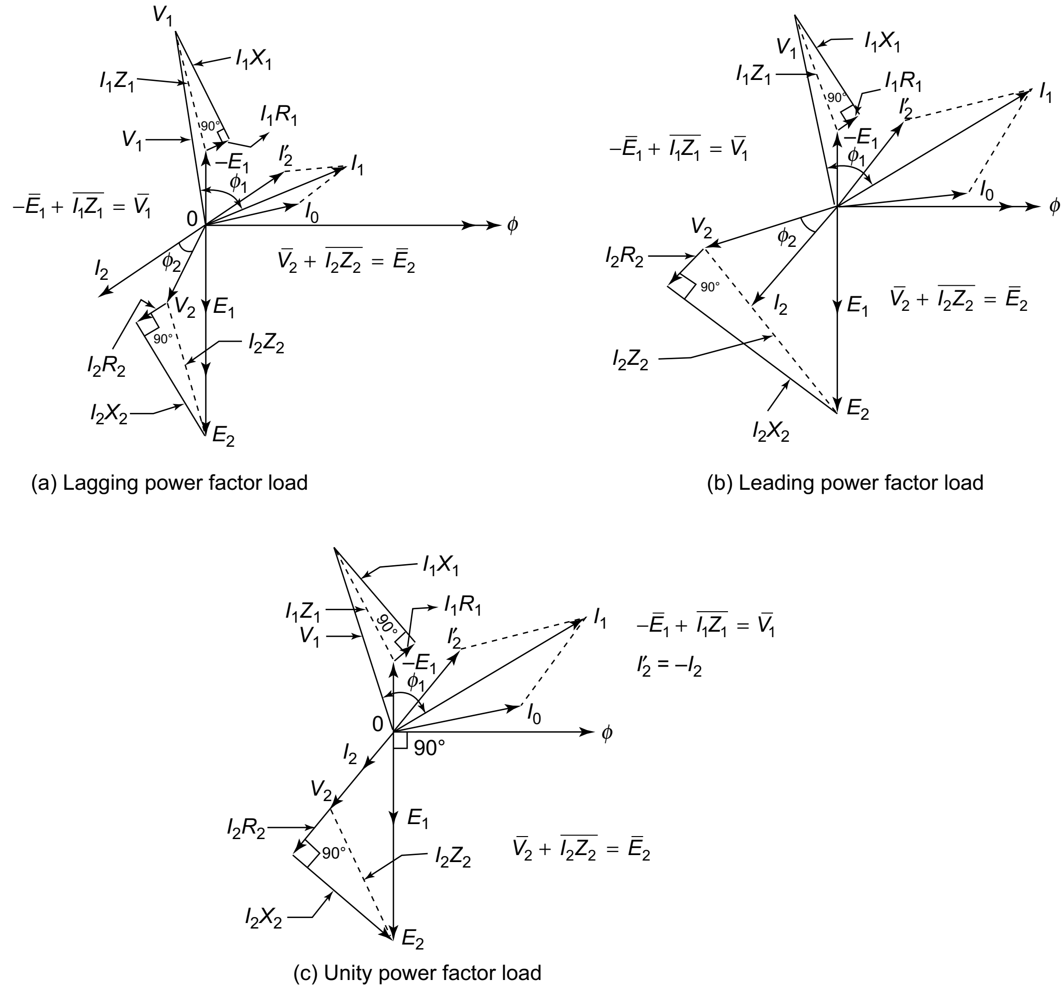


Figure 2.19 Phasor Diagram of a Transformer with Winding Impedances

2.13 Circuit Model or Equivalent Circuit of Transformer **[AU Nov/Dec, 2014]**

The equivalent circuit of a transformer is a circuit with the combination of various resistances and reactances of the primary and secondary windings, which exactly perform like a transformer. Here, the two components of no-load primary current, I_o are I_m and I_c. Since the magnetising current, I_m is required to magnetise the core and to produce the flux, it is assumed to flow through a reactance called no-load reactance, X_o. Since the active component, I_c represents the core loss, it is assumed to flow through a resistance called no-load resistance, R_o. Using this concept and Eqn. (2.10), the no-load equivalent circuit of a transformer is shown in Figure 2.20.

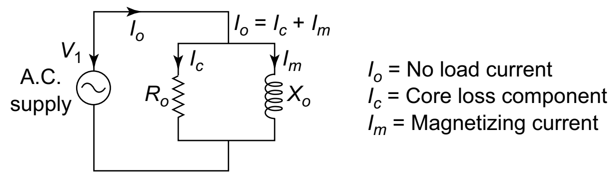


Figure 2.20 No-load Equivalent Circuit

It is noted that, both R_o and X_o are in parallel and form the exciting circuit, such that

$$R_o = \frac{V_1}{I_c} \text{ and } X_o = \frac{V_1}{I_m}$$

The other components of a practical transformer are:

(a) Leakage reactances X_1 and X_2 to represent the leakage flux in the primary and secondary windings respectively.

(b) Winding resistances R_1 and R_2 to represent the copper losses $I_1^2 R_1$ and $I_2^2 R_2$ in the primary and secondary windings respectively.

Thus, the equivalent circuit of the transformer with load impedance, Z_L, is shown in Figure 2.21.

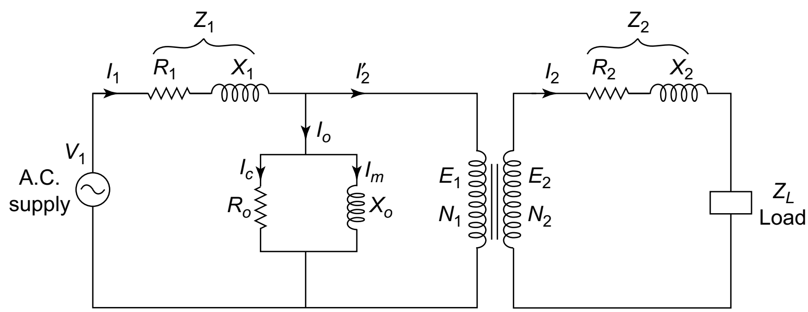


Figure 2.21 Equivalent Circuit of a Transformer at Load Condition

Similar to winding impedances, the equivalent circuit shown in Figure 2.21 can be transferred to either primary or secondary side.

When the secondary-side parameters are referred to the primary side, we get:

$$R_2' = \frac{R_2}{K^2},\ X_2' = \frac{X_2}{K^2},\ Z_2' = \frac{Z_2}{K^2} \text{ and } Z_L' = \frac{Z_L}{K^2}$$

Also, $$E_2' = \frac{E_2}{K},\ I_2' = KI_2$$

Here, K is the transformation ratio given by $\frac{N_2}{N_1}$. Therefore, the equivalent circuit of the transformer, when referred to the primary side, is shown in Figure 2.22.

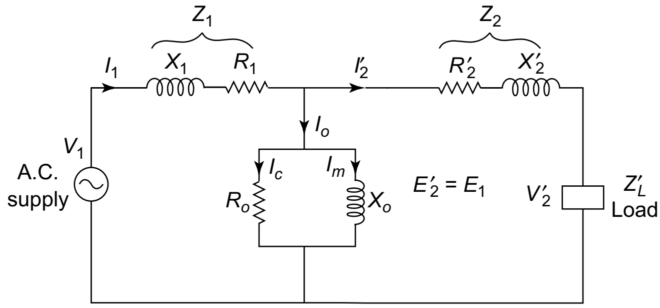


Figure 2.22 Equivalent Circuit of a Transformer when Referred to Primary Side

Similarly, when the primary-side parameters are referred to the secondary side, we get:

$$R_1' = K^2 R_1,\ X_1' = K^2 X_1 \text{ and } Z_1' = K^2 Z_1$$

Also, $$E_1' = KE_1,\ I_1' = \frac{I_1}{K} \text{ and } I_o' = \frac{I_o}{K}$$

In addition, the exciting circuit parameters are also transferred to the secondary side as $R_o' = K^2 R_o$ and $X_o' = K^2 X_o$. Therefore, the equivalent circuit of the transformer when referred to the secondary side is shown in Figure 2.23.

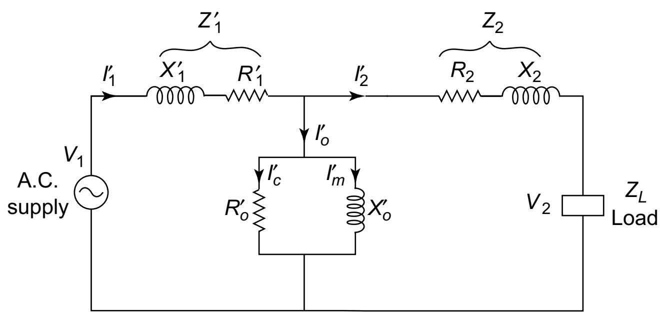


Figure 2.23 Equivalent Circuit of a Transformer when Referred to Secondary Side

Shifting the no-load branch or exciting branch can further simplify the equivalent circuits shown in Figure 2.22 and Figure 2.23. The equivalent circuit obtained by shifting the exciting branch is called approximate equivalent circuit.

2.13.1 Approximate Equivalent Circuit

If the no-load branch shown in Figure 2.22 is shifted to the left of primary-side parameters, the approximate equivalent circuit referred to primary side can be obtained as shown in Figure 2.24.

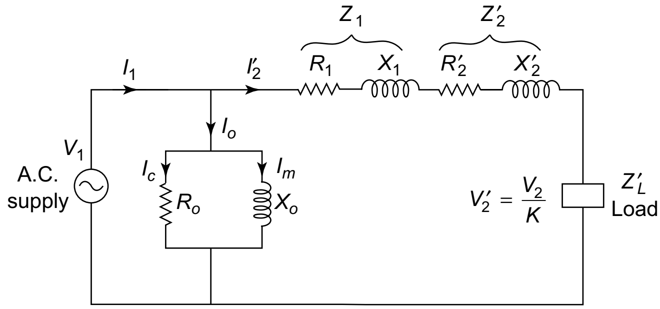


Figure 2.24 Approximate Equivalent Circuit when Referred to Primary Side

The winding resistances R_1 and R_2' can be combined to get the equivalent winding resistance when referred to primary side, R_{1e}. Similarly, the leakage reactances X_1 and X_2' can be combined to get the equivalent leakage reactance when referred to primary side, X_{1e}. Therefore, the simplified approximate equivalent circuit when referred to primary side is shown in the Figure 2.25.

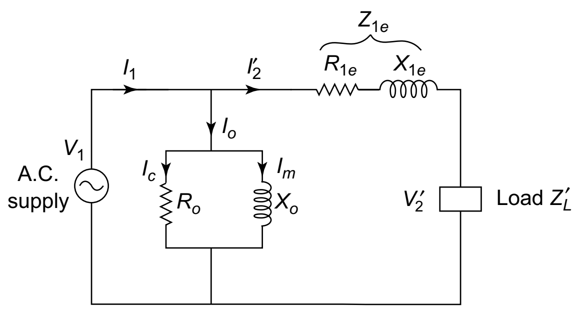


Figure 2.25 Simplified Approximate Equivalent Circuit when Referred to Primary Side

The parameters shown in the simplified approximate equivalent circuit shown in Figure 2.25 are:

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$X_{1e} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$Z_{1e} = R_{1e} + jX_{1e}$$

$$R_o = \frac{V_1}{I_c} \text{ and } X_o = \frac{V_1}{I_m}$$

In the similar way, the approximate equivalent circuit when referred to secondary side can also be obtained as shown in Figure 2.26.

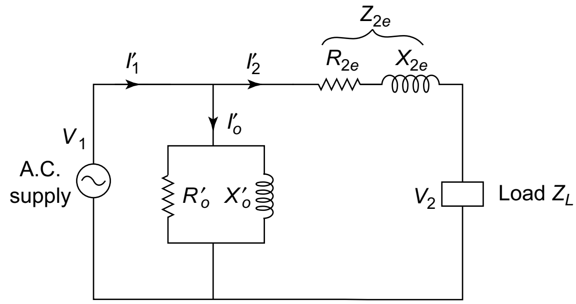


Figure 2.26 Simplified Approximate Equivalent Circuit when Referred to Secondary Side

2.14 Per Unit System

Per unit system is a tool, which scales the electrical quantities based on the base or reference value to make the calculation easier. This system is used in a circuit where the frequency of voltage variation is more. It can be used in any electrical quantity. The dimensionless per unit value of any electrical quantity is defined as the ratio of the actual value of the quantity to the base or reference value of the same quantity, as given by:

$$\text{per unit value of electrical quantity} = \frac{\text{Actual value}}{\text{Base or reference value}}$$

The base values of different electrical quantities are assumed as follows:

(i) Base voltage, V_B = rated voltage of the machine
(ii) Base current, I_B = rated current of the machine
(iii) Base impedance, $Z_B = \dfrac{V_B}{I_B}$
(iv) Base power, $(VA)_B = V_B \times I_B$

In general, $(VA)_B$ and V_B is selected first which helps in fixing the base values of other quantities.

The per unit value of impedance and current are given by:

$$Z_{\text{pu}} = \frac{Z_{\text{actual}}}{Z_B} = \frac{Z_{\text{actual}} \times I_B}{V_B} \text{ and } I_{\text{pu}} = \frac{I_{\text{actual}}}{I_{\text{B}}}$$

Similarly, for other electrical quantities, per unit value can be obtained.

2.14.1 Advantages of Per Unit System

1. If the parameters of transformer are expressed in per unit system, they lie in the same range irrespective of their ratings.
2. Helps in easier calculation.

2.15 Voltage Regulation

[AU Nov/Dec, 2016]

The voltage equation of the secondary side of the transformer with winding impedances referred to secondary side is given by

$$\overline{E}_2 = \overline{V}_2 + \overline{I_2 Z_{2e}}$$

At no-load condition, $I_2 = 0$. Therefore, the terminal voltage at no-load condition, V_{2o} is equal to the induced emf in the secondary side E_2, i.e.,

$$V_{2o} = E_2$$

When the transformer is loaded, there will be voltage drop across Z_{2e}. Due to this drop, the terminal voltage V_2 drops from its no-load value, V_{2o}. This decrease in the terminal voltage, when expressed as a fraction of the no-load terminal voltage, is called the voltage regulation of a transformer. It is also defined as the change in the secondary terminal voltage when the transformer load is reduced from full load to no load, expressed as the percentage of the rated terminal voltage at a constant supply given to the primary side. Mathematically, the voltage regulation is given by:

$$\%R = \frac{V_{2o} - V_2}{V_2} \times 100$$

$$= \frac{E_2 - V_2}{V_2} \times 100$$

where, $V_{2o} = E_2$ is the terminal voltage of the transformer at no load and V_2 is the secondary terminal voltage at a given load.

The secondary terminal voltage of the transformer, V_2, depends on both the magnitude of the load current and the power factor. When the load increases, the load current and the voltage drop across Z_{2e} increase, thereby reducing the terminal voltage V_2. For lagging power-factor load, $V_2 < E_2$ and hence the voltage regulation is positive, whereas for leading power-factor load, $V_2 > E_2$ and hence the voltage regulation is negative. It is clear that for a better performance of the transformer, the drop in the terminal voltage should be as small as possible.

The expression for voltage regulation of the transformer varies with respect to the power factor of the load. It is assumed that all the winding resistances and leakage reactances are referred to the secondary side of the transformer in deriving the expression for voltage regulation.

2.15.1 Lagging Power Factor

The phasor diagram of the secondary side of the transformer for a lagging power-factor load, with reference to V_2 is shown in Figure 2.27.

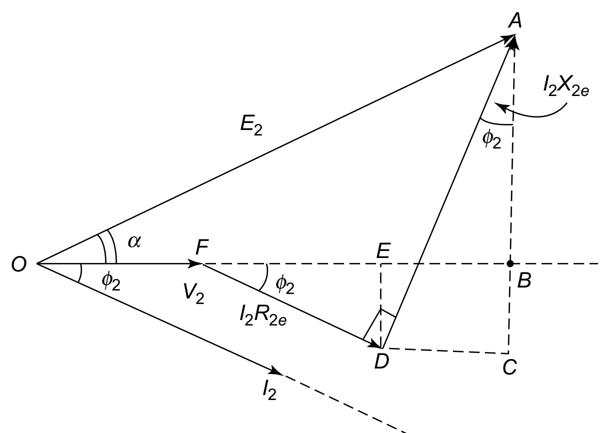


Figure 2.27 Phasor Diagram of Secondary Side for Lagging Power Factor

Using right-angled triangle OAB, we get

$$OA^2 = OB^2 + BA^2$$

Since the phase angle between V_2 and E_2 is practically very small, $BA = 0$. Therefore, $OA^2 = OB^2$.

From Figure 2.27, we get

$$OA = E_2,\ OB = OF + FE + EB,\ OF = V_2 \text{ and } EB = DC$$

Similarly, when referred to primary side, we get

$$\%R = \frac{I_1 R_{1e} \cos \phi_1 \pm I_1 X_{1e} \sin \phi_1}{V_1}$$

Hence, $$E_2^2 = (V_2 + FE + BE)^2 = (V_2 + FE + DC)^2$$

i.e., $$E_2 = (V_2 + FE + BE) = (V_2 + FE + DC) \tag{2.13}$$

Using right-angled triangle DEF, we get

$$FE = FD \cos \phi_2 = I_2 R_{2e} \cos \phi_2 \tag{2.14}$$

Similarly, using right-angled triangle ADC, we get

$$DC = AD \sin \phi_2 = I_2 X_{2e} \sin \phi_2 \tag{2.15}$$

Substituting Eqn. (2.14) and Eqn. (2.15) in Eqn. (2.13), we get

$$E_2 = V_2 + I_2 R_{2e} \cos \phi_2 + I_2 X_{2e} \sin \phi_2$$

Therefore, the voltage regulation for a lagging power-factor load is given by

$$\%R = \frac{E_2 - V_2}{V_2} = \frac{I_2 R_{2e} \cos \phi_2 + I_2 X_{2e} \sin \phi_2}{V_2} \tag{2.16}$$

2.15.2 Leading Power Factor

The phasor diagram of secondary side of the transformer for a leading power factor load with reference to V_2 is shown in Figure 2.28.

Using right-angled triangle OAE, we get

$$OA^2 = OE^2 + EA^2$$

Since the phase angle between V_2 and E_2 is practically very small, $EA = 0$.

Therefore, $OA^2 = OE^2$

From Figure 2.28, we get

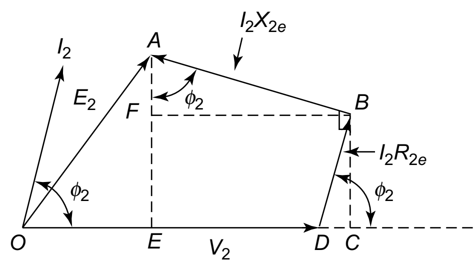


Figure 2.28 Phasor Diagram of Secondary Side for Leading Power Factor

$$OA = E_2,\ OE = OD + DC - EC,\ OD = V_2 \text{ and } EC = BF$$

Hence, $$E_2^2 = (V_2 + DC - BF)^2$$

i.e., $$E_2 = V_2 + DC - BF \tag{2.17}$$

Using right angle triangle *BCD*, we get

$$DC = DB \cos \phi_2 = I_2 R_{2e} \cos \phi_2 \tag{2.18}$$

Similarly, using right angle triangle *ABF*, we get

$$BF = BA \sin \phi_2 = I_2 X_{2e} \sin \phi_2 \tag{2.19}$$

Substituting Eqn. (2.18) and Eqn. (2.19) in Eqn. (2.17), we get

$$E_2 = V_2 + I_2 R_{2e} \cos \phi_2 - I_2 X_{2e} \sin \phi_2$$

Therefore, the voltage regulation for a leading power factor load is given by

$$\%R = \frac{E_2 - V_2}{V_2} = \frac{I_2 R_{2e} \cos \phi_2 - I_2 X_{2e} \sin \phi_2}{V_2}$$

Hence, the generalized expression for voltage regulation of the transformer is

$$\%R = \frac{I_2 R_{2e} \cos \phi_2 \pm I_2 X_{2e} \sin \phi_2}{V_2}$$

Similarly, when referred to primary side, we get

$$\%R = \frac{I_1 R_{1e} \cos \phi_1 \pm I_1 X_{1e} \sin \phi_1}{V_1}$$

where, the positive sign (+) is for lagging power factor and the negative sign (–) is for leading power factor.

2.15.3 Maximum Voltage Regulation

The condition at which the voltage regulation of the transformer is maximum, is determined as follows:

Differentiating Eqn. (2.16) with respect to ϕ_2 and equating to zero, we get

$$\frac{dR}{d\phi_2} = \frac{-I_2 R_{2e} \sin \phi_2 + I_2 X_{2e} \cos \phi_2}{V_2} = 0$$

i.e.,

$$\tan \phi_2 = \frac{X_{2e}}{R_{2e}} \text{ or } \phi_2 = \tan^{-1}\left(\frac{X_{2e}}{R_{2e}}\right)$$

Therefore, when the phase angle, $\phi_2 = \tan^{-1}\left(\frac{X_{2e}}{R_{2e}}\right)$, the voltage regulation of the transformer will be maximum. Also, the power factor at which the voltage regulation is maximum, is given by

$$\cos \phi_2 = \frac{R_{2e}}{Z_{2e}} = \frac{R_{2e}}{\sqrt{R_{2e}^2 + X_{2e}^2}}$$

2.15.4 Zero Voltage Regulation

The condition at which the voltage regulation of the transformer is zero is determined as follows:

Equating the voltage regulation of the leading power factor to zero, we get

$$\frac{I_2 R_{2e}\cos\phi_2 - I_2 X_{2e}\sin\phi_2}{V_2} = 0$$

i.e.,
$$\tan\phi_2 = \frac{R_{2e}}{X_{2e}} \text{ or } \phi_2 = \tan^{-1}\left(\frac{R_{2e}}{X_{2e}}\right)$$

Therefore, when the phase angle, $\phi_2 = \tan^{-1}\left(\frac{R_{2e}}{X_{2e}}\right)$, the voltage regulation of the transformer will be zero.

2.16 Name Plate Rating of a Transformer

The iron and copper losses depend only on the supply voltage and the current flowing through the winding respectively. Since these losses do not depend on the phase angle between the supply voltage and the current, the transformer rating expressed as a product of voltage and current is called VA rating of the transformer. Therefore, the transformer rating is expressed as kVA and not as kW. Depending on the size of the transformer, it can carry a maximum current called full-load current, I_{FL}, without overheating. Therefore, the transformer rating is given by:

$$\text{kVA} = \frac{V_{\text{rated}} \times I_{\text{FL}}}{1000}$$

The per unit impedance of a transformer, by assuming the base value of voltage and current as V_{rated} and V_{FL} respectively, is given by:

$$Z_{\text{pu}} = \frac{I_{\text{FL}} \times Z_{\text{actual}}}{V_{\text{rated}}}$$

The percentage impedance is given by:

$$\begin{aligned}\%Z &= Z_{\text{pu}} \times 100 \\ &= \frac{I_{\text{FL}} \times Z_{\text{actual}}}{V_{\text{rated}}} \times 100\end{aligned}$$

The above equation represents the percentage voltage drop under full-load condition.

Example 2.8

A 230/440 V transformer has a primary resistance and reactance of 0.25 Ω and 0.6 Ω respectively. The corresponding secondary values are 0.8 Ω and 1.8 Ω respectively. Determine the approximate secondary terminal voltage when supplying 10A at the power factor of 0.707 lagging. **[AU Nov/Dec, 2014]**

Solution

Given, $R_1 = 0.25\ \Omega$, $X_1 = 0.6\ \Omega$, $R_2 = 0.8\ \Omega$, $X_2 = 1.8\ \Omega$, $E_1 = 230$ V, $E_2 = 440$ V, $I_2 = 10$ A and $\cos\phi_2 = 0.707$.

The transformation ratio is given by:

$$K = \frac{E_2}{E_1} = \frac{440}{230} = 1.913$$

The total resistance and reactance when referred to secondary are given by:

$$R_{2e} = R_2 + K^2R_1 = 0.8 + (1.913)^2 \times 0.25 = 1.7148\ \Omega$$
$$X_{2e} = X_2 + K^2X_1 = 1.8 + (1.913)^2 \times 0.6 = 3.9957\ \Omega$$

Therefore, the approximate terminal voltage is

$$V_2 = E_2 - I_2\,[R_{2e} \cos \phi_2 + X_{2e} \sin \phi_2]$$

Here, $\cos \phi_2 = 0.707$, i.e., $\phi_2 = \cos^{-1}(0.707) = 45°$. Hence, $\sin \phi_2 = 0.707$.
Substituting the known values, we get

$$V_2 = 440 - 10 \times [1.7148 \times 0.707 + 3.9959 \times 0.707]$$
$$= 399.626\ \text{V}$$

Example 2.9

A single-phase transformer on full load has an impedance drop of 20 V and resistance drop of 10 V. Calculate the value of power factor when its regulation will be zero. **[AU April/May, 2013]**

Solution

Given, $V_R = I_2R_{2e} = 10$ V and $V_Z = I_2Z_{2e} = 20$ V

Therefore, $I_2X_{2e} = V_X = \sqrt{V_Z^2 - V_R^2} = \sqrt{20^2 - 10^2} = 17.3205$ V

If the voltage regulation is zero, we get

$$E_2 = V_2$$

Since the power factor is leading at zero voltage regulation, we get

$$\%R = \frac{I_2R_{2e} \cos \phi_2 - I_2X_{2e} \sin \phi_2}{V_2} \times 100 = 0$$

i.e., $I_2R_{2e} \cos \phi_2 - I_2X_{2e} \sin \phi_2 = 0$

or $V_R \cos \phi_2 - V_X \sin \phi_2 = 0$

Therefore, $\tan \phi_2 = \dfrac{V_R}{V_X} = \dfrac{10}{17.3205} = 0.5773$

i.e., $\phi_2 = 30°$

Therefore, the leading power factor is given by

$$\cos \phi = \cos 30° = 0.866$$

Example 2.10

A single transformer is rated at 10 kVA, 50 Hz. The secondary rated voltage is 240 V and the turns ratio is 10. The resistance and leakage reactance as referred to primary are 8.4 Ω and 13.7 Ω respectively. Find the voltage regulation at full load and power factors of 0.8 lagging, 0.8 leading and unity.

[AU Nov/Dec, 2013]

Solution

Given, $V_2 = 240$ V, kVA rating of the transformer = 10 kVA, Turns ratio, $\dfrac{N_1}{N_2} = 10$, $R_{1e} = 8.4\ \Omega$ and $X_{1e} = 13.7\ \Omega$

The transformation ratio,

$$K = \frac{N_2}{N_1} = \frac{1}{10}$$

Therefore, $K = \dfrac{V_2}{V_1}$

Substituting the known values, we get, $\dfrac{1}{10} = \dfrac{240}{V_1}$

i.e., $V_1 = 2400$ V

The full-load current that the primary winding can draw from the supply is

$$(I_1)_{FL} = \frac{\text{kVA}}{V_1} = \frac{10 \times 10^3}{2400} = 4.1667 \text{ A}$$

The general formula to determine the voltage regulation of the transformer is

$$\%R = \frac{(I_1)_{FL} \times [R_{1e} \cos \phi_1 \pm X_{1e} \sin \phi_1]}{V_1} \times 100 \text{ [+ sign for lagging and – sign for leading]}$$

(i) $\cos \phi_1 = 0.8$ lagging, $\sin \phi_1 = 0.6$

$$\%R = \frac{4.1667 \times [(8.4 \times 0.8) + (13.7 \times 0.6)]}{2400} \times 100 = 2.593\%$$

(ii) $\cos \phi_1 = 0.8$ leading, $\sin \phi_1 = 0.6$

$$\%R = \frac{4.1667[(8.4 \times 0.8) - (13.7 \times 0.6)]}{2400} \times 100 = -0.2604\%$$

(iii) $\cos \phi_1 = 1$, $\sin \phi_1 = 0$

$$\%R = \frac{4.1667[(8.4 \times 1) - (0 \times 13.7)]}{2400} \times 100 = 1.4581\%$$

Example 2.11

The primary and secondary windings of 40 kVA, 6600/250 V, single-phase transformer have primary and secondary resistances of 10 Ω and 0.02 Ω respectively. The leakage reactance of the transformer referred to the primary is 35 Ω. Calculate the full-load regulation at unity power factor and 0.8 lagging power factor. Neglect the no-load current. **[AU Nov/Dec, 2011]**

Solution

Given, $R_1 = 10\ \Omega$, $R_2 = 0.02\ \Omega$, $X_{1e} = 35\ \Omega$, Rating of the transformer = 40 kVA, $V_1 = 6600$ V and $V_2 = 250$ V

The transformation ratio is

$$K = \frac{V_2}{V_1} = 0.03787$$

The total resistance when referred to the primary side of the transformer is given by

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 10 + \frac{(0.02)^2}{(0.03787)^2} = 10.2789\ \Omega$$

The full-load current in the primary winding is

$$(I_1)_{FL} = \frac{\text{kVA}}{V_1} = \frac{40 \times 10^3}{6600} = 6.0606 \text{ A}$$

(i) $\cos \phi_1 = 1$, $\sin \phi_1 = 0$

$$\%R = \frac{(I_1)_{FL}[R_{1e} \cos \phi_1 \pm X_{1e} \sin \phi_1]}{V_1} \times 100$$

$$= \frac{6.0606 \times [(10.2789 \times 1) - (35 \times 0)]}{6600} \times 100 = 0.943\%$$

(ii) $\cos \phi_1 = 0.8$ lagging, $\sin \phi_1 = 0.6$

$$\%R = \frac{(I_1)_{FL} \times [R_{1e} \cos \phi_1 + X_{1e} \sin \phi_1]}{V_1} \times 100$$

$$= \frac{6.0606 \times [(10.2789 \times 0.8) + (35 \times 0.6)]}{6600} \times 100 = 2.683\%$$

2.17 Losses in a Transformer **[AU April/May, 2016]**

The various kinds of losses that exist in a transformer are: (i) core or iron losses (ii) ohmic or copper losses (iii) stray losses and (iv) dielectric losses. The classification of losses in a transformer is shown in Figure 2.29.

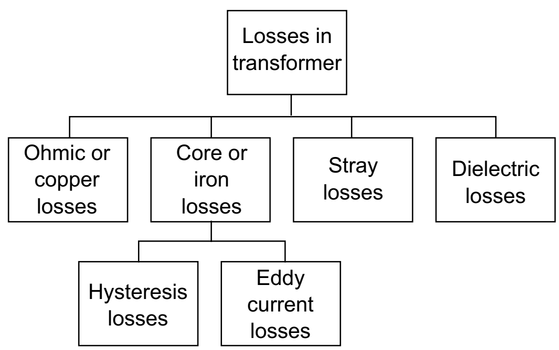


Figure 2.29 Losses in a Transformer

2.17.1 Core or Iron Losses

The losses which take place in the transformer due to the alternating magnetic flux produced in the magnetic core is called core or iron losses, P_i. These losses are further classified as hysteresis and eddy current losses.

Hysteresis Losses, P_h

Since there is generation of alternating flux in the core of the transformer, magnetisation and demagnetisation of the core takes place in an alternate manner. For each cycle of alternating flux, a hysteresis loop is obtained. Due to the effect of this hysteresis, loss of energy in the form of heat occurs in the transformer, which is called hysteresis loss and is given by

$$P_h = K_h B_m^{1.67} f v \tag{2.20}$$

where, K_h is the hysteresis constant, which varies for each material used in the transformer, B_m is the maximum flux density in Weber per square meter, f is the frequency in Hertz and v is the volume of the core in cubic metre.

Eddy Current Losses, P_e

An induced emf is generated in the core when the alternating magnetic flux links with a closed circuit. Due to this induced emf, a current called eddy current circulates within the core. Its magnitude depends on the induced emf and the winding resistance of the transformer. The I^2R losses, which occur in the transformer due to this eddy current, are called eddy current losses and are given by

$$P_e = K_e B_m^2 f^2 t^2 v \tag{2.21}$$

where, K_e is the eddy current constant and t is the thickness of the core in meter.

Since the transformer is a constant flux machine for a given supply voltage V_1 with constant frequency f, the flux density, B_m is also constant. Hence, it is clear that both the hysteresis and eddy current losses are constant for a given transformer. Therefore, the core or iron losses are also called constant losses. These constant losses can be minimized by using high-grade silicon steel material, which possesses a low hysteresis-loop with thin laminations for core construction of the transformer.

2.17.2 Ohmic or Copper Losses

The losses that occur in the transformer due to its winding resistances are called copper losses, P_c. Therefore, the total copper loss in the transformer is given by

$$P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{1e} = I_2^2 R_{2e}$$

Since the currents flowing through the windings vary with respect to the load, these losses are called variable losses.

2.17.3 Stray Losses

The losses that occur in the transformer due to the leakage of magnetic flux are called stray losses. Since the percentage of stray losses is very less when compared to iron and copper losses, they can be neglected.

2.17.4 Dielectric Loss

The losses that occur in the insulating material of the transformer are called dielectric losses, which affect the efficiency of the transformer. Similar to stray loss, the dielectric loss can be eliminated as its percentage is very small, compared to that of iron and copper losses. Therefore, the total loss in the transformer, P_T is

$$P_T = P_i + P_c$$

2.18 Efficiency of the Transformer **[AU Nov/Dec, 2016]**

Due to iron and copper losses in the transformer, the power output of the transformer is less when compared to the power supplied to it. Therefore,

Power output = Power input – Total losses

or Power input = Power output + Total losses

In general, the efficiency of the transformer is defined as the ratio of the power output to power input and is given by:

$$\eta = \frac{\text{Power output}}{\text{Power intput}}$$

i.e.,
$$\eta = \frac{\text{Power output}}{\text{Power output} + \text{Total losses}} = \frac{\text{Power output}}{\text{Power output} + P_i + P_c}$$

The power output of the transformer is given by

$$\text{Power output} = V_2 I_2 \cos \phi_2$$

where, $\cos \phi_2$ is the power factor of the load.

If the transformer supplies a load current I_2 with terminal voltage V_2, we get

$$P_c = I_2^2 R_{2e}$$

Therefore,
$$\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + I_2^2 R_{2e}}$$

If the VA rating of the transformer is $V_2 I_2$, then

$$\eta = \frac{(\text{VA rating}) \cos \phi_2}{(\text{VA rating}) \cos \phi_2 + P_i + I_2^2 R_{2e}}$$

The above expression for efficiency is for the full-load condition i.e., when a full-load current I_2 flows through the windings.

Similarly, the expression for efficiency of the transformer when it is subjected to different fractional loads can be obtained as follows:

Let $n = \dfrac{\text{Actual load}}{\text{Full load}}$. If the load connected to the transformer changes, the load current varies proportionately as

$$I_2 = n(I_2)_{\text{FL}}$$

where, $(I_2)_{\text{FL}}$ is the load current at load condition.

Similarly, the VA rating of the transformer and copper loss depending on the current also varies as

$$\text{VA rating} = n \times (\text{VA rating})_{\text{FL}}$$

and
$$P_c = n^2 (P_c)_{\text{FL}}$$

Therefore, the general expression for efficiency of the transformer is

$$\%\eta = \frac{n \times (\text{VA rating})_{\text{FL}} \cos \phi_2}{n \times (\text{VA rating})_{\text{FL}} \cos \phi_2 + P_i + n^2 (P_c)_{\text{FL}}} \times 100$$

2.18.1 Condition for Maximum Efficiency

As the load connected to the transformer increases from the no-load condition, the load current also increases, thereby increasing the efficiency of the transformer. The efficiency of the transformer attains a maximum value at a particular load current. If the load current, I_2, is further increased beyond the maximum value I_{2m}, the efficiency of the transformer starts decreasing. The plot between the efficiency and the load current is shown in Figure 2.30.

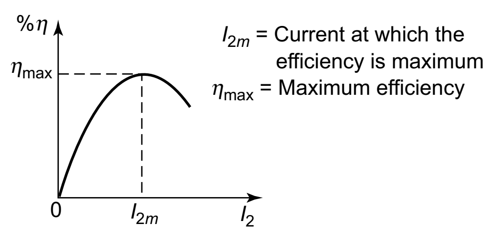


Figure 2.30 Efficiency vs. Load Current

Here, I_{2m} is the load current, at which the efficiency of the transformer is maximum, η_{max}. Assuming the terminal voltage of the transformer as constant, the only varying quantity in the efficiency equation is the load current, I_2. Therefore, the condition to obtain maximum efficiency is obtained as follows:

$$\frac{d\eta}{dI_2} = 0$$

i.e.,
$$\frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}\right] = 0$$

$$(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) \times (V_2 \cos\phi_2) - (V_2 I_2 \cos\phi_2) \times (V_2 \cos\phi_2 + 2 I_2 R_{2e}) = 0$$

Dividing the above equation by $V_2 \cos\phi_2$, we get

$$(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) - (V_2 I_2 \cos\phi_2 + 2 I_2^2 R_{2e}) = 0$$

i.e.,
$$P_i = I_2^2 R_{2e} = P_c$$

Therefore, the condition to achieve maximum efficiency in the transformer is that the copper loss must be equal to the iron loss, i.e., $P_c = P_i$.

Load Current at Maximum Efficiency, I_{2m}

If I_{2m} is the load current at maximum efficiency, then using the condition for maximum efficiency, we get

$$I_{2m}^2 R_{2e} = P_i$$

Therefore,
$$I_{2m} = \sqrt{\frac{P_i}{R_{2e}}}$$

Similarly, the load current at maximum efficiency, I_{2m}, can be obtained in terms of the full-load current, $(I_2)_{FL}$. Taking ratio of I_{2m} to $(I_2)_{FL}$, we get

$$\frac{I_{2m}}{(I_2)_{FL}} = \frac{1}{(I_2)_{FL}}\sqrt{\frac{P_i}{R_{2e}}}$$

i.e.,
$$\frac{I_{2m}}{(I_2)_{FL}} = \sqrt{\frac{P_i}{(I_2)_{FL}^2 R_{2e}}}$$

Since $(I_2)_{FL}^2 R_{2e} = (P_c)_{FL}$, we get

$$I_{2m} = (I_2)_{FL}\sqrt{\frac{P_i}{(P_c)_{FL}}} \tag{2.22}$$

The above equation shows the relation between the load-current at maximum efficiency, I_{2m} and full-load current, $(I_2)_{FL}$.

kVA Supplied at Maximum Efficiency

The kVA rating of the transformer at maximum efficiency is given by:

$$\text{kVA at } \eta_{max} = I_{2m} V_2$$

Substituting Eqn. (2.22) in the above equation, we get

$$\text{kVA at } \eta_{\max} = V_2 (I_2)_{\text{FL}} \times \sqrt{\frac{P_i}{(P_c)_{\text{FL}}}}$$

Therefore,
$$\text{kVA at } \eta_{\max} = \text{kVA rating} \times \sqrt{\frac{P_i}{(P_c)_{\text{FL}}}}$$

Also, the maximum efficiency of the transformer is

$$\%\eta_{\max} = \frac{V_2 I_{2m} \cos \phi_2}{V_2 I_{2m} \cos \phi_2 + 2P_i} \times 100$$

or
$$\%\eta_{\max} = \frac{\text{kVA for } \eta_{\max} \times \cos \phi_2}{(\text{kVA for } \eta_{\max} \times \cos \phi_2) + 2P_i} \times 100$$

Effect of Power Factor on Efficiency

The efficiency of the transformer is given by:

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$= \frac{\text{Input power} - \text{losses}}{\text{Input power}} = 1 - \frac{\text{Losses}}{\text{Input power}}$$

Here, input power = Output power + Losses = $V_2 I_2 \cos \phi_2$ + losses

Therefore,
$$\eta = 1 - \frac{\text{Losses}}{V_2 I_2 \cos \phi_2 + \text{Losses}}$$

i.e.,
$$\eta = 1 - \frac{\dfrac{\text{Losses}}{V_2 I_2}}{\cos \phi_2 + \dfrac{\text{Losses}}{V_2 I_2}}$$

If
$$\frac{\text{Losses}}{V_2 I_2} = x, \text{ then } \eta = 1 - \frac{x}{\cos \phi_2 + x}$$

i.e.,
$$\eta = 1 - \frac{\dfrac{x}{\cos \phi_2}}{1 + \dfrac{x}{\cos \phi_2}} \tag{2.23}$$

Therefore, from Eqn. (2.23), it is clear that when the power factor of the load increases, the term $\dfrac{x}{\cos \phi_2}$ decreases, thereby increasing the efficiency of the transformer. The graphical representation of the efficiency of the system versus full-load current for different power factors is shown in Figure 2.31.

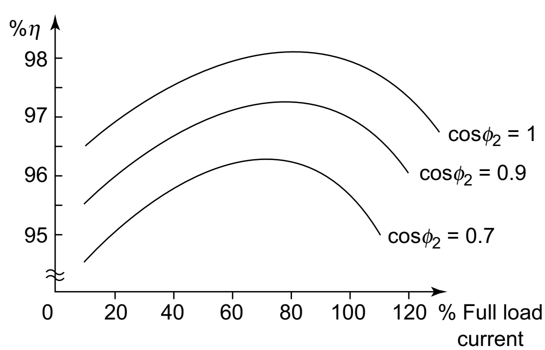


Figure 2.31 Efficiency for Different Power Factors

2.19 All-Day Efficiency **[AU Nov/Dec, 2011]**

The normal efficiency of the transformer is given by the ratio of output power to input power, which is not the true measure of the performance of distribution-transformers that serve residential and commercial loads. The true measure of distribution-transformer performance is given by all-day efficiency, which is the ratio of total energy output in kWh to total energy input in kWh.

Therefore, $$\% \text{ All day } \eta = \frac{\text{Output energy in kWh}}{\text{Input energy in kWh}} \times 100$$

Based on this all-day efficiency, the performance of various transformers is compared.

Example 2.12

A 600 kVA single-phase transformer has an efficiency of 94% both at full load and half load at unity power factor. Determine the efficiency at 75% of full load, at 0.9 power factor. **[AU April/May, 2015]**

Solution

Given, rating of the transformer = 600 kVA, $\cos \phi_2 = 1$, $\eta_{FL} = \eta_{HL} = 94\%$

The full-load efficiency of the system is

$$\%\eta_{FL} = \frac{\text{kVA} \cos \phi_2}{\text{kVA} \cos \phi_2 + P_i + (P_c)_{FL}} \times 100$$

Substituting the known values, we get

$$94 = \frac{(600 \times 10^3 \times 1)}{(600 \times 10^3 \times 1) + P_i + (P_c)_{FL}} \times 100$$

Therefore, $$P_i + (P_c)_{FL} = 38297.87 \qquad (1)$$

It is known that, $n = \frac{\text{Given load}}{\text{Full load}}$. Hence, for half load, $n = 0.5$.

Therefore, for half load,

$$\%\eta_{HL} = \frac{n \times \text{kVA} \cos \phi_2}{n \times \text{kVA} \cos \phi_2 + P_i + n^2 \times (P_c)_{FL}} \times 100$$

Substituting the known values, we get

$$94 = \frac{0.5 \times 600 \times 10^3 \times 1}{0.5 \times 600 \times 10^3 \times 1 + P_i + (0.5)^2 (P_c)_{FL}} \times 100$$

Therefore, $$P_i + 0.25(P_c)_{FL} = 19148.93 \qquad (2)$$

Solving Eqn.(1) and Eqn.(2), we get

$$(P_c)_{FL} = 25531.91 \text{ W and } P_i = 12765.95 \text{ W}$$

Therefore, the efficiency at 75% full load, i.e., when n = 0.75 and cos ϕ_2 = 0.9 is

$$\%\eta = \frac{n \times \text{kVA} \cos\phi_2}{n \times \text{kVA} \cos\phi_2 + P_i + n^2 \times (P_c)_{\text{FL}}} \times 100$$

$$= \frac{0.75 \times 600 \times 10^3 \times 0.9}{(0.75 \times 600 \times 10^3 \times 0.9) + 12765.97 + ((0.75)^2 \times 25531.91)} \times 100 = 93.72\%$$

Example 2.13

A transformer working at unity power factor has an efficiency of 90% both at full load and half load. The full-load output is 0.5 kVA. Determine the efficiency at 75% of full load. **[AU Nov/Dec, 2011]**

Solution

Given, rating of the transformer = 0.5 kVA, cos ϕ_2 = 1, $\eta_{\text{FL}} = \eta_{\text{HL}}$ = 90%

The full-load efficiency of the system is

$$\%\eta_{\text{FL}} = \frac{\text{kVA} \cos\phi_2}{\text{kVA} \cos\phi_2 + P_i + (P_c)_{\text{FL}}} \times 100$$

Substituting the known values, we get

$$90 = \frac{(0.5 \times 10^3 \times 1)}{(0.5 \times 10^3 \times 1) + P_i + (P_c)_{\text{FL}}} \times 100$$

Therefore, $P_i + (P_c)_{\text{FL}} = 55.55$ (1)

It is known that, $n = \dfrac{\text{Given load}}{\text{Full load}}$. Hence, for half load, n = 0.5.

Therefore, for half load,

$$\%\eta_{\text{HL}} = \frac{n \times \text{kVA} \cos\phi_2}{n \times \text{kVA} \cos\phi_2 + P_i + n^2 \times (P_c)_{\text{FL}}} \times 100$$

Substituting the known values, we get

$$90 = \frac{0.5 \times 0.5 \times 10^3 \times 1}{0.5 \times 0.5 \times 10^3 \times 1 + P_i + (0.5)^2 (P_c)_{\text{FL}}} \times 100$$

Therefore, $P_i + 0.25(P_c)_{\text{FL}} = 27.77$ (2)

Solving Eqn.(1) and Eqn.(2), we get

$(P_c)_{\text{FL}} = 37.03$ W and $P_i = 18.51$ W

Therefore, the efficiency at 75% full load, i.e., when n = 0.75 and cos ϕ = 1 is

$$\%\eta = \frac{n \times \text{kVA} \cos\phi_2}{n \times \text{kVA} \cos\phi_2 + P_i + n^2 \times (P_c)_{\text{FL}}} \times 100$$

$$= \frac{0.75 \times 0.5 \times 10^3 \times 1}{(0.75 \times 0.5 \times 10^3 \times 1) + 18.51 + ((0.75)^2 \times 37.03)} \times 100 = 90.50\%$$

Example 2.14

A 5 kVA, 2300/230 V, 50 Hz transformer was tested for the iron losses with normal excitation and copper losses at full load and these were found to be 40 W and 112 W respectively. Calculate the efficiencies of the transformer at 0.8 power factor and also plot the curve for efficiency vs. kVA output.

[AU April/May, 2012]

Solution

Rating of the transformer = 5 kVA, P_i = 40 W, $(P_c)_{FL}$ = 112 W, $\cos\phi_2$ = 0.8

Sl. No.	kVA output	n = Fraction of full load	New $P_c = n^2(P_c)_{FL}$	$\%\eta = \dfrac{n\text{kVA}\cos\phi_2}{n\text{kVA}\cos\phi_2 + P_i + \text{new } P_{cu}} \times 100$
1	1.25	0.25	7	95.51 %
2	2.5	0.5	28	96.71 %
3	3.75	0.75	63	96.96 %
4	5	1	112	96.33 %
5	6.25	1.25	175	95.87 %
6	7.5	1.5	252	95.35 %

The efficiency against kVA output curve is shown in Figure E2.14.

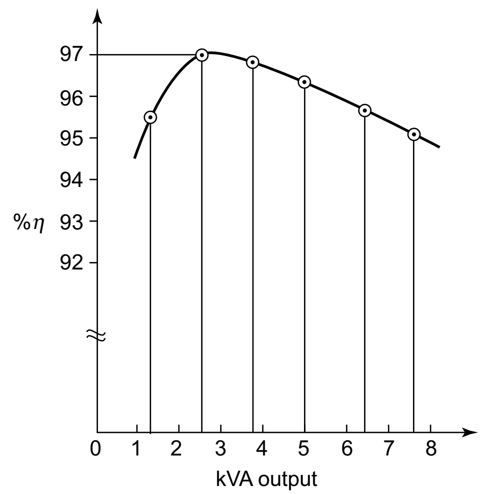


Figure E2.14

Example 2.15

A 1 kVA, 1000/200 V, single-phase transformer has an iron loss of 20 W and full-load copper loss of 50 W. Calculate the efficiency of the transformer at full load and 0.8 power factor lagging. Also, calculate the maximum efficiency of the transformer at the same power factor. **[AU Nov/Dec, 2011]**

Solution

Given P_i = 20 W, $(P_c)_{FL}$ = 50 W, $\cos\phi_2$ = 0.8 and rating of the transformer = 1 kVA

The full load efficiency of the transformer is

$$\%\eta_{FL} = \frac{\text{kVA} \times \cos\phi_2}{\text{kVA} \times \cos\phi_2 + P_i + (P_c)_{FL}} \times 100$$

$$= \frac{1 \times 10^3 \times 0.8}{(1 \times 10^3 \times 0.8) + 20 + 50} \times 100 = 91.954\%$$

The kVA rating of the transformer at maximum efficiency is

$$(\text{kVA})_{\eta_{max}} = \text{kVA} \times \sqrt{\frac{P_i}{(P_c)_{FL}}} = 1 \times \sqrt{\frac{20}{50}} = 0.6324\ \text{kVA}$$

At maximum efficiency, we get

$$(P_c)_{FL} = P_i = 20 \text{ W}$$

Therefore, the maximum efficiency at $\cos\phi_2 = 0.8$ is

$$\%\eta_{max} = \frac{\text{kVA for } \eta_{max} \times \cos\phi_2}{\text{kVA for } \eta_{max} \times \cos\phi_2 + 2P_i} \times 100$$

$$= \frac{0.6324 \times 10^3 \times 0.8}{0.6324 \times 10^3 \times 0.8 + (2 \times 20)} \times 100 = 92.673\%$$

Example 2.16

The maximum efficiency of a single-phase 11000/400 V, 550 kVA transformer is 97.5% and occurs at 80% full-load unity power factor. The impedance drop is 3.5% and the load power-factor is varied while the load current and the supply voltage are held constant at their rated values. Determine the load power-factor at which the secondary terminal voltage is minimum and find the value of the latter. **[AU Nov/Dec, 2013]**

Solution

Given, rating of the transformer = 550 kVA, η_{max} = 97.5% at 80% of full load, $\cos\phi_2 = 1$

The kVA for η_{max} is = 0.8 × 550 = 440 kVA

Therefore,
$$\%\eta_{max} = \frac{\text{kVA for } \eta_{max} \times \cos\phi_2}{\text{kVA for } \eta_{max} \times \cos\phi_2 + 2P_i} \times 100$$

Substituting the given values, we get

$$0.975 = \frac{440 \times 10^3 \times 1}{440 \times 10^3 \times 1 + 2P_i}$$

Upon solving, we get

$$P_i = 5641.02 \text{ W}$$

It is known that, at η_{max}, $P_c = P_i$. Therefore,

$$P_c|_{80\%\text{ FL}} = 5641.02 \text{ W } = (0.8)^2\,(P_c)_{FL}$$

i.e.,
$$(P_c)_{FL} = \frac{5641.02}{0.64} = 8814.10 \text{ W}$$

The per unit resistance drop is given by:

$$V_R = \frac{I_2 R_{2e}}{V_2} = \frac{I_2^2 R_{2e}}{V_2 I_2}$$

$$= \frac{(P_c)_{FL}}{(\text{VA})_{FL}} = \frac{8814.1025}{550 \times 10^3} = 0.01602 = 1.602\%$$

The per unit impedance drop, $V_Z = 3.5\%$

Therefore, the per unit reactance drop is given by

$$V_X = \sqrt{V_Z^2 - V_R^2} = \sqrt{(0.035)^2 - (0.01602)^2} = 0.0311 = 3.11\%$$

The power factor of the transformer at which terminal voltage is minimum is given by

$$\cos\phi_2 = \frac{R_{2e}}{Z_{2e}} = \frac{V_R}{V_Z}$$

Therefore, $$\cos\phi_2 = \frac{0.01602}{0.035} = 0.4577 \text{ (lagging)}$$

The voltage regulation is maximum, at which terminal voltage is minimum, and is given by

$$(\%R)_{\max} = [V_R \cos\phi_2 + V_X \sin\phi_2] \times 100$$

Substituting the given values, we get

$$(\%R)_{\max} = [0.01602 \times 0.4577 + 0.0311 \times 0.889] \times 100 = 3.5\%$$

Also, the voltage regulation is given by:

$$\%R = \frac{E_2 - V_2}{V_2} \times 100$$

i.e., $$3.5 = \frac{400 - V_2}{V_2} \times 100$$

Therefore, $V_2 = 386.47$ V

Example 2.17

A 400 kVA transformer has a primary winding resistance of 0.5 Ω and a secondary winding resistance of 0.001 Ω. The iron loss is 2.5 kW and the primary and secondary voltages are 5 kV and 320 V respectively. If the power factor of the load is 0.85, determine the efficiency of the transformer on: (i) full load and (ii) half load. **[AU April/May, 2016]**

Solution

Given $V_1 = 5$ kV, $V_2 = 320$ V, $R_1 = 0.5\ \Omega$, $R_2 = 0.001\ \Omega$, $P_i = 2.5$ kW, $\cos\phi_2 = 0.85$ and rating of the transformer = 400 kVA.

The transformation ratio is given by:

$$K = \frac{V_2}{V_1} = \frac{320}{5000} = 0.064$$

The total winding resistance, when referred to the primary side is given by

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 0.5 + 0.2441 = 0.7441\ \Omega$$

The full-load primary current is given by

$$(I_1)_{FL} = \frac{\text{kVA}}{V_1} = \frac{400 \times 10^3}{5 \times 10^3} = 80 \text{ A}$$

Therefore, $(P_c)_{FL} = (I_1)^2_{FL} \times R_{1e} = 80^2 \times 0.7441 = 4762.24$ W

(i) On full load, $P_i = 25$ kW, $\cos\phi_2 = 0.85$

$$\%\eta_{FL} = \frac{\text{kVA} \cos\phi_2}{\text{kVA} \cos\phi_2 + P_i + (P_c)_{FL}} \times 100$$

Substituting the known values, we get

$$\%\eta_{\text{FL}} = \frac{400 \times 10^3 \times 0.85}{400 \times 10^3 \times 0.85 + 2.5 \times 10^3 + 4762.24} \times 100 = 97.908\%$$

(ii) On half load, $n = 0.5$
Therefore, the efficiency of the system is:

$$\%\eta_{\text{HL}} = \frac{n\text{kVA} \cos \phi_2}{n\text{kVA} \cos \phi_2 + P_i + n^2 (P_c)_{\text{FL}}} \times 100$$

$$= \frac{0.5 \times 400 \times 10^3 \times 0.85}{(0.5 \times 400 \times 10^3 \times 0.85) + 2.5 \times 10^3 + ((0.5)^2 \times 4762.24)} \times 100 = 97.875\%$$

2.20 Determination of Parameters of Circuit Model of Transformer

[AU Nov/Dec, 2016]

There are some difficulties in direct loading of transformers to determine its parameters: (i) waste of energy is more and (ii) it is practically impossible to load large transformers. Therefore, the performance parameters of the transformer can be obtained by computing the circuit parameters, by conducing test on transformers using less power and without direct loading. This is called a *indirect loading* test, which requires a low power-supply to indicate the losses. Open-circuit and short-circuit tests are the two non-loading tests performed to obtain the transformer parameters. The different parameters obtained using these tests help in calculating the regulation and efficiency of a transformer at any load and power-factor conditions.

2.20.1 Open Circuit (OC) Test

The circuit diagram to conduct an open-circuit test is shown in Figure 2.32.

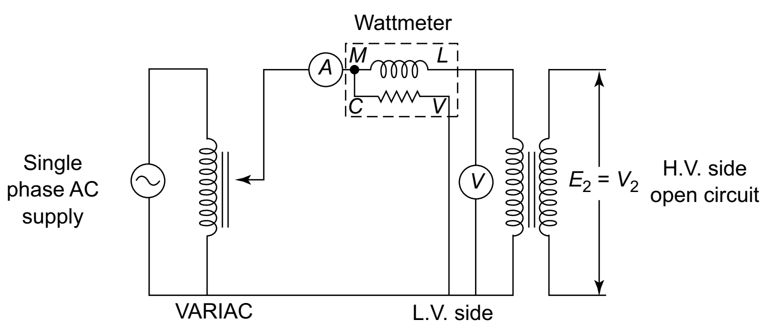


Figure 2.32 Open-circuit Test on a Transformer

Using ammeter, voltmeter and VARIAC, the primary winding of the transformer is connected to a single-phase source voltage. The secondary winding of the transformer is kept open. In general, to conduct this test, the primary side is considered as a low-voltage side and the secondary side as high-voltage side. Applying rated voltage excites the primary winding of the transformer. The rated voltage is obtained precisely using the VARIAC. The voltmeter, ammeter and wattmeter measure the rated primary voltage, input current and input power respectively. A voltmeter is used to measure the secondary voltage when the rated voltage is applied to the primary winding. The parameters observed in OC test are given in Table 2.2.

Table 2.2 *Observation Table of OC Test*

V_o in volts	I_o in amperes	W_o in watts

Since the secondary side is kept open, the ammeter connected in the primary side measures the no-load current, I_o. The two components of this current are:

$$I_m = I_o \sin \phi_o \text{ and } I_c = I_o \cos \phi_o$$

Here, $\cos \phi_o$ is the no-load power factor. Therefore, the input power is given by

$$W_o = V_o I_o \cos \phi_o$$

Since the secondary winding is kept open, $I_2 = 0$. Therefore, the additional current, which the primary winding draws from, the supply voltage is also zero i.e., $I_2 = 0$. Hence, for the circuit shown in Figure 2.32, the primary current is $I_1 = I_o$ which is very small when compared to the full load current. Since $I_2 = 0$, there is no copper loss in the secondary side of the transformer. Since the primary current drawn from the source is very small, the primary copper loss is also very small. Therefore, in OC test, the total copper loss is negligibly small. When the rated voltage is applied to the primary winding, the flux density in the core is at its maximum value, which contributes to the iron losses. Since the output power is zero and total copper loss is negligibly small, the input power contributes to the iron losses i.e., $W_o = P_i$.

Calculation from OC Test

The input power is $W_o = V_o I_o \cos \phi_o$

Using the voltmeter and ammeter readings, the no-load power factor is calculated as

$$\cos \phi_o = \frac{W_o}{V_o I_o}$$

Therefore, the two components of no-load current are determined using:

$$I_m = I_o \sin \phi_o \text{ and } I_c = I_o \cos \phi_o$$

The parameters of the exciting circuit are obtained using:

$$R_o = \frac{V_o}{I_c} \, \Omega$$

and

$$X_o = \frac{V_o}{I_m} \, \Omega$$

It is noted that the wattmeter used in this test must have a low power-factor, since $\cos \phi_o$ is very small. Also, if the voltmeter, ammeter and wattmeter are connected on the secondary side by keeping the primary side open, we get R'_o and X'_o. If the transformation ratio K is known, it is possible to obtain R_o and X_o.

2.20.2 Short Circuit (SC) Test

The circuit diagram to conduct short circuit test is shown in Figure 2.33.

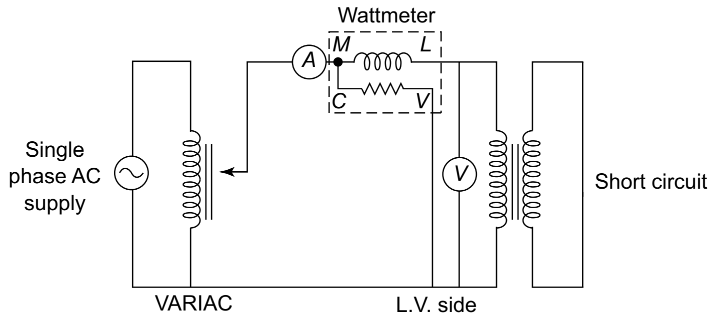


Figure 2.33 Short-circuit Test on a Transformer

Here, the secondary side is short-circuited using a thick copper wire. In short-circuit test, the high-voltage side is connected to the supply and low-voltage side is short-circuited. Due to very small resistance on the secondary side, large current flows through it, which damages the transformer due to overheating. Therefore, the primary side is supplied with a low voltage to limit the current flowing through the secondary side. A VARIAC is used to apply a low voltage to the primary side, which is sufficient for the rated current to flow through the primary side. This rated current is measured using an ammeter connected to the primary side. The readings the ammeter, voltmeter and wattmeter are tabulated, as given in Table 2.3.

Table 2.3 *Observation Table of SC Test*

V_{sc} in volts	I_{sc} in amperes	W_{sc} in watts

Since the rated current flows through the windings, the total copper loss is very large and is called full-load copper loss, $(P_c)_{FL}$. Here, the iron loss is negligible since only a small fraction of the rated voltage is applied to the primary side. Therefore, the wattmeter reading corresponds to the full-load copper loss, i.e., $W_{sc} = (P_c)_{FL}$.

Calculation from SC Test

Here, $W_{sc} = V_{sc} I_{sc} \cos \phi_{sc}$.

Therefore, the short-circuit power factor is obtained as

$$\cos \phi_{sc} = \frac{W_{sc}}{V_{sc} I_{sc}}$$

When all the parameters are referred to the primary side, the total copper loss is given by $I_{sc}^2 R_{1e}$.

Hence, $W_{sc} = I_{sc}^2 R_{1e}$

i.e., $$R_{1e} = \frac{W_{sc}}{I_{sc}^2}.$$

Also, the total impedance when referred to primary side is given by

$$Z_{1e} = \frac{V_{sc}}{I_{sc}}$$

Therefore, $X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$

If the transformation ratio, K, is known, the equivalent circuit parameters when referred to the secondary side can be obtained.

It is evident that in an SC test, the supply is given to the high-voltage side. Therefore, in a step-up transformer, using SC test, we obtain R_{2e}, Z_{2e} and X_{2e} because the supply is given to the secondary side when the primary side is short-circuited.

2.20.3 Efficiency using OC and SC Tests

The efficiency of the transformer on full load is given by

$$\%\eta = \frac{V_2(I_2)_{FL}\cos\phi_2}{V_2(I_2)_{FL}\cos\phi_2 + P_i + (P_c)_{FL}} \times 100$$

From OC and SC tests, we get

$$P_i = W_o \text{ and } (P_c)_{FL} = W_{sc}$$

Therefore, $$\%\eta = \frac{V_2(I_2)_{FL}\cos\phi_2}{V_2(I_2)_{FL}\cos\phi_2 + W_o + W_{sc}} \times 100$$

where, $\cos\phi_2$ is the load power-factor.

Similarly, for any fraction of the full load i.e., n, the efficiency is calculated as

$$\%\eta = \frac{nV_2(I_2)_{FL}\cos\phi_2}{nV_2(I_2)_{FL}\cos\phi_2 + W_o + n^2W_{sc}} \times 100$$

2.20.4 Regulation using SC Test

The voltage regulation of the transformer at full load is given by:

$$\%R = \frac{(I_2)_{FL}R_{2e}\cos\phi_2 \pm (I_2)_{FL}X_{2e}\sin\phi_2}{V_2} \times 100 \text{ (when primary side is short-circuited)}$$

or $$\%R = \frac{(I_1)_{FL}R_{1e}\cos\phi_1 \pm (I_1)_{FL}X_{1e}\sin\phi_1}{V_1} \times 100 \text{ (when secondary side is short-circuited)}$$

Similarly, for any fraction of the full load i.e., n, the voltage regulation is calculated as

$$\%R = \frac{n(I_2)_{FL}R_{2e}\cos\phi_2 \pm n(I_2)_{FL}X_{2e}\sin\phi_2}{V_2} \times 100$$

where, the positive sign (+) indicates lagging power factor and the negative sign (–) indicates leading power factor load.

Example 2.18

Calculate the full-load efficiency at a power factor of 0.8, and the voltage at the secondary terminals when supplying full-load secondary current at unity power factor, for a 4 kVA, 200/400 V, 50 Hz, single-phase transformer, of which the following are the test results:

OC test (on primary): $V = 200$ V; $I = 0.8$ A; $W = 50$ W

SC test (on secondary): $V = 17.5$ V; $I = 9$ A; $W = 50$ W **[AU April/May, 2011]**

Solution

Using OC test, we get

$$P_i = W|_{\text{OC}} = 50 \text{ W}$$

Since the SC test is conducted on the secondary side i.e., the primary side is short-circuited, the metre readings correspond to the secondary values.
Therefore,

$$Z_{2e} = \frac{V_{\text{sc}}}{I_{\text{sc}}} = 1.944\ \Omega,\ R_{2e} = \frac{W_{\text{sc}}}{I_{\text{sc}}^2} = 0.6173\ \Omega,\ X_{2e} = \sqrt{1.944^2 - 0.6173^2} = 1.8438\ \Omega$$

Also, $W|_{\text{SC}}$ = total copper loss at 9 A = 50 W. If the full-load current is 9 A, then $W|_{\text{SC}}$ corresponds to full-load copper loss i.e., $(P_c)_{\text{FL}}$.

The full-load secondary current is given by

$$(I_2)_{\text{FL}} = \frac{\text{VA}}{V_2} = \frac{4 \times 10^3}{400} = 10 \text{ A}$$

Therefore, the full-load copper loss is given by

$$(P_c)_{\text{FL}} = \left(\frac{(I_2)_{\text{FL}}}{I_{\text{sc}}}\right)^2 \times W|_{\text{SC}} = \left(\frac{10}{9}\right)^2 \times 50 = 61.7284 \text{ W}$$

(i) *Full-load efficiency at* $\cos \phi_2 = 0.8$

The efficiency of the transformer on full load is given by

$$\%\eta_{\text{FL}} = \frac{\text{VA} \cos \phi_2}{\text{VA} \cos \phi + P_i + (P_c)_{\text{FL}}} \times 100$$

Substituting the known values, we get

$$\%\eta_{\text{FL}} = \frac{4 \times 10^3 \times 0.8}{4 \times 10^3 \times 0.8 + 50 + 61.7284} \times 100 = 96.626\%$$

(ii) *Secondary voltage on full load at* $\cos \phi_2 = 1$.

The voltage drop on full load is given by

$$E_2 - V_2 = (I_2)_{\text{FL}} R_{2e} \cos \phi_2 + (I_2)_{\text{FL}} X_{2e} \sin \phi_2$$

Therefore, $E_2 - V_2 = (10 \times 0.6173 \times 1) + 0 = 6.173$ V

Hence, the terminal voltage at full load is given by

$$(V_2)_{\text{FL}} = 400 - \text{Voltage drop} = 400 - 6.173 = 393.827 \text{ V}$$

Example 2.19

Obtain the equivalent circuit of a 200/400 V, 50 Hz, single-phase transformer, of which, the test results are given below. **[AU April/May, 2013]**

OC Test: 200 V, 0.7 A, 70 W on low-voltage side
SC Test: 15 V, 10 A, 85 W on high-voltage side

Solution

From the OC test, we get

$$W_o = 70 \text{ W}, I_o = 0.7 \text{ A}, V_o = 200 \text{ V}$$

Therefore, the no-load power factor is given by

$$\cos\phi_o = \frac{W_o}{V_o I_o} = 0.5$$

The transformation ratio, $K = \dfrac{400}{200} = 2$

The two parts of no-load current are given by

$$I_c = I_o \cos\phi_o = 0.35 \text{ A}$$

and $$I_m = I_o \sin\phi_o = 0.6062 \text{ A}$$

Therefore, the exciting circuit parameters are

$$R_0 = \frac{V_1}{I_c} = \frac{200}{0.35} = 571.43\ \Omega$$

and $$X_o = \frac{V_1}{I_m} = \frac{200}{0.6062} = 329.924\ \Omega$$

From the SC test, we get

$$W_{sc} = 85 \text{ W}, I_{sc} = 10 \text{ A and } V_{sc} = 15 \text{ V}$$

Since the readings are obtained on the high-voltage side i.e., secondary side of the transformer, the circuit parameters obtained from SC test are referred to the secondary side.

Therefore, $$Z_{2e} = \frac{V_{sc}}{I_{sc}} = \frac{15}{10} = 1.5\ \Omega \text{ and } R_{2e} = \frac{W_{sc}}{(I_{sc})^2} = \frac{85}{(10)^2} = 0.85\ \Omega$$

Also, $$X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2} = 1.2359\ \Omega$$

The parameters referred to primary side are obtained as:

$$R_{1e} = \frac{R_{2e}}{K^2} = \frac{0.85}{4} = 0.2125\ \Omega, \quad X_{1e} = \frac{X_{2e}}{K^2} = \frac{1.2359}{4} = 0.3089\ \Omega$$

and $$Z_{2e} = \frac{Z_{1e}}{K^2} = \frac{1.5}{4} = 0.375\ \Omega$$

Therefore, the equivalent circuit of the transformer when referred to primary side is shown in Figure E2.19.

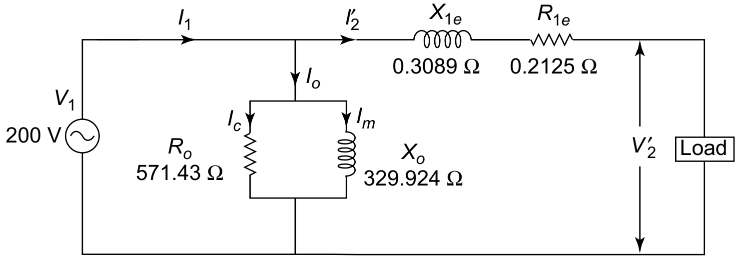


Figure E2.19

Example 2.20

A single-phase transformer with 25 kVA, 50 Hz, 2200/220 V has the following test results:
OC test: 220 V, 4.2 A 148 W on low-voltage side
SC test: 85 V, 10.5 A, 360 W on high voltage side.
Determine: (i) regulation and efficiency at power factor 0.8 lagging at full load, and (ii) power factor on short circuit. Also, obtain the approximate equivalent circuit referred to the high-voltage side.

[AU Nov/Dec, 2014]

Solution

The transformation ratio, $K = \dfrac{220}{2200} = 0.1$

From the OC test, we get

$$W_o = 148 \text{ W}, V_o = 220 \text{ V}, I_o = 4.2 \text{ A}$$

Therefore, the no-load power factor is

$$\cos\phi_o = \frac{W_o}{I_o V_o} = 0.1601$$

i.e., $\phi_o = 80.783°$ and $\sin\phi_o = 0.987$

The two components of no-load current are:

$$I_c = I_o \cos\phi_o = 0.6724 \text{ A and } I_m = I_o \sin\phi_o = 4.1454 \text{ A}$$

Since the OC test is conducted on the low-voltage side i.e., secondary side of the transformer, the exciting circuit parameters are the parameters referred to secondary side.

Therefore, the exciting circuit parameters referred to secondary side are given by:

$$R_o' = \frac{V_o}{I_c} = 327.186\ \Omega, \quad X_o' = \frac{V_o}{I_m} = 53.0708\ \Omega$$

Hence, the exciting circuit parameters referred to primary side i.e., high-voltage side are given by:

$$R_o = \frac{R_o'}{K^2} = 32.718\ \text{k}\Omega \quad \text{and} \quad X_o = \frac{X_o'}{K^2} = 5.307\ \text{k}\Omega,$$

From the SC test, we get:

$$V_{sc} = 85 \text{ V}, I_{sc} = 10.5 \text{ A and } W_{sc} = 360 \text{ W}$$

Since the SC test is conducted on high-voltage side, we get

$$Z_{1e} = \frac{V_{sc}}{I_{sc}} = 8.0952\ \Omega,\ R_{1e} = \frac{W_{sc}}{I_{sc}^2} = 3.265\ \Omega,\ X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2} = 7.4074\ \Omega$$

(i) The full-load rated current on the primary side is given by

$$(I_1)_{FL} = \frac{VA}{V_1} = \frac{25 \times 10^3}{2200} = 11.3636 \text{A}$$

Therefore, the full-load copper loss is given by

$$(P_c)_{FL} = \frac{360 \times 11.3636^2}{10.5^2} = 421.656 \text{W}$$

Iron loss = $P_i = W_o$ = 148 W

Therefore, the efficiency of the transformer is given by

$$\%\eta_{FL} = \frac{VA \cos\phi_2}{VA \cos\phi_2 + P_i + (P_c)_{FL}} \times 100$$

$$= \frac{25 \times 10^3 \times 0.8}{25 \times 10^3 \times 0.8 + 148 + 421.656} \times 100 = 97.23\%$$

$$\%R = \frac{[(I_1)_{FL} R_{1e} \cos\phi_1 + (I_1)_{FL} X_{1e} \sin\phi_1]}{V_1} \times 100$$

$$= \frac{11.3636[3.265 \times 0.8 + 7.407 \times 0.6]}{2200} \times 100 = 3.64\%$$

(ii) The power factor on short-circuit is given by

$$\cos\phi_{sc} = \frac{R_{1e}}{Z_{1e}} = 0.4033 \text{ lag}$$

The approximate equivalent circuit referred to high-voltage side is shown in Figure E2.20.

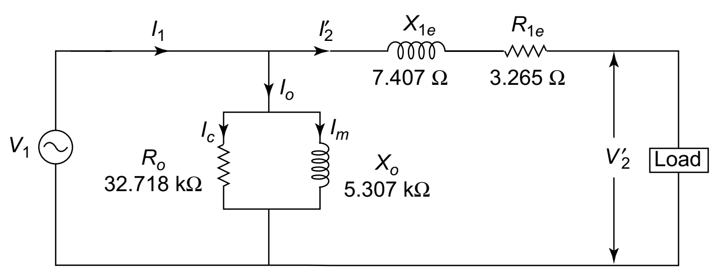


Figure E2.20

2.21 Auto-Transformer **[AU Nov/Dec, 2014]**

An auto-transformer is a special type of single-phase transformer, consisting of a single winding wound on a laminated core. Here, some part of this single winding acts as the primary winding and some part acts as the secondary winding. The number of turns of the primary and secondary windings can be varied using the switch contact. Since the output voltage of the auto-transformer can be varied by changing the number of turns of the windings, it is also known as VARIAC (VARIable AC) or voltage regulator. The principal difference between the single-phase transformer and the auto-transformer is based on the way the primary and secondary windings are interrelated. In a single-phase transformer, both primary and secondary windings are magnetically connected but electrically insulated, whereas, in an auto-transformer these windings are connected both magnetically and electrically. The schematic diagram of an auto-transformer is shown in Figure 2.34. In an auto-transformer, a continuous copper wire wound on a laminated silicon steel core is used for the windings. A single or multiple tappings or a smooth sliding brush over the winding helps in varying the turns ratio, which determines the secondary voltage of the auto-transformer. The schematic diagram of an auto-transformer with a sliding brush and with tappings are shown in Figures 2.34 (a) and (b) respectively.

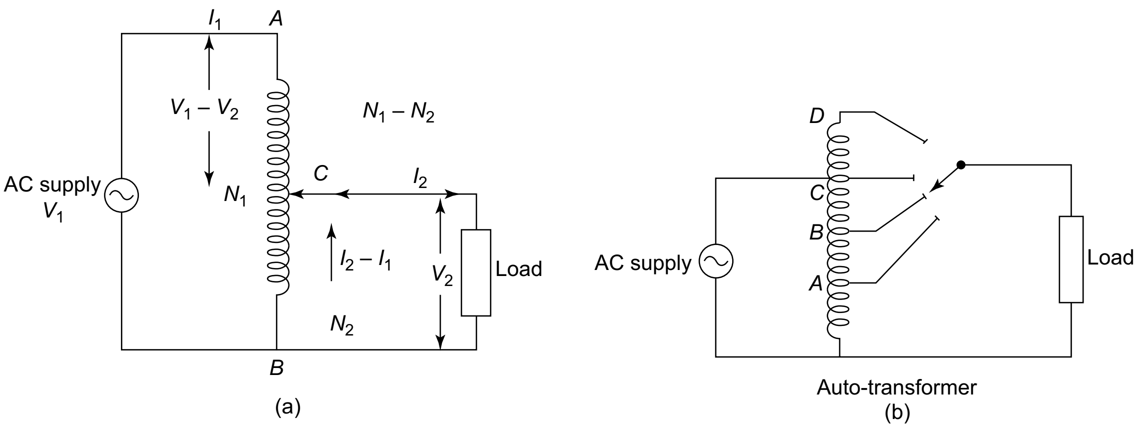


Figure 2.34 Schematic Diagram of an Auto-transformer (a) With Sliding Brush (b) With Tappings

2.21.1 Working of an Auto-transformer

The working of an auto-transformer is similar to that of a two-winding transformer, which can be seen in Figure 2.34(a). Here, AB is the primary winding of the transformer, to which the supply voltage is applied. Since a tapping is provided at point C, CB acts as a secondary winding to which the load is connected. When a supply voltage V_1 is applied to the primary winding AB, an alternating flux is set up in the core due to which an induced emf E_1 is produced in the primary winding AB. Since the secondary winding is a part of the primary winding, a part of the induced emf, E_1, is taken in it to which the load is connected. Let E_2 be the induced emf in the secondary winding. This induced emf drives the current in the secondary winding and hence the load connected to it.

Here, V_1 is the supply voltage applied to the primary winding, V_2 is the secondary voltage obtained across the load, I_1 is the primary current flowing through the primary winding AB, I_2 is the load current flowing through the load, N_1 is the number of turns in the primary winding i.e., between points A and B, and N_2 is the number of turns in the secondary winding i.e., between C and B. When there is no load, leakage reactance and losses are neglected. In this case, we have

$$E_1 = V_1 \text{ and } V_2 = E_2$$

Therefore, the transformation ratio K is given by

$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} \quad (2.24)$$

Similar to a two-winding transformer, the auto-transformer is classified as step-down and step-up auto-transformers, as shown in Figure 2.35(a) and (b) respectively. When the number of windings in the primary side is more i.e., $N_1 > N_2$, it is said to be a step-down auto-transformer. When the number of windings in the secondary side is more i.e., $N_1 < N_2$, it is said to be a step-up auto-transformer. It is known that in step-up and step-down auto-transformers, $K > 1$ and $K < 1$ respectively. Therefore, using Eqn. (2.24), we get

$I_1 > I_2$ in a step-up auto-transformer and $I_2 > I_1$ in a step-down auto-transformer. (2.25)

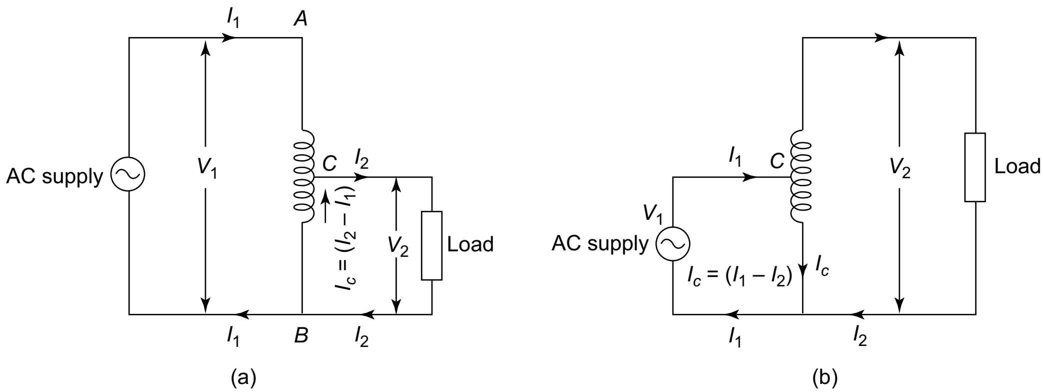


Figure 2.35 (a) Step-down and (b) Step-up Auto-transformer

The current flowing through the common winding is always the vector difference between I_1 and I_2. Therefore, using Eqn. (2.25), it can be concluded that the current through the common winding in step-down and step-up auto-transformers is $(I_2 - I_1)$ and $(I_1 - I_2)$ respectively, as shown in Figures 2.35 (a) and (b) respectively.

2.21.2 Copper Saving in Auto-transformer

The weight of the copper material used in windings is proportional to the length and area of the cross-section of the conductor used for windings. But, the length and cross-section of the conductor are directly proportional to the number of turns and to the current flowing through the conductor respectively. Using the step-down auto-transformer shown in Figure 2.35(a), the weight of copper material used in it is given by

$$W_a = \text{Weight of copper in section AC} + \text{Weight of copper in section CB}$$

where W_a is the total weight of copper material used in an auto-transformer.
But,

$$\text{Weight of copper in section AC } \alpha\, I_1(N_1 - N_2) \text{ and}$$
$$\text{Weight of copper in section CB } \alpha\, (I_2 - I_1)\, N_2$$

Therefore, $$W_a \,\alpha\, I_1(N_1 - N_2) + (I_2 - I_1)N_2 \tag{2.26}$$

Similarly, the weight of copper material used in an ordinary two-winding transformer is given by

$$W_a \,\alpha\, I_1N_1 + I_2N_2 \tag{2.27}$$

Using the above equations, we get

$$\begin{aligned}
\frac{W_a}{W_o} &= \frac{I_1(N_1 - N_2) + (I_2 - I_1)N_2}{I_1N_1 + I_2N_2} \\
&= \frac{I_1N_1 + I_2N_2 - 2I_1N_2}{I_1N_1 + I_2N_2} \\
&= \frac{I_1N_1\left(1 + \dfrac{I_2N_2}{I_1N_1} - 2\dfrac{I_1N_2}{I_1N_1}\right)}{I_1N_1\left(1 + \dfrac{I_2N_2}{I_1N_1}\right)} \\
&= \frac{(2 - 2K)}{2} \qquad \left(\text{since } \frac{N_2}{N_1} = K \text{ and } \frac{I_2}{I_1} = \frac{1}{K}\right)
\end{aligned}$$

Therefore, $\dfrac{W_a}{W_o} = 1 - K$

i.e., the weight of copper material required in a step-down auto-transformer is $(1 - K)$ times the weight of copper material required in an ordinary two-winding transformer.

Hence, the amount of copper saved in using an auto-transformer is

$$\text{Saving} = W_o - W_a = W_o - (1 - K)W_o = KW_o$$

Therefore, the amount of copper saved is K times the weight of copper used in an ordinary two-winding transformer. It is to be noted that the saving of copper will increase if K approached unity.

Similarly, the amount of copper saved in using a step-up auto-transformer is

$$\text{Saving} = \frac{W_o}{K}$$

2.21.3 VA rating of an Auto-transformer

If the auto-transformer shown in Figure 2.34 (a) is considered as an ordinary two-winding transformer with the number of turns in the primary and secondary winding as N_1 and N_2 respectively, then its voltage ratio is given by

$$a = \frac{V_1 - V_2}{V_2} = \frac{N_1}{N_2} = \frac{I_2 - I_1}{I_1} \tag{2.28}$$

But, the voltage ratio for an auto-transformer is given by

$$a' = \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}$$

i.e., $\quad a = 1 + a$

Therefore, the VA rating of the ordinary two-winding transformer is

$$(\mathrm{VA})_a = (V_1 - V_2)I_1 = V_2(I_2 - I_1) \tag{2.29}$$

Similarly, the VA rating of the auto-transformer is

$$(\mathrm{VA})_a = V_1 I_1 = V_2 I_2$$

Using Eqn. (2.28), we get

$$\frac{I_1}{I_2} = \frac{1}{1+a}$$

Substituting the above equation in Eqn. (2.29), we get

$$(\mathrm{VA})_o = V_2 I_2 \left(1 - \frac{1}{1+a}\right)$$

$$= \left(1 - \frac{1}{a'}\right)(\mathrm{VA})_a$$

or

$$(\mathrm{VA})_a = \left(\frac{1}{1 - \frac{1}{a'}}\right)(\mathrm{VA})_o \tag{2.30}$$

i.e., $(\mathrm{VA})_a > (\mathrm{VA})_o$

It is clear from Eqn. (2.30) that if the voltage ratio of the auto-transformer, a', is close to unity, then the VA rating of the auto-transformer is greater than the VA rating of the two-winding transformer.

2.21.4 Comparison between Auto-transformer and Conventional Transformer

Table 2.4 lists the comparison between an auto-transformer and a conventional transformer

Table 2.4 *Auto-transformer vs. Conventional Transformer*

Auto-transformer	Conventional transformer
It has only one winding, wound on a laminated core.	It has two separate windings, i.e., primary and secondary windings.
The windings are not electrically insulated.	The windings are electrically insulated.
Size of this transformer is small.	Size of this transformer is large.
The primary and secondary winding circuits are connected magnetically and electrically.	The primary and secondary winding circuits are connected magnetically.
It is more efficient and has better voltage regulation.	Efficiency and voltage regulation are less when compared to an auto-transformer.
Has low leakage flux and resistance.	Has high leakage flux and resistance.
Since the copper material required is less, it is very cheap and more economical.	Since the copper material required is high, it is very costly and less economical.
Loss is low.	Loss is high.
Generates variable output voltage.	Generates constant voltage.
It is mostly used as a starter in an induction motor.	It is mostly used to step-up or step-down the voltage in power system.
Also called as VARIAC	Also called as two-winding transformer

2.21.5 Advantages, Disadvantages and Applications of Auto-transformer

Advantages

1. Since the short circuit impedance of the auto-transformer is very low, it has higher efficiency and better regulation.
2. Since only one major winding is used, it is relatively smaller in size.
3. As the requirement of copper material is less, it is very cheap and more economical.
4. Requires low level of current for excitation.
5. Output voltage of the auto-transformer can be increased or decreased smoothly as per requirement.

Disadvantages

1. Since the winding is not insulated electrically, it cannot be used as an interconnection between high and low-voltage systems.
2. As it is electrically connected, the harmonics in the load get transferred to the supply.
3. The short-circuit current of the auto-transformer is high.
4. Highly sensitive to over-voltages.

Applications

1. Used as a starter in both synchronous and induction motors.
2. Since smooth variation of output voltage exists, it is used in electrical apparatus-testing labs.
3. Used as a booster to increase voltage levels in AC feeders.
4. Used as a voltage regulator.
5. Used in a power system (i.e., in transmission and distribution), audio systems and railways.

Example 2.21

A 200/400 V, 20 kVA, 50 Hz two-winding transformer is connected as an auto-transformer to a transformer from 800 V to 200 V.

(a) Determine the auto-transformer ratio a'
(b) Determine the VA rating of the auto-transformer.
(c) If the VA obtained in part (b) is connected as a load to 200 V terminals, determine the currents flowing through the load and the transformer windings.
(d) Determine the VA rating of the two-winding transformer.

[AU Nov/Dec, 2005]

Solution:

The step-down auto-transformer is shown in Figure E2.21.

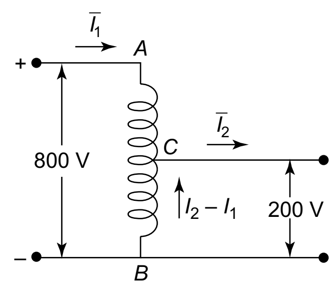


Figure E2.21

(a) Auto-transformer ratio

$$a' = \frac{800}{200} = 4$$

(b) The maximum current in the primary winding of two winding transformer is given by

$$I_1 = \frac{20 \times 10^3}{400} = 50\text{A}$$

Since the same two-winding transformer is connected as an auto-transformer, the primary current in it is given by

$I_1 = 50$ A

Therefore, the VA rating of the auto-transformer is

$(\text{VA})_a = V_1 I_1 = 800 \times 50 = 40 \times 10^3$ VA

(c) The load connected to the auto-transformer is 40×10^3 VA

Therefore, the current flowing through the load is

$$I_2 = \frac{40 \times 10^3}{200} = 200 \text{ A}$$

Using the auto-transformer voltage ratio, a', the current flowing through the load is

$I_2 = a' I_1 = 4 \times 50 = 200$ A

Hence, the current flowing through the secondary winding is

$I_2 - I_1 = 200 - 50 = 150$ A

(d) The VA rating of the two-winding transformer is given by

$(\text{VA})_o = (V_1 - V_2) I_1 = (800 - 200) \times 50 = 30 \times 10^3$ VA

Example 2.22

The primary and secondary voltages of an auto-transformer are 500 V and 400 V respectively. Draw the circuit diagram showing the current distribution in the windings when the secondary current is 200 A. Also, calculate the amount of copper saved in using an auto-transformer. **[AU Nov/Dec, 2003]**

Solution

Given, $V_1 = 500$ V, $V_2 = 400$ V and $I_2 = 200$ A

The transformation ratio of the auto-transformer is

$$K = \frac{V_2}{V_1} = \frac{400}{500} = 0.8$$

Therefore, the current drawn from the source is given by

$$I_1 = K I_2 = 0.8 \times 200 = 160 \text{ A}$$

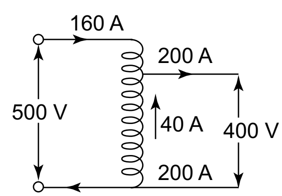


Figure E2.22

The current distribution in the windings of the auto-transformer is shown in Figure E2.22.

The ratio of copper material required in an auto-transformer to an ordinary two-winding transformer is given by

$$\frac{W_a}{W_o} = K = 0.8$$

Therefore, 80% of copper is saved in using an auto-transformer.

2.22 Three-Phase Transformer **[AU Nov/Dec, 2014]**

The transformer used to supply or transfer large amounts of power to three-phase connections, to meet the required demand economically, is called a three-phase transformer. In power systems, it is used in different stages for stepping up or stepping down higher voltages. There are numerous advantages of a three-phase transformer, when compared to a single-phase transformer.

2.22.1 Construction of a Three-phase Transformer

There are two methods to construct a three-phase transformer, as explained below:

1. ***Using a bank of three single-phase transformers:*** In this method, three single-phase transformers are connected such that the primary windings of each transformer are connected to each other. Similarly, the secondary windings of each transformer are connected to each other. The phase angle between these three single phases is 120°. In this method, if a fault occurs in any one of the single-phase transformers, the continuity of the supply is maintained by the other two single-phase transformers.
2. ***Using a single three-phase transformer:*** Here, the three-phase transformer has a single core, where all the three windings get wounded. This method of constructing a three-phase transformer is preferable since it is economical and more convenient. In this method, if a fault occurs, there will be discontinuity in the supply.

2.22.2 Types of Three Phase Transformers

The three-phase transformer can be constructed using a common magnetic core for both primary and secondary windings. Based on the type of construction of primary and secondary windings, three-phase transformers are classified as:

1. ***Core-type three-phase transformer:*** A core-type three-phase transformer is shown in Figure 2.36. Here, three limbs or legs and two yokes, which form a magnetic path, exist in the core. For each phase, both the low and high-voltage windings formed using circular cylindrical coils are concentrically wound on each limb. Here, the three-phase transformer is constructed using stack lamination. The low-voltage windings are wound near the core with suitable insulation between them, whereas, the high-voltage windings are wound over the low-voltage windings with suitable insulation between them. When the system is balanced, the magnetic flux generated in the primary windings gets added up to make the resultant flux as zero. But if the system is unbalanced, the resultant flux exists, and it circulates high current.

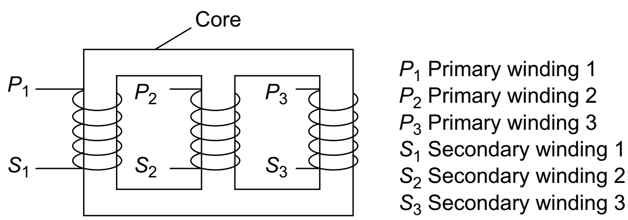


Figure 2.36 Core-type Three Phase Transformer

2. ***Shell-type three-phase transformer:*** In this type, since each phase has an individual magnetic circuit, the three phases are more independent. Here, the construction is similar to the single-phase shell-type transformer and each phase is placed on top of another, as shown in Figure 2.37. Here, the winding direction of the units 'a' and 'c' are same when compare to unit 'b'. The effect of unbalanced condition is less and each laminated core surrounds its corresponding coil.

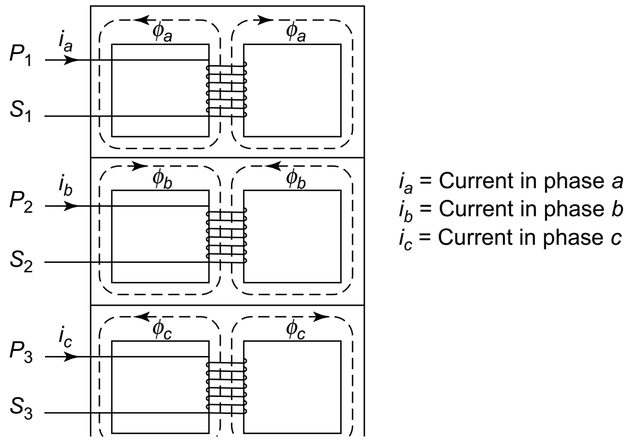


Figure 2.37 Shell-type Three-phase Transformer

2.22.3 Winding Connection

The primary and secondary windings for each phase can be either connected in star or delta, to form a complete three-phase transformer. The different possible winding connections in a three-phase transformer are explained as follows.

Star-Star (Y-Y) Connection

The primary and secondary windings are connected in star as shown in Figure 2.38(a). Since the windings are connected in star, the number of turns required per phase gets reduced, which further reduces the required insulation. In this type, the transformation ratio is given by the ratio of primary to secondary line voltages. There is no phase angle difference between the line voltages of primary and secondary windings. It is used in small three-phase transformers, if the balanced load is connected to it and can be operated at high voltages.

Advantages

1. Requires less number of turns and the insulation stress is less.
2. Economical for medium-voltage applications.
3. Mechanical strength of the windings is stronger, as it carries heavy current and hence it can withstand heavy loads and short-circuits.
4. Can be used in three-phase four-wire system, as the neutral point is available.

Disadvantages

1. If unbalanced load is connected, its performance is poor.
2. When an alternator is connected, the third harmonic present in it causes distortion in the secondary voltage.

Delta-Delta (Δ-Δ) Connection

The primary and secondary windings are connected in delta, as shown in Figure 2.38(b). The number of turns required per phase and the required insulation is more when compared to star-star connection. In this type, the transformation ratio is given by the ratio of primary to secondary line voltages. It is used in large transformers, even when an unbalance load is connected to it. Also, if any phase in this connection is disabled due to a fault, the system continues to operate in open-delta connection with reduced capacity. It can be operated only at low voltages.

Advantages

1. Since the delta connection provides the required harmonic component to I_m, the magnetic flux produced in the primary winding remains sinusoidal, which results in sinusoidal-induced voltages.
2. Unbalanced load can be used in this connection, as there will be no distortion in the secondary voltage.
3. If a bank of three single-phase transformers is used in this connection, continuous supply can be provided, even when one of the transformers fail.
4. Less winding cross-section and hence it is economical for low-voltage transformers.

Disadvantages

It is not suitable for a three-phase four-wire system due to the absence of a neutral point.

Star-Delta (Y-Δ) Connection

Here, the primary windings are connected in star with the neutral grounded and the secondary windings are connected in delta, as shown in Figure 2.38(c). In this type, the ratio of primary to secondary line voltages is $\sqrt{3}$ times the transformation ratio. The phase-angle difference between primary and secondary line voltages is 30°. It is used for stepping down the voltage at the substation level of transmission.

Advantages

1. Requires less number of turns and is more economical for large and high-voltage transformers.
2. Distortion is avoided as the neutral point is in primary.
3. Unbalanced loads can be connected to the transformer.

Disadvantages

1. A phase shift exists between the primary and secondary voltages.
2. It cannot be used in parallel with star-star or delta-delta connected transformers.

Delta-Star (Δ-Y) Connection

In this connection, the primary windings are connected in delta and the secondary windings are connected in star with the neutral grounded, as shown in Figure 2.38(d). In this type, the ratio of primary to secondary line voltages is $\frac{1}{\sqrt{3}}$ times the transformation ratio. The phase angle difference between primary and secondary line voltages is 30°. It is used for stepping up the voltage at the beginning level of transmission.

Advantages

1. Winding cross-section area is less in primary side due to delta connection.
2. It can be used in three-phase four-wire system due to the availability of neutral point.
3. No distortion in the system.
4. Economical as cost is saved due to usage of less insulation.
5. Unbalanced loads can be connected to the transformer.

Disadvantages

1. A phase shift exists between the primary and secondary voltages.
2. It cannot be used in parallel with star-star or delta-delta connected transformer.

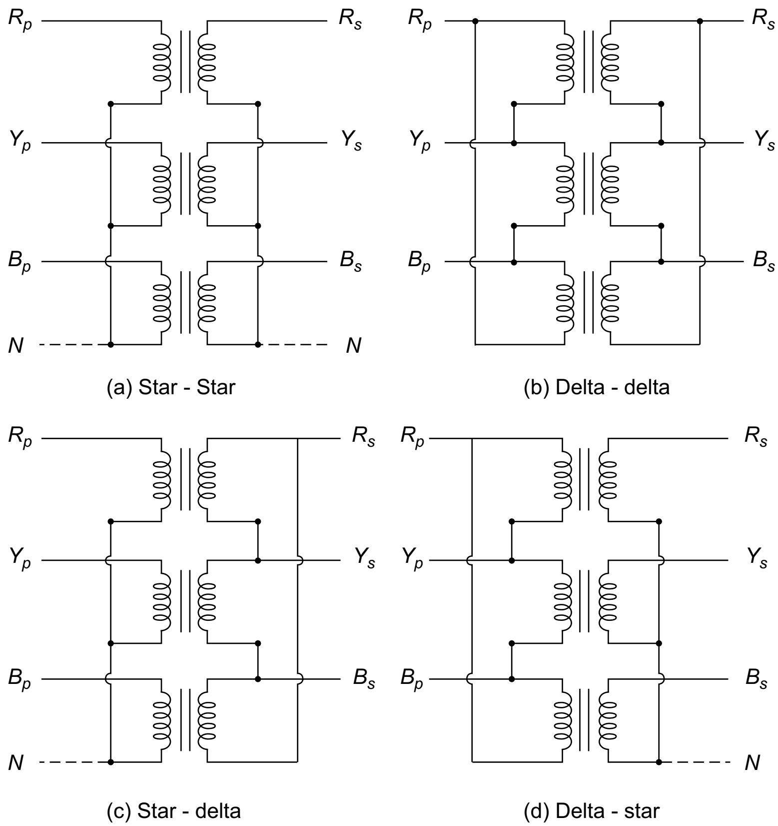


Figure 2.38 Winding Connections in a Three-phase Transformer

Special Type of Connection

These connections are used in three-phase transformers for specific purposes like fault conditions and conversion of three-phase to two-phase, as explained below:

- ***Open delta or (V-V) connection:*** Here, two transformers are used whose winding connection is shown in Figure 2.39(a). It is used to ensure the continuity of the supply when any one of the transformers is disabled due to fault. It is used in small three-phase loads, where installation of three-phase transformers is not necessary. It can carry only 57.7% of the load carried by delta-delta connection.
- ***Scott or (T - T) connection:*** Out of the two transformers used in this type, one transformer is provided with tappings at both the windings, as shown in Figure 2.39(b). The other transformer is called teaser transformer or main transformer, which is used for converting three-phase to two-phase.

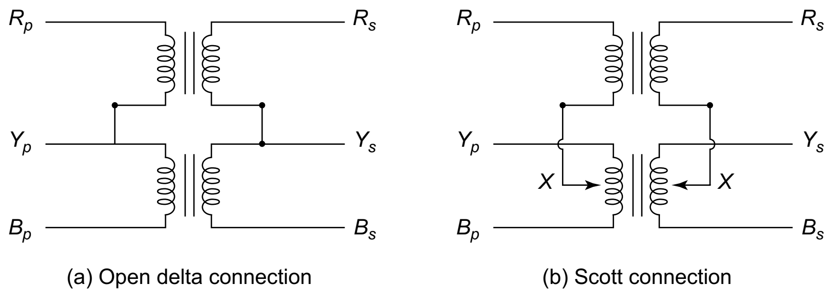


Figure 2.39 Special Type of Connection

2.23 Voltage and Current Relationship for Different Connections

[AU April/May, 2014]

The voltages and currents of primary and secondary of a three-phase transformer for different winding connections are listed in Table 2.5. The assumptions made to obtain the relation are:

1. V_L is the line voltage in the primary side.
2. I_L is the line current in the primary side.
3. K is the transformation ratio.
4. Loads are balanced resistive load with unity power factor.
5. Losses in the transformer are neglected.

Table 2.5 *Voltage and Current Relationship for Different Winding Connections*

Connection Type	Primary Side				Secondary Side			
	Line Voltage	Phase Voltage	Line Current	Phase Current	Line Voltage	Phase Voltage	Line Current	Phase Current
Star-star	V_L	$\frac{V_L}{\sqrt{3}}$	I_L	I_L	KV_L	$\frac{KV_L}{\sqrt{3}}$	$\frac{I_L}{K}$	$\frac{I_L}{K}$
Delta-delta	V_L	V_L	I_L	$\frac{I_L}{\sqrt{3}}$	KV_L	KV_L	$\frac{I_L}{K}$	$\frac{I_L}{K\sqrt{3}}$
Star-delta	V_L	$\frac{V_L}{\sqrt{3}}$	I_L	I_L	$\frac{KV_L}{\sqrt{3}}$	$\frac{KV_L}{\sqrt{3}}$	$\frac{I_L}{K}$	$\frac{I_L}{K\sqrt{3}}$
Delta-star	V_L	V_L	I_L	$\frac{I_L}{\sqrt{3}}$	KV_L	$\frac{KV_L}{\sqrt{3}}$	$\frac{I_L}{K}$	$\frac{I_L}{K}$

2.23.1 Working of a Three-phase Transformer

Consider that the primary windings of a three-phase transformer are connected in star on the cores, which are displaced by 120°, as shown in Figure 2.40. Here, for simplification purpose, only primary windings connected to the three-phase AC supply are shown. By connecting the empty leg of each core, it forms the centre leg.

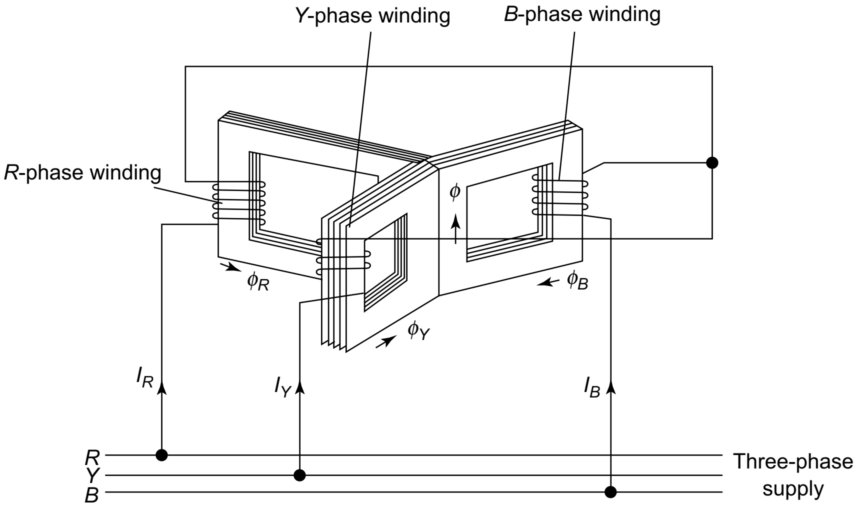


Figure 2.40 Working of a Three-phase Transformer

When the primary windings are excited using a three-phase supply, currents I_R, I_Y and I_B flow through its respective windings, which produce the magnetic fluxes, ϕ_R, ϕ_Y and ϕ_B in the respective cores. Since the centre leg is common for all the cores, the sum of all three fluxes is carried by it. These fluxes induce an emf in the primary winding and based on the principle of transformer, an emf is induced in its respective secondary winding. The phase angle between the induced emf in primary and secondary windings is based on the winding connections. The induced emf in the secondary winding drives the currents to the load connected to it. Stepping up or stepping down of primary voltage can be achieved, based on the winding connection and the number of turns in each winding.

2.23.2 Advantages and Disadvantages of a Three-phase Transformer

Advantages

1. Smaller and easier to install.
2. Requires less core material and hence it is more economical.
3. Provides higher efficiency.
4. Easier to transport, as its weight is relatively less.
5. Protective device installation is easier.
6. When compared to a bank of single-phase transformers, it requires less space for the same rating.

Disadvantages

1. Repairing cost is more.
2. When it is self-cooled, its capacity reduces.

Two Mark Questions and Answers

1. Write down the emf equations of a single-phase transformer.

[AU Nov/Dec, 2009, 2012]

The rms value of the induced emf in the primary winding is given by

$$E_1 = 4.44 f \phi_m N_1$$

Similarly, the rms value of the induced emf in the secondary winding is given by

$$E_2 = 4.44 f \phi_m N_2$$

2. Give the expression for percentage voltage regulation of a single-phase transformer.

[AU Nov/Dec, 2016]

The generalized expression for voltage regulation of a transformer is

$$\%R = \frac{I_2 R_{2e} \cos\phi_2 \pm I_2 X_{2e} \sin\phi_2}{V_2}$$

where, the positive sign (+) indicates lagging power factor and the negative sign (–) indicates leading power factor.

3. Draw the no-load phasor diagram of a transformer. **[AU Nov/Dec, 2014; April/May, 2006]**

The no-load phasor diagrams of an ideal and practical transformer, without winding resistances and leakage reactances, are shown in Figure UQ2.3 (a) and (b) respectively.

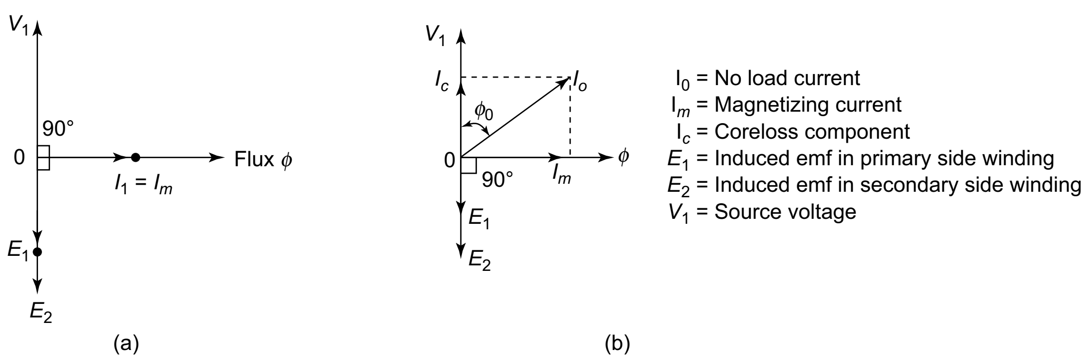


Figure UQ2.3

4. Define voltage transformation ratio of a transformer. **[AU Nov/Dec, 2012]**

It is defined as the ratio of the secondary induced voltage to the primary induced voltage and is denoted by K, i.e.,

$$K = \frac{E_2}{E_1}$$

5. What do you mean by a step-down transformer? **[AU May/June, 2013]**

It is vice-versa of a step-up transformer i.e., it transfers a low current, high AC voltage into a high current, low AC voltage. Here, $N_1 > N_2$ and $V_1 > V_2$. Therefore, the transformation ratio is less than 1 i.e., $K < 1$.

6. Draw the equivalent circuit of a transformer. **[AU May/June, 2013; Nov/Dec, 2015]**

The equivalent circuit of a transformer with load impedance, Z_L, is shown in Figure UQ2.6.

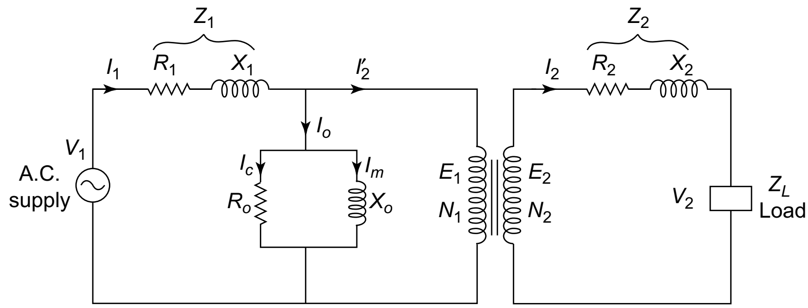


Figure UQ2.6

7. Define regulation in a transformer. **[AU Nov/Dec, 2014]**

The decrease in the terminal voltage when expressed as a fraction of the no-load terminal voltage is called voltage regulation of a transformer. Mathematically, the voltage regulation is given by:

$$\%R = \frac{V_{2o} - V_2}{V_2} \times 100$$

$$= \frac{E_2 - V_2}{V_2} \times 100$$

Where, $V_{2o} = E_2$ is the terminal voltage of the transformer at no-load condition and V_2 is the secondary terminal voltage at a given load.

8. Distinguish between core and shell-type transformers.

[AU April/May, 2015; Nov/Dec, 2008]

Refer to Table 2.1 for the differences between core and shell-type transformers.

9. What is an ideal transformer and how does it differ from a practical transformer?

[AU April/May, 2015]

Refer to Sections 2.6 and 2.8 for ideal and practical transformers.

10. Give the principle behind an auto-transformer. **[AU Nov/Dec, 2015]**

Refer to Section 2.21 for the principle behind an auto-transformer.

11. Draw the approximate equivalent circuit of a single-phase transformer and identify the various parameters. **[AU April/May, 2016]**

The approximate equivalent circuit of a single-phase transformer when referred to a primary side, is shown in Figure UQ2.11.

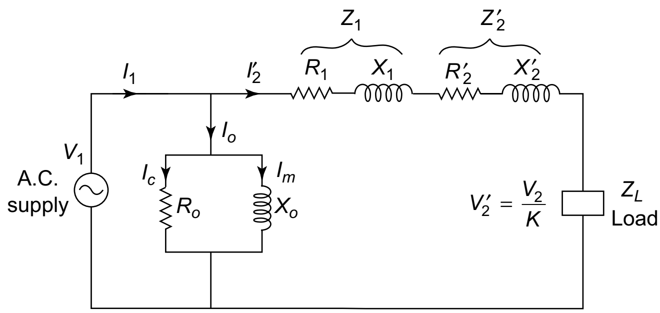


Figure UQ2.11

12. What is a transformer? [AU Nov/Dec, 2006]

A transformer is a static device used for coupling two or more electric circuits. It works on the principle of mutual induction and it transfers the electric energy from one circuit to another when there is no electrical connection between the two circuits.

13. Give the principle of a transformer. [AU April/May, 2010]

Refer to Section 2.2.1 for the principle of a transformer.

14. Why transformers are rated in kVA instead of kW? [AU April/May, 2012]

The iron and copper losses depend only on the supply voltage and the current flowing through the winding respectively. Since these losses do not depend on the phase angle between the supply voltage and the current, the transformer rating expressed as a product of voltage and current is called VA rating of the transformer. Therefore, the transformer rating is expressed as kVA and not as kW.

15. Define voltage regulation of a transformer. State its expression. [AU April/May, 2011]

The decrease in the terminal voltage when expressed as a fraction of the no-load terminal voltage is called voltage regulation of a transformer. The generalized expression for voltage regulation of the transformer is

$$\%R = \frac{I_2 R_{2e} \cos \phi_2 \pm I_2 X_{2e} \sin \phi_2}{V_2}$$

where, the positive sign (+) indicates a lagging power factor and the negative sign (–) indicates a leading power factor.

16. What is an ideal transformer? [AU April/May 2011]

Refer to Section 2.6 for ideal transformer.

17. What will happen if transformer primary is excited by DC voltage? [AU Nov/Dec, 2010]

Refer to Section 2.4 for the sequence of events that occur when the transformer primary is excited by DC voltage.

18. Give the expression for the load current when the transformer operates at its maximum efficiency. [AU Nov/Dec, 2007]

If I_{2m} is the load current at maximum efficiency, then using the condition for maximum efficiency, we get

$$I_{2m} = \sqrt{\frac{P_i}{R_{2e}}}$$

19. What are turns ratio and transformation ratio of a transformer? **[AU Nov/Dec, 2011]**

Turns ratio: It is the ratio of the number of turns in primary to number of turns in secondary, as given by

$$\text{Turns ratio} = \frac{N_1}{N_2}$$

If $N_2 > N_1$, the transformer is called a step-up transformer and if $N_2 < N_1$, the transformer is called a step-down transformer.

Transformation Ratio (K): It is defined as the ratio of the secondary induced voltage (E_2) the primary induced voltage (E_1), as given by

$$K = \frac{E_2}{E_1}$$

20. What are step-up and step-down transformers? **[AU April/May, 2013]**

(a) **Step-up transformer:** It transfers a high current, low AC voltage into a low current, high AC voltage. Here, $N_2 > N_1$ and $V_2 > V_1$. Therefore, the transformation ratio is greater than 1, i.e., $K > 1$.

(b) **Step-down transformer:** It is vice-versa of step-up transformer i.e., it transfers a low current, high AC voltage into a high current, low AC voltage. Here, $N_2 < N_1$ and $V_2 < V_1$. Therefore, the transformation ratio is less than 1 i.e., $K < 1$

21. Draw the phasor diagram of transformer on inductive load (lagging power factor). **[AU Nov/Dec, 2008]**

The phasor diagram of a transformer without and with winding impedances when an inductive load is connected is shown in Figure UQ2.22 (a) and (b) respectively.

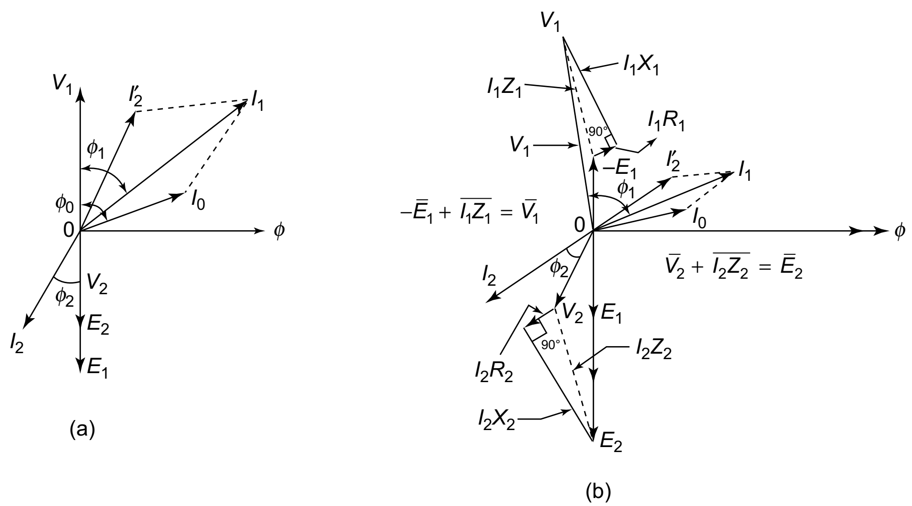


Figure UQ2.22

22. What is the purpose of conducting a short-circuit test on a transformer? **[AU Nov/Dec, 2011]**

Refer to Section 2.20 for the purpose of conducting short-circuit test on transformer.

23. State the different core constructions of a transformer. **[AU Nov/Dec, 2011]**

Refer to Section 2.2.2 for different core constructions of a transformer.

24. Give the difference in connection between OC and SC tests for a transformer. **[AU Nov/Dec, 2011]**

Refer to Section 2.20 for the OC and SC test for a transformer.

Review Questions

1. Define a static transformer and explain its principle of operation. How is energy transferred from one circuit to another? Which of the two windings is called primary and secondary?
2. Explain the construction of a single-phase transformer.
3. Distinguish between core-type and shell-type transformers. Also explain it with the help of neat sketches.
4. What are the different types of transformers?
5. Derive the emf equation of a transformer. On what factors does the induced emf in a winding depend?
6. What is meant by transformation ratio and how is it related to number of turns in primary and secondary windings of a transformer?
7. What approximate relation exists between primary and secondary currents?
8. How do you define an ideal transformer? How does an actual transformer differ from it?
9. Explain the operation of an ideal transformer at no-load condition.
10. Explain the operation of a practical transformer at no-load condition.
11. What are the two components of no-load primary current? What are the functions of magnetising current and working component? Which is generally greater?
12. Show that even when primary and secondary turns are equal, the total primary current is not equal to the secondary current. Also, show that primary current is the vector sum of two currents; one of which is relatively small but is constant and the other is large and varies with the load on the transformer secondary.
13. Explain the operation of a practical transformer with inductive, resistive and capacitive loads.
14. Discuss the effect of winding resistances and leakage reactances on a transformer.
15. Draw a complete phasor diagram for a transformer with winding resistances when the load power factor is: (i) lagging and (ii) leading. Why are both primary and secondary voltages shown in anti-phase with the applied primary voltage?
16. What is meant by the equivalent resistance of a transformer, as referred to primary or secondary?
17. Draw the circuit diagram of a transformer and also obtain the equivalent circuit and mathematical expression for a transformer.
18. Obtain the equivalent circuit of the transformer when it is referred to primary and secondary sides. Also obtain the approximate equivalent circuit of the transformer.
19. Define voltage regulation and obtain the expression for voltage regulation for lagging and leading power factors.
20. List the different losses occurring in a transformer on load. How can these losses be determined experimentally?
21. Derive an expression for the efficiency of the transformer. Also obtain the condition for maximum efficiency.
22. Discuss the effect of power factor on the efficiency of the transformer.

23. Obtain the load current and kVA supplied to the transformer at maximum efficiency.
24. Explain, with the help of circuit diagram, how are efficiency and regulation of a single-phase transformer predetermined, by conducting open-circuit and short-circuit tests?
25. Explain the working of an auto-transformer.
26. Prove that the copper material required in an auto-transformer is less when compared to the copper material required in a single-phase transformer.
27. Discuss the advantages and disadvantages of an auto-transformer.
28. Differentiate auto-transformer and conventional transformer.
29. Explain the construction and working of a three-phase transformer.
30. Explain the different winding connections in a three-phase transformer with neat diagrams.
31. Explain Scott and open-delta connections.
32. Write down the relationship between voltage and current for different connections in a three-phase transformer.
33. Discuss the advantages and disadvantages of a three-phase transformer.
34. The emf per turn of a single-phase 6.6 kV, 440 V, 50 Hz, transformer is approximately 12 V. Calculate the number of turns in the high voltage and low voltage windings and the net cross-sectional area of the core for a maximum flux density of 1.5 T. **[Ans: 550, 37 and 360 cm^2]**
35. A single-phase 50 Hz transformer has 80 turns on the primary and 400 turns on the secondary winding. The net cross-sectional area of the core is 200 sq. cm. If the primary winding is connected to a 240 V, 50 Hz supply, determine: (i) emf induced in the secondary winding and (ii) maximum flux density in the core. **[Ans: 1200 V and 0.6756 Wb/m^2]**
36. The no-load current of a transformer is 10 A, at a power factor of 0.25 lagging, when connected to a 400 V, 50 Hz supply. Determine: (i) magnetizing component of no-load current (ii) iron loss and (iii) maximum amount of flux in the core. Assume number of turns in the primary winding is 500. **[Ans: 9.6824 A, 1000 W and 3.6036 mWb]**
37. A single-phase transformer takes 10 A on no-load at a power factor of 0.2 lagging. The turns ratio is 4:1. If the load on the secondary is 200 A, at a power factor of 0.85 lagging, determine the primary current and the power factor. Neglect the voltage drop in the winding. Also draw the phasor diagram. **[Ans: 57.3246 A and 0.7762 Lagging]**
38. A single-phase transformer has 360 turns and 180 turns respectively in the secondary and primary windings. The respective resistances are 0.233 Ω and 0.067 Ω. Determine the equivalent resistance of: (i) primary in terms of secondary winding (ii) secondary in terms of primary and (iii) total resistance of the transformer in terms of primary. **[Ans: 0.268 Ω, 0.05825 Ω and 0.12525 Ω]**
39. The parameters of approximate equivalent circuit of a 4 kVA, 200/400 V, 50 Hz, single-phase transformer are: $R'_1 = 0.15\ \Omega$, $X'_1 = 0.37\ \Omega$, $R_0 = 600\ \Omega$ and $X_m = 300\ \Omega$. When a rated voltage of 200 V is applied to the primary, a current of 10 A at power factor of 0.8 lagging flows in the secondary winding. Determine: (i) current in the primary and (ii) terminal voltage at the secondary side. **[Ans: 20.67 A at -37.79^o and 386.4 V at -1.22^o]**
40. A 100 kVA, 6.6 kV/415 V single-phase transformer has an effective impedance of $(3 + j8)\ \Omega$ referred to high voltage side. Determine the full-load voltage regulation at a power factor of 0.8 lagging and a power factor of 0.8 leading. **[Ans: 1.652% and –0.5509%]**
41. A 20 kVA, 2500/250 V, single-phase transformer has the following parameters: HV winding: $R_1 = 8\ \Omega$ and $X_1 = 17\ \Omega$, LV winding: $R_2 = 0.3\ \Omega$ and $X_2 = 0.7\ \Omega$. Determine the voltage regulation and the secondary terminal voltage at full load for a power factor of 0.8 lagging and 0.8 leading. The primary voltage is held constant at 2500 V. **[Ans: 10.592% , 447.04 V (lag), –2.656%, 513.28 V(lead)]**

42. A single-phase transformer on full load has an impedance drop of 20 V and resistance drop of 10 V. Calculate the value of power factor when its regulation will be zero. **[Ans: 0.866 leading]**
43. A 200 kVA, single-phase transformer has a primary voltage of 2000 V and secondary voltage of 500 V. The supply frequency is 50 Hz. The total effective resistance and reactance referred to the primary are 0.5 Ω and 2 Ω respectively. Determine the voltage regulation of the transformer at full load unity power factor. **[Ans: 2.5 %]**
44. The primary and secondary windings of a 500 kVA transformer have resistance of 0.4 Ω and 0.001 Ω respectively. The primary and secondary voltages are 6600 V and 400 V respectively. The iron loss is 3 kW. Calculate the efficiency on full load, the load power factor being 0.8 lagging. **[Ans: 98.3143 %]**
45. A 600 kVA, single-phase transformer, while working at unity power factor, has an efficiency of 92% at full load and also at half load. Determine its efficiency when it operates at unity power factor and 60% of full load. **[Ans: 92.3282 %]**
46. Calculate the efficiency at half and full load of a 100 kVA transformer for power factors of unity and 0.8. The copper loss is 1000 W at full load and iron loss is 1000 W. **[Ans: Full load: 98.04% and 97.56%, Half load: 97.56% and 96.96%]**
47. A 100 kVA 1000/10 kV, 50 Hz, single-phase transformer has an iron loss of 1100 W. The copper loss with 5 A in the high voltage winding is 400 W. Calculate the efficiencies at: (i) 25 % (ii) 50 % and (iii) 100 % of normal load, for power factors of 1.0 and 0.8. The output terminal voltage being maintained at 10 kV. Also find the load for maximum efficiency at both power factors. **[Ans: 95.42% and 94.34% (25 %), 97.087 % and 96.385 % (50%), 97.37% and 96.735% (100%), 82.9156 kVA]**
48. A 250 kVA, single-phase transformer has 98.135% efficiency at full load and power factor of 0.8 lagging. The efficiency at half load and a power factor of 0.8 lagging is 97.751%. Determine the iron loss and full load copper loss. **[Ans: P_i = 2000.18 W and P_c = 1800.69 W]**
49. A 250 kVA single-phase transformer has an iron loss of 1.8 kW. The full-load copper loss is 2000 W. Determine: (i) efficiency at full load, with a power factor of 0.8 lagging (ii) kVA supplied at maximum efficiency and (iii) maximum efficiency at power factor of 0.8 lagging. **[Ans: 98.135%, 237.1708 kVA and 98.137%]**
50. An auto-transformer, as shown in Figure Q2.50 is used to supply a load of 2 kW at 230 V from a 400 V AC supply. Determine the currents in parts AC and BC, neglecting losses and no-load current. Also, determine the copper saving due to the use of auto-transformer instead of using a two-winding transformer. Assume purely resistive load. **[Ans: 5 A, 3.6956 A and 57.5%]**

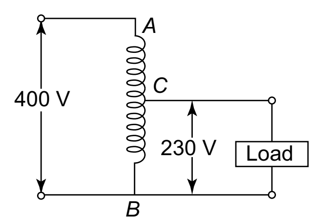


Figure Q2.50

51. Draw the equivalent circuit for a single-phase 1100/220 V transformer on which the following results were obtained:
   (i) 1100 V, 0.5 A, 55 W on primary, secondary being open-circuited.
   (ii) 10 V, 80 A, 400 W on low-voltage side, high-voltage side being short-circuited.

   Also determine the voltage regulation and efficiency for the above transformer, while supplying 100 A, at power factor of 0.8 lagging. **[Ans: 96.28% and 5.22%]**
52. Draw the equivalent circuit for a single-phase 200/400 V, 50 Hz transformer on which the following results were obtained:
   (i) 200 V, 0.7 A, 70 W on low-voltage side
   (ii) 15 V, 10 A, 85 W on high-voltage side

Also determine the secondary voltage, when delivering 5 kW, at a power factor of 0.8 lagging and the primary voltage being 200 V. **[Ans: 376.395 V]**

53. The parameters of a 10 kVA, 500/250 V, 50 Hz, single-phase transformer are as follows: primary resistance and reactance are 0.2 Ω and 0.4 Ω respectively, secondary resistance and reactance are 0.5 Ω and 0.1 Ω respectively, exciting circuit resistance and reactance are 1500 Ω and 750 Ω respectively. Determine the results of OC and SC tests.

[Ans: OC: 166.67 W, 500 V, 0.7453 A, SC: 880 W, 46.818 V, 20 A]

CHAPTER 3

DC Machines

3.1 Introduction

A highly versatile machine that operates with DC supply to generate unidirectional torque or current is known as a DC machine. It works on the principle of Faraday's law of electromagnetic induction. Fleming's left and right-hand rules are used to determine the direction of torque or current developed in the DC machine. In general, the DC machine can be constructed in many forms to use for various purposes. The size of a DC machine varies from very small machine used in a quartz crystal watch, to a giant, 75,000 kW or more rolling-mill machine. It is also easily adaptable for drives with a wide range of speed control and fast reversal. Depending on the generation of torque emf generated, a DC machine is classified as DC motor and DC generator. If the DC machine is used to generate unidirectional torque, then it is called a DC motor, whereas, if the DC machine generates unidirectional current, it is called a DC generator. In this chapter, the working principle, construction, working, governing equations of DC machines (DC generators and DC motors), their types and characteristics are discussed. In addition, a special type of motor that can run on single-phase AC or DC power supply is discussed.

3.2 DC Generator [AU Nov/Dec, 2016]

3.2.1 Working Principle

A DC generator is a dynamic DC machine, which generates electrical energy from mechanical energy. The emf induced is called dynamically induced emf. It operates on the principle of Faraday's law of electromagnetic induction, which states that whenever a current carrying conductor cuts the magnetic flux, a dynamically induced emf is generated. Its magnitude depends on the rate of change of magnetic flux linked with the conductor. If the conductor is connected to a closed circuit, then a current will flow through it. The basic elements required in a DC generator to generate a dynamically induced emf are:

- A steady magnetic field
- A current carrying conductor and
- Relative motion between conductor and magnetic field

3.2.2 Construction

The schematic diagram of a DC generator with necessary parts is shown in Figure 3.1.

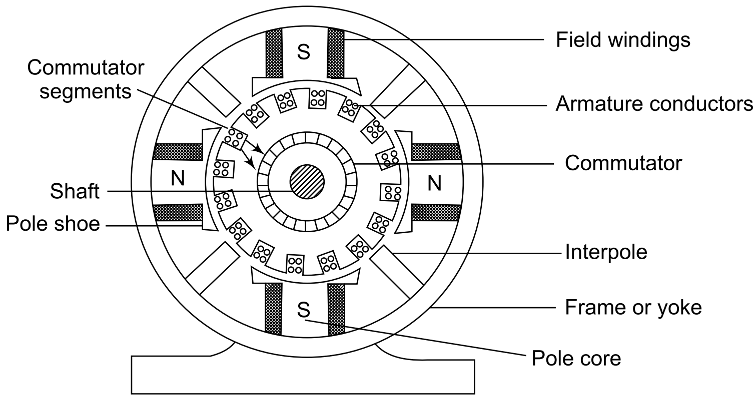


Figure 3.1 Schematic Diagram of a DC Generator

The components of a DC Generator are:

- Magnetic frame or yoke
- Pole cores and pole shoes
- Field or exciting windings
- Armature core and windings
- Commutators
- Brushes
- End housings
- Bearings
- Shaft

The above nine parts can be grouped into four major components, namely: magnetic field system, armature, commutator and brush assembly.

Magnetic Field System

The magnetic field system is a stationary or fixed part of the DC machine, where the main magnetic flux is generated. Mainframe or yoke, pole core and pole shoes and field or exciting coils are included in this system, as described below:

Magnetic Frame or Yoke

The outer frame of a DC generator, to which pole core and pole shoes are fixed, is known as yoke. In large DC generators, it is made up of cast steel and in small DC generators it is made up of cast iron. The important functions that the yoke performs are:

- It supports pole core and pole shoes.
- It provides a low reluctance path for the magnetic flux produced by the field winding.
- It protects inner parts of the DC generator.

Pole Core and Pole Shoes

The curved pole core and pole shoes are fixed to the magnetic frame or yoke with the help of bolts. The poles in the DC generator are called salient poles, since they are projected inwards. Thin cast steel or iron with or without lamination is used to make pole core and pole shoes. Using lamination can reduce Eddy current losses. The important functions that the pole core and pole shoes perform are:

- Field or exciting coils are wound around the pole core.
- Helps in uniform distribution of magnetic flux to the armature.
- Helps in increasing the cross-sectional area of the magnetic circuit, which in turn reduces the reluctance of the magnetic path.

Field Coils or Exciting Coils

The enamelled copper wire wound pole core and placed over each pole core to produce the required magnetic field is called *field or exciting coils*. It always requires a relatively small DC power to produce the required strong magnetic field. These coils are connected in such a way that when DC current is used for excitation, the poles on which they are wound attain opposite polarity i.e., the poles get magnetised to produce the required flux.

Armature

The rotating component of the DC generator is called armature and it consists of a laminated cylinder called armature core, placed over the shaft.

Armature Core

The drum or cylindrical component fixed to the rotating shaft in a DC generator is the armature core. It accommodates the armature winding in the grooves or slots provided at its outer periphery. The armature core serves the following purposes:

- Provides a place for conductors in the slots.
- Provides a low reluctance path for the magnetic flux.

In armature core, by using silicon steel material, the hysteresis loss produced due to the reversal of flux is reduced. Also, in order to reduce the eddy current loss produced due to the induced emf in the armature, lamination with 0.3 to 0.5 mm thickness stamping is used in the armature core.

Armature Windings

The insulated conductors made up of bands of steel wire are placed in the armature slots. These conductors are suitably connected by winding around the armature core, which forms the armature winding. Since the mechanical power to electrical power conversion takes place at this point, armature winding is called the heart of DC generator. Based on the way the armature conductors are connected, armature windings are classified as:

- Lap winding
- Wave winding

Lap Winding

If the armature conductors are connected in such a way that the number of poles and number of parallel paths is equal, then it is called a *lap winding*. If a DC generator has P number of poles and Z number of armature conductors, then there will be P parallel paths and Z/P conductors will be connected in series per parallel path.

Wave Winding

The armature conductors are connected in such a way that if the number of parallel paths is two, irrespective of the number of poles, then it is called *wave winding*. If a DC generator has Z number of armature conductors, then there will be two parallel paths and in each parallel path $Z/2$ number of armature conductors will be connected in series. The comparison between lap and wave windings is listed in Table 3.1.

Table 3.1 *Comparison Between Lap Winding and Wave Winding*

S. No.	Lap Winding	Wave Winding
1	The number of parallel path is equal to the total of number poles i.e., $A = P$	The number of parallel paths is equal to two i.e., $A = 2$
2	It is known as parallel or multiple winding.	It is known as two-circuit or series winding.
3	EMF induced in the machine is less.	EMF induced in the machine is more.
4	Number of brushes required is equal to the number of parallel paths.	Number of brushes required is two
5	Different types of lap winding are: simplex and duplex lap winding.	Different types of wave winding are: progressive and retrogressive wave winding.
6	Efficiency of the machine using this winding is less.	Efficiency of the machine using this winding is high.
7	Equaliser ring is the additional component while using this winding.	No such component is required.
8	Winding cost is high, since more number of conductors is required.	Winding cost is low since less number of conductors is required.

Commutators

The cylindrical wedge-shaped hard-drawn copper bars or segments, which rotate along with the armature, are called commutators. A ring shape is formed around the armature shaft using these commutator segments. Each commutator segment is insulated from each other and also from the rotating shaft. Ends of each armature coil are connected to the commutator segment. The functions that the commutator of a DC generator serves are:

- Through the brushes, it provides a connection between the rotating armature conductors and the stationary external circuit.
- The alternating current induced in the armature conductor is converted into unidirectional current in a DC generator.

Brushes

A set of carbon or graphite components attached to the rotating armature gently via commutator, connecting the external circuit to the DC generator, is called *brushes*. The main purpose of brushes is to tap the electrical power generated in the rotating armature. Brush box or holder is a metal box supporting the brushes. Springs are used to adjust the pressure exerted on the commutator by the brushes.

End Housings

The components attached to the yoke ends and which provide support to the bearings is called end housings. Both bearing and brushes get support from front housings, whereas the rear housings support only the bearings.

Bearings

Fitting a high carbon steel ball or roller bearing in the machine can reduce the friction existing between the rotating and stationary parts of the DC generator.

Shaft

The mechanical power transfer to the machine is done with the help of a mild steel shaft having maximum breaking strength. Rotating parts of the DC generator, like armature core, commutator etc., are attached to the shaft.

Terminologies Used in Windings

The different terminologies related to windings are:

- ***Conductor:*** When the armature conductor is placed in armature slot, it experiences the influence of magnetic field. Total number of conductors is denoted by Z.
- ***Turn:*** When two different conductors placed in different armature slots are connected, it forms a turn, as shown in Figure 3.2 (a). Therefore, $Z = 2 \times$ Number of turns.
- ***Coil:*** When the turns are grouped together, it forms a coil, as shown in Figure 3.2 (b). If the number of coils is N_C and number of turns per coil is N_T, then the total number of turns and total number of conductors in a DC generator can be determined.
- ***Pole pitch:*** It is defined as the distance between two adjacent poles that is given by the ratio of armature slots to the number of poles in a DC generator. If the bore diameter of the armature is known, then the pole pitch is given by

$$\text{Pole pitch} = \frac{\pi \times \text{Bore diameter of armature, } D_b}{\text{Number of poles, } P}$$

- ***Coil side:*** The conductor group on one side forms one coil side and the conductor group on other side forms the second coil side, which is shown in Figure 3.2 (c).
- ***Coil span or coil pitch:*** It is defined as the distance between two coil sides, which are joined together to form a coil. Based on the relation between coil span and pole pitch, the coils are classified as: (a) full-pitch coil, if coil span and pole pitch are equal and (b) short-chorded or short-pitched coil, if coil span is less than the pole pitch.

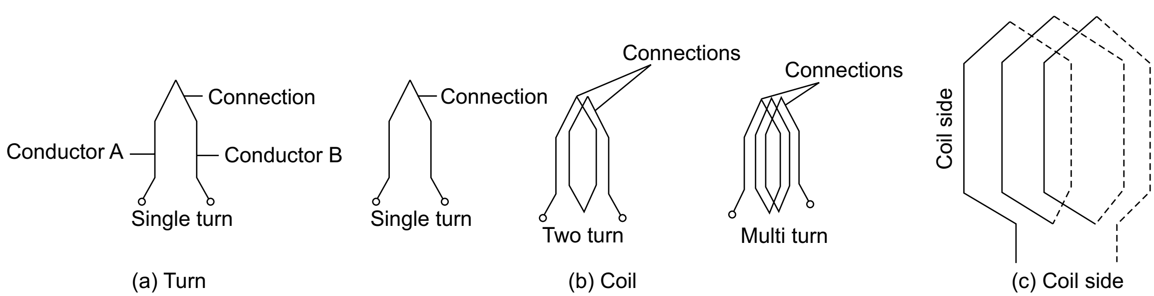


Figure 3.2 Terminologies Used in Windings

3.2.3 Working of a DC Generator

The working of a DC generator is illustrated in Figure 3.3.

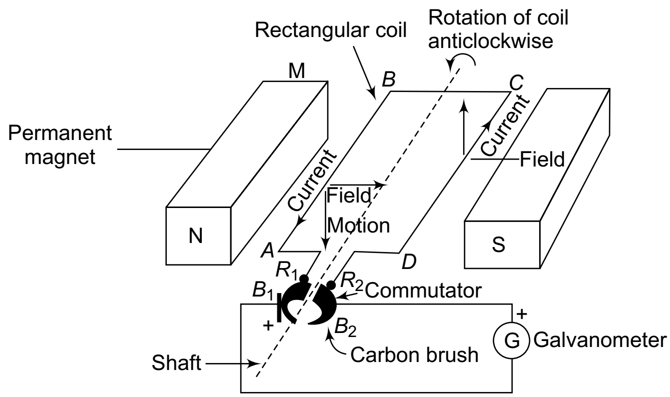


Figure 3.3 Working of a DC Generator

The basic working of a DC generator can be explained with the help of a single rectangular coil *ABCD*, as shown in Figure 3.3. This rectangular coil is placed between two opposite poles of the magnet. When the field coil wound over the magnet is excited by the DC source, the magnets get energised and magnetic flux is generated between these opposite poles. The rectangular coil connected to the shaft gets rotated in a specific direction driven by the prime mover. Therefore, the rectangular coil, when it rotates about its axis in the magnetic field i.e., when it moves from horizontal to vertical position, it cuts the magnetic field generated by the field windings. Hence, an emf is induced in both the sides of the coil i.e., on both *AB* and *CD*. Since the loop is closed, a current will be circulated through the loop. Using Fleming's right-hand rule, the direction of current can be determined. The emf generated in a DC generator is shown in Figure 3.4.

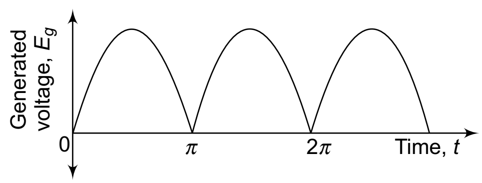


Figure 3.4 Unidirectional Emf Generated in a DC Generator

Requirement of a Split-ring Commutator

Consider an armature core, rotating in clockwise direction, which makes the current in the left conductor i.e., *AB* to move upwards and the current in the right conductor i.e., *CD* to move downwards, as shown in Figure 3.5(a). It is clear that in the load, the current is flowing from right to left, as shown in Figure 3.5(a). The direction of current in both the conductors remains same till the armature core completes a half rotation. After half-rotation of the armature core, the current through the conductors gets reversed since it is placed in the armature core i.e., conductor *AB* carries current in the downward direction and conductor *CD* carries current in the upward direction. This reversal of current in the armature conductor will make the current to flow from left to right in the load, which results in alternating current. But the output of DC generator should be unidirectional. Hence, some mechanism is required to make the current to flow through the load in the same direction, irrespective of the rotation of armature core. This can be achieved by using a split-ring commutator. When it is attached to the conductor, it rotates, thereby making the direction of current unidirectional in the load. This mechanism is shown in Figure 3.5(b).

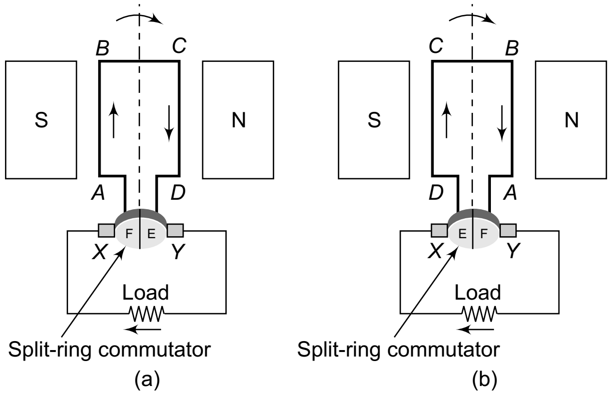


Figure 3.5 Working of a Split-ring Commutator

3.2.4 EMF Equation **[AU Nov/Dec, 2012]**

In a DC generator, when the armature core is rotated using the prime mover in the magnetic field, an induced emf is generated in the armature windings. This induced emf in the armature windings is called generated emf or armature emf, denoted as E_g. An expression for E_g is obtained as follows:

Let P be the total number of poles of the DC generator, ϕ be the flux produced per pole in Webers, Z be the total number of armature conductors, N be the armature speed in rpm, A be the number of parallel paths existing in the armature winding.

In one revolution of armature core, the total flux cut by one conductor of the armature is given by

$$d\phi_T = P\phi \tag{3.1}$$

The time taken by the armature core to complete one revolution is given by

$$dt = \frac{60}{N} \tag{3.2}$$

According to Faraday's law, the average emf induced in one armature conductor is given by

$$e_g = \frac{d\phi_T}{dt} \tag{3.3}$$

Substituting Eqns. (3.1) and (3.2) in the above equation, we get

$$e_g = \frac{P\phi N}{60} \tag{3.4}$$

Since the total number of conductors connected in series, per parallel path, is given by Z/A, the average emf induced in the armature is given by

$$E_g = e_g \times \frac{Z}{A}$$

Substituting Eqn. (3.4) in the above equation, we get the generated emf in the armature of DC generator as

$$E_g = \frac{\phi ZNP}{60A} \tag{3.5}$$

Therefore, from the above equation, it is clear that the induced emf in the DC generator is directly proportional to the speed and flux per pole. Hence, changing the direction of the magnetic field or the direction of the rotating armature core can change the polarity of the induced emf. But if both the magnetic field and armature core rotation are reversed, then the polarity of the induced emf remains the same.

Case (i) If the armature windings are lap wound, then $A = P$. Therefore, the induced emf in the DC generator is given by

$$E_g = \frac{\phi ZN}{60}$$

Case (ii) If the armature windings are wave wound, then $A = 2$. Therefore, the induced emf in the DC generator is given by

$$E_g = \frac{\phi ZNP}{120}$$

Example 3.1

An eight-pole wave-connected armature has 600 conductors and is driven at 625 rev/min. If the flux per pole is 20 mWb, determine the generated emf.

Solution

Given, $P = 8$, $Z = 600$, $N = 625$ rpm, $\phi = 20$ mWb

Since the DC generator has wave-connected armature winding, $A = 2$. It is known that the emf generated in a DC generator is given by

$$E_g = \frac{\phi ZNP}{60A}$$

Substituting the known value in the above equation, we get

$$E_g = \frac{20 \times 10^{-3} \times 8 \times 625 \times 600}{60 \times 2} = 500 \text{ V}$$

Example 3.2

A four-pole generator with wave wound armature has 51 slots, each having 24 conductors per slot. The flux per pole is 0.01 Wb. At what speed must the armature rotate to give an induced emf of 250 V? What will be the voltage developed, if the winding is lap-connected and the armature rotates at the same speed?

[AU April/May, 2008]

Solution

Given, $P = 4$, Number of slots = 51, Number of conductors per slot = 24, $\phi = 0.01$ Wb and $E_g = 250$ V

The total number of armature conductors in DC generator is

$$Z = 51 \times 24 = 1224$$

Case (i) When the DC generator has wave-connected armature winding i.e., $A = 2$.

It is known that the emf generated in a DC generator is given by

$$E_g = \frac{\phi ZNP}{60A}$$

Substituting the known values in the above equation, we get

$$250 = \frac{0.01 \times 4 \times N \times 1224}{60 \times 2}$$

Therefore, speed at which armature rotates is given by $N = 613$ rpm.

Case (ii) When the wave-connected armature winding is replaced by lap-connected winding, $A = P = 4$. It is given that the armature rotates at same speed as rotated in wave winding. Therefore, $N = 613$ rpm.

Hence, the emf generated in a DC generator with lap-connected winding is

$$E_g = \frac{0.01 \times 4 \times 613 \times 1224}{60 \times 4} = 125.05 \text{ V}$$

Example 3.3

A four-pole wave-wound DC armature has a bore diameter of 71.12 cm. It has 520 conductors and the ratio of pole arc to pole pitch is 0.63. If the armature is running at 720 rpm and the flux density in the air gap is 1.6 Wb/m^2 calculate the emf generated in the armature. Effective length of the armature conductor is 20.32 cm.

[AU Nov/Dec, 2011]

Solution

Given, Bore diameter, $D = 71.12$ cm, $\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.63$, $Z = 520$, $B = 1.6$ Wb/m^2, length of armature conductor = 20.32 cm and $N = 720$ rpm. For wave-winding, $A = 2$

The pole pitch of the DC generator is given by

$$\text{Pole pitch} = \frac{\pi D}{P}$$

i.e.,

$$\text{Pole pitch} = \frac{\pi \times 71.12}{4} = 55.8575 \text{ cm}$$

Since $\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.63$, we get

Pole arc = Pole pitch × 0.63 = 55.8575 × 0.63 = 35.1902 cm

It is known that there exists a relation between the area of the pole and pole arc, as given by

Pole area = Pole arc × Effective length of the armature conductor

Therefore, Pole area = 35.1902 × 20.32 = 715.066 cm^2

The relation between pole area and flux density is given by $B = \frac{\phi}{\text{Area}}$

Therefore, the flux produced per pole in a DC generator is given by

$$\phi = \text{Pole area} \times B = 715.066 \times 1.6 \times 10^{-5} \text{ Wb}$$

The emf generated in a DC generator is given by

$$E_g = \frac{\phi PNZ}{60A} = \frac{0.1144 \times 4 \times 720 \times 520}{60 \times 2} = 1427.71 \text{ V}$$

Example 3.4

A four-pole generator having wave-wound armature winding has 51 slots, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 rpm, assuming the flux per pole to be at 7 mWb?

[AU April/May, 2011]

Solution

Given, P = 4 Number of slots = 51, Number of armature conductors per slot = 20, ϕ = 7 mWb and N = 1500 rpm.

The total number of armature conductors in DC generator is

$$Z = 51 \times 20 = 1020$$

Since DC generator has wave-connected armature winding, we have A = 2.

The emf generated in a DC generator is given by

$$E_g = \frac{\phi ZNP}{60A} = \frac{7 \times 10^{-3} \times 4 \times 1500 \times 1020}{60 \times 2} = 357 \text{ V}$$

3.3 Armature Reaction

[AU April/May, 2010]

The effect of magnetic flux produced by the armature current on the main flux generated by poles, when it is excited by the DC source, is known as armature reaction. The armature reaction is represented in Figure 3.6. The Magnetic Neutral Axis (MNA) or plane or axis of commutation is defined as the axis along which no emf is produced in the armature conductors, since the armature conductors move parallel to the main flux. It is also defined as the axis perpendicular to the flux that passes through the armature. This axis will change if the armature rotates. Geometric Neutral Axis (GNA) is the axis perpendicular to the direction of main flux.

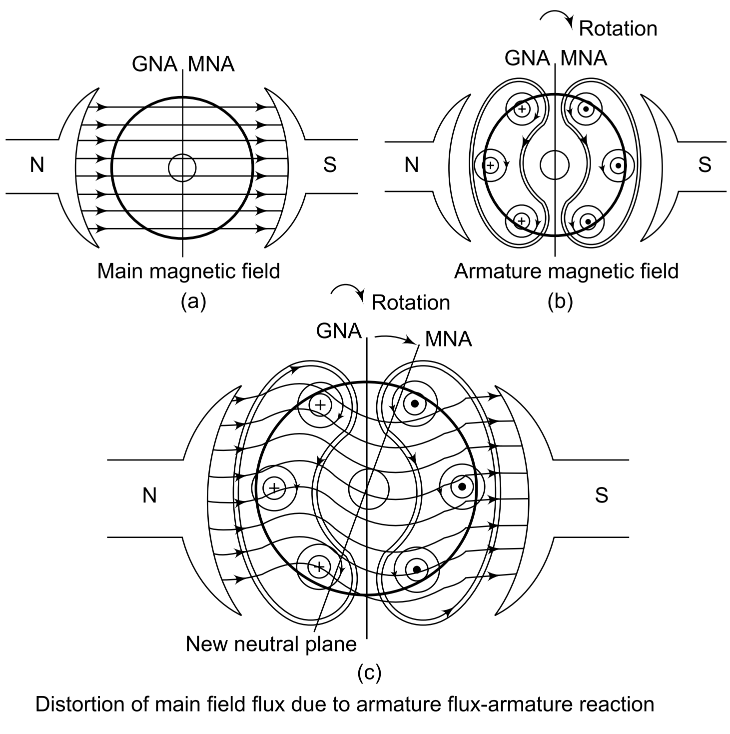


Figure 3.6 Armature Reaction in a DC Generator

When the armature circuit of the DC generator is connected to the load, the current through armature conductors is zero. Therefore, the field poles produce only one magnetic flux, called the main flux (ϕ). The symmetrical distribution of main flux about the polar axis i.e., the central line, connecting north and south poles, is shown in Figure 3.6 (a). In this case, both MNA and GNA coincide with each other. The magneto motive force (mmf) produced by the main flux, F_m, is perpendicular to the MNA.

When the current is allowed to flow through the armature winding, it produces its own flux called armature flux and the magneto motive force (mmf) produced by this armature flux is F_A. If the field winding is not excited by the DC source, the distribution of armature flux is shown in Figure 3.6 (b). In this case, F_A lies along MNA.

Now, when both the armature and main flux exist in the DC generator, the interaction between these two fluxes is shown in Figure 3.6(c). In this case, the main flux is no longer uniform. It is crowded at one pole tip and weakened at another pole tip and hence there is a shift in the MNA. This interaction between armature and main field flux causes two effects:

- Demagnetising effect: Due to this, the net flux produced in the generator gets reduced, which further reduces the generated emf and the terminal voltage of the generator.
- Cross-magnetising effect: Due to this, sparking occurs between brushes and the commutator, which results in poor commutation.

3.3.1 Methods to Reduce the Effect of Armature Reaction

The effect of armature reaction in DC generator can be reduced by two ways:

- Using compensating winding in the pole shoes
- Using interpoles

Compensating Winding

The armature flux produced in the armature due to the current flowing through armature conductors can be neutralised using concentric compensating windings placed in the pole shoes, as shown in Figure 3.7. These windings are connected in series with the armature windings in such a way that it carries the same armature current, but in the opposite direction. Since these windings carry current in opposite direction, it produces a magnetic field, which opposes the magnetic field produced in the armature, thereby neutralising the cross-magnetising effect of the armature reaction.

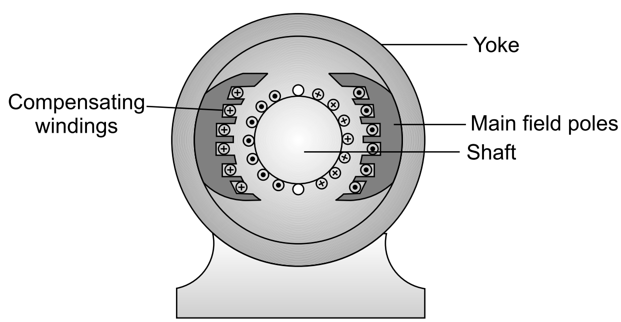


Figure 3.7 Compensating Winding Placed in Pole Shoes

Using Interpoles

Another way to reduce the effect of armature reaction is by placing small auxiliary poles, called interpoles, in between the main field poles, as shown in Figure 3.8. These poles have a few turns of large wire and are connected in series with the armature. These interpoles are wound and placed in such a way that they have the same magnetic polarity as that of the main pole but it is ahead when the direction of rotation is considered. The magnetic field generated by the interpoles produces the same effect as that of compensating winding, which cancels the armature reaction by causing a shift in the neutral plane. This shift in neutral plane is equal and opposite to the shift caused by the armature reaction.

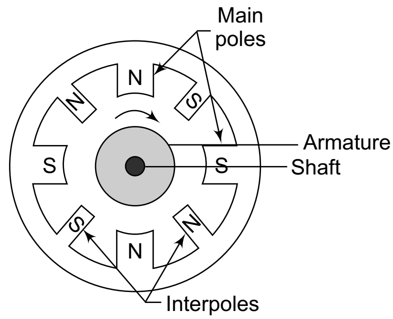


Figure 3.8 Interpoles to Reduce Armature Reaction Effect

3.4 Commutation **[AU April/May, 2010]**

Commutation is the process of reversal of current, taking place in the short-circuited armature coil. This process takes place only when the particular armature coil is passing through the interpolar axis. In general, for a lap-wound generator, the number of coils for which the commutation takes place simultaneously is P, whereas, in a wave-wound generator, it is two coil sets of $\frac{P}{2}$ number of coils, for which the commutation takes place simultaneously. The time taken for the commutation process to happen is known as the commutation period, T_C. Based on the time taken for the reversal of current to happen in a coil, the commutation is classified as:

- Ideal commutation: Occurs if the reversal of current takes place within the commutation period.
- Retarded or under or slow commutation: Occurs if the time taken for the reversal of current is greater than the commutation period.
- Over or accelerated or fast commutation: Occurs if the time taken for the reversal of current is less than the commutation period.

It is noted that the sparking occurs between the brush and the commutator segment, if the commutation is not ideal, which causes progressive damage to both the components.

3.4.1 Commutation Process

Commutation process in a DC generator is explained by assuming that the width of commutator bar and the brush are equal and negligible thickness of insulation between the commutator bars. The concept behind the commutation process can be understood by considering five instances of commutator working with respect to brushes, as shown in Figure 3.9. Here, the armature of the DC generator is rotating from left to right.

The step-by-step commutation process is given below: coil *B* in Figure 3.9 (a), carrying current from left to right, is about to be short-circuited as the brush is about to come in contact with commutator segment *a*. The total current obtained from the brush is 40 A, as each coil carries 20 A. In Figure 3.9 (b), since coil *B* has entered the commutation process, the current through it gets reduced to 10 A. But the current through the brush is always 40 A, since the other 10 A flows through commutator segment *a*. Further, if the armature is moved, as shown in Figure 3.9 (c), the brush comes in contact with half of commutator segment *a* and commutator segment *b*. Therefore, in this case, the current through coil B is zero, as the brush current is

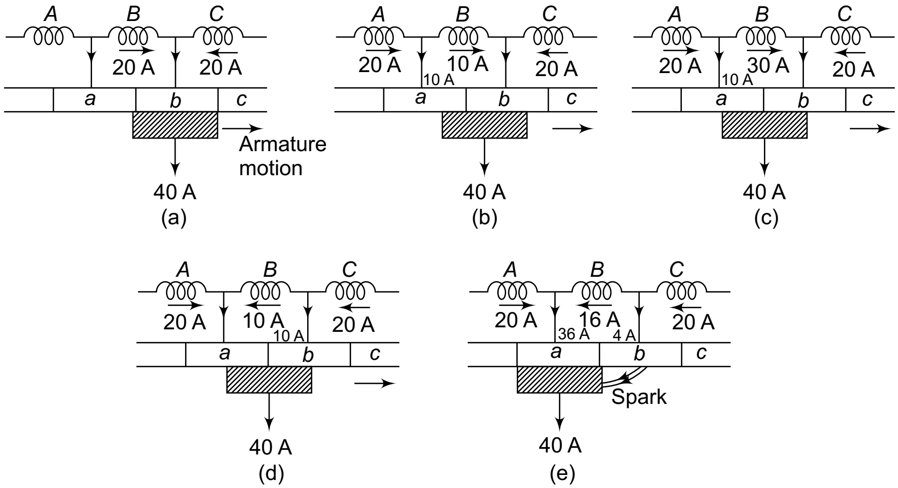


Figure 3.9 Commutation Process

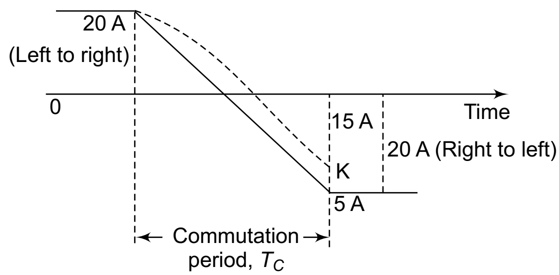


Figure 3.10 Commutation Graph

supplied by coil *C* and coil *A*. In Figure 3.9 (d), coil *B*, which was carrying current from left to right carries current from right to left as the brush comes in contact with three-fourth of the commutator segment *a*. Coil *B* will carry a current of 10 A in the opposite direction, which combines with the current in coil A, to make 40 A at the brush. Figure 3.9 (e) shows the end of commutation period. For ideal commutation, the total current carried by coil *B* should be 20 A in the reverse direction. But it is seen that only 15 A current is carried by coil *B*. This difference in current causes spark in the brush and the commutator. This process of commutation is shown in Figure 3.10.

The different factors that affect ideal commutation are the production of self-induced emf or reactance voltage in the coil undergoing commutation, as the coil possesses appreciable amount of self-inductance. The different methods to improve the commutation process are:

- Resistance Commutation: In this method, low-resistance copper brushes are replaced with high-resistance carbon brushes, as it forces the current in the short-circuited coil to change within the commutation period.
- EMF commutation: In this method, a voltage opposite to the reactance voltage is induced in the coil, which undergoes commutation process to neutralise the reactance voltage. If the injected voltage and the reactance voltage are made equal, it results in fast commutation and there will be no spark in the process. The two methods of EMF commutation are: by giving a shift in the brushes and by using interpoles between the main poles.

3.5 Types of DC Generators **[AU Nov/Dec, 2012]**

The DC generators are classified, as shown in Figure 3.11, based on the excitation given to the field windings, as:

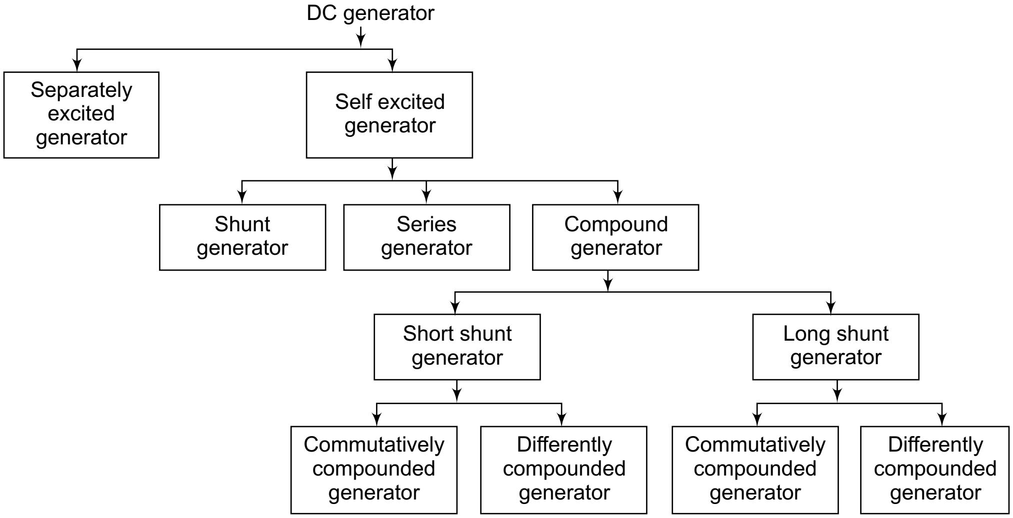


Figure 3.11 Types of DC Generators

- ***Separately excited DC generator:*** The required power for exciting the field windings is obtained from a separate DC source.
- ***Self-excited DC generator:*** The required power for exciting the field windings is obtained from the power developed in the armature of the DC generator

The self-excited DC generator is further classified, based on the connection between the field winding and armature winding, as

- DC shunt generator: The field winding is parallel to the armature winding.
- DC series generator: The field winding is in series with the armature winding.
- DC compound generator: It consists of two windings, one is connected in series and the other is connected in parallel to the armature windings. Based on these winding connections, it is further classified as:
  - Long-shunt DC compound generator
  - Short-shunt DC compound generator

If the magnetic flux generated by the series field winding aids the magnetic flux generated by the shunt field winding, the DC compound generator is known as cumulative compounded. Conversely, if the magnetic flux generated by the series field winding opposes the magnetic flux generated by the shunt field winding, the DC compound generator is known as differentially compounded. It is noted that both long-shunt and short-shunt compound generators can be either cumulative or differentially compounded.

3.6 Electrical Equivalent Circuits, Current and Voltage Equations of DC Generators

Let R_a, R_{sh} and R_{se} be the resistances of armature, shunt field and series field respectively; I_a, I_{sh} and I_{se} be the winding currents through the armature, shunt field and series field respectively; I_L be the load current, V and V_{sh} be the terminal voltage or voltage across the load and voltage across the shunt field winding respectively; V_b and V_{ar} be the voltage drop across the brush contact resistance and voltage drop due to armature reaction respectively and E_g is the emf generated at the armature of the DC generator.

Terminal Markings

In a DC generator, the symbols used to indicate shunt field windings, series field windings and armature terminals are Z and ZZ, Y and YY and A and AA, respectively.

3.6.1 Separately Excited DC Generator

If the field winding in a DC generator is energised or excited using an external DC source, it is called a separately excited DC generator. The current flowing through the field winding is called field current. The magnetic flux generated by the poles of the magnet is directly proportional to the field current in the unsaturated region of the magnetic material and remains constant in the saturated region. The electrical equivalent circuit of a separately excited DC generator is shown in Figure 3.12. The load is connected across its armature terminals.

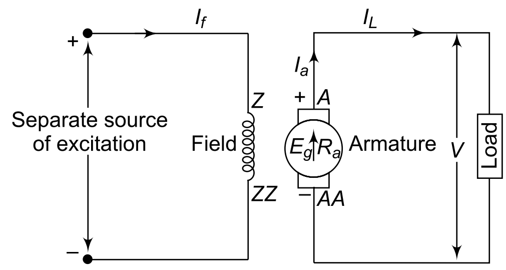


Figure 3.12 Electrical Equivalent of a Separately Excited DC Generator

From Figure 3.12, the load current equation is written as

$$I_L = I_a$$

Also, applying KVL to the armature circuit, we get

$$E_g = V + I_aR_a + V_b + V_{ar}$$

where, V_b = 2 × Voltage drop due to each brush contact since there are two brushes in the DC generator.

If V_b and V_{ar} are negligible, then

$$E_g = V + I_aR_a$$

The power developed in the DC generator is given by

$$P_d = E_gI_a$$

But, the output power of the DC generator is given by

$$P_a = VI_L = VI_a$$

3.6.2 DC Shunt Generator

If the field winding in a DC generator is connected in parallel or shunt to the armature winding, it is called a DC shunt generator. Here, the field winding is called shunt field winding. Since this field winding has many turns of thin wire with very high resistance, of the order of 100 Ω, the current, I_{sh} flowing through the shunt field winding is very small. The electrical equivalent circuit of a self-excited DC shunt generator or a DC shunt generator is shown in Figure 3.13.

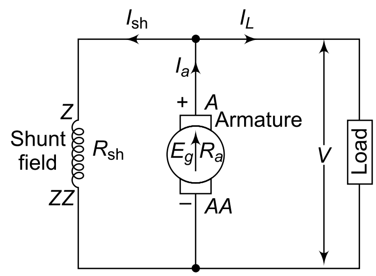


Figure 3.13 Electrical Equivalent of a DC Shunt Generator

From Figure 3.13, the relation among I_a, I_L and I_{sh} is

$$I_a = I_L + I_{sh}$$

Since the terminal voltage is available across the shunt field winding, we get

$$I_{sh} = \frac{V}{R_{sh}}$$

Applying KVL to the armature circuit, we get

$$E_g = V + I_a R_a + V_b + V_{ar}$$

If V_b and V_{ar} are negligible, then

$$E_g = V + I_a R_a$$

The power developed in the DC shunt generator is given by

$$P_d = E_g I_a$$

But, the output power of the DC shunt generator is given by

$$P_a = VI_L$$

3.6.3 DC Series Generator

If the field winding in a DC generator is connected in series with the armature winding, it is called a DC series generator. Here, the field winding is called a series field winding. Since this field winding has less number of turns of thick wire with very low resistance, of the order of 1 Ω, the current flowing through the series field winding, I_{se} is very large. The electrical equivalent circuit of a self-excited DC series generator or a DC series generator is shown in Figure 3.14.

From Figure 3.14, the current relation is written as

$$I_a = I_L = I_{se}$$

Applying KVL to the armature circuit, we get

$$E_g = V + I_a(R_a + R_{se}) + V_b + V_{ar}$$

If V_b and V_{ar} are negligible, then

$$E_g = V + I_a(R_a + R_{se})$$

The power developed in the DC series generator is given by

$$P_d = E_g I_a$$

But, the output power of the DC series generator is given by

$$P_a = VI_L = VI_a$$

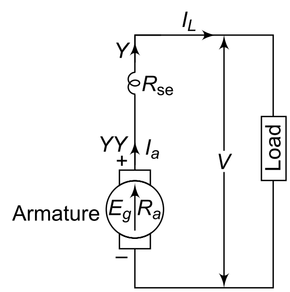


Figure 3.14 Electrical Equivalent of a DC Series Generator

In DC series generator, the machine resistance or internal resistance of the machine is the sum of armature resistance and series field winding resistance.

3.6.4 DC Compound Generator

In a DC compound generator, two sets of field winding exist on each pole: (i) series field winding with few turns of thick wire connected in series with armature winding and (ii) shunt field winding with many turns of thin wire connected in parallel with the armature winding. Since the DC generator has both shunt and series field windings, it is called a DC compound generator.

If the magnetic flux generated by the series field winding aids the magnetic flux generated by the shunt field winding, the DC generator is said to be cumulative compounded. Conversely, if the magnetic flux generated by the series field winding opposes the magnetic flux generated by the shunt field winding, the

DC generator is known as differentially compounded. Based on the connection among the series field, shunt field and armature windings, the DC compound generator is classified as:

- Long-shunt DC compound generator
- Short-shunt DC compound generator

It is noted that both long-shunt and short-shunt compound generators can be either cumulative or differentially compounded.

Long-shunt DC Compound Generator

In a compound generator, if the shunt field winding is connected in parallel to both armature and series field windings, it is called a DC long-shunt compound generator. The electrical equivalent of a DC long-shunt compound generator is shown in Figure 3.15.

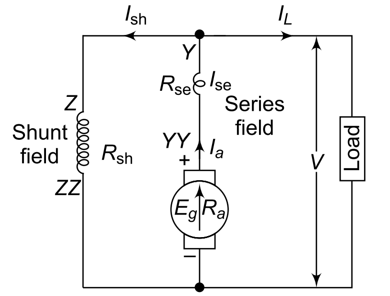


Figure 3.15 Electrical Equivalent of a DC Long-Shunt Compound Generator

From Figure 3.15, the relation among the currents in a DC long-shunt compound generator is written as:

$$I_a = I_L + I_{sh}$$
$$I_{se} = I_a$$

Since the full terminal voltage is applied across the shunt-field winding, we get

$$I_{sh} = \frac{V}{R_{sh}}$$

Applying KVL to the armature circuit, we get

$$E_g = V + I_a(R_a + R_{se}) + V_b + V_{ar}$$

If V_b and V_{ar} are negligible, then

$$E_g = V + I_a(R_a + R_{se})$$

The power developed in the DC long-shunt compound generator is given by

$$P_d = E_g I_a$$

But, the output power of the DC long-shunt compound generator is given by

$$P_a = VI_L$$

Short-shunt DC Compound Generator

In a compound generator, if the shunt field winding is connected in parallel only to the armature winding, then it is called a DC short-shunt compound generator. The electrical equivalent of DC short shunt compound generator is shown in Figure 3.16.

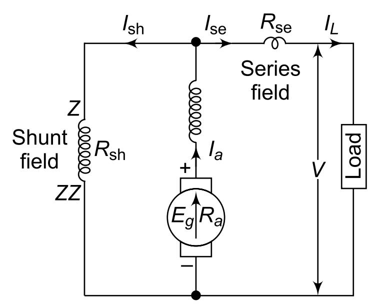


Figure 3.16 Electrical Equivalent of a DC Short-Shunt Compound Generator

From Figure 3.16, the relation among the different currents in DC long shunt compound generator is written as

$$I_L = I_{se}$$
$$I_a = I_L + I_{sh} = I_{se} + I_{sh}$$

Here, $I_{sh} = \dfrac{V_{sh}}{R_{sh}}$ and $V_{sh} = V + I_L R_{se}$

Applying KVL to the armature circuit, we get

$$E_g = V + I_a R_a + I_L R_{se} + V_b + V_{ar}$$

If V_b and V_{ar} are negligible, then

$$E_g = V + I_a R_a + I_L R_{se}$$

The power developed in the DC short shunt compound generator is given by

$$P_d = E_g I_a$$

But, the output power of the DC short shunt compound generator is given by

$$P_a = VI_L$$

The current and voltage equations of different types of DC generators are given in Table 3.2.

Table 3.2 *Current and Voltage Equations of DC generators*

Type of Generator	Armature Current	EMF Equation	Field Current
Separately excited	$I_L = I_a$	$E_g = V + I_a R_a$	I_f
DC shunt	$I_a = I_L + I_{sh}$	$E_g = V + I_a R_a$	$I_{sh} = \frac{V}{R_{sh}}$
DC series	$I_a = I_L = I_{se}$	$E_g = V + I_a (R_a + R_{se})$	$I_{se} = I_L$
DC long-shunt	$I_a = I_L + I_{sh}$	$E_g = V + I_a(R_a + R_{se})$	$I_{sh} = \frac{V}{R_{sh}}$, $I_{se} = I_a$
DC short-shunt	$I_a = I_L + I_{sh} = I_{se} + I_{sh}$	$E_g = V + I_a R_a + I_L R_{se}$	$I_{sh} = \frac{V_{sh}}{R_{sh}}$, $V_{sh} = V + I_2 R_{se}$

Example 3.5

A four-pole lap-wound DC shunt generator has a useful flux/pole of 0.6 Wb. The armature winding consists of 200 turns, each turn having a resistance of 0.003 Ω. Calculate the terminal voltage when running at 1000 rpm, if armature current is 45 A.

Solution

Given, $P = 4$, $N = 1000$ rpm, $I_a = 45$ A, $\phi = 0.6$ Wb, Number of turns = 200, Resistance per turn = 0.003 Ω
Here,
Armature resistance, $R_a = 0.003 \times 200 = 0.6\ \Omega$
and the total number of armature conductors, $Z = 2 \times$ Number of turns $= 2 \times 200 = 400$.

In DC generator, the emf generated, $E_g = \dfrac{\phi PNZ}{60A}$.

$$E_g = \frac{0.6 \times 4 \times 1000 \times 400}{60 \times 4} = 4000 \text{ V}$$

The voltage equation for a DC shunt generator is given by

$$E_g = V + I_a R_a$$

$$4000 = V + 45 \times 0.6$$

Therefore, the terminal voltage of the DC shunt generator is $V = 3973$ V

Example 3.6

A DC shunt generator has a terminal voltage of 160 V and a no-load induced emf of 168 V. The resistance of armature and field are 0.03 Ω and 20 Ω. Find the armature current, field current and load current. Neglect armature reaction. **[AU Nov/Dec, 2010]**

Solution

Given, Terminal voltage, $V = 160$ V, No-load induced emf, $E_g = 168$ V, $R_a = 0.03\ \Omega$ and $R_{sh} = 20\ \Omega$.

In a DC shunt generator, shunt field current, $I_{sh} = \dfrac{V_t}{R_{sh}} = \dfrac{160}{20} = 8\,\text{A}$

The voltage equation of a DC shunt generator is given by

$$E_g = V + I_a R_a$$

Substituting the known values in the above equation, we get

$$168 = 160 + I_a \times 0.03$$

Solving the above equation, we get the armature current as

$$I_a = 266.667 \text{ A}$$

Therefore, the load current is

$$I_L = I_a - I_{sh} = 266.667 - 8 = 258.667 \text{ A}$$

Example 3.7

A 20 kW, four-pole shunt generator has a terminal voltage of 250 V when running at 400 rpm. The armature has a resistance of 0.16 Ω and consists of 652 conductors, which are lap-wound. The diameter of the pole-shoe circle is 38 cm. The poles are 20 cm long and subtend an angle of 60° Calculate the flux density in the air gap. Neglect shunt field current. **[AU Nov/Dec, 2011]**

Solution

Given, Diameter of pole-shoe circle, $D = 38$ cm, pole length = 20 cm, subtended angle, $\theta = 60°$, $P_{\text{out}} = 20$ kW, terminal voltage, $V = 250$ V, $R_a = 0.16\ \Omega$, $P = 4$, $N = 400$ rpm, $Z = 652$.

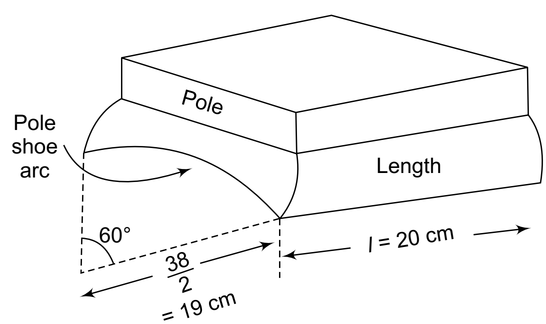


Figure E3.7

Pole arc = $R\theta = 19 \times \dfrac{\pi}{3} = 19.897\,\text{cm}$.

The relation among the pole area, pole arc and pole length is given by

$$\text{Pole area} = \text{Pole arc} \times \text{Pole length}$$

Therefore, pole area = $19.897 \times 20 = 397.935\ \text{cm}^2$

The load current, $I_L = \dfrac{P_{\text{out}}}{V} = 80\ \text{A}$.

In a DC shunt generator, $I_a = I_L + I_{sh}$. Since I_{sh} is neglected, we get

$$I_L = I_a = 80 \text{ A}$$

The voltage equation of a DC shunt generator is

$$E_g = V_t + I_a R_a$$

Substituting the known values, we get

$$E_g = 250 + 80 \times 0.16 = 262.8 \text{ V}$$

We know that, $$E_g = \frac{\phi PNZ}{60A}$$

$$262.8 = \frac{\phi \times 4 \times 400 \times 652}{60 \times 4}$$

Therefore, the flux per pole is

$$\phi = 0.06046 \text{ Wb}$$

Then, the flux density per pole is given by

$$B = \frac{\phi}{\text{Area}} = \frac{0.06046}{397.935 \times 10^{-4}} = 1.5193 \text{ Wb/m}^2$$

Example 3.8

A long-shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop. **[AU Nov/Dec, 2006]**

Solution

Given, $\boldsymbol{R_a}$ = 0.05 Ω, R_{se} = 0.03 Ω, R_{sh} = 250 Ω, I_L = 50 A, V = 500 V, V_{brush}/brush = 1 V.

The long-shunt compound DC generator with the given details is shown in Figure E3.8.

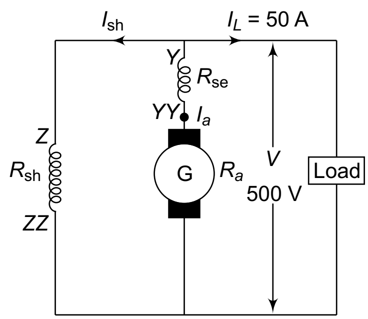


Figure E3.8

Here, $$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2 \text{ A}$$

The armature current is given by

$$I_a = I_L + I_{sh} = 50 + 2 = 52 \text{ A}$$

Considering the brush drop in a long-shunt DC compound generator, its voltage equation is given by

$$E_g = V + I_a R_{se} + I_a R_a + \text{Brush drop}$$

Since the number of brushes existing in the machine is 2, we get the generated voltage as

$$E_g = 500 + 52 \times 0.03 + 52 \times 0.05 + 1 \times 2 = 506.16 \text{ V}$$

3.7 Characteristics of DC Generators **[AU April/May, 2015]**

The explanation of the relation between the loads, excitation and terminal voltages, through graphs, for different DC generators is known as characteristics of DC generators. Studying the characteristics of DC generators help in selecting a suitable generator for a specific application. The important characteristics of DC generators are:

- **No-load characteristic or open-circuit characteristic (OCC)**
  The characteristic obtained by drawing the graph between the emf generated at no-load condition, E_0 and field current, I_f at a constant prime mover speed is known as no-load characteristic or open-circuit characteristic (OCC) or magnetisation characteristics. It gives the magnetisation curve of the material used in the magnets. In practice, the shape of the curve is same for all DC generators.
- **Internal or total characteristics**
  The characteristic obtained by drawing the graph between the emf generated in the armature E_g and armature current, I_a is known as internal or total characteristic. This characteristic is important in designing the DC generator.
- **External characteristic**
  The characteristic obtained between the terminal voltage, V and the load current I_L is known as external characteristic. It is also known as performance characteristic or voltage regulating curve. In general, this characteristic curve lies below the internal characteristic.

3.7.1 Separately Excited DC Generator

No-load Characteristic

The circuit diagram for obtaining no-load characteristic is shown in Figure 3.17 (a). A potentiometer is used to vary the field current in the DC generator. The open-circuit characteristics curve of a separately excited DC generator is shown in Figure 3.17 (b). The emf generated in the DC generator is given by Eqn. (3.5). Therefore, for a given speed of the generator, $E_g \ \alpha \ \phi$, since the other terms are constant in Eqn. (3.5).

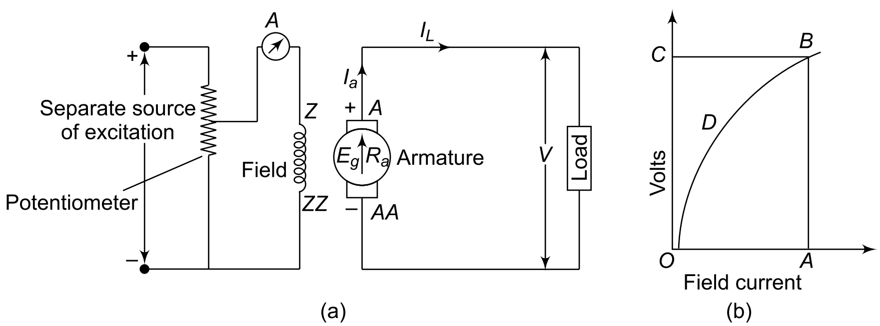


Figure 3.17 (a) Circuit Diagram of No-Load Characteristic (b) Characteristic Curve

Therefore, it is clear that if I_f increases from its initial value, the magnetic flux ϕ increases and hence induced emf, E_g also increases till the poles get saturated. This is represented as the straight-line OD in Figure 3.17 (b). Once the poles get saturated, a large increase in I_f is required to increase the induced emf, which is represented by the curve ODB in Figure 3.17 (b).

Load Characteristics

The load characteristic of the separately excited DC generator is obtained from the open-circuit characteristic, if the demagnetising effect of armature reaction and armature resistance is known. The load characteristic of a separately excited DC generator is shown in Figure 3.18.

The procedure to obtain these characteristics is given as follows:

The open-circuit characteristic of the DC generator is redrawn on the basis of field ampere-turns, as indicated by OMB in Figure 3.18. It is clear from Figure 3.18 that the field ampere-turns required to generate the rated no-load voltage, E_0 is OA. Now, when the generator is loaded, the no-load voltage gets decreased due to the demagnetising effect of armature reaction. Therefore, to generate the same no-load emf at load condition, the field ampere-turns must be increased by an amount AC. Hence, the curve LNS, which is parallel to OMB, shows the relation between the emf generated at load condition and field ampere-turns. Hence, using the voltage equation, $V = E_g - I_aR_a$, the armature resistance drop is subtracted from each point in the curve LNS to obtain the load characteristic curve QTP of a DC generator, as shown in Figure 3.18. The right-angled triangle BDE is known as drop reaction triangle.

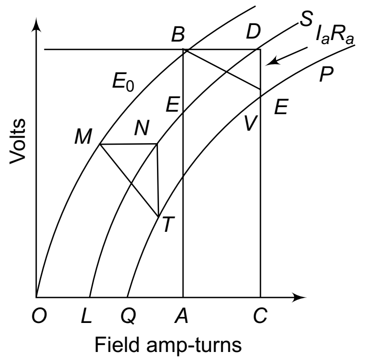


Figure 3.18 Load Characteristics of a Separately Excited DC Generator

Internal Characteristics

At no-load condition, the graph between the generated emf, E_0 and armature current, I_a is almost constant due to the absence of armature reaction and armature resistance drop and it is shown as Graph I in Figure 3.19. But when the load is increased step-by-step in a DC generator, there will be a drop in the generated emf due to the armature reaction. Therefore, if the armature reaction drop is subtracted from each point in Graph I for each load, the internal characteristics can be obtained as shown in Graph II of Figure 3.19.

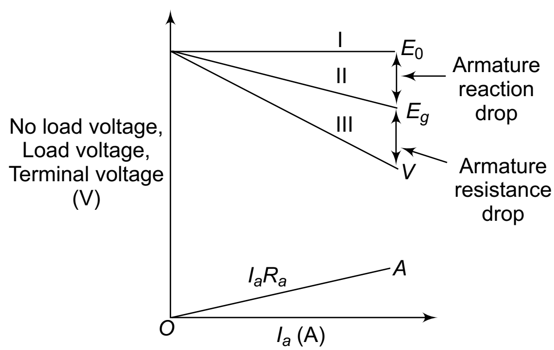


Figure 3.19 Internal and External Characteristics of a Separately Excited DC Generator

External Characteristics

The external characteristics of a separately excited DC generator are obtained by subtracting the armature resistance drop from internal characteristics at each and every point and are shown in Figure 3.19.

3.7.2 DC Shunt Generator

No-load Characteristics

The no-load characteristic of any self-excited DC generator is obtained in a similar way. The field winding is disconnected from the circuit and it is excited by a separate DC source, as shown in Figure 3.20 (a).

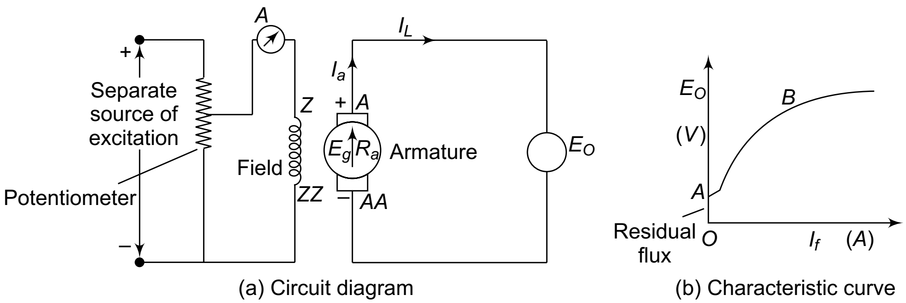


Figure 3.20 No-Load Characteristic of a DC Shunt Generator

The armature of the machine is allowed to rotate at a constant speed, N. If the field current applied to the field winding is increased with the help of a potentiometer, the no-load emf generated, E_o, starts increasing. If these are plotted, the no-load characteristic of a DC shunt generator can be obtained, as shown in Figure 3.20(b). As there exists residual magnetism in the poles, a small emf (i.e., OA) is generated even when there is no excitation given to the field windings. Hence, the no-load characteristic of a DC shunt generator does not start from zero. It can be observed that the major portion of this characteristic is linear. Beyond the saturation limit of the poles, there will not be any appreciable increase in the generated voltage even for large increase in field current. It is clear that this characteristic for a speed greater than N will lie above this curve and for a speed less than N will lie below this curve.

Voltage Build-up in a DC Shunt Generator

The loading in a DC shunt generator should be done only after the generator reaches a no-load voltage, E_0. The process by which the DC shunt generator is able to generate E_0 is known as voltage build-up process, as shown in Figure 3.21.

Here, a residual flux exists in the DC shunt generator, which generates a small emf, OA in it. This emf circulates a small current, OB, through the field winding. This current aids the residual flux and increases the flux generated by the poles. Since the flux is increased, the emf generated will increase to OC. This increase in emf circulates the larger current OD to flow through the field winding. This again increases the flux and hence the emf generated increases. This process continues till the machine generates the rated no-load emf, E_0. Beyond E_0, as the poles get saturated, there will be no further increase in the emf generated.

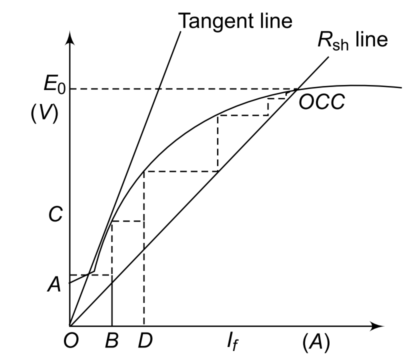


Figure 3.21 Voltage Build-up in a DC Shunt Generator

Conditions for Voltage Build-up in a DC Shunt Generator

The factors that affect the voltage build-up in a DC shunt generator are:

- ***Residual flux:*** If there is no residual flux, no emf is induced. Hence, there is no further increase in field flux and the induced emf remains zero.
- ***Reversal connection of shunt-field coil:*** Shunt-field winding should be properly connected with respect to armature windings so that the flux produced by it aids the residual flux.
- ***Shunt field circuit resistance:*** The shunt-field winding resistance R_{sh} should be equal to or less than the critical resistance, R_s for the voltage to build up.

- ***Speed of armature:*** The speed at which the armature is getting rotated must be equal to or greater than the critical speed, N_c, for the voltage to build up.

Critical Field Resistance, R_c

The maximum value of shunt field resistance with which the DC shunt generator will just build up voltage is known as critical field resistance, R_c. If the shunt field resistance is increased beyond R_c, then the DC shunt generator will fail to build up the voltage.

Critical speed, N_c

The minimum speed at which the armature must be rotated so that the DC shunt generator will just build up the voltage is known as critical speed, N_c. It is also defined as the speed at which the shunt field resistance, R_{sh}, is equal to the critical field resistance i.e., $R_{sh} = R_c$. The DC shunt generator fails to build up the voltage if the armature is rotated below this critical speed.

Determination of R_c and N_c

Both R_c and N_c can be determined from the no-load characteristics of a DC shunt generator, as shown in Figure 3.22 The no-load characteristics of a DC shunt generator at rate speed, N_R, are drawn. A tangent is drawn at the initial portion of this curve. The slope of this tangent gives the critical resistance, $R_c = \dfrac{AC}{OC}$. Now, the shunt field resistance, R_{sh} of the machine is measured and a line is drawn such that the slope of the line is R_{sh}. Any point which cuts the R_c, R_{sh} and X-axis line at A, B and C respectively is selected. Now, the critical speed is calculated as $N_c = \dfrac{BC}{AC} \times N_R$.

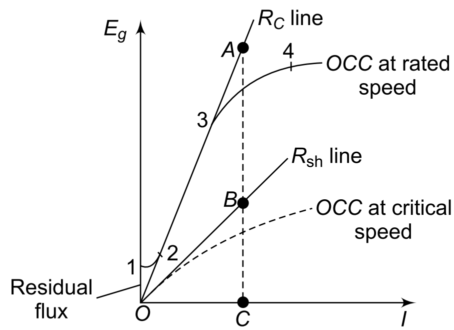


Figure 3.22 Determination of R_c and N_c

External Characteristics

The DC shunt generator is loaded once the voltage has been built up in it. During loading, the terminal voltage V drops, when there is an increase in load current. The reasons for this terminal voltage drop are:

- Armature resistance drop: Since the armature resistance consumes more voltage when the load current increases, the terminal voltage gets reduced.
- Armature reaction drop: Since the demagnetising effect of armature reaction reduces the field flux, there is a reduction in the emf generated and the terminal voltage.
- The reduction in terminal voltage due to armature resistance drop and armature reaction drop reduces the field current, which further reduces the emf generated and the terminal voltage.

The circuit diagram to obtain the external characteristics is shown in Figure 3.23. Ammeters A_1 and A_2 are used to denote the load and field currents for different load conditions. The terminal voltage across the load is given by the voltmeter reading. Therefore, the ammeter and voltmeter readings for different load conditions are noted and a graph between the terminal voltage, V and load current, I_L is drawn to obtain the external characteristics.

The external characteristic of the DC shunt generator is shown in Figure 3.24.

During normal running condition, when load increases, there will be an increase in load current and hence, there will be a decrease in terminal voltage. But if any effort is made to increase the load current beyond a certain limit, there will be drastic decrease in terminal voltage due to excessive armature reaction

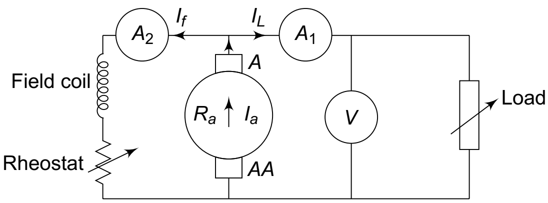


Figure 3.23 Circuit Diagram to Obtain External Characteristics

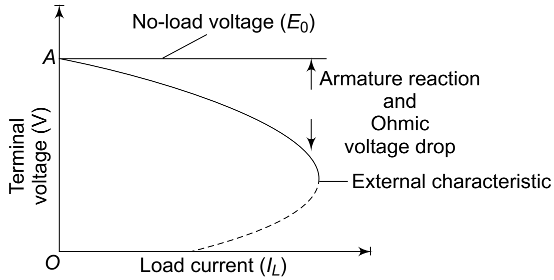


Figure 3.24 External Characteristics of a DC Shunt Generator

and Ohmic losses. Hence, beyond this limit, any further increase in load results in decreasing load current. Consequently, the external characteristic curve turns back, as shown by the dotted line, and it cuts the current axis. The terminal voltage V at that particular point is zero even when there exists a residual emf in the DC shunt generator.

Internal Characteristics

In a DC shunt generator, $I_a = I_L + I_{sh}$ and $E_g = V + I_aR_a$. Therefore, the internal characteristics curve can be obtained from external characteristic curve, as shown in Figure 3.25.

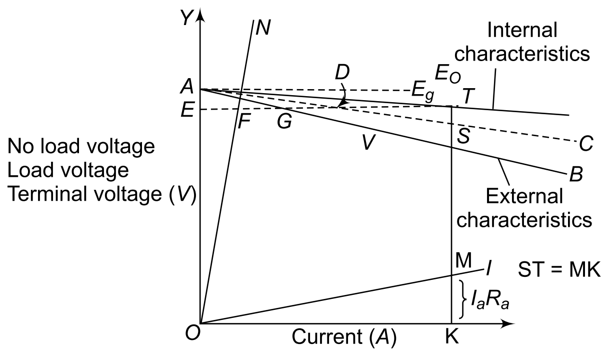


Figure 3.25 Internal Characteristic of a DC Shunt Generator

In Figure 3.25, the line AB represents the external characteristic of a DC shunt generator and OB represents R_{sh} line. The field currents for different values of V are obtained by calculating the horizontal distance between Y-axis and OB line. The armature current line AC is obtained by adding these field currents horizontally to the existing external characteristic line i.e., OB, such that GD = EF. The armature resistance drop line i.e., OI for different armature current is drawn i.e., for an armature current OK, the armature resistance drop is MK. Now, if this armature resistance drop is added to the external characteristic curve such that ST = MK, the internal characteristic curve can be obtained.

3.7.3 DC Series Generator

No-load Characteristics

The no-load characteristics of a DC series generator can be obtained with the help of the circuit shown in Figure 3.26. The procedure to obtain these characteristics is the same as that of a DC shunt generator. The circuit for obtaining the no-load characteristics of a DC series generator is shown in Figure 3.26.

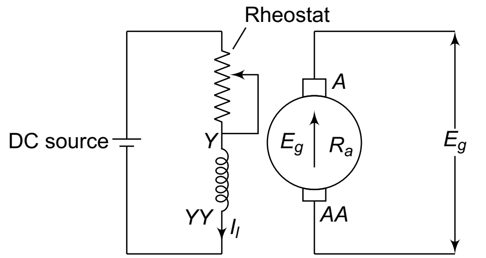


Figure 3.26 Circuit to Obtain No-Load Characteristics of a DC Series Generator

Internal Characteristics

Since the field winding is connected in series, the relation among the currents is $I_L = I_a = I_{se}$. When a DC series generator is loaded, there will be a drop in the generated voltage due to the demagnetising effect of the armature reaction. Therefore, the additional exciting current required to generate the same emf is AB. Therefore, the point B lies on the internal characteristics of the DC series generator. In a similar way, all other points in the internal characteristics can be obtained, as shown in Figure 3.27.

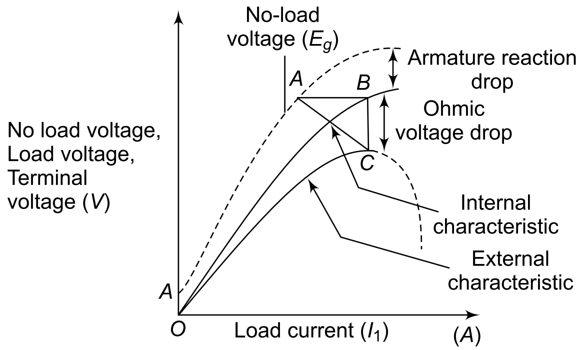


Figure 3.27 Characteristics of a DC Series Generator

External Characteristics

If the armature voltage drop, as shown by line OD, is subtracted from the internal characteristics, i.e., BC = GH, the external characteristics of a DC series generator can be obtained, as shown in Figure 3.27.

3.7.4 DC Compound Generator

The external characteristics of a DC compound generator for both cumulative and differential compounded generators are shown in Figure 3.28.

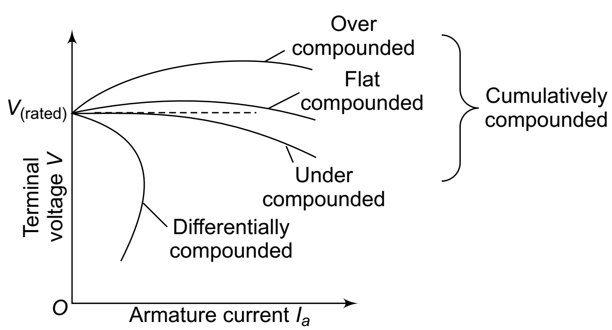


Figure 3.28 External Characteristics of a DC Compound Generator

Cumulative Compound Generator

If the series field ampere-turns are adjusted such that they induce the same voltage at both rated load and at no load, then the generator is said to be flat compounded. If the series field ampere-turns are adjusted such that the rated voltage at load condition is greater than the voltage at no-load condition, then the generator is said to be over-compounded. If the series field ampere-turns are adjusted such that the rated voltage at load condition is less than the no-load voltage, then the generator is said to be under-compounded.

If the terminal voltage of the DC generator drops very rapidly with increasing armature current, it is called differential compound generator.

3.8 Applications of DC Generators

Separately Excited DC Generator

- Since it has a very wide range of output voltage, it is used in laboratories.
- For DC motors, it is used as a supply source.

DC Shunt Generator

- It is used for excitation in AC generators or alternator.
- Since the voltage drop is very small, it is used as a source for loads, which need constant voltage.
- It acts as a source for battery charging, electroplating and electrolytic purposes.

DC Series Generator

- It is used in series arc lighting and incandescent lighting.
- It is used as a booster to compensate the voltage drop in the line during loading.
- It is used for regenerative braking of DC locomotives.

Compound Generator

- It is used to provide constant voltage at the line by proper compounding.
- Differently compounded generator is used for arc-welding purposes.
- Cumulative compounded generator is used to supply power to railway circuits, incandescent lamps, elevator motors, lighting, heavy power supply, offices, hotels, homes, schools etc.

3.9 DC Motor

3.9.1 Working Principle **[AU Nov/Dec, 2016]**

A DC motor is a machine, which converts electrical energy into mechanical energy. It is generally used in locations where it gets exposed to various environmental conditions and damages. Hence, a DC motor needs to be drip-proof, fire-proof, etc., according to the requirement. The principle of a DC motor is that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force and its direction is determined by using Fleming's left-hand rule.

3.9.2 Construction

The construction of a DC motor is exactly similar to a DC generator. The important parts of a DC motor are: yoke or frame, main field system, brushes, armatures and commutator. The functions of certain components vary with respect to a DC motor. In a DC motor, the commutator is used to convert the alternating torque produced in the armature into a unidirectional torque. A separate supply is given to armature winding to produce the required torque in the armature. The electrical power is converted into a mechanical power in the armature winding.

3.9.3 Working Principle

When the field windings wound on the poles are excited by a DC source, the poles get magnetised and a strong magnetic flux is produced. In such a magnetic field, consider a single armature coil, as shown in Figure 3.29 (a). When a DC supply is used to excite this armature coil, the current flows through the coil. Due to this, the armature coil creates its own magnetic field, as shown in Figure 3.29 (b). Here, due to the interaction between these two fields, a resultant field, F is developed, as shown in Figure 3.29 (c).

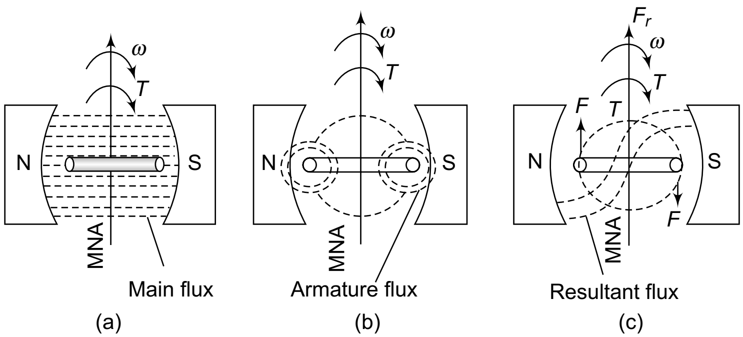


Figure 3.29 Working of a DC Motor (a) Main Field (b) Armature Field and (c) Resultant Field

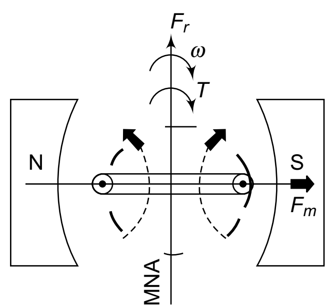


Figure 3.30 Working of a DC Motor Based on MMF

This resultant field has a tendency to align with the main field position i.e., it tries to align itself along a straight line and hence a force is exerted on the armature coil and a torque is developed, which helps in rotating the armature coil. The working of a DC motor can also be explained using the mmf developed due to these two fluxes, as shown in Figure 3.30.

The mmf developed due to the main field is given by F_m, whose direction is shown in Figure 3.30. The mmf is also developed due to the field produced in the armature when the armature windings are excited by a DC source. It is denoted as F_r, which is perpendicular to F_m. F_r tries to align with F_m and thus an electromagnetic torque, T is developed in clockwise direction and hence the armature starts rotating in the same direction with an angular speed of ω. In practice, there are Z number of armature conductors in a DC motor. Hence, when the armature windings are excited by a DC source, the armature conductors placed under the influence of a pole carry current in a particular direction, while the armature conductors placed under the influence of another pole carry current in the opposite direction indicated as '·' and 'x' respectively, in Figure 3.31.

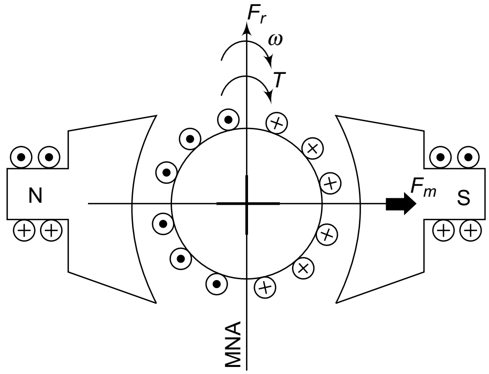


Figure 3.31 DC Motor With 'Z' Number of Armature Conductors

The electromagnetic torque developed in the armature conductors will be continuous if the direction of current in each conductor or coil side changes when it crosses the magnetic neutral axis (MNA). This reversal of direction of current can be achieved using a commutator and it helps in developing a continuous torque.

3.9.4 Back Emf in a DC Motor **[AU Nov/Dec, 2012]**

When a DC source is used to excite the armature conductor, which is placed in the main magnetic field, an electromagnetic torque is developed and hence, the armature of the DC motor starts rotating. Due to the rotation of armature, the armature conductors cut the magnetic flux of the main magnetic field and hence an induced emf is developed in the armature conductor. Since this induced emf developed in the armature conductors opposes the cause that produces it, this induced emf is known as back emf, E_b. Since this emf is induced in the armature due to the generator action, its magnitude is given by the same expression as that of the generated emf in a DC generator. Therefore, the expression for back emf, E_b is given by

$$E_b = \frac{\phi ZNP}{60\ \mathrm{A}}$$

It is noted that the supply voltage, V, applied to the armature windings will be greater than E_b.

The significance of back emf is to regulate the armature current, according to the load connected to the motor.

3.9.5 Voltage and Power Equation of DC Motor

The equivalent circuit of a DC motor armature is shown in Figure 3.32.

In a DC motor, the current flows from the line into the armature against the voltage generated in the armature. Using KVL in the circuit shown in Figure 3.32, we get

$$V = E_b + I_aR_a + V_b \tag{3.6}$$

If the drop across the brushes is negligible, we get

$$V = E_b + I_aR_a \tag{3.7}$$

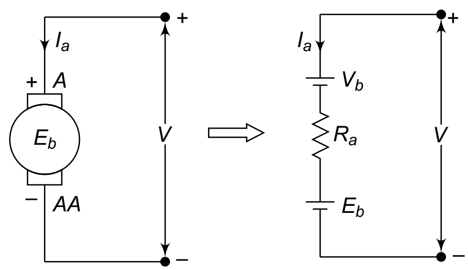


Figure 3.32 Equivalent Circuit of a DC Motor Armature

The Eqns. (3.6) and (3.7) represent the fundamental voltage equation of a DC motor.
Multiplying Eqn. (3.7) by I_a, we get

$$VI_a = E_b I_a + I_a^2 R_a \tag{3.8}$$

i.e., $$P_i = P_m + P_c$$

where, $P_i = VI_a$ is the input power supplied to the armature, $P_m = E_b I_a$ is the mechanical power developed in the armature and $P_c = I_a^2 R_a$ is the copper loss in the armature. Equation (3.8) represents the power equation of a DC motor.

Condition for Maximum Power Developed in Armature

The mechanical power developed in the armature is given by

$$P_m = VI_a - I_a^2 R_a$$

Differentiating the above equation with respect to I_a and equating it to zero, we get

$$I_a R_a = \frac{V}{2} \tag{3.9}$$

Substituting Eqn. (3.9) in Eqn. (3.7) and solving, we get

$$E_b = \frac{V}{2}$$

Therefore, the condition to develop the maximum power in the armature is given by the above equation.

Motor Efficiency

The motor efficiency, η_m is given by the ratio of mechanical power developed in the armature to the electrical power supplied to the armature. Therefore,

$$\eta_m = \frac{P_m}{P_i} = \frac{E_b I_a}{VI_a} = \frac{E_b}{V}$$

3.9.6 Torque and Speed Equation of DC Motor **[AU Nov/Dec, 2009]**

Torque Equation

In a DC motor, when a current-carrying armature conductor is placed in the magnetic field developed by the poles, an electromechanical torque is developed in the armature, T_a. It is also known as armature torque and is given by the product of the force and the radius at which this force acts.

i.e., $$T_a = F \times r \tag{3.10}$$

where, F is the force acting on the armature conductor and r is the radius of the armature.
In one revolution, the work done by the force, F is given by

$$W \cdot D = F \times 2\pi r$$

where, $2\pi r$ is the circumference of the armature.

If the armature is rotating at a speed of N rpm, the time taken to complete one revolution is $\frac{60}{N}$.
Therefore, the net mechanical power developed in the armature, P_m is given by

$$P_m = \frac{W \cdot D}{\text{time}} = \frac{F \times 2\pi r \times N}{60}$$

Using Eqn. (3.10) in the above equation, we get

$$P_m = T_a \times \frac{2\pi N}{60} = T_a \times \omega \tag{3.11}$$

where, $\omega = \dfrac{2\pi N}{60}$ is the angular velocity of the armature in radians per second.

But the mechanical power developed in the armature is given by

$$P_m = E_b I_a \tag{3.12}$$

Equating Eqns. (3.11) and (3.12), we get

$$T_a \times \frac{2\pi N}{60} = E_b I_a$$

i.e.,

$$T_a = 9.55 \times \frac{E_b I_a}{N} \tag{3.13}$$

Substituting $E_b = \dfrac{\phi ZNP}{60\text{ A}}$ in the above equation and rearranging, we get

$$T_a = \frac{\phi ZPI_a}{2\pi A} = 0.159 \times \frac{\phi ZPI_a}{A} \text{ N-m} \tag{3.14}$$

It is clear from the above equation that the armature torque is directly proportional to the product of the flux and the armature current i.e., $T_a \; \alpha \; \phi I_a$ since the other terms $0.159 \times \dfrac{ZP}{A}$ are constants. Therefore, Eqns. (3.13) and (3.14) represent the armature torque of a DC motor.

Shaft Torque (Tsh)

In a DC motor, the electromagnetic torque developed in the armature, T_a will not be available at the shaft, since a part of the torque is lost due to iron and mechanical losses. Therefore, the actual torque available at the shaft for doing useful mechanical work is known as shaft torque, T_{sh}. It is also defined as the torque available at the motor shaft and it is always less than T_a. If the speed of the motor is N rpm, then the shaft torque of the motor is given by

$$T_{sh} = \frac{\text{Output power of motor}}{\dfrac{2\pi N}{60}} = 9.55 \times \frac{\text{Output power of motor}}{N}$$

If the speed of the motor is N rps, then the shaft torque of the motor is given by

$$T_{sh} = \frac{\text{Output power of motor}}{2\pi N} = 0.159 \times \frac{\text{Output power of motor}}{N}$$

Speed Equation

Using the voltage equation and back emf equation of a DC motor, we get

$$E_b = V - I_a R_a = \frac{\phi ZNP}{60\text{ A}}$$

Therefore, the speed at which the DC motor rotates is given by

$$N = \frac{V - I_a R_a}{\phi} \times \frac{60A}{ZP} \text{ or}$$

$$N = \frac{E_b}{\phi} \times \frac{60A}{ZP}$$

It is clear from the above equation that the speed of a DC motor is directly proportional to the back emf, E_b and inversely proportional to the flux per pole, ϕ.

Speed Regulation of a DC Motor

It is defined as the change in speed when the load on the motor is reduced from rated value to zero, expressed as a percentage of rated load speed. It can also be defined as the ratio of the difference between no load and full load with respect to full load.

$$\% \text{ speed regulation} = \frac{N_{NL} - N_{FL}}{N_{FL}} \times 100$$

where, N_{NL} is the no-load speed of a DC motor and N_{FL} is the full-load speed of a DC motor.

3.10 Types of DC Motor **[AU Nov/Dec, 2014]**

The DC motors are classified, as shown in Figure 3.33, based on the excitation given to the field windings as:

- Separately excited DC motor: The required power for exciting the field windings is obtained from a separate DC source.
- Self-excited DC motor: The required power for exciting the field windings is obtained from the power supplies to the armature of the DC motor.

Further, the self-excited DC motor is classified based on the connection between the field winding and armature winding as:

- ***DC shunt motor:*** Field winding is parallel to the armature winding
- ***DC series motor:*** Field winding is in series with the armature winding
- ***DC compound motor:*** Consists of two windings; one is connected in series and other is connected in parallel to the armature windings. Based on these winding connections, it is further classified as:
  - Long-Shunt DC compound motor
  - Short-Shunt DC compound motor

If the magnetic flux generated by the series field winding aids the magnetic flux generated by the shunt-field winding, the DC compound motor is said to be cumulative compounded. Conversely, if the magnetic flux generated by the series-field winding opposes the magnetic flux generated by the shunt-field winding, the DC compound motor is known as differentially compounded. It is noted that both long-shunt and short-shunt compound motors can be either cumulative or differentially compounded.

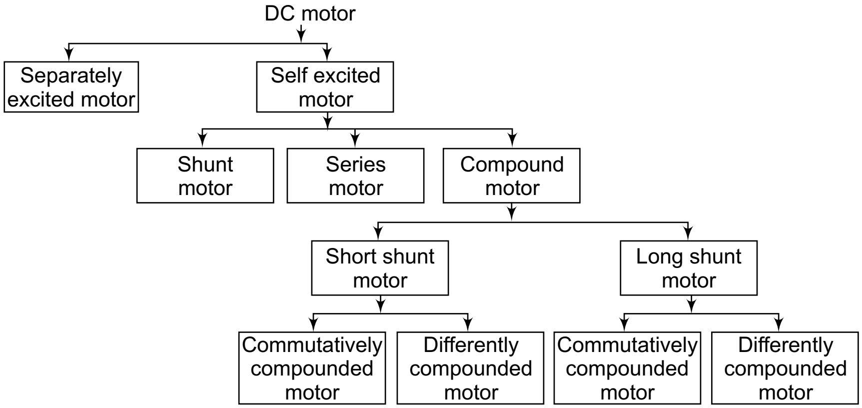


Figure 3.33 Types of DC Motors

The description of each DC motor is similar to a DC generator, which is discussed in Section 3.6. Table 3.3 lists the equivalent circuit, the voltage equation, the current equation and the flux relation for each type of DC motor.

Let ϕ_{se} and ϕ_{sh} be the fluxes produced by the series and shunt field windings respectively. Let ϕ_f and I_f be the fluxes produced by the field winding and current through the field winding in the separately excited DC motor.

Example 3.9

A shunt machine, connected to a 200 V mains, has an armature resistance of 0.15 Ω and the field resistance is 100 Ω. Find the ratio of its speed as a generator to its speed as a motor, if the line current in each case is 75 A. **[AU April/May, 2006]**

Solution

Given, $V = 200$ V, $R_a = 0.15\ \Omega$, $R_{sh} = 100\ \Omega$ and $I_L = 75$ A

Irrespective of the machine being operated as a motor or a generator, we get the shunt current as

$$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{100} = 2 \text{ A}$$

When the machine is operated as a motor, the armature current I_{am} is given by

$$I_{am} = I_L - I_{sh}$$

Substituting the known values, we get

$$I_{am} = 75 - 2 = 73 \text{ A}$$

Now, the back emf of the motor is given by

$$E_b = V - I_{am}R_a$$

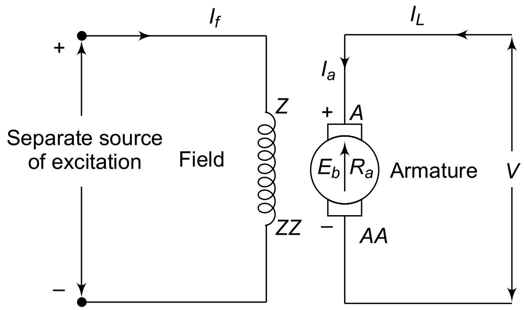
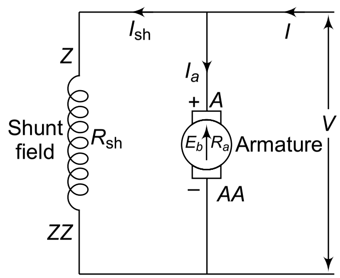
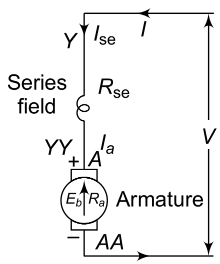
Substituting the given values, we get

$$E_b = 200 - 73 \times 0.15 = 189.05 \text{ V}$$

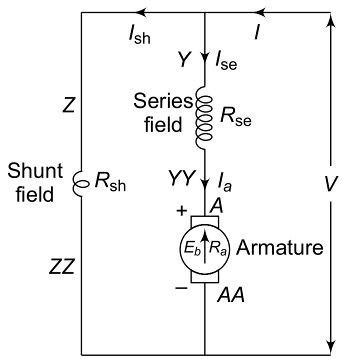
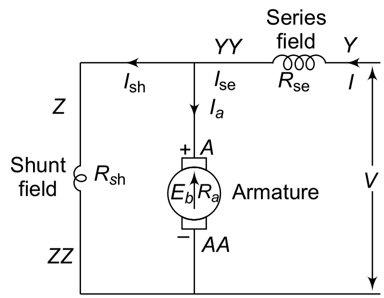
Similarly, when the machine is operated as a generator, the armature current I_{ag} is given by

$$I_{ag} = I_L + I_{sh}$$

Table 3.3 *Equivalent Circuit, Voltage and Current Equation and Flux Relation for Each Type of DC Motor*

<table>
<tr><th>Type of DC Motors</th><th>Equivalent Circuit</th><th>Voltage Equation</th><th>Current Equation</th><th>Flux Relation</th></tr>
<tr><td>Separately excited DC motor</td><td>
</td><td>$V = E_b + I_aR_a$</td><td>$I_a = I_L$</td><td>$\phi_f \, \alpha \, I_f$</td></tr>
<tr><td>DC shunt motor</td><td>
</td><td>$V = E_b + I_aR_a$</td><td>$I_L = I_a + I_{sh}$
$I_{sh} = \dfrac{V}{R_{sh}}$</td><td>$\phi_{sh} \, \alpha \, I_{sh}$</td></tr>
<tr><td>DC series motor</td><td>
</td><td>$V = E_b + I_a(R_a + R_{se})$</td><td>$I_a = I_L = I_{se}$</td><td>$\phi_{se} \, \alpha \, I_{se} \, \alpha \, I_a$</td></tr>
</table>

(*Contd.*)

Long-Shunt DC Compound Motor		$V = E_b + I_a(R_a + R_{se})$	$I_L = I_a + I_{sh}$ $I_{se} = I_a$ $I_{sh} = \frac{V}{R_{sh}}$	$\phi_{se}\ \alpha\ I_{se}\ \alpha\ I_a\ \phi_{sh}\ \alpha\ I_{sh}$
Short-Shunt DC Compound Motor		$V = E_b + I_aR_a + I_{se}R_{se}$	$I_L = I_a + I_{sh}$ $I_{se} = I_L$ $I_{sh} = \frac{V - I_{se}R_{se}}{R_{sh}}$	$\phi_{se}\ \alpha\ I_{se}\ \alpha\ I_L\ \phi_{sh}\ \alpha\ I_{sh}$

Substituting the known values, we get

$$I_{ag} = 75 + 2 = 77 \text{ A}$$

Now, the generated emf in a generator is given by

$$E_g = V + I_{ag}R_a$$

Therefore, $E_g = 200 + 77 \times 0.15 = 211.55$ V

It is known that, irrespective of a motor or a generator, $N \alpha \dfrac{E}{\phi}$. Since, in a DC shunt machine, the flux is constant, we get $N \alpha E$. Therefore, $N_m \alpha E_b$ for a motor and $N_g \alpha E_g$ for a generator. Taking ratios, we get

$$\frac{N_m}{N_g} = \frac{E_b}{E_g} = \frac{18.05}{211.55} = 0.8936$$

where, N_m is the speed of DC motor and N_g is the speed of DC generator.

Therefore, $\dfrac{N_g}{N_m} = \dfrac{1}{0.8936} = 1.119$

Example 3.10

A four-pole, 500 V DC shunt motor has 700 wave-connected conductors on its armature. The full-load armature current is 60 A and the flux per pole is 30 mWb. Calculate the full-load speed, if the rotor armature resistance is 0.2 Ω and brush drop is 1 V/brush. **[AU April/May, 2007]**

Solution

Given, $P = 4$, $V = 500$ V, $Z = 700$, $(I_a)_{FL} = 60$ A, $\phi = 30$ mWb, $R_a = 0.2\ \Omega$ and $V_{\text{brush}} = 1$ V/brush.
The back emf of DC motor is given by

$$E_b = V - (I_a)_{FL}R_a - V_{\text{brush}}$$

Since there are two brushes in a DC motor, we get

$$E_b = 500 - 60 \times 0.2 - 2 \times (1) = 486 \text{ V}$$

Since the DC shunt motor uses wave-wound armature windings, $A = 2$. Therefore, the back emf of DC shunt motor is

$$E_b = \frac{\phi PNZ}{60 \text{ A}}$$

Substituting the given values, we get

$$486 = \frac{30 \times 10^{-3} \times 4 \times N \times 700}{60 \times 2}$$

Therefore, $$N = \frac{486 \times 60 \times 2}{30 \times 10^{-3} \times 4 \times 700} = 695 \text{ rpm}$$

Example 3.11

The armature winding of a 200 V, four-pole, series motor is lap-connected. There are 280 slots and each slot has 4 conductors. The current is 45 A and the flux per pole is 18 mWb. The field resistance is 0.3 Ω;

armature resistance 0.5 Ω and the total iron and friction loss is 800 W. The pulley diameter is 0.41 m. Find the pull in Newton at the rim of the pulley. **[AU Nov/Dec, 2007]**

Solution

Given, V = 200 V, P = 4, Number of slots = 280, number of conductors per slot = 4, I_a = 45 A, ϕ = 18 mWb, R_{se} = 0.3 Ω, R_a = 0.5 Ω.

The back emf of a DC series motor is given by

$$E_b = V - I_a(R_a + R_{se})$$

Substituting the given values, we get

$$E_b = 200 - 45(0.36 + 0.5) = 164 \text{ V}$$

Since the DC series motor uses lap-wound armature winding, $A = P = 4$.

Also, the total number of armature conductors, $Z = 280 \times 4 = 1120$.

But the other expression for back emf is $E_b = \dfrac{\phi PNZ}{60\text{ A}}$.

Substituting the known values in this equation, we get

$$164 = \frac{18 \times 10^{-3} \times 4 \times N \times 1120}{60 \times 4}$$

i.e., $N = 488$ rpm

The armature torque developed in the armature is given by

$$T_a = \frac{E_b I_a}{\left(\dfrac{2\pi N}{60}\right)} = \frac{164 \times 45}{\left(\dfrac{2\pi \times 488}{60}\right)} = 144.41 \text{ Nm}$$

The torque lost due to iron and friction losses is given by

$$T_{\text{lost}} = \frac{\text{Iron and friction losses}}{\left(\dfrac{2\pi N}{60}\right)} = \frac{800}{\left(\dfrac{2\pi \times 488}{60}\right)} = 15.65 \text{ Nm}$$

Therefore, the useful torque or shaft torque available at the motor shaft is given by

$$T_{sh} = T_a - T_{\text{lost}} = 128.74 \text{ Nm}$$

This useful or shaft torque is the net torque, which is exerted on the pulley. Therefore,

$$T_{sh} = (\text{Net Pull at the rim in } N) \times (\text{Radius of pulley})$$

$$18.75 = (\text{Net pull at the rim in } N) \times \frac{0.41}{2}$$

Hence, the net pull or the force experienced at the rim is given by

$$\text{Net Pull at the rim} = 627.97 \text{ N}$$

Example 3.12

A 250 V, four-pole wave-wound DC series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 Ω. The motor takes a current of 40 A. Determine its speed and gross torque developed, if it has a flux per pole of 25 mWb. **[AU April/May, 2006]**

Solution

Given, $V = 250$ V, $P = 4$, $Z = 782$, $R_a + R_{se} = 0.75\ \Omega$, $I_a = 40$ A, $\phi = 25$ mWb

The back emf equation of a DC series motor is

$$E_b = V - I_a(R_a + R_{se})$$

Substituting the known values, we get

$$E_b = 220 \text{ V}$$

The other expression for back emf of DC motor is $E_b = \dfrac{\phi PNZ}{60\ A}$. Since the DC series motor uses wave-wound armature winding, $A = 2$. Therefore,

$$220 = \frac{25 \times 10^{-3} \times 4 \times N \times 782}{60 \times 2}$$

Therefore, $$N = \frac{220 \times 60 \times 2}{25 \times 10^{-3} \times 4 \times 782} = 338 \text{ rpm}$$

The armature torque developed in the armature is given by

$$T_a = \frac{E_b I_a}{\frac{2\pi N}{60}} = \frac{220 \times 40}{\frac{2\pi \times 338}{60}} = 248.62 \text{ Nm}$$

Example 3.13

Determine the torque developed and shaft torque of a 220 V, four-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω. **[AU April/May, 2005]**

Solution

Given, $V = 220$ V, $P = 4$, $Z = 800$, $P_{out} = 8.2$ kW, $I_L = 45$ A, $\phi = 25$ mWb and $R_a = 0.6\ \Omega$.

The armature torque developed in the armature is given by

$$T_a = \frac{1}{2\pi}\frac{PZ}{A}\phi I_a$$

In a DC series motor, $I_a = I_L = 45$ A.

Therefore, $$T_a = \frac{1}{2\pi} \times \frac{4 \times 800}{2} \times 25 \times 10^{-3} \times 45 = 286.48 \text{ Nm}$$

The back emf equation of a DC motor is

$$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193 \text{ V}$$

The other expression for back emf of DC motor is $E_b = \dfrac{\phi PNZ}{60\ A}$. Since the DC series motor uses wave-wound armature winding, $A = 2$. Therefore,

$$193 = \frac{25 \times 10^{-3} \times 4 \times N \times 800}{60 \times 2}$$

Therefore, $$N = \frac{193 \times 60 \times 2}{25 \times 10^{-3} \times 4 \times 800} = 290 \text{ rpm}$$

The shaft torque, which is available at the shaft, is given by

$$T_{sh} = \frac{P_{\text{out}}}{\omega} = \frac{8.2\times10^3}{\frac{2\pi N}{60}} = \frac{8.2\times10^3\times60}{2\pi\times290} = 270.481 \text{ Nm}$$

3.11 Characteristics of DC Motors [AU April/May, 2015]

The different characteristics of DC motors are:

- ***Electrical characteristics:*** Give the relation between T_a and I_a and also known as T_a/I_a characteristics
- ***Speed vs. armature current characteristics:*** Give the relation between N and I_a
- ***Mechanical characteristics:*** Give the relation between N and T_a and also known as N/T_a characteristics

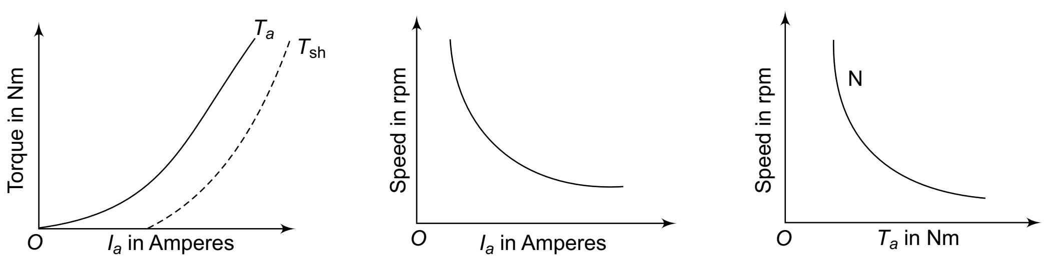
These characteristics are obtained for different DC motors, based on the back emf and armature torque equations of a DC motor. It is known that $T_a = \frac{\phi ZPI_a}{2\pi A} = 0.159\times\frac{\phi ZPI_a}{A}$, $T_a = 9.55\times\frac{E_bI_a}{N}$ and $E_b = \frac{\phi ZNP}{60\text{ A}}$.

The relations that we get using these equations are: $T_a \,\alpha\, \frac{E_bI_a}{N}$, $T_a \,\alpha\, \phi I_a$ and $N \,\alpha\, \frac{E_b}{\phi}$. Using these relations, the characteristics of different DC motors are studied. It is to be noted that ϕ denotes the flux produced by the field windings.

3.11.1 DC Series Motor

T_a/I_a Characteristics

It is known that, in a DC series motor, $I_a = I_L = I_{se}$. Here, the T_a/I_a characteristics are divided into two parts: (i) before the magnetic saturation of field flux and (ii) after saturation of field flux. In the first part, the field flux, ϕ, varies and is directly proportional to I_a i.e., $\phi \,\alpha\, I_a$. Therefore, the armature torque, $T_a \,\alpha\, I_a^2$. Hence, the T_a/I_a characteristics curve in the first-half is parabolic in nature. In the second part, once the saturation of field flux occurs, ϕ does not vary even when I_a is varied. Therefore, armature torque varies linearly with armature current i.e., $T_a \,\alpha\, I_a$. Hence, T_a/I_a characteristic curve is linear in the second part, as shown in Figure 3.34 (a). The shaft torque, T_{sh}, which is less than T_a due to losses in a DC motor, is shown as a dotted line in Figure 3.34 (a).



(a) T_a/I_a characteristics (b) N/I_a characteristic (c) N/T_a characteristic

Figure 3.34 Characteristics of a DC Series Motor

N/I_a Characteristics

Since $\phi \; \alpha \; I_a$ in a DC series motor, the relation between speed and ϕ is $N \; \alpha \; \frac{E_b}{I_a}$. If the back emf, E_b is neglected, then the speed varies inversely with the armature current, I_a. The N/I_a characteristics using these relations are shown in Figure 3.34 (b). The following inference can be made from the N/I_a characteristics shown in Figure 3.34 (b).

(a) When heavy load is connected to a DC motor, I_a becomes large and hence the speed is low, which further decreases E_b and allows more I_a to flow.
(b) Similarly, when I_L falls to a small value and since $I_a = I_L$ in a DC series motor, I_a falls to a small value and hence the speed N reaches a very high value. This is the reason why a DC series motor should not be started at no-load condition.
(c) Also, from N/I_a characteristics, we can infer that a DC series motor is a variable speed motor.

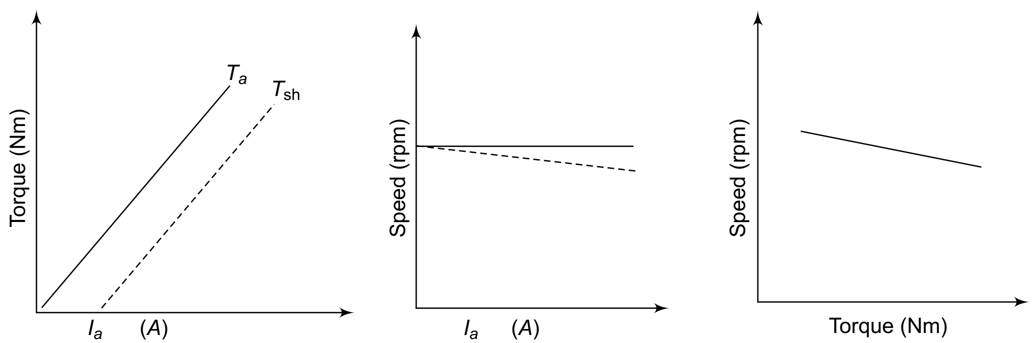
N/T_a Characteristics

The relation used to obtain this characteristic is $T_a \; \alpha \; \frac{E_b I_a}{N}$. It is clear from the equation that, when speed of the DC motor is high, the torque developed in the armature is low and vice-versa. The N/T_a characteristics curve a of DC series motor is shown in Figure 3.34 (c).

3.11.2 DC Shunt Motor

T_a/I_a Characteristics

Since the field winding in a DC shunt motor is excited by the constant supply voltage, V, the current through the field winding, I_{sh} is constant. Since the field current in the DC shunt motor is constant, the flux produced by the field is also constant i.e., ϕ is a constant value. Hence, $T_a \; \alpha \; I_a$. Therefore, the T_a/I_a characteristics curve of a DC shunt motor is linear and is shown in Figure 3.35 (a) along with T_{sh}, which is less than T_a due to losses in the DC motor.



(a) T_a/I_a characteristics (b) N/I_a characteristics (c) N/T_a characteristic

Figure 3.35 Characteristics of a DC Shunt Motor

N/I_a Characteristics

Since the field flux is constant, $N \alpha E_b$ in a DC series motor. At normal condition, E_b is almost constant but due to armature reaction and armature resistance drop, there will be a slight drop in the back emf, E_b. Hence, there will be a slight drop in the N/I_a characteristics, as shown in Figure 3.35 (b). In general, the speed of a DC shunt motor drops only to 5–15 % of the full load speed. Hence, the DC shunt motor is referred as constant speed motor.

N/T_a Characteristics

Using the above two characteristics, it is possible to get N/T_a characteristics for different values of armature current I_a, as shown in Figure 3.35 (c). It is clear that the speed of DC shunt motor decreases when the armature torque increases.

3.11.3 DC Compound Motor

The characteristics of a DC compound motor are studied based on the connection between the shunt and the series field windings. If the field windings are connected in such a way that the flux produced by the series field winding is in the direction of flux produced by the shunt field winding, then it is called a DC cumulative compound motor. But if the fluxes produced by the field windings are in the opposite direction, it is called a DC differential compound motor. The characteristics of these two compound motors are shown in Figure 3.36 and Figure 3.37.

In a DC cumulative compound motor, at heavy load, the series field winding plays a major role, while the shunt field winding prevents this motor from running at dangerously high speed at light load. In a DC differential compound motor, since the two field fluxes oppose each other, the speed of this motor remains almost constant and it increases slightly when there is an increase in load.

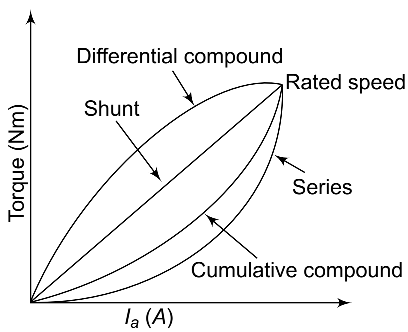


Figure 3.36 T/I_a Characteristics Curve of a DC Compound Motor

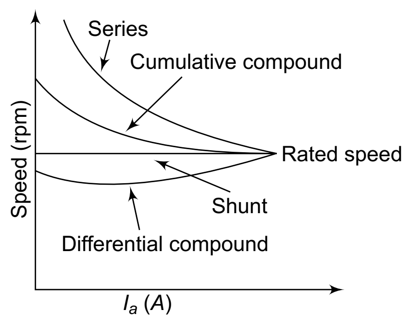


Figure 3.37 N/I_a Characteristics of a DC Compound Motor

Example 3.14

A four-pole DC motor runs at 600 rpm on full load and takes 25 A, 450 V. The armature is lap-wound with 500 conductors and the flux per pole is given by $\phi = (1.7 \times 10^{-2})I^{0.5}$ Wb, where I is the motor current. If the supply voltage and torque are halved, calculate the speed at which the motor will run. Neglect stray losses.

[AU April/May, 2012]

Solution

Given, $P = 4$ $\quad N_1 = 600$ rpm, $I_1 = 25\ A$, $V_1 = 450$ V, $Z = 500$, $\phi = 1.7 \times 10^{-2}\ I^{0.5}$ Wb.

Since lap-wound armature windings are used, $A = P$. Therefore, the back emf developed in DC motor is given by

$$E_{b1} = \frac{\phi PNZ}{60\text{A}} = \frac{1.7 \times 10^{-2} \times I_1^{0.5} \times P \times N_1 \times Z}{60\text{A}} = \frac{1.7 \times 10^{-2} \times (25)^{0.5} \times 4 \times 600 \times 500}{60 \times 4} = 425 \text{ V}$$

It is known that, $T \alpha \phi I$. It is given that $T_2 = \frac{T_1}{2}$.

Therefore, $$\frac{T_1}{T_2} = \frac{\phi_1}{\phi_2} \times \frac{I_1}{I_2}$$

$$2 = \frac{1.7 \times 10^{-2} \times (25)^{0.5}}{1.7 \times 10^{-2} \times I_2^{0.5}} \times \frac{25}{I_2} \text{ i.e., } I_2^{3/2} = \frac{125}{2}$$

Solving the above equation, we get

$$I_2 = 15.749 \text{ A}$$

The back emf developed in the DC motor is

$$E_{b1} = V_1 - I_1 R$$

Therefore, the armature resistance is

$$425 = 450 - 25\ R$$
$$R = 1\ \Omega$$

Now, the back emf developed when the torque and voltage are halved is given by

$$E_{b2} = V_2 - I_2\ R$$

Since $V_2 = \frac{V_1}{2} = 225\text{ V}$, we get

$$E_{b2} = 225 - 15.749 \times 1 = 209.25 \text{ V}$$

It is known that $N \alpha \frac{E_b}{\phi}$.

Therefore, $$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1}$$

Substituting the known values, we get

$$\frac{600}{N_2} = \frac{425}{209.251} \times \frac{1.7 \times 10^{-2} \times (15.749)^{0.5}}{1.7 \times 10^{-2} \times (25)^{0.5}}$$

Solving the above relation, we get $N_2 = 373$ rpm

Example 3.15

A series motor has an armature resistance of 0.2 Ω and a series field resistance of 0.3 Ω. It is connected to a 240V supply and at a particular load runs at 24 rev/s, when drawing 15 A from the supply. (i) Determine the generated emf at this load (ii) Calculate the speed of the motor when the load is changed such that the current is increased to 30 A. Assume that this causes doubling of the flux. **[AU Nov/Dec, 2013]**

Solution

Given, $R_a = 0.2\ \Omega$, $R_{se} = 0.3\ \Omega$, $V = 240$ V, $N_1 = 24$ rps = 1440 rpm.

Case (i) $\qquad I_{a1} = 15$ A, $\phi = \phi_1$

Therefore, the back emf developed in DC motor is given by

$$E_{b1} = V - I_{a1}(R_a + R_{se}) = 240 - 15\,(0.2 + 0.3) = 232.5 \text{ V}$$

Case (ii) $\qquad I_{a2} = 30$ A, $\phi_2 = 2\phi_1$

Therefore, the back emf developed in DC motor is given by

$$E_{b2} = V - I_{a2}(R_a + R_{se}) = 240 - 30\,(0.2 + 0.3) = 225 \text{ V}$$

It is known that, $N \,\alpha\, \dfrac{E_b}{\phi}$.

Therefore, $\qquad \dfrac{N_1}{N_2} = \dfrac{E_{b1}}{E_{b2}} \times \dfrac{\phi_2}{\phi_1}$ i.e., $\dfrac{1440}{N_2} = \dfrac{232.5}{225} \times \dfrac{2\phi_1}{\phi_1} = 2.067$

Substituting the known values and solving, we get

$$N_2 = 697 \text{ rpm} = 12 \text{ rps.}$$

Example 3.16

A DC motor operates from a 240 V supply. The armature resistance is 0.2 Ω. Determine the back emf when the armature current is 50A.

Solution

Given, $V = 240$ V, $R_a = 0.2\ \Omega$ and $I_a = 50$ A

The back emf developed in DC motor is

$$E_b = V - I_a R_a = 240 - 0.2 \times 50 = 230 \text{ V}$$

Example 3.17

A 240 V DC shunt motor has an armature resistance of 0.5 Ω and field resistance of 120 Ω. This motor drives a constant torque load and takes an armature current of 22 A at 850 rpm. If the motor speed is to be raised to 1000 rpm from 850 rpm, find the resistance that must be inserted in the shunt field circuit. Assume magnetisation curve to be a straight line. **[AU April/May, 2012]**

Solution

The two conditions given in the problem are shown in Figures E3.17 (a) and (b).

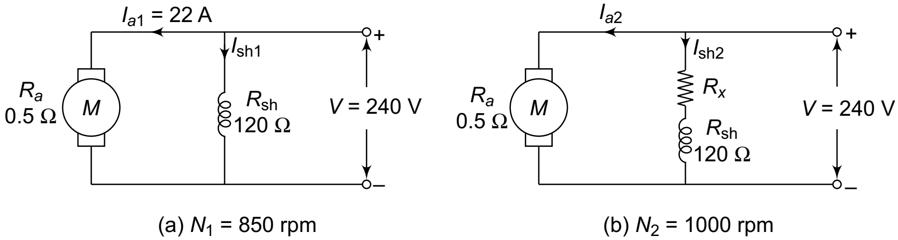


Figure E3.17

In a DC shunt motor, we have

$$T \,\alpha\, \phi I_a$$

Since $\phi \,\alpha\, I_{sh}$, $T \,\alpha\, I_{sh}\, I_a$. Therefore,

$$\frac{T_1}{T_2} = \frac{I_{sh1} I_{a1}}{I_{sh2} I_{a2}} = 1$$

Since the torque developed in both the cases is equal, we get

$$T_{sh1} I_{a1} = T_{sh2} I_{a2}$$

But $I_{sh1} = \dfrac{V}{R_{sh}} = \dfrac{240}{120} = 2$ A, and $I_{a1} = 22$ A.

Therefore, $I_{sh2} I_{a2} = 2 \times 22 = 44$

The back emf developed in the first case is given by

$$E_{b1} = V - I_{a1} R_a = 240 - 22 \times 0.5 = 229 \text{ V}$$

It is known that, $N \alpha \dfrac{E_b}{\phi} \alpha \dfrac{E_b}{I_{sh}}$.

Therefore, $$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{sh2}}{I_{sh1}}$$

Substituting the known values and solving, we get

$$\frac{850}{1000} = \frac{229}{E_{b2}} \times \frac{I_{sh2}}{2} \tag{1}$$

Now, the back emf developed and the shunt current in DC motor for case (ii) are:

$$E_{b2} = V - I_{a2} R_a = 240 - 0.5 I_{a2}, \text{ and } I_{sh2} = \frac{44}{I_{a2}}$$

Substituting the above equation in Eqn. (1), we get

$$\frac{850}{1000} = \frac{229}{240 - 0.5 I_{a2}} \times \frac{\left(\dfrac{44}{I_{a2}}\right)}{2}$$

Solving the above equation, we get

$$I_{a2} = 26.1218 \text{ A and } I_{a2} = 453.878 \text{ A.}$$

Since a large amount of current cannot pass through armature, we consider

$$I_{a2} = 26.1218 \text{ A}$$

Therefore, $$I_{sh2} = \frac{44}{I_{a2}} = \frac{44}{26.1218} = 1.6844 \text{ A}$$

But, $I_{sh2} = \dfrac{V}{R_{sh} + R_x}$ where, R_x is the extra resistance to be connected in series.

Substituting the known values and solving, we get

$$R_x = 22.4825\ \Omega$$

Example 3.18

A 230 V, 1150 rpm shunt motor is connected to a 230 V supply and delivers rated load at rated speed when drawing 38 A from the supply. The field and armature resistances are 128 Ω and 0.3 Ω respectively. Calculate the value of external resistance to be added in the armature circuit when it is desired to operate the motor at 450 rpm while delivering 120% full load electromagnetic torque. **[AU Nov/Dec, 2008]**

Solution

Given, $V = 230$ V, $N_1 = 1150$ rpm, $I_{L1} = 38$ A, $R_{sh} = 128\ \Omega$, $R_a = 0.3\ \Omega$, $N_2 = 450$ rpm.

Let R_x be the extra resistance, which is connected in series with R_a. If the torque developed in DC motor without external resistance is T_1, then $T_2 = 1.2\ T_1$.

The pictorial representation of a DC motor with and without external resistance is shown in Figures E3.18 (a) and (b).

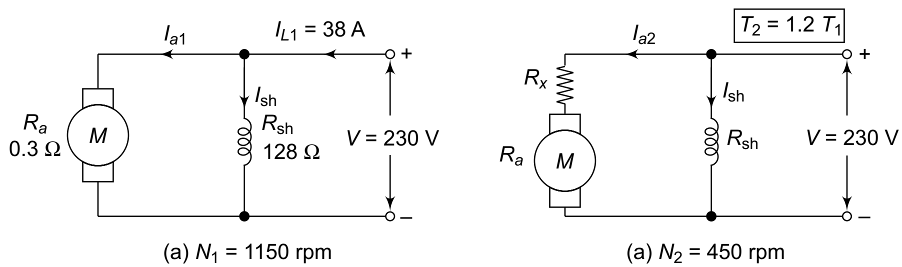


Figure E3.18

The shunt field current in both the cases is given by

$$I_{sh} = \frac{V}{R_{sh}} = \frac{230}{128} = 1.7968\ \text{A}$$

The armature current in the DC motor without external resistance is given by

$$I_{a1} = I_{L1} - I_{sh} = 38 - 1.7968 = 37.2032\ \text{A}$$

The back emf developed in the DC motor without external resistance is

$$E_{b1} = V - I_{a1}R_a = 230 - 37.2032 \times 0.3 = 218.839\ \text{V}$$

It is known that $T \alpha\ \phi I_a$. Since it is a DC shunt motor, ϕ is constant, i.e., $T \alpha\ I_a$.

Hence, $\dfrac{T_1}{T_2} = \dfrac{I_{a1}}{I_{a2}}$

Substituting the known values in the above equation and solving, we get

$$I_{a2} = 1.2 \times 37.2032 = 44.6438\ \text{A}$$

We know that $N\ \alpha\ \dfrac{E_b}{\phi}$ i.e., $N\ \alpha\ E_b$. Therefore, $\dfrac{N_1}{N_2} = \dfrac{E_{b1}}{E_{b2}}$.

Substituting the known values and solving, we get

$$E_{b2} = 85.6327\ \text{V}$$

The back emf developed in a DC motor with external resistance is given by

$$E_{b2} = V - I_{a2}\,(R_a + R_x)$$

Substituting the known values, we get

$$85.6327 = 230 - 44.6438\,(0.3 + R_x)$$

Therefore, $R_x = 2.9337\ \Omega$

Example 3.19

A 250 volt, DC shunt motor has an armature resistance of 0.25 Ω. On load, it takes an armature current of 50 A and runs at 750 rpm. If the flux of motor is reduced by 10% without changing the load torque, find the new speed of the motor. **[AU April/May, 2013]**

Solution

Given, $V = 250$ V, $R_a = 0.25\ \Omega$, $I_{a1} = 50$ A, $N_1 = 750$ rpm, $\phi_2 = 0.9\ \phi_1$, $T_1 = T_2$.
Using the relation, $T \alpha \phi I_a$, we get

$$\frac{T_1}{T_2} = \frac{\phi_1}{\phi_2} \times \frac{I_{a1}}{I_{a2}}$$

Substituting the known values in the above equation and solving, we get

$$I_{a2} = 55.555 \text{ A}$$

The back emf developed in DC motor is given by:

$$E_{b1} = V - I_{a1}R_a = 237.5 \text{ V and } E_{b2} = V - I_{a2}R_a = 236.111 \text{ V.}$$

Using the relation $N \alpha \dfrac{E_b}{\phi}$, we get

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1}$$

Substituting the known values in the above equation, we get

$$\frac{750}{N_2} = \frac{237.5}{236.11} \times \frac{0.9\phi_1}{\phi_1} = 0.905$$

Therefore, $N_2 = 829$ rpm

Example 3.20

A DC series motor developing 40 Nm torque is subjected to the conditions that make the field flux decrease by 30 % and the armature current to increase by 15 %. Calculate the new torque. **[AU Nov/Dec, 2008]**

Solution

Given, $T_1 = 40$ Nm.

Let ϕ_1 and ϕ_2 be the initial and new fluxes corresponding to the armature currents I_{a1} and I_{a2} respectively. Therefore, $\phi_2 = 0.7\ \phi_1$ and $I_{a2} = 1.15\ I_{a1}$.

Using the relation $T \,\alpha\, \phi I_a$, we get

$$\frac{T_1}{T_2} = \frac{\phi_1}{\phi_2} \times \frac{I_{a1}}{I_{a2}}$$

Substituting the known values in the above equation and solving, we get

$$T_2 = 40 \times 0.7 \times 1.15 = 32.2 \text{ Nm}$$

3.12 Speed Control of DC Motors **[AU Nov/Dec, 2014]**

The speed equation of a DC motor is given by

$$N = \frac{V - I_a R_a}{\phi} \times \frac{60A}{ZP}$$

i.e., $$N \,\alpha\, \frac{V - I_a R_a}{\phi} \text{ (since all other terms are constant)}$$

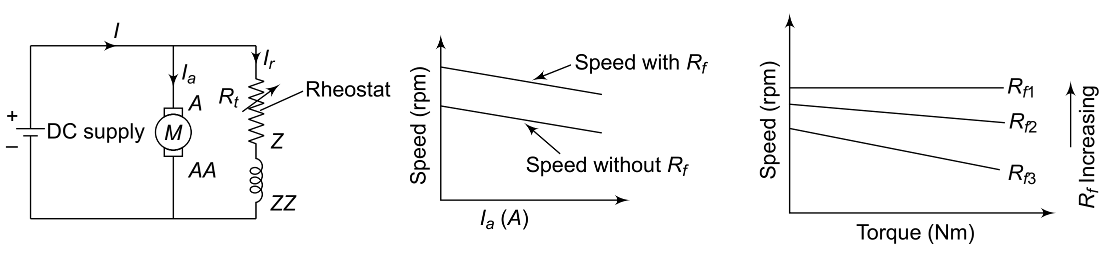
From the above equation, it is concluded that the speed of a DC motor can be controlled by the following methods:

- ***Armature resistance control method:*** Here, the armature resistance of DC motor is varied to control the speed.
- ***Flux control method:*** Here, varying the flux produced by the poles controls the speed of the DC motor.
- ***Applied voltage:*** Here, varying the excitation voltage given to the armature and field windings controls the speed of the DC motor.

3.12.1 Speed Control of a DC Shunt Motor

Flux control Method

The circuit diagram of flux control method to control the speed of a DC shunt motor is shown in Figure 3.38 (a). This method is used to control the speed of a DC shunt motor above the rated speed, by reducing the field flux, since $N \,\alpha\, \frac{1}{\phi}$. In this method, a variable resistance element called shunt field rheostat is connected in series with the shunt-field winding, which helps in reducing the flux and the field current. Since the field Ohmic loss is very small, this method is frequently used. Also, it is very simple and economic. The N/I_a characteristics and N/T_a characteristics are shown in Figures 3.38 (b) and (c) respectively.

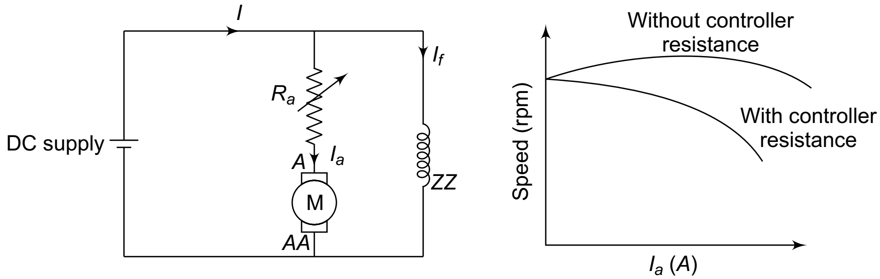


(a) Circuit Diagram (b) N/I_a Characteristics Curve (c) N/T_a Characteristics Curve

Figure 3.38 Flux Control Method of a DC Shunt Motor

Armature Resistance Control Method

In this method, including a resistance in series with R_a controls the speed of a DC shunt motor, as shown in Figure 3.39 (a). This method is used to vary the speed of a DC shunt motor, below the rated speed since $N \alpha V - I_a R_a$ i.e., the inclusion of resistance in series with R_a reduces the armature current, which further reduces the speed of a DC shunt motor. The N/I_a characteristics of a DC shunt motor with and without the extra resistance are shown in Figure 3.39 (b).



(a) Circuit Diagram (b) N/I_a Characteristics Curve

Figure 3.39 Armature Control Method of a DC Shunt Motor

Voltage Control Method

The different voltage control methods used for controlling the speed of a DC shunt motor are:

- Multiple voltage control
- Ward–Leonard system

Multiple Voltage Control

In this method, a suitable arrangement is made in such a way that the field winding gets a constant supply voltage and armature winding gets a variable voltage. The different voltages that can be used for exciting armature winding are obtained using a switch-gear. Since the speed of the motor is directly proportional to the supply voltage applied to the armature, varying the voltage applied to the armature can control the speed.

Ward–Leonard System **[AU Nov/Dec, 2010]**

The arrangement of Ward–Leonard system to control the speed of a DC shunt motor is shown in Figure 3.40. Here, M_2 is the DC shunt motor whose speed has to be controlled, M_1 can be either AC or DC motor with constant speed, which is coupled directly to generator G. As shown in Figure 3.40, the output of G_1 is fed as the input to the armature of motor M_2. The field winding of M_2 is given by a constant DC supply voltage. Therefore, varying the generator output can vary the supply voltage applied to armature of M_2. The generator output can be varied using a field regulator. Thereby, a very smooth speed control of DC shunt motor can be obtained using this method. This method is used in applications where very sensitive speed control is required. For example, elevators, electric excavators etc.

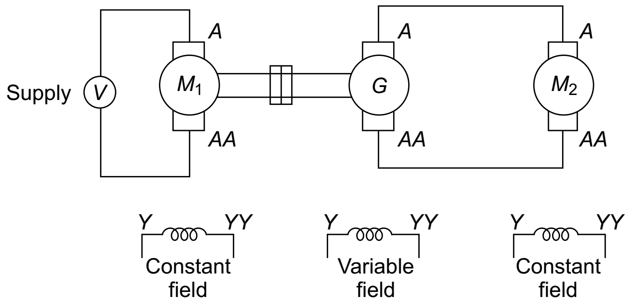


Figure 3.40 Ward–Leonard System of a DC Shunt Motor

3.12.2 Speed Control of a DC Series Motor

Flux Control Method

The different methods in which the flux produced by the field windings can be varied, thereby controlling the speed of DC series motor, are:

- Field diverter method
- Armature diverter method
- Tapped-field control method and
- Paralleling field coils

Field Diverter Method

The circuit diagram for this method is shown in Figure 3.41. In this method, a field diverter or a variable resistance is connected in parallel to the series field windings of a DC series motor. This field diverter is used to reduce the line current flowing through the series field windings. Since the field current is getting decreased, the flux developed by the poles gets lowered, which increases the speed of the motor. Using this method, the speed of a DC motor above rated speed can be obtained.

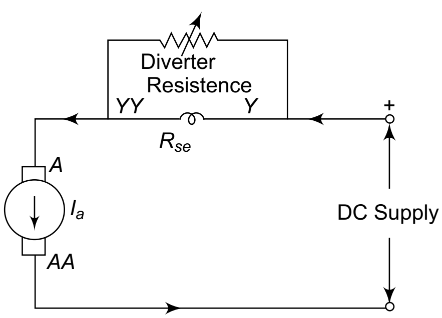


Figure 3.41 Field Diverter Method

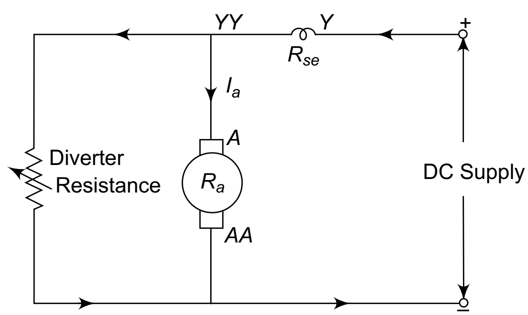


Figure 3.42 Armature Diverter Method

Armature Diverter Method

Using this method, the speed of a DC series motor below the rated speed can be obtained. This method uses armature diverter to reduce the current flowing through the armature, as shown in Figure 3.42. By varying this armature diverter, the speed control of a DC series motor can be achieved.

Tapped-field Control Method

The difference between the field diverter method and the tapped-field control method is that tapped-field windings are used in the latter, whereas a field diverter is used in the former. The circuit diagram of tapped-field control method is shown in Figure 3.43. The switch *S* is used to create a connection to the desired point in the tapped-field winding. Here, by reducing the number of turns in the field winding reduces the field flux, which in turn increases the speed of the DC series motor. Similar to field diverter method, this method can be used to control the speed above the rated speed of the DC series motor.

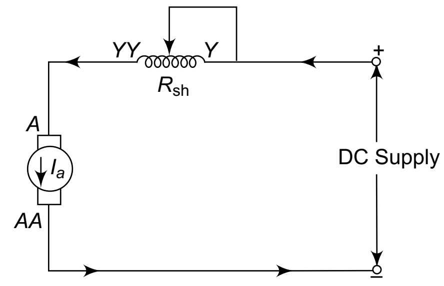


Figure 3.43 Tapped-Field Control Method

Paralleling Field Coils

In this method, a single series field winding is divided into number of parts and these sub-coils are arranged in any one way, as shown in Figure 3.44, to reduce the flux produced by the field winding, thereby increasing the speed of the DC series motor.

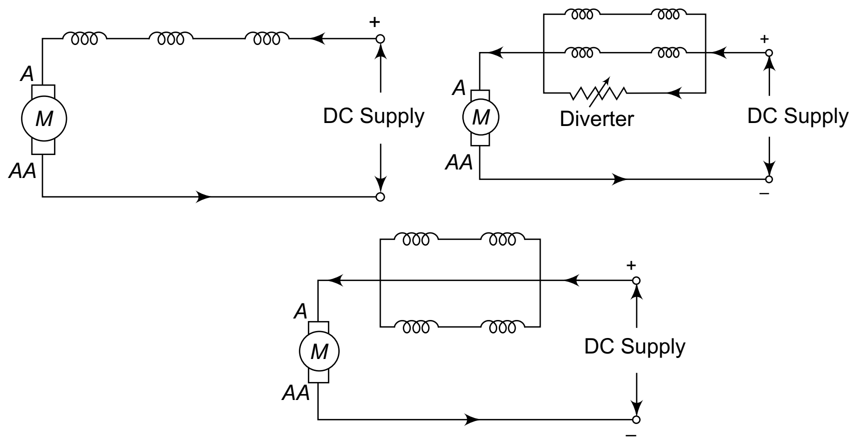


Figure 3.44 Paralleling-Field Coils

Armature Resistance Control Method

The circuit diagram for the speed control of a DC series motor, by varying armature resistance, is shown in Figure 3.45. In this method, an adjustable resistor is connected in series with the source voltage to control the speed of a DC series motor below the rated speed.

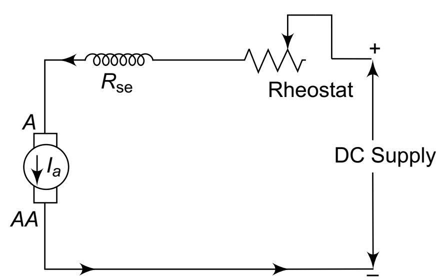


Figure 3.45 Armature Resistance Control Method

3.13 Losses in DC Machines [AU Nov/Dec, 2012]

In any practical DC machine, all the input power supplied to the machine does not get converted into the output power, as some amount of power is lost in the conversion process. This loss during the conversion reduces the efficiency of the DC machine, which is given by the ratio of output to the input power. Therefore, it becomes necessary to study the various losses occurring in a DC machine to design it with higher efficiency. The different losses that occur in the DC machine are shown in Figure 3.46.

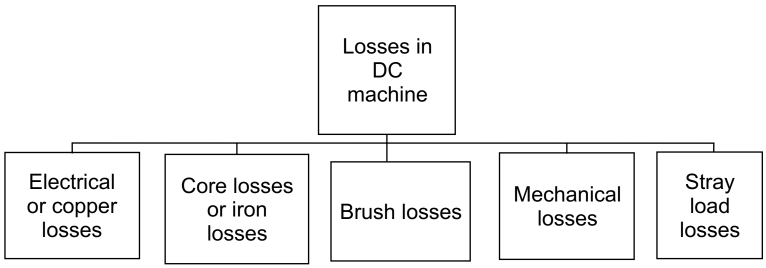


Figure 3.46 Losses in a DC Machine

Electrical or Copper Losses

Since the copper losses in a DC machine occur because of the winding resistance, they are also known as winding losses and these losses occur due the current flowing through the windings. The windings present in the DC machine are: armature, shunt and series field, interpoles and compensating windings. The losses that occur in these windings are given by:

Armature copper loss = $I_a^2 R_a$

Shunt-field copper loss = $I_{sh}^2 R_{sh}$

Series-field copper loss = $I_{se}^2 R_{se}$

Interpoles copper loss = $I_a^2 R_i$ and

Compensating winding copper loss = $I_a^2 R_c$

Where, R_i and R_c are the resistance of interpole and compensating windings respectively.
These losses contribute around 20% of the full load losses in DC machine.

Core Loss or Iron Loss or Magnetic Loss

These losses occur in the armature of a DC machine and attribute to armature rotation in the magnetic field produced by the poles. The two, different core or iron losses in a DC machine are:

- ***Hysteresis loss:*** occurs due to magnetisation reversal of the armature core, which happens when it passes through a pair of poles and it depends on the volume and grade of the material used in armature, frequency of magnetic reversals and flux density. Therefore, hysteresis loss is given by

  $$W_h = \eta B_{\max}^{1.6} f V_c$$

  where, $B_{\max}$ is the maximum flux density, η is the Steinmetz hysteresis constant, f is the frequency of magnetic reversal and V_c is the volume of the core in m^3.
- ***Eddy current loss:*** Occurs due to the large current flowing in the machine due to the self-induced emf in the armature, as given by

  $$W_e = K_e B_{\max}^2 f^2 t^2 V_c$$

  where, K_e is a constant and t is the thickness of core lamination in metres.
  These losses contribute about 20% of full load losses in the DC machine.

Mechanical Loss

The loss in a DC machine due to friction in bearings and commutator and windage loss occurring due to air friction in the armature core is known as mechanical loss. This loss is about 10 to 20% of the full load losses and it relies on the speed at which the armature core is rotated. It is noted that, for a given speed, this loss is constant.

Brush Loss

The loss that occurs at contact point of brush and takes place between the commutator and brushes is known as brush loss. This loss depends on the voltage drop across the brush contacts and I_a.

Stray Loss

Other than the above listed losses, the losses that exist in a DC machine which are difficult to determine are known as stray losses. The distortion of flux due to armature reaction and short-circuit currents occurring in the armature coil undergoing commutation are the factors considered in this loss. Also, the inaccuracies in designing and modelling of a DC machine contribute to stray losses. This loss contributes around 1% of the full load losses.

3.14 Power-Flow Diagram **[AU Nov/Dec, 2012]**

With the help of power-flow diagrams for a DC generator and a DC motor, the different losses existing in them can be clearly understood. It is known that in a DC motor, the input is the electrical power and output is the mechanical power and it is vice-versa in case of a DC generator. The power-flow diagrams of a DC generator and a DC motor are shown in Figure 3.47 and Figure 3.48 respectively.

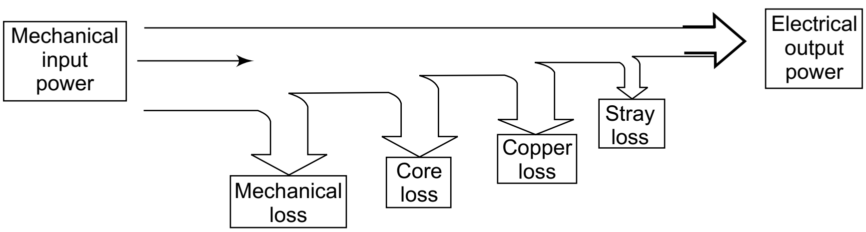


Figure 3.47 Power-flow Diagram of a DC Generator

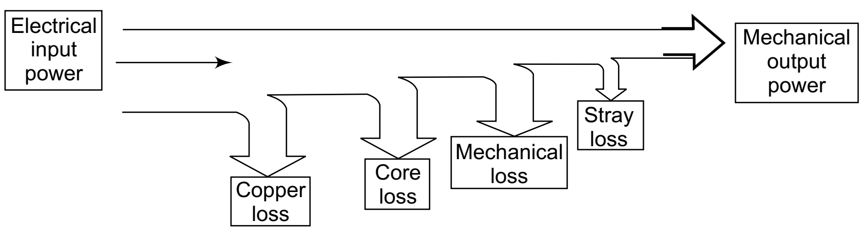


Figure 3.48 Power-flow Diagram in a DC Motor

3.15 Starting DC Motors [AU April/May, 2013]

A DC motor is started using a device called a starter and it is used to limit the starting current drawn by the motor. There are different types of starters used to start a DC motor.

3.15.1 Necessity of Starters

The voltage equation of a DC motor is given by $V = E_b + I_a R_a$. At the beginning, i.e., when the motor is in a standstill condition, $E_b = 0$. It is also known that the armature resistance is very small when compared to field windings. When the rated supply voltage is applied to a DC motor, due to negligible R_a, the current drawn by it is very high and it starts flowing through the armature. The disadvantages of having such high current are:

1. The fuse connected to the circuit gets blown and the armature winding, along with the commutator brush arrangement, gets damaged.
2. Since torque is directly proportional to I_a, a very high starting-torque will be produced and it causes a huge centrifugal force, which might throw off the armature winding.
3. A dip in the terminal voltage can be observed in the other loads connected to the same source.

Due to these effects, it is necessary that the starting current in a DC motor be kept minimum. The different starters used to start a DC motor are:

- ***Two-point starter:*** Used to start a DC series motor.
- ***Three-point starter:*** Used to start the DC shunt and DC compound motors.
- ***Four-point starter:*** Used to start the DC shunt and DC compound motors.

3.15.2 Two-point Starter

The device used to start a DC series motor is known as a two-point starter. The schematic diagram of two-point starter used in DC series motor is shown in Figure 3.49.

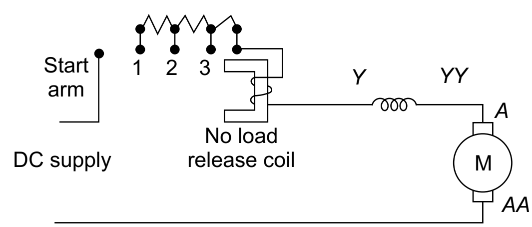


Figure 3.49 Two-point Starter

It consists of two points: start arm and no-load release coil. During the starting process, the start arm is connected to point 1, due to which the full resistance is connected in series with the armature resistance, thereby increasing the total armature resistance. Due to this movement of start arm, the armature draws a very small current. Once the DC series motor is started, the start arm is gradually moved towards right. The no-load release coil is used to hold the start arm during motor running process and leaves it when the DC motor stops or when the voltage supply is lost.

3.15.3 Three-point Starter

The schematic diagram of a three-point starter used to start a DC shunt and compound motor is shown in Figure 3.50.

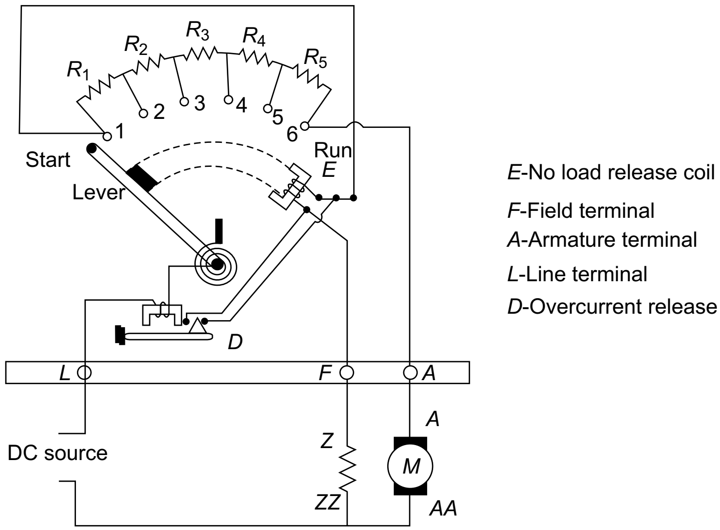


Figure 3.50 Three-point Starter

The three points or terminals in this starter are given as follows:

- L - Line terminal connected to the positive supply
- A - Armature terminal connected to the armature windings and
- F - Field terminal connected to the field terminal windings

It consists of a graded resistance, *R*, which is divided into five parts, R_1 to R_5, which is used to limit the starting current. The lever is kept in the OFF position, using a suitable arrangement and is moved manually. Electromagnet *E* is used to hold the lever in the final position. When the DC motor is switched off or when the supply voltage fails while the DC motor is running, this electromagnet is energised, which releases the lever and pulls it to its initial position. During overloading condition, the over-current release electromagnet D gets energised which in turn energises the electromagnet *E*. This will help release the lever and hence the motor is turned off.

During the starting of the DC motor, the lever is moved gradually to right such that it makes contact with position 1 and hence the whole resistances R_1 to R_5 are connected in series with the armature resistance, R_a. At this point, the field winding is directly connected to the supply. Since the total armature resistance is increased, the DC motor draws very small current. As the DC motor attains speed, the lever is moved gradually to position 6, thereby connecting the armature resistance directly to the supply. The electromagnet E holds the lever at this position and releases when there is no (or low) supply voltage.

Disadvantages of a Three-point Starter

The drawbacks of a three-point starter are as follows:

1. It is not applicable to a DC motor that has large variation of speed.
2. Reduced field current exists in the DC motor due to increase in the field resistance to increase the speed of the motor.
3. Reduced field current will make the electromagnet (*E*) to be weak and hence the lever may get released during normal operation.

Hence, to overcome these drawbacks, a four-point starter is used.

3.15.4 Four-point Starter

The schematic diagram of a four-point starter to start DC shunt and compound motors is shown in Figure 3.51.

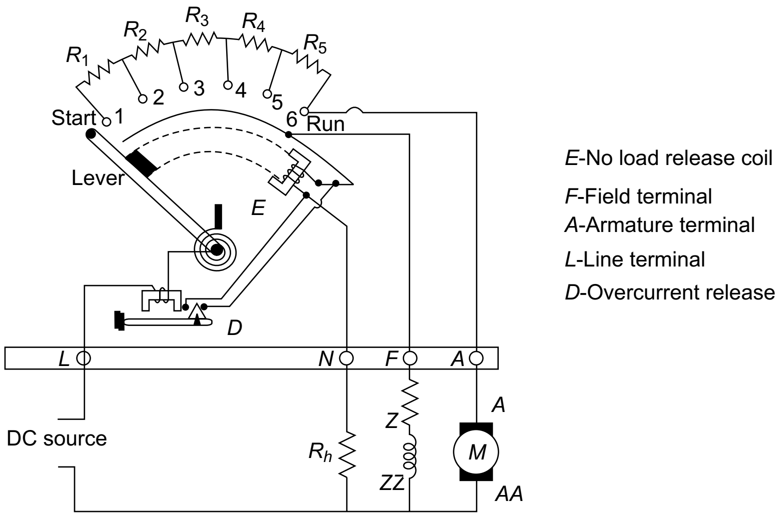


Figure 3.51 Four-point Starter

The main difference between a three-point starter and a four-point starter is that the electromagnet *E* is not connected in series with the shunt field coil. Instead, the shunt field winding is connected directly to the supply. The electromagnet *E* is connected to a current limiting resistance, R_h, which ensures that the current through electromagnet *E* does not get affected, if there is any change in current across the shunt-field winding. Therefore, the force experienced by *E* is sufficient enough to hold the lever in ON position. This type of starter is used where field rheostat is used to adjust the field current for operating the motor above rated speed.

3.16 Applications of DC Motors

3.16.1 DC Shunt Motor

1. Used in applications where constant speed is required.
2. Used in applications where adjustable speed in the range of 2 : 1, along with a medium starting-torque is required.
3. Used in lathe machine, centrifugal pumps, fans and blowers, machine tools like wood working machines, reciprocating pumps, spinning and weaving machines etc.

3.16.2 DC Series Motor

1. Used in applications where variable speed and high starting-torque are required.
2. Used in traction work, electric locomotives, rapid transit systems, trolleys, cars, cranes, conveyors etc.

3.16.3 DC Compound Motor

1. Used in applications where high starting-torque and moderately constant speed are required.
2. Used in devices like elevators, conveyors, heavy planers, rolling mills, ice machines, printing presses and air compressors.

3.17 Universal Motor [AU Nov/Dec, 2012]

A special type of motor that can be used with a single-phase AC supply or DC supply is called a universal motor. It is named so, as this motor can run both on AC as well as DC power supplies. This type of motor is a commutation-type motor and it is also known as a single-phase series motor. A high starting torque and variable speed are the characteristics of this motor. At no-load condition, this motor runs at very high speeds, exceeding 3500 rpm. Similar to a DC series motor, both field windings and armature windings are connected in series in a universal motor. The drawbacks while connecting an ordinary DC series motor to the AC supply are given below:

- Since the reactance voltage drop exists in the motor, it runs at a lower speed than that is connected to DC supply.
- When solid steel is used for forming the main frame, large eddy currents will develop in the motor.
- Power factor of the motor becomes low due to the large reactance existing in the field and armature windings.
- Excess sparking exists at the brushes.

Therefore, some modifications have to be made in a DC series motor, so that it becomes a universal motor, to enable it to operate on AC supply. These modifications are as follow:

- Material that is to be used in the field system in a DC series motor should have low hysteresis loss and should be laminated to decrease the eddy current loss.
- Field pole area is to be increased, which reduces the flux density that further reduces the iron loss and reactive voltage drop.
- Number of armature conductors, Z, is to be increased to get the required torque.

Apart from the above modifications, compensating winding is to be used in the DC series motor, which reduces the armature reaction effect and improves the commutation process. Therefore, based on the compensating winding used in the universal motor, it is classified as: (i) uncompensated type and (ii) compensated type universal motor.

3.17.1 Uncompensated-type Universal Motor

In this type of universal motor, other than the compensating winding, all modifications are implemented so that it can operate on AC supply with higher efficiency. The equivalent circuit of an uncompensated type universal motor is shown in Figure 3.52.

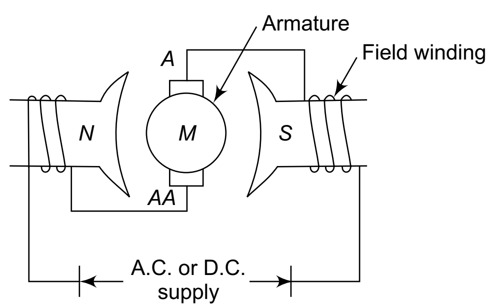


Figure 3.52 Uncompensated-type Universal Motor

3.17.2 Compensated-type Universal Motor

In this type of universal motor, the compensating winding is placed in the field poles, as shown in Figure 3.53. The distributed type field winding is used in this motor. The compensating winding used in this motor is also known as auxiliary winding and it helps reduce the reactance voltage when operated on AC supply.

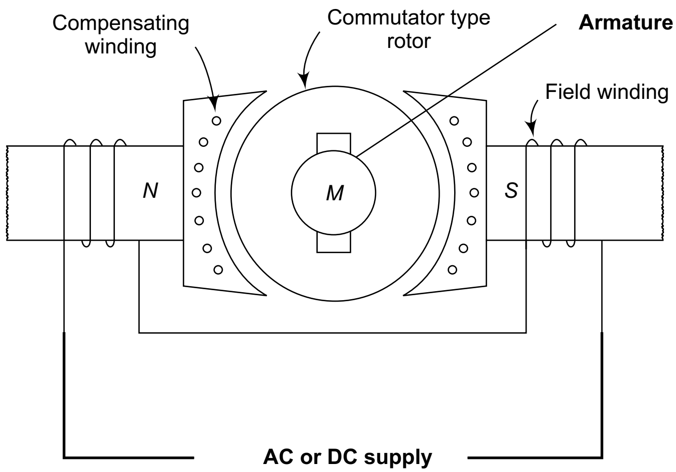


Figure 3.53 Compensated-type Universal Motor

Based on the method by which the compensating windings are connected, the compensated-type universal motor can be further classified as: (a) conductively compensated universal motor and (b) inductively compensated universal motor, whose equivalent circuits are shown in Figures 3.54 (a) and (b) respectively. It is clear from Figures 3.54 (a) and (b) that, if the compensating winding is connected in series with both armature and field windings, it is called conductively compensated universal motor and if the compensating winding is short-circuited and acts like a secondary winding in transformer, it is called inductively compensated universal motor.

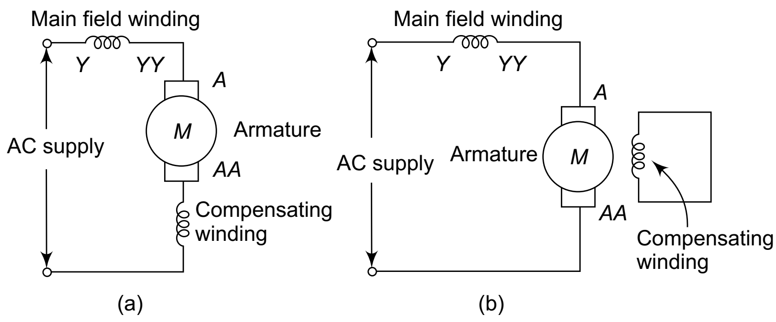


Figure 3.54 Types of compensated universal motor (a) Conductively compensated motor (b) Inductively compensated motor

In conductively compensated universal motor, the fluxes produced by the armature and compensating winding oppose each other, which further reduce the reactance of armature winding and have similar operating characteristics when operated on DC or AC supplies. In inductively compensated universal motor, the induced current in the compensating winding opposes the armature current, which further reduces the reactance of armature winding. The working of a universal motor is similar to a DC series motor. The speed control of this type of universal motor can be obtained using solid-state drives.

3.17.3 Applications of Universal Motor

1. Used in washing machines, blowers, kitchen appliances etc.
2. Used in applications where high quality of speed control is required.
3. Since the rating of universal motor is high, it is used in power drills, blenders etc.

Two Mark Questions and Answers

1. **State Faraday's law of electromagnetic induction.** **[AU Nov/Dec, 2011]**

   Faraday's law of electromagnetic induction states that whenever a current carrying conductor cuts the magnetic flux, a dynamically induced emf gets generated in the conductor.

2. **How are DC machines classified?** **[AU Nov/Dec, 2016]**

   DC machines are classified into DC motors and DC generators.

3. **What is the function of a DC generator?** **[AU Nov/Dec, 2009]**

   The DC generator is a dynamic DC machine, which generates electrical energy from mechanical energy, using Faraday's law of electromagnetic induction.

4. **List the main parts of a DC machine.** **[AU Nov/Dec, 2010]**

   The different parts of DC machine are listed in Section 3.2.2.

5. **What are the functions of a yoke? What is the choice of material for the yoke?** **[AU Nov/Dec, 2009]**

   Refer to Section 3.2.2 for the functions of a yoke and its preferred choice of material.

6. **State the functions of a commutator or specify the role of a commutator in a DC generator.** **[AU Nov/Dec, 2009; April/May, 2011]**

   The functions of a commutator are:
   - Through the brushes, it provides a connection between the rotating armature conductor and the stationary external circuit.
   - The alternating current induced in the armature conductor is converted into a unidirectional current in a DC generator.

7. **State the emf equation of a DC machine stating the meaning of each term.** **[AU Nov/Dec, 2009]**

   The emf equation of a DC machine is $\frac{\phi ZNP}{60A}$

   Where, ϕ is the flux per pole, Z is the number of armature conductors, N is the speed of the machine in rpm, P is the number of poles in the machine and A is the number of parallel paths.

8. **An eight-pole wave-connected armature has 600 conductors and is driven at 625 rpm. If the flux per pole is 20 mWb, determine the generated emf.** **[AU Nov/Dec, 2013]**

   Given, N= 625 rpm, P = 8, Z = 600 and ϕ = 20 mWb.
   Since the wave windings are wave-connected, A= 2. Therefore, the generated emf is

   $$E_g = \frac{\phi ZPN}{60A} = \frac{20 \times 10^{-3} \times 8 \times 600 \times 625}{60 \times 2} = 500 \text{ V}$$

9. What are the functions of interpoles and how are the interpole windings connected?
[AU April/May, 2011]

Refer to Section 3.3 for the functions of interpoles and their winding connection.

10. Define commutation of a DC machine. **[AU April/May, 2016]**

Refer to Section 3.4 for the commutation of a DC machine.

11. Mention the types of excitation in DC machines. Or what are the different methods of excitation of a generator? **[AU Nov/Dec, 2012]**

The different methods of excitation of a DC generator are: self-excitation and separate-excitation.

12. What is a self-excited generator? How does it get excited? **[AU April/May, 2011]**

If a separate DC source is not required for excitation, it is called a self-excited generator. Here, the required power for exciting the field winding is obtained from the power developed in the armature of the DC generator.

13. What is a DC compound generator? **[AU Nov/Dec, 2012; April/May, 2011]**

Refer to Section 3.6.4 for a DC compound generator.

14. Draw the open-circuit characteristic curve of a self-excited DC generator. **[AU Nov/Dec, 2011]**

The open-circuit characteristic of a self-excited DC generator is shown in Figure UQ3.14.

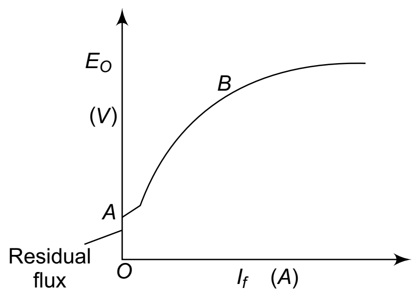


Figure UQ3.14

15. Draw the open-circuit characteristics of a DC generator. **[AU Nov/Dec, 2014]**

The open-circuit characteristics of separately and self-excited DC generators are shown in Figure UQ3.15.

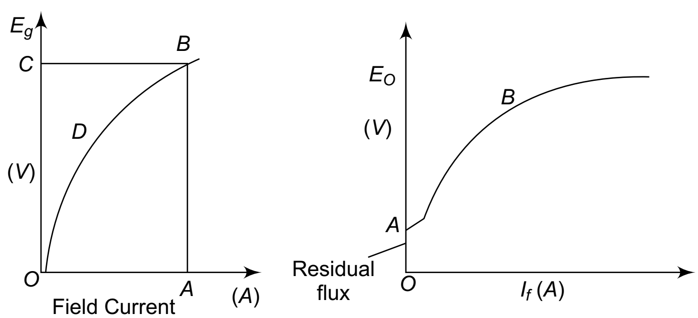


Figure UQ3.15

16. Write the necessary conditions to be satisfied for the self excited DC generator to build up emf.

Or

What are the conditions to be fulfilled for the self-excitation of a DC shunt generator?

[AU April/May, 2014; April/May, 2010; April/May 2011]

Refer to Section 3.7.2 for the conditions to be fulfilled for the self-excitation of a DC shunt generator.

17. Sketch the load characteristics of a DC series generator. **[AU April/May, 2011]**

The load characteristics of a DC series generator are shown in Figure UQ3.17.

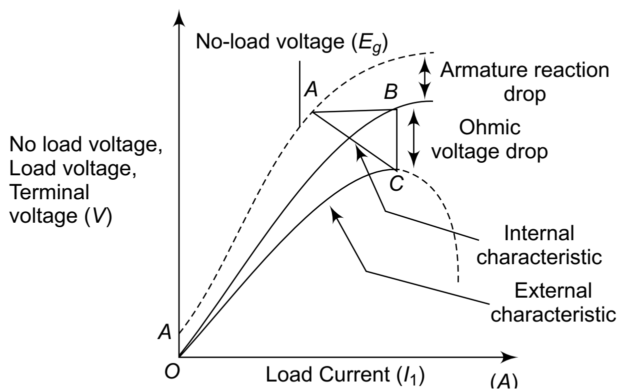


Figure UQ3.17

18. What is an electric motor? State its principle of working. **[AU Nov/Dec, 2011]**

DC motor is a dynamic DC machine, which generates mechanical energy from electrical energy. Refer to Section 3.9.1 for its working principle.

19. Write the working principle of a DC motor. **[AU Nov/Dec, 2016]**

Refer to Section 3.9.1 for the working principle of a DC motor.

20. Define back emf of a DC motor. Or what is back emf? **[AU Nov/Dec, 2012; April/May, 2012]**

Refer to Section 3.9.4 for the back emf of a DC motor.

21. What is the significance of back emf? **[AU April/May, 2013]**

Refer to Section 3.9.4 for the significance of back emf.

22. A DC motor operates from 240 V supply. The armature resistance is 0.2 Ω. Determine the back emf when the armature current is 50 A. **[AU Nov/Dec, 2013]**

Given, $R_a = 0.2\ \Omega$, $I_a = 50$ A and $V = 240$ V

Therefore, the back emf of DC motor is

$$E_b = V - I_aR_a = 240 - (50 \times 0.2) = 230 \text{ V}$$

23. A 200 V DC motor has an armature resistance of 0.06 Ω and series field resistance of 0.04 Ω. If the motor input is 20 kW, find the back emf of the motor and power developed in armature.

[AU May/June, 2016]

The line or load current of the armature is $I_L = \dfrac{P_i}{V} = \dfrac{20 \times 10^3}{200} = 100\text{ A}$. Since $I_a = I_L$, the back emf of

the motor is $E_b = V - I_aR_a = 200 - (100 \times 0.06) = 194$ V.
The power developed in armature is $E_bI_a = 194 \times 100 = 19.4$ kW.

24. For a DC motor, write the expression for speed. **[AU Nov/Dec, 2016]**

The expression for speed of a DC motor is

$$N = \frac{V - I_aR_a}{\phi} \times \frac{60A}{ZP} = \frac{E_b}{\phi} \times \frac{60A}{ZP}$$

where, V is the supply voltage, E_b is the back emf of a DC motor, I_a is the armature current, R_a is the armature resistance, ϕ is the flux per pole, Z is the number of armature conductors and A is the number of parallel paths.

25. List the types of DC motors. Give any one difference between them. Or mention the types of DC motors. **[AU Nov/Dec, 2014; Nov/Dec, 2016]**

Refer to Section 3.10 for the types of DC motors and their description.

26. Draw the speed torque characteristics of DC shunt and series motors. **[AU April/May, 2012]**

Refer to Sections 3.11.1 and 3.11.2 for the speed torque characteristics of DC series and shunt motors.

27. List the different methods of speed control of a DC shunt motor. **[AU Nov/Dec, 2011]**

Refer to Section 3.12.1 for the speed control of DC shunt motor.

28. Give the reasons for high starting-current in a DC motor. **[AU April/May, 2014]**

Refer to Section 3.15.2 for the reasons of having a high starting-current in a DC motor.

29. What is the necessity of a starter for starting a DC motor? **[AU April/May, 2012; Nov/Dec, 2010]**

Refer to Section 3.15.2 for the necessity of a starter for starting a DC motor.

30. State the various applications of DC motors. **[AU April/May, 2013]**

Refer to Section 3.16 for the various applications of DC motors.

Review Questions

1. What are the major parts of a DC generator?
2. What are the functions of yoke? Explain the choice of material for the yoke.
3. Compare lap and wave windings.
4. Define pole pitch.
5. What is a commutator?
6. State the functions of a commutator.
7. Discuss the basic concepts of emf generation in a DC machine.
8. Derive the emf equation of a DC generator.
9. State the emf equation of a DC machine.
10. What is armature reaction? What are its effects?
11. Explain the various effects of armature reaction.
12. Explain the methods to reduce the effects of the armature reaction.
13. Define commutation and commutation period.
14. Discuss in detail, the phenomenon of commutation in a DC machine.
15. Explain under-commutation in a DC machine.

16. State the methods of improving commutation.
17. Explain the voltage and current relation for a separately excited generator.
18. With schematic diagrams, explain the working principles of different types of DC generators.
19. What is meant by circuit model of a DC generator? Explain in detail.
20. The series field winding has a low resistance while the shunt field winding has a high resistance. Why?
21. Why is the external characteristic of a DC shunt generator more drooping than that of a separately excited generator?
22. State the causes of failure to excite self-excited generator and the remedies for it.
23. Draw and explain the characteristics of a DC shunt generator.
24. Draw and explain the characteristics of a DC compound generator.
25. Draw the performance characteristics of different types of DC generators and explain them.
26. State the applications of various types of DC generators.
27. What is back emf?
28. How does back emf in a DC motor make the motor self-regulating? State the significance of back emf.
29. Write the power balance equation of a motor.
30. Derive, from the first principle, an expression for the torque developed in a DC motor.
31. Draw the speed-current and torque-current characteristics of a DC series motor.
32. Draw the speed-current and torque-current characteristics of a DC shunt motor.
33. Draw and explain the characteristics of a DC series motor.
34. Draw the characteristics of a DC compound motor.
35. Explain the methods of speed control of a DC shunt motor with the help of neat diagrams.
36. Discuss in detail about shunt armature speed control of a DC shunt motor.
37. Explain in detail the various methods of speed control in a DC series motor.
38. Explain Ward–Leonard method of speed control in a DC motor.
39. Why can't a DC series motor be started on no load?
40. Give the reasons of using starters in a DC motor.
41. With a neat sketch, explain the function of a three-point starter.
42. Explain, with a neat sketch, the function of no-volt release and overload release in a three-point DC motor starter.
43. What are the various starting methods of a DC motor? Explain any one method.
44. Draw and explain the operation of a four-point starter.
45. Draw the circuit of a two-point starter and explain the principle of operation.
46. Why is a DC series motor used to start heavy loads?
47. How does a four-point starter differ from a three-point starter?
48. State the various applications of a DC motor.

CHAPTER 4

AC Machines

4.1 Introduction

The induction motor is an important class of electrical machines in our day-to-day applications. More than 85% of the motors used in industries are induction motors. Three-phase and single-phase induction motors are most widely used in industrial and domestic applications respectively. In this chapter, the working principles, constructions, equivalent circuits and torques developed by three-phase and single-phase induction motors are discussed. Also, the performance characteristics of the induction motor obtained by blocked rotor, no-load and load test are discussed. At the time of starting, the induction motor draws a large amount of current that causes damage to the equipment. Hence, a starter is needed to limit the starting current. Therefore, the necessity of a starter, along with its type, is discussed elaborately. In order to obtain constant speed, irrespective of load and a variable power-factor operation, a synchronous motor is employed. The detailed working principle, construction, emf equation, characteristics, staring methods and applications of a synchronous motor are also discussed. In addition, the working principle and construction of a stepper motor and a brushless DC motor are also discussed in this chapter.

4.2 Three-Phase Induction Motor

A three-phase induction motor is an AC motor consisting of a three-phase winding and it works on the principle of a rotating magnetic field. The magnetic field rotates at a speed known as synchronous speed. Since the induction motor rotates at a speed less than the synchronous speed, it is also called an asynchronous motor. The working principle, construction, working and characteristics of a three-phase induction motor are discussed in this section.

4.2.1 Working Principle

When a three-phase balanced AC voltage is applied to a balanced three-phase winding, a rotating magnetic field with constant magnitude and speed is produced. If a stationary conductor is placed in this magnetic field, an emf gets induced in it, which induces a current in the circuit. When a closed path is formed for the induced current to flow, an electromagnetic torque is exerted on the conductor and hence the conductor rotates. Since the three-phase induction motor works on the principle of a rotating magnetic field, it is necessary to understand the generation of the rotating magnetic field in detail.

4.2.2 Rotating Magnetic Field [AU April/May, 2011]

In a three-phase induction motor, exciting the set of stationary windings using a three-phase AC supply produces a rotating magnetic field. The current flowing through these windings produces its respective magnetic field or flux. When these three fluxes interact with each other, a resultant flux with a constant magnitude is produced. This resultant magnetic field is called a rotating magnetic field because it has constant magnitude and its axis is getting rotated without physically rotating the windings. Figure 4.1 shows the supply given to a three-phase induction motor, where three-phase windings, $R - R'$, $Y - Y'$ and $B - B'$, can be either in star or delta connection.

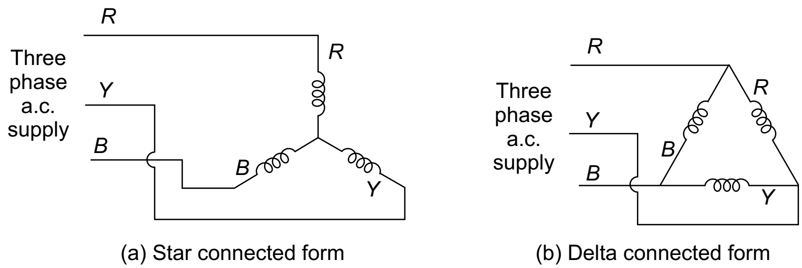


Figure 4.1 Three-phase Induction Motor with Star or Delta Connection

When an AC supply excites these windings, three alternating fluxes, displaced from each other by 120°, are produced. The fluxes generated in the windings $R - R'$, $Y - Y'$ and $B - B'$ are denoted by ϕ_R, ϕ_Y and ϕ_B respectively, and are given by:

$$\phi_R = \phi_m \sin\theta$$
$$\phi_Y = \phi_m \sin(\theta - 120°)$$
and $$\phi_B = \phi_m \sin(\theta - 240°)$$

where, ϕ_m is the maximum value of the alternating flux generated in each winding.

The waveforms of these three fluxes and their phasor diagrams are shown in Figure 4.2 (a) and (b) respectively.

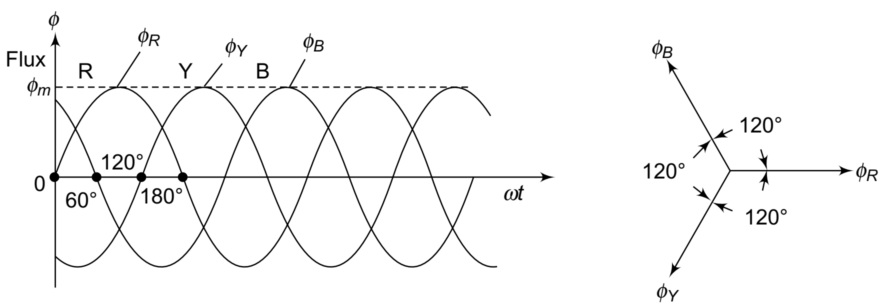


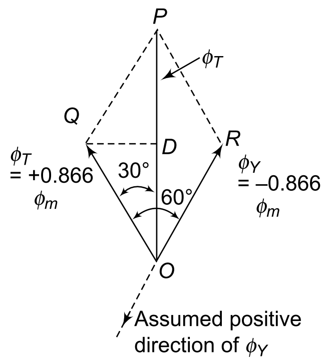
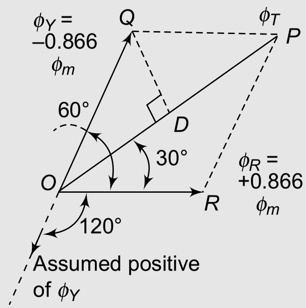
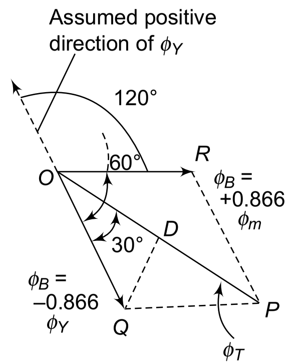
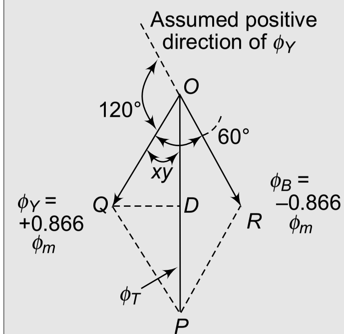
Figure 4.2 Windings of an Induction Motor (a) Flux Waveform (b) Phasor Diagram

From Figure 4.2 (a), it can be concluded that at any instant of θ, the resultant flux of the induction motor is given by the phasor addition of individual fluxes.

i.e., $$\overline{\phi_T} = \overline{\phi_R} + \overline{\phi_Y} + \overline{\phi_B}$$

The explanation of the rotation of resultant flux, $\overline{\phi_T}$ at $\theta = 0°$, 60°, 120° and 180° is given in Table 4.1

Table 4.1 *Illustration of Rotating Magnetic Field*

	$\theta = 0°$	$\theta = 60°$	$\theta = 120°$	$\theta = 180°$
Individual Flux	$\phi_R = 0$ $\phi_Y = -0.866\phi_m$ $\phi_B = +0.866\phi_m$	$\phi_R = +0.866\phi_m$ $\phi_Y = -0.866\phi_m$ $\phi_B = 0$	$\phi_R = +0.866\phi_m$ $\phi_Y = 0$ $\phi_B = -0.866\phi_m$	$\phi_R = 0$ $\phi_Y = +0.866\phi_m$ $\phi_B = -0.866\phi_m$
Phasor Diagram				
Resultant Flux	$\phi_T = 1.5\phi_m$	$\phi_T = 1.5\phi_m$	$\phi_T = 1.5\phi_m$	$\phi_T = 1.5\phi_m$

Therefore, the following inferences can be made when the individual fluxes are rotated for 180°:

- The magnitude of the resultant flux at different instances when individual fluxes interact with each other is same, and is given by $1.5\phi_m$.
- The direction of resultant magnetic flux changes at each instant and rotates with a certain speed.

Since the resultant flux, $\overline{\phi}_T$ rotates for each instant of θ, the magnetic field generated by the stator windings is called rotating magnetic field.

4.2.3 Speed of Rotating Magnetic Field

The speed in rpm at which the rotating magnetic field rotates is called synchronous speed, denoted by N_{syn}. If the supply frequency, f and the number of poles, P are known, then the synchronous speed is given by

$$N_{\text{syn}} = \frac{120f}{P} \text{rpm} \tag{4.1}$$

The synchronous speed in rps, n_{syn} is given by

$$n_{\text{syn}} = \frac{N_{\text{syn}}}{60} = \frac{2f}{P}$$

4.2.4 Direction of Rotating Magnetic Field

The rotating magnetic field (RMF) produced by the stator windings can be rotated either in clockwise or anti-clockwise direction. When any two stator-windings of an induction motor are interchanged, the direction of the rotating magnetic field gets reversed. This concept of direction of rotating magnetic field is shown in Figures 4.3 (a) and (b).

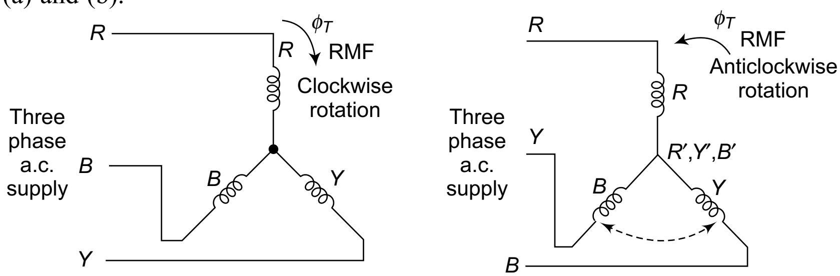


Figure 4.3 Direction of Rotating Magnetic Field (a) Clockwise Direction (b) Anti-clockwise Direction

4.2.5 Construction of a Three-phase Induction Motor

[AU Nov/Dec, 2014; April/May, 2014]

The two important parts of a three-phase induction motor are: (i) stationary three-phase windings, called stator and (ii) rotating component, called rotor. The rotor is connected to the mechanical load through a shaft. The schematic representation of a three-phase induction motor is shown in Figure 4.4.

Stator

It is the stationary part of the three-phase induction motor. The outer, solid, circular, steel metal part of the stator is called a yoke or frame. Also, it has a laminated cylindrical drum with insulated stampings, called the stator drum. These silicon-steel stampings are insulated from each other, with about 0.5 mm thickness, to reduce the iron losses and hysteresis losses. These stampings are embossed together to build the stator drum and fitted in a yoke or frame. Slots are provided in the stampings to carry the required number of stator conductors. These conductors are connected in series to form balanced three-phase windings called stator windings, which are star or delta-connected windings. These windings are wound on a definite number of poles. The stator windings, when excited using a three-phase AC supply, produce the required rotating magnetic field.

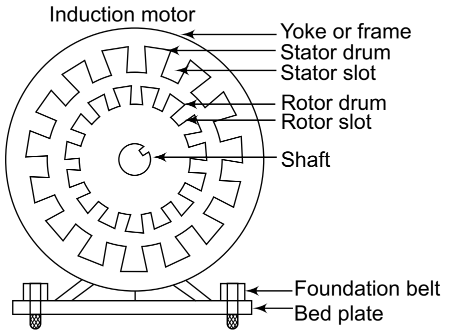


Figure 4.4 Schematic Representation of a Three-phase Induction Motor

Rotor

It is the rotating part of the three-phase induction motor and is placed inside the stator. This cast iron rotor is cylindrical, laminated and provided with slots to carry rotor conductors or windings. The air gap between the stator and the rotor is kept as low as possible. The two different types of rotors are: (i) squirrel-cage rotor and (ii) slip-ring or wound rotor.

Squirrel-cage Rotor

The construction of a squirrel-cage rotor and its symbolic representation is shown in Figures 4.5 (a) and (b) respectively. In this type, the rotor core is cylindrical in shape with slots to carry the bar-shaped uninsulated copper or aluminium rotor conductors. Using a conducting copper end-ring, these conductors are permanently shorted at its ends to provide good mechanical strength. Since the closed electrical circuit resembles a cage, this rotor is called a squirrel-cage rotor or a short-circuited rotor.

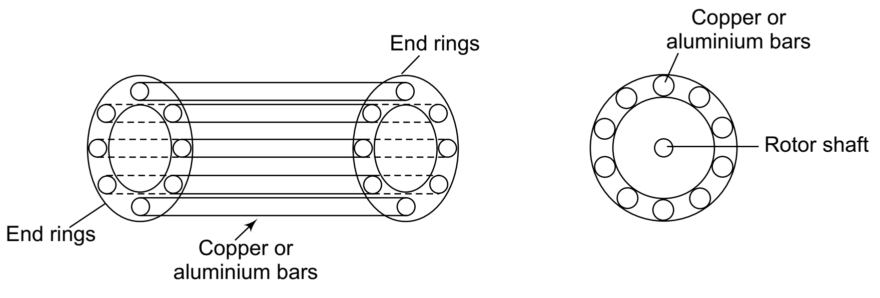


Figure 4.5 Squirrel-cage Rotor (a) Construction (b) Symbolic Representation

As the conductors are permanently shorted, the total rotor resistance of the rotor is minimum and no external resistance can be added in the rotor resistance. In general, the slots are skewed, as shown in Figure 4.6, rather than in parallel to the shaft.

The reasons for skewing the slots in the squirrel-cage rotor are:

- Smooth rotor operation is possible.
- Magnetic locking between stator and rotor gets reduced.
- Effective transformation ratio between stator and rotor increases.

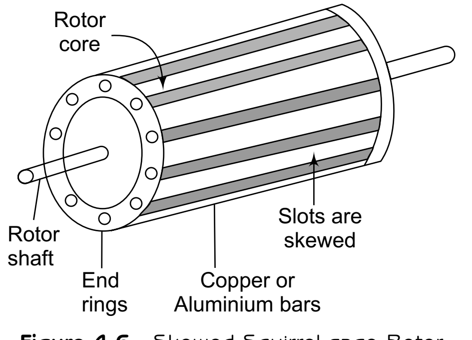


Figure 4.6 Skewed Squirrel-cage Rotor

Slip-ring or Wound-ring Rotor

The schematic diagram of a slip-ring rotor is shown in Figure 4.7. Similar to the stator of a three-phase induction motor, the rotor conductors placed in the rotor slots are electrically connected to form a balanced three-phase winding. Same number of stator poles is used in the rotor to carry the rotor windings. These star or delta-connected rotor windings are permanently connected to a slip ring and brush assembly, which are mounted on the shaft. During running condition of the motor, these slip rings are used to create a short-circuit condition by connecting a metal collar. In this rotor, the external resistances can be added in series with rotor resistance per phase through slip and brush assembly and hence, the total rotor resistance per phase can be controlled.

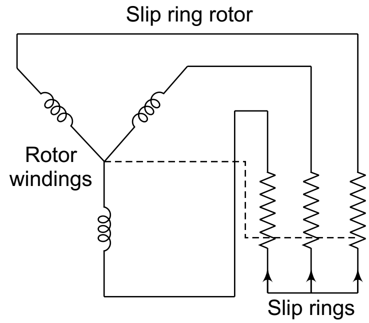


Figure 4.7 Slip-ring Rotor

Slip Ring and Brush Assembly

The slip ring and brush assembly are used to connect the rotor, which is rotating continuously, to the stationary external circuit. The three slip-rings are mounted on the shaft where the three-phase rotor winding is rotating and each slip ring is connected to an individual winding. The stationary brushes make contact with these slip rings, as shown in Figure 4.7. Hence, the rotating windings are available at the brushes, to which the external circuit can be connected. Also, the voltage can be supplied to the rotor windings by connecting the supply to the brushes.

4.2.6 Comparison between Slip-ring and Squirrel-cage Rotor **[AU Nov/Dec, 2009]**

The comparison between slip-ring and squirrel-cage rotors is listed in Table 4.2.

Table 4.2 *Comparison Between Slip-ring and Squirrel-cage Rotors*

Slip-ring Rotor	Squirrel-cage Rotor
The rotor of the motor is constructed as a slip-ring type.	The rotor of the motor is a squirrel-cage type.
Has a three-phase winding, similar to a stator.	Has bar-shaped rotor conductor and is shorted using end rings.
Also called phase-wound rotor.	Also called cage motor.
Complicated construction and costly.	Simple construction and cheap.
Possible to add external resistance.	It is not possible to add external resistance.
Requires slip ring and brush assembly to connect the rotor to the external circuit.	Slip ring and brush assembly is not required.
The rotor resistance starter can be used.	Rotor resistance starter cannot be used.
High starting-torque and low starting-current.	Low starting-torque and high starting-torque.
Requires frequent maintenance due to the existence of brushes.	Requires less maintenance.
Rotor copper loss is high.	Rotor copper loss is less.
Efficiency of the motor is low.	Efficiency of the motor is high.
Used in applications like lifts, hoists etc., where high starting-torque is required.	Used in lathe machines, fans, blowers, profiting machines, etc.

4.2.7 Working of a Three-phase Induction Motor [AU Nov/Dec, 2014; April/May, 2011]

When a three-phase balanced AC voltage is applied across the balanced three-phase stator winding, a rotating magnetic field, which rotates at synchronous speed, N_{syn}, is generated and it passes through the stator, air gap and rotor, as shown in Figure 4.8 (a). When the time-varying rotating magnetic field links with the stationary rotor conductors, an emf is induced in the rotor. When the rotor is connected to an external circuit, a current called rotor current flows through the rotor conductors, as shown in Figure 4.8 (b). It is assumed that the rotating magnetic field is rotating in a clockwise direction and the current flows inside the rotor conductor that is indicated as '×' in Figure 4.8 (b). It is obvious that the current carrying conductor generates its magnetic flux by itself. Hence, a flux called rotor flux is produced, whose direction is determined using Fleming's rule, as shown in Figure 4.8 (c). Since there are two fluxes, an interaction between these fluxes is possible, as shown in Figure 4.8 (d), where at the right of the rotor conductor, the two fluxes cancel each other and at the left of the rotor conductor, the two fluxes are added up. Therefore, low and high flux densities are seen at the right and left of the rotor conductor. Since the high flux-density area exerts a push on the rotor conductor, it starts revolving from left to right, as shown in Figure 4.8(d). The direction of rotation of rotor conductor is same as the direction of the rotating magnetic field. In other words, according to Lenz's law, the nature of the rotor-induced current is to oppose the cause producing it, i.e., the rotating magnetic field. Therefore, the rotor rotates in the same direction as that of the rotating magnetic field with speed N that is less than N_{syn} i.e., $N < N_{syn}$ and the rotor conductor tries to match up the speed of the rotating magnetic field but never reaches N_{syn}.

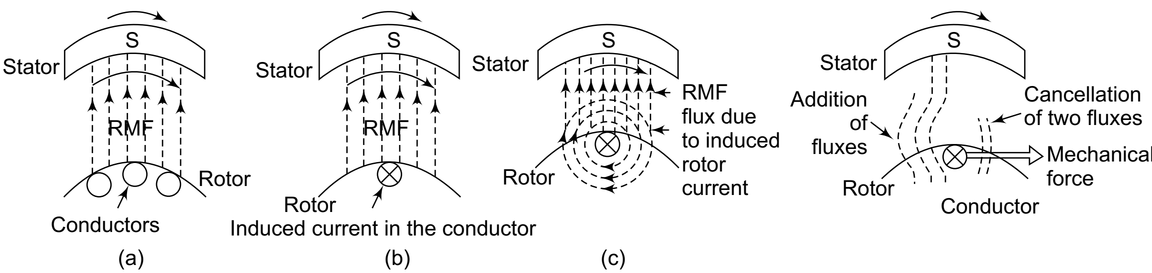


Figure 4.8 Working of a Three-phase Induction Motor

4.2.8 Reason for $N < N_{syn}$

The following sequence of events will happen if the rotor conductor rotates with speed N_{syn} i.e., when $N = N_{syn}$:

- The relative motion between the rotor and the rotating magnetic field becomes zero i.e., $N_{syn} - N = 0$
- Since the relative speed is zero, no emf will be induced in the rotor and there will be no rotor current or rotor flux.
- Therefore, no torque will be produced on the rotor and eventually the induction motor stops.

Therefore, when the rotor is rotating at N_{syn}, the motor eventually stops. Hence, the rotor that rotates at speed less than N_{syn} is called sub-synchronous speed.

4.3 Slip of an Induction Motor

The difference between the synchronous or rotating magnetic-field speed, N_{syn} and sub-synchronous or rotor speed, N is called slip speed. Therefore,

$$\text{slip speed} = N_{syn} - N \text{ rpm}$$

The magnitude of induced emf, rotor current and torque developed in the rotor are decided based on this slip speed.

The slip is defined as the difference between the synchronous speed, N_{syn} and the rotor or motor speed, N expressed as a fraction of N_{syn}. It is also called absolute or fractional slip, denoted by s. Therefore,

$$s = \frac{N_{syn} - N}{N_{syn}} \tag{4.2}$$

Also, the percentage slip is given by

$$\%s = \frac{N_{syn} - N}{N_{syn}} \times 100 \tag{4.3}$$

Using Eqn. (4.2), the motor speed is given by

$$N = N_{syn}(1 - s) \tag{4.4}$$

4.3.1 Inference From Slip, *s*

- Since the motor speed, N is zero at start, the slip $s = 1$. This is the maximum slip value achieved by the induction motor.
- If the motor speed is equal to N_{syn}, the slip $s = 0$. But in an induction motor, the motor speed is always less than N_{syn}. Therefore, the slip of an induction motor can neither be zero nor less than zero.
- The slip of an induction motor at full-load condition is called *full-load slip*.
- Generally, the slip of an induction motor lies in the range of 0.01 to 0.05 i.e., 1 % to 5 %.

4.4 Effect of Slip on Rotor Parameters **[AU Nov/Dec, 2010]**

Similar to a transformer, till the slip of the induction motor is 1 i.e., $s = 1$, the frequency of the induced emf in the rotor is equal to the frequency of the voltage supplied for exciting stator windings i.e., stator voltage. As the speed of the induction motor increases, slip starts decreasing and for each value of motor speed, a slip exists. Therefore, the frequency of induced emf in rotor starts decreasing due to slip. Hence, due to this variation in rotor frequency, the following rotor parameters also get affected:

- Rotor frequency
- Magnitude of rotor induced emf
- Rotor resistance and rotor reactance
- Rotor power-factor and
- Rotor current

The variation of these rotor parameters with respect to slip is discussed below.

4.4.1 Rotor Frequency, f_r

At the start of the induction motor, $N = 0$ and $s = 1$. Since the relative speed between the synchronous speed and motor speed is $(N_{syn} - N)$, the magnitude of induced emf in the rotor is more and its frequency is same as that of the supply voltage. But when the motor gains speed, the relative speed $N_{syn} - N$ starts decreasing, which further reduces the magnitude of the induced emf and its frequency. If f_r is the rotor frequency of the induced emf in running condition, then there exists a relation between the slip speed i.e., $N_{syn} - N$ and f_r, similar to Eqn. (4.1), as given by

$$N_{syn} - N = \frac{120 f_r}{P} \tag{4.5}$$

where, P is the number of rotor poles that is equal to the number of stator poles.
Taking ratio of Eqn. (4.5) to Eqn. (4.1), we get

$$\frac{N_{syn} - N}{N_{syn}} = \frac{f_r}{f}$$

Using Eqn. (4.2) in the above equation, we get

$$\frac{f_r}{f} = s$$

i.e.,

$$f_r = sf \tag{4.6}$$

Hence, the frequency of rotor-induced emf in running condition is slip times the supply voltage frequency. Since the slip of the induction motor is in the range of 1% to 5%, the rotor frequency is very small in running condition.

4.4.2 Magnitude of Rotor-induced emf

Since the rotating magnetic field is rotating at N_{syn}, an emf E_2 is induced in the rotor at standstill condition. Here, the induced emf is directly proportional to the speed of the machine responsible for it, as represented by

$$E_2 \ \alpha \ N_{syn} \tag{4.7}$$

Similarly, at running condition, when the motor gains speed, the relative speed between N_{syn} and N i.e., $N_{syn} - N$ is responsible for the emf induced in the rotor, E_{2r}. Therefore,

$$E_{2r} \ \alpha \ N_{syn} - N \tag{4.8}$$

Dividing Eqn. (4.8) by Eqn. (4.7), we get

$$\frac{E_{2r}}{E_2} = \frac{N_{syn} - N}{N_{syn}}$$

Using Eqn. (4.2) in the above equation, we get

$$\frac{E_{2r}}{E_2} = s$$

i.e.,

$$E_{2r} = sE_2 \tag{4.9}$$

Hence, the magnitude of rotor-induced emf in running condition is slip times the magnitude of induced emf in standstill condition.

4.4.3 Rotor Resistance and Rotor Reactance

The rotor winding has its own resistance and inductance i.e., the rotor winding has its own impedance. The squirrel cage rotor has a very small resistance whereas the slip ring motor has large resistance respectively. Generally, the rotor resistance in a squirrel-cage motor is neglected and the rotor resistance in a slip-ring motor can be varied using external resistance. Here, R_2 and X_2 are the resistance and reactance of the rotor per phase in ohms (Ω).

i.e., $$X_2 = 2\pi f L_2 \tag{4.10}$$

where, L_2 is the inductance of the rotor per phase and f is the frequency of the induced emf in rotor at standstill condition.

It is known that the frequency of induced emf in rotor varies when the speed of the motor increases. Therefore, at running condition, the reactance of the rotor per phase is given by

$$X_{2r} = 2\pi f_r L_2$$

Substituting Eqn. (4.6) in the above equation, we get

$$X_{2r} = 2\pi s f L_2$$

Using Eqn. (4.10) in the above equation, we get

$$X_{2r} = sX_2 \tag{4.11}$$

It is clear from the above equation that the rotor reactance in running condition decreases when compared to the reactance value in standstill condition. Since the rotor resistance is independent of the frequency, its values are same in both standstill and running conditions.

Therefore, the rotor impedances per phase in standstill and running conditions are represented as Z_2 and Z_{2r}. They are given by:

$$Z_2 = R_2 + jX_2 \text{ i.e., } |Z_2| = \sqrt{R_2^2 + X_2^2}\ \Omega/\text{ph} \tag{4.12}$$

and $$Z_{2r} = R_2 + jX_{2r} \text{ i.e., } Z_{2r} = \sqrt{R_2^2 + X_{2r}^2}\ \Omega/\text{ph} \tag{4.13}$$

Using Eqn. (4.11), we get

$$Z_{2r} = R_2 + jsX_2 \text{ i.e., } Z_{2r} = \sqrt{R_2^2 + (sX_2)^2}\ \Omega/\text{ph} \tag{4.14}$$

4.4.4 Rotor Power-factor

The impedance triangles of the rotor in standstill and running condition are shown in Figures 4.9 (a) and (b) respectively.

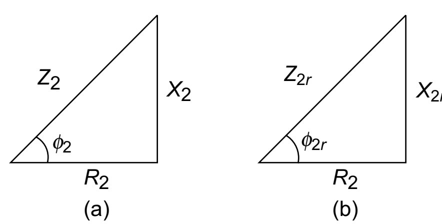


Figure 4.9 Rotor Impedance Triangle in (a) Standstill Condition (b) Running Condition

Therefore, the rotor power-factors in standstill and running conditions are given by:

$$\cos\phi_2 = \frac{R_2}{|Z_2|} \quad \text{and} \quad \cos\phi_{2r} = \frac{R_2}{|Z_{2r}|}$$

Substituting Eqns. (4.12) and (4.14) in the above equation, we get:

$$\cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \quad \text{and} \quad \cos\phi_{2r} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \tag{4.15}$$

Here, the rotor power-factor is lagging since the rotor winding is inductive in nature.

4.4.5 Rotor Current

The rotor current per phase in standstill and running conditions are denoted by I_2 and I_{2r} respectively. Using Ohm's law, the rotor current per phase in standstill condition is given by

$$I_2 = \frac{E_2}{|Z_2|} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \tag{4.16}$$

Therefore, the equivalent circuit of a rotor in standstill condition using the above equation is shown in Figure 4.10 (a).

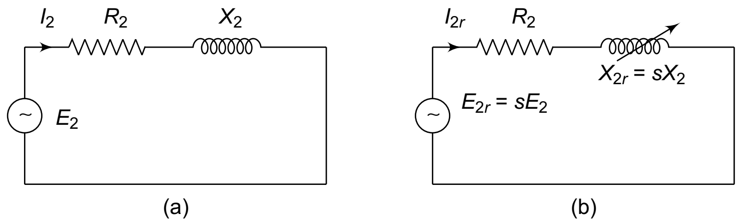


Figure 4.10 Equivalent Circuit of Rotor at (a) Standstill Condition (b) Running Condition

Similarly, the rotor current per phase in running condition is given by

$$I_{2r} = \frac{E_{2r}}{|Z_{2r}|} = \frac{E_{2r}}{\sqrt{R_2^2 + X_{2r}^2}}$$

Substituting Eqn. (4.9) and Eqn. (4.14) in the above equation, we get

$$I_{2r} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \tag{4.17}$$

Therefore, the equivalent circuit of a rotor in running condition, using Eqn. (4.17), is shown in Figure 4.10 (b).

The different rotor parameters in running and standstill conditions are listed in Table 4.3.

Table 4.3 *Rotor parameters in running and standstill conditions*

Rotor Parameter	At Standstill Condition	At Running Condition
Rotor Frequency	f	$f_r = sf$
Magnitude of Rotor-induced emf	E_2	$E_{2r} = sE_2$
Rotor Resistance	R_2	R_2
Rotor Reactance	$X_2 = 2\pi f L_2$	$X_{2r} = sX_2 = 2\pi f_r L_2$
Rotor Impedance	$Z_2 = R_2 + jX_2$	$Z_{2r} = R_2 + jX_{2r} = R_2 + jsX_2$
Rotor Power Factor	$\cos\phi_2 = \frac{R_2}{\lvert Z_2\rvert}$	$\cos\phi_{2r} = \frac{R_2}{\lvert Z_{2r}\rvert}$
Rotor Current	$I_2 = \frac{E_2}{\lvert Z_2\rvert} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$	$I_{2r} = \frac{E_{2r}}{\lvert Z_{2r}\rvert} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$

4.5 Three-Phase Induction Motor vs Transformer [AU April/May, 2003]

The comparison of a three-phase induction motor with a transformer is listed in Table 4.4.

Table 4.4 *Comparison Between a Three-phase Induction Motor and a Transformer*

Three-phase Induction Motor	Transformer
Stator and rotor are the two parts of an induction motor.	Primary and secondary windings are the two parts of a transformer.
AC supply is given to the stator	AC supply is given to the primary winding
Also called rotating transformer	Also called stationary transformer
A distinct air gap exists between the stator and the rotor.	No air gap exists between the stator and the rotor
Frequencies of induced emf and current in the stator and the rotor vary with respect to slip	Frequencies of induced emf and current in the primary and secondary windings are same
Part of the energy in the rotor circuit is electrical and some part is converted into mechanical form.	The total energy in the secondary winding is in electrical form.
If E_1 and E_2 are the stator and rotor emfs per phase, then the transformation ratio is $K = \frac{E_2}{E_1}$	If E_1 and E_2 are the primary and secondary induced emfs per phase, then the transformation ratio is $K = \frac{E_2}{E_1}$
$K = \frac{\text{Rotor turns/ph}}{\text{Stator turns/ph}}$	$K = \frac{\text{Number of turns in secondary winding}}{\text{Number of turns in primary winding}}$

Example 4.1

A three-phase six-pole 50 Hz squirrel-cage induction motor is running with a slip of 4%. Determine the speed of the rotating field relative to the stator winding, motor speed and frequency of emf induced in the rotor. **[AU Nov/Dec, 2006]**

Solution

Given, $P = 6$, $f = 50$ Hz and $s = 4\% = 0.04$

(i) The speed of rotating field with respect to stator winding is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

(ii) The speed of the motor is given by

$$N = N_{\text{syn}}(1 - s) = 1000(1 - 0.04) = 960 \text{ rpm}$$

(iii) The frequency of emf induced in the rotor is given by

$$f_r = sf = 0.04 \times 50 = 2 \text{ Hz}$$

Example 4.2

A 373 kW three-phase 440 V 50 Hz induction motor has a speed of 950 rpm on full load. The motor has 6 poles. Determine slip of induction motor and the number of complete alternations made by the rotor voltage per minute. **[AU April/May, 2007]**

Solution

Given, $P = 6, f = 50$ Hz and $N = 950$ rpm

The synchronous speed of the induction motor or the speed of stator is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm.}$$

The slip of the induction motor is given by

$$s = \frac{N_{\text{syn}} - N}{N_{\text{syn}}} = \frac{1000 - 950}{1000} = 0.05 = 5\%$$

The stator frequency is given by

$$f_s = sf = 0.05 \times 50 = 2.5 \text{ Hz} = 2.5 \text{ cycles/sec.}$$

Here, the induced voltage in the rotor has the same frequency as the supply or stator frequency.

Therefore, the number of complete alternations made by the rotor voltage per minute is = 2.5 × 60 = 150

Example 4.3

A 1000 V 50 Hz three-phase induction motor has a star-connected stator. The ratio of stator to rotor turns is 3.6 and the standstill impedance of rotor per phase is $0.01 + j0.2\ \Omega$. Determine: (i) rotor current at start (ii) rotor power-factor at start (iii) rotor current at slip of 3%, and (iv) external resistance per phase in the rotor circuit to limit starting rotor current to 200A. **[AU April/May, 2003]**

Solution

Given, $V_L = 1000$ V, $f = 50$ Hz, $Z_2 = 0.01 + j0.2\ \Omega$ and $K_1 = \dfrac{1}{K} = \dfrac{\text{Stator turns}}{\text{Rotor turns}} = 3.6.$

Since $Z_2 = R_2 + jX_2$, we get $R_2 = 0.01\ \Omega$ and $X_2 = 0.2\ \Omega$.

The induced emf in stator per phase is given by

$$E_1 = \frac{1000}{\sqrt{3}} = 577.35 \text{ V per phase}$$

We know that, $K_1 = \dfrac{1}{K} = \dfrac{E_1}{E_2}$.

Therefore, $E_2 = \dfrac{577.35}{3.6} = 160.37$ V per phase

(i) At start, the slip of the induction motor is 1, i.e., $s = 1$

Therefore, the rotor current at start is given by

$$I_2 = \frac{E_2}{Z_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{160.37}{\sqrt{(0.01)^2 + (0.2)^2}} = 800.85 \text{ A}$$

(ii) The rotor power-factor at start is given by

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{0.01}{\sqrt{(0.01)^2 + (0.2)^2}} = 0.045 \text{ lagging}$$

(iii) When the rotor is rotating with slip, $s = 0.03$, the rotor current is given by

$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{0.03 \times 160.37}{\sqrt{(0.01)^2 + (0.03 \times 0.2)^2}} = 412.54 \text{ A}$$

Consider R_{ex} is the external resistance per phase, added in series with the rotor to limit the rotor current to 200 A. Hence, the total rotor resistance per phase is $R_2' = R_2 + R_{ex}$. Therefore,

$$I_2 = \frac{E_2}{\sqrt{(R_2')^2 + (X_2)^2}}$$

i.e., $$200 = \frac{160.37}{\sqrt{(R_2')^2 + (0.2)^2}}$$

Hence, $$\sqrt{(R_2')^2 + (0.2)^2} = 0.8018$$

or, $$(R_2')^2 + 0.04 = 0.643$$

Therefore, $$R_2' = 0.7765\ \Omega$$

Since $R_2' = R_2 + R_{\text{ex}}$, the external resistance to be added in series with the rotor resistance is given by

$$R_{\text{ex}} = R_2' - R_2 = 0.7765 - 0.01 = 0.7665\ \Omega \text{ per phase}$$

Example 4.4

For a three-phase star-connected four-pole 50 Hz induction motor, the ratio of stator to rotor turns is 4. At a certain load, its speed is 1450 rpm when connected to 415 V supply. Determine: (i) frequency of rotor emf in running condition, (ii) magnitude of induced emf in the rotor at standstill and (iii) magnitude of induced emf in the rotor under running condition.

Solution

Given, $K_1 = \dfrac{1}{K} = 4$, $P = 4$, $f = 50$ Hz, $N = 1450$ rpm, $E_{1L} = 415$ V

The synchronous speed of the induction motor is

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Therefore, $$s = \frac{N_{\text{syn}} - N}{N_{\text{syn}}} = \frac{1500 - 1450}{1500} = 0.033 \text{ i.e., } 3.33\%$$

(i) The frequency of rotor emf in running condition is

$$f_r = s\,f = 0.03 \times 50 = 1.5 \text{ Hz}$$

(ii) To determine the magnitude of induced emf in the rotor at standstill condition:
The phase voltage of the emf induced in the stator is

$$E_{1\text{ph}} = \frac{E_{1L}}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

It is known that, $\dfrac{E_{2\text{ph}}}{E_{1\text{ph}}} = \dfrac{\text{Rotor turns}}{\text{Stator turns}} = \text{K}$

Therefore, $\dfrac{E_{2\text{ph}}}{E_{1\text{ph}}} = \dfrac{1}{4}$

Hence, the rotor-induced emf on standstill condition is

$$E_{2\text{ph}} = \frac{1}{4} \times 239.6 = 59.9 \text{ V}$$

(iii) The magnitude of the induced emf in the rotor under running condition is

$$E_{2r} = sE_2 = 0.03 \times 59.9 = 1.797 \text{ V}$$

4.6 TORQUE EQUATION

[AU Nov/Dec, 2012]

Similar to a DC motor, the torque developed in the rotor of the induction motor depends on: (i) flux per stator pole, ϕ, (ii) rotor current, I_{2r} and (iii) power factor of the rotor, $\cos \phi_{2r}$.

Therefore, $$T \,\alpha\, \phi I_{2r} \cos \phi_{2r} \tag{4.18}$$

It is known that the flux produced in the stator, ϕ, is directly proportional to the stator voltage, E_1 i.e., $\phi \,\alpha\, E_1$.

Hence, $$T \,\alpha\, E_1 I_{2r} \cos \phi_{2r}$$

We know that the transformation ratio is $K = \dfrac{E_1}{E_2}$ i.e.,. $E_1 = KE_2$.

Substituting $E_1 = KE_2$ in the above equation, we get

$$T \,\alpha\, KE_2 I_{2r} \cos \phi_{2r} \tag{4.19}$$

Substituting Eqn. (4.15) and Eqn. (4.17) in the above equation, we get

$$T \,\alpha\, KE_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Therefore, $$T = kE_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

where, k is the proportionality constant given by $k = \dfrac{3}{2\pi n_{\text{syn}}}$, and n_{syn} is the synchronous speed in rps.

Hence, the torque developed in the rotor during running condition becomes

$$T = \frac{ksE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi n_{\text{syn}}} \times \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \tag{4.20}$$

Therefore, if the slip at any particular load and standstill rotor parameters is known, the torque developed in the rotor can be obtained.

4.6.1 Starting Torque, T_{st}

The torque developed in the rotor at start is called the starting torque. At start, $N = 0$ and $s = 1$. Hence, the starting torque, T_{st} of the induction motor can be obtained by substituting $s = 1$ in Eqn. (4.20). Therefore,

$$T_{st} = \frac{kE_2^2 R_2}{R_2^2 + X_2^2} = \frac{3}{2\pi n_{syn}} \times \frac{E_2^2 R_2}{R_2^2 + X_2^2} \tag{4.21}$$

Therefore, by varying the rotor resistance, the starting torque of the induction motor can be controlled.

Full-load Torque

The torque developed in the rotor of the induction motor at full-load condition is called full-load torque and is denoted by T_{FL}. The expression for T_{FL} is obtained by substituting the slip at full-load condition, s_{FL} in Eqn. (4.20).

Therefore,
$$T_{FL} = \frac{ks_{FL} E_2^2 R_2}{R_2^2 + (s_{FL} X_2)^2} = \frac{3}{2\pi n_{syn}} \times \frac{s_{FL} E_2^2 R_2}{R_2^2 + (s_{FL} X_2)^2} \tag{4.22}$$

4.6.2 Condition for Maximum Running Torque **[AU April/May, 2010; Nov/Dec, 2008]**

It is evident from Eqn. (4.20) that the only parameter that affects the running torque is slip, s, as the other parameters R_2, X_2, E_2 and n_{syn} are constant. Here, the induced voltage in rotor E_2 is constant because it is obtained using the transformation ratio, K, and stator voltage, E_1. Hence, the condition to obtain the maximum starting torque is determined by differentiating Eqn. (4.20), with respect to s and equating it to zero.

$$\frac{dT}{ds} = 0$$

i.e.,
$$\frac{(ksE_2^2 R_2) \times (2sX_2^2) - (R_2^2 + s^2 X_2^2) \times (kE_2^2 R_2)}{(R_2^2 + s^2 X_2^2)} = 0$$

i.e.,
$$2ks^2 E_2^2 R_2 X_2^2 - kE_2^2 R_2^3 - ks^2 E_2^2 R_2 X_2^2 = 0$$

i.e.,
$$ks^2 E_2^2 R_2 X_2^2 = kE_2^2 R_2^3$$

Solving the above equation, we get

$$s^2 = \frac{R_2^2}{X_2^2} \quad \text{or} \quad s = \pm \frac{R_2}{X_2}$$

Since the slip of the induction motor cannot be negative, the condition to achieve maximum running torque is given by

$$s_m = \frac{R_2}{X_2} \tag{4.23}$$

where, s_m is the slip of the induction motor at which the running torque is maximum.

4.6.3 Magnitude of Maximum Running Torque

The magnitude of maximum running torque T_{max} is obtained by substituting Eqn. (4.23) in Eqn. (4.20).

i.e., $$T_{max} = T\big|_{s=s_m} = \frac{3}{2\pi n_{syn}} \times \left.\frac{s_m E_2^2 R_2}{R_2^2 + (s_m X_2)^2}\right|_{s=s_m}$$

Substituting $s_m = \dfrac{R_2}{X_2}$ in the above equation, we get

$$T_{max} = \frac{\dfrac{3}{2\pi n_{syn}}\left(\dfrac{R_2}{X_2}\right) E_2^2 R_2}{R_2^2 + \left(\dfrac{R_2}{X_2} \times X_2\right)^2}$$

Therefore, $$T_{max} = \frac{3}{2\pi n_{syn}} \times \frac{E_2^2}{2X_2} = \frac{kE_2^2}{2X_2} \tag{4.24}$$

It is clear from Eqn. (4.24) that the maximum torque T_{max}, depends only on the rotor reactance and not on the rotor resistance. From Eqn. (4.23), it is noted that the slip at which the maximum torque is obtained depends on rotor resistance and rotor reactance.

4.6.4 Condition for Maximum Starting Torque

It is clear from Eqn. (4.21) that by varying the rotor resistance R_2, the starting torque, T_{st} of the induction motor can be controlled. Therefore, to determine the condition to achieve maximum starting torque, the starting torque represented by Eqn. (4.21) is differentiated with respect to R_2 and equated to zero.

i.e., $$\frac{dT_{st}}{dR_2} = 0$$

Substituting Eqn. (4.21) in the above equation, we get

$$\frac{(kE_2^2 \times 2R_2) - (R_2^2 + X_2^2) \times kE_2^2}{(R_2^2 + X_2^2)^2} = 0$$

i.e., $$2kE_2^2 R_2 - kE_2^2 R_2^2 - kE_2^2 X_2^2 = 0$$

i.e., $$kE_2^2 R_2^2 = kE_2^2 X_2^2$$

Upon solving, we get

$$R_2 = X_2 \tag{4.25}$$

Therefore, when the rotor resistance is equal to rotor reactance, maximum starting torque is achieved in the induction motor.

4.6.5 Magnitude of Maximum Starting Torque

Substituting Eqn. (4.25) in Eqn. (4.21), we get the maximum starting torque as

$$(T_{st})_{max} = \frac{3}{2\pi n_{syn}} \times \frac{E_2^2 R_2}{R_2^2 + R_2^2}$$

Upon solving, we get

$$(T_{\text{st}})_{\max} = \frac{3}{2\pi n_{\text{syn}}} \times \frac{E_2^2}{2R_2} = \frac{kE_2^2}{2R_2} \tag{4.26}$$

where, $k = \dfrac{3}{2\pi n_{\text{syn}}}$.

4.6.6 Torque Ratios **[AU April/May, 2012]**

Using different torques, like full-load torque, starting torque and maximum torque, the performance of induction motor is expressed. Therefore, it is necessary to obtain the various torque ratios among these.

Ratio of Full-load Torque to Maximum Torque

Dividing Eqn. (4.22) by Eqn. (4.24), we get the ratio of full-load torque to maximum torque as

$$\frac{T_{\text{FL}}}{T_{\max}} = \frac{s_{\text{FL}} E_2^2 R_2}{(R_2^2 + (s_{\text{FL}} X_2)^2)} \times \frac{(R_2^2 + (s_{\text{m}} X_2)^2)}{s_m E_2^2 R_2}$$

$$= \frac{s_{\text{FL}}}{s_m} \times \frac{(R_2^2 + (s_{\text{m}} X_2)^2)}{(R_2^2 + (s_{\text{FL}} X_2)^2)}$$

$$= \frac{s_{\text{FL}}}{s_m} \times \frac{\left(\dfrac{R_2^2}{X_2^2} + s_m^2\right)}{\left(\dfrac{R_2^2}{X_2^2} + s_{\text{FL}}^2\right)}$$

It is known that $s_m = \dfrac{R_2}{X_2}$.

Therefore, $$\frac{T_{\text{FL}}}{T_{\max}} = \frac{s_{\text{FL}}}{s_m} \times \frac{(2s_m^2)}{(s_m^2 + s_{\text{FL}}^2)} = \frac{2s_{FL} s_m}{s_m^2 + s_{FL}^2} \tag{4.27}$$

Ratio of Starting Torque to Maximum Torque

Dividing Eqn. (4.21) by Eqn. (4.24), we get the ratio of starting torque to maximum torque as:

$$\frac{T_{\text{st}}}{T_{\max}} = \frac{E_2^2 R_2}{(R_2^2 + X_2^2)} \times \frac{(R_2^2 + (s_m X_2)^2)}{s_m E_2^2 R_2}$$

$$= \frac{(R_2^2 + (s_m X_2)^2)}{s_m (R_2^2 + X_2^2)}$$

$$= \frac{\left(\dfrac{R_2^2}{X_2^2} + s_m^2\right)}{s_m\left(\dfrac{R_2^2}{X_2^2} + 1\right)}$$

We know that $s_m = \frac{R_2}{X_2}$.

Therefore, $$\frac{T_{st}}{T_{max}} = \frac{(s_m^2 + s_m^2)}{s_m(s_m^2+1)} = \frac{2s_m}{s_m^2+1} \tag{4.28}$$

4.6.7 Torque-Slip Characteristics [AU Nov/Dec, 2011; April/May, 2007]

The characteristics of the induction motor obtained by plotting the torque against slip when the motor is operated from a standstill position to a synchronous running position are called torque-slip characteristics i.e., the slip of the induction motor varies from $s = 1$ to $s = 0$. In general, since E_2 is constant, the torque of the induction motor is given by

$$T \,\alpha\, \frac{sR_2}{R_2^2 + (sX_2)^2} \tag{4.29}$$

Based on the slip value, the torque-slip characteristics are divided into two regions as: (i) low-slip region and (ii) high-slip region.

Low-slip Region

In low-slip region, the slip s is very small and hence $R_2 >> sX_2$. Therefore, neglecting sX_2 term in Eqn. (4.29), we get

$$T \,\alpha\, \frac{s}{R_2}$$

i.e., $$T \,\alpha\, s \text{ (since } R_2 \text{ is constant)} \tag{4.30}$$

Therefore, in low-slip region, the torque is directly proportional to slip s and it varies linearly. Hence, the torque-slip characteristic is a straight line in this region. In this region, as the load increases, the speed of the motor decreases. This decrease in speed increases the slip and as a result the torque increases. This region is also called stable region of operation.

High-slip Region

In high-slip region, the slip, s, is very high and hence $R_2 << sX_2$. Therefore, neglecting R_2 term in the denominator of Eqn. (4.29), we get

$$T \,\alpha\, \frac{sR_2}{(sX_2)^2}$$

i.e., $$T \,\alpha\, \frac{1}{s} \text{ (Since } R_2 \text{ and } X_2 \text{ is constant)} \tag{4.31}$$

Therefore, in high-slip region, the torque is inversely proportional to slip, s, and it varies linearly. Hence, the torque-slip characteristic is rectangular hyperbolic in this region.

Here, as the load increases, speed of the motor decreases. This decrease in speed increases the slip and hence, the torque decreases. Due to extra loading effect, there is further decrease in speed and increase in slip. This increase in slip causes the torque to decrease further and eventually motor comes to a halt position i.e., standstill condition. Therefore, the induction motor cannot be operated at any point in this region and hence it is called unstable region of operation.

The complete torque–slip characteristic curve of an induction motor is shown in Figure 4.11.

In low-slip or stable region, as the load increases, the slip increases and the torque increases linearly till maximum torque, T_{max} is achieved. The slip at which T_{max} is achieved is s_m. If the load is increased beyond T_{max}, the motor shifts to the high-slip or unstable region of operation and hence, the induction motor comes to a standstill condition at such a load. Therefore, the maximum torque developed in the motor is also called breakdown torque or pull out torque.

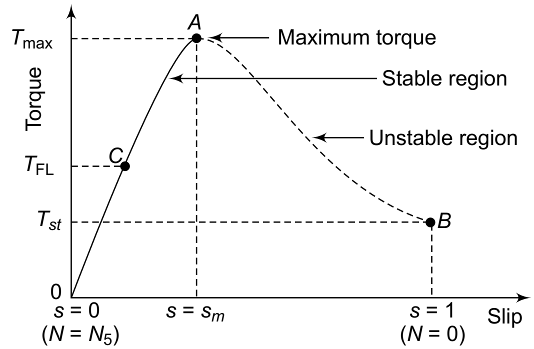


Figure 4.11 Torque–Slip Characteristic Curve

Full-load condition is the load at which the current drawn by the motor is within its safe limits. The torque developed at this condition is called full-load torque and is denoted by T_{FL}. It is clear from Figure 4.11 that the load can be increased beyond full-load condition till maximum torque is achieved. But if the motor is operated continuously in the region *C* to *A*, there is a possibility of breakdown of winding insulation due to large current.

Therefore, the region OC corresponds to full-load condition in which the induction motor is operated safely for longer duration. The region CA is used to achieve maximum torque in which the induction motor is operated only for a short duration of time.

4.6.8 Speed-Torque Characteristics **[AU Nov/Dec, 2008]**

The speed-torque characteristic curve of an induction motor is shown in Figure 4.12. When the motor is running at synchronous speed i.e., $N = N_{syn}$, the relative speed and the slip becomes zero and hence the torque developed in the motor is zero. Also, when the motor is in a standstill position i.e., $N = 0$, the relative speed is maximum i.e., $s = 1$ and therefore, a torque called starting torque, T_{st} is developed in the motor.

The speed at which the motor is running at no-load condition is N_{NL} and the torque developed at this condition is zero. When the induction motor is loaded, the motor speed gets reduced and hence a torque is developed in the motor. When the motor is fully loaded, the torque developed in the motor is T_{FL} and the speed at which the motor runs is N_{FL}. It is seen that the drop in motor speed from T_{NL} to N_{FL} is very minimum, in the range of 4% to 6%, and hence, the three-phase induction motor is called a constant-speed motor.

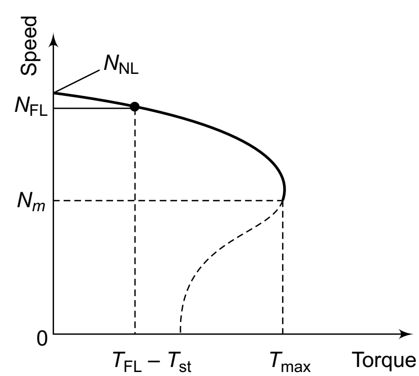


Figure 4.12 Speed-Torque Characteristic Curve

The induction motor can be loaded till the maximum torque, T_{max} is developed in the motor. The motor speed at which T_{max} is developed in the motor is N_m. If the motor is further loaded, the motor enters the unstable region and comes to a standstill position, where the speed is zero and torque developed is T_{st}. This unstable region is shown as a dotted line in Figure 4.12.

4.7 Effect of Change in Rotor Resistance on Torque

In a slip-ring induction motor, the extra resistance can be added externally to control the value of rotor resistance and thereby the torque developed in the rotor is controlled. The different torque equations of induction motor are:

$$T_{st}\ \alpha\ \frac{E_2^2 R_2}{\sqrt{R_2^2 + X_2^2}},\ T\ \alpha\ \frac{sE_2^2 R_2}{\sqrt{R_2^2 + (sX_2)^2}} \text{ and } T_{max}\ \alpha\ \frac{E_2^2}{2X_2} \tag{4.32}$$

Also, the slips corresponding to the torques given in the above equations are:

$$s = 1,\ s = \frac{N_{syn} - N}{N_{syn}} \text{ and } s_m = \frac{R_2}{X_2} \tag{4.33}$$

Hence, the change in rotor resistance does not affect the slip and T_{max}. But the other parameters, including s_m change. The pictorial representation of the effect of change in rotor resistance on torque–slip characteristics is shown in Figure 4.13.

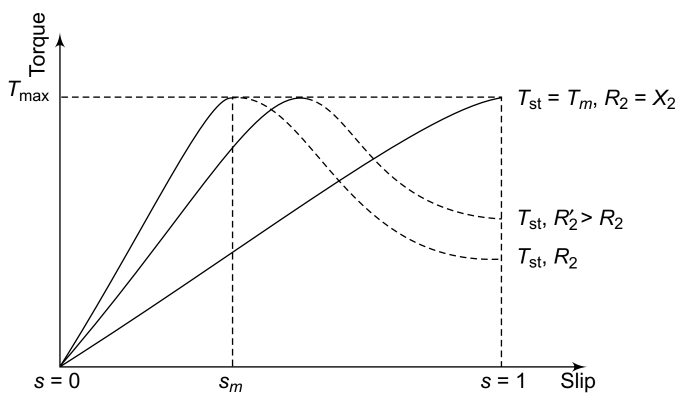


Figure 4.13 Effect of Rotor Resistance on Torque–Slip Characteristics

Here, the slip at a standstill condition is 1. Therefore, if the starting and maximum torques are same, then the slip s_m is equal to 1. Hence, the required condition for the starting and maximum torques to be same is $R_2 = X_2$.

4.8 Effect of Change in Rotor Reactance on Torque

Similar to the rotor resistance R_2, the rotor reactance X_2 can be changed, which will affect the torque and slip of the induction motor. From Eqns. (4.32) and (4.33), it is clear that if the rotor reactance X_2 changes, there will be a change in the torque and slip. The torque-slip characteristics of a three-phase induction motor for different values of X_2 are shown in Figure 4.14.

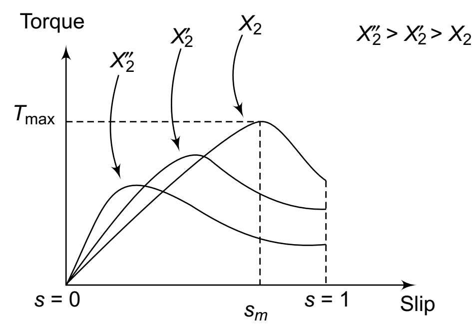


Figure 4.14 Effect of Rotor Reactance on Torque–Slip Characteristics

Example 4.5

A three-phase 400V 50Hz four-pole induction motor has star-connected stator winding. The rotor resistance and reactance per phase are 0.1 Ω and 1 Ω respectively. The full-load speed is 1440 rpm. Determine the synchronous speed, slip, rotor emf per phase and the torque developed by the motor on full-load. Assume the ratio of stator to rotor turns as 1.75. **[AU April/May, 2004]**

Solution

Given, $V_L = 400$ V, $f = 50$ Hz, $P = 4$, $R_2 = 0.1\ \Omega$, $X_2 = 1\ \Omega$, $K_1 = \dfrac{1}{K} = 1.75$ and $N_{\text{FL}} = 1440$ rpm.
The synchronous speed of the induction motor is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Therefore, the slip is given by

$$s = \frac{N_{\text{syn}} - N}{N_{\text{syn}}} = \frac{1500 - 1440}{1500} = 0.04 = 4\%$$

The induced emf in the stator per phase is given by

$$E_1 = \frac{400}{\sqrt{3}} = 230.94 \text{ V per phase}$$

Since $K_1 = \dfrac{E_1}{E_2}$, the induced emf in rotor per phase is given by

$$E_2 = \frac{230.94}{1.75} = 131.965 \text{ V per phase}$$

The torque developed in the rotor at full-load condition is given by

$$T = \frac{k\, sE_2^2 R_2}{(R_2^2 + (sX_2)^2)}$$

where, $k = \dfrac{3}{2\pi n_{\text{syn}}}$ and $n_{\text{syn}} = \dfrac{N_{\text{syn}}}{60} = \dfrac{1500}{60} = 25$ rps

Therefore,

$$T = \frac{3}{2\pi n_{\text{syn}}} \times \frac{sE_2^2 R_2}{(R_2^2 + (sX_2)^2)} = \frac{3}{2\pi \times 25} \times \frac{0.04 \times (131.965)^2 \times 0.1}{[(0.1)^2 + (0.04 \times 1)^2]} = 114.6887 \text{ Nm}$$

Example 4.6

A three-phase 50 Hz 440 V four-pole induction motor develops half the rated torque at 1490 rpm. With the applied voltage magnitude remaining at the rated value, what should be its frequency if the motor has to develop the same torque at 1600 rpm? Neglect stator and rotor winding resistances, leakage reactance and iron losses. **[AU Nov/Dec, 2011]**

Solution

Given, f_1 = 50 Hz, P = 4, N_1 = 1490 rpm and N_2 = 1600 rpm
The torque developed in the rotor is given by

$$T = \frac{ksE_2^2 R_2}{(R_2^2 + (sX_2)^2)}$$

We know that, $E_2 \,\alpha\, V$ which is constant. Therefore, the torque developed in the rotor by neglecting R_2 and X_2 is given by

$$T \,\alpha\, s \tag{1}$$

When N_1 = 1490 rpm, the synchronous speed is given by

$$N_{\text{syn1}} = \frac{120 f_1}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Therefore, the slip is given by

$$s_1 = \frac{N_{\text{syn1}} - N_1}{N_{\text{syn1}}} = \frac{1500 - 1490}{1500} = 0.00667$$

Using Eqn. (1), we get

$$\frac{T_1}{T_2} = \frac{s_1}{s_2} \tag{2}$$

where, T_1 and s_1 are the torque developed and slip of the motor respectively, when N_1 = 1490 rpm; T_2 and s_2 are the torque developed and slip of the motor respectively, when N_2 = 1600 rpm.
Since $T_1 = T_2$ and using Eqn. (2), we get

$$s_2 = s_1 = 0.00667$$

But the slip of the induction motor when N_2 = 1600 rpm is given by

$$s_2 = \frac{N_{syn2} - N_2}{N_{syn2}}$$

where, N_{s2} is the synchronous speed of induction motor when N_2 = 1600 rpm.

Therefore, $$0.00666 = \frac{N_{\text{syn2}} - 1600}{N_{\text{syn2}}}$$

Upon solving, we get

$$N_{\text{syn2}} = 1611 \text{ rpm}$$

The relation between N_{syn2} and f_2 is given by

$$N_{\text{syn2}} = \frac{120 f_2}{P}$$

Therefore, $$f_2 = \frac{N_{\text{syn2}} \times P}{120} = \frac{1611 \times 4}{120} = 53.7 \text{ Hz}$$

Example 4.7

A four-pole 50 Hz three-phase induction motor has a rotor resistance of 0.24 Ω per phase and standstill reactance of 0.6 Ω per phase. Determine the speed at which the maximum torque is developed.

[AU April/May, 2006]

Solution

Given, $P = 4$, $f = 50$ Hz, $R_2 = 0.024\ \Omega$ and $X_2 = 0.6\ \Omega$

The synchronous speed of the induction motor is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

The slip of the induction motor, at which the maximum torque $T_{\max}$ is developed, is given by

$$s_m = \frac{R_2}{X_2} = \frac{0.024}{0.6} = 0.04 = 4\%$$

Therefore, the speed of the rotor at $T_{\max}$ is given by

$$N_m = N_{\text{syn}}(1 - s_m) = 1500 \times (1 - 0.04) = 1440 \text{ rpm.}$$

Example 4.8

A 220 V four-pole 50 Hz three-phase induction motor has rotor resistance and standstill reactance of 0.1 Ω and 0.9 Ω respectively. The ratio of stator to rotor turns is 1.72 and full-load slip is 5%. Determine the full-load torque, output power in horsepower (HP), maximum torque and speed at maximum torque. Neglect stator impedance. **[AU Nov/Dec, 2008]**

Solution

Given, $V_L = 220$ V, $P = 4$, $f = 50$ Hz, $R_2 = 0.1\ \Omega$, $X_2 = 0.9\ \Omega$, $K_1 = 1.72$ and $s_{\text{FL}} = 5\% = 0.05$

The induced emf in stator per phase is given by

$$E_1 = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}} = 127.017 \text{ V per phase}$$

Since $K_1 = \dfrac{E_1}{E_2} = 1.72$, the induced emf in rotor per phase is given by

$$E_2 = \frac{E_1}{K_1} = \frac{127.017}{1.72} = 73.8471 \text{ V per phase}$$

The synchronous speed of the motor is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Also, the synchronous speed in rps is given by

$$n_{\text{syn}} = \frac{N_s}{60} = \frac{1500}{60} = 25 \text{ rps}$$

The torque developed in the rotor at full-load condition is given by

$$T_{FL} = \frac{3}{2\pi n_{syn}} \times \frac{s_{FL} E_2^2 R_2}{R_2^2 + (s_{FL} X_2)^2} = \frac{3}{2\pi \times 25} \times \frac{0.05 \times (73.8471)^2 \times 0.1}{[(0.1)^2 + (0.05 \times 0.9)^2]} = 43.3065 \text{ Nm}$$

The speed of the rotor at full-load condition is given by

$$N = N_{syn}(1 - s_{FL}) = 1500(1 - 0.05) = 1425 \text{ rpm}$$

The output power developed in the motor at full-load condition is given by

$$P_{out} = T_{FL} \times \omega = T_{FL} \times \frac{2\pi N}{60} = \frac{43.3065 \times 2\pi \times 1425}{60} = 6.4624 \text{ kW}$$

Since 1 HP = 735.5 W, the output power developed in the motor in HP is

$$P_{out} = \frac{6.4624 \times 10^3}{735.5} = 8.78 \text{ HP}$$

The slip at maximum torque is given by

$$s_m = \frac{R_2}{X_2} = \frac{0.1}{0.9} = 0.1111$$

Therefore, the speed at which maximum torque is developed in the motor is given by

$$N_m = N_{syn}(1 - s_m) = 1500(1 - 0.1111) = 1333 \text{ rpm}$$

Therefore, the maximum torque developed in the motor is given by

$$T_{max} = \frac{3}{2\pi n_{syn}} \times \frac{E_2^2}{2X_2} = \frac{3}{2\pi \times 25} \times \frac{(73.8471)^2}{2 \times 0.9} = 57.8623 \text{ Nm}$$

Example 4.9

A six-pole three-phase 50 Hz induction motor develops a maximum torque of 30 Nm at 960 rpm. Determine the torque exerted by the motor at 5% slip. The rotor resistance per phase is 0.6 Ω.

[AU April/May, 2010]

Solution

Given, $P = 6$, $f = 50$ Hz, $T_{max} = 30$ Nm, $N_m = 960$ rpm, $s = 5\% = 0.05$ and $R_2 = 0.6\ \Omega$

The synchronous speed is given by

$$N_{syn} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

The slip at which maximum torque is developed is given by

$$s_m = \frac{N_{syn} - N_m}{N_{syn}} = \frac{1000 - 960}{1000} = 0.04 = 4\%$$

We know that, $s_m = \dfrac{R_2}{X_2}$. Therefore, the rotor reactance is given by

$$X_2 = \frac{R_2}{s_m} = \frac{0.6}{0.04} = 15\ \Omega$$

The maximum torque developed in the motor is given by

$$T_m = \frac{kE_2^2}{2X_2}$$

$$k = \frac{3}{2\pi n_{\text{syn}}} = \frac{3\times 60}{2\pi N_{\text{syn}}} = \frac{180}{2\pi\times 1000} = 0.02866$$

Therefore, $$30 = \frac{0.02866\times E_2^2}{2\times 15}$$

i.e., $$E_2^2 = \frac{2\times 15\times 30}{0.02866} = 31402.65$$

Therefore, E_2 = 177.207 V

The torque developed in the motor when slip s = 5% is given by

$$T = \frac{k\,sE_2^2\,R_2}{R_2^2 + (sX_2)^2} = \frac{0.02866\times 177.207^2\times 0.05\times 0.6}{[(0.6)^2 + (0.05\times 15)^2]} = 29.2683\ \text{Nm}$$

Example 4.10

The starting torque and maximum torque of a three-phase induction motor are 140% and 220% of the full-load torque. Neglecting stator resistance and assuming constant rotor resistance, determine: (i) slip at maximum torque, (ii) full-load slip and (iii) rotor current at starting in terms of full-load rotor current.

[AU April/May, 2012]

Solution

Given, $T_{\max} = 2.2T_{\text{FL}}$ and $T_{\text{st}} = 1.4\ T_{\text{FL}}$

We know that the slip at maximum torque is $s_m = \dfrac{R_2}{X_2}$ and the slip at the starting of the induction motor is 1 i.e., s = 1. After substituting the slip equation, the maximum starting torque is given by:

$$T_{\max}\ \alpha\ \frac{E_2^2}{2X_2} \text{ and } T_{\text{st}}\ \alpha\ \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

The full-load torque of the induction motor is given by

$$T_{\text{FL}} = \frac{s_{\text{FL}}\,E_2^2 R_2}{R_2^2 + (s_{\text{FL}} X_2)^2}$$

where, s_{FL} is the full-load slip.

Since $T_{\max} = 2.2T_{FL}$, we get

$$\frac{E_2^2}{2X_2} = 2.2 \times \frac{s_{FL}E_2^2R_2}{R_2^2 + (s_{FL}X_2)^2}$$

Therefore, $$R_2^2 + (s_{FL}X_2)^2 = 4.4s_{FL}\,R_2X_2 \quad (1)$$

Similarly, since $T_{st} = 1.4T_{FL}$, we get

$$\frac{E_2^2}{R_2^2 + X_2^2} = 1.4 \times \frac{s_{FL}\,E_2^2R_2}{R_2^2 + (s_{FL}X_2)^2}$$

Therefore, $$R_2^2 + (s_{FL}X_2)^2 = 1.4s_{FL}R_2^2X_2^2 \quad (2)$$

Using Eqns. (1) and (2), we get

$$4.4s_{FL}R_2X_2 = 1.4s_{FL}(R_2^2 + X_2^2)$$

Upon solving the above equation, we get

$$\left(\frac{R_2}{X_2}\right)^2 + 1 = 3.1428\left(\frac{R_2}{X_2}\right)$$

Assuming $\frac{R_2}{X_2} = a$ in the above equation, we get

$$a^2 - 3.1428a + 1 = 0$$

Solving the above quadratic equation, we get

$$a = 0.359 \text{ and } a = 2.7835$$

Since R_2 cannot be greater than X_2, the higher value of a is neglected.

Hence, $a = \frac{R_2}{X_2} = 0.359$.

(i) Slip at maximum torque, $s_m = \frac{R_2}{X_2} = 0.359 = 35.9\%$

(ii) To determine full-load slip, s:
Dividing Eqn. (1) by X_2^2 on both sides, we get

$$\left(\frac{R_2}{X_2}\right)^2 + s_{FL}^2 = 4.4s_{FL}\left(\frac{R_2}{X_2}\right)$$

Since $\frac{R_2}{X_2} = 0.359$, we get

$$(0.359)^2 + s_{FL}^2 = 4.4 \times s_{FL} \times 0.359$$

i.e. $$s_{FL}^2 - 1.5796\,s_{FL} + 0.1288 = 0$$

Upon solving the above equation, we get

$$s_{FL} = 0.0863 \text{ or } 1.4934$$

Since the slip cannot be greater than 1, $s_{FL} = 0.0863 = 8.63\%$.

(iii) The torque developed in the motor is directly proportional to the rotor current and power factor of the rotor. That is,

$$T \,\alpha\, I_{2r} \cos \phi_{2r}$$

Since $\cos \phi_{2r} = \dfrac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$, we get

$$T \,\alpha\, I_{2r} \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Therefore, the ratio of starting torque, T_{st} to full-load torque, T_{FL} is given by

$$\frac{T_{st}}{T_{FL}} = \frac{(I_{2r})_{st} R_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{\sqrt{R_2^2 + (s_{FL} X_2)^2}}{(I_{2r})_{FL} R_2} \quad \text{(since } s = 1 \text{ at start)} \tag{3}$$

where, $(I_{2r})_{st}$ is the rotor current at start and $(I_{2r})_{FL}$ is the full-load rotor current.

Since $T_{st} = 1.4 T_{FL}$ or $\dfrac{T_{st}}{T_{FL}} = 1.4$, Eqn. (3) becomes,

$$1.4 = \frac{(I_{2r})_{st}}{(I_{2r})_{FL}} \times \frac{\sqrt{R_2^2 + (s_{FL} X_2)^2}}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{(I_{2r})_{st}}{(I_{2r})_{FL}} \times \frac{\sqrt{\left(\dfrac{R_2}{X_2}\right)^2 + (s_{FL})^2}}{\sqrt{\left(\dfrac{R_2}{X_2}\right)^2 + 1}} = \frac{(I_{2r})_{st}}{(I_{2r})_{FL}} \times \frac{\sqrt{s_m^2 + (s_{FL})^2}}{\sqrt{s_m^2 + 1}}$$

Rearranging the above equation, we get

$$\frac{(I_{2r})_{st}}{(I_{2r})_{FL}} = \frac{1.4\sqrt{s_m^2 + 1}}{\sqrt{s_m^2 + (s_{FL})^2}}$$

Substituting the known values, we get

$$\frac{(I_{2r})_{st}}{(I_{2r})_{FL}} = \frac{1.4\sqrt{(0.359)^2 + 1}}{\sqrt{(0.359)^2 + (0.0863)^2}} = 4.028$$

Therefore, the rotor starting current in terms of full-load rotor current is

$$(I_{2r})_{st} = 4.028 (I_{2r})_{FL}$$

Example 4.11

A three-phase 50 Hz 400 V four-pole star-connected induction motor has a rotor resistance and reactance per phase equal to 0.02 Ω and 0.2 Ω respectively. The ratio of stator to rotor turns is 2. Determine: (i) starting torque (ii) slip at which maximum torque will occur (iii) speed at which maximum torque will occur (iv) maximum torque and (v) full-load torque if full-load slip is 4%.

Solution

Given, $P = 4$, $f = 50$ Hz, $K_1 = \dfrac{1}{K} = 2$, $R_2 = 0.02\ \Omega$, $X_2 = 0.2\ \Omega$ and $E_{1L} = 400$ V

The synchronous speed of the induction motor is

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

The phase voltage of the emf induced in the stator is

$$E_{1\text{ph}} = \frac{E_{1\text{line}}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

Since $\dfrac{E_{2\text{ph}}}{E_{1\text{ph}}} = \dfrac{\text{Rotor turns}}{\text{Stator turns}} = \text{K}$, the phase voltage of the emf induced in the rotor is

$$E_2 = E_{2\text{ph}} = \frac{1}{2} \times E_{1\text{ph}} = \frac{230.94}{2} = 115.47 \text{ V}$$

(i) The starting torque of the induction motor is

$$T_{\text{st}} = \frac{k\,E_2^2 R_2}{R_2^2 + X_2^2}$$

where, $k = \dfrac{3}{2\pi n_{\text{syn}}}$ and $n_{\text{syn}} = \dfrac{N_{\text{syn}}}{60} = 25$ rps

Substituting the values, we get

$$T_{\text{st}} = \frac{\dfrac{3}{2\pi \times 25} \times (115.47)^2 \times 0.02}{(0.02)^2 + (0.2)^2} = 126 \text{ Nm}$$

(ii) The slip at which maximum torque occurs is

$$s_m = \frac{R_2}{X_2} = \frac{0.02}{0.2} = 0.1 \text{ i.e. } 10\%$$

(iii) The speed at which maximum torque occurs is given by

$$N = N_{\text{sym}}(1 - s_m) = 1500 \times (1 - 0.1) = 1350 \text{ rpm}$$

(iv) The maximum torque developed in the motor is

$$T_{\max} = \frac{k\,E_2^2}{2X_2} = \frac{\dfrac{3}{2\pi \times 25} \times (115.47)^2}{2 \times 0.2} = 636.33 \text{ Nm}$$

(v) When $s_{FL} = 4\% = 0.04$, the full-load torque developed in the motor is

$$T_{FL} = \frac{k\, s_{FL}\, E_2^2}{R_2^2 + (s_{FL} X_2)^2} = \frac{\dfrac{3}{2\pi \times 25} \times 0.04 \times (115.47)^2 \times 0.02}{(0.02)^2 + (0.04 \times 0.2)^2} = 438.85 \text{ Nm}$$

4.9 Losses in Induction Motors

The different losses in a three-phase induction motors are explained below:

4.9.1 Constant Losses

The core and mechanical losses that occur in the induction motor constitute the constant losses. Hysteresis and eddy current losses that occur in both stator and rotor core depend on the frequency and constitute the core or iron losses. Since the stator or supply frequency is greater than the rotor frequency, the iron losses in the stator are high. Hence, the rotor iron losses are neglected during the running condition. Using laminated high-grade silicon steel material in the induction motor can reduce the constant losses.

Mechanical losses of the induction motor include the frictional losses in the bearings and windings. Since they are very small in the running condition, these losses are also considered as constant losses.

4.9.2 Variable Losses

Copper losses that occur due to the stator and rotor resistances present in their respective windings are called variable losses, which depend on the current flowing through the windings.

The current supplied to the stator winding is constant and hence, the stator copper loss is combined with the stator iron loss to form the total stator loss of the induction motor. But in rotor, the rotor copper loss of the induction motor is given by

$$P_c = 3 I_{2r}^2 R_2 \tag{4.34}$$

where, I_{2r} is the rotor current in running condition, as given by Eqn. (4.17) and R_2 is the rotor resistance per phase in the induction motor.

4.10 Power Flow in Induction Motor **[AU Nov/Dec, 2005]**

The three-phase induction motor converts the three-phase stator voltage to mechanical energy. Due to the existence of losses in the induction motor, all the electrical input is not converted to mechanical energy. Let P_{in} be the net input power supplied to the stator of the induction motor. It is also called stator input, which is given by

$$P_{in} = \sqrt{3} V_L I_L \cos\phi \tag{4.35}$$

where, V_L and I_L are the line voltage and line current respectively and cos ϕ is the power factor of the motor. The power input to the stator (P_{in}) is utilized to serve stator iron loss, stator copper loss and power input to the rotor P_2

$$P_m = \text{stator iron loss} + \text{stator copper loss} + P_2$$

Now, $$P_2 = P_m - \text{stator iron loss} - \text{stator copper loss} \tag{4.36}$$

The rotor of the induction motor does not convert the entire power input to the mechanical power output, as it has to supply the rotor copper losses, P_c, present in the system. Therefore, the mechanical power developed in the rotor after supplying the rotor copper loss is given by

$$P_m = P_2 - P_c \tag{4.37}$$

Substituting Eqn. (4.34) in the above equation, we get

$$P_m = P_2 - 3I_{2r}^2 R_2 \tag{4.38}$$

The rotor input P_2 supplies only the rotor copper loss since the rotor iron loss is neglected. Similarly, the mechanical power developed in the rotor is not fully available at the load connected to the shaft because there are mechanical losses in the induction motor. Therefore, the mechanical power available at the shaft or load is called net output power or shaft power, P_{out} as given by

$$P_{out} = P_m - \text{mechanical loss} \tag{4.39}$$

The power-flow diagram of a three-phase induction motor is shown in Figure 4.15.

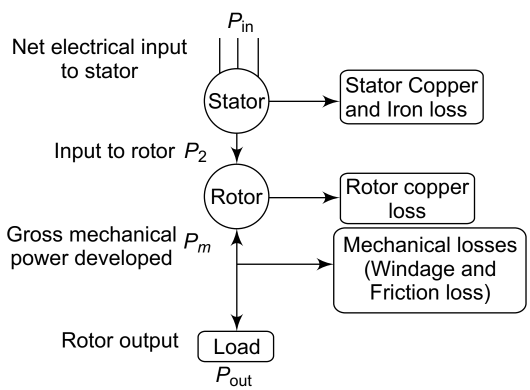


Figure 4.15 Power-flow Diagram of a Three-phase Induction Motor

4.11 Relation Between P_2, P_c and P_m **[AU April/May, 2012; Nov/Dec, 2010]**

The power is directly proportional to the product of the torque developed and the angular velocity, ω, responsible for generating the power. Using this concept, the relations among P_2, P_c and P_m are derived as follows: Consider that T is the gross torque developed in the motor. Due to the rotation of rotating magnetic field, the rotor input is developed, which is given by

$$P_2 = T \times \omega_{syn}$$

Substituting $\omega_{\text{syn}} = \dfrac{2\pi N_{\text{syn}}}{60}$, we get

$$P_2 = T \times \frac{2\pi N_{\text{syn}}}{60} \tag{4.40}$$

Since the speed of rotor is N, the mechanical power developed in the rotor due to its rotation is given by

$$P_m = T \times \omega$$

where, $\omega = \dfrac{2\pi N}{60}$.

Therefore,
$$P_m = T \times \frac{2\pi N}{60} \tag{4.41}$$

From Eqn. (4.37), we have

$$P_c = P_2 - P_m$$

Using Eqns. (4.40) and (4.41), we get

$$P_c = T \times \frac{2\pi N_{\text{syn}}}{60} - T \times \frac{2\pi N}{60} = T \times \frac{2\pi}{60}(N_{\text{syn}} - N) \tag{4.42}$$

Dividing Eqn. (4.42) by Eqn. (4.40), we get

$$\frac{P_c}{P_2} = \frac{N_{\text{syn}} - N}{N_{\text{syn}}}$$

Since slip, $s = \dfrac{N_{\text{syn}} - N}{N_{\text{syn}}}$, we get

$$\frac{P_c}{P_2} = s$$

Therefore,
$$P_c = sP_2 \tag{4.43}$$

Similarly, dividing Eqn. (4.41) by Eqn. (4.40), we get

$$\frac{P_m}{P_2} = \frac{N}{N_{\text{syn}}}$$

Since $N = N_{\text{syn}}(1 - s)$, we get

$$\frac{P_m}{P_2} = \frac{N_{\text{syn}}(1-s)}{N_{\text{syn}}}$$

Therefore,
$$\frac{P_m}{P_2} = (1 - s)$$

or
$$P_m = (1 - s)P_2 \tag{4.44}$$

Using Eqns. (4.43) and (4.44), we get

$$P_2 : P_c : P_m = 1 : s : (1 - s) \tag{4.45}$$

The above equation represents the relation among P_2, P_c and P_{m} and different ratios among them can also be obtained using this equation.

Shaft Torque or Useful Torque

The torque available at the shaft is called shaft torque or useful torque, T_{sh} and is given by

$$T_{\text{sh}} = \frac{P_{\text{out}}}{\omega} = \frac{P_{\text{out}}}{\left(\dfrac{2\pi N}{60}\right)}$$

4.11.1 Derivation of Constant *k* in the Torque Equation

The torque developed in the motor during running condition is given by

$$T = \frac{ksE_2^2 R_2}{R_2^2 + (sX_2)^2} \tag{4.46}$$

Substituting Eqn. (4.17) in Eqn. (4.34), we get

The rotor copper loss, $P_c = 3 \times \left(\dfrac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \right)^2 \times R_2$

i.e., $$P_c = \frac{3s^2 E_2^2 R_2}{R_2^2 + (sX_2)^2} \tag{4.47}$$

From Eqn. (4.45), we have

$$P_m = \frac{1 - s}{s} P_c$$

Substituting Eqn. (4.41) and Eqn. (4.47) in the above equation, we get

$$T \times \frac{2\pi N}{60} = \frac{(1 - s)}{s} \times \frac{3s^2 E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

i.e., $$T = \frac{60}{2\pi N} \times \frac{3(1 - s)sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Substituting $N = N_{\text{syn}}(1 - s)$ in the above equation, we get

$$T = \frac{60}{2\pi N_{\text{syn}}(1 - s)} \times \frac{3(1 - s)sE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{3}{\dfrac{2\pi N_{\text{syn}}}{60}} \times \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

i.e., $$T = \frac{3}{2\pi n_{\text{syn}}} \times \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \tag{4.48}$$

Comparing Eqn. (4.46) and Eqn. (4.48), we get

$$k = \frac{3}{2\pi n_{syn}} = \frac{3}{\dfrac{2\pi N_{syn}}{60}}.$$

4.12 Efficiency of an Induction Motor **[AU April/May, 2012; Nov/Dec, 2008]**

Since there are different power stages in the induction motor, as shown in Figure 4.15, different efficiencies in the induction motor can be obtained as given below.

4.12.1 Rotor Efficiency, η_r

The rotor efficiency η_r is defined as the ratio of the mechanical power developed in the rotor to the net input power supplied to the rotor i.e., ratio of rotor output to rotor input. Therefore,

$$\eta_r = \frac{P_m}{P_2} \tag{4.49}$$

4.12.2 Net Motor Efficiency, η

The net motor efficiency η is defined as the ratio of the mechanical power available at the shaft or load to the net electrical power supplied to the motor or stator input. Therefore,

$$\eta = \frac{P_{out}}{P_{in}} \tag{4.50}$$

But the efficiency characteristics of the induction motor are based on the net motor efficiency only. The motor efficiency obtained for different output power P_{out} is shown in Figure 4.16.

The net motor efficiency will reach its maximum value only when the constant and variable losses become equal. At no load, the net motor efficiency is low since the motor consumes only small current. When the motor is loaded, the motor consumes more current and hence the copper or variable loss increases. When these losses attain the same value as that of constant losses, the net motor efficiency reaches its maximum value. If the motor is further loaded beyond this point, variable losses become greater and reduce the motor efficiency, as shown in Figure 4.16.

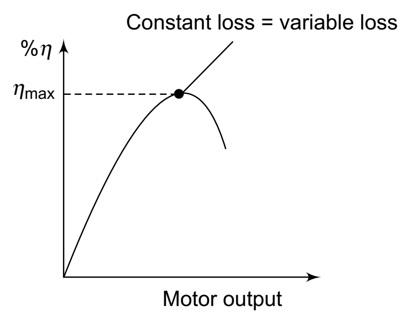


Figure 4.16 Efficiency Characteristics of an Induction Motor

4.13 Equivalent Circuit of an Induction Motor **[AU April/May, 2014; Nov/Dec, 2011]**

The induction motor can be considered as a two-winding transformer, with the stator acting as primary and the rotor acting as secondary. Similar to a two-winding transformer, the induction motor on no-load condition draws a current called no-load current I_o, to supply the air-gap flux and constant or iron losses. Hence, the no-load current contains two components: (i) Magnetising component, I_m, to supply the air-gap flux and (ii) Active component, I_c, to supply the iron loss. Therefore,

$$I_o = I_m + I_c \tag{4.51}$$

If R_o and X_o are used to represent the iron loss and air-gap flux respectively, then

$$R_o = \frac{V_1}{I_c} \text{ and } X_o = \frac{V_1}{I_m} \tag{4.52}$$

Where, V_1 is the supply voltage.

If R_1 and X_1 are the stator resistance and reactance per phase respectively, then similar to the two-winding transformer, the equivalent circuit of a three-phase induction motor in running condition can be obtained, as shown in Figure 4.17.

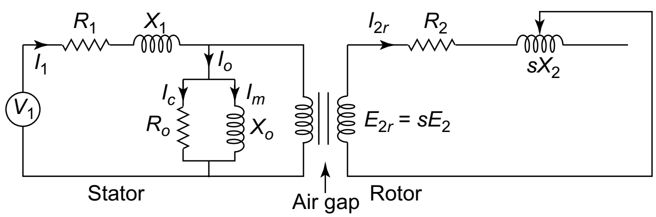


Figure 4.17 Equivalent Circuit of a Three-Phase Induction Motor

We know that the rotor current in running condition is given by

$$I_{2r} = \frac{E_{2r}}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}}$$

Hence, the rotor reactance remains constant and the rotor resistance varies in the running condition. This varying rotor resistance can be represented as:

$$\frac{R_2}{s} = \frac{R_2}{s} + R_2 - R_2$$

i.e.,
$$\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$$

Therefore, the variable rotor resistance has two parts, namely:

- Constant rotor resistance, R_2 and
- Electrical equivalent of mechanical load is given by, $R_L = R_2\left(\frac{1}{s} - 1\right)$

Hence, the equivalent circuit of a rotor can be represented in three different ways, as shown in Figures 4.18 (a), (b) and (c).

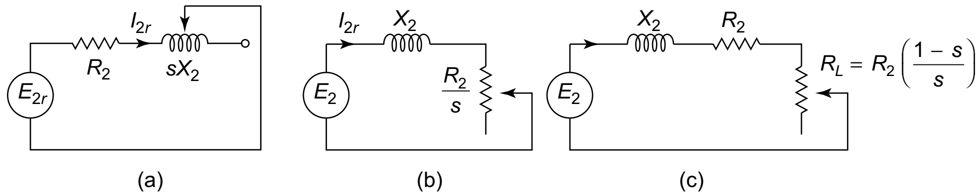


Figure 4.18 Equivalent Circuit of Rotor With (a) Varying X_2 (b) Varying R_2 (c) Load Resistance R_L

Similar to a two-winding transformer, the equivalent circuit of the induction motor can be referred to both stator and rotor sides.

4.13.1 Equivalent Circuit Referred to Stator Side

The equivalent circuit of an induction motor when referred to stator side using the transformation ratio, K, is shown in Figure 4.19.

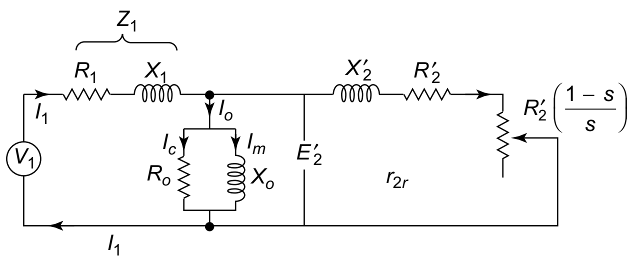


Figure 4.19 Equivalent Circuit of an Induction Motor Referred to Stator Side

Here,

$$E_2' = \frac{E_2}{K},\ I_{2r}' = KI_{2r},\ X_2' = \frac{X_2}{K^2},\ R_2' = \frac{R_2}{K^2} \text{ and } R_L' = \frac{R_L}{K^2} = \frac{R_2}{K^2}\left(\frac{1-s}{s}\right) = R_2'\left(\frac{1-s}{s}\right) \tag{4.53}$$

4.13.2 Equivalent Circuit Referred to Rotor Side

The equivalent circuit of an induction motor when referred to rotor side, using the transformation ratio, K, is shown in Figure 4.20.

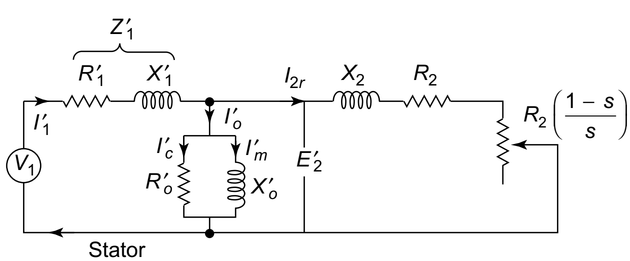


Figure 4.20 Equivalent Circuit of an Induction Motor Referred to Rotor Side

Here, $$E_1' = KE_1,\ I_1' = \frac{I_1}{K},\ X_1' = K^2X_1,\ R_1' = K^2R_1,\ X_o' = K^2X_o \text{ and } R_o' = K^2R_o \tag{4.54}$$

4.13.3 Approximate Equivalent Circuit

The approximate equivalent circuit of an induction motor, when referred to stator side is shown in Figure 4.21.

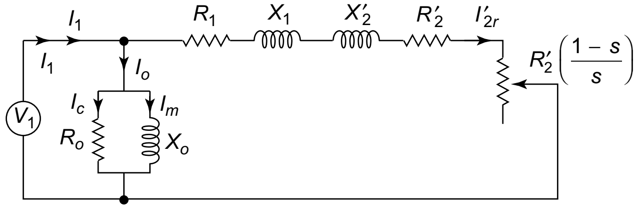


Figure 4.21 Approximate Equivalent Circuit of an Induction Motor

If R_{1e} and X_{1e} are the equivalent resistance and reactance of an induction motor when referred to stator side, then

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} \text{ and } X_{1e} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} \tag{4.55}$$

Using the above equation, the total approximate equivalent circuit of an induction motor can be obtained, as shown in Figure 4.22.

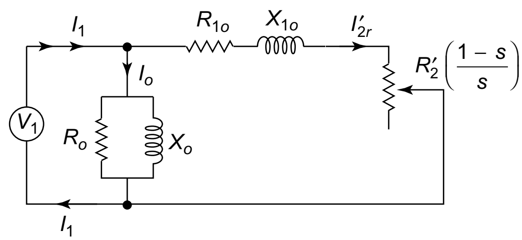


Figure 4.22 Total Approximate Equivalent Circuit of an Induction Motor

4.13.4 Power Equations from Equivalent Circuit

Using the total approximate equivalent circuit shown in Figure 4.22, the power equations are obtained as:

Input power, $P_{\text{in}} = 3V_1I_1 \cos\phi$

Where, V_1 is the phase voltage and I_1 is the phase current of the supply.

Stator core or iron loss, $P_{\text{si}} = I_m^2 R_o$

Stator copper loss, $P_{\text{sc}} = 3I_1^2R_1$

Rotor copper loss, $$P_c = 3 \times (I_{2r}')^2 R_2' \tag{4.56}$$

Rotor input, $$P_2 = \frac{P_c}{s} = \frac{3 \times (I_{2r}')^2 R_2'}{s} \tag{4.57}$$

Mechanical power developed in rotor,

$$P_m = (1-s)P_2 = \frac{3(1-s) \times (I_{2r}')^2 R_2'}{s} = 3(I_{2r}')^2 R_L' \tag{4.58}$$

4.13.5 Maximum Power Output

The approximate equivalent circuit of an induction motor, by neglecting the exciting branch, is shown in Figure 4.23.

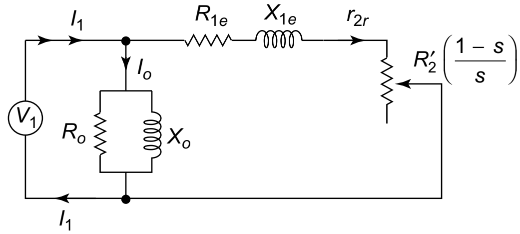


Figure 4.23 Approximate Equivalent Circuit by Neglecting the Exciting Branch

From Figure 4.23, we get

$$I_1 = I_{2r}' = \frac{V_1}{Z_T} = \frac{V_1}{\sqrt{(R_{1e} + R_L')^2 + X_{1e}^2}}$$

Therefore, the power supplied to the load is given by

$$P_{\text{out}} = 3 \times (I_{2r}')^2 \times R_L'$$

i.e., $$P_{\text{out}} = 3 \times \frac{V_1^2}{(R_{1e} + R_L')^2 + X_{1e}^2} \times R_L' \tag{4.59}$$

Differentiating the above equation with respect to R_L' and equating it to zero gives the condition for maximum power output.

i.e., $$\frac{dP_{\text{out}}}{dR_L'} = 0$$

Substituting Eqn. (4.59) in the above equation, we get

$$\frac{((R_{1e} + R_L')^2 + X_{1e}^2) \times (3V_1^2) - 3V_1^2 R_L' \times (2(R_{1e} + R_L'))}{[(R_{1e} + R_L')^2 + X_{1e}^2]^2} = 0$$

Upon solving, we get

$$R_{1e}^2 + X_{1e}^2 = (R_L')^2$$

If $Z_{1e} = R_{1e} + jX_{1e}$ is the total leakage impedance of the motor when referred to a stator, then the condition to obtain the maximum power output is given by

$$Z_{1e}^2 = (R_L')^2$$

i.e., $$R_L' = Z_{1e} \tag{4.60}$$

Here, if the equivalent load resistance when referred to stator is equal to the total leakage impedance of the motor, the power delivered to the load is maximum.
Substituting Eqn. (4.53) in Eqn. (4.60), we get

$$\frac{R_2'(1-s)}{s} = Z_{1e}$$

i.e., $$sZ_{1e} = R_2' - sR_2'$$

Therefore, the slip of the motor at maximum output is

$$s = \frac{R_2'}{R_2' + Z_{1e}} \tag{4.61}$$

Substituting Eqn. (4.60) in Eqn. (4.59), we get

$$(P_{out})_{max} = 3 \times \frac{V_1^2}{(R_{1e}+Z_{1e})^2 + X_{1e}^2} \times Z_{1e} = 3 \times \frac{V_1^2}{R_{1e}^2 + 2R_{1e}Z_{1e} + Z_{1e}^2 + X_{1e}^2} \times Z_{1e}$$

Since $R_{1e}^2 + X_{1e}^2 = Z_{1e}^2$, we get

$$(P_{out})_{max} = 3 \times \frac{V_1^2}{2(R_{1e}+Z_{1e})} \tag{4.62}$$

which represents the magnitude of the maximum power output of the induction motor.

4.13.6 Synchronous Watt

The torque developed in the motor using P_2 is given by

$$T = \frac{P_2}{\frac{2\pi N_{syn}}{60}}$$

If synchronous watt is the new unit of torque, then it is defined as the torque developed in the motor when the rotor input power across the air gap is 1 W at synchronous speed N_{syn}.

Therefore, 1 synchronous watt = $\frac{60}{2\pi N_{syn}}$ Nm

or $\quad 1 \text{ N m} = \frac{2\pi N_{syn}}{60}$ synchronous watt

Example 4.12

The active power input to a 415 V 50 Hz six-pole three-phase induction motor running at 970 rpm is 41 kW. The input power-factor is 0.9. The stator losses amount to 1.1 kW and the mechanical losses total to 1.2 kW. Calculate line current, slip, rotor copper loss, mechanical power output and efficiency.

[AU Nov/Dec, 2004]

Solution

Given, V_L = 415 V, f = 50 Hz, P = 6, N = 970 rpm, P_{in} = 4 lkW, Stator loss = 1.1 kW, cos ϕ = 0.9, and mechanical loss = 1.2 kW.

(i) The input power supplied to the motor is given by

$$P_{in} = \sqrt{3} V_L I_L \cos\phi$$

Substituting the known values, we get

$$41 \times 10^3 = \sqrt{3} \times 415 \times I_L \times 0.9$$

Solving the above equation, we get the line current of the motor as

$$I_L = 63.3771 \text{ A}$$

(ii) The synchronous speed of the induction motor is given by

$$N_{syn} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm.}$$

Therefore, the slip of the induction motor is given by

$$s = \frac{N_{\text{syn}} - N}{N_{\text{syn}}} = \frac{1000 - 970}{1000} = 0.03 = 3\%$$

(iii) We power input to the rotor is given by

$$P_2 = P_{\text{in}} - \text{Stator losses} = 41 - 1.1 = 39.9 \text{ kW}$$

We know that $P_2 : P_c : P_m = 1 : s : 1 - s$.

Therefore, $\frac{P_2}{P_c} = \frac{1}{s}$ and $\frac{P_c}{P_m} = \frac{s}{1-s}$

Where, P_c is the rotor copper loss in kW and P_m is the mechanical power developed in the rotor in kW. Hence, the rotor copper loss is given by

$$P_c = sP_2 = 0.03 \times 39.9 = 1.197 \text{ kW}$$

(iv) Using $P_2 : P_c : P_m = 1 : s : 1 - s$, we get the mechanical power developed in the rotor as

$$P_m = \frac{(1-s)P_c}{s} = \frac{(1-0.03) \times 1.197}{0.03} = 38.703 \text{ kW}$$

Therefore, the mechanical power output developed in the rotor is given by

$$P_{\text{out}} = P_m - \text{Mechanical losses} = 38.703 - 1.2 = 37.503 \text{ kW}$$

(v) The efficiency of the induction motor is given by

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100 = \frac{37.503}{41} \times 100 = 91.4707\ \%$$

Example 4.13

A three-phase four-pole 50 Hz induction motor runs at a speed of 1440 rpm. When the total torque is 70 Nm, determine: (i) total input to the rotor and (ii) total rotor copper loss. **[AU Nov/Dec, 2008]**

Solution

Given, $P = 4$, $f = 50$ Hz, $N = 1440$ rpm and $T = 70$ Nm

The synchronous speed of the induction motor is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Therefore, the slip of the induction motor is given by

$$s = \frac{N_{\text{syn}} - N}{N_{\text{syn}}} = \frac{1500 - 1440}{1500} = 0.04 = 4\%$$

The total mechanical power output developed in the induction motor is given by

$$P_m = T \times \omega = T \times \frac{2\pi N}{60} = \frac{70 \times 2\pi \times 1440}{60} = 10555.7513 \text{ W} = 10.555 \text{ kW}$$

(i) We know that $P_2 : P_c : P_m = 1 : s : 1 - s$. Hence,

$$\frac{P_2}{P_m} = \frac{1}{1-s}$$

i.e.,

$$\frac{P_2}{10555.7513} = \frac{1}{(1-0.04)}$$

Therefore, the total power input applied to the rotor is given by

$$P_2 = 10.9955 \text{ kW} = 11 \text{ kW}$$

(ii) Similarly, using $P_2 : P_c : P_m = 1 : s : 1 - s$, we get

$$\frac{P_c}{P_m} = \frac{s}{1-s}$$

Therefore, the total rotor copper loss is given by

$$P_c = 10555.7513 \times \frac{0.04}{(1-0.04)} = 439.823 \text{ W}$$

Example 4.14

An induction motor has an efficiency of 90% when delivering an output of 37 kW. At this load, the stator copper loss and rotor copper loss, each is equal to the iron loss. The mechanical losses are one third of the no-load. Calculate the slip. **[AU April/May, 2012]**

Solution

Given, $\eta = 90\% = 0.9$ and $P_{\text{out}} = 37$ kW

Since $\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$, the input power is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{37}{0.9} = 41.11 \text{ kW}$$

Let X represent the stator copper loss, rotor copper loss and iron loss.

i.e., $X = P_{cs} = P_c = P_i$

We know that, the no-load loss is the summation of mechanical and iron losses. If Y represents the mechanical loss, then

$$Y = \frac{1}{3} \times \text{no-load loss}$$

and no-load loss $= Y + X$

Therefore, $Y = \frac{1}{3} \times [Y + X]$

i.e., $Y = 0.5X$

The total loss in the induction motor is given by

$$P_T = P_i + P_{cs} + P_c + \text{Mechanical loss}$$

i.e., $P_T = X + X + X + 0.5X = 3.5X$ (1)

But the total loss in the induction motor is given by

$$P_T = P_{\text{in}} - P_{\text{out}} = 41.111 \times 10^3 - 37 \times 10^3 = 4.11 \text{ kW}$$

Substituting $P_T = 4.11$ kW in Eqn. (1) and solving, we get

$X = 1174.6$ kW (2)

Therefore, the rotor copper loss, iron loss and stator copper loss are equal to 1174. 6 kW.

The output power developed in the motor is

$$P_{out} = P_m - \text{Mechanical loss} = 37 \times 10^3 + 0.5X$$

We know that, $P_2 : P_c : P_m = 1 : s : 1 - s$.

Therefore, $$\frac{P_c}{P_m} = \frac{s}{1-s}$$

Substituting the known values, we get

$$\frac{X}{37 \times 10^3 \times 0.5X} = \frac{s}{1-s} \tag{3}$$

Substituting Eqn. (2) in Eqn. (3), we get

$$\frac{1174.6}{37 \times 10^3 \times 0.5 \times 1174.6} = \frac{s}{1-s}$$

Upon solving, we get the slip of the induction motor as

$$s = 0.0303 = 3.03\%$$

Example 4.15

A 415 V three-phase 50 Hz four-pole star-connected motor runs at 24 rev/s on full load. The rotor resistance and reactance per phase are 0.35 Ω and 3.5 Ω respectively, and the effective rotor-stator turns ratio is 0.85:1. Determine the: (i) synchronous speed (ii) slip (iii) full-load torque (iv) power output, if mechanical loss is 770W (v) maximum torque (vi) speed at which maximum torque occurs and (vii) starting torque. **[AU Nov/Dec, 2013]**

Solution

Given, $V_L = 415$ V, $f = 50$ Hz, $P = 4$, $n_{FL} = 24$ rps, $R_2 = 0.3\ \Omega$, $X_2 = 3.5\ \Omega$, $K = \dfrac{1}{0.85} = 1.176$.

The induced emf per phase in the stator is given by

$$E_1 = \frac{V_L}{\sqrt{3}} = 239.6\text{V},$$

Using $K = \dfrac{E_1}{E_2}$, we get

$$E_2 = \frac{E_1}{K} = \frac{239.6}{1.176} = 203.66 \text{ V}$$

(i) The synchronous speed of the induction motor is given by

$$N_{syn} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

The full-load speed of the rotor in rpm is given by

$$N_{FL} = n_{FL} \times 60 = 1440 \text{ rpm}$$

(ii) The slip of the induction motor is

$$s_{FL} = \frac{N_s - N_{FL}}{N_s} = \frac{1500 - 1440}{1500} = 0.04 = 4\%$$

(iii) The full-load torque of the induction motor is given by

$$T_{FL} = \frac{3}{2\pi n_{syn}} \times \frac{s_{FL} E_2^2 R_2}{R_2^2 + (s_{FL} X_2)^2} = \frac{3}{2\pi \times 25} \times \frac{0.04 \times 203.66^2 \times 0.35}{[0.35^2 + (0.04 \times 3.5)^2]} = 78.045 \text{ Nm}$$

(iv) The mechanical power output in the rotor is given by

$$P_m = T_{FL} \times \frac{2\pi N}{60} = \frac{78.045 \times 2\pi \times 1440}{60} = 11768.908 \text{W}$$

Therefore, the output power of the induction motor is

$$P_{out} = P_m - \text{Mechanical loss} = 11768908 - 770 = 10998908 \text{ W}$$

(v) The maximum torque developed in the motor is given by

$$T_{max} = \frac{3}{2\pi n_{syn}} \times \frac{E_2^2}{2X_2} = \frac{3}{2\pi \times 25} \times \frac{203.66^2}{2 \times 3.5} = 113.1657 \text{ Nm}$$

(vi) The slip, at which the maximum torque occurs, is given by

$$s_m = \frac{R_2}{X_2} = \frac{0.35}{3.5} = 0.1$$

Therefore, the speed of the rotor at which maximum torque occurs is given by

$$N_m = N_{syn}(1 - s_m) = 1500 \times (1 - 0.1) = 1.350 \text{ rpm.}$$

(vii) The starting torque is given by

$$T_{st} = \frac{3}{2\pi n_{syn}} \times \frac{E_2^2 R_2}{R_2^2 + X_2^2} = \frac{3}{2\pi \times 25} \times \frac{203.66^2 \times 0.35}{[0.35^2 + 3.5^2]} = 22.41 \text{ Nm}$$

Example 4.16

The power supplied to a three-phase induction motor is 32 kW and the stator losses are 120 W. If the slip is 5%, determine the: (i) rotor copper loss (ii) total mechanical power developed by the rotor (iii) output power of the motor if friction and windage losses are 750 W and (iv) efficiency of the motor, neglecting rotor iron loss. **[AU Nov/Dec, 2013]**

Solution

Given P_{in} = 32 kW, stator loss = 120 W and s = 5% = 0.05

The input to the rotor of induction motor is given by

$$P_2 = P_{in} - \text{stator loss} = 32 \times 10^3 - 1200 = 30.8 \text{ kW}$$

(i) We know that, $P_2 : P_c : P_m = 1 : s : 1 - s$

Using $\frac{P_2}{P_c} = \frac{1}{s}$, we get the rotor copper loss as

$$P_c = sP_2 = 0.05 \times 30800 = 1540 \text{ W}$$

(ii) The mechanical power developed in the rotor is given by

$$P_m = P_2 - P_c = 30.8 \times 10^3 - 1540 = 29.26 \text{ kW}$$

(iii) The output power of the induction motor is given by

$$P_{out} = P_m - \text{Friction loss} = 29.26 \times 10^3 - 750 = 28.51 \text{ kW}$$

(iv) The efficiency of induction motor is given by

$$\%\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{28510}{32 \times 10^3} \times 100 = 89.09\%$$

Example 4.17

A single-phase equivalent circuit of a six-pole squirrel-cage induction motor that operates from a 220V line voltage at 50 Hz is given below. Calculate the stator current, output power, torque and efficiency at a slip of 2.5%. The fixed winding and friction losses are 350W. Neglect the core loss. **[AU Nov/Dec, 2011]**

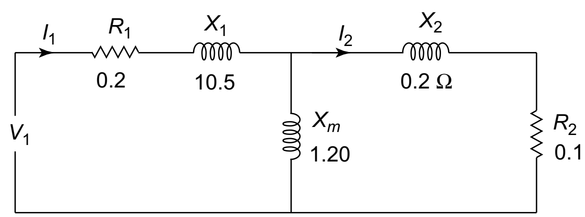


Figure E4.17(a)

Solution

Given, V_L = 220 V, f = 50 Hz, P= 6 and s = 2.5% = 0.025

When the induction motor is in running condition, its equivalent circuit is shown in Figure E4.15 (b).

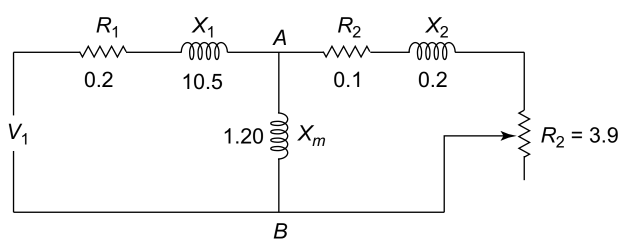


Figure E4.17(b)

The total impedance of the induction motor is obtained as follows:

The impedance across the terminals AB is given by

$$Z_{AB} = (jX_m) \,||\left(R_2 + \frac{R_2(1-s)}{s} + jX_2\right)$$

$$= (j20) \,||\, (4 + j\,0.2) = \frac{(20\angle 90°)\,(4.005\angle 2.86°)}{(4 + j0.2 + j20)} = \frac{80.1\angle 92.86°}{20.592\angle 78.8°}$$

$$= 3.8898\angle 14.06° = 3.7732 + j0.945\ \Omega$$

Hence, the total impedance of the induction motor is given by

$$Z_T = R_1 + j\,X_1 + Z_{AB} = 0.2 + j\,0.5 + 3.7732 + j\,0.945$$
$$= 3.9732 + j1.445\ \Omega = 4.2278\ \angle 19.985°\ \Omega$$

The stator current is given by

$$I_1 = \frac{V_{\text{ph}}\angle 0°}{Z_T} = \frac{\frac{220}{\sqrt{3}}\angle 0°}{4.2278\angle 19.985°} = 30.043\angle -19.985°\ \text{A}$$

Hence, the stator copper loss is

$$P_{cs} = 3I_1^2 R_1 = 3 \times 30.043^2 \times 0.2 = 541.5598\ \text{W}$$

The total input power of the motor is given by

$$P_{\text{in}} = \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 220 \times 30.043 \times \cos(19.985°) = 10.758\ \text{kW}$$

The rotor input is given by

$$P_2 = P_{\text{in}} - \text{Stator losses} = 10.217\ \text{kW}$$

It is known that, $P_2 : P_c : P_m = 1 : s : 1 - s$.

Therefore, $\dfrac{P_2}{P_m} = \dfrac{1}{1-s}$

i.e., $P_m = (1 - s)\,P_2 = (1 - 0.025) \times 10.217 = 9.9615\ \text{kW}$

Hence, the total power output of the motor is

$$P_{\text{out}} = P_m - \text{Friction loss} = 9.9615 \times 10^3 - 350 = 9.611\ \text{kW}$$

The synchronous speed of the motor is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1200\ \text{rpm}$$

Hence, the speed of the motor is given by

$$N = N_{\text{syn}}\,(1 - s) = 1000\,(1 - 0.025) = 975\ \text{rpm}$$

Therefore, the torque developed at the shaft is given by

$$T = \frac{P_{\text{out}}}{\omega} = \frac{P_{\text{out}}}{\left(\dfrac{2\pi N}{60}\right)} = \frac{9.611 \times 10^3 \times 60}{2\pi \times 975} = 94.1315\ \text{Nm}$$

Hence, the efficiency is given by

$$\%\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100 = \frac{9.611}{10.758} \times 100 = 89.33\%$$

4.14 Phasor Diagram of an Induction Motor

Similar to a two-winding transformer, the phasor diagram of an induction motor can be drawn. The different equations that are required to draw the phasor diagram are given below:

Since the supply voltage has to counter-balance the self-induced emf in the stator side and its losses, the supply voltage in vector form is given by

$$\overline{V_1} = -\overline{E_1} + \overline{I_1 R_1} + j\overline{I_1 X_1} = -\overline{E_1} + \overline{I_1 Z_1}$$

As the rotor is short-circuited, the emf induced in the rotor during running condition is necessary to supply the impedance drop, which is given by

$$\overline{E}_{2r} = \overline{I_{2r} R_2} + j\overline{I_{2r} X_{2r}} = \overline{I_{2r} Z_{2r}}$$

The total current drawn by the stator has to supply the exciting branch and reflected as rotor current on the stator side. Therefore,

$$\overline{I_1} = \overline{I_o} + \overline{I'_{2r}}$$

Where, $\overline{I_o}$ is the no-load current, as given by $\overline{I_o} = \overline{I_c} + \overline{I_m}$

$\overline{I'_{2r}}$ is the reflected rotor current in the stator side, as given by $\overline{I'_{2r}} = K\overline{I}_{2r}$.

Using the steps given in section 2.12, the phasor diagram of the three-phase induction motor is shown in Figure 4.24

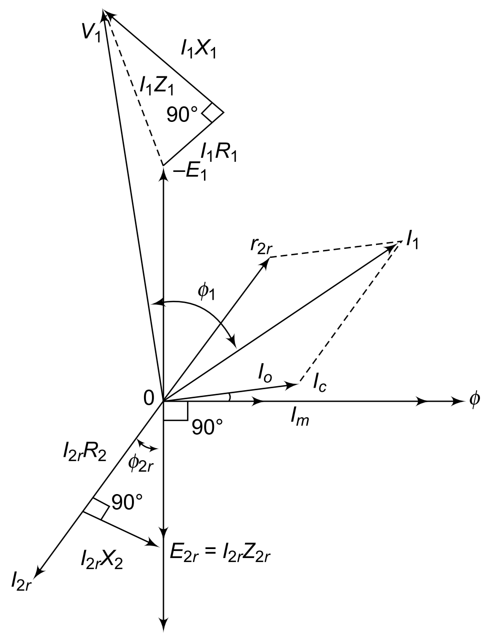


Figure 4.24 Phasor Diagram of a Three-phase Induction Motor

4.15 Starters

A device used to start, reverse and protect a machine by controlling the use of electrical power is called a starter. Since it is connected in series with the machine, it helps in decreasing the starting current flowing into the motor. In general, the starter consists of a connector and an overload unit to measure the current and thereby it controls the current. The necessity of a starter and different types of starters used in three-phase induction motor are discussed below.

4.15.1 Necessity of Starters

It is known that the rotor current in running condition is given by

$$I_{2r} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

The necessity of a starter in a three-phase induction motor is explained below:

- When the motor is in a standstill position, slip s = 1. Hence, the magnitude of E_{2r} is more and since the rotor is short-circuited, a large rotor current circulates through the rotor.

- Due to a large rotor current, the stator draws high current from the supply and it is 5 to 8 times greater than the full-load current.
- Because of this high stator-current, the possibility of damage in the motor winding is high.
- In addition, the line-voltage drop in the system is high. Due to this, the other equipment connected in the same line gets a voltage spike and affects the equipment.

Due to these reasons, the stator current at the start of the induction motor should be kept small. Starters are used to limit this stator current and provide protection to the three-phase induction motor.

4.15.2 Types of Starters

The different types of starters used in three-phase induction motors are:

- Stator resistance starter
- Auto-transformer starter
- Star-delta starter
- Rotor resistance starter and
- Direct-on-line starter

Stator Resistance Starter

The schematic diagram of an induction motor with stator resistance starter is shown in Figure 4.25.

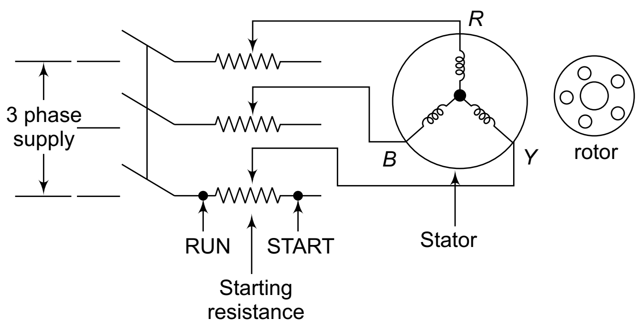


Figure 4.25 Stator Resistance Starter

The external resistance, called starting resistance, is added in series with the stator windings to apply reduced voltage to the stator. The START and RUN positions indicate that the starting resistance is kept at maximum and minimum positions respectively. Therefore, at start, due to a large voltage drop across the starting resistance, a reduced voltage is applied to the stator winding. Hence, the starting current of the induction motor is reduced.

The relation between the starting and full-load torques, while using stator resistance starter is given by

$$\frac{T_{\mathrm{st}}}{T_{\mathrm{FL}}} = x^2 \left(\frac{I_{\mathrm{st}}}{I_{\mathrm{FL}}}\right) s_{\mathrm{FL}}$$

Where, x is the factor by which the stator voltage is reduced and I_{st} is the starting current.

Advantages and Disadvantages of Stator Resistance Starter

Advantages

1. Simple construction
2. Can be used for both star and delta-connected stator windings
3. Cost is less

Disadvantages

1. Power loss in the motor is high
2. Starting torque gets reduced

Auto-transformer Starter

The schematic diagram of induction motor with an auto-transformer starter is shown in Figure 4.26.

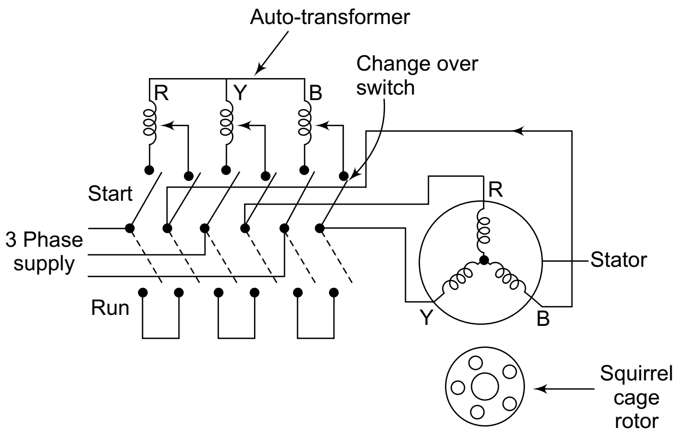


Figure 4.26 Auto-transformer Starter

Using this starter, a reduced voltage is applied to the stator. During start, a three-phase start-connected auto-transformer is connected to the stator and using the tapping in the auto-transformer, a reduced voltage is applied to the stator. When the motor reaches 80% of the synchronous speed, the auto-transformer is disconnected using a change-over switch. Since an additional component like a relay is required to operate the change-over switch, this starter is expensive. If x is the tapping used in an auto-transformer, then the ratio of starting to full-load torques is given by

$$\frac{T_{st}}{T_{FL}} = x^2 \left(\frac{I_{st}}{I_{FL}} \right) s_{FL}$$

Star–Delta Starter

The schematic diagram of an induction motor with a star–delta starter is shown in Figure 4.27.

As the name of the starter suggests, during start condition, the stator windings are connected in start connection such that the voltage applied to the stator winding is reduced by $\frac{1}{\sqrt{3}}$. When the motor attains a certain speed, using triple-pole double-throw (TPDT) or a change-over switch, the stator windings are connected in

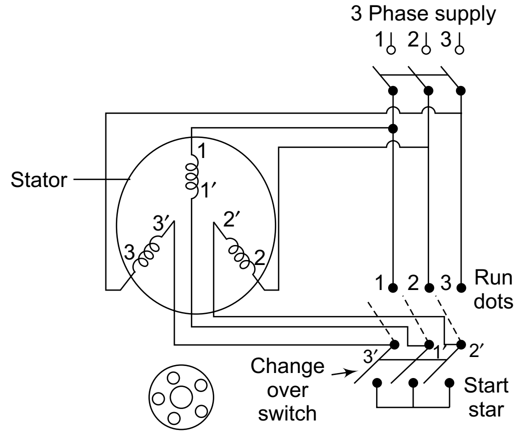


Figure 4.27 Star–Delta Starter

delta so that a rated voltage is applied to the stator windings. A relay is used to operate the TPDT switch. This is the cheapest starter used in an induction motor. The ratio of starting to full-load torque is given by

$$\frac{T_{\mathrm{st}}}{T_{\mathrm{FL}}}=\frac{1}{3}\times\left(\frac{I_{\mathrm{st}}}{I_{\mathrm{FL}}}\right)s_{\mathrm{FL}}$$

Rotor Resistance Starter

The schematic diagram of an induction motor with rotor resistance starter is shown in Figure 4.28.

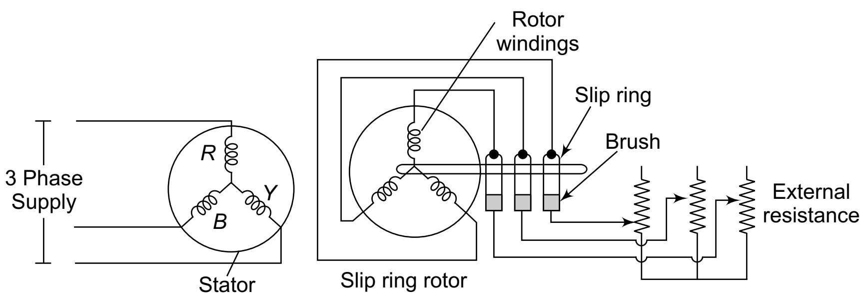


Figure 4.28 Rotor Resistance Starter

In this type of starter, rated voltage is applied to the stator windings. At start, maximum external resistance is added to the rotor circuit to decrease the rotor current flowing through the motor. Once the motor attains speed, it is brought to the minimum position. The main benefit of this type of starter is that it increases the starting torque of the motor and decreases the starting current. It can be used only in a slip-ring induction motor, because there is no provision of adding external resistance in a squirrel-cage induction motor.

Direct-on-line Starter

The schematic diagram of an induction motor with direct-on-line (DOL) starter is shown in Figure 4.29.

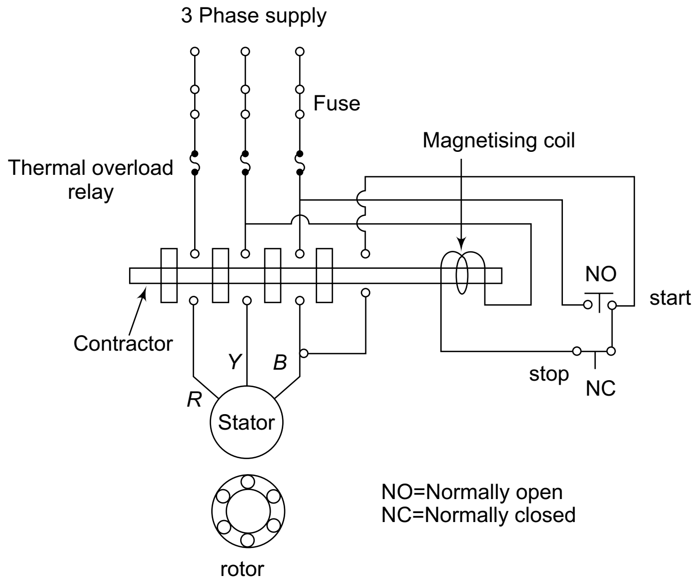


Figure 4.29 Direct-on-line Starter

The main purpose of this starter is to protect the induction motor from different abnormal conditions like over-voltage, low voltage etc. It is used in the induction motor whose capacity is less than 5 Horsepower. Since the starting current in this capacity motor is less, a DOL starter applies rated voltage to the stator winding. The normal open and normal close contacts in a DOL starter represent the normally open switch and normally closed switch. These contacts are used to start and stop the induction motor respectively.

4.15.3 Comparison of Different Starters

The comparison of different starters used in induction motors is listed in Table 4.5.

Table 4.5 *Comparison of Different Starters*

Category	Type of Starter				
	Stator Resistance	Auto-transformer	Star–Delta	Rotor Resistance	DOL
Voltage Applied to Stator	Reduced voltage	Reduced voltage	$\frac{1}{\sqrt{3}}$	Reduced voltage	Normal voltage
Initial Cost and Maintenance	Less	More	Least	More	Less
Operation	Easier	Difficult	Easier	Difficult	Easier
Rotor Type	Squirrel cage	Squirrel cage and Slip ring	Squirrel cage and Slip ring	Slip ring	Squirrel cage
Starting Torque	Less	Less	Medium	More	Low
Capacity of Motor Used	Less	Moderate to high	Moderate	Moderate to high	Less

4.16 Speed Control of a Three-Phase Induction Motor

It is known that a three-phase induction motor is a constant-speed motor, whose speed cannot be controlled using rheostats. When the speed varies, it affects the motor performance in terms of its power factor, efficiency etc.

The motor speed and torque equation of an induction motor are given by

$$N = N_{\text{syn}}(1 - s) \quad \text{and} \quad T \;\alpha\; \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \tag{4.63}$$

From the above equation, it is seen that by varying either the synchronous speed or slip of induction motor, the motor speed can be controlled. Also, if the slip of induction motor varies, the torque developed in the motor varies. Therefore, to maintain constant torque at constant load condition, $R_2$and X_2can be varied.

4.16.1 Speed Control Methods

The speed of induction motor can be controlled using different methods. These methods are classified into two categories, as follows:

1. ***From stator side:***
   - Supply frequency control or V/f control
   - Supply voltage control
   - Controlling number of stator poles
   - Adding rheostats in stator circuit
2. ***From rotor side:***
   - Adding external resistance in rotor circuit
   - Cascade control
   - Injecting slip frequency voltage into the rotor circuit

4.16.2 Supply Frequency Control or V/f Control

The synchronous speed of the induction motor is given by

$$N_{\text{syn}} = \frac{120f}{P}$$

From the above equation, it is clear that by controlling the supply frequency f, the speed of the induction motor can be controlled. Similar to a transformer, the air-gap flux in the induction motor is given by

$$\phi_{ap} = \frac{1}{4.44K_1 T_{\text{ph1}}}\left(\frac{V}{f}\right)$$

where, K_1 is the stator winding constant and T_{ph1} is the stator turns per phase.

Hence, if the supply frequency is changed to control the speed of the induction motor, the air-gap flux changes and it results in saturation of stator and rotor cores. Due to this saturation, the no-load current increases, which damages the stator windings. Therefore, to keep the air-gap flux constant, the supply voltage is varied. Hence, by varying the supply voltage, the ratio V/f is maintained constant, thereby ensuring a constant air-gap flux in controlling the speed of motor. The electronic method for V/f control is shown in Figure 4.30.

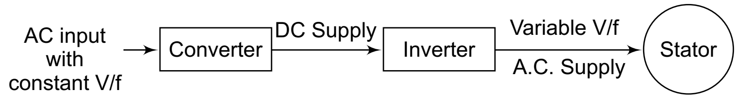


Figure 4.30 Electronic Method for *V/f* Control

It is noted that the supply voltage in this method is a variable voltage and variable frequency supply. Converter and inverter are the devices used in this scheme to maintain constant air-gap flux, by maintaining *V/f* as constant. The torque–slip characteristics of this method, by varying *f* and maintaining *V/f* as constant, is shown in Figure 4.31.

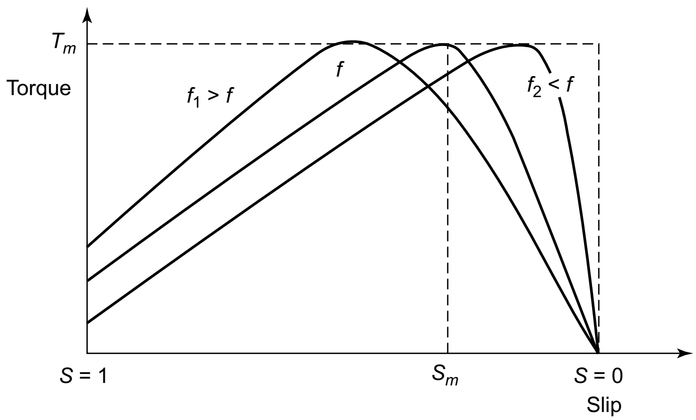


Figure 4.31 Torque–slip Characteristics Using *V/f* Method

4.16.3 Supply Voltage Control

It is known that by varying the supply voltage, the emf induced in the rotor E_2 can be varied, since $E_2 \alpha E_1 \alpha V$. Therefore, the torque equation of Eqn. (4.63) becomes,

$$T \alpha \frac{sV^2 R_2}{R_2^2 + (sX_2)^2}$$

Since the induction motor is normally operated in low-slip region, the term sX_2 is neglected, as $sX_2 << R_2$. Hence,

$$T \alpha \frac{sV^2}{R_2}$$

For a constant R_2, if the supply voltage is reduced, the torque developed in the motor decreases. But, to supply the same load, increasing the slip in induction motor increases the torque developed in the motor. Increase in slip indicates the decrease in speed of the motor. Therefore, the motor is able to develop the required torque at a lower speed and the speed–torque characteristics of the motor using this speed control are shown in Figure 4.32.

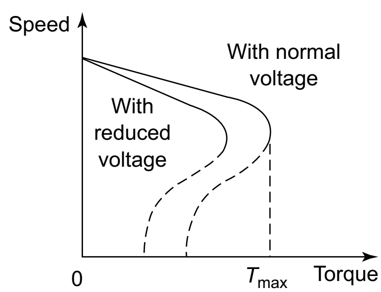


Figure 4.32 Speed–Torque Characteristics for Different *V*

4.16.4 Controlling Number of Poles

In this method, varying the number of stator poles controls the speed of the induction motor. Hence, this method is also called as pole changing method. Using this method, different speeds can be obtained in the motor. The number of stator poles can be changed using: (i) Consequent poles method (ii) Multiple stator-winding method and (iii) Pole amplitude modulation method. The speed–torque characteristics of a three-phase induction motor by varying the number of stator poles are shown in Figure 4.33.

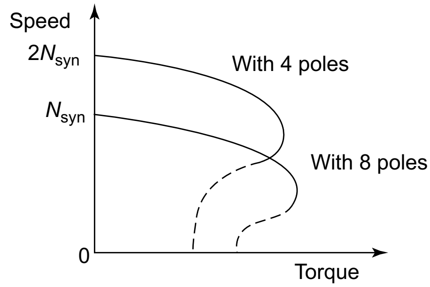


Figure 4.33 Speed–Torque Characteristics for Different Stator

4.16.5 Adding Rheostats in Stator Circuit

The schematic diagram of a three-phase induction motor with rheostat in the stator circuit to control the speed is shown in Figure 4.34. Using this method, a reduced voltage is applied to the stator, as there will be a drop in the external rheostats connected to the circuit. Since there is a direct relation between the applied voltage and the speed, the speed of the motor reduces.

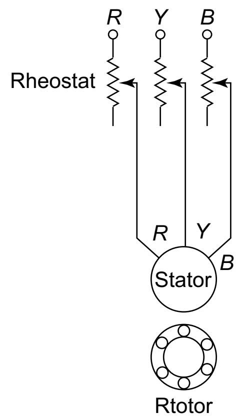


Figure 4.34 Addition of External Rheostats in the Stator Circuit

4.16.6 Adding External Resistance in Rotor Circuit

It is applicable only to slip-ring induction motors. In low-slip region, the torque developed in the motor is given by

$$T \,\alpha\, \frac{s}{R_2} \quad \text{(since } sX_2 << R_2\text{, the term } sX_2 \text{ is neglected)}$$

For a constant supply voltage, if adding external rheostats increases the rotor resistance, the torque developed in the motor decreases. But to supply the same load, the increasing the slip of induction motor increases the torque developed in the motor, which decreases the speed of the motor. Therefore, the motor is able to develop the required torque at a lower speed and the speed–torque characteristics of the motor using this speed control are shown in Figure 4.35.

This type of speed control is rarely used, due to the following reasons:

- Speed greater than normal speed is not possible.
- Rotor copper loss increases if the external resistance is added to the circuit and hence efficiency of the motor is less.
- It is applicable only to slip-ring induction motors.
- Extra component is to be added to the motor for cooling purpose and hence this method becomes expensive.

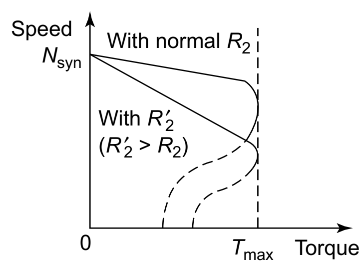


Figure 4.35 Speed–Torque Characteristics for Different R_2

4.16.7 Cascade Control

The schematic diagram of cascade control of a three-phase induction motor is shown in Figure 4.36.

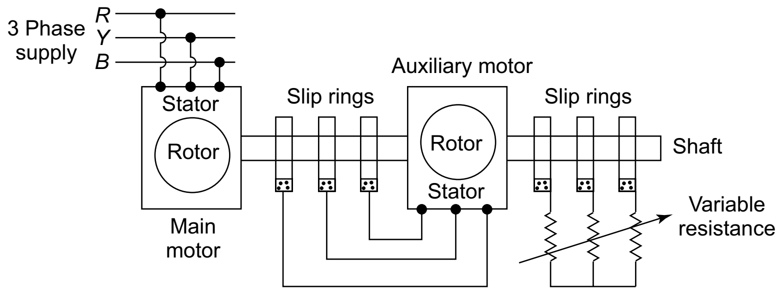


Figure 4.36 Cascade Speed Control of an Induction Motor

In this method, two induction motors—main and auxiliary motors—are mounted on the same shaft. A slip-ring rotor is used in the main motor while the auxiliary motor can be either squirrel-cage or slip-ring rotor. Main supply is given to the stator winding of the main motor. While stator winding of auxiliary motor receives a voltage obtained at a slip frequency from the slip rings of the main motor. This process of connecting two motors is called cascading of motors. The cascading of motors can be either cumulative cascading or differential cascading. If torque developed in the motors act in the same direction it is called cumulative cascading. If torque developed in the motors act in the opposite direction it is called differential cascading. Using this method, the different speeds of auxiliary induction motor are:

$$N = \frac{120f}{P_A + P_B} \quad \text{for cumulative cascading}$$

and $$N = \frac{120f}{P_A - P_B} \quad \text{for differential cascading}$$

4.16.8 Injecting Slip Frequency EMF to the Rotor Circuit

This method is used to control the motor speed by injecting a voltage in the rotor circuit. Since the rotor frequency is slip times the supply frequency, the injected voltage to the circuit must be at slip frequency. This injected voltage can either oppose or aid the induced emf in the rotor circuit. If the injected voltage opposes the induced emf in the rotor, the effective rotor resistance increases and if the injected voltage aids the induced emf in the rotor, the effective rotor resistance decreases. Hence, the rotor resistance can either be increased or decreased, thereby controlling the speed of the motor.

4.17 Single-Phase Induction Motor

A single-phase AC supply is commonly used as general-purpose equipment in residential and commercial loads. Therefore, the motor that works using a single-phase AC supply is called a single-phase induction motor and is more commonly used when compared to DC motor. Since this motor uses a single-phase AC supply, it has very small power rating. Most commonly used applications are: small toys, small fans, hair dryers etc.

4.17.1 Construction of Single-phase Induction Motor

Similar to a three-phase induction motor, the single-phase induction motor has two main components: a stator and a rotor. The stator and rotor construction of the single-phase induction motor is similar to the three-phase induction motor, as discussed in Section 4.2.5. Also, only squirrel-cage rotor is used in a single-phase induction motor and its construction is explained in Section 4.2.5. The schematic representation of a single-phase induction motor is shown in Figure 4.37.

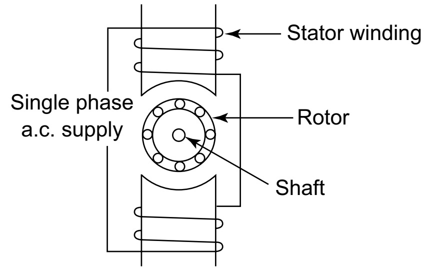


Figure 4.37 Single-phase Induction Motor

4.17.2 Working of a Single-phase Induction Motor

A single-phase induction motor must have two fluxes for its operation. A rotating magnetic flux called main flux is produced in the stator winding when a single-phase AC supply is supplied to the stator windings. When the main flux links with the stationary rotor conductors, using the transformer action, an emf is induced in the rotor. Since the squirrel-cage rotor is closed at the end, the induced emf drives the current through the rotor. As this rotor current flows through its conductor, a flux called rotor flux is produced. When these two fluxes i.e., main and rotor fluxes interact, a torque is developed in the rotor and hence the rotor rotates.

The major differences between a DC motor and a single-phase induction motor are: (i) two supplies are required in a DC motor when compared to a single supply in AC motor; (ii) DC motors are self-starting while single-phase induction motors are not self starting. Double-revolving field theory is used to explain the reason why a single-phase induction motor is not self-starting.

4.17.3 Double Revolving Field Theory **[AU April/May, 2014; Nov/Dec, 2011]**

It states that any alternating quantity is resolved into two rotating components such that the magnitude of these components is exactly half the magnitude of the original alternating quantity and these two components rotate in opposite direction. Consider that the maximum magnitude of alternating flux ϕ_1, produced by exciting the stator windings in a single-phase induction motor is $\phi_{1\max}$.

According to double revolving field theory, this flux ϕ_1 is resolved into two components each with magnitude $\phi_{1\max}/2$. They are called forward and backward components, represented as ϕ_f and ϕ_b respectively. The speed of these two components is N_{syn} and it depends on the frequency and the number of stator poles. The components ϕ_f and ϕ_b are rotated in anti-clockwise and clockwise directions respectively, so that the resultant of these components gives the instantaneous value of the stator flux at any instant.

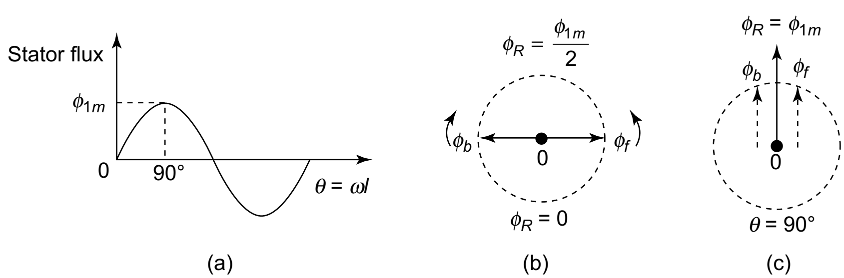


Figure 4.38 Stator Flux (a) Original Quantity (b) At Start (c) At θ= 90°

The original stator flux ϕ_1, produced due to the excitation of stator windings, is shown in Figure 4.38 (a). Consider two instants to prove that the single-phase induction motor is not a self-starting motor. First, at start, the two components ϕ_f and ϕ_b, are shown opposite to each other, as shown in Figure 4.38 (b), so that the resultant flux is zero and second at $\theta = 90°$, where the two components ϕ_f and ϕ_b, are shown in the same direction, as shown in Figure 4.38 (c), so that the resultant flux is maximum.

Since these two components rotate at speed N_{syn}, the rotor conductors disturb it and hence an emf is induced in the rotor that further circulates rotor current in the circuit. This rotor current, when passing through the rotor conductor, produces rotor flux. The rotor flux that interacts with ϕ_f produces a torque in the clockwise direction, and that interacts with ϕ_b produces a torque in the anti-clockwise direction. Since at start, these torque act opposite to each other, the resultant torque is zero. As the resultant torque is zero, the rotor does not rotate and it proves that the single-phase induction motor is not self-starting. These two opposite torques and their resultant torque at different motor speed is shown in Figure 4.39.

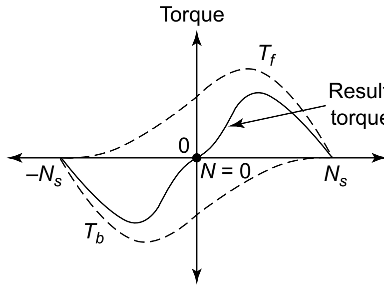


Figure 4.39 Forward, Backward and Resultant Torque at Different Speeds

When a rotor is given an initial rotation, the torque developed in the motor increases and hence the motor starts rotating in the direction in which the rotor is initially rotated. However, rotating the rotor initially is not practically possible. Therefore, some modifications are required in the construction of a single-phase induction motor to make them self-starting.

4.18 Types of Single-Phase Induction Motors

An arrangement is to be provided in the single-phase induction motor to make the alternating stator flux as rotating so that the stator flux rotates in a particular direction. Hence, the torque produced in the single-phase induction motor will be unidirectional. Therefore, the single-phase induction motor becomes self-starting under the influence of a rotating stator magnetic field. Based on the method of producing a rotating magnetic field, the single-phase induction motors are classified as:

- Split-phase induction motor
- Capacitor induction motor
  - Capacitor-start induction motor
  - Capacitor-start capacitor-run induction motor
- Shaded pole induction motor

The necessary condition for a rotating magnetic field to be generated in a single-phase induction motor is to have a minimum of two alternating fluxes with phase difference α between them, as shown in Figure 4.40.

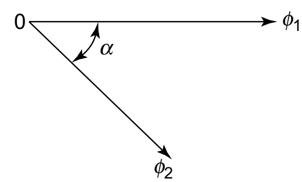


Figure 4.40 Alternating Fluxes with α as Phase Difference

Due to interaction of these two fluxes, a resultant flux is produced. Since the fluxes are alternating, the resultant flux will change its direction at every instant. This results in a resultant flux rotating in a particular direction. Therefore, in all the types of single-phase induction motors, suitable arrangement is made to generate an additional flux so that the required rotating magnetic field is produced. Here, if the phase angle α is more, the starting torque of the single-phase induction motor is high.

4.18.1 Split-phase Induction Motor **[AU Nov/Dec, 2012]**

In this type of single-phase induction motor, the stator carries two windings: main and auxiliary or starting windings. Here, the main winding is purely inductive and the auxiliary winding is highly resistive, since it carries a series resistance. The circuit diagram of a split-phase induction motor is shown in Figure 4.41.

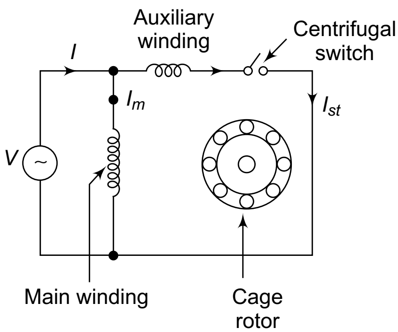


Figure 4.41 Circuit Diagram of a Split-phase Induction Motor

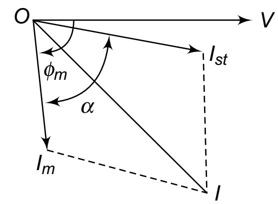


Figure 4.42 Phasor Diagram of a Split-phase Induction Motor

Consider that the current carried by the main and starting windings are I_m and I_{st}. The phasor diagram of these currents, with respect to the supply voltage, is shown in Figure 4.42. Since the main winding is purely inductive, the current I_m lags the voltage by ϕ_m and since the auxiliary windings are highly resistive, the current I_{st} lags the voltage by a very small angle. Hence, the phase difference between I_m and I_{st} is more i.e., α is high.

These currents, while flowing through their respective windings, produce two fluxes, which are displaced by phase angle α. Therefore, the resultant of these fluxes produces a rotating magnetic field and the induction motor gets started due to the production of a starting torque. The split-phase induction motor has a centrifugal switch that is used to disconnect the auxiliary windings from the induction motor when the motor attains 75 to 80% of synchronous speed. The torque–speed characteristic curve of a split-phase induction motor is shown in Figure 4.43.

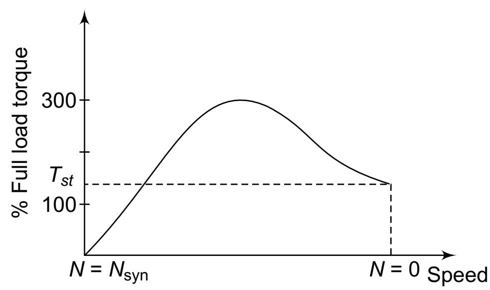


Figure 4.43 Torque–Speed Characteristics of a Split-phase Induction Motor

The starting torque of split-phase induction motor is poor, which is 125% to 150% of T_{FL}. Here, changing the terminals of either starting or main windings can vary the direction of motor speed.

Applications

Low starting-current and moderate starting-torque are the characteristics of a split-phase induction motor. Therefore, it is used in easily started loads like fans, blowers, grinders, centrifugal pumps, washing machines, oil burners, office equipment etc.

4.18.2 Capacitor Induction Motor [AU Nov/Dec, 2010; April/May, 2008]

The only difference between the split-phase and the capacitor induction motor is the centrifugal switch connected in series with the capacitor. Due to the presence of the capacitor in series with the starting winding, a leading current I_{st} is drawn by the starting winding so that the phase difference between I_{st} and I_m is more when compared to a split-phase induction motor. The phasor diagram of the capacitor induction motor is shown in Figure 4.44.

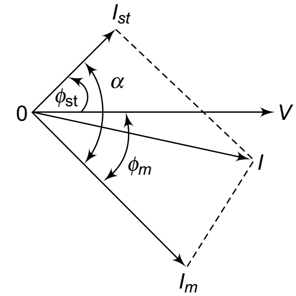


Figure 4.44 Phasor Diagram of a Capacitor Induction Motor

This type of induction motor is classified based on the capacitor that remains in the circuit permanently or disconnected, as:

- Capacitor-start induction motor
- Capacitor-start capacitor-run induction motor

Capacitor-start Induction Motor

The circuit diagram of the capacitor-start induction motor is shown in Figure 4.45.

Since the phase angle, α, between the currents is very large, the starting torque proportional to α is also very high. In addition, when the motor reaches 75 to 80% of the synchronous speed, using a centrifugal switch, the starting winding and the capacitor are disconnected from the main winding. Since the capacitor remains in the motor only at the beginning, it is called a capacitor-start induction motor.

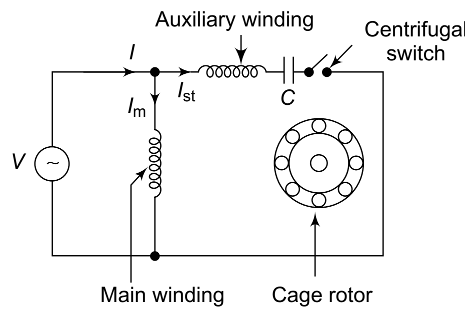


Figure 4.45 Capacitor-start Induction Motor

Capacitor-start Capacitor-run Induction Motor

The construction of this motor is similar to a capacitor-run induction motor, except for the absence of a centrifugal switch in the motor, as it exists throughout the motor operation. The schematic diagram of a capacitor-start capacitor-run induction motor is shown in Figure 4.46. Since the capacitor exists throughout the motor operation, the power factor of the motor is improved.

The capacitor value is chosen in such a way that it should compromise between the best starting and running conditions. Hence, the capacitor value is chosen such that the starting torque is only 50 to 100% of full-load torque T_{FL}, which is less when compared to a capacitor-start induction motor.

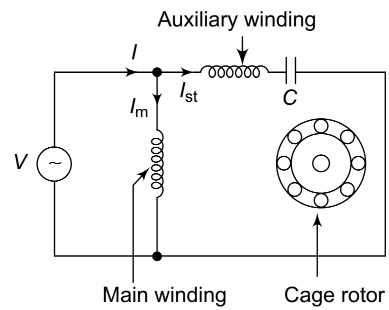


Figure 4.46 Capacitor-start Capacitor-run Induction Motor

Since capacitor is added to the circuit, this motor is costly when compared to a split-phase induction motor. The torque–speed characteristics of a capacitor induction motor are shown in Figure 4.47.

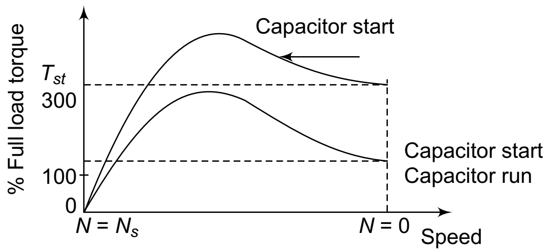


Figure 4.47 Torque–Speed Characteristics of a Capacitor Induction Motor

Applications

Since the capacitor-start induction motor has a high starting torque, it is used in hard starting loads like compressors, conveyors, grinders, fans, blowers, refrigerators, air conditioners etc. Also, the capacitor-start capacitor-run motor is used in ceiling fans, blowers and air-circulation equipment, where the requirement of starting torque is less.

4.18.3 Shaded Pole Induction Motor **[AU Nov/Dec, 2010; April/May, 2009]**

In a shaded pole induction motor, the stator has projected salient poles where the stator windings are wound. Here, each projected pole is provided with a copper band. The schematic diagram of a shaded pole induction motor and the enlarged view of a projected pole are shown in Figures 4.48 (a) and (b) respectively.

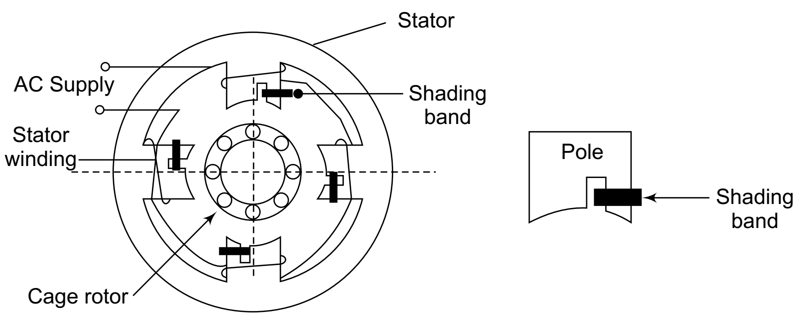


Figure 4.48 Shaded Pole Induction Motor (a) Schematic Diagram (b) Projected Pole

When a single-phase AC supply is provided to the stator winding, a rotating magnetic field is produced due to a copper band provided to the poles. The explanation of production of rotating magnetic field is given below:

When the current flows through the stator winding, a flux called main flux is produced, as shown in Figure 4.49.

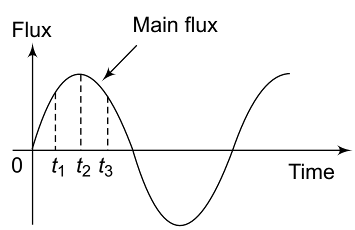


Figure 4.49 Main Flux of Induction Motor

Also, when the current flows through the copper band, a flux called shaded-ring flux is produced. Here, the magnetic axis lies at the position where there is more flux. When the magnetic axis shifts, a rotating magnetic field is produced. The concept of shifting of magnetic axis is shown in Figure 4.50.

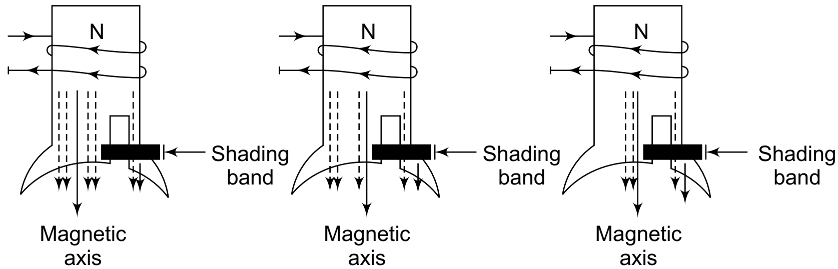


Figure 4.50 Magnetic Axis at (a) t= t_1 (b) t= t_2 and (c) t = t_3

Since the rate of change of current is high, main flux is also very high at $t = t_1$. Using the transformer principle, large emf is induced in the copper band that circulates large current. Due to this current, a flux called shaded-ring flux is produced and it opposes the main flux. Therefore, there is flux crowding in the non-shaded part when compared to the shaded part. Hence, the magnetic axis aligns in the non-shaded part, as shown in Figure 4.50 (a).

At instant $t = t_2$, the current and flux reach the maximum value, and the rate of change of flux is zero. Hence, the shaded-ring flux is minimum and is neglected. Therefore, the main flux is unaffected, and it is distributed uniformly, aligning with the magnetic axis at the centre of the pole, as shown in Figure 4.50(b).

At instant $t = t_3$, the current and flux start decreasing. Since the rate of change of flux is high, a large emf is induced in the copper band and hence a current starts flowing in the copper band. Due to the production of shaded-ring flux, it opposes the main flux. Since the main flux is decreasing, the shaded ring flux acts in the same direction. Thus, the flux is crowded in the shaded part. Therefore, the magnetic axis aligns as shown in Figure 4.50(c). This sequence is repeated and hence the rotating magnetic field is produced. The starting torque of the shaded pole induction motor is 40 to 50% of full-load torque, T_{FL}, and the torque–speed characteristic curve is shown in Figure 4.51.

Though the construction of a single-phase induction motor is simple, it is rarely used due to the following disadvantages:

1. Poor starting-torque and very low power-factor when compared to other types.
2. Due to the presence of a copper band, I^2R losses are high and hence the efficiency is very low.
3. Difficult to change the direction of induction motor.
4. The size and power rating of these motors are very small.

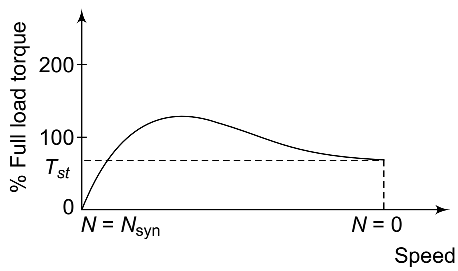


Figure 4.51 Torque–Speed Characteristics of a Shaded Pole Induction Motor

Applications

The single-phase induction motor has the following characteristics: very low starting-torque, low power-factor and low efficiency. Hence, it used in small fans, advertising displays, film projectors, record players, gramophones, hair dryers, photo-copying machines etc.

4.19 Equivalent Circuit of a Single-Phase Induction Motor

According to double revolving field theory, it is assumed that the single-phase induction motor consists of one stator winding and two imaginary rotor windings, where one winding is rotating in forward direction or in the direction of the rotating magnetic field with slip, s, and other winding is rotating in backward direction with slip $(2 - s)$. The equivalent circuit of a single-phase induction motor is obtained by considering core loss and without core loss.

4.19.1 Without Core Loss

Consider that R_2 and X_2 are the total rotor resistance and the rotor reactance with reference to the stator. Since the rotor has two windings, the rotor resistance of forward and backward field rotors is r_2/s and $r_2/(2 - s)$ respectively. Similarly, the rotor reactance of forward and backward field rotors is $x_2/2$, since the slip has no effect on the rotor reactance. Consider that X_o is the total magnetising reactance of the induction motor, such that $x_o/2$ is the magnetising reactance. The equivalent circuit of a single-phase induction motor without core loss is shown in Figure 4.52.

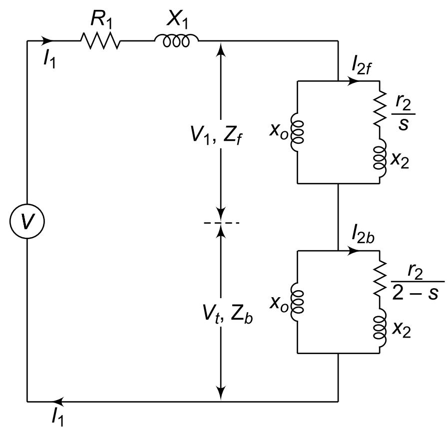


Figure 4.52 Equivalent Circuit Without Core Loss

The equations governing the single-phase induction motor represented in Figure 4.52 are given below:

Impedance of forward rotor, $Z_f = \dfrac{jx_o \times \left(\dfrac{r_2}{s} + jx_2\right)}{\dfrac{r_2}{s} + j(x_o + x_2)}$

Impedance of backward rotor, $Z_b = \dfrac{jx_o \times \left(\dfrac{r_2}{2-s} + jx_2\right)}{\dfrac{r_2}{2-s} + j(x_o + x_2)}$

Equivalent impedance of rotor, $Z_{eq} = Z_f + Z_b$
Voltage across the forward rotor, $V_f = I_f Z_f$
Voltage across the backward rotor, $V_b = I_b Z_b$

Current through the forward rotor, $I_{2f} = \dfrac{I_1 \times Z_f}{\left(\dfrac{r_2}{s} + jx_2\right)}$

Current through the backward rotor, $I_{2b} = \dfrac{I_1 \times Z_b}{\left(\dfrac{r_2}{2-s} + jx_2\right)}$

Power input to the forward rotor, $P_{2f} = \dfrac{I_{2f}^2}{\left(\dfrac{r_2}{s}\right)}$

Power input to the backward rotor, $P_{2b} = \dfrac{I_{2b}^2}{\left(\dfrac{r_2}{2-s}\right)}$

Net power input to the rotor, $P_{2n} = P_{2f} - P_{2b}$
Mechanical power developed in rotor, $P_m = (1 - s)\, P_{2n}$
Power output of the motor, $P_{out} = P_m$ − mechanical loss − core loss

Forward torque, $T_f = \dfrac{P_f}{\left(\dfrac{2\pi N}{60}\right)}$

Backward torque, $T_b = \dfrac{P_b}{\left(\dfrac{2\pi N}{60}\right)}$

Net torque developed in the rotor, $T_n = T_f - T_b$

Therefore, the shaft torque available at the shaft, $T_{sh} = \dfrac{P_{out}}{\left(\dfrac{2\pi N}{60}\right)}$

The above equations are obtained for a single-phase induction motor, by considering the three-phase induction motor concept.

4.19.2 With Core Loss

The equivalent circuit of a single-phase induction motor with core loss is shown in Figure 4.53. Here, the core loss is connected in parallel to the magnetising reactance. Therefore, the equations listed in section 4.19.1 are applicable to this circuit, except the impedance of the forward and backward field rotor changes, as given below:

Equivalent impedance of exciting branch in forward and field rotor, $Z_0 = r_o \parallel jx_o$.

Therefore, $Z_f = Z_0 \parallel \left(\dfrac{r_2}{s} + jx_2\right)$ and $Z_b = Z_0 \parallel \left(\dfrac{r_2}{2-s} + jx_2\right)$.

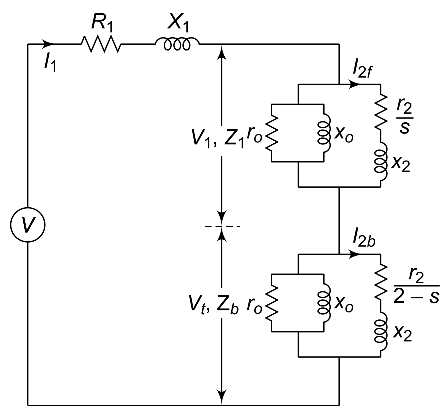


Figure 4.53 Equivalent Circuit With Core Loss

4.20 Alternator or a Three-Phase AC Generator

The AC machine, which generates an alternating emf, is called a three-phase AC generator or an alternator. Since the alternator rotates at a synchronous speed, N_s, it is also called a synchronous generator. In general, an alternator is of three-phase type, as a three-phase power system has more advantages when compared to a single-phase power system. This generator is mainly used to generate large power at the power stations.

4.20.1 Working Principle

An alternator generates the emf using the principle of electromagnetic induction, which states that, if a stationary conductor is placed in a moving magnetic field, an emf is induced in it, as shown in Figure 4.54.

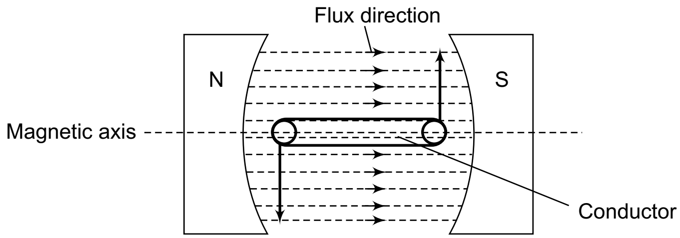


Figure 4.54 Principle of an Alternator

4.20.2 Construction of an Alternator **[AU Nov/Dec, 2010; April/May, 2009]**

Similar to an induction motor, the two important parts of an alternator are the stator and the rotor, where the stationary part of the machine that carries the armature winding is called a stator and the rotating part of the machine that produces the field is called a rotor. Here, the output or the induced emf is generated in the stator, whereas the main field required to generate the induced emf is produced at the rotor. The construction of an AC generator or an alternator is shown in Figure 4.55.

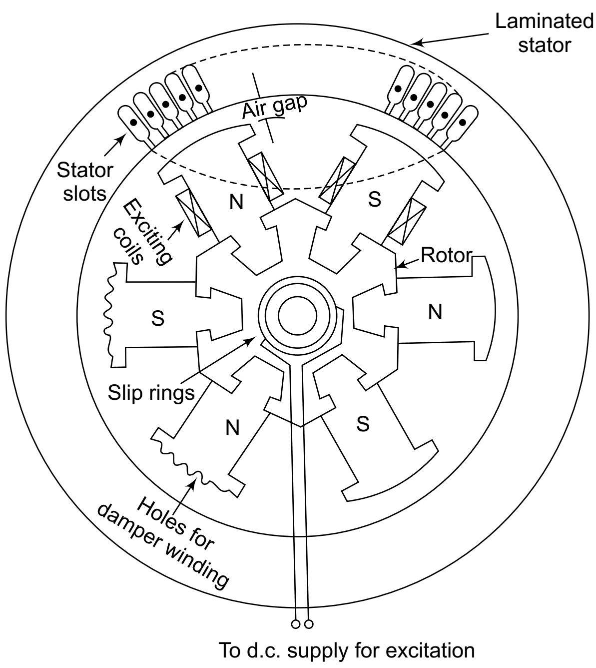


Figure 4.55 Schematic Diagram of an Alternator

Stator

The stator is the stationary part of the alternator and it consists of different parts like stator frame, stator core and stator windings. Stator frame is the outer cover of the alternator, made up of a cast iron or mild steel-frame and helps in protecting the inner parts of the alternator and supports the stator core. In addition, it provides a closed path for the magnetic flux to pass through stator windings. The stator or armature core is made of laminated steel or magnetic iron sheets. The armature core is slotted in the inner periphery to accommodate the stator or armature winding. It also provides a path for the magnetic flux. The armature core is laminated to reduce the constant losses in the alternator. The three-phase balanced star-connected stator or armature winding may be single layered or double layered. Since the windings are balanced, the number of turns and the size of wire used in the windings should be the same. The induced emf in the alternator is brought out of the alternator using the stator windings. Ventilation is provided using the holes in the stator frame. The stator construction of an alternator is shown in Figure 4.56.

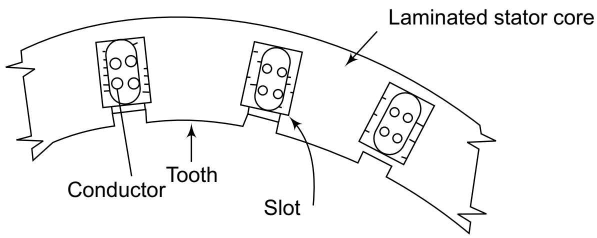


Figure 4.56 Construction of a Stator

Rotor

The rotating part of the alternator is called a rotor and it is like a flywheel with alternate *N* and S-pole electromagnets on it. These electromagnets are magnetised using DC excitation. Since the rotor is rotating, the excitation voltage is given through slip rings and brushes, which are fixed on the frame. The two different types of rotors used in an alternator are:

Salient or Projecting Pole

This type of rotor consists of even number of heavy-iron poles projecting from the rotor core surface. The typical construction of a salient pole rotor is shown in Figure 4.57.

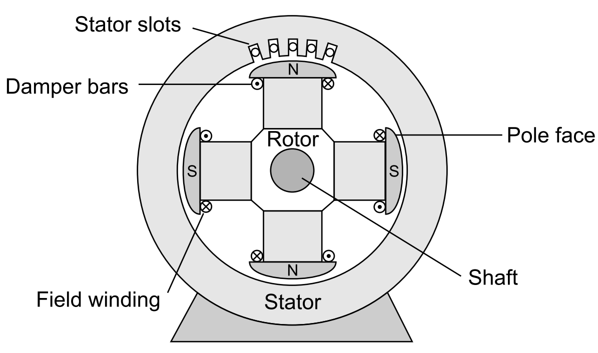


Figure 4.57 Salient or Projecting Pole Rotor

It is primarily used in alternators that rotate at low and medium speeds. This type of rotor construction is used as a prime mover in hydraulic and internal combustion turbines. The axial length of the rotor is less whereas the diameter of the rotor is high. The field or rotor windings are provided on the pole face and excited by the DC supply through slip rings. Steel spider attached to the shaft helps in providing a path for the magnetic flux.

Smooth Cylindrical or non-Salient Pole

In this type, the poles are formed when the current flows through the rotor or field windings. When compared to a salient rotor, it has small rotor diameter and large axial length. It is used in driving steam turbines or turbo alternators that rotate at high speeds. It is made up of cast iron and is cylindrical in shape. The rotor or field windings are placed in the slots provided at the outer periphery and wound in such a way that alternate *N* and *S* poles are formed when it is excited. A uniform air-gap is maintained between the stator and the rotor. The construction of a smooth cylindrical rotor is shown in Figure 4.58.

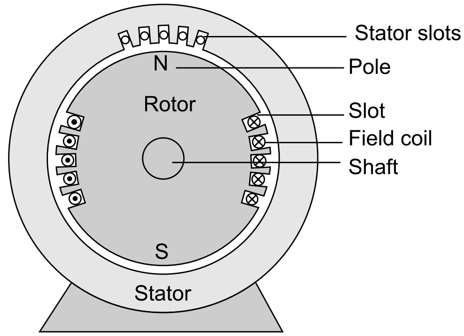


Figure 4.58 Smooth Cylindrical Rotor

Comparison Between Salient and Non-salient Pole Rotors

The comparison between salient and non-salient pole motors is given in Table 4.6

Table 4.6 *Comparison Between Salient and Non-salient Pole Rotors*

Salient Pole Rotor	Non-salient Pole Rotor
Also called projecting-pole rotor	Also called smooth cylindrical rotor
Consists of projecting poles	Consists of cylindrical type poles
Non-uniform gap between rotor and stator	Uniform air gap between rotor and stator
Rotor diameter is high	Rotor diameter is less
Axial length of the rotor is less	Axial length of the rotor is high
Mainly used in low or medium-speed alternators	Mainly used in high-speed alternators
Damper windings are provided in the slots in the poles	Damper winding is absent
Rating of the alternator is less	Rating of the alternator is more

4.20.3 Rotating Field Windings vs. Rotating Armature

In the above construction of an alternator, the field windings are rotating and the armature conductors are stationary. It is possible to construct an alternator with rotating armature and stationary field-windings. But this type of construction is not practically used due to the following advantages of rotating field and stationary armature.

1. More space can be provided to accommodate conductors that help in generating very high voltages.
2. Easier to protect the conductors if armature is stationary.
3. Connecting the stationary armature to the load is easier and it avoids the usage of slip ring and brush assembly. But, slip ring and brushes are required to excite the rotor windings.
4. Sparking problems at the slip rings can be avoided.
5. Since the field system has low inertia, it is easier to be rotated instead of rotating the high-inertia armature conductors.
6. Overall construction of the machine becomes simple. Also, for a given alternator size, the output is high when compared to an alternator with rotating armature and stationary field.
7. Initial cost is less when compared to rotating armature and stationary-field alternator because the number of brushes and slip rings required is less.
8. Good ventilation system can be provided if the armature conductors are stationary.

Terminologies used in Windings

The different terminologies related to windings are:

- ***Conductor:*** The armature conductor placed in an armature slot experiences the influence of magnetic field. The total number of conductors is denoted by Z.
- ***Turn:*** When two different conductors placed in different armature slots are connected, it forms a turn, as shown in Figure 4.59 (a). Therefore, $Z = 2 \times$ Number of turns.
- ***Coil:*** When the turns are grouped together, it forms a coil, as shown in Figure 4.59 (b). If the number of coils is N_C and the number of turns per coil is N_T, then the total number of turns and total number of conductors in the alternator can be determined.
- ***Pole pitch:*** It is defined as the distance between two adjacent poles. It is given by the ratio of armature slots to the number of poles in the alternator.

$$\text{Pole pitch, } n = \frac{\text{Number of slots}}{\text{Number of poles}}$$

- ***Coil side:*** The conductor group on one side forms one coil-side and the conductor group on the other side forms the second coil-side, as shown in Figure 4.59 (c).

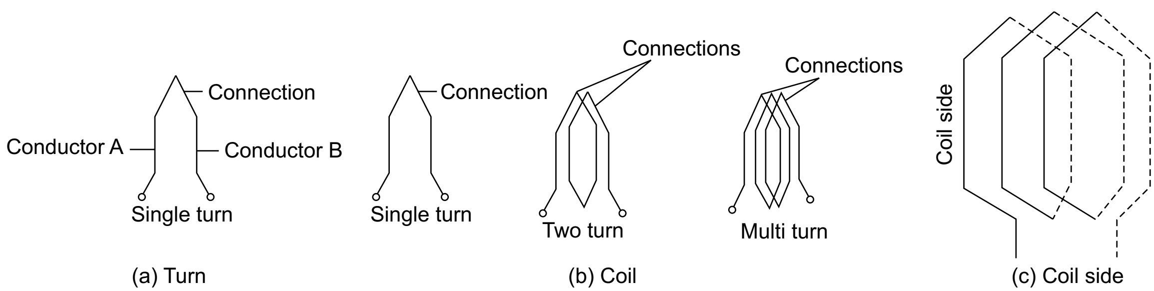


Figure 4.59 Terminologies Used in Windings

- ***Coil span or coil pitch:*** It is defined as the distance between two coil sides, which are joined together to form a coil. Based on the relation between coil span and pole pitch, the coils are classified as: (a) full-pitch coil, if coil span and pole pitch are equal and (b) short-chorded or short-pitched coil, if coil span is less than the pole pitch.
- ***Slot angle, β:*** It is defined as the phase difference in electrical degrees contributed by one slot. It is given by

$$\beta = \frac{180^\circ}{n}$$

It is noted that there is a relation between the mechanical and electrical degrees, which is given by

$$1^\circ \text{ mechanical} = \left(\frac{P}{2}\right)^\circ \text{ electrical}$$

Types of Armature Winding

- ***Single and double-layer winding:*** If one coil side is placed in an armature slot, then it is called a single-layer winding and if two coil sides are placed in an armature slot, it is called a double-layer winding.

- ***Full-pitch and short-pitch winding:*** If a coil side in a slot is connected to the other coil side, which is placed at a distance of one pole-pitch, then it is called a full-pitch winding. If the distance between two coil sides, placed in two different slots, is less than one pole-pitch, then it is called a short-pitch winding.
- ***Concentrated and distributed winding:*** If the conductors or coils belonging to a phase are placed in one slot placed under one pole, it is called a concentrated winding. If the conductors or coils belonging to a phase are placed in many slots placed under one pole, it is called a distributed winding.

4.20.4 Working of Alternator **[AU April/May, 2013]**

When the field windings are energised using a DC supply through slip rings, alternate N and S poles are generated in the case of a smooth rotor and hence magnetic flux is produced. The prime movers are used to rotate the rotor and field windings. As the rotor rotates, the stationary armature conductors are cut by the magnetic flux. Due to the principle of electromagnetic induction, an emf is induced in the stator conductors. Since the rotor poles are alternative in nature, the induced emfs in the stator conductors are also alternating in nature and their directions are given by Fleming's rule. The frequency of the induced emf depends on the number of N and S poles that the armature conductors pass in one second. Therefore, the frequency of induced emf in the stationary armature conductors is given by

$$f = \frac{PN_s}{120} \tag{4.64}$$

4.21 EMF Equation of an Alternator **[AU April/May, 2011; Nov/Dec, 2007]**

In an alternator, when the stationary armature conductor cuts the magnetic flux generated by the rotor, which is rotated using the prime mover, an emf is induced in the armature conductor. This induced emf is called generated emf, denoted by E_{ph}. The derivation of E_{ph} is obtained as follows:

Consider that P is the total number of poles of the alternator, ϕ is the flux produced per pole in Weber, Z is the total number of armature conductors, N_{sym} is the rotor speed or synchronous speed in rpm and Z_{ph} is the number of armature conductors per phase, connected in series.

In one revolution of armature core, the total flux cut by one conductor of the armature is given by

$$\phi_T = P\phi \tag{4.65}$$

The time taken by the rotor to complete one revolution is given by

$$t = \frac{60}{N_{\text{syn}}} \tag{4.66}$$

According to Faraday's law, the average emf induced in one armature conductor is given by

$$e_{\text{ph}} = \frac{\phi_T}{t} \tag{4.67}$$

Substituting Eqns. (4.65) and (4.66) in the above equation, we get

$$e_{\text{ph}} = \frac{P\phi N_{\text{syn}}}{60} \tag{4.68}$$

Using Eqn. (4.64), we get

$$e_{\text{ph}} = 2\,f\phi$$

Therefore, the emf induced per turn is given by

$$e_{T\text{ph}} = 2 \times e_{\text{ph}}$$

i.e., $$e_{T\text{ph}} = 4\,f\phi$$

If T_{ph} is the total number of turns connected in series such that $T_{\text{ph}} = \dfrac{Z_{\text{ph}}}{2}$. Then, the net average emf nduced per phase is given by

$$(E_{\text{ph}})_{\text{avg}} = 4\,f\phi\,T_{\text{ph}}$$

In an AC circuit, the RMS value of the induced emf is given by

$$E_{\text{ph}} = 1.11 \times (E_{\text{ph}})_{\text{avg}}$$

i.e., $$E_{\text{ph}} = 4.44\,f\phi T_{\text{ph}} \tag{4.69}$$

which is the basic induced emf per phase of an alternator.
Therefore, the generalised emf equation of an alternator is given by

$$E_{\text{ph}} = 4.44\,K_C K_d f\phi T_{\text{ph}}$$

where, K_C is the pitch coil factor and K_d is the distribution factor.

4.21.1 Pitch-coil factor, K_C

It is given by the ratio of emf induced when the coil is short-pitched to the emf induced when the coil is full-pitched. It is denoted by K_C, which is always less than one. It is represented by

$$K_C = \cos\left(\frac{\alpha}{2}\right)$$

where m is the short-pitch angle i.e., the angle by which the coil is short-pitched.

4.21.2 Distribution factor, K_d

The factor by which the emf induced in the armature gets reduced due to coil distribution is called distribution factor. It is denoted by $\boldsymbol{K_d}$, which is always less than one. It is represented by

$$K_d = \frac{\sin\left(\dfrac{m\beta}{2}\right)}{m\sin\left(\dfrac{\beta}{2}\right)}$$

Where, m is the number of slots per pole per phase and β is the slot angle as given by $\beta = \dfrac{180^\circ}{n}$, where n is the number of slots per pole.

It is noted that for a full-pitch coil and concentrated winding, $K_C = 1$ and $K_d = 1$.

Note: If the armature winding is star-connected, the line voltage of the alternator is given by

$$E_L = \sqrt{3}E_{\text{ph}}$$

Similarly, if the armature winding is delta-connected, the line voltage of the alternator is given by

$$E_L = E_{\text{ph}}$$

4.22 Parameters of Armature Winding

The different parameters of an armature winding placed in stator slots are:

- Armature resistance, R_a
- Armature leakage reactance, X_a
- Armature reaction reactance, X_{ar}

Armature Resistance R_a

The armature winding, placed in the armature slots of the stator, has its own resistance called armature resistance, R_a. For each phase, the effective armature resistance is denoted by $R_{a\text{ph}}$ or R_a Ω/ph. It is known that the armature winding in an alternator can be connected either in star or delta connection. If R_{RY} is the armature resistance between two phases, then the armature resistance is given by

$$R_a = \frac{R_{RY}}{2}\ \Omega/\text{ph} \quad \text{for star-connection}$$

and
$$R_a = \frac{3}{2} R_{RY}\ \Omega/\text{ph} \quad \text{for delta-connection}$$

Armature Leakage Reactance, X_a

Here, an emf is induced in the armature conductor when the magnetic flux is cut by it. When a closed circuit is provided, the current starts flowing through the armature and hence an armature flux is produced. It is clear that some part of the armature flux completes its path through the air gap and this flux is called leakage flux, which makes the armature winding more inductive. Hence, the armature winding, in addition to R_a, possesses a leakage reactance, X_a, where $X_a >> R_a$. If L is the leakage inductance per phase of the armature winding per phase, then leakage reactance per phase, X_a is given by $X_a = 2\pi f L$ Ω/ph.

Armature Reaction Reactance, X_{ar}

The interaction between the armature flux and main flux is called armature reaction. This armature reaction depends not only on the current flowing through the armature winding but also on the power factor of the load connected to it. The detailed analysis of the armature reaction in an alternator using a phasor diagram and waveform is explained, by assuming the following points:

- The main flux, ϕ_m, is taken as the reference point.
- The emf induced in the armature, E_{ph}, lags the main flux ϕ_m by 90°.
- Due to E_{ph}, a current called armature current I_o is induced in it.
- The angle between E_{ph} and I_a depends on the power factor of the load i.e., lagging or leading or unity power-factor.
- Due to the current I_a, a flux called armature flux is produced, which is in phase with the armature current, I_a.

Using these points, the phasor diagrams of the alternator for unity, lagging and leading power-factor loads are shown in Figures 4.60 (a) to (c) respectively.

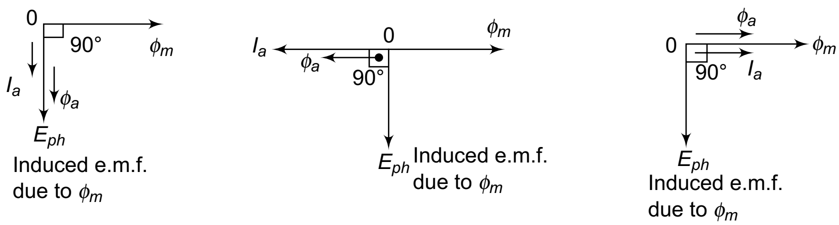


Figure 4.60 Phasor Diagrams of an Alternator for (a) Unity (b) Lagging and (c) Leading Power-factor Load

The phase difference between the main flux and armature flux is 90° for unity power-factor load, 180° for lagging power-factor load and in-phase for leading power-factor load. The main and armature flux waveforms depicting this relation for unity, lagging and leading power-factor loads are shown in Figures 4.61 (a) to (c) respectively.

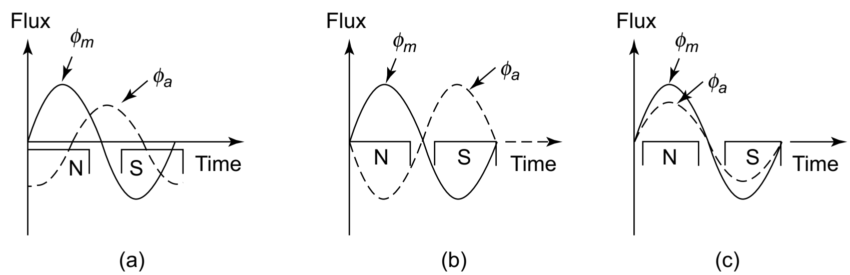


Figure 4.61 Armature and Main Flux Waveforms for (a) Unity (b) Lagging and (c) Leading Power-factor Loads

Therefore, the effects of armature flux on main field flux are:

- Distorting effect, called cross-magnetising effect, of armature reaction for unity power-factor, as it causes a drop in the terminal voltage of the alternator.
- Demagnetising effect of the armature reaction for lagging power-factor, as ϕ_a cancels the main flux. This effect causes a drop in the terminal voltage and this drop is higher when compared to the drop in unity power-factor load.
- Cross-magnetising effect of the armature reaction for leading power-factor, as ϕ_a supports the main flux. This effect increases in the terminal voltage of the alternator.

It is noted that in all the power factors, due to armature reaction, there is a change in the terminal voltage, which is considered as a drop across a fictitious reactance called armature reactance, X_{ar} Ω/ph.

4.23 Synchronous Reactance and Synchronous Impedance

Synchronous reactance of the armature winding is defined as the sum of the fictitious armature reaction reactance and the leakage reactance. It is denoted by X_s. Therefore,

$$X_s = X_a + X_{ar}\ \Omega/\text{ph}$$

Synchronous impedance of the armature winding is obtained by combining the synchronous reactance and armature resistance. It is denoted by Z_s. Therefore,

$$Z_s = R_a + jX_s\ \Omega/\text{ph}$$

4.24 Equivalent Circuit of an Alternator

The armature winding in an alternator has: (i) armature resistance, R_a, (ii) armature reactance , X_L and (iii) armature reaction reactance, X_{ar}. Therefore, if the emf induced in the armature E_{ph} gets reduced to E'_{ph} due to the drop across X_{ar}, which is due to armature reaction. When the current flows through the armature, there is a drop across R_a and X_a. Hence, the terminal voltage V is less than E'_{ph}. This concept is explained by drawing the equivalent circuit of an alternator, as shown in Figure 4.62.

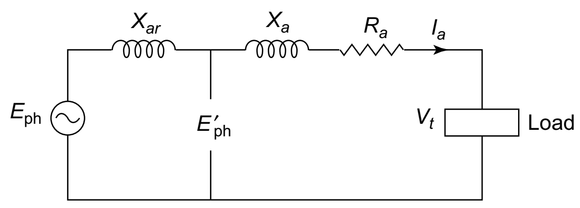


Figure 4.62 Equivalent Circuit of an Alternator

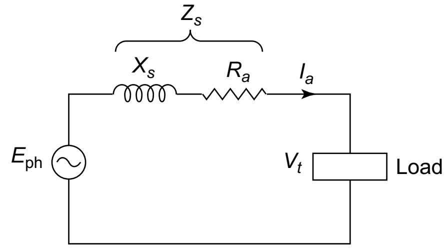


Figure 4.63 Equivalent Circuit of an Alternator with X_s and Z_s

Also, the equivalent circuit of an alternator using synchronous reactance and armature resistance is shown in Figure 4.63.

4.25 Voltage Equation of the Alternator

Since the emf induced in the alternator has to supply the terminal voltage in addition to the drop across the armature resistance, the armature leakage reactance and the armature reaction reactance, the emf induced in the alternator in vector form is given by:

$$\bar{E}_{ph} = \bar{V}_t + \overline{I_a R_a} + \overline{I_a X_a} + \overline{I_a X_{ar}}$$

i.e.,

$$\bar{E}_{ph} = \bar{V}_t + \overline{I_a R_a} + \overline{I_a X_s}$$

which is the voltage equation of the alternator.

Using this voltage equation and phasor diagram for different power-factor loads, the relation between $\bar{E}_{ph}$ and $\bar{V}_t$ is obtained as described next.

4.26 Phasor Diagram Of The Loaded Alternator

The following are the steps to draw a phasor diagram of the loaded alternator:

- Armature current, I_a, is taken as the reference phasor.
- Using the relation between V_t and I_a i.e., based on the power factor of the load, the vector V_t is drawn.
- The drop across the armature resistance R_a, is in phase with I_a and is drawn from the terminal voltage end-point.
- Then, the drop across the synchronous impedance is considered. Since it is purely inductive, the drop is drawn such that it leads the current by 90°.
- Finally, the induced emf is drawn from the origin.

Using the above points, the phasor diagrams for the lagging, leading and unity power-factor loads are drawn, as shown in Figures 4.64(a) to (c) respectively.

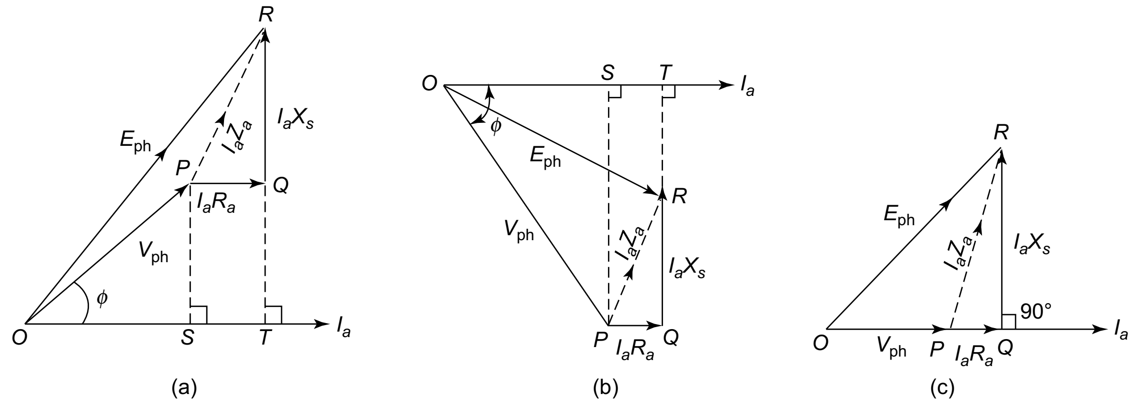


Figure 4.64 Phasor Diagrams of a Loaded Alternator (a) Lagging (b) Leading and (c) Unity Power-Factor

Using the concept of right-angled triangle, we get the generalised expression for induced emf as

$$E_{ph} = \sqrt{(V_t \cos\phi + I_a R_a)^2 + (V_{ph} \sin\phi \pm I_a X_s)^2}$$

where, (+) is for lagging power-factor load and (–) is for leading power-factor load.
Since cos $\phi = 1$ in a unity power-factor load, the expression for induced emf is given by

$$E_{ph} = \sqrt{(V_t + I_a R_a)^2 + (I_a X_s)^2}$$

4.27 Voltage Regulation of an Alternator **[AU April/May, 2013]**

The ratio of change in the terminal voltage of the alternator to the rated terminal voltage when full load is removed by keeping the excitation and motor speed as constant is called voltage regulation. It is given by

$$\%\ \text{Regulation} = \frac{V_{ph} - E_{ph}}{V_{ph}}$$

The voltage regulation depends on the armature current and also on the power factor of the load. The load characteristics of the alternator between the terminal voltage and armature current for different power factors are shown in Figure 4.65.

The different methods to obtain the voltage regulation of an alternator are:

- Direct-loading method
- Synchronous-impedance method or emf method
- Ampere-turns method or mmf method
- Zero power-factor method or potier triangle method
- ASA-modified form of mmf method
- Two-reaction theory

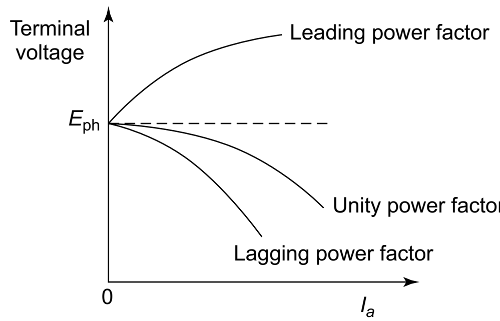


Figure 4.65 Load Characteristics of the Alternator

Example 4.18

A twelve-pole three-phase alternator, driven at a speed of 500 rpm, supplies power to an eight-pole three-phase induction motor. If the slip of the motor at full load is 3%, calculate the full-load speed of the motor.

[AU April/May, 2013]

Solution

Given, P = 12 and N_s = 500 rpm

The speed of the alternator is given by

$$N_{\text{syn}} = \frac{120f}{P}$$

Therefore,
$$f = \frac{N_{\text{syn}} \times P}{120} = \frac{500 \times 12}{120} = 50\text{Hz}$$

For an induction motor, P = 12 and s = 3% = 0.03. Therefore, the synchronous speed of an induction motor is given by

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

Therefore, the full-load speed of the motor is given by

$$N = N_{\text{syn}}(1 - s) = 750\,(1 - 0.03) = 727.5 \text{ rpm}$$

Example 4.19

Calculate the distribution factor for a four-pole 36 slots single-layer three-phase winding of an alternator.

[AU Nov/Dec, 2007]

Solution

Given, Total number of slots = 36 and P = 4.

Therefore, the number of slots per pole is given by

$$n = \frac{36}{4} = 9$$

The slot angle is given by

$$\beta = \frac{180^\circ}{n} = \frac{180^\circ}{9} = 20^\circ$$

The slots per pole per phase is given by

$$m = \frac{n}{3} = \frac{9}{3} = 3$$

Therefore, the distribution factor is given by

$$K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\left(\frac{3 \times 20^\circ}{2}\right)}{3\sin\left(\frac{20^\circ}{2}\right)} = 0.9597$$

Example 4.20

Calculate the breadth or distribution factor for a machine having 9 slots per pole in the following cases: (i) three-phase winding with 120° phase groups and (ii) three-phase winding with 60° phase groups.

[AU Nov/Dec, 2003]

Solution

Given, $n = 9$

Therefore, the slot angle is given by

$$\beta = \frac{180°}{n} = 20°$$

Case (i) The slots per pole per phase is given by

$$m = \frac{120°}{20°} = 6$$

Therefore, the distribution factor is given by

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\frac{(6\times 20°)}{2}}{6\sin\left(\frac{20°}{2}\right)} = 0.8312$$

Case (ii) The slots per pole per phase is given by

$$m = \frac{60°}{20°} = 3$$

Therefore, the distribution factor is given by

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)} = \frac{\sin\frac{(3\times 20°)}{2}}{6\sin\left(\frac{20°}{2}\right)} = 0.959$$

Example 4.21

A three-phase twelve-pole synchronous machine has a star-connected full-pitch distributed winding with 108 slots and 12 conductors per slot. The flux per pole is 50 mWb. The speed of rotation is 500 rpm. Determine the frequency, phase emf and the line emf. **[AU Nov/Dec, 2006]**

Solution

Given, $P = 12$ Number of slots = 108, Number of conductors per slot = 12, $\phi = 50$ mWb and $N_s = 500$ rpm

Since the armature is a full-pitch winding, $K_c = 1$

The synchronous speed of the alternator is given by

$$N_{\text{syn}} = \frac{120f}{p}$$

Rearranging the above equation, we get

$$f = \frac{N_{\text{syn}} \times P}{120} = \frac{500 \times 12}{120} = 50\text{Hz}$$

The number of slots per pole is given by

$$n = \frac{\text{slots}}{\text{pole}} = \frac{108}{12} = 9$$

The number of slots per pole per phase is given by

$$m = \frac{n}{3} = \frac{9}{3} = 3$$

The slot angle is given by

$$\beta = \frac{180°}{n} = \frac{180°}{9} = 20°$$

Hence, the distribution factor is given by

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \frac{\sin\frac{(3 \times 20°)}{2}}{6 \sin\left(\frac{20°}{2}\right)} = 0.9597$$

The total number of armature conductors is given by

$$Z = \text{Slots} \times \text{conductors slot} = 180 \times 12 = 1296$$

Therefore, the number of conductors per phase is given by

$$Z_{\text{ph}} = \frac{Z}{3} = 432$$

Hence, the number of turns per phase is given by

$$T_{\text{ph}} = \frac{Z_{\text{ph}}}{2} = 216$$

Therefore, the phase emf induced in the alternator is given by

$$E_{\text{ph}} = 4.44\ K_c K_d\ \phi f T_{\text{ph}} = 4.44 \times 1 \times 0.9597 \times 50 \times 10^{-3} \times 50 \times 216$$

i.e., $E_{\text{ph}} = 2300.98$ V which is the phase emf of the alternator.

Since the armature windings are star-connected, the line emf is given by

$$E_{line} = \sqrt{3} E_{\text{ph}} = 3985.408 \text{ V}$$

Example 4.22

A 250 kVA 3300 V star-connected three-phase synchronous generator has resistance and reactance per phase of 0.25 Ω and 3.5 Ω respectively. Calculate the voltage regulation at full load, 0.8 power-factor lagging.

[AU Nov/Dec, 2005]

Solution

Given, V_L = 3300 V, VA = 250 kVA, R_a = 0.25 Ω, X_s = 3.5 Ω and cos ϕ = 0.8

We know that, $VA = \sqrt{3}\, V_L\, I_L$.

Therefore, $250 \times 10^3 = \sqrt{3}\, V_L\, I_L$

Solving the above equation, we get

$$I_L = 43.7386 \text{ A}$$

Since the armature windings are star-connected, $I_{a\text{ph}} = I_L = 43.7386$ A.

Also, $V_{\text{ph}} = \dfrac{V_L}{\sqrt{3}} = 1905.2558$ V

Therefore, the emf induced in the alternator for a lagging power-factor is given by

$$E_{ph}^2 = (V_{\text{ph}} \cos\phi + I_a R_a)^2 + (V_{\text{ph}} \sin\phi + I_a X_s)^2$$
$$= (1905.2558 \times 0.8 + 43.7386 \times 0.25)^2 + (1905.2558 \times 0.6 + 43.7386 \times 3.5)^2$$

i.e., $E_{\text{ph}} = 2009 \cdot 2006$ V

Therefore, the voltage regulation at full load, 0.8 power-factor lagging is given by

$$\% R = \frac{E_{\text{ph}} - V_{\text{ph}}}{V_{\text{ph}}} \times 100 = 5.4556\%$$

Example 4.23

A 60 kVA 220 V 50 Hz single-phase alternator has an effective armature resistance of 0.016 Ω and an armature leakage reactance of 0.07 Ω. Compute the voltage induced in the armature when the alternator is delivering rated current at a load power-factor of unity and at 0.7 power-factor lagging. **[AU April/May, 2013]**

Solution

Given, VA = 60 kVA, V = 220 V, R_a = 0.016 Ω, X_s = 0.07 Ω and cos ϕ = 0.7 lagging

Since the alternator is a single-phase alternator, the values given here are considered as phase values. Therefore,

$$VA = VI_a$$
$$60 \times 10^3 = 220 \times I_a$$

i.e. $I_a = 272.72$ A

(i) When the power factor is unity i.e., cos ϕ = 1, sin ϕ = 0
Therefore, the voltage induced in the alternator is given by

$$(E_{\text{ph}})^2 = (V_{\text{ph}} \cos\phi + I_a R_a)^2 + (V_{\text{ph}} \sin\phi + I_a X_s)^2$$
$$= (220 \times 1 + 272.72 \times 0.016)^2 + (0 + 272.72 \times 0.07)^2$$

i.e., $E_{\text{ph}} = 225.174$ V

(ii) When the power factor is 0.7 i.e., cos ϕ = 0.7 lag, sin ϕ = 0.7141
Therefore, the voltage induced in the alternator is given by

$$(E_{\text{ph}})^2 = (V_{\text{ph}} \cos\phi + I_a R_a)^2 + (V_{\text{ph}} \sin\phi + I_a X_s)^2$$
$$= (220 \times 0.7 + 272.72 \times 0.016)^2 + (220 \times 0.7141 + 272.72 \times 0.07)^2 = 56122.75$$

i.e., $E_{\text{ph}} = 236.9024$ V.

Example 4.24

A four-pole 50 Hz star-connected alternator has 15 slots per pole and each slot has 10 conductors. All the conductors of each phase are connected in series and the winding factor is 0.95. When running on no load for a certain flux per pole, the terminal emf was 1825 V. If the windings are lap-connected like in a DC generator, what will be the emf between the brushes for the same speed and flux per pole? Assume sinusoidal distribution of flux.

Solution

Given, $P = 4, f = 50$ Hz, $k_d = 0.95$, $k_c = 1$, Number of slots per pole = 15, Number of conductors per slot = 10 and $E_L = 1825$ V

Therefore, the phase voltage of the induced emf is

$$E_{\text{ph}} = \frac{E_L}{\sqrt{3}} = \frac{1825}{\sqrt{3}} = 1053.69 \text{ V}$$

The total number of slots in the alternator = 15 × 4 = 60
Therefore, the number of slots per phase is given by

$$\text{Number of slots per phase } = \frac{60}{3} = 20$$

The number of turns per slot = $\dfrac{10}{2} = 5$

Therefore, number of turns per phase is

$$T_{\text{ph}} = 20 \times 5 = 100$$

The generalised equation of the emf of the generator is

$$E_{\text{ph}} = 4.44\, K_d K_c f \phi T_{\text{ph}}$$

Substituting the given values, we get

$$1053.69 = 4.44 \times 0.95 \times 1 \times 50 \times \phi \times 100$$

Therefore, the flux produced per pole is given by

$$\phi = \frac{1053.69}{(4.44 \times 0.95 \times 50 \times 100)} = 49.97 \text{ mWb.}$$

The total number of armature conductors is given by

$$Z = 60 \times 10 = 600.$$

The speed of the DC generator is given by

$$N = \frac{120 f}{P} = 120 \times \frac{50}{4} = 1500 \text{ rpm.}$$

Since the windings are lap-connected, $A = P = 4$.

When it is used as a DC generator, the emf between the brushes for the same speed and flux per pole is given by

$$E_g = \left(\frac{\Phi ZN}{60}\right) \times \left(\frac{P}{A}\right) V$$

Therefore, $$E_g = \frac{49.97 \times 10^{-3} \times 600 \times 1500}{60} \times \frac{4}{4} = 750 \text{ V}.$$

4.28 Synchronous Motor

When a three-phase supply is given to the stator of a three-phase alternator, it works as a motor. Now, if an electromagnet is present in the rotating magnetic field, the electromagnet is magnetically locked with this field and rotates at the same speed. These machines are called synchronous motors. This machine produces a steady-state torque at constant speed and frequency. The rotor speed of this machine is equal to the synchronous speed of the stator magnetic field and it is used in situations where constant speed drive is required. The speed of this motor is constant irrespective of the load. Its speed changes for an instant at the time of loading and regains its original speed shortly.

4.28.1 Types of Synchronous Motors

The synchronous motors are classified into two types, namely:

1. Single-phase synchronous motors
   - Reluctance motor
   - Hysteresis motor
2. Three-phase synchronous motors

In synchronous motors, the speed of the rotor is same as the rotating magnetic field. Basically, it is a fixed-speed motor with only one speed, which is synchronous speed and there is no intermediate speed. In other words, it is in synchronism with the supply frequency. Synchronous speed is given by

$$N_{\text{syn}} = \frac{120 \times f}{P}$$

where, N_{syn} is the synchronous speed of the motor, P is the number of poles and f is the frequency.

4.28.2 Construction **[AU April/May, 2007]**

Similar to an alternator, the synchronous motor has a three-phase winding on the stator and a DC field winding on the rotor. The basic construction of a synchronous motor consists of two parts. The schematic representation of a three-phase synchronous motor is shown in Figure 4.66.

Stator

It is the stationary part of the synchronous motor and it is built with a stack of laminated steel sheets. The inner periphery of the stator consists of slots to accommodate armature winding. The armature winding is star-connected and the neutral conductor is connected to ground.

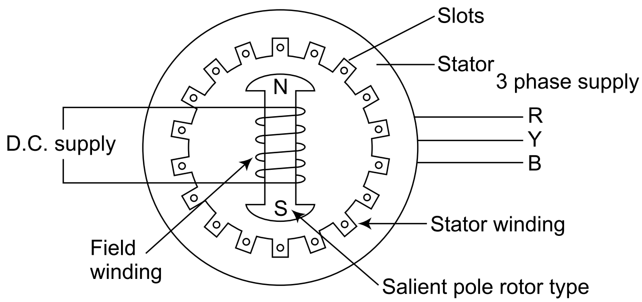


Figure 4.66 Schematic Representation of a Three-phase Synchronous Motor

Rotor

It carries a field winding and it is excited by a separate DC supply through the slip-ring arrangement. Rotor construction is classified into two types, namely:

- Salient or projected pole type
- Non-salient or cylindrical pole type

Practically, most of the motors use salient or projected pole type of construction.

The detailed description of these rotors is given in Section 4.20.2.

4.28.3 Working Principle **[AU April/May, 2007]**

It is a doubly excited machine i.e., two electrical inputs are provided to it. The three-phase stator winding is fed by a three-phase supply and the rotor is provided with a DC supply. The three-phase currents in the stator windings produce a rotating magnetic flux in the air gap at synchronous speed and the rotor carrying the DC supply produces a constant flux. The rotor has a tendency to align with the rotating field produced by the stator at all times, in order to present the path of least reluctance. Thus, if the field is rotating, the rotor will tend to rotate with the field and experiences interlocking between these two magnetic fields.

Consider a two-pole stator machine excited by an AC supply with the poles, N_s and S_s, and their positions marked at p and q respectively. The stator produces a rotating magnetic field at synchronous speed in clockwise direction, as shown in Figure 4.67(a). The rotor produces a constant magnetic field excited by a DC supply, with poles N_r and S_r, as shown in Figure 4.67(b). When two like-poles N_s and N_r, as well as S_s and S_r are brought nearer to each other, they will experience a repulsive force within each other and the rotor tends to rotate in anti-clockwise direction.

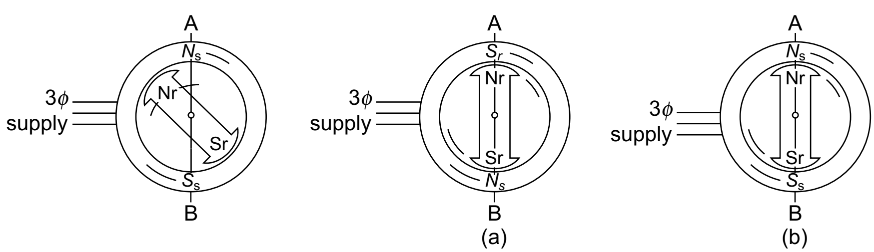


Figure 4.67 Synchronous Motor for Various Rotor Positions (a) Repulsion of Rotor in Anti-clockwise Direction (b) and (c) Attraction of Unlike Poles in Clockwise Direction

During the next half-period, the position of the stator poles gets interchanged i.e., N_s and S_s shift their positions to p and q respectively. Under these conditions, N_s attracts S_r and S_s attracts N_r. Hence, the rotor tends to rotate in clockwise direction, which is reverse of the first direction. Here, it is concluded that due to rapid and continuous interchange of stator poles, the rotor is subjected to a rapidly reversing torque. Owing to the larger rotor inertia, it cannot instantaneously respond to such quick-reversing torque. Therefore, the rotor remains in a standstill condition and the motor is not self-starting in nature.

The stator and rotor poles are attracting each other, as shown in Figure 4.67 (c). The rotor is not in standstill but rotating in clockwise direction. The rotor poles shift their positions along with the stator poles and they will experince a continuous unidirectional torque in the clockwise direction, as shown in Figure 4.67 (b).

4.28.4 Synchronous Motor: Not Self-Starting

Without energising the rotor, the synchronous motor is started and it is speeded upto synchronous speed. Upon reaching the synchronous speed, the rotor is excited by the DC source and it is magnetially locked in position with the stator. During this condition, the rotor poles get engaged with stator poles and both run at synchronous speed in the same direction. Due to this interlocking, the stator and rotor poles either run at synchronuous speed or not at all.

However, the arrangement of the stator and the rotor poles is not an absolutely rigid one. When the load on motor is increased, the rotor tends to fall back in phase by load or coupling angle α, but still continues to run at synchronous speed. The torque developed by the motor depends on this load angle, as shown in Figure 4.68.

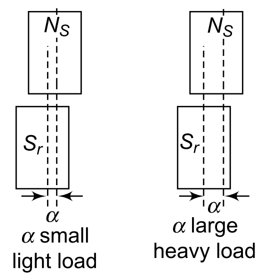


Figure 4.68 Load Angle for Light and Heavy-load Conditions

4.28.5 Procedure for Starting a Synchronous Motor **[AU April/May, 2013]**

The procedure to start a synchronous motor is as follows:

1. Field winding of the motor is short-ciruited.
2. Stator winding is applied a reduced voltage using an auto-transformer, when the motor is starting up.
3. Once the motor speed reaches a steady state, a weak DC excitation is applied by removing the short-ciruited field winding.
4. Upon reaching a sufficient excitation level, the machine will be interlocked and pulled into synchronism.
5. Rated voltage is applied across the stator terminals.
6. Based on the desired power factor, the DC excitation of the motor is adjusted.

4.29 Methods of Starting Synchronous Motors **[AU Nov/Dec, 2014]**

Synchronous motors are not self-starting in nature and it is necessary to rotate the rotor at a speed nearer to synchronous speed. Various methods to start synchronous motors are given below.

4.29.1 Using Pony Motors

A pony motor is a small induction motor used as an external device to rotate the rotor to attain synchronous speed and then the DC excitation to the rotor is switched on subsequently. After establishment of synchronism, the pony motor is decoupled from the system and the main motor continues to rotate at its synchronous speed.

4.29.2 Using Damper Winding

Along with the main field winding, an additional compensating winding consisting of copper bars is placed in the slots and these bars are short-circuited at its end rings. These windings are called damper windings. The short-circuited winding acts as a squirrel-cage rotor winding of an induction motor. The schematic representation of a damper winding is shown in Figure 4.69.

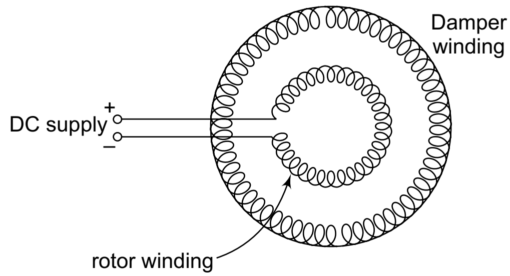


Figure 4.69 Schematic Representation of a Damper Winding

Upon excitation of the stator by a three-phase supply, the motor starts rotating as an induction motor at sub-synchronous speed and the DC supply is excited to the field winding. Once the field winding is excited, the motor is pulled into synchronism and starts rotating at its synchronous speed. During this condition, the relative motion between the damper winding and the rotating magnetic field becomes zero. Now, the motor behaves as a synchronous motor and it does not induce any emf in the damper winding. Therefore, at the time of starting, the damper winding is used to run the motor as an induction motor and afterwards, it goes out of the circuit.

4.29.3 As a Slip-ring Induction Motor

Due to the nature of squirrel-cage induction motor, the above method does not provide high starting torque at the time of starting a synchronous motor. In order to overcome the above limitation, it is designed to form a three-phase star or delta-connected winding and their terminals are brought out through slip rings.

An external rheostat is connected in series with the rotor circuit and gradually cut off once the motor speeds up. This arrangement limits the high inrush starting-current and attains a high starting-torque at the starting time. Once the motor reaches a near synchronous speed, the rotor is excited by the DC supply. The motor gets pulled into synchronism and starts rotating at synchronous speed. Then the slip rings will short-circuit the damper winding.

The schematic diagram of a slip-ring arrangement for synchronous motor starting is shown in Figure 4.70.

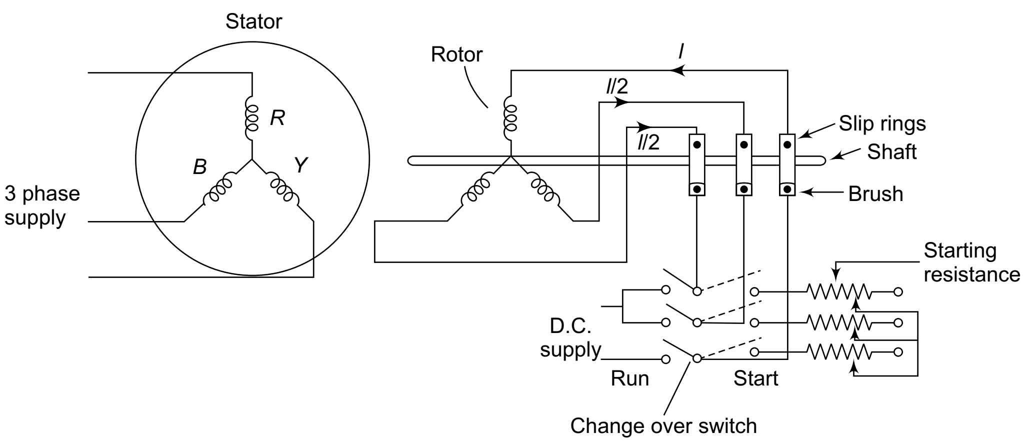


Figure 4.70 Schematic Diagram of a Slip-ring Arrangement for Synchronous Motor Starting

4.29.4 Using Small DC Machine Coupled to it

In this method, large synchronous motors are coupled with a DC motor and that is used to rotate the synchronous motors at their synchronous speed. After attaining synchronous speed, the excitation is provided to the rotor and it behaves as a synchronous motor. During this condition, the coupled DC motor acts as a DC generator (exciter). The field of the synchronous motor is then excited by this DC generator itself.

4.30 Behaviour of a Synchronous Motor on Loading [AU Nov/Dec, 2013]

It is known that, in a DC or induction motor, speed change occurs when it is loaded. But in the case of a synchronous motor, the speed remains constant irrespective of the load condition. The response of the synchronous motor when the load is changed is studied as follows:

Similar to a DC motor, the armature in synchronous motor develops an emf called back emf that opposes the supply voltage, which is given by

$$E_{b\text{ph}} = 4.44\, K_c K_d \phi f T_{\text{ph}} \tag{4.70}$$

Since the speed of the synchronous motor is always constant, the frequency remains constant. Therefore, from the above equation, it is clear that the magnitude of back emf is directly proportional to the flux ϕ produced by the rotor.

i.e., $$E_{b\text{ph}}\ \alpha \phi$$

Hence, the back emf depends on the excitation provided to the field winding. Also, it is not dependant on the speed because it is practically constant.

Since the construction of an alternator and a synchronous motor are similar, the impedance of the stator is called synchronous impedance, Z_s consisting of stator winding resistance per phase, R_a and synchronous reactance per phase, X_s. Therefore,

$$Z_s = R_a + jX_s\ \Omega$$

Therefore, the voltage equation of the synchronous motor in phasor form is given by

$$\bar{V}_{\text{ph}} = \bar{E}_{b\text{ph}} + \bar{I}_{a\text{ph}} \bar{Z}_s$$

i.e., $$\bar{I}_{a\text{ph}} = \frac{\bar{V}_{\text{ph}} - \bar{E}_{b\text{ph}}}{Z_s} \tag{4.71}$$

where, V_{ph} is the supply voltage per phase.

4.30.1 Ideal Condition on No-Load

The ideal condition on no-load of the synchronous motor is assumed by neglecting various losses. The back emf magnitude is adjusted to V_{ph} by controlling the excitation i.e., flux generated by the field winding. Therefore, at no-load condition,

$$V_{\text{ph}} = E_{b\text{ph}}$$

In this condition, magnetic locking occurs between stator and rotor so that the magnetic axes of both the stator and the rotor coincide with each other, as shown in Figure 4.71. The magnetic locking occurs only under ideal condition.

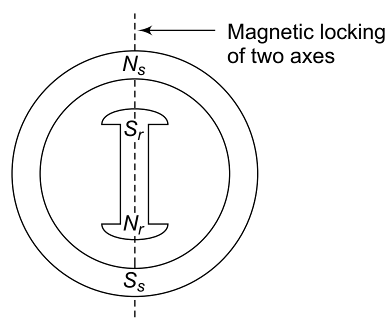


Figure 4.71 Magnetic Locking Under No-load Condition

Since $|E_{bph}| = |V_{ph}|$ and E_{bph} opposes the supply voltage, V_{ph}, the phasor diagram of this ideal case is shown in Figure 4.72.

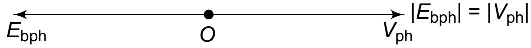


Figure 4.72 Phasor Diagram on No-load Losses

Therefore, using Eqn. (4.71), we get

$$\overline{I}_{aph} = 0$$

In practical case, the above condition is not possible since the current has to be drawn by the motor to supply constant losses.

4.30.2 Synchronous Motor on No Load (With Losses)

Due to presence of constant losses, the magnetic locking between stator and rotor occurs in such a way that a small angle difference exists between them, as shown in Figure 4.73, such that the rotor lags behind stator by a small angle, δ. This angle decides the current required to produce the torque to supply various losses.

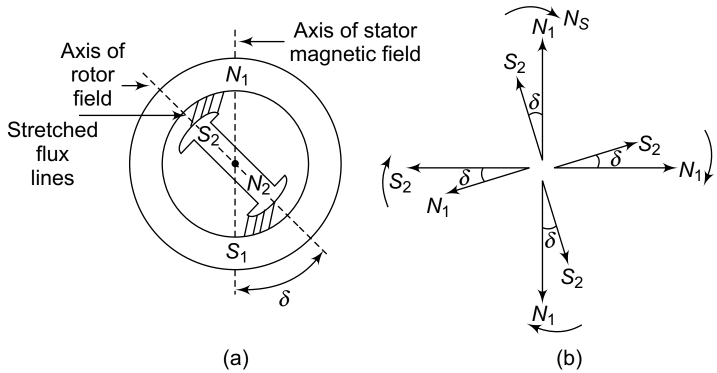


Figure 4.73 Magnetic Locking Under Practical Condition

This angle δ is called load angle or power angle or coupling angle or torque angle or angle of retardation. It is noted that, in this condition, $|E_{bph}| = |V_{bph}|$, but E_{bph} will not be located exactly opposite to V_{bph}, as it is displaced by an angle δ, as shown in Figure 4.74.

Since this vector difference is non-zero, a resultant phasor OB occurs, as shown in Figure 4.74. The resultant phasor decides the current I_{aph} to be drawn from the supply, to produce the torque to meet the constant losses. Since the constant losses are very small, the current drawn by the motor is also very small on no load.

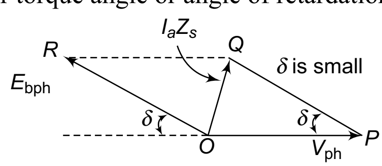


Figure 4.74 Phasor Diagram for No-Load Condition with Losses

4.30.3 Synchronous Motor on Load

When the load on the synchronous motor increases, the load angle, δ, increases but speed remains constant. As δ increases, the vector difference $\overline{V}_{ph} - \overline{E}_{bph}$ increases. Since the synchronous impedance is constant, the current, $I_{a\text{ph}}$, drawn by the motor increases as load increases and it produces the necessary torque to satisfy the increase in load.

The phasor diagrams depicting the increase in $E_{R\text{ph}}$ as load increases are shown in Figures 4.75 (a) and (b).

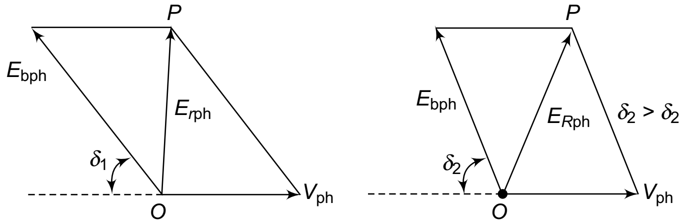


Figure 4.75 Synchronous Motor (a) Light Load and (b) Heavy Load

From the above discussion, it is clear that the torque developed in the synchronous motor depends on the load angle, δ. When the load angle δ increases, magnetic locking between rotor and stator gets weakened as the magnetic flux is increased. When δ increases to 90°, the stretched flux lines get broken and there will be no magnetic locking between the stator and rotor i.e., the motor no longer runs at synchronous speed. Hence, the torque developed at $\delta = 90°$ is the maximum torque that the synchronous motor can produce by maintaining magnetic locking between rotor and stator. The maximum torque developed is called pull-out torque and the torque–load angle relationship is shown in Figure 4.76.

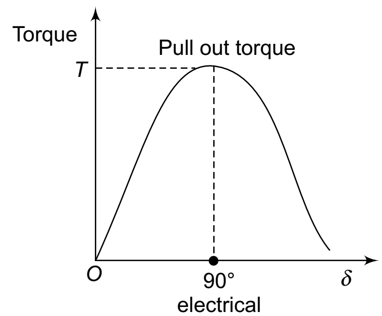


Figure 4.76 Torque–Angle Characteristic

4.31 Phasor Diagram

Consider a phasor diagram with normal excitation i.e., such that current flowing through the field winding will produce flux that will adjust magnitude of $E_{b\text{ph}}$ same as V_{ph}.

Here, δ is the load angle corresponding to the load on the motor. So, from the exact opposing position of $E_{b\text{ph}}$ with respect to V_{ph}, $E_{b\text{ph}}$ gets displaced by the angle δ.

The vector difference of $E_{b\text{ph}}$ and V_{ph} gives the phasor, which represents $I_a Z_s$ called $E_{R\text{ph}}$.

Here, $Z_s = R_a + jX_s\ \Omega$

Where, R_a is the resistance of the stator per phase and

X_s is the synchronous reactance of the stator per phase.

i.e., $$\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right)$$

and $$|Z_s| = \sqrt{R_a^2 + R_s^2}\ \Omega$$

This angle 'θ' is called internal machine angle or an impedance angle.

The significance of θ is evident from that fact that phasor I_{aph} lags behind E_{Rph} i.e., $I_a Z_s$ by an angle θ. The current always lags in case of an inductive impedance, with respect to voltage drop across that impedance. So, the phasor I_{aph} can be shown lagging with respect to E_{Rph}, by angle θ. Practically, R_a is very small compared to X_a and therefore θ tends to 90°.

Note: The power factor at which the motor is running gets decided by the angle between V_{ph} and I_{aph}, shown in Figure 4.77. This angle is denoted as ϕ and is called power-factor angle.

$$\phi = V_{ph} \wedge I_{aph} = \text{power-factor angle}$$

and cos ϕ = Power factor at which motor is operated.

The nature of this power factor is lagging if I_{aph} lags V_{ph} by angle ϕ. Whereas, it is leading if I_{aph} leads V_{ph} by angle ϕ. Its phasor diagram is shown in Figure 4.77.

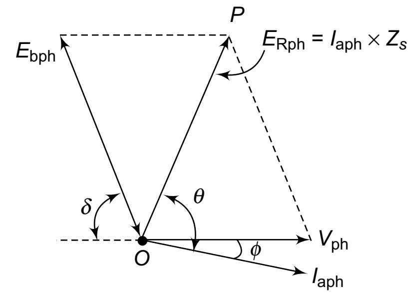


Figure 4.77 Phasor Diagram Under Normal Working Condition

4.32 V and Inverted-V Curves

The excitation of a synchronous motor varies with the magnitude of the armature current and it has a large magnitude for low and high-excitation values. The over-excited and under-excited motor runs with leading and lagging power-factors respectively. The excitation at which the magnitude of the induced emf is less than the applied voltage ($E_{bph} < V_{ph}$) is called under-excitation. Similarly, when the excitation changes in such a way that if the magnitude of the induced emf is less than the applied voltage ($E_{bph} < V_{ph}$), then it is called over-excitation.

In between the under and over-excitation values, the magnitude of current reaches minimum value I_{amin}, at unity power-factor. The excitation at this point is called critical excitation, where I_{aph} is in phase with V_{ph}.

The variation of armature current I_a, with different excitation values, is shown in Figure 4.78 (a). The excitation can be varied by changing the field current of the motor. A graph is drawn between armature current, I_a, of the motor against the field current, I_f, for various loading conditions. Since the shape of the curve looks like the alphabet 'V', these curves are called V curves of the synchronous motor.

Similarly, a collection of inverted V-curves is obtained by plotting power factor, cos ϕ against the field current, I_f for various loading conditions. Since the shape of these curves looks like an inverted 'V', these curves are called inverted-V curves, as shown in Figure 4.78 (b).

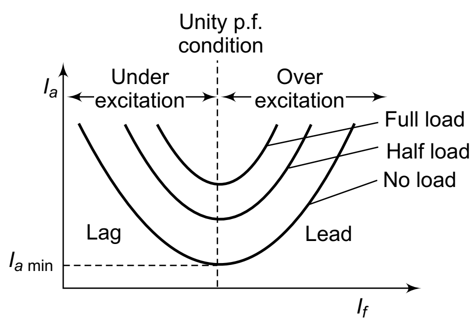


Figure 4.78(a) V-Curves for Various Loads

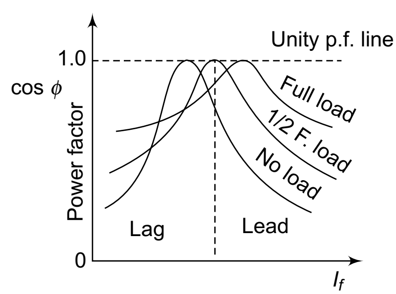


Figure 4.78 (b) Inverted V-Curves for Various Loads

4.33 Expression for Back EMF or Induced EMF Per Phase in a Synchronous Motor [AU April/May, 2012]

4.33.1 Case (i) Under-excitation, $E_{bph} < V_{ph}$

When $E_{bph} < V_{ph}$, the field excitation of motor is said to be under-excited and it has lagging power-factor. For this case,

$$Z_s = R_a + jX_s = |Z_s| \angle\theta\,\Omega$$

$$\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right)$$

$E_{Rph} \wedge I_{aph} = \theta$, I_a lags always by angle θ.

V_{ph} = Phase voltage applied

E_{Rph} = Back emf induced per phase

$E_{Rph} = I_a \times Z_s$ V per phase

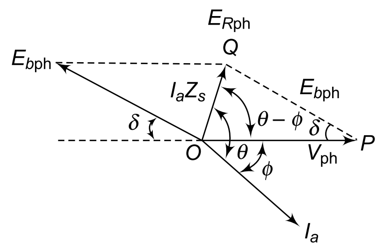


Figure 4.79 Phasor Diagram for Under-excited Condition

The power factor be cos ϕ, (lagging), as the machine is under-excited and the phasor diagram is shown in Figure 4.79.

Applying cosine rule to ΔOQP,

$$E_{bph}^2 = V_{ph}^2 + E_{Rph}^2 - 2V_{ph}\,E_{Rph}(\theta - \phi) \tag{4.72}$$

where, $E_{Rph} = I_{aph} \times Z_s$.

Appling sine rule to ΔOQR

$$\frac{E_{bph}}{\sin(\theta - \phi)} = \frac{E_{Rph}}{\sin\delta}$$

$$\sin\delta = \frac{E_{Rph}\sin(\theta - \phi)}{E_{bph}}$$

If E_{bph} is calculated, load angle δ can be determined by using sine rule.

4.33.2 Case (ii) Over-excitation, $E_{bph} > V_{ph}$

Here, power factor is leading in nature.

$E_{Rph} \wedge I_{aph} = \theta$

$V_{ph} \wedge I_{aph} = \phi$

The phasor diagram corresponding to over-excitation is shown in Figure 4.80.

Applying cosine rule to ΔOPQ,

$$E_{bph}^2 = V_{ph}^2 + E_{Rph}^2 - 2V_{ph}\,E_{Rph}\cos(\theta + \phi)$$

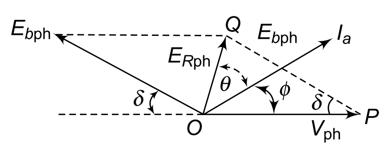


Figure 4.80 Phasor Diagram for Over-excited Condition

Since $(\theta + \phi) > 90°$, $\cos(\theta + \phi)$ becomes negative. Hence, for leading power-factor, $E_{bph} > V_{ph}$

$$\sin\delta = \frac{E_{Rph}\sin(\theta + \phi)}{E_{bph}}$$

Therefore, the load angle δ can be calculated once E_{bph} is known.

4.33.3 Case (iii) Critical excitation

In this case, $E_{bph} \approx V_{ph}$, but power factor of a synchronous motor is unity.

i.e., $\cos\phi = 1$ $\phi = 0°$

i.e., V_{ph} and I_{aph} are in phase.

and $E_{Rph} \wedge I_{aph} = \theta$

The phasor diagram corresponding to critical excitation is shown in Figure 4.81.

Where, $E_{Rph} = I_{aph} \times Z_s$

Thus, in general, the induced emf can be obtained by

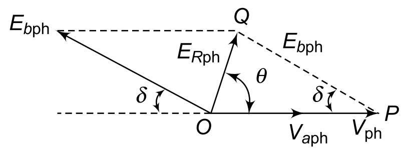


Figure 4.81 Phasor Diagram for Critical Excitation

$$E_{bph}^2 = V_{ph}^2 + E_{Rph}^2 - 2V_{ph}E_{Rph}\cos(\theta \pm \phi)$$

where, positive sign (+) indicates lagging power-factor and negative sign (–) indicates leading power-factor.

4.34 Power Flow in Synchronous Motor **[AU Nov/Dec, 2014; April/May, 2012]**

The power input to the star-connected three-phase synchronous motor is given by

$$P_{in} = \sqrt{3}V_L I_L \cos\phi = 3 \times V_{ph} I_{ph} \cos\phi$$

where, V_L is the line voltage of the supply, I_L is the line current drawn by the motor and $\cos\phi$ is the power factor of the motor.

The stator copper loss of the motor due to the per-phase resistance of the stator winding, R_a is given by

$$\text{Stator copper loss} = 3 \times I_{aph}^2 R_a$$

Where, I_{aph} is the current per phase.

Therefore, the gross mechanical power developed by the motor is given by

$$P_m = P_{in} - \text{Stator copper loss}$$

This gross mechanical power helps in developing a gross mechanical torque in the motor. Hence, the gross mechanical torque is given by

$$T_g = \frac{P_m \times 60}{2\pi N_{syn}}$$

The mechanical power developed in the rotor is given by

$$P_m = 3 \times E_{bph} \times I_{aph} \times \cos(\phi \pm \delta)$$

where E_{aph} is the back emf,

I_{aph} is the per-phase current,

cos ($\phi \pm \delta$) is the cosine of the angle between E_{bph} and I_{aph},

negative sign (–) sign for lagging power-factor and

positive sign (+) for leading power-factor and

δ is for unity power factor

Therefore, the mechanical power developed in the rotor is given by

$$P_m = 3\, E_{bph}\, I_{aph} \cos(\phi \pm \delta)$$

Hence, the net output of the motor is given by

$$P_{out} = P_m - \text{mechanical loss}$$

The net torque or shaft torque available at the shaft is given by

$$T_{sh} = \frac{P_{out} \times 60}{2\pi N_{syn}}$$

Therefore, the overall efficiency of the motor is given by

$$\%\eta = \frac{P_{out}}{P_{in}} \times 100$$

4.35 Power Developed by a Synchronous Motor

The power developed by a synchronous motor, running on leading power-factor, is obtained by considering the phasor diagram shown in Figure 4.82.

In Figure 4.82, the supply voltage per phase, V_{ph}, is represented by the line OP, armature current is I_a, back emf E_{bph} is represented by the line PQ and the resultant voltage $E_{Rph} = I_a Z_s$, is represented by line OQ. I_a leads the supply voltage V_{ph} by ϕ and lags behind E_{Rph} by an angle $\theta = \tan^{-1}(X_s/R_a)$. Line RS is drawn at angle of θ to PQ. RP and ST are perpendicular to RS.

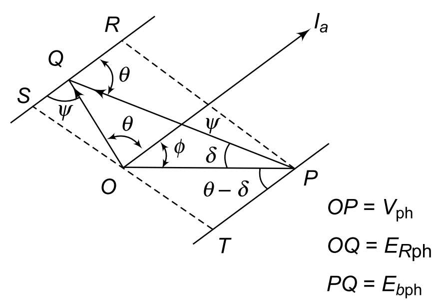


Figure 4.82 Phasor Diagram of a Synchronous Motor Running on Leading Power Factor

$OQ = E_{Rph} = I_a Z_s$, $\angle OQS = \psi$, angle between E_{bph} and I_a is ψ.

Therefore, the mechanical per-phase power developed is given by,

$$P_m = E_{bph} I_a \cos\psi \tag{4.73}$$

Using the triangle OQS, we get

$$QS = OQ \cos\psi = I_a Z_s \cos\psi,\; OS = OQ \sin\psi = I_a Z_s \sin\psi$$

$$QR = E_{bph} \cos\theta,\; PT = OP \cos(\theta - \delta) = V_{ph} \cos(\theta - \delta) \tag{4.74}$$

$$QS = RS - QR = PT - QR \tag{4.75}$$

Substituting Eqn. (4.74) in Eqn. (4.75), we get

$$I_a Z_s \cos\psi = V_{ph} \cos(\theta - \delta) - E_{bph} \cos\theta \tag{4.76}$$

Substituting Eqn. (4.76) in Eqn. (4.73), we get

$$P_m = E_{bph}\left[\frac{V_{ph}}{Z_s}\cos(\theta-\delta) - \frac{E_{bph}}{Z_s}\cos\theta\right]$$

i.e., $$P_m = \left[\frac{E_{bph}V_{ph}}{Z_s}\cos(\theta-\delta) - \frac{E_{bph}^2}{Z_s}\cos\theta\right] \tag{4.77}$$

which is the expression for the mechanical power developed in the synchronous motor.
Therefore, the gross torque developed in the synchronous motor is given by:

$$\text{Gross torque, } T_g = \frac{P_m}{\omega} = \frac{P_m}{\left(\dfrac{2\pi N_s}{60}\right)}$$

i.e., $$T_g = \frac{P_m}{\omega} = \frac{60P_m}{2\pi N_s} = \frac{9.55P_m}{N_s} \tag{4.78}$$

4.36 Comparison Between Synchronous and Induction Motors [AU April/May, 2013]

The comparison between synchronous and induction motors is given in Table 4.7.

Table 4.7 *Comparison Between Synchronous and Induction Motors*

Sl. No.	Synchronous Motor	Induction Motor
1.	Requires damper winding for self-starting.	Inherently self-starting.
2.	Separate DC source is required for motor excitation.	Rotor windings are not fed by separate source and it is excited by the induced emf.
3.	Runs at a constant speed, irrespective of the load.	Speed of the motor decreases with increase in load.
4.	By changing excitation, the motor can be operated over a wide range of power factors.	Motor always runs with lagging power-factor.
5.	Can be used as a synchronous condenser for power-factor correction.	Not possible with this motor.
7.	Very sensitive to sudden load change results in hunting.	No hunting phenomenon exists in this motor.
8.	Complex in construction and expensive.	Simple in construction , rugged, low maintenance, cheap particularly in case of squirrel-cage motor.

Appliations of Synchronous Motors

The applications of synchronous motors are:

1. *Power-factor correction*
   Over-excited synchronous motor delivers a leading power-factor. It is used as a power-factor correcting device in systems equipped with induction motors, like welding machines, flouresent lamps etc.

2. *Constant-speed application*
   Due to constant speed characteristics, high efficiency and high speed, it is used in centrifugal pumps, reciporating compressors, paper mills and metal-rolling mills etc.
3. *Voltage regulation*
   Especially for long-distance transmission lines, receiving end voltage drastically varies based on the load condition and it developes over and under-voltages. Inorder to contain these voltages within the limit, excitation of the synchronous motor is adjusted accordingly.

Example 4.25

A 6.6 kV star-connected three-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20Ω per phase. Neglecting resistance, when the input is 1000 kW, the power factor is 0.8 lagging. Determine the power factor when the input is increased to 1400 kW.

[AU April/May, 2012]

Solution

Given, $V_L = 6.6$ kV, $Z_s = j20\Omega = 20\angle 90°\ \Omega$, $\theta = 90°$, $P_{\text{in}} = 1000$ kW and $\cos\phi = 0.8$ lagging

The input power supplied to the motor is given by:

$$P_{\text{in}} = \sqrt{3}\, V_L I_L \cos\phi$$

$$1000 \times 10^3 = I_L \times 6.6 \times 10^3 \times 0.8 \times \sqrt{3} \tag{1}$$

i.e., $I_L = 109.346$ A

The phase voltage of the supply is given by

$$V_{\text{ph}} = \frac{V_L}{\sqrt{3}} = \frac{6.6 \times 10^3}{\sqrt{3}} = 3810.511 \text{ V}$$

The phase current is given by

$$I_{\text{ph}} = I_L = 109.346 \text{ A}$$

Since $\cos\phi = 0.8$, $\phi = 36.899°$, lagging.

The back emf voltage generated in the motor is given by

$$E_{b\text{ph}}^2 = V_{\text{ph}}^2 + (I_{a\text{ph}}\, Z_s)^2 - 2V_{\text{ph}}\, I_{a\text{ph}} Z_s \cos(\theta - \phi)$$

$$= (3810.511)^2 + (109.346 \times 20)^2 - 2 \times 3810.511 \times 109.346 \times 20 \times \cos(90 - 36.899)$$

$$= 9302670.8$$

i.e., $E_{b\text{ph}} = 3050.028$ V

The back emf of the motor i.e., $E_{b\text{ph}}$ remains same, because the excitation is kept constant.

Therefore, for the input power, $P_{\text{in}} = 1400$ kW and using Eqn. (1), we get

$$1400 \times 10^3 = \sqrt{3} \times 6.6 \times 10^3\, I_{L2} \cos\phi_2$$

Solving the above equation, we get

$$I_{a2} \cos\phi_2 = I_{L2} \cos\phi_2 = 122.4684 \tag{2}$$

In general, if the armature current is resolved into components, we get

$$I_{a2} = I_{a2} \cos \phi_2 \pm I_{a2} \sin \phi_2$$

Since the motor has lagging power-factor load, we get

$$I_{a2} = I_{a2} \cos \phi_2 - I_{a2} \sin \phi_2 \quad (3)$$

Substituting Eqn. (3) in Eqn. (2), we get

$$I_{a2} = 122.4684 - jI_y \quad (4)$$

The phasor diagram of the synchronous motor with lagging power-factor is shown in Figure E4.25.

Therefore, the resultant voltage is given by:

$$E_{Rph} = I_{a2}Z_s$$
$$= (122.4684 - jI_y)(0 + j20)$$
$$= 20I_y + j2449368$$

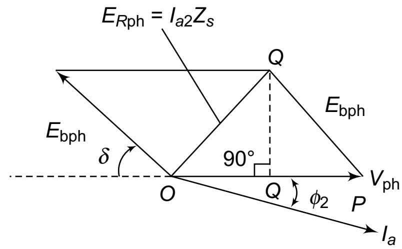


Figure E4.25

Using the above equation and Figure E4.25, we get

$$OC = x \text{ component of } E_{Rph} = 20\ I_y$$

and $\quad BC = y \text{ component of } E_{Rph} = 2449.368$

Using triangle ABC, we get:

$$AB^2 = AC^2 + BC^2$$

i.e., $\quad AB^2 = (OA - OC)^2 + BC^2$

Therefore, $E_{bph}^2 = (V_{ph} - 20\,I_y)^2 + (2449.368)^2$

i.e., $\quad (3050.028)^2 = (3810.511 - 20I_y)^2 + (2449.368)^2$

Solving the above equation, we get

$$I_y = 99.651 \text{ A} \quad (5)$$

Substituting Eqn. (5) in Eqn. (4), we get

$$I_{a2} = 122.4684 - j99.651 \text{ A} = 157.888 \angle -39.13° \text{ A}$$

Therefore, the power factor when the input is increased to 1400 kW is

$$\cos \phi_2 = \cos(-39.13°) = 0.7757 \text{ lagging}$$

Example 4.26

A salient pole star-connected synchronous motor, rated at 187 kVA, 3ϕ, 2300V, 47A, 50Hz, 187.5 rpm has an effective resistance of 1.5 Ω and a synchronous reactance of 20Ω per phase. Determine the internal power developed by the motor when it is operating at rated current and 0.8 power-factor leading.

[AU April/May, 07]

Solution

Given, $V_L = 2300$ V, $I_{aph} = 47$ A, $R_a = 1.5\ \Omega$, $X_s = 20\ \Omega$ and $\cos \phi = 0.8$ leading

The synchronous impedance of the motor is given by

$$Z_s = R_a + jX_s = 1.5 + j20\ \Omega = 20.056 \angle 85.71°\ \Omega$$

Therefore, $\theta = 85.71°$

Using $\cos\phi = 0.8$, we get

$$\phi = \cos^{-1} 0.8 = 36.8698°$$

Since the armatures winding is star-connected, the phase voltage of the supply is given by

$$V_{\text{ph}} = \frac{V_L}{\sqrt{3}} = 1327.9056 \text{ V}$$

Also, the resultant emf is given by

$$E_{R\text{ph}} = I_{a\text{ph}} \times Z_s = 47 \times 20.056 = 942.632 \text{ V}$$

The back emf of the synchronous motor is

$$E_{b\text{ph}}^2 = V_{\text{ph}}^2 + E_{R\text{ph}}^2 - 2V_{\text{ph}}\, E_{R\text{ph}} \cos(\theta + \phi)$$

Therefore,

$$E_{b\text{ph}} = \sqrt{(1327.9056)^2 + (942.632)^2 - 2 \times 1327.9056 \times 942.632 \times \cos(36.86° + 85.71°)}$$
$$= 2000 \text{ V}$$

The mechanical power developed in the motor is given by

$$P_m = \text{Input power} \quad P_{\text{in}} = \text{Armature copper losses}$$

Therefore, the internal power developed by the motor is

$$P_m = \sqrt{3}\, V_L I_L \cos\phi - 3 \times I_a^2 R_a$$
$$= \sqrt{3} \times 2300 \times 47 \times 0.8 - 3 \times 47^2 \times 1.5 = 139.8472 \text{ kW}$$

Example 4.27

The efficiency of a three-phase 400 V star-connected synchronous motor is 92% and it takes 22 A at full-load unity power-factor. Determine the back emf generated and the total mechanical power developed in kW for full load and 0.8 power-factor lagging. The synchronous impedance per phase is $(0.3 + j4)\Omega$.

[AU April/May, 2012]

Solution

Given, $V_L = 400$ V, $\eta = 92\%$, $I_{a1} = 22$ A, $\cos\phi_1 = 1$, and $\cos\phi_2 = 0.8$ lag

The synchronous impedance of the motor is

$$Z_s = 0.3 + j4\Omega = 4.0112 \angle 85.71° \ \Omega$$

Therefore, $\theta = 85.71°$.

Since the input power of the motor is same, we get

$$P_{\text{in}} = \sqrt{3}\, V_L I_{L1} \cos\phi_1 = \sqrt{3}\, V_L I_{L2} \cos\phi_2$$

i.e., $$I_{L1} \cos\phi_1 = I_{L2} \cos\phi_2$$

As the armature windings of the motor are star-connected, $I_{L1} = I_{a1}$ and $I_{L2} = I_{a2}$.

Therefore,

$$I_{a1}\cos\phi_1 = I_{a2}\cos\phi_2$$
$$22 \times 1 = I_{a2} \times 0.8$$

i.e., $$I_{a2} = 27.5\text{ A}$$

Using $\cos\phi_2 = 0.8$, we get $\phi_2 = 36.869°$
Therefore, the back emf generated in the synchronous motor is given by

$$E_{b\text{ph}}^2 = V_{\text{ph}}^2 + (I_{a2}Z_s)^2 - 2V_{\text{ph}}\,I_a Z_s\cos(\theta - \phi)$$
$$= \left(\frac{400}{\sqrt{3}}\right)^2 + (27.5 \times 4.0112)^2 - 2 \times \frac{400}{\sqrt{3}} \times 27.5 \times 4.0112 \times \cos(85.71° - 36.86°)$$
$$= 31975$$

i.e., $$E_{b\text{ph}} = 178.8156\text{ V}$$

Therefore, the line voltage of the back emf is given by

$$E_{bL} = \sqrt{3}\,E_{b\text{ph}} = 309.7177\text{ V}$$

The total copper loss of the motor is

$$P_c = 3\,I_{a2}^2\;R_a = 3 \times 27.5^2 \times 0.3 = 680.625\text{ W}$$

Hence, the mechanical power developed in the motor is

$$P_m = P_{\text{in}} - \text{Total copper loss}$$

i.e., $$P_m = \sqrt{3}V_L I_{L2}\cos\phi_2 - P_c$$

Substituting the values, we get the total mechanical power as

$$P_m = (\sqrt{3} \times 400 \times 27.5 \times 0.8) - 680.625 = 14.5614\text{ kW}$$

Example 4.28

A three-phase 1000 kVA 11000 V star-connected synchronous motor has an armature resistance and reactance per phase of 3.5 Ω and 40 Ω respectively. Determine the induced emf and angular retardation of the rotor when fully loaded, at: (i) unity power-factor (ii) 0.8 power-factor lagging and (iii) 0.8 power-factor leading.

Solution

Given, $P = 1000$ kVA, $V_L = 11000$ V, $R_a = 3.5\ \Omega$ and $X_s = 40\ \Omega$

We know that, $P = \sqrt{3}V_L I_L$. Therefore, the full-load armature current is

$$I_L = \frac{P}{\sqrt{3}\,V_L} = \frac{1000 \times 10^3}{\sqrt{3} \times 11000} = 52.5\text{ A}$$

Since the windings are star-connected, $I_a = I_L = 52.5$ A.

The phase voltage of the supply is $V_{\text{ph}} = \dfrac{V_L}{\sqrt{3}} = \dfrac{11000}{\sqrt{3}} = 6351\text{ V}$.

The drop across the armature resistance and armature reactance per phase is

$$V_{Ra} = I_a R_a = 52.5 \times 3.5 = 184 \text{ V}$$
$$V_{Xs} = I_a X_s = 52.5 \times 40 = 2100 \text{ V}$$

Hence, the drop across the armature impedance per phase is given by

$$E_{R\text{ph}} = V_{Zs} = \sqrt{V_{Ra}^2 + V_{Xs}^2} = \sqrt{184^2 + 2100^2} = 2100 \text{ V}$$

The impedance angle is given by

$$\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right) = \tan^{-1}\left(\frac{40}{3.5}\right) = 85°$$

(i) At unity power-factor:

i.e., $\cos\phi = 1$ or $\phi = 0°$.

The phasor diagram of the synchronous motor is shown in Figure E4.28(a).

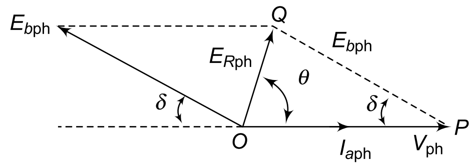


Figure E4.28(a)

The induced emf in the synchronous motor is given by

$$E_{b\text{ph}}^2 = V_{\text{ph}}^2 + E_{R\text{ph}}^2 - 2V_{\text{ph}}\,E_{R\text{ph}}\cos(\theta \pm \phi)$$

Substituting the values, we get

$$E_{b\text{ph}}^2 = (6351)^2 + (2100)^2 - (2 \times 6351 \times 2100 \times \cos 85°) = 42419169$$

i.e., $E_{b\text{ph}} = 6513$ V.

Therefore, the line voltage of the back emf induced in the motor is

$$E_{bL} = E_{b\text{ph}} \times \sqrt{3} = 6513 \times \sqrt{3} = 11280 \text{ V}$$

From ΔOPQ, we get

$$\frac{E_{b\text{ph}}}{\sin\theta} = \frac{E_{R\text{ph}}}{\sin\delta}$$

$$\sin\delta = \frac{E_{R\text{ph}}\sin\theta}{E_{b\text{ph}}}$$

i.e.,

Substituting the values, we get

$$\sin\delta = 2100 \times \frac{\sin 85°}{6513} = 0.3212$$

Therefore, the angular retardation of the rotor at unity power-factor is

$$\delta = \sin^{-1}(0.3212) = 18.4°.$$

(ii) At 0.8 power-factor lagging:

i.e., $\cos\phi = 0.8$ or $\phi = \cos^{-1}(0.8) = 36.53°$.

Therefore, the phasor diagram of the synchronous motor is shown in Figure E4.28(b).

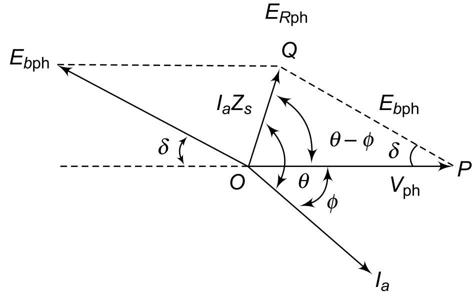


Figure E4.28(b)

The back emf generated in the synchronous motor is given by

$$E_{b\text{ph}}^2 = V_{\text{ph}}^2 + E_{R\text{ph}}^2 - 2V_{\text{ph}}\, E_{R\text{ph}} \cos(\theta - \phi)$$

Substituting the values, we get

$$E_{b\text{ph}}^2 = (6351)^2 + (2100)^2 - (2 \times 6351 \times 2100 \times \cos(85 - 36.53)°) = 26936100$$

i.e., $E_{b\text{ph}} = 5190$ V.

Therefore, the line voltage of the back emf induced in the motor is

$$E_{bL} = E_{b\text{ph}} \times \sqrt{3} = 5190 \times \sqrt{3} = 8989 \text{ V}$$

From ΔOPQ, we get

$$\frac{E_{b\text{ph}}}{\sin(\theta - \phi)} = \frac{E_{R\text{ph}}}{\sin\delta}$$

i.e.,
$$\sin\delta = \frac{E_{R\text{ph}} \sin(\theta - \phi)}{E_{b\text{ph}}}$$

Substituting the values, we get

$$\sin\delta = 2100 \times \frac{\sin(85 - 36.53)}{5190} = 0.3012$$

Therefore, the angular retardation of the rotor at power-factor 0.8 lagging is

$$\delta = \sin^{-1}(0.3012) = 17.32°.$$

(iii) At 0.8 power-factor leading:

i.e., $\cos\phi = 0.8$ or $\phi = \cos^{-1}(0.8) = 36.53°$.

Therefore, the phasor diagram of the synchronous motor is shown in Figure E4.28(c).

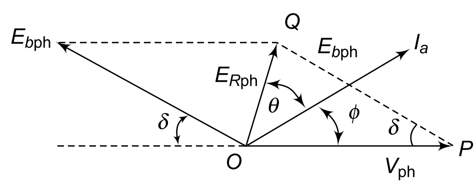


Figure E4.28(c)

The induced emf in the synchronous motor is given by

$$E_{b\text{ph}}^2 = V_{\text{ph}}^2 + E_{R\text{ph}}^2 - 2V_{\text{ph}}\, E_{R\text{ph}} \cos(\theta + \phi)$$

Substituting the values, we get

$$E_{b\text{ph}}^2 = (6351)^2 + (2100)^2 - (2 \times 6351 \times 2100 \times \cos(85 + 36.53)°) = 58828900$$

i.e., $E_{b\text{ph}} = 7670$ V.

Therefore, the line voltage of the back emf induced in the motor is

$$E_{bL} = E_{b\text{ph}} \times \sqrt{3} = 7670 \times \sqrt{3} = 13280$$

From ΔOPQ, we get

$$\frac{E_{b\text{ph}}}{\sin(\theta + \phi)} = \frac{E_{R\text{ph}}}{\sin\delta}$$

i.e.,
$$\sin\delta = \frac{E_{R\text{ph}} \sin(\theta + \phi)}{E_{b\text{ph}}}$$

Substituting the values, we get

$$\sin\delta = 2100 \times \frac{\sin(85 + 36.53)}{7670} = 0.2325$$

Hence, the angular retardation of the rotor at power-factor 0.8 leading is

$$\delta = \sin^{-1}(0.2325) = 13.27°$$

4.37 Stepper Motor

The stepper motor has gained more importance in recent years because it can be easily interfaced with digital circuits. A special type of synchronous motor designed to rotate through a specific angle for each applied electrical pulse is known as a stepper motor. The specific angle through which the stepper motor rotates is called step. The electrical pulses are received from the control unit of the stepper motor. Stepper motor is used along with electronic switching devices to switch the control windings according to the command received. The number of steps per revolution and the rate at which the pulses are applied determine the rotational rate.

The stepper motor completes a full rotation by sequencing through a series of discrete rotational steps (stepwise rotation). Each step position is an equilibrium position, so that without further excitation, the rotor position stays at the latest step. Thus, a train of input pulses, each of which causes an advance of one step, achieves continuous rotation.

Classification of Stepper Motor

The stepper motor can be classified depending on:

(i) The type of rotor:
  - (a) Variable-reluctance stepper motor
  - (b) Permanent-magnet stepper motor
  - (c) Hybrid stepper motor

(ii) The windings on the stator
  - (a) Two-phase stepper motor
  - (b) Three-phase stepper motor
  - (c) Four-phase stepper motor

4.37.1 Variable-Reluctance Stepper Motor

The variable-reluctance stepper motor has a single or several stacks of stators and rotors. The stators have a common frame, while the rotor has a common shaft.

The stator of this stepper motor has six laminated poles with exciting windings wound for three-phases. Slotted steel lamination is used in making the rotor. The number of poles on the stator and the rotor are different and it gives the variable-reluctance motor the ability to rotate in both directions and for self-starting. The stator and the rotor of this type of stepper motor have toothed structures. The longitudinal cross-sectional view of a three-stack variable-reluctance stepper motor is shown in Figure 4.83.

The difference in angular displacement of the stator and the rotor, when the teeth of the rotor are perfectly aligned, is given by

$$\alpha = \frac{360°}{nT}$$

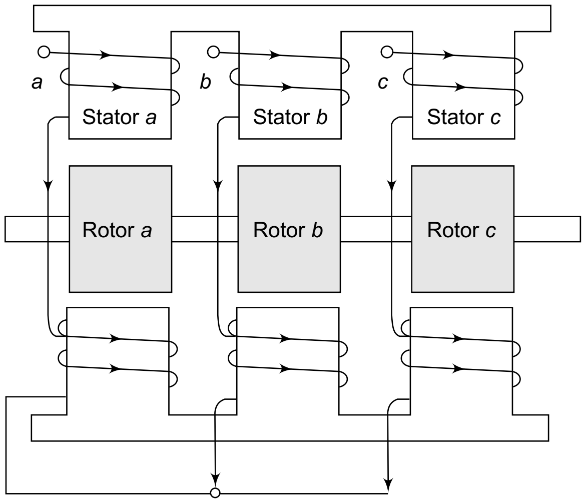


Figure 4.83 Three-stack Variable-Reluctance Stepper Motor

Where, n is the number of stacks and T is the number of rotor teeth.

The schematic diagram explaining the concept of separation between the stator and the rotor is shown in Figure 4.84.

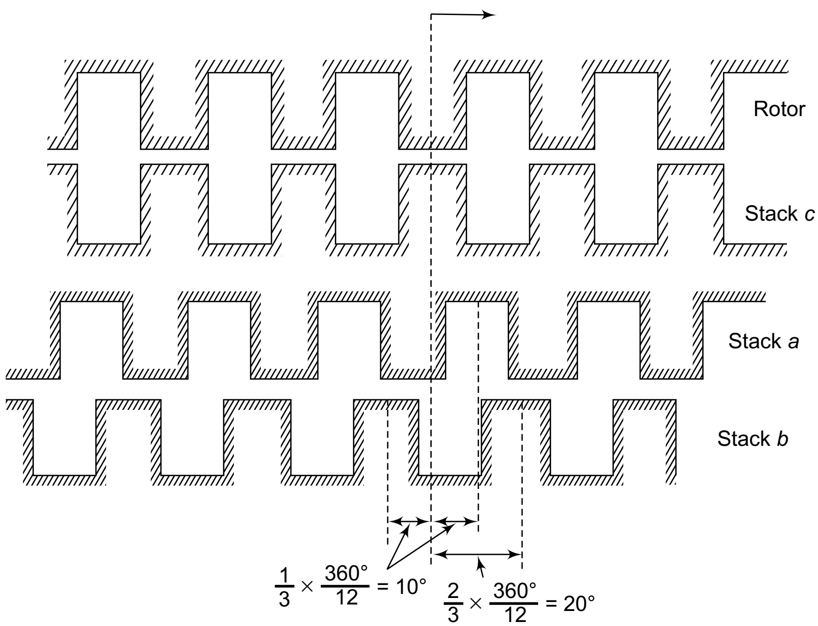


Figure 4.84 Separation Between Stator and Rotor

The schematic diagram of a variable-reluctance stepper motor and its driving circuit are shown in Figures 4.85 (a) and (b) respectively.

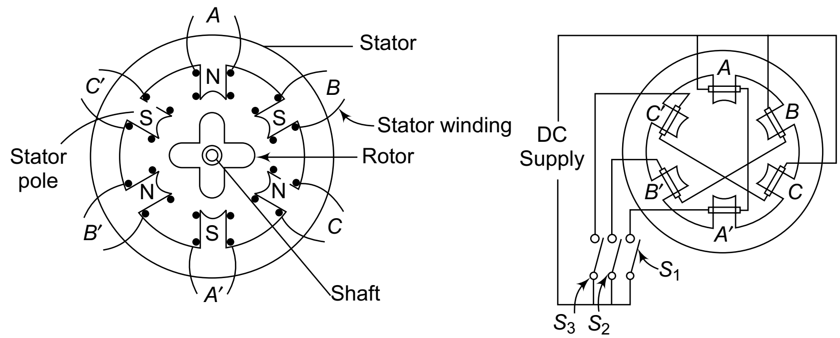


Figure 4.85 Variable-Reluctance Stepper Motor (a) Schematic Diagram (b) Driving Circuit

The working principle of this stepper motor is based on the different reluctance positions of the rotor with respect to the stator. When any one stator-phase is excited, a magnetic field whose axis lies along the poles is produced. Then the rotor rotates in a particular direction so that the reluctance position between the stator and the rotor is minimum and in such position, the magnetic field axis of the stator passes through any two rotor-poles. The working of a variable-reluctance stepper motor when the phases A, B and C are energised in this sequence, using switches S_1, S_2 and S_3, is shown in Figures 4.86 (a) to (c) respectively.

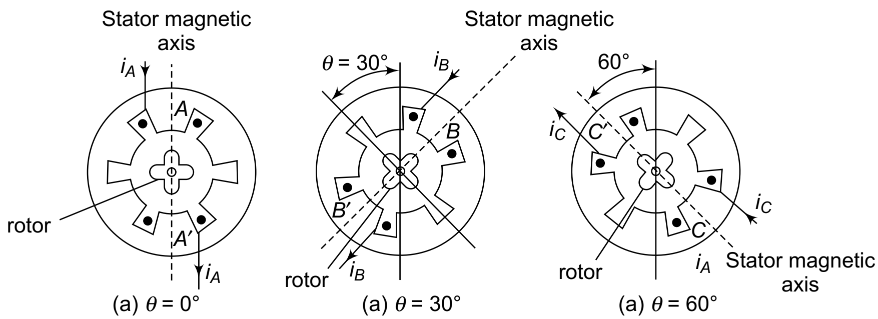


Figure 4.86 Working of a Variable-Reluctance Stepper Motor

When phase A is excited using switch S_1, a vertical magnetic axis is formed along the poles of phase A. Then the rotor rotates and adjusts itself to a minimum reluctance position i.e., rotor axis gets matched with stator magnetic axis through any two poles and this position is shown in Figure 4.86 (a). Similarly, Figures 4.86 (b) and (c) indicate the rotor position and the stator magnetic axis, when the phases B and C are energised respectively.

The torque acting on the rotor, when the current i flows through the stator and any one of the phases is energised, is given by

$$T_m = \frac{1}{2} i^2 \frac{dL}{d\theta}$$

where, L is the inductance of a phase at an angle θ.

Since the torque is proportional to the square of the phase current, the rotor rotation is independent of the direction of the current, i. But changing the sequence of the phase, which is energised using switches, can change the direction of rotation.

4.37.2 Permanent-Magnet Stepper Motor

In this type of stepper motor, the stator has salient poles, which carry control windings. A phase is created when the two control windings are connected in series. In this type, the rotor is made in the form of a spider cast integral permanent magnet or assembled permanent magnets. The schematic diagram of a permanent-magnet stepper motor and its driving circuit are shown in Figures 4.87 (a) and (b) respectively.

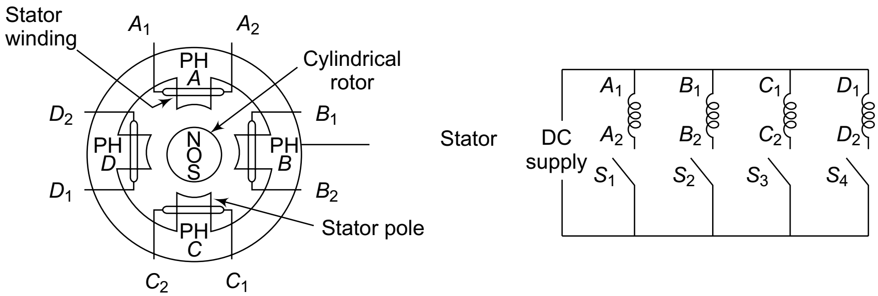


Figure 4.87 Permanent-Magnet Stepper Motor (a) Schematic Diagram (b) Driving Circuit

The working of this stepper motor is explained when the phases *A*, *B*, *C* and *D* are energised using switches S_1, S_2, S_3 and S_4, as shown in Figures 4.88 (a) to (d) respectively.

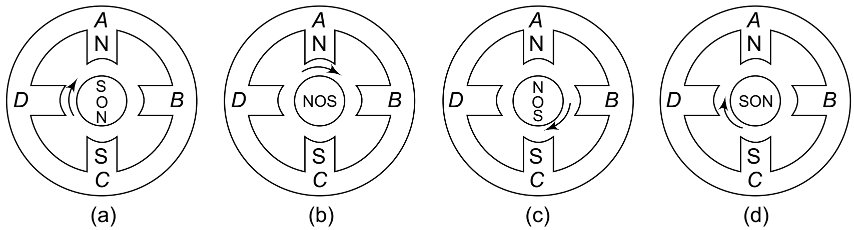


Figure 4.88 Working of a Permanent-Magnet Stepper Motor

The permanent magnet stepper motor operates at larger steps of up to 90°, at a maximum response rate of 300 pps.

4.37.3 Hybrid Stepper Motor

The hybrid stepper motor is similar to the permanent-magnet stepper motor with the constructional features of the rotor adopted from variable-reluctance stepper motor. The teeth on the stack of the rotor at both ends are of different polarities. Thus, the two sets of teeth in the rotor are displaced from each other by one-half of the tooth pitch (pole pitch). The constructional features of this type of stepper motor are shown in Figures. 4.89 (a) and (b).

4.37.4 Operation of a Stepper Motor

Depending on the step angle by which the stepper motor rotates, the operation of stepper motor can be classified into:

- Full-step operation
- Half-step operation and
- Micro-step operation

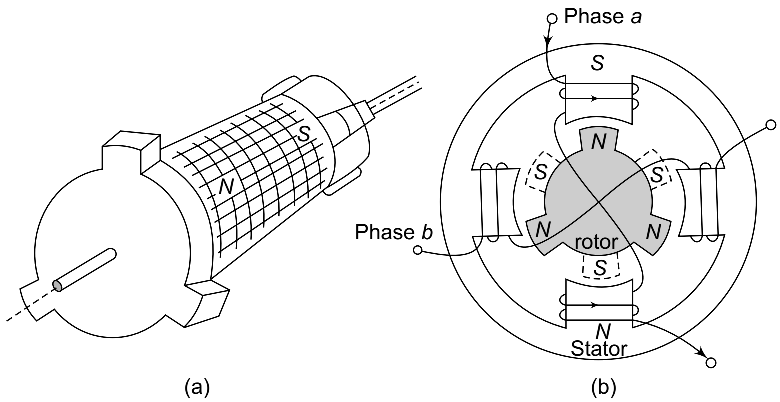


Figure 4.89 Constructional Details of a Hybrid Stepper Motor

Full-step Operation

In this type of operation, the stepper motor moves one full-step for each of the input pulses applied.

$$\text{Full step} = \frac{360^\circ}{N_R \times N_S}$$

Where, N_R is the number of rotor poles and
N_S is the number of stator pole-pairs.

Half-step Operation

In this type, the stepper motor moves one-half of the full step for each of the input pulses applied. In the full-step operation, the motor rotates at X° for each input pulse, and in the half-step operation, the motor will rotate at $\frac{X^\circ}{2}$ for each input pulse.

Micro-step Operation

In this type of operation, the stepper motor moves through angles of 1/10, 1/16, 1/32 and 1/125 of a full step. The major advantage of this type of operation is that it provides a much finer resolution.

4.37.5 Advantages and Applications of a Stepper Motor

Advantages

The advantages of stepper motors are:
1. Compatible with digital systems.
2. No sensor is required to sense the speed and position.
3. Motors are available in larger power ratings at reduced cost.
4. Motors are available with the torque range of 0.5 mNm to 100 Nm.
5. Used to upgrade mechanical systems for a greater precision and production rate.

Applications

The stepper motors are used in:

1. Computer peripherals like printers and disk-drives, X-Y plotters, scientific instruments and machine tools
2. Quartz-crystal watches
3. Many supporting roles in the manufacture of packaged foods, commercial end products and even in the production of science-fiction movies.

4.38 Brushless Direct Current (BLDC) Motor

The Brushless Direct Current (BLDC) motor is a derivative of the DC motor and it shares the same torque and speed performance characteristics. The major difference between BLDC and DC motor is the use of brushes for commutation. The DC motor assembly contains a physical commutator, which changes the motor phase at the appropriate times to produce the required torque, whereas a BLDC motor does not have brushes and electrical current powers a permanent magnet of the motor through an electronically commutated system to produce the required torque. It is highly reliable since it does not have any brushes to wear out and replace and it has longer life-expectancy, when operated at its rated conditions. Moreover, it has tremendous benefits for long-term applications.

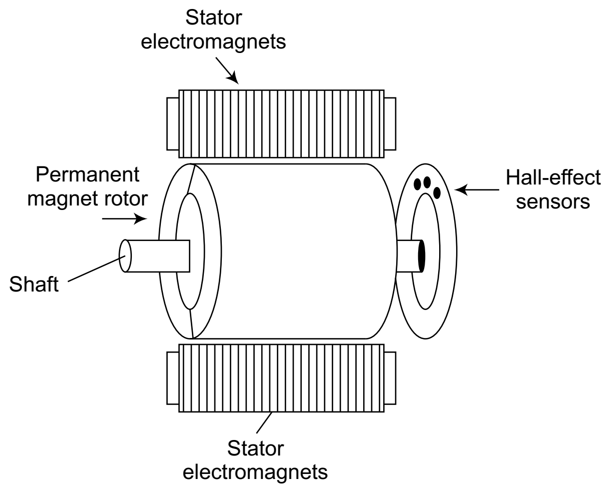
4.38.1 Construction of a BLDC Motor

The construction of a BLDC motor is similar to that of a three-phase induction motor and a conventional DC motor and it is shown in Figures 4.90(a) and (b). The motor consists of four primary parts, namely: Permanent-magnet rotor; Stator; Stator windings; and Hall-Effect sensors.

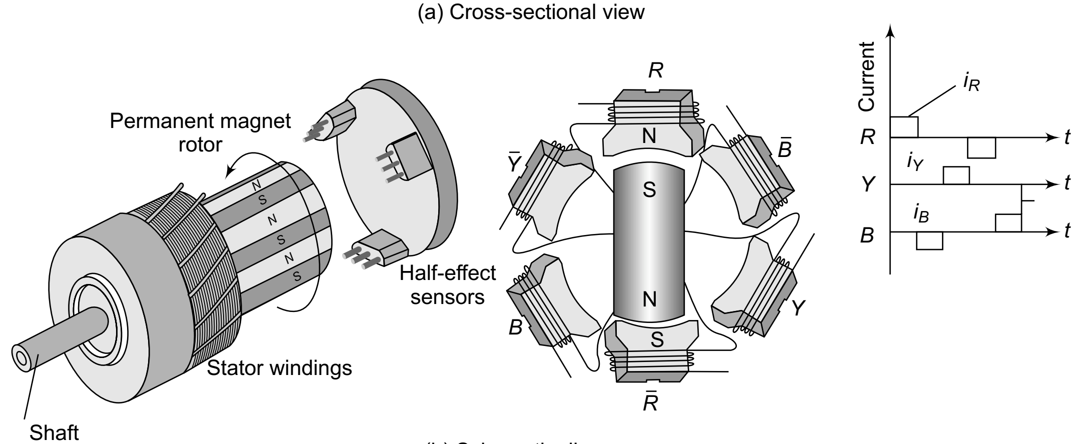
Stator electromagnets of a BLDC motor consist of stacked steel laminations to carry the stator windings. The windings are placed in slots in such a way that it axially cuts along the inner periphery of the stator. The windings are arranged in either star or delta-connected forms. However, most of the BLDC motors have three-phase star-connected stator windings. Each winding in the stator is constructed with numerous interconnected coils, where one or more coils are placed in each slot. In order to form an even number of poles, each of these stator windings is distributed over the entire stator periphery of the motor.

In a BLDC motor, the rotor is made up of a permanent magnet. Based on core configurations, the rotor is constructed as a circular core with the permanent magnet on the periphery, a circular core with rectangular magnets, etc. Depending on the application, the number of poles in the rotor can vary from 2 to 8 pole-pairs and the magnets are placed with alternate north and south poles. In order to achieve maximum torque, the flux density of the material should be high in the rotor. Rare-earth alloy magnets such as Samarium-Cobalt (SmCo), Neodymium (Nd), and Ferrite and Boron (NdFeB) are commonly used for new designs. Normally, Ferrite magnets are inexpensive but suffer from low flux-density for a given volume.

In order to synchronise the stator armature excitation with the rotor position, a Hall sensor is used. This sensor is embedded in the stator and senses the position of the rotor. Before energising a particular stator winding, acknowledgment of rotor position is necessary in this motor. Normally, three Hall sensors are embedded into the stator. Whenever the rotor poles pass near this sensor, it will generate Low and High signals. Based on the combination of this sensor's response, the excitation sequence to the stator winding is determined and energised.



(a) Cross-sectional view



(b) Schematic diagram

Figure 4.90 Construction of a BLDC Motor

4.38.2 Working of a BLDC Motor

The principle and operation of a BLDC motor is similar to that of a brushed DC motor. According to the Lorentz force law, whenever a current carrying conductor is placed in a magnetic field, it experiences a force. As a result of this force, the magnet will experience an equal and opposite force. In case of stationary current-carrying conductors, the permanent magnet moves in the motor. The stator coil becomes an electromagnet, when it is switched electrically by a supply source and starts producing a uniform magnetic field in the air gap. Even though the supply is a DC source, by electrical switching, it generates an AC voltage waveform with trapezoidal shape and in turn starts producing a uniform field in the air gap. Due to the interaction between the stator electromagnets and the permanent-magnet rotor, the rotor continues to rotate.

The working principle of a BLDC motor can be understood with the help of diagrams shown in Figures 4.91(a) and (b).

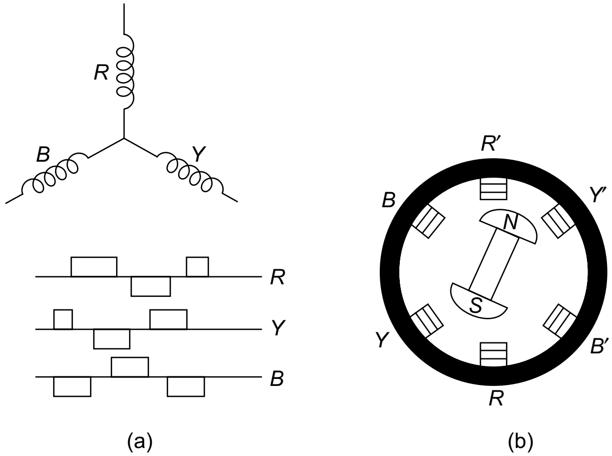


Figure 4.91 Working Principle of a BLDC Motor (a) Switching Sequence (b) Stator Electromagnets and Permanent Magnet Rotor

The stator windings of the motor are excited based on different switching sequences. Due to the switching of windings as high and low states, the corresponding winding is energised as north and south poles. The north and south poles of the permanent-magnet rotor will align with the stator poles, causing the motor to rotate. The motor produces torque because of the development of attraction forces when north–south or south–north alignment occurs. Similarly, it produces repulsion forces when north–north or south–south alignment occurs. By this way, the motor rotates in a clockwise direction based on the switching sequence applied to the coils.

4.38.3 Classification of BLDC Motors

The BLDC motors are classified based on two different parameters.

Depending on the Arrangement of the Permanent Magnet Rotor and Stator Electromagnets

Outer Rotor Design

In an outer rotor BLDC design, the windings are located in the core of the motor and the rotor permanent magnets surround the stator windings, as shown in Figure 4.92.

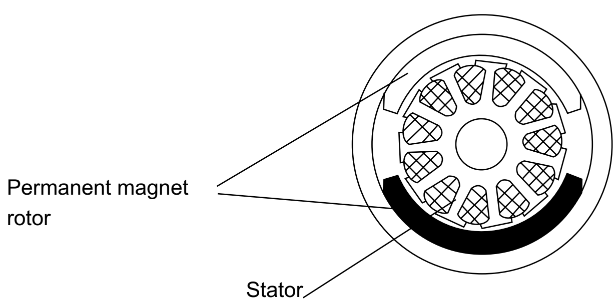


Figure 4.92 Outer Rotor Design

Here, the rotor permanent magnet acts as an insulator and thereby reduces the rate of heat dissipation from the motor. Therefore, outer rotor design motor is mainly used in lower duty cycles or lower-rated current applications. The main advantage of an outer rotor design is its motor, which offers relatively low cogging torque.

Inner Rotor Design

In an inner rotor BLDC design, the stator windings surround the permanent-magnet rotor and are affixed to the motor's housing, as shown Figure 4.93.

The main advantage of this type of rotor construction is that it is capable of dissipating heat easily. For this reason, the majority of BLDC motors use inner rotor design. Moreover, an inner rotor design offers lower rotor inertia.

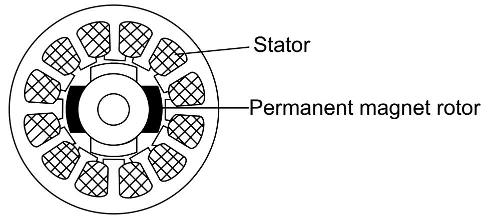


Figure 4.93 Inner Rotor Design

Based on Physical Configurations of Stator Windings, as:

(i) Single-phase motor
(ii) Two-phase motor and
(iii) Three-phase motor

4.38.4 Advantages, Disadvantages and Applications of BLDC Motor

Advantages

1. Effective operation over a wide range of speeds with its rated load current.
2. Has high efficiency due to the presence of a permanent-magnet rotor.
3. Due to the absence of brushes, it operates at high speed even in loaded and unloaded conditions.
4. It is small in size and less in weight. It has high output power-to-size ratio.
5. It has reduced size with far superior thermal characteristics.
6. Higher dynamic response due to low inertia.
7. Less electromagnetic interference
8. Higher speed operation is possible with low electric noise.
9. Less maintenance cost due to the absence of brushes.

Disadvantages

1. Requires complex drive circuitry for its operation.
2. Needs additional Hall sensors
3. Control of BLDC motor requires expensive electronic controllers.

Applications

The BLDC motor is used for a wide variety of applications, such as:

1. Computer hard-disk drives
2. Electric vehicles, hybrid vehicles, and electric bicycles
3. Industrial robots
4. Domestic appliances, such as washing machines, fans, and dryers
5. Compressors, pumps and blowers etc.

Two Marks Questions and Answers

1. **State the principle of a three-phase induction motor. [AU Nov/Dec, 2014; April/May, 2011]**
   Refer to section 4.2.1 for the principle of a three-phase induction motor.
2. **A three-phase induction motor does not run at synchronous speed. Why? [AU April/May, 2011; Nov/Dec, 2011]**
   Refer to section 4.2.8 to understand why a three-phase induction motor does not run at synchronous speed.
3. **Write down the relation between speed and frequency. [AU Nov/Dec, 2012]**
   The relation between the speed and frequency is given by

$$N_{\text{syn}} = \frac{120f}{P}$$

4. **How to reverse the direction of rotation of a three-phase induction motor? [AU April/May, 2012]**
   Refer to section 4.2.4 to reverse the direction of rotation of a three-phase induction motor.
5. **Why are the slots on the induction motors are usually skewed? [AU April/May, 2006]**
   Refer to section 4.2.5 for the reasons of skewing the slots in an induction motor.
6. **Explain why at synchronous speed, the torque developed by the induction motor is zero. [AU April/May, 2009]**
   Refer to section 4.2.8 for the reason why the torque developed in the motor is zero at synchronous speed.
7. **What is slip of an induction motor? State its expression. [AU April/May, 2013; Nov/Dec, 2012]**
   Refer to section 4.3 for the slip of an induction motor and its expression.
8. **What is the slip at start? How slip affects the rotor frequency? [AU Nov/Dec, 2006]**
   Refer to section 4.3 for the slip at start. Also, refer to section 4.4 for the effect of slip on rotor frequency.
9. **Why does the reactance of a three-phase induction motor greatly vary between the starting and running conditions? [AU April/May, 2005]**
   Refer to section 4.4 for the reasons why the reactance of a three-phase induction motor greatly varies between the starting and running conditions.
10. **Draw the torque–slip characteristics of a three-phase induction motor and show the various regions of operation. [AU April/May, 2013]**
    Refer to section 4.6.7 for the torque–slip characteristics of a three-phase induction motor and its various regions of operation.
11. **Draw the equivalent circuit of a three-phase induction motor. [AU April/May, 2012]**
    Refer to section 4.13 for the equivalent circuit of a three-phase induction motor.
12. **A six-pole 50 Hz three-phase induction motor runs at 800 rpm at full load. Determine the value of slip at this load condition. [AU Nov/Dec, 2004]**
    Given, $P = 6$, $f = 50$ Hz and $N = 800$ rpm

The synchronous speed of the induction motor is

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Therefore, the slip of the induction motor is

$$\%s = \frac{N_{\text{syn}} - N}{N_{\text{syn}}} \times 100 = \frac{1000 - 800}{1000} \times 100 = 20\%$$

i.e., s = 0.02

13. What is the speed of the rotor field in space? **[AU Nov/Dec, 2008]**

The speed at which the rotor field rotates in space is the rotor speed, which is denoted as N.

14. A four-pole three-phase induction motor operates from a supply of frequency 50 Hz. Determine the speed at which the magnetic field of the stator is rotating. **[AU Nov/Dec, 2007]**

Given, $P = 4$ and $f = 50$ Hz

The speed at which the magnetic field of the stator is rotating is called synchronous speed, which is denoted by N_{syn}. Therefore,

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

15. In which type of motor can external resistance be introduced in the rotor circuit? What is the effect of it? **[AU April/May, 2005]**

Refer to section 4.2.5 for the motor type to which the external resistance can be added.

16. A six-pole three-phase induction motor operating on a 50 Hz supply has a rotor emf frequency of 2 Hz. Determine the rotor speed. **[AU Nov/Dec, 2011]**

Given, $P = 6$, $f = 50$ Hz and $f_r = 2$ Hz

The synchronous speed of the motor is

$$N_{\text{syn}} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Since $f_r = sf$, we get

$$s = \frac{f_r}{f} = \frac{2}{50} = 0.04 = 4\%$$

Therefore, the rotor speed of induction motor is

$$N = N_{\text{syn}} \times (1 - s) = 1000 \times (1 - 0.04) = 960 \text{ rpm}$$

17. State the expression for the power-factor of an induction motor. **[AU Nov/Dec, 2011]**

The power-factor of an induction motor at standstill and running conditions is given by

$$\cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \quad \text{and} \quad \cos\phi_{2r} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

18. Determine the % slip of an induction motor with synchronous speed 2000 rpm and the motor is running at 1850 rpm. [AU Nov/Dec, 2011]

Given, $N_{syn} = 2000$ rpm and $N = 1850$ rpm

The slip of the induction motor is

$$s = \frac{N_{syn} - N}{N_{syn}} = \frac{2000 - 1850}{2000} = 0.075 = 7.5\%$$

19. Draw the torque–speed curve of the induction motor. [AU April/May, 2012]

The torque–speed curve of the induction motor is shown in Figure UQ4.19.

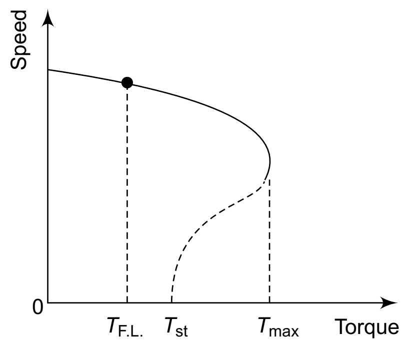


Figure UQ4.19

20. A stator winding supplied from a three-phase 60 Hz system is required to produce a magnetic flux rotating at 900 rev/min. Determine the number of poles. [AU Nov/Dec, 2013]

Given, $f = 60$ Hz and $N_{syn} = 900$ rpm

The number of poles is given by

$$P = \frac{120f}{N_{syn}} = \frac{120 \times 60}{900} = 8$$

21. Distinguish between squirrel cage and slip-ring induction motor. [AU April/May, 2014]

Refer to Table 4.2 for the differences between squirrel-cage and slip-ring induction motors.

22. Draw the power-flow diagram of a three-phase induction motor. [AU April/May, 2005]

The power-flow diagram of a three-phase induction motor is shown in Figure UQ4.22.

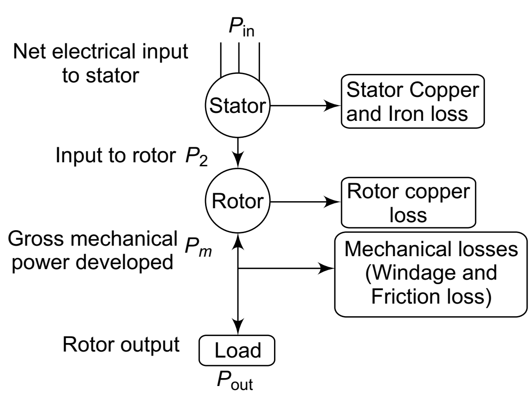


Figure UQ4.22

23. Why is a single-phase induction motor not self-starting? [AU Nov/Dec, 2011; Nov/Dec, 2010]

Refer to section 4.17 for the reason why a single-phase induction motor is not self-starting.

24. Where are split-phase motors used? [AU Nov/Dec, 2006]

Refer to section 4.18.1 for the usage of split-phase motors.

25. List out four applications of single-phase induction motors. [AU Nov/Dec, 2011; April/May, 2008]

Refer to section 4.18 for the applications of single-phase induction motors.

26. Name the starting methods for single-phase induction motors. [AU April/May, 2014; April/May, 2011; Nov/Dec, 2010]

Refer to section 4.18 for the different starting methods of single-phase induction motors.

27. Why are centrifugal switches provided on many single-phase induction motors? [AU April/May, 2011]

Refer to section 4.18.1 for the reasons why centrifugal switches are required in single-phase induction motors.

28. Name the types of alternators. [AU Nov/Dec, 2014]

The different types of alternator are salient pole alternator and non-salient pole alternator.

29. Define voltage regulation of alternator and its expression in terms of winding parameters. [AU April/May, 2003]

Refer to section 4.27 for the voltage regulation of an alternator and its expression.

30. Define pitch factor as applied to alternator. [AU Nov/Dec, 2011]

It is given by the ratio of emf induced when the coil is short-pitched to the emf induced when the coil is full-pitched. It is denoted by K_C, which is always less than one. It is given by:

$$K_C = \cos\left(\frac{\alpha}{2}\right)$$

where, α is the short-pitch angle i.e., the angle by which the coil is short-pitched.

31. Define distribution factor as applied to alternator. State its expression. [AU Nov/Dec, 2008]

The factor by which the emf induced in the armature gets reduced due to coil distribution is called the distribution factor. It is denoted by K_d, which is always less than one. It is given by:

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$$

where, m is the number of slots per pole per phase and β is the slot angle, which is given by $\beta = \dfrac{180°}{n}$, n is the number of slots per pole.

32. Write down the emf equation of an alternator. [AU Nov/Dec, 2010]

The generalised emf equation of an alternator is

$$E_{\text{ph}} = 4.44\, K_C K_d f \phi T_{\text{ph}}$$

33. What are the factors that contribute to reduction in terminal voltage of a loaded alternator? **[AU April/May, 2005]**

The factors that contribute to reduction in terminal voltage of a loaded alternator are:

- Armature reaction
- Load current and
- Load power-factor

34. Calculate the distribution factor for a 36 slots four-pole single-layer three-phase winding of an alternator. **[AU Nov/Dec, 2007]**

Given, Number of slots = 36 and $P = 4$

The number of slots per pole per phase is given by

$$m = \frac{36}{4 \times 3} = 3$$

The number of slots per pole is

$$n = \frac{36}{4} = 9$$

The slot angle is

$$\beta = \frac{180^\circ}{n} = 20^\circ$$

Therefore, the distribution factor of the alternator is given by

$$K_d = \frac{\sin\left(\dfrac{m\beta}{2}\right)}{m\sin\left(\dfrac{\beta}{2}\right)} = \frac{\sin\left(\dfrac{3 \times 20^\circ}{2}\right)}{3 \times \sin\left(\dfrac{20^\circ}{2}\right)} = 0.9597$$

35. Define voltage regulation. Name two methods used to determine voltage regulation of an alternator. **[AU Nov/Dec, 2012]**

Refer to section 4.27 for voltage regulation and the different methods in determining it.

36. Draw the phasor diagram of a loaded alternator with leading power-factor.

[AU Nov/Dec, 2008]

The phasor diagram of the loaded alternator with leading power-factor is shown in Figure UQ4.36.

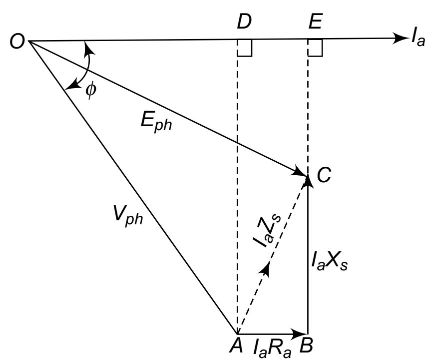


Figure UQ.4. 36

37. State any four advantages of rotating field and stationary armature. [AU Nov/Dec, 2011]

Refer to section 4.20.3 for the advantages of rotating field and stationary armature.

38. State the difference between salient and cylindrical type of rotors. [AU April/May, 2011]

Refer to section 4.20.2 for the difference between salient and cylindrical type of rotors.

39. Determine the speed at which the six-pole alternator is driven to obtain the frequency of emf induced to be 50 Hz. [AU April/May, 2014]

Given, $P = 6$ and $f = 50$ Hz

The speed at which the alternator is to be driven is called synchronous speed and is given by

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

40. What are the causes of faulty starting of a synchronous motor? [AU April/May, 2010]

Refer to section 4.28.4 for the causes of faulty starting of a synchronous motor.

41. How can a synchronous motor be made self-starting? [AU Nov/Dec, 2011]

Refer to section 4.28.5 for the procedure to turn a synchronous motor into a self-starting motor.

42. State the characteristic features of a synchronous motor. [AU Nov/Dec, 2011]

Refer to section 4.28 for the characteristic features of a synchronous motor.

43. What are the different excitations of a synchronous motor? [AU Nov/Dec, 2011]

The different excitations of a synchronous motor are: normal excitation, under-excitation and over-excitation.

44. State the applications of a synchronous motor. [AU Nov/Dec, 2013]

Refer to section 4.36 for the applications of a synchronous motor.

45. Why is the synchronous motor called so? [AU April/May, 2013]

Since the motor rotates at a synchronous speed, the motor is called a synchronous motor.

46. What is a stepper motor? [AU April/May, 2009]

A special type of synchronous motor designed to rotate through a specific angle for each applied electrical pulse is known as a stepper motor.

47. What are the different types of stepper motors? [AU Nov/Dec, 2010]

The different types of stepper motors are variable-reluctance, permanent-magnet and hybrid stepper motor.

48. List the applications of stepper motors. [AU April/May, 2007]

Stepper motors are used in:

- Computer peripherals like printers, disk drives etc., X-Y plotters, scientific instruments and machine tools.
- Quartz-crystal watches
- Many supporting roles in the manufacture of packaged food, commercial end-products and even in the production of science-fiction movies.

49. What is a brushless DC motor? **[AU Nov/Dec, 2009]**

The Brushless Direct Current (BLDC) motor is a derivative of the DC motor and it shares the same torque and speed performance curve characteristics. The major difference between a BLDC and a DC motor is the use of brushes for commutation.

50. What are advantages and disadvantages of a BLDC motor? **[AU Nov/Dec, 2008]**

Refer to section 4.38.4 for the advantages and disadvantages of a BLDC motor.

Review Questions

1. State the working principle of a three-phase induction motor.
2. Explain how a revolving magnetic field is produced in a three-phase induction motor.
3. What is the speed of a revolving magnetic field? Can it be reversed? How?
4. Explain the construction and working of a three-phase induction motor.
5. Why are the slip ring and brush assembly required in a slip-ring motor?
6. Compare slip-ring and squirrel-cage rotors in an induction motor.
7. Is it possible for the rotor to run at synchronous speed? Why?
8. Define slip and percentage slip of an induction motor.
9. What is the significance of slip?
10. Explain the effect of slip on rotor parameters.
11. Compare the rotor parameters at standstill and at running conditions.
12. Derive the: (i) running torque and (ii) starting torque equations of an induction motor.
13. What is the condition for maximum running torque and its magnitude in an induction motor?
14. Obtain the equation for different torque ratios in an induction motor.
15. Explain the torque–slip and speed–torque characteristics of an induction motor.
16. How does the torque in an induction motor change with respect to rotor resistance and rotor reactance?
17. What are the different losses in an induction motor? Explain.
18. Discuss the power flow in an induction motor.
19. Derive the relation between P_2, P_c and P_m.
20. Define shaft torque and synchronous watts.
21. Prove that the constant in the torque developed is equal to $\dfrac{3}{2\pi n_s}$.
22. Define rotor and net motor efficiency. Also, explain its characteristics with respect to power output.
23. Obtain the equivalent circuit of a three-phase induction motor.
24. Derive the power equation using the equivalent circuit of an induction motor and determine the maximum power output.
25. Explain the phasor diagram of an induction motor.
26. Explain the necessity of starters and the different types of starters used in induction motors.
27. Compare different types of starters used in induction motors.
28. How can the induction motor speed be controlled? Explain.
29. Explain the construction and working of a single-phase induction motor.
30. Discuss double revolving field theory.
31. What are the different types of single-phase induction motors? Explain them.
32. Obtain the equivalent circuit of a single-phase induction motor.

33. Explain the construction and working of an alternator.
34. Distinguish between salient pole and smooth cylindrical rotor. Discuss their point of relative merits and demerits.
35. Give reasons for the preference of revolving field alternators over revolving armature alternators.
36. Discuss the different terminologies used in armature windings. Also, explain the different types of armature windings.
37. Define pitch coil factor and distribution factor.
38. Derive the emf equation of an alternator.
39. Discuss the different parameters of an armature winding.
40. Define synchronous reactance and synchronous impedance.
41. Obtain the equivalent circuit and voltage equation of an alternator.
42. Draw the phasor diagram of a loaded alternator.
43. Explain the construction and working of a synchronous motor.
44. Why synchronous motor is not a self-starting motor?
45. Explain the procedure and method to start a synchronous motor.
46. Discuss the behaviour of a synchronous motor on loading.
47. What are V and inverted-V curves?
48. Derive the expression for back emf in a synchronous motor.
49. Explain the power flow in a synchronous motor.
50. What is the power developed in a synchronous motor?
51. How does a synchronous motor differ from an induction motor?
52. Discuss the applications of a synchronous motor.
53. What do you mean by a stepper motor?
54. Discuss the different types of stepper motors.
55. Explain the operation of a stepper motor.
56. What are the advantages of a stepper motor?
57. Describe the working of any one type of stepper motor.
58. Discuss brushless DC motor.
59. A three-phase 50 Hz induction motor has 2 poles. If the slip is 2% at a certain load, determine the speed of the motor. **[Ans: 2940 rpm]**
60. A three-phase four-pole 50 Hz induction motor is running at 1440 rpm. Determine the slip speed and slip. **[Ans: 60 rpm and 4 %]**
61. A four-pole three-phase 50 Hz induction motor runs at a speed of 1470 rpm speed. Determine the frequency of the induced emf in the rotor under this condition. **[Ans: 1 Hz]**
62. For a four-pole three-phase star-connected stator, 50 Hz induction motor, the ratio of stator to rotor turns is 2. On a certain load, its speed is observed to be 1455 rpm when connected to 415 V supply. Calculate: (i) frequency of rotor emf in running condition, (ii) magnitude of induced emf in the rotor at standstill condition and (iii) magnitude of induced emf in the rotor under running condition.
[Hint: Convert line voltage to phase voltage][Ans: (i) 1.5 Hz, (ii) 119.8 V (iii) 3.594 V]
63. A three-phase 400 V 50 Hz four-pole induction motor has star-connected stator winding. The rotor resistance and reactance are 0.1 Ω and 1 Ω respectively. The full-load speed is 1440 rpm. Calculate the torque developed on full-load by the motor. Assume stator to rotor ratio as 2. **[Ans: 87.81 Nm]**
64. A 400 V four-pole three-phase 50 Hz star-connected induction motor has a rotor resistance and reactance per phase as 0.01 Ω and 0.1 Ω respectively. Determine: (i) starting torque (ii) slip at which maximum torque will occur (iii) speed at which maximum torque will occur (iv) maximum torque and (v) full-load torque if full-load slip is 4%. Assume ratio of stator to rotor turns as 4.
[Ans: (i) 63.031 Nm (ii) 10% (iii) 1350 rpm (iv) 318.16 Nm (v) 219.52 Nm]

65. A 746 kW three-phase 50 Hz sixteen-pole induction motor has a rotor impedance of $(0.02 + j0.15)\ \Omega$ at standstill. Full-load torque is obtained at 300 rpm. Calculate: (i) ratio of maximum to full-load torque (ii) speed at maximum torque (iii) rotor resistance to be added to get maximum starting torque and (iv) ratio of maximum torque to starting torque.
[Ans: (i) 1.817 (ii) 325 rpm (iii) 0.13 Ω and (iv) 3.8176]
66. A four-pole 50 Hz 7.46 kW motor at rated voltage and frequency has a starting torque of 160 % and a maximum torque of 200 % of full-load torque. Determine: (i) full-load speed and (ii) speed at maximum torque. **[Ans: (i) 1299.15 rpm and (ii) 750 rpm]**
67. A six-pole 50 Hz three-phase induction motor has a rotor resistance of 0.25 Ω per phase and a maximum torque of 10 Nm at 875 rpm. Calculate: (i) torque when the slip is 5% and (ii) the resistance to be added to the rotor circuit to obtain 60% of the maximum torque at starting. Explain why two values are obtained for this resistance. Which value will be used? The stator impedance is assumed to be negligible. **[Ans: (i) 6.8965 Nm and (ii) 0.4166 Ω]**
68. An eight-pole 50 Hz three-phase induction motor is running at 4% slip when delivering full-load torque. It has standstill rotor resistance of 0.1 Ω and reactance of 0.6 Ω per phase. Calculate the speed of the motor if an additional resistance of 0.5 Ω per phase is inserted in the rotor circuit. Assume full-load torque remains constant. **[Ans: 570 rpm]**
69. A 415 V three-phase 50 Hz four-pole star-connected induction motor runs at 24 rev / sec on full load. The rotor resistance and reactance per phase are 0.35 Ω and 3.5 Ω respectively and the effective rotor–stator turns ratio is 0.85. Calculate: (i) slip (ii) full-load torque (iii) power output if the mechanical losses amount to 770 W (iv) maximum torque (v) speed at which the maximum torque occurs and (vi) starting torque.
[Ans: (i) 4% (ii) 78.0455 Nm (iii) 10998.99 W (iv) 113.166 Nm (v) 10% (vi) 22.3965 Nm]
70. A 100 kW (output) 3300 V three-phase star-connected induction motor has a synchronous speed of 500 rpm. The full-load slip is 1.8% and full-load power factor is 0.85. Stator copper loss, iron loss and rotational loss of the motor are 2440 W, 3500 W and 1200 W respectively. Calculate: (i) rotor copper loss (ii) line current and (iii) full-load efficiency.
[Ans: (i) 1.855 kW (ii) 22.4353 A and (iii) 91.743%]
71. An induction motor has an efficiency of 0.9 when the shaft load is 45 kW. At this load, the stator ohmic loss and the rotor ohmic loss are both equal to the iron loss. The mechanical loss is one third of the no-load losses. Neglect ohmic losses at no-load. Calculate the slip. **[Ans: 3.5%]**
72. The real power input to a 415 V 50 Hz six-pole three-phase induction motor running at 970 rpm is 41 kW. The input power factor is 0.9. The stator losses amount to 1.1 kW and the total mechanical loss is 1.2 kW. Calculate: (i) line current (ii) slip (iii) rotor copper loss (iv) mechanical power output and (v) efficiency. **[Ans: (i) 63.3771 A (ii) 3% (iii) 1.197 kW (iv) 37.503 kW and (v) 91.47%]**
73. A three-phase 50 Hz sixteen-pole star-connected alternator has a stator winding with 144 slots with 10 conductors per slot. The magnetic flux per pole is 0.03 Wb and is sinusoidally distributed in space. The coil pitch of the winding is 8 slots. Estimate the emf induced between the lines of the alternator.
[Ans: 2616.5535 V]
74. A three-phase sixteen-pole alternator has a star-connected winding, with 144 slots and 10 conductors per slot. The flux per pole is 0.04 Wb and is distributed sinusoidally. The speed is 375 rpm. Determine the frequency, phase emf and line emf. The coil span is 120 degree electrical.
[Ans: 50 Hz, 1771.24 V and 3067.8789 V]
75. Determine the no-load phase and line voltage of a star-connected three-phase, six-pole alternator which runs at 1200 rpm, having flux per pole of 0.1 Wb sinusoidally distributed. Its stator has 54 slots with double-layer winding. Each coil has eight turns and coil chorded by 1 slot.
[Ans: 3625.98 V and 6.28 kV]

76. A three-phase star-connected alternator is rated at 1500 kVA, 12000 V. The armature effective resistance per phase and synchronous reactance per phase are 2 Ω and 35 Ω respectively. Calculate the percentage regulation for a load of 1200 kW at a power factor of 0.8 lagging. **[Ans: 26.65 %]**
77. A three-phase star-connected alternator has an open-circuit line voltage of 6599 V. The armature resistance and synchronous reactance are 0.6 Ω and 6 Ω per phase respectively. Determine terminal voltage, voltage regulation and delta (δ), if load current is 180 A at a power factor of: (i) 0.9 lagging and (ii) 0.9 leading. **[Ans: (i) For lagging power-factor: 3128.21 V , 5418.21 V, 21.79%, 6.83°**
(ii) For leading power-factor: 3879.75 V, 6719.92 V, –1.79%, 7.16°]

CHAPTER 5

Measurement and Instrumentation

5.1 Introduction

The process of measuring a quantity is known as measurement and the apparatus used to measure the quantities like voltage, current, power, energy, resistance and so on, are called *measuring instruments*. The quantity to be measured using the measuring instrument is called *measurand*. The measurement of a measurand is the result of comparison between the unknown quantity to be measured and the standard quantity. The measuring instruments indicate, record, display or integrate the electrical quantity to be measured using direct or comparison method for a specified period. The operation and working principles of measuring instruments are discussed in this chapter.

Further, this chapter deals with the classification of transducers, construction and operation of electrical transducers, displacement transducers and mechanical transducers.

5.2 Essential Requirements of Measuring Instruments **[AU Nov/Dec, 2009]**

The necessary or essential requirements for any measuring instruments are:

- When the instrument is used in the circuit, its conditions should not be altered and therefore the quantity to be measured goes unaffected.
- It should consume low power.
- It should possess very high efficiency and high sensitivity.
- The output should be linearly proportional to the input.
- It should be less affected by the noise, modifiable and properly priced.

5.3 Elements of The Measuring Instruments **[AU Nov/Dec, 2013]**

The elements present in measuring instruments are shown in Figure 5.1.

5.3.1 Primary Sensing Unit

The first unit in the measurement system, which detects the measurand, is known as the primary sensing unit. It helps in transferring the measurand to a variable-conversion unit for further processing. For example, liquid or mercury in glass thermometer act as a primary sensing unit. Displacement or voltage is the output of the primary sensing unit.

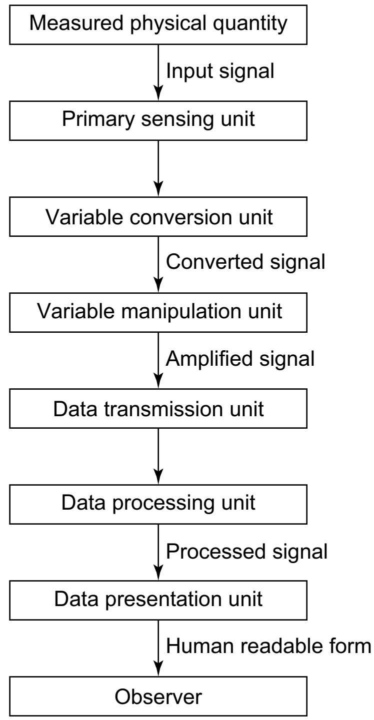


Figure 5.1 Units of the Measuring Instruments

5.3.2 Variable-conversion Unit

The conversion of primary sensing unit output to more suitable variables while preserving the information is achieved with the help of this unit. Hence, it can be called an intermediate transducer. Generally, a variable conversion unit is not required in most of the measuring instruments. In some measuring instruments, more than one variable-conversion unit is required and in some cases the primary sensing unit and the variable-conversion unit are combined to form a single unit called a transducer.

5.3.3 Variable-manipulation Unit

It is the intermediate stage of the measuring system, where the numerical value of the signal gets modified i.e., it manipulates the signal presented to this unit without affecting the original nature of the signal. It can be placed either before or after the variable-conversion unit. It helps in improving the output quality of the measurement system by removing the random signals like noise.

5.3.4 Data Transmission Unit

When the functional units of the measuring system are spatially separated, the data transmission unit acts as a communication link for transmitting the signals from one unit to another. This unit is mandatory when the system is operated remotely. Some of the common data-transmission units used are cables, wireless antennae, transducers, telemetry systems and so on.

5.3.5 Data Processing Unit

This unit is used to process the data obtained from either the variable-manipulation unit or data-transmission unit and to produce suitable output to be presented to the experimenter. In addition, it is used to compare the measured value with the standard value to produce the required output.

5.3.6 Data Presentation Unit

This unit is used for communicating the measured quantity to the experimenter, which could be used for controlling and analysing purposes.

Let us consider the following two examples to demonstrate each unit in the measuring system.

Example 1: Thermometer

In this case, the thermometer bulb containing mercury acts as the primary sensing unit and a variable-conversion unit. It senses the temperature, which is the input quantity. With increase in temperature, mercury in the bulb expands and its volume increases. Therefore, the temperature signal is converted into volume displacement. As the mercury expands, it moves through the capillary tube in the thermometer stem, integrated to the bulb. With the cross-section area of the capillary being constant, the volume signal is converted into linear distance signal. The capillary, thus, has the role of signal manipulation and data transportation units. The final data presentation stage consists of the scale on the thermometer stem, which is calibrated to give the indication of the temperature signal applied to the thermometer bulb. A restriction bend is provided in the clinical thermometers at the junction of the bulb and the capillary, which does not allow the back flow of mercury to the bulb once it has expanded to the capillary. Thus, the restriction in the capillary acts as the data storage function of the instrument.

Example 2: Bourdon-tube Pressure Gauge

The Bourdon tube used to measure pressure is shown in Figure 5.2.

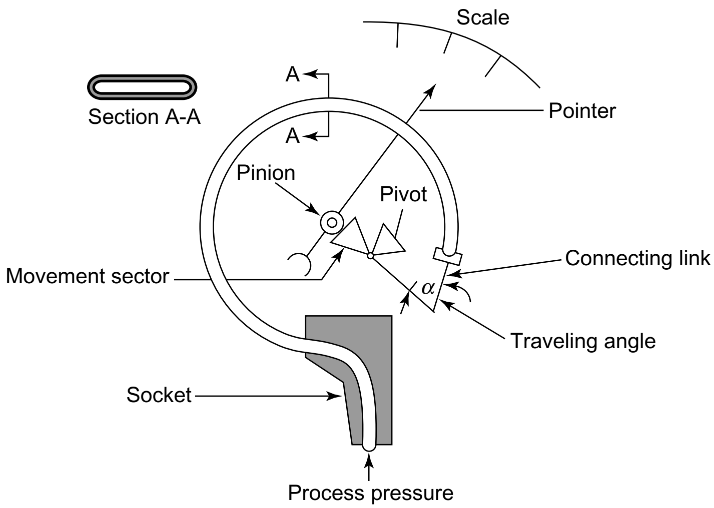


Figure 5.2 Bourdon-Tube Pressure Gauge

The different units present in the Bourdon tube are:

- Primary sensing unit : Bourdon tube
- Variable-conversion unit : Bourdon tube
- Variable-processing (manipulating) unit : Gearing
- Data presentation unit : Pointer & Scale

5.4 Types of Electrical and Electronic Measuring Instruments

5.4.1 Basic Classification

The basic classifications of measuring instruments are:

- ***Mechanical instrument:*** It is a reliable instrument to measure under static and stable conditions but does not respond faster to the dynamic and transient conditions.
- ***Electrical instrument*:** In this instrument, electrical quantities are used to indicate the output of detector. It is more rapid when compared to a mechanical instrument but depends on the mechanical meter movement as an indicating device.
- ***Electronic instrument*:** A rapid response can be obtained by using this instrument.

5.4.2 Other Classifications

Other classifications of measuring instruments are:

- ***Absolute or primary instruments:*** The instruments that give the magnitude of the measurement quantity in terms of physical constants of the instrument and deflections are called absolute or primary instruments. Also, these instruments do not require any comparison with the standard instrument. They are generally not used in laboratories and are seldom used by electricians and engineers. The time required to determine the magnitude of the measuring quantity is high. Examples of such instruments are: tangent galvanometer, Rayleigh current balance, absolute electrometer and so on.
- ***Secondary instruments:*** The instruments used to measure the quantity only by the output indicated by the instruments are known as secondary instruments. These instruments are calibrated by comparing with the absolute or primary instrument. They are usually preferred to absolute instruments, as the former takes less time to compute the output. These instruments are generally used in laboratories. Some of the widely used secondary instruments are: ammeters, voltmeters, wattmeters, energy meters (watt-hour meters), ampere-hour meters and so on.

The secondary instruments are further classified based on:

- **Various effects used to measure electrical quantity**
  - ***Magnetic effect:*** Used in ammeter, voltmeter, wattmeter etc.
  - ***Thermal effect:*** Used in ammeter and voltmeter
  - ***Chemical effect:*** Used in DC ampere hour meter
  - ***Electrostatic effect:*** Used in voltmeter
  - ***Electromagnetic induction effect:*** Used in AC ammeter, voltmeter, wattmeter etc.
- **Nature of the instrument operation**
  - ***Indicating instrument:*** Indicate the quantity to be measured using a pointer, which moves over a scale. Eg: ammeter, voltmeter and so on
  - ***Recording instruments:*** Record the quantity that is continuously varying with respect to time. They can make a permanent record of the indication.
  - ***Integrating instruments:*** Record the consumption of total quantity of electricity, energy and so on.
  - ***Displaying instruments:*** These instruments measure the electrical quantities in the form of waves on the screen. Eg: Oscilloscope

- **Nature of quantity that can be measured**
  - ***DC instruments:*** Measure only DC quantities
  - ***AC instruments:*** Measure only AC quantities
  - ***Both DC and AC instruments:*** Measure both DC and AC quantities
- **Methods used**
  - ***Direct measuring instruments:*** Convert the energy of the measured quantity directly into a form, which actuates the instrument and the value of the unknown quantity, is indicated or measured or recorded directly.
  - ***Comparison measuring instruments:*** Measure the unknown quantity with the help of comparison with the standard quantity. Example: AC Bridge.

5.5 Principles of Electrical Indicating Instruments

An indicating instrument essentially consists of a moving system and a stationary system. A pointer is attached to the moving system, which indicates the electrical quantity to be measured, on a graduated scale. In order to ensure the proper operation of the indicating instruments, the following torques are required:

- Deflecting or operating torque
- Controlling or restoring torque
- Damping torque

5.5.1 Deflecting Torque

The deflecting torque acts on the moving system of the instrument to give the required deflection and indicates the corresponding electrical quantity to be measured on a graduated scale. It exists as long as the instrument is connected to the supply. It is produced by any one of the following effects:

- ***Magnetic effect:*** When a current-carrying conductor is placed in a uniform magnetic field, it experiences a force, which causes the conductor to move. Example: moving iron attraction and repulsion type, permanent-magnet moving coil instrument.
- ***Thermal effect:*** When the current to be measured is allowed to flow through a small element, heat gets generated, which causes rise in temperature and it is then converted to an emf. Example: hot-wire instrument, thermocouple instrument.
- ***Electrostatic effect:*** When two charged plates are placed together, a force is exerted between them, which makes any one plate to move.
- ***Induction effect:*** When a non-magnetic conducting disc is placed in a magnetic field produced by an electromagnet, an emf gets induced in it.
- ***Hall effect:*** If a current-carrying bar of semiconducting material is placed in a uniform magnetic field, an emf is produced between the two edges of conductor.

5.5.2 Controlling Torque

The controlling torque is produced by a spring or gravity, which opposes the deflecting torque. The pointer comes to rest at a particular position corresponding to the electrical quantity to be measured, when these two torques are equal. This torque is always present in the instrument whether it is connected to the supply or not. The controlling torque increases with the deflection of the moving system. The controlling torque is also essential to bring back the moving system to its *initial* or *rest* or *zero* position, once the instrument is disconnected from the supply. The control torque can be produced using spring or gravity as explained below:

Spring Control

Two helical springs of rectangular cross-sections are connected to the spindle of the moving system, as shown in Figure 5.3. With the movement of the pointer, the springs get twisted in the opposite direction, which affects the moving system. In spring-controlled instruments, the scale is linear if the deflecting torque is proportional to the quantity being measured.

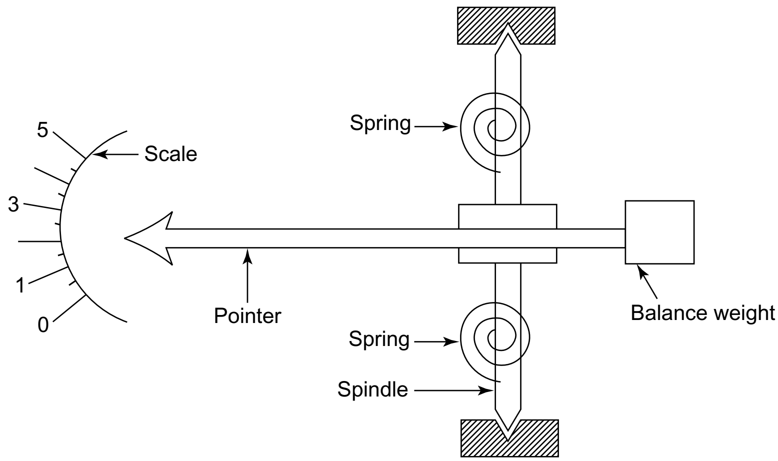


Figure 5.3 Spring Control

Gravity Control

In this method, small weights, which can be adjusted, are added to the moving system, as shown in Figure 5.4. When the pointer deflects, this weight also takes a deflected position.

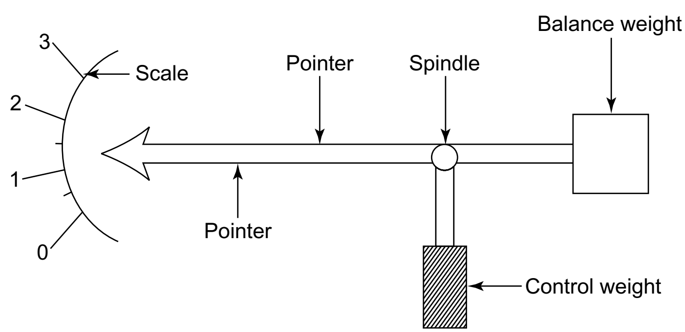


Figure 5.4 Gravity Control

The required controlling torque is produced by the gravitational force, which is acting on the moving weight. The instrument using this method of producing controlling torque has the following disadvantages:

1. Non-uniform scale will be present in the instruments i.e., the scale will be crowded near the minimum limit and uniform near the maximum limit.
2. The instrument can be used only in the vertical position.

5.5.3 Damping Torque

The torque that is used to reduce the oscillations of the pointer and to bring it to the final deflected position is known as damping torque. It acts on the pointer only when the instrument is in operation. If sufficient damping torque is not produced, the pointer makes under-damped oscillations before reaching the steady deflection. If the damping torque is more than the required value, the pointer becomes sluggish and it takes longer than the required time to reach the final deflection, as shown in Figure 5.5. Critical damping or dead beat is the condition where the magnitude of damping torque is sufficient enough to make the pointer to read the correct reading without passing or oscillating about it.

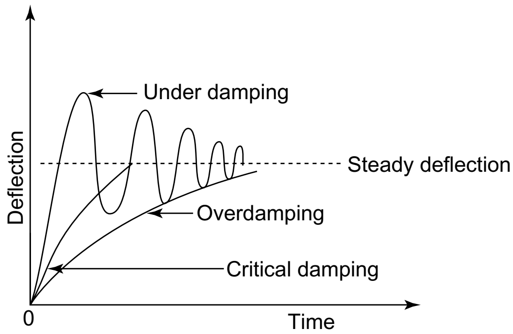


Figure 5.5 Characteristics of Damping Torque

The required damping torque can be produced by the following methods:

Air-friction Damping

The different methods of producing damping torque using air are shown in Figures 5.6 (a) and (b). Figure 5.6 (a) shows the arrangement where a piston, attached to the spindle of the moving system, moves inside the air chamber provided with a very small clearance between the piston and the chamber. When the deflecting torque acts on the moving system, the suction and compression actions on the air inside the air chamber produce the necessary damping torque.

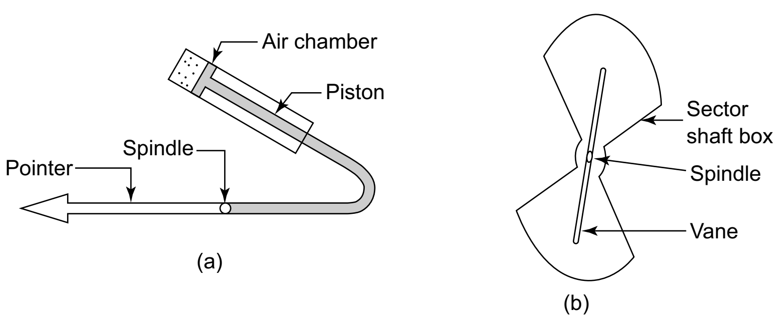


Figure 5.6 Air-Friction Damping

Another arrangement is shown in Figure 5.6(b). There is a sector-shaped box, containing air. This box moves a pair of vanes attached to the spindle of the instrument. The movement of the vanes in the air produces the required damping torque. Both the box and the vanes are made of aluminium.

Eddy current Damping

A thin disc of a conducting but non-magnetic material (like copper or aluminium) is mounted on the spindle, which carries the moving system and the pointer, as shown in Figure 5.7. The position

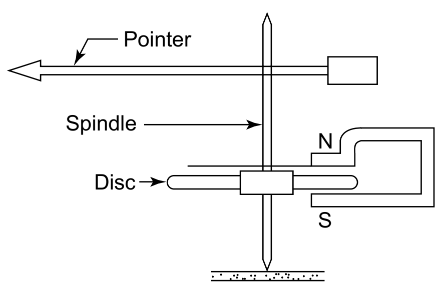


Figure 5.7 Eddy Current Damping

of the disc is such that while in motion, it cuts the magnetic flux produced by the permanent magnet and eddy currents are produced in the disc. These currents flow in such a direction that the motion of the disc is opposed. Thus, the required damping torque is produced.

5.6 Types of Indicating Instruments

The indicating instruments are of different types, as listed below:

- Moving-iron instruments
- Moving-coil instruments
- Electrothermic instruments
- Electrostatic type instruments.
- Induction instruments
- Rectifier type instruments.

5.7 Moving-iron Instruments

Moving-iron instruments are generally used to measure the flow of alternating voltage and current with the help of moving iron. When compared to other AC instruments, the moving-iron instruments are precise, low-priced and rugged. The basic principle of the moving-iron instrument is that when an iron piece is brought near the magnet, it gets attracted towards the magnet. This movement of the soft-iron piece is used to measure the current or voltage, which produced the magnetic field. The strength of magnetic field, which depends on the magnitude of current passing through the magnet, decides the force of attraction of the iron piece. The two different types of moving-iron instruments are:

- Moving-iron instrument–attraction type
- Moving-iron instrument–repulsion type

5.7.1 Construction of a moving-iron instrument

The different components existing in the moving-iron instruments are briefly described below:

- ***Moving element:*** A soft-iron piece, in the form of a vane or a rod or a plate, is placed in such a way that it gets moved freely in the magnetic field.
- ***Stationary coil:*** Used to generate the magnetic field when it is excited by the voltage or current flowing through the coil, whose magnitude is to be measured. Also, it is used to magnetise the moving element. The strength of the magnetic field increases or decreases based on the current flowing through it.
- ***Spring or weight:*** It is used to provide the control torque.
- ***Damping device:*** It consists of an air chamber and a moving vane attached to the instrument spindle and is used to generate the damping torque, which is normally pneumatic.
- ***Aluminium pointer:*** It is used to indicate the movement produced by the deflecting torque over a graduated scale.

In addition to the above components, in repulsion type, there exists another soft-iron piece in the form of a vane or a rod or a plate, which is fixed and magnetised with the same polarity of the moving element.

5.7.2 Torque Equation in Moving-iron Instrument

At any instant of time, let I be the current flowing through the coil, which has a self-inductance L and produces a deflection θ in the needle. The deflection θ, can also be known as the angular position of the soft-iron piece. Therefore, the initial energy stored in the coil in the form of magnetic field is given by

$$E_i = \frac{1}{2} L I^2 \tag{5.1}$$

Let $d\theta$ be the increment in the deflection indicated in the instrument using the deflecting torque, T_d, when a small increment in current, dI is supplied to the coil. Therefore, the mechanical work done due to such deflection is given by

$$W_m = T_d \times d\theta \tag{5.2}$$

Also, due to this change in current dI, there will be a change in inductance dL. Therefore, the final energy stored in the coil is given by

$$E_f = \frac{1}{2}(L + dL)(I + dI)^2 \tag{5.3}$$

Therefore, the change in energy stored in the coil is given by

$$dE = E_f - E_i$$

Substituting Eqn. (5.3) and Eqn. (5.1) in the above equation, we get

$$dE = \frac{1}{2}(L + dL)(I + dI)^2 - \frac{1}{2} L I^2$$

Expanding the above equation and eliminating the higher order terms, we get

$$dE = LIdI + \frac{1}{2} I^2 dL \tag{5.4}$$

In addition, the emf induced in the coil also increases due to this change in current, as given by

$$e = \frac{d(LI)}{dt} = I\frac{dL}{dt} + L\frac{dI}{dt} \tag{5.5}$$

Therefore, the electrical energy supplied by the source is given by

$$E_s = eIdt$$

Substituting Eqn. (5.5) in the above equation, we get

$$E_s = I^2 dL + LIdI \tag{5.6}$$

According to the law of conservation of energy, the electrical energy supplied by the source is converted into stored energy in the coil and the mechanical work done for deflection of needle in the instruments. Therefore,

$$E_s = dE + W_m$$

Substituting Eqn. (5.4) and Eqn. (5.2) in the above equation and solving, we get

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \tag{5.7}$$

From the above equation, it is clear that the deflecting torque depends on the rate of change of inductance with the angular position of the soft-iron piece and the square of the RMS current flowing through the coil. The controlling torque, T_C, provided by the spring arrangement in the instrument is given by

$$T_C = k_s \theta \tag{5.8}$$

where, k_s is the spring constant.

In equilibrium state, the deflecting and controlling torques are equal, as given by

$$T_d = T_C$$

Substituting Eqn. (5.7) and Eqn. (5.8) in the above equation, we get the deflection of the needle or the angular position of the soft-iron piece as

$$\theta = \frac{1}{2k_s} I^2 \frac{dL}{d\theta}$$

From the above equation, it is clear that the deflection is proportional to the square of the current flowing through the coil and is independent of the direction of current.

5.7.3 Moving-Iron Instrument-Attraction Type

The basic working principle of attraction type of moving-iron instrument is that, when a soft-iron piece is brought near to the magnet, the magnet attracts it. The schematic diagram of the attraction type moving-iron instrument is shown in Figure 5.8.

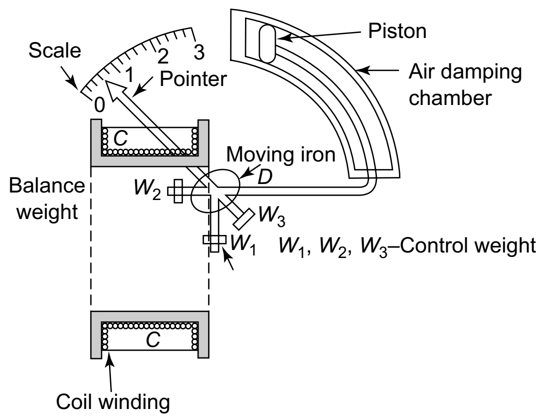


Figure 5.8 Moving-Iron Instrument-Attraction Type

Construction

This type of instrument consists of a fixed coil *C*, which is flat with a narrow slot-like opening and a moving-iron piece *D*, which is a flat disc mounted on the spindle supported by the jewel bearings. The pointer used to indicate the alternating current or voltage moves over a graduated scale and is fixed with the spindle. The range of the alternating current or voltage measured by the instrument is directly proportional to the number of turns in the fixed coil. Springs are used to provide the controlling torque. But in a vertically-mounted instrument, the gravity control can be used to provide the controlling torque. Air friction, with the help of a light aluminium piston that fixed to the moving system and moves in a fixed chamber, is used to provide the damping torque.

Working Principle

When the measuring instrument is connected to the circuit, the current starts flowing through the coil and generates a magnetic field. Now, the coil behaves like a magnet, thereby attracting the soft-iron piece towards the centre of the coil, where the flux density is maximum. As a result, the spindle and the pointer attached to the spindle move from their initial positions and give a proportional deflection due to deflecting torque. If the current flowing through the coil is reversed, then the direction of the magnetic field, and hence the polarity formed in the soft-iron piece, get reversed. Hence, there will be no change in the direction of deflecting torque. Therefore, this instrument can be used to measure both DC and AC quantities.

5.7.4 Moving-Iron Instrument-Repulsion Type

The basic working principle of a repulsion type moving-iron instrument is that, when two soft-iron pieces are magnetised to the same polarity, a force of repulsion exists between them, which cause the movement.

In this instrument, there are two pieces of soft-iron inside the coil: one is fixed and the other is movable. If one of the two soft-iron pieces is made to move, the existing repulsive force makes the other soft-iron piece to move. This movement is used to measure the current or voltage, which produces the magnetic field. The two different designs of repulsion type instruments are:

- Radial vane type
- Co-axial or concentric vane type

Radial-vane Type

The schematic diagram of a radial vane type repulsive moving-iron instrument is shown in Figure 5.9. When compared to other moving-iron instruments, this is more sensitive and has a linear scale.

Construction

In this type, the radial strips of soft-iron piece are used and are placed within the coil. The fixed soft-iron piece is attached to the coil and the movable one is attached to the spindle of the instrument. Using the deflecting torque, the pointer attached to the moving-iron moves over the scale. The controlling torque is produced by spring mechanism, and the air-friction damping provides the damping torque.

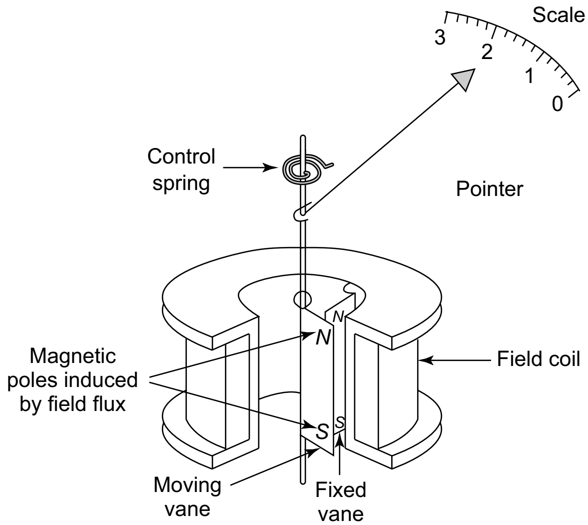


Figure 5.9 Radial Vane Type of Moving-Iron Instrument

Working Principle

The magnetic field, which magnetises both the soft-iron pieces, is produced when the current starts flowing through the operating coil. Hence, a repulsive force exists between these two soft-iron pieces. This repulsive force, when acting on the moving iron, pushes away from its initial position. Thus, the spindle attached to the moving-iron moves and hence the pointer gives a proportional deflection. Now, even when the alternating current flows through the coil, a repulsive force always exists between the two soft-iron pieces. Therefore, the deflection of the pointer is always in the same direction and is directly proportional to the actual current. Hence, this instrument can be used to measure both AC and DC quantities.

Coaxial or Concentric Vane Type

The schematic diagram of a concentric vane type repulsive moving-iron instrument is shown in Figure 5.10. In this type of instrument, the fixed and moving vanes are sections of co-axial cylinders, as shown in Figure 5.10. This type of instrument is moderately sensitive and has low square response. Thus, the scale of the instrument is non-uniform in nature.

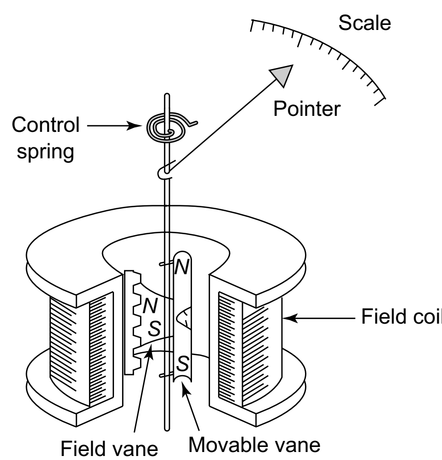


Figure 5.10 Coaxial Vane Type of Moving-Iron Instrument

Construction

The instrument has two concentric vanes or soft-irons. One is fixed to the coil frame rigidly while the other rotates coaxially inside the fixed vane. The shaft, which holds the pointer, is attached to the coaxial moving vane. The controlling torque is provided by the spring or gravity arrangement, and pneumatic or air-damping arrangement provides the damping torque.

Working Principle

The soft-iron pieces or vanes are magnetised to the same polarity by the magnetic field produced due to the current flowing through the coil. A repulsive force, which exists between the two vanes, causes the movable system to rotate. The movable vane attached to the pivoted shaft causes the rotation of shaft. The pointer attached to the shaft shows deflection that is proportional to the current flowing through the coil. Similar to the other moving-iron instruments, whatever may be the direction of the current in the coil, the deflection in this instrument will be in the same direction.

5.7.5 Errors in Moving-iron Instruments

The different types of error in moving-iron instruments are:

- **Error with both AC and DC work**
  - ***Hysteresis error:*** Its readings will not be same for a particular current value during rise and fall in current.
  - ***Stray magnetic-field error:*** It occurs due to the weakness of the operating magnetic field, which can be eliminated by placing or shielding the instrument in an iron case or in a steel case respectively.
  - ***Temperature error:*** Increase in temperature affects the instrument resistance and stiffness of the spring.
  - ***Friction error:*** It occurs due to the friction in the moving parts of the instrument.
- **Error with AC work**
  - ***Frequency error:*** It occurs due to change in reactance of the coil and due to change in eddy current.
  - ***Error due to reactance of the instrument coil:*** It occurs due to change in frequency and causes serious error in measurement.
  - ***Error due to eddy current:*** It affects both the voltmeter and the ammeter.
  - ***Error due to waveform:*** It occurs due to change in flux waveform.

5.7.6 Advantages, Disadvantages and Applications of Moving-iron Instrument

Advantages

1. As the instrument is independent of the direction of current, it can be used for both DC and AC circuits.
2. Since the components present in the instrument are simple and require less number of turns, they are cheap.
3. It is robust in nature because of its simple construction.
4. High operating-torque is available in this instrument.
5. Reasonably accurate measurements are possible with these instruments.

6. It can withstand overload momentarily.
7. It can be used in low frequency and high-power circuits.
8. Frictional error existing in the instrument is very less, as it has high torque-to-weight ratio.
9. Possible to extend the range of the instrument.

Disadvantages

1. Because of pneumatic damping, the scale of the instrument is not uniform, which results in less accurate readings.
2. The instrument is not very sensitive.
3. Errors exist due to hysteresis, frequency and stray magnetic field.
4. There exists difference in calibrating AC and DC instruments.
5. High power consumption
6. Increase in temperature increases the resistance of coil and decreases the stiffness of the spring, and permeability of soft-iron, which affects the reading.

Applications

1. Heavy-current moving-iron ammeter
2. Moving-iron voltmeter
3. Moving-iron power factor meter
4. Moving-iron synchroscope

Example 5.1

The inductance of a moving iron instrument is given by $L = (16 + 8\theta - \theta^2)$ μH, where θ is the deflection in radians from zero position. If the spring constant of the instrument is 16×10^{-6} Nm/rad, calculate the deflection for a current of 8 A.

Solution

Given, the inductance of the system, $L = (16 + 8\theta - \theta^2)$ μH and spring constant, $k_s = 16 \times 10^{-6}$ Nm/rad.
The rate of change of inductance with respect to deflection is given by

$$\frac{dL}{d\theta} = (8 - 2\theta)\ \mu\text{H/rad} = (8 - 2\theta) \times 10^{-6}\ \text{H/rad}$$

Using the deflecting torque derivation, we get

$$\theta = \frac{1}{2k_s} I^2 \frac{dL}{d\theta}$$

Substituting the known values, we get

$$\theta = \frac{1}{2 \times 16 \times 10^{-6}} \times 8 \times 8 \times (8 - 2\theta) \times 10^{-6}$$

Solving the above equation, we get

$$\theta = 3.2\ \text{rad}$$

5.8 Moving-coil Instruments

Moving-coil instruments are mainly used in measuring DC quantities. When fed through appropriate rectifiers, this instrument can be used in measuring AC quantities. The basic principle of moving-coil instruments is that, when a current-carrying coil is placed in a magnetic field, a force or torque is exerted on it, which moves the coil away from the magnetic field. This movement of the coil helps in measuring current or voltage. The different types of moving coil instruments are:

- Permanent magnet moving coil instrument (PMMC): used for DC
- Dynamometer type: used for both AC and DC

5.8.1 Permanent Magnet Moving Coil Instrument (PMMC)

If the permanent magnet is used in the instrument for creating the stationary magnetic field in which current-carrying coil moves, it is known as the permanent magnet moving coil (PMMC) instrument. The PMMC instrument, which gives accurate results while measuring DC quantities, works on the principle that the torque is exerted on the moving coil placed in the magnetic field generated using permanent magnet.

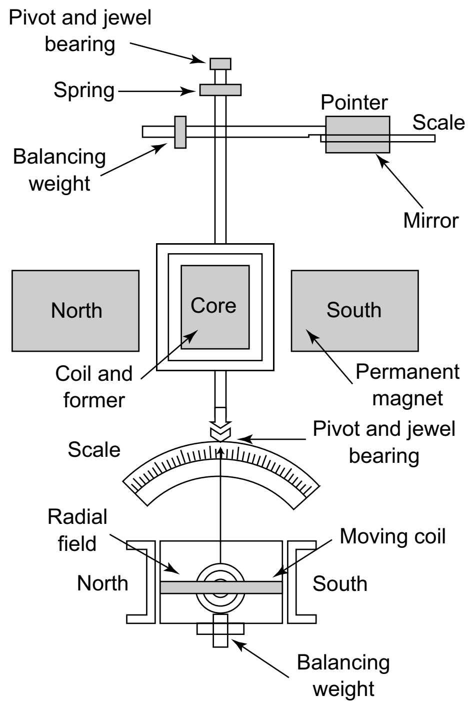


Figure 5.11 PMMC Instrument

Construction

The different components existing in a PMMC instrument are:

- Moving Coil
- Magnet System
- Control Spring
- Damping
- Pointer and Scale

The schematic diagram of a PMMC instrument is shown in Figure 5.11.

- ***Moving coil:*** It is the current-carrying part in a PMMC instrument, which moves in the magnetic field produced by the permanent magnet. It is made up of many turns of copper and is mounted on an aluminium rectangular former placed between the poles of the magnet. The aluminium rectangular former, which is pivoted on jewel-bearing, is used to increase the magnetic field between the air gap of the poles. The current flowing through the coil deflects it and this deflection is used to measure the magnitude of the current or voltage. When PMMC is used as a voltmeter, the moving coil is wound on the metallic frame, which provides the required electromagnetic damping and when PMMC is used as an ammeter, the moving coil is wound on non-magnetic former, as the turns of the coil are effectively shorted using an ammeter shunt. Thus, the moving coil in a PMMC instrument provides the electromagnetic damping.
- ***Magnet System:*** In a PMMC instrument, a simple U-shaped permanent magnet, made up of Alcomax or Alnico, having high coercive force and high field intensities, helps in generating the magnetic field. The magnetic field is made radial, uniform and boring a soft-iron cylinder between the poles of the permanent magnet increases its strength.

- ***Controlling torque:*** The two-control phosphorous-bronze springs mounted on the jewel-bearing are used to provide the controlling torque in a PMMC instrument. It also helps in providing the path for the current to flow in and out of the moving coil.
- ***Damping torque:*** The movement of aluminium former in the permanent magnet field provides the required damping torque, which is used for keeping the coil movement in rest. This damping torque is induced due to the development of eddy current in the former due to its movement.
- ***Pointer & Scale:*** The lightweight pointer, which gets easily deflected, is carried by the spindle, moves through a graduated scale and is linked with the moving coil. The pointer notices the deflection of the coil and its magnitude is shown on the scale. The lightweight material used in the pointer is to avoid parallax error.

Working Principle

When the current starts flowing through the moving coil, a magnetic field gets generated, which is proportional to the current. Based on the electromagnetic action between the current-carrying coil and the permanent magnetic field, a deflecting torque is developed. When the controlling torque provided by the two springs matches with the deflecting torque or at balanced condition, the moving coil gets stopped. The pointer attached to the moving coil measures the amount of electrical quantity passing through the coil, by determining the angular displacement of the coil against a fixed reference, called a scale. The damping torque prevents further oscillation of the coil i.e., after the balanced condition.

5.8.2 Torque in PMMC Instrument

The deflecting torque equation for a PMMC Instrument is given by

$$T_d = NBLId = GI \tag{5.9}$$

where, N is the number of turns in the moving coil, B is the magnetic flux density existing between the permanent magnet poles, L is the length and d is the breadth of the moving coil and $G = NBLd$ is the constant.
The magnitude of the controlling torque provided by the spring is given by

$$T_C = k_s\theta \tag{5.10}$$

where, k_s is the spring constant and θ is the angular movement made by the moving coil in radians.
At steady-state condition, deflecting and controlling torques shall be equal as given by

$$T_d = T_C$$

Substituting Eqn. (5.9) and Eqn. (5.10) in the above equation, we get

$$GI = k_s\theta$$

Therefore, the angular displacement made by the moving coil is given by

$$\theta = \frac{G}{k_s} I \tag{5.11}$$

It is clear from the above equation that, the deflection in a PMMC instrument is directly proportional to the current flowing in the moving-coil, due to which the meter scale used for the measurement of current/voltage is linear in this instrument.

5.8.3 Errors in PMMC Instrument

The errors that usually occur in PMMC instruments are:

- ***Frictional error:*** It is a serious error for sensitive instruments designed for low operating-torque and can be eliminated by using a light moving system and large deflecting-torque.
- ***Temperature error:*** It occurs due to heating of working and other coils, which affects the instrument resistance and stiffness of the control spring. Using a resistance of negligible temperature coefficient along with the PMMC instrument can eliminate this error.
- ***Error owing to weakening of permanent magnet:*** It occurs due to ageing of the instrument and can be reduced by proper attention and reasonable care.
- ***Stray magnetic-field error:*** It occurs due to stray magnetic-field and can be reduced by using an iron case or a thin iron-shield for the instrument
- ***Thermo-electric error:*** It predominantly exists when a PMMC is used as an ammeter and can be reduced by using an alloy of small thermo-electric power to copper with careful treatment of protection.
- ***Observational error:*** It occurs due to misreading of the scale, parallax in reading or a zero error, which can be reduced by re-calibrating the instrument, placing a mirror under the scale and correctly adjusting zero-setting respectively.

5.8.4 Advantages, Disadvantages and Applications of PMMC Instrument

Advantages

1. Presence of uniform scale.
2. Low power-consumption.
3. High torque-to-weight ratio and hence the accuracy is high.
4. Using multipliers and shunts, a single device can be used to measure electrical quantity of different ranges.
5. Shelf-shielding magnet is used for aerospace applications.
6. Absence of hysteresis loss.
7. Effective and efficient eddy current damping.
8. As strong permanent magnet exists, the instrument is less affected by stray magnetic fields.
9. Due to presence of spring control, it can be used in vertical and horizontal positions.
10. Possesses high sensitivity.

Disadvantages

1. It can be used only for the direct current as the rapid variation in alternating quantity cannot be shown by the pointer
2. It is costly when compared to moving-coil instruments.
3. With ageing of the instrument, the possibility of occurrence of error is high.
4. Occurrence of errors due to friction, mechanical unbalance, resistance temperature coefficient etc.

Applications

The PMMC instrument can be used as:

1. ***Ammeter:*** Moving coil is connected across a suitable low shunt-resistance.
2. ***Voltmeter:*** Moving coil is connected in series with high resistance.
3. ***Galvanometer:*** Used to measure small current along with its direction and strength.
4. ***Ohm Meter:*** Used to measure resistance of the electric circuit.

5.9 Electro Dynamometer Type

An electrodynamometer-type instrument is used for the measurement of AC and DC quantities, unlike a PMMC instrument, which can only be used for the measurement of DC quantities. The electrodynamometer-type instrument is a transfer instrument, which is calibrated with a DC source; and without using any modifications, the same instrument can be used for AC measurements with same accuracy as that of DC measurements. This type of instrument is often used in AC voltmeters and ammeters with high accuracy and with small modifications, it can be used as wattmeter for measuring the power.

The electrodynamometer-type instrument, which is similar to a PMMC type instrument except for the permanent magnet used in PMMC-type instrument, is replaced with another fixed coil that generates the necessary magnetic field. The reason for which the PMMC cannot be used for AC quantities is used as a working principle in electrodynamometer-type instrument. In order to read the AC quantities using a moving-coil instrument, the magnetic field existing in the instrument must change along with the change in AC quantities, which is not possible in a PMMC instrument.

5.9.1 Construction

The schematic diagram of an electrodynamometer-type instrument is shown in Figure 5.12.

The various parts of the electrodynamometer-type instrument are:

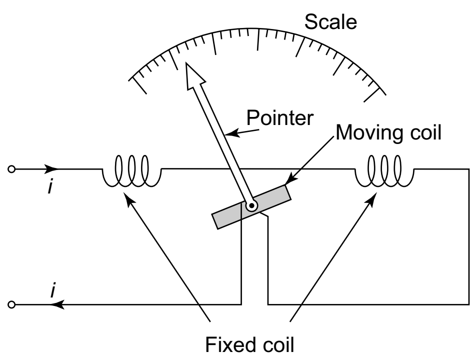


Figure 5.12 Electrodynamometer-type Instrument

Fixed Coil

This air core varnished coil is used to generate the necessary magnetic field required for the operation of the instrument and a uniform magnetic field is obtained near the centre of the coil, as the fixed coil is divided into two sections. If the instrument is used as an ammeter or a voltmeter, the fixed coils are wound with thin wire and heavy wire respectively. They are clamped in place against the coil support, which is made up of ceramic.

Moving Coil

The moving air-cored light and rigid coil is wound as a self-sustaining coil or on a non-metallic former.

Controlling Torque

The controlling torque by the spring acts as leads to the moving coil.

Moving System

The moving coil is mounted on an aluminium spindle, which carries the counter weights and pointer. In some cases, a suspension is used if high accuracy is desired.

Damping Torque

A pair of aluminium vanes, which are attached to the spindle at the bottom, provides the damping torque using air friction.

Shielding

To prevent the effect of earth's magnetic field on the reading, the instrument is shielded by enclosing it in a casing made with a high-permeability alloy.

Cases and Scales

These instruments used in laboratory are usually contained in a rigid, polished wooden or metal case and are supported by adjustable levelling screws. A proper levelling of these components can be provided by the spirit level.

5.9.2 Working Principle

When the electrodynamometer instrument is used as an ammeter, both the fixed and moving coils are connected in series to carry the same current. To limit the current is flowing through these coils, a suitable shunt-resistance is connected to these coils. But, when the electrodynamometer instrument is used as a voltmeter, the fixed and moving coils are connected in series with high non-inductive resistance. When it is used to measure the power, the fixed coil and moving coil act as the current and voltage coil, connected in series with the load and across the supply terminals respectively.

When the current starts flowing through both the coils, a magnetic field is produced. The magnetic field produced by the fixed coil is proportional to the load current and the magnetic field produced by the moving coil is proportional to the voltage. Now, the deflecting torque is produced due to the interaction of these two fields, and the deflection indicated by the pointer is proportional to the power supplied to the load. The connections of an electrodynamometer instrument as an ammeter, a voltmeter and a wattmeter are shown in Figures 5.13 (a), (b) and (c).

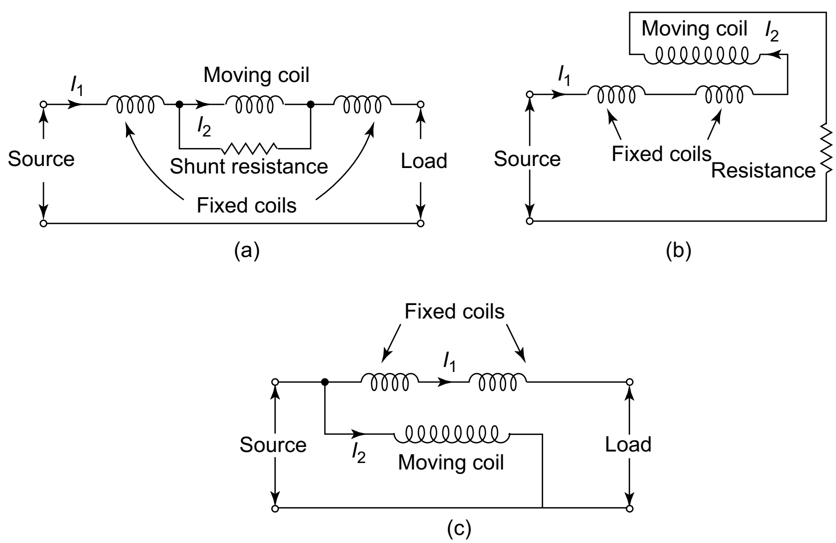


Figure 5.13 Electrodynamometer as (a) Ammeter (b) Voltmeter and (c) Wattmeter

5.9.3 Torque Equation

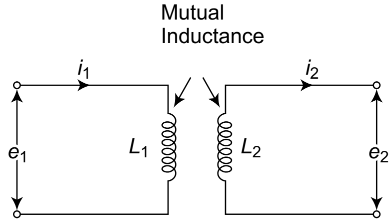


Figure 5.14 Equivalent Circuit of an Electrodynamometer Instrument

Let i_1 and i_2 be the instantaneous currents flowing through the fixed and moving coils respectively; L_1 and L_2 be the self-inductances of the fixed and moving coils respectively and M be the mutual inductance existing between the fixed and moving coils. The equivalent circuit of an electrodynamometer instrument is shown in Figure 5.14.

The flux linkages of coil 1 and coil 2 are given by

$$\phi_1 = L_1 i_1 + M_1 i_2 \text{ and } \phi_2 = L_2 i_2 + M_2 i_1 \tag{5.12}$$

Now, the induced emfs in the fixed and moving coils are given by

$$e_1 = \frac{d\phi_1}{dt} \text{ and } e_2 = \frac{d\phi_2}{dt} \tag{5.13}$$

The electrical input energy is given by

$$e_i = e_1 i_1 dt + e_2 i_2 dt$$

Using Eqn. (5.13) in the above equation, we get

$$e_i = i_1 d\phi_1 + i_2 d\phi_2$$

Substituting Eqn. (5.12) in the above equation, we get

$$e_i = i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1) \tag{5.14}$$

$$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1$$

The energy stored in the magnetic field due to L_1, L_2 and M is given by

$$e_s = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M$$

The change in stored energy is given by

$$de_s = d\left[\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M\right]$$

$$= i_1 L_1 di_1 + \frac{1}{2} i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \tag{5.15}$$

According to the principle of conservation of energy, we get

Input energy = Energy stored + Mechanical energy

Therefore, Mechanical energy = Input energy – Energy stored
Substituting Eqn. (5.14) and Eqn. (5.15) in the above equation, we get

$$\text{Mechanical energy} = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM$$

Since the self-inductances L_1 and L_2 are constant, $dL_1 = dL_2 = 0$. Therefore, the mechanical energy = $i_1 i_2 dM$.

If T_d is the instantaneous deflecting torque and $d\theta$ is the change in deflection, then the mechanical work done is given by $T_d d\theta$. It is known that the mechanical energy is equal to the mechanical work done, and is given by

$$i_1 i_2 dM = T_d d\theta$$

Therefore, the instantaneous deflecting torque of the instrument becomes

$$T_d = i_1 i_2 \frac{dM}{d\theta}$$

DC Operation

In DC operation, $i_1 = I_1$ and $i_2 = I_2$. Therefore, the deflecting torque is given by

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

If the controlling torque, $T_C = k_s\theta$ is provided by the springs, then at the balanced condition, we get

$$T_d = T_C$$

Therefore, $k_s\theta = I_1 I_2 \frac{dM}{d\theta}$. Hence, the deflection of the pointer or the moving coil is

$$\theta = \frac{1}{k_s} I_1 I_2 \frac{dM}{d\theta}$$

From the above equation, it is clear that in DC operation, the deflection of the moving coil is proportional to the fixed and moving-coil currents and the rate of change of mutual inductance.

AC Operation

In AC operation, the total deflecting torque over a cycle is obtained as

$$T_d = \frac{1}{T}\int_0^T i_1 i_2 \frac{dM}{d\theta} dt \tag{5.16}$$

where, T is the time period of AC quantity.

If the fixed and moving-coil currents are sinusoidal and is displaced by a phase angle, then

$$i_1 = I_{m1} \sin \omega t \text{ and } I_2 = I_{m2} \sin(\omega t - \phi) \tag{5.17}$$

where, I_{m1} and I_{m2} are the maximum values of respective currents.

Substituting Eqn. (5.16) in Eqn. (5.17), we get

$$T_d = \frac{1}{T}\frac{dM}{d\theta}\int_0^T I_{m1} I_{m2} \sin(\omega t)\sin(\omega t - \phi)\, d\omega t$$

Solving the above equation, we get

$$T_d = \frac{I_{m1} I_{m2}}{2}\frac{dM}{d\theta}\cos\phi$$

Let I_1 and I_2 be the RMS values of the currents and they are given by $I_1 = \frac{I_{m1}}{\sqrt{2}}$ and $I_2 = \frac{I_{m1}}{\sqrt{2}}$. Substituting this in the above equation, we get

$$T_d = I_1 I_2 \frac{dM}{d\theta}\cos\phi$$

If the controlling torque, $T_C = k_s\theta$ is provided by the springs, then at the balanced condition, we get

$$T_d = T_C$$

Therefore, $K_S\theta = I_1 I_2 \dfrac{dM}{d\theta}\cos\phi$. Hence, the deflection of the pointer or the moving coil is

$$\theta = \frac{1}{k_s} I_1 I_2 \cos\phi \frac{dM}{d\theta}$$

From the above equation, it is clear that in AC operation, the deflection of the moving coil is proportional to the fixed and moving-coil currents, cosine of the phase angle and the rate of change of mutual inductance.

5.9.4 Errors in Electrodynamometer Instrument

The main sources of error in a dynamometer type of instruments are:

1. ***Torque-to-weight ratio:*** Large number of turns provided in the air-cored coils increase the magnetic field strength to produce a reasonable deflecting torque, leading to a small torque-to-weight ratio, which in turn leads to large frictional losses when compared to other instruments.
2. ***Frequency:*** The frequency error, which leads to changes in the self-inductance of the coils, is due to: (a) change in ratio of coils and (b) change in the eddy currents set up instrument. Having equal time-constants in the coils can reduce these errors.
3. ***Eddy current:*** The eddy current produced in the instrument interacts with the main current, which reducing the deflecting torque and causes an error in the instrument. Using metals with high resistivity in the instrument can reduce this error.
4. ***External magnetic fields:*** It interacts with the instrument's magnetic field and causes error in the deflection. Using shields in the instrument can reduce this error.
5. ***Temperature change:*** Heavy currents carried by coils produce heat and leads to change in the resistance of the coil, which causes errors in the instrument. Using temperature-compensating resistors in the instrument can reduce this error.

5.9.5 Advantages, Disadvantages and Applications of Electrodynamometer Instrument

Advantages

1. Free from hysteresis and eddy current losses as it has air-cored coils.
2. Has precision-grade security.
3. Can be used on both AC and DC systems.
4. Free from hysteresis errors.
5. Low power-consumption.
6. Light in weight.

Disadvantages

1. Has frictional errors, which reduce sensitivity of the instrument.
2. Non-uniformity exists in the scale.
3. Requirement of screening process to avoid the stray-field effect.
4. More expensive when compared to PMMC or MI type instruments.
5. Low torque-to-weight ratio.
6. Sensitive to overloads and mechanical impacts.

Applications

The electrodynamometer instrument can be used as:

1. Ammeter: Same current will pass through the coils.
2. Voltmeter: Coils are connected in series with high resistance.
3. Wattmeter

5.9.6 Comparison between Moving-Coil and Moving-Iron Instrument

Table 5.1 lists the comparison between the moving-coil and moving-iron instruments.

Table 5.1 *Comparison between the moving-coil and moving-iron instrument*

Moving-Coil Instrument	Moving-Iron Instrument
More accurate	Less accurate
Costly	Cheap
Uniformly distributed reading scale	Absence of uniformity in the scale
Very sensitive	Robust in construction
Power consumption is low	Power consumption is high
Uses eddy current damping	Uses air-friction damping
Can be used only for DC	Can be used on AC and DC
Spring arrangement provides the controlling torque	Gravity or spring arrangement provides the controlling torque
Deflection is directly proportional to current, $\theta \alpha I$	Deflection is directly proportional to square of the current, $\theta \alpha I^2$
Errors are set due to ageing of control springs, permanent magnet	Errors are set due to hysteresis and stray fields

5.10 Electrothermic Instruments

The instruments whose operation depends on the heating effect of the current to be measured are known as thermal or electrothermic instruments. With the help of these instruments, it is possible to measure the current at high frequencies with great accuracy. These instruments are used to measure the current at higher frequencies and at moderate frequencies and it is also used for precise voltage measurements. The different types of electrothermic instruments are:

- Hot-wire instrument: Based on the property that when current passes through the wire, it gets heated up and expands.
- Thermocouple instrument: Based on the principle of thermocouple.

5.10.1 Hot-Wire Instrument

The instrument, which uses the heating effect of the electrical quantity to be measured for knowing their magnitude, is known as a hot-wire instrument. Its working principle is that the length of the wire gets increased because of the heating effect when the electrical quantity is allowed to flow through it. This instrument can be used for both the AC and DC quantities.

Construction

The schematic diagram of the hot-wire instrument is shown in Figure 5.15.

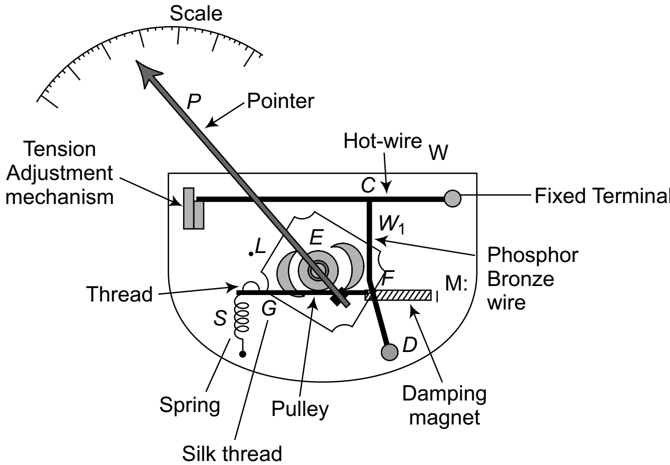


Figure 5.15 Hot-wire Instrument

The current whose magnitude is to be determined is passed through a very fine hot-wire with a diameter of the order of 0.1 mm. The hot-wire used in this instrument is made up of a platinum-iridium alloy, as it has the capability to withstand high temperatures without oxidation or deterioration of wire. The hot-wire is stretched between the fixed terminal and a tension adjustment mechanism. One end of the wire, W_1, which is made up of phosphor-bronze, is connected to the hot-wire at a particular point C, and the other end is connected to a fixed point, D. A fine silk thread, G, is connected between the spring and point F and is wound around the pulley E. The contraction of hot-wire takes place with the help of spring S. The point F and spring S are fixed in this instrument. An aluminium disc L, a pointer P and pulley E are mounted on the spindle. The aluminium disc L, which rotates between the poles of the permanent magnet M, provides the eddy current damping to the instrument.

Working Principle

When the current whose quantity to be measured is allowed to flow through the hot-wire, it expands due to the heating effect. This heating effect increases the sag of the hot-wire, W, and the sag of the wire W_1 attached to W. This sag in wire W_1 is taken up the silk thread, which allows the spring S to pull after it passes through the pulley. Since the spring S pulls the thread, the pulley moves and the pointer attached to the pulley gets deflected. The deflection produced in the pointer is directly proportional to the expansion and contraction of the wire, which is directly proportional to the square of the RMS value of the current. The pointer regains its original position with the help of the spring.

Torque Equation of Hot-wire Instrument

The amount of heat H generated in the hot-wire, W is given by

$$H = I_{\text{RMS}}^2 R$$

where, I_{RMS} is the RMS value of the current flowing through the hot-wire W and R is the internal resistance of the wire.

Therefore, the deflecting torque T_d developed in the instrument is proportional to the square of RMS value of the current flowing through the wire, as given by

$$T_d \,\alpha\, I_{\text{RMS}}^2$$

The controlling torque provided by the spring is given by

$$T_C = k_s \theta$$

Since, at balanced condition, both the deflecting and controlling torque are equal, we get

$$T_C = T_d$$

Substituting the known values in the above equation, we get the deflection of the pointer as

$$\theta \,\alpha\, I_{\text{RMS}}^2$$

It is evident from the above equation that the hot-wire instrument has non-uniform scale and the reading is independent of the frequency and waveform.

Advantages and Disadvantages of Hot-wire Instrument

Advantages

1. The hot-wire instrument can be used in AC and DC systems, as the deflection is independent of frequency and waveform of the current to be measured.
2. Instrument is free from stray magnetic field, as it operates based on heating effect.
3. Requires same calibration for AC and DC systems.
4. No significant error exists based on temperature.
5. Can be used for measuring current at very high frequencies.
6. High accuracy, very simple and cheap in cost.

Disadvantages

1. As a thin wire is used in this instrument, it cannot withstand overloads.
2. Does not have uniform scale.
3. Consumes very high power.
4. Very delicate instrument.
5. Response time of the instrument is high.
6. Presence of hysteresis error i.e., it does not provide same deflection for ascending and descending values of current.
7. Reading depends on atmospheric temperature.

5.10.2 Thermocouple Instrument

The thermocouple instrument is used to measure the temperature in addition to the AC and DC values of current and voltage. Its working principle is that, if two dissimilar metals such as iron and a copper-nickel alloy are joined together to form a thermocouple junction and are subjected to change in temperature, a thermo-electric emf is generated, which is proportional to the difference in temperature, that is indicated on the PMMC instrument.

Construction

The schematic diagram of a thermocouple instrument is shown in Figure 5.16. The essential components of thermocouple instrument are: (a) heater element (b) thermocouple and (c) an indicating instrument like a PMMC.

- ***Heater element:*** A fine wire made up of an alloy with zero temperature-coefficient of resistance and a very small cross-sectional area that carries the current to be measured forms the heater element. The materials used for heater element are non-magnetic, like constantan or platinum-iridium alloy. This type of arrangement is used to reduce the skin effect.

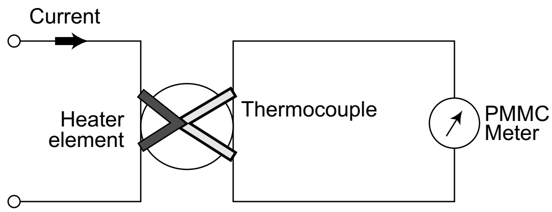


Figure 5.16 Thermocouple Instrument

- ***Thermocouple:*** A junction where two dissimilar metals like iron and copper-nickel alloy are joined is known as thermocouple. It will be in contact with the heater element subjected to temperature rise due to the flow of current, and they produce an emf proportional to the rms value of the current, as indicated by the PMMC meter. The different types of contact between the heater element and the thermocouple are: (i) contact type (ii) compensated type, (iii) non-contact type and (iv) bridge type.
- ***Indicating instrument:*** A very sensitive indicating instrument like a PMMC is used in this instrument to sense the emf generated at the thermocouple. The combination of heater element along with thermocouple is called a thermo-element.

Working of Thermocouple Instrument

The current whose magnitude is to be measured is allowed to flow through the heater element. This produces heat in the heater element, and hence a thermo-electric emf is induced in the output terminals of the thermocouple. The PMMC instrument connected across the thermocouple measures the emf whose magnitude is proportional to the rms value of the current and causes deflection in the instrument for the measurement of current.

Deflection Equation in Thermocouple Instrument

The thermoelectric emf generated in a thermocouple, which is parabolic in nature is proportional to the difference of temperatures of hot and cold junctions existing in the thermocouple, as given by

$$e = a(T_1 - T_2) + b(T_1 - T_2)^2 \tag{5.18}$$

where, a, b are the constants of depending metals and T_1 and T_2 are the temperatures at hot and cold junctions respectively. The constant a varies between 40 to 50 microvolt per °C difference of temperature and the constant b varies between few tenths or hundredths of a microvolt per square (°C).

Therefore, $e = a\Delta t + b(\Delta T)^2$ (5.19)

Where, the temperature difference is given by $\Delta T = T_1 - T_2$
Since the heat produced is directly proportional to square of the rms value of the current, the difference in temperature is directly proportional to I^2R where I is the rms value of the current and R is the resistance of the heater element as given by

$$\Delta T \propto I^2R \text{ i.e., } \Delta T = K_1 I^2 R \tag{5.20}$$

In practice, the value of constant b is very small and can be neglected. Therefore,

$$e = a\Delta T = aK_1 I^2 R \tag{5.21}$$

This emf drives the PMMC instrument to cause the deflection, which is directly proportional to this emf, as given by

$$\theta \propto e$$

i.e., $$\theta = K_2 e = K_1 K_2 a I^2 R = K_3 I^2 \tag{5.22}$$

where $K_3 = K_1\ K_2\ a\ R$ = constant

Advantages, Disadvantages and Applications of Thermocouple Instrument

Advantages

1. The rms value of voltage and current is indicated directly.
2. Free from the stray magnetic field.
3. Used for a wide range of current measurement.
4. High sensitivity and high accuracy.
5. Used in calibrating the potentiometer with the help of the standard cell.
6. Unaffected by frequency, since it has very small inductance and capacitance.
7. Since it is free from frequency error, it can be used for a wide range of frequency.

Disadvantages

1. Small overload capacity due to the square-law response of the instrument.
2. Large amount of heat is produced at large currents, which increases the temperature to its maximum limit.
3. Poor thermal conversion efficiency.
4. High power losses.
5. Careful handling of PMMC instrument is required

Applications

1. Used for measurement of current from power frequencies i.e., up to 100 MHz.
2. Used in pyrometers.

5.11 Electrostatic Type Instrument

The instruments used to measure the AC or DC quantities with the help of electric field effect are known as electrostatic instruments i.e., the operating principle of these instruments is based on the interaction between the electrodes, which carry opposite electric charges. Generally, these instruments are used for measuring high voltages, but in some cases, they are used to measure low voltages and powers of a given circuit. There exist two types of movements in electrostatic instruments i.e., the force of attraction or repulsion between the charge-carrying electrodes causes linear motion and rotary motion.

Based on the movement of electrodes, the two different types of electrostatic instruments are:

- Linear electrostatic instrument
- Rotary electrostatic instrument

5.11.1 Linear Electrostatic Instrument

The schematic diagram of a linear electrostatic instrument is shown in Figure 5.17.

This instrument has a fixed plate and a movable plate. The movable plate is attached to the spring to provide the controlling torque. The pointer is attached to the moving plate, which moves on a non-uniform scale. When these two plates are charged by a high voltage battery, as shown in Figure 5.17, the fixed plate gets a negative charge, while the movable plate gets a positive charge. It is known that there exists a force of attraction between the opposite charges and hence the movable plate moves towards the fixed plate until it gains maximum electrostatic energy. The pointer, which is attached to the moving plate, moves accordingly. The spring attached to the moving plate provides the controlling torque, thereby halting the pointer at the reading, which corresponds to the magnitude of high voltage applied between the plates.

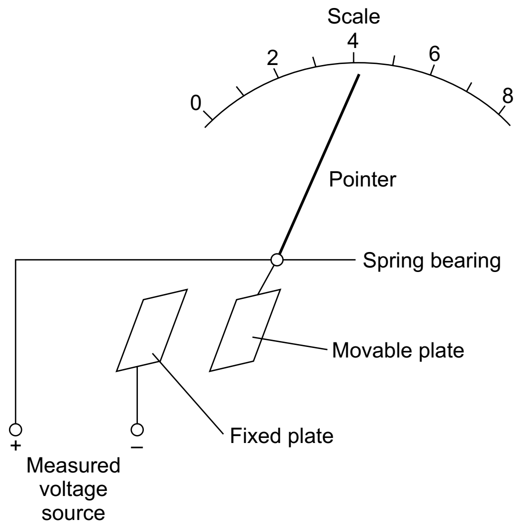


Figure 5.17 Linear Electrostatic Instruments

Deflection Equation of Linear Electrostatic Instrument

If C is the capacitance between the plates in Farad and V is the applied voltage between the plates, then the initial energy stored in the plates is given by

$$E_i = \frac{1}{2}CV^2 \text{ joules} \tag{5.23}$$

If there is a small increment in the applied voltage of magnitude dV, then the moving plate moves through a small distance dx towards the fixed plate. It is known that, the capacitive current flows in the circuit if the voltage is increased and it is given by

$$i = \frac{dq}{dt}$$

where, q is the charge between the plates i.e., $q = CV$.

Therefore, $$i = \frac{CdV}{dt} + V\frac{dC}{dt} \tag{5.24}$$

Now, the final energy stored in the plates is given by

$$E_f = Vidt$$

Substituting Eqn. (5.24) in the above equation, we get

$$E_f = CVdV + V^2dC$$

Since there is a change in capacitance between the plates, the final energy stored in the plates is also given by

$$E_f = \frac{1}{2}(C + dC)(V + dV)^2 \tag{5.25}$$

Therefore, the change in stored energy in the plates is given by

$$\Delta E = E_f - E_i$$

Substituting Eqns. (5.23) and (5.25) in the above equation and neglecting higher-order terms, we get

$$\Delta E = \frac{1}{2}V^2dC + CVdV$$

The mechanical work done in the system when there is an increment in the voltage applied is

$$W_m = Fdx$$

where, F is the force of attraction existing between the plates and dx is the linear distance travelled by the moving plate.

According to the law of conservation of energy, we get

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + Fx$$

Therefore, the force of attraction existing between the plates is given by

$$F = \frac{1}{2}V^2\frac{dC}{dx}$$

If the controlling force provided by the spring is given by $F_c = k_s x$, then at equilibrium point, the distance travelled by the plate is given by

$$x = \frac{1}{2k_s}V^2\frac{dC}{dx}$$

Since the distance is proportional to the square of the voltage to be measured, the instrument can be used for both AC and DC quantities. The instrument exhibits a square-law response and hence the scale is non-uniform.

5.11.2 Rotary Electrostatic Instrument

The schematic diagram of a rotary electrostatic instrument is shown in Figure 5.18.

The construction and working of a rotary electrostatic instrument are same except that the plates are fixed on the shaft and due to the force of attraction or repulsion there exist an angular displacement θ instead of linear displacement.

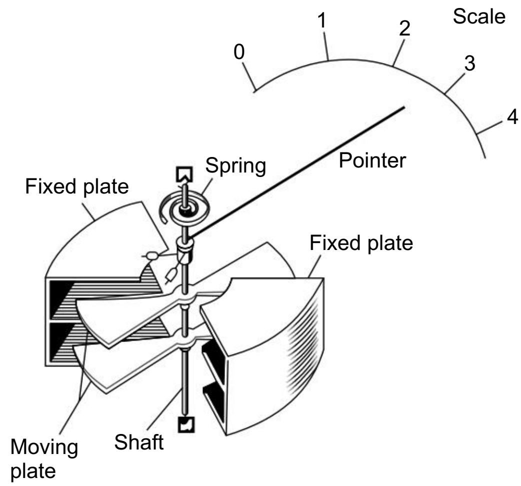


Figure 5.18 Rotary Electrostatic Instrument

Torque Equation of Rotary Electrostatic Instrument

Similar to linear electrostatic instrument, the deflecting torque experienced by the moving plate is given by

$$T_d = \frac{1}{2}V^2\frac{dC}{d\theta}$$

where, $d\theta$ is the change in angular displacement of the moving plate when the voltage is applied to it.

Therefore, the angular deflection experienced by the instrument is given by

$$\theta = \frac{1}{2}\frac{V^2}{k_s}\frac{dC}{d\theta}$$

Advantages and Disadvantages of Electrostatic Instrument

Advantages

1. Can be used in both AC and DC.
2. Absence of frequency and hysteresis error.
3. As the instrument works on electrical effect, there is no stray magnetic field.
4. Used for measuring higher voltages.
5. Low power-consumption.

Disadvantages

1. Non-uniform scale.
2. Size of the instrument is large.
3. Cost of the instrument is high.
4. Small operating force.

5.12 Induction Type Instruments

Induction-type instruments have a wide range of applications. They are used to measure AC quantities and can be used as either an ammeter or a voltmeter or a wattmeter. But, it is widely used as a wattmeter or an energy meter.

When a conducting material in the form of a drum or a disc is placed in an alternating magnetic field, eddy current gets induced in the material. The eddy current is proportional to the magnetic field produced by the electrical quantity, which is to be measured. The interaction between this eddy current and the magnetic field develops the deflecting torque, which acts on the conductor material and makes it to rotate. Based on the way in which the deflecting torque is produced in the induction type instrument, it is classified as:

- Shaded-pole induction type
- Split-phase induction type

5.12.1 Shaded-pole Induction Type Instrument

The schematic diagram of a shaded-pole induction type instrument is shown in Figure 5.19. The shaded-pole induction type instrument uses a single winding to produce a flux in the electromagnet, which has an

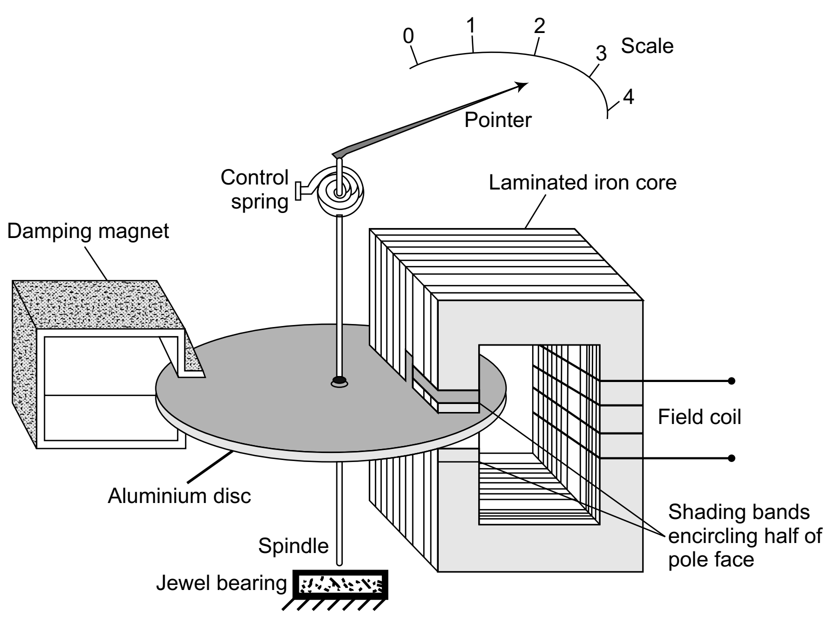


Figure 5.19 Shaded-pole Induction Type Instrument

air gap in one limb. But, it is known that the induction-type instrument works on the principle of interaction between the two fluxes. To produce another flux, which differs in phase by 90° with the main flux, a copper-shading band, which acts as a single-turn short-circuited secondary winding of the transformer is placed in the pole faces of the electromagnet. Therefore, there exist two fluxes of shaded and un-shaded portions of the electromagnet. The damping magnet provides the damping torque required for the instrument. A metallic aluminium disc rotates between the pole faces of the electromagnet and the damping magnet. The aluminium disc is mounted on pivots and jewel bearings. Two spiral springs, which are wound in opposite direction to each other, provide the controlling torque. The spiral springs, the pointer and the scale are mounted on the spindle.

Working Principle

When the electrical quantity to be measured is allowed to pass through the coil in the electromagnet, total flux ϕ is produced in the electromagnet. Due to the presence of copper-shading band, the total flux gets split into two fluxes i.e., flux through un-shaded portion and other through the shaded portion, ϕ_1. This flux ϕ_1 induces an emf in the copper band which lags ϕ_1 by 90°. Due to this induced emf, a current i is allowed to flow through the copper band, which will produce its own magnetic field ϕ_2', which is in phase with current i. The total flux given in the shaded portion of the pole, ϕ_2, is the vector sum of ϕ_1 and ϕ_2', which lags behind ϕ_1 by θ, whose value will be 40° to 60° to produce an effective deflecting torque.

The fluxes ϕ_1 and ϕ_2 induce emfs e_1 and e_2 in the aluminium discs, which lag by 90° with their respective fluxes. These induced emfs induce eddy currents i_1 and i_2 in the discs, which lag behind the respective voltages by a small angle due to the presence of inductance. Hence, these two torques act on the aluminium disc and produce the operating or deflecting torque. The pointer attached to the aluminium disc deflects based on the resultant deflecting torque, which measures the electrical quantity to be measured.

5.12.2 Split Phase Induction Type Instrument

The schematic diagram of a split-phase induction type instrument is shown in Figure 5.20.

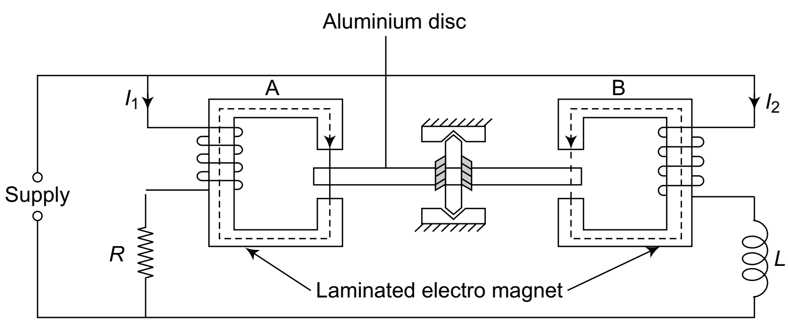


Figure 5.20 Split-phase Induction Type Instrument

This is also called a Ferraris-type instrument. It consists of two laminated magnets A and B, with air gap in one limb, which are placed very closely with an aluminium disc in between. A non-inductive resistance R is connected in series with the coil of magnet A and an inductive coil L is connected in series with the coil of magnet B to create a phase difference of 90° with the fluxes in the respective magnets. The aluminium disc is mounted on pivots and jewel bearings. The spring arrangement is used to provide the required controlling torque. The spiral springs, the pointer and the scale are mounted on the spindle. The working of a split-phase induction type instrument is same as that of the shaded-pole induction type, except for the fact that the two fluxes required to develop the necessary deflecting torque are produced by the electromagnets *A* and *B*.

Torque Equation in Induction Type Instrument

The deflecting torque produced in the induction type instrument is directly proportional to the two fluxes, as given by

$$T_d \alpha \phi_1 \phi_2 \sin \alpha$$

where, α is the phase angle difference between the two fluxes.
Since the same current produces the two fluxes, the deflection torque is given by

$$T_d \alpha I^2$$

The controlling torque produced by the spring arrangement in the instrument is given by

$$T_C \alpha k_s \theta$$

Since, at equilibrium condition, the deflecting and controlling torque are equal, the deflection produced in the instrument based on the electrical quantity to be measured is given by

$$\theta \alpha I^2$$

5.12.3 Errors in Induction Type Instrument

The two different errors, which exist in induction type instruments are:

1. ***Frequency error:*** Since the deflecting torque depends on the frequency, large error will be present in the reading if the instrument is not calibrated to the same frequency of the electrical quantity to be measured. This type of error can be compensated by using an inductive shunt in case of ammeters.
2. ***Temperature error:*** Errors exist in the instrument due to the variation of temperature caused by the eddy current. These errors can be compensated by using a shunt in case of ammeters and a combination of shunt and swamping resistance in case of voltmeters

5.12.4 Advantages, Disadvantages and Applications of Induction Type Instrument

Advantages

1. Effective and efficient damping.
2. Full-scale deflection of more than 200^o can be obtained.
3. Absence of moving iron.
4. High torque-to-weight ratio.
5. No electrical contact exists between the moving element and the circuit.
6. High accuracy with a wide load range.
7. As the operating fields are large, there exists very less stray magnetic field.

Disadvantages

1. Power consumption is large.
2. Variation in temperature and frequency may cause serious errors if necessary compensations are not provided.
3. Applicable only to AC supply, as it is based on induction principle.
4. Scale is non-uniform.
5. Cost is high.

Applications

It can be used as an ammeter, a voltmeter, a wattmeter and a watt-hour meter to measure AC current, voltage, power and energy respectively.

5.13 Rectifier Type Instruments

The instruments used to measure AC voltages and currents with the help of rectifier element and a PMMC instrument are known as rectifier type instruments. Rectifier element is a device, which converts AC quantity to a DC quantity; and PMMC is an instrument, which can sense only DC quantities. Usually, the AC meters are calibrated in such a way that it shows the RMS value or peak value of the input. Therefore, it is necessary that the rectifier type instrument should be calibrated in such a way that the PMMC instrument reads either the RMS value or peak value of the AC quantity. Hence, a careful calibration of rectifier type instrument is necessary.

5.13.1 Calibration of Rectifier Type Instruments

The relation between the RMS and peak value of the pure sinusoidal AC quantity is given by

$$V_{\mathrm{RMS}} = 0.707\ V_m$$

where, V_{RMS} is the RMS value of the AC quantity and V_m is the maximum value of the AC quantity.
Similarly, the relation between the average and peak values of pure sinusoidal AC quantity is given by

$$V_{av} = 0.636\ V_m$$

where, V_{av} is the average value of the AC quantity.

The form factor and peak or crest factor of the pure sinusoidal AC quantity is given by

$$K_f = \frac{\text{RMS Value}}{\text{Average Value}} = 1.11$$

and $$K_p = \frac{\text{Maximum Value}}{\text{RMS Value}} = 1.414$$

where, K_f is the form factor and K_p is the peak or crest factor.

Therefore, if the rectifier type instrument has to read the RMS value of the AC quantity, then the instrument must be calibrated by form factor, K_f and if the same instrument has to read the peak value of AC quantity, then the instrument must be calibrated by peak or crest factor, K_p.

5.13.2 Types of Rectifier Type Instruments

Based on the rectifier element, the different types of rectifier type instruments are:

- AC voltmeter using half-wave and full-wave rectifiers
- AC ammeter using full-wave rectifier

AC Voltmeter using Half-wave and Full-wave Rectifiers

The schematic diagrams of AC voltmeter using half-wave rectifier and full-wave rectifier are shown in Figures 5.21 (a) and (b) respectively.

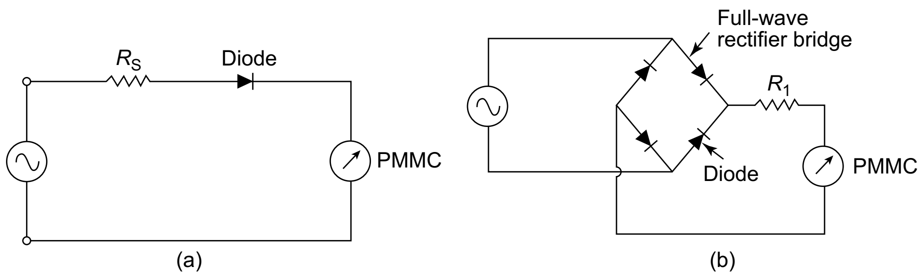


Figure 5.21 AC Voltmeter Using (a) Half-wave and (b) Full-wave Rectifiers

The working of the above instruments can be studied by knowing the working of diode and PMMC instrument. The input and output waveforms for the circuit diagram shown in Figures 5.21(a) and (b) are shown in Figures 5.22 (a) and (b) respectively.

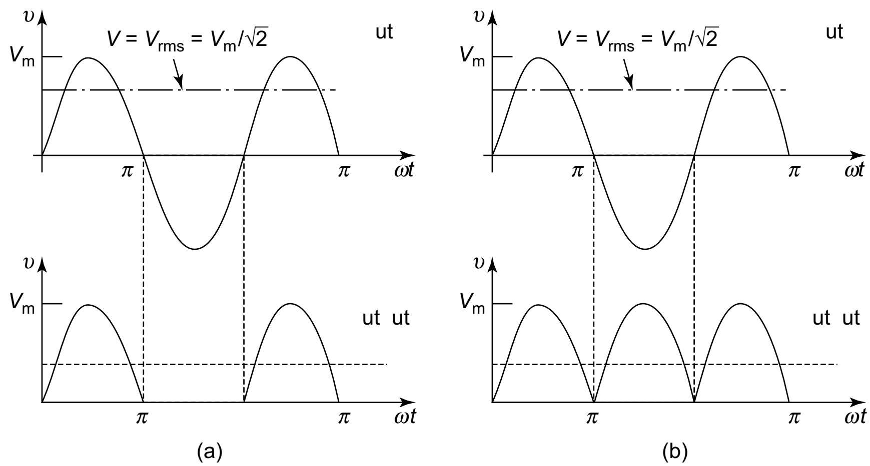


Figure 5.22 Waveforms of (a) Half-wave and (b) Full-wave Rectifiers

AC Ammeter using Full-wave Rectifier

The schematic diagram of an AC ammeter using a full-wave rectifier, along with its operation, is shown in Figure 5.23.

The expression for the deflecting torque and the angle of deflection of the pointer for the rectifier type instruments is same as that of PMMC instruments.

5.13.3 Advantages, Disadvantages and Applications of Rectifier Type Instrument

Advantages

1. More sensitive when compared to other instruments.
2. Possible to extend the frequency range of operation.
3. Uniform scale
4. Low power-consumption.

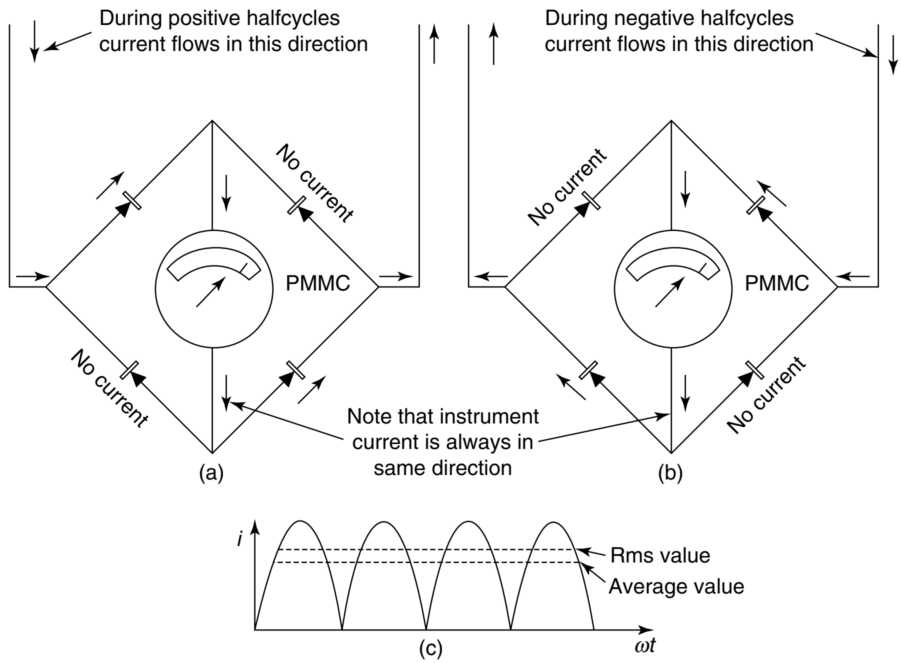


Figure 5.23 AC Ammeter Using Full-wave Rectifier

Disadvantages

1. Accurate only on the waveforms for which they are calibrated.
2. As it is more temperature sensitive, readings are affected by large variations in temperature.
3. Since the rectifier element becomes too bulky, it is not suitable for measuring large currents.
4. Also, providing shunts is impracticable due to rectifier characteristics.
5. They have low operating value of current and voltage

Applications

1. Best suitable for measuring AC voltages in the range of 50–250 V.
2. Can be used as a micro-ammeter or low milli-ammeter, whose range is up to 10–15 mA.
3. Used in measuring high-impedance circuits at low and audio frequencies.
4. Commonly used in communication circuits because of their high sensitivity and low power-consumption.

The various types of indicating instruments along with their suitability for the type of measurements, types of control, types of damping, special features and applications are given in the Table 5.2.

Table 5.2 *Different Indicating Instruments*

S.No	Type of Instrument	Suitability of Measurements	Type of Control	Type of Damping
1	Moving iron	DC or AC measurements (current & voltage)	Spring or gravity control	Air friction
2	Moving-coil Permanent magnet type	DC Measurements (current &voltage only)	Spring	Eddy current

(*Contd.*)

3	Moving-coil Dynamometer	DC or AC measurements (current, voltage & power)	Spring	Air friction
4	Thermal Hot-wire	DC or AC measurements (current and voltage)	Spring	Eddy current
5	Thermocouple			
6	Electrostatic	DC&AC measurement (voltage and current)	Gravity or spring	Air friction
7	Induction	AC measurements only (current, voltage, power and energy)	Spring	Eddy current
8	Rectifier	DC or AC (current and voltage)	Spring	Eddy current

5.14 Ammeter

5.14.1 Permanent Magnet Moving Coil (PMMC)

In a PMMC meter, as shown in Figure 5.24, there is a coil suspended in the magnetic field of a permanent magnet in the shape of a horse shoe. The coil is suspended so that it can rotate freely in the magnetic field. When a DC current flows in the coil, an electromagnetic torque (EM) is developed and the coil is deflected. The EM torque is counter-balanced by the mechanical torque of the control springs attached to it so that the angular position of the movable coil is indicated by a pointer against a fixed reference called a scale.

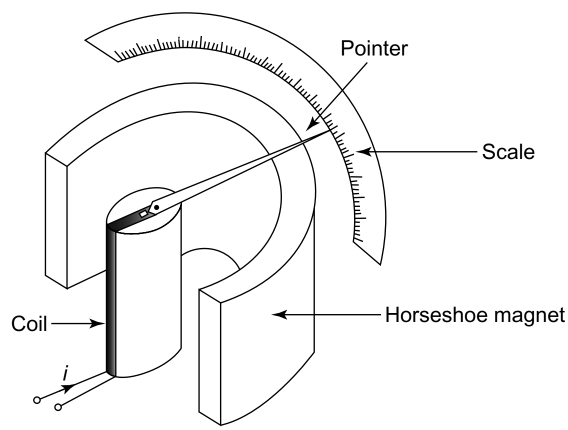


Figure 5.24 Basic PMMC instrument

The equation for the developed torque is

$$T = B \cdot A \cdot I \cdot N$$

where, T is the torque in Newton-Metre (N-m), B is the flux density in the air gap in Weber/square metre or Tesla, A is the effective coil area in square metre (m^2), I is the current in the movable coil in ampere (A) and N is the number of turns of the coil.

The basic PMMC movement is often called d'Arsonval movement, named after its inventor.

5.14.2 DC Ammeter

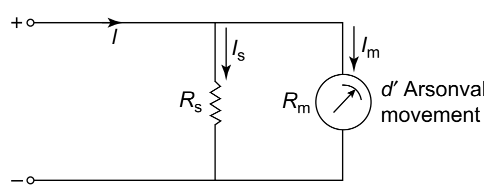


Figure 5.25 Basic DC Ammeter Circuit

Referring to Figure 5.25, the DC ammeter is constructed using the basic movement. But the instrument with the basic movement alone can carry only small current for full-scale deflection because the coil winding is small and light. When large currents are to be measured, it is necessary to bypass the excess current through a resistance called "shunt", so that the current through the coil of the basic movement does not exceed its maximum limit.

The resistance of the shunt can be calculated by applying conventional circuit analysis. Since the shunt resistance is in parallel with the meter movement, the voltage drop across the shunt and movement must be same.

$$V_{\text{shunt}} = V_{\text{movement}}$$

Therefore, $$I_s R_s = I_m R_m \text{ and } R_s = \frac{I_m R_m}{I_s}$$

Since $$I_s = I - I_m,\ R_s = \frac{I_m R_m}{I - I_m}$$

5.14.3 Multi-range Ammeter

Having a number of shunts, selected by a range switch, can extend the current range of the DC ammeter.

Figure 5.26 shows the diagram of a multi-range ammeter. The circuit has three shunts, R_a, R_b, and R_c, which are placed in parallel with the basic movement to give three different current ranges. Switch S is a multi-position make-before-break type. Hence, the movement of the pointer does not get damaged because the range of movement is always restricted by the shunt.

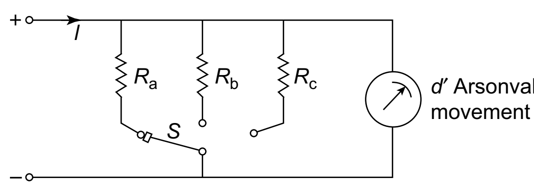


Figure 5.26 Multi-range Ammeter

5.14.4 Ayrton Shunt

Ayrton or universal shunt eliminates the possibility of having the meter in the circuit without a shunt and is shown in Figure 5.27.

For example, the d'Arsonval movement is selected with the internal resistance R_m = 50 W and full-scale deflection current I_{fsd} = 1 mA. If this ammeter is extended with the current ranges of I_1 = 1 A, I_2 = 5 A, I_3 = 10 A, the following design procedures are to be followed.

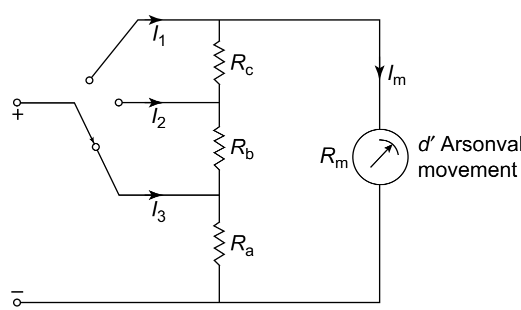


Figure 5.27 Ayrton Shunt

For the (0–I_1) range, R_a, R_b and R_c are included and are in parallel with the 50 W movement. Since I_{fsd} is 1 mA, the shunt will be required to pass a current of 1 A–1 mA = 999 mA. Voltage drop across the basic movement V_{m} is 1 mA × 50 W = 50 mV. The voltage across the shunt and the basic movement is the same.

Therefore, $$R_a + R_b + R_c = V_m/(I_1 - I_{fsd}) = \frac{1 \times 10^{-3} \times 50}{999 \times 10^{-3}}$$

Similarly, for (0–I_2) range, R_a and R_b are included and are in parallel with the basic movement and R_c is in series with R_m. Therefore,

$$R_a + R_b = \frac{I_{fsd}(R_c + R_m)}{I_2 - I_{fsd}}$$

For the (0–I_3) range, R_a alone serves as the shunt, and R_b and R_c are in series with R_m.

Therefore, $R_a = \dfrac{I_{fsd}(R_b + R_c + R_m)}{I_3 - I_{fsd}}$

Solving the three simultaneous equations, the values of R_a, R_b and R_c can be calculated.

5.15 DC Voltmeter

Basic d'Arsonval movement is used to measure DC voltage, but voltage across the meter, $V_m = I_{fsd}.\ R_m$, is small. If the voltage to be measured is greater than V_m, the excess voltage is allowed to drop across the series resistance or "multiplier" so that the current through the basic movement is not exceeding the value of full-scale deflection current.

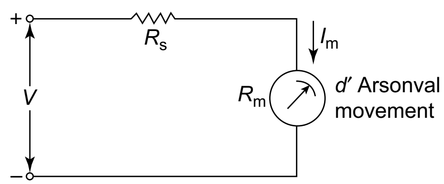


Figure 5.28 Basic DC Voltmeter

The basic DC voltmeter is shown in Figure 5.28. The DC voltmeter measures the potential difference between two points in a DC circuit and is, therefore, connected across a source of emf or a circuit component with correct polarity.

$$V = I_m\,(R_s + R_m)$$

$$R_s = \frac{V - I_m R_m}{i_m}$$

The multiplier is usually mounted inside the case of the voltmeter for a moderate range of up to 500 V. For higher voltages, the multiplier may be mounted separately outside the case, on a pair of binding posts to avoid excess heating inside the case.

5.15.1 Multi-range Voltmeter

The addition of a number of multipliers, together with a range switch, provides the instrument with a workable number of voltage ranges. Figure 5.29 shows a multi-range voltmeter using a four-position switch and four multipliers, R_1, R_2, R_3, and R_4, for the voltage ranges V_1, V_2, V_3 and V_4 respectively. The values of the multiplier can be calculated using the method shown earlier.

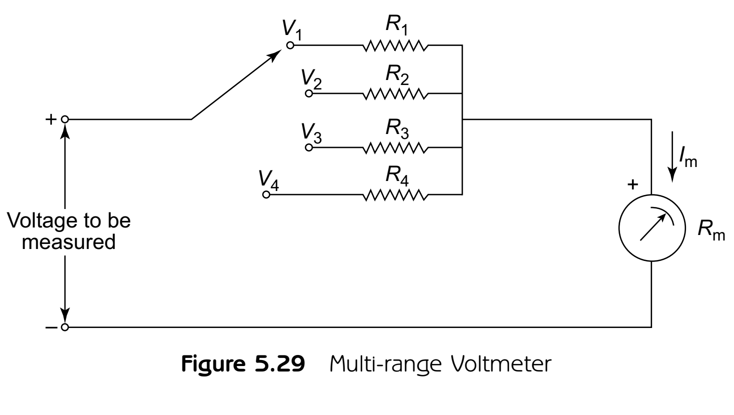


Figure 5.29 Multi-range Voltmeter

In an alternate method, as shown in Figure 5.30, multipliers are connected in a series string and the range selector switches the appropriate amount of resistance in series with the movement. This system has the advantage that all multipliers except the first have standard resistance values and can be obtained commercially in precision tolerances. The low-range multiplier, R_4, is the only special resistor that must be manufactured to meet the specific circuit requirements.

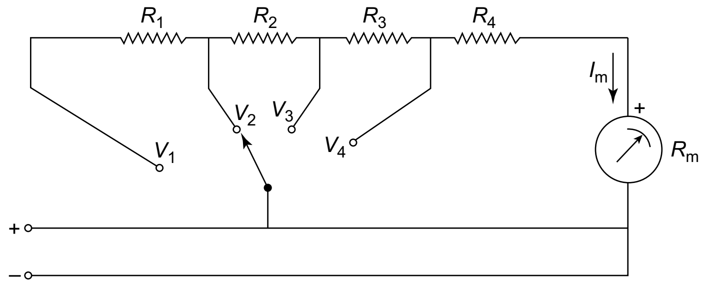


Figure 5.30 An Alternate Multi-range Voltmeter

5.16 Digital Voltmeter (DVM)

The Digital VoltMeter (DVM) is a voltage-indicating device. The main use of a DVM is to measure voltage between two points. It displays DC or AC voltage as discrete numerals. It is a useful laboratory instrument for several applications. It is also a useful building block of digital instrumentation systems. The utility of a DVM can be easily extended for multiple functions, such as in a Digital Multimeter (DMM) by the addition of simple auxiliary hardware. The DVM is often used in data processing systems.

An ideal voltmeter has an infinitely high input resistance so that it does not draw any current from the circuit. Consider a meter that has a low input resistance of 1000 Ω. It cannot give an accurate value of the voltage across a resistance of the same magnitude because the meter shunts the resistance. Therefore, it is important to measure the loading effect of a voltmeter in terms of ohm per volt.

The block diagram of a digital voltmeter is shown in Figure 5.31. It has three stages: (i) signal preparation, (ii) analogue-to-digital conversion, and (iii) display unit.

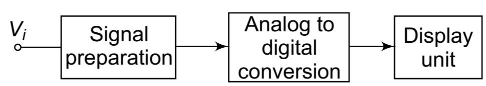


Figure 5.31 Block diagram of Digital Voltmeter

The signal-preparation stage or input circuit modifies the signal amplitude according to the requirement and it also protects the source from loading. Here, a resistive attenuator is used to decrease the large incoming signal and an amplifier is used to amplify the small incoming signal to the measurable range.

The input circuit for DVM, using an operational amplifier is shown in Figure 5.32. The operational amplifier is a multi-stage integrated circuit. Amplification is controlled with negative feedback, since amplifier gain 'A' is proportional to the ratio of the feedback and input resistors.

$$A = -\frac{R_f}{R_i}$$

In the next two stages, the analogue input signals are typically converted into digital signals in the form of binary or Binary Coded Decimal (BCD) data and suitably displayed in the display unit. The block diagram of a dual-slope A/D converter with a display unit is shown in Figure 5.33.

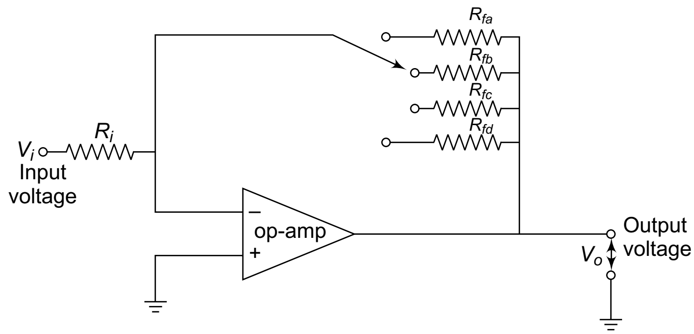


Figure 5.32 Input Circuit for Digital Voltmeter

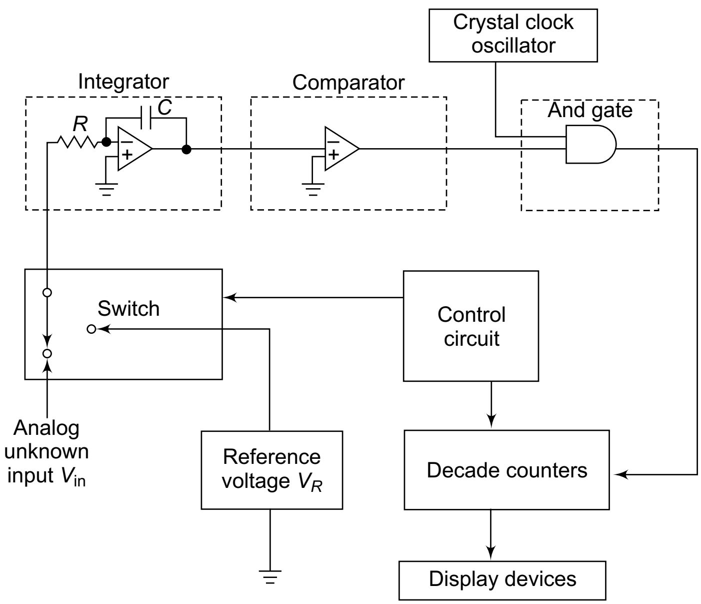


Figure 5.33 Block Diagram of a Dual-slope A/D Converter with Display

An unknown voltage V_{in} is given to the input of the integrator through the selection switch for a known time period T. The output from the integrator is given by the equation,

$$V_C = -\frac{V_{in}}{RC}T$$

Here, R and C are the resistance and capacitance values in the integrator. As the input voltage V_{in} is positive, the integrator output will be a negative ramp, as shown in Figure 5.33. The output from the integrator is compared with zero-volt reference in the comparator. The output from the comparator will be a positive voltage. For the entire time period T, the AND gate is opened and during this period, the pulses from the crystal clock oscillator are counted in the counter.

At the end of the known time-period T, the control circuit resets the counters and at the same time, it activates the selection switch so that the negative known reference voltage V_R is applied to the input of the integrator. As shown in Figure 5.34, the output from the integrator will be a positive ramp, given by the equation

$$V_C = \frac{V_R}{RC} t$$

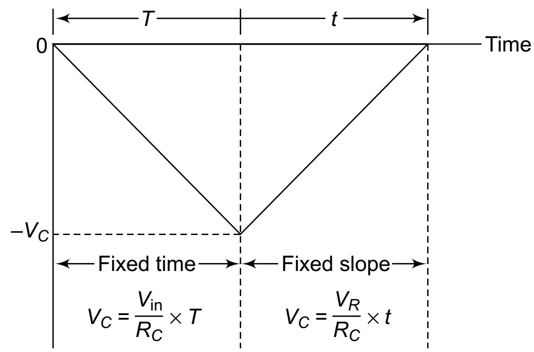


Figure 5.34 Output of Integrator

At the end of time period t, V_C is equal to zero volt. All this time, the counter is counting and hence, t can be known. Since the integrator output begins at zero volt, integrates down to $-V_C$ and then integrates back up to zero volt, the two equations given for V_C can be equated as

$$\frac{V_{\text{in}}}{RC} T = \frac{V_R}{RC} t$$

Therefore, $$V_{\text{in}} = \frac{V_R}{T} t$$

Since V_R and T are known values, the unknown input voltage V_{in}, which is the content of the counter, is proportional to the variable time-period t.

A complete DVM on a single chip (IC) is available. They include A/D conversion circuitry and the necessary timing, counting and display circuitry. Examples are the low-power $3\frac{1}{2}$-digit ICL 7136 and the $4\frac{1}{4}$-digit ICL 7129; both use LCD 7-segment displays and run from a single 9-V battery.

The merits of DVM over other voltmeter types are:

1. Greater speed
2. Higher accuracy and resolution
3. No parallax error
4. Reduced human error
5. Compatibility with other digital equipment for further processing and recording.

5.17 Ohmmeter

There are two types of ohmmeters, namely: series-type ohmmeter and shunt-type ohmmeter.

5.17.1 Series-type Ohmmeter

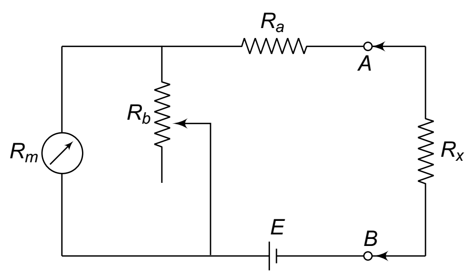


Figure 5.35 Simple Single-range Series Ohmmeter

The series-type ohmmeter circuit is shown in Figure 5.35, where the meter internal resistance R_m, is connected in series with the current-limiting resistance R_a and a battery E, with a pair of terminals A and B. The unknown resistance R_x can be connected to this terminal. The meter current I_m depends on the value of R_x. As other values are constant, the meter reading is proportional to the value of R_x.

When the terminals of A and B are shorted ($R_x = 0$), the meter current I_m will be maximum, causing the meter to reach its full-scale deflection ($I_m = I_{\text{fsd}}$) and hence, this position is marked as "0 Ω" on the meter scale. For reverse condition, when the terminals A & B are open ($R_x = \infty$), the meter current $I_m = 0$ and hence, this position is marked as '∞' on the meter scale. Similarly, by connecting different known values of R_x to the terminals A and B, the intermediate readings are marked on the meter scale. It is to be noted that the half-scale position resistance R_h is marked when the unknown resistance is equal to the total internal resistance of the ohmmeter.

In the design of the meter shown in Figure 5.35, the following parameters are considered:

R_h = the value of 'R', which cause the half scale deflection
R_a = current limiting resistor
R_b = zero adjust resistor
R_m = internal resistance of the movement
E = internal battery

The half-scale deflection resistance is given by

$$R_h = R_a + \frac{R_b R_m}{R_b + R_m} \tag{5.26}$$

For producing a full-scale deflection, the battery current is expressed by

$$I_h = \frac{E}{2R_h} \tag{5.27}$$

For producing full-scale deflection, the battery current should be doubled.
Therefore,

$$I_t = 2I_h = \frac{E}{R_h} \tag{5.28}$$

The shunt current through R_b is

$$I_b = I_t - I_{fsd} \tag{5.29}$$

The shunt voltage (E_{sh}) is equal to the voltage across the movement. Therefore,

$$E_{sh} = E_m \text{ or } I_b R_b = I_{fsd} R_m$$

and

$$R_b = \frac{I_{fsd} R_m}{I_b} \tag{5.30}$$

Substituting Eqn. (5.29) into Eqn. (5.30) and using Eqn. (5.28), we get

$$R_b = \frac{I_{fsd} R_m}{I_t - I_{fsd}} = \frac{I_{fsd} R_m R_h}{E - I_{fsd} R_h} \tag{5.31}$$

Upon solving Eqn. (5.26) for R_a, we get

$$R_a = R_h - \frac{R_b R_m}{R_b + R_m} \tag{5.32}$$

Substituting Eqn. (5.31) into Eqn. (5.32) and solving for R_a, we get

$$R_a = R_h - \frac{I_{fsd} R_m R_h}{E}$$

5.17.2 Shunt-type Ohmmeter

Figure 5.36 shows the circuit diagram of a shunt-type ohmmeter. Here, the internal battery E is connected in series with the adjustable resistor 'R_a' and a d'Arsonval movement. There are two terminals A and B, connected in parallel with the meter. The unknown resistance R_x is connected across the terminals A and B. A switch 'S' is used for disconnecting the battery when the ohmmeter is not in use.

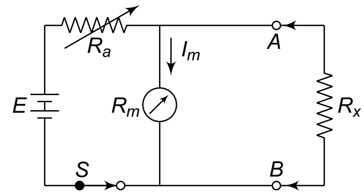


Figure 5.36 Shunt-type Ohmmeter

When the unknown resistor $R_x = 0$, i.e., the terminals A and B are shorted, the meter current $I_m = 0$ (no current). When $R_x = \infty$, i.e., the terminals A and B are open, the meter current will be maximum (full-scale deflection current).

The range of shunt-type ohmmeter is normally used to measure low-value resistors. As shown in Figure 5.36, when $R_x = \infty$, the full-scale meter current becomes

$$I_{fsd} = \frac{E}{R_a + R_m}$$

Solving for the current-limiting resistor, R_a, we get

$$R_a = \frac{E}{I_{fsd}} - R_m \tag{5.33}$$

When any value of resistor R_x is connected across the terminals A and B, the meter current decreases and it is given by

$$I_m = \left\{\frac{E}{R_a + [R_m R_x/(R_m + R_x)]}\right\}\frac{R_x}{R_m + R_x}$$

i.e.,
$$I_m = \frac{ER_x}{R_a R_m + R_x(R_a + R_m)} \tag{5.34}$$

The ratio of meter current to the full-scale deflection current is given by

$$S = \frac{I_m}{I_{fsd}} = \frac{R_x(R_a + R_m)}{R_a(R_m + R_x) + R_m R_x}$$

i.e.,
$$S = \frac{R_x(R_a + R_m)}{R_x(R_a + R_m) + R_a R_m} \tag{5.35}$$

Here, $\dfrac{R_a R_m}{R_a + R_m} = R_p$

Therefore, $S = \dfrac{R_x}{R_x + R_p}$ (5.36)

When $I_m = 0.5 I_{fsd}$, the meter current I_m given in Eqn. (5.34) reduced to

$$0.5 I_{fsd} = \frac{ER_h}{R_a R_m + R_h(R_a + R_m)}$$

where, R_h is the resistance required for half-scale deflection. For determining the relative scale value for the given value of R_a, the half-scale reading may be found by diving equation and solving for R_h,

$$R_h = \frac{R_a R_m}{R_a + R_m} \tag{5.37}$$

5.18 Digital Multimeter

[AU April/May, 2015]

An instrument used to measure voltage, current and resistance is known as a multimeter. There are two types of multimeters, namely: analogue and digital. Of these two types, the digital multimeter is commonly used in laboratories and workshops because of its high input-resistance, greater accuracy, better resolution and easy readability. The DMM combines in one case the instruments for the measurements of voltage, current and resistance. The block diagram of a digital multimeter is shown in Figure 5.37.

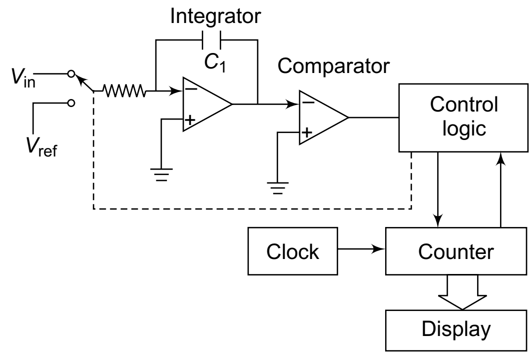


Figure 5.37 Block Diagram of a Digital Multimeter

5.18.1 Measurement of Voltage

The principle used in a digital voltmeter is used in a DMM for the measurement of voltage.

5.18.2 Measurement of Current

A series of current-sensing resistors are used to measure either DC or AC current. The current to be measured is passed through one of the sensing resistors and the DMM digitises the voltage developed across the resistor.

For example, referring to Figure 5.38, the output voltage of a current to voltage converter is given by

$$V_0 = -I_s R_f$$

where, R_f is the known resistance. The output voltage V_0, which is proportional to the unknown source current I_s is applied to DVM section of DMM and the value of current I_s is displayed.

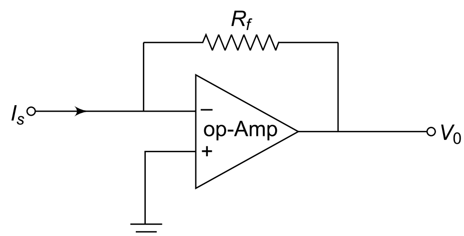


Figure 5.38 Current-to-Voltage Converter

5.18.3 Measurement of Resistance

The DMM measures the resistance by applying a known current from an internal current source to the unknown resistance and then digitising the resulting voltage developed.

For example, referring to Figure 5.39, the output voltage of a scale changer is given by

$$V_0 = \frac{-R_f}{R_i} V_i$$

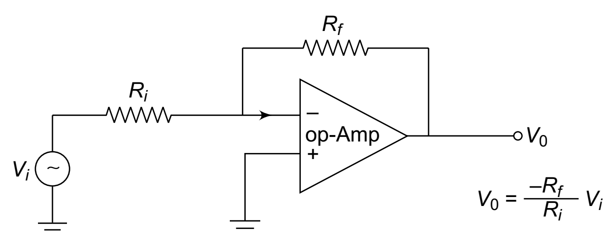


Figure 5.39 Scale Changer

where, V_i and R_i are the known parameters.

The output voltage, which is proportional to the unknown resistance R_f, is applied to DVM section of DMM and the value of unknown resistance R_f is displayed.

Most of the DMMs are similar in terms of voltage, current and resistance measurements. They differ only in terms of accuracy, selection of ranges, and AC bandwidth. There are some DMMs that have built-in capacitance measuring circuitry. Most DMMs have protection from input overload by using circuit breakers, fuses, auto-ranging and diode clipper circuit. The display used can be either Liquid Crystal Display (LCD) or Light Emitting Diode (LED) display.

5.18.4 Applications

A DMM is typically used for measurement of voltage, current and resistance. It is also used to test whether the diode, transistor or SCR is good or faulty and to check circuit continuity. For example, to check a diode, the resistance is measured in one direction and then in the other direction. In the forward-biased direction, a low resistance is indicated and in the reverse-biased direction, a high resistance is indicated.

5.19 Cathode Ray Oscilloscope

The CRO is a versatile electronic testing and measuring instrument that allows the amplitude of the signal, which may be voltage, current, power, etc., to be displayed primarily as a function of time. It is used for voltage, frequency, and phase-angle measurement and also for examining the waveforms, from DC or very-low frequency to very-high frequencies.

Figure 5.40 shows the basic block diagram of a CRO. It comprises of the following main sections: (i) Horizontal and vertical voltage amplifiers, (ii) power-supply circuits, and (iii) Cathode-ray Tube (CRT).

5.19.1 Vertical and Horizontal Voltage Amplifiers

These amplifiers are connected between the input terminals and the deflection plates. The function of the amplifiers is to increase the deflection sensitivity for weak input voltages. The input signal is fed through a calibrated attenuator and a wideband high-gain vertical amplifier to the vertical deflection plates of the CRT. The horizontal amplifier, which is connected to the horizontal plates of the CRT, is fed from an internally generated time base, usually a saw-tooth waveform generator, or alternatively the horizontal amplifier can also be fed from an externally connected X input. A portion of the input signal applied to the vertical plates triggers the horizontal sweep (saw-tooth) signal. A finite amount of time (in the range of seconds) is elapsed before the saw-tooth waveform is applied to the horizontal plates. Hence, to observe the starting edge of the input signal fully, it should be delayed by the same amount of time in the delay line.

5.19.2 Power-supply Circuits

The power-supply unit provides high voltages required by the CRT to generate and accelerate the electron beam, in addition to supplying the required operating voltage for the other circuits of the oscilloscope. The CRT requires high voltages, of the order of a few thousand volts, for acceleration and a low voltage for the heater of the electron gun, which emits electrons. The CRO has various control switches on the panel. The respective control knobs can adjust the intensity of the spot and focus.

5.19.3 Cathode-Ray Tube (CRT)

The CRT is the heart of the oscilloscope. It is a vacuum tube of special geometrical shape and converts an electrical signal into a visual one. A heated cathode emits electrons, which are accelerated to a high velocity and are brought to focus on a fluorescent screen. When the electron beam strikes the screen of the CRT,

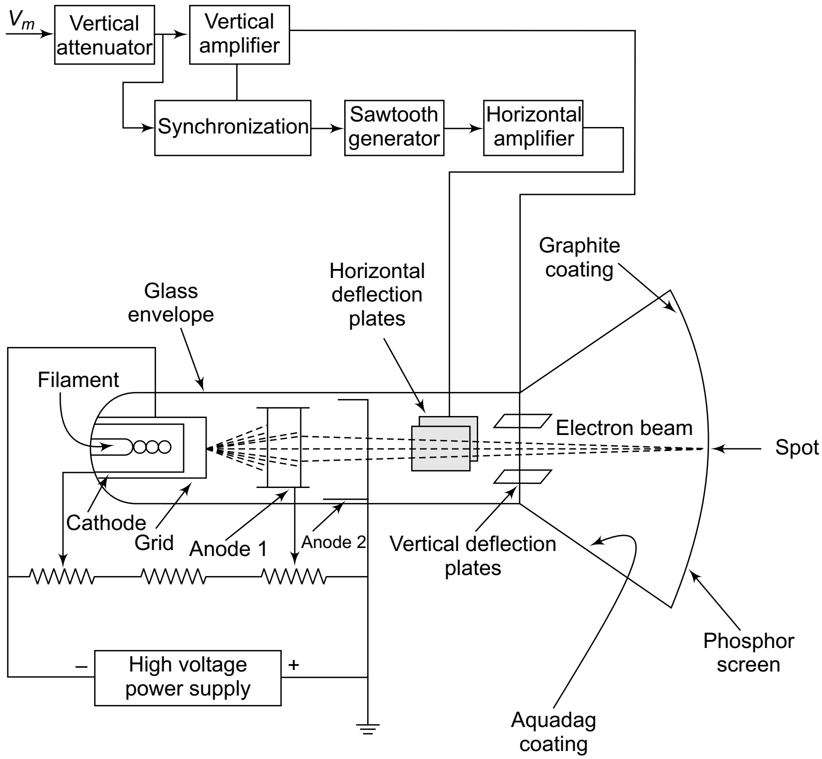


Figure 5.40 Schematic Diagram of a CRO

a spot light is produced. The electron beam, on its journey, is deflected in response to the electrical signal under study. As a result, the waveform of the electrical signal is displayed. As shown in Figure 5.40, the CRT has various parts, which are described below.

Glass Envelope and Screen

It houses the electron gun, vertical and horizontal plates, and a screen on the conical front-end. The inner walls of the CRT between neck and screen are usually coated with a conducting material (graphite) called acquadag. This conductive coating is electrically connected to the accelerating anode so that the electrons, which accidentally strike the wall, are returned to the anode. It prevents the wall of the tube from charging to a high negative potential.

The screen is coated with a suitable fluorescent material, depending on the required colour of the spot. Some of the substances, which give characteristic fluorescent colours, are:

- Zinc orthosilicate: Green (used in CRT for general purpose)
- Calcium tungstate: Blue (used in CRT for fast photography)
- Zinc sulphide or
- Zinc cadmium sulphate: White (used in television receiver tubes).

Electron Gun

It produces a focused beam of electrons. It consists of an indirectly heated cathode, a control grid, a focusing anode and an accelerating anode. The control grid is at a negative potential with respect to the cathode, whereas the two anodes are maintained at a high positive potential with respect to the cathode. These two anodes act as electrostatic lens to converge the electron beam at a point on the screen. The cathode consists of a nickel cylinder, coated with an oxide coating that provides plenty of electrons. The control grid encloses the cathode and consists of a metal cylinder with a tiny circular opening to keep the electron beam small in size. The focusing anode focuses the electron beam to a sharp point by controlling the positive potential on it. The positive potential (about 10,000 V) on the accelerating anode is much higher than that on the focusing anode so that, this anode accelerates the narrow beam to high velocity. Therefore, the electron gun generates a narrow, accelerated beam of electrons, which produces a spot of light when it strikes the screen.

Deflection Plates

The electron beam comes under the influence of vertical and horizontal deflection plates before it strikes the screen.

When no voltage is applied to the vertical deflection plates, the electron beam produces a spot of light at the centre of the screen. If the upper plate is positive with respect to the lower plate, the electron beam is deflected upwards and strikes the screen above its centre. If the upper plate is negative with respect to the lower plate, the electron beam is deflected downwards and strikes the screen below its centre. Thus, the electron beam is made to move up and down vertically, by controlling the voltage on the vertical plates, thereby producing spots of light on the screen.

When a sinusoidal voltage is applied to the vertical deflection plates, the upper plate is positive during the positive half cycle and negative during the negative half cycle, thereby producing a continuous vertical line on the screen.

The electron beam is made to move horizontally from side-to-side, at a uniform rate by applying a saw-tooth wave, which varies linearly with time across the horizontal deflection plates.

Thus, the spot of light can be moved all over the surface of the screen by the simultaneous action of both vertical and horizontal deflection plates. In order to get the exact pattern of the signal on the screen, the signal voltage is given to the vertical deflection plates and saw-tooth wave to the horizontal deflection plates.

Types of CRTs

Conventionally, CRTs form the basis of cathode-ray oscilloscopes (CROs), TVs and consoles/monitors. They are useful in displaying numeric, alphanumeric and graphic displays with high resolution. These are of two types:

- Electrostatic (used in CROs)
- Electromagnetic (used in TVs)

There are also storage CRTs using digital storage, mesh storage, phosphor storage, and transfer storage. Flat CRTs are also available.

5.19.4 Special Oscilloscopes

Some special oscilloscopes, which are discussed briefly in the following section, are designed for specific applications.

Dual-beam Oscilloscope

A dual-beam oscilloscope is useful to observe two signals simultaneously and compare their waveforms. Figure 5.41 shows the block diagram of a dual-beam CRO. It has two completely separate electron guns, two sets of vertical deflecting plates (Y-plates) and a single set of horizontal deflecting plates (X-plates). Both the channels have a common time-base but have completely independent pre-amplifiers, delay lines and vertical amplifiers. Only one beam can be synchronised at one time because the horizontal sweep is common for both signals. In order to lock the two signals on the CRT screen, the two signals must have the same frequency and phase or must be related harmonically. The dual-beam CRO is used to observe both the input and output signals of the amplifier under test. Signal *A* may be the input signal and signal *B* may be the output signal from the amplifier.

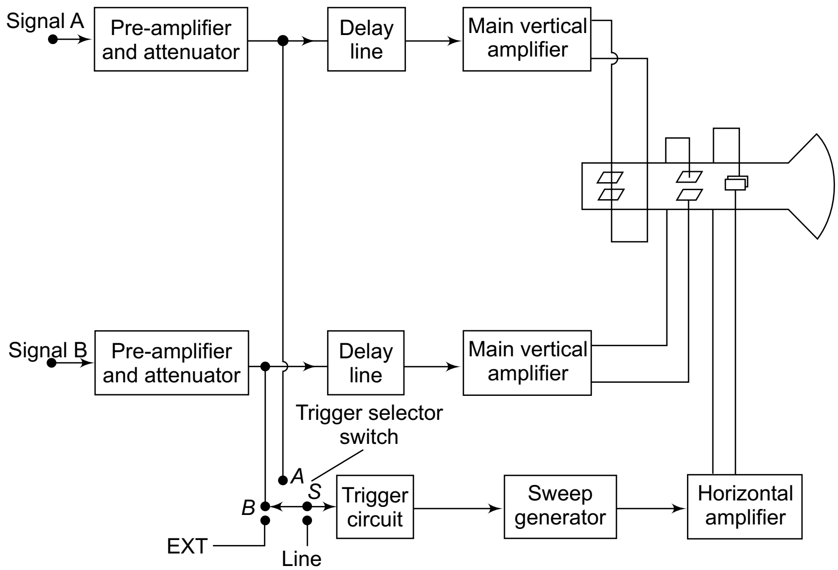


Figure 5.41 Dual-beam Oscilloscope

Dual-trace CRO

The function of a dual-trace CRO is similar to that of a dual-beam CRO but this CRO has a single electron-gun. A single electron beam is split into two beams by means of an electronic switch. The two signals are displayed simultaneously. Figure 5.42 shows a block diagram of the two vertical input channels of a dual-trace CRO. Each channel has its own calibrated input attenuator and positioning control. Therefore, the amplitude of each signal can be independently adjusted. The electronic switch alternately connects the two input channels to the vertical amplifier. The signals pass through the common vertical channel or vertical amplifier. Two channels share the horizontal channels on a time basis.

The dual-trace oscilloscopes have four modes of operation, namely: A, B, alternate and chopped. In the A or B modes, only the input at that channel is displayed. In the alternate mode, the inputs are displayed on alternate traces. The switching rate of the electronic switch is synchronised to the sweep rate, so that the CRT spot traces the channel A signal on one sweep and the channel B signal on the succeeding sweep. This mode of operation is generally preferred when displaying relatively high-frequency signals. In the chopped mode, the electronic switch is free-running at the rate of 100–500 kHz, entirely independent of the frequency of the sweep generator. The switch successively connects small segments of A and B waveforms alternately to the main vertical amplifier at a relatively fast chopping rate of 500 kHz. If the sweep rate is low, the chopped mode is normally used as the alternate mode and would provide a display with considerable flicker.

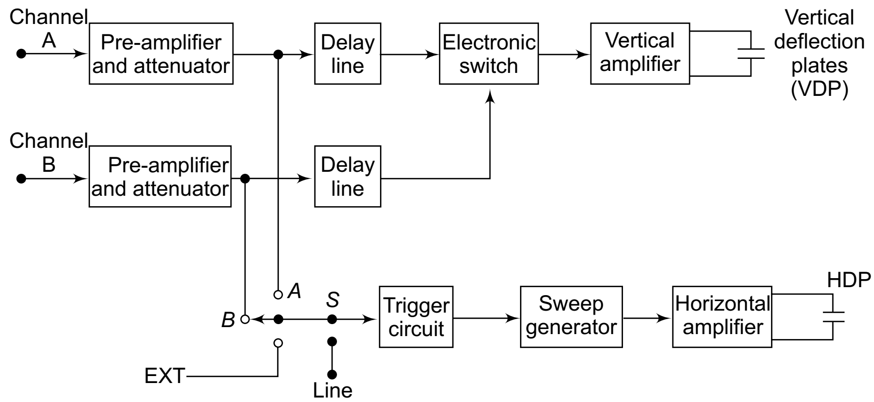


Figure 5.42 Block Diagram of the Input Channels of a Dual-trace Oscilloscope

Storage Oscilloscopes **[AU Nov/Dec, 2014]**

A limitation in conventional CROs is an event that occurs only once, will disappear from the screen after a relatively short interval of time, as the persistence of the phosphor on the screen ranges only from a few milliseconds to several seconds. In a storage CRO, the display is retained for a much longer time, sometimes even for some hours, after the image was first traced on the screen. This retention feature is, therefore, useful in the study of waveforms, which have very low frequency. In a conventional CRO, the start of such a display will fade out before the end is reached. The analogue storage oscilloscopes use the phenomenon of secondary electron emission to build up and store electrostatic charges on the surface of an insulated target.

The block diagram of a basic Digital Storage Oscilloscope (DSO) is illustrated in Figure 5.43. The input signal is digitised and stored in memory in digital form. In this digital form, it is capable of being analysed to produce a variety of information about the input signal. The digital data is reconstructed in analogue form to view the display on the CRT. In order to ensure that no information is lost, the sampling rate must be at least twice the highest frequency of the input signal. The digital oscilloscope is primarily limited in speed by the digitising capacity of the analogue-to-digital converter. Digital oscilloscope is capable of an infinite storage time using its digital memory. A crystal clock generates a time-base in a digital oscilloscope.

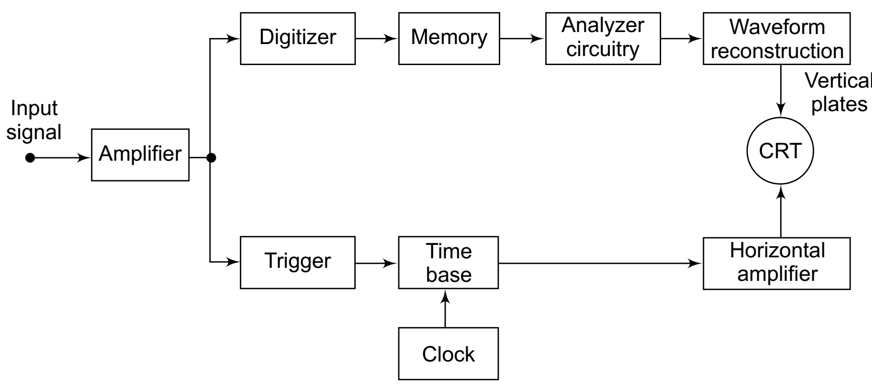


Figure 5.43 Block Diagram of a Digital Storage Oscilloscope

In addition, digital storage oscilloscopes are available in processing and non-processing types. The processing-types have built-in computing facilities and take advantage of the fact that all the data is already in digital form. DSO is also capable of operating in a look-back mode like waveform recorder. If it is triggered, it prints out the stored result on to a hardcopy recorder or disk storage.

Sampling Oscilloscope

A sampling CRO is used to examine very fast signals using instruments having bandwidth of several orders lower. The gain–bandwidth relationship of the vertical amplifier limits the frequency range of signals, which can be displayed on a CRO. As shown in Figure 5.44, samples of the input waveform are taken at different portions of the waveform over successive cycles, with one sample taken per cycle, and each sample slightly delayed with respect to the preceding sample. Then the total picture is stretched, amplified by relatively low bandwidth amplifiers and displayed as a continuous waveform on the screen. The disadvantage of a sampling CRO is that it can only make measurements on repetitive waveform signals.

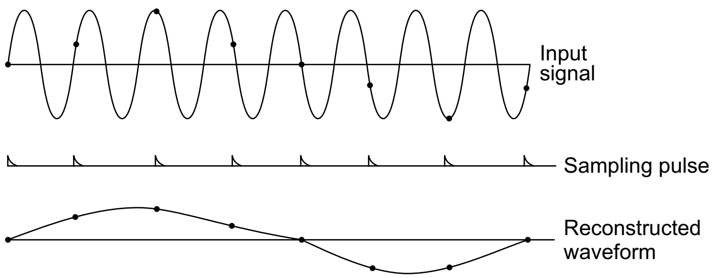


Figure 5.44 Principle of Sampling Oscilloscope

The sampling technique transforms the high-frequency input signal into lower frequency domain where conventional low-frequency circuit is capable of producing a highly effective display. The sample frequency used in sampling CROs can be as low as $\frac{1}{100}$ of the signal frequency and hence a signal frequency of 1 GHz needs an amplifier bandwidth of 10 MHz.

5.19.5 Applications of CRO

The modulation index of Amplitude Modulation (AM) waves can be measured using a CRO. The voltage–current characteristics of a *PN* junction diode and transistor, and characteristics of a transformer core can be displayed on CRO. Some more applications are discussed below.

Measurement of Voltage

If the signal is applied to the vertical deflection plates only, a vertical line appears on the screen. The height of the line is proportional to peak voltage of the applied signal. The amplitude of the signal can be measured by applying the signal to the vertical plates and the sweep is applied to the horizontal plates using internal sweep circuitry. The vertical scale on the CRT screen is marked in centimetres. Each centimetre is further subdivided into 5 parts so that each part represents 0.2 cm. If the peak amplitude of the waveform is 1.7 cm and the scale selected by the dial setting is 1 V/cm, then the amplitude of the signal is 1.4 cm × 1 V/cm = 1.4V.

Measurement of Current

When a current is to be measured, it is passed through a known resistance and the voltage across it is measured.

Measurement of Frequency

- ***Using signal waveform:*** The signal for which the frequency (f) is to be measured is given to the vertical input. The number of divisions occupied by one complete cycle of the waveform is measured. The number of divisions multiplied by the time-base setting in seconds is equal to the time period (T) of one cycle. The frequency (f) of the waveform is inverse of the time period T, i.e., $f = 1/T$.
- ***Using Lissajous figure:*** If sinusoidal voltages are applied to both vertical and horizontal inputs of CRO, some interesting figures are displayed, which are known as *Lissajous figures*. Two sine waves of the same frequency produce a Lissajous figure, which may be a straight line, an ellipse, or a circle, depending on the phase and amplitude of the two signals. Two sine waves of equal amplitudes but different frequencies will produce a figure from which the relationship between the two frequencies can be understood. For example, Figure 5.45(a) shows that the vertical input signal has twice the frequency of the horizontal input signal. Similarly, Figure 5.45 (b) indicates that horizontal input signal has twice the frequency of the vertical one. Figure 5.45 (c) shows three loops indicating that vertical input signal has thrice the frequency of the horizontal one.

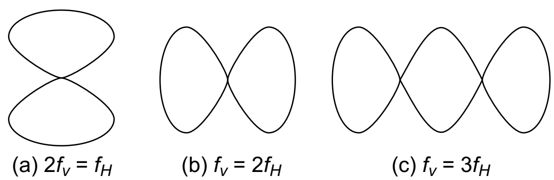


Figure 5.45 Frequency Measurement (Lissajous Figures) (a) $2f_V = f_H$ (b) $f_V = 2f_H$ & (c) $f_V = 3f_H$

A known frequency (f_H) is applied to horizontal input and unknown frequency (f_V) to the vertical input. Then a Lissajous pattern with loops is obtained. The unknown frequency (f_V) can be measured by the following relationship.

$$\frac{f_V}{f_H} = \frac{\text{No. of loops cut by horizontal line}}{\text{No. of loops cut by vertical line}}$$

Measurement of Phase Difference

The phase difference between two sinusoidal signals of same frequency can be calculated from the amplitudes A and B of the Lissajous pattern (an ellipse) shown in Figure 5.46. The phase difference (deg), $q = \sin^{-1} (A/B)$. Lissajous figures are formed when two sine waves are applied simultaneously to the vertical and horizontal deflecting plates of a CRO. In general, the shape of the Lissajous figures depends on amplitude, phase difference and ratio of frequency of the two waves. Two sine waves of the same frequency and amplitude may produce a straight line, an ellipse or a circle, depending on their phase difference, as shown in Figure 5.47.

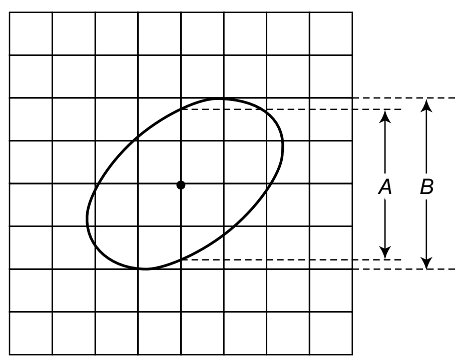


Figure 5.46 Phase Difference Measurement

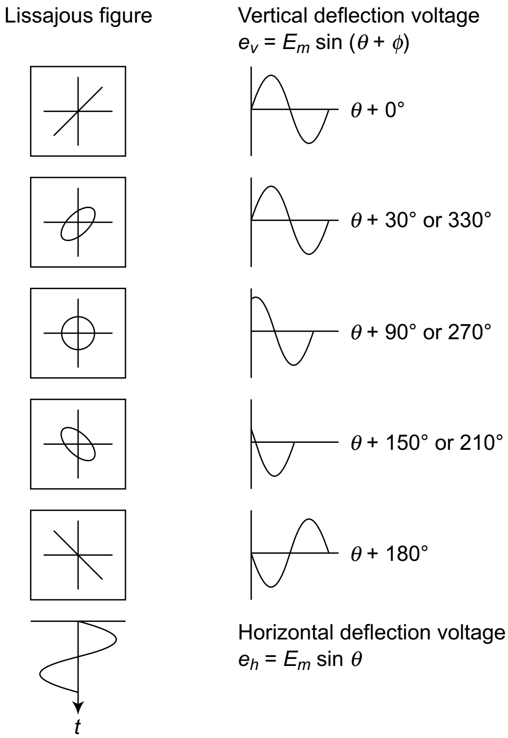


Figure 5.47 Lissajous Figure Depending on the Phase Difference of the Two Waves

Test of Distortion on Amplifier

CRO is useful to measure the distortion using Lissajous figures. Figure 5.48 shows the connections for testing the frequency distortion of an amplifier network. The audio oscillator is adjusted to a known frequency and is connected to the deflecting plates $x - x'$. The input signal obtained at the output of the amplifier is connected to the deflecting plates $y - y'$. If the amplifier produces higher harmonics of the input frequency due to the non-linearity of the active device used, the CRO screen shows the loops in the Lissajous figure, which indicates the presence of distortion. A straight-line display indicates the absence of distortion in the amplifier

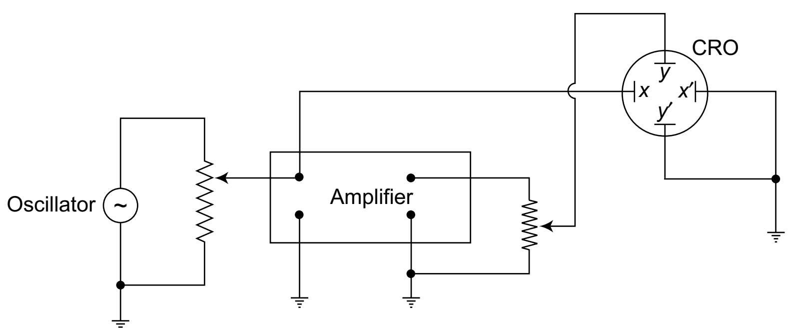


Figure 5.48 Distortion Test on Amplifier

5.20 Static and Dynamic Characteristics of Instruments [AU April/May, 2012]

In a measuring instrument, it is necessary to have a better understanding of all the parameters involved in defining its characteristics. The performance characteristics of the measuring instrument, which decide the overall performance, can be divided into two distinct categories: static and dynamic characteristics. Under the circumstances where the quantities to be measured are either constant or vary very slowly with respect to time, a set of characteristics is defined to give a meaningful description about the measurement quality. These characteristics are called *static characteristics* of measurement system, which are to be considered when the measurement system or instrument is used under a static condition. But in practice, many quantities, to be measured, vary rapidly with respect to time. In such cases, a set of characteristics is to be defined based on the dynamic relationship existing between the input and output. These characteristics, which are normally done using differential equations, constitute the *dynamic characteristics* of the measurement system.

5.20.1 Static Characteristics of Instruments [AU April/May, 2015]

The characteristics, which are defined for the instruments measuring constant quantities or slowly varying with respect to time, are called **static characteristics**. These characteristics give a meaningful description about the measurement quality without interfering the dynamic descriptions, which use differential equations. The various static characteristics of instruments are:

- Scale Range and scale span
- True value
- Accuracy
- Precision
- Static error and static correction
- Sensitivity
- Linearity
- Scale readability
- Reproducibility and Repeatability
- Resolution
- Threshold
- Drift
- Stability
- Tolerance
- Dead zone and dead time
- Hysteresis
- Noise
- Loading effect

Scale Range and Scale Span

The value to be measured is indicated on a scale using a pointer in an analogue instrument or using digital values in a digital instrument. Each instrument has a maximum and minimum limit within which the instrument is designed to measure, indicate or record a physical quantity. This region between the maximum and minimum limits is called the *scale range* of the instrument, given by its limits. It is the most important factor in the instrument. If the maximum and minimum values that the instrument can measure is X_{max} units and X_{min} units and the calibration is continuous between these points, then the instrument scale range is between X_{min} and X_{max}. *Scale Span* represents the algebraic difference between X_{max} and X_{min} of the instrument, which is given by

$$\text{Span} = X_{max} - X_{min}$$

For example, if a voltmeter is calibrated between 0 V to 10 V, then the scale range of voltmeter is 0 V to 10 V and the scale span is given by 10 – 0 = 10 V.

Frequency range is another factor that has to be considered in determining the scale range of the instrument. It is defined as the frequencies over which the instrument measures the quantity with a specified degree of accuracy. For example, a moving-iron instrument may have a 0–250 V range and a 0–135 Hz frequency range.

True Value

True value of the quantity is defined as the average value of an infinite number of measured values during which the average deviation of various factors tends to zero. But, in practice, it is impossible to realise due to several factors like lags, loading effect, noise and so on.

Accuracy

The uncertainty existing in the measured value is expressed in terms of accuracy, precision and error. Accuracy of a measured value of a quantity is defined as the closeness of the measured value obtained using the instrument used to measure the true value of the same quantity. It depends on the accuracy of the instrument itself, variation of the quantity, which is to be measured, observer accuracy and so on. The accuracy of an instrument may be expressed as: (a) point accuracy (b) percentage of true value or (c) percentage of scale range.

Point Accuracy

The accuracy of the instrument, which is specified only at one particular point on the instrument scale, is called point accuracy. This type of accuracy does not provide information about the instrument at any other points in the instrument scale i.e., it does not provide the general accuracy of the instrument.

Accuracy as Percentage of True Value

The accuracy when represented as the percentage of true value of the quantity being measured is the finest way of visualising the accuracy of the instrument. For example, this type of accuracy is expressed as +0.5% or –0.5% of true value.

Accuracy as Percentage of Scale Range

The accuracy can be expressed as the percentage of scale range, only when the instrument has a uniform scale.

Precision

Generally, there exists confusion between accuracy and precision. Precision comes from the term precise, which indicates clearly or sharply defined, and it is a measure of reproducibility of the measurements or a degree of agreement within a measurement group. The two characteristics of the precision term are:

- ***Conformity:*** An instrument reading consistently a particular quantity as 2.4 M instead of the true value 23456789 due to absence of proper scale.
- ***Number of significant figures:*** It is used to obtain the precision of the instrument in which the reading is expressed and conveys the actual information about the magnitude and measurement precision of the quantity. The mathematical expression for precision is

  $$P = 1 - \left| \frac{X_n - X_{an}}{X_{an}} \right|$$

  where, X_n is the value of nth measurement and X_{an} is the average of measurement set values

Static Error and Static Correction

- ***Static error:*** It is the difference between the measured value and true value of the quantity, as given by

  $$E_s = A_m - A_t$$

  where, A_m is the measured value of the quantity and A_t is the true value of the quantity. Since the static error does not indicate the accuracy of the instrument precisely, relative static error is defined.

The relative static error is defined as the ratio of the absolute static error to the true value of the quantity under measurement and is given by

$$E_r = \frac{E_s}{A_t}$$

- ***Static correction:*** It is the difference between the true value and measured value of the quantity, as given by

$$\delta C = A_t - A_m$$

Sensitivity

It is defined as the ratio of the change in the output of an instrument to a change in the quantity to be measured. Mathematically it is expressed as,

$$\text{Sensitivity} = \frac{\text{Change in output}}{\text{Change in input}}$$

If the input–output relation of the instrument is linear, then the slope of the curve represents the sensitivity. If the relation is not linear, then the sensitivity varies with respect to the input. Inverse sensitivity or deflection factor is given by the reciprocal of sensitivity.

Linearity

It is defined as the ability of the instrument to reproduce the true input–output characteristics symmetrically and linearly. The percentage of non-linearity existing in the instrument is given by

$$\%\ \text{non} - \text{linearity} = \frac{\text{Maximum deviation of output from true curve}}{\text{Full scale reading}}$$

The representation of linearity of the instrument is represented in Figure 5.49.

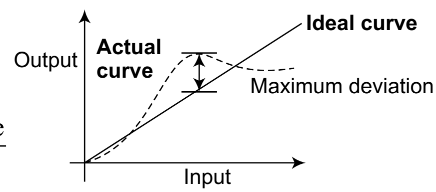


Figure 5.49 Representation of Linearity Between Input and Output

Scale readability

In analogue instruments, the closeness to which the scale can be read is known as scale readability and it depends on factors such as: (a) number of graduations (b) spacing between the graduations (c) size of the pointer and (d) discriminating power of the observer.

Reproducibility and Repeatability

- ***Reproducibility:*** It is the degree of closeness with which a given value may be repeatedly measured using the same instrument, under different conditions like changes in the method of measurement, observer, measuring instrument location, conditions of use and time of measurement. It is specified in terms of scale readings over a given period of time.
- ***Repeatability:*** It is the instrument characteristic which describes the closeness with which a given value is repeatedly measured on the same instrument, at the same location, by the same observer, under the same measurement conditions and when the same input is given to the instrument repetitively over a particular time. It is specified as a variation in scale reading.

Resolution or Discrimination of the Instrument

The smallest change in input which is required to obtain a change in output or smallest measurable input change is known as resolution i.e., when the input is slowly increased from some arbitrary non-zero value, there will be no change in the output till an increment is achieved.

Threshold

The minimum value of input quantity required to change the output reading from zero is known as threshold. It is defined as the minimum value below which there exists no output signal or smallest measurable input.

Drift

It is the measure of deviation in the instrument output for a particular period. If an instrument has no drift, then it has the capability of producing the same reading at different times when there is a variation in the measured variable. The factors which contribute towards the drift are: (i) wear and tear (ii) mechanical vibration (iii) Stress developed in the instrument components (iv) variation in temperature (v) stray electric and magnetic fields and (vi) thermal emf. The different types of drift are:

Zero drift: If all the instrument outputs shift by the same amount, it is known as zero drift and it occurs due to pointer shift, slippage or permanent set.

Span drift: If the instrument output changes proportionately from zero, it is called span drift and it occurs due to the change in spring gradient.

Zonal drift: If the drift occurs only for a particular portion, it is called zonal drift.

The different types of drift are shown in Figures 5.50(a) and (b).

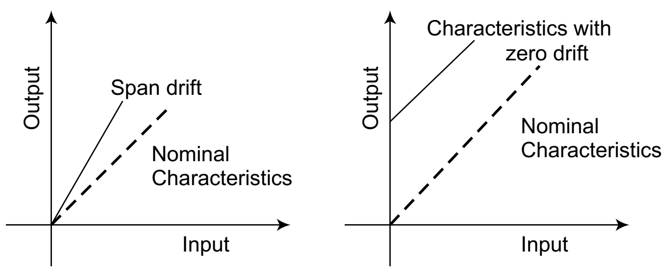


Figure 5.50 (a) Span Drift (b) Zero Drift

Stability

It is the measure of capability of the instrument to maintain standard of performance over a prolonged period of time. The instrument will have zero stability if the instrument restores to zero when the input quantity reaches zero, while other conditions remain the same.

Tolerance

The maximum error, which can be expected in the measured value, is known as tolerance.

Dead Zone and Dead Time

- ***Dead zone:*** A region in the input where there is no output is known as dead zone or dead space or neutral zone, which is shown in Figure 5.51. It is also defined as the largest change required in the input variable to make the instrument respond.
- **Dead time:** The time taken by the instrument to respond after the change in input variable has taken place is known as dead time.

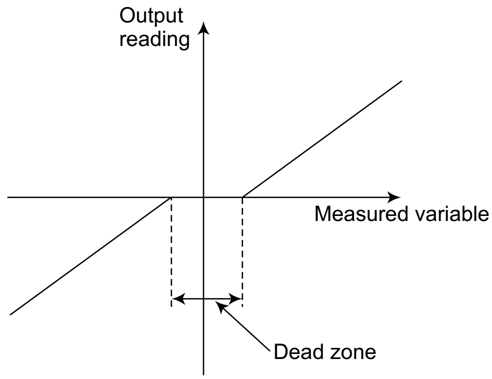


Figure 5.51 Dead Zone

Hysteresis

The phenomenon that occurs in the measuring instrument, which shows different characteristics during loading and unloading, is known as hysteresis, as shown in Figure 5.52. It occurs in the instrument due to mechanical friction, motion in bearing, magnetic and thermal effect.

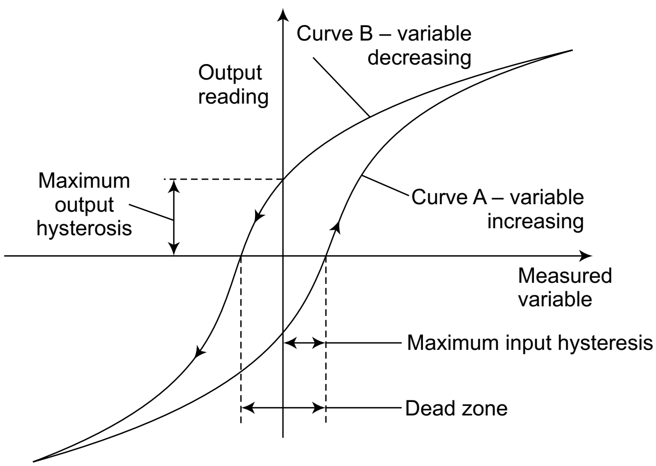


Figure 5.52 Hysteresis

Noise

Random fluctuation in a signal, which does not convey any information or an error or undesired random disturbance in the useful signal, is known as noise. The common sources of noise are stray electrical and magnetic fields, mechanical shocks and vibrations.

Loading Effect

The loading effect, which occurs due to both electrical and mechanical elements, is the alteration caused in the voltage, current etc. when the measurement is made.

5.20.2 Dynamic Characteristics of Instruments [AU Nov/Dec, 2010]

The static characteristics of measuring instruments described in the previous section are for the instruments subjected to non-varying inputs. However, in practice, the input varies from instant to instant and so does the output. The behaviour of the system subjected to varying inputs is known as dynamic response and its characteristics are known as dynamic characteristics. In general, the step, ramp and frequency response of the measuring instrument determine the dynamic characteristics of the measuring instrument. The different dynamic characteristics of measuring instruments are:

- Speed of response
- Response time
- Lag
- Fidelity
- Dynamic error
- Time constant

Speed of Response

It is defined as the rapidity with which an instrument responds to changes in input quantity or the quantity to be measured.

Response Time or Settling Time

It is the time required by the instrument to settle to its final steady state value after the input quantity is applied. For example, in a step-input function, the response time is the time taken by the instrument to settle at a specified percentage of the output, after the application of the input.

Lag

The delay existing in the dynamic response of the instrument when a change in input quantity is applied is known as lag. Though its value is very small, it becomes important for high-speed measurements. Therefore, in the high-speed measurement systems, the time lag should be minimum. The two different types of lag are:

Retardation type: In this type of measuring lag, the output is obtained immediately after a change in measured quantity has occurred.

- ***Time delay:*** In this type of measuring lag, the output is obtained after a dead zone.

Fidelity

It is the capability of the instrument to reproduce the output in the same form as the input is known as fidelity. It is also defined as the degree to which an instrument indicates a change in the input quantity without any dynamic error.

Dynamic Error

The difference between the true value of the time varying quantity, which is changing with time and the output value indicated by the instrument, if no static error is assumed, is known as dynamic error. Generally, the total dynamic error of the instrument is given by the combination of its fidelity and the time lag between input and output of the system.

Time Constant

It is defined as the time taken to reach 63.2 % of the final output value. If the system has less time constant, it indicates that the final output value will be attained earlier.

5.21 Errors in Measurement **[AU May/June, 2012]**

The error in measurement is defined as the difference between the true or actual value and the measured value. The different sources of error in measurement are as follows:

- Gross error
- Systematic error
- Random error

The systematic error is further classified as shown in Figure 5.53.

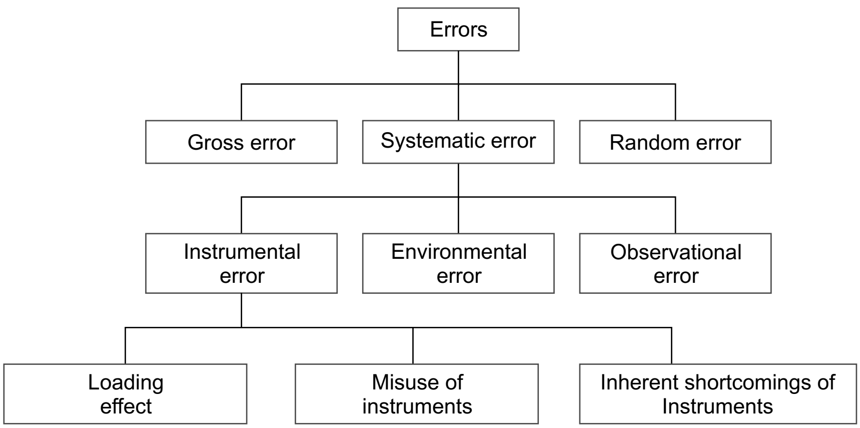


Figure 5.53 Sources of Errors

5.21.1 Gross Error

An error that is caused by an experimenter while reading, recording and calculating the measurement result is known as gross error. For example, the experimenter can possibly read the temperature as 21.5° C while the actual value is 31.5° C or the experimenter can possibly read the value as 34.5° C and instead might record it as 35.4° C. These types of errors are called gross errors. Gross error can be avoided by taking more care, while reading and recording the data and by taking more readings of the quantity under measurement by a different experimenters.

5.21.2 Systematic Error

An error that occurs due to the presence of a fault in the measuring instrument is known as systematic error. Correcting the measurement instrument can rectify this error.

Further, these errors are classified into different categories as:

- Instrumental Error

- Environmental Error
- Observational Error

Instrumental Error

An error that arises due to faulty construction and calibration of the measuring instrument is known as instrumental error. The main reasons behind these instrumental errors are:
- Inherent shortcomings of instrument
- Misuse of instrument
- Loading effect

Inherent Shortcomings of Instrument

These types of errors exist in the instrument inherently due to its mechanical structure. These errors occur due to manufacturing, measurement, calibration or operation of the measuring instruments and causes it to read too low or too high of the measurand. For example, if the spring of a permanent magnet has become weak, that particular device will always read very high.

This type of error can be avoided by carefully planning the measurement procedure, applying correction factors after finding these errors and carefully recalibrating the instrument.

Misuse of Instruments

When a good instrument is operated in an unintelligent manner, it results in these types of errors. Examples of these types of errors are: failing to adjust the zero of instruments, poor initial setting and so on. Though this improper usage of instrument does not lead to a permanent damage to it, it causes errors in the measurement.

Loading Effect

The most common type of error caused by the instrument due to improper usage is known as loading effect. A well-calibrated voltmeter will give a wrong reading when it is connected across a high-resistance circuit and will give dependable reading when it is connected across the low resistance circuit. Using the meters intelligently can eliminate this error. For example, when measuring a low resistance, using ammeter–voltmeter method, a voltmeter that has a very high resistance should be used.

Environmental Error

An error that occurs due to some external condition of the measuring instrument is known as environmental error. The external conditions that affect the measuring instrument are: temperature, pressure, humidity, dust, vibration due to magnetic or electrostatic field and so on.

These errors can be eliminated or reduced by using the following methods:
- Keeping the conditions like humidity and temperature, as constant as possible using some techniques.
- Using equipment that does not get disturbed by these external conditions.
- More care should be taken to ensure that there exists no external electrostatic or magnetic fields around the measuring instrument.
- Applying computed corrections.

Observational Error

An error caused due to the wrong observation of the reading in the measuring instrument is known as an observational error. There exist different sources of this error and parallax error is the most common error. For example, as the pointer in any instrument resets slightly above the surface of the scale, this error will not occur, unless the vision line of the experimenter is exactly above the pointer. This error can be minimised by using a highly accurate meter provided with mirrored scale.

5.21.3 Random Error

An error caused by sudden change in the experimental conditions, noise, or tiredness of the experimenter is known as random error, and it can be either positive or negative. This error will still remain even after elimination of other types of errors. Hence, such type of an error is also called a residual error and some examples of this type of error are: humidity change, temperature change, voltage fluctuations and so on. Taking the average of a large number of readings can help reduce this error.

5.22 Introduction to Transducer

A transducer is a device that converts energy from one form to another form. This energy may be electrical, mechanical, chemical, optical or thermal. Transducers may be classified according to their applications, methods of energy conversion, nature of the output signal, and so on. All these classifications usually result in overlapping areas. A sharp distinction among the types of transducers is difficult. A transducer that gives electrical energy as the output is known as an *electrical transducer*. The output electrical signal could be voltage, current, or frequency, and the production of these signals is based upon resistive, capacitive, inductive effects, etc. For measuring non-electrical quantities, a detector is used, which usually converts the physical quantity into a displacement that activates the electrical transducer. The *displacement transducers*, namely, capacitive, potentiometric, photoelectric (phototube) and piezoelectric, use the principle of converting a mechanical force into displacement and then into electrical parameters. Here, the mechanical elements used for converting this applied force into displacement are called force-summing devices.

5.23 Classification of Transducers

The transducers are classified:

- **On the basis of transduction form:**
  Based on how the input quantity or measurand is converted, it is further classified as
  - Resistive transducer
  - Capacitive transducer
  - Inductive transducer

  Example: piezoelectric, thermoelectric, magneto restrictive and so on.
- **Primary and secondary transducers**
  Element which makes direct contact to the measurand or the physical quantity is called a primary transducer and the transducers which convert the output of primary transducer to electrical output are called secondary transducers.

  Example: In Bourdon-tube pressure-gauge device, the primary transducer is the Bourdon tube and secondary transducer is an LVDT (Linear variable differential transformer)
- **Passive and active transducers**
  Active transducers, also known as self-generating type, develop their own voltage or current as the output signal. The energy required for production of this output signal is obtained from the physical

phenomenon being measured. Passive transducers, also known as externally powered transducers, derive the power required for energy conversion from an external power source. However, they may also absorb some energy from the physical phenomenon under study. A few examples of active and passive transducers are given in Table 5.3.

Table 5.3 *Active and Passive transducers*

Active Transducers	Passive Transducers
Thermocouple	**Resistance**
Piezoelectric transducer	Potentiometric device
Photovoltaic (Photojunction) cell	Resistance strain gauge
Moving coil generator	Resistance thermometer
Photoelectric (Photoemission) cell	Thermistor
	Photoconductive cell
	Inductance
	Linear Variable Differential Transformer (LVDT)
	Capacitance
	Voltage and current
	Devices using Hall effect
	Photoemissive cell
	Photomultiplier tube

Opto-electronic transducers, such as photoconductive cells, photovoltaic cells, solar cells, phototubes, and photomultiplier tubes use the principle of converting light energy into electrical energy.

(a) Analogue and digital transducer

Transducers that convert the input quantity into an analogue output are known as analogue transducers. Example: Strain gauge, LVDT, thermocouple, thermistor.

Transducers that convert the input quantity into an electrical output in the form of pulses are known as digital transducers.

(b) Transducer and inverse transducer

A device that converts a non-electrical quantity into an electrical quantity is known as a transducer and the device that converts an electrical quantity to a non-electrical quantity is known as an inverse transducer.

Some of the basic requirements of a transducer are given as follows: **[AU Nov/Dec, 2011]**

- ***Linearity:*** The input–output characteristics of the transducer should be linear.
- ***Ruggedness:*** The transducer should withstand overloads, with measures for overload protection.
- ***Repeatability:*** The transducer should produce identical output signals when the same input signal is applied at different times under the same environmental conditions.
- ***High stability and reliability:*** The output from the transducer should not be affected by temperature, vibration, and other environmental variations, and there should be minimum errors in measurements.
- ***Good dynamic response:*** In industrial, aerospace, and biological applications, the input to the transducer will not be static but dynamic in nature, i.e., the input will vary with time. The transducer should respond to the changes in input as quickly as possible.

- ***Convenient instrumentation:*** The transducer should produce a sufficiently high analogue output signal with high signal-to-noise ratio, so that the output can be measured either directly or after suitable amplification.
- ***Good mechanical characteristics:*** The transducer, under working conditions, will be subjected to various mechanical strains. Such external forces should not introduce any deformity and affect the performance of the transducer.

Of the many effects that are used in transducers, the principal effects used are variation of resistance, inductance, capacitance, piezoelectric effect and thermal effects which are described in the following sections.

5.24 Resistive Transducer [AU Nov/Dec, 2011]

A transducer that converts the change in resistance of the material into an electrical signal with respect to environmental conditions is known as a resistive transducer. This transducer can be used to change resistance in both AC and DC devices. The resistive transducer is used for measuring physical quantities like temperature, displacement, vibration and so on. In general, the resistance of the material is given by

$$R = \frac{\rho l}{A}$$

where, R is the resistance in ohms, A is the cross-sectional area of the conductor in metre square, L is the length of the conductor in metres and ρ is the resistivity of the conductor in materials in ohm metre.

The classification of resistive transducers is based on the variation of any one of the quantities i.e., length, area or resistivity of the metal. The different types of resistance transducers are:

- Potentiometers
- Strain gauges
- Resistance thermometers
- Thermistors

5.24.1 Potentiometric Transducer [AU Nov/Dec, 2012]

The basic circuit of a potentiometric transducer is shown in Figure 5.54. A potentiometric transducer consists of a resistance element that is contacted by a movable slider. A force-summing member is used to move the slider, thereby changing the resistance and correspondingly the output voltage changes. The same principle can be used to vary the resistance in a bridge circuit. This transducer has high electric efficiency and provides a sufficient output to permit control operations without further amplification.

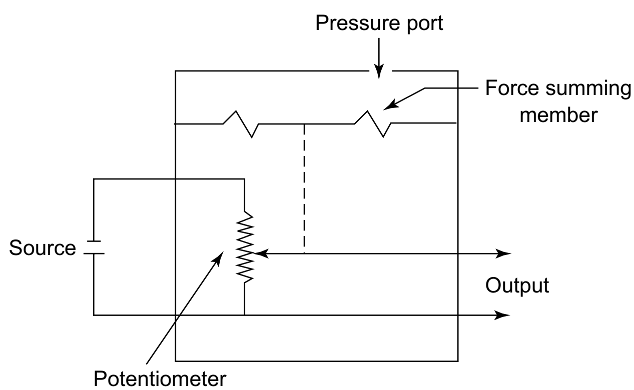


Figure 5.54 Potentiometric Transducers

The advantages, disadvantages and applications of potentiometric transducer are discussed below:

Advantages

1. It is cheap, simple to operate and has a high resolution.
2. It is very useful in applications where there are no severe requirements.
3. It helps in measuring large amplitudes or displacement.
4. Since it has a high electrical efficiency, it is used in control applications.

Disadvantages

1. It requires a large force to move its contacts.
2. Sliding contacts can get contaminated, worn out, and there is a possibility of misalignment and generation of noise.
3. Life-time of this transducer is limited.

Applications

This transducer is used in:

1. A voltage divider to obtain an adjustable output voltage.
2. Audio-control devices for frequency attenuation, to adjust loudness and so on.
3. Televisions to control brightness, contrast and colour response.
4. Measuring the displacement

5.24.2 Electrical Strain Gauges **[AU Nov/Dec, 2014]**

If a metal conductor is stretched or compressed, its resistance changes because of dimensional changes (length and cross-sectional area) and resistivity change. If a wire is under tension and increases its length from l to $l + \Delta l$, i.e., the strain $S = \frac{\Delta l}{l}$, then its resistance increases from R to $R + \Delta R$.

The sensitivity of a strain gauge is described in terms of a characteristic called the gauge factor G, defined as the unit change in resistance per unit change in length, i.e.,

$$G = \frac{\Delta R/R}{\Delta l/l} = \frac{\Delta R/R}{S}$$

Unbonded Strain Gauges

The schematic diagram of a typical displacement transducer wherein the measuring forces are transmitted to the platform containing the unbonded wire structure by means of a force rod is shown in Figure 5.55 (a) and (b). The resistance wires have equal lengths.

When an external force is applied to the strain gauge, the armature moves in the direction indicated. Elements A and D increase in length, whereas, elements B and C decrease in length. The change in resistance of the four wires is proportional to their change in length and this change can be measured with a Wheatstone bridge, as shown in Figure 5.55(c).

Thus, the external force causes variation in resistance of the wires, unbalancing the bridge and causing an output voltage V_a proportional to the pressure. The bridge is balanced if

$$\frac{R_A}{R_C} = \frac{R_B}{R_D}$$

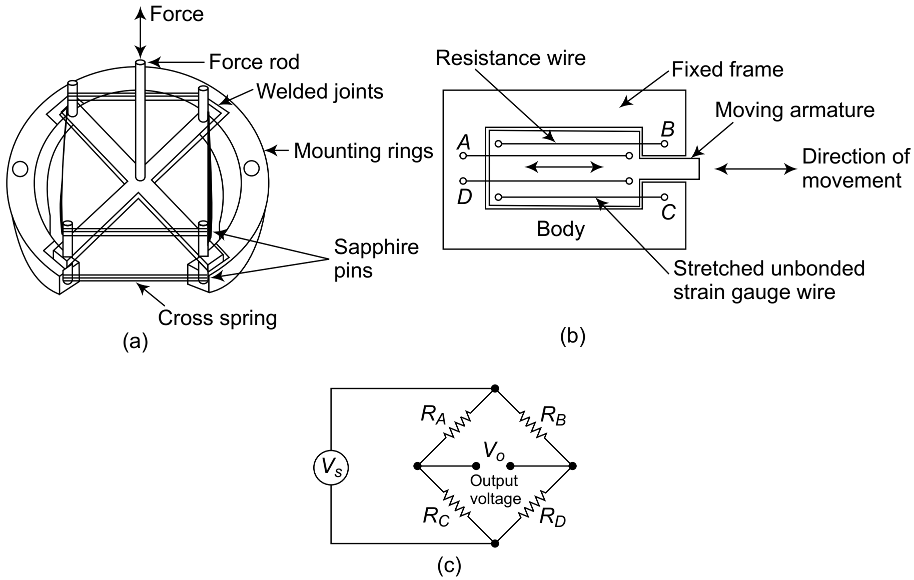


Figure 5.55 Unbonded Strain Gauge

Bonded Strain Gauge

A bonded-wire strain gauge consists of a grid of fine resistance wire of a diameter of about 25 mm. The wire is cemented to a base. The base may be a thin sheet of paper or a very thin Bakelite sheet. The wire is covered with a thin sheet of material so that it is not damaged mechanically. The base is bonded to the structure under study with an adhesive material. It acts as a bonding material. It permits a good transfer of strain from base to wires. The commonly used types of bonded strain gauges are shown in Figure 5.56. The advantages, disadvantages and applications of strain gauges are given below:

Advantages

1. No moving part exists in the system.
2. Device is small and inexpensive.
3. It has faster response time.

Disadvantages

1. Non-linear characteristics exist in the transducer.
2. Transducer needs to be calibrated.
3. Very sensitive to environmental condition.
4. Has very long term-drift.

Applications

1. Used in measuring normal strains in any desired direction.
2. Can be used in measuring shear strain using some special arrangements.
3. Possible to read the reading remotely.

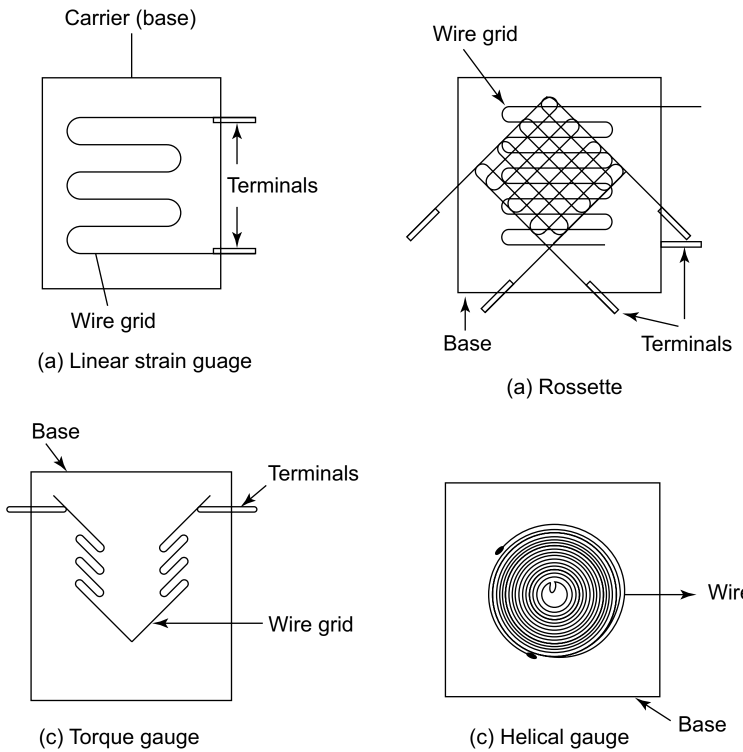


Figure 5.56 Bonded Strain Gauges

4. Can be used in measuring static and dynamic strains.
5. Can be used in measuring vibration, torque, bending, deflection, compression and tension.

5.24.3 Resistance Thermometer **[AU Nov/Dec, 2011]**

The resistance of most electrical conductors varies with temperature, according to the relation

$$R = R_0(1 + aT + bT^2 + \dots)$$

Where, R_0 is the resistance at temperature T_0 (at 0°C), R is the resistance at T, and a and b are constants.

Over a small temperature range, depending on the material, the above equation reduces to

$$R = R_0(1 + \alpha T)$$

Where, α is the temperature coefficient of resistance.

Important properties of materials used for resistance thermometers are: (i) high temperature-coefficient of resistance, (ii) stable properties so that the resistance characteristic does not drift with repeated heating and cooling or mechanical strain, and (iii) a high resistivity to permit the construction of small sensors. The variation of resistivity with temperature of some of the materials used for resistance thermometers is shown in Figure 5.57. From the figure, it can be seen that tungsten has a suitable temperature coefficient of resistance but is brittle and difficult to form. Copper has a low resistivity and is generally confined to applications where the sensor size is not restricted. Both platinum and nickel are widely used because they are relatively easy to obtain in pure state.

Platinum has an advantage over nickel, as its temperature coefficient of resistance is linear over a larger temperature range. The resistance–temperature relationship for platinum resistance elements is determined from the Callendar equations:

$$T = \frac{100(R_T - R_0)}{R_{100} - R_0} + d\left(\frac{T}{100} - 1\right)\frac{T}{100}$$

Where, R_T is the resistance at temperature T, R_0 is the resistance at 0°C, R_{100} is the resistance at 100°C and d is the Callendar constant, which is approximately 1.5.

The construction of an industrial platinum resistance thermometer is shown in Figure 5.58.

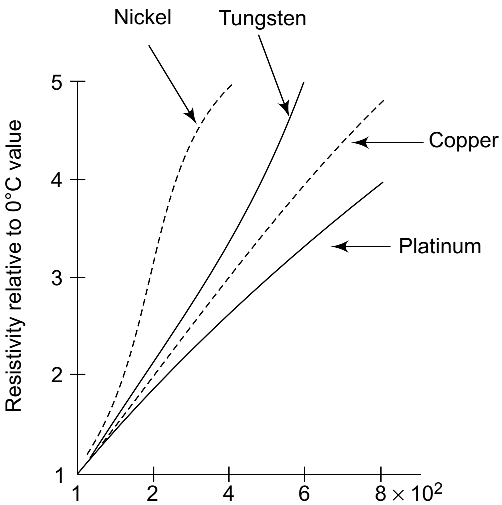


Figure 5.57 Variation of Resistivity with Temperature of Materials Used for Resistance Thermometer

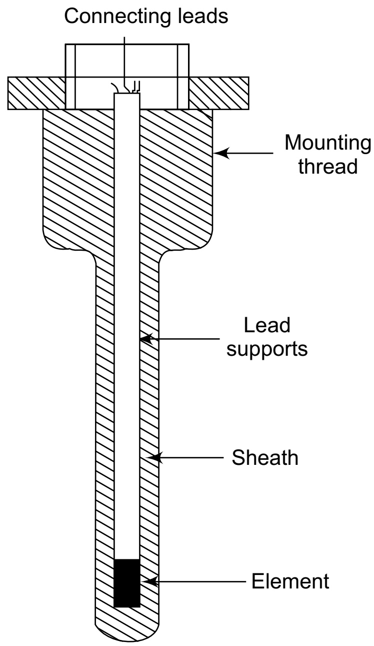


Figure 5.58 Industrial Platinum Resistance Thermometer

Advantages, Disadvantages and Applications of Resistance Thermometers

Advantages

1. Accurate measurement of quantity is possible.
2. Direct operation of indicators and recorders is possible.
3. Easily to install and replace.
4. Possible to measure differential temperature.
5. Wide range of temperature can be measured i.e., from -20° C to + 650° C.
6. Smaller in size and is suitable for remote indication.

Disadvantages

1. External power source is necessary for its operation.
2. Comparatively expensive when compared to other transducers.
3. Self-heating problem exists in the transducer.

Applications

Commonly used in aerospace, analytical equipment, food-service equipment, and semiconductor equipment.

5.24.4 Thermistor **[AU Nov/Dec, 2014]**

A thermistor, or a thermal resistor, is a two-terminal semiconductor device whose resistance is temperature sensitive. The value of such resistors decreases with increase in temperature. Materials employed in the manufacture of the thermistors include oxides of cobalt, nickel, copper, iron, uranium and manganese. The symbol for a thermistor is shown in Figure 5.59(a).

The thermistor has a very high temperature-coefficient of resistance, of the order of 3 to 5% per °C, making it an ideal temperature transducer. The temperature coefficient of resistance is normally negative. The resistance at any temperature T, is given approximately by

$$R_T = R_0 \exp \beta\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

where, R_T is the thermistor resistance at temperature $T(K)$, R_0 is the thermistor resistance at temperature $T_0(K)$ and β is a constant determined by calibration.

At high temperatures, this equation reduces to

$$R_T = R_0 \exp\left(\frac{\beta}{T}\right)$$

The resistance–temperature characteristic is shown in Figure 5.59 (b). The curve is non-linear and the drop in resistance from 5000 Ω to 10 Ω occurs for an increase in temperature from 20°C to 100°C. The temperature of the device can be changed internally or externally. An increase in current through the device will raise its temperature, carrying a drop in its terminal resistance. Any externally applied heat source will result in an increase in its body temperature and drop in resistance. This type of action (internal or external) lends itself well to control mechanisms.

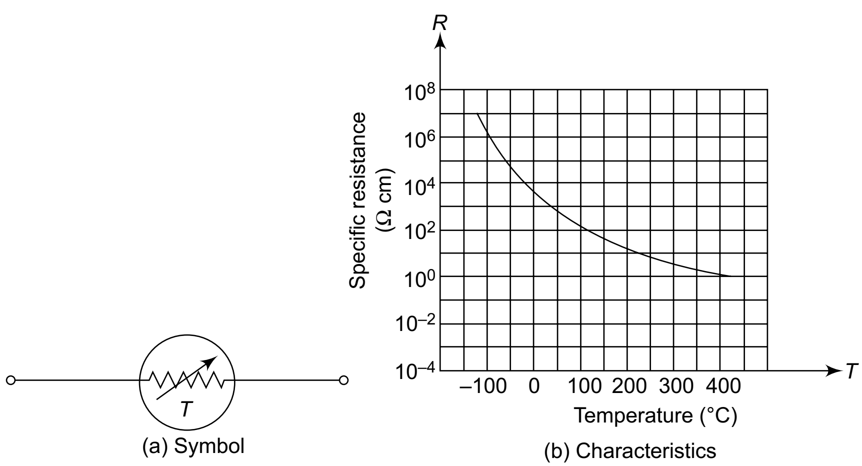


Figure 5.59 Symbol and Resistance–Temperature Characteristics of a Thermistor

Three useful parameters for characterising thermistors are: the time constant, dissipation constant, and the resistance ratio. The time constant is the time for a thermistor to change its resistance by 63% of its initial value, for zero power dissipation. Typical values of time-constant range from 1–50 s.

The dissipation factor is the power necessary to increase the temperature of a thermistor by 1°C. Typical values of dissipation factor range from 1 mW/°C to 10 mW/°C.

Resistance ratio is the ratio of the resistance at 25°C to that at 125°C. Its range is approximately 3–60.

Advantages, Disadvantages and Applications of Thermistors

Advantages

1. Compact, low cost and longer life-time.
2. Has good stability of the system.
3. Has faster response i.e., from seconds to minutes.
4. More sensitive when compared to other temperature sensors.
5. Compatible with many devices.
6. Easy to interface with the external circuits.

Disadvantages

1. Requires shielding.
2. Requires an input power to activate.
3. Low excitation current is required to avoid self-heating.
4. Not suitable for large temperature range.
5. Non-linear resistance temperature characteristics.

Applications

The applications of thermistor are:

1. Measurement of temperature.
2. Control of temperature.
3. Temperature compensation.
4. Measuring voltage and power at high frequencies, thermal conductivity, level, flow and pressure of liquids and composition of gases.
5. Used in measuring vacuum and to provide time-delay.

Comparison between RTD and Thermistor

The comparison between RTD and thermistor is given in Table 5.4

Table 5.4 *Comparison between RTD and thermistor*

RTD	THERMISTOR
It is made up of metals.	Thermistor is made up of semiconductor materials.
Since metals have a positive temperature coefficient (PTC) of resistance, its resistance is directly proportional to temperature	Since semiconductor materials have a negative temperature coefficient (NTC) of resistance, its resistance is inversely proportional to temperature.
Has linear resistance temperature characteristics	Has non-linear resistance temperature characteristics
Less sensitive to temperature.	Highly sensitive to temperature.
Has wide operating range i.e., –200°C to 650°C	Has a narrow operating range i.e., –100°C to 300°C

(*Contd.*)

Larger in size.	Smaller in size.
Costlier when compared to a thermistor.	Cheaper when compared to RTD
Has low self-resistance.	Has high self-resistance.
Provides high degree of accuracy and long-term stability.	Provides an accuracy of ± 0.01°C.
Used in laboratory and industrial applications.	Used in dynamic temperature measurement.

Example 5.2

A thermistor has a resistance of 3980 Ω at the ice point (0°C) and 794 Ω at 50° C. The resistance–temperature relationship is given by $R_T = aR_0 e^{\frac{b}{T}}$. Calculate the constants *a* and *b*. Calculate the range of resistance to be measured in case the temperature varies from 40° to 100° C.

Solution

The resistance at ice point, R_0 = 3980 Ω
Absolute temperature at ice point 273 K

$$3980 = a \times 3980 \times e^{\frac{b}{273}} \quad \text{or} \quad 1 = a.e^{\frac{b}{273}} \tag{1}$$

Resistance at 50° C is R_T = 794 Ω
Absolute temperature corresponding to 50° C is T = 273 + 50° = 323 K.

Hence,
$$794 = a \times 3980 e^{\frac{b}{323}} = 3980\, a\, e^{\frac{b}{323}} \tag{2}$$

Solving (1) and (2), we have $a = 30 \times 10^{-6}$ and $b = 2845$
Absolute temperature at 40° C = 273 + 40 = 313 K

$$\text{Resistance at } 40°\text{ C} = 30 \times 10^{-6} \times 3980 \times e^{\frac{2845}{313}} = 1060\ \Omega$$

Absolute temperature at 100° C = 273 + 100 = 373 K

$$\text{Resistance at } 100°\text{ C} = 30 \times 10^{-6} \times 3980 e^{\frac{2845}{373}} = 245\ \Omega$$

Thus, the range of resistance is 1060 Ω to 245 Ω.

5.25 Inductive Transducer **[AU Nov/Dec, 2011]**

When a force is applied to a ferromagnetic armature, the air gap, as shown in Figure 5.60, is changed, thereby varying the reluctance of the magnetic circuit. Thus, the applied force is measured by the change of inductance in a single coil. The inductive transducer enables static and dynamic measurements. Its drawback is that it has limited frequency response.

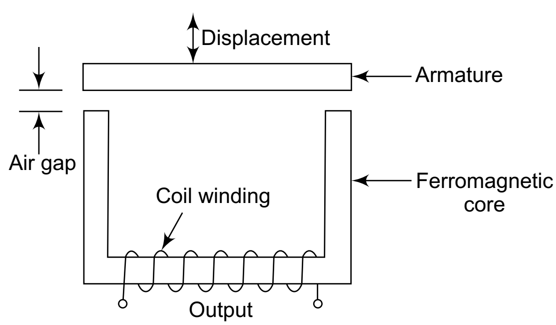


Figure 5.60 Inductive Transducers

5.25.1 Linear Variable Differential Transformer (LVDT) [AU April/May, 2015]

The most widely used inductance transducer is the Linear Variable Differential Transformer (LVDT), as shown in Figure 5.61(a). It consists of a primary coil and two exactly similar secondary coils with a rod-shaped magnetic core positioned centrally, inside the coil. An alternating current is fed into the primary, and voltages V_{o1} and V_{o2} are induced in the secondary coils. As these coils are connected in series opposition, the output voltage is $V_o = V_{o1} - V_{o2}$. If the core is placed ideally in the central position (null position or reference position), $V_{o1} = V_{o2}$ and hence, the output voltage $V_o = 0$. In practice, due to incomplete balance, a residual voltage usually remains with the core in this position. As shown in Figure 5.61 (a), when the core is displaced from the null position, the induced voltage in the secondary towards which the core has moved increases, while in the other the secondary voltage decreases. This results in a differential voltage output from the transformer.

The output voltage produced by the displacement of the core is linear over a considerable range, as shown in Figure 5.61(b) but flattens out at both ends, and the voltage phase changes by 180°, as the core moves through the centre position.

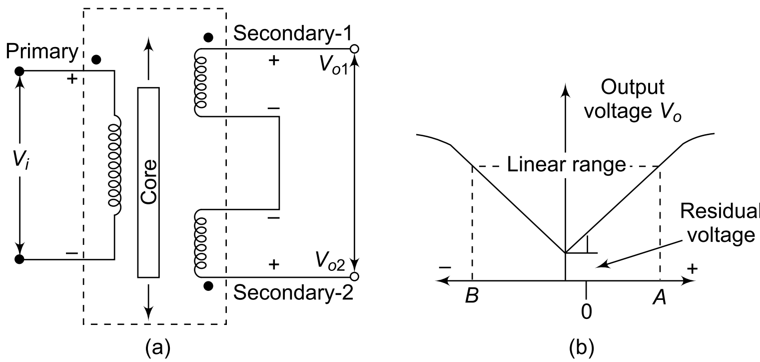


Figure 5.61 Linear Variable Differential Transformer

LVDT provides continuous resolution and shows low hysteresis and hence, repeatability is excellent under all conditions. As there are no sliding contacts, there is less friction and less noise. It is sensitive to vibrations and temperature. The receiving instrument must be selected to operate on AC signals or a demodulator network must be used if a DC output is required.

Advantages, Disadvantages and Applications of LVDT

Advantages

1. Can be used in wide range of applications
2. Presence of a linear relationship in the instrument
3. High sensitivity and output
4. High resolution, high sensitivity and good repeatability
5. Consumes less power
6. Produces low hysteresis
7. Low frictional losses

Disadvantages

1. Requires large displacement to get considerable differential output.
2. Very sensitive to stray magnetic field and hence requires shielding.

3. Temperature and vibrations affect output of the transducer.
4. The dynamic response is being controlled mechanically.

Applications

1. Used to measure displacement with ranging from a few mm to cm.
2. Can be used as primary and secondary transducers.
3. Used in combination with Bourdon tube to measure pressure.
4. Mostly used in servomechanisms and other industrial applications.

5.25.2 Rotary Variable Differential Transformer (RVDT) **[AU Nov/Dec, 2014]**

Rotary Variable Differential Transformer or RVDT is an electromechanical transducer, which senses the angular displacement of the conductor and gives a linear output proportional to it i.e., it provides a variable AC output voltage proportional to the angular displacement of the input shaft. It is similar to LVDT, except that its core is in cam shape and moves between the windings by means of a shaft. The output signal of RVDT is linear within a specified range over the angular displacement when it is energised using a fixed AC source. The schematic diagram of RVDT is shown in Figure 5.62.

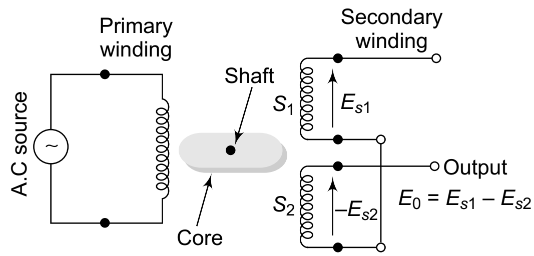


Figure 5.62 Schematic Diagram of RVDT

The RVDT consists of one primary winding and two secondary windings. The emf induced in the two secondary windings is a function of rotary displacement of the core around the shaft and both the secondary windings are placed in such a way that the emf induced is 180^0 out of phase with each other.

Working

The working of RVDT is similar to the operation of LVDT. According to the angular movement of the shaft, three differential conditions are formed.

Condition 1

When shaft is at null position, as shown in Figure 5.62, the emf induced in both the secondary windings are equal but opposite in phase. Therefore, the differential output taken from the secondary windings is zero and is explained mathematically using the following equation:

$$E_{s1} = E_{s2}$$

Where, E_{s1} is the emf induced in the first secondary winding and E_{s2} is the emf induced in the other secondary winding.

Therefore, the resultant output voltage is given by $E_0 = E_{s1} - E_{s2} = 0$.

Condition 2

When the shaft starts rotating in the clockwise direction, more portion of the core comes in contact with secondary winding S_1 when compared to S_2. Hence, the emf induced across the secondary winding S_1 is more than the emf induced across the secondary winding S_2 i.e.,$E_{s1} > E_{s2}$. Therefore, the differential output drawn from these windings is positive.

i.e., $$E_0 = E_{s1} - E_{s2} = \text{positive}$$

Condition 3

When the shaft starts rotating in the anti-clockwise direction, more portion of the core comes in contact with secondary winding S_2, when compared to S_2. Hence, the emf induced across the secondary winding S_2 is more than the emf induced across the secondary winding S_1 i.e., $E_{s1} < E_{s2}$. Therefore, the differential output drawn from these windings is negative

i.e., $$E_0 = E_{s1} - E_{s2} = \text{negative}$$

The curve between the magnitude of differential output voltage and angular displacement is shown in Figure 5.63. The curve is linear for small angular displacements and beyond this range, it starts to deviate from the straight line. In practice, there will be some residual voltage in RVDT when the core is kept at null position.

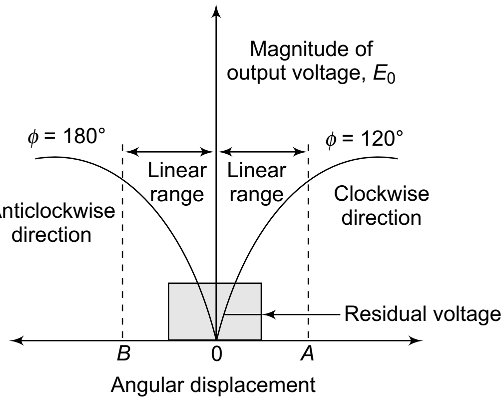


Figure 5.63 Input–Output Curve of RVDT

Advantages, Disadvantage and Applications of RVDT

The advantages, disadvantage and applications of RVDT are given below:

Advantages

1. Low cost due to popularity in application.
2. Solid and robust construction, which helps in operating it at different environmental conditions.
3. High accuracy and reliability can be achieved, as there is no frictional resistance.
4. Hysteresis is negligible.

Disadvantage

The RVDT provides linear output only for certain range of angular displacement.

Applications

The RVDT is used in:

1. Flight control actuation / navigation
2. Fuel-control valves
3. Cockpit controls
4. Signal conditioning as RVDT conditioner
5. Actuator feedback

5.26 Capacitive Transducers **[AU Nov / Dec, 2014]**

The capacitance of a parallel-plate capacitor is given by

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

where, A is the area of each plate in m^2, d is the distance between parallel plates in m, ε_0 is the dielectric constant (permittivity) of free space in F/m and ε_r is the relative dielectric constant (permittivity).

The capacitance is directly proportional to the area of the plate (A) and inversely proportional to the distance between the parallel plates (d). Obviously, any variation in A or d causes a corresponding variation in the capacitance. This principle of variation in d is used in the capacitive transducer, as shown in Figure 5.64.

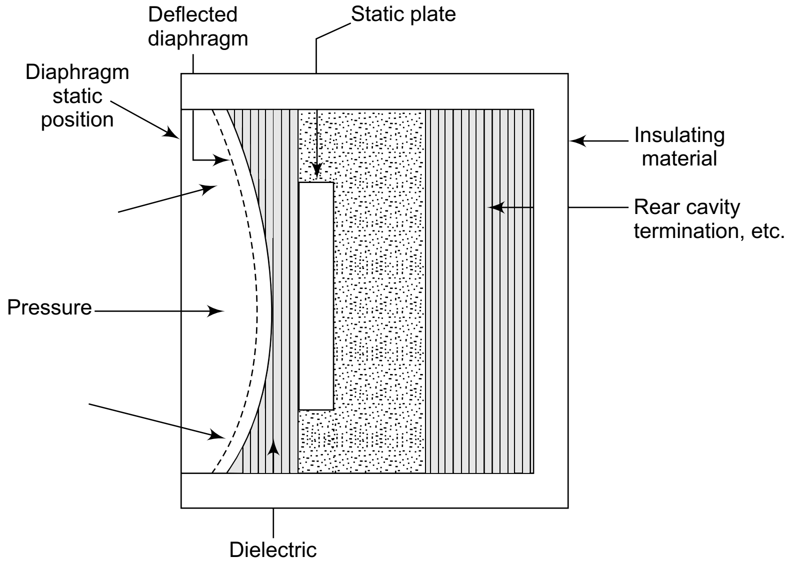


Figure 5.64 Capacitive Transducers

When a force is applied to a diaphragm, which acts as one plate of a capacitor, the distance between the diaphragm and the static plate is changed. The resulting change in capacitance can be measured with an AC bridge or an oscillator circuit in which an electric counter can measure the change in frequency and which is a measure of the magnitude of the applied force. In a capacitor microphone, the same principle is used, where the sound pressure varies the capacitance between the fixed plate and a movable diaphragm. The capacitive transducer can measure static and dynamic changes. The drawback of this transducer is its sensitivity to temperature variations.

5.26.1 Advantages, Disadvantages and Applications of Capacitive Transducers

Advantages

1. Requires external force for operation, which makes it useful for small systems.
2. Highly sensitive and has good resolution.
3. Good frequency response.
4. Requires small power for its operation.
5. High input impedance decreases the loading effect.
6. It requires an external force for operation and hence very useful for small systems.

Disadvantages

1. Requires insulation.
2. Requires earthing to avoid stray magnetic field.
3. Sensitive to temperature changes, dust particles and moisture.
4. Presence of non-linear characteristics
5. Associates complex instrumentation circuitry.

Applications

1. Helps in measuring linear and angular displacement.
2. Used to measure force and pressure
3. Used as pressure transducer where change in dielectric constant occurs.
4. Measurement of humidity in the gases, volume, density and so on.

5.27 Thermoelectric Transducer **[AU April/May, 2011]**

A temperature transducer, which converts thermal energy into electrical energy, is known as a thermoelectric transducer. Thermocouple is the most commonly used thermoelectric transducer. Thermocouple, a type of primary transducer, is used for measuring temperature, where the change in temperature arising from two dissimilar metals is converted into an electrical energy. Thermoelectric phenomena like Seebeck effect, Peltier effect and Thompson effect are used to describe the thermocouple behaviour.

5.27.1 Seebeck Effect

This effect was introduced by Prof. Seebeck in 1821, which states that if two wires of different metals, like copper and iron, are joined together to form a closed circuit with two junctions and if those junctions are maintained at different temperatures, then an electric current will flow through the closed circuit i.e., the current will flow from copper to iron in the hot junction and from iron to copper in the cold junction. The explanation of Seebeck effect is shown in Figure 5.65 (a). In addition, it also says that an emf called Seebeck emf, which is directly proportional to the change in temperature, appears across the open circuit if the copper wire is cut at a particular point.

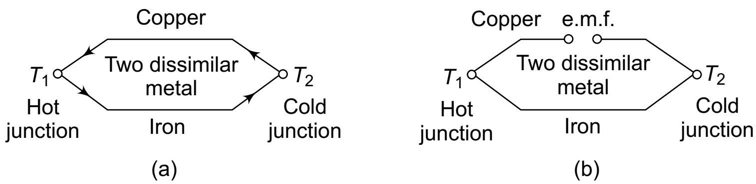


Figure 5.65 Seebeck Effect (a) Flow of Current (b) Emf

5.27.2 Peltier Effect

Professor Peltier introduced this effect, which is a reverse of Seebeck effect, in 1824. It states that if two wires of dissimilar metals form two junctions when an external voltage source is connected, as shown in Figure 5.66, then the current starts flowing through both the junctions. It also states that the heat is absorbed at a junction where the current is flowing from copper to iron, making the junction T_1 hot, and heat is liberated at a junction where the current is flowing from iron to copper, making the junction T_2 cold.

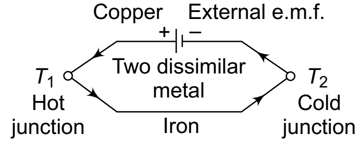


Figure 5.66 Peltier effect

5.27.3 Thompson Effect

It is a reversible heat flow effect, which was introduced by Professor Thompson. It states that when a current flows through the copper conductor with a thermal gradient along its length, then heat is released at a junction where the current and heat flow are in the same direction and heat is absorbed at a junction where different directions exist for current and heat flow.

5.27.4 Construction of a Thermocouple

Two dissimilar metals, when joined together to form two junctions T_1 and T_2, form a thermocouple, as shown in Figure 5.67 (a). Usually, T_2 is kept at constant reference temperature and is referred as cold junction or reference junction. The temperature which is to be measured is subjected to T_1 and hence it is referred as hot junction or measuring junction. When there is a temperature difference between T_1 and T_2, an emf that is proportional to the temperature gradient gets generated and can be measured using any meter or recorder, as shown Figure 5.67 (b).

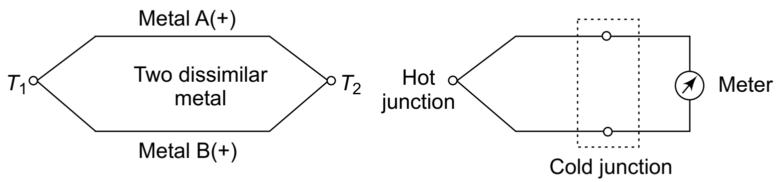


Figure 5.67 Thermocouple Circuit (a) Schematic Diagram (b) Practical Circuit

Generally, the junction in the thermocouple is formed in two ways: twisted weld and butt weld. In twisted weld, two large sized wires are twisted and welded together with several turns to give mechanical strength, while in butt weld, two comparatively small wires are fused together into a round bend. The normal sizes of metals are: 0.5 mm diameter for noble metals and 1.5 to 3 mm diameter for base metals.

5.27.5 Types of Thermocouples

Based on the materials used in the thermocouple and the range of temperature it can measure, there are different types of thermocouples listed in Table 5.5.

Table 5.5 *Types of Thermocouples*

Type of Thermocouples	Material Used	Temperature Range
T	Copper - constantan	–250°C to 400°C
J	Iron - constantan	–200°C to 850°C
K	Chromel - Alumel	–200°C to 110°C
E	Chromel - Constantan	–200°C to 850°C
S	Platinum - Platinum rhodium	0°C to 1400°C
—	Tungsten - molybdenum	0°C to 2700°C
—	Tungsten-Rhenium	0°C to 2600°C

Advantages, Disadvantages and Applications of Thermocouples

Advantages

1. Rugged construction
2. Covers wide range of temperature: –270°C to 2700°C
3. Most suitable for temperature measurement in industrial furnaces
4. Cheaper in cost
5. Easy to check the calibration
6. Offers good reproducibility
7. High response speed and good accuracy

Disadvantages

1. To have high accuracy, it is necessary to have cold junction compensation.
2. Non-linear characteristics exist between induced emf and temperature.
3. Possible to have stray voltage pickup.
4. Signal amplification is required for many applications.

Applications

1. Testing temperatures associated with different process plants e.g. chemical production, petroleum refineries, heating appliance safety, food industries, steel, iron and aluminium industries, plastics and resin industries.
2. Suitable for low temperature and cryogenic applications.
3. Temperature profiling in ovens, furnaces and kilns.
4. Temperature measurement of gas turbine and engine exhausts.

5.28 Piezoelectric Transducer **[AU May/June, 2011]**

If the dimensions of asymmetrical crystalline materials, such as quartz, Rochelle salt and barium titanite, are changed by the application of a mechanical force, the crystal produces an emf. This property is used in piezoelectric transducers. The basic circuit of a piezoelectric transducer is shown in Figure 5.68. Here, a crystal is placed between a solid base and the force-summing member. An externally applied force gives pressure to the top of the crystal. Hence, it produces an emf across the crystal, which is proportional to the magnitude of the applied pressure. As this transducer has a very good high-frequency response, it is used in high-frequency accelerometers. As it needs no external power source, it is called a self-generating transducer. The main drawbacks are that it cannot measure static conditions and the output voltage is affected by temperature variations of the crystal.

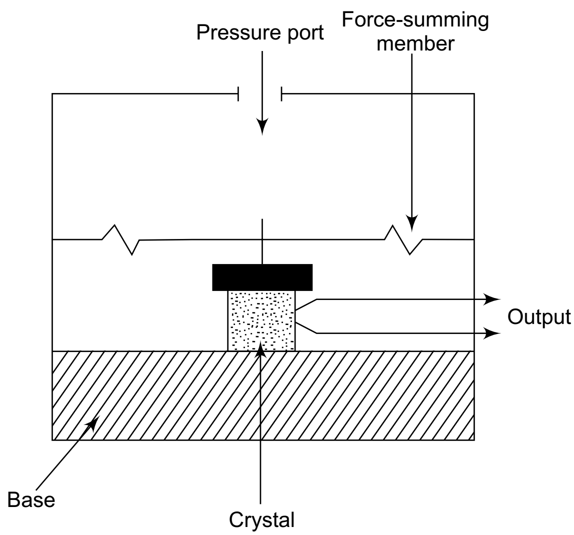


Figure 5.68 Piezoelectric Transducer

5.28.1 Advantages, Disadvantages and Applications of Piezoelectric Transducer

Advantages

1. Available in desired shape
2. Has rugged construction and it is smaller in size
3. Has good frequency response and negligible phase-shift

Disadvantages

1. Used in dynamic measurement only
2. Highly sensitive to temperature
3. Since some crystals are water-soluble, it might get dissolved in highly humid environment

Applications

1. Helps in stabilising electronic oscillators
2. Used in measuring surface roughness, accelerometer and vibration pickup
3. Used in industrial cleansing apparatus and in underwater detection systems
4. Used in spark-ignition engine, electronic watches and record players
5. Used as a sensing element e.g., piezoelectric microphones
6. Used in ultrasound imaging, chemical and biological sensors

5.29 Photoelectric Transducer

This is an optoelectronic or optical transducer, shown in Figure 5.69. It uses a phototube and a light source, separated by a small window, whose aperture is controlled by the force-summing device. The quantity of incident light on the photosensitive cathode is varied according to the externally applied force, thereby changing the anode current. This device measures both static and dynamic phenomena and it has high efficiency. It does not respond to high frequency light variation.

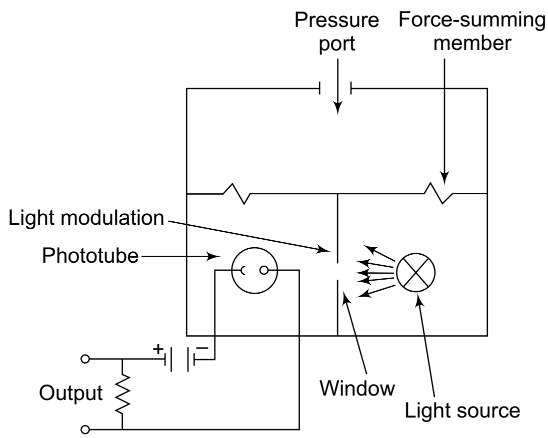


Figure 5.69 Photoelectric Transducer

5.29.1 Advantages, Disadvantages and Application of Photoelectric Transducer

Advantages

1. Capability to sense all the possible materials
2. Has long duration of life
3. Highly sensitive and reliable
4. Has very quick response time
5. Less cost

Disadvantages

1. Gets affected by atmospheric conditions.
2. Sensing range gets affected by target colour and reflexivity,

Application

Used in packaging, material handling and parts detection.

5.30 Hall Effect Transducers

When a transverse magnetic field B is applied to a specimen (thin strip of metal or semiconductor) carrying current I, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon is known as the *Hall effect*.

A Hall-effect measurement experimentally confirms the validity of the concept that it is possible for two independent types of charge carriers, electrons and holes, to exist in a semiconductor.

The schematic arrangement of the semiconductor, the magnetic field and the current flow pertaining to Hall effect are shown in Figure 5.70. Under equilibrium condition, the electric field intensity, E, due to the Hall effect must exert a force on the carrier of charge, q, which just balances the magnetic force, i.e.,

$$qE = Bqv_d$$

where, v_d is the drift velocity. Also, the electric field intensity due to Hall effect is given by

$$E = \frac{V_H}{d}$$

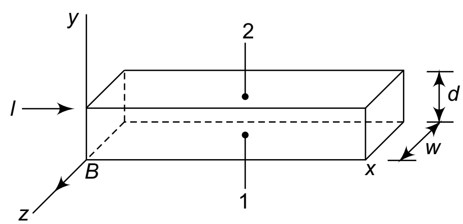


Figure 5.70 Schematic Diagram to Observe Hall Effect

where, d is the distance between surfaces 1 and 2, and V_H is the Hall voltage appearing between surfaces 1 and 2. In an N-type semiconductor, electrons carry the current and these electrons will be forced downward towards side 1, which becomes negatively charged with respect to side 2.

The current density (J) is related to charge density (r) by

$$J = \rho v_d$$

Further, the current density (J) is related to current (I) by

$$J = \frac{I}{\text{Area}} = \frac{I}{wd}$$

where, w is the width of the specimen in the direction of magnetic field (B).
Combining the above relations, we get

$$V_H = Ed = Bv_d d = \frac{BJd}{\rho} = \frac{BI}{\rho w}$$

The Hall coefficient, R_H, is defined by

$$R_H = \frac{1}{\rho}$$

so that $V_H = \frac{R_H}{w} BI$. A measurement of the Hall coefficient R_H, determines not only the sign of the charge carriers but also their concentration. The Hall coefficient for a *P*-type semiconductor is positive, whereas it is negative for an *N*-type semiconductor. This is true because the Hall voltage in a *P*-type semiconductor is of opposite polarity to that in an *N*-type semiconductor.

The advantage of Hall-effect transducers is that they are non-contact devices with high resolution and small size.

The Hall effect is used to find whether a semiconductor is *N* or *P*-type and to determine the carrier concentration. If the terminal 2 becomes charged positively with respect to terminal 1, the semiconductor must be *N*-type and $\rho = pq$, where n is the electron concentration. On the other hand, if the polarity of V_H is positive at terminal 1 with respect to terminal 2, the semiconductor must be *P*-type and $\rho = pq$, where p is the hole concentration.

The mobility (m) can also be calculated with simultaneous measurement of the conductivity (s). The conductivity and the mobility are related by the equation $\sigma = \rho\mu$ or $\mu = \sigma R_H$.

Therefore, the conductivity for *N*-type semiconductor is $\sigma = nq\mu_n$ and for *P*-type semiconductor, $\sigma = pq\mu_n$, where μ_n is the electron mobility and μ_p is the hole mobility.

Thus, if the conductivity of a semiconductor is also measured along with R_H, then mobility can be determined from the following relations.

For *N*-type semiconductor, $\mu_n = \frac{\sigma}{nq} = \sigma R_H$

and for *P*-type semiconductor, $\mu_p = \frac{\sigma}{pq} = \sigma R_H$

Since V_H is proportional to B for a given current I, Hall effect can be used to measure the AC power and the strength of magnetic field and sense the angular position of static magnetic fields in a magnetic field meter. It is also used in an instrument called Hall-effect multiplier, which gives the output proportional to the product of two input signals. If I is made proportional to one of the inputs and B is made proportional to the second signal, then from the equation, $V_H = \frac{BI}{\rho w}$, V_H will be proportional to the product of two inputs. Hall devices for such applications are made from a thin wafer or film of Indium Antimonide (InSb) or Indium Arsenide. As the material has very high electron mobility, it has high Hall coefficient and high sensitivity.

An electrical current can be controlled by a magnetic field because the magnetic field changes the resistances of some elements with which it comes in contact. In the magnetic bubble memory, while read-out, the Hall effect element is passed over the bubble. Hence, a change in current of the circuit will create, say, a *one*. If there is no bubble, there will be a *zero* and there will be no current change in the output circuit. The read-in device would have an opposite effect, wherein the Hall device creates a magnetic field when supplied with a pulse of current. This, in turn, creates a little domain and then a magnetic bubble is created.

Some of the other applications are in measurement of velocity, rpm, sorting, limit sensing, and non-contact current measurements.

5.30.1 Advantages, Disadvantages and Applications of Hall Effect Transducers

Advantages

1. High-speed operation over 100 kHz is possible, whereas at high frequencies, the inductive or capacitive sensor output begins to distort.

2. As there is no wear and friction due to non-contact operation, the number of operating cycles is unlimited.
3. When packed, it is immune to dust, air and water, whereas dust triggers a capacitive sensor.
4. It can measure zero speed.
5. Highly repeatable operation.
6. Capable of measuring large current.

Disadvantages

1. Gets affected by external interfering magnetic field.
2. There exists a large temperature drift.
3. Large offset voltage.

Applications

1. Used in converting magnetic flux to electric transducer.
2. Used as current sensor.
3. Automotive fuel-level indicator.
4. Spacecraft propulsion.
5. Used in brushless DC motor to sense the position of the rotor.
6. Used in measuring power, current and displacement.

5.31 Mechanical Tranducers

A transducer that converts one form of physical quantity to another is known as a mechanical transducer. This transducer is the primary transducer, which acts as an input to the electrical or secondary transducer. It can measure the physical quantities such as pressure, force, displacement, flow rate etc. The common mechanical transducers, which convert one form of physical quantity to another form, are:

- ***Flat spiral spring***: It can produce the controlling torque in the instruments to measure electrical quantities.
- ***Torsion bar of shaft:*** The primary sensing element in torque meter, which is used to measure the torque, is known torsion bar of shaft. Its deflection or twist is directly proportional to the torque applied and hence its deformation is used to measure the torque.
- ***Proving ring:*** It is used to measure force, weight or load. It causes a deflection, which is further measured with the help of electrical transducer.
- ***Spring flexure pivot:*** It is a frictionless device used in measurements. The sensitivity of the device is almost constant for an angular displacement that is less than 15^{o}.
- ***Bourdon tube:*** The different forms of Bourdon tubes are: (i) C type, (ii) spiral, (iii) twisted tube and (iv) helical, as shown in Figure 5.71. It is made of brass or phosphor bronze or beryllium copper or steel. It is made out of an elliptically sectioned elastic tube, which is bent to form the above-mentioned shapes. One end of the tube is closed and other end is opened for the liquid to enter. When the liquid, whose pressure is to be determined, enters the tube, a movement which can be measured is caused in the free end. It is normally used to measure gauge pressure.

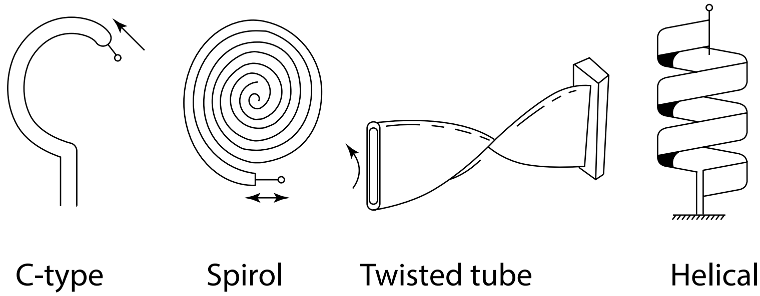


Figure 5.71 Bourdon Tubes

- ***Diaphragm:*** Flat and corrugated diaphragms, shown in Figure 5.72, are used to measure pressure by determining the displacement of the diaphragm. The pressure to be measured is applied at one side of the diaphragm and the other side is rigidly fixed. This type of arrangement causes deflection at the centre of the diaphragm, which is directly proportional to the pressure applied.

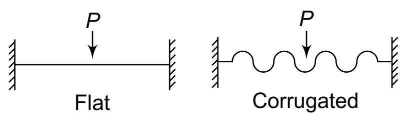


Figure 5.72 Diaphragms

Deformation occurs in the flat diaphragm, when a pressure P is applied to it, as shown in Figure 5.73. The relation between the pressure P applied and the displacement d_m at the centre of the diaphragm is given by

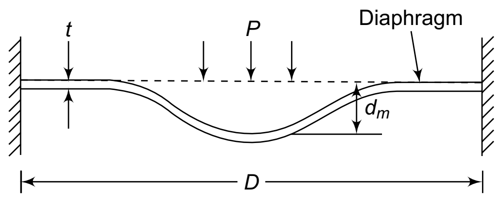


Figure 5.73 Flat Diaphragm with Pressure P Applied

$$P = \frac{256Et^3 d_m}{2(1-v^2)D^4} \text{ N/m}^2$$

where, E is the Young's modulus in N/m^2, t is the thickness of the diaphragm in m, D is the diameter of the diaphragm in m, d_m is the deflection at the centre of the diaphragm in m and v is the Poisson's ratio.

The above relation between pressure P and d_m is linear for $d_m \leq 0.5t$ and non-linear in other cases.

- ***Bellow:*** A thin-walled tube, whose thickness is approximately 0.1 mm and having a corrugated shape, is known as bellow and is made up of a single piece of special brass or stainless steel. Figure 5.74 shows a simple bellow. It is also known as pressure-activated spring and its displacement for a particular pressure depends on its type and the thickness of the material used.

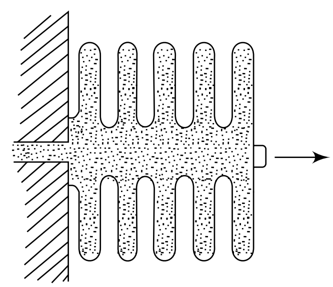


Figure 5.74 Bellow

- ***Temperature detector:*** The different principles used for detecting temperature are:
  - ***Using bimetallic strip:*** It consists of two different metals with different coefficients of thermal expansion, which are joint together. When the strip is heated due to expansion of metal, a deflection made by the bimetallic strip is converted into the movement of the pointer to indicate the temperature.

- ***Thermocouple:*** Temperature is detected using the thermoelectric emf generated between two metals.
- ***Resistive thermometer and thermistor:*** Temperature is detected by changing the resistance of the material used in it.

- ***Hydro-pneumatic device:*** This device is used to measure the flow and it works on the principle of simple float or a hydrometer .

Two Mark Questions and Answers

1. **Define 'error' in measurement.** **[AU April/May, 2013; Nov/Dec, 2014]**

   The error in measurement is defined as the difference between the true or actual value and the measured value.

2. **What is a transducer?** **[AU Nov/Dec, 2014; April/May, 2011]**

   A transducer is a device, which converts energy from one form to another form. This energy may be electrical, mechanical, chemical, optical or thermal.

3. **What is piezoelectric effect?** **[AU April/May, 2015; Nov/Dec, 2011]**

   If the dimensions of asymmetrical crystalline materials, such as quartz, Rochelle salt and barium titanite, are changed by the application of a mechanical force, the crystal produces an emf. This property is used in piezoelectric transducers.

4. **What are the basic elements of a generalised measurement system?** **[AU Nov/Dec, 2013]**

   Refer to section 5.3 for the basic elements of a generalised measurement system.

5. **List any four static characteristics of a measuring system.** **[AU April/May, 2012]**

   Refer to section 5.16.1 for the static characteristics of a measuring system.

6. **Define the term 'accuracy'.** **[AU April/May, 2012]**

   Accuracy of a measured value of a quantity is defined as the closeness of the measured value obtained using an instrument to the true value of the same quantity. It depends on the accuracy of the instrument itself, variation of the quantity that is to be measured, and observer accuracy etc.

7. **Define the term 'precision'.** **[AU April/May, 2012]**

   Precision comes from the term precise, which means clearly or sharply defined, and it is a measure of reproducibility of the measurements or a degree of agreement within a measurement group.

8. **Differentiate zero drift and span drift** **[AU April/May, 2014]**

   Zero drift: If all the instrument outputs shift by the same amount, it is known as zero drift and it occurs due to pointer shift, slippage or permanent set.
   Span drift: If the instrument output changes proportionately from zero, it is called span drift and it occurs due to the change in spring gradient.

9. **What is measurement and how is it classified?** **[AU April/May, 2014]**

   The process of measuring the quantity is known as measurement and the apparatus used to measure quantities like voltage, current, power, energy, resistance and so on, is called a *measuring instrument*.
   Refer to section 5.4 for the classification of measuring instruments.

10. Mention the basic requirements of measuring instruments. **[AU Nov/Dec, 2014]**

The necessary or essential requirements for any measuring instrument are:

(i) When the instrument is used in the circuit, its conditions should not be altered and therefore the quantity to be measured goes unaffected.
(ii) It should consume as low power as possible.
(iii) It should possess a very high efficiency and high sensitivity.
(iv) The output should be linearly proportional to the input.
(v) It should be less affected by the noise, modifiable and properly priced.

11. What is meant by dynamic characteristics of instruments? **AU April/May, 2011]**

Refer to section 5.16.2 for the dynamic characteristics of instruments.

12. Define 'static error'. How are static errors classified? **[AU April/May, 2010]**

Static error is the difference between the measured value and true value of the quantity, as given by

$$E_s = A_m - A_t$$

where, A_m is the measured value of the quantity and A_t is the true value of the quantity.
Refer to section 5.17 for classification of static errors.

13. Distinguish between reproducibility and repeatability. **[AU April/May, 2012]**

Reproducibility is the degree of closeness with which a given value may be repeatedly measured, using the same instrument under different conditions, like changes in the method of measurement, observer, measuring instrument location, conditions of use and time of measurement. Reproducibility is specified in terms of scale readings over a given period of time, whereas the repeatability is the instrument characteristic, which describes the closeness with which a given value is repeatedly measured on the same instrument, at the same location, by the same observer, under the same measurement conditions and when the same input is given to the instrument repetitively over a particular time. It is specified as a variation in scale reading.

14. What is a primary sensing element? **[AU Nov/Dec, 2012]**

The first unit in the measurement system, which detects the measurand, is known as a primary sensing unit. It helps in transferring the measurand to a variable conversion unit for further processing. For example, liquid or mercury in a glass thermometer acts as primary sensing unit. Displacement or voltage is the output in the primary sensing unit.

15. With one example, explain instrumental errors. **[AU Nov/Dec, 2012]**

Refer to section 5.17.2 for instrumental errors.

16. How are the absolute and relative errors expressed mathematically? **[AU Nov/Dec, 2012]**

Absolute error: $E_s = A_m - A_t$

Relative error: $E_r = \dfrac{E_s}{A_t}$

Where, A_m is the measured value of the quantity and A_t is the true value of the quantity.

17. What is drift? **[AU Nov/Dec, 2011]**

Drift is the measure of deviation in the instrument output for a particular period.

18. Distinguish between active and passive transducer. **[AU Nov/Dec, 2013]**

Refer to section 5.19 for the differences between active and passive transducer.

19. How are transducers classified on the basis of principle of transduction? [AU April/May, 2010]

On the basis of principle of transduction, the transducers are classified as

(i) Resistive transducer
(ii) Capacitive transducer
(iii) Inductive transducer

Example: piezoelectric, thermoelectric, magneto restrictive and so on

20. List the factors to be considered for selecting a transducer. [AU April/May, 2012]

The factors to be considered in selecting a transducer are:

(i) Linearity
(ii) Ruggedness
(iii) Repeatability
(iv) High stability and reliability
(v) Good dynamic response
(vi) Convenient instrumentation
(vii) Good mechanical characteristics

21. Define 'gauge factor' of a strain gauge. [AU April/May, 2012]

The sensitivity of a strain gauge, described in terms of a characteristic called the gauge factor G, is defined as the unit change in resistance per unit change in length, i.e., $G = \frac{\Delta R/R}{\Delta l/l} = \frac{\Delta R/R}{S}$

22. Mention the uses of capacitive transducer. [AU Nov/Dec, 2011]

Uses of capacitive transducer are:

(i) It helps in measuring linear and angular displacement.
(ii) It is used to measure force and pressure.
(iii) It is used as a pressure transducer where change in dielectric constant occurs.
(iv) It is used to measure the humidity in the gases, volume and density.

Review Questions

1. Define measurement, measuring instrument and measurand.
2. What are the essential requirements of measuring instrument?
3. What are the basic elements of a generalised measurement system?
4. What is measurement and how it is classified?
5. What are the basic requirements of an indicating instrument? Briefly discuss them.
6. Why is damping torque necessary in indicating instruments? What are the methods of producing the same? Explain with necessary sketches.
7. Justify the necessity of controlling torque in indicating instruments. Discuss the various methods of producing the same. Compare them.
8. What are the types of indicating instruments?
9. What is the working principle of a moving iron instrument? Explain with necessary equations.
10. What to you understand by attraction type and repulsion type instruments? What are the important differences between moving coil and moving iron instruments?
11. What are the advantages, disadvantages and applications of moving iron instruments?

12. Describe the working principle of a moving coil instrument with necessary equations.
13. Explain the working of a dynamometer type instrument. What are the specific requirements of the same when it is used as a wattmeter?
14. What are the advantages, disadvantages and applications of moving coil instruments?
15. Describe the working of an electrothermic instrument with necessary equations.
16. How are electrostatic instruments used to measure electrical quantities?
17. Explain the operation of induction and rectifier type instruments.
18. Compare different types of indicating instruments.
19. List the advantages, disadvantages and applications of different indicating instruments.
20. What is the basic movement of an ammeter?
21. What are the requirements of a shunt?
22. Design the value of shunt resistance required for using a 100 mA meter movement with an internal resistance of 50 Ω for measuring 0–500 mA. **[Ans: 10.002 mΩ]**
23. Design a multi-range ammeter with ranges of (0–1 A), (0–5 A), (0–15 A) and (0–125 A) employing individual shunt in each. A d'Arsonval movement with an internal resistance of 250 Ω and a full-scale current of 5 mA is available.
[Ans: R_a = 0.5025 Ω, R_b = 0.2503 Ω, R_c = 0.0834 Ω and R_d = 0.0100 Ω]
24. Design an Ayrton shunt multi-range ammeter for the specifications given in previous question.
[Ans: R_a = 0.0101 Ω, R_b = 0.074 Ω and R_d = 2.2475 Ω]
25. Explain a basic DC voltmeter.
26. Convert a basic d'Arsonval movement with an internal resistance of 250 Ω and a full-scale deflection current of 5 mA into a multi-range DC voltmeter with voltage ranges (0–2.5 V), (0–10 V), (0–25 V) and (0–50 V).
[Ans: R_4 = 250 Ω, R_3 = 1500 Ω, R_2 = 3000 Ω and R_1 = 5000 Ω]
27. State the advantages of a DVM over an analogue meter.
28. Explain the basic principle of a digital voltmeter.
29. An ohmmeter uses a 50 Ω basic movement requiring a full-scale current of 1 mA. The internal battery voltage is 3 V. The desired scale marking for half-scale deflection is 2000 W. Calculate the values of R_a and R_b. If battery voltage is dropped by 20% due to aging, calculate the value of R_b to compensate the drop.
[Ans: (a) R_a = 1966.7 W, R_b = 100 W; (b) R_b = 250 W]
30. The shunt type ohmmeter uses a 10 mA basic d'Arsonval movement with an internal resistance 250 W. The battery voltage is 3 V. It is desired to modify the circuit by adding an appropriate resistor R_{sh} across the movement, so that the instrument will indicate 0.5 Ω at the midpoint on its scale. Calculate: (a) the value of the shunt resistor R_{sh} (b) the value of the current-limiting resistor, R_a.
[Ans: R_{sh} = 0.501 Ω, R_a = 0.35 Ω]
31. Explain the operation of a basic digital multimeter with the help of a block diagram.
32. Describe the working of a CRO with the help of a block diagram.
33. Explain how frequency and phase can be measured using a CRO.
34. What is the speciality of a dual-beam CRO? Explain its working with a block diagram.
35. Describe with the help of a neat block diagram, the working principle of a dual-trace CRO.
36. What are the special features of storage oscilloscopes?
37. Explain the principle of operation of a digital storage oscilloscope.
38. How does the sampling CRO increase the apparent frequency response of an oscilloscope?
39. Describe the applications of CRO.
40. Describe the distortion test on amplifiers using a CRO.

41. Explain the static characteristics of instruments.
42. List the dynamic characteristics of instruments and explain them.
43. Define errors in measurement.
44. What is a transducer? Briefly describe any one of the displacement transducers.
45. What are active and passive transducers? Why are they called so?
46. What are the basic requirements of a transducer?
47. Discuss resistive transducer with necessary diagrams.
48. What is meant by gauge factor of a strain gauge?
49. Discuss with suitable diagrams the salient features of unbonded and bonded strain gauges.
50. Explain the principle of operation of a resistance thermometer.
51. What is a thermocouple?
52. How is a thermocouple used for temperature measurement?
53. Compare RTD and thermistor.
54. Discuss the working principle of an inductive transducer.
55. Explain in detail the working of a linear variable differential transformer.
56. Describe the construction features of a linear variable differential transformer.
57. Explain the working of rotary variable differential transfer, with a neat sketch.
58. Explain the transduction principle used in the capacitor transducer.
59. Explain Hall effect. How can Hall effect be used to determine some of the properties of a semiconductor?
60. Describe the applications of Hall effect.
61. List the advantages, disadvantages and applications of different resistive transducers.
62. Write short notes on:
    (i) Thermistor
    (ii) Photoelectric transducer
    (iii) Piezoelectric transducer
    (iv) Thermoelectric transducer
63. List the advantages, disadvantages and applications of inductive, capacitive, thermoelectric, piezoelectric, photoelectric and Hall-Effect transducers.
64. Explain mechanical transducers.